31. A large wooden plate of area 10 m^2 floating on the surface of a river is made to move horizontally with a speed of 2 m/s by applying a tangential force. If the river is 1 m deep and the water in contact with the bed is stationary, find the tangential force needed to keep the plate moving. Coefficient of viscosity of water at the temperature of the river $= 10^{-2}$ poise.

32. The velocity of water in a river is 18 km/h near the surface. If the river is 5 m deep, find the shearing stress between the horizontal layers of water. The coefficient of viscosity of water $= 10^{-2}$ poise.

33. Water rises up in a glass capillary upto a height of 9.0 cm, while mercury falls down by 3.4 cm in the same capillary. Assume angles of contact for water glass and mercury glass $0°$ and $135°$ respectively. Determine the ratio of surface tension of mercury and water ($\cos 135° = -0.71$).

34. Two equal spherical bubbles of radius a each coalesce to form a spherical bubble of radius b. If P is the atmospheric pressure, show that the surface tension of the bubble is $\dfrac{P(2a^3 - b^3)}{4(b^2 - 2a^2)}$.

35. A tube of insufficient length is immersed in water (surface tension $= 0.07$ N/m) with 1 cm of it projecting vertically upwards outside the water. What is the radius of meniscus ? Radius of tube $= 1$ mm.

36. A glass capillary sealed at the upper end is of length 0.11 m and internal diameter 2×10^{-5} m. The tube is immersed vertically into a liquid of surface tension 5.06×10^{-2} N/m.

To what length has the capillary to be immersed so that the liquid levels inside and outside the capillary become the same ? What will happen to the water levels inside the capillary if the seal is now broken ?

Objective Questions

Single Correct Option

1. When a sphere falling in a viscous fluid attains a terminal velocity, then
(a) the net force acting on the sphere is zero
(b) the drag force balances the buoyant force
(c) the drag force balances the weight of the sphere
(d) the buoyant force balances the weight and drag force

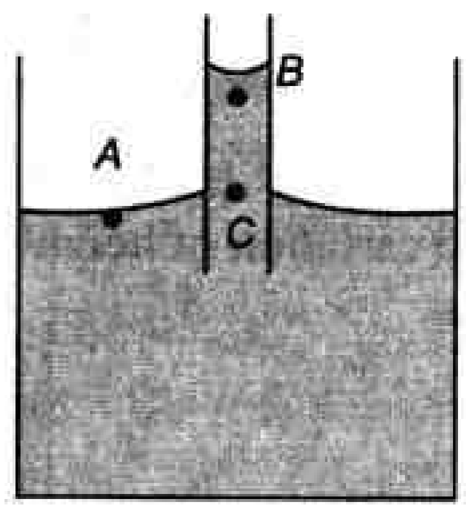
2. Which one of the following represents the correct dimensions of the quantity : $x = \dfrac{\eta}{\rho}$, where $\eta =$ coefficient of viscosity and $\rho =$ the density of a liquid?
(a) $[ML^{-2}T^{-1}]$ (b) $[ML^{-4}T^{-2}]$ (c) $[ML^{-5}T^{-2}]$ (d) $[M^0L^2T^{-1}]$

3. Viscosity of liquids
(a) increases with increase in temperature
(b) is independent of temperature
(c) decreases with decrease in temperature
(d) decreases with increase in temperature

4. Two soap bubbles in vacuum of radius 3 cm and 4 cm coalesce to form a single bubble under isothermal conditions. Then the radius of bigger bubble is

(a) 7 cm (b) $\frac{12}{7}$ cm (c) 12 cm (d) 5 cm

5. A capillary tube is dipped in a liquid. Let pressures at points A, B and C be P_A, P_B and P_C respectively then

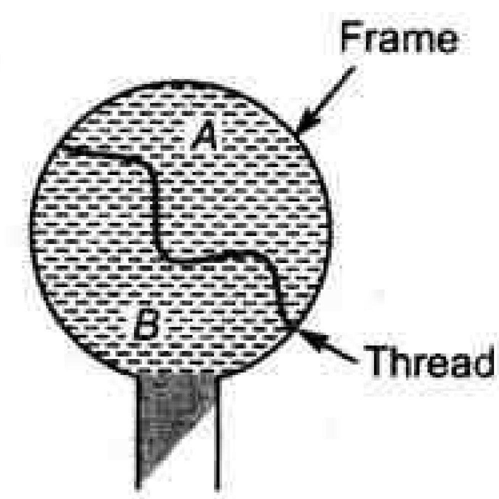


(a) $P_A = P_B = P_C$ (b) $P_A = P_B < P_C$ (c) $P_A = P_C < P_B$ (d) $P_A = P_C > P_B$

6. A small ball (mass m) falling under gravity in a viscous medium experiences a drag force proportional to the instantaneous speed u such that $F_{\text{drag}} = ku$. Then the terminal speed of ball within viscous medium is

(a) $\frac{k}{mg}$ (b) $\frac{mg}{k}$ (c) $\sqrt{\frac{mg}{k}}$ (d) None of these

7. A thread is tied slightly loose to a wire frame as in figure and the frame is dipped into a soap solution and taken out. The frame is completely covered with the film. When the portion A is punctured with a pin, the thread



(a) becomes concave towards A
(b) becomes convex towards A
(c) either (a) or (b) depending on the size of A with respect to B
(d) remains in the initial position

8. There is a small hole at the bottom of tank filled with water. If total pressure at the bottom is 3 atm ($1 \text{ atm} = 10^5 \text{ Nm}^{-2}$), then velocity of water flowing from hole is

(a) $\sqrt{400}$ ms^{-1} (b) $\sqrt{600}$ ms^{-1} (c) $\sqrt{60}$ ms^{-1} (d) None of these

9. A steel ball of mass m falls in a viscous liquid with terminal velocity v, then the steel ball of mass 8 m will fall in the same liquid with terminal velocity

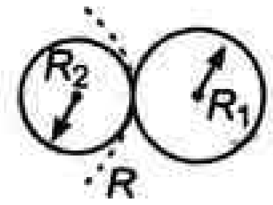
(a) v (b) $4v$ (c) $8v$ (d) $16\sqrt{2}v$

10. A liquid flows between two parallel plates along the x-axis. The difference between the velocity of two layers separated by the distance dy is dv. If A is the area of each plate, then Newton's law of viscosity may be written as

(a) $F = -\eta A \frac{dv}{dx}$ (b) $F = +\eta A \frac{dv}{dx}$ (c) $F = -\eta A \frac{dv}{dy}$ (d) $F = +\eta A \frac{dv}{dy}$

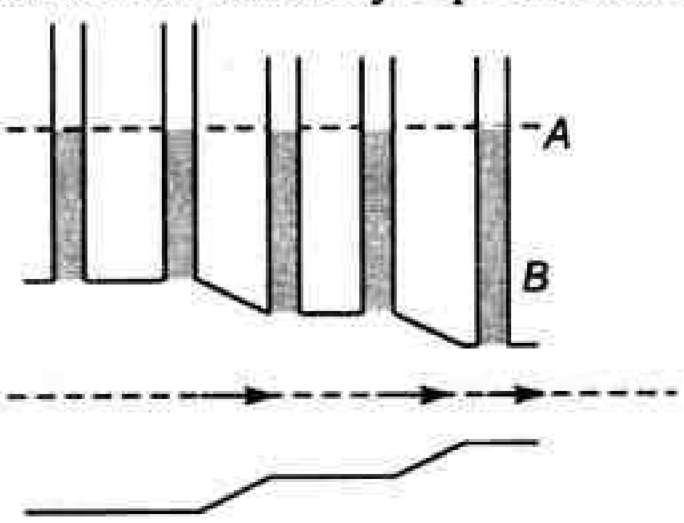
11. The work done to split a liquid drop of radius R into N identical drops is (take σ as the surface tension of the liquid)
(a) $4\pi R^2(N^{1/3}-1)\sigma$ (b) $4\pi R^2 N\sigma$ (c) $4\pi R^2(N^{1/2}-1)$ (d) None of these

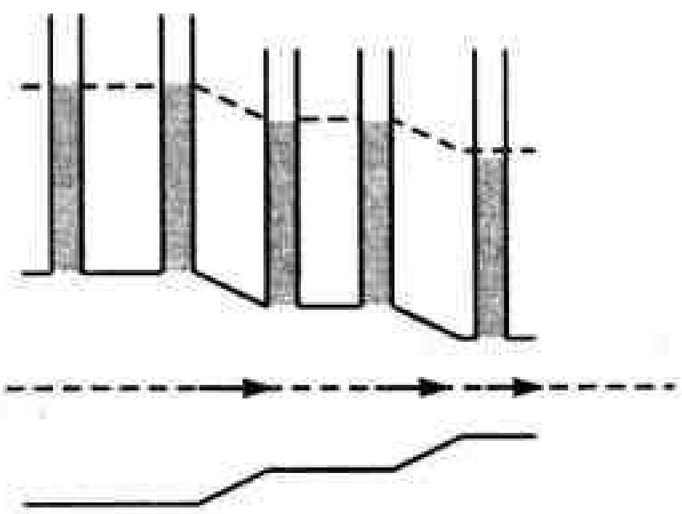
12. Two soap bubbles of different radii R_1 and $R_2(<R_1)$ coalesce to form an interface of radius R as shown in figure. The correct value of R is

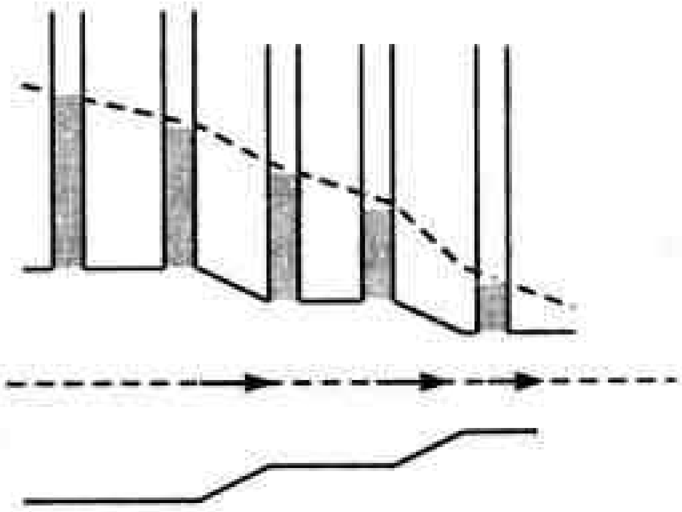


(a) $R=R_1-R_2$ (b) $R=\dfrac{R_1+R_2}{2}$ (c) $\dfrac{1}{R}=\dfrac{1}{R_2}-\dfrac{1}{R_1}$ (d) $\dfrac{1}{R}=\dfrac{1}{R_1}+\dfrac{1}{R_2}$

13. A viscous liquid flows through a horizontal pipe of varying cross-sectional area. Identify the option which correctly represents the variation of height of rise of liquid in each vertical tube

(a) 

(b) 

(c) 

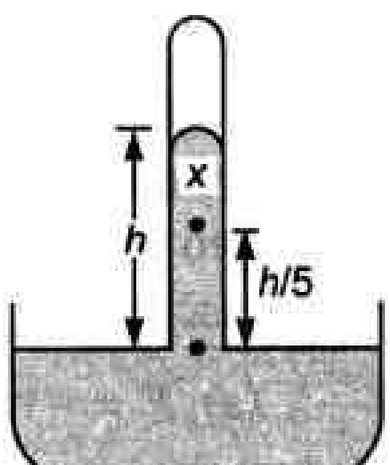
(d) None of these

14. The terminal velocity of a rain drop is 30 cm/s. If the viscosity of air is 1.8×10^{-5} Nsm^{-2}. The radius of rain drop is
(a) 1 μm (b) 0.5 mm (c) 0.05 mm (d) 1 mm

15. If a capillary tube is dipped and the liquid levels inside and outside the tube are same, then the angle of contact is
(a) zero (b) 90° (c) 45° (d) Cannot be obtained

16. Uniform speed of 2 cm diameter ball is 20 cm/s in a viscous liquid. Then, the speed of 1 cm diameter ball in the same liquid is
(a) 5 cms^{-1} (b) 10 cms^{-1} (c) 40 cms^{-1} (d) 80 cms^{-1}

17. The height of mercury barometer is h when the atmospheric pressure is 10^5 Pa. The pressure at x in the shown diagram is

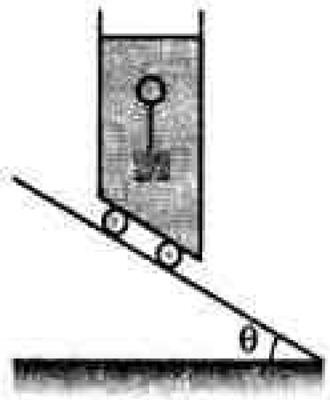


(a) 10^5 Pa
(b) 0.8×10^5 Pa
(c) 0.2×10^5 Pa
(d) 120×10^5 Pa

18. A body floats in water with its one-third volume above the surface. The same body floats in a liquid with one third volume immersed. The density of the liquid is
(a) 9 times more than that of water (b) 2 times more than that of water
(c) 3 times more than that of water (d) 1.5 times more than that of water

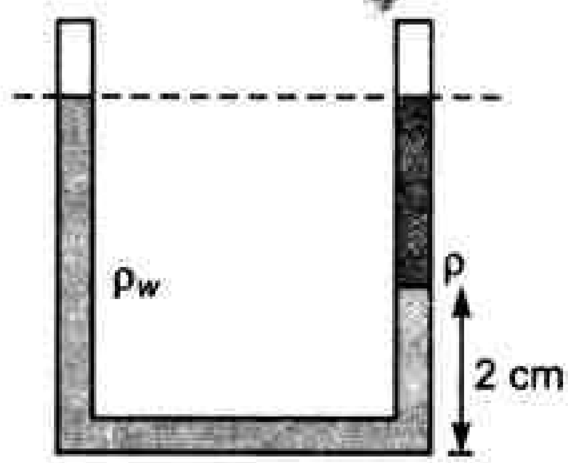
19. A piece of ice is floating in a beaker containing thick sugar solution of water. As the ice melts, the total level of the liquid
(a) increases (b) decreases (c) remains unchanged (d) insufficient data

20. A body floats in completely immersed condition in water as shown in figure. As the whole system is allowed to slide down freely along the inclined surface, the magnitude of buoyant force



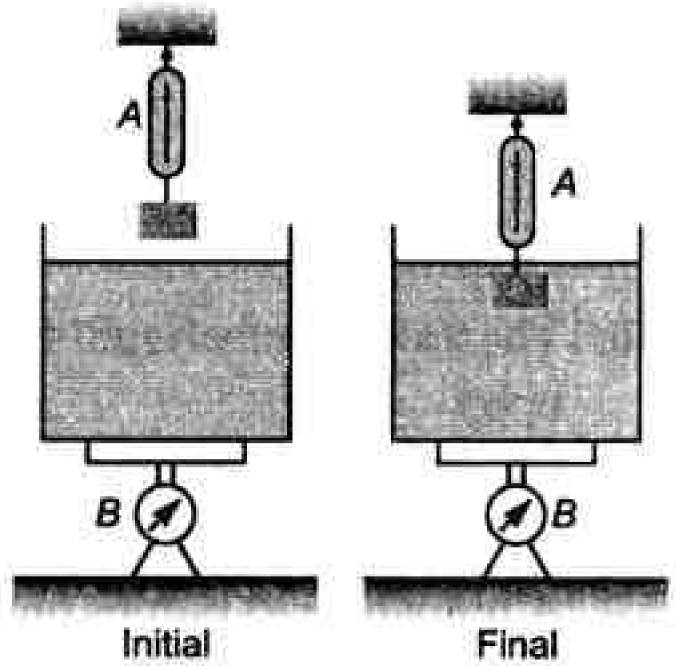
(a) remains unchanged
(b) increases
(c) decreases
(d) becomes zero

21. The figure represents a U-tube of uniform cross-section filled with two immiscible liquids. One is water with density ρ_w and the other liquid is of density ρ. The liquid interface lies 2 cm above the base. The relation between ρ and ρ_w is



(a) $\rho = \rho_w$
(b) $\rho = 1.02\,\rho_w$
(c) $\rho = 1.2\,\rho_w$
(d) None of the above

22. For the arrangement shown in figure, initially the balance A and B reads F_1 and F_2 respectively and $F_1 > F_2$. Finally when the block is immersed in the liquid then the readings of balance A and B are f_1 and f_2 respectively. Identify the statement which is not always (where F is some force) correct statement

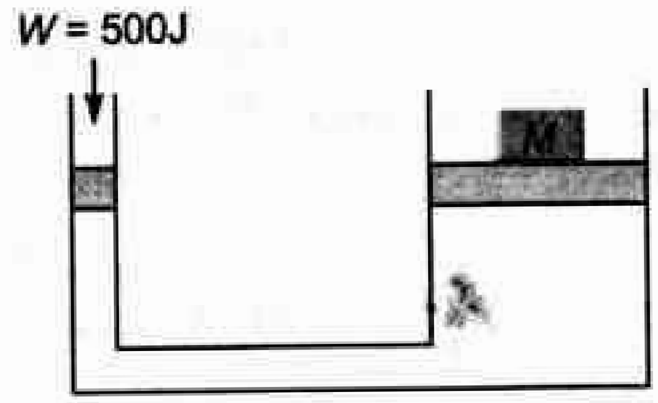


(a) $f_1 > f_2$
(b) $F_1 - F > F_2 + F$
(c) $f_1 + f_2 = F_1 + F_2$
(d) None of the above

23. When a tap is closed, the manometer attached to the pipe reads 3.5×10^5 Nm^{-2}. When the tap is opened, the reading of manometer falls to 3.0×10^5 Nm^{-2}. The velocity of water in the pipe is
(a) 0.1 ms^{-1} (b) 1 ms^{-1} (c) 5 ms^{-1} (d) 10 ms^{-1}

24. A balloon of mass M descends with an acceleration a_0. The mass that must be thrown out in order to give the balloon an equal upward acceleration will be
(a) $\dfrac{Ma_0}{g}$ (b) $\dfrac{2Ma_0}{g}$ (c) $\dfrac{2Ma_0}{g + a_0}$ (d) $\dfrac{M(g + a_0)}{a_0}$

25. The hydraulic press shown in the figure is used to raise the mass M through a height of 0.5 cm be performing 500 J of work at the small piston. The diameter of the large piston is 10 cm, while that of the smaller one is 2 cm. The mass M is



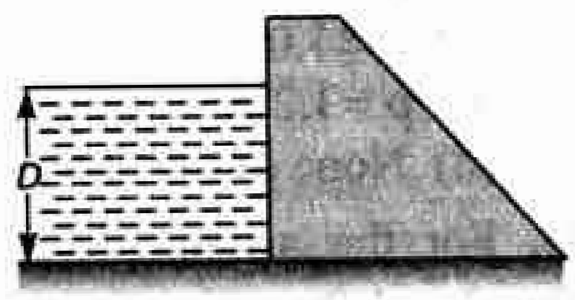
(a) 100 kg (b) 10^4 kg (c) 10^3 kg (d) 10^5 kg

26. When equal volumes of two substances are mixed, the specific gravity of the mixture is 4. When equal weights of the same substances are mixed, the specific gravity of the mixture is 3. The specific gravities of the two substances could be
(a) 6 and 2 (b) 3 and 4 (c) 2.5 and 3.5 (d) 5 and 3

27. A block of ice of total area A and thickness 0.5 m is floating in water. In order to just support a man of mass 100 kg, the area A should be (the specific gravity of ice is 0.9)
(a) $2.2\ m^2$ (b) $20\ m^2$ (c) $2222\ m^2$ (d) $20\times10^4\ m^2$

28. A piece of gold ($\rho = 19.3\ g/cm^3$) has a cavity in it. It weights 38.2 g in air and 36.2 g in water. The volume of the cavity in gold is
(a) $0.2\ cm^3$ (b) $0.04\ cm^3$ (c) $0.02\ cm^3$ (d) $0.01\ cm^3$

29. Water stands at a depth D behind the vertical upstream face of a dam as shown in the figure. The force exerted on the dam by water per unit width is

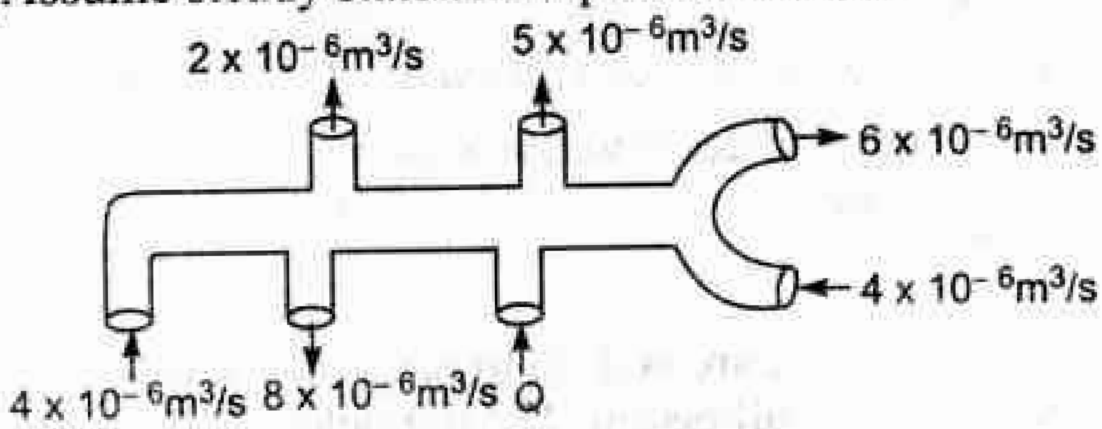


(a) $\frac{1}{3}\rho g D^2$ (b) $\frac{1}{2}\rho g D^2$
(c) $\frac{1}{3}\rho g D$ (d) $\frac{1}{2}(\rho g D)^2$

30. The volume of a liquid flowing per second out of an orifice at the bottom of a tank does not depend upon
(a) the height of the liquid above the orifice (b) the acceleration due to gravity
(c) the density of the liquid (d) the area of the orifice

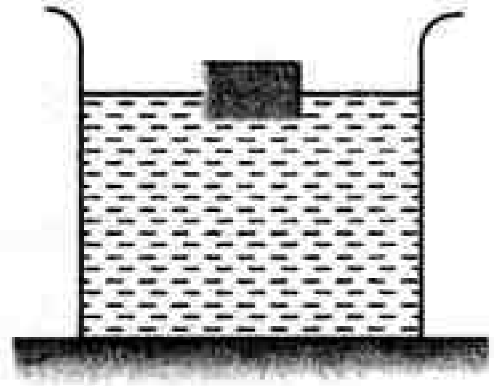
31. A human heart pumps out 60 mL of blood at an average pressure of 100 mm of mercury and makes 72 heart beats per minute. Its pumping power is
(a) 0.98 W (b) 1.96 W (c) 0.49 W (d) 1.47 W

32. The pipe shows the volume flow rate of an ideal liquid at certain time and its direction. What is the value of Q in m^3/s (Assume steady state and equal area of cross section at each opening)



(a) 10×10^{-6} (b) 11×10^{-6} (c) 13×10^{-6} (d) 18×10^{-6}

33. A uniform cube of mass M is floating on the surface of a liquid with three fourth of its volume immersed in the liquid (density $= \rho$). The length of the side of the cube is equal to



(a) $(4M/3\rho)^{2/3}$ (b) $(M/3\rho)^{2/3}$
(c) $(M/4\rho)^{2/3}$ (d) None of these

34. Water rises to a height of 10 cm in a certain capillary tube. An another identical tube when dipped in mercury the level of mercury is depressed by 3.42 cm. Density of mercury is 13.6 g/cc. The angle of contact for water in contact with glass is $0°$ and mercury in contact with glass is $135°$. The ratio of surface tension of water to that of Hg is
(a) 1 : 3 (b) 1 : 4 (c) 1 : 5.5 (d) 1 : 6.5

35. A capillary glass tube records a rise of 20 cm when dipped in water. When the area of cross-section of the tube is reduced to half of the former value, water will rise to a height of
(a) $10\sqrt{2}$ cm (b) 10 cm (c) 20 cm (d) $20\sqrt{2}$ cm

36. A cylindrical vessel open at the top is 20 cm high and 10 cm in diameter. A circular hole of cross sectional area $1\ cm^2$ is cut at the centre of the bottom of the vessel. Water flows from a tube above it into the vessel at the rate of $10^4\ cm^3/s$. The height of water in the vessel under steady state is (Take $g = 10\ m/s^2$)
(a) 20 cm (b) 15 cm (c) 10 cm (d) 5 cm

37. A horizontal pipeline carries water in a streamline flow. At a point along the tube where the cross sectional area is $10^{-2}\ m^2$, the water velocity is 2 m/s and the pressure is 8000 Pa. The pressure of water at another point where cross sectional area is $0.5 \times 10^{-2}\ m^2$ is
(a) 4000 Pa (b) 1000 Pa (c) 2000 Pa (d) 3000 Pa

38. Eight spherical rain drops of the same mass and radius are falling down with a terminal speed of $6\ cms^{-1}$. If they coalesce to form one big drop, what will be its terminal speed? Neglect the buoyancy due to air
(a) $1.5\ cms^{-1}$ (b) $6\ cms^{-1}$ (c) $24\ cms^{-1}$ (d) $32\ cms^{-1}$

For **JEE Advanced**

Assertion and Reason

Directions : *Choose the correct option.*

*(a) If both **Assertion** and **Reason** are true and the **Reason** is correct explanation of the **Assertion.***
*(b) If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion.***
*(c) If **Assertion** is true, but the **Reason** is false.*
*(d) If **Assertion** is false but the **Reason** is true.*

1. Assertion : Pressure is a vector quantity.

Reason : Pressure $P = \frac{F}{A}$. Here F, the force is a vector quantity.

2. **Assertion :** Surface tension $\left(T = \frac{F}{l}\right)$ is not a vector quantity.

Reason : Direction of force is specified.

3. **Assertion :** At depth 'h' below the water surface pressure is P. Then at depth '$2h$' pressure will be $2P$. (Ignore density variation).

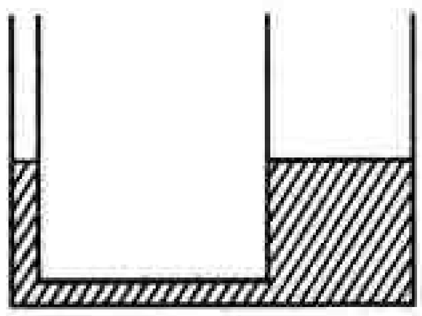
Reason : With depth pressure increases linearly.

4. **Assertion :** Weight of solid in air is W and in water is $\frac{2W}{3}$. Then relative density of solid is 3.0.

Reason : Relative density of any solid is given by :

$$\text{RD} = \frac{\text{Weight in air}}{\text{Change in weight in water}}$$

5. **Assertion :** Water is filled in a U-tube of different cross-sectional area on two sides as shown in figure. Now equal amount of oil (RD = 0.5) is poured on two sides. Level of water on both sides will remain unchanged.

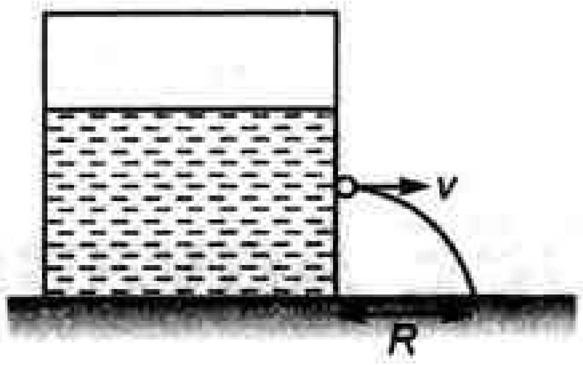


Reason : Same weight of oil poured on two sides will produce different pressures.

6. **Assertion :** An ideal fluid is flowing through a pipe. Speed of fluid particles is more at places where pressure is low.

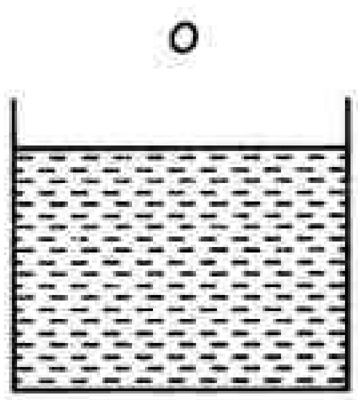
Reason : Bernoulli's theorem can be derived from work-energy theorem.

7. **Assertion :** In the figure shown v and R will increase if pressures above the liquid surface inside the chamber is increased.



Reason : Value of v or R is independent of density of liquid.

8. **Assertion :** A ball is dropped from a certain height above the free surface of an ideal fluid. When the ball enters the liquid it may accelerate or retard.

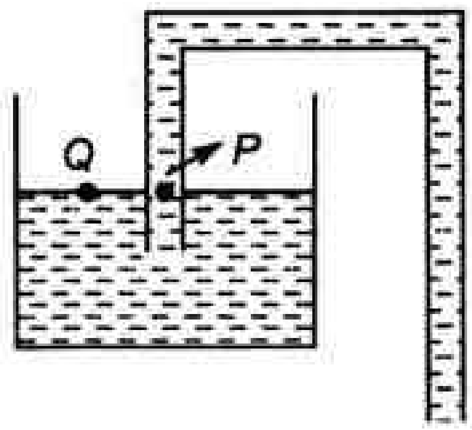


Reason : Ball accelerates or retards it all depends on the density of ball and the density of liquid.

9. **Assertion :** On moon barometer height will be six times compared to the height or earth.

Reason : Value of 'g' on moon's surface is $\frac{1}{6}$ the value of 'g' on earth's surface.

10. **Assertion :** In the siphon shown in figure, pressure at P is equal to atmospheric pressure.



Reason : Pressure at Q is atmospheric pressure and points P and Q are at same levels.

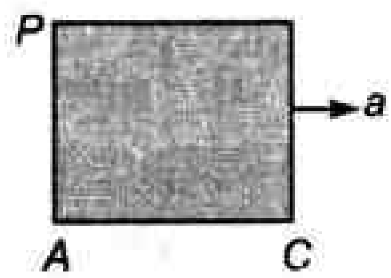
11. **Assertion :** Force of buoyancy due to atmosphere on a small body is almost zero (or negligible).

Reason : If a body is completely submerged in a fluid, then buoyant force is zero.

Objective Questions

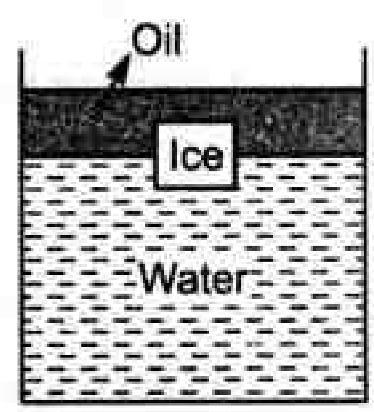
Single Correct Option

1. A cubical open vessel of side 5 m filled with a liquid is accelerated with an acceleration a. The value of a so that pressure at mid point of AC is equal to atmospheric pressure is



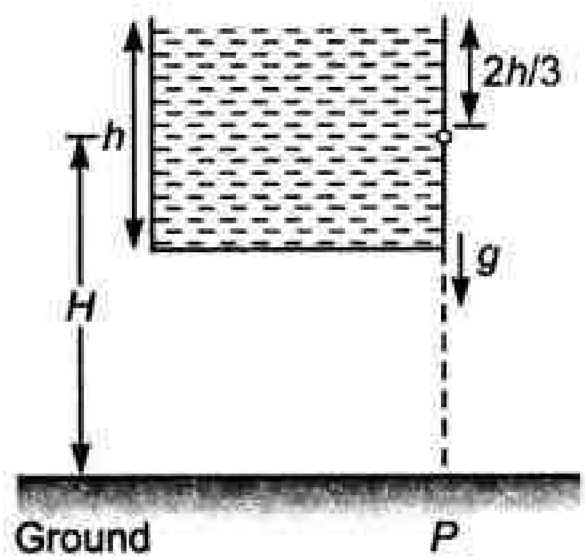
(a) g (b) $2g$
(c) $g/2$ (d) $2g/5$

2. An ice cube is floating in water above which a layer of lighter oil is poured. As the ice melts completely, the level of interface and the upper most level of oil will respectively



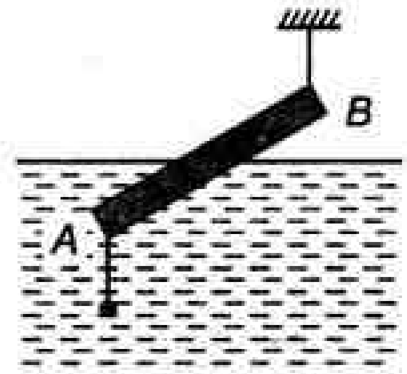
(a) rise and fall
(b) fall and rise
(c) not change and no change
(d) not change and fall

3. An open vessel full of water is falling freely under gravity. There is a small hole in one face of the vessel as shown in the figure. The water which comes out from the hole at the instant when hole is at height H above the ground, strikes the ground at a distance of x from P. Which of the following is correct for the situation described?



(a) The value of x is $2\sqrt{\frac{2hH}{3}}$
(b) The value of x is $\sqrt{\frac{4hH}{3}}$
(c) The value of x can't be computed from information provided
(d) The question is irrevalent as no water comes out from the hole

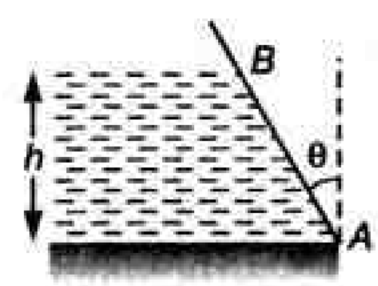
4. A uniform rod AB, 12 m long weighing 24 kg, is supported at end B by a flexible light string and a lead weight (of very small size) of 12 kg attached at end A.



The rod floats in water with one-half of its length submerged. For this situation, mark out the correct statement.
[Take $g = 10\ \text{m/s}^2$, density of water $= 1000\ \text{kg/m}^3$]
(a) The tension in the string is 36 g
(b) The tension in the string is 12 g
(c) The volume of the rod is $6.4 \times 10^{-2}\ \text{m}^3$
(d) The point of application of the buoyancy force is passing through C (centre of mass of rod)

5. A water hose pipe of cross-section area $5\ \text{cm}^2$ is used to fill a tank of 120 L. It has been observed that it takes 2 min to fill the tank. Now, a nozzle with an opening of cross-section area $1\ \text{cm}^2$ is attached to the hose. The nozzle is held so that water is projected horizontally from a point 1 m above the ground. The horizontal distance over which the water can be projected is
(Take $g = 10\ \text{m/s}^2$)
(a) 3 m (b) 8 m (c) 4.47 m (d) 8.64 m

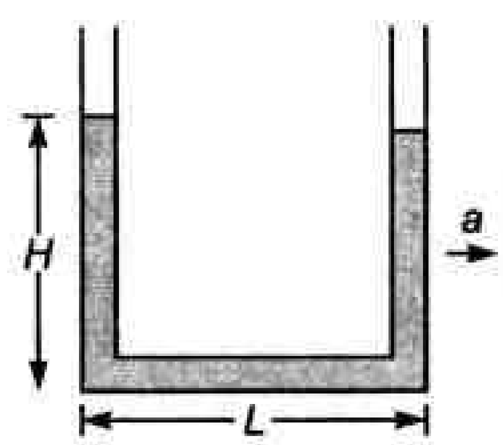
6. The height of water in a vessel is h. The vessel wall of width b is at an angle θ to the vertical. The net force exerted by the water on the wall is



(a) $\frac{1}{3}\rho b h^2 g \cos\theta$ (b) $\frac{1}{2} b h^2 \rho g$
(c) $\frac{1}{2}\rho b h^2 g \sec\theta$ (d) zero

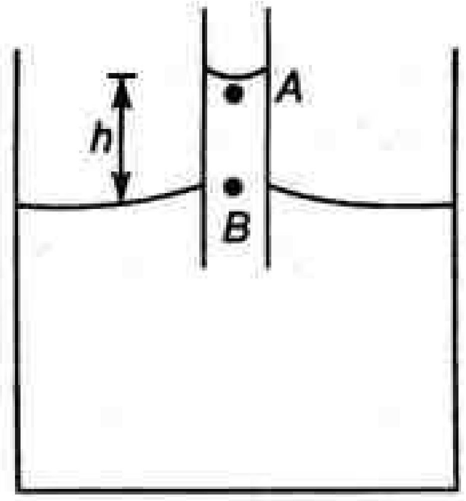
7. A body of density ρ is dropped from rest from a height h into a lake of density $\sigma\ (\sigma > \rho)$. The maximum depth the body sinks inside the liquid is (neglect viscous effect of liquid)
(a) $\frac{h\rho}{\sigma - \rho}$ (b) $\frac{h\sigma}{\sigma - \rho}$ (c) $\frac{h\rho}{\sigma}$ (d) $\frac{h\sigma}{\rho}$

8. A liquid stands at the plane level in the U-tube when at rest. If areas of cross-section of both the limbs are equal, what will be the difference in heights h of the liquid in the two limbs of U-tube, when the system is given an acceleration a in horizontal direction towards right as shown?



