In addition to geostationary equatorial orbits, there are two more orbits which are being used for communication. These are

(a) **Polar circular orbit** This orbit passes over or very close to the poles. It is approximately at a height of 1000 km from earth.

(b) **Highly elliptical (inclined) orbits**

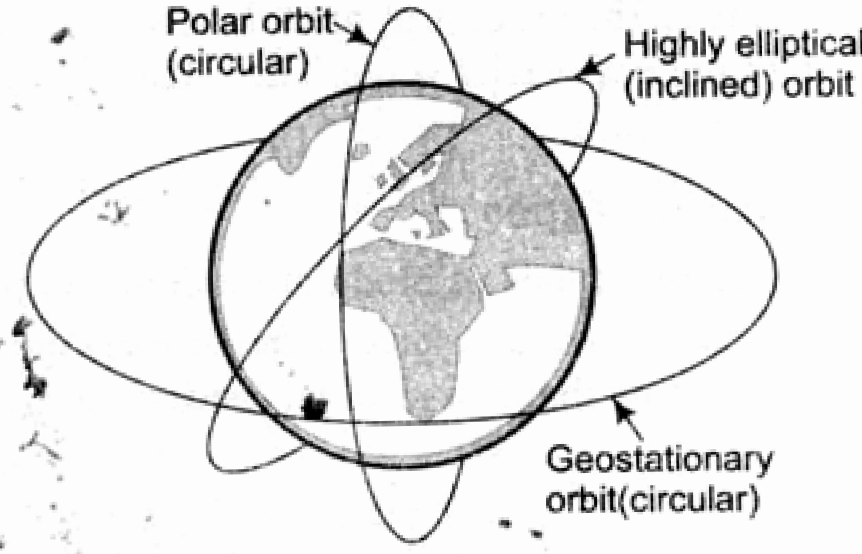


Fig. 33.8. A schematic diagram of various satellite orbits used in satellite communication

Remote sensing

Remote sensing is an application of satellite communication. It is the art of obtaining information about an object or area acquired by a sensor that is not in direct contact with the target of investigation. Any photography is a kind of remote sensing. If we want to cover large areas for which information is required, we have to take photographs from larger distances. This is called aerial photography. Town and country planning can also be done by remote sensing.

A satellite equipped with appropriate sensors is used for remote sensing. Taking photograph of any object relies on the reflected wave from the object. We use visible light in normal photography. In principle, waves of any wavelength in the electromagnetic spectrum can be used for this purpose by using suitable sensors.

Some applications of remote sensing include meteorology (development of weather systems and weather forecasting), climatology (monitoring climate changes), and oceanography etc.

33.6 Modulation

In this section we will discuss in detail about modulation. What is it ? What is the need of modulation or how the modulation is done etc.

No signal in general is a single frequency signal but it spreads over a range of frequencies called the signal bandwidth. Suppose we wish to transmit an electronic signal in the Audio Frequency (20 Hz-20 kHz) range over a long distance. Can we do it ? No it cannot because of the following problems.

(i) **Size of antenna** For transmitting a signal we need an antenna. This antenna should have a size comparable to the wavelength of the signal. For an electromagnetic wave of frequency 20 kHz, the wavelength is 15 km. Obviously such a long antenna is not possible and hence direct transmission of such signal is not practical.

(ii) **Effective power radiated by anantenna** Power radiated by an antenna $\propto \frac{1}{(\lambda)^2}$.

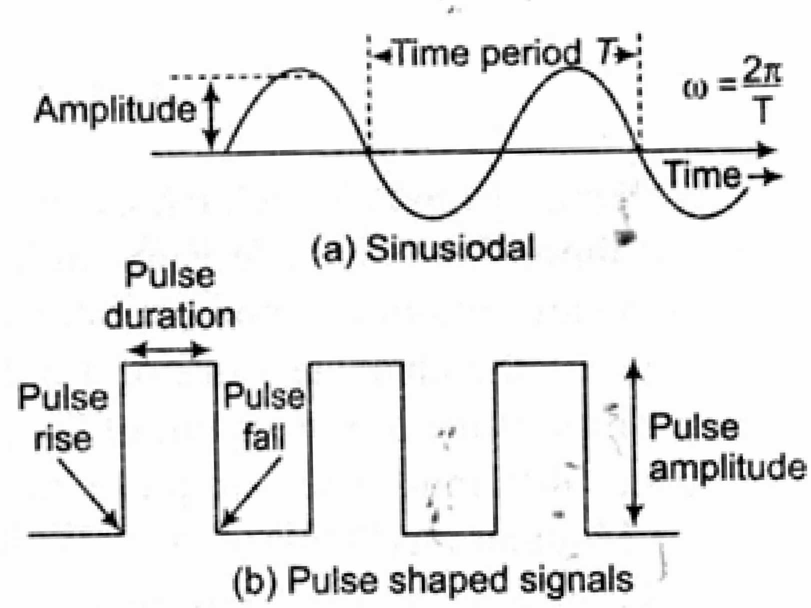


Fig. 33.9.

Therefore power radiated by large wavelength would be small. For good transmission we require high power and hence need of high frequency transmission is required.

(iii) **Mixing up of signal from different transmitters** Another problem in transmitting baseband signals directly is of intermixing of different signals. Suppose many people are talking at the same time or many transmitters are transmitting baseband information signals simultaneously. All these signals will get mixed and there is no simple way to distinguish between them. A possible solution to all above problems is using communication at high frequencies and allotting a band of frequencies to each message signal for its transmission.

Thus in the process of modulation the original low frequency information signal is attached with the high frequency carrier wave. The carrier wave may be continuous (sinusoidal) or in the form of pulses as shown in figure.

Modulation Types

Different types of modulation depend upon the specific characteristic of the carrier wave which is being varied in accordance with the message signal.

We know that a sinusoidal carrier wave can be expressed as

$$E = E_0 \sin(\omega t + \phi)$$

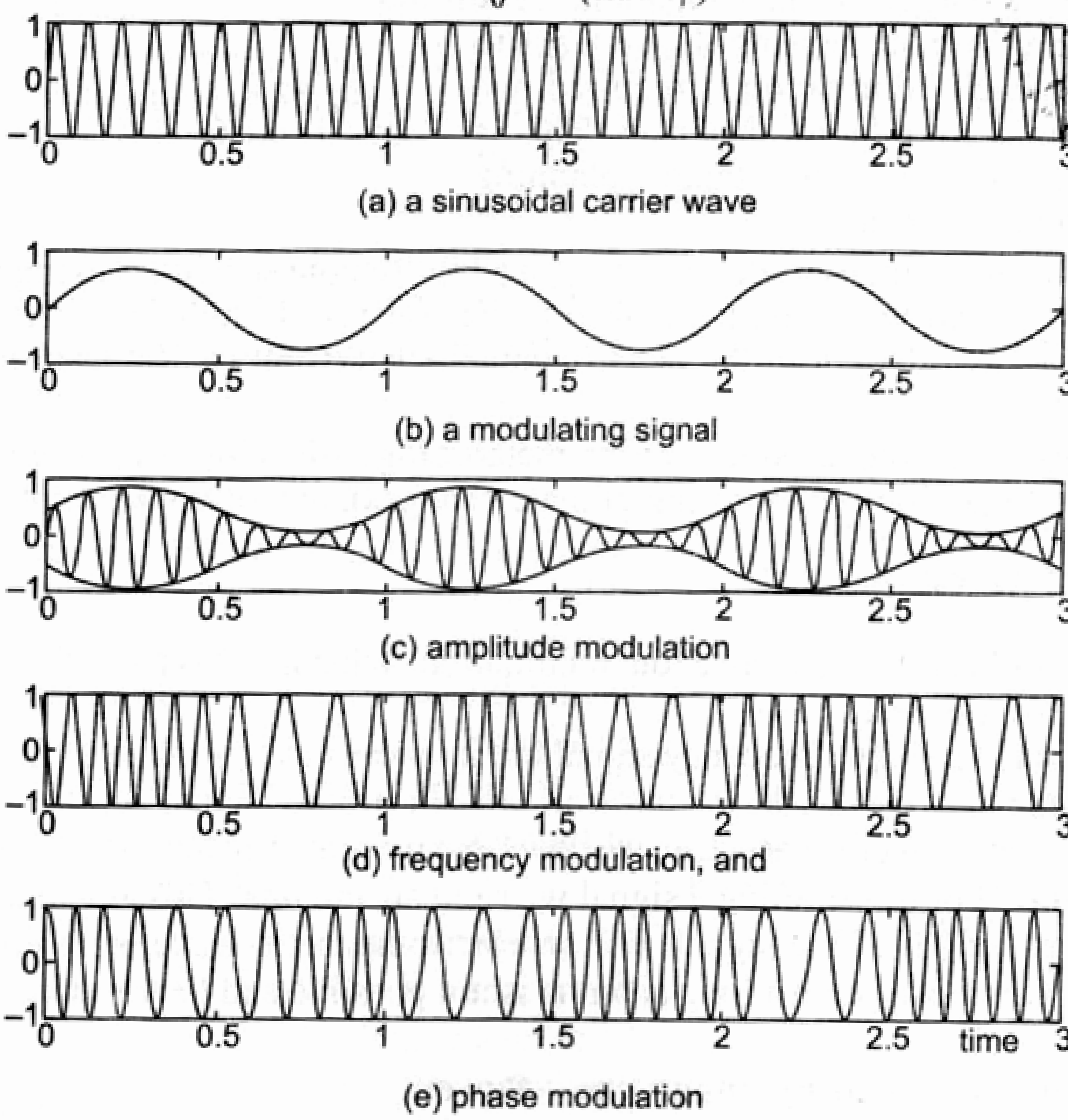


Fig. 33.10. Modulation of a carrier wave

The three distinct characteristics are Amplitude (E_0), angular frequency (ω) and phase angle (ϕ). Either of these three characteristics can be varied in accordance with the signal. The three types of modulation are, amplitude modulation, frequency modulation and phase modulation.

Similarly, the characteristics of a pulse are, Pulse amplitude, pulse duration or pulse width and pulse position (time of rise or fall of the pulse amplitude).

Hence, different types of pulse modulation are, Pulse Amplitude Modulation (PAM), Pulse Duration Modulation (PDM) or Pulse Width Modulation (PWM) and Pulse Position modulation (PPM).

In this chapter, we shall confine to amplitude modulation (of continuous wave or sinusoidal wave) only.

33.7 Amplitude Modulation

In this type of modulation, the amplitude of the carrier signal varies in accordance with the information signal. The high frequency carrier wave (Fig.a) is superimposed on low frequency information signal (Fig. b). As a result, in the amplitude modulated carrier wave, amplitude no longer remains constant, but its envelope has similar sinusoidal variation as that of the low frequency or modulating signal. The carrier wave frequency ranges from 0.5 to 2.0 MHz. AM signals are noisy because electrical noise signals significantly affect this.

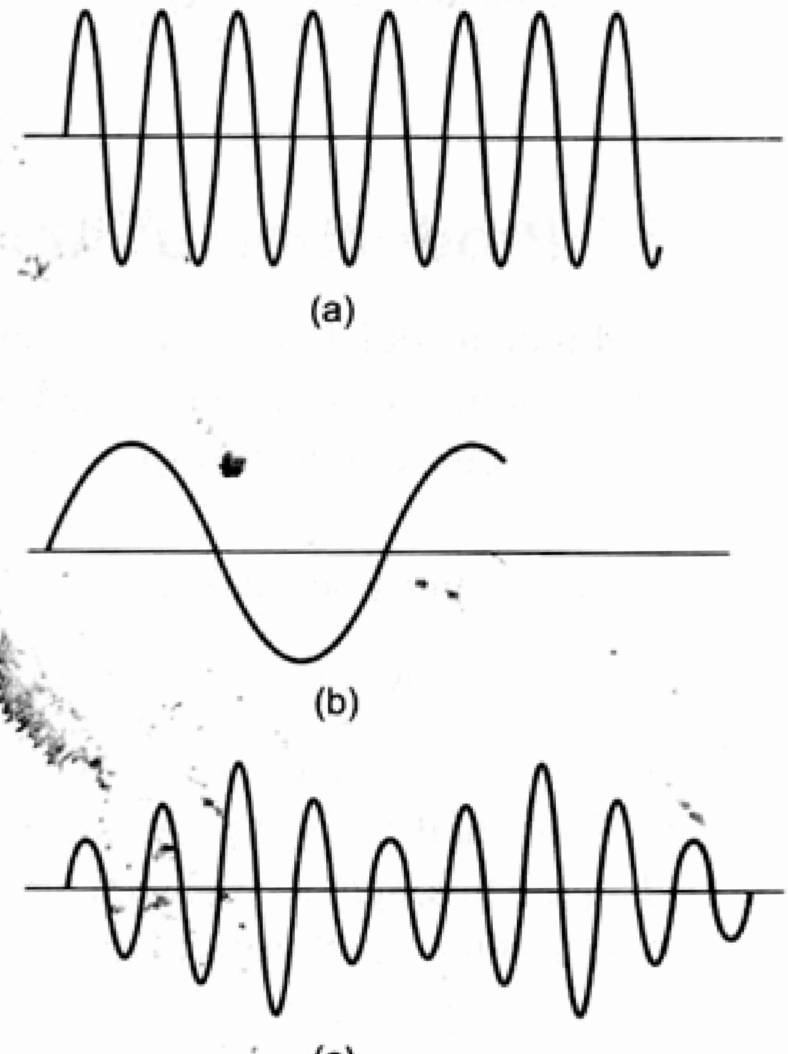


Fig. 33.11

Let $C = A_c \sin \omega_c t$ represents a carrier wave and $S = A_s \sin \omega_s t$ represents the signal wave

Then after making calculations we see that the modulated signal wave equation can be written as

$$m = A_c \sin \omega_c t + \frac{\mu A_c}{2} \cos (\omega_c - \omega_s) t - \frac{\mu A_c}{2} \cos (\omega_c + \omega_s) t \quad \text{...(i)}$$

where, $\mu = \frac{A_s}{A_c}$ is called the **modulation index.** In practice, μ is kept ≤ 1 to avoid distortion.

In Eq. (i), $\omega_c - \omega_s$ and $\omega_c + \omega_s$ are respectively called the lower side and upper side frequencies.

The modulated signal therefore consists of the carrier wave of frequency ω_c plus two sinusoidal waves each with a frequency slightly different from ω_c, known as side bands.

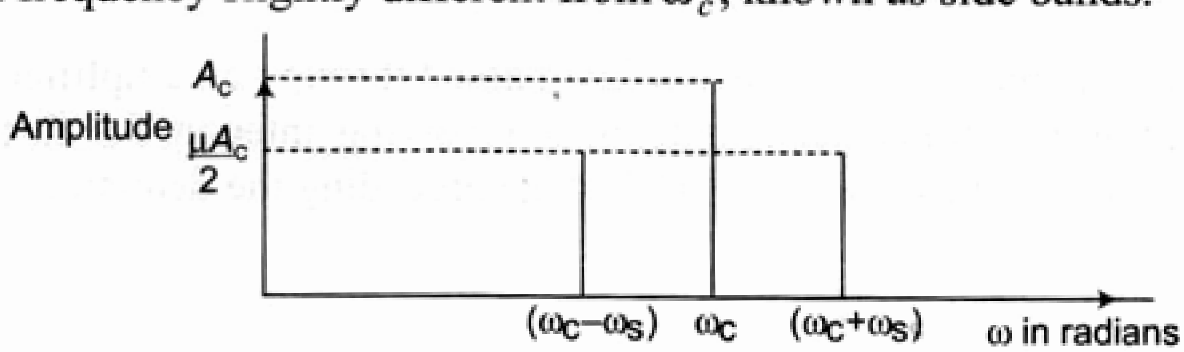


Fig. 33.12. A plot of amplitude *versus* ω for an amplitude modulated signal

Sample Example 33.2 *A message signal of frequency 10 kHz and peak voltage of 10 volts is used to modulate a carrier wave of frequency 1 MHz and peak voltage of 20 volts. Determine (a) modulation index, (b) the side bands produced.*

Solution (a) **Modulation Index**

$$\mu = \frac{A_s}{A_c} = \frac{10}{20} = 0.5$$ **Ans.**

(b) **Side Bands**

1 MHz = 1000 kHz

The side bands are, $(\omega_c - \omega_s)$ and $(\omega_c + \omega_s)$ or we can write $(f_c - f_s)$ and $(f_c + f_s)$

$$\therefore \qquad (f_c - f_s) = (1000 - 10) = 990 \text{ kHz And}$$
$$(f_c + f_s) = (1000 + 10) = 1010 \text{ kHz}$$

Ans.

33.8 Production of Amplitude Modulated Wave

Amplitude modulation can be produced by a variety of methods. A simple method is shown in the blocks diagram of following figure.

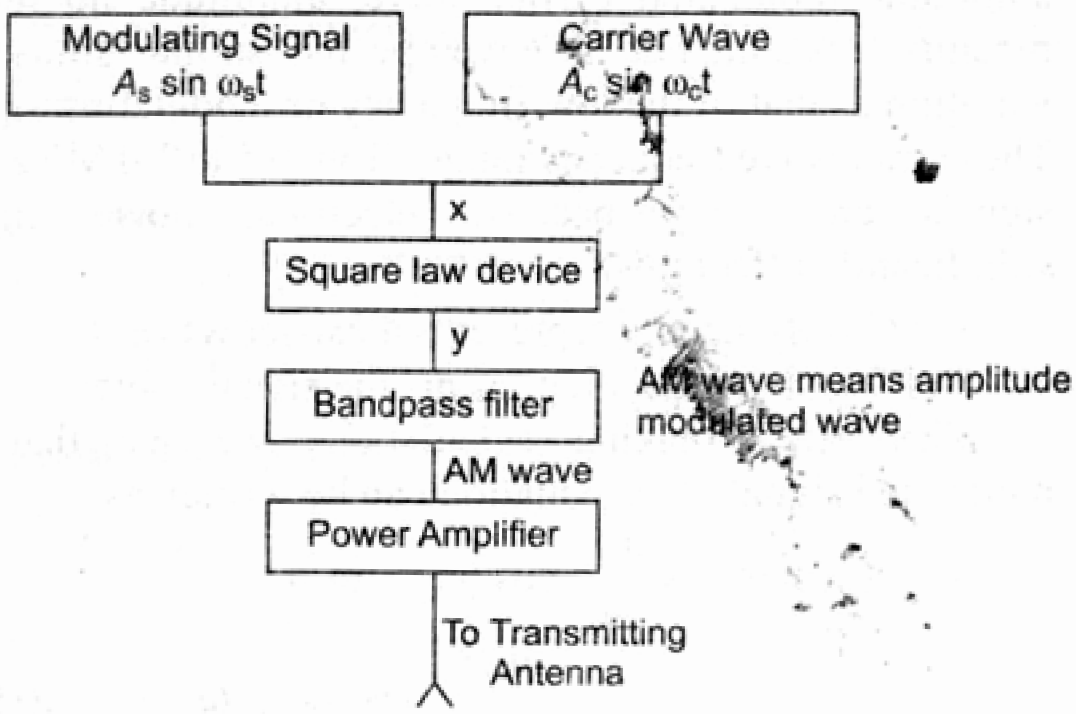


Fig.33.13

In the y function shown in block diagram there is a DC term $\frac{C}{2}(A_C^2 + A_S^2)$ and sinusoids of frequencies $\omega_S, 2\omega_S, 2\omega_C, \omega_c - \omega_S$ and $\omega_C + \omega_S$. As shown in block diagram this signal y is passed through a band pass filter which rejects dc and the sinusoids of frequencies ω_S, $2\omega_S$ and $2\omega_C$. After bandpass filter, the frequencies remaining are $\omega_C, \omega_C - \omega_S$ and $\omega_C + \omega_S$. The output (AM wave) of the band pass filter therefore is of the same form as Eq. (i) of previous article.

It is further to be noticed that this modulated signal cannot be transmitted as such. This signal is passed through a power amplifier and then the signal is fed to transmitter antenna.

33.9 Detection of Amplitude Modulated Wave

Signal received from the receiving antenna is first passed through an amplifier because the signal becomes weak in travelling from transmitting antenna to receiving antenna. For further processing, the signal is passed through intermediate frequency (IF) stage preceding the detection.

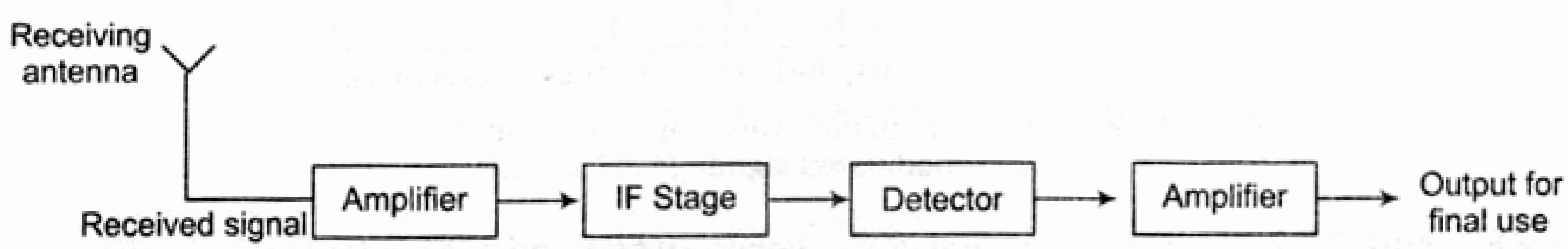


Fig. 33.14. Block diagram of a receiver

At this stage the carrier frequency is usually changed to a lower frequency. The output signal from detector may not be strong enough. So it is further passed through an amplifier for final use. The block diagram of all steps is shown in above figure.

Inside the Detector

Detection is the process of recovering the signal from the carrier wave. In the previous two articles we have seen that the modulated carrier wave contains the frequencies ω_C, $\omega_C + \omega_S$ and $\omega_C - \omega_S$. In order to obtain the original message signal S $(= A_S \sin \omega_S t)$ of angular frequency ω_S a simple method is shown in the form of a block diagram as shown below.

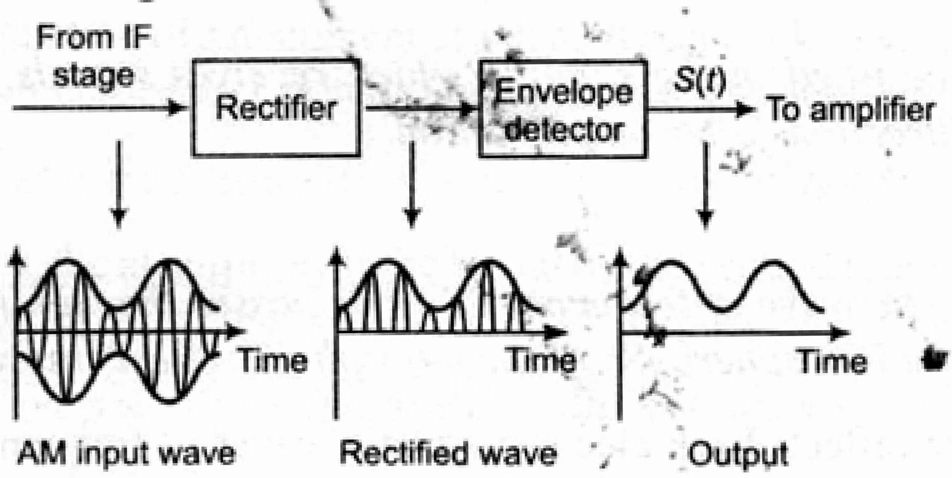


Fig. 33.15. Block diagram of a detector for AM signal

Note *that the quantity on y-axis can be current or voltage.*

Extra Points

1. **The Internet** Everyone is well aware with internet. It has billions of users worldwide. It was started in 1960's and opened for public use in 1990's. Its applications include, E-mail, file transfer, websites, E-commerce and chating etc.
2. **Facsimile (FAX)** It first scans the image of contents of a document. Then those are converted into electronic signals. These electronic signals are sent to another FAX machine using telephone lines. At the destination, signals are reconverted into a replica of the original document.

Solved Examples

Example 1 *Name the device fitted in the satellite which receives signals from Earth station and transmits them in different directions after amplification.*

Solution Transponder.

Example 2 *An electromagnetic wave of frequency 28 MHz passes through the lower atmosphere of Earth and gets incident on the ionosphere. Shall the ionosphere reflects these waves?*

Solution Yes. The ionosphere reflects back electromagnetic waves of frequency less than 30 MHz.

Example 3 *Which waves constitute amplitude-modulated band?*

Solution Electromagnetic waves of frequency less than 30 MHz constitute amplitude-modulated band.

Example 4 *Give the frequency ranges of the following (i) High frequency band (HF) (ii) Vary high frequency band (VHF) (iii) Ultra high frequency band (iv) Super high frequency band.*

Solution (i) 3 MHz to 30 MHz (ii) 30 MHz to 300 MHz (iii) 300 MHz to 3000 MHz

(iv) 3000 MHz to 30,000 MHz.

Example 5 *State the two functions performed by a modem.*

Solution (i) Modulation (ii) Demodulation.

Example 6 *Why is the transmission of signals using ground waves restricted up to a frequency of 1500 kHz ?*

Solution This is because at frequencies higher than 1500 kHz, there is an increase in the absorption of signal by the ground.

Example 7 *How does the effective power radiated by an antenna vary with wavelength?*

Solution Power radiated by an antenna $\propto \left(\frac{1}{\lambda}\right)^2$.

Example 8 *Why is it necessary to use satellites for long distance TV transmission?*

Solution Television signals are not properly reflected by the ionosphere. So, reflection is affected by satellites.

Example 9 *Why long distance radio broadcasts use shortwave bands ?*

Solution This is because ionosphere reflects waves in these bands.

Example 10 *What is a channel bandwidth ?*

Solution Channel bandwidth is the range of frequencies that a system can transmit with efficient fidelity.

Example 11 *Give any one difference between FAX and e-mail systems of communication.*

Solution Electronic reproduction of a document at a distant place is known as FAX. In e-mail system, message can be created, processed and stored. Such facilities are not there in Fax system.

Example 12 *Why ground wave propagation is not suitable for high frequency?*

Solution At high frequency, the absorption of the signal by the ground is appreciable. So, ground wave propagation is not suitable for high frequency.

Example 13 *What is the purpose of modulating a signal in transmission?*

Solution A low frequency signal cannot be transmitted to long distances because of many practical difficulties. On the other hand, effective transmission is possible at high frequencies. So modulation is always done in communication systems.

Example 14 *What is a transducer?*

Solution A device which converts energy in one form to another is called a transducer.

Example 15 *Why do we need a higher band width for transmission of music compared to that for commercial telephone communication?*

Solution As compared to speech signals in telephone communication, the music signals are more complex and correspond to higher frequency range.

Example 16 *From which layer of the atmosphere, radio waves are reflected back?*

Solution The electromagnetic waves of radio frequencies are reflected by ionosphere.

Example 17 *Why sky waves are not used in the transmission of television signals?*

Solution The television signals have frequencies in 100-200 MHz range. As the ionosphere cannot reflect radio waves of frequency greater than 40 MHz back to the earth, the sky waves cannot be used in the transmission of TV signals.

Example 18 *Why are short waves used in long distance broadcasts?*

Solution The short waves (wavelength less than 200 m or frequencies greater than 1,00 kHz) are absorbed by the earth due to their high frequency but are effectively reflected by Flayer in ionosphere. After reflection from the ionosphere, the short waves reach the surface of earth back only at a large distance from the transmitter. For this reason, short waves are used in long distance transmission.

Example 19 *Define the term critical frequency in relation to sky wave propagation of electromagnetic waves.*

Solution The highest value of the frequency of radio waves, which on being radiated towards the ionosphere at some angle are reflected back to the earth is called critical frequency.

Example 20 *What mode of communication is employed for transmission of TV signals?*

Solution Space wave communication.

EXERCISES

Single Correct Option

1. Three waves A, B and C of frequencies 1600 kHz, 5 MHz and 60 MHz, respectively are to be transmitted from one place to another. Which of the following is the most appropriate mode of communication?
   (a) A is transmitted *via* space wave while B and C transmitted *via* sky wave.
   (b) A is transmitted *via* ground wave, B via sky wave and C *via* space wave.
   (c) B and C are transmitted *via* ground wave while A is transmitted via sky wave.
   (d) B is transmitted *via* ground wave while A and C are transmitted *via* space wave.

2. A 100 m long antenna is mounted on a 500 m tall building. The complex can become a transmission tower for waves with λ
   (a) ~ 400 *m* (b) ~ 25 *m* (c) ~ 150 *m* (d) ~ 2400 *m*

3. A speech signal of 3 kHz is used to modulate a carrier signal of frequency 1 MHz, using amplitude modulation. The frequencies of the side bands will be
   (a) 1.003 MHz and 0.997 MHz (b) 3001 kHz and 2997 kHz
   (c) 1003 kHz and 1000 kHz (d) 1 MHz and 0.997 MHz

4. A message signal of frequency ω_m is superposed on a carrier wave of frequency ω_c to get an amplitude modulated wave (AM). The frequency of the AM wave will be
   (a) ω_m (b) ω_c (c) $\frac{\omega_c + \omega_m}{2}$ (d) $\frac{\omega_c - \omega_m}{2}$

5. A basic communication system consists of
   (A) transmitter (B) information source (C) user of information (D) channel
   (E) receiver
   Choose the correct sequence in which these are arranged in a basic communication system.
   (a) ABCDE (b) BADEC (c) BDACE (d) BEADC

6. Which of the following frequencies will be suitable for beyond the horizon communication using sky waves?
   (a) 10 kHz (b) 10 MHz (c) 1 GHz (d) 1000 GHz

7. Frequencies in the UHF range normally propagate by means of
   (a) ground waves (b) sky waves (c) surface waves (d) space waves

8. Digital signals
   (i) do not provide a continuous set of values
   (ii) represent values as discrete steps
   (iii) can utilize binary system and
   (iv) can utilize decimal as well as binary systems

Which of the above statements are true?

(a) (i) and (ii) only
(b) (ii) and (iii) only
(c) (i), (ii) and (iii) but not (iv)
(d) All of (i), (ii), (iii) and (iv)

More than One Correct Options

9. A TV transmission tower has a height of 240 m. Signals broadcast from this tower will be received by LOS communication at a distance of (assume the radius of earth to be 6.4×10^6 m)

(a) 100 km
(b) 24 km
(c) 55 km
(d) 50 km

10. An audio signal of 15 kHz frequency cannot be transmitted over long distances without modulation because

(a) the size of the required antenna would be at least 5 km which is not convenient
(b) the audio signal can not be transmitted through sky waves
(c) the size of the required antenna would be at least 20 km, which is not convenient
(d) effective power transmitted would be very low, if the size of the antenna is less than 5 km

11. Audio sine waves of 3 kHz frequency are used to amplitude modulate a carrier signal of 1.5 MHz. Which of the following statements are true?

(a) The side band frequencies are 1506 kHz and 1494 kHz
(b) The bandwidth required for amplitude modulation is 6 kHz
(c) The bandwidth required for amplitude modulation is 3 MHz
(d) The side band frequencies are 1503 kHz and 1497 kHz

12. In amplitude modulation, the modulation index μ, is kept less than or equal to 1 because

(a) $\mu > 1$ will result in interference between carrier frequency and message frequency, resulting into distortion.
(b) $\mu > 1$ will result in overlapping of both side bands resulting into loss of information.
(c) $\mu > 1$ will result in change in phase between carrier signal and message signal.
(d) $\mu > 1$ indicates amplitude of message signal greater than amplitude of carrier signal resulting into distortion.

Subjective Questions

13. Compute the LC product of a tuned amplifier circuit required to generate a carrier wave of 1 MHz for amplitude modulation.

14. A carrier wave of peak voltage 12 V is used to transmit a message signal. What should be the peak voltage of the modulating signal in order to have a modulation index of 75%?

