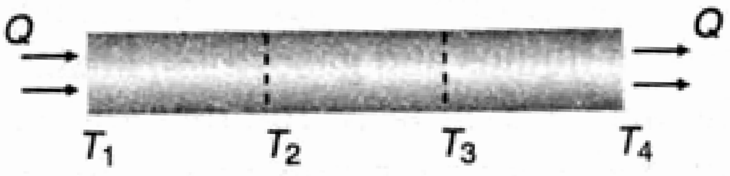
Hence, in the figure :



Insulated rod in steady state

Fig. 19.5

$$T_1 = \text{constant}, \quad T_2 = \text{constant etc.}$$

and $$T_1 > T_2 > T_3 > T_4$$

Now, a natural question arises, why the temperature of whole rod not becomes equal when heat is being continuously supplied? The answer is : there must be a temperature difference in the rod for the heat flow, same as we require a potential difference across a resistance for the current flow through it.

In steady state, the temperature varies linearly with distance along the rod if it is insulated.

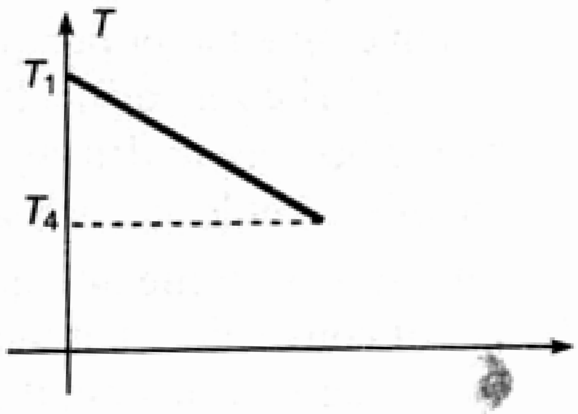


Fig. 19.6

2. Comparing equation number (iii), *i.e.*, heat current

$$H = \frac{dQ}{dt} = \frac{\Delta T}{R} \qquad \left(\text{where } R = \frac{l}{kA}\right)$$

with the equation, of current flow through a resistance,

$$i = \frac{dq}{dt} = \frac{\Delta V}{R} \qquad \left(\text{where } R = \frac{l}{\sigma A}\right)$$

We find the following similarities in heat flow through a rod and current flow through a resistance.

Table 19.1

Heat flow through a conducting rod	Current flow through a resistance
Heat current $H = \frac{dQ}{dt}$ = rate of heat flow	Electric current $i = \frac{dq}{dt}$ = rate of charge flow
$H = \frac{\Delta T}{R} = \frac{\text{TD}}{R}$	$i = \frac{\Delta V}{R} = \frac{\text{PD}}{R}$
$R = \frac{l}{kA}$	$R = \frac{l}{\sigma A}$
k = thermal conductivity	σ = electrical conductivity

From the above table it is evident that flow of heat through rods in series and parallel is analogous to the flow of current through resistances in series and parallel. This analogy is of great importance in solving complicated problems of heat conduction.

Convection

Although conduction does occur in liquids and gases also, heat is transported in these media mostly by convection. In this process, the actual motion of the material is responsible for the heat transfer. Familiar examples include hot-air and hot-water home heating systems, the cooling system of an automobile engine and the flow of blood in the body.

You probably have warmed your hands by holding them over an open flame. In this situation, the air directly above the flame is heated and expands. As a result, the density of this air decreases and then air rises. When the movement results from differences in density, as with air around fire, it is referred to as **natural convection.** Air flow at a beach is an example of natural convection. When the heated substance is forced to move by a fan or pump, the process in called **forced convection.** If it were not for convection currents, it would be very difficult to boil water. As water is heated in a kettle, the heated water expands and rises to the top because its density is lowered. At the same time, the denser, cool water at the surface sinks to the bottom of the kettle and is heated. Heating a room by a radiator is an example of forced convection.

It is possible to write an equation for the thermal energy transported by convection and define a coefficient of convection, but the analysis of practical problems is very difficult and will not be treated here. To some approximation, the heat transferred from a body to its surroundings is proportional to the area of the body and to the difference in temperature between the body and the surrounding fluid.

Radiation

The third means of energy transfer is radiation which does not require a medium. The best known example of this process is the radiation from sun. All objects radiate energy continuously in the form of electromagnetic waves. The rate at which an object radiates energy is proportional to the fourth power of its absolute temperature. This is known as the **Stefan's law** and is expressed in equation form as

$$P = \sigma A e T^4$$

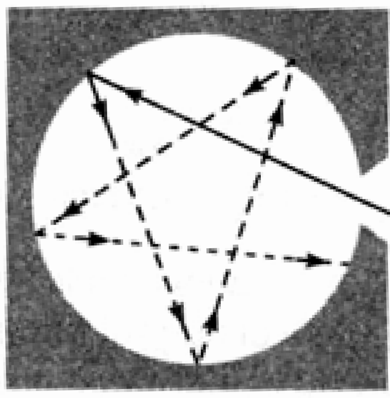
Here P is the power in watts (J/s) radiated by the object, A is the surface area in m^2, e is a fraction between 0 and 1 called the **emissivity** of the object and σ is a universal constant called **Stefan's constant**, which has the value

$$\sigma = 5.67 \times 10^{-8}\ W/m^2\text{-}K^4$$

Now, let us define few terms before studying the other topics.

(a) Perfectly black body

A body that absorbs all the radiation incident upon it and has an emissivity equal to 1 is called a perfectly black body. A black body is also an ideal radiator. It implies that if a black body and an identical another body are kept at the same temperature, then the black body will radiate maximum power as is obvious from equation $P = eA\sigma T^4$ also. Because $e = 1$ for a perfectly black body while for any other body $e < 1$.



Cavity approximating an ideal black body. Radiation entering the cavity has little chance of leaving before it is completely absorbed.

Fig. 19.7

Materials like black velvet or lamp black come close to being ideal black bodies, but the best practical realization of an ideal black body is a small hole leading into a cavity, as this absorbs 98% of the radiation incident on them.

(b) Absorptive power 'a'

"It is defined as the ratio of the radiant energy absorbed by it in a given time to the total radiant energy incident on it in the same interval of time."

$$a = \frac{\text{energy absorbed}}{\text{energy incident}}$$

As a perfectly black body absorbs all radiations incident on it, the absorptive power of a perfectly black body is maximum and unity.

(c) Spectral absorptive power 'a_λ'

The absorptive power 'a' refers to radiations of all wavelengths (or the total energy) while the spectral absorptive power is the ratio of radiant energy absorbed by a surface to the radiant energy incident on it for a particular wavelength λ. It may have different values for different wavelengths for a given surface. Let us take an example, suppose $a = 0.6$, $a_\lambda = 0.4$ for 1000 Å and $a_\lambda = 0.7$ for 2000 Å for a given surface. Then it means that this surface will absorbs only 60% of the total radiant energy incident on it. Similarly it absorbs 40% of the energy incident on it corresponding to 1000 Å and 70% corresponding to 2000 Å. The spectral absorptive power a_λ is related to absorptive power a through the relation

$$a = \int_0^\infty a_\lambda \, d\lambda$$

(d) Emissive power 'e'

(Don't confuse it with the emissivity e which is different from it, although both have the same symbols e).

"For a given surface it is defined as the radiant energy emitted per second per unit area of the surface." It has the units of W/m^2 or $J/s\text{-}m^2$. For a black body $e = \sigma T^4$.

(e) Spectral emissive power 'e_λ'

"It is emissive power for a particular wavelength λ." Thus,

$$e = \int_0^\infty e_\lambda \, d\lambda$$

Kirchhoff's law : "According to this law the ratio of emissive power to absorptive power is same for all surfaces at the same temperature."

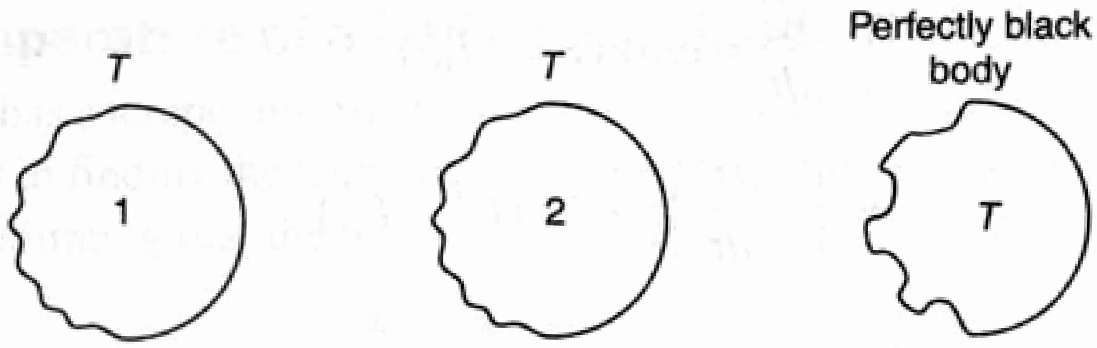


Fig. 19.8

Hence, $$\frac{e_1}{a_1} = \frac{e_2}{a_2} = \left(\frac{e}{a}\right)_{\text{Perfectly black body}}$$

but $$(a)_{\text{black body}} = 1$$

and $$(e)_{\text{black body}} = E \quad \text{(say)}$$

Then, $$\left(\frac{e}{a}\right)_{\text{for any surface}} = \text{constant} = E$$

Similarly, for a particular wavelength λ,

$$\left(\frac{e_\lambda}{a_\lambda}\right)_{\text{for any body}} = E_\lambda$$

Here, E = emissive power of black body at temperature T

$= \sigma T^4$

From the above expression, we can see that

$$e_\lambda \propto a_\lambda$$

i.e., **good absorbers for a particular wavelength are also good emitters of the same wavelength.**

Cooling by radiation

Consider a hot body at temperature T placed in an environment at a lower temperature T_0. The body emits more radiation than it absorbs and cools down while the surroundings absorb radiation from the body and warm up. The body is losing energy by emitting radiations at a rate.

$$P_1 = eA\sigma T^4$$

and is receiving energy by absorbing radiations at a rate

$$P_2 = aA\sigma T_0^4$$

Here, 'a' is a pure number between 0 and 1 indicating the relative ability of the surface to absorb radiation from its surroundings. Note that this 'a' is different from the absorptive power 'a'. In thermal equilibrium, both the body and the surrounding have the same temperature (say T_c) and,

$$P_1 = P_2$$

or $$eA\sigma T_c^4 = aA\sigma T_c^4$$

or $$e = a$$

Thus, when $T > T_0$, the net rate of heat transfer from the body to the surroundings is,

$$\frac{dQ}{dt} = eA\sigma\,(T^4 - T_0^4)$$

or $$mc\left(-\frac{dT}{dt}\right) = eA\sigma\,(T^4 - T_0^4)$$

⇒ Rate of cooling

$$\left(-\frac{dT}{dt}\right) = \frac{eA\sigma}{mc}(T^4 - T_0^4)$$

or $$-\frac{dT}{dt} \propto (T^4 - T_0^4)$$

Newton's law of cooling

According to this law, if the temperature T of the body is not very different from that of the surroundings T_0, then rate of cooling $-\frac{dT}{dt}$ is proportional to the temperature difference between them.

To prove it let us assume that

$$T = T_0 + \Delta T$$

So that

$$T^4 = (T_0 + \Delta T)^4 = T_0^4 \left(1 + \frac{\Delta T}{T_0}\right)^4$$

$$\approx T_0^4 \left(1 + \frac{4\Delta T}{T_0}\right) \qquad \text{(from binomial expansion)}$$

∴

$$(T^4 - T_0^4) = 4T_0^3\,(\Delta T)$$

or

$$(T^4 - T_0^4) \propto \Delta T \qquad (\text{as } T_0 = \text{constant})$$

Now, we have already shown that rate of cooling

$$\left(-\frac{dT}{dt}\right) \propto (T^4 - T_0^4)$$

and here we have shown that

$$(T^4 - T_0^4) \propto \Delta T,$$

if the temperature difference is small.

Thus, rate of cooling

$$-\frac{dT}{dt} \propto \Delta T \quad \text{or} \quad -\frac{d\theta}{dt} \propto \Delta\theta$$

as

$$dT = d\theta \quad \text{or} \quad \Delta T = \Delta\theta$$

Variation of temperature of a body according to Newton's law

Suppose a body has a temperature θ_i at time $t = 0$. It is placed in an atmosphere whose temperature is θ_0. We are interested in finding the temperature of the body at time t, assuming Newton's law of cooling to hold good or by assuming that the temperature difference is small. As per this law,

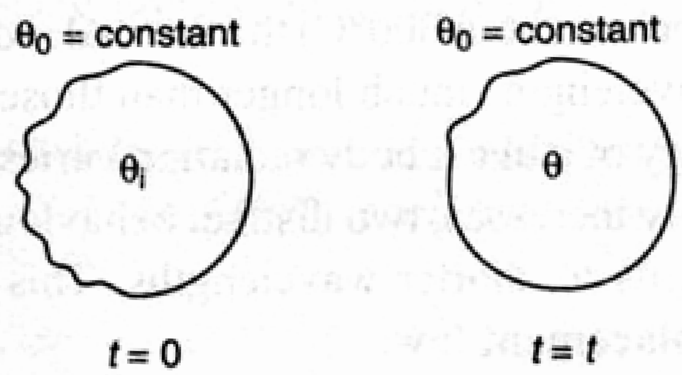


Fig. 19.9

rate of cooling $\propto$ temperature difference

or $$\left(-\frac{d\theta}{dt}\right)=\left(\frac{eA\sigma}{mc}\right)(4\theta_0^3)(\theta-\theta_0)$$

or $$\left(-\frac{d\theta}{dt}\right)=\alpha(\theta-\theta_0).$$

Here $$\alpha=\left(\frac{4eA\sigma\theta_0^3}{mc}\right) \text{ is a constant}$$

$\therefore$ $$\int_{\theta_i}^{\theta}\frac{d\theta}{\theta-\theta_0}=-\alpha\int_0^t dt$$

$\therefore$ $$\theta=\theta_0+(\theta_i-\theta_0)e^{-\alpha t}$$

From this expression we see that $\theta=\theta_i$ at $t=0$ and $\theta=\theta_0$ at $t=\infty$, *i.e.*, temperature of the body varies exponentially with time from θ_i to θ_0 $(<\theta_i)$. The temperature versus time graph is as shown in figure.

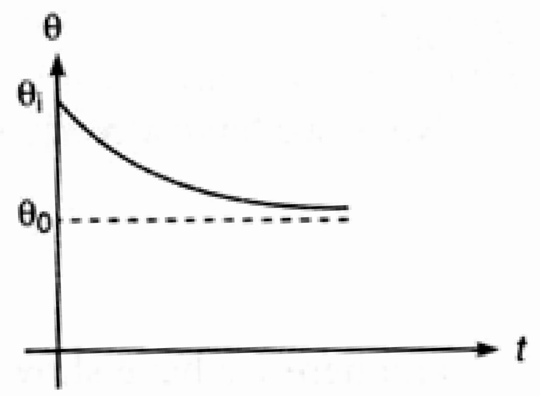


Fig. 19.10

Note *If the body cools by radiation from θ_1 to θ_2 in time t, then taking the approximation*

$$\left(-\frac{d\theta}{dt}\right)=\frac{\theta_1-\theta_2}{t} \quad \text{and} \quad \theta=\theta_{av}=\left(\frac{\theta_1+\theta_2}{2}\right)$$

The equation $\left(-\frac{d\theta}{dt}\right)=\alpha(\theta-\theta_0)$ *becomes*

$$\frac{\theta_1-\theta_2}{t}=\alpha\left(\frac{\theta_1+\theta_2}{2}-\theta_0\right)$$

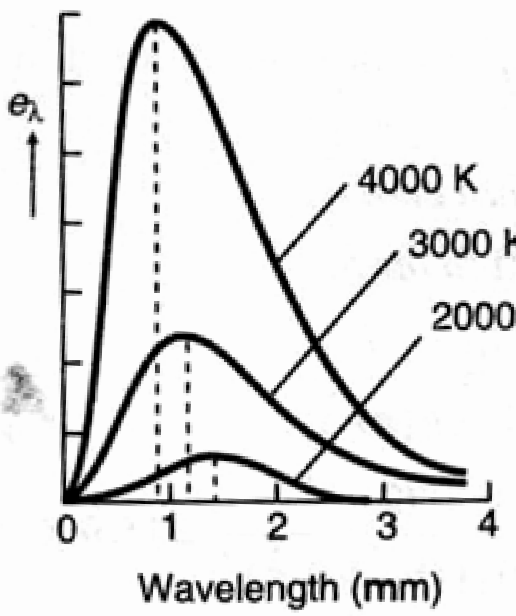
This form of the law helps in solving numerical problems related to **Newton's law of cooling.**

Wien's displacement law

At ordinary temperatures (below about 600°C) the thermal radiation emitted by a body is not visible, most of it is concentrated in wavelengths much longer than those of visible light.

Figure shows how the energy of a black body radiation varies with temperature and wavelength. As the temperature of the black body increases, two distinct behaviours are observed. The first effect is that the peak of the distribution shifts to shorter wavelengths. This shift is found to obey the following relationship called **Wien's displacement law.**

$$\lambda_{max}T=b$$



Power of black body radiation *versus* wavelength at three temperatures. Note that the amount of radiation emitted (the area under a curve) increase with increasing temperature.

Fig. 19.11

Here, b is a constant called Wien's constant. The value of this constant in SI unit is 2.898×10^{-3} m-K. Thus,

$$\lambda_{\max} \propto \frac{1}{T}$$

Here, $\lambda_{\max}$ is the wavelength corresponding to the maximum spectral emissive power e_λ.

The second effect is that the total amount of energy the black body emits per unit area per unit time $(= \sigma T^4)$ increases with fourth power of absolute temperature T. This is also known as the emissive power. We know

$$e = \int_0^\infty e_\lambda \, d\lambda = \text{Area under } e_\lambda\text{-}\lambda \text{ graph} = \sigma T^4$$

or

$$\text{Area} \propto T^4$$

$$A_2 = (2)^4 A_1 = 16 A_1$$

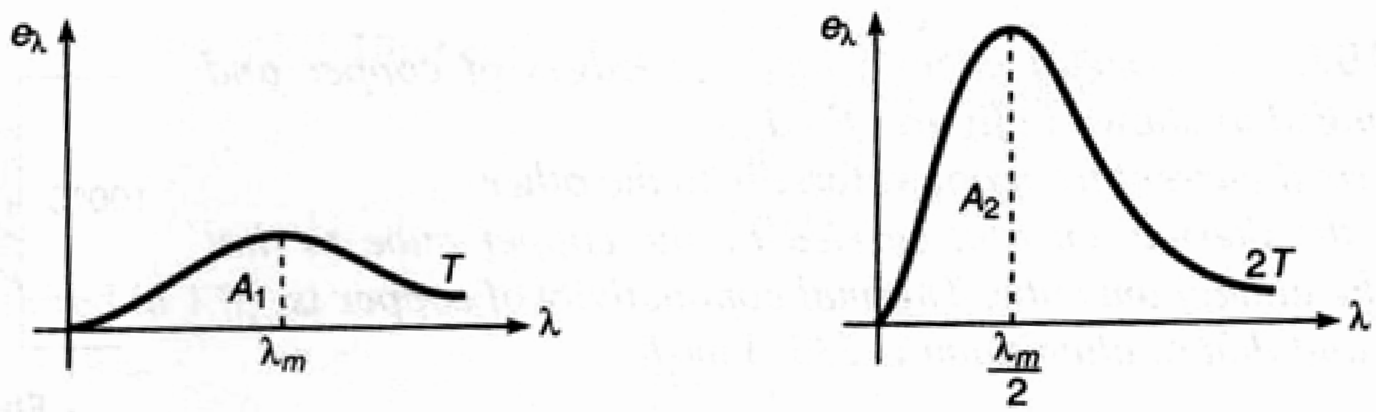


Fig. 19.12

Thus, if the temperature of the black body is made two fold, $\lambda_{\max}$ remains half while the area becomes 16 times.

Sample Example 19.5 *A copper rod 2 m long has a circular cross section of radius 1 cm. One end is kept at 100°C and the other at 0°C, and the surface is insulated so that negligible heat is lost through the surface. Find :*

(a) the thermal resistance of the bar

(b) the thermal current H

(c) the temperature gradient $\frac{dT}{dx}$ *and*

(d) the temperature 25 cm from the hot end.

Thermal conductivity of copper is 401 W/m-K.

Solution (a) Thermal resistance $R = \frac{l}{kA} = \frac{l}{k(\pi r^2)}$

or $$R = \frac{(2)}{(401)(\pi)(10^{-2})^2} = 15.9 \text{ K/W}$$ **Ans.**

(b) $$\text{Thermal current, } H = \frac{\Delta T}{R} = \frac{\Delta\theta}{R} = \frac{100}{15.9}$$

or $$H = 6.3 \text{ W}$$ **Ans.**

(c) Temperature gradient

$$= \frac{0-100}{2} = -50 \text{ K/m} = -50°\text{C/m}$$ **Ans.**

(d) Let θ_c° be the temperature at 25 cm from the hot end, then

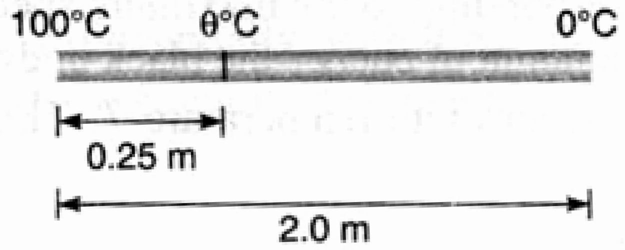


Fig. 19.13

$$(\theta - 100) = (\text{temperature gradient}) \times (\text{distance})$$

or $$\theta - 100 = (-50)(0.25)$$

or $$\theta = 87.5°\text{C}$$ **Ans.**

Sample Example 19.6 *Two metal cubes with 3 cm-edges of copper and aluminium are arranged as shown in figure. Find :*

(a) the total thermal current from one reservoir to the other

(b) the ratio of the thermal current carried by the copper cube to that carried by the aluminium cube. Thermal conductivity of copper is 401 W/m-K and that of aluminium is 237 W/m-K.

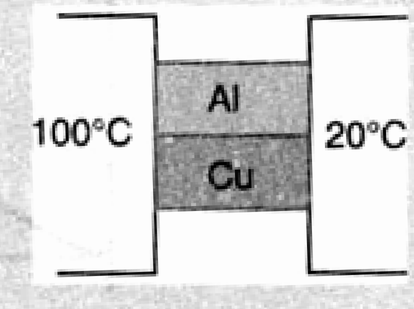


Fig. 19.14

Solution (a) Thermal resistance of aluminium cube $R_1 = \frac{l}{kA}$

or $$R_1 = \frac{(3.0\times10^{-2})}{(237)(3.0\times10^{-2})^2} = 0.14 \text{ K/W}$$

and thermal resistance of copper cube $R_2 = \frac{l}{kA}$

or $$R_2 = \frac{(3.0\times10^{-2})}{(401)(3.0\times10^{-2})^2} = 0.08 \text{ K/W}$$

As these two resistances are in parallel, their equivalent resistance will be

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

$$= \frac{(0.14)(0.08)}{(0.14) + (0.08)}$$

$$= 0.05 \text{ K/W}$$

$\therefore$ $$\text{Thermal current } H = \frac{\text{Temperature difference}}{\text{Thermal resistance}}$$

$$= \frac{(100-20)}{0.05} = 1.6 \times 10^3 \text{ W}$$

(b) In parallel thermal current distributes in the inverse ratio of resistance. Hence,

$$\frac{H_{\text{Cu}}}{H_{\text{Al}}} = \frac{R_{\text{Al}}}{R_{\text{Cu}}} = \frac{R_1}{R_2} = \frac{0.14}{0.08} = 1.75$$

Sample Example 19.7 *One end of a copper rod of length 1 m and area of cross section 4.0×10^{-4} m^2 is maintained at 100°C. At the other end of the rod ice is kept at 0°C. Neglecting the loss of heat from the surroundings find the mass of ice melted in 1 h. Given $k_{Cu} = 401$ W/m-K and $L_f = 3.35 \times 10^5$ J/kg.*

Solution Thermal resistance of the rod,

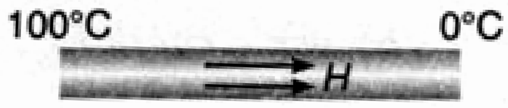


Fig. 19.15

$$R = \frac{l}{kA} = \frac{1.0}{(401)(4 \times 10^{-4})} = 6.23 \text{ K/W}$$

$\therefore$ $$\text{Heat current } H = \frac{\text{Temperature difference}}{\text{Thermal resistance}}$$

$$= \frac{(100-0)}{6.23} = 16 \text{ W}$$

Heat transferred in 1 h,

$$Q = Ht \qquad \left(H = \frac{Q}{t}\right)$$

$$= (16)(3600) = 57600 \text{ J}$$

Now, let m mass of ice melts in 1 h, then

$$m = \frac{Q}{L} \qquad (Q = mL)$$

$$= \frac{57600}{3.35 \times 10^5} = 0.172 \text{ kg} \quad \text{or} \quad 172 \text{ g}$$

Sample Example 19.8 *A body cools in 10 minutes from 60°C to 40°C. What will be its temperature after next 10 minutes? The temperature of the surroundings is 10°C.*

Solution According to Newton's law of cooling

$$\left(\frac{\theta_1-\theta_2}{t}\right)=\alpha\left[\left(\frac{\theta_1+\theta_2}{2}\right)-\theta_0\right]$$

For the given conditions,

$$\frac{60-40}{10}=\alpha\left[\frac{60+40}{2}-10\right] \quad \text{...(i)}$$

Let θ be the temperature after next 10 minutes. Then,

$$\frac{40-\theta}{10}=\alpha\left[\frac{40+\theta}{2}-10\right] \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$\theta=28°\text{C}$$

Sample Example 19.9 *Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are same. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiancy from B is shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from A by 1.0 μm. If the temperature of A is 5802 K, calculate*

(a) the temperature of B, *(b) wavelength λ_B.*

Solution (a)

$$P_A=P_B$$

∴

$$e_A\sigma A_A T_A^4=e_B\sigma A_B T_B^4$$

∴

$$T_B=\left(\frac{e_A}{e_B}\right)^{1/4}T_A \quad (\text{as } A_A=A_B)$$

Substituting the values

$$T_B=\left(\frac{0.01}{0.81}\right)^{1/4}(5802)=1934\text{ K}$$

(b) According to Wein's displacement law,

$$\lambda_A T_A=\lambda_B T_B$$

∴

$$\lambda_B=\left(\frac{5802}{1934}\right)\lambda_A$$

or

$$\lambda_B=3\lambda_A$$

Also,

$$\lambda_B-\lambda_A=1\,\mu\text{m}$$

or

$$\lambda_B-\left(\frac{1}{3}\right)\lambda_B=1\,\mu\text{m}$$

or

$$\lambda_B=1.5\,\mu\text{m}$$

Introductory Exercise 19.2

1. Suppose a liquid in a container is heated at the top rather than at the bottom. What is the main process by which the rest of the liquid becomes hot?
2. The inner and outer surfaces of a hollow spherical shell of inner radius 'a' and outer radius 'b' are maintained at temperatures T_1 and T_2 ($< T_1$). The thermal conductivity of material of the shell is k. Find the rate of heat flow from inner to outer surface.
3. Show that the SI units of thermal conductivity are W/m-K.
4. A carpenter builds an outer house wall with a layer of wood 2.0 cm thick on the outside and a layer of an insulation 3.5 cm thick as the inside wall surface. The wood has $k = 0.08$ W/m-K and the insulation has $k = 0.01$ W/m-K. The interior surface temperature is 19° C and the exterior surface temperature is − 10° C.
   (a) What is the temperature at the plane where the wood meets the insulation?
   (b) What is the rate of heat flow per square metre through this wall?
5. A pot with a steel bottom 1.2 cm thick rests on a hot stove. The area of the bottom of the pot is 0.150 m^2. The water inside the pot is at 100°C and 0.440 kg are evaporated every 5.0 minute. Find the temperature of the lower surface of the pot, which is in contact with the stove. Take $L_v = 2.256 \times 10^6$ J/kg and $k_{steel} = 50.2$ W/m-K.
6. A layer of ice of thickness y is on the surface of a lake. The air is at a constant temperature $-\theta$° C and the ice water interface is at 0°C. Show that the rate at which the thickness increases is given by,
$$\frac{dy}{dt} = \frac{k\theta}{L\rho y}$$
   where k is the thermal conductivity of the ice, L the latent heat of fusion and ρ is the density of the ice.
7. The emissivity of tungsten is 0.4. A tungsten sphere with a radius of 4.0 cm is suspended within a large evacuated enclosure whose walls are at 300 K. What power input is required to maintain the sphere at a temperature of 3000 K if heat conduction along supports is neglected? Take $\sigma = 5.67 \times 10^{-8}$ W/m^2-K^4.
8. Find SI units of thermal resistance.

