Fundamental of Mathematics - II

Mathematics Question Bank

Fundamentals of Mathematics

Exercise-1

Marked Questions may have for Revision Questions.

OBJECTIVE QUESTIONS

Section (A) : Polynomial inequalities

A-1 Number of integers satisfying $x^2 \le 4$ and $x \ge -4$
(1) 3 (2) 5 (3) 0 (4) 2

A-2 Number of positive integers satisfying $\frac{x^2-4x+3}{2x+1} < 3$ is
(1) 10 (2) 9 (3) 8 (4) 7

A-3 Number of integers satisfying the relation $\frac{14x}{x+1} - \frac{9x-30}{x-4} < 0$.
(1) 6 (2) 4 (3) 2 (4)3

A-4 Number of positive integers satisfying. $\frac{7}{(x-2)(x-3)} + \frac{9}{(x-3)} + 1 < 0$
(1) 3 (2) 0 (3) 4 (4) 5

A-5. Number of integer values of x satisfying $-5 \le x < 10$ and $0 \le x \le 15$ is
(1) 10 (2) 11 (3) 12 (4) 13

A-6. The number of positive integers satisfying the inequality $\frac{x^2-1}{2x+5} < 3$ is
(1) 10 (2) 9 (3) 8 (4) 7

A-7. The complete set of values of 'x' which satisfy the inequations : $5x + 2 < 3x + 8$ and $\frac{x+2}{x-1} < 4$ is
(1) $(-\infty, 1)$ (2) $(2, 3)$ (3) $(-\infty, 3)$ (4) $(-\infty, 1) \cup (2, 3)$

A-8. The number of the integral solutions of $x^2 + 9 < (x + 3)^2 < 8x + 25$ is :
(1) 1 (2) 2 (3) 3 (4) 5

A-9. The complete solution set of the inequality $\frac{x^4-3x^3+2x^2}{x^2-x-30} \ge 0$ is:
(1) $(-\infty, -5) \cup (1, 2) \cup (6, \infty) \cup \{0\}$ (2) $(-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$
(3) $(-\infty, -5] \cup [1, 2] \cup [6, \infty) \cup \{0\}$ (4) $(-\infty, -5) \cup (1, 2) \cup (6, \infty) \cup \{0\}$

A-10. Number of positive integral values of x satisfying the inequality

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$$\frac{(x-4)^{2013}.(x+8)^{2014}\ (x+1)}{x^{2016}(x-2)^{3}\ .(x+3)^{5}\ .(x-6)\ (x+9)^{2012}} \le 0 \text{ is}$$

(1) 0 (2) 1 (3) 2 (4) 3

A-11. Number of non-negative integral values of x satisfying the inequality $\frac{2}{x^2-x+1}-\frac{1}{x+1}-\frac{2x-1}{x^3+1} \geq 0$ is
(1) 0 (2) 1 (3) 2 (4) 3

Section (B) : Logarithm identities, properties and graphs

B-1. If $a^4 \cdot b^5 = 1$ then the value of $\log_a (a^5b^4)$ equals
(1) 9/5 (2) 4 (3) 5 (4) 8/5

B-2. $\frac{1}{1+\log_b a+\log_b c}+\frac{1}{1+\log_c a+\log_c b}+\frac{1}{1+\log_a b+\log_a c}$ has the value equal to
(1) abc (2) $\frac{1}{abc}$ (3) 0 (4) 1

B-3. $\frac{1}{\log_{\sqrt{bc}} abc}+\frac{1}{\log_{\sqrt{ca}} abc}+\frac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to :
(1) 1/2 (2) 1 (3) 2 (4) 4

B-4. $(\log_2 10) . (\log_2 80) – (\log_2 5) . (\log_2 160)$ is equal to :
(1) $\log_2 5$ (2) $\log_2 20$ (3) $\log_2 10$ (4) $\log_2 16$

B-5. The ratio $\frac{2^{\log_{2^{1/4}} a}-3^{\log_{27}\left(a^2+1\right)^3}-2a}{7^{4\log_{49} a}-a-1}$ simplifies to :
(1) $a^2 - a - 1$ (2) $a^2 + a - 1$ (3) $a^2 - a + 1$ (4) $a^2 + a + 1$

B-6. Let $x = 2^{\log 3}$ and $y = 3^{\log 2}$ where base of the logarithm is 10, then which one of the following holds good ?
(1) 2x < y (2) 2y < x (3) 3x = 2y (4) y = x

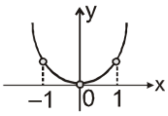
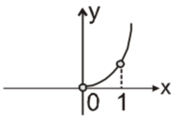
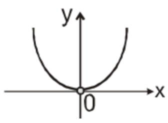
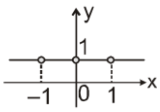
B-7. If $\log_a (ab) = x$, then $\log_b (ab)$ is equal to
(1) $\frac{1}{x}$ (2) $\frac{x}{1+x}$ (3) $\frac{x}{1-x}$ (4) $\frac{x}{x-1}$

B-8. Which one of the following is the smallest ?
(1) $\log_{10}\pi$ (2) $\sqrt{\log_{10}\pi^2}$ (3) $\left(\frac{1}{\log_{10}\pi}\right)^3$ (4) $\left(\frac{1}{\log_{10}\sqrt{\pi}}\right)$