15. Which of the following would produce analog signals and which would produce digital signals?

(i) A vibrating tuning fork
(ii) Musical sound due to a vibrating sitar string
(iii) Light pulse
(iv) Output of NAND gate

16. Two waves A and B of frequencies 2 MHz and 3 MHz, respectively are beamed in the same direction for communication *via* sky wave. Which one of these is likely to travel longer distance in the ionosphere before suffering total internal reflection?
17. The maximum amplitude of an AM wave is found to be 15 V while its minimum amplitude is found to be 3V. What is the modulation index?
18. Why is a AM signal likely to be more noisy than a FM signal upon transmission through a channel?
19. Is it necessary for a transmitting antenna to be at the same height as that of the receiving antenna for line of sight communication? A TV transmitting antenna is 81 m tall. How much service area can it cover, if the receiving antenna is at the ground level?
20. A TV transmission tower antenna is at a height of 20 m. How much service area can it cover if the receiving antenna is (i) at ground level, (ii) at a height of 25 m? Calculate the percentage increase in area covered in case (ii) relative to case (i).
21. If the whole earth is to be connected by LOS communication using space waves (no restriction of antenna size or tower height), what is the minimum number of antennas required? Calculate the tower height of these antennas in terms of earth's radius?
22. The maximum frequency for reflection of sky waves from a certain layer of the ionosphere is found to be $f_{max} = 9(N_{max})^{1/2}$, where N_{max} is the maximum electron density at that layer of the ionosphere. On a certain day it is observed that signals of frequencies higher than 5 MHz are not received by reflection from the F_1 layer of the ionosphere while signals of frequencies higher than 8 MHz are not received by reflection from the F_2 layer of the ionosphere. Estimate the maximum electron densities of the F_1 and F_2 layers on that day.
23. A 50 MHz sky wave takes 4.04 ms to reach a receiver via re-transmission from a satellite 600 km above earth's surface. Assuming re-transmission time by satellite negligible, find the distance between source and receiver. If communication between the two was to be done by Line of Sight (LOS) method, what should be the size of transmitting antenna?

ANSWERS

Exercises

1. (b) **2.** (a) **3.** (a) **4.** (b) **5.** (b) **6.** (b) **7.** (d) **8.** (d) **9.** (b,c,d) **10.** (a,b,d) **11.** (b,d)

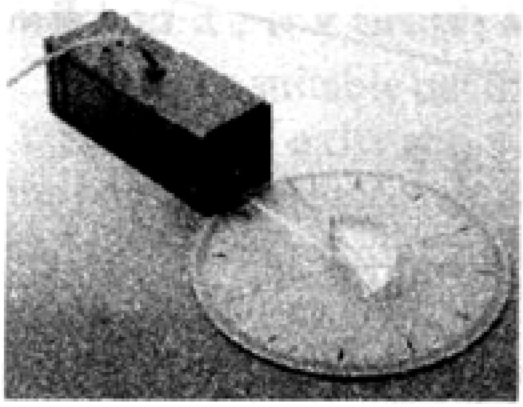
12. (b,d) **13.** $2.53 \times 10^{-14}\ s^2$ **14.** 9 V **15.** (i) analog (ii) analog (iii) digital (iv) digital

16. 3 MHz **17.** 2/3 **18.** See the hints

19. No, 3258 km^2 **20.** (i) 803.84 km^2 (ii) 3608.52 km^2, 348.9%

21. Six, h = radius of earth **22.** $3.086 \times 10^{11}\ m^{-3}$, $7.9 \times 10^{11}\ m^{-3}$

23. 170 km, 565 m



EXPERIMENTAL SKILLS

Chapter Contents

1. Focal length of a convex mirror

Since a convex mirror always forms a virtual image so its focal length cannot be found directly. For this purpose, indirect method is used as described below.

An auxiliary convex lens L is introduced between the convex mirror M and object needle O. Now the object needle is kept at a distance which is roughly 1.5 times the focal length of the convex lens. The position of the convex mirror is now adjusted such that a real inverted image of object needle O is formed at O itself. This will be possible only when the rays incident on the convex mirror are incident normally on it and hence retrace their path and when produced rightward must pass through the centre of curvature C of the mirror.

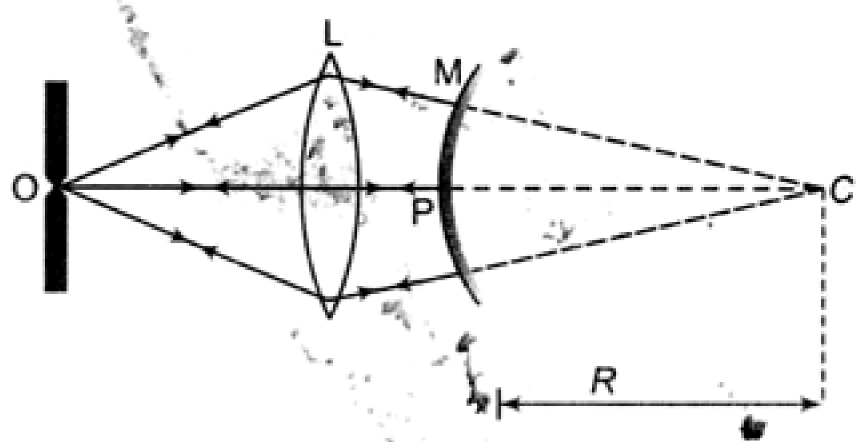


Fig. 1

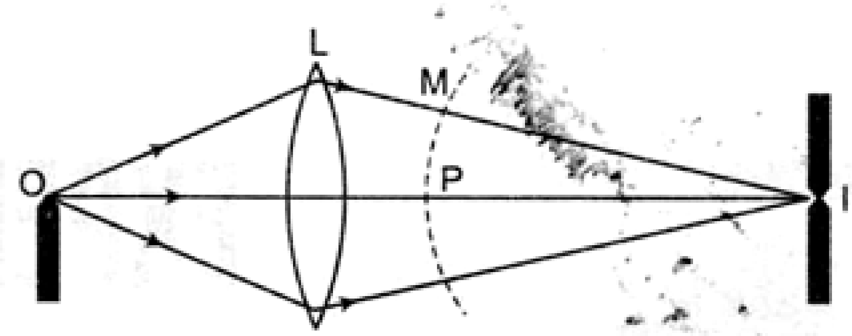


Fig. 2

To locate the position of C, convex mirror is removed (without disturbing the object needle O and convex lens L). An image needle I is put behind the convex lens and moved to a position at which there is no parallax between tip of inverted image of object needle and tip of the image needle. Position of image needle I gives position of centre of curvature C of the mirror M.

Then, $$PC = PI = R$$

and $$f = \frac{R}{2} = \frac{PI}{2}$$

2. Focal length of concave mirror

We can calculate the focal length of a concave mirror by using following three graphical methods.

Method 1: u-v Graph Method

Let us select a suitable but the same scale to represent u along–X axis and v along–Y axis. According to sign convention, in this case, u and v both are negative. Plot the various points for different sets of values of u and v from the observation table. The graph then comes out to be a rectangular hyperbola as shown.

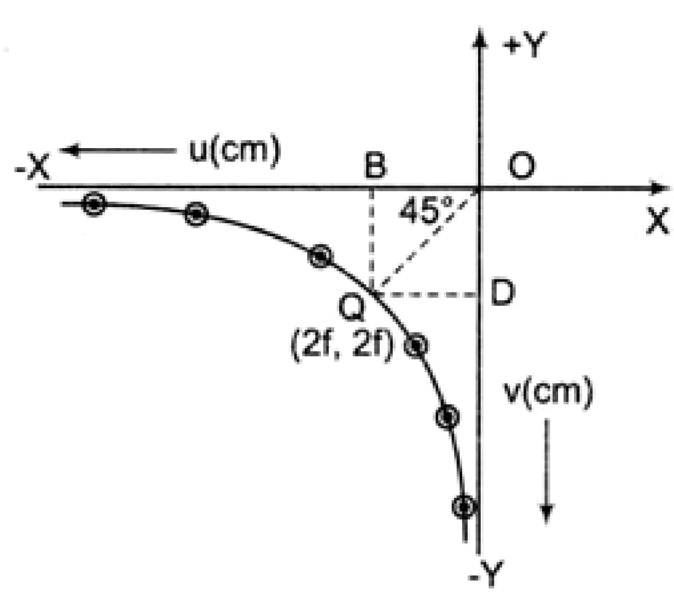


Fig. 3

Draw a line OQ making an angle of 45° with either axis and meeting the curve at point Q. Draw QB and QD perpendicular on X and Y axes respectively. The values of u and v will be same for point Q. So the coordinates of point Q must be $(2f, 2f)$. This is due to the fact that for a concave mirror u and v are equal only when the object is placed at the centre of curvature. So,

$$u = v = R = 2f$$

From mirror formula applied to point Q

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{2}{R}$$

Since $u = v$

$$\therefore \quad \frac{1}{f} = \frac{2}{u} = \frac{2}{v}$$

$$\therefore \quad f = \frac{u}{2} \text{ or } \frac{v}{2}$$

Hence, half the values of either coordinate of Q (i.e., distance OD or OB) gives the focal length of the concave mirror.

$$f = -\frac{OD}{2} = -\text{....cm} = f_1 \text{ (say)}$$

Also $$f = -\frac{OB}{2} = -\text{....cm} = f_2 \text{ (say)}$$

So, mean value of $$f = \left(\frac{f_1 + f_2}{2}\right) = -\text{....cm}$$

Method 2 : u-v Graph Method

Select a suitable but the same scale to represent u along- X axis (or X' axis) and v along $-Y$ axis (or Y' axis). Mark the points at distances $u_1, u_2, u_3, ...$etc. along the OX' axis and the corresponding points at distances $v_1, v_2, v_3......$etc. along the OY' axis for different sets of observations.

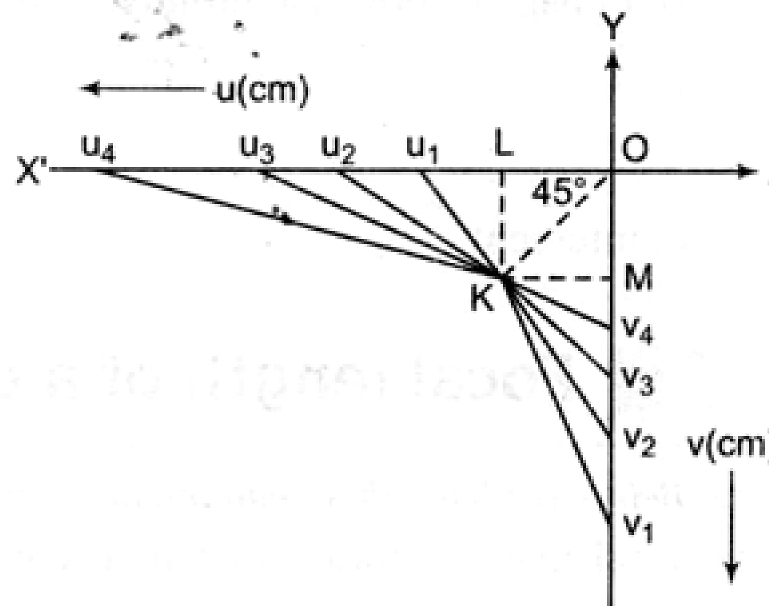


Fig. 4

Draw straight lines joining u_1 with v_1, u_2 with v_2, u_3 with v_3 etc. These lines will intersect at a point K as shown in the graph. Draw KL and KM perpendiculars on X' and Y' axes respectively, then

$$OL = OM = -f$$

$$\therefore \quad f = -\text{..........cm}$$

The above argument can be justified from the fact that the mirror formula, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ is satisfied by extreme values (f, ∞) and (∞, f). The straight lines corresponding to extreme values intersect at a point K having co-ordinates (f, f). For line LK, $u = f$, $v \to \infty$ and for line MK, $v = f$, $u \to \infty$.

Method 3 : $\frac{1}{u}$ and $\frac{1}{v}$ Graph Method

Select a suitable but the same scale to represent $\frac{1}{u}$ along $-X$ axis (or X' axis) and $\frac{1}{v}$ along $-Y$ axis (or Y' axis). By sign conventions, both $\frac{1}{u}$ and $\frac{1}{v}$ are negative (for real images). For different sets of values of $\frac{1}{u}$ and $\frac{1}{v}$ plot the graph, which comes out to be a straight line as shown in figure.

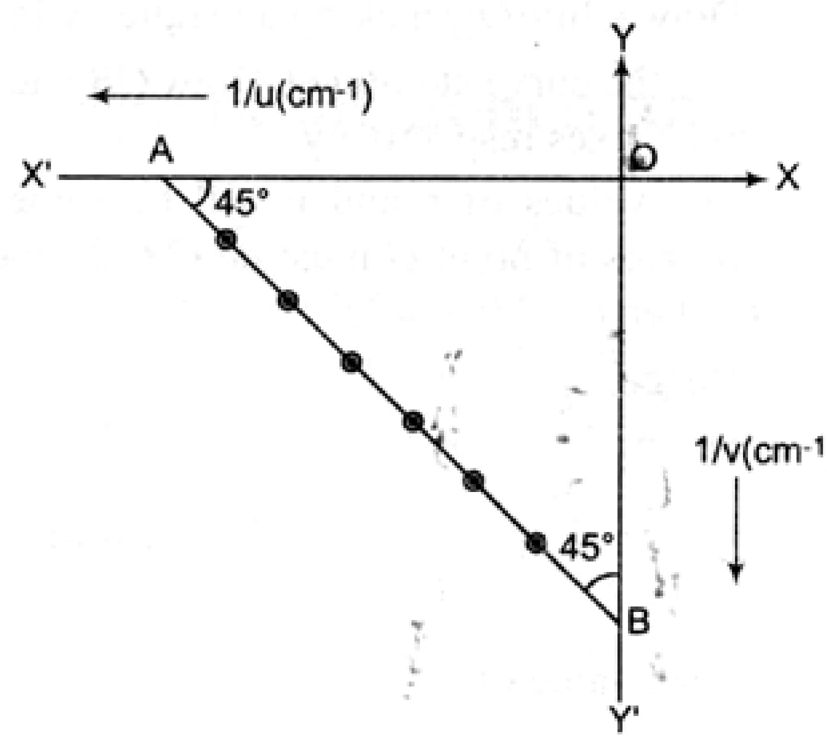


Fig. 5

The straight line cuts the two axes X' and Y' at an angle of 45° at points A and B respectively and also has equal intercepts for both the axis. Measure the distance OA and OB.

The focal length, $$f = -\frac{1}{OA} = -\frac{1}{OB} = f_0 \text{ (say)}$$

$\therefore$ $$f = -f_0 \text{ cm}$$

The above agreement can be explained as follows.

From mirror formula, $$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

(a) If object is placed at infinity, then $$u \to \infty$$

$\therefore$ $$\frac{1}{v} = \frac{1}{f}$$

So, intercept $$OB = \frac{1}{v} = \frac{1}{f}$$

(b) If image is formed at infinity, then $$v \to \infty$$

$\therefore$ $$\frac{1}{u} = \frac{1}{f}$$

So, intercept $$OA = \frac{1}{u} = \frac{1}{f}$$

3. Focal length of a convex lens using parallax method

Note *Before studying this, read the previous article carefully. Many points are common in both.*

We can calculate the focal length of a convex lens by using following three graphical methods.

Method 1: u-v Graph Method

Let us select a convinient but identical scale to represent u along $-X$ axis (or X' axis) and v along Y axis. According to sign conventions, in this case, u is negative and v is positive. Plot the various points for different sets of values of u and v. The graph comes out to be a rectangular hyperbola as shown.

Draw a line OQ making an angle of 45° with either axis and meeting the curve at point Q. Draw QB and QC perpendicular on X' and Y axes respectively.

The values of u and v will be same for point Q. So the coordinates of point Q must be $(2f, 2f)$, because for a convex lens, when $u = 2f$, $v = 2f$.

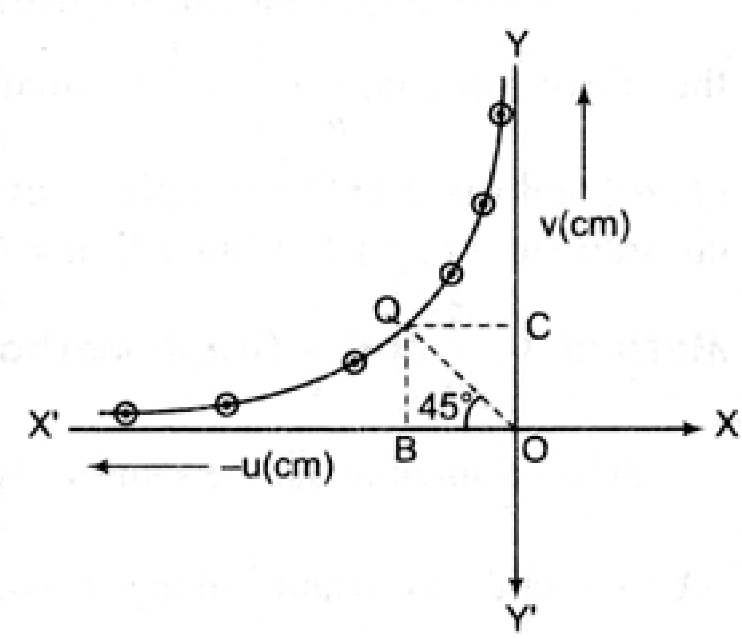


Fig. 6

Hence, $$QB = QC = 2f$$

or $$OC = OB = 2f$$

$\therefore$ $$f = \frac{OB}{2} = f_1 \text{ and } f = \frac{OC}{2} = f_2$$

So, mean value of $$f = \frac{f_1 + f_2}{2} \text{ cm}$$

Method 2: u-v Graph Method

Let us again select the same convenient scale to represent u along $-X$ axis (or X' axis) and v along Y axis. Mark the points at distances $u_1, u_2, u_3, \ldots\ldots$, along X' axis and the correcponding points at distances $v_1, v_2, v_3, \ldots\ldots$ along Y axis for different sets of observations.

Draw straight lines joining u_1 with v_1, u_2 with v_2, u_3 and v_3 and so on. These lines will intersect at a point K as shown in graph.

Draw KL and KM perpendiculars on X' and Y axes respectively. Then

$OL = OM = f$ (in cm) (the focal length of convex lens)

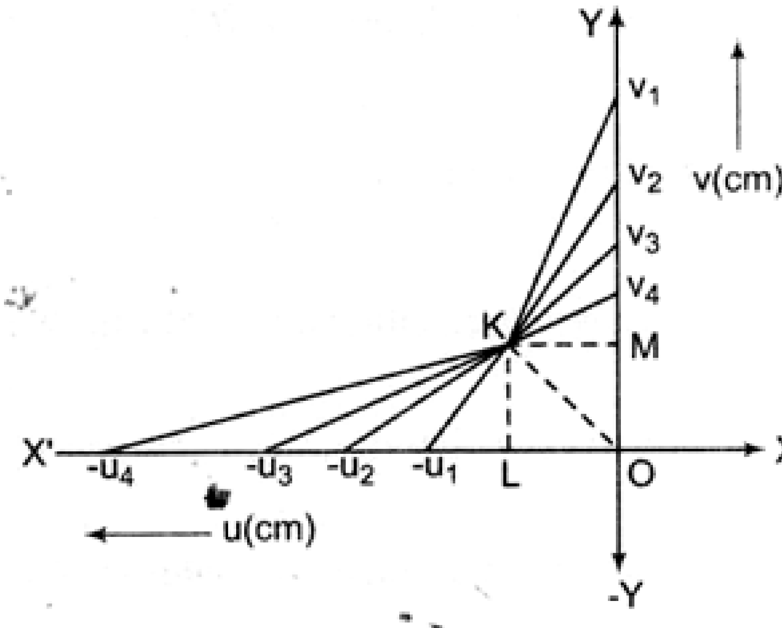


Fig. 7

Method 3: $\frac{1}{u}$ and $\frac{1}{v}$ Graph Method

Select a suitable but the same scale to represent $\frac{1}{u}$ along $-X$ axis or X' axis and $\frac{1}{v}$ along Y axis. According to sign conventions $\frac{1}{u}$ is negative and $\frac{1}{v}$ is positive. Let us plot the various points for different sets of values of $\frac{1}{u}$ and $\frac{1}{v}$. The graph comes out to be a straight line as shown.

The straight line cuts the two axes X' and Y at an angle of 45° at points P and Q respectively and make equal intercepts on both the axes. The distance OP and OQ are measured from the graph.

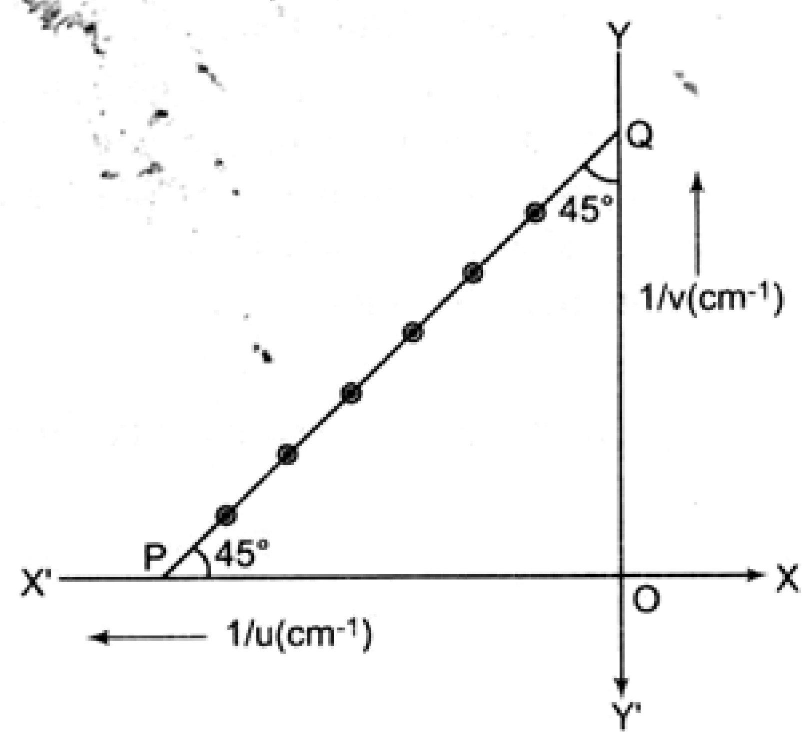


Fig. 8

Then focal length is

$$\frac{1}{OP} = \frac{1}{OQ} = f_1 \text{ (in cm)} \qquad \text{(focal length of convex lens)}$$

The focal length of the convex lens can also be calculated by another method called Displacement Method. This method has been discussed in the chapter of Ray optics.

4. Plot of angle of deviation *vs* angle of incidence for a triangular prism

Consider a monochromatic ray EF to be incident on the face AB of prism ABC of refracting angle A at angle of incidence i. The ray is refracted along FG, r_1 being angle of refraction. The ray FG is incident on the face AC at angle of incidence r_2 and is refracted in air along GH. Thus GH is the emergent ray and e is the angle of emergence. The angle between incident ray EF and emergent ray GH (produced backwards) is called angle of deviation δ.

$$\delta = (i - r_1) + (e - r_2)$$

$$\therefore \qquad \delta = (i + e) - (r_1 + r_2) \qquad \ldots(i)$$

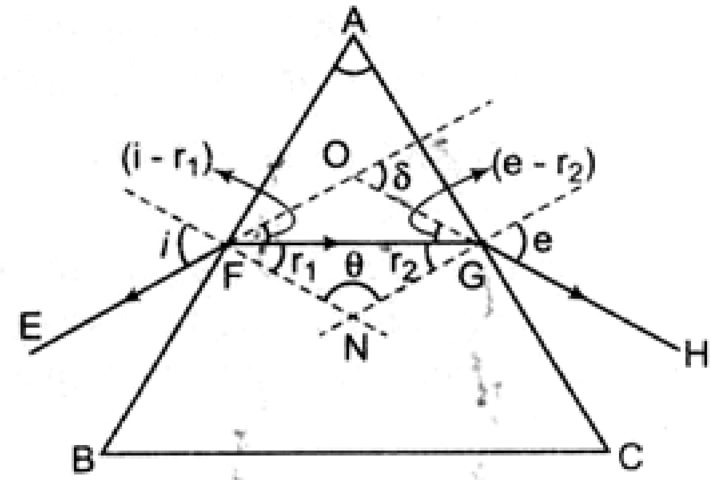


Fig. 9

Also in quadrilateral $AFNG$, $A+90^\circ+\theta+90^\circ=360^\circ$

$\therefore$ $A+\theta=180^\circ$ ...(ii)

And in triangle FGN,

$$r_1+r_2+\theta=180^\circ \quad \text{...(iii)}$$

Comparing equations (ii) and (iii), we get

$$A=r_1+r_2 \quad \text{...(iv)}$$

From (i), we get

$$\delta=i+e-A \quad \text{...(v)}$$

If μ is the refractive index of material of prism, then from Snell's law

$$\mu=\frac{\sin i}{\sin r_1}=\frac{\sin e}{\sin r_2}$$

Plot of δ *vs* i and Calculation of μ

If angle of incidence is changed, the angle of deviation δ also changes. As angle of incidence is increased angle of deviation decreases, then attains a minimum value called angle of minimum deviation δ_m. Now as i is further increased δ also starts increasing.

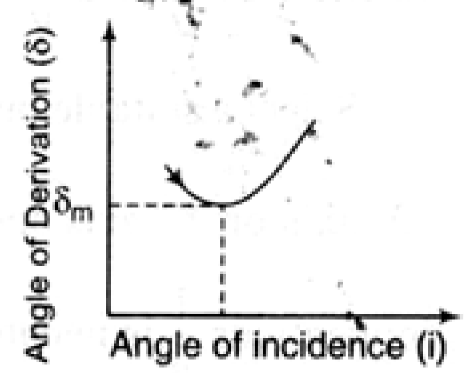


Fig. 10

At the minimum deviation,

$$i=e \quad \text{and} \quad r_1=r_2=r \quad \text{(say)}$$

Then from equations (iv) and (v), we get

$$r+r=A \qquad \therefore \qquad r=\frac{A}{2}$$

$$i+i=A+\delta_m \qquad \therefore \qquad i=\frac{A+\delta_m}{2}$$

So, the refractive index of material of prism is

$$\mu=\frac{\sin i}{\sin r}=\frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

5. Refractive index of a glass slab using a travelling microscope

The bottom surface of a vessel containing a refracting liquid appears to be raised, such that apparent depth is less than the real depth. Refractive index of refracting liquid is defined as the ratio of real depth to the apparent depth.

Mathematically, refractive index, $\mu=\dfrac{\text{real depth}}{\text{apparent depth}}$ (For almost normal incidence)

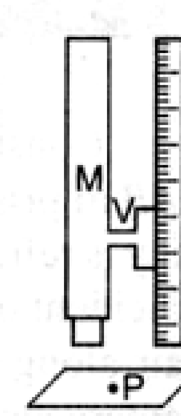


Fig. 11

To measure the real depth and apparent depth, we take the help of a travelling microscope, a travelling microscope is a compound microscope that is fitted vertically on a vertical scale. It can be moved up and down and has a Vernier scale V moving along the main scale M.

In any position, the reading is taken by combining main scale and vernier scale reading.

Figure shows the experimental arragnement.

Procedure Followed :

Step 1 : Make a cross mark with black ink on the base of the microscope. The mark will be now called as point P

Step 2 : Make the microscope vertical and focus it on the cross at P, so that there is no parallax between the cross-wires and the image of the mark P.

Step 3 : Note the main scale reading and the vernier scale readings (R_1) on the vertical scale.

Step 4 : Now place a glass slab over the mark P.

Step 5 : Raise the microscope upwards and focus it on the image of P.

Step 6 : Note the reading (R_2) on the vertical scale as done earlier.

Step 7 : Sprinkle few particles of lycopodium powder on the surface of the slab.

Step 8 : Raise the microscope further upwards and focus it on the lycopodium powder particle on the surface of slab.

Step 9 : Note the reading (R_3) on the vertical scale again.

Step 10 : Repeat above steps with other glass slabs of different thickness.

Step 11 : Record observations in tabular form as shown below.

Table 1

S. No.	Reading on vertical scale			Real thickness $(R_1 - R_3)$ cm	Apparent thickness $(R_2 - R_3)$ cm	Refractive index $\mu = \frac{R_1 - R_3}{R_2 - R_3}$
	R_1 (cm)	R_2 (cm)	R_3 (cm)			
1.	....	....	....	....	....	....
2.	....	....	....	....	....	....
3.	...	....	....	....	....	....

So, finally after all measurements are made, we get $\mu = \dfrac{R_1 - R_3}{R_2 - R_3} = \dfrac{R_3 - R_1}{R_3 - R_2}$

6. Characteristic curves of a *p-n* junction diode in forward and reverse bias

Forward Bias Characteristic

Figure shows the experimental arrangement for studying the characteristic curve of a *p-n* junction when it is forward biased. A battery is connected across the *p-n* junction diode through a potentiometer (or rheostat) so that the voltage applied to the diode can be changed. The milliammeter measures the current through the diode and the voltmeter measures the voltage across the diode. For different values of voltages, the value of current is noted. A graph is plotted between V and I, as shown in figure. This voltage-current graph is called forward characteristic graph of *p-n* junction diode.

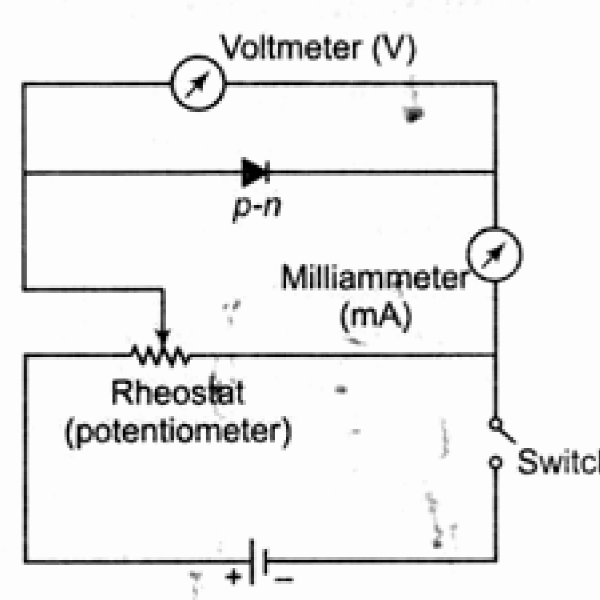


Fig. 12

Important Features of the Graph

(a) The *V-I* graph is not a straight line i.e., a junction diode does not obey Ohm's law.

(b) Initially, the current increases very slowly almost negligibly, till the voltage across the diode crosses a certain value, called the thre shold-voltage or cut-in-voltage. The value of the cut-in voltage is about 0.2 V for a *Ge* diode and 0.7 V for a *Si* diode. Before this characteristic voltage, the depletion layer plays a dominant role in controlling the motion of charge carriers.

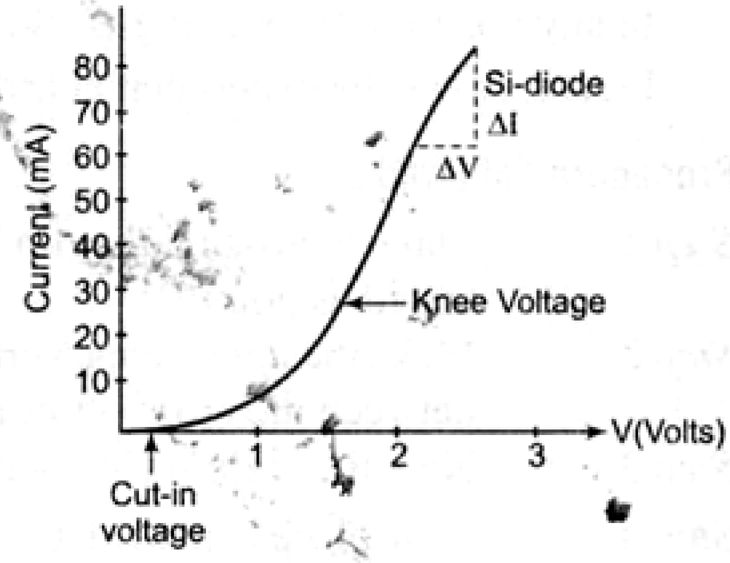


Fig. 13 Forward characteristic of a junction diode

(c) After the cut-in voltage, the diode current increases rapidly even for a very small increase in the diode voltage. Here the majority charge carriers feel negligible resistance at the junction i.e., the resistance across the junction is quite low. The forward voltage at which the current through the junction starts increasing rapidly is called the **Knee Voltage.**

Reverse Bias Characteristic

Figure shows the experimental arrangement for studying characteristic curve of a *p-n* junction when it is reverse biased.

Here a microammeter is used to measure the small currents through the reverse biased diode. *V-I* graph of the type shown in figure is obtained. It is called reverse characteristic of the junction diode.

