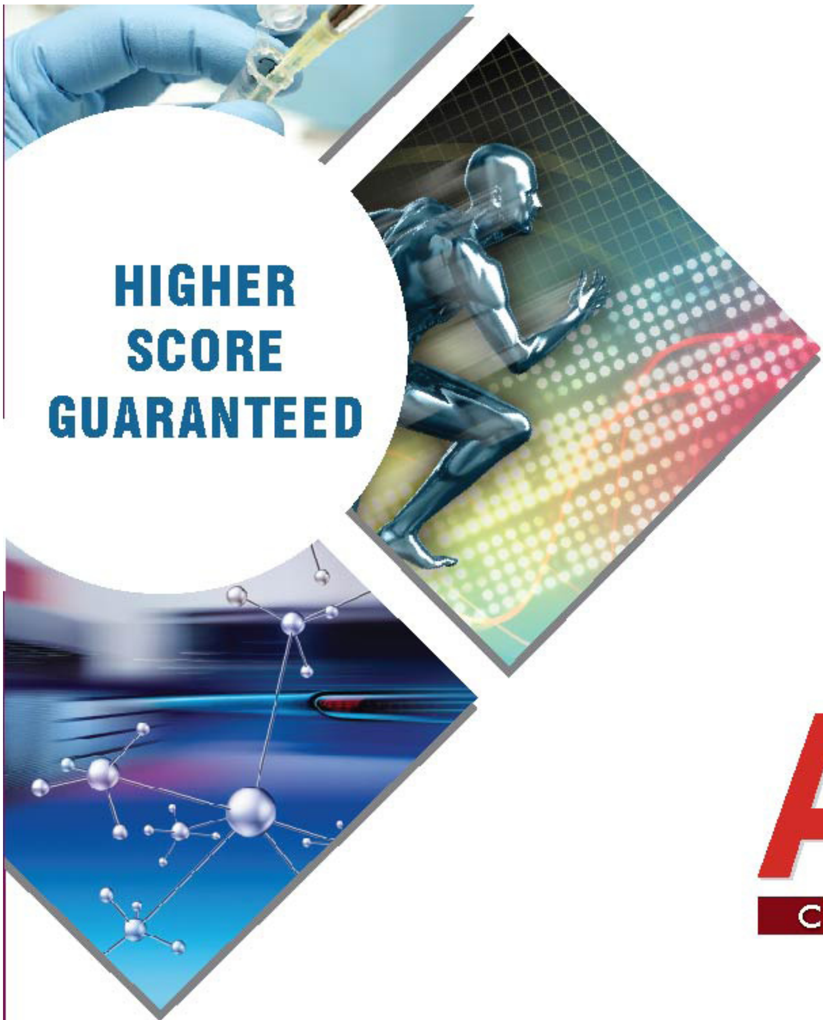

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CONTENTS

MATHEMATICS

SYLLABUS*

PHYSICS

SECTION A

Unit - I: Physics and Measurement

Physics, technology and society, S. I. units, fundamental and derived units, least count, accuracy and precision of measuring instruments, errors in measurement, significant figures.

Dimensions of physical quantities, dimensional analysis and its applications.

Unit - II: Kinematics

Frame of reference, motion in a straight line: position-time graph, speed and velocity, uniform and non-uniform motion, average speed and instantaneous velocity. Uniformly accelerated motion, velocity-time, position-time graphs, relations for uniformly accelerated motion. Scalars and vectors, vector addition and subtraction, zero vector, scalar and vector products, unit vector, resolution of a vector, relative velocity, motion in a plane, projectile motion, uniform circular motion.

Unit - III: Laws of Motion

Force and inertia, Newton's first law of motion, momentum, Newton's second law of motion, impulse, Newton's third law of motion, law of conservation of linear momentum and its applications, equilibrium of concurrent forces.

Static and kinetic friction, laws of friction, rolling friction. Dynamics of uniform circular motion: centripetal force and its applications.

Unit - IV: Work, Energy and Power

Work done by a constant force and a variable force, kinetic and potential energies, work-energy theorem, power.

Potential energy of a spring, conservation of mechanical energy, conservative and non-conservative forces, elastic and inelastic collisions in one and two dimensions.

Unit - V: Rotational Motion

Centre of mass of a two-particle system, centre of mass of a rigid body, basic concepts of rotational motion, moment of a force, torque, angular momentum, conservation of angular momentum and its applications, moment of inertia, radius of gyration, values of moments of inertia for simple geometrical objects, parallel and perpendicular axes theorems and their applications.

Rigid body rotation, equations of rotational motion.

Unit - VI: Gravitation

The universal law of gravitation.

Acceleration due to gravity and its variation with altitude and depth.

Kepler's laws of planetary motion.

Gravitational potential energy, gravitational potential.

Escape velocity, orbital velocity of a satellite, geostationary satellites.

* For latest information, please refer JEE 2013 Information Bulletin

Unit - VII: Properties of Solids and Liquids

Elastic behaviour, stress-strain relationship, Hooke's law, Young's modulus, bulk modulus, modulus of rigidity.

Pressure due to a fluid column, Pascal's law and its applications.

Viscosity, Stokes's law, terminal velocity, streamline and turbulent flow, Reynolds number, Bernoulli's principle and its applications.

Surface energy and surface tension, angle of contact, application of surface tension - drops, bubbles and capillary rise.

Heat, temperature, thermal expansion, specific heat capacity, calorimetry, change of state, latent heat.

Heat transfer-conduction, convection and radiation, Newton's law of cooling.

Unit - VIII: Thermodynamics

Thermal equilibrium, zeroth law of thermodynamics, concept of temperature, heat, work and internal energy, first law of thermodynamics.

Second law of thermodynamics, reversible and irreversible processes, Carnot engine and its efficiency.

Unit - IX: Kinetic Theory of Gases

Equation of state of a perfect gas, work done on compressing a gas.

Kinetic theory of gases - assumptions, concept of pressure, kinetic energy and temperature, rms speed of gas molecules, degrees of freedom, law of equipartition of energy, applications to specific heat capacities of gases, mean free path, Avogadro's number.

Unit - X: Oscillations and Waves

Periodic motion - period, frequency, displacement as a function of time, periodic functions, simple harmonic motion (S.H.M.) and its equation, phase, oscillations of a spring - restoring force and force constant, energy in S.H.M. - kinetic and potential energies, simple pendulum - derivation of expression for its time period, free, forced and damped oscillations, resonance.

Wave motion, longitudinal and transverse waves, speed of a wave, displacement relation for a progressive wave, principle of superposition of waves, reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, beats, Doppler effect in sound.

Unit - XI: Electrostatics

Electric charges, conservation of charge, Coulomb's law-forces between two point charges, forces between multiple charges, superposition principle and continuous charge distribution.

Electric field, electric field due to a point charge, electric field lines, electric dipole, electric field due to a dipole, torque on a dipole in a uniform electric field.

Electric flux, Gauss's law and its applications to find field due to infinitely long, uniformly charged straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell.

Electric potential and its calculation for a point charge, electric dipole and system of charges, equipotential surfaces, electrical potential energy of a system of two point charges in an electrostatic field.

Conductors and insulators, dielectrics and electric polarization, capacitor, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, energy stored in a capacitor.

Unit - XII: Current Electricity

Electric current, drift velocity, Ohm's law, electrical resistance, resistances of different materials, V-I characteristics of ohmic and non-ohmic conductors, electrical energy and power, electrical resistivity,

colour code for resistors, series and parallel combinations of resistors, temperature dependence of resistance.
Electric cell and its internal resistance, potential difference and emf of a cell, combination of cells in series and in parallel.
Kirchhoff's laws and their applications, Wheatstone bridge, metre bridge.
Potentiometer - principle and its applications.

Unit - XIII: Magnetic Effects of Current and Magnetism
Biot - Savart law and its application to current carrying circular loop.
Ampere's law and its applications to infinitely long current carrying straight wire and solenoid. Force on a moving charge in uniform magnetic and electric fields, cyclotron.
Force on a current-carrying conductor in a uniform magnetic field, force between two parallel current-carrying conductors-definition of ampere, torque experienced by a current loop in uniform magnetic field, moving coil galvanometer, its current sensitivity and conversion to ammeter and voltmeter. Current loop as a magnetic dipole and its magnetic dipole moment, bar magnet as an equivalent solenoid, magnetic field lines, earth's magnetic field and magnetic elements, para-, dia- and ferro- magnetic substances .
Magnetic susceptibility and permeability, hysteresis, electromagnets and permanent magnets.

Unit - XIV: Electromagnetic Induction and Alternating Currents
Electromagnetic induction, Faraday's law, induced emf and current, Lenz's law, Eddy currents, self and mutual inductance.
Alternating currents, peak and rms value of alternating current/ voltage, reactance and impedance, LCR series circuit, resonance, quality factor, power in AC circuits, wattless current.
AC generator and transformer.

Unit - XV: Electromagnetic Waves
Electromagnetic waves and their characteristics, transverse nature of electromagnetic waves, electromagnetic spectrum (radio waves, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays). Applications of electromagnetic waves..

Unit - XVI: Optics
Reflection and refraction of light at plane and spherical surfaces, mirror formula, total internal reflection and its applications, deviation and dispersion of light by a prism, lens formula, magnification, power of a lens, combination of thin lenses in contact, microscope and astronomical telescope (reflecting and refracting) and their magnifying powers.
Wave optics - wavefront and Huygens's principle, laws of reflection and refraction using Huygens's principle, interference, Young's double slit experiment and expression for fringe width, coherent sources and sustained interference of light, diffraction due to a single slit, width of central maximum, resolving power of microscopes and astronomical telescopes, polarisation, plane polarized light, Brewster's law, uses of plane polarized light and polaroids.

Unit - XVII: Dual Nature of Matter and Radiation
Dual nature of radiation, photoelectric effect, Hertz and Lenard's observations, Einstein's photoelectric equation, particle nature of light.
Matter waves-wave nature of particle, de Broglie relation, Davisson-Germer experiment.

Unit - XVIII: Atoms and Nuclei

Alpha-particle scattering experiment, Rutherford's model of atom, Bohr model, energy levels, hydrogen spectrum.

Composition and size of nucleus, atomic masses, isotopes, isobars, isotones, radioactivity-alpha, beta and gamma particles/rays and their properties, radioactive decay law, mass-energy relation, mass defect, binding energy per nucleon and its variation with mass number, nuclear fission and fusion.

Unit - XIX: Electronic Devices

Semiconductors, semiconductor diode - I-V characteristics in forward and reverse bias, diode as a rectifier, I-V characteristics of LED, photodiode, solar cell. Zener diode, Zener diode as a voltage regulator, junction transistor, transistor action, characteristics of a transistor, transistor as an amplifier (common emitter configuration) and oscillator, logic gates (OR, AND, NOT, NAND and NOR), transistor as a switch.

Unit - XX: Communication Systems

Propagation of electromagnetic waves in the atmosphere, sky and space wave propagation, need for modulation, amplitude and frequency modulation, bandwidth of signals, bandwidth of transmission medium, basic elements of a communication system (Block Diagram only).

SECTION B

Unit XXi : Experimental Skills

Familiarity with the basic approach and observations of the experiments and activities:

- Vernier callipers-its use to measure internal and external diameter and depth of a vessel.
- Screw gauge-its use to determine thickness/diameter of thin sheet/wire.
- Simple Pendulum-dissipation of energy by plotting a graph between square of amplitude and time.
- Metre Scale - mass of a given object by principle of moments.
- Young's modulus of elasticity of the material of a metallic wire.
- Surface tension of water by capillary rise and effect of detergents.
- Co-efficient of Viscosity of a given viscous liquid by measuring terminal velocity of a given spherical body.
- Plotting a cooling curve for the relationship between the temperature of a hot body and time.
- Speed of sound in air at room temperature using a resonance tube.
- Specific heat capacity of a given (i) solid and (ii) liquid by method of mixtures.
- Resistivity of the material of a given wire using metre bridge.
- Resistance of a given wire using Ohm's law.
- Potentiometer –
  (i) Comparison of emf of two primary cells.
  (ii) Determination of internal resistance of a cell.
- Resistance and figure of merit of a galvanometer by half deflection method.
- Focal length of:
  (i) Convex mirror (ii) Concave mirror, and (iii) Convex lens

- Using parallax method. Plot of angle of deviation vs angle of incidence for a triangular prism.
- Refractive index of a glass slab using a travelling microscope.
- Characteristic curves of a p-n junction diode in forward and reverse bias.
- Characteristic curves of a Zener diode and finding reverse break down voltage.
- Characteristic curves of a transistor and finding current gain and voltage gain.
- Identification of Diode, LED, Transistor, IC, Resistor, Capacitor from mixed collection of such items.
- Using multimeter to:
  (i) Identify base of a transistor
  (ii) Distinguish between npn and pnp type transistor
  (iii) See the unidirectional flow of current in case of a diode and an LED.
  (iv) Check the correctness or otherwise of a given electronic component (diode, transistor or IC).

CHEMISTRY

Section - A (Physical Chemistry)

UNIT - 1: SOME BASIC CONCEPTS IN CHEMISTRY
Matter and its nature, Dalton's atomic theory, concept of atom, molecule, element and compound, physical quantities and their measurements in chemistry, precision and accuracy, significant figures, S.I. units, dimensional analysis, Laws of chemical combination, atomic and molecular masses, mole concept, molar mass, percentage composition, empirical and molecular formulae, chemical equations and stoichiometry.

UNIT - 2: STATES OF MATTER
Classification of matter into solid, liquid and gaseous states.

Gaseous State - Measurable properties of gases, Gas laws - Boyle's law, Charle's law, Graham's law of diffusion, Avogadro's law, Dalton's law of partial pressure, concept of absolute scale of temperature, Ideal gas equation, kinetic theory of gases (only postulates), concept of average, root mean square and most probable velocities, real gases, deviation from Ideal behaviour, compressibility factor, van der Waals equation, liquefaction of gases, critical constants.

Liquid State - Properties of liquids - vapour pressure, viscosity and surface tension and effect of temperature on them (qualitative treatment only).

Solid State - Classification of solids - molecular, ionic, covalent and metallic solids, amorphous and crystalline solids (elementary idea), Bragg's Law and its applications, unit cell and lattices, packing in solids (fcc, bcc and hcp lattices), voids, calculations involving unit cell parameters, imperfection in solids, electrical, magnetic and dielectric properties.

UNIT - 3: ATOMIC STRUCTURE
Discovery of subatomic particles (electron, proton and neutron), Thomson and Rutherford atomic models and their limitations, nature of electromagnetic radiation, photoelectric effect, spectrum of

hydrogen atom, Bohr model of hydrogen atom - its postulates, derivation of the relations for energy of the electron and radii of the different orbits, limitations of Bohr's model, dual nature of matter, de-Broglie's relationship, Heisenberg uncertainty principle, elementary ideas of quantum mechanics, quantum mechanical model of atom, its important features, ϕ and ϕ_2, concept of atomic orbitals as one electron wave functions, variation of ϕ and ϕ_2 with r for 1s and 2s orbitals, various quantum numbers (principal, angular momentum and magnetic quantum numbers) and their significance, shapes of s, p and d - orbitals, electron spin and spin quantum number, rules for filling electrons in orbitals – Aufbau principle, Pauli's exclusion principle and Hund's rule, electronic configuration of elements, extra stability of half-filled and completely filled orbitals.

UNIT - 4: CHEMICAL BONDING AND MOLECULAR STRUCTURE

Kossel - Lewis approach to chemical bond formation, concept of ionic and covalent bonds.

Ionic Bonding - Formation of ionic bonds, factors affecting the formation of ionic bonds, calculation of lattice enthalpy.

Covalent Bonding - concept of electronegativity, Fajan's rule, dipole moment, Valence Shell Electron Pair Repulsion (VSEPR) theory and shapes of simple molecules.

Quantum mechanical approach to covalent bonding - valence bond theory - its important features, concept of hybridization involving s, p and d orbitals, Resonance.

Molecular Orbital Theory - its important features, LCAOs, types of molecular orbitals (bonding, antibonding), sigma and pi-bonds, molecular orbital electronic configurations of homonuclear diatomic molecules, concept of bond order, bond length and bond energy.

Elementary idea of metallic bonding, hydrogen bonding and its applications.

UNIT - 5: CHEMICAL THERMODYNAMICS

Fundamentals of thermodynamics: system and surroundings, extensive and intensive properties, state functions, types of processes.

First law of thermodynamics - Concept of work, heat internal energy and enthalpy, heat capacity, molar heat capacity, Hess's law of constant heat summation, enthalpies of bond dissociation, combustion, formation, atomization, sublimation, phase transition, hydration, ionization and solution.

Second law of thermodynamics - Spontaneity of processes, ΔS of the universe and ΔG of the system as criteria for spontaneity, $\Delta G°$ (standard Gibbs energy change) and equilibrium constant.

UNIT - 6: SOLUTIONS

Different methods for expressing concentration of solution - molality, molarity, mole fraction, percentage (by volume and mass both), vapour pressure of solutions and Raoult's law - Ideal and non-ideal solutions, vapour pressure - composition plots for ideal and non-ideal solutions, colligative properties of dilute solutions - relative lowering of vapour pressure, depression of freezing point, elevation of boiling point and osmotic pressure, determination of molecular mass using colligative properties, abnormal value of molar mass, van't Hoff factor and its significance.

UNIT - 7: EQUILIBRIUM

Meaning of equilibrium, concept of dynamic equilibrium.

Equilibria involving physical processes - Solid - liquid, liquid - gas and solid - gas equilibria, Henry's law, general characteristics of equilibrium involving physical processes.

Equilibria involving chemical processes - Law of chemical equilibrium, equilibrium constants (K_p and K_c) and their significance, significance of ΔG and $\Delta G°$ in chemical equilibria, factors affecting equilibrium concentration, pressure, temperature, effect of catalyst, Le Chatelier's principle.

Ionic equilibrium - Weak and strong electrolytes, ionization of electrolytes, various concepts of acids and bases (Arrhenius, Bronsted - Lowry and Lewis) and their ionization, acid - base equilibria (including multistage ionization) and ionization constants, ionization of water, pH scale, common ion effect, hydrolysis of salts and pH of their solutions, solubility of sparingly soluble salts and solubility products, buffer solutions. .

UNIT - 8 : REDOX REACTIONS AND ELECTROCHEMISTRY

Electronic concepts of oxidation and reduction, redox reactions, oxidation number, rules for assigning oxidation number, balancing of redox reactions.

Electrolytic and metallic conduction, conductance in electrolytic solutions, specific and molar conductivities and their variation with concentration: Kohlrausch's law and its applications.

Electrochemical cells - electrolytic and galvanic cells, different types of electrodes, electrode potentials including standard electrode potential, half - cell and cell reactions, emf of a galvanic cell and its measurement, Nernst equation and its applications, relationship between cell potential and Gibbs' energy change, dry cell and lead accumulator, fuel cells, corrosion and its prevention.

UNIT - 9 : CHEMICAL KINETICS

Rate of a chemical reaction, factors affecting the rate of reactions concentration, temperature, pressure and catalyst, elementary and complex reactions, order and molecularity of reactions, rate law, rate constant and its units, differential and integral forms of zero and first order reactions, their characteristics and half - lives, effect of temperature on rate of reactions - Arrhenius theory, activation energy and its calculation, collision theory of bimolecular gaseous reactions (no derivation).

UNIT - 10 : SURFACE CHEMISTRY

Adsorption - Physisorption and chemisorption and their characteristics, factors affecting adsorption of gases on solids, Freundlich and Langmuir adsorption isotherms, adsorption from solutions.

Catalysis - Homogeneous and heterogeneous, activity and selectivity of solid catalysts, enzyme catalysis and its mechanism.

Colloidal state - distinction among true solutions, colloids and suspensions, classification of colloids - lyophilic, lyophobic, multi molecular, macromolecular and associated colloids (micelles), preparation and properties of colloids - Tyndall effect, Brownian movement, electrophoresis, dialysis, coagulation and flocculation, emulsions and their characteristics.

Section - B (Inorganic Chemistry)

UNIT - 11: CLASSIFICATION OF ELEMENTS AND PERIODICITY IN PROPERTIES

Modern periodic law and present form of the periodic table, s, p, d and f block elements, periodic trends in properties of elementsatomic and ionic radii, ionization enthalpy, electron gain enthalpy, valence, oxidation states and chemical reactivity.

UNIT - 12: GENERAL PRINCIPLES AND PROCESSES OF ISOLATION OF METALS

Modes of occurrence of elements in nature, minerals, ores, steps involved in the extraction of metals - concentration, reduction (chemical and electrolytic methods) and refining with special reference to the extraction of Al, Cu, Zn and Fe, thermodynamic and electrochemical principles involved in the extraction of metals.

UNIT - 13: HYDROGEN

Position of hydrogen in periodic table, isotopes, preparation, properties and uses of hydrogen, physical and chemical properties of water and heavy water, structure, preparation, reactions and uses of hydrogen peroxide, classification of hydrides - ionic, covalent and interstitial, hydrogen as a fuel.

UNIT - 14: *s* - BLOCK ELEMENTS (ALKALI AND ALKALINE EARTH METALS)

Group - 1 and 2 Elements

General introduction, electronic configuration and general trends in physical and chemical properties of elements, anomalous properties of the first element of each group, diagonal relationships.

Preparation and properties of some important compounds - sodium carbonate, sodium chloride, sodium hydroxide and sodium hydrogen carbonate, Industrial uses of lime, limestone, Plaster of Paris and cement, Biological significance of Na, K, Mg and Ca.

UNIT - 15: *p* - BLOCK ELEMENTS Group - 13 to Group 18 Elements

General Introduction - Electronic configuration and general trends in physical and chemical properties of elements across the periods and down the groups, unique behaviour of the first element in each group.

Group - 13

Preparation, properties and uses of boron and aluminium, structure, properties and uses of borax, boric acid, diborane, boron trifluoride, aluminium chloride and alums.

Group - 14

Tendency for catenation, structure, properties and uses of allotropes and oxides of carbon, silicon tetrachloride, silicates, zeolites and silicones.

Group - 15

Properties and uses of nitrogen and phosphorus, allotropic forms of phosphorus, preparation, properties, structure and uses of ammonia, nitric acid, phosphine and phosphorus halides, (PCl_3, PCl_5), structures of oxides and oxoacids of nitrogen and phosphorus.

Group - 16

Preparation, properties, structures and uses of dioxygen and ozone, allotropic forms of sulphur, preparation, properties, structures and uses of sulphur dioxide, sulphuric acid (including its industrial preparation), Structures of oxoacids of sulphur.

Group - 17

Preparation, properties and uses of chlorine and hydrochloric acid, trends in the acidic nature of hydrogen halides, structures of interhalogen compounds and oxides and oxoacids of halogens.

Group - 18

Occurrence and uses of noble gases, structures of fluorides and oxides of xenon.

UNIT - 16: *d* - and *f* - BLOCK ELEMENTS

Transition Elements

General introduction, electronic configuration, occurrence and characteristics, general trends in properties of the first row transition elements - physical properties, ionization enthalpy, oxidation states, atomic radii, colour, catalytic behaviour, magnetic properties, complex formation, interstitial compounds, alloy formation, preparation, properties and uses of $K_2Cr_2O_7$ and $KMnO_4$.

Inner Transition Elements

Lanthanoids - Electronic configuration, oxidation states, chemical reactivity and lanthanoid contraction.

Actinoids - Electronic configuration and oxidation states.

UNIT - 17: COORDINATION COMPOUNDS

Introduction to coordination compounds, Werner's theory, ligands, coordination number, denticity, chelation, IUPAC nomenclature of mononuclear coordination compounds, isomerism, bonding valence bond approach and basic ideas of crystal field theory, colour and magnetic properties, importance of coordination compounds (in qualitative analysis, extraction of metals and in biological systems).

UNIT - 18: ENVIRONMENTAL CHEMISTRY

Environmental Pollution - Atmospheric, water and soil. **Atmospheric pollution** - tropospheric and stratospheric.

Tropospheric pollutants - Gaseous pollutants: oxides of carbon, nitrogen and sulphur, hydrocarbons, their sources, harmful effects and prevention, green house effect and global warming, acid rain.

Particulate pollutants - Smoke, dust, smog, fumes, mist, their sources, harmful effects and prevention.

Stratospheric pollution - Formation and breakdown of ozone, depletion of ozone layer - its mechanism and effects.

Water Pollution - Major pollutants such as, pathogens, organic wastes and chemical pollutants, their harmful effects and prevention.

Soil Pollution - Major pollutants like pesticides (insecticides, herbicides and fungicides), their harmful effects and prevention.

Strategies to control environmental pollution.

SECTION - C (Organic Chemistry)

UNIT - 19: PURIFICATION AND CHARACTERISATION OF ORGANIC COMPOUNDS

Purification - Crystallization, sublimation, distillation, differential extraction and chromatography - principles and their applications.

Qualitative analysis - Detection of nitrogen, sulphur, phosphorus and halogens.

Quantitative analysis (Basic principles only) - Estimation of carbon, hydrogen, nitrogen, halogens, sulphur, phosphorus.

Calculations of empirical formulae and molecular formulae, numerical problems in organic quantitative analysis.

UNIT - 20: SOME BASIC PRINCIPLES OF ORGANIC CHEMISTRY

Tetravalency of carbon, shapes of simple molecules - hybridization (s and p), classification of organic compounds based on functional groups: – C – C – , – C = C - and those containing halogens, oxygen, nitrogen and sulphur, homologous series, Isomerism - structural and stereoisomerism.
nomenclature (trivial and IUPAC)

Covalent bond fission - Homolytic and heterolytic: free radicals, carbocations and carbanions, stability of carbocations and free radicals, electrophiles and nucleophiles.

Electronic displacement in a covalent bond - Inductive effect, electromeric effect, resonance and hyperconjugation.

Common types of organic reactions - Substitution, addition, elimination and rearrangement.

UNIT - 21: HYDROCARBONS

Classification, isomerism, IUPAC nomenclature, general methods of preparation, properties and reactions.

Alkanes - Conformations: Sawhorse and Newman projections (of ethane), mechanism of halogenation of alkanes.

Alkenes - Geometrical isomerism, mechanism of electrophilic addition: addition of hydrogen, halogens, water, hydrogen halides (Markownikoff's and peroxide effect*),* ozonolysis, oxidation, and polymerization.

Alkynes - Acidic character, addition of hydrogen, halogens, water and hydrogen halides, polymerization.**Aromatic hydrocarbons** - Nomenclature, benzene - structure and aromaticity, mechanism of electrophilic substitution: halogenation, nitration, Friedel – Craft's alkylation and acylation, directive influence of functional group in mono-substituted benzene.

UNIT - 22: ORGANIC COMPOUNDS CONTAINING HALOGENS

General methods of preparation, properties and reactions, nature of C-X bond, mechanisms of substitution reactions. Uses/environmental effects of chloroform, iodoform, freons and DDT.

UNIT - 23: ORGANIC COMPOUNDS CONTAINING OXYGEN

General methods of preparation, properties, reactions and uses.

Alcohols - Identification of primary, secondary and tertiary alcohols, mechanism of dehydration.

Phenols - Acidic nature, electrophilic substitution reactions: halogenation, nitration and sulphonation, Reimer - Tiemann reaction.

Ethers - Structure.

Aldehyde and Ketones - Nature of carbonyl group, nucleophilic addition to >C=O group, relative reactivities of aldehydes and ketones, important reactions such as - nucleophilic addition reactions (addition of HCN, NH_3 and its derivatives), Grignard reagent, oxidation, reduction (Wolff Kishner and Clemmensen), acidity of α-hydrogen, aldol condensation, Cannizzaro reaction, haloform reaction, chemical tests to distinguish between aldehydes and ketones.

Carboxylic acid - Acidic strength and factors affecting it.

UNIT - 24: ORGANIC COMPOUNDS CONTAINING NITROGEN

General methods of preparation, properties, reactions and uses.

Amines - Nomenclature, classification, structure basic character and identification of primary, secondary and tertiary amines and their basic character.

Diazonium Salts - Importance in synthetic organic chemistry.

UNIT - 25: POLYMERS

General introduction and classification of polymers, general methods of polymerization - addition and condensation, copolymerization, natural and synthetic rubber and vulcanization, some important polymers with emphasis on their monomers and uses - polythene, nylon, polyester and bakelite.

UNIT - 26: BIOMOLECULES

General introduction and importance of biomolecules.

Carbohydrates - Classification: aldoses and ketoses, monosaccharides (glucose and fructose), constituent monosaccharides of oligosaccharides (sucrose, lactose, maltose) and polysaccharides (starch, cellulose, glycogen).

Proteins - Elementary Idea of α - amino acids, peptide bond, polypeptides, proteins - primary, secondary, tertiary and quaternary structure (qualitative idea only), denaturation of proteins, enzymes.

Vitamins - Classification and functions.

Nucleic acids - Chemical constitution of DNA and RNA, biological functions of nucleic acids.

UNIT - 27: CHEMISTRY IN EVERYDAY LIFE

Chemicals in medicines - Analgesics, tranquilizers, antiseptics, disinfectants, antimicrobials, antifertility drugs, antibiotics, antacids, antihistamins - their meaning and common examples.

Chemicals in food - Preservatives, artificial sweetening agents - common examples.

Cleansing agents - Soaps and detergents, cleansing action.

UNIT - 28: PRINCIPLES RELATED TO PRACTICAL CHEMISTRY

Detection of extra elements (N, S, halogens) in organic compounds, detection of the following functional groups: hydroxyl (alcoholic and phenolic), carbonyl (aldehyde and ketone), carboxyl and amino groups in organic compounds.

Chemistry involved in the preparation of the following:

Inorganic compounds - Mohr's salt, potash alum.

Organic compounds - Acetanilide, p-nitroacetanilide, aniline yellow, iodoform.

Chemistry involved in the titrimetric exercises - Acids, bases and the use of indicators, oxalic acid vs $KMnO_4$, Mohr's salt vs $KMnO_4$.

Chemical principles involved in the qualitative salt analysis:

Cations - Pb^{2+}, Cu^{2+}, Al^{3+}, Fe^{3+}, Zn^{2+}, Ni^{2+}, Ca^{2+}, Ba^{2+}, Mg^{2+}, NH_4^+.

Anions – CO_3^{2-}, S^{2-}, SO_4^{2-}, NO_2^-, NO_3^-, Cl^-, Br^-, I^- (insoluble salts excluded).

Chemical principles involved in the following experiments:

1. Enthalpy of solution of $CuSO_4$
2. Enthalpy of neutralization of strong acid and strong base.
3. Preparation of lyophilic and lyophobic sols.
4. Kinetic study of reaction of iodide ion with hydrogen peroxide at room temperature.

MATHEMATICS

UNIT - 1 : SETS, RELATIONS AND FUNCTIONS

Sets and their representation, union, intersection and complement of sets and their algebraic properties, power set, relations, types of relations, equivalence relations, functions, one-one, into and onto functions, composition of functions.

UNIT - 2 : COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Complex numbers as ordered pairs of reals, representation of complex numbers in the form a + *i*b and their representation in a plane, Argand diagram, algebra of complex numbers, modulus and argument (or amplitude) of a complex number, square root of a complex number, triangle inequality, quadratic equations in real and complex number system and their solutions, relation between roots and coefficients, nature of roots, formation of quadratic equations with given roots.

UNIT - 3 : MATRICES AND DETERMINANTS

Matrices, algebra of matrices, types of matrices, determinants and matrices of order two and three. Properties of determinants, evaluation of determinants, area of triangles using determinants. Adjoint and evaluation of inverse of a square matrix using determinants and elementary transformations, Test of consistency and solution of simultaneous linear equations in two or three variables using determinants and matrices.

UNIT - 4 : PERMUTATIONS AND COMBINATIONS

Fundamental principle of counting, permutation as an arrangement and combination as selection, Meaning of $P(n,r)$ and $C(n,r)$, simple applications.

UNIT - 5 : MATHEMATICAL INDUCTION

Principle of Mathematical Induction and its simple applications.

UNIT - 6 : BINOMIAL THEOREM AND ITS SIMPLE APPLICATIONS

Binomial theorem for a positive integral index, general term and middle term, properties of Binomial coefficients and simple applications.

UNIT - 7 : SEQUENCES AND SERIES

Arithmetic and Geometric progressions, insertion of arithmetic, geometric means between two given numbers. Relation between A.M. and G.M. Sum upto n terms of special series: $\sum n, \sum n^2, \sum n^3$. Arithmetic - Geometric progression.

UNIT - 8 : LIMITS, CONTINUITY AND DIFFERENTIABILITY

Real - valued functions, algebra of functions, polynomials, rational, trigonometric, logarithmic and exponential functions, inverse functions. Graphs of simple functions. Limits, continuity and differentiability. Differentiation of the sum, difference, product and quotient of two functions. Differentiation of trigonometric, inverse trigonometric, logarithmic, exponential, composite and implicit functions, derivatives of order upto two. Rolle's and Lagrange's Mean value theorems. Applications of derivatives: Rate of change of quantities, monotonic - increasing and decreasing functions, Maxima and minima of functions of one variable, tangents and normals.

UNIT - 9 : INTEGRAL CALCULUS

Integral as an anti - derivative. Fundamental integrals involving algebraic, trigonometric, exponential and logarithmic functions. Integration by substitution, by parts and by partial fractions. Integration using trigonometric identities.

Evaluation of simple integrals of the type3

$$\frac{dx}{x^2 \pm a^2}, \frac{dx}{x^2 \pm a^2}, \frac{dx}{a^2 - x^2}, \frac{dx}{a^2 - x^2}, \frac{dx}{ax^2 + bx + c}, \frac{dx}{ax^2 + bx + c},$$

$$\frac{(px+q)dx}{ax^2 + bx + c}, \frac{(px+q)dx}{ax^2 + bx + c}, \frac{(px+q)dx}{ax^2 + dx + c}$$

$\int \sqrt{a^2 \pm x^2 dx}$ and $\int \sqrt{x^2 - a^2 dx}$

Integral as limit of a sum. Fundamental theorem of calculus. Properties of definite integrals. Evaluation of definite integrals, determining areas of the regions bounded by simple curves in standard form.

UNIT - 10: DIFFERENTIAL EQUATIONS

Ordinary differential equations, their order and degree. Formation of differential equations. Solution of differential equations by the method of separation of variables, solution of homogeneous and linear differential equations of the type:

$$\frac{dy}{dx} + p(x)y = q(x)$$

UNIT - 11: COORDINATE GEOMETRY

Cartesian system of rectangular coordinates in a plane, distance formula, section formula, locus and its equation, translation of axes, slope of a line, parallel and perpendicular lines, intercepts of a line on the coordinate axes.

Straight lines - Various forms of equations of a line, intersection of lines, angles between two lines, conditions for concurrence of three lines, distance of a point from a line, equations of internal and external bisectors of angles between two lines, coordinates of centroid, orthocentre and circumcentre of a triangle, equation of family of lines passing through the point of intersection of two lines.

Circles, conic sections - Standard form of equation of a circle, general form of the equation of a circle, its radius and centre, equation of a circle when the end points of a diameter are given, points of intersection of a line and a circle with the centre at the origin and condition for a line to be tangent to a circle, equation of the tangent. Sections of cones, equations of conic sections (parabola, ellipse and hyperbola) in standard forms, condition for $y = mx + c$ to be a tangent and point(s) of tangency.

UNIT - 12: THREE DIMENSIONAL GEOMETRY

Coordinates of a point in space, distance between two points, section formula, direction ratios and direction cosines, angle between two intersecting lines. Skew lines, the shortest distance between them and its equation. Equations of a line and a plane in different forms, intersection of a line and a plane, coplanar lines.

UNIT - 13: VECTOR ALGEBRA

Vectors and scalars, addition of vectors, components of a vector in two dimensions and three dimensional space, scalar and vector products, scalar and vector triple product.

UNIT - 14: STATISTICS AND PROBABILITY

Measures of Dispersion - Calculation of mean, median, mode of grouped and ungrouped data. Calculation of standard deviation, variance and mean deviation for grouped and ungrouped data.

Probability - Probability of an event, addition and multiplication theorems of probability, Baye's theorem, probability distribution of a random variate, Bernoulli trials and Binomial distribution.

UNIT - 15: TRIGONOMETRY

Trigonometrical identities and equations. Trigonometrical functions. Inverse trigonometrical functions and their properties. Heights and Distances.

UNIT - 16: MATHEMATICAL REASONING

Statements, logical operations and, or, implies, implied by, if and only if. Understanding of tautology, contradiction, converse and contrapositive.

From the graph shown with each chapters, it is evident that the average chances of questions coming in the AIEEE examination is different for different units. The probability of questions being asked in the examination is maximum for the following units.

Physics

• Electrostatics • Thermodynamics • Oscillations and Waves • Electromagnetic Induction and Alternating Current • Atoms and Nuclei

Chemistry

• Organic Compounds Containing Oxygen • Solutions • Equilibrium • Chemical Thermodynamics • Some Basic Principles of Organic Chemistry • p-Block Elements (Group 13 to 18)

Mathematics

• Limits, Continuity and Differentiability • Co-ordinate Geometry • Statistics and Probability • Matrices and Determinants • Integral Calculus

CHAPTER
1

Units and Measurements

1. Identify the pair whose dimensions are equal.
   (a) torque and work (b) stress and energy
   (c) force and stress (d) force and work.
   (2002)

2. Dimensions of $\frac{1}{\mu_0 \varepsilon_0}$, where symbols have their usual meaning, are
   (a) $[L^{-1}T]$ (b) $[L^{-2}T^2]$
   (c) $[L^2T^{-2}]$ (d) $[LT^{-1}]$.
   (2003)

3. The physical quantities not having same dimensions are
   (a) torque and work
   (b) momentum and Planck's constant
   (c) stress and Young's modulus
   (d) speed and $(\mu_0 \varepsilon_0)^{-1/2}$.
   (2003)

4. Which one of the following represents the correct dimensions of the coefficient of viscosity?
   (a) $ML^{-1}T^{-2}$ (b) MLT^{-1}
   (c) $ML^{-1}T^{-1}$ (d) $ML^{-2}T^{-2}$.
   (2004)

5. Out of the following pairs which one does not have identical dimensions is
   (a) moment of inertia and moment of a force
   (b) work and torque
   (c) angular momentum and Planck's constant
   (d) impulse and momentum
   (2005)

6. Which of the following units denotes the dimensions ML^2/Q^2, where Q denotes the electric charge?
   (a) weber (Wb) (b) Wb/m^2
   (c) henry (H) (d) H/m^2.
   (2006)

7. The dimension of magnetic field in M, L, T and C (coulomb) is given as
   (a) $MT^{-2}C^{-1}$ (b) $MLT^{-1}C^{-1}$
   (c) MT^2C^{-2} (d) $MT^{-1}C^{-1}$.
   (2008)

Answer Key

1. (a)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (a)	**6.** (c)
7. (d)					

1. **(a) :** Torque and work have the same dimensions.

2. **(c) :** Velocity of light in vacuum $=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$

or $[LT^{-1}]=\left[\frac{1}{\sqrt{\mu_0\varepsilon_0}}\right]$

or $[L^2T^{-2}]=\left[\frac{1}{\mu_0\varepsilon_0}\right]$

$\therefore$ Dimensions of $\frac{1}{\mu_0\varepsilon_0}=[L^2T^{-2}]$.

3. **(b) :** [Momentum] = $[MLT^{-1}]$
[Planck's constant] = $[ML^2T^{-1}]$
Momentum and Planck's constant do not have same dimensions.

4. **(c) :** Viscous force $F = 6\pi\eta rv$

$\therefore$ $\eta=\frac{F}{6\pi rv}$

or $[\eta]=\frac{[F]}{[r][v]}$

or $[\eta]=\frac{[MLT^{-2}]}{[L][LT^{-1}]}$

or $[\eta] = [ML^{-1}\ T^{-1}]$.

5. **(a) :** Moment of inertia $(I) = mr^2$
$\therefore$ $[I] = [ML^2]$
Moment of force $(C) = r\ F$
$\therefore$ $[C] = [r][F] = [L][MLT^{-2}]$
or $[C] = [ML^2\ T^{-2}]$
Moment of inertia and moment of a force do not have identical dimensions.

6. **(c) :** $[ML^2Q^{-2}] = [ML^2\ A^{-2}\ T^{-2}]$
$[Wb] = [ML^2\ T^{-2}\ A^{-1}]$

$\left[\frac{Wb}{m^2}\right]=[MT^{-2}A^{-1}]$

$[henry] = [ML^2\ T^{-2}\ A^{-2}]$

$\left[\frac{H}{m^2}\right]=[MT^{-2}A^{-2}]$

Obviously henry (H) has dimensions $\frac{ML^2}{Q^2}$.

