30.6 The Bohr Hydrogen Atom

After Neils Bohr obtained his doctorate in 1911, he worked under Rutherford for a while. In 1913, he presented a model of the hydrogen atom, which has one electron. He postulated that an electron moves only in certain circular orbits, called stationary orbits. In stationary orbits electron does not emit radiation, contrary to the predictions of classical electromagnetic theory. According to Bohr, there is a definite energy associated with each stable orbit and an atom radiaties energy only when it makes a transition from one of these orbits to another. The energy is radiated in the form of a photon with energy and frequency given by,

$$\Delta E = hf = E_i - E_f \qquad \ldots\text{(i)}$$

Bohr found that the magnitude of the electron's angular momentum is quantized, and this magnitude for the electron must be integral multiple of $\frac{h}{2\pi}$. The magnitude of the angular momentum is $L = mvr$ for a particle with mass m moving with speed v in a circle of radius r. So, according to Bohr's postulate,

$$mvr = \frac{nh}{2\pi} \qquad (n = 1, 2, 3 \ldots)$$

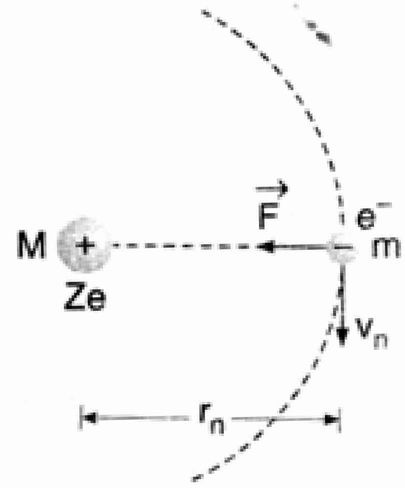


Fig. 30.4

Each value of n corresponds to a permitted value of the orbit radius, which we will denote by r_n and the corresponding speed v_n. The value of n for each orbit is called **principal quantum number** for the orbit. Thus,

$$mv_nr_n = \frac{nh}{2\pi} \qquad \ldots\text{(ii)}$$

According to Newton's second law a radially inward centripetal force of magnitude $F = \frac{mv^2}{r_n}$ is needed to the electron which is being provided by the electrical attraction between the positive proton and the negative electron.

Thus,

$$\frac{mv_n^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2} \qquad \ldots\text{(iii)}$$

Solving Eqs. (ii) and (iii), we get

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2} \quad \begin{pmatrix} n\text{th orbit radius} \\ \text{in Bohr model} \end{pmatrix} \qquad \ldots\text{(iv)}$$

and

$$v_n = \frac{e^2}{2\varepsilon_0 nh} \quad \begin{pmatrix} n\text{th orbital speed} \\ \text{in Bohr model} \end{pmatrix} \qquad \ldots\text{(v)}$$

The smallest orbit radius corresponds to $n = 1$. We'll denote this minimum radius, called the **Bohr radius** as a_0. Thus,

$$a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2}$$

Substituting values of ε_0, h, π, m and e, we get

$$a_0 = 0.529 \times 10^{-10}\ \text{m} = 0.529\ \text{Å} \qquad \ldots\text{(vi)}$$

Eq. (iv), in terms of a_0 can be written as,

$$r_n = n^2 a_0 \quad \text{or} \quad r_n \propto n^2 \qquad \text{...(vii)}$$

Similarly, substituting values of e, ε_0 and h with $n = 1$ in Eq. (v), we get

$$v_1 = 2.19 \times 10^6 \text{ m/s} \approx \frac{c}{137} \qquad \text{...(viii)}$$

This is the greatest possible speed of the electron in the hydrogen atom. Which is approximately equal to $c/137$ where c is the speed of light in vacuum.

Eq. (v), in terms of v_1 can be written as,

$$v_n = \frac{v_1}{n} \quad \text{or} \quad v_n \propto \frac{1}{n} \qquad \text{...(ix)}$$

Energy levels : Kinetic and potential energies K_n and U_n in n^{th} orbit are,

$$K_n = \frac{1}{2} m v_n^2 = \frac{me^4}{8 \varepsilon_0^2 n^2 h^2}$$

and

$$U_n = -\frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_n} = -\frac{me^4}{4 \varepsilon_0^2 n^2 h^2}$$

The total energy E_n is the sum of the kinetic and potential energies.

$$E_n = K_n + U_n = -\frac{me^4}{8 \varepsilon_0^2 n^2 h^2}$$

Substituting values of m, e, ε_0 and h with $n = 1$, we get the least energy of the atom in first orbit, which is -13.6 eV. Hence,

$$E_1 = -13.6 \text{ eV} \qquad \text{...(x)}$$

and

$$E_n = \frac{E_1}{n^2} = -\frac{13.6}{n^2} \text{ eV} \qquad \text{...(xi)}$$

Substituting $n = 2, 3, 4 \ldots$, etc., we get energies of atom in different orbits.

$$E_2 = -3.40 \text{ eV}, \; E_3 = -1.51 \text{ eV}, \ldots E_\infty = 0$$

Ionization energy of the hydrogen atom is the energy required to remove the electron completely. In ground state ($n = 1$) energy of atom is -13.6 eV and energy corresponding to $n = \infty$ is zero. Hence, energy required to remove the electron from ground state is 13.6 eV.

Emission spectrum of hydrogen atom

Under normal conditions the single electron in hydrogen atom stays in ground state ($n = 1$). It is excited to some higher energy state when it acquires some energy from external source. But it hardly stays there for more than 10^{-8} second.

A photon corresponding to a particular spectrum line is emitted when an atom makes a transition from a state in an excited level to a state in a lower excited level or the ground level.

Let n_i be the initial and n_f the final energy state, then depending on the final energy state following series are observed in the emission spectrum of hydrogen atom.

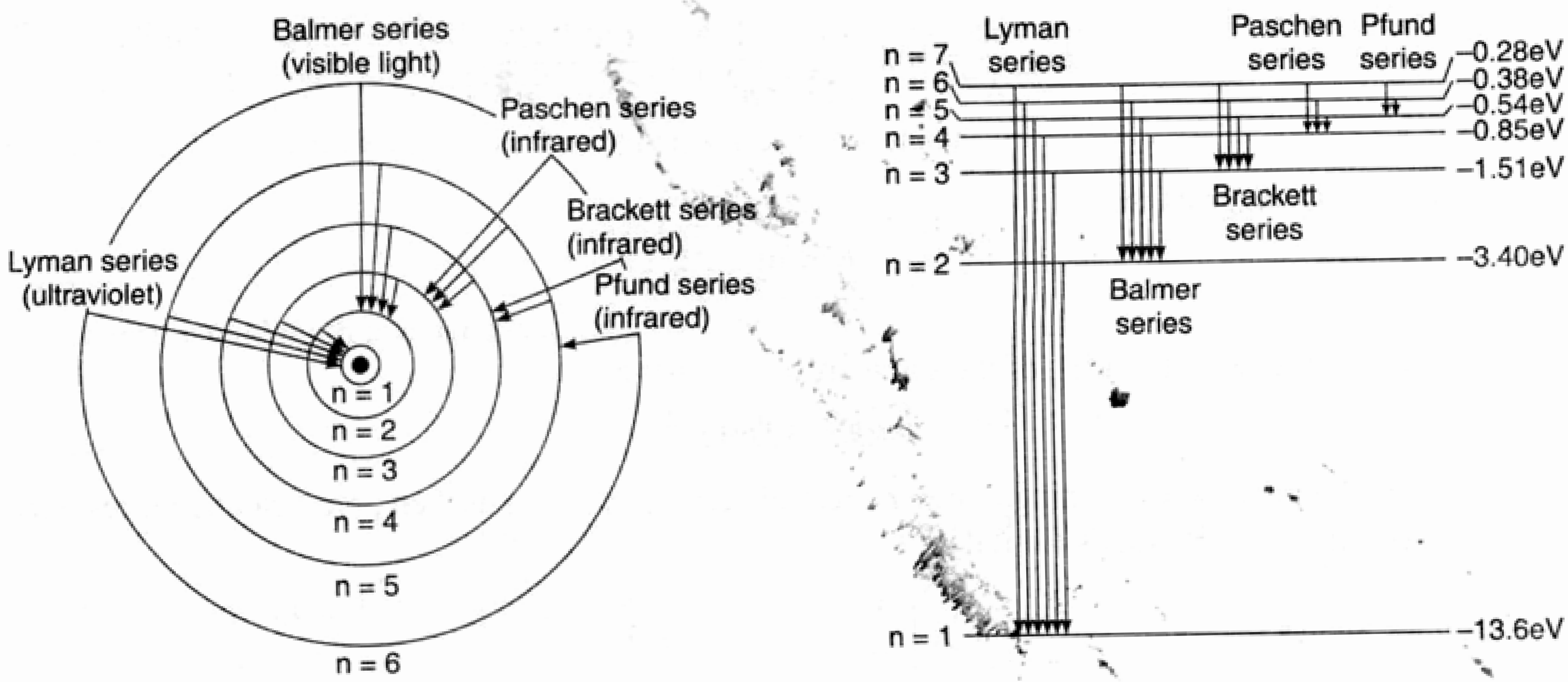


Fig. 30.5

For the Lyman series $n_f = 1$, for Balmer series $n_f = 2$ and so on. The relation of the various spectral series to the energy levels and to electron orbits is shown in figure.

Wavelength of Photon Emitted in De-excitation

According to Bohr when an atom makes a transition from one energy level to a lower level it emits a photon with energy equal to the energy difference between the initial and final levels. If E_i is the initial energy of the atom before such a transition, E_f is its final energy after the transition, and the photon's energy is $hf = \frac{hc}{\lambda}$, then conservation of energy gives,

$$hf = \frac{hc}{\lambda} = E_i - E_f \qquad \text{(energy of emitted photon)} \quad \ldots\text{(xii)}$$

By 1913, the spectrum of hydrogen had been studied intensively. The visible line with longest wavelength, or lowest frequency is in the red and is called H_α, the next line, in the blue-green is called H_β and so on.

In 1885, Johann Balmer, a swiss teacher found a formula that gives the wave lengths of these lines. This is now called the Balmer series. The Balmer's formula is,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad \ldots\text{(xiii)}$$

Here, $n = 3, 4, 5 \ldots$, etc.

R = Rydberg constant $= 1.097 \times 10^7 \text{ m}^{-1}$

and λ is the wavelength of light/photon emitted during transition.

For $n = 3$, we obtain the wavelength of H_α line. Similarly, for $n = 4$, we obtain the wavelength of H_β line. For $n = \infty$, the smallest wavelength ($= 3646$ Å) of this series is obtained. Using the relation, $E = \frac{hc}{\lambda}$ we can find the photon energies corresponding to the wavelength of the Balmar series. Multiplying Eq. (xiii) by hc, we find

$$E = \frac{hc}{\lambda} = hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \frac{Rhc}{2^2} - \frac{Rhc}{n^2}$$

$$= E_n - E_2$$

This formula suggests that,

$$E_n = -\frac{Rhc}{n^2}, n = 1, 2, 3 \ldots \qquad \ldots(\text{xiv})$$

Comparing this with Eq. (xi), of the same article, we have

$$Rhc = 13.60 \text{ eV} \qquad \ldots(\text{xv})$$

The wavelengths corresponding to other spectral series (Lyman, Paschen , etc.) can be represented by formulas similar to Balmer's formula.

Lyman Series : $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), n = 2, 3, 4 \ldots$

Paschen Series : $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), n = 4, 5, 6 \ldots$

Brackett Series : $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right), n = 5, 6, 7 \ldots$

Pfund Series : $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right), n = 6, 7, 8 \ldots$

The Lyman series is in the ultraviolet, and the Paschen, Brackett and Pfund series are in the infrared region.

Sample Example 30.3 *Calculate (a)the wavelength and (b) the frequency of the H_β line of the Balmer series for hydrogen.*

Solution (a) H_β line of Balmer series corresponds to the transition from $n = 4$ to $n = 2$ level. Using Eq. (xiii), the corresponding wavelength for H_β line is,

$$\frac{1}{\lambda} = (1.097 \times 10^7)\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$$= 0.2056 \times 10^7$$

$\therefore$ $$\lambda = 4.9 \times 10^{-7} \text{ m}$$ **Ans.**

(b) $$f = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{4.9 \times 10^{-7}}$$

$$= 6.12 \times 10^{14} \text{ Hz}$$ **Ans.**

Sample Example 30.4 *Find the largest and shortest wavelengths in the Lyman series for hydrogen. In what region of the electromagnetic spectrum does each series lie?*

Solution The transition equation for Lyman series is given by,

$$\frac{1}{\lambda}=R\left(\frac{1}{1^2}-\frac{1}{n^2}\right),\quad n=2,3,\ldots$$

The largest wavelength is corresponding to $n=2$.

$$\therefore \qquad \frac{1}{\lambda_{\max}}=1.097\times10^7\left(\frac{1}{1}-\frac{1}{4}\right)=0.823\times10^7$$

$$\therefore \qquad \lambda_{\max}=1.2154\times10^{-7}\text{ m}=1215\text{ Å}$$ **Ans.**

The shortest wavelength corresponds to $n=\infty$

$$\therefore \qquad \frac{1}{\lambda_{\min}}=1.097\times10^7\left(\frac{1}{1}-\frac{1}{\infty}\right)$$

or $$\lambda_{\min}=0.911\times10^{-7}\text{ m}=911\text{ Å}$$ **Ans.**

Both of these wavelengths lie in ultraviolet (UV) region of electromagnetic spectrum.

30.7 Hydrogen Like Atoms

The Bohr model of hydrogen can be extended to hydrogen like atoms, *i.e.*, one electron atoms, such as singly ionized helium (He^+), doubly ionized lithium (Li^{+2}) and so on. In such atoms, the nuclear charge is $+ze$, where z is the atomic number, equal to the number of protons in the nucleus.

The effect in the previous analysis is to replace e^2 everywhere by ze^2. Thus, the equations for, r_n, v_n and E_n are altered as under:

$$r_n=\frac{\varepsilon_0 n^2h^2}{\pi mze^2}=\frac{n^2}{z}\cdot a_0 \quad\text{or}\quad r_n\propto\frac{n^2}{z} \qquad \ldots\text{(i)}$$

where $a_0=0.529$ Å (radius of first orbit of H)

$$v_n=\frac{ze^2}{2\varepsilon_0 nh}=\frac{z}{n}v_1 \quad\text{or}\quad v_n\propto\frac{z}{n} \qquad \ldots\text{(ii)}$$

where $v_1=2.19\times10^6$ m/s (speed of electron in first orbit of H)

$$E_n=-\frac{mz^2e^4}{8\varepsilon_0^2n^2h^2}=\frac{z^2}{n^2}E_1 \quad\text{or}\quad E_n\propto\frac{z^2}{n^2} \qquad \ldots\text{(iii)}$$

where $E_1=-13.60$ eV (energy of atom in first orbit of H)

Fig. 30.6 compares the energy levels of H and He^+ which has $z=2$. H and He^+ have many spectrum lines that have almost the same wavelengths.

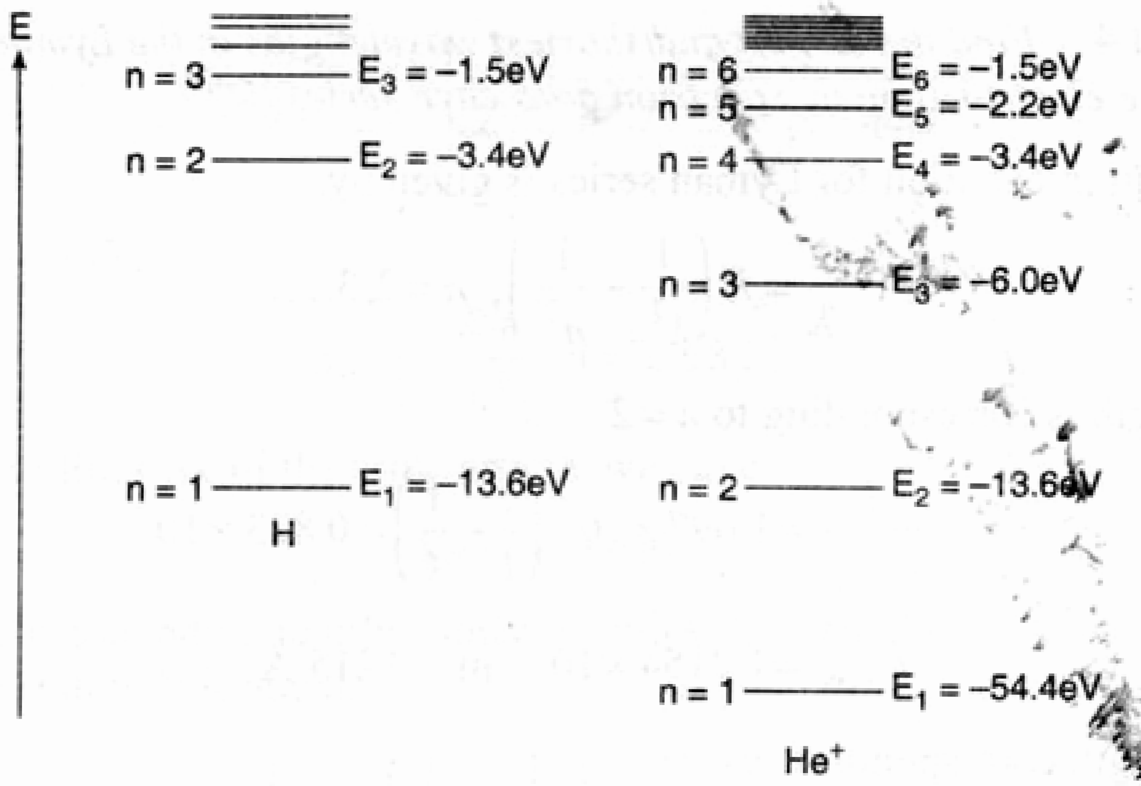


Fig. 30.6 Energy levels of H and He^+. Because of the additional factor Z^2 in the energy expression, the energy of the He^+ ion with a given n is almost exactly four times that of the H atom with the same n. There are small differences (of the order of 0.05%) because the reduced masses are slightly different.

Sample Example 30.5 *Using the known values for hydrogen atom, calculate*
(a) radius of third orbit for Li^{+2}
(b) speed of electron in fourth orbit for He^+.

Solution (a) $z = 3$ for Li^{+2}. Further we know that $r_n = \frac{n^2}{z} a_0$

substituting, $n = 3, z = 3$ and $a_0 = 0.529$ Å

we have r_3 for $Li^{+2} = \frac{(3)^2}{(3)} (0.529)$ Å $= 1.587$ Å **Ans.**

(b) $z = 2$ for He^+. Also we know that

$$v_n = \frac{z}{n} v_1$$

Substituting $n = 4$, $z = 2$ and $v_1 = 2.19 \times 10^6$ m/s

we get, v_4 for $He^+ = \left(\frac{2}{4}\right)(2.19 \times 10^6)$m/s

$$= 1.095 \times 10^6 \text{ m/s}$$ **Ans.**

Extra Points

- Total number of emission lines from some excited state n_1 to another energy state n_2 ($< n_1$) is given by $\frac{(n_1 - n_2)(n_1 - n_2 + 1)}{2}$.

  For example total number of lines from $n_1 = n$ to $n_2 = 1$ are $\frac{n(n-1)}{2}$.

- As the principal quantum number n is increased in hydrogen and hydrogen like atoms, some quantities are decreased and some are increased. The table given below shows which quantities are increased and which are decreased.

Table 30.1

Increased	Decreased
Radius	Speed
Potential energy	Kinetic energy
Total energy	Angular speed
Time period	
Angular momentum	

- Whenever the force obeys inverse square law $\left(F \propto \frac{1}{r^2}\right)$ and potential energy is inversely proportional to r $\left(U \propto \frac{1}{r}\right)$, kinetic energy ($K$), potential energy ($U$) and total energy ($E$) have the following relationships.

$$K = \frac{|U|}{2} \quad \text{and} \quad E = -K = \frac{U}{2}$$

  If force is not proportional to $\frac{1}{r^2}$ or potential energy is not proportional to $\frac{1}{r}$, the above relations do not hold good. In JEE problems, this situation arises at two places, in an atom (between nucleus and electron) and in solar system (between sun and planet).

  See sample example number 30.7.

- Total energy of a closed system is always negative and the modulus of this is the binding energy of the system. For instance, suppose a system has a total energy of –100 J. It means that this system will separate if 100 J of energy is supplied to this. Hence, binding energy of this system is 100 J. Thus, total energy of an open system is either zero or greater than zero.

- Kinetic energy of a particle can't be negative, while the potential energy can be zero, positive or negative. It basically depends on the reference point where we have taken it zero. It is customary to take zero potential energy when the electron is at infinite distance from the nucleus. In some problem suppose we take zero potential energy in first orbit ($U_1 = 0$), then the modulus of actual potential energy in first orbit (when reference point was at infinity) is added in U and E in all energy states, while K remains unchanged. See sample example number 30.6.

Sample Example 30.6 *Find the kinetic energy, potential energy and total energy in first and second orbit of hydrogen atom if potential energy in first orbit is taken to be zero.*

Solution $E_1 = -13.60\,\text{eV}$ $\quad K_1 = -E_1 = 13.60\,\text{eV}$, $\quad U_1 = 2E_1 = -27.20\,\text{eV}$

$E_2 = \dfrac{E_1}{(2)^2} = -3.40\,\text{eV}$ $\quad K_2 = 3.40\,\text{eV}$ $\quad$ and $\quad U_2 = -6.80\,\text{eV}$

Now, $U_1 = 0$, *i.e.*, potential energy has been increased by 27.20 eV. So, we will increase U and E in all energy states by 27.20 eV while kinetic energy will remain unchanged. Changed values in tabular form are as under.

Table 30.2

Orbit	K (eV)	U (eV)	E (eV)
First	13.60	0	13.60
Second	3.40	20.40	23.80

Sample Example 30.7 *A small particle of mass m moves in such a way that the potential energy $U = ar^2$ where a is constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, find the radius of n^{th} allowed orbit.*

Solution The force at a distance r is,

$$F = -\frac{dU}{dr} = -2ar$$

Suppose r be the radius of n^{th} orbit. Then the necessary centripetal force is provided by the above force. Thus,

$$\frac{mv^2}{r} = 2ar \qquad \text{...(i)}$$

Further, the quantization of angular momentum gives,

$$mvr = \frac{nh}{2\pi} \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii) for r, we get

$$r = \left(\frac{n^2h^2}{8\,am\,\pi^2}\right)^{1/4} \qquad \textbf{Ans.}$$

30.8 X-Rays

Electromagnetic radiation with wavelengths from 0.1 Å to 100 Å falls into the category of X-rays. The boundaries of this category are not sharp. The shorter wavelength end overlaps gamma rays and the longer wavelength end overlaps ultraviolet rays. Photoelectric effect (will be discussed later) provides convincing evidence that photons of light can transfer energy to electrons. Is the inverse process also possible? That is, can part or all of the kinetic energy of a moving electron be converted into a photon? Yes, it is possible. In 1895 **Wilhelm Roentgen** found that a highly penetrating radiation of unknown nature is produced when fast moving electrons strike a target of high atomic number and high melting point. These radiations were given a name X-rays as their nature was unknown (in mathematics an unknown quantity is normally designated by X). Later it was discovered that these are high energy photons (or electromagnetic waves)

Production of X-Rays : Figure shows a diagram of a X-ray tube, called the coolidge tube. A cathode (a plate connected to negative terminal of a battery), heated by a filament through which an electric current is passed, supplies electrons by thermionic emission. The high potential difference V maintained between the cathode and a metallic target accelerate the electrons toward the latter. The face of the target is at an angle relative to the electron beam, and the X-rays that leave the target pass through the side of the tube. The tube is evacuated to permit the electrons to get to the target unimpeded.

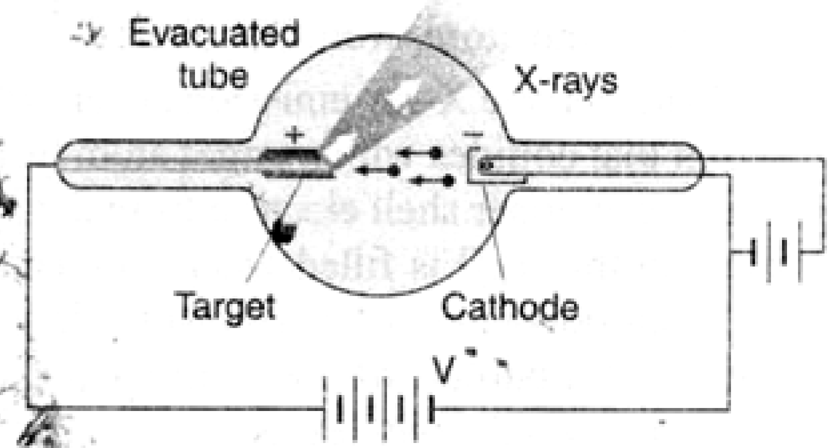


Fig. 30.7 An X-ray tube. The higher the accelerating voltage V, the faster the electrons and the shorter the wavelengths of the X-rays.

Continuous and characteristic X-rays : X-rays so produced by the coolidge tube are of two types, continuous and characteristic. While the former depends only on the accelerating voltage V the later depends on the target used.

Continuous X-rays : Electromagnetic theory predicts that an accelerated electric charge will radiate electromagnetic waves, and a rapidly moving electrons when suddenly brought to rest is certainly accelerated (of course negative). X-rays produced under these circumstances is given the German name **bremsstrahlung** (braking radiation). Energy loss due to bremsstrahlung is more important for electrons than for heavier particles because electrons are more violently accelerated when passing near nuclei in their paths. The continuous X-rays (or bremsstrahlung X-rays) produced at a given accelerating potential V vary in wavelength, but none has a wavelength shorter than a certain value λ_{min}. This minimum wavelength corresponds to the maximum energy of the X-rays which in turn is equal to the maximum kinetic energy qV or eV of the striking electrons. Thus,

$$\frac{hc}{\lambda_{min}} = eV \qquad \text{or} \qquad \lambda_{min} = \frac{hc}{eV}$$

After substituting values of h, c and e we obtain the following simple formula for λ_{min}.

$$\lambda_{min} \text{ (in Å)} = \frac{12375}{V \text{(in volts)}} \qquad \ldots\text{(i)}$$

Increasing V decreases λ_{min}. This wavelength is also known as the cutoff wavelength or the threshold wavelength.