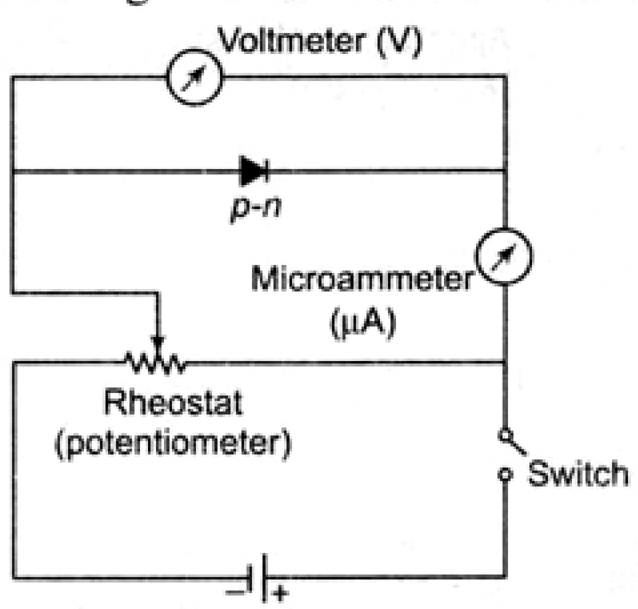


Fig. 14 Circuit for studying V-I characteristic of a reverse biased diode

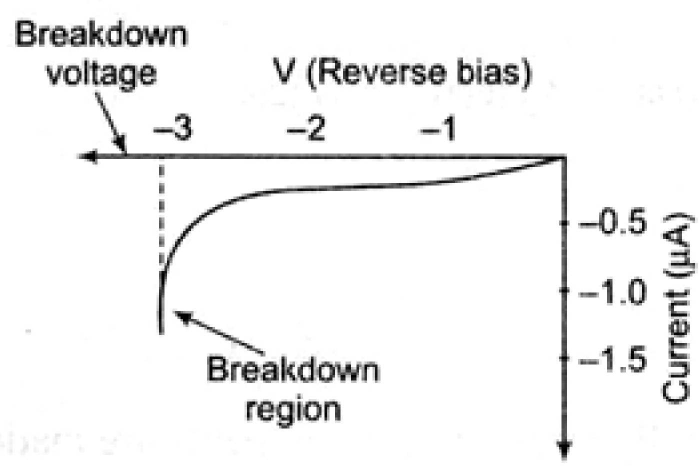


Fig. 15 Reverse characteristic of a junction diode

• Important Features of the Graph

- When the diode is reverse bias voltage produces a very small current, about a few microamperes which almost remains constant with increase in voltage. This small current is called reverse saturation current. It is due to the drift of minority charge carriers (a few holes in *n*-region and a few electrons in *p*-region) across the junction.
- When the reverse voltage across the $p-n$ junction reaches a sufficiently high value, the reverse current suddenly increases to a large value. This voltage at which breakdown of the junction diode occurs is called **Zener breakdown voltage** or peak-inverse voltage of the diode. It ranges from as low as 1 to 2 V to several hundred volts, depending on the dopant density and the depletion layer.

Note *A junction diode offers a very small resistance when forward biased and has a very large resistance when reverse biased i.e., the diode can conduct current well only in one direction. This property is used to convert a.c. into d.c. The conversion of a.c. into d.c. is called rectification.*

7. Characteristic curves of a zener diode and finding reverse breakdown voltage

Zener Diode

A junction diode specially designed to operate only in the reverse breakdown region continuously (without getting damaged) is called a Zener diode. Zener diodes with different breakdown voltages can be obtained by changing the doping concentrations of p– and n–sides which, in turn, change the width of depletion layer and also the barrier field across the junction. The symbol of a Zener diode is shown in figure.

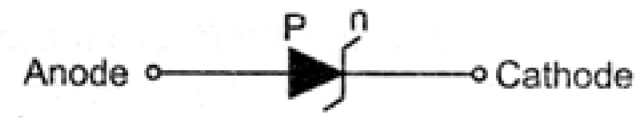


Fig. 16 Symbol for Zener diode

Zener Diode As A Voltage Regulator

When a Zener diode is operated in the reverse breakdown region, the voltage across it remains practically constant (equal to the breakdown voltage V_z) for a large change in the reverse current. The use of Zener diode as a d.c. voltage regulator is based on this fact.

Working

Figure, shows the circuit for using zener diode as a voltage regulator. Here the zener diode is connected in reverse bias to a source of fluctuating d.c. (e.g., the output from a rectifier) through a resistor R. Thus the voltage gets divided between R and zener diode. The output is obtained across the load resistance R_L, connected in parallel with the zener diode.

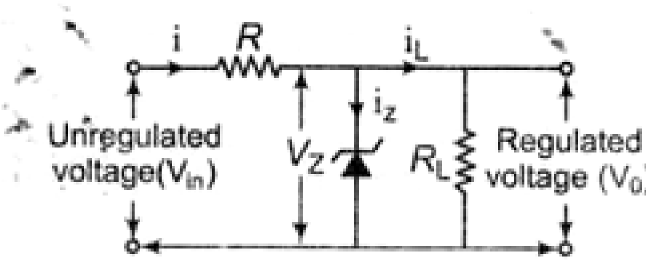


Fig. 17 Zener diode as a voltage regulator

The value of the series resistance R is so chosen that the diode operates in the reverse breakdown region under the Zener breakdown voltage V_Z. If i is the current drawn from the supply, i_Z the current through the Zener diode and i_L the current through the load resistance, then

$$i = i_Z + i_L$$

If R_Z is the resistance of Zener diode, then

$$V_0 = V_Z = i_Z R_Z = i_L R_L$$

Applying Kirchhoff's loop rule to the mesh containing R, Zener diode and the supply voltage V_{in}, we get

$$Ri + V_Z = V_{in}$$

$$\therefore \quad V_Z = V_{in} - Ri$$

Case I : When $V_{in} < V_Z$, almost no current flows through the Zener diode and $V_0 = V_{in}$.

Case II : When $V_{in} = V_Z$, the junction diode operates in the breakdown region and the output voltage $V_0 = V_Z = (V_{in} - Ri)$ becomes constant.

Case III : When $V_{in} > V_Z$, it does not increase V_0 or V_Z but merely increases the voltage drop across R due to the sharp increase in i_Z (or i). Thus the series resistance R absorbs the voltage fluctuations to provide a constant voltage across the load.

Figure shows the graph of the output voltage V_0 versus input voltage V_{in} for a Zener diode. Clearly, the output voltage remains constant after the reverse breakdown voltage V_Z.

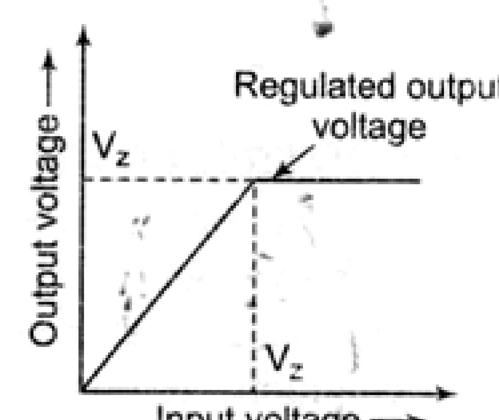


Fig. 18 Graph of V_0 versus V_{in} for a Zener diode

8. Characteristic curves of a transistor and finding current gain and voltage gain

Three Configurations of A Transistor

A transistor is a three element device. One terminal has to be always common to the input as well as the output circuits. This terminal is connected to the ground and serves as a reference point for the entire circuit. So a transistor can be used in one of the following three configurations :

(a) Common base (CB) mode

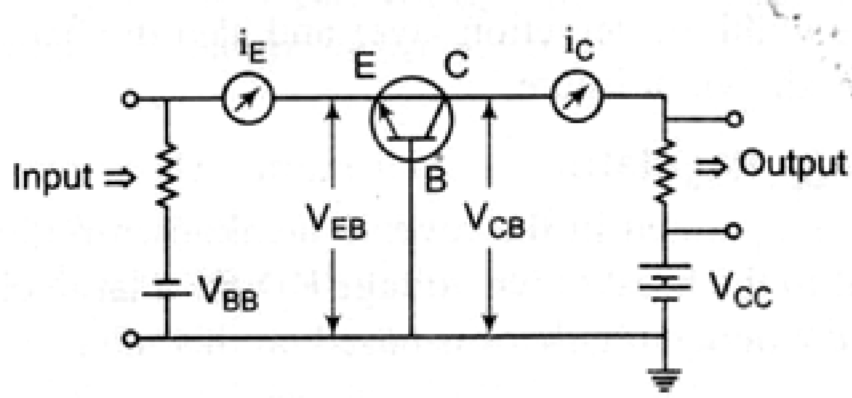


Fig. 19 Common base connections for *n-p-n* transistor

(b) Common emitter (CE) mode

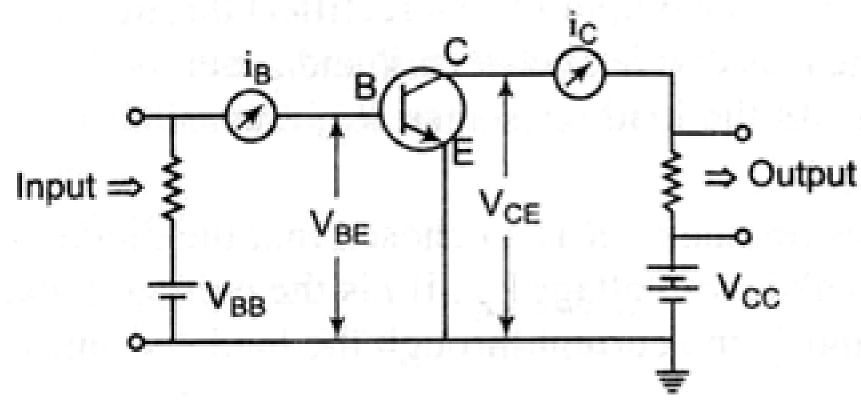


Fig. 20 Common emitter connections for *n-p-n* transistor

(c) Common collector (CC) mode

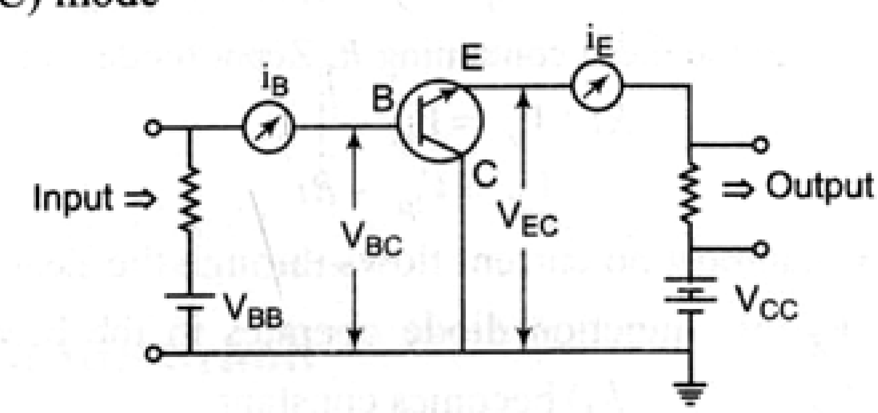


Fig. 21 Common collector connections for *n-p-n* transistor

Common Emitter Characteristics

The common emitter characteristics are the graphs drawn between appropriate voltages and currents for a transistor when its emitter is taken as the common terminal and grounded (zero potential), base is the input terminal and collector is the output terminal.

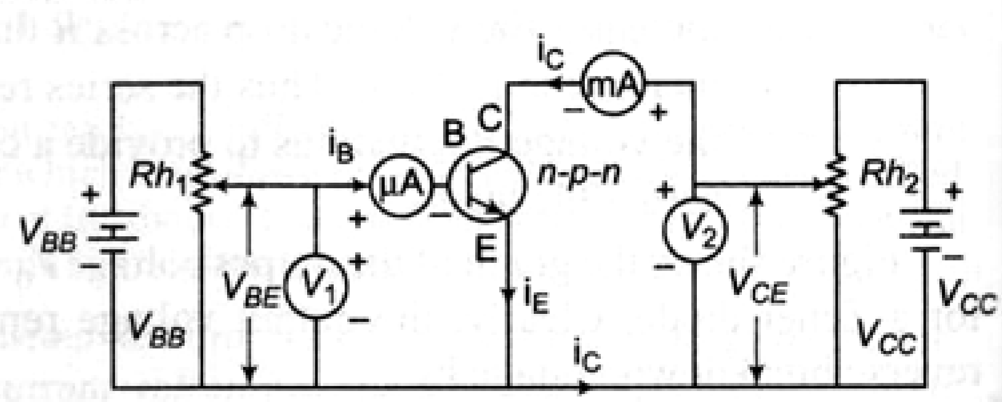


Fig. 22 Circuit for studying the common emitter characteristics of an *n-p-n* transistor

Figure shows the circuit diagram for studying the common emitter characteristics of an *n-p-n* transistor. The emitter base junction is forward biased by means of battery V_{BB} through rheostat Rh_1. The emitter collector circuit is reverse biased by means of battery V_{CC} through rheostat Rh_2. The base emitter voltage V_{BE} and the collector emitter voltage V_{CE} are measured by high resistance voltmeters. The base current i_B is measured by a microammeter and the collector current i_C by a milliammeter. Three types of characteristic curves are studied.

Input Chacteristic

A graph showing the variation of base current i_B with base emitter voltage V_{BE} at constant collector emitter voltage V_{CE} is called the input characteristic of the transistor. Two such curves for two different collector emitter voltages have been plotted in figure.

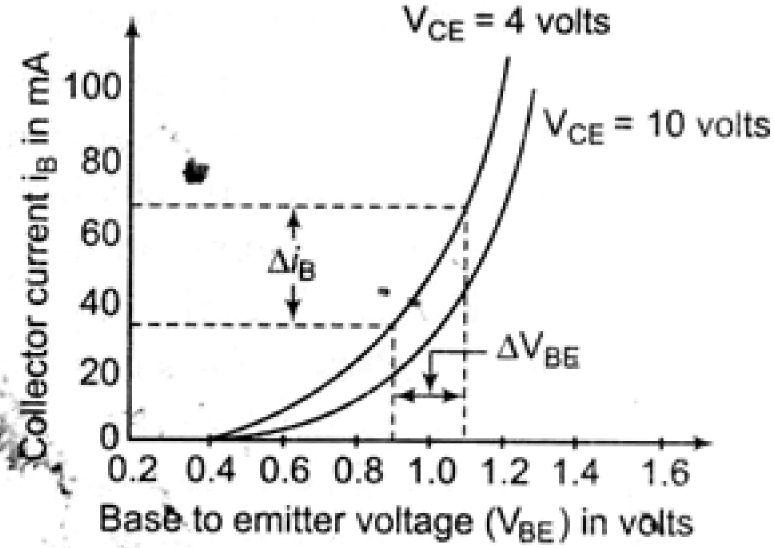


Fig. 23 Input characteristics of CE *n-p-n* transistor

A study of these curves reveals the following facts :

(a) As long as V_{BE} is less than the barrier voltage, the base current i_B is small as in the case of forward biased diode.

(b) When the base emitter voltage V_{BE} exceeds the barrier voltage, the base current i_B increases sharply with a small increaes in V_{BE} as in the case of a forward biased diode.

(c) The value of i_B is much smaller than that in a normal diode. More than 95% majority emitter carriers (electrons in *n-p-n* and holes in *p-n-p* transistor) go to the collector to constitute the collector current i_C.

The input resistance r_1 of the transistor in *CE* configuration is defined as the ratio of the small change in base-emitter voltage to the corresponding small change in the base current, when the collector emitter voltage is kept fixed. Thus

$$r_1 = \left[\frac{\Delta V_{BE}}{\Delta i_B}\right]_{V_{CE} = \text{constant}}$$

As input characteristic is non-linear, so r_1 varies. At any point of the curve, r_1 is equal to the slope of the tangent to the curve.

Output Characteristic

A graph showing the variation of collector current i_C with collector emitter voltage V_{CE} at constant base current i_B is called the output characteristic of the transistor. Figure shows such curves for different values of i_B.

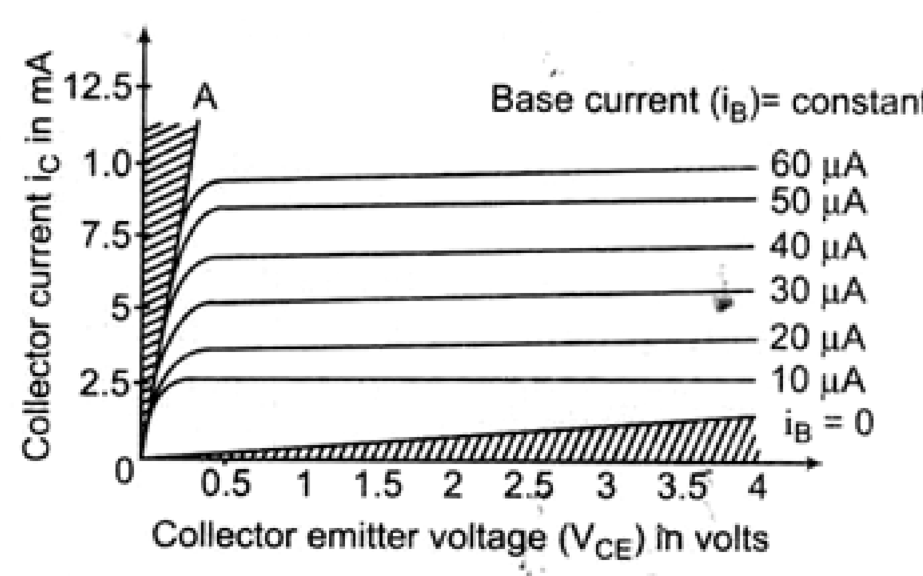


Fig. 24 Output characteristic of CE *n-p-n* transistor

A study of these curves reveals the following features:

(a) When the voltage V_{CE} increases from 0 to about 0.5 V, the collector current i_C increases rapidly. The value of V_{CE} upto which i_C increases rapidly is called knee voltage.

(b) Once the voltage V_{CE} exceeds the voltage V_{BE} (so that the collector base junction is reverse biased).

The output current i_C varies very slowly but linearly with V_{CE} for a given base current i_B i.e., beyond the knee voltage the output resistance of the transistor is high.

(c) Larger the value of i_B larger is the value of i_C for a given V_{CE}.

Three regions of the output characteristic :

(i) The shaded region towards the left of line OA is called saturation region and the line OA is called saturation line. Here $V_{CE} < V_{BE}$. Both the junctions are forward biased. Here i_C does not depend on the input current i_B.

(ii) The shaded region lying below the curve for $i_B = 0$ is called cut-off region. In this region, both the junctions are reverse biased. Here $i_C = 0$. In the shaded regions, the transistor works as switch, it turns over rapidly from OFF state for which $i_C = 0$ (cut-off) to the ON state for which i_C is maximum (saturation state).

(iii) The non-shaded central region of the output characteristic is called active region. In this region, the emitter base junction is forward biased and the collector base junction is reverse biased. A transistor works as an audio amplifier in this region.

The output resistance r_0 of a transitor in CE configuration is defined as the ratio of the small change in the collector-emitter voltage to the corresponding change in the collector current when the base current is kept constant. Thus

$$r_0 = \left[\frac{\Delta V_{CE}}{\Delta i_C}\right]_{i_B = \text{constant}}$$

Transfer Characteristic

It is a graph showing the variation of collector current i_C with base current i_B at constant collector-emitter voltage V_{CE}. As shown in figure, the transfer characteristic of a transistor is almost a straight line.

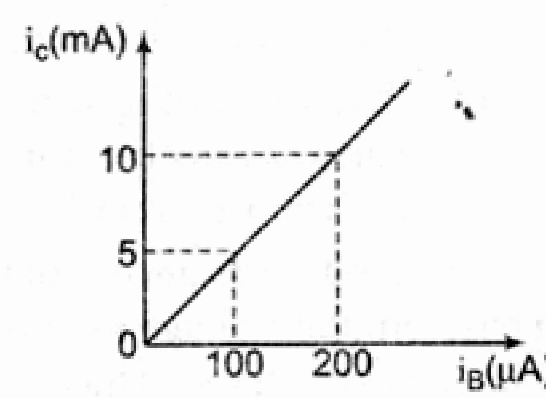


Fig. 25 Transfer characteristic of CE *n-p-n* transfer

Current gain and Voltage gain

(i) In Common base Amplifier

(a) **Current gain** Also called AC current gain (α_{ac}), is defined as the ratio of the change in the collector current to the change in the emitter current at constant collector-base voltage.

Thus, α_{ac} or simply $\alpha = \dfrac{\Delta i_c}{\Delta i_e}$ (V_{CB} = constant)

α is slightly less than 1.

(b) **Voltage gain** It is defined as the ratio of change in the output voltage to the change in the input voltage. It is denoted by A_V. Thus,

$$A_V = \frac{\Delta i_c \times R_{out}}{\Delta i_e \times R_{in}}$$

but $\dfrac{\Delta i_c}{\Delta i_e} = \alpha$, the current gain. $\therefore \quad A_V = \dfrac{\alpha\, R_{out}}{R_{in}}$

Since, $R_{out} >> R_{in}$, A_V is quite high, although α is slightly less than 1.

(ii) In common emitter amplifier

(a) **Current gain** Also called ac current gain (β_{ac}), is defined as the ratio of the change in the collector current to the change in the base current at constant collector to emitter voltage.

$$\beta_{ac} \text{ or simply } \beta = \left(\frac{\Delta i_c}{\Delta i_b}\right) \qquad (V_{CE} = \text{constant})$$

(b) **Voltage gain** It is defined as the ratio of the change in the output voltage to the change in the input voltage. It is denoted by A_V. Thus,

$$A_V = \frac{\Delta i_c \times R_{out}}{\Delta i_b \times R_{in}} \text{ or } A_V = \beta\left(\frac{R_{out}}{R_{in}}\right)$$

9. Identification of Diode, LED, Transistor, IC, Resistor, Capacitor from mixed collection of such items

Identification Based on Appearance

(a) **A diode** is a two terminal device. It conducts current only under forward bias and does not conduct under reverse bias. It does not emit light while conducting.

(b) **LED (Light emitting diode):** It is a two terminal device. It conducts current under forward bias and does not conduct under reverse bias. It emits light while conducting.

(c) **Transistor:** It is a three terminal device. The terminals represent emitter (E), base (B) and collector (C).

(d) **IC (Integrated Circuit):** It is a multi-terminal device in the form of a chip. Hence any item with more than four legs must be an I.C.

(e) **Resistor:** It is a two terminal device. It conducts under either forward bias or under reverse bias. (in fact there is no forward or reverse bias for a resistor). It conducts even when operated under A.C. voltage.

(f) **Capacitor:** It is a two terminal device. It does not conduct with either forward bias or with reverse bias. (Hence it does not conduct with D.C. voltage). However, it conducts with A.C. voltage.

Identification Based on Working

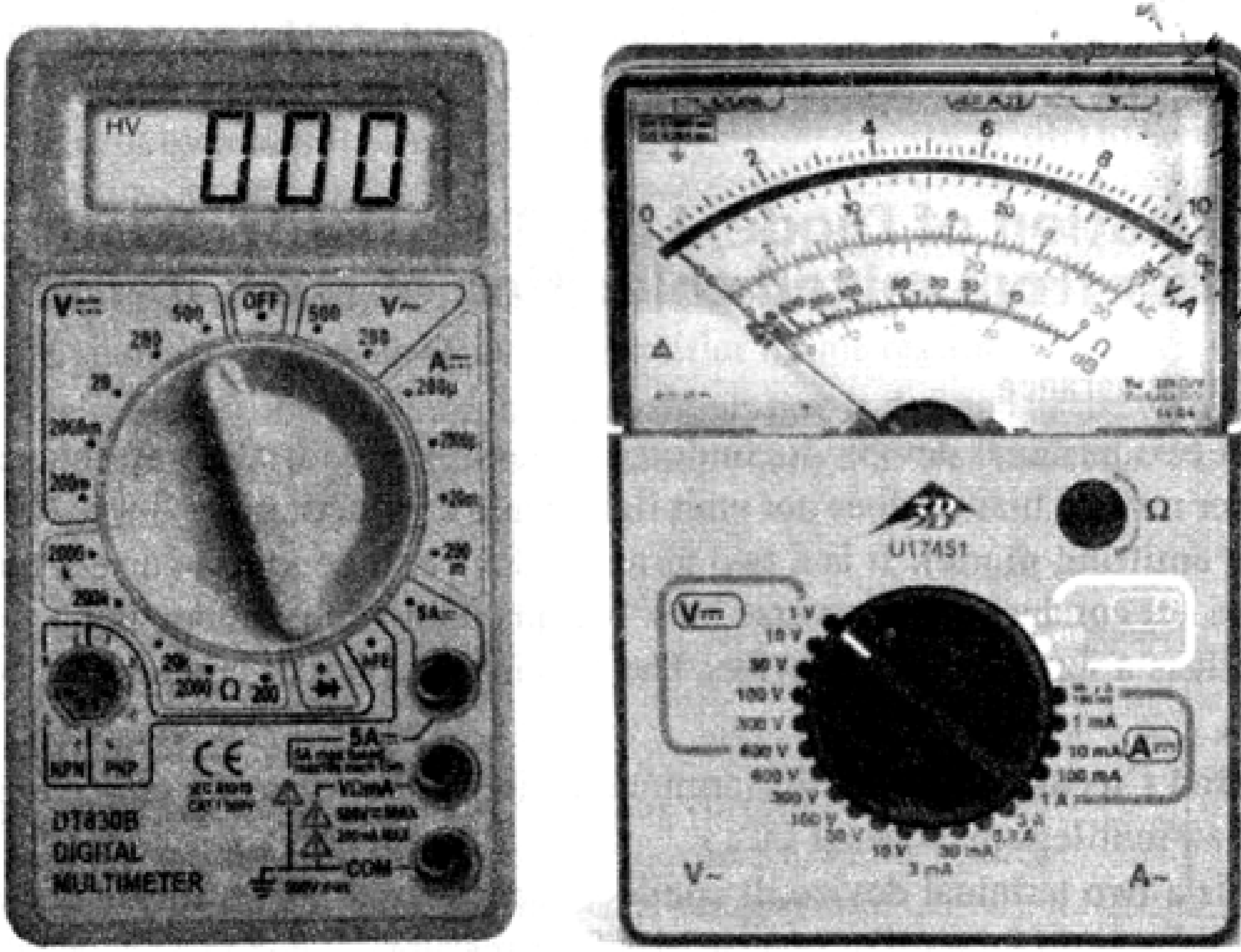
The various steps are as follows :

1. Make a series circuit with a battery eliminator, a reversing key, the given item and the multimeter with range set in milliamperes. Switch on the battery eliminator and watch the movement of the multimeter pointer.
2. If pointer moves when voltage is applied in one way and does not move when reversed and there is no light emission, the item is a **Diode**.
3. If pointer moves when voltage is applied in one way and does not move when reversed and there is light emission, the item is a **LED**.
4. If pointer moves when voltage is applied in one way and also when reversed, the item is **Resistor**.
5. If pointer does not move when voltage is applied in one way and also when reversed, the item is a **Capacitor**.

10. Multimeters

Multimeters are very useful test instruments. By operating a multi-position switch on the meter they can be set to be an Ammeter, a Voltmeter and an Ohm meter. For this reason it is also called an AVO meter. They have the choice of AC or DC. There are many ranges present, so that it can measure quantities ranging from 10^{-6} units to 10^{6} units. The rotation of a knob changes the section and the range in particular section.

Two Types of multimeters are available in the market, digital and analogue.



(a) Digital Multimeter **(b)** Analogue Multimeter

Fig. 26

Using Multimeter-To Identify Base of A Transistor

Transistor is a device used to send the signal from low resistance path to high resistance path. It is a three terminal device. Physically the middle terminal of the transistor is known as base. The base region is thin in size in comparision to other two regions. Doping of the base region is very low.

In open circuit when the resistance is measured with multimeter between two terminals and the resistance is low the terminals are emitter and base. If the resistance is high the terminals are base and collector. In closed circuit if the current to the terminal is very low that is the base.

Using Multimeter-To Distinguish Between *n-p-n* And *p-n-p* Type Transistor

A transistor can be assumed to consist of two back to back connected *p-n* junction diodes. So by using the concept that a *pn* junction conducts current only when it is forward biased, and it blocks the current if reverse bias, we can check whether the given transistor is *n-p-n* or *p-n-p*.

The forward bias resistance for a transistor is low. Usually it is around 900 Ω or 1 kΩ. And reverse bias resistance of a transistor in very high, its value is around $M\Omega$.

First adjust the multimeter in resistance mode and choose two terminals among 3 terminals of transistor to be connected to the two probes of multimeter, so that multimeter shows low resistance (around 1 $k\ \Omega$). Then those terminals selected are said to be forward biased and terminal connected to red probe is *p*-type and that connected to black probe is *n*-type. Now reconnect the black probe to third terminal of transistor and check the resistance, we will study two cases.

***Case* I :** If the resistance is low (around 1 $k\Omega$) then again the connection is said to be forward biased. Then the terminal connected to black probe is *n*-type and hence we can say that the given transistor is *n-p-n*.

***Case* II :** If the resistance is high (around $M\Omega$) then the terminal connected to the black probe is *p*-type, and we can say that the given transistor is *n-p-n*.

Using Multimeter : To check the correctness or otherwise of a given electronic component (diode, transistor or IC)

To Check Whether Diode Is In Working Order (Or Not), The Following Procedure is Adopted

(a) The multimeter must be set as resistance meter.

(b) Note the deflection by touching its probes to two ends of the given diode.

(c) Now, by reversing the probes, deflection must be observed.

(d) If the deflection in multimeter is large in both the cases (or) small in both the cases then diode is spoiled.

(e) If the deflection is large in one case and small in the other case, the diode is in working order.

To Check Whether A Transistor Is Working Or Spoiled, The Following Procedure Is Carried Out

(a) The resistance of transistor between emitter and base should be measured and deflection must be noted.

(b) By reversing the probes deflection must be observed.

(c) The procedure above should be repeated by touching the base and collector.

(d) If the resistance in both cases is low (*i.e.*, large deflection), transistor is spoiled.

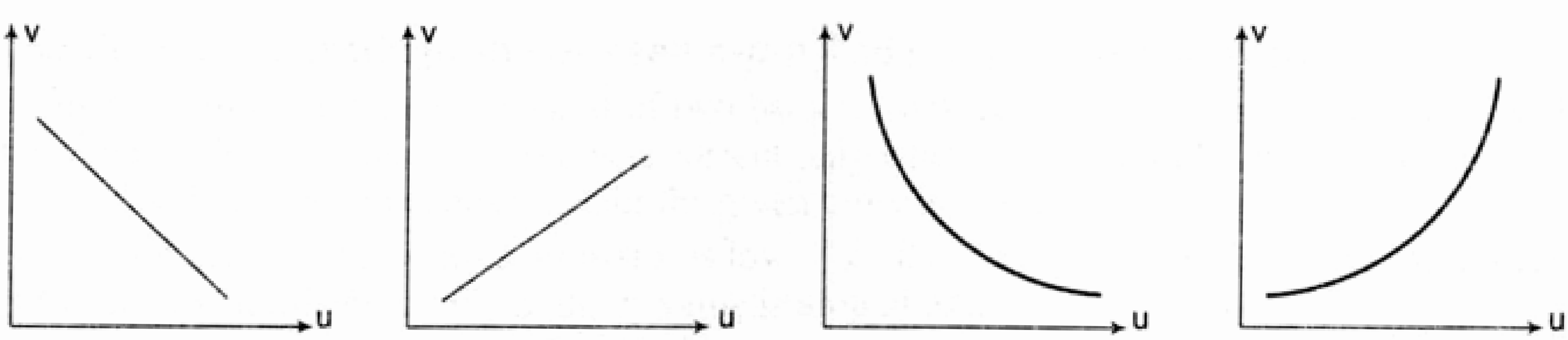
(e) If the resistance is low in one direction (*i.e.*, large deflection) and high in the other direction (*i.e.*, small deflection), the transistor is in working order.