Extra Points

- The general expression for the heat involved in phase change is
$$Q = \pm mL$$
  The plus sign (heat entering) is used when the material melts, the minus sign (heat leaving) is used when it freezes.
- For JEE remember the heat equations in differential forms as under :
$$\frac{dQ}{dt} = mc\frac{dT}{dt}$$
  and
$$\frac{dQ}{dt} = \pm L\frac{dm}{dt}$$
  Here, $\frac{dQ}{dt}$ is the time rate of heat transfer,
  $\frac{dT}{dt}$ is the rate of change of temperature

and $\frac{dm}{dt}$ is the rate of mass transfer from one phase of matter to the other.

- While solving the problems of heat flow, remember the following equation :

$$\frac{dQ}{dt}=\frac{\text{T.D.}}{R}=mc\cdot\frac{dT}{dt}=L\cdot\frac{dm}{dt}$$

of these we will have to choose the appropriate relation according to the problem.

For instance, one end of a rod of length l, area A and thermal conductivity k is maintained at 100°C and at the other end ice is melting at 0°C. We are interested in finding the mass of ice which transforms into water in unit time. For this, we will take

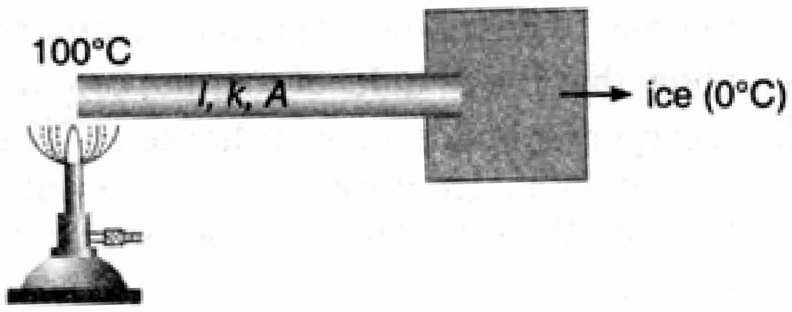


Fig. 19.16

$$\frac{\text{T.D.}}{R}=L_f\frac{dm}{dt}$$

$\therefore$ $$\left(\frac{dm}{dt}\right)=\frac{\text{T.D}}{(L_f)(R)}$$ = rate of ice which transforms into water in unit time.

Here, T. D. = temperature difference

$R=\frac{l}{kA}$ and L_f =latent heat of fusion

- **Growth of ice on ponds :** When temperature of the atmosphere falls below 0°C, the water in the pond starts freezing. Let at time t thickness of ice in the pond is y and atmospheric temperature is $-T$°C. The temperature of water in contact with the lower surface of ice will be 0°C. Using

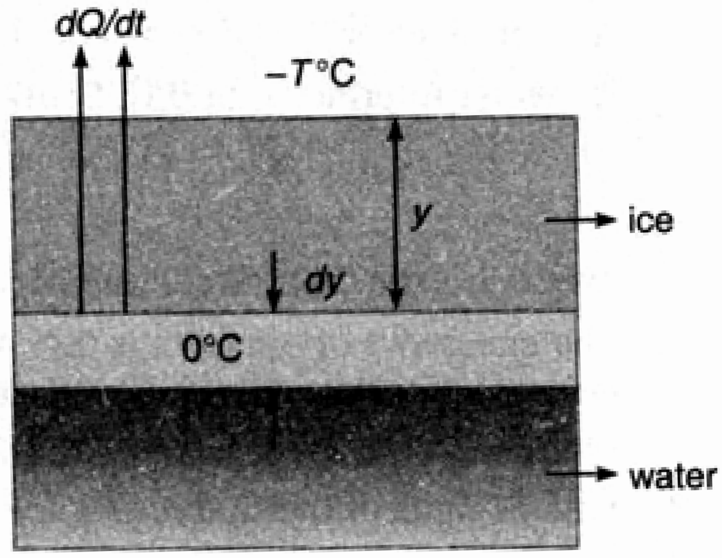


Fig. 19.17

$$\frac{dQ}{dt}=L_f\left(\frac{dm}{dt}\right)$$

or $$\frac{\text{T.D.}}{R}=L_f\frac{d}{dt}\{A\rho y\}\quad (A = \text{area of pond})$$

$\therefore$ $$\frac{[0-(-T)]}{(y/kA)}=L_fA\rho\cdot\frac{dy}{dt}$$ (k = thermal conductivity of ice)

$\therefore$ $$\frac{dy}{dt}=\frac{kT}{\rho L_f}\cdot\frac{1}{y}$$

and hence time taken by ice to grow a thickness y,

$$t=\frac{\rho L_f}{kT}\int_0^y y\,dy$$

or $$t=\frac{1}{2}\frac{\rho L_f}{kT}y^2$$

Time does not depend on the area of pond.

Solved Examples

For JEE Main

Example 1 *5 g of water at 30°C and 5 g of ice at – 20°C are mixed together in a calorimeter. Find the final temperature of mixture. Water equivalent of calorimeter is negligible, specific heat of ice = 0.5 cal/g°C and latent heat of ice = 80 cal/g.*

Solution In this case heat is given by water and taken by ice

Heat available with water to cool from 30° C to 0° C

$$= ms\Delta\theta = 5 \times 1 \times 30 = 150 \text{ cal}$$

Heat required by 5 g ice to increase its temperature up to 0°C

$$ms\Delta\theta = 5 \times 0.5 \times 20 = 50 \text{ cal}$$

Out of 150 cal heat available, 50 cal is used for increasing temperature of ice from – 20° C to 0° C. The remaining heat 100 cal is used for melting the ice.

If mass of ice melted is m g then

$$m \times 80 = 100 \quad \Rightarrow \quad m = 1.25 \text{ g}$$

Thus, 1.25 g ice out of 5 g melts and mixture of ice and water is at 0° C.

Example 2 *A bullet of mass 10 g moving with a speed of 20 m/s hits an ice block of mass 990 g kept on a frictionless floor and gets stuck in it. How much ice will melt if 50% of the lost kinetic energy goes to ice? (Temperature of ice block = 0°C).*

Solution Velocity of bullet + ice block,

$$V = \frac{(10 \text{ g}) \times (20 \text{ m/s})}{1000 \text{ g}} = 0.2 \text{ m/s}$$

$$\text{Loss of KE} = \frac{1}{2} mv^2 - \frac{1}{2}(m + M)V^2$$

$$= \frac{1}{2}[0.01 \times (20)^2 - 1 \times (0.2)^2]$$

$$= \frac{1}{2}[4 - 0.04] = 1.98 \text{ J}$$

$$\therefore \quad \text{Heat received by ice block} = \frac{1.98}{4.2 \times 2} \text{ cal}$$

$$= 0.24 \text{ cal}$$

$$\therefore \quad \text{Mass of ice melted} = \frac{(0.24 \text{ cal})}{(80 \text{ cal/g})}$$

$$= 0.003 \text{ g}$$

Example 3 *The temperature of equal masses of three different liquids A, B and C are 12°C, 19°C and 28°C respectively. The temperature when A and B are mixed is 16°C and when B and C are mixed it is 23°C. What should be the temperature when A and C are mixed ?*

Solution Let m be the mass of each liquid and S_A, S_B, S_C specific heats of liquids A, B and C respectively. When A and B are mixed. The final temperature is 16° C.

$\therefore$ Heat gained by A = heat lost by B

i.e., $$m S_A\ (16-12) = m S_B\ (19-16)$$

i.e., $$S_B = \frac{4}{3} S_A \quad \text{...(i)}$$

When B and C are mixed. Heat gained by B = heat lost by C

i.e., $$m S_B\ (23-19) = m\, S_C\ (28-23)$$

i.e., $$S_C = \frac{4}{5} S_B \quad \text{...(ii)}$$

From Eqs. (i) and (ii)

$$S_C = \frac{4}{5} \times \frac{4}{3} S_A = \frac{16}{15} S_A$$

When A and C are mixed, let the final temperature be θ

$\therefore$ $$m S_A\ (\theta - 12) = m\, S_C\ (28 - \theta)$$

i.e., $$\theta - 12 = \frac{16}{15}(28 - \theta)$$

By solving, we get $$\theta = \frac{628}{31} = 20.26°\text{C}$$

Example 4 *At* 1 *atmospheric pressure,* 1.000 *g of water having a volume of* 1.000 cm^3 *becomes* 1671 cm^3 *of steam when boiled. The heat of vaporization of water at* 1 *atmosphere is* 539 *cal/g. What is the change in internal energy during the process?*

Solution Heat spent during vaporisation

$$Q = mL = 1.000 \times 539 = 539\,\text{cal}$$

Work done $$W = P(V_v - V_l)$$
$$= 1.013 \times 10^5 \times (1671 - 1.000) \times 10^{-6}$$
$$= 169.2\text{ J} = \frac{169.2}{4.18}\text{ cal} = 40.5\text{ cal}$$

$\therefore$ Change in internal energy

$\therefore$ $$U = 539\text{ cal} - 40.5\text{ cal} = 498.5\text{ cal}$$

Example 5 *At* 1 *atmospheric pressure,* 1.000 *g of water having a volume of* 1.000 cm^3 *becomes* 1.091 cm^3 *of ice on freezing. The heat of fusion of water at 1 atmosphere is* 80.0 *cal/g. What is the change in internal energy during the process?*

Solution Heat given out during freezing

$$Q = -mL = -1 \times 80 = -80 \text{ cal}$$

External work done

$$W = P(V_{ice} - V_{water})$$
$$= 1.013 \times 10^5 \times (1.091 - 1.000) \times 10^{-6}$$
$$= 9.22 \times 10^{-3} \text{ J}$$
$$= \frac{9.22 \times 10^{-3}}{4.18} \text{ cal} = 0.0022 \text{ cal}$$

∴ Change in internal energy

$$\Delta U = Q - W = -80 - 0.0022 = -80.0022 \text{ cal}$$ **Ans.**

Example 6 *Two bodies A and B have thermal emissivities of* 0.01 *and* 0.81 *respectively. The outer surface area of the two bodies are the same. The two bodies emit total radiant power at the same rate. The wavelengths* λ_A *and* λ_B *corresponding to maximum spectral radiancy in the radiation from A and B respectively differ by* 1.00 μm. *If the temperature of A is* 5802 K, *Find (a) the temperature of B (b) and* λ_B.

Solution Given $e_A = 0.01, e_B = 0.81$ and $T_A = 5802\text{K}$

From Wien's displacement law

$$\lambda_m T = \text{constant} \quad \therefore \quad \lambda_A T_A = \lambda_B T_B$$

Power radiated $P = e\sigma T^4 A$ as $P_1 = P_2$ and $A_1 = A_2$

we have

$$e_A T_A^4 = e_B T_B^4$$

∴

$$T_B = \left(\frac{e_A}{e_B}\right)^{1/4} T_A = \left(\frac{0.01}{0.81}\right)^{1/4} \times 5802$$
$$= 1934 \text{ K}$$

as

$$T_B < T_A, \lambda_B > \lambda_A$$

∴

$$\lambda_B - \lambda_A = 1 \, \mu\text{m} \quad \text{(given)}$$
$$\lambda_B - \lambda_A = 1 \times 10^{-6} \text{ m} \quad \text{...(i)}$$

and

$$\frac{\lambda_B}{\lambda_A} = \frac{T_A}{T_B} = \frac{5802}{1934} = 3$$

∴

$$\lambda_B = 3\lambda_A \quad \text{...(ii)}$$

From Eqs. (i) and (ii)

$$\lambda_B = 1.5 \times 10^{-6} \text{ m}$$ **Ans.**

Example 7 *Two plates each of area A, thickness* L_1 *and* L_2 *thermal conductivities* K_1 *and* K_2 *respectively are joined to form a single plate of thickness* $(L_1 + L_2)$. *If the temperatures of the free surfaces are* T_1 *and* T_2. *Calculate*

(a) Rate of flow of heat
(b) Temperature of interface and
(c) Euivalent thermal conductivity.

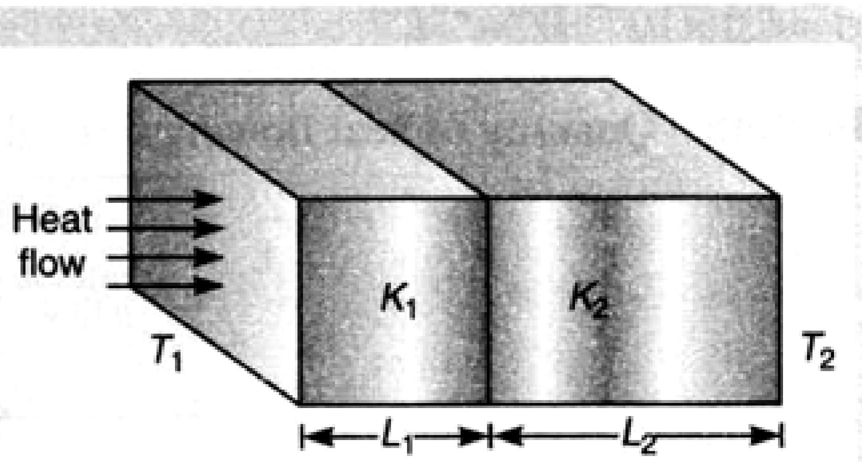


Fig. 19.18

Solution (a) If the thermal resistance of the two plates are R_1 and R_2 respectively then as plates are in series.

$$R_S = R_1 + R_2 = \frac{L_1}{AK_1} + \frac{L_2}{AK_2} \quad \text{as } R = \frac{L}{KA}$$

and so
$$H = \frac{dQ}{dt} = \frac{\Delta\theta}{R} = \frac{(T_1 - T_2)}{(R_1 + R_2)} = \frac{A(T_1 - T_2)}{\left[\frac{L_1}{K_1} + \frac{L_2}{K_2}\right]}$$

(b) If T is the common temperature of interface then as in series rate of flow of heat remains same. *i.e.*, $H = H_1\ (= H_2)$

$$\frac{T_1 - T_2}{R_1 + R_2} = \frac{T_1 - T}{R_1},$$

i.e.,
$$T = \frac{T_1 R_2 + T_2 R_1}{(R_1 + R_2)}$$

or
$$T = \frac{\left[T_1 \frac{L_2}{K_2} + T_2 \frac{L_1}{K_1}\right]}{\left[\frac{L_1}{K_1} + \frac{L_2}{K_2}\right]} \quad \text{as } R = \frac{L}{KA}$$

(c) If K is the equivalent conductivity of composite slab, *i.e.*, slab of thickness $L_1 + L_2$ and cross-sectional area A, then as in series

$$R_S = R_1 + R_2 \quad \text{or} \quad \frac{(L_1 + L_2)}{AK_{eq}} = R_1 + R_2$$

i.e.,
$$K_{eq} = \frac{L_1 + L_2}{A(R_1 + R_2)} = \frac{L_1 + L_2}{\left[\frac{L_1}{K_1} + \frac{L_2}{K_2}\right]} \quad \text{as } R = \frac{L}{KA}$$

Example 8 *One end of a rod of length 20 cm is inserted in a furnace at 800 K. The sides of the rod are covered with an insulating material and the other end emits radiation like a black body. The temperature of this end is 750 K in the steady state. The temperature of the surrounding air is 300 K. Assuming radiation to be the only important mode of energy transfer between the surrounding and the open end of the rod, find the thermal conductivity of the rod. Stefan constant* $\sigma = 6.0 \times 10^{-8}\ W/m^2\text{-}K^4$.

Solution Quantity of heat flowing through the rod per second in steady state:

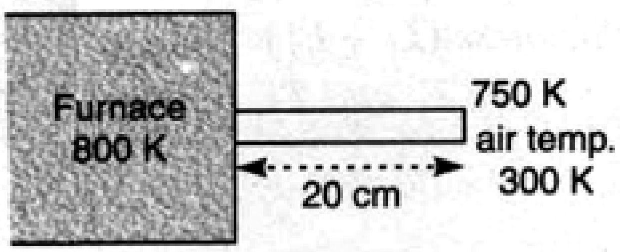


Fig. 19.19

$$\frac{dQ}{dt} = \frac{K.A.d\theta}{x} \quad ...(i)$$

Quantity of heat radiated from the end of the rod per second in steady state:

$$\frac{dQ}{dt} = A\sigma(T^4 - T_0^4) \quad ...(ii)$$

From Eqs. (i) and (ii),

$$\frac{K.d\theta}{x} = \sigma(T^4 - T_0^4)$$

$$\frac{K \times 50}{0.2} = 6.0 \times 10^{-8}[(7.5)^4 - (3)^4] \times 10^8$$

or $$K = 74 \text{ W/mK}$$ **Ans.**

For **JEE Advanced**

Example 1 *Three rods of material x and three rods of material y are connected as shown in figure. All the rods are of identical length and cross-sectional area. If the end A is maintained at 60°C and the junction E at 10°C, calculate temperature of junctions B, C and D. The thermal conductivity of x is 0.92 cal/cm-s°C and that of y is 0.46 cal/cm-s°C.*

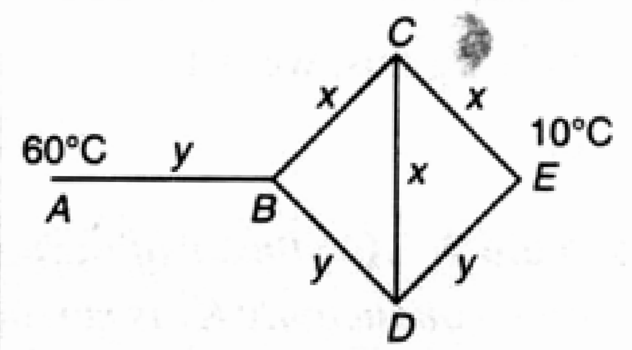


Fig. 19.20

Solution Thermal resistance $R = \frac{l}{kA}$

$\therefore$ $$\frac{R_x}{R_y} = \frac{k_y}{k_x} \quad (\text{as } l_x = l_y \text{ and } A_x = A_y)$$

$$= \frac{0.46}{0.92} = \frac{1}{2}$$

So, if $R_x = R$ then $R_y = 2R$

CEDB forms a balanced Wheatstone bridge, *i.e.*, $T_C = T_D$ and no heat flows through *CD*

$\therefore$ $$\frac{1}{R_{BE}} = \frac{1}{R+R} + \frac{1}{2R+2R}$$

or $$R_{BE} = \frac{4}{3}R$$

The total resistance between *A* and *E* will be,

$$R_{AE} = R_{AB} + R_{BE} = 2R + \frac{4}{3}R = \frac{10}{3}R$$

$\therefore$ Heat current between *A* and *E* is

$$H = \frac{(\Delta T)_{AE}}{R_{AE}} = \frac{(60-10)}{(10/3)R} = \frac{15}{R}$$

Now, if T_B is the temperature at B,

$$H_{AB} = \frac{(\Delta T)_{AB}}{R_{AB}}$$

or
$$\frac{15}{R} = \frac{60 - T_B}{2R}$$

or
$$T_B = 30°\text{C}$$ **Ans.**

Further,

$$H_{AB} = H_{BC} + H_{BD}$$

or
$$\frac{15}{R} = \frac{30 - T_C}{R} + \frac{30 - T_D}{2R} \qquad [T_C = T_D = T \text{ (say)}]$$

or
$$15 = (30 - T) + \frac{(30 - T)}{2}$$

Solving this, we get
$$T = 20°\text{C}$$

or
$$T_C = T_D = 20°\text{C}$$ **Ans.**

Example 2 *A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by cylindrical shell of inner radius R and outer radius 2R made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and system is in steady state. What is the effective thermal conductivity of the system?*

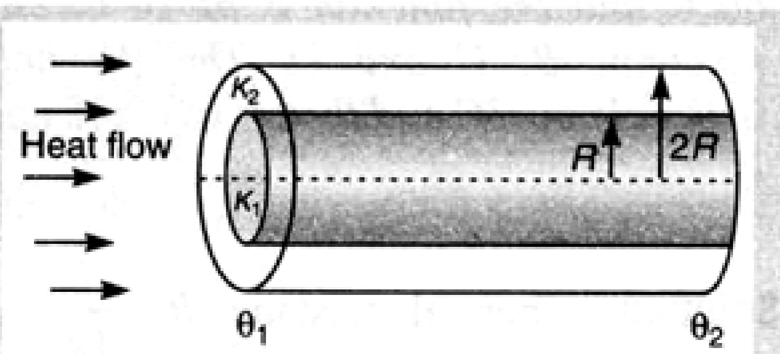


Fig. 19.21

Solution In this situation a rod of length L and area of cross-section πR^2 and another of same length L and area of cross-section $\pi[(2R)^2 - R^2] = 3\pi R^2$ will conduct heat simultaneously so total heat flowing per sec will be

$$\frac{dQ}{dt} = \frac{dQ_1}{dt} + \frac{dQ_2}{dt}$$

$$= \frac{K_1 \pi R^2 (\theta_1 - \theta_2)}{L} + \frac{K_2 3\pi R^2 (\theta_1 - \theta_2)}{L} \qquad \text{...(i)}$$

Now, if the equivalent conductivity is K. Then,

$$\frac{dQ}{dt} = K \frac{4\pi R^2 (\theta_1 - \theta_2)}{L} \qquad [\text{as } A = \pi(2R)^2] \text{ ...(ii)}$$

So, from Eqs. (i) and (ii), we have

$$4K = K_1 + 3K_2 \quad i.e., K = \frac{(K_1 + 3K_2)}{4}$$

Example 3 *A hollow sphere of glass whose external and internal radii are 11 cm and 9 cm respectively is completely filled with ice at 0°C and placed in a bath of boiling water. How long will it take for the ice to melt completely? Given that density of ice* $=0.9\ g/cm^3$*, latent heat of fusion of ice* $=80\ cal/g$ *and thermal conductivity of glass* $=0.002\ cal/cm\text{-}s°C$.