7. **(d) :** Lorentz force $=|\vec{F}|=|q\vec{v}\times\vec{B}|$

$\therefore$ $[B]=\frac{[F]}{[q][v]}=\frac{MLT^{-2}}{C\times LT^{-1}}=\frac{MLT^{-2}}{CLT^{-1}}=[MT^{-1}C^{-1}]$

CHAPTER 2

Description of Motion in One Dimension

1. If a body looses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest?
(a) 1 cm (b) 2 cm
(c) 3 cm (d) 4 cm.
(2002)

2. Speeds of two identical cars are u and $4u$ at a specific instant. If the same deceleration is applied on both the cars, the ratio of the respective distances in which the two cars are stopped from that instant is
(a) 1 : 1 (b) 1 : 4
(c) 1 : 8 (d) 1 : 16.
(2002)

3. From a building two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then
(a) $v_B > v_A$
(b) $v_A = v_B$
(c) $v_A > v_B$
(d) their velocities depend on their masses.
(2002)

4. A car moving with a speed of 50 km/hr, can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is
(a) 12 m (b) 18 m
(c) 24 m (d) 6 m.
(2003)

5. A ball is released from the top of a tower of height h metre. It takes T second to reach the ground. What is the position of the ball in $T/3$ second?
(a) $h/9$ metre from the ground
(b) $7h/9$ metre from the ground
(c) $8h/9$ metre from the ground
(d) $17h/18$ metre from the ground.
(2004)

6. An automobile travelling with a speed of 60 km/h, can brake to stop within a distance of 20 m. If the car is going twice as fast, *i.e.* 120 km/h, the stopping distance will be
(a) 20 m (b) 40 m
(c) 60 m (d) 80 m.
(2004)

7. The relation between time t and distance x is $t = ax^2 + bx$ where a and b are constants. The acceleration is
(a) $-2av^3$ (b) $2av^2$
(c) $-2av^2$ (d) $2bv^3$
(2005)

8. A car, starting from rest, accelerates at the rate f through a distance s, then continues at constant speed for time t and then decelerates at the rate $f/2$ to come to rest. If the total distance traversed is 15 s, then
(a) $s = \frac{1}{2} f t^2$ (b) $s = \frac{1}{4} f t^2$
(c) $s = ft$ (d) $s = \frac{1}{6} f t^2$
(2005)

9. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 m/s^2. He reaches the ground with a speed of 3 m/s. At what height, did he bail out?

(a) 293 m (b) 111 m
(c) 91 m (d) 182 m

(2005)

10. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as

(a) t^3 (b) t^2
(c) t (d) $t^{1/2}$.

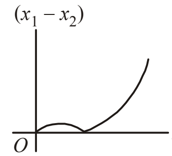
(2006)

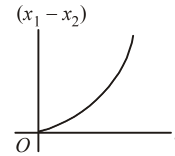
11. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is

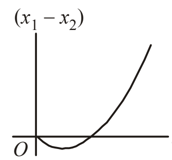
(a) $v_0 + g/2 + f$ (b) $v_0 + 2g + 3f$
(c) $v_0 + g/2 + f/3$ (d) $v_0 + g + f$

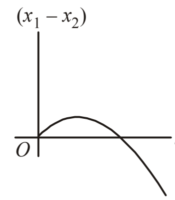
(2007)

12. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by x_2 (t) after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t?

(a)


(b)


(c)


(d)


(2008)

Answer Key

1.	(a)	**2.**	(d)	**3.**	(b)	**4.**	(c)	**5.**	(c)	**6.**	(d)
7.	(a)	**8.**	no option	**9.**	(a)	**10.**	(b)	**11.**	(c)	**12.**	(c)

1. **(a) :** For first part of penetration, by equation of motion,

$$\left(\frac{u}{2}\right)^2 = u^2 - 2a(3)$$

or $3u^2 = 24a \Rightarrow u^2 = 8a$ .... (i)

For latter part of penetration,

$$0 = \left(\frac{u}{2}\right)^2 - 2ax$$

or $u^2 = 8ax$ .... (ii)

From (i) and (ii)

$8ax = 8a \Rightarrow x = 1$ cm.

2. **(d) :** Both are given the same deceleration simultaneously and both finally stop.

Formula relevant to motion : $u^2 = 2\ as$

$\therefore$ For first car, $s_1 = \dfrac{u^2}{2a}$

For second car, $s_2 = \dfrac{(4u)^2}{2a} = \dfrac{16u^2}{2a}$

$\therefore \quad \dfrac{s_1}{s_2} = \dfrac{1}{16}$.

3. **(b) :** Ball A projected upwards with velocity u falls back to building top with velocity u downwards. It completes its journey to ground under gravity.

$\therefore \quad v_A^2 = u^2 + 2gh$ ....(i)

Ball B starts with downwards velocity u and reaches ground after travelling a vertical distance h

$\therefore \quad v_B^2 = u^2 + 2gh$ ....(ii)

From (i) and (ii)

$v_A = v_B$.

4. **(c) :** For first case,

$$u_1 = 50\frac{\text{km}}{\text{hour}} = \frac{50\times 1000}{60\times 60} = \frac{125}{9}\frac{\text{m}}{\text{sec}}$$

$\therefore$ Acceleration

$$a = -\frac{u_1^2}{2s_1} = -\left(\frac{125}{9}\right)^2 \times \frac{1}{2\times 6} = -16 \text{ m/sec}^2$$

For second case,

$$u_2 = 100\frac{\text{km}}{\text{hour}} = \frac{100\times 1000}{60\times 60} = \frac{250}{9}\frac{\text{m}}{\text{sec}}$$

$$\therefore \quad s_2 = \frac{-u_2^2}{2a} = \frac{-1}{2}\left(\frac{250}{9}\right)^2 \times \left(-\frac{1}{16}\right) = 24 \text{ m}$$

or $s_2 = 24$ m.

5. **(c) :** Equation of motion : $s = ut + \frac{1}{2}gt^2$

$\therefore \quad h = 0 + \frac{1}{2}gT^2$

or $2h = gT^2$ ..... (i)

After $T/3$ sec,

$$s = 0 + \frac{1}{2}\times g\left(\frac{T}{3}\right)^2 = \frac{gT^2}{18}$$

or $18\ s = gT^2$ .... (ii)

From (i) and (ii),

$18\ s = 2h$

or $s = \dfrac{h}{9}$ m from top.

$\therefore$ Height from ground $= h - \dfrac{h}{9} = \dfrac{8h}{9}$ m.

6. **(d) :** Let a be the retardation for both the vehicles

For automobile, $v^2 = u^2 - 2\ as$

$\therefore \quad u_1^2 - 2as_1 = 0$

$\Rightarrow \quad u_1^2 = 2as_1$

Similarly for car, $u_2^2 = 2as_2$

$$\therefore \quad \left(\frac{u_2}{u_1}\right)^2 = \frac{s_2}{s_1} \Rightarrow \left(\frac{120}{60}\right)^2 = \frac{s_2}{20}$$

or $s_2 = 80$ m.

7. **(a) :** $t = ax^2 + bx$

Differentiate the equation with respect to t

$$\therefore \quad 1 = 2ax\frac{dx}{dt} + b\frac{dx}{dt}$$

or $1 = 2axv + bv \quad$ as $\dfrac{dx}{dt} = v$

or $v = \dfrac{1}{2ax+b}$

or $\dfrac{dv}{dt} = \dfrac{-2a(dx/dt)}{(2ax+b)^2} = -2av\times v^2$

or Acceleration $= -2av^3$.

8. For first part of journey, $s = s_1$,

$s_1 = \frac{1}{2}f\,t_1^2 = s$ .... (i)

$v = f\,t_1$ .... (ii)

For second part of journey,

$s_2 = vt$

or $s_2 = f\,t_1\,t$ .... (iii)

For the third part of journey,

$$s_3 = \frac{1}{2}\left(\frac{f}{2}\right)(2t_1)^2$$

or $s_3 = \frac{1}{2} \times \frac{4f\,t_1^2}{2}$

or $s_3 = 2s_1 = 2s$ .... (iv)

$s_1 + s_2 + s_3 = 15s$

or $s + f\,t_1 t + 2s = 15s$

or $f\,t_1 t = 12s$ .... (v)

From (i) and (v),

$$\frac{s}{12s} = \frac{ft_1^2}{2 \times ft_1 t}$$

or $t_1 = \frac{t}{6}$

or $s = \frac{1}{2}ft_1^2 = \frac{1}{2}f\left(\frac{t}{6}\right)^2 = \frac{ft^2}{72}$

or $s = \frac{ft^2}{72}$

None of the given options provide this answer.

9. **(a) :** Initially, the parachutist falls under gravity

$\therefore$ $u^2 = 2ah = 2 \times 9.8 \times 50 = 980\ m^2s^{-2}$

He reaches the ground with speed

$= 3$ m/s, $a = -2\ ms^{-2}$

$\therefore$ $(3)^2 = u^2 - 2 \times 2 \times h_1$

or $9 = 980 - 4\,h_1$

or $h_1 = \frac{971}{4}$

or $h_1 = 242.75$ m

$\therefore$ Total height $= 50 + 242.75$

$= 292.75$

$= 293$ m.

10. **(b) :** $v = \alpha\sqrt{x}$

or $\frac{dx}{dt} = \alpha\sqrt{x}$

or $\frac{dx}{\sqrt{x}} = \alpha\,dt$

or $\int\frac{dx}{\sqrt{x}} = \alpha\int dt$

or $2x^{1/2} = \alpha t$

or $x = \left(\frac{\alpha}{2}\right)^2 t^2$

or displacement is proportional to t^2.

11. **(c) :** Given : velocity $v = v_0 + gt + ft^2$

$\therefore$ $v = \frac{dx}{dt}$ or $\int_0^x dx = \int_0^t v dt$

or $x = \int_0^t (v_0 + gt + ft^2)dt$

$$x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3} + C$$

where C is the constant of integration

Given : $x = 0$, $t = 0$. $\therefore$ $C = 0$

or $x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}$

At $t = 1$ sec

$\therefore$ $x = v_0 + \frac{g}{2} + \frac{f}{3}$.

12. **(c) :** As $u = 0$, $v_1 = at$, $v_2 =$ constant for the other particle. Initially both are zero. Relative velocity of particle 1 w.r.t. 2 is velocity of 1 – velocity of 2. At first the velocity of first particle is less than that of 2. Then the distance travelled by particle 1 increases as

$x_1 = (1/2)\,at_1^2$. For the second it is proportional to t. Therefore it is a parabola after crossing x-axis again. Curve (c) satisfies this.

CHAPTER 3

Description of Motion in 2 and 3 Dimension

1. Two forces are such that the sum of their magnitudes is 18 N and their resultant is 12 N which is perpendicular to the smaller force. Then the magnitudes of the forces are
(a) 12 N, 6 N (b) 13 N, 5 N
(c) 10 N, 8 N (d) 16 N, 2 N.
(2002)

2. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground ? [g = 10 m/s^2, sin30° = 1/2, cos30° = $\sqrt{3}/2$]
(a) 5.20 m (b) 4.33 m
(c) 2.60 m (d) 8.66 m.
(2003)

3. The co-ordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by
(a) $3t\sqrt{\alpha^2+\beta^2}$ (b) $3t^2\sqrt{\alpha^2+\beta^2}$
(c) $t^2\sqrt{\alpha^2+\beta^2}$ (d) $\sqrt{\alpha^2+\beta^2}$.
(2003)

4. If $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$, then the angle between A and B is
(a) π (b) π/3
(c) π/2 (d) π/4.
(2004)

5. A projectile can have the same range R for two angles of projection. If T_1 and T_2 be the time of flights in the two cases, then the product of the two time of flights is directly proportional to
(a) $1/R^2$ (b) $1/R$
(c) R (d) R^2.
(2004)

6. Which of the following statements is false for a particle moving in a circle with a constant angular speed?
(a) The velocity vector is tangent to the circle.
(b) The acceleration vector is tangent to the circle
(c) the acceleration vector points to the centre of the circle
(d) the velocity and acceleration vectors are perpendicular to each other.
(2004)

7. A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection?
(a) yes, 60° (b) yes, 30°
(c) no (d) yes, 45°.
(2004)

8. A particle is moving eastwards with a velocity of 5 m/s. In 10 s the velocity changes to 5 m/s northwards. The average acceleration in this time is
(a) zero
(b) $\frac{1}{\sqrt{2}}$ ms^{-2} towards north-west
(c) $\frac{1}{\sqrt{2}}$ ms^{-2} towards north-east

(d) $\frac{1}{2}$ ms^{-2} towards north

(2005)

9. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the time of flights in the two cases, then the product of the two time of flights is proportional to

(a) $1/R$
(b) R
(c) R^2
(d) $1/R^2$.

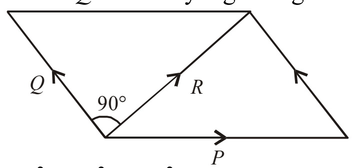
(2005)

Answer Key

1.	(b)	**2.**	(d)	**3.**	(b)	**4.**	(a)	**5.**	(c)	**6.**	(b)
7.	(a)	**8.**	(b)	**9.**	(b)						

1. **(b) :** Resultant R is perpendicular to smaller force Q and $(P + Q) = 18$ N

$\therefore$ $P^2 = Q^2 + R^2$ by right angled triangle



or $(P^2 - Q^2) = R^2$

or $(P + Q)(P - Q) = R^2$

or $(18)(P - Q) = (12)^2$ $[\because P + Q = 18]$

or $(P - Q) = 8$

Hence $P = 13$ N and $Q = 5$ N.

2. **(d) :** Height of building = 10 m

The ball projected from the roof of building will be back to roof - height of 10 m after covering the maximum horizontal range.

Maximum horizontal range $(R) = \dfrac{u^2 \sin 2\theta}{g}$

or $R = \dfrac{(10)^2 \times \sin 60°}{10} = 10 \times 0.866$

or $R = 8.66$ m.

3. **(b) :** $\because$ $x = \alpha t^3$

$\therefore$ $\dfrac{dx}{dt} = 3\alpha t^2 \Rightarrow v_x = 3\alpha t^2$

Again $y = \beta t^3$

$\therefore$ $\dfrac{dy}{dt} \Rightarrow v_y = 3\beta t^2$ $\therefore$ $v^2 = v_x^2 + v_y^2$

or $v^2 = (3\alpha t^2)^2 + (3\beta t^2)^2 = (3t^2)^2 (\alpha^2 + \beta^2)$

or $v = 3t^2\sqrt{\alpha^2 + \beta^2}$.

4. **(a) :** $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$

or $AB \sin\theta\, \hat{n} = AB \sin(-\theta)\, \hat{n}$

or $\sin\theta = -\sin\theta$

or $2 \sin\theta = 0$

or $\theta = 0, \pi, 2\pi.....$

$\therefore$ $\theta = \pi$.

5. **(c) :** Range is same for angles of projection θ and $(90° - \theta)$

$\therefore$ $T_1 = \dfrac{2u \sin\theta}{g}$ and $T_2 = \dfrac{2u \sin(90° - \theta)}{g}$

$\therefore$ $T_1 T_2 = \dfrac{4u^2 \sin\theta \cos\theta}{g^2} = \dfrac{2}{g} \times \left(\dfrac{u^2 \sin 2\theta}{g}\right) = \dfrac{2R}{g}$

$\therefore$ $T_1\ T_2$ is proportional to R.

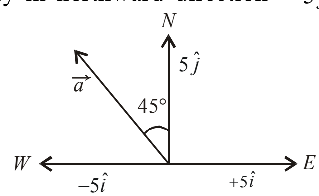
6. **(b) :** The acceleration vector acts along the radius of the circle. The given statement is false.

7. **(a) :** The person will catch the ball if his speed and horizontal speed of the ball are same

$= v_0 \cos\theta = \dfrac{v_0}{2} \Rightarrow \cos\theta = \dfrac{1}{2} = \cos 60°$ $\therefore$ $\theta = 60°$.

8. **(b) :** Velocity in eastward direction $= 5\hat{i}$

velocity in northward direction $= 5\hat{j}$



$\therefore$ Acceleration $\vec{a} = \dfrac{5\hat{j} - 5\hat{i}}{10}$

or $\vec{a} = \dfrac{1}{2}\hat{j} - \dfrac{1}{2}\hat{i}$ or $|\vec{a}| = \sqrt{\left(\dfrac{1}{2}\right)^2 + \left(-\dfrac{1}{2}\right)^2}$

or $|\vec{a}| = \dfrac{1}{\sqrt{2}}$ ms^{-2} towards north-west.

9. **(b) :** Range is same for angles of projection θ and $(90 - \theta)$

$\therefore$ $t_1 = \dfrac{2u \sin\theta}{g}$ and $t_2 = \dfrac{2u \sin(90 - \theta)}{g}$

$\therefore$ $t_1 t_2 = \dfrac{4u^2 \sin\theta \cos\theta}{g^2} = \dfrac{2}{g} \times \left(\dfrac{u^2 \sin 2\theta}{g}\right) = \dfrac{2R}{g}$

$\therefore$ $t_1\ t_2$ is proportional to R.

CHAPTER
4

Laws of Motion

1. The minimum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is

(a) 60 (b) 30 (c) 15 (d) 25.

(2002)

2. A lift is moving down with acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively

(a) g, g (b) $g - a, g - a$

(c) $g - a, g$ (d) a, g.

(2002)

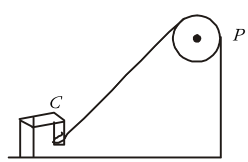
3. When forces F_1, F_2, F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. If the force F_1 is now removed then the acceleration of the particle is

(a) F_1/m (b) F_2F_3/mF_1

(c) $(F_2 - F_3)/m$ (d) F_2/m.

(2002)

4. One end of a massless rope, which passes over a massless and frictionless pulley P is tied to a hook C while the other end is free. Maximum tension that the rope can bear is 960 N. With what value of maximum safe acceleration (in ms^{-2}) can a man of 60 kg climb on the rope?



(a) 16 (b) 6 (c) 4 (d) 8.

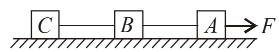
(2002)

5. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$, then the ratio of the masses is

(a) 8 : 1 (b) 9 : 7 (c) 4 : 3 (d) 5 : 3.

(2002)

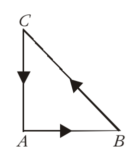
6. Three identical blocks of masses m = 2 kg are drawn by a force F = 10.2 N with an acceleration of 0.6 ms^{-2} on a frictionless surface, then what is the tension (in N) in the string between the blocks B and C?



(a) 9.2 (b) 7.8 (c) 4 (d) 9.8

(2002)

7. Three forces start acting simultaneously on a particle moving with velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle ABC (as shown). The particle will now move with velocity



(a) less than $\vec{v}$

(b) greater than $\vec{v}$

(c) $|\vec{v}|$ in the direction of the largest force BC

(d) $\vec{v}$, remaining unchanged.

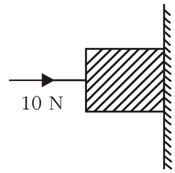
(2003)

8. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of 5 m/s^2, the reading of the spring balance will be

(a) 24 N (b) 74 N (c) 15 N (d) 49 N.

(2003)

9. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is



(a) 20 N (b) 50 N (c) 100 N (d) 2 N.

(2003)

10. A marble block of mass 2 kg lying on ice when given a velocity of 6 m/s is stopped by friction in 10 s. Then the coefficient of friction is

(a) 0.02 (b) 0.03 (c) 0.06 (d) 0.01.

(2003)

11. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force P is applied at the free end of the rope, the force exerted by the rope on the block is

(a) $\frac{Pm}{M+m}$ (b) $\frac{Pm}{M-m}$

(c) P (d) $\frac{PM}{M+m}$.

(2003)

12. A light spring balance hangs from the hook of the other light spring balance and a block of mass M kg hangs from the former one. Then the true statement about the scale reading is

(a) both the scales read M kg each

(b) the scale of the lower one reads M kg and of the upper one zero

(c) the reading of the two scales can be anything but the sum of the reading will be M kg

(d) both the scales read $M/2$ kg.

(2003)

13. A rocket with a lift-off mass 3.5×10^4 kg is blasted upwards with an initial acceleration of 10 m/s^2. Then the initial thrust of the blast is

(a) 3.5×10^5 N (b) 7.0×10^5 N

(c) 14.0×10^5 N (d) 1.75×10^5 N.

(2003)

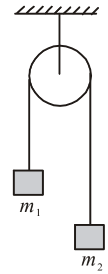
14. A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most?

(a) one (b) four (c) two (d) three.

(2004)

15. Two masses $m_1 = 5$ kg and $m_2 = 4.8$ kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses when lift free to move?

(g = 9.8 m/s^2)



(a) 0.2 m/s^2

(b) 9.8 m/s^2

(c) 5 m/s^2

(d) 4.8 m/s^2.

(2004)

16. A block rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass of the block (in kg) is

(take g = 10 m/s^2)

(a) 2.0 (b) 4.0 (c) 1.6 (d) 2.5

(2004)

17. An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two particles situated on the inner and outer parts of the ring, F_1/F_2 is

(a) 1 (b) $\frac{R_1}{R_2}$

(c) $\frac{R_2}{R_1}$ (d) $\left(\frac{R_1}{R_2}\right)^2$

(2005)

18. A smooth block is released at rest on a 45° incline and then slides a distance d. The time taken to slide is n times as much to slide on rough incline than on a smooth incline. The coefficient of friction is

(a) $\mu_s = 1 - \frac{1}{n^2}$ (b) $\mu_s = \sqrt{1 - \frac{1}{n^2}}$

(c) $\mu_k = 1 - \frac{1}{n^2}$ (d) $\mu_k = \sqrt{1 - \frac{1}{n^2}}$

(2005)

19. The upper half of an inclined plane with inclination ϕ is perfectly smooth while the lower half is rough. A body starting from rest at the top will again come

to rest at the bottom if the coefficient of friction for the lower half is given by

(a) $2\tan\phi$ (b) $\tan\phi$ (c) $2\sin\phi$ (d) $2\cos\phi$

(2005)

20. A bullet fired into a fixed target loses half its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest assuming that it faces constant resistance to motion?

(a) 1.5 cm (b) 1.0 cm (c) 3.0 cm (d) 2.0 cm

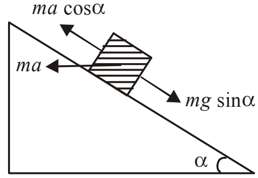
(2005)

21. A particle of mass 0.3 kg is subjected to a force $F = -kx$ with $k = 15$ N/m. What will be its initial acceleration if it is released from a point 20 cm away from the origin?

(a) 5 m/s^2 (b) 10 m/s^2 (c) 3 m/s^2 (d) 15 m/s^2

(2005)

22. A block is kept on a frictionless inclined surface with angle of inclination α. The incline is given an acceleration a to keep the block stationary. Then a is equal to



(a) g

(b) $g\tan\alpha$

(c) $g/\tan\alpha$ (d) $g\,\mathrm{cosec}\,\alpha$

(2005)

23. Consider a car moving on a straight road with a speed of 100 m/s. The distance at which car can be stopped is [$\mu_k = 0.5$]

(a) 100 m (b) 400 m

(c) 800 m (d) 1000 m

(2005)

24. A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching process is completed in 0.1 s, the force of the blow exerted by the ball on the hand of the player is equal to

(a) 300 N (b) 150 N (c) 3 N (d) 30 N.

(2006)

25. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m which applying the force and the ball goes upto 2 m height further, find the magnitude of the force. Consider $g = 10$ m/s^2

(a) 22 N (b) 4 N (c) 16 N (d) 20 N.

(2006)

26. A block of mass m is connected to another block of mass M by a spring (massless) of spring constant k. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then a constant force F starts acting on the block of mass M to pull it. Find the force of the block of mass m.

(a) $\dfrac{MF}{(m+M)}$ (b) $\dfrac{mF}{M}$

(c) $\dfrac{(M+m)F}{m}$ (d) $\dfrac{mF}{(m+M)}$

(2007)

27. A body of mass $m = 3.513$ kg is moving along the x-axis with a speed of 5.00 ms^{-1}. The magnitude of its momentum is recorded as

(a) 17.57 kg ms^{-1} (b) 17.6 kg ms^{-1}

(c) 17.565 kg ms^{-1} (d) 17.56 kg ms^{-1}.

(2008)

Answer Key

1.	(b)	**2.**	(c)	**3.**	(a)	**4.**	(b)	**5.**	(b)	**6.**	(b)
7.	(d)	**8.**	(a)	**9.**	(d)	**10.**	(c)	**11.**	(d)	**12.**	(a)
13.	(a)	**14.**	(d)	**15.**	(a)	**16.**	(a)	**17.**	(b)	**18.**	(c)
19.	(a)	**20.**	(b)	**21.**	(b)	**22.**	(b)	**23.**	(d)	**24.**	(d)
25.	(d)	**26.**	(d)	**27.**	(a)						

1. **(b) :** For no skidding along curved track, $v = \sqrt{\mu R g}$

$\therefore \quad v = \sqrt{0.6 \times 150 \times 10} = 30 \dfrac{\text{m}}{\text{s}}$.

2. **(c) :** For observer in the lift, acceleration = $(g - a)$
For observer standing outside, acceleration = g.

3. **(a) :** F_2 and F_3 have a resultant equivalent to F_1

$\therefore \quad$ Acceleration $= \dfrac{F_1}{m}$.

4. **(b) :** $T - 60g = 60a$
or $960 - (60 \times 10) = 60a$
or $60a = 360$
or $a = 6 \text{ ms}^{-2}$.

5. **(b) :** $\dfrac{a}{g} = \dfrac{(m_1 - m_2)}{(m_1 + m_2)}$

$\therefore \quad \dfrac{1}{8} = \dfrac{(m_1 - m_2)}{(m_1 + m_2)}$

or $\quad \dfrac{m_1}{m_2} = \dfrac{9}{7}$.

6. **(b) :** $\because$ Force = mass × acceleration
$\therefore \quad F - T_{AB} = ma$
and $T_{AB} - T_{BC} = ma$
$\therefore \quad T_{BC} = F - 2\,ma$
or $\quad T_{BC} = 10.2 - (2 \times 2 \times 0.6)$
or $\quad T_{BC} = 7.8$ N.

7. **(d) :** By triangle of forces, the particle will be in equilibrium under the three forces. Obviously the resultant force on the particle will be zero. Consequently the acceleration will be zero. Hence the particle velocity remains unchanged at $\vec{v}$.

8. **(a) :** When lift is standing, $W_1 = mg$
When the lift descends with acceleration a, $W_2 = m\,(g - a)$

$\therefore \quad \dfrac{W_2}{W_1} = \dfrac{m(g-a)}{mg} = \dfrac{9.8 - 5}{9.8} = \dfrac{4.8}{9.8}$

or $\quad W_2 = W_1 \times \dfrac{4.8}{9.8} = \dfrac{49 \times 4.8}{9.8} = 24$ N.

9. **(d) :** Weight of the block is balanced by force of friction
$\therefore$ Weight of the block $= \mu R = 0.2 \times 10 = 2$ N.

10. **(c) :** Frictional force provides the retarding force
$\therefore \quad \mu\, mg = ma$

or $\quad \mu = \dfrac{a}{g} = \dfrac{u/t}{g} = \dfrac{6/10}{10} = 0.06$.

11. **(d) :** Acceleration of block $(a) = \dfrac{\text{Force applied}}{\text{Total mass}}$

or $\quad a = \dfrac{P}{(M+m)}$

$\therefore \quad$ Force on block
$= \text{Mass of block} \times a = \dfrac{MP}{(M+m)}$.

12. **(a) :** Both the scales read M kg each.

13. **(a) :** Initial thrust = (Lift - off mass) × acceleration
$= (3.5 \times 10^4) \times (10) = 3.5 \times 10^5$ N.

14. **(d) :** Suppose he can fire n bullets per second
$\therefore \quad$ Force = Change in momentum per second

$144 = n \times \left(\dfrac{40}{1000}\right) \times (1200)$

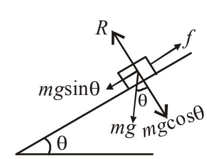
or $\quad n = \dfrac{144 \times 1000}{40 \times 1200}$

or $\quad n = 3$.

15. **(a) :** $\dfrac{a}{g} = \dfrac{(m_1 - m_2)}{(m_1 + m_2)} = \dfrac{(5 - 4.8)}{(5 + 4.8)} = \dfrac{0.2}{9.8}$

or $\quad a = g \times \dfrac{0.2}{9.8} = \dfrac{9.8 \times 0.2}{9.8} = 0.2 \text{ ms}^{-2}$.

16. **(a) :** For equilibrium of block,



$f = mg\sin\theta$
$\therefore \quad 10 = m \times 10 \times \sin 30°$
or $\quad m = 2$ kg.

17. **(b) :** Centripetal force on particle $= mR\omega^2$

$\therefore \quad \frac{F_1}{F_2} = \frac{mR_1\omega^2}{mR_2\omega^2} = \frac{R_1}{R_2}$.

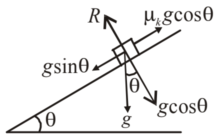
18. **(c) :** Component of g down the plane = $g\sin\theta$

$\therefore$ For smooth plane,

$d = \frac{1}{2}(g\sin\theta)t^2$ .... (i)

For rough plane,

Frictional retardation up the plane = μ_k ($g\cos\theta$)



$\therefore \quad d = \frac{1}{2}(g\sin\theta - \mu_k g\cos\theta)(nt)^2$

$\therefore \quad \frac{1}{2}(g\sin\theta)t^2 = \frac{1}{2}(g\sin\theta - \mu_k g\cos\theta)n^2t^2$

or $\quad \sin\theta = n^2 (\sin\theta - \mu_k\cos\theta)$

Putting $\theta = 45°$

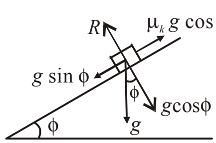
or $\quad \sin 45° = n^2 (\sin 45° - \mu_k \cos 45°)$

or $\quad \frac{1}{\sqrt{2}} = \frac{n^2}{\sqrt{2}}(1-\mu_k)$

or $\quad \mu_k = 1 - \frac{1}{n^2}$.

19. **(a) :** For upper half smooth incline, component of g down the incline = $g\sin\phi$

$\therefore \quad v^2 = 2(g\sin\phi)\frac{l}{2}$



For lower half rough incline, frictional retardation = $\mu_k g\cos\phi$

$\therefore$ Resultant acceleration = $g\sin\phi - \mu_k g\cos\phi$

$\therefore \quad 0 = v^2 + 2\,(g\sin\phi - \mu_k g\cos\phi)\frac{l}{2}$

or $\quad 0 = 2(g\sin\phi)\frac{l}{2} + 2g(\sin\phi - \mu_k\cos\phi)\frac{l}{2}$

or $\quad 0 = \sin\phi + \sin\phi - \mu_k\cos\phi$

or $\quad \mu_k\cos\phi = 2\sin\phi$

or $\quad \mu_k = 2\tan\phi$.

20. **(b) :** For first part of penetration, by equation of motion,

$$\left(\frac{u}{2}\right)^2 = (u)^2 - 2f(3)$$

or $\quad 3u^2 = 24f$ .... (i)

For latter part of penetration,

$$0 = \left(\frac{u}{2}\right)^2 - 2fx$$

or $\quad u^2 = 8fx$ .... (ii)

From (i) and (ii)

$3 \times (8fx) = 24f$

or $\quad x = 1$ cm.

21. **(b) :** $F = -kx$

or $\quad F = -15 \times \left(\frac{20}{100}\right) = -3$ N

Initial acceleration is over come by retarding force.

or $\quad m \times$ (acceleration a) = 3

or $\quad a = \frac{3}{m} = \frac{3}{0.3} = 10 \text{ ms}^{-2}$.

22. **(b) :** The incline is given an acceleration a. Acceleration of the block is to the right. Pseudo acceleration a acts on block to the left. Equate resolved parts of a and g along incline.

$\therefore \quad ma\cos\alpha = mg\sin\alpha$

or $\quad a = g\tan\alpha$.

23. **(d) :** Retardation due to friction = μg

$\because \quad v^2 = u^2 + 2as$

$\therefore \quad 0 = (100)^2 - 2(\mu g)s$

or $\quad 2\,\mu gs = 100 \times 100$

or $\quad s = \frac{100 \times 100}{2 \times 0.5 \times 10} = 1000$ m.

24. **(d) :** Force × time = Impulse = Change of momentum

$\therefore \quad \text{Force} = \frac{\text{Impulse}}{\text{time}} = \frac{3}{0.1} = 30$ N.

25. **(d) :** Work done by hand = Potential energy of the ball

$\therefore \quad FS = mgh \Rightarrow F = \frac{mgh}{s} = \frac{0.2 \times 10 \times 2}{0.2} = 20$ N.

26. (d) : Acceleration of the system $a = \frac{F}{m+M}$

Force on block of mass $m = ma = \frac{mF}{m+M}$.

27. **(a) :** Momentum is mv.

$m = 3.513$ kg ; $v = 5.00$ m/s

$\therefore \quad mv = 17.57 \text{ m s}^{-1}$

Because the values will be accurate up to second decimel place only, 17.565 = 17.57.

CHAPTER 5

Work, Energy and Power

1. A ball whose kinetic energy is E, is projected at an angle of 45° to the horizontal. The kinetic energy of the ball at the highest point of its flight will be
(a) E (b) $E/\sqrt{2}$ (c) $E/2$ (d) zero.
(2002)

2. If mass-energy equivalence is taken into account, when water is cooled to form ice, the mass of water should
(a) increase (b) remain unchanged
(c) decrease
(d) first increase then decrease.
(2002)

3. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is
(a) 16 J (b) 8 J
(c) 32 J (d) 24 J.
(2002)

4. Consider the following two statements.
A. Linear momentum of a system of particles is zero.
B. Kinetic energy of a system of particles is zero.
Then
(a) A does not imply B and B does not imply A
(b) A implies B but B does not imply A
(c) A does not imply B but B implies A
(d) A implies B and B implies A.
(2003)

5. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to
(a) $t^{3/4}$ (b) $t^{3/2}$
(c) $t^{1/4}$ (d) $t^{1/2}$.
(2003)

6. A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is
(a) 12.50 N-m (b) 18.75 N-m
(c) 25.00 N-m (d) 6.25 N-m.
(2003)

7. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to
(a) x^2 (b) e^x
(c) x (d) $\log_e x$.
(2004)

8. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle, the motion of the particle takes place in a plane. It follows that
(a) its velocity is constant
(b) its acceleration is constant
(c) its kinetic energy is constant
(d) it moves in a straight line.
(2004)

9. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table?
(a) 7.2 J (b) 3.6 J
(c) 120 J (d) 1200 J.
(2004)

10. A force $\vec{F} = (5\hat{i} + 3\hat{j} + 2\hat{k})$ N is applied over a particle which displaces it from its origin to the point $\vec{r} = (2\hat{i} - \hat{j})$ m. The work done on the particle in joule is

(a) –7 (b) +7
(c) +10 (d) +13.

(2004)

11. A body of mass m, accelerates uniformly from rest to v_1 in time t_1. The instantaneous power delivered to the body as a function of time t is

(a) $\frac{mv_1 t}{t_1}$ (b) $\frac{mv_1^2 t}{t_1^2}$

(c) $\frac{mv_1 t^2}{t_1}$ (d) $\frac{mv_1^2 t}{t_1}$.

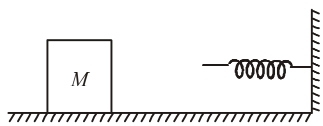
(2004)

12. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is

(a) 10 m/s (b) 34 m/s (c) 40 m/s (d) 20 m/s

(2005)

13. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant K and compresses it by length L. The maximum momentum of the block after collision is

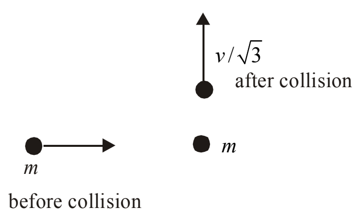


(a) zero (b) $\frac{ML^2}{K}$

(c) $\sqrt{MK}\,L$ (d) $\frac{KL^2}{2M}$

(2005)

14. A mass m moves with a velocity v and collides inelastically with another identical mass. After collision the first mass moves with velocity in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision



(a) $\frac{2}{\sqrt{3}}v$ (b) $\frac{v}{\sqrt{3}}$

(c) v (d) $\sqrt{3}v$

(2005)

15. A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time is given by

(a) $\frac{1}{2}\frac{mv^2}{T^2}t$ (b) $\frac{1}{2}\frac{mv^2}{T^2}t^2$

(c) $\frac{mv^2}{T^2}\cdot t$ (d) $\frac{mv^2}{T^2}\cdot t^2$

(2005)

16. A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string making an angle of 45° with the initial vertical direction is

(a) $Mg(\sqrt{2}-1)$ (b) $Mg(\sqrt{2}+1)$

(c) $Mg\sqrt{2}$ (d) $\frac{Mg}{\sqrt{2}}$.

(2006)

17. A bomb of mass 16 kg at rest explodes into two pieces of masses of 4 kg and 12 kg. The velocity of the 12 kg mass is 4 ms^{-1}. The kinetic energy of the other mass is

(a) 96 J (b) 144 J
(c) 288 J (d) 192 J.

(2006)

18. A particle of mass 100 g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is

(a) 0.5 J (b) –0.5 J
(c) –1.25 J (d) 1.25 J.

(2006)

19. The potential energy of a 1 kg particle free to move along the x-axis is given by

$$V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right) \text{J}.$$

The total mechanical energy of the particle 2 J. Then, the maximum speed (in m/s) is

(a) 2 (b) $3/\sqrt{2}$

(c) $\sqrt{2}$ (d) $1/\sqrt{2}$.

(2006)

20. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes a uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10,000 N/m. The spring compresses by

(a) 8.5 cm (b) 5.5 cm
(c) 2.5 cm (d) 11.0 cm

(2007)

21. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is

(a) $K/2$ (b) K (c) zero (d) $K/4$

(2007)

22. An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range

(a) 2,000 J - 5,000 J (b) 200 J - 500 J
(c) 2×10^5 J - 3×10^5 J
(d) 20,000 J - 50,000 J.

23. A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is

(a) 0.34 J (b) 0.16 J
(c) 1.00 J (d) 0.67 J.

(2008)

Answer Key

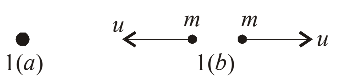
1.	(c)	**2.**	(c)	**3.**	(b)	**4.**	(c)	**5.**	(b)	**6.**	(b)
7.	(a)	**8.**	(c)	**9.**	(b)	**10.**	(b)	**11.**	(b)	**12.**	(b)
13.	(c)	**14.**	(a)	**15.**	(c)	**16.**	(a)	**17.**	(c)	**18.**	(c)
19.	(b)	**20.**	(b)	**21.**	(d)	**22.**	(a)	**23.**	(d)		

1. **(c) :** Kinetic energy point of projection $(E)=\frac{1}{2}mu^2$
At highest point velocity = $u\cos\theta$
$\therefore$ Kinetic energy at highest point
$$=\frac{1}{2}m(u\cos\theta)^2$$
$$=\frac{1}{2}mu^2\cos^2 45°$$
$$=\frac{E}{2}.$$

2. **(c) :** When water is cooled to form ice, its thermal energy decreases. By mass energy equivalent, mass should decrease.

3. **(b) :** $W=\int_{x_1}^{x_2}Fdx=\int_{0.05}^{0.15}kx\,dx$
$$\therefore\quad W=\int_{0.05}^{0.15}800x dx=\frac{800}{2}\left[x^2\right]_{0.05}^{0.15}$$
$$=400\left[(0.15)^2-(0.05)^2\right]$$
or $W = 8$ J.

4. **(c) :** A system of particles implies that one is discussing total momentum and total energy.



1(*a*) explodes
Total momentum = 0
But total kinetic energy = $2\left(\frac{1}{2}\right)mu^2$
But if total kinetic energy = 0, velocities are zero.
Here A is true, but B is not true.
A does not imply B, but B implies A.

