EXERCISES

For JEE Main

Subjective Questions

Speed of Longitudinal Waves

1. A person standing between two parallel hills fires a gun. He hears the first echo after 3/2 s, and a second echo after 5/2 s. If speed of sound is 332 m/s, calculate the distance between the hills. When will he hear the third echo ?
2. Using the fact that hydrogen gas consists of diatomic molecules with $M = 2$ kg/K-mol. Find the speed of sound in hydrogen at 27° C.
3. Helium is a monoatomic gas that has a density of $0.179\ \text{kg/m}^3$ at a pressure of 76 cm of mercury and a temperature of 0° C. Find the speed of compressional waves (sound) in helium at this temperature and pressure.
4. (a) In a liquid with density $1300\ \text{kg/m}^3$, longitudinal waves with frequency 400 Hz are found to have wavelength 8.00 m. Calculate the bulk modulus of the liquid.
   (b) A metal bar with a length of 1.50 m has density $6400\ \text{kg/m}^3$. Longitudinal sound waves take 3.90×10^{-4} s to travel from one end of the bar to the other. What is Young's modulus for this metal ?
5. What must be the stress (F/A) in a stretched wire of a material whose Young's modulus is Y for the speed of longitudinal waves equal to 30 times the speed of transverse waves?
6. A gas is a mixture of two parts by volume of hydrogen and one part by volume of nitrogen. If the velocity of sound in hydrogen at 0° C is 1300 m/s. Find the velocity of sound in the gaseous mixture at 27° C.

Intensity and Sound Level

7. About how many times more intense will the normal ear perceive a sound of $10^{-6}\ \text{W/m}^2$ than one of $10^{-9}\ \text{W/m}^2$?
8. The explosion of a fire cracker in the air at a height of 40 m produces a 100 dB sound level at ground below. What is the instantaneous total radiated power? Assuming that it radiates as a point source.
9. (a) What is the intensity of a 60 dB sound ?
   (b) If the sound level is 60 dB close to a speaker that has an area of $120\ \text{cm}^2$. What is the acoustic power output of the speaker?
10. (a) By what factor must the sound intensity be increased to increase the sound intensity level by 13.0 dB ?
    (b) Explain why you do not need to know the original sound intensity?
11. The speed of a certain compressional wave in air at standard temperature and pressure is 330 m/s. A point source of frequency 300 Hz radiates energy uniformly in all directions at the rate of 5 watt.

(a) What is the intensity of the wave at a distance of 20 m from the source?
(b) What is the amplitude of the wave there? [Density of air at STP $=1.29\ kg/m^3$]

12. What is the amplitude of motion for the air in the path of a 60 dB, 800 Hz sound wave ? Assume that $\rho_{air}=1.29\ kg/m^3$ and $v=330\ m/s$.

13. A rock band gives rise to an average sound level of 102 dB at a distance of 20 m from the centre of the band. As an approximation, assume that the band radiates sound equally into a sphere. What is the sound power output of the band ?

14. If it were possible to generate a sinusoidal 300 Hz sound wave in air that has a displacement amplitude of 0.200 mm. What would be the sound level of the wave ? (Assume $v=330\ m/s$ and $\rho_{air}=1.29\ kg/m^3$)

15. (a) A longitudinal wave propagating in a water-filled pipe has intensity $3.00\times10^{-6}\ W/m^2$ and frequency 3400 Hz. Find the amplitude A and wavelength λ of the wave. Water has density $1000\ kg/m^3$ and bulk modulus 2.18×10^9 Pa. (b) If the pipe is filled with air at pressure 1.00×10^5 Pa and density $1.20\ kg/m^3$, what will be the amplitude A and wavelength λ of a longitudinal wave with the same intensity and frequency as in part (a)? (c) In which fluid is the amplitude larger, water or air? What is the ratio of the two amplitudes? Why is this ratio so different from one ?

16. For a person with normal hearing, the faintest sound that can be heard at a frequency of 400 Hz has a pressure amplitude of about 6.0×10^{-5} Pa. Calculate the corresponding intensity and sound intensity level at 20°C.

Organ Pipes

17. The fundamental frequency of an open pipe is 594 Hz. What is the fundamental frequency if one end is closed ?

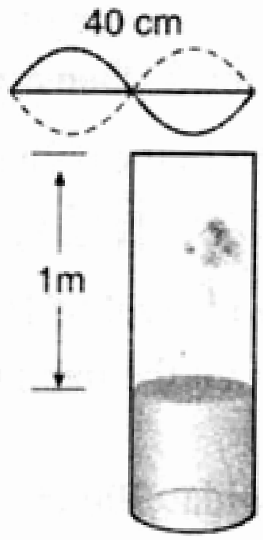
18. Find the fundamental frequency and the frequency of the first three overtones of a pipe 45.0 cm long. (a) If the pipe is open at both ends. (b) If the pipe is closed at one end. Use $v=344\ m/s$.

19. A uniform tube of length 60 cm stands vertically with its lower end dipping into water. First two air column lengths above water are 15 cm and 45 cm, when the tube responds to a vibrating fork of frequency 500 Hz. Find the lowest frequency to which the tube will respond when it is open at both ends.

20. Write the equation for the fundamental standing sound waves in a tube that is open at both ends. If the tube is 80 cm long and speed of the wave is 330 m / s. Represent the amplitude of the wave at an antinode by A.

21. A long glass tube is held vertically, dipping into water, while a tuning fork of frequency 512 Hz is repeatedly struck and held over the open end. Strong resonance is obtained, when the length of the tube above the surface of water is 50 cm and again 84 cm, but not at any intermediate point. Find the speed of sound in air and next length of the air column for resonance.

22. A wire of length 40 cm which has a mass of 4 g oscillates in its second harmonic and sets the air column in the tube to vibrations in its fundamental mode as shown in figure. Assuming the speed of sound in air as 340 m/s, find the tension in the wire.



23. In a resonance tube experiment to determine the speed of sound in air, a pipe of diameter 5 cm is used. The column in pipe resonates with a tuning fork of frequency 480 Hz when the minimum length of the air column is 16 cm. Find the speed of sound in air column at room temperature.

24. The air column in a pipe closed at one end is made to vibrate in its second overtone by tuning fork of frequency 440 Hz. The speed of sound in air is 330 m / s. End corrections may be neglected. Let P_0 denote the mean pressure at any point in the pipe, and ΔP_0 the maximum amplitude of pressure variation.
    (a) Find the length L of the air column.
    (b) What is the amplitude of pressure variation at the middle of the column ?
    (c) What are the maximum and minimum pressure at the open end of the pipe ?
    (d) What are the maximum and minimum pressure at the closed end of the pipe ?

25. On a day when the speed of sound is 345 m/s, the fundamental frequency of a closed organ pipe is 220 Hz. (a) How long is this closed pipe ? (b) The second overtone of this pipe has the same wavelength as the third harmonic of an open pipe. How long is the open pipe ?

26. A closed organ pipe is sounded near a guitar, causing one of the strings to vibrate with large amplitude. We vary the tension of the string until we find the maximum amplitude. The string is 80% as long as the closed pipe. If both the pipe and the string vibrate at their fundamental frequency, calculate the ratio of the wave speed on the string to the speed of sound in air.

Beats and Doppler Effect

27. A police siren emits a sinusoidal wave with frequency $f_S = 300$ Hz. The speed of sound is 340 m/s. (a) Find the wavelength of the waves if the siren is at rest in the air. (b) If the siren is moving at 30 m/s, find the wavelength of the waves ahead of and behind the source.

28. Two identical violin strings, when in tune and stretched with the same tension, have a fundamental frequency of 440.0 Hz. One of the string is retuned by adjusting its tension. When this is done, 1.5 beats per second are heard when both strings are plucked simultaneously. (a) What are the possible fundamental frequencies of the retuned string? (b) By what fractional amount was the string tension changed if it was (i) increased (ii) decreased ?

29. A swimming duck paddles the water with its feet once every 1.6 s, producing surface waves with this period. The duck is moving at constant speed in a pond where the speed of surface waves is 0.32 m/s, and the crests of the waves ahead of the duck are spaced 0.12 m apart. (a) What is the duck's speed? (b) How far apart are the crests behind the duck?

30. A railroad train is travelling at 30.0 m/s in still air. The frequency of the note emitted by the train whistle is 262 Hz. What frequency is heard by a passenger on a train moving in the opposite direction to the first at 18.0 m/s and (a) approaching the first? (b) receding from the first? Speed of sound in air = 340 m/s.

31. A boy is walking away from a wall at a speed of 1.0 m/s in a direction at right angles to the wall. As he walks, he blows a whistle steadily. An observer towards whom the boy is walking hears 4.0 beats per second. If the speed of sound is 340 m/s, what is the frequency of the whistle?

32. The first overtone of an open organ pipe beats with the first overtone of a closed organ pipe with a beat frequency of 2.2 Hz. The fundamental frequency of the closed organ pipe is 110 Hz. Find the lengths of the pipes. Speed of sound in air $v = 330$ m/s.

33. A tuning fork P of unknown frequency gives 7 beats in 2 seconds with another tuning fork Q. When Q runs towards a wall with a speed of 5 m/s it gives 5 beats per second with its echo. On loading P with wax, it gives 5 beats per second with Q. What is the frequency of P? Assume speed of sound = 332 m/s.

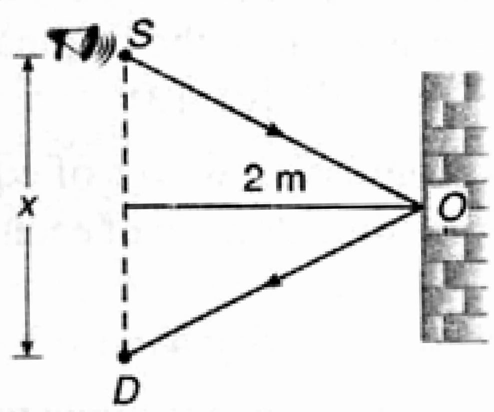
34. A stationary observer receives sonic oscillations from two tuning forks one of which approaches and the other recedes with the same velocity. As this takes place, the observer hears the beats of frequency $f = 2.0$ Hz. Find the velocity of each tuning fork if their oscillation frequency is $f_0 = 680$ Hz and the velocity of sound in air is $v = 340$ m/s.

Interference in Sound

35. Sound waves from a tuning fork A reach a point P by two separate paths ABP and ACP. When ACP is greater than ABP by 11.5 cm, there is silence at P. When the difference is 23 cm the sound becomes loudest at P and when 34.5 cm there is silence again and so on. Calculate the minimum frequency of the fork if the velocity of sound is taken to be 331.2 m/s.

36. Two loudspeakers S_1 and S_2 each emit sounds of frequency 220 Hz uniformly in all directions. S_1 has an acoustic output of 1.2×10^{-3} W and S_2 has 1.8×10^{-3} W. S_1 and S_2 vibrate in phase. Consider a point P such that $S_1P = 0.75$ m and $S_2P = 3$ m. How are the phases arriving at P related? What is the intensity at P when both S_1 and S_2 are on? Speed of sound in air is 330 m/s.

37. A source of sound emitting waves at 360 Hz is placed in front of a vertical wall, at a distance 2 m from it. A detector is also placed in front of the wall at the same distance from it. Find the minimum distance between the source and the detector for which the detector detects a maximum of sound. Take speed of sound in air = 360 m/s.



Objective Questions

Single Correct Option

1. Velocity of sound in vacuum is
   (a) equal to 330 m/s (b) greater than 330 m/s
   (c) less than 330 m/s (d) None of these
2. Longitudinal waves are possible in
   (a) solids (b) liquids (c) gases (d) All of these
3. The temperature at which the velocity of sound in oxygen will be the same as that of nitrogen at 15°C is
   (a) 112°C (b) 72°C (c) 56°C (d) 17°C
4. A closed organ pipe is excited to vibrate in the third overtone. It is observed that there are
   (a) three nodes and three antinodes (b) three nodes and four antinodes
   (c) four nodes and three antinodes (d) four nodes and four antinodes
5. When temperature is increased, the frequency of organ pipe
   (a) increases (b) decreases (c) remains same (d) Nothing can be said
6. When a sound wave travels from water to air, it
   (a) bends towards normal (b) bends away from normal
   (c) may bend in any direction (d) data insufficient
7. A closed organ pipe and an open organ pipe are tuned to the same fundamental frequency. The ratio of their lengths is
   (a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1
8. A sonometer wire under a tension of 10 kg weight is in unison with a tuning fork of frequency 320 Hz. To make the wire vibrate in unison with a tuning fork of frequency 256 Hz, the tension should be altered by
   (a) 3.6 kg decreased (b) 3.6 kg increased (c) 6.4 kg decreased (d) 6.4 kg increased
9. A tuning fork of frequency 256 Hz is moving towards a wall with a velocity of 5 m/s. If the speed of sound is 330 m/s, then the number of beats heard per second by a stationary observer lying between tuning fork and the wall is
   (a) 2 (b) 4 (c) zero (d) 8
10. Two sound waves of wavelength 1 m and 1.01 m in a gas produce 10 beats in 3 s. The velocity of sound in the gas is
    (a) 330 m/s (b) 337 m/s (c) 360 m/s (d) 300 m/s
11. When a source is going away from a stationary observer with the velocity equal to that of sound in air, then the frequency heard by observer is n times the original frequency. The value of n is
    (a) 0.5 (b) 0.25 (c) 1.0 (d) no sound is heard
12. When interference is produced by two progressive waves of equal frequencies, then the maximum intensity of the resulting sound are N times the intensity of each of the component waves. The value of N is
    (a) 1 (b) 2 (c) 4 (d) 8
13. A tuning fork of frequency 500 Hz is sounded on a resonance tube. The first and second resonances are obtained at 17 cm and 52 cm. The velocity of sound is
    (a) 170 m/s (b) 350 m/s (c) 520 m/s (d) 850 m/s

14. A vehicle, with a horn of frequency n is moving with a velocity of 30 m/s in a direction perpendicular to the straight line joining the observer and the vehicle. The observer perceives the sound to have a frequency $(n + n_1)$. If the sound velocity in air is 300 m/s, then

(a) $n_1 = 10n$ (b) $n_1 = 0$ (c) $n_1 = 0.1n$ (d) $n_1 = -0.1n$

15. How many frequencies below 1 kHz of natural oscillations of air column will be produced if a pipe of length 1 m is closed at one end ? [velocity of sound in air is 340 m/s]

(a) 5 (b) 6 (c) 4 (d) 8

16. A sound source emits frequency of 180 Hz when moving towards a rigid wall with speed 5 m/s and an observer is moving away from wall with speed 5 m/s. Both source and observer moves on a straight line which is perpendicular to the wall. The number of beats per second heard by the observer will be [speed of sound = 355 m/s]

(a) 5 beats/s (b) 10 beats/s (c) 6 beats/s (d) 8 beats/s

17. Two sound waves of wavelengths λ_1 and λ_2 $(\lambda_2 > \lambda_1)$ produce n beats/s, the speed of sound is

(a) $\dfrac{n\lambda_1\lambda_2}{\lambda_2 - \lambda_1}$ (b) $n\left(\dfrac{1}{\lambda_1} - \dfrac{1}{\lambda_2}\right)$ (c) $n(\lambda_2 - \lambda_1)$ (d) $n(\lambda_2 + \lambda_1)$

18. *A, B* and *C* are three tuning forks. Frequency of *A* is 350 Hz. Beats produced by *A* and *B* are 5/s and by *B* and *C* are 4/s. When a wax is put on *A* beat frequency between *A* and *B* is 2 Hz and between *A* and *C* is 6 Hz. Then frequency of *B* and *C* respectively are

(a) 355 Hz, 349 Hz (b) 345 Hz, 341 Hz (c) 355 Hz, 341 Hz (d) 345 Hz, 349 Hz

19. The first resonance length of a resonance tube is 40 cm and the second resonance length is 122 cm. The third resonance length of the tube will be

(a) 200 cm (b) 202 cm (c) 203 cm (d) 204 cm

20. Two identical wires are stretched by the same tension of 100 N and each emits a note of frequency 200 Hz. If the tension in one wire is increased by 1 N, then the beat frequency is

(a) 2 Hz (b) $\dfrac{1}{2}$ Hz (c) 1 Hz (d) None

21. A tuning fork of frequency 340 Hz is sounded above an organ pipe of length 120 cm. Water is now slowly poured in it. The minimum height of water column required for resonance is (speed of sound in air = 340 m/s)

(a) 25 cm (b) 95 cm (c) 75 cm (d) 45 cm

22. In a closed end pipe of length 105 cm, standing waves are set up corresponding to the third overtone. What distance from the closed end, amongst the following is a pressure node ?

(a) 20 cm (b) 60 cm (c) 85 cm (d) 45 cm

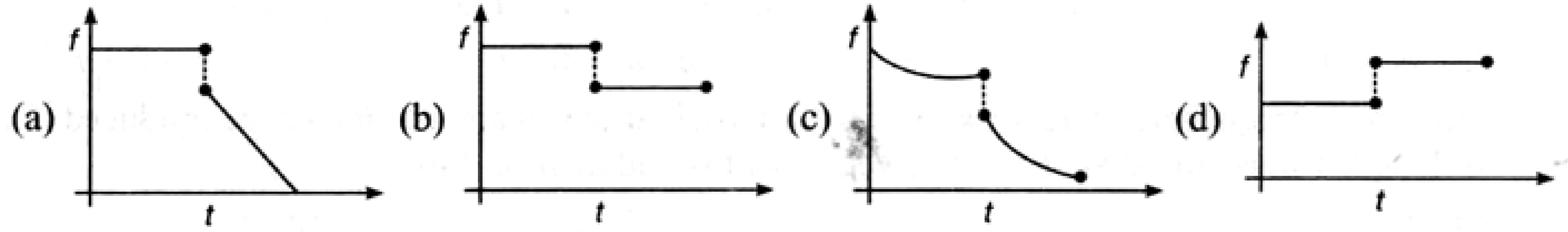
23. If the fundamental frequency of a pipe closed at one end is 512 Hz. The frequency of a pipe of the same dimension but open at both ends will be

(a) 1024 Hz (b) 512 Hz (c) 256 Hz (d) 128 Hz

24. Oxygen is 16 times heavier than hydrogen. At NTP equal volume of hydrogen and oxygen are mixed. The ratio of speed of sound in the mixture to that in hydrogen is

(a) $\sqrt{8}$ (b) $\sqrt{\dfrac{1}{8}}$ (c) $\sqrt{\dfrac{2}{17}}$ (d) $\sqrt{\dfrac{32}{17}}$

25. A train is moving towards a stationary observer. Which of the following curve best represents the frequency received by observer f as a function of time?



26. A closed organ pipe and an open pipe of same length produce 4 beats when they are set into vibrations simultaneously. If the length of each of them were twice their initial lengths, the number of beats produced will be

(a) 2 (b) 4 (c) 1 (d) 8

27. One train is approaching an observer at rest and another train is receding from him with the same velocity 4 m/s. Both trains blow whistles of same frequency of 243 Hz. The beat frequency in Hz as heard by the observer is (speed of sound in air $= 320$ m/s)

(a) 10 (b) 6 (c) 4 (d) 1

28. Speed of sound in air is 320 m/s. A pipe closed at one end has a length of 1 m and there is another pipe open at both ends having a length of 1.6 m. Neglecting end corrections, both the air columns in the pipes can resonate for sound of frequency

(a) 80 Hz (b) 240 Hz (c) 320 Hz (d) 400 Hz

29. Four sources of sound each of sound level 10 dB are sounded together in phase, the resultant intensity level will be ($\log_{10} 2 = 0.3$)

(a) 40 dB (b) 26 dB (c) 22 dB (d) 13 dB

30. A longitudinal sound wave given by $P = 2.5 \sin \frac{\pi}{2}(x - 600t)$ (P is in N/m^2 and x is in metre and t is in second) is sent down a closed organ pipe. If the pipe vibrates in its second overtone, the length of the pipe is

(a) 6 m (b) 8 m (c) 5 m (d) 10 m

31. Sound waves of frequency 600 Hz fall normally on perfectly reflecting wall. The distance from the wall at which the air particles have the maximum amplitude of vibration is

(speed of sound in air $= 330$ m/s)

(a) 13.75 cm (b) 40.25 cm (c) 70.5 cm (d) 60.75 cm

32. The wavelength of two sound waves are 49 cm and 50 cm respectively. If the room temperature is 30°C then the number of beats produced by them is approximately

(velocity of sound in air at 30°C $= 332$ m/s)

(a) 6 (b) 10 (c) 13 (d) 18

33. Two persons A and B, each carrying a source of frequency 300 Hz, are standing a few metre apart. A starts moving towards B with velocity 30 m/s. If the speed of sound is 300 m/s, which of the following is true?

(a) Number of beats heard by A is higher than that heard by B

(b) The number of beats heard by B are 30 Hz

(c) Both (a) and (b) are correct

(d) Both (a) and (b) are wrong

34. A fixed source of sound emitting a certain frequency appears as f_a when the observer is approaching the source with speed v_0 and f_r when the observer recedes from the source with the same speed. The frequency of the source is

(a) $\frac{f_r + f_a}{2}$ (b) $\frac{f_r - f_a}{2}$ (c) $\sqrt{f_a f_r}$ (d) $\frac{2f_r f_a}{f_r + f_a}$

For **JEE Advanced**

Assertion and Reason

Directions : *Choose the correct option.*

(*a*) *If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion.***

(*b*) *If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion.***

(*c*) *If **Assertion** is true, but the **Reason** is false.*

(*d*) *If **Assertion** is false but the **Reason** is true.*

1. **Assertion :** A closed pipe and an open organ pipe are of same length. Then neither of their frequencies can be same.
Reason : In the above case fundamental frequency of closed pipe will be two times the fundamental frequency of open pipe.

2. **Assertion :** A sound source is approaching towards a stationary observer along the line joining them. Then apparent frequency to the observer will go on increasing.
Reason : If there is no relative motion between source and observer apparent frequency is equal to the actual frequency.

3. **Assertion :** In longitudinal wave pressure is maximum at a point where displacement is zero.
Reason : There is a phase difference of $\frac{\pi}{2}$ between $y(x, t)$ and $\Delta P(x, t)$ equation in case of longitudinal wave.

4. **Assertion :** A train is approaching towards a hill. The driver of the train will hear beats.
Reason : Apparent frequency of reflected sound observed by driver will be more than the frequency of direct sound observed by him.

5. **Assertion :** Sound level increases linearly with intensity of sound.
Reason : If intensity of sound is doubled sound level increases approximately 3 dB.

6. **Assertion :** Speed of sound in gases is independent of pressure of gas.
Reason : With increase in temperature of gas speed of sound will increase.

7. **Assertion :** Beat frequency between two tuning forks A and B is 4 Hz. Frequency of A is greater than the frequency of B. When A is loaded with wax, beat frequency may increase or decrease.
Reason : When a tuning fork is loaded with wax its frequency decreases.

8. **Assertion :** Two successive frequencies of an organ pipe are 450 Hz and 750 Hz. Then this pipe is a closed pipe.
Reason : Fundamental frequency of this pipe is 150 Hz.

9. **Assertion :** Fundamental frequency of a narrow pipe is more.
Reason : According to Laplace end correction if radius of pipe is less, frequency should be more.

10. **Assertion :** In the experiment of finding speed of sound by resonance tube method, as the level of water is lowered, wavelength increases.
Reason : By lowering the water level number of loops increases.

