Fundamental of Mathematics - II

Exercise-1

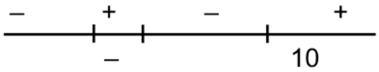
OBJECTIVE QUESTIONS

Section – (A) Polynomial inequalities

A-1 **Sol.** $x^2 \le 4$ $\Rightarrow$ $x \in [-2, 2]$
$x \ge -4$ $\quad x = \{-2, -1, 0, 1, 2\}$

A-2 **Sol.** $\dfrac{x^2 - 4x + 3}{2x + 1} - 3 < 0$

$\dfrac{x^2 - 10x}{2x + 1} < 0$

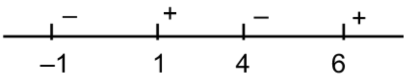


{1, 2, 3, 4, 5, 6, 7, 8, 9}

A-3 **Sol.** $\dfrac{14x(x-4) - (9x-30)(x+1)}{(x+1)(x-4)} < 0$

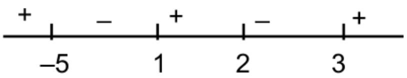
$\dfrac{14x^2 - 56x - (9x^2 - 21x - 30)}{(x+1)(x-4)} < 0$

$\dfrac{(x-1)(x-6)}{(x+1)(x-4)} < 0$



$\Rightarrow x \in \{0, 5\}$

A-4 **Sol.** $\dfrac{7 + 9(x-2) + (x-2)(x-3)}{(x-2)(x-3)} < 0$ $\Rightarrow \dfrac{x^2 + 4x - 5}{(x-2)(x-3)} < 0$ $\Rightarrow \dfrac{(x-1)(x-5)}{(x-2)(x-3)} < 0$



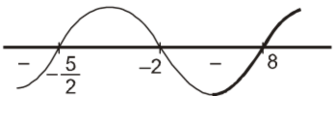
No positive Integers

A-5. **Sol.** $-5 \le x < 10 \Rightarrow x = -5, -4, -3, ...9$
and $0 \le x \le 15$ means $x = 0, 1, 2, ...9, 10, 11, 12, ...15$
Required integers values of x are 0,1, 2, 3, 4,........9 $\Rightarrow$ Number of integers values of x = 10

A-6. **Sol.** $\dfrac{x^2 - 1}{2x + 5} - 3 < 0$ $\Rightarrow$ $\dfrac{x^2 - 1 - 6x - 15}{2x + 5} < 0$ $\Rightarrow$ $\dfrac{x^2 - 6x - 16}{2x + 5} < 0$ $\Rightarrow$

$\dfrac{x^2 - 8x + 2x - 16}{\left(x + \frac{5}{2}\right)} < 0$

$\Rightarrow \dfrac{x(x-8) + 2(x-8)}{x + \frac{5}{2}} < 0$ $\Rightarrow \dfrac{(x-8)(x-2)}{x + \frac{5}{2}} < 0$



$x \in (-\infty, -5/2) \cup (-2, 8)$

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A-7. **Sol.** $5x + 2 < 3x + 8 \Rightarrow 2x < 6 \Rightarrow x < 3$ ...(i)

$\frac{x+2}{x-1} < 4 \Rightarrow \frac{x+2}{x-1} - 4 < 0 \Rightarrow \frac{-3x+6}{x-1} < 0$

$\Rightarrow \frac{x-2}{x-1} > 0 \Rightarrow x \in (-\infty, 1) \cup (2, \infty)$ ...(ii)

Taking intersection of (i) and (ii) $x \in (-\infty, -1) \cup (2, 3)$

Sol. $5x + 2 < 3x + 8 \Rightarrow 2x < 6 \Rightarrow x < 3$ ...(i)

$\frac{x+2}{x-1} < 4 \Rightarrow \frac{x+2}{x-1} - 4 < 0 \Rightarrow \frac{-3x+6}{x-1} < 0$

$\Rightarrow \frac{x-2}{x-1} > 0 \Rightarrow x \in (-\infty, 1) \cup (2, \infty)$ ...(ii)

i) ∩ (ii) , $x \in (-\infty, -1) \cup (2, 3)$

A-8. **Sol.** $x^2 + 9 < (x + 3)^2 < 8x + 25$

$\Rightarrow (x + 3)^2 > x^2 + 9 \Rightarrow x > 0$ ....(i)

and $(x + 3)^2 < 8x + 25 \Rightarrow x^2 - 2x - 16 < 0$

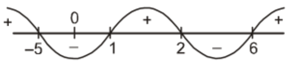
$\Rightarrow x \in (1 - \sqrt{17}, 1 + \sqrt{17})$ .....(ii)

(i) ∩ (ii) $\Rightarrow x \in (0, 1 + \sqrt{17})$

Noumber of integers = 5

A-9. **Sol.** $\frac{x^2(x^2-3x+2)}{x^2-x-30} \geq 0 \Rightarrow \frac{x^2(x-1)(x-2)}{(x+5)(x-6)} \geq 0$

$x \neq -5, 6$



$x \in (-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$

A-10. **Sol.** $x \in (-\infty, -9) \cup (-9, -3) \cup [-1, 0) \cup (0, 2) [4, 6)$

so +ve integral solution

A-11. **Sol.** $\frac{2}{x^2-x+1} - \frac{1}{x+1} - \frac{2x-1}{x^3+1} \geq 0$

$\frac{2x+2-x^2+x-1}{x^3+1} - \frac{(2x-1)}{x^2+1} \geq 0 = \frac{3x+1-x^2-2x+1}{x^3+1} \geq 0 = \frac{-x^2+x+2}{x^3+1} \geq 0 = \frac{x^2-x-2}{x^3+1} \leq 0$

$= \frac{(x+1)(x-2)}{(x+1)(x^2-x+1)} \leq 0 \Rightarrow \frac{(x-2)}{(x^2-x+1)} \leq 0$

required value of x, {0, 1, 2}

Section (B) : Logarithm identities, properties and graphs

B-1. **Sol.** $a^4b^5 = 1 \Rightarrow$ log w.r.t. a → $4 + 5\log_a b = \log_a 1 \Rightarrow \log_a b = -\frac{4}{5}$

Now $\log_a(a^5b^4) = 5 + 4\log_a b = 5 + 4\left(-\frac{4}{5}\right) = \frac{25-16}{5} = \frac{9}{5}$

B-2. Sol. $\frac{1}{1+\log_b a+\log_b c}+\frac{1}{1+\log_c a+\log_c b}+\frac{1}{1+\log_a b+\log_a c}$

$=\frac{1}{\log_b abc}+\frac{1}{\log_c abc}+\frac{1}{\log_a abc}$

$= \log_{abc} b + \log_{abc} c + \log_{abc} a = \log_{abc} abc = 1$

B-3. Sol. $\frac{1}{\log_{\sqrt{bc}} abc}+\frac{1}{\log_{\sqrt{ca}} abc}+\frac{1}{\log_{\sqrt{ab}} abc}$

$= \log_{abc}\sqrt{bc}+\log_{abc}\sqrt{ca}+\log_{abc}\sqrt{ab}=\log_{abc} abc = 1$

B-4. Sol. $\log_2(5\times 2).\log_2(2^4\times 5)-(\log_2 5)\log_2(2^5\times 5)$

$= (\log_2 5 + 1)(4 + \log_2 5) - \log_2 5(5 + \log_2 5)$, let $\log_2 5 = t$

$= (t + 1)(4 + t) - t(5 + t) = t^2 + 5t + 4 - 5t - t^2 = 4 = 4\log_2 2 = \log_2 16$

B-5. Sol. $y = \frac{2^{\log_{2^{1/4}} a}-3^{\log_{27}(a^2+1)^3}-2a}{7^{4\log_{49} a}-a-1}$ $\Rightarrow$ $2^{\log_{2^{1/4}} a} = 2^{4\log_2 a} = a^4$

$3^{\log_{27}(a^2+1)^3} = 3^{\log_3(a^2+1)} = a^2 + 1$ $\Rightarrow$ $7^{4\log_{49} a} = 7^{2\log_7 a} = a^2$

$\therefore$ $y = \frac{a^4-(a^2+1+2a)}{a^2-a-1} = \frac{a^4-(a+1)^2}{a^2-a-1} = a^2 + a + 1$

B-6. Sol. $x = 2^{\log 3}$, $y = 3^{\log 2} = 2^{\log 3} = x$

B-7. Sol. $\log_a(ab) = x$ $\Rightarrow$ $1 + \log_a b = x$ $\Rightarrow$ $\log_a b = x - 1$ $\Rightarrow$ $\log_b a = \frac{1}{x-1}$

Now $\log_b(ab) = 1 + \log_b a = 1 + \frac{1}{x-1} = \frac{x-1+1}{x-1} = \frac{x}{x-1}$

B-8. Sol. $\log_{10}\pi$ is quantity lie between 0 to 1.

B-9. Sol. $\log_p \log_p (p)^{\frac{1}{p^n}} = \log_p\left(\frac{1}{p}\right)^n = -\log_p p^n = -n$ independent of p.

B-10. Sol. $\log_{10}(\log_2 3) + \log_{10}(\log_3 4) + \log_{10}(\log_4 5) + \ldots\ldots + \log_{10}(\log_{1023} 1024) = \log_{10}(\log_2 1024)$

$= \log_{10}(\log_2 2^{10}) = \log_{10}(10) = 1$

B-11. Sol. Clearly Domain is $x > 0$ and $x \neq 1$

Section (C) : Logarithm equation and inequalities

C-1 Sol. Let $\log_2 x = t$

$\frac{1-t/2}{1+t}\frac{-1}{2} \le 0$

$\frac{1-2t}{1+t} \le 0$ $\Rightarrow$ $\frac{2t-1}{t+1} \ge 0$

$\log_2 x \le -1$ or $\log_2 x \ge \frac{1}{2}$ $\Rightarrow x \le \frac{1}{2}$ or $x \ge \sqrt{2}$

C-2. **Sol.** $\log_p(\log_q(\log_r x)) = 0$ $\Rightarrow$ $\log_q(\log_r x) = b$ $\Rightarrow$ $\log_r x = q \Rightarrow$ $x = r^q$ ......(i)
and $\log_q(\log_r(\log_p x)) = 0$
$\Rightarrow$ $\log_r(\log_p x) = 1$ $\Rightarrow$ $\log_p x = r$ $\Rightarrow$ $x = p^r$ ......(ii)
from (i) and (ii) $p^r = r^q$
(i) (ii) $p^r = r^q$
$\Rightarrow$ $p = r^{q/r}$

C-3. **Sol.** $2\log_{10}x - \log_{10}(2x - 75) = 2$ $\Rightarrow$ $\frac{x^2}{2x-75} = 10^2 = 100$
$\Rightarrow x^2 - 200x + 7500 = 0 \Rightarrow x = 50, x = 150$
sum = 200

C-4. **Sol.** $\log_x \log_{18}\left(\sqrt{2}+2\sqrt{2}\right) = \frac{1}{3}$ $\Rightarrow$ $\log_x \log_{18}\left(\sqrt{18}\right) = \frac{1}{3}$ $\Rightarrow$ $\log_x \frac{1}{2} = \frac{1}{3}$
$\Rightarrow$ $x^{1/3} = \frac{1}{2}$ $\Rightarrow$ $x = \frac{1}{8}$ $\Rightarrow$ 1000 x = 125.