Characteristic X-rays : The X-ray spectrum typically consists of a broad continuous band containing a series of sharp lines, as shown in Fig. 30.8.

As discussed above the continuous spectrum is the result of collisions between incoming electrons and atoms in the target. The kinetic energy lost by the electrons during the collisions emerges as the energy of the X-ray photons radiated from the target.

The sharp lines superimposed on the continuous spectrum are known as **characteristic X-rays** because they are characteristic of the target material. They were discovered in 1908, but their origin remained unexplained until the details of atomic structure, particularly the shell structure of the atom, were discovered.

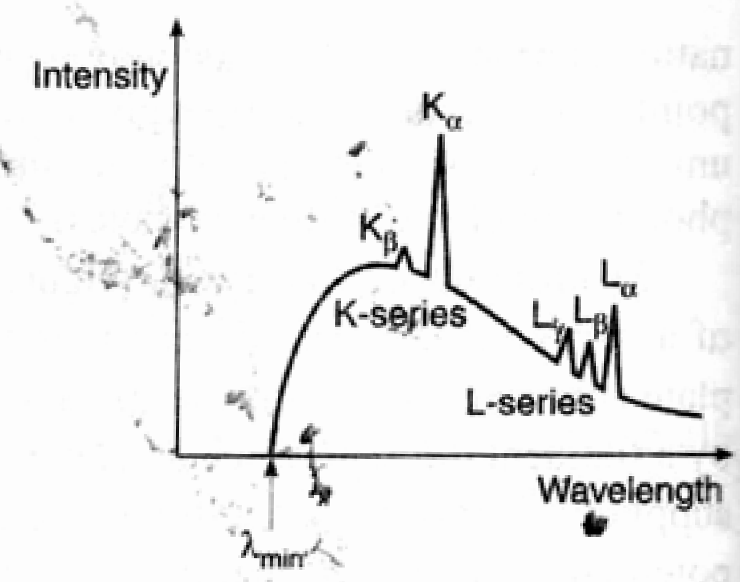


Fig. 30.8 X-ray spectrum

Characteristic X-ray emission occurs when a bombarding electron that collides with a target atom has sufficient energy to remove an inner shell electron from the atom. The vacancy created in the shell is filled when an electron from a higher level drops down into it. This transition is accompanied by the emission of a photon whose energy equals the difference in energy between the two levels.

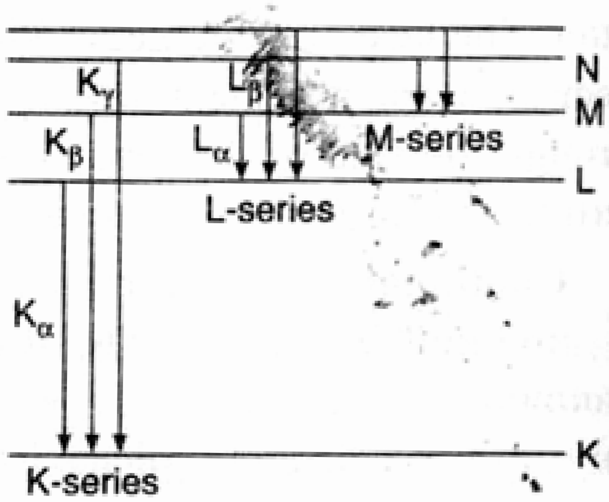


Fig. 30.9

Let us assume that the incoming electron his dislodged an atomic electron from the innermost shell-the K shell. If the vacancy is filled by an electron droping from the next higher shell-the L shell-the photon emitted has an energy corresponding to the K_α characteristic X-ray line. If the vacancy is filled by an electron dropping from the M shell, the K_β line is produced. An L_α line is produced as an electron drops from the M shell to the L-shell, and an L_β line is produced by a transition from the N-shell to the L-shell.

Moseley's Law for Characteristic Spectrum

Although multielectron atoms cannot be analyzed with the Bohr model, Henery G.J. Moseley in 1914 made an effort towards this. Moseley measured the frequencies of characteristic X-rays from a large number of elements and plotted the square root of the frequency $\sqrt{f}$ against the atomic number z of the element. He discovered that the plot is very close to a straight line. He plotted the square root of the frequency of the K_α line versus the atomic number z.

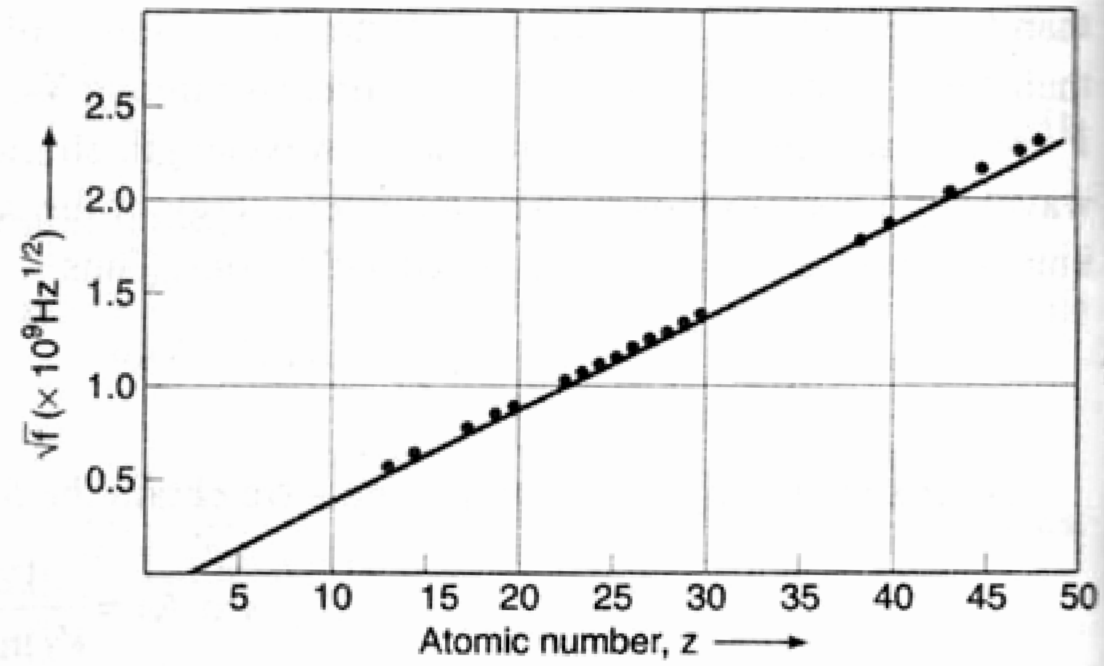


Fig. 30.10 A plot of the square root of the frequency of the K_α lines versus atomic number using Moseley's data.

As figure shows, Moseley's plot did not pass through the origin. Let us see why. It can be understood from Gauss's law. Consider an atom of atomic number Z in which one of the two electrons in the K-shell has been ejected. Imagine that we draw a Gaussian sphere just inside the most probable radius of the L-electrons. The effective charge inside the Gaussian surface is the positive nuclear charge and one negative charge due to the single K-electron. If we ignore the interactions between L-electrons, a single L electron behaves as if it experiences an electric field due to a charge $(Z-1)$ e enclosed by the Gaussian surface.

Thus, Moseley's law of the frequency of K_α line is,

$$\sqrt{f_{K_\alpha}} = a\,(Z-1) \qquad \ldots\text{(ii)}$$

where a is a constant that can be related to Bohr's theory.
The above law in general can be stated as under,

$$\sqrt{f} = a\,(Z-b) \qquad \ldots\text{(iii)}$$

For K_α line, $$\Delta E = hf = Rhc\,(Z-1)^2\left(\frac{1}{1^1}-\frac{1}{2^2}\right)$$

or $$\sqrt{f} = \sqrt{\frac{3Rc}{4}}\,(Z-1) \quad \text{or} \quad a = \sqrt{\frac{3Rc}{4}} \quad \text{and} \quad b=1$$

After substituting values of R and c, we get

$$a = 4.98\times 10^7\ (\text{Hz})^{1/2}$$

Eq. (iii) can also be written as,

$$f = a^2\,(Z-b)^2 \qquad \ldots\text{(iv)}$$

For K_α line, $$a^2 = \frac{3Rc}{4} = (2.48\times 10^{15}\ \text{Hz}) \quad \text{and} \quad b=1$$

Hence, $$f_{K_\alpha} = (2.48\times 10^{15}\ \text{Hz})\,(Z-1)^2$$

Note *We have studied above Moseley's law only for K_α line for which $b = 1$ and $a = 4.98\times 10^7\ (\text{Hz})^{1/2}$. For other spectral lines (K_β, K_γ, L_α etc.) the theory requires a good enough discussion and I personally feel it is not required at all as far as JEE problems are concerned. Even though for interested readers a brief idea has been given below.*

Screening Effect (exclusively for interested readers) : The energy levels, in general, depend on principal quantum number (n) and orbital quantum number (l). *Let us take sodium ($Z=11$) as an example. According to Gauss's law, for any spherically symmetric charge distribution the electric field magnitude at a distance r* from the centre is $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_{encl}}{r^2}$, where q_{encl} is the total charge enclosed within a sphere with radius r. Mentally remove the outer (valence) electron from a sodium atom. What you have left is a spherically symmetric collection of 10 electrons (filling the K and L shells) and 11 protons. So,

$$q_{encl} = -10e + 11e = +e$$

If the eleventh is completely outside this collection of charges, it is attracted by an effective charge of $+e$, not $+11e$.

This effect is called **screening**, the 10 electrons screen 10 of the 11 protons leaving an effective net charge of $+e$. In general, an electron that spends all its time completely outside a positive charge $z_{\text{eff}}\,e$ has energy levels given by the hydrogen expression with e^2 replaced by $z_{\text{eff}}\,e^2$. *i.e.*,

$$E_n = -\frac{z_{\text{eff}}^2}{n^2}(13.6\ \text{eV}) \quad \text{(energy levels with screening)}$$

If the eleventh electron in the sodium atom is completely outside the remaining charge distribution, then $z_{\text{eff}} = 1$.

We can estimate the frequency of K_α X-ray photons using the concept of screening. A K_α X-ray photon is emitted when an electron in the L shell ($n=2$) drops down to fill a hole in the K-shell ($n=1$). As the electron drops down, it is attracted by the z protons in the nucleus screened by one remaining electron in the K-shell. Thus,

$$z_{eff} = (Z-1) \qquad n_i = 2 \quad \text{and} \quad n_f = 1$$

The energy before transition is,

$$E_i = -\frac{(Z-1)^2}{2^2}(13.6 \text{ eV}) = -(Z-1)^2 (3.4 \text{ eV})$$

and energy after transition is,

$$E_f = -\frac{(Z-1)^2}{1^2}(13.6 \text{ eV}) = -(Z-1)^2 (13.6 \text{ eV})$$

The energy of the K_α X-ray photon is

$$E_{K_\alpha} = E_i - E_f = (Z-1)^2 (10.2 \text{ eV})$$

The frequency of K_α X-ray photon is therefore,

$$f_{K_\alpha} = \frac{E_{K_\alpha}}{h} = \frac{(Z-1)^2 (10.2 \text{ eV})}{(4.136 \times 10^{-15} \text{ eV-s})} = (2.47 \times 10^{15} \text{ Hz})(Z-1)^2$$

This relation agrees almost exactly with Moseley's experimental law.

Sample Example 30.8 *Find the cutoff wavelength for the continuous X-rays coming from an X-ray tube operating at 40 kV.*

Solution Cutoff wavelength λ_{min} is given by,

$$\lambda_{min} (\text{in Å}) = \frac{12375}{V(\text{in volts})} = \frac{12375}{40 \times 10^3}$$

$$= 0.31 \text{ Å}$$ **Ans.**

Sample Example 30.9 *Use Moseley's law with $b = 1$ to find the frequency of the K_α X-rays of La ($Z = 57$) if the frequency of the K_α X-rays of Cu ($Z = 29$) is known to be 1.88×10^{18} Hz.*

Solution Using the equation, $\sqrt{f} = a(Z-b)$ $(b = 1)$

$$\frac{f_{La}}{f_{Cu}} = \left(\frac{Z_{La} - 1}{Z_{Cu} - 1}\right)^2$$

or

$$f_{La} = f_{Cu}\left(\frac{Z_{La} - 1}{Z_{Cu} - 1}\right)^2$$

$$= 1.88 \times 10^{18}\left(\frac{57-1}{29-1}\right)^2$$

$$= 7.52 \times 10^{18} \text{ Hz}$$ **Ans.**

• Important Points

1. The target (or anode) used in the Coolidge tube should be of high melting point. This is because less than 0.5% of the kinetic energy of the electrons is converted into X-rays. The rest of the kinetic energy becomes internal energy of the target which simultaneously has to be kept cool by circulating oil or water.
2. Atomic number of the target should be high. This is because X-rays are high energy photons and as we have seen above energy of the X-rays increases as z increases.
3. X-rays are basically electromagnetic waves. So they possess all the properties of electromagnetic waves.

Sample Example 30.10 *Determine the energy of the characteristic X-ray (K_β) emitted from a tungsten ($z = 74$) target when an electron drops from the M shell ($n = 3$) to a vacancy in the K-shell ($n = 1$).*

Solution Energy associated with the electron in the K-shell is approximately

$$E_K = -(74-1)^2 (13.6\,\text{eV}) = -72474\,\text{eV}$$

An electron in the M-shell is subject to an effective nuclear charge that depends on the number of electrons in the $n = 1$ and $n = 2$ states because these electrons shield the M electrons from the nucleus. Because there are eight electrons in the $n = 2$ state and one remaining in the $n = 1$ state, roughly nine electrons shield M electrons from the nucleus,

so
$$z_{\text{eff}} = z - 9$$

Hence, the energy associated with an electron in the M shell is,

$$E_M = \frac{-13.6\, z_{\text{eff}}^2}{3^2}\,\text{eV} = \frac{-13.6\,(z-9)^2}{3^2}\,\text{eV}$$

$$= -\frac{(13.6)\,(74-9)^2}{9}\,\text{eV} = -6384\,\text{eV}$$

Therefore, emitted X-ray has an energy equal to

$$E_M - E_K = \{-6384 - (-72474)\}\,\text{eV} = 66090\,\text{eV}$$ **Ans.**

Introductory Exercise 30.1

1. The wavelength for $n = 3$ to $n = 2$ transition of the hydrogen atom is 656.3 nm. What are the wavelengths for this same transition in (a) positronium, which consists of an electron and a positron (b) singly ionized helium (Note: A positron is a positively charged electron).
2. Find the longest wavelength present in the Balmer series of hydrogen.
3. (a) Find the frequencies of revolution of electrons in $n = 1$ and $n = 2$ Bohr orbits.
   (b) What is the frequency of the photon emitted when an electron in an $n = 2$ orbit drops to an $n = 1$ orbit?
   (c) An electron typically spends about 10^{-8} s in an excited state before it drops to a lower state by emitting a photon. How many revolutions does an electron in an $n = 2$ Bohr orbit make in 1.00×10^{-8} s?
4. A **muon** is an unstable elementary particle whose mass is $207\,m_e$ and whose charge is either ^+e or ^-e. A negative muon (μ^-) can be captured by a nucleus to form a muonic atom.
   (a) A proton captures a μ^-. Find the radius of the first Bohr orbit of this atom.
   (b) Find the ionization energy of the atom.

Note *Attempt this question after reading the whole chapter.*

5. Find the de-Broglie wavelengths of
   (a) a 46 g golf ball with a velocity of 30 m/s. (b) an electron with a velocity of 10^7 m/s.
6. (a) A gas of hydrogen atoms in their ground state is bombarded by electrons with kinetic energy 12.5 eV. What emitted wavelengths would you expect to see?
   (b) What if the electrons were replaced by photons of same energy?
7. For a given element the wavelength of the K_α line is 0.71 nm and of the K_β line it is 0.63 nm. Use this information to find wavelength of the L_α line.
8. The energy of the $n = 2$ state in a given element is $E_2 = -2870$ eV. Given that the wavelengths of the K_α and K_β lines are 0.71 nm and 0.63 nm, respectively determine the energies E_1 and E_3.

9. The energy levels of a certain atom are shown in figure. If a photon of frequency f is emitted when there is an electron transition from $5E$ to E, what frequencies of photons could be produced by other energy level transitions?

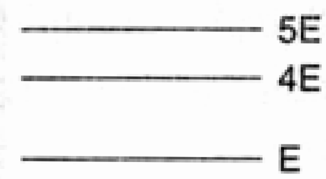


Fig. 30.11

10. An excited atom of mass m and initial speed v emits a photon in its direction of motion. If $v << c$, use the requirement that linear momentum and energy must both be conserved to show that the frequency of the photon is higher by $\frac{\Delta f}{f} \approx \frac{v}{c}$ than it would have been if the atom had been at rest.

30.9 Emission of Electrons

At room temperature the free electrons move randomly within the conductor, but they don't leave the surface of the conductor due to attraction of positive charges. Some external energy is required to emit electrons from a metal surface. Minimum energy is required to emit the electrons which are just on the surface of the conductor. This minimum energy is called the **work function** (denoted by W) of the conductor. Work function is the property of the metallic surface.

The energy required to liberate an electron from metal surface may arise from various sources such as heat, light, electric field etc. Depending on the nature of source of energy, the following methods are possible.

(i) **Thermionic emission :** The energy to the free electrons can be given by heating the metal. The electrons so emitted are known as **thermions**.

(ii) **Field emission :** When a conductor is put under strong electric field the free electrons on it experience an electric force in the opposite direction of field. Beyond a certain limit electrons start coming out of the metal surface. Emission of electrons from a metal surface by this method is called the field emission.

(iii) **Secondary emission :** Emission of electrons from a metal surface by the bombardment of high speed electrons or other particles is known as secondary emission.

(iv) **Photoelectric emission :** Emission of free electrons from a metal surface by falling light (or any other electromagnetic wave which has an energy greater than the work function of the metal) is called photoelectric emission. The electrons so emitted are called **photoelectrons**.

30.10 Photoelectric Effect

When light of an appropriate frequency (or correspondingly of an appropriate wavelength) is incident on a metallic surface, electrons are liberated from the surface. This observation is known as **photoelectric effect**. Photoelectric effect was first observed in 1887 by Hertz. For photoemission to take place, energy of incident light photons should be greater than or equal to the work function of the metal.

or $$E \geq W \qquad \text{...(i)}$$

∴ $$hf \geq W$$

or $$f \geq \frac{W}{h}$$

Here, $\frac{W}{h}$ is the minimum frequency required for the emission of electrons. This is known as threshold frequency f_0.

Thus, $$f_0 = \frac{W}{h} \quad \text{(threshold frequency)} \qquad \text{...(ii)}$$

Further Eq. (i) can be written as, $$\frac{hc}{\lambda} \geq W$$

or $$\lambda \leq \frac{hc}{W}$$

Here, $\frac{hc}{W}$ is the largest wavelength beyond which photoemission does not take place. This is called the threshold wavelength λ_0.

Thus, $$\lambda_0 = \frac{hc}{W} \quad \text{(threshold wavelength)} \qquad \text{...(iii)}$$

Hence, for the photoemission to take place either of the following conditions must be satisfied.

$$E \geq W \quad \text{or} \quad f \geq f_0 \quad \text{or} \quad \lambda \leq \lambda_0 \qquad \text{...(iv)}$$

Stopping Potential and Maximum Kinetic Energy of Photoelectrons

When the frequency f of the incident light is greater than the threshold frequency, some electrons are emitted from the metal with substantial initial speeds. Suppose E is the energy of light incident on a metal surface and W $(< E)$ the work function of metal. As minimum energy is required to extract electrons from the surface, they will have the maximum kinetic energy which is $E - W$.

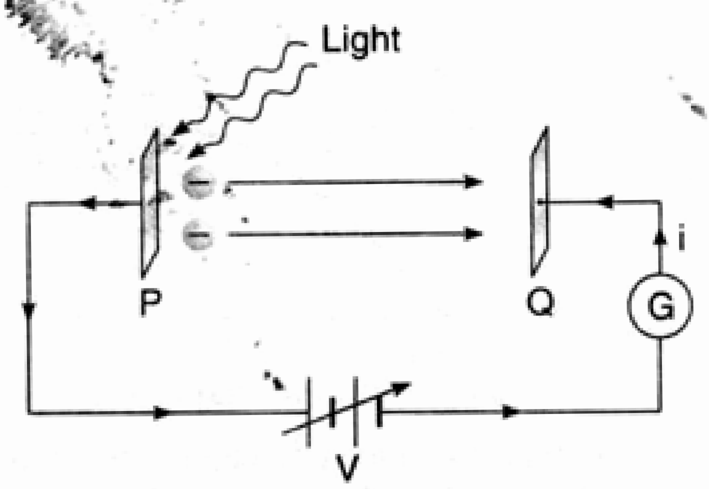


Fig. 30.12

Thus, $$K_{max} = E - W \qquad \text{...(v)}$$

This value K_{max} can experimentally be found by keeping the metal plate P (from which electrons are emitting) at higher potential relative to an another plate Q placed infront of P. Some electrons after emitting from plate P, reach the plate Q despite the fact that Q is at lower potential and it is repelling the electrons from reaching in itself. This is because the electrons emitted from plate P possess some kinetic energy and due to this energy they reach the plate Q and current i flows in the circuit in the direction shown in figure.

As the potential V is increased, the force of repulsion to the electrons gets increased and less number of electrons reach the plate Q and current in the circuit gets decreased. At a certain value V_0 electrons having maximum kinetic energy (K_{max}) also get stopped and current in the circuit becomes zero. This is called the **stopping potential**.

As an electron moves from P to Q, the potential decreases by V_0 and negative work $-eV_0$ is done on the (negatively charged) electron, the most energetic electron leaves plate P with kinetic energy $K_{max} = \frac{1}{2} mv_{max}^2$ and has zero kinetic energy at Q. Using the work energy theorem, we have

$$W_{ext} = -eV_0 = \Delta K = 0 - K_{max}$$

or $$K_{max} = \frac{1}{2} mv_{max}^2 = eV_0 \qquad \text{...(vi)}$$

Photoelectric Current

Figure shows an apparatus used to study the variation of photo current i with the intensity and frequency of light falling on metal plate P. Photoelectrons are emitted from plate P which are being attracted by the positive plate Q and a photoelectric current i flows in the circuit, which can be measured by the galvanometer G.

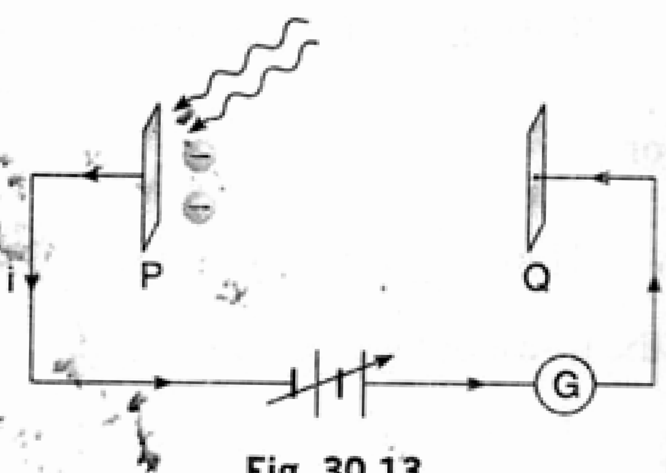
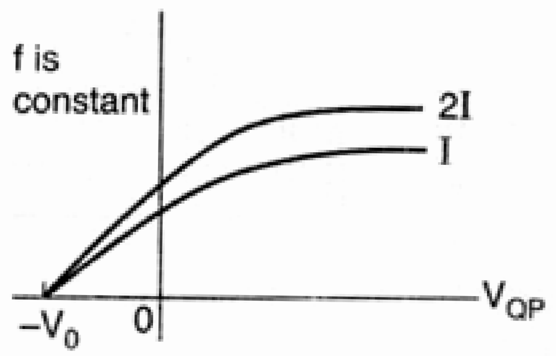


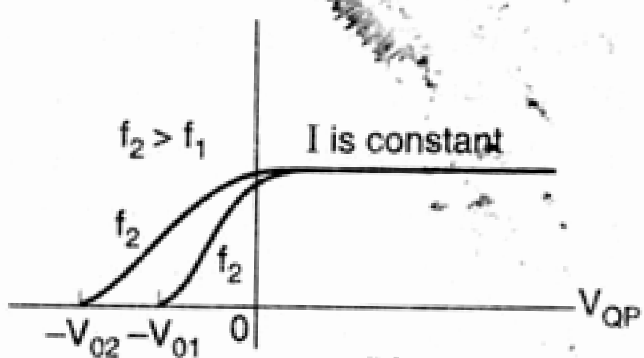
Fig. 30.13

Figure (a) shows graphs of photocurrent as a function of potential difference V_{QP} for light of constant frequency and two different intensities. When V_{QP} is sufficiently large and positive the current becomes constant, showing that all the emitted electrons are being collected by the anode plate Q. The stopping potential difference $-V_0$ needed to reduce the current to zero is shown. If the intensity of light is increased, (or we can say the number of photons incident per unit area per unit time is increased) while its



(a)

Photocurrent i as a function of the potential V_{QP} of the anode with respect to the cathode for a constant light frequency f, the stopping potential V_0 is independent of the light intensity I.



(b)

Photocurrent i as a function of the potential V_{QP} of an anode with respect to a cathode for two different light frequencies f_1 and f_2 with the same intensity. The stopping potential V_0 (and therefore the maximum kinetic energy of the photoelectrons) increases linearly with frequency.

Fig. 30.14

frequency is kept the same, the current becomes constant at a higher value, showing that more electrons are being emitted per unit time. But the stopping potential is found to be the same.

Figure (b) shows current as a function of potential difference for two different frequencies, with the same intensity in each case. We see that when the frequency of the incident monochromatic light is increased, the stopping potential V_0 gets increased, Of course, V_0 turns out to be a linear function of the frequency f.

