Optics-&-Modern-Physics

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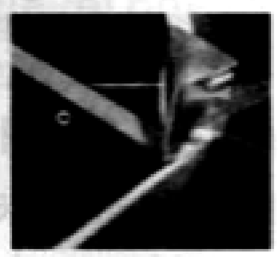

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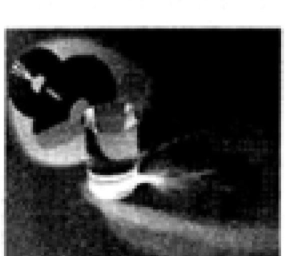

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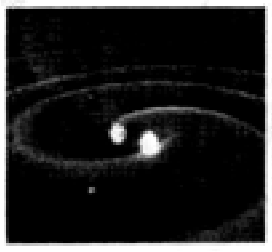

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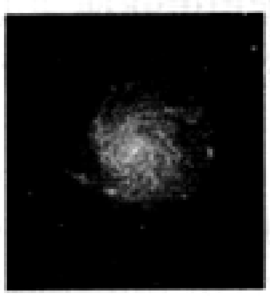

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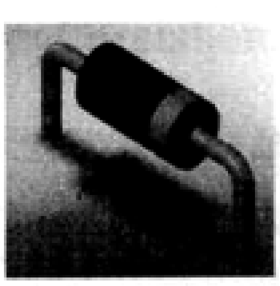

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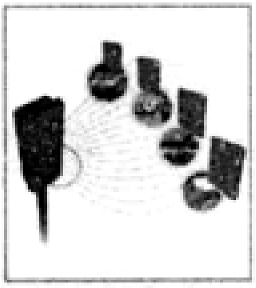

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Syllabus *of* JEE Main

Electromagnetic Waves

Electromagnetic waves and their characteristics. Transverse nature of electromagnetic waves.

Electromagnetic spectrum (radio waves, microwaves, infrared, visible, ultraviolet, X-rays, γ- rays). Applications of e.m. waves.

Optics

Reflection and refraction of light at plane and spherical surfaces, Mirror formula, Total internal reflection and its applications, Deviation and Dispersion of light by a prism, Lens formula, Magnification, Power of a lens, Combination of thin lenses in contact, Microscope and astronomical telescope (reflecting and refracting) and their magnifying powers.

Wave Optics

Wavefront and Huygens' principle, Laws of reflection and refraction using Huygen's principle. Interference, Young's double slit experiment and expression for fringe width. Diffraction due to a single slit, width of central maximum. Resolving power of microscopes and astronomical telescopes, Polarisation, Plane polarized light; Brewster's law, uses of plane polarized light and polaroids.

Dual Nature of Matter and Radiation

Dual nature of radiation. Photoelectric effect, Hertz and Lenard's observations; Einstein's photoelectric equation; Particle nature of light. Matter waves-wave nature of particle, de-Broglie relation. Davisson-Germer experiment.

Atoms and Nuclei

α-particle scattering experiment; Rutherford's model of atom; Bohr model, Energy levels, Hydrogen spectrum. Composition and size of nucleus, Atomic masses, Isotopes, Isobars; Isotones. Radioactivity - α, β and γ particles/rays and their properties; radioactive decay law. Mass-energy relation, Mass defect; Binding energy per nucleon and its variation with mass number, Nuclear fission and fusion.

Electronic Devices

Semiconductors; semiconductor diode: *I-V* characteristics in forward and reverse bias; Diode as a rectifier; *I-V* characteristics of LED, photodiode, solar cell and Zener diode; Zener diode as a voltage regulator. Junction Transistor, transistor action, Characteristics of a transistor; transistor as an amplifier (common emitter configuration) and oscillator. Logic gates (OR, AND, NOT, NAND and NOR). Transistor as a switch.

Communication Systems

Propagation of electromagnetic waves in the atmosphere; Sky and space wave propagation, Need for modulation, Amplitude and frequency modulation, Bandwidth of signals, Bandwidth of transmission medium, Basic elements of a communication system (block diagram only).

Experimental Skills

- Focal length of
  (i) Convex mirror (ii) Concave mirror and (iii) Convex lens using parallax method.
- Plot of angle of deviation *vs* angle of incidence for a triangular prism.
- Refractive index of a glass slab using a travelling microscope.
- Characteristic curves of a *p-n* junction diode in forward and reverse bias.
- Characteristic curves of a Zener diode and finding reverse break down voltage.
- Characteristic curves of a transistor and finding current gain and voltage gain.
- Identification of Diode, LED, Transistor, IC, Resistor, Capacitor from mixed collection of such items.
- Using multimeter to
  (i) Identify base of a transistor
  (ii) Distinguish between *n-p-n* and *p-n-p* type transistor
  (iii) See the unidirectional flow of current in case of a diode and an LED
  (iv) Check the correctness or otherwise of a given electronic component (diode, transistor or IC).

JEE *Advanced*

General

Focal length of a concave mirror and a convex lens using U V method.

Optics

Rectilinear propagation of light, Reflection and refraction at plane and spherical surfaces, Total internal reflection, Deviation and dispersion of light by a prism, Thin lenses, Combinations of mirrors and thin lenses, Magnification.

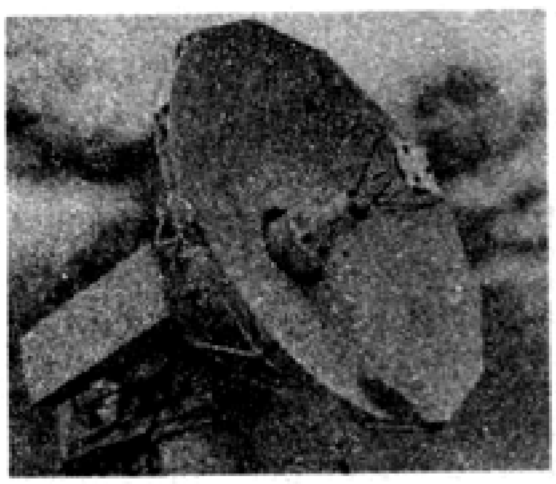
Wave Nature of Light

Huygens' principle, Interference limited to Young's double-slit experiment.

Modern physics

Atomic nucleus, α, β and γ radiations, Law of radioactive decay, Decay constant, Half-life and mean life, Binding energy and its calculation, Fission and fusion processes, Energy calculation in these processes.

Photoelectric effect, Bohr's theory of hydrogen-like atoms, Characteristic and continuous X-rays, Moseley's law, de-Broglie wavelength of matter waves.



26

ELECTROMAGNETIC WAVES

Chapter Contents

26.1 Introduction
26.2 Displacement Current
26.3 Electromagnetic Waves
26.4 Electromagnetic Spectrum

26.1 Introduction

Earlier we have learned that a time varying magnetic field produces an electric field. Is the converse also true? Does a time varying electric field can produce a magnetic field? James Clerk Maxwell argued that not only an electric current but also a time-varying electric field generates magnetic field.

Maxwell formulated a set of equations (known as Maxwell's equations) involving electric and magnetic fields. Maxwell's equations and Lorentz force formula make all the basic laws of electromagnetism.

The most important outcome of Maxwell's equations is the existence of electromagnetic waves.

The changing electric and magnetic fields form the basis of electromagnetic waves. A combination of time varying electric and magnetic fields (referred as electromagnetic wave) propagate in space very close to the speed of light ($= 3 \times 10^8$ m/s) obtained from optical measurements. We shall take a brief discussion of electromagnetic waves mainly developed by Maxwell around 1864.

26.2 Displacement Current

An electric current produces magnetic field. Value of magnetic field (due to an electric current) at some point can be obtained by Biot-Savart law or Ampere's circuital law.

We have stated Ampere's law as :

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \,(i) \qquad \text{...(i)}$$

where left hand side of this equation is the line integral of magnetic field over a closed path and i is the electric current crossing the surface bounded by that closed path.

Ampere's law in this form is not valid if the electric field at the surface varies with time. For an example if we place a magnetic needle in between the plates of a capacitor during its charging or discharging then it deflects. Although there is no current between the plates, so magnetic field should be zero. Hence, the needle should not show any deflection. But deflection of needle shows that there is a magnetic field in the region between plates of capacitor during charging or discharging. So, there must be some other source (other than current) of magnetic field. This other source is nothing but the changing electric field. Because at the time of charging or discharging of capacitor electric field between the plates changes.

The relation between the changing electric field and the magnetic field resulting from it is given by

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt} \qquad \text{...(ii)}$$

Here ϕ_E is the flux of the electric field through the area bounded by the closed path along which line integral of **B** is calculated.

Combining Eqs. (i) and (ii) we can make a general expression of Ampere's circuital law and that is

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

$$= \mu_0 \left(i + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

or

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \,(i + i_d) \qquad \text{...(iii)}$$

Here,

$$i_d = \varepsilon_0 \frac{d\phi_E}{dt} \qquad \text{...(iv)}$$

is called the **displacement current** and which is produced by the change in electric field. The current due to flow of charge is often called **conduction current** and is denoted by i_c . Thus Eq. (iii) can also be written as

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \,(i_c + i_d) \qquad \text{...(v)}$$

Example

In the figure a capacitor is charged by a battery through a resistance R. Charging of capacitor will be exponential. A time varying charging current i flows in the circuit (due to flow of charge) till charging continues. A time varying electric field is also produced between the plates. This causes a displacement current i_d between the plates. There is no current between the plates due to flow of charge, as medium between the plates is insulator.

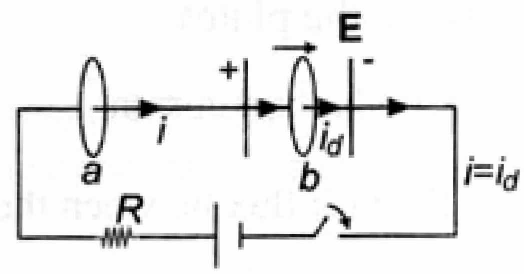


Fig. 26.1

Consider two closed paths a and b as shown in figure. Ampere's circuital law in these two paths is

Along path *a*

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 (i)$$

or

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 (i_c)$$

Along path *b*

$$\oint \mathbf{B}\ d\mathbf{l} = \mu_0 (i_d)$$

Here $i_d = \varepsilon_0 \dfrac{d\phi_E}{dt}$ is in the direction shown in figure. In sample example 26.1 we have shown that

$$i_c = i_d$$

• Important Points

Faraday's law of electromagnetic induction says that changing magnetic field gives rise to an electric field and its line integral (= *emf*) is given by the equation

$$\oint \mathbf{E}\cdot d\mathbf{l} = -\frac{d\phi_B}{dt}$$

A changing electric field gives rise to a magnetic field is the symmetrical counterpart of Faraday's law. This is a consequence of the displacement current and given by

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0\varepsilon_0 \frac{d\phi_E}{dt} = \mu_0 (i_d)$$

Thus time dependent electric and magnetic fields give rise to each other.

Maxwell's Equations

1. $\oint \mathbf{E}\cdot d\mathbf{s} = q_{in}/\varepsilon_0$ (Gauss's law for electricity)
2. $\oint \mathbf{B}\cdot d\mathbf{s} = 0$ (Gauss's law for magnetism)
3. $\oint \mathbf{E}\cdot d\mathbf{l} = -\dfrac{d\phi_B}{dt}$ (Faraday's law)
4. $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0(i_c + i_d) = \mu_0 i_c + \mu_0\varepsilon_0 \dfrac{d\phi_E}{dt}$ (Ampere-Maxwell's law)

Sample Example 26.1 *During charging of a capacitor show that the displacement current between the plates is equal to the conduction current in the connecting wire.*

Solution Let A is the area of plates, q is the charge on capacitor at some instant and d the separation between the plates.

Conduction current $$i_c = \frac{dq}{dt} \quad ...(i)$$

Electric flux between the plates,

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q/A}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$$

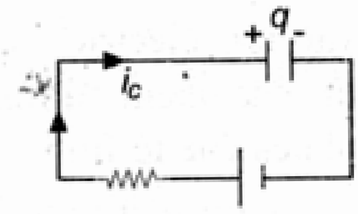


Fig. 26.2

The flux of the electric field through the given area is

$$\phi_E = EA = \left(\frac{q}{A\,\varepsilon_0}\right)A = \frac{q}{\varepsilon_0}$$

$$\therefore \quad \frac{d\phi_E}{dt} = \frac{1}{\varepsilon_0}\left(\frac{dq}{dt}\right)$$

Displacement current,

$$i_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0\left[\frac{1}{\varepsilon_0}\cdot\frac{dq}{dt}\right]$$

or $$i_d = \frac{dq}{dt} \quad ...(ii)$$

From Eqs. (i) and (ii) we can see that,

$$i_c = i_d$$

Hence Proved

Introductory Exercise 26.1

1. The parallel-plate capacitor with plate area A and separation between the plates d, is charged by a constant current i. Consider a plane surface of area $A/2$ parallel to the plates and drawn symmetrically between the plates. Find the displacement current through this area.

26.3 Electromagnetic Waves

Stationary charges produce only electric field. Charges in uniform motion (or steady currents) produce both electric and magnetic fields. **Accelerated charges radiate electromagnetic waves.** It is an important result of Maxwell's theory. Thus, an accelerated charge produces all three electric field, magnetic field and electromagnetic waves.

Consider an oscillating charged particle. Let f is the frequency of its oscillations. This oscillating charged particle produces an oscillating electric field (of same frequency f). Now, this oscillating electric field becomes a source of oscillating magnetic field (Ampere-Maxwell's law). This oscillating magnetic field again becomes a source of oscillating electric field (Faraday's law) and so on.

The oscillating electric and magnetic fields regenerate each other and electromagnetic wave propagates through the space. The frequency of the electromagnetic wave is equal to the frequency of oscillation of the charge.

Frequency of visible light is of the order of 10^{14} Hz, while the maximum frequency that we can get with modern electronic circuits is of the order of 10^{11} Hz. Therefore it is difficult to experimentally demonstrate the production of visible light. **Hertz's** experiment (in 1887) demonstrated the production of electromagnetic waves of low frequency (in radio wave region). **Jagdish Chandra Bose** succeeded in producing the electromagnetic waves of much higher frequency in the laboratory.

• Key Points of Electromagnetic Waves

1. When electromagnetic waves propogate in space then electric and magnetic fields oscillate in mutually perpendicular directions. Further, they are perpendicular to the direction of propogation of electromagnetic wave also.

2. Consider a plane electromagnetic wave propagating along the z-direction. The electric field E_x is along the x-axis and varies sinusoidally. The magnetic field B_y is along the y-axis and again varies sinusoidally. We can write E_x and B_y as,

$$E_x = E_0 \sin(\omega t - kz)$$

and
$$B_y = B_0 \sin(\omega t - kz)$$

Thus, electromagnetic wave travels in the direction of $\mathbf{E} \times \mathbf{B}$.

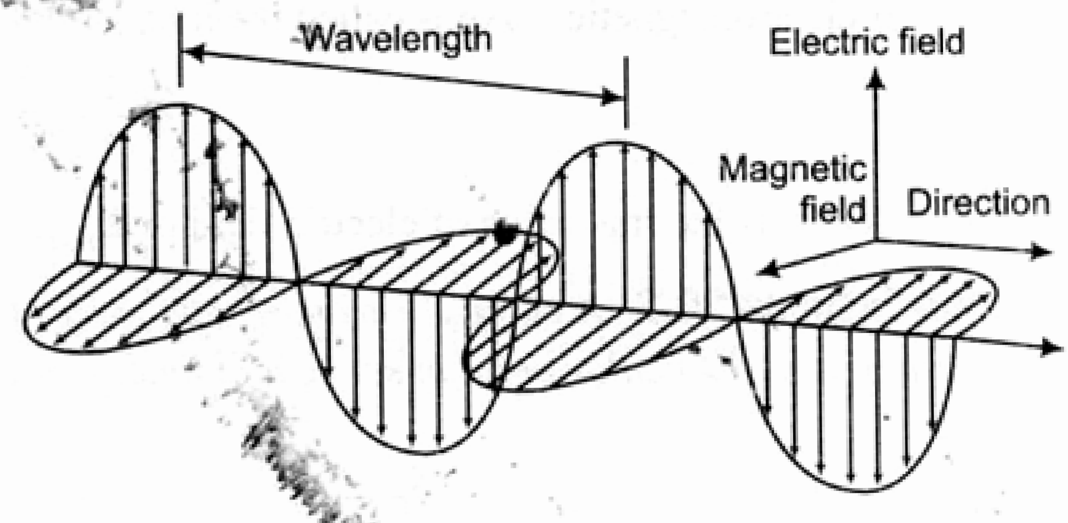


Fig. 26.3

3. From Maxwell's equations and the knowledge of waves we can write following expressions,

$$k = 2\pi/\lambda \quad \text{and} \quad \omega = 2\pi f$$

Speed of light (in vacuum)

$$c = \frac{\omega}{k} = f\lambda = \frac{E_0}{B_0} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

where f is the frequency of electromagnetic wave and λ its wavelength.

4. Unlike a mechanical wave (like sound wave) an electromagnetic wave does not require any material medium for the oscillations of electric and magnetic fields. They can travel in vacuum also. Oscillations of electric and magnetic fields are self sustaining in free space or vacuum.

5. In a material medium (like glass, water etc.) electric and magnetic fields are different from the external fields. They are described in terms of permittivity ε and magnetic permeability μ.
In Maxwell's equations ε_0 and μ_0 are thus replaced by ε and μ and the velocity of light becomes,

$$v = \frac{1}{\sqrt{\varepsilon\mu}}$$

Thus, the velocity of light depends on electric and magnetic properties of the medium.

6. Like other waves, electromagnetic waves also carry energy and momentum.In previous chapters we have studied that,

energy density in electric field $= \frac{1}{2}\varepsilon_0 E^2$ and energy density in magnetic field $= \frac{B^2}{2\mu_0}$

An electromagnetic wave contains both electric and magnetic field. Therefore energy density is associated with both the fields.

7. Consider a plane perpendicular to the direction of propagation of the electromagnetic wave. If the total energy transferred to a surface in time t is E, then total momentum delivered to this surface for complete absorption is

$$\Delta p = \frac{E}{c} \quad \text{(complete absorption)}$$

If the wave is totally reflected, the momentum delivered is,

$$\Delta p = \frac{2E}{c} \qquad \text{(completely reflected)}$$

8. The energy transferred per unit area per unit time perpendicular to the direction of propagation of electromagnetic wave is called the intensity of wave. It is given by

$$I = \frac{1}{2}\varepsilon_0 E^2 c$$

Here E is the rms value of electric field or E_{rms}.

Sample Example 26.2 *A plane electromagnetic wave of frequency 25 MHz travels in free space along the x-direction. At a particular point in space and time, E = (6.3 $\hat{\mathbf{j}}$) V/m. What is* **B** *at this point?*

Solution

$$c = \frac{E_0}{B_0} \quad \text{or} \quad \frac{E}{B}$$

$$\therefore \quad B = \frac{E}{c}$$

Substituting the values in SI units,

$$B = \frac{6.3}{3 \times 10^8}$$

$$= 2.1 \times 10^{-8}\ \text{T}$$

From the relation,

$$\mathbf{c} = \mathbf{E} \times \mathbf{B}$$

We can see that B is along positive z–direction. Because, **E** is along $\hat{\mathbf{j}}$ direction and **c** along $\hat{\mathbf{i}}$ direction.

$$\therefore \quad \mathbf{B} = (2.1 \times 10^{-8}\ \text{T})\hat{\mathbf{k}}$$ **Ans.**

Sample Example 26.3 *The magnetic field in a plane electromagnetic wave is given by $B_y = 2 \times 10^{-7} T$ sin $(0.5 \times 10^3 x + 1.5 \times 10^{11}\ t)\ T$ (a) What is the wavelength and frequency of the wave?(b) Write an expression for the electric field.)*

Solution (a) **Wavelength**

From the given equation we can see that,

$$k = 0.5 \times 10^3\ \text{m}^{-1}$$

but

$$k = \frac{2\pi}{\lambda}$$

$$\therefore \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.5 \times 10^3}$$

$$= 1.25 \times 10^{-2}\ \text{m}$$ **Ans.**

Frequency

Angular frequency

$$\omega = 1.5 \times 10^{11}\ \text{rad/s}$$

But

$$\omega = 2\pi f$$

$$\therefore \qquad f = \frac{\omega}{2\pi}$$

$$= \frac{1.5 \times 10^{11}}{2\pi}$$

$$= 2.39 \times 10^{10} \text{ Hz}$$ **Ans.**

(b)
$$c = \frac{E_0}{B_0}$$

$$\therefore \qquad E_0 = cB_0$$

$$= (3.0 \times 10^8)(2 \times 10^{-7})$$

$$= 60 \text{ V/m}$$

From the relation,

$$\mathbf{c} = \mathbf{E} \times \mathbf{B}$$

We can see that **E** is along z-direction.

$$\therefore \qquad E_z = 60 \sin (0.5 \times 10^3 \, x + 1.5 \times 10^{11} \, t) \text{ V/m}$$ **Ans.**

Sample Example 26.4 *Light with an energy flux of* 18 W/cm² *falls on a non-reflecting surface at normal incidence. If the surface has an area of* 20 cm², *find the average force exerted on the surface during a* 30 *minute time span.*

Solution Total energy incident on the given surface in the given time interval is,

$$E = (18 \times 10^4 \text{ W/m}^2)(20 \times 10^{-4} \text{ m}^2)(30 \times 60 \text{s})$$

$$= 6.48 \times 10^5 \text{ J}$$

Therefore, the total momentum transferred to the given surface for complete absorption is,

$$\Delta P = \frac{E}{c}$$

$$= \frac{6.48 \times 10^5}{3.0 \times 10^8}$$

$$= 2.16 \times 10^{-3} \text{ kg-m/s}$$

$$\therefore \qquad F_{av} = \frac{\Delta p}{\Delta t}$$

$$= \frac{2.16 \times 10^{-3}}{30 \times 60}$$

$$= 1.2 \times 10^{-6} \text{ N}$$ **Ans.**

Sample Example 26.5 *In the above example what is the average force if surfae is perfectly reflecting?*

Solution (a) If the surface is perfectly reflecting then,

$$\Delta p = \frac{2E}{c}$$

Therefore average force is doubled or

$$F_{av} = 2.4 \times 10^{-6} \text{ N}$$ **Ans.**

Sample Example 26.6 *Calculate the electric and magnetic fields produced by the radiation coming from a* 100 W *bulb at a distance of* 3 m. *Assume that the efficiency of the bulb is* 2.5 % *and it is a point source.*

Solution Intensity at a distance r from a point source of power P is given by

$$I = \frac{P}{4\pi r^2}$$

So, intensity at a distance of 3 m from the bulb with 2.5% efficiency will be

$$I = \frac{100}{4\pi (3)^2} \times \frac{2.5}{100} = 0.022 \text{ W/m}^2$$

Half of the intensity is provided by electric field and half by magnetic field.

$$\therefore \quad I_E = \frac{I}{2} = 0.011 \text{ W/m}^2$$

But, I_E is given by $\frac{1}{2}\varepsilon_0 E^2 c$

$$\therefore \quad I_E = \frac{1}{2}\varepsilon_0 E^2 c$$

or
$$E = \sqrt{\frac{2I_E}{\varepsilon_0 c}}$$

Substituting the values we have,

$$E = \sqrt{\frac{2 \times 0.011}{(8.85 \times 10^{-12})(3 \times 10^8)}}$$

$$= 2.9 \text{ V/m}$$ **Ans.**

Note *that this is actually the rms value of electric field.*

From the equation, $c = E/B$

$$B = \frac{E}{c} = \frac{2.9}{3.0 \times 10^8} = 9.6 \times 10^{-9} \text{ T}$$ **Ans.**

This is again the rms value of magnetic field.

Introductory Exercise 26.2

1. Show that the unit of $\frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ is m/s.
2. A capacitor is connected to an alternating-current source. Is there a magnetic field between the plates?
3. The sunlight reaching the earth has maximum electric field of 810 V m^{-1}. What is the maximum magnetic field in this light?
4. The electric field in an electromagnetic wave is given by
$$E = (50 \text{ N C}^{-1}) \sin \omega (t - x/c)$$
Find the energy contained in a cylinder of cross-section 10 cm^2 and length 50 cm along the x-axis.

26.4 Electromagnetic Spectrum

The basic source of electromagnetic wave is an accelerated charge. This produces the changing electric and magnetic fields which constitute an electromagnetic wave. An electromagnetic wave may have its wavelength varying from zero to infinity. Not all of them are known till date. Today we are familiar with electromagnetic waves having wavelengths as small as 30 fm ($1\text{ fm} = 10^{-15}$ m) to as large as 30 km. The boundaries separating different regions of spectrum are not sharply defined, with visible light ranging from 4000 Å to 7000 Å. An approximate range of wavelengths associated with each colour are violet (4000 Å – 4500 Å), blue (4500 Å – 5200 Å), green (5200 Å – 5600 Å), yellow (5600 Å – 6000 Å), orange (6000 Å – 6250 Å) and red (6250 Å – 7000 Å).

The classification of electromagnetic waves according to frequency or wavelength is called electromagnetic spectrum. Table below gives range of wavelengths and frequencies for different waves

S.No.	Type	Wavelength Range	Frequency Range
1.	Radio Waves	> 0.1 m	$< 3 \times 10^9$ Hz
2.	Micro Waves	0.1 m to 1 mm	3×10^9 Hz to 3×10^{11} Hz
3.	Infra-Red	1 mm to 7000 Å	3×10^{11} Hz to 4.3×10^{14} Hz
4.	Visible Light	7000 Å to 4000 Å	4.3×10^{14} Hz to 7.5×10^{14} Hz
5.	Ultraviolet	4000 Å to 10 Å	7.5×10^{14} Hz to 3×10^{17} Hz
6.	X-Rays	10 Å to 0.01 Å	3×10^{17} Hz to 3×10^{20} Hz
7.	Gamma Rays	< 0.01 Å	$> 3 \times 10^{20}$ Hz

Note *In the above table wavelength is decreasing from top to bottom. But frequency is increasing. Now let us discuss them in brief in the order of increasing wavelength.*

1. **Gamma Rays** These high frequency radiations are usually produced in nuclear reactions and also emitted by radioactive nuclei. They are used in medicines to destroy cancer cells.
2. **X-Rays** X-rays were discovered in 1895 by W.Roentgen. These are produced by the rapid deceleration of electrons that bombard a heavy metal target. These are also produced byelectronictransitions between the energy levels in an atom. X-rays are used as a diagnostic tool in medicine and as a treatment for certain forms of cancer.
3. **Ultraviolet Rays** Ultraviolet radiation is produced by special lamps and very hot bodies. The sun is an important source of ultraviolet light. If plays an important role in the production of vitamin-D. But prolonged doses of UV radiation can induce skin cancers in human beings. Ozone layer in atmosphere at an altitude of about 40-50 km plays a protective role in this regarding. Depletion of this layer by chlorofluorocarbon (CFC) gas (such as freon) is a matter of international concern now a a days.
4. **Visible Light** It is most familiar form of electromagnetic waves.Our eye is sensitive to visible light. Visible light emitted or reflected from objects around us provides information about world. Process of photosynthesis in plants needs visible light. Visible light is produced by the transition of electrons in an atom from one energy level to other.