Solution

In steady state, rate of heat flow $H=\dfrac{4\pi k r_1 r_2 \Delta T}{r_2-r_1}$

Substituting the values, $H=\dfrac{(4)(\pi)(0.002)(11)(9)(100-0)}{(11-9)}$

or $\dfrac{dQ}{dt}=124.4\ \text{cal/s}$

This rate should be equal to, $L\dfrac{dm}{dt}$

$\therefore$ $\left(\dfrac{dm}{dt}\right)=\dfrac{dQ/dt}{L}=\dfrac{124.4}{80}=1.555\ \text{g/s}$

Total mass of ice,
$$m=\rho_{ice}\,(4\pi r_1^2)$$
$$=(0.9)(4)(\pi)(9)^2$$
$$=916\ \text{g}$$

$\therefore$ Time taken for the ice to melt completely

$$t=\frac{m}{(dm/dt)}=\frac{916}{1.555}=589\ \text{s}$$ **Ans.**

Example 4 *A point source of heat of power P is placed at the centre of a spherical shell of mean radius R. The material of the shell has thermal conductivity k. Calculate the thickness of the shell if temperature difference between the outer and inner surfaces of the shell in steady state is T.*

Solution Consider a concentric spherical shell of radius r and thickness dr as shown in figure. In steady state, the rate of heat flow (heat current) through this shell will be,

$$H=\frac{\Delta T}{R}=\frac{\dfrac{(-d\theta)}{dr}}{(k)(4\pi r^2)} \qquad \left(R=\frac{l}{kA}\right)$$

or $$H=-(4\pi k r^2)\frac{d\theta}{dr}$$

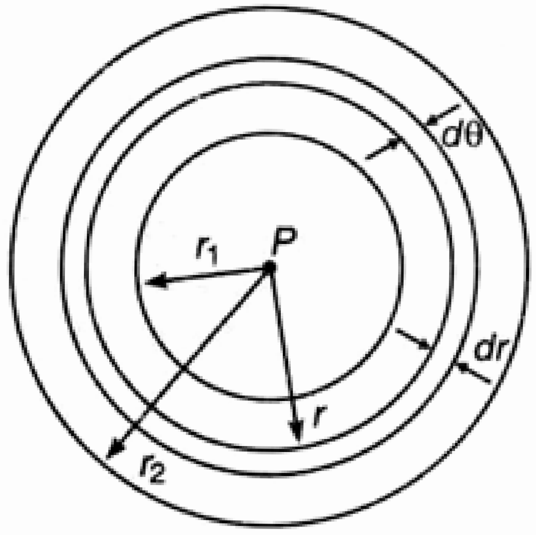


Fig. 19.22

Here, negative sign is used because with increase in r, θ decreases.

$\therefore$ $$\int_{r_1}^{r_2}\frac{dr}{r^2}=-\frac{4\pi k}{H}\int_{\theta_1}^{\theta_2}d\theta$$

This equation gives,
$$H = \frac{4\pi k r_1 r_2 (\theta_1 - \theta_2)}{(r_2 - r_1)}$$

In steady state,
$$H = P, \quad r_1 r_2 \approx R^2 \quad \text{and} \quad \theta_1 - \theta_2 = T$$

$\therefore$ Thickness of shell,
$$r_2 - r_1 = \frac{4\pi k R^2 T}{P}$$
Ans.

Example 5 *A steam pipe of radius 5 cm carries steam at 100°C. The pipe is covered by a jacket of insulating material 2 cm thick having a thermal conductivity 0.07 W/m-K. If the temperature at the outer wall of the pipe jacket is 20°C, how much heat is lost through the jacket per metre length in an hour?*

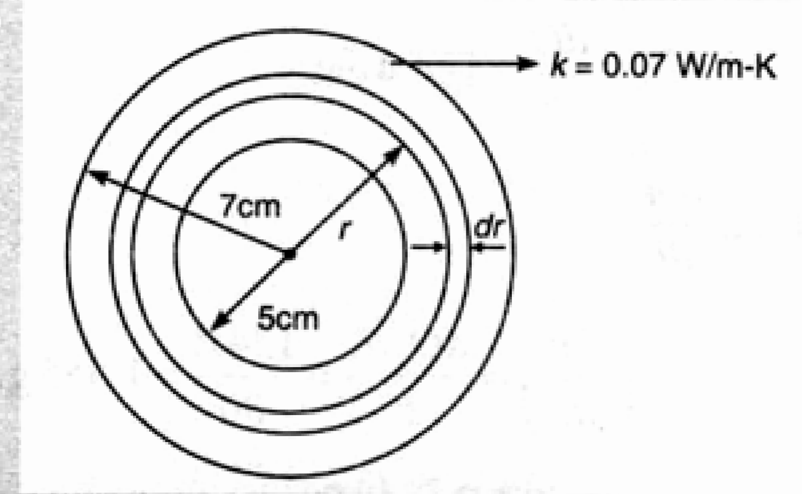


Fig. 19.23

Solution Thermal resistance per metre length of an element at distance r of thickness dr is
$$dR = \frac{dr}{k\,(2\pi r)} \qquad \left(R = \frac{l}{kA}\right)$$

$\therefore$
$$\text{Total resistance } R = \int_{r_1 = 5\text{ cm}}^{r_2 = 7\text{ cm}} dR$$
$$= \frac{1}{2\pi k} \int_{5.0\times10^{-2}\text{ m}}^{7.0\times10^{-2}\text{ m}} \frac{dr}{r}$$
$$= \frac{1}{2\pi k} \ln\left(\frac{7}{5}\right)$$
$$= \frac{1}{(2\pi)(0.07)} \ln(1.4)$$
$$= 0.765\text{ K/W}$$
$$\text{Heat current } H = \frac{\text{Temperature difference}}{\text{Thermal resistance}}$$
$$= \frac{(100-20)}{0.765} = 104.6\text{ W}$$

$\therefore$
$$\text{Heat lost in one hour} = \text{Heat current} \times \text{time}$$
$$= (104.6)(3600)\text{ J}$$
$$= 3.76 \times 10^5\text{ J}$$
Ans.

EXERCISES

For JEE Main

Subjective Questions

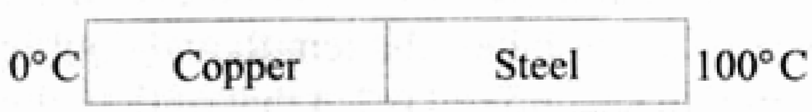
Calorimetry

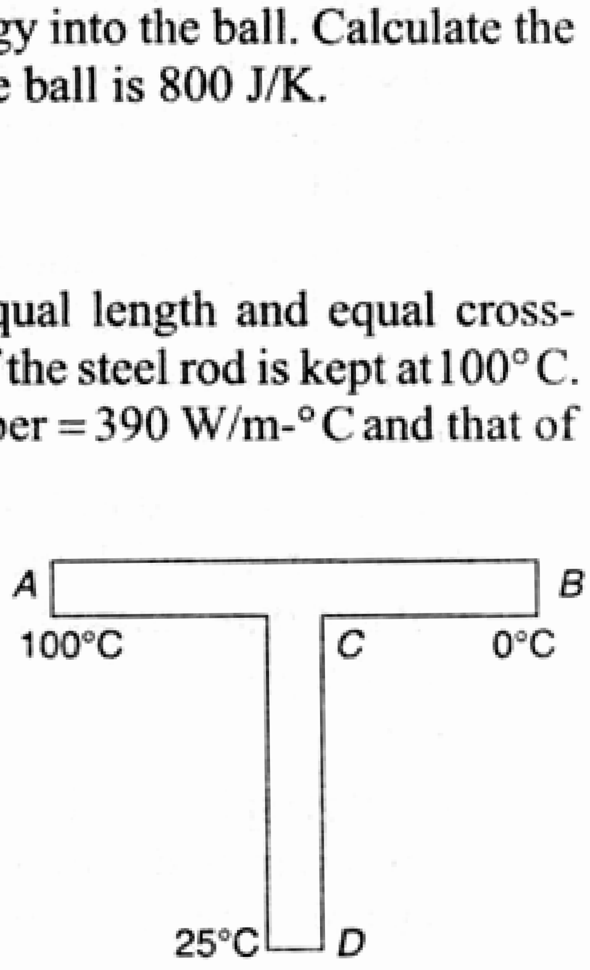
1. 10 g ice at 0°C is converted into steam at 100°C. Find total heat required. ($L_f = 80$ cal/g, $S_w = 1$ cal/g °C, $L_v = 540$ cal/g)
2. 15 g ice at 0°C is mixed with 10 g water at 40°C. Find the temperature of mixture. Also, find mass of water and ice in the mixture.
3. Three liquids P, Q and R are given. 4 kg of P at 60°C and 1 kg of R at 50°C when mixed produce a resultant temperature 55°C. A mixture of 1 kg of P at 60°C and 1 kg of Q at 50°C shows a temperature of 55°C. What will be the resulting temperature when 1 kg of Q at 60°C is mixed with 1 kg of R at 50°C?
4. A certain amount of ice is supplied heat at a constant rate for 7 minutes. For the first one minute the temperature rises uniformly with time. Then it remains constant for the next 4 minutes and again the temperature rises at uniform rate for the last two minutes. Calculate the final temperature at the end of seven minutes. (Given: L of ice $= 336 \times 10^3$ J/kg and specific heat of water = 4200 J/kg-K)
5. A lead bullet penetrates into a solid object and melts. Assuming that 50% of its kinetic energy was used to heat it, calculate the initial speed of the bullet. The initial temperature of the bullet is 27°C and its melting point is 327°C. Latent heat of fusion of lead $= 2.5 \times 10^4$ J/kg and specific heat capacity of lead = 125 J / kg-K.
6. A ball is dropped on a floor from a height of 2.0 m. After the collision it rises upto a height of 1.5 m. Assume that 40% of the mechanical energy lost goes as thermal energy into the ball. Calculate the rise in the temperature of the ball in the collision. Specific heat of the ball is 800 J/K.

Heat Transfer

(a) Conduction

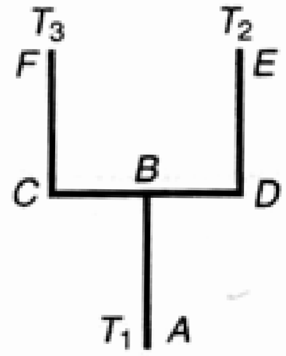
7. Figure shows a copper rod joined to a steel rod. The rods have equal length and equal cross-sectional area. The free end of the copper rod is kept at 0°C and that of the steel rod is kept at 100°C. Find the temperature at the junction of the rods. Conductivity of copper = 390 W/m-°C and that of steel = 46 W/m-°C.





8. A rod CD of thermal resistance 5.0 K/W is joined at the middle of an identical rod AB as shown in figure. The ends A, B and D are maintained at 100°C and 25°C respectively. Find the heat current in CD.

9. Four identical rods AB, CD, CF and DE are joined as shown in figure. The length, cross sectional area and thermal conductivity of each rod are l, A and K respectively. The ends A, E and F are maintained at temperatures T_1, T_2 and T_3 respectively. Assuming no loss of heat to the atmosphere. Find the temperature at B, the mid point of CD.



10. Three rods each of same length and cross section are joined in series. The thermal conductivity of the materials are $k, 2k$ and $3k$ respectively. If one end is kept at $200°C$ and the other at $100°C$. What would be the temperature of the junctions in the steady state? Assume that no heat is lost due to radiation from the sides of the rods.

11. The ends of a copper rod of length 1 m and area of cross-section $1\,cm^2$ are maintained at $0°C$ and $100°C$. At the centre of the rod there is a source of heat of power 25 W. Calculate the temperature gradient in the two halves of the rod in steady state. Thermal conductivity of copper is $400\,W\,m^{-1}K^{-1}$.

(b) Radiation

12. A thin square steel plate 10 cm on a side is heated in a black smith's forge to temperature of $800°C$. If the emissivity is 0.60, what is the total rate of radiation of energy. ?

13. A copper sphere is suspended in an evacuated chamber maintained at 300 K. The sphere is maintained at a constant temperature of 500 K by heating it electrically . A total of 210 W of electric power is needed to do it. When the surface of the copper sphere is completely blackened, 700 W is needed to maintain the same temperature of the sphere. Calculate the emissivity of copper.

14. A liquid takes 5 minutes to cool from $80°C$ to $50°C$. How much time will it take to cool from $60°C$ to $30°C$? The temperature of surrounding is $20°C$.

Objective Questions

Single Correct Option

1. A wall has two layers A and B each made of different materials. The layer A is 10 cm thick and B is 20 cm thick. The thermal conductivity of A is thrice that of B. Under thermal equilibrium temperature difference across the wall is 35°C. The difference of temperature across the layer A is
(a) 30°C (b) 14°C (c) 8.75°C (d) 5°C

2. The intensity of radiation emitted by the sun has its maximum value at a wavelength of 510 nm and that emitted by the North star has the maximum value at 350 nm. If these stars behave like black bodies, then the ratio of the surface temperatures of the sun and the north star is
(a) 1.46 (b) 0.69 (c) 1.21 (d) 0.83

3. A cylindrical rod with one end in a steam chamber and the other end in ice results in melting of 0.1 g of ice per second. If the rod is replaced by another with half the length and double the radius of the first and if the thermal conductivity of material of second rod is 1/4 that of first, the rate at which ice melts in g/s will be
(a) 0.4 (b) 0.05 (c) 0.2 (d) 0.1

4. A hollow sphere of inner radius a and outer radius $2a$ is made of a material of thermal conductivity K. It is surrounded by another hollow sphere of inner radius $2a$ and outside radius $3a$ made of same material of thermal conductivity K. The inside of smaller sphere is maintained at 0°C and the

outside of bigger sphere at 100°C. The system is in steady state. The temperature of the interface will be

(a) 50°C (b) 60°C (c) 75°C (d) 25°C

5. Two sheets of thickness d and $3d$, are touching each other. The temperature just outside the thinner sheet is T_1 and on the side of the thicker sheet is T_3. The interface temperature is T_2. T_1, T_2 and T_3 are in arithmetic progression. The ratio of thermal conductivity of thinner sheet to thicker sheet is

(a) 1 : 3 (b) 3 : 1 (c) 2 : 3 (d) 3 : 9

6. The end of two rods of different material with their thermal conductivities, area of cross-section and lengths all in the ratio 1 : 2 are maintained at the same temperature difference. If the rate of flow of heat in the first rod is 4 cal/s. Then in the second rod rate of heat flow in cal/s will be

(a) 1 (b) 2 (c) 8 (d) 16

7. A long rod has one end at 0°C and other end at a high temperature. The coefficient of thermal conductivity varies with distance from the low temperature end as $k = k_0(1 + ax)$, where $k_0 = 10^2$ SI unit and $a = 1\,\text{m}^{-1}$. At what distance from the first end the temperature will be 100°C ? The area of cross-section is $1\,\text{cm}^2$ and rate of heat conduction is 1 W.

(a) 2.7 m (b) 1.7 m (c) 3 m (d) 1.5 m

8. For an enclosure maintained at 2000 K, the maximum radiation occurs at wavelength λ_m. If the temperature is raised to 3000 K, the peak will shift to

(a) $0.5\,\lambda_m$ (b) λ_m (c) $\frac{2}{3}\lambda_m$ (d) $\frac{3}{2}\lambda_m$

9. 1 g of ice is mixed with 1 g of steam. After thermal equilibrium is achieved, the temperature of the mixture is

(a) 100°C (b) 55°C (c) 75°C (d) 0°C

10. A substance cools from 75°C to 70°C in T_1 minute, from 70°C to 65°C in T_2 minute and from 65°C to 60°C in T_3 minute, then

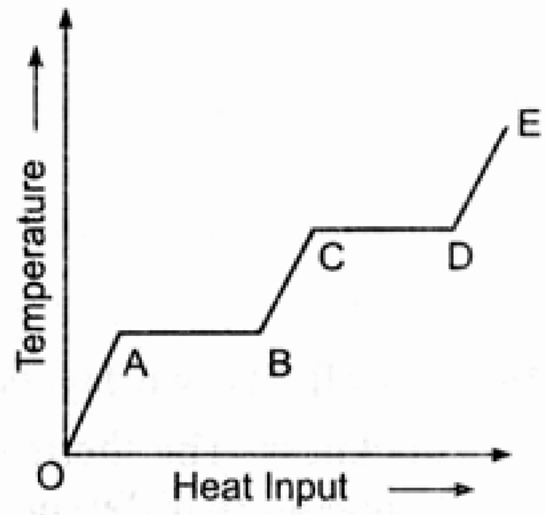
(a) $T_1 = T_2 = T_3$ (b) $T_1 < T_2 < T_3$ (c) $T_1 > T_2 > T_3$ (d) $T_1 < T_2 > T_3$

11. Two ends of rods of length L and radius R of the same material are kept at the same temperature. Which of the following rods conducts the maximum heat ?

(a) $L = 50\,\text{cm}$, $R = 1\,\text{cm}$ (b) $L = 100\,\text{cm}$, $R = 2\,\text{cm}$

(c) $L = 25\,\text{cm}$, $R = 0.5\,\text{cm}$ (d) $L = 75\,\text{cm}$, $R = 1.5\,\text{cm}$

12. A solid material is supplied heat at a constant rate. The temperature of material is changing with heat input as shown in the figure. What does the slope of DE represent ?



(a) Latent heat of liquid

(b) Latent heat of vaporization

(c) Heat capacity of vapour

(d) Inverse of heat capacity of vapour

13. The specific heat of a metal at low temperatures varies according to $S = aT^3$, where a is a constant and T is absolute temperature. The heat energy needed to raise unit mass of the metal from temperature $T = 1$ K to $T = 2$ K is

(a) $3a$ (b) $\frac{15a}{4}$ (c) $\frac{2a}{3}$ (d) $\frac{13a}{4}$

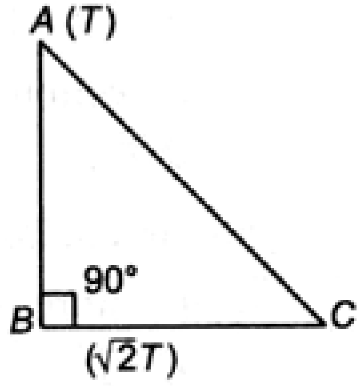
14. Two rods are of same material and having same length and area. If heat ΔQ flows through them for 12 min when they are joined side by side. If now both the rods are joined in parallel, then the same amount of heat ΔQ will flow in

(a) 24 min (b) 3 min (c) 12 min (d) 6 min

15. Two liquids are at temperatures 20°C and 40°C. When same mass of both of them is mixed, the temperature of the mixture is 32°C. What is the ratio of their specific heats?

(a) 1/3 (b) 2/5 (c) 3/2 (d) 2/3

16. Three rods of identical cross-sectional area and made from the same metal form the sides of an isosceles triangle ABC right angled at B as shown in the figure. The points A and B are maintained at temperatures T and $\sqrt{2}\,T$ respectively in the steady state. Assuming that only heat conduction takes place, temperature of point C will be



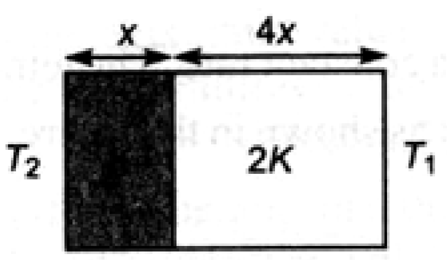
(a) $\frac{T}{\sqrt{2}-1}$ (b) $\frac{T}{\sqrt{2}+1}$

(c) $\frac{3T}{\sqrt{2}+1}$ (d) $\frac{T}{\sqrt{3}(\sqrt{2}-1)}$

17. A kettle with 2 litre water at 27°C is heated by operating coil heater of power 1 kW. The heat is lost to the atmosphere at constant rate 160 J/s, when its lid is open. In how much time will water heated to 77°C with the lid open? (specific heat of water = 4.2 kJ/°C-kg)

(a) 8 min 20 s (b) 6 min 2 s (c) 14 min (d) 7 min

18. The temperature of the two outer surfaces of a composite slab consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$ respectively are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab in steady state is $\frac{AK(T_2 - T_1)}{x} f$. where f is equal to



(a) 1 (b) 1/2 (c) 2/3 (d) 1/3

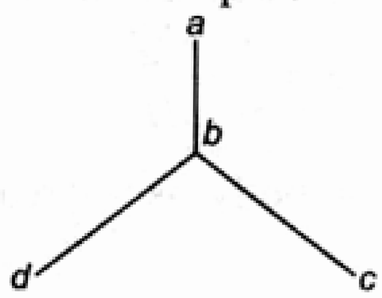
19. A wall has two layers A and B each made of different materials. Both the layers have the same thickness. The thermal conductivity of material A is twice of B. Under thermal equilibrium the temperature difference across the layer B is 36°C. The temperature difference across layer A is

(a) 6°C (b) 12°C (c) 18°C (d) 24°C

More than One Correct Options

20. A solid sphere and a hollow sphere of the same material and of equal radii are heated to the same temperature
(a) both will emit equal amount of radiation per unit time in the beginning
(b) both will absorb equal amount of radiation per second from the surrounding in the beginning
(c) the initial rate of cooling will be the same for both the spheres
(d) the two spheres will have equal temperatures at any instant

21. Three identical conducting rods are connected as shown in figure. Given that $\theta_a = 40°C$, $\theta_c = 30°C$ and $\theta_d = 20°C$. Choose the correct options



(a) temperature of junction b is 15°C
(b) temperature of junction b is 30°C
(c) heat will flow from c to b
(d) heat will flow from b to d

22. Two liquids of specific heat ratio 1 : 2 are at temperatures 2θ and θ
(a) if equal amounts of them are mixed, then temperature of mixture is $1.5\,\theta$
(b) if equal amounts of them are mixed, then temperature of mixture is $\frac{4}{3}\theta$
(c) for their equal amounts, the ratio of heat capacities is 1 : 1
(d) for their equal amounts, the ratio of their heat capacities is 1 : 2

23. Two conducting rods when connected between two points at constant but different temperatures separately the rate of heat flow through them is q_1 and q_2
(a) When they are connected in series the net rate of heat flow will be $q_1 + q_2$
(b) When they are connected in series, the net rate of heat flow is $\frac{q_1 q_2}{q_1 + q_2}$
(c) When they are connected in parallel, the net rate of heat flow is $q_1 + q_2$
(d) When they are connected in parallel, the net rate of heat flow is $\frac{q_1 q_2}{q_1 + q_2}$

24. Choose the correct options.
(a) Good absorbers of a particular wavelength are good emitters of same wavelength. This statement was given by Kirchhoff
(b) At low temperature of a body the rate of cooling is directly proportional to temperature of the body. This statement was given by the Newton
(c) Emissive power of a perfectly black body is 1
(d) Absorptive power of a perfectly black body is 1

For **JEE Advanced**

Assertion and Reason

Directions : *Choose the correct option.*

(a) *If both **Assertion and Reason** are true and the **Reason** is correct explanation of the **Assertion.***

(b) *If both **Assertion** and **Reason** are true but **Reason** is not the correct explanation of **Assertion.***

(c) *If **Assertion** is true, but the **Reason** is false.*

(d) *If **Assertion** is false but the **Reason** is true.*

1. **Assertion :** Specific heat of any substance remains constant at all temperatures.

   Reason : It is given by $S = \frac{1}{m} \cdot \frac{dQ}{dT}$

2. **Assertion :** When temperature of a body is increased, in radiant energy number of low wavelength photons get increased.

   Reason : According to Wien's displacement law $\lambda_m \propto \frac{1}{T}$.

3. **Assertion :** Warming a room by a heat blower is an example of forced convection.

   Reason : Natural convection takes place due to gravity.

4. **Assertion :** A conducting rod is placed between boiling water and ice. If rod is broken into two equal parts and two parts are connected side by side, then rate of melting of ice will increase to four times.

   Reason : Thermal resistance will become four times.

5. **Assertion :** A normal body can radiate energy more than a perfectly black body.

   Reason : A perfectly black body is always black in colour.

6. **Assertion :** According to Newton's law, good conductors of electricity are also good conductors of heat.

   Reason : At a given temperature, $e_\lambda \propto a_\lambda$ for any body.

7. **Assertion :** Good conductors of electricity are also good conductors of heat due to large number of free electrons.

   Reason : It is easy to conduct heat from free electrons.

8. **Assertion :** Emissivity of any body (e) is equal to its absorptive power (a).

   Reason : Both the quantities are dimensionless.

9. **Assertion :** Heat is supplied at constant rate from one end of a conducting rod. In steady state, temperature of all points of the rod become uniform.

   Reason : In steady state temperature of rod does not increase.

10. **Assertion :** A solid sphere and a hollow sphere of same material and same radius are kept at same temperatures in atmosphere. Rate of cooling of hollow sphere will be more.

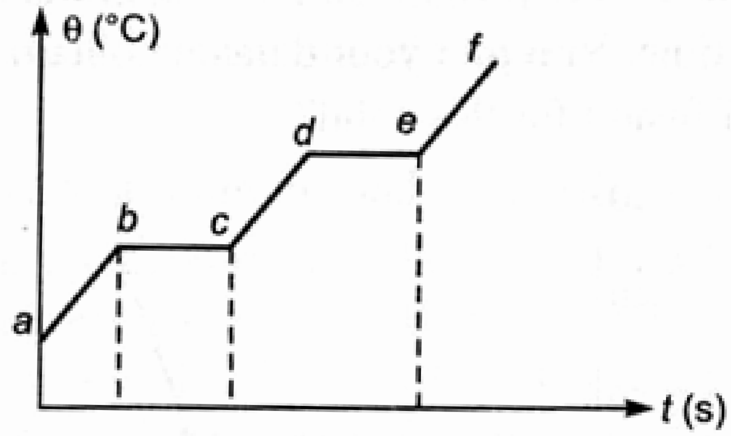
    Reason : If all other conditions are same, then rate of cooling is inversely proportional to mass of body.

Match the Columns

1. Match the following two columns.

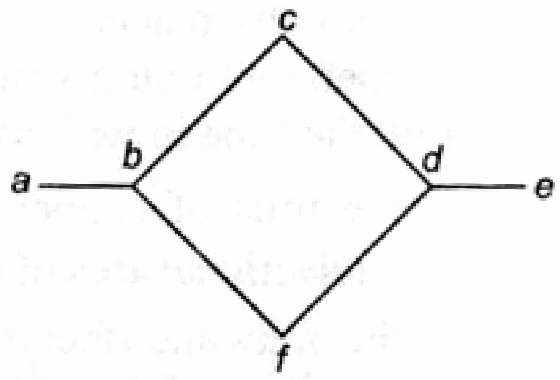
Column I	Column II
(a) Stefan's constant	(p) $[L\theta]$
(b) Wien's constant	(q) $[ML^2T^{-3}\theta^{-2}]$
(c) Emissive power	(r) $[MT^{-3}]$
(d) Thermal resistance	(s) None of these

2. Heat is supplied to a substance in solid state. Its temperature varies with time as shown in figure. Match the following two columns.