Note *There major features of the photoelectric effect could not be explained by the wave theory of light which were later explained by Einstein's photon theory.*

(i) *Wave theory suggests that the kinetic energy of the photoelectrons should increase with the increase in intensity of light. However, Eq. (iv), $K_{max} = eV_0$ suggests that it is independent of the intensity of light.*

(ii) *According to wave theory, the photoelectric effect should occur for any frequency of the light, provided that the light is intense enough. However Eq. (iv) suggests that photo emission is possible only when frequency of incident light is either greater than or equal to the threshold frequency f_0.*

(iii) *If the energy to the photo electrons is obtained by soaking up from the incident wave, it is not likely that the effective target area for an electron in the metal is much more than a few atomic diameters. (see example 29.11) between the impinging of the light on the surface and the ejection of the photo electrons. During this interval the electron should be "soaking up" energy from the beam until it had accumulated enough energy to escape. However no detectable time lag has ever been measured.*

Sample Example 30.11 *A metal plate is placed 5 m from a monochromatic light source whose power output is 10^{-3} W. Consider that a given ejected photoelectron may collect its energy from a circular area of the plate as large as ten atomic diameters (10^{-9} m) in radius. The energy required to remove an electron through the metal surface is about 5.0 eV. Assuming light to be a wave, how long would it take for such a 'target' to soak up this much energy from such a light source.*

Solution The target area is $S_1 = \pi\,(10^{-9})^2 = \pi \times 10^{-18}\ \text{m}^2$. The area of a 5 m sphere centered on the light source is, $S_2 = 4\pi\,(5)^2 = 100\,\pi\ \text{m}^2$. Thus, if the light source radiates uniformly in all directions the rate P at which energy falls on the target is given by,

$$P = (10^{-3}\ \text{watt})\left(\frac{S_1}{S_2}\right) = (10^{-3})\left(\frac{\pi \times 10^{-18}}{100 \times \pi}\right) = 10^{-23}\ \text{J/s}$$

Assuming that all power is absorbed, the required time is,

$$t = \left(\frac{5\ \text{eV}}{10^{-23}\ \text{J/s}}\right)\left(\frac{1.6 \times 10^{-19}\ \text{J}}{1\text{eV}}\right) \approx 20\ \text{h}$$ **Ans.**

Einstein's photon theory

Einstein succeeded in explaining the photoelectric effect by making a remarkable assumption, that the energy in a light beam travels through space in concentrated bundles, called photons. The energy E of a single photon is given by

$$E = hf$$

Applying the photon concept to the photoelectric effect, Einstein wrote

$$hf = W + K_{max} \qquad \text{(already discussed)}$$

Consider how Einstein's photon hypothesis meets the three objections raised against the wave theory interpretation of the photoelectric effect.

As for objection 1 (the lack of dependence of K_{max} on the intensity of illumination), doubling the light intensity merely doubles the number of photons and thus doubles the photoelectric current, it does not change the energy of the individual photons

Objection 2 (the existence of a cutoff frequency) follows from equation $hf = W + K_{max}$. If K_{max} equals zero, We have $hf_0 = W$ which asserts that the photon has just enough energy to eject the photoelectrons and none extra to appear as kinetic energy. The quantity W is called the work function of the substance. If f is reduced below f_0, the individual photons, no matter how many of them there are (that is, no matter how intense the illumination), will not have enough energy to eject photo electrons.

Objection 3 (the absence of a time lag) follows from the photon theory because the required energy is supplied in a concentrated bundle. It is not spread uniformly over a large area, as in the wave theory. Although the photon hypothesis certainly fits the facts of photoelectricity, it seems to be in direct conflict with the wave theory of light. Out modern view of the nature of light is that it has a dual character, behaving like a wave under some circumstances and like a particle, or photon, under others.

Graph between K_{max} and f

Let us plot a graph between maximum kinetic energy K_{max} of photoelectrons and frequency f of incident light. The equation between K_{max} and f is,

$$K_{max} = hf - W$$

comparing it with $y = mx + c$, the graph between K_{max} and f is a straight line with positive slope and negative intercept.

From the graph we can note the following points.

(i) $K_{max} = 0$ at $f = f_0$

(ii) Slope of the straight line is h, a universal constant. *i.e.*, if graph is plotted for two different metals 1 and 2, slope of both the lines is same.

(iii) The negative intercept of the line is W, the work function, which is characteristic of a metal, *i.e.*, intercepts for two different metals will be different. Further,

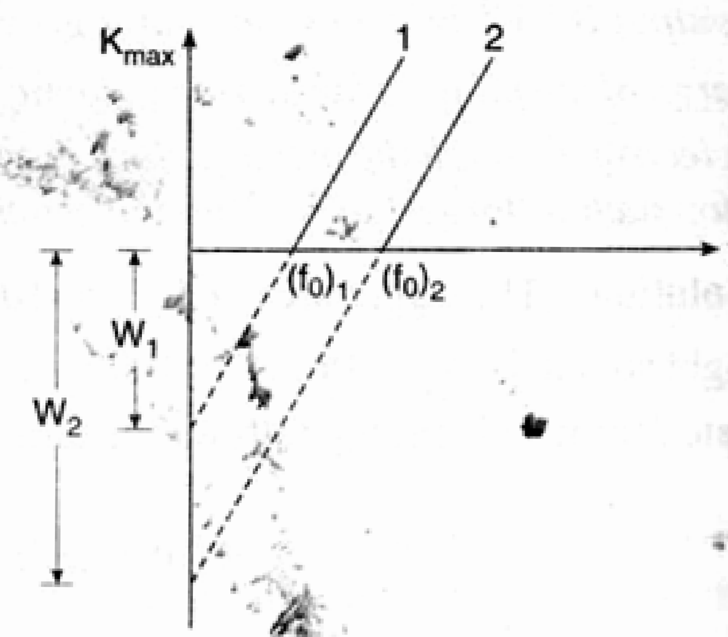


Fig. 30.15

$$W_2 > W_1 \quad \therefore \quad (f_0)_2 > (f_0)_1 \qquad \text{as } W = hf_0$$

Here $\quad f_0$ = threshold frequency

Graph between V_0 and f

Let us now plot a graph between the stopping potential V_0 and the incident frequency f. The equation between them is,

$$eV_0 = hf - W$$

or $\quad V_0 = \left(\frac{h}{e}\right)f - \left(\frac{W}{e}\right)$

Again comparing with $y = mx + c$, the graph between V_0 and f is a straight line with positive slope $\frac{h}{e}$ (a universal constant) and negative intercept $\frac{W}{e}$ (which depends on the metal). The corresponding graph is shown in figure.

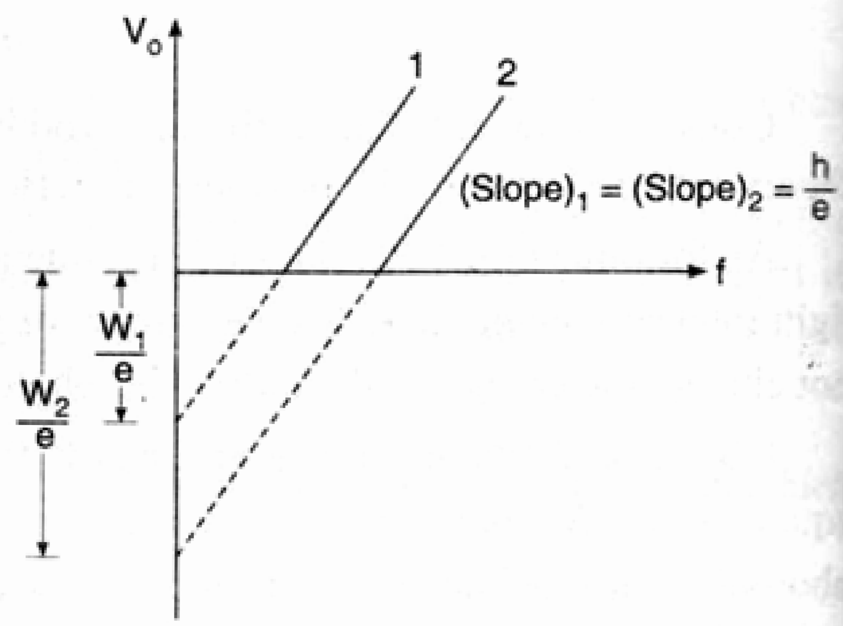


Fig. 30.16

Sample Example 30.12 *The photoelectric work function of potassium is 2.3 eV. If light having a wavelength of 2800 Å falls on potassium, find*

(a) the kinetic energy in electron volts of the most energetic electrons ejected.

(b) the stopping potential in volts

Solution Given, $W = 2.3\,\text{eV}$ $\lambda = 2800\,\text{Å}$

$$\therefore \quad E\ (\text{in eV}) = \frac{12375}{\lambda\ (\text{in Å})} = \frac{12375}{2800} = 4.4\ \text{eV}$$

(a)
$$K_{max} = E - W$$
$$= (4.4 - 2.3)\ \text{eV}$$
$$= 2.1\ \text{eV} \qquad \textbf{Ans.}$$

(b)
$$K_{max} = eV_0$$
$$\therefore \quad 2.1\,\text{eV} = eV_0$$
or
$$V_0 = 2.1\ \text{volt} \qquad \textbf{Ans.}$$

Introductory Exercise 30.2

1. A silver ball is suspended by a string in a vacuum chamber and ultraviolet light of wavelength 2000 Å is directed at it. What electrical potential will the ball acquire as a result? Work function of silver is 4.3 eV.
2. 1.5 mW of 400 nm light is directed at a photoelectric cell. If 0.1% of the incident photons produce photo electrons, find the current in the cell.
3. Is it correct to say that K_{max} is proportional to f? If not, what would a correct statement of the relationship between K_{max} and f?
4. Light of wavelength 2000 Å is incident on a metal surface of work function 3.0 eV. Find the minimum and maximum kinetic energy of the photoelectrons.
5. When a metal is illuminated with light of frequency f the maximum kinetic energy of the photoelectrons is 1.2 eV. When the frequency is increased by 50% the maximum kinetic energy increases to 4.2 eV. What is the threshold frequency for this metal.

30.11 Reduced Mass

In our earlier discussion we have assumed that the nucleus (a proton in case of hydrogen atom) remains at rest. With this assumption the values of the Rydberg constant R and the ionization energy of hydrogen predicted by Bohr's analysis are within 0.1% of the measured values.

Rather the proton and electron both revolve in circular orbits about their common centre of mass. We can take the motion of the nucleus into account simply by replacing the mass of electron m by the reduced mass μ of the electron and the nucleus.

Here
$$\mu = \frac{Mm}{M+m} \qquad \text{...(i)}$$

Where M = mass of nucleus. The reduced mass can also be written as,

$$\mu = \frac{m}{1 + \dfrac{m}{M}}$$

Now, when $M >> m$, $\frac{m}{M} \to 0$ or $\mu \to m$

For ordinary hydrogen we let $M = 1836.2$ m. Substituting in Eq. (i), we get $\mu = 0.99946$ m when this value is used instead of the electron mass m in the Bohr equations, the predicted values are well within 0.1% of the measured values.

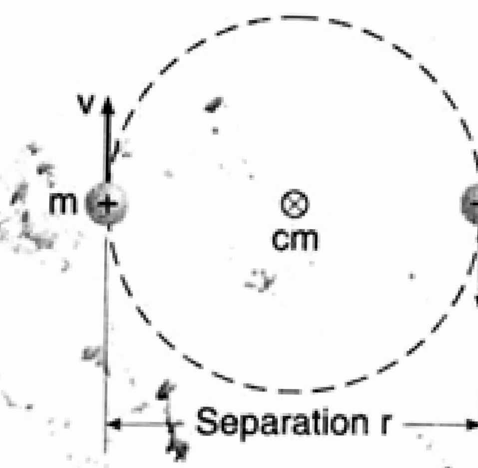


Fig. 30.17 Applying the Bohr model to positronium. The electron and the positron revolve about their common centre of mass, which is located midway between them because they have equal mass.

The concept of reduced mass has other applications. A positron has the same rest mass as an electron but a charge $+e$.

A positronium atom consists of an electron and a positron, each with mass m, in orbit around their common centre of mass. This structure lasts only about 10^{-6} s before two particles annihiliate (combine) one another and disappear, but this is enough time to study the positronium spectrum. The reduced mass is $m/2$, so the energy levels and photon frequencies have exactly half the values for the simple Bohr model with infinite proton mass.

Now, let us prove why m is replaced by the reduced mass μ when motion of nucleus (proton) is also to be considered.

In Fig. 30.18, both the nucleus (mass $= M$, charge $= e$) and electron (mass $= m$, charge $= e$) revolve about their centre of mass (CM) with same angular velocity (ω) but different linear speeds. Let r_1 and r_2 be the distance of CM from proton and electron.

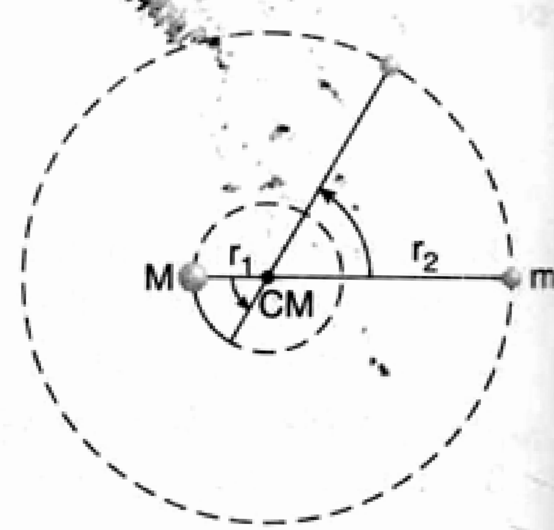


Fig. 30.18

Let r be the distance between the proton and the electron. Then,

$$Mr_1 = mr_2 \quad \text{...(ii)}$$

$$r_1 + r_2 = r \quad \text{...(iii)}$$

$$\therefore \quad r_1 = \frac{mr}{M+m} \quad \text{and} \quad r_2 = \frac{Mr}{M+m} \quad \text{...(iv)}$$

Centripetal force to the electron is provided by the electrostatic force. So,

$$mr_2\omega^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

or

$$m\left(\frac{Mr}{M+r}\right)\omega^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r^2}$$

or

$$\left(\frac{Mm}{M+m}\right)r^3\,\omega^2 = \frac{e^2}{4\pi\varepsilon_0}$$

or

$$\mu r^3\omega^2 = \frac{e^2}{4\pi\varepsilon_0} \quad \text{...(v)}$$

Where

$$\frac{Mm}{M+m} = \mu$$

Moment of inertia of atom about CM,

$$I = Mr_1^2 + mr_2^2 = \left(\frac{Mm}{M+M}\right)r^2 = \mu r^2 \quad \text{...(vi)}$$

According to Bohr's theory, $$\frac{nh}{2\pi} = I\omega$$

or $$\mu r^2\omega = \frac{nh}{2\pi} \quad \text{...(vii)}$$

Solving Eqs. (v) and (vii) for r, we get

$$r = \frac{\varepsilon_0 n^2 h^2}{\pi \mu e^2} \quad \text{...(ix)}$$

Comparing this equation with Eq. (iv) of article 30.6 we see that m has been replaced by μ: Further electrical potential energy of the system,

$$U = \frac{-e^2}{4\pi\varepsilon_0 r}$$

and kinetic energy, $$K = \frac{1}{2} I\omega^2 = \frac{1}{2}\mu r^2\omega^2$$

From Eq. (v), $$\omega^2 = \frac{e^2}{4\pi\varepsilon_0 \mu r^3}$$

$\therefore$ $$K = \frac{e^2}{8\pi\varepsilon_0 r}$$

$\therefore$ Total energy of the system, $$E = K + U = -\frac{e^2}{8\pi\varepsilon_0 r}$$

Substituting value of r from Eq. (ix), we have

$$E = -\frac{\mu e^4}{8\varepsilon_0^2 n^2 h^2} \quad \text{...(x)}$$

The expression for E_n without considering the motion of proton is $E_n = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}$, *i.e.*, m is replaced by μ while considering the motion of proton.

Note *(i) Variation of r_n, v_n and E_n with mass of electron is as under,*

$$r_n \propto \frac{1}{m}, \quad v_n = \text{independent of } m \quad \text{and} \quad E_n \propto m$$

Sometimes the electron is replaced by some another particle which has a charge $-e$ but mass different from the mass of electron. Here, two cases are possible.

Case 1. Let say mass of the replaced particle is x times the mass of the electron and nucleus is still very heavy compared to the replaced particle, *i.e.*, the motion of the nucleus is not to be considered. In this case r_n will become $\frac{1}{x}$ times, v_n will remain unchanged and E_n becomes x times.

Case 2. In this case motion of nucleus is also to be considered, *i.e.*, mass of the replaced particle is comparable to the mass of the nucleus. In this case the mass of the electron is replaced by the reduced mass of the nucleus and the replaced particle. Let say the reduced mass is y times the mass of the electron. Then, r_n will become $\frac{1}{y}$ times, v_n remains unchanged and E_n becomes y-times.

(ii) Reduced mass $\mu = \frac{m_1 m_2}{m_1 + m_2}$ of m_1 and m_2 is less than both the masses.

Solved Examples

For **JEE Main**

Example 1 *When a beam of 10.6 eV photons of intensity $2.0\ W/m^2$ falls on a platinum surface of area $1.0\times10^{-4}\ m^2$ and work function 5.6 eV, 0.53% of the incident photons eject photo electrons. Find the number of photoelectrons emitted per second and their minimum and maximum energies (in eV). Take $1\ eV = 1.6\times10^{-19}\ J$.*

Solution Number of photoelectrons emitted per second

$$=\frac{\text{(Intensity) (Area)}}{\text{(Energy of each photon)}}\times\frac{0.53}{100}$$

$$=\frac{(2.0)(1.0\times10^{-4})}{(10.6\times1.6\times10^{-19})}\times\frac{0.53}{100}$$

$$=6.25\times10^{11}$$ **Ans.**

Minimum kinetic energy of photoelectrons,

$$K_{\min}=0$$

and maximum kinetic energy is, $K_{\max}=E-W=(10.6-5.6)\,\text{eV}$

$$=5.0\,\text{eV}$$ **Ans.**

Example 2 *If an X-ray tube operates at the voltage of 10 kV, find the ratio of the de-Broglie wavelength of the incident electrons to the shortest wavelength of X-rays produced. The specific charge of electron is $1.8\times10^{11}\ C/kg$.*

Solution de-Broglie wavelength when a charge q is accelerated by a potential difference of V volts is

$$\lambda_b=\frac{h}{\sqrt{2qVm}} \quad \text{...(i)}$$

For cutoff wavelength of X-rays, we have $qV=\dfrac{hc}{\lambda_m}$

or $$\lambda_m=\frac{hc}{qV} \quad \text{...(ii)}$$

From Eqs. (i) and (ii) $$\frac{\lambda_b}{\lambda_m}=\frac{\sqrt{\dfrac{qV}{2m}}}{c}$$

For electron $\dfrac{q}{m}=1.8\times10^{11}$ C/kg (given). Substituting the values the desired ratio is

$$\frac{\lambda_b}{\lambda_m} = \frac{\sqrt{\frac{1.8\times10^{11}\times10\times10^3}{2}}}{3\times10^8}$$

$$=0.1$$ **Ans.**

Example 3 *A doubly ionized lithium atom is hydrogen like with atomic number 3. Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit. The ionization energy of the hydrogen atom is 13.6 eV.*

Solution $$E_n = -\frac{z^2}{n^2}(13.6\,\text{eV})$$

By putting $z=3$, we have
$$E_n = -\frac{122.4}{n^2}\,\text{eV}$$
$$E_1 = -\frac{122.4}{(1)^2} = -122.4\,\text{eV}$$
and
$$E_3 = -\frac{122.4}{(3)^2} = -13.6\,\text{eV}$$
$\therefore$
$$\Delta E = E_3 - E_1 = 108.8\,\text{eV}$$
The corresponding wavelength is
$$\lambda = \frac{12375}{\Delta E\ (\text{in eV})}\,\text{Å} = \frac{12375}{108.8}\,\text{Å}$$
$$=113.74\,\text{Å}$$ **Ans.**

Example 4 *The wavelength of the first line of Lyman series for hydrogen is identical to that of the second line of Balmer series for some hydrogen like ion x. Calculate energies of the first four levels, of x.*

Solution Wavelength of the first line of Lyman series for hydrogen atom will be given by the equation
$$\frac{1}{\lambda_1} = R\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = \frac{3R}{4} \quad \text{...(i)}$$
The wavelength of second Balmer line for hydrogen like ion X is
$$\frac{1}{\lambda_2} = Rz^2\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = \frac{3Rz^2}{16} \quad \text{...(ii)}$$
Given that
$$\lambda_1 = \lambda_2 \quad \text{or} \quad \frac{1}{\lambda_1} = \frac{1}{\lambda_2}$$
i.e.,
$$\frac{3R}{4} = \frac{3Rz^2}{16}$$
$\therefore$
$$z=2$$
i.e., x ion is He^+. The energies of first four levels of x are,
$$E_1 = -(13.6)\,z^2 = -54.4\,\text{eV}$$

$$E_2 = \frac{E_1}{(2)^2} = -13.6\,\text{eV}$$

$$E_3 = \frac{E_1}{(3)^2} = -6.04\,\text{eV}$$

and $$E_4 = \frac{E_1}{(4)^2} = -3.4\,\text{eV}$$ **Ans.**

Example 5 *In Moseley's equation $\sqrt{f} = a\,(Z - b)$, a and b are constant. Find their values with the help of the following data.*

Element	Z	Wavelength of K_α X-rays
Mo	42	0.71 Å
Co	27	1.785 Å

Solution $$\sqrt{f} = a\,(Z - b)$$

or $$\sqrt{\frac{c}{\lambda_1}} = a\,(Z_1 - b) \qquad \ldots\text{(i)}$$

and $$\sqrt{\frac{c}{\lambda_2}} = a\,(Z_2 - b) \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have

$$\sqrt{c}\left[\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right] = a\,(Z_1 - Z_2) \qquad \ldots\text{(iii)}$$

Solving above three equations with $c = 3.0 \times 10^8$ m/s, $\lambda_1 = 0.71 \times 10^{-10}$ m
$\lambda_2 = 1.785 \times 10^{-10}$ m, $Z_1 = 42$ and $Z_2 = 27$, we get

$$a = 5 \times 10^7\ (\text{Hz})^{1/2} \quad \text{and} \quad b = 1.37$$ **Ans.**

Example 6 *A monochromatic light source of frequency illuminates a metallic surface and ejects photoelectrons. The photo electrons having maximum energy are just able to ionize the hydrogen atoms in ground state. When the whole experiment is repeated with an incident radiation of frequency $\frac{5}{6}f$, the photoelectrons so emitted are able to excite the hydrogen atom beam which then emits a radiation of wavelength 1215 Å.*

(a) What is the frequency of radiation?
(b) Find the work function of the metal.

Solution (a) Using Einstein's equation of photoelectric effect,

$$K_{\text{max}} = hf - W$$

Here $$K_{\text{max}} = 13.6\,\text{eV}$$

∴ $$hf - W = 13.6\,\text{eV} \qquad \ldots\text{(i)}$$

Further, $$h\left(\frac{5}{6}f\right)-W=\frac{12375}{1215}=10.2\,\text{eV} \qquad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we have

$$\frac{hf}{6}=3.4\,\text{eV}$$

or $$f=\frac{(6)\,(3.4)\,(1.6\times10^{-19})}{(6.63\times10^{-34})}=4.92\times10^{15}\ \text{Hz}$$ **Ans.**

(b) $$W=hf-13.6 \qquad \text{[from Eq. (i)]}$$
$$=6\,(3.4)-13.6$$
$$=6.8\,\text{eV}$$ **Ans.**

Example 7 *A hydrogen-like atom of atomic number z is in an excited state of quantum number 2n. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Find n, z and the ground state energy (in eV) for this atom. Also, calculate the minimum energy (in eV) that can be emitted by this atom during de-excitation. Ground state energy of hydrogen atom is 13.6 eV.*

Solution Given $E_{2n}-E_1=204\,\text{eV}$

$$\therefore \qquad (13.6)z^2\left(1-\frac{1}{4n^2}\right)=204 \qquad \ldots\text{(i)}$$

$$E_{2n}-E_n=40.8\,\text{eV}$$

$$\therefore \qquad 13.6\,z^2\left(\frac{1}{n^2}-\frac{1}{4n^2}\right)=40.8 \qquad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we have $n=2$ and $z=4$ **Ans.**

$$E_1=(-13.6)\,z^2\ \text{eV}\ =(-13.6)\,(4)^2\ \text{eV}$$
$$=-217.6\,\text{eV}$$ **Ans.**

During de-excitation, minimum energy emitted is,

$$E_{\min}=E_{2n}-E_{2n-1}=E_4-E_3$$
$$=\frac{-217.6}{4^2}-\left(\frac{-217.6}{3^2}\right)$$
$$=10.58\,\text{eV}$$ **Ans.**

Example 8 *Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of the fastest photo-electrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV. Find:*

(a) the energy of the photons causing the photo-electric emission.
(b) the quantum number of the two levels involved in the emission of these photons.

(c) *the change in the angular momentum of the electron in the hydrogen atom in the above transition.*

(d) *the recoil speed of the emitting atom assuming it to be at rest before the transition (Take mass of hydrogen* $= 1.6 \times 10^{-27}$ *kg)*

Solution Given $K_{max} = 0.73$ eV and $W = 1.82$ eV.