C-5. **Sol.** $3^{2\log_3 x} - 2x - 3 = 0$ $\Rightarrow$ $x^2 - 2x - 3 = 0$
$(x - 3)(x + 1) = 0$ $\Rightarrow$ $x = 3, x = -1$ but $x \ne -1$
$\therefore$ x = 3.

C-6. **Sol.** $\sqrt{\log_{10}(-x)} = \log_{10}|x|$ $\Rightarrow$ $-x > 0$ $\Rightarrow$ $\sqrt{\log_{10}(-x)}$ $x < 0$
$\therefore$ $|x| = -x$ $\Rightarrow$ $= \log_{10}(-x)$
$\log_{10}(-x)\left(\log_{10}(-x) - 1\right) = 0$
$\log_{10}(-x) = 0$ $\log_{10}(-x) = 1$
$\Rightarrow$ $-x = 1$ $-x = 10$
$x = -1$ $x = -10$.

C-7. **Sol.** $\log_{\sin\frac{\pi}{3}}(x^2 - 3x + 2) \ge 2$ $\Rightarrow$ $x^2 - 3x + 2 \le \left(\frac{\sqrt{3}}{2}\right)^2$ $\Rightarrow$ $4x^2 - 12x + 8 \le 3$
$\Rightarrow$ $4x^2 - 12x + 5 \le 0$ $\Rightarrow$ $(2x - 5)(2x - 1) \le 0$ $\Rightarrow$ $x \in \left[\frac{1}{2}, \frac{5}{2}\right]$
But domain $x^2 - 3x + 2 > 0$ $\Rightarrow$ $(x - 1)(x - 2) > 0$ $\Rightarrow$ $x \in (-\infty, 1) \cup (2, \infty)$
Hence $x \in \left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$

C-8. **Sol.** $\log_{0.3}(x - 1) < \log_{0.09}(x - 1)$; $\log_{0.3}(x - 1) < \frac{\log_{0.3}(x-1)}{2}$
$\Rightarrow$ $\log_{0.3}(x - 1) < 0 \Rightarrow$ $x - 1 > 1$ $\Rightarrow$ $x > 2$

C-9. **Sol.** $2 - \log_2(x^2 + 3x) \ge 0$ $\Rightarrow$ $\log_2(x^2 + 3x) \le 2$
$x^2 + 3x > 0$ $\Rightarrow$ $x \in (-\infty, -3) \cup (0, \infty)$ .....(i)
and $x^2 + 3x \le 4$
$\Rightarrow$ $(x - 1)(x + 4) \le 0$ $\Rightarrow$ $x \in [-4, 1]$ .....(ii)

(i) ∩ (ii) ⇒ $x \in [-4, -3) \cup (0, 1]$

C-10. **Sol.** $\log_{0.5} \log_5 (x^2 - 4) > \log_{0.5} 1$; $\log_{0.5} \log_5 (x^2 - 4) > 0$

⇒ $x^2 - 4 > 0$ ⇒ $x \in (-\infty, -2) \cup (2, \infty)$ ....(i)

$\log_5 (x^2 - 4) > 0$ ⇒ $x^2 - 5 > 0$

⇒ $x \in (-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$ ....(ii)

$\log_5 (x^2 - 4) < 1$

⇒ $x^2 - 9 < 0$ ⇒ $x \in (-3, 3)$ ....(iii)

(i) ∩ (ii) ∩ (iii) ⇒ $x \in (\sqrt{5} -3,) \cup (\sqrt{5}, 3)$

C-11. **Sol.** $\left(\frac{1}{2}\right)^{x^2-2x} < \left(\frac{1}{2}\right)^{2}$ ⇒ $x^2 - 2x > 2$

⇒ $x^2 - 2x - 2 > 0$ ⇒ $x \in (-\infty, 2-\sqrt{3}) \cup (2 + \sqrt{3}, \infty)$

Section (D) : Modulus Functions, Equation, Inequalities

D-1. **Sol.** $|x^2-4x + 3|$ ⇒ $x^2 - 4x + 3$ $(x-1)(x-3) \ge 0$

$-(x^2-4x+3)$ $(x-1)(x-3)<0$

∴ $y = x^2-4x+3+x=7$ $x \in (-\infty,1] \cup [3,\infty)$

$x^2-4x+3+7-x=0$ $x \in (1,3)$

D-2 **Sol.** y =

$-1 - 2x$	$-\infty < x < -3$
5	$-3 \le x < 0$
$5 - 2x$	$0 \le x < 2$
1	$2 \le x < 3$
$2x - 5$	$x \ge 3$

Solve for $y = 7x + 1$ ⇒ $x = 4/9$

D-3. **Sol.** If p is solution, then –p ao will be another sol.

D-4 **Sol.** Let $|x-1| = t$; $t^2 - 3t + 2 = 0$ ⇒ $t = 1,2$

$|x-1| = 1$ $|x-1| = 2$

$x = 0,2$ $x = -1, 3$

D-5 **Sol.** Minimum will be at $x = 1$ $f(1) = 2$

D-6 **Sol.** $|x-4| = 5$ or $|x-4| = 3$

$x = 9, -1$ $x = 7, 1$

D-7 **Sol.** $|x^3 - 9x^2 + 26x -24| = |x-2|\,|x-3|\,|x-4|$

When $x \in I$; above 3 are consecutive positive integers, hence multiplication can never be a prime number

D-8. **Sol.** y =

$-2x + 1$	$x \in (-\infty,-1)$
3	$x \in [-1, 2)$
$2x-1$	$x \in [2, \infty)$

$y \ge 3$ $\forall x \in R$

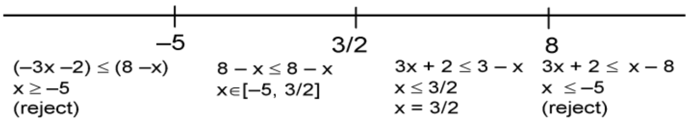
D-9. **Sol.** $-1 \le |x -2|-1 \le 1$

$0 \le |x-2| \le 2$

$|x-2| \ge 0$ and $|x-2| \le 2$

$x \in R$ $0 \le x \le 4$

D-10. **Ans.** $x \in [-5, 3/2]$



Sol.

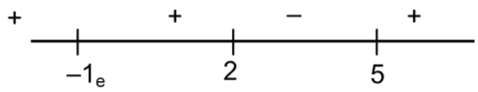
D-11. **Ans.** $x \in [2, 5] \cup \{-1\}$

Sol. $|-x^2 + 4x + 5| + |x^2 - x - 2| = |3x + 3|$

$|a| + |b| = |a + b| \Rightarrow ab \geq 0$

$(x^2 - 4x - 5)(x^2 - x - 2) \leq 0$

$(x+1)^2 (x-5)(x-2) \leq 0$



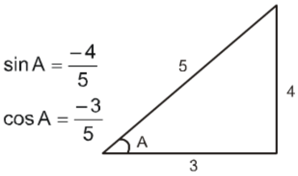
D-12. **Sol.**

Case- I	**Case- II**
$\log_2 x \geq 0$	$\log_2 x < 0$
$x \geq 1$	$0 < x < 1$
$\log_2 x \geq 2/3$	$-\log_2 x \geq 2$
$x \geq 2^{\frac{2}{3}}$	$x \leq \frac{1}{4}$ $\quad x \in \left(0, \frac{1}{4}\right]$

Section (E) :Trigonometric ratio and identities

E-1. **Sol.** $\tan A = \frac{4}{3}$ $\Rightarrow$ $A \to$ III[rd] quadrant

$5 \sin 2A + 3 \sin A + 4 \cos A = 10 \sin A \cos A + 3 \sin A + 4 \cos A$



$= 10 \sin A \cos A + 3 \sin A + 4 \cos A = 0$

E-2 **Sol.** $\frac{2\sin 8\theta \cos\theta - 2\sin 6\theta \cos 3\theta}{2\cos 2\theta \cos\theta - 2\sin 3\theta \sin 4\theta} = \frac{\sin 9\theta + \sin 7\theta - \sin 9\theta - \sin 3\theta}{\cos 3\theta + \cos\theta - \cos\theta + \cos 7\theta}$

$= \frac{2\sin 2\theta \cos 5\theta}{2\cos 5\theta \cos 2\theta} = \tan 2\theta$

E-3. **Sol.** $0 < \theta < \frac{\pi}{4}$ $\Rightarrow$ LHS $= \sqrt{2 + \sqrt{2 \times 2\cos^2 2\theta}} = \sqrt{2 + 2|\cos 2\theta|}$

$0 < \theta < \frac{\pi}{4}$ $\Rightarrow$ $0 < 2\theta < \frac{\pi}{2}$

$\therefore$ $\cos 2\theta$ is + ve $\Rightarrow$ $2|\cos\theta| = 2\cos\theta$ $\because$ $\theta \in (0, \pi/4)$

E-4. **Sol.** LHS $= \left\{-\left(\dfrac{1-\tan^2\left(\dfrac{\alpha-\pi}{4}\right)}{1+\tan^2\left(\dfrac{\alpha-\pi}{4}\right)}\right)+\cos\dfrac{\alpha}{2}\cot 4\alpha\right\}\sec\dfrac{9\alpha}{2}$

$= \left\{-\cos\left(\dfrac{\alpha-\pi}{2}\right)+\cos\dfrac{\alpha}{2}\cot 4\alpha\right\}\sec\dfrac{9\alpha}{2} = \left\{-\sin\dfrac{\alpha}{2}+\dfrac{\cos\dfrac{\alpha}{2}\cos 4\alpha}{\sin 4\alpha}\right\}\sec\dfrac{9\alpha}{2}$

