Exercise-3

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Sol. The given differential equation is

$$\left(1+3\frac{dy}{dx}\right)^{2/3} = 4\left(\frac{d^3y}{dx^3}\right)$$

This equation can be re written as

$$\left(1+3\frac{dy}{dx}\right)^{2} = 4\left(\frac{d^3y}{dx^3}\right)^3$$

This show that the order and degree of given equation are 3 and 3 respectively.

2. Sol. Since $\frac{d^2y}{dx^2} = e^{-2x}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{e^{-2x}}{-2} + c$

$\Rightarrow \quad y = \frac{e^{-2x}}{4} + cx + d$

3. Sol. General equation of parabola whose axis is x-axis, is

$$y^2 = 4a(x + h)$$

On differentiating w.r.t.x, we get

$$2y\frac{dy}{dx} = 4a \Rightarrow y\frac{dy}{dx} = 2a$$

Again, differentiating, we get

$$y\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} = 0$$

This is a differential equation whose degree and order are 1 and 2 respectively.

4. Sol. $(1+ y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$

$\Rightarrow (1+ y^2)\frac{dx}{dy} + x = e^{\tan^{-1}y}$

$\Rightarrow \frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{e^{\tan^{-1}y}}{1+y^2}$

I.F. $= e^{\int P\,dy} = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

Therefore, required solution is

$xe^{\tan^{-1}y} = \int e^{\tan^{-1}y}.\frac{e^{\tan^{-1}y}}{1+y^2}dy + k_1$

$\Rightarrow \quad xe^{\tan^{-1}y} = \int \frac{e^{2\tan^{-1}y}}{1+y^2}.\ dy + k_1$

$\Rightarrow \quad xe^{\tan^{-1}y} = \frac{1}{2}e^{2\tan^{-1}y} + k_1$

$\Rightarrow \quad 2x\,e^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$

5. **Sol.** Given equation of family of curves is

$x^2 + y^2 - 2ay = 0$ ......(i)

On differentiating w.r.t.x, we get

$2x + 2yy' - 2ay' = 0$

$\Rightarrow 2x + 2yy' = 2ay'$

$\Rightarrow \frac{2x + 2yy'}{y'} = 2a$ ....(ii)

from Equation (i)

$2a = \frac{x^2 + y^2}{y}$

On putting this value in Equation (ii),

$\frac{2x + 2yy'}{y'} = \frac{x^2 + y^2}{y}$

$\Rightarrow 2xy + 2y^2y' = x^2y' + y^2y'$

$\Rightarrow (x^2 - y^2)y' = 2xy$

6. **Sol.** $y\,dx + (x + x^2y)dy = 0$

$\Rightarrow y\,dx + x\,dy = -x^2y\,dy$

$\Rightarrow \frac{y\,dx + x\,dy}{x^2y^2} = -\frac{1}{y}dy$

$\Rightarrow d\left(-\frac{1}{xy}\right) = -\frac{1}{y}dy$

On integrating,we get

$-\frac{1}{xy} = -\log y + c$

$\Rightarrow -\frac{1}{xy} + \log y = c$

7. **Sol.** The differential equation of a family of curves of n parameters is a differential equation of n order.

Equation of family of curves is

$y^2 = 2c(x + \sqrt{c})$ ... (i)

On differentiating equation (i) w.r.t. x, then

$2yy_1 = 2c$

$\Rightarrow c = yy_1$

On putting the value of c in equation (i), we get

$y^2 = 2yy_1(x + \sqrt{yy_1})$

$\Rightarrow (y^2 - 2yy_1 x)^2 = 4(yy_1)^3$

Hence, the degree and order of above equation are 3 and 1 respectively.

8. **Sol.** $x\frac{dy}{dx} = y(\ell n y - \ell n x + 1)$

$\therefore \frac{dy}{dx} = \left(\frac{y}{x}\right)\left(\ell n\left(\frac{y}{x}\right) + 1\right)$

Now, put $\frac{y}{x} = t$

$\Rightarrow \quad y = tx \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}$

$\therefore \quad t + x\frac{dt}{dx} = t\,\ell n t + t$

$\Rightarrow \quad \frac{dt}{t \sin t} = \frac{dx}{x} \quad \Rightarrow \quad \ell n(\ell n\, t) = \ell nx + \ell nc \quad \Rightarrow \quad \ell nt = cx$

$\Rightarrow \quad \ell n\left(\frac{y}{x}\right) = cx$

9. Sol. The given equation is $Ax^2 + By^2 = 1$

On differentiating w.r.t.x we get

$2Ax + 2By\frac{dy}{dx} = 0$ ....(i)

On again differentiating, we get

$2A + 2B\left\{\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2}\right\} = 0$ ....(ii)

On solving Eqs. (i) and (ii), we get

$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - \frac{y}{x}.\frac{dy}{dx} = 0$

This is the required differential equation whose order is 2 and degree is 1.

10. Sol. General equation of all such circle which pass through the origin and whose centre lie on x axis, is

$x^2 + y^2 + 2gx = 0$ ....(i)

on differentiating w.r.t.x, we get

$2x + 2y\frac{dy}{dx} + 2g = 0$

$2g = -\left(2x + 2y\frac{dy}{dx}\right)$

On putting the value of 2g in Eq.(i), we get

$x^2 + y^2 + \left(-2x - 2y\frac{dy}{dx}\right)x = 0$

$\Rightarrow \quad x^2 + y^2 - 2x^2 - 2xy\frac{dy}{dx} = 0$

$\Rightarrow \quad y^2 = x^2 + 2xy\frac{dy}{dx}$

Which is required equation.

11*. Sol. Equation of normal is $Y - y = -\frac{dx}{dy}(X - x)$

$\Rightarrow \quad G = \left(x + y\frac{dy}{dx},\ 0\right)$

According to question

$\Rightarrow \quad y\frac{dy}{dx} = x \quad$ or $\quad y\frac{dy}{dx} = -3x$

$\Rightarrow \quad y\,dy = xdx \quad$ or $\quad y\,dy = -3x\,dx$

$\Rightarrow \quad \frac{y^2}{2} = \frac{x^2}{2} + c \quad$ or $\quad \frac{y^2}{2} = -\frac{3x^2}{2} + c$

$\Rightarrow \quad x^2 - y^2 = -2c \quad$ or $\quad 3x^2 + y^2 = 2c$

12. Sol. Given equation can be rewritten as $\frac{dy}{dx} - \frac{1}{x}.y = 1$

Now, I.F. = $e^{-\int \frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$

∴ Requirerd solution is $y\left(\frac{1}{x}\right) = \int \frac{1}{x}dx = \log x + c$