EXERCISES

Experimental Skills & General Physics

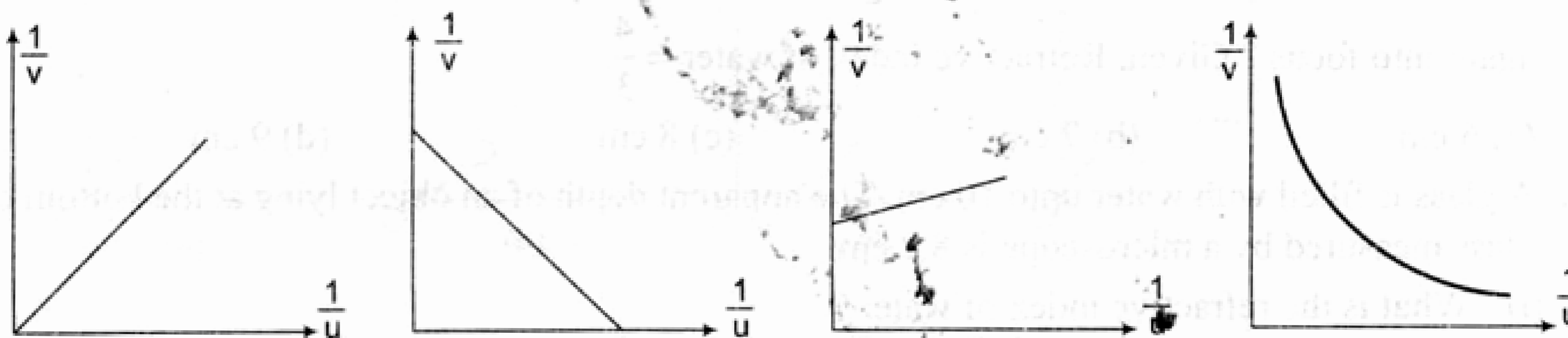
Optics

1. How will you distinguish between two given spherical mirrors, as concave or convex ?
2. What is meant by 'parallax' ?
3. How is the parallax removed ?
4. Why is a concave mirror of large focal length often used as shaving glass ?
5. For what purpose does a surgeon use a concave morror ?
6. What type of mirrors are used in headlights of vehicles and in searchlights ?
7. Which mirror is used as a driving mirror in automobiles and why ?
8. Can you determine the focal length of a convex mirror by plotting $\frac{1}{v}$ *versus* $\frac{1}{u}$.
9. In determining focal length of a convex mirror using auxillary convex lens, can a convex lens of any focal length be used ?
10. What is spherical aberration ?
11. Which method is more accurate in the determination of f for a concave mirror.
    (i) u vs. v, or (ii) $\frac{1}{u}$ vs. $\frac{1}{v}$ graphs ?
12. An object is placed in front of the convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the two mirrors is 10 cm, it is found that there is no parallax between the images formed by the two mirrors. Then the focal length of the convex mirror is
    (a) 12.5cm (b) - 12.5 cm (c) 25 cm (d) 75 cm
13. In the determination of focal length of convex mirror by using a convex lens the object is generally placed from the convex lens at a distance x times the focal length of the convex lens, then x equals
    (a) 4 (b) 0.25 (c) 1.5 (d) 0.5
14. In an experiment to find focal length of a concave mirror, a graph is drawn between the magnitudes of u and v. The graph looks like



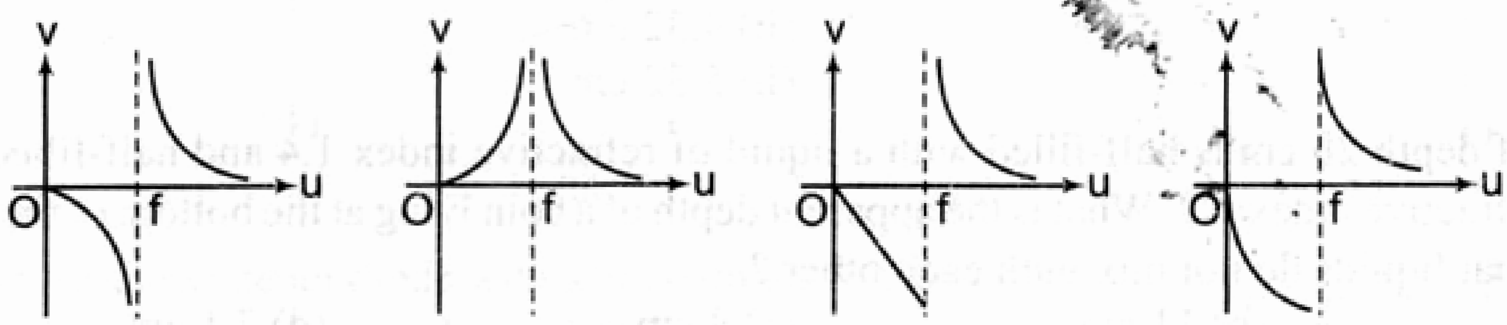
(a) (b) (c) (d)

15. Which of the following graph is applicable between $\frac{1}{v}$ and $\frac{1}{u}$ for a concave mirror ?



(a) (b) (c) (d)

16. The graph showing correctly the variation of image distance (v) as a function of object distance (u) in case of a concave mirror is



(a) (b) (c) (d)

17. How do you distinguish a convex lens from a concave lens ?

18. What is meant by principal axis of a lens.

19. If a lens is plano-convex, what is the radius of curvature of its plane surface ? Where does the centre of curvature of its plane surface lie ?

20. Then, how do you define principal axis for a plano-convex lens ?

21. A graph is drawn with $\frac{1}{u}$ along x-axis and $\frac{1}{v}$ along the y-axis. If the intercept on the x-axis is $0.5\ m^{-1}$, the focal length of the lens is (in meter)
(a) 2.00 (b) 0.50 (c) 0.20 (d) 1.00

22. What happens to a ray of light when it passes through a rectangular glass slab ?

23. Does the lateral displacement depend upon the thickness of the slab ?

24. What causes earlier rise of the sun than the actual rise.

25. What causes formation of a rainbow after rains ?

26. If a plane glass plate is placed over letters of different colours, then which coloured letters appears to be most raised
(a) blue (b) green (c) violet (d) red

27. A small pin fixed on a table top is viewed from above from a distance of 50 cm. The distance by which the pin appears to be raised, if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table is (Refractive index of glass $= 1.5$)
(a) 1 cm (b) 2 cm (c) 3 cm (d) 5 cm

28. A mark is made at the bottom of a beaker and a microscope is focussed on it. The microscope is then raised through 1.5 cm. To what height water must be poured into the beaker to bring the mark again into focus ? Given, Refractive index of water $= \frac{4}{3}$.

(a) 6 cm (b) 7 cm (c) 8 cm (d) 9 cm

29. A glass is filled with water upto 10 cm. The apparent depth of an object lying at the bottom of the glass measured by a microscope is 8.2 cm.

(i) What is the refractive index of water ?

(ii) If water is replaced by a liquid of refractive index 1.7 upto the same height, by what distance would the microscope have to be moved to focus the object again ?

(a) (i) 1.2 (ii) 2.32 cm
(b) (i) 1.2 (ii) 3.32 cm
(c) (i) 1.4 (ii) 4.32 cm
(d) (i) 1.4 (ii) 5.32 cm

30. A beaker of depth 20 cm is half-filled with a liquid of refractive index 1.4 and half-filled with a liquid of refractive index 1.7. What is the apparent depth of a coin lying at the bottom of the beaker, assuming that liquids do not mix with each other ?

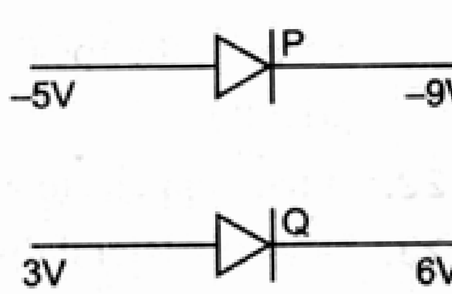
(a) 13 cm (b) 11 cm (c) 9 cm (d) 7.1 cm

Modern Physics

31. The diffusion dominant mechanisms for motion of charge carriers in forward and reverse biased silicon p - n junction are

(a) drift in foward bias, diffusion in reverse bias
(b) diffusion in forward bias, drift in reverse bias
(c) diffusion in both forward and reverse bias
(d) drift in both forward and reverse bias

32. Terminal potentials of two diodes P and Q are as shown in figure. Then which of the following is correct?



(a) Both are reverse biased
(b) P is forward biased and Q is reverse biased
(c) P is reverse biased and Q is forward biased
(d) both are forward biased

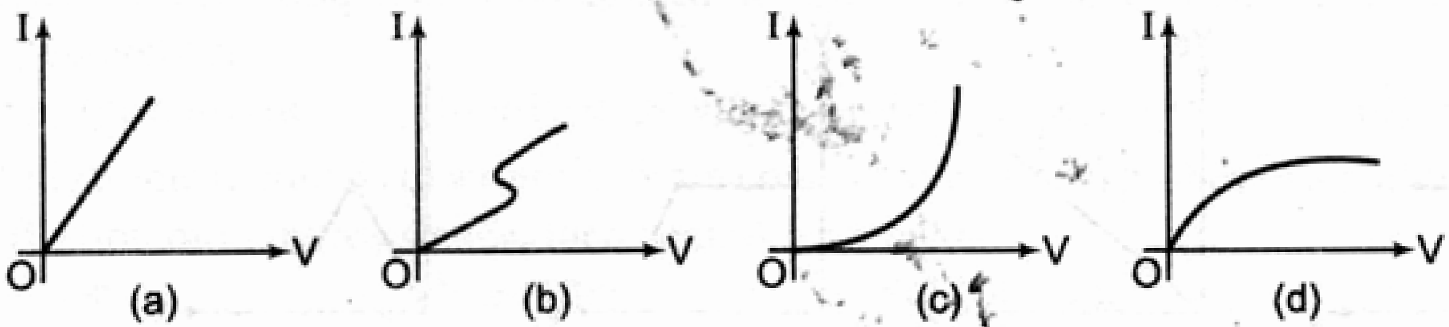
33. The cause of the potential barrier in a p - n junction diode is

(a) depletion of positive charges near the junction
(b) concentration of positive charges near the junction
(c) concentration of negative charges near the junction
(d) concentration of positive and negative charges near the junction

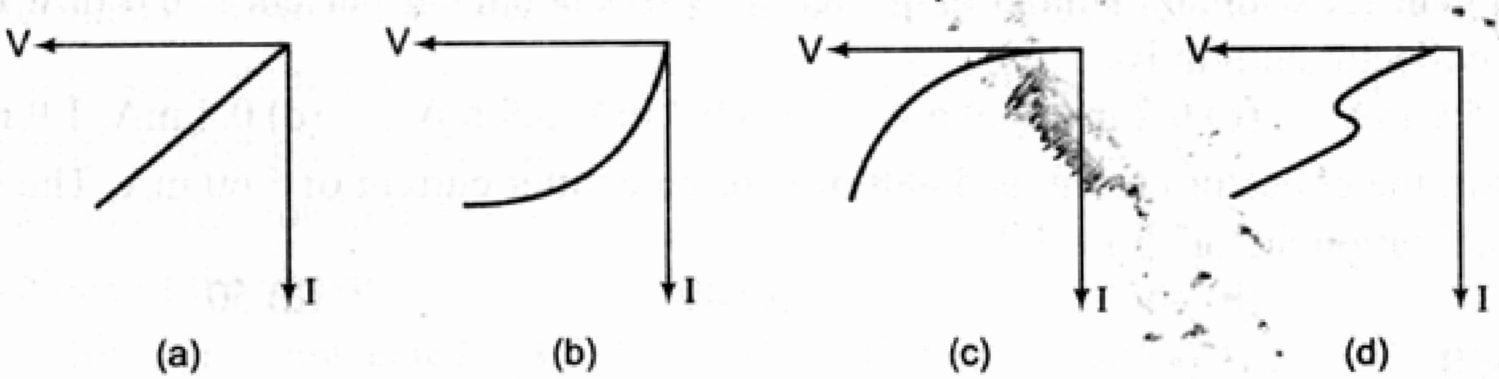
34. The reverse voltage at which the current increases steeply is called

(a) threshold voltage (b) knee voltage
(c) breakdown voltage (d) stopping voltage

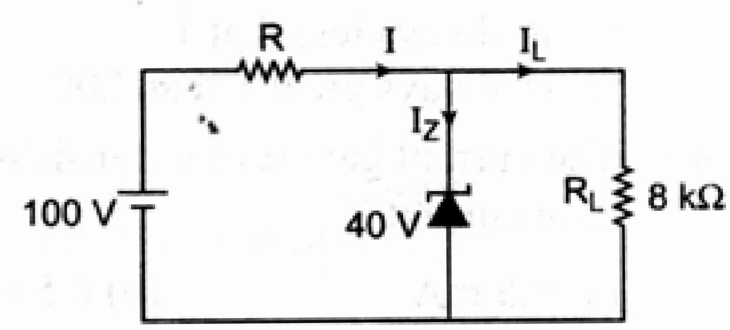
35. The forward biased characteristics of a p - n junction diode is :



36. The reverse biased characteristics of a p - n junction diode is :



37. Figure shows zener diode with a breakdown voltage of 40 V connected to a 100 Volt D.C. source with a series resistance R and a load resistance (R_L). The output voltage across R_L is



(a) 40 V
(b) 20 V
(c) 50 V
(d) 60 V

38. Zener breakdown occurs in junction which is

(a) heavily doped and has wide depletion layer
(b) lightly doped and has wide depletion layer
(c) moderately doped and has narrow depletion layer
(d) heavily doped and has narrow depletion layer

39. At breakdown region of zener diode which of the following quantities does not change much ?

(a) Voltage
(b) Current
(c) Dynamic impedance
(d) Capacitance

40. The avalanche breakdown occurs at

(a) higher reverse voltage
(b) lower reverse voltage
(c) lower forward voltage
(d) higher forward voltage

41. Zener diode is always used in

(a) forward bias mode
(b) reverse bias mode
(c) may be forward or reverse mode
(d) unbiased mode

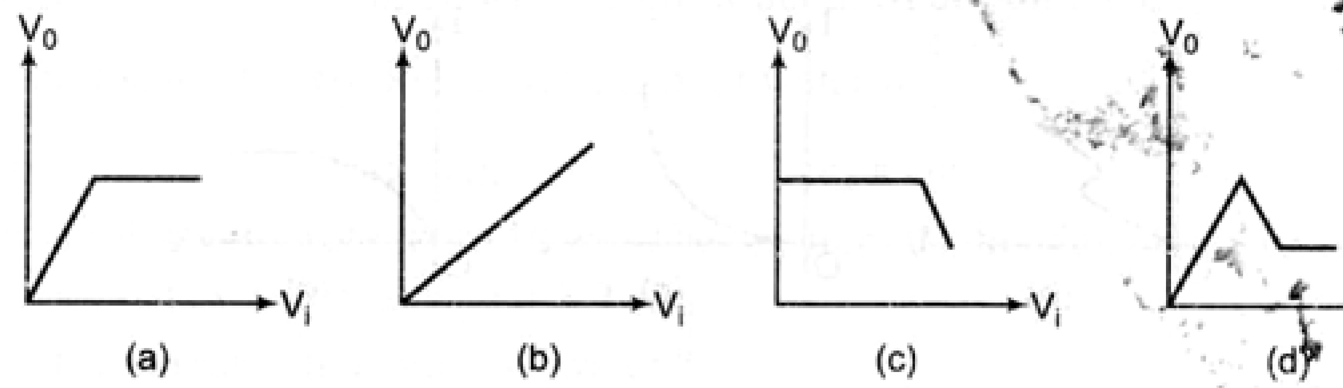
42. Zener diode is used as a / an

(a) amplifier
(b) rectifier
(c) oscillator
(d) voltage regulator

43. In the zener diode circuit, total current is i, load current is i_L, zener current is i_Z then

(a) $i_Z = i_L$
(b) $i_Z = i + i_L$
(c) $i_Z = i - i_L$
(d) $i_Z - i_L = i$

44. In a voltage regulating circuit of Zener diode, the graph of output voltage V_0 versus input voltaage V_i is



45. The current gain for common emitter amplifier is 59. If the emitter current is 6.0 mA, the base current and collector current is
(a) 0.1 mA, 5.9 mA (b) 0.2 mA, 6.9 mA (c) 0.3 mA, 3.9 mA (d) 0.4 mA, 1.9 mA

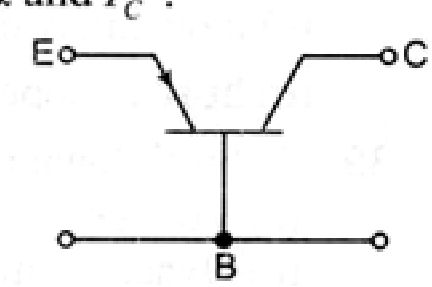
46. In a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of current amplification factor (β) will be
(a) 51 (b) 48 (c) 49 (d) 50

47. The value of β
(a) is always less that 1
(b) lies between 20 and 200
(c) is always greater than 200
(d) is always infinity

48. The current gain α of a transistor is 0.95. The change in emitter current is 10 mA. The change in base current is
(a) 9.5 mA (b) 0.5 mA (c) 10.5 mA (d) $\left(\frac{200}{19}\right)$ mA

49. In a common-emitter transistor amplifier circuit $\beta = 100$, input resistance $R_1 = 1\,\text{k}\Omega$, output resistance $R_2 = 10\,\text{k}\Omega$. The voltage gain of circuit is
(a) 100 (b) 1000 (c) 10 (d) 5000

50. For circuit shown in figure $I_E = 4$ mA, $I_B = 40\,\mu$A. What are the values of α and I_C ?
(a) 0.99, 3.96 mA
(b) 1.01, 4.04 mA
(c) 0.97, 4.04 mA
(d) 0.99, 4.04 mA



51. While using a transistor as an amplifier
(a) the collector junction is forward biased and emitter junction reverse biased.
(b) the collector junction is reverse biased and emitter junction is forward biased.
(c) both the junctions are forward biased.
(d) both the junctions are reverse biased.

52. For a transistor the value of $\alpha = 0.9$, the value of β is
(a) 1 (b) 100 (c) 90 (d) 9

53. Multimeter is used to
(a) check whether a given diode or a transistor is in working order
(b) identify the base of a transistor and terminals of an *IC*.
(c) distinguish between *p-n-p* and *n-p-n* transistor
(d) verify all the above

54. While checking the nature of components of a transistor, if multimeter shows low resistance then nature of the components connected to the positive and negative terminals of multimeter respectively are

(a) p- type, n-type (b) n-type, p-type (c) n-type, n-type (d) p-type, p-type

55. The extreme terminals of a transistor are connected with two probes of a multimeter. If it show less deflection or more resistance, then central terminal is

(a) base (b) emitter (c) collector (d) any one

56. If a semi conductor device has more than three legs (or) pins, it is a / an:

(a) transistor (b) junction diode (c) zener diode (d) integrated circuit

57. Multimeter ends are connected to emitter base of a working order transistor, then multimeter shows

(a) large deflection in one direction and small deflection in other direction

(b) large deflection in both cases

(c) small deflection in both cases

(d) None of the above

58. A multimeter is connected to LED, then deflection before and after reversing the probes is

(a) same in both cases

(b) large and then falls to zero

(c) almost zero in both cases

(d) very small in one case and very large in reverse case or vice versa and emits light

59. If the diode is spoiled then multimeter shows

(a) large deflection in one case

(b) small deflection in reverse case

(c) deflection is large in one case and small in reverse case

(d) large or small deflection in both cases

60. How do we find whether a diode is working or not by using a multimeter ?

(a) Forward bias resistance is more

(b) Reverse bias resistance is less

(c) In both the cases resistance is more

(d) Forward bias resistance is less and reverse bias resistance more

ANSWERS

Experimental Skills & General Physics

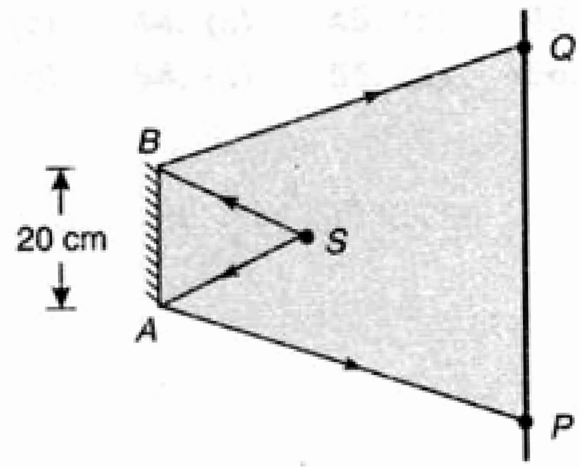
12. (d)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (a)	**21.** (a)	**26.** (c)	**27.** (d)	**28.** (a)	**29.** (a)
30. (a)	**31.** (a)	**32.** (b)	**33.** (d)	**34.** (c)	**35.** (c)	**36.** (c)	**37.** (a)	**38.** (d)	**39.** (a)
40. (a)	**41.** (b)	**42.** (d)	**43.** (c)	**44.** (a)	**45.** (a)	**46.** (c)	**47.** (b)	**48.** (b)	**49.** (b)
50. (a)	**51.** (b)	**52.** (d)	**53.** (d)	**54.** (a)	**55.** (a)	**56.** (d)	**57.** (a)	**58.** (d)	**59.** (d)
60. (d)									

HINTS & SOLUTIONS

Chapter 27 Reflection of Light

JEE Advanced (Subjective Questions)

1. Insect can see the image of source S in the mirror, so far as it remains in field of view of image overlapping with the road.

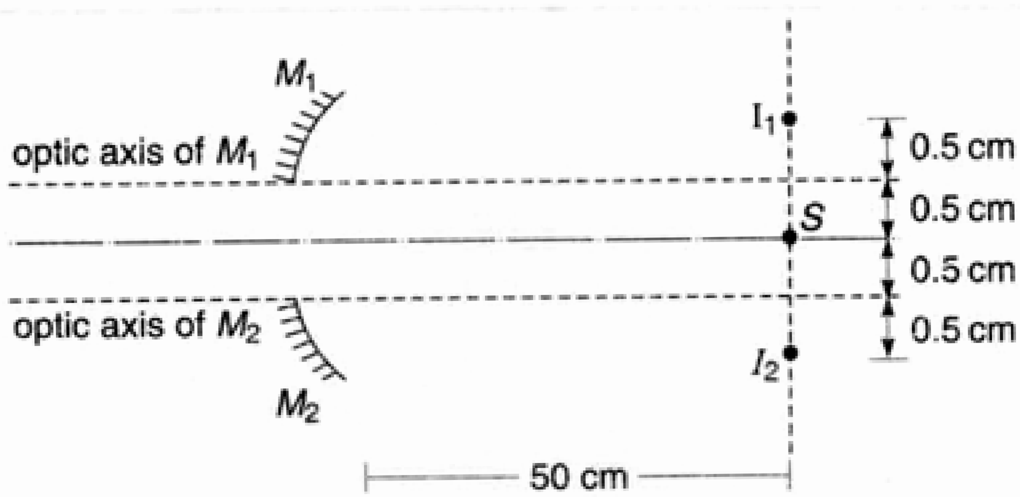


Shaded portion is the field of view, which overlaps with the road upto length PQ.

By geometry we can see that, $PQ = 3AB = 60$ cm

$\therefore$ $$t = \frac{\text{distance}}{\text{speed}} = \frac{60}{10} = 6 \text{ s}$$ **Ans.**

2. Using mirror formula, $\left(\frac{1}{v} + \frac{1}{u} = \frac{1}{f}\right)$



$$\frac{1}{v} - \frac{1}{50} = \frac{-1}{25}$$

$\therefore$ $$v = -50 \text{ cm}$$

$$m = -\frac{v}{u} = -1$$ **Ans.**

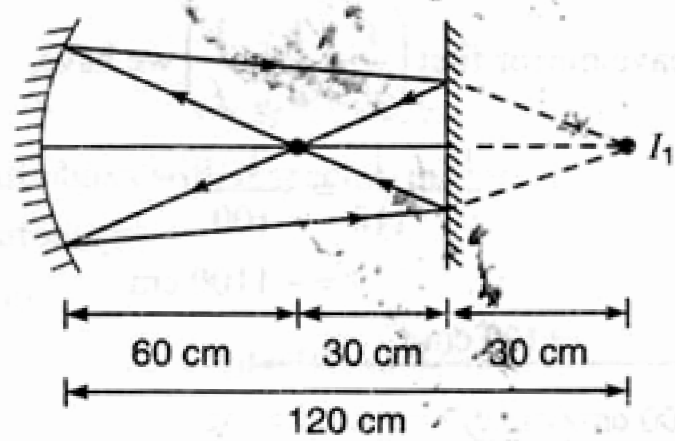
3. Using mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ for concave mirror first, we have

$$\frac{1}{v} - \frac{1}{60} = \frac{1}{-40}$$ $\left(f = \frac{R}{2}\right)$

or $$v = -120 \text{ cm}$$

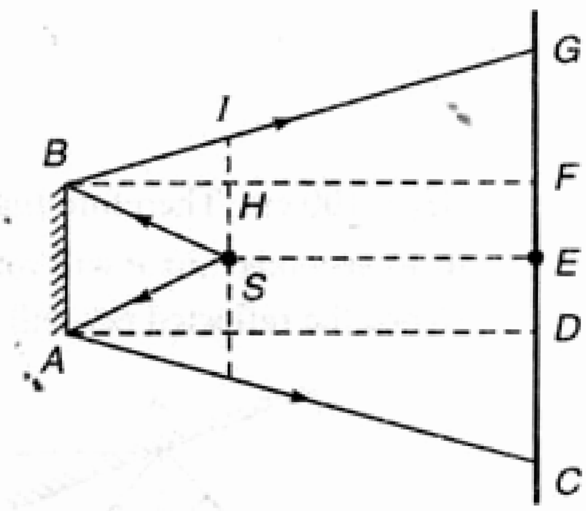
First image I_1 at 120 cm from concave mirror will act as virtual object for plane mirror. Plane mirror will form real image of I_1 at S.

Ray diagram is shown in figure.



Distance between two mirrors is 90 cm. **Ans.**

4.



$$\frac{FG}{BF} = \frac{IH}{BH} = \frac{HS}{BH}$$

$\therefore$

$$FG = (BF)\left(\frac{HS}{BH}\right)$$

$$= (5)\left(\frac{1.0}{0.5}\right)$$

$$= 10 \text{ m}$$

$$FC = 2 + 10 = 12 \text{ m}$$

The boy has dropped himself at point F. So, his velocity is 20 m/s in upward direction.

Let us first find the time to move from F to topmost point and then from topmost point to point C. From

$$s = ut + \frac{1}{2}at^2 \text{ we have}$$

$$-12 = (20t) + \frac{1}{2}(-10)t^2$$

Solving this equation we get, $t_1 = 4.53$ s.

Velocity of boy at point G,

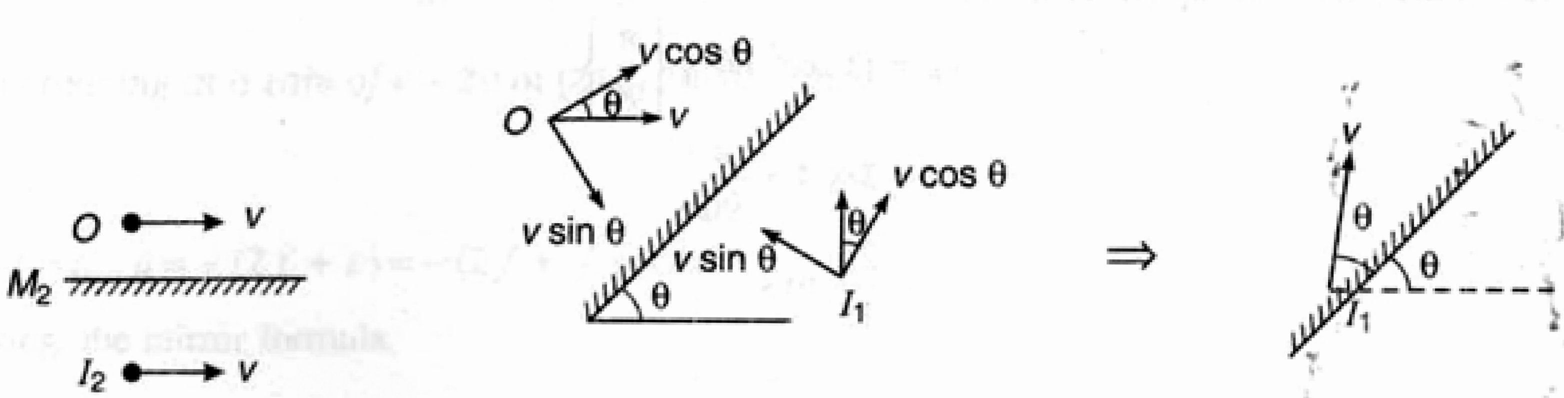
$$v = \sqrt{(20)^2 - 2 \times 10 \times 10} = 14.14 \text{ m/s} \qquad (v^2 = u^2 - 2gh)$$

Time taken to move the boy from G to topmost point and then from topmost point to G will be,

$$t_2 = \frac{2v}{g} = 2.83 \text{ s}$$

$\therefore$ The required time is : $t = t_1 - t_2 = 1.7$ s **Ans.**

5. Angle between v_{I_1} and v_{I_2} is 2θ. Their magnitudes is v.



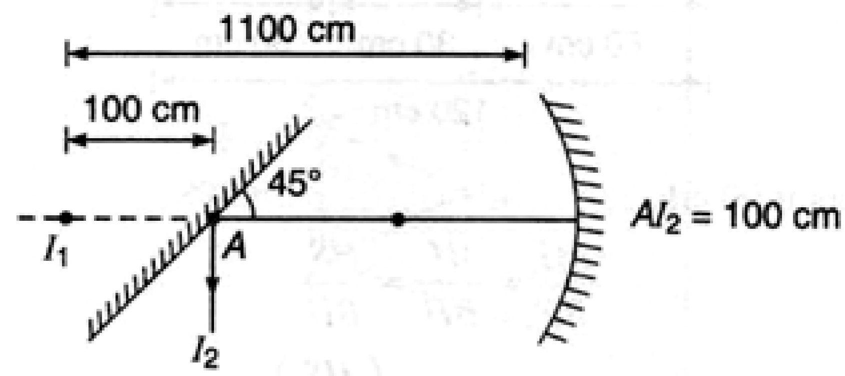
$$\therefore \qquad v_{I_1I_2} = |\vec{v}_{I_1} - \vec{v}_{I_2}| = \sqrt{v^2 + v^2 - 2vv\cos 2\theta}$$

$$= 2v\sin\theta$$ **Ans.**

6. Applying mirror formula for concave mirror first $\left(\frac{1}{v} + \frac{1}{u} = \frac{1}{f}\right)$ we have,

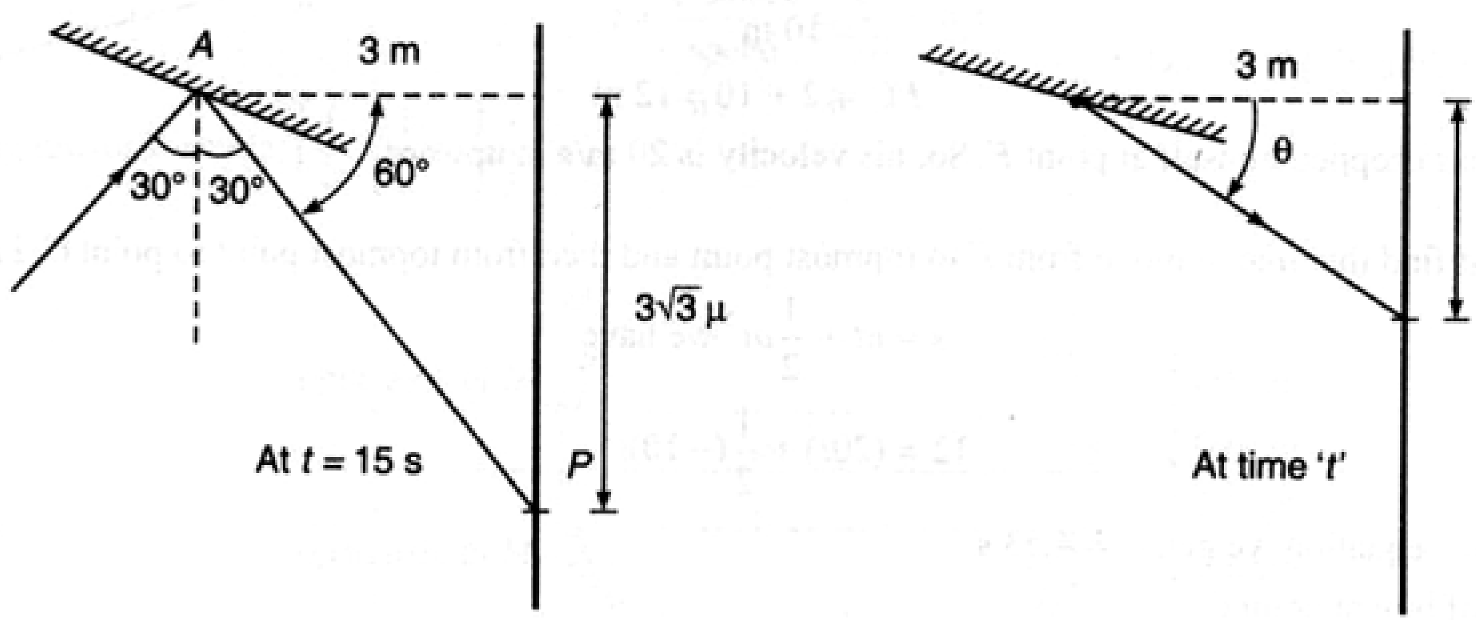
$$\frac{1}{v} - \frac{1}{110} = \frac{1}{-100}$$

$$v = -1100 \text{ cm}$$



$AI_1 = 100$ cm. Therefore final image will be real and at distance 100 cm below point A at I_2.