$= \dfrac{1}{\sin 4\alpha}\left[\cos 4\alpha\cos\dfrac{\alpha}{2}-\sin 4\alpha\sin\dfrac{\alpha}{2}\right]\sec\dfrac{9\alpha}{2} = \dfrac{1}{\sin 4\alpha}\times\cos\dfrac{9\alpha}{2}.\sec\dfrac{9\alpha}{2}$ = cosec 4α = RHS

E-5. **Sol.** $\dfrac{\sin 24^\circ\cos 6^\circ-\sin 6^\circ\cos 24^\circ}{\sin 21^\circ\cos 39^\circ-\sin 39^\circ\cos 21^\circ} = \dfrac{\sin(24^\circ-6^\circ)}{\sin(21^\circ-39^\circ)} = \dfrac{\sin 18^\circ}{\sin(-18^\circ)} = -1$

E-6. **Sol.** $\dfrac{\tan(180^\circ-25^\circ)-\tan(90^\circ+25^\circ)}{1+(\tan(180^\circ-25^\circ)\tan(90^\circ+25^\circ)} = \dfrac{-\tan 25^\circ+\dfrac{1}{\tan 25^\circ}}{2} = \dfrac{1-x^2}{2x}$

E-7. **Sol.** 3A = 2A + A ⇒ tan3A = tan(2A + A) ⇒ tan3A = $\dfrac{\tan 2A+\tan A}{1-\tan 2A\tan A}$

⇒ tan3A – tanA tan 2A tan3A = tan2A + tanA

⇒ tan3A – tan2A – tanA = tanA tan2A tan 3A

E-8. **Sol.** $\alpha\in\left[\dfrac{\pi}{2},\pi\right] \Rightarrow \dfrac{\alpha}{2}\in\left[\dfrac{\pi}{4},\dfrac{\pi}{2}\right]$

$\sqrt{1+\sin\alpha}-\sqrt{1-\sin\alpha} = \sqrt{\left(\sin\dfrac{\alpha}{2}+\cos\dfrac{\alpha}{2}\right)^2}-\sqrt{\left(\sin\dfrac{\alpha}{2}-\cos\dfrac{\alpha}{2}\right)^2} = \left|\sin\dfrac{\alpha}{2}+\cos\dfrac{\alpha}{2}\right|-\left|\sin\dfrac{\alpha}{2}-\cos\dfrac{\alpha}{2}\right|$

$= \sin\dfrac{\alpha}{2}+\cos\dfrac{\alpha}{2}-\sin\dfrac{\alpha}{2}+\cos\dfrac{\alpha}{2} = 2\cos\dfrac{\alpha}{2}\left(\because \text{ for } \dfrac{\alpha}{2}\in\left[\dfrac{\pi}{4},\dfrac{\pi}{2}\right], \sin\dfrac{\alpha}{2}>\cos\dfrac{\alpha}{2}\right)$

E-9. **Sol.** cos (540°– θ) – sin (630°– θ) = – cos θ + cos θ = 0

E-10. **Sol.** $\dfrac{1}{2}$ = (2 sin 12° . sin48° . sin 54°) = $\dfrac{1}{2}$ [(cos 36° – cos 60°) sin 54°]

$= \dfrac{1}{2}\left[\sin^2 54^\circ-\dfrac{1}{2}\sin 54^\circ\right]$

$= \dfrac{1}{4}\left[2\dfrac{(\sqrt{5}+1)^2}{16}-\dfrac{(\sqrt{5}+1)}{4}\right] = \dfrac{1}{4}\left[\dfrac{5+1+2\sqrt{5}}{8}-\dfrac{(\sqrt{5}+1)}{4}\right] = \dfrac{1}{32}\left[6+2\sqrt{5}-2\sqrt{5}-2\right] = \dfrac{1}{8}$

E-11. **Sol.** $\dfrac{2\sin x}{\sin x(3-4\sin^2 x)}+\dfrac{\tan x(1-3\tan^2 x)}{\tan x(3-\tan^2 x)}$

$\dfrac{2}{3-4\sin^2 x}+\dfrac{\cos^2 x-3\sin^2 x}{3\cos^2 x-\sin^2 x} = \dfrac{2}{3-4\sin^2 x}+\dfrac{1-\sin^2 x-3\sin^2 x}{3-3\sin^2 x-\sin^2 x} = \dfrac{2+1-4\sin^2 x}{3-4\sin^2 x} = 1$

E-12. Sol. LHS = $\sin^2 x$ + sin (y + z) sin(y – z) = $\sin^2 x$ + sin(y+z) sin(π – x) = sinx [sin(π – (y –z)) + sin(y + z)]
= sinx . 2 siny cos z = 2 sinx siny cosz

E-13. Sol. LHS = 2sin $\left(\frac{\pi}{2}-z\right)$ cos (x – y) + 2 sin z cos z ; x + y = $\left(\frac{\pi}{2}-z\right)$
⇒ 2 cos z {cos(x – y) + cos(x + y)} (∵ z = – (x + y))
= 2 cos z × 2cosx cosy = 4 cos x cos y cos z

E-14. Sol. LHS = 2sin(A + B) cos(A – B) + 2 sin C cos C (∵ A + B = – C)
= 2sinC {– cos(A – B) + cos(A + B)} = 2sinC { 2 sinA sin (– B)} = – 4 sinA sinB sinC

E-15. Sol. LHS = sin θ + sin (θ + φ) + sin (θ +2φ) + ... + sin(θ + (n –1) φ)

= $\frac{\sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}}\cdot\sin\left(\frac{2\theta+(n-1)\phi}{2}\right)$

∵ $\phi=\frac{2\pi}{n}$ (External angle of regular polygon)

So LHS = $\frac{\sin\frac{n}{2}(2\pi/n)}{\sin(\pi/n)}\sin\left(\frac{2\theta+\frac{(n-1)2\pi}{n}}{2}\right)$ =0 = RHS

E-16. Sol. By using Series formulae

$\cos A\cos 2A\cos 2^2A\cos 2^3A..........\cos 2^{n-1}A=\frac{\sin 2^nA}{2^n\sin A}$

LHS = $\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7}=-\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}=-\frac{\sin\frac{8\pi}{7}}{2^3.\sin\frac{\pi}{7}}=\frac{1}{8}$ = RHS

E-17. Sol. $\cos\frac{\pi}{19}+\cos\frac{3\pi}{19}+\cos\frac{5\pi}{19}+......+\cos\frac{17\pi}{19}$ A = $\frac{\pi}{19}$, D = $\frac{2\pi}{19}$, n = 9

∵cos A + cos (A + D) + cos(A + 2D) + ... + cos (A + (n – 1)D) = $\frac{\sin\left(\frac{nD}{2}\right)}{\sin\frac{D}{2}}$.

$\cos\left(\frac{2A+(n-1)D}{2}\right)=\frac{\sin 9\times\frac{\pi}{19}}{\sin\frac{\pi}{19}}\times\cos\left(\frac{\frac{\pi}{19}+\frac{17\pi}{19}}{2}\right)=\frac{\sin\frac{9\pi}{19}}{\sin\frac{\pi}{19}}\times\cos\frac{9\pi}{19}$ **Ans.**

E-18. Sol. Let y = cosx .cos $\left(\frac{2\pi}{3}+x\right)\cos\left(\frac{2\pi}{3}-x\right)$

y = $\frac{1}{2}\cos x\left[\cos\frac{4\pi}{3}+\cos 2x\right]$ ⇒ y = $\frac{1}{2}\cos x\left[\frac{-1+2\cos 2x}{2}\right]$

y = $\frac{1}{4}\left[2\cos 2x\cos x-\cos x\right]$ ⇒ y = $\frac{1}{4}$ [cos 3x + cos x – cosx]

$y = \frac{1}{4}\cos 3x$ $\quad\because\quad -1 \le \cos 3x \le 1$

$y_{min} = -\frac{1}{4}$ and $y_{max} = \frac{1}{4}$

E-19. Sol. $y = \cos^2\left(\frac{\pi}{4}+x\right) + (\sin x - \cos x)^2 = \cos^2\left(\frac{\pi}{4}+x\right) + 2\left(\cos^2\left(\frac{\pi}{4}+x\right)\right)$

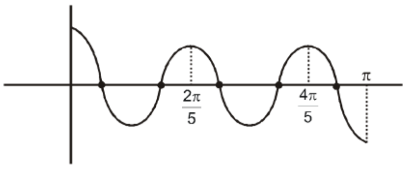
$y = 3\cos^2\left(\frac{\pi}{4}+x\right)$ $\quad\because\quad 0 \le \cos^2\theta \le 1 \quad\Rightarrow\quad y_{max} = 3.1 = 3 \quad\Rightarrow\quad y_{min} = 0$

E-20. Sol. $\cos^2x + \sec^2x + 3\sec^2x$
$\ge 2 + 3$
≥ 5

Section (F) :Trigonometric equation and inequalities

F-1. Sol. Obvious

F-2. Sol. $\sin x \,.\, \tan 4x = \cos x \Rightarrow \sin x \sin 4x = \cos x \cos 4x$
$\Rightarrow \cos 5x = 0 \Rightarrow$ five solutions.