(a) $$E = K_{max} + W = (0.73 + 1.82)\text{ eV} = 2.55\text{ eV}$$ **Ans.**

(b) $$E_n = -\frac{13.6}{n^2}\text{ eV}$$ (for hydrogen atom)

$\therefore$ $E_1 = -13.6$ eV, $E_2 = -3.4$ eV, $E_3 = -1.51$ eV and $E_4 = -0.85$ eV

Clearly $$E_4 - E_2 = -0.85 - (-3.4) = 2.55\text{ eV}$$

Hence, quantum levels involved are 4 and 2. **Ans.**

(c) $$\Delta L = L_4 - L_2 = 4\left(\frac{h}{2\pi}\right) - 2\left(\frac{h}{2\pi}\right)$$

$$= \frac{h}{\pi}$$ **Ans.**

(d) If p = linear momentum

Then $$p = \frac{E}{c} = \frac{2.55 \times 1.6 \times 10^{-19}}{3.0 \times 10^8} = 1.36 \times 10^{-27}\text{ kg-m/s}$$

If v = recoil speed of hydrogen atom of mass M then
from conservation of linear momentum $p = Mv$

$\therefore$ $$v = \frac{p}{M} = \frac{1.36 \times 10^{-27}}{1.6 \times 10^{-27}} = 0.85\text{ m/s}$$ **Ans.**

Example 9 *Stopping potential of* 24, 10, 110 *and* 115 kV *are measured for photoelectrons emitted from a certain element when it is radiated with monochromatic X-ray. If this element is used as a target in an X-ray tube, what will be the wavelength of* K_α *line?*

Solution Stopping potentials are 24, 100, 110 and 115 kV. *i.e.*, if the electrons are emitted from conduction band, maximum kinetic energy of photoelectrons would be 115×10^3 eV. If they are emitted from next inner shell maximum kinetic energy of photoelectrons would be 110×10^3 eV and so on.

For photoelectrons of L shell it would be 100×10^3 3 eV and for K shell it is 24×10^3 eV. Therefore, difference between energy of L shell and K-shell is,

$$\Delta E = E_L - E_K = (100 - 24) \times 10^3\text{ eV}$$
$$= 76 \times 10^3\text{ eV}$$

$\therefore$ Wavelength of K_α line (transition of electron from L-shell to K-shell) is,

$$\lambda_{K_\alpha}\text{ (in Å)} = \frac{12375}{\Delta E\text{ (in eV)}}$$
$$= \frac{12375}{76 \times 10^3}$$
$$= 0.163\text{ Å}$$ **Ans.**

For **JEE Advanced**

Example 1 *A moving hydrogen atom makes a head on collision with a stationary hydrogen atom. Before collision both atoms are in ground state and after collision they move together. What is the minimum value of the kinetic energy of the moving hydrogen atom, such that one of the atoms reaches one of the excitation state?*

Solution Let K be the kinetic energy of the moving hydrogen atom and K', the kinetic energy of combined mass after collision.

From conservation of linear momentum,

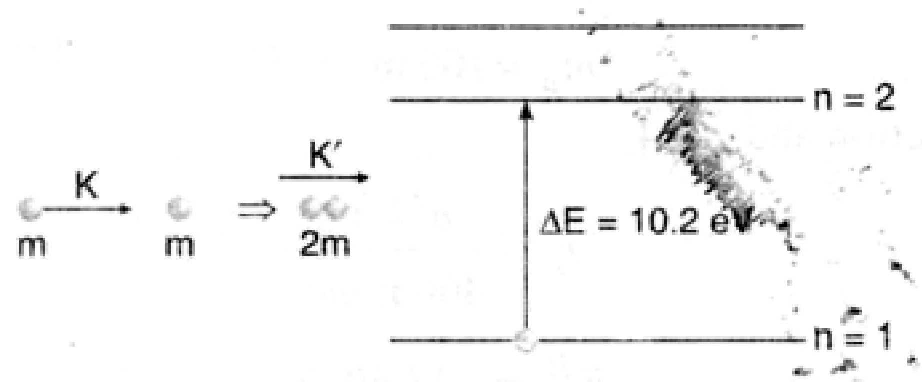


Fig. 30.19

$$p = p' \quad \text{or} \quad \sqrt{2Km} = \sqrt{2K'(2m)}$$

or $$K = 2K' \qquad \text{...(i)}$$

From conservation of energy, $$K = K' + \Delta E \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get $$\Delta E = \frac{K}{2}$$

Now, minimum value of ΔE for hydrogen atom is 10.2 eV.

or $$\Delta E \geq 10.2\ \text{eV}$$

$\therefore$ $$\frac{K}{2} \geq 10.2$$

$\therefore$ $$K \geq 20.4\ \text{eV}$$

Therefore, the minimum kinetic energy of moving hydrogen is 20.4 eV. **Ans.**

Example 2 *An imaginary particle has a charge equal to that of an electron and mass* 100 *times the mass of the electron. It moves in a circular orbit around a nucleus of charge* + 4e. *Take the mass of the nucleus to be infinite. Assuming that the Bohr's model is applicable to this system.*

(a) *Derive an expression for the radius of* n^{th} *Bohr orbit.*

(b) *Find the wavelength of the radiation emitted when the particle jumps from fourth orbit to the second orbit.*

Solution (a) We have

$$\frac{m_p v^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{ze^2}{r_n^2} \qquad \text{...(i)}$$

The quantization of angular momentum gives,

$$m_p v r_n = \frac{nh}{2\pi} \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$r = \frac{n^2 h^2 \varepsilon_0}{z\pi m_p e^2}$$

Substituting $m_p = 100\text{ m}$

where m = mass of electron and $z = 4$

we get,

$$r_n = \frac{n^2 h^2 \varepsilon_0}{400\,\pi\, me^2}$$

Ans.

(b) As we know,

$$E_1^H = -13.60\,\text{eV}$$

and

$$E_n \propto \left(\frac{z^2}{n^2}\right) m$$

For the given particle,

$$E_4 = \frac{(-13.60)\,(4)^2}{(4)^2} \times 100 = -1360\,\text{eV}$$

and

$$E_2 = \frac{(-13.60)\,(4)^2}{(2)^2} \times 100 = -5440\,\text{eV}$$

$$\Delta E = E_4 - E_2 = 4080\,\text{eV}$$

$$\therefore \qquad \lambda\ (\text{in Å}) = \frac{12375}{\Delta E\ (\text{in eV})} = \frac{12375}{4080}$$

$$= \mathbf{3.0\ Å}$$

Ans.

Example 3 *The energy levels of a hypothetical one electron atom are given by*

$$E_n = -\frac{18.0}{n^2}\,eV$$

where $n = 1, 2, 3, \ldots$

(a) Compute the four lowest energy levels and construct the energy level diagram.

(b) What is the excitation potential of the stage $n = 2$?

(c) What wavelengths (Å) can be emitted when these atoms in the ground state are bombarded by electrons that have been accelerated through a potential difference of 16.2 V?

(d) If these atoms are in the ground state, can they absorb radiation having a wavelength of 2000 Å?

(e) What is the photoelectric threshold wavelength of this atom?

Solution (a)
$$E_1 = \frac{-18.0}{(1)^2} = -18.0\,\text{eV}$$
$$E_2 = \frac{-18.0}{(2)^2} = -4.5\,\text{eV}$$
$$E_3 = \frac{-18.0}{(3)^2} = -2.0\,\text{eV}$$
and
$$E_4 = \frac{-18.0}{(4)^2} = -1.125\,\text{eV}$$

The energy level diagram is shown below :

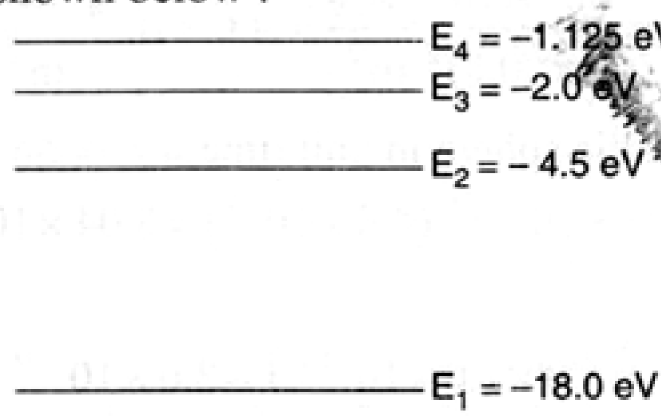


Fig. 30.20

(b) The excitation potential of stage $n = 2$ is, $18.0 - 4.5 = 13.5$ volt **Ans.**

(c) Energy of the electron accelerated by a potential difference of 16.2 V is 16.2 eV. With this energy the electron can excite the atom from $n = 1$ to $n = 3$ as
$$E_4 - E_1 = -1.125 - (-18.0) = 16.875\,\text{eV} > 16.2\,\text{eV}$$
and
$$E_3 - E_1 = -2.0 - (-18.0) = 16.0\,\text{eV} < 16.2\,\text{eV}$$
Now,
$$\lambda_{32} = \frac{12375}{E_3 - E_2} = \frac{12375}{-2.0 - (-4.5)}$$
$$= 4950\,\text{Å}$$ **Ans.**
$$\lambda_{31} = \frac{12375}{E_3 - E_1} = \frac{12375}{16} = 773\,\text{Å}$$ **Ans.**
and
$$\lambda_{21} = \frac{12375}{E_2 - E_1} = \frac{12375}{-4.5 - (-18.0)}$$
$$= 917\,\text{Å}$$ **Ans.**

(d) No, the energy corresponding to $\lambda = 2000$ Å is,
$$E = \frac{12375}{2000} = 6.1875\,\text{eV}$$ **Ans.**

The minimum excitation energy is 13.5 eV ($n = 1$ to $n = 2$).

(e) Threshold wavelength for photoemission to take place from such an atom is,
$$\lambda_{min} = \frac{12375}{18} = 687.5\,\text{Å}$$ **Ans.**

Example 4 *In a photoelectric effect setup, a point source of light of power 3.2×10^{-3} W emits monoenergetic photons of energy 5 eV. The source is located at a distance of 0.8 m from the centre of a stationary metallic sphere of work function 3 eV and of radius 8×10^{-3}. The efficiency of photoelectron emission is one for every 10^6 incident photons. Sphere is initially neutral, and that the photoelectrons are instantly swept away after emission.*

(a) *Calculate the number of photoelectrons emitted per second.*

(b) *It is observed that the photoelectrons emission stops at a certain time t after the light source is switched on. Evaluate time t.*

Solution (a) Intensity of light at a distance 0.8 m from the source

$$I = \frac{(3.2\times10^{-3}\ \text{J/s})}{4\pi\,(0.8)^2\ \text{m}^2} \approx 4.0\times10^{-4}\ \frac{\text{W}}{\text{m}^2}$$

∴ Energy incident on the metallic sphere in unit time

$$E_1 = \pi\,(8\times10^{-3})^2\,(4.0\times10^{-4}) = 8.04\times10^{-8}\ \text{W}$$

Energy of one single photon

$$E_2 = 5.0\times1.6\times10^{-19}\ \text{J} = 8.0\times10^{-19}\ \text{J}$$

Therefore, total number of photons incident on the sphere per second

$$n_1 = \frac{E_1}{E_2} = \frac{8.04\times10^{-8}}{8.0\times10^{-19}} \approx 10^{11}$$

Since, the efficiency of photoelectric emission is one for every 10^6. Hence, total number of photoelectrons per second,

$$n_2 = \frac{n_1}{10^6} = \frac{10^{11}}{10^6} = 10^5$$ **Ans.**

(b) Maximum kinetic energy of photoelectrons

$$K_{max} = E - W = 2\,\text{eV}$$

∴ Stopping potential $V_0 = 2$ volt.

Total positive charge on the sphere after time t is,

$$q = (n_2 e)t = (10^5)\,(1.6\times10^{-19})t$$

$$= (1.6\times10^{-14})\,t$$

Potential on the sphere $$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$$

Photoemission will stop when this potential becomes the stopping potential.

i.e., $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{(1.6\times10^{-14})\,t}{r} = 2$$

or $$\frac{(9\times10^{9})(1.6\times10^{-14})\,t}{(8\times10^{-3})}=2$$

or $$t=111\ \text{second}$$ **Ans.**

Example 5 *In a photocell the plates P and Q have a separation of 5 cm, which are connected through a galvanometer without any cell. Bichromatic light of wavelengths 4000 Å and 6000 Å are incident on plate Q whose work function is 2.39 eV. If a uniform magnetic field B exists parallel to the plates, find the minimum value of B for which the galvanometer shows zero deflection.*

Solution Energy of photons corresponding to light of wavelength $\lambda_1 = 4000$ Å is

$$E_1=\frac{12375}{4000}=3.1\ \text{eV}$$

and that corresponding to $\lambda_2=6000$ Å is,

$$E_2=\frac{12375}{6000}=2.06\ \text{eV}$$

As $E_2 < W$ and $E_1 > W$ (W = Work function)

Photoelectric emission is possible with λ_1 only.

Photoelectrons experience magnetic force and move along a circular path. The galvanometer will

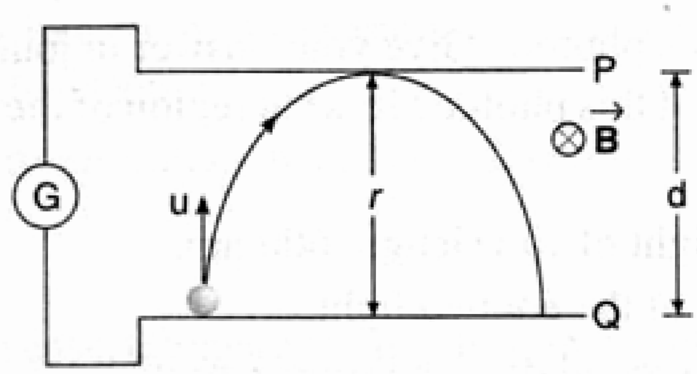


Fig. 30.21

indicate zero deflection if the photoelectrons just complete semi-circular path before reaching the plate *P*.

Thus, $d=r=5\ \text{cm}$ $\quad\therefore\ r=5\ \text{cm}=0.05\ \text{m}$

Further $$r=\frac{mv}{Bq}=\frac{\sqrt{2Km}}{Bq}$$

$\therefore$ $$B_{\min}=\frac{\sqrt{2Km}}{rq}$$

Here $$K=E_1-W=(3.1-2.39)=0.71\ \text{eV}$$

Substituting the values, we have

$$B_{\min}=\frac{\sqrt{2\times0.71\times1.6\times10^{-19}\times9.109\times10^{-31}}}{(0.05)(1.6\times10^{-19})}$$

$$=5.68\times10^{-5}\ \text{T}$$ **Ans.**

EXERCISES

For JEE Main

Subjective Questions

Note *You can take approximations in the answers.*

$h = 6.62 \times 10^{-34}$ J-s, $c = 3.0 \times 10^8$ m/s, $m_e = 9.1 \times 10^{-31}$ kg and $1\text{ eV} = 1.6 \times 10^{-19}$ J

Electromagnetic Waves

1. Find the energy, the mass and the momentum of a photon of ultraviolet radiation of 280 nm wavelength.
2. A small plate of a metal is placed at a distance of 2 m from a monochromatic light source of wavelength 4.8×10^{-7} m and power 1.0 watt. The light falls normally on the plate. Find the number of photons striking the metal plate per square metre per second.
3. A photon has momentum of magnitude 8.24×10^{-28} kg-m/s.
   (a) What is the energy of this photon? Give your answer in joules and in electron volts.
   (b) What is the wavelength of this photon? In what region of the electromagnetic spectrum does it lay?
4. A 75 W light source emits light of wavelength 600 nm.
   (a) Calculate the frequency of the emitted light.
   (b) How many photons per second does the source emit?
5. An excited nucleus emits a gamma-ray photon with energy of 2.45 MeV.
   (a) What is the photon frequency? (b) What is the photon wavelength?
6. (a) A proton is moving at a speed much less than the speed of light. It has kinetic energy K_1 and momentum p_1. If the momentum of the proton is doubled, so $p_2 = 2p_1$, how is its new kinetic energy K_2 related to K_1?
   (b) A photon with energy E_1 has momentum p_1. If another photon has momentum p_2 that is twice p_1, how is the energy E_2 of the second photon related to E_1?

Momentum and Radiation Pressure

7. A parallel beam of monochromatic light of wavelength 500 nm is incident normally on a perfectly absorbing surface. The power through any cross-section of the beam is 10 W. Find
   (a) the number of photons absorbed per second by the surface and
   (b) the force exerted by the light beam on the surface.
8. A beam of white light is incident normally on a plane surface absorbing 70% of the light and reflecting the rest. If the incident beam carries 10 W of power, find the force exerted by it on the surface.

9. A parallel beam of monochromatic light of wavelength 663 nm is incident on a totally reflecting plane mirror. The angle of incidence is 60° and the number of photons striking the mirror per second is 1.0×10^{19}. Calculate the force exerted by the light beam on the mirror.

10. A 100 W light bulb is placed at the centre of a spherical chamber of radius 20 cm. Assume that 60% of the energy supplied to the bulb is converted into light and that the surface of the chamber is perfectly absorbing. Find the pressure exerted by the light on the surface of the chamber.

de-Broglie Wavelength

11. **Wavelength of Bullet**. Calculate the de-Broglie wavelength of a 5.00 g bullet that is moving at 340 m/s. Will it exhibit wave like properties?

12. (a) An electron moves with a speed of 4.70×10^6 m/s. What is its de-Broglie wavelength?

(b) A proton moves with the same speed. Determine its de-Broglie wavelength.

13. An electron has a de-Broglie wavelength of 2.80×10^{-10} m. Determine

(a) the magnitude of its momentum,

(b) its kinetic energy (in joule and in electron volt).

14. Find de-Broglie wavelength corresponding to the root-mean square velocity of hydrogen molecules at room temperature (20°C).

15. An electron, in a hydrogen-like atom, is in excited state. It has a total energy of –3.4 eV, find the de-Broglie wavelength of the electron.

16. In the Bohr model of the hydrogen atom, what is the de-Broglie wavelength for the electron when it is in

(a) the $n = 1$ level?

(b) the $n = 4$ level? In each case, compare the de-Broglie wavelength to the circumference $2\pi r_n$ of the orbit.

Bohr's Atomic Model and Emission Spectrum

17. Find the ionization energy of a doubly ionized lithium atom.

18. The total energy of an electron in the first excited state of the hydrogen atom is –3.4 eV.

(a) What is the kinetic energy of the electron in this state?

(b) What is the potential energy of the electron in this state?

(c) Which of the answers above would change if the choice of the zero of potential energy is changed?

19. The binding energy of an electron in the ground state of He atom is equal to $E_0 = 24.6$ eV. Find the energy required to remove both electrons from the atom.

20. A hydrogen atom is in a state with energy -1.51 eV. In the Bohr model, what is the angular momentum of the electron in the atom, with respect to an axis at the nucleus?

21. Hydrogen atom in its ground state is excited by means of monochromatic radiation of wavelength 1023 Å. How many different lines are possible in the resulting spectrum? Calculate the longest wavelength among them. You may assume the ionization energy of hydrogen atom as 13.6 eV.

22. A doubly ionized lithium atom is hydrogen-like with atomic number 3. Find the wavelength of the radiation require to excite the electron in Li^{++} from the first to the third Bohr orbit (ionization energy of the hydrogen atom equals 13.6 eV).

23. Find the quantum number n corresponding to nth excited state of He^{+} ion if on transition to the ground state the ion emits two photons in succession with wavelengths 108.5 nm and 30.4 nm. The ionization energy of the hydrogen atom is 13.6 eV.

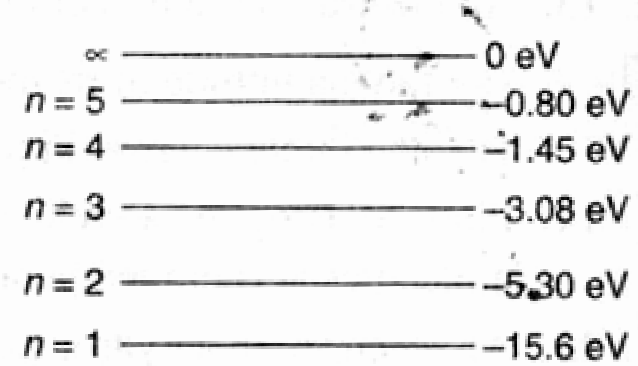
24. A hydrogen like atom (described by the Bohr model) is observed to emit ten wavelengths, originating from all possible transitions between a group of levels. These levels have energies between −0.85 eV and −0.544 eV (including both these values).

(a) Find the atomic number of the atom.

(b) Calculate the smallest wavelength emitted in these transitions.

(Take ground state energy of hydrogen atom = − 13.6 eV)

25. The energy levels of a hypothetical one electron atom are shown in the figure.



(a) Find the ionization potential of this atom.

(b) Find the short wavelength limit of the series terminating at $n = 2$.

(c) Find the excitation potential for the state $n = 3$.

(d) Find wave number of the photon emitted for the transition $n = 3$ to $n = 1$.

26. (a) An atom initially in an energy level with $E = -6.52$ eV absorbs a photon that has wavelength 860 nm. What is the internal energy of the atom after it absorbs the photon? (b) An atom initially in an energy level with $E = -2.68$ eV emits a photon that has wavelength 420 nm. What is the internal energy of the atom after it emits the photon?

27. A small particle of mass m moves in such a way that the potential energy $U = \frac{1}{2} m^2 \omega^2 r^2$ where ω is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, show that radius of the nth allowed orbit is proportional to $\sqrt{n}$.

X-Rays

28. Wavelength of K_α line of an element is λ_0. Find wavelength of K_β line for the same element.

29. X-rays are produced in an X-ray tube by electrons accelerated through an electric potential difference of 50.0 kV. An electron makes three collisions in the target coming to rest and loses half its remaining kinetic energy in each of the first two collisions. Determine the wavelength of the resulting photons. (Neglecting the recoil of the heavy target atoms).

30. From what material is the anode of an X-ray tube made, if the K_α-line wavelength of the characteristic spectrum is 0.76 Å?

31. A voltage applied to an X-ray tube being increased $\eta = 1.5$ times, the short wave limit of an X-ray continuous spectrum shifts by $\Delta\lambda = 26$ pm. Find the initial voltage applied to the tube.

32. The K_α X-rays of aluminium ($Z = 13$) and zinc ($Z = 30$) have wavelengths 887 pm and 146 pm respectively. Use Moseley's equation $\sqrt{v} = a(Z - b)$ to find the wavelength of the K_a X-ray of iron ($Z = 26$).

33. Characteristic X-ray of frequency 4.2×10^{18} Hz are produced when transitions from L shell take place in a certain target material. Use Moseley's law and determine the atomic number of the target material. Given Rydberg constant $R = 1.1 \times 10^7 \text{ m}^{-1}$.

34. The electric current in an X-ray tube operating at 40 kV is 10 mA. Assume that on an average 1% of the total kinetic energy of the electrons hitting the target are converted into X-rays.

(a) What is the total power emitted as X-rays and

(b) How much heat is produced in the target every second?

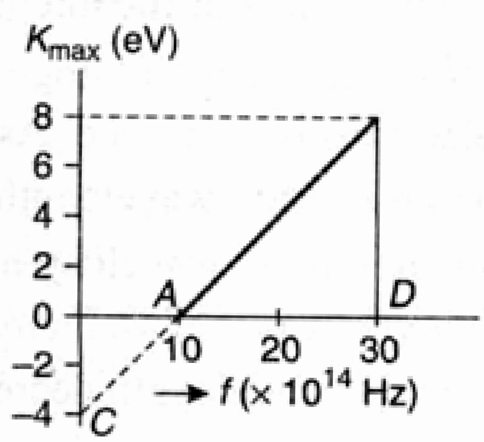
Photoelectric Effect

35. The stopping potential for the photoelectrons emitted from a metal surface of work function 1.7 eV is 10.4 V. Find the wavelength of the radiation used. Also identity the energy levels in hydrogen atom, which will emit this wavelength.

36. What will be the maximum kinetic energy of the photoelectrons ejected from magnesium (for which the work function $W = 3.7$ eV) when irradiated by ultraviolet light of frequency 1.5×10^{15} sec^{-1}.

37. A metallic surface is irradiated with monochromatic light of variable wavelength. Above a wavelength of 5000 Å, no photoelectrons are emitted from the surface. With an unknown wavelength, stopping potential of 3 V is necessary to eliminate the photocurrent. Find the unknown wavelength.

38. A graph regarding photoelectric effect is shown between the maximum kinetic energy of electrons and the frequency of the incident light . On the basis of data as shown in the graph, calculate :



(a) Threshold frequency,

(b) Work function,

(c) Planck's constant

39. A metallic surface is illuminated alternatively with light of wavelengths 3000 Å and 6000 Å. It is observed that the maximum speeds of the photoelectrons under these illuminations are in the ratio 3 : 1. Calculate the work function of the metal and the maximum speed of the photoelectrons in two cases.

40. When a beam of 10.6 eV photons of intensity 2.0 Wm^{-2} falls on a platinum surface of area 1.0×10^{-4} m^2 and work function 5.6 eV, 0.53% of the incident photons eject photoelectrons. Find the number of photoelectrons emitted per second and their minimum and maximum energies (in eV).

41. Light of wavelength 180 nm ejects photoelectrons from a plate of metal whose work function is 2 eV. If a uniform magnetic field of 5×10^{-5} T be applied parallel to the plate, what would be the radius of the path followed by electrons ejected normally from the plate with maximum energy.

42. Light described at a place by the equation $E = (100 \text{ V/m}) [\sin(5 \times 10^{15} \text{ s}^{-1})t + \sin(8 \times 10^{15} \text{ s}^{-1})t]$ falls on a metal surface having work function 2.0 eV. Calculate the maximum kinetic energy of the photoelectrons.

43. The electric field associated with a light wave is given by $E = E_0 \sin [(1.57 \times 10^7 \text{ m}^{-1}) (x - ct)]$. Find the stopping potential when this light is used in an experiment on photoelectric effect with the similar having work function 1.9 eV.

Objective Questions

Single Correct Option

1. According to Einstein's photoelectric equation, the plot of the maximum kinetic energy of the emitted photoelectrons from a metal versus frequency of the incident radiation gives a straight line whose slope

(a) depends on the nature of metal used
(b) depends on the intensity of radiation
(c) depends on both intensity of radiation and the nature of metal used
(d) is the same for all metals and independent of the intensity of radiation

2. The flilament current in the electron gun of a Coolidge tube is increased while the potential difference used to accelerate the electrons is decreased. As a result, in the emitted radiation

(a) the intensity decreases while the minimum wavelength increases
(b) the intensity increases while the minimum wavelength decreases
(c) the intensity as well as the minimum wavelength increase
(d) the intensity as well as the minimum wavelength decrease

3. The velocity of the electron in the first Bohr orbit as compared to that of light is about

(a) 1/300 (b) 1/500 (c) 1/137 (d) 1/187

4. $_{86}A^{222} \rightarrow {}_{84}B^{210}$. In this reaction how many α and β particles are emitted ?

