Introductory Exercise 28.1

1. There is a dust particle on a glass slab of thickness t and refractive index $\mu = 1.5$. When seen from one side of the slab, the dust particle appears at a distance 6 cm. From other side it appears at 4 cm. Find the thickness t of the glass slab.
2. Given that ${}_1\mu_2 = 4/3$, ${}_2\mu_3 = 3/2$. Find ${}_1\mu_3$.
3. What happens to the frequency, wavelengths and speed of light that crosses from a medium with index of refraction μ_1 to one with index of refraction μ_2?
4. A monochromatic light beam of frequency 6.0×10^{14} Hz crosses from air into a transparent material where its wavelength is measured to be 300 nm. What is the index of refraction of the material?

(d) Refraction from a Spherical Surface

Consider two transparent media having indices of refraction μ_1 and μ_2, where the boundary between the two media is a spherical surface of radius R. We assume that $\mu_1 < \mu_2$. Let us consider a single ray leaving point O and focussing at point I. Snell's law applied to this refracted ray gives,

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$$

Because θ_1 and θ_2 are assumed to be small, we can use the small angle approximation

$$\sin\theta \approx \theta$$

(angles in radians) and say that

$$\mu_1\theta_1 = \mu_2\theta_2 \quad \text{...(i)}$$

From the geometry shown in the figure,

$$\theta_1 = \alpha + \beta \quad \text{...(ii)}$$

and

$$\beta = \theta_2 + \gamma \quad \text{...(iii)}$$

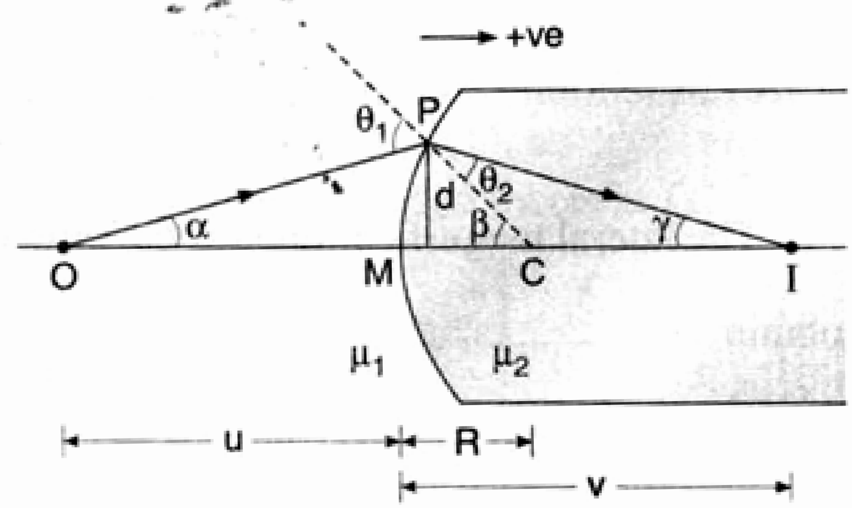


Fig. 28.14

Eqs. (i) and (iii) can be combined to express θ_2 in terms of α and β. Substituting the resulting expression into Eq. (ii), then yields

$$\beta = \frac{\mu_1}{\mu_2}(\alpha + \beta) + \gamma$$

so

$$\mu_1\alpha + \mu_2\gamma = (\mu_2 - \mu_1)\beta \quad \text{...(iv)}$$

Since, the arc PM (of length S) subtends an angle β at the centre of curvature,

$$\beta = \frac{S}{R}$$

Also in the paraxial approximation $\alpha = \frac{S}{u}$ and $\gamma = \frac{S}{v}$

Using these expressions in Eq. (iv) with proper signs, we are left with,

$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

or

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad \text{...(v)}$$

Although the formula (v) is derived for a particular situation, it is valid for all other situations of refraction at a single spherical surface.

Lateral Magnification : The lateral magnification may be obtained with the help of the adjacent figure, where two rays from the tip of an object of height h_0 meet at the corresponding point on an image of height h_i. One ray passes through the centre of curvature of the spherical surface so its direction is unchanged. The path of the second ray is obtained from Snell's law. With the paraxial approximation,

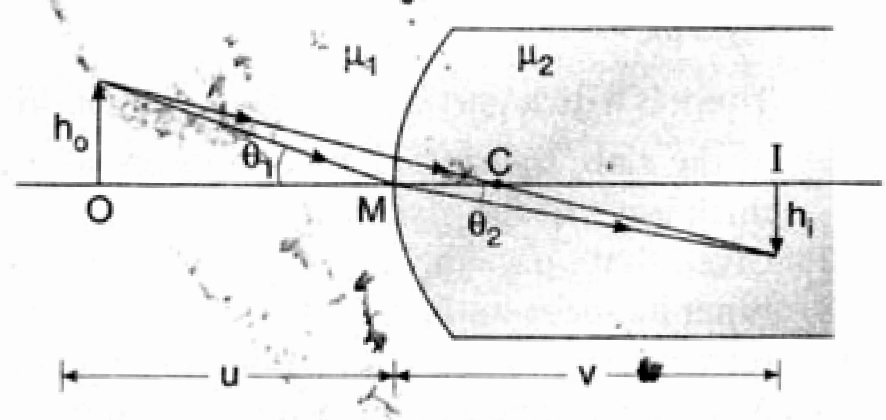


Fig. 28.15

$$\sin\theta_1 \approx \frac{h_o}{u} \qquad \text{and} \qquad \sin\theta_2 \approx \frac{h_i}{v}$$

Combining these equations with Snell's law then gives,

$$\mu_1\left(\frac{h_o}{u}\right)=\mu_2\left(\frac{h_i}{v}\right)$$

or

$$\frac{h_i}{h_o}=\left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v}{u}\right) \qquad \text{...(vi)}$$

The lateral magnification m is the ratio of the image height to the object height or $\frac{h_i}{h_o}$. We therefore, obtain

$$m=\left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v}{u}\right) \qquad \text{...(vii)}$$

Note *Here $v = +$ ve, $u = -$ ve, $h_i = -$ ve and $h_o = +$ ve (distances measured above the axis are taken positive). So, if we put these sign conventions in Eq. (vi), we obtain the same result viz,. $m = \frac{\mu_1}{\mu_2}\frac{v}{u}$.*

Sample Example 28.5 *A glass sphere of radius $R = 10$ cm is kept inside water. A point object O is placed at 20 cm from A as shown in figure. Find the position and nature of the image when seen from other side of the sphere. Also draw the ray diagram. Given, $\mu_g = 3/2$ and $\mu_w = 4/3$.*

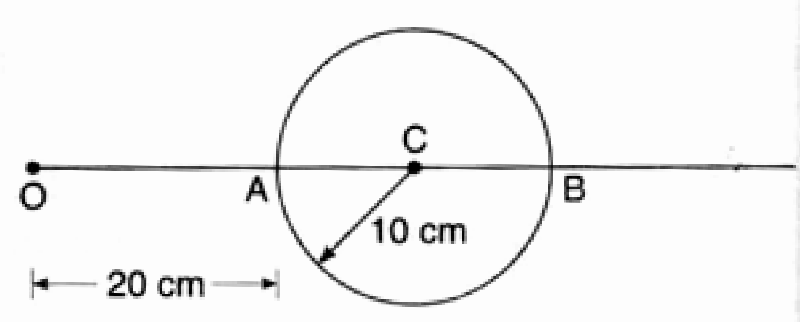


Fig. 28.16

Solution A ray of light starting from O gets refracted twice. The ray of light is travelling in a direction from left to right. Hence, the distances measured in this direction are taken positive. Applying $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$, twice with proper signs.

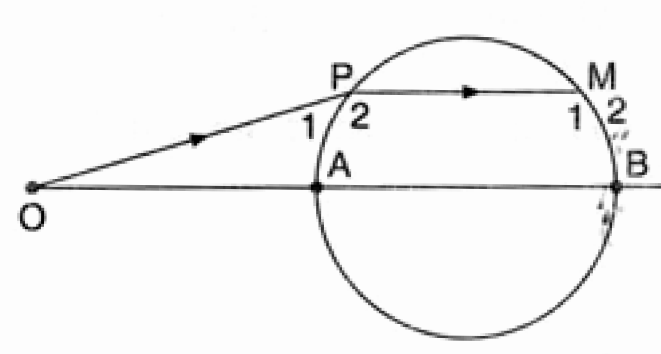


Fig. 28.17

We have,

$$\frac{3/2}{AI_1}-\frac{4/3}{-20}=\frac{3/2-4/3}{10}$$

or

$$AI_1 = -30\,\text{cm}$$

Now, the first image I_1, acts as an object for the second surface, where

$$BI_1 = u = -(30+20) = -50\text{ cm}$$

$$\therefore \quad \frac{4/3}{BI_2} - \frac{3/2}{-50} = \frac{4/3 - 3/2}{-10}$$

$\therefore$ $BI_2 = -100\text{cm}$ *i.e.*, the final image I_2 is **virtual** and is formed at a distance 100 cm (towards left) from B. The ray diagram is as shown.

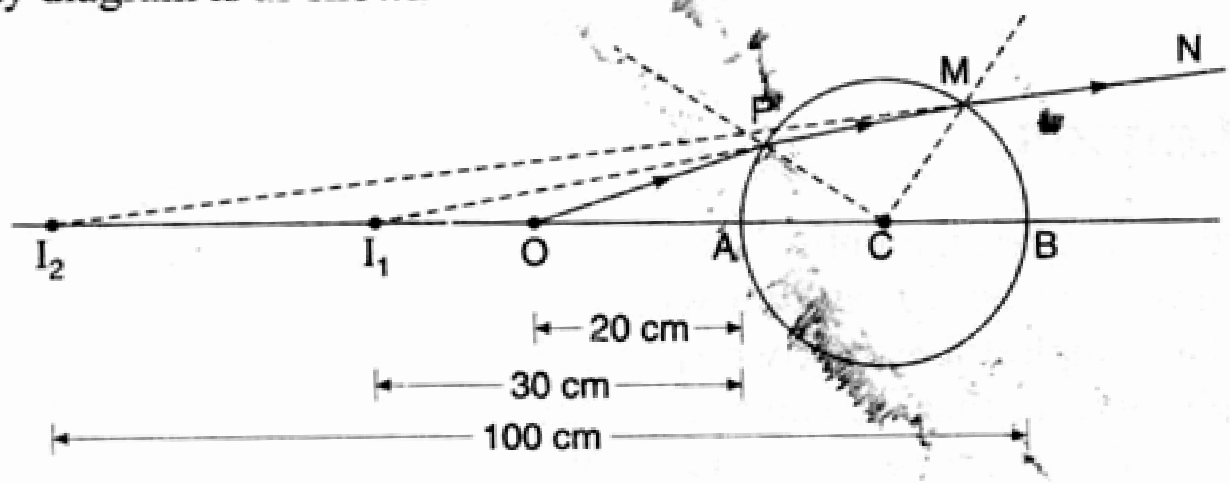


Fig. 28.18

Following points should be noted while drawing the ray diagram.

(i) At P the ray travels from rare to a denser medium. Hence, it will bend towards normal PC. At M, it travels from a denser to a rare medium, hence, moves away from the normal MC.

(ii) PM ray when extended backwards meets at I_1 and MN ray when extended meets at I_2.

Note *The refraction formula* $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ *can also be applied to plane refracting surfaces with* $R = \infty$. *Let us derive* $d_{app} = \frac{d_{actual}}{\mu}$ *using this.*

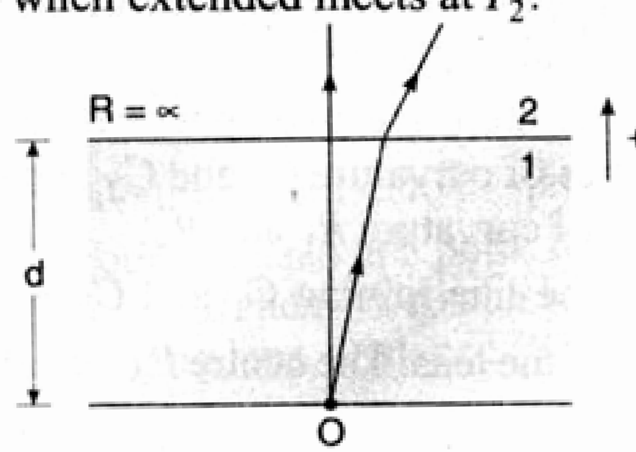


Fig. 28.19

Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

With proper sign and values, we have

$$\frac{1}{v} - \frac{\mu}{-d} = \frac{1-\mu}{\infty} = 0 \quad or \quad v = -\frac{d}{\mu}$$

i.e., image of object O is formed at a distance $\frac{d}{\mu}$ *on same side.*

or $$d_{app} = \frac{d_{actual}}{\mu}$$

Introductory Exercise 28.2

1. A glass sphere ($\mu = 1.5$) with a radius of 15.0 cm has a tiny air bubble 5 cm above its centre. The sphere is viewed looking down along the extended radius containing the bubble. What is the apparent depth of the bubble below the surface of the sphere?
2. One end of a long glass rod ($\mu = 1.5$) is formed into a convex surface of radius 6.0 cm. An object is positioned in air along the axis of the rod. Find the image positions corresponding to object distances of
   (a) 20.0 cm (b) 10.0 cm (c) 3.0 cm
   from the end of the rod.

3. A dust particle is inside a sphere of refractive index $\frac{4}{3}$. If the dust particle is 10.0 cm from the wall of the 15.0 cm radius bowl, where does it appear to an observer outside the bowl.
4. A parallel beam of light enters a clear plastic bead 2.50 cm in diameter and index 1.44. At what point beyond the bead are these rays brought to a focus?
5. The left end of a long glass rod of index 1.6350 is grounded and polished to a convex spherical surface of radius 2.50 cm. A small object is located in the air and on the axis 9.0 cm from the vertex. Find the lateral magnification.

28.2 Thin Lenses

A lens is one of the most familiar optical devices for a human being. A lens is an optical system with two refracting surfaces. The simplest lens has two spherical surfaces close enough together that we can neglect the distance between them (the thickness of the lens). We call this a **thin lens.**

Lenses are of two basic types **convex** which are thicker in the middle than at the edges and **concave** for which the reverse holds.

Figure shows examples of both types bounded by spherical or plane surfaces.

As there are two spherical surfaces, there are two centres of curvature C_1 and C_2 and correspondingly two radii of curvature R_1 and R_2.

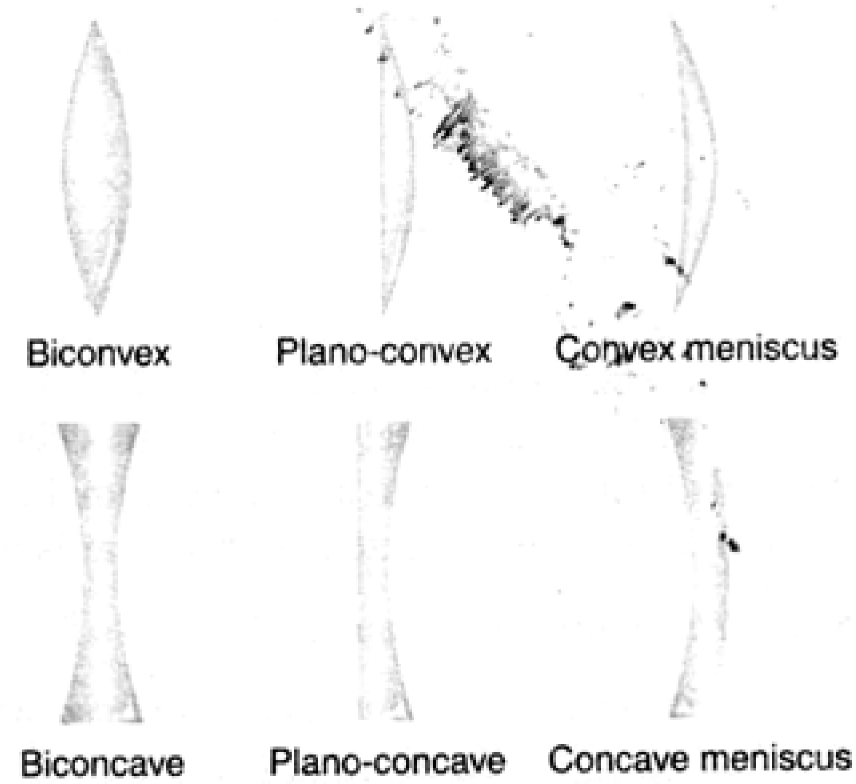


Fig. 28.20 Types of lens.

The line joining C_1 and C_2 is called the **principal axis** of the lens. The centre P of the thin lens which lies on the principal axis, is called the **optical centre.**

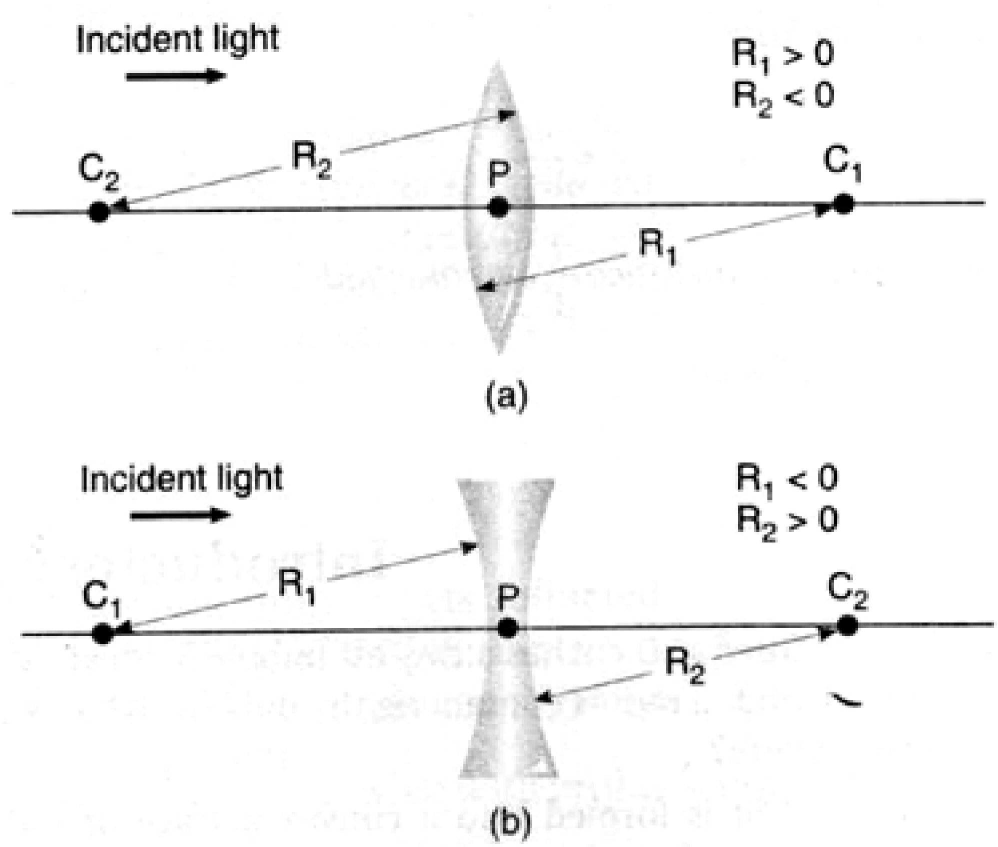


Fig. 28.21 (a) A converging thin lens and (b) a diverging thin-lens.

Focus :

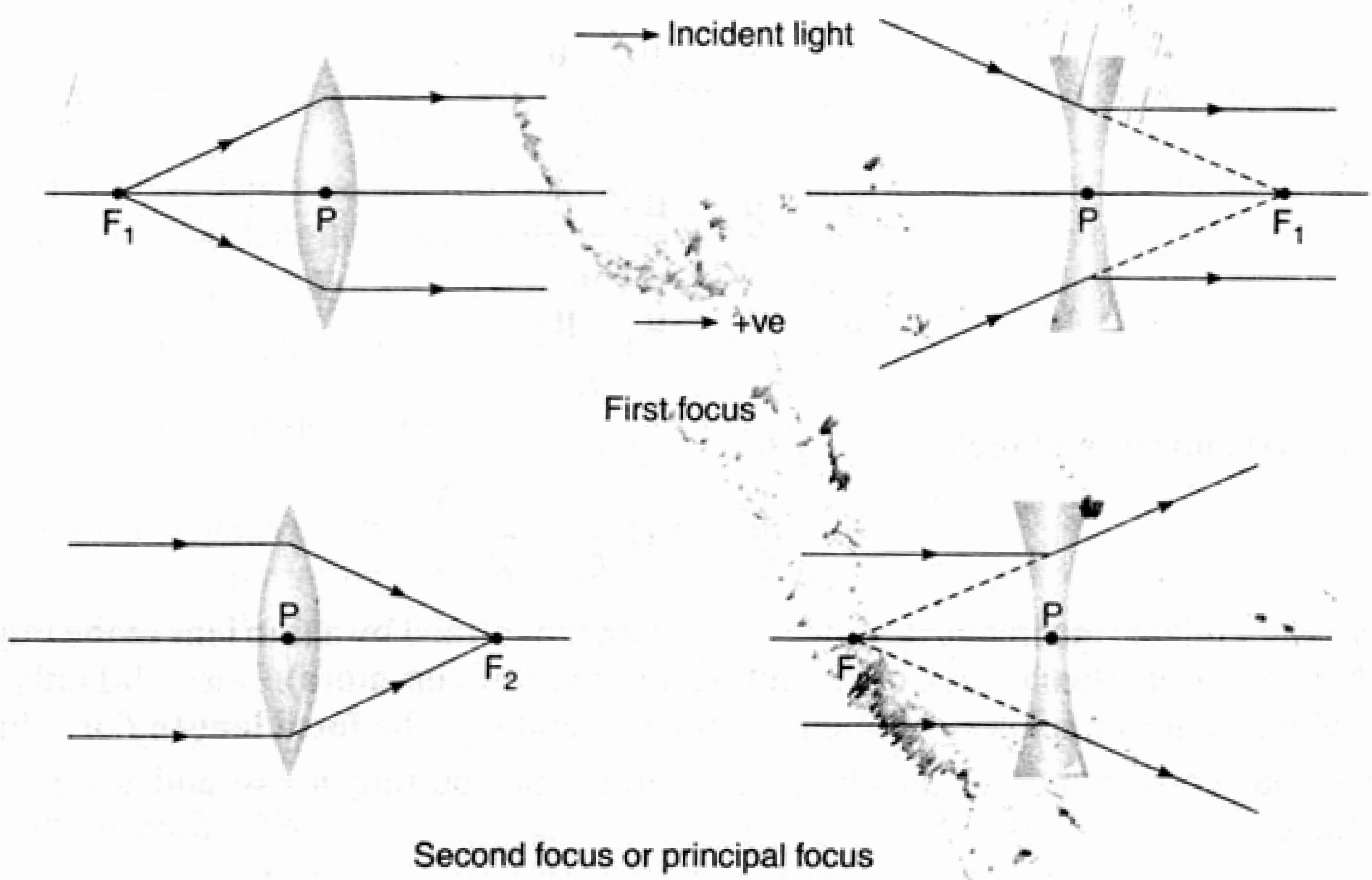


Fig. 28.22

Unlike a mirror, a lens has two foci.

First focus (F_1) : It is defined as a point at which if an object (real in case of a convex lens and virtual for concave) is placed, the image of this object is formed at infinity. Or we can say, rays passing through F_1 become parallel to the principal axis after refraction from the lens. The distance PF_1 is the first focal length f_1.

Second focus or principal focus (F_2) : A narrow beam of light travelling parallel to the principal axis either converge (in case of a convex lens) or diverge (in case of a concave lens) at a point F_2 after refraction from the lens. This point F_2 is called the second or principal focus. If the rays converge at F_2, the lens is said a converging lens and if they diverge, they are called diverging lens. Distance PF_2 is the second focal length f_2.

Note *(i) From the figure we can see that f_1 is negative for a convex lens and positive for a concave lens. But f_2 is positive for convex lens and negative for concave lens.*

(ii) $|f_1| = |f_2|$ if the media on the two sides of a thin lens have same refractive index.

(iii) We are mainly concerned with the second focus f_2. Thus, wherever we write the focal length f, it means the second or principal focal length. Thus, $f = f_2$ and hence, f is positive for a convex lens and negative for a concave lens.

Lens maker's formula and lens formula

Consider an object O placed at a distance u from a convex lens as shown in figure. Let its image I after two refractions from spherical surfaces of radii R_1 (positive) and R_2 (negative) be formed at a distance v from the lens. Let v_1 be the distance of image formed by refraction from the refracting surface of radius R_1. This image acts as an object for the second surface. Using,

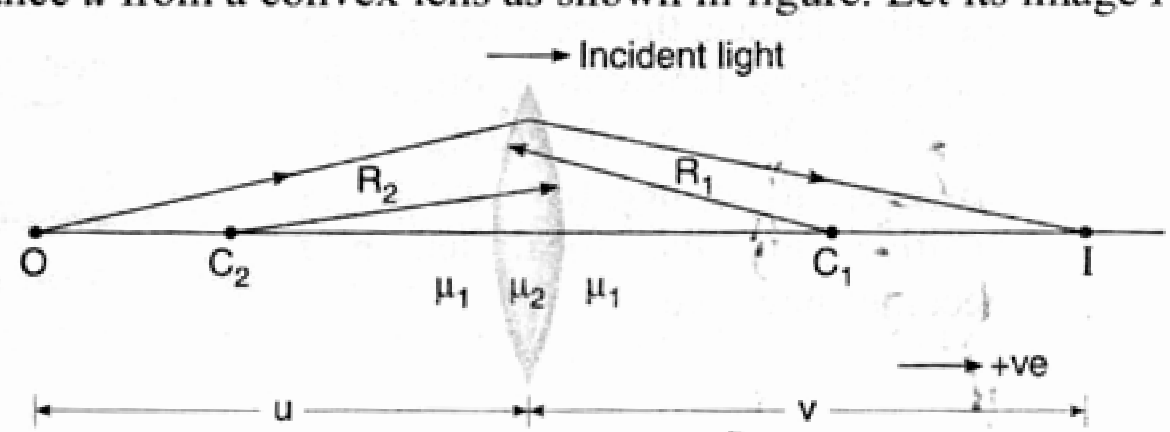


Fig. 28.23

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

twice, we have

or
$$\frac{\mu_2}{v_1}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R_1} \quad \text{...(i)}$$

and
$$\frac{\mu_1}{v}-\frac{\mu_2}{v_1}=\frac{\mu_1-\mu_2}{-R_2} \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii) and then simplifying, we get

$$\frac{1}{v}-\frac{1}{u}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \text{...(iii)}$$

This expression relates the image distance v of the image formed by a thin lens to the object distance u and to the thin lens properties (index of refraction and radii of curvature). It is valid only for paraxial rays and only when the lens thickness is much less then R_1 and R_2. The **focal length** f of a thin lens is the image distance that corresponds to an object at infinity. So, putting $u=\infty$ and $v=f$ in the above equation, we have

$$\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \text{...(iv)}$$

If the refractive index of the material of the lens is μ and it is placed in air, $\mu_2=\mu$ and $\mu_1=1$ so that Eq. (iv) becomes

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \text{...(v)}$$

This is called the **lens maker's formula** because it can be used to determine the values of R_1 and R_2 that are needed for a given refractive index and a desired focal length f.

Combining Eqs. (iii) and (v), we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \text{...(vi)}$$

Which is known as the **lens formula.** Following conclusions can be drawn from Eqs. (iv), (v) and (vi).

1. For a converging lens, R_1 is positive and R_2 is negative. Therefore, $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ in Eq. (v) comes out a positive quantity and if the lens is placed in air, $(\mu-1)$ is also a positive quantity. Hence, the focal length f of a converging lens comes out to be positive. For a diverging lens however, R_1 is negative and R_2 is positive and the focal length f becomes negative.

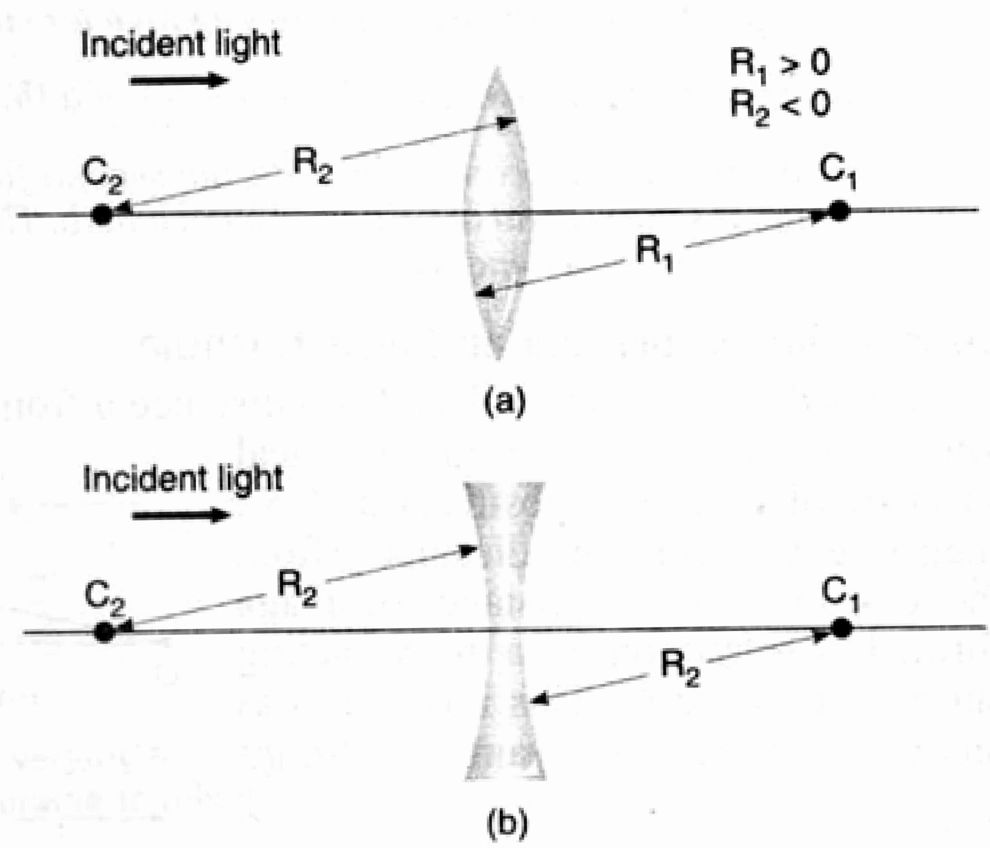


Fig. 28.24

2. Focal length of a mirror $\left(f_M = \frac{R}{2}\right)$ depends only upon the radius of curvature R while that of a lens [Eq. (iv)] depends on μ_1, μ_2, R_1 and R_2. Thus, if a lens and a mirror are immersed in some liquid, the focal length of lens would change while that of the mirror will remain unchanged.
3. Suppose $\mu_2 < \mu_1$ in Eq. (iv), *i.e.*, refractive index of the medium (in which lens is placed) is more than the refractive index of the material of the lens, then $\left(\frac{\mu_2}{\mu_1} - 1\right)$ becomes a negative quantity, *i.e.*, the lens changes its behaviour. A converging lens behaves as a diverging lens and *vice-versa*. An air bubble in water seems as a convex lens but behaves as a concave (diverging) lens.