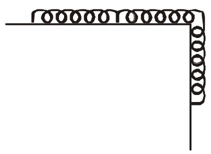
5. **(b) :** Power =
$$\frac{\text{Work}}{\text{Time}}=\frac{\text{Force}\times\text{distance}}{\text{Time}}=\text{Force}\times\text{velocity}$$
$\therefore$ Force × velocity = constant (K)
or $(ma)\,(at)=K$
or $a=\left(\frac{K}{mt}\right)^{1/2}$
$\because\quad s=\frac{1}{2}at^2$
$$\therefore\quad s=\frac{1}{2}\left(\frac{K}{mt}\right)^{1/2}t^2=\frac{1}{2}\left(\frac{K}{m}\right)^{1/2}t^{3/2}$$
or s is proportional to $t^{3/2}$.

6. **(b) :** Force constant of spring $(k)=F/x$
or $F=kx$
$\therefore$ $dW=kxdx$
$$\text{or}\quad \int dW=\int_{0.05}^{0.1}kx\,dx=\frac{k}{2}\left[(0.1)^2-(0.05)^2\right]$$
$$=\frac{k}{2}\times[0.01-0.0025]$$
or Workdone
$$=\frac{(5\times10^3)}{2}\times(0.0075)=18.75\text{ N-m}.$$

7. **(a) :** Given : Retardation $\propto$ displacement
or $\frac{dv}{dt}=kx$
or $\left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right)=kx$
or $dv\,(v)=kx\,dx$
or $\int_{v_1}^{v_2}vdv=k\int_0^x x\,dx$
or $\frac{v_2^2}{2}-\frac{v_1^2}{2}=\frac{kx^2}{2}$
or $\frac{mv_2^2}{2}-\frac{mv_1^2}{2}=\frac{mkx^2}{2}$
or $(K_2-K_1)=\frac{mk}{2}x^2$
or Loss of kinetic energy is proportional to x^2.

8. **(c) :** No work is done when a force of constant magnitude always acts at right angles to the velocity of a particle when the motion of the particle takes place in a plane.
Hence kinetic energy of the particle remains constant.

9. **(b) :** The centre of mass of the hanging part is at 0.3 m from table



mass of hanging part $= \frac{4 \times 0.6}{2} = 1.2$ kg

$\therefore \quad W = mgh$

$= 1.2 \times 10 \times 0.3$

$= 3.6$ J.

10. **(b) :** Work done $= \vec{F} \cdot \vec{r}$

or work done $= (5\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (2\hat{i} - \hat{j})$

or work done $= 10 - 3 = 7$ J.

11. **(b) :** Acceleration $a = \frac{v_1}{t_1}$

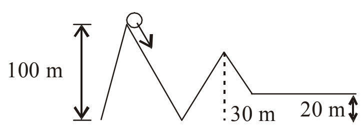
$\therefore$ velocity (v) $= 0 + at = \frac{v_1}{t_1}t$

$\therefore$ Power P = Force × velocity = $m\,a\,v$

or $P = m\left(\frac{v_1}{t_1}\right) \times \left(\frac{v_1 t}{t_1}\right) = \frac{mv_1^2 t}{t_1^2}$.

12. **(b) :** $mgh = \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right)$

$= \frac{1}{2}mv^2 \cdot \frac{7}{5}$



$\therefore \frac{1}{2}mv^2 \left(\frac{7}{5}\right) = mg \times 80$

or $v^2 = 2 \times 10 \times 80 \times \frac{5}{7} = 1600 \times \frac{5}{7}$

or $v = 34$ m/s.

13. **(c) :** Elastic energy stored in spring $= \frac{1}{2}KL^2$

$\therefore$ kinetic energy of block $E = \frac{1}{2}KL^2$

Since $p^2 = 2ME$

$\therefore \quad p = \sqrt{2ME} = \sqrt{\frac{2M \times KL^2}{2}} = \sqrt{MK}\,L$.

14. **(a) :** Let v_1 = speed of second mass after collision

Momentum is conserved

Along X-axis, $mv_1\cos\theta = mv$ ....(i)

Along Y-axis, $mv_1\sin\theta = \frac{mv}{\sqrt{3}}$ .... (ii)

From (i) and (ii)

$\therefore \quad (mv_1\cos\theta)^2 + (mv_1\sin\theta)^2 = (mv)^2 + \left(\frac{mv}{\sqrt{3}}\right)^2$

or $m^2v_1^2 = \frac{4m^2v^2}{3}$

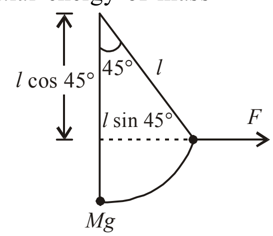
or $v_1 = \frac{2}{\sqrt{3}}v$.

15. **(c) :** Power = Force × velocity

$= (ma)\,(v) = (ma)\,(at) = ma^2t$

or Power $= m\left(\frac{v}{T}\right)^2(t) = \frac{mv^2}{T^2}t$

16. **(a) :** Work done in displacement is equal to gain in potential energy of mass



Work done $= F \times l\sin 45° = \frac{Fl}{\sqrt{2}}$

Gain in potential energy = $Mg(l - l\cos45°)$

$= Mgl\left(1 - \frac{1}{\sqrt{2}}\right)$

$\therefore \quad \frac{Fl}{\sqrt{2}} = \frac{Mgl(\sqrt{2} - 1)}{\sqrt{2}}$

or $F = Mg(\sqrt{2} - 1)$.

17. **(c) :** Linear momentum is conserved

$\therefore \quad 0 = m_1v_1 + m_2v_2 = (12 \times 4) + (4 \times v_2)$

or $4v_2 = -48 \Rightarrow v_2 = -12$ m/s

$\therefore$ Kinetic energy of mass $m_2 = \frac{1}{2}m_2v_2^2$

$= \frac{1}{2} \times 4 \times (-12)^2 = 288$ J.

18. **(c) :** Kinetic energy at projection point is converted into potential energy of the particle during rise. Potential energy measures the workdone against the force of gravity during rise.

$\therefore$ (– work done) = Kinetic energy $= \frac{1}{2}mv^2$

or (– work done)

$$= \frac{1}{2} \times \left(\frac{100}{1000}\right)(5)^2 = \frac{5 \times 5}{2 \times 10} = 1.25 \text{ J}$$

$\therefore$ Work done by force of gravity = – 1.25.

19. **(b) :** Total energy $E_T = 2$ J. It is fixed.
For maximum speed, kinetic energy is maximum
The potential energy should therefore be minimum.

$\because \quad V(x) = \frac{x^4}{4} - \frac{x^2}{2}$

or $\quad \frac{dV}{dx} = \frac{4x^3}{4} - \frac{2x}{2} = x^3 - x = x(x^2 - 1)$

For V to be minimum, $\frac{dV}{dx} = 0$

$\therefore \quad x(x^2 - 1) = 0$, or $\quad x = 0, \pm 1$

At $\quad x = 0$, $V(x) = 0$

At $\quad x = \pm 1$, $V(x) = -\frac{1}{4}$ J

$\therefore \quad$ (Kinetic energy)$_{max}$ = $E_T - V_{min}$

or $\quad$ (Kinetic energy)$_{max}$ $= 2 - \left(-\frac{1}{4}\right) = \frac{9}{4}$ J

or $\quad \frac{1}{2} m v_m^2 = \frac{9}{4}$

or $\quad v_m^2 = \frac{9 \times 2}{m \times 4} = \frac{9 \times 2}{1 \times 4} = \frac{9}{2}$

$\therefore \quad v_m = \frac{3}{\sqrt{2}}$ m/s.

20. **(b) :** Let the spring be compressed by x
Initial kinetic energy of the mass = potential energy of the spring + work done due to friction

$$\frac{1}{2} \times 2 \times 4^2 = \frac{1}{2} \times 10000 \times x^2 + 15x$$

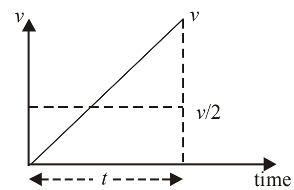
or $\quad 5000\, x^2 + 15x - 16 = 0$

or $\quad x = 0.055$ m = 5.5 cm.

21. **(d) :** The kinetic energy of a particle is K
At highest point velocity has its horizontal component. Therefore kinetic energy of a particle at highest point is

$$K_H = K\cos^2\theta = K\cos^2 60° = \frac{K}{4}.$$

22. **(a) :** $\bar{v} = v/2$ is average velocity
$s = 100$ m, $t = 10$ s. $\quad \therefore$ $(v/2) = 10$ m/s.
$v_{average} = (v/2) = 10$ m/s.



Assuming an athelete has about 50 to 100 kg, his kinetic energy would have been $\frac{1}{2} m v_{av}^2$.
$(1/2) m v_{av}^2 = (1/2) \times 50 \times 100 = 2500$ J.
For 100 kg, $(1/2) \times 100 \times 100 = 5000$ J.
It could be in the range 2000 to 5000 J.

23. **(d) :** By the law of conservation of momentum
$mu = (M + m)v$

$0.50 \times 2.00 = (1 + 0.50)\, v$, $\quad \frac{1.00}{1.50} = v$

Initial K.E. = $(1/2) \times 0.50 \times (2.00)^2 = 1.00$ J.

$$\text{Final K.E.} = \frac{1}{2} \times 1.50 \times \frac{1.00^2}{(1.50)^2} = \frac{1.00}{3.00} = 0.33$$

$\therefore$ Loss of energy = 1.00 – 0.33 = 0.67 J.

CHAPTER
6

Rotational Motion and Moment of Inertia

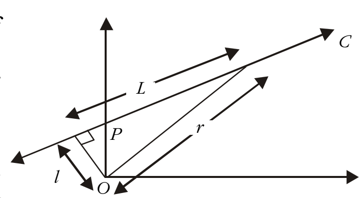
1. Two identical particles move towards each other with velocity $2v$ and v respectively. The velocity of centre of mass is
(a) v (b) $v/3$ (c) $v/2$ (d) zero.
(2002)

2. Initial angular velocity of a circular disc of mass M is ω_1. Then two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?
(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$
(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$.
(2002)

3. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then maximum acceleration down the plane is for (no rolling)
(a) solid sphere (b) hollow sphere
(c) ring (d) all same.
(2002)

4. Moment of inertia of a circular wire of mass M and radius R about its diameter is
(a) $MR^2/2$ (b) MR^2 (c) $2MR^2$ (d) $MR^2/4$.
(2002)

5. A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about P ?



(a) mvL (b) mvl (c) mvr (d) zero.
(2002)

6. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius $4R$ is made from an iron plate of thickness $t/4$. Then the relation between the moment of inertia I_X and I_Y is
(a) $I_Y = 32I_X$ (b) $I_Y = 16I_X$
(c) $I_Y = I_X$ (d) $I_Y = 64I_X$.
(2003)

7. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is
(a) $L/4$ (b) $2L$ (c) $4L$ (d) $L/2$.
(2003)

8. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$ and $\vec{T}$ be the torque of this force about the origin. Then
(a) $\vec{r}\cdot\vec{T}=0$ and $\vec{F}\cdot\vec{T}\neq 0$
(b) $\vec{r}\cdot\vec{T}\neq 0$ and $\vec{F}\cdot\vec{T}=0$
(c) $\vec{r}\cdot\vec{T}\neq 0$ and $\vec{F}\cdot\vec{T}\neq 0$
(d) $\vec{r}\cdot\vec{T}=0$ and $\vec{F}\cdot\vec{T}=0$
(2003)

9. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected?
(a) moment of inertia
(b) angular momentum
(c) angular velocity
(d) rotational kinetic energy.
(2004)

10. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that

(a) $I_A = I_B$ (b) $I_A > I_B$

(c) $I_A < I_B$ (d) $I_A/I_B = d_A/d_B$

where d_A and d_B are their densities.

(2004)

11. The moment of inertia of a uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the center is

(a) Mr^2 (b) $\frac{1}{2}Mr^2$

(c) $\frac{1}{4}Mr^2$ (d) $\frac{2}{5}Mr^2$

(2005)

12. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}M$ and body C of mass $\frac{2}{3}M$. The center of mass of bodies B and C taken together shifts compared to that of body A towards

(a) body C

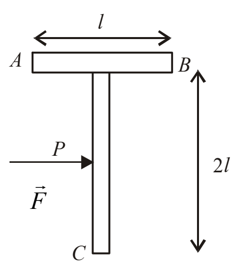
(b) body B

(c) depends on height of breaking

(d) does not shift

(2005)

13. A T shaped object with dimensions shown in the figure, is lying on a smooth floor. A force $\vec{F}$ is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C.



(a) $\frac{4}{3}l$ (b) l

(c) $\frac{3}{4}l$ (d) $\frac{3}{2}l$

(2005)

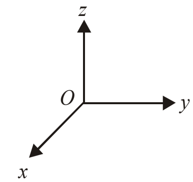
14. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved, so as to keep the centre of mass at the same position?

(a) d (b) $\frac{m_2}{m_1}d$

(c) $\frac{m_1}{m_1+m_2}d$ (d) $\frac{m_1}{m_2}d$.

(2006)

15. A force of $-F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point $(1, -1)$ is



(a) $-F(\hat{i}-\hat{j})$

(b) $F(\hat{i}-\hat{j})$

(c) $-F(\hat{i}+\hat{j})$

(d) $F(\hat{i}+\hat{j})$.

(2006)

16. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity $\omega' =$

(a) $\frac{\omega m}{(m+2M)}$ (b) $\frac{\omega(m+2M)}{m}$

(c) $\frac{\omega(m-2M)}{(m+2M)}$ (d) $\frac{\omega m}{(m+M)}$.

(2006)

17. Four point masses, each of value m, are placed at the corners of a square $ABCD$ of side l. The moment of inertia of this system about an axis through A and parallel to BD is

(a) ml^2 (b) $2ml^2$

(c) $\sqrt{3}\,ml^2$ (d) $3ml^2$.

(2006)

18. A circular disc of radius R is removed from a bigger circular disc of radius $2R$ such that the circumferences of the discs coincide. The centre of mass of the new disc is α/R from the centre of the bigger disc. The value of α is

(a) 1/4 (b) 1/3

(c) 1/2 (d) 1/6.

(2007)

19. A round uniform body of radius R, mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is

(a) $\dfrac{g\sin\theta}{1-MR^2/I}$ (b) $\dfrac{g\sin\theta}{1+I/MR^2}$

(c) $\dfrac{g\sin\theta}{1+MR^2/I}$ (d) $\dfrac{g\sin\theta}{1-I/MR^2}$

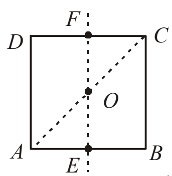
(2007)

20. Angular momentum of the particle rotating with a central force is constant due to

(a) constant torque
(b) constant force
(c) constant linear momentum
(d) zero torque

(2007)

21. For the given uniform square lamina $ABCD$, whose centre is O,



(a) $I_{AC}=\sqrt{2}I_{EF}$
(b) $\sqrt{2}I_{AC}=I_{EF}$
(c) $I_{AD}=3I_{EF}$
(d) $I_{AC}=I_{EF}$

(2007)

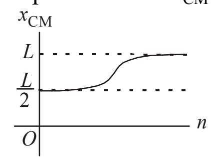
22. Consider a uniform square plate of side a and mass m. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

(a) $\dfrac{2}{3}ma^2$ (b) $\dfrac{5}{6}ma^2$

(c) $\dfrac{1}{12}ma^2$ (d) $\dfrac{7}{12}ma^2$

(2008)

23. A thin rod of length L is lying along the x-axis with its ends at $x = 0$ and $x = L$. Its linear density (mass/length) varies with x as $k(x/L)^n$ where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against n, which of the following graphs best approximates the dependence of x_{CM} on n?

(a)


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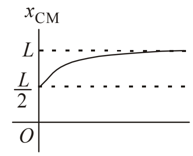

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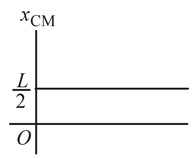
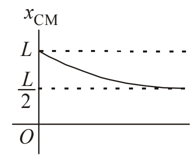


(d)


(2008)

Answer Key

1.	(c)	**2.**	(c)	**3.**	(d)	**4.**	(a)	**5.**	(d)	**6.**	(d)
7.	(a)	**8.**	(d)	**9.**	(b)	**10.**	(c)	**11.**	(b)	**12.**	(d)
13.	(a)	**14.**	(d)	**15.**	(d)	**16.**	(a)	**17.**	(d)	**18.**	
19.	(b)	**20.**	(d)	**21.**	(d)	**22.**	(a)	**23.**	(b)		

1. **(c) :** $v_c = \dfrac{m_1v_1 + m_2v_2}{m_1 + m_2}$

or $v_c = \dfrac{m(2v) + m(-v)}{m+m} = \dfrac{v}{2}$.

2. **(c) :** Angular momentum of the system is conserved

$\therefore \quad \frac{1}{2}MR^2\ \omega_1 = 2mR^2\omega + \frac{1}{2}MR^2\omega$

or $M\omega_1 = (4m + M)\omega$

or $\omega = \dfrac{M\omega_1}{M+4m}$.

3. **(d) :** The bodies slide along inclined plane. They do not roll. Acceleration for each body down the plane = $g\sin\theta$. It is the same for each body.

4. **(a) :** A circular wire behaves like a ring

M.I. about its diameter $= \dfrac{MR^2}{2}$.

5. **(d) :** The particle moves with linear velocity v along line PC. The line of motion is through P. Hence angular momentum is zero.

6. **(d) :** Mass of disc $X = (\pi R^2 t)\sigma$ where σ = density

$\therefore \quad I_X = \dfrac{MR^2}{2} = \dfrac{(\pi R^2 t\sigma)R^2}{2} = \dfrac{\pi R^4 \sigma t}{2}$

Similarly, $I_Y = \dfrac{(\text{Mass})(4R)^2}{2} = \dfrac{\pi(4R)^2}{2}\dfrac{t}{4}\sigma \times 16R^2$

or $I_Y = 32\pi R^4 t\sigma$

$\therefore \quad \dfrac{I_X}{I_Y} = \dfrac{\pi R^4 \sigma t}{2} \times \dfrac{1}{32\pi R^4 \sigma t} = \dfrac{1}{64}$

$\therefore \quad I_Y = 64\ I_X$.

7. **(a) :** Angular momentum $L = I\omega$

Rotational kinetic energy $(K) = \frac{1}{2}I\omega^2$

$\therefore \quad \dfrac{L}{K} = \dfrac{I\omega \times 2}{I\omega^2} = \dfrac{2}{\omega} \Rightarrow L = \dfrac{2K}{\omega}$

or $\dfrac{L_1}{L_2} = \dfrac{K_1}{K_2} \times \dfrac{\omega_2}{\omega_1} = 2 \times 2 = 4$

$\therefore \quad L_2 = \dfrac{L_1}{4} = \dfrac{L}{4}$.

8. **(d) :** $\because \quad \vec{T} = \vec{r} \times \vec{F}$

$\therefore \quad \vec{r} \cdot \vec{T} = \vec{r} \cdot (\vec{r} \times \vec{F}) = 0$

Also $\vec{F} \cdot \vec{T} = \vec{F} \cdot (\vec{r} \times \vec{F}) = 0$.

9. **(b) :** Free space implies that no external torque is operating on the sphere. Internal changes are responsible for increase in radius of sphere. Here the law of conservation of angular momentum applies to the system.

10. **(c) :** For solid sphere, $I_A = \frac{2}{5}MR^2$

For hollow sphere, $I_B = \frac{2}{3}MR^2$

$\therefore \quad \dfrac{I_A}{I_B} = \dfrac{2MR^2}{5} \times \dfrac{3}{2MR^2} = \dfrac{3}{5}$

or $I_A < I_B$.

11. **(b) :** $I = \dfrac{(\text{Mass of semicircular disc}) \times r^2}{2}$

or $I = \dfrac{Mr^2}{2}$.

12. **(d) :** The centre of mass of bodies B and C taken together does not shift as no external force is applied horizontally.

13. **(a) :** It is a case of translation motion without rotation. The force should act at the centre of mass

$Y_{\text{cm}} = \dfrac{(m \times 2l) + (2m \times l)}{m + 2m} = \dfrac{4l}{3}$.

14. **(d) :** Let m_2 be moved by x so as to keep the centre of mass at the same position

$\therefore \quad m_1 d + m_2(-x) = 0$

or $m_1 d = m_2 x$ or $x = \dfrac{m_1}{m_2}d$.

15. **(d) :** Torque $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{F} = -F\hat{k},\ \vec{r} = \hat{i} - \hat{j}$

$$\therefore \quad \vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 0 & 0 & -F \end{vmatrix}$$

$$= \hat{i}F - \hat{j}(-F) = F(\hat{i}+\hat{j}).$$

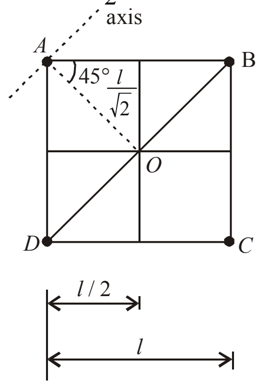
16. **(a) :** Angular momentum is conserved

$\therefore \quad L_1 = L_2$

$\therefore \quad mR^2\,\omega = (mR^2 + 2\,MR^2)\,\omega' = R^2\,(m + 2M)\,\omega'$

or $\quad \omega' = \dfrac{m\omega}{m+2M}.$

17. **(d) :** $AO\cos 45° = \dfrac{l}{2} \qquad \therefore \quad AO \times \dfrac{1}{\sqrt{2}} = \dfrac{l}{2}$



or $\quad AO = \dfrac{l}{\sqrt{2}}$

$I = I_D + I_B + I_C \quad$ or $\quad I = \dfrac{2ml^2}{2} + m\left(\dfrac{2l}{\sqrt{2}}\right)^2$

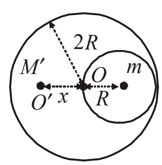
$$I = \frac{2ml^2}{2} + \frac{4ml^2}{2}$$

or $\quad I = \dfrac{6ml^2}{2} = 3ml^2.$

18. $(M' + m) = M = \pi\,(2R)^2\cdot\sigma$

where σ = mass per unit area

$m = \sigma R^2\cdot\sigma,\ M' = 3\pi R^2\sigma$



$$\frac{3\pi R^2\sigma\cdot x + \pi R^2\sigma\cdot R}{M} = 0$$

Because for the full disc, the centre of mass is at the centre O.

$\Rightarrow \quad x = -\dfrac{R}{3} \quad = \alpha R. \quad \therefore \ |\alpha| = \left|\dfrac{-1}{3}\right|.$

The centre of mass is at $R/3$ to the left on the diameter of the original disc.

The question should be at a distance αR and not α/R.

19. **(b) :** Acceleration of a uniform body of radius R and mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal is given by

$$a = \frac{g\sin\theta}{1+\dfrac{I}{MR^2}}.$$

20. **(d) :** Central forces passes through axis of rotation so torque is zero.

If no external torque is acting on a particle, the angular momentum of a particle is constant.

21. **(d) :** By perpendicular axes theorem,

$$I_{EF} = M\frac{a^2+b^2}{12} = \frac{M(a^2+a^2)}{12} = M\frac{2a^2}{12}$$

$$I_z = \frac{M(2a^2)}{12} + \frac{M(2a^2)}{12} = \frac{Ma^2}{3}.$$

By perpendicular axes theorem,

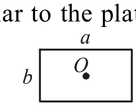
$$I_{AC} + I_{BD} = I_z \ \Rightarrow \ I_{AC} = \frac{I_z}{2} = \frac{Ma^2}{6}$$

By the same theorem $I_{EF} = \dfrac{I_z}{2} = \dfrac{Ma^2}{6}$

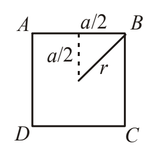
$\therefore \quad I_{AC} = I_{EF}.$

22. **(a) :** For a rectangular sheet moment of inertia passing through O, perpendicular to the plate is

$$I_0 = M\left(\frac{a^2+b^2}{12}\right)$$



for square plate it is $\dfrac{Ma^2}{6}.$



$$r = \sqrt{\frac{a^2}{4}+\frac{a^2}{4}} = \frac{a}{\sqrt{2}}. \quad \therefore \ r^2 = \frac{a^2}{2}$$

$\therefore \quad I$ about B parallel to the axis through O is

$$I_o + Md^2 = \frac{Ma^2}{6} + \frac{Ma^2}{2} = \frac{4Ma^2}{6}$$

$$I = \frac{2}{3}Ma^2$$

23. **(b) :** $x_{\text{C.M}} = \dfrac{\int_0^L \left(\dfrac{k}{L^n}\cdot x^n\cdot dx\right)x}{\int_0^L \dfrac{k}{L^n}\cdot x^n\cdot dx}$

$$\Rightarrow \quad x_{\text{C.M}} = \frac{\int_0^L x^{n+1} dx}{\int_0^L x^n dx} = \frac{L^{n+2}}{n+2} \cdot \frac{(n+1)}{L^{n+1}}$$

$$\Rightarrow \quad x_{\text{C.M}} = \frac{L(n+1)}{(n+2)}$$

The variation of the centre of mass with x is given by

$$\frac{dx}{dn} = L\left\{\frac{(n+2)1-(n+1)}{(n+2)^2}\right\} = \frac{L}{(n+2)^2}$$

If the rod has the same density as at $x = 0$ *i.e.*, $n = 0$, therefore uniform, the centre of mass would have been at $L/2$. As the density increases with length, the centre of mass shifts towards the right. Therefore it can only be (b).

CHAPTER

7

Gravitation

1. If suddenly the gravitational force of attraction between Earth and a satellite revolving around it becomes zero, then the satellite will
(a) continue to move in its orbit with same velocity
(b) move tangentially to the original orbit in the same velocity
(c) become stationary in its orbit
(d) move towards the earth.
(2002)

2. Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is
(a) $GMm/12R^2$ (b) $GMm/3R^2$
(c) $GMm/8R$ (d) $GMm/6R$.
(2002)

3. The kinetic energy needed to project a body of mass m from the earth surface (radius R) to infinity is
(a) $mgR/2$ (b) $2mgR$
(c) mgR (d) $mgR/4$.
(2002)

4. The escape velocity of a body depends upon mass as
(a) m^0 (b) m^1 (c) m^2 (d) m^3.
(2002)

5. The time period of a satellite of earth is 5 hour. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become
(a) 10 hour (b) 80 hour
(c) 40 hour (d) 20 hour.
(2003)

6. Two spherical bodies of mass M and $5M$ and radii R and $2R$ respectively are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is
(a) $2.5R$ (b) $4.5R$ (c) $7.5R$ (d) $1.5R$.
(2003)

7. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, the escape velocity will be
(a) $11\sqrt{2}$ km/s (b) 22 km/s
(c) 11 km/s (d) $11/\sqrt{2}$ m/s.
(2003)

8. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is
(a) gx (b) $\frac{gR}{R-x}$
(c) $\frac{gR^2}{R+x}$ (d) $\left(\frac{gR^2}{R+x}\right)^{1/2}$.
(2004)

9. The time period of an earth satellite in circular orbit is independent of
(a) the mass of the satellite
(b) radius of its orbit
(c) both the mass and radius of the orbit
(d) neither the mass of the satellite nor the radius of its orbit.
(2004)

10. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth is

(a) $2\ mgR$ (b) $\frac{1}{2}mgR$

(c) $\frac{1}{4}mgR$ (d) mgR.

(2004)

11. Suppose the gravitational force varies inversely as the n^{th} power of distance. Then the time period of a planet in circular orbit of radius R around the sun will be proportional to

(a) $R^{\left(\frac{n+1}{2}\right)}$ (b) $R^{\left(\frac{n-1}{2}\right)}$

(c) R^n (d) $R^{\left(\frac{n-2}{2}\right)}$.

(2004)

12. The change in the value of g at a height h above the surface of the earth is the same as at a depth d below the surface of earth. When both d and h are much smaller than the radius of earth, then which of the following is correct?

(a) $d = 2h$ (b) $d = h$

(c) $d = h/2$ (d) $d = 3h/2$

(2005)

13. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere.
(you may take $G = 6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$)

(a) 6.67×10^{-9} J (b) 6.67×10^{-10} J

(c) 13.34×10^{-10} J (d) 3.33×10^{-10} J

(2005)

14. Average density of the earth

(a) is directly proportional to g

(b) is inversely proportional to g

(c) does not depend on g

(d) is a complex function of g

(2005)

15. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be

(a) 0.11 km s^{-1} (b) 1.1 km s^{-1}

(c) 11 km s^{-1} (d) 110 km s^{-1}

(2008)

16. *Directions : The following question contains statement-1 and statement-2. Of the four choices given, choose the one that best describes the two statements.*

(a) Statement-1 is true, statement-2 is false.

(b) Statement-1 is false, statement-2 is true.

(c) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(d) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.

Statement-1 : For a mass M kept at the centre of a cube of side a, the flux of gravitational field passing through its sides is $4\pi GM$.

Statement-2 : If the direction of a field due to a point source is radial and its dependence on the distance r from the source is given as $1/r^2$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.

(2008)

Answer Key

1.	(b)	**2.**	(d)	**3.**	(c)	**4.**	(a)	**5.**	(c)	**6.**	(c)
7.	(c)	**8.**	(d)	**9.**	(a)	**10.**	(b)	**11.**	(a)	**12.**	(a)
13.	(b)	**14.**	(a)	**15.**	(d)	**16.**	(c)				

1. **(b) :** The centripetal and centrifugal forces disappear, the satellite has the tangential velocity and it will move in a straight line
Compare Lorentzian force on charges in the cyclotron.

2. **(d) :** Energy = $(P.E.)_{3R} - (P.E.)_{2R}$
$$= -\frac{GmM}{3R} - \left(-\frac{GmM}{2R}\right) = +\frac{GmM}{6R}.$$

3. **(c) :** Escape velocity $v_e = \sqrt{2gR}$
$\therefore$ Kinetic energy
$$= \frac{1}{2}mv_e^2 = \frac{1}{2}m \times 2gR = mgR.$$

4. **(a) :** Escape velocity $= \sqrt{2gR} = \sqrt{\frac{2GM_e}{R}}$
Escape velocity does not depend on mass of body which escapes or it depends on m^0.

5. **(c) :** According to Kepler's law $T^2 \propto r^3$
$$\therefore \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{4}\right)^3 = \frac{1}{64} \quad \text{or} \quad \frac{T_1}{T_2} = \frac{1}{8}$$
or $T_2 = 8T_1 = 8 \times 5 = 40$ hour.

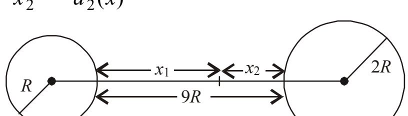
6. **(c) :** Let the spheres collide after time t, when the smaller sphere covered distance x_1 and bigger sphere covered distance x_2.
The gravitational force acting between two spheres depends on the distance which is a variable quantity.

The gravitational force, $F(x) = \dfrac{GM \times 5M}{(12R-x)^2}$

Acceleration of smaller body, $a_1(x) = \dfrac{G \times 5M}{(12R-x)^2}$

Acceleration of bigger body, $a_2(x) = \dfrac{GM}{(12R-x)^2}$

From equation of motion,
$$x_1 = \frac{1}{2}a_1(x)t^2 \text{ and } x_2 = \frac{1}{2}a_2(x)t^2$$
$$\Rightarrow \frac{x_1}{x_2} = \frac{a_1(x)}{a_2(x)} = 5 \Rightarrow x_1 = 5x_2$$



We know that $x_1 + x_2 = 9R$
$$x_1 + \frac{x_1}{5} = 9R \qquad \therefore x_1 = \frac{45R}{6} = 7.5R$$
Therefore the two spheres collide when the smaller sphere covered the distance of $7.5R$.

7. **(c) :** The escape velocity of a body does not depend on the angle of projection from earth.
It is 11 km/sec.

8. **(d) :** For a satellite
centripetal force = Gravitational force
$$\therefore \frac{mv_0^2}{(R+x)} = \frac{GMm}{(R+x)^2}$$
$$\text{or } v_0^2 = \frac{GM}{(R+x)} = \frac{gR^2}{(R+x)} \qquad \left[\because g = \frac{GM}{R^2}\right]$$
$$\text{or } v_0 = \sqrt{\frac{gR^2}{R+x}}.$$

9. **(a) :** For a satellite
Centripetal force = Gravitational force
$$\therefore mR\omega^2 = \frac{GmM_e}{R^2} \qquad \text{where } R = r_e + h$$
$$\text{or } \omega = \sqrt{\frac{GM_e}{R^3}} = \sqrt{\frac{GM_e}{(r_e+h)^3}}$$
$$\therefore T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{(r_e+h)^3}{GM_e}}$$
$\therefore$ T is independent of mass (m) of satellite.

10. **(b) :** Force on object $= \dfrac{GMm}{x^2}$ at x from centre of earth.
$$\therefore \text{Work done} = \frac{GMm}{x^2}dx$$
$$\therefore \int \text{Work done} = GMm\int_R^{2R}\frac{dx}{x^2}$$
$\therefore$ Potential energy gained
$$= GMm\left[-\frac{1}{x}\right]_R^{2R} = \frac{GMm \times 1}{2R}$$
$\therefore$ Gain in P.E.
$$= \frac{1}{2}mR\left(\frac{GM}{R^2}\right) = \frac{1}{2}mgR \qquad \left[\because g = \frac{GM}{R^2}\right].$$

11. (a) : For motion of a planet in circular orbit, Centripetal force = Gravitational force

$$\therefore \quad mR\omega^2 = \frac{GMm}{R^n} \quad \text{or} \quad \omega = \sqrt{\frac{GM}{R^{n+1}}}$$

$$\therefore \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R^{n+1}}{GM}} = \frac{2\pi}{\sqrt{GM}} R^{\left(\frac{n+1}{2}\right)}$$

$\therefore$ T is proportional to $R^{\left(\frac{n+1}{2}\right)}$.

12. (a) : At height, $g_h = g\left(1 - \frac{2h}{R}\right)$ where $h << R$

$$\text{or} \quad g - g_h = \frac{2hg}{R} \quad \text{or} \quad \Delta g_h = \frac{2hg}{R} \quad \text{........ (i)}$$

At depth, $g_d = g\left(1 - \frac{d}{R}\right)$ where $d << R$

$$\text{or} \quad g - g_d = \frac{dg}{R}$$

$$\text{or} \quad \Delta g_d = \frac{dg}{R} \quad \text{........ (ii)}$$

From (i) and (ii), when $\Delta g_h = \Delta g_d$

$$\frac{2hg}{R} = \frac{dg}{R} \quad \text{or} \quad d = 2h.$$

13. (b) : Gravitational force $F = \frac{Gm_1m_2}{R^2}$

$$\therefore \quad dW = FdR = \frac{Gm_1m_2}{R^2}dR$$

$$\therefore \quad \int_0^W dW = Gm_1m_2\int_R^\infty \frac{dR}{R^2} = Gm_1m_2\left[-\frac{1}{R}\right]_R^\infty$$

$$= \frac{Gm_1m_2}{R}$$

$\therefore$ Workdone

$$= \frac{(6.67\times10^{-11})\times(100)\times(10\times10^{-3})}{10\times10^{-2}}$$

$$= 6.67 \times 10^{-10} \text{ J.}$$

14. (a) : $\text{Density} = \frac{\text{Mass of earth}}{\text{Volume of earth}}$

$$\rho = \frac{M}{(4/3)\pi R^3} = \frac{3M}{4\pi R^3} \quad \text{........ (i)}$$

$$g = \frac{GM}{R^2} \quad \text{........ (ii)}$$

$$\therefore \quad \frac{\rho}{g} = \frac{3M}{4\pi R^3}\times\frac{R^2}{GM} = \frac{3}{4\pi RG} \quad \text{or} \quad \rho = \frac{3}{4\pi RG}g$$

$\therefore$ Average density is directly proportional to g.

15. (d) : $v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$ for the earth

$v_e = 11 \text{ kms}^{-1}$

Mass of the planet = $10\ M_e$.

Radius of the planet = $R/10$.

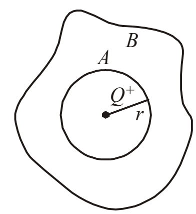
$$\therefore \quad v_e = \sqrt{\frac{2GM\times10}{R/10}} = 10\times11 = 110 \text{ km s}^{-1}$$

16. (c) : Let A be the Gaussian surface enclosing a spherical charge Q.

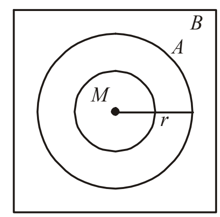
$$\vec{E}\cdot4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 . r^2}$$

$$\text{Flux } \phi = \vec{E}\cdot4\pi r^2 = \frac{Q}{\varepsilon_0}$$



Every line passing through A, has to pass through B, whether B is a cube or any surface. It is only for Gaussian surface, the lines of field should be normal. Assuming the mass is a point mass.



$$\vec{g}, \text{gravitational field} = -\frac{GM}{r^2}$$

$$\text{Flux } \phi_g = |\vec{g}\cdot4\pi r^2| = \frac{4\pi r^2\cdot GM}{r^2} = 4\pi GM.$$

Here B is a cube. As explained earlier, whatever be the shape, all the lines passing through A are passing through B, although all the lines are not normal.

Statement-2 is correct because when the shape of the earth is spherical, area of the Gaussian surface is $4\pi r^2$. This ensures inverse square law.

CHAPTER
8

Properties of Matter

1. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom is
(a) 10 (b) 20 (c) 25.5 (d) 5.
(2002)

2. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is
(a) 0.2 J (b) 10 J (c) 20 J (d) 0.1 J.
(2003)

3. A wire fixed at the upper end stretches by length l by applying a force F. The work done in stretching is
(a) $F/2l$ (b) Fl (c) $2Fl$ (d) $Fl/2$.
(2004)

4. Spherical balls of radius R are falling in a viscous fluid of viscosity η with a velocity v. The retarding viscous force acting on the spherical ball is
(a) directly proportional to R but inversely proportional to v
(b) directly proportional to both radius R and velocity v
(c) inversely proportional to both radius R and velocity v
(d) inversely proportional to R but directly proportional to velocity v.
(2004)

5. If two soap bubbles of different radii are connected by a tube,
(a) air flows from the bigger bubble to the smaller bubble till the sizes become equal
(b) air flows from bigger bubble to the smaller bubble till the sizes are interchanged
(c) air flows from the smaller bubble to the bigger
(d) there is no flow of air.
(2004)

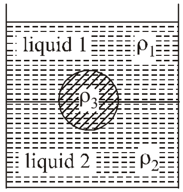
6. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator the length of water column in the capillary tube will be
(a) 4 cm (b) 20 cm (c) 8 cm (d) 10 cm
(2005)

7. If S is stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is
(a) $2Y/S$ (b) $S/2Y$
(c) $2S^2Y$ (d) $\dfrac{S^2}{2Y}$
(2005)

8. If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3) find the terminal speed of a sphere of silver (density 10.5 kg/m^3) of the same size in the same liquid
(a) 0.2 m/s (b) 0.4 m/s
(c) 0.133 m/s (d) 0.1 m/s.
(2006)

9. A wire elongates by l mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be (in mm)
(a) $l/2$ (b) l (c) $2l$ (d) zero.
(2006)

10. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2 respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for ρ_1, ρ_2 and ρ_3?



(a) $\rho_1 < \rho_3 < \rho_2$ (b) $\rho_3 < \rho_1 < \rho_2$
(c) $\rho_1 > \rho_3 > \rho_2$ (d) $\rho_1 < \rho_2 < \rho_3$

(2008)

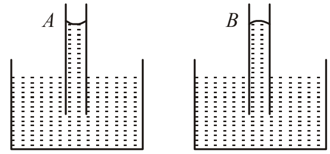
11. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_2 ($\rho_2 < \rho_1$). Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, *i.e.*, $F_{\text{viscous}} = -kv^2$ ($k > 0$). The terminal speed of the ball is

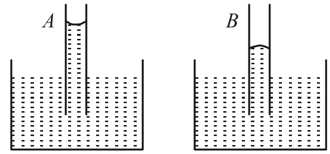
(a) $\dfrac{Vg(\rho_1 - \rho_2)}{k}$ (b) $\sqrt{\dfrac{Vg(\rho_1 - \rho_2)}{k}}$

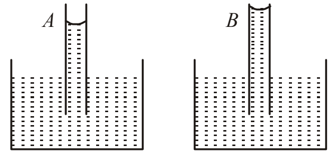
(c) $\dfrac{Vg\rho_1}{k}$ (d) $\sqrt{\dfrac{Vg\rho_1}{k}}$

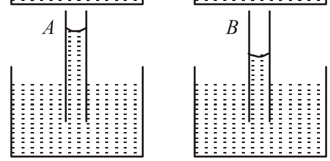
(2008)

12. A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes?

(a)


(b)


(c)


(d)


(2008)

Answer Key

1. (b)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (c)	**6.** (b)	**7.** (d)	**8.** (d)	**9.** (b)
10. (a)	**11.** (b)	**12.** (d)						

1. **(b) :** $v = \sqrt{2gh} = \sqrt{2\times10\times20} = 20$ m/s.