Since, (y) = 1 $\Rightarrow$ 1 = log1 + c $\Rightarrow$ c =1

∴ y = x log x + x

13. Sol. $(x - h)^2 + (y - 2)^2 = 25$

$\Rightarrow 2(x - h) + 2(y - 2)\frac{dy}{dx} = 0$

$\Rightarrow (x - h) = -(y - 2)\frac{dy}{dx}$

Substituting in (1), we have

$(y - 2)^2\left(\frac{dy}{dx}\right)^2 + (y - 2)^2 = 25$

$(y - 2)^2 y'^2 = 25 - (y - 2)^2$

14. Sol. $y = c_1 e^{c_2 x}$

$y' = c_1c_2 e^{c_2 x} \quad \Rightarrow \quad c_2 = \frac{y'}{y}$

$y'' = c_1c_2^2 e^{c_2 x} \quad \Rightarrow \quad y'' = y . \left(\frac{y'}{y}\right)^2 \quad \Rightarrow \quad yy'' = (y')^2$

15. Ans. (4)

Sol. $\cos x\, dy - y \sin x\, dx = - y^2\, dx$

$\cos x\, dy + y\, d(\cos x) = - y^2\, dx$

$\frac{d(y\cos x)}{y^2\cos^2 x} = -\frac{dx}{\cos^2 x}$

$-\frac{1}{y\cos x} = - \tan x + c$

$- \sec x = y(- \tan x + c)$

$\sec x = y(\tan x + k)$

Hence correct option is (4)

or lgh fodYi (4) gSA

16. Sol. (2)

$\frac{dv(t)}{dt} = k(T - t)$

$\int dv(t) = \int(-kT)dt + \int ktdt$

$V(t) = -kTt + k\frac{t^2}{2} + c$

at t = 0 C = I

$V(T) = -kTt + \frac{kt^2}{2} + I$

Now at t = T

$V(T) = -k\,T^2 + k\frac{T^2}{2} + I$

$V(T) = I - \frac{1}{2}kT^2$ **Ans.**

17. $\frac{dy}{dx} = y + 3$

$\frac{dy}{y+3} = dx$

$\ell n(y+3) = x + c$

given at x = 0 , y = 2

$\ell n5 = c$

$\therefore \ell n(y+3) = x + \ell n5$

$\ell n\left(\frac{y+3}{5}\right) = x$

$y + 3 = 5e^x$

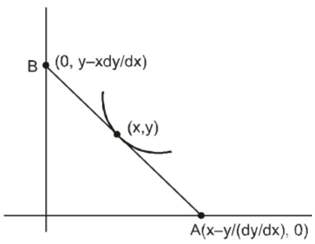
$y = 5e^x - 3$

$\therefore y(\ell n2) = 5e^{\ell n2} - 3 = 7$ **Ans.**

18. **Sol.** **(2)**

$Y - y = \frac{dy}{dx}(X - x)$

X-intercept is $\left(x - \frac{y}{dy/dx},\ 0\right)$



Y- intercept is $\left(0,\ y - \frac{xdy}{dx}\right)$

According to statment

$x - \frac{y}{dy/dx} = 2x$ and $y - \frac{xdy}{dx} = 2y$

$\frac{-y}{\frac{dy}{dx}} = x$ $\qquad \frac{-xdy}{dx} = y$

$\frac{dx}{x} + \frac{dy}{y} = 0$

$\ell ny = -\ell nx + \ell nc$

$y = \frac{c}{x} \Rightarrow c = 6$

Hence $y = \frac{6}{x}$

19. Sol. (3)

$\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

I.F. = $e^{\int \frac{1}{y^2}dy} = e^{-\frac{1}{y}}$

so $x \cdot e^{-\frac{1}{y}} = \int \frac{1}{y^3} e^{-\frac{1}{y}} dy$

Let $\frac{-1}{y} = t$

$\Rightarrow \frac{1}{y^2} dy = dt$

$\Rightarrow I = -\int te^t dt = e^t - te^t = e^{-\frac{1}{y}} + \frac{1}{y} e^{-\frac{1}{y}} + c$

$\Rightarrow xe^{-\frac{1}{y}} = e^{-\frac{1}{y}} + \frac{1}{y} e^{-\frac{1}{y}} + c$

$\Rightarrow x = 1 + \frac{1}{y} + c.e^{1/y}$

since $y(1) = 1$

$\therefore c = -\frac{1}{e}$

$\Rightarrow x = 1 + \frac{1}{y} - \frac{1}{e}.e^{1/y}$

20. Sol. Ans. (1)

$2\frac{dp(t)}{900 - p(t)} = -dt$

$-2\ell n(900 - p(t)) = -t + c$

when $t = 0$, $p(0) = 850$

$-2\ell n(50) = c$ $\therefore$ $2\ell n\left(\frac{50}{900 - p(t)}\right) = -t$

$900 - p(t) = 50\, e^{t/2}$

$p(t) = 900 - 50\, e^{t/2}$

let $p(t_1) = 0$

$0 = 900 - 50\, e^{\frac{t_1}{2}}$ ∴ $t_1 = 2\ell n\, 18$

21. **Sol.** **(3)**

$dP = (100 - 12\sqrt{x})dx$

By integrating

$\int dP = \int (100 - 12\sqrt{x})dx$

$P = 100x - 8x^{3/2} + C$

When x = 0 then P = 2000

$\Rightarrow C = 2000$

Now when x = 25 then P is

$P = 100 \times 25 - 8 \times (25)^{3/2} + 2000$

$= 2500 - 8 \times 125 + 2000$

$= 4500 - 1000$

$\Rightarrow P = 3500$

22. **Sol.** **Ans.** **(3)**

$p'(t) = \frac{1}{2} p(t) - 200$

$p'(t) - \frac{1}{2} p(t) = -200$

$I.F = e^{-\frac{1}{2}t}$

Hence solutions is

$p(t)\, e^{-\frac{1}{2}t} = \int -200\, e^{-t/2} dt = 400\, e^{-\frac{1}{2}t} + C$

or $p(t) = 400 + Ce^{t/2}$

Since p(0) = 100

$\Rightarrow 100 = 400 + C \Rightarrow C = -300$

Thus $p(t) = 400 - 300\, e^{t/2}$.