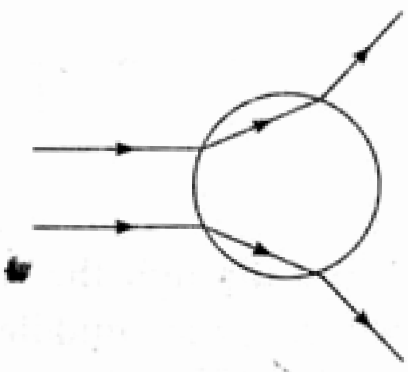


Fig. 28.25 Air bubble in water diverges the parallel beam of light incident

Sample Example 28.6 *Focal length of a convex lens in air is 10 cm. Find its focal length in water. Given that* $\mu_g = 3/2$ *and* $\mu_w = 4/3$.

Solution
$$\frac{1}{f_{\text{air}}} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(i)}$$

and
$$\frac{1}{f_{\text{water}}} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get
$$\frac{f_{\text{water}}}{f_{\text{air}}} = \frac{(\mu_g - 1)}{(\mu_g/\mu_w - 1)}$$

Substituting the values, we have
$$f_{\text{water}} = \frac{(3/2 - 1)}{\left(\frac{3/2}{4/3} - 1\right)} f_{\text{air}}$$
$$= 4 f_{\text{air}} = 4 \times 10 = 40\,\text{cm}$$ **Ans.**

Note Students can remember the result $f_{\text{water}} = 4 f_{\text{air}}$, if $\mu_g = 3/2$ and $\mu_w = 4/3$.

Images formed by thin lenses

Information as to the position and nature of the image in any case can be obtained either from a ray diagram or by calculation.

(a) Ray diagram : To construct the image of a small object perpendicular to the axis of a lens, two of the following three rays are drawn from the top of the object.

1. A ray parallel to the principal axis after refraction passes through the principal focus or appears to diverge from it.

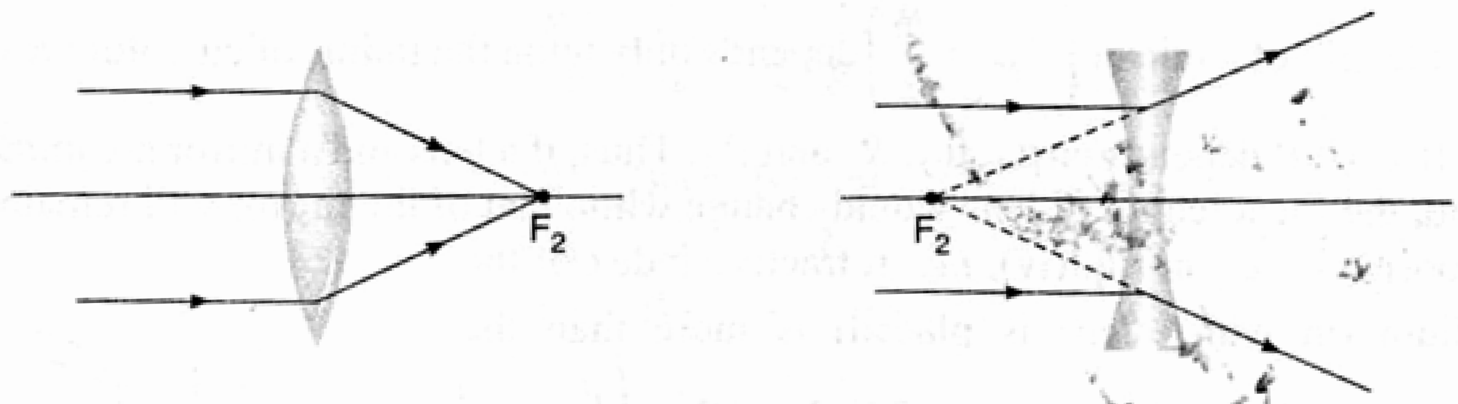


Fig. 28.26

2. A ray through the optical centre P passes undeviated because the middle of the lens acts like a thin parallel-sided slab.

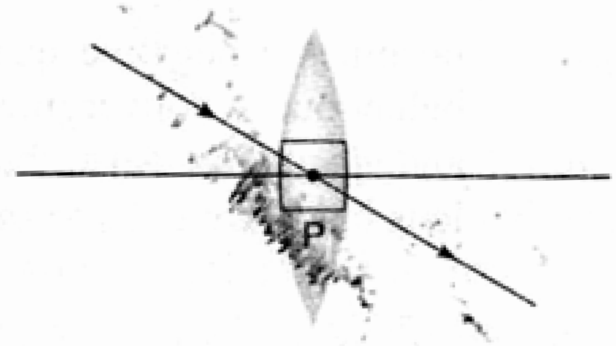


Fig. 28.27

3. A ray passing through the first focus F_1 become parallel to the principal axis after refraction.

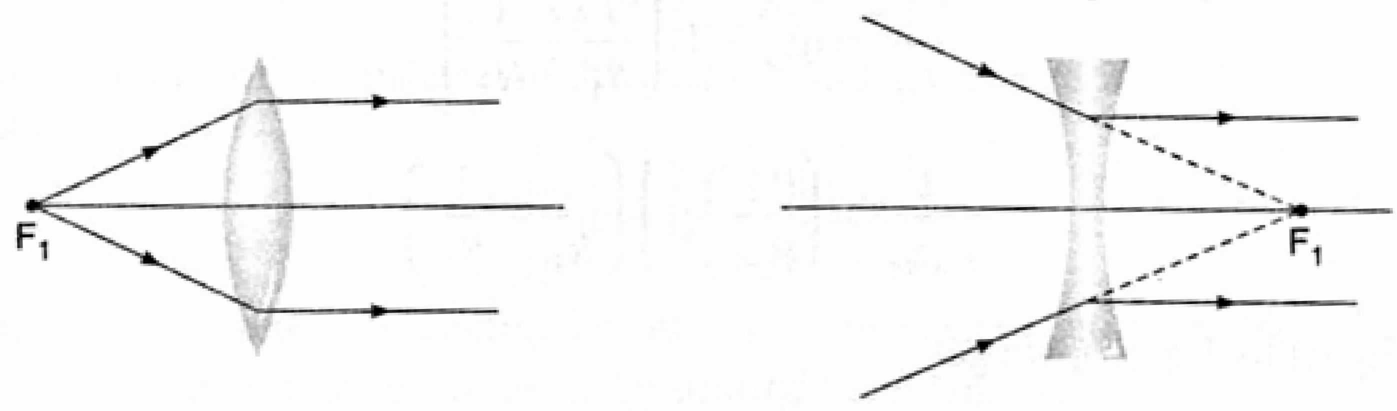
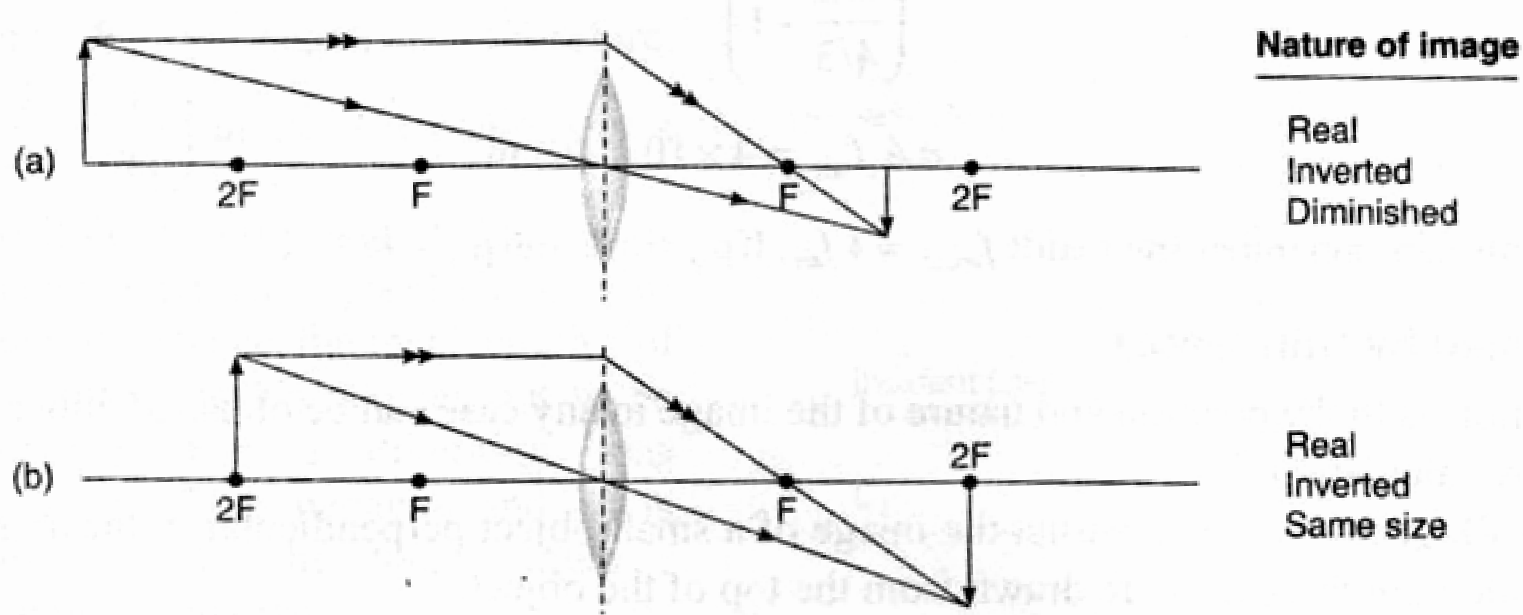


Fig. 28.28

Keeping the above three points in mind we can show that image formed by a concave lens is always virtual, erect and diminished (like a convex mirror) while the nature of image in case of a convex lens depends on the position of object. The ray diagrams for a convex and a concave lens are shown below. For a convex lens image is virtual when object lies between F and P. In all other cases it is real.



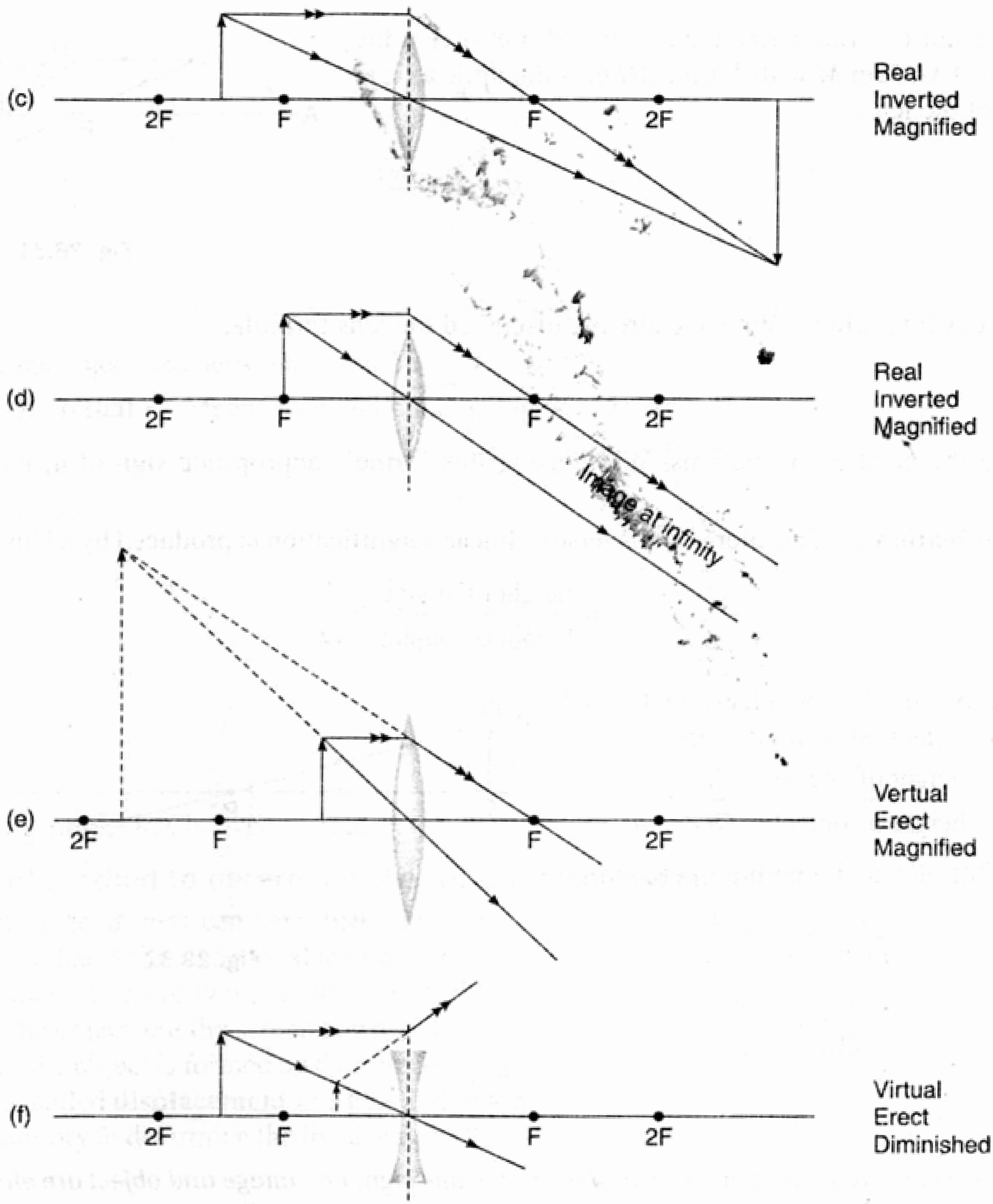


Fig. 28.29 Ray diagrams for a convex lens (a–e) and a concave lens (f).

Sample Example 28.7 *An image I is formed of point object O by a lens whose optic axis is AB as shown in figure.*

(a) State whether it is a convex lens or concave?

(b) Draw a ray diagram to locate the lens and its focus.

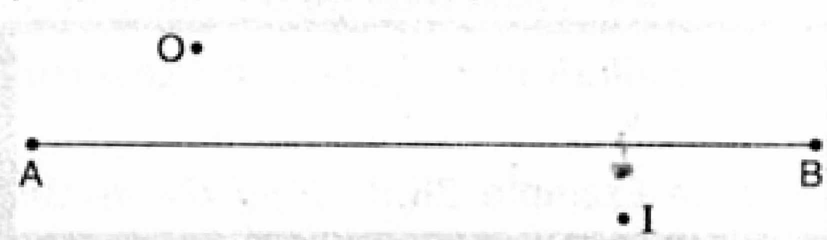


Fig. 28.30

Solution (a) (i) Concave lens always forms an erect image. The given image *I* is on the other side of the optic axis. Hence, the lens is **convex.**

(ii) Join *O* with *I*. Line *OI* cuts the optic axis *AB* at pole (*P*) of the lens. The dotted line shows the position of lens.

From point O, draw a line parallel to AB. Let it cuts the dotted line at M. Join M with I. Line MI cuts the optic axis at focus (F) of the lens.

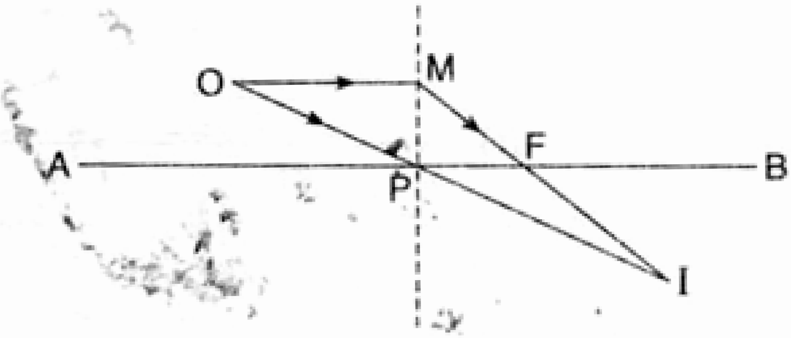


Fig. 28.31

(b) Lens formula : We have already discussed the lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

for finding the position of the lens. While using this formula appropriate sign of u, v and f must be included.

Magnification : The lateral, transverse or linear magnification m produced by a lens is defined by,

$$m = \frac{\text{height of image}}{\text{height of object}} = \frac{I}{O}$$

A real image II' of an object OO' formed by a convex lens is shown in figure.

$$\frac{\text{height of image}}{\text{height of object}} = \frac{II'}{OO'} = \frac{v}{u}$$

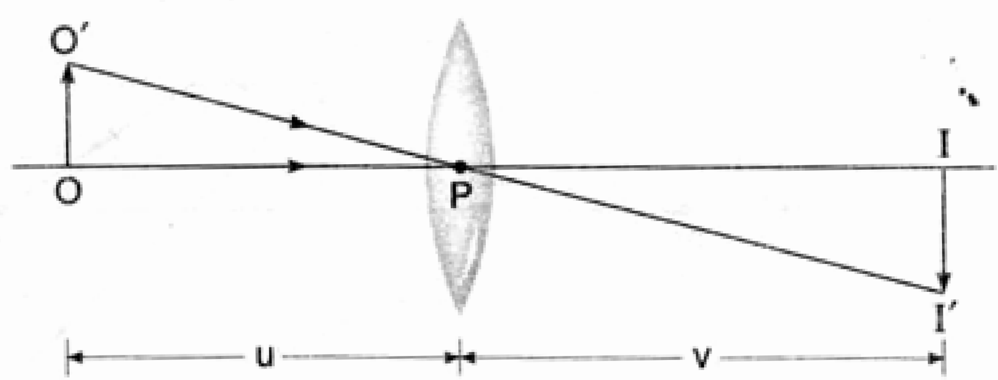


Fig. 28.32

Substituting v and u with proper sign,

$$\frac{II'}{OO'} = \frac{-I}{O} = \frac{v}{-u}$$

or $$\frac{I}{O} = m = \frac{v}{u}$$

Thus, $$m = \frac{v}{u}$$

Note *Suppose m is positive it means v and u are of same sign, i.e., image and object are on the same side (left side), which implies that the image of a real object is virtual. Thus,*
$m = +2$, means image is virtual, erect and two times magnified and $|v| = 2|u|$.
Similarly $m = -\frac{1}{2}$ means image is real, inverted and diminished and $|v| = \frac{1}{2}|u|$.

Sample Example 28.8 *Find the distance of an object from a convex lens if image is two times magnified. Focal length of the lens is 10 cm.*

Solution Convex lens forms both type of images real as well as virtual. Since, nature of the image is not mentioned in the question, we will have to consider both the cases.

When image is real : Means v is positive and u is negative with $|v| = 2|u|$. Thus if

$$u = -x \quad \text{then} \quad v = 2x \quad \text{and} \quad f = 10\,\text{cm}$$

Substituting in $$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

We have $$\frac{1}{2x}+\frac{1}{x}=\frac{1}{10}$$

or $$\frac{3}{2x}=\frac{1}{10}$$

$\therefore$ $$x=15\text{ cm}$$ **Ans.**

$x=15$ cm, means object lies between F and 2 F.

When image is virtual : Means v and u both are negative. So let,

$$u=-y \quad \text{then} \quad v=-2y \quad \text{and} \quad f=10\text{ cm}$$

Substituting in, $$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

We have, $$\frac{1}{-2y}+\frac{1}{y}=\frac{1}{10}$$

or $$\frac{1}{2y}=\frac{1}{10}$$

$\therefore$ $$y=5\text{ cm}$$ **Ans.**

$y=5$ cm, means object lies between F and P.

Displacement method to determine the focal length of a convex lens

If the distance d between an object and screen is greater than 4 times the focal length of a convex lens, then there are two positions of the lens between the object and the screen at which a sharp image of the object is formed on the screen. This method is called **displacement method** and is used in laboratory to determine the focal length of convex lens.

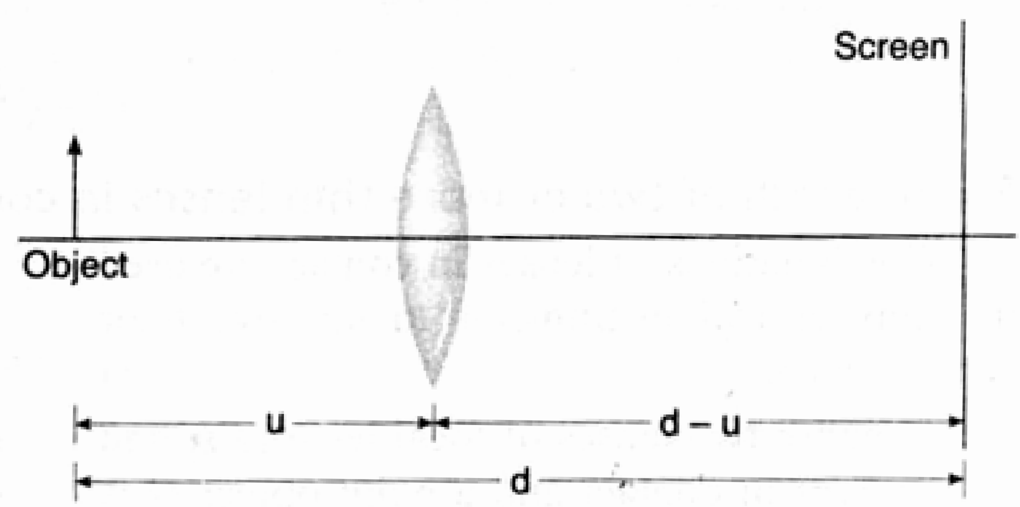


Fig. 28.33

To prove this, let us take an object placed at a distance u from a convex lens of focal length f. The distance of image from the lens $v=(d-u)$. From the lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

We have, $$\frac{1}{d-u}-\frac{1}{-u}=\frac{1}{f}$$

or $$u^2-du+df=0$$

$\therefore$ $$u=\frac{d\pm\sqrt{d(d-4f)}}{2}$$

Now, there are following possibilities:

(i) If $d < 4f$, then u is imaginary.
So, physically no position of the lens is possible.

(ii) If $d = 4f$ then $u = \frac{d}{2} = 2f$. So only one position is possible. From here we can see that the **minimum distance between an object and its real image in case of a convex lens is $4f$.**

(iii) If $d > 4f$, there are two positions of lens at distances $\frac{d + \sqrt{d(d-4f)}}{2}$ and $\frac{d - \sqrt{d(d-4f)}}{2}$ for which real image is formed on the screen.

(iv) Suppose I_1 is the image length in one position of the object and I_2 the image length in second position, then object length O is given by,

$$O = \sqrt{I_1 I_2}$$

This can be proved as under:

$$|u_1| = \frac{d + \sqrt{d(d-4f)}}{2} \qquad \therefore \quad |v_1| = d - |u_1| = \frac{d - \sqrt{d(d-4f)}}{2}$$

$$|u_2| = \frac{d - \sqrt{d(d-4f)}}{2} \qquad \therefore \quad |v_2| = d - |u_2| = \frac{d + \sqrt{d(d-4f)}}{2}$$

Now,

$$|m_1 m_2| = \frac{I_1}{O} \times \frac{I_2}{O} = \frac{|v_1|}{|u_1|} \times \frac{|v_2|}{|u_2|}$$

Substituting the values, we get

$$\frac{I_1 I_2}{O^2} = 1$$

or

$$O = \sqrt{I_1 I_2}$$

Hence Proved.

Focal length of two or more thin lenses in contact

Combinations of lenses in contact are used in many optical instruments to improve their performance.

Suppose two lenses of focal lengths f_1 and f_2 are kept in contact and a point object O is placed at a distance u from the combination. The first image (say I_1) after refraction from the first lens is formed at a distance v_1 (whatever may be the sign of v_1) from the combination. This image I_1 acts as an object for the second lens and let v be the distance of the final image from the combination. Applying the lens formula,

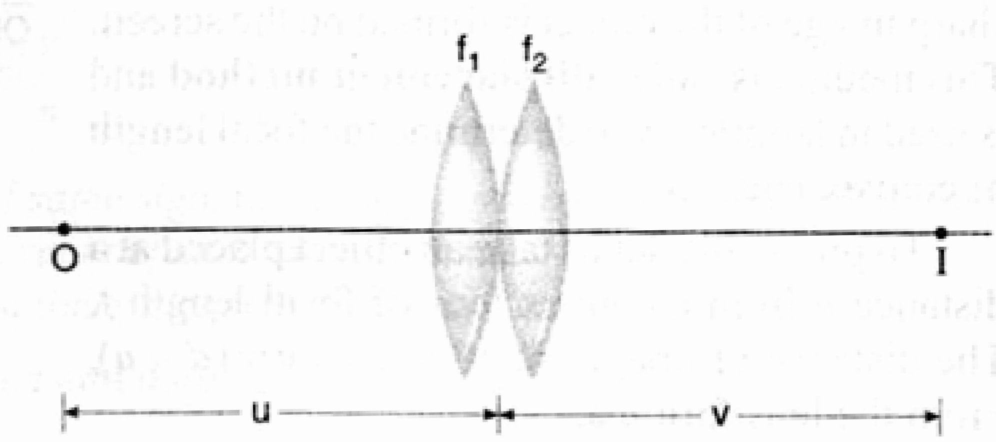


Fig. 28.34

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

For the two lenses, we have

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \qquad \ldots \text{(i)}$$

and $$\frac{1}{v}-\frac{1}{v_1}=\frac{1}{f_2} \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we have $$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{F} \text{ (say)}$$

Here, F is the equivalent focal length of the combination. Thus,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

Similarly for more than two lenses in contact, the equivalent focal length is given by the formula,

$$\frac{1}{F}=\sum_{i=1}^{n}\frac{1}{f_i}$$

Note *Here, f_1, f_2 etc., are to be substituted with sign.*

Sample Example 28.9 *A thin plano-convex lens of focal length f is split into two halves. One of the halves is shifted along the optical axis as shown in figure. The separation between object and image planes is 1.8 m. The magnification of the image, formed by one of the half lens is 2. Find the focal length of the lens and separation between the two halves. Draw the ray diagram for image formation.*

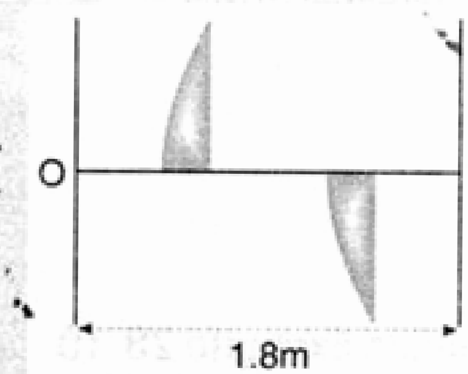


Fig. 28.35

Solution For both the halves, position of object and image is same. Only difference is of magnification. Magnification for one of the halves is given as 2 (>1). This can be for the first one, because for this, $|v|>|u|$. Therefore, magnification, $|m|=|v/u|>1$.

So, for the first half

$$|v/u|=2 \quad \text{or} \quad |v|=2|u|$$

Let $$u=-x, \quad \text{then} \quad v=+2x$$

and $$|u|+|v|=1.8 \text{ m}$$

i.e., $$3x=1.8 \text{ m} \quad \text{or} \quad x=0.6 \text{ m}$$

Hence, $$u=-0.6 \text{ m} \quad \text{and} \quad v=+1.2 \text{ m}$$

Using $$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{1}{1.2}-\frac{1}{-0.6}=\frac{1}{0.4}$$

$\therefore$ $$f=0.4 \text{ m}$$ **Ans.**

For the second half $$\frac{1}{f}=\frac{1}{1.2-d}-\frac{1}{-(0.6+d)}$$

or $$\frac{1}{0.4}=\frac{1}{1.2-d}+\frac{1}{(0.6+d)}$$

Solving this, we get $$d=0.6 \text{ m}$$ **Ans.**

Magnification for the second half will be

$$m_2 = \frac{v}{u} = \frac{0.6}{-(1.2)} = -\frac{1}{2}$$

and magnification for the first half is $m_1 = \frac{v}{u} = \frac{1.2}{-(0.6)} = -2$

The ray diagram is as follows:

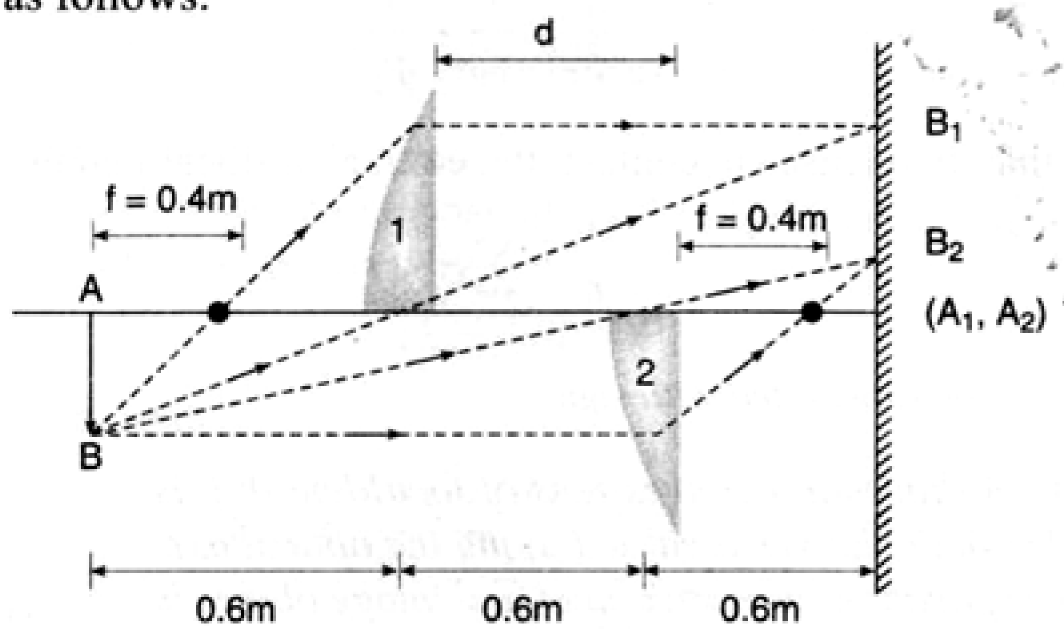


Fig. 28.36

Sample Example 28.10 *A converging lens of focal length* 5.0 *cm is placed in contact with a diverging lens of focal length* 10.0 *cm. Find the combined focal length of the system.*

Solution Here, $f_1 = +5.0\,\text{cm}$, and $f_2 = -10.0\,\text{cm}$

Therefore, the combined focal length F is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{5.0} - \frac{1}{10.0} = +\frac{1}{10.0}$$

$\therefore$ $F = +10.0\,\text{cm}$ **Ans.**

i.e., the combination behaves as a converging lens of focal length 10.0 cm.