Objective Questions

Single Correct Option

1. A plane wave of sound travelling in air is incident upon a plane water surface. The angle of incidence is 60°. If velocity of sound in air and water are 330 m/s and 1400 m/s, then the wave undergoes
(a) refraction only
(b) reflection only
(c) Both reflection and refraction
(d) neither reflection nor refraction

2. An organ pipe of 3.9 π m long, open at both ends is driven to third harmonic standing wave. If the amplitude of pressure oscillation is 1% of mean atmospheric pressure $[p_0 = 10^5 \text{ N/m}^2]$. The maximum displacement of particle from mean position will be
[Given velocity of sound = 200 m/s and density of air = 1.3 kg/m^3]
(a) 2.5 cm (b) 5 cm (c) 1 cm (d) 2 cm

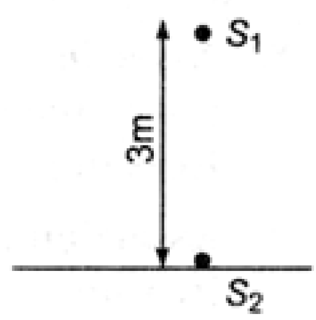
3. A plane sound wave passes from medium 1 into medium 2. The speed of sound in medium 1 is 200 m/s and in medium 2 is 100 m/s. The ratio of amplitude of the transmitted wave to that of incident wave is
(a) $\frac{3}{4}$ (b) $\frac{4}{5}$ (c) $\frac{5}{6}$ (d) $\frac{2}{3}$

4. Two sources of sound are moving in opposite directions with velocities v_1 and v_2 $(v_1 > v_2)$. Both are moving away from a stationary observer. The frequency of both the sources is 1700 Hz. What is the value of $(v_1 - v_2)$ so that the beat frequency observed by the observer is 10 Hz? $v_{\text{sound}} = 340$ m/s and assume that v_1 and v_2 both are very much less than v_{sound}.
(a) 1 m/s (b) 2 m/s (c) 3 m/s (d) 4 m/s

5. A sounding body emitting a frequency of 150 Hz is dropped from a height. During its fall under gravity it crosses a balloon moving upwards with a constant velocity of 2 m/s one second after it started to fall. The difference in the frequency observed by the man in balloon just before and just after crossing the body will be (velocity of sound = 300 m/s, $g = 10$ m/s^2)
(a) 12 (b) 6 (c) 8 (d) 4

6. A closed organ pipe has length L. The air in it is vibrating in third overtone with maximum amplitude a. The amplitude at distance $\frac{L}{7}$ from closed end of the pipe is
(a) 0 (b) a (c) $\frac{a}{2}$ (d) Data insufficient

7. S_1 and S_2 are two coherent sources of sound having no initial phase difference. The velocity of sound is 330 m/s. No maxima will be formed on the line passing through S_2 and perpendicular to the line joining S_1 and S_2. If the frequency of both the sources is



(a) 330 Hz
(b) 120 Hz
(c) 100 Hz
(d) 220 Hz

8. A source is moving with constant speed $v_s = 20$ m/s towards a stationary observer due east of the source. Wind is blowing at the speed of 20 m/s at 60° north of east. The source has frequency 500 Hz. Speed of sound = 300 m/s. The frequency registered by the observer is approximately

(a) 541 Hz (b) 552 Hz (c) 534 Hz (d) 517 Hz

9. A car travelling towards a hill at 10 m/s sounds its horn which has a frequency 500 Hz. This is heard in a second car travelling behind the first car in the same direction with speed 20 m/s. The sound can also be heard in the second car by reflection of sound from the hill. The beat frequency heard by the driver of the second car will be (speed of sound in air = 340 m/s)

(a) 31 Hz (b) 24 Hz (c) 21 Hz (d) 34 Hz

10. Two sounding bodies are producing progressive waves given by $y_1 = 2\sin(400\pi t)$ and $y_2 = \sin(404\pi t)$ where t is in second, which superpose near the ears of a person. The person will hear

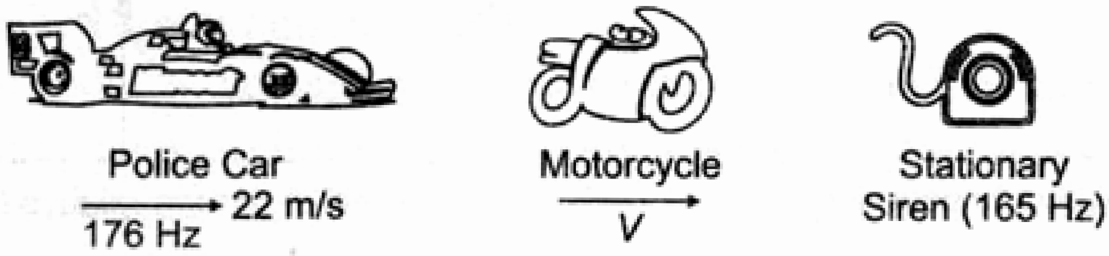
(a) 2 beats/s with intensity ratio 9/4 between maxima and minima
(b) 2 beats/s with intensity ratio 9 between maxima and minima
(c) 4 beats/s with intensity ratio 16 between maxima and minima
(d) 4 beats/s with intensity ratio 16/9 between maxima and minima

11. The air in a closed tube 34 cm long is vibrating with two nodes and two antinodes and its temperature is 51°C. What is the wavelength of the waves produced in air outside the tube, when the temperature of air is 16°C?

(a) 42.8 cm (b) 68 cm (c) 17 cm (d) 102 cm

12. A police car moving at 22 m/s, chase a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motorcyclist, if he does not observe any beats. (velocity of sound in air = 330 m/s)

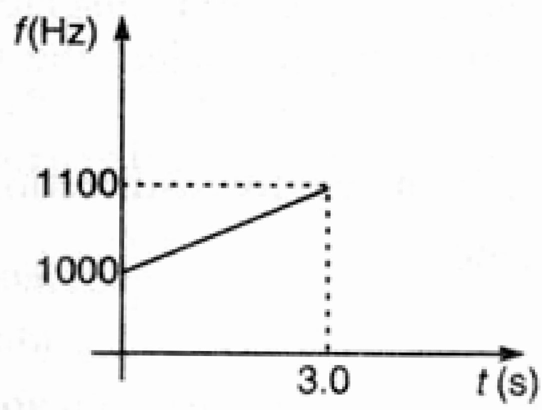
(a) 33 m/s (b) 22 m/s (c) zero (d) 11 m/s



13. A closed organ pipe resonates in its fundamental mode at a frequency of 200 Hz with O_2 in the pipe at a certain temperature. If the pipe now contains 2 moles of O_2 and 3 moles of ozone, then what will be the fundamental frequency of same pipe at same temperature?

(a) 268.23 Hz (b) 175.4 Hz (c) 149.45 Hz (d) None of these

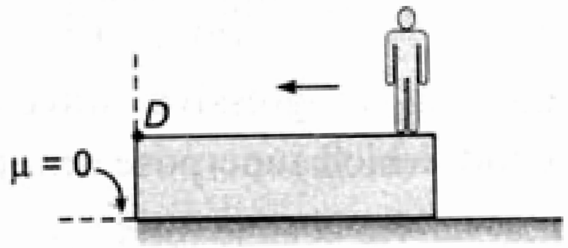
14. A detector is released from rest over a source of sound of frequency $f_0 = 10^3$ Hz. The frequency observed by the detector at time t is plotted in the graph. The speed of sound in air is ($g = 10$ m/s^2)



(a) 330 m/s (b) 350 m/s (c) 300 m/s (d) 310 m/s

Passage : (Q. 15 - Q. 17)

A man of mass 50 kg is running on a plank of mass 150 kg with speed of 8 m/s relative to plank as shown in the figure (both were initially at rest and the velocity of man with respect to ground any how remains constant). Plank is placed on smooth horizontal surface. The man, while running whistles with frequency f_0. A detector (D) placed on plank detects frequency. The man jumps off with same velocity (w.r.t. to ground) from point D and slides on the smooth horizontal surface [Assume coefficient of friction between man and horizontal is zero]. The speed of sound in still medium is 330 m/s. Answer following questions on the basis of above situations.



15. The frequency of sound detected by detector D, before man jumps off the plank is

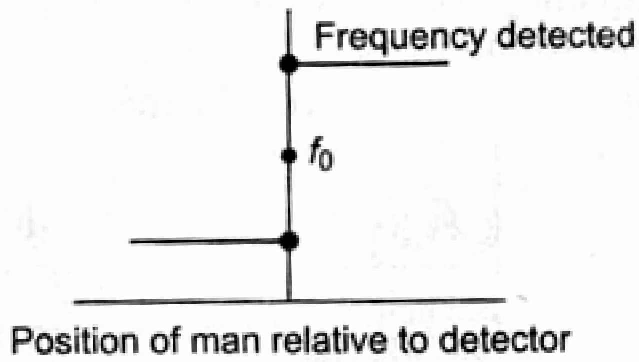
(a) $\frac{332}{324} f_0$ (b) $\frac{330}{322} f_0$ (c) $\frac{328}{336} f_0$ (d) $\frac{330}{338} f_0$

16. The frequency of sound detected by detector D, after man jumps off the plank is

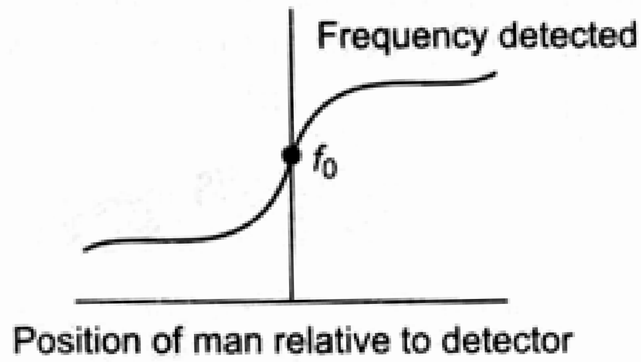
(a) $\frac{332}{324} f_0$ (b) $\frac{330}{322} f_0$ (c) $\frac{328}{336} f_0$ (d) $\frac{330}{338} f_0$

17. Choose the correct plot between the frequency detected by detector versus position of the man relative to detector

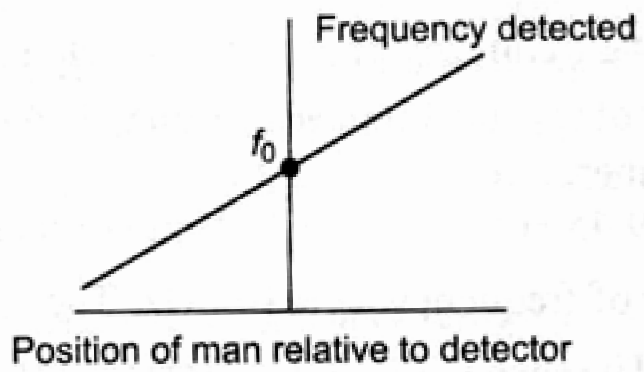
(a)



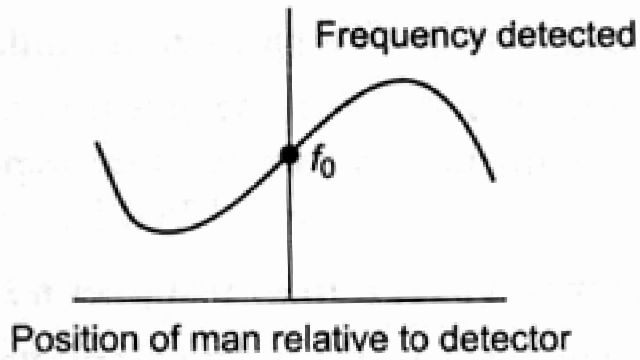
(b)



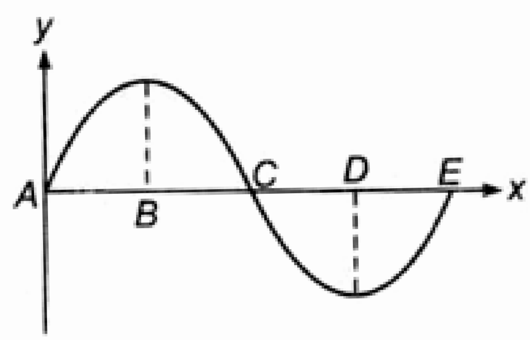
(c)



(d)



18. Sound waves are travelling along positive x-direction. Displacement of particle at any time t is as shown in figure. Select the wrong statement.



(a) Particle located at E has its velocity in negative x-direction

(b) Particle located at D has zero velocity

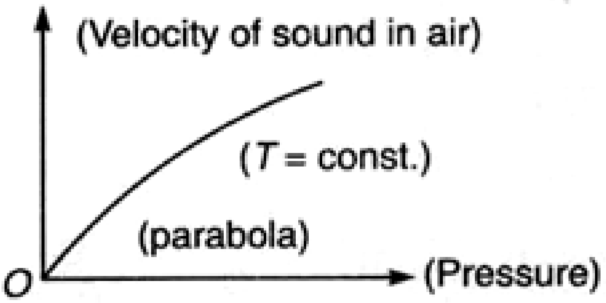
(c) Both (a) and (b) are correct

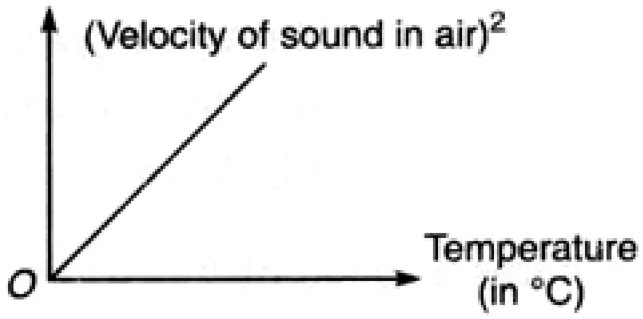
(d) Both (a) and (b) are wrong

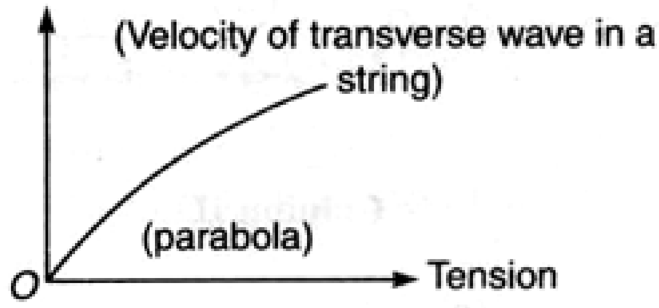
More than One Correct Options

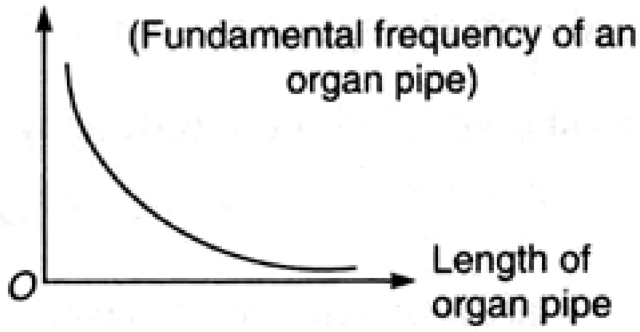
19. An air column in a pipe, which is closed at one end, is in resonance with a vibrating tuning fork of frequency 264 Hz. If $v = 330$ m/s, the length of the column in cm is (are)
(a) 31.25 (b) 62.50 (c) 93.75 (d) 125

20. Which of the following is/are correct?

(a)


(b)


(c)


(d)


21. Choose the correct options for longitudinal wave
(a) maximum pressure variation is BAk
(b) maximum density variation is ρAk
(c) pressure equation and density equation are in phase
(d) pressure equation and displacement equation are out of phase

22. Second overtone frequency of a closed pipe and fourth harmonic frequency of an open pipe are same. Then choose the correct options.
(a) Fundamental frequency of closed pipe is more than the fundamental frequency of open pipe
(b) First overtone frequency of closed pipe is more than the first overtone frequency of open pipe
(c) Fifteenth harmonic frequency of closed pipe is equal to twelfth harmonic frequency of open pipe
(d) Tenth harmonic frequency of closed pipe is equal to eighth harmonic frequency of open pipe

23. For fundamental frequency f of a closed pipe, choose the correct options.
(a) If radius of pipe is increased f will decrease
(b) If temperature is increased f will increase
(c) If molecular mass of the gas filled in the pipe is increased f will decrease.
(d) If pressure of gas (filled in the pipe) is increased without change in temperature, f will remain unchanged

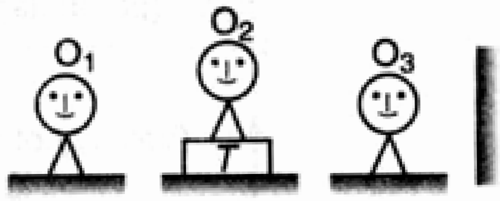
24. A source is approaching towards an observer with constant speed along the line joining them. After crossing the observer, source recedes from observer with same speed. Let f is apparent frequency heard by observer. Then
(a) f will increase during approaching
(b) f will decrease during receding
(c) f will remain constant during approaching
(d) f will remain constant during receding

Match the Columns

1. Fundamental frequency of an open organ pipe is f. Match the following two columns for a closed pipe of double the length.

Column I	Column II
(a) Fundamental frequency	(p) $1.25 f$
(b) Second overtone frequency	(q) f
(c) Third harmonic frequency	(r) $0.75 f$
(d) First overtone frequency	(s) None

2. A train T horns a sound of frequency f. It is moving towards a wall with speed $\frac{1}{4}$ th the speed of sound. There are three observers O_1, O_2 and O_3 as shown. Match the following two columns :



Column I	Column II
(a) Beat frequency observed to O_1	(p) $\frac{2}{3} f$
(b) Beat frequency observed to O_2	(q) $\frac{8}{15} f$
(c) Beat frequency observed to O_3	(r) None
(d) If train moves in opposite direction with same speed then beat frequency observed to O_3	(s) Zero

3. A tuning fork is placed near a vibrating stretched wire. A boy standing near the two hears a beat frequency f. It is known that frequency of tuning fork is greater than frequency of stretched wire. Match the following two columns.

Column I	Column II
(a) If tuning fork is loaded with wax.	(p) beat frequency must increase.
(b) If prongs of tuning fork are filed.	(q) beat frequency must decrease.
(c) If tension in stretched wire is increased.	(r) beat frequency may increase.
(d) If tension in stretched wire is decreased	(s) beat frequency may decrease.

4. I represents intensity of sound wave, A the amplitude and r the distance from the source. Then match the following two columns.

Column I	Column II
(a) Intensity due to a point source.	(p) Proportional to $r^{-1/2}$
(b) Amplitude due to a point source.	(q) Proportional to r^{-1}
(c) Intensity due to a line source.	(r) Proportional to r^{-2}
(d) Amplitude due to a line source.	(s) Proportional to r^{-4}

5. Equation of longitudinal stationary wave in second overtone mode in a closed organ pipe is

$$y = (4\ \text{mm}) \sin \pi x \cos \pi t$$

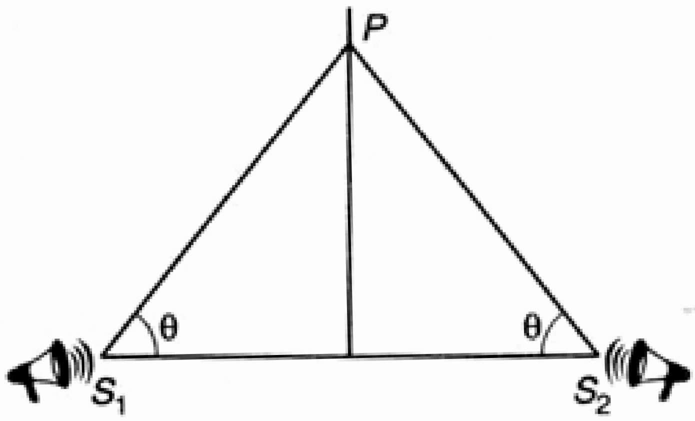
Here x is in metre and t in second. Then match the following two columns :

Column I	Column II
(a) Length of pipe	(p) 1 m
(b) Wavelength	(q) 1.5 m
(c) Distance of displacement node from the closed end	(r) 2.0 m
(d) Distance of pressure node from the closed end	(s) None

Subjective Questions

1. A train of length l is moving with a constant speed v along a circular track of radius R. The engine of the train emits a sound of frequency f. Find the frequency heard by a guard at the rear end of the train.
2. A 3 m long organ pipe open at both ends is driven to third harmonic standing wave. If the amplitude of pressure oscillations is 1 per cent of mean atmospheric pressure ($P_0 = 10^5\ \text{N/m}^2$). Find the amplitude of particle displacement and density oscillations. Speed of sound $v = 332\ \text{m/s}$ and density of air $\rho = 1.03\ \text{kg/m}^3$.
3. A siren creates a sound level of 60 dB at a location 500 m from the speaker. The siren is powered by a battery that delivers a total energy of 1.0 kJ. Assuming that the efficiency of siren is 30%, determine the total time the siren can sound.
4. A cylinder of length 1 m is divided by a thin perfectly flexible diaphragm in the middle. It is closed by similar flexible diaphragms at the ends. The two chambers into which it is divided contain hydrogen and oxygen. The two diaphragms are set in vibrations of same frequency. What is the minimum frequency of these diaphragms for which the middle diaphragm will be motionless ? Velocity of sound in hydrogen is 1100 m/s and that in oxygen is 300 m/s.
5. A conveyor belt moves to the right with speed $v = 300\ \text{m/min}$. A very fast pieman puts pies on the belt at a rate of 20 per minute and they are received at the other end by a pieeater.
   (a) If the pieman is stationary find the spacing x between the pies and the frequency with which they are received by the stationary pieeater.
   (b) The pieman now walks with speed 30 m/min towards the receiver while continuing to put pies on the belt at 20 per minute. Find the spacing of the pies and the frequency with which they are received by the stationary pieeater.
6. A point sound source is situated in a medium of bulk modulus $1.6 \times 10^5\ \text{N/m}^2$. An observer standing at a distance 10 m from the source writes down the equation for the wave as $y = A \sin (15\pi x - 6000\,\pi t)$. Here y and x are in metres and t is in second. The maximum pressure amplitude received to the observer's ear is 24π Pa, then find :
   (a) the density of the medium,
   (b) the displacement amplitude A of the waves received by the observer and
   (c) the power of the sound source.

7. Two sources of sound S_1 and S_2 vibrate at the same frequency and are in phase. The intensity of sound detected at a point P (as shown in figure) is I_0.



(a) If $\theta = 45°$ what will be the intensity of sound detected at this point if one of the sources is switched off?

(b) What will be intensity of sound detected at P if $\theta = 60°$ and both the sources are now switched on?

8. Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monoatomic gas of molar mass M_A. Pipe B is open at one end and closed at the other end and is filled with a diatomic gas of molar mass M_B. Both gases are at the same temperature.

(a) If the frequency of the second harmonic of the fundamental mode in pipe A is equal to the frequency of the third harmonic of the fundamental mode in pipe B, determine the value of $\frac{M_A}{M_B}$.

(b) Now, the open end of pipe B is also closed (so that the pipe is closed at both ends.) Find the ratio of the fundamental frequency in pipe A to that in pipe B.

9. A boat is travelling in a river with a speed 10 m/s along the stream flowing with a speed 2 m/s. From this boat a sound transmitter is lowered into the river through a rigid support. The wavelength of the sound emitted from the transmitter inside the water is 14.45 mm. Assume that attenuation of sound in water and air is negligible.

(a) What will be the frequency detected by a receiver kept inside the river downstream?

(b) The transmitter and the receiver are now pulled up into the air. The air is blowing with a speed 5 m/s in the direction opposite the river stream. Determine the frequency of the sound detected by the receiver.

(Temperature of the air and water = 20° C; Density of river water = 10^3 kg/m^3; Bulk modulus of the water $= 2.088 \times 10^9$ Pa; Gas constant $R = 8.31$ J/mol-K; Mean molecular mass of air $= 28.8 \times 10^{-3}$ kg per mol and C_P/C_V for air = 1.4)

10. A string 25 cm long and having a mass of 2.5 g is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string decrease the beat frequency. If the speed of sound in air is 320 m/s, find the tension in the string.

11. A source emits sound waves of frequency 1000 Hz. The source moves to the right with a speed of 32 m/s relative to ground. On the right a reflecting surface moves towards left with a speed of 64 m/s relative to the ground. The speed of sound in air is 332 m/s. Find :

(a) the wavelength of sound in ahead of the source,

(b) the number of waves arriving per second which meets the reflecting surface,

(c) the speed of reflected waves and

(d) the wavelength of reflected waves.

ANSWERS

Introductory Exercise 16.1

1. 1.4×10^5 N/m^2 **2.** 7.25 cm, 72.5 m **3.** (a) Zero (b) 3.63×10^{-6} m
4. 1.04×10^{-5} m

Introductory Exercise 16.2

1. 819°C **2.** 20.06 m/s **3.** 3.6×10^9 Pa **4.** 315 m/s

Introductory Exercise 16.3

1. (a) 4.67 Pa (b) 2.64×10^{-2} W/m^2 (c) 104 dB **2.** 7.9 **3.** 20 dB
4. Faintest (a) 4.49×10^{-13} W/m^2, −3.48 dB (b) 1.43×10^{-11} m
Loudest (a) 0.881 W/m^2, +119 dB (b) 2.01×10^{-5} m

Introductory Exercise 16.4

1. 1375 Hz **2.** (a) 11.7 cm (b) 180° **3.** $\lambda = 2\sqrt{4(H+h)^2 + d^2} - 2\sqrt{4H^2 + d^2}$
4. (a) 8.14° (b) 16.5° (c) 3 Maxima beyond the $\theta = 0°$ maximum
5. (a) 0 (b) $2I_0$ (c) $4I_0$ **6.** (a) 0 (b) 66 dB (c) 63 dB
7. (a) $I_1 = 19.9\,\mu$W/m^2, $I_2 = 8.84\,\mu$W/m^2 (b) $55.3\,\mu$W/m^2 (c) $2.2\,\mu$W/m^2 (d) $28.7\,\mu$W/m^2

Introductory Exercise 16.5

1. (a) 0.392 m (b) 0.470 m
2. (a) Fundamental 0.8 m, first overtone 0.267 m, 0.8 m, second overtone 0.16 m, 0.48 m, 0.8 m
(b) Fundamental 0, first overtone 0, 0.533 m. Second overtone 0, 0.32 m, 0.64 m
3. (a) closed (b) 5, 7 (c) 1.075 m **4.** Diatomic **5.** 352 m/s

Introductory Exercise 16.6

1. 252 Hz **2.** 387 Hz **3.** 1.02 **4.** 0.4 cm

Introductory Exercise 16.7

2. (a) 1.3 m (b) 262 Hz **3.** No **4.** (a) 0.628 m (b) 0.748 m (c) 548 Hz (d) 460 Hz
5. 274 Hz

For JEE Main

Subjective Questions

1. 10.664 m, 4 s **2.** 1321 m/s **3.** 972 m/s
4. (a) 1.33×10^{10} Pa (b) 9.47×10^{10} Pa **5.** $Y/900$ **6.** 591 m/s **7.** Two times
8. 201 W **9.** (a) 10^{-6} W/m^2 (b) 1.2×10^{-8}W **10.** (a) 20
11. (a) 9.95×10^{-4} W/m^2 (b) 1.15×10^{-6} m **12.** 13.6 nm **13.** 80 W **14.** 134.4 dB
15. (a) 9.44×10^{-11}m, 0.43 m (b) 5.66×10^{-9} m, 0.434 m (d) air, $Aa/A\omega = 60$
16. 4.5×10^{-12} W/m^2, 6.53 dB **17.** 297 Hz
18. (a) 382.2Hz, 764.4 Hz, 1146.7 Hz (b) 191.1Hz, 573.3 Hz, 955.5 Hz
19. 250 Hz **20.** $y = A\cos(3.93x)\sin(1297t)$ **21.** 348.16 m/s, 118 cm **22.** 11.56 N
23. 336 m/s **24.** (a) $\frac{15}{16}$ m (b) $\pm\frac{\Delta P_0}{\sqrt{2}}$ (c) $P_{max} = P_{min} = P_0$ (d) $P_{max} = P_0 + \Delta P_0$, $P_{min} = P_0 - \Delta P_0$

25. (a) 0.392 m (b) 0.470 m **26.** 0.40 **27.** (a) 1.13 m (b) 1.03 m, 1.23 m
28. (a) 441.5 Hz, 438.5 Hz (b) (i) + 0.68% (ii) –0.68% **29.** (a) 0.245 m/s (b) 0.904 m
30. (a) 302 Hz (b) 228 Hz **31.** 680 Hz
32. Length of closed organ pipe is $l_1 = 0.75$ m while length of open pipe is either $l_2 = 0.99$ m or 1.0067 m.
33. 160 Hz **34.** 0.5 m/s **35.** 1440 Hz **36.** $\phi_1 = \pi$, $\phi_2 = 4\pi$, Resultant power = 6.0×10^{-5} W
37. 7.5 m

Objective Questions

1.(d)	**2.**(d)	**3.**(c)	**4.**(d)	**5.**(a)	**6.**(a)	**7.**(a)	**8.**(a)	**9.**(c)	**10.**(b)
11.(a)	**12.**(c)	**13.**(b)	**14.**(b)	**15.**(a)	**16.**(a)	**17.**(a)	**18.**(b)	**19.**(d)	**20.**(c)
21.(d)	**22.**(d)	**23.**(a)	**24.**(c)	**25.**(b)	**26.**(a)	**27.**(b)	**28.**(d)	**29.**(c)	**30.**(c)
31.(a)	**32.**(c)	**33.**(d)	**34.**(a)						

For JEE Advanced

Assertion and Reason

1. (c)	**2.** (d)	**3.** (d)	**4.** (a)	**5.** (d)	**6.** (d)	**7.** (b)	**8.** (b)	**9.** (a)	**10.** (d)

Objective Questions

1.(b)	**2.**(a)	**3.**(d)	**4.**(b)	**5.**(a)	**6.**(b)	**7.**(c)	**8.**(c)	**9.**(a)	**10.**(b)
11.(a)	**12.**(b)	**13.**(b)	**14.**(c)	**15.**(a)	**16.**(c)	**17.**(a)	**18.**(c)		

More than One Correct Options

19. (a,c) **20.** (c,d) **21.** (a,b,c) **22.** (b,c,d) **23.** (a,b,c,d) **24.** (c,d)

Match the Columns

1. (a) → s (b) → p (c) → r (d) → r
2. (a) → q (b) → p (c) → s (d) → s
3. (a) → r,s (b) → p (c) → r, s (d) → p
4. (a) → r (b) → q (c) → q (d) → p
5. (a) → s (b) → r (c) → p,r (d) → q

Subjective Questions

1. f **2.** 0.28 cm, 9.0×10^{-3} kg/m^3 **3.** 95.5 s **4.** 1650 Hz
5. (a) 15 m, 20 min^{-1} (b) 13.5 m, 22.22 min^{-1} **6.** (a) 1 kg/m^3 (b) 10μm (c) 288 π^3 W
7. (a) $\frac{l_0}{4}$ (b) l_0 **8.** (a) $\frac{400}{189}$ (b) $\frac{3}{4}$ **9.** (a) 1.0069×10^5 Hz (b) 1.0304×10^5 Hz
10. 27.04 N **11.** (a) 0.3 m (b) 1320 (c) 332 m/ss (d) 0.2 m

17

THERMOMETRY, THERMAL EXPANSION & KINETIC THEORY OF GASES

Chapter Contents

17.1 Thermometers and The Celsius Temperature Scale

Thermometers are devices that are used to measure temperatures. All thermometers are based on the principle that some physical properties of a system change as the system's temperature changes. Some physical properties that change with temperature are

(1) the volume of a liquid
(2) the length of a solid
(3) the pressure of a gas at constant volume
(4) the volume of a gas at constant pressure and
(5) the electric resistance of a conductor.