F-3. Sol. $\sin 7x + \sin 4x + \sin x = 0 \quad\Rightarrow\quad 2\sin 4x \cos 3x + \sin 4x = 0$

$\Rightarrow \sin 4x = 0$ or $\cos 3x = -\frac{1}{2} \quad\Rightarrow\quad 4x = n\pi$ or $3x = 2n\pi \pm \frac{2\pi}{3}$

$\Rightarrow x = \frac{n\pi}{4}, \frac{2n\pi}{3} \pm \frac{2\pi}{9} = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{9}, \frac{4\pi}{9}$

F-4. Sol. $\sin x + \sin 5x = \sin 2x + \sin 4x \quad\Rightarrow\quad 2\sin 3x \cos 2x = 2\sin 3x \cos x$
$\Rightarrow \sin 3x = 0$ or $\cos 2x = \cos x \Rightarrow \quad 3x = n\pi$ or $2x = 2n\pi$

$\Rightarrow x = \frac{n\pi}{3}, \; 2n\pi, \; \frac{2n\pi}{3}, \quad x = \frac{n\pi}{3} \Rightarrow$ (It includes all three possible)

F-5. Sol. $\cos 2\theta + 3\cos\theta = 0 \Rightarrow 2\cos^2\theta + 3\cos\theta - 1 = 0 \Rightarrow \cos\theta = \frac{3 \pm \sqrt{9+8}}{4} = \frac{-3 \pm \sqrt{17}}{4}$

As $-1 \le \cos\theta \le 1 \quad\therefore\quad \cos\theta = \frac{-3+\sqrt{17}}{4}$ only

$\Rightarrow \theta = 2n\pi \pm \alpha$ where $\cos\alpha = \frac{\sqrt{17}-3}{4}$

F-6. Sol. $\tan\theta = -1 \quad\Rightarrow\quad \theta = \frac{3\pi}{4}, \frac{7\pi}{4}$ in $[0, 2\pi]$

$\cos\theta = \frac{1}{\sqrt{2}}$ ⇒ $\theta = \frac{\pi}{4}, \frac{7\pi}{4}$ in $[0, 2\pi]$

common value is $x = \frac{7\pi}{4}$ general solution is $2n\pi + \frac{7\pi}{4}$, $n \in I$.
(Reject)

F-7. **Sol.** $\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$ ⇒ $3 \tan 3x = 3$

⇒ $\tan 3x = 1$ ⇒ $x = \frac{n\pi}{3} + \frac{\pi}{12}$, $n \in I$

Exercise-2

1. **Sol.** $\frac{(x+1)(x-1)(e^x-1)}{(x-1)} \le 0$

$+ \quad -1 \quad - \quad 0 \quad + \quad 1 \quad +$ $[-1,0]$

2. **Sol.** $\frac{6x^2 - 5x - 3}{x^2 - 2x + 6} - 4 \le 0$ ⇒ $\frac{2x^2 + 3x - 27}{x^2 - 2x + 6} \le 0$

denominator $x^2 - 2x + 6 > 0$ $\forall$ $x \in R$ $(\because D < 0)$

then $2x^2 + 3x - 27 \le 0$ ⇒ $(2x + 9)(x - 3) \le 0$

$-\frac{9}{2} \le x \le 3$ ⇒ $0 \le x^2 \le \frac{81}{4}$

$(4x^2)_{max} = 4\left(-\frac{9}{2}\right)^2 = 81$ ⇒ $(4x^2)_{min} = 4(0) = 0$

3. **Sol.** $\frac{4\alpha}{\alpha^2 + 1} \ge 1$ ⇒ $\alpha^2 - 4\alpha + 1 \le 0$ ⇒ $\alpha + \frac{1}{\alpha} \le 4$

Now $\alpha + \frac{1}{\alpha} = 1$. But $\alpha + \frac{1}{\alpha} \ge 2$
so reject

and $\alpha +$ $= 3$ ⇒ $\alpha^2 - 3\alpha + 1 = 0$ ⇒ $\alpha = \frac{3 \pm \sqrt{9-4}}{2}$

⇒ $\alpha = \frac{3 \pm \sqrt{5}}{2}$

two real values

4. **Sol.** $y = \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots\infty}}}$

$y^2 = 6 + y \Rightarrow y = 3$

$\log_2 \log_9 3 = 1$

5. **Sol.** $\log_a 625 = I \Rightarrow 5^4 = a^I$

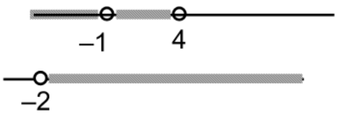
a = 5, 25, 625

6. **Sol.** $xy = 15$; $\frac{y-x}{xy} = \frac{2}{15} \Rightarrow y–x = 2$
$y = 5$; $x = 3$

7. **Sol.** $b = a^2$, $c = b^2$, $\frac{c}{a} = 3^3 \Rightarrow$ $c = 27a \Rightarrow$ $b^2 = 27a$ $\Rightarrow$ $a^4 = 27a$
$\Rightarrow$ $a = 3, a > 0$
$c = 81, b = 9$
$\therefore$ $a + b + c = 3 + 9 + 81 = 93$

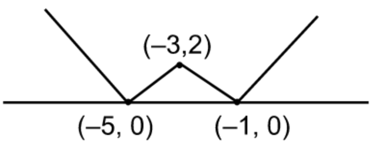
8. **Sol.** $a^{(\log_3 7)^2} = \left(a^{\log_3 7}\right)^{\log_3 7} = 27^{\log_3 7} = 27^{\log_3 7} = 7^3 = 343$
$b^{(\log_7 11)^2} = \left(b^{\log_7 11}\right)^{\log_7 11} = 49^{\log_7 11} = 11^{\log_7 49} = 121$
$c^{(\log_{11} 25)^2} = \left(c^{\log_{11} 25}\right)^{\log_{11} 25} = \left(\sqrt{11}\right)^{\log_{11} 25} = 25^{\log_{11}\sqrt{11}} = 5$
hence the sum is $343 + 121 + 5 = 469$

9. **Sol.** $\frac{\ell n(x+2)}{x^2-3x-4} \le 0 \Rightarrow$



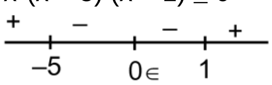
10. **Sol.** $\log_{(0.2)^2}(x-1) \ge \log_{0.2}(x-1)$
$\log_{0.2}(x-1) \ge 2\log_{0.2}(x-1)$
$(x-1) \le (x-1)^2$
$\Rightarrow x \ge 2$

11. **Sol.** Domain $\frac{x-1}{x+2} > 0 \Rightarrow x < -2$ or $x > 1$
Since $\log_9 10 > 1 \Rightarrow \log_2\left(\frac{x-1}{x+2}\right) > 1 \Rightarrow \left(\frac{x-1}{x+2}\right) > 2$
$\Rightarrow \frac{x+5}{x+2} < 0 \Rightarrow x \in (-5, -2)$ $\Rightarrow$ integers = {– 4,–3}

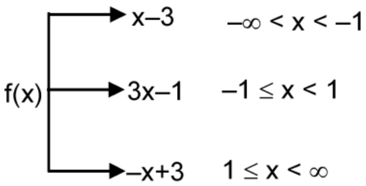


12. **Sol.**
P = 2
no. of values = 1

13. **Sol.** $|a| + |b| = |a + b| \Rightarrow a.b. \ge 0$
$x(x + 5)(x)(1 - x) \ge 0$
$x^2(x + 5)(x - 1) \le 0$



$x \in [-5,1] \Rightarrow \{-5, -4,-3,-2,-1,0,1\}$

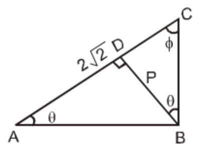


14. Sol.
max at $x = 1$ $f(1) = 2 - 2|1 - 1| = 2$

15. Sol. $x = 2 + |y|$
$|x + 1| + y = 5$
$|2 + |y| + 1| + y = 5 \Rightarrow 3 + |y| + y = 5$
$y + |y| = 2$
$y > 0 \Rightarrow y = 1$; $x = 3$

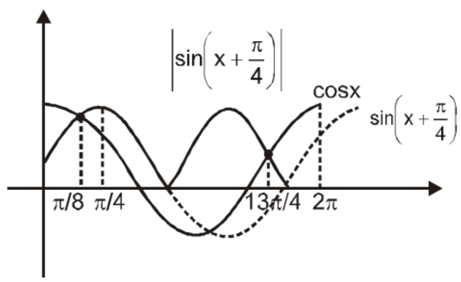
16. Sol. $\because$ $\tan A < 0$ and () $A + B + C = 180°$
$\Rightarrow A > 90°$ $\Rightarrow B + C < 90°$ $\Rightarrow \tan(B + C) > 0$
$\Rightarrow \dfrac{\tan B + \tan C}{1 - \tan B \tan C} > 0$ $\Rightarrow 1 - \tan B \tan C > 0$ $\Rightarrow \tan B \tan C < 1$

17. Sol. $AC = 2\sqrt{2}P$ $\Rightarrow$ $\dfrac{P}{AD} = \dfrac{DC}{P} = \tan\theta$
$\because$ $AD + DC = 2P$ $\Rightarrow$ $+ P\tan\theta = 2P$
$\Rightarrow \dfrac{\cos^2\theta + \sin^2\theta}{2\sin\theta \cos\theta} = \sqrt{2}$ $\Rightarrow$ $\sin 2\theta = \dfrac{1}{\sqrt{2}}$ $\Rightarrow$ $\theta = \dfrac{\pi}{8}$



So, $\varphi = \pi - \left(\dfrac{\pi}{2} + \dfrac{\pi}{8}\right)$ $\Rightarrow$ $\varphi = \dfrac{3\pi}{8}$

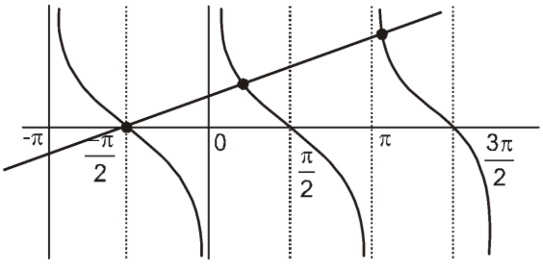
18. Sol. $2\cos x = \sqrt{2 + 2\sin 2x}$ $\Rightarrow \sqrt{2}\cos x = \sqrt{1 + \sin 2x} = |\sin x + \cos x|$
$\Rightarrow \cos x = \left|\dfrac{1}{\sqrt{2}}(\sin x + \cos x)\right|$ $\Rightarrow$ $\cos x = \left|\sin\left(x + \dfrac{\pi}{4}\right)\right|$
see from graph or we can put values given in options to verify.



19. Ans. 3

Sol. $\cot x = \dfrac{\pi}{2} + x$; $x \in \left[-\pi, \dfrac{3\pi}{2}\right]$

let y = cot x & y = $\frac{\pi}{2}$ + x



3 solutions.

20. **Sol.** $\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0 \Rightarrow 2\cos 4\theta \cos 2\theta + (1 + \cos 4\theta) = 0$

$\Rightarrow 2\cos 4\theta \cos 2\theta + 2\cos^2 2\theta = 0 \Rightarrow 2\cos 2\theta (\cos 4\theta + \cos 2\theta) = 0$

$\Rightarrow \cos 2\theta (2\cos 3\theta \cos\theta) = 0 \quad \Rightarrow \theta = (2n+1)\frac{\pi}{2}, \quad (2n+1)\frac{\pi}{4}, \quad (2n+1)\frac{\pi}{6}$

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

A-1 **Ans. (1)**

Sol. Since $0 < \sqrt{13} - \sqrt{12} < 1$ $\therefore$ $\log_{10}(\sqrt{13} - \sqrt{12}) < 0$

Since $0 < \sqrt{14} - \sqrt{13} < 1$ $\therefore$ $\log_{0.1}(\sqrt{14} - \sqrt{13}) > 0$

A-2 **Ans. (1)**

Sol. Statement-1 : $y = \log_{1/3}(x^2 - 4x + 5)$ is max.

when $x^2 - 4x + 5$ is min.