(a) $\frac{g}{a}\frac{L^2}{H}$ (b) $\frac{La}{g}$
(c) $\frac{L^2}{H}\frac{a}{g}$ (d) $\frac{Lg}{a}$

9. A liquid of density ρ and surface tension σ rises in a capillary tube of inner radius R. The angle of contact between the liquid and the glass is θ. The point A lies just below the meniscus in the tube and the point B lies at the outside level of liquid in the beaker as shown in figure. The pressure at A is



(a) $p_B - \rho g h$ (b) $p_B - \dfrac{2\sigma\cos\theta}{R}$ (c) $p_{atm} - \dfrac{2\sigma\cos\theta}{R}$ (d) All of these

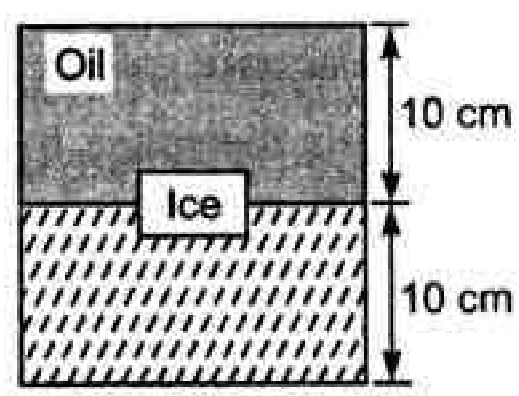
10. A large open tank has two holes in the wall. One is a square hole of side L at a depth h from the top and the other is a circular hole of radius R at a depth $4h$ from the top. When the tank is completely filled with water, quantities of water flowing out per second from both holes are the same. Then R is equal to

(a) $\dfrac{L}{\sqrt{2\pi}}$ (b) $2\pi L$ (c) L (d) $\dfrac{L}{2\pi}$

11. Two identical cylindrical vessels with their bases at the same level each contain a liquid of density ρ. The area of either base is A but in one vessel the liquid height is h_1 and in the other liquid height is $h_2 (h_2 < h_1)$. If the two vessels are connected, the work done by gravity in equalizing the levels is

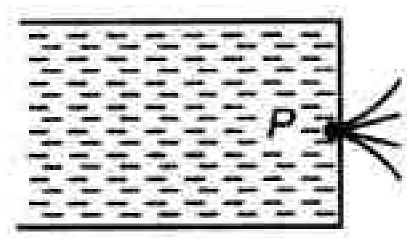
(a) $\dfrac{1}{2}(h_1 - h_2)^2 A\rho g$ (b) $\dfrac{1}{2}(h_1 + h_2)A\rho g$ (c) $\dfrac{1}{2}(h_1^2 - h_2^2)A\rho g$ (d) $\dfrac{1}{4}(h_1 - h_2)^2 A\rho g$

12. A cubical block of side 10 cm floats at the interface of an oil and water as shown in the figure. The density of oil is 0.6 g cm^{-3} and the lower face of ice cube is 2 cm below the interface. The pressure above that of the atmosphere at the lower face of the block is



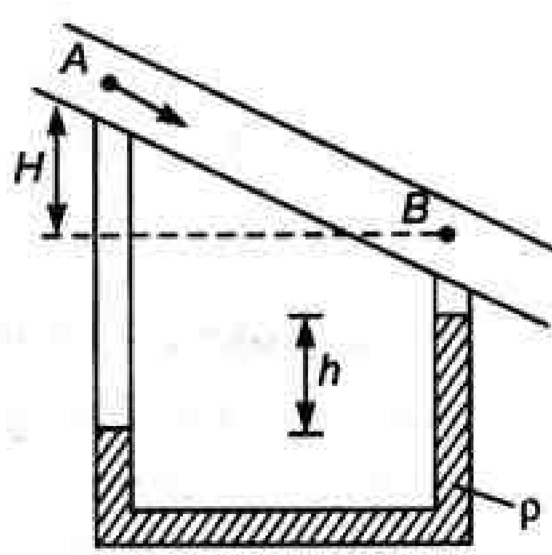
(a) 200 Pa (b) 620 Pa
(c) 900 Pa (d) 800 Pa

13. A leakage begins in water tank at position P as shown in the figure. The initial gauage pressure (pressure above that of the atmosphere) at P was $5 \times 10^5 \text{ N/m}^2$. If the density of water is 1000 kg/m^3 the initial velocity with which water gushes out is



(a) 3.2 ms^{-1} (b) 32 ms^{-1}
(c) 28 ms^{-1} (d) 2.8 ms^{-1}

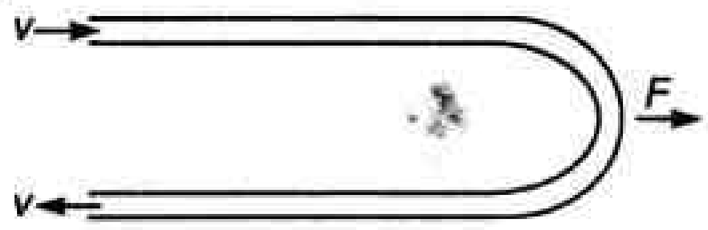
14. The figure shows a pipe of uniform cross-section inclined in a vertical plane. A U-tube manometer is connected between the points A and B. If the liquid of density ρ_0 flows with velocity v_0 in the pipe. Then the reading h of the manometer is



(a) $h = 0$ (b) $h = \dfrac{v_0^2}{2g}$

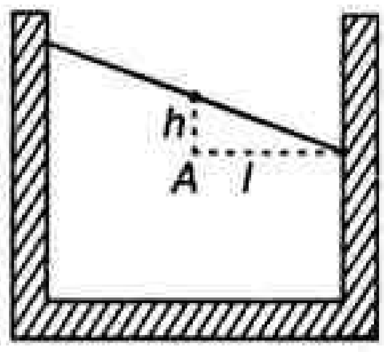
(c) $h = \dfrac{\rho_0}{\rho}\left(\dfrac{v_0^2}{2g}\right)$ (d) $h = \dfrac{\rho_0 H}{\rho - \rho_0}$

15. A horizontal tube of uniform cross-sectional area A is bent in the form of U as shown in figure. If the liquid of density ρ enters and leaves the tube with velocity v then the external force F required to hold the bend stationary is



(a) $F = 0$ (b) $\rho A v^2$ (c) $2\rho A v^2$ (d) $\frac{1}{2}\rho A v^2$

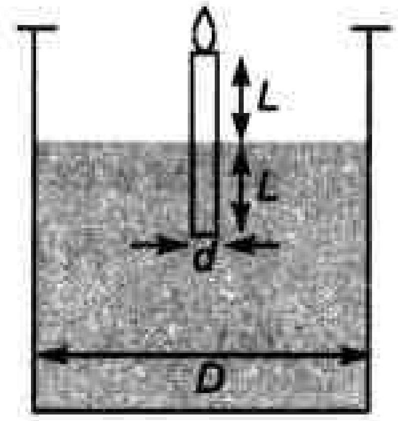
16. A rectangular container moves with an acceleration a along the positive direction as shown in figure. The pressure at the point A in excess of the atmospheric pressure p_0 is (take ρ as the density of liquid)



(a) ρgh (b) ρal

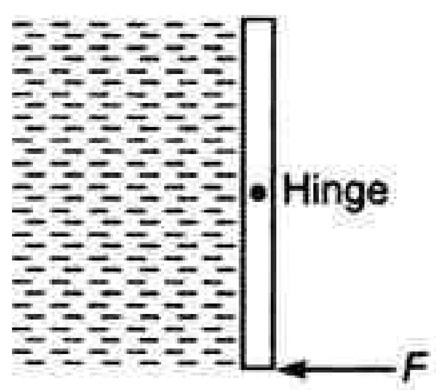
(c) $\rho\,(gh + al)$ (d) Both (a) and (c)

17. A candle of diameter d is floating on a liquid in a cylindrical container of diameter D ($D >> d$) as shown in figure. It it is burning at the rate of 2 cm/h. Then the top of the candle will



(a) remain at the same height
(b) fall at the rate of 1 cm/h
(c) fall at the rate of 2 cm/h
(d) go up at the rate of 1 cm/h

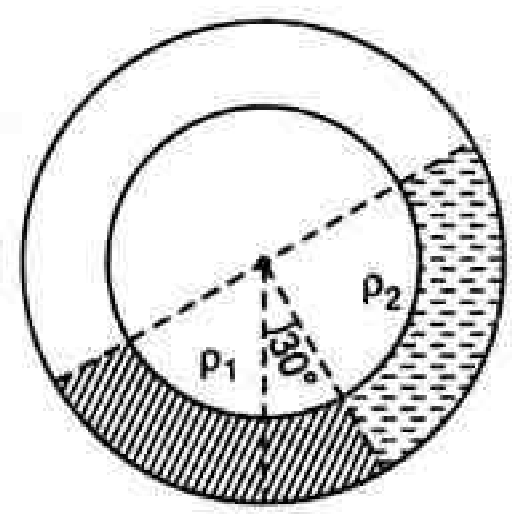
18. A square gate of size 1 m × 1 m is hinged at its mid point. A fluid of density ρ fills the space to the left of the gate. The force F required to hold the gate stationary is



(a) $\frac{\rho g}{3}$ (b) $\frac{1}{2}\rho g$

(c) $\frac{\rho g}{6}$ (d) None of these

19. A thin uniform circular tube is kept in a vertical plane. Equal volumes of two immiscible liquids whose densities are ρ_1 and ρ_2 fill half of the tube as shown. In equilibrium the radius passing through the interface makes an angle of 30° with vertical. The ratio of densities (ρ_1/ρ_2) is equal to



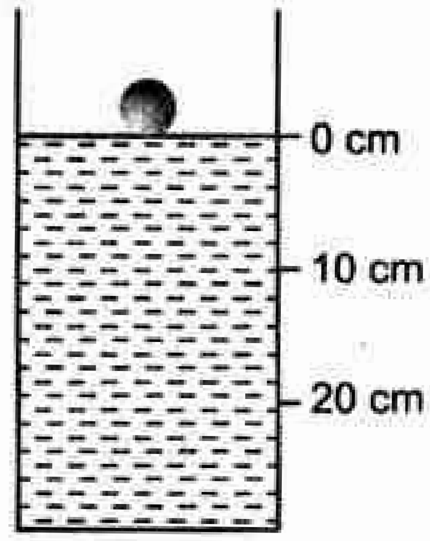
(a) $\frac{\sqrt{3}-1}{2-\sqrt{3}}$ (b) $\frac{\sqrt{3}+1}{2+\sqrt{3}}$

(c) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (d) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

20. A plate moves normally with the speed v_1 towards a horizontal jet of water of uniform area of cross-section. The jet discharges water at the rate of volume V per second at a speed of v_2. The density of water is ρ. Assume that water splashes along the surface of the plate ar right angles to the original motion. The magnitude of the force acting on the plate due to the jet of water is

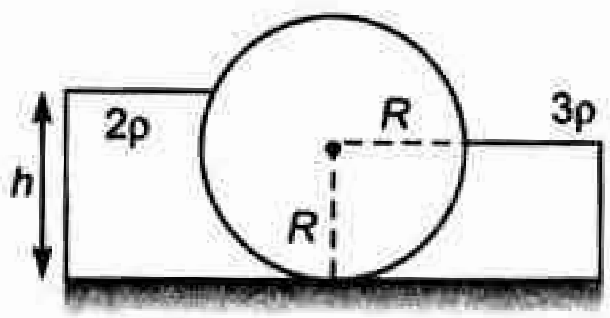
(a) $\rho V v_1$ (b) $\rho\left(\frac{V}{v_2}\right)(v_1+v_2)^2$ (c) $\frac{\rho V}{v_1+v_2}(v_1)^2$ (d) $\rho V(v_1+v_2)$

21. A spherical ball of density ρ and radius 0.003 m is dropped into a tube containing a viscous fluid up to the 0 cm mark as shown in the figure. Viscosity of the fluid = 1.26 N-s/m^2 and its density $\rho_L = \frac{\rho}{2} = 1260\ \text{kg/m}^3$. Assume the ball reaches a terminal speed at 10 cm mark. The time taken by the ball to travel the distance between the 10 cm and 20 cm mark is $(g = 10\ \text{m/s}^2)$



(a) 2 s (b) 1 s
(c) 0.5 s (d) 5 s

22. In the figure shown, the heavy cylinder (radius R) resting on a smooth surface separates two liquids of densities 2ρ and 3ρ. The height h for the equilibrium of cylinder must be

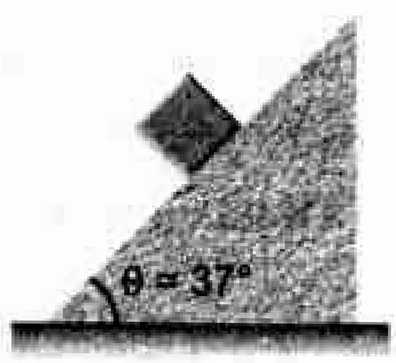


(a) $3R/2$ (b) $R\sqrt{\frac{3}{2}}$ (c) $R\sqrt{2}$ (d) None of these

23. A U-tube having horizontal arm of length 20 cm, has uniform cross-sectional area $= 1\ \text{cm}^2$. It is filled with water of volume 60 cc. What volume of a liquid of density 4g/cc should be poured from one side into the U-tube so that no water is left in the horizontal arm of the tube?

(a) 60 cc (b) 45 cc (c) 50 cc (d) 35 cc

24. A cubical block of side a and density ρ slides over a fixed inclined plane with constant velocity v. There is a thin film of viscous fluid of thickness t between the plane and the block. Then the coefficient of viscosity of the film will be



(a) $\frac{3\rho agt}{5v}$ b) $\frac{4\rho agt}{5v}$ (c) $\frac{\rho agt}{v}$ (d) None of these

25. A water barrel stands on a table of height h. If a small hole is punched in the side of the barrel at its base, it is found that the stream of water strikes the ground at a horizontal distance R from the barrel. The depth of water in the barrel is

(a) $\frac{R}{2}$ (b) $\frac{R^2}{4h}$ (c) $\frac{R^2}{h}$ (d) $\frac{h}{2}$

26. A spring balance reads 10 kg when a bucket of water is suspended from it. What will be the reading of the balance when an iron piece of mass 7.2 kg suspended by a string is immersed with half its volume inside the water in the bucket? Relative density of iron is 7.2?

(a) 10 kg (b) 10.5 kg (c) 13.6 kg (d) 17.2 kg

27. Three points A, B and C on a steady flow of a non viscous and incompressible fluid are observed. The pressure, velocity and height of the points A, B and C are (2, 3, 1), (1, 2, 2) and (4, 1, 2) respectively. Density of the fluid is $1\ \text{kgm}^{-3}$ and all other parameters are given in SI units. Then which of the following is correct? ($g = 10\ \text{ms}^{-2}$).

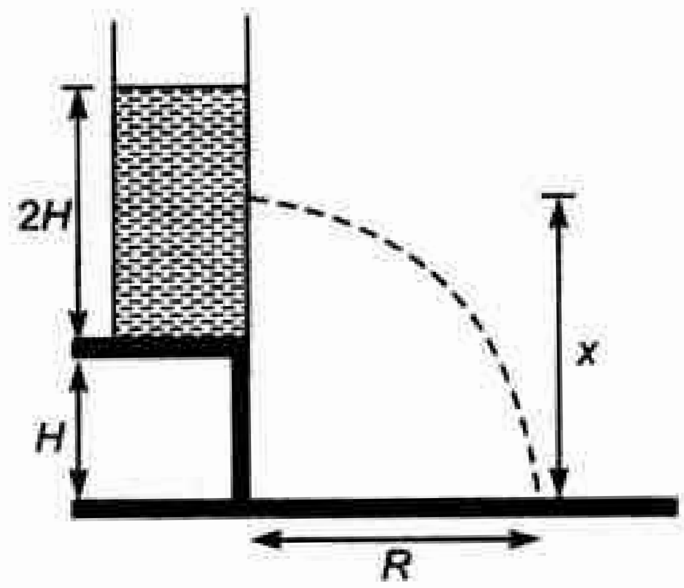
(a) points A and B lie on the same stream line
(b) point B and C lie on same stream line
(c) point C and A lie on same stream line
(d) None of the above

28. A body of density ρ is dropped from rest from height 'h' (from the surface of water) into a lake of density of water σ ($\sigma > \rho$). Neglecting all dissipative effects, the acceleration of body while it is in the lake is

(a) $g\left(\frac{\sigma}{\rho}-1\right)$ upwards (b) $g\left(\frac{\sigma}{\rho}-1\right)$ downwards

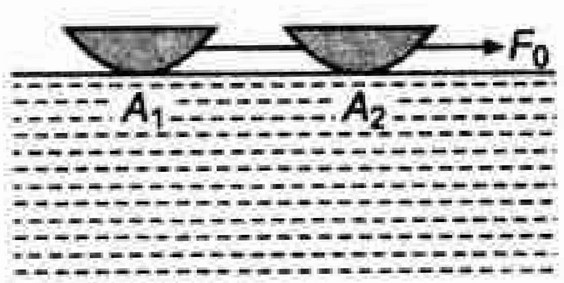
(c) $g\left(\frac{\sigma}{\rho}\right)$ upwards (d) $g\left(\frac{\sigma}{\rho}\right)$ downwards

29. A tank is filed up to a height $2H$ with a liquid and is placed on a platform of height H from the ground. The distance x from the ground where a small hole is punched to get the maximum range R is



(a) H
(b) $1.25\ H$
(c) $1.5\ H$
(d) $2\ H$

30. Two boats of base areas A_1 and A_2, connected by a string are being pulled by an external force F_0. The viscosity of water is η and depth of the water body is H. When the system attains a constant speed, the tension in the thread will be



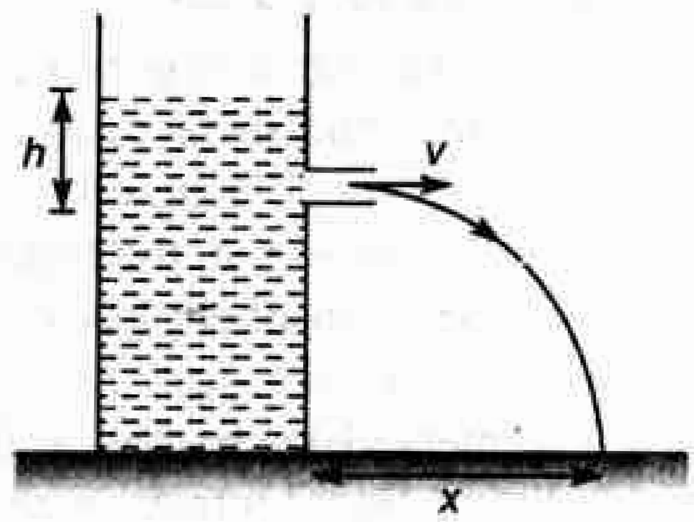
(a) zero (b) $F_0 \dfrac{A_2}{(A_1 + A_2)}$

(c) $F_0 \dfrac{A_1}{(A_1 + A_2)}$ (d) None of these

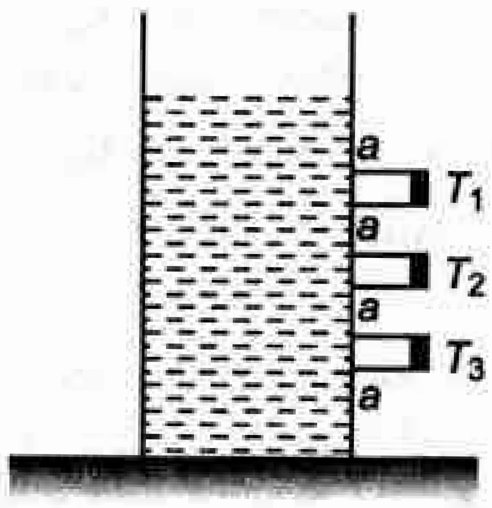
31. A U-tube is partially filled with water. Oil which does not mix with water is next poured into one side, until water rises by 25 cm on the other side. If the density of oil is $0.8\ \text{g/cm}^3$, the oil level will stand higher than the water level by

(a) 6.25 cm (b) 12.50 cm (c) 31.75 cm (d) 25 cm

Passage : (Q 32 to Q 34)



The spouting can is something used to demonstrate the variation of pressure with depth. When the corks are removed from the tubes in the side of the can, water flows out with a speed that depends on the depth. In a certain can, three tubes T_1, T_2 and T_3 are set at equal distances a above the base of the can. When water contained in this can is allowed to come out of the tubes the distances on the horizontal surface are measured as x_1, x_2 and x_3.



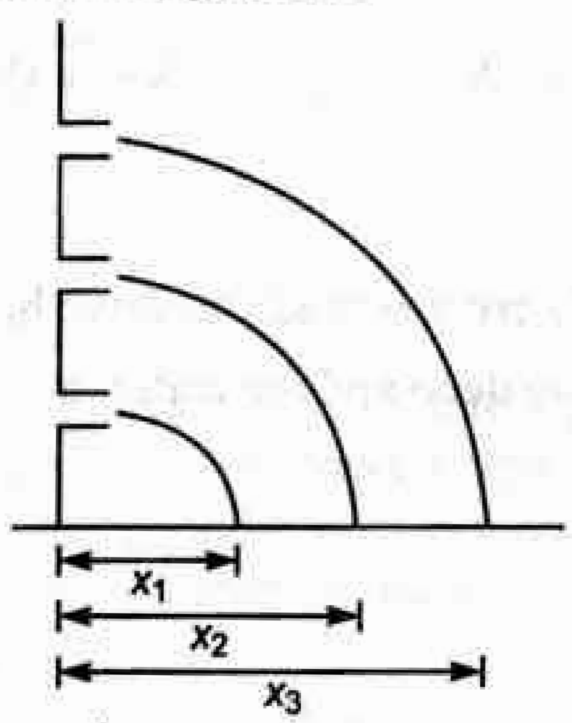
32. Speed of efflux is

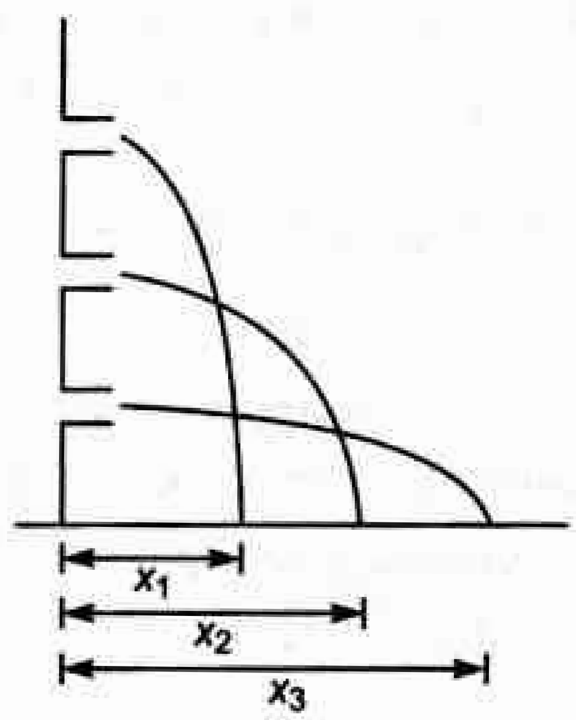
(a) $\sqrt{3gh}$ (b) $\sqrt{2gh}$ (c) $\sqrt{gh}$ (d) $\frac{1}{2}\sqrt{2gh}$

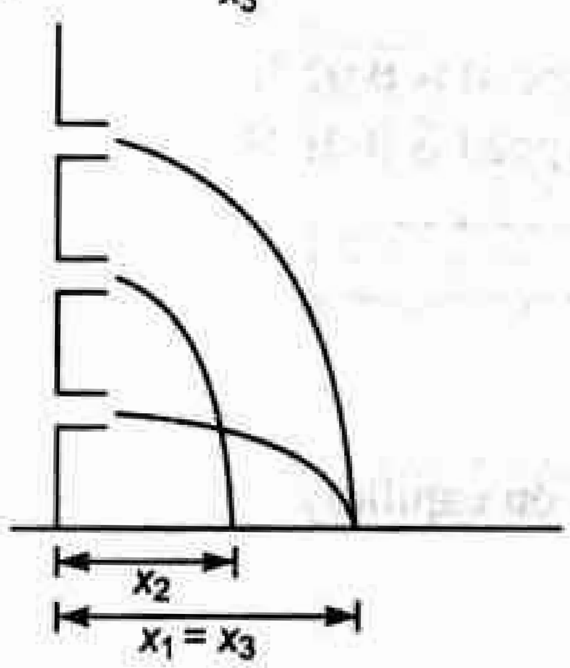
33. Distance x_3 is given by

(a) $\sqrt{3}\,a$ (b) $\sqrt{2}\,a$ (c) $\frac{1}{2}\sqrt{3}\,a$ (d) $2\sqrt{3}\,a$

34. The correct sketch is

(a)


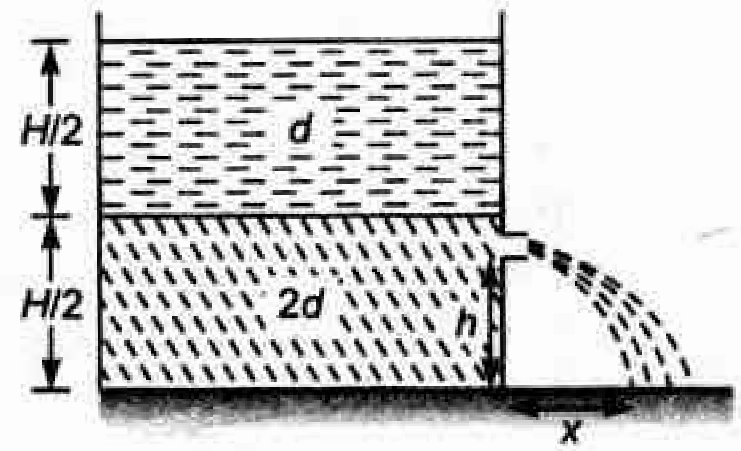
(b)


(c)


(d) None of these

Passage : (Q 35 to Q 38)

A container of large uniform cross-sectional area A resting on a horizontal surface, holds, two immiscible, non-viscous and incompressible liquids of densities d and $2d$ each of height $H/2$ as shown in the figure. The lower density liquid is open to the atmosphere having pressure P_0. A homogeneous solid cylinder of length $L(L < H/2)$, cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid.



The cylinder is then removed and the original arrangement is restored. A tiny hole of area $s(s << A)$ is punched on the vertical side of the container at a height $h(h < H/2)$. As a result of this, liquid starts flowing out of the hole with a range x on the horizontal surface.

35. The density D of the material of the floating cylinder is

(a) $5d/4$ (b) $3d/4$ (c) $4d/5$ (d) $4d/3$

36. The total pressure with cylinder, at the bottom of the container is

(a) $P_0 + \dfrac{(6L+H)}{4}dg$ (b) $P_0 + \dfrac{(L+6H)}{4}dg$ (c) $P_0 + \dfrac{(L+3H)}{4}dg$ (d) $P_0 + \dfrac{(L+2H)}{4}dg$

37. The initial speed of efflux without cylinder is

(a) $v = \sqrt{\dfrac{g}{3}[3H+4h]}$

(b) $v = \sqrt{\dfrac{g}{2}[4H-3h]}$

(c) $v = \sqrt{\dfrac{g}{2}[3H-4h]}$

(d) None of these

38. The horizontal distance traveled by the liquid, initially, is

(a) $\sqrt{(3H+4h)h}$ (b) $\sqrt{(3h+4H)h}$ (c) $\sqrt{(3H-4h)h}$ (d) $\sqrt{(3H-3h)h}$

More than One Correct Options

1. A large wooden plate of area $10\ \text{m}^2$ floating on the surface of a river is made to move horizontally with a speed of 2 m/s by applying a tangential force. River is 1 m deep and the water in contact with the bed is stationary. Then choose correct statement.

(coefficient of viscosity of water $= 10^{-3}$ N-s/m^2)

(a) velocity gradient is $2\,\text{s}^{-1}$

(b) velocity gradient is $1\,\text{s}^{-1}$

(c) force required to keep the plate moving with constant speed is 0.02 N

(d) force required to keep the plate moving with constant speed is 0.01 N

2. Choose the correct options

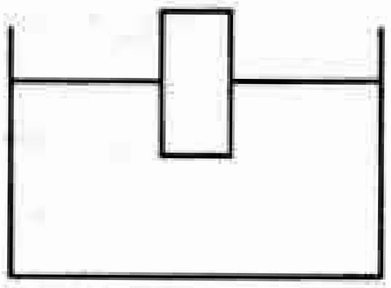
(a) viscosity of liquids increases with temperature

(b) viscosity of gases increases with temperature

(c) surface tension of liquids decreases with temperature

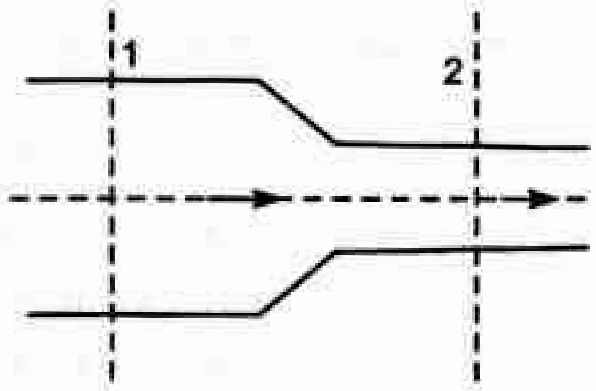
(d) for angle of contact $\theta = 0°$, liquid neither rises nor falls on capillary

3. A plank is floating in a non-viscous liquid as shown. Choose the correct options.



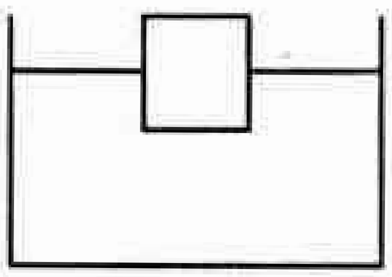
(a) Equilibrium of plank is stable in vertical direction
(b) For small oscillations of plank in vertical direction motion is simple harmonic
(c) Even if oscillations are large, motion is simple harmonic
(d) On vertical displacement motion is periodic but not simple harmonic

4. A non-viscous incompressible liquid is flowing from a horizontal pipe of non-uniform cross section as shown. Choose the correct options.



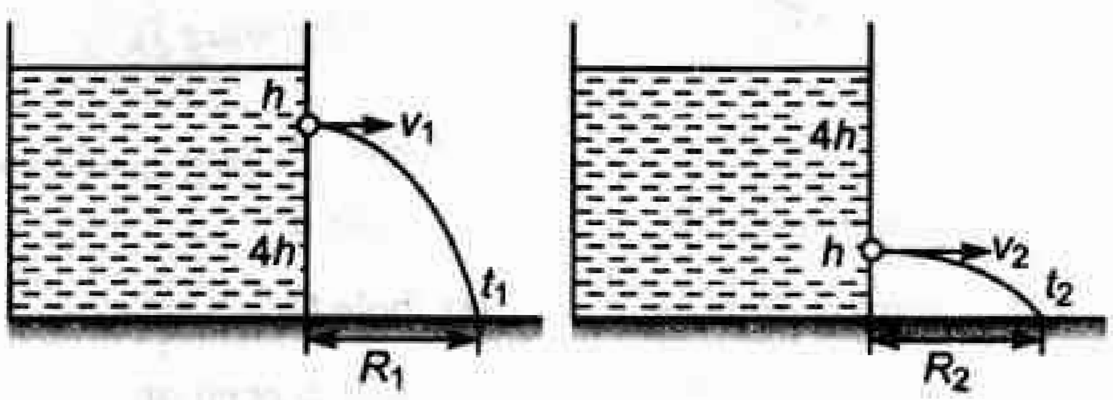
(a) speed of liquid at section-2 is more
(b) volume of liquid flowing per second from section-2 is more
(c) mass of liquid flowing per second at both the sections is same
(d) pressure at section-2 is less

5. A plank is floating in a liquid as shown. Fraction f of its volume is immersed. Choose the correct options.



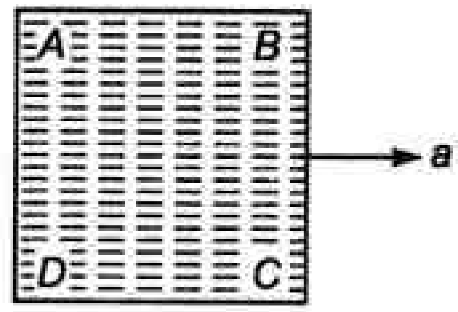
(a) If the system is taken to a place where atmospheric pressure is more, 'f' will increase
(b) In above condition f will remain unchanged
(c) If temperature is increased and expansion of only liquid is considered f will increase
(d) If temperature is increased and expansion of only plank is considered f will decrease

6. In two figures



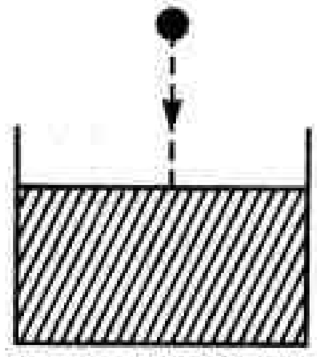
(a) $v_1/v_2 = 1/2$ (b) $t_1/t_2 = 2/1$ (c) $R_1/R_2 = 1$ (d) $v_1/v_2 = 1/4$

7. A liquid is filled in a container as shown in figure. Container is accelerated towards right. There are four points A, B, C and D in the liquid. Choose the correct options.



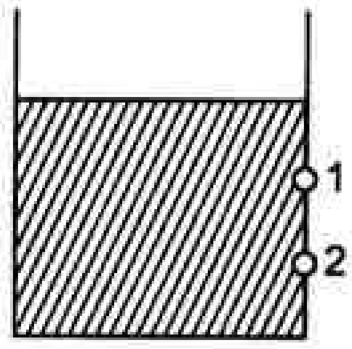
(a) $P_A > P_B$ (b) $P_C > P_A$
(c) $P_D > P_B$ (d) $P_A > P_C$

8. A ball of density ρ is dropped from a height on the surface of a non-viscous liquid of density 2ρ. Choose the correct options.



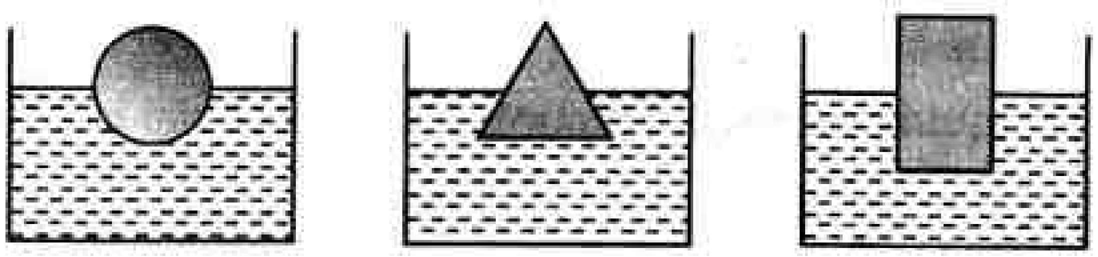
(a) Motion of ball is periodic but not simple harmonic
(b) Acceleration of ball in air and in liquid are equal
(c) Magnitude of upthrust in the liquid is two times the weight of ball
(d) Net force on ball in air and in liquid are equal and opposite

9. Two holes 1 and 2 are made at depths h and $16h$ respectively. Both the holes are circular but radius of hole-1 is two times.



(a) Initially equal volumes of liquid will flow from both the holes in unit time
(b) Initially more volume of liquid will flow from hole-2 per unit time
(c) After some time more volume of liquid will flow from hole-1 per unit time
(d) After some time more volume of liquid will flow from hole-2 per unit time

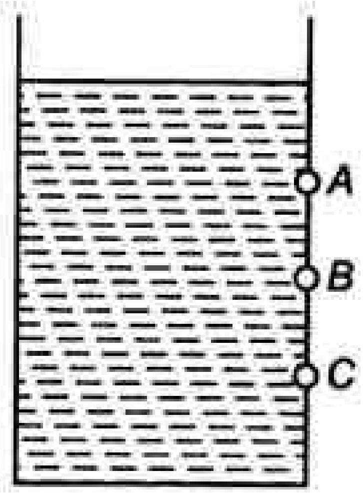
10. A solid sphere, a cone and a cylinder are floating in water. All have same mass, density and radius. Let f_1, f_2 and f_3 are the fraction of their volumes inside the water and h_1, h_2 and h_3 depths inside water. Then



(a) $f_1 = f_2 = f_3$ (b) $f_3 > f_2 > f_1$ (c) $h_3 < h_1$ (d) $h_3 < h_2$

Match the Columns

1. Three holes A, B and C are made at depths 1m, 2 m and 5 m as shown. Total height of liquid in the container is 8 m. Let v is the speed with which liquid comes out of the hole and R the range on ground. Match the following two columns.



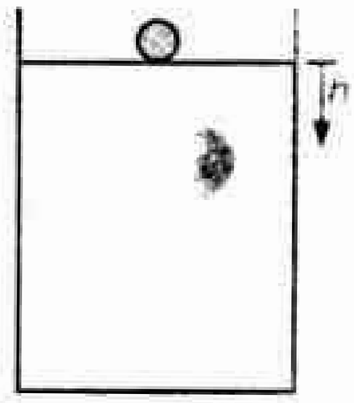
Column I	Column II
(a) v is maximum for	(p) hole A
(b) v is minimum for	(q) hole B
(c) R is maximum for	(r) hole C
(d) R is minimum for	(s) will depend on density of liquid

2. Match the following two columns.

Column I	Column II
(a) When temperature is increased	(p) Upthrust on a floating solid of constant volume will increase
(b) When density of liquid is increased	(q) Upthrust on a floating solid of constant volume will decrease
(c) When density of solid is increased	(r) Viscosity of gas will decrease
(d) When atmospheric pressure is increased	(s) None

3. A ball of density ρ is released from the surface of a liquid whose density varies with depth h as.
$\rho_l = \alpha h$

Here α is a positive constant. Match the following two columns. (liquid is ideal)

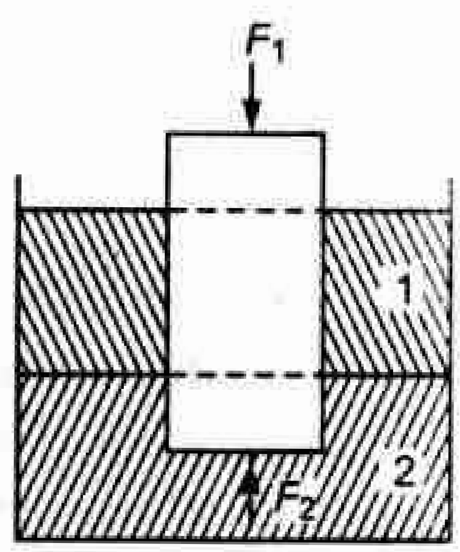


Column I	Column II
(a) Upthrust on ball	(p) will continuously decrease
(b) Speed of ball	(q) will continuously increase
(c) Net force on ball	(r) first increase then decrease
(d) Gravitational potential energy of ball	(s) first decrease then increase

4. Match the following two columns.

Column I	Column II
(a) Surface tension	(p) $[ML^{-1}T^{-2}]$
(b) Coefficient of viscosity	(q) $[L^3T^{-1}]$
(c) Energy density	(r) $[MT^{-2}]$
(d) Volume flow rate	(s) $[ML^{-1}T^{-1}]$

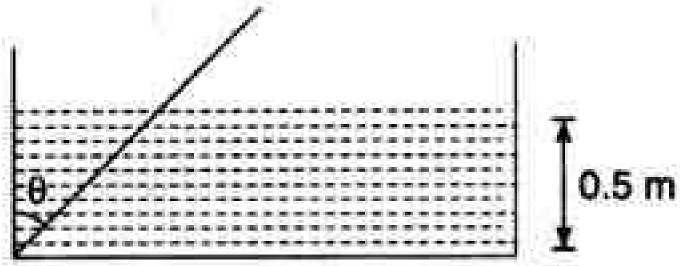
5. A cylinder of weight W is floating in two liquids as shown in figure. Net force on cylinder from top is F_1 and force on cylinder from the bottom is F_2. Match the following two columns.