B-9. The expression $\log_p \underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\ldots\ldots\ldots\ldots \sqrt[p]{p}}}}}_{n \text{ radical sign}}$, where $p \geq 2$, $p \in N$; $n \in N$ when simplified is
(1) independent of p (2) independent of p and of n
(3) dependent on both p and n (4) positive

B-10. $\log_{10} (\log_2 3) + \log_{10} (\log_3 4) + \log_{10} (\log_4 5) + ... + \log_{10}(\log_{1023} 1024)$ simplifies to
(1) a composite (2) a prime number
(3) rational which is not an integer (4) an integer

B-11. The correct graph of $y = x^{\log_x x^2}$ is

(1) 
(2) 
(3) 
(4) 

Section (C) : Logarithm equation and inequalities

C-1 Number of positive integer(s) which do not satisfy $\frac{1-\log_4 x}{1+\log_2 x} \le \frac{1}{2}$.

(1) 2 (2) 3 (3) 1 (4) 4

C-2. $10^{\log_p(\log_q(\log_r x))}$ = 1 and $\log_q\left(\log_r(\log_p x)\right)$ = 0 then 'p' equals

(1) $r_{q/r}$ (2) rq (3) 1 (4) $r_{r/q}$

C-3. The sum of all the solutions to the equation $2 \log_{10} x - \log_{10}(2x - 75) = 2$

(1) 30 (2) 350 (3) 75 (4) 200

C-4. If $\log_x \log_{18} \left(\sqrt{2} + \sqrt{8}\right) = \frac{1}{3}$. Then the value of 1000 x is equal to

(1) 8 (2) 1/8 (3) 1/125 (4) 125

C-5. If $3^{2\log_3 x} - 2x - 3 = 0$, then the number of values of 'x' satisfying the equation is

(1) zero (2) 1 (3) 2 (4) more than 2

C-6. Number of real solutions of the equation $\sqrt{\log_{10}(-x)} = \log_{10}\sqrt{x^2}$ is :

(1) zero (2) exactly 1 (3) exactly 2 (4) 4

C-7. The solution set of the inequality $\log_{\sin\left(\frac{\pi}{3}\right)} (x^2 - 3x + 2) \ge 2$ is

(1) $\left(\frac{1}{2}, 2\right)$ (2) $\left(1, \frac{5}{2}\right)$

(3) $\left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$ (4) $\left(\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right)$

C-8. If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval

(1) $(2, \infty)$ (2) $(1, 2)$ (3) $(-2, -1)$ (4) $(-\infty, -1)$

C-9. Solution set of the inequality $2 - \log_2(x_2 + 3x) \ge 0$ is :

(1) $[-4, 1]$ (2) $[-4, -3) \cup (0, 1]$

(3) $(-\infty, -3) \cup (1, \infty)$ (4) $(-\infty, -4) \cup [1, \infty)$

C-10. If $\log_{0.5} \log_5 (x_2 - 4) > \log_{0.5} 1$, then 'x' lies in the interval

(1) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3)$ (2) $(-3, -\sqrt{5}) \cup (, 3\sqrt{5})$

(3) $(\sqrt{5}, 3\sqrt{5})$ (4) φ

C-11. The set of all solutions of the inequality $(1/2)^{x^2-2x} < 1/4$ contains the set
(1) $(-\infty, 0)$ (2) $(-\infty, 1)$ (3) $(1, \infty)$ (4) $(3, \infty)$

Section (D) : Modulus Functions, Equation, Inequalities

D-1. Solution of the equation $|x^2 - 4x + 3| + x = 7$ is
(1) – 1,4 (2) 1,4 (3) – 1,–4 (4) 1,–4

D-2 Solution of the equation $|x - 2| + ||x| - 3| = 7x + 1$ is
(1) $\frac{9}{4}$ (2) $\frac{4}{9}$ (3) $-\frac{4}{9}$ (4) $-\frac{9}{4}$

D-3. Sum of solution of the equation $|x|^3-15x^2-8|x|-11=0$ is
(1) 15 (2) 30 (3) 0 (4) –15

D-4 Number of real roots of the equation $|x-1|^2 - 3|x-1| + 2 = 0$ is :
(1) 1 (2) 2 (3) 3 (4) 4

D-5 Minimum value of $f(x) =|x| + |x-1| + |x-2|$ is equal to
(1) 3 (2) 4 (3) 5 (4) 2

D-6 If $||x-4| -4| = 1$ then sum of values of x is
(1) 16 (2) 24 (3) 0 (4) 3

D-7 If $|x^3 - 9x^2 + 26x -24|$ is a prime number then number of possible integral value of x is
(1) 1 (2) 2 (3) 0 (4) 3

D-8. $|x-2| + |x+1| \geq 3$, then complets solution set of this inequation is :
(1) $[1, \infty)$ (2) $(-\infty, -2]$ (3) R (4) $[-2, 1]$

D-9. The complete solution set of $||x-2| -1| \leq 1$ is
(1) [0, 2) (2) [0, 4] (3) [–1, 3] (4) [1, 3]

D-10. The complete solution set of $|2x-3| + |x-5| \leq |x-8|$ is
(1) $\left[-5, \frac{3}{2}\right]$ (2) $(-\infty, -5]$ (3) $\left[\frac{3}{2}, \infty\right]$ (4) $(-\infty, -5] \cup \left[\frac{3}{2}, \infty\right)$