2. **(d) :** Elastic energy per unit volume

$= \frac{1}{2}\times$ stress $\times$ strain

$\therefore$ Elastic energy

$= \frac{1}{2}\times$ stress $\times$ strain $\times$ volume

$= \frac{1}{2}\times\frac{F}{A}\times\frac{\Delta L}{L}\times(AL)$

$= \frac{1}{2}F\Delta L = \frac{1}{2}\times200\times10^{-3} = 0.1$ J .

3. **(d) :** Young's modulus $Y = \frac{FL}{Al}$ .... (i)

$\therefore \quad F = \frac{YAl}{L}$

or $\quad dW = F\,dl = \frac{YAl(dl)}{L}$

or $\quad \int dW = \frac{YA}{L}\int_0^l l\,dl = \frac{YAl^2}{2L}$

or $\quad$ Workdone $= \frac{YAl^2}{2L}$ .... (ii)

From (i) and (ii)

Workdone $= \frac{Fl}{2}$.

4. **(b) :** Retarding viscous force $= 6\pi\eta Rv$ obviously option (b) holds goods.

5. **(c) :** Pressure inside the bubble $= P_0 + \frac{4T}{r}$
Smaller the radius, greater will be the pressure. Air flows from higher pressure to lower pressure. Hence air flows from the smaller bubble to the bigger.

6. **(b) :** In a freely falling elevator $g = 0$
Water will rise to the full length i.e., 20 cm to tube.

7. **(d) :** Energy stored per unit volume

$= \frac{1}{2}\times$ stress $\times$ strain

$= \frac{\text{Stress}\times\text{stress}}{2Y} = \frac{S^2}{2Y}$.

8. **(d) :** Terminal velocity $= v$
viscous force upwards = weight of sphere downwards

or $\quad 6\pi\eta rv = \left(\frac{4}{3}\pi r^3\right)(\rho-\sigma)g$

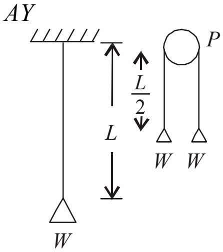
For gold and silver spheres falling in viscous liquid,

$\therefore \quad \frac{v_g}{v_s} = \frac{\rho_g-\sigma}{\rho_s-\sigma} = \frac{19.5-1.5}{10.5-1.5} = \frac{18}{9} = \frac{2}{1}$

or $\quad v_s = \frac{v_g}{2} = \frac{0.2}{2} = 0.1$ m/s.

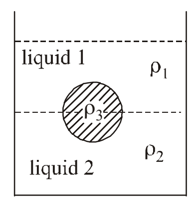
9. **(b) :** $Y = \frac{\text{Force}\times L}{A\times l} = \frac{WL}{Al}$

$\therefore \quad l = \frac{WL}{AY}$



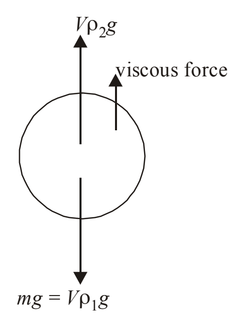
Due to pulley arrangement, the length of wire is $L/2$ on each side and so the elongation will be $l/2$. For both sides, elongation $= l$.

10. **(a) :** The liquid 1 is over liquid 2. Therefore $\rho_1 < \rho_2$. If ρ_3 had been greater than ρ_2, it will not be partially inside but anywhere inside liquid 2 if $\rho_3 = \rho_2$ or it would have sunk totally if ρ_3 had been greater than ρ_2.



$\therefore \quad \rho_1 < \rho_3 < \rho_2$.

11. **(b) :** The forces acting on the solid ball when it is falling through a liquid are mg downwards, thrust by Archimedes principle upwards and the force due to the force of friction also acting upwards. The viscous force rapidly increases with velocity,

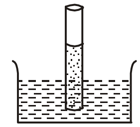


attaining a maximum when the ball reaches the terminal velocity. Then the acceleration is zero. $mg - V\rho_2 g - kv^2 = ma$ where V is volume, v is the terminal velocity.

When the ball is moving with terminal velocity $a = 0$. Therefore $V\rho_1 g - V\rho_2 g - kv^2 = 0$.

$$\Rightarrow v = \sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}.$$

12. (d) : The force acting upwards $2\pi rT \simeq h\pi r^2\rho g$, the force acting down or $T \propto h$ without making finer corrections. Soap reduces the surface tension of water. The height of liquid supported decreases. But it is also a wetting agent. Therefore the meniscus will not be convex as in mercury. Therefore (d).



Oscillations

1. If a spring has time period T, and is cut into n equal parts, then the time period of each part will be
(a) $T\sqrt{n}$ (b) $T/\sqrt{n}$ (c) nT (d) T.
(2002)

2. In a simple harmonic oscillator, at the mean position
(a) kinetic energy is minimum, potential energy is maximum
(b) both kinetic and potential energies are maximum
(c) kinetic energy is maximum, potential energy is minimum
(d) both kinetic and potential energies are minimum.
(2002)

3. A child swinging on a swing in sitting position, stands up, then the time period of the swing will
(a) increase
(b) decrease
(c) reamains same
(d) increases if the child is long and decreases if the child is short.
(2002)

4. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes $5T/3$. Then the ratio of m/M is
(a) 3/5 (b) 25/9
(c) 16/9 (d) 5/3.
(2003)

5. Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities, during oscillations, are equal, the ratio of amplitudes of A and B is
(a) $\sqrt{k_1/k_2}$ (b) k_2/k_1
(c) $\sqrt{k_2/k_1}$ (d) k_1/k_2.
(2003)

6. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is
(a) 11% (b) 21% (c) 42% (d) 10%.
(2003)

7. A body executes simple harmonic motion. The potential energy (P.E.), the kinetic energy (K.E.) and total energy (T.E.) are measured as function of displacement x. Which of the following statement is true?
(a) K.E. is maximum when $x = 0$
(b) T.E. is zero when $x = 0$
(c) K.E. is maximum when x is maximum
(d) P.E. is maximum when $x = 0$.
(2003)

8. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is $(4/3) \times 1000$ kg/m^3. What relationship between t and t_0 is true?
(a) $t = t_0$ (b) $t = t_0/2$ (c) $t = 2t_0$ (d) $t = 4t_0$.
(2003)

9. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then

(a) $T = t_1 + t_2$ (b) $T^2 = t_1^2 + t_2^2$

(c) $T^{-1} = t_1^{-1} + t_2^{-1}$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$.

(2004)

10. The total energy of a particle, executing simple harmonic motion is

(a) $\propto x$

(b) $\propto x^2$

(c) independent of x

(d) $\propto x^{1/2}$

where x is the displacement from the mean position.

(2004)

11. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force $F(t)$ proportional to $\cos\omega t$ ($\omega \neq \omega_0$) is applied to the oscillator. The time displacement of the oscillator will be proportional to

(a) $\dfrac{m}{\omega_0^2 - \omega^2}$ (b) $\dfrac{1}{m(\omega_0^2 - \omega^2)}$

(c) $\dfrac{1}{m(\omega_0^2 + \omega^2)}$ (d) $\dfrac{m}{\omega_0^2 + \omega^2}$.

(2004)

12. In forced oscillation of a particle the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force, then

(a) $\omega_1 = \omega_2$

(b) $\omega_1 > \omega_2$

(c) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large

(d) $\omega_1 < \omega_2$

(2004)

13. The function $\sin^2(\omega t)$ represents

(a) a simple harmonic motion with a period $2\pi/\omega$

(b) a simple harmonic motion with a period π/ω

(c) a periodic, but not simple harmonic motion with a period $2\pi/\omega$

(d) a periodic, but not simple harmonic motion with a period π/ω

(2005)

14. Two simple harmonic motions are represented by the equations $y_1 = 0.1\sin\left(100\pi t + \dfrac{\pi}{3}\right)$ and $y_2 = 0.1\cos\pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is

(a) $-\pi/3$ (b) $\pi/6$

(c) $-\pi/6$ (d) $\pi/3$.

(2005)

15. If a simple harmonic motion is represented by $\dfrac{d^2x}{dt^2} + \alpha x = 0$, its time period is

(a) $2\pi\alpha$ (b) $2\pi\sqrt{\alpha}$

(c) $2\pi/\alpha$ (d) $2\pi/\sqrt{\alpha}$

(2005)

16. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would

(a) remain unchanged

(b) increase towards a saturation value

(c) first increase and then decrease to the original value

(d) first decrease and then increase to the original value

(2005)

17. Starting from the origin, a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy by 75% of the total energy?

(a) $\dfrac{1}{12}$ s (b) $\dfrac{1}{6}$ s (c) $\dfrac{1}{4}$ s (d) $\dfrac{1}{3}$ s.

(2006)

18. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is

(a) 100 s (b) 0.01 s (c) 10 s (d) 0.1 s.

(2006)

19. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time

(a) at the highest position of the platform

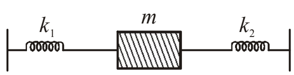
(b) at the mean position of the platform

(c) for an amplitude of $\frac{g}{\omega^2}$

(d) for an amplitude of $\frac{g^2}{\omega^2}$.

(2006)

20. Two springs, of force constants k_1 and k_2 are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes



(a) $2f$ (b) $f/2$
(c) $f/4$ (d) $4f$

(2007)

21. A particle of mass m executes simple harmonic motion with amplitude a and frequency υ. The average kinetic energy during its motion from the position of equilibrium to the end is

(a) $2\pi^2 m a^2 \upsilon^2$ (b) $\pi^2 ma^2\upsilon^2$
(c) $\frac{1}{4}ma^2\upsilon^2$ (d) $4\pi^2 ma^2\upsilon^2$

(2007)

22. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metre. The time at which the maximum speed first occurs is

(a) 0.25 s (b) 0.5 s
(c) 0.75 s (d) 0.125 s

(2007)

23. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A\cos(\omega t + \delta)$, then

(a) $A = x_0\omega^2, \delta = 3\pi/4$ (b) $A = x_0, \delta = -\pi/4$
(c) $A = x_0\omega^2, \delta = \pi/4$ (d) $A = x_0\omega^2, \delta = -\pi/4$

(2007)

Answer Key

1.	(b)	**2.**	(c)	**3.**	(b)	**4.**	(c)	**5.**	(c)	**6.**	(d)
7.	(a)	**8.**	(c)	**9.**	(b)	**10.**	(c)	**11.**	(b)	**12.**	(a)
13.	(d)	**14.**	(c)	**15.**	(d)	**16.**	(c)	**17.**	(b)	**18.**	(b)
19.	(c)	**20.**	(a)	**21.**	(b)	**22.**	(b)	**23.**	(a)		

1. **(b) :** For a spring, $T = 2\pi\sqrt{\frac{m}{k}}$

For each piece, spring constant = nk

$\therefore \quad T' = 2\pi\sqrt{\frac{m}{nk}}$

$\therefore \quad T' = 2\pi\sqrt{\frac{m}{k}} \times \frac{1}{\sqrt{n}} = \frac{T}{\sqrt{n}}.$

2. **(c) :** In a simple harmonic oscillator, kinetic energy is maximum and potential energy is minimum at mean position.

3. **(b) :** Time period will decrease.

When the child stands up, the centre of gravity is shifted upwards and so length of swing decreases. $T = 2\pi\sqrt{l/g}$.

4. **(c) :** Initially, $T = 2\pi\sqrt{M/k}$

Finally, $\frac{5T}{3} = 2\pi\sqrt{\frac{M+m}{k}}$

$\therefore \quad \frac{5}{3} \times 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M+m}{k}}$

or $\quad \frac{25}{9}\frac{M}{k} = \frac{M+m}{k}$

or $\quad 9m + 9M = 25M$

or $\quad \frac{m}{M} = \frac{16}{9}.$

5. **(c) :** Maximum velocity under simple harmonic motion $v_m = a\omega$

$\therefore \quad v_m = \frac{2\pi a}{T} = (2\pi a)\left(\frac{1}{T}\right) = (2\pi a)\left(\frac{1}{2\pi}\sqrt{\frac{k}{m}}\right)$

or $\quad v_m = a\sqrt{\frac{k}{m}}$

$\because \quad (v_m)_A = (v_m)_B$

$\therefore \quad a_1\sqrt{\frac{k_1}{m}} = a_2\sqrt{\frac{k_2}{m}} \Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}.$

6. **(d) :** Let the lengths of pendulum be $(100l)$ and $(121l)$

$\therefore \quad \frac{T'}{T} = \sqrt{\frac{121}{100}} = \frac{11}{10}$

$\therefore \quad$ Fractional change $= \frac{T'-T}{T} = \frac{11-10}{10} = \frac{1}{10}$

$\therefore \quad$ Percentage change = 10%.

7. **(a) :** Kinetic energy is maximum at $x = 0$.

8. **(c) :** $t_0 = 2\pi\sqrt{l/g}$ .... (i)

Due to upthrust of water on the top, its apparent weight decreases

upthrust = weight of liquid displaced

$\therefore$ Effective weight $= mg - (V\sigma g) = V\rho g - V\sigma g$

$V\rho g' = Vg(\rho - \sigma)$, where σ is density of water

or $\quad g' = g\left(\frac{\rho-\sigma}{\rho}\right)$

$\therefore \quad t = 2\pi\sqrt{l/g'} = 2\pi\sqrt{\frac{l\rho}{g(\rho-\sigma)}}$ ....(ii)

$\therefore \quad \frac{t}{t_0} = \sqrt{\frac{l\rho}{g(\rho-\sigma)} \times \frac{g}{l}} = \sqrt{\frac{\rho}{\rho-\sigma}}$

$$= \sqrt{\frac{4\times1000/3}{\left(\frac{4000}{3}-1000\right)}} = 2$$

or $\quad t = t_0 \times 2 = 2t_0$.

9. **(b) :** When springs are in series, $k = \frac{k_1k_2}{k_1+k_2}$

For first spring, $t_1 = 2\pi\sqrt{\frac{m}{k_1}}$

For second spring $t_2 = 2\pi\sqrt{\frac{m}{k_2}}$

$\therefore \quad t_1^2 + t_2^2 = \frac{4\pi^2 m}{k_1} + \frac{4\pi^2 m}{k_2} = 4\pi^2 m\left(\frac{k_1+k_2}{k_1k_2}\right)$

or $\quad t_1^2 + t_2^2 = \left[2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}\right]^2$

or $\quad t_1^2 + t_2^2 = T^2.$

10. **(c) :** Under simple harmonic motion, total energy

$$= \frac{1}{2}ma^2\omega^2$$

Total energy is independent of x.

11. **(b) :** In case of forced oscillations,

$x = a\sin(\omega t + \phi)$ where $a = \frac{F_0/m}{\omega_0^2 - \omega^2}$

$\therefore$ x is proportional to $\dfrac{1}{m(\omega_0^2 - \omega^2)}$.

12. **(a) :** In case of forced oscillations

(i) The amplitude is maximum at resonance

$\therefore$ Natural frequency = Frequency of force = ω_1

(ii) The energy is maximum at resonance

$\therefore$ Natural frequency = Frequency of force = ω_2

$\therefore$ From (i) and (ii),

$\omega_1 = \omega_2$.

13. **(d) :** $y = \sin^2 \omega t = \dfrac{1-\cos 2\omega t}{2} = \dfrac{1}{2} - \dfrac{\cos 2\omega t}{2}$

It is a periodic motion but it is not SHM

$\therefore$ Angular speed = 2ω

$\therefore$ Period $T = \dfrac{2\pi}{\text{angular speed}} = \dfrac{2\pi}{2\omega} = \dfrac{\pi}{\omega}$

Hence option (d) represents the answer.

14. **(c) :** $v_1 = \dfrac{d}{dt}(y_1) = (0.1\times 100\pi)\cos\left(100\pi t + \dfrac{\pi}{3}\right)$

$v_2 = \dfrac{d}{dt}(y_2) = (-0.1\times\pi)\sin \pi t$

$= (0.1\times\pi)\cos\left(\pi t + \dfrac{\pi}{2}\right)$

$\therefore$ $\Delta\phi = \dfrac{\pi}{3} - \dfrac{\pi}{2} = -\dfrac{\pi}{6}$.

15. **(d) :** Standard differential equation of SHM is

$\dfrac{d^2x}{dt^2} + \omega^2 x = 0$

Given equation is $\dfrac{d^2x}{dt^2} + \alpha x = 0$

$\therefore$ $\omega^2 = \alpha$

or $\omega = \sqrt{\alpha}$

$\therefore$ $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\sqrt{\alpha}}$.

16. **(c) :** For a pendulum, $T = 2\pi\sqrt{\dfrac{l}{g}}$ where l is measured upto centre of gravity. The centre of gravity of system is at centre of sphere when hole is plugged. When unplugged, water drains out. Centre of gravity goes on descending. When the bob becomes empty, centre of gravity is restored to centre.

$\therefore$ Length of pendulum first increases, then decreases to original value.

$\therefore$ T would first increase and then decrease to the original value.

17. **(b) :** During simple harmonic motion,

Kinetic energy

$$= \frac{1}{2}mv^2 = \frac{1}{2}m\,(a\,\omega\cos\omega t)^2$$

Total energy $E = \dfrac{1}{2}ma^2\omega^2$

$\because$ (Kinetic energy) $= \dfrac{75}{100}(E)$

or $\dfrac{1}{2}ma^2\omega^2\cos^2\omega t = \dfrac{75}{100}\times\dfrac{1}{2}ma^2\omega^2$

or $\cos^2\omega t = \dfrac{3}{4} \Rightarrow \cos\omega t = \dfrac{\sqrt{3}}{2} = \cos\dfrac{\pi}{6}$

$\therefore$ $\omega t = \dfrac{\pi}{6}$

or $t = \dfrac{\pi}{6\omega} = \dfrac{\pi}{6(2\pi/T)} = \dfrac{2\pi}{6\times 2\pi} = \dfrac{1}{6}$ sec.

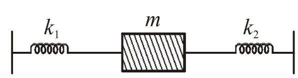
18. **(b) :** Maximum velocity $v_m = a\omega = a\left(\dfrac{2\pi}{T}\right)$

$\therefore$ $T = \dfrac{2\pi a}{v_m} = 2\times\dfrac{22}{7}\times\dfrac{(7\times 10^{-3})}{4.4}$

$= 10^{-2}$ sec $= 0.01$ sec.

19. **(c) :** In vertical simple harmonic motion, maximum acceleration ($a\omega^2$) and so the maximum force ($ma\omega^2$) will be at extreme positions. At highest position, force will be towards mean position and so it will be downwards. At lowest position, force will be towards mean position and so it will be upwards. This is opposite to weight direction of the coin. The coin will leave contact will the platform for the first time when $m(a\omega^2) \geq mg$ at the lowest position of the platform.

20. **(a) :** In the given figure two springs are connected in parallel. Therefore the effective spring constant is given by



$k_{eff} = k_1 + k_2$

Frequency of oscillation,

$f = \dfrac{1}{2\pi}\sqrt{\dfrac{k_{eff}}{m}} = \dfrac{1}{2\pi}\sqrt{\dfrac{k_1+k_2}{m}}$ ... (i)

As k_1 and k_2 are increased four times

New frequency,

$f' = \dfrac{1}{2\pi}\sqrt{\dfrac{4(k_1+k_2)}{m}} = 2f$ (using (i).

21. **(b) :** For a particle to execute simple harmonic motion its displacement at any time t is given by $x(t) = a(\cos \omega t + \phi)$

where, a = amplitude, ω = angular frequency, ϕ = phase constant.

Let us choose $\phi = 0$

$\therefore \quad x(t) = a\cos\omega t$

Velocity of a particle $v = \frac{dx}{dt} = -a\,\omega\sin\omega t$

Kinetic energy of a particle is $K = \frac{1}{2}mv^2$

$$= \frac{1}{2}ma^2\omega^2\sin^2\omega t$$

Average kinetic energy $< K >$

$$= <\frac{1}{2}ma^2\omega^2\sin^2\omega t>$$

$$= \frac{1}{2}m\omega^2a^2 <\sin^2\omega t>$$

$$= \frac{1}{2}m\omega^2a^2\left(\frac{1}{2}\right)$$

$$\left[\because \ <\sin^2\theta> = \frac{1}{2}\right]$$

$$= \frac{1}{4}ma^2(2\pi\upsilon)^2 \qquad [\because \ \omega = 2\pi\upsilon]$$

$= \pi^2 ma^2 \upsilon^2$.

22. **(b) :** Given : displacement $x = 2 \times 10^{-2} \cos \pi t$

Velocity $v = \frac{dx}{dt} = -2\times10^{-2}\pi\sin\pi t$

For the first time when $v = v_{max}$, $\sin \pi t = 1$

or $\quad \sin\pi t = \sin\frac{\pi}{2} \quad$ or $\quad \pi t = \frac{\pi}{2}$

or $\quad t = \frac{1}{2}\text{ s} = 0.5\text{ s}.$

23. **(a) :** Given : $x = x_0\cos\left(\omega t - \frac{\pi}{4}\right)$ ... (i)

Acceleration $a = A \cos (\omega t + \delta)$ ... (ii)

Velocity $v = \frac{dx}{dt}$

$v = -x_0\omega\sin\left(\omega t - \frac{\pi}{4}\right)$ ... (iii)

Acceleration $a = \frac{dv}{dt}$

$$= -x_0\omega^2\cos\left(\omega t - \frac{\pi}{4}\right) = x_0\omega^2\cos\left[\pi + (\omega t - \frac{\pi}{4})\right]$$

$= x_0\omega^2\cos\left[\omega t + \frac{3\pi}{4}\right]$ ... (iv)

Compare (iv) with (ii), we get

$A = x_0\omega^2,\ \delta = \frac{3\pi}{4}$.

CHAPTER

10

Waves

1. Tube A has both ends open while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tube A and B is
(a) 1 : 2 (b) 1 : 4
(c) 2 : 1 (d) 4 : 1.
(2002)

2. A tuning fork arrangement (pair) produces 4 beats/sec with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats/sec. The frequency of the unknown fork is
(a) 286 cps (b) 292 cps
(c) 294 cps (d) 288 cps.
(2002)

3. A wave $y = a\sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then the equation of the unknown wave is
(a) $y = a\sin(\omega t + kx)$ (b) $y = -a\sin(\omega t + kx)$
(c) $y = a\sin(\omega t - kx)$ (d) $y = -a\sin(\omega t - kx)$.
(2002)

4. Length of a string tied to two rigid supports is 40 cm. Maximum length (wavelength in cm) of a stationary wave produced on it is
(a) 20 (b) 80
(c) 40 (d) 120.
(2002)

5. When temperature increases, the frequency of a tuning fork
(a) increases
(b) decreases
(c) remains same
(d) increases or decreases depending on the material.
(2002)

6. The displacement y of a wave travelling in the x-direction is given by
$$y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right) \text{ metre,}$$
where x is expressed in metre and t in second. The speed of the wave-motion, in ms^{-1} is
(a) 300 (b) 600
(c) 1200 (d) 200.
(2003)

7. The displacement of a particle varies according to the relation $x = 4(\cos\pi t + \sin\pi t)$. The amplitude of the particle is
(a) -4 (b) 4
(c) $4\sqrt{2}$ (d) 8.
(2003)

8. A metal wire of linear mass density of 9.8 g/m is stretched with a tension of 10 kg-wt between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a permanent magnet, and it vibrates in resonance when carrying an alternating current of frequency υ. The frequency υ of the alternating source is
(a) 50 Hz (b) 100 Hz
(c) 200 Hz (d) 25 Hz.
(2003)

9. A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was

(a) (256 + 2) Hz (b) (256 – 2) Hz
(c) (256 – 5) Hz (d) (256 + 5) Hz.

(2003)

10. The displacement y of a particle in a medium can be expressed as:
$y = 10^{-6}\sin(100t + 20x + \pi/4)$ m, where t is in second and x in meter. The speed of the wave is
(a) 2000 m/s (b) 5 m/s
(c) 20 m/s (d) 5π m/s.

(2004)

11. When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now, some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard. if the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2?
(a) 196 Hz (b) 204 Hz
(c) 200 Hz (d) 202 Hz

(2005)

12. An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency?
(a) 5% (b) 20%
(c) zero (d) 0.5%

(2005)

13. A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with speed v ms^{-1}. The velocity of sound in air is 300 ms^{-1}. If the person can hear frequencies upto a maximum of 10000 Hz, the maximum value of v upto which he can hear the whistle is
(a) 30 ms^{-1} (b) $15\sqrt{2}$ ms^{-1}
(c) $15/\sqrt{2}$ ms^{-1} (d) 15 ms^{-1}.

(2006)

14. A string is stretched between fixed points separated by 75 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is
(a) 10.5 Hz (b) 105 Hz
(c) 1.05 Hz (d) 1050 Hz.

(2006)

15. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of
(a) 100 (b) 1000
(c) 10000 (d) 10

(2007)

16. The speed of sound in oxygen (O_2) at a certain temperature is 460 ms^{-1}. The speed of sound in helium (He) at the same temperature will be (assume both gases to be ideal)
(a) 330 ms^{-1} (b) 460 ms^{-1}
(c) 500 ms^{-1} (d) 650 ms^{-1}.

(2008)

17. A wave travelling along the x-axis is described by the equation $y(x, t) = 0.005\cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are
(a) $\alpha = 12.50\pi,\ \beta = \frac{\pi}{2.0}$ (b) $\alpha = 25.00\pi,\ \beta = \pi$
(c) $\alpha = \frac{0.08}{\pi},\ \beta = \frac{2.0}{\pi}$ (d) $\alpha = \frac{0.04}{\pi},\ \beta = \frac{1.0}{\pi}$

(2008)

Answer Key

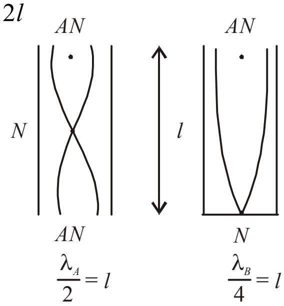
1.	(c)	**2.**	(b)	**3.**	(b)	**4.**	(b)	**5.**	(b)	**6.**	(a)
7.	(c)	**8.**	(a)	**9.**	(c)	**10.**	(b)	**11.**	(a)	**12.**	(b)
13.	(d)	**14.**	(b)	**15.**	(a)	**16.**	No option	**17.**	(b)		

1. **(c) :** In tube A, $\lambda_A = 2l$
In tube B, $\lambda_B = 4l$

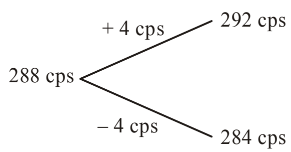
$\therefore\ \upsilon_A = \dfrac{v}{\lambda_A} = \dfrac{v}{2l}$

$\upsilon_B = \dfrac{v}{\lambda_B} = \dfrac{v}{4l}$

$\therefore\ \dfrac{\upsilon_A}{\upsilon_B} = \dfrac{2}{1}.$



2. **(b) :** The wax decreases the frequency of unknown fork. The possible unknown frequencies are (288 + 4)cps and (288 − 4) cps.



Wax reduces 284 cps and so beats should increases. It is not given in the question. This frequency is ruled out. Wax reduced 292 cps and so beats should decrease. It is given that the beats decrease to 2 from 4.
Hence unknown fork has frequency 292 cps.

3. **(b) :** Consider option (*a*)
Stationary wave :
$Y = a\sin(\omega t + kx) + a\sin(\omega t - kx)$
when $x = 0$, Y is not zero. The option is not acceptable.
Consider option (*b*)
Stationary wave :
$Y = a\sin(\omega t - kx) - a\sin(\omega t + kx)$
At $x = 0$, $Y = a\sin\omega t - a\sin\omega t$ = zero
This option holds good
Option (*c*) gives $Y = 2a\sin(\omega t - kx)$
At $x = 0$, Y is not zero
Option (*d*) gives $Y = 0$
Hence only option (b) holds good.

4. **(b) :** $\dfrac{\lambda_{\max}}{2} = 40 \Rightarrow \lambda_{\max} = 80\text{ cm}.$

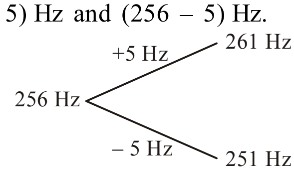
5. **(b) :** When temperature increases, l increases
Hence frequency decreases.

6. **(a) :** Given wave equation :
$$y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right)\text{m}$$
Standard wave equation : $y = a\sin(\omega t - kx + \phi)$
Compare them
Angular speed = ω = 600 sec^{-1}
Propagation constant = k = 2 m^{-1}
$$\frac{\omega}{k} = \frac{2\pi\upsilon}{2\pi/\lambda} = \upsilon\lambda = \text{velocity}$$
$$\therefore\ \text{velocity} = \frac{\omega}{k} = \frac{600}{2} = 300\,\text{m/sec}.$$

7. **(c) :** $x = 4(\cos\pi t + \sin\pi t)$
$$= 4\times\sqrt{2}\left[\frac{1}{\sqrt{2}}\cos\pi t + \frac{1}{\sqrt{2}}\sin\pi t\right]$$
or $$x = 4\sqrt{2}\left[\sin\frac{\pi}{4}\cos\pi t + \cos\frac{\pi}{4}\sin\pi t\right]$$
$$= 4\sqrt{2}\sin\left(\pi t + \frac{\pi}{4}\right)$$
Hence amplitude $= 4\sqrt{2}$.

8. **(a) :** At resonance, frequency of vibration of wire become equal to frequency of a.c.
For vibration of wire, $\upsilon = \dfrac{1}{2l}\sqrt{\dfrac{T}{\mu}}$
$$\therefore\ \upsilon = \frac{1}{2\times 1}\sqrt{\frac{10\times 9.8}{9.8\times 10^{-3}}} = \frac{100}{2} = 50\text{ Hz.}$$

9. **(c) :** The possible frequencies of piano are (256 + 5) Hz and (256 − 5) Hz.



For piano string, $\upsilon = \dfrac{1}{2l}\sqrt{\dfrac{T}{\mu}}$
When tension T increases, υ increases
(i) If 261 Hz increases, beats/sec increase. This is not given in the question.
(ii) If 251 Hz increases due to tension, beats per second decrease. This is given in the question.
Hence frequency of piano = (256 − 5) Hz.

10. **(b) :** Given wave equation :

$y = 10^{-6}\sin\left(100t + 20x + \frac{\pi}{4}\right)$m

Standard equation : $y = a\sin(\omega t + kx + \phi)$

Compare the two

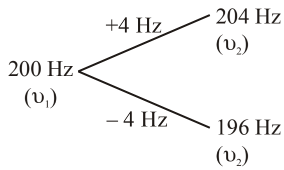
$\therefore\ \omega = 100$ and $k = 20$

$\therefore\ \frac{\omega}{k} = \frac{100}{20} \Rightarrow \frac{2\pi n}{2\pi/\lambda} = n\lambda = v = 5$

$\therefore\ v = 5$ m/s.

11. **(a) :** Let the two frequencies be υ_1 and υ_2

υ_2 may be either 204 Hz or 196 Hz.



As mass of second fork increases, υ_2 decreases. If υ_2 = 204 Hz, a decrease in υ_2 decreases beats/sec. But this is not given in question

If υ_2 = 196 Hz, a decrease in υ_2 increased beats/sec.

This is given in the question when beats increase to 6

$\therefore$ Original frequency of second fork = 196 Hz.

12. **(b) :** By Doppler's effect

$\frac{\upsilon'}{\upsilon} = \frac{v_s + v_O}{v_s}$ (where v_s is the velocity of sound)

$= \frac{v + (v/5)}{v} = \frac{6}{5}$

$\therefore$ Fractional increase

$$= \frac{\upsilon' - \upsilon}{\upsilon} = \left(\frac{\upsilon'}{\upsilon} - 1\right) = \left(\frac{6}{5} - 1\right) = \frac{1}{5}$$

$\therefore$ Percentage increase $= \frac{100}{5} = 20\%$.

13. **(d) :** $\frac{\upsilon'}{\upsilon} = \frac{v_s}{v_s - v}$

where v_s is the velocity of sound in air.

$\frac{10000}{9500} = \frac{300}{300 - v}$

$\Rightarrow (300 - v) = 285 \Rightarrow v = 15$ m/s.

14. **(b) :** Let the successive loops formed be p and $(p + 1)$ for frequencies 315 Hz and 420 Hz

$\therefore\ \upsilon = \frac{p}{2l}\sqrt{\frac{T}{\mu}} = \frac{pv}{2l}$

$\therefore\ \frac{pv}{2l} = 315$ Hz and $\frac{(p+1)v}{2l} = 420$ Hz

or $\frac{(p+1)v}{2l} - \frac{pv}{2l} = 420 - 315$

or $\frac{v}{2l} = 105 \Rightarrow \frac{1 \times v}{2l} = 105$ Hz

$p = 1$ for fundamental mode of vibration of string.

$\therefore$ Lowest resonant frequency = 105 Hz.

15. **(a) :** $L_1 = 10\log\left(\frac{I_1}{I_0}\right)$; $L_2 = 10\log\left(\frac{I_2}{I_0}\right)$

$\therefore\ L_1 - L_2 = 10\log\left(\frac{I_1}{I_0}\right) - 10\log\left(\frac{I_2}{I_0}\right)$

or $\Delta L = 10\log\left(\frac{I_1}{I_2}\right)$ or $20\text{ dB} = 10\log\left(\frac{I_1}{I_2}\right)$

or $10^2 = \frac{I_1}{I_2}$ or $I_2 = \frac{I_1}{100}$.

16.* $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

γ for $O_2 = 1 + 2/5 = 1.4$;

γ for He = 1 + 2/3 = 5/3

$\frac{v_2}{v_1} = \left(\sqrt{\frac{\gamma_{He}}{4} \times \frac{32}{\gamma_{O_2}}}\right) \times 460$

$= 460 \times \sqrt{\frac{5}{3} \times \frac{1}{4} \times \frac{32 \times 5}{7}} = 1420$ m/s.

* The value of the speed of sound in He should have been 965 m/s and that of O_2, about 320 m/s. The value of the velocity given for O_2 is quite high. Option not given.

17. **(b) :** The wave travelling along the x-axis is given by $y(x, t) = 0.005\cos(\alpha x - \beta t)$.

Therefore $\alpha = k = \frac{2\pi}{\lambda}$. As $\lambda = 0.08$ m.

$\therefore\ \alpha = \frac{2\pi}{0.08} = \frac{\pi}{0.04} \Rightarrow \alpha = \frac{\pi}{4} \times 100.00 = 25.00\pi$.

$\omega = \beta \Rightarrow \frac{2\pi}{2.0} = \beta \Rightarrow \pi$

$\therefore\ \alpha = 25.00\,\pi,\ \beta = \pi$

CHAPTER

11

Heat and Thermodynamics

1. Heat given to a body which raises its temperature by 1°C is
(a) water equivalent
(b) thermal capacity
(c) specific heat
(d) temperature gradient.

(2002)

2. Which statement is incorrect?
(a) all reversible cycles have same efficiency
(b) reversible cycle has more efficiency than an irreversible one
(c) Carnot cycle is a reversible one
(d) Carnot cycle has the maximum efficiency in all cycles.

(2002)

3. Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will
(a) increase
(b) decrease
(c) remain same
(d) decrease for some, while increase for others.

(2002)

4. At what temperature is the r.m.s. velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C?
(a) 80 K (b) –73 K (c) 3 K (d) 20 K.

(2002)

5. Even Carnot engine cannot give 100% efficiency because we cannot
(a) prevent radiation
(b) find ideal sources
(c) reach absolute zero temperature
(d) eliminate friction.

(2002)

6. 1 mole of a gas with $\gamma = 7/5$ is mixed with 1 mole of a gas with $\gamma = 5/3$, then the value of γ for the resulting mixture is
(a) 7/5 (b) 2/5 (c) 24/16 (d) 12/7.

(2002)

7. "Heat cannot by itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of
(a) second law of thermodynamics
(b) conservation of momentum
(c) conservation of mass
(d) first law of thermodynamics.

(2003)

8. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_P/C_V for the gas is
(a) 4/3 (b) 2 (c) 5/3 (d) 3/2.

(2003)

9. Which of the following parameters does not characterize the thermodynamic state of matter?
(a) temperature (b) pressure
(c) work (d) volume.

(2003)

10. A Carnot engine takes 3×10^6 cal of heat from a reservoir at 627°C, and gives it to a sink at 27°C. The work done by the engine is
(a) 4.2×10^6 J (b) 8.4×10^6 J
(c) 16.8×10^6 J (d) zero.

(2003)

11. One mole of ideal monoatomic gas (γ = 5/3) is mixed with one mole of diatomic gas (γ = 7/5). What is γ for the mixture? γ denotes the ratio of specific heat at constant pressure, to that at constant volume.

(a) 3/2 (b) 23/15 (c) 35/23 (d) 4/3.

(2004)

12. Which of the following statements is correct for any thermodynamic system?

(a) The internal energy changes in all processes.
(b) Internal energy and entropy are state functions.
(c) The change in entropy can never be zero.
(d) The work done in an adiabatic process is always zero.

(2004)

13. Two thermally insulated vessels 1 and 2 are filled with air at temperatures (T_1, T_2), volume (V_1, V_2) and pressure (P_1, P_2) respectively. If the valve joining the two vessels is opened, the temperature inside the vessel at equilibrium will be

(a) $T_1 + T_2$ (b) $(T_1 + T_2)/2$

(c) $\dfrac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_2+P_2V_2T_1}$ (d) $\dfrac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_1+P_2V_2T_2}$.

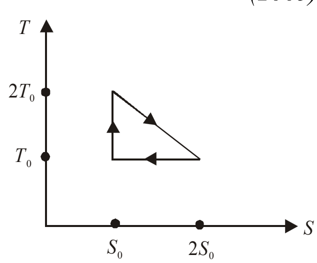
(2004)

14. Which of the following is incorrect regarding the first law of thermodynamics?

(a) It introduces the concept of the internal energy
(b) It introduces the concept of entropy
(c) It is not applicable to any cyclic process
(d) It is a restatement of the principle of conservation of energy

(2005)

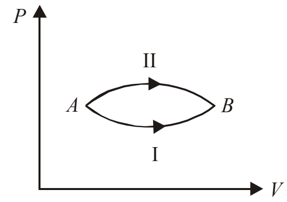
15. The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is



(a) 1/3
(b) 2/3 (c) 1/2
(d) 1/4

(2005)

16. A system goes from A to B *via* two processes I and II as shown in figure. If ΔU_1 and ΔU_2 are the changes in internal energies in the processes I and II respectively, then



(a) $\Delta U_2 > \Delta U_1$ (b) $\Delta U_2 < \Delta U_2$
(c) $\Delta U_1 = \Delta U_2$
(d) relation between ΔU_1 and ΔU_2 can not be determined

(2005)

17. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio C_P/C_V of the mixture is

(a) 1.4 (b) 1.54 (c) 1.59 (d) 1.62

(2005)

18. The work of 146 kJ is performed in order to compress one kilo mole of gas adiabatically and in this process the temperature of the gas increases by 7ºC. The gas is (R = 8.3 J mol^{-1} K^{-1})

(a) monoatomic
(b) diatomic (c) triatomic
(d) a mixture of monoatomic and diatomic.

(2006)

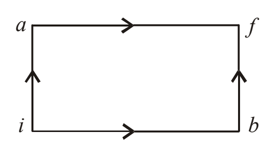
19. Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0, while Box B contains one mole of helium at temperature (7/3) T_0. The boxes are then put into thermal contact with each other and heat flows between them until the gases reach a common final temperature. (Ignore the heat capacity of boxes). Then, the final temperature of the gases, T_f, in terms of T_0 is

(a) $T_f = \frac{5}{2}T_0$ (b) $T_f = \frac{3}{7}T_0$

(c) $T_f = \frac{7}{3}T_0$ (d) $T_f = \frac{3}{2}T_0$.