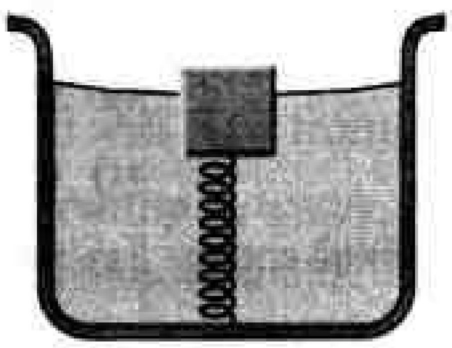
Column I	Column II
(a) Net force on cylinder from liquid-1	(p) Zero
(b) $F_2 - F_1$	(q) W
(c) Net force on cylinder from liquid-2	(r) Net upthrust
(d) Net force on cylinder from atmosphere	(s) None of these

Subjective Questions

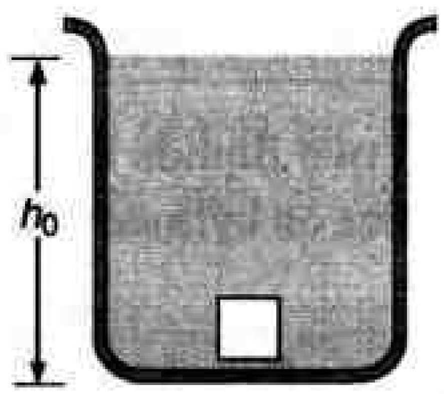
1. A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in figure. The tank is filled with water upto a height of 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position. (Exclude the case $\theta = 0$).



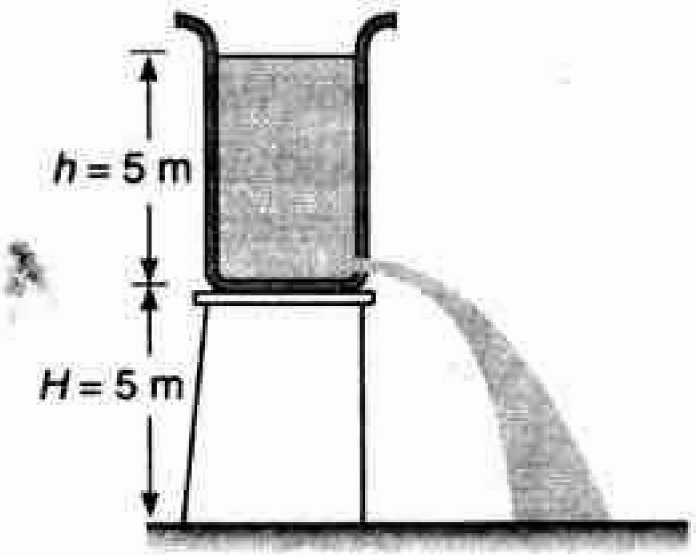
2. A cubical block of wood of edge 3 cm floats in water. The lower surface of the cube just touches the free end of a vertical spring fixed at the bottom of the pot. Find the maximum weight that can be put on the block without wetting the new weight. Density of wood $= 800\,\text{kg/m}^3$ and spring constant of the spring $= 50$ N/m. (Take $g = 10\,\text{m/s}^2$)



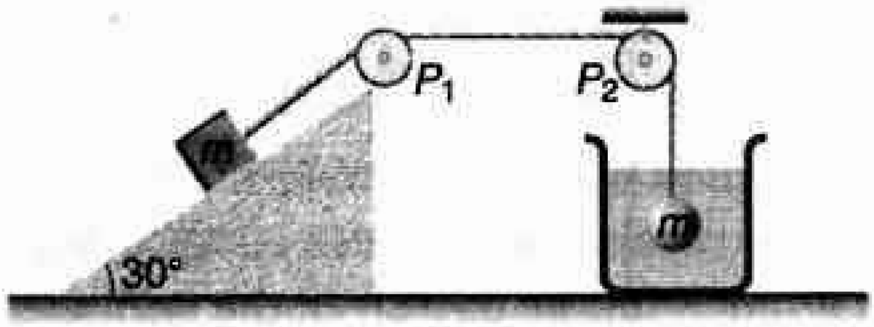
3. Figure shows a container having liquid of variable density. The density of liquid varies as $\rho = \rho_0\left(4 - \frac{3h}{h_0}\right)$. Here, h_0 and ρ_0 are constants and h is measured from bottom of the container. A solid block of small dimensions whose density is $\frac{5}{2}\rho_0$ and mass m is released from bottom of the tank. Prove that the block will execute simple harmonic motion. Find the frequency of oscillation.



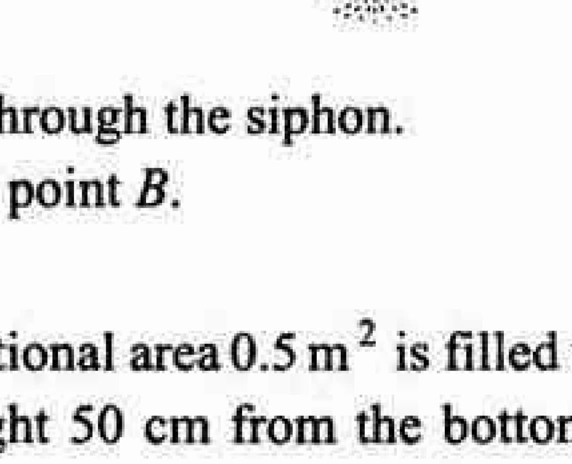
4. A cylindrical tank 1 m in radius rests on a platform 5 m high. Initially the tank is filled with water to a height of 5 m. A plug whose area is $10^{-4}\,\text{m}^2$ is removed from an orifice on the side of the tank at the bottom. Calculate (a) initial speed with which the water flows from the orifice, (b) initial speed with which water strikes the ground, (c) time taken to empty the tank to half its original value. $(g = 10\,\text{m/s}^2)$



5. A block of mass m is kept over a fixed smooth wedge. Block is attached to a sphere of same mass through fixed massless pullies P_1 and P_2. Sphere is dipped inside the water as shown. If specific gravity of material of sphere is 2. Find the acceleration of sphere.

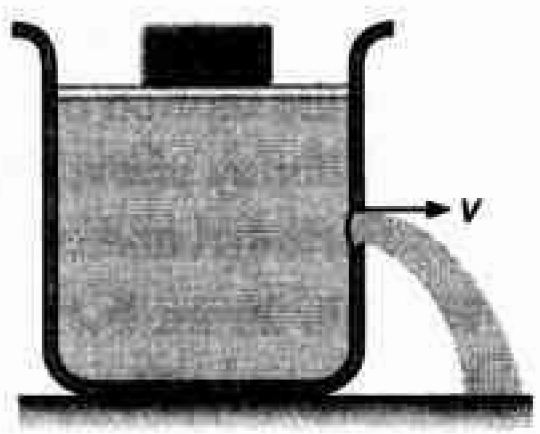


6. A cubic body floats on mercury with 0.25 fraction of its volume below the surface. What fraction of the volume of the body will be immersed in the mercury if a layer of water poured on top of the mercury covers the body completely ?
7. A siphon tube is discharging a liquid of specific gravity 0.9 from a reservoir as shown in the figure.

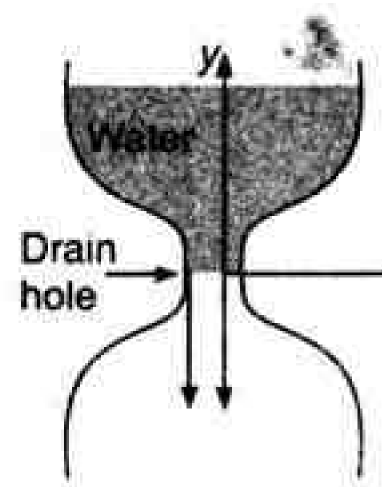


(a) Find the velocity of the liquid through the siphon.
(b) Find the pressure at the highest point B.
(c) Find the pressure at point C.

8. A long cylindrical tank of cross-sectional area $0.5\ m^2$ is filled with water. It has a small hole at a height 50 cm from the bottom. A movable piston of cross-sectional area almost equal to $0.5\ m^2$ is fitted on the top of the tank such that it can slide in the tank freely. A load of 20 kg is applied on the top of the water by piston, as shown in the figure. Calculate the speed of the water jet with which it hits the surface when piston is 1 m above the bottom. (Ignore the mass of the piston).

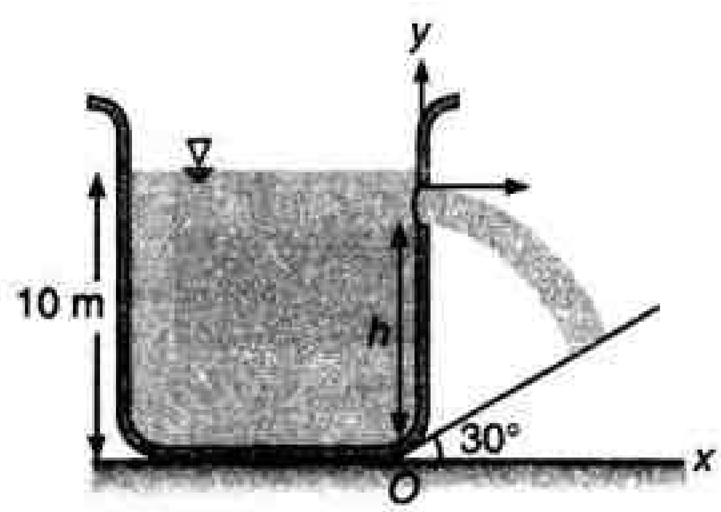


9. The shape of an ancient water clock jug is such that water level descends at a constant rate at all times. If the water level falls by 4 cm every hour, determine the shape of the jar, *i.e.*, specify x as a function of y. The radius of drain hole is 2 mm and can be assumed to be very small compared to x.



10. A spring is attached to the bottom of an empty swimming pool, with the axis of the spring oriented vertically. An 8.00 kg block of wood ($\rho = 840$ kg/m^3) is fixed to the top of the spring and compresses it. Then the pool is filled with water, completely covering the block. The spring is now observed to be stretched twice as much as it had been compressed. Determine the percentage of the block's total volume that is hollow. Ignore any air in the hollow space.

11. A rectangular tank of height 10 m filled with water, is placed near the bottom of a plane inclined at an angle 30° with horizontal. At height h from bottom a small hole is made (as shown in figure) such that the stream coming out from hole, strikes the inclined plane normally. Calculate h.



12. A ball of density d is dropped onto a horizontal solid surface. It bounces elastically from the surface and returns to its original position in a time t_1. Next, the ball is released and it falls through the same height before striking the surface of a liquid of density d_L.
    (a) If $d < d_L$, obtain an expression (in terms of d, t_1 and d_L) for the time t_2 the ball takes to come back to the position from which it was released.
    (b) Is the motion of the ball simple harmonic ?
    (c) If $d = d_L$, how does the speed of the ball depend on its depth inside the liquid ? Neglect all frictional and other dissipative forces. Assume the depth of the liquid to be large.

13. There is an air bubble of radius 1.0 mm in a liquid of surface tension 0.075 N/m and density 1000 kg/m^3. The bubble is at a depth of 10 cm below the free surface. By what amount is the pressure inside the bubble greater than the atmospheric pressure ? (Take $g = 9.8$ m/s^2)

14. A metal sphere of radius 1 mm and mass 50 mg falls vertically in glycerine. Find (a) the viscous force exerted by the glycerine on the sphere when the speed of the sphere is 1 cm/s, (b) the hydrostatic force exerted by the glycerine on the sphere and (c) the terminal velocity with which the sphere will move down without acceleration. Density of glycerine $= 1260\ \text{kg/m}^3$ and its coefficient of viscosity at room temperature $= 8.0$ poise.

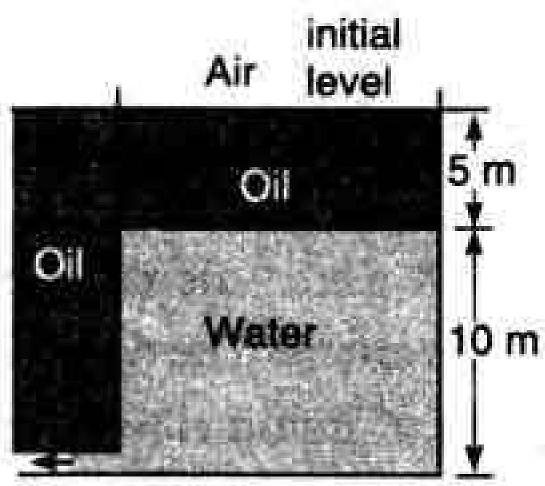
15. A wire forming a loop is dipped into soap solution and taken out, so that a film of soap solution is formed. A loop of 6.28 cm long thread is gently put on the film and the film is pricked with a needle inside the loop. The thread loop takes the shape of a circle. Find the tension in the thread. Surface tension of soap solution $= 0.030$ N/m.

16. A cylindrical vessel is filled with water upto a height of 1 m. The cross-sectional area of the orifice at the bottom is (1/400) that of the vessel.

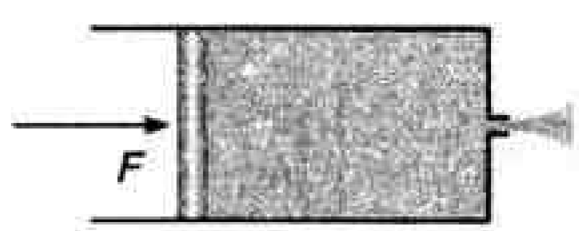
(a) What is the time required to empty the tank through the orifice at the bottom?

(b) What is the time required for the same amount of water to flow out if the water level in tank is maintained always at a height of 1 m from orifice?

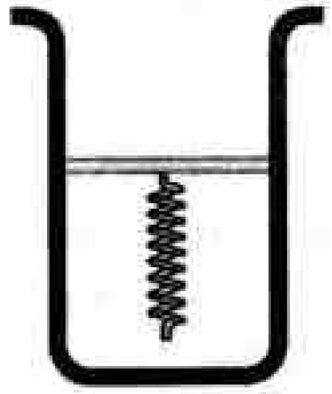
17. A tank having a small circular hole contains oil on top of water. It is immersed in a large tank of the same oil. Water flows through the hole. What is the velocity of this flow initially? When the flow stops, what would be the position of the oil-water interface in the tank from the bottom. The specific gravity of oil is 0.5.



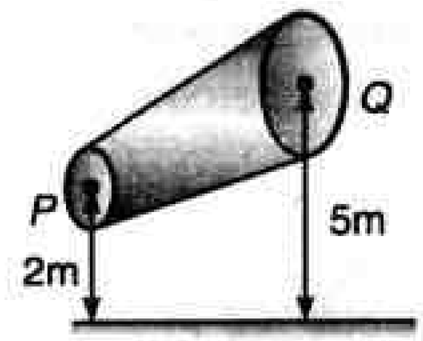
18. What work should be done in order to squeeze all water from a horizontally located cylinder (figure) during the time t by means of a constant force acting on the piston? The volume of water in the cylinder is equal to V, the cross-sectional area of the orifice is s, with s being considerably less than the piston area. The friction and viscosity are negligibly small. Density of water is ρ.



19. A cylinder is fitted with a piston, beneath which is a spring, as in the figure. The cylinder is open at the top. Friction is absent. The spring constant of the spring is 3600 N/m. The piston has a negligible mass and a radius of 0.025 m. (a) When air beneath the piston is completely pumped out, how much does the atmospheric pressure cause the spring to compress? (b) How much work does the atmospheric pressure do in compressing the spring?



20. A non-viscous liquid of constant density 1000 kg/m^3 flows in a streamline motion along a tube of variable cross-section. The tube is kept inclined in the vertical plane as shown in the figure. The area of cross-section of the tube at two points P and Q at heights of 2 m and 5 m are respectively 4×10^{-3} m^2 and 8×10^{-3} m^2. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q. Take $g = 9.8$ m/s^2.

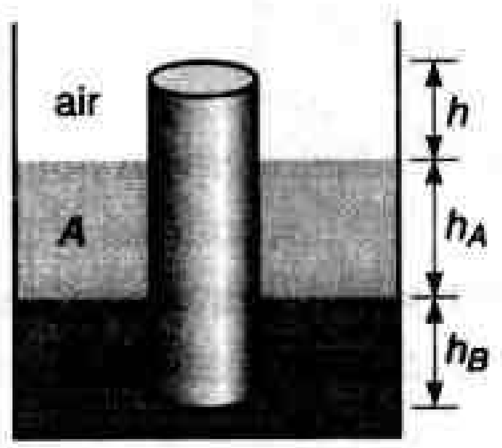


21. A uniform solid cylinder of density 0.8 g/cm^3 floats in equilibrium in a combination of two non-mixing liquids A and B with its axis vertical. The densities of the liquids A and B are 0.7 g/cm^3 and 1.2 g/cm^3, respectively. The height of liquid A is $h_A = 1.2$ cm. The length of the part of the cylinder immersed in liquid B is $h_B = 0.8$ cm.

(a) Find the total force exerted by liquid A on the cylinder.

(b) Find h, the length of the part of the cylinder in air.

(c) The cylinder is depressed in such a way that its top surface is just below the upper surface of liquid A and is then released. Find the acceleration of the cylinder immediately after it is released.



ANSWERS

Introductory Exercise 13.1

1. B 2. 6.92×10^5 Pa 3. 0.8
4. Mercury will rise in the arm containing sprit. The difference in level is 0.221 cm
5. (i) Absolute pressure = 96 cm of Hg, Gauge pressure = 20 cm of Hg for (a),
   absolute pressure = 58 cm of Hg, gauge pressure = – 18 cm of Hg for (b)
   (ii) Mercury would rise in the left limb such that the difference in the levels in the two limbs becomes 19 cm.

Introductory Exercise 13.2

1. 3.5×10^3 kg/m^3 2. 7 g/cm^3, 3 g/cm^3 3. It will float at the same level 4. Zero 5. False
6. 80 kg 7. It will remain same 8. 17 cm 9. 600 kg/m^3, 705 kg/m^3 10. 6.03 g

Introductory Exercise 13.3

1. 1.44×10^{-5} J 2. $8\pi R^2 T$ 3. It will increase by $8\pi r^2 T$ 4. The capillaries formed in threads disappear
5. (a) 2.25 m/s (b) zero 6. (a) 7.0×10^{-4} m^3/s (b) 2.24×10^4 Pa (c) 1.5×10^{-3} m^3/s

For JEE Main

Subjective Questions

2. 3840 kg/m^3 3. 28 cm^3 4. 23.7 N 5. 0.33 m^3 6. (a) 2×10^{-6} m^3, 5000 kg/m^3 (b) 750 kg/m^3
7. 0.206 N 8. 32 mm in mercury and 28 mm in water 9. (a) 14.7 m/s^2 (b) 0.63 s
10. $T = 19.6$ N, $V = 32 \times 10^{-3}$ m^3 11. (a) 24 N (b) 12 m/s^2 12. (a) 10.9 cm (b) 2.5 cm
13. $\tan\theta = \dfrac{\rho - \sigma}{\rho + \sigma}$ 14. 31 N
15. 12.6 cm 16. (a) $\dfrac{\rho g l h^2}{2}$ (b) $\dfrac{\rho g l h^3}{6}$ (c) $\dfrac{h}{3}$ 17. (a) 1470 Pa (b) 13.9 cm 18. 18 cm
19. 0.58 cm 20. 6.7 kPa, 8.2 kPa 21. $R^2/4h$ 22. $v = \sqrt{\dfrac{2F}{A\rho}}$ 23. 12.3 m/s
24. $v_B \approx 3.3$ m/s 25. 146 cm^3/s 26. 2.13 cm 27. 0.87 mm/s 28. 19.5 mm of Hg
29. $v' = (2)^{2/3} v$ 30. $v = \dfrac{u_0 \ln(R_2/r)}{\ln(R_2/R)}$ 31. $F = 0.02$ N 32. 10^{-3} N/m^2 33. $T_m/T_w = 7.23$
35. 1.4 mm 36. $h = 1$ cm

Objective Questions

1. (a)	2. (d)	3. (d)	4. (d)	5. (d)	6. (d)	7. (a)	8. (a)	9. (b)	10. (c)
11. (a)	12. (c)	13. (c)	14. (c)	15. (b)	16. (a)	17. (b)	18. (b)	19. (a)	20. (c)
21. (a)	22. (a)	23. (d)	24. (c)	25. (b)	26. (a)	27. (b)	28. (c)	29. (b)	30. (c)
31. (a)	32. (c)	33. (d)	34. (d)	35. (d)	36. (d)	37. (c)	38. (c)		

For JEE Advanced

Assertion and Reason

1. (d) **2.** (a) **3.** (d) **4.** (a) **5.** (d) **6.** (d) **7.** (b) **8.** (a) **9.** (d) **10.** (d)
11. (c)

Objective Questions

1. (b) **2.** (a) **3.** (d) **4.** (c) **5.** (c) **6.** (c) **7.** (a) **8.** (b) **9.** (d) **10.** (a)
11. (d) **12.** (d) **13.** (b) **14.** (a) **15.** (c) **16.** (a) **17.** (b) **18.** (c) **19.** (d) **20.** (d)
21. (d) **22.** (b) **23.** (d) **24.** (a) **25.** (b) **26.** (b) **27.** (d) **28.** (a) **29.** (c) **30.** (c)
31. (b) **32.** (b) **33.** (d) **34.** (d) **35.** (a) **36.** (b) **37.** (c) **38.** (c)

More than One Correct Options

1. (a,c) **2.** (b,c) **3.** (a,b,c) **4.** (a,c,d) **5.** (b,c,d) **6.** (a,b,c) **7.** (a,c)
8. (a,c,d) **9.** (a,d) **10.** (a,c,d)

Match the Columns

1. (a) → r (b) → p (c) → r (d) → p

2. (a) → s (b) → s (c) → p (d) → s

3. (a) → r (b) → r (c) → s (d) → s

4. (a) → r (b) → s (c) → p (d) → q

5. (a) → p (b) → q,r (c) → s (d) → s

Subjective Questions

1. 45° **2.** 0.354 N **3.** $\frac{1}{2\pi}\sqrt{\frac{6g}{5h_0}}$ **4.** (a) 10 m/s (b) 14.1 m/s (c) 9200 s

5. zero **6.** 0.19 **7.** (a) 9.9 m/s (b) 4.36×10^4 Pa (c) 6.6×10^4 Pa **8.** 4.51 m/s

9. $y = 0.4x^4$ **10.** 60.41% **11.** 8.33 m

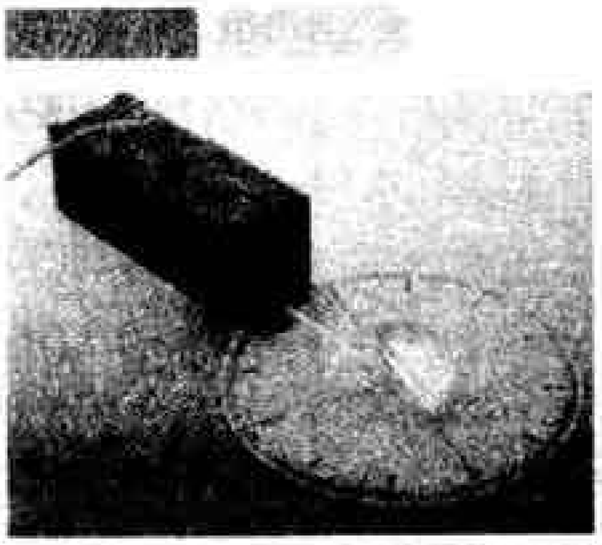
12. (a) $\frac{t_1 d_L}{d_L - d}$ (b) No (c) The ball will continue to move with constant velocity $v = \frac{gt_1}{2}$ inside the liquid.

13. 1130 Pa **14.** (a) 1.5×10^{-4} N (b) 5.2×10^{-5} N (c) 32.5 m/s **15.** 3.0×10^{-4} N

16. (a) 3 min (b) 1.5 min **17.** 9.8 m/s, 5.0 m

18. $\frac{1}{2}\frac{\rho V^3}{s^2 t^2}$ **19.** (a) 5.5 cm (b) 5.445 J **20.** 29025 J/m^3, 29400 J/m^3

21. (a) zero (b) 0.25 cm (c) $\frac{g}{6}$ (upwards)



EXPERIMENTAL SKILLS

Chapter Contents

- Simple Pendulum
- Young's Modulus by Searle's Method
- Mass of given Object by Principle of Moments
- Terminal Velocity and Coefficient of Viscosity

1. Simple Pendulum

(a) Determination of g using simple pendulum

Object

To investigate the motion of a simple pendulum and to derive a value for g, the acceleration due to gravity.

Apparatus and Material Required

A clamp stand, a split cork, thread, bob, Vernier Callipers, stop watch etc.

Formula

The time period 'T' of a simple pendulum of length L is given by the relation :

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{...(i)}$$

Squaring the Eq. (i) and on rearranging, we get :

$$g = 4\pi^2 \frac{L}{T^2} \quad \text{...(ii)}$$

Therefore, the Eq. (ii) suggests that the average value of L/T^2 (for different lengths L and their corresponding square of time periods, T^2) when multiplied by $4\pi^2$ would give us the value of g at the place of experiment.

Procedure

(a) Measure, record and average a reasonable number of measurements of the period T for 6 to 8 different lengths.

(b) Draw a graph between L and T^2. The graph will be straight line.

(c) The slope of L *vs* T^2 graph plotted by taking L along y-axis gives the average value of $\frac{L}{T^2}$.

(d) Now, $g = (4\pi^2)$ (Slope of L - T^2 graph)

Note *An important observation to be made is that the mass of the object that swings in the pendulum has no effect on the results (if air resistance is neglected). This is because the period of oscillation of the pendulum depends only on the length L of the pendulum and the acceleration due to gravity g at that point.*

(b) Dissipation of energy by plotting a graph between square of amplitude and time

As we have seen in article 11.9, that amplitude of oscillation (in case of spring-block system) decreases exponentially with time in case of damped oscillations. So,

$$A(t) = A_0 e^{-\alpha t}$$

$$\therefore \quad A^2 = A_0^2 e^{-2\alpha t}$$

A^2 *versus* t graph is as shown in figure.

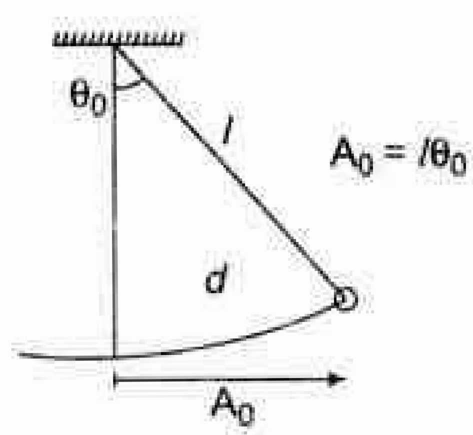


Fig. 1

Energy in SHM is given by :

$$E = \frac{1}{2} KA^2$$

$\therefore$ Dissipation of energy $= E_i - E_t$

$$= \frac{1}{2} KA_0^2 - \frac{1}{2} KA^2 = \frac{1}{2} K (A_0^2 - A^2) \quad \text{...(i)}$$

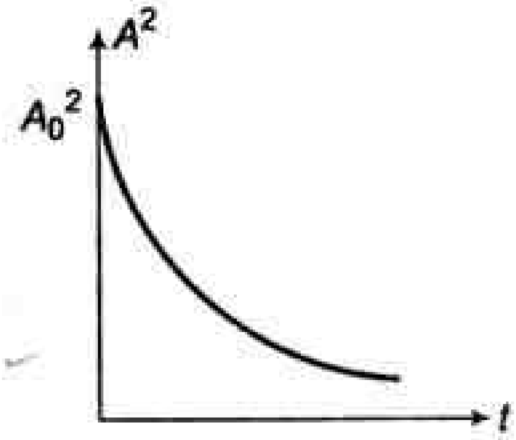


Fig. 2

For small oscillations motion of pendulum is almost linear. Therefore

$$T = 2\pi \sqrt{\frac{l}{g}} = 2\pi \sqrt{\frac{m}{K}} \text{ or } K = \frac{mg}{l}$$

Substituting in Eq. (i) we have,

Dissipation of energy $= \frac{mg}{2l} (A_0^2 - A^2)$ ...(ii)

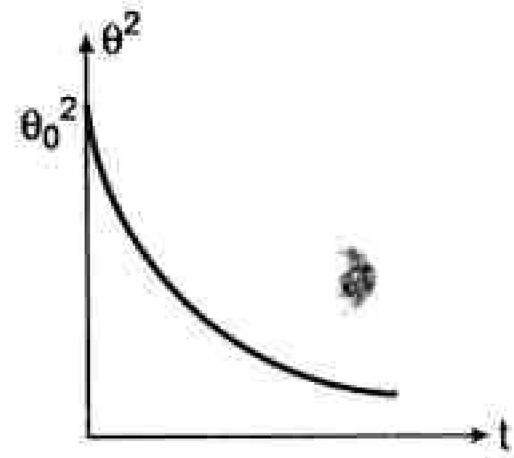


Fig. 3

Further, $A_0 = l\theta_0$ and $A = l\theta$

Substituting in equation (ii) :

$$\text{dissipation of energy} = \frac{mgl}{2} (\theta_0^2 - \theta^2)$$

Here, θ_0 = initial angular amplitude and

θ = angular amplitude at time t

θ will also decrease exponentially. Therefore θ *versus* t graph is also like the previous graph as shown in figure.

2. Young's Modulus By Searle's Method

Object

To determine Young's modulus of elasticity of the material of a given wire by using Searle's apparatus.

Apparatus and Material Required

Searle's apparatus, two long steel wires A and B, screw gauge, a metre rod, slotted weights and hanger.

Formula

It consists of two metal frames P and Q hinged together, such that they can have only vertical relative motion. A spirit level S.L. is supported at one end on a rigid cross bar frame whose other end rests on the tip of a micrometer screw C. If there is any relative motion between the two frames, the spirit level no longer remains horizontal and the bubble is displaced in the spirit level.

To bring the bubble back to its original position, the screw has to be moved up or down. The distance through which the screw has to be moved gives the relative motion between the two frames.

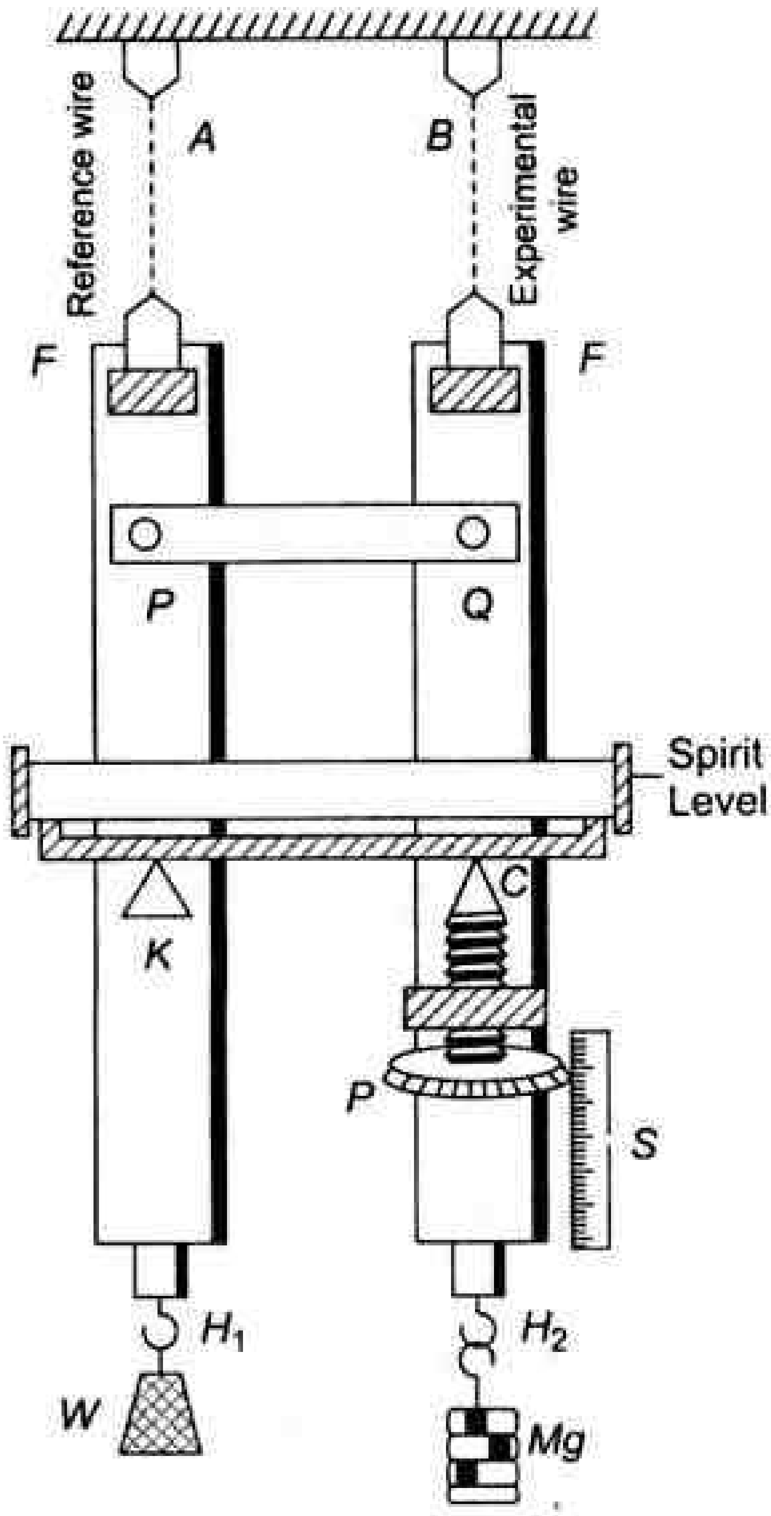


Fig. 4

The frames are suspended by two identical long wires of the same material, from the same rigid horizontal support. Wire B is the experimental wire and the wire A acts simply as a reference wire. The frames are provided with hooks H_1 and H_2 at their ends from which weights are suspended. The hook H_1 attached to the frame of the reference wire carries a constant weight W to keep the wire taut. To the hook H_2 of the experimental wire (*i.e.*, wire B), is attached a hanger over which slotted weights can be placed to apply the stretching force, Mg.

Suppose a wire of length L and cross-section A is stretched by an amount l by a force F acting along its length, then

$$\text{Stress} = \frac{F}{A}$$

and
$$\text{Strain} = \frac{l}{L}$$

$\therefore$ Young's modulus
$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$\therefore$
$$Y = \frac{F}{A} \times \frac{L}{l} = \frac{Mg}{\pi r^2} \cdot \frac{L}{l} \qquad \text{...(i)}$$

where,
$$F = Mg \quad \text{and} \quad A = \pi r^2$$

With the increase in load attached to the experimental wire it elongates. The elongation in the wire for a fixed increase in load can be measured with micrometer screw so that air bubble in the spirit level lies at the middle of the spirit level. Spirit level will be placed on the bar connecting the two wires. Air bubble will be disturbed from the middle of spirit level when the experimental wire elongates due to increase in the load attached to it.

Average elongation l for a load M is determined. If L is the length and r is radius of the wire then Young's modulus Y of the material of the given wire is $Y = \dfrac{MgL}{\pi r^2 l}$.

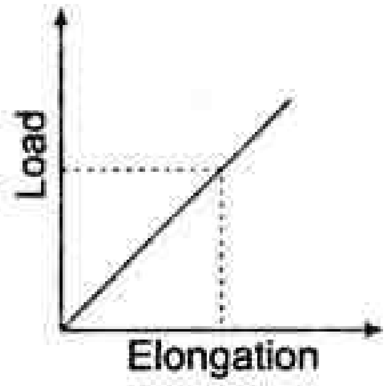


Fig. 5

$$\text{Slope of the shown graph} = \frac{\text{Load}}{\text{Elongation}} = \frac{Mg}{l}$$

If elongations are measured for different values of load Mg then graph drawn by taking load M on y-axis and elongation l on x-axis will give $\dfrac{Mg}{l}$ value by the slope of this graph.

Now in Eq. (i), L and r of the experimental wire will be known to us. Therefore, we can find the value of Y from this equation.

3. Mass of given object by principle of moments

Object

To find mass of a given object by the principle of moments.