D-11. If $|x^2 - 4x -5| + |x^2 -x -2| = 3\,|x +1|$ then set of all real values of x is
(1) [2, 5] (2) $[2, 5] \cup \{-1\}$ (3) $[-2, 1] \cup (4, \infty]$ (4) $(-\infty, -2] \cup [1, 4]$

D-12. The complete solution set of $2|\log_2 x| + \log_2 x \geq 2$
(1) $x \geq 2^{\frac{2}{3}}$ (2) $x \leq \frac{1}{4}$ (3) Both A and B (4) $\left[2^{\frac{2}{3}}, \infty\right) \cup \left(0, \frac{1}{4}\right]$

Section (E) :Trigonometric ratio and identities

E-1. If A lies in the third quadrant and 3 tan A – 4 = 0, then 5 sin 2A + 3sinA + 4 cosA is equal to
(1) 0 (2) $-\frac{24}{5}$ (3) $\frac{24}{5}$ (4) $\frac{48}{5}$

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E-2 $\frac{\sin 8\theta \cos\theta - \sin 6\theta \cos 3\theta}{\cos 2\theta \cos\theta - \sin 3\theta \sin 4\theta}$ is equal to

(1) $\tan\theta$ (2) $\cot 2\theta$ (3) $\cot\theta$ (4) $\tan 2\theta$

E-3. If $0 < \theta < \pi/4$, then $\sqrt{2+\sqrt{2(1+\cos 4\theta)}}$ is equal to
(1) $2\cos\theta$ (2) $2\sin\theta$ (3) $-2\cos\theta$ (4) $-2\sin\theta$

E-4. $\left\{\frac{1-\cot^2\left(\frac{\alpha-\pi}{4}\right)}{1+\cot^2\left(\frac{\alpha-\pi}{4}\right)} + \cos\frac{\alpha}{2}\cot 4\alpha\right\}\sec\frac{9\alpha}{2}$ is equal to
(1) $-\text{cosec } 4\alpha$ (2) $\text{cosec } 4\alpha$ (3) $\sec 4\alpha$ (4) $-\sec 4\alpha$

E-5. The value of is $\frac{\sin 24^\circ \cos 6^\circ - \sin 6^\circ \sin 66^\circ}{\sin 21^\circ \cos 39^\circ - \cos 51^\circ \sin 69^\circ}$

(1) -1 (2) 1 (3) 2 (4) 0

E-6. If $\tan 25^\circ = x$, then $\frac{\tan 155^\circ - \tan 115^\circ}{1+\tan 155^\circ \tan 115^\circ}$ is equal to
(1) $\frac{1-x^2}{2x}$ (2) $\frac{1+x^2}{2x}$ (3) $\frac{1+x^2}{1-x^2}$ (4) $\frac{1-x^2}{1+x^2}$

E-7. The value of tan 3A – tan 2A – tan A is equal to
(1) tan 3A tan 2A tan A
(2) – tan 3A tan 2A tan A
(3) tan A tan 2A – tan 2A tan 3A – tan 3A tan A
(4) tan A tan 2A + tan 2A tan 3A + tan 3A tan A

E-8. If $\alpha \in \left[\frac{\pi}{2}, \pi\right]$ then the value of $\sqrt{1+\sin\alpha}$ - $\sqrt{1-\sin\alpha}$ is equal to:

(1) $2\cos\frac{\alpha}{2}$ (2) $2\sin\frac{\alpha}{2}$ (3) 2 (4) 1

E-9. $\cos(540^\circ - \theta) - \sin(630^\circ - \theta)$ is equal to
(1) 0 (2) $2\cos\theta$ (3) $2\sin\theta$ (4) $\sin\theta - \cos\theta$

E-10. The numerical value of $\sin 12^\circ . \sin 48^\circ . \sin 54^\circ$ is equal to
(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{1}{16}$ (4) $\frac{1}{8}$

E-11 The value of $\frac{2\sin x}{\sin 3x} + \frac{\tan x}{\tan 3x}$ is
(1) 0 (2) 2 (3) 1 (4) 3

E-12. If x + y = π + z, then $\sin^2 x + \sin^2 y - \sin^2 z$ is equal to
(1) sinx siny cosz (2) 2cosx siny cosz (3) 2sinx siny cosz (4) sinx cosy cosz

E-13. If x + y + z = $\frac{\pi}{2}$ then the value of sin 2x + sin 2y + sin 2z is equal to
(1)4cosx cosy cosz (2) 4sinx cosy cosz (3) 4cosx siny sinz (4)4sinx siny sinz

E-14. If A + B + C = 0° then the value of sin 2A + sin 2B + sin 2C is equal to
(1) – 4 cos A cos B cos C (2) – 4 cos A sin B sin C
(3) – 4 sin A cos B cos C (4) – 4 sin A sin B sin C

E-15. If φ is the exterior angle of a regular polygon of n sides and θ is any constant, then the value of sin θ + sin (θ + φ) + sin (θ +2φ) + ... up to n terms is equal to
(1) 1 (2) 0 (3) – 1 (4) depends on θ and φ

E-16. The value of $\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7}$ is
(1) $\frac{1}{4}$ (2) $\frac{1}{8}$ (3) $\frac{1}{16}$ (4) 1