Power of an optical instrument : By optical power of an instrument (whether it is a lens, mirror or a refractive surface) we mean the ability of the instrument to deviate the path of rays passing through it. If the instrument converges the rays parallel to the principal axis its power is said positive and if it diverges the rays it is said a negative power.

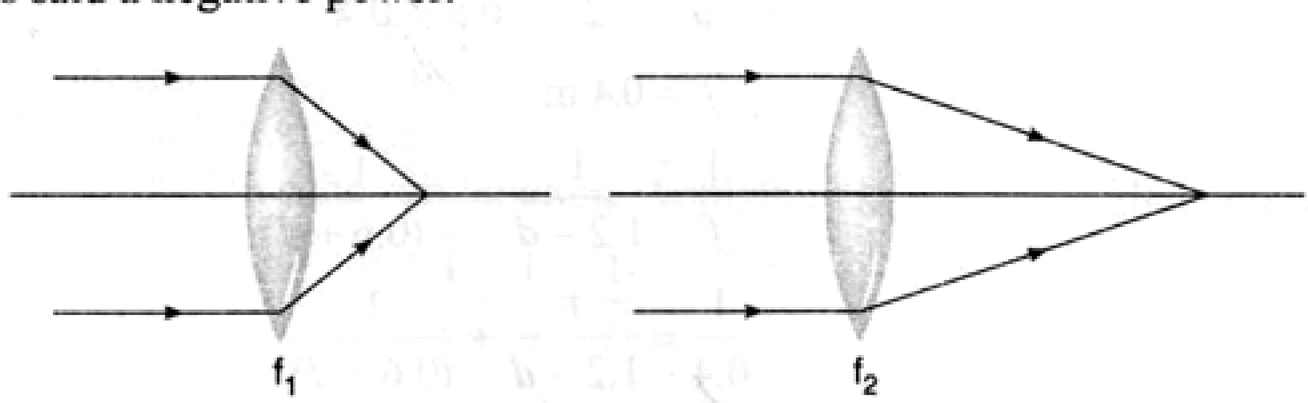


Fig. 28.37

The shorter the focal length of a lens (or a mirror) the more it converges or diverges light. As shown in the figure,

$$f_1 < f_2$$

and hence the power $P_1 > P_2$, as bending of light in case 1 is more than that of case 2. For a lens,

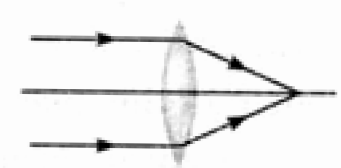
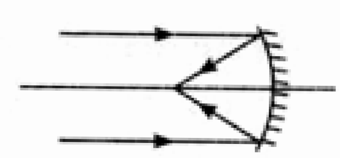
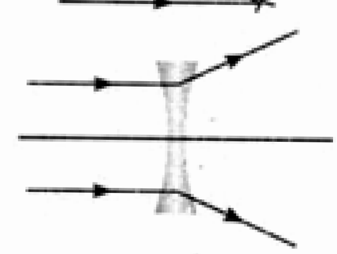
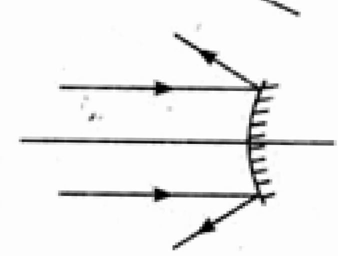
$$P \text{ (in dioptre)} = \frac{1}{f \text{ (metre)}}$$

and for a mirror,

$$P \text{ (in dioptre)} = \frac{-1}{f \text{ (metre)}}$$

Following table gives the sign of P and f for different type of lens and mirror.

Table 28.1

Nature of lens/mirror	Focal length (f)	Power $P_L = \frac{1}{f}, P_M = -\frac{1}{f}$	Converging/ diverging	Ray diagram
Convex lens	+ ve	+ ve	converging	
Concave mirror	− ve	+ ve	converging	
Concave lens	− ve	− ve	diverging	
Convex mirror	+ ve	− ve	diverging	

Thus, convex lens and concave mirror have positive power or they are converging in nature. Concave lens and convex mirror have negative power or they are diverging in nature.

Sample Example 28.11 *A spherical convex surface separates object and image space of refractive index 1.0 and $\frac{4}{3}$. If radius of curvature of the surface is 10 cm, find its power.*

Solution Let us see where does the parallel rays converge (or diverge) on the principal axis. Let us call it the focus and the corresponding length the focal length f. Using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ with proper values and signs, we have

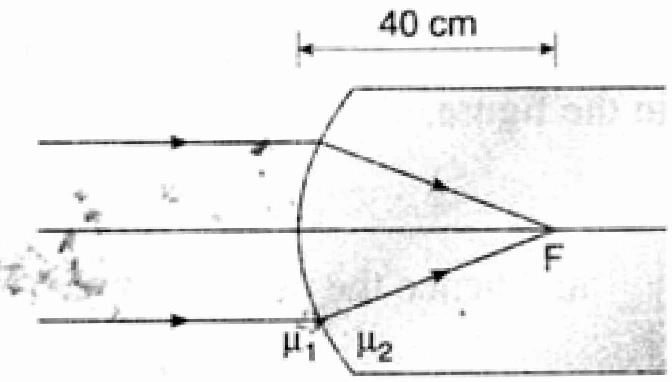


Fig. 28.38

$$\frac{4/3}{f} - \frac{1.0}{\infty} = \frac{4/3 - 1.0}{+10}$$

or $$f = 40\text{ cm} = 0.4\text{ m}$$

Since, the rays are converging, its power should be positive. Hence,

$$P\text{ (in dioptre)} = \frac{+1}{f\text{ (metre)}} = \frac{1}{0.4}$$

or $$P = 2.5\text{ dioptre}$$ **Ans.**

Introductory Exercise 28.3

1. When an object is placed 60 cm in front of a diverging lens, a virtual image is formed 20 cm from the lens. The lens is made of a material of refractive index $\mu = 1.65$ and its two spherical surfaces have the same radius of curvature. What is the value of this radius?
2. A converging lens has a focal length of 30 cm. Rays from a 2.0 cm high filament that pass through the lens form a virtual image at a distance of 50 cm from the lens. Where is the filament located? What is the height of the image?
3. Show that the focal length of a thin lens is not changed when the lens is rotated so that the left and the right surfaces are interchanged.
4. As an object is moved from the surface of a thin converging lens to a focal point, over what range does the image distance vary?
5. A diverging lens is made of material with refractive index 1.3 and has identical concave surfaces of radius 20 cm. The lens is immersed in a transparent medium with refractive index 1.8.
   (a) What is now the focal length of the lens?
   (b) What is the minimum distance that an immersed object must be from the lens so that a real image is formed?
6. An object is located 20 cm to the left of a converging lens with $f = 10$ cm A second identical lens is placed to the right of the first lens and then moved until the image it produces is identical in size and orientation to the object. What is the separation between the lenses.
7. Suppose an object has thickness du so that it extends from object distance u to $u + du$. Prove that the thickness dv of its image is given by $\left(-\frac{v^2}{u^2}\right) du$, so the longitudinal magnification $\frac{dv}{du} = -m^2$, where m is the lateral magnification.
8. Two thin similar convex glass pieces are joined together front to front, with its rear portion silvered such that a sharp image is formed 0.2 m for an object at infinity. When the air between the glass pieces is replaced by water $\left(\mu = \frac{4}{3}\right)$, find the position of image.

9. When a pin is moved along the principal axis of a small concave mirror, the image position coincides with the object at a point 0.5 m from the mirror. If the mirror is placed at a depth of 0.2 m in a transparent liquid, the same phenomenon occurs when the pin is placed 0.4 m from the mirror. Find the refractive index of the liquid.

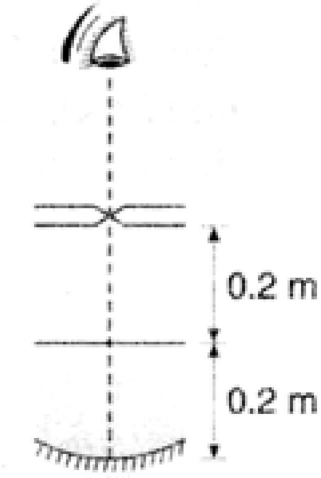


Fig. 28.39

10. When a lens is inserted between an object and a screen which are a fixed distance apart the size of the image is either 6 cm or $\frac{2}{3}$ cm. Find size of the object.
11. A lens of focal length 12 cm forms an upright image three times the size of a real object. Find the distance in cm between the object and images.
12. The distance between an object and its upright image is 20 cm. If the magnification is 0.5, what is the focal length of the lens that is being used to form the image?
13. A thin lens of focal length + 10.0 cm lies on a horizontal plane mirror. How far above the lens should an object be held if its image is to coincide with the object?

28.3 Total Internal Reflection (TIR)

Figure shows the reflection and refraction of a light ray at the interface between a denser and a rare medium, whose refractive indices are μ_D and μ_R. Angle of incidence in denser medium is i and angle of refraction is r.

From Snell's law ($\mu \sin i =$ constant), we may write

$$\mu_D \sin i = \mu_R \sin r$$

or

$$\frac{\mu_D}{\mu_R} \sin i = \sin r$$

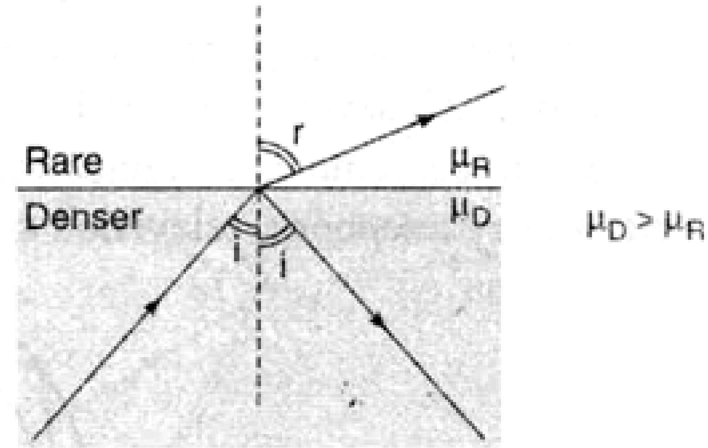


Fig. 28.40

The right hand side of this equation is a sine function that has a range from 0 to 1. The left hand side must therefore, have the same range, *i.e.*,

$$1 \geq \frac{\mu_D}{\mu_R} \sin i \geq 0 \quad \text{or} \quad \frac{\mu_R}{\mu_D} \geq \sin i \geq 0$$

or

$$\theta_c \geq i \geq 0$$

Here,

$$\theta_c = \sin^{-1}\left(\frac{\mu_R}{\mu_D}\right) \qquad \ldots\text{(i)}$$

is called the **critical angle.** When the angle of incidence exceeds θ_c, no refracted beam is observed and the incident beam is completely reflected at the boundary. This phenomenon known as **total internal reflection (TIR)**, only occurs when the light travels from a denser medium to a rare medium. When the rare medium is air,

$$\mu_R = 1 \quad \text{and} \quad \mu_D = \mu$$

$$\therefore \qquad \theta_c = \sin^{-1}\left(\frac{1}{\mu}\right) \qquad \text{...(ii)}$$

TIR has following applications.

(i) Totally reflecting prisms : Refractive index of crown glass is 3/2. Hence,

$$\theta_c = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{2}{3}\right) \approx 42°$$

A ray OA incident normally on face PQ of a crown glass prism suffers TIR at face PR since, the angle of incidence in the optically denser medium is 45°. A bright ray AB emerges at right angles to face QR. The prism thus, reflects the ray through 90°. Light can be reflected through 180° and an erect image can be obtained of an inverted one if the prism is arranged as shown in figure (b).

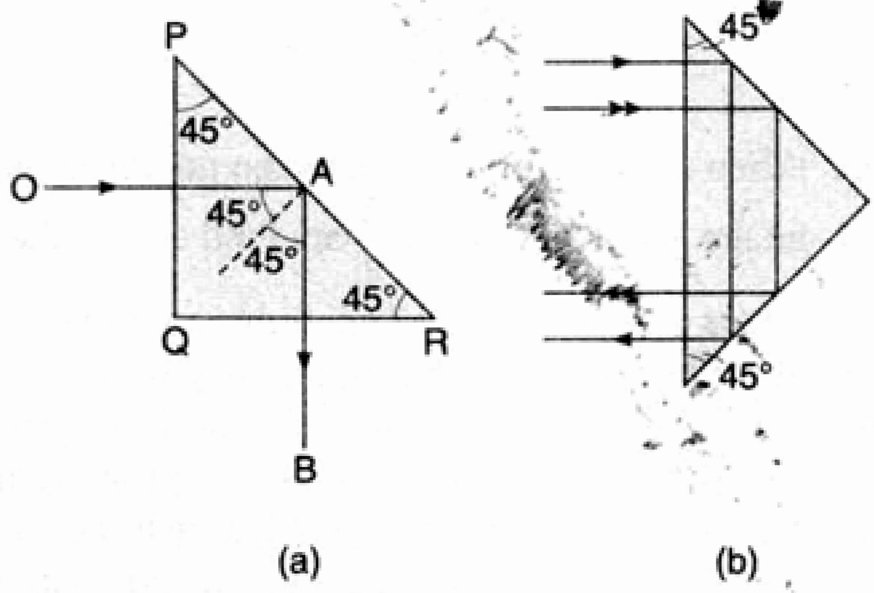


Fig. 28.41 Prism reflectors

(ii) Optical fibres : Light can be confined within a bent glass rod by TIR and so 'piped' along a twisted path as in figure. The beam is reflected from side to side practically without loss (except for that due to absorption in the glass) and emerges only at the end of the rod where it strikes the surface almost normally, *i.e.*, at an angle less than the critical angle. A single, very thin, solid glass fibre behaves in the same way and if several thousand are taped together a flexible light pipe is obtained that can be used, for example in medicine and engineering to illuminate an inaccessible spot. Optical fibres are now a days used to carry telephone, television and computer signals from one place to the other.

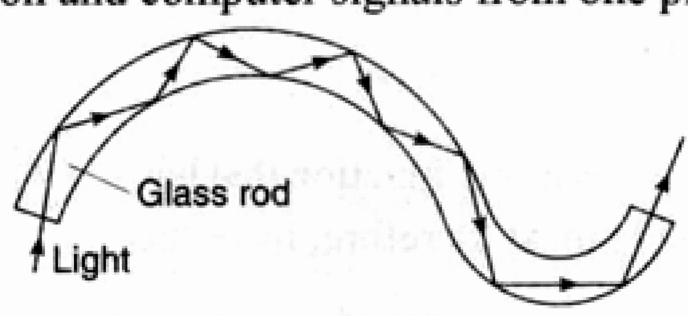


Fig. 28.42 Principle of an optical fibre.

Note *As we have seen*

$$\theta_c = \sin^{-1}\left(\frac{\mu_R}{\mu_D}\right)$$

Suppose we have two sets of media 1 *and* 2 *and*

$$\left(\frac{\mu_R}{\mu_D}\right)_1 < \left(\frac{\mu_R}{\mu_D}\right)_2$$

then $$(\theta_c)_1 < (\theta_c)_2$$

So, a ray of light has more chances to have TIR in case 1.

Sample Example 28.12 *An isotropic point source is placed at a depth h below the water surface. A floating opaque disc is placed on the surface of water so that the bulb is not visible from the surface. What is the minimum radius of the disc? Take refractive index of water* $= \mu$.

Solution

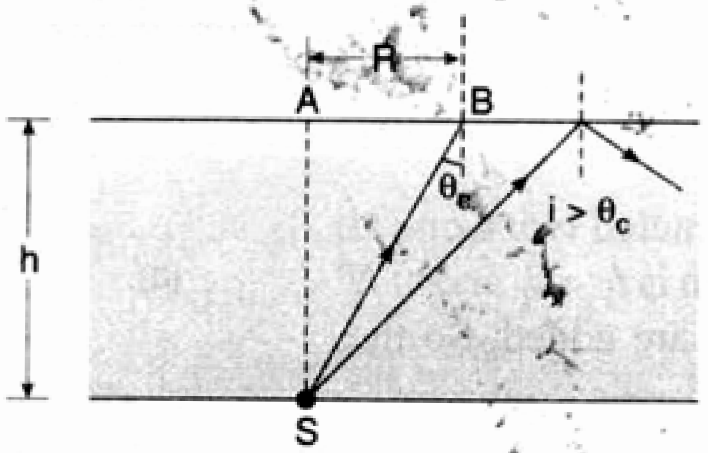


Fig. 28.43

As shown in figure light from the source will not emerge out of water if $i > \theta_c$.

Therefore, minimum radius R corresponds to $i = \theta_c$

In ΔSAB,

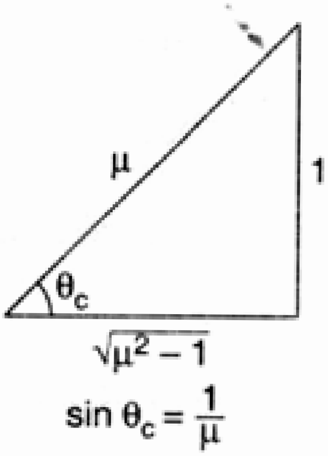


Fig. 28.44

$$\frac{R}{h} = \tan\theta_c$$

$\therefore$ $$R = h\tan\theta_c$$

or $$R = \frac{h}{\sqrt{\mu^2 - 1}}$$ **Ans.**

28.4 Refraction Through Prism

A prism has two plane surfaces AB and AC inclined to each other as shown in figure. $\angle A$ is called the **angle of prism** or **refracting angle.**

The importance of the prism really depends on the fact that the angle of deviation suffered by light at the first refracting surface, say AB (in 2-dimensional figure) is not cancelled out by the deviation at the second surface AC (as it is in a parallel glass slab), but is added to it. This is why it can be used in a spectrometer, an instrument for analysing light into its component colours.

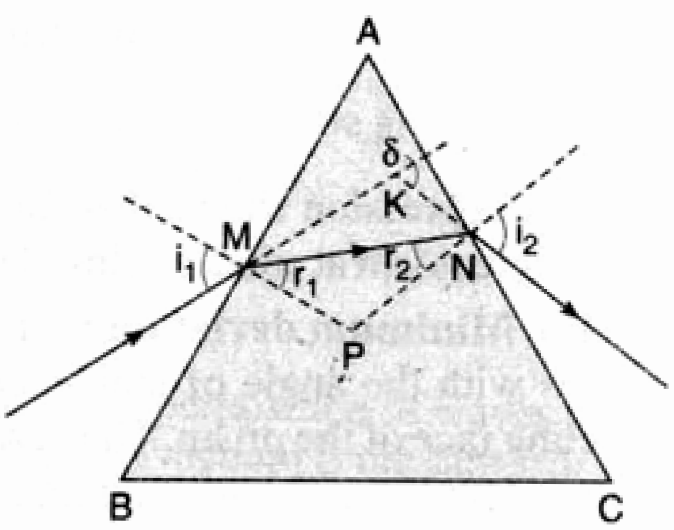


Fig. 28.45

General Formulae

(i) In quadrilateral $AMPN$, $$\angle AMP + \angle ANP = 180°$$

$\therefore$ $$A + \angle MPN = 180° \quad \ldots\text{(i)}$$

In triangle MNP, $$r_1 + r_2 + \angle MPN = 180° \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have $$r_1 + r_2 = A \quad \ldots\text{(iii)}$$

(ii) Deviation : Deviation δ means angle between incident ray and emergent ray.

In reflection

$$\delta = 180 - 2i = 180 - 2r$$

in refraction

$$\delta = |i - r|$$

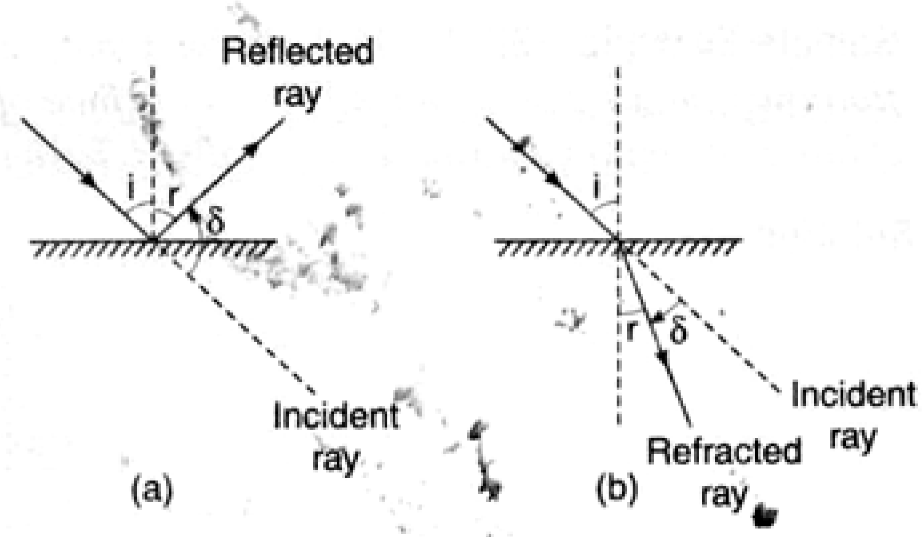


Fig. 28.46

In prism a ray of light gets refracted twice one at M and other at N. At M its deviation is $i_1 - r_1$ and at N it is $i_2 - r_2$. These two deviations are added. So the net deviation is,

$$\delta = (i_1 - r_1) + (i_2 - r_2) = (i_1 + i_2) - (r_1 + r_2) = (i_1 + i_2) - A$$

Thus, $$\delta = (i_1 + i_2) - A \quad \text{...(iv)}$$

(iii) If A and i_1 are small : The expression for the deviation in this case is basically used for developing the lens theory. Consider a ray falling almost normally in air on a prism of small angle A (less than about 6° or 0.1 radian) so that angle i_1 is small. Now $\mu = \frac{\sin i_1}{\sin r_1}$, therefore, r_1 will also be small. Hence, since sine of a small angle is nearly equal to the angle in radians, we have

$$i_1 = \mu r_1$$

Also, $A = r_1 + r_2$ and so if A and r_1 are small r_2 and i_2 will also be small. From $\mu = \frac{\sin i_2}{\sin r_2}$, we can say

$$i_2 = \mu r_2$$

Substituting these values in Eq. (iv), we have

$$\delta = (\mu r_1 + \mu r_2) - A = \mu(r_1 + r_2) - A = \mu A - A$$

or $$\delta = (\mu - 1) A \quad \text{...(v)}$$

This expression shows that for a given angle A all rays entering a small angle prism at small angles of incidence suffer the same deviation.

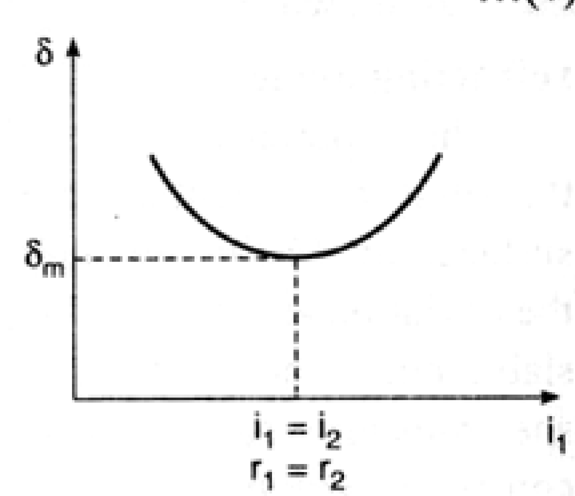


Fig. 28.47

(iv) Minimum deviation : It is found that the angle of deviation δ varies with the angle of incidence i_1 of the ray incident on the first refracting face of the prism. The variation is shown in figure and for one angle of incidence it has a minimum value δ_{min}. At this value the ray passes symmetrically through the prism (a fact that can be proved theoretically as well as be shown experimentally), *i.e.*, the angle of emergence of the ray from the second face equals the angle of incidence of the ray on the first face.

$$i_2 = i_1 = i \quad \text{...(vi)}$$

It therefore, follows that

$$r_1 = r_2 = r \quad \text{...(vii)}$$

From Eqs. (iii) and (vii) $$r = \frac{A}{2}$$

Further at $$\delta = \delta_m = (i + i) - A$$

or $$i = \frac{A + \delta_m}{2} \qquad ...(viii)$$

$\therefore$ $$\mu = \frac{\sin i}{\sin r}$$

or $$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin \frac{A}{2}} \qquad ...(ix)$$

(v) Condition of no emergence : In this section we want to find the condition such that a ray of light entering the face AB does not come out of the face AC for any value of angle i_1, *i.e.*, TIR takes place on AC

$$r_1 + r_2 = A$$

$\therefore$ $$r_2 = A - r_1$$

or $$(r_2)_{\min} = A - (r_1)_{\max} \qquad ...(x)$$

Now, r_1 will be maximum when i_1 is maximum and maximum value of i_1 can be 90°.

Hence, $$\mu = \frac{\sin(i_1)_{\max}}{\sin(r_1)_{\max}} = \frac{\sin 90°}{\sin (r_1)_{\max}}$$

$\therefore$ $$\sin(r_1)_{\max} = \frac{1}{\mu} = \sin \theta_c$$

$\therefore$ $$(r_1)_{\max} = \theta_c$$

$\therefore$ From Eq. (x), $$(r_2)_{\min} = A - \theta_c \qquad ...(xi)$$

Now, if minimum value of r_2 is greater than θ_c then obviously all values of r_2 will be greater than θ_c and TIR will take place under all conditions. Thus, the condition of no emergence is,

$$(r_2)_{\min} > \theta_c \quad \text{or} \quad A - \theta_c > \theta_c$$

or $$A > 2\theta_c \qquad ...(xii)$$

(vi) Dispersion and deviation of light by a prism : White light is a superposition of waves with wavelengths extending throughout the visible spectrum. The speed of light in vacuum is the same for all wavelengths, but the speed in a material substance is different for different wavelengths. Therefore, the index of refraction of a material depends on wavelength. In most materials the value of refractive index μ decreases with increasing wavelength.