23. **Ans.** **(3)**

Sol. $\frac{dy}{dx} + \frac{y}{x\log x} = 2$ at x = 1 ; y = 0

I.F. = $e^{\int \frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$

$y(\log x) = \int 2(\log x)dx$

$y(\log x) = 2[x\log x - x] + c$

at x = 1, c = 2

x = e

$y = 2(e - e) + 2$

$y = 2$

24. **Ans.** **(3)**

Sol. $y(1 + xy)\, dx = xdy$

$ydx - xdy + xy^2dx = 0$

$y^2 d\left(\frac{x}{y}\right) + xy^2dx = 0$

$\frac{x}{y} + \frac{x^2}{2} = C$ ....(i)

(1, –1) satisfies $-1 + \frac{1}{2} = C \Rightarrow C = -\frac{1}{2}$

Put in (i) $x = -\frac{1}{2}$

$\frac{-\frac{1}{2}}{y} + \frac{\frac{1}{4}}{2} = -\frac{1}{2} \Rightarrow \frac{-1}{2y} = \frac{-1}{2} - \frac{1}{8}$

$\frac{1}{2y} = \frac{5}{8}$

$y = \frac{4}{5}$

25. **Ans. (1)**

Sol. $\frac{dy}{dx} = \frac{-(y+1)\cos x}{2+\sin x}$

$\int \frac{dy}{y+1} = -\int \frac{\cos x}{2+\sin x}dx$

$\ell n(y + 1) = -\ell n(2 + \sin x) + c$

$(y + 1)(2 + \sin x) = A$; for $x = 0, y = 1 \Rightarrow A = 4$

$\therefore (y + 1)(2 + \sin x) = 4$

for $x = \frac{\pi}{2} \Rightarrow y = \frac{1}{3}$

26. **Sol. (1)**

$\frac{dy}{dx} + \cot x\, y = 4x \operatorname{cosec} x$

I.F. = $e^{\int \cot x dx} = \sin x$

$y (\sin x) = \int 4x \operatorname{cosec} x . \sin x dx + C$

$y \sin x = 2x^2 + C$

$\because y\left(\frac{\pi}{2}\right) = 0$

$C = \frac{-\pi^2}{2}$

$y \sin x = 2x^2 - \frac{-\pi^2}{2}$

so $\left(\frac{\pi}{6}\right) = 2\left(\frac{2\pi^2}{36} - \frac{\pi^2}{2}\right) = 2\pi^2\left(\frac{1}{18} - \frac{1}{2}\right) = -\frac{8\pi^2}{9}$

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. **Sol.** We can write the differential equation as $\frac{dy}{dt} - \frac{t}{1+t} y = \frac{1}{1+t}$ (1)

This is a linear differential equation and its I.F. = $\exp\left(-\int \frac{t}{1+t} dt\right)$

$= \exp[-t + \ell n (1 + t)] = e^{-t} (1 + t)$

Multiplying both the sides of (1) by $e^{-t} (1 + t)$ we get

$e^{-t}(1 + t)\frac{dy}{dt} - te^{-t}y = e^{-t}$

$\Rightarrow \quad \frac{d}{dt}[e^{-t}(1 + t)y] = e^{-t}$

$\Rightarrow \quad e^{-t}(1 + t)y = \int e^{-t}\,dt$

$\Rightarrow \quad e^{-t}(1 + t)y = -e^{-t} + C$

$\Rightarrow \quad y = \frac{-1}{1+t} + \frac{Ce^{t}}{1+t}$

When t = 0, y = – 1

$\therefore \quad -1 = -1 + C \quad \Rightarrow \quad C = 0$

Thus, $y = \frac{-1}{1+t}$

$\Rightarrow \quad y(1) = \frac{-1}{2}$

2. **Sol.** sin x dy + y cos x dx = – cos x dx – 2dy

þ d(y sin x) = d(–sin x) – 2d (y)

þ y sin x = – sin x – 2y + c

Since y(0) = 1 $\Rightarrow$ c = 2.

Thus, $y = \frac{2-\sin x}{2+\sin x}$. Now $y\left(\frac{\pi}{2}\right) = \frac{1}{3}$

3. **Sol.** $\lim\limits_{t\to x}\frac{t^2f(x)-x^2f(t)}{t-x}=1 \quad \frac{0}{0}$ form

using L.Hospital $\lim\limits_{t\to x}\frac{2tf(x)-x^2f'(t)}{1}=1$ (differentiating with respect to t)

So $2x\,f(x) - x^2 f'(x) = 1$ $\quad y = f(x)$

So $2xy - x^2\frac{dy}{dx} = 1$

$x^2\frac{dy}{dx} - 2xy = -1$

$\frac{dy}{dx} - \frac{2y}{x} = \frac{-1}{x^2}$

This is linear differential equation.

Integrating factor = $e^{\int -\frac{2}{x}dx} = \frac{1}{x^2}$

So $\frac{y}{x^2} = \int \frac{-1}{x^4}dx$

$\frac{y}{x^2} = \frac{1}{3x^3} + c$ $\quad \therefore$ f(1) = 1 $\Rightarrow$ $c = \frac{2}{3}$

$y = \frac{1}{3x} + \frac{2}{3}x^2$

4. **Sol.** $\int \frac{y}{\sqrt{1-y^2}}\,dy = x + c$

⇒ $-\sqrt{1-y^2} = x + c$

⇒ $1 - y^2 = (x + c)^2$

⇒ $(x + c)^2 + y^2 = 1$

5. **Sol.** **(A)**

Given slope at (x, y) is

$\frac{dy}{dx} = \frac{y}{x} + \sec(y/x)$

let $\frac{y}{x} = t \Rightarrow y = xt \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}$

$t + x\frac{dt}{dx} = t + \sec(t)$

$\int \cos t\, dt = \int \frac{1}{x} dx$

$\sin t = \ell n\, x + c$

$\sin(y/x) = \ell n\, x + c$

This curve passes through (1, π/6)

$\sin(\pi/6) = \ell n(1) + c \Rightarrow c = 1/2$

$\sin(y/x) = \ell n\, x + 1/2$

6. **Ans.** **(B)**

Sol. I.F. = $e^{\int \frac{x}{x^2-1} dx} = e^{\frac{1}{2}\int \frac{2x}{x^2-1} dx} = e^{\frac{1}{2}\ell n|x^2-1|} = e^{\frac{1}{2}\ell n(1-x^2)} = \sqrt{1-x^2}$

∴ $y\sqrt{1-x^2} = \int \frac{x^4+2x}{\sqrt{1-x^2}} \times \sqrt{1^2-x^2}\, dx + c$

$y\sqrt{1^2-x^2} = \frac{x^5}{5} + x^2 + c$

x = 0, y = 0 ⇒ c = 0

$y = \frac{\frac{x^5}{5}+x^2}{\sqrt{1-x^2}}$

∴ $I = \int_0^{\frac{\sqrt{3}}{2}} \left(\frac{\frac{x^5}{5}+x^2}{\sqrt{1-x^2}} + \frac{\frac{-x^5}{5}+x^2}{\sqrt{1-x^2}} \right) dx$

$= 2\int_0^{\frac{\sqrt{3}}{2}} \frac{x^2}{\sqrt{1-x^2}} dx$

$x = \sin\theta$

$dx = \cos\theta\, d\theta$

$= 2\int_0^{\frac{\pi}{3}} \frac{\sin^2\theta\cos\theta}{\cos\theta} d\theta$

$$= \int_0^{\frac{\pi}{3}} (1-\cos 2\theta)\,d\theta = \left(\theta - \frac{1}{2}\sin 2\theta\right)\Big|_0^{\frac{\pi}{3}} = \frac{\pi}{3} - \frac{1}{2}\times \sin\frac{2\pi}{3} = \frac{\pi}{3} - \frac{1}{2}\times\frac{\sqrt{3}}{2} = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$$