A common thermometer in everyday use consists of a mass of liquid, usually mercury or alcohol that expands in a glass capillary tube when heated. In this case the physical property is the **change in volume of the liquid**. Any temperature change is proportional to the change in length of the liquid column. The thermometer can be calibrated accordingly. On the celcius temperature scale, a thermometer is usually calibrated between 0°C (called the ice point of water) and 100°C (called the steam point of water). Once the liquid levels in the thermometer have been established at these two points, the distance between the two points is divided into 100 equal segments to create the celsius scale.

Thus, each segment denotes a change in temperature of one celsius degree (1°C). A practical problem in this type of thermometer is that readings may vary for two different liquids. When one thermometer reads a temperature, for example 40°C the other may indicate a slightly different value. This discrepancies between thermometers are especially large at temperatures far from the calibration points. To surmount this problem we need a universal thermometer whose readings are independent of the substance used in it. The gas thermometer used in the next article meets this requirement.

17.2 The Constant Volume Gas Thermometer and The Absolute Temperature Scale

The physical property used by the constant volume gas thermometer is the **change in pressure of a gas at constant volume.**

The pressure versus temperature graph for a typical gas taken with a constant volume is shown in figure. The two dots represent the two reference temperatures namely, the ice and steam points of water. The line connecting them serves as a calibration curve for unknown temperatures. Experiments show that the thermometer readings are nearly independent of the type of gas used as long as the gas pressure is low and the temperature is well above the point at which the gas liquifies.

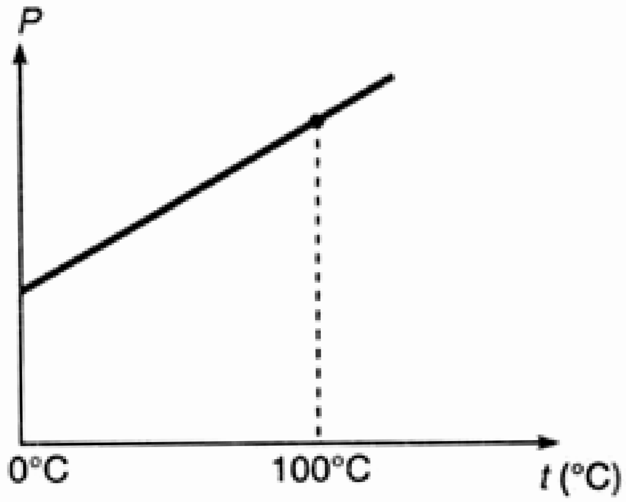


Fig. 17.1

If you extend the curves shown in figure toward negative temperatures, you find, in every case, that the pressure is zero when the temperature is –273.15° C. This significant temperature is used as the basis for the **absolute temperature scale**, which sets –273.15° C as its zero point.

This temperature is often referred to as **absolute zero**. The size of a degree on the absolute temperature scale is identical to the size of a degree on the Celsius scale. Thus, the conversion between these temperatures is

$$T_C = T - 273.15 \quad \text{...(i)}$$

In 1954, by the International committee on weights and measures, the **triple point of water** was chosen as the reference temperature for this new scale. The triple point of water is the single combination of temperature and pressure at which liquid water, gaseous water and ice (solid water) coexist in equilibrium. This triple point occurs at a temperature of approximately 0.01°C and a pressure of 4.58 mm of mercury. On the new scale, which uses the unit **kelvin**, the temperature of water at the triple point was set at 273.16 kelvin, abbreviated as 273.16 K. (No degree sign is used with the unit kelvin).

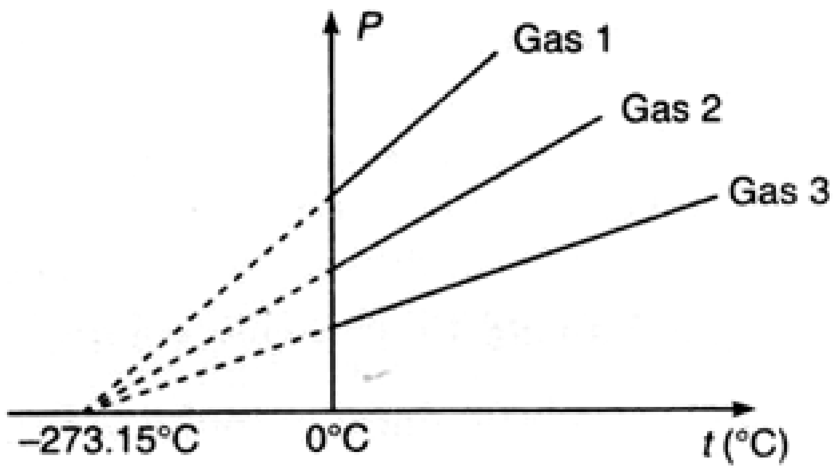


Fig. 17.2

This new absolute temperature scale (also called the kelvin scale) employs the SI unit of absolute temperature, the kelvin which is defined to be "$\frac{1}{273.16}$ of the difference between absolute zero and the temperature of the triple point of water".

The Celsius, Fahrenheit and Kelvin Temperature Scales

Equation (i) shows the relation between the temperatures in celsius scales and kelvin scale. Because the size of a degree is the same on the two scales, a temperature difference of 10°C is equal to a temperature difference of 10 K. The two scales differ only in the choice of the zero point. The ice point temperature on the kelvin scale, 273.15 K, corresponds to 0.00°C and the kelvin steam point 373.15 K, is equivalent to 100.00°C.

A common temperature scale in everyday use in US is the **Fahrenheit scale**. The ice point in this scale is 32°F and the steam point is 212°F. The distance between these two points are divided in 180 equal parts. The relation between celsius scale and fahrenheit scale is as derived below :

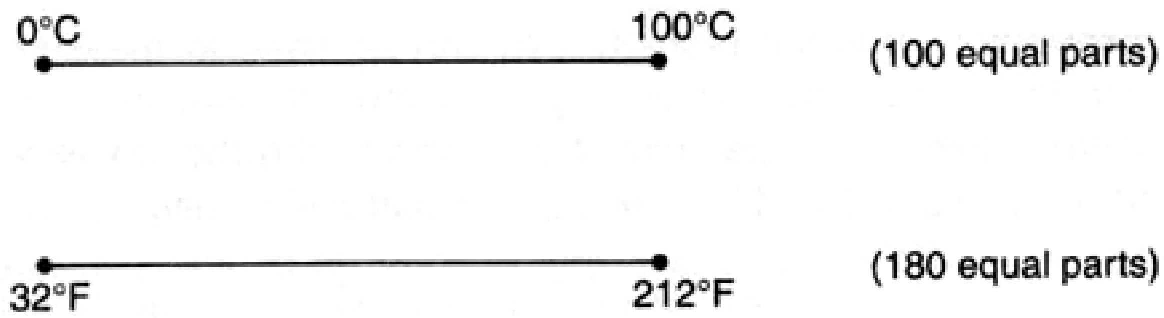
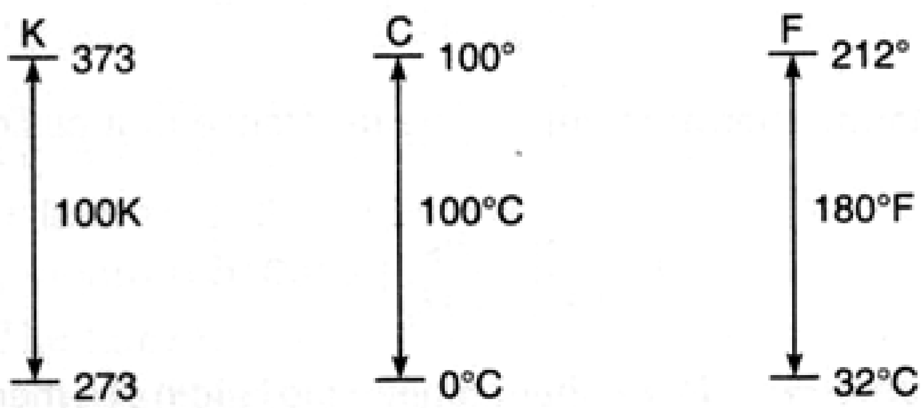


Fig. 17.3

100 parts of celsius scale = 180 parts of fahrenheit scale

$\therefore$ 1 part of celsius scale $= \frac{9}{5}$ parts of fahrenheit scale



Relation among Kelvin, Celsius and Fahrenheit temperature scales

Fig. 17.4

Hence, $$T_F = 32 + \frac{9}{5} T_C \quad ...(ii)$$

Further, $$\Delta T_C = \Delta T = \frac{5}{9} \Delta T_F \quad ...(iii)$$

Sample Example 17.1 *Express a temperature of 60°F in degrees celsius and in kelvins.*

Solution Substituting $T_F = 60°$ F in Eq. (ii)

$$T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(60° - 32°) = 15.55°\text{C}$$ **Ans.**

From Eq. (i)

$$T = T_C + 273.15 = 15.55°\text{C} + 273.15 = 288.7\ \text{K}$$ **Ans.**

Sample Example 17.2 *The temperature of an iron piece is heated from 30°C to 90°C. What is the change in its temperature on the fahrenheit scale and on the kelvin scale?*

Solution $$\Delta T_C = 90°\text{C} - 30°\text{C} = 60°\text{C}$$

Using Eq. (iii),

$$\Delta T_F = \frac{9}{5}\Delta T_C = \frac{9}{5}(60°\text{C}) = 108°\ \text{F}$$ **Ans.**

and $$\Delta T = \Delta T_C = 60\ \text{K}$$ **Ans.**

• *Important points in* THERMOMETERS

1. **Different Thermometers**

Thermometric property : It is the property that can be used to measure the temperature. It is represented by any physical quantity such as length, volume, pressure and resistance etc., which varies linearly with a certain range of temperature. Let X denote the thermometric physical quantity and X_0, X_{100} and X_t be its values at 0°C, 100°C and t °C respectively. Then,

$$t = \left(\frac{X_t - X_0}{X_{100} - X_0}\right) \times 100°\ \text{C}$$

(i) Constant volume gas thermometer : The pressure of a gas at constant volume is the thermometric property. Therefore,

$$t = \left(\frac{P_t - P_0}{P_{100} - P_0}\right) \times 100°\ \text{C}$$

(ii) Platinum resistance thermometer : The resistance of a platinum wire is the thermometric property. Hence,

$$t = \left(\frac{R_t - R_0}{R_{100} - R_0}\right) \times 100°\ \text{C}$$

(iii) Mercury thermometer : In this thermometer the length of a mercury column from some fixed point is taken as thermometric property. Thus,

$$t = \left(\frac{l_t - l_0}{l_{100} - l_0}\right) \times 100°\ \text{C}$$

2. Two other thermometers, commonly used are **thermocouple thermometer** and **total radiation pyrometer.**
3. **Total radiation pyrometer** is used to measure very high temperatures. When a body is at a high temperature, it glows brightly and the radiation emitted per second from unit area of the surface of the body is proportional to the fourth power of the absolute temperature of the body. If this radiation is measured by some device, the temperature of the body is calculated. This is the principle of a total radiation pyrometer. The main advantage of this thermometer is that the experimental body is not kept in contact with it. Hence, there is no definite higher limit of its temperature-range. It can measure temperature from 800°C to 3000°C–4000°C. However it cannot be used to measure temperatures below 800°C because at low temperatures the emission of radiation is so poor that it cannot be measured directly.
4. **Ranges of different thermometers :**

Thermometer	Lower Limit	Upper Limit
Mercury Thermometer	–30°C	300°C
Gas Thermometer	–268°C	1500°C
Platinum Resistance Thermometer	–200°C	1200°C
Thermocouple Thermometer	–200°C	1600°C
Radiation Thermometer	800°C	No limit

5. **Reaumer's Scale :** Other than Celsius, Fahrenheit and Kelvin temperature scales **Reaumer's Scale** was designed by Reaumer in 1730. The lower fixed point is 0°R representing melting point of ice. The upper fixed point is 80°R, which represents boiling point of water. The distance between the two fixed points is divided into 80 equal parts. Each part represents 1°R. If T_C, T_F and T_R are temperature values of a body on Celsius scale, Fahrenheit scale and Reaumer scale respectively, then,

$$\frac{T_C - 0}{100} = \frac{T_F - 32}{180} = \frac{T_R - 0}{80}$$

6. A substance is found to exist in three states solid, liquid and gas. For each substance there is a set of temperature and pressure at which all the three states may coexist. This is called **triple point** of that substance. For water, the values of pressure and temperature corresponding to triple point are 4.58 mm of Hg and 273.16°K.

17.3 Quantity of Heat

When a cold body is brought in contact with a hot body, the cold body warms up and the hot body cools down as they approach thermal equilibrium. Fundamentally a transfer of energy takes place from one substance to the other. This type of energy transfer that takes place solely because of a temperature difference is called **heat flow or heat transfer** and energy transfer in this way is called **heat.**

Water can be warmed up by vigorous stirring with a paddle wheel. The paddle wheel adds energy to the water by doing work on it. The same temperature change can also be caused by putting the water in contact with some hotter body. Hence, this interaction must also involve an energy exchange. Before exploring the relation between heat and mechanical energy let us define a unit of quantity of heat.

"One **calorie** (1 cal) is defined as the amount of heat required to raise the temperature of one gram of water from 14.5°C to 15.5°C."

Experiments have shown that

$$1\,\text{cal} = 4.186\,\text{J}$$

Similarly $$1\,\text{kcal} = 1000\,\text{cal} = 4186\,\text{J}$$

The calorie is not a fundamental SI unit.

17.4 Thermal Expansion

Most substances expand when they are heated. Thermal expansion is a consequence of the change in average separation between the constituent atoms of an object. Atoms of an object can be imagined to be connected to one another by stiff springs as shown in figure. At ordinary temperatures, the atoms in a solid oscillate about their equilibrium positions with an amplitude of approximately 10^{-11} m. The average spacing between the atoms is about 10^{-10} m. As the temperature of solid increases, the atoms oscillate with greater amplitudes, as a result the average separation between them increases, consequently the object expands.

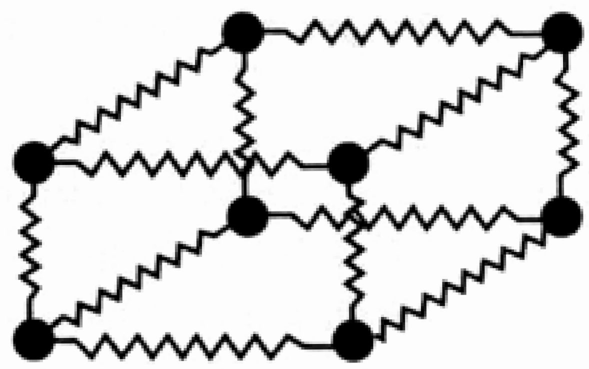
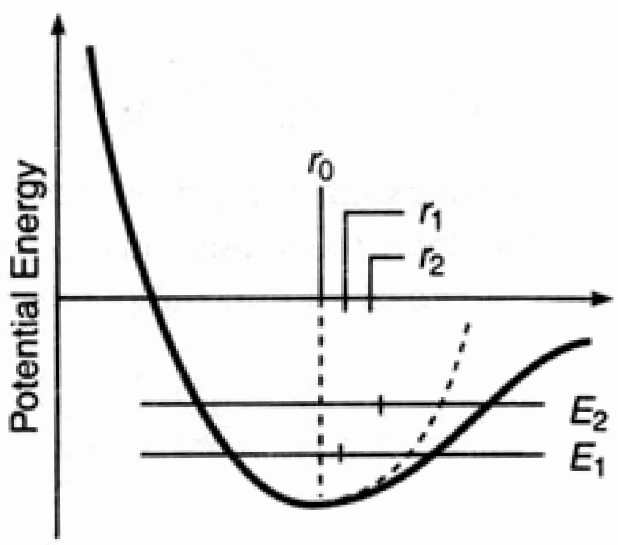


Fig. 17.5

More precisely, thermal expansion arises from the asymmetrical nature of the potential energy curve.

At the atomic level, thermal expansion may be understood by considering how the potential energy of the atoms varies with distance. The equilibrium position of an atom will be at the minimum of the potential energy well if the well is symmetric. At a given temperature each atom vibrates about its equilibrium position and its average position remains at the minimum point. If the shape of the well is not symmetrical, as shown in figure, the average position of an atom will not be at the minimum point. When



The potential energy of an atom. Thermal expansion arises because the "wall" is not symmetrical about the equilibrium position r_0. As the temperature rises, the energy of the atom changes. The average position r when the energy is E_2 is not the same as that when the E_1.

Fig. 17.6

the temperature is raised the amplitude of the vibrations increases and the average position is located at a greater interatomic separation. This increased separation is manifested as expansion of the material.

Linear expansion

Suppose that the temperature of a thin rod of length l is changed from T to $T + \Delta T$. It is found experimentally that, if ΔT is not too large, the corresponding change in length Δl of the rod is directly proportional to ΔT and l. Thus,

$$\Delta l \propto \Delta T \quad \text{and} \quad \Delta l \propto l$$

Introducing a proportionality constant α (which is different for different materials) we may write Δl as

$$\Delta l = l\alpha\Delta T \quad \text{...(i)}$$

Here, the constant α is called the **coefficient of linear expansion** of the material of the rod and its units are K^{-1} or $[(°C)^{-1}]$. Remember that $\Delta T = \Delta T_C$.

Actually, α does depend slightly on the temperature, but its variation is usually small enough to be negligible, even over a temperature range of 100°C. We will always assume that α is a constant.

Volume expansion

Because the linear dimensions of an object change with temperature, it follows that surface area and volume change as well. Just as with linear expansion, experiments show that if the temperature change ΔT is not too great (less than 100°C or so), the increase in volume ΔV is proportional to both the temperature change ΔT and the initial volume V. Thus,

$$\Delta V \propto \Delta T \quad \text{and} \quad \Delta V \propto V$$

Introducing a proportionality constant γ, we may write ΔV as,

$$\Delta V = V \times \gamma \times \Delta T \quad \text{... (ii)}$$

Here, γ is called the **coefficient of volume expansion**. The units of γ are K^{-1} or $(°C)^{-1}$.

Relation between γ and α

For an isotropic solid (which has the same value of α in all directions) $\gamma = 3\alpha$. To see that $\gamma = 3\alpha$ for a solid, consider a cube of length l and volume $V = l^3$.

When the temperature of the cube is increased by dT, the side length increases by dl and the volume increases by an amount dV given by

$$dV = \left(\frac{dV}{dl}\right) \cdot dl = 3l^2 \cdot dl$$

Now, $\quad dl = l\alpha dT$

$\therefore \quad dV = 3l^3\alpha dT = (3\alpha)VdT$

This is consistent with Eq. (ii),

$$dV = \gamma V dT, \text{ only if}$$

$$\gamma = 3\alpha \quad \text{...(iii)}$$

Average values of α and γ for some materials are listed in Table 17.1. You can check the relation $\gamma = 3\alpha$, for the materials given in the table.

Table 17.1

Material	α [K^{-1} or $(°C)^{-1}$]	γ [K^{-1} or $(°C)^{-1}$]
Steel	1.2×10^{-5}	3.6×10^{-5}
Copper	1.7×10^{-5}	5.1×10^{-5}
Brass	2.0×10^{-5}	6.0×10^{-5}
Aluminium	2.4×10^{-5}	7.2×10^{-5}

The anomalous expansion of water

Most liquids also expand when their temperatures increase. Their expansion can also be described by Eq. (ii). The volume expansion coefficients for liquids are about 100 times larger than those for solids.

Some substances contract when heated over a certain temperature range. The most common example is water.

Figure shows how the volume of 1 gm of water varies with temperature at atmospheric pressure. The volume decreases as the temperature is raised from 0°C to about 4°C, at which point the volume is a minimum and the density is a maximum (1000 kg/m^3). Above 4°C, water expands with increasing temperature like most substances.

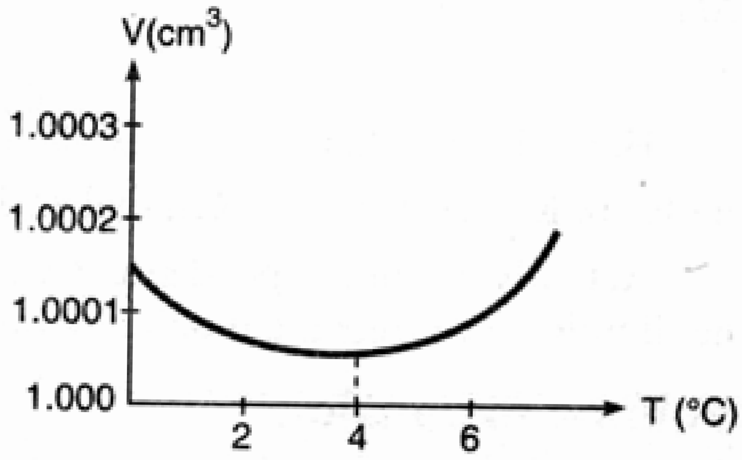


Fig. 17.7

This anomalous behaviour of water causes ice to form first at the surface of a lake in cold weather. As winter approaches, the water temperature increases initially at the surface. The water there sinks because of its increased density. Consequently, the surface reaches 0°C first and the lake becomes covered with ice. Aquatic life is able to survive the cold winter as the lake bottom remains unfrozen at a temperature of about 4°C.

• *Important points in* THERMAL EXPANSION

1. If a solid object has a hole in it, what happens to the size of the hole, when the temperature of the object increases? A common misconception is that if the object expands, the hole will shrink because material expands into the hole. But the truth is that if the object expands, the hole will expand too, because every linear dimension of an object changes in the same way when the temperature changes.