Column I	Column II
(a) Slope of line *ab*	(p) *de*
(b) Length of line *bc*	(q) *cd*
(c) Solid + liquid state	(r) directly proportional to mass
(d) Only liquid state	(s) None of these

3. Six identical conducting rods are connected as shown in figure. In steady state temperature of point *a* is fixed at 100°C and temperature of *e* at −80°C. Match the following two columns.



Column I	Column II
(a) Temperature of *b*	(p) 10°C
(b) Temperature of *c*	(q) 40°C
(c) Temperature of *f*	(r) − 20°
(d) Temperature of *d*	(s) None of these

4. Three liquids *A*, *B* and *C* having same specific heats have masses *m*, 2*m* and 3*m*. Their temperatures are, θ, 2θ and 3θ respectively. For temperature of mixture, match the following two columns.

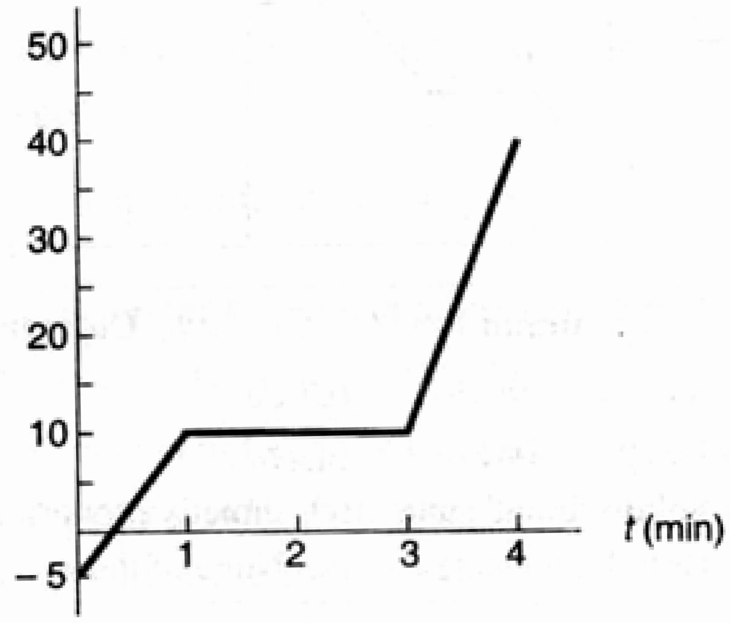
Column I	Column II
(a) When *A* and *B* are mixed	(p) $\frac{5}{2}\theta$
(b) When *A* and *C* are mixed	(q) $\frac{5}{3}\theta$
(c) When *B* and *C* are mixed	(r) $\frac{7}{3}\theta$
(d) When *A*, *B* and *C* all are mixed	(s) $\frac{13}{5}\theta$

5. Match the following two columns.

Column I	Column II
(a) Specific heat	(p) watt
(b) Heat capacity	(q) J/kg-°C
(c) Heat current	(r) J/sec
(d) Latent heat	(s) none

Subjective Questions

1. As a physicist, you put heat into a 500 g solid sample at the rate of 10.0 kJ/min, while recording its temperature as a function of time. You plot your data and obtain the graph shown in figure.
(a) What is the latent heat of fusion for this solid?



(b) What is the specific heat of solid state of the material?

2. A hot body placed in air is cooled according to Newton's law of cooling, the rate of decrease of temperature being k times the temperature difference from the surroundings. Starting from $t = 0$, find the time in which the body will lose half the maximum temperature it can lose.

3. Three rods of copper, brass and steel are welded together to form a Y-shaped structure. The cross-sectional area of each rod is 4 cm^2. The end of copper rod is maintained at 100° C and the ends of the brass and steel rods at 80° C and 60° C respectively. Assume that there is no loss of heat from the surfaces of the rods. The lengths of rods are : copper 46 cm, brass 13 cm and steel 12 cm.
(a) What is the temperature of the junction point?
(b) What is the heat current in the copper rod ?
$$k(\text{copper}) = 0.92,\ k(\text{steel}) = 0.12 \quad \text{and} \quad k(\text{brass}) = 0.26 \text{ cal/cm-s-}^\circ\text{C}$$

4. Ice at 0° C is added to 200 g of water initially at 70° C in a vacuum flask. When 50 g of ice has been added and has all melted the temperature of the flask and contents is 40° C. When a further 80 g of ice has been added and has all melted the temperature of the whole becomes 10° C. Find the latent heat of fusion of ice.

5. A copper cube of mass 200 g slides down a rough inclined plane of inclination 37° at a constant speed. Assuming that the loss in mechanical energy goes into the copper block as thermal energy. Find the increase in temperature of the block as it slides down through 60cm. Specific heat capacity of copper is equal to 420 J / kg-K. (Take $g = 10\text{ m/s}^2$)

6. A cylindrical block of length 0.4 m and area of cross-section 0.04 m^2 is placed coaxially on a thin metal disc of mass 0.4 kg and of the same cross section. The upper face of the cylinder is maintained at a constant temperature of 400 K and the initial temperature of the disc is 300 K. If the thermal conductivity of the material of the cylinder is 10 watt/m-K and the specific heat of the material of the disc is 600 J/kg-K, how long will it take for the temperature of the disc to increase to 350 K? Assume for purpose of calculation the thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder.

7. A metallic cylindrical vessel whose inner and outer radii are r_1 and r_2 is filled with ice at 0°C. The mass of the ice in the cylinder is m. Circular portions of the cylinder is sealed with completely adiabatic walls. The vessel is kept in air. Temperature of the air is 50° C. How long will it take for the ice to melt completely. Thermal conductivity of the cylinder is k and its length is l. Latent heat of fusion is L.

8. An electric heater is placed inside a room of total wall area 137 m^2 to maintain the temperature inside at 20° C . The outside temperature is −10° C. The walls are made of three composite materials. The inner most layer is made of wood of thickness 2.5 cm the middle layer is of cement of thickness 1 cm and the exterior layer is of brick of thickness 2.5 cm. Find the power of electric heater assuming that there is no heat losses through the floor and ceiling . The thermal conductivities of wood, cement and brick are 0.125 W/m°C, 1.5 W/m°C and 1.0 W/m°C respectively.

9. A 2 m long wire of resistance 4 Ω and diameter 0.64 mm is coated with plastic insulation of thickness 0.06 mm. A current of 5 A flows through the wire. Find the temperature difference across the insulation in the steady state. Thermal conductivity of plastic $= 0.16 \times 10^{-2}$ cal/s cm°C.

10. Two chunks of metal with heat capacities C_1 and C_2 are interconnected by a rod of length l and cross-sectional area A and fairly low conductivity k. The whole system is thermally insulated from the environment. At a moment $t = 0$, the temperature difference between two chunks of metal equals $(\Delta T)_0$. Assuming the heat capacity of the rod to be negligible, find the temperature difference between the chunks as a function of time .

11. A rod of length l with thermally insulated lateral surface consists of material whose heat conductivity coefficient varies with temperature as $k = a / T$, where a is a constant. The ends of the rod are kept at temperatures T_1 and T_2. Find the function $T(x)$ where x is the distance from the end whose temperature is T_1.

12. One end of a uniform brass rod 20 cm long and 10 cm^2 cross-sectional area is kept at 100°C. The other end is in perfect thermal contact with another rod of identical cross-section and length 10 cm. The free end of this rod is kept in melting ice and when the steady state has been reached, it is found that 360 g of ice melts per hour. Calculate the thermal conductivity of the rod, given that the thermal conductivity of brass is 0.25 cal/s cm °C and $L = 80$ cal/g.

13. Heat flows radially outward through a spherical shell of outside radius R_2 and inner radius R_1. The temperature of inner surface of shell is θ_1 and that of outer is θ_2. At what radial distance from centre of shell the temperature is just half way between θ_1 and θ_2?

ANSWERS

Introductory Exercise 19.1

1. 0°C, mass of ice is 54 g and that of water is 286 g **2.** 20.25°C **3.** 80°C **4.** 87.5 g
5. 1.2×10^4 kg/s

Introductory Exercise 19.2

1. Conduction **2.** $\frac{dQ}{dt} = 4\pi kab\left(\frac{T_1 - T_2}{b - a}\right)$ **4.** (a) −8.1°C (b) 7.7 W/m² **5.** 105°C
7. 3.7×10^4 W **8.** KW^{-1}

For JEE Main

Subjective Questions

1. 7200 cal **2.** 0°C, $m_w = 15$ g, $m_i = 10$ g **3.** 52°C **4.** 40°C **5.** 500 m/s
6. 2.5×10^{-3}°C **7.** 10.6°C **8.** 4 W **9.** $\frac{3T_1 + 2(T_2 + T_3)}{7}$ **10.** 145.5°C, 118.2°C
11. 412°C/m, 212°C/m **12.** 900 W **13.** 0.3 **14.** 9 min.

Objective Questions

1.(d) 2.(b) 3.(c) 4.(c) 5.(a) 6.(c) 7.(b) 8.(c) 9.(a) 10.(b)
11.(b) 12.(d) 13.(b) 14.(b) 15.(d) 16.(c) 17.(a) 18.(d) 19.(c)

More than One Correct Options

20.(a,b) 21.(b,d) 22.(b,d) 23.(b,c) 24.(a,d)

For JEE Advanced

Assertion and Reason

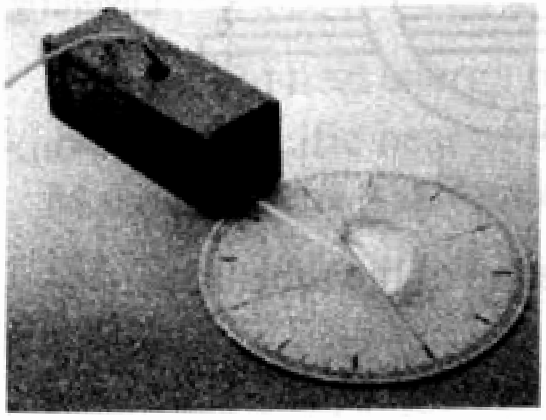
1. (d) 2. (a) 3. (b) 4. (c) 5. (c) 6. (d) 7. (a) 8. (b) 9. (d) 10. (a or b)

Match The Columns

1. (a) → s (b) → p (c) → r (d) → s
2. (a) → s (b) → r (c) → s (d) → q
3. (a) → q (b) → p (c) → p (d) → r
4. (a) → q (b) → p (c) → s (d) → r
5. (a) → q (b) → s (c) → p,r (d) → s

Subjective Questions

1. (a) 40 kJ/kg (b) 1.33 kJ/kg-°C **2.** $\ln(2)/k$ **3.** (a) 84°C (b) 1.28 cal/s **4.** 90 cal/g
5. 8.6×10^{-3}°C **6.** 166 s **7.** $t = mL\ln(r_2/r_1)/100\pi kl$ **8.** 17647 W **9.** 2.23°C
10. $\Delta T = (\Delta T)_0 e^{-\alpha t}$, where $\alpha = \frac{kA(C_1 + C_2)}{lC_1C_2}$ **11.** $T = T_1\left(\frac{T_2}{T_1}\right)^{x/l}$ **12.** 0.222 cal/cm-s-°C
13. $\frac{2R_1R_2}{R_1 + R_2}$



EXPERIMENTAL SKILLS

Chapter Contents

1. Speed of Sound Using Resonance Tube

Apparatus

Figure shows a resonance tube. It consists of a long vertical glass tube T. A metre scale S (graduated in mm) is fixed adjacent to this tube. The zero of the scale coincides with the upper end of the tube. The lower end of the tube T is connected to a reservoir R of water tube through a pipe P. The water level in the tube can be adjusted by the adjustabel screws attached with the reservoir. The vertical adjustment of the tube can be made with the help of levelling screws. For fine adjustments of the water level in the tube, the pinchcock is used.

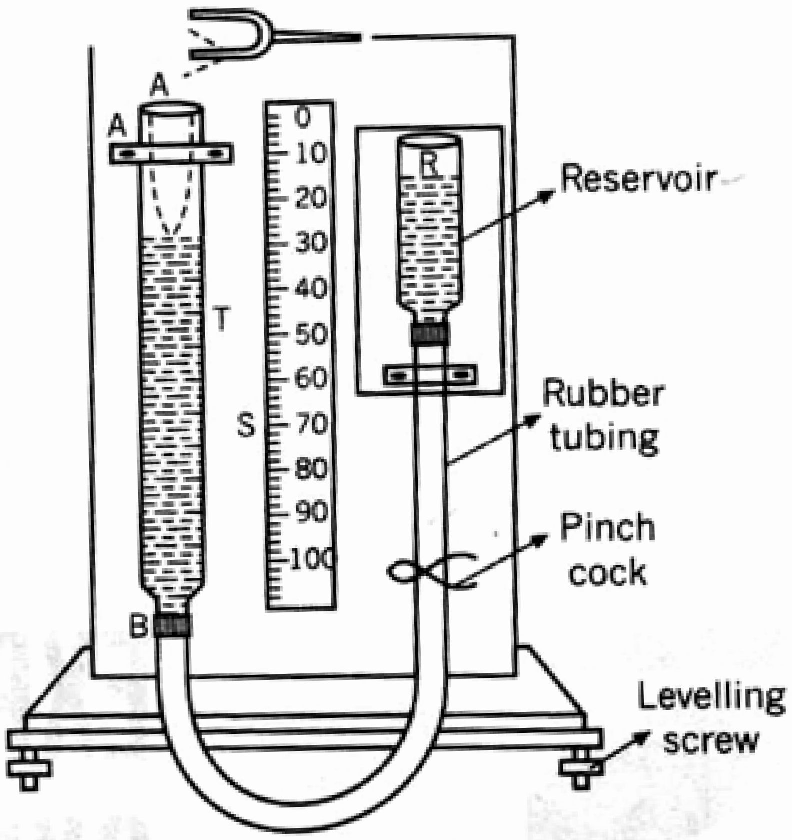


Fig. 1

Principle

If a fibrating tuning fork (of known frequency) is held over the open end of the resonance tube T, then resonance is obtained at some position as the level of water is lowered. If e is the end correction of the tube and l_1 is the length from the water level to the top of the tube, then

$$l_1 + e = \frac{\lambda}{4}$$

$$= \frac{1}{4}\left(\frac{v}{f}\right) \quad \ldots\text{(i)}$$

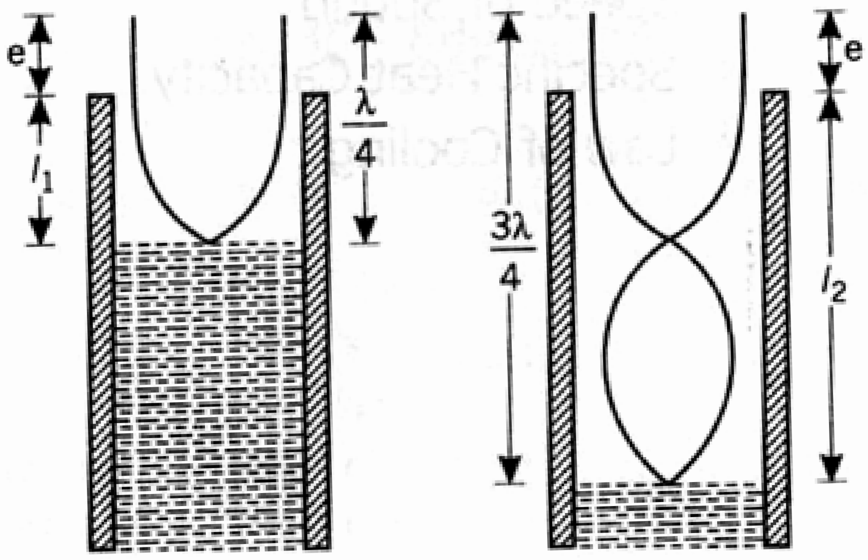


Fig. 2

Here v is the speed of sound in air and f is the frequency of tuning fork (or air column). Now, the water level is further lowered until a resonance is again obtained. If l_2 is the new length of air column.

Then

$$l_2 + e = \frac{3\lambda}{4}$$

$$= \frac{3}{4}\left(\frac{v}{f}\right) \quad \ldots\text{(ii)}$$

Subtracting Eq. (i) from Eq. (ii), we get

$$l_2 - l_1 = \frac{1}{2}\left(\frac{v}{f}\right)$$

or $$v = 2f\,(l_2 - l_1) \quad \text{...(iii)}$$

So, from Eq. (iii) we can find speed of sound v.

Note *that we have nothing to do with the end correction e, as far as v is concerned.*

2. Specific Heat Capacity

(i) Determination of specific heat capacity of a given solid

Specific Heat

"Amount of heat energy required to raise the temperature of unit mass of a solid by one Kelvin or 1°C is known as specific heat of the solid."

Mathematical Expression

If ΔQ is the amount of heat required to raise the temperature of m kg of the solid through ΔT Kelvin. Then

$$\Delta Q \propto m \quad \text{...(i)}$$

$$\Delta Q \propto \Delta T \quad \text{...(ii)}$$

or $$\Delta Q = mc\Delta T$$

where c is the constant called specific heat of solid.

or $$c = \Delta Q / m\Delta T$$

Unit of Specific Heat

Unit of specific heat is J/kg-K in S.I. system. Experiments show that specific heat of a particular material varies with temperature.

Determination of Specific Heat

Specific heat of a solid can be determined by the **"Method of Mixture"** using the concept of the **"law of Heat Exchange"** *i.e.,*

Heat lost by hot body = Heat gained by cold body

The method of mixture is based on the fact that when a hot solid is mixed with a cold body, the hot body loses heat and the cold body absorbs heat until thermal equilibrium is attained. At equilibrium, final temperature of mixture is measured. The specific heat of the solid is calculated with the help of the law of heat exchange.

Let

Mass of solid = m_s kg

Mass of liquid = m_l kg

Mass of calorie meter = m_c kg

Initial temperature of solid = T_s K
Initial temperature of liquid = $T_1 K$
Initial temperature of the calorie meter = $T_c K$
Specific heat of solid = c_s
Specific heat of liquid = c_1
Specific heat of the material of the calorie meter = c_c
Final temperature of the mixture = TK

According to the law of heat exchange

$$Q_{\text{Lost by solid}} = Q_{\text{Gained by Liquid}} + Q_{\text{Gained by calorie meter}}$$

$$M_s c_s (T_s - T) = m_1 c_1 (T - T_1) + m_c c_c (T - T_c)$$

$$c_s = \frac{m_1 c_1 (T - T_1) + m_c c_c (T - T_c)}{M_s (T_s - T)}$$

Which is the required value of specific heat of solid in J/kg K.

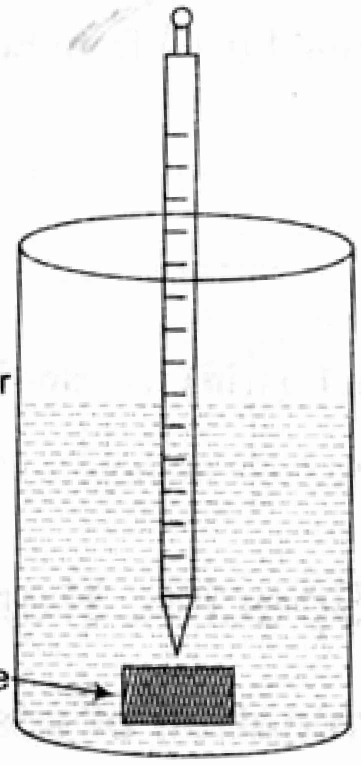


Fig. 3

(ii) Determination of specific heat capacity of the given liquid by the method of mixtures.

To determine the specific heat capacity of a liquid by the method of mixtures a solid of known specific heat capacity is taken and the given liquid is taken in the calorimeter in place of water. Suppose a solid of mass m_s and specific heat capacity c_s is heated to T_2 °C and then mixed with m_1 mass of liquid of specific heat capacity c_1 at temperature T_1. The temperature of the mixture is T. Then

Heat lost by the solid = $m_s c_s (T_2 - T)$
Heat gained by the liquid plus calorimeter = $(m_1 c_1 + m_c c_c)(T - T_1)$
By Law of heat exchange,

$$\text{Heat Lost} = \text{Heat Gained}$$

$$\therefore \quad m_s c_s (T_2 - T) = (m_1 c_1 + m_c c_c)(T - T_1)$$

From this equation we calculate the value of c_1. However the procedure remains exactly the same as done previously.

3. Law of Cooling

Aim

To study the relationship between temperature of a hot body (allowed to cool) and time, by plotting a cooling curve.

Theory

According to Newton's law of cooling, the rate at which a hot body loses heat is directly proportional to the excess of temperature of hot body over that of its surroundings (provided the difference of temperature is not too large). This law can be expressed mathematically as follows :

Consider a body of mass *m* and specific heat *s* losing heat (at temperature θ), at the rate of $\frac{dQ}{dt}$. Let θ_0 be the temperature of its surroundings. Then according to Newton's law of cooling,

$$\frac{dQ}{dt} = -ms\frac{d\theta}{dt} \propto (\theta - \theta_0) \text{ or } \frac{dQ}{dt} = -ms\frac{d\theta}{dt} = k(\theta - \theta_0) \qquad ...(i)$$

$$\therefore \quad \frac{d\theta}{dt} = -\frac{k}{ms}(\theta - \theta_0) \qquad ...(ii)$$

where *k* is the constant of proportionality. The equation (ii) can be expressed in the form

$$\frac{d\theta}{dt} = -K(\theta - \theta_0) \qquad ...(iii)$$

where *K* is another constant given as

$$K = \frac{k}{ms}$$

Further modifying the relation (iii) as

$$\frac{d\theta}{\theta - \theta_0} = -Kdt$$

or

$$\int \frac{d\theta}{\theta - \theta_0} = -K\int dt \qquad ...(iv)$$

or

$$\log_e(\theta - \theta_0) = -Kt + C \qquad ...(v)$$

where *C* is constant of integration. Equation (iv) shows that the graph between $\log_e(\theta - \theta_0)$ and *t* will be a straight line. This has to be borne in mind that this law holds for excess of temperatures of the body over that of its surroundings, maximum up to about 30°C.

If initial temperature of body is θ_i and temperature at time *t* is θ, then equation (iv) can be written as :

$$\int_{\theta_i}^{\theta} \frac{d\theta}{\theta - \theta_0} = -K\int_0^t dt$$

Solving these equation we get,

$\theta = \theta_0 + (\theta_i - \theta_0)e^{-Kt}$ i.e., θ *versus* *t* graph is exponentially decreasing.

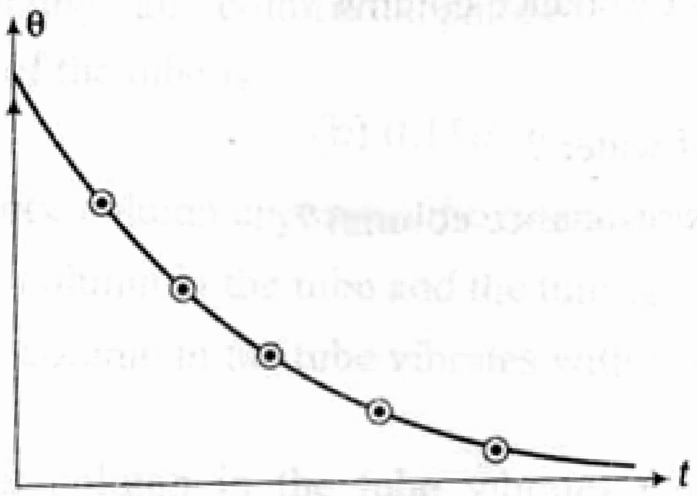


Fig. 4

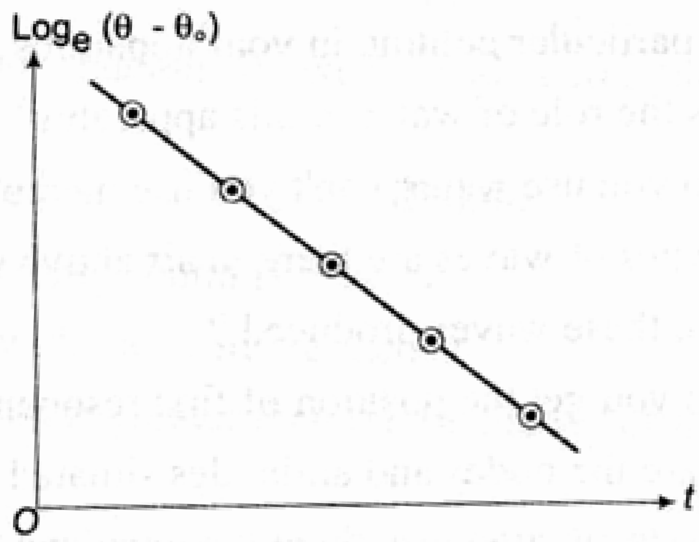


Fig. 5

EXERCISES

Experimental Skills & General Physics

Sound

1. Do the prongs and stem of a tuning fork execute same type of vibrations ?
2. Is the frequency of these vibrations different ?
3. What about the amplitude of two kinds of vibrations ?
4. How does the frequency of tuning fork vary with increase of length of the prong ?
5. Suppose you have been given two tuning forks of the same metal on which, marks of the frequency have disappeared. How will you detect the one with higher frequency ?
6. If the prongs of the fork are rubbed with a file slightly, will it affect the frequency ?
7. If a prong of the tuning fork is loaded with wax, will its frequency change ?
8. Why are the forks made of some standard frequencies like 256, 288, 320, 341.5, 384, 426.6, 484, 512, Hz etc. ?
9. What is the significance of the letters of English alphabet engraved on the tuning forks ?
10. What do you mean by a note ? How does it differ from a tone ?
11. The vibrations of a fork stop when its prongs are touched but they do not stop if the stem is touched. Why?
12. Why a tuning fork has two prongs ?
13. Why should a tuning fork not be struck with a great force ?
14. Why do both the prongs vibrate when we strike only one ?
15. Can sound also travel in vacuum ?
16. In which medium is the velocity of sound higher, in oxygen or hydrogen ?
17. Which particular column in your apparatus, do you call resonance column ?
18. What is the role of water in this apparatus ?
19. Why do you use water; can't you use mercury instead of water ?
20. What types of waves are there in air above water in this resonance column ?
21. How are these waves produced ?
22. How do you get the position of first resonance ?
23. Where are the nodes and antinodes situated ?
24. Is antinode situated exactly at the open end ?
25. How can you get wavelength of sound by this first resonating length ?
26. Can we use a resonance tube of square cross-section for the experiment ?