Apparatus

A metre rod is supported on a sharp wedge at its centre of gravity G. This metre rod is balanced (i.e., remains horizontal) with an unknown mass m, of

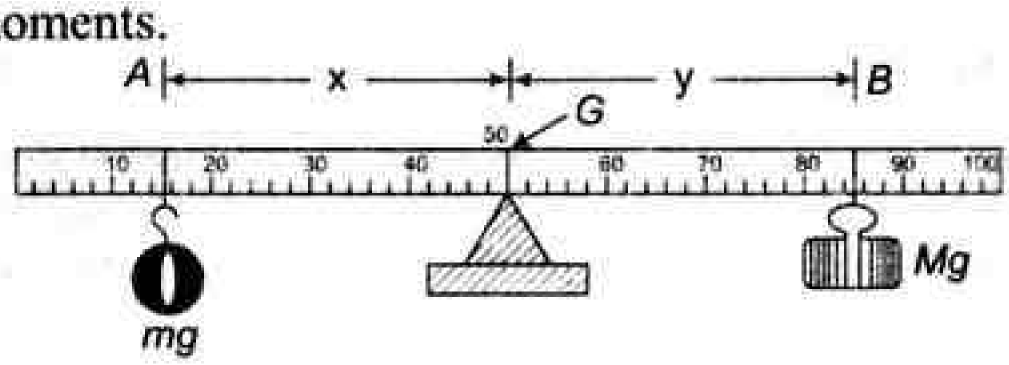


Fig. 6

weight $W = mg$, suspended from the left hand side and a known standard mass M suspended from the right hand side of the knife edge as shown in figure.

Formula

When a body is in rotational equilibrium, the sum of clockwise moments about any point is equal to the sum of anticlockwise moments about that point.

On applying the principle of moments, the rod will be horizontal or in rotational equilibrium if

Sum of anticlockwise moments = Sum of clockwise moments

$$mg \times x = Mg \times y$$

$$\Rightarrow \quad m = M\left(\frac{y}{x}\right)$$

4. Terminal Velocity and coefficient of viscosity

To Find Coefficient of viscosity by measuring terminal velocity

For a liquid the viscosity can be determined by measuring the terminal velocity of solid sphere falling through the same liquid of column of large depth. In the figure, the apparatus used for the purpose is shown. It has a glass cylinder that contains the experimental liquid for which coefficient of viscosity has to be measured. A spherical metal ball is dropped from a point (say C). There are three equidistant marks P, Q and R on the cylinder well below the point C.

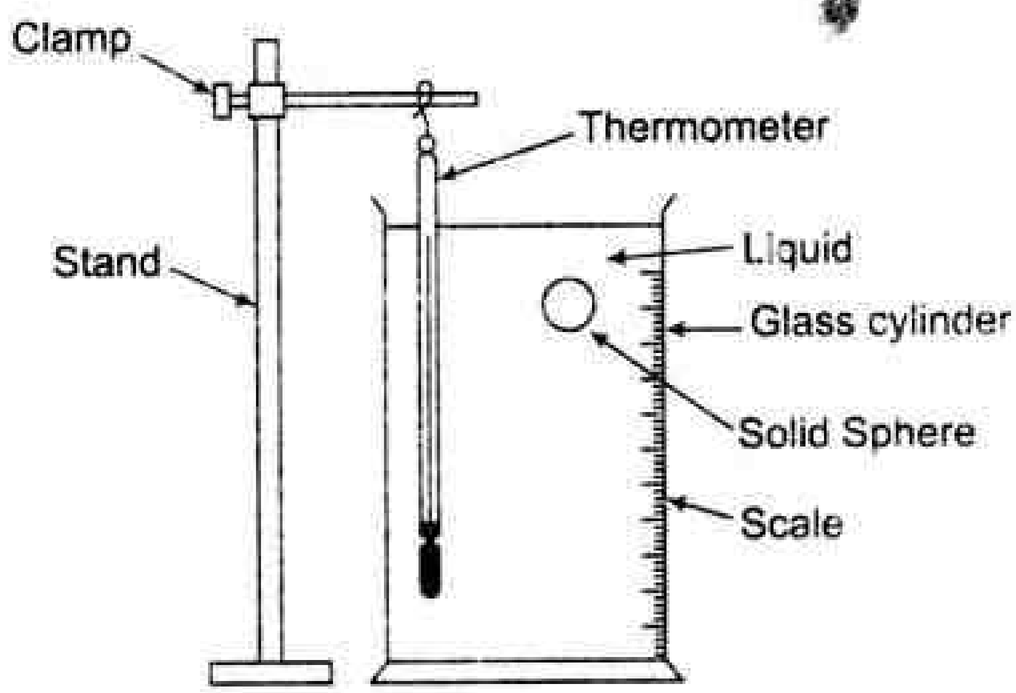


Fig. 8

The time interval taken by the ball to pass through the length PQ and through the length QR are noted with the help of a stop watch. If these two intervals are not equal the same process will be repeated with balls of small size until two time intervals are equal. If two time intervals are equal the velocity achieved by the metal ball before passing through the mark P is called terminal velocity which is constant. The radius of the ball is determined by a screw gauge. Mass of the ball is calculated by weighing it. The length $PQ = QR$ is measured by using a scale.

In article 13.10 we have derived the formula of terminal velocity :

$$v_T = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

where, ρ = density of metal ball and
σ = density of liquid.

Therefore, the coefficient of viscosity at a given temperature (which can be measured by the thermometer) can be determined by using the formula $\eta = \frac{2}{9}\frac{r^2}{v_T}(\rho-\sigma)g$.

EXERCISES

Experimental Skills & General Physics

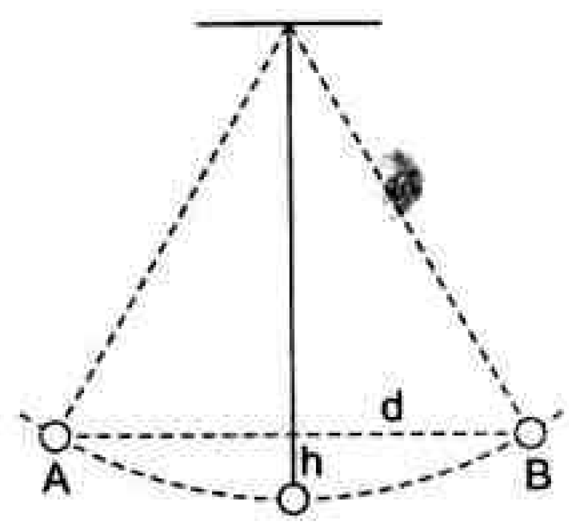
Simple Pendulum

1. What is an ideal simple pendulum ?
2. Is there any difference between gravity and acceleration due to gravity ?
3. What is the difference between g and G.
4. What is meant by effective length of the pendulum ?
5. What is a second's pendulum ?
6. What is the approximate effective length of a second's pendulum ?
7. Will the time period of a simple pendulum change if we take it from Earth to the surface of Moon ?
8. Why does it increase ?
9. How does the value of g vary from place to place on the surface of the Earth ?
10. Is g a scalar or a vector quantity ?
11. Why should the amplitude be small for a simple pendulum experiment ?
12. Does the time period depend upon the mass, the size and the material of the bob ?
13. What will happen to the time period if the bob is made to oscillate in water (neglect viscosity)?
14. How will the amplitude of oscillation change, if the pendulum oscillates in vacuum ?
15. Why do you avoid rotational motion of the bob during the course of your experiment ?
16. What type of graph do you expect between (i) L and T and (ii) L and T^2 ?
17. Why do the pendulum clocks go slow in summer and fast in winter ?
18. Why do we use Invar material for the pendulum of good clocks ?
19. A simple pendulum has a bob which is a hollow sphere full of sand and oscillates with certain period. If all that sand is drained out through a hole at its bottom, then its period
    (a) Increase (b) Decreases (c) Remains same (d) Is zero
20. The second's pendulum is taken from earth to moon, to keep the time period constant
    (a) the length of the second's pendulum should be decreased
    (b) the length of the second's pendulum should be increased
    (c) the amplitude should increase
    (d) the amplitude should decrease
21. The length of a second's pendulum at the surface of earth is 1 m. The length of second pendulum at the surface of moon where g is $(1/6)^{th}$ that at earth's surface is
    (a) $\frac{1}{6}m$ (b) $6\ m$ (c) $\frac{1}{36}m$ (d) $36\ m$

22. The time period of a second's pendulum is 2 sec. The spherical bob which is empty from inside has a mass 50 g. This is now replaced by another solid bob of same radius but having a different mass of 100 g. The new time period will be
(a) 4 s (b) 1 s (c) 2 s (d) 8 s

23. The time period of a simple pendulum is doubled, when
(a) its length is doubled
(b) mass of bob is doubled
(c) length is made four times
(d) the mass of bob and length of pendulum are double

24. Figure shows an oscillating simple pendulum. The mass of the bob is m. The potential energy of the bob, at B, in terms of d and l is $(d >> h)$:



(a) $\frac{mgd^2}{l}$
(b) $\frac{mgd^2}{2l}$
(c) $\frac{mgl^2}{d}$
(d) $\frac{mgl^3}{d^2}$

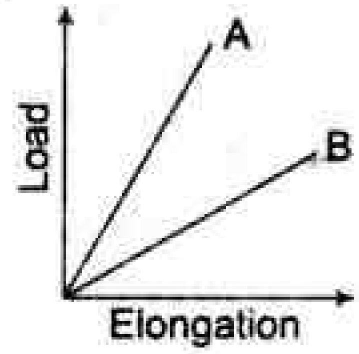
Elasticity

25. How do you differentiate between elastic and plastic bodies ?
26. What is meant by elastic limit ?
27. What is yield point ?
28. What is breaking point ?
29. What is a breaking stress ?
30. Is there any effect of temperature change on the value of Y ?
31. Why do you use long wires in searle's experiment ?
32. Why is it advisable not to take readings of micrometer screw immediately after loading or unloading ?
33. Radius of the experiment wire is to be measured with _______ in Searle's experiment to determine Y.
(a) Ordinary scale (b) Vernier calliper (c) Spherometer (d) Screw gauge

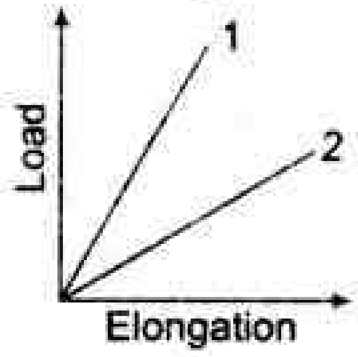
34. Three identical wires A, B and C made of different materials attached with same load have $Y_A = 20 \times 10^{11}\ \text{Nm}^{-2}$, $Y_B = 2 \times 10^{11}\ \text{Nm}^{-2}$ and $Y_C = 0.2 \times 10^{11}\ \text{Nm}^{-2}$. The wire having maximum elongation is
(a) A
(b) B
(c) C
(d) same elongation for all three wires

35. Approximation to perfectly elastic body is :
(a) quartz fibre (b) optical fibre (c) human bone (d) copper rod

36. The dimensions of the wires A and B are the same but materials are different. If the graphs for load versus elongation for those two wires are as shown, Young's modulus is less for
(a) A
(b) B
(c) same for both



37. Load-elongation graph for two wires of same material is as shown in the figure. If their lengths are same which of the two wires A and B is thin
(a) 2
(b) 1
(c) both are of same radius



38. Which of the following is wrong regarding Searle's apparatus method in finding Young's modulus of a given wire ?
(a) Average elongation of wire will be determined with a particular load while increasing the load and decreasing the load.
(b) Reference wire will be just taut and experimental wire will undergo for elongation.
(c) Air bubble in the spirit level will be disturbed from the central position due to relative displacement between the wires due to elongation.
(d) Average elongation of the wires is to be determined by increasing the load attached to both the wires.

39. A wire of area of cross-section $10^{-6} m^2$ is increased in length by 0.1%. The tension produced is 1000 N. The Young's modulus of wire is
(a) 10^{12} Nm^{-2} (b) 10^{11} Nm^{-2} (c) 10^{10} Nm^{-2} (d) 10^{9} Nm^{-2}

40. Which of the following is most elastic ?
(a) rubber (b) wet clay (c) plastic (d) steel

41. A wire can support a load W without breaking. It is cut into equal parts. The maximum load that each part can support is
(a) $\frac{W}{4}$ (b) $\frac{W}{2}$ (c) W (d) $2W$

42. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied on all ?
(a) length = 50 cm, diameter = 0.5 mm (b) length = 100 cm, diameter = 1 mm
(c) length = 300 cm, diameter = 2 mm (d) length = 300 cm, diameter = 3 mm

43. Two wires of copper have lengths in the ratio 1 : 2 and radii in the ratio 2 : 1. Their Young's modulli are in the ratio
(a) 1 : 1 (b) 1 : 8 (c) 8 : 1 (d) 1 : 4

44. Young's modulus for a perfectly rigid body is
(a) 0 (b) 1 (c) ∞ (d) none of these

45. Three wires A, B and C made of different materials, have same length and cross-section. They are stretched by the same force. It is found that wire A is stretched least and comes back to its original length when the force is withdrawn. Wire B is stretched more than A and also comes back to its original length. Wire C is stretched most but does not return to its original length. Which of the three materials has the largest Young's modulus ?

(a) A (b) B

(c) C (d) No conclusion can be drawn from the given information

Principle of Moments

46. What is the principle of moments ?

47. The bottom of a ship is made heavy. Why ?

48. Why does a girl lean towards right while carrying a bag in her left hand ?

49. Some heavy boxes are to be loaded along with some empty boxes on a cart. Which boxes should be put in the cart first and why ?

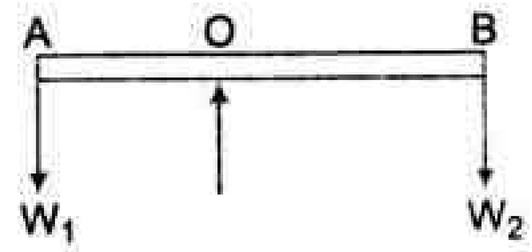
50. Standing is not allowed in upper deck of a double decker bus. Why ?

51. Why cannot we rise from a chair without bending a little forward ?

52. For determining the mass of a given body using a meter scale by principle of moments, the mass of weight in the paper pan is 30 g and the length of the weight arm is 20 cm. If the fixed length of the power arm is 25 cm, then the unknown mass is

(a) 18 g (b) 20 g (c) 24 g (d) 28 g

53. A rigid rod AB shown in figure has negligible mass. A force W_1 of 20 N acts on end A. The rod is pivoted about O such that $AO = 1.2$ m and $OB = 1.6$ m. The force W_2 at B which will keep the rod in equilibrium is



(a) 5 N (b) 10 N

(c) 15 N (d) 20 N

54. A balance is made of a rigid rod free to rotate about a point not at the centre of the rod. When an unknown mass m is placed in the left hand pan, it is balanced by a mass m_1 placed in the right hand pan, and similarly when the mass m is placed in the right hand pan, it is balanced by a mass m_2 in the left hand pan. Neglecting the masses of the pans, m is

(a) $\frac{1}{2}(m_1 + m_2)$ (b) $\sqrt{m_1 m_2}$ (c) $\frac{1}{2}\sqrt{m_1 m_2}$ (d) $\sqrt{\frac{m_1^2 + m_2^2}{2}}$

55. A false balance has equal arms. An object weights x when placed in one pan and y in the other pan. The true weight of the object is equal to

(a) $\sqrt{xy}$ (b) $\frac{x+y}{2}$ (c) $(x + y)$ (d) $\frac{\sqrt{x^2 + y^2}}{2}$

Surface Tension and Viscosity

56. Why are the small spheres wetted in glycerine before making them to fall through glycerine ?

57. Why are the small spheres centrally dropped in the glass jar ?

58. Why do bubbles of air or gas rise up through water or any other liquid ?

59. Why does the water not rise in an ordinary glass tube as it does in a capillary tube ?

60. What formula are you using to determine the surface tension ?

61. While measuring the height of water column in the capillary, up to what point of meniscus do you measure the height ?

62. Which of the following is a better cleaning agent ?
(a) Cold water (b) Hot water (c) Soap water (d) Hot soap water

63. A capillary tube is dipped in water and the top is closed then the rise of water
(a) increases (b) decreases (c) remains same (d) water does not rise

64. If a capillary tube of insufficient length is used, the liquid
(a) rises and over flows slowly (b) does not overflow
(c) rises and comes out like fountain (d) does not rise into the tube

65. What happens to the length of liquid column in the capillary tube if it is inclined with respect to vertical ?
(a) Decreases (b) Increases (c) Remains same (d) Overflows

66. h is the height of the liquid column in the capillary tube when it is clamped vertically. If the tube is making an angle (θ) with vertical. Then length of the liquid column (l) is
(a) $l = h\cos\theta$ (b) $l = \dfrac{h\cos\theta}{2}$ (c) $l = 2h\cos\theta$ (d) $l = \dfrac{h}{\cos\theta}$

67. Two students A and B determine the surface tension in summer and winter as T_A and T_B respectively (other conditions are same in the experiment). Then :
(a) $T_A > T_B$ (b) $T_A < T_B$ (c) $T_A = T_B$ (d) $T_A = \dfrac{T_B}{2}$

68. A student A does the capillary rise experiment at equator and gets the surface tension value T_A. Another student does the same experiment in same conditions at poles and gets the surface tension value T_B. Then:
(a) $T_A > T_B$ (b) $T_A < T_B$ (c) $T_A = T_B$ (d) None of these

69. A clean capillary tube is dipped vertically in a beaker containing water and the water rises 6cm in it. If another capillary tube of same radius and length 3 cm is dipped vertically in the same beaker containing water (assume angle of contact of water is 0 °C) then the water
(a) will flow out like a fountain
(b) will rise to a height of 3 cm only and angle of contacts is 0 °C
(c) will rise to a height of 3 cm only and angle of contact is 60 °C
(d) will not rise at all

70. When a partially soluable impurity is added to a liquid, then the surface tension
(a) increases (b) decreases (c) remains same (d) cannot be measured

71. When a completely soluable impurity is added to a liquid, then the surface tension
(a) increases (b) decreases (c) remains same (d) become zero

72. If temperature is increased, then angle of contact
(a) decreases (b) increases (c) become zero (d) remains same

73. If radius of the tube is decreased, then the height of the liquid column
(a) increases (b) decreases
(c) unchanged (d) may increase or decrease

74. If the angle of contact is 90° then the liquid level in a capillary tube will
(a) increase (b) decrease
(c) neither rise nor fall (d) may increase or decrease

75. What happens to the surface tension if detergents are added to the water ?
(a) increases (b) decreases (c) Remains same (d) Become zero

76. What happens to the angle of contact if detergents are added to the water ?
(a) decreases (b) increases (c) become zero (d) remains same

77. A liquid rises to a height h in a capillary tube on the earth. The height to which the same liquid would rise in the same tube on the moon is about
(a) $6h$ (b) h (c) $\frac{h}{6}$ (d) None of these

78. At critical temperature the surface tension of a liquid
(a) is zero (b) is infinity
(c) is the same as that at any other temperature (d) cannot be determined

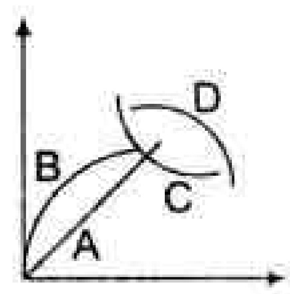
79. A liquid will not wet the surface of a solid if the angle of contact is
(a) acute (b) obtuse (c) zero (d) $\frac{\pi}{2}$

80. A water-proofing agent changes the angle of contact from
(a) an obtuse to an acute value (b) an acute to an obtuse value
(c) an obtuse value to $\frac{\pi}{2}$ (d) an acute value to $\frac{\pi}{2}$

81. A rain drop falls near the surface of the earth with almost uniform velocity because
(a) its weight is negligible
(b) the force of surface tension balances its weight
(c) the viscous drag and the upthrust due to air balances its weight
(d) the drops are charged and atmospheric electric field balances its weight

82. A solid sphere falls with a terminal velocity of 20 ms^{-1} in air. If it is allowed to fall in vacuum
(a) Terminal velocity will be 20 ms^{-1}
(b) Terminal velocity will be less than 20 ms^{-1}
(c) Terminal velocity will be greater than 20 ms^{-1}
(d) There will be no terminal velocity

83. A spherical ball is dropped in a long column of a viscous liquid. The speed of the ball as a function of time may be best represented by the graph



(a) A (b) B
(c) C (d) D

84. Eight spherical rain drops of the same mass and radius are falling down with a terminal speed v. If they coalesce to form one big drop. Its terminal speed will be (neglect buoyancy due to air)
(a) $\frac{v}{4}$ (b) v (c) $4v$ (d) $8v$

85. A spherical steel ball released at the top of a long column of glycerine of length L, falls through a distance $L/2$ with accelerated motion and remaining distance $L/2$ with a uniform velocity. If t_1 and t_2 denote the times taken to cover the first and second half and w_1 and w_2 are the respective works done against gravity in the two halves then
(a) $t_1 < t_2, w_1 > w_2$ (b) $t_1 > t_2, w_1 < w_2$ (c) $t_1 = t_2, w_1 = w_2$ (d) $t_1 > t_2, w_1 = w_2$

86. A tiny sphere of mass m and density ρ is dropped in a tall jar of glycerine of density σ. When the sphere acquires terminal velocity, the magnitude of the viscous force acting on it is

(a) $\frac{mg\rho}{\sigma}$ (b) $\frac{mg\sigma}{\rho}$ (c) $mg\left(1-\frac{\sigma}{\rho}\right)$ (d) $mg\left(1-\frac{\rho}{\sigma}\right)$

87. The viscous force on a spherical body moving through a fluid depends upon

(i) the mass of the body (ii) the radius of the body
(iii) the velocity of the body (iv) the viscosity of the fluid
(a) (i), (ii) and (iii) are true (b) (i), (iii) and (iv) are true
(c) (ii), (iii) and (iv) are true (d) (ii) and (iv) are true

88. When the upthrust on a body is negligible compared to its weight, the terminal velocity of a small spherical body falling through a viscous liquid depends upon

(i) the density of the body (ii) the diameter of the body
(iii) the viscosity of liquid (iv) the acceleration due to gravity
(a) (i) and (ii) are true (b) (ii) and (iii) are true
(c) (ii), (iii) and (iv) are true (d) All are true

89. A small metal sphere of radius r and density ρ falls from rest in a viscous liquid of density σ and coefficient of viscosity η. Due to friction heat is produced. The expression for the rate of production of heat when the sphere has acquired the terminal velocity is

(a) $\left[\frac{8\pi g}{27\eta}(\rho-\sigma)^2\right]r^5$ (b) $\left[\frac{8\pi g^2}{27\eta}(\rho-\sigma)^2\right]r^5$

(c) $\left[\frac{8\pi g^2}{27\eta}(\rho-\sigma)\right]r^5$ (d) $\left[\frac{8\pi g^2}{27\eta^2}(\rho-\sigma)\right]r^5$

90. A steel ball of density ρ_1 and radius r falls vertically through a liquid of density ρ_2. Assume that viscous force acting on the ball is $F = Krv$, Where k is a constant and v is velocity. The terminal velocity of the ball is

(a) $\frac{4\pi gr^2(\rho_1-\rho_2)}{3K}$ (b) $\frac{4\pi r^2(\rho_1-\rho_2)}{3gK}$ (c) $\frac{4\pi(\rho_1+\rho_2)}{3gr^2K}$ (d) $\frac{4\pi gr}{3K}(\rho_1+\rho_2)$

91. For a body falling with terminal velocity, the net force on it is

(a) buoyant force
(b) weight of the body
(c) difference of viscous force and weight of the body
(d) zero

92. An air bubble of radius 1.0 cm rises with a constant speed of 3.5 mms^{-1} through a liquid of density $1.75\times10^3\ \text{kgm}^{-3}$. Neglecting the density of air, the coefficient of viscosity of the liquid is (in $\text{kgm}^{-1}\ \text{s}^{-1}$)

(a) 54.5 (b) 109 (c) 163.5 (d) 218

ANSWERS

Experimental Skills & General Physics

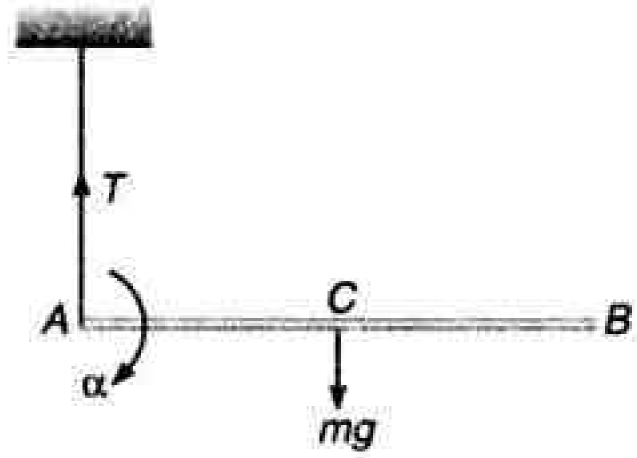
19. (c)	**20.** (a)	**21.** (a)	**22.** (c)	**23.** (c)	**24.**(b)	**33.**(d)	**34.**(c)	**35.**(a)	**36.**(b)
37. (a)	**38.** (d)	**39.** (a)	**40.** (d)	**41.** (c)	**42.**(a)	**43.**(a)	**44.**(c)	**45.**(a)	**52.**(c)
53. (c)	**54.** (b)	**55.** (b)	**62.** (d)	**63.** (c)	**64.**(b)	**65.**(b)	**66.**(d)	**67.**(b)	**68.**(b)
69. (c)	**70.** (b)	**71.** (a)	**72.** (a)	**73.** (a)	**74.**(c)	**75.**(b)	**76.**(a)	**77.**(a)	**78.**(a)
79. (b)	**80.** (b)	**81.**(c)	**82.**(d)	**83.** (b)	**84.**(c)	**85.**(d)	**86.**(c)	**87.**(c)	**88.**(d)
89. (b)	**90.** (a)	**91.**(d)	**92.**(b)						

HINTS & SOLUTIONS

Chapter 9 Mechanics of Rotational Motion

JEE Advanced (Subjective Questions)

1. Torque about bottommost point in each case is clockwise.



2. $$\alpha = \frac{\tau}{I} = \frac{mg\left(\frac{l}{2}\right)}{\frac{ml^2}{3}} = \frac{3}{2}\frac{g}{l}$$

(a) $$a_B = l\alpha = \frac{3g}{2} \quad \textbf{Ans.}$$

(b) $$a_C = \frac{l}{2}\alpha = \frac{3g}{4} \quad \textbf{Ans.}$$

(c) $$a_C = \frac{mg - T}{m} \quad \text{or} \quad \frac{3g}{4} = \frac{mg - T}{m} \quad \therefore \; T = \frac{mg}{4} \quad \textbf{Ans.}$$

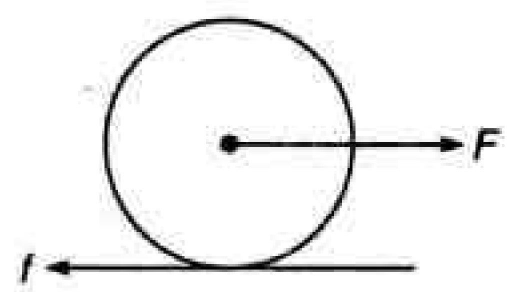
3. $$mgh = K_R + K_T = \frac{3}{4}mv^2$$

Here $h = s\sin\theta$

$\therefore$ $$gs\sin\theta = \frac{3}{4}v^2$$

or $$s = \frac{3v^2}{4g\sin\theta} = \frac{3 \times (2.0)^2}{4 \times 9.8 \times \frac{1}{2}} = 0.612 \text{ m} \quad \textbf{Ans.}$$

4. $I = MR^2$



For pure rolling to take place. $$a = R\alpha$$

or $$\frac{F - f}{M} = R\left(\frac{f.R}{MR^2}\right) = \frac{f}{M}$$

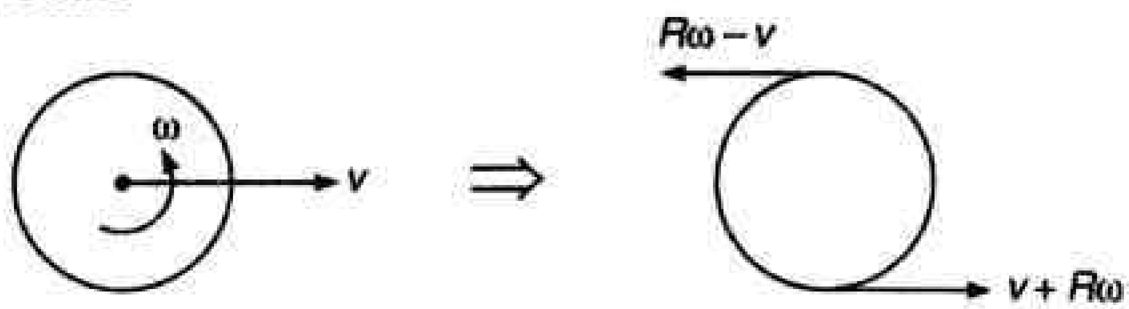
$\therefore$ $$f = \frac{F}{2} \quad \textbf{Ans.}$$

and $$a = \frac{F - f}{M} = \frac{F}{2M} \quad \textbf{Ans.}$$

5. $$v + R\omega = 1.5 \quad \ldots\text{(i)}$$

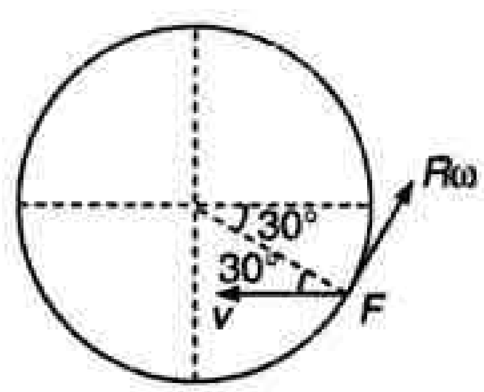
and $$R\omega - v = 3.0 \quad \ldots\text{(ii)}$$

From Eq. (i) and (ii), we have



$$R\omega = 2.25 \text{ m/s} \quad \text{and} \quad v = -\,0.75 \text{ m/s}$$

Thus, velocity of point C is 0.75 m/s (towards left).



$$v_F = \sqrt{v^2 + (R\omega)^2 + 2v(R\omega)\cos(90 + 30)}$$

$$= \sqrt{0.5625 + 5.0625 + 2 \times 0.75 \times 2.25 \times \left(-\frac{1}{2}\right)}$$

$$= 1.98 \text{ m/s}$$ **Ans.**

6.
$$a = \frac{Mg - T}{M} \quad \ldots(i)$$

$$\alpha = \frac{TR}{\frac{1}{2}MR^2} = \frac{2T}{MR} \quad \ldots(ii)$$

$$a = R\alpha \quad \ldots(iii)$$

Solving these three equations, we get

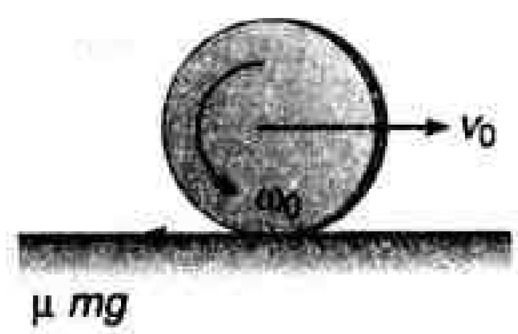
$$a = \frac{2g}{3} \quad \text{and} \quad T = \frac{Mg}{3}$$ **Ans.**

7. From conservation of mechanical energy, decrease in gravitational PE = increase in rotational KE

or
$$mg(R) = \left[\frac{1}{2}MR^2 + mR^2\right]\left(\frac{1}{2}\omega^2\right)$$

or
$$\omega = \sqrt{\frac{4mg}{(2m + M)R}}$$ **Ans.**

8. Initially there is forward slipping. Therefore, friction is backwards and maximum. Let velocity becomes zero in time t_1 and angular velocity becomes zero in time t_2.



Then,
$$0 = v_0 - at_1$$

or
$$t_1 = \frac{v_0}{a} = \frac{v_0}{\mu g} \quad \ldots(i)$$

and
$$0 = \omega_0 - \alpha t_2 \quad \text{or} \quad t_2 = \frac{\omega_0}{\alpha}$$

Here,
$$\alpha = \frac{\mu mgR}{\frac{1}{2}mR^2} = \frac{2\mu g}{R}$$

$\therefore$
$$t_2 = \frac{\omega_0 R}{2\mu g} \quad \ldots(ii)$$

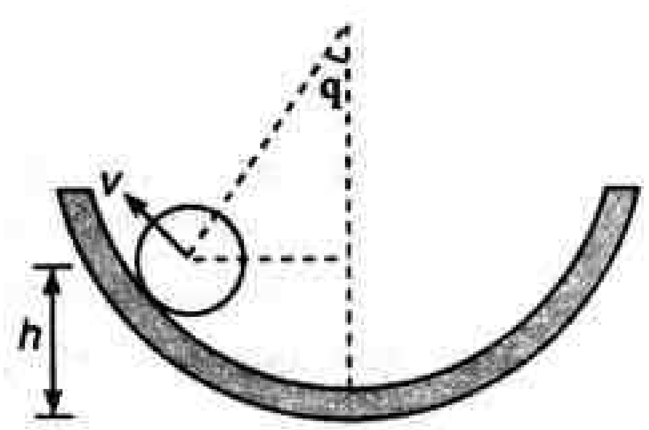
Disk will return back when
$$t_2 > t_1$$

or
$$\frac{\omega_0 R}{2\mu g} > \frac{v_0}{\mu g}$$

or
$$\omega_0 > \frac{2v_0}{R}$$ **Ans.**

9.
$$h = (R - r)(1 - \cos\theta) \quad \ldots(i)$$

Kinetic energy at angle θ is, $K = \frac{7}{5}\left(\frac{1}{2}mv_0^2\right) - mgh$



$\because$ In case of pure rolling $\quad K_T = \frac{5}{7}K$

$\therefore$
$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 - \frac{5}{7}mgh$$

$\therefore$
$$v^2 = v_0^2 - \frac{10}{7}gh \quad \ldots(ii)$$

Equation of motion at angle θ is,

$$N - mg\cos\theta = \frac{mv^2}{(R-r)}$$

$\therefore$
$$N = mg\cos\theta + \frac{m}{(R-r)}\left(v_0^2 - \frac{10}{7}gh\right)$$

Substituting value of h from Eq. (i)

$$N = mg\cos\theta + \left(\frac{m}{R-r}\right)\left\{v_0^2 - \frac{10}{7}g(R-r)(1-\cos\theta)\right\}$$

$$= \frac{mg}{7}(17\cos\theta - 10) + \frac{mv_0^2}{(R-r)}$$ **Ans.**

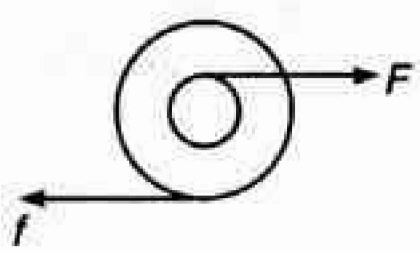
Force of friction,
$$f = \frac{mg\sin\theta}{1 + \dfrac{mr^2}{I}}$$ (for pure rolling to take place)

$$= \frac{mg\sin\theta}{1 + \dfrac{5}{2}}$$ $\left(I = \frac{2}{5}mr^2\right)$

$$= \frac{2}{7}mg\sin\theta$$ **Ans.**

10. (a) For pure rolling to take place,

$$a = R\alpha$$



or
$$\frac{F-f}{m} = R\left[\frac{Fr + fR}{\frac{1}{2}mR^2}\right]$$

Solving this equation, we get

$$f = \frac{2}{3}\left(\frac{1}{2} - \frac{r}{R}\right)F$$ **Ans.**

(b) Acceleration
$$a = \frac{F-f}{m}$$

Substituting value of f from part (a), we get

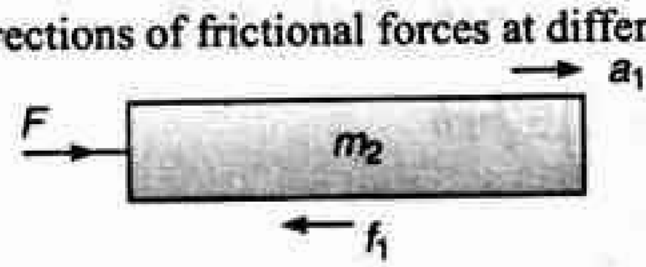
$$a = \frac{2F}{3mR}(R+r)$$ **Ans.**

(c) $a > \frac{F}{m}$ if

$$\frac{2}{3R}(R+r) > 1 \quad \text{or} \quad r > \frac{R}{2}$$ **Ans.**

(d) In this case force of friction is in forward direction. **Ans.**

11. We can choose any arbitrary directions of frictional forces at different contacts.



In the final answer the negative value will show the opposite directions.

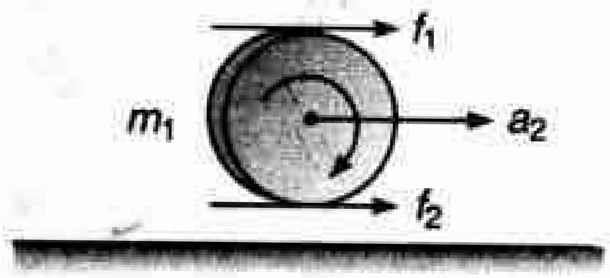
Let f_1 = friction between plank and cylinder $\quad$ f_2 = friction between cylinder and ground

a_1 = acceleration of plank $\quad$ a_2 = acceleration of centre of mass of cylinder

and α = angular acceleration of cylinder about is COM.

Directions of f_1 and f_2 are as shown here :

Since, there is no slipping anywhere



$\therefore$ $$a_1 = 2a_2 \qquad \text{...(i)}$$

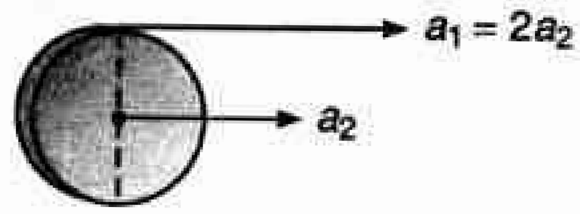
(Acceleration of plank = acceleration of top point of cylinder)

$$a_1 = \frac{F - f_1}{m_2} \qquad \text{...(ii)}$$

$$a_2 = \frac{f_1 + f_2}{m_1} \qquad \text{...(iii)}$$

$$\alpha = \frac{(f_1 - f_2)R}{I}$$

(I = moment of inertia of cylinder about COM)



$\therefore$ $$\alpha = \frac{(f_1 - f_2)R}{\frac{1}{2}m_1 R^2}$$

$$\alpha = \frac{2(f_1 - f_2)}{m_1 R} \qquad \text{...(iv)}$$

$$a_2 = R\alpha = \frac{2(f_1 - f_2)}{m_1} \qquad \text{...(v)}$$

(Acceleration of bottommost point of cylinder = 0)

(a) Solving Eqs. (i), (ii), (iii) and (v), we get

$$a_1 = \frac{8F}{3m_1 + 8m_2} \quad \text{and} \quad a_2 = \frac{4F}{3m_1 + 8m_2}$$

(b) $$f_1 = \frac{3m_1F}{3m_1 + 8m_2}$$

$$f_2 = \frac{m_1F}{3m_1 + 8m_2}$$

Since, all quantities are positive, they are correctly shown in figures.