(2006)

20. When a system is taken from state i to state f along the path iaf, it is found that Q = 50 cal and W = 20 cal. Along the path ibf Q = 36 cal. W along the path ibf is



(a) 14 cal (b) 6 cal

(c) 16 cal (d) 66 cal

(2007)

21. A Carnot engine, having an efficiency of $\eta = 1/10$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is

(a) 100 J (b) 99 J

(c) 90 J (d) 1 J

(2007)

22. If C_P and C_V denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then

(a) $C_P - C_V = 28R$ (b) $C_P - C_V = R/28$

(c) $C_P - C_V = R/14$ (d) $C_P - C_V = R$

(2007)

23. An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure P_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure P_2 and temperature T_2. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be

(a) $\dfrac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_1 + P_2V_2T_2}$ (b) $\dfrac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$

(c) $\dfrac{P_1V_1T_1 + P_2V_2T_2}{P_1V_1 + P_2V_2}$ (d) $\dfrac{P_1V_1T_2 + P_2V_2T_1}{P_1V_1 + P_2V_2}$

(2008)

Answer Key

1.	(b)	**2.**	(a)	**3.**	(c)	**4.**	(d)	**5.**	(c)	**6.**	(c)
7.	(a)	**8.**	(d)	**9.**	(c)	**10.**	(b)	**11.**	(a)	**12.**	(b)
13.	(c)	**14.**	(b, c)	**15.**	(a)	**16.**	(c)	**17.**	(d)	**18.**	(b)
19.	(d)	**20.**	(b)	**21.**	(c)	**22.**	(b)	**23.**	(b)		

1. **(b) :** Thermal capacity.
2. **(a) :** All reversible cycles do not have same efficiency.
3. **(c) :** It is the relative velocities between molecules that is important. Root mean square velocities are different from lateral translation.
4. **(d) :** $v_{\text{rms}} = \sqrt{\dfrac{RT}{M}}$

   $\therefore \quad (v_{\text{rms}})_{O_2} = (v_{\text{rms}})_{H_2}$

   or $\sqrt{\dfrac{273+47}{32}} = \sqrt{\dfrac{T}{2}} \Rightarrow T = 20$ K .
5. **(c) :** We cannot reach absolute zero temperature.
6. **(c) :** For mixture of gases

   $$\frac{n_1+n_2}{\gamma_m-1} = \frac{n_1}{\gamma_1-1} + \frac{n_2}{\gamma_2-1}$$

   $$\frac{1+1}{\gamma_m-1} = \frac{1}{\left(\frac{7}{5}-1\right)} + \frac{1}{\left(\frac{5}{3}-1\right)}$$

   $$\frac{2}{\gamma_m-1} = \frac{5}{2}+\frac{3}{2}$$

   or $\dfrac{2}{\gamma_m-1} = \dfrac{8}{2}$

   or $8\gamma_m - 8 = 4$

   or $8\gamma_m = 12$

   or $\gamma_m = \dfrac{12}{8} = \dfrac{24}{16}$
7. **(a) :** Second law of thermodynamics.
8. **(d) :** In an adiabatic process, $T^{\gamma} = (\text{constant})P^{\gamma-1}$

   or $T^{\gamma/\gamma-1}$ = (constant) P

   Given T^3 = (constant) P

   $\therefore \quad \dfrac{\gamma}{\gamma-1} = 3 \Rightarrow 3\gamma - 3 = \gamma$

   or $2\gamma = 3 \Rightarrow \gamma = 3/2$

   N.B For monoatomic gas, $\gamma = \dfrac{5}{3} = 1.67$

   For diatomic gas, $\gamma = \dfrac{7}{5} = 1.4$

   when $\gamma = 1.5$, the gas must be a suitable mixture of monoatomic and diatomic gases

   $\therefore \quad \gamma = 3/2$.
9. **(c) :** The work does not characterize the thermodynamic state of matter.
10. **(b) :** Efficiency $= 1-\dfrac{T_2}{T_1} = 1-\dfrac{300}{900} = 1-\dfrac{1}{3} = \dfrac{2}{3}$

    Heat energy = 3×10^6 cal = $3 \times 10^6 \times 4.2$ J

    $\therefore$ Workdone by engine = (Heat energy) $\times$ (efficiency)

    $= (3\times10^6\times4.2)\times\dfrac{2}{3}$ J

    $= 8.4 \times 10^6$ J.
11. **(a) :** For mixture of gases,

    $$\frac{n_1+n_2}{\gamma_m-1} = \frac{n_1}{\gamma_1-1} + \frac{n_2}{\gamma_2-1}$$

    or $\dfrac{1+1}{\gamma_m-1} = \dfrac{1}{\frac{5}{3}-1} + \dfrac{1}{\frac{7}{5}-1}$

    or $\dfrac{2}{\gamma_m-1} = \dfrac{3}{2}+\dfrac{5}{2} = 4 \Rightarrow \gamma_m - 1 = 0.5$

    $\therefore \quad \gamma_m = 1.5 = 3/2$.
12. **(b) :** Internal energy and entropy are state functions.
13. **(c) :** For the system, the total number of moles remains constant.

    $$\therefore \quad \frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2} = \frac{P(V_1+V_2)}{RT}$$

    or $T = \dfrac{P(V_1+V_2)T_1T_2}{P_1V_1T_2+P_2V_2T_1}$

    Apply Boyle's law

    $P_1V_1 + P_2V_2 = P(V_1 + V_2)$

$\therefore \quad T = \frac{(P_1V_1 + P_2V_2)T_1T_2}{(P_1V_1T_2 + P_2V_2T_1)}.$

14. (b, c) : Statements (b) and (c) are incorrect regarding the first law of thermodynamics.

15. (a) : Efficiency $\eta = 1 - \frac{Q_2}{Q_1}$

$Q_2 = T_0\ (2S_0 - S_0) = T_0\ S_0$

$Q_1 = T_0S_0 + \frac{T_0S_0}{2} = \frac{3}{2}T_0S_0$

$\therefore \quad \eta = 1 - \frac{T_0S_0 \times 2}{3T_0S_0} = 1 - \frac{2}{3} = \frac{1}{3}.$

16. (c) : $\Delta U_1 = \Delta U_2$, because the change in internal energy depends only upon the initial and final states A and B.

17. (d) : For 16 g of helium, $n_1 = \frac{16}{4} = 4$

For 16 g of oxygen, $n_2 = \frac{16}{32} = \frac{1}{2}$

For mixture of gases,

$C_V = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2}$ where $C_V = \frac{f}{2}R$

$C_P = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1 + n_2}$ where $C_P = \left(\frac{f}{2} + 1\right)R$

For helium, $f = 3$, $n_1 = 4$

For oxygen, $f = 5$, $n_2 = 1/2$

$$\therefore \quad \frac{C_P}{C_V} = \frac{\left(4 \times \frac{5}{2}R\right) + \left(\frac{1}{2} \times \frac{7}{2}R\right)}{\left(4 \times \frac{3}{2}R\right) + \left(\frac{1}{2} \times \frac{5}{2}R\right)} = \frac{47}{29} = 1.62.$$

18. (b) : According to first law of thermodynamics

$\Delta Q = \Delta U + \Delta W$

For an adiabatic process, $\Delta Q = 0$

$\therefore \quad 0 = \Delta U + \Delta W$

or $\quad \Delta U = -\Delta W$

or $\quad nC_V\,\Delta T = -\Delta W$

or $\quad C_V = \frac{-\Delta W}{n\Delta T} = \frac{-(-146) \times 10^3}{(1 \times 10^3) \times 7}$

$= 20.8\ \text{J mol}^{-1}\ \text{K}^{-1}$

For diatomic gas,

$C_V = \frac{5}{2}R = \frac{5}{2} \times 8.3 = 20.8\ \text{J mol}^{-1}\ \text{K}^{-1}$

Hence the gas is diatomic.

19. (d) : $\Delta U = 0$

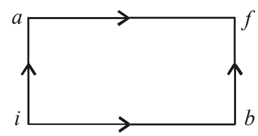
$\therefore \quad 1 \times \left(\frac{5}{2}R\right)(T_f - T_0) + 1 \times \frac{3}{2}R\left(T_f - \frac{7}{3}T_0\right) = 0$

or $\quad 5T_f - 5T_0 + 3T_f - 7T_0 = 0$

or $\quad 8T_f = 12T_0$

or $\quad T_f = \frac{3}{2}T_0.$

20. (b) : According to first law of thermodynamics for the path iaf,



$Q_{iaf} = \Delta U_{iaf} + W_{iaf}$

or $\quad \Delta U_{iaf} = Q_{iaf} - W_{iaf}$

$= 50 - 20 = 30$ cal

For the path ibf,

$Q_{ibf} = \Delta U_{ibf} + W_{ibf}$

Since $\Delta U_{iaf} = \Delta U_{ibf}$, change in internal energy are path independent.

$Q_{ibf} = \Delta U_{iaf} + W_{ibf}$

$\therefore \quad W_{ibf} = Q_{ibf} - \Delta U_{iaf} = 36 - 30 = 6$ cal.

21. (c) : For Carnot engine efficiency $\eta = \frac{Q_H - Q_L}{Q_L}$

Coefficient of performance of a refrigerator

$\beta = \frac{1 - \eta}{\eta}$

$\beta = \frac{1 - \frac{1}{10}}{1/10} = 9$

Also $\beta = \frac{Q_L}{W}$ (where W is the work done)

or $\quad Q_L = \beta \times W = 9 \times 10 = 90$ J.

22. (b) : Molar heat capacity = Molar mass × specific heat capacity

So, the molar heat capacities at constant pressure and constant volume will be $28C_P$ and $28C_V$ respectively

$\therefore \quad 28C_P - 28C_V = R \quad$ or $\quad C_P - C_V = \frac{R}{28}.$

23. (b) : As this is a simple mixing of gas, even if adiabatic conditions are satisfied, $PV = nRT$ for adiabatic as well as isothermal changes. The total number of molecules is conserved.

$$\therefore\ n_1 = \frac{P_1V_1}{R_1T_1}, n_2 = \frac{P_2V_2}{RT_2}$$

Final state $= (n_1 + n_2)RT$

$$(n_1 + n_2) = \frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2} = \frac{T_2P_1V_1 + T_1P_2V_2}{RT_1T_2}$$

$$T = \frac{T_1n_1 + T_2n_2}{n_1 + n_2},\ T = \frac{T_1T_2(P_1V_1 + P_2V_2)}{T_2P_1V_1 + T_1P_2V_2}$$

Transfer of Heat

1. Which of the following is more close to a black body?
(a) black board paint (b) green leaves
(c) black holes (d) red roses.
(2002)

2. Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K respectively. The ratio of the energy radiated per second by the first sphere to that by the second is
(a) 1 : 1 (b) 16 : 1 (c) 4 : 1 (d) 1 : 9.
(2002)

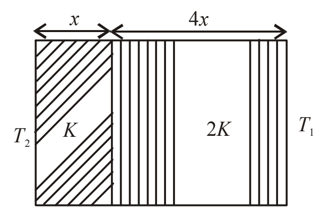
3. The earth radiates in the infra-red region of the spectrum. The wavelength of the maximum intensity of the spectrum is correctly given by
(a) Rayleigh Jeans law
(b) Planck's law of radiation
(c) Stefan's law of radiation
(d) Wien's law.
(2003)

4. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and the surroundings, and n is equal to
(a) two (b) three (c) four (d) one.
(2003)

5. If the temperature of the sun were to increase from T to $2T$ and its radius from R to $2R$, then the ratio of the radiant energy received on earth to what it was previously will be
(a) 4 (b) 16 (c) 32 (d) 64.
(2004)

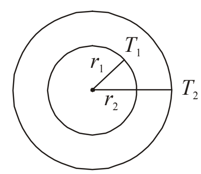
6. A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is
(a) E/c (b) $2E/c$
(c) Ec (d) E/c^2.
(2004)

7. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab, in a steady state is $\left(\dfrac{A(T_2 - T_1)K}{x}\right)f$, with f equal to



(a) 1 (b) 1/2
(c) 2/3 (d) 1/3.
(2004)

8. The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperatures T_1 and T_2, respectively. The radial rate of flow of heat in a substance between the two concentric spheres is proportional to



(a) $\dfrac{r_1 r_2}{(r_2 - r_1)}$ (b) $(r_2 - r_1)$
(c) $\dfrac{(r_2 - r_1)}{r_1 r_2}$ (d) $\ln\left(\dfrac{r_2}{r_1}\right)$
(2005)

9. Assuming the sun to be a spherical body of radius R at a temperature of T K, evaluate the total radiant power, incident on earth, at a distance r from the sun.

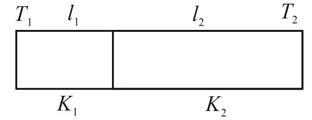
(a) $\frac{R^2\sigma T^4}{r^2}$ (b) $\frac{4\pi r_0^2 R^2\sigma T^4}{r^2}$

(c) $\frac{\pi r_0^2 R^2\sigma T^4}{r^2}$ (d) $\frac{r_0^2 R^2\sigma T^4}{4\pi r^2}$.

where r_0 is the radius of the earth and σ is Stefan's constant.

(2006)

10. One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of lengths l_1 and l_2 and thermal conductivities K_1 and K_2 respectively. The temperature at the interface of the two sections is



(a) $\frac{(K_1l_1T_1 + K_2l_2T_2)}{(K_1l_1 + K_2l_2)}$ (b) $\frac{(K_2l_2T_1 + K_1l_1T_2)}{(K_1l_1 + K_2l_2)}$

(c) $\frac{(K_2l_1T_1 + K_1l_2T_2)}{(K_2l_1 + K_1l_2)}$ (d) $\frac{(K_1l_2T_1 + K_2l_1T_2)}{(K_1l_2 + K_2l_1)}$

(2007)

Answer Key

1.	(a)	**2.**	(a)	**3.**	(d)	**4.**	(d)	**5.**	(d)	**6.**	(b)
7.	(d)	**8.**	(a)	**9.**	(c)	**10.**	(d)				

1. **(a) :** A good absorber is a good emitter but black holes do not emit all radiations.

2. **(a) :** Energy radiated
$E = \sigma T^4 \times (\text{area } 4\pi R^2) \times \text{time} \times e$
$$\frac{E_1}{E_2} = \frac{(4000)^4 \times (1)^2 \times 1 \times 4\pi\sigma e}{(2000)^4 \times (4)^2 \times 1 \times 4\pi\sigma e} = \frac{1}{1}.$$

3. **(d) :** Wien's law

4. **(d) :** According to Newton's law of cooling, rate of cooling is proportional to $\Delta\theta$
$\therefore \quad (\Delta\theta)^n = (\Delta\theta)$ or $n = 1$.

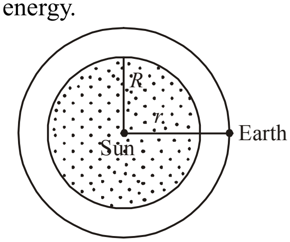
5. **(d) :** According to Stefan's law,
Radiant energy $E = (\sigma T^4) \times \text{area} \times \text{time}$
$$\therefore \quad \frac{E_2}{E_1} = \frac{\sigma(2T)^4 \times 4\pi(2R)^2 \times t}{\sigma T^4 \times (4\pi R)^2 \times t} = 16 \times 4$$
$$\therefore \quad \frac{E_2}{E_1} = 64.$$

6. **(b) :** Initial momentum $= E/c$
Final momentum $= -E/c$
$$\therefore \quad \text{Change of momentum} = \frac{E}{c} - \left(-\frac{E}{c}\right) = \frac{2E}{c}$$
$$\therefore \quad \text{Momentum transferred to surface} = \frac{2E}{c}.$$

7. **(d) :** From first surface, $Q_1 = \dfrac{KA(T_2 - T)t}{x}$
From second surface, $Q_2 = \dfrac{(2K)A(T - T_1)t}{(4x)}$
At steady state,
$$Q_1 = Q_2 \Rightarrow \frac{KA(T_2 - T)t}{x} = \frac{2KA(T - T_1)t}{4x}$$
or $\quad 2(T_2 - T) = (T - T_1)$
or $\quad T = \dfrac{2T_2 + T_1}{3}$
$$\therefore \quad Q_1 = \frac{KA}{x}\left[T_2 - \frac{2T_2 + T_1}{3}\right]t$$
or $\quad \left[\dfrac{A(T_2 - T_1)K}{x}\right]f = \dfrac{KA}{x}\left[\dfrac{T_2 - T_1}{3}\right] \times 1$
or $\quad f = \dfrac{1}{3}.$

8. **(a) :** For conduction from inner sphere to outer one,
$$dQ = -KA\frac{dT}{dr} \times (\text{time } dt)$$
or $\quad \dfrac{dQ}{dt} = -K \times (4\pi r^2)\dfrac{dT}{dr}$
$\therefore \quad$ Radial rate of flow $Q = -4\pi K r^2 \dfrac{dT}{dr}$
$$\therefore \quad Q\int_{r_1}^{r_2} \frac{dr}{r^2} = -4\pi K \int_{T_1}^{T_2} dT$$
or
or $\quad Q\left[\dfrac{r_1 - r_2}{r_1 r_2}\right] = 4\pi K\left[T_2 - T_1\right]$
or $\quad Q = \dfrac{4\pi K(T_1 - T_2)r_1 r_2}{(r_2 - r_1)}$
$\therefore \quad Q$ is proportional to $\left(\dfrac{r_1 r_2}{r_2 - r_1}\right)$.

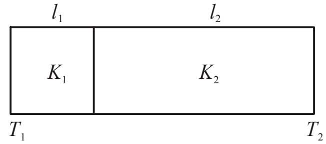
9. **(c) :** Energy radiated by sun, according to Stefan's law,
$E = \sigma T^4 \times (\text{area } 4\pi R^2)\ (\text{time})$
This energy is spread around sun in space, in a sphere of radius r. Earth (E) in space receives part of this energy.



$$\frac{\text{Energy}}{\text{Area of envelope}} = \frac{\sigma T^4 \times 4\pi R^2 \times \text{time}}{4\pi r^2}$$
Energy incident per unit area on earth
$$= \frac{\sigma T^4 R^2 \times \text{time}}{r^2}$$
$\therefore \quad$ Power incident per unit area on earth
$$= \left(\frac{R^2 \sigma T^4}{r^2}\right)$$

$\therefore$ Power incident on earth $= \pi r_0^2 \times \frac{R^2 \sigma T^4}{r^2}$

10. (d) : Let T be the temperature of the interface.



Since two section of rod are in series, rate of flow of heat in them will be equal

$\therefore \quad \frac{K_1 A [T_1 - T]}{l_1} = \frac{K_2 A [T - T_2]}{l_2}$

or $\quad K_1 l_2 (T_1 - T) = K_2 l_1 (T - T_2)$

or $\quad T(K_1 l_2 + K_2 l_1) = K_1 l_2 T_1 + K_2 l_1 T_2$

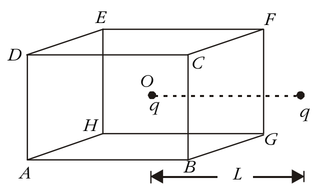
or $\quad T = \frac{K_1 l_2 T_1 + K_2 l_1 T_2}{K_1 l_2 + K_2 l_1}$.

CHAPTER 13

Electrostatics

1. On moving a charge of 20 coulomb by 2 cm, 2 J of work is done, then the potential difference between the points is
(a) 0.1 V (b) 8 V (c) 2 V (d) 0.5 V.
(2002)

2. A charged particle q is placed at the centre O of cube of length L ($ABCDEFGH$). Another same charge q is placed at a distance L from O. Then the electric flux through $ABCD$ is



(a) $q/4\pi\varepsilon_0 L$ (b) zero
(c) $q/2\pi\varepsilon_0 L$ (d) $q/3\pi\varepsilon_0 L$.
(2002)

3. If there are n capacitors in parallel connected to V volt source, then the energy stored is equal to
(a) CV (b) $\frac{1}{2}nCV^2$
(c) CV^2 (d) $\frac{1}{2n}CV^2$.
(2002)

4. If a charge q is placed at the centre of the line joining two equal charges Q such that the system is in equilibrium then the value of q is
(a) $Q/2$ (b) $-Q/2$ (c) $Q/4$ (d) $-Q/4$.
(2002)

5. Capacitance (in F) of a spherical conductor with radius 1 m is
(a) 1.1×10^{-10} (b) 10^{-6}
(c) 9×10^{-9} (d) 10^{-3}.
(2002)

6. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2, the electric charge inside the surface will be
(a) $(\phi_2 - \phi_1)\varepsilon_0$ (b) $(\phi_1 + \phi_2)/\varepsilon_0$
(c) $(\phi_2 - \phi_1)/\varepsilon_0$ (d) $(\phi_1 + \phi_2)\varepsilon_0$.
(2003)

7. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor
(a) decreases
(b) remains unchanged
(c) becomes infinite
(d) increases.
(2003)

8. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P at a distance $R/2$ from the centre of the shell is
(a) $\frac{2Q}{4\pi\varepsilon_0 R}$ (b) $\frac{2Q}{4\pi\varepsilon_0 R} - \frac{2q}{4\pi\varepsilon_0 R}$
(c) $\frac{2Q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 R}$ (d) $\frac{(q+Q)}{4\pi\varepsilon_0}\frac{2}{R}$.
(2003)

9. The work done in placing a charge of 8×10^{-18} coulomb on a condenser of capacity 100 micro-farad is
(a) 16×10^{-32} joule (b) 3.1×10^{-26} joule
(c) 4×10^{-10} joule (d) 32×10^{-32} joule.
(2003)

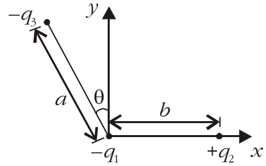
10. Three charges $-q_1$, $+q_2$ and $-q_3$ are placed as shown in the figure. The x-component of the force on $-q_1$ is proportional to

(a) $\frac{q_2}{b^2}-\frac{q_3}{a^2}\cos\theta$

(b) $\frac{q_2}{b^2}+\frac{q_3}{a^2}\sin\theta$

(c) $\frac{q_2}{b^2}+\frac{q_3}{a^2}\cos\theta$

(d) $\frac{q_2}{b^2}-\frac{q_3}{a^2}\sin\theta$



(2003)

11. Two spherical conductors B and C having equal radii and carrying equal charges in them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that of B but uncharged is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is

(a) $F/4$ (b) $3F/4$ (c) $F/8$ (d) $3F/8$.

(2004)

12. A charged particle q is shot towards another charged particle Q which is fixed, with a speed v. It approaches Q upto a closest distance r and then returns. If q were given a speed $2v$, the closest distances of approach would be

(a) r (b) $2r$ (c) $r/2$ (d) $r/4$.

(2004)

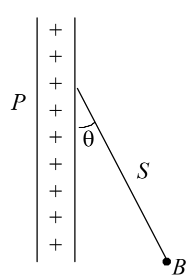
13. Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium the value of q is

(a) $-\frac{Q}{4}(1+2\sqrt{2})$ (b) $\frac{Q}{4}(1+2\sqrt{2})$

(c) $-\frac{Q}{2}(1+2\sqrt{2})$ (d) $\frac{Q}{2}(1+2\sqrt{2})$.

(2004)

14. A charged ball B hangs from a silk thread S, which makes an angle θ with a large charged conducting sheet P, as shown in the figure. The surface charge density σ of the sheet is proportional to



(a) $\sin\theta$

(b) $\tan\theta$

(c) $\cos\theta$ (d) $\cot\theta$

(2005)

15. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$ respectively. The location of a point on the x axis at which the net electric field due to these two point charges is zero is

(a) $8L$ (b) $4L$ (c) $2L$ (d) $L/4$

(2005)

16. Two thin wire rings each having a radius R are placed at a distance d apart with their axes coinciding. The charges on the two rings are $+Q$ and $-Q$. The potential difference between the centers of the two rings is

(a) zero

(b) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$

(c) $\frac{QR}{4\pi\varepsilon_0 d^2}$

(d) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$

(2005)

17. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is C then the resultant capacitance is

(a) C (b) nC

(c) $(n-1)C$ (d) $(n+1)C$

(2005)

18. A fully charged capacitor has a capacitance C. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity s and mass m. If the temperature of the block is raised by ΔT, the potential difference V across the capacitance is

(a) $\frac{ms\Delta T}{C}$ (b) $\sqrt{\frac{2ms\Delta T}{C}}$

(c) $\sqrt{\frac{2mC\Delta T}{s}}$ (d) $\frac{mC\Delta T}{s}$

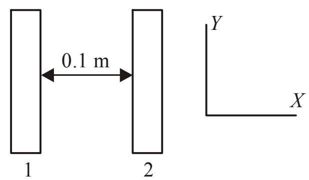
(2005)

19. A electric dipole is placed at an angle of 30º to a non-uniform electric field. The dipole will experience

(a) a torque only
(b) a translational force only in the direction of the field
(c) a translational force only in a direction normal to the direction of the field
(d) a torque as well as a translational force.

(2006)

20. Two insulating plates are both uniformaly charged in such a way that the potential difference between them is $V_2 - V_1 = 20$ V. (*i.e.* plate 2 is at a higher potential). The plates are separated by $d = 0.1$ m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate 1. What is its speed when it hits plate 2? ($e = 1.6 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg)



(a) 32×10^{-19} m/s
(b) 2.65×10^{6} m/s
(c) 7.02×10^{12} m/s
(d) 1.87×10^{6} m/s.

(2006)

21. Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire then in equilibrium condition, the ratio of the magnitude of the electric fields at the surface of spheres A and B is

(a) 1 : 4
(b) 4 : 1
(c) 1 : 2
(d) 2 : 1.

(2006)

22. An electric charge 10^{-3} μC is placed at the origin (0, 0) of $X - Y$ co-ordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2,0) respectively. The potential difference between the points A and B will be

(a) 4.5 volt
(b) 9 volt
(c) zero
(d) 2 volt

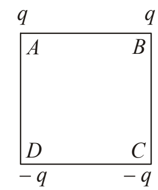
(2007)

23. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be

(a) 1/2
(b) 1
(c) 2
(d) 1/4

(2007)

24. Charges are placed on the vertices of a square as shown. Let $\vec{E}$ be the electric field and V the potential at the centre. If the charges on A and B are interchanged with those on D and C respectively, then



(a) $\vec{E}$ changes, V remains unchanged
(b) $\vec{E}$ remains unchanged, V changes
(c) both $\vec{E}$ and V change
(d) $\vec{E}$ and V remain unchanged

(2007)

25. The potential at a point x (measured in μm) due to some charges situated on the x-axis is given by

$$V(x) = 20/(x^2 - 4) \text{ volt}$$

The electric field E at $x = 4$ μm is given by

(a) (10/9) volt/μm and in the +ve x direction
(b) (5/3) volt/μm and in the –ve x direction
(c) (5/3) volt/μm and in the +ve x direction
(d) (10/9) volt/μm in the –ve x direction

(2007)

26. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volt. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is

(a) zero
(b) $\frac{1}{2}(K - 1)\, CV^2$
(c) $\dfrac{CV^2(K-1)}{K}$
(d) $(K - 1)\, CV^2$

(2007)

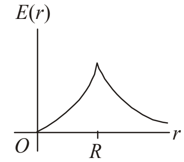
27. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is d. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $k_1 = 3$ and thickness $d/3$ while the other one has dielectric constant $k_2 = 6$

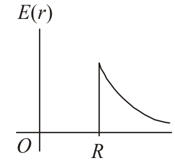
and thickness $2d/3$. Capacitance of the capacitor is now

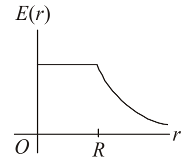
(a) 20.25 pF (b) 1.8 pF
(c) 45 pF (d) 40.5 pF.

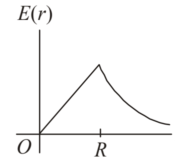
(2008)

28. A thin spherical shell of radius R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field $E(r)$ produced by the shell in the range $0 \le r < \infty$, where r is the distance from the centre of the shell?

(a) 

(b) 

(c) 

(d) 

(2008)

Answer Key

1.	(a)	**2.**	No option	**3.**	(b)	**4.**	(d)	**5.**	(a)	**6.**	(a)
7.	(b)	**8.**	(c)	**9.**	(d)	**10.**	(b)	**11.**	(d)	**12.**	(d)
13.	(b)	**14.**	(b)	**15.**	(c)	**16.**	(d)	**17.**	(c)	**18.**	(b)
19.	(d)	**20.**	(b)	**21.**	(d)	**22.**	(c)	**23.**	(a)	**24.**	(a)
25.	(a)	**26.**	(a)	**27.**	(d)	**28.**	(b)				

1. **(a) :** $W = QV$

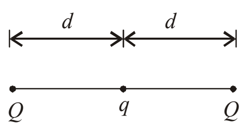
$\therefore \quad V = \frac{W}{Q} = \frac{2}{20} = 0.1$ volt.

2. Electric flux through $ABCD$ = zero for the charge placed outside the box as the charged enclosed is zero. But for the charge inside the cube, it is $\frac{q}{\varepsilon_0}$ through all the surfaces. For one surface, it is $\frac{q}{6\varepsilon_0}$. (Option not given).

3. **(b) :** Total capacity = nC

$\therefore \quad$ Energy $= \frac{1}{2}nCV^2$

4. **(d) :** When the system of three charges is in equilibrium, $\frac{Q \times q}{4\pi\varepsilon_0 d^2} + \frac{Q \times Q}{4\pi\varepsilon_0 (2d)^2} = 0$



or $\quad q = -\frac{Q}{4}$.

5. **(a) :** $C = 4\pi\varepsilon_0 R = \frac{1}{9 \times 10^9} = 1.1 \times 10^{-10}$ F .

6. **(a) :** According to Gauss theorem,

$$(\phi_2 - \phi_1) = \frac{Q}{\varepsilon_0} \Rightarrow Q = (\phi_2 - \phi_1)\varepsilon_0 .$$

The flux enters the enclosure if one has a negative charge ($-q_2$) and flux goes out if one has a +ve charge ($+q_1$). As one does not know whether $\phi_1 > \phi_2$, $\phi_2 > \phi_1$, $Q = q_1 \sim q_2$

7. **(b) :** Aluminium is a good conductor. Its sheet introduced between the plates of a capacitor is of negligible thickness. The capacity remains unchanged.

With air as dielectric, $C = \frac{\varepsilon_0 A}{d}$

With space partially filled, $C' = \frac{\varepsilon_0 A}{(d-t)} = \frac{\varepsilon_0 A}{d} = C$.

8. **(c) :** Potential at any internal point of charged shell $= \frac{q}{4\pi\varepsilon_0 R}$

Potential at P due to Q at centre $= \frac{1}{4\pi\varepsilon_0}\frac{2Q}{R}$

$\therefore \quad$ Total potential point

$$= \frac{q}{4\pi\varepsilon_0 R} + \frac{2Q}{4\pi\varepsilon_0 R} = \frac{1}{4\pi\varepsilon_0 R}(q + 2Q)$$

9. **(d) :** Energy of condenser

$$= \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} \times \frac{(8 \times 10^{-18})^2}{(100 \times 10^{-6})} = 32 \times 10^{-32} \text{ J}$$

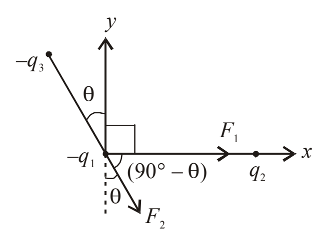
10. **(b) :** Force on $(-q_1)$ due to $q_2 = \frac{-q_1 q_2}{4\pi\varepsilon_0 b^2}$

$\therefore \quad F_1 = \frac{q_1 q_2}{4\pi\varepsilon_0 b^2}$ along $(q_1 q_2)$

Force on $(-q_1)$ due to $(-q_3) = \frac{(-q_1)(-q_3)}{4\pi\varepsilon_0 a^2}$

$F_2 = \frac{q_1 q_3}{4\pi\varepsilon_0 a^2}$ as shown

F_2 makes an angle of $(90° - \theta)$ with $(q_1 q_2)$



Resolved part of F_2 along $q_1 q_2$
$= F_2 \cos(90° - \theta)$

$= \frac{q_1 q_3 \sin\theta}{4\pi\varepsilon_0 a^2}$ along $(q_1 q_2)$

$\therefore \quad$ Total force on $(-q_1)$

$$= \left[\frac{q_1 q_2}{4\pi\varepsilon_0 b^2} + \frac{q_1 q_3 \sin\theta}{4\pi\varepsilon_0 a^2}\right] \text{along } x\text{-axis}$$

$\therefore$ x-component of force $\propto \left[\dfrac{q_2}{b^2} + \dfrac{q_3}{a^2}\sin\theta\right]$.

11. (d) : Initially, $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{d^2}$ ....(i)

when the third equal conductor touches B, the charge of B is shared equally between them

$\therefore$ Charge on $B = \dfrac{q}{2}$ = charge on third conductor

Now this third conductor with charge $\left(\dfrac{q}{2}\right)$ touches C, their total charge $\left(q + \dfrac{q}{2}\right)$ is equally shared between them.

$\therefore$ Charge on

$C = \dfrac{3q}{4}$ = Charge of third conductor

$\therefore$ New force between B and C

$$= \frac{1}{4\pi\varepsilon_0 d^2}\left(\frac{q}{2}\times\frac{3q}{4}\right) = \frac{3}{8}F$$

12. (d) : Energy is conserved in the phenomenon

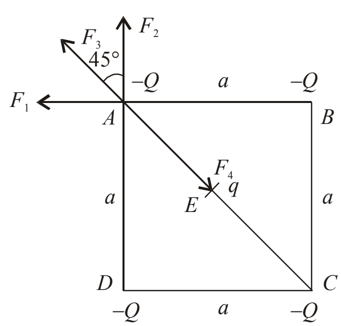
Initially, $\dfrac{1}{2}mv^2 = \dfrac{kqQ}{r}$ ....(i)

Finally, $\dfrac{1}{2}m(2v)^2 = \dfrac{kqQ}{r_1}$ ....(ii)

$\therefore$ From (i) and (ii)

$\dfrac{1}{4} = \dfrac{r_1}{r} \Rightarrow r_1 = \dfrac{r}{4}$.

13. (b) : Consider the four forces F_1, F_2, F_3 and F_4 acting on charge $(-Q)$ placed at A.



Distance $CA = \sqrt{2}\,a$

Distance $EA = \dfrac{\sqrt{2}\,a}{2} = \dfrac{a}{\sqrt{2}}$

For equilibrium, consider forces along DA and equate the resultant to zero

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\frac{Q\times Q}{(DA)^2} + \frac{1}{4\pi\varepsilon_0}\frac{Q\times Q}{(CA)^2}\cos 45^\circ$$

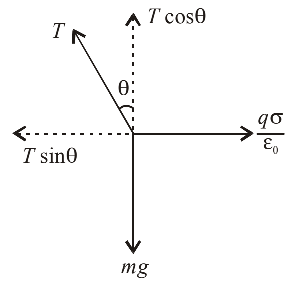
$$-\frac{1}{4\pi\varepsilon_0}\frac{Q\times q}{(EA)^2}\cos 45^\circ = 0$$

or $\dfrac{Q}{a^2} + \dfrac{Q}{2a^2}\times\dfrac{1}{\sqrt{2}} - \dfrac{q}{a^2/2}\times\dfrac{1}{\sqrt{2}} = 0$

or $Q\left[1 + \dfrac{1}{2\sqrt{2}}\right] = q\sqrt{2}$

or $q = \dfrac{Q}{\sqrt{2}}\left[\dfrac{2\sqrt{2}+1}{2\sqrt{2}}\right] = \dfrac{Q}{4}(1 + 2\sqrt{2})$.

14. (b) : $T\sin\theta = \sigma q/\varepsilon_0$

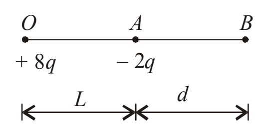


$T\cos\theta = mg$

$\therefore$ $\tan\theta = \dfrac{\sigma q}{\varepsilon_0 mg}$

$\therefore$ σ is proportional to $\tan\theta$.

15. (c) : Resultant intensity = 0



$$\frac{1}{4\pi\varepsilon_0}\frac{8q}{(L+d)^2} - \frac{1}{4\pi\varepsilon_0}\frac{2q}{d^2} = 0$$

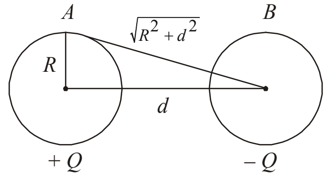
or $(L + d)^2 = 4d^2$

or $d = L$

$\therefore$ Distance from origin = $2L$.

16. (d) : $V_A = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R} - \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{\sqrt{R^2+d^2}}$

$V_B = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(-Q)}{R} + \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{\sqrt{R^2+d^2}}$



$\therefore \quad V_A - V_B = \dfrac{1\times Q}{4\pi\varepsilon_0}\left[\dfrac{2}{R} - \dfrac{2}{\sqrt{R^2+d^2}}\right]$

$= \dfrac{Q}{2\pi\varepsilon_0}\left[\dfrac{1}{R} - \dfrac{1}{\sqrt{R^2+d^2}}\right].$

17. **(c) :** n plates connected alternately give rise to $(n-1)$ capacitors connected in parallel

$\therefore$ Resultant capacitance = $(n-1)C$.

18. **(b) :** Energy of capacitor = Heat energy of block

$\therefore \quad \dfrac{1}{2}CV^2 = ms\Delta T$

or $\quad V = \sqrt{\dfrac{2ms\,\Delta T}{C}}.$

19. **(d) :** In a non-uniform electric field, the dipole will experience a torque as well as a translational force.

20. **(b) :** An electron on plate 1 has electrostatic potential energy. When it moves, potential energy is converted into kinetic energy

$\therefore$ Kinetic energy = Electrostatic potential energy

or $\quad \dfrac{1}{2}mv^2 = e\Delta V$

or $\quad v = \sqrt{\dfrac{2e\times\Delta V}{m}} = \sqrt{\dfrac{2\times1.6\times10^{-19}\times20}{9.11\times10^{-31}}}$

or $\quad v = 2.65 \times 10^6$ m/s.

21. **(d) :** When the spherical conductors are connected by a conducting wire, charge is redistributed and the spheres attain a common potential V.

$\therefore$ Intensity $E_A = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_A}{R_A^2}$

or $\quad E_A = \dfrac{1\times C_AV}{4\pi\varepsilon_0R_A^2} = \dfrac{(4\pi\varepsilon_0R_A)V}{4\pi\varepsilon_0R_A^2} = \dfrac{V}{R_A}$

Similarly $E_B = \dfrac{V}{R_B}$

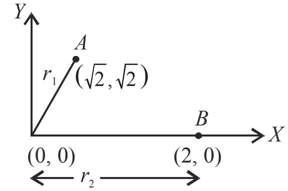
$\therefore \quad \dfrac{E_A}{E_B} = \dfrac{R_B}{R_A} = \dfrac{2}{1}.$

22. **(c) :** $\vec{r}_1 = \sqrt{2}\hat{i} + \sqrt{2}\hat{j}$

$|\vec{r}_1| = r_1 = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = 2$

$\vec{r}_2 = 2\hat{i} + 0\hat{j}$

or $\quad |\vec{r}_2| = r_2 = 2$



Potential at point A is

$V_A = \dfrac{1q}{4\pi\varepsilon_0 r_1}$

$= \dfrac{1}{4\pi\varepsilon_0}\dfrac{10^{-3}\times10^{-6}}{2}$

Potential at point B is

$V_B = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r_2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{10^{-3}\times10^{-6}}{2}$

$\therefore \quad V_A - V_B = 0.$

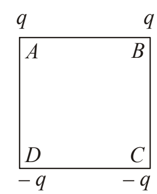
23. **(a) :** Let E be emf of the battery

Work done by the battery $W = CE^2$

Energy stored in the capacitor $U = \dfrac{1}{2}CE^2$

$\therefore \quad \dfrac{U}{W} = \dfrac{\frac{1}{2}CE^2}{CE^2} = \dfrac{1}{2}.$

24. **(a) :** "Unit positive charge" will be repelled by A and B and attracted by $-q$ and $-q$ downwards in the same direction. If they are exchanged, the direction of the field will be opposite. In the case of potential, as it is a scalar, they cancel each other whatever may be their position.



$\therefore$ Field is affected but not the potential.

25. **(a) :** Given : Potential $V(x) = \dfrac{20}{x^2-4}$

Electric field $E = \dfrac{-dV}{dx} = \dfrac{-d}{dx}\left(\dfrac{20}{x^2-4}\right) = \dfrac{40x}{(x^2-4)^2}$

At $x = 4$ μm

$\therefore \quad E = \dfrac{40\times4}{[16-4]^2} = \dfrac{160}{144} = \dfrac{10}{9}$ V/μm.