(a) $6\alpha, 3\beta$ (b) $3\alpha, 4\beta$ (c) $4\alpha, 3\beta$ (d) $3\alpha, 6\beta$

5. An X-ray tube is operated at 20 kV. The cut off wavelength is
(a) 0.89 Å (b) 0.75 Å (c) 0.62 Å (d) None of these

6. An X-ray tube is operated at 18 kV. The maximum velocity of electron striking the target is
(a) 8×10^7 m/s (b) 6×10^7 m/s (c) 5×10^7 m/s (d) None of these

7. What is the ratio of de-Broglie wavelength of electron in the second and third Bohr orbits in the hydrogen atoms ?
(a) 2/3 (b) 3/2 (c) 4/3 (d) 3/4

8. The energy of a hydrogen like atom (or ion) in its ground state is – 122.4 eV. It may be
(a) hydrogen atom (b) He^+ (c) Li^{2+} (d) Be^{3+}

9. The operating potential in an X-ray tube is increased by 2%. The percentage change in the cut off wavelength is
(a) 1% increase (b) 2% increase (c) 2% decrease (d) 1% decrease

10. The energy of an atom or ion in the first excited state is –13.6 eV. It may be
(a) He^+ (b) Li^{++} (c) hydrogen (d) deuterium

11. In order that the short wavelength limit of the continuous X-ray spectrum be 1 Å, the potential difference through which an electron must be accelerated is
(a) 124 kV (b) 1.24 kV (c) 12.4 kV (d) 1240 kV

12. The momentum of an a X-ray photon with $\lambda = 0.5$ Å is
(a) 13.26×10^{-26} kg m/s (b) 1.326×10^{-26} kg m/s
(c) 13.26×10^{-24} kg m/s (d) 13.26×10^{-22} kg m/s

13. The work function of a substance is 1.6 eV. The longest wavelength of light that can produce photo emission from the substance is
(a) 7750 Å (b) 3875 Å (c) 5800 Å (d) 2900 Å

14. Find the binding energy of an electron in the ground state of a hydrogen like atom in whose spectrum the third Balmer line is equal to 108.5 nm.
(a) 54.4 eV (b) 13.6 eV (c) 112.4 eV (d) None of these

15. Let the potential energy of hydrogen atom in the ground state be zero. Then its energy in the first excited state will be
(a) 10.2 eV (b) 13.6 eV (c) 23.8 eV (d) 27.2 eV

16. Light of wavelength 330 nm falling on a piece of metal ejects electrons with sufficient energy with requires voltage V_0 to prevent them from reaching a collector. In the same setup, light of wavelength 220 nm, ejects electrons which requires twice the voltage V_0 to stop them in reaching a collector. The numerical value of voltage V_0 is
(a) $\frac{16}{15}V$ (b) $\frac{15}{16}V$ (c) $\frac{15}{8}V$ (d) $\frac{8}{15}V$

17. Maximum kinetic energy of a photoelectron is E when the wavelength of incident light is λ. If energy becomes four times when wavelength is reduced to one-third, then work function of the metal is
(a) $\frac{3hc}{\lambda}$ (b) $\frac{hc}{3\lambda}$ (c) $\frac{hc}{\lambda}$ (d) $\frac{hc}{2\lambda}$

18. If the frequency of K_α X-ray emitted from the element with atomic number 31 is f, then the frequency of K_α X-ray emitted from the element with atomic number 51 would be

(a) $\frac{5f}{3}$ (b) $\frac{51f}{31}$ (c) $\frac{9f}{25}$ (d) $\frac{25f}{9}$

19. According to Moseley's law the ratio of the slope of graph between $\sqrt{f}$ and Z for K_β and K_α is

(a) $\sqrt{\frac{32}{27}}$ (b) $\sqrt{\frac{27}{32}}$ (c) $\sqrt{\frac{5}{36}}$ (d) $\sqrt{\frac{36}{5}}$

20. If the electron in hydrogen orbit jumps from third orbit to second orbit, the wavelength of the emitted radiation is given by

(a) $\lambda = \frac{R}{6}$ (b) $\lambda = \frac{5}{R}$ (c) $\lambda = \frac{36}{5R}$ (d) $\lambda = \frac{5R}{36}$

21. A potential of 10,000 V is applied across an X-ray tube. Find the ratio of de-Broglie wavelength associated with incident electrons to the minimum wavelength associated with X-rays.

(Given $e/m = 1.8 \times 10^{11}$ C/kg for electrons)

(a) 10 (b) 20 (c) 1/10 (d) 1/20

22. When a metallic surface is illuminated with monochromatic light of wavelength λ, the stopping potential is $5\,V_0$. When the same surface is illuminated with the light of wavelength 3λ, the stopping potential is V_0. Then the work function of the metallic surface is

(a) $hc/6\lambda$ (b) $hc/5\lambda$ (c) $hc/4\lambda$ (d) $2hc/4\lambda$

23. The threshold frequency for a certain photosensitive metal is ν_0. When it is illuminated by light of frequency $\nu = 2\nu_0$, the stopping potential for photoelectric current is V_0. What will be the stopping potential when the same metal is illuminated by light of frequency $\nu = 3\nu_0$

(a) $1.5\,V_0$ (b) $2\,V_0$ (c) $2.5\,V_0$ (d) $3\,V_0$

24. The frequency of the first line in Lyman series in the hydrogen spectrum is ν. What is the frequency of the corresponding line in the spectrum of doubly ionized Lithium?

(a) v (b) $3\,v$ (c) $9\,v$ (d) $2\,v$

25. Which energy state of doubly ionized Lithium (Li^{++}) has the same energy as that of the ground state of Hydrogen?

(a) $n = 1$ (b) $n = 2$ (c) $n = 3$ (d) $n = 4$

26. Two identical photocathodes receive light of frequencies ν_1 and ν_2. If the velocities of the photoelectrons (of mass m) coming out are v_1 and v_2 respectively, then

(a) $v_1 - v_2 = \left[\frac{2h}{m}(\nu_1 - \nu_2)\right]^{1/2}$ (b) $v_1^2 - v_2^2 = \frac{2h}{m}(\nu_1 - \nu_2)$

(c) $v_1 + v_2 = \left[\frac{2h}{m}(\nu_1 - \nu_2)\right]^{1/2}$ (d) $v_1^2 + v_2^2 = \frac{2h}{m}(\nu_1 - \nu_2)$

27. The longest wavelength of the Lyman series for Hydrogen atom is the same as the wavelength of a certain line in the spectrum of He^+ when the electron makes a transition from $n \to 2$. The value of n is

(a) 3 (b) 4 (c) 5 (d) 6

28. The wavelength of the K_α- line for the Uranium is $(Z = 92)$ $(R = 1.0973 \times 10^7 \text{ m}^{-1})$

(a) 1.5 Å (b) 0.5 Å (c) 0.15 Å (d) 2.0 Å

29. The frequencies of K_α, K_β and L_α X-rays of a material are γ_1, γ_2 and γ_3 respectively. Which of the following relation holds good ?

(a) $\gamma_2 = \sqrt{\gamma_1 \gamma_3}$ (b) $\gamma_2 = \gamma_1 + \gamma_3$ (c) $\gamma_2 = \frac{\gamma_1 + \gamma_3}{2}$ (d) $\gamma_3 = \sqrt{\gamma_1 \gamma_2}$

30. A proton and an α-particle are accelerated through same potential difference. Then the ratio of de-Broglie wavelength of proton and α-particle is

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $2\sqrt{2}$ (d) None of these

31. If E_1, E_2 and E_3 represent respectively the kinetic energies of an electron , an alpha particle and a proton respectively each having same de-Broglie wavelength then

(a) $E_1 > E_3 > E_2$ (b) $E_2 > E_3 > E_1$ (c) $E_1 > E_2 > E_3$ (d) $E_1 = E_2 = E_3$

32. If the potential energy of a hydrogen atom in the ground state is assumed to be zero, then total energy of $n = \infty$ is equal to

(a) 13.6 eV (b) 27.2 eV (c) zero (d) None of these

33. A 1000 W transmitter works at a frequency of 880 kHz. The number of photons emitted per second is

(a) 1.7×10^{28} (b) 1.7×10^{30} (c) 1.7×10^{23} (d) 1.7×10^{25}

34. Electromagnetic radiation of wavelength 3000 Å is incident on an isolated platinum surface of work function 6.30 eV. Due to the radiation the

(a) sphere becomes positively charged

(b) sphere becomes negatively charged

(c) sphere remains neutral

(d) maximum kinetic energy of the ejected photoelectrons would be 2.03 eV

35. The energy of a hydrogen atom in its ground state is – 13.6 eV. The energy of the level corresponding to the quantum number $n = 5$ is

(a) – 0.54 eV (b) – 5.40 eV (c) – 0.85 eV (d) – 2.72 eV

36. Ultraviolet radiation of 6.2 eV falls on an aluminum surface (work function = 4.2 eV). The kinetic energy in joule of the fastest electrons emitted is

(a) 3.2×10^{-21} (b) 3.2×10^{-19} (c) 3.2×10^{-17} (d) 3.2×10^{-15}

37. What should be the velocity of an electron so that its momentum becomes equal to that of a photon of wavelength 5200 Å?

(a) 700 m/s (b) 1000 m/s (c) 1400 m/s (d) 2800 m/s

38. Photoelectric work function of a metal is 1 eV. Light of wavelength $\lambda = 3000$ Å falls on it. The photoelectrons come out with maximum velocity

(a) 10 m/s (b) 10^3 m/s (c) 10^4 m/s (d) 10^6 m/s

For **JEE Advanced**

Assertion and Reason

Directions : *Choose the correct option.*

(*a*) *If both* ***Assertion*** *and* ***Reason*** *are true and the* ***Reason*** *is correct explanation of the* ***Assertion.***
(*b*) *If both* ***Assertion*** *and* ***Reason*** *are true but* ***Reason*** *is not the correct explanation of* ***Assertion.***
(*c*) *If* ***Assertion*** *is true, but the* ***Reason*** *is false.*
(*d*) *If* ***Assertion*** *is false but the* ***Reason*** *is true.*

1. **Assertion :** X-rays can't be deflected by electric or magnetic fields.

   Reason : These are electromagnetic waves.

2. **Assertion :** If wavelength of light is doubled, energy and momentum of photons are reduced to half.

   Reason : By increasing the wavelength, speed of photons will decrease.

3. **Assertion :** We can increase the staturation current in photoelectric experiment even without increasing the intensity of light.

   Reason : Intensity can be increased by increasing the frequency of incident photons.

4. **Assertion :** Photoelectric effect proves the particle nature of light.

   Reason : Photoemission starts as soon as light is incident on the metal surface, provided frequency of incident light is greater than or equal to the threshold frequency.

5. **Assertion :** During de-excitation from $n = 6$ to $n = 3$ total six emission lines may be obtained.

   Reason : From $n = n$ to $n = 1$, total $\frac{n(n-1)}{2}$ emission lines are obtained.

6. **Assertion :** If frequency of incident light is doubled, the stopping potential will also become two times.

   Reason : Stopping, potential is given by

$$V_0 = \frac{h}{e}(\nu - \nu_0)$$

7. **Assertion :** X-rays can't be obtained in the emission spectrum of hydrogen atom.

   Reason : Maximum energy of photons emitted from hydrogen spectrum is 13.6 eV.

8. **Assertion :** If applied potential difference in coolidge tube is increased, then difference between K_α wavelength and cut off wavelength will increase.

   Reason : Cut off wavelength is inversely proportional to the applied potential difference in coolidge tube.

9. **Assertion :** In $n = 2$, energy of electron in hydrogen like atoms is more compared to $n = 1$.

   Reason : Electrostatic potential energy in $n = 2$ is more.

10. **Assertion :** In continuous X-ray spectrum all wavelengths can be obtained.

    Reason : Accelerated (or retarded) charged particles radiate energy. This is the cause of production of continuous X-rays.

Objective Questions

Single Correct Option

1. If we assume only gravitational attraction between proton and electron in hydrogen atom and the Bohr's quantization rule to be followed, then the expression for the ground state energy of the atom will be (the mass of proton is M and that of electron is m.)
(a) $\frac{G^2M^2m^2}{h^2}$ (b) $-\frac{2\pi^2G^2M^2m^3}{h^2}$ (c) $-\frac{2\pi^2GM^2m^3}{h^2}$ (d) None of these

2. An electron in a hydrogen atom makes a transition from first excited state to ground state. The magnetic moment due to circulating electron
(a) increases two times (b) decreases two times
(c) increases four times (d) remains same

3. The excitation energy of a hydrogen-like ion to its first excited state is 40.8 eV. The energy needed to remove the electron from the ion in the ground state is
(a) 54.4 eV (b) 62.6 eV (c) 72.6 eV (d) 58.6 eV

4. An electron in a hydrogen atom makes a transition from first excited state to ground state. The equivalent current due to circulating electron
(a) increases 4 times (b) decreases 4 times
(c) increases 8 times (d) decreases 8 times

5. In a sample of hydrogen like atoms all of which are in ground state, a photon beam containing photons of various energies is passed. In absorption spectrum, five dark lines are observed. The number of bright lines in the emission spectrum will be
(assume that all transitions take place)
(a) 21 (b) 10 (c) 15 (d) None of these

6. Let A_n be the area enclosed by the nth orbit in a hydrogen atom. The graph of $\ln(A_n/A_1)$ against $\ln(n)$
(a) will not pass through origin (b) will be a straight line with slope 4
(c) will be a rectangular hyperbola (d) will be a parabola

7. In the hydrogen atom, an electron makes a transition from $n=2$ to $n=1$. The magnetic field produced by the circulating electron at the nucleus
(a) decreases 16 times (b) increases 4 times (c) decreases 4 times (d) increases 32 times

8. A stationary hydrogen atom emits photon corresponding to the first line of Lymann series. If R is the Rydberg's constant and M is the mass of the atom, then the velocity acquired by the atom is
(a) $\frac{3Rh}{4M}$ (b) $\frac{4M}{3Rh}$ (c) $\frac{Rh}{4M}$ (d) $\frac{4M}{Rh}$

9. Light wave described by the equation 200 V/m $\sin(1.5\times10^{15}\text{ s}^{-1})t\cos(0.5\times10^{15}\text{ s}^{-1})t$ falls on metal surface having work function 2.0 eV. Then the maximum kinetic energy of photoelectrons is
(a) 3.27 eV (b) 2.2 eV (c) 2.85 eV (d) None of these

10. A hydrogen like atom is excited using a radiation. Consequently six spectral lines are observed in the spectrum. The wavelength of emission radiation is found to be equal or smaller than the radiation used. This concludes that the gas was initially at
(a) ground state (b) first excited state (c) second excited state (d) third excited state

11. The time period of the electron in the ground state of hydrogen atom is two times the time period of the electron in the first excited state of a certain hydrogen like atom (Atomic Number Z). The value of Z is

(a) 2 (b) 3 (c) 4 (d) None of these

12. The wavelengths of K_α X-rays from lead isotopes Pb^{204}, Pb^{206} and Pb^{208} are λ_1, λ_2 and λ_3 respectively. Choose the correct alternative.

(a) $\lambda_1 < \lambda_2 < \lambda_3$ (b) $\lambda_1 > \lambda_2 > \lambda_3$ (c) $\lambda_1 = \lambda_2 = \lambda_3$ (d) None of these

13. In case of hydrogen atom, whenever a photon is emitted in the Balmer series,

(a) there is a probability of emitting another photon in the Lyman series
(b) there is a probability of emitting another photon of wavelength 1215 Å
(c) the wavelength of radiation emitted in Lymen series is always shorter than the wavelength emitted in the Balmer series
(d) All of the above

14. An electron of kinetic energy K collides elastically with a stationary hydrogen atom in the ground state. Then

(a) $K > 13.6$ eV (b) $K > 10.2$ eV (c) $K < 10.2$ eV (d) data insufficient

15. In a stationary hydrogen atom, an electron jumps from $n = 3$ to $n = 1$. The recoil speed of the hydrogen atom is about

(a) 4 m/s (b) 4 cm/s (c) 4 mm/s (d) 4×10^{-4} m/s

16. An X-ray tube is operating at 150 kV and 10 mA. If only 1% of the electric power supplied is converted into X-rays, the rate at which the target is heated in calorie per second is

(a) 3.55 (b) 35.5 (c) 355 (d) 3550

17. An electron revolves round a nucleus of atomic number Z. If 32.4 eV of energy is required to excite an electron from the $n = 3$ state to $n = 4$ state, then the value of Z is

(a) 5 (b) 6 (c) 4 (d) 7

18. If the de-Broglie wavelength of a proton is 10^{-13} m, the electric potential through which it must have been accelerated is

(a) 4.07×10^4 V (b) 8.15×10^4 V (c) 8.15×10^3 V (d) 4.07×10^5 V

19. If E_n and L_n denote the total energy and the angular momentum of an electron in the nth orbit of Bohr atom, then

(a) $E_n \propto L_n$ (b) $E_n \propto \dfrac{1}{L_n}$ (c) $E_n \propto L_n^2$ (d) $E_n \propto \dfrac{1}{L_n^2}$

20. An orbital electron in the ground state of hydrogen has the magnetic moment μ_1. This orbital electron is excited to 3rd excited state by some energy transfer to the hydrogen atom. The new magnetic moment of the electron is μ_2, then

(a) $\mu_1 = 4\mu_2$ (b) $2\mu_1 = \mu_2$ (c) $16\mu_1 = \mu_2$ (d) $4\mu_1 = \mu_2$

21. A moving hydrogen atom makes a head-on collision with a stationary hydrogen atom. Before collision, both atoms are in ground state and after collision they move together. The minimum value of the kinetic energy of the moving hydrogen atom, such that one of the atoms reaches one of the excitation state is

(a) 20.4 eV (b) 10.2 eV (c) 54.4 eV (d) 13.6 eV

22. In an excited state of hydrogen like atom an electron has total energy of – 3.4 eV. If the kinetic energy of the electron is E and its de-Broglie wavelength is λ, then
(a) $\lambda = 6.6$ Å (b) $E = 3.4$ eV (c) Both are correct (d) Both are wrong

Passage : (Q. 23 to Q. 25)

When a surface is irradiated with light of wavelength 4950 *Å, a photocurrent appears which vanishes if a retarding potential greater than* 0.6 *volt is applied across the phototube. When a second source of light is used, it is found that the critical retarding potential is changed to* 1.1 *volt.*

23. The work function of the emitting surface is
(a) 2.2 eV (b) 1.5 eV (c) 1.9 eV (d) 1.1 eV

24. The wavelength of the second source is
(a) 6150 Å (b) 5150 Å (c) 4111 Å (d) 4500 Å

25. If the photoelectrons (after emission from the source) are subjected to a magnetic field of 10 tesla, the two retarding potentials would
(a) uniformly increase (b) uniformly decrease (c) remain the same (d) None of these

Passage : (Q. 26 to Q. 28)

In an experimental set up to study the photoelectric effect a point-source of light of power 3.2×10^{-3} *W was taken. The source can emit mono energetic photons of energy* 5 *eV and is located at a distance of* 0.8 *m from the centre of a stationary metallic sphere of work function* 3.0 *eV. The radius of the sphere is* $r = 8 \times 10^{-3}$ *m. The efficiency of photoelectric emission is one for every* 10^6 *incident photons.*

Based on the information given above answer the questions given below.

(Assume that the sphere is isolated and photo-electrons are instantly swept away after the emission).

26. de-Broglie wavelength of the fastest moving photo-electron is
(a) 6.63 Å (b) 8.69 Å (c) 2 Å (d) 5.26 Å

27. It was observed that after some time emission of photo-electrons from the sphere stopped. Charge on the sphere when the photon emission stops is
(a) $16\pi\varepsilon_0 r$ Coulomb (b) $8\pi\varepsilon_0 r$ Coulomb (c) $15\pi\varepsilon_0 r$ Coulomb (d) $20\pi\varepsilon_0 r$ Coulomb

28. Time after which photo-electric emission stops is
(a) 100 s (b) 121 s (c) 111 s (d) 141 s

More than One Correct Options

1. If the potential difference of coolidge tube producing X-ray is increased, then choose the correct option (s)
(a) the interval between $\lambda_{k\alpha}$ and $\lambda_{k\beta}$ increases
(b) the interval between $\lambda_{k\alpha}$ and λ_0 increases
(c) the interval between $\lambda_{k\beta}$ and λ_0 increases
(d) λ_0 does not change

Here λ_0 is cut off wavelength and λ_{K_α} and λ_{K_β} are wavelengths of K_α and K_β characteristic X-rays.

2. In Bohr model of the hydrogen atom, let R, v and E represent the radius of the orbit, speed of the electron and the total energy of the electron respectively. Which of the following quantities are directly proportional to the quantum number n ?

(a) vR (b) RE (c) $\frac{v}{E}$ (d) $\frac{R}{E}$

3. The magnitude of angular momentum, orbital radius and time period of revolution of an electron in a hydrogen atom corresponding to the quantum number n are L, r and T respectively. Which of the following statements is/are correct?

(a) $\frac{rL}{T}$ is independent of n (b) $\frac{L}{T} \propto \frac{1}{n^2}$

(c) $\frac{T}{r} \propto n$ (d) $Lr \propto \frac{1}{n^3}$

4. In which of the following cases the heavier of the two particles has a smaller de-Broglie wavelength? The two particles

(a) move with the same speed
(b) move with the same linear momentum
(c) move with the same kinetic energy
(d) have the same change of potential energy in a conservative field

5. Hydrogen atom absorbs radiations of wavelength λ_0 and consequently emit radiations of 6 different wavelengths, of which two wavelengths are longer than λ_0. Choose the correct alternative(s).

(a) The final excited state of the atoms is $n = 4$
(b) The initial state of the atoms is $n = 2$
(c) The initial state of the atoms is $n = 3$
(d) There are three transitions belonging to Lyman series

6. In coolidge tube, if f and λ represent the frequency and wavelength of K_α line for a metal of atomic number Z, then identify the statement which represents a straight line

(a) $\sqrt{f}$ *versus* Z (b) $\frac{1}{\sqrt{\lambda}}$ *versus* Z (c) $\ln f$ *versus* $\ln Z$ (d) $\ln \sqrt{f}$ *versus* Z

Match the Columns

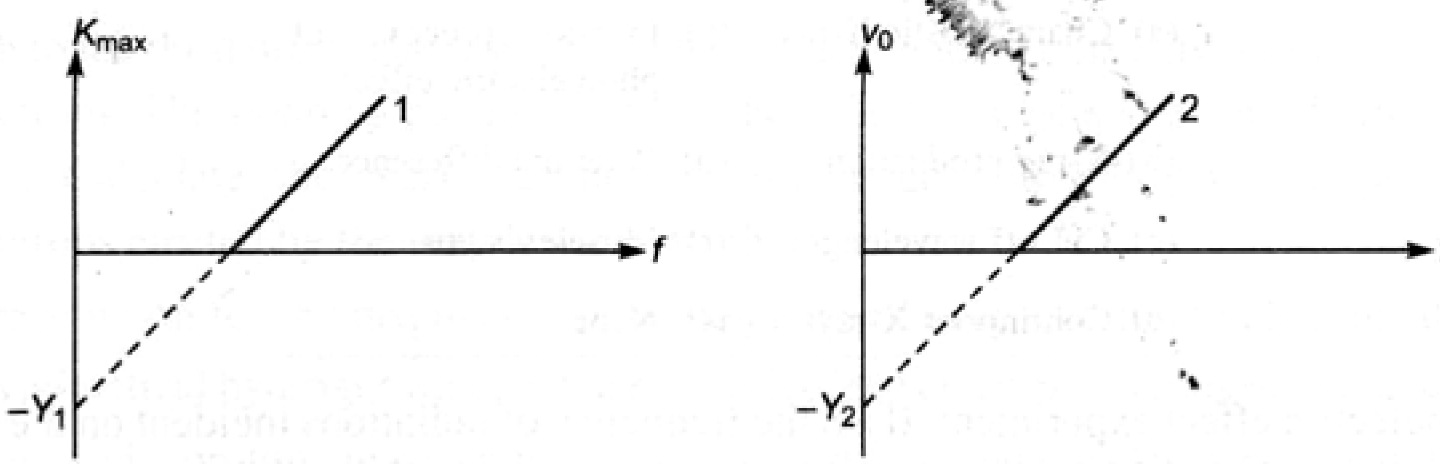
1. Match the following two columns for hydrogen spectrum.

Column I	Column II
(a) Lymen series	(p) infrared region
(b) Balmer series	(q) visible region
(c) Paschen series	(r) ultraviolet region
(d) Brackett series	(s) X-rays

2. Ionization energy from first excited state of hydrogen atom is E. Match the following two columns for He^+ atom.

Column I	Column II
(a) Ionization energy from ground state	(p) $4E$
(b) Electrostatic potential energy in first excited state.	(q) $-16E$
(c) Kinetic energy of electron in ground state.	(r) $-8E$
(d) Ionization energy from first excited state.	(s) $16E$

3.



Maximum kinetic energy versus frequency of incident light and stopping potential versus frequency of incident light graphs are shown in figure. Match the following two columns.

Column I	Column II
(a) Slope of line-1	(p) h/e
(b) Slope of line-2	(q) h
(c) Y_1	(r) W
(d) Y_2	(s) W/e

Here, h = Planck's constant, $e = 1.6 \times 10^{-19}$ C and W = work function.

4. For hydrogen and hydrogen type atoms, match the following two columns.

Column I	Column II
(a) Time period	(p) Proportional to n/Z
(b) Angular momentum	(q) Proportional to n^2/Z
(c) Speed	(r) Proportional to n^3/Z^2
(d) Radius	(s) None of these

5. In hydrogen atom wavelength of second line of Balmer series is λ. Match the following two columns corresponding to the wavelength.