27. Why do we use a long tube ?

28. How do you keep the vibrating tuning fork near the open end of the tube ?

29. What do you mean by second resonance ?

30. Why is the second resonance found feebler than the first ?

31. Where are the nodes and antinodes in this case ?

32. What will be the wavelength of sound in this case ?

33. Can't you eliminate this end correction ?

34. Can you also determine the velocity of sound by this method ?

35. If n is the frequency of the tuning fork used to excite the air column in resonating air column apparatus, l_1 first resonating length and l_2 second resonating length, then the velocity of sound in air is given by the formula

(a) $v = n(l_2 - l_1)$ (b) $v = n(l_1 - l_2)$
(c) $v = 2n(l_2 - l_1)$ (d) $v = 2n(l_1 - l_2)$

36. In resonating air column apparatus while comparing the frequencies of the two tuning forks, if

l_1 is the 1st resonating length of air column with 1st fork of frequency n_1

l_2 is the 2nd resonating length of air column with 1st fork of frequency n_1

l_1' is the 1st resonating length of air column with 2nd fork of frequency n_2

l_2' is the 2nd resonating length of air column with 2nd fork of frequency n_2

Then :

(a) $\frac{n_1}{n_2} = \frac{l_2 - l_1}{l_2' - l_1'}$ (b) $\frac{n_1}{n_2} = \frac{l_2' - l_2}{l_1' - l_1}$ (c) $\frac{n_1}{n_2} = \frac{l_1' - l_1}{l_2' - l_2}$ (d) $\frac{n_1}{n_2} = \frac{l_2' - l_1'}{l_2 - l_1}$

37. The end correction (e) is (l_1 = length of air column at first resonance and l_2 is length of air column at second resonance)

(a) $e = \frac{l_2 - 3l_1}{2}$ (b) $e = \frac{l_1 - 3l_2}{2}$ (c) $e = \frac{l_2 - 2l_1}{2}$ (d) $e = \frac{l_1 - 3l_2}{2}$

38. In the resonating air column experiment, l_1 represents 1st resonating length and l_2 represents 1 2nd resonating length, the relation between 1st and 2nd resonating lengths is

(a) $l_2 = l_1$ (b) $l_2 = 2l_1$ (c) $l_2 = 3l_1$ (d) $l_2 = 4l_1$

39. In resonating air column apparatus the approximate relation between end correction and diameter of the tube is

(a) $0.3d$ (b) $0.15d$ (c) $0.6d$ (d) $0.4d$

40. In resonance column apparatus the reason for hearing booming sound is because

(a) the air column in the tube and the tuning fork vibrate with the same frequency

(b) the air column in the tube vibrates with frequency which is greater than the frequency of the tuning fork

(c) the air column in the tube vibrates with frequency which is less than the frequency of the tuning fork.

(d) velocity of sound in air column is greater than the velocity of sound in atmospheric air

41. The resonating lengths of an air column in the first and second modes of vibration are 37 cm and 97 cm. If velocity of sound is 300 ms^{-1} the frequency of the tuning fork is

(a) 1000 Hz (b) 500 Hz (c) 250 Hz (d) 125 Hz

42. The end correction of a resonance tube is 1 cm. If shortest resonating length is 15 cm, the next resonating length will be

(a) 47 cm (b) 45 cm (c) 50 cm (d) 33 cm

43. A tuning fork of frequency 340 Hz is excited and held above a cylindrical tube of length 120 cm. It is slowly filled with water. The minimum height of water column required for resonance to be first heard (Velocity of sound = 340 ms^{-1}) is

(a) 25 cm (b) 75 cm (c) 45 cm (d) 105 cm

44. Two unknown frequency tuning forks are used in resonance column apparatus. When only first tuning fork is excited the 1st and 2nd resonating lengths noted are 10 cm and 30 cm respectively. When only second tuning fork is excited the 1st and 2nd resonating lengths noted are 30 cm and 90 cm respectively. The ratio of the frequency of the 1st to 2nd tuning fork is

(a) 1 : 3 (b) 1 : 2 (c) 3 : 1 (d) 2 : 1

45. In a resonance tube, the first resonance is obtained when the level of water in the tube is at 16 cm from the open end. Neglecting end correction, the next resonance will be obtained when the level of water from the open end is

(a) 24 cm (b) 32 cm (c) 48 cm (d) 64 cm

Heat

46. Is specific heat of a substance is a constant quantity ?

47. What is meant by thermal capacity of a body ?

48. What are the units of water equivalent and thermal capacity of a body ?

49. Why do we use generally a calorimeter made of copper ?

50. In an experiment to determine the specific heat of aluminium, piece of aluminium weighing 500g is heated to 100 °C. It is then quickly transferred into a copper calorimeter of mass 500g containing 300g of water at 30 °C. The final temperature of the mixture is found to be 46.8 °C. If specific heat of copper is 0.093 cal $g^{-1}\,°C^{-1}$, then the specific heat of aluminium is

(a) 0.11 cal $g^{-1}\,°C^{-1}$ (b) 0.22 cal $g^{-1}\,°C^{-1}$ (c) 0.33 cal $g^{-1}\,°C^{-1}$ (d) 0.44 cal $g^{-1}\,°C^{-1}$

51. The mass of a copper calorimeter is 40 g and its specific heat in SI units is 4.2×10^2 J $kg^{-1}\,°C^{-1}$. The thermal capacity is

(a) 4 J $°C^{-1}$ (b) 18.6 J (c) 16.8 J/kg (d) 16.8J $°C^{-1}$

52. When 0.2 kg of brass at 100 °C is dropped into 0.5 kg of water at 20 °C, the resulting temperature is 23 °C. The specific heat of brass is

(a) 0.41×10^3 $Jkg^{-1}\,°C^{-1}$ (b) 0.41×10^2 $Jkg^{-1}\,°C^{-1}$

(c) 0.41×10^4 $Jkg^{-1}\,°C^{-1}$ (d) 0.41 $Jkg^{-1}\,°C^{-1}$

53. In an experiment to determine the specific heat of a metal, a 0.20 kg block of the metal at $150\,^\circ C$ is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing $150\,cm^3$ of water at $27\,^\circ C$. The final temperature is $40\,^\circ C$. The specific heat of the metal is

(a) $0.1\ Jg^{-1}\ ^\circ C^{-1}$ (b) $0.2\ Jg^{-1}\ ^\circ C^{-1}$

(c) $0.3\ cal\ g^{-1}\ ^\circ C^{-1}$ (d) $0.1\ cal\ g^{-1}\ ^\circ C^{-1}$

54. Is Newton's law of cooling true for all differences of temperature between the body losing heat and that of its surroundings ?

55. How do you express this law mathematically ?

56. What is the shape of the graph of $\log(\theta - \theta_0)$ *versus* t ?

ANSWERS

Experimental Skills & General Physics

35.(c)	36.(d)	37.(a)	38.(c)	39.(a)	40.(a)	41.(c)	42.(a)	43.(c)	44.(c)
45.(c)	50.(b)	51.(c)	52.(a)	53.(d)					

HINTS & SOLUTIONS

Chapter 14 Wave Motion

JEE Advanced (Subjective Questions)

1. (a)
$$v_p = -v\left(\frac{dy}{dx}\right)_P$$

As v_p and $(\text{slope})_P$ are both positive, v must be negative. Hence, the wave is moving in negative x-axis.

(b)
$$y = A\sin(\omega t - kx + \phi) \quad \ldots\text{(i)}$$
$$k = \frac{2\pi}{\lambda} = \frac{\pi}{2}\ \text{cm}^{-1}$$
$$A = 4\times10^{-3}\ \text{m} = 0.4\ \text{cm}$$

At $t = 0, x = 0$, slope $\frac{dy}{dx} = +\ \text{ve}$

$\therefore \quad v_P = -v(\text{slope}) = +\ \text{ve}$

Further at $\quad t = 0, x = 0,\ y = +\ \text{ve}$

$\therefore \quad \phi = \frac{\pi}{4}$

Further, $\quad 20\sqrt{3} = -v\tan 60°$

$\therefore \quad v = -20\ \text{cm/s}$

$$f = \frac{v}{\lambda} = 5\ \text{Hz}$$

$\therefore \quad \omega = 2\pi f = 10\pi$

$\therefore \quad y = (0.4\ \text{cm})\sin\left(10\pi t + \frac{\pi}{2}x + \frac{\pi}{4}\right)$ **Ans.**

(c)
$$P = 2\pi^2A^2f^2\mu v$$

$\therefore$ Energy carried per cycle $\quad E = PT = \frac{P}{f} = 2\pi^2A^2f\mu v$

Substituting the values, we have $\quad E = 1.6\times10^{-5}\ \text{J}$ **Ans.**

2. (a) $v = \sqrt{\frac{T}{\rho S}} = \sqrt{\frac{64}{12.5\times10^3\times0.8\times10^{-6}}} = 80\ \text{m/s}$ **Ans.**

(b)
$$\omega = 2\pi f = 2\pi(20) = 40\pi\ \text{rad/s}$$
$$k = \frac{\omega}{v} = \frac{40\pi}{80} = \frac{\pi}{2}\ \text{m}^{-1}$$

$\therefore \quad y = (1.0\ \text{cm})\cos\left[(40\pi s^{-1})t - \left(\frac{\pi}{2}m^{-1}\right)x\right]$ **Ans.**

(c) Substituting $x = 0.5$ m and $t = 0.05$ s, we get

$y = \frac{1}{\sqrt{2}}\ \text{cm}$ **Ans.**

(d) Particle velocity at time t.

$$v_P = \frac{\partial y}{\partial t} = -(40\pi\ \text{cm/s})\sin\left[(40\pi s^{-1})t - \left(\frac{\pi}{2}\ \text{m}^{-1}\right)x\right]$$

Substituting $x = 0.5$ m and $t = 0.05$ s, we get

$v_P = 89\ \text{cm/s}$ **Ans.**

3.
$$k_{eff} = 2k = 1.0 \text{ N/m}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}} = \frac{10}{2\pi} \text{ s}^{-1}$$

$$v = 0.1 \text{ m/s}, \qquad A = 0.02 \text{ m}$$

$$\lambda = \frac{v}{f} = \frac{0.1}{\left(\frac{10}{2\pi}\right)} = \frac{2\pi}{100} \text{ m}$$

$$y = A\cos\left(\frac{2\pi}{\lambda}\right)(vt - x)$$

$$= 0.02\cos 100(0.1t - x)$$

$$= 0.02\cos(10t - 100x) \text{ m}$$ **Ans.**

The distance between two successive maximas

$$= \lambda = \frac{2\pi}{100} = 0.0628 \text{ m}$$ **Ans.**

4. (a) Dimensions of A and Y are same. Similary dimensions of a and x are same.

(b) As the wave is travelling towards positive x-axis, there should be netative sign between term of x and term of t.

Further, speed of wave $v = \dfrac{\text{coefficient of } t}{\text{coefficient of } x}$

$\therefore$ coefficient of $t = (v) \times$ coefficient of x.

5. From the given figure we can see that :

(a) Amplitude $A = 1.0$ mm

(b) Wavelength $\lambda = 4$ cm

(c) Wave number $k = \dfrac{2\pi}{\lambda} = 1.57 \text{ cm}^{-1} \approx 1.6 \text{ cm}^{-1}$

(d) Frequency $f = \dfrac{v}{\lambda} = \dfrac{20}{4} = 5$ Hz

6. $v = \sqrt{\dfrac{T}{\rho S}}$ or $v \propto \dfrac{1}{\sqrt{\rho}}$

$\therefore$
$$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

$\therefore$
$$\frac{\rho_1}{\rho_2} = \left(\frac{v_2}{v_1}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

7. $v_1 = \sqrt{\dfrac{T_1}{\mu_1}} = \sqrt{\dfrac{4.8}{1.2 \times 10^{-2}}} = 20$ m/s

$$v_2 = \sqrt{\frac{T_2}{\mu_2}} = \sqrt{\frac{7.5}{1.2 \times 10^{-2}}} = 25 \text{ m/s}$$

Pulses will meet when $x_A = x_B$

or
$$20t = 25(t - 0.02)$$

$\therefore$
$$t = 0.1 \text{ s}$$

and
$$x_A \text{ or } x_B = 20 \times 0.1 = 2 \text{ m}$$

8. (a) $P = \frac{1}{2}\rho\omega^2 A^2 sv$ or $A = \frac{1}{\omega}\sqrt{\frac{2P}{\rho sv}}$ ...(i)

Here $\rho s = \mu =$ mass per unit length $= \frac{6 \times 10^{-3}}{8}$ kg/m

$$\omega = 2\pi f = \frac{2\pi v}{\lambda} = \frac{2\pi \times 30}{0.2}$$

Substituting these values in Eq. (i) we have,

$$A = \frac{0.2}{2\pi \times 30}\sqrt{\frac{2 \times 50 \times 8}{6 \times 10^{-3} \times 30}} = 0.0707 \text{ m}$$

$$= 7.07 \text{ cm}$$

(b) $P \propto v\omega^2$ or $P \propto v(v^2)$

or $$P \propto v^3$$

When wave speed is double, power will become eight times.

9. $-dT = (dm)x\omega^2 = \left(\frac{m}{L}.dx\right)x\omega^2$

or $$-\int_0^T dT = \frac{m}{L}\omega^2 \int_{x=L}^{x} x \, . \, dx$$

∴ $$-T = \frac{m}{L}\omega^2\left(\frac{x^2}{2} - \frac{L^2}{2}\right)$$

or $$T = \frac{m\omega^2}{2L}(L^2 - x^2)$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\frac{m\omega^2}{2L}(L^2 - x^2)}{m/L}} = \omega\sqrt{\frac{L^2 - x^2}{2}}$$

or $$\frac{dx}{dt} = \frac{\omega}{\sqrt{2}}\sqrt{L^2 - x^2}$$

∴ $$\int_0^t dt = \frac{\sqrt{2}}{\omega}\int_0^L \frac{dx}{\sqrt{L^2 - x^2}}$$

∴ $$t = \frac{\sqrt{2}}{\omega}\left[\sin^{-1}\left(\frac{x}{L}\right)\right]_0^L = \frac{\sqrt{2}}{\omega}\cdot\frac{\pi}{2} = \frac{\pi}{\sqrt{2}\omega}$$ **Ans.**

Chapter 15 Superposition of Waves

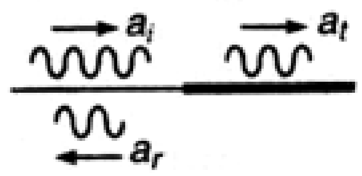
JEE Advanced (Subjective Questions)

1. (a) $v_1 = \sqrt{T/\mu_1}$, $v_2 = \sqrt{F/4\mu_1} = \frac{1}{2}\sqrt{F/\mu_1}$, $v_3 = \sqrt{F/(\mu_1/4)} = 2\sqrt{F/\mu_1}$

$\therefore$ $$t = t_1 + t_2 + t_3 = \frac{L}{v_1} + \frac{L}{v_2} + \frac{L}{v_3} = \frac{7L}{2}\sqrt{\frac{\mu_1}{F}}$$ **Ans.**

2. Let a_i and a_r be the amplitudes of incident and reflected waves.

Then $$\frac{a_i + a_r}{a_i - a_r} = 6$$ (given)



Hence $$\frac{a_r}{a_i} = \frac{5}{7}$$

Now $$\frac{E_r}{E_i} = \left(\frac{a_r}{a_i}\right)^2 = \left(\frac{5}{7}\right)^2 = 0.51$$

or percentage of energy reflected is

$$100 \times \frac{E_r}{E_i} = 51\%.$$

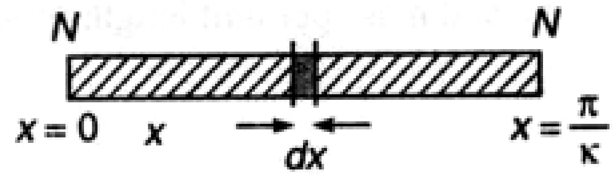
So, percentage of energy transmitted will be (100 – 51)% or 49%. **Ans.**

3. Amplitude at a distance x is

$$A = a \sin kx$$

First node can be obtained at $x = 0$,

and the second at $x = \pi/k$



At position x, mass of the element PQ is

$$dm = (\rho S)dx$$

its amplitude is $A = a \sin kx$

Hence mechanical energy stored in this element is

(energy of particle in SHM) $$dE = \frac{1}{2}(dm)A^2\omega^2$$

or $$dE = \frac{1}{2}(\rho S A^2 \omega^2).\, dx$$

$$= \frac{1}{2}(\rho S a^2 \omega^2 \sin^2 kx)\, dx$$

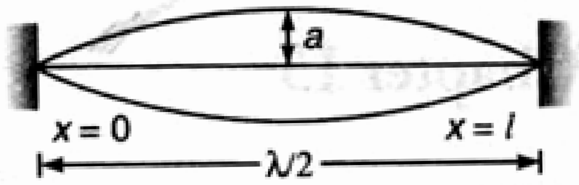
Therefore, total energy stored between two adjacent nodes will be

$$E = \int_{x=0}^{x=\pi/k} dE$$

Solving this, we get

$$E = \frac{\pi S \rho \omega^2 a^2}{4k}$$ **Ans.**

4. $l = \frac{\lambda}{2}$ or $\lambda = 2l, k = \frac{2\pi}{\lambda} = \frac{\pi}{l}$



The amplitude at a distance x from $x = 0$ is given by

$$A = a \sin kx$$

Total mechanical energy at x of length dx is

$$dE = \frac{1}{2}(dm)A^2\omega^2$$

$$= \frac{1}{2}(\mu dx)(a \sin kx)^2(2\pi f)^2$$

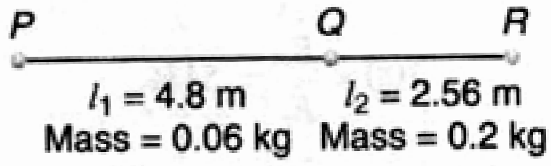
or $$dE = 2\pi^2 \mu f^2 a^2 \sin^2 kx\, dx \quad \text{...(i)}$$

Here $$f = \frac{v^2}{\lambda^2} = \frac{\left(\frac{T}{\mu}\right)}{(4l^2)} \text{ and } k = \frac{\pi}{l}$$

Substituting these values in Eq. (i) and integrating it from $x = 0$ to $x = l$, we get total energy of string.

$$E = \frac{\pi^2 a^2 T}{4l}$$ **Ans.**

5. Tension $T = 80$ N



Amplitude of incident wave, $A_i = 3.5$ cm
Mass per unit length of wire PQ is

$$m_1 = \frac{0.06}{4.8} = \frac{1}{80} \text{ kg/m}$$

and mass per unit length of wire QR is

$$m_2 = \frac{0.2}{2.56} = \frac{1}{12.8} \text{ kg/m}$$

(a) Speed of wave in wire PQ is

$$v_1 = \sqrt{T/m_1} = \sqrt{\frac{80}{1/80}} = 80 \text{ m/s}$$

and speed of wave in wire QR is

$$v_2 = \sqrt{T/m_2} = \sqrt{\frac{80}{1/12.8}} = 32 \text{ m/s}$$

∴ Time teken by the wave pulse to reach from P to R is

$$t = \frac{4.8}{v_1} + \frac{2.56}{v_2} = \left(\frac{4.8}{80} + \frac{2.56}{32}\right) \text{s} = 0.14 \text{ s}$$ **Ans.**

(b) The expressions for reflected and transmitted amplitudes (A_r and A_t) in terms of v_1, v_2 and A_i are as follows

$$A_r = \frac{v_2 - v_1}{v_2 + v_1} A_i \text{ and } A_t = \frac{2v_2}{v_1 + v_2} A_i$$

Substituting the values, we get

$$A_r = \left(\frac{32 - 80}{32 + 80}\right)(3.5) = -1.5 \text{ cm}$$

i.e., the amplitude of reflected wave will be 1.5 cm. Negative sign of A_r indicates that there will be a phase change of π in reflected wave.

Similarly

$$A_t = \left(\frac{2 \times 32}{32 + 80}\right)(3.5) = 2.0 \text{ cm}$$

i.e., the amplitude of transmitted wave will be 2.0 cm.

The expressions of A_r and A_t are derived as below.

Derivation

Suppose the incident wave of amplitude A_i and angular frequency ω is travelling in positive x-direction with velocity v_1 then we can write

$$y_i = A_i \sin \omega[t - x/v_1] \quad \text{...(i)}$$

In reflected as well as transmitted wave, ω will not change, therefore, we can write

$$y_r = A_r \sin \omega [t + x/v_1] \quad \text{(ii)}$$

and $$y_t = A_t \sin \omega [t - x/v_2] \quad \text{...(iii)}$$

Now as wave is continuous, so at the boundary ($x = 0$).

Continuity of displacement requires

$$y_i + y_r = y_t \text{ for } x = 0$$

Substituting from (i), (i) and (iii) in the above, we get

$$A_i + A_r = A_t \quad \text{...(iv)}$$

Also at the boundary, slope of wave will be continuous, *i.e.*,

$$\frac{\partial y_i}{\delta x} + \frac{\partial y_r}{\delta x} = \frac{\partial y_t}{\partial x} \quad \text{for } x = 0$$

which gives

$$A_i - A_r = \left(\frac{v_1}{v_2}\right) A_t \quad \text{...(v)}$$

Solving Eq. (iv) and (v) for A_r and A_t we get the required equations, *i.e.*,

$$A_r = \frac{v_2 - v_l}{v_2 + v_l} A_i$$

and $$A_t = \frac{2v_2}{v_2 + v_l} A_i$$

6. **When A is a node :** Suppose n_1 and n_2 are the complete loops formed on left and right side of point A. Then

$$f_1 = f_2$$

or $$n_1\left(\frac{v_1}{2L}\right) = n_2\left(\frac{v_2}{2L}\right)$$

or $$\frac{n_1}{n_2} = \left(\frac{v_2}{v_1}\right) = \sqrt{\frac{\mu_1}{\mu_2}} = \frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \ldots, \text{etc.} \quad \left(\text{as } v \propto \frac{1}{\sqrt{\mu}}\right)$$

$\therefore$ Possible frequencies are,

$$\frac{v_1}{2L}, 2\left(\frac{v_1}{2L}\right), \frac{3v_1}{2L}, \ldots, \text{etc.} \quad \left(v_1 = \sqrt{\frac{T}{\mu}}\right)$$

or $$\frac{1}{2L}\sqrt{\frac{T}{\mu}}, \frac{1}{L}\sqrt{\frac{T}{\mu}}, \frac{3}{2L}\sqrt{\frac{T}{2\mu}}, \ldots, \text{etc.}$$

When A is an antinode : Suppose n_1 and n_2 are complete loops on left and right side of point A,

$$n_1\frac{\lambda_1}{2}+\frac{\lambda_1}{4}=L \quad \text{or} \quad f_1=\frac{v_1}{L}\left(\frac{n_1}{2}+\frac{1}{4}\right)$$

$$n_2\frac{\lambda_2}{2}+\frac{\lambda_2}{4}=L \quad \text{or} \quad f_2=\frac{v_2}{L}\left(\frac{n_2}{2}+\frac{1}{4}\right)$$

Substituting, $f_1 = f_2$, we get

$$\frac{2n_1+1}{2n_2+1}=\frac{1}{3}$$

$\therefore$ For

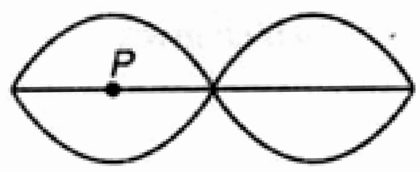
$$n_1 = 1, \quad n_2 = 4$$
$$n_1 = 2, \quad n_2 = 7$$
$$n_1 = 3, \quad n_2 = 10 \quad \text{etc.}$$
$$\ldots \quad \ldots \quad \ldots$$

Therefore, the possible frequencies are

$$\frac{v_1}{L}\left(\frac{1}{2}+\frac{1}{4}\right), \frac{v_1}{L}\left(\frac{2}{2}+\frac{1}{4}\right), \frac{v_1}{L}\left(\frac{3}{2}+\frac{1}{4}\right), \ldots, \text{etc.}$$

or $$\frac{3}{4L}\sqrt{\frac{T}{\mu}}, \frac{5}{4L}\sqrt{\frac{T}{\mu}}, \frac{7}{4L}\sqrt{\frac{T}{\mu}}, \ldots, \text{etc.}$$