If a beam of white light, which contains all colours, is sent through the prism, it is separated into a spectrum of colours. The spreading of light into its colour components is called **dispersion.**

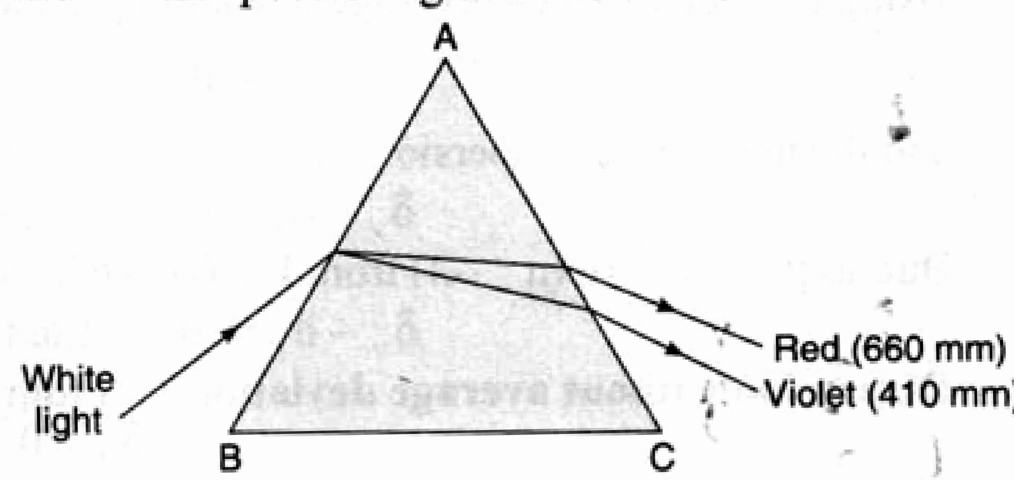


Fig. 28.48

Dispersive Power

When a beam of white light is passed through a prism of transparent material light of different wavelengths are deviated by different amounts. If δ_r, δ_y and δ_v are the deviations for red, yellow and violet components then average deviation is measured by δ_y as yellow light falls in between red and violet. $\delta_v - \delta_r$ is called **angular dispersion.** The **dispersive power** of a material is defined as the ratio of angular dispersion to the average deviation when a white beam of light is passed through it. It is denoted by ω. As we know

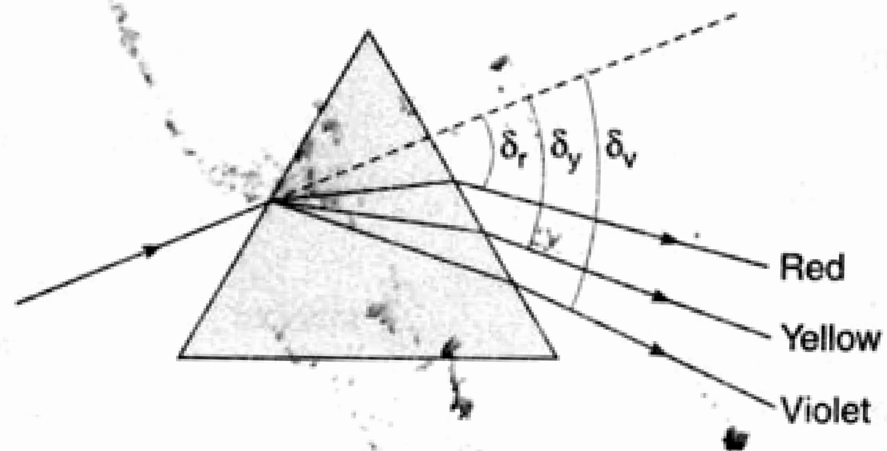


Fig. 28.49

$$\delta = (\mu - 1) A$$

This equation is valid when A and i are small. Suppose, a beam of white light is passed through such a prism, the deviation of red, yellow and violet light are

$$\delta_r = (\mu_r - 1)A, \qquad \delta_y = (\mu_y - 1)A$$

and
$$\delta_v = (\mu_v - 1) A$$

The angular dispersion is $\delta_v - \delta_r = (\mu_v - \mu_r) A$ and the average deviation is $\delta_y = (\mu_y - 1) A$. Thus, the dispersive power of the medium is,

$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} \qquad \text{...(i)}$$

Dispersion without average deviation and average deviation without dispersion

Figure shows two prisms of refracting angles A and A' and dispersive powers ω and ω' respectively. They are placed in contact in such a way that the two refracting angles are reversed with respect to each other. A ray of light passes through the combination as shown. The deviation produced by the two prisms are,

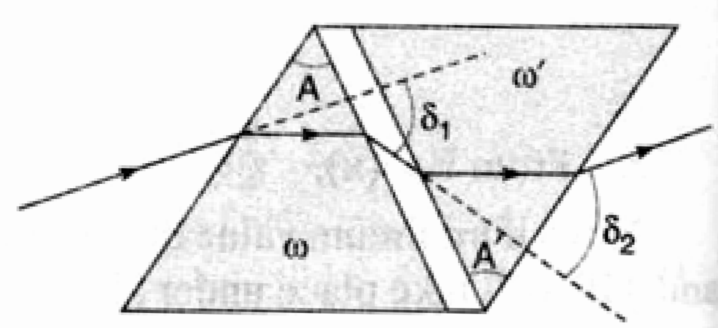


Fig. 28.50

$$\delta_1 = (\mu - 1) A$$

and
$$\delta_2 = (\mu' - 1) A'$$

As the two deviations are opposite to each other, the net deviation is,

$$\delta = \delta_1 - \delta_2 = (\mu - 1)A - (\mu' - 1) A' \qquad \text{...(ii)}$$

Using this equation, the average deviation produced by the combination if white light is passed is,

$$\delta_y = (\mu_y - 1)A - (\mu'_y - 1) A' \qquad \text{...(iii)}$$

and the net angular dispersion is,

$$\delta_v - \delta_r = (\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A'$$

But, as $\mu_v - \mu_Y = \omega(\mu_y - 1)$ from Eq. (i), we have

$$\delta_v - \delta_r = (\mu_y - 1)\omega A - (\mu'_y - 1)\omega' A' \qquad \text{...(iv)}$$

Dispersion without average deviation : From Eq. (iii),

$$\delta_Y = 0 \quad \text{if}$$

$$\frac{A}{A'} = \frac{\mu'_y - 1}{\mu_y - 1} \qquad \text{...(v)}$$

This is the required condition of dispersion without average deviation. Using this in Eq. (iv) the net angular dispersion produced is:

$$\delta_v - \delta_r = (\mu_y - 1)A(\omega - \omega')$$

Average deviation without dispersion : From Eq. (iv),

$$\delta_v - \delta_r = 0 \text{ if}$$

$$\frac{A}{A'} = \frac{(\mu'_y - 1)\omega'}{(\mu_y - 1)\omega} = \frac{\mu'_v - \mu'_r}{\mu_v - \mu_r} \quad \text{...(vi)}$$

This is the required condition of average deviation without dispersion. Using the above condition in Eq. (iii) the net average deviation is,

$$\delta_Y = (\mu_y - 1)A\left(1 - \frac{\omega}{\omega'}\right)$$

Sample Example 28.13 *An isosceles glass prism has one of its faces coated with silver. A ray of light is incident normally on the other face (which is equal to the silvered face). The ray of light is reflected twice on the same sized faces and then emerges through the base of the prism perpendicularly. Find angles of prism.*

Note *Most of the problems of prims are easily solved by drawing proper ray diagram and then applying laws of geometry with the basic knowledge of prism formulae.*

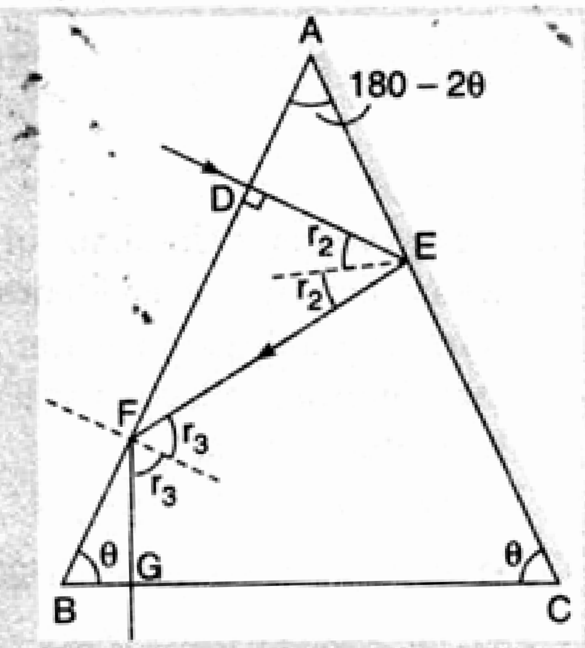


Fig. 28.51

Solution

$$r_1 = 0 \quad \therefore \quad r_2 = A = 180° - 2\theta \quad \text{...(i)}$$

$$\angle DFE = 180° - 90° - 2r_2$$

$$= 180° - 90° - 360° + 4\theta$$

$$= 4\theta - 270° \quad \text{...(ii)}$$

$\therefore$

$$r_3 = 90° - \angle DFE = 360° - 4\theta \quad \text{...(iii)}$$

$$\angle BFG = 90° - \theta = 90° - r_3$$

or

$$r_3 = \theta \quad \text{...(iv)}$$

From Eqs. (iii) and (iv)

$$5\theta = 360°$$

$\therefore$

$$\theta = 72°$$

and

$$180° - 2\theta = 36°$$

$\therefore$ Angles of prism are 72°, 72° and 36°. **Ans.**

Introductory Exercise 28.4

1. The prism shown in figure has a refractive index of 1.60 and the angles A are 30°. Two light rays P and Q are parallel as they enter the prism. What is the angle between them after they emerge? [$\sin^{-1}(0.8) = 53°$]

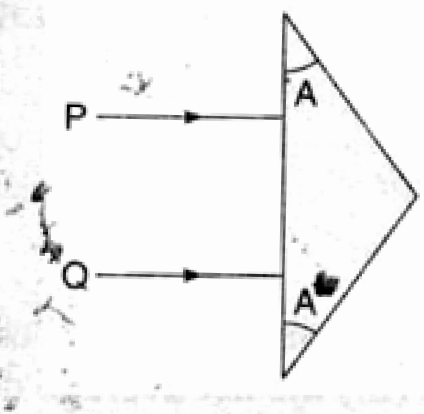


Fig. 28.52

2. Light is incident normally on the short face of a 30° – 60° – 90° prism. A liquid is poured on the hypotenuse of the prism. If the refractive index of the prism is $\sqrt{3}$, find the maximum refractive index of the liquid so that light is totally reflected.

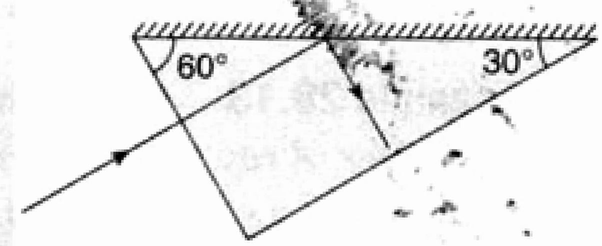


Fig. 28.53

3. A glass vessel in the shape of a triangular prism is filled with water, and light is incident normally on the face XY. If the refractive indices for water and glass are 4/3 and 3/2 respectively, total internal reflection will occur at the glass-air surface XZ only for $\sin\theta$ greater than

A 1/2 **B** 2/3 **C** 3/4
D 8/9 **E** 16/27.

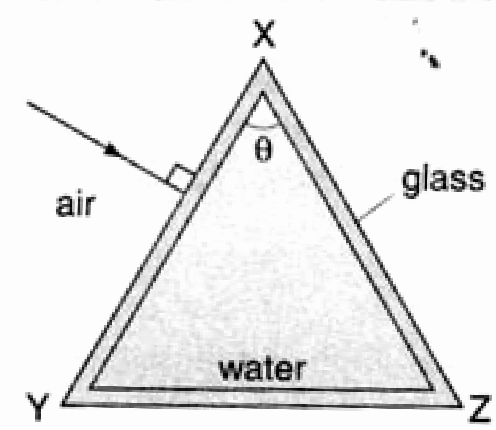


Fig. 28.54

4. A parallel beam of light is incident on a prism shown in figure. Through what angle should the mirror be rotated so that light returns back to its original path? Refractive index of prism is 1.5.

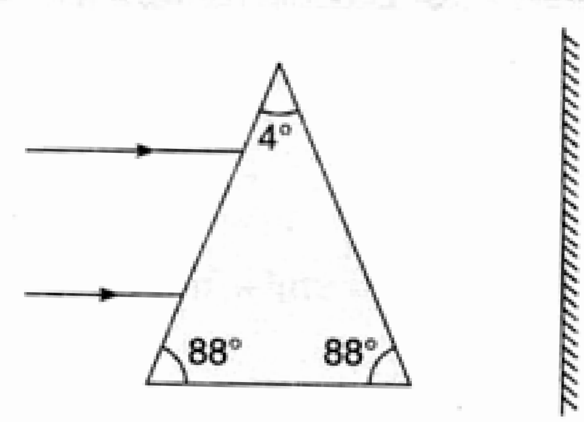


Fig. 28.55

5. A light ray going through a prism with the angle of prism 60°, is found to deviate at least by 30°. What is the range of the refractive index of the prism?
6. A ray of light falls normally on a refracting face of a prism. Find the angle of prism if the ray just fails to emerge from the prism ($\mu = 3/2$).
7. A ray of light is incident at an angle of 60° on one face of a prism which has an angle of 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges and calculate the refractive index of the material of prism.

8. A ray of light passing through a prism having refractive index $\sqrt{2}$ suffers minimum deviation. It is found that the angle of incidence is double the angle of refraction within the prism. What is the angle of prism?

9. A ray of light undergoes deviation of 30° when incident on an equilateral prism of refractive index $\sqrt{2}$. What is the angle subtended by the ray inside the prism with the base of the prism?

10. Light is incident at an angle i on one planar end of a transparent cylindrical rod of refractive index μ. Find the least value of μ so that the light entering the rod does not emerge from the curved surface of the rod irrespective of the value of i.

Fig. 28.56

11. The refractive index of the material of a prism of refracting angle 45° is 1.6 for a certain monochromatic ray. What will be the minimum angle of incidence of this ray on the prism so that no TIR takes place as the ray comes out of the prism.

Extra Points

- **Under what conditions does an image coincides with object :** Real image coincide with object only when there is a mirror and the rays fall normally on the mirror. In case of a spherical mirror it is possible when object for the mirror lies at it centre of curvature, *i.e.*, at a distance $2f$ from the mirror. If the virtual image coincides with the object then there should be no mirror at all. Many possibilities may exist in this case. One of them is shown in figure.

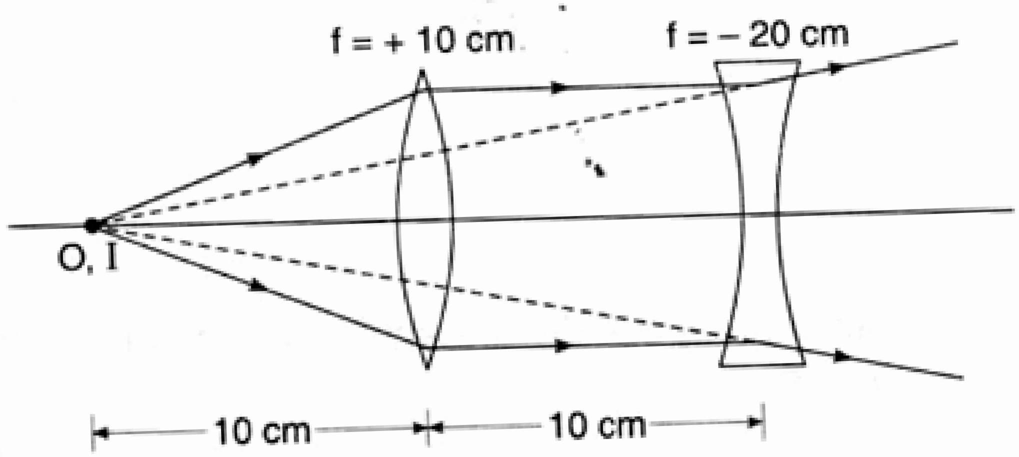


Fig. 28.57

- **To find the position of image when one face of a lens is silvered :** Let us make a formula to find the position of image under such condition.

  Ray of light is first refracted, then reflected and then again refracted. In first two steps light is travelling from left to right and in the last one direction of light is reversed. But we will take one sign convention, *i.e.*, left to right as positive and in the last step will take v, u and R as negative.

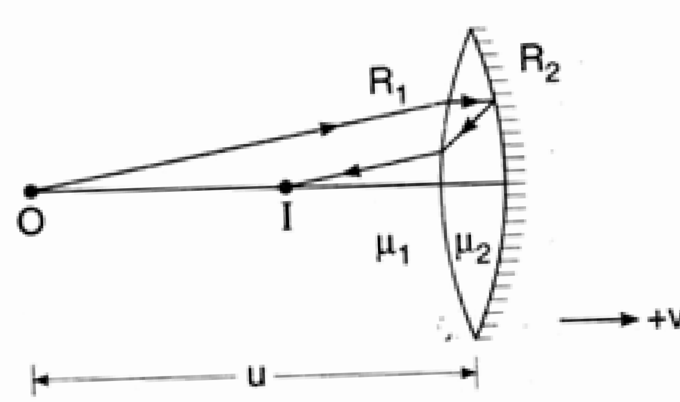


Fig. 28.58

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \quad \text{...(i)}$$

$$\frac{1}{v_2} + \frac{1}{v_1} = \frac{2}{R_2} \quad \text{...(ii)}$$

$$\frac{\mu_1}{-v} - \frac{\mu_2}{-v_2} = \frac{\mu_1 - \mu_2}{-R_1} \quad \text{...(iii)}$$

Solving Eqs. (i), (ii) and (iii), we get

$$\frac{1}{v} + \frac{1}{u} = \frac{2(\mu_2/\mu_1)}{R_2} - \frac{2(\mu_2/\mu_1 - 1)}{R_1} \quad \text{...(iv)}$$

This is the desired formula for finding position of image for the given situation.

Note *The given system finally behaves as a mirror. Whose focal length can be found by comparing Eq. (iv) with mirror formula $1/v + 1/u = 1/f$.*

$$\frac{1}{f} = \frac{2(\mu_2/\mu_1)}{R_2} - \frac{2(\mu_2/\mu_1 - 1)}{R_1} \qquad \text{...(v)}$$

- To find the total deviation of a ray of light, its sense of rotation (clockwise or anticlockwise) should be kept in mind in each reflection and refraction and they should be added and subtracted accordingly.

  For example, in the figure shown

  $\delta_A = i_1 - r_1$ (clockwise)

  $\delta_B = i_2 - r_1$ (anticlockwise)

  $\delta_C = 180° - 2i_2$ (anticlockwise)

  $\therefore \quad \delta_{Total} = \delta_A - \delta_B - \delta_C$

  $= (i_1 - r_1) - (i_2 - r_1) - (180° - 2i_2)$

  (clockwise)

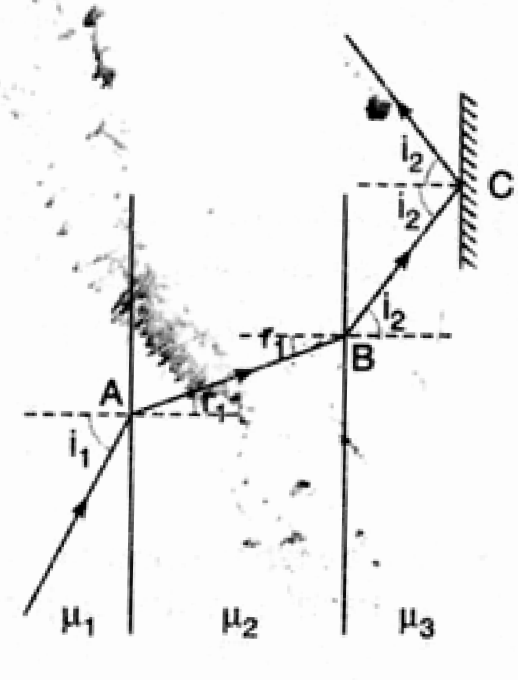


Fig. 28.59

- Equation $r_1 + r_2 = A$ can be applied at any of the three vertices. For example in the figure shown, $r_1 + r_2 = B$.

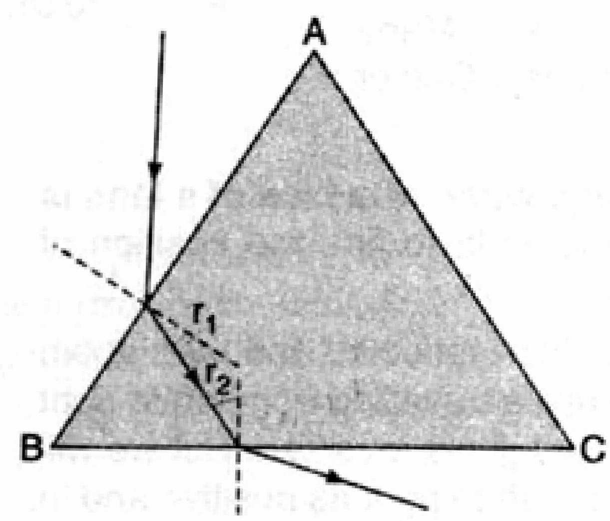


Fig. 28.60

- Sometimes a part of a prism is given and we keep on thinking whether how should we proceed? To solve such problems first complete the prism then solve as the problems of prism are solved.

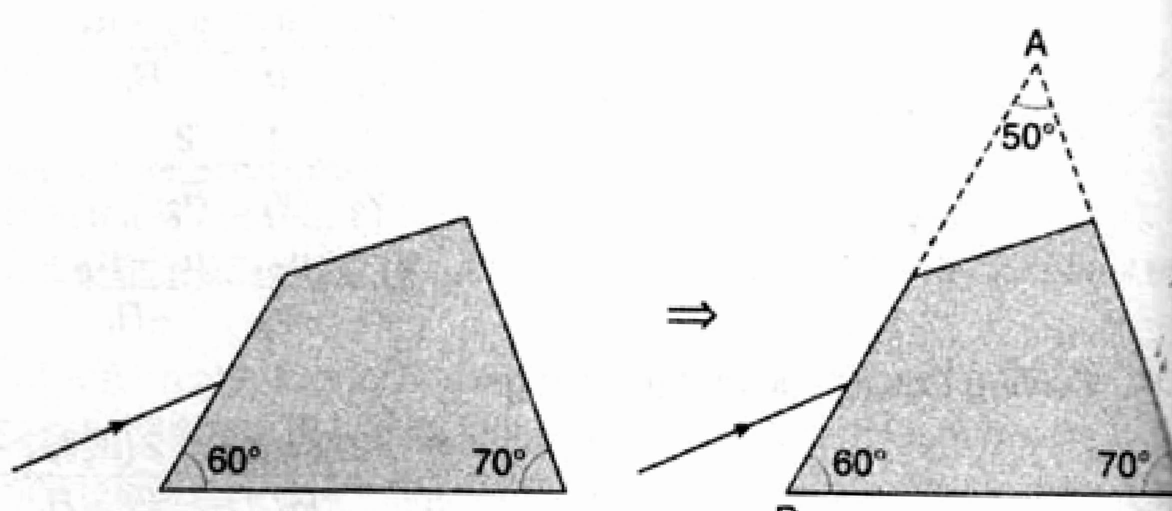


Fig. 28.61

- A lens made of three different materials have three focal lengths. Thus, for a given object there are three images.

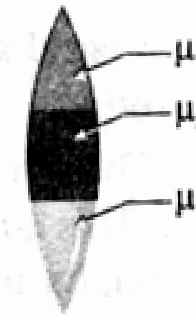


Fig. 28.62

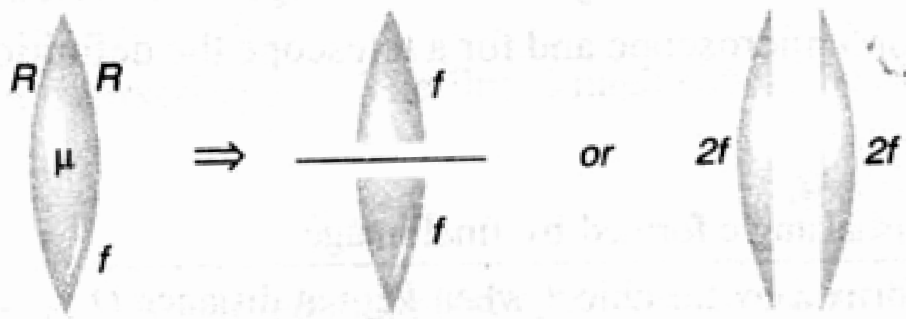
- 

Fig. 28.63

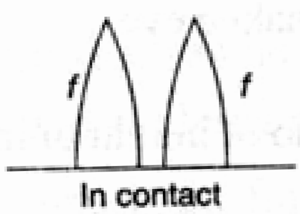
- 

Fig. 28.64

The resultant focal length in this case is $\frac{f}{2}$.

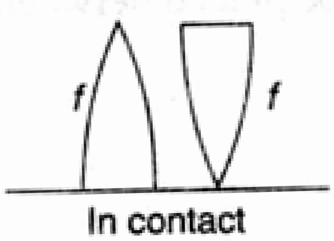
- 

Fig. 28.65

The resultant focal length in this case is ∞ or power is zero. This is because optic axes of both parts have been inverted.

27.5 Optical Instruments

Optical instruments are used to assist the eye in viewing an object. Our eye lens has a power to adjust its focal length to see the nearer objects. This process of adjusting focal length is called accommodation. However if the object is brought too close to the eye, the focal length cannot be adjusted to form the image on the retina. Thus, there is a minimum distance for the clear vision of an object. This distance is called **least distance of distinct vision (+D).** For normal eye this distance is generally taken to be 25 cm.

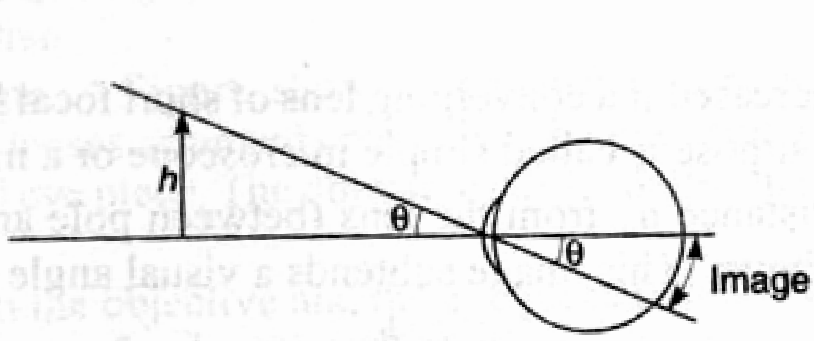


Fig. 28.66

Visual angle : The size of an object as sensed by us is related to the size of the image formed on the retina.

The size of the image on the retina is roughly proportional to the angle subtended by the object on the eye. This angle is known as the visual angle. Optical instruments are used to increase this angle artificially in order to improve the clarity.

Magnifying power (M) : Magnifying power is the factor by which the image on the retina can be enlarged by using the microscope or telescope. For a microscope and for a telescope the definition of M is slightly different.

For a microscope,

$$M = \frac{\text{Visual angle formed by final image}}{\text{Visual angle formed by the object when kept at distance } D}$$

For a telescope,

$$M = \frac{\text{Visual angle formed by final image}}{\text{Visual angle subtended by the object directly when seen from naked eye}}$$

Note that M is different from linear magnification $m \left(= \pm \frac{v}{u} \right)$ which is the ratio of height of image to that of object. While M is the ratio of apparent increase in size of image seen by the eye. Unit of M is X. Thus, we write an angular magnification of 10 as $10X$.

Simple microscope : To view an object with naked eyes, the object must be placed between D and infinity. The maximum angle is subtended when it is placed at D.

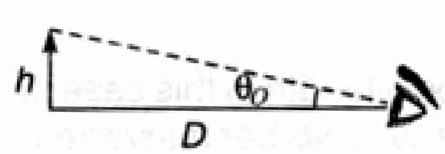


Fig. 28.67

Say this angle is θ_o. Then, $\theta_o = \frac{h}{D}$

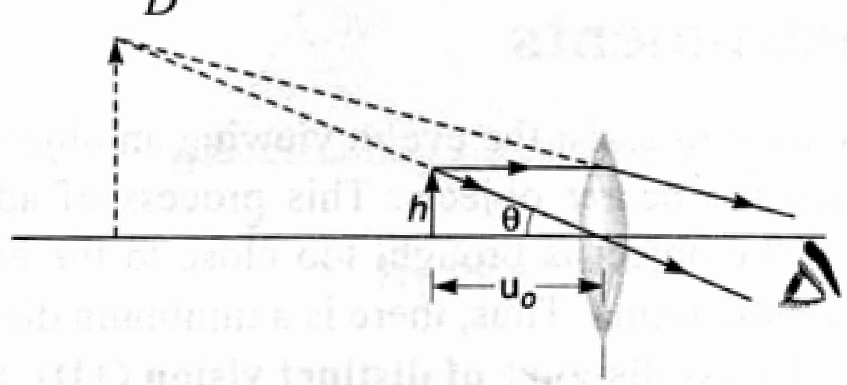


Fig. 28.68

This angle can be further increased if a converging lens of short focal length is placed just infront of the eye. The lens used for this purpose is called simple microscope or a magnifier.

The object is placed at a distance u_o from the lens (between pole and focus of lens). The virtual magnified image is formed as shown. This image subtends a visual angle say θ on the eye. Then,

$$\theta = \frac{h}{u_o}$$

From the definition of magnifying power for a microscope,

$$M = \frac{\theta}{\theta_o} = \frac{h/u_o}{h/D}$$

$\therefore$
$$M = \frac{D}{u_o}$$

For relaxed eye : The final image should be at infinity. Thus, $u_o = f$

$\therefore$
$$M_\infty = \frac{D}{f}$$

This is also called magnifying power for normal adjustment.
We can see that $M_\infty > 1$ if $f < D$.

Magnifying power when final image is at D : In the above case we saw that M is equal $\frac{D}{f}$. The magnification can be made large by chosing the focal length f small.

The magnifying power can be increased in an another way be moving the object still closer to the lens. Suppose, the final virtual image is formed at a distance D. Then, from the equation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we have

$$\frac{1}{-D} + \frac{1}{u_o} = \frac{1}{f}$$

or
$$\frac{1}{u_o} = \frac{1}{D} + \frac{1}{f}$$

Substituting this value in the equation $M = \frac{D}{u_o}$, we have

$$M_D = 1 + \frac{D}{f}$$

Note (i) *That $M_D > M_\infty$, i.e., when final image is formed at 25 cm, angular magnification is increased but eye is most strained. On the other hand when final image is at infinity, angular magnification is slightly less but eye is relaxed. So, the choice is yours whether you want to see bigger size with strained eye or smaller size with relaxed eye.*

(ii) *That M can be increased by decreasing f, but due to several other aberrations the image becomes too defective at large magnification with a simple microscope. Roughly speaking a magnification upto 4 is trouble free.*

Compound Microscope : Figure shows a simplified version of a compound microscope. It consists of two converging lenses arranged coaxially. The one facing the object is called objective and the one close to eye is called eye piece. The objective has a smaller aperture and smaller focal length than those of the eye piece.

The separation between the objective and the eye piece (called the length of the microscope L) can be varied by appropriate screws fixed on the panel of microscope.