Positive sign indicate E is +ve x direction.

26. **(a) :** The potential energy of a charged capacitor

$$U_i = \frac{q^2}{2C}$$

where U_i is the initial potential energy.

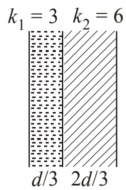
If a dielectric slab is slowly introduced, the energy

$$= \frac{q^2}{2KC}$$

Once is taken out, again the energy increases to the old value.

Therefore after it is taken out, the potential energy come back to the old value. Total work done = zero.

27. (d) : $C = \frac{\varepsilon_0 A}{d} = 9 \times 10^{-12}$ F



With dielectric, $C = \frac{\varepsilon_0 kA}{d}$

$C_1 = \frac{\varepsilon_0 A \cdot 3}{d/3} = 9C$;

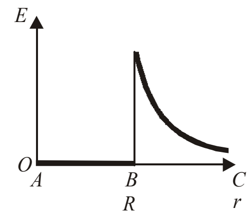
$C_2 = \frac{\varepsilon_0 A \cdot 6}{2d/3} = 9C$

$\therefore\ C_{\text{total}} = \frac{C_1 C_2}{C_1 + C_2}$ as they are in series.

$= \frac{9C \times 9C}{18C} = \frac{9}{2} \times C$ or $\frac{9}{2} \times 9 \times 10^{-12}$ F

$\Rightarrow\ C_{\text{total}} = 40.5$ pF.

28. (b) : The electric field for a uniformly charged spherical shell is given in the figure. Inside the shell, the field is zero and it is maximum at the surface and then decreases $\propto 1/r^2$.



$E = \frac{Q}{4\pi\varepsilon_0 \cdot r^2}$ outside shell and zero inside.

CHAPTER 14

Current Electricity

1. The length of a wire of a potentiometer is 100 cm, and the e.m.f. of its standard cell is E volt. It is employed to measure the e.m.f. of a battery whose internal resistance is 0.5 Ω. If the balance point is obtained at l = 30 cm from the positive end, the e.m.f. of the battery is

(a) $\frac{30E}{100.5}$

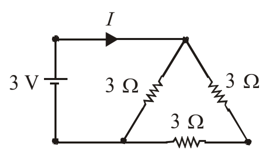
(b) $\frac{30E}{100-0.5}$

(c) $\frac{30E}{100}-0.5i$, where i is the current in the potentiometer wire.

(d) $\frac{30E}{100}$.

(2003)

2. A 3 volt battery with negligible internal resistance is connected in a circuit as shown in the figure. The current I, in the circuit will be



(a) 1 A
(b) 1.5 A
(c) 2 A
(d) (1/3) A

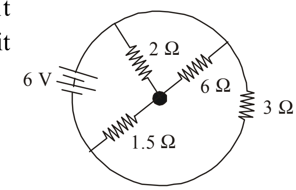
(2003)

3. The length of a given cylindrical wire is increased by 100%. Due to the consequent decrease in diameter the change in the resistance of the wire will be

(a) 200% (b) 100%
(c) 50% (d) 300%.

(2003)

4. The total current supplied to the circuit by the battery is



(a) 1 A
(b) 2 A
(c) 4 A
(d) 6 A.

(2004)

5. The resistance of the series combination of two resistances is S. When they are joined in parallel the total resistance is P. If $S = nP$, then the minimum possible value of n is

(a) 4 (b) 3
(c) 2 (d) 1.

(2004)

6. An electric current is passed through a circuit containing two wires of the same material, connected in parallel. If the lengths and radii of the wires are in the ratio of 4/3 and 2/3, then the ratio of the currents passing through the wire will be

(a) 3 (b) 1/3
(c) 8/9 (d) 2.

(2004)

7. In a metre bridge experiment null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y. If $X < Y$, then where will be the new position of the null point from the same end, if one decides to balance

a resistance of $4X$ against Y?

(a) 50 cm (b) 80 cm
(c) 40 cm (d) 70 cm.

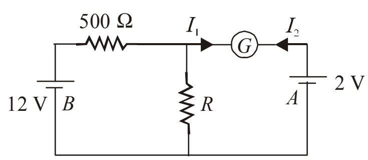
(2004)

8. The thermistors are usually made of

(a) metals with low temperature coefficient of resistivity
(b) metals with high temperature coefficient of resistivity
(c) metal oxides with high temperature coefficient of resistivity
(d) semiconducting materials having low temperature coefficient of resistivity.

(2004)

9. In the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be



(a) 500 Ω (b) 1000 Ω
(c) 200 Ω (d) 100 Ω

(2005)

10. Two sources of equal emf are connected to an external resistance R. The internal resistances of the two sources are R_1 and R_2 ($R_2 > R_1$). If the potential difference across the source having internal resistance R_2 is zero, then

(a) $R = \frac{R_1R_2}{R_1 + R_2}$ (b) $R = \frac{R_1R_2}{R_2 - R_1}$

(c) $R = R_2\frac{(R_1 + R_2)}{(R_2 - R_1)}$ (d) $R = R_2 - R_1$

(2005)

11. In a potentiometer experiment the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of 2 Ω, the balancing length becomes 120 cm. The internal resistance of the cell is

(a) 4 Ω (b) 2 Ω
(c) 1 Ω (d) 0.5 Ω

(2005)

12. The Kirchhoff's first law $(\sum i = 0)$ and second law $(\sum iR = \sum E)$, where the symbols have their usual meanings, are respectively based on

(a) conservation of charge, conservation of energy
(b) conservation of charge, conservation of momentum
(c) conservation of energy, conservation of charge
(d) conservation of momentum, conservation of charge.

(2006)

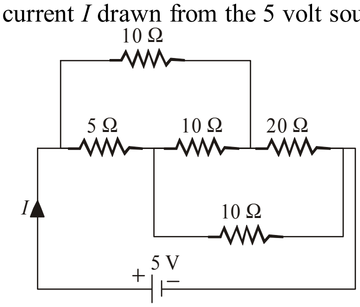
13. In a Wheatstone's bridge, three resistance P, Q and R connected in the three arms and the fourth arm is formed by two resistance S_1 and S_2 connected in parallel. The condition for bridge to be balanced will be

(a) $\frac{P}{Q} = \frac{R}{S_1 + S_2}$ (b) $\frac{P}{Q} = \frac{2R}{S_1 + S_2}$

(c) $\frac{P}{Q} = \frac{R(S_1 + S_2)}{S_1S_2}$ (d) $\frac{P}{Q} = \frac{R(S_1 + S_2)}{2S_1S_2}$.

(2006)

14. The current I drawn from the 5 volt source will be



(a) 0.17 A (b) 0.33 A
(c) 0.5 A (d) 0.67 A.

(2006)

15. The resistance of a bulb filament is 100 Ω at a temperature of 100ºC. If its temperature coefficient of resistance be 0.005 per ºC, its resistance will become 200 Ω at a temperature of

(a) 200ºC (b) 300ºC
(c) 400ºC (d) 500ºC.

(2006)

16. A material B has twice the specific resistance of

A. A circular wire made of B has twice the diameter of a wire made of A. Then for the two wires to have the same resistance, the ratio l_B/l_A of their respective lengths must be

(a) 2 (b) 1

(c) 1/2 (d) 1/4.

(2006)

17. The resistance of a wire is 5 ohm at 50°C and 6 ohm at 100°C. The resistance of the wire at 0°C will be

(a) 3 ohm (b) 2 ohm

(c) 1 ohm (d) 4 ohm

(2007)

Directions : Questions 18 and 19 are based on the following paragraph.

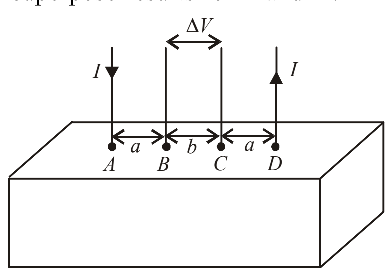
Consider a block of conducting material of resistivity ρ shown in the figure. Current I enters at A and leaves from D. We apply superposition principle to find voltage ΔV developed between B and C. The calculation is done in the following steps:

(i) Take current I entering from A and assume it to spread over a hemispherical surface in the block.

(ii) Calculate field $E(r)$ at distance r from A by using Ohm's law $E = \rho j$, where j is the current per unit area at r.

(iii) From the r dependence of $E(r)$, obtain the potential $V(r)$ at r.

(iv) Repeat (i), (ii) and (iii) for current I leaving D and superpose results for A and D.



18. ΔV measured between B and C is

(a) $\dfrac{\rho I}{2\pi(a-b)}$ (b) $\dfrac{\rho I}{\pi a}-\dfrac{\rho I}{\pi(a+b)}$

(c) $\dfrac{\rho I}{a}-\dfrac{\rho I}{(a+b)}$ (d) $\dfrac{\rho I}{2\pi a}-\dfrac{\rho I}{2\pi(a+b)}$

(2008)

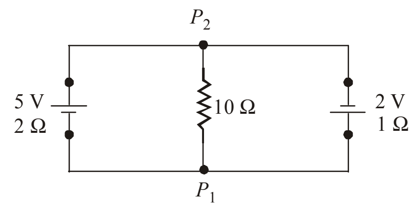
19. For current entering at A, the electric field at a distance r from A is

(a) $\dfrac{\rho I}{4\pi r^2}$ (b) $\dfrac{\rho I}{8\pi r^2}$

(c) $\dfrac{\rho I}{r^2}$ (d) $\dfrac{\rho I}{2\pi r^2}$

(2008)

20. A 5 V battery with internal resistance 2 Ω and 2 V battery with internal resistance 1 Ω are connected to a 10 Ω resistor as shown in the figure. The current in the 10 Ω resistor is

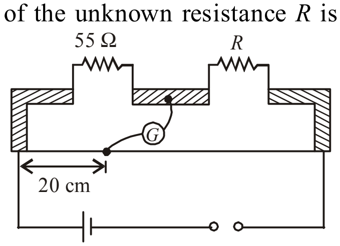


(a) 0.27 A P_1 to P_2 (b) 0.27 A P_2 to P_1

(c) 0.03 A P_1 to P_2 (d) 0.03 A P_2 to P_1.

(2008)

21. Shown in the figure below is a meter-bridge set up with null deflection in the galvanometer. The value of the unknown resistance R is



(a) 55 Ω (b) 13.75 Ω

(c) 220 Ω (d) 110 Ω.

(2008)

Answer Key

1.	(c)	**2.**	(b)	**3.**	(d)	**4.**	(c)	**5.**	(a)	**6.**	(b)
7.	(a)	**8.**	(c)	**9.**	(d)	**10.**	(d)	**11.**	(b)	**12.**	(a)
13.	(c)	**14.**	(c)	**15.**	(c)	**16.**	(a)	**17.**	(d)	**18.**	(d)
19.	(d)	**20.**	(d)	**21.**	(c)						

1. **(c) :** Potential gradient along wire $= \frac{E}{100} \frac{\text{volt}}{\text{cm}}$

$\therefore \quad K = \frac{E}{100} \frac{\text{volt}}{\text{cm}}$

For battery $V = E' - ir$, where E' is emf of battery.

or $K \times 30 = E' - ir$, where current i is drawn from battery

or $\frac{E \times 30}{100} = E' + 0.5i$ or $E' = \frac{30E}{100} - 0.5i$

2. **(b) :** Equivalent resistance $= \frac{(3+3) \times 3}{(3+3)+3} = \frac{18}{9} = 2\ \Omega$

$\therefore$ Current $I = \frac{V}{R} = \frac{3}{2} = 1.5$ A.

3. **(d) :** Let the length of the wire be l, radius of the wire be r

$\therefore$ Resistance $R = \rho \frac{l}{\pi r^2}$ ρ = resistivity of the wire

Now l is increased by 100% $\therefore l' = l + \frac{100}{100} l = 2l$

As length is increased, its radius is going to be decreased in such a way that the volume of the cylinder remains constant.

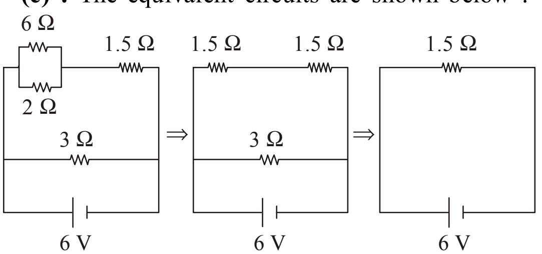
$\pi r^2 \times l = \pi r'^2 \times l' \Rightarrow r'^2 = \frac{r^2 \times l}{l'} = \frac{r^2 \times l}{2l} = \frac{r^2}{2}$

$\therefore$ The new resistance $R'^2 = \rho \frac{l'}{\pi r'^2} = \rho \frac{2l}{\pi \times \frac{r^2}{2}} = 4R$

$\therefore$ Change in resistance $= R' - R = 3R$

$\therefore$ % change $= \frac{3R}{R} \times 100\% = 300\%$.

4. **(c) :** The equivalent circuits are shown below :



$I = \frac{6}{1.5} = 4$ A.

5. **(a) :** In series combination, $S = (R_1 + R_2)$

In parallel combination, $P = \frac{R_1 R_2}{(R_1 + R_2)}$

$\because \quad S = nP$

$\therefore \quad (R_1 + R_2) = n \frac{R_1 R_2}{(R_1 + R_2)}$

$\therefore \quad (R_1 + R_2)^2 = nR_1R_2$

For minimum value, $R_1 = R_2 = R$

$\therefore \quad (R + R)^2 = n(R \times R) \Rightarrow 4R^2 = nR^2$

or $n = 4$.

6. **(b) :** Potential difference is same when the wires are put in parallel

$V = I_1 R_1 = I_1 \times \frac{\rho l_1}{\pi r_1^2}$

Again $V = I_2 R_2 = I_2 \times \frac{\rho l_2}{\pi r_2^2}$

$\therefore \quad \frac{I_1 \times \rho l_1}{\pi r_1^2} = \frac{I_2 \times \rho l_2}{\pi r_2^2} \Rightarrow \frac{I_1}{I_2} = \left(\frac{l_2}{l_1}\right)\left(\frac{r_1}{r_2}\right)^2$

or $\frac{I_1}{I_2} = \left(\frac{3}{4}\right)\left(\frac{2}{3}\right)^2 = \frac{3 \times 4}{4 \times 9} = \frac{1}{3}$.

7. **(a) :** For meter bridge experiment,

$\frac{R_1}{R_2} = \frac{l_1}{l_2} = \frac{l_1}{(100 - l_1)}$

In the first case, $\frac{X}{Y} = \frac{20}{100 - 20} = \frac{20}{80} = \frac{1}{4}$

In the second case,

$\frac{4X}{Y} = \frac{l}{(100 - l)} \Rightarrow \frac{4}{4} = \frac{l}{100 - l} \Rightarrow l = 50$ cm.

8. **(c) :** Thermistors are made of metal oxides with high temperature co-efficient of resistivity.

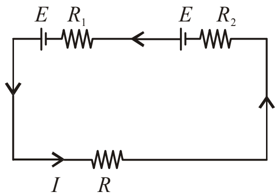
9. **(d) :** For zero deflection in galvanometer, $I_1 = I_2$

or $\frac{12}{500 + R} = \frac{2}{R}$.

$\Rightarrow 12R = 1000 + 2R \Rightarrow R = 100\ \Omega$

10. **(d) :** $I = \frac{2E}{R_1 + R_2 + R}$

$\because \quad E - IR_2 = 0$ (Given)



$\therefore \quad E = IR_2$

or $\quad E = \dfrac{2ER_2}{R_1 + R_2 + R}$

or $\quad R_1 + R_2 + R = 2R_2$

or $\quad R = R_2 - R_1$.

11. **(b) :** The internal resistance of a cell is given by

$$r = R\left(\frac{l_1}{l_2} - 1\right) = R\left(\frac{l_1 - l_2}{l_2}\right)$$

$$\therefore \quad r = 2\left[\frac{240 - 120}{120}\right] = 2\ \Omega.$$

12. **(a) :** Kirchhoff's first law [$\Sigma\ i = 0$] is based on conservation of charge

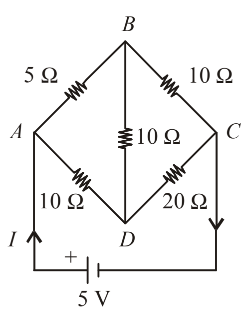
Kirchhoff's second law ($\Sigma\ i\ R = \Sigma\ E$) is based on conservation of energy.

13. **(c) :** For balanced Wheatstone's bridge, $\dfrac{P}{Q} = \dfrac{R}{S}$

$\because \quad S = \dfrac{S_1 S_2}{S_1 + S_2}$ ($\because S_1$ and S_2 are in parallel)

$\therefore \quad \dfrac{P}{Q} = \dfrac{R(S_1 + S_2)}{S_1 S_2}$.

14. **(c) :** The equivalent circuit is a balanced Wheatstone's bridge. Hence no current flows through arm BD.



AB and BC are in series

$\therefore \quad R_{ABC} = 5 + 10 = 15\ \Omega$

AD and DC are in series

$\therefore \quad R_{ADC} = 10 + 20 = 30\ \Omega$

ABC and ADC are in parallel

$$\therefore \quad R_{eq} = \frac{(R_{ABC})(R_{ADC})}{(R_{ABC} + R_{ADC})}$$

$$\text{or} \quad R_{eq} = \frac{15 \times 30}{15 + 30} = \frac{15 \times 30}{45} = 10\ \Omega$$

$$\therefore \quad \text{Current } I = \frac{E}{R_{eq}} = \frac{5}{10} = 0.5\ \text{A}.$$

15. **(c) :** Given : $R_{100} = 100\ \Omega$

$\alpha = 0.005^\circ\text{C}^{-1}$

$R_t = 200\ \Omega$

$\therefore \quad R_{100} = R_0[1 + 0.005\ (100)]$

or $\quad 100 = R_0[1 + 0.005 \times 100]$ ....(i)

$R_t = R_0[1 + 0.005t]$

$200 = R_0[1 + 0.005t]$ ....(ii)

Divide (i) by (ii), we get

$$\frac{100}{200} = \frac{[1 + 0.005 \times 100]}{[1 + 0.005t]}$$

$1 + 0.005t = 2 + 1$

or $\quad t = 400^\circ$C.

16. **(a) :** Resistance of a wire $R = \dfrac{\rho l}{\pi r^2} = \dfrac{\rho l \times 4}{\pi D^2}$

$\because \quad R_A = R_B$

$$\therefore \quad \frac{4\rho_A l_A}{\pi D_A^2} = \frac{4\rho_B l_B}{\pi D_B^2}$$

$$\text{or} \quad \frac{l_B}{l_A} = \left(\frac{\rho_A}{\rho_B}\right)\left(\frac{D_B}{D_A}\right)^2$$

$$= \left(\frac{\rho_A}{2\rho_A}\right)\left(\frac{2D_A}{D_A}\right)^2 = \frac{4}{2} = \frac{2}{1}$$

17. **(d) :** Given : $R_{50} = 5\ \Omega$, $R_{100} = 6\ \Omega$

$R_t = R_0(1 + \alpha t)$

where R_t = resistance of a wire at t°C, R_0 = resistance of a wire at 0°C, α = temperature coefficient of resistance.

$\therefore \quad R_{50} = R_0\ [1 + \alpha\ 50]$

and $R_{100} = R_0\ [1 + \alpha\ 100]$

or $\quad R_{50} - R_0 = R_0\alpha(50)$ ... (i)

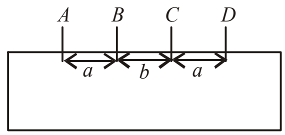
$R_{100} - R_0 = R_0\alpha\ (100)$ ... (ii)

Divide (i) by (ii), we get

$$\frac{5-R_0}{6-R_0} = \frac{1}{2} \quad \text{or} \quad 10-2R_0 = 6-R_0$$

or $R_0 = 4\ \Omega$.

18. (d) : Current is spread over an area $2\pi r^2$. The current I is a surface current.



Current density, $j = \frac{I}{2\pi r^2}$

Resistance $= \frac{\rho l}{\text{area}} = \frac{\rho r}{2\pi r^2}$

$E = I\rho/2\pi r^2$.

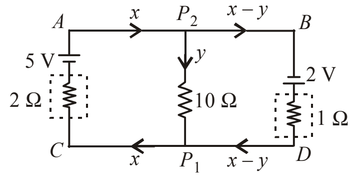
$$V_B - V_C = \Delta V = \int_{a+b}^{a} -E dr$$

$$\Rightarrow \Delta V = \frac{-I\rho}{2\pi}\int_{a+b}^{a}\frac{1}{r^2}dr = \frac{-I\rho}{2\pi}\left[-\frac{1}{r}\right]_{a+b}^{a}$$

$$\Delta V = \frac{I\rho}{2\pi}\left[\frac{1}{a} - \frac{1}{a+b}\right].$$

19. (d) : $j \times \rho = E$. $\therefore$ $E = \frac{I\rho}{2\pi r^2}$

20. (d) :



Applying Kirchhoff's law for the loops AP_2P_1CA and $P_2BDP_1P_2$, one gets

$-10y - 2x + 5 = 0$

$\Rightarrow$ $2x + 10y = 5$ ...(i)

$+ 2 - 1(x - y) + 10 \cdot y = 0$

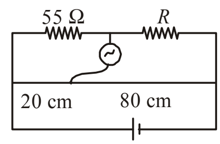
$+x - 11y = 2$ ...(ii)

$\Rightarrow 2x - 22y = 4$ ...(iii) = (ii) × 2

(i) – (iii) gives $32y = 1$

$\Rightarrow$ $y = \frac{1}{32}$ A $= 0.03$ A from P_2 to P_1.

21. (c) :



This is a Wheatstone bridge.

If ρ_l is the resistance per unit length (in cm)

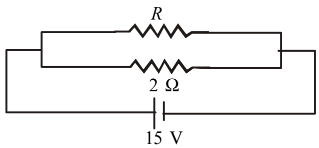
$$\frac{20\rho_l}{55} = \frac{80\rho_l}{R} \quad \text{or} \quad R = \frac{80 \times 55}{20} = 220\ \Omega.$$

CHAPTER

15

Thermal and Chemical Effects of Current

1. If in the circuit, power dissipation is 150 W, then R is



(a) 2 Ω
(b) 6 Ω
(c) 5 Ω
(d) 4 Ω.

(2002)

2. A wire when connected to 220 V mains supply has power dissipation P_1. Now the wire is cut into two equal pieces which are connected in parallel to the same supply. Power dissipation in this case is P_2. Then $P_2 : P_1$ is

(a) 1 (b) 4 (c) 2 (d) 3.

(2002)

3. If θ_i is the inversion temperature, θ_n is the neutral temperature, θ_c is the temperature of the cold junction, then

(a) $\theta_i + \theta_c = \theta_n$ (b) $\theta_i - \theta_c = 2\theta_n$

(c) $\dfrac{\theta_i + \theta_c}{2} = \theta_n$ (d) $\theta_c - \theta_i = 2\theta_n$.

(2002)

4. The mass of a product liberated on anode in an electrochemical cell depends on

(a) $(It)^{1/2}$ (b) It (c) I/t (d) I^2t.

(where t is the time period for which the current is passed).

(2002)

5. The thermo e.m.f. of a thermo-couple is 25 μV/°C at room temperature. A galvanometer of 40 ohm resistance, capable of detecting current as low as 10^{-5} A, is connected with the thermocouple. The smallest temperature difference that can be detected by this system is

(a) 16°C (b) 12°C

(c) 8°C (d) 20°C.

(2003)

6. The negative Zn pole of a Daniell cell, sending a constant current through a circuit, decreases in mass by 0.13 g in 30 minutes. If the electrochemical equivalent of Zn and Cu are 32.5 and 31.5 respectively, the increase in the mass of the positive Cu pole in this time is

(a) 0.180 g (b) 0.141 g

(c) 0.126 g (d) 0.242 g.

(2003)

7. A 220 volt, 1000 watt bulb is connected across a 110 volt mains supply. The power consumed will be

(a) 750 watt (b) 500 watt

(c) 250 watt (d) 1000 watt.

(2003)

8. Time taken by a 836 W heater to heat one litre of water from 10°C to 40°C is

(a) 50 s (b) 100 s (c) 150 s (d) 200 s.

(2004)

9. The thermo emf of a thermocouple varies with the temperature θ of the hot junction as $E = a\theta + b\theta^2$ in volt where the ratio a/b is 700°C. If the cold junction is kept at 0°C, then the neutral temperature is

(a) 700°C (b) 350°C

(c) 1400°C

(d) no neutral temperature is possible for this thermocouple.

(2004)

10. The electrochemical equivalent of a metal is 3.3×10^{-7} kg per coulomb. The mass of the metal liberated at the cathode when a 3 A current is passed for 2 second will be

(a) 19.8×10^{-7} kg (b) 9.9×10^{-7} kg
(c) 6.6×10^{-7} kg (d) 1.1×10^{-7} kg.

(2004)

11. A heater coil is cut into two equal parts and only one part is now used in the heater. The heat generated will now be

(a) one fourth (b) halved
(c) doubled (d) four times

(2005)

12. Two voltameters, one of copper and another of silver, are joined in parallel. When a total charge q flows through the voltmeters, equal amount of metals are deposited. If the electrochemical equivalents of copper and silver are z_1 and z_2 respectively the charge which flows through the silver voltameter is

(a) $q\frac{z_1}{z_2}$ (b) $q\frac{z_2}{z_1}$

(c) $\frac{q}{1+\frac{z_1}{z_2}}$ (d) $\frac{q}{1+\frac{z_2}{z_1}}$

(2005)

13. The resistance of hot tungsten filament is about 10 times the cold resistance. What will be the resistance of 100 W and 200 V lamp when not in use?

(a) 400 Ω (b) 200 Ω
(c) 40 Ω (d) 20 Ω

(2005)

14. An energy source will supply a constant current into the load if its internal resistance is

(a) zero
(b) non-zero but less than the resistance of the load
(c) equal to the resistance of the load
(d) very large as compared to the load resistance

(2005)

15. A thermocouple is made from two metals, antimony and bismuth. If one junction of the couple is kept hot and the other is kept cold then, an electric current will

(a) flow from antimony to bismuth at the cold junction
(b) flow from antimony to bismuth at the hot junction
(c) flow from bismuth to antimony at the cold junction
(d) not flow through the thermocouple.

(2006)

16. An electric bulb is rated 220 volt - 100 watt. The power consumed by it when operated on 110 volt will be

(a) 50 watt (b) 75 watt
(c) 40 watt (d) 25 watt.

(2006)

Answer Key

1.	(b)	**2.**	(b)	**3.**	(c)	**4.**	(b)	**5.**	(a)	**6.**	(c)
7.	(c)	**8.**	(c)	**9.**	(d)	**10.**	(a)	**11.**	(c)	**12.**	(d)
13.	(c)	**14.**	(a)	**15.**	(a)	**16.**	(d)				

1. **(b) :** Power $= \dfrac{V^2}{R}$

$\therefore \quad 150 = \dfrac{(15)^2}{R} + \dfrac{(15)^2}{2}$

$= \dfrac{225}{R} + \dfrac{225}{2} \Rightarrow R = 6\ \Omega$.

2. **(b) :** $P_1 = \dfrac{V^2}{R}$

when connected in parallel,

$R_{eq} = \dfrac{(R/2)\times(R/2)}{\dfrac{R}{2}+\dfrac{R}{2}} = \dfrac{R}{4}$

$\therefore \quad P_2 = \dfrac{V^2}{R/4} = 4\dfrac{V^2}{R} = 4P_1$

$\therefore \quad \dfrac{P_2}{P_1} = 4.$

3. **(c) :** $\theta_c + \theta_i = 2\theta_n \Rightarrow \dfrac{\theta_i + \theta_c}{2} = \theta_n$.

4. **(b) :** According to Faraday's laws, $m \propto It$.

5. **(a) :** Let the smallest temperature be θ°C

$\therefore$ Thermo emf = $(25 \times 10^{-6})\ \theta$ volt

Potential difference across galvanometer =

$IR = 10^{-5} \times 40 = 4 \times 10^{-4}$ volt

$\therefore \quad (25 \times 10^{-6})\theta = 4 \times 10^{-4}$

$\therefore \quad \theta = \dfrac{4\times10^{-4}}{25\times10^{-6}} = 16°\text{C}.$

6. **(c) :** According to Faraday's laws of electrolysis,

$\dfrac{m_{Zn}}{m_{Cu}} = \dfrac{Z_{Zn}}{Z_{Cu}}$ when i and t are same

$\therefore \quad \dfrac{0.13}{m_{Cu}} = \dfrac{32.5}{31.5}$

$\Rightarrow \quad m_{Cu} = \dfrac{0.13\times31.5}{32.5} = 0.126$ g

7. **(c) :** Resistance of bulb $= \dfrac{V^2}{P} = \dfrac{(220)^2}{1000} = 48.4\ \Omega$.

Required power

$= \dfrac{V^2}{R} = \dfrac{(110)^2}{48.4} = \dfrac{110\times110}{48.4} = 250$ W.

8. **(c) :** Electrical energy is converted into heat energy

$\therefore \quad 836 \times t = 1000 \times 1 \times (40 - 10) \times (4.18)$

$[\because 4.18\ \text{J} = 1\,\text{cal}]$

or $\quad t = \dfrac{1000\times30\times4.18}{836} = 150$ sec.

9. **(d) :** $E = a\theta + b\theta^2$

$\therefore \quad \dfrac{dE}{d\theta} = a + 2b\theta$

At neutral temperature (θ_n), $\dfrac{dE}{d\theta} = 0$

or $\quad 0 = a + 2b\theta_n$

or $\quad \theta_n = -\dfrac{a}{2b} = -\dfrac{1}{2}\times(700) = -350°\text{C}$

Neutral temperature is calculated to be –350°C

Since temperature of cold junction is 0°C, no neutral temperature is possible for this thermocouple.

10. **(a) :** $m = Z\ i\ t$

or $\quad m = (3.3 \times 10^{-7}) \times (3) \times (2)$

$= 19.8 \times 10^{-7}$ kg.

11. **(c) :** Resistance of full coil = R

Resistance of each half piece = $R/2$

$\therefore \quad \dfrac{H_2}{H_1} = \dfrac{V^2 t}{R/2} \times \dfrac{R}{V^2 t} = \dfrac{2}{1}$

$\therefore \quad H_2 = 2H_1$

Heat generated will now be doubled.

12. **(d) :** The voltameters are joined in parallel

Mass deposited = $z_1 q_1 = z_2 q_2$

$\therefore \quad \dfrac{q_1}{q_2} = \dfrac{z_2}{z_1} \Rightarrow \dfrac{q_1 + q_2}{q_2} = \dfrac{z_1 + z_2}{z_1}$

$\Rightarrow \dfrac{q}{q_2} = \left(1 + \dfrac{z_2}{z_1}\right)$

or $\quad q_2 = \dfrac{q}{\left(1 + \dfrac{z_2}{z_1}\right)}$.

13. **(c) :** Resistance of hot tungsten

$= \dfrac{V^2}{P} = \dfrac{(200)^2}{100} = 400\ \Omega$

Resistance when not in use $= \dfrac{400}{10} = 40\ \Omega$.

14. **(a) :** If internal resistance is zero, the energy source will supply a constant current.

15. **(a) :** Antimony-Bismuth couple is ABC couple. It means that current flows from A to B at cold junction.

16. **(d) :** Resistance of the bulb

$$(R) = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\ \Omega$$

Power across 110 volt $= \frac{(110)^2}{484}$

$$\therefore \quad \text{Power} = \frac{110 \times 110}{484} = 25\ \text{W}.$$

CHAPTER 16

Magnetic Effect of Current

1. If an ammeter is to be used in place of a voltmeter, the we must connect with the ammeter a
   (a) low resistance in parallel
   (b) high resistance in parallel
   (c) high resistance in series
   (d) low resistance in series.
   (2002)

2. If in a circular coil A of radius R, current I is flowing and in another coil B of radius $2R$ a current $2I$ is flowing, then the ratio of the magnetic fields, B_A and B_B, produced by them will be
   (a) 1 (b) 2 (c) 1/2 (d) 4.
   (2002)

3. If an electron and a proton having same momenta enter perpendicular to a magnetic field, then
   (a) curved path of electron and proton will be same (ignoring the sense of revolution)
   (b) they will move undeflected
   (c) curved path of electron is more curved than that of the proton
   (d) path of proton is more curved.
   (2002)

4. If a current is passed through a spring then the spring will
   (a) expand (b) compress
   (c) remains same (d) none of these.
   (2002)

5. The time period of a charged particle undergoing a circular motion in a uniform magnetic field is independent of its
   (a) speed (b) mass
   (c) charge (d) magnetic induction.
   (2002)

6. A particle of mass M and charge Q moving with velocity $\vec{v}$ describes a circular path of radius R when subjected to a uniform transverse magnetic field of induction B. The work done by the field when the particle completes one full circle is
   (a) $\left(\frac{Mv^2}{R}\right)2\pi R$ (b) zero
   (c) $BQ\ 2\pi R$ (d) $BQv\ 2\pi R$.
   (2003)

7. A particle of charge -16×10^{-18} coulomb moving with velocity 10 ms^{-1} along the x-axis enters a region where a magnetic field of induction B is along the y-axis, and an electric field of magnitude 10^4 V/m is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is
   (a) 10^3 Wb/m^2 (b) 10^5 Wb/m^2
   (c) 10^{16} Wb/m^2 (d) 10^{-3} Wb/m^2.
   (2003)

8. An ammeter reads upto 1 ampere. Its internal resistance is 0.81 ohm. To increase the range to 10 A the value of the required shunt is
   (a) 0.03 Ω (b) 0.3 Ω
   (c) 0.9 Ω (d) 0.09 Ω.
   (2003)

9. A current i ampere flows along an infinitely long straight thin walled tube, then the magnetic induction at any point inside the tube is
   (a) infinite (b) zero
   (c) $\frac{\mu_0}{4\pi}\cdot\frac{2i}{r}$ tesla (d) $\frac{2i}{r}$ tesla .
   (2004)

10. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be

(a) nB (b) n^2B
(c) $2nB$ (d) $2n^2B$.

(2004)

11. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is 54 μT. What will be its value at the centre of the loop?

(a) 250 μT (b) 150 μT
(c) 125 μT (d) 75 μT.

(2004)

12. Two long conductors, separated by a distance d carry current I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one of them is increased to two times and its direction is reversed. The distance is also increased to $3d$. The new value of the force between them is

(a) $-2F$ (b) $F/3$ (c) $-2F/3$ (d) $-F/3$.

(2004)

13. Two thin long, parallel wires, separated by a distance d carry a current of i A in the same direction. They will

(a) attract each other with a force of $\frac{\mu_0 i^2}{(2\pi d^2)}$

(b) repel each other with a force of $\frac{\mu_0 i^2}{(2\pi d^2)}$

(c) attract each other with a force of $\frac{\mu_0 i^2}{(2\pi d)}$

(d) repel each other with a force of $\frac{\mu_0 i^2}{(2\pi d)}$

(2005)

14. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milliampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 volt, the resistance in ohms needed to be connected in series with the coil will be

(a) 99995 (b) 9995
(c) 10^3 (d) 10^5

(2005)

15. Two concentric coils each of radius equal to 2π cm are placed at right angles to each other. 3 ampere and 4 ampere are the currents flowing in each coil respectively. The magnetic induction in weber/m^2 at the center of the coils will be ($\mu_0 = 4\pi \times 10^{-7}$ Wb/A-m)

(a) 5×10^{-5} (b) 7×10^{-5}
(c) 12×10^{-5} (d) 10^{-5}

(2005)

16. A charged particle of mass m and charge q travels on a circular path of radius r that is perpendicular to a magnetic field B. The time taken by the particle to complete one revolution is

(a) $\frac{2\pi qB}{m}$ (b) $\frac{2\pi m}{qB}$
(c) $\frac{2\pi mq}{B}$ (d) $\frac{2\pi mq}{B}$

(2005)

17. A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity then

(a) it will turn towards right of direction of motion
(b) it will turn towards left of direction of motion
(c) its velocity will decrease
(d) its velocity will increase

(2005)

18. In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be a

(a) circle (b) helix
(c) straight line (d) ellipse.

(2006)

19. A long solenoid has 200 turns per cm and carries a current i. The magnetic field at its centre is 6.28×10^{-2} weber/m^2. Another long solenoid has 100 turns per cm and it carries a current $i/3$. The value of the magnetic field at its centre is

(a) 1.05×10^{-4} Wb/m^2 (b) 1.05×10^{-2} Wb/m^2
(c) 1.05×10^{-5} Wb/m^2 (d) 1.05×10^{-3} Wb/m^2.

(2006)

20. A long straight wire of radius a carries a steady current i. The current is uniformly distributed across its cross section. The ratio of the magnetic field at $a/2$ and $2a$ is

(a) 1/2 (b) 1/4
(c) 4 (d) 1

(2007)

21. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then

(a) the magnetic field at all points inside the pipe is the same, but not zero
(b) the magnetic field is zero only on the axis of the pipe
(c) the magnetic field is different at different points inside the pipe
(d) the magnetic field at any point inside the pipe is zero

(2007)

22. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields $\vec{E}$ and $\vec{B}$ with a velocity $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$, and comes out without any change in magnitude or direction of $\vec{v}$. Then

(a) $\vec{v} = \vec{B} \times \vec{E} / E^2$ (b) $\vec{v} = \vec{E} \times \vec{B} / B^2$
(c) $\vec{v} = \vec{B} \times \vec{E} / B^2$ (d) $\vec{v} = \vec{E} \times \vec{B} / E^2$

(2007)

23. A charged particle moves through a magnetic field perpendicular to its direction. Then

(a) kinetic energy changes but the momentum is constant
(b) the momentum changes but the kinetic energy is constant
(c) both momentum and kinetic energy of the particle are not constant
(d) both, momentum and kinetic energy of the particle are constant

(2007)

24. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance d from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by

(a) $\frac{\mu_0}{2\pi d}(I_1^2 + I_2^2)$ (b) $\frac{\mu_0}{2\pi}\left(\frac{I_1 + I_2}{d}\right)^{\frac{1}{2}}$

(c) $\frac{\mu_0}{2\pi d}(I_1^2 + I_2^2)^{\frac{1}{2}}$ (d) $\frac{\mu_0}{2\pi d}(I_1 + I_2)$

(2007)

25. A horizontal overhead powerline is at a height of 4 m from the ground and carries a current of 100 A from east to west. The magnetic field directly below it on the ground is

($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})

(a) 2.5×10^{-7} T northward
(b) 2.5×10^{-7} T southward
(c) 5×10^{-6} T northward
(d) 5×10^{-6} T southward.

(2008)

Answer Key

1.	(c)	**2.**	(a)	**3.**	(a)	**4.**	(b)	**5.**	(a)	**6.**	(b)
7.	(a)	**8.**	(d)	**9.**	(b)	**10.**	(b)	**11.**	(a)	**12.**	(c)
13.	(c)	**14.**	(b)	**15.**	(a)	**16.**	(b)	**17.**	(c)	**18.**	(c)
19.	(b)	**20.**	(d)	**21.**	(d)	**22.**	(b)	**23.**	(b,c)	**24.**	(c)
25.	(d)										

1. **(c) :** High resistance in series with a galvanometer converts it into a voltmeter.

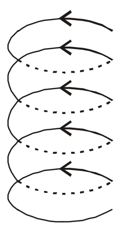
2. **(a) :** $B = \frac{\mu_0}{4\pi}\frac{2\pi I}{R} = \frac{\mu_0}{2}\frac{I}{R}$

$\therefore \quad \frac{B_A}{B_B} = \frac{I_A}{I_B} \times \frac{R_B}{R_A} = \left(\frac{1}{2}\right)\left(\frac{2}{1}\right) = 1$

3. **(a) :** $Bqv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{Bq} = \frac{p}{Bq}$

r will be same for electron and proton as p, B and q are of same magnitude.

4. **(b) :** The spring will compress. It will be on account of force of attraction between two adjacent turns carrying currents in the same direction.



5. **(a) :** $mR\omega^2 = BqR\omega \Rightarrow \omega = \frac{Bq}{m} \Rightarrow T = \frac{2\pi m}{Bq}$

T is independent of speed.

6. **(b) :** Workdone by the field = zero.

7. **(a) :** Particle travels along x-axis. Hence $v_y = v_z = 0$
Field of induction B is along y-axis. $B_x = B_z = 0$
Electric field is along the negative z-axis.