7. Ans. (A)

Sol. $f'(x) = 2 - \frac{f(x)}{x}$ or $\frac{dy}{dx} + \frac{y}{x} = 2$

I.F. $= e^{\int \frac{1}{x}dx} = e^{\ell nx} = x$

solution is $y \times x = \int 2x\,dx = x^2 + c$

or $y = x + \frac{c}{x}$ & $c \neq 0$ as $f(1) \neq 1$

(A) $\lim_{x\to 0^+} f'\left(\frac{1}{x}\right) = \lim_{x\to 0^+} (1 - cx^2) = 1$

(B) $\lim_{x\to 0^+} x\, f\left(\frac{1}{x}\right) = \lim_{x\to 0^+} 1 + cx^2 = 1$

(C) $\lim_{x\to 0^+} x^2 f'(x) = \lim_{x\to 0^+} x^2 - c = -c \neq 0$

(D) $\lim_{x\to 0^+} f(x) \to +\infty$ or $-\infty$

Hence only (A) is correct

8. Ans. (A,D)

Sol. $[(x + 2)(x + 2 + y)]\frac{dy}{dx} - y^2 = 0 \Rightarrow y = (x + 2)t$

$\frac{dy}{dx} = (x + 2).\frac{dt}{dx} + t \Rightarrow ((x + 2)(x + 2 + (x + 2)t)\left((x+2)\frac{dt}{dx} + t\right) - (x + 2)^2.t^2 = 0$

$(x + 2)^2 = 0$ or $(1 + t)\left((x+2)\frac{dt}{dx} + t\right) - t^2 = 0 \Rightarrow (x + 2)(1 + t)\frac{dt}{dx} + t = 0$

$\left(\frac{1+t}{t}\right)dt = -\frac{dx}{x+2}$

$\ell nt + t = -\ell n(x + 2) + c \Rightarrow \ell n\left(\frac{y}{x+2}\right) + \left(\frac{y}{x+2}\right) = -\ell n(x + 2) + c$

$\ell ny - \ell n(x + 2) + \frac{y}{x+2} = -\ell n(x + 2) + c \Rightarrow \ell ny + \frac{y}{x+2} = c$

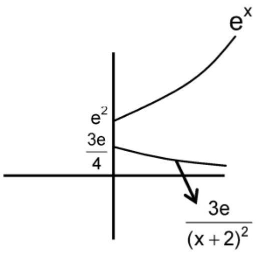
$\ell n3 + \frac{3}{3} = + c \Rightarrow c = \ell n3 + 1 \Rightarrow \ell ny + \frac{y}{x+2} = \ell n3e$

(A) $\ell ny + \frac{y}{x+2} = \ell n3e \Rightarrow \ell n(x + 2) + 1 = \ell n3 + 1$

$\Rightarrow$ one solution ,

(C) $\ln(x + 2)^2 + \frac{(x+2)^2}{x+2} = \ell n3 + 1 \Rightarrow 2\ell n(x + 2) + (x + 2) = \ell n3e$

$(x + 2)^2 e^{(x + 2)} = 3e \Rightarrow e^{x + 2} = \frac{3e}{(x+2)^2}$



no solution

(D) $y = (x + 3)^2 \Rightarrow \ell n(x + 3)^2 + \frac{(x+3)^2}{x+2} = \ell n3 + 1$

$2\ell n(x + 3) + \frac{(x+2)^2+1+2(x+2)}{x+2} = \ell n3 + 1$

$g(x) = 2\ell n(x + 3) + (x + 2) + 2 + \frac{1}{(x+2)} - \ell n3 - 1$

$g'(x) = \frac{2}{(x+3)} + 1 + 0 - \frac{1}{(x+2)^2} = \frac{2(x+2)^2-(x+3)}{(x+3)(x+2)^2} + 1 = \frac{2x^2+8x+8-x-3}{(x+3)(x+2)^2} + 1 > 0$

g(x) increasing$\Rightarrow$ $g(0) = 2\ell n3 + 2 + 1 + \frac{1}{2} - \ell n3 = \ell n3 + \frac{7}{2}$

which is positive, thus no solution

9. **Ans. (B)**

Sol. $\frac{dy}{dx} = \frac{\left(\sqrt{4+\sqrt{9+x}}\right)^{-1}}{8\sqrt{x}\sqrt{9+\sqrt{x}}}$

$dy = \frac{1}{\sqrt{4+\sqrt{9+\sqrt{x}}}} . \frac{1}{\sqrt{9+\sqrt{x}}} . \frac{1}{8\sqrt{x}} dx$

Let $4 + \sqrt{9+\sqrt{x}} = t \Rightarrow \frac{1}{2\sqrt{9+\sqrt{x}}} \times \frac{1}{2\sqrt{x}} dx = dt$

$\int dy = \int \frac{1}{\sqrt{t}}.\frac{1}{2}dt$

$y = \sqrt{t} + c$

$y = \sqrt{4+\sqrt{9+\sqrt{x}}} + c$

at x = 0 , $y = \sqrt{7}$

$\Rightarrow \sqrt{7} = \sqrt{7} + c \Rightarrow c = 0$

$y = \sqrt{4+\sqrt{9+\sqrt{x}}}$

at x = 256 $\Rightarrow$ $y = \sqrt{4+\sqrt{9+\sqrt{256}}} = 3$

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

1. **Sol.** **Obvious**

2. **Sol.** **Obvious**

3. **Sol.** $y = k_1.2^{k_2}.2^x + \frac{1+\tan k_3 x}{1-\tan k_3 x} \times \frac{1-\tan k_3 x}{1+\tan k_3 x} + 2$

$y = \lambda_1 2^x + 1 + 2$

there is exactly one essential arbitrary constant so order is one

4. **Sol.** Equation of circle be $(x-1)^2 + y^2 + \lambda(x-1) = 0$ ....(1)

$2(x-1) + 2y\frac{dy}{dx} + \lambda = 0$

$\lambda = -\{2(x-1) + 2y(\frac{dy}{dx})\}$

from (1) $(x-1)^2 + y^2 = \{2(x-1) + 2y\frac{dy}{dx}\}(x-1)$

$y^2 = (x-1)^2 + 2y(x-1)\frac{dy}{dx}$

5. **Sol.** $y = k(x+k)^3$ ......(i)

$\frac{dy}{dx} = 3k(x+k)^2$ ....(ii)

from (i) & (ii)

$\frac{y}{dy/dx} = \frac{x+k}{3} \Rightarrow x+k = \frac{3y}{dy/dx}$

substitute value of k is (i)

$y = \left(\frac{3y}{dy/dx} - x\right)\left(\frac{3y}{dy/dx}\right)^3$

$y.\left(\frac{dy}{dx}\right)^4 = (3y)^3\left(3y - \frac{xdy}{dx}\right)$

6. **Sol.** Equation of parabolas $(x-\alpha)^2 = a(y-\beta)$ $2(x-\alpha) = a\frac{dy}{dx}$