Column I	Column II
(a) First line of Balmer series	(p) $(27/20)\lambda$
(b) Third line of Balmer series	(q) $(\lambda/4)$
(c) First line of Lymen series	(r) $(25/12)\lambda$
(d) Second line of Lymen series	(s) None of these

6. Match the following (Give most appropriate one matching)

Column I	Column II
(a) Characteristic X-ray	(p) Inverse process of photoelectric effect
(b) X-ray production	(q) Potential difference
(c) Cut-off wavelength	(r) Moseley's law
(d) Continuous X-ray	(s) None

7. In a photoelectric effect experiment. If f is the frequency of radiations incident on the metal surface and I is the intensity of the incident radiations, then match the following.

Column I	Column II
(a) If f is increased keeping I and work function constant	(p) Stopping potential increases
(b) If distance between cathode and anode is increased	(q) Saturation current increases
(c) If I is increased keeping f and work function constant	(r) Maximum kinetic energy of photoelectron increases
(d) Work function is decreased keeping f and I constant	(s) Stopping potential remains same

Subjective Questions

1. A hydrogen like atom of atomic number Z is in an excited state of quantum number $2n$. It can emit a photon of maximum energy 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Find n, Z and the ground state energy (in eV) for this atom. Ground state energy of hydrogen atom is –13.6 eV.
2. A beam of light consists of four wavelengths 4000 Å, 4800 Å, 6000 Å and 7000 Å, each of intensity 1.5×10^{-3} Wm^{-2}. The beam falls normally over an area 10^{-4} m^2 on a clean metallic surface of work function 1.9 eV. Assuming no loss of light energy, calculate the number of photoelectrons liberated per second. Assume that each capable photon emits an electron.

3. A source emits monochromatic light of frequency 5.5×10^{14} Hz at a rate of 0.1 W. Of the photons given out, 0.15% fall on the cathode of a photocell which gives a current of $6\,\mu A$ in an external cricuit.
   (a) Find the energy of a photon.
   (b) Find the number of photons leaving the source per second.
   (c) Find the percentage of the photons falling on the cathode which produce photoelectrons.
4. The hydrogen atom in its ground state is excited by means of monochromatic radiation. Its resulting spectrum has six different lines. These radiations are incident on a metal plate. It is observed that only two of them are responsible for photoelectric effect. If the ratio of maximum kinetic energy of photoelectrons in the two cases is 5 then find the work function of the metal.
5. Electrons in hydrogen like atoms ($Z = 3$) make transitions from the fifth to the fourth orbit and from the fourth to the third orbit. The resulting radiation are incident normally on a metal plate and eject photoelectrons. The stopping potential for the photoelectrons ejected by the shorter wavelength is 3.95 volts. Calculate the work function of the metal and the stopping potential for the photoelectrons ejected by the longer wavelength.
6. Find an expression for the magnetic dipole moment and magnetic field induction at the centre of Bohr's hypothetical hydrogen atom in the n^{th} orbit of the electron in terms of universal constant.
7. An electron and a proton are separated by a large distance and the electron approaches the proton with a kinetic energy of 2 eV. If the electron is captured by the proton to form a hydrogen atom in the ground state, what wavelength photon would be given off ?
8. Hydrogen gas in the atomic state is excited to an energy level such that the electrostatic potential energy of H-atom becomes -1.7 eV. Now, a photoelectric plate having work function $W = 2.3$ eV is exposed to the emission spectra of this gas. Assuming all the transitions to be possible, find the minimum de-Broglie wavelength of the ejected photoelectrons.
9. A gas of hydrogen like atoms can absorb radiation of 68 eV. Consequently, the atom emits radiations of only three different wavelengths. All the wavelengths are equal or smaller than that of the abosrbed photon.
   (a) Determine the initial state of the gas atoms.
   (b) Identify the gas atoms.
   (c) Find the minimum wavelength of the emitted radiations.
   (d) Find the ionization energy and the respective wavelength for the gas atoms.
10. A photon with energy of 4.9 eV ejects photoelectrons from tungsten. When the ejected electron enters a constant magnetic field of strength $B = 2.5$ m T at an angle of 60° with the field direction, the maximum pitch of the helix described by the electron is found to be 2.7 mm. Find the work function of the metal in electron-volt. Given that specific charge of electron is 1.76×10^{11} C/kg.

11. For a certain hypothetical one-electron atom, the wavelength (in Å) for the spectral lines for transitions originating at $n = p$ and terminating at $n = 1$ are given by

$$\lambda = \frac{1500p^2}{p^2 - 1} \quad \text{where} \quad p = 2, 3, 4$$

(a) Find the wavelength of the least energetic and the most energetic photons in this series.
(b) Construct an energy level diagram for this element showing the energies of the lowest three levels.
(c) What is the ionization potential of this element?

12. A photocell is operating in saturation mode with a photocurrent 4.8 mA when a monochromatic radiation of wavelength 3000 Å and power of 1 mW is incident. When another monochromatic radiation of wavelength 1650 Å and power 5 mW is incident, it is observed that maximum velocity of photoelectron increases to two times. Assuming efficiency of photoelectron generation per incident photon to be same for both the cases, calculate,
(a) threshold wavelength for the cell
(b) saturation current in second case
(c) efficiency of photoelectron generation per incident photon

13. Wavelengths belonging to Balmer series for hydrogen atom lying in the range of 450 nm to 750 nm were used to eject photoelectrons from a metal surface whose work function is 2.0 eV. Find (in eV) the maximum kinetic energy of the emitted photoelectrons.

14. Assume that the de-Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance d between the atoms of the array is 2 Å. A similar standing wave is again formed if d is increased to 2.5 Å but not for any intermediate value of d. Find the energy of the electron in eV and the least value of d for which the standing wave of the type described above can form.

15. The negative muon has a charge equal to that of an electron but a mass that is 207 times as great. Consider hydrogen like atom consisting of a proton and a muon.
(a) What is the reduced mass of the atom?
(b) What is the ground-level energy (in eV)?
(c) What is the wavelength of the radiation emitted in the transition from the $n = 2$ level to the $n = 1$ level?

16. Assume a hypothetical hydrogen atom in which the potential energy between electron and proton at separation r is given by $U = [k \ln r - (k/2)]$ where k is a constant. For such a hypothetical hydrogen atom, calculate the radius of n^{th} Bohr's orbit and energy levels.

17. An electron is orbiting in a circular orbit of radius r under the influence of a constant magnetic field of strength B. Assuming that Bohr's postulate regarding the quantisation of angular momentum holds good for this electron, find
(a) the allowed values of the radius r of the orbit.
(b) the kinetic energy of the electron in orbit

(c) the potential energy of interaction between the magnetic moment of the orbital current due to the electron moving in its orbit and the magnetic field B.

(d) the total energy of the allowed energy levels.

(e) the total magnetic flux due to the magnetic field B passing through the n^{th} orbit.

(Assume that the charge on the electron is $-e$ and the mass of the electron is m).

18. A mixture of hydrogen atoms (in their ground state) and hydrogen like ions (in their first excited state) are being excited by electrons which have been accelerated by same potential difference V volts. After excitation when they come directly into ground state, the wavelengths of emitted light are found in the ratio 5 : 1. Then find:

(a) The minimum value of V for which both the atoms get excited after collision with electrons.

(b) Atomic number of other ion.

(c) The energy of emitted light.

19. When a surface is irradiated with light of $\lambda = 4950$ Å a photocurrent appears which vanishes if a retarding potential 0.6 V is applied. When a different source of light is used it is found that critical retarding potential is changed to 1.1 volt. Find the work function of emitting surface and wavelength of second source. If photoelectrons after emission from surface are subjected to a magnetic field of 10 tesla, what changes will be observed in the above two retarding potentials?

20. In an experiment on photoelectric effect light of wavelength 400 nm is incident on a metal plate at the rate of 5 W. The potential of the collector plate is made sufficiently positive with respect to emitter so that the current reaches the saturation value. Assuming that on the average one out of every 10^6 photons is able to eject a photoelectron, find the photocurrent in the circuit.

21. A light beam of wavelength 400 nm is incident on a metal of work function 2.2 eV. A particular electron absorbs a photon and makes 2 collisions before coming out of the metal

(a) Assuming that 10% of existing energy is lost to the metal in each collision find the final kinetic energy of this electron as it comes out of the metal.

(b) Under the same assumptions find the maximum number of collisions the electron should suffer before it becomes unable to come out of the metal.

ANSWERS

Introductory Exercise 30.1

1. (a) 1.31 μm, (b) 164 nm
2. 656 nm
3. (a) 6.58×10^{15} Hz, 0.823×10^{15} Hz; (b) 2.46×10^{15} Hz; (c) 8.23×10^{6} revolutions
4. (a) 2.55×10^{-13} m, (b) 2.81 keV
5. (a) 4.8×10^{-34} m, (b) 7.3×10^{-11} m
6. (a) 102 nm, 122 nm, 653 nm (b) No lines
7. 5.59 nm
8. $E_1 = -4613$ eV, $E_3 = -2650$ eV
9. $\frac{3f}{4}, \frac{f}{4}$

Introductory Exercise 30.2

1. 1.9 V 2. 0.48 μA 3. $K_{max} \propto (f - f_0)$ 4. Zero, 3.19 eV 5. 1.16×10^{15} Hz

For JEE Main

Subjective Questions

1. 4.6 eV, 8.2×10^{-36} kg, $2.45\times 10^{-27}\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$ 2. 4.82×10^{16} per $m^2\cdot s$
3. (a) 2.47×10^{-19} J = 1.54 eV (b) 804 nm, infrared
4. (a) 5.0×10^{14} Hz (b) 2.3×10^{20} photons/s 5. (a) 5.92×10^{20} Hz (b) 5.06×10^{-13} m
6. (a) $K_2 = 4K_1$ (b) $E_2 = 2E_1$ 7. (a) 2.52×10^{19} (b) 3.33×10^{-8} N
8. 4.3×10^{-8} N 9. 10^{-8} N 10. 4.0×10^{-7} N/m^2 11. 3.90×10^{-34} m, No
12. (a) 1.55×10^{-10} m (b) 8.44×10^{-14} m
13. (a) $2.37\times 10^{-24}\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$ (b) 3.07×10^{-18} J = 19.2 eV 14. 1.04 Å 15. 6.663 Å
16. (a) 3.32×10^{-10} m (b) 1.33×10^{-9} m 17. 122.4 eV
18. (a) 3.4 eV (b) –6.8 eV (c) potential energy will change 19. 79 eV
20. 3.16×10^{-34} kg-m^2/s 21. 3, 1023 Å 22. 113.74 Å 23. $n = 5$
24. (a) $z = 4$ (b) $\lambda_{min} = 40441$ Å
25. (a) 15.6 Volt (b) 2335 Å (c) 12.52 V (d) $1.01\times 10^{7}\ m^{-1}$
26. (a) –5.08 eV (b) –5.63 eV 28. $\frac{27}{32}\lambda_0$ 29. 49.5 pm, 99.0 pm 30. $z \approx 41$
31. 15865 V 32. 198 pm 33. $z = 42$ 34. (a) 4 W (b) 396 J/s
35. 1022 Å, $n = 3$ to $n = 1$ 36. 2.48 eV 37. 2260 Å
38. (a) 10^{15} Hz (b) 4 eV (c) 6.4×10^{-34} J-s
39. 1.81 eV, 9.0×10^{5} m/s, 3.0×10^{5} m/s 40. $6.25\times 10^{11} s^{-1}$, zero, 5.0 eV
41. 0.148 m 42. 3.27 eV 43. 1.2 V

Objective Questions

1. (d)	2. (c)	3. (c)	4. (b)	5. (c)	6. (a)	7. (a)	8. (c)	9. (c)	10. (a)
11. (c)	12. (c)	13. (a)	14. (a)	15. (c)	16. (c)	17. (b)	18. (d)	19. (a)	20. (c)
21. (c)	22. (a)	23. (b)	24. (c)	25. (c)	26. (b)	27. (b)	28. (c)	29. (b)	30. (c)
31. (a)	32. (b)	33. (b)	34. (c)	35. (a)	36. (b)	37. (c)	38. (d)		

For JEE Advanced

Assertion and Reason

1. (a) **2.** (c) **3.** (a,b) **4.** (a) **5.** (a,b) **6.** (d) **7.** (a,b) **8.** (b) **9.** (b) **10.** (d)

Objective Questions

1. (b) **2.** (b) **3.** (a) **4.** (c) **5.** (c) **6.** (b) **7.** (d) **8.** (a) **9.** (d) **10.** (c)
11. (c) **12.** (c) **13.** (d) **14.** (c) **15.** (a) **16.** (c) **17.** (d) **18.** (b) **19.** (d) **20.** (d)
21. (a) **22.** (c) **23.** (c) **24.** (c) **25.** (c) **26.** (b) **27.** (b) **28.** (c)

More than One Correct Options

1.(b,c) **2.**(a,c) **3.**(a,b,c) **4.**(a,c) **5.**(a,b,d) **6.**(a,b,c)

Match the Columns

1. (a) → r (b) → q (c) → p (d) → p
2. (a) → s (b) → r (c) → s (d) → p
3. (a) → q (b) → p (c) → r (d) → s
4. (a) → r (b) → s (c) → s (d) → q
5. (a) → p (b) → s (c) → q (d) → s
6. (a) → r (b) → p (c) → q (d) → q
7. (a) → p,r (b) → s (c) → q,s (d) → p,r

Subjective Questions

1. $n = 2, z = 4, -217.6$ eV **2.** 1.12×10^{12} **3.** (a) 2.27 eV (b) 2.75×10^{17} (c) 9%

4. $W = 11.925$ eV **5.** 2 eV, 0.754 V **6.** $\frac{neh}{4\pi m}, \frac{\mu_0 \pi m^2 e^7}{8\varepsilon_0 h^5 n^5}$ **7.** 793.3 Å **8.** 3.8 Å

9. (a) $n_i = 2$ (b) $z = 6$ (c) 28.43 Å (d) 489.6 eV, 25.3 Å **10.** 4.5 eV

11. (a) 2000 Å, 1500 Å (b) $E_1 = -8.25$ eV, $E_2 = -2.05$ eV and $E_3 = -0.95$ eV (c) 8.25 V

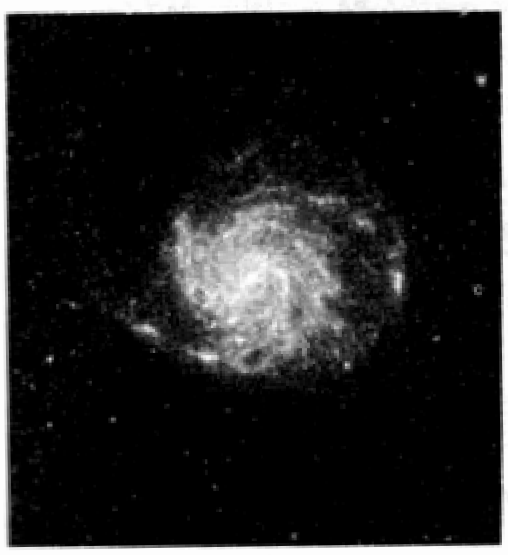
12. (a) 4125 Å (b) 34 μA (c) 5.1% **13.** 0.55 eV **14.** 150 eV, 0.5 Å

15. (a) 1.69×10^{-28} kg (b) -2.53 keV (c) 0.653 nm **16.** $r_n = \frac{nh}{2\pi\sqrt{mk}}$, $E_n = k\ln\left\{\frac{nh}{2\pi\sqrt{mk}}\right\}$

17. (a) $r_n = \sqrt{\frac{nh}{2\pi Be}}$ (b) $K = \frac{nhBe}{4\pi m}$ (c) $U = \frac{nheB}{4\pi m}$ (d) $E = \frac{nheB}{2\pi m}$ (e) $\frac{nh}{2e}$

18. (a) 10.2 volt (b) $Z = 2$ (c) 10.2 eV and 51 eV **19.** 1.9 eV, 4125 Å, No change is observed

20. 1.6μA **21.** (a) 0.31 eV (b) 4



31

MODERN PHYSICS-II

Chapter Contents

31.1 Nuclear Stability and Radioactivity
31.2 The Radioactive Decay Law
31.3 Successive Disintegration
31.4 Equivalence of Mass and Energy
31.5 Binding Energy and Nuclear Stability
31.6 Nuclear Fission (Divide and Conquer)
31.7 Nuclear Fusion
31.8 Q-Value of a Nuclear Reaction (Optional)

31.1 Nuclear Stability and Radioactivity

Among about 1500 known nuclides, less than 260 are stable. The others are unstable that decay to form other nuclides by emitting α and β-particles and γ-electromagnetic waves. This process is called radioactivity. It was discovered in 1896 by Henry Becquerel.

Whilst the chemical properties of an atom are governed entirely by the number of protons in the nucleus (*i.e.*, the proton number Z), the stability of an atom appears to depend on both the number of protons and the number of neutrons. For light nuclei, the greatest stability is achieved when the numbers of protons and neutrons are approximately equal ($N \approx Z$).

For heavier nuclei, instability caused by electrostatic repulsion between the protons is minimized when there are more neutrons than protons.

Figure shows a plot of N versus Z for the stable nuclei. For mass numbers upto about $A = 40$, we see that $N \approx Z$. 40Ca is the heaviest stable nucleus for which $N = Z$. For larger values of Z, the (short range) nuclear force is unable to hold the nucleus together against the (long-range) electrical repulsion of the protons unless the number of neutrons exceeds the number of protons. At Bi ($Z = 83$, $A = 209$), the neutron excess is $N - Z = 43$. There are no stable nuclides with $Z > 83$.

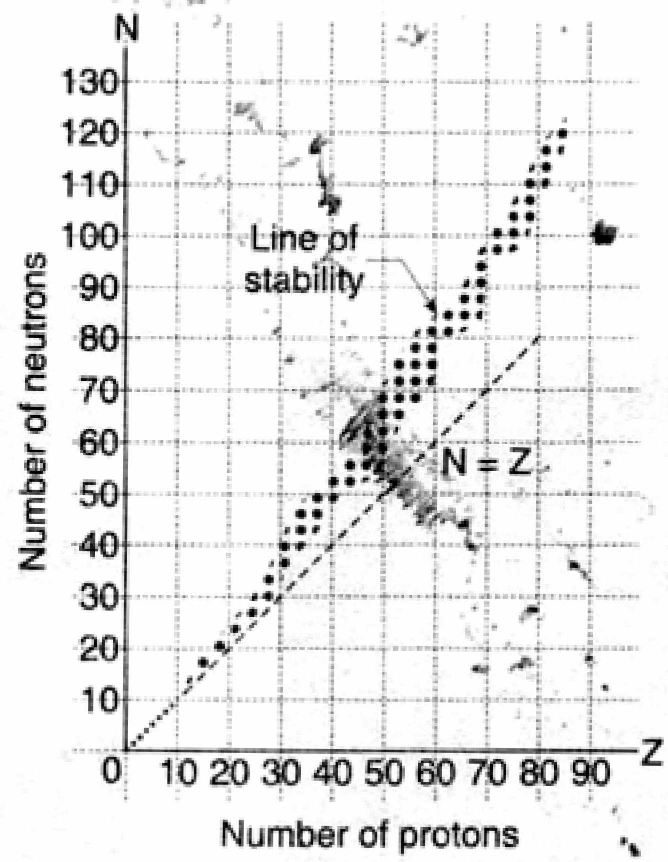


Fig. 31.1 The stable nuclides plotted on a graph of neutron number, N, versus proton number, Z. Note that for heavier nuclides, N is larger relative to Z. The stable nuclides group along a curve called the line of stability.

The nuclide $^{209}_{83}$Bi is the heaviest stable nucleus.

Atoms are radioactive if their nuclei are unstable and spontaneously (and randomly) emit various particles, the α, β and/or γ radiations. When naturally occuring nuclei are unstable, we call the phenomena **natural radioactivity.** Other nuclei can be transformed into radioactive nuclei by various means, typically involving irradiation by neutrons, this is called **artificial radioactivity.**

A radioactive nucleus is called a **parent nucleus,** the nucleus resulting from its decay by particle emission is called **daughter nucleus.** Daughter nuclei also might be granddaughter nuclei, an so on. There are no son or grandson nuclei. For unstable nuclides and radioactivity following points can be made.

(i) Disintegrations tend to produce new nuclides near the stability line and continue until a stable nuclide is formed.

(ii) Radioactivity is a nuclear property, *i.e.*, α, β and γ emission take place from the nucleus.

(iii) Nuclear processes involve huge amount of energy so the particle emission rate is independent of temperature and pressure. The rate depends solely on the concentration of the number of atoms of the radioactive substance.

(iv) A radioactive substance is either an α-emitter or a β-emitter. γ-rays emit with both.

Alpha Decay

An alpha particle is a helium nucleus. Thus a nucleus emitting an alpha particle loses two protons and two neutrons. Therefore, the atomic number Z decreases by 2, the mass number A decreases by 4 and the neutron number N decreases by 2. The decay can be written as,

$$^{A}_{Z}X^{A} = {}^{A-4}_{Z-2}Y + {}^{4}_{2}\text{He}$$

where X is the parent nucleus and Y the daughter nucleus. As examples U^{238} and Ra^{226} are both alpha emitters and decay according to,

$$^{238}_{92}\text{U} \longrightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$$

$$^{226}_{88}\text{Ra} \longrightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He}$$

As a general rule in any decay sum of mass numbers A and atomic numbers Z must be the same on both sides.

Note that a nuclide below the stability line in Fig. 31.1 disintegrates in such a way that its proton number decreases and its neutron to proton ratio increases. In heavy nuclides this can occur by alpha emission.

If the original nucleus has a mass number A that is 4 times an integer, the daughter nucleus and all those in the chain will also have mass numbers equal to 4 times an integer. (Because in α-decay A decreases by 4 and in β-decay it remains the same). Similarly, if the mass number of the original nucleus is $4n+1$, where n is an integer, all the nuclei in the decay chain will have mass numbers given by $4n+1$ with n decreasing by 1 in each α-decay. We can see therefore, that there are four possible α-decay chains, depending on whether A equals $4n$, $4n+1$, $4n+2$ or $4n+3$ where n is an integer.

Series $4n+1$ is now not found. Because its longest lived member (other than the stable end product Bi^{209}) is Np^{237} which has a half life of only 2×10^{6} years. Because this is much less than the age of the earth this series has disappeared.

Figure shows the Uranium $(4n+2)$ series.

The series branches at Bi^{214} which decays either by α-decay to TI^{210} or β-decay to Po^{214}. The branches meet at the lead isotope Pb^{210}. Table 31.1 lists the four radioactive series.

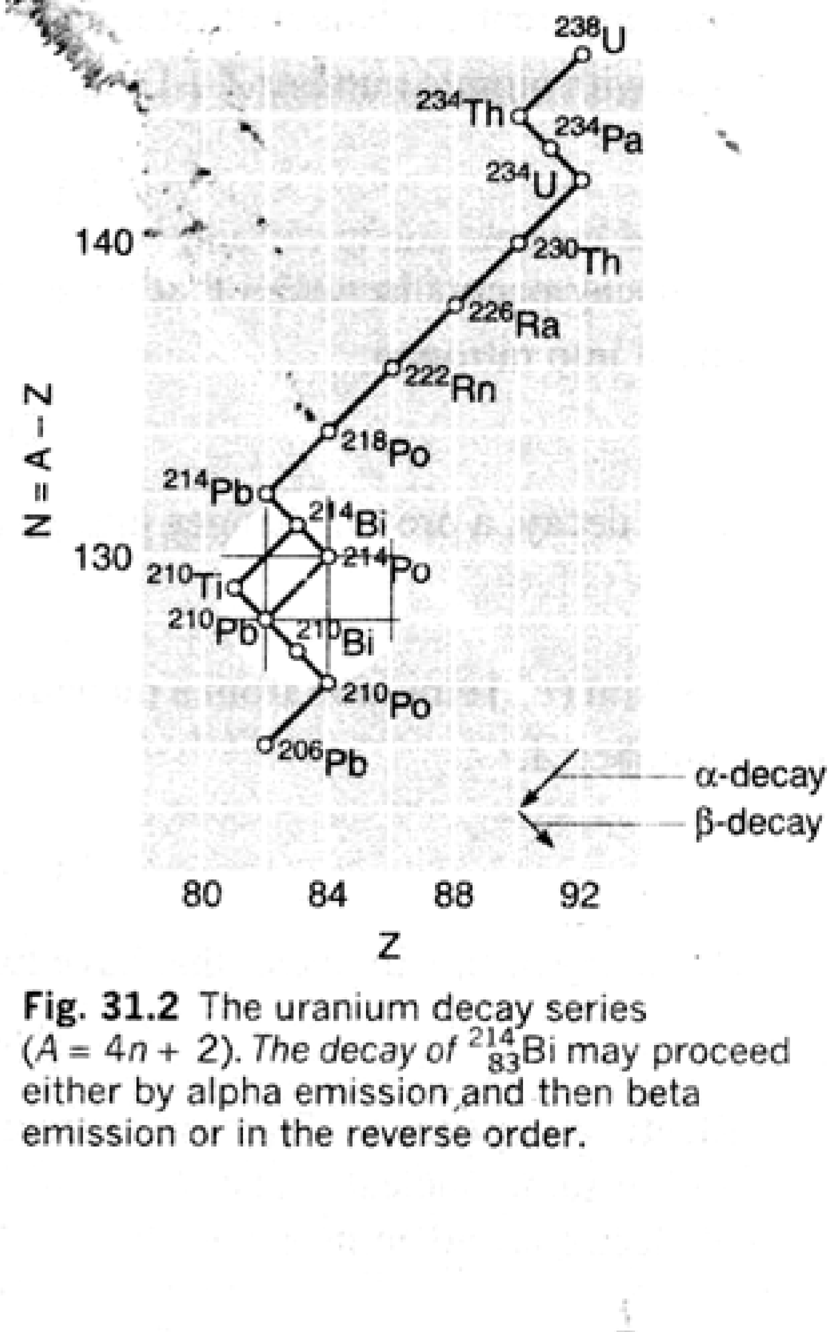


Fig. 31.2 The uranium decay series ($A = 4n + 2$). *The decay of* $^{214}_{83}$Bi may proceed either by alpha emission and then beta emission or in the reverse order.