E-17. The value of $\cos\frac{\pi}{19} + \cos\frac{3\pi}{19} + \cos\frac{5\pi}{19} + ... + \cos\frac{17\pi}{19}$ is equal to:
(1) 1/2 (2) 0 (3) 1 (4) 2

E-18. The maximum and minimum values of $\cos x \cos\left(\frac{2\pi}{3}+x\right)\cos\left(\frac{2\pi}{3}-x\right)$, respectively are
(1) 1,–1 (2) 1, $\frac{1}{4}$ (3) $\frac{1}{2}, \frac{1}{4}$ (4) $\frac{1}{4}, -\frac{1}{4}$

E-19. The maximum and minimum values of trigonometric function $\cos^2\left(\frac{\pi}{4}+x\right)+(\sin x - \cos x)^2$ respectively are
(1) 3, 0 (2) 2,0 (3) 3,–1 (4) 3,1

E-20. Range of function $f(x) = \cos^2 x + 4\sec^2 x$ is
(1) [4, ∞) (2) [0, ∞) (3) [5, ∞) (4) (0, ∞)

Section (F) :Trigonometric equation and inequalities

F-1. Solution set of the trigonometric equation $\sin^2 n\theta - \sin^2(n-1)\theta = \sin^2\theta$, where n is constant and n ≠ 0, 1 is
(1) $m\pi, m \in I$ or $\frac{m\pi}{n-1}, m \in I$ or $\left(m+\frac{1}{4}\right)\frac{\pi}{n}, m \in I$
(2) $m\pi, m \in I$ or $\frac{2m\pi}{n-1}, m \in I$ or $\left(m+\frac{1}{2}\right)\frac{\pi}{n}, m \in I$
(3) $m\pi, m \in I$ or $\frac{m\pi}{n+1}, m \in I$ or $\left(m-\frac{1}{2}\right)\frac{\pi}{n}, m \in I$
(4) $m\pi, m \in I$ or $\frac{m\pi}{n-1}, m \in I$ or $\left(m+\frac{1}{2}\right)\frac{\pi}{n}, m \in I$

F-2. Total number of solutions of equation sinx . tan4x = cosx belonging to (–π, 2π) are :
(1) 4 (2) 7 (3) 8 (4) 15

F-3. If $x \in \left[0, \frac{\pi}{2}\right]$, the number of solutions of the equation $\sin 7x + \sin 4x + \sin x = 0$ is:
(1) 3 (2) 5 (3) 6 (4) 4

F-4. The general solution of equation $\sin x + \sin 5x = \sin 2x + \sin 4x$ is:
(1) $\frac{n\pi}{2}$; $n \in I$ (2) $\frac{n\pi}{5}$; $n \in I$ (3) $\frac{n\pi}{3}$; $n \in I$ (4) $\frac{2n\pi}{3}$; $n \in I$

F-5. If $\cos 2\theta + 3\cos\theta = 0$, then
(1) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{\sqrt{17}-3}{4}\right)$ (2) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{-\sqrt{17}-3}{4}\right)$
(3) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{\pm\sqrt{17}-3}{4}\right)$ (4) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{\sqrt{17}+3}{4}\right)$

F-6. The most general solution of $\tan\theta = -1$ and $\cos\theta = \frac{1}{\sqrt{2}}$ is :
(1) $n\pi + \frac{7\pi}{4}$, $n \in I$ (2) $n\pi + (-1)^n \frac{7\pi}{4}$, $n \in I$
(3) $2n\pi + \frac{7\pi}{4}$, $n \in I$ (4) $2n\pi + \frac{3\pi}{4}$, $n \in I$

F-7. The general solution of the equation $\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$ is
(1) $\frac{n\pi}{4} + \frac{\pi}{12}$, $n \in I$ (2) $\frac{n\pi}{3} + \frac{\pi}{6}$, $n \in I$ (3) $\frac{n\pi}{3} + \frac{\pi}{12}$, $n \in I$ (4) $\frac{n\pi}{3} + \frac{\pi}{4}$, $n \in I$

Exercise-2

Marked Questions may have for Revision Questions.

PART - I : OBJECTIVE QUESTIONS

1. Solution of inequality $\frac{(x^2-1)(e^x-1)}{(x-1)} \leq 0$ is
(1) [0,1) (2) [0,2) (3) [–1,0] (4) (–∞,1]

2. If $\frac{6x^2-5x-3}{x^2-2x+6} \leq 4$, then the least and the highest values of $4x^2$ are:
(1) 0 & 81 (2) 9 & 81 (3) 36 & 81 (4) 0 & 9

3. If $\frac{4\alpha}{\alpha^2+1} \geq 1$ and $\alpha + \frac{1}{\alpha}$ is an odd integer then number of possible values of α is
(1) 1 (2) 2 (3) 3 (4) 4

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4. The value of $\left|\log_2 \log_9 \sqrt{6+\sqrt{6+\sqrt{6+\ldots\infty}}}\right|$ is

(1) 1 (2) 2 (3) 3 (4) 4

5. How many positive integers 'a' have the property that $\log_a 625$ is a positive integer

(1) 4 (2) 3 (3) 2 (4) More than 4

6. Let the equation $\frac{1}{x} - \frac{1}{y} = \frac{2}{15}$ and $\log_3 x + \log_3 y = 1 + \log_3 5$ then $x^2 + y^2$ is