$2 = a\frac{d^2y}{dx^2}$

7. **Sol.** $\frac{dy}{dx} = \cot x \cot y$

$\int \tan y dy = \int \cot x dx$

$\ell n \sec y = \ell n \sin x + \ell n c$

$\sec y = c \sin x$

8. **Sol.** $\frac{dy}{dx} = \frac{y^2 - y - 2}{x^2 + 2x - 3}$

$\frac{dy}{(y-2)(y+1)} = \frac{dy}{(x+3)(x-1)}$

$\Rightarrow \int \frac{dy}{(y-2)(y+1)} = \int \frac{dy}{(x+3)(x-1)}$

$\Rightarrow \frac{1}{3}\int\left(\frac{1}{y-2} - \frac{1}{y+1}\right)dy = \frac{1}{4}\int\left(\frac{1}{x-1} - \frac{1}{x+3}\right)dx$

$\Rightarrow \frac{1}{3}\ell n\left|\frac{y-2}{y+1}\right| = \frac{1}{4}\ell n\left|\frac{x-1}{x+3}\right| + c$

9. **Sol.** $\frac{dy}{dx} = \ell n(x+1)$

$\int dy = \int \ell n(x+1)dx$

$y = x\ell n(x+1) - \int \frac{x}{x+1}dx$

$= x\ell n(x+1) - \int (1 - \frac{1}{x+1})dx$

$= x\ell n(x+1) - x + \ell n(x+1) + c$

Now at $x = 0$, $y = 3$

$3 = 0 + c \Rightarrow c = 3$

$\therefore y = (x+1)\ \ell n|x+1| - x + 3$

10. **Sol.** $\frac{dy}{dx} = e^{x+y} \Rightarrow \frac{dy}{dx} = e^x.e^y$

$\int e^{-y}dy = \int e^x dx$

$-e_{-y} = e_x + c$

$e_x + e_{-y} = c$

11. **Sol.** $\frac{dy}{dx} = (2x+y)^2$

Let $2x+y = t$

$2 + \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} - 2 = t^2$

$\frac{dt}{t^2+2} = dx$

$\frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}} = x + c$

$\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{2x+y}{\sqrt{2}}\right) = x + c$

12. **Sol.** Let $x+y+1 = t_2$

$1+\frac{dy}{dx}=\frac{2tdt}{dx}$

$\left(\frac{2tdt}{dx}-1\right)t=t^2-2$

$\frac{dt}{dx}=\frac{t^2+t-2}{2t^2}$ or $\int\frac{2t^2}{t^2+t-2}dt=\int dx$

or $\int\left[1+\frac{1}{3(t-1)}-\frac{4}{3(t+2)}\right]dt=\int dx$

or $2\left[t+\frac{1}{3}\ell n\,|t-1|-\frac{4}{3}\ell n\,|t+2|\right]=x+c$ where $t=\sqrt{x+y+1}$

13. **Sol.** $\frac{dx}{dy}=x+y+1 \Rightarrow \frac{dx}{dy}-x=y+1$

I .F = $e^{-\int dy}$ = e^{-y}

solution of differntial equation is

$xe^{-y}=\int(y+1)e^{-y}dy+c$

$\Rightarrow x=ce^{y}-y-2$

14. **Sol.** $\frac{dy}{dx}=\frac{y/x}{1-2\sqrt{y/x}}$

Let $y=vx \Rightarrow \frac{dy}{dx}=x\frac{dv}{dx}+v$

$\therefore x\frac{dv}{dx}=\frac{v}{1-2\sqrt{v}}-v=\frac{2v^{3/2}}{1-2\sqrt{v}}$

$\frac{1-2\sqrt{v}}{2v^{3/2}}dv=\frac{dx}{x}$

on intergration

$-v^{-1/2}-\ell nv=\ell nx+c$

or $-\sqrt{\frac{x}{y}}-\ell ny+\ell nx=c+\ell nx$

or $\ell ny+\sqrt{\frac{x}{y}}=c$

15. **Sol.** $\sin\left(\frac{y}{x}\right)\frac{dy}{dx}=\frac{y}{x}\sin\frac{y}{x}-2$

Let $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dy}{dx}$

$\sin v\left(v+x\frac{dv}{dx}\right)$ = vsinv –2

$x\sin v\,\frac{dv}{dx}=-2$

$\sin v\,dv=-2\,\frac{dx}{x}$

$-\cos v=-2\ell nx+c \Rightarrow \cos v=2\ell nx+c$

$\Rightarrow \cos\frac{y}{x} = \ell n x^2 + c$

16. **Sol.** (2x+y+4) dy = (x–2y+3) dx

2(xdy+ydx) + ydy+4dy = xdx + 3 dx

2d(xy) + ydy + 4dy = xdx +3dx

on intergrating

$2xy+\frac{y^2}{2}+4y = \frac{x^2}{2}+3x+c$

17. **Sol.** $\frac{dx}{dy}+\frac{x^2-xy+y^2}{y^2}=0$

$\Rightarrow \frac{dx}{dy}+\left(\frac{x}{y}\right)^2-\left(\frac{x}{y}\right)+1=0$

Put x =vy $\Rightarrow \frac{dx}{dy} = v+y\frac{dv}{dy}$

x=vy

Now $v+y\frac{dv}{dy}+v^2-v+1 = 0$

$\Rightarrow \frac{dv}{v^2+1}+\frac{dy}{y} = 0$

$\Rightarrow \int\frac{dv}{v^2+1}+\int\frac{dy}{y} = 0$

$\Rightarrow \tan^{-1}(v)+\ell ny+c =0$

$\Rightarrow \tan^{-1}\left(\frac{x}{y}\right)+\ell ny+c =0$

18. **Sol.** $\frac{dy}{dx}+\frac{1}{x}=\frac{e^y}{x^2}$

$e^{-y}\frac{dy}{dx}+\frac{1}{x}e^{-y}=\frac{1}{x^2}$

Put $e^{-y}= t \Rightarrow -e^{-y}\frac{dy}{dx}=\frac{dt}{dx}$

$\Rightarrow -\frac{dt}{dx}+\frac{t}{x}=\frac{1}{x^2} \Rightarrow \frac{dt}{dx}-\frac{t}{x}=-\frac{1}{x^2}$

I.F. = $e^{\int-\frac{1}{x}dx}=e^{-\ell nx}=\frac{1}{x} \Rightarrow \frac{t}{x}=\int-\frac{1}{x^3} \Rightarrow \frac{t}{x}=\frac{1}{2x^2}+c \Rightarrow \frac{e^{-y}}{x}=\frac{1}{2x^2}+c \Rightarrow 2xe^{-y} = cx^2+1$

19. **Sol.** $\frac{dy}{dx}=\frac{y}{2y\ell ny+y-x} \Rightarrow \frac{dx}{dy}+\frac{x}{y}=(2\ell ny+1)$