Table 31.1 Four Radioactive Series.

Mass Numbers	Series	Parent	Half-Life, Years	Stable Product
$4n$	Thorium	$^{232}_{90}$Th	1.39×10^{10}	$^{208}_{82}$Pb
$4n+1$	Neptunium	$^{237}_{93}$Np	2.25×10^{6}	$^{209}_{83}$Bi
$4n+2$	Uranium	$^{238}_{92}$U	4.47×10^{9}	$^{206}_{82}$Pb
$4n+3$	Actinium	$^{235}_{92}$U	7.07×10^{8}	$^{207}_{82}$Pb

Beta Decay

Beta decay can involve the emission of either electrons or positrons. A positron is a form of antimatter. Which has a charge equal to $+e$ and mass equal to that of an electron. The electrons or positrons emitted in β-decay do not exist inside the nucleus. They are only created at the time of emission, just as photons are created when an atom makes a transition from a higher to a lower energy state.

In β^- decay a neutron in the nucleus is transformed into a proton, an electron and an antineutrino.

$$n \longrightarrow P + e^- + \bar{\nu}$$

To conserve energy and momentum in the process, the emission of an antineutrino ($\bar{\nu}$) (alongwith proton and electron) was first suggested by W. Pauli in 1930, but it was first observed experimentally in 1957.

Thus a parent nucleus with atomic number Z and mass number A decays by β^- emission into a daughter with atomic number $Z+1$ and the same mass number A.

$${}^{A}_{Z}X \xrightarrow{\beta^-} {}_{Z+1}^{A}Y$$

β^- decay occurs in nuclei that have too many neutrons. An example of β^- decay is the decay of carbon-14 into nitrogen,

$${}^{14}_{6}\text{C} \longrightarrow {}^{14}_{7}\text{N} + e^- + \bar{\nu}$$

In β^+ decay, a proton changes into a neutron with the emission of a positron (and a neutrino)

$$P \longrightarrow n + e^+ + \nu$$

Positron (e^+) emission from a nucleus decreases the atomic number Z by 1 while keeping the same mass number A.

$${}^{A}_{Z}X \xrightarrow{\beta^+} {}_{Z-1}^{A}Y$$

B^+ decay occurs in nuclei that have two few neutrons. A typical β^+ decay is,

$${}^{13}_{7}\text{N} \longrightarrow {}^{13}_{6}\text{C} + e^+ + \nu$$

Electron capture : Electron capture is competitive with positron emission since both processes lead to the same nuclear transformation. This occurs when a parent nucleus captures one of its own orbital electrons and emits a neutrino.

$${}^{A}_{Z}X + e^{-1} \longrightarrow {}_{Z-1}^{A}Y + \nu$$

In most cases, it is a K-shell electron that is captured, and for this reason the process is referred to as ***K*-capture.** One example is the capture of an electron by ${}_4\text{Be}^7$

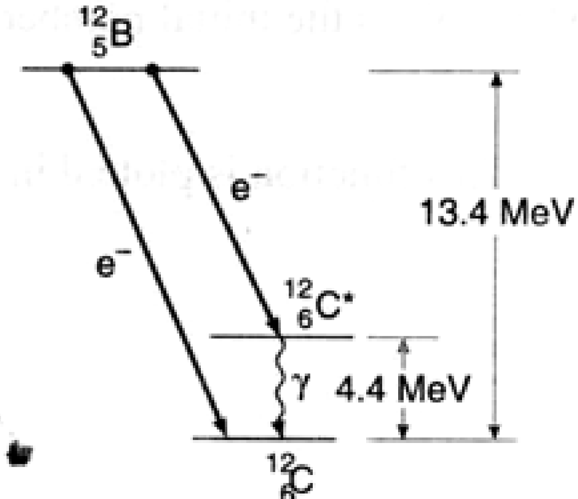
$${}^{7}_{4}\text{Be} + e^- \longrightarrow {}^{7}_{3}\text{Li} + \nu$$

Gamma Decay

Very often a nucleus that undergoes radioactive decay (α or β decay) is left in an excited energy state (analogous to the excited states of the orbiting electrons, except that the energy levels associated with the nucleus have much larger energy differences than those involved with the atomic electrons). The typical half-life of an excited nuclear state is 10^{-10} s. The excited nucleus (X^*) then undergoes to a lower energy state, by emitting a high energy photon, called the γ-ray photon. The following sequence of events represents a typical situation in which γ-decay occurs.

$${}^{12}_{5}B \longrightarrow {}^{12}_{6}C^* + e^- + \bar{\nu}$$

$${}^{12}_{6}C^* \longrightarrow {}^{12}_{6}C + \gamma$$



Gamma decay

Fig. 31.3

Figure shows decay of B^{12} nucleus, which undergoes β-decay to either of two levels of C^{12}. It can either decay directly to the ground state of C^{12} by emitting a 13.4 MeV electron or undergo β-decay to an excited state of ${}^{12}_{6}C^*$ followed by γ-decay to the ground state. The latter process results in the emission of a 9.0 MeV electron and a 4.4 MeV photon. The various pathways by which a radioactive nucleus can undergo decay are summarized in Table 31.2.

Note *In both α and β decay, the Z value of a nucleus changes and the nucleus of one element becomes the nucleus of a different element. In γ-decay, the element does not change, the nucleus merely goes from an excited state to a less excited state.*

Table 31.2 Various Decay Pathways

Alpha decay	${}^{A}_{Z}X^{A} \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$
Beta decay (β^-)	${}^{A}_{Z}X \longrightarrow {}_{Z+1}^{A}Y + e^- + \bar{\nu}$
Beta decay (β^+)	${}^{A}_{Z}X \longrightarrow {}_{Z-1}^{A}Y + e^+ + \nu$
Electron capture	${}^{A}_{Z}X + e^- \longrightarrow {}_{Z-1}^{A}Y + \nu$
Gamma decay	${}^{A}_{Z}X^* \longrightarrow {}^{A}_{Z}X + \gamma$

31.2 The Radioactive Decay Law

Radioactive decay is a random process. Each decay is an independent event and one cannot tell when a particular nucleus will decay. When a particular nucleus decays, it is transformed into another nuclide, which may or may not be radioactive. When there is a very large number of nuclei in a sample, the rate of decay is proportional to the number of nuclei, N, that are present,

$$\left(-\frac{dN}{dt}\right) \propto N \quad \text{or} \quad \left(-\frac{dN}{dt}\right) = \lambda N$$

where λ is called the **decay constant.** This equation may be expressed in the form $\frac{dN}{N} = -\lambda dt$ and integrated,

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt \quad \text{or} \quad \ln\left(\frac{N}{N_0}\right) = -\lambda t$$

where N_0 is the initial number of parent nuclei at $t = 0$. The number that survive at time t is therefore,

$$N = N_0 e^{-\lambda t} \quad \text{...(i)}$$

This function is plotted in Fig. 31.4.

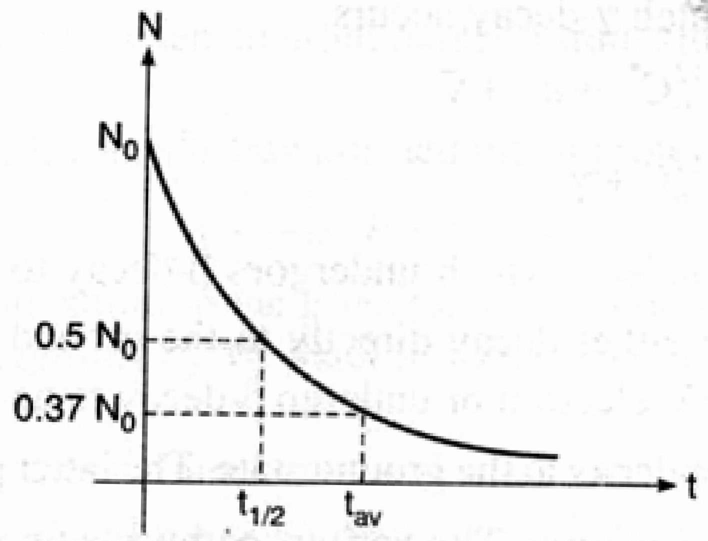


Fig. 31.4

Half-Life : The time required for the number of parent nuclei to fall to 50% is called half-life $t_{1/2}$ and may be related to λ as follows. Since,

$$0.5N_0 = N_0 e^{-\lambda t_{1/2}}$$

We have

$$\lambda t_{1/2} = \ln(2) = 0.693$$

∴

$$t_{1/2} = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda} \quad \text{...(ii)}$$

Mean life : The average or mean life t_{av} is the reciprocal of the decay constant.

$$t_{av} = \frac{1}{\lambda} \quad \text{...(iii)}$$

The mean life is analogous to the time constant in the exponential decrease in the charge on a capacitor in an RC circuit. After a time equal to the mean life time, the number of radioactive nuclei and the decay rate have each decreased to 37% of their original values.

Activity of a radioactive substance

The decay rate R of a radioactive substance is the number of decays per second. And as we have seen above

$$-\frac{dN}{dt} \propto N \quad \text{or} \quad -\frac{dN}{dt} = \lambda N$$

Thus,

$$R = -\frac{dN}{dt} \quad \text{or} \quad R \propto N$$

or

$$R = \lambda N \quad \text{or} \quad R = \lambda N_0 e^{-\lambda t}$$

or

$$R = R_0 e^{-\lambda t} \quad \text{...(iv)}$$

where $R_0 = \lambda N_0$ is the activity of the radioactive substance at time $t = 0$. The activity versus time graph is shown in Fig. 31.5.

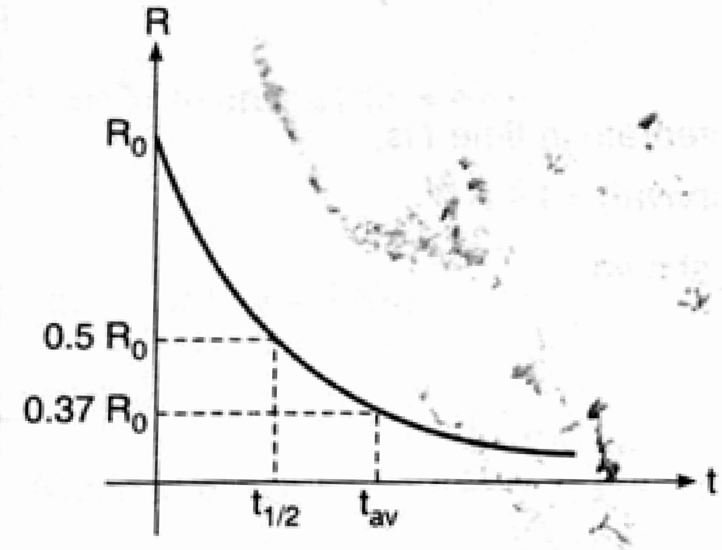


Fig. 31.5

Thus, the number of nuclei and hence the activity of the radioactive substance decrease exponentially with time.

Units of activity : The SI unit for the decay rate is the becqueral (Bq), but the curie (Ci) and rutherford (rd) are often used in practice.

$$1\,\text{Bq} = 1\,\text{decays/s}, \quad 1\,\text{Ci} = 3.7 \times 10^{10}\,\text{Bq} \quad \text{and} \quad 1\,\text{rd} = 10^6\,\text{Bq}$$

• *Important Points in Radioactivity*

1. After n half lives,

   (a) number of nuclei left $= N_0\left(\frac{1}{2}\right)^n$

   (b) fraction of nuclei left $= \left(\frac{1}{2}\right)^n$ and

   (c) percentage of nuclei left $= 100\left(\frac{1}{2}\right)^2$

2. Number of nuclei decayed after time t,

$$= N_0 - N$$
$$= N_0 - N_0 e^{-\lambda t} = N_0(1 - e^{-\lambda t})$$

   The corresponding graph is as shown in figure.

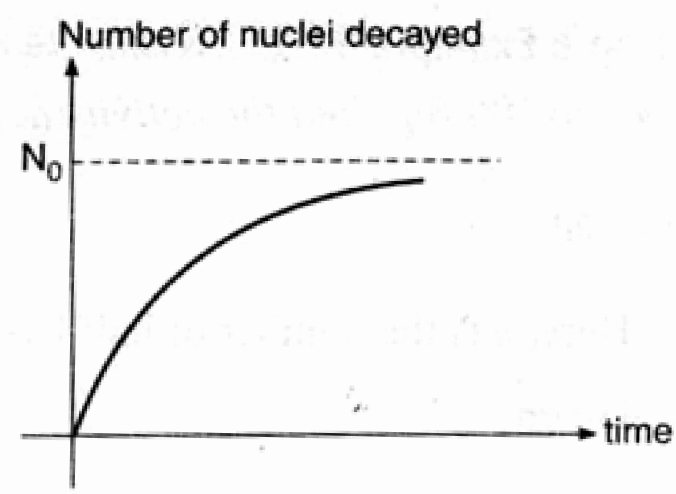


Fig. 31.6

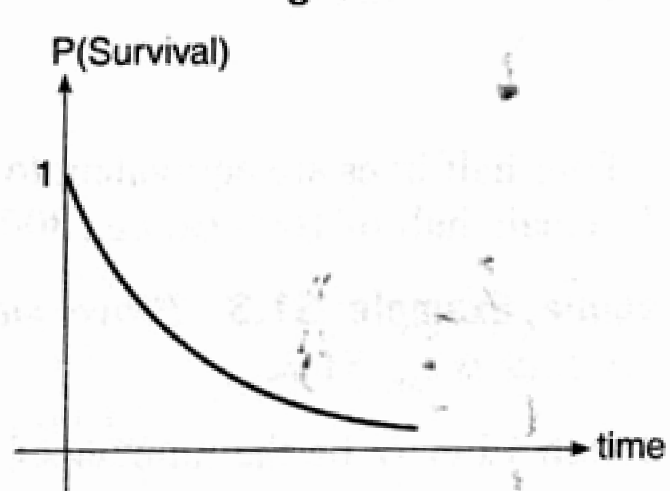


Fig. 31.7

3. Probability of a nucleus for survival of time t,

$$P\,(\text{survival}) = \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$$

   The corresponding graph is shown in figure.

4. Probability of a nucleus to disintegrate in time t is,

$$P\text{ (disintegration)} = 1 - P\text{ (survival)} = 1 - e^{-\lambda t}$$

The corresponding graph is as shown.

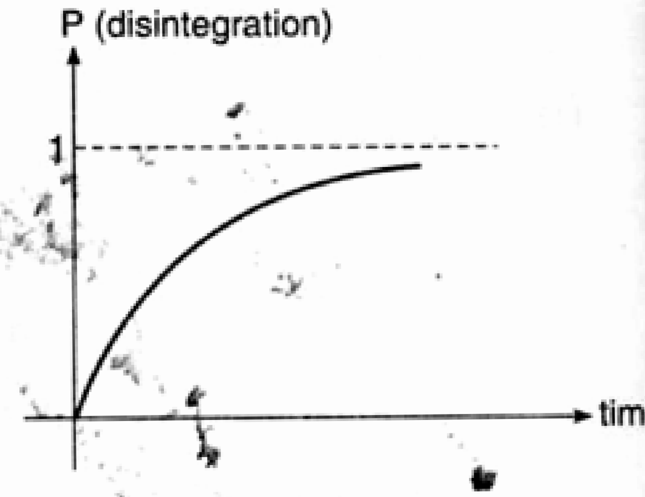


Fig. 31.8

5. Half life and mean life are related to each other by the relation,

$$t_{1/2} = 0.693\, t_{av} \quad \text{or} \quad t_{av} = 1.44 t_{1/2}$$

6. As we said in point number (2), number of nuclei decayed in time t are $N_0(1 - e^{-\lambda t})$. This expression involves power of e.

So, to avoid it we can use,

$$\Delta N = \lambda N \Delta t$$

where, ΔN are the number of nuclei decayed in time Δt, at the instant when total number of nuclei are N. But this can be applied only when $\Delta t << t_{1/2}$.

7. In same interval of time, equal percentage (or fraction) of nuclei are decayed (or left undecayed).

Sample Example 31.1 *At time $t = 0$, number of nuclei of a radioactive substance are 100. At $t = 1$ s these numbers become 90. Find the number of nuclei at $t = 2$ s.*

Solution In 1 second 90% of the nuclei have remained undecayed, so in another 1 second 90% of 90 *i.e.*, 81 nuclei will remain undecayed.

Sample Example 31.2 *At time $t = 0$, activity of a radioactive substance is 1600 Bq, at $t = 8$ s activity remains 100 Bq. Find the activity at $t = 2$ s.*

Solution

$$R = R_0\left(\frac{1}{2}\right)^n$$

Here, n is the number of half lives.

Given,

$$R = \frac{R_0}{16}$$

$\therefore$

$$\frac{R_0}{16} = R_0\left(\frac{1}{2}\right)^n$$

or

$$n = 4$$

Four half lives are equivalent to 8 s. Hence, 2 s is equal to one half-life. So in one half-life activity will remain half of 1600 Bq *i.e.*, 800 Bq.

Sample Example 31.3 *Prove mathematically that mean life or average life of a radioactive substance is $t_{av} = 1/\lambda$.*

Solution Let N be the number of atoms that exist at time t. Between t and $t + dt$ let dN atoms are decayed, then

$$\text{Mean or average life} = \frac{\int_{N_0}^{0} t\,dN}{\int_{N_0}^{0} dN}$$

Further, $-\dfrac{dN}{dt} = \lambda N$ or $dN = -\lambda N dt$

$$\therefore \quad \text{Mean average life} = \frac{-\int_0^\infty t\,\lambda N dt}{-N_0}$$

But $N = N_0 e^{-\lambda t}$. Hence

$$\text{Mean life} = \frac{\int_0^\infty t\lambda N_0 e^{-\lambda t} dt}{N_0}$$

This integration is done by parts. The result is,

$$t_{av} = \frac{1}{\lambda}$$

Hence proved.

Sample Example 31.4 *Uranium ores on the earth at the present time typically have a composition consisting of 99.3% of the isotope* $_{92}U^{238}$ *and 0.7% of the isotope* $_{92}U^{235}$. *The half lives of these isotopes are* $4.47 \times 10^9\ yr$ *and* $7.04 \times 10^8\ yr$ *respectively. If these isotopes were equally abundant when the earth was formed, estimate the age of the earth.*

Solution Let N_0 be number of atoms of each isotope at the time of formation of the earth ($t = 0$) and N_1 and N_2 the number of atoms at present ($t = t$). Then

$$N_1 = N_0 e^{-\lambda_1 t} \quad \text{...(i)}$$

and

$$N_2 = N_0 e^{-\lambda_2 t} \quad \text{...(ii)}$$

$$\therefore \quad \frac{N_1}{N_2} = e^{(\lambda_2 - \lambda_1)t} \quad \text{...(iii)}$$

Further it is given that

$$\frac{N_1}{N_2} = \frac{99.3}{0.7} \quad \text{...(iv)}$$

Equating Eqs. (iii) and (iv) and taking log both sides, we have

$$(\lambda_2 - \lambda_1)\,t = \ln\left(\frac{99.3}{0.7}\right)$$

$$\therefore \quad t = \left(\frac{1}{\lambda_2 - \lambda_1}\right) \ln\left(\frac{99.3}{0.7}\right)$$

Substituting the values, we have

$$t=\frac{1}{\dfrac{0.693}{7.04\times10^{8}}-\dfrac{0.693}{4.47\times10^{9}}}\ln\left(\frac{99.3}{0.7}\right)$$

or $$t=5.97\times10^{9}\ \text{yr}$$ **Ans.**

Introductory Exercise 31.1

1. Activity of a radioactive substance decreases from 8000 Bq to 1000 Bq in 9 days. What is the half-life and average life of the radioactive substance?
2. A radioactive substance has a half-life of 64.8 h. A sample containing this isotope has an initial activity ($t=0$) of 40 μCi. Calculate the number of nuclei that decay in the time interval between $t_1=10.0$ h and $t_2=12.0$ h.
3. A freshly prepared sample of a certain radioactive isotope has an activity of 10 mCi. After 4.0 h its activity is 8.00 mCi.
   (a) Find the decay constant and half life
   (b) How many atoms of the isotope were contained in the freshly prepared sample?
   (c) What is the sample's activity 30.0 h after it is prepared?
4. A radioactive substance contains 10^{15} atoms and has an activity of 6.0×10^{11} Bq. What is its half-life?
5. Two radioactive elements X and Y have half-life periods of 50 minutes and 100 minutes respectively. Initially both of them contain equal number of atoms. Find the ratio of atoms left N_X/N_Y after 200 minutes.

31.3 Successive Disintegration

Suppose a parent radioactive nucleus A (decay constant $=\lambda_a$) has number of atoms N_0 at time $t=0$. After disintegration it converts into a nucleus B (decay constant $=\lambda_b$) which is further radioactive. Initially ($t=0$), number of atoms of B are zero. We are interested in finding N_b, the number of atoms of B

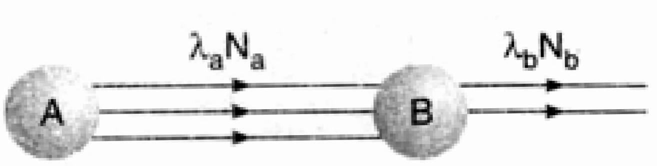


Fig. 31.9

at time t.

At $t=0$	N_0	0
At $t=t$	$N_a=N_0e^{-\lambda_a t}$	$N_b=?$

At time t, net rate of formation of B = rate of disintegration of A – rate of disintegration of B.

$\therefore$ $$\frac{dN_b}{dt}=\lambda_a N_a-\lambda_b N_b$$

or $$\frac{dN_b}{dt}=\lambda_a N_0e^{-\lambda_a t}-\lambda_b N_b \qquad (\text{as } N_a=N_0e^{-\lambda_a t})$$

or $$dN_b+\lambda_b N_b\,dt=\lambda_a N_0e^{-\lambda_a t}$$

Multiplying this equation by $e^{\lambda_b t}$, we have

$$e^{\lambda_b t} dN_b + e^{\lambda_b t} \lambda_b N_b \, dt = \lambda_a N_0 e^{(\lambda_b - \lambda_a)t}$$

$$\therefore \qquad d\{N_b e^{\lambda_b t}\} = \lambda_a N_0 e^{(\lambda_b - \lambda_a)t} dt$$

Integrating both sides, we get

$$N_b e^{\lambda_b t} = \left(\frac{\lambda_a}{\lambda_b - \lambda_a} \right) N_0 e^{(\lambda_b - \lambda_a)t} + c \qquad \text{...(i)}$$

where c is the constant of integration, which can be found as under.

At time, $t = 0$, $N_b = 0$

$$\therefore \qquad c = -\left(\frac{\lambda_a}{\lambda_b - \lambda_a} \right) N_0$$

Substituting this value in Eq. (i), we have

$$N_b = \frac{N_0 \lambda_a}{\lambda_b - \lambda_a} (e^{-\lambda_a t} - e^{-\lambda_b t}) \qquad \text{...(ii)}$$

Now following conclusions may be drawn from the above discussion.

(1) From Eq. (ii) we can see that $N_b = 0$ at time $t = 0$ (it was given) and at $t = \infty$ (because B is also radioactive)

(2) N_a will continuously decrease while N_b will first increase (until $\lambda_a N_a > \lambda_b N_b$), reaches to a maximum value (when $\lambda_a N_a = \lambda_b N_b$) and then decreases (when $\lambda_b N_b > \lambda_a N_a$). The two graphs for N_a and N_b with time are shown below.

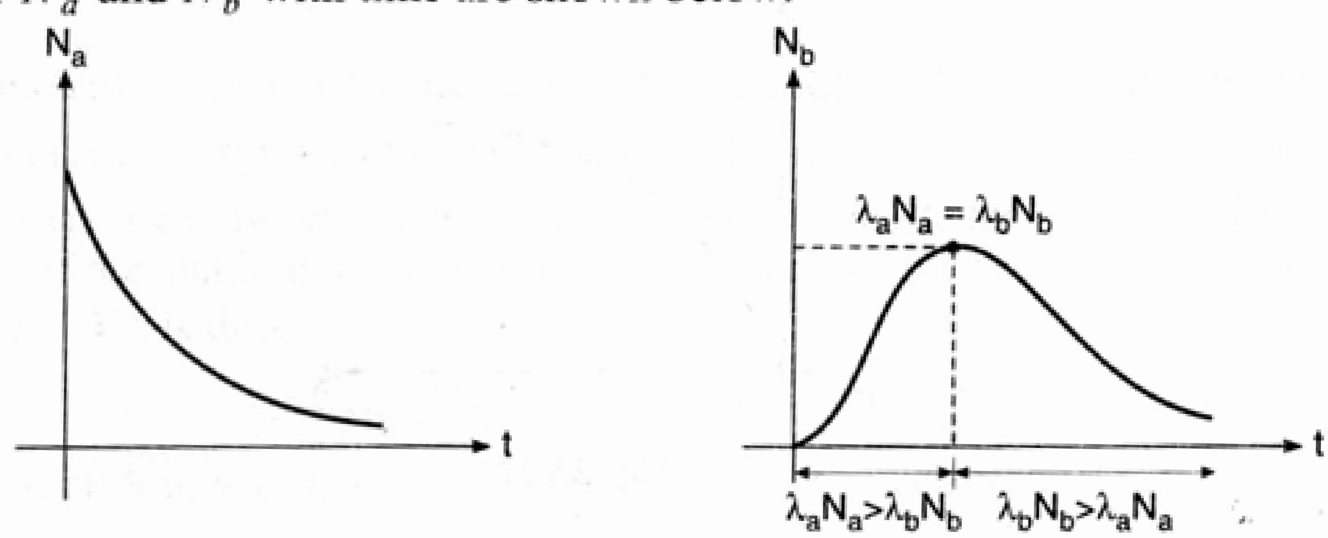


Fig. 31.10

(3) From equation number (ii) it seems as if λ_b should be greater than λ_a for this equation to hold good but it is not so. Because if $\lambda_b > \lambda_a$ then $e^{-\lambda_a t} > e^{-\lambda_b t}$ and N_b will be positive and if $\lambda_a > \lambda_b$ then $e^{-\lambda_a t} < e^{-\lambda_b t}$ and again N_b is positive.