5. **Infrared Radiation** Infrared rays also sometimes referred as heat waves are produced by hot bodies. They are perceived by us as heat. In most of the materials water molecules are present. These molecules readily absorb infrared rays. After absorption, their thermal motion increases, i.e., they heat up and heat their surroundings. Infrared rays are used for early detection of tumors. Infrared detectors are also used to observe growth of crops and for military purposes.

6. **Microwaves** Microwaves may be generated by the oscillations of electrons in a device called klystron. Microwave ovens are used in kitchens. In microwave ovens frequency of the microwaves is selected to match the resonant frequency of water molecules so that energy from the waves is transferred to the kinetic energy of the molecules. This raises the temperature of any food containing water.

7. **Radio Waves** Radio waves are generated when charges are accelerating through conducting wires. They are generated in $L-C$ oscillators and are used in radio and television communication systems.

Extra Points

1. Our eyes are sensitive to visible light (λ between 4000 Å to 7000 Å). Similarly, different animals are sensitive to different range of wavelengths. For example, snakes can detect infrared waves.

2. The basic difference between different types of electromagnetic waves lies in their frequencies and wavelengths. All of them travel with same speed in vacuum. Further they differ in their mode of interaction with matter.

   For example infrared waves vibrate the electrons, atoms and molecules of a substance. These vibrations increase the internal energy and temperature of the substance. This is why infrared waves are also called heat waves.

3. Electromagnetic waves interact with matter via their electric and magnetic fields, which set in oscillation with the charges present in all matter. This interaction depends on the wavelength of the electromagnetic wave and the nature of atoms or molecules in the medium.

4. **Microwave Oven** In electromagnetic spectrum frequency and energy of microwaves is smaller than the visible light. Water content is required for cooking food in microwave oven. Almost all food items contain some water content. Microwaves interact with water molecules and atoms via their electric and magnetic fields. Temperature of water molecules increases by this. These water molecules share this energy with neighboring food molecules, heating up the food.

   Procelain vessel are used for cooking food in microwave oven. Because its large molecules vibrate and rotate with much smaller frequencies and do not get heated up. We can not use metal vessels. Metal vessels interact with microwaves. These vessels may melt due to heating.

Solved Examples

Example 1 *Long distance radio broadcasts use short-wave bands. Explain why?*

Solution Short radio waves are reflected by ionosphere.

Example 2 *It is necessary to use satellites for long distance TV transmission. Explain why?*

Solution TV waves (part of radio waves) range form 54 MHz to 890 MHz. Unlike short-wave bands (used in radio broadcasts) which are reflected by ionosphere, TV waves are not properly reflected by ionosphere. This is why, satellites are used for long distance TV transmission.

Example 3 *The ozone layer on the top of the stratosphere is crucial for human survival. Explain why ?*

Solution Ozone layer protect ourselves from ultraviolet radiations. Over exposure to UV radiation can cause skin cancer in human beings. Ozone layer absorbs UV radiations. But unfortunately over use of Chlorofluoro Carbon Gases (CFCs) is depleting this ozone layer and it is a matter of international concern now a days.

Example 4 *Optical and radiotelescopes are built on ground but X-ray astronomy is possible only from satellites orbiting the earth. Explain why?*

Solution Visible and radiowaves can penetrate the atmosphere, while X-rays are absorbed by the atmosphere. This is why X-ray telescopes are installed in satellites orbiting the earth.

Example 5 *If the earth did not have an atmosphere, would its average surface temperature be higher or lower than what it is now?*

Solution Due to presence of atmosphere green house effect takes place. Heat radiated by earth is trapped due to green house effect. In the absence of atmosphere, temperature of the earth would be lower because the greenhouse effect of the atmosphere would be absent.

Example 6 *Some scientists have predicted the a global nuclear war on the earth would be followed by a severe nuclear winter with a devastating effect on life on earth. What might be the basis of this prediction?*

Solution After nuclear war, clouds would perhaps cover the atmosphere of earth preventing solar light from reaching many parts of earth. This would cause a winter.

Example 7 *Why is the orientation of the portable radio with respect to broadcasting station important?*

Solution Electromagnetic waves are plane polarised, so the receiving antenna should be parallel to electric and magnetic part of wave.

Example 8 *A plane electromagnetic wave propagating in the x-direction has a wavelength of 5.0 mm. The electric field is in the y-direction and its maximum magnitude is 30 Vm^{-1}. Write suitable equations for the electric and magnetic fields as a function of x and t.*

Solution Given, $\lambda = 5 \text{ mm} = 5 \times 10^{-3} \text{m}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{5 \times 10^{-3}} \text{ m}^{-1} = 1257 \text{ m}^{-1}$$

From the equation, $$c = \frac{\omega}{k}$$

$$\omega = ck = (3 \times 10^{8})(1257)$$
$$= 3.77 \times 10^{11} \text{ rad/s}$$
$$E_0 = 30 \text{ V/m}$$
$$c = \frac{E_0}{B_0}$$

$\therefore$ $$B_0 = \frac{E_0}{c} = \frac{30}{3 \times 10^{8}} = 10^{-7} \text{ T}$$

now, $$E_y = E_0 \sin(\omega t - kx)$$
$$= (30 \text{ V/m}) \sin[(3.77 \times 10^{11} \text{ s}^{-1})t - (1257 \text{ m}^{-1})x]$$

and $$B_z = (10^{-7} \text{ T}) \sin[(3.77 \times 10^{11} \text{ s}^{-1})t - (1257 \text{ m}^{-1})x]$$ **Ans.**

Example 9 *A light beam travelling in the x-direction is described by the electric field $E_y = (300 \text{ Vm}^{-1}) \sin \omega (t - x/c)$. An electron is constrained to move along the y-direction with a speed of $2.0 \times 10^7 \text{ ms}^{-1}$. Find the maximum electric force and the maximum magnetic force on the electron.*

Solution **Maximum Electric Force**

Maximum electric field $E_0 = 300 \text{ V/m}$

$\therefore$ Maximum electric force $= qE_0$

$$= (1.6 \times 10^{-19})(300)$$
$$= 4.8 \times 10^{-17} \text{ N}$$ **Ans.**

Maximum Magnetic Force

From the equation, $$c = \frac{E_0}{B_0}$$

Maximum magnetic field $$B_0 = \frac{E_0}{c}$$

or $$B_0 = \frac{300}{3.0 \times 10^{8}} = 10^{-6} \text{ T}$$

$\therefore$ Maximum magnetic force $= B_0 qv \sin 90° = B_0 qv$

Substituting the values we have,

Maximum magnetic force $= (10^{-6})(1.6 \times 10^{-19})(2.0 \times 10^{7})$

$$= 3.2 \times 10^{-18} \text{ N}$$ **Ans.**

Example 10 *A parallel-plate capacitor having plate-area A and plate separation d is joined to a battery of emf V and internal resistance R at t = 0. Consider a plane surface of area A/2, parallel to the plates and situated symmetrically between them. Find the displacement current through this surface as a function of time. [The charge on the capacitor at time t is given by $q = CV(1 - e^{-t/\tau})$, where $\tau = CR$]*

Solution Given, $q = CV(1 - e^{-t/\tau})$

$\therefore$ Surface charge density $\sigma = \frac{q}{A} = \frac{CV}{A}(1 - e^{-t/\tau})$

Electric field between the plates of capacitor,

$$E = \frac{\sigma}{\varepsilon_0} = \frac{CV}{\varepsilon_0 A}(1 - e^{-t/\tau})$$

Electric flux from the given area,

$$\phi_E = \frac{EA}{2} = \frac{CV}{2\varepsilon_0}(1 - e^{-t/\tau})$$

Displacement current, $i_d = \varepsilon_0 \frac{d\phi_E}{dt}$

or, $i_d = \varepsilon_0 \frac{d}{dt}\left[\frac{CV}{2\varepsilon_0}(1 - e^{-t/\tau})\right] = \frac{CV}{2\tau} e^{-t/\tau}$

Substituting, $\tau = CR$

we have, $i_d = \frac{V}{2R} e^{-t/CR}$

Again substituting, $C = \frac{\varepsilon_0 A}{d}$

$$i_d = \frac{V}{2R} e^{-\frac{td}{\varepsilon_0 AR}}$$ **Ans.**

Example 11 *About 5% of the power of a 100 W light bulb is converted to visible radiation. What is the average intensity of visible radiation*

(a) at a distance of 1 m from the bulb?

(b) at a distance of 10 m?

Assume that the radiation is emitted isotropically and neglect reflection.

Solution Effective power (energy radiated per second)

$= 5\%$ of 100 W

$P = 5$ W.

This energy will distribute on a sphere. At a distance r from the point source, area on which light is incident is,

$$S = 4\pi r^2$$

$\therefore$ Intensity at distance r from the point source,

$$I = \frac{P}{S} = \frac{5}{4\pi r^2} = \text{Energy incident per unit area per unit time}$$

Fig. 26.4

(a) At $r = 1$ m

$$I = \frac{5}{4\pi\,(1)^2}$$

$$= 0.4\ \text{W/m}^2$$ **Ans.**

(b) At $r = 10$ m

$$I = \frac{5}{4\pi\,(10)^2}$$

$$= 0.004\ \text{W/m}^2$$ **Ans.**

Example 12 *Suppose that the electric field of an electromagnetic wave in vacuum is* $E = \{(3.0\ N/C)\cos[(1.8\ \text{rad/m})\ y + (5.4\times10^6\ \text{rad/s})\ t]\}\,\hat{\mathbf{i}}$.

(a) What is the direction of propagation of wave?

(b) What is the wavelength λ?

(c) What is the frequency f?

(d) What is the amplitude of the magnetic field of the wave?

(e) Write an expression for the magnetic field of the wave.

Solution (a) From the knowledge of wave we can see that electromagnetic wave is travelling along negative y-direction, as $\omega\,t$ and $k\,y$ both are positive.

(b) $k = 1.8$ rad/m

$$k = \frac{2\pi}{\lambda}$$

$\therefore$ $$\lambda = \frac{2\pi}{k} = \frac{2\pi}{1.8} = 3.5\ \text{m}$$ **Ans.**

(c) $\omega = 5.4\times10^6$ rad/s

$$\omega = 2\pi f$$

$\therefore$ $$f = \frac{\omega}{2\pi}$$

$$= \frac{5.4\times10^6}{2\pi}$$

$$= 8.6\times10^5\ \text{Hz}$$ **Ans.**

(d) $E_0 = 3.0$ N/C

From the relation, $$c = \frac{E_0}{B_0}$$

we have,

$$B_0 = \frac{E_0}{c} = \frac{3.0}{3.0 \times 10^8}$$

$$= 10^{-8}\ \text{T}$$ **Ans.**

(e) **E** is along $\hat{\mathbf{i}}$ direction, wave is travelling along negative y-direction. Therefore oscillations of **B** are along z-directions, or

$$\mathbf{B} = (10^{-8}\ \text{T}) \cos [(1.8\ \text{rad/m})\ y + (5.4 \times 10^6\ \text{rad/s})\ t]\ \hat{\mathbf{k}}$$ **Ans.**

Example 13 *A parallel plate capacitor made of circular plates each of radius $R = 6.0$ cm has a capacitance $C = 100$ pF. The capacitor is connected to a 230 V AC supply with a (angular) frequency of 300 rad/s.*

(a) What is the rms value of the conduction current?

(b) Is the conduction current equal to the displacement current?

(c) Determine the amplitude of B at a point 3.0 cm from the axis between the plates.

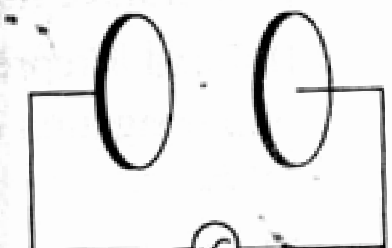


Fig. 26.5

Solution (a) Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{300 \times 100 \times 10^{-12}} = \frac{10^8}{3}\ \Omega$$

There is only capacitance in the circuit.

$$\therefore \quad i_{rms} = \frac{V_{rms}}{X_C} = \frac{230}{(10^8/3)}$$

$$= 6.9 \times 10^{-6}\ \text{A}$$ **Ans.**

(b) Yes, the derivation in sample example 26.1 is true even if current is alternating.

(c) Here, i_d is displacement current and i the conduction current. Magnetic field at a distance r from the axis,

$$B = \frac{\mu_0}{2\pi} \frac{i_d}{R^2} r$$

$$\therefore \quad B_{rms} = \frac{\mu_0}{2\pi} \frac{i_{rms}}{R^2} r \qquad (i_d = i = i_{rms})$$

Substituting the values we have,

$$B_{rms} = \frac{(2 \times 10^{-7})(6.9 \times 10^{-6})}{(6 \times 10^{-2})^2} (3 \times 10^{-2})$$

$$= 1.15 \times 10^{-11}\ \text{T}$$

$$\therefore \quad B_0 = \sqrt{2}\ B_{rms}$$

$$= (\sqrt{2})(1.15 \times 10^{-11})\ \text{T}$$

$$= 1.63 \times 10^{-11}\ \text{T}$$ **Ans.**

EXERCISES

Single Correct Option

1. One requires 11eV of energy to dissociate a carbon monoxide molecule into carbon and oxygen atoms. The minimum frequency of the appropriate electromagnetic radiation to achieve the dissociation lies in
(a) visible region (b) infrared region (c) ultraviolet region (d) microwave region

2. If **E** and **B** represent electric and magnetic field vectors of the electromagnetic wave, the direction of propagation of electromagnetic wave is along
(a) **E** (b) **B** (c) **B** × **E** (d) **E** × **B**

3. The ratio of contributions made by the electric field and magnetic field components to the intensity of an EM wave is
(a) $c:1$ (b) $c^2:1$ (c) $1:1$ (d) $\sqrt{c}:1$

4. Light with an energy flux of 20 W/cm^2 falls on a non-reflecting surface at normal incidence. If the surface has an area of 30 cm^2. The total momentum delivered (for complete absorption) during 30 minutes is
(a) 36×10^{-5} kg - m/s (b) 36×10^{-4} kg - m/s
(c) 1.08×10^{4} kg - m/s (d) 1.08×10^{7} kg - m/s

More than One Correct Options

5. A plane electromagnetic wave propagating along x-direction can have the following pairs of **E** and **B**
(a) E_x, B_y (b) E_y, B_z (c) B_x, E_y (d) E_z, B_y

6. The source of electromagnetic waves can be a charge
(a) moving with a constant velocity (b) moving in a circular orbit
(c) at rest (d) falling in an electric field

7. An electromagnetic wave of intensity I falls on a surface kept in vacuum and exerts radiation pressure P on it. Which of the following are true?
(a) Radiation pressure is I/c if the wave is totally absorbed.
(b) Radiation pressure is I/c if the wave is totally reflected.
(c) Radiation pressure is $2I/c$ if the wave is totally reflected.
(d) Radiation pressure is in the range $I/c < P < 2I/c$ for real surfaces.

8. A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz. The electromagnetic waves produced
(a) will have frequency of 10^9 Hz (b) will have frequency of 2×10^9 Hz
(c) will have a wavelength of 0.3 m (d) fall in the region of radiowaves

Subjective Questions

9. Can an electromagnetic wave be deflected by an electric field? By a magnetic field?

10. What physical quantity is the same for X-rays of wavelength 10^{-10}m, red light of wavelength 6800 Å and radiowaves of wavelength 500 m?

11. A plane electromagnetic wave travels in vacuum along z-direction. What can you say about the directions of its electric and magnetic field vectors. If the frequency of the wave is 30 MHz, what is its wavelength?

12. A radio can tune into any station in the 7.5 MHz to 12 MHz band. What is the corresponding wavelength of band?

13. A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz. What is the frequency of the electromagnetic waves produced by the oscillator?

14. The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is $B_0 = 510$ nT. What is the amplitude of the electric field part of the wave?

15. Figure shows a capacitor made of two circular plates each of radius 12 cm, and separated by 5.0 cm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15 A.

(a) Calculate the capacitance and the rate of change of potential difference between the plates.
(b) Obtain the displacement current across the plates.
(c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.

16. Suppose that the electric field amplitude of an electromagnetic wave is $E_0 = 120$ N/C and that its frequency is 50.0 MHz. (a) Determine B_0, ω, k, and λ, (b) Find expressions for **E** and **B**.

17. A variable frequency ac source is connected to a capacitor. How will the displacement current change with decrease in frequency?

18. A laser beam has intensity 2.5×10^{14} Wm^{-2}. Find the amplitudes of electric and magnetic fields in the beam.

19. In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{10} Hz and amplitude 48 Vm^{-1}.

(a) What is the wavelength of the wave?
(b) What is the amplitude of the oscillating magnetic field?
(c) Show that the average energy density of the **E** field equals the average energy density of the **B** field. [$c = 3 \times 10^8$ m s^{-1}]

20. The charge on a parallel plate capacitor varies as $q = q_0 \cos 2\pi ft$. The plates are very large and close together (area $= A$, separation $= d$). Neglecting the edge effects, find the displacement current through the capacitor?

ANSWERS

Introductory Exercise 26.1

1. $i/2$

Introductory Exercise 26.2

2. Yes **3.** 2.7 μT **4.** 5.55×10^{-12} J

Exercises

1. (c) **2.** (d) **3.** (c) **4.** (b) **5.** (b,d) **6.** (b,d) **7.** (a,c,d) **8.** (a,c,d) **9.** No, No

10. The speed in vacuum is the same for all

11. **E** and **B** lie in x-y plane and are mutually perpendicular, 10 m

12. Wavelength band from 40 m to 25 m **13.** 10^9 Hz **14.** 153 N/C

15. (a) 8.0 pF, , 1.87×10^{10} Vs^{-1} (b) 0.15 A

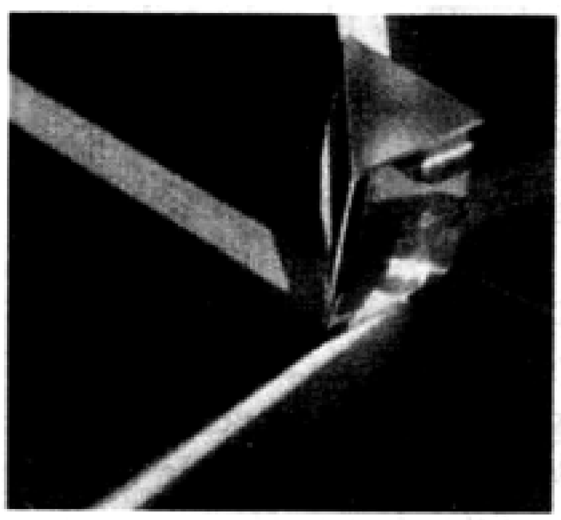
(c) Yes, provided by current we mean the sum of conduction and displacement currents.

16. (a) 400 nT, 3.14×10^8 rad/s , 1.05 rad/m , 6.00 m (b) **E** = (120 N/C) sin [(1.05 rad/m)] x – (3.14×10^8 rad/s)t]

B = (400 nT) sin [(1.05 rad/m)] x – (3.14 $\times 10^8$ rad/s) t]

17. Displacement current will decrease **18.** 4.3×10^8 N/C , 1.44 T

19. (a) 1.5×10^{-2} m (b) 1.6×10^{-7} T **20.** $-2\pi q_0 f \sin 2\pi ft$



27

REFLECTION OF LIGHT

Chapter Contents

27.1 Introduction
27.2 The Nature of Light
27.3 Few General Points of Geometric Optics
27.4 Reflection of Light

27.1 Introduction

The branch of physics called **optics** deals with the behaviour of light and other electromagnetic waves. Light is the principal means by which we gain knowledge of the world. Consequently the nature of light has been the source of one of the longest debates in the history of science.

Electromagnetic radiation with wavelengths in the range of about 4000 Å to 7000 Å, to which eye is sensitive is called light.

Our investigation of light will revolve around two questions of fundamental importance (1) What is the nature of light and (2) How does it behave under various circumstances? The answers to these two questions can be found in Maxwell's field equations (which is out of JEE syllabus). These equations predict the existence of electromagnetic waves that travel at the speed of light. They also describe how these waves behave. Interestingly, not all light phenomena can be explained by Maxwell's theory. Experiments performed at the beginning of this century showed that light also has corpuscular, or particle like properties.

In the present and next two chapters we investigate the behaviour of a beam of light when it encounters simple optical devices like mirrors, lenses and apertures. Under many circumstances, the wavelength of light is negligible compared with the dimensions of the device as in the case of ordinary mirrors and lenses. A light beam can then be treated as a ray whose propagation is governed by simple geometric rules. The part of optics that deals with such phenomena is known as **geometric optics.** However, if the wavelength is not negligible compared with the dimensions of the device (for example a very narrow slit), the ray approximation becomes invalid and we have to examine the behaviour of light in terms of its wave properties. This study is known as **physical optics.**

27.2 The Nature of Light

The question whether light is a wave or a particle has a very interesting and long history. Early theories considered light to be a stream of particles which emanated from a source and caused the sensation of vision upon entering the eye. The most influential proponent of this particle theory of light was **Newton.** Using it, he was able to explain the laws of reflection and refraction. The chief proponents of the wave theory of light propagation were **Christian Hygens** and **Robert Hooke.** Hygen's using his wave theory was also able to explain reflection and refraction. Newton saw the virtues of the wave theory of light particularly as it explained the colours formed by thin films, which Newton studied extensively. However, he rejected the wave theory because of the observed straight line propagation of light. Because of Newton's great reputation and authority, this reluctant rejection of the wave theory of light, based on lack of evidence of diffraction was strictly adhered to by Newton's followers. Newton's particle theory of light was accepted for more than a century.

In 1801 **Thomas Young** revived the wave theory of light. He was one of the first to introduce the idea of interference as a wave phenomenon in both light and sound. His observation of interference with light was a clear demonstration of the wave nature of light. Young's work went unnoticed by the scientific community for more than a decade.

Fresnel performed extensive experiments on interference and diffraction and put the wave theory on a mathematical basis. He showed, that the rectilinear propagation of light is a result of very short wavelength of visible light. In 1850 **Jean Foucault** measured the speed of light in water and showed that it is less than that in air, thus ruling out Newton's particle theory according to whom the speed of light is more in water.

But the drama was not yet over. The climax came when the wave theory of light failed to explain the photoelectric effect invented by **Albert Einstein** in 1905. He himself explained it on the basis of particle nature of light. An amicable understanding was ultimately reached in accepting that light has **dual nature.** It can behave as particles as well as waves depending on its interaction with the surrounding. Later it was found that even the well established particles such as electrons also have a dual character and can show interference and diffraction under suitable conditions.

Electromagnetic Waves

In Chapter 14 we saw that a wave travelling along x-axis with a speed v satisfies the wave equation

$$\frac{\partial^2 y}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad \ldots(i)$$

Maxwell was able to show that time dependent electric and magnetic fields also satisfy the wave equation. The changing electric and magnetic fields form the basis of electromagnetic waves. In free space, far from the source of the fields, the fields satisfy **Maxwell wave equations** :

$$\frac{\partial^2 E}{\partial x^2}=\mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2} \quad \ldots(ii)$$

$$\frac{\partial^2 B}{\partial x^2}=\mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2} \quad \ldots(iii)$$

On comparing these with the standard wave equation, we see that the electromagnetic wave speed is

$$c=\frac{1}{\sqrt{\varepsilon_0\mu_0}} \quad \ldots(iv)$$

When the values $\mu_0 = 4\pi\times10^{-7}$ H/m and $\varepsilon_0 = 8.85\times10^{-12}$ F/m are inserted, we find

$$c=3.00\times10^8 \text{ m/s}$$

This is speed of light in vacuum.

The simplest plane wave solutions to Eqs. (ii) and (iii) are

$$E = E_0 \sin(\omega t - kx) \quad \ldots(v)$$

$$B = B_0 \sin(\omega t - kx) \quad \ldots(vi)$$

From these equations we see that at any point E and B are in phase. The electric and magnetic fields in a plane electromagnetic wave are perpendicular to each other and also perpendicular to the direction of propagation of light as shown in figure. They are transverse electromagnetic waves. The magnitudes of the fields are related by

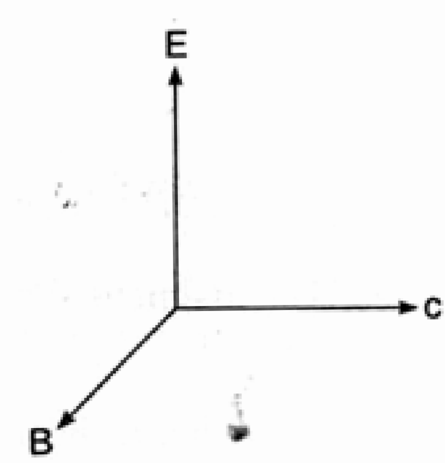


Fig. 27.1

$$c=\frac{E}{B} \quad \text{or} \quad E = cB \quad \ldots(vii)$$

According to the thinking of the 19th century, the constants μ_0 and ε_0 referred to properties of the **ether**, the medium through which the electromagnetic waves were assumed to propagate. This is not our present thinking. The ether does not exists and electromagnetic waves do not require any medium in which to propagate. However, when they travel through a substance, the fields do interact with charges in the medium. The strength of the interaction is related to the permittivity ε and the permeability μ of the substance. As a result the speed of light in medium is reduced to $\frac{1}{\sqrt{\varepsilon\mu}}$. Hence,

$$v = \frac{1}{\sqrt{\varepsilon\mu}} \quad \text{...(viii)}$$

The ratio of c and v ($< c$) is known as the refractive index of the substance. This is a pure ratio which has a value greater than or equal to one. Thus,

$$\text{Refractive index} = \frac{c}{v} \quad \text{...(ix)}$$

Sample Example 27.1 *The magnetic field of an electromagnetic wave in a substance is given by*

$$B = (2 \times 10^{-6}\ T) \cos[\pi(0.04\,x + 10^7\,t)]$$

Find the refractive index of the substance.

Solution Comparing the given equation with the standard wave equation

$$B = B_0 \cos(\omega t + kx)$$

We have, $\omega = \pi \times 10^7$ rad/s and $k = \pi \times (0.04)\ \text{m}^{-1}$

$\therefore$ Speed of electromagnetic wave in this medium is

$$v = \frac{\omega}{k} = 2.5 \times 10^8\ \text{m/s}$$

Now, refractive index of substance $= \frac{c}{v} = \frac{3.0 \times 10^8}{2.5 \times 10^8}$

$$= 1.2$$

Ans.

Introductory Exercise 27.1

1. Show that the unit of $\frac{1}{\sqrt{\varepsilon_0\mu_0}}$ is m/s.
2. The magnetic field in a plane electromagnetic wave is given by (SI units)
$$B_y = (2 \times 10^{-7}\ \text{T}) \sin[500x + 1.5 \times 10^{11}\,t]$$
   (a) What is the wavelength and frequency of the wave?
   (b) Write an expression for the electric field vector.

27.3 Few General Points of Geometric Optics

Here are few general points which I consider are important before studying the geometric optics.