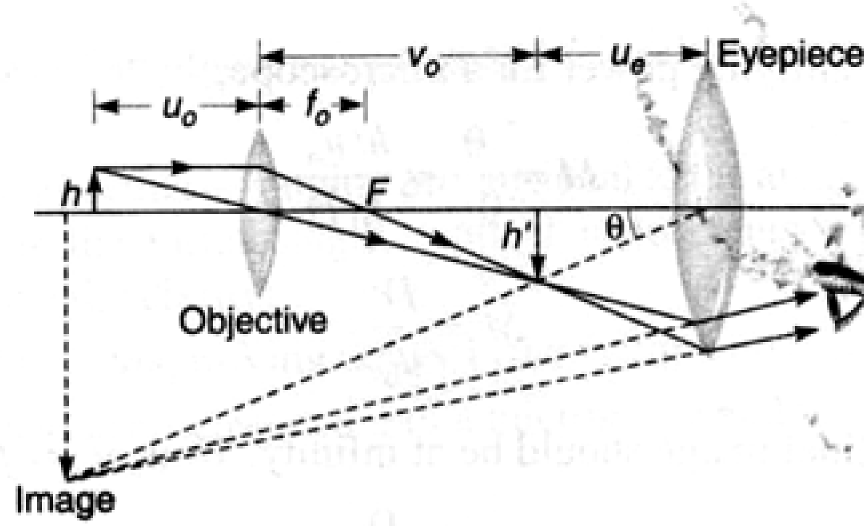


Fig. 28.69

The object is placed beyond first focus of objective, so that an inverted and real image (intermediate image) is formed by the objective. This intermediate image acts as an object for the eye piece and lies between first focus and pole of eye piece. The final magnified virtual image is formed by the eye piece. Let θ be the angle subtended by the final image on the eye, then,

$$\theta = \frac{h'}{u_e}$$

Here, h' is the height of the first image and u_e is its distance from the eye piece.

Further $$\theta_o = \frac{h}{D}$$

∴ Magnifying power of the compound microscope will be,

$$M = \frac{\theta}{\theta_o} = \frac{h'}{u_e} \times \frac{D}{h} = \left(\frac{h'}{h}\right)\left(\frac{D}{u_e}\right)$$

Here, $\frac{h'}{h}$ is the linear magnification by the objective. Thus,

$$\frac{h'}{h} = |m_o| = \frac{v_o}{u_o}$$

∴ $$M = \frac{v_o}{u_o}\left(\frac{D}{u_e}\right)$$

Length of the microscope will be,

$$L = v_o + u_e$$

For relaxed eye : For relaxed eye final image should be at infinity. Or,

$$u_e = f_e$$

∴ $$M_\infty = \frac{v_o}{u_o}\frac{D}{f_e}$$

and $$L_\infty = v_o + f_e$$

Final image at *D* : When the final image (by eye piece) is formed at *D*. Then by the formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we have,

$$\frac{1}{-D}+\frac{1}{u_e}=\frac{1}{f_e}$$

∴ $$\frac{1}{u_e}=\frac{1}{D}+\frac{1}{f_e}$$

or $$u_e=\frac{Df_e}{D+f_e}$$

Thus, $$M_D=\frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right)$$

and $$L_D=v_o+\frac{Df_e}{D+f_e}$$

Telescopes : A microscope is used to view the objects placed closed to it. To look at distant objects such as a star, a planet or a distant tree etc., we use telescopes. There are three types of telescopes in use.

(i) Astronomical telescope,
(ii) Terrestrial telescope and
(iii) Galilean telescope.

(i) Astronomical telescope : Figure shows the construction and working of an astronomical telescope.

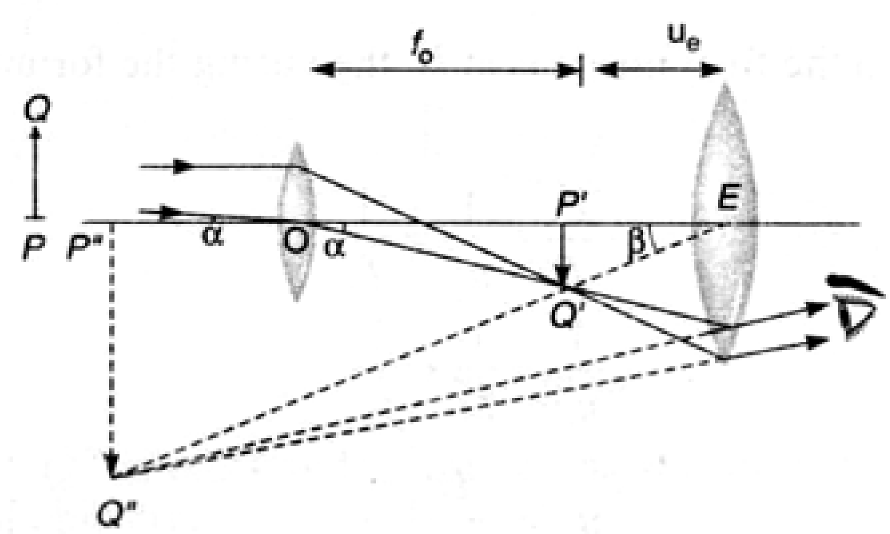


Fig. 28.70

It consists of two converging lenses placed coaxially. The one facing the distant object is called the objective and has a large aperture and large focal length. The other is called the eye piece, as the eye is placed closed to it. The eye piece tube can slide within the objective tube, so that the separation between the objective and the eye piece may be varied.

Magnifying power : Although a telescope can also be used to view the objects of few kilometers away but the magnifying power calculated below is for the case when object is at infinity. Rays coming from the object in that case will be almost parallel.

The image formed by objective will be at its second focus. This image called the intermediate image will act as the object for eye piece. This usually lies between pole and first focus of eye piece. So, that eye piece forms a virtual and magnified image of it.

$|\alpha|$ = angle subtended by object on objective (or you can say at eye)

$$= \frac{P'Q'}{f_o}$$

$|\beta|$ = angle subtended by final image at eye piece (or at eye)

$$= \frac{P'\theta'}{u_e}$$

From the definition of magnifying power (for telescope),

$$M = \frac{|\beta|}{|\alpha|} = \frac{f_o}{u_e}$$

or

$$M = \frac{f_o}{u_e}$$

and length of telescope,

$$L = f_o + u_e$$

For relaxed eye : For relaxed eye, intermediate image should lie at first focus of eye piece or

$$u_e = f_e$$

$\therefore$

$$M_\infty = \frac{f_o}{f_e} \quad \text{and} \quad L_\infty = f_o + f_e$$

Final image at *D* : When the final image is at *D*, then using the formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ for eye piece we have,

$$\frac{1}{-D} + \frac{1}{u_e} = \frac{1}{f_e}$$

$\therefore$

$$\frac{1}{u_e} = \frac{1}{D} + \frac{1}{f_e}$$

or

$$u_e = \frac{Df_e}{D + f_e}$$

Therefore,

$$M_D = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$$

and

$$L_D = f_o + \frac{Df_e}{D + f_e}$$

(ii) Terrestrial telescope : In an astronomical telescope, the final image is inverted with respect to the object. To remove this difficulty, a convex lens of focal length *f* is included between the objective and the eye piece in such a way that the focal plane of the objective is a distance 2*f* away from this lens.

The role of the intermediate lens *L* is only to invert the image. The magnification produced by it is – 1. The formulae of *M* does not change at all. They remain as it is, as were derived for astronomical telescope. The length of telescope will however increase by 4*f*. Here, you should note that we are talking only about magnitude of *M*. Thus,

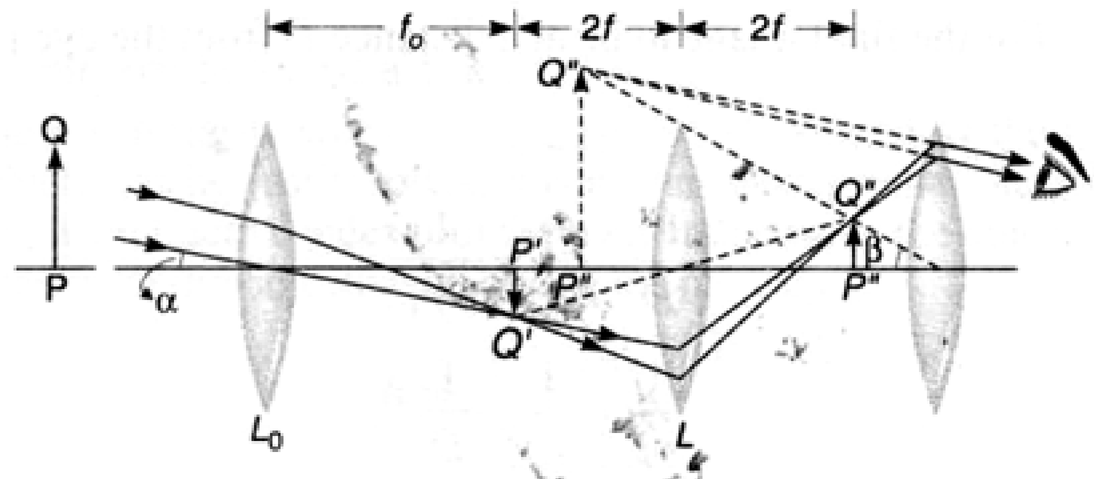


Fig. 28.71

$$M_\infty = \frac{f_o}{f_e} \quad \text{and} \quad M_D = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$$

$$L_\infty = f_o + 4f + f_e \quad \text{and} \quad L_D = f_o + 4f + \frac{Df_e}{D + f_e}$$

(iii) Galilean telescope : Figure shows a simple model of Galilean telescope. A convergent lens is used as the objective and a divergent lens as the eye piece. The objective lens forms a real and inverted image $P'Q'$ but the divergent lens comes in between. This intermediate image acts as virtual object for

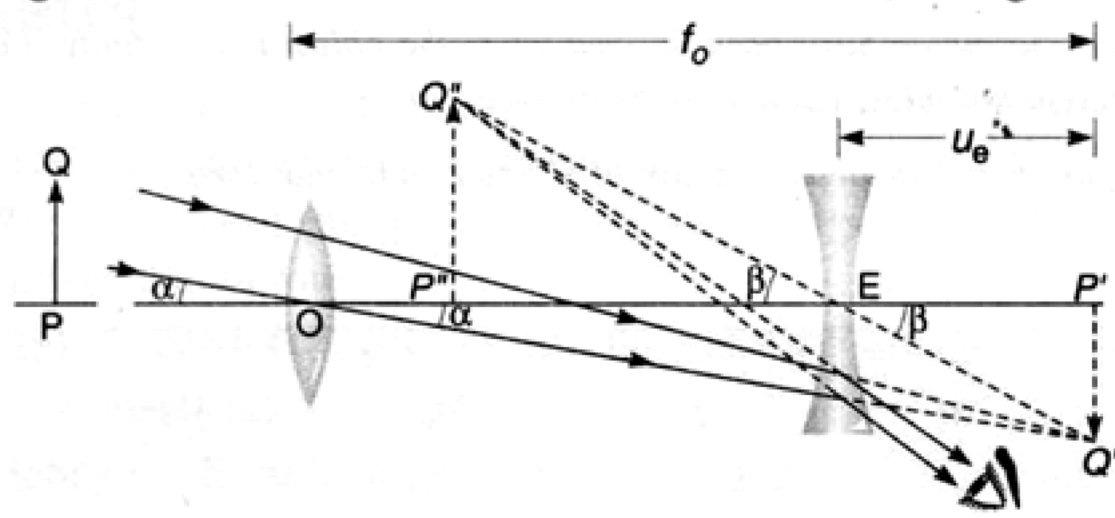


Fig. 28.72

eye piece. Final image $P''Q''$ is erect and magnified as shown in figure. The intermediate image is formed at second focus of objective.

Magnifying power : From the figure, we can see that

$$|\alpha| = \frac{P'Q'}{f_o} \quad \text{and} \quad |\beta| = \frac{P'Q'}{u_e}$$

From the definition of magnifying power for telescope,

$$M = \frac{|\beta|}{|\alpha|} = \frac{f_o}{u_e}$$

and length of the telescope, $\quad L = f_o - u_e$

For relaxed eye : For relaxed eye intermediate image should lie at first focus of eye piece. Or,

$$u_e = f_e$$

Hence, $$M_\infty = \frac{f_o}{f_e} \quad \text{and} \quad L_\infty = f_o - f_e$$

Final image at D : For the final image to be at a distance D from the eye piece, we have from the formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{-D}-\frac{1}{u_e}=\frac{1}{-f_e}$$

$\therefore$
$$\frac{1}{u_e}=\frac{1}{f_e}-\frac{1}{D}$$

or
$$u_e=\frac{Df_e}{D-f_e}$$

Thus,
$$M_D=\frac{f_o}{f_e}\left(1-\frac{f_e}{D}\right)$$

and
$$L_D=f_o-\frac{f_eD}{D-f_e}$$

Note *(i) In all above formulae of M, we are considering only the magnitude of M.*

(ii) For telescopes, formulae have been derived when the object is at infinity. For the object at some finite distance different formulae will have to be derived.

(iii) Given below are formulae derived above of M and L in tabular form.

Table 28.2

Name of Optical Instruments	M	L	M_∞	M_D	L_∞	L_D
Simple Microscope	$\frac{D}{u_o}$	—	$\frac{D}{f}$	$1+\frac{D}{f}$	—	—
Compound Microscope	$\frac{v_o}{u_o}\frac{D}{u_e}$	v_o+u_e	$\frac{v_o}{u_o}\frac{D}{f_e}$	$\frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right)$	v_o+f_e	$v_o+\frac{Df_e}{D+f_e}$
Astronomical Telescope	$\frac{f_o}{u_e}$	f_o+u_e	$\frac{f_o}{f_e}$	$\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$	f_o+f_e	$f_o+\frac{Df_e}{D+f_e}$
Terrestrial Telescope	— do —	f_o+4f+u_e	— do —	— do —	f_o+4f+f_e	$f_o+4f+\frac{Df_e}{D+f_e}$
Galilean Telescope	$\frac{f_o}{u_e}$	f_o-u_e	$\frac{f_o}{f_e}$	$\frac{f_o}{f_e}\left(1-\frac{f_e}{D}\right)$	f_o-f_e	$f_o-\frac{f_eD}{D-f_e}$

Resolving power of a microscope and a telescope

Microscope : The resolving power of a microscope is defined as the reciprocal of the distance between two objects which can be just resolved when seen through the microscope. It depends on the wavelength λ of the light, the refractive index μ of the medium between the object and the objective and the angle θ subtended by a radius of the objective on one of the objects.

$$R = \frac{1}{\Delta d} = \frac{2\mu \sin\theta}{\lambda}$$

To increase R, objective and object are immersed in oil.

Telescope : The resolving power of a telescope is defined as the reciprocal of the angular separation between two distant objects which are just resolved by a telescope.

It is given by,

$$R = \frac{1}{\Delta\theta} = \frac{a}{1.22\lambda}$$

Here, a is the diameter of the objective. That is why, telescopes with larger objective aperture are used.

28.6 Photometry

The branch of optics which deals with the light emitting capacity of a source and the illumination produced by it on a surface is called photometry. Let us first study few definitions.

(i) **Radiant flux (R) :** The total energy radiated by a source per second is called the radiant flux. The SI unit of radiant flux is watt.

(ii) **Luminous flux (ϕ) :** The total light energy emitted by a source per second is called luminous flux (ϕ). Its unit is lumen (lm).

(iii) **Luminous efficiency :** Total luminous flux per unit radiant flux is called luminous efficiency. Thus,

$$\text{Luminous efficiency } \eta = \frac{\text{Total luminous flux}}{\text{Total radiant flux}} = \frac{\phi}{R}$$

The unit of luminous efficiency is lumen/watt.

(iv) **Luminous intensity (L) :** To understand luminous intensity let us first discuss the solid angle.

Solid angle (ω) : The solid angle measures the angular divergence of a cone and is defined as,

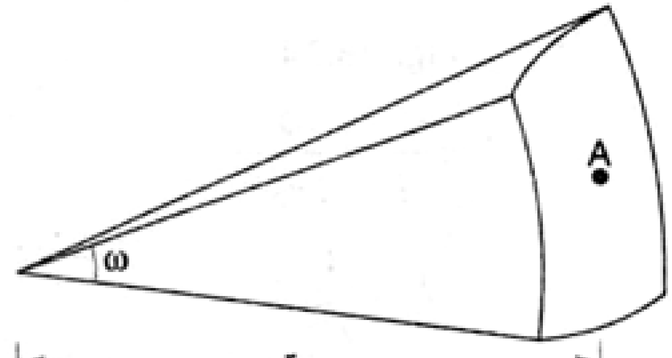


Fig. 28.73

$$\omega = \frac{A}{r^2}$$

Here, A is the area intercepted by the cone on a sphere of radius r centred at the apex of the cone. The solid angle does not depend on the radius of the sphere. The SI unit of solid angle is steradian.

Total solid angle around a point source,

$$\omega = \frac{A}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi \text{ steradian}$$

Luminuous intensity (L) : In a given direction it is defined as the luminous flux per unit solid angle. Thus,

$$L = \frac{\Delta\phi}{\Delta\omega}$$

The SI unit of luminous intensity is lumen per steradian and is called **candela (cd).**

For an isotropic point source,

$$L = \frac{\phi_{\text{Total}}}{\omega_{\text{Total}}} = \frac{\phi}{4\pi}$$

or $$\phi = 4\pi L$$

Note *That candela is one of the seven base units in international system of units.*

(v) Illuminance (I) : "The luminous flux incident per unit area of a surface is called illuminance or intensity of illumination." Thus,

$$I = \frac{\Delta\phi}{\Delta A}$$

The SI unit of illuminance is lumen/m^2 and is called **lux (lx).** Another unit is phot.

$$1 \text{ phot} = 1 \frac{\text{lm}}{\text{cm}^2}$$

For a point source : At a distance r from the point source,

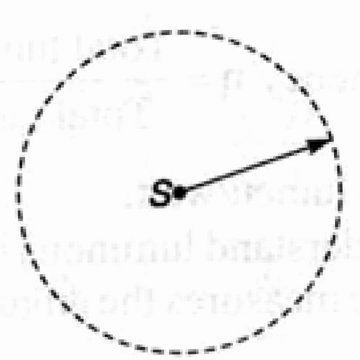


Fig. 28.74

$$I = \frac{\phi}{A} = \frac{\phi}{4\pi r^2}$$

or $$I \propto \frac{1}{r^2}$$

For a line source : At a distance r from the line source,

$$I = \frac{\phi}{2\pi r l}$$

or $$I \propto \frac{1}{r}$$

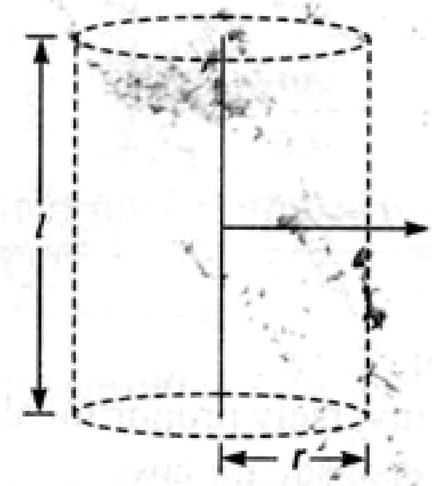


Fig. 28.75

For a parallel beam of light : For a parallel beam of light normal area does not change with distance. So, illuminance is independent of r.

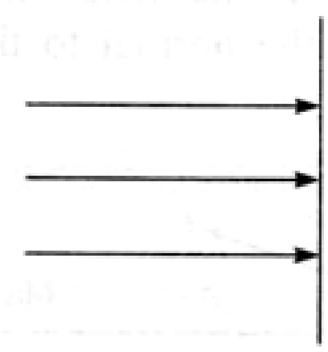


Fig. 28.76

Relation between luminous intensity and illuminance : Suppose, luminous flux $d\phi$ is incident on area dA at an angle of incidence θ as shown in figure. Then by definition,

$$I = \frac{d\phi}{dA} \quad \text{and} \quad L = \frac{d\phi}{d\omega}$$

From these two relations,

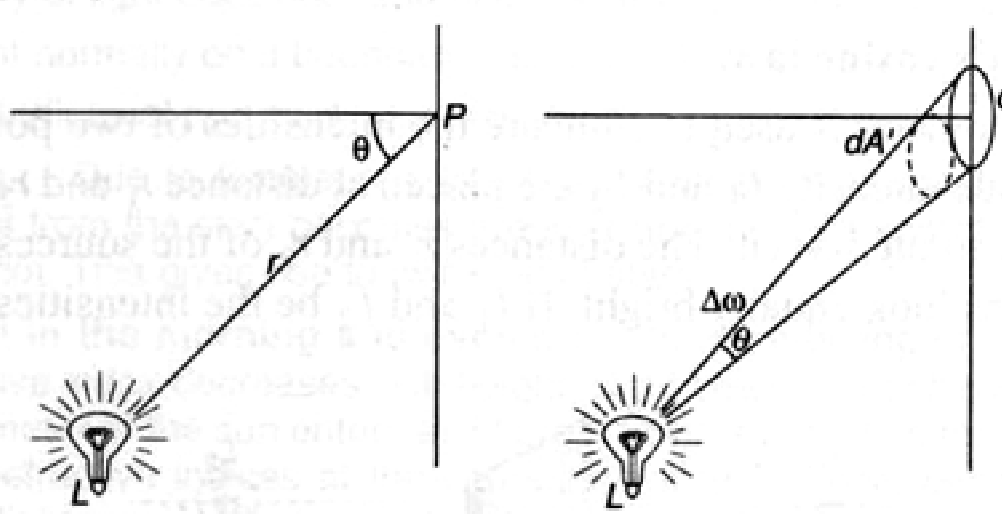


Fig. 28.77

$$I = L \cdot \frac{d\omega}{dA}$$

But for an isotropic point source,

$$d\omega = \frac{dA'}{r^2} = \frac{dA\cos\theta}{r^2}$$

or

$$\frac{d\omega}{dA} = \frac{\cos\theta}{r^2}$$

Hence,

$$I = \frac{L\cos\theta}{r^2} \quad \text{or} \quad I \propto \frac{1}{r^2}$$

From the above equation we note that,

(i) the illuminance of a small area is inversely proportional to the square of the distance of the area from the source. This is known as the inverse square law.

(ii) the illuminance of a small area is proportional to $\cos\theta$, where θ is the angle made by the normal to the area with the direction of incident radiation.

Lambert's cosine law : An ideal point source emits radiation uniformly in all directions. In general sources are extended and such a source has different luminous intensity in different directions. The intensity is maximum (say I_0) along the normal to the surface and decreases as we consider directions away from this normal.

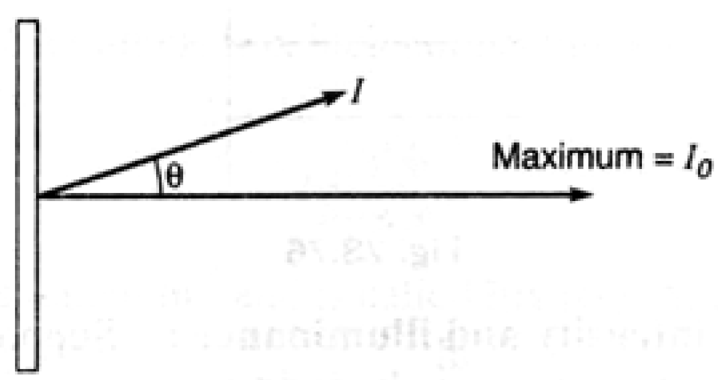


Fig. 28.78

In a direction making an angle θ with the normal the intensity is,

$$I = I_0 \cos\theta$$

This is called **Lambert's cosine law.**

Photometer : A photometer is used to compare the intensities of two point sources.

Two sources of luminous intensity L_1 and L_2 are placed at distance r_1 and r_2 from the screen, so that their flux are perpendicular to the screen. The distances r_1 and r_2 of the sources from the screens are so adjusted that the two screens look equally bright. If I_1 and I_2 be the intensities of the sources we must have,

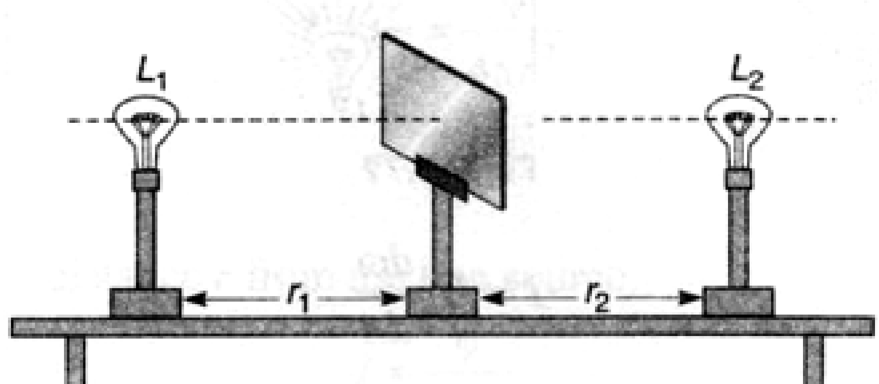


Fig. 28.79

$$I_1 = I_2$$

or

$$\frac{L_1}{r_1^2} = \frac{L_2}{r_2^2}$$

$\therefore$

$$\frac{L_1}{L_2} = \left(\frac{r_1}{r_2}\right)^2$$

Note *The terms/formulae used in photometry are listed below :*

(i) Radiant flux (R) has the unit watt.

(ii) Luminous flux (ϕ) has the unit Lumen (lm).

(iii) Luminous efficiency $\eta = \frac{\phi}{R}$ *(lumen/watt).*

(iv) Luminous intensity $L = \frac{d\phi}{d\omega}$ *candela (cd).*

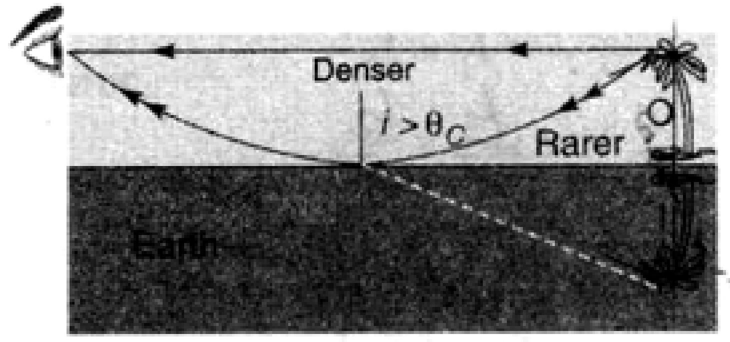
(v) Illuminance $I = \frac{d\phi}{dA}$ *lux (lx).*

(vi) $I = \frac{L\cos\theta}{r^2}$

(vii) $I = I_0 \cos\theta$.

• *Extra points*

1. Eye is most sensitive to yellow-green light ($\lambda = 5550$ Å).
2. Photographic plate is most sensitive to blue and least to red.
3. Persistance of eye is $\frac{1}{10}$ sec, *i.e.*, if time interval between two successive light pulses is lesser than 0.1 second, eye cannot distinguish them separately.
4. Frequency of visible light is of the order of 10^{15} Hz.
5. Colour of light is determined by its frequency and not the wavelength. During refraction of light frequency and colour of light do not change.
6. During refraction, ray of light does not bend under following two conditions.
   (i) If light is incident normally on a boundary, *i.e.*, $\angle i = 0$.
   (ii) If the refractive indices of two media are equal, *i.e.*, $\mu_1 = \mu_2$.
7. **Twinkling of stars :** Due to fluctuations in refractive index of atmosphere the refraction of light (reaching to our eye from the star) becomes irregular and the light sometimes reaches the eye and sometimes it does not. This gives rise to twinkling of stars.
8. **Oval shape of sun in the morning and evening :** In the morning or evening, the sun is at the horizon. The refractive index decreases with height. Light reaching earth's atmosphere from different parts of vertical diameter of the sun enters at different heights in earth's atmosphere and so travels in media of different refractive indices at the same instant and hence, bends unequally. Due to this unequal bending of light from vertical diameter, the image of the sun gets distorted and it appears oval and larger. However, at noon when the sun is overhead, then due to normal incidence there will be no bending and the sun will appear circular.
9. The sparkling of diamond is due to total internal reflection inside it.
10. **Mirage :** Mirage in deserts is caused by total internal reflection.



(A) Mirage

Fig. 28.80

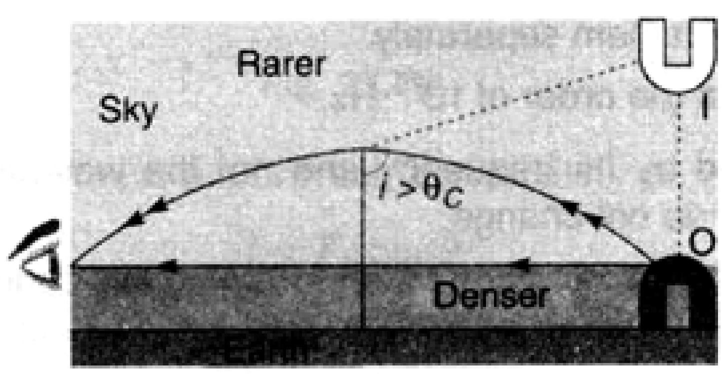
Due to heating of the earth, the refractive index of air near the surface of earth becomes lesser than above it. Light from distant objects reaches the surface of earth with $i > \theta_c$, so that total internal reflection will take place and we see the image of an object along with the object as seen in figure, creating an illusion of water near the object.

11. Duration of sun's visibility : In the absence of atmosphere the sun will be visible for its positions from M to E as shown in figure. However in presence of atmosphere, due to **total internal reflection,** the sun will become visible even when it is below the horizon.

Fig. 28.81

12. Looming : It is also due to **total internal reflection**. This phenomenon is observed in cold deserts and opposite to that of mirage.

13. If an object is placed between two parallel mirrors ($\theta = 0°$) the number of images formed will be infinite but of decreasing intensity.



(B) Looming

Fig. 28.82

14. Number of images : If there are two plane mirrors inclined to each other at an angle θ, the number of images formed are determined as follows:

(i) $n = (m - 1)$ for all positions of object, where $m = \dfrac{360}{\theta}$ is an even integer.

(ii) $n = m$, where $m = \dfrac{360}{\theta}$ is an odd integer and object is not on the bisector of mirrors.

(iii) $n = (m - 1)$, where $m = \dfrac{360}{\theta}$ is an odd integer and object is on the bisector of mirrors.

(iv) If $\dfrac{360}{\theta}$ is a fraction, the number of images formed will be equal to its integral part.