$E_x = E_y = 0$

$\therefore$ Net force on particle $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Resolve the motion along the three coordinate axis

$\therefore \quad a_x = \frac{F_x}{m} = \frac{q}{m}(E_x + v_y B_z - v_z B_y)$

$a_y = \frac{F_y}{m} = \frac{q}{m}(E_y + v_z B_x - v_x B_z)$

$a_z = \frac{F_z}{m} = \frac{q}{m}(E_z + v_x B_y - v_y B_x)$

Since $E_x = E_y = 0$, $v_y = v_z = 0$, $B_x = B_z = 0$

$\therefore \quad a_x = a_y = 0$, $a_z = \frac{q}{m}(-E_z + v_x B_y)$

Again $a_z = 0$ as the particle traverse through the region undeflected

$\therefore \quad E_z = v_x B_y$

or $\quad B_y = \frac{E_z}{v_x} = \frac{10^4}{10} = 10^3 \frac{\text{Wb}}{\text{m}^2}$.

8. **(d) :** $\frac{S}{S+G} = \frac{I_g}{I} \Rightarrow S = \frac{I_g G}{I - I_g}$

$\therefore \quad S = \frac{1 \times 0.81}{10-1} = \frac{0.81}{9} = 0.09\ \Omega$ in parallel.

9. **(b) :** Magnetic field will be zero inside the straight thin walled tube according to ampere's theorem.

10. **(b) :** Initially, r_1 = radius of coil = $l/2\pi$

$\therefore \quad B = \frac{\mu_0 i}{2r_1} = \frac{2\mu_0 i\pi}{2l}$

Finally, r_2 = radius of coil $= \frac{l}{2\pi n}$

$\therefore \quad B' = \frac{\mu_0 i \times n}{2r_2} = \frac{n\mu_0 i \times 2\pi n}{2l} = \frac{2\mu_0 i n^2 \pi}{2l}$

$\therefore \quad \frac{B'}{B} = \frac{2\mu_0 i n^2 \pi}{2l} \times \frac{2l}{2\mu_0 i \pi} = n^2$

$\therefore \quad B' = n^2 B$.

11. **(a) :** Field along axis of coil $B = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$

At the centre of coil, $B' = \frac{\mu_0 i}{2R}$

$\therefore \quad \frac{B'}{B} = \frac{\mu_0 i}{2R} \times \frac{2(R^2 + x^2)^{3/2}}{\mu_0 i R^2} = \frac{(R^2 + x^2)^{3/2}}{R^3}$

$\therefore \quad B' = \frac{B \times (R^2 + x^2)^{3/2}}{R^3}$

$= \frac{54 \times [(3)^2 + (4)^2]^{3/2}}{(3)^3} = \frac{54 \times 125}{27}$

or $\quad B' = 250\ \mu\text{T}$.

12. **(c) :** Initially, $F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} l$

Finally, $F' = \frac{\mu_0}{2\pi}\frac{(-2I_1)(I_2)}{3d} l$

$\therefore \quad \frac{F'}{F} = \frac{-\mu_0}{2\pi}\frac{2I_1I_2l}{3d}\times\frac{2\pi d}{\mu_0 I_1 I_2 l} = -\frac{2}{3}$

$\therefore \quad F' = -2F/3.$

13. **(c) :** Force of attraction between wires $= \frac{\mu_0 i^2 L}{2\pi d}$.

N.B. The options do not mention L, perhaps by slip.

14. **(b) :** $V_{max} = \frac{150}{2} = 75 \text{ mV}$

$I_{max} = \frac{150}{10} = 15 \text{ mA} = I_g$

Resistance of galvanometer $G = 75/15 = 5\ \Omega$

For conversion into a voltmeter, a high resistance should be connected in series with the galvanometer

$V = I_g(G+R) = \frac{15}{1000}(5+R) \Rightarrow 150 = 15\frac{(5+R)}{1000}$

or $\quad 5 + R = \frac{150\times 1000}{15} = 10000$

$\therefore \quad R = 9995\ \Omega.$

15. **(a) :** Magnetic induction at centre of one coil

$B_1 = \frac{\mu_0 i_1}{2r}$

Similarly $B_2 = \frac{\mu_0 i_2}{2r}$

$\therefore \quad B^2 = B_1^2 + B_2^2 = \left(\frac{\mu_0 i_1}{2r}\right)^2 + \left(\frac{\mu_0 i_2}{2r}\right)^2$

$= \frac{\mu_0{}^2}{4r^2}(i_1^2 + i_2^2)$

$\therefore \quad B = \frac{\mu_0}{2r}\sqrt{i_1^2 + i_2^2}$

$= \frac{4\pi\times 10^{-7}}{2\times(2\pi\times 10^{-2})}\sqrt{(3)^2+(4)^2}$

or $\quad B = 5 \times 10^{-5}$ Wb/m^2.

16. **(b) :** $T = \frac{2\pi}{\omega} = \frac{2\pi r}{v}$ ....(i)

$\because$ centripetal force = magnetic force

$\therefore \quad \frac{mv^2}{r} = qvB \Rightarrow v = \frac{qBr}{m}$ ....(ii)

From (i) and (ii)

$\therefore \quad T = \frac{2\pi r\times m}{qBr} = \frac{2\pi m}{qB}.$

17. **(c) :** Magnetic field applied parallel to motion of electron exerts no force on it as $\theta = 0$ and force $= Bev\sin\theta$ = zero

Electric field opposes motion of electron which carries a negative charge

$\therefore$ velocity of electron decreases.

18. **(c) :** Magnetic field exerts a force

$= Bev\sin\theta = Bev\sin 0 = 0$

Electric field exerts force along a straight line

The path of charged particle will be a straight line.

19. **(b) :** In first case, $B_1 = \mu_0 n_1 I_1$

In second case, $B_2 = \mu_0 n_2 I_2$

$\therefore \quad \frac{B_2}{B_1} = \frac{n_2}{n_1}\times\frac{I_2}{I_1} = \frac{100}{200}\times\frac{i/3}{i} = \frac{1}{6}$

$\therefore \quad B_2 = \frac{B_1}{6} = \frac{6.28\times 10^{-2}}{6}$

$= 1.05 \times 10^{-2}$ Wb/m^2.

20. **(d) :** Uniform current is flowing. Current enclosed in the 1st ampèrean path is $\frac{I\cdot\pi r_1^2}{\pi R^2} = \frac{I r_1^2}{R^2}$

$\therefore \quad B = \frac{\mu_0 \times \text{current}}{\text{path}} = \frac{\mu_0\cdot I r_1^2}{2\pi r_1 R^2} = \frac{\mu_0 I r_1}{2\pi R^2}$

Magnetic induction at a distance $r_2 = \frac{\mu_0\cdot I}{2\pi r_2}$

$\therefore \quad \frac{B_1}{B_2} = \frac{r_1 r_2}{R^2} = \frac{\frac{a}{2}\cdot 2a}{a^2} = 1.$

21. **(d) :** Magnetic field is shielded and no current is inside the pipe to apply Ampère's law. (Compare to electric field inside a hollow sphere).

22. **(b) :** When $\vec{E}$ and $\vec{B}$ are perpendicular and velocity has no changes then $qE = qvB$ *i.e.*, $v = \frac{E}{B}$. The two forces oppose each other if v is along $\vec{E}\times\vec{B}$ *i.e.*,

$\vec{v} = \frac{\vec{E}\times\vec{B}}{B^2}$

As $\vec{E}$ and $\vec{B}$ are perpendicular to each other

$$\frac{\vec{E}\times\vec{B}}{B^2} = \frac{EB\sin 90°}{B^2} = \frac{E}{B}$$

For historic and standard experiments like Thomson's e/m value, if v is given only as E/B, it would have been better from the pedagogic view, although the answer is numerically correct.

23. **(b, c) :** Due to Lorentzian force, $F = qv \times B$, When a charged particle enters a field with its velocity perpendicular to the magnetic field, the motion is circular with $qvB = \frac{mv^2}{r}$. v constantly changes its direction (but not the magnitude). Therefore its tangential momentum changes its direction but its energy remains the same $\left(\frac{1}{2}I\omega^2 = \text{constant}\right)$. Therefore the answer is (b).

If angular momentum is taken, $I\omega$ is a constant.

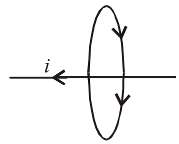
As $\frac{1}{2}I\omega^2$ is also constant, (c) is the answer.

* *The questions could have been more specific, whether by "momentum" it is meant tangential momentum or angular momentum.*

24. **(c) :** The field at the same point at the same distance from the mutually perpendicular wires carrying current will be having the same magnitude but in perpendicular directions.

$\therefore \quad B = \sqrt{B_1^2 + B_2^2}$

$\therefore \quad B = \frac{\mu_0}{2\pi d}(I_1^2 + I_2^2)^{1/2}$.

25.* (d) : 

By Ampere's theorem, $\vec{B} \cdot 2\pi d = \mu_0 i$

$$\vec{B} = \frac{\mu_0 i}{2\pi d} = \frac{4\pi \times 10^{-7} \times 100\,\text{A}}{2\pi \times 4\,\text{m}} = 50 \times 10^{-7}\,\text{T}$$

$\Rightarrow \quad B = 5 \times 10^{-6}$ T southwards.

* *It is assumed that this is a direct current. If it is a.c, the current at the given instant is in the given direction.*

CHAPTER
17

Magnetism

1. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60°. The torque needed to maintain the needle in this position will be
   (a) $\sqrt{3}W$ (b) W
   (c) $\left(\frac{\sqrt{3}}{2}\right)W$ (d) $2W$.
   (2003)

2. The magnetic lines of force inside a bar magnet
   (a) are from north-pole to south-pole of the magnet
   (b) do not exist
   (c) depend upon the area of cross-section of the bar magnet
   (d) are from south-pole to north-pole of the magnet.
   (2003)

3. Curie temperature is the temperature above which
   (a) a ferromagnetic material becomes paramagnetic
   (b) a paramagnetic material becomes diamagnetic
   (c) a ferromagnetic material becomes diamagnetic
   (d) a paramagnetic material becomes ferromagnetic.
   (2003)

4. A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T', the ratio $\frac{T'}{T}$ is
   (a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2}$
   (c) 2 (d) $\frac{1}{4}$.
   (2003)

5. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2 s. The magnet is cut along its length into three equal parts and three parts are then placed on each other with their like poles together. The time period of this combination will be
   (a) 2 s (b) $\frac{2}{3}$ s
   (c) $(2\sqrt{3})$ s (d) $\left(\frac{2}{\sqrt{3}}\right)$ s
   (2004)

6. The materials suitable for making electromagnets should have
   (a) high retentivity and high coercivity
   (b) low retentivity and low coercivity
   (c) high retentivity and low coercivity
   (d) low retentivity and high coercivity.
   (2004)

7. A magnetic needle is kept in a non-uniform magnetic field. It experiences
   (a) a force and a torque
   (b) a force but not a torque
   (c) a torque but not a force
   (d) neither a force nor a torque
   (2005)

8. Needles N_1, N_2 and N_3 are made of a ferromagnetic,

a paramagnetic and a diamagnetic substance respectively. A magnet when brought close to them will

(a) attract all three of them
(b) attract N_1 and N_2 strongly but repel N_3
(c) attract N_1 strongly, N_2 weakly and repel N_3 weakly
(d) attract N_1 strongly, but repel N_2 and N_3 weakly.

(2006)

9. Relative permittivity and permeability of a material are ε_r and μ_r, respectively. Which of the following values of these quantities are allowed for a diamagnetic mateiral?

(a) $\varepsilon_r = 1.5,\ \mu_r = 1.5$
(b) $\varepsilon_r = 0.5,\ \mu_r = 1.5$
(c) $\varepsilon_r = 1.5,\ \mu_r = 0.5$
(d) $\varepsilon_r = 0.5,\ \mu_r = 0.5$

(2008)

Answer Key

1.	(a)	**2.**	(d)	**3.**	(a)	**4.**	(b)	**5.**	(b)	**6.**	(b)
7.	(a)	**8.**	(c)	**9.**	(c)						

EXPLANATIONS

1. **(a) :** $W = -MB(\cos\theta_2 - \cos\theta_1)$

$$= -MB(\cos 60° - \cos 0) = \frac{MB}{2}$$

$\therefore \quad MB = 2W \quad$ ....(i)

Torque $= MB \sin 60° = (2W) \sin 60°$

$$= \frac{2W \times \sqrt{3}}{2} = \sqrt{3}\,W.$$

2. **(d) :** The magnetic lines of force inside a bar magnet are from south pole to north pole of magnet.

3. **(a) :** A ferromagnetic material becomes paramagnetic above Curie temperature.

4. **(b) :** For an oscillating magnet, $T = 2\pi\sqrt{\frac{I}{MB}}$

where $I = ml^2/12$, $M = xl$, x = pole strength When the magnet is divided into 2 equal parts, the magnetic dipole moment

$$M' = \text{Pole strength} \times \text{length} = \frac{x \times l}{2} = \frac{M}{2} \quad .....(i)$$

$$I' = \frac{\text{Mass} \times (\text{length})^2}{12}$$

$$= \frac{(m/2)(l/2)^2}{12} = \frac{ml^2}{12 \times 8} = \frac{I}{8} \quad .....(ii)$$

$\therefore$ Time period $T' = 2\pi\sqrt{\frac{I'}{M'B}}$

$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{I'}{M'} \times \frac{M}{I}} = \sqrt{\frac{I'}{I} \times \frac{M}{M'}} \quad ....(iii)$$

$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{1}{8} \times \frac{2}{1}} = \frac{1}{2}.$$

5. **(b) :** For a vibrating magnet, $T = 2\pi\sqrt{\frac{I}{MB}}$

where $I = ml^2/12$, $M = xl$, x = pole strength of magnet

$$I' = \left(\frac{m}{3}\right)\left(\frac{l}{3}\right)^2 \times \frac{3}{12} = \frac{ml^2}{9 \times 12} = \frac{I}{9}$$

(For three pieces together)

$$M' = (x)\left(\frac{l}{3}\right) \times 3 = xl = M$$

(For three pieces together)

$$\therefore \quad T' = 2\pi\sqrt{\frac{I'}{M'B}} = 2\pi\sqrt{\frac{I/9}{MB}}$$

$$= \frac{1}{3} \times 2\pi\sqrt{\frac{I}{MB}} = \frac{T}{3}$$

$$\therefore \quad T' = \frac{T}{3} = \frac{2}{3}\text{sec}.$$

6. **(b) :** Materials of low retentivity and low coercivity are suitable for making electromagnets.

7. **(a) :** A force and a torque act on a magnetic needle kept in a non-uniform magnetic field.

8. **(c) :** Magnet will attract N_1 strongly, N_2 weakly and repel N_3 weakly.

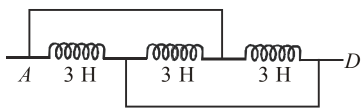
9. **(c) :** The values of relative permeability of diamagnetic materials are slightly less than 1 and ε_r is quite high. According to the table given, one takes

$\varepsilon_r = 1.5$ and $\mu_r = 0.5$. Then the choice (c) is correct.

CHAPTER 18

Electromagnetic Induction and Alternating Current

1. The inductance between A and D is
(a) 3.66 H
(b) 9 H
(c) 0.66 H
(d) 1 H.

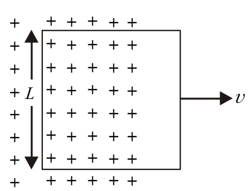


(2002)

2. The power factor of an AC circuit having resistance (R) and inductance (L) connected in series and an angular velocity ω is
(a) $R/\omega L$ (b) $R/(R^2 + \omega^2L^2)^{1/2}$
(c) $\omega L/R$ (d) $R/(R^2 - \omega^2L^2)^{1/2}$.
(2002)

3. In a transformer, number of turns in the primary coil are 140 and that in the secondary coil are 280. If current in primary coil is 4 A, then that in the secondary coil is
(a) 4 A (b) 2 A (c) 6 A (d) 10 A.
(2002)

4. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing perpendicular and into the plane at the loop exists everywhere with half the loop outside the field, as shown in figure. The induced emf is



(a) zero (b) RvB (c) vBL/R (d) vBL.
(2002)

5. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon
(a) the rates at which currents are changing in the two coils
(b) relative position and orientation of the two coils
(c) the materials of the wires of the coils
(d) the currents in the two coils.
(2003)

6. When the current changes from +2 A to –2 A in 0.05 second, an e.m.f. of 8 V is induced in a coil. The coefficient of self-induction of the coil is
(a) 0.2 H (b) 0.4 H
(c) 0.8 H (d) 0.1 H.
(2003)

7. In an oscillating LC circuit the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic field is
(a) $Q/2$ (b) $Q/\sqrt{3}$
(c) $Q/\sqrt{2}$ (d) Q.
(2003)

8. The core of any transformer is laminated so as to
(a) reduce the energy loss due to eddy currents
(b) make it light weight
(c) make it robust & strong
(d) increase the secondary voltage.
(2003)

9. In an LCR series a.c. circuit, the voltage across each of the components, L, C and R is 50 V. The voltage across the LC combination will be
(a) 50 V (b) $50\sqrt{2}$ V
(c) 100 V (d) 0 V (zero).
(2004)

10. Alternating current cannot be measured by D.C. ammeter because

(a) A.C. cannot pass through D.C. ammeter
(b) A.C. changes direction
(c) average value of current for complete cycle is zero
(d) D.C. ammeter will get damaged.

(2004)

11. A coil having n turns and resistance $R\ \Omega$ is connected with a galvanometer of resistance $4R\ \Omega$. This combination is moved in time t seconds from a magnetic field W_1 weber to W_2 weber. The induced current in the circuit is

(a) $-\dfrac{W_2 - W_1}{5Rnt}$ (b) $-\dfrac{n(W_2 - W_1)}{5Rt}$
(c) $-\dfrac{(W_2 - W_1)}{Rnt}$ (d) $-\dfrac{n(W_2 - W_1)}{Rt}$.

(2004)

12. In a uniform magnetic field of induction B a wire in the form of a semicircle of radius r rotates about the diameter of the circle with angular frequency ω. The axis of rotation is perpendicular to the field. If the total resistance of the circuit is R the mean power generated per period of rotation is

(a) $\dfrac{B\pi r^2\omega}{2R}$ (b) $\dfrac{(B\pi r^2\omega)^2}{8R}$
(c) $\dfrac{(B\pi r\omega)^2}{2R}$ (d) $\dfrac{(B\pi r\omega^2)^2}{8R}$.

(2004)

13. In a LCR circuit capacitance is changed from C to $2C$. For the resonant frequency to remain unchanged, the inductance should be changed from L to

(a) $4L$ (b) $2L$
(c) $L/2$ (d) $L/4$.

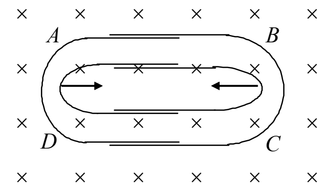
(2004)

14. A metal conductor of length 1 m rotates vertically about one of its ends at angular velocity 5 radian per second. If the horizontal component of earth's magnetic field is 0.2×10^{-4} T, then the e.m.f. developed between the two ends of the conductor is

(a) 5 μV (b) 50 μV
(c) 5 mV (d) 50 mV.

(2004)

15. One conducting U tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed v, then the emf induced in the circuit in terms of B, l and v where l is the width of each tube, will be



(a) zero (b) $2Blv$ (c) Blv (d) $-Blv$

(2005)

16. A coil of inductance 300 mH and resistance 2 Ω is connected to a source of voltage 2 V. The current reaches half of its steady state value in

(a) 0.15 s (b) 0.3 s (c) 0.05 s (d) 0.1 s

(2005)

17. The self inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of

(a) 1 μF (b) 2 μF (c) 4 μF (d) 8 μF

(2005)

18. A circuit has a resistance of 12 ohm and an impedance of 15 ohm. The power factor of the circuit will be

(a) 1.25 (b) 0.125 (c) 0.8 (d) 0.4

(2005)

19. The phase difference between the alternating current and emf is $\pi/2$. Which of the following cannot be the constituent of the circuit?

(a) LC (b) L alone (c) C alone (d) R, L

(2005)

20. In a series resonant LCR circuit, the voltage across R is 100 volts and $R = 1$ kΩ with $C = 2$ μF. The resonant frequency ω is 200 rad/s. At resonance the voltage across L is

(a) 4×10^{-3} V (b) 2.5×10^{-2} V
(c) 40 V (d) 250 V.

(2006)

21. In an AC generator, a coil with N turns, all of the same area A and total resistance R, rotates with frequency ω in a magnetic field B. The maximum value of emf generated in the coil is

(a) $NAB\omega$ (b) $NABR\omega$

(c) NAB (d) $NABR$

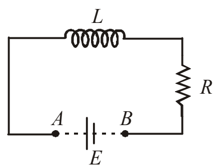
(2006)

22. The flux linked with a coil at any instant t is given by $\phi = 10t^2 - 50t + 250$. The induced emf at $t = 3$ s is

(a) 190 V (b) –190 V
(c) –10 V (d) 10 V.

(2006)

23. An inductor ($L = 100$ mH), a resistor ($R = 100\ \Omega$) and a battery ($E = 100$ V) are initially connected in series as shown in the figure. After a long time the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is



(a) 1 A (b) $(1/e)$ A
(c) e A (d) 0.1 A.

(2006)

24. In an a.c. circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by

(a) $P = \sqrt{2}E_0 I_0$ (b) $P = \frac{E_0 I_0}{\sqrt{2}}$
(c) $P =$ zero (d) $P = \frac{E_0 I_0}{2}$

(2007)

25. An ideal coil of 10 H is connected in series with a resistance of 5 Ω and a battery of 5 V. 2 second after the connection is made, the current flowing in ampere in the circuit is

(a) $(1 - e^{-1})$ (b) $(1 - e)$
(c) e (d) e^{-1}

(2007)

26. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area $A = 10\ \text{cm}^2$ and length = 20 cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is

($\mu_0 = 4\pi \times 10^{-7}$ T mA^{-1})

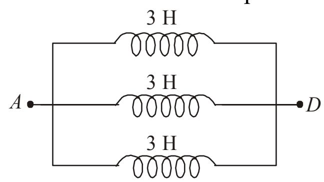
(a) $2.4\pi \times 10^{-4}$ H (b) $2.4\pi \times 10^{-5}$ H
(c) $4.8\pi \times 10^{-4}$ H (d) $4.8\pi \times 10^{-5}$ H.

(2008)

Answer Key

1.	(d)	**2.**	(b)	**3.**	(b)	**4.**	(d)	**5.**	(c)	**6.**	(d)
7.	(c)	**8.**	(a)	**9.**	(d)	**10.**	(c)	**11.**	(b)	**12.**	(b)
13.	(c)	**14.**	(b)	**15.**	(a)	**16.**	(d)	**17.**	(a)	**18.**	(c)
19.	(d)	**20.**	(d)	**21.**	(a)	**22.**	(c)	**23.**	(b)	**24.**	(c)
25.	(a)	**26.**	(a)								

1. **(d) :** Three inductors are in parallel



$\therefore \quad \frac{1}{L_{(eq)}} = \frac{1}{3}+\frac{1}{3}+\frac{1}{3} = \frac{3}{3} = \frac{1}{1}$

$\therefore \quad L_{(eq)} = 1$ H.

2. **(b) :** Power factor $= \frac{R}{\sqrt{R^2 + L^2\omega^2}}$.

3. **(b) :** $I_2 N_2 = I_1 N_1$ for a transformer

$\therefore \quad I_2 = \frac{I_1 N_1}{N_2} = \frac{4\times 140}{280} = 2$ A .

4. **(d) :** Induced emf = vBL.

5. **(c) :** Mutual inductance between two coils depends on the materials of the wires of the coils.

6. **(d) :** $L = \frac{-e}{di/dt} = \frac{-8\times 0.05}{-4} = 0.1$ H .

7. **(c) :** Let Q denote maximum charge on capacitor. Let q denote charge when energy is equally shared

$\therefore \quad \frac{1}{2}\left(\frac{1}{2}\frac{Q^2}{C}\right) = \frac{1}{2}\frac{q^2}{C} \Rightarrow Q^2 = 2q^2$

$\therefore \quad q = Q/\sqrt{2}$.

8. **(a) :** The energy loss due to eddy currents is reduced by using laminated core in a transformer.

9. **(d) :** In an LCR series a.c. circuit, the voltages across components L and C are in opposite phase. The voltage across LC combination will be zero.

10. **(c) :** Average value of A.C. for complete cycle is zero. Hence A.C. can not be measured by D.C. ammeter.

11. **(b) :** Induced current $I = \frac{-n}{R'}\frac{d\phi}{dt} = \frac{-n}{R'}\frac{dW}{dt}$ where $\phi = W$ = flux × per unit turn of the coil

$\therefore \quad I = -\frac{1}{(R+4R)}\frac{n(W_2 - W_1)}{t} = -\frac{n(W_2-W_1)}{5Rt}$.

12. **(b) :** Magnetic flux linked

$= BA\cos\omega t = \frac{B\pi r^2 \cos\omega t}{2}$

$\therefore \quad$ Induced emf $e = \frac{-d\phi}{dt} = \frac{-1}{2}B\pi r^2 \omega \sin\omega t$

$\therefore \quad$ Power $= \frac{e^2}{R} = \frac{B^2\pi^2 r^4 \omega^2 \sin^2 \omega t}{4R}$

$= \frac{(B\pi r^2\omega)^2}{4R}\sin^2\omega t$

$\because \quad \langle\sin^2 \omega t\rangle = 1/2$

$\therefore \quad$ Mean power generated

$= \frac{(B\pi r^2\omega)^2}{4R}\times\frac{1}{2} = \frac{(B\pi r^2\omega)^2}{8R}$.

13. **(c) :** At resonance, $\omega = \frac{1}{\sqrt{LC}}$

when ω is constant,

$\therefore \quad \frac{1}{L_1C_1} = \frac{1}{L_2C_2} \Rightarrow \frac{1}{LC} = \frac{1}{L_2(2C)} = \frac{1}{2L_2C}$

$\therefore \quad L_2 = L/2$.

14. **(b) :** Induced emf

$= \frac{1}{2}B\omega l^2 = \frac{1}{2}\times(0.2\times10^{-4})(5)(1)^2$

$\therefore \quad$ Induced emf $= \frac{10^{-4}}{2} = \frac{100\times10^{-6}}{2} = 50\ \mu$V.

15. **(a) :** The emf induced in the circuit is zero because the two emf induced are equal and opposite when one U tube slides inside another tube.

16. **(d) :** During growth of charge in an inductance,

$I = I_0\,(1 - e^{-Rt/L})$

or $\quad \frac{I_0}{2} = I_0(1 - e^{-Rt/L})$

or $\quad e^{-Rt/L} = \frac{1}{2} = 2^{-1}$

or $\quad \frac{Rt}{L} = \ln 2 \Rightarrow t = \frac{L}{R}\ln 2$

$t = \frac{300\times10^{-3}}{2}\times(0.693)$

or $\quad t = 0.1$ sec.

17. **(a) :** For maximum power, $L\omega = \frac{1}{C\omega}$

$\therefore \quad C = \frac{1}{L\omega^2} = \frac{1}{10\times(2\pi\times 50)^2}$

$= \frac{1}{10\times 10^4 \times (\pi)^2} = 10^{-6}$ F

or $\quad C = 1\ \mu$F.

18. (c) : Power factor $\cos\phi = \frac{R}{Z} = \frac{12}{15} = 0.8.$

19. (d) : R and L cause phase difference to lie between 0 and $\pi/2$ but never 0 and $\pi/2$ at extremities.

20. (d) : Current $I = \frac{E}{Z}$

where $E = \sqrt{V_R^2 + (V_L - V_C)^2}$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

At resonance, $X_L = X_C$

$\therefore \quad Z = R$

Again at resonance, $V_L = V_C$

$\therefore \quad E = V_R$

$\therefore \quad I = \frac{V_R}{R} = \frac{100}{1\times 10^3} = 0.1$ A

$\therefore \quad V_L = I L\omega = \frac{I}{C\omega} = \frac{0.1}{(2\times 10^{-6})\times(200)}$

$\therefore \quad V_L = 250$ volt.

21. (a) : In an a.c. generator, maximum emf = $NAB\omega$.

22. (c) : $\phi = 10t^2 - 50t + 250$

$\therefore \quad \frac{d\phi}{dt} = 20t - 50$

Induced emf $e = \frac{-d\phi}{dt}$

or $e = -(20t - 50) = -[(20 \times 3) - 50] = -10$ volt

or $e = -10$ volt.

23. (b) : Maximum current $I_0 = \frac{E}{R} = \frac{100}{100} = 1$ A

The current decays for 1 millisecond = 1×10^{-3} sec

During decay, $I = I_0 e^{-tR/L}$

$I = (1)e^{\frac{(-1\times 10^{-3})\times 100}{100\times 10^{-3}}}$

or $\quad I = e^{-1} = \frac{1}{e}$ A.

24. (c) : Given : $E = E_0 \sin \omega t$

$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

Since the phase difference (ϕ) between voltage and current is $\frac{\pi}{2}$.

$\therefore \quad$ Power factor $\cos\phi = \cos\frac{\pi}{2} = 0$

Power consumption = $E_{rms} I_{rms} \cos\phi = 0$.

25. (a) : During the growth of current in LR circuit is given by $I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$

or $\quad I = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{5}{5}\left(1 - e^{-\frac{5}{10}\times 2}\right)$

$I = (1 - e^{-1})$.

26. (a) : $M = \mu_0 n_1 n_2 \pi r_1^2 l$.

From $\phi_2 = \pi r_1^2 (\mu_0 n i) n_2 l$.

$A = \pi r_1^2 = 10$ cm^2, $l = 20$ cm, $\quad N_1 = 300$, $N_2 = 400$.

$M = \frac{\mu_0 N_1 N_2 A}{l} = \frac{4\pi\times 10^{-7}\times 300\times 400\times 10\times 10^{-4}}{0.20}$

$= 2.4\pi \times 10^{-4}$ H

Ray Optics

1. If two mirrors are kept at 60° to each other, then the number of images formed by them is
(a) 5 (b) 6 (c) 7 (d) 8.
(2002)

2. Wavelength of light used in an optical instrument are λ_1 = 4000 Å and λ_2 = 5000 Å, then ratio of their respective resolving powers (corresponding to λ_1 and λ_2) is
(a) 16 : 25 (b) 9 : 1 (c) 4 : 5 (d) 5 : 4.
(2002)

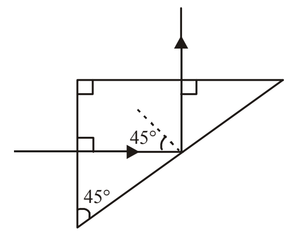
3. Which of the following is used in optical fibres?
(a) total internal reflection
(b) scattering
(c) diffraction
(d) refraction.
(2002)

4. An astronomical telescope has a large aperture to
(a) reduce spherical aberration
(b) have high resolution
(c) increase span of observation
(d) have low dispersion.
(2002)

5. The image formed by an objective of a compound microscope is
(a) virtual and diminished
(b) real and diminished
(c) real and enlarged
(d) virtual and enlarged.
(2003)

6. To get three images of a single object, one should have two plane mirrors at an angle of
(a) 60° (b) 90°
(c) 120° (d) 30°.
(2003)

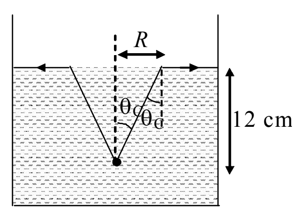
7. A light ray is incident perpendicular to one face of a 90° prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45°, we conclude that the refractive index n



(a) $n < \frac{1}{\sqrt{2}}$ (b) $n > \sqrt{2}$
(c) $n > \frac{1}{\sqrt{2}}$ (d) $n < \sqrt{2}$.
(2004)

8. A plano convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now this lens has been used to form the image of an object. At what distance from this lens an object be placed in order to have a real image of the size of the object?
(a) 20 cm (b) 30 cm (c) 60 cm (d) 80 cm.
(2004)

9. A fish looking up through the water sees the outside world contained in a circular horizon. If the refractive index of water is 4/3 and the fish is 12 cm below the surface, the radius of this circle in cm is



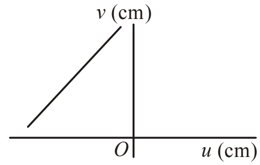
(a) $36\sqrt{5}$ (b) $4\sqrt{5}$
(c) $36\sqrt{7}$ (d) $36/\sqrt{7}$
(2005)

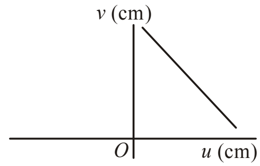
10. A thin glass (refractive index 1.5) lens has optical power of –5 D in air. Its optical power in a liquid medium with refractive index 1.6 will be
(a) 25 D (b) –25 D (c) 1 D (d) –1 D
(2005)

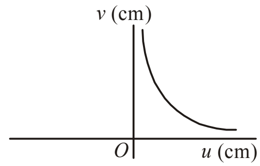
11. The refractive index of glass is 1.520 for red light and 1.525 for blue light. Let D_1 and D_2 be angles of minimum deviation for red and blue light respectively in a prism of this glass. Then
(a) $D_1 > D_2$ (b) $D_1 < D_2$
(c) $D_1 = D_2$
(d) D_1 can be less than or greater than depending upon the angle of prism.
(2006)

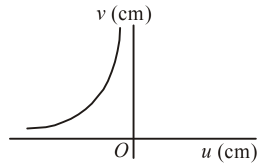
12. Two lenses of power –15 D and +5 D are in contact with each other. The focal length of the combination is
(a) + 10 cm (b) –20 cm
(c) –10 cm (d) + 20 cm
(2007)

13. A student measures the focal length of a convex lens by putting an object pin at a distance u from the lens and measuring the distance v of the image pin. The graph between u and v plotted by the student should look like

(a)


(b)


(c)


(d)


(2008)

Answer Key

1.	(a)	**2.**	(d)	**3.**	(a)	**4.**	(b)	**5.**	(c)	**6.**	(b)
7.	(b)	**8.**	(a)	**9.**	(d)	**10.**	no option	**11.**	(b)	**12.**	(c)
13.	(d)										

1. **(a) :** $n = \frac{360°}{\theta°} - 1 = \frac{360°}{60°} - 1 = 5.$

2. **(d) :** Resolving power is proportional to λ^{-1}

$\therefore \quad \frac{R.P. \text{ for } \lambda_1}{R.P. \text{ for } \lambda_2} = \frac{\lambda_2}{\lambda_1} = \frac{5000}{4000} = \frac{5}{4}.$

3. **(a) :** Total internal reflection is used in optical fibres.

4. **(b) :** Large aperture leads to high resolution of telescope.

5. **(c) :** The objective of compound microscope forms a real and enlarged image.

6. **(b) :** $n = \frac{360°}{\theta^0} - 1$

$\therefore \quad 3 = \frac{360°}{\theta°} - 1 \Rightarrow 4\theta° = 360° \Rightarrow \theta° = 90°.$

7. **(b) :** Total internal reflection occurs in a denser medium when light is incident at surface of separation at angle exceeding critical angle of the medium.
Given : $i = 45°$ in the medium and total internal reflection occurs at the glass air interface

$\therefore \quad n > \frac{1}{\sin C} > \frac{1}{\sin 45°} > \sqrt{2}.$

8. **(a) :** A plano-convex lens behaves like a concave mirror when its curved surface is silvered.
$\therefore$ F of concave mirror so formed

$= \frac{R}{2\mu} = \frac{30}{2 \times 1.5} = 10 \text{ cm}$

To form an image of object size, the object should be placed at ($2F$) of the concave mirror.
$\therefore$ Distance of object from lens = $2 \times F$ = 2×10 = 20 cm.

9. **(d) :** For total internal reflection,

$\mu = \frac{1}{\sin \theta_C} \Rightarrow \sin \theta_C = \frac{1}{\mu} = \frac{3}{4}$

$\therefore \quad \tan \theta_c = \frac{\sin \theta_c}{\sqrt{1 - \sin^2 \theta_c}}$

$= \frac{3/4}{\sqrt{1 - \frac{9}{16}}} = \frac{3}{4} \times \frac{4}{\sqrt{7}} = \frac{3}{\sqrt{7}}$

$\therefore \quad \frac{R}{12} = \frac{3}{\sqrt{7}} \Rightarrow R = \frac{36}{\sqrt{7}} \text{ cm}.$

10. $\frac{1}{f_a} = ({}^a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f_l} = ({}^l\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\therefore \quad \frac{f_a}{f_l} = \frac{({}^l\mu_g - 1)}{({}^a\mu_g - 1)} = \frac{(\mu_g/\mu_l) - 1}{(\mu_g - 1)}$

$= \frac{\mu_g - \mu_l}{\mu_l(\mu_g - 1)} = \frac{1.5 - 1.6}{1.6\,(1.5 - 1)}$

or $\quad \frac{P_l}{P_a} = -\frac{0.1}{1.6 \times 0.5} = \frac{-1}{8}$

$\Rightarrow \quad P_l = -\frac{P_a}{8} = -\frac{(-5)}{8} = \frac{5}{8}$

or Optical power in liquid medium $= \frac{5}{8}$ Dipotre

N.B. : This answer is not given in the four options provided in the question.

11. **(b) :** Angle of minimum deviation $D = A(\mu - 1)$

$\frac{D_1 \text{ for red}}{D_2 \text{ for blue}} = \frac{\mu_R - 1}{\mu_B - 1}$

Since $\mu_B > \mu_R$,

$\therefore \quad \frac{D_1}{D_2} < 1$

$\therefore \quad D_1 < D_2.$

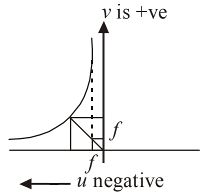
12. **(c) :** Power of combination = $P_1 + P_2$
= – 15 D + 5 D = – 10 D.

Focal length of combination $F=\frac{1}{P}=\frac{1}{-10\text{ D}}$

$= -0.1 \text{ m} = -10 \text{ cm}.$

13. **(d) :** According to the new cartesian system used in schools, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ for a convex lens.

u has to be negative.



If $v = \infty$, $u = f$ and if $u = \infty$, $v = f$.

A parallel beam ($u = \infty$) is focussed at f and if the object is at f, the rays are parallel. The point which meets the curve at $u = v$ gives $2f$. Therefore v is +ve, u is negative, both are symmetrical and this curve satisfies all the conditions for a convex lens.

CHAPTER

20

Wave Optics

1. To demonstrate the phenomenon of interference we require two sources which emit radiation of
   (a) nearly the same frequency
   (b) the same frequency
   (c) different wavelength
   (d) the same frequency and having a definite phase relationship.
   (2003)

2. The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double-slit experiment is
   (a) infinite (b) five (c) three (d) zero.
   (2004)

3. The angle of incidence at which reflected light in totally polarized for reflection from air to glass (refractive index n), is
   (a) $\sin^{-1}(n)$ (b) $\sin^{-1}(1/n)$
   (c) $\tan^{-1}(1/n)$ (d) $\tan^{-1}(n)$.
   (2004)

4. A Young's double slit experiment uses a monochromatic source. The shape of the interference fringes formed on a screen
   (a) straight line (b) parabola
   (c) hyperbola (d) circle
   (2005)

5. If I_0 is the intensity of the principal maximum in the single slit diffraction pattern, then what will be its intensity when the slit width is doubled?
   (a) I_0 (b) $I_0/2$ (c) $2I_0$ (d) $4I_0$
   (2005)

6. When an unpolarized light of intensity I_0 is incident on a polarizing sheet, the intensity of the light which does not get transmitted is
   (a) zero (b) I_0
   (c) $\frac{1}{2}I_0$ (d) $\frac{1}{4}I_0$
   (2005)

7. Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 mm. Approximately, what is the maximum distance at which these dots can be resolved by the eye? [Take wavelength of light = 500 nm]
   (a) 6 m (b) 3 m (c) 5 m (d) 1 m
   (2005)

8. In a Young's double slit experiment the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of light used) is I. If I_0 denotes the maximum intensity, $\frac{I}{I_0}$ is equal to
   (a) $\frac{3}{4}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$
   (2007)

Answer Key

1.	(d)	**2.**	(b)	**3.**	(d)	**4.**	(a)	**5.**	(a)	**6.**	(c)
7.	(c)	**8.**	(a)								

1. **(d) :** For interference phenomenon, two sources should emit radiation of the same frequency and having a definite phase relationship.