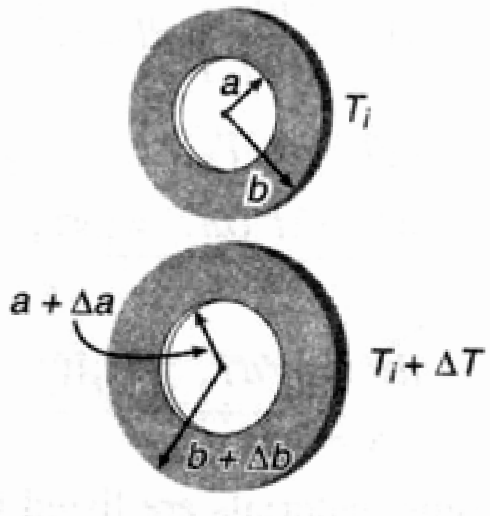


Fig. 17.8

2. **Expansion of a bimetallic strip :** As table 17.1 indicates, each substance has its own characteristic average coefficient of expansion. For example, when the temperatures of a brass rod

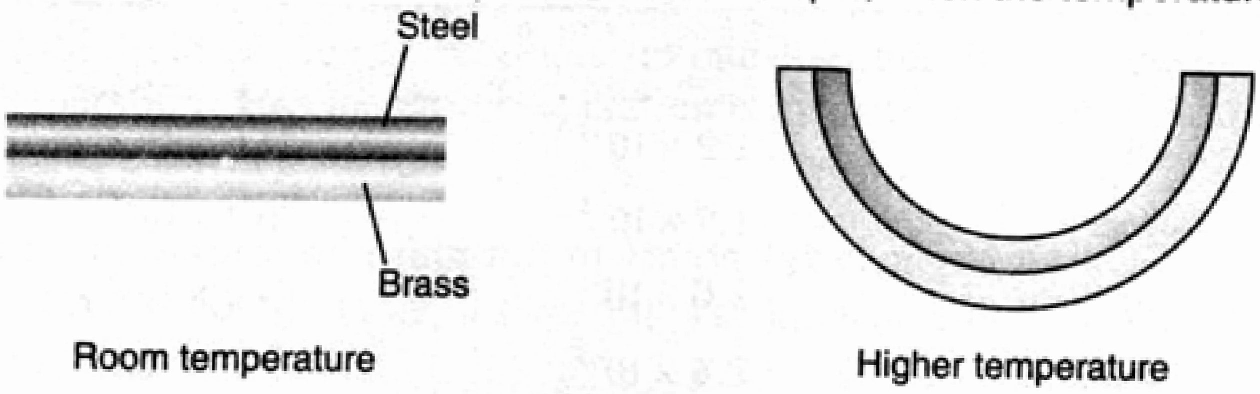


Fig. 17.9

and a steel rod of equal length are raised by the same amount from some common initial value, the brass rod expands more than the steel rod because brass has a greater average coefficient of expansion than steel. Such type of bimetallic strip is found in practical devices such as **thermostats** to break or make electrical contact.

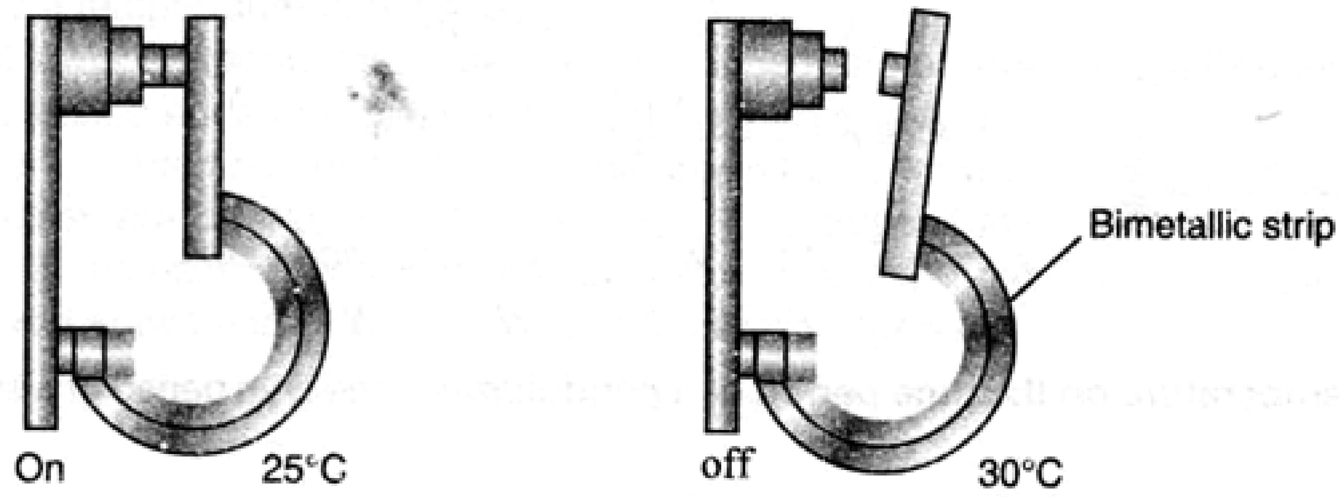


Fig. 17.10

3. **Variation of density with temperature :** Most substances expand when they are heated, *i.e.*, volume of a given mass of a substance increases on heating, so the density should decrease $\left(\text{as } \rho \propto \frac{1}{V}\right)$. Let us see how the density (ρ) varies with increase in temperature.

$$\rho = \frac{m}{V}$$

or $$\rho \propto \frac{1}{V} \qquad \text{(for a given mass)}$$

$\therefore$ $$\frac{\rho'}{\rho} = \frac{V}{V'} = \frac{V}{V + \Delta V} = \frac{V}{V + \gamma V \Delta T} = \frac{1}{1 + \gamma \Delta T}$$

$\therefore$ $$\rho' = \frac{\rho}{1 + \gamma \Delta T}$$

This expression can also be written as,

$$\rho' = \rho (1 + \gamma \Delta T)^{-1}$$

As γ is small, $$(1 + \gamma \Delta T)^{-1} \approx 1 - \gamma \Delta T$$

$\therefore$ $$\rho' \approx \rho (1 - \gamma \Delta T)$$

4. **Effect of temperature on upthrust :** When a solid body is completely immersed in a liquid its apparent weight gets decreased due to an upthrust acting on it by the liquid. The apparent weight is given by,

$$w_{app} = w - F$$

Here $F = \text{upthrust} = V_S \rho_L g$

where V_S = volume of solid and ρ_L = density of liquid

Now, as the temperature is increased V_S increases while ρ_L decreases. So, F may increase or decrease (or may remain constant also) depending upon the condition that which factor dominates on the other. We can write

$$F \propto V_S \rho_L$$

or $$\frac{F'}{F}=\frac{V_S'}{V_S}\cdot\frac{\rho_L'}{\rho_L}=\frac{(V_S+\Delta V_S)}{V_S}\cdot\left(\frac{1}{1+\gamma_L\Delta T}\right)$$

$$=\left(\frac{V_S+\gamma_S V_S\Delta T}{V_S}\right)\left(\frac{1}{1+\gamma_L\Delta T}\right)$$

or $$F'=F\left(\frac{1+\gamma_s\Delta T}{1+\gamma_L\Delta T}\right)$$

Now, if $\gamma_S>\gamma_L, F'>F$ or $w' < w$ and *vice-versa*.

And if $\gamma_S=\gamma_L$, $F'=F$ or $W'=W$

5. **Effect of temperature on the time period of a pendulum :** The time period of a simple pendulum is given by

$$T=2\pi\sqrt{\frac{l}{g}}$$

or $$T\propto\sqrt{l}$$

As the temperature is increased length of the pendulum and hence, time period gets increased or a pendulum clock becomes slow and it loses the time.

$$\frac{T'}{T}=\sqrt{\frac{l'}{l}}=\sqrt{\frac{l+\Delta l}{l}}$$

Here, we put $\Delta l=l\alpha\Delta\theta$ in place of $l\alpha\Delta T$ so as to avoid the confusion with change in time period. Thus,

$$\frac{T'}{T}=\sqrt{\frac{l+l\alpha\Delta\theta}{l}}=(1+\alpha\Delta\theta)^{1/2}$$

or $$T'\approx T\left(1+\frac{1}{2}\alpha\Delta\theta\right)$$

or $$\Delta T=T'-T=\frac{1}{2}T\alpha\Delta\theta$$

Time lost in time t (by a pendulum clock whose actual time period is T and the changed time period at some higher temperature is T') is

$$\Delta t=\left(\frac{\Delta T}{T'}\right)t$$

Similarly, if the temperature is decreased the length and hence, the time period gets decreased. A pendulum clock in this case runs fast and it gains the time.

$$\frac{T'}{T}=\sqrt{\frac{l'}{l}}=\sqrt{\frac{l-l\alpha\Delta\theta}{l}}\approx 1-\frac{1}{2}\alpha\Delta\theta$$

or $$T'=T\left(1-\frac{1}{2}\alpha\Delta\theta\right)$$

$$\Delta T=T-T'=\frac{1}{2}T\alpha\Delta\theta$$

and time gained in time t is the same, *i.e.*,

$$\Delta t=\left(\frac{\Delta T}{T'}\right)t$$

6. At some higher temperature a scale will expand and scale reading will be lesser than true values, so that

$$\text{true value} = \text{scale reading } (1 + \alpha \Delta T)$$

Here ΔT is the temperature difference.
However, at lower temperatures scale reading will be more or true value will be less.

7. When a rod whose ends are rigidly fixed such as to prevent from expansion or contraction undergoes a change in temperature, thermal stresses are developed in the rod. This is because, if the temperature is increased, the rod has a tendency to expand but since, it is fixed at two ends, the rod exerts a force on supports.

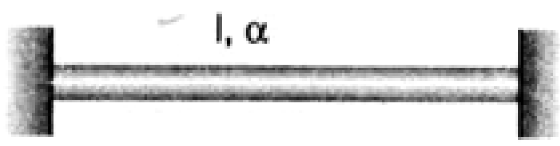


Fig. 17.11

$$\text{Thermal strain} = \frac{\Delta l}{l} = \alpha \, . \, \Delta T$$

So $\quad$ thermal stress $= (Y)(\text{thermal strain}) = Y\alpha\Delta T$

or $\quad$ force on supports $F = A\,(\text{stress}) = YA\,\alpha\Delta T$

Here, $\quad Y$ = Young's modulus of elasticity of the rod.

$$F = YA\alpha\Delta T$$

8. Expansion of liquids : For heating a liquid it has to be put in some container. When the liquid is heated, the container will also expand. We define coefficient of apparent expansion of a liquid as the apparent increase in **volume** per unit original volume per °C rise in temperature. It is represented by γ_a. Thus,

$$\gamma_a = \gamma_r - \gamma_g$$

Here, γ_r = coefficient of real expansion of a liquid

and γ_g = coefficient of cubical expansion of the container

Sample Example 17.3 *A steel ruler exactly* 20 cm *long is graduated to give correct measurements at* 20°C.

(a) Will it give readings that are too long or too short at lower temperatures?

(b) What will be the actual length of the ruler be when it is used in the desert at a temperature of $40°C$? $\alpha_{steel} = 1.2 \times 10^{-5}\ (°C)^{-1}$.

Solution (a) If the temperature decreases, the length of the ruler also decreases through thermal contraction. Below 20°C, each centimetre division is actually somewhat shorter than 1.0 cm, so the steel ruler gives readings that are too long.

(b) At 40°C, the increase in length of the ruler is

$$\Delta l = l\alpha\Delta T$$

$$= (20)\,(1.2 \times 10^{-5})\,(40° - 20°)$$

$$= 0.48 \times 10^{-2} \text{ cm}$$

$\therefore$ The actual length of the ruler is,

$$l' = l + \Delta l = 20.0048 \text{ cm}$$

Ans.

Sample Example 17.4 *Find the coefficient of volume expansion for an ideal gas at constant pressure.*

Solution For an ideal gas

$$PV = nRT$$

As P is constant, we have

$$P \cdot dV = nRdT$$

$$\therefore \quad \frac{dV}{dT} = \frac{nR}{P}$$

or

$$\gamma = \frac{1}{V} \cdot \frac{dV}{dT} = \frac{nR}{PV} = \frac{nR}{nRT} = \frac{1}{T}$$

$$\therefore \quad \gamma = \frac{1}{T}$$ **Ans.**

Sample Example 17.5 *The scale on a steel metre stick is calibrated at 15°C. What is the error in the reading of 60 cm at 27°C?* $\alpha_{steel} = 1.2 \times 10^{-5}\ (°C)^{-1}$.

Solution At higher temperatures actual reading is more than the scale reading. The error in the reading will be

$$\Delta l = \text{(scale reading)}\ (\alpha)\ (\Delta T)$$

$$= (60)\ (1.2 \times 10^{-5})\ (27° - 15°)$$

$$= 0.00864 \text{ cm}$$ **Ans.**

Sample Example 17.6 *A second's pendulum clock has a steel wire. The clock is calibrated at 20°C. How much time does the clock lose or gain in one week when the temperature is increased to 30°C?* $\alpha_{steel} = 1.2 \times 10^{-5} (°C)^{-1}$.

Solution The time period of second's pendulum is 2 second. As the temperature increases length and hence, time period increases. Clock becomes slow and it loses the time. The change in time period is

$$\Delta T = \frac{1}{2} T \alpha \Delta\theta$$

$$= \left(\frac{1}{2}\right)(2)\ (1.2 \times 10^{-5})\ (30° - 20°)$$

$$= 1.2 \times 10^{-4} \text{ s}$$

$\therefore$ New time period is,

$$T' = T + \Delta T = (2 + 1.2 \times 10^{-4})$$

$$= 2.00012 \text{ s}$$

$\therefore$ Time lost in one week

$$\Delta t = \left(\frac{\Delta T}{T'}\right) t = \frac{(1.2 \times 10^{-4})}{(2.00012)} (7 \times 24 \times 3600)$$

$$= 36.28 \text{ s}$$ **Ans.**

Introductory Exercise 17.1

1. What is the value of
   (a) 0°F in celsius scale?
   (b) 0 K on Fahrenheit scale?
2. At what temperature is the Fahrenheit scale reading equal to
   (a) twice
   (b) half of Celsius?
3. A Faulty thermometer reads 5°C in melting ice and 99°C in steam. Find the correct temperature in °F when this faulty thermometer reads 52°C.
4. At what temperature the Fahrenheit and Kelvin scales of temperature give the same reading?
5. At what temperature the Fahrenheit and Celsius scales of temperature give the same reading?
6. A pendulum clock of time period 2 sec gives the correct time at 30°C. The pendulum is made of iron. How many seconds will it lose or gain per day when the temperature falls to 0°C? $\alpha_{Fe} = 1.2 \times 10^{-5} (°C)^{-1}$.
7. A block of wood is floating in water at 0°C. The temperature of water is slowly raised from 0°C to 10°C. How will the percentage of volume of block above water level change with rise in temperature?
8. A piece of metal floats on mercury. The coefficient of volume expansion of metal and mercury are γ_1 and γ_2 respectively. If the temperature of both mercury and metal are increased by an amount ΔT, by what factor does the fraction of the volume of the metal submerged in mercury changes?
9. A brass disc fits snugly in a hole in a steel plate. Should you heat or cool the system to loosen the disc from the hole? Given that $\alpha_B > \alpha_{Fe}$.
10. Show that the volume thermal expansion coefficient for an ideal gas at constant pressure is $\frac{1}{T}$.

17.5 Concept of an Ideal Gas

A gas has no shape and size and can be contained in a vessel of any size or shape. It expands indefinitely and uniformly to fill the available space. It exerts pressure on its surroundings.

The gases whose molecules are point masses (mass without volume) and do not attract each other are called **ideal** or **perfect** gases. It is a hypothetical concept which can't exist in reality. The gases such as hydrogen, oxygen or helium which cannot be liquified easily are called **permanent gases**. An actual gas behaves as ideal gas most closely at low pressure and high temperature.

17.6 Gas Laws

Assuming permanent gases to be ideal, through experiments, it was established that gases irrespective of their nature obey the following laws :

(a) Boyle's law

According to this law, for a given mass of a gas the volume of a gas at constant temperature (called **isothermal** process) is inversely proportional to its pressure, *i.e.*,

$$V \propto \frac{1}{P} \qquad (T = \text{constant})$$

or

$$PV = \text{constant}$$

or

$$P_i V_i = P_f V_f$$

Thus, P-V graph in an isothermal process is a rectangular hyperbola. Or PV *versus* P or V graph is a straight line parallel to P or V axis.

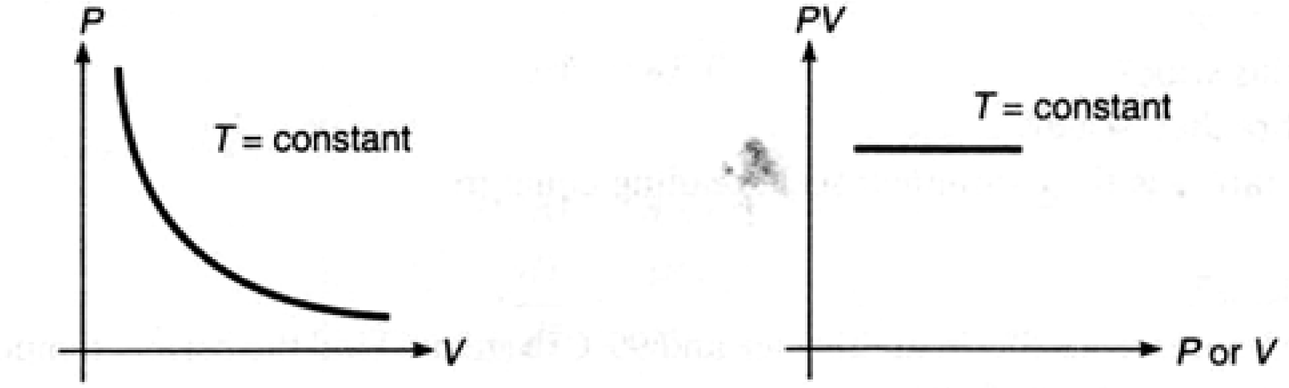


Fig. 17.12

(b) Charle's law

According to this law, for a given mass of a gas the volume of a gas at constant pressure (called **isobaric** process) is directly proportional to its absolute temperature, *i.e.*,

$$V \propto T \qquad (P = \text{constant})$$

or
$$\frac{V}{T} = \text{constant}$$

or
$$\frac{V_i}{T_i} = \frac{V_f}{T_f}$$

Thus, V-T graph in an isobaric process is a straight line passing through origin. Or V/T versus V or T graph is a straight line parallel to V or T axis.

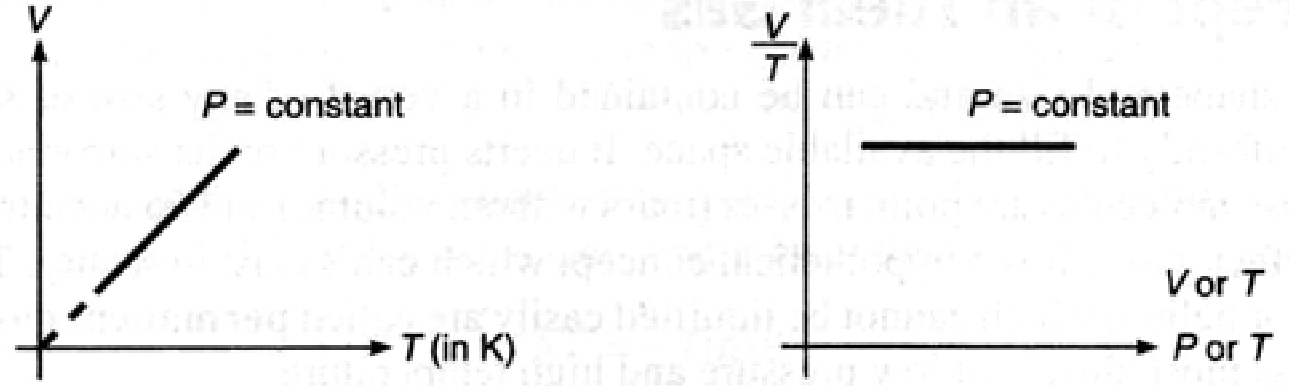


Fig. 17.13

(c) Gay Lussac's law or Pressure law

According to this law, for a given mass of a gas the pressure of a gas at constant volume (called **isochoric** process) is directly proportional to its absolute temperature, *i.e.*,

$$P \propto T \qquad (V = \text{constant})$$

or
$$\frac{P}{T} = \text{constant}$$

or
$$\frac{P_i}{T_i} = \frac{P_f}{T_f}$$

Thus, P-T graph in an isochoric process is a straight line passing through origin or P/T versus P or T graph is a straight line parallel to P or T axis.

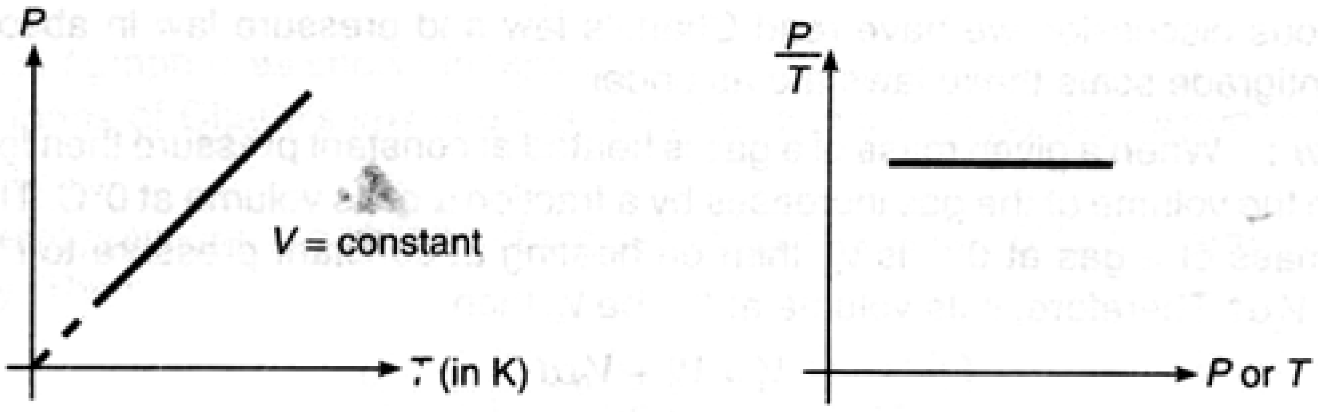


Fig. 17.14

(d) Avogadro's law

According to this law, at same temperature and pressure equal volumes of all gases contain equal number of molecules.

17.7 Ideal Gas Equation

All the above four laws can be written in one single equation known as ideal gas equation. According to this equation.

$$PV = nRT = \frac{m}{M}RT$$

In this equation n = number of moles of the gas

$$= \frac{m}{M}$$

m = total mass of the gas,

M = molecular mass of the gas

and R = Universal gas constant

$= 8.31 \text{ J/mol-K}$

$= 2.0 \text{ cal/mol-K}$

The above four laws can be derived from this single equation. For example, for a given mass of a gas (m = constant)

PV = constant at constant temperature (Boyle's law)

$\frac{P}{T}$ = constant at constant volume (Pressure law)

$\frac{V}{T}$ = constant at constant pressure (Charle's law)

and if P, V and T are constants then

$$n = \text{constant for all gases.}$$

And since, equal number of moles contain equal number of molecules. So, at constant pressure, volume and temperature all gases will contain equal number of molecules. Which is nothing but Avogadro's law.

• *Important Points in* GAS LAWS

1. In our previous discussion we have read Charle's law and pressure law in absolute temperature scale. In centigrade scale these laws are as under :

Charle's law : When a given mass of a gas is heated at constant pressure then for each 1°C rise in temperature the volume of the gas increases by a fraction α of its volume at 0°C. Thus, if the volume of a given mass of a gas at 0°C is V_0, then on heating at constant pressure to t°C its volume will increase by $V_0\alpha t$. Therefore, if its volume at t° be V_t, then

$$V_t = V_0 + V_0\alpha t$$

or
$$V_t = V_0(1 + \alpha t)$$

Here α is called the 'volume coefficient' of the gas. For all gases the experimental value of α is nearly $\frac{1}{273}$ per °C.

$\therefore$
$$V_t = V_0\left(1 + \frac{t}{273}\right)$$

Thus, V_t *versus* t graph is a straight line with slope $\frac{V_0}{273}$ and positive intercept V_0. Further $V_t = 0$ at $t = -273°$.

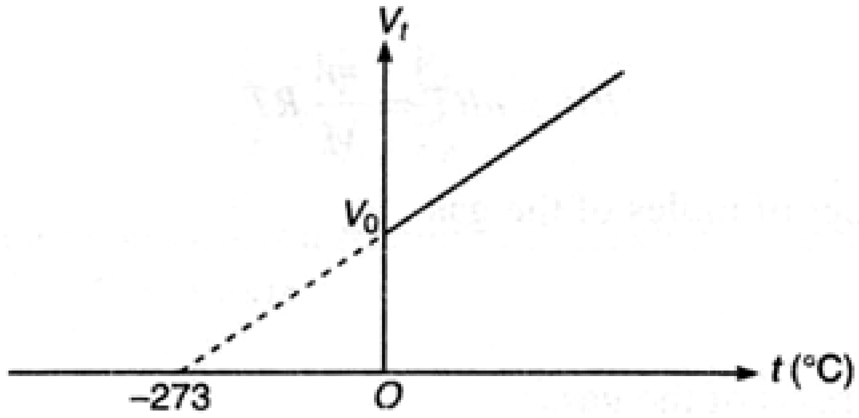


Fig. 17.15

Pressure law : According to this law, when a given mass of a gas is heated at constant volume then for each 1°C rise in temperature, the pressure of the gas increases by a fraction β of its pressure at 0°C. Thus, if the pressure of a given mass of a gas at 0°C be P_0, then on heating at constant volume to t°, its pressure will increase by $P_0\beta t$. Therefore, if its pressure at t° be P_t, then

$$P_t = P_0 + P_0\beta t$$

or
$$P_t = P_0(1 + \beta t)$$

Here β is called the 'pressure coefficient' of the gas. For all gases the experimental value of β is also $\frac{1}{273}$ per°C.