Note *The number of images formed by two mutually perpendicular ($\theta = 90°$) mirrors will be 3. All these three images will lie on a circle with centre at C, the point of intersection of mirrors M_1 and M_2 and whose radius is equal to distance between C and the object O.*

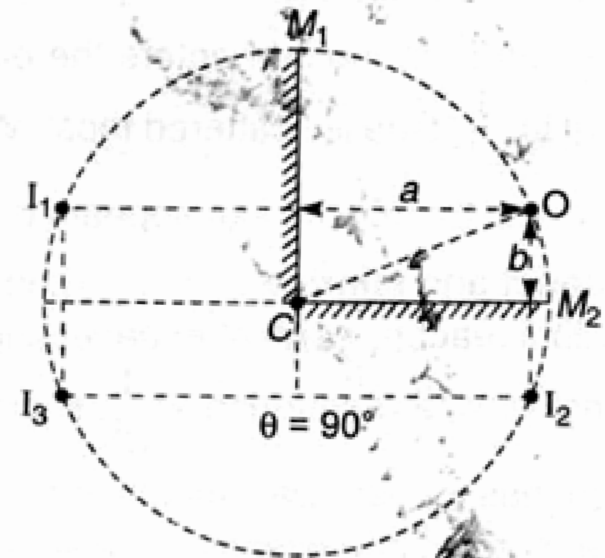


Fig. 27.83

Example: Find total number of images formed by two mirrors inclined to each other at an angle $\theta = 60°$.

Solution: $\frac{360}{\theta} = \frac{360}{60} = 6 = m$

Since, m is an even integer, total number of images will be $m - 1$ or 5 for all positions of the object between the mirrors.

15. In case of spherical mirrors if object distance x_1 and image distance x_2 are measured from focus instead of pole, $u = (f + x_1)$ and $v = (f + x_2)$ the mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ reduces to, } \frac{1}{f + x_2} + \frac{1}{f + x_1} = \frac{1}{f}$$

which on simplification gives, $$x_1 x_2 = f^2$$

This formula is called **Newton's formula.**

This formula applies to a lens also, but in that case x_1 is the object distance from first focus and x_2 the image distance from second focus.

16. In case of a **lens camera:**

(i) Time of exposure $\propto \frac{1}{(\text{Aperture})^2}$

(ii) The ratio of focal length to the aperature of lens is called ***f*-number** of the camera.

$$\therefore \quad f\text{-number} = \frac{\text{Focal length}}{\text{Aperture}}$$

If *f*-number of a camera is *f*/11, it implies that aperture is $\frac{1}{11}$ of its focal length. Movie Cameras have very low *f*-number such as *f*/1.5.

17. Scattering of light : If the molecules of a medium after absorbing incoming radiations (light) emit them in all possible directions, the process is called scattering. In scattering if the wavelength of radiation remains unchanged the scattering is called elastic otherwise inelastic.

Rayleigh has shown, theoretically that in case of elastic scattering of light by molecules, the intensity of scattered light depends on both nature of molecules and wavelength of light. According to him,

Intensity of scattered light $\propto \frac{1}{\lambda^4}$

Raman effect was based on inelastic scattering. For this **C.V. Raman** was awarded the Noble Prize in 1930.

Scattering helps us in understanding the following:

Why sky is Blue : When white light from the sun enters the earth's atmosphere scattering takes place. As scattering is proportional to $\frac{1}{\lambda^4}$, blue is scattered most. When we look at the sky we receive scattered light which is rich in blue and hence, the sky appears blue.

Why sun appears red during sunset and sunrise : In the morning and evening when sun is at the horizon, due to oblique incidence light reaches earth after traversing maximum path in the atmosphere and so suffers maximum scattering. Now, as scattering $\propto \frac{1}{\lambda^4}$, shorter wavelengths are scattered most leaving the longer one. As red light has longest wavelength in the visible region, it is scattered least. This is why sun appears red in the morning and evening. The same reason is why red light is used for danger signals.

18. **Defects of images :** Actual image formed by an optical system is usually imperfect. The defects of images are called **aberrations.** The defect may be due to light or optical system. If the defect is due to light it is called **chromatic aberration,** and if due to optical system, **monochromatic aberration.**

(a) Chromatic aberration : The image of an object formed by a lens is usually coloured and blurred. This defect of image is called chromatic aberration. This defect arises due to the fact that focal length of a lens is different for different colours. For a lens,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

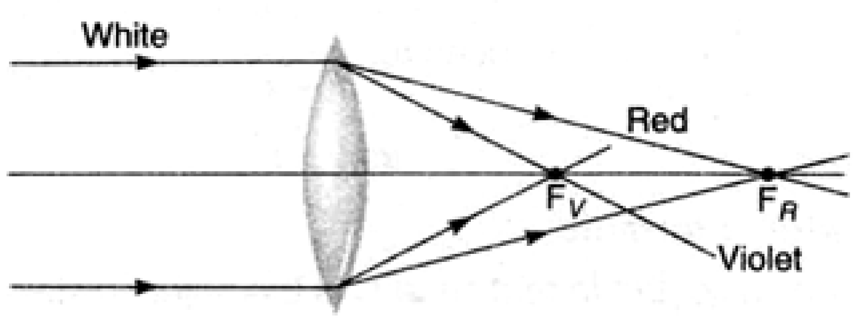


Fig. 28.84

As μ is maximum for violet while minimum for red, violet is focused nearest to the lens while red farthest from it.

The difference between f_R and f_v is a measure of longitudinal chromatic aberration. Thus,

$$L.C.A. = f_R - f_V$$

This can also be written as,

$$f_R - f_V = -df$$

where, $$df = f_V - f_R$$

Differentiating the equation,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

We find, $$\frac{-df}{f^2} = d\mu\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $$\frac{-df}{f} = \frac{d\mu}{\mu - 1} = \omega \qquad \left[\text{where } \omega = \frac{d\mu}{\mu - 1} = \text{dispersive power}\right]$$

Thus, $$\text{LCA} = -df = f\omega$$

For a single lens neither f nor ω can be zero. Thus, we cannot have a single lens free from chromatic aberration.

Note Dispersive power is also written as,

$$\omega = \frac{\mu_V - \mu_R}{\mu_Y - 1} \text{ Here } \mu_Y = \frac{\mu_V + \mu_R}{2}$$

Condition of achromatism : To get achromatism, we use a pair of two lenses in contact. For two thin lenses in contact we have,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\therefore \quad \frac{-dF}{F^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2}$$

The combination will be free from chromatic aberration if,

$$dF = 0$$

$$\therefore \quad \frac{df_1}{f_1^2} + \frac{df_2}{f_2^2} = 0$$

$$\therefore \quad \frac{\omega_1 f_1}{f_1^2} + \frac{\omega_2 f_2}{f_2^2} = 0$$

$$\therefore \quad \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

This is the **condition of achromatism.** From the condition of achromatism, following conclusions can be drawn:

(i) As ω_1 and ω_2 are positive quantities, f_1 and f_2 should have opposite signs, *i.e.*, if one lens is convex, the other must be concave.

(ii) If $\omega_1 = \omega_2$, means both the lenses are of same material. Then,

$$\frac{1}{f_1} + \frac{1}{f_2} = 0 \quad \text{or} \quad \frac{1}{F} = 0 \quad \text{or} \quad F = \infty$$

Thus, the combination behaves as a plane glass plate. So, we can conclude that both the lenses should be of different materials or,

$$\omega_1 \neq \omega_2$$

(iii) Dispersive power of crown glass (ω_C) is less than that of flint glass (ω_F).

(iv) If we want the combination to behave as a convergent lens then convex lens should have lesser focal length or its dispersive power should be more. Thus, convex lens should be made of flint glass and concave lens of crown. Thus, combination is converging if convex is made of flint glass and concave of crown. Similarly for the combination to behave as diverging lens, convex is made of crown glass and concave of flint glass.

(b) Monochromatic aberration : This is the defect in image due to optical system. Monochromatic aberration is of many types such as, spherical, coma, distortion, curvature and antigmatism. Here we shall limit ourselves to spherical aberration only.

Spherical aberration : Spherical aberration arises due to spherical nature of lens (or mirror). The paraxial rays (close to optic axis) get focused at I_P and **marginal rays** (away from the optic axis) are focused at I_M. Thus, image of a point object O is not a point.

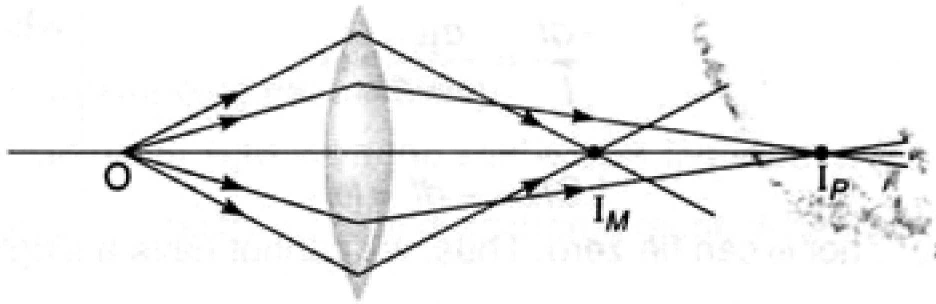


Fig. 28.85

The inability of the lens to form a point image of an axial point object is called spherical aberration. Spherical aberration can never be eliminated but can be minimised by the following methods:

(i) By using stops : By using stops either paraxial or marginal rays are cut-off.

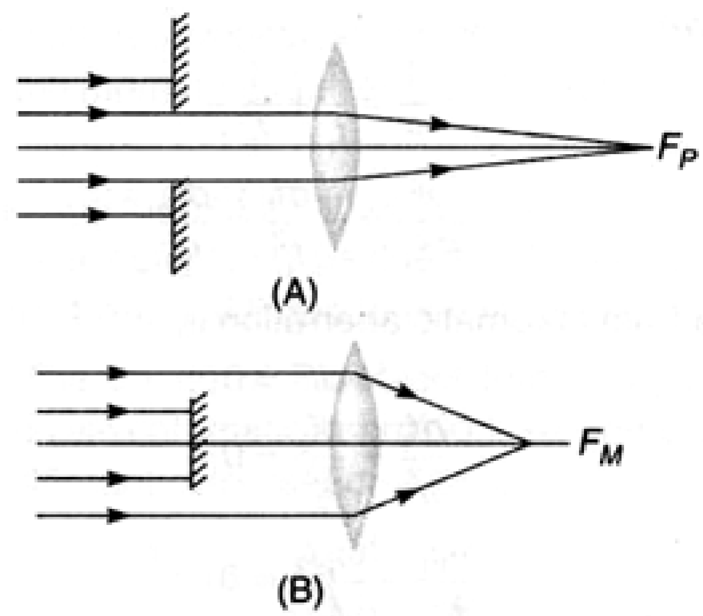


Fig. 28.86

(ii) Using two thin lenses separated by a distance : Two thin lenses separated by a distance $d = f_2 - f_1$ has the minimum spherical aberration.

(iii) Using parabolic mirrors : If spherical mirror is replaced by parabolic mirror, spherical aberration is minimised.

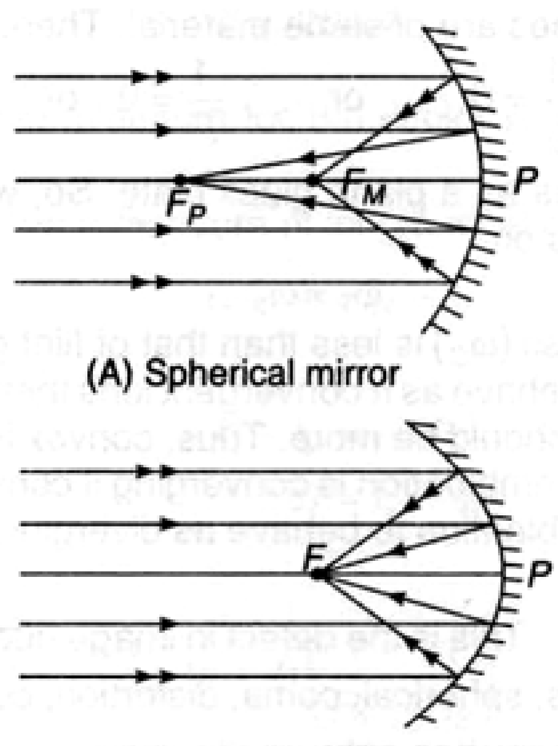
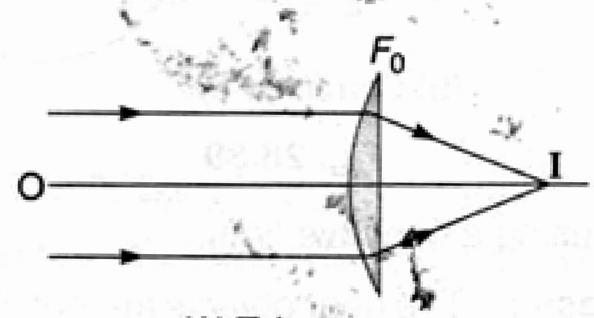


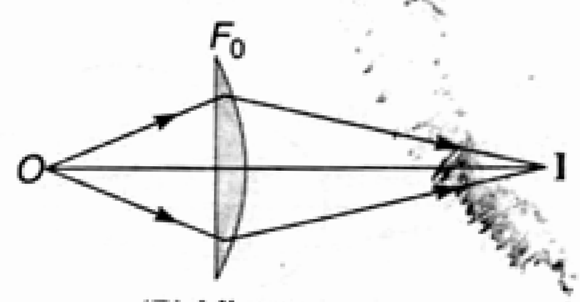
Fig. 28.87

(iv) Using lens of large focal length : It has been found that spherical aberration varies inversely as the cube of the focal length. So, if f is large, spherical aberration will be reduced.

(v) Using plano-convex lens : In case of plano- convex lens spherical aberration is minimised, if its curved surface faces the incident or emergent ray whichever is more parallel.



(A) Telescope



(B) Microscope

Fig. 28.88

This is why in telescope the curved surface faces the object while in microscope curved surface is towards the image.

(vi) Using crossed lens : For a single lens with object at infinity, spherical aberration is found to be minimum when R_1 and R_2 have the following ratio,

$$\frac{R_1}{R_2} = \frac{2\mu^2 - \mu - 4}{\mu\,(2\mu + 1)}$$

A lens which satisfies this condition is called a crossed lens.

Defects of vision : Regarding eye following points are worthnoting:

(i) The human eye is most sensitive to yellow-green light ($\lambda = 5550$ Å).

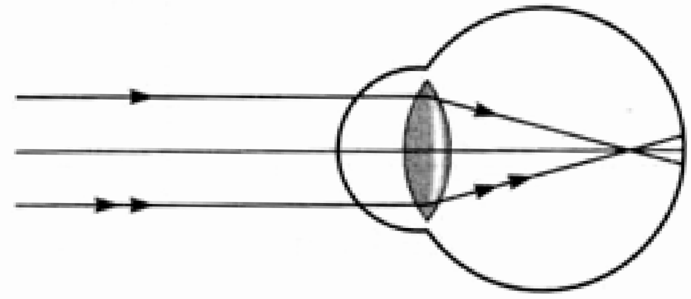
(ii) The persistance of vision is $\frac{1}{10}$ sec, *i.e.*, if time interval between two consecutive light pulses is less than 0.1 sec, eye cannot distinguish them separately.

(iii) By the eye lens, real, inverted and diminished image is formed at retina.

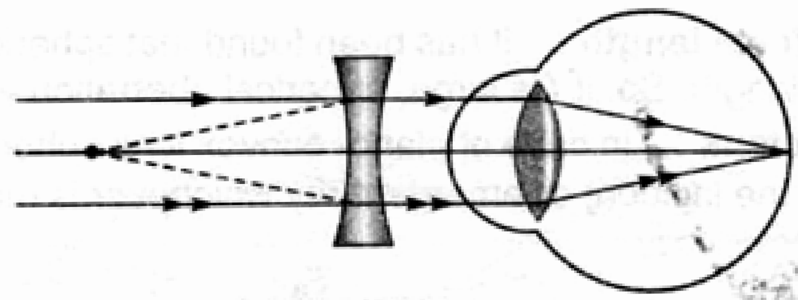
(iv) While testing your eye through reading chart if doctor finds it to 6/12, it implies that you can read a letter from 6 m which the normal eye can read from 12 m. Thus, 6/6 means normal eye sight.

The common defects of vision are as follows:

(i) Myopia or short-sightedness : Distant objects are not clearly visible in this defect. The image of distant object is formed before the retina.



(A) Defective-eye

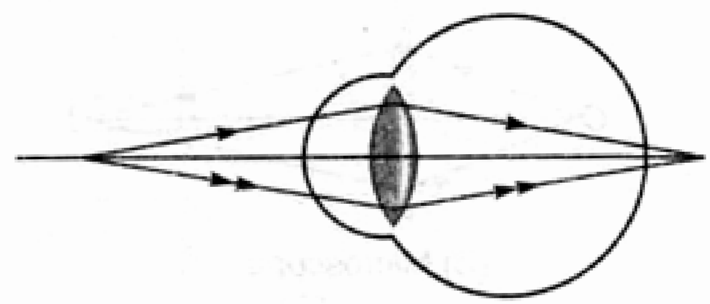


(B) Corrected-eye

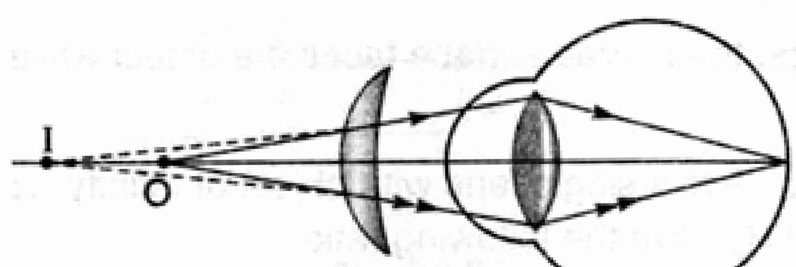
Fig. 28.89

The defect can be remedied by using a concave lens.

(ii) Hyperopia or far-sightedness : The near objects are not clearly visible. Image of near object is formed behind the retina.



(A) Defected-eye



(B) Corrected-eye

Fig. 27.90

This defect is remedied by using a convex lens.

(iii) Presbyopia : In it both near and far objects are not clearly visible. This is remedied either by using two separate lenses or by using single spectacle having bifocal lenses.

(iv) Astigmatism : In this defect eye cannot see objects in two orthogonal (perpendicular) directions clearly simultaneously. This defect is remedied by using cylindrical lens.

Solved Examples

For JEE Main

Example 1 *A ray of light falls on a glass plate of refractive index $\mu = 1.5$. What is the angle of incidence of the ray if the angle between the reflected and refracted rays is 90°?*

Solution In the figure $r = 90 - i$

From Snell's law,

$$1.5 = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin (90 - i)} = \tan i$$

$\therefore$ $$i = \tan^{-1} (1.5) = 56.3°$$ **Ans.**

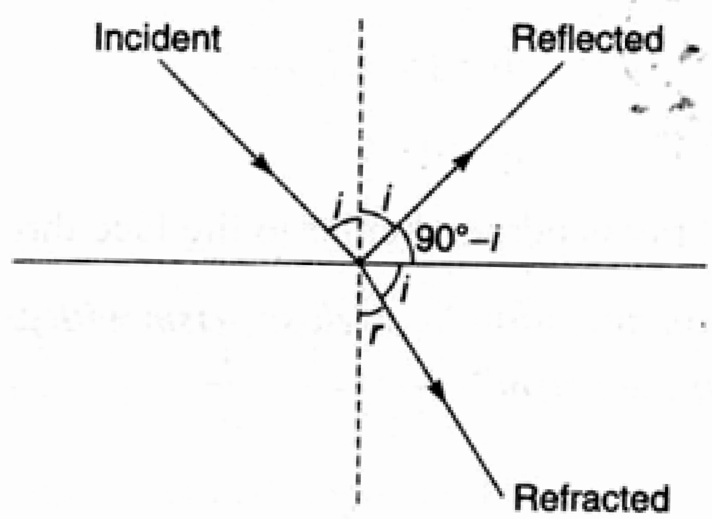


Fig. 28.91

Example 2 *A pile 4 m high driven into the bottom of a lake is 1 m above the water. Determine the length of the shadow of the pile on the bottom of the lake if the sun rays make an angle of 45° with the water surface. The refractive index of water is 4/3.*

Solution From Snell's law,

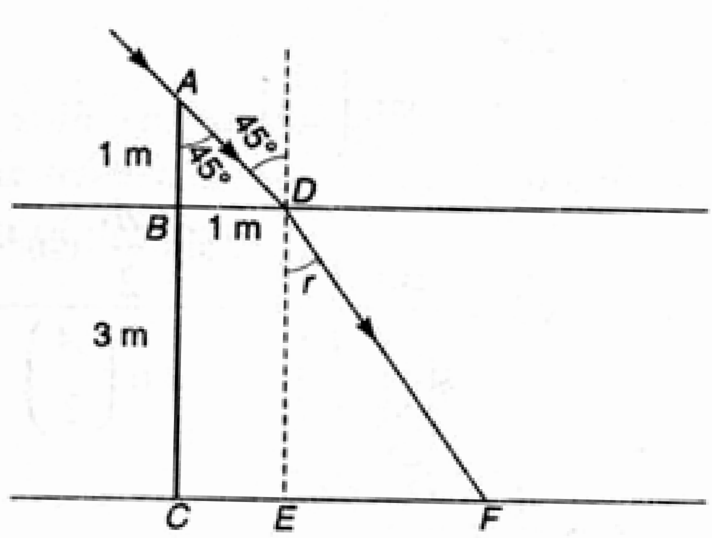


Fig. 28.92

$$\frac{4}{3} = \frac{\sin 45°}{\sin r}$$

Solving this equation we get,

$$r = 32°$$

Further, $EF = (DE)\tan r$

$$= (3)\tan 32°$$

$$= 1.88 \text{ m}$$

$\therefore$ Total length of shadow $L = CF$

or $L = (1 + 1.88) \text{ m} = 2.88 \text{ m}$ **Ans.**

Example 3 *A ray of light is incident at an angle of 60° on the face of a prism having refracting angle 30°. The ray emerging out of the prism makes an angle 30° with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges.*

Solution Given, $i_1 = 60°$, $A = 30°$ and $\delta = 30°$.

From the relation,

$$\delta = (i_1 + i_2) - A$$

we have, $i_2 = 0$

This means that the emergent ray is perpendicular to the face through which it emerges. **Ans.**

Example 4 *The angle of minimum deviation for a glass prism with $\mu = \sqrt{3}$ equals the refracting angle of the prism. What is the angle of the prism?*

Solution Given $A = \delta_m$

Using,

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

we have,

$$\sqrt{3} = \frac{\sin\left(\frac{A + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

or

$$\sqrt{3} = \frac{\sin A}{\sin\left(\frac{A}{2}\right)} = \frac{2\sin\frac{A}{2}\cdot\cos\frac{A}{2}}{\sin\left(\frac{A}{2}\right)}$$

$\therefore$ $$\cos\frac{A}{2} = \frac{\sqrt{3}}{2}$$

$\therefore$ $$\frac{A}{2} = 30°$$

or $A = 60°$ **Ans.**

Example 5 *The distance between two point sources of light is 24 cm. Find out where would you place a converging lens of focal length 9 cm, so that the images of both the sources are formed at the same point.*

Solution

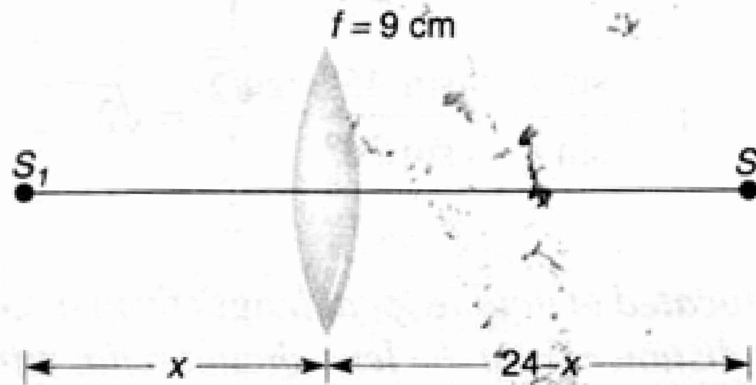


Fig. 28.93

For S_1 : $$\frac{1}{v_1}-\frac{1}{-x}=\frac{1}{9}$$

$\therefore$ $$\frac{1}{v_1}=\frac{1}{9}-\frac{1}{x} \quad \text{...(i)}$$

For S_2 : $$\frac{1}{v_2}-\frac{1}{-(24-x)}=\frac{1}{9}$$

$\therefore$ $$\frac{1}{v_2}=\frac{1}{9}-\frac{1}{24-x} \quad \text{...(ii)}$$

Since, sign convention for S_1 and S_2 is just opposite. Hence,

$$v_1=-v_2$$

or $$\frac{1}{v_1}=-\frac{1}{v_2}$$

$\therefore$ $$\frac{1}{9}-\frac{1}{x}=\frac{1}{24-x}-\frac{1}{9}$$

Solving this equation we get, $x=6$ cm. Therefore, the lens should be kept at a distance of 6 cm from either of the object. **Ans.**

Example 6 *One face of a prism with a refractive angle of 30° is coated with silver. A ray incident on another face at an angle of 45° is refracted and reflected from the silver coated face and retraces its path. What is the refractive index of the prism?*

Solution

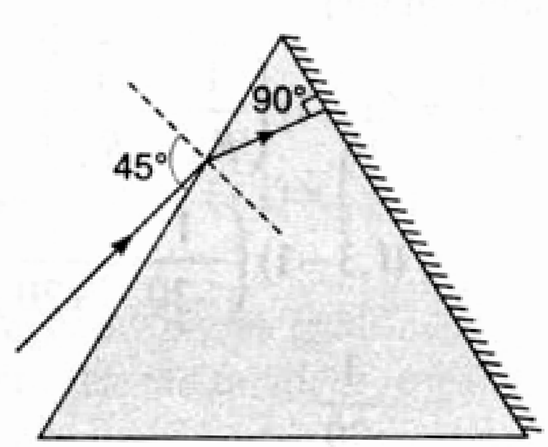


Fig. 28.94

Given $A = 30°$, $i_1 = 45°$ and $r_2 = 0$

Since, $$r_1 + r_2 = A$$

$\therefore$ $$r_1 = A = 30°$$

Now refractive index of the prism,

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin 45°}{\sin 30°} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \sqrt{2}$$ **Ans.**

Example 7 *A source of light is located at double focal length from a convergent lens. The focal length of the lens is $f = 30$ cm. At what distance from the lens should a flat mirror be placed, so that the rays reflected from the mirror are parallel after passing through the lens for the second time?*

Solution

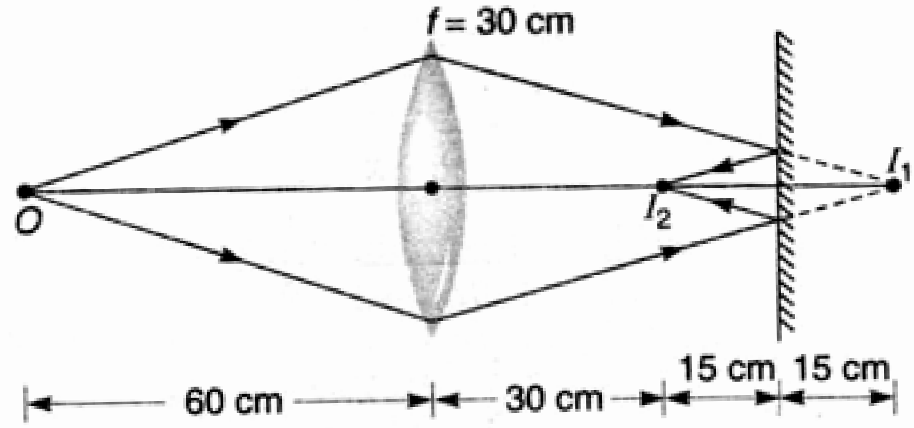


Fig. 28.95

Object is at a distance of $2f = 60$ cm from the lens. Image formed by lens I_1, should be at a distance 60 cm from the lens. Now I_2, the image formed by plane mirror should lie at focus or at a distance of 30 cm from the lens. Hence, the mirror should be placed at distance 45 cm from the lens as shown in figure.

Example 8 *Two equi-convex lenses of focal lengths 30 cm and 70 cm, made of material of refractive index = 1.5, are held in contact coaxially by a rubber band round their edges. A liquid of refractive index 1.3 is introduced in the space between the lenses filling it completely. Find the position of the image of a luminous point object placed on the axis of the combination lens at a distance of 90 cm from it.*

Solution $$|R_1| = |R_2| = f_1 = 30 \text{ cm}$$ (As $\mu = 1.5$)

Similarly, $$|R_3| = |R_4| = f_2 = 70 \text{ cm}$$

The focal length of the liquid lens (in air).

$$\frac{1}{f_3} = (\mu - 1)\left(\frac{1}{R_2} - \frac{1}{R_3}\right)$$

$$= (1.3 - 1)\left(\frac{1}{-30} - \frac{1}{+70}\right)$$

$$= -\frac{1}{70}$$

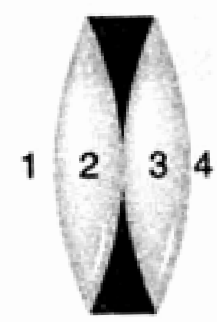


Fig. 28.96

Further, equivalent focal length of the combination,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}$$

$$=\frac{1}{30}+\frac{1}{70}-\frac{1}{70}=\frac{1}{30}$$

Using the lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{F}$, we have

$$\frac{1}{v}-\frac{1}{-90}=\frac{1}{30}$$

$\therefore$ $$v=+45 \text{ cm}$$

Thus, image will be formed at a distance of 45 cm from the combination.