1. Normally the object in kept on the left hand side of the optical instrument (mirror, lens etc.), *i.e.*, the ray of light travels from left to right. Sometimes it may happen that the light is travelling in opposite direction. See the figure.

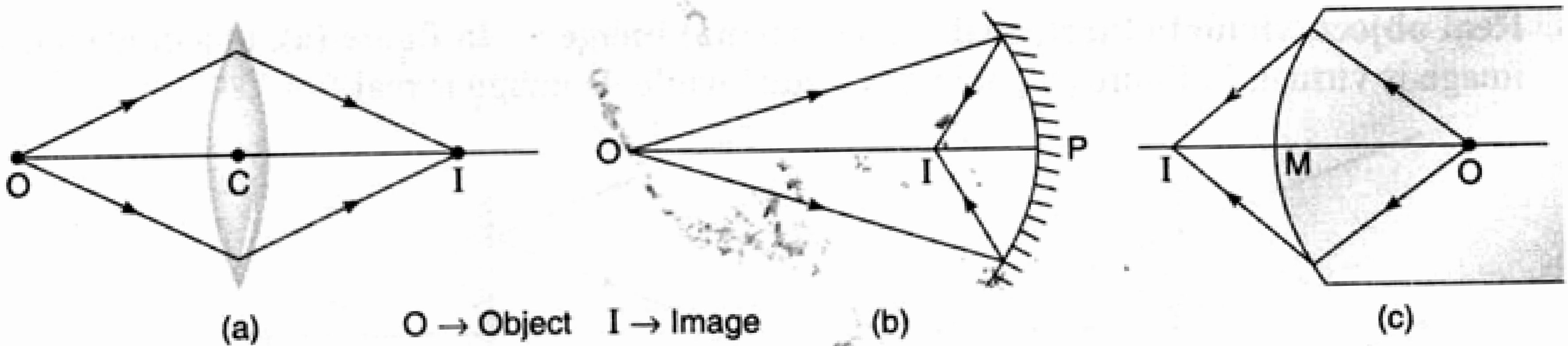


Fig. 27.2

In figures (a) and (b) light is travelling from left to right and in figure (c) it is travelling from right to left.

2. Whenever a silvered surface comes on the path of a ray of light it returns from there, otherwise it keeps on moving forwards.

3. **Sign convention :** The distances measured along the incident light are taken as positive while the distances against incident light are taken as negative. For example, in figures (a) and (b) the incident light travels from left to right. So the distances measured in this direction are positive. While in figure (c) the incident light travels from right to left. So in this case this direction will be positive. Distances are measured from pole of the mirror [point P in figure (b)], optical centre of the lens [point C in figure (a)] and the centre of the refracting surface [point M in figure (c)]. It may happen in some problem that sign convention does not remain same for the whole problem.

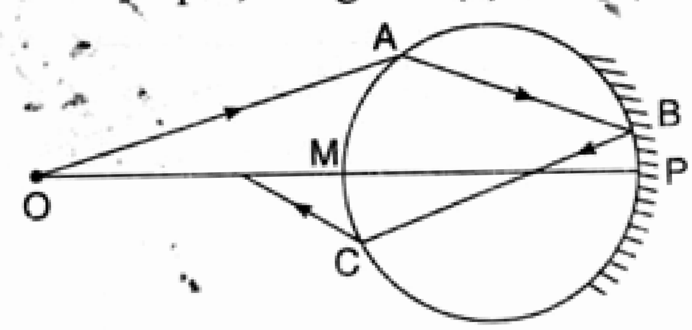


Fig. 27.3

For example, in the figure 27.3 shown, the ray of light emanating from O first undergoes refraction at A, then reflection at B and then finally refraction at C. For refraction and reflection at A and B the incident light is travelling from left to right, so distances measured along this direction are positive. For final refraction at C the incident light travels from right to left, so now the sign convention will change or right to left is positive.

4. Object distance (from P, C or M along the optic axis) is shown by u and image distance by v.

5. Image at infinity means rays after refraction or reflection have become parallel to the optic axis. If a screen is placed directly in between these parallel rays no image will be formed on the screen. But if a converging lens (convex) is placed on the path of the parallel rays and a screen is placed at the focus of the lens, image will be formed on the screen. Sometimes our eye plays the role of this converging lens and the retina is the screen.

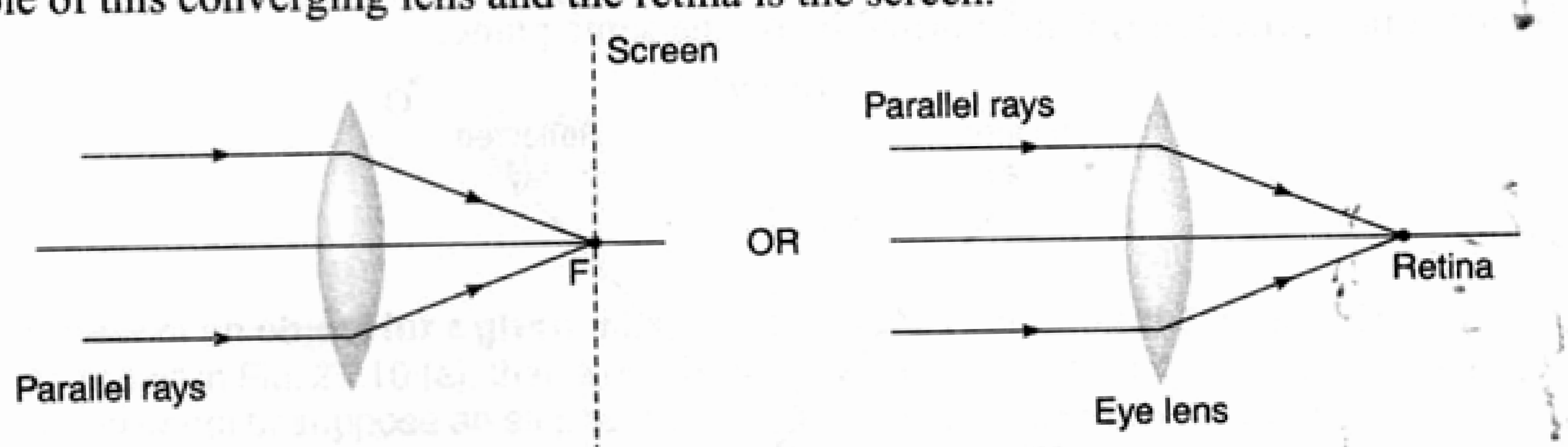


Fig. 27.4

6. **Real object, virtual object, real image, virtual image :** In figure (a), object is real, while image is virtual. In figure (b), object is virtual while its image is real.

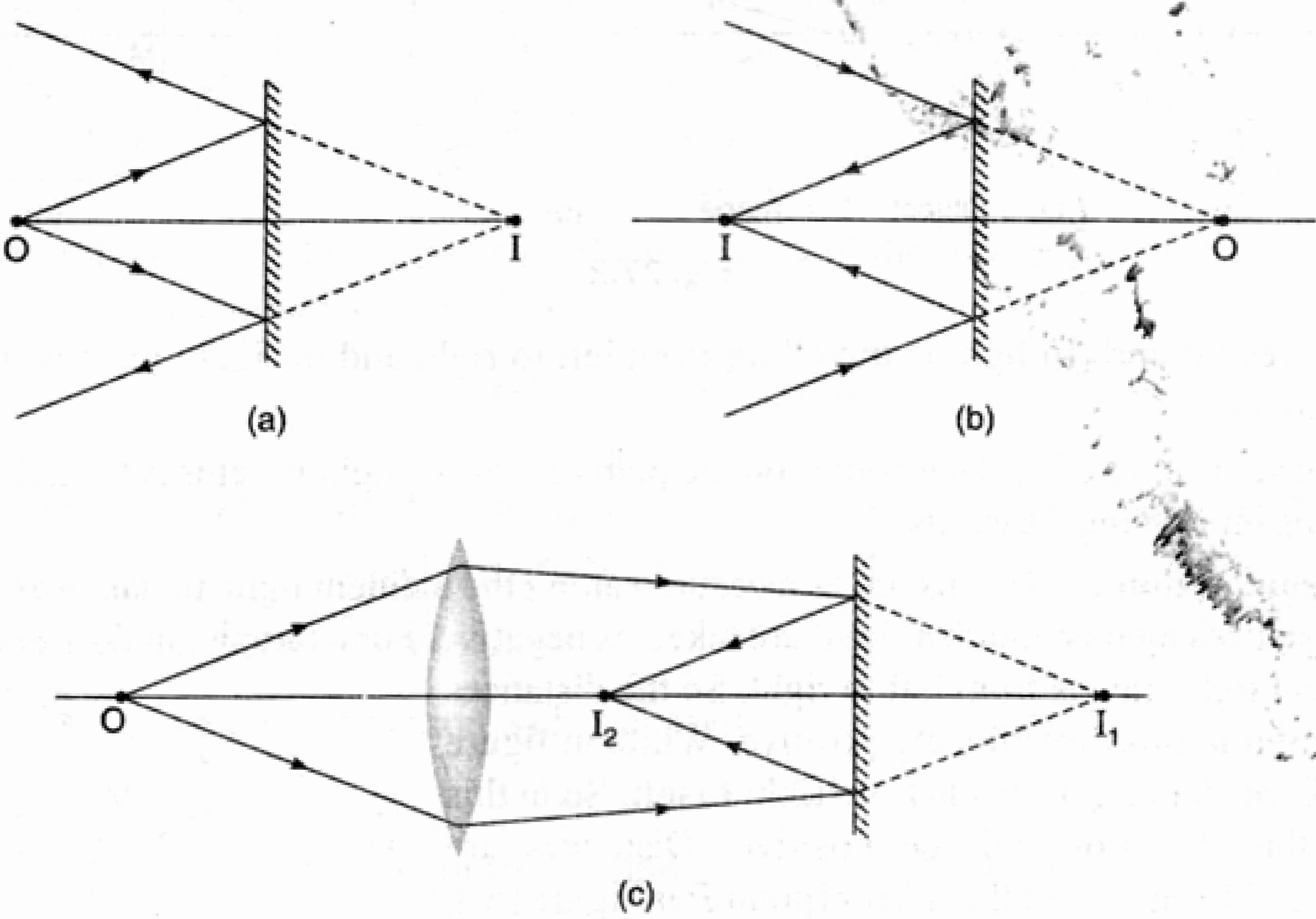


Fig. 27.5

In figure (c), the object O is real. Its image formed by the lens (*i.e.*, I_1) is real. But it acts as a virtual object for mirror which forms its real image I_2.

Note *The virtual images cannot be taken on screen. But they can be seen by our eye. Because our eye lens forms their real image on our retina. Thus, if we put a screen at I in figure (a) no image will be formed on it. At the same time if we put the screen at I in figure (b), image will be formed.*

27.4 Reflection of Light

When waves of any type strike the interface between two optical materials, new waves are generated which move away from the barrier. Experimentally it is found that the rays corresponding to the incident and reflected waves make equal angles with the normal to the interface and that the reflected ray lies in the plane of incidence formed by the incident ray and the normal. Thus, the two laws of reflection can be summarised as under:

(1) $\angle i = \angle r$

(2) Incident ray, reflected ray and normal lie on the same plane.

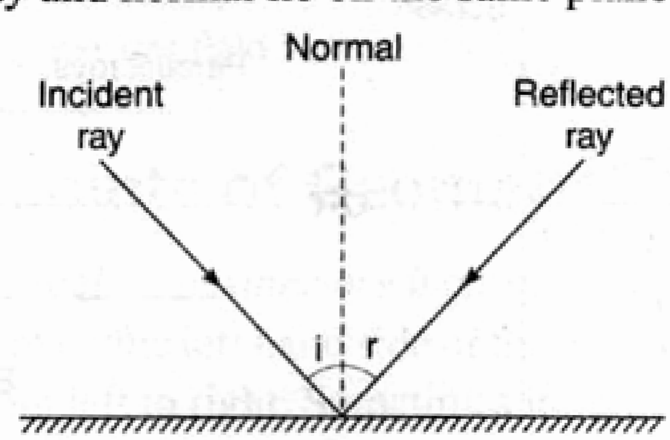


Fig. 27.6

Note *The above two laws of reflection can be applied to the reflecting surfaces which are not even horizontal. The following figures illustrates the point.*

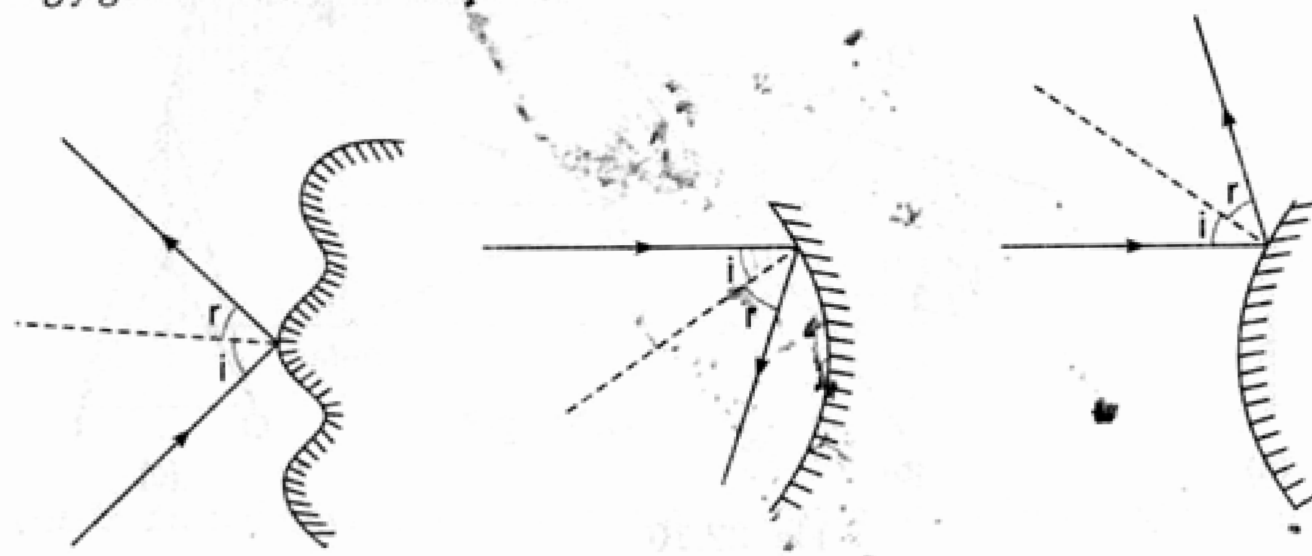


Fig. 27.7

Reflection from a Plane surface (or Plane mirror)

Almost everybody is familiar with the image formed by a plane mirror. If the object is real, the image formed by a plane mirror is virtual, erect, of same size and at the same distance from the mirror. The ray diagram of the image of a point object and of an extended object is as shown below.

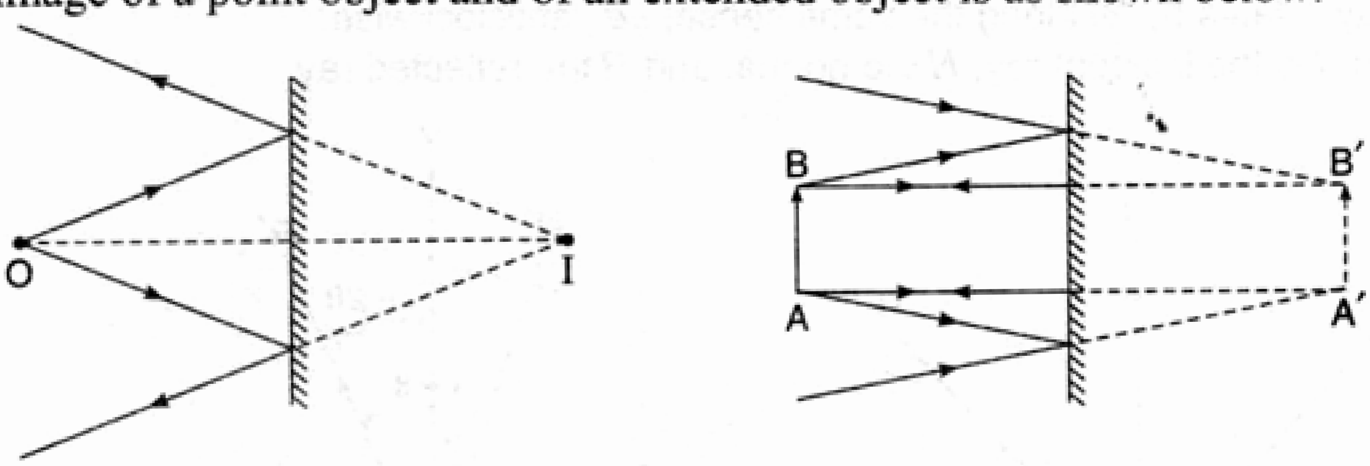
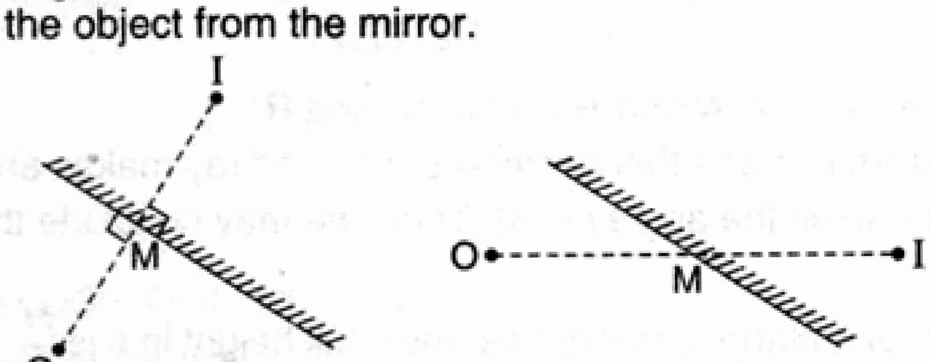


Fig. 27.8

• *Important points in* REFLECTION FROM PLANE MIRROR

1. To find the location of image of an object from an inclined plane mirror, you have to see the perpendicular distance of the object from the mirror.



Correct
OM = MI

Wrong
OM = MI

Fig. 27.9

2. **Field of view of an object for a given mirror :** Suppose a point object *O* is placed in front of a small mirror as shown in Fig. 27.10 (a), then a question arises in mind whether this mirror will form the image of this object or not or suppose an elephant is standing in front of a small mirror, will the mirror form the image of the elephant or not. The answer is yes, it will form. A mirror whatever may be the size of it

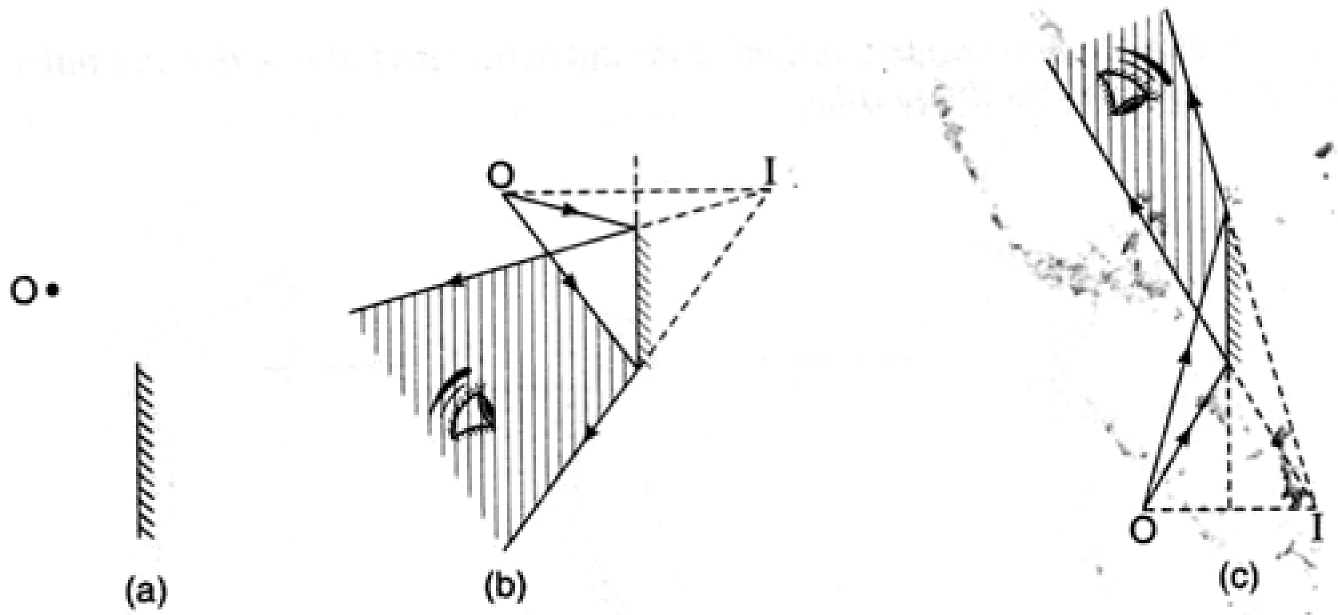


Fig. 27.10

forms the images of all objects lying in front of it. But every object has its own field of view for the given mirror. The field of view is the region between the extreme reflected rays and depends on the location of the object in front of the mirror. If our eye lies in the field of view then only we can see the image of the object otherwise not. The field of view of an object placed at different locations in front of a plane mirror are shown in Fig. 27.10 (b) and (c).

3. Suppose a mirror is rotated by an angle θ (say anticlockwise), keeping the incident ray fixed then the reflected ray rotates by 2θ along the same sense, *i.e.*, anticlockwise.
   In figure (a), I is the incident ray, N the normal and R the reflected ray.

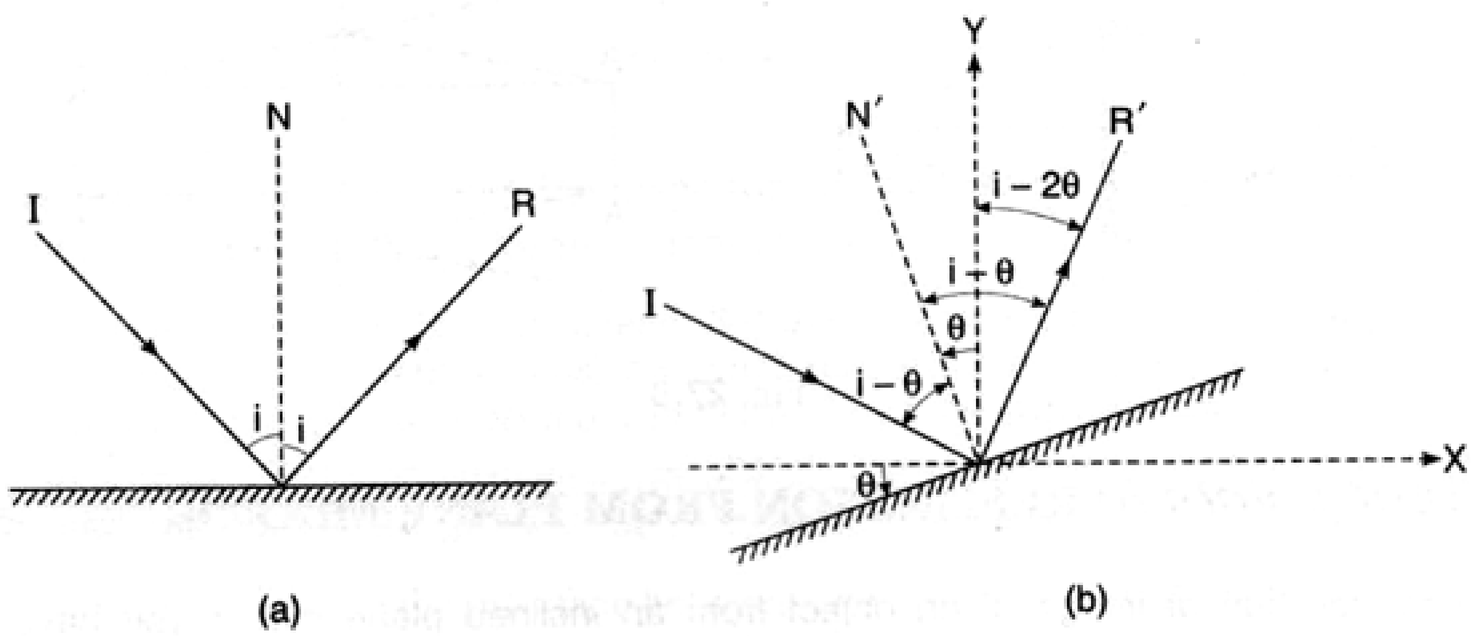


Fig. 27.11

   In figure (b), I remains as it is. N and R shift to N' and R'.
   From the two figures we can see that earlier the reflected ray makes an angle i with y-axis while after rotating the mirror it makes the angle $i - 2\theta$. Thus, we may conclude that the reflected ray has been rotated by angle 2θ.

4. The minimum length of a plane mirror to see one's full height in it is $\frac{H}{2}$, where H is the height of man.
   But the mirror should be placed in a fixed position which is shown in Fig. 27.12.
   A ray starting from head (A) after reflecting from upper end of the mirror (F) reaches the eye at C. Similarly the ray starting from the foot (E) after reflecting from the lower end (G) also reaches the eye at C. In two similar triangles ABF and BFC, $AB = BC = x$ (say), Similarly in triangles CDG and DGE,
   $$CD = DE = y \text{ (say)}$$
   Now, we can see that height of the man is $2(x + y)$ and that of mirror is $(x + y)$, *i.e.*, height of the mirror is half the height of the man.

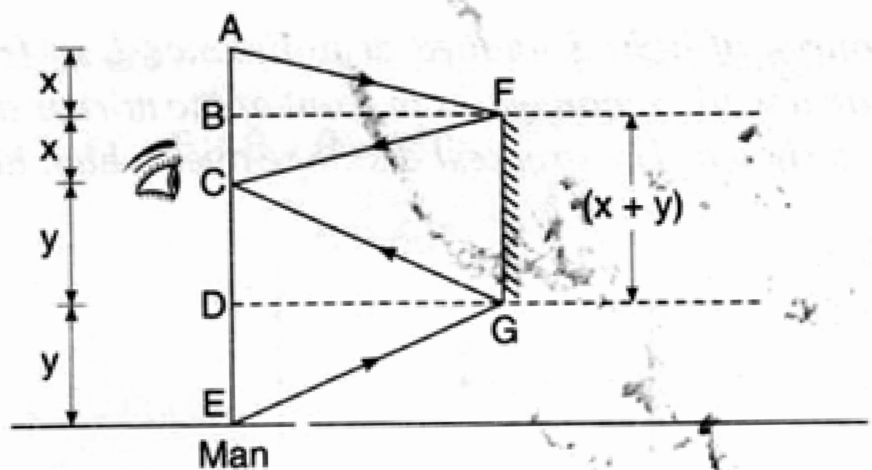


Fig. 27.12

Note *The mirror can be placed anywhere between the centre line BF (of AC) and DG (of CE).*

5. A man is standing exactly at midway between a wall and a mirror and he want to see the full height of the wall (behind him) in a plane mirror (in front of him). The minimum length of mirror in this case should be $\frac{H}{3}$, where H is the height of wall. The ray diagram in this case is drawn in Fig. 27.13.