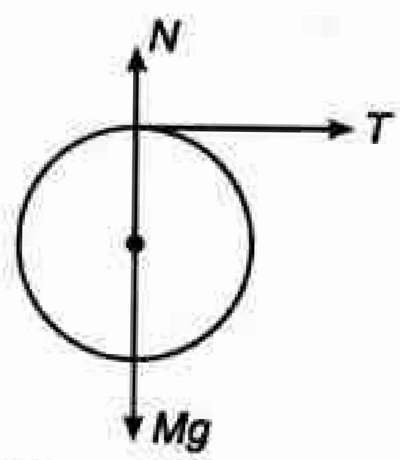
Note *Above calculations have been done at* $t = 0$ *when* $\omega = 0$.

12. If α be the angular acceleration of the hoop and a be the acceleration of its centre, acceleration of m would be $\alpha r + a$.
Here, $Tr = I\alpha$ [where I = moment of inertia of the hoop about the horizontal axis passing through its centre]

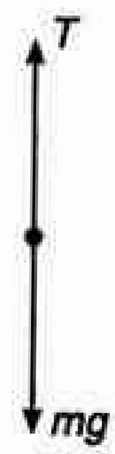
Also, $T = Ma$ and $mg - T = m[a + \alpha r]$

Solving, we get

$$a = \frac{mg}{[M + 2m]} = \frac{2}{1.4} = 1.43 \text{ m/s}^2$$



F.B.D. of hoop



F.B.D. of block of mass m

Hence, $$T = 1.43 \text{ N}$$

and $$\alpha = \frac{Tr}{I} = \frac{T}{Mr} = 7.15 \text{ rad/s}^2$$

13. Initially the cylinder will slip on the plank, therefore kinetic friction will act between the cylinder and the plank.

$$a_c = -\frac{\mu mg}{m} = -\mu g$$

$$a_p = +\frac{\mu mg}{2m} = +\frac{\mu g}{2}$$

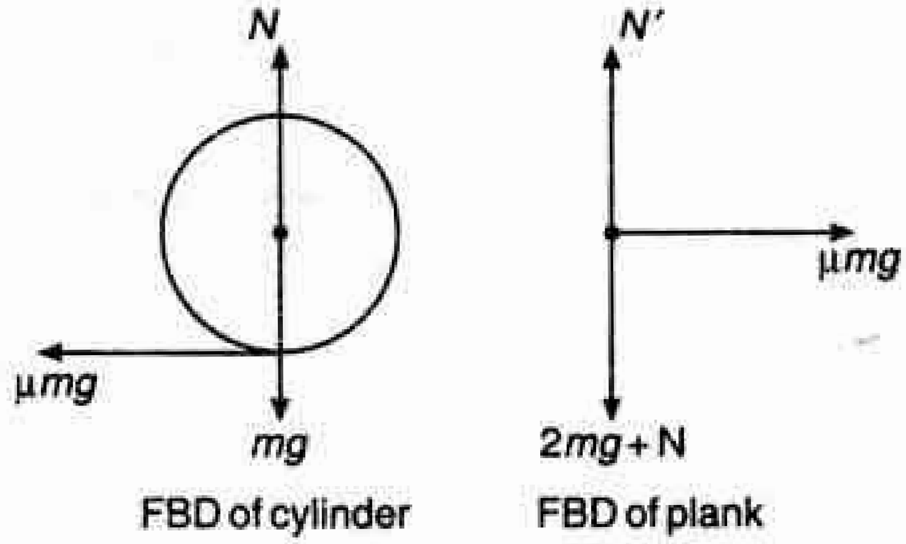
$$\alpha_c = +\frac{(\mu mg)(R)}{(mR^2/2)} = +\frac{2\mu g}{R}$$

For pure rolling,

$$v_p = v_c - R\omega_c$$

$\therefore$ $$\frac{\mu g}{2} t = v_0 - \mu g t - (R)\left(\frac{2\mu g}{R}\right)(t)$$

$\therefore$ $$t = \frac{v_0}{3.5\mu g} = \frac{7}{3.5 \times 0.1 \times 10} = 2\text{ s}$$



FBD of cylinder   FBD of plank

$\therefore$ $$s_c - s_p = v_0 t - \frac{1}{2} \times (\mu g)(t^2) - \frac{1}{2}\left(\frac{\mu g}{2}\right)(t^2)$$

$$= (7 \times 2) - \frac{1}{2}(0.1)(10)(4) - \frac{1}{2}\left(\frac{0.1 \times 10}{2}\right)(4) = 11\text{ m}$$

Also, $$v_c - v_P = (v_0 - \mu g t) - \left(\frac{\mu g}{2}\right)(t)$$

$$= 7 - 0.1 \times 10 \times 2 - \frac{0.1 \times 10 \times 2}{2} = 4\text{ m/s}$$

Hence, the remaining distance $(12 - 11 = 1\text{ m})$ is travelled in a time,

$$t' = \frac{1.0}{4} = 0.25\text{ s}$$

$\therefore$ Total time = 2 + 0.25 = 2.25 s

14. In rolling without sliding on a stationary ground, work done by friction is zero. Hence

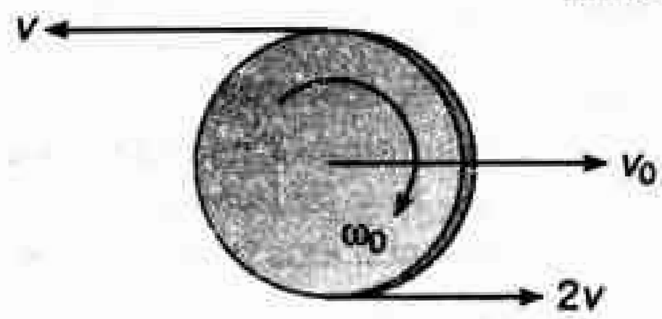
work done by the applied force = change in kinetic energy

$\therefore$ $$(30)(0.25) = \frac{1}{2} \times 9 \times v^2 + 2\left[\frac{1}{2} \times 6 \times v^2 + \frac{1}{2} \times \frac{1}{2} \times 6 \times r^2 \times \frac{v^2}{r^2}\right]$$

or $$7.5 = 13.5v^2$$

$\therefore$ $$v = 0.745\text{ m/s}$$ **Ans.**

15. Let v_0 be the linear velocity and ω_0 the angular velocity of the disc as shown in figure then,



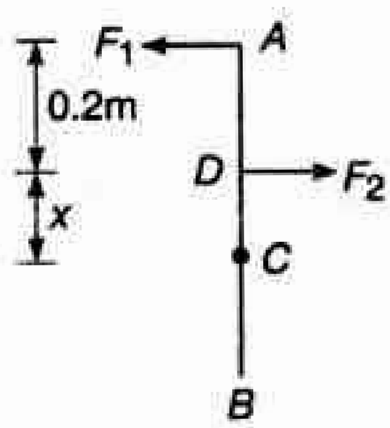
$$v_0 - r\omega_0 = 2v \qquad \text{...(i)}$$

and $$v_0 + r\omega_0 = -v \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we have $$\omega_0 = -\frac{3}{2}\frac{v}{r}$$

Hence, the angular velocity of disc is $\frac{3}{2}\frac{v}{r}$ anticlockwise. **Ans.**

16. Let x be the distance of centre point C of rod from D. Then,



$$F_2 - F_1 = ma \quad \text{or} \quad F_1 = 3\text{ N}$$

Further, $$\tau_c = 0$$

$\therefore$ $$F_2 x = F_1(0.2 + x)$$

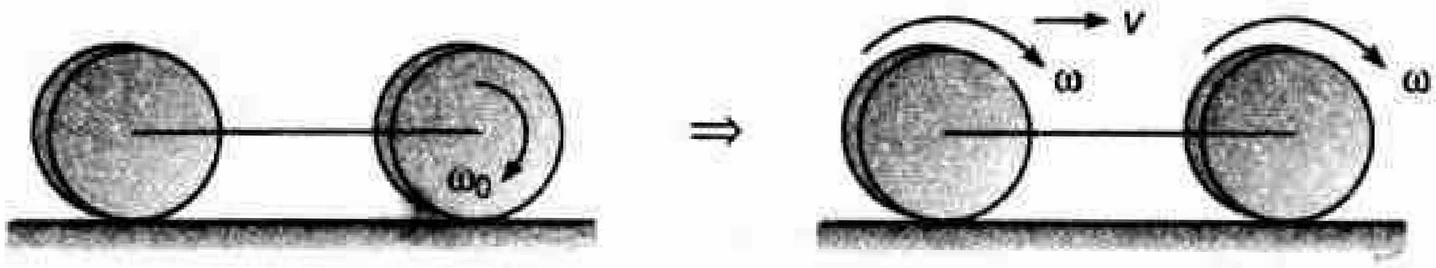
$$5x = F_1(0.2 + x)$$

$\therefore$ $$5x = 3(0.2 + x)$$

or $$x = 0.3\text{ m}$$

$\therefore$ Length of rod = 2 (x + 0.2) = 1.0 m **Ans.**

17. $$L_i = L_f \quad \text{(about bottommost point)}$$



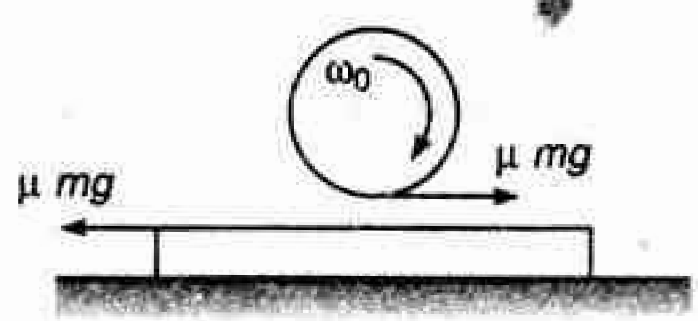
$\therefore$ $$I\omega_0 = 2[I\omega + mRv]$$

or $$\left(\frac{1}{2}mR^2\right)\omega_0 = 2\left[\frac{1}{2}mR^2\omega + mR(\omega R)\right]$$

$\therefore$ $$\omega = \frac{\omega_0}{6} \qquad \textbf{Ans.}$$

and $$v = \omega R = \frac{\omega_0 R}{6} \qquad \textbf{Ans.}$$

18. Let, a_1 = linear acceleration of sphere (towards right), a_2 = linear acceleration of plank (towards left) and α = angular retardation of sphere



$$a_1 = a_2 = \frac{\mu mg}{m} = \mu g$$

$$\alpha = \frac{\mu mgr}{\frac{2}{5}mr^2} = \frac{5}{2}\frac{\mu g}{r}$$

Let pure rolling starts after time 't'. Then $\omega r - v = v$

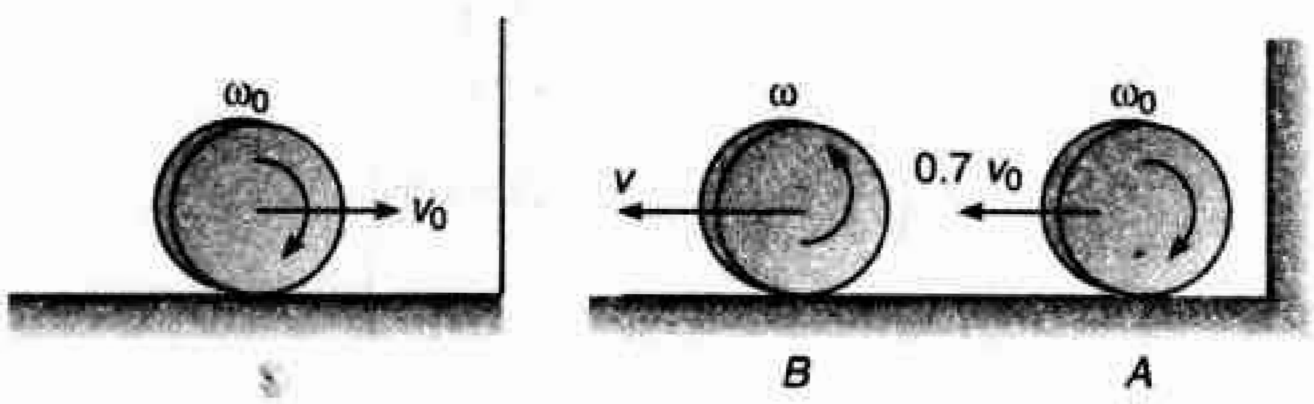
$\therefore$ $$\omega r = 2v$$

$$(\omega_0 - \alpha t)r = 2(a_1 t)$$

Substituting the values, $$t = \frac{2}{9}\frac{\omega_0 r}{\mu g}$$

$\therefore$ $$s = \frac{1}{2}(a_2)t^2 = \frac{2\omega_0^2 r^2}{81\mu g} \qquad \textbf{Ans.}$$

19. Between A and B, there is forward slipping. Therefore, friction will be maximum and backwards (rightwards). At point B where $v = R\omega$, ball starts rolling without slipping and force of friction becomes zero.
From conservation of angular momentum between points A and B about bottommost point (because torque of friction about this point is zero)



$$L_A = L_B$$

$\therefore$ $$m(0.7v_0)R - I\omega_0 = mvR + I\omega$$

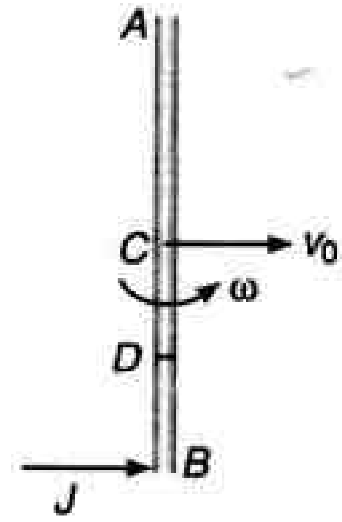
Substituting $\omega_0 = \frac{v_0}{R}$, $\omega = \frac{v}{R}$ and $I = \frac{2}{5}mR^2$, we get

$$v = \frac{3}{14}v_0 = \left(\frac{3}{14}\right)(7)\ \text{m/s} = 1.5\ \text{m/s} \qquad \textbf{Ans.}$$

20. Let J be the linear impulse applied at B and ω the angular speed of rod.

$$J = mv_0 \qquad \text{...(i)}$$

$$J\left(\frac{l}{2}\right) = \frac{ml^2}{12} \cdot \omega \qquad \text{...(ii)}$$



Solving these two equations, $\omega = \dfrac{6v_0}{l}$

Linear speed of D (mid-point of CB) relative to C,

$$v = \omega\left(\frac{l}{4}\right) = \frac{3}{2}v_0$$

$\therefore$ Force exerted by upper half on the lower half,

$$F = \frac{\left(\frac{m}{2}\right)v^2}{\left(\frac{l}{4}\right)}$$

Substituting $v = \dfrac{3}{2}v_0$, we get $F = \dfrac{9}{2}\dfrac{mv_0^2}{l}$ **Ans.**

21.
$$\frac{I}{mR^2} = \frac{2}{5} = 0.4 \text{ for sphere}$$

$$= \frac{1}{2} = 0.5 \text{ for disc and} = 1 \text{ for hoop}$$

$$s = \frac{2}{\sin 30^\circ} = 4 \text{ m}$$

For sphere :

$$a = \frac{g \sin\theta}{1 + \frac{I}{mR^2}} = \frac{9.8 \times \frac{1}{2}}{1 + 0.4} = 3.5 \text{ m/s}$$

$\therefore$
$$v = \sqrt{2as} = \sqrt{2 \times 3.5 \times 4} = 5.29 \text{ m/s}$$

$$f = \frac{mg \sin\theta}{1 + mR^2/I} = \frac{3 \times 9.8 \times \frac{1}{2}}{1 + \left(\frac{1}{0.4}\right)} = 4.2 \text{ N}$$

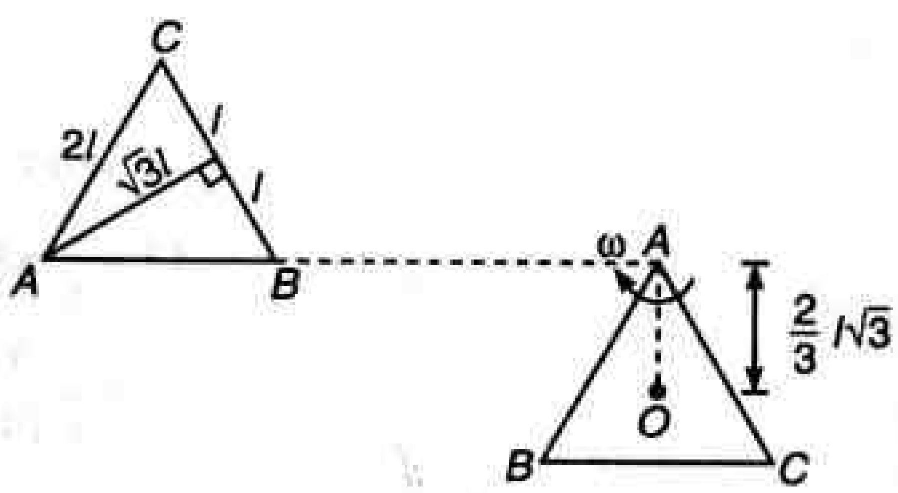
$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 4}{3.5}} = 1.51 \text{ second}$$

Similarly the values for disk and hoop can be obtained.

22.
$$I_A = I_{AB} + I_{AC} + I_{BC}$$

$$= \frac{4}{3}ml^2 + \frac{4}{3}ml^2 + \left\{\frac{1}{3}ml^2 + m(l\sqrt{3})^2\right\}$$

$$= 6\,ml^2$$



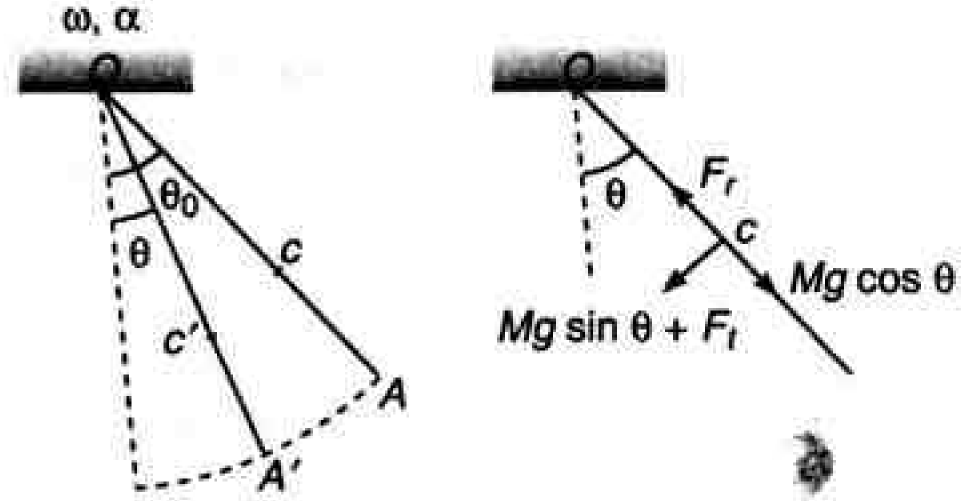
If ω is the angular velocity in the second position, then using conservation of mechanical energy, we have

of COM : $h_i = +\dfrac{\sqrt{3}l}{3}$ and $h_f = -\dfrac{2\sqrt{3}}{3}l$

$$3mg\left(\frac{l\sqrt{3}}{3}\right) = \frac{1}{2}(6ml^2)\omega^2 + 3mg\left(\frac{-2l\sqrt{3}}{3}\right) \quad \text{or} \quad \omega = \sqrt{\frac{g\sqrt{3}}{l}}$$

Now, velocity of C at this instant is $2l\omega$ or $2\sqrt{gl\sqrt{3}}$ and maximum. **Ans.**

23. (i) C is the centre of mass of the rod. Let ω be the angular speed of rod about point O at angle θ. From conservation of mechanical energy,



$$Mg\frac{L}{2}(\cos\theta - \cos\theta_0) = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$$

$\therefore$ $$\omega^2 = \frac{3g}{L}(\cos\theta - \cos\theta_0) \quad \text{...(i)}$$

Now, $$F_r - Mg\cos\theta = M\left(\frac{L}{2}\right)\omega^2 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$F_r = \frac{1}{2}Mg(5\cos\theta - 3\cos\theta_0)$$ **Hence proved.**

(ii) Angular acceleration of rod at this instant,

$$\alpha = \frac{\tau}{I} = \frac{Mg\dfrac{L}{2}\sin\theta}{\dfrac{ML^2}{3}} = \frac{3}{2}\frac{g\sin\theta}{L}$$

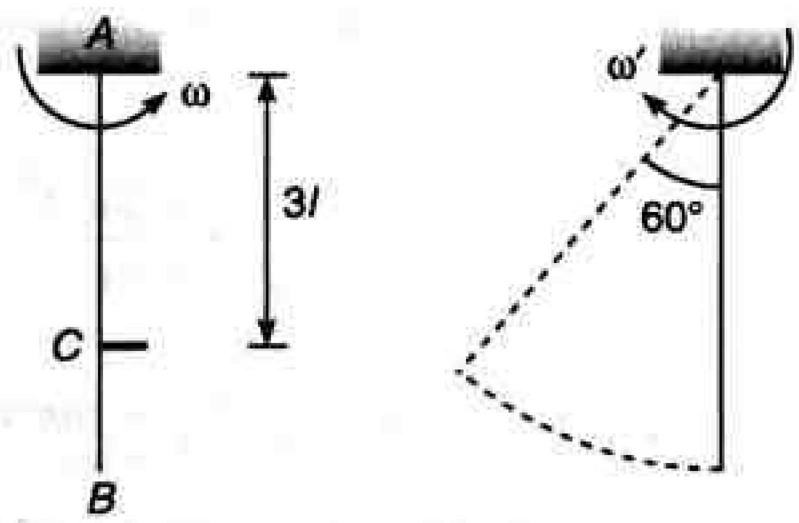
Tangential acceleration of COM, $$a_t = (\alpha)\left(\frac{L}{2}\right) = \frac{3}{4}g\sin\theta \quad \text{...(iii)}$$

Now, $$F_t + Mg\sin\theta = Ma_t \quad \text{...(iv)}$$

From Eqs. (iii) and (iv), we get $$F_t = -\frac{1}{4}Mg\sin\theta$$ **Hence proved.**

Here negative sign implies that direction of F_t is opposite to the component $Mg\sin\theta$.

24. (a) From conservation of mechanical energy.



$$(3m)(g)(2l) = \frac{1}{2}I\omega^2 = \frac{1}{2}\left[\frac{(3m)(4l)^2}{3}\right]\omega^2 = 8ml^2\omega^2$$

$\therefore$ $$\omega = \frac{1}{2}\sqrt{\frac{3g}{l}}$$

Applying, angular impulse = change in angular momentum

$$J(3l) = I\omega$$

or $$3Jl = (16ml^2)\left(\frac{1}{2}\sqrt{\frac{3g}{l}}\right)$$

$\therefore$ $$J = \frac{8}{3}ml\sqrt{\frac{3g}{l}}$$

or $$J = \frac{8}{3}m\sqrt{3gl}$$ **Ans.**

(b) Let ω' be the angular speed in opposite direction. Again applying conservation of mechanical energy,

$$(3m)(g)(l) = \frac{1}{2}I(\omega')^2 = 8ml^2(\omega')^2$$

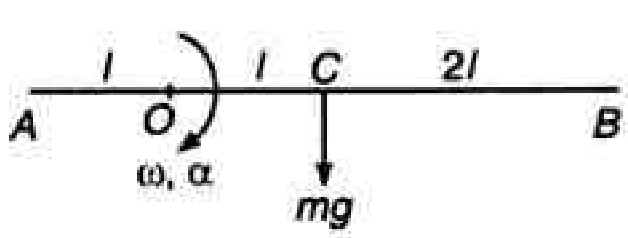
$\therefore$ $$\omega' = \frac{1}{2\sqrt{2}}\sqrt{\frac{3g}{l}}$$

Now, applying, angular impulse = change in angular momentum

$\therefore$ $$J(3l) = I(\omega + \omega') = (16ml^2)\frac{1}{2}\sqrt{\frac{3g}{l}}\left(1 + \frac{1}{\sqrt{2}}\right)$$

$\therefore$ $$J = \frac{4}{3}m\sqrt{6gl}\,(\sqrt{2} + 1)$$ **Ans.**

25. $$\alpha = \frac{mgl}{\frac{m(4l)^2}{12} + ml^2} = \frac{3}{7}\cdot\frac{g}{l}$$



$\therefore$ $$(a_C)_V = l\alpha = \frac{3}{7}g$$ (downwards)

Let V be the vertical reaction (upwards) at axis, then

$$mg - V = ma_C = \frac{3mg}{7}$$

$\therefore$ $$V = \frac{4}{7}mg$$ ...(i)

If H be the horizontal reaction (towards CO) at axis, then

$$H = ml\omega^2$$ ...(ii)

$\therefore$ Total reaction at axis,

$$N = \sqrt{H^2 + V^2} = \frac{4}{7}mg\sqrt{1 + \left(\frac{7l\omega^2}{4g}\right)^2}$$ **Ans.**

(b) $$a_C = \sqrt{(a_C)_V^2 + (l\omega^2)^2} = \sqrt{\left(\frac{3g}{7}\right)^2 + (l\omega^2)^2}$$ **Ans.**

(c) Let ω' be the angular speed of the rod when it becomes vertical for the first time. Then from conservation of mechanical energy,

$$\frac{1}{2}I(\omega'^2 - \omega^2) = mgl$$

$\therefore$ $$\omega'^2 = \omega^2 + \frac{2mgl}{I}$$

$$= \omega^2 + \frac{2mgl}{\frac{7}{3}ml^2} = \omega^2 + \frac{6}{7}\frac{g}{l}$$

Acceleration of centre of mass at this instance will be,

$$a_C = l\omega'^2 = l\omega^2 + \frac{6g}{7}$$ **Ans.**

Let V be the reaction (upwards) at axis at this instant, then,

$$V - mg = ma_C = ml\omega^2 + \frac{6mg}{7}$$

$\therefore$ $$V = \frac{13}{7}mg + ml\omega^2$$ **Ans.**

(d) From conservation of mechanical energy,

$$mgl = \frac{1}{2}I\omega_{\min}^2$$

$\therefore$ $$\omega_{\min} = \sqrt{\frac{2mgl}{I}} = \sqrt{\frac{2mgl}{\frac{7}{3}ml^2}} = \sqrt{\frac{6}{7}\frac{g}{l}}$$ **Ans.**

26. Linear momentum, angular momentum and kinetic energy are conserved in the process.
From conservation of linear momentum,

$$Mv' = mv$$

or $$v' = \frac{m}{M}v \quad \text{...(i)}$$

Conservation of angular momentum gives,

$$mvd = \left(\frac{Ml^2}{12}\right)\omega$$

or $$\omega = \left(\frac{12mvd}{Ml^2}\right) \quad \text{...(ii)}$$

Collision is elastic. Hence,

$$e = 1$$

or relative speed of approach = relative speed of separation

$\therefore$ $$v = v' + d\omega$$

Substituting the values, we have $$v = \frac{m}{M}v + \frac{12mvd^2}{Ml^2}$$

Solving it, we get $$m = \frac{Ml^2}{12d^2 + l^2}$$ **Ans.**

27. Let v = linear velocity of rod after impact (upwards),

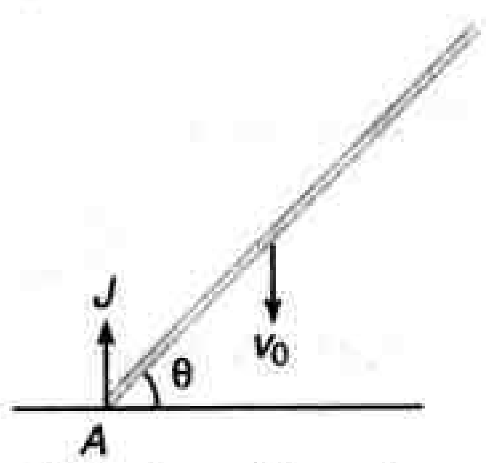
ω = angular velocity of rod

and J = linear impulse at A during impact

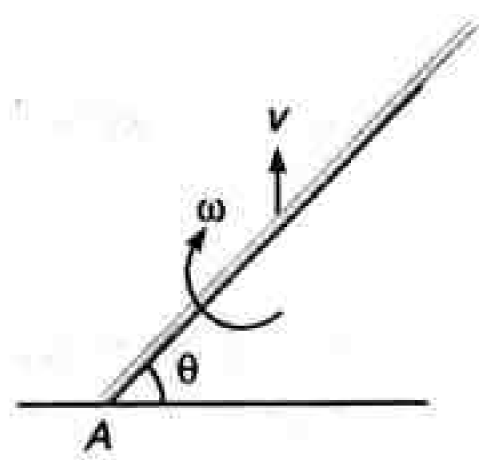
Then, $J = \Delta P = P_f - P_i$

$J = mv - (-mv_0)$

$\therefore$ $J = m(v + v_0)$ ...(i)



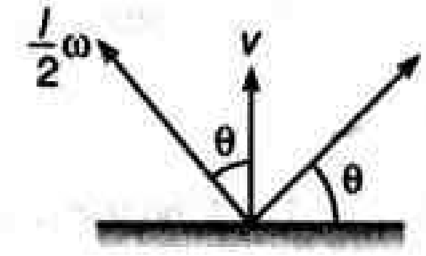
At the time of impact



After impact

Angular impulse = ΔL

$\therefore$ $$J\left(\frac{l}{2}\cos\theta\right) = I\omega = \frac{ml^2}{12}\omega \qquad \text{...(ii)}$$



Collision is elastic ($e = 1$)

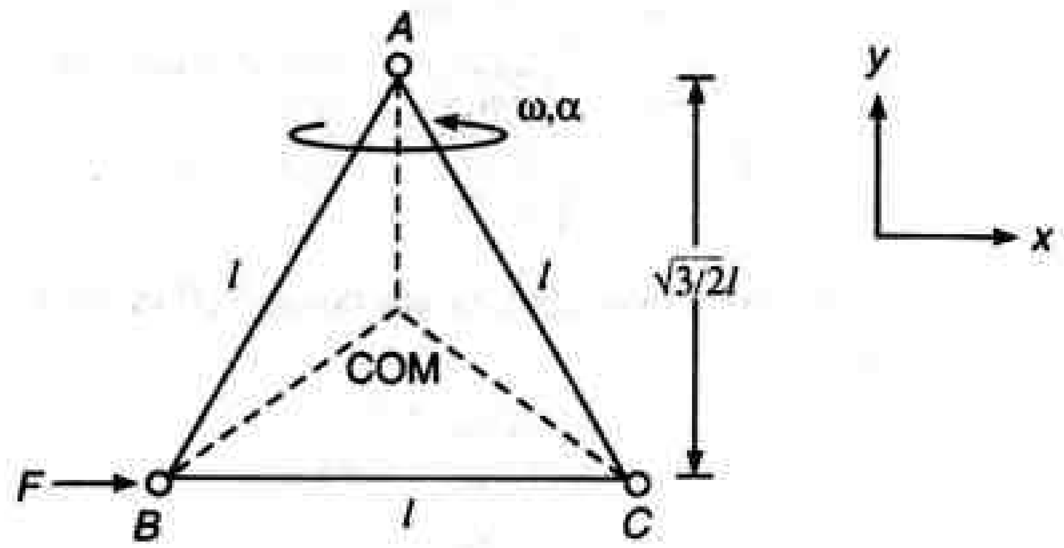
$\therefore$ Relative speed of approach = Relative speed of separation at point of impact

$$v_0 = v + \frac{l}{2}\omega\cos\theta \qquad \text{...(iii)}$$

Solving above equations, we get $\omega = \dfrac{6v_0\cos\theta}{l(1+3\cos^2\theta)}$ **Ans.**

28. (a) The distance of centre of mass (COM) of the system about point A will be :

$$r = \frac{l}{\sqrt{3}}$$



Therefore, the magnitude of horizontal force exerted by the hinge on the body is

F = centripetal force

or $$F = (3m)r\omega^2$$

or $$F = (3m)\left(\frac{l}{\sqrt{3}}\right)\omega^2$$

or $$F = \sqrt{3}\,ml\omega^2$$

(b) Angular acceleration of system about point A is

$$\alpha = \frac{\tau_A}{I_A}$$

$$= \frac{(F)\left(\frac{\sqrt{3}}{2}l\right)}{2ml^2}$$

$$= \frac{\sqrt{3}F}{4ml}$$

Now, acceleration of COM along x-axis is

$$a_x = r\alpha = \left(\frac{l}{\sqrt{3}}\right)\left(\frac{\sqrt{3}F}{4ml}\right) \text{ or } a_x = \frac{F}{4m}$$

Now, let F_x be the force applied by the hinge along x-axis.

Then,

$$F_x + F = (3m)a_x$$

or $$F_x + F = (3m)\left(\frac{F}{4m}\right)$$

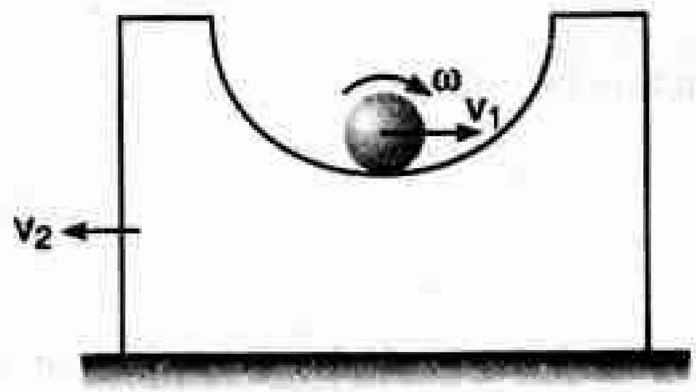
or $$F_x + F = \frac{3}{4}F \text{ or } F_x = -\frac{F}{4}$$

Further if F_y be the force applied by the hinge along y-axis. Then,

$$F_y = \text{centripetal force}$$

or $$F_y = \sqrt{3}\, ml\omega^2$$

29. From conservation of linear momentum



$$mv_1 = Mv_2 \quad \text{...(i)}$$

$\therefore$ Velocity of cylinder axis relative to block $v_r = v_1 + v_2$ ...(ii)

Applying conservation of mechanical energy,

$$mg(R-r) = \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_2^2 \quad \text{...(iii)}$$

Here, $$I = \frac{1}{2}mr^2 \quad \text{and} \quad \omega = \frac{v_r}{r}$$

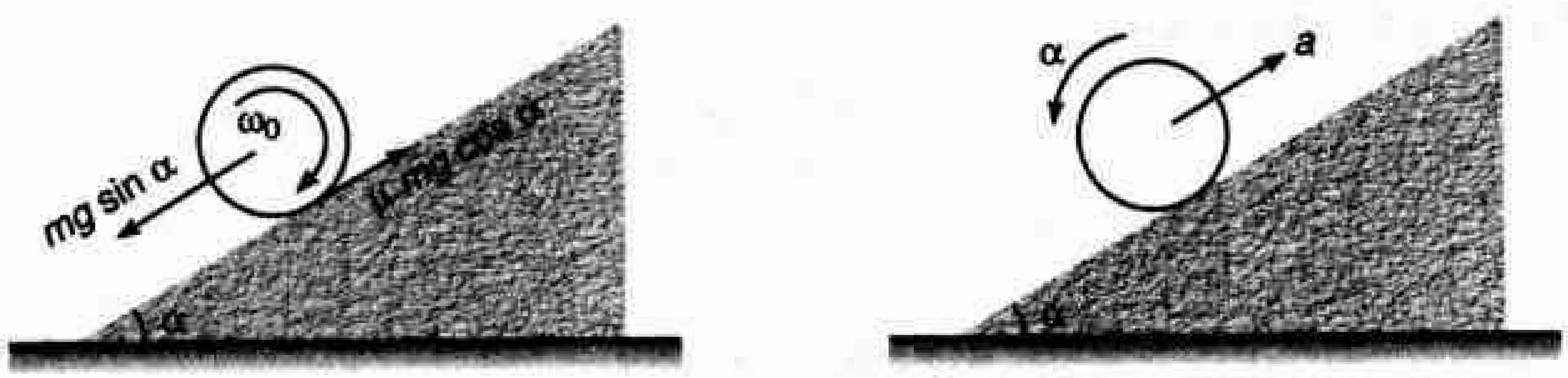
Solving the above equations with given data, we get

$$v_1 = 2.0 \text{ m/s} \quad \text{and} \quad v_2 = 1.5 \text{ m/s}$$ **Ans.**

Further, $$N - mg = \frac{mv_r^2}{R-r}$$

$\therefore$ $$N = mg + \frac{mv_r^2}{R-r} = (0.5)(10) + \frac{(0.5)(3.5)^2}{0.525} = 16.67 \text{ N}$$ **Ans.**

30. Given $\mu > \tan\alpha \Rightarrow \mu mg\cos\alpha > mg\sin\alpha$



$$a = (\mu g\cos\alpha - g\sin\alpha)$$

$$\alpha = \frac{(\mu mg\cos\alpha)\,r}{\frac{1}{2}mr^2} = \frac{2\mu g\cos\alpha}{r}$$

Slipping will stop when,

$$v = r\omega$$

or $$at = r(\omega_0 - \alpha t)$$

$$\therefore \qquad t = \frac{r\omega_0}{a + r\alpha} = \left(\frac{r\omega_0}{3\mu g\cos\alpha - g\sin\alpha}\right)$$

$$d_1 = \frac{1}{2}at^2 = \frac{1}{2}(\mu g\cos\alpha - g\sin\alpha)\left(\frac{r\omega_0}{3\mu g\cos\alpha - g\sin\alpha}\right)^2$$

$$= \frac{r^2\omega_0^2(\mu\cos\alpha - \sin\alpha)}{2g(3\mu\cos\alpha - \sin\alpha)^2} \qquad \textbf{Ans.}$$

$$v = at = (\mu g\cos\alpha - g\sin\alpha)\left(\frac{r\omega_0}{3\mu g\cos\alpha - g\sin\alpha}\right) = \frac{r\omega_0(\mu\cos\alpha - \sin\alpha)}{(3\mu\cos\alpha - \sin\alpha)}$$

Once slipping is stopped, retardation in cylinder,

$$a' = \frac{g\sin\alpha}{1 + \dfrac{I}{mr^2}} = \frac{g\sin\alpha}{1 + \dfrac{1}{2}} = \frac{2}{3}g\sin\alpha$$

$$d_2 = \frac{v^2}{2a'} = \frac{3r^2\omega_0^2(\mu\cos\alpha - \sin\alpha)^2}{(3\mu\cos\alpha - \sin\alpha)^2(4g\sin\alpha)}$$

$$\therefore \qquad d_{\max} = d_1 + d_2$$

$$= \frac{r^2\omega_0^2(\mu\cos\alpha - \sin\alpha)}{2g(3\mu\cos\alpha - \sin\alpha)^2}\left[1 + \frac{3(\mu\cos\alpha - \sin\alpha)}{2\sin\alpha}\right]$$

$$= \frac{r^2\omega_0^2(\mu\cos\alpha - \sin\alpha)}{4g\sin\alpha(3\mu\cos\alpha - \sin\alpha)} \qquad \textbf{Ans.}$$

Note *Once slipping was stopped, pure rolling continues if*

$$\mu > \frac{\tan\alpha}{1 + \dfrac{mr^2}{I}}$$

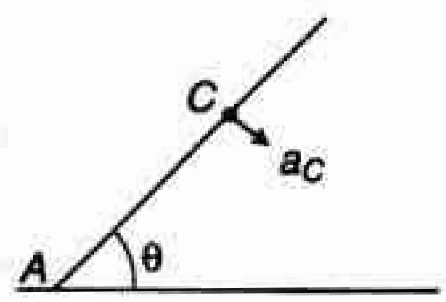
or

$$\mu > \frac{\tan\alpha}{1 + 2} \quad \text{or} \quad \mu > \frac{\tan\alpha}{3}$$

and already in the question it is given that $\mu > \tan\alpha$. That's why we have taken $a' = \frac{2}{3}g\sin\alpha$.