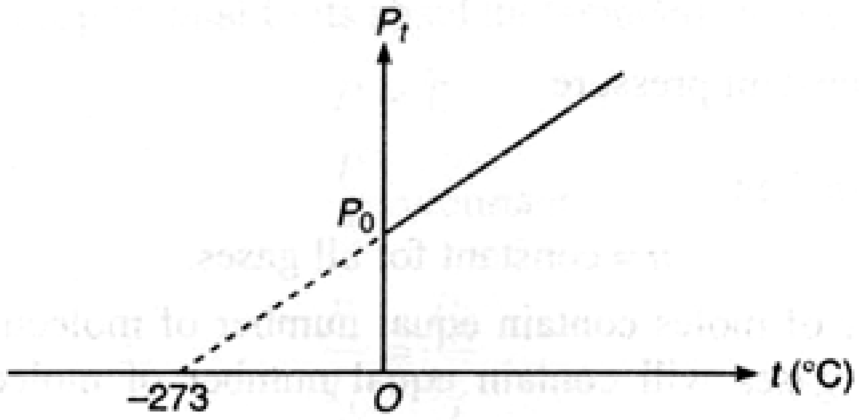


Fig. 17.16

$\therefore \qquad P_t = P_0\left(1 + \frac{t}{273}\right)$

The P_t versus t graph is as shown in figure.

2. The above forms of Charle's law and pressure law can be simply expressed in terms of absolute temperature.

Let at constant pressure, the volume of a given mass of a gas at 0°C, $t_1°$ and $t_2°$ be V_0, V_1 and V_2 respectively. Then,

$$V_1 = V_0\left(1 + \frac{t_1}{273}\right) = V_0\left(\frac{273 + t_1}{273}\right)$$

$$V_2 = V_0\left(1 + \frac{t_2}{273}\right) = V_0\left(\frac{273 + t_2}{273}\right)$$

$$\therefore \qquad \frac{V_1}{V_2} = \frac{273 + t_1}{273 + t_2} = \frac{T_1}{T_2}$$

where T_1 and T_2 are the absolute temperatures corresponding to $t_1°$ and $t_2°$. Hence,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

or $\qquad \frac{V}{T} = \text{constant}$

or $\qquad V \propto T$

This is the form of Charle's law which we have already studied in article 17.6. In the similar manner we can prove the pressure law.

3. Under isobaric conditions (P = constant), V-T graph is a straight line passing through origin (where T is in kelvin). The slope of this line is $\left(\frac{nR}{P}\right)$ as $V = \left(\frac{nR}{P}\right)T$ or slope of the line is directly proportional to $\frac{n}{P}$.

$$\text{slope} = \frac{nR}{P} \qquad \text{or} \qquad \text{slope} \propto \frac{n}{P}$$

Similarly, under isochoric conditions (V = constant), P-T graph is a straight line passing through origin whose slope is $\frac{nR}{V}$ or slope is directly proportional to $\frac{n}{V}$.

4. **Density of a gas :** The ideal gas equation is,

$$PV = nRT = \frac{m}{M}RT$$

$$\therefore \qquad \frac{m}{V} = \rho = \frac{PM}{RT} \qquad (\rho = \text{density})$$

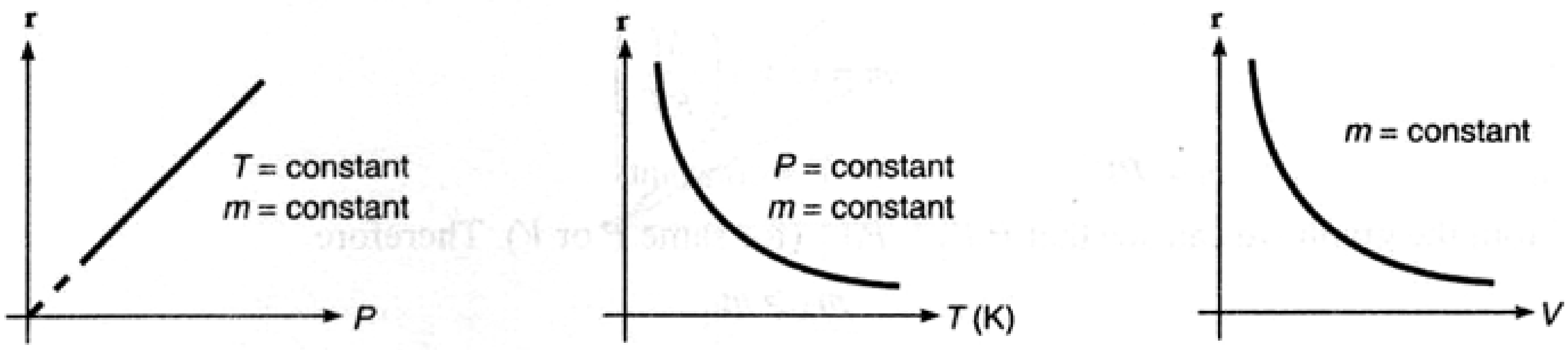


Fig. 17.17

$$\therefore \qquad \rho = \frac{PM}{RT}$$

From this equation we can see that ρ-P graph is straight line passing through origin at constant temperature ($\rho \propto P$) for a given gas and ρ-T graph is rectangular hyperbola at constant pressure $\left(\rho \propto \frac{1}{T}\right)$. Similarly for a given mass of a gas ρ-V graph is a rectangular hyperbola $\left(\rho \propto \frac{1}{V}\right)$.

Sample Example 17.7 *P-V diagrams of same mass of a gas are drawn at two different temperatures T_1 and T_2. Explain whether $T_1 > T_2$ or $T_2 > T_1$.*

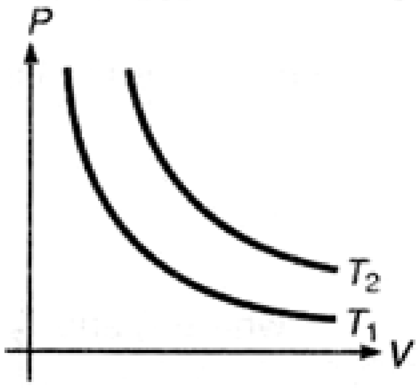


Fig. 17.18

Solution The ideal gas equation is,

$$PV = nRT$$

or $$T = \frac{PV}{nR}$$

$T \propto PV$ if number of moles of the gas are kept constant. Here mass of the gas is constant, which implies that number of moles are constant, *i.e.*, $T \propto PV$. In the given diagram product of P and V for T_2 is more than T_1 at all points (keeping either P or V same for both graphs). Hence,

$$T_2 > T_1$$ **Ans.**

Sample Example 17.8 *The P-V diagram of two different masses m_1 and m_2 are drawn (as shown) at constant temperature T. State whether $m_1 > m_2$ or $m_2 > m_1$?*

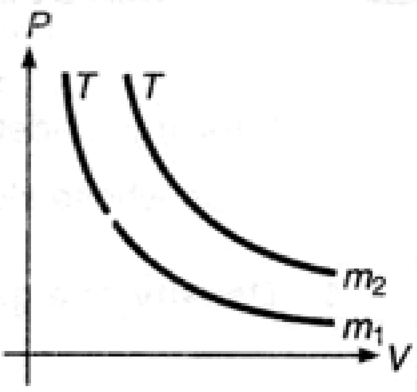


Fig. 17.19

Solution

$$PV = nRT = \frac{m}{M}RT$$

$$\therefore \qquad m = (PV)\left(\frac{M}{RT}\right)$$

or $\qquad m \propto PV \qquad$ if $T =$ constant

From the graph we can see that $P_2V_2 > P_1V_1$ (for same P or V). Therefore,

$$m_2 > m_1$$ **Ans.**

Sample Example 17.9 *The P-T graph for the given mass of an ideal gas is shown in figure. What inference can be drawn regarding the change in volume (whether it is constant, increasing or decreasing)?*

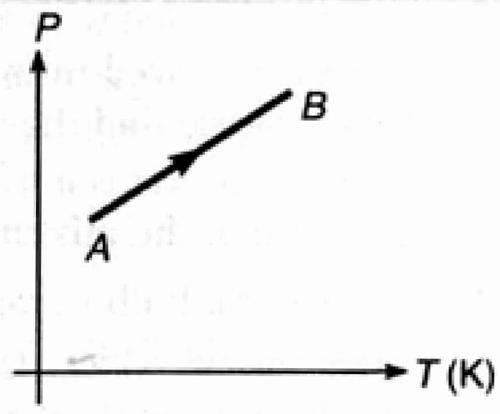


Fig. 17.20

HOW TO PROCEED Definitely it is not constant. Because when volume is constant, *P-T* graph is a straight line passing through origin. The given line does not pass through origin, hence, volume is not constant.

$$V = (nR)\left(\frac{T}{P}\right)$$

Now, to see the volume of the gas we will have to see whether $\frac{T}{P}$ is increasing or decreasing.

Solution From the given graph we can write the *P-T* equation as,

$$P = aT + b \qquad (y = mx + c)$$

here a and b are positive constants. Further,

$$\frac{P}{T} = a + \frac{b}{T}$$

Now, $T_B > T_A \quad \therefore \quad \frac{b}{T_B} < \frac{b}{T_A}$ or $\left(\frac{P}{T}\right)_B < \left(\frac{P}{T}\right)_A$

or $$\left(\frac{T}{P}\right)_B > \left(\frac{P}{T}\right)_A$$

or $$V_B > V_A$$ **Ans.**

Thus, as we move from A to B, volume of the gas is increasing.

Introductory Exercise 17.2

1. From the graph for an ideal gas state whether m_1 or m_2 is greater?

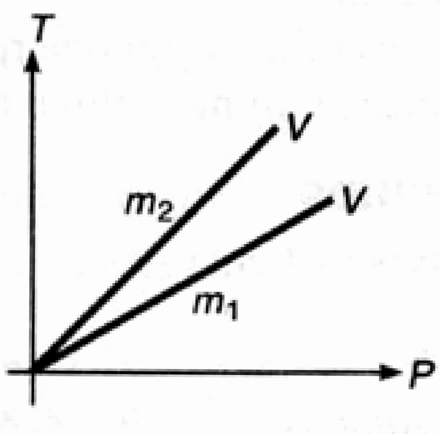


Fig. 17.21

2. A vessel is filled with an ideal gas at a pressure of 20 atm and is at a temperature of 27°C. One-half of the mass is removed from the vessel and the temperature of the remaining gas is increased to 87°C. At this temperature find the pressure of the gas.
3. A vessel contains a mixture of 7 g of nitrogen and 11 g of carbondioxide at temperature $T = 290$ K. If pressure of the mixture is 1 atm ($= 1.01 \times 10^5$ N / m^2), calculate its density ($R = 8.31$ J / mol - K).
4. An electric bulb of volume 250 cm^3 was sealed off during manufacture at a pressure of 10^{-3} mm of mercury at 27°C. Compute the number of air molecules contained in the bulb. Given that $R = 8.31$ J / mol - K and $N_A = 6.02 \times 10^{23}$ per mol.
5. State whether $P_1 > P_2$ or $P_2 > P_1$ for given mass of a gas?

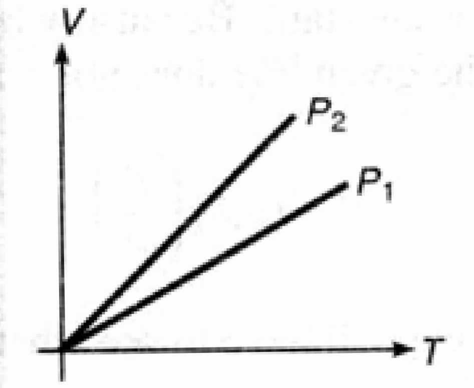


Fig. 17.22

6. For a given mass of a gas what is the shape of P *versus* $\frac{1}{V}$ graph at constant temperature?

17.8 Degree of Freedom (f)

The term degree of freedom refers to the number of possible independent ways in which a system can have energy.

For example : In figure (a) block has one degree of freedom, because it is confined to move in a

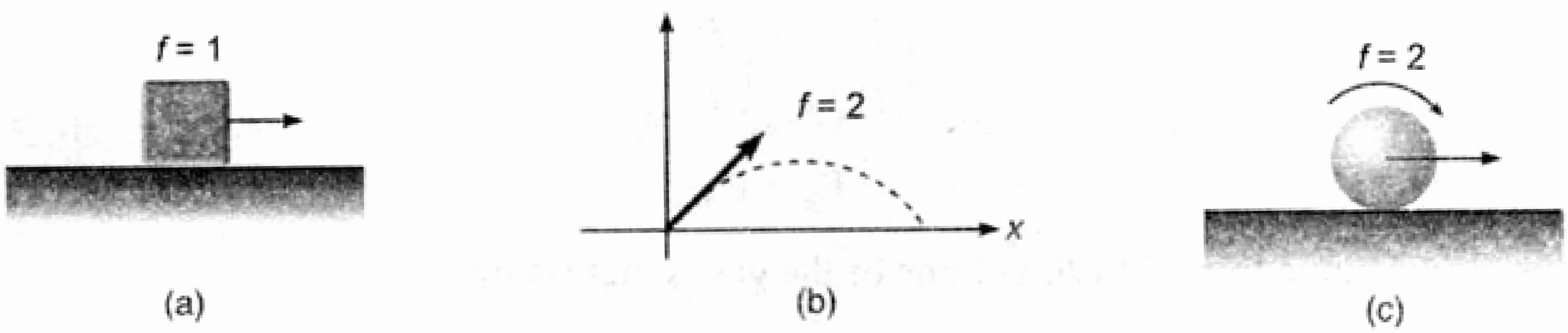


Fig. 17.23

straight line and has only one translational degree of freedom.

In figure (b), the projectile has two degrees of freedom because it is confined to move in a plane and so it has two translational degrees of freedom.

In figure (c), the sphere has two degrees of freedom one rotational and another translational.

Similarly a particle free to move in space will have three translational degrees of freedom.

Degree of freedom of gas molecules

A gas molecule can have following types of energies :

(i) translational kinetic energy
(ii) rotational kinetic energy
(iii) vibrational energy (potential + kinetic)

Vibrational energy

The forces between different atoms of a gas molecule may be visualized by imagining every atom as being connected to its neighbours by springs. Each atom can vibrate along the line joining the atoms. Energy associated with this is called vibrational energy.

Degree of freedom of monoatomic gas

A monoatomic gas molecule (like He) consists of a single atom. It can have translational motion in any direction in space. Thus, it has 3 translational degrees of freedom.

$$f = 3 \quad \text{(all translational)}$$

It can also rotate but due to its small moment of inertia, rotational kinetic energy is neglected.

Degree of freedom of a diatomic and linear polyatomic gas

The molecules of a diatomic and linear polyatomic gas (like O_2, CO_2 and H_2) cannot only move bodily but also rotate about any one of the three co-ordinate axes as shown in figure. However, its moment of inertia about the axis joining the two atoms (x-axis) is negligible. Hence, it can have only two rotational degrees of freedom. Thus, a diatomic molecule has 5 degrees of freedom : 3 translational and 2 rotational. At sufficiently high temperatures it has vibrational energy as well providing it two more degrees of freedom (one vibrational kinetic energy and another vibrational potential energy). Thus, at high temperatures a diatomic molecule has 7 degrees of freedom, 3 translational, 2 rotational and 2 vibrational. Thus,

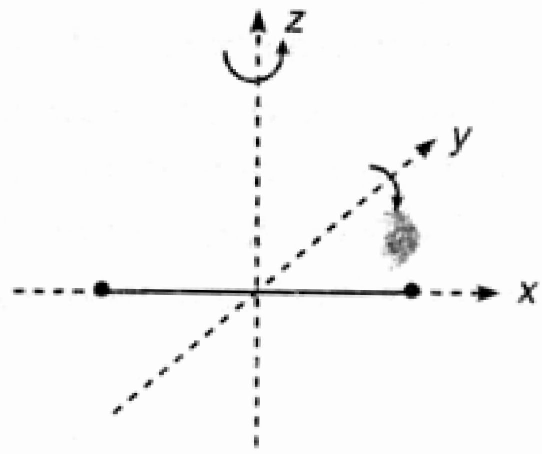


Fig. 17.24

$$f = 5$$

(3 translational + 2 rotational) at room temperatures

and $$f = 7$$

(3 translational + 2 rotational + 2 vibrational) at high temperatures.

Degree of freedom of nonlinear polyatomic gas

A nonlinear polyatomic molecule (such as NH_3) can rotate about any of three co-ordinate axes. Hence, it has 6 degrees of freedom 3 translational and 3 rotational. At room temperatures a polyatomic gas molecule has vibrational energy greater than that of a diatomic gas. But at high enough temperatures it is also significant. So it has 8 degrees of freedom 3 rotational, 3 translational and 2 vibrational. Thus,

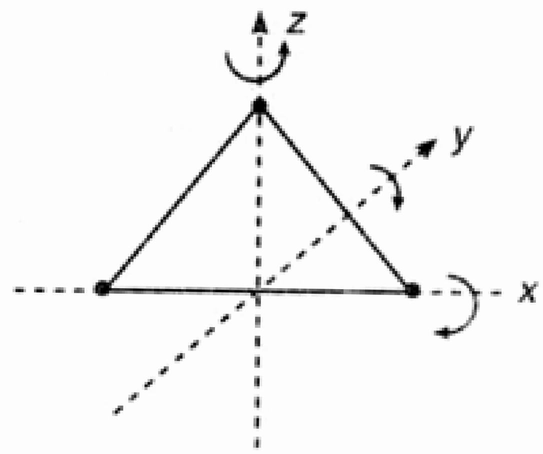


Fig. 17.25

$$f = 6$$

(3 translational + 3 rotational) at room temperatures

and $$f = 8$$

(3 translational + 3 rotational + 2 vibrational) at high temperatures.

Degree of freedom of a solid

An atom in a solid has no degrees of freedom for translational and rotational motion. At high temperatures due to vibration along 3 axes it has $3 \times 2 = 6$ degrees of freedom.

$$f = 6 \quad \text{(all vibrational) at high temperatures}$$

Note *(i) Degrees of freedom of a diatomic and polyatomic gas depends on temperature and since there is no clear cut demarcation line above which vibrational energy become significant. Moreover, this temperature varies from gas to gas. On the other hand for a monoatomic gas there is no such confusion. Degree of freedom here is 3 at all temperatures. Unless and until stated in the question you can take $f = 3$ for monoatomic gas, $f = 5$ for a diatomic gas and $f = 6$ for a non-linear polyatomic gas.*

(ii) When a diatomic or polyatomic gas dissociates into atoms it behaves as a monoatomic gas. Whose degrees of freedom are changed accordingly.

17.9 Internal Energy of an Ideal Gas

Suppose a gas is contained in a closed vessel as shown in figure. If the container as a whole is moving with some speed then this motion is called the **ordered motion** of the gas. Source of this motion is some external force. The zig zag motion of gas molecules within the vessel is known as the **disordered motion**. This motion is directly related to the temperature of the gas. As the temperature is increased, the disordered motion of the gas molecules gets fast. The internal energy (U) of the gas is concerned only with its disordered motion. It is in no way concerned with its ordered motion. When the temperature of the gas is increased, its disordered motion and hence its internal energy is increased.

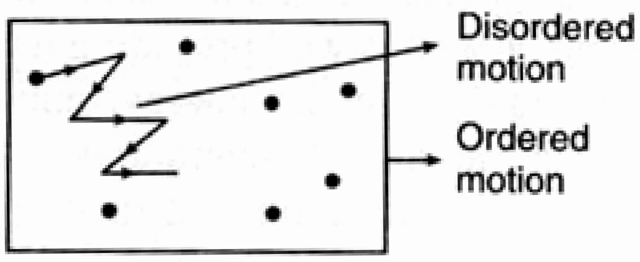
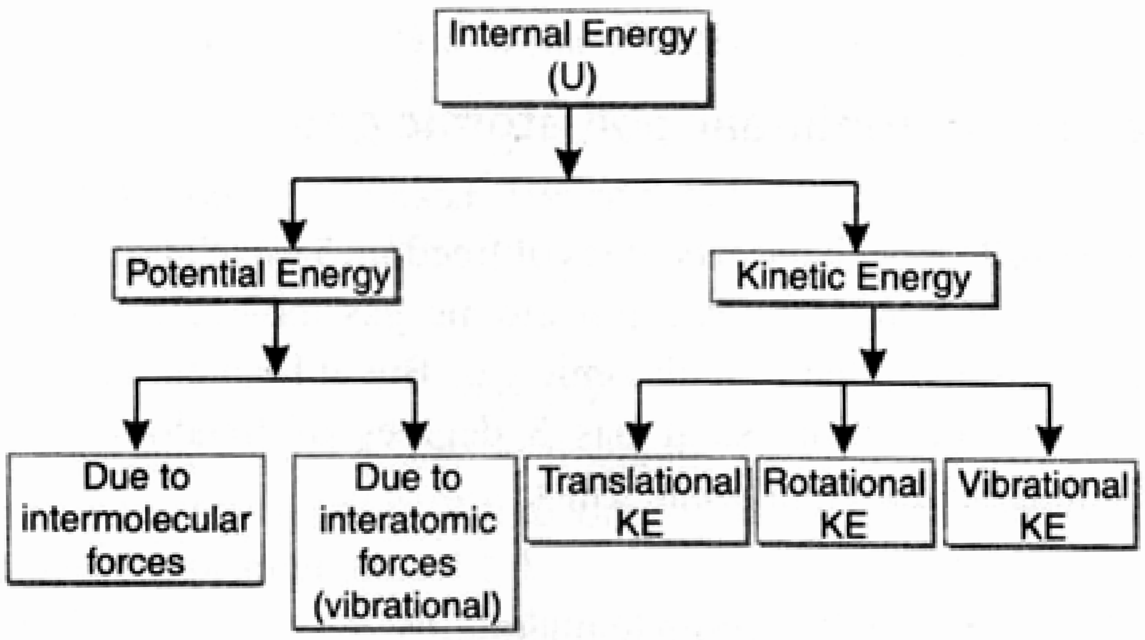


Fig. 17.26

Intermolecular forces in an ideal gas is zero. Thus, P.E. due to intermolecular forces of an ideal gas is zero. A monoatomic gas is having a single atom. Hence its vibrational energy is zero. For dia



and polyatomic gases vibrational energy is significant only at high temperatures. So, they also have only translational and rotational K.E. We may thus conclude that at room temperature the internal energy of an ideal gas (whether it is mono, dia or poly) consists of only translational and rotational K.E. Thus,

$$U \text{ (of an ideal gas)} = K_T + K_R \qquad \text{at room temperatures.}$$

Later in the next article we will see that K_T (translational KE) and K_R (rotational KE) depends on T only. They are directly proportional to the absolute temperature of the gas. Thus, **internal energy of an ideal gas depends only on its absolute temperature (T) and is directly proportional to T.**

Or, $$U \propto T$$

17.10 Law of Equipartition of Energy

An ideal gas is just like an ideal father. As an ideal father distributes whole of its assets equally among his children. Same is the case with an ideal gas. It distributes its internal energy equally in all degrees of freedom. In each degree of freedom energy of one mole of an ideal gas is $\frac{1}{2}RT$ where T is the absolute temperature of the gas. Thus, if f be the number of degrees of freedom, the internal energy of 1 mole of the gas will be $\frac{f}{2}RT$ or internal energy of n moles of the gas will be $\frac{n}{2}fRT$. Thus,

$$U = \frac{n}{2}fRT \quad \text{...(i)}$$

For a monoatomic gas, $f = 3$.

Therefore, $$U = \frac{3}{2}RT \quad \text{(for 1 mole of a monoatomic gas)}$$

For a dia and linear polyatomic gas at low temperatures, $f = 5$, so,

$$U = \frac{5}{2}RT \quad \text{(for 1 mole)}$$

and for nonlinear polyatomic gas at low temperatures, $f = 6$, so

$$U = \frac{6}{2}RT = 3RT \quad \text{(for 1 mole)}$$

Note *From Eq. (i) we can see that internal energy of an ideal gas depends only on its temperature and which is directly proportional to its absolute temperature T. In an isothermal process T = constant. Therefore, the internal energy of the gas does not change or $dU = 0$.*

17.11 Molar Heat Capacity

"Molar heat capacity C is the heat required to raise the temperature of 1 mole of a gas by 1°C (or 1 K)." Thus,

$$C = \frac{\Delta Q}{n\Delta T} \quad \text{or} \quad \Delta Q = nC\Delta T$$

For a gas the value of C depends on the process through which its temperature is raised.

For example, in an isothermal process $\Delta T = 0$ or $C_{iso} = \infty$. In an adiabatic process (we will discuss it later) $\Delta Q = 0$. Hence, $C_{adi} = 0$. Thus, molar heat capacity of a gas varies from 0 to ∞ depending on the process. In general experiments are made either at constant volume or at constant pressure. In case of solids and liquids, due to small thermal expansion, the difference in measured values of molar heat capacities is very small and is usually neglected. However, in case of gases molar heat capacity at constant volume C_V is quite different from that at constant pressure C_P. Later in the next chapter we will derive the following relations, for an ideal gas

$$C_V = \frac{dU}{dT} = \frac{f}{2}R = \frac{R}{\gamma - 1}$$

$$C_P = C_V + R$$

$$\gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$$

Here U is the internal energy of one mole of the gas. The most general expression for C in the process PV^x = constant is,

$$C = \frac{R}{\gamma - 1} + \frac{R}{1 - x}$$ (we will derive it later)

For example : For isobaric process P = constant or $x = 0$ and

$$C = C_P = \frac{R}{\gamma - 1} + R = C_V + R$$

For isothermal process, PV = constant or $x = 1$

$\therefore$ $C = \infty$ and

for adiabatic process PV^γ = constant or $x = \gamma$

$\therefore$ $C = 0$

Values of f, U, C_V, C_P and γ for different gases are shown in table 17.2.

Table 17.2

Nature of gas	f	$U = \frac{f}{2}RT$	$C_V = dU/dT = \frac{f}{2}R$	$C_P = C_V + R$	$\gamma = C_P/C_V = 1 + \frac{2}{f}$
Monoatomic	3	$\frac{3}{2}RT$	$\frac{3}{2}R$	$\frac{5}{2}R$	1.67
Dia and linear polyatomic	5	$\frac{5}{2}RT$	$\frac{5}{2}R$	$\frac{7}{2}R$	1.4
Non-linear polyatomic	6	$3RT$	$3R$	$4R$	1.33

17.12 Kinetic Theory of Gases

We have studied the mechanics of single particles. When we approach the mechanics associated with the many particles in systems such as gases, liquids and solids, we are faced with analyzing the dynamics of a huge number of particles. The dynamics of such many particle systems is called statistical mechanics.