In triangles HBI and IBC, $HI = IC = x$ (say). Now, in triangles HBI and ABF,

$$\frac{AF}{HI} = \frac{FB}{BI}$$

or

$$\frac{AF}{x} = \frac{2d}{d}$$

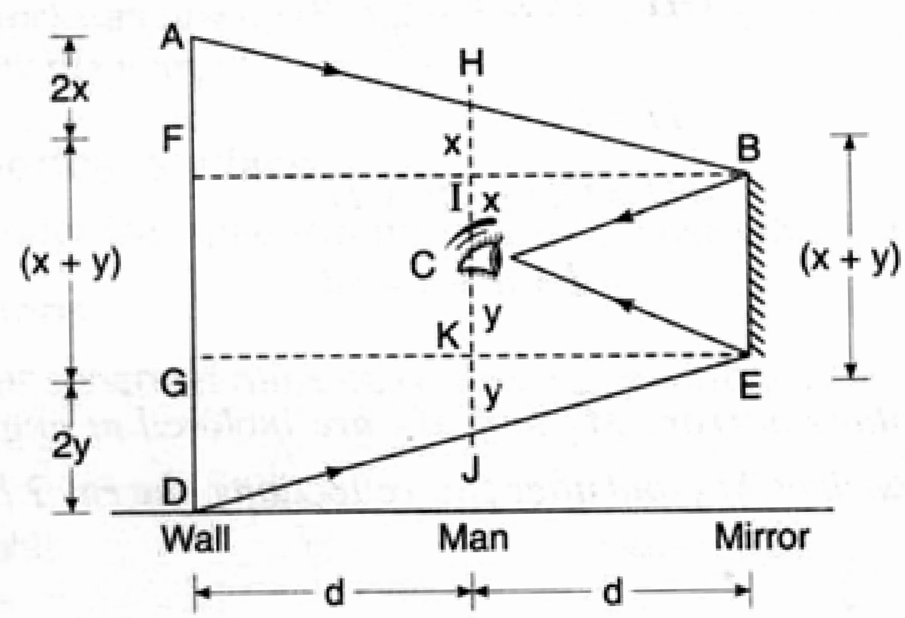


Fig. 27.13

or

$$AF = 2x$$

Similarly we can prove that $DG = 2y$ if, $CK = KJ = y$

Now, we can see that height of the wall is $3(x + y)$ while that of the mirror is $(x + y)$.

Sample Example 27.2 *A point source of light S, placed at a distance L in front of the centre of a mirror of width d, hangs vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 2L from it as shown. The greatest distance over which he can see the image of the light source in the mirror is*

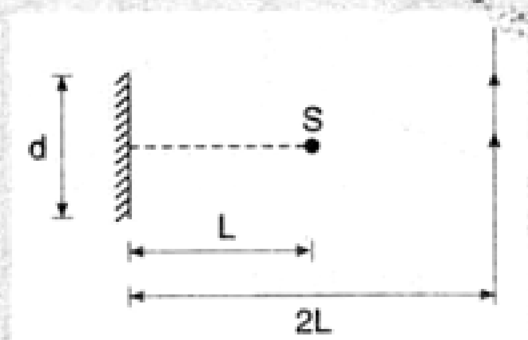


Fig. 27.14

(a) $d/2$ *(b)* d *(c)* $2d$ *(d)* $3d$.

Solution (d) The ray diagram will be as shown in Fig. 27.15.

$$HI = AB = d$$

$$DS = CD = \frac{d}{2}$$

Since, $$AH = 2AD$$

$\therefore$ $$GH = 2\,CD = 2\frac{d}{2} = d$$

Similarly $$IJ = d$$

$\therefore$ $$GJ = GH + HI + IJ$$

$$= d + d + d = 3d$$

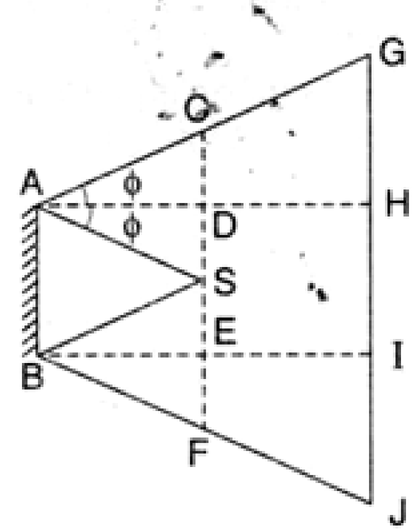


Fig. 27.15

Sample Example 27.3 *Two plane mirrors M_1 and M_2 are inclined at angle θ as shown. A ray of light 1, which is parallel to M_1 strikes M_2 and after two reflections, the ray 2 becomes parallel to M_2. Find the angle θ.*

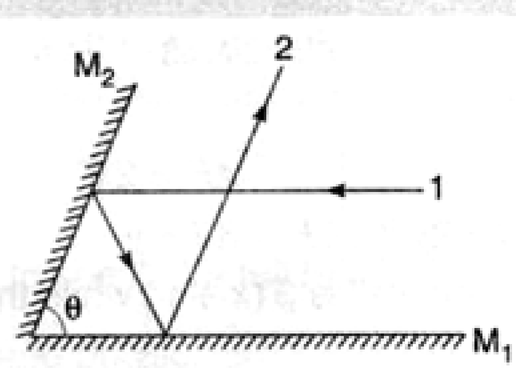


Fig. 27.16

Solution Different angles are as shown in Fig. 27.17. In triangle *ABC*,

$$\theta + \theta + \theta = 180°$$

$$\therefore \qquad \theta = 60° \qquad \textbf{Ans.}$$

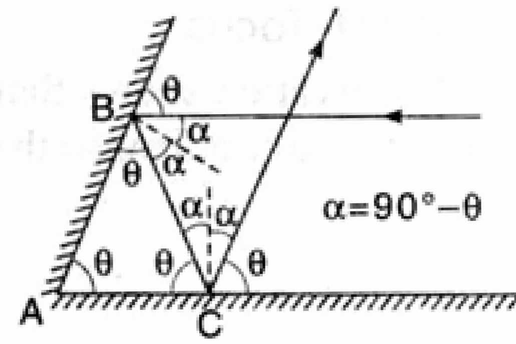


Fig. 27.17

Introductory Exercise 27.2

1. Prove that for any value of angle *i*, rays 1 and 2 are parallel.

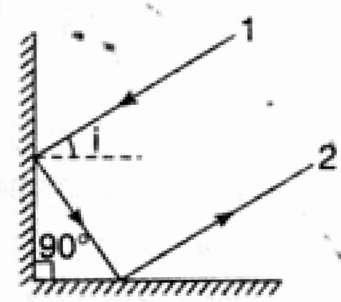


Fig. 27.18

2. A man approaches a vertical plane mirror at speed of 2 m/s. At what rate does he approach his image?
3. A pole of height 4 m is kept in front of a vertical plane mirror of length 2 m. The lower end of the mirror is at a height of 6 m from the ground. The horizontal distance between the mirror and the pole is 2 m. Upto what minimum and maximum heights a man can see the image of top of the pole at a horizontal distance of 4 m (from the mirror) standing on the same horizontal line which is passing through the pole and the horizontal point below the mirror?

Reflection from a Spherical Surface

We shall mainly consider the spherical mirrors, *i.e.*, those which are part of a spherical surface.

(a) Terms and Definitions

There are two types of spherical mirrors, concave and convex.

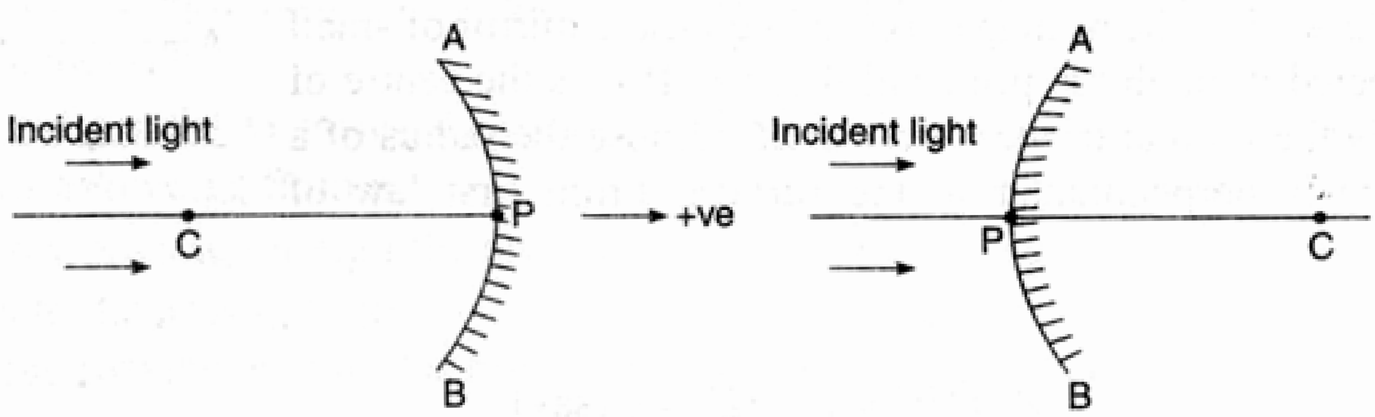


Fig. 27.19

Centre *C* of the sphere of which the mirror is a part is called the **centre of curvature** of the mirror and *P* the centre of the mirror surface, is called the **pole.** The line *CP* produced is the **principal axis** and *AB* is the **aperture** of the mirror. The distance *CP* is called the **radius of curvature (*R*).** All distances are measured from point *P*. We can see from the two figures that *R* is positive for convex mirror and negative for concave mirror.

Principal focus

Observation shows that a narrow beam of rays, parallel and near to the principal axis, is reflected from a concave mirror so that all rays converge to a point F on the principal axis. F is called the **principal**

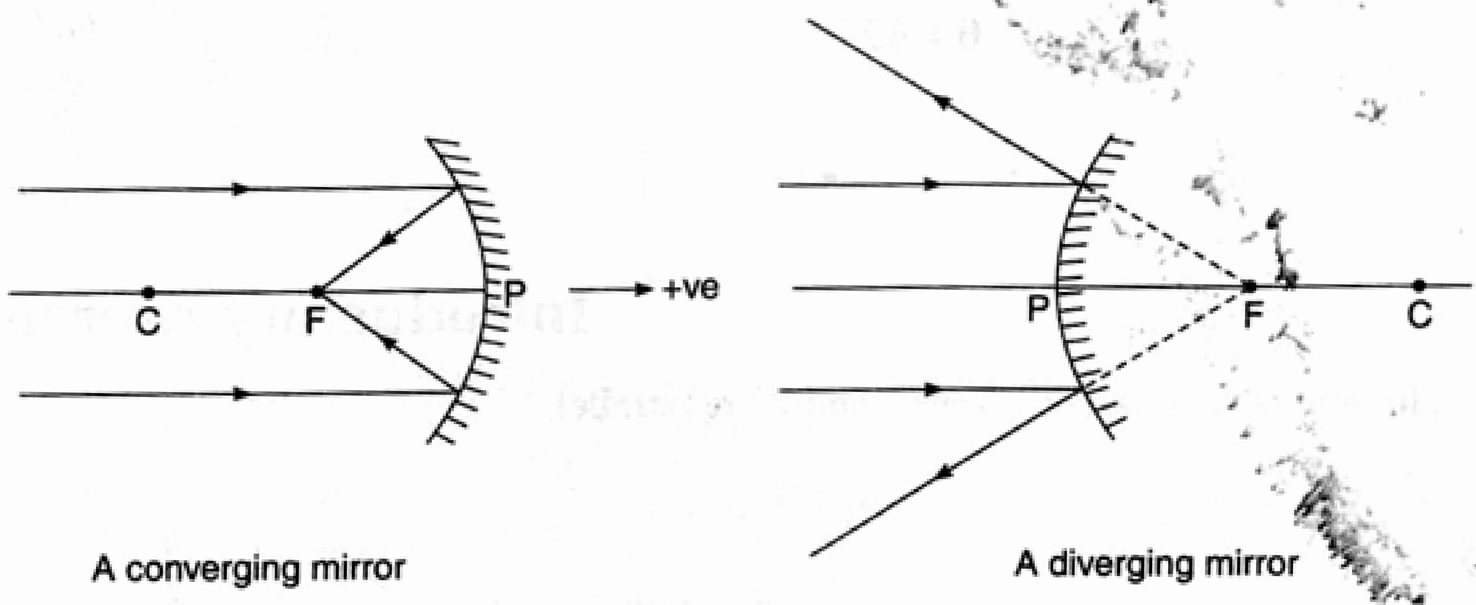


Fig. 27.20

focus of the mirror and it is a **real** focus, since, light actually passes through it. Concave mirrors are also known as converging mirrors because of their action on a parallel beam of light. They are used in car head-lights, search-lights and telescopes. A narrow beam of rays, parallel and near to the principal axis, falling on a convex mirror is reflected to form a divergent beam which appears to come from a point F behind the mirror. A convex mirror thus has a virtual principal focus. It is also called a diverging mirror. The distance FP is called the **focal length (f)** of the mirror. Further, we can see that f is negative for a concave mirror and positive for convex mirror. Later we will see that $f = R/2$.

Paraxial rays : Rays which are close to the principal axis and make small angles with it, *i.e.*, they are nearly parallel to the axis, are called paraxial rays. Our treatment of spherical mirrors will be restricted to such rays which means we shall consider only mirrors of small aperture. In diagrams, however, they will be made larger for clarity.

(b) Relation between f and R

A ray AM parallel to the principal axis of a concave mirror of small aperture is reflected through the principal focus F. If C is the centre of curvature, CM is the normal to the mirror at M because the radius of a spherical surface is perpendicular to the surface. From first law of reflection,

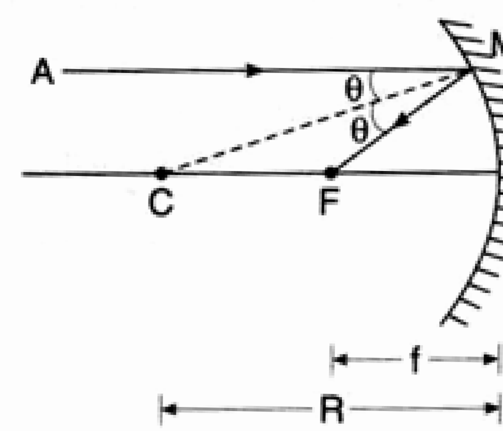


Fig. 27.21

$$\angle i = \angle r$$

or $$\angle AMC = \angle CMF = \theta \text{ (say)}$$

But $$\angle AMC = \angle MCF \text{ (alternate angles)}$$

$\therefore$ $$\angle CMF = \angle MCF$$

Therefore, $\Delta\, FCM$ is thus isosceles with $FC = FM$. The rays are paraxial and so M is very close to P. Therefore,

$$FM \approx FP$$

$\therefore$ $$FC = FP$$

or $$FP = \frac{1}{2} CP$$

or $$f = \frac{R}{2}$$

EXERCISE Prove the above relation for convex mirror.

(c) Images formed by Spherical Mirrors

In general position of image and its nature (*i.e.*, whether it is real or virtual, erect or inverted, magnified or diminished) depend on the distance of object from the mirror.

Information about the image can be obtained either by drawing a **ray diagram** or by calculation using **formulae**.

Ray diagrams : We shall consider the small objects and mirrors of small aperture so that all rays are paraxial. To construct the image of a point object two of the following four rays are drawn passing through the object. To construct the image of an extended object the image of two end points is only drawn. The image of a point object lying on principles axis is formed on the principal axis itself. The four rays are as under:

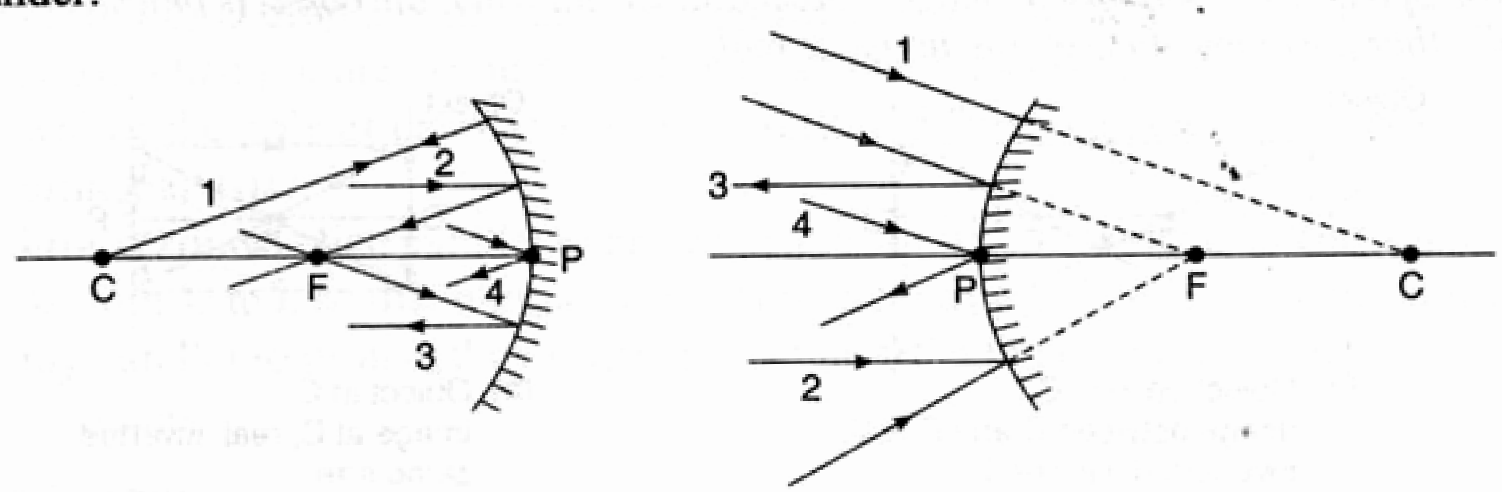


Fig. 27.22

Ray 1. A ray through the centre of curvature which strikes the mirror normally and is reflected back along the same path.

Ray 2. A ray parallel to principal axis after reflection either actually passes through the principal focus F or appears to diverge from it.

Ray 3. A ray passing through the principal focus F or a ray which appears to converge at F is reflected parallel to the principal axis.

Ray 4. A ray striking at pole P is reflected symmetrically back in the opposite side.

Note 1. *Image formed by convex mirror is always virtual, erect and diminished no matter where the object is.*

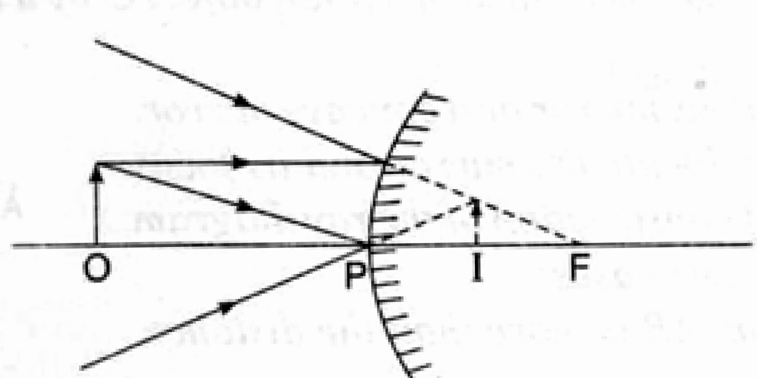


Fig. 27.23

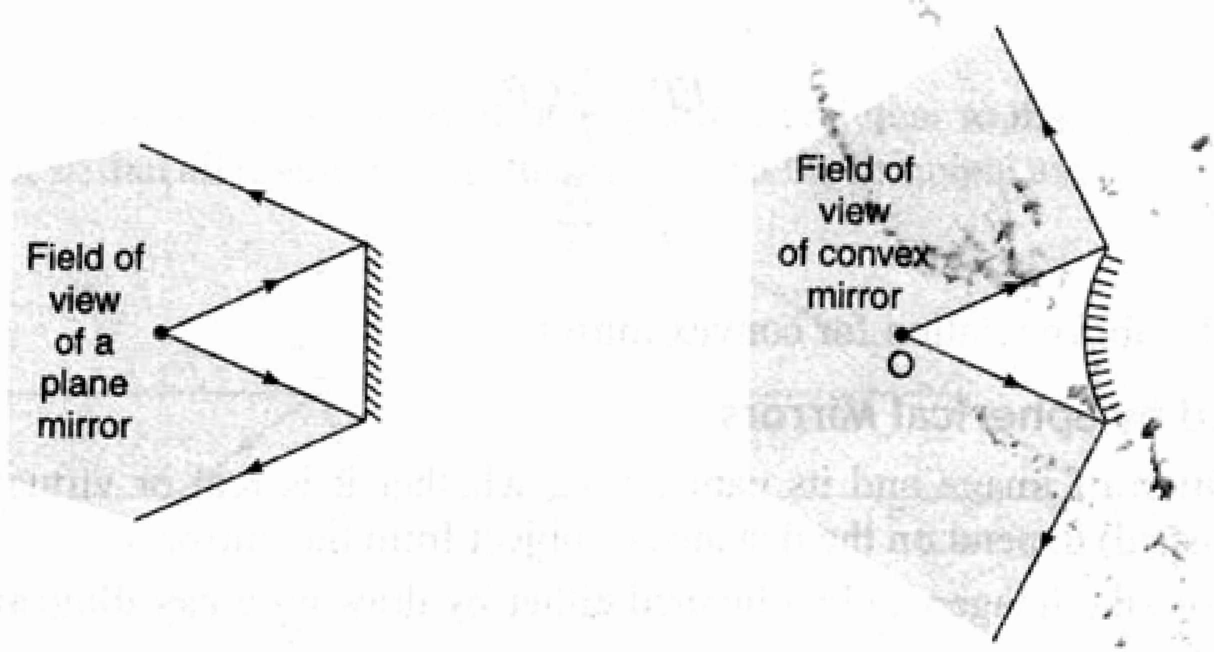
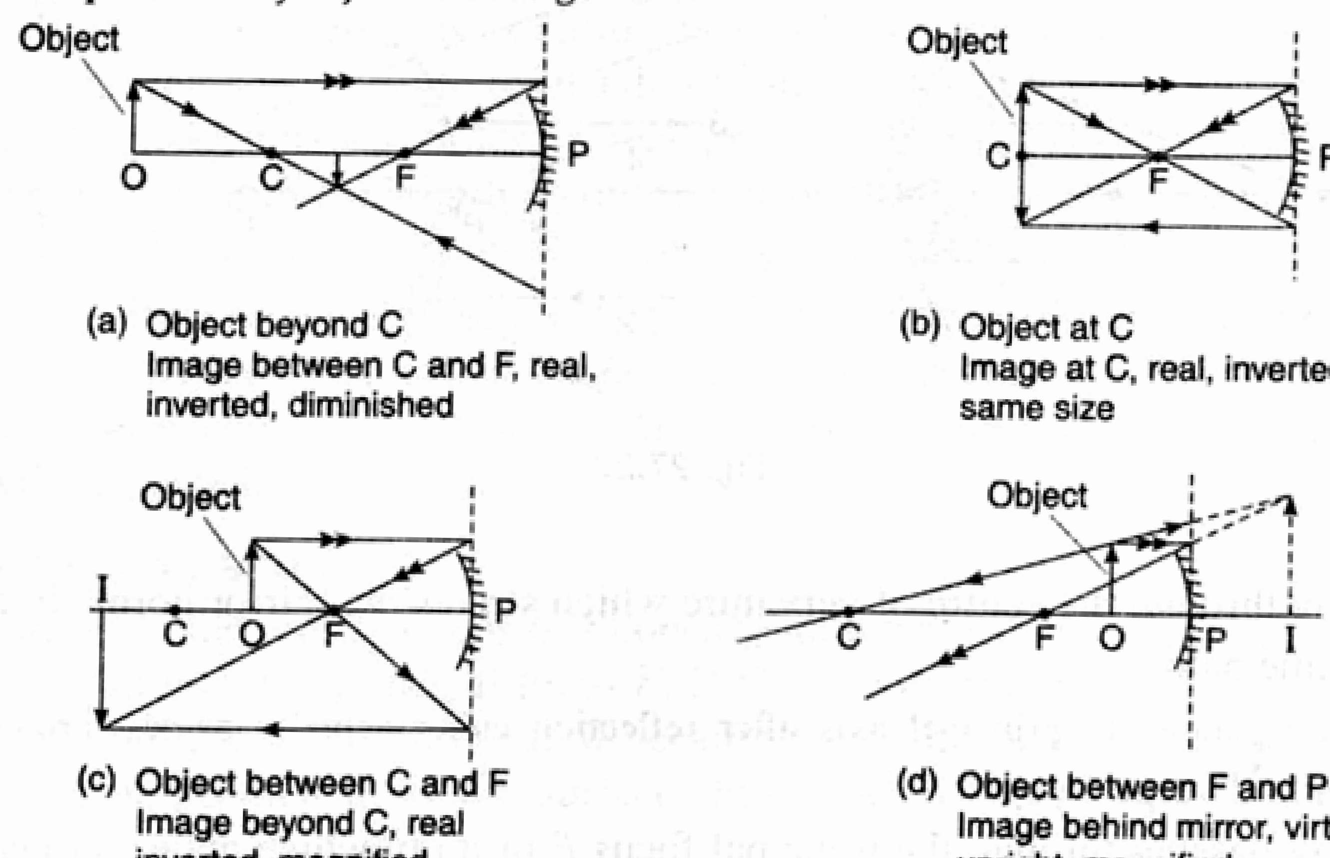


Fig. 27.24

*Fig. 27.24 shows that convex mirror gives a wider **field of view** than a plane mirror, convex mirrors are therefore, used as rear view mirrors in cars or scooters. Although they make the estimation of distances more difficult but still they are preferred because there is only a small movement of the image for a large movement of the object.*

2. *In case of a concave mirror the image is erect and virtual when the object is placed between F and P. In all other positions of object the image is real.*



(a) Object beyond C
Image between C and F, real, inverted, diminished

(b) Object at C
Image at C, real, inverted, same size

(c) Object between C and F
Image beyond C, real inverted, magnified

(d) Object between F and P
Image behind mirror, virtual upright, magnified

Fig. 27.25

Sample Example 27.4 *An image I is formed of a point object O by a mirror whose principal axis is AB as shown in figure.*

(a) *State whether it is a convex mirror or a concave mirror.*

(b) *Draw a ray diagram to locate the mirror and its focus. Write down the steps of construction of the ray diagram. Consider the possible two cases:*

(1) *When distance of I from AB is more than the distance of O from AB and*

(2) *When distance of O from AB is more than the distance of I from AB*

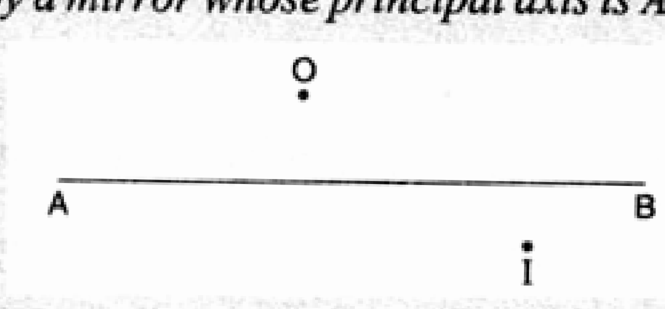


Fig. 27.26

Solution

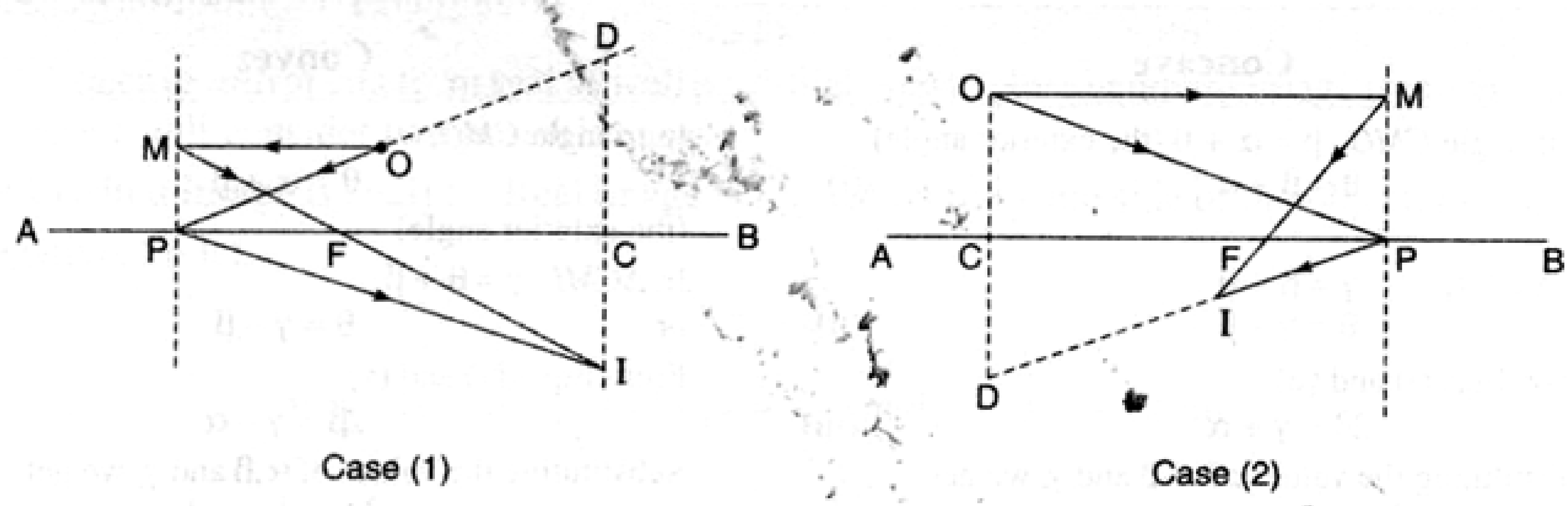


Fig. 27.27

(a) As the image is on the opposite side of the principal axis, the mirror is concave. Because convex mirror always forms an erect image.