31. Point A is momentarily at rest.

$$\alpha = \frac{mg\dfrac{l}{2}\cos\theta}{\dfrac{ml^2}{3}} = \frac{3}{2}\frac{g\cos\theta}{l}$$



$$\therefore \qquad a_C = \frac{l}{2}\alpha = \frac{3}{4}g\cos\theta$$

Now $\qquad \mu N = ma_x \quad \text{or} \quad \mu N = ma_C\sin\theta$

or $\qquad \mu N = \frac{3}{4}mg\sin\theta\cos\theta \qquad \ldots\text{(i)}$

Further, $\qquad mg - N = ma_y$

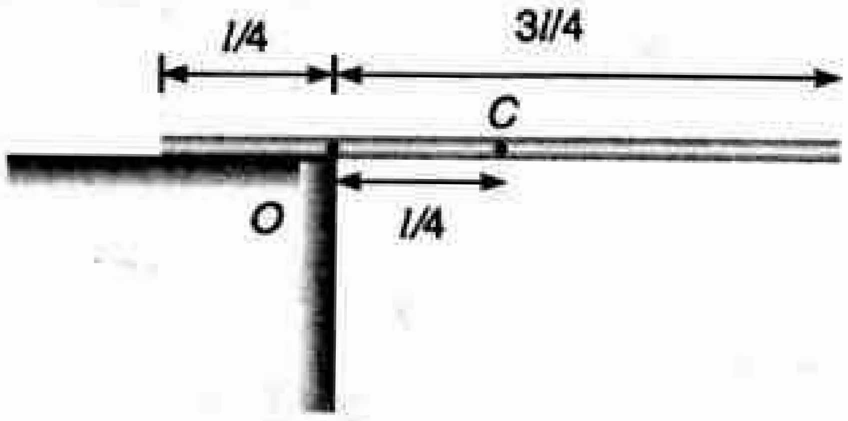
or $\qquad N = mg - ma_C\cos\theta$

or $\qquad N = mg - \frac{3}{4}mg\cos^2\theta \qquad \ldots\text{(ii)}$

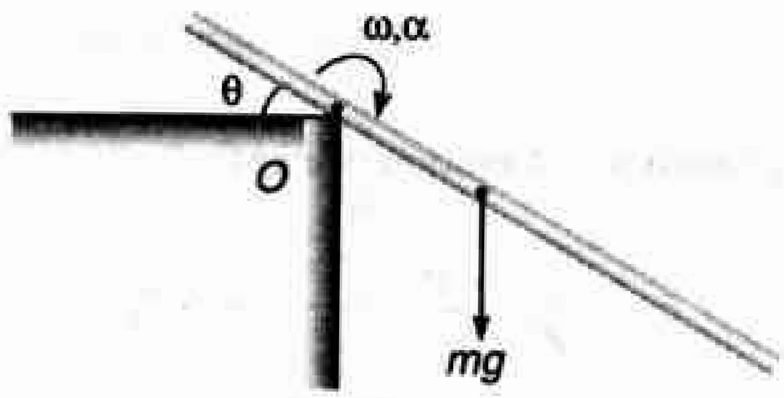
Dividing Eq. (i) by (ii), we have $\mu = \dfrac{\dfrac{3}{4}\sin\theta\cos\theta}{1-\dfrac{3}{4}\cos^2\theta} = \dfrac{3\sin\theta\cos\theta}{4-3\cos^2\theta}$

$$= \frac{3\sin\theta\cos\theta}{1+3\sin^2\theta}$$ **Proved.**

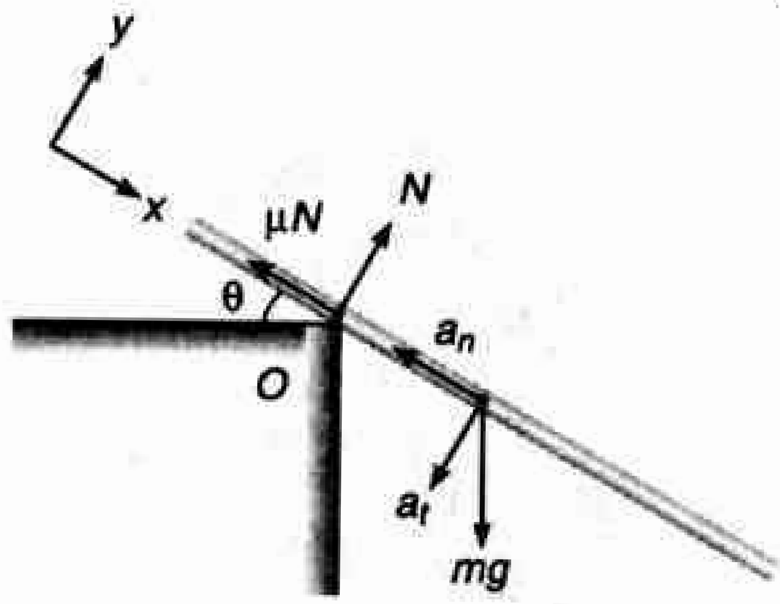
32. Figure (a) and (b) :



(a)



(b)



(c)

ω : Decrease in gravitational potential energy = increase in rotational kinetic energy

$\therefore$ $$mg\frac{l}{4}\sin\theta = \frac{1}{2}I_0\omega^2 = \frac{1}{2}\left[\frac{ml^2}{12}+m\left(\frac{l}{4}\right)^2\right]\omega^2$$

$\therefore$ $$\omega = \sqrt{\left[\frac{24g\sin\theta}{7l}\right]} \qquad \text{...(i)}$$

α : $$\alpha = \frac{\tau}{I} = \frac{mg\dfrac{l}{4}\cos\theta}{\left[\dfrac{ml^2}{12}+m\left(\dfrac{l}{4}\right)^2\right]} = \frac{12\,g\cos\theta}{7l} \qquad \text{...(ii)}$$

$$\Sigma F_y = ma_y \text{ or } mg\cos\theta - N = ma_t$$

or $$N = mg\cos\theta - ma_t = mg\cos\theta - m\frac{l}{4}\alpha$$

Substituting value of α from Eq. (ii), we get

$$N = \frac{4}{7}mg\cos\theta \qquad \text{...(iii)}$$

Rod begins to slip when :

$$\mu N - mg \sin\theta = ma_n$$

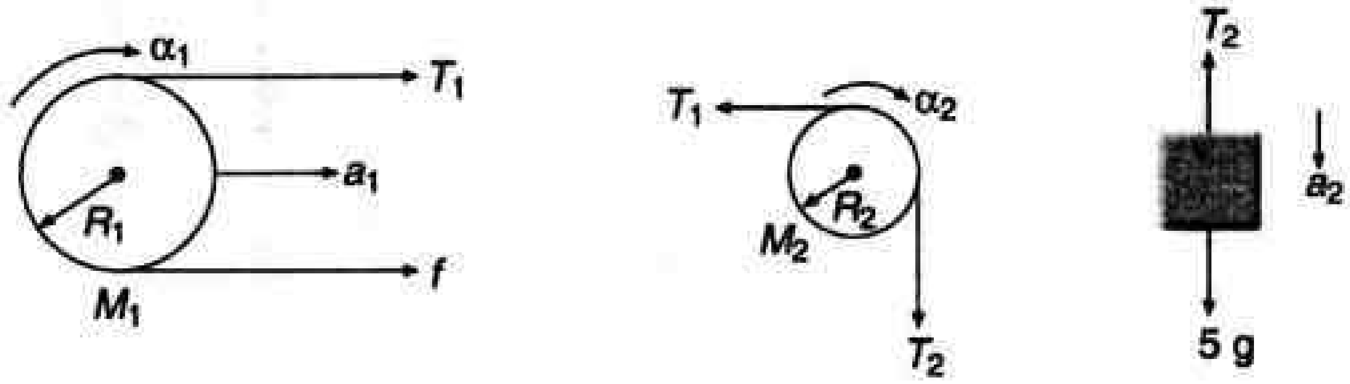
or
$$\frac{4}{7}\mu mg \cos\theta - mg\sin\theta = m\frac{l}{4}\omega^2$$

Substitution value of ω from Eq. (i), we get

$$\tan\theta = \frac{4\mu}{13}$$

$\therefore$
$$\theta = \tan^{-1}\left(\frac{4\mu}{13}\right)$$
Ans.

33. Writing equations of motion, we get



$$5g - T_2 = 5a_2 \quad \text{...(i)}$$

$$\frac{(T_2 - T_1)R_2}{\frac{1}{2}M_2R_2^2} = \alpha_2 \quad \text{...(ii)}$$

$$T_1 + f = M_1a_1 \quad \text{...(iii)}$$

$$\frac{(T_1 - f)R_1}{\frac{1}{2}M_1R_1^2} = \alpha_1 \quad \text{...(iv)}$$

$$a_1 = R_1\alpha_1 \quad \text{...(v)}$$

$$a_1 + R_1\alpha_1 = R_2\alpha_2 \quad \text{...(vi)}$$

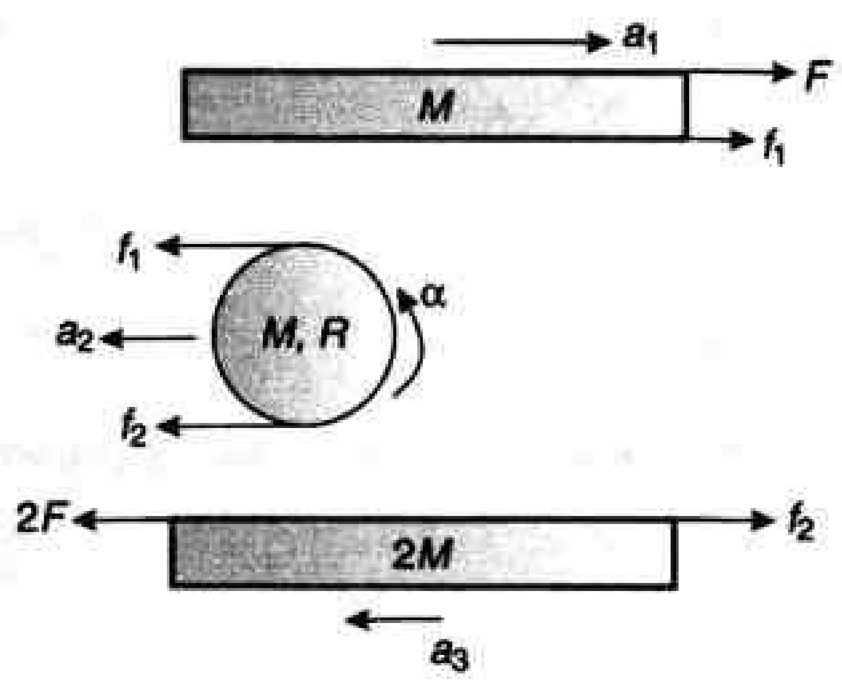
$$R_2\alpha_2 = a_2 \quad \text{...(vii)}$$

We have seven unknowns, $T_1, T_2, a_1, a_2, \alpha_1, \alpha_2$ and f solving above equations, we get

$$a_2 = \frac{4}{11}g = 3.6 \text{ m/s}^2$$
Ans.

$$v = a_2t = \frac{4gt}{11}$$
Ans.

34. Equations of motion are,



$$F + f_1 = Ma_1 \quad \text{...(i)}$$

$$f_1 + f_2 = Ma_2 \quad \text{...(ii)}$$

$$2F - f_2 = 2Ma_3 \quad \text{...(iii)}$$

$$\alpha = \frac{(f_1 - f_2)R}{\frac{1}{2}MR^2}$$

or
$$\alpha = \frac{2(f_1 - f_2)}{MR} \quad \text{...(iv)}$$

For no slipping condition,

$$a_2 + R\alpha = -a_1 \quad \text{...(v)}$$

and
$$a_2 - R\alpha = a_3 \quad \text{...(vi)}$$

We have six unknowns, f_1, f_2, a_1, a_2, a_3 and α. Solving the above six equations, we get

$$a_1 = \frac{21F}{26M} \quad \text{and} \quad a_2 = \frac{F}{26M} \qquad \textbf{Ans.}$$

35. Angular velocity : From conservation of mechanical energy,

decrease in gravitational PE = increase in rotational KE

or
$$mgr \sin 60° + mg\,(2r \sin 60°) = \frac{1}{2}\left[\frac{3mr^2}{2} + m\,(2r)^2\right]\omega^2$$

$\therefore$
$$\frac{3\sqrt{3}mgr}{2} = \frac{11}{4}\,mr^2\omega^2$$

$\therefore$
$$\omega = \sqrt{\frac{6g\sqrt{3}}{11r}} \qquad \textbf{Ans.}$$

Angular acceleration : $\alpha = \dfrac{\tau}{I} = \dfrac{mgr\cos 60° + mg\,(2r\cos 60°)}{\left[\dfrac{3mr^2}{2} + m\,(2r)^2\right]} = \dfrac{\dfrac{3mgr}{2}}{\dfrac{11}{2}mr^2} = \dfrac{3g}{11r}$ **Ans.**

36.
$$\vec{\mathbf{a}}_B = \vec{\mathbf{a}}_0 + \vec{\mathbf{a}}_{B/0}$$

Here, $\vec{\mathbf{a}}_{B/0}$ has two components a_t (tangential acceleration) and a_n (normal acceleration)

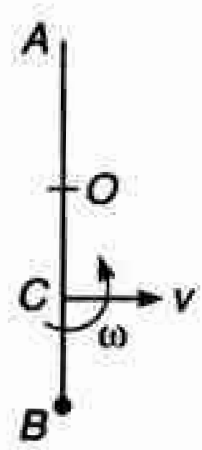
$$a_t = r\alpha = (0.3)(5) = 1.5 \text{ m/s}^2$$
$$a_n = r\omega^2 = (0.3)(4)^2 = 4.8 \text{ m/s}^2$$

and
$$a_0 = 2 \text{ m/s}^2$$

$\therefore$
$$a_B = \sqrt{(\Sigma a_x)^2 + (\Sigma a_y)^2}$$
$$= \sqrt{(2 + 4.8\cos 45° - 1.5\cos 45°)^2 + (4.8\sin 45° + 1.5\sin 45°)^2}$$
$$= 6.21 \text{ m/s}^2$$

37. C is the COM of $(M + m)$

$$BC = \left(\frac{M}{M+m}\right)\left(\frac{l}{2}\right) \quad \text{and} \quad OC = \left(\frac{m}{M+m}\right)\left(\frac{l}{2}\right)$$



From conservation of linear momentum,

$$(M + m)\,v = mv_0$$

or
$$v = \left(\frac{m}{M+m}\right)v_0 \quad \text{...(i)}$$

From conservation of angular momentum about point C we have,

$$mv_0(BC) = I\omega$$

or
$$\frac{mMv_0 l}{2\,(M+m)} = \left[m\left(\frac{M}{M+m}\right)^2\left(\frac{l}{4}\right)^2 + \frac{Ml^2}{12} + M\left(\frac{m}{M+m}\right)^2\left(\frac{l^2}{4}\right)\right]\omega$$

Putting $\dfrac{mv_0}{M+m} = v$ from Eq. (i), we have

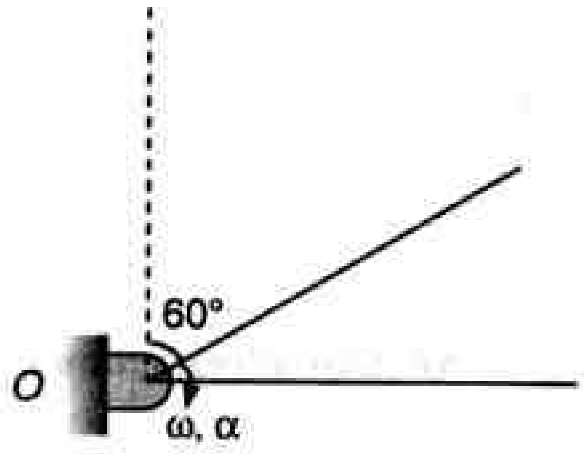
$$\frac{v}{\omega} = \frac{l}{6}\left[\frac{4m+M}{M+m}\right]$$

Now a point (say P) at a distance $x = \dfrac{v}{\omega}$, from C (towards O) will be at rest. Hence, distance of point P from boy at B will be

$$BP = BC + x$$

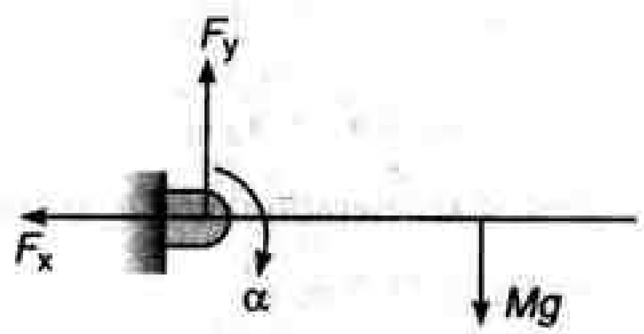
$$= \left(\frac{M}{M+m}\right)\left(\frac{l}{2}\right) + \frac{l}{6}\left[\frac{4m+M}{M+m}\right] = \frac{2l}{3}$$ **Ans.**

38. Let ω be the angular velocity and α the angular acceleration of rod in horizontal position. Then

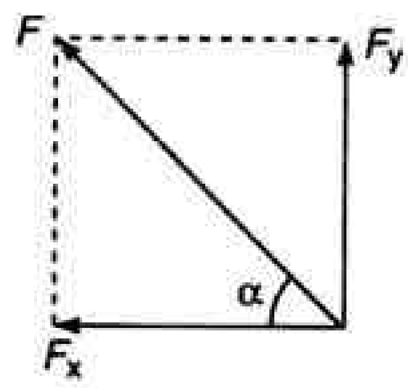


$$\alpha = \frac{(Mg)\dfrac{l}{2}}{\dfrac{Ml^2}{3}} = \frac{3}{2}\frac{g}{l} \qquad \text{...(i)}$$

$$\frac{1}{2}\left(\frac{Ml^2}{3}\right)\omega^2 = Mg\,\frac{l}{4}$$



$\therefore$ $$\omega = \frac{3}{2}\cdot\frac{g}{l} \qquad \text{...(ii)}$$



$$F_x = M\left(\frac{l}{2}\right)\omega^2 = M\left(\frac{l}{2}\right)\left(\frac{3g}{2l}\right) = \frac{3}{4}Mg$$

$$Mg - F_y = M(\alpha)\left(\frac{l}{2}\right)$$

$$F_y = Mg - \frac{3}{4}Mg = \frac{Mg}{4}$$

$\therefore$ $$F = \sqrt{F_x^2 + F_y^2} = \frac{\sqrt{10}}{4}Mg$$ **Ans.**

$$\tan\alpha = \frac{F_y}{F_x} = \frac{(Mg/4)}{(3Mg/4)} = \frac{1}{3}$$

$\therefore$ $$\alpha = \tan^{-1}\left(\frac{1}{3}\right)$$ **Ans.**

Chapter 10 Gravitation

JEE Advanced (Subjective Questions)

1. $W = \Delta U = U_f - U_i$

$$= 3\left[\frac{-Gmm}{2a}\right] - 3\left[-\frac{Gmm}{a}\right] = \frac{3\,Gmm}{2a} = \frac{3Gm^2}{2a}$$ **Ans.**

2. $h = \dfrac{u^2}{2g_e}$

$\therefore$ $$u = \sqrt{2g_e h} \quad \text{...(i)}$$

For the asked planet this u should be equal to the escape velocity from its surface.

$\therefore$ $$\sqrt{2g_e h} = \sqrt{2g_p R_p}$$

or $$g_e h = g_p R_p$$

$$\frac{GM_e}{R_e^2}.h = \frac{GM_p}{R_p^2}.R_p$$

or $$\frac{\left(\frac{4}{3}\pi R_e^3\right)\rho h}{R_e^2} = \frac{\left(\frac{4}{3}\pi R_p^3\right)\rho R_p}{R_p^2}$$

or $$R_p = \sqrt{R_e h} = \sqrt{(6.41\times 10^6)(1.5)}$$

$$= 3.1\times 10^3 \text{ m}$$ **Ans.**

3. (a) $v_o = \dfrac{v_e}{2}$

$\therefore$ $$\sqrt{\frac{GM}{r}} = \frac{\sqrt{\frac{2GM}{R}}}{2}$$

$\therefore$ $$r = 2R$$

or $h = r - R = R$ or height = radius of earth.

(b) Increase in kinetic energy = decrease in potential energy

$\therefore$ $$\frac{1}{2}mv^2 = \frac{mgh}{1+\frac{h}{R}}$$

$\therefore$ $$v = \sqrt{\frac{2hg}{1+\frac{h}{R}}}$$

Substituting the values we have,

$$v = \sqrt{\frac{2\times 9.81\times 6400\times 10^3}{1+\frac{R}{R}}} = 7924 \text{ m/s}$$

$$\approx 7.92 \text{ km/s}$$ **Ans.**

4. (i) At point A, field strength due to shell will be zero.

Net field is only due to metal sphere. Distance between centre of metal sphere and point A is $4R$.

$$\therefore \qquad E_A = \frac{G(m)}{(4R)^2} = \frac{Gm}{16R^2}$$

(ii) At point B, net field is due to both, due to shell and due to metal sphere.

$$\therefore \qquad E_B = \frac{Gm}{(5R)^2} + \frac{Gm}{(6R)^2} = \frac{61\,Gm}{900\,R^2}$$ **Ans.**

5. Radius of hollow spehre is $\frac{R}{2}$, so mass in this hollow portion would had been, $\frac{M}{8}$.

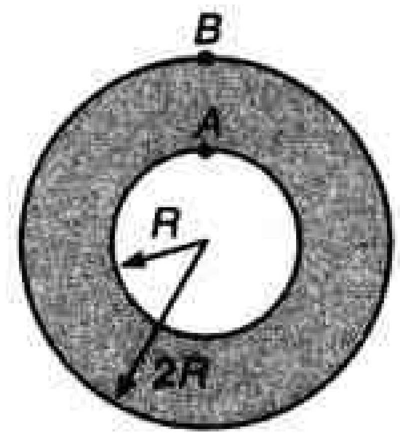
Now net force on m due to whole sphere = force due to remaining mass + force due to cavity mass.

$\therefore$ Force due to remaining mass = force due to whole sphere − force due to cavity mass

$$= \frac{G\,Mm}{d^2} - \frac{G\,Mm}{8(d - R/2)^2}$$

$$= \frac{G\,Mm}{d^2}\left[1 - \frac{1}{8\left(1 - \frac{R}{2d}\right)^2}\right]$$

6. Let m_1 be the mass of the core and m_2 the mass of outer shell.



$$g_A = g_B \quad \text{(given)}$$

Then $$\frac{Gm_1}{R^2} = \frac{G(m_1 + m_2)}{(2R)^2}$$

$$\therefore \qquad 4m_1 = (m_1 + m_2)$$

or $$4\left\{\frac{4}{3}\pi R^3 \rho_1\right\} = \frac{4}{3}\pi R^3 \cdot \rho_1 + \left\{\frac{4}{3}\pi (2R)^3 - \frac{4}{3}\pi R^3\right\}\rho_2$$

$$\therefore \qquad 4\rho_1 = \rho_1 + 7\rho_2$$

$$\therefore \qquad \frac{\rho_1}{\rho_2} = \frac{7}{3}$$ **Ans.**

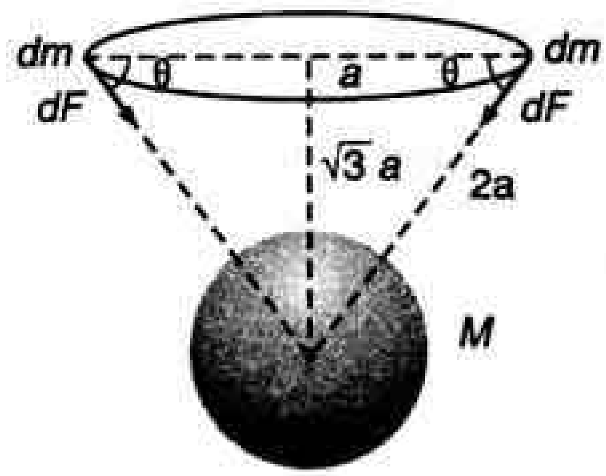
7. Total mechanical energy of a satellite in an elliptical orbit of semi major axis 'a' is $-\frac{G\,Mm}{2a}$

$$E = K + U$$

$$\therefore \qquad -\frac{G\,Mm}{2a} = \frac{1}{2}mv^2 - \frac{G\,Mm}{r}$$

or $$v^2 = GM\left[\frac{2}{r} - \frac{1}{a}\right]$$ **Hence Proved**

8. dF = force on a small mass 'dm' of the ring by the sphere.



$$\text{Net force on ring} = \Sigma(dF \sin\theta) \text{ or } \int dF \sin\theta$$

$$= \Sigma \frac{GM(dm)}{(2a)^2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}GM}{8a^2}\,\Sigma(dm)$$

But $\Sigma(dm) = m$, the mass of whole ring.

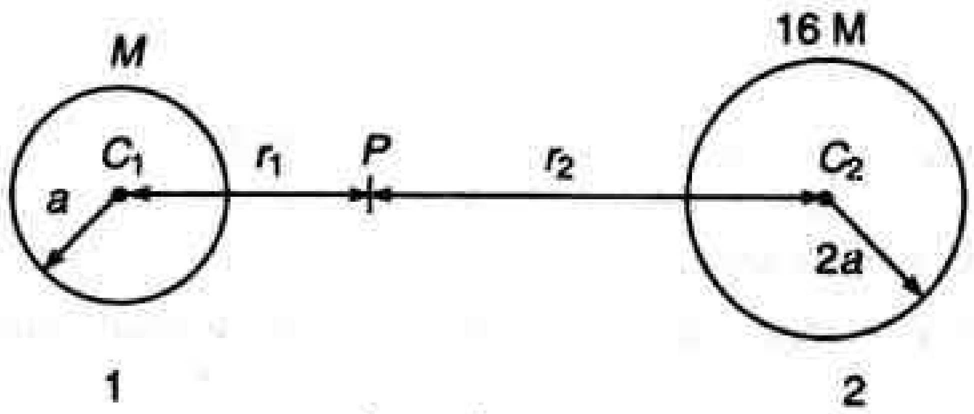
$\therefore$ $$\text{Net force} = \frac{\sqrt{3}GMm}{8a^2}$$ **Ans.**

9. Let there are two stars 1 and 2 as shown below :

Let P is a point between C_1 and C_2, where gravitational field strength is zero. Or at P field strength due to star 1 is equal and opposite to the field strength due to star 2. Hence,

$$\frac{GM}{r_1^2} = \frac{G(16M)}{r_2^2}$$

or $$\frac{r_2}{r_1} = 4 \quad \text{also} \quad r_1 + r_2 = 10a$$



$\therefore$ $$r_2 = \left(\frac{4}{4+1}\right)(10a) = 8a$$

and $$r_1 = 2a$$

Now, the body of mass m is projected from the surface of larger star towards the smaller one. Between C_2 and P it is attracted towards 2 and between C_1 and P it will be attracted towards 1. Therefore, the body should be projected to just cross point P because beyond that the particle is attracted towards the smaller star itself.

From conservation of mechanical energy $\frac{1}{2}mv_{\min}^2$ = Potential energy of the body at P

– Potential energy at the surface of the larger star.

$\therefore$ $$\frac{1}{2}mv_{\min}^2 = \left[-\frac{GMm}{r_1} - \frac{16GMm}{r_2}\right] - \left[-\frac{GMm}{10a-2a} - \frac{16GMm}{2a}\right]$$

$$= \left[-\frac{GMm}{2a} - \frac{16GMm}{8a}\right] - \left[-\frac{GMm}{8a} - \frac{8GMm}{a}\right]$$

or $$\frac{1}{2}mv_{\min}^2 = \left(\frac{45}{8}\right)\frac{GMm}{a}$$

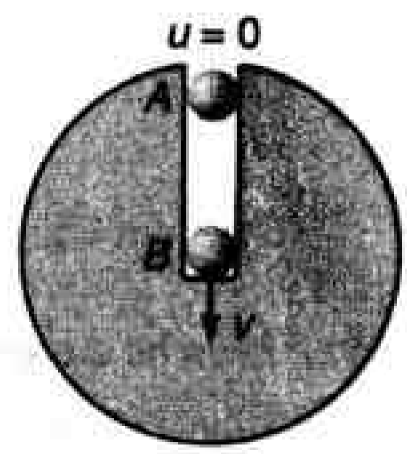
$\therefore$ $$v_{\min} = \frac{3\sqrt{5}}{2}\left(\sqrt{\frac{GM}{a}}\right)$$ **Ans.**

10. Let mass of the ball be m.

$$\frac{1}{2}mv^2 = m(V_A - V_B) = m\left[-\frac{GM}{R} - \left(-1.5\frac{GM}{R}\right)\right]$$

$$= \frac{GMm}{2R}$$

$\therefore$ $$v = \sqrt{\frac{GM}{R}}$$



Velocity of ball just after collision, $$v' = ev = \frac{1}{2}\sqrt{\frac{GM}{R}}$$

Let r be the distance from the centre upto where the ball reaches after collision. Then,

$$\frac{1}{2}mv'^2 = m\,[V(r) - V(\text{centre})]$$

or $$\frac{1}{8}\frac{GMm}{R} = m\left[\frac{3GM}{2R} - \frac{GM}{R^3}\left(\frac{3R^2}{2} - \frac{r^2}{2}\right)\right] \quad \text{or} \quad \frac{1}{8} = \frac{3}{2} - \frac{3}{2} + \frac{r^2}{2R^2}$$

$\therefore$ $$\frac{r^2}{R^2} = \frac{1}{4} \quad \text{or} \quad r = \frac{R}{2}$$

$\therefore$ The desired distance,

$$s = R + \frac{R}{2} + \frac{R}{2} = 2R$$ **Ans.**

11. Applying conservation of mechanical energy,

Increase in kinetic energy = decrease in gravitational potential energy

or $$\frac{1}{2}m_0v^2 = U_B - U_A = m_0(V_B - V_A)$$

$\therefore$ $$v = \sqrt{2(V_B - V_A)} \qquad \text{...(i)}$$

Potential at A : V_A = potential due to complete sphere – potential due to cavity

$$= -\frac{1.5\,GM}{R} - \left[-\frac{Gm}{R/2}\right] = \frac{2Gm}{R} - \frac{1.5\,GM}{R}$$

Here, $$m = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3\rho = \frac{\pi\rho R^3}{6} \quad \text{and} \quad M = \frac{4}{3}\pi R^3\rho$$

Substituting the values, we get

$$V_A = \frac{G}{R}\left[\frac{\pi\rho R^3}{3} - 2\pi\rho R^3\right] = -\frac{5}{3}\pi G\rho R^2$$

Potential at B :

$$V_B = -\frac{GM}{R^3}\left[1.5R^2 - 0.5\left(\frac{R}{2}\right)^2\right] + \frac{1.5Gm}{R/2} = -\frac{11}{8}\frac{GM}{R} + \frac{3Gm}{R}$$

$$= \frac{G}{R}\left[\frac{\pi\rho R^3}{2} - \frac{11}{6}\cdot\pi\rho R^3\right] = -\frac{4}{3}\pi G\rho R^2$$

$\therefore$ $$V_B - V_A = \frac{1}{3}\pi G\rho R^2$$

So, from Eq. (i) $$v = \sqrt{\frac{2}{3}\pi G\rho R^2}$$ **Ans.**

12. Let v be the speed of the projectile at highest point and $r_{\max}$ its distance from the centre of the earth. Applying conservation of angular momentum and mechanical energy,

$$mv_0 \sin\alpha = mvr_{\max} \qquad \text{...(i)}$$

$$\frac{1}{2}mv_0^2 - \frac{GM_e m}{R_e} = \frac{1}{2}mv^2 - \frac{GM_e m}{r_{max}} \quad \ldots(ii)$$

Solving these two equations with the given data we get,

$$r_{max} = \frac{3R_e}{2}$$

or the maximum height $h_{max} = r_{max} - R_e = \dfrac{R_e}{2}$ **Ans.**

13. (a) Let x be the displacement of ring. Then displacement of the particle is $x_0 - x$, or $(3.0 - x)$ m. Centre of mass will not move. Hence,

$$(5.4 \times 10^9)x = (6 \times 10^8)(3 - x)$$

Solving, we get $x = 0.3$ m **Ans.**

(b) Apply conservation of linear momentum and conservation of mechanical energy.

14. (a) Mean radius of planet, $m_2 = \dfrac{r_1 + r_2}{2} = 1.4 \times 10^8$ km

Now, $T \propto r^{3/2}$

$\therefore$ Time period of m_2: $T_2 = T_1\left(\dfrac{1.4 \times 10^8}{10^8}\right)^{3/2}$

or $T_2 = 2\,(1.4)^{3/2} = 3.31$ year **Ans..**

(b) For m_2, point P is perigce position. So speed at this point is greater than orbital speed for circular orbit.