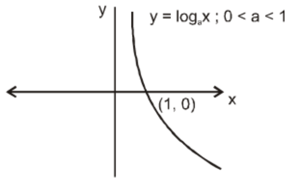
Let $f(x) = x^2 - 4x + 5$

$\Rightarrow$ $(x - 2)^2 + 1$

$f(x)_{min} = 1$

$y_{max} = \log_{1/3}1 = 0$

Statement-1 is true



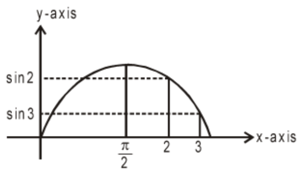
Statement-2 : $\log_a x \le 0$ for $x \ge 1$, $0 < a < 1$

$\therefore$ Statement-2 is true and

correct explanation for statement-1

A-3. **Ans. (1)**

Sol.



A-4. **Ans. (1)**

Sol. If a & b are of same sign then $|a + b| = |a| + |b|$ $ab \ge 0$

$\therefore$ $(x - 2)(x - 7) \ge 0$ $\Rightarrow$ $x \le 2$ or $x \ge 7$

$\because$ $2x - 9 = (x - 2) + (x - 7)$

Section (B) : MATCH THE COLUMN

B-1. **Ans.** (A) → (p), (B) → (r), (C) → (p), (D) → (q)

B-2. Ans. (A) → (p), (B) → (s), (C) → (r), (D) → (q)

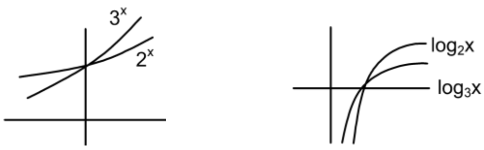
Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. **Sol.** $N = \dfrac{\log_3 135}{\log_{15} 3} - \dfrac{\log_3 5}{\log_{405} 3} = (\log_3 27 + \log_3 5)(\log_3 15) - \log_3 5 . \log_3 405$

$= (3+\log_3 5)\,(1+\log_3 5) - \log_3 5\ \log_3(81 \times 5)$

$= (3 + \log_3 5)\ (1 + \log_3 5) - \log_3 5(4 + \log_3 5) = 3$

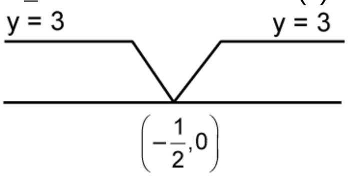
C-2. **Sol.**



C-3. **Sol.** $-\infty < x < -2 \quad \Rightarrow \quad f(x) = 3$

$-2 \le x < 1 \quad f(x) = |2x + 1|$

$1 \le x < \infty \quad f(x) = 3$



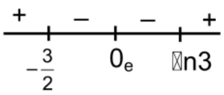
C-4. **Sol.** $\cos 15x = \sin 5x \Rightarrow \quad \cos 15x = \cos\left(\frac{\pi}{2} - 5x\right)$ or $\cos\left(\frac{3\pi}{2} + 5x\right)$

$15x = 2n\pi \pm \left(\frac{\pi}{2} - 5x\right)$ or $15x = 2n\pi \pm \left(\frac{3\pi}{2} + 5x\right)$

$\Rightarrow \quad x = \frac{n\pi}{10} + \frac{\pi}{40}, n \in I, \quad x = \frac{n\pi}{5} + \frac{3\pi}{20}, n \in I$

and $x = \frac{n\pi}{5} - \frac{\pi}{20}, n \in I \quad$ and $x = \frac{n\pi}{10} - \frac{3\pi}{40}, n \in I$

C-5. **Sol.**



$x \in \left(-\infty, -\frac{3}{2}\right] \cup (\ell n3, \infty) \cup \{0\}$

Exercise-3

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. **Sol.** $\because \quad \alpha$ is a root of $25 \cos^2\theta + 5 \cos\theta - 12 = 0$.

$\therefore \quad 25 \cos^2\alpha + 5 \cos\alpha - 12 = 0$

$\Rightarrow \quad \cos\alpha = -\frac{4}{5}, \frac{3}{5} \qquad$ But $\frac{\pi}{2} < \alpha < \pi$ (IInd quadrant)

$\therefore \cos\alpha = \frac{-4}{5}$ and $\sin\alpha = \frac{3}{5}$

$\Rightarrow \quad \sin 2\alpha = 2 \sin\alpha \cos\alpha = -$

2. **Sol.** $\because \quad |c| > \sqrt{a^2 + b^2}$

$\Rightarrow$ $c < -\sqrt{a^2+b^2}$ & $c > \sqrt{a^2+b^2}$

But $-\sqrt{a^2+b^2} \le a\sin x + b\cos x \le \sqrt{a^2+b^2}$ .... (i)

& $a\sin x + b\cos x = c$ .... (ii)

$\therefore$ from (i) & (ii)

no slution

3. Sol. $y = \sin^2\theta + \mathrm{cosec}^2\theta$

$= (\sin\theta - \mathrm{cosec}\,\theta)^2 + 2$

$\Rightarrow$ $y \ge 2$, $\theta \ne 0$

4. Sol. $\sin(\alpha+\beta) = 1$, $\Rightarrow$ $\alpha + \beta = \frac{\pi}{2}$ ..... (i)

$\sin(\alpha-\beta) = \frac{1}{2}$, $\Rightarrow$ $\alpha - \beta = \frac{\pi}{6}$ ..... (ii)

on solving (i) & (ii) (i) (ii)

$\alpha = \frac{\pi}{3}$, $\beta = \frac{\pi}{6}$

$\therefore$ $\tan(\alpha+2\beta).\tan(2\alpha+\beta) = \tan\left(\frac{2\pi}{3}\right)\tan\left(\frac{5\pi}{6}\right)$

5. Sol. $\tan\theta = -\frac{4}{3}$ $\Rightarrow$ $\theta \in$ IInd or IVth quadrant

$\therefore$ $\sin\theta = \frac{4}{5}$ or $-\frac{4}{5}$

6. Sol. $\frac{1-\tan^2 15^o}{1+\tan^2 15^o} = \cos 30^o = \frac{\sqrt{3}}{2}$

7. Sol. $\because$ $\sin^2\theta \le 1$

$\therefore$ $\frac{4xy}{(x+y)^2} \le 1$ $\Rightarrow$ $x^2 + y^2 + 2xy - 4xy \ge 0$

$\Rightarrow$ $(x-y)^2 \ge 0$

which is true for all real values of x & y

provided $x + y \ne 0$, otherwise $\frac{4xy}{(x+y)^2}$ will be meanigless.

8. Sol. $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$

$\Rightarrow$ $u^2 = a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta + 2\sqrt{a^2\cos^2\theta + b^2\sin^2\theta} \times \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$

$\Rightarrow$ $u^2 = (a^2 + b^2) + 2\sqrt{\{a^2 + (b^2-a^2)\sin^2\theta\} \times \{a^2 + (b^2-a^2)\cos^2\theta\}}$

$\Rightarrow$ $u^2 = (a^2+b^2) + 2\sqrt{a^4 + a^2(b^2-a^2) + (b^2-a^2)^2\sin^2\theta\cos^2\theta}$

$\Rightarrow$ $u^2 = (a^2+b^2) + 2\sqrt{a^2b^2 + \left(\frac{b^2-a^2}{2}\right)^2\sin^2 2\theta}$

∴ $\min(u_2) = a_2 + b_2 + 2ab = (a+b)^2$

and $\max(u_2) = a_2 + b_2 + (a^2+b^2) = 2(a^2+b^2)$

Now $\max(u_2) - \min(u_2) = (a-b)_2$

9. **Sol.** $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$

squaring and adding, we get

$\sin_2\alpha + \sin_2\beta + 2\sin\alpha\sin\beta + \cos_2\alpha + \cos_2\beta + 2\cos\alpha \cdot \cos\beta$

$= \left(-\frac{21}{65}\right)^2 + \left(-\frac{27}{65}\right)^2$

$\Rightarrow 2 + 2\cos(\alpha-\beta) = \frac{1170}{4225}$

$\Rightarrow \cos_2\left(\frac{\alpha-\beta}{2}\right) = \frac{1170}{4\times4225} = \frac{9}{130}$

$\Rightarrow \cos\left(\frac{\alpha-\beta}{2}\right) = \frac{-3}{\sqrt{130}}$ $(\because \pi < \alpha-\beta < 3\pi \Rightarrow \frac{\pi}{2} < \left(\frac{\alpha-\beta}{2}\right) < \frac{3\pi}{2})$

10. **Sol.** $\cos x + \sin x = \frac{1}{2}$

∴ $\frac{1-\tan^2 x/2}{1+\tan^2 x/2} + \frac{2\tan x/2}{1+\tan^2 x/2} = \frac{1}{2}$, Let $\tan\frac{x}{2} = t$

$\frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2} = \frac{1}{2} \Rightarrow 3t_2 - 4t - 1 = 0$

∴ $t = \frac{2\pm\sqrt{7}}{3}$

as $0 < x < \pi \Rightarrow 0 < \frac{x}{2} < \frac{\pi}{2}$

∴ $\tan\frac{x}{2}$ is positive

∴ $t = \tan\frac{x}{2} = \frac{2+\sqrt{7}}{3}$

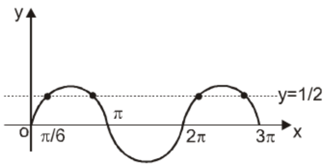
Now $\tan x = \frac{2\tan x/2}{1-\tan^2 x/2} = \frac{2t}{1-t^2}$

$\Rightarrow \tan x = \frac{2\left(\frac{2+\sqrt{7}}{3}\right)}{1-\left(\frac{2+\sqrt{7}}{3}\right)^2} = -\left(\frac{4+\sqrt{7}}{3}\right)$

11. **Sol.** Given equation is $2\sin_2 x + 5\sin x - 3 = 0$

$\Rightarrow (2\sin x - 1)(\sin x + 3) = 0$

$\Rightarrow \sin x = \frac{1}{2}$ $(\because \sin x \neq -3)$



It is clear from figure that the curve intersect the line at four points in the given interval.
Hence, number of solutions are 4.