(1) 31 (2) 32 (3) 33 (4) 34

7. If $\log_a b = 2$; $\log_b c = 2$ and $\log_3 c = 3 + \log_3 a$ then (a + b + c) equals

(1) 90 (2) 93 (3) 102 (4) 243

8. If a, b, c are positive real numbers such that $a^{\log_3 7} = 27$; $b^{\log_7 11} = 49$ and $c^{\log_{11} 25} = \sqrt{11}$. The value of $\left(a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}\right)$ equals

(1) 489 (2) 469 (3) 464 (4) 400

9. The solution set of the inequality $\frac{\ell n(x+2)}{x^2-3x-4} \le 0$ is

(1) $(-\infty, 0] \cup (4, \infty)$ (2) $(-2, 0] \cup (4, \infty)$ (3) $(-1, 0] \cup (4, \infty)$ (4) $(-2, -1) \cup (-1, 4)$

10. Solve $\log_{0.04}(x-1) \ge \log_{0.2}(x-1)$

(1) $x \le 1$ or $x \ge 2$ (2) $x = 1$; $x \ge 2$ (3) $x \ge 2$ (4) $x \in R$

11. If $\log_{\log_9 10}\left(\log_2\left(\frac{x-1}{x+2}\right)\right) > 0$ then number of integers satisfying above inequation is

(1) 3 (2) 2 (3) 1 (4) 0

12. If the equation $||x+3| - 2| = p$ where p is a constant integer has exactly three distinct solutions, then the number of integral values of p is

(1) 0 (2) 1 (3) 2 (4) 4

13. Number of integers satisfying the equation $|x^2 + 5x| + |x - x^2| = |6x|$ is

(1) 3 (2) 5 (3) 7 (4) 9

14. Maximum value of $f(x) = |x + 1| - 2|x - 1|$ is

(1) 3 (2) 2 (3) 1 (4) 0

15. If $|x + 1| + y = 5$ and $x - |y| = 2$, find x + y

(1) 3 (2) 4 (3) 2 (4) 1

16. In a triangle ABC if tan A < 0 then:

(1) $\tan B . \tan C > 1$ (2) $\tan B . \tan C < 1$

(3) $\tan B . \tan C = 1$ (4) Data insufficient

17. In a right angled triangle the hypotenuse is 2 times the perpendicular drawn from the opposite vertex. Then the other acute angles of the triangle are

(1) $\frac{\pi}{3}$ & $\frac{\pi}{6}\pi$ (2) $\frac{\pi}{8}$ & $\frac{3\pi}{8}$ (3) $\frac{\pi}{4}$ & $\frac{\pi}{4}$ (4) $\frac{\pi}{5}$ & $\frac{3\pi}{10}$

18. The principal solution set of the equation 2 cos x = $\sqrt{2+2\sin 2x}$ is

(1) $\left\{\frac{\pi}{8}, \frac{13\pi}{8}\right\}$ (2) $\left\{\frac{\pi}{4}, \frac{13\pi}{8}\right\}$ (3) $\left\{\frac{\pi}{4}, \frac{13\pi}{10}\right\}$ (4) $\left\{\frac{\pi}{8}, \frac{13\pi}{10}\right\}$

19. The number of roots of the equation cot x = $\frac{\pi}{2}$ + x in $\left[-\pi, \frac{3\pi}{2}\right]$ is

(1) 4 (2) 2 (3) 3 (4) 5

20. The number of all values of $\theta \in [0, 10.5]$ satisfying the equation

$\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$ is

(1) 16 (2) 17 (3) 18 (4) 19

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true.
(2) Statement-I is true, but Statement-II is false.
(3) Statement-I is false, but Statement-II is true.
(4) Both the statements are false.

A-1 **Statement-I :** $\log_{10}(\sqrt{13}-\sqrt{12}) < \log_{0.1}(\sqrt{14}-\sqrt{13})$

Statement-II : (i) If a > 1, then $x > 1 \Rightarrow \log_a x > 0$ and $0 < x < 1 \Rightarrow \log_a x < 0$

(ii) If $0 < a < 1$, then $x > 1 \Rightarrow \log_a x < 0$ and $0 < x < 1 \Rightarrow \log_a x > 0$

A-2 **Statement-I** : Maximum value of $\log_{1/3}(x^2 - 4x + 5)$ is '0'.

Statement-II : $\log_a x \le 0$ for $x \ge 1$ and $0 < a < 1$.

A-3. **Statement-I :** sin 2 > sin 3

Statement-II : If x, y $\in \left(\frac{\pi}{2}, \pi\right)$, x < y, then sin x > sin y

A-4. **Statement-I :** If $|x-2| + |x-7| = |2x-9|$, then $x \le 2$ or $x \ge 7$

Statement-II : $|x-a| + |x-b| = b-a$ has infinitely many solution, for a < b.

Section (B) : MATCH THE COLUMN

B-1.