(b) Two different cases are shown in figure. Steps are as under :

(i) From I or O drop a perpendicular on principal axis, such that $CI = CD$ or $OC = CD$.

(ii) Draw a line joining D and O or D and I so that it meets the principal axis at P. The point P will be the pole of the mirror as a ray reflected from the pole is always symmetrical about principal axis.

(iii) From O draw a line parallel to principal axis towards the mirror so that it meets the mirror at M. Join M to I, so that it intersects the principal axis at F. F is the focus of the mirror as any ray parallel to principal axis after reflection from the mirror intersects the principal axis at the focus.

The Mirror Formula

In figures 27.28 (a) and (b) a ray OM from a point object O on the principal axis is reflected at M so that the angle θ, made by the incident and reflected rays with the normal CM are equal. A ray OP strikes the mirror normally and is reflected back along PO. The intersection I of the reflected rays MI and PO in figure (a) gives a **real** point image of O and in figure (b) gives a **virtual** point image of O. Let α, β and γ be the angles as shown. As the rays are paraxial, these angles are small, we can take $\alpha \approx \tan \alpha = \dfrac{MP}{OP}$, $\beta = \dfrac{MP}{CP}$ and $\gamma = \dfrac{MP}{IP}$.

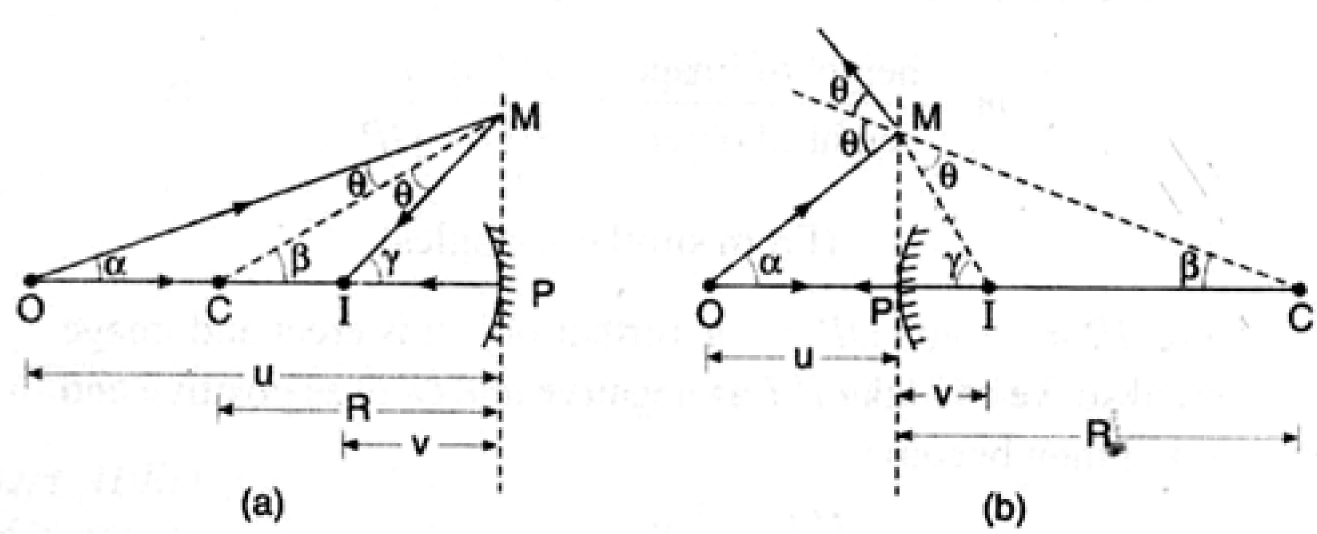


Fig. 27.28

Now, let us take the two figures simultaneously

Concave

In triangle CMO, $\beta = \alpha + \theta$ (the exterior angle)

or $\theta = \beta - \alpha$ ...(i)

In ΔCMI, $\gamma = \beta + \theta$

$\therefore$ $\theta = \gamma - \beta$ ...(ii)

From Eqs. (i) and (ii)

$2\beta = \gamma + \alpha$ ...(iii)

Substituting the values of α, β and γ, we get

$$\frac{2}{CP} = \frac{1}{IP} + \frac{1}{OP} \quad \text{...(A)}$$

If we now substitute the values with sign, $CP = -R$, $IP = -v$ and $OP = -u$

we get, $$\frac{2}{R} = \frac{1}{v} + \frac{1}{u}$$

or $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{as} \quad \left(f = \frac{R}{2}\right)$$

Convex

In triangle CMO,

$\theta = \alpha + \beta$ ...(iv)

(the exterior angle)

In ΔCMI $\gamma = \theta + \beta$

or $\theta = \gamma - \beta$ ...(v)

From Eqs. (iv) and (v)

$2\beta = \gamma - \alpha$ ...(vi)

Substituting the values of α, β and γ, we get

$$\frac{2}{CP} = \frac{1}{IP} - \frac{1}{OP} \quad \text{...(B)}$$

i.e., If we now substitute the values with sign, *i.e.*, $CP = +R$, $IP = +v$ and $OP = -u$, we get

$$\frac{2}{R} = \frac{1}{v} + \frac{1}{u}$$

or $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{as} \quad \left(f = \frac{R}{2}\right)$$

Note *Out of u, v and f values of two will be known to us and we will be asked to find the third. In such type of problems two cases are possible. Case 1 is when signs of all three will be known to us from the given information. Under this condition substitute all three with sign, then answer (i.e., the third quantity) will come without sign. Only numerical value of the unknown comes. Case 2 is when the sign of third unknown quantity is not known to us. Under such situation substitute only the known quantities with sign. Sign with numerical value will automatically come in the answer.*

Magnification

The lateral, transverse or linear magnification m is defined as,

$$m = \frac{\text{height of image}}{\text{height of object}} = \frac{I'I}{O'O} = \frac{IP}{OP} \quad \text{...(i)}$$

(From similar triangles)

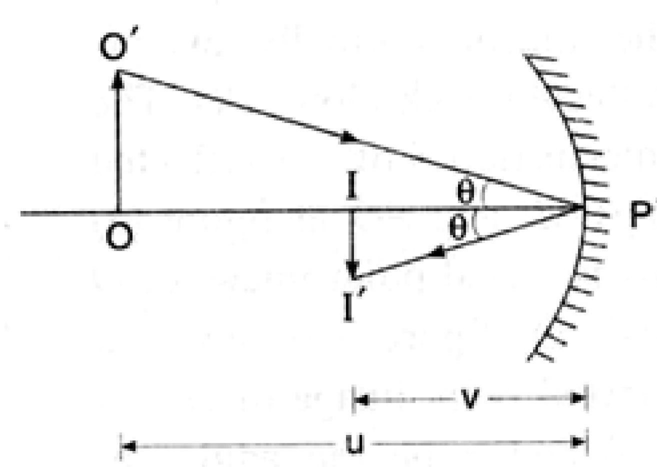


Fig. 27.29

Here, $IP = -v$ and $OP = -u$, further object is erect and image is inverted so we can take $I'I$ as negative and $O'O$ as positive and Eq. (i) will then become

$$\frac{I'I}{O'O} = -\frac{v}{u} \quad \text{or} \quad m = -\frac{v}{u}$$

Note *We have derived* $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ *and* $m = -\frac{v}{u}$ *for special cases of the position of object but the same result can be derived for other cases.*

Sample Example 27.5 *Find the distance of object from a concave mirror of focal length 10 cm so that image size is four times the size of the object.*

Solution Concave mirror can form real as well as virtual image. Here nature of image is not given in the question. So we will consider two possible cases.

***Case* 1 (when image is real) :** Real image is formed on the same side of the object, *i.e.*, *u*, *v* and *f* all are negative. So let,

$$u = -x$$

then $v = -4x$ as $\left|\frac{v}{u}\right| = |m| = 4$ and $f = -10$ cm

Substituting in, $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

we have $$\frac{1}{-4x} - \frac{1}{x} = \frac{1}{-10}$$

or $$\frac{5}{4x} = \frac{1}{10}$$

$\therefore$ $$x = 12.5 \text{ cm}$$ **Ans.**

Note *$|x| > |f|$ and we know that in case of a concave mirror, image is real when object lies beyond F.*

***Case* 2 (When image is virtual) :** In case of a mirror image is virtual, when it is formed behind the mirror, *i.e.*, *u* and *f* are negative, while *v* is positive. So let

$$u = -y$$

then $v = +4y$ and $f = -10$ cm

Substituting in $$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

We have $$\frac{1}{4y} - \frac{1}{y} = \frac{1}{-10}$$

or $$\frac{3}{4y} = \frac{1}{10}$$

or $$y = 7.5 \text{ cm}$$ **Ans.**

Note *Here, $|y| < |f|$, as we know that image is virtual when the object lies between F and P.*

Sample Example 27.6 *An object ABED is placed in front of a concave mirror beyond centre of curvature C as shown in figure. State the shape of the image.*

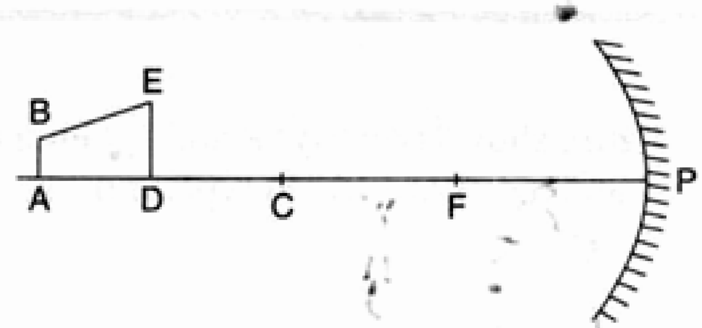


Fig. 27.30

Solution Object is placed beyond C. Hence, the image will be real and it will lie between C and F. Further u, v and f all are negative, hence the mirror formula will become

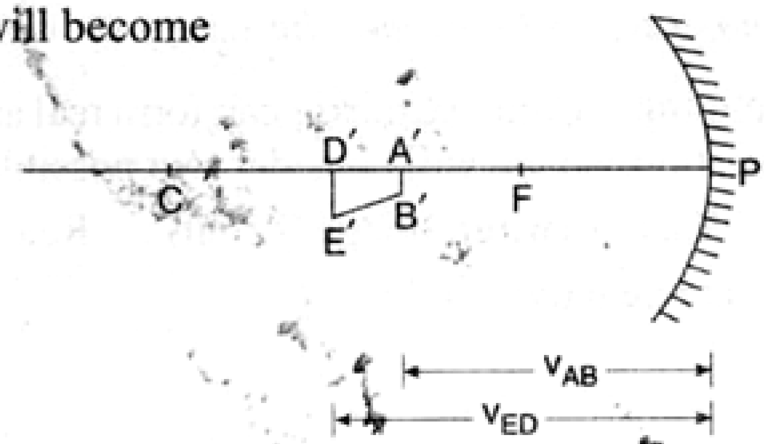


Fig. 27.31

$$-\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$$

or
$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{u-f}{uf}$$

or
$$v=\frac{f}{1-\frac{f}{u}}$$

Now
$$u_{AB}>u_{ED}$$

∴
$$v_{AB}<v_{ED}$$

and
$$|m_{AB}|<|m_{ED}| \qquad \left(\text{as } m=-\frac{v}{u}\right)$$

Therefore, shape of the image will be as shown in figure 27.31.

Also note that $v_{AB}<u_{AB}$ and $v_{ED}<u_{ED}$,

So, $|m_{AB}|<1$ *and* $|m_{ED}|<1$

Introductory Exercise 27.3

1. Assume that a certain spherical mirror has a focal length of – 10.0 cm. Locate and describe the image for object distances of
(a) 25.0 cm
(b) 10.0 cm
(c) 5.0 cm.

2. A ball is dropped from rest 3.0 m directly above the vertex of a concave mirror that has a radius of 1.0 m and lies in a horizontal plane.
(a) Describe the motion of ball's image in the mirror.
(b) At what time do the ball and its image coincide?

3. A spherical mirror is to be used to form on a screen 5.0 m from the object an image five times the size of the object.
(a) Describe the type of mirror required.
(b) Where should the mirror be positioned relative to the object?

4. Figure shows two rays P and Q being reflected by a mirror and going as P' and Q'. State which type of mirror is this?

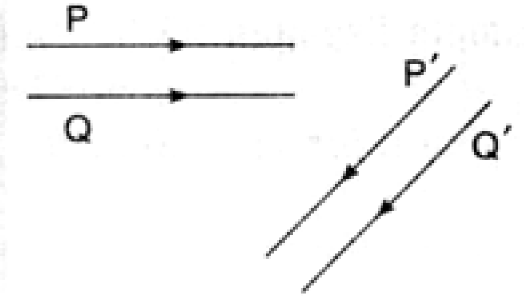


Fig. 27.32

5. The following table shows object distance, object size, and mirror focal length (all in centimeters) for various situations. Use ray tracing to determine the image size and location for each case.

Object Distance	Object Size	Focal Length
40	2.0	20
10	1.0	20
30	1.5	−30
20	2.0	−40

6. Complete the following table for spherical mirrors. All distances are in centimeters.

Mirror Type	Radius	Focal Length	Object Distance	Image Distance	Lateral Magnification	Real / Virtual Image
		− 25	40			
			20	− 30		
		+ 15		+ 30		
			20		− 2.0	
	+ 20			− 40		
		− 10			+ 2.0	

Extra Points

- For spherical mirrors

$$m = -\frac{v}{u}$$

Positive value of m means v and u are of opposite sign. So if u is negative then v is positive and *vice-versa*. Thus, if $m = +2$ for a real object, it means image is virtual, erect and two times greater in size. Similarly $m = -\frac{1}{2}$ means image is real, inverted and $\frac{1}{2}$ times in size (that of object).

- **Method of finding coordinates of image of a point object if the coordinates of object are known :** Suppose coordinates of a point object (x_0, y_0) with respect to the coordinate axes shown in figure are known to us. The coordinates of image (x_i, y_i) can be obtained using

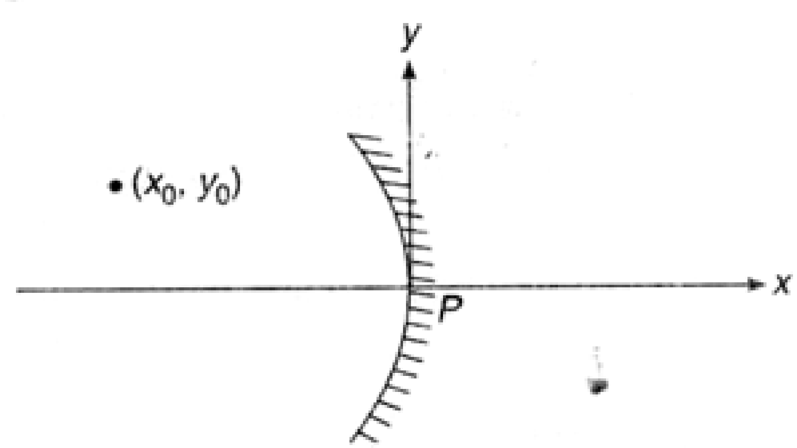


Fig. 27.33

$$\frac{1}{x_0} + \frac{1}{x_i} = \frac{1}{f} \qquad \left(\frac{1}{v} + \frac{1}{u} = \frac{1}{f}\right)$$

or $$x_i = \frac{fx_0}{x_0 - f} \qquad \ldots(\text{i})$$

Similarly $$m = \frac{y_i}{y_0} = -\frac{v}{u} = -\frac{x_i}{x_0}$$

or $$y_i = \frac{f y_0}{f - x_0} \qquad \ldots(\text{ii})$$

- **For Concave Mirror**

S. No.	Position of object	Details of image
1.	At ∞	At F, real, inverted $\|m\| << 1$
2.	Between C and ∞	Between F and C, real, inverted, $\|m\|<1$
3.	At C	At C, real, inverted, $\|m\|=1$
4.	Between F and C	Between C and ∞, real, inverted, $\|m\|>1$
5.	At F	At infinity, real, inverted, $\|m\|>>1$
6.	Between F and P	Behind the mirror, virtual, erect $\|m\|>1$

For Convex Mirror

S. No.	Position of object	Details of images
1.	At infinity	At F, virtual, erect, $\|m\|<<1$
2.	In front of mirror	Between P and F, virtual, erect, $\|m\|<1$

- If an object is placed with its length along the principal axis, then so called **longitudinal magnification** becomes,

$$m_L = \frac{I}{O} = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) = -\frac{dv}{du}$$ (for small objects)

From, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ we have

$$-v^{-2}dv - u^{-2}du = 0 \quad \text{or} \quad \frac{dv}{du} = -\left(\frac{v}{u}\right)^2$$

or

$$m_L = -\frac{dv}{du} = \left(\frac{v}{u}\right)^2 = m^2$$

- If we differentiate the mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

with respect to time, we get

$$-v^{-2}.\frac{dv}{dt} - u^{-2}\frac{du}{dt} = 0$$ (as f = constant)

or

$$\frac{dv}{dt} = -\left(\frac{v^2}{u^2}\right)\frac{du}{dt}$$ ...(iii)

Here, $\frac{du}{dt}$ is the rate by which u is changing. Or it is the object speed if mirror is stationary. Similarly, $\frac{dv}{dt}$ is the rate by which v (distance between image and mirror) is changing. Or it is image speed if mirror is stationary. So if at a known values of v and u, the object speed is given, we can find the image speed from the above formula.

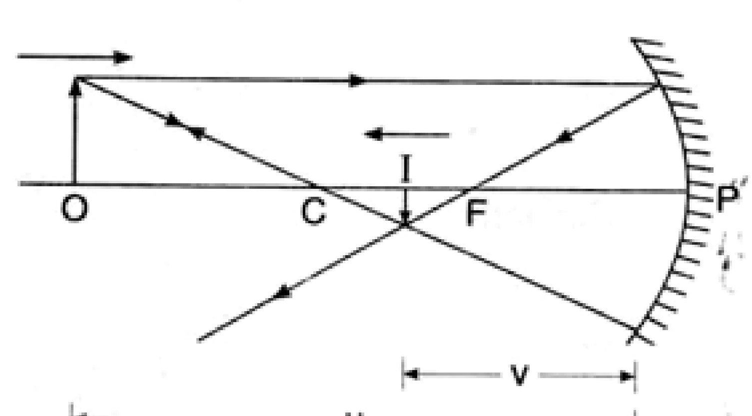


Fig. 27.34

Let us take an example for a concave mirror.

Suppose the object is moved from infinity towards focus.

As u is decreasing therefore,

$$\left(-\frac{du}{dt}\right) = \text{rate of decrease of } u \qquad \text{(speed of object)}$$

Therefore,
$$\left(\frac{dv}{dt}\right) = \text{rate of increase of } v \qquad \text{(speed of image)}$$

Further $v < u$ (when the object lies between ∞ and C)

$$\therefore \quad \left(\frac{dv}{dt}\right) < \left(-\frac{du}{dt}\right) \qquad \text{[from Eq. (iii)]}$$

Hence, as the object is moved towards mirror the image (which is real) will recede from the mirror with speed less than the speed of object.

When the object is at C, image is also at C, *i.e.*, $v = u$ or $\left(\frac{dv}{dt}\right) = \left(-\frac{du}{dt}\right)$. Hence, speed of image is equal to the speed of object. When the object lies between C and F, $v > u$, *i.e.*, image speed is more than the object speed when object comes inside F, image becomes virtual *i.e.*, u and f are negative while v is positive.

Hence,
$$\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}$$

or
$$\frac{1}{u} - \frac{1}{v} = \frac{1}{f}$$

or
$$-u^{-2}\left(\frac{du}{dt}\right) - v^{-2}\left(\frac{dv}{dt}\right) = 0$$

$$\left(-\frac{dv}{dt}\right) = \left(\frac{v^2}{u^2}\right)\left(-\frac{du}{dt}\right)$$

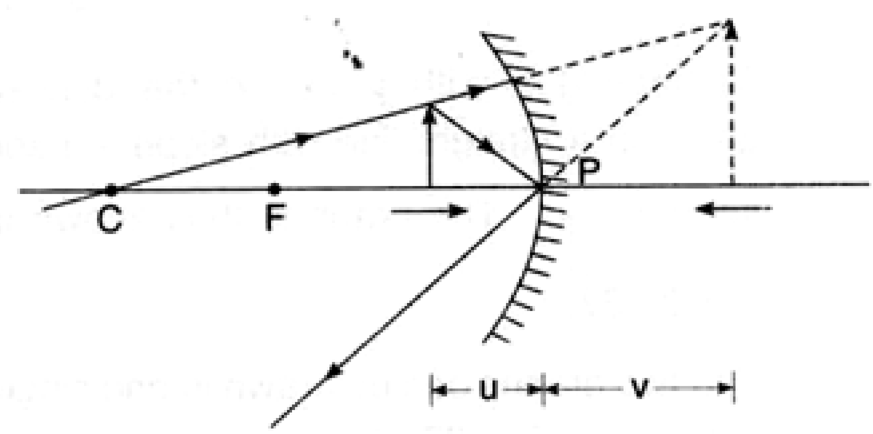


Fig. 27.35

Now as u is further decreased, v also decreases to keep $\frac{1}{f}$ constant. So, $-\frac{du}{dt}$ is rate with which object is approaching towards mirror and $\left(-\frac{dv}{dt}\right)$ is rate by which image is approaching towards mirror. Further in this case we know that image is always enlarged or $v > u$. Therefore, image speed is more than the object speed. Thus, we may conclude the above discussion as under:

> When an object is moved from $-\infty$ to F, the image (real) moves from F to $-\infty$ and then when the object is further moved from F to P image (now virtual) moves from $+\infty$ to P.

Note *When the object is either at centre of curvature C or at pole P, the two speeds are equal. When the object is at pole it can be assumed as if the image is forming by a plane mirror due to the small aperture of the mirror.*

EXERCISE Do the above exercise with a convex mirror.

- **Graph between $\frac{1}{v}$ *versus* $\frac{1}{u}$:** Let us first take the case of a concave mirror. Here, two cases are possible.

Case 1. When the image is real, *i.e.*, object lies between F and infinity. In such a situation u, v and f are negative. Hence, the mirror formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \quad \text{becomes}$$

$$-\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$$

or again $$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

or $$\frac{1}{v}=-\frac{1}{u}+\frac{1}{f}$$

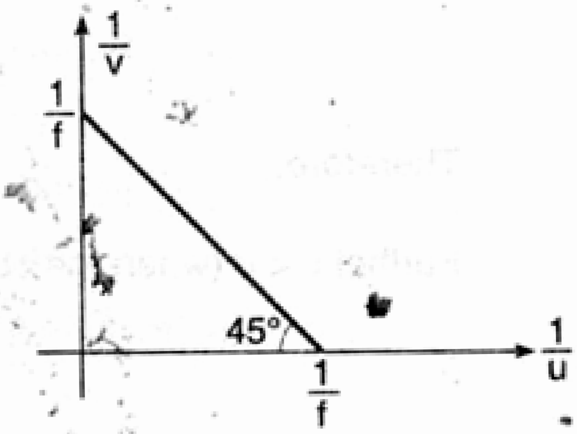


Fig. 27.36

Comparing with $y=-x+c$, the desired graph will be a straight line with slope -1 and slope equal to $\frac{1}{f}$.

The corresponding graph is shown in Fig. 27.36.

Case 2. When the image is virtual, *i.e.*, object lies between F and P. Under such situation u and f are negative while v is positive. The mirror formula thus becomes

$$\frac{1}{v}=\frac{1}{u}-\frac{1}{f}$$

Comparing it with $y=x-c$ the desired graph is a straight line with slope $+1$ and intercept $-\frac{1}{f}$. The graph is thus shown in Fig. 27.37.

The two graphs can be drawn in one single graph as in Fig. 27.38.