The game involved in studying a system with a large number of particles is similar to what happens after every physics test. Of course we are interested in our individual marks, but we also want to know the class average.

The kinetic theory that we study in this article is a special aspect of the statistical mechanics of large number of particles. We begin with the simplest model for a monoatomic ideal gas, a dilute gas whose particles are single atoms rather than molecules.

Macroscopic variables of a gas are pressure, volume and temperature and microscopic properties are speed of gas molecules, momentum of molecules, etc. Kinetic theory of gases relates the microscopic properties to macroscopic properties. Further more, the kinetic theory provides us with a physical basis for our understanding of the concept of pressure and temperature.

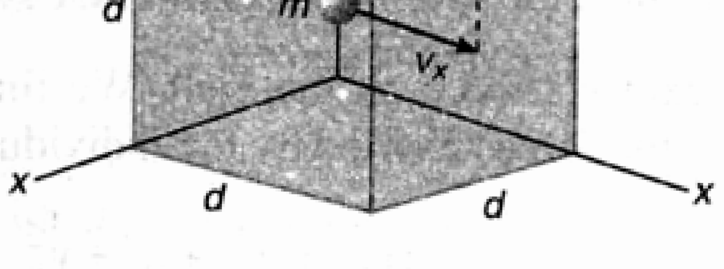
The ideal gas approximation

We make the following assumptions while describing an ideal gas :

1. The number of particles in the gas is very large.
2. The volume V containing the gas is much larger than the total volume actually occupied by the gas particles themselves.
3. The dynamics of the particles is governed by Newton's laws of motion.
4. The particles are equally likely to be moving in any direction.
5. The gas particles interact with each other and with the walls of the container only via elastic collisions.
6. The particles of the gas are identical and indistinguishable.

The pressure of an ideal gas

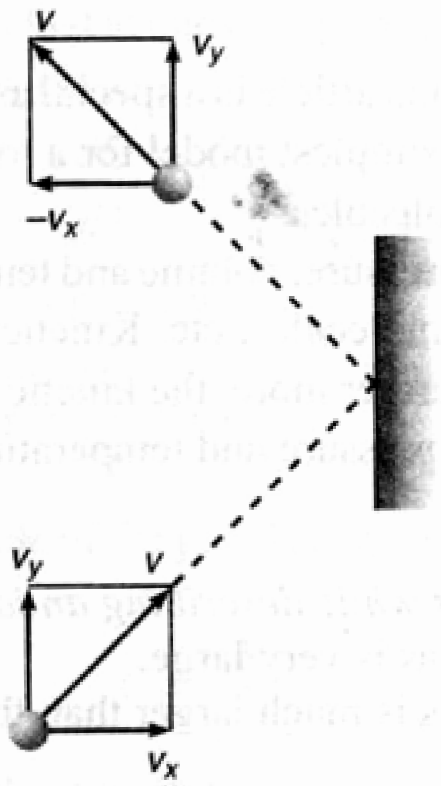
Consider an ideal gas consisting of N molecules in a container of volume V. The container is a cube with edges of length d. Consider the collision of one molecule moving with a velocity $\vec{v}$ toward the right hand face of the cube. The molecule has velocity components v_x, v_y and v_z. Previously we used m to represent the mass of a sample, but in this article we shall use m to represent the mass of one molecule. As the molecule collides with the wall elastically its x-component of velocity is reversed, while its y and z components of velocity remain unaltered. Because the



A cubical box with sides of length d containing an ideal gas. The molecule shown moves with velocity v.

Fig. 17.27

x-component of the momentum of the molecule is mv_x before the collision and $-mv_x$ after the collision, the change in momentum of the molecule is



A molecule makes an elastic collision with the wall of the container. Its *x* component of momentum is reversed, while its *y* component remains unchanged. In this construction, we assume that the molecule moves in the *xy* plane.

Fig. 17.28

$$\Delta p_x = -mv_x - (mv_x) = -2mv_x$$

Applying impulse = change in momentum to the molecule

$$F\Delta t = \Delta p_x = -2mv_x$$

where F is the magnitude of the average force exerted by the wall on the molecule in time Δt. For the molecules to collide twice with the same wall, it must travel a distance $2d$ in the x-direction. Therefore, the time interval between two collisions with the same wall is $\Delta t = \dfrac{2d}{v_x}$. Over a time interval that is long compared with Δt, the average force exerted on the molecules for each collision is

$$F = \frac{-2mv_x}{\Delta t} = \frac{-2mv_x}{2d/v_x} = \frac{-mv_x^2}{d}$$

According to Newton's third law, the average force exerted by the molecule on the wall is, $\dfrac{mv_x^2}{d}$.

Each molecule of the gas exerts a force on the wall. We find the total force exerted by all the molecules on the wall by adding the forces exerted by the individual molecules.

$$\therefore \quad F_{\text{wall}} = \frac{m}{d}\left(v_{x_1}^2 + v_{x_2}^2 + \ldots + v_{x_N}^2\right)$$

This can also be written as,

$$F_{\text{wall}} = \frac{Nm}{d}\bar{v}_x^2$$

where $$\bar{v}_x^2 = \frac{v_{x_1}^2 + v_{x_2}^2 + \ldots + v_{x_N}^2}{N}$$

Since the velocity has three components v_x, v_y and v_z, we can have

$$\bar{v}^2 = \bar{v}_x^2 + \bar{v}_y^2 + \bar{v}_z^2 \qquad (\text{as } v^2 = v_x^2 + v_y^2 + v_z^2)$$

Because the motion is completely random, the average values $\bar{v}_x^2$, $\bar{v}_y^2$ and $\bar{v}_z^2$ are equal to each other. So,

$$\bar{v}^2 = 3\bar{v}_x^2 \quad \text{or} \quad \bar{v}_x^2 = \frac{1}{3}\bar{v}^2$$

Therefore, $$F_{\text{wall}} = \frac{N}{3}\left(\frac{m\bar{v}^2}{d}\right)$$

∴ Pressure on the wall

$$P = \frac{F_{\text{wall}}}{A} = \frac{F_{\text{wall}}}{d^2} = \frac{1}{3}\left(\frac{N}{d^3}m\bar{v}^2\right)$$

$$= \frac{1}{3}\left(\frac{N}{V}\right)m\bar{v}^2 = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{1}{2}m\bar{v}^2\right)$$

∴ $$P = \frac{1}{3}\frac{mN}{V}\bar{v}^2 = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{1}{2}m\bar{v}^2\right) \qquad \ldots\text{(i)}$$

This result indicates that the pressure is proportional to the number of molecules per unit volume (N/V) and to the average translational kinetic energy of the molecules $\frac{1}{2}m\bar{v}^2$. This result relates the large scale quantity (macroscopic) of pressure to an atomic quantity (microscopic)—the average value of the square of the molecular speed. The above equation varifies some features of pressure with which you are probably familiar. One way to increase the pressure inside a container is to increase the number of molecules per unit volume in the container.

The meaning of the absolute temperature

Rewriting Eq. (i) in the more familiar form

$$PV = \frac{2}{3}N\left(\frac{1}{2}m\bar{v}^2\right)$$

Let us now compare it with the ideal gas equation

$$PV = nRT$$

$$nRT = \frac{2}{3}N\left(\frac{1}{2}m\bar{v}^2\right)$$

Here $$n = \frac{N}{N_A} \qquad (N_A = \text{Avogadro number})$$

$\therefore$ $$T = \frac{2}{3}\left(\frac{N_A}{R}\right)\left(\frac{1}{2}m\bar{v}^2\right)$$

or $$T = \frac{2}{3k}\left(\frac{1}{2}m\bar{v}^2\right) \qquad \ldots\text{(ii)}$$

where k is **Boltzmann's constant** which has the value

$$k = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ J/K}$$

By rearranging Eq. (ii) we can relate the translational molecular kinetic energy to the temperature

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$

That is, the average translational kinetic energy per molecule is $\frac{3}{2}kT$. Because

$$\bar{v}_x^2 = \frac{1}{3}\bar{v}^2, \quad \text{it follows that}$$

$$\frac{1}{2}m\bar{v}_x^2 = \frac{1}{2}kT$$

In the similar manner it follows that

$$\frac{1}{2}m\bar{v}_y^2 = \frac{1}{2}kT \quad \text{and} \quad \frac{1}{2}m\bar{v}_z^2 = \frac{1}{2}kT$$

Thus, in each translational degree of freedom one gas molecule has an energy $\frac{1}{2}kT$. One mole of a gas has N_A number of molecules. Thus, one mole of the gas has an energy $\frac{1}{2}(kN_A)T = \frac{1}{2}RT$ in each degree of freedom. Which is nothing but the law of equipartition of energy. The total translational kinetic energy of one mole of an ideal gas is therefore, $\frac{3}{2}RT$.

$$(\text{KE})_{\text{Trans}} = \frac{3}{2}RT \quad \text{(of one mole)}$$

Root mean square speed

The square root of $\bar{\mathbf{v}}^2$ is called the root mean square (rms) speed of the molecules. From Eq. (ii) we obtain, for the rms speed

$$v_{\text{rms}} = \sqrt{\bar{v}^2} = \sqrt{\frac{3kT}{m}}$$

Using $$k = \frac{R}{N_A}, \quad mN_A = M \quad \text{and} \quad \frac{RT}{M} = \frac{P}{\rho}$$

we can write,

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}}$$

Mean speed or average speed

The particles of a gas have a range of speeds. The average speed is found by taking the average of the speeds of all the particles at a given instant. Remember that the speed is a positive scalar since it is the magnitude of the velocity.

$$v_{av} = \frac{v_1 + v_2 + \ldots + v_N}{N}$$

From Maxwellian speed distribution law, we can show that

$$v_{av} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8P}{\pi\rho}}$$

Most probable speed

This is defined as the speed which is possessed by maximum fraction of total number of molecules of the gas. For example, if speeds of 10 molecules of a gas are, 1, 2, 2, 3, 3, 3, 4, 5, 6, 6 km/s, then the most probable speed is 3 km/s, as maximum fraction of total molecules possess this speed. Again from Maxwellian speed distribution law (out of JEE syllabus)

$$v_{mp} = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2P}{\rho}}$$

Note

1. *In the above expressions of v_{rms}, v_{av} and v_{mp}, M is the molar mass in kilogram per mole. For example, molar mass of hydrogen is 2×10^{-3} kg/mol.*
2. $v_{rms} > v_{av} > v_{mp}$ *(RAM)*
3. $v_{rms} : v_{av} : v_{mp} = \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2}$

   and since, $\frac{8}{\pi} \approx 2.5$, *we have* $v_{rms} : v_{av} : v_{mp} = \sqrt{3} : \sqrt{2.5} : \sqrt{2}$

Sample Example 17.10 *A tank used for filling helium balloons has a volume of 0.3 m^3 and contains 2.0 mol of helium gas at 20.0°C. Assuming that the helium behaves like an ideal gas.*

(a) What is the total translational kinetic energy of the molecules of the gas?

(b) What is the average kinetic energy per molecule?

Solution (a) Using $(KE)_{Trans} = \frac{3}{2} nRT$

with $n = 2.0$ mol and $T = 293$ K, we find that

$$(KE)_{Trans} = \frac{3}{2}(2.0)(8.31)(293)$$

$$= 7.3 \times 10^3 \text{ J}$$ **Ans.**

(b) The average kinetic energy per molecule is $\frac{3}{2}kT$.

or
$$\frac{1}{2}m\bar{v}^2 = \frac{1}{2}m\bar{v}^2_{rms} = \frac{3}{2}kT$$
$$= \frac{3}{2}(1.38 \times 10^{-23})(293)$$
$$= 6.07 \times 10^{-21} \text{ J}$$
Ans.

Sample Example 17.11 *Consider an 1100 particles gas system with speeds distribution as follows :*
1000 *particles each with speed* 100 *m/s*
2000 *particles each with speed* 200 *m/s*
4000 *particles each with speed* 300 *m/s*
3000 *particles each with speed* 400 *m/s and* 1000 *particles each with speed* 500 *m/s*
Find the average speed, and rms speed.

Solution The average speed is:
$$v_{av} = \frac{(1000)(100) + (2000)(200) + (4000)(300) + (3000)(400) + (1000)(500)}{1100}$$
$$= 309 \text{ m/s}$$
Ans.

The rms speed is :
$$v_{rms} = \sqrt{\frac{(1000)(100)^2 + (2000)(200)^2 + (4000)(300)^2 + (3000)(400)^2 + (1000)(500)^2}{1100}}$$
$$= 328 \text{ m/s}$$
Ans.

Note *Here* $\frac{v_{rms}}{v_{av}} \neq \sqrt{\frac{3}{8/\pi}}$ *as values and gas molecules are arbitrarily taken.*

Sample Example 17.12 *Calculate the change in internal energy of* 3.0 *mol of helium gas when its temperature is increased by* 2.0 *K.*

Solution Helium is a monoatomic gas. Internal energy of n moles of the gas is,
$$U = \frac{3}{2}nRT$$
$$\therefore \quad \Delta U = \frac{3}{2}nR(\Delta T)$$

Substituting the values,
$$\Delta U = \left(\frac{3}{2}\right)(3)(8.31)(2.0) = 74.8 \text{ J}$$
Ans.

Sample Example 17.13 *In a crude model of a rotating diatomic molecule of chlorine (Cl_2), the two Cl atoms are 2.0×10^{-10} m apart and rotate about their centre of mass with angular speed $\omega = 2.0 \times 10^{12}$ rad/s. What is the rotational kinetic energy of one molecules of Cl_2, which has a molar mass of 70.0 g/mol?*

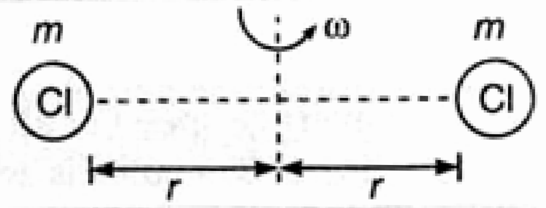


Fig. 17.29

Solution Moment of inertia,

$$I = 2(mr^2) = 2mr^2$$

Here $$m = \frac{70 \times 10^{-3}}{2 \times 6.02 \times 10^{23}} = 5.81 \times 10^{-26} \text{ kg}$$

and $$r = \frac{2.0 \times 10^{-10}}{2} = 1.0 \times 10^{-10} \text{ m}$$

$\therefore$ $$I = 2(5.81 \times 10^{-26})(1.0 \times 10^{-10})^2$$
$$= 1.16 \times 10^{-45} \text{ kg-m}^2$$

$\therefore$ $$K_R = \frac{1}{2} I\omega^2$$
$$= \frac{1}{2} \times (1.16 \times 10^{-45}) \times (2.0 \times 10^{12})^2$$
$$= 2.32 \times 10^{-21} \text{ J}$$ **Ans.**

Note *At $T = 300$ K, rotational K.E. should be equal to* $\frac{1}{2}kT = \frac{1}{2} \times (1.38 \times 10^{-23}) \times (300) = 2.07 \times 10^{-21}$ J

Sample Example 17.14 *Prove that the pressure of an ideal gas is numerically equal to two third of the mean translational kinetic energy per unit volume of the gas.*

Solution Translational KE per unit volume

$$E = \frac{1}{2} (\text{mass per unit volume}) (\bar{v}^2)$$
$$= \frac{1}{2}(\rho)\left(\frac{3P}{\rho}\right) = \frac{3}{2}P$$

or $$P = \frac{2}{3}E$$ **Hence Proved.**

Note *Students are advised to remember this result. In this expression E is the translational KE per unit volume.*

Introductory Exercise 17.3

1. The average speed of all the molecules in a gas at a given instant is not zero, whereas the average velocity of all the molecules is zero. Explain why?
2. A sample of helium gas is at a temperature of 300 K and a pressure of 0.5 atm. What is the average kinetic energy of a molecule of a gas?
3. A sample of helium and neon gases has a temperature of 300 K and pressure of 1.0 atm. The molar mass of helium is 4.0 g/mol and that of neon is 20.2 g/mol.
   (a) Find the rms speed of the helium atoms and of the neon atoms.
   (b) What is the average kinetic energy per atom of each gas?
4. At what temperature will the particles in a sample of helium gas have an rms speed of 1.0 km/s?
5. At 0°C and 1.0 atm ($= 1.01 \times 10^5 \text{ N/m}^2$) pressure the densities of air, oxygen and nitrogen are 1.293 kg/m^3, 1.429 kg/m^3 and 1.251 kg/m^3 respectively. Calculate the percentage of nitrogen in the air from these data, assuming only these two gases to be present.
6. An air bubble of 20 cm^3 volume is at the bottom of a lake 40 meters deep where the temperature is 4°C. The bubble rises to the surface which is at a temperature of 20°C. Take the temperature to be the same as that of the surrounding water and find its volume just before it reaches the surface.
7. If the water molecules in 1.0 g of water were distributed uniformly over the surface of earth, how many such molecules would there be in 1.0 cm^2 of earth's surface?
8. For a certain gas the heat capacity at constant pressure is greater than that at constant volume by 29.1 J /K.
   (a) How many moles of the gas are there?
   (b) If the gas is monoatomic, what are heat capacities at constant volume and pressure ?
   (c) If the gas molecules are diatomic which rotate but do not vibrate, what are heat capacities at constant volume and at constant pressure.
9. The heat capacity at constant volume of a sample of a monoatomic gas is 35 J/K. Find :
   (a) the number of moles
   (b) the internal energy at 0°C
   (c) the molor heat capacity at constant pressure.
10. For any distribution of speeds $v_{rms} \geq v_{av}$. Is this statement true or false?

Extra Points

- Pressure exerted by an ideal gas is numerically equal to two-third of the mean kinetic energy of translation per unit volume of the gas. Thus,

$$P = \frac{2}{3} E$$

- **Mean Free Path :** Every gas consists of a very large number of molecules. These molecules are in a state of continuous rapid and random motion. They undergo perfectly elastic collisions against one another. Therefore, path of a single gas molecule consists of a series of short zig-zag paths of different lengths.

  The mean free path of a gas molecule is the average distance between two successive collisions. It is represented by λ.

$$\lambda = \frac{kT}{\sqrt{2}\pi\sigma^2 p}$$

Here, σ = diameter of the molecule

k = Boltzmann's constant

- **Avagadro's Hypothesis :** At constant temperature and pressure equal volumes of different gases contain equal number of molecules. In 1 gm-mole of any gas there are 6.02×10^{23} molecules of that gas. This is called Avagadro's number. Thus,

$$N = 6.02 \times 10^{23} \text{ per gm-mole}$$

Therefore, the number of molecules in mass m of the substance :

$$\text{Number of molecules } = nN = \frac{m}{M} \times N$$

- **Dalton' Law of Partial Pressure :** According to this law if the gases filled in a vessel do not react chemically, then the combined pressure of all the gases is due to the partial pressure of the molecules of the individual gases. If $P_1, P_2, \ldots$ represent the partial pressures of the different gases, then the total pressure is,

$$P = P_1 + P_2 \ldots$$

- **Van der Waal's Equation :** Experiments have proved that real gases deviate largely from ideal behaviour. The reason of this deviation is two wrong assumptions in the kinetic theory of gases.
  (i) The size of the molecules is much smaller in comparison to the volume of the gas, hence, it may be neglected.
  (ii) Molecules do not exert intermolecular force on each other.
  Van der Waal made corrections for these assumptions and gave a new equation. This equation is known as Van der Waal's equation for real gases.
  (i) Correction for the finite size of molecules : Molecules occupy some volume. Therefore, the volume in which they perform thermal motion is less than the observed volume of the gas. It is represented by $(V - b)$. Here, b is a constant which depends on the effective size and number of molecules of the gas. Therefore, we should use $(V - b)$ in place of V in gas equation.
  (ii) Correction for intermolecular attraction : Due to the intermolecular force between gas molecules the molecules which are very near to the wall experiences a net inward force. Due to this inward force there is a decrease in momentum of the particles of a gas. Thus, the pressure exerted by real gas molecules is less than the pressure exerted by the molecules of an ideal gas.
  So, we use $\left(P + \frac{a}{V^2}\right)$ in place of P in gas equation. Here, again a is a constant.
  Van der Waal's equation of state for real gases thus becomes,

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

- **Critical Temperature, Pressure and Volume :** Gases can't be liquified above a temperature called critical temperature (T_C) however large the pressure may be. The pressure required to liquify the gas at critical temperature is called critical pressure (P_C) and the volume of the gas at critical temperature and pressure is called critical volume (V_C). Value of critical constants in terms of Van der Waal's constants 'a' and 'b' are as under :

$$V_C = 3b, \quad P_C = \frac{a}{27b^2} \quad \text{and} \quad T_C = \frac{8a}{27Rb}$$

Further, $\frac{RT_C}{P_C V_C} = \frac{8}{3}$ is called critical coefficient and is same for all gases.

Detailed Discussion on molar heat capacity (can be read only for personal interest):

- For monoatomic gases value of C_V is $\frac{3}{2}R$. No variation is observed in this. So, value of C_V, C_P, $C_P - C_V$ and γ comes out to be same for different monoatomic gases (Table 17.3).

Table 17.3

Gas	C_P	C_V	$C_P - C_V$	$\gamma = C_P/C_V$
Monoatomic Gases				
He	20.8	12.5	8.33	1.67
Ar	20.8	12.5	8.33	1.67
Diatomic Gases				
H_2	28.8	20.4	8.33	1.41
N_2	29.1	20.8	8.33	1.40
O_2	29.4	21.1	8.33	1.40
CO	29.3	21.0	8.33	1.40
Cl_2	34.7	25.7	8.96	1.35
Polyatomic Gases				
CO_2	37.0	28.5	8.50	1.30
SO_2	40.4	31.4	9.00	1.29
H_2O	35.4	27.0	8.37	1.30
CH_4	35.5	27.1	8.41	1.31

*All values except that for water were obtained at 300 K. SI units are used for C_P and C_V.

For dia and polyatomic gases these values are not equal for different gases. These values vary from gas to gas. Even for one gas values are different at different temperatures.

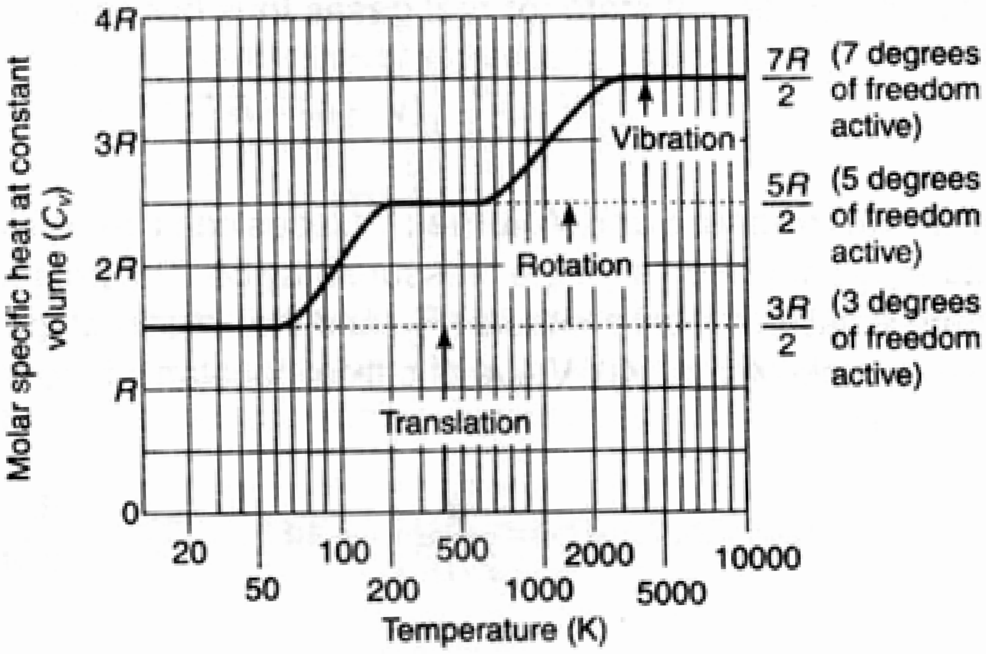


Fig. 17.30

Figure illustrates the variation in the molar specific heat (at constant volume) for H_2 over a wide range in temperatures. (Note that T is drawn on a logrithmic scale). Below about 100 K, C_V is $\frac{3R}{2}$ which is characteristic of three translational degrees of freedom. At room temperature (300 K) it is $\frac{5R}{2}$ which includes the two rotational degrees of freedom. It seems, therefore, that at low temperatures, rotation is not allowed. At high temperatures, C_V starts to rise toward the value $\frac{7R}{2}$. Thus, the vibrational degrees of freedom contribute only at these high temperatures. In table 17.3 the large values of C_V for some polyatomic molecules show the contributions of vibrational energy. In addition, a molecule with three or more atoms that are not in a straight line has three, not two, rotational degrees of freedom.

From this discussion we expect heat capacities to be temperature-dependent, generally increasing with increasing temperature.

- **Solids :** In crystalline solids (monoatomic), the atoms are arranged in a three dimensional array, called a lattice. Each atom in a lattice can vibrate along three mutually perpendicular directions, each of which has two degrees of freedom. One corresponding to vibrations KE and the other vibrational PE Thus, each atom has a total of six degrees of freedom. The volume of a solid does not change significantly with temperature, and so there is little difference between C_V and C_P for a solid. The molar heat capacity is expected to be,

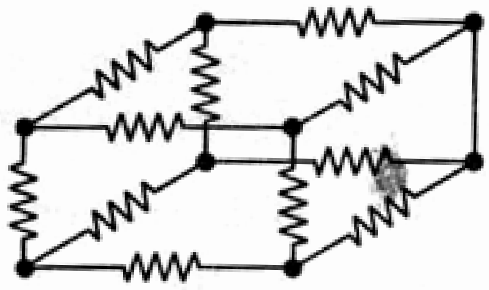


Fig. 17.31

$$C = \frac{f}{2}R = \frac{6}{2}R$$

or $$C = 3R \quad \text{(ideal monoatomic solid)}$$

Its numerical value is $C \approx 25$ J/mol-K ≈ 6 cal/mol-K. This result was first found experimentally by **Dulong and Petit**.