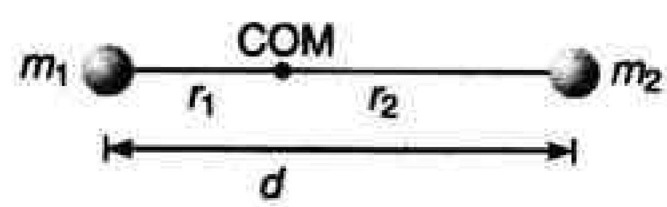
$\therefore$ $U_{m_2} = U_{m_1}$

$\therefore$ $E_{m_2} > E_{m_1}$

$K_{m_2} > K_{m_1}$

(c) vr = constant

15. (a) $r_1 + r_2 = d$ ...(i)



$m_1 r_1 = m_2 r_2$ ...(ii)

Solving these two equations we get,

$$r_1 = \left(\frac{m_2}{m_1 + m_2}\right)d \quad \text{or} \quad r_2 = \left(\frac{m_1}{m_1 + m_2}\right)d$$

The centripetal force is provided by gravitational force,

$$m_1 r_1 \omega^2 = m_2 r_2 \omega^2 = \frac{Gm_1 m_2}{d^2}$$

Solving these equations, we get $\omega = \sqrt{\dfrac{G(m_1 + m_2)}{d^3}}$

$\therefore$ $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{d^3}{G(m_1 + m_2)}}$ **Ans.**

(b)
$$\frac{K_1}{K_2} = \frac{\frac{1}{2} I_1 \omega^2}{\frac{1}{2} I_2 \omega^2} = \frac{I_1}{I_2} = \frac{m_1 r_1^2}{m_2 r_2^2}$$

$$= \left(\frac{m_1}{m_2}\right)\left(\frac{r_1}{r_2}\right)^2 = \left(\frac{m_1}{m_2}\right)\left(\frac{m_2}{m_1}\right)^2 = \frac{m_2}{m_1}$$ **Ans.**

(c)
$$\frac{L_1}{L_2} = \frac{I_1 \omega}{I_2 \omega} = \frac{I_1}{I_2} = \frac{m_2}{m_1}$$ **Ans.**

(d)
$$L = L_1 + L_2 = (I_1 + I_2)\omega$$

$$= (m_1 r_1^2 + m_2 r_2^2)\,\omega$$

$$= \left[\frac{m_1 m_2^2 d^2}{(m_1 + m_2)^2} + \frac{m_2 m_1^2 d^2}{(m_1 + m_2)^2}\right]\omega$$

$$= \mu \omega d^2$$ **Ans.**

where
$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \text{reduced mass}$$

(e)
$$K = \frac{1}{2}(I_1 + I_2)\omega^2 = \frac{1}{2}\mu \omega^2 d^2$$ **Ans.**

Chapter 11 Simple Harmonic Motion

JEE Advanced (Subjective Questions)

1. $$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{100}{4}} = \frac{2.5}{\pi} = 0.8 \text{ Hz}$$ **Ans.**

with 1 kg mass, $$f_0 = \frac{1}{2\pi}\sqrt{\frac{100}{1}} = \frac{5}{\pi} \text{ Hz}$$

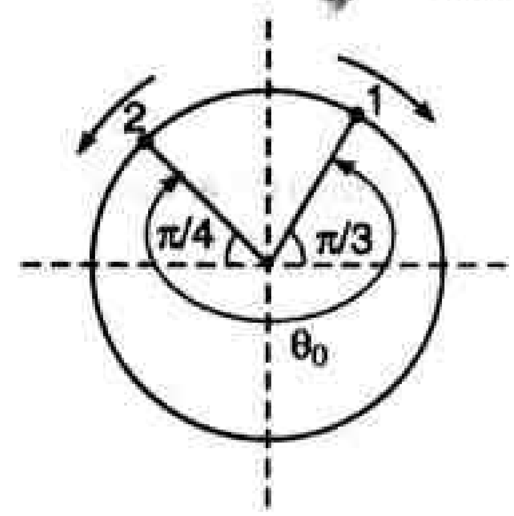
Further, from conservation of linear momentum (at mean position)

$$\omega A = \frac{1}{4}\omega_0 A_0$$

or $$fA = \frac{1}{4} f_0 A_0 \text{ or } A = \frac{f_0 A_0}{4f} - \frac{(5/\pi)(0.1)}{(4)(2.5/\pi)}$$

$$= 0.05 \text{ m}$$ **Ans.**

2. (a) $\theta_0 = \omega_1 t + \omega_2 t$



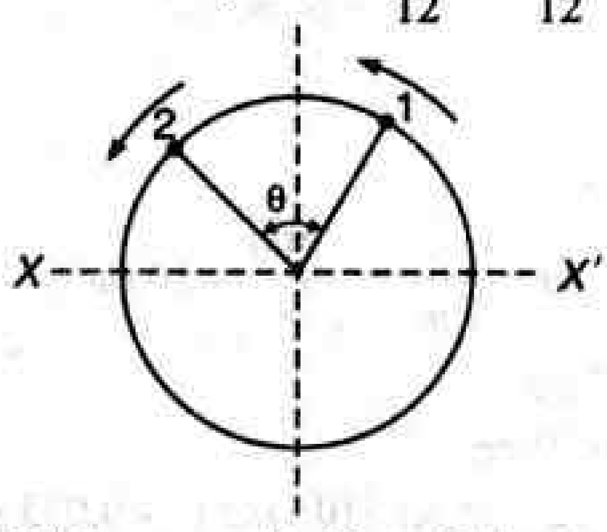
but $$\omega_1 = \omega_2 = \frac{2\pi}{T}$$

and $$\theta_0 = \pi + \frac{\pi}{3} + \frac{\pi}{4} = \frac{19}{12}\pi$$

$\therefore$ $$\frac{19\pi}{12} = \frac{2\pi}{T}t + \frac{2\pi}{T}t$$

or $$t = \frac{19}{48}T$$

(b) $$\theta = 2\pi - \theta_0 = 2\pi - \frac{19}{12}\pi = \frac{5\pi}{12}$$



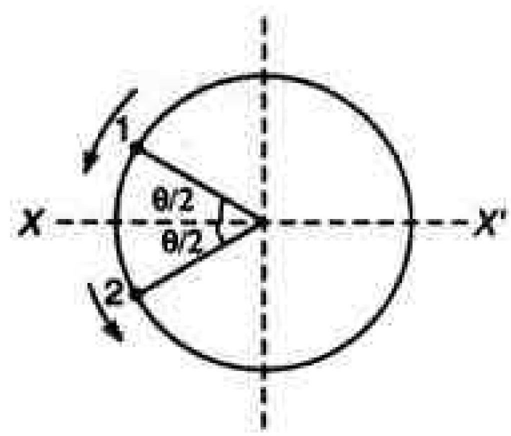
Two particles will collide when line XX' becomes the line of bisector of angle θ.

$\therefore$ Any one of the particles (say-2) has rotated an angle

$$\omega t = \pi/4 + \theta/2$$

or $$\frac{2\pi}{T}t = \frac{\pi}{4} + \frac{5\pi}{24} = \frac{11\pi}{24}$$

$\therefore$ $$t = \frac{11T}{48}$$



3. $1.5 = \omega A = 2A$

$\therefore$ $$A = 0.75 \text{ m}$$

Further, the particle will start its journey from its mean position in downward direction (– ve direction).

4. (a) $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{600}{2+1}} = 10\sqrt{2}$ rad/s

From conservation of linear momentum (at mean position) velocity of combined mass will be 2 m/s. This is the maximum velocity of combined mass.

$\therefore$ $$v = \omega A$$

or $$A = \frac{v}{\omega} = \frac{2}{10\sqrt{2}} \text{ m} = 0.141 \text{ m} = 14.1 \text{ cm}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{10\sqrt{2}} = 0.44 \text{ s}$$

(b) If the collision is elastic,

$$\omega' = \sqrt{\frac{600}{2}} = 10\sqrt{3} \text{ rad/s}$$

In elastic collision,

$$v_2' = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)v_2 + \left(\frac{2m_1}{m_1 + m_2}\right)v_1$$

$$= \left(\frac{2\times 1}{1+2}\right)(6) = 4 \text{ m/s} \qquad (\text{As } v_2 = 0)$$

This is maximum velocity.

$\therefore$ $$v_2' = \omega' A'$$

or $$A' = \frac{v_2'}{\omega'} = \frac{4}{10\sqrt{3}} = 0.23 \text{ m} = 23 \text{ cm}$$

$$T' = \frac{2\pi}{\omega'} = \frac{2\pi}{10\sqrt{3}} = 0.36 \text{ s}$$

(c) In both cases journey is started from mean position

$\therefore$ $$x = \pm A \sin \omega t$$

will be the displacement-time equation.
For impulse we can apply the equation.
Impulse = change in linear momemtum.

5. $$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{400}{4}} = 10 \text{ rad/s}$$

Let t_1 be the time from $x = 0$ to $x = 12$ cm
and t_2 the time from $x = 0$ to $x = 9$ cm. Then,

$$12 = 15 \sin (10t_1) \text{ or } t_1 = 0.093 \text{ s}$$
$$9 = 15 \sin (10t_2) \text{ or } t_2 = 0.064 \text{ s}$$

$\therefore$ Total time $$= t_1 + t_2 = 0.157 \text{ s}$$

6. (a) $$y = a(1 - \cos \omega t)$$

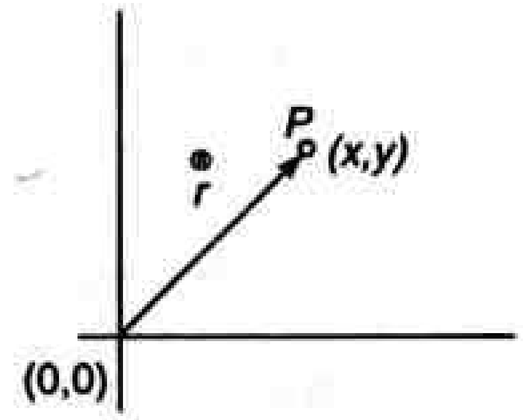
$$\frac{d^2y}{dt^2} = a\,\omega^2 \cos \omega t$$

$$N - mg = m \cdot \frac{d^2y}{dt^2} \quad \text{or} \quad N = mg + ma\omega^2 \cos \omega t$$

or $$N = m\,(g + a\omega^2 \cos \omega t)$$ **Ans.**

(b) $$\left(\frac{d^2y}{dt^2}\right)_{\text{max}} = a\omega^2 \quad \text{or} \quad a\omega^2 = g$$

$$\therefore \qquad a = \frac{g}{\omega^2} = \frac{980}{(11)^2} = 8.1 \text{ cm}$$ **Ans.**



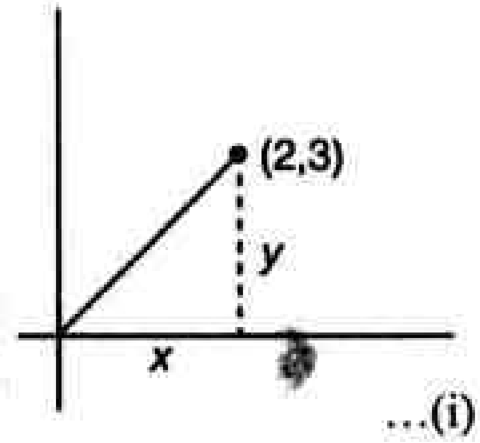
7. $$\vec{F} = -kx\hat{i} - ky\hat{j}$$

$$\vec{F} = 0 \quad \text{at} \quad (0, 0)$$

When it is displaced to a point P whose position vector is

$$\vec{r} = x\hat{i} + y\hat{j}$$

Force on it is $$\vec{F} = -k(x\hat{i} + y\hat{j}) = -k\vec{r}$$



Since, $\vec{F} \propto -\vec{r}$, motion is simple harmonic. At $t = 0$ particle is at (2, 3)

$$\frac{y}{x} = \frac{3}{2}$$

or $$2y - 3x = 0$$

i.e., the particle will oscillate simple harmonically along this line.

8. In equilibrium, $$mg \sin\theta = kx_0 \qquad \ldots\text{(i)}$$

When displaced by x,

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}k(x + x_0)^2 - mgx \sin\theta$$

Since, $$E = \text{constant}$$

$$\frac{dE}{dt} = 0$$

$$0 = mv\left(\frac{dv}{dt}\right) + I\omega\left(\frac{d\omega}{dt}\right) + k(x + x_0)\frac{dx}{dt} - mg \sin\theta \frac{dx}{dt}$$

Substituting, $$\frac{dv}{dt} = a, \quad \omega = \frac{v}{R}, \quad I = \frac{1}{2}mR^2$$

$$\frac{d\omega}{dt} = \alpha = \frac{a}{R}, \quad \frac{dx}{dt} = v \quad \text{and} \quad kx_0 = mg \sin\theta$$

We get, $$3ma = -2kx$$

$$\therefore \qquad f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{x}\right|} = \frac{1}{2\pi}\sqrt{\frac{2k}{3m}}$$

Substituting the values, $$f = \frac{1}{2\pi}\sqrt{\frac{2 \times 200}{\frac{3 \times 100}{9.8}}} = 0.56 \text{ Hz}$$ **Ans.**

9. In the displaced position,

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}k(2x)^2$$

$$I = \frac{1}{2}mR^2$$

and $$\omega = \frac{v}{R}$$

$$\therefore \qquad E = \frac{3}{4}mv^2 + 2kx^2$$

$$E = \text{constant}$$

$$\therefore \qquad \frac{dE}{dt} = 0$$

or $$\frac{3}{2}mv\frac{dv}{dt} + 4\,kx\frac{dx}{dt}$$

Substituting, $$\frac{dx}{dt} = v \quad \text{and} \quad \frac{dv}{dt} = a$$

$$a = -\frac{8k}{3m}\,.\,x$$

Comparing with, $$a = -\omega^2 x$$

We have $$\omega = \sqrt{\frac{8k}{3m}} = \sqrt{\frac{8\times 1000}{\frac{3\times 100}{9.8}}} = 16.16\text{ rad/s}$$

$\therefore$ $$\theta = \theta_0 \cos \omega t$$

or $$\theta = 0.4 \cos(16.16t)$$ **Ans.**

10. Similar to Q. 9

11. (a) $$\omega^2 = \frac{k}{m}$$

$\therefore$ $$k = m\omega^2 = (1)(10)^2 = 100\text{ N/m}$$

At $t = 0$, block of mass m is at mean position ($x = 10$ cm) and moving towards positive x-direction with velocity $A\omega$ or 30 cm/s.

From conservation of linear momentum,

$$(M + m)v = M(30) - m(30)$$

Substituting the values, we have

$$v = 15\text{ cm/s} \quad \text{or} \quad 0.15\text{ m/s}$$

From conservation of mechanical energy,

$$\frac{1}{2}(M + m)v^2 = \frac{1}{2}kA^2$$

$\therefore$ $$A = \left(\sqrt{\frac{M + m}{k}}\right)v = \left(\frac{4}{100}\right)^{1/2}(0.15)$$

$$= 0.03\text{ m}$$

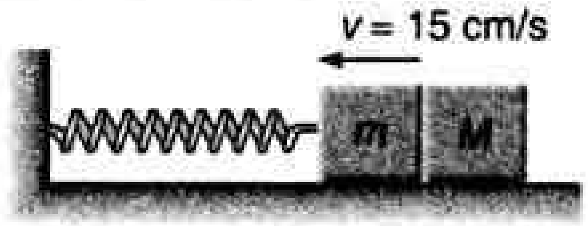
or $$A = 3\text{ cm}$$ **Ans.**

(b) $$\omega' = \sqrt{\frac{k}{M + m}} = \sqrt{\frac{100}{4}} = 5\text{ rad/s}$$

$\therefore$ $$x' = 10 - 3\sin 5t$$

(c) $$\Delta E = \frac{1}{2}(1)(0.3)^2 + \frac{1}{2}(3)(0.3)^2 - \frac{1}{2}(4)(0.15)^2$$

$$= 0.135\text{ J}$$ **Ans.**



12. Let F be the restoring force (extra tension) on block m when displaced by x from its equilibrium position.

$$x = 2x_1 + 2x_2 = 2\left[\frac{2F}{k_1} + \frac{2F}{k_2}\right] = 4F\left(\frac{k_1 + k_2}{k_1 k_2}\right)$$

or $$F = -\frac{k_1 k_2}{4(k_1 + k_2)}x$$

$\therefore$ $$a = -\frac{k_1k_2}{4m\,(k_1+k_2)}\,x$$

$$\omega = \sqrt{\frac{k_1k_2}{4m\,(k_1+k_2)}}$$ **Ans.**

13. (a) $$f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{\text{total mass}}} = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m_1+m_2}}$$

(b) Suppose the system is displaced towards left by a distance x.

Restoring force on m_1 :

$$F = m_1\omega^2 x \qquad \text{(towards right)}$$

$$= m_1\left(\frac{k_1+k_2}{m_1+m_2}\right)x$$

Friction f on it will be towards right if,

$$k_1x < F$$

or $$k_1x < m_1\left(\frac{k_1+k_2}{m_1+m_2}\right)x$$

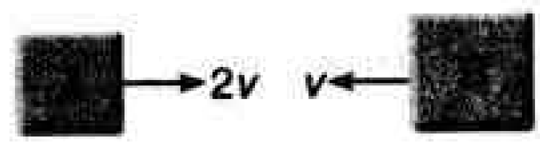
or $$\frac{k_1}{k_2} < \frac{m_1}{m_2}$$ **Ans.**

(c) $$k_1A_m + \mu m_2 g = m_1\left(\frac{k_1+k_2}{m_1+m_2}\right)A_m$$

$$A_m\left(\frac{m_1k_1+m_1k_2}{m_1+m_2} - k_1\right) = \mu\, m_2 g$$

or $$A_m = \frac{\mu\,(m_1+m_2)m_2 g}{m_1k_2 - m_2k_1}$$ **Ans.**

14. (a) $$\frac{1}{2}kx_0^2 = \frac{1}{2}\mu v_r^2$$



Here, $$\mu = \text{reduced mass} = \frac{6\times 3}{6+3} = 2\text{ kg}$$

$\therefore$ $$v_r = \sqrt{\frac{k}{\mu}}\,x_0 = \left(\sqrt{\frac{200}{2}}\right)(3\times 10^{-2})$$

$$= 0.3\text{ m/s} = 2v + v$$

$\therefore$ $$v = 0.1\text{ m/s}$$

$\therefore$ $$v_1 = 0.2\text{ m/s} \quad \text{and} \quad v_2 = 0.1\text{ m/s}$$ **Ans.**

Angular frequency $$\omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{200}{2}} = 10\text{ rad/s}$$

(b) $$v_{\text{com}} = \frac{m_1v_1 + m_2v_2 + m_3v_3}{m_1+m_2+m_3}$$

$$= \frac{(3)(0.2) - (6)(0.1) + 3(0.4)}{3+6+3}$$

$$= 0.1\text{ m/s} \quad \text{(towards right)}$$ **Ans.**

(c) After collision velocity of combined blocks $(A + C)$

$$v_0 = \frac{(3 \times 0.2) + (3)(0.4)}{3 + 3} = 0.3 \text{ m/s}$$

and velocity of block B is $\quad v_2 = 0.1 \text{ m/s}$

The spring will compress till velocity of all the blocks become equal to the centre of mass. Applying conservation of mechanical energy,

$$\frac{1}{2}(3 + 3)(0.3)^2 + \frac{1}{2}(6)(0.1)^2 = \frac{1}{2}(3 + 3 + 6)(0.1)^2 + \frac{1}{2}kA^2$$

Solving this we get, $\quad A = 0.048 \text{ m}$

or $\quad A = 4.8 \text{ cm}$ **Ans.**

(d)
$$\Delta E = \frac{1}{2}(3)(0.4)^2 + \frac{1}{2}(3)(0.2)^2 - \frac{1}{2}(3 + 3)(0.3)^2$$

$$= 0.24 + 0.06 - 0.27 = 0.03 \text{ J}$$ **Ans.**

15. Restoring torgue $\tau = -(kl\theta)l - \left(k\frac{l}{2}\theta\right)\frac{l}{2} = -\frac{5}{4}kl^2\theta$

Now,
$$\left(\frac{ml^2}{3}\right)\alpha = -\left(\frac{5}{4}kl^2\right)\theta$$

$$f = \frac{1}{2\pi}\sqrt{\left|\frac{\alpha}{\theta}\right|} = \frac{1}{2\pi}\sqrt{\frac{15k}{4m}}$$

16. When the mass m is displaced from its mean position by a distance x, let F be the restoring (extra tension) force produced in the string.By this extra tension further elongaion in the springs are $\frac{2F}{k_1}, \frac{2F}{k_2}, \frac{2F}{k_3}$ and $\frac{2F}{k_4}$ respectively.

Then,
$$x = 2\left(\frac{2F}{k_2}\right) + 2\left(\frac{2F}{k_2}\right) + 2\left(\frac{2F}{k_3}\right) + 2\left(\frac{2F}{k_4}\right)$$

or
$$F\left(\frac{4}{k_1} + \frac{4}{k_2} + \frac{4}{k_3} + \frac{4}{k_4}\right) = -x$$

Here netative sign shows the restoring nature of force.

$$a = -\frac{x}{m\left(\frac{4}{k_1} + \frac{4}{k_2} + \frac{4}{k_3} + \frac{4}{k_4}\right)}$$

$$T = 2\pi\sqrt{\left|\frac{x}{a}\right|}$$

$$= 4\pi\sqrt{m\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4}\right)}$$ **Ans.**

17. Let F be the extra tension in the string, when the block is displaced x from its mean position.

Extension in spring-2 is $x_2 = \frac{F}{k_2}$

Extension is spring-1 is $x_1 = \frac{2F}{k_1}$

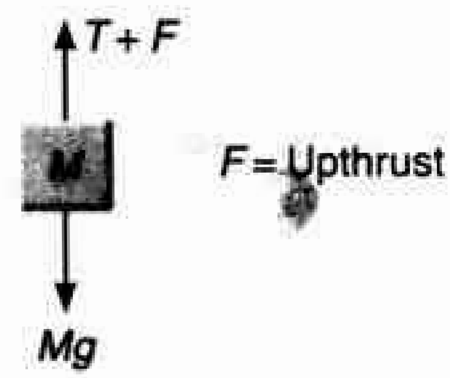
$$x = 2x_1 + x_2 = \frac{4F}{k_1} + \frac{F}{k_2}$$

Extra tension F will become restoring force for the block. Therefore above equation can be written as,

$$F = -\left(\frac{1}{\frac{4}{k_1} + \frac{1}{k_2}}\right)x = \left(\frac{k_1 k_2}{4k_2 + k_1}\right)x$$

or $$k_e = \frac{k_1 k_2}{4k_2 + k_1}$$

$$T = 2\pi\sqrt{\frac{m}{k_e}} = 2\pi\sqrt{\frac{m(4k_2 + k_1)}{k_1 k_2}}$$ **Ans.**



18. In equilibrium, $$T + F = Mg \qquad \text{...(i)}$$

When the block is further depressed by x, weight Mg remains unchanged, upthrust F increases by ρAxg and let ΔT be the increase in tension.

If a is the acceleration of block then,

$$\Delta T + \rho Axg = Ma \qquad \text{...(ii)}$$

Restoring torque on the cylinder,

$$\tau = \left[\frac{kx}{2}\frac{R}{2} - \Delta T\,R\right] = \left[\frac{kxR}{4} - (Ma - \rho Axg)\,R\right]$$

$$\frac{1}{2}MR^2\alpha = \left[\frac{kR^2\theta}{4} - (MR\alpha - \rho AgR\theta)\,R\right]$$

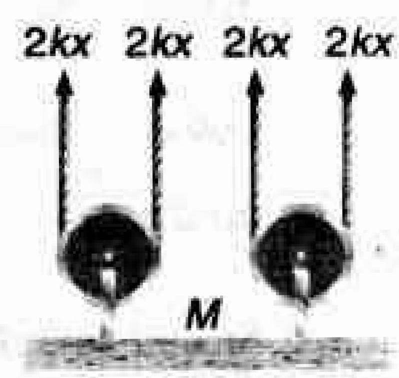
or $$\frac{3}{2}MR^2\alpha = \left[\frac{kR^2}{4} + \rho AgR^2\right]\theta$$

or $$\alpha = \frac{-\left[\frac{k}{4} + \rho Ag\right]}{\frac{3}{2}M}\theta$$

Here negative sign has been used for restoring nature of torque.

$\therefore$ $$f = \frac{1}{2\pi}\sqrt{\left|\frac{\alpha}{\theta}\right|}$$

$$= \frac{1}{2\pi}\sqrt{\frac{k + 4\rho Ag}{6M}}$$ **Ans.**



19. If the mass M is displaced by x from its mean position each spring further stretches by $2x$.

Net restoring force $$F = -8kx$$

$\therefore$ $$M \cdot a = -8kx$$

$$f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{x}\right|}$$

$$= \frac{1}{2\pi}\sqrt{\frac{8k}{M}} = \frac{1}{\pi}\sqrt{\frac{2k}{M}}$$ **Ans.**

Chapter 12 Elasticity

JEE Advanced (Subjective Questions)

1. $\Delta l = \dfrac{Fl}{AY} = \dfrac{Fl}{(\pi d^2/4)Y}$

$\therefore$ $$d = 2\sqrt{\frac{Fl}{\pi Y.\Delta l}} = 2\sqrt{\frac{400 \times 3}{\pi \times 2.1 \times 10^{11} \times 0.2 \times 10^{-2}}} = 1.91 \times 10^{-3}\text{ m} = 1.91\text{ mm}$$ **Ans.**

2. $m(g + a) = \dfrac{1}{3}(\sigma A)$

$\therefore$ $$a = \frac{\sigma A}{3m} - g = \frac{3.0 \times 10^8 \times 4 \times 10^{-4}}{3 \times 900} - 9.8 = 34.64\text{ m/s}^2$$ **Ans.**

3. $\dfrac{\sigma \times \pi d^2}{4} = mg$

$\therefore$ $$d = \sqrt{\frac{4mg}{\sigma\pi}} = \sqrt{\frac{4 \times 10 \times 9.8}{1.5 \times 10^8 \times \pi}} = 9.12 \times 10^{-4}\text{ m}$$
$$= 0.912\text{ mm}$$ **Ans.**

4. Change in length due to own weight of a wire

$$\Delta l = \frac{mgl}{2AY} = \frac{(lA\rho)gl}{2AY} = \frac{\rho g l^2}{2Y} = \frac{8000 \times 9.8 \times (5)^2}{2 \times 2 \times 10^{11}}$$
$$= 4.9 \times 10^{-6}\text{ m}$$ **Ans.**

Potential energy stored $= \dfrac{1}{2}\left(\dfrac{YA}{l}\right)(\Delta l)^2$

$$= \frac{1}{2} \times 2.0 \times 10^{11} \times \pi \times \frac{(6 \times 10^{-3})^2 \times (4.9 \times 10^{-6})^2}{5}$$
$$= 5.43 \times 10^{-5}\text{ J}$$ **Ans.**

5. Let T_1 be the tension in the upper string and T_2 on lower string. Let m be the new mass.

On upper string $T_{max} = \sigma_1 A_1 = (8 \times 10^8 \times 0.006 \times 10^{-4}) = (m_1 + m_2 + m_{max})g$

Substituting the values we have,

$$m_{max} = 18\text{ kg}$$

Further on lower string

$$T_{max} = \sigma_2 A_2 = (8 \times 10^8 \times 0.003 \times 10^{-4}) = (m_1 + m_{max})g$$

Substituting the values we have,

$$m_{max} = 14\text{ kg}$$

Lower of the two is 14 kg, therefore answer is 14 kg. **Ans.**

6. (a) $\sigma = \dfrac{T}{A}$

Since T and A both are same, $\dfrac{\sigma_1}{\sigma_2} = \dfrac{1}{1} = 1$

(b) Strain $= \dfrac{\text{Stress}}{Y}$ or Strain $\propto \dfrac{1}{Y}$

7. $\rho' = \dfrac{\rho}{1 - \dfrac{dP}{B}} = \rho\left(1 - \dfrac{dP}{B}\right)^{-1} \approx \rho\left(1 + \dfrac{dP}{B}\right)$

or $$\Delta\rho = \rho' - \rho = \frac{\rho dP}{B}$$

Here, $$dP = h\rho g$$

$\therefore$ $$\Delta\rho = \frac{\rho^2 hg}{B} = \frac{(1030)^2(400)(9.8)}{2\times 10^9} = 2.0 \text{ kg/m}^3$$ **Ans.**

8. $\Delta l = \dfrac{FL}{AY}$ and $\Delta l' = \dfrac{F' l}{AY}$

Decrease in length $\Delta l - \Delta l' = \dfrac{(F - F')l}{AY}$

But $F - F' = \text{Upthrust} = V\rho g$

$\therefore$ $$\text{Decrease in length} = \frac{V\rho g l}{AY} = \frac{(10^{-3})(800)(9.8)(3)}{(\pi/4)(0.4\times 10^{-3})^2(8\times 10^{10})} = 2.34\times 10^{-3} \text{ m}$$

$$= 2.34 \text{ mm}$$ **Ans.**

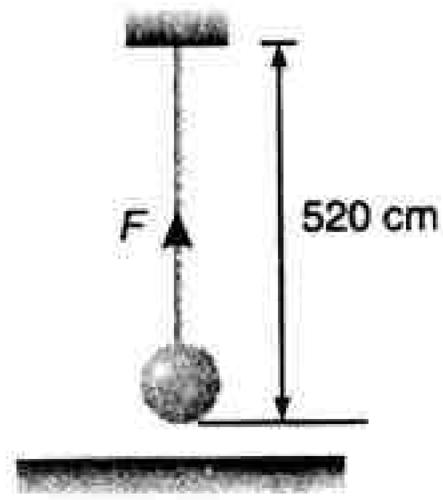
9. $B = \dfrac{-dP}{(dV/V)}$, $dP = \rho g h$ and $-(dV/V)\times 100 = 0.1$ or $(dV/V) = 0.001$

$\therefore$ $$B = \frac{\rho g h}{0.001} = 1000\,\rho g h$$

$$= (1000)\times(10)^3(9.8)(180)$$

$$= 1.76\times 10^9 \text{ N/m}^2$$ **Ans.**

10. $\Delta l = \dfrac{Fl}{AY}$



$$\Delta l = (521 - 520) = 1 \text{ cm} = 10^{-2} \text{ m}$$

$$F = \frac{\Delta l AY}{l} = 10^{-2}\times\frac{\pi}{4}\frac{(4\times 10^{-3})^2}{5}\times 2\times 10^{11}$$

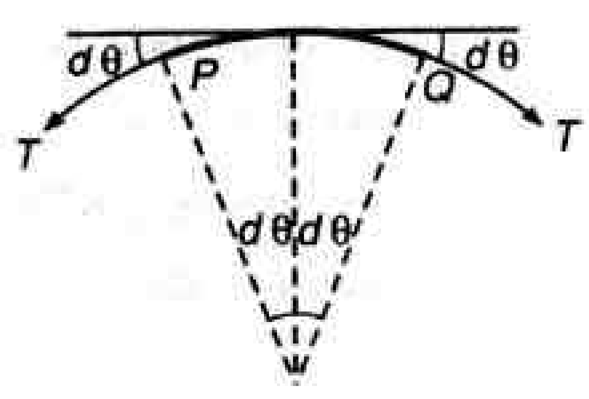
$$= 5026 \text{ N}$$

Now at lowest position, $F - mg = \dfrac{mv^2}{R}$

$\therefore$ $$v = \sqrt{\frac{FR - mgR}{m}}$$

$$= \sqrt{\frac{5026\times 5.1 - 25\times 9.8\times 5.1}{25}} = 31.23 \text{ m/s}$$ **Ans.**

11. $2T\sin(d\theta)$ will provide necessary centripetal force to section PQ of the ring.



$\therefore$ $$2T\sin(d\theta) = (dm)R\omega^2 = (2R\cdot d\theta)(A\rho)R(2\pi\nu)^2$$

For small angle $$\sin d\theta \approx d\theta.$$

Substituting we get,

$$T = 4\pi^2 A\rho R^2\nu^2$$

Now, $$\Delta l = \frac{Tl}{AY} = \frac{(4\pi^2 A\rho R^2 v^2)(2\pi R)}{(A)Y} = \frac{8\pi^3 \rho R^3 v^2}{Y}$$

Further, $$l = 2\pi R$$

$\therefore$ $$\Delta R = \frac{\Delta l}{2\pi}$$

or $$\Delta R = \frac{4\pi^2 \rho R^3 v^2}{Y}$$ **Ans.**

12. $$T - mg = mR\omega^2$$ (At lowest point)

$\therefore$ $$T = mg + mR\,(2\pi f)^2 = (6 \times 9.8) + (6 \times 0.6)(2\pi \times 2)^2 = 627.28 \text{ N}$$

Now, $$\Delta l = \frac{Tl}{AY} = \frac{(627.28)(0.6)}{(0.05 \times 10^{-4})(2 \times 10^{11})} = 3.8 \times 10^{-4} \text{ m}$$ **Ans.**

13. $$2T_C + T_S = mg \quad \text{...(i)}$$

$$\Delta l_C = \Delta l_S$$

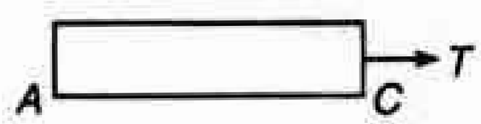
or $$\frac{T_C l}{AY_C} = \frac{T_S l}{AY_S}$$

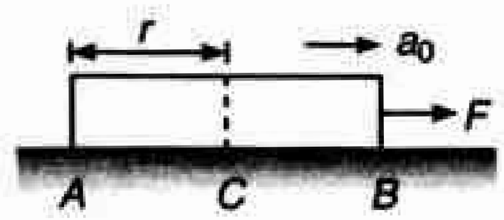
or $$\frac{T_C}{T_S} = \frac{1}{2}$$ (as $Y_S = 2Y_C$) ...(ii)

Solving these two equations, we get $$T_C = \frac{mg}{4}$$

and $$T_S = \frac{mg}{2}$$ **Ans.**

14. (a) Free body diagram of AC,





$$T = \text{(mass) (acceleration)} = \rho r S a_0$$

$\therefore$ $$\text{stress} = \frac{T}{S} = \rho r a_0$$

At $$r = \frac{L}{2}, \quad \text{stress} = \frac{\rho L a_0}{2}$$ **Ans.**

(b) $$\text{Total elongation} = \int_0^L \frac{(\text{stress})\,dr}{Y}$$

$$= \int_0^L \frac{(\rho r a_0)\,dr}{Y} = \frac{\rho a_0 L^2}{2Y}$$ **Ans.**

15. Given :

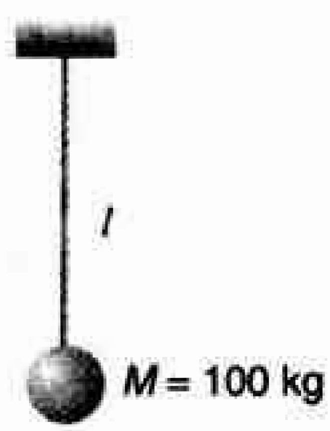
Length of the wire, $l = 5$ m

Radius of the wire, $r = 2 \times 10^{-3}$ m

Density of wire, $\rho = 7860$ kg/m^3

Young's modulus,

$$Y = 2.1 \times 10^{11} \text{ N/m}^2$$



and Specific heat, $S = 420$ J/kg-K

Mass of wire, $m = $ (density) (volume)

$$= (\rho)(\pi r^2 l)$$

$$= (7860)(\pi)(2 \times 10^{-3})^2 (5) \text{ kg}$$

$$= 0.494 \text{ kg}$$

Elastic potential energy stored in the wire, $U = \frac{1}{2}$ (stress) (strain) (volume) $\left[\frac{\text{energy}}{\text{volume}} = \frac{1}{2} \times \text{stress} \times \text{strain}\right]$

or
$$U = \frac{1}{2}\left(\frac{Mg}{\pi r^2}\right)\left(\frac{\Delta l}{l}\right)(\pi r^2 l)$$

$$= \frac{1}{2}(Mg)\cdot \Delta l \qquad \left(\Delta l = \frac{Fl}{AY}\right)$$

$$= \frac{1}{2}(Mg)\frac{(Mgl)}{(\pi r^2)Y}$$

$$= \frac{1}{2}\frac{M^2 g^2 l}{\pi r^2 Y}$$

Substituting the values, we have

$$U = \frac{1}{2}\frac{(100)^2 (10)^2 (5)}{(3.14)(2 \times 10^{-3})^2 (2.1 \times 10^{11})}$$

$$= 0.9478 \text{ J}$$

When the bob gets snapped, this energy is utilised in raising the temperature of the wire.

So,
$$U = ms\,\Delta\theta$$

$\therefore$
$$\Delta\theta = \frac{U}{ms} = \frac{0.9478}{0.494(420)} \, ^\circ\text{C or K}$$

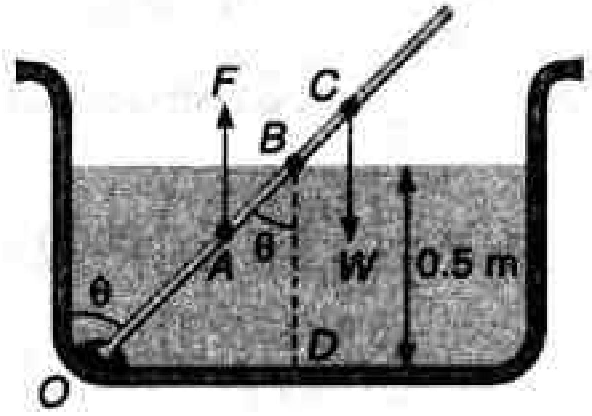
$$\Delta\theta = 4.568 \times 10^{-3} \, ^\circ\text{C}$$

Chapter 13 Fluid Mechanics

JEE Advanced (Subjective Questions)

1. W = weight

F = Upthrust



$$OA = \frac{OB}{2} = \frac{0.5 \sec\theta}{2} = 0.25 \sec\theta$$

About point O, clockwise moment of W = anticlockwise moment of F.

$\therefore$ $$W\left(\frac{L}{2}\sin\theta\right) = F(OA \sin\theta) = F(0.25 \sin\theta.\sec\theta)$$

Given $L = 1$ m

$\therefore$ $$\cos\theta = \frac{F}{2W} = \frac{(0.5 \sec\theta)(A)(1.0)g}{2(1)(A)((0.5)g}$$

or $$\cos^2\theta = \frac{1}{2}, \cos\theta = \frac{1}{\sqrt{2}}$$

$\therefore$ $$\theta = 45°$$ **Ans.**

2. Fraction of volume immersed before putting the new weight $= \frac{\rho_{block}}{\rho_{water}}$

$$= \frac{800}{1000} = 0.8$$

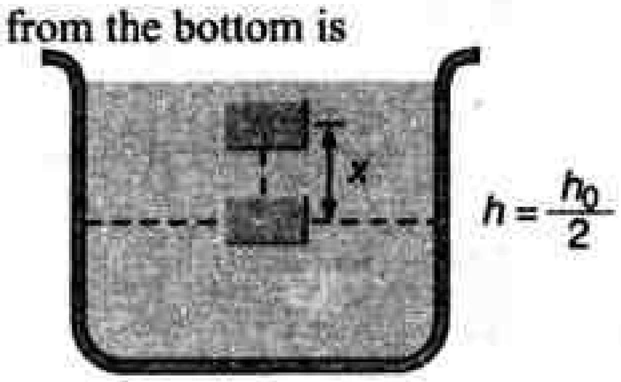
i.e., 20% of 3 cm or 0.6 cm is above water. Let W is the new weight, then spring will be compressed by 0.6 cm.

$\therefore$ W + weight of block = Upthrust on whole volume of block + spring force

or $$W = (3\times10^{-2})^3 \times 1000 \times 10 + 50 \times (0.6 \times 10^{-2}) - (3\times10^{-2})^3 \times 800 \times 10$$

$\therefore$ $$W = 0.354 \text{ N}$$ **Ans.**

3. Net force on the block at a height h from the bottom is



$$F_{net} = \text{upthrust} - \text{weight} \qquad \text{(upwards)}$$

$$= \left(\frac{m}{\frac{5}{2}\rho_0}\right)\rho_0\left(4 - \frac{3h}{h_0}\right)g - mg$$

$$F_{net} = 0 \text{ at } h = \frac{h_0}{2}$$

So, $h = \frac{h_0}{2}$ is the equilibrium position of the block.