7. $$\frac{L}{4}=\frac{\lambda}{4}$$



$\therefore$ $$L = \lambda$$

In the next higher mode there will be total 6 loops and the desired frequency is

$$\left(\frac{6}{2}\right)(100) = 300 \text{ Hz}$$ **Ans.**

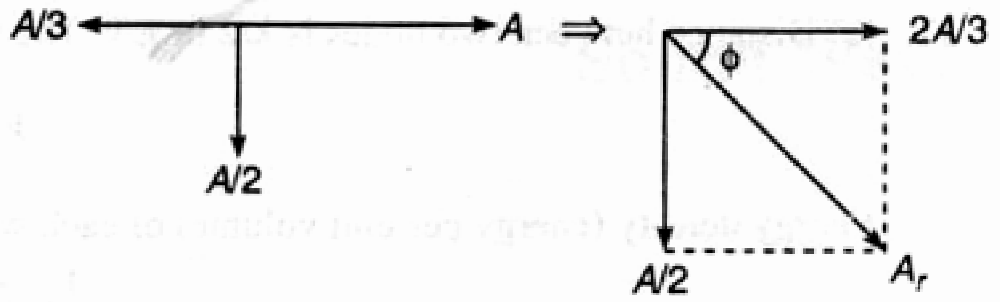
8. $$k=\frac{\omega}{v}=\frac{\pi}{3}$$

$$y_1 = 0.06\,(\pi t - kx) = 0.06 \sin\left(\pi t - \frac{\pi}{3}\times 12\right)$$
$$= 0.06 \sin(\pi t - 4\pi)$$

Similarly, $$y_2 = 0.02 \sin(\pi t - kx') = 0.02 \sin\left(\pi t - \frac{\pi}{3}\times 8\right)$$
$$= 0.02 \sin\left(\pi t - \frac{8\pi}{3}\right)$$

$$y = y_1 + y_2$$
$$= 0.06 \sin \pi t \cos 4\pi - 0.06 \cos \pi t \sin 4\pi + 0.02 \sin \pi t \cos \frac{8\pi}{3} - 0.02 \cos \pi t \sin \frac{8\pi}{3}$$
$$= 0.05 \sin \pi t - 0.0173 \cos \pi t$$ **Ans.**

9. Resultant amplitude $A_r = \sqrt{\left(\frac{2A}{3}\right)^2 + \left(\frac{A}{2}\right)^2}$



$$= \frac{5}{6} A \quad \textbf{Ans.}$$

$$\tan \phi = -\frac{A/2}{2A/3} = -\frac{3}{4}$$

or $\phi = -\tan^{-1}(3/4)$ **Ans.**

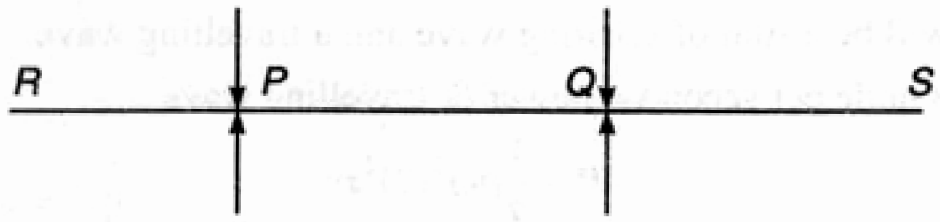
10. Speed of longitudinal waves in the rod

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{1.6 \times 10^{11}}{2500}} = 8000 \text{ m/s}$$

At the clamped position nodes will be formed. Between the clamps integer number of loops will be formed. Hence,

$$n_1 \frac{\lambda}{2} = 80$$

or $n_1 \lambda = 160$ ...(i)



Between P and R, P is a fixed end and R is the free end. It means the number of loops between P and R will be odd multiple of $\frac{\lambda}{4}$. Then

$$\frac{(2n_2 - 1)}{2} \frac{\lambda}{2} = 5$$

or $(2n_2 - 1)\lambda = 20$ ...(ii)

Also between Q and S

$$(2n_3 - 1)\lambda = 60 \quad \text{...(iii)}$$

From Eqs. (i) and (ii)

$$\frac{n_1}{2n_2 - 1} = \frac{160}{20} = 8 \quad \text{...(iv)}$$

and from Eqs. (i) and (iii)

$$\frac{n_1}{2n_3 - 1} = \frac{160}{60} = \frac{8}{3} \quad \text{...(v)}$$

For minimum frequency n_1, n_2 and n_3 should be least from Eqs. (iv) and (v)

We get, $n_1 = 8,\ n_2 = 1,\ n_3 = 2$

$$\lambda = \frac{20}{2n_2 - 1} = 20 \text{ cm} \quad \text{[from Eq. (ii)]}$$

$$= 0.2 \text{ m}$$

$\therefore$ $f_{min} = \frac{v}{\lambda} = \frac{8000}{0.2} = 40 \text{ kHz}$ **Ans.**

Next higher frequency corresponds to

$$n_1 = 24,\ n_2 = 2 \text{ and } n_3 = 5$$

and $f = 120 \text{ kHz}$ **Ans.**

11. (a) Distance between two nodes is $\lambda/2$ or π/k. The volume of string between two nodes is therefore,

$$V = \frac{\pi}{k} s \quad \text{...(i)}$$

Energy density (energy per unit volume) of each wave will be,

$$u_1 = \frac{1}{2}\rho\omega^2(8)^2 = 32\,\rho\omega^2$$

and

$$u_2 = \frac{1}{2}\rho\omega^2(6)^2 = 18\,\rho\omega^2$$

∴ Total mechanical energy between two consecutive nodes will be,

$$E = (u_1 + u_2)\,V$$

$$= 50\frac{\pi}{k}\rho\omega^2 s$$

(b)

$$y = y_1 + y_2$$

$$= 8 \sin(\omega t - kx) + 6 \sin(\omega t + kx)$$

$$= 2 \sin(\omega t - kx) + \{6 \sin(\omega t + kx) + 6 \sin(\omega t + kx)\}$$

$$= 2 \sin(\omega t - kx) + 12 \cos kx \sin \omega t$$

Thus, the resultant wave will be a sum of standing wave and a travelling wave.

Energy crossing through a node per second = power or travelling wave

∴

$$P = \frac{1}{2}\rho\omega^2(2)^2 sv$$

$$= \frac{1}{2}\rho\omega^2(4)(s)\left(\frac{\omega}{k}\right)$$

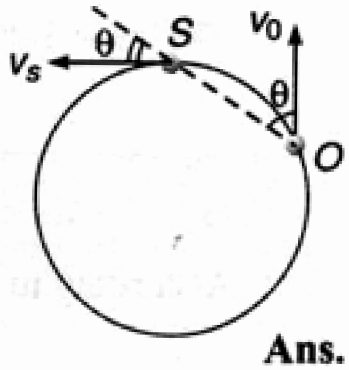
$$= \frac{2\rho\omega^3 s}{k}$$

Chapter 16 Sound Waves

JEE Advanced (Subjective Questions)

1. $v_s = v_0 = v$

Let u be the speed of sound. Then

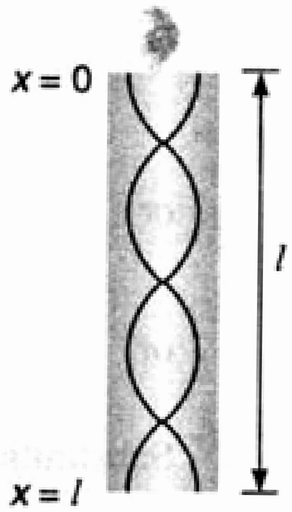


$$f' = f\left(\frac{u + v_0 \cos\theta}{u + v_s \cos\theta}\right)$$

$$= f\left(\frac{u + v \cos\theta}{u + v \cos\theta}\right)$$

or $$f' = f$$ **Ans.**

2. Given length of pipe $l = 3$ m



Third harmonic implies that $3\left(\frac{\lambda}{2}\right) = l$

or $$\lambda = \frac{2l}{3} = \frac{2 \times 3}{3} = 2 \text{ m}$$

The angular frequency is $\omega = 2\pi f$

$$= \frac{2\pi v}{\lambda} = \frac{(2\pi)(332)}{2}$$

or $$\omega = 332\pi \text{ rad/s}$$

The particle displacement $y(x, t)$ can be written as

$$y(x, t) = A \cos kx \sin \omega t$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(2l/3)} = \frac{3\pi}{l}$$

and $$\omega = kv = \frac{3\pi v}{l} \qquad \left(v = \frac{\omega}{k}\right)$$

∴ $$y(x, t) = A \cos\left(\frac{3\pi x}{l}\right) \cdot \sin\left(\frac{3\pi v}{l}\right) t$$

The longitudinal oscillations of an air column can be viewed as oscillations of particle displacement or pressure wave or density wave. Pressure variation is related to particle displacement as

$$P(x, t) = -B\frac{\partial y}{\partial x} \qquad (B = \text{Bulk modulus})$$

$$= \left(\frac{3BA\pi}{l}\right) \sin\left(\frac{3\pi x}{l}\right) \sin\left(\frac{3\pi v}{l}\right) t$$

The amplitude of pressure variation is

$$P_{max} = \frac{3BA\pi}{l}$$

$$v = \sqrt{\frac{B}{\rho}} \text{ or } B = \rho v^2$$

$$\therefore \qquad P_{max} = \frac{3\rho v^2 A\pi}{l}$$

or
$$A = \frac{P_{max} l}{3\rho v^2 \pi}$$

Here $P_{max} = 1\%$ or $P_0 = 10^3\ \text{N/m}^2$

Substituting the values

$$A = \frac{(10^3)(3)}{(3)(1.03)(332)^2\pi}$$

$$= 0.0028 \text{ m}$$

or
$$A = 0.28 \text{ cm}$$ **Ans.**

According to definition of Bulk modulus (B)

$$B = \frac{-dP}{(dV/V)} \qquad \text{...(i)}$$

$$\text{Volume} = \frac{\text{mass}}{\text{density}} \text{ or } V = \frac{m}{\rho}$$

or
$$dV = -\frac{m}{\rho^2}.d\rho = -\frac{V.d\rho}{\rho}$$

or
$$\frac{dV}{V} = -\frac{d\rho}{\rho}$$

Substituting in Eq. (i), we get

$$d\rho = \frac{\rho\,(dP)}{B}$$

or amplitude of density oscillation is

$$d\rho_{max} = \frac{\rho}{B}.P_{max}$$

$$= \frac{P_{max}}{v^2} \qquad \left(\frac{\rho}{B} = \frac{1}{v^2}\right)$$

$$= \frac{10^3}{(332)^2}$$

$$= 9 \times 10^{-3} \text{ kg/m}^3$$

3. Sound level (in dB)

$$L = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

where
$$I_0 = 10^{-12} \text{ W/m}^2$$

$$L = 60 \text{ dB}$$

Hence
$$I = (10^6)I_0 = 10^{-6} \text{ W/m}^2$$

Intensity,
$$I = \frac{\text{Power}}{\text{Area}} = \frac{P}{4\pi r^2}$$

$\Rightarrow$ $$P = I(4\pi r^2)$$

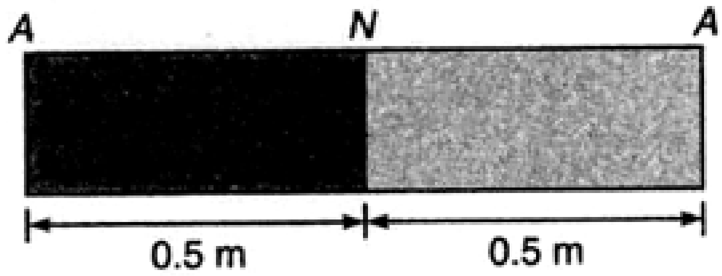
$$P = (10^{-6})(4\pi)(500)^2 = 3.14\text{ W}$$

$\therefore$ Time, $$t = \frac{(1.0\times10^3)\left(\frac{30}{100}\right)}{3.14}$$

$$t = 95.5\text{ s}$$

4. Let n_1 harmonic of the chamber containing H_2 is equal to n_2 harmonic of the chamber containing O_2. Then

$$n_1\left(\frac{v_{H_2}}{4l}\right) = n_2\left(\frac{v_{O_2}}{4l}\right)$$



$\therefore$ $$\frac{n_1}{n_2} = \frac{v_{O_2}}{v_{H_2}} = \frac{300}{1100} = \frac{3}{11}$$

$\therefore$ $$f_{min} = 3\left(\frac{v_{H_2}}{4l}\right) = 3\left(\frac{1100}{4\times0.5}\right) = 1650\text{ Hz}$$ **Ans.**

5. This problem is a Doppler-effect analogy.

(a) Here, $$f = 20\text{ min}^{-1}$$
$$v = 300\text{ m/min}$$
$$v_s = 0\text{ and } v_0 = 0$$

Spacing between the pies $= \frac{300}{20} = 15$ m **Ans.**

and $$f' = f = 20\text{ min}^{-1}$$

(b) $v_s = 30$ m/min

Spacing between the pies will be

$$\frac{300-30}{20} \quad\text{or}\quad 13.5\text{ m}$$

and $$f' = f\left(\frac{v}{v-v_s}\right) = (20)\left(\frac{300}{300-30}\right)$$
$$= 22.22\text{ min}^{-1}$$ **Ans.**

6. (a) Comparing with the equation of a travelling wave

$$y = a\sin(kx-\omega t)$$
$$k = 15\pi \quad\text{and}\quad \omega = 6000\pi$$

$\therefore$ velocity of the sound $v = \frac{\omega}{k}$

$$= \frac{6000\pi}{15\pi} = 400\text{ ms}^{-1}$$

as $$v = \sqrt{\frac{B}{\rho}}$$

Hence, $$\rho = \frac{B}{v^2} = \frac{1.6\times10^5}{(400)^2} = 1\text{ kg/m}^3$$ **Ans.**

(b) Pressure amplitude $P_0 = BAk$

Hence
$$A = \frac{P_0}{Bk}$$
$$= \frac{24\pi}{1.6 \times 10^5 \times 15\pi}$$
$$= 10^{-5}\ \text{m} = 10\mu\text{m}$$ **Ans.**

(c) Intensity received by the person
$$I = \frac{W}{4\pi R^2} = \frac{W}{4\pi (10)^2}$$

Also,
$$I = \frac{p_0^2}{2\rho v}$$
$$\frac{W}{4\pi (10)^2} = \frac{(24\pi)^2}{2 \times 1 \times 400}$$
$$W = 288\pi^3\ \text{W}$$ **Ans.**

7. (a) Path difference and hence phase difference at P from both the sources is 0°, whether $\theta = 45°$, or $\theta = 60°$. So, both the wave will interfere constructively. Or maximum intensity will be obtained. From
$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 \text{ we have}$$
$$I_0 = (\sqrt{I} + \sqrt{I})^2 = 4I \quad (I_1 = I_2 = I, \text{say})$$
$$\therefore \quad I = I_0/4$$

When one source is switched off, no interference will be obtained. Intensity will be due to a single source, or $I_0/4$.

(b) At $\theta = 60°$, also maximum intensity or I_0 will be observed.

8. (a) Frequency of second harmonic in pipe A = frequency of third harmonic in pipe B
$$\therefore \quad 2\left(\frac{v_A}{2l_A}\right) = 3\left(\frac{v_B}{4l_B}\right)$$

or
$$\frac{v_A}{v_B} = \frac{3}{4}$$
$$\frac{\sqrt{\dfrac{\gamma_A R T_A}{M_A}}}{\sqrt{\dfrac{\gamma_B R T_B}{M_B}}} = \frac{3}{4}$$

or
$$\sqrt{\frac{\gamma_A}{\gamma_B}}\sqrt{\frac{M_B}{M_A}} = \frac{3}{4} \qquad (\text{as } T_A = T_B)$$
$$\therefore \quad \frac{M_A}{M_B} = \frac{\gamma_A}{\gamma_B}\left(\frac{16}{9}\right)$$
$$= \left(\frac{5/3}{7/5}\right)\left(\frac{16}{9}\right)$$

or $$\frac{M_A}{M_B} = \left(\frac{25}{21}\right)\left(\frac{16}{9}\right) = \frac{400}{189}$$

(b) Ratio of fundamental frequency in pipe A and in pipe B is :

$$\frac{f_A}{f_B} = \frac{v_A/2l_A}{v_B/2l_B}$$

$$= \frac{v_A}{v_B} \qquad (\text{as } l_A = l_B)$$

$$= \sqrt{\frac{\frac{\gamma_A RT_A}{M_A}}{\frac{\gamma_B RT_B}{M_B}}}$$

$$= \sqrt{\frac{\gamma_A}{\gamma_B} \cdot \frac{M_B}{M_A}} \qquad (\text{as } T_A = T_B)$$

Substituting $\frac{M_B}{M_A} = \frac{189}{400}$ from part (a), we get

$$\frac{f_A}{f_B} = \sqrt{\frac{25}{21} \times \frac{189}{400}} = \frac{3}{4}$$ **Ans.**

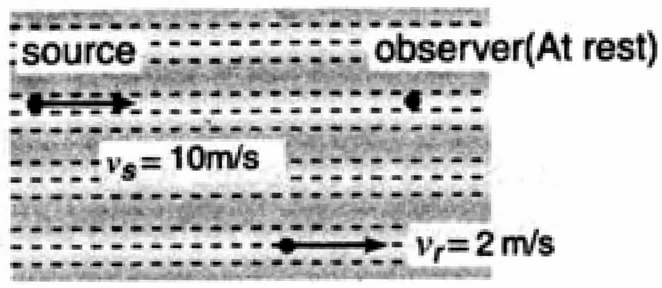
9. Velocity of sound in water is

$$v_w = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.088 \times 10^9}{10^3}} = 1445 \text{ m/s}$$

Frequency of sound in water will be

$$f_0 = \frac{v_w}{\lambda_w} = \frac{1445}{14.45 \times 10^{-3}} \text{ Hz}$$

$$f_0 = 10^5 \text{ Hz}$$

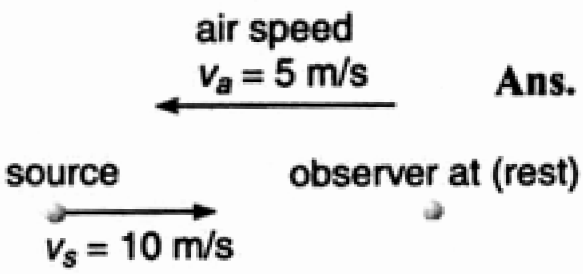


(a) Frequency of sound detected by receiver (observer) at rest would be

$$f_1 = f_0\left(\frac{v_w + v_r}{v_w + v_r - v_s}\right)$$

$$= 10^5\left(\frac{1445 + 2}{1445 + 2 - 10}\right) \text{ Hz}$$

$$f_1 = 1.0069 \times 10^5 \text{ Hz}$$ **Ans.**



(b) Velocity of sound in air is

$$v_a = \sqrt{\frac{\gamma RT}{M}}$$

$$= \sqrt{\frac{(1.4)(8.31)(20 + 273)}{28.8 \times 10^{-3}}}$$

$$= 344 \text{ m/s}$$

Frequency does not depend on the medium. Therefore, frequency in air is also $f_0 = 10^5$ Hz.

$\therefore$ Frequency of sound detected by receiver (observer) in air would be

$$f_2 = f_0\left(\frac{v_a - v_w}{v_a - v_w - v_s}\right)$$

$$= 10^5\left[\frac{344 - 5}{344 - 5 - 10}\right]\text{Hz}$$

$$f_2 = 1.0304 \times 10^5 \text{ Hz}$$ **Ans.**

10. Frequency of fundamental mode of closed pipe

$$f_1 = \frac{v}{4l} = 200 \text{ Hz}$$

Decreasing the tension in the string decrease the beat frequency.

Hence the first overtone frequency of the string should be 208 Hz (not 192 Hz)

$\therefore$ $$208 = \frac{1}{l}\sqrt{\frac{T}{\mu}}$$

$\therefore$ $$T = \mu \cdot l^2(208)$$

$$= \left(\frac{2.5 \times 10^{-3}}{0.25}\right)(0.25)^2(208)^2$$

$$= 27.04 \text{ N}$$ **Ans.**

11. (a) Wavelength of sound ahead of the source is :

$$\lambda' = \frac{v - v_s}{f} = \frac{332 - 32}{1000} = 0.3 \text{ m}$$ **Ans.**

(b) $$f' = f\left(\frac{v + v_0}{v - v_s}\right) = 1000\left(\frac{332 + 64}{332 - 32}\right)$$

$$= 1320 \text{ Hz}$$ **Ans.**

(c) Speed of reflected wave will remain 332 m/s. **Ans.**

(d) Wavelength of reflected wave.

$$\lambda'' = \frac{v - v_0}{f'} = \frac{332 - 64}{1320} = 0.2 \text{ m}$$ **Ans.**

Chapter 17 Thermometry, Thermal Expansion and Kinetic Theory of Gases

JEE Advanced (Subjective Questions)

1. (a) $v_{rms} = \sqrt{\dfrac{3RT}{M}} = \sqrt{\dfrac{3 \times 8.31 \times 300}{28 \times 10^{-3}}} = 517$ m/s

Pressure = force per unit area = change in momentum per unit area.

$\therefore$ $$P = 2mnv_{rms}$$

Here n = number of molecules striking per unit area per second.

$\therefore$ $$n = \frac{P}{2mv_{rms}} = \frac{P}{2\left(\dfrac{M}{N}\right)v_{rms}} = \frac{PN}{2Mv_{rms}} = \frac{(2 \times 1.01 \times 10^5)(6.02 \times 10^{26})}{2 \times 28 \times 517}$$

$$= 4.2 \times 10^{27}$$ **Ans.**

(b) $\frac{1}{2}mv^2 = nC_V\Delta T$

$$v = \sqrt{\frac{2nC_V\Delta T}{m}}$$ (m = mass of gas)

$$= \sqrt{\frac{2C_V\Delta T}{M}}$$ $\left(\dfrac{n}{m} = \dfrac{1}{M}\right)$

$$= \sqrt{\frac{2 \times \frac{5}{2} \times 8.31 \times 2}{28 \times 10^{-3}}} = 54.5 \text{ m/s}$$ **Ans.**

2. In the process PV^x = constant

$$C = \frac{R}{\gamma - 1} + \frac{R}{1 - x}, \text{ Given } C = R \text{ and } \gamma = 7/5$$

substituting we get,

$$x = \frac{5}{3}$$

Now, $$PV^{5/3} = \text{constant}$$

or $$P \propto \frac{1}{(V)^{5/3}}$$

By increasing volume to two times pressure will decrease $(2)^{5/3}$ times.

$$v_{rms} \propto \sqrt{T} \quad \text{or} \quad v_{rms} \propto \sqrt{PV}$$

or v_{rms} will become $\sqrt{\dfrac{(2)}{(2)^{5/3}}}$ times

or v_{rms} will become $(2)^{-1/3}$ times or $\dfrac{1}{(2)^{1/3}}$ times.

Now,
$$P \propto (\text{no. of collisions})\,(v_{rms})$$
$$\frac{1}{(2)^{5/3}} \propto (\text{no. of collisions})\,\frac{1}{(2)^{1/3}}$$

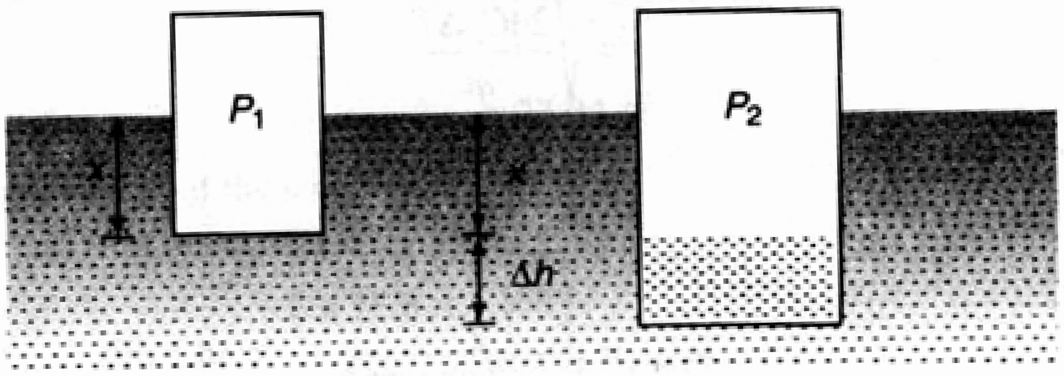
or number of collisions will decrease $(2)^{4/3}$ times. **Ans.**

3.
$$P_1V = n_1RT$$
$$P_2V = n_2RT \qquad (T = 273\text{ K})$$
$\therefore$
$$(P_1 - P_2)V = (n_1 - n_2)RT = \left(\frac{m_1 - m_2}{M}\right)RT$$
$$(\Delta P)V = \frac{\Delta m}{M}RT \qquad \ldots(i)$$

In the initial condition (at STP)
$$\frac{RT}{M} = \frac{P_0}{\rho} \qquad \ldots(ii)$$
$$(T = 273\text{ K})$$

From Eqs. (i) and (ii), we get
$$\Delta m = \frac{\rho V\,\Delta P}{P_0} = \frac{1.2 \times 0.4 \times 0.24}{1.0} = 0.1152\text{ kg}$$
$$= 115.2\text{ g}$$ **Ans.**

4. P_2 = pressure at depth x



$$P_1V_1 = P_2V_2$$
or
$$P_1(Ah) = P_2(A)(h - \Delta h)$$
$\therefore$
$$P_1 = \frac{P_2(h - \Delta h)}{h} \qquad \ldots(i)$$

Initially
$$mg = \rho g x A$$

Now,
$$P_2 = P_0 + \rho g x = P_0 + \frac{mg}{A}$$

Substituting in Eq. (i), we have
$$P_1 = \left(P_0 + \frac{mg}{A}\right)\left(1 - \frac{\Delta h}{h}\right)$$ **Ans.**

5. (a) In 1 sec, molecules make 500 hit in a cubical vessel of side 1m. Therefore
$$v_{rms} = 1000\text{ m/s}$$

Because between two successive collisions a molecule will travel 2m.