Figure shows that the Dulong and Petit law is obeyed quite well at high (> 250 K) temperatures. At low temperatures, the heat capacities decreases.

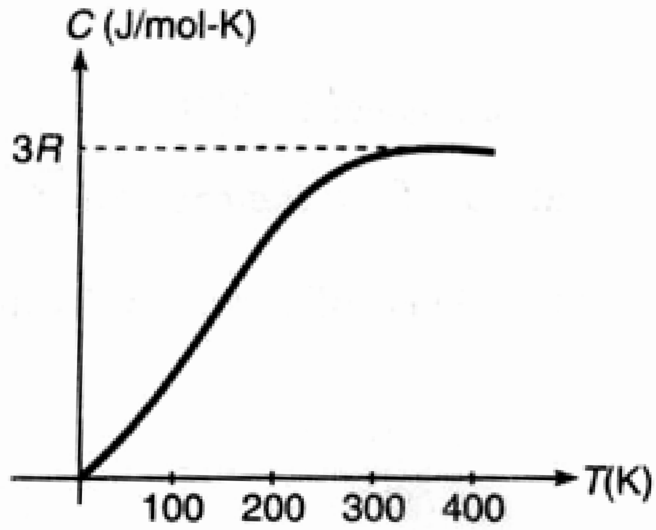


Fig. 17.32

Solved Examples

For JEE Main

Example 1 *A platinum resistance thermometer reads* $0°C$ *when its resistance is* $80\,\Omega$ *and* $100°C$ *when its resistance is* $90\,\Omega$*. Find the temperature at which the resistance is* $86\,\Omega$*.*

Solution The temperature on the platinum scale is

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100° = \frac{86-80}{90-80} \times 100°\text{C} = 60°\text{C}$$

Example 2 *The steam point and the ice point of a mercury thermometer are marked as* $80°$ *and* $10°$*. At what temperature on centigrade scale the reading of this thermometer will be* $59°$ *?*

Solution Let the relation between the thermometer reading and centigrade be $y = ax + b$

given at $x = 100$, $y = 80$ and at $x = 0$, $y = 10$

$\therefore$ $$80 = 100\,a + b, 10 = b \quad \Rightarrow \quad a = 0.7$$

Now, we have to find x when $y = 59$

$\therefore$ $$59 = 0.7x + b \quad \Rightarrow \quad x = 70$$

$\therefore$ The answer is $70°$C

Example 3 *Find the rms speed of hydrogen molecules at room temperature* $(= 300\,K)$.

Solution Mass of 1 mole of hydrogen gas

$$= 2\text{ g} = 2 \times 10^{-3}\text{ kg}$$

$\Rightarrow$ $$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 300}{2 \times 10^{-3}}}$$

$$= 1.93 \times 10^3\text{ m/s}$$ **Ans.**

Example 4 *Find the temperature at which oxygen molecules would have the same rms speed as of hydrogen molecules at* 300 *K.*

Solution If T be the corresponding temperature,

$$\sqrt{\frac{3RT}{M_{\text{O}}}} = \sqrt{\frac{3R\,(300)}{M_{\text{H}}}} \quad \Rightarrow \quad T = (300)\left(\frac{M_{\text{O}}}{M_{\text{H}}}\right)$$

$$= 4800\text{ K}$$

Example 5 *A sphere of diameter 7 cm and mass 266.5 g floats in a bath of liquid. As the temperature is raised, the sphere just sinks at a temperature of* $35°$ *C. If the density of the liquid at* $0°$ *C is* $1.527\ g/cm^3$ *find the coefficient of cubical expansion of the liquid.*

Solution The sphere will sink in the liquid at 35°C, when its density becomes equal to the density of liquid at 35° C.

The density of sphere,

$$\rho_{35} = \frac{266.5}{\frac{4}{3}\times\left(\frac{22}{7}\right)\times\left(\frac{7}{2}\right)^3} = 1.483 \text{ g/cm}^3$$

Now

$$\rho_0 = \rho_{35}\,[1+\gamma\Delta T)$$

$$1.527 = 1.483\,[1+\gamma\times 35]$$

$$1.029 = 1+\gamma\times 35$$

$$\gamma = \frac{1.029-1}{35} = 0.00083/°\text{C}$$

Example 6 *4 g hydrogen is mixed with 11.2 litre of He at STP in a container of volume 20 litre. If the final temperature is 300 K find the pressure.*

Solution 4 g hydrogen = 2 moles hydrogen.

$$11.2 \text{ litre He at STP} = \frac{1}{2} \text{ mole of He}$$

$$P = P_{\text{H}} + P_{\text{He}} = (n_{\text{H}} + n_{\text{He}})\frac{RT}{V}$$

$$= \left(2+\frac{1}{2}\right)\frac{8.31\times(300\text{ K})}{(20\times10^{-3})\text{ m}^3}$$

$$= 3.12\times10^5 \text{ N/m}^2$$ **Ans.**

Example 7 *One mole of an ideal monoatomic gas is taken at a temperature of 300 K. Its volume is doubled keeping its pressure constant. Find the change in internal energy.*

Solution Since, pressure is constant

$\therefore$ $$V \propto T$$

$\therefore$ $$\frac{V_i}{T_i} = \frac{V_f}{T_f}$$

$\therefore$ $$T_f = \frac{V_f}{V_i}T_i \Rightarrow T_f = 2T_i = 600\text{K}$$

$\therefore$ $$\Delta U = \frac{f}{2}n.R\Delta T = \frac{3}{2}R\,(600-300) = 450R$$

Example 8 *A beam of particles each of mass m_0 and speed v is directed along the x-axis. The beam strikes an area 1 mm square with 10^{15} particles striking per second. Find the pressure on the area due to the beam if the particles stick to the area when they hit. Evaluate for an electron beam in a television tube where $m_0 = 9.1 \times 10^{-31}$ kg and $v = 8 \times 10^7$ m/s.*

Solution Each electron exerts an impulse of $m_0 v$ when it strikes and sticks to the surface. This times the number striking in unit time divided by area is pressure. Hence,

$$P = \frac{10^{15}}{(10^{-3})^2} m_0 v = 10^{21} \; m_0 v$$

Substituting the given values, we have

$$P = (9.1 \times 10^{-31})(8 \times 10^7)(10^{21}) = 0.073 \text{ N/m}^2$$ **Ans.**

Example 9 *Two perfect monoatomic gases at absolute temperatures T_1 and T_2 are mixed. There is no loss of energy. Find the temperature of the mixture if the number of moles in the gases are n_1 and n_2.*

Solution From energy conservation principle,

$$E_i = E_f$$

or

$$E_1 + E_2 = E$$

$\therefore$

$$n_1\left(\frac{3}{2}RT_1\right) + n_2\left(\frac{3}{2}RT_2\right) = (n_1 + n_2)\left(\frac{3}{2}RT\right)$$

$\therefore$

$$T = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}$$ **Ans.**

Example 10 *A glass beaker holds exactly 1 lt at 0°C*

(a) What is its volume at 50°C?

(b) If the beaker is filled with mercury at 0°C, what volume of mercury overflows when the temperature is 50°C? $\alpha_g = 8.3 \times 10^{-6}$ per °C and $\gamma_{Hg} = 1.82 \times 10^{-4}$ per °C.

Solution (a) The volume of beaker after the temperature change is,

$$V_{\text{beaker}} = V_0 \,(1 + 3\alpha_g \Delta\theta)$$
$$= (1)\,[1 + 3 \times 8.3 \times 10^{-6} \times 50]$$
$$= 1.001 \text{ lt}$$ **Ans.**

(b) Volume of mercury at 50°C is

$$V_{\text{mercury}} = V_0 \,(1 + \gamma_{Hg} \Delta\theta)$$
$$= (1)\,[1 + 1.82 \times 10^{-4} \times 50] = 1.009 \text{ lt}$$

The overflow is thus $1.009 - 1.001 = 0.008$ lt or 8 ml **Ans.**

For JEE Advanced

Example 1 *A cubical box of side 1 m contains helium gas (atomic weight 4) at a pressure of 100 N/m^2. During an observation time of 1 second, an atom travelling with the root-mean-square speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. Take $R = \frac{25}{3}$ J/mol-K and $k = 1.38 \times 10^{-23}$ J/K.*

(a) Evaluate the temperature of the gas.
(b) Evaluate the average kinetic energy per atom.
(c) Evaluate the total mass of helium gas in the box.

Solution Volume of the box $= 1\ \text{m}^3$, pressure of the gas $= 100\ \text{N/m}^2$. Let T be the temperature of the gas.

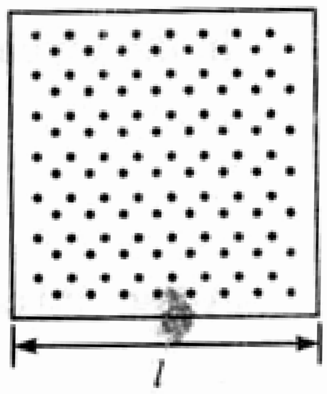


Fig. 17.33

(a) Time between two consecutive collisions with one wall $= \frac{1}{500}$ sec

This time should be equal to $\frac{2l}{v_{\text{rms}}}$, where l is the side of the cube.

$$\frac{2l}{v_{\text{rms}}} = \frac{1}{500}$$

or $$v_{\text{rms}} = 1000\ \text{m/s} \quad (\text{as } l = 1\ \text{m})$$

$\therefore$ $$\sqrt{\frac{3RT}{M}} = 1000$$

or $$T = \frac{(1000)^2 M}{3R} = \frac{(10^6)(4 \times 10^{-3})}{3\left(\frac{25}{3}\right)} = 160\ \text{K}$$ **Ans.**

(b) Average kinetic energy per atom $= \frac{3}{2} kT$

$$= \frac{3}{2}(1.38 \times 10^{-23})(160)\ \text{J}$$

$$= 3.312 \times 10^{-21}\ \text{J}$$ **Ans.**

(c) From $PV = nRT = \frac{m}{M} RT$

we get mass of helium gas in the box,

$$m = \frac{PVM}{RT}$$

Substituting the values, we get

$$m = \frac{(100)(1)(4 \times 10^{-3})}{\left(\frac{25}{3}\right)(160)}$$

$$= 3.0 \times 10^{-4}\ \text{kg}$$ **Ans.**

Example 2 *An ideal diatomic gas with* $C_V = \frac{5R}{2}$ *occupies a volume* V_i *at a pressure* P_i. *The gas undergoes a process in which the pressure is proportional to the volume. At the end of the process, it is found that the rms speed of the gas molecules has doubled from its initial value. Determine the amount of energy transferred to the gas by heat.*

Solution Given that, $P \propto V$ or $PV^{-1} = \text{constant}$

As we know, molar heat capacity in the process $PV^x = \text{constant}$ is,

$$C = \frac{R}{\gamma - 1} + \frac{R}{1-x} = C_V + \frac{R}{1-x}$$

In the given problem, $C_V = \frac{5R}{2}$ and $x = -1$

$$\therefore \quad C = \frac{5R}{2} + \frac{R}{2} = 3R \qquad \text{...(i)}$$

At the end of the process rms speed is doubled, *i.e.*, temperature has become four times ($v_{\text{rms}} \propto \sqrt{T}$).

Now,

$$\Delta Q = nC\Delta T$$
$$= nC\,(T_f - T_i)$$
$$= nC\,(4T_i - T_i)$$
$$= 3T_i nC$$
$$= (3T_i)(n)(3R)$$
$$= 9\,(nRT_i)$$

or

$$\Delta Q = 9P_iV_i$$ **Ans.**

Example 3 *1 g mole of oxygen at 27°C and 1 atmospheric pressure is enclosed in a vessel.*

(a) *Assuming the molecules to be moving with* v_{rms}, *find the number of collisions per second which the molecules make with one square metre area of the vessel wall.*

(b) *The vessel is next thermally insulated and moved with a constant speed* v_0. *It is then suddenly stopped. The process results in a rise of temperature of the gas by 1°C. Calculate the speed* v_0.

[$k = 1.38 \times 10^{-23}$ *J/K and* $N_A = 6.02 \times 10^{23}$ */mol*]

Solution (a) Mass of one oxygen molecule

$$m = \frac{M}{N_A}$$
$$= \frac{32}{6.02 \times 10^{23}}\ \text{g}$$
$$= 5.316 \times 10^{-23}\ \text{g}$$
$$= 5.316 \times 10^{-26}\ \text{kg}$$

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

$$= \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{5.316 \times 10^{-26}}}$$

$$= 483.35 \text{ m/s}$$

Change in momentum per collision

$$\Delta p = mv_{\text{rms}} - (-mv_{\text{rms}}) = 2mv_{\text{rms}}$$
$$= (2)(5.316 \times 10^{-26})(483.35)$$
$$= 5.14 \times 10^{-23} \text{ kg-m/s}$$

Now, suppose n particles strike per second

$$F = n\Delta p = (n)(5.14 \times 10^{-23}) \text{ N} \qquad \left(F_{\text{ext}} = \frac{dP}{dt}\right)$$

Now, as $\quad P = \frac{F}{A}$, for unit area $F = P$

$\therefore \quad (n)(5.14 \times 10^{-23}) = 1.01 \times 10^5$

or $\quad n = 1.965 \times 10^{27}$ per second **Ans.**

(b) When the vessel is stopped the ordered motion of the vessel converts into disordered motion and temperature of the gas is increased.

$$\therefore \quad \frac{1}{2} mv_0^2 = \Delta U \qquad \ldots\text{(i)}$$

$$U = \frac{5}{2} RT \qquad (\text{for } O_2)$$

$$\therefore \quad \Delta U = \frac{5}{2} R\Delta T$$

Here m is not the mass of one gas molecule but it is the mass of the whole gas.

$$m = \text{ mass of 1 mol} = 32 \times 10^{-3} \text{ kg}$$

Substituting these values in Eq. (i)

$$v_0 = \sqrt{\frac{5R\Delta T}{m}}$$

$$= \sqrt{\frac{5 \times 8.31 \times 1}{32 \times 10^{-3}}}$$

$$= 36 \text{ m/s}$$ **Ans.**

Example 4 *Given : Avogadro's number* $N = 6.02 \times 10^{23}$ *and Boltzmann's constant* $k = 1.38 \times 10^{-23}$ *J/K. Calculate*

(a) the average kinetic energy of translation of the molecules of an ideal gas at $0°C$ *and at* $100°C$.
(b) also calculate the corresponding energies per mole of the gas.

Solution (a) According to the kinetic theory, the average kinetic energy of translation per molecule of an ideal gas at kelvin temperature T is $\left(\frac{3}{2}\right)kT$, where k is Boltzmann's constant.

At $0°C$ ($T = 273$ K), the kinetic energy of translation $= \frac{3}{2}kT$

$$= \frac{3}{2} \times (1.38 \times 10^{-23}) \times 273 = 5.65 \times 10^{-21} \text{ J/molecule}$$

At $100°C$ ($T = 373$ K), the energy is

$$\frac{3}{2} \times (1.38 \times 10^{-23}) \times 373 = 7.72 \times 10^{-21} \text{ J/molecule}$$

(b) 1 mole of gas contains N $(= 6.02 \times 10^{23})$ molecules. Therefore, at $0°C$, the kinetic energy of translation of 1 mole of the gas is

$$= (5.65 \times 10^{-21})(6.02 \times 10^{23}) \approx 3401 \text{ J/mol and at } 100°\text{C}.$$

the kinetic energy of translation of 1 mole of gas is

$$= (7.72 \times 10^{-21})(6.02 \times 10^{23}) \approx 4647 \text{ J/mol}$$

Example 5 *An air bubble starts rising from the bottom of a lake. Its diameter is 3.6 mm at the bottom and 4 mm at the surface. The depth of the lake is 250 cm and the temperature at the surface is 40°C. What is the temperature at the bottom of the lake? Given atmospheric pressure = 76 cm of Hg and $g = 980$ cm/s^2.*

Solution At the bottom of the lake, volume of the bubble

$$V_1 = \frac{4}{3}\pi r_1^3 = \frac{4}{3}\pi(0.18)^3 \text{ cm}^3$$

Pressure on the bubble P_1 = Atmospheric pressure + Pressure due to a column of 250 cm of water

$$= 76 \times 13.6 \times 980 + 250 \times 1 \times 980$$

$$= (76 \times 13.6 + 250)\, 980 \text{ dyne/cm}^2$$

At the surface of the lake, volume of the bubble

$$V_2 = \frac{4}{3}\pi r_2^3 = \frac{4}{3}\pi(0.2)^3 \text{ cm}^3$$

Pressure on the bubble

$$P_2 = \text{atmospheric pressure}$$

$$= (76 \times 13.6 \times 980) \text{ dyne/cm}^2$$

$$T_2 = 273 + 40°\text{C} = 313° \text{ K}$$

Now
$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

or
$$\frac{(76 \times 13.6 + 250) 980 \times \left(\frac{4}{3}\right)\pi(0.18)^3}{T_1}$$

$$= \frac{(76 \times 13.6) \times 980 \left(\frac{4}{3}\right) \pi (0.2)^3}{313}$$

or $\quad T_1 = 283.37 \text{ K}$

$\therefore \quad T_1 = 283.37 - 273 = 10.37°\text{C}$

Example 6 *P-V diagram of n moles of an ideal gas is as shown in figure. Find the maximum temperature between A and B.*

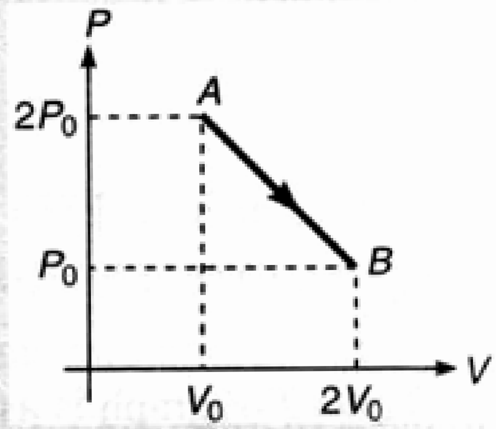


Fig. 17.34

HOW TO PROCEED For given number of moles of a gas,

$$T \propto PV \qquad (PV = nRT)$$

Although $(PV)_A = (PV)_B$ or $T_A = T_B$, still it is not an isothermal process. Because in isothermal process *P-V* graph is a rectangular hyperbola while it is a straight line. So, to see the behaviour of temperature first we will find either *T-V* equation or *T-P* equation and from that equation we can judge how the temperature varies. From the graph first we will write *P-V* equation, then we will convert it either in *T-V* equation or in *T-P* equation.

Solution From the graph the *P-V* equation can be written as,

$$P = -\left(\frac{P_0}{V_0}\right)V + 3P_0 \qquad (y = -mx + c)$$

or
$$PV = -\left(\frac{P_0}{V_0}\right)V^2 + 3P_0V$$

or
$$nRT = 3P_0V - \left(\frac{P_0}{V_0}\right)V^2 \qquad (\text{as } PV = nRT)$$

or
$$T = \frac{1}{nR}\left[3P_0V - \left(\frac{P_0}{V_0}\right)V^2\right]$$

This is the required *T-V* equation. This is quadratic in *V*. Hence, *T-V* graph is a parabola. Now, to find maximum or minimum value of *T* we can substitute.

$$\frac{dT}{dV} = 0$$

or
$$3P_0 - \left(\frac{2P_0}{V_0}\right)V = 0$$

or $$V = \frac{3}{2}V_0$$

Further $\frac{d^2T}{dV^2}$ is negative at $V = \frac{3}{2}V_0$

Hence, T is maximum at $V = \frac{3}{2}V_0$ and this maximum value is,

$$T_{max} = \frac{1}{nR}\left[(3P_0)\left(\frac{3V_0}{2}\right) - \left(\frac{P_0}{V_0}\right)\left(\frac{3V_0}{2}\right)^2\right]$$

or $$T_{max} = \frac{9P_0V_0}{4nR}$$ **Ans.**

Thus, T-V graph is as shown in figure.

$$T_A = T_B = \frac{2P_0V_0}{nR}$$

and $$T_{max} = \frac{9P_0V_0}{4nR}$$

$$= 2.25\frac{P_0V_0}{nR}$$

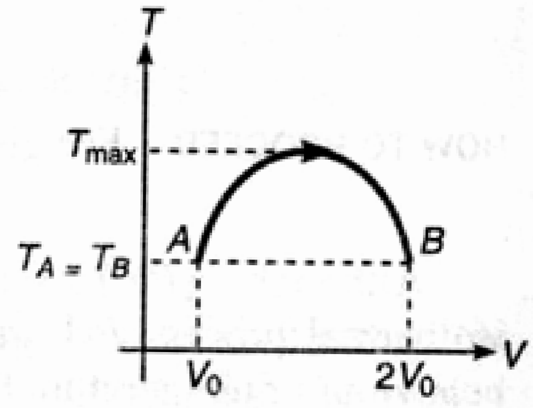


Fig. 17.35

Note *Most of the problems of T_{max}, P_{max} and V_{max} are solved by differentiation. Sometimes graph will be given and sometimes direct equation will be given. For Problem, for P_{max} you will require either P-V or P-T equation.*

EXERCISES

For JEE Main

Subjective Questions

Temperature Scales

1. Change each of the given temperatures to the Celsius and Kelvin scales: $68°$ F, $5°$ F and $176°$ F.
2. Change each of the given temperatures to the Fahrenheit and Rankine scales: $30°$C, $5°$C and $-20°$C.
3. At what temperature do the Celsius and Fahrenheit readings have the same numerical value?
4. You work in a materials testing lab and your boss tells you to increase the temperature of a sample by $40.0°$C. The only thermometer you can find at your workbench reads in °F. If the initial temperature of the sample is $68.2°$ F. What is its temperature in °F when the desired temperature increase has been achieved?
5. The steam point and the ice point of a mercury thermometer are marked as $80°$ and $20°$. What will be the temperature in centigrade mercury scale when this thermometer reads $32°$?

Thermometers

6. The pressure of the gas in a constant volume gas thermometer is 80 cm of mercury in melting ice at 1 atm. When the bulb is placed in a liquid, the pressure becomes 160 cm of mercury. Find the temperature of the liquid.
7. The resistances of a platinum resistance thermometer at the ice point, the steam point and the boiling point of sulphur are 2.50, 3.50 and 6.50 Ω respectively. Find the boiling point of sulphur on the platinum scale. The ice point and the steam point measure $0°$ and $100°$ respectively.
8. In a constant volume gas thermometer, the pressure of the working gas is measured by the difference in the levels of mercury in the two arms of a *U*-tube connected to the gas at one end. When the bulb is placed at the room temperature $27.0°$C, the mercury column in the arm open to atmosphere stands 5.00 cm above the level of mercury in the other arm. When the bulb is placed in a hot liquid, the difference of mercury levels becomes 45.0 cm. Calculate the temperature of the liquid.(Atmospheric pressure = 75.0 cm of mercury.)

Thermal Expansion

9. An iron ball has a diameter of 6 cm and is 0.010 mm too large to pass through a hole in a brass plate when the ball and plate are at a temperature of $30°$C. At what temperature, the same for ball and plate, will the ball just pass through the hole ?
10. (a) An aluminum measuring rod which is correct at $5°$C, measures a certain distance as 88.42 cm at 35 °C. Determine the error in measuring the distance due to the expansion of the rod. (b) If this aluminum rod measures a length of steel as 88.42 cm at $35°$C, what is the correct length of the steel at $35°$C?
11. A steel tape is calibrated at $20°$C. On a cold day when the temperature is $-15°$C, what will be the percentage error in the tape ?

12. A steel wire of 2.0 mm^2 cross-section is held straight (but under no tension) by attaching it firmly to two points a distance 1.50 m apart at 30 °C. If the temperature now decreases to $-10°C$ and if the two points remain fixed, what will be the tension in the wire ? For steel, $Y = 20{,}0000$ MPa.

13. A metallic bob weighs 50 g in air. If it is immersed in a liquid at a temperature of 25° C, it weighs 45 g. When the temperature of the liquid is raised to 100° C, it weighs 45.1 g. Calculate the coefficient of cubical expansion of the liquid. Given that coefficient of cubical expansion of the metal is $12 \times 10^{-6}\ °C^{-1}$.

Gas Laws and Ideal Gas Equation

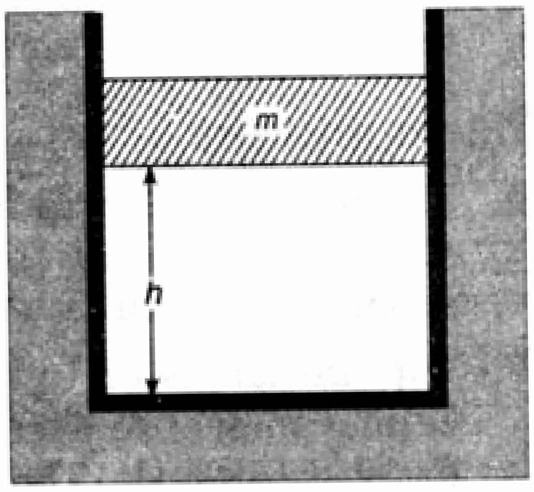
14. Find the mass (in kilogram) of an ammonia molecule NH_3.

15. An ideal gas exerts a pressure of 1.52 MPa when its temperature is 298.15 K and its volume is $10^{-2}\ m^3$. (a) How many moles of gas are there? (b) What is the mass density if the gas is molecular hydrogen? (c) What is the mass density if the gas is oxygen?

16. A compressor pumps 70 L of air into a 6 L tank with the temperature remaining unchanged. If all the air is originally at 1 atm. What is the final absolute pressure of the air in the tank?