12. **Sol.** $2\{\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)\} + 3 = 0$
$(\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2 = 0$
$\sum \cos\alpha = 0 = \sum \sin\alpha$

13. **Ans.** **(1)**

Sol. $\tan 2\alpha = \tan((\alpha+\beta) + (\alpha-\beta)) = \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)} = \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}} = \frac{(9+5)4}{48-15} = \frac{14\times 4}{33} = \frac{56}{33}$
Hence correct option is (1)

14. **Sol.** **(1)**
$A = \sin^2 x + \cos^4 x$
$= \sin^2 x + (1 - \sin^2 x)^2$
$= \sin^4 x - \sin^2 x + 1$
$= \left(\sin^2 x - \frac{1}{2}\right)^2 + \frac{3}{4}$
$= \frac{3}{4} \leq A \leq 1$

15. **Sol.** **Ans. (2)**
$3\sin P + 4\cos Q = 6$ ...(i)
$4\sin Q + 3\cos P = 1$ ...(ii)

Squaring and adding (i) & (ii) we get $\sin(P + Q) = \frac{1}{2}$

$\Rightarrow$ $P + Q = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$\Rightarrow$ $R = \frac{5\pi}{6}$ or $\frac{\pi}{6}$
or

If $R = \frac{5\pi}{6}$ then $0 < P, Q < \frac{\pi}{6}$

$\Rightarrow$ $\cos Q < 1$ and $\sin P < \frac{1}{2}$

$\Rightarrow$ $3 \sin P + 4 \cos Q < \frac{11}{2}$

So $R = \frac{\pi}{6}$

16. **Sol.** **(2)**

Given expression

$$= \frac{\sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} + \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A - \sin A}$$

$$= \frac{1}{\sin A - \cos A}\left\{\frac{\sin^3 A - \cos^3 A}{\cos A \ \sin A}\right\} = \frac{\sin^2 A + \sin A \cos A + \cos^2 A}{\sin A \ \cos A} = 1 + \sec A \operatorname{cosec} A$$

17. **Sol.** **Ans.** **(2)**

$f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$

$f_4 - f_6 = \frac{1}{4}(\sin^4 x + \cos^4 x) - \frac{1}{6}(\sin^6 x + \cos^6 x)$

$= \frac{1}{4}(1 - 2\sin^2 x \cos^2 x) - \frac{1}{6}(1 - 3\sin^2 x \cos^2 x)$

$\frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

18. **Ans.** **(2)**

Sol. $0 \le x < 2\pi$

$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$(\cos x + \cos 4x) + (\cos 2x + \cos 3x) = 0$

$2 \cos \frac{5x}{2} \cos \frac{3x}{2} + 2 \cos \frac{5x}{2} \cos \frac{x}{2} = 0$

$2 \cos \frac{5x}{2} \left[2 \cos x \cos \frac{x}{2}\right] = 0$

$\cos \frac{5x}{2} = 0$ or $\cos x = 0$ or $\cos \frac{x}{2} = 0$

$x = \frac{(2n+1)\pi}{5}$ or $x = (2n + 1)\frac{\pi}{2}$ or $x = (2n + 1)\pi$

$x = \left\{\frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$

Number of solution is 7

19. **Ans.** **(4)**

Sol. $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$

$x^2 - 5x + 5 = 1$	$x^2 + 4x - 60 = 0$	$x^2 - 5x + 5 = -1$
$x^2 - 5x + 4 = 0$	$x = -10, x = 6$	$x^2 - 5x + 6 = 0$
$x = 1, x = 4$		$x = 2, 3$

x = 1, 2, 4, 6, –10

12. Ans. (4)

Sol. $5(\tan^2x - \cos^2x) = 2\cos2x + 9$

$5\left(\tan^2 x - \frac{1}{1+\tan^2 x}\right) = 2\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right) + 9$

$5(\tan^4x + \tan^2x - 1) = 2 - 2\tan^2x + 9 + 9\tan^2x$

$5\tan^4x - 2\tan^2x - 16 = 0$

$5\tan^4x - 10\tan^2x + 8\tan^2x - 16 = 0$

$5\tan^2x(\tan^2x - 2) + 8(\tan^2x - 2) = 0$

$(5\tan^2x + 8)(\tan^2x - 2) = 0$

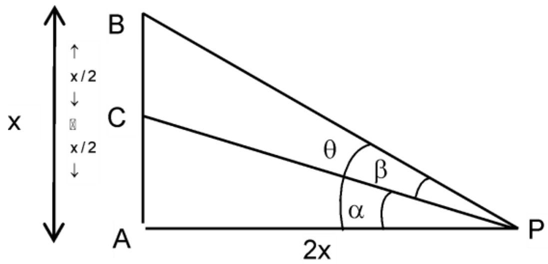
$\tan^2x = 2$

$\cos2x = \frac{1-2}{1+2} = -\frac{1}{3}$

$\cos4x = 2\cos^2 2x - 1 = -\frac{7}{9}$

13. Ans. (3) TRIGO.

Sol.



$\tan\theta = \frac{1}{2}$, $\tan\alpha = \frac{1}{4}$, $\tan\beta = y$

$\tan\theta = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\frac{1}{2} = \frac{\frac{1}{4} + y}{1 - \frac{y}{4}} \Rightarrow \frac{1}{2} = \frac{1+4y}{4-y}$

$4 - y = 2 + 8y$

$\frac{2}{9} = y$

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. **Sol.** $\frac{1}{2}\log_2(x-1) = \log_2(x-3) \Rightarrow \sqrt{x-1} = x - 3$

$(x-1) = x^2 - 6x + 9 \Rightarrow x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$ but () $x \neq 2$

∴ $x = 5$

2. Sol. $\theta \in \left(0, \frac{\pi}{4}\right)$

∵ $\tan\theta \uparrow$ in $\theta \in \left(0, \frac{\pi}{4}\right)$ and $0 < \tan\theta < 1$

$\cot\theta \downarrow$ in $\theta \in \left(0, \frac{\pi}{4}\right)$ and $\cot\theta > 1$

Let $\tan\theta = 1 - \lambda_1$ and $\cot\theta = 1 + \lambda_2$ where λ_1 and λ_2 are very small and positive, then

$t_1 = (1-\lambda_1)^{1-\lambda_1}$, $t_2 = (1-\lambda_1)^{1+\lambda_2}$, $t_3 = (1+\lambda_2)^{1-\lambda_1}$, $t_4 = (1+\lambda_2)^{(1+\lambda_2)}$

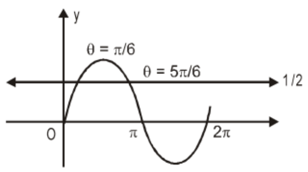
∴ $t_4 > t_3 > t_1 > t_2$

OR

∵ $\tan\theta \uparrow$ in $\theta \in \left(0, \frac{\pi}{4}\right)$ and $0 < \tan\theta < 1$

$\cot\theta \downarrow$ in $\theta \in \left(0, \frac{\pi}{4}\right)$ and $\cot\theta > 1$ think only above and conculude result.

3.

Sol.


$2\sin^2\theta - 5\sin\theta + 2 > 0$

$\Rightarrow (\sin\theta - 2)(2\sin\theta - 1) > 0$

$\Rightarrow \sin\theta < \frac{1}{2}$ [∴ $-1 \le \sin\theta \le 1$]

From graph, we get $\theta \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$

4. Ans. (A) → (p), (r), (s) ; (B) → (q), (s) ; (C) → (q), (s) ; (D) → (p), (r), (s)

Sol. Given $f(x) = \frac{x^2-6x+5}{x^2-5x+6}$

(i) when $0 < f(x) < 1$ then $0 < \frac{x^2-6x+5}{(x^2-5x+6)} < 1$

So $\frac{(x-5)(x-1)}{(x-2)(x-3)} > 0$ and $\frac{x^2-6x+5}{x^2-5x+6} - 1 < 0$ $\Rightarrow \frac{(x+1)}{(x-2)(x-3)} > 0$

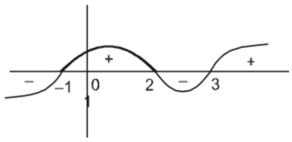
$\Rightarrow$ $x \in (-1, 1) \cup (5, 0)$

(ii) when $f(x) < 0$

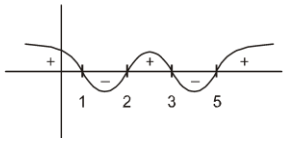
$\frac{(x-1)(x-5)}{(x-2)(x-3)} < 0$

⇒ $x \in (1, 2) \cup (3, 5)$

(iii) $f(x) > 0, x \in (-\infty, 1) \cup (2, 3) \cup (5, \infty)$



(iv) $f(x) < 1 \Rightarrow$ > 0



$x \in (-1, 2) \cup (3, \infty)$

(A) $-1 < x < 1$, f(x) satisfies p, q, s

(B) $1 < x < 2$, f(x) satisfies q, s

(C) $3 < x < 5$, f(x) satisfies q, s

(D) $x > 5$, f(x) satisfies p, r, s

5. Ans. (C)

Sol. $(2x)^{\ell n2} = (3y)^{\ell n3}$

$\Rightarrow \ell n2\ \ell n(2x) = \ell n3\ \ell n(3y) = \ell n3\ (\ell n3 + \ell ny)$ .... (1)

ao $3^{\ell nx} = 2^{\ell ny}$

$\Rightarrow \ell nx\ \ell n3 = \ell ny\ \ell n2$ .... (2)

by (1) $\Rightarrow \ell n2\ \ell n(2x) = \ell n3\ (\ell n3 + \ell ny) \Rightarrow \ell n\,2\,.\,\ell n\,(2x) = \ell n3 \left\{\ell n3 + \frac{\ell nx\ell n3}{\ell n2}\right\}$

$\Rightarrow \ell n^2 2\ \ell n2x = \ell n^2 3\ (\ell n2 + \ell nx) \Rightarrow \left(\ell n^2 2 - \ell n^2 3\right)(\ell n2x) = 0 \Rightarrow \ell n2x = 0 \Rightarrow x = \frac{1}{2}$

6. Sol. Ans (4)

Let $\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots\ldots}} = t \Rightarrow \sqrt{4-\frac{1}{3\sqrt{2}}t} = t \Rightarrow 4 - \frac{1}{3\sqrt{2}}t = t^2 \Rightarrow$

$t^2 + \frac{1}{3\sqrt{2}}t - 4 = 0 \Rightarrow 3\sqrt{2}\ t^2 + t - 12\sqrt{2} = 0 \Rightarrow t = \frac{-1 \pm \sqrt{1+4\times 3\sqrt{2}\times 12\sqrt{2}}}{2\times 3\sqrt{2}} = \frac{-1\pm 17}{2\times 3\sqrt{2}}$