Using
$$v_{rms} = \sqrt{\frac{3RT}{M}}$$
$\therefore$
$$T = \frac{Mv_{rms}^2}{3R} = \frac{(4 \times 10^{-3})(10^3)^2}{3 \times 8.31} = 160\text{ K}$$ **Ans.**

(b) Average kinetic energy per atom $= \frac{3}{2}kT$ (monoatomic)

$$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 160 \text{ J}$$

$$= 3.31 \times 10^{-21} \text{ J}$$ **Ans.**

(c) $PV = nRT = \frac{m}{M}RT$

$\therefore$ $$m = \frac{PMV}{RT} = \frac{(100)(4 \times 10^{-3})(1)^3}{8.31 \times 160} = 3.0 \times 10^{-4} \text{ kg}$$ **Ans.**

6. Under isothermal conditions :

$$P_1V_1 = P_2V_2$$

$$V_2 = \left(\frac{P_1}{P_2}\right)V_1$$

P_1 = pressure at depth 11 m

$$= P_0 + \rho gh = (1.01 \times 10^5) + (10^3 \times 10 \times 11) = 2.11 \times 10^5 \text{ N/m}^2$$

$$P_2 = P_0 = 1.01 \times 10^5 \text{ N/m}^2$$

$\therefore$ $$V_2 = \left(\frac{2.11 \times 10^5}{1.01 \times 10^5}\right)V = 2.089\, V$$

7. $$P = \frac{RT}{V} \qquad (n = 1)$$

or $$P = \frac{R}{V}(T_0 + \alpha V^2) \qquad \ldots\text{(i)}$$

For minimum attainable pressure

$$\frac{dP}{dV} = 0 \text{ or } \frac{-RT_0}{V^2} + \alpha R = 0$$

or $$V = \sqrt{\frac{T_0}{\alpha}}$$

At this volume we can see that $\frac{d^2P}{dV^2}$ is positive or P is minimum.

From Eq. (i)

$$P_{\min} = \frac{RT_0}{\sqrt{T_0/\alpha}} + \alpha R\sqrt{T_0/\alpha} = 2R\sqrt{\alpha\, T_0}$$ **Ans.**

8. At 50°C, density of solid = density of liquid.

At 0°C, fraction submerged $\left(\frac{\rho_s}{\rho_l}\right)_{0°C} \times 100$ ...(i)

$$(\rho)_{50} = \frac{\rho_0}{1 + 50\gamma}$$

$\therefore$ $$\rho_0 = \rho_{50}(1 + 50\gamma)$$

Substituting in Eq. (i) we have

$$\% \text{ of fraction submerged} = \left(\frac{1+50\gamma_s}{1+50\gamma_l}\right)\times 100 = \left(\frac{1+0.3\times 10^{-5}}{1+8\times 10^{-5}}\right)\times 100$$

$$= 99.99\%$$ **Ans.**

9. $$\rho_{100} g h_{100} = \rho_0 g h_0 \quad \text{or} \quad \frac{\rho_{100}}{\rho_0} = \frac{h_0}{h_{100}}$$

$$\left(\frac{1}{1+100\gamma}\right) = \frac{39.2}{40}$$

Solving this equation we get :

$$\gamma = 2.0\times 10^{-4} \text{ per °C}$$ **Ans.**

10. Let Δl be the change in length. (Let $\Delta l_s > \Delta l > \Delta l_a$)

$$\text{Strain in steel} = \frac{\Delta l_s - \Delta l}{l_0}$$

and $$\text{strain in aluminum} = \frac{\Delta l - \Delta l_a}{l_0}$$

In equilibrium : $$2F_s = F_a$$

$$2\left[\frac{\Delta l_s - \Delta l}{l_0}\right] Y_s A = \left[\frac{\Delta l - \Delta l_a}{l_0}\right] Y_a A$$

or $$2(l_0\alpha_s\theta - \Delta l)Y_s = (\Delta l - l_0\alpha_a\theta)Ya$$

Solving this equation we get,

$$\Delta l = \left(\frac{2\alpha_s Y_s + \alpha_a Y_a}{2Ys + Y_a}\right) l_0\theta$$

$$\therefore \quad \text{Total length} = l_0 + \Delta l = l_0\left[1 + \left(\frac{\alpha_a Y_a + 2\alpha_s Y_s}{2Y_s + Y_a}\right)\theta\right]$$ **Ans.**

11. From $\Delta l = l\alpha\Delta\theta$ we have,

$$0.05 = 25\alpha_A(100)$$

$$\therefore \quad \alpha_A = 0.00002 \text{ per°C}$$

$$0.04 = 40\,\alpha_B(100)$$

$$\alpha_B = 0.00001 \text{ per°C}$$

In third case let l is the length of rod A. Then length of rod B will be $(50 - l)$ cm.

$$\Delta l = \Delta l_1 + \Delta l_2$$

or $$0.03 = l(0.00002)(50) + (50 - l)(0.00001)(50)$$

Solving we get $l = 10$ cm and $50 - l = 40$ cm **Ans.**

Chapter 18 First Law of Thermodynamics

JEE Advanced (Subjective Questions)

1. First process is isobaric

$\therefore$ $$\Delta Q_1 = nC_V\Delta T + P\Delta V$$

Second process is isochoric

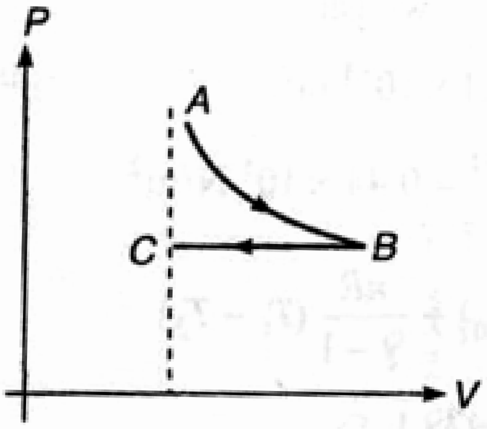
$\therefore$ $$\Delta Q_2 = nC_V\Delta T$$

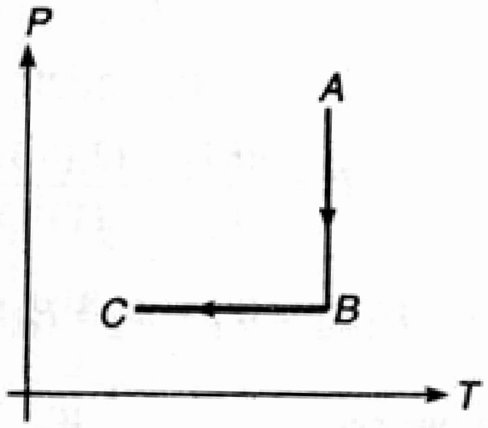
$$\Delta Q_1 - \Delta Q_2 = P\Delta V = \left[P_0 + \frac{mg}{A}\right][Ax] = (P_0A + mg)x$$

$$= [10^5 \times 60 \times 10^{-4} + 8 \times 10](0.2)$$

$$= 136 \text{ J}$$ **Ans.**

2. (a)





(b) For the process AB

$$\frac{P_0V_0}{T_0} = \frac{P_B(2V_0)}{T_0}$$

$\therefore$ $$P_B = \frac{P_0}{2}$$

$$\Delta U = 0$$

$$Q = W + \Delta U = nRT \ln \frac{V_B}{V_A} = 3RT_0 \ln 2$$

For the process BC

$$\frac{2V_0}{T_0} = \frac{V_0}{T_C}$$

$\therefore$ $$T_C = \frac{T_0}{2}$$

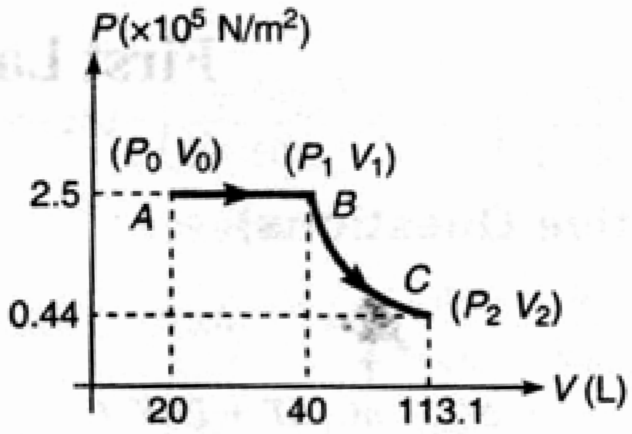
$$W = nR\Delta T = 3R\left(\frac{T_0}{2} - T_0\right) = -\frac{3}{2}RT_0$$

$$Q = nC_P\Delta T = -\frac{21}{4}RT_0$$

$\therefore$ $$W_{\text{Total}} = 3RT_0 \ln(2) - \frac{3}{2}RT_0$$

(c) $$Q_{\text{Total}} = 3RT_0 \ln(2) - \frac{21}{4}RT_0$$

3. (a)



(b)
$$P_0 = \frac{nRT_0}{V_0} = \frac{2 \times 8.31 \times 300}{20 \times 10^{-3}} = 2.5 \times 10^5 \text{ N/m}^2$$
$$P_1 = P_0 = 2.5 \times 10^5 \text{ N/m}^2$$
$$V_1 = 2V = 40 \times 10^{-3} \text{ m}^3$$

Process *AB* : $V \propto T$ $\quad \therefore \; T_1 = 2T_0 = 600$ K

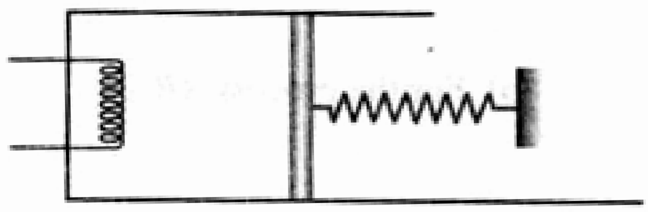
Process *BC* : Using $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$, we get
$$V_2 = 2\sqrt{2}V_1 = 113.1 \times 10^{-3} \text{ m}^3$$ **Ans.**

and
$$P_2 = \frac{nRT_2}{V_2} = \frac{(2)(8.31)(300)}{113.1 \times 10^{-3}} = 0.44 \times 10^5 \text{ N/m}^2$$ **Ans.**

(c)
$$W_{\text{Total}} = W_1 + W_2 = P_0(V_1 - V_0) + \frac{nR}{\gamma - 1}(T_1 - T_2)$$

Substituting the values, we get $W_{\text{Total}} = 12479$ J **Ans.**

4. (a)
$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$



$$T_2 = \left(\frac{P_2V_2T_1}{P_1V_1}\right) \quad \text{...(i)}$$

Here, $P_1 = 1.0 \times 10^5 \text{ N/m}^2, \; V_1 = 2.4 \times 10^{-3} \text{ m}^3, \; T_1 = 300$ K
$$P_2 = P_1 + \frac{kx}{A} = 1.0 \times 10^5 + \frac{8000 \times 0.1}{8 \times 10^{-3}} = 2.0 \times 10^5 \text{ N/m}^2$$
$$V_2 = V_1 + Ax = 2.4 \times 10^{-3} + 8 \times 10^{-3} \times 0.1 = 3.2 \times 10^{-3} \text{ m}^3$$

Substituting in Eq. (i), we get
$$T_2 = 800 \text{ K}$$ **Ans.**

(b) Heat supplied by the heater.
$$Q = W + \Delta U$$

Here,
$$\Delta U = nC_V\,\Delta T = \left(\frac{P_1V_1}{RT_1}\right)\left(\frac{3}{2}R\right)(800 - 300)$$
$$= \frac{(1.0 \times 10^5)(2.4 \times 10^{-3})(1.5)(500)}{(300)}$$
$$= 600 \text{ J}$$
$$W = \frac{1}{2}kx^2 + P_1\Delta V$$

$$= \frac{1}{2} \times (8000)(0.1)^2 + (1.0 \times 10^5)(0.1)(8 \times 10^{-3})$$

$$= (40 + 80) \text{ J}$$

$$= 120 \text{ J}$$

$\therefore$ $Q = 600 + 120 = 720$ J **Ans.**

5. $$U \propto \sqrt{V}$$

As $$U \propto T$$

$\therefore$ $$T \propto V^{1/2}$$

or $$TV^{-1/2} = \text{constant}$$

or $$PV^{1/2} = \text{constant}$$

Comparing with PV^x = constant, we have

$$x = \frac{1}{2}$$

$\therefore$ Molar specific heat $$C = \frac{R}{\gamma - 1} + \frac{R}{1 - x}$$

$$= \frac{R}{7/5 - 1} + \frac{R}{1 - 1/2} \qquad (\gamma = 7/5)$$

$$= \frac{5}{2}R + 2R = \frac{9R}{2}$$ **Ans.**

$$Q = nC\Delta T$$

$$\Delta U = nC_V\Delta T$$

$\therefore$ $$W = Q - \Delta U = n(C - C_V)\Delta T$$

$$\frac{W}{\Delta U} = \frac{C - C_V}{C_V}$$

$\therefore$ $$W = \left(\frac{C - C_V}{C_V}\right)\Delta U = \left(\frac{9/2 - 5/2}{5/2}\right)(100) = 80 \text{ J}$$ **Ans.**

6. $$(PV)_A = (PV)_C = 3P_0V_0$$

$\therefore$ $$T_A = T_C$$

or $$\Delta T = 0$$

$\therefore$ $$\Delta U_{ABC} = 0$$

$$Q_{ABC} = W_{ABC} = \text{Area under the graph}$$

$$= -2P_0V_0$$ **Ans.**

i.e., Heat is released during the process.

7. $$dQ = dU + dW$$

$$C\,dT = C_V\,dT + P\,dV$$

$$(C_V + 3aT^2)\,dT = C_V\,dT + P\,dV$$

$\therefore$ $$3aT^2\,dT = P\,dV = \left(\frac{RT}{V}\right).dV$$

$\therefore$ $$\left(\frac{3a}{R}\right)T\,dT = \frac{dV}{V}$$

Integrating, we get

$$\left(\frac{3aT^2}{2R}\right) = \ln V - \ln C$$

$$V = Ce^{\frac{3aT^2}{2R}} \quad \text{or} \quad Ve^{-\frac{3aT^2}{2R}} = \text{constant}$$

8. (a) $$P = \alpha T^{1/2}$$

or $$PT^{-1/2} = \text{constant}$$

$$P^{1/2}V^{-1/2} = \text{constant}$$

$$PV^{-1} = \text{constant}$$

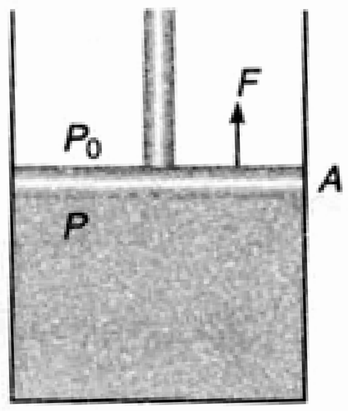
$\therefore$ $$x = -1$$

$$\Delta W = \left(\frac{R}{1-x}\right)(\Delta T) = \left(\frac{8.31}{2}\right)(50)$$

$$= 207.75 \text{ J}$$ **Ans.**

(b) $$C = C_V + \frac{R}{1-x} = \frac{3}{2}R + \frac{R}{2} = 2R$$ **Ans.**

9. $$F + PA = P_0A$$



$$F = (P_0 - P)A$$

$$W = \int_V^{2V} (P_0 - P)A\, dx$$

$$= \int_V^{2V} P_0\, dV - \int_V^{2V} P\, dV$$

$$= P_0V - \int_V^{2V} RT\left(\frac{dV}{V}\right)$$

$$= P_0V - RT \ln(2)$$

$$= RT - RT \ln(2)$$

$$= RT(1 - \ln 2)$$ **Ans.**

10. (a) $$P \propto \frac{1}{T} \text{ or } PT = \text{constant}$$

$\therefore$ $$P(PV)) = \text{constant}$$

or $$PV^{1/2} = \text{constant}$$

In the process PV^x = constant, molar heat capacity is,

$$C = C_V + \frac{R}{1-x}$$

or $$C = \frac{3}{2}R + \frac{R}{1-\frac{1}{2}}$$

$$= \frac{3}{2}R + 2R = \frac{7}{2}R$$ **Ans.**

(b) $W = Q - \Delta U = nC\Delta T - nC_V\Delta T = n(C - C_V)\Delta T$

$$= 2\left[\frac{7}{2}R - \frac{3}{2}R\right](T_2 - T_1) = 4R(T_2 - T_1)$$ **Ans.**

11. $V = \frac{a}{T}$ or VT = constant or $V(PV)$ = constant

$\therefore$ $$PV^2 = \text{constant}$$

In the process PV^x = constant, molar heat capacity is

$$C = \frac{R}{\gamma - 1} + \frac{R}{1 - x}$$

Here $$x = 2$$

$\therefore$ $$C = \frac{R}{\gamma - 1} + \frac{R}{1 - 2} = \left(\frac{2 - \gamma}{\gamma - 1}\right) R$$

Now $$Q = nC\Delta T = (1)\left(\frac{2 - \gamma}{\gamma - 1}\right) R\Delta T = \left(\frac{2 - \gamma}{\gamma - 1}\right) R\Delta T$$ **Ans.**

12. Process AB is isochoric (V = constant). Hence

$\Delta W_{AB} = 0$

$$\Delta W_{BCD} = P_0V_0 + \frac{\pi}{2}(P_0)\left(\frac{V_0}{2}\right)$$

$$= \left(\frac{\pi}{4} + 1\right) P_0V_0$$

$$\Delta W_{DA} = -\frac{1}{2}\left(\frac{P_0}{2} + P_0\right)(2V_0 - V_0)$$

$$= -\frac{3}{4} P_0V_0$$

$$\Delta U_{AB} = nC_V\Delta T = (2)\left(\frac{3}{2}R\right)(T_B - T_A) \qquad \left(n = 2, C_V = \frac{3}{2}R\right)$$

$$= 3R\left(\frac{P_0V_0}{2R} - \frac{P_0V_0}{4R}\right)$$

$$= \frac{3}{4} P_0V_0 = \Delta Q_{AB} \qquad \left(T = \frac{PV}{nR}\right)$$

$$\Delta U_{BCD} = nC_V\Delta T = (2)\left(\frac{3}{2}R\right)(T_D - T_B)$$

$$= (3R)\left(\frac{2P_0V_0}{2R} - \frac{P_0V_0}{2R}\right) = \frac{3}{2} P_0V_0$$

Hence $$\Delta Q_{BCD} = \Delta U_{BCD} + \Delta W_{BCD}$$

$$= \left(\frac{\pi}{4} + \frac{5}{2}\right) P_0V_0$$

$$\Delta U_{DA} = nC_V\Delta T$$

$$= (2)\left(\frac{3}{2}R\right)(T_A - T_D)$$

$$= (3R)\left(\frac{P_0V_0}{4R} - \frac{2P_0V_0}{2R}\right)$$

$$= -\frac{9}{4}P_0V_0$$

$\therefore$

$$\Delta Q_{DA} = \Delta U_{DA} + \Delta W_{DA}$$
$$= -\frac{9}{4}P_0V_0 - \frac{3}{4}P_0V_0$$
$$= -3P_0V_0$$

Net work done is

$$W_{net} = \left(\frac{\pi}{4} + 1 - \frac{3}{4}\right)P_0V_0$$
$$= 1.04P_0V_0$$

and heat absorbed is

$$Q_{ab} = \Delta Q_{+ve}$$
$$= \left(\frac{3}{4} + \frac{\pi}{4} + \frac{5}{2}\right)P_0V_0 = 4.03P_0V_0$$

Hence efficiency of the cycle is

$$\eta = \frac{W_{net}}{Q_{ab}} \times 100$$
$$= \frac{1.04\,P_0V_0}{4.03P_0V_0} \times 100 = 25.8\%$$ **Ans.**

13. $$P = \frac{\alpha T - \beta T^2}{V}$$ (P = constant)

Hence $$V = \frac{\alpha T - \beta T^2}{P}$$

or $$dV = \left(\frac{\alpha - 2\beta T}{P}\right) dT$$

$$W = \int P dV = \int_{T_1}^{T_2} P\left(\frac{\alpha - 2\beta T}{P}\right) dT$$

or $$W = [\alpha T - \beta T^2]_{T_1}^{T_2}$$
$$= \alpha(T_2 - T_1) - \beta(T_2^2 - T_1^2)$$ **Ans.**

14. (a) First law of thermodynamics for the given process from state 1 to state 2

$$Q_{12} - W_{12} = U_2 - U_1$$

Here, $$Q_{12} = +\,10P_0V_0 \text{ joule}$$

$W_{12} = 0$ (Volume remains constant)

$$U_2 - U_1 = nC_V\,(T_2 - T_1)$$
$$nC_V\,(T_2 - T_1) = 10P_0V_0$$

For an ideal gas

$$P_0V_0 = nRT_0$$

and $$C_P - C_V = R$$

$\therefore$ $$C_V = C_P - R = \frac{5R}{2} - R = \frac{3R}{2}$$

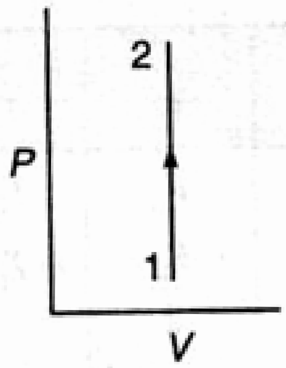
$$\therefore \quad n\left(\frac{3R}{2}\right)(T_2 - T_0) = 10nRT_0$$

$$T_2 = \frac{23}{3}T_0$$

As $P \propto T$ for constant volume

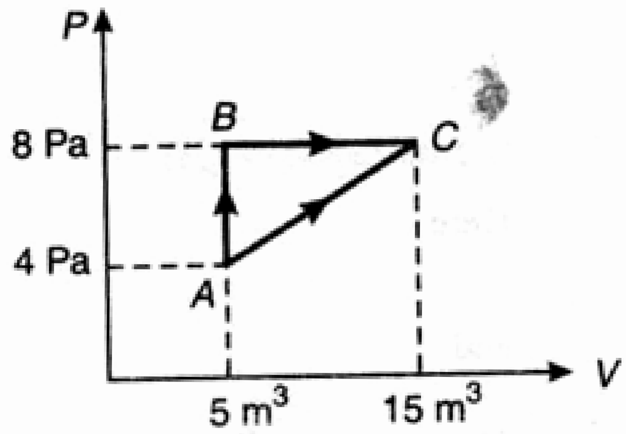
$$P_2 = \frac{23}{3}P_0$$

(b)



15. (a) W_{AC} is less than W_{ABC} as area under graph is less.

(b) For process A to C



$$Q = 200 \text{ J}$$

Work done

$$W_{AC} = \text{area under } AC$$

$$= \frac{1}{2}(8 + 4) \times 10 = 60 \text{ J}$$

From first law of thermodynamics.

$$\Delta U = Q - W_{AC}$$

$$U_C - U_A = 200 - 60$$

$$\therefore \quad U_C = U_A + 140$$

$$= 10 + 140 = 150 \text{ J}$$ **Ans.**

(c) From A to B

$$U_B = 20 \text{ J}$$

$$\therefore \quad \Delta U = Q - W_{AB}$$

$$U_B - U_A = Q - 0$$

$$20 - 10 = Q$$

$$\therefore \quad Q = 10 \text{ J}$$ **Ans.**

16. **Process *A-B*** is an isothermal process *i.e.* T = constant.

Hence $P \propto \frac{1}{V}$ or P - V graph will a rectangular hyperbola with increasing P and decreasing V.

$\rho \propto \frac{1}{V}$. Hence ρ - V graph is also a rectangular hyperbola with decreasing V and hence increasing ρ.

$$\rho \propto P \qquad \left[\rho = \frac{PM}{RT}\right]$$

Hence ρ - P graph will be a straight line passing through origin, with increasing ρ and P.

Process *B* - *C* is an isochoric process, because P - T graph is a straight line passing through origin

i.e. V = constant

Hence P - V graph will be a straight line parallel to P-axis with increasing P.

Since V = constant hence ρ will also be constant

Hence ρ-V graph will be a dot.

ρ-P graph will be a straight line parallel to P-axis with increasing P, because

$$\rho = \text{constant}$$

Process *C-D* is inverse of A-B and D-A is inverse of B-C.