Example 9 *Two thin converging lenses are placed on a common axis, so that the centre of one of them coincides with the focus of the other. An object is placed at a distance twice the focal length from the left-hand lens. Where will its image be? What is the lateral magnification? The focal of each lens is f.*

Solution

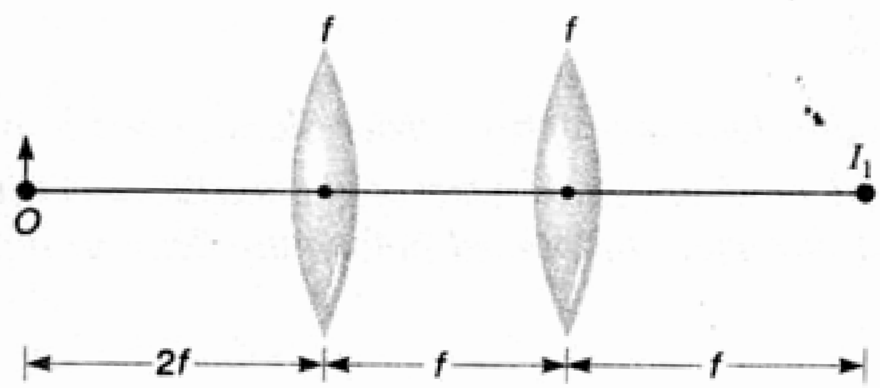


Fig. 28.97

The image formed by first lens will be at a distance $2f$ with lateral magnification $m_1=-1$. For the second lens this image will behave as a virtual object. Using the lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we have,

$$\frac{1}{v}-\frac{1}{f}=\frac{1}{f}$$

$\therefore$ $$v=\frac{f}{2}$$

$$m_2=\frac{v_2}{u_2}=\frac{(f/2)}{f}=\frac{1}{2}$$

Therefore, final image is formed at a distance $\frac{f}{2}$ from the second lens with total lateral magnification,

$$m=m_1\times m_2=(-1)\times\left(\frac{1}{2}\right)=-\frac{1}{2}$$ **Ans.**

Example 10 *The refracting angle of a glass prism is 30°. A ray is incident onto one of the faces perpendicular to it. Find the angle δ between the incident ray and the ray that leaves the prism. The refractive index of glass is μ = 1.5.*

Solution Given, $A = 30°, \mu = 1.5$ and $i_1 = 0°$

Since, $i_1 = 0°$, therefore, r_1 is also equal to 0°.

Further, since, $r_1 + r_2 = A$

$$\therefore \quad r_2 = A = 30°$$

Using,
$$\mu = \frac{\sin i_2}{\sin r_2}$$

we have,
$$1.5 = \frac{\sin i_2}{\sin 30°}$$

or
$$\sin i_2 = 1.5 \sin 30° = 1.5 \times \frac{1}{2} = 0.75$$

$$\therefore \quad i_2 = \sin^{-1}(0.75) = 48.6°$$

Now, the deviation, $\delta = (i_1 + i_2) - A$

$$= (0 + 48.6) - 30$$

or
$$\delta = 18.6°$$ **Ans.**

For JEE Advanced

Example 1 *A biconvex thin lens is prepared from glass of refractive index 3/2. The two bounding surfaces have equal radii of 25 cm each. One of the surfaces is silvered from outside to make it reflecting. Where should an object be placed before this lens so that the image coincides with the object.*

Solution Refer extra points.

Here, $R_1 = +25\text{ cm}, R_2 = -25\text{ cm}, \mu_1 = 1$ and $\mu_2 = 3/2$

Image coincides with object, hence, $u = v = -x$ (say)

Substituting in Eq. (iv), we have

$$\frac{1}{-x} - \frac{1}{x} = \frac{2(3/2)}{-25} - \frac{2(3/2-1)}{25}$$

or
$$\frac{2}{x} = \frac{3}{25} + \frac{1}{25} = \frac{4}{25}$$

$$\therefore \quad x = 12.5\text{ cm}$$ **Ans.**

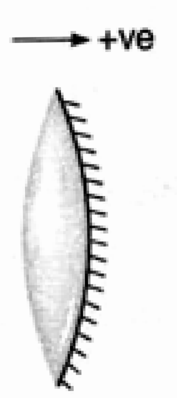


Fig. 28.98

Hence, the object should be placed at a distance 12.5 cm in front of the silvered lens.

Example 2 *A point source of light is placed at a distance h below the surface of a large and deep lake. Show that the fraction f of light that escapes directly from water surface is independent of h and is given by,*

$$f = \frac{[1 - \sqrt{1 - 1/\mu^2}]}{2}$$

Solution Due to TIR, light will be reflected back into the water if $i > \theta_c$. So, only that portion of incident light will escape which passes through the cone of angle $\theta = 2\theta_c$.

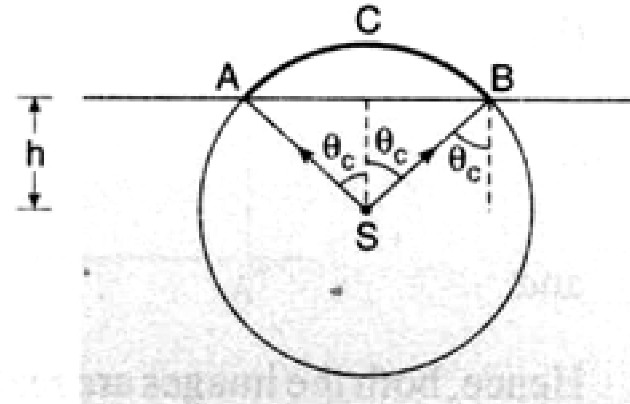


Fig. 28.99

So, the fraction of light escaping

$$f = \frac{\text{area } ACB}{\text{total area of sphere}}$$

$$= \frac{2\pi R^2(1-\cos\theta_c)}{4\pi R^2} = \frac{1-\cos\theta_c}{2}$$

Now, as f depends on θ_c and which depends only on μ, it is independent of h.

Proved.

Further $$\cos\theta_c = \frac{\sqrt{\mu^2-1}}{\mu} = \sqrt{1-1/\mu^2}$$

$\therefore$ $$f = \frac{1-\sqrt{1-1/\mu^2}}{2}$$ **Ans.**

Note *Area of ACB* $= 2\pi R^2(1-\cos\theta_c)$ *can be obtained by integration.*

Example 3 *An object is 5.0 m to the left of a flat screen. A converging lens for which the focal length is* $f = 0.8$ *m is placed between object and screen.*

(a) Show that two lens positions exist that form images on the screen and determine how far these positions are from the object?

(b) How do the two images differ from each other?

Solution (a) Using the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

We have, $$\frac{1}{5.0-u} - \frac{1}{-u} = \frac{1}{0.8}$$

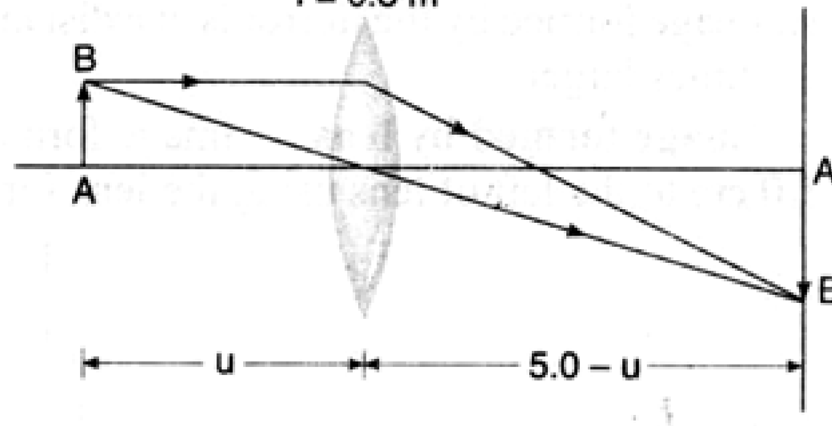


Fig. 28.100

or $$\frac{1}{5-u} + \frac{1}{u} = 1.25$$

$\therefore$ $$u + 5 - u = 1.25\,u\,(5-u)$$

or $$1.25\,u^2 - 6.25\,u + 5 = 0$$

$\therefore$ $$u = \frac{6.25 \pm \sqrt{39.0625-25}}{2.5}$$

or $$u = 4 \text{ m} \quad \text{and} \quad 1 \text{ m}$$ **Ans.**

Both the values are real, which means there exist two positions of lens that form images of object on the screen.

(b) $$m = \frac{v}{u}$$

∴ $$m_1 = \frac{(5.0 - 4.0)}{(-4.0)} = -0.25$$

and $$m_2 = \frac{(5.0 - 1.0)}{(-1.0)} = -4.00$$

Hence, both the images are real and inverted, the first has magnification – 0.25 and the second – 4.00 **Ans.**

Example 4 *The object is midway between the lens and the mirror. The mirror's radius of curvature is 20.0 cm and the lens has a focal length of – 16.7 cm. Considering only the rays that leaves the object and travels first toward the mirror, locate the final image formed by this system. Is this image real or virtual? Is it upright or inverted? What is the overall magnification?*

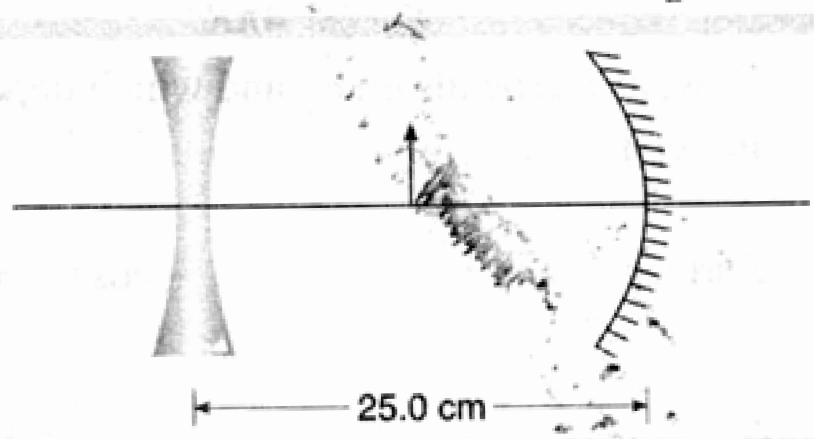


Fig. 28.101

Solution **Image formed by mirror :** Using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{R} \qquad \left(\text{as } f = \frac{R}{2}\right)$$

We have, $$\frac{1}{v_1} + \frac{1}{-12.5} = \frac{2}{-20}$$

∴ $$v_1 = -50 \text{ cm}$$

$$m_1 = -\frac{v}{u} = -\frac{(-50)}{(-12.5)} = -4$$

i.e., image formed by the mirror is at a distance of 50 cm from the mirror to the left of it. It is inverted and four times larger.

Image formed by lens : Image formed by mirror acts as an object for lens. It is at a distance of 25.0 cm to the left of lens using the lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

We have, $$\frac{1}{v_2} - \frac{1}{25} = \frac{1}{-16.7}$$

∴ $$v_2 = -50.3 \text{ cm}$$

and $$m_2 = \frac{v}{u} = \frac{-50.3}{25} = -2.012$$

overall magnification is $$m = m_1 \times m_2 = 8.048$$

Thus, the final image is at a distance 25.3 cm to the right of the mirror, virtual, upright enlarged and 8.048 times. Positions of the two images are shown in figure.

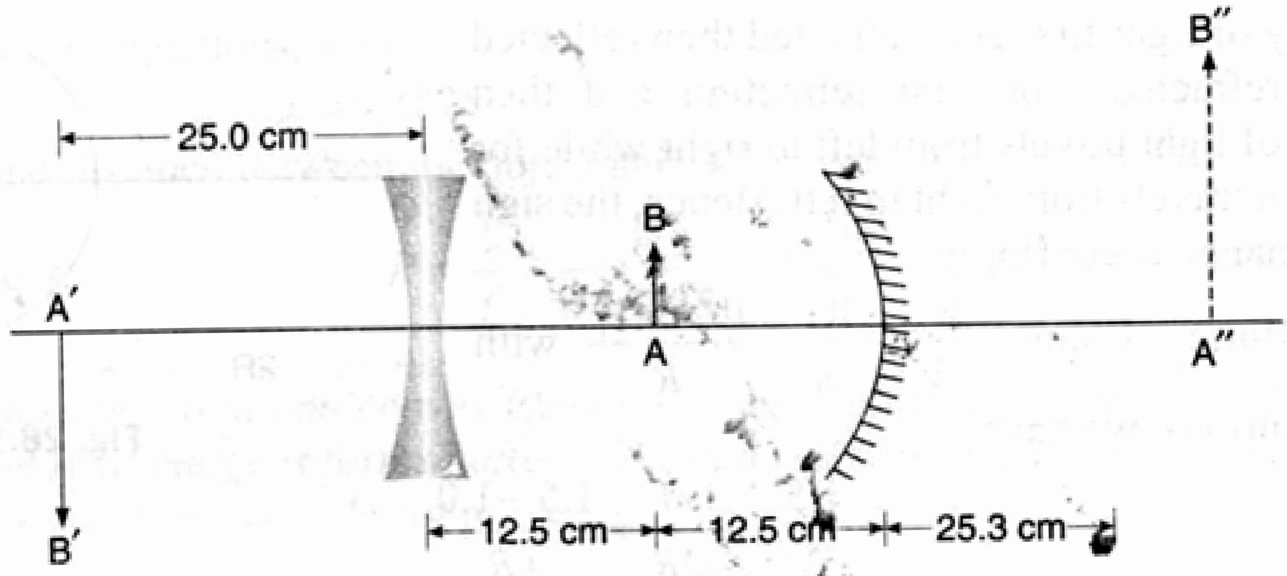


Fig. 28.102

Example 5 *An object is placed 12 cm to the left of a diverging lens of focal length −6.0 cm. A converging lens with a focal length of 12.0 cm is placed at a distance d to the right of the diverging lens. Find the distance d that corresponds to a final image at infinity.*

Solution

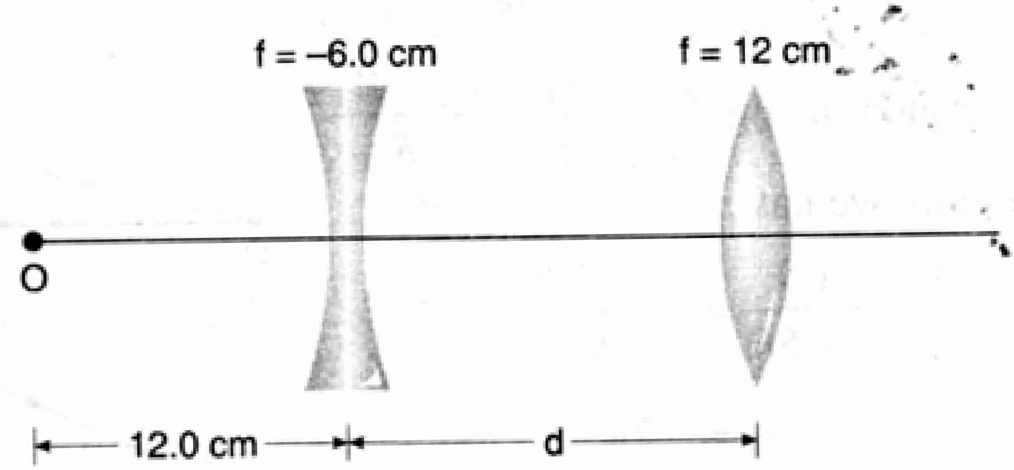


Fig. 28.103

Applying lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ twice we have,

$$\frac{1}{v_1} - \frac{1}{-12} = \frac{1}{-6} \quad \text{...(i)}$$

$$\frac{1}{\infty} - \frac{1}{v_1 - d} = \frac{1}{12} \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we have

$$v_1 = -4\,\text{cm}$$

and $$d = 8\,\text{cm}$$ **Ans.**

Example 6 *A solid glass sphere with radius R and an index of refraction 1.5 is silvered over one hemisphere. A small object is located on the axis of the sphere at a distance 2R to the left of the vertex of the unsilvered hemisphere. Find the position of final image after all refractions and reflections have taken place.*

Solution The ray of light first gets refracted then reflected and then again refracted. For first refraction and then reflection the ray of light travels from left to right while for the last refraction it travels from right to left. Hence, the sign convention will change accordingly.

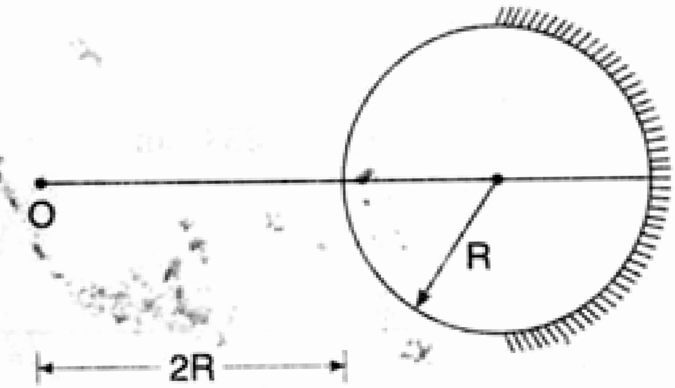


Fig. 28.104

First refraction : Using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ with proper sign conventions, we have

$$\frac{1.5}{v_1} - \frac{1.0}{-2R} = \frac{1.5 - 1.0}{+R}$$

$\therefore$
$$v_1 = \infty$$

Second reflection : Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{R}$ with proper sign conventions, we have

$$\frac{1}{v_2} + \frac{1}{\infty} = -\frac{2}{R} \qquad \therefore \quad v_2 = -\frac{R}{2}$$

Third refraction : Again using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ with reversed sign convention, we have

$$\frac{1.0}{v_3} - \frac{1.5}{-1.5R} = \frac{1.0 - 1.5}{-R}$$

or
$$v_3 = -2R$$

i.e., final image is formed on the vertex of the silvered face

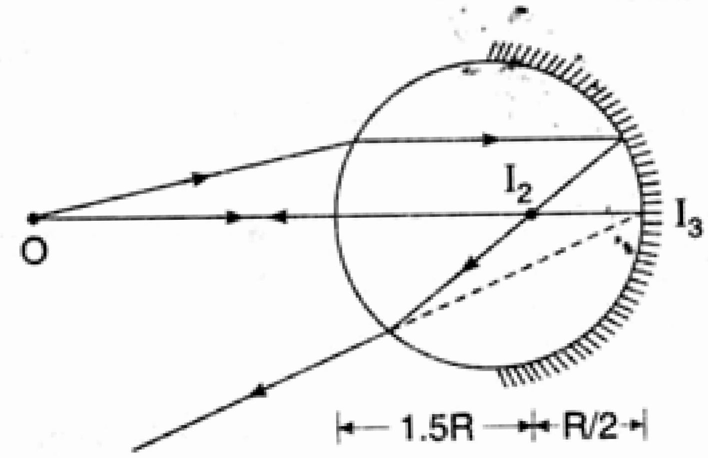


Fig. 28.105

Example 7 *A converging lens forms a five fold magnified image of an object. The screen is moved towards the object by a distance d = 0.5 m, and the lens is shifted so that the image has the same size as the object. Find the lens power and the initial distance between the object and the screen.*

Solution In the first case image is five times magnified. Hence, $|v| = 5|u|$. In the second case image and object are of equal size. Hence, $|v| = |u|$. From the two figures,

$$6x = 2y + d$$

or
$$6x - 2y = 0.5 \qquad \text{...(i)}$$

Using the lens formula for both the cases,

$$\frac{1}{5x} - \frac{1}{-x} = \frac{1}{f}$$

or
$$\frac{6}{5x} = \frac{1}{f} \qquad \text{...(ii)}$$

$$\frac{1}{y} - \frac{1}{-y} = \frac{1}{f}$$

or
$$\frac{2}{y} = \frac{1}{f} \qquad \text{...(iii)}$$

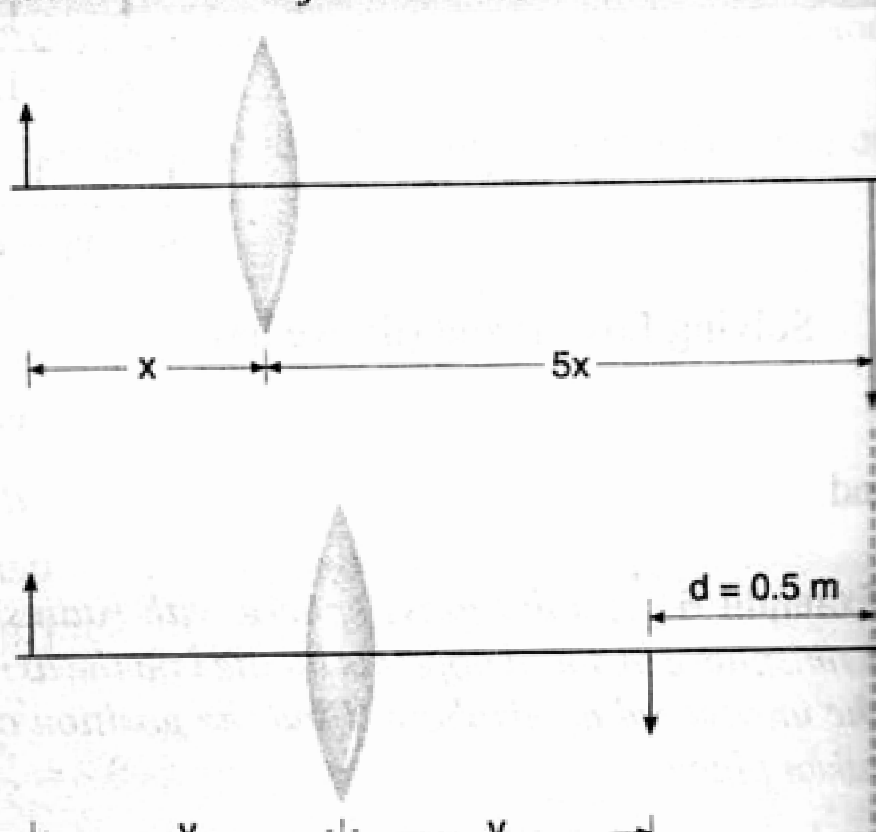


Fig. 28.106

Solving these three equations, we get

$$x = 0.1875\ \text{m} \quad \text{and} \quad f = 0.15625\ \text{m}$$

Therefore, initial distance between the object and the screen $= 6x = 1.125$ m **Ans.**

Power of the lens, $$P = \frac{1}{f} = \frac{1}{0.15625} = 6.4\ \text{D}$$ **Ans.**

Example 8 *Surfaces of a thin equiconvex glass lens have radius of curvature R. Paraxial rays are incident on it. If the final image is formed after n internal reflections, calculate distance of this image from pole of the lens. Refractive index of glass is μ.*

Solution The rays will first get refracted, then n-times reflected and finally again refracted. So, using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ for first refraction, we have

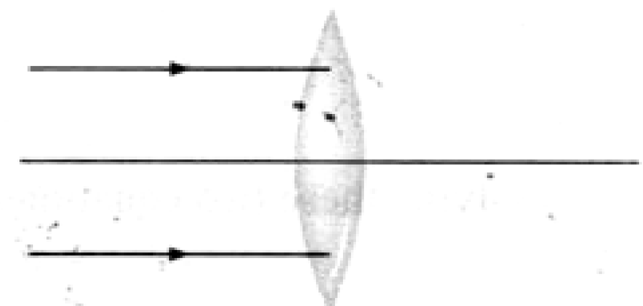


Fig. 28.107

$$\frac{\mu}{v_i} - \frac{1}{\infty} = \frac{\mu - 1}{R}$$

$$\therefore \quad v_i = \left(\frac{\mu}{\mu - 1}\right) R$$

For first reflection, let us use $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{R}$

$$\therefore \quad \frac{1}{v_1} + \left(\frac{\mu - 1}{\mu R}\right) = \frac{-2}{R} \quad \text{or} \quad \frac{1}{v_1} = -\left(\frac{3\mu - 1}{\mu R}\right)$$

For second reflection $$\frac{1}{v_2} + \frac{3\mu - 1}{\mu R} = \frac{-2}{R} \quad \text{or} \quad \frac{1}{v_2} = -\left(\frac{5\mu - 1}{\mu R}\right)$$

Similarly after n^{th} reflections, $$\frac{1}{v_n} = -\left[\frac{(2n+1)\mu - 1}{\mu R}\right]$$

Finally using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, we have

$$\frac{1}{v_f} - \left\{\frac{(2n+1)\mu - 1}{R}\right\} = \frac{1 - \mu}{-R}$$

$$v_f = \frac{R}{2(\mu n + \mu - 1)}$$ **Ans.**

Optical Instruments and Photometry

Example 1 *An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and eye piece is 36 cm and the final image is formed at infinity. Determine the focal length of objective and eye piece.*

Solution For final image at infinity,

$$M_\infty = \frac{f_o}{f_e} \quad \text{and} \quad L_\infty = f_o + f_e$$

$\therefore$ $$5 = \frac{f_o}{f_e} \qquad \ldots\text{(i)}$$

and $$36 = f_o + f_e \qquad \ldots\text{(ii)}$$

Solving these two equations, we have

$$f_o = 30\,\text{cm} \quad \text{and} \quad f_e = 6\,\text{cm}$$ **Ans.**

Example 2 *A telescope has an objective of focal length 50 cm and an eye piece of focal length 5 cm. The least distance of distinct vision is 25 cm. The telescope is focused for distinct vision on a scale 2 m away from the objective. Calculate* (a) *magnification produced and* (b) *separation between objective and eye piece.*

Solution Given, $f_o = 50\,\text{cm}$ and $f_e = 5\,\text{cm}$

Note *Here object is placed at finite distance from the objective. Hence, formulae derived for angular magnification M cannot be applied directly as they have been devired for the object to be at infinity. Here it will be difficult to find angular magnification. So, only linear magnification can be obtained.*

For objective : $$\frac{1}{v_o} - \frac{1}{-200} = \frac{1}{50}$$

$\therefore$ $$v_o = \frac{200}{3}\,\text{cm}$$

$$m_o = \frac{v_o}{u_o} = \frac{(200/3)}{-200} = -\frac{1}{3}$$

For eye piece : $$\frac{1}{-25} - \frac{1}{u_e} = \frac{1}{5}$$

$\therefore$ $$u_e = -\frac{25}{6}\,\text{cm}$$

and $$m_e = \frac{v_e}{u_e} = \frac{-25}{-(25/6)} = 6$$

(a) Magnification, $$m = m_o \times m_e = -2$$ **Ans.**

(b) Separation between objective and eye piece,

$$L = v_o + |u_e| = \frac{200}{3} + \frac{25}{6} = \frac{425}{6} = 70.83\,\text{cm}$$ **Ans.**

Example 3 *The minimum luminous flux required to operate a certain photoelectric circuit is 3 lumen. The diameter of the sensitive surface of the photocell is 8 cm. There is a point source of light located 1 m above the cell. What is the minimum luminous intensity of the source required for operating the circuit?*

Solution From the relation,

$$I = \frac{L\cos\theta}{r^2}$$

We have, $I = L$ as $\theta = 0°$ and $r = 1$ m

Further, ϕ (the radiant flux) $= I \times A$

or $\phi = (L) \times \pi\,(4 \times 10^{-2})^2 = 16\pi L \times 10^{-4}$ lm

According to given problem for operation of the circuit,

$$\phi > 3 \text{ lm}$$

$\therefore$ $16\pi L \times 10^{-4} > 3$

or $L > 597$

$\therefore$ $L_{min} = 597$ cd **Ans.**

EXERCISES

For JEE Main

Note *In different books refractive index has been represented by the symbol n and μ. So in our book we have used both symbols at different places.*

Subjective Questions

Refraction of Light

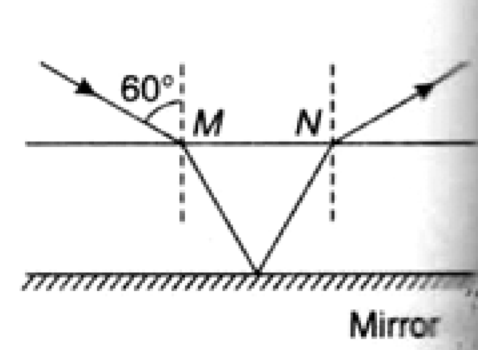
1. A ray of light falls on a glass plate of refractive index $n = 1.5$. What is the angle of incidence of the ray if the angle between the reflected and refracted rays is 90°?
2. The laws of reflection or refraction are the same for sound as for light. The index of refraction of a medium (for sound) is defined as the ratio of the speed of sound in air 343 m/s to the speed of sound in the medium.
   (a) What is the index of refraction (for sound) of water ($v = 1498$ m/s)?
   (b) What is the critical angle θ, for total reflection of sound from water?
3. Light from a sodium lamp ($\lambda_0 = 589$ nm) passes through a tank of glycerin (refractive index $= 1.47$) 20 m long in a time t_1. If it takes a time t_2 to transverse the same tank when filled with carbon disulfide (index $= 1.63$), determine the difference $t_2 - t_1$.
4. A light beam of wavelength 600 nm in air passes through film 1 ($n_1 = 1.2$) of thickness 1.0 μm, then through film 2 (air) of thickness 1.5 μm, and finally through film 3 ($n_3 = 1.8$) of thickness 1.0 μm.
   (a) Which film does the light cross in the least time, and what is that least time?
   (b) What are the total number of wavelengths (at any instant) across all three films together?
5. A plane harmonic infrared wave travelling through a transparent medium is given by

$$E_x(y, t) = E_{0x} \sin 2\pi \left(\frac{y}{5 \times 10^{-7}} - 3 \times 10^{14} t \right)$$

in SI units. Determine refractive index of the medium at that frequency, and the vacuum wavelength of the disturbance.

Refraction from Plane and Spherical Surface

6. A plate with plane parallel faces having refractive index 1.8 rests on a plane mirror. A light ray is incident on the upper face of the plate at 60°. How far from the entry point will the ray emerge after reflection by the mirror. The plate is 6 cm thick?