7. In 15 seconds mirror will rotate 15° in clockwise direction.
Hence the reflected ray will rotate 30° in clockwise direction.



$$y = 3\tan\theta$$

$$\therefore \qquad \frac{dy}{dt} = (3\sec^2\theta).\frac{d\theta}{dt} \qquad \text{...(i)}$$

Here $\qquad \frac{dy}{dt} = v_P, \frac{d\theta}{dt} = 2° \text{ per second} = \frac{2\times\pi}{180} = \frac{\pi}{90}$ rad per second.

At $\qquad t = 15$ s and $\theta = 60°$

Substituting the values in Eq. (i) we have,

$$v_P = \{3\sec^2 60°\}\left\{\frac{\pi}{90}\right\}$$

$$= 3\times 4\times\frac{\pi}{90}$$

$$= \frac{2\pi}{15} \text{ m/s}$$ **Ans.**

8. (a) Differentiating the mirror formula, (with respect to time)

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

We get velocity of image

$$v_I=(m)^2 v_O \qquad \ldots\text{(i)}$$

Here v_O = relative velocity of object with respect to mirror

v_I = relative velocity of image.

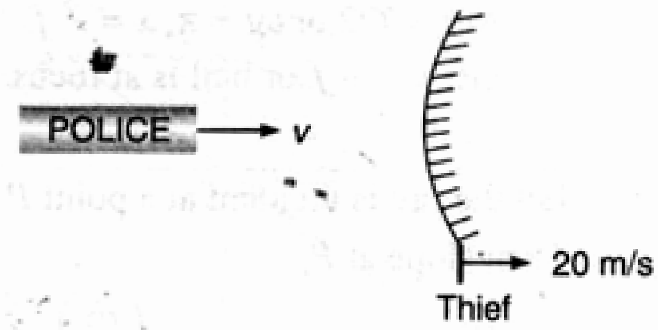
m = linear magnification

Here,

$$v_O=(v-20)\ \text{m/s}$$

$$v_I=1\ \text{cm/s}=0.01\ \text{m/s}$$

$$m=\frac{1}{10}$$



Substituting in Eq. (i) we have,

$$0.01=\left(\frac{1}{10}\right)^2(v-20)$$

$\therefore$ $$v=21\ \text{m/s}$$

(b) $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

Multiplying with u we get,

$$\frac{u}{v}+1=\frac{u}{f}$$

or $$\frac{1}{m}=\frac{u}{f}-1=\frac{u-f}{f}$$

$\therefore$ $$m=\frac{f}{u-f}$$

Differentiating we have,

$$\left(\frac{dm}{dt}\right)=-\frac{f}{(u-f)^2}\cdot\frac{du}{dt} \qquad \ldots\text{(ii)}$$

Using mirror formula to find u with magnification $m=\frac{1}{10}$ we get,

$$\frac{1}{u/10}-\frac{1}{u}=\frac{1}{10}$$

or $$u=90\ \text{m} \qquad \text{(with sign } u=-90\ \text{m)}$$

Substituting in Eq. (ii) we have,

$$\frac{dm}{dt}=-\frac{(10)}{(-90-10)^2}(-1)=10^{-3}\ \text{per second}$$ **Ans.**

Note *u is decreasing at a rate of* $v-20$ *or* $(21-20)$ *or* 1 m/s

$\therefore$ $$\frac{du}{dt}=-1\ \text{m/s}$$

9. (a) At $t=t$, $u=-(2f+x)=-(2f+f\cos\omega t)$

Using, the mirror formula, $$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

We have $$\frac{1}{v} - \frac{1}{2f + f\cos\omega t} = \frac{-1}{f}$$

$\therefore$ $$v = -\left(\frac{2 + \cos\omega t}{1 + \cos\omega t}\right) f$$

i.e., distance of image from mirror at time t is $\left(\frac{2 + \cos\omega t}{1 + \cos\omega t}\right) f$ **Ans.**

(b) Ball coincides with its image at centre of curvature, *i.e.*, at $x = 0$.

(c) At $t = T/2$ or $\omega t = \pi$, $x = -f$

i.e., $u = -f$ or ball is at focus. So, its image is at ∞

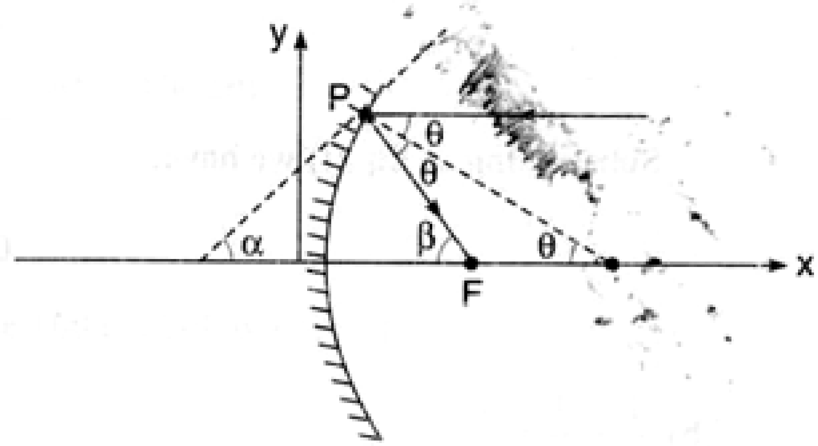
$$m = \infty$$

10. Let the ray is incident at a point $P = (x_1, y_1)$ on the mirror.

Then slope at P,

$$\tan\alpha = \left(\frac{dy}{dx}\right)_{(x_1 y_1)} = \frac{1}{4by_1} \qquad \text{...(i)}$$

$$\alpha = 90 - \theta \quad \text{and} \quad \beta = 2\theta$$



Now, the reflected ray is passing through $P(x_1 y_1)$ and has a slope $-\tan\beta$. Hence, the equation will be,

$$\left(\frac{y - y_1}{x - x_1}\right) = -\tan\beta = -\tan 2\theta = \frac{-2\tan\theta}{1 - \tan^2\theta}$$

$\therefore$ $$y - y_1 = \frac{2\cot\alpha}{1 + \cot^2\alpha}(x_1 - x) \qquad \text{...(ii)}$$

Further, $$x_1 = 4by_1^2 \qquad \text{...(iii)}$$

At F, $$x = 0 \qquad \text{...(iv)}$$

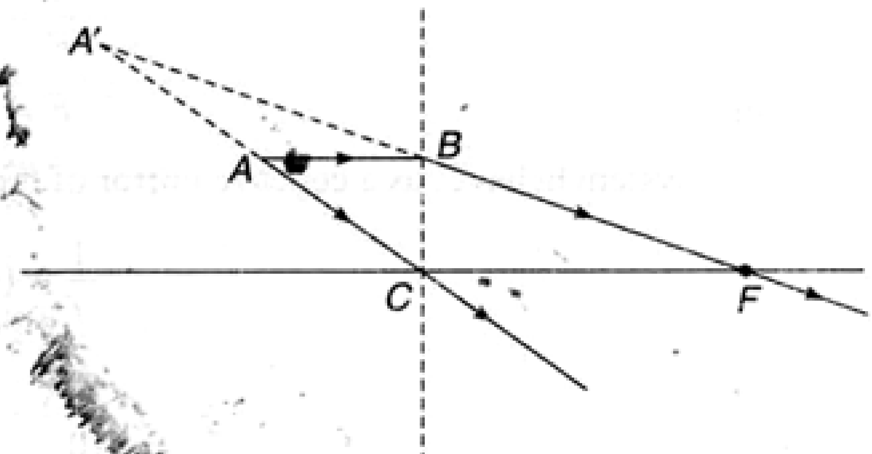
From Eq. (i) to Eq. (iv), we get $$x = \frac{1}{8b}$$

It shows that the co-ordinates of F are unique $\left(\frac{1}{8b}, 0\right)$. Hence, the reflected ray passed through one focus and the focal length is $\frac{1}{8b}$. **Hence proved.**

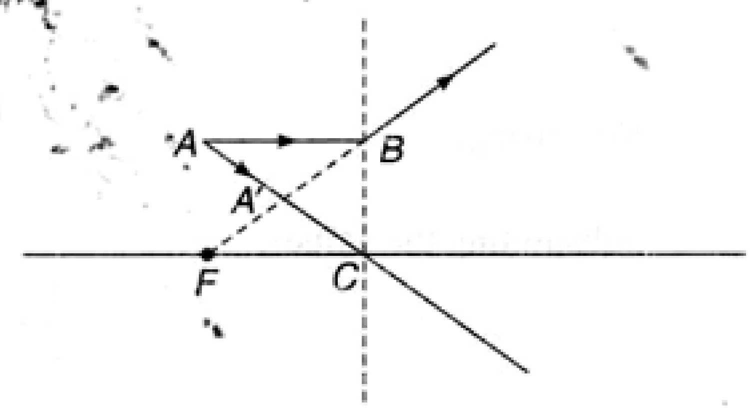
Chapter 28 Refraction of Light

JEE Advanced (Subjective Questions)

1. First draw a ray AA' until it intersects with the principal optical axis and find the centre of the lens C. Since, the virtual image is magnified, the lens is convex. Draw a ray AB parallel to the principal optical axis. It is refracted by the lens so that it passes through its focus and its continuation passes through the virtual image. The ray $A'B$ intersects the principal optical axis at point F, the focus of the lens.



2. The paths of the rays are shown in figure. Since, the virtual image is diminished, the lens is concave.



3. (a) $OA = 8.0$ cm

$$\therefore \quad AI_1 = (n_g)(OA) = \left(\frac{8}{5}\right)(8.0) = 12.8 \text{ cm}$$

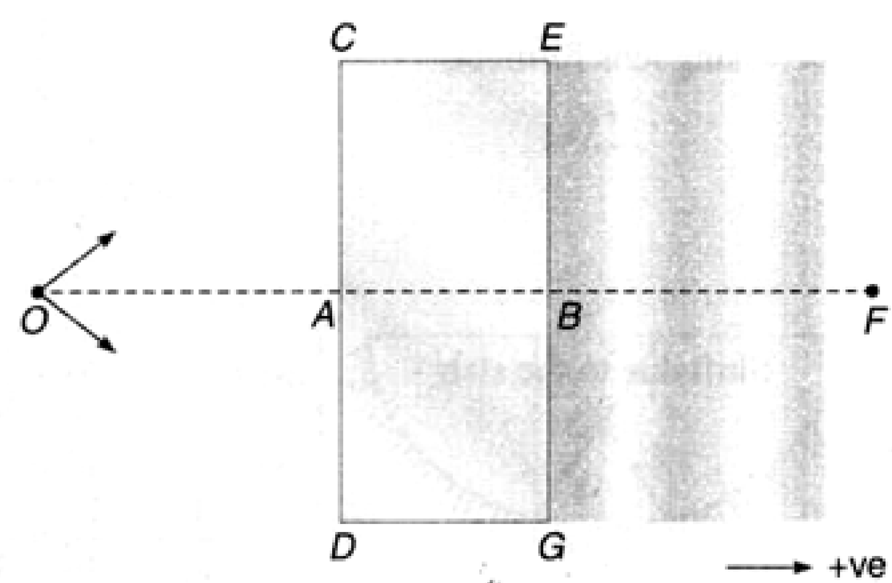
For refraction at EG ($R = \infty$), using

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\therefore \quad \frac{4/3}{BI_2} - \frac{8/5}{-(12.8+3)} = 0$$

$$\therefore \quad BI_2 = -(15.8)(4/3)(5/8) = -13.2 \text{ cm}$$

$$\therefore \quad FI_2 = 13.2 + 6.8 = 20.0 \text{ cm}$$ **Ans.**



(b) **For face EF :**

$$\frac{8/5}{BI_1} - \frac{4/3}{-6.8} = 0$$

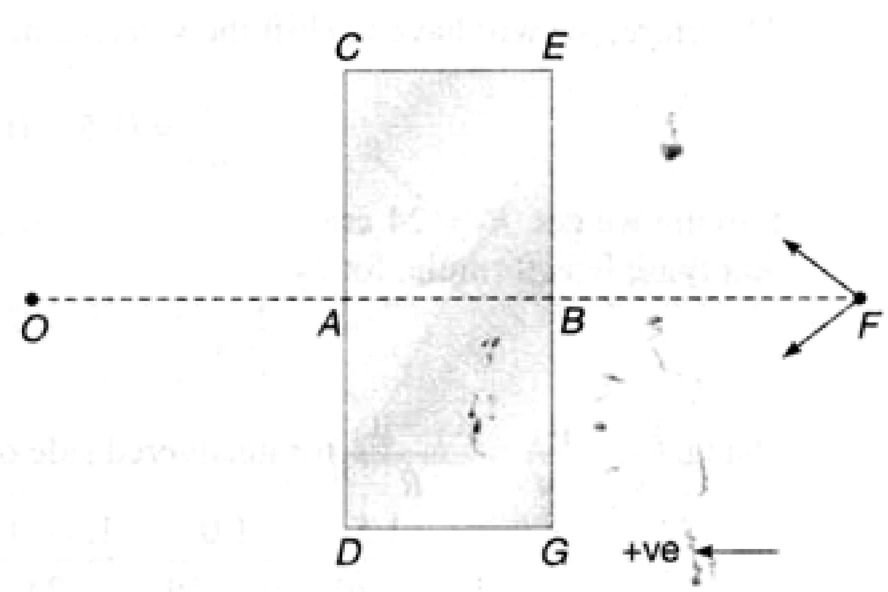
$$\therefore \quad BI_1 = -(6.8)(8/5)(3/4) = -8.16 \text{ cm}$$

For face CD :

$$\frac{1.0}{AI_2} - \frac{8/5}{-11.16} = 0$$

$$\therefore \quad AI_2 = -(11.16)(5/8) = -6.975 \text{ cm}$$

$$\therefore \quad FI_2 = 8 + 6.975 = 14.975 \text{ cm}$$ **Ans.**



4. The system behaves like a mirror of focal length given by,

$$\frac{1}{F}=\frac{2(n_2/n_1)}{R_2}-\frac{2(n_2/n_1-1)}{R_1}$$

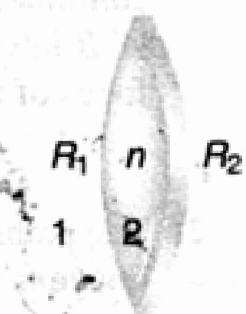
Substituting the values with proper sign.

$$\frac{1}{F}=\frac{2\times 4/3}{-20}\qquad (R_1=\infty)$$

or $$F=-7.5\text{ cm}$$ **Ans.**

i.e., system behaves as a concave mirror of focal length 7.5 cm.

5.



$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{10}=(1.5-1)\left(\frac{1}{R_1}-\frac{1}{-10}\right)$$

$\therefore$ $$R_1=+10\text{ cm}$$

Now using, $$\frac{1}{v}+\frac{1}{u}=\frac{2(n_2/n_1)}{R_2}-\frac{2(n_2/n_1-1)}{R_1}$$

Substituting the values,

$$\frac{1}{v}-\frac{1}{-15}=\frac{2(1.5)}{-10}-\frac{2(1.5-1)}{+10}$$

$\therefore$ $$v=-2.14\text{ cm}$$ **Ans.**

6. Using lens formula, $$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v_1}-\frac{1}{-40}=\frac{1}{30}$$

$\therefore$ $$v_1=120\text{ cm}$$

Shift due to the slab $$\Delta x=\left(1-\frac{1}{\mu}\right)d=\left(1-\frac{1}{1.8}\right)9=4\text{ cm}$$

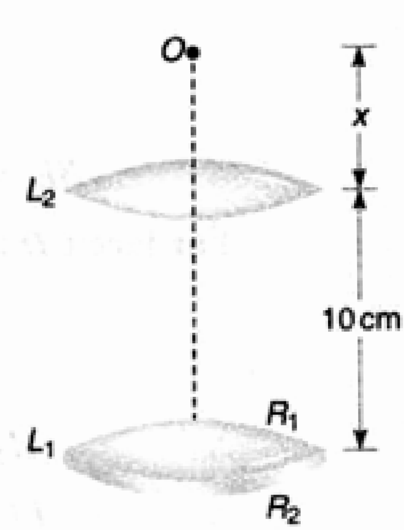
$\therefore$ $$u'=-(40-\Delta x)=-36\text{ cm}$$

$\therefore$ $$\frac{1}{v_2}-\frac{1}{-36}=\frac{1}{30}$$

$\therefore$ $$v_2=180\text{ cm}$$

Therefore, we will have to shift the screen a distance $x=v_2-v_1=60$ cm away from lens. **Ans.**

7.



$$\frac{1}{40}=(1.5-1)\left(\frac{1}{120}+\frac{1}{R_1}\right)$$

Solving we get, $R_2=24$ cm

Applying lens formula, for L_2

$$\frac{1}{v_1}+\frac{1}{x}=\frac{1}{20}\qquad \ldots\text{(i)}$$

Using, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ for unsilvered side of L_1.

$$\frac{1.5}{-120}-\frac{1.0}{v_1-10}=\frac{1.5-1.0}{24}\qquad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we get $$x=10\text{ cm}$$ **Ans.**

8. ***Case* I :**

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\therefore \quad \frac{1}{v_1}-\frac{1}{30}=\frac{1}{-10}$$

$$\therefore \quad v_1=-15 \text{ cm}$$

***Case* II :**

$$\text{Shift}=\left(1-\frac{1}{\mu}\right)t=\left(1-\frac{1}{1.5}\right)6=2 \text{ cm}$$

$$\therefore \quad \frac{1}{v_2}-\frac{1}{28}=\frac{1}{-10}$$

$$\therefore \quad v_2=-15.55 \text{ cm}$$

$$\therefore \quad \Delta v=0.55 \text{ cm}$$ **Ans.**

9. Using, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$, twice with $u=\infty$, we have

$$\frac{1.5}{v_1}=\frac{1.5-1.4}{+20} \quad \ldots(i)$$

$$\frac{1.6}{v_2}-\frac{1.5}{v_1}=\frac{1.6-1.5}{-20} \quad \ldots(ii)$$

Solving Eqs. (i) and (ii), we get $f=v_2=\infty$

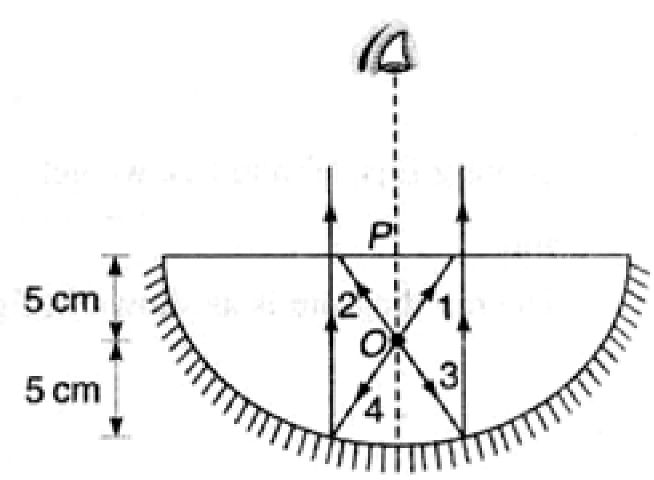
i.e., the system behaves like a glass plate.

10. First image will be formed by direct rays 1 and 2, etc.

$$PI_1=\frac{PO}{\mu}=\frac{5}{1.5}=3.33 \text{ cm}$$ **Ans.**

Second image will be formed by reflected rays 3 and 4, etc.

Object is placed at the focus of the mirror. Hence, I_2 is formed at infinity.



11. (a) Applying Snell's law at D.

$$\left(\frac{4}{3}\right)\sin i=\left(\frac{3}{2}\right)\sin 30°$$

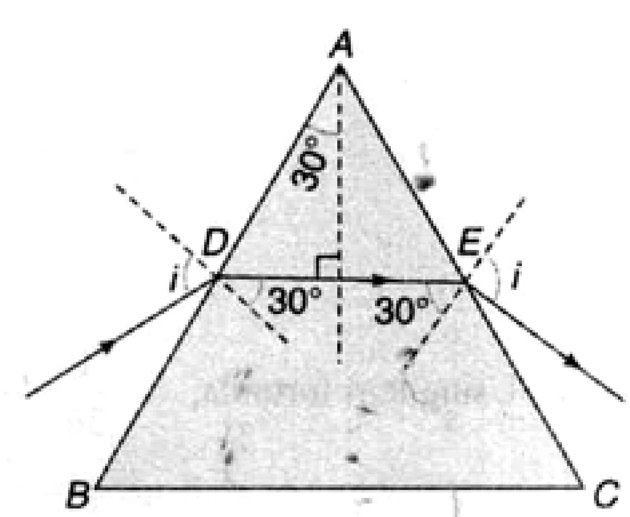
$$\therefore \quad i=34.2°$$ **Ans.**

(b)

$$\delta=\delta_D+\delta_E=2\delta_D$$

$$=2(i-30°)$$

$$=8.4°$$ **Ans.**

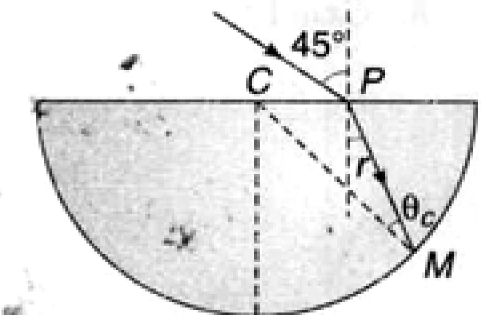


12.

$$\sqrt{2} = \frac{\sin 45°}{\sin r}$$

$$r = 30°$$

$$\theta_c = \sin^{-1}\left(\frac{1}{\mu}\right) = 45°$$



Applying, Sine law, in ΔCPM

$$\frac{CP}{\sin \theta_c} = \frac{CM}{\sin (90° + r)}$$

$\therefore$ $$\frac{CP}{(1/\sqrt{2})} = \frac{R}{\cos r}$$ (R = radius)

$\therefore$ $$CP = \sqrt{\frac{2}{3}}R$$

As we move away from C, angle PMC will increase. Therefore, $CP \not> \sqrt{\frac{2}{3}}R$. Same is the case on left side of C.

13.

$$\frac{1}{20} = (1.5 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$\therefore$ $$R = 20 \text{ cm}$$

Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ twice with the condition that rays must fall normally on the concave mirror.

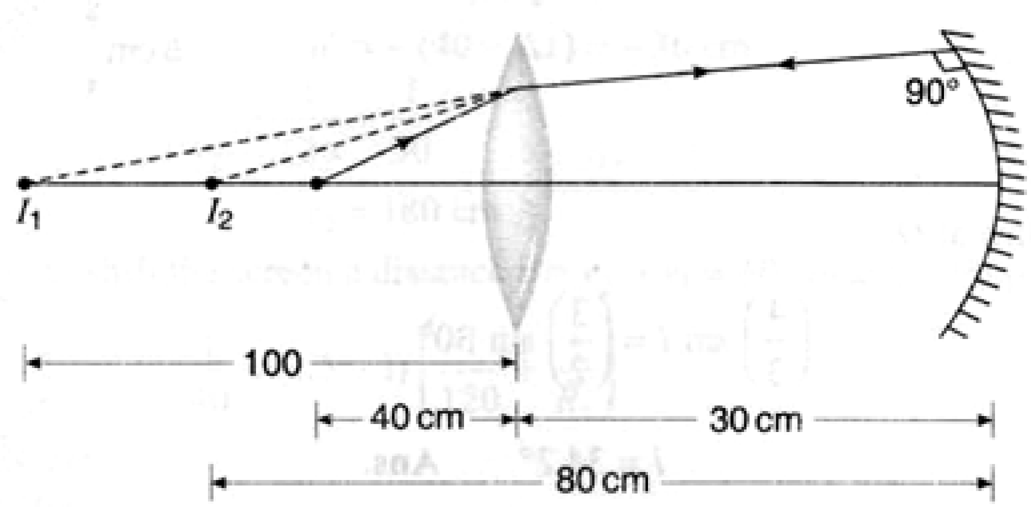
$$\frac{1.5}{v_1} - \frac{1.2}{-40} = \frac{1.5 - 1.2}{+20} \quad \text{...(i)}$$

$$\frac{2.0}{d - 80} - \frac{1.5}{v_1} = \frac{2.0 - 1.5}{-20} \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get $d = 30$ cm **Ans.**

and $v_1 = -100$ cm

The ray diagram is as shown in figure.



14. Using lens formula, $$\frac{1}{36} - \frac{1}{-45} = \frac{1}{f}$$

$\therefore$ $$f = 20 \text{ cm}$$

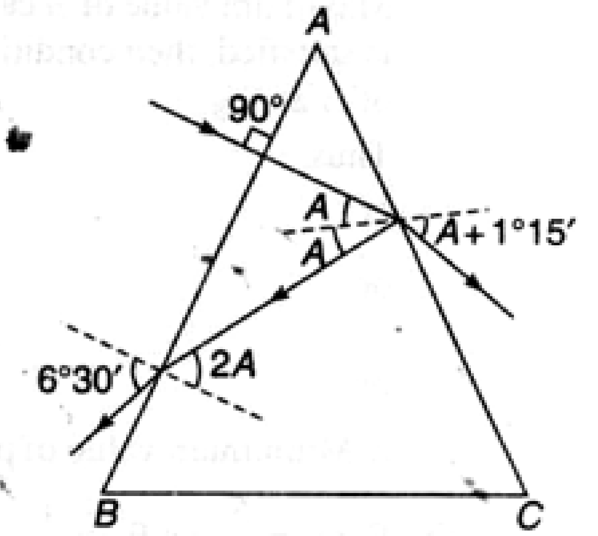
In the second case, let μ be the refractive index of the liquid, then

$$\frac{1}{48} - \frac{1}{-\left(5 + \frac{40}{\mu}\right)} = \frac{1}{20}$$

Solving this we get $\mu = 1.37$ **Ans.**

15. As the angles are small we can take,

$$\sin\theta \approx \theta$$

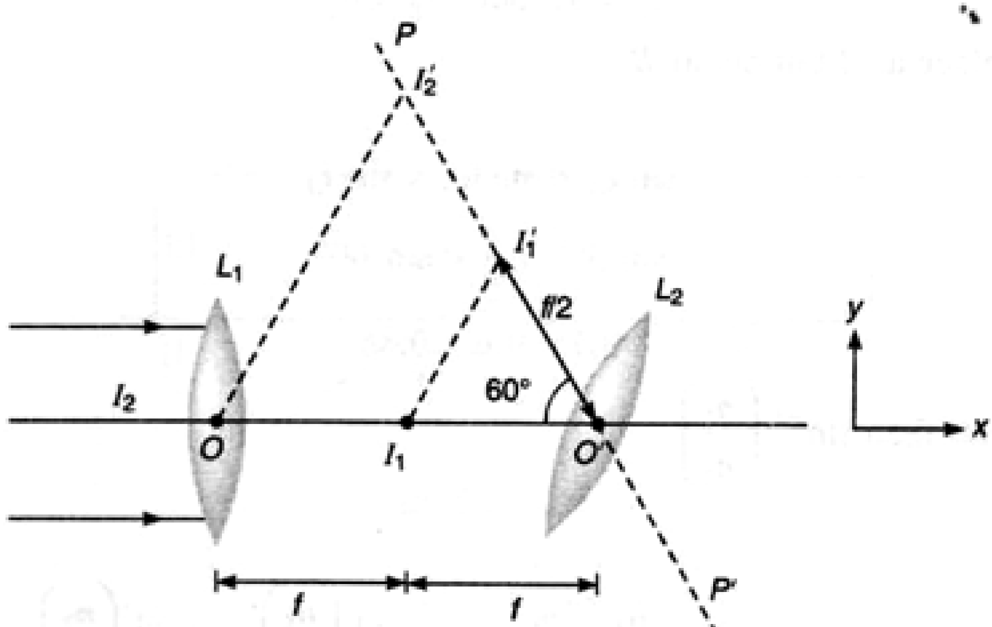


Now, $\mu = \frac{A + 1°15'}{A} = \frac{6°30'}{2A}$

Solving this equation, we get

$A = 2°$ and $\mu = 1.62$ **Ans.**

16. Using lens formula for L_2.



$$\frac{1}{v} - \frac{1}{-f/2} = \frac{1}{f}$$

or $v = -f$

This f length will be along PP' from point O' (towards P).

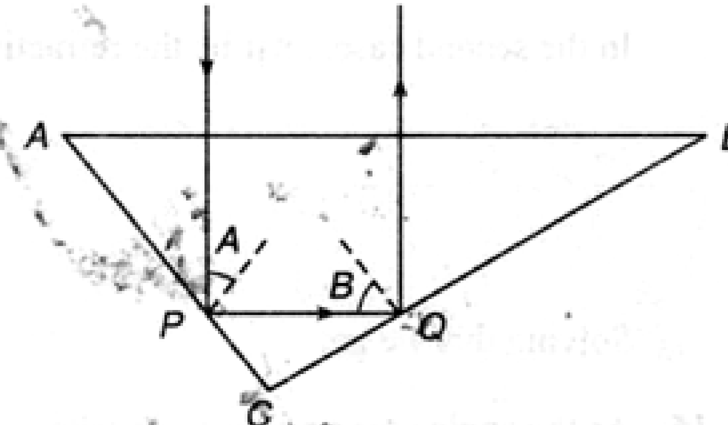
$\therefore$ $O'I_2' = f$

On x-axis this distance will be $f \sec 60° = 2f$.

Since $OO' = 2f$. Therefore image will be formed at origin.

Note *A ray passing through O and then O′ goes undeviated.*

Therefore I_1 and I_2 both should be on this line, which is also the x-axis. That's why for final image we have taken projection of $O'I_2'$ on x-axis.



17. (a) At P, angle of incidence $i_A = A$

and at Q, angle of incidence $i_B = B$

If TIR satisfies for the smaller angle of incidence than for larger angle of incidence is automatically satisfied.

$$B \le A$$

$$\therefore \quad i_B \le i_A$$

Maximum value of B can be 45°. Therefore, if condition of TIR is satisfied, then condition of TIR will be satisfied for all value of i_A and i_B

Thus, $$45° \ge \theta_c$$

or $$\sin 45° \ge \sin \theta_c$$

or $$\frac{1}{\sqrt{2}} \ge \frac{1}{\mu}$$

or $$\mu \ge \sqrt{2}$$

$\therefore$ Miniumum value of μ is $\sqrt{2}$.

(b) For $\mu = \frac{5}{3}$, $\sin \theta_c = \frac{1}{\mu} = \sin^{-1}\left(\frac{3}{5}\right) \approx 37°$

If $$B = 30°, \quad \text{then} \quad i_B = 30°$$

then $$A = 60° \quad \text{or} \quad i_A = 60°$$

$$i_A > \theta_c \quad \text{but} \quad i_B < \theta_c$$

i.e., TIR will take place at A but not at B.

Or we can write :

$$\sin i_B < \sin \theta_c < \sin i_A$$

or $$\sin 30° < \frac{3}{5} < \sin 60°$$

or $$0.5 < 0.6 < 0.86$$

18. Given θ is slightly greater than $\sin^{-1}\left(\frac{n_1}{n_2}\right)$

(i) When $n_3 < n_1$:

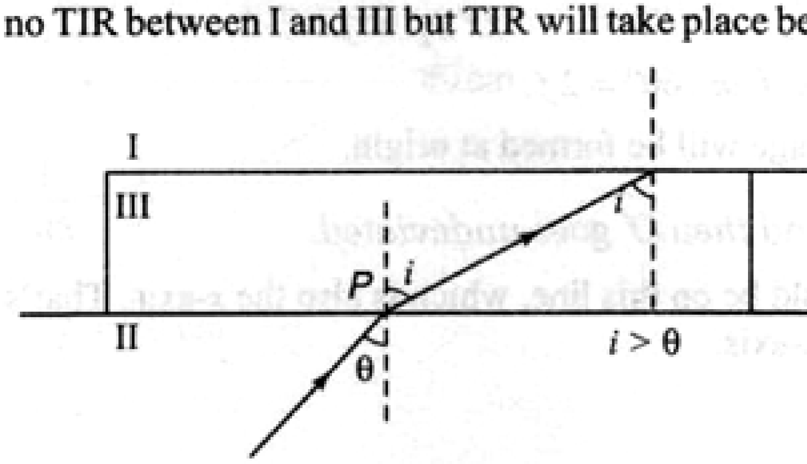
i.e., $$n_3 < n_1 < n_2 \quad \text{or} \quad \frac{n_3}{n_2} < \frac{n_1}{n_2} \quad \text{or} \quad \sin^{-1}\left(\frac{n_3}{n_2}\right) < \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Hence, critical angle for III and II will be less than the critical angle for II and I. So, it TIR is taking place between I and II, then TIR will definitely take place between I and III.