Column – I	Column – II
(A) Number of solutions of $\sin^2\theta = \frac{1}{3}$	(p) 4

in $[-\pi, \pi]$

(B)	Number of solutions of sin x. = cos x in $(0, \pi)$	(q)	0
(C)	Number of solutions of equation $(1 - \tan\theta)(1 + \tan\theta) = 0$ where $\theta \in [-\pi, \pi]$	(r)	1
(D)	Number of solutions of tanx.cosx = 1 in $[-\pi, \pi]$	(s)	2

B-2. Let $f(x) = \dfrac{x^2 - 5x + 4}{x^2 - 5x + 6}$

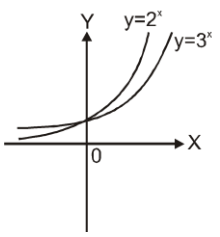
	Column – I		**Column – II**
(A)	$f(x) > 0$	(p)	$(-\infty,1) \cup (2,3) \cup (4,\infty)$
(B)	$f(x) < 1$	(q)	$(1,2) \cup (3,4)$
(C)	$f(x) > 1$	(r)	$(2,3)$
(D)	$f(x) < 0$	(s)	$(-\infty,2) \cup (3,\infty)$

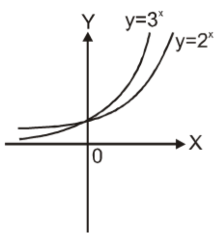
Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

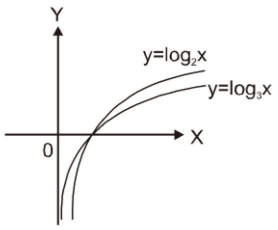
C-1. Let $N = \dfrac{\log_3 135}{\log_{15} 3} - \dfrac{\log_3 5}{\log_{405} 3}$. Then N is :

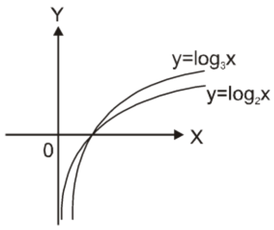
(1) a natural number (2) a prime number (3) a rational number (4) an integer

C-2. Which of the following is correct :

(1) 

(2) 

(3) 

(4) 

C-3. Consider $f(x) = ||x - 1| - |x + 2|| = P$.

(1) If P = 0 then f(x) has exactly one solution (2) If P =1 then f(x) has exactly 2 solution

(3) If P = 3 then f(x) has infinite solution (4) If P = 4 then f(x) has no solution

C-4. cos 15 x = sin 5x if

(1) $x = -\frac{\pi}{20} + \frac{n\pi}{5}, n \in I$

(2) $x = \frac{\pi}{40} + \frac{n\pi}{10}, n \in I$

(3) $x \frac{3\pi}{20} = + \frac{n\pi}{5}, n \in I$

(4) $x = \frac{3\pi}{40} - + \frac{n\pi}{10}, n \in I$

C-5. What values of 'x' satisfying $\frac{x^2(2x+3)(\sin x+3)}{(e^x-3)(x^2-x+2)} \geq 0$

(1) e
(2) π
(3) e^2
(4) $\ell n6$

Exercise-3

Marked Questions may have for Revision Questions.
*** Marked Questions may have more than one correct option.**

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. If α is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$, $\frac{\pi}{2} < \alpha < \pi$, then $\sin 2\alpha$ is equal to

[AIEEE- 2002, (3, –1), 225]

(1) $\frac{24}{25}$ (2) $-\frac{24}{25}$ (3) $\frac{13}{18}$ (4) $-\frac{13}{18}$

2. The equation $a\sin x + b\cos x = c$, where $|c| > \sqrt{a^2+b^2}$ has **[AIEEE- 2002, (3, –1), 225]**

(1) a unique solution (2) infinite number of solutions

(3) no solution (4) None of the above

3. If $y = \sin^2\theta + \mathrm{cosec}^2\theta$, $\theta \neq 0$, then **[AIEEE- 2002, (3, –1), 225]**

(1) $y = 0$ (2) $y \leq 2$ (3) $y \geq -2$ (4) $y \geq 2$

4. If $\sin(\alpha + \beta) = 1$, $\sin(\alpha - \beta) = \frac{1}{2}$ then $\tan(\alpha + 2\beta)\tan(2\alpha + \beta)$ is equal to **[AIEEE- 2002, (3, –1), 225]**

(1) 1 (2) –1 (3) 0 (4) None of these

5. If $\tan\theta = -\frac{4}{3}$ then $\sin\theta$ is **[AIEEE- 2002, (3, –1), 225]**

(1) $-\frac{4}{5}$ but not $\frac{4}{5}$ (2) $-\frac{4}{5}$ or $\frac{4}{5}$ (3) $\frac{4}{5}$ but not $-\frac{4}{5}$ (4) None of these

6. The value of $\frac{1-\tan^2 15^\circ}{1+\tan^2 15^\circ}$ is **[AIEEE- 2002, (3, –1), 225]**

(1) 1 (2) $\sqrt{3}$ (3) $\frac{\sqrt{3}}{2}$ (4) 2

7. $\sin^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if **[AIEEE- 2002, (3, –1), 225]**

(1) $x - y \neq 0$ (2) $x = y, x \neq 0$ (3) $x = y$ (4) $x \neq 0, y \neq 0$

8. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$, then the difference between the maximum and minimum values of u^2 is given by **[AIEEE- 2004, (3, –1), 225]**

(1) $2(a^2 + b^2)$ (2) $2\sqrt{a^2 + b^2}$ (3) $(a + b)^2$ (4) $(a - b)^2$

9. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin \alpha + \sin \beta = -\frac{21}{65}$ and $\cos \alpha + \cos \beta = -\frac{27}{65}$, then the value of $\cos\left(\frac{\alpha - \beta}{2}\right)$ is **[AIEEE- 2004, (3, –1), 225]**