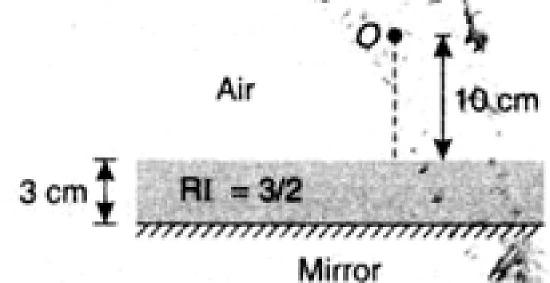


7. A pile 4 m high driven into the bottom of a lake is 1 m above the water. Determine the length of the shadow of the pile on the bottom of the lake if the sun rays make an angle of 45° with the water surface. The refractive index of water is 4/3.

8. An object is at a distance of $d = 2.5$ cm from the surface of a glass sphere with a radius $R = 10$ cm. Find the position of the final image produced by the sphere. The refractive index of glass is $\mu = 1.5$.

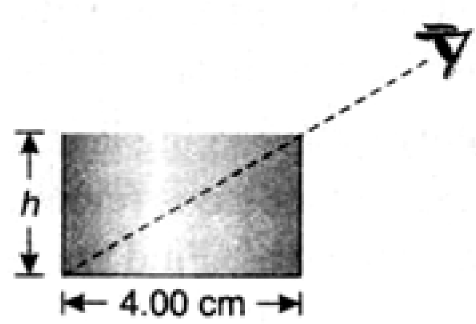
9. An air bubble is seen inside a solid sphere of glass ($n = 1.5$) of 4.0 cm diameter at a distance of 1.0 cm from the surface of the sphere (on seeing along the diameter). Determine the real position of the bubble inside the sphere.

10. Find the position of final image of an object O as shown in figure.

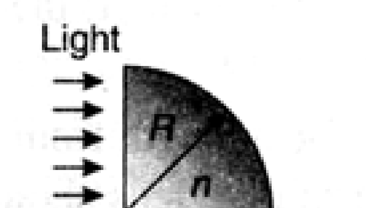


11. One face of a rectangular glass plate 6 cm thick is silvered. An object held 8 cm infront of the unsilvered face forms an image 6 cm behind the silvered face. Find the refractive index of glass. Consider all the three steps.

12. A shallow glass dish is 4.00 cm wide at the bottom, as shown in figure. When an observer's eye is positioned as shown, the observer sees the edge of the bottom of the empty dish. When this dish is filled with water, the observer sees the centre of the bottom of the dish. Find the height of the dish $\mu_w = 4/3$.

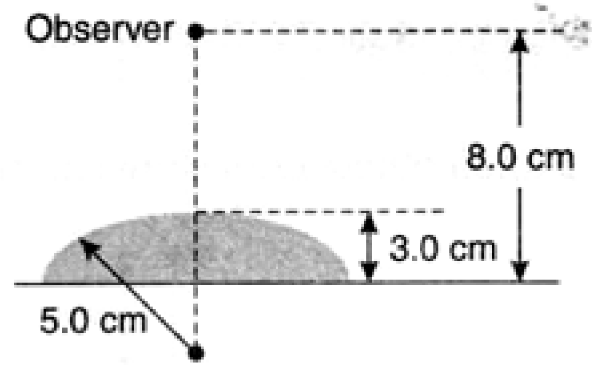


13. A glass prism in the shape of a quarter-cylinder lies on a horizontal table. A uniform, horizontal light beam falls on its vertical plane surface, as shown in the figure. If the radius of the cylinder is $R = 5$ cm and the refractive index of the glass is $n = 1.5$, where, on the table beyond the cylinder, will a path of light be found?



14. A glass sphere with 10 cm radius has a 5 cm radius spherical hole at its centre. A narrow beam of parallel light is directed radially into the sphere. Where, if anywhere, will the sphere produce an image? The index of refraction of the glass is 1.50.

15. A glass sphere has a radius of 5.0 cm and a refractive index of 1.6. A paperweight is constructed by slicing through the sphere on a plate that is 2.0 cm from the centre of the sphere and perpendicular to a radius of the sphere that passes through the centre of the circle formed by the intersection of the plane and the sphere. The paperweight is placed on a table and viewed from directly above an observer who is 8.0 cm from the tabletop, as shown in figure. When viewed through the paperweight, how far away does the tabletop appear to the observer?



16. A fish is rising up vertically inside a pond with velocity 4 cm/s, and notices a bird, which is diving downward and its velocity appears to be 16 cm/s (to the fish). What is the real velocity of the diving bird, if refractive index of water is 4/3 ?

Thin Lenses

17. A lens with a focal length of 16 cm produces a sharp image of an object in two positions, which are 60 cm apart. Find the distance from the object to the screen.
18. Two glasses with refractive indices of 1.5 and 1.7 are used to make two identical double convex lenses.
    (a) Find the ratio between their focal lengths.
    (b) How will each of these lenses act on a ray parallel to its optical axis if the lenses are submerged into a transparent liquid with a refractive index of 1.6?
19. A converging beam of rays is incident on a diverging lens. Having passed through the lens the rays intersect at a point 15 cm from the lens. If the lens is removed, the point where the rays meet, move 5 cm closer to the mounting that holds the lens. Find the focal length of the lens.
20. The distance between two point sources of light is 24 cm. Find out where would you place a converging lens of focal length of 9 cm, so that the images of both the sources are formed at the same point.
21. Two thin converging lenses are placed on a common axis so that the centre of one of them coincides with the focus of the other. An object is placed at a distance twice the focal length from the left-hand lens. Where will its image be? What is the lateral magnification? The focal length of each lens is f.
22. A source of light is located at double focal length from a convergent lens. The focal length of the lens is $f = 30$ cm. At what distance from the lens should a flat mirror be placed so that the rays reflected from the mirror are parallel after passing through the lens for the second time?
23. A parallel beam of rays is incident on a convergent lens with a focal length of 40 cm. Where a divergent lens with a focal length of 15 cm be placed for the beam of rays to remain parallel after passing through the two lenes.
24. An optical system consists of two convergent lenses with focal lengths $f_1 = 20$ cm and $f_2 = 10$ cm. The distance between the lenses is $d = 30$ cm. An object is placed at a distance of 30 cm from the first lens. At what distance from the second lens will the image be obtained?

25. Determine the position of the image produced by an optical system consisting of a concave mirror with a focal length of 10 cm and a convergent lens with a focal length of 20 cm. The distance from the mirror to the lens is 30 cm and from the lens to the object is 40 cm. Consider only two steps. Plot the image.

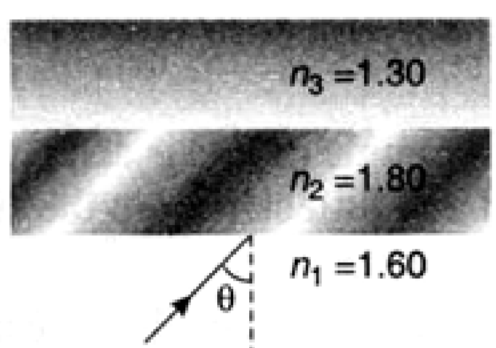
26. A parallel beam of light is incident on a system consisting of three thin lenses with a common optical axis. The focal lengths of the lenses are equal to $f_1 = +10\,\text{cm}$ and $f_2 = -20\,\text{cm}$, and $f_3 = +9\,\text{cm}$ respectively. The distance between the first and the second lens is 15 cm and between the second and the third is 5 cm. Find the position of the point at which the beam converges when it leaves the system of lenses.

27. Two equi-convex lenses of focal lengths 30 cm and 70 cm, made of material of refractive index $= 1.5$, are held in contact coaxially by a rubber band round their edges. A liquid of refractive index 1.3 is introduced in the space between the lenses filling it completely. Find the position of the image of a luminous point object placed on the axis of the combination lens at distance of 90 cm from it.

Total Internal Reflection

28. If the speed of light in ice is 2.3×10^8 m/s, what is its index of refraction? What is the critical angle of incidence for light going from ice to air?

29. In figure, light refracts from material 1 into a thin layer of material 2, crosses that layer, and then is incident at the critical angle on the interface between materials 2 and 3.



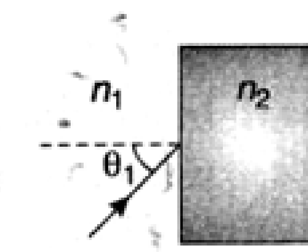
(a) What is the angle θ?

(b) If θ is decreased, is there refraction of light into material 3?

30. A point source of light S is placed at the bottom of a vessel containing a liquid of refractive index 5/3. A person is viewing the source from above the surface. There is an opaque disc of radius 1 cm floating on the surface. The centre of the disc lies vertically above the source S. The liquid from the vessel is gradually drained out through a tap. What is the maximum height of the liquid for which the source cannot at all be seen from above?

31. A ray of light travelling in glass ($\mu_g = 3/2$) is incident on a horizontal glass-air surface at the critical angle θ_C. If a thin layer of water ($\mu_w = 4/3$) is now poured on the glass-air surface, at what angle will the ray of light emerges into water at glass-water surface?

32. A ray of light is incident on the left vertical face of glass cube of refractive index n_2, as shown in figure. The plane of incidence is the plane of the page, and the cube is surrounded by liquid (refractive index $= n_1$). What is the largest angle of incidence θ_1 for which total internal reflection occurs at the top surface?



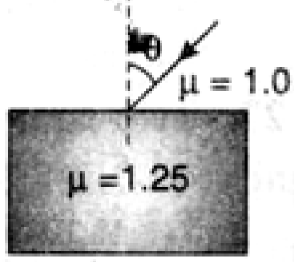
33. Light is incident from glass $\left(\mu_g = \frac{3}{2}\right)$ to water $\left(\mu_w = \frac{4}{3}\right)$. Find the range of the angle of deviation for which there are two angles of incidence.

34. A point source is placed at a depth h below the surface of water (refractive index $= \mu$).

(a) Show that light escapes through a cricular area on the water surface with its centre directly above the point source.

(b) Find the angle subtended by a radius of the area on the source.

35. Consider the situation shown in figure. Find the maximum angle θ for which the light suffers total internal reflection at the left vertical surface.



Prism

36. The angle of minimum deviation for a glass prism with $n = \sqrt{3}$ equals the refracting angle of the prism. What is the angle of the prism?

37. The refracting angle of a glass prism is 30°. A ray is incident onto one of the faces perpendicular to it. Find the angle δ between the incident ray and the ray that leaves the prism. The refractive index of glass is $n = 1.5$.

38. The perpendicular faces of a right isosceles prism are coated with silver. Prove that the rays incident at an arbitrary angle on the hypotenuse face will emerge from the prism parallel to the initial direction.

39. One face of a prism with a refractive angle of 30° is coated with silver. A ray incident on another face at an angle of 45° is refracted and reflected from the silver coated face and ratraces its path. What is the refractive index of the prism?

40. In an isosceles prism of angle 45°, it is found that when the angle of incidence is same as the prism angle the emergent ray grazes the emergent surface. Find the refractive index of the material of the prism. For what angle of incidence the angle of deviation will be minimum?

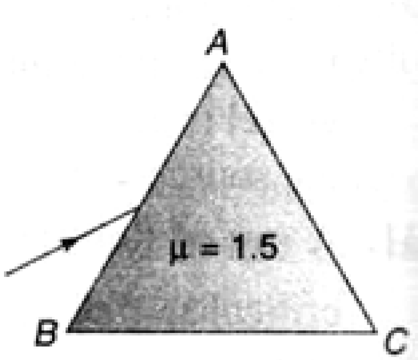
41. A ray incident on the face of a prism is refracted and escapes through an adjacent face. What is the maximum permissible angle of the prism, if it is made of glass with a refractive index of $\mu = 1.5$?

42. In an equilateral prism of $\mu = 1.5$, the condition for minimum deviation is fulfilled. If face AC is polished

(a) Find the net deviation.

(b) If the system is placed in water what will be the net deviation?

Refractive index of water $= \frac{4}{3}$.



Deviation and Dispersion by a Prism

43. In a cartain spectrum produced by a glass prism of dispersive power 0.0305, it is found that the refractive index for the red ray is 1.645 and that for the violet ray is 1.665. What is the refractive index for the yellow ray?

44. An achromatic lens-doublet is formed by placing in contact a convex lens of focal length 20 cm and a concave lens of focal length 30 cm. The dispersive power of the material of the convex lens is 0.18.

(a) Determine the dispersive power of the material of the concave lens.

(b) Calculate the focal length of the lens-doublet.

45. An achromatic convergent lens of focal length 150 cm is made by combining flint and crown glass lenses. Calculate the focal lengths of both the lenses and point out which one is divergent if the ratio of the dispersive power of flint and crown glasses is 3 : 2.

46. The index of refraction of heavy flint glass is 1.68 at 434 nm and 1.65 at 671 nm. Calculate the difference in the angle of deviation of blue (434 nm) and red (671 nm) light incident at 65° on one side of a heavy-flint glass prism with apex angle 60°.

Objective Questions

Single Correct Option

1. An endoscope is employed by a physician to view the internal parts of body organ. It is based on the principle of

(a) refraction (b) reflection
(c) total internal reflection (d) dispersion

2. Refractive index μ is given as $\mu = A + \frac{B}{\lambda^2}$ where A and B are constants and λ is wavelength, then dimensions of B are same as that of

(a) wavelength (b) volume (c) pressure (d) area

3. A plane glass slab is placed over various coloured letters. The letter which appears to be raised the least is

(a) violet (b) yellow (c) red (d) green

4. Critical angle of light passing from glass to air is least for

(a) red (b) green (c) yellow (d) violet

5. The power in dioptre of an equi-convex lens with radii of curvature of 10 cm and refractive index 1.6 is

(a) +12 (b) +18 (c) +1.2 (d) +1.8

6. The refractive index of water is 4/3. The speed of light in water is

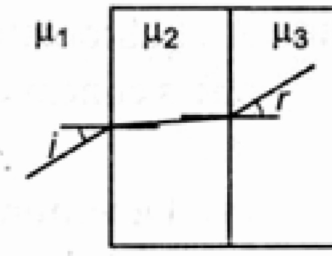
(a) 1.50×10^8 m/s (b) 1.78×10^8 m/s (c) 2.25×10^8 m/s (d) 2.67×10^8 m/s

7. White light is incident from under water on the water-air interface. If the angle of incidence is slowly increased from zero, the emergent beam coming out into the air will turn from

(a) white to violet (b) white to red (c) white to black (d) None of these

8. When light enters from air to water, then its

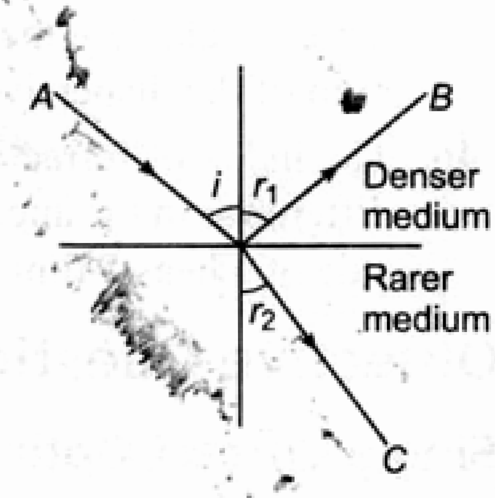
(a) frequency increases and speed decreases
(b) frequency is same, but the wavelength is smaller in water than in air
(c) frequency is same but the wavelength in water is greater than in air
(d) frequency decreases and wavelength is smaller in water than in air



9. In the figure shown $\frac{\sin i}{\sin r}$ is equal to

(a) $\frac{\mu_2^2}{\mu_3 \mu_1}$ (b) $\frac{\mu_3}{\mu_1}$

(c) $\frac{\mu_3 \mu_1}{\mu_2^2}$ (d) $\frac{\mu_1}{\mu_3}$



10. In figure, the reflected ray B makes an angle 90° with the ray C. If i, r_1 and r_2 are the angles of incidence, reflection and refraction, respectively. Then the critical angle of the medium is

(a) $\sin^{-1}(\tan i)$

(b) $\sin^{-1}(\cot i)$

(c) r_1

(d) r_2

11. A prism of apex angle $A = 60°$ has the refractive index $\mu = \sqrt{2}$. The angle of incidence for minimum deviation is

(a) 30° (b) 45° (c) 60° (d) None of these

12. A thin equi-convex lens is made of glass of refractive index 1.5 and its focal length is 0.2 m. If it acts as a concave lens of 0.5 m focal length when dipped in a liquid, the refractive index of the liquid is

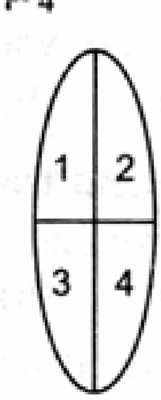
(a) $\frac{17}{8}$ (b) $\frac{15}{8}$ (c) $\frac{13}{8}$ (d) $\frac{9}{8}$

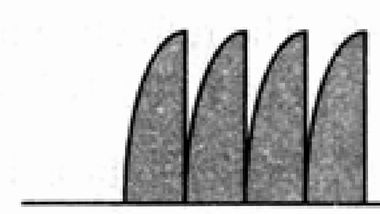
13. A ray of light, travelling in a medium of refractive index μ, is incident at an angle i on a composite transparent plate consisting of three plates of refractive indices μ_1, μ_2 and μ_3. The ray emerges from the composite plate into a medium of refractive index μ_4 at angle x. Then

(a) $\sin x = \sin i$ (b) $\sin x = \frac{\mu}{\mu_4} \sin i$

(c) $\sin x = \frac{\mu_4}{\mu} \sin i$ (d) $\sin x = \frac{\mu_1 \mu_3 \mu}{\mu_2 \mu_2 \mu_4} \sin i$

14. The given equi-convex lens is broken into four parts and rearranged as shown. If the initial focal length is f then after rearrangement, the equivalent focal length is





(a) f

(b) $f/2$

(c) $f/4$

(d) $4f$

15. A thin convergent glass lens ($\mu_g = 15$) has a power of + 5.0 D. When this lens is immersed in a liquid of refractive index μ_l, it acts as a divergent lens of focal length 100 cm. The value of μ_l is

(a) 4/3 (b) 5/3 (c) 5/4 (d) 6/5

16. Two convex lenses of focal length 10 cm and 20 cm respectively placed coaxially and are separated by some distance d. The whole system behaves like a concave lens. One of the possible value of d is

(a) 15 cm (b) 20 cm (c) 25 cm (d) 40 cm

17. A prism can have a maximum refracting angle of (θ_c = critical angle for the material of prism)

(a) 60° (b) θ_c

(c) $2\theta_c$ (d) slightly less than 180°

18. A ray of light is incident at small angle I on the surface of prism of small angle A and emerges normally from the opposite surface. If the refractive index of the material of the prism is μ, the angle of incidence is nearly equal to

(a) $\frac{A}{\mu}$ (b) $\frac{A}{2\mu}$ (c) μA (d) $\mu A/2$

19. The refractive angle of a prism is A, and the refractive index of the material of the prism is $\cot(A/2)$. The angle of minimum deviation is

(a) $180° - 3A$ (b) $180° + 2A$ (c) $90° - A$ (d) $180° - 2A$

20. A prism of refractive index $\sqrt{2}$ has refractive angle 60°. In order that a ray suffers minimum deviation it should be incident at an angle of

(a) 45° (b) 90° (c) 30° (d) None

21. The focal length of a combination of two lenses is doubled if the separation between them is doubled. If the separation is increased to 4 times, the magnitude of focal length is

(a) doubled (b) quadrupled (c) halved (d) same

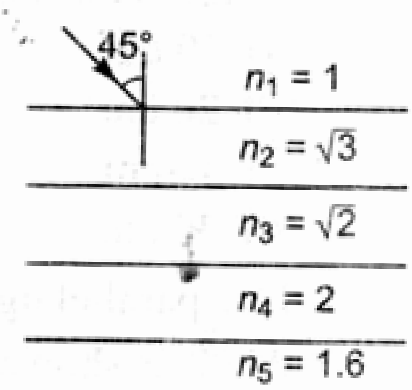
22. A convexo-concave convergent lens is made of glass of refractive index 1.5 and focal length 24 cm. Radius of curvature for one surface is double than that of the other. Then radii of curvature for the two surfaces are (in cm)

(a) 6, 12 (b) 12, 24 (c) 3, 6 (d) 18, 36

23. An optical system consists of a thin convex lens of focal length 30 cm and a plane mirror placed 15 cm behind the lens. An object is placed 15 cm in front of the lens. The distance of the final image from the object is

(a) 60 cm (b) 30 cm (c) 75 cm (d) 45 cm

24. In the figure shown the angle made by the light ray with the normal in the medium of refractive index $\sqrt{2}$ is



(a) 30°

(b) 60°

(c) 90°

(d) None of the above

25. For refraction through a small angled prism, the angle of minimum deviation

(a) increases with increase in refractive index of a prism

(b) will be 2δ for a ray of refractive index 2.4 if it is δ for a ray of refractive index 1.2

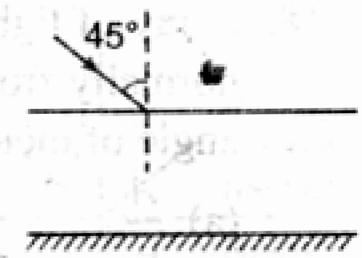
(c) is directly proportional to the angle of the prism

(d) will decrease with increase in refractive index of the prism

26. A ray of light passes from vacuum into a medium of refractive index n. If the angle of incidence is twice the angle of refraction, then the angle of incidence is
(a) $\cos^{-1}(n/2)$ (b) $\sin^{-1}(n/2)$ (c) $2\cos^{-1}(n/2)$ (d) $2\sin^{-1}(n/2)$

27. A thin convex lens of focal length 30 cm is placed in front of a plane mirror. An object is placed at a distance x from the lens (not in between lens and mirror) so that its final image coincide with itself. Then, the value of x is
(a) 15 cm (b) 30 cm (c) 60 cm (d) Insufficient data

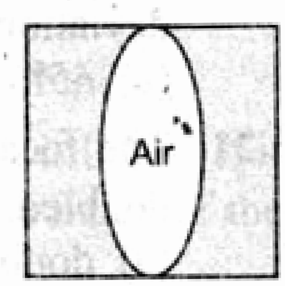
28. One side of a glass slab is silvered as shown in the figure. A ray of light is incident on the other side at angle of incidence 45°. Refractive index of glass is given as $\sqrt{2}$. The deflection suffered by the ray when it comes out of the slab is



(a) 90° (b) 180°
(c) 120° (d) 45°

29. A prism has refractive index $\sqrt{\frac{3}{2}}$ and refractive angle 90°. Find the minimum deviation produced by prism
(a) 60° (b) 45° (c) 30° (d) 15°

30. In figure, an air lens of radius of curvature of each surface equal to 10 cm is cut in a cylinder of glass of refractive index 1.5. The focal length and the nature of lens are



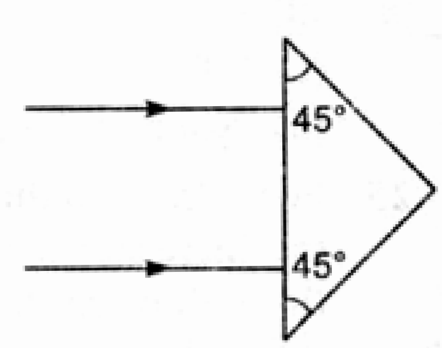
(a) 15 cm diverging (b) 15 cm converging
(c) 10 cm diverging (d) 10 cm converging

31. A point object is placed at a distance of 12 cm from a convex lens of focal length 10 cm. On the other side of the lens, a convex mirror is placed at a distance of 10 cm from the lens such that the image formed by the combination coincides with the object itself. The focal length of the convex mirror is
(a) 20 cm (b) 25 cm (c) 15 cm (d) 30 cm

32. An object, a convex lens of focal length 20 cm and a plane mirror are arranged as shown in the figure. How far behind the mirror is the second image formed ?

(a) 30 cm
(b) 20 cm
(c) 40 cm
(d) 50 cm

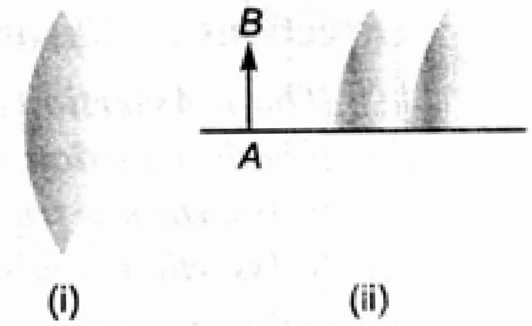
33. Two parallel light rays pass through an isosceles prism of refractive index $\sqrt{3/2}$ as shown in figure. The angle between the two emergent rays is



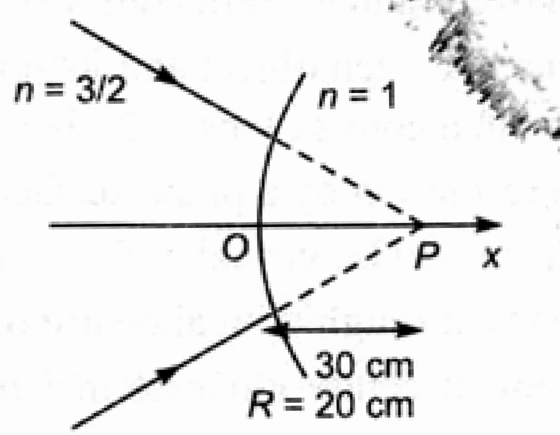
(a) 15°
(b) 30°
(c) 45°
(d) 60°

34. A prism having refractive index $\sqrt{2}$ and refractive angle 30° has one of the refractive surfaces polished. A beam of light incident on the other surface will trace its path if the angle of incidence is
(a) 0° (b) 30° (c) 45° (d) 60°

35. In Fig. (i), a lens of focal length 10 cm is shown. It is cut into two parts and placed as shown in Fig. (ii). An object AB of height 1 cm is placed at a distance of 7.5 cm. The height of the image will be
(a) 2 cm (b) 1 cm
(c) 1.5 cm (d) 3 cm



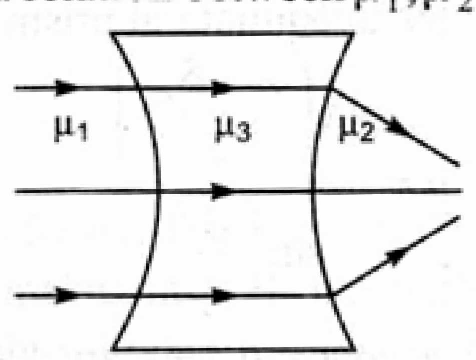
36. The image for the converging beam after refraction through the curved surface is formed at



(a) $x = 40$ cm (b) $x = 40/3$ cm (c) $x = -40/3$ cm (d) $x = 20$ cm

37. A concavo convex lens is made of glass of refractive index 1.5. The radii of curvature of its two surfaces are 30 cm and 50 cm. Its focal length when placed in a liquid of refractive index 1.4 is
(a) 200 cm (b) 500 cm (c) 800 cm (d) 1050 cm

38. From the figure shown establish a relation between μ_1, μ_2 and μ_3



(a) $\mu_1 < \mu_2 < \mu_3$ (b) $\mu_3 < \mu_2 ; \mu_3 = \mu_1$
(c) $\mu_3 > \mu_2 ; \mu_3 = \mu_1$ (d) None of these

39. When light of wavelength λ is incident on an equilateral prism, kept on its minimum deviation position, it is found that the angle of deviation equals the angle of the prism itself. The refractive index of the material of the prism for the wavelength λ is
(a) $\sqrt{3}$ (b) $\sqrt{3/2}$ (c) 2 (d) $\sqrt{2}$

For **JEE Advanced**

Assertion and Reason

Directions : *Choose the correct option.*

(*a*) *If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion.***
(*b*) *If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion.***
(*c*) *If **Assertion** is true, but the **Reason** is false.*
(*d*) *If **Assertion** is false but the **Reason** is true.*

1. **Assertion :** There is a glass slab between Ram and Shyam. Then, Ram appears nearer to Shyam as compared to the actual distance between them.
Reason : Ray of light starting from Ram will undergo two times refraction before reaching Shyam.

2. **Assertion :** Minimum distance between object and its real image by a convex lens is $4f$.
Reason : If object distance from a convex lens is $2f$ then its image distance is also $2f$.

3. **Assertion :** In case of single refraction by a plane surface image and object are on same side.
Reason : If object is real, image will be virtual and *vice-versa.*

4. **Assertion :** Ray of light passing through optical centre of a lens goes undeviated.
Reason : Ray falls normal at optical centre and in normal incidence, there is no deviation of light.

5. **Assertion :** In displacement method of finding focal length of a convex lens its magnification in one position of lens is +2, then magnification in another position of lens should be $-\frac{1}{2}$.
Reason : This method can't be applied for a concave lens.

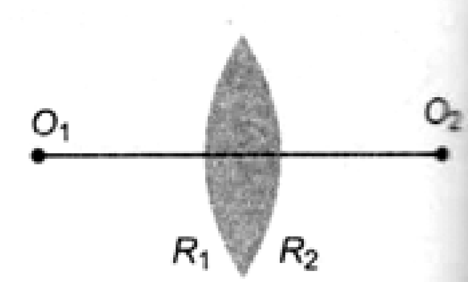
6. **Assertion :** If object is placed at infinity, then a virtual image will be formed at first focus of a concave lens.
Reason : First focal length of a concave lens is positive.

7. **Assertion :** Minimum deviation by an equilateral prism of refractive index $\sqrt{2}$ is 30°.
Reason : It is from the relation, $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$

8. **Assertion :** A convex lens and a concave lens are kept in contact. They will behave as a combination of diverging lens if focal length of convex lens is more.
Reason : Power of a concave lens is always less than the power of a convex lens, as power of concave lens is negative whereas power of convex lens is positive.

9. **Assertion :** Image of an object is of same size by a convex lens . If a glass slab is placed between object and lens, image will become magnified.
Reason : By inserting the slab, image may be real or virtual.

10. **Assertion :** In the figure shown $|R_1|>|R_2|$. Two point objects O_1 and O_2 are kept at same distance from the lens. Image distance of O_1 from the lens will be more compared to the image distance of O_2.



Reason : If medium on two sides of the lens is different, we cannot apply lens formulae directly.