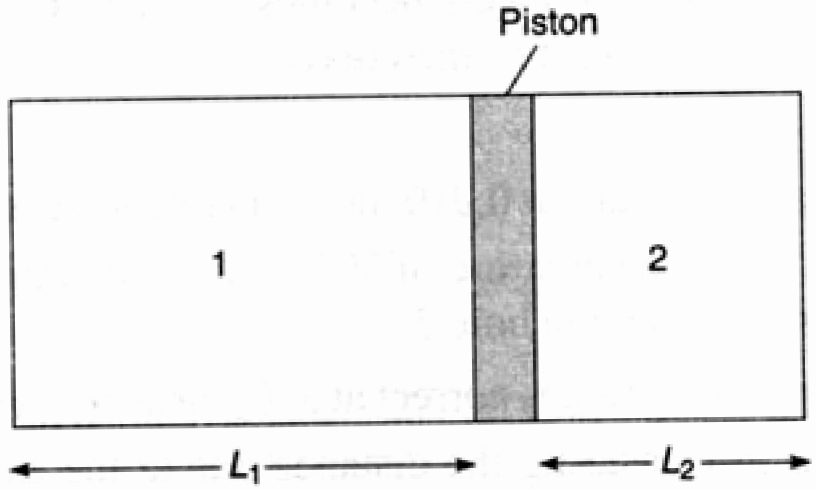
17. A partially inflated balloon contains 500 m^3 of helium at 27° C and 1 atm pressure. What is the volume of the helium at an altitude of 18000 ft, where the pressure is 0.5 atm and the temperature is −3° C?

18. A cylinder whose inside diameter is 4.00 cm contains air compressed by a piston of mass $m = 13.0$ kg which can slide freely in the cylinder. The entire arrangement is immersed in a water bath whose temperature can be controlled. The system is initially in equilibrium at temperature $t_i = 20°C$. The initial height of the piston above the bottom of the cylinder is $h_i = 4.00$ cm.



The temperature of the water bath is gradually increased to a final temperature $t_f = 100°C$. Calculate the final height h_f of the piston.

19. The closed cylinder shown in figure has a freely moving piston separating chambers 1 and 2. Chamber 1 contains 25 mg of N_2 gas and chamber 2 contains 40 mg of helium gas. When equilibrium is established what will be the ratio L_1 / L_2? What is the ratio of the number of moles of N_2 to the number of moles of He? (Molecular weights of N_2 and He are 28 and 4).



20. Two gases occupy two containers A and B. The gas in A of volume 0.11 m^3 exerts a pressure of 1.38 MPa. The gas in B of volume 0.16 m^3 exerts a pressure of 0.69 MPa. Two containers are united by a tube of negligible volume and the gases are allowed to intermingle. What is the final pressure in the container if the temperature remains constant?

21. A glass bulb of volume $400\,cm^3$ is connected to another of volume $200\,cm^3$ by means of a tube of negligible volume. The bulbs contain dry air and are both at a common temperature and pressure of 20°C and 1.00 atm. The larger bulb is immersed in steam at 100°C and the smaller in melting ice at 0°C. Find the final common pressure.

22. The condition called standard temperature and pressure (STP) for a gas is defined as temperature of $0°C = 273.15$ K and a pressure of $1\ atm = 1.013 \times 10^5$ Pa. If you want to keep a mole of an ideal gas in your room at STP, how big a container do you need?

23. A large cylindrical tank contains $0.750\,m^3$ of nitrogen gas at 27°C and 1.50×10^5 Pa (absolute pressure). The tank has a tightfitting piston that allows the volume to be changed. What will be the pressure if the volume is decreased to $0.480\,m^3$ and the temperature is increased to 157°C.

24. A vessel of volume 5 litres contains 1.4 g of N_2 and 0.4 g of He at 1500 K. If 30% of the nitrogen molecules are dissociated into atoms then find the gas pressure.

Degree of Freedom, Internal Energy and Molar Heat Capacity

25. Temperature of diatomic gas is 300 K. If moment of inertia of its molecules is 8.28×10^{-38} g-cm^2. Calculate their root mean square angular velocity.

26. How many degrees of freedom have the gas molecules, if under standard conditions the gas density is $\rho = 1.3\ kg/m^3$ and velocity of sound propagation on it is $v = 330$ m/s?

27. Three moles of an ideal gas having $\gamma = 1.67$ are mixed with 2 moles of another ideal gas having $\gamma = 1.4$. Find the equivalent value of γ for the mixture.

28. If the kinetic energy of the molecules in 5 litres of helium at 2 atm is E. What is the kinetic energy of molecules in 15 litres of oxygen at 3 atm in terms of E?

29. Find the number of degrees of freedom of molecules in a gas. Whose molar heat capacity
(a) at constant pressure $C_P = 29\ J\ mol^{-1} K^{-1}$
(b) $C = 29\ J\ mol^{-1}\ K^{-1}$ in the process PT = constant.

30. In a certain gas $\frac{2}{5}$th of the energy of molecules is associated with the rotation of molecules and the rest of it is associated with the motion of the centre of mass.
(a) What is the average translational energy of one such molecule when the temperature is 27°C?
(b) How much energy must be supplied to one mole of this gas at constant volume to raise the temperature by 1°C?

31. A mixture contains 1 mole of helium ($C_P = 2.5\,R, C_V = 1.5\,R$) and 1 mole of hydrogen ($C_P = 3.5R, C_V = 2.5\,R$). Calculate the values of C_P, C_V and γ for the mixture.

32. Two ideal gases have the same value of $C_P/C_V = \gamma$. What will be the value of this ratio for a mixture of the two such gases in the ratio 1 : 2?

33. An ideal gas ($C_P / C_V = \gamma$) is taken through a process in which the pressure and the volume vary as $P = aV^b$. Find the value of b for which the specific heat capacity in the process is zero.

34. An ideal gas is taken through a process in which the pressure and the volume are changed according to the equation $P = kV$. Show that the molar heat capacity of the gas for the process is given by $C = C_V + \frac{R}{2}$.

Kinetic Theory of Gases

35. Calculate the root mean square speed of hydrogen molecules at 373.15 K.

36. Five gas molecules chosen at random are found to have speed of 500, 600, 700, 800 and 900 m/s. Find the rms speed. Is it the same as the average speed?

37. The pressure of a gas in a 100 mL container is 200 kPa and the average translational kinetic energy of each gas particle is 6×10^{-26} J. Find the number of gas particles in the container. How many moles are there in the container?

38. One gram mole NO_2 at $57°C$ and 2 atm pressure is kept in a vessel. Assuming the molecules to be moving with rms velocity. Find the number of collisions per second which the molecules make with one square meter area of the vessel wall.

39. A 2.00 mL volume container contains 50 mg of gas at a pressure of 100 kPa. The mass of each gas particle is 8.0×10^{-26} kg. Find the average translational kinetic energy of each particle.

40. Call the rms speed of the molecules in an ideal gas v_0 at temperature T_0 and pressure p_0. Find the speed if (a) the temperature is raised from $T_0 = 293$ K to 573 K (b) the pressure is doubled and $T = T_0$ (c) the molecular weight of each of the gas molecules is tripled.

41. At what temperature is the "effective" speed of gaseous hydrogen molecules (molecular weight = 2) equal to that of oxygen molecules (molecular weight = 32) at 47°C?

42. At what temperature is v_{rms} of H_2 molecules equal to the escape speed from earth's surface. What is the corresponding temperature for escape of hydrogen from moon's surface? Given $g_m = 1.6\ \text{m/s}^2$, $R_e = 6367$ km and $R_m = 1750$ km.

43. (a) What is the average translational kinetic energy of a molecule of an ideal gas at temperature of 27°C?

(b) What is the total random translational kinetic energy of the molecules in one mole of this gas?

(c) What is the rms speed of oxygen molecules at this temperature?

Objective Questions

Single Correct Option

1. The average velocity of molecules of a gas of molecular weight M at temperature T is

(a) $\sqrt{\frac{3RT}{M}}$ (b) $\sqrt{\frac{8RT}{\pi M}}$ (c) $\sqrt{\frac{2RT}{M}}$ (d) zero

2. Four particles have velocities 1, 0, 2 and 3 m/s. The root mean square velocity of the particles (definition wise) is

(a) 3.5 m/s (b) $\sqrt{3.5}$ m/s (c) 1.5 m/s (d) $\sqrt{\frac{14}{3}}$ m/s

3. A steel rod of length 1 m is heated from 25° to 75°C keeping its length constant. The longitudinal strain developed in the rod is (Given coefficient of linear expansion of steel $= 12 \times 10^{-6}$/°C)

(a) 6×10^{-4} (b) -6×10^{-5} (c) -6×10^{-4} (d) zero

4. In a process the pressure of a gas remains constant. If the temperature is doubled, then the change in the volume will be

(a) 100% (b) 200% (c) 50% (d) 25%

5. The average kinetic energy of the molecules of an ideal gas at 10°C has the value E. The temperature at which the kinetic energy of the same gas becomes $2E$ is

(a) 5°C (b) 10°C (c) 40°C (d) None of these

6. A polyatomic gas with n degrees of freedom has a mean energy per molecule given by

(a) $\frac{n}{2}RT$ (b) $\frac{1}{2}RT$ (c) $\frac{n}{2}kT$ (d) $\frac{1}{2}kT$

7. The temperature of an ideal gas is increased from 27°C to 927°C. The rms speed of its molecules becomes

(a) twice (b) half (c) four times (d) one-fourth

8. In case of hydrogen and oxygen at NTP, which of the following is the same for both?

(a) Average linear momentum per molecule (b) Average KE per molecule
(c) KE per unit volume (d) KE per unit mass

9. Two thermally insulated vessels 1 and 2 are filled with air at temperatures (T_1, T_2), volumes (V_1, V_2) and pressures (P_1, P_2) respectively. If the valve joining the two vessels is opened, the temperature inside the vessel at equilibrium will be (P = common pressure)

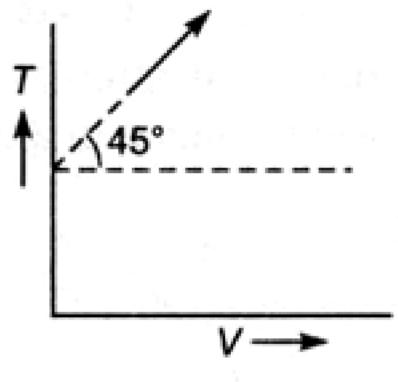
(a) $T_1 + T_2$ (b) $(T_1 + T_2)/2$

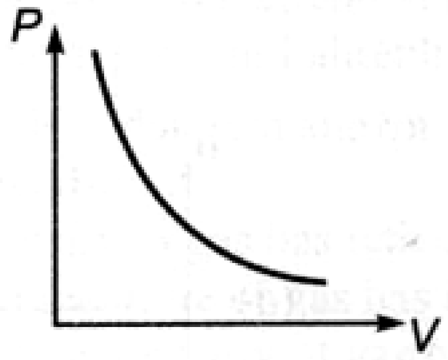
(c) $\frac{T_1T_2P(V_1+V_2)}{P_1V_1T_2+P_2V_2T_1}$ (d) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_2+P_2V_2T_1}$

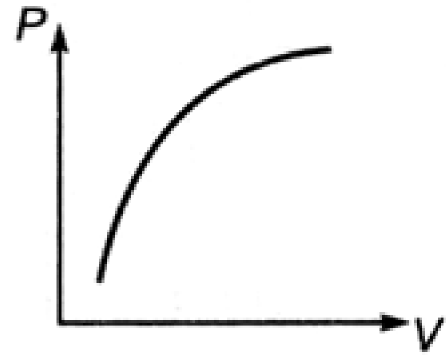
10. Two marks on a glass rod 10 cm apart are found to increase their distance by 0.08 mm when the rod is heated from 0°C to 100°C. A flask made of the same glass as that of rod measures a volume of 100 cc at 0°C. The volume it measures at 100°C in cc is

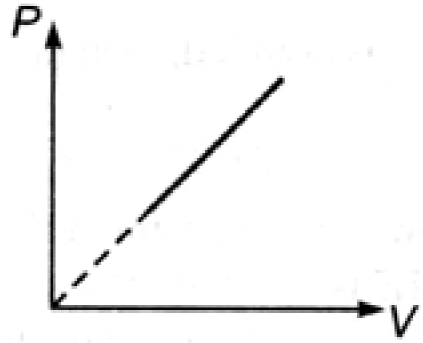
(a) 100.24 (b) 100.12 (c) 100.36 (d) 100.48

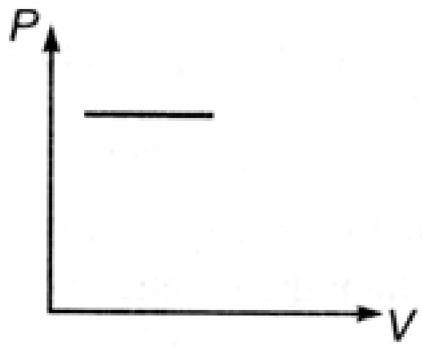
11. The given curve represents the variation of temperature as a function of volume for one mole of an ideal gas. Which of the following curves best represents the variation of pressure as a function of volume?



(a) 

(b) 

(c) 

(d) 

12. A gas is found to obey the law P^2V = constant. The initial temperature and volume are T_0 and V_0. If the gas expands to a volume $3V_0$, then the final temperature becomes

(a) $\sqrt{3}\,T_0$ (b) $\sqrt{2}\,T_0$ (c) $\dfrac{T_0}{\sqrt{3}}$ (d) $\dfrac{T_0}{\sqrt{2}}$

13. Air fills a room in winter at 7°C and in summer at 37°C. If the pressure is the same in winter and summer, the ratio of the weight of the air filled in winter and that in summer is

(a) 2.2 (b) 1.75 (c) 1.1 (d) 3.3

14. Three closed vessels *A*, *B* and *C* are at the same temperature *T* and contain gases which obey Maxwell distribution law of velocities. Vessel *A* contains O_2, *B* only N_2 and *C* mixture of equal quantities of O_2 and N_2. If the average speed of the O_2 molecules in vessel *A* is v_1 that of N_2 molecules in vessel *B* is v_2, then the average speed of the O_2 molecules in vessel *C* is

(a) $\dfrac{(v_1+v_2)}{2}$ (b) v_1 (c) $\sqrt{v_1 v_2}$ (d) None of these

15. In a very good vacuum system in the laboratory, the vacuum attained was 10^{-13} atm. If the temperature of the system was 300 K, the number of molecules present in a volume of 1 cm^3 is

(a) 2.4×10^6 (b) 24 (c) 2.4×10^9 (d) zero

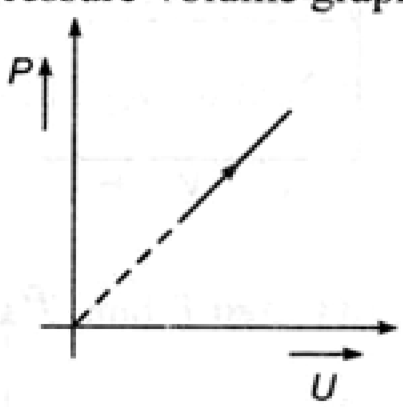
16. If nitrogen gas molecule goes straight up with its rms speed at 0°C from the surface of the earth and there are no collisions with other molecules, then it will rise to an approximate height of

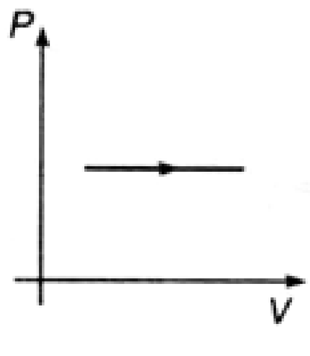
(a) 8 km (b) 12 km (c) 12 m (d) 8 m

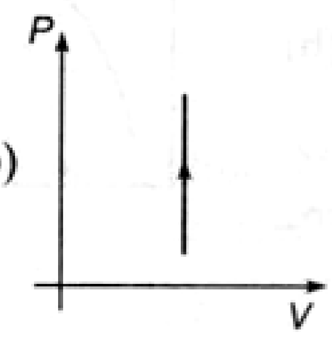
17. The coefficient of linear expansion of steel and brass are 11×10^{-6}/°C and 19×10^{-6}/°C respectively. If their difference in lengths at all temperatures has to kept constant at 30 cm, their lengths at 0°C should be

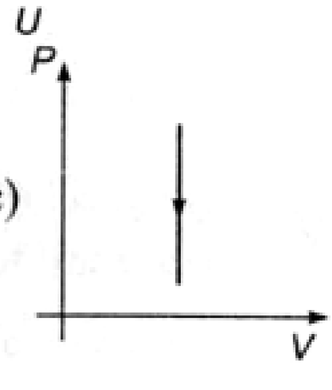
(a) 71.25 cm and 41.25 cm (b) 82 cm and 52 cm
(c) 92 cm and 62 cm (d) 62.25 cm and 32.25 cm

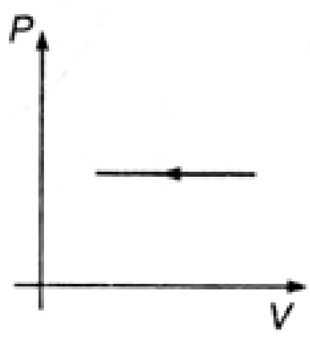
18. The given *P*-*U* graph shows the variation of internal energy of an ideal gas with increase in pressure. Which of the following pressure-volume graph is equivalent to this graph?



(a) 

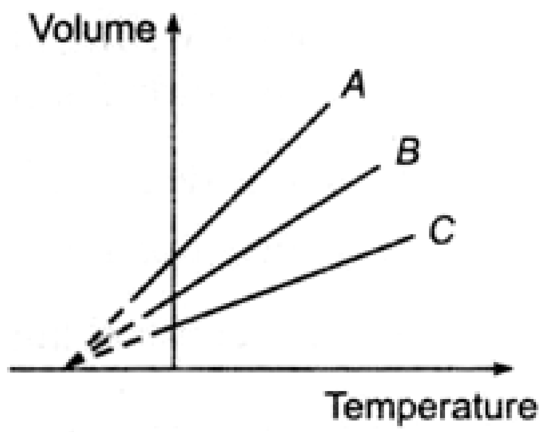
(b) 

(c) 

(d) 

19. The expansion of an ideal gas of mass m at a constant pressure P is given by the straight line B. Then, the expansion of the same ideal gas of mass $2m$ at a pressure $2P$ is given by the straight line

(a) C
(b) A
(c) B
(d) data insufficient



20. 28 g of N_2 gas is contained in a flask at a pressure of 10 atm and at a temperature of 57°C. It is found that due to leakage in the flask, the pressure is reduced to half and the temperature to 27°C. The quantity of N_2 gas that leaked out is

(a) 11/20 g (b) 20/11 g (c) 5/63 g (d) 63/5 g

21. A mixture of 4g of hydrogen and 8 g of helium at NTP has a density about

(a) 0.22 kg/m^3 (b) 0.62 kg/m^3 (c) 1.12 kg/m^3 (d) 0.13 kg/m^3

22. The pressure (P) and the density (ρ) of given mass of a gas expressed by Boyle's law, $P = K\rho$ holds true

(a) for any gas under any condition (b) for some gases under any conditions
(c) only if the temperature is kept constant (d) none of these

More than One Correct Options

23. During an experiment, an ideal gas is found to obey a condition $\frac{P^2}{\rho}$ = constant. (ρ = density of the gas). The gas is initially at temperature T, pressure P and density ρ. The gas expands such that density changes to $\rho/2$.

(a) The pressure of the gas changes to $\sqrt{2}\,P$
(b) The temperature of the gas changes to $\sqrt{2}\,T$
(c) The graph of the above process on P-T diagram is parabola
(d) The graph of the above process on P-T diagram is hyperbola

24. During an experiment, an ideal gas is found to obey a condition VP^2 = constant. The gas is initially at a temperature T, pressure P and volume V. The gas expands to volume $4V$.

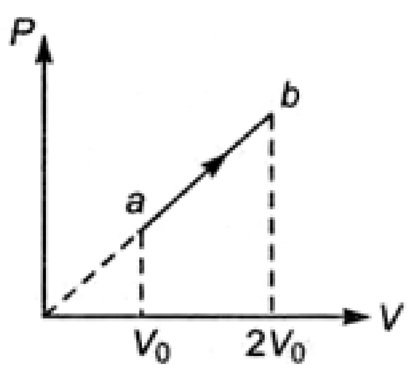
(a) The pressure of gas changes to $\frac{P}{2}$
(b) The temperature of gas changes to $4T$
(c) The graph of the above process on P-T diagram is parabola
(d) The graph of the above process on P-T diagram is hyperbola

25. Find the correct options

(a) Ice point in Fahrenheit scale is 32°F (b) Ice point in Fahrenheit scale is 98.8°F
(c) Steam point in Fahrenheit scale is 212°F (d) Steam point in Fahrenheit scale is 252°F

26. In the P-V diagram shown in figure, choose the correct options for the process a-b:

(a) density of gas has reduced to half
(b) temperature of gas has increased to two times
(c) internal energy of gas has increased to four times
(d) T-V graph is a parabola passing through origin



27. Choose the **wrong** options

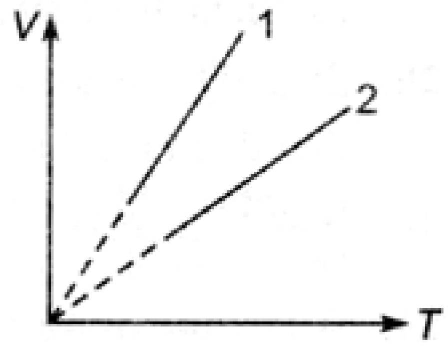
(a) Translational kinetic energy of all ideal gases at same temperature is same

(b) In one degree of freedom all ideal gases has internal energy $= \frac{1}{2}RT$

(c) Translational kinetic energy of all ideal gases is three

(d) Translational kinetic energy of all ideal gases is $\frac{3}{2}RT$

28. Along the line-1, mass of gas is m_1 and pressure is P_1. Along the line-2 mass of same gas is m_2 and pressure is P_2. Choose the correct options.



(a) m_1 may be less than m_2

(b) m_2 may be less than m_1

(c) P_1 may be less than P_2

(d) P_2 may be less than P_1

29. Choose the correct options

(a) In $P = \frac{m}{M}RT$, m is mass of gas per unit volume

(b) In $PV = \frac{m}{M}RT$, m is mass of one molecule of gas

(c) In $P = \frac{1}{3}\frac{mN}{V}v_{rms}^2$, m is total mass of gas.

(d) In $v_{rms} = \sqrt{\frac{3kT}{m}}$, m is mass of one molecule of gas

For **JEE Advanced**

Assertion and Reason

Directions : *Choose the correct option.*

*(a) If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion.***

*(b) If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion.***

*(c) If **Assertion** is true, but the **Reason** is false.*

*(d) If **Assertion** is false but the **Reason** is true.*

1. Assertion : Straight line on P-T graph for an ideal gas represents isochoric process.
Reason : If $P \propto T, V =$ constant.

2. Assertion : Vibrational kinetic energy is insignificant at low temperatures.
Reason : Interatomic forces are responsible for vibrational kinetic energy.

3. Assertion : In the formula $P = \frac{2}{3}E$, the term E represents translational kinetic energy per unit volume of gas.
Reason : In case of monoatomic gas translational kinetic energy and total kinetic energy are equal.

4. Assertion : If a gas container is placed in a mooving train, the temperature of gas will increase.
Reason : Kinetic energy of gas molecules will increase.

5. Assertion : According to the law of equipartition of energy, internal energy of an ideal gas at a given temperature, is equally distributed in translational and rotational kinetic energies.
Reason : Rotational kinetic energy of a monoatomic gas is zero.

6. **Assertion :** Real gases behave as ideal gases most closely at low pressure and high temperature.
Reason : Intermolecular force between ideal gas molecules is assumed to be zero.

7. **Assertion :** A glass of water is filled at 4°C. Water will overflow, if temperature is increased or decreased. (Ignore expansion of glass).
Reason : Density of water is minimum at 4°C.

8. **Assertion :** If pressure of an ideal gas is doubled and volume is halved, then its internal energy will remain unchanged.
Reason : Internal energy of an ideal gas is a function of temperature only.

9. **Assertion :** In equation $P = \frac{1}{3} \alpha v_{rms}^2$, the term α represents density of gas.
Reason : $v_{rms} = \sqrt{\frac{3RT}{M}}$.

10. **Assertion :** In isobaric process, V-T graph is a straight line passing through origin. Slope of this line is directly proportional to mass of the gas. V is taken on y-axis.
Reason : $V = \left(\frac{nR}{P}\right)T$
$\therefore$ slope $\propto n$ or slope $\propto m$

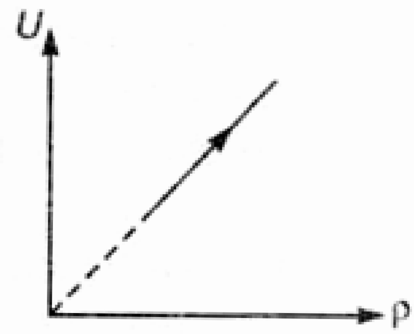
Match the Columns

1. Match the following two columns for 2 moles of a diatomic gas at room temperature T.

Column I	Column II
(a) translational kinetic energy	(p) $2RT$
(b) rotational kinetic energy	(q) $4RT$
(c) potential energy	(r) $3RT$
(d) total internal energy	(s) None

2. In the graph shown, U is the internal energy of gas and ρ the density. Corresponding to given graph match the following two columns.

Column I	Column II
(a) Pressure	(p) is constant
(b) Volume	(q) is increasing
(c) Temperature	(r) is decreasing
(d) Ratio T/V	(s) data insufficient



3. At a given temperature T

$v_1 = \sqrt{\frac{x_1 RT}{M}}$ = rms speed of gas molecules, $v_2 = \sqrt{\frac{x_2 RT}{M}}$ = average speed of gas molecules

$v_3 = \sqrt{\frac{x_3 RT}{M}}$ = most probable speed of gas molecules

$v_4 = \sqrt{\frac{x_4 RT}{M}}$ = speed of sound