(ii) When $n_3 > n_1$: Now two cases may arise :

Case I : $$n_1 < n_3 < n_2$$

In this case there will be no TIR between I and III but TIR will take place between III and II. This is because :



Ray of light first enters from II to III *i.e.,* form denser to rarer.

$$\therefore \quad i > \theta$$

Applying Snell's law at P:

$$n_2 \sin\theta = n_3 \sin i$$

or
$$\sin i = \left(\frac{n_2}{n_3}\right)\sin\theta$$

Since, $\sin\theta$ is slightly greater than $\frac{n_1}{n_2}$

$\therefore$ $\sin i$ is slightly greater than $\frac{n_2}{n_3} \times \frac{n_1}{n_2}$ or $\frac{n_1}{n_3}$

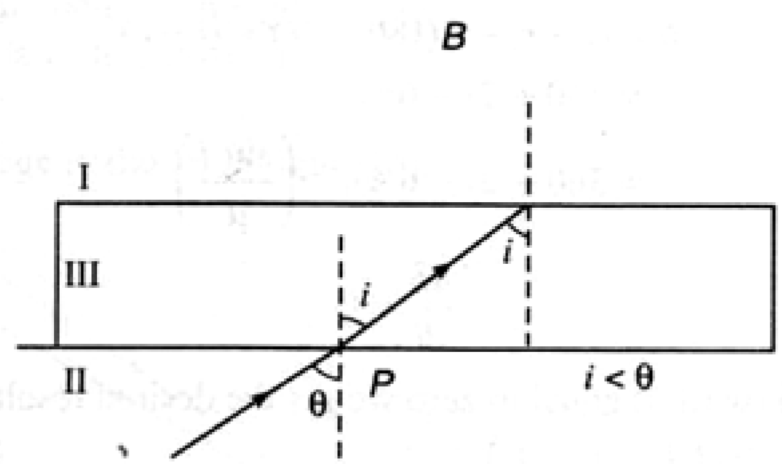
but $\frac{n_1}{n_3}$ is nothing but $\sin(\theta_c)_{\text{I, III}}$

$\therefore$ $\sin(i)$ is slightly greater than $\sin(\theta_c)_{\text{I, III}}$

or TIR will now take place on I and III and the ray will be reflected back.

Case II : $\quad n_1 < n_2 < n_3$

This time while moving from II to III, ray of light will bend towards normal. Again applying Snell's law at P :



$$n_2 \sin\theta = n_3 \sin i$$

$$\sin i = \frac{n_2}{n_3}\sin\theta$$

Since, $\sin\theta$ slightly greater than $\frac{n_1}{n_2}$

$\sin i$ will be slightly greater than $\frac{n_2}{n_3} \times \frac{n_1}{n_2}$ or $\frac{n_1}{n_3}$

but $\quad \frac{n_1}{n_3}$ is $\sin(\theta_c)_{\text{I, III}}$

i.e., $\quad \sin i > \sin(\theta_c)_{\text{I, III}}$

or $\quad i > (\theta_c)_{\text{I, III}}$

Therefore, TIR will again take place between I and III and the ray will be reflected back.

Note *Case* II *and case* II *of* $n_3 > n_1$ *can be explained by one equation only. But two cases are deliberately formed for better understanding of refraction, Snell's law and total internal reflection (TIR).*

19. Since, the vessel is cubical, $\angle GDE = 45°$

and $$GE = ED = h \text{ (say)}$$

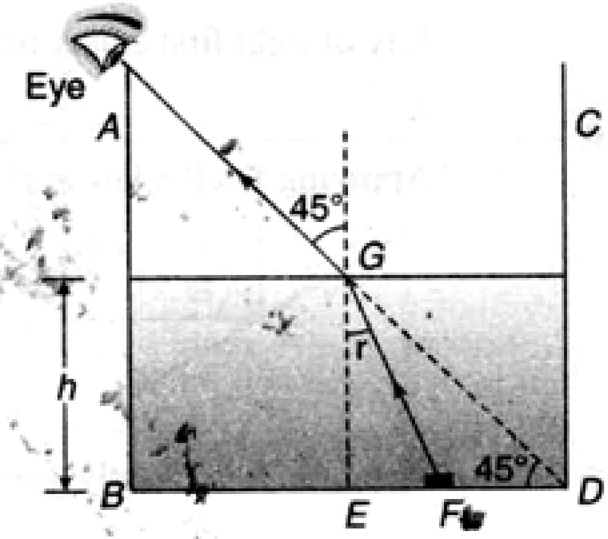
then $$EF = ED - FD = (h - 10)$$

Further, $$\frac{4}{3} = \frac{\sin 45°}{\sin r} \quad \therefore \quad r = 32°$$

Now $$\frac{EF}{GE} = \tan r = \tan 32°$$

$\therefore$ $$\frac{h - 10}{h} = 0.62$$

Solving this, we get $$h = 26.65 \text{ cm}$$ **Ans.**

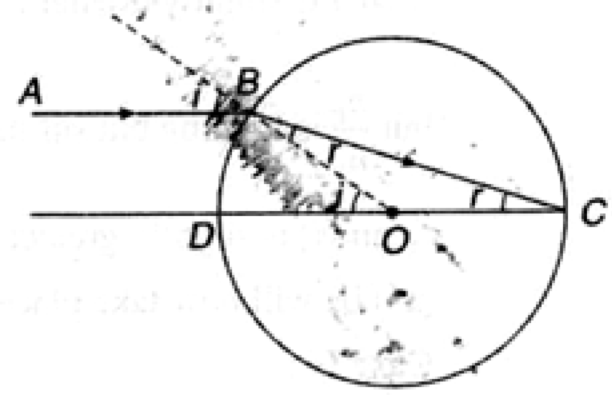


20. $$BO = OC$$

$\therefore$ $$\angle OBC = \angle BCO = r \text{ (say)}$$

Let angle of incidence be i $\quad i = r + r = 2r \quad$ (external angle)

$\therefore$ $$\mu = \frac{\sin i}{\sin r} = \frac{\sin 2r}{\sin r} \approx \frac{2r}{r} = 2$$ **Ans.**



21. $\delta_{\text{Total}} = \delta_{\text{Refraction}} + 2\delta_{\text{Reflection}} + \delta_{\text{Refraction}}$

or $$\delta = (i - r) + 2(180 - 2r) + (i - r)$$
$$= 360 + 2i - 6r$$
$$= 360 + 2i - 6\sin^{-1}\left(\frac{\sin i}{\mu}\right)$$

For deviation to be minimum, $$\frac{d\delta}{di} = 0$$

By putting first derivative of δ (w.r.t. i) equal to zero we get the desired result.

22. (a) For refraction at first half lens $\left(\frac{1}{v} - \frac{1}{u} = \frac{1}{f}\right)$

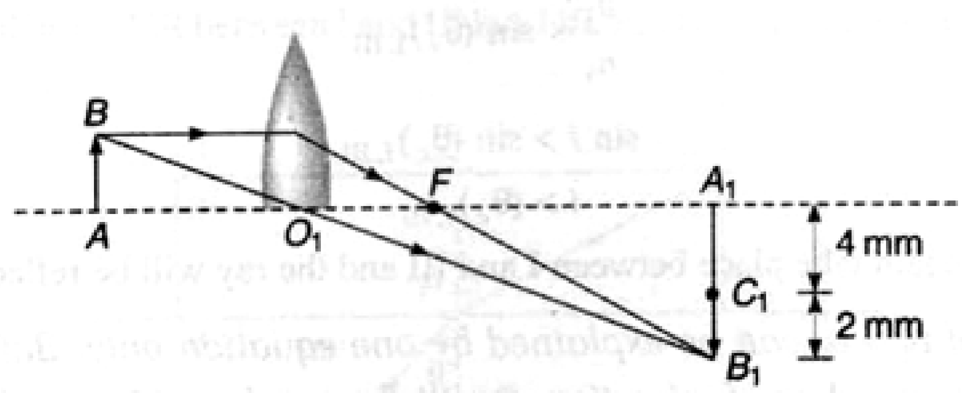
$$\frac{1}{v} - \frac{1}{-20} = \frac{1}{15}$$

$\therefore$ $$v = 60 \text{ cm}$$

Magnification, $$m = \frac{v}{u} = \frac{60}{-20} = -3$$

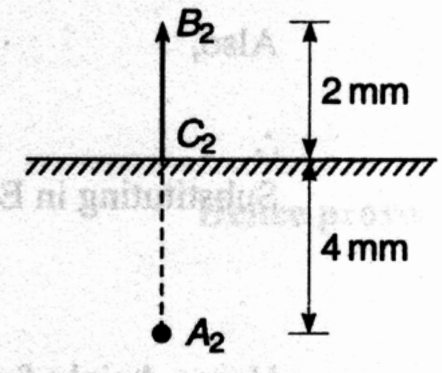
The image formed by first half lens is shown in figure (a).

$AB = 2$ mm, $\quad A_1B_1 = 6$ mm, $\quad AO_1 = 20$ cm, $\quad O_1F = 15$ cm and $O_1A_1 = 60$ cm.



(a)

Point B_1 is 6 mm below the principal axis of the lenses. Plane mirror is 4 mm below it. Hence, 4 mm length of A_1B_1 (*i.e.*, A_1C_1) acts as real object for mirror. Mirror forms its virtual image A_2C_2. 2 mm length of A_1B_1 (*i.e.*, C_1B_1) acts as virtual object for mirror. Real image C_2B_2 is formed of this part. Image formed by plane mirror is shown in figure (b).



(b)

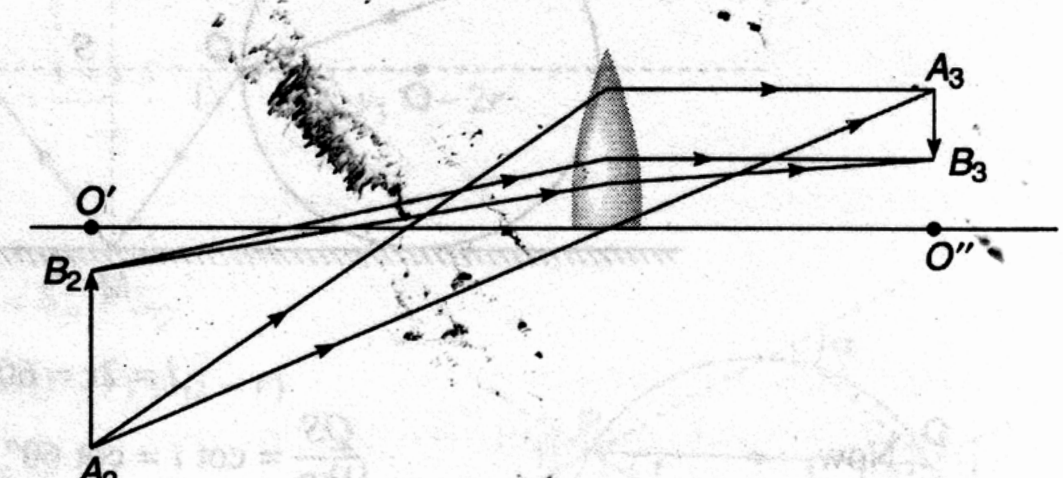
For the second half lens, $\quad \dfrac{1}{v} - \dfrac{1}{-60} = \dfrac{1}{15}$

$\therefore \quad v = +20$

$$m = \frac{v}{u} = \frac{20}{-60} = -\frac{1}{3}$$

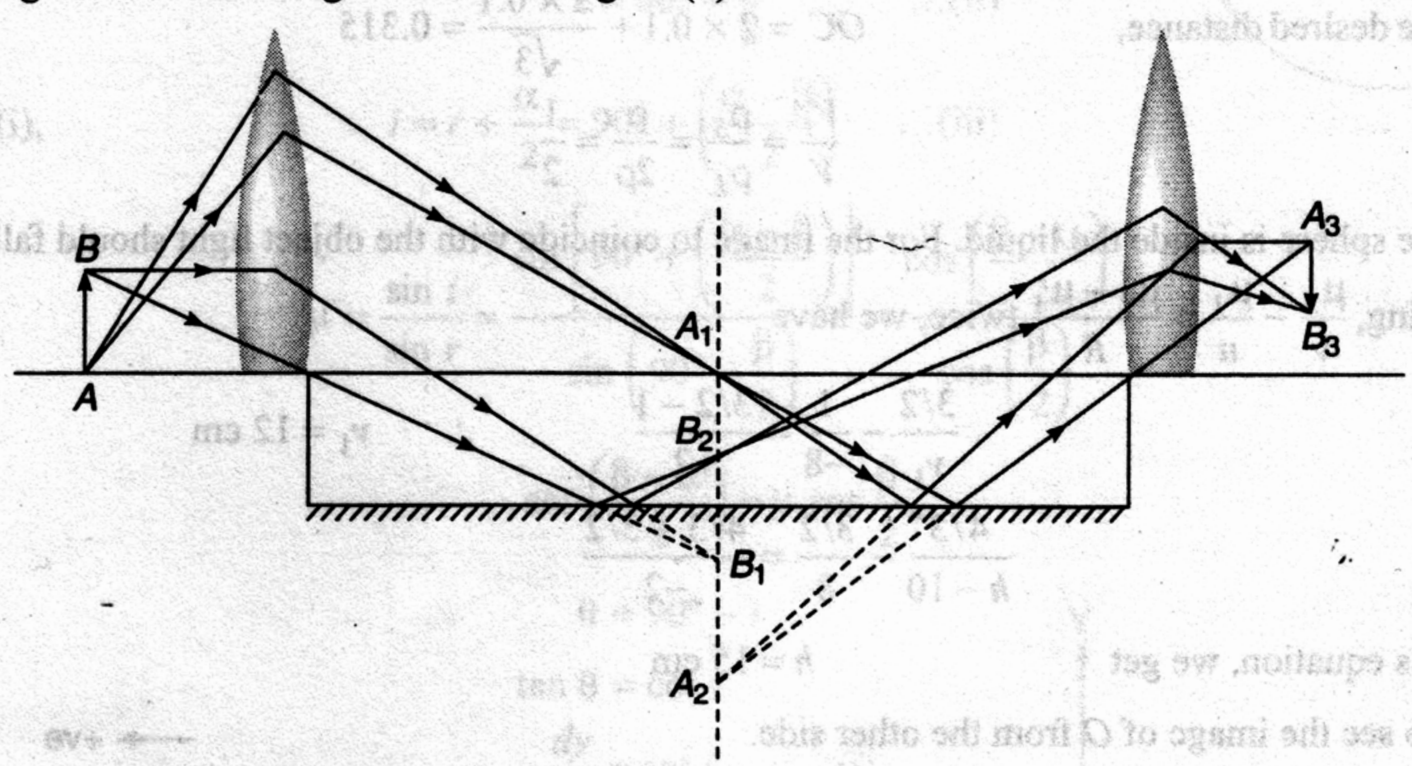
So, length of final image $A_3B_3 = \dfrac{1}{3} A_2B_2 = 2$ mm.

Point B_2 is 2 mm below the optic axis of second half lens. Hence, its image B_3 is formed 2/3 mm above the principal axis. Similarly, point A_2 is 8 mm below the principal axis. Hence, its image is 8/3 mm above it. Therefore, image is at a distance of 20 cm behind the second half lens and at a distance of 2/3 mm above the principal axis. The size of image is 2 mm and is inverted as compared to the given object. Image formed by second half lens is shown in figure (c).



(c)

(b) Ray diagram for final image is shown in figure (d).



(d)

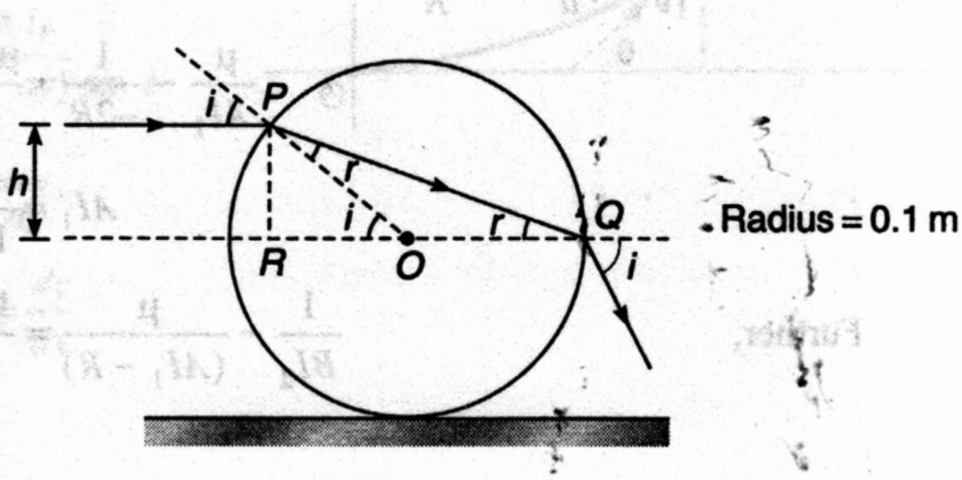
23. (i) $PO = OQ \quad \therefore \quad \angle OPQ = \angle OQP = r$ (say)

Also, $\quad i = r + r = 2r$

In, ΔPOR, $\quad h = OP \sin i = 0.1 \sin i$

$\quad = 0.1 \sin 2r$

or $\quad h = 0.2 \sin r \cos r \quad \ldots$(i)



Also, $$\sqrt{3}=\frac{\sin i}{\sin r}=\frac{2\sin r\cos r}{\sin r}=2\cos r$$

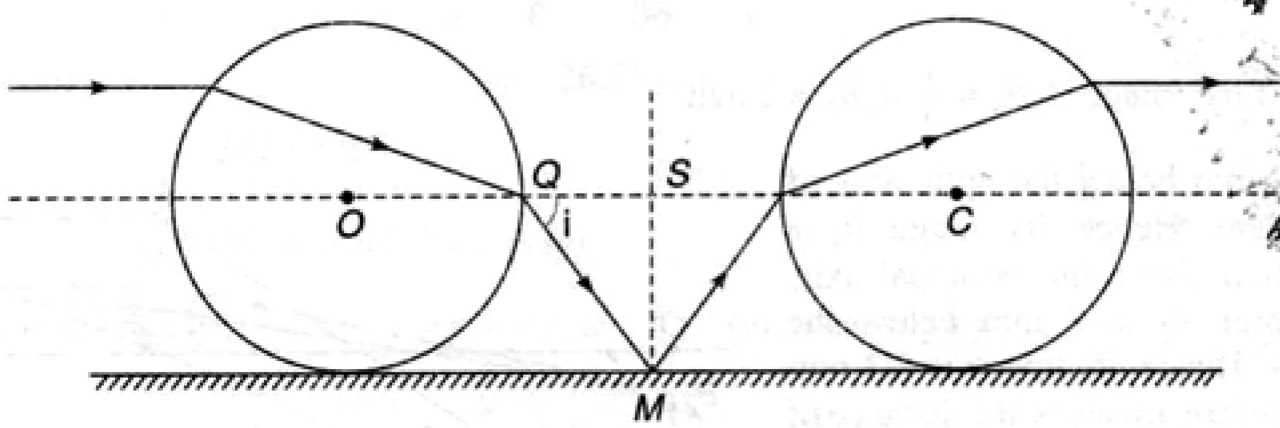
$\therefore$ $$r=30^\circ$$

Substituting in Eq. (i), we get

$$h=0.2\times\frac{1}{2}\times\frac{\sqrt{3}}{2}=0.086\text{ m}$$

Hence, height from the mirror is $0.1+0.086=0.186$ m

(ii) Use the principle of reversibility.



$$i=2r=60^\circ$$

Now, $$\frac{QS}{MS}=\cot i=\cot 60^\circ=\frac{1}{\sqrt{3}}$$

$\therefore$ $$QS=\frac{MS}{\sqrt{3}}=\frac{0.1}{\sqrt{3}}$$

$\therefore$ The desired distance, $$OC=2\times 0.1+\frac{2\times 0.1}{\sqrt{3}}=0.315$$ **Ans.**

24. $$\frac{V_i}{V}=\frac{\rho_S}{\rho_L}=\frac{\rho}{2\rho}=\frac{1}{2}$$

i.e., half the sphere is inside the liquid. For the image to coincide with the object light should fall normally on the sphere. Using, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ twice, we have

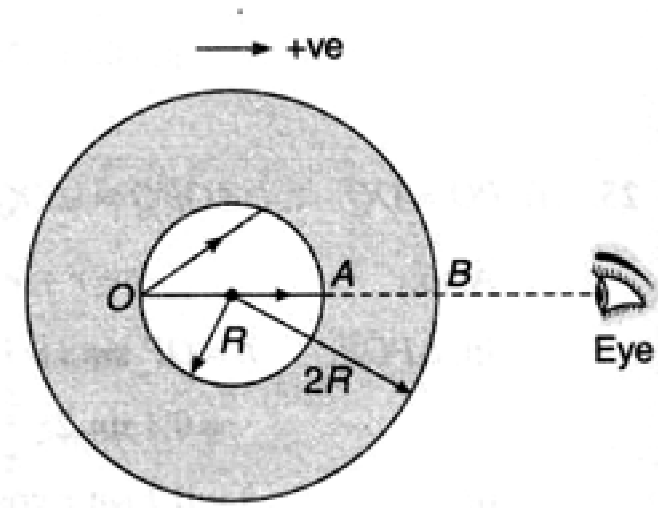
$$\frac{3/2}{v_1}-\frac{1}{-8}=\frac{3/2-1}{+2}\qquad\therefore\qquad v_1=12\text{ cm}$$

Further, $$\frac{4/3}{h-10}-\frac{3/2}{8}=\frac{4/3-3/2}{-2}$$

Solving this equation, we get $$h=15\text{ cm}$$ **Ans.**

25. We have to see the image of O from the other side.

Applying, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ twice, we have



$$\frac{\mu}{AI_1}-\frac{1}{-2R}=\frac{\mu-1}{-R}$$

$\therefore$ $$AI_1=\frac{2\mu R}{1-2\mu}$$

Further, $$\frac{1}{BI_2}-\frac{\mu}{(AI_1-R)}=\frac{1-\mu}{-R}$$

Solving this equation, we get $$BI_2 = -\frac{2R(4\mu - 1)}{3\mu - 1}$$

$\therefore$ Distance between the final image and object is

$$d = 3R - \frac{2R(4\mu - 1)}{3\mu - 1} = \frac{(\mu - 1)R}{(3\mu - 1)}$$ **Hence proved.**

26. First refraction, $$\frac{1.5}{v_1} - \frac{1}{-2r} = \frac{1.5 - 1}{r}$$

$\therefore$ $$v_1 = \infty$$

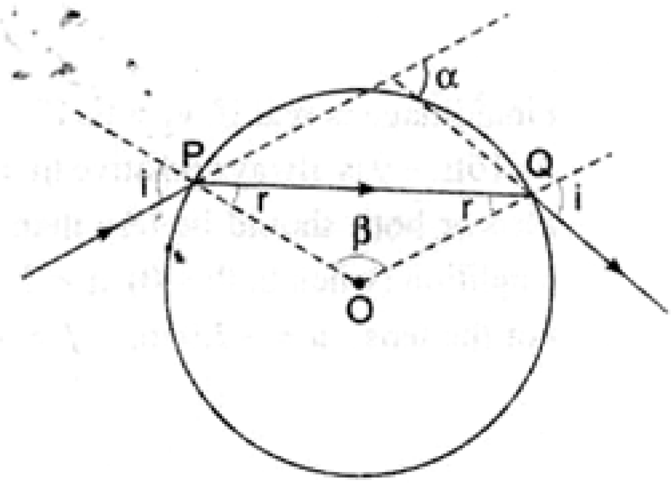
i.e., rays become parallel to the principal axis.

So, $$v_2 = \frac{r}{2}$$ (from pole of the mirror)

Second refraction, $$\frac{1}{v_3} - \frac{1.5}{-\frac{3r}{2}} = \frac{1 - 1.5}{-r}$$ $\therefore v_3 = -2r$

i.e., final image is formed at pole of the mirror.

27. $$\delta_{\text{Total}} = \delta_P + \delta_Q$$



$\therefore$ $$\alpha = (i - r) + (i - r)$$

or $$i - r = \frac{\alpha}{2} \quad \text{...(i)}$$

Further, in ΔOPQ, $$r + r + \beta = 180°$$

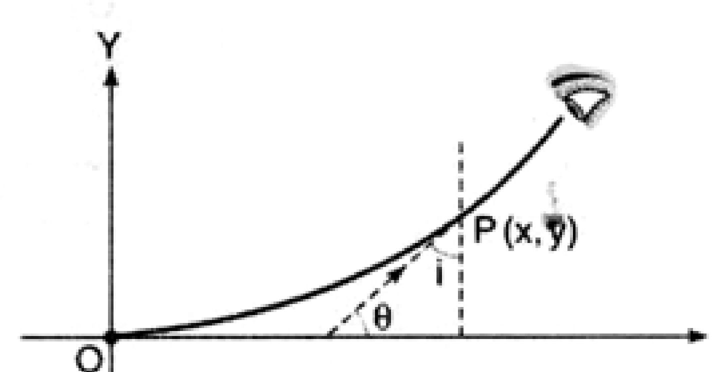
$\therefore$ $$r = 90° - \frac{\beta}{2} \quad \text{...(ii)}$$

From Eq. (i), $$i = r + \frac{\alpha}{2} = 90° + \left(\frac{\alpha - \beta}{2}\right) \quad \text{...(iii)}$$

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin\left[90° + \left(\frac{\alpha - \beta}{2}\right)\right]}{\sin\left(90° - \frac{\beta}{2}\right)} = \frac{\cos\left(\frac{\beta - \alpha}{2}\right)}{\cos\left(\frac{\beta}{2}\right)}$$

or $$\cos\left(\frac{\beta - \alpha}{2}\right) = \mu \cos\frac{\beta}{2}$$ **Hence Proved**

28. $$\theta = 90° - i$$



$$\tan\theta = \cot i$$

or $$\frac{dy}{dx} = \cot i \quad \text{...(i)}$$

$$\mu_0 \sin i_0 = \mu_P \sin i_P$$

(i) $$\sin 90° = (\sqrt{1 + ay}) \sin i$$

$\therefore$ $$\sin i = \frac{1}{\sqrt{1 + ay}}$$

$\therefore$ $$\cot i = \sqrt{ay} = \frac{dy}{dx}$$

$\therefore$ $$\int_0^y \frac{dy}{\sqrt{ay}} = \int_0^x dx$$

or $$x = 2\sqrt{\frac{y}{a}}$$

Substituting $y = 2\text{ m}$ and $a = 2.0 \times 10^{-6}\text{ m}^{-1}$

We get $$x_{max} = 2000\text{ m} = 2\text{ km}$$ **Ans.**

29. Applying, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ twice, we have

$$\frac{\mu}{v_1} - \frac{1}{-2R} = \frac{\mu - 1}{R}$$

$\therefore$ $$v_1 = \frac{2\mu R}{2\mu - 3}$$

Further, $$\frac{1}{v_2} - \frac{\mu}{-(3R - v_1)} = \frac{1-\mu}{R/2}$$

$\therefore$ $$v_2 = \frac{R(9 - 4\mu)}{(10\mu - 9)(\mu - 2)}$$ **Ans.**

Final image is real if, $v_2 > 0$.

As $10\mu - 9$ is always positive ($\mu > 1$). Therefore, for $v_2 > 0$, either $(9 - 4\mu)$ and $(\mu - 2)$ both should be greater than zero or both should be less than zero. For the first condition (when both > 0) $2 < \mu < 2.25$ and for the second condition (when both < 0), $\mu < 2$ and $\mu > 2.25$ which is not possible. Hence, μ should lie between 2 and 2.25.

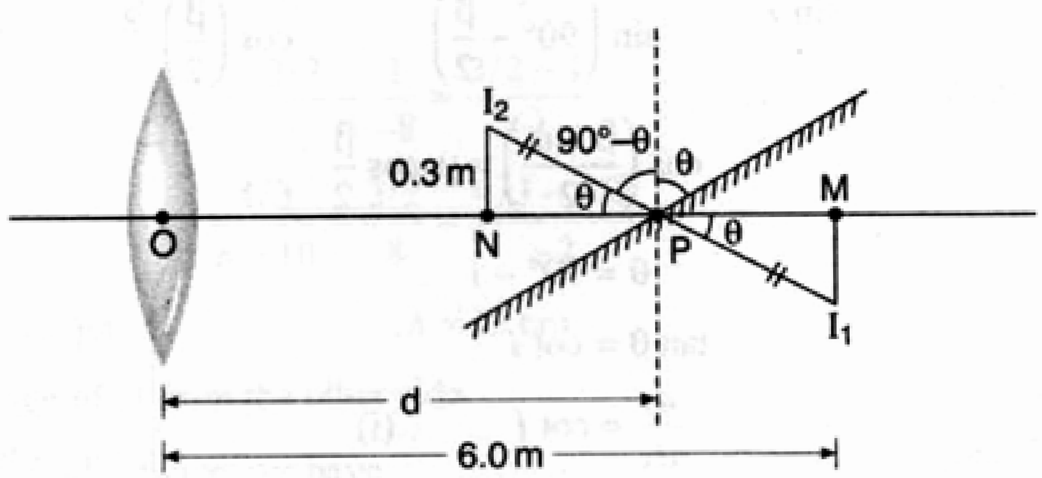
30. For the lens, $u = -2.0$ m, $f = +1.5$ m

$\therefore$ $$\frac{1}{v} - \frac{1}{-2.0} = \frac{1}{1.5}$$

or $$v = 6.0\text{ m}$$

$$m = \frac{6.0}{-2.0} = -3.0$$

Therefore, y-coordinate of image formed by lens is $m(0.1) = -0.3$ m.



$$\frac{0.3}{NP} = \tan\theta = 0.3$$

$\therefore$ $$NP = MP = 1.0\text{ m}$$

or $$d = 6.0 - 1.0 = 5.0\text{ m}$$ **Ans.**

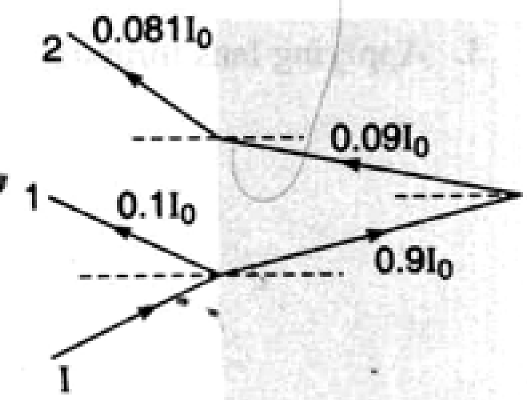
and x-coordinate of final image I_2 is,

$$x = d - 1.0 = 4.0\text{ m}$$ **Ans.**

Chapter 29 Interference and Diffraction of Light

JEE Advanced (Subjective Questions)

1. $I_1 = 0.1 I_0$, $\quad I_2 = 0.081 I_0$

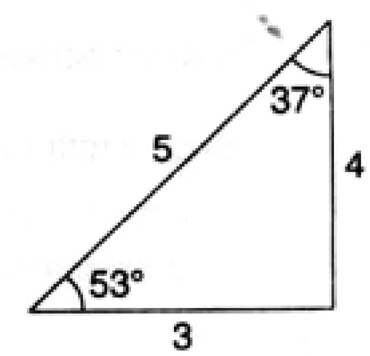


$\therefore \quad \sqrt{\dfrac{I_1}{I_2}} = \dfrac{10}{9}$

$\therefore \quad \dfrac{I_{max}}{I_{min}} = \left(\dfrac{\sqrt{I_1/I_2}+1}{\sqrt{I_1/I_2}-1}\right)^2 = (19)^2$

$= 361$ **Ans.**

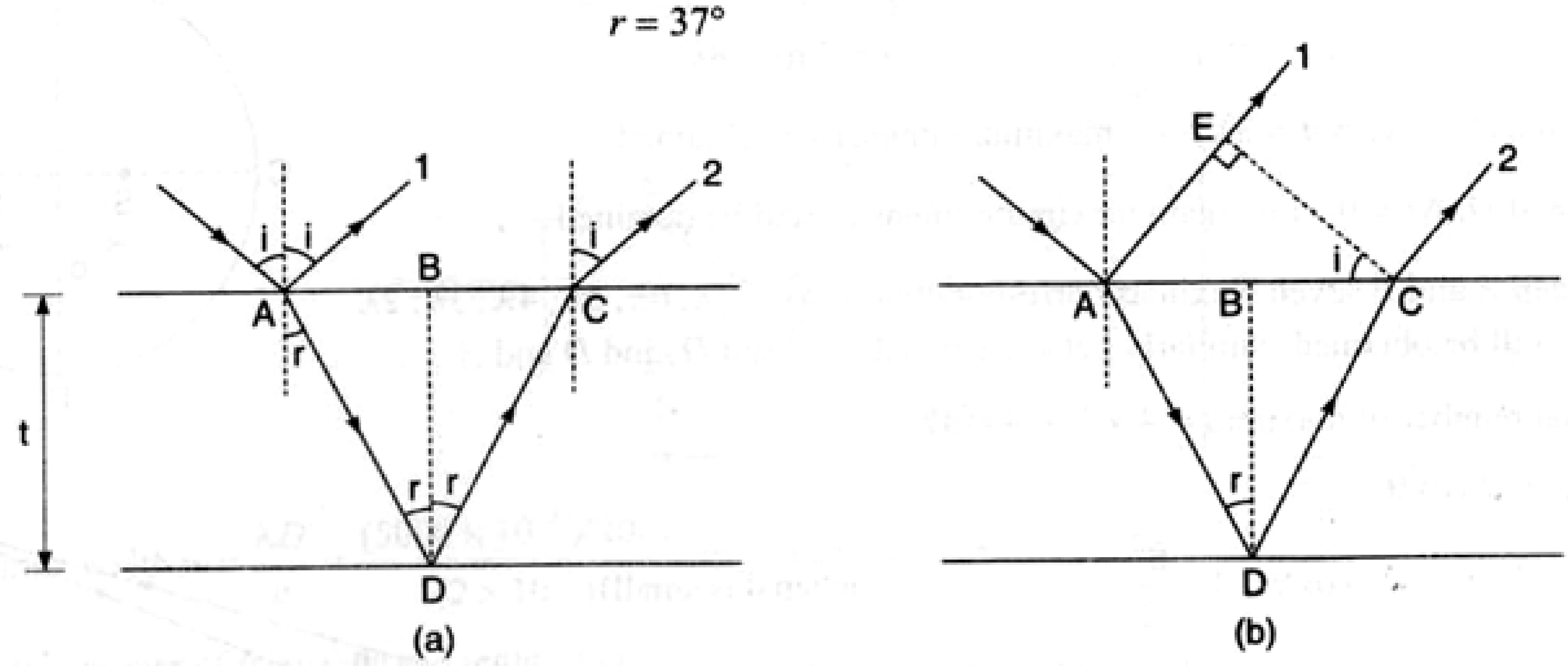
2. $\mu = \dfrac{\sin i}{\sin r}$



$\therefore \quad \dfrac{4}{3} = \dfrac{\sin 53°}{\sin r} = \dfrac{4/5}{\sin r}$

$\therefore \quad \sin r = \dfrac{3}{5}$

$\therefore \quad r = 37°$



Refer figure (a) :

$$\Delta x_1 = \text{between 2 and 1} = 2\,(AD)$$
$$= 2BD \sec r = 2t \sec r$$

Their optical path $\Delta x_1 = 2\mu t \sec r$.