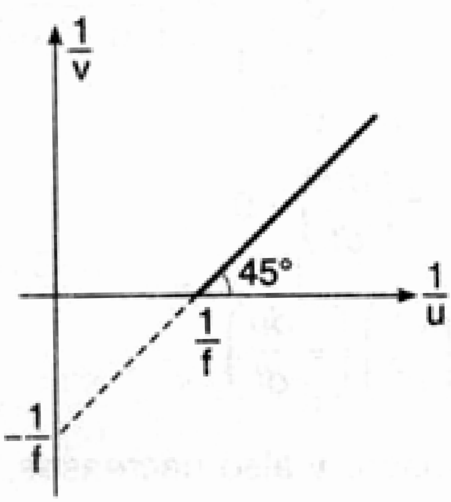


Fig. 27.37

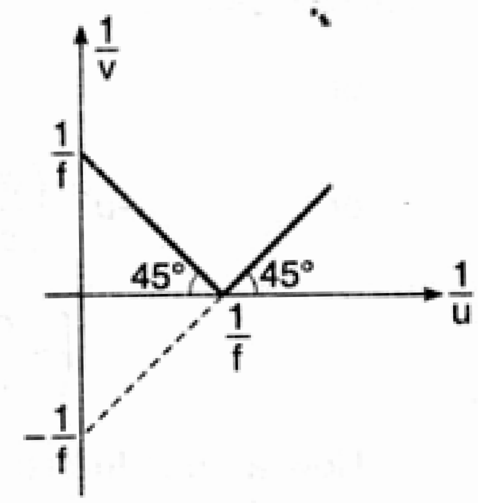


Fig. 27.38

Note *Here $\frac{1}{u}$ and $\frac{1}{v}$ are really the magnitudes of $\frac{1}{u}$ and $\frac{1}{v}$ (i.e., without sign)*

*For a **convex mirror** image is always virtual, i.e., u is negative while v and f are positive. Hence, the mirror formula becomes,*

$$\frac{1}{v}=\frac{1}{u}+\frac{1}{f}$$

Comparing with $y=x+c$, the desired graph is a straight line of slope $+1$ and intercept $\frac{1}{f}$. The graph is thus shown in Fig. 27.39.

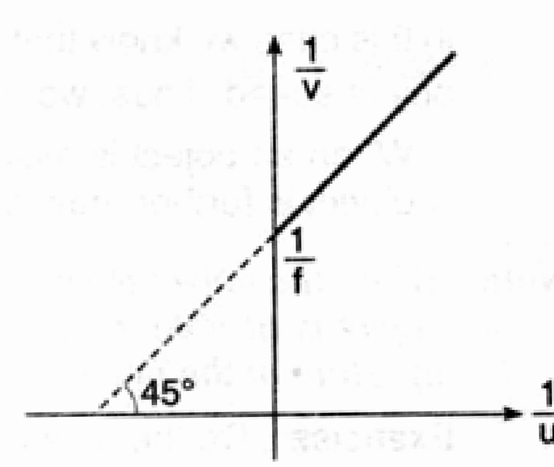


Fig. 27.39

Solved Examples

Example 1 *An object is 30.0 cm from a spherical mirror, along the central axis. The absolute value of lateral magnification is $\frac{1}{2}$. The image produced is inverted. What is the focal length of the mirror?*

Solution Image is inverted, so it is real u and v both are negative. Magnification is $\frac{1}{2}$, therefore, $v = \frac{u}{2}$.

Given, $u = -30$ cm, $v = -15$ cm

Using the mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

We have,

$$\frac{1}{f} = \frac{1}{-15} - \frac{1}{30} = -\frac{1}{10}$$

$\therefore$ $$f = -10 \text{ cm}$$ **Ans.**

Example 2 *A concave mirror has a radius of curvature of 24 cm. How far is an object from the mirror if an image is formed that is :*

(a) virtual and 3.0 times the size of the object,
(b) real and 3.0 times the size of the object and
(c) real and 1/3 the size of the object?

Solution Given $R = -24$ cm (concave mirror)

Hence,

$$f = \frac{R}{2} = -12 \text{ cm}$$

(a) Image is virtual and 3 times larger. Hence, u is negative and v is positive. Simultaneously, $|v| = 3|u|$. So let,

$$u = -x$$

then

$$v = +3x$$

Substituting in the mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we have

$$\frac{1}{3x} - \frac{1}{x} = \frac{1}{-12}$$

$\therefore$ $$x = 8 \text{ cm}$$

Therefore, object distance is 8 cm. **Ans.**

(b) Image is real and three times larger. Hence, u and v both are negative and $|v| = 3|u|$. So let,

$$u = -x$$

then

$$v = -3x$$

Substituting in mirror formula, we have

$$\frac{1}{-3x} - \frac{1}{x} = -\frac{1}{12}$$

or $$x = 16 \text{ cm}$$

∴ Object distance should be 16 cm. **Ans.**

(c) Image is real and $\frac{1}{3}$rd the size of object. Hence, both u and v are negative and $|v| = \frac{|u|}{3}$.

So let, $$u = -x$$

then $$v = -\frac{x}{3}$$

Substituting in the mirror formula, we have

$$-\frac{3}{x} - \frac{1}{x} = -\frac{1}{12}$$

∴ $$x = 48 \text{ cm.}$$

∴ Object distance should be 48 cm. **Ans.**

Example 3 *A ray of light is incident on a plane mirror along a vector* $\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$. *The normal on incidence point is along* $\hat{\mathbf{i}} + \hat{\mathbf{j}}$. *Find a unit vector along the reflected ray.*

Solution Reflection of a ray of light is just like an elastic collision of a ball with a horizontal ground. Component of incident ray along the inside normal gets reversed while the component perpendicular to it remains unchanged. Thus the component of incident ray vector $\vec{\mathbf{A}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ parallel to normal, *i.e.*, $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ gets reversed while perpendicular to it, *i.e.*, $-\hat{\mathbf{k}}$ remains unchanged. Thus, the reflected ray can be written as,

$$\vec{\mathbf{R}} = -\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$$

∴ A unit vector along the reflected ray will be,

$$\hat{\mathbf{r}} = \frac{\vec{\mathbf{R}}}{R} = \frac{-\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{3}}$$

or $$\hat{\mathbf{r}} = -\frac{1}{\sqrt{3}}(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$$ **Ans.**

Note *In this problem normal is inside the mirror surface. Think why?*

Example 4 *A gun of mass m_1 fires a bullet of mass m_2 with a horizontal speed v_0. The gun is fitted with a concave mirror of focal length f facing towards a receding bullet. Find the speed of separations of the bullet and the image just after the gun was fired.*

Solution Let v_1 be the speed of gun (or mirror) just after the firing of bullet. From conservation of linear momentum,

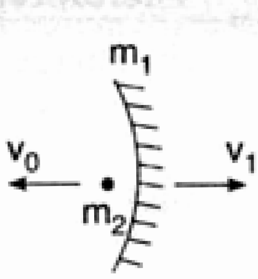


Fig. 27.40

$$m_2 v_0 = m_1 v_1$$

or $$v_1 = \frac{m_2 v_0}{m_1} \quad \ldots\text{(i)}$$

Now, $\frac{du}{dt}$ = rate at which distance between mirror and bullet is increasing $= v_1 + v_0$ ...(ii)

We have already read in extra points that :

$$\therefore \quad \frac{dv}{dt} = \left(\frac{v^2}{u^2}\right)\frac{du}{dt}$$

Here, $\frac{v^2}{u^2} = m^2 = 1$ (as at the time of firing bullet is at pole).

$$\therefore \quad \frac{dv}{dt} = \frac{du}{dt} = v_1 + v_0 \quad \ldots\text{(iii)}$$

Here, $\frac{dv}{dt}$ is the rate at which distance between image (of bullet) and mirror is increasing. So, if v_2 is the absolute velocity of image (towards right) then,

$$v_2 - v_1 = \frac{dv}{dt} = v_1 + v_0$$

or $$v_2 = 2\,v_1 + v_0 \quad \ldots\text{(iv)}$$

Therefore, speed of separation of bullet and image will be,

$$v_r = v_2 + v_0 = 2v_1 + v_0 + v_0$$

or $$v_r = 2\,(v_1 + v_0)$$

Substituting value of v_1 from Eq. (i) we have,

$$v_r = 2\left(1 + \frac{m_2}{m_1}\right)v_0 \quad \textbf{Ans.}$$

EXERCISES

For JEE Main

Subjective Questions

Note *You can take approximations in the answers.*

Reflection from Plane Surface (Plane Mirror)

1. A candle 4.85 cm tall is 39.2 cm to the left of a plane mirror. Where does the mirror form the image, and what is the height of this image?
2. A plane mirror lies face up, making an angle of 15° with the horizontal. A ray of light shines down vertically on the mirror. What is the angle of incidence? What will the angle between the reflected ray and the horizontal be?
3. Two plane mirrors are placed parallel to each other and 40 cm apart. An object is placed 10 cm from one mirror. What is the distance from the object to the image for each of the five images that are closest to the object?
4. If an object is placed between two parallel mirrors, an infinite number of images result. Suppose that the mirrors are a distance $2b$ apart and the object is put at the midpoint between the mirrors. Find the distances of the images from the object.
5. Show that a ray of light reflected from a plane mirror rotates through an angle 2θ when the mirror is rotated through an angle θ about its axis perpendicular to both the incident ray and the normal to the surface.
6. Two plane mirrors each 1.6 m long, are facing each other. The distance between the mirrors is 20 cm. A light incident on one end of one of the mirrors at an angle of incidence of 30°. How many times is the ray reflected before it reaches the other end?
7. Two plane mirrors are inclined to each other at an angle θ. A ray of light is reflected first at one mirror and then at the other. Find the total deviation of the ray.

Reflection from Spherical Surface (Convex and Concave Mirror)

8. An object 6.0 mm is placed 16.5 cm to the left of the vertex of a concave spherical mirror having a radius of curvature of 22.0 cm.
   (a) Draw principal ray diagram showing formation of the image.
   (b) Determine the position, size, orientation, and nature (real or virtual) of the image.
9. An object 9.0 mm tall is placed 12.0 cm to the left of the vertex of a convex spherical mirror whose radius of curvature has a magnitude of 20.0 cm.
   (a) Draw a principal ray diagram showing formation of the image.
   (b) Determine the position, size, orientation, and nature (real or virtural) of the image.
10. How far should an object be from a concave spherical mirror of radius 36 cm to form a real image one-ninth its size?

11. An object is 30.0 cm from a spherical mirror, along the central axis. The absolute value of the lateral magnification is $\frac{1}{2}$. The image produced is inverted. What is the focal length of the mirror?

12. A concave mirror has a radius of curvature of 24 cm. How far is an object from the mirror if an image is formed that is
(a) virtual and 3.0 times the size of the object,
(b) real and 3.0 times the size of the object and
(c) real and one-third the size of the object?

13. As the position of an object in front of a concave spherical mirror of 0.25 m focal length is varied, the position of the image varies. Plot the image distance as a function of the object distance letting the latter change from 0 to $+\infty$. Where is the image real? Where virtual?

14. An object is placed 42 cm, in front of a concave mirror of focal length 21 cm. Light from the concave mirror is reflected onto a small plane mirror 21 cm in front of the concave mirror. Where is the final image?

15. Prove that for spherical mirrors the product of the distance of the object and the image to the principal focus is always equal to the square of the principal focal length.

16. Convex and concave mirrors have the same radii of curvature R. The distance between the mirrors is $2R$. At what point on the common optical axis of the mirrors should a point source of light A be placed for the rays to coverage at the point A after being reflected first on the convex and then on the concave mirror?

Objective Questions

Single Correct Option

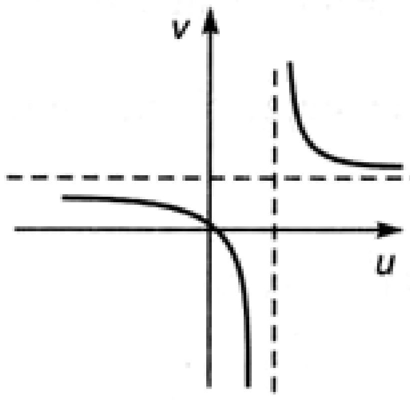
1. A plane mirror reflects a beam of light to form a real image. The incident beam should be
(a) parallel
(b) convergent
(c) divergent
(d) not possible

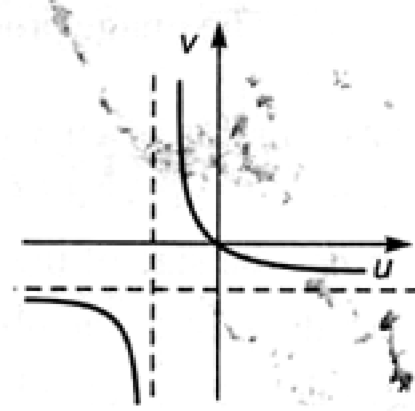
2. When an object lies at the focus of a concave mirror, then the position of the image formed and its magnification are
(a) pole and unity
(b) infinity and unity
(c) infinity and infinity
(d) centre of curvature and unity

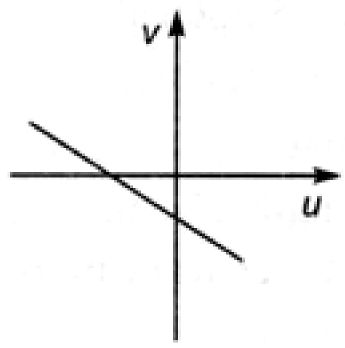
3. Two plane mirrors are inclined to each other at 90°. A ray of light is incident on one mirror. The ray will undergo a total deviation of
(a) 180°
(b) 90°
(c) 45°
(d) Data insufficient

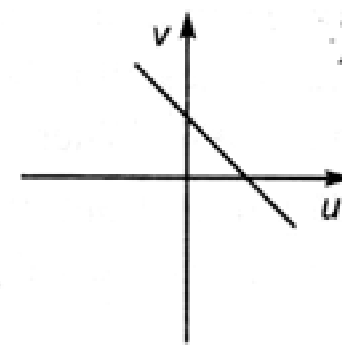
4. A concave mirror cannot form
(a) virutal image of virtual object
(b) virtual image of real object
(c) real image of real object
(d) real image of virtual object

5. Which of the following is correct graph between u and v for a concave mirror for normal sign convention?

(a) 

(b) 

(c) 

(d) 

6. Two plane mirrors are inclined at 70°. A ray incident on one mirror at incidence angle θ, after reflection falls on the second mirror and is reflected from there parallel to the first mirror. The value of θ is
(a) 50° (b) 45° (c) 30° (d) 25°

7. The radius of curvature of a convex mirror is 60 cm. When an object is placed at A its image is formed at B. If the size of image is half that of the object, then the distance between A and B is
(a) 30 cm (b) 60 cm (c) 45 cm (d) 90 cm

8. A boy of height 1.5 m with his eye level at 1.4 m stands before a plane mirror of length 0.75 m fixed on the wall. The height of the lower edge of the mirror above the floor is 0.8 m. Then
(a) the boy will see his full image
(b) the boy cannot see his hair
(c) the body cannot see his feet
(d) the boy can see neither his hair nor his feet

9. A spherical mirror forms an erect image three times the size of the object. If the distance between the object and the image is 80 cm, the nature and the focal length of the mirror are
(a) concave, 30 cm
(b) convex, 30 cm
(c) concave, 15 cm
(d) convex, 15 cm

10. A convex mirror of focal length f produces an image $(1/n)^{\text{th}}$ of the size of the object. The distance of the obejct from the mirror is
(a) nf (b) f/n (c) $(n+1)\,f$ (d) $(n-1)\,f$

11. An object is moving towards a concave mirror of focal length 24 cm. When it is at a distance of 60 cm from the mirror its speed is 9 cm/s. The speed of its image at that instant, is
(a) 4 cm/s towards the mirror
(b) 6 cm/s towards the mirror
(c) 4 cm/s away from the mirror
(d) 6 cm/s away from the mirror

12. All the following statements are correct except (for real objects)
(a) the magnification produced by a convex mirror is always less than one
(b) a virtual, erect and same sized image can be obtained using a plane mirror
(c) a virtual, erect, magnified image can be formed using a concave mirror
(d) a real, inverted same sized image can be formed using a convex mirror

13. A particle moves perpendicularly towards a plane mirror with a constant speed of 4 cm/s. What is the speed of the image observed by an observer moving with 2 cm/s along the same direction? Mirror is also moving with a speed of 10 cm/s in the opposite direction.
(All speeds are with respect to ground frame of reference)
(a) 4 cm/s (b) 12 cm/s (c) 14 cm/s (d) 26 cm/s

For **JEE Advanced**

Assertion and Reason

Directions : *Choose the correct option.*

(*a*) *If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion**.*
(*b*) *If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion**.*
(*c*) *If **Assertion** is true, but the **Reason** is false.*
(*d*) *If **Assertion** is false but the **Reason** is true.*

1. Assertion : A convex mirror can never make a real image.
Reason : For all real objects image formed by a convex mirror is virtual.

2. Assertion : Focal length of a convex mirror is 20 cm. If a real object is placed at distance 20 cm from the mirror, its virtual erect and diminished image will be formed.
Reason : If a virtual object is placed at 20 cm distance its image is formed at infinity.

3. Assertion : In case of concave mirror if a point object is moving towards the mirror along its principal axis, then its image will move away from the mirror.
Reason : In case of reflection (along the principal axis of mirror) object and image always travel in opposite directions.

4. Assertion : Real view mirror of vehicles is a convex mirror.
Reason : It never makes real image of real objects.

5. Assertion : If magnification of a real object is -2. Then it is definitely a concave mirror.
Reason : Only concave mirror can make real images of real objects.

6. Assertion : Any ray of light suffers a deviation of $(180°-2i)$ after one reflection.
Reason : For normal incidence of light deviation is zero.

7. Assertion : Two plane mirrors kept at right angles deviate any ray of light by 180° after two reflections.
Reason : The above condition is satisfied only for angle of incidence $i = 45°$.

8. Assertion : In reflection from a denser medium, any ray of light suffers a phase difference of π.
Reason : Denser medium is that medium in which speed of wave is less.

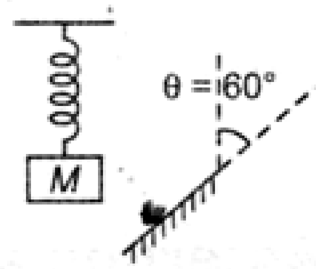
9. Assertion : For real objects, image formed by a convex mirror always lies between pole and focus.
Reason : When object moves from pole to infinity its image will move from pole to focus.

10. Assertion : Light converges on a virutal object.
Reason : Virtual obejct is always behind a mirror.

Objective Questions

Single Option Correct

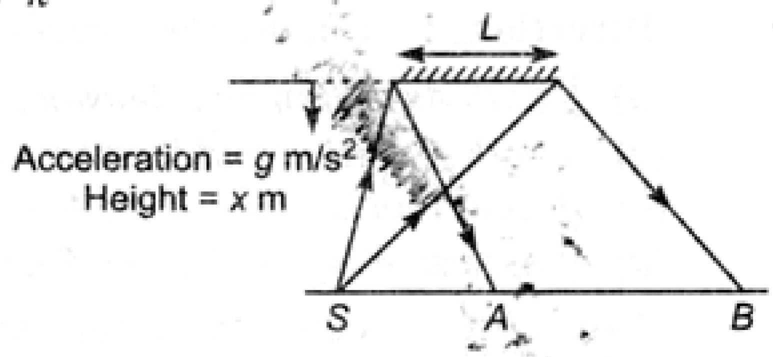
1. An insect of negligible mass is sitting on a block of mass M, tied with a spring of force constant k. The block performs simple harmonic motion with amplitude A in front of a plane mirror placed as shown. The maximum speed of insect relative to its image will be



(a) $A\sqrt{\frac{k}{M}}$ (b) $\frac{A\sqrt{3}}{2}\sqrt{\frac{k}{M}}$

(c) $A\sqrt{3}\sqrt{\frac{k}{M}}$ (d) $2A\sqrt{\frac{M}{k}}$

2. A plane mirror is falling vertically as shown in the figure. If S is a point source of light, the rate of increase of the length AB is

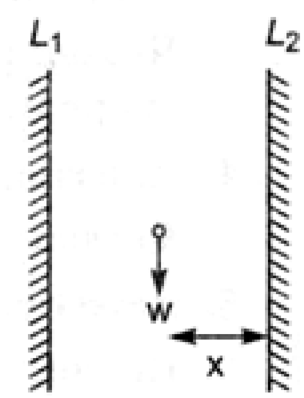


(a) directly proportional to x
(b) constant but not zero
(c) inversely proportional to x
(d) zero

3. A point object is placed at a distance of 10 cm and its real image is formed at a distance of 20 cm from a concave mirror. If the object is moved by 0.1 cm towards the mirror. The image will shift by about

(a) 0.4 cm away from the mirror (b) 0.4 cm towards the mirror
(c) 0.8 cm away from the mirror (d) 0.8 cm towards the mirror

4. Two plane mirrors L_1 and L_2 are parallel to each other and 3 m apart. A person standing x m from the right mirror L_2 looks into this mirror and sees a series of images. The distance between the first and second image is 4 m. Then the value of x is



(a) 2 m (b) 1.5 m
(c) 1 m (d) 2.5 m

5. A piece of wire bent into an L shape with upright and horizontal portion of equal lengths 10 cm each is placed with the horizontal portion along the axis of the concave mirror towards pole of mirror whose radius of curvature is 10 cm. If the bend is 20 cm from the pole of the mirror, then the ratio of the lengths of the images of the upright and horizontal portion of the wire is

(a) 1 : 2 (b) 1 : 3 (c) 1 : 1 (d) 2 : 1

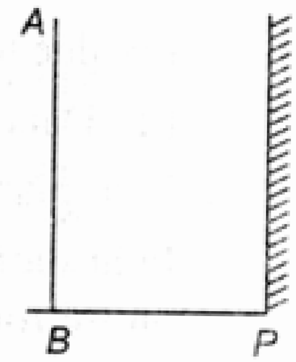
6. A point object at 15 cm from a concave mirror of radius of curvature 20 cm is made to oscillate along the principal axis with amplitude 2 mm. The amplitude of its image will be

(a) 2 mm (b) 4 mm (c) 8 mm (d) None of these

7. A ray of light falls on a plane mirror. When the mirror is turned, about an axis at right angles to the plane of mirror by 20° the angle between the incident ray and new reflected ray is 45°. The angle between the incident ray and original reflected ray was therefore

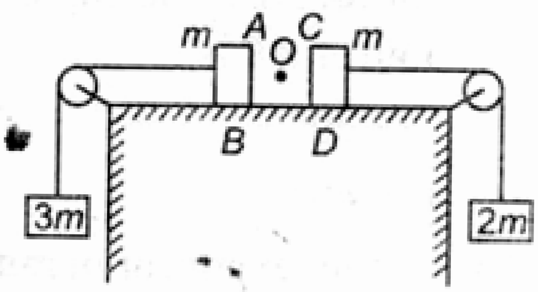
(a) 35° or 50° (b) 25° or 65° (c) 85° or 5° (d) 75° or 30°

8. A person AB of height 170 cm is standing in front of a plane mirror. His eyes are at height 164 cm. At what distance from P should a hole be made in mirror so that he cannot see his hair



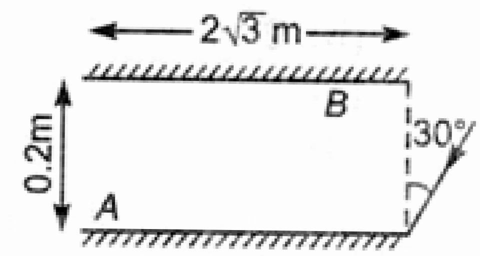
(a) 167 cm
(b) 161 cm
(c) 163 cm
(d) 165 cm

9. Two blocks each of masses m lie on a smooth table. They are attached to two other masses as shown in figure. The pulleys and strings are light. An object O is kept at rest on the table. The sides AB and CD of the two blocks are made reflecting. The acceleration of two images formed in those two reflecting surfaces with respect to each other is



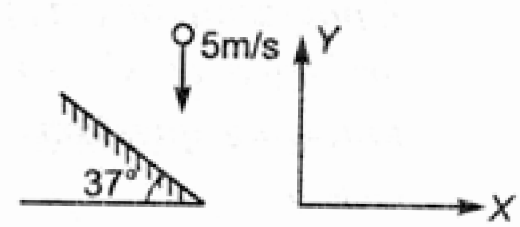
(a) $\frac{5g}{6}$
(b) $\frac{5g}{3}$
(c) $\frac{17g}{12}$
(d) $\frac{17g}{6}$

10. Two plane mirrors A and B are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle of 30° at a point just inside one end of A. The number of times the ray undergoes reflections (including the first one) before it emerges out is



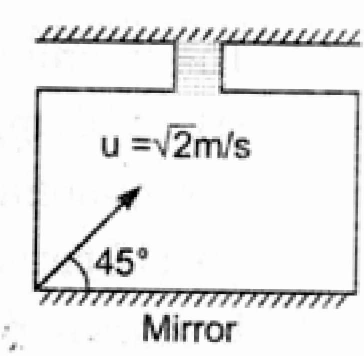
(a) 29
(b) 30
(c) 31
(d) 32

11. An object O is just about to strike a perfectly reflecting inclined plane of inclination 37°. Its velocity is 5 m/s. Find the velocity of its image.



(a) $3\hat{i}+4\hat{j}$
(b) $4\hat{i}+3\hat{j}$
(c) $4.8\hat{i}+1.4\hat{j}$
(d) $1.4\hat{i}+4.8\hat{j}$

12. An elevator at rest which is at 10th floor of a building is having a plane mirror fixed to its floor. A particle is projected with a speed $\sqrt{2}$ m/s and at 45° with the horizontal as shown in the figure. At the very instant of projection, the cable of the elevator breaks and the elevator starts falling freely. What will be the separation between the particle and is image 0.5 s after the instant of projection?