(1) $\frac{-3}{\sqrt{130}}$ (2) $\frac{3}{\sqrt{130}}$ (3) $\frac{6}{65}$ (4) $\frac{-6}{65}$

10. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is **[AIEEE-2006 (3, – 1), 165]**

(1) $\frac{4-\sqrt{7}}{3}$ (2) $-\left(\frac{4+\sqrt{7}}{3}\right)$ (3) $\frac{1+\sqrt{7}}{4}$ (4) $\frac{1-\sqrt{7}}{4}$

11. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2 \sin^2 x + 5 \sin x - 3 = 0$ is **[AIEEE 2006 (3, – 1), 165]**

(1) 6 (2) 1 (3) 2 (4) 4

12. Let A and B denote the statements **[AIEEE 2009 (4, –1), 144]**

A : $\cos \alpha + \cos \beta + \cos \gamma = 0$

B : $\sin \alpha + \sin \beta + \sin \gamma = 0$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then :

(1) A is false and B is true (2) both A and B are true

(3) both A and B are false (4) A is true and B is false

13. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha$ = **[AIEEE 2010 (4, –1), 144]**

(1) $\frac{56}{33}$ (2) $\frac{19}{12}$ (3) $\frac{20}{7}$ (4) $\frac{25}{16}$

14. If $A = \sin^2 x + \cos^4 x$, then for all real x : **[AIEEE 2011 (4, –1), 120]**

(1) $\frac{3}{4} \le A \le 1$ (2) $\frac{13}{16} \le A \le 1$ (3) $1 \le A \le 2$ (4) $\frac{3}{4} \le A \le \frac{13}{16}$

15. In a ΔPQR, if $3 \sin P + 4 \cos Q = 6$ and $4 \sin Q + 3 \cos P = 1$, then the angle R is equal to : **[AIEEE-2012, (4, –1)/120]**

(1) $\frac{5\pi}{6}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$

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MATHEMATICS

16. The expression $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$ can be written as : **[AIEEE - 2013, (4, –1),120]**

(1) sinA cosA + 1 (2) secA cosecA + 1 (3) tanA + cotA (4) secA + cosecA

17. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals **[JEE(Main) 2014, (4, – 1), 120]**

(1) $\frac{1}{4}$ (2) $\frac{1}{12}$ (3) $\frac{1}{6}$ (4) $\frac{1}{3}$

18. If $0 \leq x < 2\pi$, then the number of real values of x, which satisfy the equation

cosx + cos2x + cos3x + cos4x = 0, is **[JEE(Main) 2016, (4, – 1), 120]**

(1) 5 (2) 7 (3) 9 (4) 3

19. The sum of all real values of x satisfying the equation $(x^2-5x+5)^{x^2+4x-60} = 1$ is

[JEE(Main) 2016, (4, – 1), 120]

(1) – 4 (2) 6 (3) 5 (4) 3

20. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of cos4x is : **[JEE(Main) 2017, (4, – 1), 120]**

(1) $\frac{-3}{5}$ (2) $\frac{1}{3}$ (3) $\frac{2}{9}$ (4) $-\frac{7}{9}$

21. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan\beta$ is equal to

[JEE(Main) 2017, (4, – 1), 120]

(1) $\frac{6}{7}$ (2) $\frac{1}{4}$ (3) $\frac{2}{9}$ (4) $\frac{4}{9}$

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. The number of solution(s) of $\log_4(x-1) = \log_2(x-3)$ is/are **[IIT-JEE-2002, Scr., (1, 0)/90]**

(A) 3 (B) 1 (C) 2 (D) 0

2. Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then

[IIT-JEE - 2006 , Main - (3, –1), 184]

(A) $t_1 > t_2 > t_3 > t_4$ (B) $t_2 < t_1 < t_3 < t_4$ (C) $t_3 > t_1 > t_2 > t_4$ (D) $t_2 > t_3 > t_1 > t_4$

3. If $0 < \theta < 2\pi$, then the intervals of values of θ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, is

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[IIT-JEE-2006, Stage-1, (3,–1)/84]

(A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (B) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$

(C) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (D) $\left(\frac{41\pi}{48}, \pi\right)$

4. Let $f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6}$ **[IIT-JEE 2007, Paper-2, (6, 0), 81]**

Column – I	**Column – II**
(A) If $-1 < x < 1$, then f(x) satisfies	(p) $0 < f(x) < 1$
(B) If $1 < x < 2$, then f(x) satisfies	(q) $f(x) < 0$
(C) If $3 < x < 5$, then f(x) satisfies	(r) $f(x) > 0$
(D) If $x > 5$, then f(x) satisfies	(s) $f(x) < 1$

5. Let (x_0, y_0) be the solution of the following equations

$$(2x)^{\ell n2} = (3y)^{\ell n3}$$
$$3^{\ell nx} = 2^{\ell ny}.$$

Then x_0 is **[IIT-JEE 2011, Paper-1, (3, –1), 80]**

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6

6. The value of $6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}....}}}\right)$ is **[IIT-JEE 2012, Paper-1, (4, 0), 70]**

7.* If $3^x = 4^{x-1}$, then x = **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**