Refer figure (b):

$$\Delta x_2 = AC \sin i = (2t \tan r) \sin i$$

$$\therefore \quad (\Delta x)_{net} = \Delta x_1 - \Delta x_2 = 2\mu t \sec r - 2t (\tan r)(\sin i)$$

$$= 2 \times \frac{4}{3} \times t \times \frac{5}{4} - 2 \times t \times \frac{3}{4} \times \frac{4}{5}$$

$$= \frac{32}{15} t$$

Phase difference between 1 and 2 is π.

$\therefore$ For constructive interference

$$\frac{32}{15}t = \frac{\lambda}{2}$$

or
$$t = \frac{15\lambda}{64} = \frac{15 \times 0.6}{64} = 0.14\ \mu m \qquad \textbf{Ans.}$$

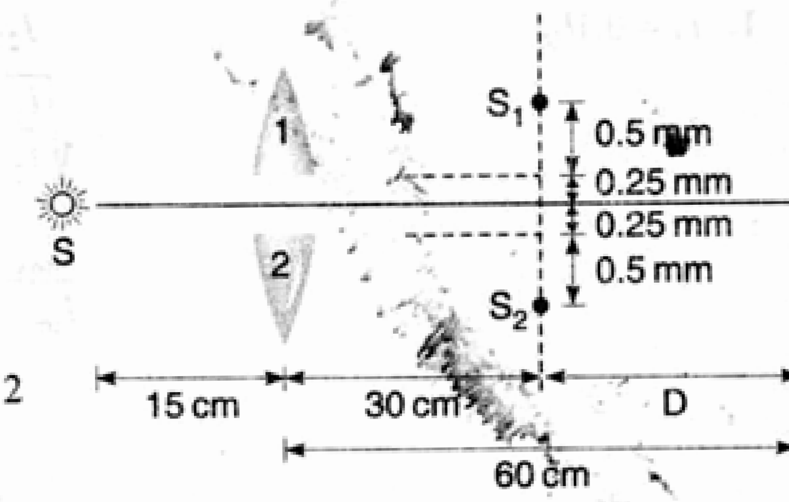
3. Applying lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{15} = \frac{1}{10}$$

$\therefore$
$$v = 30\text{ cm}$$

$$m = \frac{v}{u} = \frac{30}{-15} = -2$$



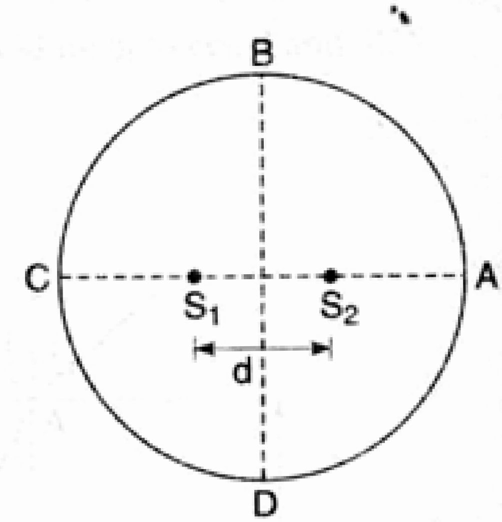
Distance between two slits. $d = 1.5$ mm, $D = 30$ cm

Fringe width
$$w = \frac{\lambda D}{d} = \frac{(5.0 \times 10^{-7})(0.3)}{(1.5 \times 10^{-3})} = 10^{-4}\text{ m}$$

$$= 0.1\text{ mm} \qquad \textbf{Ans.}$$

4.
$$\lambda = 0.25\text{ m}$$

$$d = 2\text{ m} = 8\lambda$$



At A and C, $\Delta x = d = 8\lambda$ *i.e.*, maximum intensity is obtained.

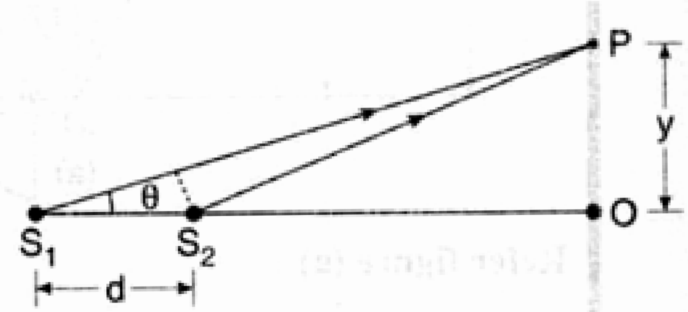
At B and D, $\Delta x = 0$ *i.e.*, again maximum intensity will be obtained.

Between A and B seven maximas corresponding to $\Delta x = 7\lambda, 6\lambda, 5\lambda, 4\lambda, 3\lambda, 2\lambda$ and λ will be obtained. Similarly between B and C, C and D, and D and A.

$\therefore$ Total number of maximas $= 4 \times 7 + 4 = 32$ **Ans.**

5. (a) $\Delta x = d\cos\theta$

$$\cos\theta = 1 - \frac{\theta^2}{2} \qquad \text{(when } \theta \text{ is small)}$$



$\therefore$
$$\Delta x = d\left(1 - \frac{\theta^2}{2}\right) = d\left(1 - \frac{y^2}{2D^2}\right)$$

For n^{th} maxima $\Delta x = n\lambda$

$\therefore$ y = radius of n^{th} bright ring $= D\sqrt{2\left(1 - \frac{n\lambda}{d}\right)}$ **Ans.**

(b)
$$d = 1000\lambda$$

At O,
$$\Delta x = d = 1000\lambda$$

i.e., at O, 1000^{th} order maxima is obtained.

Substituting $n = 998$ in
$$y = D\sqrt{2\left(1 - \frac{n\lambda}{d}\right)}$$

We get the radius of second closest ring

$$r = 6.32 \text{ cm}$$ **Ans.**

(c) $n = 998$ **Ans.**

6. (a) The optical path difference between the two waves arriving at P is

$$\Delta x = \frac{y_1 d}{D_1} + \frac{y_2 d}{D_2} = \frac{(1)(10)}{10^3} + \frac{(5)(10)}{2 \times 10^3}$$

$$= 3.5 \times 10^{-2} \text{ mm} = 0.035 \text{ mm}$$

As, $\Delta x = 70\lambda$

$\therefore$ 70^{th} order maxima is obtained at P.

(b) At O, $\Delta x = \frac{y_1 d}{D_1} = 10^{-2} \text{ mm} = 0.01 \text{ mm}$

As $\Delta x = 20\lambda$

$\therefore$ 20^{th} order maxima is obtained at O.

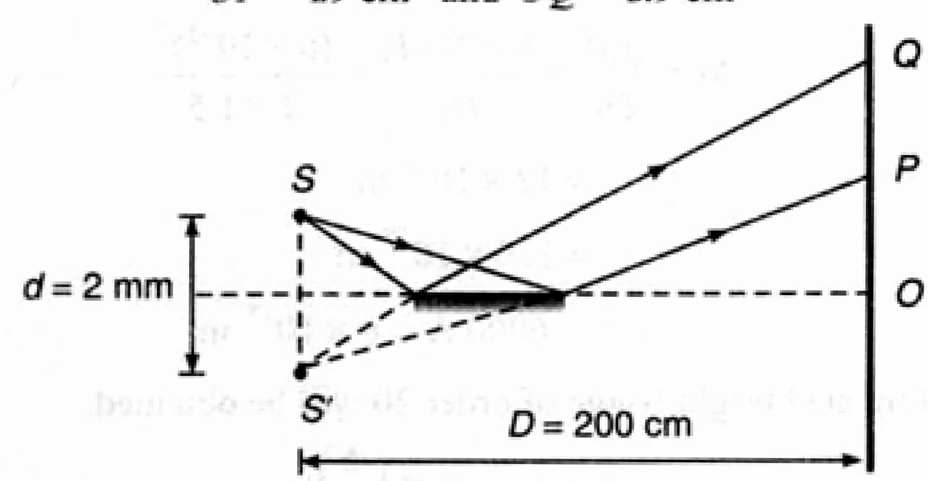
(c) $(\mu - 1)\, t = 0.01 \text{ mm}$

$\therefore$ $t = \frac{0.01}{1.5 - 1} = 0.02 \text{ mm} = 20\,\mu\text{m}$ **Ans.**

Since the pattern has to be shifted upwards, therefore, the film must be placed infront of S_1.

7. Interference will be obtained between direct rays from S and reflected rays from S' (image of S on mirror). Since the reflected rays will lie between region P and Q on the screen. So, interference is obtained in this region only. From geometry will can show that,

$$OP = 1.9 \text{ cm} \quad \text{and} \quad OQ = 3.9 \text{ cm}$$ **Ans.**

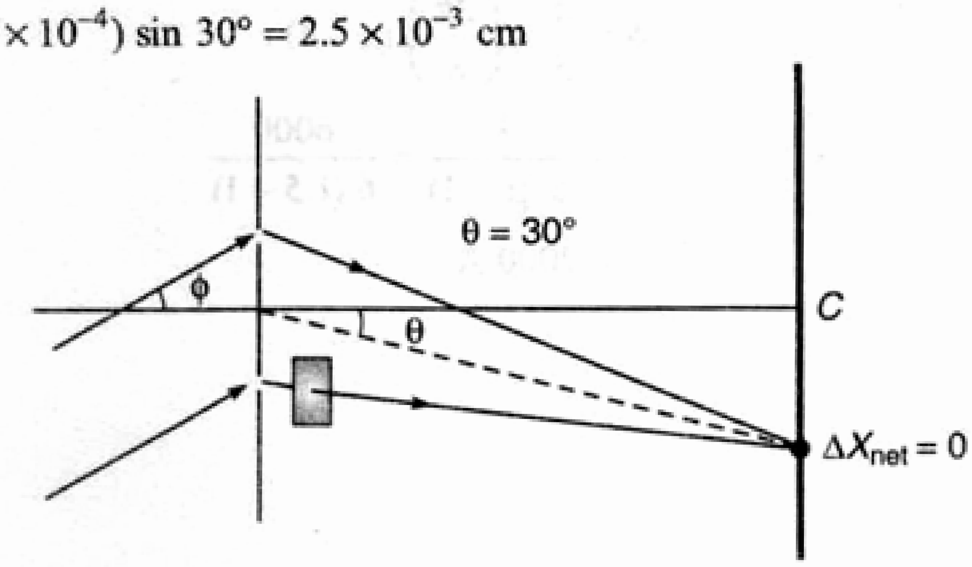


Fringe width $w = \frac{\lambda D}{d} = \frac{(5000 \times 10^{-8})(200)}{(2 \times 10^{-1})} \text{ cm} = 0.05 \text{ cm}$

Total number of fringes in the region PQ,

$$n = \frac{PQ}{w} = \frac{2}{0.05} = 40$$ **Ans.**

8. (a) $d \sin \phi = \Delta x_1 = (50 \times 10^{-4}) \sin 30° = 2.5 \times 10^{-3}$ cm



$$\Delta x_2 = (\mu - 1)t = \left(\frac{3}{2} - 1\right)(0.01) = 5.0 \times 10^{-3} \text{ cm}$$

$$\Delta x_2 - \Delta x_1 = 2.5 \times 10^{-3} \text{ cm} = \Delta x_1 \text{ (also)}$$

$\therefore$ Central maxima will be obtained at $\theta = 30°$ below C.

(b) **At C :** $\Delta X_{\text{net}} = 2.5 \times 10^{-3}$ cm $= n\lambda$

$\therefore$ $$n = \frac{2.5 \times 10^{-3}}{\lambda} = \frac{2.5 \times 10^{-3}}{500 \times 10^{-7}} = 50$$ **Ans.**

(c) Number of fringes that will pass if we remove the slab

$$= \frac{\text{Path difference due to slab}}{\lambda}$$

$$= \frac{5 \times 10^{-3}}{500 \times 10^{-7}} = 100$$ **Ans.**

9. (a)

$$(\Delta x)_{\text{net}} = 0$$

$\therefore$ $$\frac{y_1 d}{D_1} = \frac{y_2 d}{D_2}$$

$\therefore$ $$\frac{d/2}{1.5} = \frac{y}{2.0}$$

or $$y = \frac{d}{1.5} = \frac{6}{1.5} = 4 \text{ mm}$$ **Ans.**

(b) At O, net path difference,

$$\Delta x = \frac{y_1 d}{D_1} = \frac{(d/2)(d)}{D_1} = \frac{(6 \times 10^{-3})^2}{2 \times 1.5}$$

$$= 12 \times 10^{-6} \text{ m}$$

$$= 120 \times 10^{-7} \text{ m}$$

$$\lambda = 6000 \text{ Å} = 6 \times 10^{-7} \text{ m}$$

As $\Delta x = 20\lambda$, therefore at O bright fringe of order 20 will be obtained.

(c) $$I = I_{\max} \cos^2\left(\frac{\phi}{2}\right)$$

$$\frac{3}{4} I_{\max} = I_{\max} \cos^2\left(\frac{\phi}{2}\right)$$

$\therefore$ $$\frac{\phi}{2} = \frac{\pi}{6}$$

$$\phi = \frac{\pi}{3} = \left(\frac{2\pi}{\lambda}\right)(\mu - 1)\, t$$

$\therefore$ $$t = \frac{\lambda}{6(\mu - 1)} = \frac{6000}{6(1.5 - 1)}$$

$$= 2000 \text{ Å}$$ **Ans.**

10. (a)

$$\left(1-\frac{1}{\mu}\right)t=\frac{3\lambda}{\mu}$$

$\therefore$

$$t=\frac{3\lambda}{(\mu-1)}=\frac{3\times 0.78}{1.3-1}=7.8\,\mu\text{m}$$ **Ans.**

(b) **Upwards:**

$$\frac{yd}{D}-\left(1-\frac{1}{\mu}\right)t=\frac{4\lambda}{\mu}$$

Solving, we get

$$y=4.2\text{ mm}$$ **Ans.**

Downwards:

$$t\left(1-\frac{1}{\mu}\right)+\frac{yd}{D}=\frac{4\lambda}{\mu}$$

Solving, we get

$$y=0.6\text{ mm}$$ **Ans.**

Chapter 30 Modern Physics-I

JEE Advanced (Subjective Questions)

1. $$E_{2n} - E_1 = 204$$

or $$(-13.6)\left(\frac{Z^2}{4n^2}\right) + (13.6)(Z^2) = 204 \quad \text{...(i)}$$

Further, $$E_{2n} - E_n = 40.8$$

or, $$(-13.6)\left(\frac{Z^2}{4n^2}\right) + (13.6)\left(\frac{Z^2}{n^2}\right) = 40.8 \quad \text{...(ii)}$$

Solving these two equations we get, $n = 2$ and $Z = 4$ **Ans.**

Ground state energy $E_1 = (-13.6)(Z^2) = (-13.6)(4)^2 = -217.6$ eV **Ans.**

2. Threshold wavelength $\lambda_0 = \frac{12375}{1.9}$ Å $= 6513$ Å

Only first three wavelengths can emit photoelectrons. Let energy corresponding to their photons be E_1, E_2 and E_3. Then :

$$E_1 = \frac{hc}{\lambda_1} = \frac{6.62 \times 10^{-34} \times 3.0 \times 10^8}{4000 \times 10^{-10}} = 5.0 \times 10^{-19} \text{ J}$$

$$E_2 = \frac{6.62 \times 10^{-34} \times 3.0 \times 10^8}{4800 \times 10^{-10}} = 4.17 \times 10^{-19} \text{ J}$$

$$E_3 = \frac{6.62 \times 10^{-34} \times 3.0 \times 10^8}{6000 \times 10^{-10}} = 3.33 \times 10^{-19} \text{ J}$$

Energy incident corresponding to each wavelength = Intensity × Area

$$E = 1.5 \times 10^{-3} \times 10^{-4}$$

$$E = 1.5 \times 10^{-7} \text{ J} \quad \text{(per second)}$$

$$n = n_1 + n_2 + n_3 = \frac{E}{E_1} + \frac{E}{E_2} + \frac{E}{E_3}$$

$$= \frac{1.5 \times 10^{-7}}{5.0 \times 10^{-19}} + \frac{1.5 \times 10^{-7}}{4.17 \times 10^{-19}} + \frac{1.5 \times 10^{-7}}{3.33 \times 10^{-19}}$$

$$= 1.12 \times 10^{12}$$ **Ans.**

3. (a) $$E = hf = (6.6 \times 10^{-34})(5.5 \times 10^{14}) = 36.3 \times 10^{-20} \text{ J}$$

$$= 2.27 \text{ eV}$$ **Ans.**

(b) Number of photons leaving the source per second,

$$n = \frac{P}{E} = \frac{0.1}{36.3 \times 10^{-20}} = 2.75 \times 10^{17}$$ **Ans.**

(c) Number of photons falling on cathode per sec

$$n_1 = \frac{0.15}{100} \times 2.75 \times 10^{17} = 4.125 \times 10^{14}$$

Number of photoelectrons emitting per second

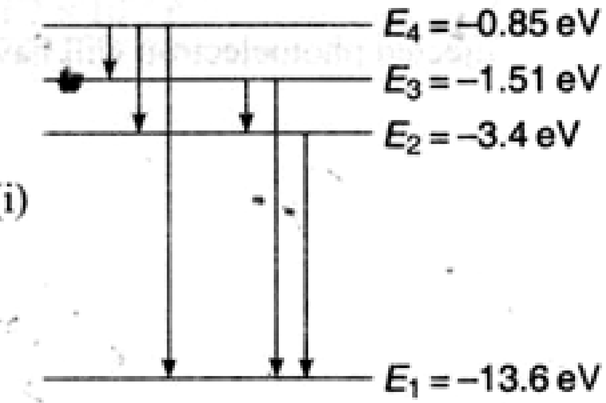
$$n_2 = \frac{6 \times 10^{-6}}{1.6 \times 10^{-19}} = 3.75 \times 10^{13}$$

$\therefore$

$$\% = \frac{n_2}{n_1} \times 100$$

$$= \frac{3.75 \times 10^{13}}{4.125 \times 10^{14}} \times 100 = 9\%$$ **Ans.**

4. $$\frac{K_1}{K_2} = 5$$



$\therefore$

$$\frac{\Delta E_1 - W}{\Delta E_2 - W} = 5 \qquad \text{...(i)}$$

Here, $\Delta E_1 = E_4 - E_1 = 12.75$ eV and $\Delta E_2 = E_3 - E_1 = 12.09$ eV

Substituting in Eq. (i) and solving we get

$$W = 11.925 \text{ eV}$$ **Ans.**

5. **For shorter wavelength:** $$\Delta E = E_4 - E_3 = \frac{(-13.6)(3)^2}{(4)^2} - \left[\frac{(-13.6)(3)^2}{(3)^2}\right] = 5.95 \text{ eV}$$

$$W = E - K_{\max} = (5.95 - 3.95) \text{ eV} = 2 \text{ eV}$$

For longer wavelength:

$$\Delta E = E_5 - E_4 = \frac{(-13.6)(3)^2}{(5)^2} - \left[\frac{(-13.6)(3)^2}{(4)^2}\right] = 2.754 \text{ eV}$$

$\therefore$ $$K_{\max} = E - W = 0.754 \text{ eV}$$

or stopping potential is 0.754 volt. **Ans.**

6. Magnetic moment $$\mu = NiA = \left(\frac{e}{T}\right)(\pi r^2)$$

or $$\mu = \left(\frac{e}{2\pi r/v}\right)(\pi r^2) = \frac{evr}{2} \qquad \text{...(i)}$$

We know that $$mvr = \frac{nh}{2\pi} \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), $$\mu = \frac{neh}{4\pi m}$$ **Ans.**

Magnetic induction, $$B = \frac{\mu_0 i}{2r} = \frac{\mu_0 e}{2rT}$$

or $$B = \frac{\mu_0 ev}{(2r)(2\pi r)} = \frac{\mu_0 ev}{4\pi r^2} \qquad \text{...(iii)}$$

From Newton's second law, $$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r} \quad \text{or} \quad v^2 = \frac{e^2}{4\pi\varepsilon_0 mr} \qquad \text{...(iv)}$$

Solving all these equations, we get $$B = \frac{\mu_0 \pi m^2 e^7}{8\varepsilon_0 h^5 n^5}$$ **Ans.**

7. Energy of electron in ground state of hydrogen atom is – 13.6 eV. Earlier it had a kinetic energy of 2 eV. Therefore, energy of photon released during formation of hydrogen atom,

$$\Delta E = 2 - (-13.6) = 15.6 \text{ eV}$$

$\therefore$ $$\lambda = \frac{12375}{\Delta E} = \frac{12375}{15.6} = 793.3 \text{ Å}$$ **Ans.**

8. $$U = -1.7 \text{ eV}$$

$\therefore$ $$E = \frac{U}{2} = -0.85 \text{ eV} = \frac{-13.6}{n^2} \quad \therefore \quad n = 4$$

Ejected photoelectron will have minimum de-Broglie wavelength corresponding to transition from $n = 4$ to $n = 1$

$$\Delta E = E_4 - E_1 = -0.85 - (-13.6) = 12.75 \text{ eV}$$

$$K_{max} = \Delta E - W = 10.45 \text{ eV}$$

$\therefore$ $$\lambda = \sqrt{\frac{150}{10.45}} \text{ Å}$$ (for an electron)

$$= 3.8 \text{ Å}$$ **Ans.**

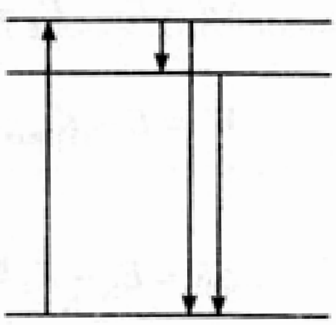
9. (a) $$\frac{n(n-1)}{2} = 3$$

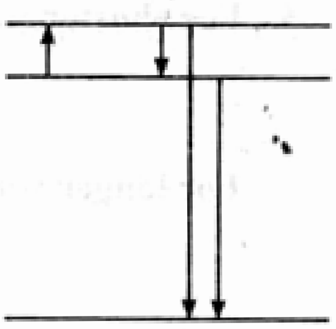
$\therefore$ $$n = 3$$

i.e., after excitation atom jumps to second excited state. Hence $n_f = 3$. So n_i can be 1 or 2.

If $n_i = 1$ then energy emitted is either equal to, or less than the energy absorbed. Hence the emitted wavelength is either equal to or greater than the absorbed wavelength. Hence $n_i \neq 1$.

If $n_i = 2$, then $E_e \geq E_a$. Hence $\lambda_e \leq \lambda_b$





$\therefore$ $$n_i = 2$$ **Ans.**

(b) $$E_3 - E_2 = 68 \text{ eV}$$

$\therefore$ $$(13.6)(Z^2)\left(\frac{1}{4} - \frac{1}{9}\right) = 68$$

$\therefore$ $$Z = 6$$ **Ans.**

(c) $$\lambda_{min} = \frac{12375}{E_3 - E_1} = \frac{12375}{(13.6)(6)^2\left(1 - \frac{1}{9}\right)} = 28.43 \text{ Å}$$ **Ans.**

(d) Ionization energy = $(13.6)(6)^2 = 489.6$ eV **Ans.**

$$\lambda = \frac{12375}{489.6} = 25.3 \text{ Å}$$ **Ans.**

10. Pitch of helical path $p = (v\cos\theta)\,T = \frac{vT}{2}$. (as $\theta = 60°$)

$$T = \frac{2\pi m}{Bq} = \frac{2\pi}{B\alpha} \qquad \left(\alpha = \frac{q}{m}\right)$$

$\therefore$ $$p = \frac{\pi v}{B\alpha}$$

or $$v = \frac{B\alpha p}{\pi} \qquad \text{...(i)}$$

$$KE = \frac{1}{2}mv^2 = E - W.$$

$$\therefore \quad W = E - \frac{1}{2}mv^2 \qquad \text{...(ii)}$$

Substituting value of v from Eq. (i) in Eq. (ii) we get

$$W = 4.9 - \frac{1}{2} \times \frac{9.1 \times 10^{-31} \times (2.5 \times 10^{-3})^2 \, (1.76 \times 10^{11})^2 \, (2.7 \times 10^{-3})^2}{\pi^2 \times 1.6 \times 10^{-19}}$$

$$= (4.9 - 0.4) \text{ eV}$$

$$= 4.5 \text{ eV}$$ **Ans.**

11. (a) $$\lambda = 1500\left(\frac{1}{1 - 1/p^2}\right)$$

λ_{max} corresponds to least energetic photon with $p = 2$.

$$\therefore \quad \lambda_{max} = 1500\left(\frac{1}{1 - 1/4}\right) = 2000 \text{ Å}$$ **Ans.**

λ_{min} corresponds to most energetic photon with $p = \infty$

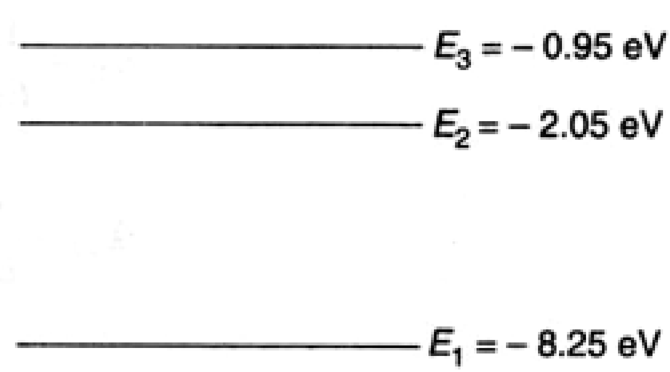
$$\therefore \quad \lambda_{min} = 1500 \text{ Å}$$ **Ans.**

(b) $$\lambda_{\infty - 1} = 1500 \text{ Å}$$

$$\therefore \quad E_\infty - E_1 = \frac{12375}{1500} \text{ eV} = 8.25 \text{ eV}$$

$$\therefore \quad E_1 = -8.25 \text{ eV} \qquad (\text{as } E_\infty = 0)$$

$$\lambda_{2-1} = 2000 \text{ Å}$$



$$\therefore \quad E_2 - E_1 = \frac{12375}{2000} \text{ eV} = 6.2 \text{ eV}$$

$$\therefore \quad E_2 = -2.05 \text{ eV}$$

Similarly $$\lambda_{31} = 1500\left(\frac{1}{1 - 1/9}\right) = 1687.5 \text{ Å}$$

$$\therefore \quad E_3 - E_1 = \frac{12375}{1687.5} \text{ eV} = 7.3 \text{ eV}$$

$$\therefore \quad E_3 = -0.95 \text{ eV}$$

(c) Ionization potential = 8.25 volt **Ans.**

12. (a) $$K_1 = \frac{12375}{3000} - W \qquad \text{...(i)}$$

$$K_2 = \frac{12375}{1650} - W \qquad \text{...(ii)}$$

$$v_2 = 2v_1 \quad \therefore \quad K_2 = 4K_1 \qquad \text{...(iii)}$$

Solving these equations, we get $W = 3$ eV

$\therefore$ Threshold wavelength $$\lambda_0 = \frac{12375}{3} = 4125 \text{ Å}$$ **Ans.**

(b) $$E_2 = \frac{12375}{1650} = 7.5 \text{ eV} = 12 \times 10^{-19} \text{ J}$$

Therefore, number of photons incident per second

$$n_2 = \frac{P_2}{E_2} = \frac{5.0 \times 10^{-3}}{12 \times 10^{-19}} = 4.17 \times 10^{15} \text{ per second}$$

Number of electrons emitted per second ($\eta = 5.1\%$) $= \frac{5.1}{100} \times 4.17 \times 10^{15}$

$$= 2.13 \times 10^{14} \text{ per second}$$

$\therefore$ Saturation current in second case $i = (2.13 \times 10^{14})(1.6 \times 10^{-19})$ amp

$$= 3.4 \times 10^{-5} \text{ A}$$

$$= 34 \ \mu\text{A}$$ **Ans.**

(c) Energy of photon in first case $= \frac{12375}{3000} = 4.125$ eV

or $$E_1 = 6.6 \times 10^{-19} \text{ J}$$

Rate of incident photons $= \frac{P_1}{E_1} = \frac{10^{-3}}{6.6 \times 10^{-19}} = 1.52 \times 10^{15}$ per second

Number of electrons ejected $= \frac{4.8 \times 10^{-3}}{1.6 \times 10^{-19}}$ per second

$= 3.0 \times 10^{16}$ per second

$\therefore$ Efficiency of photoelectron generation $= \frac{1.52 \times 10^{15}}{3.0 \times 10^{16}} \times 100 = 5.1\%$ **Ans.**

13. Balmer Series :

$$\lambda_{32} = \frac{12375}{E_3 - E_2} = \frac{12375}{(13.6)\left(\frac{1}{4} - \frac{1}{9}\right)} = 6551 \text{ Å} = 655.1 \text{ nm}$$

$$\lambda_{42} = \frac{12375}{E_4 - E_2} = \frac{12375}{(13.6)\left(\frac{1}{4} - \frac{1}{16}\right)} = 4853 \text{ Å} = 485.3 \text{ nm}$$

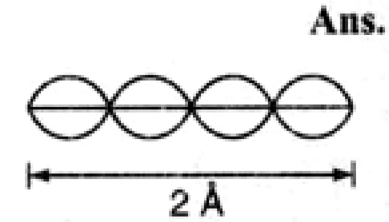
$$\lambda_{52} = \frac{12375}{E_5 - E_2} = \frac{12375}{(13.6)\left(\frac{1}{4} - \frac{1}{25}\right)} = 4333 \text{ Å} = 433.3 \text{ nm}$$

First two lie in the given range. Of these λ_{42} corresponds to more energy.

$$E = E_4 - E_2 = (13.6)\left(\frac{1}{4} - \frac{1}{16}\right) = 2.55 \text{ eV}$$

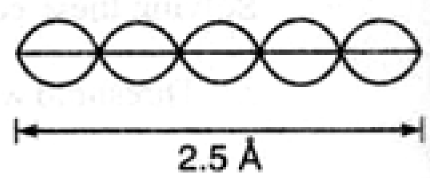
$\therefore$ $$K_{max} = E - W = (2.55 - 2.0) \text{ eV} = 0.55 \text{ eV}$$ **Ans.**

14. From the theory of standing wave we can say that



$$\frac{\lambda}{2} = (2.5 - 2.0) = 0.5 \text{ Å} \text{ or } \lambda = 1 \text{ Å}$$

Therefore least value of d required will correspond to a single loop.