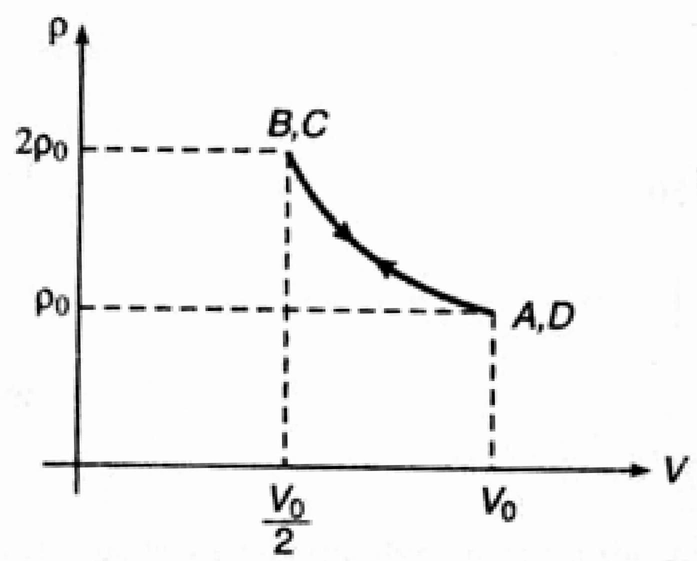
Different values of P, V, T and ρ in tabular form are shown below

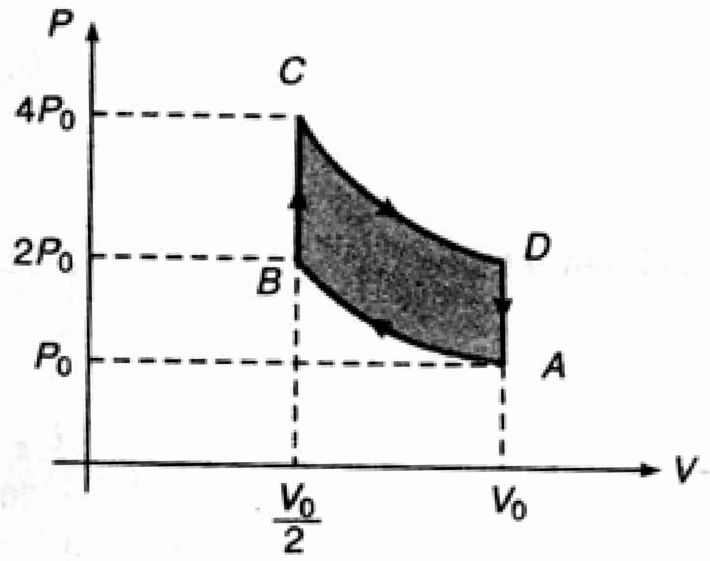
	P	V	T	ρ
A	P_0	V_0	T_0	ρ_0
B	$2P_0$	$\frac{V_0}{2}$	T_0	$2\rho_0$
C	$4P_0$	$\frac{V_0}{2}$	$2T_0$	$2\rho_0$
D	$2P_0$	V_0	$2T_0$	ρ_0

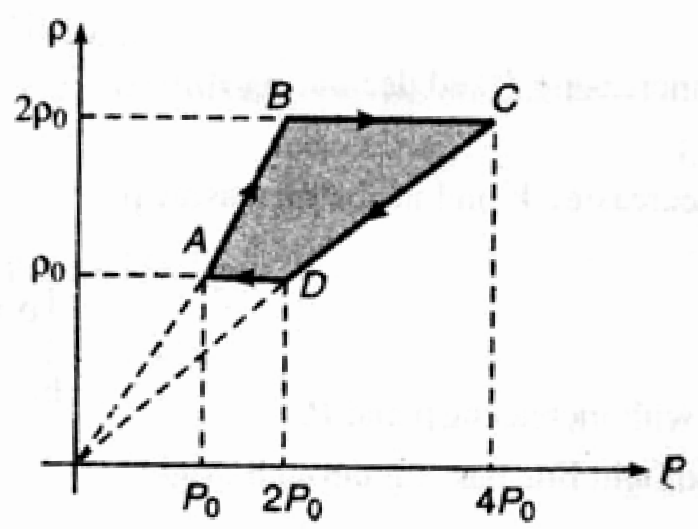
Here $$V_0 = nR\left(\frac{T_0}{P_0}\right)$$

and $$\rho_0 = \frac{P_0 M}{RT_0}$$

The corresponding graphs are as follows :







Chapter 19 Calorimetry and Heat Transfer

JEE Advanced (Subjective Questions)

(a) Between $t = 1$ min to $t = 3$ min, there is no rise in the temperature of substance. Therefore solid melts in this time.

$$L = \frac{Q}{m} = \frac{Ht}{m} = \frac{10 \times 2}{0.5} = 40 \text{ kJ/kg}$$

(b) From $Q = ms\Delta T$ or $s = \dfrac{Q}{m\Delta T}$

Specific heat in solid state $s = \dfrac{10 \times 1}{0.5 \times 15} = 1.33$ kJ/kg-°C

2. Let T_0 be the temperature of surrounding and T the temperature of hot body at some instant. Then :

$$-\frac{dT}{dt} = k(T - T_0)$$

or $$\int_{T_m}^{T} \frac{dT}{T - T_0} = -k \int_0^t dT \qquad (T_m = \text{temperature at } t = 0)$$

Solving this equation, we get

$$T = T_0 + (T_m - T_0)e^{-kt} \qquad \text{(ii)}$$

Maximum temperature it can loose is $(T_m - T_0)$

From Eq. (i)

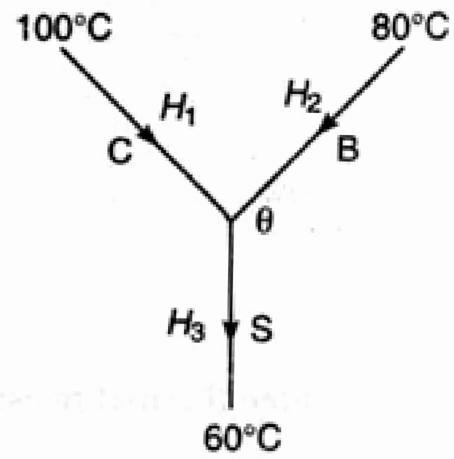
$$T - T_0 = (T_m - T_0)e^{-kt}$$

Given that $$T - T_0 = \frac{T_m - T_0}{2} = (T_m - T_0)e^{-kt}$$

Solving this equation we get

$$t = \frac{\ln(2)}{k} \qquad \textbf{Ans.}$$

3. Let θ be the temperature of junction. Then :



$$H_1 + H_2 = H_3$$

$$\therefore \quad \frac{100 - \theta}{(46/0.92A)} + \frac{80 - \theta}{(13/0.26A)} = \frac{\theta - 60}{(12/0.12A)}$$

Solving this equation we have, $\theta = 84°C$

(b) Heat current in copper rod $H_1 = \dfrac{100 - \theta}{(46/0.92 \times 4)} = 1.28$ cal/s **Ans.**

4. Let heat capacity of flask be C and latent heat of fusion of ice L.

Then, $$C(70 - 40) + 200 \times 1 \times (70 - 40) = 50L + 50 \times 1 \times (40 - 0)$$

or $$3C - 5L = -400 \qquad \ldots\text{(i)}$$

Further, $$C(40 - 10) + 250 \times 1 \times (40 - 10) = 80L + 80 \times 1 \times (10 - 0)$$

or $$3C - 8L = -670 \qquad \ldots\text{(ii)}$$

Solving Eq. (i) and (ii), we have,

$$L = 90 \text{ cal/g} \qquad \textbf{Ans.}$$

5. $$ms\Delta\theta = \text{work done against friction} = (\mu mg\cos\theta).d \qquad (\text{but } \mu mg\cos\theta = mg\sin\theta)$$

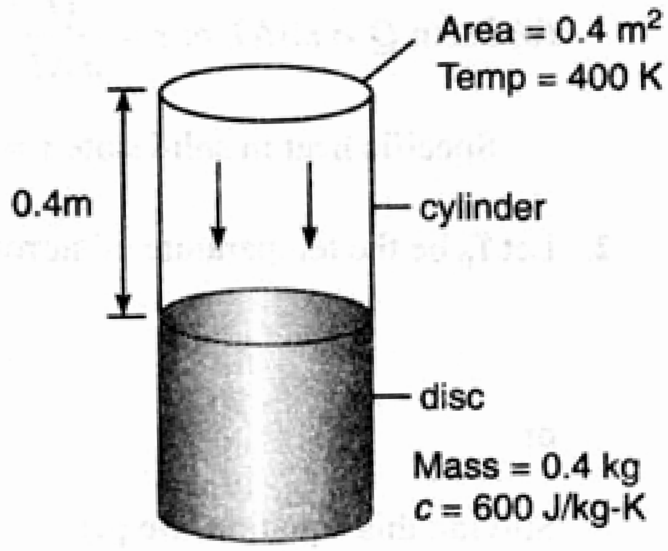
$$\Delta\theta = \frac{(\mu g\cos\theta)d}{s} = \frac{(g\sin\theta)d}{s}$$

$$= \frac{(10)\left(\frac{3}{5}\right)(0.6)}{420}$$

$$= 8.57\times10^{-3}\ °C$$

$$\approx 8.6\times10^{-3}\ °C$$ **Ans.**

6. Using $\dfrac{dQ}{dt} = mc\dfrac{d\theta}{dt} = \dfrac{KA(\theta_1-\theta_2)}{l}$ we have



$$mc\left(\frac{d\theta}{dt}\right) = \frac{KA(400-\theta)}{0.4}$$

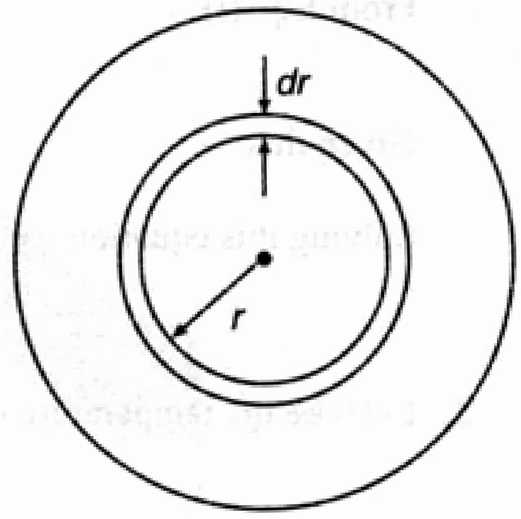
$$(0.4)(600)\frac{d\theta}{dt} = \frac{(10)(0.04)(400-\theta)}{0.4}$$

$$\therefore \quad \left(\frac{d\theta}{400-\theta}\right) = \frac{1}{240}dt$$

$$\therefore \quad \int_0^t dt = 240\int_{300}^{350}\frac{d\theta}{400-\theta}$$

Solving this, we get $t \approx 166$ s **Ans.**

7. Consider a differential cylinder



$$H = \frac{dQ}{dt} = kA\frac{d\theta}{dr} = (2\pi krl)\frac{d\theta}{dr}$$

$$\therefore \quad \frac{H}{2\pi kl}\int_{r_1}^{r_2}\frac{dr}{r} = \int_0^{50} d\theta$$

or $$\frac{H}{2\pi kl}\ln\left(\frac{r_2}{r_1}\right) = 50$$

$$\therefore \quad H = \frac{100\pi kl}{\ln(r_2/r_1)}$$

Now $$Ht = mL$$

$$\therefore \quad t = \frac{mL}{H} = \frac{mL\ln(r_2/r_1)}{100\pi kl}$$ **Ans.**

8. Three thermal resistances are in series. $\left(R = \dfrac{l}{KA}\right)$

$$\therefore \quad R = R_1 + R_2 + R_3 = \frac{2.5\times10^{-2}}{0.125\times137} + \frac{1.0\times10^{-2}}{1.5\times137} + \frac{2.5\times10^{-2}}{1.0\times137}$$

$$= 0.0017\frac{°C-s}{J}$$

Now heat current $$H = \frac{\text{Temperature difference}}{\text{Net thermal resistance}}$$

$$= \frac{30}{0.0017} = 17647\ \text{W}.$$

9. Thermal resistance of plastic coating : $\left(R_t = \dfrac{l}{KA}\right)$

$$R_t = \frac{t}{K(\pi d)l} \qquad (d = \text{diameter})$$

$$= \frac{0.06 \times 10^{-3}}{(0.16 \times 10^{-2} \times 4.18 \times 10^{2})(\pi)(0.64 \times 10^{-3})(2)} = 0.0223\ ^\circ\text{C-s/J}$$

Now

$$i^2 R_e = \frac{\text{TD}}{R_t}$$

$\therefore$

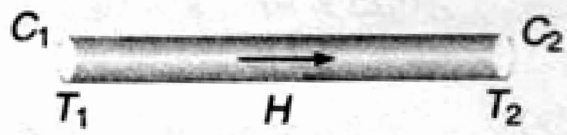
$$\text{TD} = i^2 R_e R_t$$

$$= (5)^2(4)(0.0223)$$

$$= 2.23^\circ\text{C}$$

Ans.

10. Let T_1 be the temperature of C_1 and T_2 the temperature of C_2 at some instant of time. Further let T be the temperature difference at that instant.



Then,

$$C_1\left(-\frac{dT_1}{dt}\right) = H = \frac{T}{l/KA} = \frac{KA}{l}(T) \qquad T = T_1 - T_2$$

and

$$C_2\left(+\frac{dT_2}{dt}\right) = H = \frac{T}{l/KA} = \frac{KA}{l}(T) \qquad \therefore -\frac{dT}{dt} = -\frac{dT_1}{dt} + \frac{dT_2}{dt}$$

$\therefore$

$$-\frac{dT_1}{dt} = \frac{KA}{lC_1}(T) \quad \text{and} \quad +\frac{dT_2}{dt} = \frac{KA}{lC_2}$$

Further,

$$-\frac{dT}{dt} = -\frac{dT_1}{dt} + \frac{dT_2}{dt} = \frac{KA(C_1 + C_2)}{lC_1C_1}.T$$

or

$$\int_{\Delta T_0}^{T} \frac{dT}{T} = -\frac{KA(C_1 + C_2)}{lC_1C_2}\int_0^t dt$$

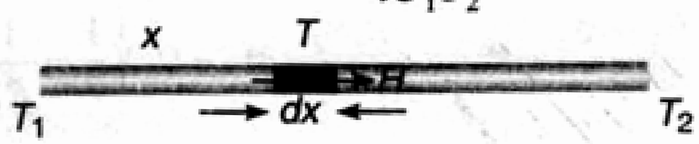
Solving we get :

$$T = \Delta T_0 e^{-\alpha t}$$

where

$$\alpha = \frac{KA(C_1 + C_2)}{lC_1C_2}$$

11.



$$H = \frac{\text{TD}}{R} = \frac{(-dT)}{(dx)/KA} = -\left(\frac{dT}{dx}\right)\left(\frac{a}{T}A\right) = \text{constant} \qquad \ldots\text{(i)}$$

$\therefore$

$$\int_{T_1}^{T_2} -\frac{dT}{T} = \frac{H}{aA}\int_0^l dx$$

$$\ln\left(\frac{T_1}{T_2}\right) = \frac{Hl}{aA} \quad \text{or} \quad H = \frac{aA}{l}\ln(T_1/T_2)$$

Substituting in Eq. (i), we have

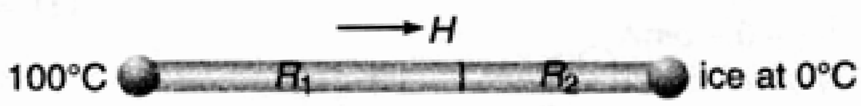
$$\frac{aA}{l}\ln\left(\frac{T_1}{T_2}\right) = -\left(\frac{dT}{dx}\right).\frac{aA}{T}$$

or $$\int_{T_1}^{T}\frac{dT}{T}=-\frac{\ln\left(\frac{T_1}{T_2}\right)}{l}\int_0^x dx$$

∴ $$\ln\left(\frac{T}{T_1}\right)=-\frac{x}{l}\ln(T_1/T_2)=\ln\left(\frac{T_1}{T_2}\right)^{-\frac{x}{l}}$$

or $$\frac{T}{T_1}=\left(\frac{T_1}{T_2}\right)^{-\frac{x}{l}} \quad \text{or} \quad T=T_1\left(\frac{T_1}{T_2}\right)^{-\frac{x}{l}}=T_1\left(\frac{T_2}{T_1}\right)^{\frac{x}{l}}$$ **Ans.**

12. $$L\left(\frac{dm}{dt}\right)=\frac{\text{TD}}{R_1+R_2}$$



or $$\frac{80\times 360}{3600}=\frac{100}{\frac{20}{0.25\times 10}+\frac{10}{k\times 10}}$$

Solving this equation we get :

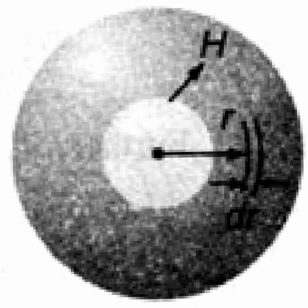
$$k=0.222 \text{ cal/cm-s-°C}$$ **Ans.**

13. $$H=\frac{-d\theta}{dr/K(4\pi r^2)}=\left(-\frac{d\theta}{dr}\right)(4\pi kr^2) \qquad \text{...(i)}$$

or $$\int_{R_1}^{R_2}\frac{dr}{r^2}=-\frac{4\pi k}{H}\cdot\int_{\theta_1}^{\theta_2}d\theta$$

or $$\frac{R_2-R_1}{R_1R_2}=\frac{4\pi k}{H}(\theta_1-\theta_2)$$

∴ $$H=\frac{4\pi k(R_1R_2)(\theta_1-\theta_2)}{R_2-R_1}$$



Substituting this value of H in Eq. (i) we have,

$$\frac{R_1R_2}{R_2-R_1}(\theta_1-\theta_2)=-\frac{d\theta}{dr}(r^2)$$

∴ $$\int_{\theta_1}^{\left(\frac{(\theta_1+\theta_2)}{2}\right)}d\theta=-\frac{R_1R_2}{R_2-R_1}(\theta_1-\theta_2)\int_{R_1}^{r}\frac{dr}{r^2}$$

or $$\frac{\theta_1+\theta_2}{2}-\theta_1=\frac{R_1R_2}{R_2-R_1}(\theta_1-\theta_2)\left[\frac{1}{r}-\frac{1}{R_1}\right]$$

or $$\frac{\theta_1-\theta_2}{2}=\frac{R_1R_2}{R_2-R_1}(\theta_1-\theta_2)\left(\frac{1}{R_1}-\frac{1}{r}\right)$$

$$\frac{1}{R_1}-\frac{1}{r}=\frac{R_2-R_1}{2R_1R_2}$$

$$\frac{1}{r}=\frac{1}{R_1}-\frac{R_2-R_1}{2R_1R_2}=\frac{R_1+R_2}{2R_1R_2}$$

or $$r=\frac{2R_1R_2}{R_1+R_2}$$ **Ans.**

Experimental Skills

1. No, prongs execute transverse vibrations as the vibrations and the stem executes longitudinal vibrations.
2. No, frequency is exactly the same for both kinds of vibrations.
3. Amplitudes in the two cases are different ; prongs have larger amplitudes than that of the stem.
4. The frequency of a tuning fork decreases with the increase in length of its prongs.
5. The one which has smaller and thicker prongs will be of higher frequency.
6. The frequency will increase.
7. Yes, its frequency will decrease.
8. These are the frequencies identical with those of the major diatonic scale which is a musical scale (C, D, E, F, G, A, B, C_1). Frequency ratio of the first and eight note is 1 : 2.
9. It signifies the note of the major diatonic scale (C, D, E, F, G, A, B, C_1). It means that the letter C stands for first note of the scale and corresponds to a frequency of 256 Hz Similarly D corresponds to a frequency of 288 Hz and so on.
10. A tone is a simple sound resulting from a pure harmonic motion. On the other hand a note is a complex sound made up of a complex periodic motion as obtained by the superposition of a number of pure simple harmonic motions.
11. This is because the prongs are vibrating perpendicular to their length whereas the stem is vibrating along its length.
12. (i) If there is only one prong, then the vibrations will die out quickly as and when the stem is touched and (ii) we want to have a node in between the two antinodes which exist at the free ends of the two prongs.
13. If we strike a tuning fork with a great force, a pure note may not be produced, but overtones may be produced.
14. Energy from one prong is transmitted to the other prong through the body of the tuning fork and as such the other prong also starts vibrating.
15. No, sound can't travel in vacuum because sound needs a material medium for its propagation.
16. In hydrogen its velocity is higher because velocity of sound is inversely proportional to square root of the density of the medium. Since hydrogen is a lighter medium than oxygen, sound will have a higher velocity in hydrogen.
17. The air column above water over which the vibrating tuning fork is held, is the resonance column.
18. It acts as a rigid wall for the reflection of sound waves. We can also adjust the length of air column by adjusting the level of water.
19. Yes, mercury can be used; rather mercury will be better in the sense that air above it will not become wet. But mercury is costly and dangerous for health. That's why water is used.
20. Longitudinal stationary waves.
21. A vibrating tuning fork is placed near the end of the resonance column which gives rise to the longitudinal waves in air column. These waves are reflected from the water surface. Thus due to superposition of the direct and reflected waves, stationary waves are formed.
22. The vibrating tuning fork is placed near the open end of the resonance column. The length of air column between the open end and the water surface is increased starting from a minimum, such that loudness of sound increases to a quite large extent. The length of the air column above the water surface up to the end of the tube is the first resonating length.
23. Node is at the water surface and antinode is at the open end of the tube.
24. No, it is slightly above the open end.

25. The distance between a node and the nearest antinode is given as $\lambda/4$, where λ is the wavelength of the sound. Here in this case, this distance is $l_1 + x$, where l_1 is the length of the air column above the water surface and $x = 0.3\ d$, is known as end correction.
Therefore,

$$\lambda/4 = l_1 + x$$

or

$$\lambda = 4\ (l_1 + x)$$

26. Yes, a long tube having cross-section of any shape will serve the purpose.

27. In order to obtain two resonance positions which enable us to eliminate end correction.

28. It is kept horizontally just above the open end of the resonating tube in such a manner that the direction of vibrations of the prongs of the tuning fork is along the length of the air column.

29. When after first resonance is reached, if we increase the length of air column, a situation of resonance is again reached when the resonating length is approximately thrice the first resonant length. This is called second resonance.

30. In the case of second resonance, energy gets distributed over a larger region and as such second resonance becomes feebler.

31. In between the node at the water surface and antinode at the open end of the tube, there is one more node and one more antinode. These, node and antinode, are at distances $l_2/3$ and $2l_2/3$ from the open end respectively (neglecting the end correction), where l_2 is the length of the resonating column in the second case.

32. In this case

$$\frac{3\lambda}{4} = l_2 + x \text{ or } \lambda = \frac{4}{3}\ (l_2 + x)$$

where x is the end correction.

33. Yes, we can. If we take both the positions of resonance then we get

$$\frac{3\lambda}{4} - \frac{\lambda}{4} = (l_2 + x) - (l_1 - x)$$

or

$$\frac{\lambda}{2} = l_2 + l_1 \text{ or } \lambda = 2(l_2 - l_1)$$

and hence λ so obtained is free from the end correction.

34. Yes. Once λ of waves in air column is known and frequency f of the tuning fork is the same as that of resonating air column then by using the relation $v = f\lambda$, v can be determined.

36. $n_1 = \dfrac{v}{2(l_2 - l_1)}$

$$n_2 = \frac{v}{2\ (l_2' - l_1')} \quad \therefore \quad \frac{n_1}{n_2} = \frac{l_2' - l_1'}{l_2 - l_1}$$

41. $v = 2n\ (l_2 - l_1)$

$$300 = 2n\left(\frac{97 - 37}{100}\right) \quad \therefore \quad 300 = 2n\left(\frac{60}{100}\right)$$

$\therefore$ $$n = 250 \text{ Hz}$$ **Ans.**

42. $e = \dfrac{l_2 - 3l_1}{2}$

$\therefore$ $$e = 1 \text{ cm} = \frac{1}{100} \text{ m}$$

$\therefore$ $$\frac{1}{100} = \frac{l_2 - 3\ (0.15)}{2}$$

$\therefore$ $$l_2 = 0.47 \text{ m}$$

$\therefore$ $$l = 47 \text{ cm}$$ **Ans.**

43. Maximum length of air column $l = \frac{3\lambda}{4}$

$\therefore$ $$l = \frac{3v}{4f}$$

where $v = 340 \text{ ms}^{-1}$ and $f = 340$ Hz

$\therefore$ $$l = \frac{3}{4} \text{ m} = 0.75 \text{ m} = 75 \text{ cm}$$

So, length of water column is $l_{water} = 120 \text{ cm} - 75 \text{ cm} = 45 \text{ cm}$ **Ans.**

44. $\frac{n_1}{n_2} = \frac{l_2' - l_1'}{l_2 - l_1} = \frac{90-30}{30-10} = \frac{3}{1}$

45. $l_2 = 3l_1 = 48$ cm

46. No, the specific heat of a substance changes with temperature.

47. Thermal capacity of a body is the amount of heat required to raise the temperature of the whole body through 1 °C.

Thermal capacity = Mass × Specific heat = $m \times s$

48. Thermal capacity has the unit calorie/gm and water equivalent gm.

49. Since specific heat of copper is small, so for a certain amount of heat transferred, rise in temperature is large and the change in temperature can be measured accurately.

50. Heat lost by aluminium = $500 \times c \times (100 - 46.8)$ cal

$\therefore$ Heat lost = 26600 c

Heat gained by water and calorimeter = $300 \times 1 \times (46.8 - 30) + 500 \times 0.093 \times (46.8 - 30)$

$\therefore$ Heat gained = 5040 + 781.2 = 5821.2

Now, heat lost = Heat gained

$\therefore$ $$26600 \text{ c} = 5821.2$$

$\therefore$ $$c \approx 0.22 \text{ cal g}^{-1} (^\circ\text{C})^{-1}$$

51. Thermal capacity = $(0.04 \text{ kg})(4.2 \times 10^2 \text{ J kg}^{-1} \,^\circ\text{C}^{-1})$

$= 16.8$ J/ kg **Ans.**

52. Heat lost = Heat gained

$\therefore$ $$m_1 c_1 \Delta T_1 = m_2 c_2 \Delta T_2$$

$\therefore$ $$c_1 = \frac{m_2 c_2 \Delta T_2}{m_1 \Delta T_1} = \frac{0.5 \times 4.2 \times 10^3 \times 3}{0.2 \times 77} \text{ J kg}^{-1} \,^\circ\text{C}^{-1}$$

$\therefore$ $$c_1 = 0.41 \times 10^3 \text{ J kg}^{-1} \,^\circ\text{C}^{-1}$$ **Ans.**

53. Heat lost = Heat gained

$$0.20 \times 10^3 \times c(150 - 40)$$

$$= 150 \times 1 \times (40 - 27) + 0.025 \times 10^3 \times (40 - 27) \qquad \{\because c_{water} = 1 \text{ cal g}^{-1} (^\circ\text{C})^{-1}\}$$

$\therefore$ $$22000\, c = 1950 + 325$$

$\therefore$ $$22000\, c = 2275$$

$\therefore$ $$c = 0.10 \text{ cal g}^{-1} (^\circ\text{C})^{-1}$$ **Ans.**

54. No, it holds good only when the difference in temperatures is small, in the range (20°C – 30°C).

55. $\frac{dQ}{dt} \propto (\theta - \theta_0)$

where the symbols have their usual meanings.

56. It is a straight line.