$t = \frac{16}{6\sqrt{2}}, \frac{-18}{6\sqrt{2}} \Rightarrow t = \frac{8}{3\sqrt{2}}, \frac{-3}{\sqrt{2}}$ and $\frac{-3}{\sqrt{2}}$ is rejected

so $6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}}\times\frac{8}{3\sqrt{2}}\right) = 6 + \log_{3/2}\left(\frac{4}{9}\right) = 6 + \log_{3/2}\left(\left(\frac{2}{3}\right)^2\right) = 6 - 2 = 4$

7.* Sol. (A, B, C)

$3^x = 4^{x-1} \Rightarrow x = (x-1)\log_3 4 \Rightarrow x(1 - 2\log_3 2) = -2\log_3 2$

$x = \frac{2\log_3 2}{2\log_3 2 - 1}$ **Ans. (A)**

Again x $\log_2 3 = (x-1)\cdot 2$ $\Rightarrow x(\log_2 3 - 2) = -2 \Rightarrow x = \frac{2}{2-\log_2 3}$

Ans. (B)

$x = \frac{1}{1-\frac{1}{2}\log_2 3} = \frac{1}{1-\log_4 3}$ **Ans.(C)**

8. **Ans.** 2

Sol. $f(\theta) = \frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta} = \frac{1}{\frac{1-\cos 2\theta}{2} + \frac{3}{2}\sin 2\theta + \frac{5(1+\cos 2\theta)}{2}} = \frac{2}{6+3\sin 2\theta + 4\cos 2\theta}$

$\therefore$ $f(\theta)_{max} = \frac{2}{6-5} = 2$

9. **Ans.** 3

Sol. $\tan\theta = \cot 5\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{\cos 5\theta}{\sin 5\theta}$ $\Rightarrow \cos 6\theta = 0 \Rightarrow 6\theta = (2n+1)\frac{\pi}{2}$ $\Rightarrow \theta = (2n+1)\frac{\pi}{12}$; $n \in I$

$\Rightarrow$ $\theta = -\frac{5\pi}{12}, -\frac{\pi}{4}, -\frac{\pi}{12}, \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}$ ....(1)

$\sin 2\theta = \cos 4\theta$

$\Rightarrow$ $\sin 2\theta = 1 - 2\sin^2 2\theta$ $\Rightarrow$ $2\sin^2 2\theta + \sin 2\theta - 1 = 0$ $\Rightarrow$ $\sin 2\theta = -1, \frac{1}{2}$

$\Rightarrow$ $2\theta = (4m-1)\frac{\pi}{2}, p\pi + (-1)^p\frac{\pi}{6}$ $\Rightarrow$ $\theta = (4m-1)\frac{\pi}{4}, + (-1)^p\frac{\pi}{12}$; m, p $\in$ I

$\Rightarrow$ $\theta = -\frac{\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$ ....(2)

From (1) & (2) (1) $\cap$ (2) $\theta \in \left\{-\frac{\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}\right\}$

Number of solution is 3.

10. **Ans.** **(D)**

Sol. P = {θ : sin θ – cos θ = $\sqrt{2}$ cos θ}

$\sin\theta = (\sqrt{2}+1)\cos\theta$ $\Rightarrow$ $\tan\theta = \sqrt{2} + 1$ $\Rightarrow$ $\theta = n\pi +$; $n \in I$

Q = {θ : sin θ + cos θ = sin θ}

$\therefore$ $\cos\theta = (\sqrt{2}-1)\sin\theta$ $\Rightarrow$ $\tan\theta = = +1$ $\Rightarrow$ $\theta = n\pi + \frac{3\pi}{8}$; $n \in I$

$\therefore$ P = Q

11. **Ans.** **8**

Sol. $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$

⇒ $\frac{5}{4}\cos^2 2x + 1 - \frac{1}{2}\sin^2 2x + 1 - \frac{3}{4}\sin^2 2x = 2$

⇒ $\cos^2 2x = \sin^2 2x$

⇒ $\tan^2 2x = 1$

Now $2x \in [0, 4\pi]$ ⇒ $x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

so number of solution

12 Ans. (C)

Sol. $\sqrt{3}$ secx + cosecx + 2 (tanx – cotx) = 0 ⇒ $\sqrt{3}$ sinx + cosx + 2 ($\sin^2x - \cos^2x$) = 0

$\sqrt{3}$ sinx + cosx – 2cos2x = 0 ⇒ $\sin\left(x + \frac{\pi}{3}\right) = \cos 2x$

cos (π/3 – x) = cos2x ⇒ 2x = 2nπ ± (π/3 – x)

$x = \frac{2n\pi}{3} + \frac{\pi}{9}$ or $x = 2n\pi - \frac{\pi}{3}$.

– 100º – 60º + 20º +140º = 0

13. Ans. (C)

Sol.
$$\sum_{k=1}^{13} \frac{\sin\left[\left(\frac{\pi}{4} + \frac{k\pi}{6}\right) - \left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right)\right]}{\sin\frac{\pi}{6}\left(\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)\sin\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right)\right)} = 2\sum_{k=1}^{13}\left(\cot\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right) - \cot\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)\right)$$

$$= 2\left(\cot\frac{\pi}{4} - \cot\left(\frac{\pi}{4} + \frac{13\pi}{6}\right)\right) = 2\left(1 - \cot\left(\frac{29\pi}{12}\right)\right) = 2\left(1 - \cot\left(\frac{5\pi}{12}\right)\right) = 2\,(1-(2-\sqrt{3})) = 2\,(-1 + \sqrt{3})$$

$= 2\,(\sqrt{3} - 1)$

Fundamental of Mathematics - II

MATHEMATICS

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

1. **Sol.** $p = 2 + \frac{1}{p}$ ⇒ $p^2 - 2p - 1 = 0$ ⇒ $p = 1+\sqrt{2}$

$\log_{\sqrt{2}-1}\left(\sqrt{2}+1\right) = -1$

2. Sol. If $y = \log_a b$ ⇒ $a > 1$ and $b > 1$ then $y > 0$

and If $0 < a < 1$ and $0 < b < 1$ then $y > 0$

3. **Sol.** $y = (\sin^2x + \cos^2x)^3 - 3\sin^2x\cos^2x\,(\sin^2x + \cos^2x) = 1-\frac{3}{4}\sin^2 2x$, $\left[\frac{1}{4},\ 1\right]$

4. **Sol.** $x^2+\frac{1}{x^2} + 2 = 4$ ⇒ $x^2+\frac{1}{x^2} = 2$ ⇒ $x^4+\frac{1}{x^4}+2 = 4$ ⇒ $x^4+\frac{1}{x^4} = 2$

5. **Sol.** $|x - 3| + 2 = 5$ or $|x-3| + 2 = -5$ (reject)

$|x-3| = 3$ ⇒ $(x-3) = \pm 3$ ⇒ $x = 0, 6$

6. **Sol.** $\tan(100^\circ + 125^\circ) = 1 = \frac{\tan\left(100^\circ\right)+\tan\left(125^\circ\right)}{1-\tan\left(100^\circ\right)\tan\left(125^\circ\right)}$

7. **Sol.** $2x^2 + 5x + 27 > 0$ ⇒ $x \in R$

$2x - 1 > 0$ ⇒ $x > 1/2$

$\log_4(2x^2 + 5x + 27) \geq \log_2(2x-1)$

$\log_2(2x^2 + 5x + 27) \geq \log_2(2x-1)^2$

$2x^2 + 5x + 27 \geq 4x^2 + 1 - 4x$ ⇒ $2x^2 - 9x - 26 \leq 0$

$2x^2 + 4x - 13x - 26 \leq 0$

+   –   +

–2   13/2

Ans. $x \in \{1, 2, 3, 4, 5, 6\}$

8. **Sol.** Let $a = \log 2$; $b = \log 5$

$a^3 + 3ab(a+b) + b^3 = (a + b)^3 = 1$

9. **Sol.** $2x = n\pi + x$ $(0, 3\pi)$

$x = n\pi$ ⇒ $x = \{\pi, 2\pi\}$

10. **Sol.** Since $t > 0$,

$\left|t-\frac{2}{t}\right| < 1 < \left(t+\frac{2}{t}\right)$

$-1 < t - \frac{2}{t} < 1$

$t^2 - t + 2 > 0 \Rightarrow (t \in R)$

$-t < t^2 - 2 < t$

$t^2 - t - 2 < 0$ and $t^2 + t - 2 > 0$

$(t + 1)(t - 2) < 0$ $\quad$ $(t - 1)(t + 2) > 0$

$t \in (-1, 2)$ $\quad$ $t \in (-\infty, -2) \cup (1, \infty)$

Final Ans: $t \in (1, 2)$

11. **Sol.** case -I $\quad x \geq \frac{11}{3} \quad \Rightarrow \quad 4x = 25 \Rightarrow x = \frac{25}{4}$

case -II $\quad x < 11/3$

$\Rightarrow \quad 11 - 2x = 14 \quad \Rightarrow \quad 2x = -3$

$\Rightarrow \quad x = -3/2 \quad$ Sum $\frac{19}{4}$

12. **Sol.** case -I $\quad x < -5 \quad -2x - 3 \geq 12 \quad \Rightarrow \quad x \leq -15/2$

case -II $\quad -5 \leq x < 2 \quad 7 \geq 12 \quad \Rightarrow \quad x \in \varphi$

case -III $\quad x \geq 2$

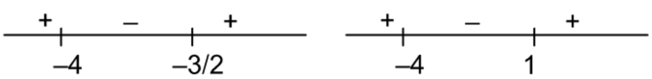
$2x + 3 \geq 12 \quad \Rightarrow \quad x \geq 9/2$

$x \in \left(-\infty, \frac{-15}{2}\right] \cup \left(\frac{9}{2}, \infty\right]$

13. **Sol.** $-1 \leq \frac{3x+2}{x+4} \leq 1$

$\frac{3x+2}{x+4} + 1 \geq 0$ and $\frac{3x+2}{x+4} - 1 \leq 0$

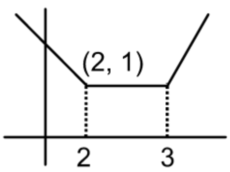
$\frac{4x+6}{x+4} \geq 0$ and $\frac{2x-2}{x+4} \leq 0$



$x \in (-\infty, -4) \cup \left[-\frac{3}{2}, \infty\right)$ $\quad$ $x \in (-4, \ 1]$