$\therefore$ $$d_{min} = \frac{\lambda}{2} = 0.5 \text{ Å}$$ **Ans.**

Further for de-Broglie wavelength of an electron,

$$\lambda = \sqrt{\frac{150}{K(\text{in eV})}}\ \text{Å}$$

$$\lambda = 1\ \text{Å}$$

$\therefore$ $$K = 150\ \text{eV}$$ **Ans.**

15. (a) Reduced mass $$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{(1837 m_e)(207 m_e)}{1837 m_e + 207 m_e}$$

$$= 186 m_e$$

$$= 186 \times 9.1 \times 10^{-31}$$

$$= 1.69 \times 10^{-28}\ \text{kg}$$ **Ans.**

(b) $E_n \propto m$

Here reduced mass is 186 times mass of electron. Hence ground state energy will also be 186 times that of hydrogen atom.

$\therefore$ $$E_1 = 186(-13.6)\ \text{eV} = -2529.6\ \text{eV} \approx -2.53\ \text{keV}$$ **Ans.**

(c) $$E_2 = 186\,(-3.4)\ \text{eV} = -632.4\ \text{eV}$$

$\therefore$ $$\Delta E_{21} = 1897.2\ \text{eV}$$

$\therefore$ $$\lambda_{21} = \frac{12375}{\Delta E_{21}}\ \text{Å} = \frac{12375}{1897.2}\ \text{Å} = 6.53\ \text{Å} \approx 0.653\ \text{nm}$$ **Ans.**

16. Force of interaction between electron and proton is

$$F = -\frac{dU}{dr} = \frac{-k}{r}$$

Force is negative. It means there is attraction between the particles and they are bound to each other. This force provides the necessary centripetal force for the electron.

$\therefore$ $$\frac{mv^2}{r} = \frac{k}{r} \quad \text{...(i)}$$

According to Bohr's assumption,

$$mvr = n\frac{h}{2\pi} \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$r = \frac{nh}{2\pi\sqrt{mk}} \quad \text{and} \quad v = \sqrt{\frac{k}{m}}$$

$\therefore$ $$E = U + \frac{1}{2}mv^2 = k\ln r - \frac{k}{2} + \frac{k}{2} = k\ln r$$

Thus, $$r_n = \frac{nh}{2\pi\sqrt{mk}} \quad \text{and} \quad E_n = k\ln\left\{\frac{nh}{2\pi\sqrt{mk}}\right\}$$ **Ans.**

17. (a) $$r = \frac{mv}{Be} \quad \text{...(i)}$$

$$mvr = \frac{nh}{2\pi} \quad \text{...(ii)}$$

Solving these two equations, we get

$$r = \sqrt{\frac{nh}{2\pi Be}} \quad \text{and} \quad v = \sqrt{\frac{nhBe}{2\pi m^2}}$$

(b) $$K = \frac{1}{2} mv^2 = \frac{nhBe}{4\pi m}$$ **Ans.**

(c) $$M = iA = \left(\frac{e}{T}\right)(\pi r^2) = \frac{e}{\left(\frac{2\pi r}{v}\right)}(\pi r^2) = \frac{evr}{2}$$

$$= \frac{e}{2}\sqrt{\frac{nh}{2\pi Be}}\sqrt{\frac{nhBe}{2\pi m^2}} = \frac{nhe}{4\pi m}$$

$$U = -MB\cos 180° = \frac{nheB}{4\pi m}$$

Note *Angle between $\vec{M}$ and $\vec{B}$ will be 180°. Think why?*

(d) $$E = U + K = \frac{nheB}{2\pi m}$$

(e) $$|\phi| = B\pi r^2 = \frac{nh}{2e}$$

18. (a) and (b) When hydrogen atom is excited.

$$eV = E_0\left(\frac{1}{1} - \frac{1}{n^2}\right) \qquad \text{...(i)}$$

When ion is excited,

$$eV = E_0 Z^2\left[\frac{1}{2^2} - \frac{1}{n_1^2}\right] \qquad \text{...(ii)}$$

Wavelength of emitted light

$$\frac{hc}{\lambda_1} = E_0\left(\frac{1}{1} - \frac{1}{n^2}\right) \qquad \text{...(iii)}$$

$$\frac{hc}{\lambda_2} = E_0 Z^2\left(\frac{1}{1} - \frac{1}{n_1^2}\right) \qquad \text{...(iv)}$$

Further it is given that

$$\frac{\lambda_1}{\lambda_2} = \frac{5}{1} \qquad \text{...(v)}$$

Solving the above equations, we get

$$Z = 2, \quad n = 2, \; n_1 = 4 \quad \text{and} \quad V = 10.2 \text{ volt}$$ **Ans.**

(c) Energy of emitted photon by the hydrogen atom $= E_2 - E_1$

$$= 10.2 \text{ eV}$$ **Ans.**

and by the ion $= E_4 - E_1 = (13.6)(2)^2\left(1 - \frac{1}{16}\right)$

$$= 51 \text{ eV}$$ **Ans.**

19. $$0.6 = \frac{12375}{4950} - W \qquad \text{...(i)}$$

$$1.1 = \frac{12375}{\lambda} - W \qquad \text{...(ii)}$$

Solving above two equations, we get

$$W = 1.9 \text{ eV} \quad \text{and} \quad \lambda = 4125 \text{ Å}$$ **Ans.**

20.
$$E = \frac{12375}{4000} = 3.1 \text{ eV}$$

Number of photoelectrons emitted per second

$$n = \left(\frac{1}{10^6}\right)\left(\frac{5}{3.1 \times 1.6 \times 10^{-19}}\right) = 1.0 \times 10^{13} \text{ per second}$$

$\therefore$
$$i = (ne) = 1.0 \times 10^{13} \times 1.6 \times 10^{-19} = 1.6 \times 10^{-6} \text{ A}$$
$$= 1.6\,\mu\text{A}$$
Ans.

21. (a)
$$E = \frac{12375}{4000} = 3.1 \text{ eV}$$

Energy of electron after first collision

$$E_1 = 90\% \text{ of } E = 2.79 \text{ eV}$$ (as 10% is lost)

Energy of electron after second collision

$$E_2 = 90\% \text{ of } E_1 = 2.51 \text{ eV}$$

KE of this electron after emitting from the metal surface = (2.51 – 2.2) eV = 0.31 eV **Ans.**

(b) Energy after third collision,

$$E_3 = 90\% \text{ of } E_2 = 2.26 \text{ eV}$$

Similarly,
$$E_4 = 90\% \text{ of } E_3 = 2.03 \text{ eV}$$

So, after four collisions it becomes unable for the electrons to come out of the metal.

Chapter 31 Modern Physics-II

JEE Advanced (Subjective Questions)

1.

$$N = \frac{R}{\lambda} = \frac{10^9}{\frac{0.693}{14.3 \times 3600}} = 7.43 \times 10^{13}$$

Now, $$\frac{dN}{dt} = q - \lambda N \quad \text{or} \quad \int_0^N \frac{dN}{q - \lambda N} = \int_0^t dt$$

$\therefore$ $$N = \frac{q}{\lambda}(1 - e^{-\lambda t})$$

Substituting the values, $$7.43 \times 10^{13} = \frac{2 \times 10^9}{\frac{0.693}{14.3 \times 3600}}[1 - e^{-(0.693/14.3 \times 3600)t}]$$

Solving this equation we get $$t = 14.3 \text{ h}$$ **Ans.**

2. (a)

	A	B	C
At $t = 0$	N_0	0	0
At t	N_1	N_2	N_3

Here, $$N_1 = N_0 e^{-\lambda t} \qquad \text{...(i)}$$

$$\frac{dN_2}{dt} = \lambda (N_1 - N_2)$$

or $$\frac{dN_2}{dt} = \lambda N_0 e^{-\lambda t} - \lambda N_2$$

or $$dN_2 + \lambda N_2 dt = \lambda N_0 e^{-\lambda t}$$

$\therefore$ $$e^{\lambda t} dN_2 + \lambda N_2 e^{\lambda t} dt = \lambda N_0 dt$$

or $$d(N_2 e^{\lambda t}) = \lambda N_0 dt$$

$\therefore$ $$N_2 e^{\lambda t} = \lambda N_0 t + c$$

At $t = 0$, $N_2 = 0$, $\therefore c = 0$

$\therefore$ $$N_2 = \lambda N_0 (te^{-\lambda t})$$

(b) Activity of B is, $$R_2 = \lambda N_2 = \lambda^2 N_0 (te^{-\lambda t})$$

For maximum activity $\frac{dR_2}{dt} = 0$

$\therefore$ $$t = \frac{1}{\lambda}$$ **Ans.**

$\therefore$ $$R_{max} = \frac{\lambda N_0}{e}$$ **Ans.**

3. (a) Let at time t, number of radioactive nuclei are N.

Net rate of formation of nuclei of A

$$\frac{dN}{dt} = \alpha - \lambda N$$

or
$$\frac{dN}{\alpha - \lambda N} = dt$$

or
$$\int_{N_0}^{N} \frac{dN}{\alpha - \lambda N} = \int_0^t dt$$

Solving this equation, we get
$$N = \frac{1}{\lambda}[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}] \quad \text{...(i)}$$

(b) (i) Substituting $\alpha = 2\lambda N_0$ and $t = t_{1/2} = \frac{\ln(2)}{\lambda}$ in Eq. (i) we get,
$$N = \frac{3}{2}N_0$$

(ii) Substituting $\alpha = 2\lambda N_0$ and $t \to \infty$ in Eq. (i), we get
$$N = \frac{\alpha}{\lambda} = 2N_0$$

or
$$N = 2N_0$$

4. λ = Disintegration constant
$$\frac{0.693}{t_{1/2}} = \frac{0.693}{15}\ \text{h}^{-1}$$
$$= 0.05462\ \text{h}^{-1}$$

Let R_0 = initial activity = 1 microcurie = 3.7×19^4 disintegrations per second.

r = Activity in 1 cm^3 of blood at $t = 5$ h

$= \frac{296}{90}$ disintegration per second

= 4.93 disintegration per second, and

R = Activity of whole blood at time $t = 5$ h

Total volume of blood should be
$$V = \frac{R}{r}$$
$$= \frac{R_0 e^{-\lambda t}}{r}$$

Substituting the value, we have
$$V = \left(\frac{3.7 \times 10^4}{4.93}\right)e^{-(0.0462)(5)}\ \text{cm}^3$$
$$V = 5.95 \times 10^3\ \text{cm}^3$$

or
$$V = 5.95\ \text{L}$$

5. (a) Let R_{A_0} and R_{B_0} be the initial activities of A and B. Then
$$R_{A_0} + R_{B_0} = 10^{10}\ \text{dps} \quad \text{...(i)}$$

Activity of A after time t = 20 days (two half lives of A) is
$$R_A = \left(\frac{1}{2}\right)^2 R_{A_0} = 0.25\, R_{A_0}$$

Similarly activity of B after $t = 20$ days (four half lives of B) is

$$R_B = \left(\frac{1}{2}\right)^4 R_{B_0} = 0.0625\, R_{B_0}$$

Now it is given that

$$R_A + R_B = 20\% \text{ of } 10^{10}$$

or $$0.25\, R_{A_0} + 0.0625\, R_{B_0} = 0.2 \times 10^{10} \text{ dps} \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$R_{A_0} = 0.73 \times 10^{10} \text{ dps}$$

and $$R_{B_0} = 0.27 \times 10^{10} \text{ dps}$$

(b) $$\frac{R_{A_0}}{R_{B_0}} = \frac{\lambda_A N_{A_0}}{\lambda_B N_{B_0}} = \frac{(t_{1/2})_B}{(t_{1/2})_A} \cdot \frac{N_{A_0}}{N_{B_0}}$$

$\therefore$ $$\frac{N_{A_0}}{N_{B_0}} = \left(\frac{R_{A_0}}{R_{B_0}}\right) \frac{(t_{1/2})_A}{(t_{1/2})_B}$$

$$= \left(\frac{0.73}{0.27}\right)\left(\frac{10}{5}\right) = 5.4$$

6. Let N be the number of radionuclei at any time t. Then

$\therefore$ net rate of formation of nuclei at time t is

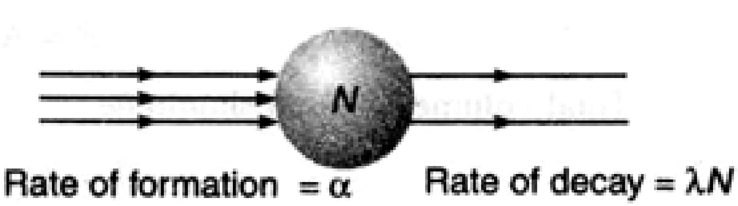
$$\frac{dN}{dt} = \alpha - \lambda N \quad \text{or} \quad \int_0^N \frac{dN}{\alpha - \lambda N} = \int_0^t dt$$

or $$N = \frac{\alpha}{\lambda}(1 - e^{-\lambda t})$$

Number of nuclei formed in time $t = \alpha t$

and number of nuclei left after time

$$t = \frac{\alpha}{\lambda}(1 - e^{-\lambda t})$$



Therefore, number of nuclei disintegrated in time $t = \alpha t - \dfrac{\alpha}{\lambda}(1 - e^{-\lambda t})$

$\therefore$ eneryg released till time

$$t = E_0\left[\alpha t - \frac{\alpha}{\lambda}(1 - e^{-\lambda t})\right]$$

But only 20% of it is used in raising the temperature of water.

So $$0.2\, E_0\left[\alpha t - \frac{\alpha}{\lambda}(1 - e^{-\lambda t})\right] = Q$$

where $Q = ms\Delta\theta$

$\therefore$ $\Delta\theta$ = increase in temperature of water $= \dfrac{Q}{ms}$

$\therefore$ $$D\theta = \frac{0.2\, E_0\left[at - \frac{a}{l}(1 - e^{-\lambda t})\right]}{ms}$$

7. (a) Let at time $t = t$, number of nuclei of Y and Z are N_Y and N_Z. Then

Rate equations of the populations of X, Y and Z are

$$\left(\frac{dN_X}{dt}\right) = -\lambda_X N_X \quad \text{...(i)}$$

$$\left(\frac{dN_Y}{dt}\right) = \lambda_X N_X - \lambda_Y N_Y \quad \text{...(ii)}$$

and

$$\left(\frac{dN_Z}{dt}\right) = \lambda_Y N_Y \quad \text{...(iii)}$$

(b) Given $N_Y(t) = \frac{N_0 \lambda_X}{\lambda_X - \lambda_Y}[e^{-\lambda_Y t} - e^{-\lambda_X t}]$

For N_Y to be maximum

$$\frac{dN_Y(t)}{dt} = 0$$

i.e.,

$$\lambda_X N_X = \lambda_Y N_Y \quad \text{...(iv)} \quad \text{[from Eq. (ii)]}$$

or

$$\lambda_X(N_0 e^{-\lambda_X t}) = \lambda_Y \frac{N_0 \lambda_Y}{\lambda_X - \lambda_Y}[e^{-\lambda_Y t} - e^{-\lambda_X t}]$$

or

$$\frac{\lambda_X - \lambda_Y}{\lambda_Y} = \frac{e^{-\lambda_Y t}}{e^{-\lambda_Y t}} - 1$$

$$\frac{\lambda_X}{\lambda_Y} = e^{(\lambda_X - \lambda_Y)t}$$

or

$$(\lambda_X - \lambda_Y)t \ln(e) = \ln\left(\frac{\lambda_Y}{\lambda_Y}\right)$$

or

$$t = \frac{1}{\lambda_X - \lambda_Y} \ln\left(\frac{\lambda_X}{\lambda_Y}\right)$$

Substituting the values of λ_X and λ_Y, we have

$$t = \frac{1}{(0.1 - 1/30)} \ln\left(\frac{0.1}{1/30}\right) = 15 \ln(3)$$

or $t = 16.48$ s.

(c) The population of X at this moment,

$$N_X = N_0 e^{-\lambda_X t}$$

$$= (10^{20}) e^{-(0.1)(16.48)}$$

$$N_X = 1.92 \times 10^{19}$$

$$N_Y = \frac{N_X \lambda_Y}{\lambda_Y} \quad \text{[From Eq. (iv)]}$$

$$= (1.92 \times 10^{19}) \frac{(0.1)}{1/30}$$

$$= 5.76 \times 10^{19}$$

$$N_Z = N_0 - N_X - N_Y$$

$$= 10^{20} - 1.92 \times 10^{19} - 5.76 \times 10^{19}$$

or

$$N_Z = 2.32 \times 10^{19}$$

8. We have for B

$$\frac{dN_B}{dt} = P - \lambda_2 N_B$$

$\Rightarrow$ $$\int_0^{N_B} \frac{dN_B}{P - \lambda_2 N_B} = \int_0^t dt$$

$\Rightarrow$ $$\ln\left(\frac{P - \lambda_2 N_B}{P}\right) = -\lambda_2 t$$

$\Rightarrow$ $$N_B = \frac{P(1 - e^{-\lambda_2 t})}{\lambda_2}$$

The number of nuclei of A after time t is

$$N_A = N_0 e^{-\lambda_1 t}$$

Thus $$\frac{dN_c}{dt} = \lambda_1 N_A + \lambda_2 N_B$$

$\Rightarrow$ $$\frac{dN_c}{dt} = \lambda_1 N_0 e^{-\lambda_1 t} + P(1 - e^{-\lambda_2 t})$$

$\Rightarrow$ $$N_c = N_0(1 - e^{-\lambda_1 t}) + P\left(t + \frac{e^{-\lambda_2 t} - 1}{\lambda_2}\right)$$

9. $${}^{210}_{84}\text{Po} \longrightarrow {}^{206}_{82}\text{Pb} + {}^{4}_{2}\text{He}$$

$$\Delta m = 0.00564 \text{ amu}$$

Energy liberated per reaction = $(\Delta m)931$ MeV= 8.4×10^{-13} J

Electrical energy produced = 8.4×10^{-14} J

Let m g of ^{210}Po is required to produce the desired energy.

$$N = \frac{m}{210} \times 6 \times 10^{23}$$

$$\lambda = \frac{0.693}{t_{1/2}} = 0.005 \text{ per day}$$

$$\left(-\frac{dN}{dt}\right) = \lambda N = \frac{(0.005)(6 \times 10^{23} \text{ m})}{210} \text{ per day}$$

$\therefore$ Electrical energy produced per day

$$= \frac{(0.005)(6 \times 10^{23} \text{ m})}{210} \times 8.4 \times 10^{-14} \text{ J}$$

This is equal to 1.2×10^7 J (given)

$\therefore$ $$m = 10 \text{ g}$$ **Ans.**

Activity at the end of 693 days is, $R = \dfrac{0.005 \times 6 \times 10^{23} \times 10}{210} = \dfrac{10^{21}}{7}$ per day $= R_0\left(\dfrac{1}{2}\right)^n$

Here, n = number of half lives $= \dfrac{693}{138.6} = 5$

$\therefore$ $$R_0 = R(2)^5 = 32 \times \frac{10^{21}}{7} = 4.57 \times 10^{21} \text{ per day}$$ **Ans.**

10. $$\Delta m = 2(\text{mass of } {}_1H^2) - (\text{mass of } {}_2He^4) = 0.0256 \text{ u}$$

$\therefore$ $$E = 0.0256 \times 931.5 \text{ MeV} = 23.85 \text{ MeV}$$

Total energy required per day $= 200 \times 10^6 \times 24 \times 3600 \text{ J} = 1.728 \times 10^{13} \text{ J}$

Let m be the mass of deuterium required. Then energy required for reactor.

$$= \left(\frac{25}{100}\right)\left(\frac{m/2}{2}\right)(6.02 \times 10^{23})(23.85 \times 1.6 \times 10^{-13})$$

This should be equal to 1.728×10^{13} J

$\therefore$ $$m = \frac{4 \times 100 \times 1.728 \times 10^{13}}{25 \times 6.02 \times 10^{23} \times 23.85 \times 1.6 \times 10^{-13}} \text{ g} = 120.35 \text{ g}$$ **Ans.**

11. The reaction involved in α-decay is

$$^{248}_{96}\text{Cm} \rightarrow {}^{244}_{94}\text{Pu} + {}^{4}_{2}\text{He}$$

Mass defect

$$\Delta m = \text{mass of } ^{248}_{96}\text{Cm} - \text{mass of } ^{244}_{94}\text{Pu} - \text{mass of } ^{4}_{2}\text{He}$$
$$(248.072220 - 244.064100 - 4.002603) \text{ u} = 0.005517 \text{ u}$$

Therefore, energy released in α-decay will be :

$$E_\alpha = (0.0005517 \times 931) \text{MeV} = 5.136 \text{ MeV}$$

Similarly, $$E_{\text{fission}} = 200 \text{ MeV (given)}$$

Mean life is given as :

$$t_{\text{mean}} = 10^{13} \text{ s} = \frac{1}{\lambda}$$

$\therefore$ Disintegration constant $\lambda = 10^{-13} \text{ s}^{-1}$

Rate of decay the moment when number of nuclei are

$$N = (10^{20})$$
$$\lambda N = (10^{-13})(10^{20})$$
$$= 10^7 \text{ disintegration per second}$$

Of these disintegrations, 8% are in fission and 92% are in α-decay.

Therefore,

energy released per second

$$= (0.08 \times 10^7 \times 200 + 0.92 \times 10^7 \times 5.136) \text{ MeV}$$
$$= 2.074 \times 10^8 \text{ MeV}$$

$\therefore$ Power output (in watt) = energy released per second (J/s)

$$= (2.074 \times 10^8)(1.6 \times 10^{-3})$$

$\therefore$ $$\text{Power output} = 3.32 \times 10^{-5} \text{ W}$$

12. (i) At $t = 0$, probabilities of getting α and β particles are same. This implies that initial activity of both is equal. Say it is R_0.

Activity after $t = 1620$ s

$$R_1 = R_0 \left(\frac{1}{2}\right)^{1620/405} = \frac{R_0}{16}$$

and $$R_2 = R_0\left(\frac{1}{2}\right)^{1620/1620} = \frac{R_0}{2}$$

$$\text{Total activity } R = R_1 + R_2 = \frac{9}{16}R_0$$

$$\text{Probability of getting } \alpha\text{-particles} = \frac{R_1}{R} = \frac{1}{9}$$

and $$\text{probability of getting } \beta\text{-particles} = \frac{R_2}{R} = \frac{8}{9}$$ **Ans.**

(ii) $R_{01} = R_{02}$

$\therefore$ $$\frac{N_{01}}{T_1} = \frac{N_{02}}{T_2}$$

$\therefore$ $$\frac{N_{01}}{N_{02}} = \frac{1}{4}$$

Let N_0 be the total number of nuclei at $t = 0$.

then, $$N_{01} = \frac{N_0}{5} \quad \text{and} \quad N_{02} = \frac{4N_0}{5}$$

Given, that $$N_1 + N_2 = \frac{N_0}{2}$$

or $$\frac{N_0}{5}\left(\frac{1}{2}\right)^{t/405} + \frac{4N_0}{5}\left(\frac{1}{2}\right)^{t/1620} = \frac{N_0}{2}$$

Let $$\left(\frac{1}{2}\right)^{t/1620} = x$$

Then, above equation becomes, $x^4 + 4x - 2.5 = 0$

$\therefore$ $$x = 0.594$$

or $$\left(\frac{1}{2}\right)^{t/1620} = 0.594$$

Solving it we get $t = 1215$ s **Ans.**

13. $$N = \frac{10^{-3}}{210} \times 6.02 \times 10^{23} = 2.87 \times 10^{18}$$

During one mean life period 63.8% nuclei are decayed. Hence, energy released

$$E = 0.638 \times 2.87 \times 10^{18} \times 5.3 \times 1.6 \times 10^{-13}\text{ J} = 1.55 \times 10^{6}\text{ J}$$ **Ans.**

14. $$R_0 = \lambda N = \frac{0.693}{14.3 \times 3600 \times 24} \times 6.02 \times 10^{23}\text{ per sec} = 3.37 \times 10^{17}\text{ per sec}$$

After 70 hours activity, $R = R_0 e^{-\lambda t} = (3.37 \times 10^{17})e^{-(0.693/14.3 \times 24)(70)} = 2.92 \times 10^{17}$ per sec

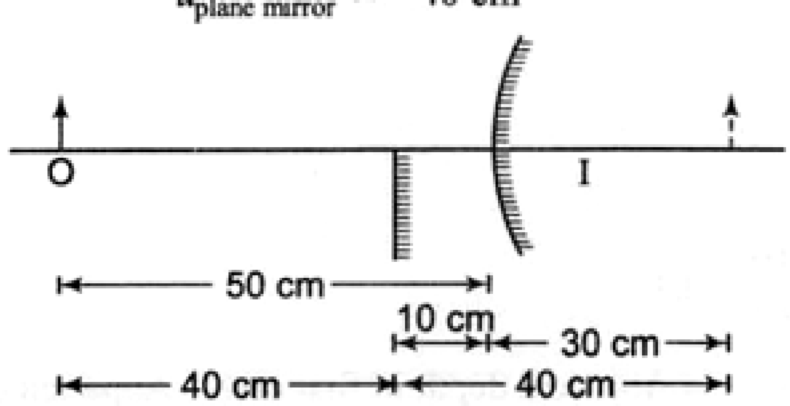
In fruits activity was observed $1\,\mu$Ci or 3.7×10^4 per sec. Therefore, percentage of activity transmitted from root to the fruit.

$$= \frac{3.7 \times 10^4}{2.92 \times 10^{17}} \times 100 = 1.26 \times 10^{-11}\ \%$$ **Ans.**

Experimental Skills

1. Take one of the two given mirrors in your hand and see your face in it. If the image of your face is smaller in size for all positions of the mirror, then it is a convex mirror othewise concave.
2. The relative or apparent shift between two objects (placed at different distances from the eye), when the eye is moved sideways, is called parallax.
3. When the two objects are placed at the same distance from the eye and the eye is moved sideways, there is no relative shift between them. When they are at diffetent distances, the nearer object moves in a direction opposite to that of the eye and the farther one in the direction of the eye. Once their relative positions are known, they can be brought to the same position by shifting them suitably. When the two objects occupy the same position in space with respect to the eye, then the apparent shift disappears and parallax is said to be removed.
4. When a concave mirror is held near the face, the face lies between its pole and focus. Thus, an errect and enlarged image is formed. An enlarged image of the face helps in having a better and a closer shave.
5. A concave mirror having a small aperture is used by a surgeon to throw a sharp and narrow beam of light into the ear, nose and throat or the eye of the patients for the medical check up.
6. Concave mirrors are used as reflectors in headlights of automobiles. Searchlights which are meant for throwing light to large distances, also use concave mirrors. The source of light is placed just at the focus of the concave mirror. A parallel beam of light is thus obtained by regular reflection from the surface of the concave mirror.
7. Convex mirror. The image formed by a convex mirror is always erect and diminished in size. It also has a wider field of view. These two properties of the convex mirror enable the driver to see if any vehicle is approaching him from behind of what is happening over a large area behind the vehicle.
8. No, since a convex mirror does not form a real image, v cannot be determined on an optical bench.
9. No, the focal length of convex lens should be greater than the focal length of the convex mirror.
10. It is a defect of image formation. The paraxial and marginal rays starting from a point object after reflection from the mirror do not meet at a single point. This means, there is no point image for a point object. This defect is called spherical aberration.
11. $\frac{1}{v}$ vs. $\frac{1}{u}$ method, because the graph in this case is a straight line.
12. Both images coincide. Therefore :

$$u_{\text{plane mirror}} = -40 \text{ cm}$$



$$u_{\text{convex mirror}} = -50 \text{ cm}$$

$$v_{\text{plane mirror}} = 40 \text{ cm}$$

$$v_{\text{convex mirror}} = (40 - 10) = 30 \text{ cm}$$

Since, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ $\quad\therefore\quad$ $\frac{1}{30} + \frac{1}{-50} = \frac{1}{f}$

$\therefore$ $\frac{1}{f} = \frac{1}{30} - \frac{1}{50}$

$$\therefore \quad \frac{1}{f} = \frac{50 - 30}{1500}$$

$$\therefore \quad f = \frac{1500}{20} = 75 \text{ cm}$$ **Ans.**

16. For object placed beyond F or for all object distances $> f$, the concave mirror forms a real image, however for object lying between F and P i.e., $(u < f)$, the image formed is a magnified virtual image.

17. A convex lens is thicker in the middle than at the edges whereas a concave lens is thinner in the middle than at the edges.

18. The straight line passing through the centres of curvature of the curved surfaces of the lens is called the principal axis of the lens.

19. Radius of curvature of a plane surface is infinity because the centre of curvature of a plane lies at infinity.

20. When one surface of a lens is plane and the other one is curved, the principal axis is defined as that straight line which is perpendicular to the plane surface but passes through the centre or curvature of the curved surface.

21. $f = \dfrac{1}{\text{Intercept}} = \dfrac{1}{0.5} = 2 \text{ m}$

22. On emergence from the slab, the incident ray undergoes a lateral diaplacement. It means that the incident ray and the emergent ray are parallel to each other but not coincident.

24. Phenomenon of refraction of light.

25. Suspended water droplets in the atmosphere act as prisms, causing the phenomena of dispersion and total internal reflection of sunlight.

26. $d_{app} = \dfrac{d_{\text{actual}}}{\mu}$

μ_{violet} is maximum.

27. Normal shift in the position of the pin is

$$d = 1\left(1 - \frac{1}{\mu}\right)$$

The answer is independent of the location of the slab.

28. Since shift

$$d = t\left(1 - \frac{1}{\mu}\right)$$

$$\therefore \quad 1.5 = t\left[1 - \frac{1}{4/3}\right]$$

$$\therefore \quad t = 6 \text{ cm}$$

29. (i)
$$\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{10}{8.2} = 1.2$$

(ii) Distance through which microscope is to be moved is Δx = Apparent depth in first case – Apparent depth in second case

$$\therefore \quad \Delta x = 8.2 - \frac{10}{1.7} = 2.32 \text{ cm}$$

30. Apparent depth $= \dfrac{d_1}{\mu_1} + \dfrac{d_2}{\mu_2} = \dfrac{10}{1.4} + \dfrac{10}{1.7} = 13 \text{ cm}$

37. The output voltage across R_L will be equal to the voltage across the zener diode i.e., 40 V.

45. Since,

$$I_e = I_b + I_c$$

$$\therefore \quad 6 = I_b + I_c$$

Also,
$$\frac{I_c}{I_b} = 59$$

$\therefore$
$$I_e = I_b + 59\, I_b$$

$\therefore$
$$60\, I_b = 6 \text{ mA}$$

$\therefore$
$$I_b = \frac{1}{10} \text{ A} = 0.1 \text{ mA}$$

$$I_c = 59\, I_b = 5.9 \text{ mA}$$

46. $\Delta i_b = \Delta i_e - \Delta i_c = (5.60 - 5.488) \text{ mA} = 0.112 \text{ mA}$

Now,
$$\beta = \frac{\Delta i_c}{\Delta i_b} = \frac{5.488}{0.112} = 49$$

48.
$$\alpha = \frac{\Delta I_C}{\Delta I_E} = \frac{\Delta I_E - \Delta I_B}{\Delta I_B}$$

$\therefore$
$$0.95 = \frac{10 - \Delta I_B}{10}$$

$\therefore$
$$\Delta I_B = 0.5 \text{ mA}$$

49. $A_V = \beta \dfrac{R_2}{R_1} = 100 \times \dfrac{10 \times 10^3}{1 \times 10^3} = 1000$

50. $I_C = I_E - I_B = 4 \text{ mA} - 40\,\mu\text{A} = (4 - 0.04) = 3.96 \text{ mA}$

$$\alpha = \frac{I_C}{I_E} = \frac{3.96}{4} = 0.99$$

52. $\beta = \dfrac{\alpha}{1 - \alpha} = \dfrac{0.9}{1 - 0.9} = 9$