Sample Example 31.5 *A radio nuclide X is produced at constant rate α. At time $t = 0$, number of nuclei of X are zero. Find*

(a) the maximum number of nuclei of X.

(b) the number of nuclei at time t.

Decay constant of X is λ.

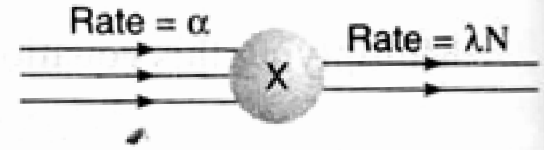


Fig. 31.11

Solution (a) Let N be the number of nuclei of X at time t.

Rate of formation of $X = \alpha$ (given)

Rate of disintegration $= \lambda N$

Number of nuclei of X will increase until both the rates will become equal. Therefore,

$$\alpha = \lambda N_{max}$$

$$\therefore \quad N_{max} = \frac{\alpha}{\lambda} \qquad \textbf{Ans.}$$

(b) Net rate of formation of X at time t is,

$$\frac{dN}{dt} = \alpha - \lambda N$$

$$\therefore \quad \frac{dN}{\alpha - \lambda N} = dt$$

Integrating with proper limits, we have

$$\int_0^N \frac{dN}{\alpha - \lambda N} = \int_0^t dt$$

or
$$N = \frac{\alpha}{\lambda}(1 - e^{-\lambda t}) \qquad \textbf{Ans.}$$

This expression shows that number of nuclei of X are increasing exponentially from 0 to $\frac{\alpha}{\lambda}$.

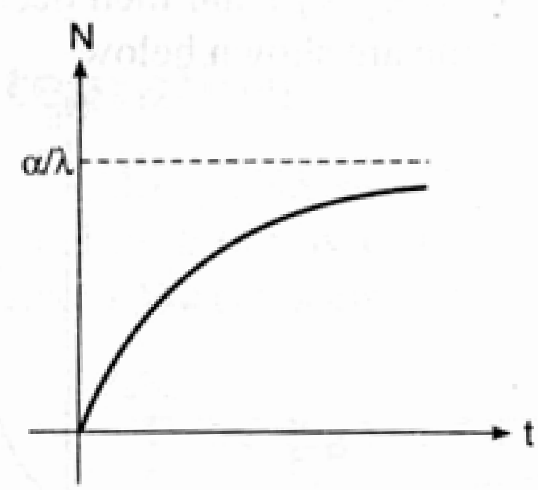


Fig. 31.12

31.4 Equivalence of Mass and Energy

In 1905, while developing his special theory of relativity, Einstein made the suggestion that energy and mass are equivalent. He predicted that if the energy of a body changes by an amount E, its mass changes by an amount m given by the equation,

$$E = mc^2$$

where c is the speed of light. Everyday examples of energy gain are much too small to produce detectable changes of mass. But in nuclear physics this plays an important role. Mass appears as energy and the two can be regarded as equivalent. In nuclear physics mass is measured in **unified atomic mass units (u)**, 1 u being one-twelfth of the mass of carbon-12 atom and equals 1.66×10^{-27} kg. It can readily be shown using $E = mc^2$ that, 1 u mass has energy 931.5 MeV

Thus, $$1\ u \equiv 931.5\ MeV$$

or $$c^2 = 931.5\ MeV/u$$

A unit of energy may therefore be considered to be a unit of mass. For example, the electron has a rest mass of about 0.5 MeV.

If the principle of conservation of energy is to hold for nuclear reactions it is clear that mass and energy must be regarded as equivalent. The implication of $E = mc^2$ is that any reaction producing an appreciable mass decrease is a possible source of energy.

Sample Example 31.6 *Find the increase in mass of water when 1.0 kg of water absorbs 4.2×10^3 J of energy to produce a temperature rise of 1 K.*

Solution
$$m = \frac{E}{c^2} = \frac{4.2 \times 10^3}{(3.0 \times 10^8)^2}\ kg$$
$$= 4.7 \times 10^{-14}\ kg$$ **Ans.**

31.5 Binding Energy and Nuclear Stability

The existence of a stable nucleus means that the nucleons (protons and neutrons) are in a bound state. Since the protons in a nucleus experience strong electrical repulsion, there must exist a stronger attractive force that holds the nucleus together. The **nuclear force** is a short range interaction that extends only to about 2 fm. (In contrast, the electromagnetic interaction is a long-range interaction). An important feature of the nuclear force is that it is essentially the same for all nucleons, independent of charge.

The **binding energy** (E_b) of a nucleus is the energy required to completely separate the nucleons. The origin of the binding energy may be understood with the help of mass-energy relation, $\Delta E = \Delta mc^2$, where Δm is the difference between the total mass of the separated nucleons and the mass of the stable nucleus. The mass of the stable nucleus is less than the sum of the mass of its nucleons. The binding energy of a nuclide $_zX^A$ is thus,

$$E_b = [zm_p + (A - z)\, m_N - m_X]\, c^2 \qquad \text{...(i)}$$

where m_p = mass of proton, m_N = mass of neutron and m_X = mass of nucleus.

Note *(1) $\Delta m = [zm_p + (A - z)\, m_N - m_X]$ is called the **mass defect**. This much mass is lost during the formation of a nucleus. Energy $\Delta E = (\Delta m)\, c^2$ is liberated during the making of the nucleus. This is the energy due to which nucleons are bound together. So, to break the nucleus in its constituent nucleons this much energy has to be given to the nucleus.*

(2) It is better to write Eq. (i) as under,

$$E_b = [zm_H + (A - z)\, m_N - m_A]\, c^2 \qquad \text{...(ii)}$$

where m_H is the mass of H atom and m_A the atomic mass. By using the masses of H atoms rather than protons, masses of the electrons in the atom cancel out. We do this because it is atomic masses that are measured directly by mass spectrometer. A slight error is made by doing so but that is negligible.

*(3) **Stability :** Although nuclides with z values upto z = 92 (Uranium) occur naturally, not all of these are stable. The nuclide $^{209}_{83}Bi$ is the heaviest stable nucleus. Even though uranium is not stable, however, its long lived isotope ^{238}U, has a half-life of some 4 billion year.*

Binding energy per nucleon : If the binding energy of a nucleus is divided by its mass number, the binding energy per nucleon is obtained. A plot of binding energy per nucleon E_b/A as a function of mass number A for various stable nuclei is shown in figure.

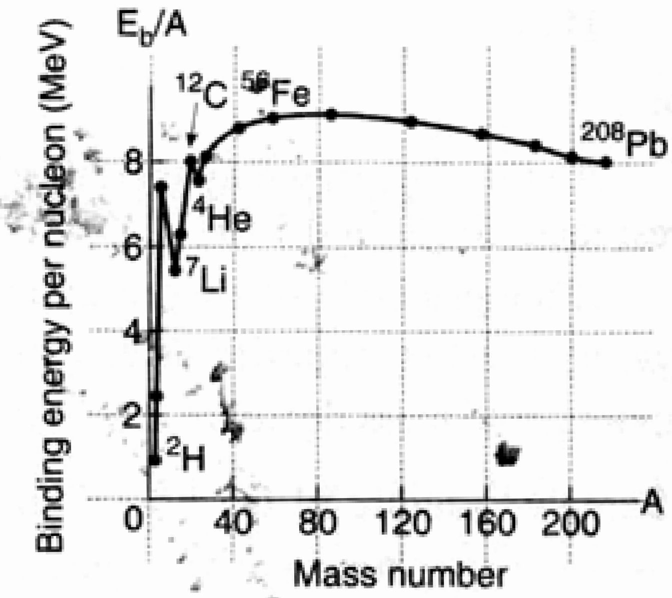


Fig. 31.13 The binding energy per nucleon, E_b/A, as a function of the mass number A.

Note *That it is the binding energy per nucleon which is more important for stability of a nucleus rather than the total binding energy.*

Following conclusions can be drawn from the above graph.

(1) The greater the binding energy per nucleon the more stable is the nucleus. The curve reaches a maximum of about 8.75 MeV in the vicinity of $^{56}_{26}Fe$ and then gradually falls to 7.6 MeV for $^{238}_{92}U$.

(2) In a nuclear reaction energy is released if total binding energy is increasing. Let us take an example.

Suppose a nucleus X, which has total binding energy of 100 MeV converts into some another nucleus Y which has total binding energy 120 MeV. Then in this process 20 MeV energy will be released. This is bacause 100 MeV energy has already been released during the formation of X while in case of Y it is 120 MeV. So the remaining 20 MeV will be released now.

Energy is released if ΣE_b is increasing.

(3) ΣE_b in a nuclear process is increased if binding energy per nucleon of the daughter products gets increased. Let us take an example. Consider a nucleus X $(A_X = 100)$ breaks in two lighter nuclei Y $(A_Y = 60)$ and $Z(A_Z = 40)$.

$$X \rightarrow Y + Z$$

Binding energy per nucleon of these three are say, 7 MeV, 7.5 MeV and 8.0 MeV. Then total binding energy of X is $100 \times 7 = 700$ MeV and that of $Y + Z$ is $(60 \times 7.5) + (40 \times 8.0) = 770$ MeV. So in this process 70 MeV energy will be released.

(4) Binding energy per nucleon is increased if two or more lighter nuclei combine to form a heavier nucleus. This process is called **nuclear fusion.**

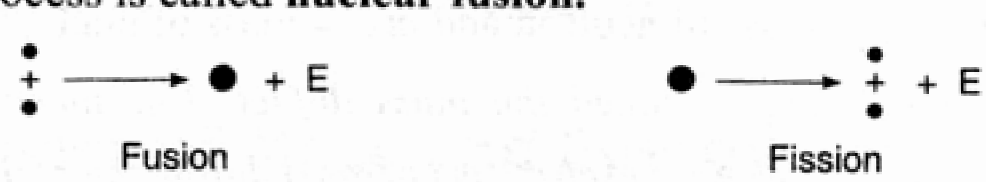


Fig. 31.14

In **nuclear fission** a heavy nucleus splits into two or more lighter nuclei of almost equal mass.

In both the processes E_b/A is increasing. Thus, energy will be released.

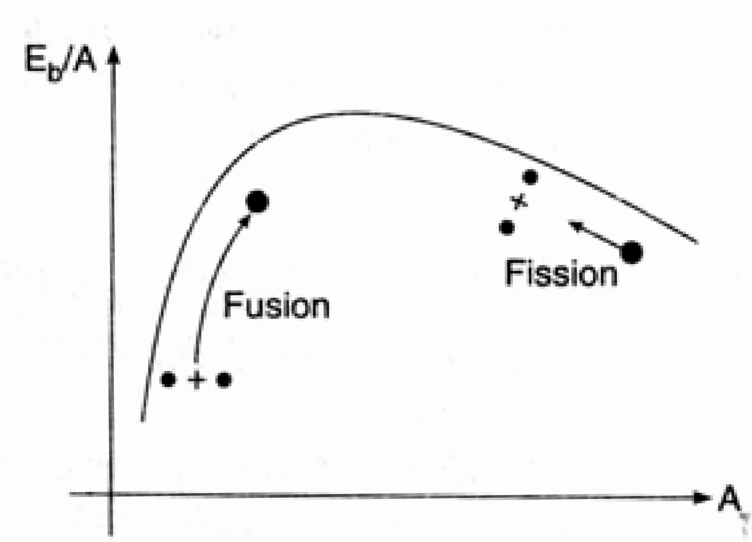


Fig. 31.15

31.6 Nuclear Fission (Divide and Conquer)

As we saw in the above article nuclear fission occurs when a heavy nucleus such as ^{235}U, splits into two lighter nuclei. In nuclear fission the combined mass of the daughter nuclei is less than the mass of the parent nucleus. The difference is called the mass defect. Fission is initiated when a heavy nucleus captures a thermal neutron (slow neutrons). Multiplying the mass defect by c^2 gives the numerical value of the released energy. Energy is released because the binding energy per nucleon of the daughter nuclei is about 1 MeV greater than that of the parent nucleus.

The fission of ^{235}U by thermal neutrons can be represented by the equation,

$$^1_0n + ^{235}_{92}\text{U} \longrightarrow ^{236}_{92}\text{U}^* \longrightarrow X + Y + \text{neutrons}$$

where ^{236}U* is an intermediate excited state that lasts only for 10^{-12} s before breaking into nuclei X and Y, which are called fission fragments. In any fission equation there are many combinations of X and Y that satisfy the requirements of conservation of energy and charge with uranium, for example, there are about 90 daughter nuclei that can be formed.

Fission also results in the production of several neutrons, typically two or three. On the average, about 2.5 neutrons are released per event. A typical fission reaction for uranium is

$$^1_0n + ^{235}_{92}\text{U} \longrightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3(^1_0n)$$

About 200 MeV is released in the fission of a heavy nucleus. The fission energy appears mostly as kinetic energy of the fission fragments (*e.g.*, barium and krypton nuclei) which fly apart at great speed. The kinetic energy of the fission neutrons also makes a slight contribution. In addition one or both of the large fragments are highly radioactive and small amount of energy takes the form of beta and gamma radiation.

Chain Reaction : Shortly after nuclear fission was discovered, it was realized that, the fission neutrons can cause further fission of ^{235}U and a chain reaction can be maintained.

In practice only a proportion of the fission neutrons is available for new fissions since, some are lost by escaping from the surface of the uranium before colliding with another nucleus. The ratio of neutrons escaping to those causing fission decreases as the size of the piece of uranium-235 increases and there is a **critical size** (about the size of a cricket ball) which must be attained before a chain reaction can start.

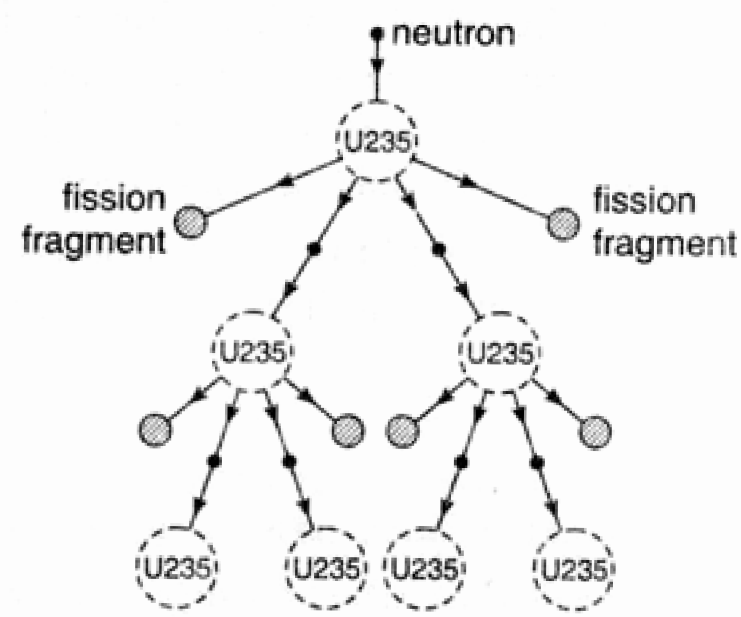


Fig. 31.16 A chain reaction

In the **'atomic bomb'** an increasing uncontrolled chain reaction occurs in a very short time when two pieces of Uranium-235 are rapidly brought together to form a mass greater than the critical size.

Nuclear Reactors : In a nuclear reactor the chain reaction is steady and controlled so that on average only one neutron from each fission produces another fission. The reaction rate is adjusted by inserting neutron-absorbing rods of boron steel into the Uranium 235.

Graphite core is used as a **moderator** to slow down the neutrons. Natural Uranium contains over 99% of ^{238}U and less than 1% of ^{235}U. The former captures the medium speed fission neutrons without fissioning. It fissions with very fast neutrons. On

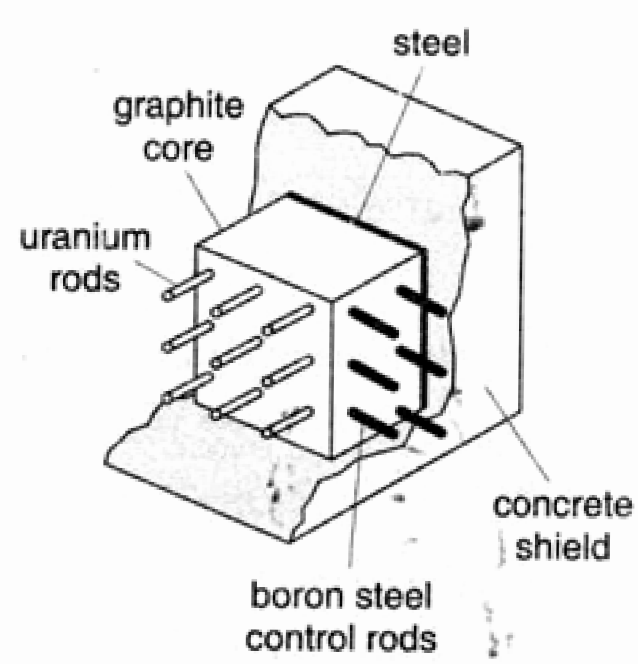


Fig. 31.17 Nuclear reactor

the other hand ^{235}U (and plutonium-239) fissions with slow neutrons and the job of moderator is to slow down the fission neutrons very quickly so that most escape capture by ^{238}U and then cause the fission of ^{235}U.

A bombarding particle gives up most energy when it has an elastic collision with a particle of similar mass. For neutrons, hydrogen atoms would be most effective but they absorb the neutrons. But deuterium (in heavy water) and carbon (as graphite) are both suitable as moderator.

To control the power level **control rods** are used. These rods are made of materials such as cadmium, that are very efficient in absorbing neutrons.

The first nuclear reactor was built by Enrico Fermi and his team at the University of Chicago in 1942.

31.7 Nuclear Fusion

Binding energy for light nuclei ($A < 20$) is much smaller than the binding energy for heavier nuclei. This suggests a process that is the reverse of fission. When two light nuclei combine to form a heavier nucleus, the process is called nuclear fusion. The union of light nuclei into heavier nuclei also lead to a transfer of mass and a consequent liberation of energy. Such a reaction has been achieved in **'hydrogen bomb'** and it is believed to be the principal source of the sun's energy.

A reaction with heavy hydrogen or deuterium which yields 3.3 MeV per fusion is,

$$^2_1H + ^2_1H \rightarrow ^3_2He + ^1_0n$$

By comparison with the 200 MeV per fission of ^{235}U this seems small, but per unit mass of material it is not. Fusion of two deuterium nuclei, *i.e.*, deuterons, will only occur if they overcome their mutual electrostatic repulsion. This may happen, if they collide at very high speed when, for example, they are raised to a very high temperature ($10^8 - 10^9$ K). So, much high temperature is obtained by using an atomic (fission) bomb to trigger off fusion.

If a controlled fusion reaction can be achieved an almost unlimited supply of energy will become available from deuterium in the water of the oceans.

Introductory Exercise 31.2

1. When fission occurs, several neutrons are released and the fission fragments are beta radioactive. why?

2. (a) How much mass is lost per day by a nuclear reactor operated at a 10^9 watt power level?

(b) If each fission releases 200 MeV, how many fissions occur per second to yield this power level?

3. Find energy released in the alpha decay

$$^{238}_{92}U \longrightarrow ^{234}_{90}Th + ^4_2He$$

Given

$$M(^{238}_{92}U) = 238.050784 \text{ u}$$
$$M(^{234}_{90}Th) = 234.043593 \text{ u}$$
$$M(^4_2He) = 4.002602 \text{ u}$$

4. Complete the nuclear reactions.

(a) $^6_3Li + ? \longrightarrow ^7_4Be + ^1_0n$

(b) $^{35}_{17}Cl + ? \longrightarrow ^{32}_{16}S + ^4_2He$

(c) $^9_4Be + ^4_2He \longrightarrow 3(^4_2He) + ?$

(d) $^{79}_{35}Br + ^2_1H \longrightarrow ? + 2(^1_0n)$

31.8 Q-Value of a Nuclear Reaction (Optional)

Consider a nuclear reaction in which a target nucleus X is bombarded by a particle a resulting in a daughter nucleus Y and a particle b.

$$a + X \rightarrow Y + b$$

Sometimes this reaction is written as,

$$X\ (a,\ b)Y$$

The reaction energy Q associated with a nuclear reaction is defined as the total energy released as a result of the reaction. Thus,

$$Q = (M_a + M_X - M_Y - M_b)c^2$$

A reaction for which Q is positive is called **exothermic.** A reaction for which Q is negative is called **endothermic.**

In an exothermic reaction, the total mass of incoming particles is greater than that of the outgoing particles and the Q value is positive. If the total mass of the incoming particles is less than that of the outgoing particles, energy is required for reaction to take-place and the reaction is said to be endothermic. Thus, an endothermic reaction does not occur unless the bombarding particle has a kinetic energy greater than $|Q|$. The minimum energy necessary for such a reaction to occur is called **threshold energy K_{th}**. The threshold energy is somewhat greater than $|Q|$ because the outgoing particles must have some kinetic energy to conserve momentum.

Thus, $\quad K_{th} > |Q| \quad$ (in endothermic reaction)

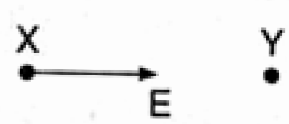


Fig. 31.18

Consider a bombarding particle X of mass m_1 and a target Y of mass m_2 (at rest). The threshold energy of X for endothermic reaction (negative value of Q) to take place is,

$$K_{th} = |Q|\left(\frac{m_1}{m_2} + 1\right)$$

Sample Example 31.7 *Find the minimum kinetic energy of an α-particle to cause the reaction* $^{14}N\ (\alpha,\ p)^{17}O$. *The masses of* ^{14}N, ^{4}He, ^{1}H *and* ^{17}O *are respectively* 14.00307 *u*, 4.00260 *u*, 1.00783 *u and* 16.99913 *u*.

Solution Since, the masses are given in atomic mass units, it is easiest to proceed by finding the mass difference between reactants and products in the same units and then multiplying by 931.5 MeV/u. Thus, we have

$$Q = (14.00307\text{ u} + 4.00260\text{ u} - 1.00783\text{ u} - 16.99913\text{ u})\left(931.5\frac{\text{MeV}}{\text{u}}\right)$$

$$= -1.20\text{ MeV}$$

Q value is negative. It means reaction is endothermic.

So, the minimum kinetic energy of α-particle to initiate this reaction would be,

$$K_{min} = |Q|\left(\frac{m_\alpha}{m_N} + 1\right)$$

$$= (1.20)\left(\frac{4.00260}{14.00307} + 1\right)$$

$$= 1.54\text{ MeV}$$ **Ans.**

Sample Example 31.8 *Neon-23 decays in the following way*

$$^{23}_{10}\text{Ne} \longrightarrow {}^{23}_{11}\text{Na} + {}^{0}_{-1}e + \bar{\nu}$$

Find the minimum and maximum kinetic energy that the beta particle ($^{0}_{-1}e$) can have. The atomic masses of ^{23}Ne *and* ^{23}Na *are* 22.9945 u *and* 22.9898 u*, respectively.*

Solution Here, atomic masses are given (not the nuclear masses), but still we can use them for calculating the mass defect because mass of electrons get cancelled both sides. Thus,

Mass defect $\Delta m = (22.9945 - 22.9898) = 0.0047\text{ u}$

$\therefore$ $Q = (0.0047\text{ u})\,(931.5\text{ MeV/u})$

$= 4.4\text{ MeV}$

Hence, the energy of beta particles can range from 0 to 4.4 MeV. **Ans.**

Classification of Nuclei :

The nuclei have been divided in isotopes, isobars and isotones on the basis of number of protons (atomic number) or the total number of nucleons (mass number).

Isotopes : The elements having the same number of protons but different number of neutrons are called isotopes. In other words isotopes have same value of atomic number (Z) but different value of mass number (A). Almost every element has isotopes. Because of the same atomic number isotopes of an element have the same place in the periodic table. The isotopes of some elements are given below.

Element	Its isotopes	Number of Protons	Number of neutrons
Hydrogen	$_1H^1$	1	0
	$_1H^2$	1	1
	$_1H^3$	1	2
Oxygen	$_8O^{16}$	8	8
	$_8O^{17}$	8	9
	$_8O^{18}$	8	10
Chlorine	$_{17}Cl^{35}$	17	18
	$_{17}Cl^{37}$	17	20
Uranium	$_{92}U^{235}$	92	143
	$_{92}U^{238}$	92	146

In nature, the isotopes of chlorine ($_{17}Cl^{35}$ and $_{17}Cl^{37}$) are found in the ratio 75.4% and 24.6%. When chlorine is prepared in laboratory, its atomic mass is found to be,

$$M = (35 \times 0.754) + (37 \times 0.246) = 35.5$$

Note *Since, the isotopes have the same atomic number, they have the same chemical properties. Their physical properties are different as they have different mass numbers. Two isotopes, thus cannot be separated by chemical method, but they can be separated from the physical methods.*

Isobars : The elements having the same mass number (A) but different atomic number (Z) are called isobars. They have different places in periodic table. Their chemical (as well as physical) properties are different.

$_1H^3$ and $_2He^3$, $_8O^{17}$ and $_9F^{17}$ are examples of isobars.

Isotones : Elements having the equal number of neutrons ($A - Z$) are called isotones.

$_3Li^7$ and $_4Be^8$, $_1H^3$ and $_2He^4$ are examples of isotones.

Nuclear Forces : In nucleus the positively charged protons and the uncharged neutrons are held together in an extremely small space ($\approx 10^{-15}$ m) inspite of the strong electrostatic repulsion between the protons. Obviously there are some strong attractive forces operating within the nucleus between the nucleons. The nuclear forces are nonelectric and non gravitational forces. These forces are extremely short-range forces. They become operative only when the distance between two nucleons is a small multiple of 10^{-15} m. They do not exist when the distance is appreciably larger than 10^{-15} m and become repulsive when the distance is appreciably smaller than 10^{-15} m. Nuclear forces between protons and protons between neutrons and neutrons and between protons and neutrons are all essentially the same in magnitude. Thus we can say that nuclear forces are charge independent.