(a) 0.5 m
(b) 1 m
(c) 2 m
(d) 1.5 m

13. A plane mirror is moving with velocity $4\hat{i}+4\hat{j}+8\hat{k}$. A point object in front of the mirror moves with a velocity $3\hat{i}+4\hat{j}+5\hat{k}$. Here $\hat{k}$ is along the normal to the plane mirror and facing towards the object. The velocity of the image is

(a) $-3\hat{i}-4\hat{j}+5\hat{k}$
(b) $3\hat{i}+4\hat{j}+11\hat{k}$
(c) $-4\hat{i}+5\hat{j}+11\hat{k}$
(d) $7\hat{i}+9\hat{j}+3\hat{k}$

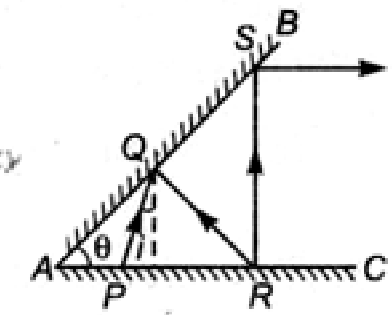
14. Point A (0, 1 cm) and B (12 cm, 5 cm) are the coordiantes of object and image. x-axis is the principle axis of the mirror. Then this object image pair is

(a) due to a convex mirror of focal length 2.5 cm
(b) due to a concave mirror having its pole at (2 cm, 0)

(c) due to a concave mirror having its pole at (– 2 cm, 0)

(d) Data is insufficient

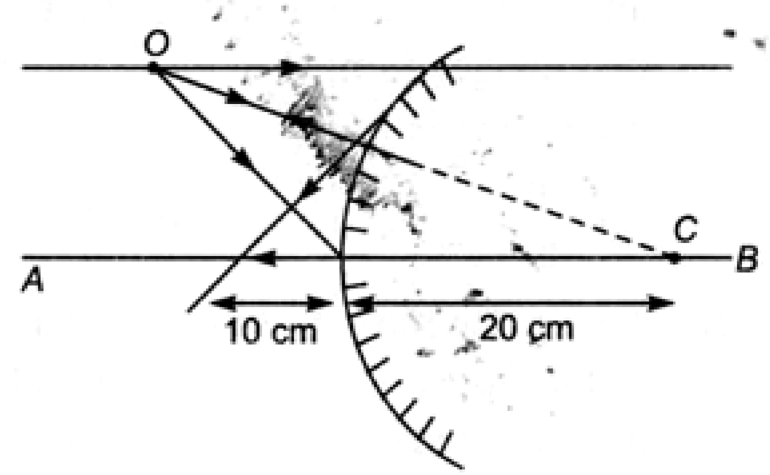
15. Two plane mirrors AB and AC are inclined at an angle $\theta = 20°$. A ray of light starting from point P is incident at point Q on the mirror AB, then at R on mirror AC and again on S on AB. Finally the ray ST goes parallel to mirror AC. The angle which the ray makes with the normal at point Q on mirror AB is



(a) 20° (b) 30°

(c) 40° (d) 60°

16. A convex mirror of radius of curavature 20 cm is shown in figure. An object O is placed in front of this mirror. Its ray diagram is shown. How many mistakes are there in the ray diagram (AB is principal axis)



(a) 3

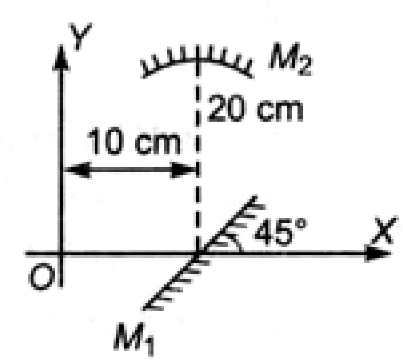
(b) 2

(c) 1

(d) 0.

Passage : (Q. 17 to Q. 20)

A plane mirror (M_1) *and a concave mirror* (M_2) *of focal length* 10 *cm are arranged as shown in figure. An object is kept at origin. Answer the following questions. (consider image formed by single reflection in all cases)*



17. The co-ordinates of image formed by plane mirror are

(a) (– 20 cm, 0) (b) (10 cm, – 60 cm) (c) (10 cm, – 10 cm) (d) (10 cm, 10 cm)

18. The co-ordinates of image formed by concave mirror are

(a) (10 cm, – 40 cm) (b) (10 cm, – 60 cm) (c) (10 cm, 8 cm) (d) None of these

19. If concave mirror is replaced by convex mirror of same focal legnth, then coordinates of image formed by M_2 will be

(a) (10 cm, 12 cm) (b) (10 cm, 22 cm) (c) (10 cm, 8 cm) (d) None of these

20. If concave mirror is replaced by another plane mirror peralllel to x-axis then co-ordinates of image formed by M_2 are

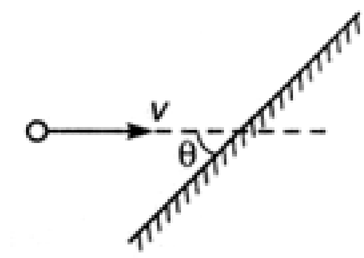
(a) (40 cm, 20 cm) (b) (20 cm, 40 cm) (c) (– 20 cm, 20 cm) (d) None of these

More than One Correct Options

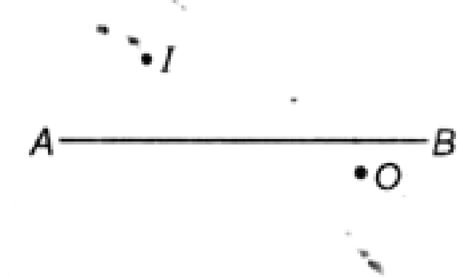
1. The image formed by a concave mirror is twice the size of the object. The focal length of the mirror is 20 cm. The distance of the object from the mirror is/are

(a) 10 cm (b) 30 cm (c) 25 cm (d) 15 cm

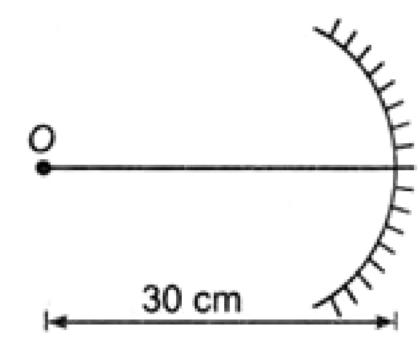
2. Magnitude of focal length of a spherical mirror is f and magnitude of linear magnification is $\frac{1}{2}$
   (a) If image is inverted it is a concave mirror (b) If image is erect, it is a convex mirror
   (c) Object distance from the mirror may be $3f$ (d) Object distance from the mirror may be f
3. A point object is moving towards a plane mirror as shown in figure. Choose the correct options.
   (a) Speed of image is also v
   (b) Image velocity will also make an angle θ with mirror
   (c) Relative velocity between object and image is $2v$
   (d) Relative velocity between object and image is $2v \sin\theta$



4. AB is the principal axis of a spherical mirror. I is the point image corresponding to point object O. Choose the correct options
   (a) Mirror is lying to the right hand side of O
   (b) Focus of mirror is lying to the right hand side of O
   (c) Centre of curvature of mirror is lying to the right hand side of O
   (d) Centre of curvature of mirror is lying between I and O



5. A point object is placed on the principal axis of a concave mirror of focal length − 20 cm. At this instant object is given a velocity v towards the axis (event-1) or perpendicular to axis (event-2). Then speed of image
   (a) In event-1 is $2v$ (b) In event-1 is $4v$
   (c) In event-2 is $2v$ (d) In event-2 is $4v$



6. A point object is placed at equal distance $3f$ infront of a concave mirror, a convex mirrror and a plane mirror separately (event-1). Now the distance is decreased to $1.5\ f$ from all the three mirrors (event-2). Magnitude of focal length of convex mirror and concave mirror is f. Then choose the correct options.
   (a) Maximum distance of object in event-1 from the mirror is from plane mirror
   (b) Minimum distance of object in event-1 from the mirror is from convex mirror
   (c) Maximum distance of object in event-2 from the mirror is from concave mirror
   (d) Minimum distance of object in event-2 from the mirror is from plane mirror

Match the Columns

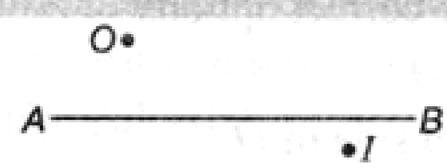
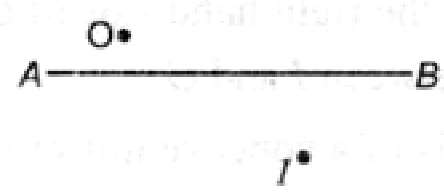
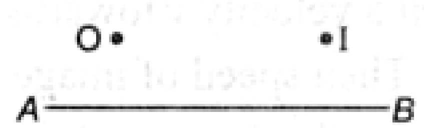
1. For real objects match the following two columns corresponding to linear magnification m given in column I.

Column I	Column II
(a) $m = -2$	(p) convex mirror
(b) $m = -\frac{1}{2}$	(q) concave mirror
(c) $m = +2$	(r) real image
(d) $m = +\frac{1}{2}$	(s) virtual image

2. For virtual objects match the following two columns.

Column I	Column II
(a) Plane mirror	(p) only real image
(b) Convex mirror	(q) only virtual image
(c) Concave mirror	(r) may be real or virtual image

3. Principal axis of a mirror (AB), a point object O and its image I are shown in column-I. Match it with column-II

Column I	Column II
(a) 	(p) plane mirror
(b) 	(q) convex mirror
(c) 	(r) concave mirror
(d) 	(s) Not possible

Note *All objects in column I are real.*

4. Focal length of a concave mirror M_1 is -20 cm and focal length of a convex mirror M_2 is $+20$ cm. A point object is placed at distance X in front of M_1 or M_2. Match the following two columns.

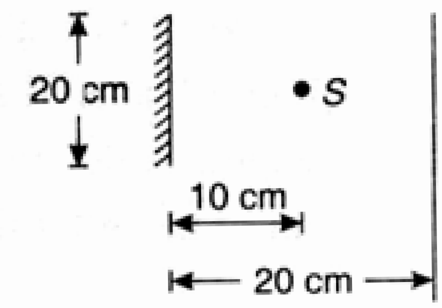
Column I	Column II
(a) $X = 20$, mirror is M_1	(p) image at infinity
(b) $X = 20$, mirror is M_2	(q) image is real
(c) $X = 30$, mirror is M_1	(r) image is virtual
(d) $X = 30$, mirror is M_2	(s) image is magnified

5. Focal length of a concave mirror is -20 cm. Match the object distance given in column II corresponding to magnification (only magnitude) given is column I.

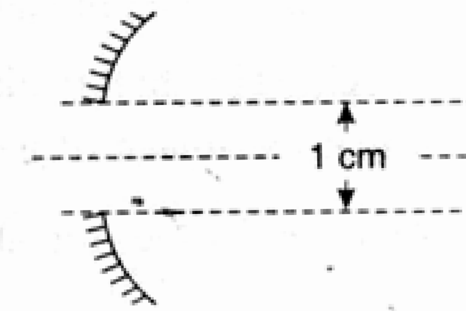
Column I	Column II
(a) 2	(p) 10 cm
(b) 1/2	(q) 30 cm
(c) 1	(r) 20 cm
(d) 1/4	(s) None of these

Subjective Questions

1. A point source of light S is placed at a distance 10 cm infront of the centre of a mirror of width 20 cm suspended vertically on a wall. An insect walks with a speed 10 cm/s infront of the mirror along a line parallel to the mirror at a distance 20 cm from it as shown in figure. Find the maximum time during which the insect can see the image of the source S in the mirror.

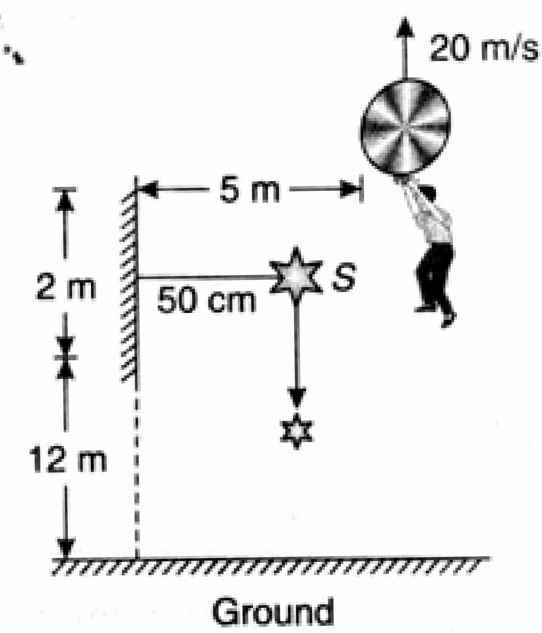


2. A concave mirror forms the real image of a point source lying on the optical axis at a distance of 50 cm from the mirror. The focal length of the mirror is 25 cm. The mirror is cut in two and its halves are drawn a distance of 1 cm apart in a direction perpendicular to the optical axis. How will the image formed by the halves of the mirror be arranged?

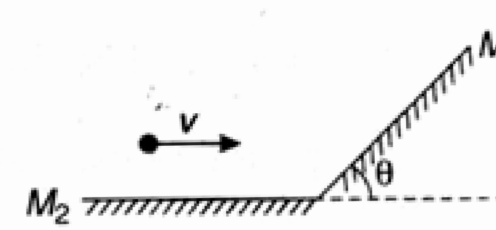


3. A point source of light S is placed on the major optical axis of the concave mirror at a distance of 60 cm. At what distance from the concave mirror should a flat mirror be placed for the rays to converage again at the point S having been reflected from the concave mirror and then from the flat one? Will the position of the point where the rays meet change if they are first reflected from the flat mirror? The radius of the concave mirror is 80 cm.

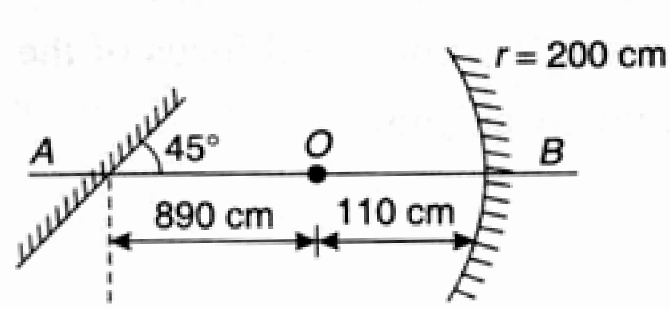
4. A balloon is moving upwards with a speed of 20 m/s. When it is at a height of 14 m from ground infront of a plane mirror in situation as shown in figure, a boy drops himself from the balloon. Find the time duration for which he will see the image of source S placed symmetrically before plane mirror during free fall.



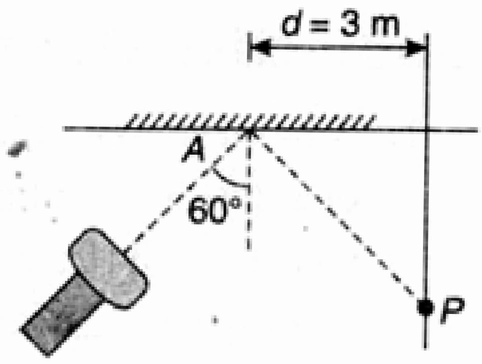
5. A point object is moving with a speed v before an arrangement of two mirrors as shown in figure. Find the velocity of image in mirror M_1 with respect to image in mirror M_2.



6. A plane mirror and a concave mirror are arranged as shown in figure and O is a point object. Find the position of image formed by two reflections, first one taking place at concave mirror.



7. Figure shows a torch producing a straight light beam falling on a plane mirror at an angle 60°. The reflected beam makes a spot P on the screen along Y-axis. If at $t = 0$, mirror starts rotating about the hinge A with an angular velocity $\omega = 1°$ per second clockwise. Find the speed of the spot on screen after time $t = 15$ s.



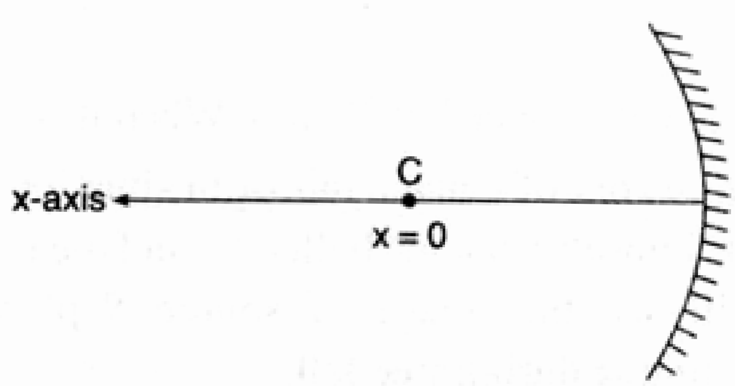
8. A thief is running away in a car with velocity of 20 m/s. A police jeep is following him, which is sighted by thief in his rear view mirror, which is a convex mirror of focal length 10 m. He observes that the image of jeep is moving towards him with a velocity of 1 cm/s. If the magnification of mirror for the jeep at that time is $\frac{1}{10}$. Find :

(a) the actual speed of jeep,

(b) rate at which magnification is changing.

Assume the police jeep is on axis of the mirror.

9. A ball swings back and forth infront of a concave mirror. The motion of the ball is described approximately by the equation $x = f \cos \omega t$, where f is the focal length of the mirror and x is measured along the axis of mirror. The origin is taken at the centre of curvature of the mirror.

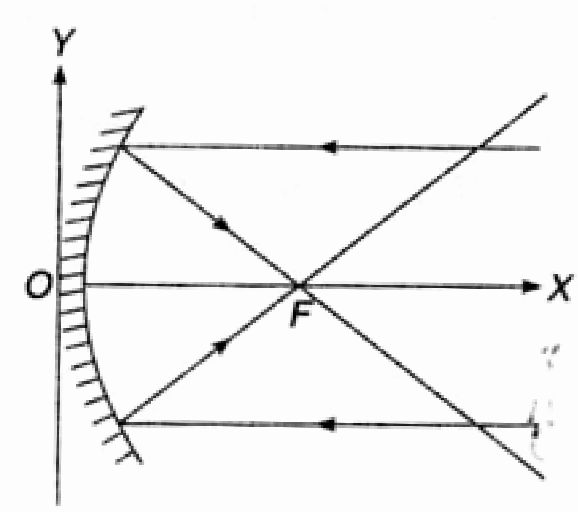


(a) Derive an expression for the distance from the mirror of the image of the swinging ball.

(b) At what point does the ball appear to coincide with its image?

(c) What will be the lateral magnification of the image of the ball at time $t = \frac{T}{2}$, where T is time period of oscillation?

10. Show that a parallel bundle of light rays parallel to the x-axis and incident on a parabolic reflecting surface given by $x = 2by^2$, will pass through a single point called focus of the reflecting surface. Also, find the focal length.



ANSWERS

Introductory Exercise 27.1

2. (a) 1.26 cm, 2.39×10^{10} Hz (b) $E_z = (60\text{ V/m}) \sin(500x + 1.5 \times 10^{11} t)$

Introductory Exercise 27.2

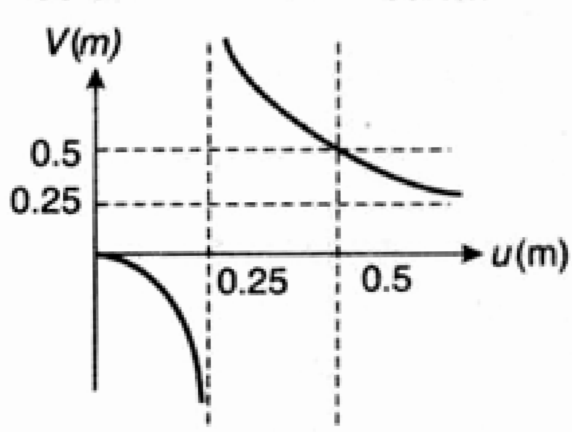
2. 4 m/s 3. 10 m, 16 m

Introductory Exercise 27.3

1. (a) – 16.7 cm, real (b) ∞ (c) + 10.0 cm, virtual
2. (a) A real image moves from – 0.6 m to – ∞, then a virtual image moves from + ∞ to 0.
   (b) 0.639 s and 0.782 s.
3. (a) A concave mirror with radius of curvature 2.08 m (b) 1.25 m from the object.
4. Plane mirror

For JEE Main

Subjective Questions

1. 39.2 cm to the right of mirror, 4.85 cm 2. 15°, 60°
3. 20 cm, 60 cm, 80 cm, 100 cm, 140 cm
4. The images are at $2nb$ from the object with n an integer. 6. 14 7. $360° - 2\theta$
8. (b) 33.0 cm to the left of vertex 1.20 cm tall, inverted, real
9. (b) 5.46 cm to the right of vertex, 4.09 mm tall, erect, virtual
10. 180 cm 11. $f = -10$ cm 12. (a) 8 cm (b) 16 cm (c) 48 cm
13. 

14. 21 cm in front of plane mirror 16. At a distance $\left(\frac{\sqrt{3}+1}{2}\right)R$ from convex mirror

Objective Questions

1. (b) 2. (c) 3. (a) 4. (a) 5. (b) 6. (a) 7. (c) 8. (c) 9. (a) 10. (d)
11. (c) 12. (d) 13. (d)

For JEE Advanced

Assertion and Reason

1. (d) 2. (b) 3. (a) 4. (b) 5. (a or b) 6. (c) 7. (c) 8. (b) 9. (a,b) 10. (b)

Objective Questions

1.(c) 2.(d) 3.(a) 4.(c) 5.(c) 6.(c) 7.(c) 8.(a) 9.(c) 10.(c)
11.(c) 12.(b) 13.(b) 14.(b) 15.(b) 16.(b) 17.(c) 18.(d) 19.(d) 20.(d)

More than One Correct Options

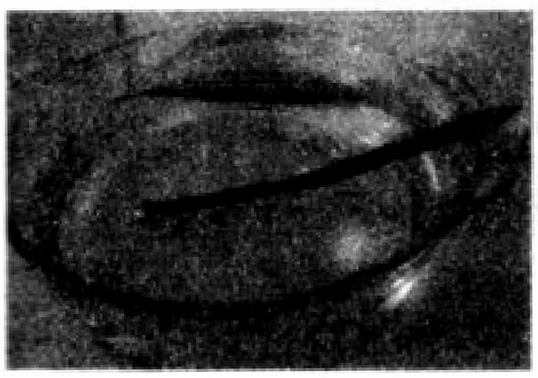
1.(a,b) 2.(a,b,c,d) 3.(a,b,d) 4.(a,b,d) 5.(b,c) 6.(a,b,c)

Match the Columns

1. (a) → q,r (b) → q,r (c) → q,s (d) → p,s
2. (a) → p (b) → r (c) → p
3. (a) → r (b) → r (c) → p (d) → r
4. (a) → p,s (b) → r (c) → q (d) → r
5. (a) → p,q (b) → s (c) → s (d) → s

Subjective Questions

1. 6 s 2. At a distance of 50 cm from mirror and 2 cm from each other
3. 90 cm, Yes 4. 1.7 s 5. $2v\sin\theta$ 6. 100 cm vertically below A 7. $\frac{2\pi}{15}$ m/s
8. (a) 21 m/s (b) 10^{-3}/s 9. (a) Distance = $\left(\frac{2+\cos\omega t}{1+\cos\omega t}\right)f$ (b) At $x = 0$ (c) $m = \infty$
10. $f = \frac{1}{8b}$



28

REFRACTION OF LIGHT

Chapter Contents

28.1 Introduction
28.2 Thin Lenses
28.3 Total Internal Reflection (TIR)
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28.1 Refraction of Light

(a) Laws of Refraction

When light passes from one medium, say air, to another, say glass, part is reflected back into the first medium and the rest passes into the second medium. When it passes into the second medium, its direction of travel is changed. It either bends towards the normal or away from the normal. This phenomenon is known as **refraction**. *There are two laws of refraction:*

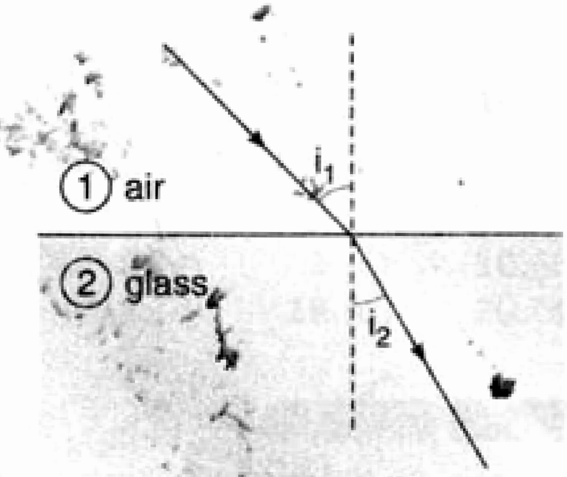

Fig. 28.1

1. For two particular media, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant, *i.e.*,

$$\frac{\sin i_1}{\sin i_2} = \text{constant} \quad \text{...(i)}$$

This is known as **Snell's law**.

2. The incident ray, the reflected ray and the refracted ray all lie in the same plane.

Note *(i) The constant ratio* $\frac{\sin i_1}{\sin i_2}$ *is called the* ***refractive index*** *for light passing from the first to the second medium (or refractive index of 2 with respect to 1). It is denoted by* ${}_1\mu_2$. *Thus,*

$${}_1\mu_2 = \frac{\sin i_1}{\sin i_2} \quad \text{...(ii)}$$

(ii) If medium 1 is a vacuum (or, in practice air) we refer ${}_1\mu_2$ *as the* ***absolute refractive index*** *of medium 2 and denote it by* μ_2 *or simply* μ *(if no other medium is there).*

(iii) Now, we can write Snell's law as,

$$\mu \sin i = \text{constant} \quad \text{...(iii)}$$

For two media,

$$\mu_1 \sin i_1 = \mu_2 \sin i_2$$

or

$$\frac{\mu_2}{\mu_1} = \frac{\sin i_1}{\sin i_2} = {}_1\mu_2 \quad \text{...(iv)}$$

(iv) From Eq. (iii) we can see that $i_1 > i_2$ *if* $\mu_2 > \mu_1$, *i.e., if a ray of light passes from a rare to a denser medium it bends towards normal and vice-versa.*

(v) Eq. (iv) can be written as,

$${}_1\mu_2 = \frac{\sin i_1}{\sin i_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{\mu_2}{\mu_1} \quad \text{...(v)}$$

Here, v_1 is the speed of light in medium 1 and v_2 in medium 2. Similarly, λ_1 and λ_2 are the corresponding wavelengths.

If $\mu_2 > \mu_1$ then $v_1 > v_2$ and $\lambda_1 > \lambda_2$, *i.e.*, in a rare medium speed and hence, wavelength of light is more.