I.F. $e^{\int\frac{1}{y}dy}=e^{\ell ny}=y$

solution is $yx = \int(2\ell ny+1)ydy+c$

$xy = y^2\ell ny +c$

20. **Sol.** I.F. = $e^{\int \cot x dx} = e^{\ell n \sin x} = \sin x$

solution is,

ysinx = $\int \sin^2 x$ dx+c

ysinx = $\frac{1}{2}\int (1-\cos 2x)dx + c$

ysinx= $\frac{1}{4}(2x - \sin 2x) + c$

21. **Sol.** $\frac{dx}{dy} = -\left(\frac{1+x(1+y^2)}{y+y^3}\right) = -\left[\frac{1}{y(y^2+1)} + \frac{x(1+y^2)}{y(1+y^2)}\right]$

$\frac{dx}{dy} + \frac{x}{y} = \frac{-1}{y(1+y^2)}$

I.F. = $e^{\int \frac{1}{y}dy} = y$

The solution is

xy = – $\int \frac{1}{y(1+y^2)}$ .y dy+c

or xy = – $\tan^{-1}$y+c

22. **Sol.** $\frac{1}{x^3}\frac{dx}{dy} + \frac{1}{yx^2} = 1$

Let $\frac{1}{x^2} = t$ ⇒ $-\frac{2}{x^3}\frac{dx}{dy} = \frac{dt}{dy}$

$-\frac{1}{2}\frac{dt}{dy}$ + $\frac{t}{y}$ = 1 or $\frac{dt}{dy}$ – $\frac{2}{y}$ t = –2

I.F. = $e^{-\int \frac{2}{y}dy}$ = $e^{-2\ell n y}$ = $\frac{1}{y^2}$

Solution is, t $\left(\frac{1}{y^2}\right)$ = – 2 $\int \frac{1}{y^2}dy + c$

$\frac{t}{y^2}$ = $\frac{2}{y} + c$

$2x^2$ y + c x^2y^2 = 1

23. **Sol.** $\frac{dy}{dx} = \frac{2xy}{x^2 - 1 - 2y}$ ⇒x^2dy–(1+2y) dy=2xydx

or 2xydx–x^2dy= – (1+2y) dy

or $\frac{yd(x^2) - x^2dy}{y^2} = -\left(\frac{1}{y^2} + \frac{2}{y}\right)dy$

$d\left(\frac{x^2}{y}\right)$ = $-\left(\frac{1}{y^2} + \frac{2}{y}\right)dy$

Integrating , $\frac{x^2}{y} = \frac{1}{y} - 2\ell n y + c$

24. **Sol.** xy^4dx+ydx = xdy

xdx+ $\frac{ydx - xdy}{y^4}$ = 0

or $x^3dx + \left(\frac{x}{y}\right)^2 d\left(\frac{x}{y}\right) = 0$

integration , $\frac{x^4}{4} + \frac{1}{3}\left(\frac{x}{y}\right)^3 = c$ $\Rightarrow 3x^4y^3+4x^3=cy^3$

25. Sol. $\frac{xdx + ydy}{(x^2+y^2)^2} = \left(\frac{ydx - xdy}{y^2}\right) \times \frac{y^2}{x^2}$

or $\int \frac{d(x^2+y^2)}{(x^2+y^2)^2} = -2\int d\left(\frac{y}{x}\right)$

$-\frac{1}{(x^2+y^2)} = -2\frac{y}{x} + c$ or $\frac{2y}{x} = \frac{1}{x^2+y^2} + c$

26. Sol. The equation of tangent a the point A(x,f(x) is

Y–f(x) = f'(x) (X–x)

P(o,f(x) – x f'(x))

The slope of perpendicular line through P is

$\frac{f(x) - xf'(x)}{-1} = -\frac{1}{f'(x)}$

or f(x) f'(x) – x(f'(x))^2= 1

or $\frac{ydy}{dx} = x\left(\frac{dy}{dx}\right)^2 + 1$

27. Sol. Equation of normal at P is Y–y = – $\frac{dx}{dy}$ (X–x)

$\Rightarrow \frac{1}{2}\left(y\frac{dy}{dx} + x\right)\left(x\frac{dy}{dx} + y\right) = 1$

$y^2\left(\frac{dy}{dx}\right)^2 + 2(xy-1)\frac{dy}{dx} + x^2 = 0$

28. Sol. $\frac{dv}{dt} = -k(4\pi r^2)$

Put v = $\frac{4}{3}\pi r^3$ or $\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

Hence $\frac{dr}{dt}$ = – k

29. Sol. $\frac{dv(t)}{dt} = -k(T - t^2)$

$\int dv(t) = \int -k(T - t^2)dt$

v(t) = – k (Tt– $\frac{t^3}{3}$)+c

v(0) = I ⇒ I = c

v(t) = – k (Tt– $\frac{t^3}{3}$)+I

$v(T) = -k\left(T^2 - \frac{T^3}{3}\right) + I$

30. **Sol.** $\frac{dy}{dx} = x\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3\frac{d^2y}{dx^2} - 8\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)$

$0 = \frac{d^2y}{dx^2}\left(x + 3 - 8\frac{dy}{dx}\right)$

$\frac{d^2y}{dx^2} = 0$ or $8\frac{dy}{dx} = x+3$

$\frac{dy}{dx} = c$

genered solution is $y = c\,x + 3c - 4c^2$

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

1. **Sol.** $Y - y = m(X - x)$

Y-intercept ($X = 0$)

$Y = y - mx$

Given that $y - mx = x^3$ $\Rightarrow$ $x\frac{dy}{dx} - y = -x^3$

$\Rightarrow$ $\frac{dy}{dx} - \frac{y}{x} = -x^2$

Intergrating factor (I.F.) = $e^{-\int \frac{1}{x}dx} = \frac{1}{x}$

solution $y \cdot \frac{1}{x} = \int \frac{1}{x} \cdot (-x^2)\, dx$ $\Rightarrow$ $f(x) = y = -\frac{x^3}{2} + cx$

Given $f(1) = 1$ $\Rightarrow$ $c = \frac{3}{2}$

$\therefore$ $f(x) = -\frac{x^3}{2} + \frac{3x}{2}$ $\Rightarrow$ $f(-3) = 9$

2.* **Sol.** $f''(x) = f'(x) + e^x$

Integration both sides w.r.t., x

$f'(x) = f(x) + e^x + c$

put $x = 0$ $\Rightarrow$ $1 = 0 + 1 + c$ $\Rightarrow$ $c = 0$

so, $\frac{dy}{dx} - y = e^x$

$\Rightarrow$ $y(\text{I.F.}) = \int e^x . (\text{I.F.})dx$

$ye^{-x} = \int dx$

$ye^{-x} = x + c$

$\Rightarrow$ $f(x) = xe^x + ce^x$

$f(0) = 0 + c \quad \Rightarrow \quad c = 0$

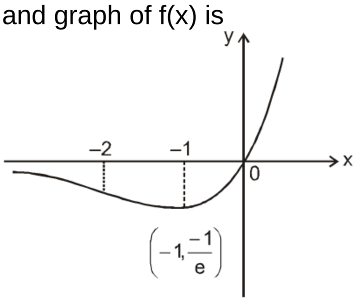
$f(x) = xe^x$

Now $f'(x) = xe^x + e^x$

$f'(x) = (x + 1)e^x$

$f''(x) = (x + 2)e^x$

and graph of f(x) is



3.* **Sol.** $f(\theta) = \frac{d}{d\theta}\left(\frac{\operatorname{cosec}^2\theta}{2}\int_0^\theta \frac{1}{2}\sec^2 x dx\right) = \frac{d}{d\theta}\left(\frac{1}{4}\operatorname{cosec}^2\theta.\tan\theta\right)$