(A) $\frac{2\log_3 2}{2\log_3 2 - 1}$ (B) $\frac{2}{2 - \log_2 3}$ (C) $\frac{1}{1 - \log_4 3}$ (D) $\frac{2\log_2 3}{2\log_2 3 - 1}$

8. The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

[IIT-JEE-2010, Paper-1, (3, 0)/84]

9. The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for n = 0, ±1, ± 2 and tanθ = cot 5θ as well as sin 2θ = cos 4θ is **[IIT-JEE-2010, Paper-1, (3, 0)/84]**

10. Let P = {θ : sin θ – cos θ = $\sqrt{2}$ cos θ} and Q = {θ : sin θ + cos θ = $\sqrt{2}$ sin θ} be two sets. Then

(A) $P \subset Q$ and $Q - P \neq \varnothing$ (B) $Q \not\subset P$

(C) $P \not\subset Q$ (D) P = Q **[IIT-JEE 2011, Paper-1, (3, –1), 80]**

11. The number of distinct solutions of the equation **[JEE (Advanced) 2015, P-1 (4, 0) /88]**

$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is

12 Let S = $\left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation

$\sqrt{3}$ sec x + cosec x + 2(tan x – cot x) = 0 in the set S is equal to

[JEE (Advanced) 2016, Paper-1, (3, –1)/62]

(A) $-\frac{7\pi}{9}$ (B) $-\frac{2\pi}{9}$ (C) 0 (D) $\frac{5\pi}{9}$

13. The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$ is equal to **[JEE (Advanced) 2016, Paper-1, (3, –1)/62]**

(A) $3-\sqrt{3}$ (B) $2(3-\sqrt{3})$ (C) $2(\sqrt{3}-1)$ (D) $2(2+\sqrt{3})$

Answers

EXERCISE # 1

PART-I

Section (A) :

A-1 (2) **A-2** (2) **A-3** (3) **A-4** (2) **A-5.** (1) **A-6.** (4) **A-7.** (4)

A-8. (4) **A-9.** (2) **A-10.** (4) **A-11.** (4)

Section (B) :

B-1. (1) **B-2.** (4) **B-3.** (2) **B-4.** (4) **B-5.** (4) **B-6.** (4) **B-7.** (4)

B-8. (1) **B-9.** (1) **B-10.** (4) **B-11.** (2)

Section (C) :

C-1 (3) **C-2.** (1) **C-3.** (4) **C-4.** (4) **C-5.** (2) **C-6.** (3) **C-7.** (3)

C-8. (1) **C-9.** (2) **C-10.** (1) **C-11.** (4)

Section (D) :

D-1. (1) **D-2** (2) **D-3.** (3) **D-4** (4) **D-5** (4) **D-6** (1) **D-7** (3)

D-8. (3) **D-9.** (2) **D-10.** (1) **D-11.** (2) **D-12.** (4)

Section (E) :

E-1. (1) **E-2** (4) **E-3.** (1) **E-4.** (2) **E-5.** (1) **E-6.** (1) **E-7.** (1)

E-8. (1) **E-9.** (1) **E-10.** (4) **E-11** (3) **E-12.** (3) **E-13.** (1) **E-14** (4)

E-15. (2) **E-16.** (2) **E-17.** (1) **E-18.** (4) **E-19.** (1) **E-20.** (3)

Section (F) :

F-1. (4) **F-2.** (4) **F-3.** (2) **F-4.** (3) **F-5.** (1) **F-6.** (3) **F-7.** (3)

EXERCISE # 2

Fundamental of Mathematics - II

PART-I

1.	(3)	**2.**	(1)	**3.**	(2)	**4.**	(1)	**5.**	(2)	**6.**	(4)	**7.**	(2)
8.	(2)	**9.**	(4)	**10.**	(3)	**11.**	(2)	**12.**	(2)	**13.**	(3)	**14.**	(2)
15.	(2)	**16.**	(2)	**17.**	(2)	**18.**	(1)	**19.**	(3)	**20.**	(2)		

PART-II

Section (A) :

A-1 (1) **A-2** (1) **A-3.** (1) **A-4.** (1)

Section (B) :

B-1. (A) → (p), (B) → (r), (C) → (p), (D) → (q)

B-2. (A) → (p), (B) → (s), (C) → (r), (D) → (q)

Section (C) :

C-1. (1,2,3,4) **C-2.** (2,3) **C-3.** (1,2,3,4)

C-4. (1,2,3,4) **C-5.** (1,3,4)

EXERCISE # 3

PART-I

1.	(2)	**2.**	(3)	**3.**	(4)	**4.**	(1)	**5.**	(2)	**6.**	(3)	**7.**	(3)
8.	(4)	**9.**	(1)	**10.**	(2)	**11.**	(4)	**12.**	(2)	**13.**	(1)	**14.**	(1)
15.	(2)	**16.**	(2)	**17.**	(2)	**18.**	(2)	**19.**	(4)	**20.**	(4)	**21.**	(3)

PART-II

1. (B) **2.** (B) **3.** (A)

4. (A) → (p), (r), (s) ; (B) → (q), (s) ; (C) → (q), (s) ; (D) → (p), (r), (s)

5. (C) **6.** (4) **7.** (ABC) **8.** 2 **9.** 3 **10.** (D) **11.** 8

12 (C) **13.** (C)