Final ; $\left[-\frac{3}{2}, \ 1\right]$

14. **Sol.**



15. **Sol.** $(x+4)(x+2) \geq 0$

$(-\infty, -4] \cup [-2, \infty)$

$a = 4$; $b = -2$

16. **Sol.** Let $\log_5 x = t$ $\Rightarrow$ $t + \frac{1}{t} = \frac{10}{3}$

$t = \frac{1}{3}, 3$ $\Rightarrow$ $\log_5 x = 3, \frac{1}{3}$ $\Rightarrow$ $x = 125, 5^{\frac{1}{3}}$

17. **Sol.** $(x+1)^2 = 1 - x$

$x^2 + 3x = 0$

$x(x+3) = 0$

$x = 0$; $x = -3$

(reject)

18. **Sol.** $4^{2x-1} > 4^x$

$2x - 1 > x \Rightarrow x > 1$

19. **Sol.** $a\cos\theta + b\sin\theta = 4$

$a\sin\theta - b\cos\theta = 3$

Square and add : $a^2 + b^2 = 25$

20. **Sol.** $S_1 = S_3$ $\Rightarrow$ $\theta + 2\theta + 3\theta = n\pi$

$\theta = \frac{n\pi}{6}$

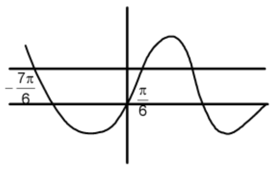
but $\theta \neq (2k+1)\frac{\pi}{2}, (2k+1)\frac{\pi}{4}, (2k+1)\frac{\pi}{6}$

21. **Sol.** In $[0, 2\pi]$ only solution possible for such equations is $\frac{5\pi}{6}$

22. **Sol.** $5(\sin x + \cos x) + (\sin x + \cos x)^2 = 0$

$(\sin x + \cos x)(5 + \sin x + \cos x) = 0$ $\tan x = -1$

23. Sol.



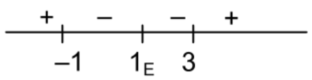
24. **Sol.** $[\cos(60+10)\cos(60-10)\cos 10]\cos 30°$

$\frac{\cos^2 30°}{4} = \frac{3}{16}$

25. **sol.** $y = 3 + 15\sin x + 20\cos x$

$\pm\sqrt{a^2 + b^2} = \pm\sqrt{225 + 400} = \pm 25$

$y \in [-22, 28]$



26. **sol.**

$x \in (\infty, -1] \cup \{1\} \cup [3, \infty)$

27. **Sol.** $(2 - \log 2)(2 + \log 2) + (\log 2)^2 = 4$

28. **Sol.** $\log_{1000} 4 = \frac{2}{3} \log_{10} 2 = \frac{2}{3} a$

$\log_{100} 27 = \frac{3}{2} \log_{10} 3 = \frac{3}{2} b$

29. Sol. Take log both side

$(1 + \log x) \log x = 1 + \log x$

$\Rightarrow$ $\log x = -1$ or ;k $\log x = 1$

30. **Sol.** **Case-I** $x < 2$

(+ ve) > (– ve)

True

Case-II $2 \le x \le 8$

squaring both side

$8 - x > x^2 - 4x + 4$

$x^2 - 3x - 4 < 0$

$(x - 4)(x + 1) < 0$

$x \in [2, 4]$

Final : $x \in (-\infty, 4]$

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

1. **Sol.** $18x - x^2 - 77 > 0$ $\Rightarrow$ $(x-11)(x-7) < 0$ $x \in (7, 11)$
$\log_5 \{\log_3(18x - x^2 - 77)\} > 0$
$\log_3 (18x - x^2 - 77) > 1$
$18x - x^2 - 80 > 0$
$(x-10)(x-8) < 0$ $\Rightarrow$ $x \in (8, 10)$
domain $x \in (8, 10)$

2. **Sol.** $3x^2 - 10x + 3 = 0$ $\Rightarrow$ $x = \frac{1}{3}$, 3(reject) {as it will make $\frac{0}{0}$ from}
$|x-3| = 1$ $\Rightarrow$ $x = 2, 4$

3. **Sol.** $\alpha + M = 8$ and α and M are twin prime
Case-I – I : M = 5 $\alpha = 3$
$\log_5 N = 3 + \beta$ $\Rightarrow N \in [125, 625)$

Case-2 – II : M = 3 ; $\alpha = 5$
$\log_3 N = \alpha + \beta$ $\Rightarrow$ $N \in [125, 729)$

4. **Sol.** $\frac{3\pi}{4} < \alpha < \pi$, then $\sqrt{2\cot\alpha + \frac{1}{\sin^2\alpha}} = \sqrt{\operatorname{cosec}^2\alpha + 2\cot\alpha} = \sqrt{\cot^2\alpha + 1 + 2\cot\alpha}$

$= \sqrt{(1+\cot\alpha)^2}$ $\because$ $\frac{3\pi}{4} < \alpha < \pi$ $= -(1 + \cot\alpha)$

5. **Sol.** $a_1 + a_2 \cos 2x + a_3 \sin^2 x = 0$ $\Rightarrow$ $a_1 + a_2 (1 - 2\sin^2 x) + a_3 \sin^2 x = 0$
$\Rightarrow$ $(a_1 + a_2) + \sin^2 x\,(a_3 - 2a_2) = 0$ is an identity $\Rightarrow a_1 + a_2 = 0$ & $a_3 - 2a_2 = 0$
$\Rightarrow$ $\frac{a_1}{-1} = \frac{a_2}{+1} = \frac{a_3}{+2}$ $\therefore$ infinite triplets are possible

6. **Sol.** $\cos A \cos B \cos C = \lambda (\cos 3A + \cos 3B + \cos 3C)$
$\Rightarrow$ $\cos A \cos B \cos C = \lambda (4\cos^3 A - 3\cos A + 4\cos^3 B - 3\cos B + 4\cos^3 C - 3\cos C)$
$\Rightarrow$ $\cos A \cos B \cos C = \lambda (4(\cos^3 A + \cos^3 B + \cos^3 C) - 3 \times 0)$
$\because$ $\cos A + \cos B + \cos C = 0 \Rightarrow$ $\cos^3 A + \cos^3 B + \cos^3 C = 3\cos A \cos B \cos C$

$\Rightarrow$ $\cos A \cos B \cos C = 12\lambda \cos A \cos B \cos C \Rightarrow$ $\lambda = \frac{1}{12}$

7. **Sol.** $\log_3 x^2 - 2\log_x 9 = 7$
$x - 1 = 1$ or $\log_3 x^2 - 2\log_x 9 = 7$

$x = 2$ $2\log_3 x - \frac{4}{\log_3 x} = 7$

$\log_3 x = -\frac{1}{2}, 4$
$x = 3^{-1/2}$ (reject) ; 3^4

8. **Sol.** $P_n - P_{n-2} = \cos_n\theta + \sin_n\theta - \cos_{n-2}\theta - \sin_{n-2}\theta = \cos_{n-2}\theta(\cos_2\theta - 1) + \sin_{n-2}\theta(\sin_2\theta - 1)$

$= \cos_{n-2}\theta(-\sin_2\theta) + \sin_{n-2}\theta(-\cos_2\theta) = (-\sin_2\theta\cos_2\theta)\{\cos_{n-4}\theta + \sin_{n-4}\theta\}$

$= (-\sin_2\theta\cos_2\theta)P_{n-4}$

put $n = 4 \Rightarrow P_4 - P_2 = (-\sin_2\theta\cos_2\theta)P_0 \Rightarrow P_4 = P_2 - 2\sin_2\theta\cos_2\theta$

$= 1 - 2\sin_2\theta\cos_2\theta$

similarly we can prove the other result ao.

10. **Sol.** Let $\log_{10}M = P$

$||x - 3| - P| = 4$

$|x - 3| = P + 4$ and $P - 4$

(9) for 2 solutions 2, $P + 4 > 0$ and $P - 4 < 0$

$-4 < P < 4$

$-4 \log M < 4$

$10^{-4} < M < 10^4$

(10) For 4 solutions : 4, $P - 4 > 0$

$P > 4$

$\log M > 4$

$M > 10^4$

11. **Sol.** $\dfrac{\sqrt{(x-8)(2-x)}}{\log_{0.3}\left(\dfrac{10}{7}(\log_2 5 - 1)\right)} \geq 0$

For $\sqrt{(x-8)(2-x)}$ to be defined

(i) $(x - 8)(2 - x) \geq 0$

$(x - 2)(x - 8) \leq 0 \Rightarrow 2 \leq x \leq 8$

Now Let say $y = \log_{0.3}\dfrac{10}{7}(\log_2 5 - \log_2 2) = \log_{0.3}\dfrac{10}{7}(\log_2 5/2)$

Let $y < 0$ (assume) then $\log_{0.3}\dfrac{10}{7}(\log_2 5/2) < 0$

$\Rightarrow \dfrac{10}{7}\log_2 5/2 > 1 \Rightarrow \log_2 5/2 > \dfrac{7}{10} \Rightarrow \dfrac{5}{2} > 2_{(7/10)}$ which is true

So $y < 0$

so denominator is – ve and numerator is +ve, so inequality is not satisfied,

thus $\sqrt{(x-8)(2-x)} = 0$

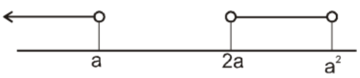
$x = 2, 8$ .....(i)

Now $2_{x-3} > 31$

$\Rightarrow (x - 3) > \log_2 31 \Rightarrow x > 3 + \log_2 2_{4.9}$ (approx)

$\Rightarrow x > 7.9 \Rightarrow x = 8$

12. **Sol.**



so blfy, $a - 1 + a_2 - 2a - 1 = 18$

$a = 5, -4 \quad \therefore \quad a = 5$

Sol. **13 and 14** Given $x = \sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}}$

$\Rightarrow$ $x^3 = \left(2+\sqrt{5}\right) + \left(2-\sqrt{5}\right) + 3.\sqrt[3]{(2+\sqrt{5})\,(2-\sqrt{5})}\ \left(\sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}}\right)$

$\Rightarrow$ $x_3 = 4 + 3.\sqrt[3]{4-5}\ (x)$ $\Rightarrow$ $x_3 = 4 - 3x$

$\Rightarrow$ $x_3 + 3x - 4 = 0$ $\Rightarrow$ $x_2 + 3x = 4$

$\Rightarrow$ $(x - 1)\ (x_2 + x + 4) = 0$

$(D < 0)$

Hence $x = 1$

15. **Sol.** Minimum of function is obtained at mid values $x = 5$.