$f(\theta) = \frac{1}{2}\frac{d}{d\theta}(\operatorname{cosec}(2\theta))$

$f(\theta) = -\operatorname{cosec}2\theta . \cot 2\theta$

Now $\frac{df}{d\theta} - 2f(\theta).\tan 2\theta = 2\operatorname{cosec}^3 2\theta + 2\operatorname{cosec}2\theta + 2\operatorname{cosec}2\theta \cot^2 2\theta$

$= 4\operatorname{cosec}^3 2\theta$

4.* **Sol.** $(x - \alpha)^2 + (y - \alpha)^2 = r^2$

$x^2 + y^2 - 2\alpha x - 2\alpha y + 2\alpha^2 - r^2 = 0$

$2x + 2yy' - 2\alpha - 2\alpha y' = 0$ ...(i)

$\Rightarrow$ $\alpha = \frac{x + yy'}{1 + y'}$ ...(ii)

again diff. w.r.t.

$2 + 2(y')^2 + 2yy'' - 2\alpha y''\left(\frac{x + yy'}{1 + y'}\right) = 0$

$\Rightarrow$ $1 + (y')^2 + yy'' - \left(\frac{x + yy'}{1 + y'}\right)y'' = 0$

$\Rightarrow$ $1 + y' + (y')^2 + (y')^3 + yy'' + yy'y'' - xy'' - yy'y'' = 0$

$\Rightarrow$ $(y - x)y'' + (1 + y + (y')^2)\, y' + 1 = 0$

$\Rightarrow$ $P = y - x,\ Q = 1 + y' + (y')^2$

Ans. (2,3)

Note : P & Q will not be unique function as

$Py'' + Qy' + Ry' - Ry' + 1 = 0$

$\Rightarrow$ $\frac{Py'}{1 - Ry''} + \frac{Qy'}{1 - Ry'} + 1, = 0$ Hence new P & Q can be obtained.

So it can be a controversial problem.

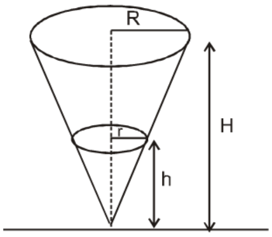
5. **Sol.** $\frac{R}{H} = \frac{r}{h}$

$h = \frac{Hr}{R}$

$\frac{dV}{dt} = -k\pi r^2$

$\Rightarrow \frac{d}{dt}\left[\frac{1}{3}\pi r^2 h\right] = -k\pi r^2$

$\Rightarrow \frac{d}{dt}\left(\frac{r^3 H}{R}\right) = -3k\pi r^2$



$\Rightarrow \frac{dr}{dt} = -k\frac{R}{H}$

$\int_R^0 dr = -\frac{kR}{H}\int_0^t dt$

$\Rightarrow -R = \frac{-kR}{H}t \Rightarrow t = \frac{H}{k}$

6*. **Sol.** Equation of the tangent is

$$Y - y = \frac{dy}{dx}(X - x)$$

Given $\frac{BP}{AP} = \frac{3}{1}$ so that $y = \frac{y - x\frac{dy}{dx}}{3+1}$

$\Rightarrow \frac{dx}{x} = -\frac{dy}{3y} \Rightarrow x\frac{dy}{dx} + 3y = 0$

$\Rightarrow \ln x = -\frac{1}{3}\ln y - \ln c \Rightarrow \ln x^3 = -(\ln cy)$

$\Rightarrow \frac{1}{x^3} = cy$. Given f(1) = 1 $\Rightarrow$ c = 1

$\therefore y = \frac{1}{x^3}$

curve passes through $\left(2, \frac{1}{8}\right)$, $m_{Normal} = \frac{1}{3}$

$y - 1 = +\frac{1}{3}(x - 1)$

$\Rightarrow 3y - x = 2$

7. **Sol.** $x^2 y_1^2 + xy\, y_1 - 6y^2 = 0$

It is quadratic equation in y_1

$$y_1 = \frac{-xy \pm \sqrt{x^2y^2 + 24y^2x^2}}{2x^2} = \frac{-xy \pm 5xy}{2x^2}$$

$y_1 = -\frac{3y}{x}$	$y_1 = \frac{2y}{x}$
$\frac{dy}{dx} = \frac{-3y}{x}$	$\frac{dy}{dx} = \frac{2y}{x}$
$-\frac{dy}{y} = 3\frac{dx}{x}$	$\frac{dy}{dx} = \frac{2y}{x}$
$-\ell n\, y = 3\,\ell nx + \ell nc$	$\ln y = 2\,\ell nx + \ell nc$
$x^3 y = C$	$y = cx^2$

Option (3)

$\ell n\, y = c + 2\ell nx$

$\frac{1}{2}\ell n\, y = \ell n\, c_1 + \ell nx$

8. **Sol.** $\frac{dm}{dx} = m$

$\ell nm = x \quad \Rightarrow \quad m = c_1e^x$

$\frac{dy}{dx} = c_1e^x$

$y = c_1e^x + c_2$

9. **Sol.** Let $t = \frac{d^2y}{dx^2}$

$\frac{dt}{dx} = 8t$

$\ell nt = 8x + c$ here at $x = 0$, $c = 0$

$t = e^{8x} \Rightarrow \frac{d^2y}{dx^2} = e^{8x}$

now put $m = \frac{dy}{dx}$

$\frac{dm}{dx} = e^{8x}$

$m = \frac{e^{8x}}{8} + c_1$

at $x = 0$, $c_1 = \frac{-1}{8}$ $\quad x = 0\ c_1 = \frac{-1}{8}$

$\frac{dy}{dx} = \frac{e^{8x}}{8} - \frac{1}{8}$

$y = \frac{e^{8x}}{64} - \frac{x}{8} + c_2$

at x = 0, $c_2 = \frac{7}{64}$

$\therefore \quad y = \frac{e^{8x}}{64} - \frac{x}{8} + \frac{7}{64} \quad \Rightarrow \quad 64y = e^{8x} - 8x + 7$