For $h > \frac{h_0}{2}$, weight > upthrust

i.e., net force is downwards and for $h < \frac{h_0}{2}$

weight < upthrust

i.e., net force is upwards

For upward displacement x from mean position, net downward force is

$$F = -\left[\left(\frac{m}{\frac{5}{2}\rho_0}\right)\rho_0\left\{4 - \frac{3(h+x)}{h_0}\right\}g - mg\right]$$

$$\left(h = \frac{h_0}{2}\right)$$

$$\therefore \qquad F = -\frac{6mg}{5h_0}x \qquad \ldots\text{(i)}$$

(because at $h = \frac{h_0}{2}$ upthrust and weight are equal)

Since $$F \propto -x$$

oscillations are simple harmonic in nature. Rewriting eqution (i)

$$ma = -\frac{6mgx}{5h_0}$$

or $$a = -\frac{6g}{5h_0}x$$

$$\therefore \qquad f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{x}\right|}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{6g}{5h_0}}$$ **Ans.**

4. (a) $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 5}$

$$= 10 \text{ m/s}$$

(b) From conservation of energy,

$$v'^2 = v^2 + 2gH = 100 + 2 \times 10 \times 5 = 200$$

$$v' = 14.1 \text{ m/s}$$

(c) $$t = \frac{2A}{a\sqrt{2g}}\left[\sqrt{H} - \sqrt{\frac{H}{2}}\right]$$

$$= \frac{2 \times \pi \times (1)^2}{10^{-4}\sqrt{2 \times 10}}[\sqrt{5} - \sqrt{2.5}] = 9200 \text{ s}$$

5. Writing equation of motion for the block :

$$T - mg \sin 30° = ma \quad \text{...(i)}$$

For the sphere :

$$\text{Weight} - \text{Buoyant force} - T = ma \quad \text{...(ii)}$$

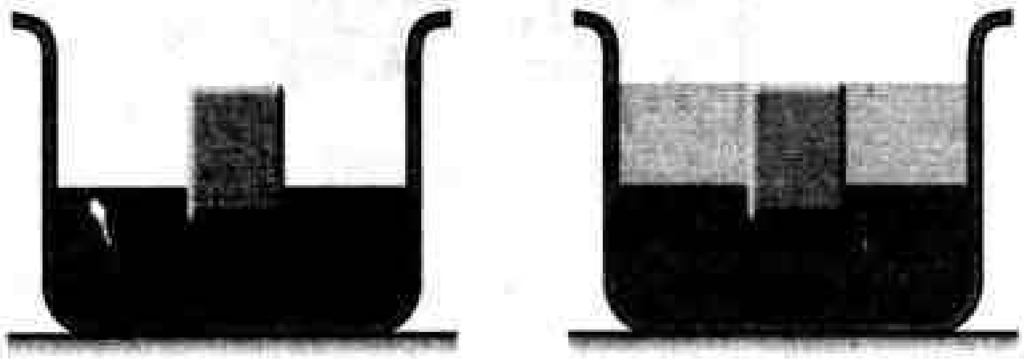
or
$$mg - \frac{mg}{2} - T = ma$$

Solving, we get
$$a = 0$$

6.
$$W = (0.25)V\rho_{Hg}\, g \quad \text{...(i)}$$

Let x fraction of volume is immersed in mercury in the second case. Then,

$$W = xV\rho_{Hg}g + (1-x)\,V\rho_w g \quad \text{...(ii)}$$



Equating Eqs. (i) and (ii), we have

$$\frac{\rho_{Hg}}{4} = x\rho_{Hg} + (1-x)\rho_w$$

$\therefore$
$$12.6x = 2.4 \quad \text{or} \quad x = 0.19$$ **Ans.**

7. (a)
$$P_A - \frac{1}{2}\rho v^2 + \rho g h = P_D$$

But
$$P_A = P_D = P_0$$

$\therefore$
$$\frac{1}{2}\rho v^2 = \rho g h$$

$\therefore$
$$v = \sqrt{2gh}$$

Here,
$$h = (4+1) = 5 \text{ m}$$

$\therefore$
$$v = \sqrt{2 \times 9.8 \times 5} = 9.9 \text{ m/s}$$

(b) Applying Bernoulli's equation at A and B,

$$P_A + 0 + 0 = P_B + \frac{1}{2}\rho v^2 + \rho g(1.5)$$

or
$$P_0 = P_B + \frac{1}{2}\rho v^2 + 1.5\rho g$$

$\therefore$
$$P_B = 1.01 \times 10^5 - \frac{1}{2} \times 900 \times (9.9)^2 - 1.5 \times 900 \times 9.8$$
$$= 4.36 \times 10^4 \text{ Pa}$$ **Ans.**

(c) Applying Bernoulli's equation at A and C,

$$P_0 = P_C + \frac{1}{2}\rho v^2 - \rho g(1.0)$$

$\therefore$
$$P_C = P_0 + \rho g - \frac{1}{2}\rho v^2$$
$$= 1.01 \times 10^5 + 900 \times 9.8 - \frac{1}{2} \times 900 \times (9.9)^2$$
$$= 6.6 \times 10^4 \text{ Pa}$$ **Ans.**

8.
$$\frac{1}{2}\rho v^2 = \rho g h + \frac{mg}{A}$$

Here, $h = 1.0 - 0.5 = 0.5$ m, A = Area of piston = 0.5 m^2

$\therefore$
$$v = \sqrt{2gh + \frac{2mg}{\rho A}} = \sqrt{2 \times 9.8 \times 0.5 + \frac{2 \times 20 \times 9.8}{10^3 \times 0.5}}$$
$$= 3.25 \text{ m/s}$$

$\therefore$ Speed with which it hits the surface is
$$v' = \sqrt{v^2 + 2gh'} = \sqrt{(3.25)^2 + (2 \times 9.8 \times 0.5)}$$
$$= 4.51 \text{ m/s}$$ **Ans.**

9.
$$a\sqrt{2gy} = \pi x^2 \left(-\frac{dy}{dt}\right) \quad \text{...(i)}$$

Here,
$$-\frac{dy}{dt} = \frac{4 \times 10^{-2}}{3600} = 1.11 \times 10^{-5} \text{ m/s}$$
$$a = \pi r^2 = \pi\ (2 \times 10^{-3})^2$$
$$= 1.26 \times 10^{-5} \text{ m}^2$$

Substituting these values in Eq. (i), we have
$$(1.26 \times 10^{-5})\sqrt{2 \times 9.8 \times y} = \pi(1.11 \times 10^{-5})x^2$$

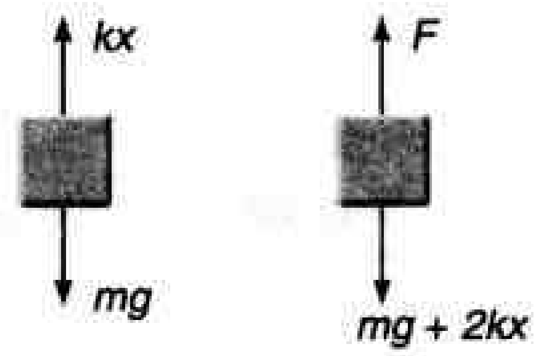
or
$$y = 0.4x^4$$

This is the desired x-y relation.

10. Initially, $\qquad kx = mg \qquad$ ...(i)

In the second case, $\qquad F = mg + 2kx$

or $\qquad kx = \dfrac{F - mg}{2} \qquad$ ...(ii)



From Eqs. (i) and (ii)
$$mg = \frac{F - mg}{2} \quad \text{or} \quad F = 3\ mg$$

Let V be the total volume then,
$$V\rho_w g = 3mg$$

$\therefore$
$$V = \frac{3mg}{\rho_w g} = \frac{(3)(8)}{10^3} \text{ m}^3 = 0.024 \text{ m}^3$$
$$\text{Volume of wood} = \frac{\text{mass}}{\text{density}} = \frac{8}{840} = 0.0095$$

$\therefore$
$$\text{Volume of cavity} = 0.024 - 0.0095 = 0.0145$$

$\therefore$
$$\text{Percentage volume of cavity} = \frac{0.0145}{0.024} \times 100 = 60.41\%$$

11.
$$v = \sqrt{2g(10 - h)} \quad \text{...(i)}$$

Component of its velocity parallel to the plane is $v \cos 30°$.
Let the stream strikes the plane after time t. Then

$$0 = v\cos 30° - g\sin 30°\, t$$

$\therefore$ $$t = \frac{v\cot 30°}{g}$$

Further $$x = vt = \frac{v^2\cot 30°}{g} = \sqrt{3}y$$

or $$\frac{v^2\cot 30°}{g} = \sqrt{3}\left(h - \frac{1}{2}gt^2\right)$$

$\therefore$ $$\frac{\sqrt{3}v^2}{g} = \sqrt{3}\left(h - \frac{g}{2}\frac{v^2\cot^2 30°}{g^2}\right) \quad \text{or} \quad \frac{v^2}{g} = h - \frac{3}{2}\frac{v^2}{g}$$

$\therefore$ $$\frac{5}{2}\frac{v^2}{g} = h \quad \text{or} \quad 5(10 - h) = h$$

$\therefore$ $$h = 8.33 \text{ m}$$ **Ans.**

12. In elastic collision with the surface, direction of velocity is reversed but its magnitude remains the same. Therefore, time of fall = time of rise.

or time of fall $= \frac{t_1}{2}$

Hence, velocity of the ball just before it collides with liquid is

$$v = g\frac{t_1}{2} \qquad \text{...(i)}$$

Retardation inside the liquid

$$a = \frac{\text{upthrust} - \text{weight}}{\text{mass}}$$

$$= \frac{Vd_Lg - Vdg}{Vd} = \left(\frac{d_L - d}{d}\right)g \qquad \text{...(ii)}$$

Time taken to come to rest under this retardation will be

$$t = \frac{v}{a} = \frac{gt_1}{2a} = \frac{gt_1}{2\left(\frac{d_L - d}{d}\right)g} = \frac{dt_1}{2(d_L - d)}$$

Same will be the time to come back on the liquid surface.

Therefore,

(a) t_2 = time the ball takes to came back to the position from where it was released

$$= t_1 + 2t$$

$$= t_1 + \frac{dt_1}{d_L - d}$$

$$= t_1\left[1 + \frac{d}{d_L - d}\right]$$

or $$t_2 = \frac{t_1 d_L}{d_L - d}$$

(b) The motion of the ball is periodic but not simple harmonic because the acceleration of the ball is g in air and $\left(\frac{d_L - d}{d}\right) g$ inside the liquid which is not proportional to the displacement, which is necessary and sufficient condition for SHM.

(c) When $d_L = d$, retardation or accelaration inside the liquid becomes zero (upthrust = weight). Therefore, the ball will continue to move with constant velocity $v = \frac{gt_1}{2}$ inside the liquid.

13. Pressure inside the bubble $= P_0 + \rho gh + \frac{2T}{r}$

$\therefore$ Amount of pressure inside the bubble greater than the atmospheric pressure $= \rho gh + \frac{2T}{r}$

Substituting the values we get,

$$\Delta P = 10^3 \times 9.8 \times 0.1 + \frac{2 \times 0.075}{10^{-3}}$$

$$= 980 + 150 = 1130 \text{ N/m}^2$$

14. (a)

$$F_r = 6\pi\eta rv$$

$$= 6\pi(0.8)(10^{-3})(10^{-2}) = 1.5 \times 10^{-4} \text{ N}$$ **Ans.**

(b)

$$\text{Hydrostatic force} = \text{Upthrust}$$

$$= \left(\frac{4}{3}\pi r^3\right)\rho g$$

$$= \frac{4}{3}\pi\,(10^{-3})^3 \times 1260 \times 9.8$$

$$= 5.2 \times 10^{-5} \text{ N}$$ **Ans.**

(c) At terminal velocity :

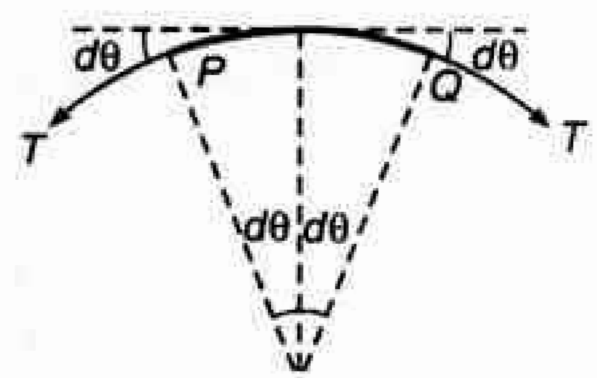
$$W = \text{Upthrust} + \text{viscous force}$$

or $$(50 \times 10^{-3} \times 9.8) = (5.2 \times 10^{-5}) + 6\pi(0.8)(10^{-3})\, v_T$$

Solving we get $$v_T = 32.5 \text{ m/s}$$ **Ans.**

15. The loop will take circular shape after pricking. Radius of which is given by the relation.

$$l = 2\pi R \text{ or } R = \frac{l}{2\pi} = \frac{6.28}{2 \times 3.14} = 1 \text{ cm} = 10^{-2} \text{ m}$$



$2T \sin (d\theta)$ force in inward direction is balaced by surface tension force in outward direction.

$\therefore$ $$2T \sin (d\theta) = (\text{Surface tension}) \times (\text{length of arc})$$

For small angles, $$\sin d\theta \approx d\theta$$

$\therefore$ $$2T\, d\theta = S(2R d\theta)$$ (S = Surface tension)

$\therefore$ $$T = SR = (0.030)(10^{-2}) = 3.0 \times 10^{-4} \text{ N}$$ **Ans.**

16. (a) Time taken to empty the tank (has been derived in theory) is :

$$t = \frac{2A}{a\sqrt{2g}}\sqrt{H}$$

Given, $$\frac{A}{a} = 400$$

Substituting the values we have,

$$t = \frac{2 \times 400}{\sqrt{2 \times 9.8}} \sqrt{1} = 180 \text{ s} = 3 \text{ min}$$ **Ans.**

(b) Rate of flow of water $Q = a\sqrt{2gH}$ = constant

Total volume of water $V = AH$

$\therefore$ Time take to empty the tank with constant rate

$$t = \frac{V}{Q} = \frac{AH}{a\sqrt{2gH}}$$

$$= \frac{400 \times 1}{\sqrt{2 \times 9.8 \times 1}} = 90 \text{ s} = 1.5 \text{ min}$$ **Ans.**

17. (a) $$\Delta P = h(\rho_w - \rho_0)g = (10)(1000 - 500)9.8 = 49000 \text{ N/m}^2$$

Now, $$\Delta P = \frac{1}{2}\rho_w v^2$$

$\therefore$ $$v = \sqrt{\frac{2\Delta P}{\rho_w}} = \sqrt{\frac{2 \times 49000}{1000}} = 9.8 \text{ m/s}$$ **Ans.**

The flow will stop when,

(b) $$(10 + 5)\rho_0 g = 5\rho_0 g + h\rho_w g$$

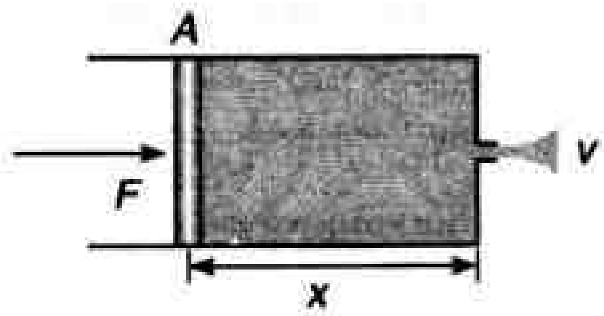
$$10\rho_0 = h\rho_w$$

$\therefore$ $$h = \frac{10 \times 500}{1000} = 5 \text{ m}$$

i.e., flow will stop when the water-oil interface is at a height of 5.0 m. **Ans.**

18. $$\frac{F}{A} = \frac{1}{2}\rho v^2 \quad \text{or} \quad F = \frac{1}{2}\rho A v^2 \quad \text{...(i)}$$

Here, v is the velocity of liquid, with which it comes out of the hole.



Further $$V = Ax \quad \text{...(ii)}$$

$$t = \frac{V}{sv} \quad \text{...(iii)}$$

and $$W = F \cdot x \quad \text{...(iv)}$$

From the above four equations, $$W = \left(\frac{1}{2} A\rho v^2\right)\left(\frac{V}{A}\right)$$

$$= \frac{1}{2}\rho \frac{V^2}{s^2 t^2} \cdot V = \frac{1}{2}\frac{\rho V^3}{s^2 t^2}$$ **Ans.**

19. (a) $$P_0 A = kx$$

$\therefore$ $$x = \frac{P_0 A}{k} = \frac{(P_0)(\pi r^2)}{k}$$

$$= \frac{(1.01 \times 10^5)(\pi)(0.025)^2}{3600} = 0.055 \text{ m}$$

$$= 5.5 \text{ cm}$$ **Ans.**

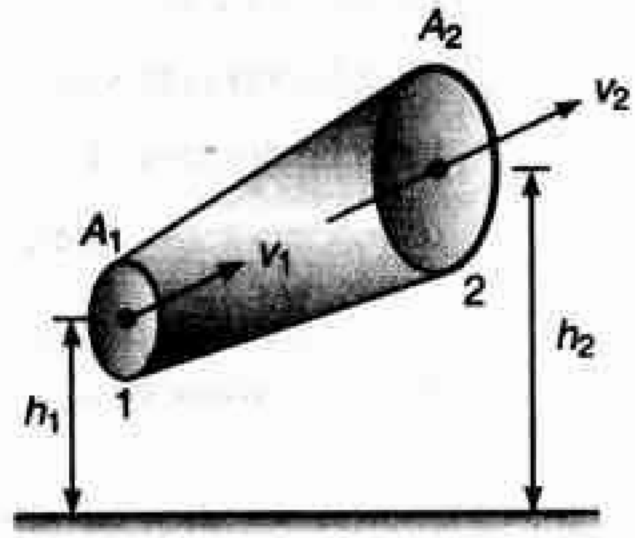
(b) Work done by atmospheric pressure

$$W = \frac{1}{2}kx^2 = \frac{1}{2}(3600)(0.055)^2$$

$$= 5.445 \text{ J}$$ **Ans.**

20. Given : $A_1 = 4 \times 10^{-3}\ \text{m}^2, A_2 = 8 \times 10^{-3}\ \text{m}^2$,

$$h_1 = 2\ \text{m},\ h_2 = 5\ \text{m}$$
$$v_1 = 1\ \text{m/s and}\ \rho = 10^3\ \text{kg/m}^3$$



From continuity equation, we have

$$A_1 v_1 = A_2 v_2 \text{ or } v_2 = \left(\frac{A_1}{A_2}\right) v_1$$

or
$$v_2 = \left(\frac{4 \times 10^{-3}}{8 \times 10^{-3}}\right)(1\ \text{m/s})$$

$$v_2 = \frac{1}{2}\ \text{m/s}$$

Applying Bernoulli's equation at section 1 and 2

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

or
$$P_1 - P_2 = \rho g(h_2 - h_1) + \frac{1}{2}\rho(v_2^2 - v_1^2) \qquad \text{...(i)}$$

(i) Work done per unit volume by the pressure as the fluid flows from P to Q.

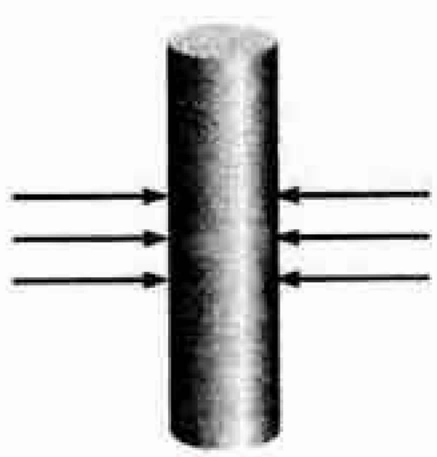
$$W_1 = P_1 - P_2$$
$$= \rho g(h_2 - h_1) + \frac{1}{2}\rho(v_2^2 - v_1^2) \qquad \text{[From Eq. (i)]}$$
$$= \left\{(10^3)(9.8)(5-2) + \frac{1}{2}(10)^3\left(\frac{1}{4} - 1\right)\right\}\ \text{J/m}^3$$
$$= [29400 - 375]\ \text{J/m}^3$$
$$= 29025\ \text{J/m}^3$$

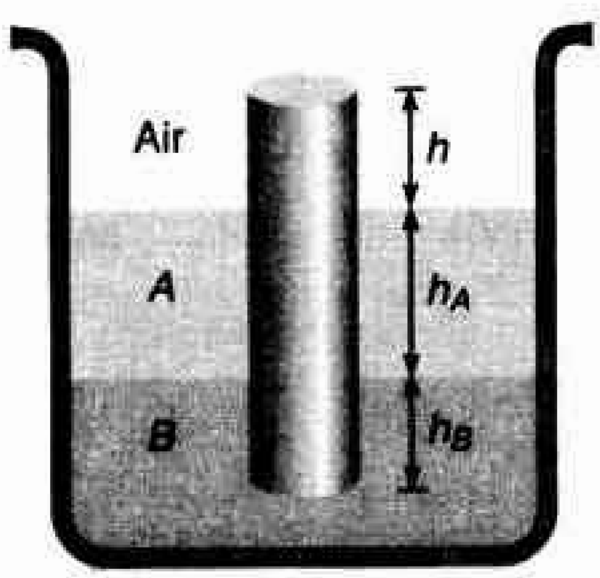
(ii) Work done per unit volume by the gravity as the fluid flows from P to Q.

$$W_2 = \rho g(h_2 - h_1) = \{(10^3)(9.8)(5-2)\}\ \text{J/m}^3$$

or
$$W_2 = 29400\ \text{J/m}^3$$

21. (a) Liquid A is applying the hydrostatic force on cylinder from all the sides. So, net force is zero.





(b) In equilibrium :

Weight of cylinder = Net upthrust on the cylinder

Let s be the area of cross-sectionof the cylinder, then

weight = $(s)(h + h_A + h_B)\rho_{\text{cylinder}}g$ and upthrust on the cylinder

= upthrust due to liquid A + upthrust due to liquid $B = sh_A\rho_A g + sh_B\rho_B g$

Equating these two

$$s(h + h_A + h_B)\rho_{\text{cylinder}}g = sh_A\rho_A g + sh_B\rho_B g$$

or
$$(h + h_A + h_B)\rho_{\text{cylinder}} = h_A\rho_A + h_B\rho_B$$

Substituting

$$h_A = 1.2 \text{ cm}, h_B = 0.8 \text{ cm} \quad \text{and} \quad \rho_A = 0.7 \text{ g/cm}^3$$

$$\rho_B = 1.2 \text{ g/cm}^3 \quad \text{and} \quad \rho_{\text{cylinder}} = 0.8 \text{ g/cm}^3$$

In the above equation, we get

$$h = 0.25 \text{ cm}$$

(c) Net upward force = extra upthrust

$$= sh\rho_B g$$

$\therefore$
$$\text{Net acceleration } a = \frac{\text{force}}{\text{mass of cylinder}}$$

or
$$a = \frac{sh\rho_B g}{s(h + h_A + h_B)\rho_{\text{cylinder}}}$$

or
$$a = \frac{h\rho_B g}{(h + h_A + h_B)\rho_{\text{cylinder}}}$$

Substituting the values of h, h_A h_B, ρ_B and ρ_{cylinder}, we get

$$a = \frac{g}{6} \qquad \text{(upwards)}$$

Experimental Skills

1. An ideal simple pendulum consists of a heavy point mass suspended by a weightless and inextensible string.
2. Yes, gravity is the force with which a body is attracted by earth and other term is simply the acceleration produced by this force in the body.
3. g is the acceleration due to gravity (due to attraction of the Earth). Its value will be different for different planets. On the other hand, G is a universal gravitational constant.
4. It is the distance between the point of suspension and the centre of gravity of the metallic bob.
5. A pendulum which has a time period of two seconds is called a second's pendulum.
6. About 100 cm.
7. Yes, it changes. It increases.
8. Because the value of g on the surface of moon decreases. Value of g on moon is one-sixth the value of g on the earth.
9. The value of g is maximum at the poles and it goes on decreasing as we go towards the equator. It is minimum at the equator.
10. g is a vector quantity.
11. Because $T = 2\pi\sqrt{L/g}$ and this relation is based on the assumption that $\sin\theta \cong \theta$ which is true only for small amplitude, such that value of θ lies between 0° to about 8°.
12. No, the time period does not depend on any of the given three properties of the bob.
13. The time period increases because the value of g decreases due to upthrust created by water.
14. The pendulum will continue oscillating forever without any decrease in amplitude if the vacuum is perfect.
15. On account of rotational motion of the bob, there will be a twist in the thread and the twist will affect the time period.
16. (i) parabolic and (ii) straight line.
17. The length of the pendulum used in clocks increases in summer and hence T increases whereas in winter, the length of the pendulum decreases, so T decreases. T increases means clock goes slow and vice versa.
18. Invar is an alloy which has a very small coefficient of linear thermal expansion. Hence the time period does not change appreciably with the change of temperature.
19. During the draining of the sand the period first increases due to change in effective length, then decreases and finally attains a value that it had when the sphere was full of sand.
20. $g_{\text{moon}} = \dfrac{g_{\text{earth}}}{6} = \dfrac{g}{6}$

$$T = \text{constant}$$

$\therefore$ l should be made $\dfrac{l}{6}$ at moon (because $T = 2\pi\sqrt{\dfrac{l}{g}} = 2\pi\sqrt{\dfrac{l/6}{g/6}}$)

21. $T = 2\pi\sqrt{\dfrac{l}{g}}$

$$T' = T = 2\pi\sqrt{\frac{l'}{g/6}} = 2\pi\sqrt{\frac{l}{g}}$$

$\therefore$ $6l' = l$

$\therefore$ $l' = \dfrac{l}{6} = \dfrac{1}{6}\,m$ **Ans.**

22. $T = 2\pi\sqrt{\frac{l_{\text{eff}}}{g}}$

Period will remain the same as long as effective length is constant.

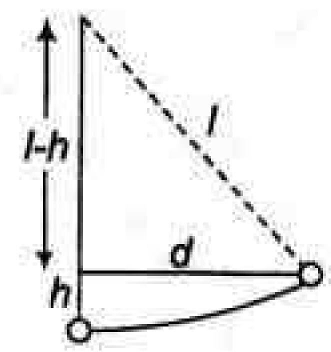
23. $T = 2\pi\sqrt{\frac{l}{g}}$

$$2T = 2\pi\sqrt{\frac{l'}{g}}$$

$\therefore$ $$l' = 4l$$ **Ans.**

24. $l^2 = (l-h)^2 + d^2$

or $$l^2 = l^2 + h^2 - 2lh + d^2$$



Since $d >> h$

$\therefore$ $$2lh = d^2$$

$\therefore$ $$h = \frac{d^2}{2l}$$

Potential energy of bob is

$$U = mgh = \frac{mgd^2}{2l}$$ **Ans.**

25. Those bodies which completely recover after the removal of deforming force are known as elastic bodies and those which do not show any tendency to recover after theremoval of deforming forces are called perfectly plastic bodies. In reality, there is no perfectly elastic or plastic body in the world.
26. It is the limiting value of the stress upto which the direct proportionality between stress and strain is maintained.
27. It is a point on the stress-strain curve at which the wire begins to flow and thins out uniformly even without any increase in the load.
28. After yield point in stress-strain curve, when the wire goes on increasing in length without increase in load, a stage comes at which the wire ultimately breaks. The point at which the wire ultimately breaks is called the breaking point.
29. The minimum value of the load with which the wire breaks is called the breaking load or breaking weight. The breaking load per unit area is called the breaking stress.
30. Yes, the value of Y decreases with the rise of temperature.
31. Long wires are used for getting large extension for a given load. When extension produced is large, then the measurements will also be more accurate.
32. One should wait for some time so that the wire attains the full extension and the wire also attains the room temperature.

34. $\Delta l \propto \frac{1}{Y}$

36. $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\text{Load / Area}}{\text{Elongation / Original length}}$

$\therefore$ $$Y \propto \frac{\text{Load}}{\text{Elongation}} \text{ (= Slope)}$$

$$(\text{Slope})_A > (\text{Slope})_B$$

$\therefore$ $$Y_A > Y_B$$

37. $Y_1 = Y_2$ (same material)

$\therefore$ $$\frac{W_1/A_1}{l_1/L_1} = \frac{W_2/A_2}{l_2/L_2}$$

Also, $L_1 = L_2$

$$\therefore \quad \frac{\left(\dfrac{W_1}{l_1}\right)}{\left(\dfrac{W_2}{l_2}\right)} = \frac{A_1}{A_2}$$

$$\therefore \quad \frac{(\text{Slope})_1}{(\text{Slope})_2} = \frac{A_1}{A_2}$$

Since $(\text{Slope})_1 > (\text{Slope})_2$

$$\therefore \quad \frac{A_1}{A_2} > 1$$

$$\therefore \quad A_1 > A_2$$

39. $Y = \dfrac{F/A}{\Delta l/L}$

$$\therefore \quad Y = \frac{1000/10^{-6}}{\left(\dfrac{0.1}{100}\right)}$$

$$\therefore \quad Y = \frac{10^9 \times 100}{0.1}$$

$$\therefore \quad Y = 10^{12}\ \text{Nm}^{-2}$$

41. Breaking stress depends upon the area of cross-section and material not on the length.

42. $\Delta l = \dfrac{Fl}{AY}$

or $$\Delta l \propto \frac{l}{A}$$

or $$\Delta l \propto \frac{l}{d^2}$$

43. Young's Modulus is the property of the material and changes only with the change in material but is independent of the dimensions.

45. For wires of different materials

$$Y \propto \frac{1}{\Delta l}$$

46. For a body in equilibrium under the action of several forces acting in one plane, 'the sum of clockwise moments is equal to the sum of anticlockwise moments' or algebraic sum of moments about a fulcrum of all the forces acting in a plane on the body is zero.

47. The bottom of a ship is made heavy so that its centre of gravity remains low. This ensures the stability of its equilibrium.

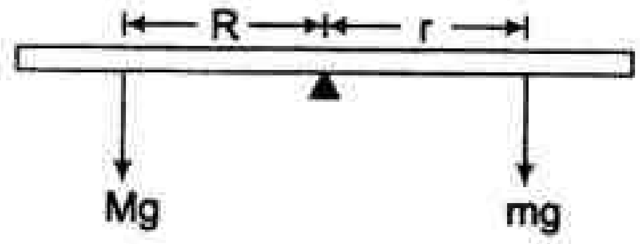
48. When the girl carries a bag in her left hand, her CG shifts towards left. In order to bring it in the middle (for stability of equilibrium), the girl has to lean towards her right.

49. The heavy boxes should be loaded first so that the CG of the loaded cart remains low. This ensures stability of equilibrium.

50. When the passengers stand in the upper deck, the CG of the loaded bus is raised which makes it less stable.

51. Our weight exerts a torque about our feet. This makes difficult for us to rise from the chair. When we bend forward, the CG of our body comes above our feet. The torque due to our weight becomes zero and we can easily rise from the chair.

52. $M = 30\,g$, $R = 20$ cm



$$r = 25 \text{ cm}$$

$$m = \frac{MR}{r}$$

$\therefore$
$$m = \frac{30 \times 20}{25}\,g$$

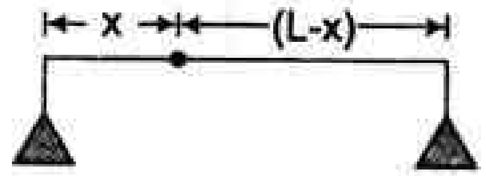
$\therefore$
$$m = 24\,g$$ **Ans.**

53. For rotational equilibrium :

$$20 \times 1.2 - W_2 \times 1.6 = 0$$

or
$$W_2 = \frac{20 \times 1.2}{1.6}\,N = 15\,N$$ **Ans.**

54. When m is placed in left pan, m_1 is placed in right pan. So,



$mgx = m_1g\,(L - x)$ ...(i)

When m is placed in right pan, m_2 is placed in left pan. So,

$m_2gx = mg\,(L - x)$ ...(ii)

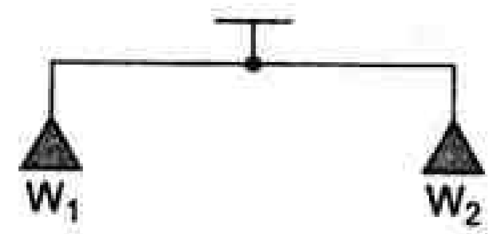
Dividing (i) by (ii), we get

$$m^2 = m_1m_2$$

$\therefore$
$$m = \sqrt{m_1m_2}$$ **Ans.**

55. Since the balance has equal arms and it is a false balance, so the pans must be having different weights.

Let w_1 and w_2 be the weights of left pan and right pan respectively.



Let w be the true weight of object. When placed in left pan it weights x i.e. a total weight of $w + w_1$ on left balances a total weight of $w_2 + x$ on the right pan.

$\therefore\ w + w_1 = w_2 + x$ ...(i)

Similarly it weights y when placed in right pan i.e.

$y + w_1 = w + w_2$ ...(ii)

Solving these two equations we get :

$$w = \frac{x + y}{2}$$ **Ans.**

56. This is done to avoid the formation of air bubbles.

57. The velocity of the fall of the balls is influenced by the proximity of the walls of the vessel. So the balls should be centrally dropped.

58. A net force is experienced by bubbles of air or gas in the upward direction because buoyant force is more than the weight of the air bubbles. As such the air or gas bubbles attain terminal velocity in the upward direction.

59. The weight of the water column rise in the tube is balanced by the surface tension force of water. A capillary tube has a very fine bore and as a result of which the height of water that rises up becomes large. In case of a tube of wide bore the height becomes so small that it is not even perceptible.

60. The formula for surface tension is

$$T = \frac{rh\rho g}{2}$$

where T is surface tension of water, r is the radius of capillary tube, h is the height to which water rises in the capillary, ρ is the density of water and g is the acceleration due to gravity.

61. It is measured up to the lowest point of the meniscus.

65. Vertical height h to which the liquid rises in the capillary is same. So, length of liquid column must increase.

67. Surface tension decreases with temperature. So,

$$T_A < T_B$$

68. $T = \dfrac{rh\rho g}{2\cos\theta}$

$$g_{\text{equator}} < g_{\text{pole}}$$

$\therefore$ $$T_{\text{equator}} < T_{\text{pole}}$$

$\therefore$ $$T_A < T_B$$

82. In vacuum, there will be no viscous drag as well as upthrust.

84. $\dfrac{4}{3}\pi R^3 = 8\left(\dfrac{4}{3}\pi r^3\right)$

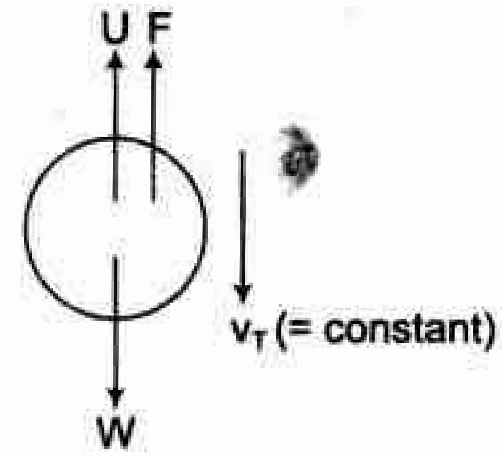
$$R = 2r$$

$$V_T \propto r^2$$

$\therefore$ $$v'_T = 4v_T = 4v$$ **Ans.**

85. For the first half, velocity will be less than the terminal velocity. While in the second half, velocity will be constant and equal to terminal velocity.

86. At $v = v_T$



$$U + F = W$$

$\therefore$ $$F = W - U = W\left(1 - \frac{U}{W}\right)$$

$\therefore$ $$F = W\left(1 - \frac{V\sigma g}{V\rho g}\right)$$

$\therefore$ $$F = mg\left(1 - \frac{\sigma}{\rho}\right)$$

87. $F = 6\pi\eta r v$

88. $\sigma << \rho$

$\therefore$ $$v_T = \frac{2r^2}{9\eta}\rho g$$

89. Rate of production of heat is actually the power dissipation

$$P = Fv$$

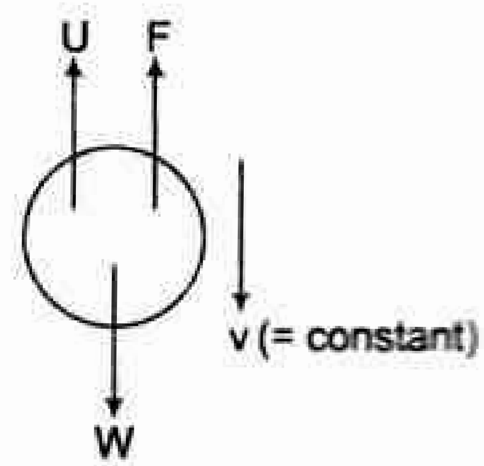
$\therefore$ $$P = 6\pi\eta r v_T^2$$

$\therefore$ $$P = 6\pi\eta r\left[\frac{2}{9}\frac{r^2}{\eta}(\rho - \sigma)g\right]^2$$

$\therefore$ $$P = 6\pi\eta r\left(\frac{4}{81}\right)\frac{r^4}{\eta^2}(\rho - \sigma)^2 g^2$$

$\therefore$ $$P = \frac{8\pi g^2}{27\eta}(\rho - \sigma)^2 r^5$$ **Ans.**

90. $W = \dfrac{4}{3}\pi r^3 \rho_1 g$



$$U = \frac{4}{3}\pi r^3 \rho_2 g$$

$$F = Krv$$

At velocity equal to terminal velocity, the steel ball in equilibrium. So

$$U + F = W$$

$\therefore$ $$F = W - U$$

$$\therefore \qquad Krv = \frac{4}{3}\pi r^3 (\rho_1 - \rho_2)g$$

$$\therefore \qquad v = \frac{4\pi r^2}{3K}(\rho_1 - \rho_2)g \qquad \textbf{Ans.}$$

92. We have,

$$\eta = \frac{2}{9}\frac{r^2(\sigma - \rho)g}{v_T}$$

where σ is the density of liquid and ρ is the density of air.

$$\therefore \qquad \eta = \frac{2}{9}\frac{(0.01)^2 \times 1.75 \times 10^3 \times 9.8}{3.5 \times 10^{-3}}$$

$$\eta = 109 \text{ Nsm}^{-2} \qquad \textbf{Ans.}$$