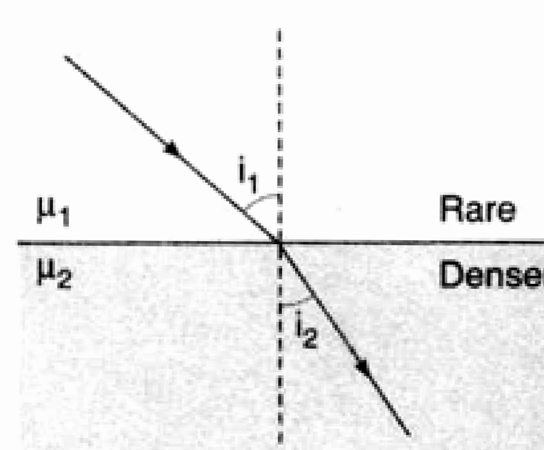

Fig. 28.2

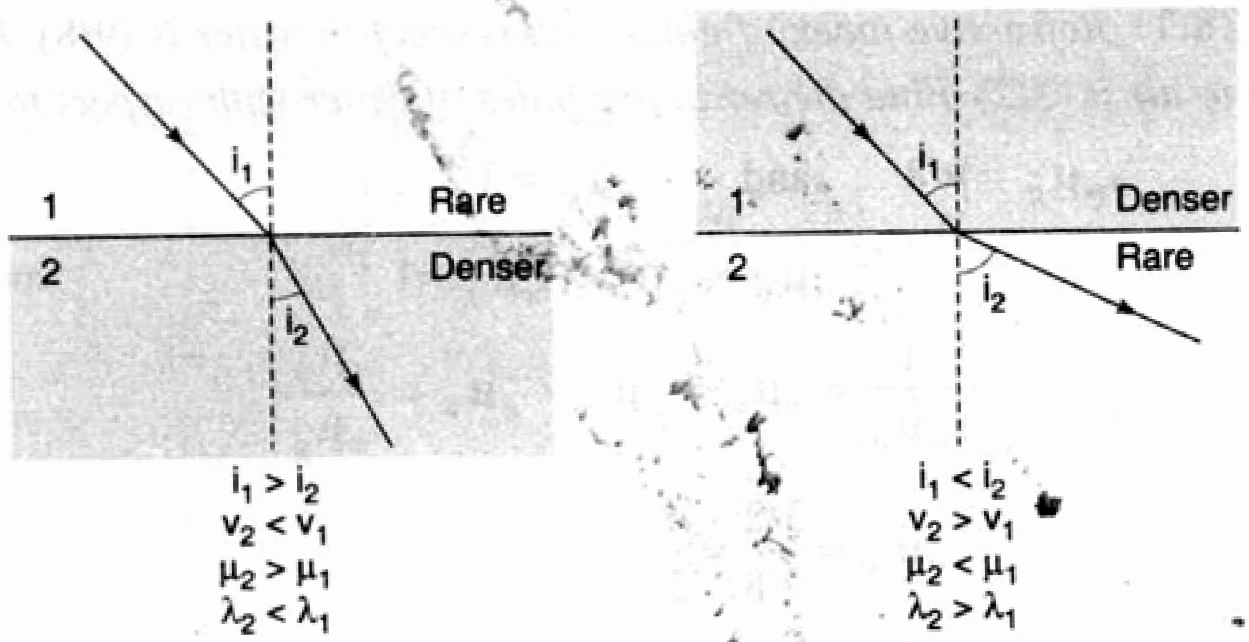

Fig. 28.3

(vi) In general speed of light in any medium is less than its speed in vacuum. It is convenient to define refractive index μ of a medium as,

$$\mu = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}} = \frac{c}{v}$$

(vii) As a ray of light moves from medium 1 to medium 2, its wavelength changes but its frequency remains constant.

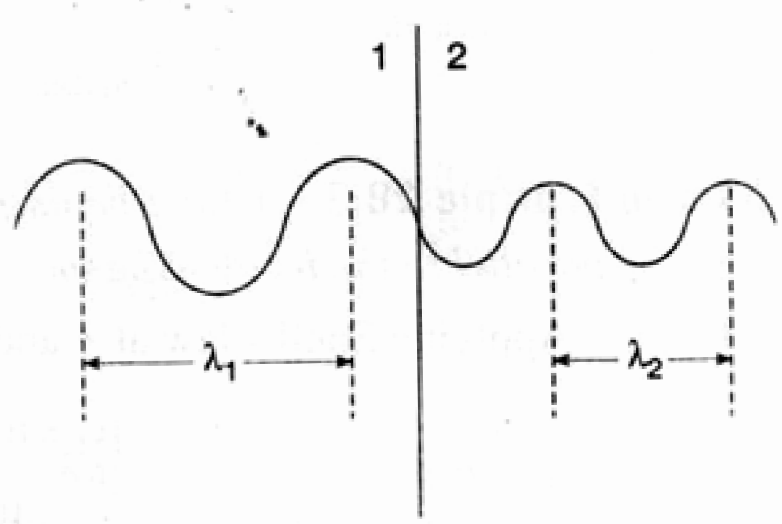

Fig. 28.4

$$\mu_2 > \mu_1, v_1 > v_2, \lambda_1 > \lambda_2$$

(viii) ${}_1\mu_2 = \frac{\mu_2}{\mu_1}$ and ${}_2\mu_1 = \frac{\mu_1}{\mu_2}$

$\therefore$ $${}_1\mu_2 = \frac{1}{{}_2\mu_1}$$

(ix) ${}_1\mu_2 = \frac{\mu_2}{\mu_1}$, ${}_2\mu_3 = \frac{\mu_3}{\mu_2}$ and ${}_3\mu_1 = \frac{\mu_1}{\mu_3}$

$\therefore$ $${}_1\mu_2 \times {}_2\mu_3 \times {}_3\mu_1 = 1$$

(x) Experiments show that if the boundaries of the media are **parallel** the emergent ray *CD* although laterally displaced, is parallel to the incident ray *AB* if $\mu_1 = \mu_5$. We can also directly apply the Snell's law ($\mu \sin i$ = constant) in medium 1 and 5, *i.e.*,

$$\mu_1 \sin i_1 = \mu_5 \sin i_5$$

So, $i_1 = i_5$ if $\mu_1 = \mu_5$

If any of the boundary is not parallel we cannot use this law directly by jumping the intervening media.

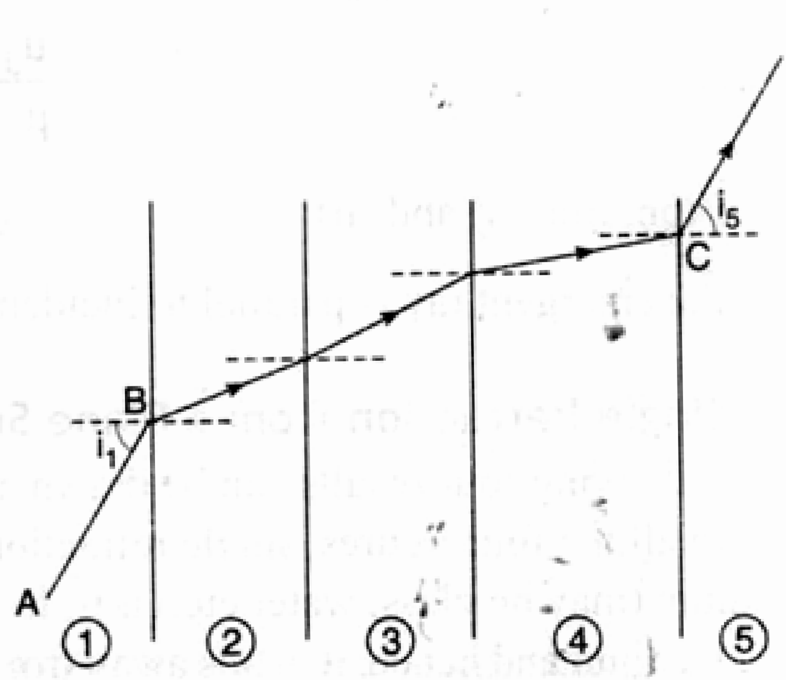

Fig. 28.5

Sample Example 28.1 *Refractive index of glass with respect to water is (9/8). Refractive index of glass with respect to air is (3/2). Find the refractive index of water with respect to air.*

Solution Given, ${}_w\mu_g = 9/8$ and ${}_a\mu_g = 3/2$

As,

$$ {}_a\mu_g \times {}_g\mu_w \times {}_w\mu_a = 1 $$

$$ \therefore \quad \frac{1}{{}_w\mu_a} = {}_a\mu_w = {}_a\mu_g \times {}_g\mu_w = \frac{{}_a\mu_g}{{}_w\mu_g} $$

$$ \therefore \quad {}_a\mu_w = \frac{3/2}{9/8} = \frac{4}{3} $$ **Ans.**

Sample Example 28.2 *(a) Find the speed of light of wavelength $\lambda = 780$ nm (in air) in a medium of refractive index $\mu = 1.55$.*
(b) What is the wavelength of this light in the given medium?

Solution (a)

$$ v = \frac{c}{\mu} = \frac{3.0 \times 10^8}{1.55} = 1.94 \times 10^8 \text{ m/s} $$ **Ans.**

(b)

$$ \lambda_{\text{medium}} = \frac{\lambda_{\text{air}}}{\mu} = \frac{780}{1.55} = 503 \text{ nm} $$ **Ans.**

Sample Example 28.3 *A light beam passes from medium 1 to medium 2. Show that the emerging beam is parallel to the incident beam.*

Solution Applying Snell's law at *A* and *B*,

$$ \mu_1 \sin i_1 = \mu_2 \sin i_2 $$

or

$$ \frac{\mu_1}{\mu_2} = \frac{\sin i_2}{\sin i_1} \qquad \ldots\text{(i)} $$

Similarly

$$ \mu_2 \sin i_2 = \mu_1 \sin i_3 $$

$$ \therefore \quad \frac{\mu_1}{\mu_2} = \frac{\sin i_2}{\sin i_3} \qquad \ldots\text{(ii)} $$

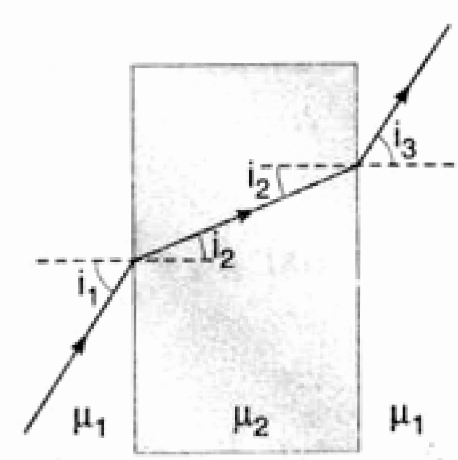

Fig. 28.6

From Eqs. (i) and (ii)

$$ i_3 = i_1 $$

i.e., the emergent ray is parallel to incident ray. **Proved**

(b) Single Refraction from a Plane Surface

Following four results can be drawn after refraction from a plane surface.

In all the four figures, single refraction is taking place through a plane surface. Refractive index of medium (may be glass, water etc.) is μ. In figures (a) and (d) the ray of light is travelling from denser to rare medium and hence, it bends away from the normal. In figures (b) and (c) the ray of light is travelling from a rare to a denser medium and hence, it bends towards the normal. Now, let us take the four figures individually.

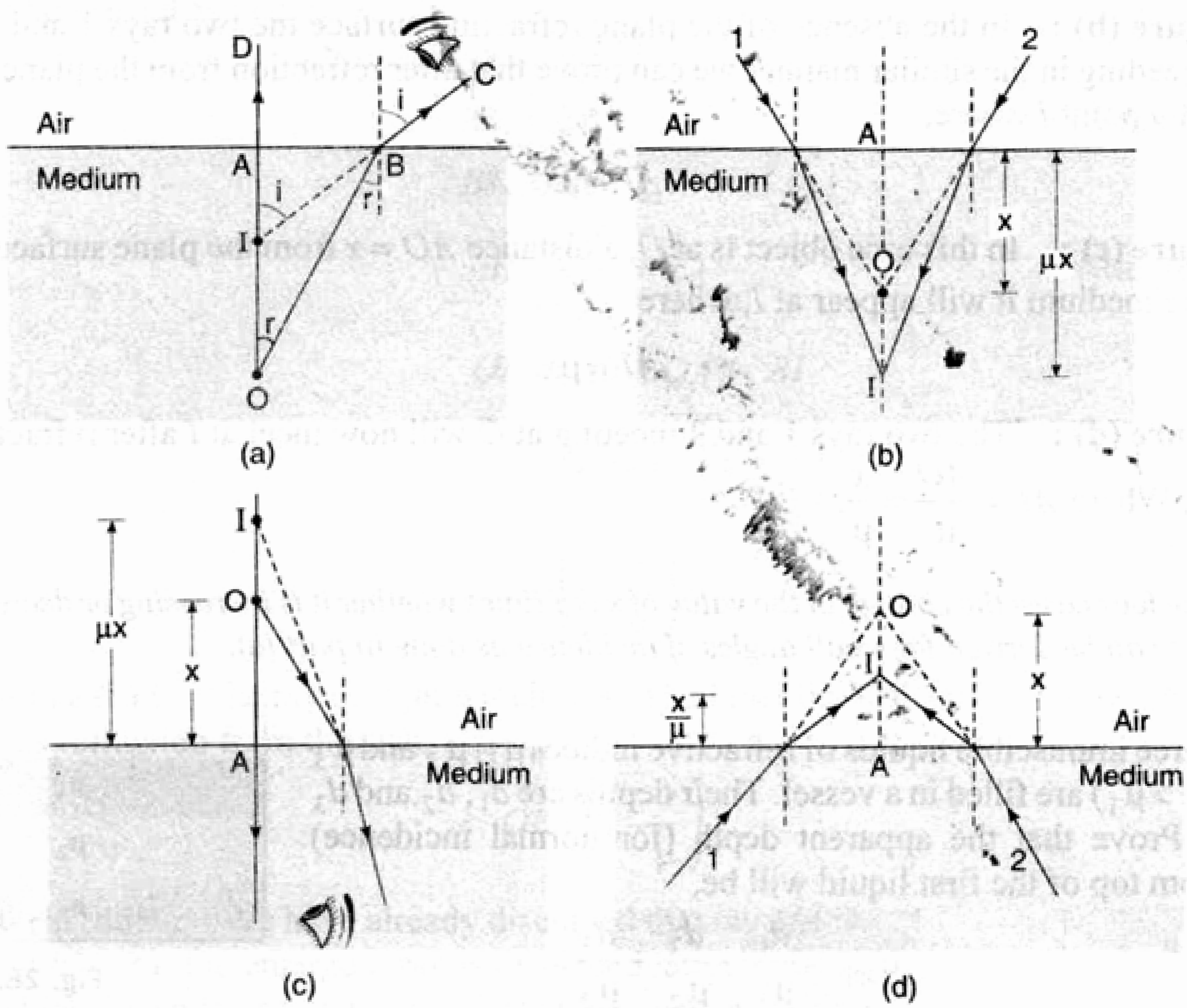

Fig. 28.7

Refer figure (a) : Object O is placed at a distance x from A. Ray OA, which falls normally on the plane surface, passes undeviated as AB. Ray OB, which falls at angle r (with the normal) on the plane surface, bends away from the normal and passes as BC in air. Rays AD and BC meet at I after extending these two rays backwards. Let BC makes an angle i (> r) with normal.

In the figure $\angle AOB$ will be r and $\angle AIB$ is i. For normal incidence (*i.e.*, small angles of i and r)

$$\sin i \approx \tan i = \frac{AB}{AI} \qquad \text{...(i)}$$

and

$$\sin r \approx \tan r = \frac{AB}{AO} \qquad \text{...(ii)}$$

Dividing Eq. (i) by (ii), we have $\dfrac{\sin i}{\sin r} = \dfrac{AO}{AI}$

or

$$\mu = \frac{AO}{AI} \qquad \left(\text{as } \frac{\sin i}{\sin r} = \mu\right)$$

$$\therefore \quad AI = \frac{AO}{\mu} = \frac{x}{\mu}$$

If point O is at a depth of d from a water surface, then the above result is also sometimes written as,

$$d_{\text{apparent}} = \frac{d_{\text{actual}}}{\mu}$$

or the apparent depth is μ times less than the actual depth.

Refer figure (b) : In the absence of the plane refracting surface the two rays 1 and 2 would had met at O. Proceeding in the similar manner we can prove that after refraction from the plane surface they will now meet a point I where,

$$AI = \mu x \qquad \text{(if } AO = x\text{)}$$

Refer figure (c) : In this case object is at O, a distance $AO = x$ from the plane surface. When seen from inside the medium it will appear at I, where

$$AI = \mu x$$

Refer figure (d) : The two rays 1 and 2 meeting at O will now meet at I after refraction from the plane surface. Where $AI = \frac{AO}{\mu} = \frac{x}{\mu}$.

Note *In all the four cases the change in the value of x is μ times whether it is increasing or decreasing. All the relations can be derived for small angles of incidence as done in part (a).*

EXERCISE Three immiscible liquids of refractive indices μ_1, μ_2 and μ_3 (with $\mu_3 > \mu_2 > \mu_1$) are filled in a vessel. Their depths are d_1, d_2 and d_3 respectively. Prove that the apparent depth (for normal incidence) when seen from top of the first liquid will be,

$$d_{app} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3}$$

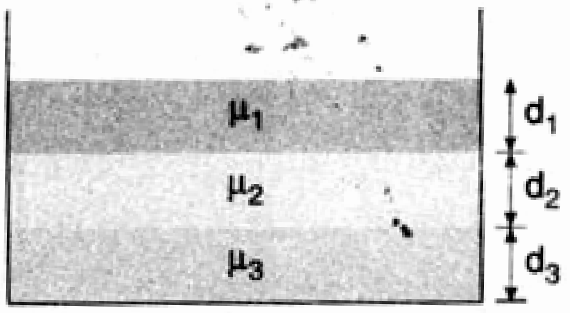

Fig. 28.8

(c) Shift due to a Glass Slab (Double Refraction from Plane Surfaces)

(i) Normal Shift : Here, again two cases are possible.

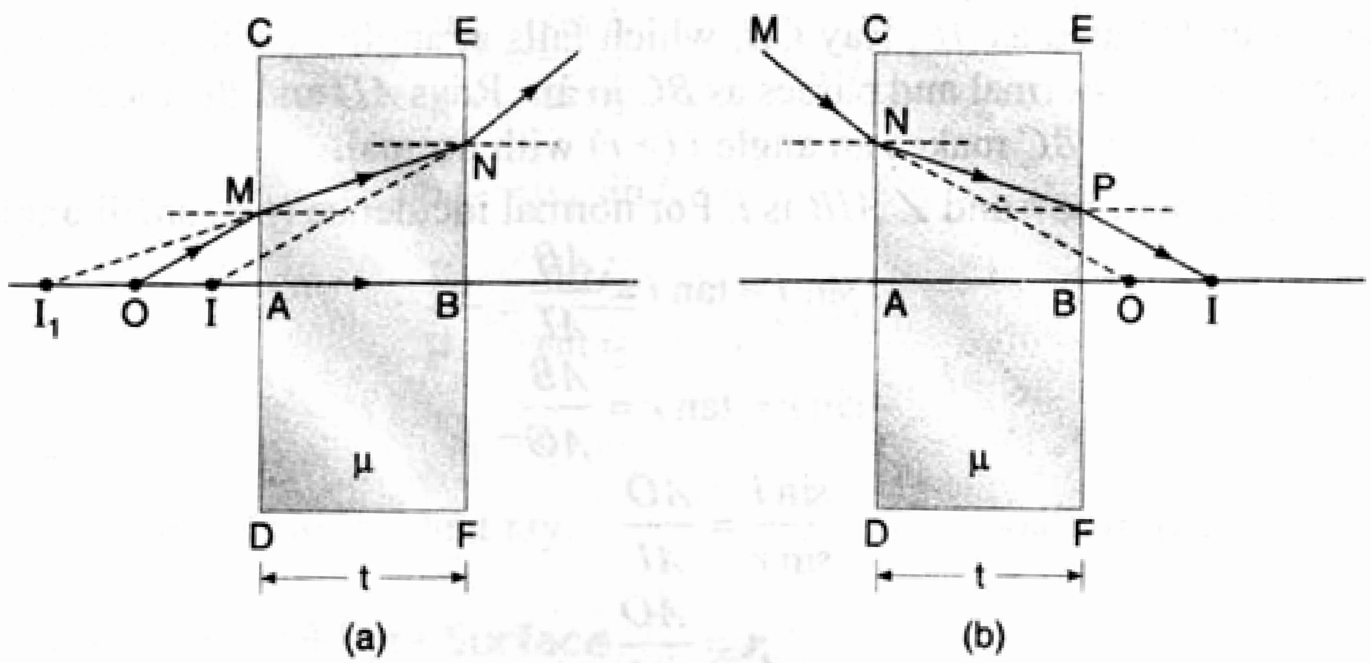

Fig. 28.9

Refer figure (a) : An object is placed at O. Plane surface CD forms its image (virtual) at I_1. This image acts as object for EF which finally forms the image (virtual) at I. Distance OI is called the normal shift and its value is,

$$OI = \left(1 - \frac{1}{\mu}\right)t$$

This can be proved as under:

Let $OA = x$

then $AI_1 = \mu x$ (Refraction from CD)

$$BI_1 = \mu x + t$$

$$BI = \frac{BI_1}{\mu} = x + \frac{t}{\mu} \quad \text{(Refraction from } EF\text{)}$$

$$\therefore \quad OI = (AB + OA) - BI$$

$$= (t + x) - \left(x + \frac{t}{\mu}\right)$$

$$= \left(1 - \frac{1}{\mu}\right)t$$

Hence Proved.

Refer figure (b) : The ray of light which would had met line AB at O will now meet this line at I after two times refraction from the slab. Here

$$OI = \left(1 - \frac{1}{\mu}\right)t$$

(ii) Lateral Shift : We have already discussed that ray MA is parallel to ray BN. But the emergent ray is displaced laterally by a distance d, which depends on μ, t and i and its value is given by the relation,

$$d = t\left(1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}}\right)\sin i$$

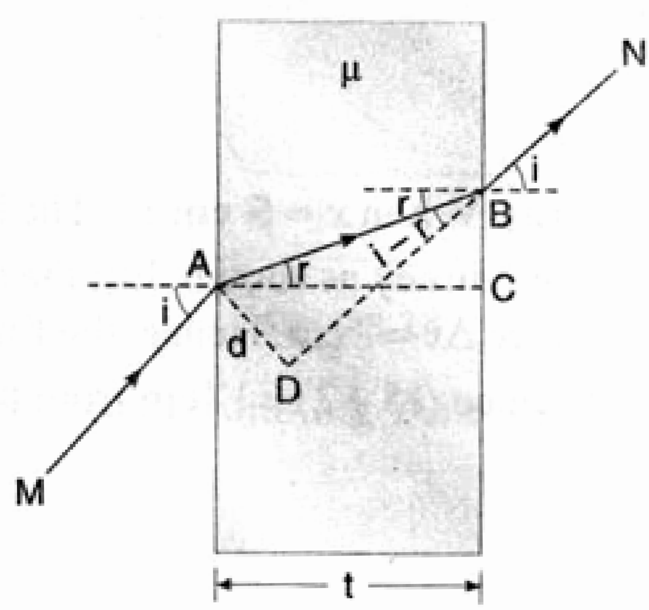

Fig. 28.10

Proof : $AB = \frac{AC}{\cos r} = \frac{t}{\cos r}$ (as $AC = t$)

Now, $d = AB \sin(i - r)$

$$= \frac{t}{\cos r}[\sin i \cos r - \cos i \sin r]$$

or $d = t[\sin i - \cos i \tan r]$ ...(i)

Further $\mu = \frac{\sin i}{\sin r}$ or $\sin r = \frac{\sin i}{\mu}$

$$\therefore \quad \tan r = \frac{\sin i}{\sqrt{\mu^2 - \sin^2 i}}$$

Substituting in Eq. (i), we get $d = \left[1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}}\right] t \sin i$ **Hence Proved.**

EXERCISE Show that for small angles of incidence, $d = ti\left(\frac{\mu - 1}{\mu}\right)$.

Sample Example 28.4 *A point object O is placed in front of a concave mirror of focal length* 10 *cm. A glass slab of refractive index* $\mu = 3/2$ *and thickness* 6 cm *is inserted between object and mirror. Find the position of final image when the distance x shown in figure is:*

(a) 5 *cm* (b) 20 *cm.*

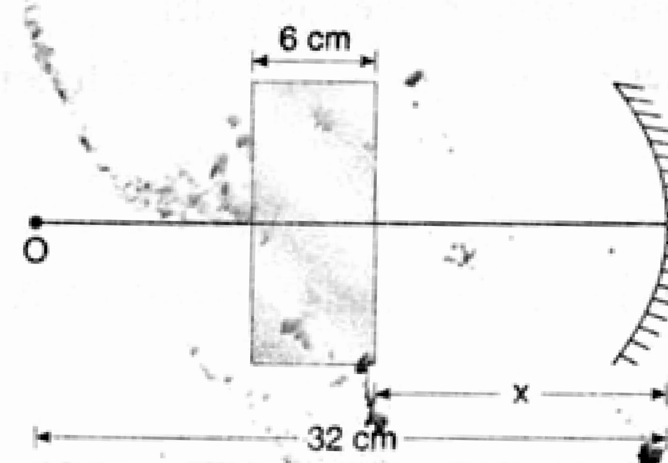

Fig. 28.11

Solution As we have read in the above article the normal shift produced by a glass slab is,

$$\Delta x = \left(1 - \frac{1}{\mu}\right)t = \left(1 - \frac{2}{3}\right)(6) = 2\text{cm}$$

i.e., for the mirror the object is placed at a distance $(32 - \Delta x) = 30$ cm from it. Applying mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

or

$$\frac{1}{v} - \frac{1}{30} = -\frac{1}{10}$$

or

$$v = -15\text{ cm}$$

(a) When $x = 5$ cm : The light falls on the slab on its return journey as shown. But the slab will again shift it by a distance $\Delta x = 2$ cm. Hence, the final **real** image is formed at a distance $(15 + 2) = 17$ cm from the mirror.

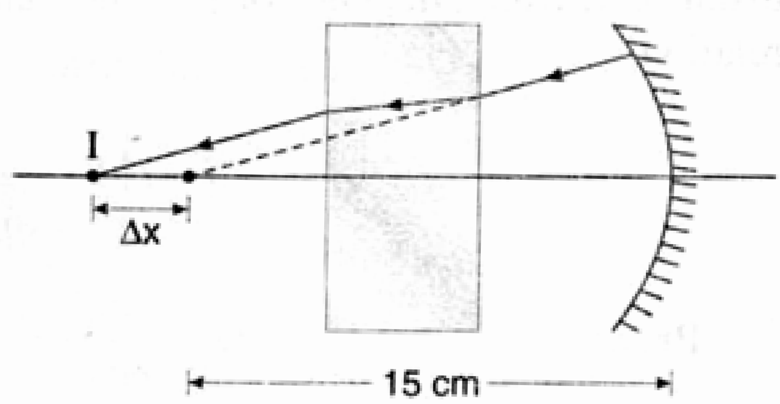

Fig. 28.12

(b) When $x = 20$ cm : This time also the final image is at a distance 17 cm from the mirror but it is **virtual** as shown.

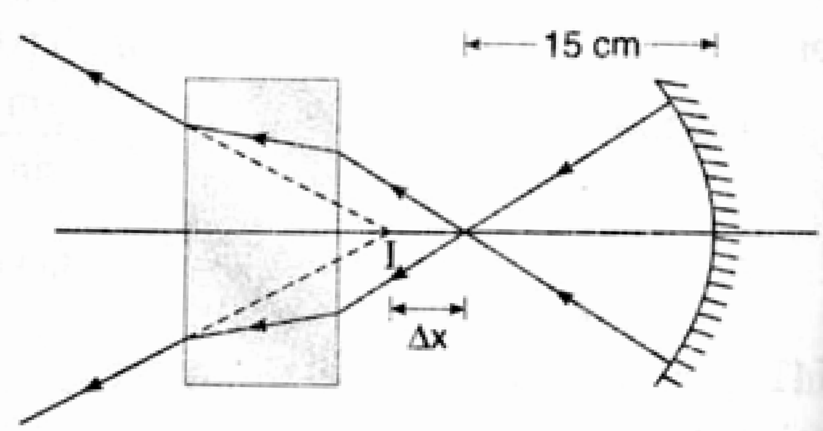

Fig. 28.13