10. Sol. $y_1y_3 = 3y_2^2$

$\frac{y_3}{y_2} = 3\frac{y_2}{y_1} \quad \Rightarrow \quad \ell n\, y_2 = 3\ell n\, y_1 + \ell n\, c$

$y_2 = cy_1^3$

$\frac{y_2}{y_1^2} = cy_1$

$-\frac{1}{y_1} = cy + c_2$

$\frac{dx}{dy} = -cy - c_2$

$x = -\frac{cy^2}{2} - c_2y + c_3$

$\therefore \quad x = A_1y^2 + A_2y + A_3$

11. Sol. $\frac{xdx - ydy}{xdy - ydx} = \sqrt{\frac{1 + x^2 - y^2}{x^2 - y^2}}$

$x = r\sec\theta$

$y = r\tan\theta$

$x^2 - y^2 = r^2$

$dx = r\sec\theta\tan\theta\, d\theta + \sec\theta\, dr$

$dy = r\sec^2\theta\, d\theta + \tan\theta\, dr$

$\Rightarrow \quad \frac{r\ \ d\ \ r}{r^2\sec\theta\ \ d\theta} = \sqrt{\frac{1+r^2}{r^2}}$

$\Rightarrow \quad \int\frac{dr}{\sqrt{1+r^2}} = \int\sec\theta\ \ d\theta \quad \Rightarrow \quad \ell n|r + \sqrt{1+r^2}| = \ell n|\sec\theta + \tan\theta| + \ell nc$

$\Rightarrow \quad r + \sqrt{1+r^2} = c(\sec\theta + \tan\theta)$

on solving $\sqrt{x^2 - y^2} + \sqrt{1 + x^2 - y^2} = \frac{c\ \ (x+y)}{\sqrt{x^2 - y^2}}$

12. Sol. $ax^2 + 2hxy + by^2 = 1$

order : 3

13. Sol. $\frac{dy}{dx} = \frac{(x+y)+1}{2(x+y)+1}$

put $\quad x + y = t, \ \ 1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \quad \frac{dt}{dx} = \frac{t+1}{2t+1} + 1 = \frac{3t+2}{2t+1}$

$\Rightarrow \quad \int\frac{2t+1}{3t+2}\, dt = \int dx$

or $\frac{2t}{3} - \frac{1}{9}\ell n\,(3t + 2) = x + c$

or $6(x + y) - \ell n\,(3x + 3y + 2) = 9x + c$

or $\ell n\,(3x + 3y + 2) = 6y - 3x + c$

since it passes throw (0, 0) hence equation of curve is $6y - 3x = \ell n\left|\frac{3x+3y+2}{2}\right|$

14. **Sol.** $e^{(d^3y/dx^3)^2} = 1 + \left(\frac{d^3y}{dx^3}\right)^2 + \frac{\left(\frac{d^3x}{dx^3}\right)^4}{2\,!} + \ldots\ldots$

here power of highest order is not defined, hence degree is not defined.

15. **Sol.** $y\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y\frac{dy}{dx} - x = 0$

$y\frac{dy}{dx}\left(\frac{dy}{dx} - 1\right) + x\left(\frac{dy}{dx} - 1\right) = 0$

$\left(y\frac{dy}{dx} + x\right)\left(\frac{dy}{dx} - 1\right) = 0$

∴ either $ydy + xdx = 0$ or $dy - dx = 0$

since the curves pass through the point (3, 4)

∴ $x^2 + y^2 = 25$ or $x - y + 1 = 0$

16. **Sol.** $P(x, y) = ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

$\frac{dP}{dx} = 0 \Rightarrow 2ax + 2hy + 2\,g = 0$ $\qquad ax + hy + g = 0$

$\frac{dP}{dy} = 0 \Rightarrow 2by + 2hx + 2f = 0$ $\qquad by + hx + f = 0$

$\Rightarrow h = g = f = 0$

conic : $ax^2 + by^2 + c = 0 \Rightarrow x^2 + \left(\frac{b}{a}\right)y^2 + \frac{c}{a} = 0$ order = 2

17. **Sol.** **(1)** $\frac{dy}{dx} = -c_1 \sin x + \sqrt{c_2}\cos x$

$\frac{d^2y}{dx^2} = -c_1\cos x - \sqrt{c_2}\sin x = 2 - y$

$\frac{d^2y}{dx^2} + y - 2 = 0$

(2) $\frac{dy}{dx} = \cos x\,\frac{\sec^2 x/2}{2\tan\frac{x}{2}} - \sin x\,\ell n\left(\tan\frac{x}{2}\right)$

$\frac{dy}{dx} = \cot x - \sin x\,\ell n\left(\tan\frac{x}{2}\right)$

$$\frac{d^2y}{dx^2} = -\text{cosec}^2 x - \left(\sin x.\frac{1}{\sin x} + \cos x \ \ell n \ \left(\tan\frac{x}{2}\right)\right)$$

$$\frac{d^2y}{dx^2} = -\cot^2 x - 2 - \cos x \ \ell n\left(\tan\frac{x}{2}\right)$$

$$\frac{d^2y}{dx^2} + y + \cot^2 x = 0$$

(3) $\frac{dy}{dx} = \cos x - \sin x$

$$\frac{d^2y}{dx^2} = -\cos x - \sin x$$

$$\frac{d^2y}{dx^2} + y + \cot^2 x = \cot^2 x$$

18. Sol. Here, slope of tangent

$$\frac{dy}{dx} = \frac{(x+1)^2 + y - 3}{(x+1)}$$

$\Rightarrow$ $\frac{dy}{dx} = (x + 1) + \frac{(y-3)}{(x+1)}$, put $x + 1 = X$ and $y - 3 = Y$ $\left(\text{here } \frac{dy}{dx} = \frac{dY}{dX}\right)$

$\therefore$ $\frac{dY}{dX} = X + \frac{Y}{X}$ $\Rightarrow$ $\frac{dY}{dX} - \frac{1}{X} Y = X$

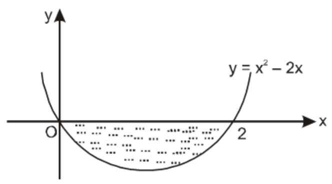
where integrating factor

$$= e^{\int -\frac{1}{x} dX} = e^{-\ell n X} = \frac{1}{X}$$

$\therefore$ Solution is,

$$Y . \frac{1}{X} = \int X \cdot \frac{1}{X} dX + c \Rightarrow \frac{Y}{X} = X + c$$

$y - 3 = (x + 1)^2 + c(x + 1)$, which passes through (2, 0)



$-3 = 9 + 3c$

$\Rightarrow$ $c = -4$

$\therefore$ Required curve

$y = (x + 1)^2 - 4(x + 1) + 3$

$\Rightarrow$ $y = x^2 - 2x$

Drawing curve

Thus, required area

$$= \left| \int_0^2 (x^2 - 2x) \ dx \right| = \left| \left(\frac{x^3}{3} - x^2\right)_0^2 \right| = \frac{4}{3} \text{ sq. units}$$