2. **(b) :** For interference maxima, $d\sin\theta = n\lambda$

$\therefore \quad 2\lambda\sin\theta = n\lambda$

or $\quad \sin\theta = \frac{n}{2}$

This equation is satisfied if $n = -2, -1, 0, 1, 2$. $\sin\theta$ is never greater than (+1), less than (–1)

$\therefore$ Maximum number of maxima can be five.

3. **(d) :** According to Brewster's law of polarization, $n = \tan i_p$ where i_p is angle of incidence

$\therefore \quad i_p = \tan^{-1}(n)$.

4. **(a) :** Straight line fringes are formed on screen.

5. **(a) :** For diffraction pattern

$I = I_0\left(\frac{\sin\phi}{\phi}\right)^2$ where ϕ denotes path difference

For principal maxima, $\phi = 0$. Hence $\left(\frac{\sin\phi}{\phi}\right) = 1$

Hence intensity remains constant at I_0

$I = I_0\,(1) = I_0$.

6. **(c) :** Intensity of polarized light $= I_0/2$

$\therefore$ Intensity of light not transmitted

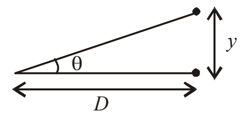
$$= I_0 - \frac{I_0}{2} = \frac{I_0}{2}$$

7. **(c) :** Resolution limit $= \frac{1.22\lambda}{d}$

Again resolution limit $= \sin\theta = \theta = \frac{y}{D}$

$\therefore \quad \frac{y}{D} = \frac{1.22\lambda}{d}$

or $\quad D = \frac{yd}{1.22\lambda}$



or $\quad D = \frac{(10^{-3})\times(3\times10^{-3})}{(1.22)\times(5\times10^{-7})} = \frac{30}{6.1} \approx 5$ m.

8. **(a) :** In Young's double slit experiment intensity at a point is given by

$I = I_0\cos^2\left(\frac{\phi}{2}\right)$

where ϕ = phase difference, I_0 = maximum intensity

or $\quad \frac{I}{I_0} = \cos^2\left(\frac{\phi}{2}\right) \quad$ ... (i)

Phase difference $\phi = \frac{2\pi}{\lambda} \times$ path difference

$\therefore \quad \phi = \frac{2\pi}{\lambda}\times\frac{\lambda}{6}$ or $\phi = \frac{\pi}{3} \quad$ ... (ii)

Substitute eqn. (ii) in eqn. (i), we get

$\frac{I}{I_0} = \cos^2\left(\frac{\pi}{6}\right)$ or $\frac{I}{I_0} = \frac{3}{4}$.

CHAPTER 21

Electromagnetic Waves

1. Infrared radiation is detected by
   (a) spectrometer (b) pyrometer
   (c) nanometer (d) photometer.
   (2002)
2. Electromagnetic waves are transverse in nature is evident by
   (a) polarization (b) interference
   (c) reflection (d) diffraction.
   (2002)
3. Which of the following are not electromagnetic waves?
   (a) cosmic rays (b) gamma rays
   (c) β-rays (d) X-rays.
   (2002)
4. Consider telecommunication through optical fibres. Which of the following statements is not true?
   (a) Optical fibres can be of graded refractive index.
   (b) Optical fibres are subject to electromagnetic interference from outside.
   (c) Optical fibres have extremely low transmission loss.
   (d) Optical fibres may have homogeneous core with a suitable cladding.
   (2003)
5. An electromagnetic wave of frequency $\upsilon = 3.0$ MHz passes from vacuum into a dielectric medium with permitivity $\varepsilon = 4.0$. Then
   (a) wavelength is doubled and the frequency remains unchanged
   (b) wavelength is doubled and frequency becomes half
   (c) wavelength is halved and frequency remains unchanged
   (d) wavelength and frequency both remain unchanged.
   (2004)
6. The rms value of the electric field of the light coming from the sun is 720 N/C. The average total energy density of the electromagnetic wave is
   (a) 3.3×10^{-3} J/m^3 (b) 4.58×10^{-6} J/m^3
   (c) 6.37×10^{-9} J/m^3 (d) 81.35×10^{-12} J/m^3.
   (2006)

Answer Key

1.	(b)	**2.**	(a)	**3.**	(c)	**4.**	(b)	**5.**	(c)	**6.**	(b)

1. **(b) :** Infrared radiation produces thermal effect and is detected by pyrometer.
2. **(a) :** Polarization proves the transverse nature of electromagnetic waves.
3. **(c) :** β-rays are not electromagnetic waves.
4. **(b) :** Optical fibres are subject to electromagnetic intereference from outside.
5. **(c) :** During propogation of a wave from one medium to another, frequency remains constant and wavelength changes

   $\mu = \sqrt{\frac{\varepsilon}{\varepsilon_0}} = \sqrt{4} = 2$

   Since $\mu \propto \frac{1}{\lambda}$

   $\therefore$ Wavelength is halved

   Hence option (c) holds good.
6. **(b) :** $u = \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 + \frac{1}{2\mu_0} B_{\text{rms}}^2$

   $= \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 + \frac{1}{2\mu_0}\left(\frac{E_{\text{rms}}^2}{c^2}\right)$

   $= \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 + \frac{1}{2\mu_0} E_{\text{rms}}^2 \varepsilon_0 \mu_0$

   $= \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 + \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 = \varepsilon_0 E_{\text{rms}}^2$

   $= (8.85 \times 10^{-12}) \times (720)^2$

   $= 4.58 \times 10^{-6}$ Jm^{-3}.

CHAPTER

22

Electrons and Photons

1. Sodium and copper have work functions 2.3 eV and 4.5 eV respectively. Then the ratio of the wavelengths is nearest to
(a) 1 : 2 (b) 4 : 1 (c) 2 : 1 (d) 1 : 4.
(2002)

2. Two identical photocathodes receive light of frequencies f_1 and f_2. If the velocities of the photoelectrons (of mass m) coming out are respectively v_1 and v_2, then
(a) $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$
(b) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{\frac{1}{2}}$
(c) $v_1^2 + v_2^2 = \frac{2h}{m}(f_1 + f_2)$
(d) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{\frac{1}{2}}$.
(2003)

3. According to Einstein's photoelectric equation, the plot of the kinetic energy of the emitted photo electrons from a metal *vs* the frequency, of the incident radiation gives a straight line whose slope
(a) depends on the nature of the metal used
(b) depends on the intensity of the radiation
(c) depends both on the intensity of the radiation and the metal used
(d) is the same for all metals and independent of the intensity of the radiation.
(2004)

4. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately
(a) 540 nm (b) 400 nm
(c) 310 nm (d) 220 nm.
(2004)

5. A charged oil drop is suspended in a uniform field of 3×10^4 V/m so that it neither falls nor rises. The charge on the drop will be (take the mass of the charge = 9.9×10^{-15} kg and g = 10 m/s^2)
(a) 3.3×10^{-18} C (b) 3.2×10^{-18} C
(c) 1.6×10^{-18} C (d) 4.8×10^{-18} C.
(2004)

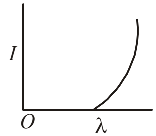
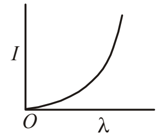
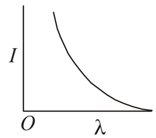
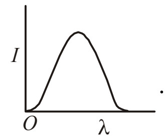
6. A photocell is illuminated by a small bright source placed 1 m away. When the same source of light is placed (1/2) m away, the number of electrons emitted by photocathode would
(a) decrease by a factor of 2
(b) increase by a factor of 2
(c) decrease by a factor of 4
(d) increase by a factor of 4
(2005)

7. If the kinetic energy of a free electron doubles, its de Broglie wavelength changes by the factor
(a) $1/\sqrt{2}$ (b) $\sqrt{2}$ (c) 1/2 (d) 2.
(2005)

8. The time by a photoelectron to come out after the photon strikes is approximately
(a) 10^{-1} s (b) 10^{-4} s
(c) 10^{-10} s (d) 10^{-16} s.
(2006)

9. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV, and the stopping potential for a radiation incident on this surface 5 V. The incident radiation lies in
(a) X-ray region (b) ultra-violet region
(c) infra-red region (d) visible region.
(2006)

10. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows

(a) 
(b) 
(c) 
(d) 

(2006)

11. Photon of frequency υ has a momentum associated with it. If c is the velocity of light, the momentum is

(a) $h\upsilon/c$ (b) υ/c
(c) $h\upsilon c$ (d) $h\upsilon/c^2$

(2007)

12. If g_E and g_M are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio

$$\frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}} \text{ to be}$$

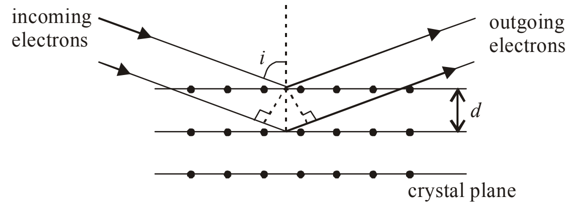
(a) g_M/g_E (b) 1
(c) 0 (d) g_E/g_M

(2007)

Directions : Questions 13, 14 and 15 are based on the following paragraph.

Wave property of electrons implies that they will show diffraction effects. Davisson and Germer demonstrated this by diffracting electrons from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively (see figure).

(2008)



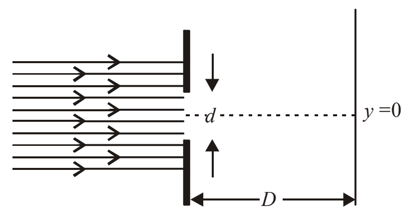
13. Electrons accelerated by potential V are diffracted from a crystal. If $d = 1$ Å and $i = 30°$, V should be about ($h = 6.6 \times 10^{-34}$ Js, $m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C)

(a) 1000 V (b) 2000 V (c) 50 V (d) 500 V.

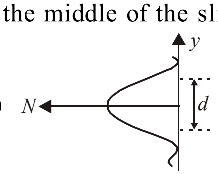
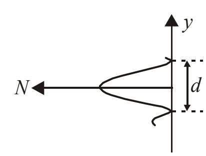
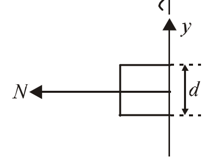
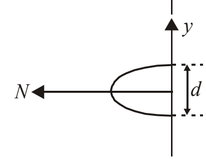
14. If a strong diffraction peak is observed when electrons are incident at an angle i from the normal to the crystal planes with distance d between them (see figure), de Broglie wavelength λ_{dB} of electrons can be calculated by the relationship (n is an integer)

(a) $d \cos i = n\lambda_{dB}$ (b) $d \sin i = n\lambda_{dB}$
(c) $2d \cos i = n\lambda_{dB}$ (d) $2d \sin i = n\lambda_{dB}$

15. In an experiment, electrons are made to pass through a narrow slit of width d comparable to their de Broglie wavelength. They are detected on a screen at a distance D from the slit.



Which of the following graphs can be expected to represent the number of electrons N detected as a function of the detector position y ($y = 0$ corresponds to the middle of the slit)?

(a) 
(b) 
(c) 
(d) 

Answer Key

1.	(c)	**2.**	(a)	**3.**	(d)	**4.**	(c)	**5.**	(a)	**6.**	(d)
7.	(a)	**8.**	(c)	**9.**	(b)	**10.**	(c)	**11.**	(a)	**12.**	(b)
13.	(c)	**14.**	(c)	**15.**	(a)						

1. **(c) :** Work function $= hc/\lambda$

$$\frac{W_{Na}}{W_{Cu}} = \frac{4.5}{2.3} = \frac{2}{1}.$$

2. **(a) :** For photoelectric effect, according to Einstein's equation,
Kinetic energy of emitted electron = hf – (work function ϕ)

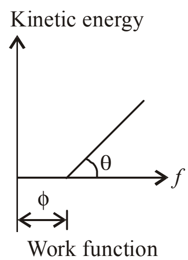
$$\therefore \quad \frac{1}{2}mv_1^2 = hf_1 - \phi$$

$$\frac{1}{2}mv_2^2 = hf_2 - \phi$$

$$\therefore \quad \frac{1}{2}m(v_1^2 - v_2^2) = h(f_1 - f_2)$$

$$\therefore \quad v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2).$$

3. **(d) :** According to Einstein's equation,
Kinetic energy $= hf - \phi$ where kinetic energy and f(frequency) are variables, compare it with equation, $y = mx + c$



$\therefore$ slope of line $= h$
h is Planck's constant.
Hence the slope is same for all metals and independent of the intensity of radiation.
Option (d) represents the answer.

4. **(c) :** Let λ_m = Longest wavelength of light

$$\therefore \quad \frac{hc}{\lambda_m} = \phi(\text{work function})$$

$$\therefore \quad \lambda_m = \frac{hc}{\phi} = \frac{(6.63\times10^{-34})\times(3\times10^{8})}{4.0\times1.6\times10^{-19}}$$

or $\lambda_m = 310$ nm.

5. **(a) :** For equilibrium of charged oil drop,

$$qE = mg$$

$$\therefore \quad q = \frac{mg}{E} = \frac{(9.9\times10^{-15})\times10}{(3\times10^{4})} = 3.3\times10^{-18}\text{ C}.$$

6. **(d) :** $I = \dfrac{P \text{ of source}}{4\pi(\text{distance})^2} = \dfrac{P}{4\pi d^2}$

Here, we assume light to spread uniformly in all directions.
Number of photo-electrons emitted from a surface depend on intensity of light I falling on it. Thus the number of electrons emitted n depends directly on I. P remains constant as the source is the same.

$$\therefore \quad \frac{I_2}{I_1} = \frac{n_2}{n_1} \Rightarrow \frac{P_2}{P_1}\left(\frac{d_1}{d_2}\right)^2 = \frac{n_2}{n_1}$$

$$\therefore \quad \frac{n_2}{n_1} = \left(\frac{P}{P}\right)\left(\frac{1}{1/2}\right)^2 = \frac{4}{1}.$$

7. **(a) :** de Broglie wavelength $\lambda = h/p = h/\sqrt{(2mK)}$

$\therefore \quad \lambda = \dfrac{h}{\sqrt{2mK}}$ where K = kinetic energy of particle

$$\therefore \quad \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{K_1}{K_2}} = \sqrt{\frac{K_1}{2K_1}} = \frac{1}{\sqrt{2}}.$$

8. **(c) :** Emission of photo-electron starts from the surface after incidence of photons in about 10^{-10} sec.

9. **(b) :** For photo-electron emission,
(Incident energy E) = $(\text{K.E.})_{max}$ + (Work function ϕ)

or $E = K_m + \phi$
or $E = 5 + 6.2 = 11.2$ eV
$= 11.2 \times (1.6 \times 10^{-19})$ J

$$\therefore \quad \frac{hc}{\lambda} = 11.2\times1.6\times10^{-19}$$

$$\text{or} \quad \lambda = \frac{(6.63\times10^{-34})\times(3\times10^{8})}{11.2\times1.6\times10^{-19}}\text{ m}$$

or $\lambda = 1110 \times 10^{-10}$ m $= 1110$ Å.
The incident radiation lies in ultraviolet region.

10. **(c) :** The graph (c) depicts the variation of λ with I.

11. **(a) :** Energy of a photon $E = h\upsilon$ ... (i)
Also $E = pc$ ... (ii)
where p is the momentum of a photon
From (i) and (ii), we get

$$h\upsilon = pc \quad \text{or} \quad p = \frac{h\upsilon}{c}.$$

12. **(b) :** Since electronic charge (1.6×10^{-19} C) universal constant. It does not depend on g.
$\therefore$ Electronic charge on the moon = electronic charge on the earth

or $$\frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}} = 1.$$

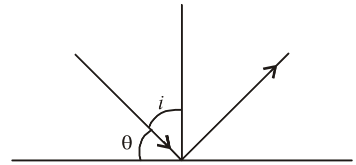
13. **(c) :** For electron diffraction, $d = 1$ Å, $i = 30°$
i.e., grazing angle $\theta = 60°$, $h = 6.6 \times 10^{-34}$ J s.
$m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C.
Bragg's equation for X-rays, which is also used in electron diffraction gives $n\lambda = 2d \sin\theta$.

$$\therefore\ \lambda = \frac{2 \times 1(\text{Å}) \times \sin 60°}{1} \text{(assuming first order)}$$

$$\lambda = \sqrt{3}\,\text{Å}, \quad \sqrt{V} = \frac{(12.27 \times 10^{-10})}{\sqrt{3} \times 10^{-10}}$$

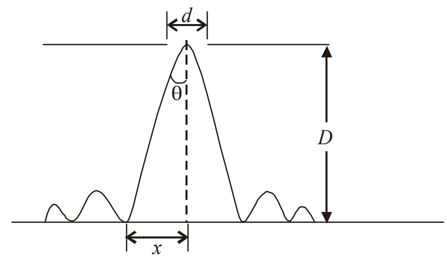
$V = 50.18$ Volt.

14. **(c) :** Bragg's relation $n\lambda = 2d \sin\theta$ for having an intensity maximum for diffraction pattern.



But as the angle of incidence is given,
$n\lambda = 2d \cos i$ is the formula for finding a peak.

15. **(a) :** The electron diffraction pattern from a single slit will be as shown below.



$$d \sin\theta = \frac{\lambda}{2\pi}$$

The line of maximum intensity for the zeroth order will exceed d very much.

CHAPTER
23

Atoms, Molecules and Nuclei

1. At a specific instant emission of radioactive compound is deflected in a magnetic field. The compound can emit
   (i) electrons (ii) protons
   (iii) He^{2+} (iv) neutrons
   The emission at the instant can be
   (a) i, ii, iii (b) i, ii, iii, iv
   (c) iv (d) ii, iii.
   (2002)

2. If 13.6 eV energy is required to ionize the hydrogen atom, then the energy required to remove an electron from $n = 2$ is
   (a) 10.2 eV (b) 0 eV
   (c) 3.4 eV (d) 6.8 eV.
   (2002)

3. If N_0 is the original mass of the substance of half-life period $t_{1/2} = 5$ years, then the amount of substance left after 15 years is
   (a) $N_0/8$ (b) $N_0/16$ (c) $N_0/2$ (d) $N_0/4$.
   (2002)

4. Which of the following radiations has the least wavelength?
   (a) γ-rays (b) β-rays
   (c) α-rays (d) X-rays.
   (2003)

5. When U^{238} nucleus originally at rest, decays by emitting an alpha particle having a speed u, the recoil speed of the residual nucleus is
   (a) $\frac{4u}{238}$ (b) $-\frac{4u}{234}$ (c) $\frac{4u}{234}$ (d) $-\frac{4u}{238}$.
   (2003)

6. A radioactive sample at any instant has its disintegration rate 5000 disintegrations per minute. After 5 minutes, the rate is 1250 disintegrations per minute. Then, the decay constant (per minute) is
   (a) 0.4 ln 2 (b) 0.2 ln 2
   (c) 0.1 ln 2 (d) 0.8 ln 2.
   (2003)

7. A nucleus with $Z = 92$ emits the following in a sequence: $\alpha, \alpha, \beta^-, \beta^-, \alpha, \alpha, \alpha, \alpha, \beta^-, \beta^-, \alpha, \beta^+, \beta^+, \alpha$. The Z of the resulting nucleus is
   (a) 76 (b) 78 (c) 82 (d) 74.
   (2003)

8. Which of the following cannot be emitted by radioactive substances during their decay?
   (a) protons (b) neutrinos
   (c) helium nuclei (d) electrons.
   (2003)

9. In the nuclear fusion reaction, $^2_1H + ^3_1H \rightarrow ^4_2He + n$ given that the repulsive potential energy between the two nuclei is $\sim 7.7 \times 10^{-14}$ J, the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K]
   (a) 10^7 K (b) 10^5 K (c) 10^3 K (d) 10^9 K.
   (2003)

10. Which of the following atoms has the lowest ionization potential?
    (a) $^{14}_7N$ (b) $^{133}_{55}Cs$ (c) $^{40}_{18}Ar$ (d) $^{16}_8O$.
    (2003)

11. The wavelengths involved in the spectrum of deuterium (^2_1D) are slightly different from that of hydrogen spectrum, because
    (a) size of the two nuclei are different
    (b) nuclear forces are different in the two cases
    (c) masses of the two nuclei are different

(d) attraction between the electron and the nucleus is different in the two cases.

(2003)

12. If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{++} is

(a) 30.6 eV (b) 13.6 eV
(c) 3.4 eV (d) 122.4 eV.

(2003)

13. A nucleus disintegrates into two nuclear parts which have their velocities in the ratio 2 : 1. The ratio of their nuclear sizes will be

(a) $2^{1/3} : 1$ (b) $1 : 3^{1/2}$ (c) $3^{1/2} : 1$ (d) $1 : 2^{1/3}$.

(2004)

14. The binding energy per nucleon of deuteron $\left({}^{2}_{1}\mathrm{H}\right)$ and helium nucleus $\left({}^{4}_{2}\mathrm{He}\right)$ is 1.1 MeV and 7 MeV respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is

(a) 13.9 MeV (b) 26.9 MeV
(c) 23.6 MeV (d) 19.2 MeV.

(2004)

15. An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of the closest approach is of the order of

(a) 1 Å (b) 10^{-10} cm
(c) 10^{-12} cm (d) 10^{-15} cm.

(2004)

16. If radius of the ${}^{27}_{13}\mathrm{Al}$ nucleus is estimated to be 3.6 fermi then the radius of ${}^{125}_{52}\mathrm{Al}$ nucleus be nearly

(a) 4 fermi (b) 5 fermi
(c) 6 fermi (d) 8 fermi

(2005)

17. The intensity of gamma radiation from a given source is I. On passing through 36 mm of lead, it is reduced to $I/8$. The thickness of lead which will reduce the intensity to $I/2$ will be

(a) 18 mm (b) 12 mm
(c) 6 mm (d) 9 mm

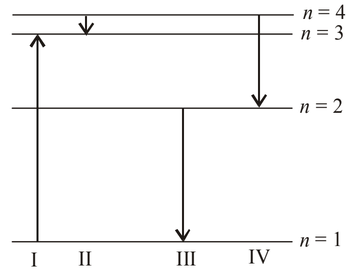
(2005)

18. Starting with a sample of pure ${}^{66}\mathrm{Cu}$, 7/8 of it decays into Zn in 15 minutes. The corresponding half-life is

(a) 5 minutes (b) $7\frac{1}{2}$ minutes
(c) 10 minutes (d) 14 minutes

(2005)

19. The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy?



(a) I (b) II (c) III (d) IV

(2005)

20. A nuclear transformation is denoted by $X(n, \alpha)\,{}^{7}_{3}\mathrm{Li}$. Which of the following is the nucleus of element X?

(a) ${}^{9}_{5}\mathrm{B}$ (b) ${}^{11}_{4}\mathrm{Be}$ (c) ${}^{12}_{6}\mathrm{C}$ (d) ${}^{10}_{5}\mathrm{B}$

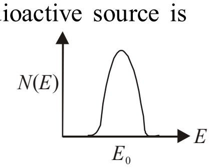
(2005)

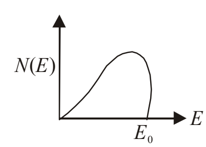
21. An alpha nucleus of energy $\frac{1}{2}mv^2$ bombards a heavy nuclear target of charge Ze. Then the distance of closest approach for the alpha nucleus with be proportional to

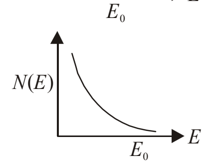
(a) $1/Ze$ (b) v^2 (c) $1/m$ (d) $1/v^4$.

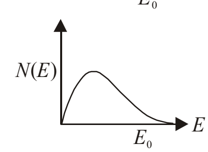
(2006)

22. The energy spectrum of β-particles [number $N(E)$ as a function of β-energy E] emitted from a radioactive source is

(a)


(b)


(c)


(d)


(2006)

23. When ${}_{3}\mathrm{Li}^{7}$ nuclei are bombarded by protons, and the resultant nuclei are ${}_{4}\mathrm{Be}^{8}$, the emitted particles will be

(a) neutrons (b) alpha particles
(c) beta particles (d) gamma photons.

(2006)

24. The 'rad' is the correct unit used to report the measurement of
(a) the rate of decay of radioactive source
(b) the ability of a beam of gamma ray photons to produce ions in a target
(c) the energy delivered by radiation to target
(d) the biological effect of radiation.

(2006)

25. If the binding energy per nucleon in ${}^{7}_{3}\text{Li}$ and ${}^{4}_{2}\text{He}$ nuclei are 5.60 MeV and 7.06 MeV respectively, then in the reaction : $p + {}^{7}_{3}\text{Li} \rightarrow 2\,{}^{4}_{2}\text{He}$, energy of proton must be
(a) 39.2 MeV (b) 28.24 MeV]
(c) 17.28 MeV (d) 1.46 MeV.

(2006)

26. If M_O is the mass of an oxygen isotope ${}_8O^{17}$, M_P and M_N are the masses of a proton and a neutron respectively, the nuclear binding energy of the isotope is
(a) $(M_O - 17\,M_N)\,c^2$ (b) $(M_O - 8\,M_P)\,c^2$
(c) $(M_O - 8\,M_P - 9\,M_N)\,c^2$
(d) $M_O\,c^2$

(2007)

27. In gamma ray emission from a nucleus
(a) only the proton number changes
(b) both the neutron number and the proton number change
(c) there is no change in the proton number and the neutron number
(d) only the neutron number changes

(2007)

28. The half-life period of a radio-active element X is same as the mean life time of another radio-active element Y. Initially they have the same number of atoms. Then
(a) X and Y decay at same rate always
(b) X will decay faster than Y
(c) Y will decay faster than X
(d) X and Y have same decay rate initially

(2007)

29. Which of the following transitions in hydrogen atoms emit photons of highest frequency ?
(a) $n = 1$ to $n = 2$ (b) $n = 2$ to $n = 6$
(c) $n = 6$ to $n = 2$ (d) $n = 2$ to $n = 1$

(2007)

Directions : *Question 30 contains statement-1 and statement-2. Of the four choices given, choose the one that best describes the two statements.*
(a) Statement-1 is true, statement-2 is false.
(b) Statement-1 is false, statement-2 is true.
(c) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.
(d) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.

30. **Statement-1 :** Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion.
Statement-2 : For heavy nuclei, binding energy per nucleon increases with increasing Z while for light nuclei it decreases with increasing Z.

(2008)

31. Suppose an electron is attracted towards the origin by a force k/r where k is a constant and r is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the n^{th} orbital of the electron is found to be r_n and the kinetic energy of the electron to be T_n. Then which of the following is true?
(a) $T_n \propto \frac{1}{n},\ r_n \propto n^2$ (b) $T_n \propto \frac{1}{n^2},\ r_n \propto n^2$
(c) T_n independent of n, $r_n \propto n$
(d) $T_n \propto \frac{1}{n},\ r_n \propto n$

(2008)

Answer Key

1.	(a)	**2.**	(c)	**3.**	(a)	**4.**	(a)	**5.**	(b)	**6.**	(a)
7.	(b)	**8.**	(a)	**9.**	(d)	**10.**	(b)	**11.**	(c)	**12.**	(a)
13.	(d)	**14.**	(c)	**15.**	(c)	**16.**	(c)	**17.**	(b)	**18.**	(a)
19.	(c)	**20.**	(d)	**21.**	(c)	**22.**	(d)	**23.**	(d)	**24.**	(d)
25.	(c)	**26.**	(c)	**27.**	(c)	**28.**	(b)	**29.**	(d)	**30.**	(a)
31.	(c)										

1. **(a) :** Neutrons are electrically neutral. They are not deflected by magnetic field
Hence (a) represents the answer.

2. **(c) :** $E_n = \frac{13.6}{n^2} \Rightarrow E_2 = \frac{13.6}{(2)^2} = 3.4 \text{ eV}$

3. **(a) :** $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T} = \left(\frac{1}{2}\right)^{15/5} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$
$\therefore$ $N = N_0/8$.

4. **(a) :** Gamma rays have the least wavelength.

5. **(b) :** Linear momentum is conserved
α-particle = ${}_2^4He$
$U^{238} \rightarrow X^{234} + He^4$
$\therefore$ $(238 \times 0) = (238 \times v) + 4u$
or $v = -\frac{4u}{234}$.

6. **(a) :** Let decay constant per minute = λ
Disintegration rate, inially = 5000
$\therefore$ $N_0\lambda = 5000$ .... (i)
Disintegration rate, finally = 1250
$\therefore$ $N\lambda = 1250$ .... (ii)
$\therefore$ $\frac{N\lambda}{N_0\lambda} = \frac{1250}{5000} = \frac{1}{4}$
or $\frac{N}{N_0} = \frac{1}{4} \Rightarrow \frac{N_0 e^{-5\lambda}}{N_0} = \frac{1}{4} \Rightarrow e^{-5\lambda} = (4)^{-1}$
$\therefore$ $5\lambda = \ln 4 = 2\ln 2$
$\therefore$ $\lambda = \frac{2}{5}\ln 2 = 0.4 \ln 2.$

7. **(b) :** The nucleus emits 8α particles *i.e.*, $8({}_2He^4)$
$\therefore$ Decrease in $Z = 8 \times 2 = 16$ .... (i)
Four β^- particles are emitted *i.e.*, $4({}_{-1}\beta^0)$
$\therefore$ Increase in $Z = 4 \times 1 = 4$ .... (ii)
2 positrons are emitted i.e., $2({}_1\beta^0)$
$\therefore$ Decrease in $Z = 2 \times 1 = 2$ ......(iii)
$\therefore$ Z of resultant nucleus = 92 – 16 + 4 – 2 = 78.

8. **(a) :** Protons are not emitted during radioactive decay.

9. **(d) :** At temperature T, molecules of a gas acquire a kinetic energy $= \frac{3}{2}kT$ where k = Boltzmann's constant
$\therefore$ To initiate the fusion reaction
$\frac{3}{2}kT = 7.7 \times 10^{-14}$ J
$\therefore$ $T = \frac{7.7 \times 10^{-14} \times 2}{3 \times 1.38 \times 10^{-23}} = 3.7 \times 10^9$ K.

10. **(b) :** ${}_{55}^{133}Cs$ has the lowest ionization potential. Of the four atoms given, Cs has the largest size. Electrons in the outer most orbit are at large distance from nucleus in a large-size atom. Hence the ionization potential is the least.

11. **(c) :** Masses of ${}_1H^1$ and ${}_1D^2$ are different. Hence the corresponding wavelengths are different.

12. **(a) :** Energy
$$E_2 = \frac{-Z^2 E_0}{n^2} = \frac{-(3)^2 \times 13.6}{(2)^2} = -30.6 \text{ eV}$$
$\therefore$ energy required = 30.6 eV.

13 **(d) :** Momentum is conserved during disintegration
$\therefore$ $m_1 v_1 = m_2 v_2$ ....(i)
For an atom, $R = R_0 A^{1/3}$
$\therefore$ $\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$
$= \left(\frac{m_1}{m_2}\right)^{1/3} = \left(\frac{m_2 v_2}{m_2 v_1}\right)^{1/3}$, from (i)
$\therefore$ $\frac{R_1}{R_2} = \left(\frac{1}{2}\right)^{1/3} = \frac{1}{2^{1/3}}$

14. **(c) :** Total binding energy for (each deuteron)
= 2 × 1.1 = 2.2 MeV
Total binding energy for helium = 4 × 7 = 28 MeV
$\therefore$ Energy released = 28 –(2 × 2.2)
= 28 – 4.4 = 23.6 MeV.

15. **(c) :** Kinetic energy is converted into potential energy at closest approach
$\therefore$ K.E. = P.E.

$\therefore \quad 5\text{ MeV} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r}$

or $\quad 5\times10^6 \times e = \dfrac{(9\times10^9)\times(92e)(2e)}{r}$

or $\quad r = \dfrac{9\times10^9\times92\times2\times e}{5\times10^6}$

$= \dfrac{9\times10^9\times92\times2\times(1.6\times10^{-19})}{5\times10^6}$

$\therefore \quad r = 5.3\times10^{-14}$ m $= 5.3\times10^{-12}$ cm.

16. **(c) :** R is proportional to $A^{1/3}$ where A is mass number

$3.6 = R_0\ (27)^{1/3} = 3R_0$, for $^{27}_{13}\text{Al}$.

Again $R = R_0\ (125)^{1/3}$, for $^{125}_{52}\text{Al}$

$\therefore \quad R = \dfrac{(3\cdot6)}{3}\times5 = 6$ fermi.

17. **(b) :** $\because\ I = I_0\,e^{-kx} \Rightarrow \dfrac{I}{I_0} = e^{-kx}$

$\therefore \quad \ln\left(\dfrac{I}{I_0}\right) = -kx$

In first case

$\ln\left(\dfrac{1}{8}\right) = -k\times36$

$\ln(2^{-3}) = -k\times36$

or $3\ln2 = k\times36$ ....(i)

In second case, $\ln\left(\dfrac{1}{2}\right) = -k\times x$

or $\quad \ln(2^{-1}) = -kx$

or $\quad \ln2 = kx$ .... (ii)

From (i) and (ii)

$3\times(kx) = k\times36$

or $\quad x = 12$ mm.

18. **(a) :** $\dfrac{N}{N_0} = \left(\dfrac{1}{2}\right)^{t/T}$

$\therefore \quad \dfrac{1}{8} = \left(\dfrac{1}{2}\right)^{15/T} \Rightarrow \left(\dfrac{1}{2}\right)^3 = \left(\dfrac{1}{2}\right)^{15/T}$

$\therefore \quad \dfrac{15}{T} = 3 \Rightarrow T = 5$ min.

19. **(c) :** I is showing absorption photon.

From rest of three, III having maximum energy from

$\Delta E \propto \left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$.

20. **(d) :** The nuclear transformation is given by

$^{A}_{Z}X + ^{1}_{0}n \rightarrow ^{4}_{2}\text{He} + ^{7}_{3}\text{Li}$

According to conservation of mass number

$A + 1 = 4 + 7$

or $\quad A = 10$

According to conservation of charge number

$Z + 0 \rightarrow 2 + 3$

or $\quad Z = 5$

So the nucleus of the element be $^{10}_{5}\text{B}$

21. **(c) :** For closest approach, kinetic energy is converted into potential energy

$\therefore \quad \dfrac{1}{2}mv^2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r_0} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(Ze)(2e)}{r_0}$

or $\quad r_0 = \dfrac{4Ze^2}{4\pi\varepsilon_0 mv^2} = \dfrac{Ze^2}{\pi\varepsilon_0 v^2}\left(\dfrac{1}{m}\right)$

or $\quad r_0$ is proportional to $\left(\dfrac{1}{m}\right)$.

22. **(d) :** Graph (d) represents the variation.

23. **(d) :** $_3\text{Li}^7 + {_1}\text{H}^1 \rightarrow {_4}\text{Be}^8 + {_Z}X^A$

Z for the unknown X nucleus $= (3 + 1) - 4 = 0$

A for the unknown X nucleus $= (7 + 1) - 8 = 0$

Hence particle emitted has zero Z and zero A

It is a gamma photon.

24. **(d) :** The 'rad' the biological effect of radiation.

25. **(c) :** Binding energy of

$^{7}_{3}\text{Li} = 7\times5.60 = 39.2$ MeV

Binding energy of $^{4}_{2}\text{He} = 4\times7.06 = 28.24$ MeV

$\therefore$ Energy of proton = Energy of $\left[2(^{4}_{2}\text{He}) - ^{7}_{3}\text{Li}\right]$

$= 2\times28.24 - 39.2$

$= 17.28$ MeV.

26. **(c) :** Binding energy $= [ZM_P + (A - Z)\,M_N - M]c^2$

$= [8M_P + (17 - 8)\,M_N - M_O]c^2$

$= (8M_P + 9M_N - M_O)c^2$

[But the option given is negative of this].

27. **(c) :** γ-ray emission takes place due to deexcitation of the nucleus. Therefore during γ-ray emission, there is no change in the proton and neutron number.

28. **(b) :** $T_{1/2}$, half life of $X = \tau_Y$, mean life of Y

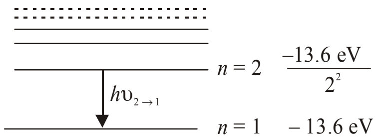
$$\frac{\ln 2}{\lambda_X} = \frac{1}{\lambda_Y} \Rightarrow \lambda_X = \lambda_Y \ln 2$$

$\lambda_X > \lambda_Y$

$\therefore \quad A_X = A_0 e^{-\lambda_X t}\ ; \quad A_Y = A_0 e^{-\lambda_Y t}$

X will decay faster than Y.

29. (d) :



$$h\upsilon_{2\to1} = -13.6\left(\frac{1}{2^2} - \frac{1}{1^2}\right) \text{ eV}$$

$$= +13.6 \times \frac{3}{4} \text{eV} = 10.2 \text{ eV}.$$

Emission is $n = 2 \to n = 1$ *i.e.*, higher n to lower n Transition from lower to higher levels are absorption lines.

$$-13.6\left(\frac{1}{6^2} - \frac{1}{2^2}\right) = +13.6 \times \frac{2}{9}$$

This is <$E_{n=2} \to E_{n=1}$.

30. (a) : Statement-1 states that energy is released when heavy nuclei undergo fission and light nuclei undergo fusion is correct. Statement-2 is wrong The binding energy per nucleon, B/A, starts at a small value, rises to a maximum at ^{62}Ni, then decreases to 7.5 MeV for the heavy nuclei. The answer is (a).

31. (c) : Supposing that the force of attraction in Bohr atom does not follow inverse square law but inversely proportional to r,

$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$ would have been $= \frac{mv^2}{r}$

$$\therefore\ mv^2 = \frac{e^2}{4\pi\varepsilon_0} = k \Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}k.$$

This is independent of n.

From $mvr_n = \frac{nh}{2\pi}$,

as mv is independent of r, $r_n \propto n$.

CHAPTER
24

Solids and Semiconductor Devices

1. At absolute zero, Si acts as
(a) non-metal (b) metal
(c) insulator (d) none of these.
(2002)

2. By increasing the temperature, the specific resistance of a conductor and a semiconductor
(a) increases for both
(b) decreases for both
(c) increases, decreases
(d) decreases, increases.
(2002)

3. The energy band gap is maximum in
(a) metals (b) superconductors
(c) insulators (d) semiconductors.
(2002)

4. The part of a transistor which is most heavily doped to produce large number of majority carriers is
(a) emitter
(b) base
(c) collector
(d) can be any of the above three.
(2002)

5. Formation of covalent bonds in compounds exhibits
(a) wave nature of electron
(b) particle nature of electron
(c) both wave and particle nature of electron
(d) none of these.
(2002)

6. A strip of copper and another germanium are cooled from room temperature to 80 K. The resistance of
(a) each of these decreases
(b) copper strip increases and that of germanium decreases
(c) copper strip decreases and that of germanium increases
(d) each of these increases.
(2003)

7. The difference in the variation of resistance with temperature in a metal and a semiconductor arises essentially due to the difference in the
(a) crystal structure
(b) variation of the number of charge carriers with temperature
(c) type of bonding
(d) variation of scattering mechanism with temperature.
(2003)

8. In the middle of the depletion layer of a reverse-biased *p-n* junction, the
(a) electric field is zero
(b) potential is maximum
(c) electric field is maximum
(d) potential is zero.
(2003)

9. When *npn* transistor is used as an amplifier
(a) electrons move from base to collector
(b) holes move from emitter to base
(c) electrons move from collector to base
(d) holes move from base to emitter.
(2004)

10. For a transistor amplifier in common emitter configuration for load impedance of 1 kΩ (h_{fe} = 50 and h_{oe} = 25) the current gain is
(a) –5.2 (b) –15.7
(c) – 24.8 (d) – 48.78.
(2004)

11. A piece of copper and another of germanium are cooled from room temperature to 77 K, the resistance of
(a) each of them increases
(b) each of them decreases
(c) copper decreases and germanium increases
(d) copper increases and germanium decreases.
(2004)

12. The manifestation of band structure in solids is due to
(a) Heisenberg's uncertainty principle
(b) Pauli's exclusion principle
(c) Bohr's correspondence principle
(d) Boltzmann's law
(2004)

13. When *p-n* junction diode is forward biased, then
(a) the depletion region is reduced and barrier height is increased
(b) the depletion region is widened and barrier height is reduced.
(c) both the depletion region and barrier height are reduced
(d) both the depletion region and barrier height are increased
(2004)

14. The electrical conductivity of a semiconductor increases when electromagnetic radiation of wavelength shorter than 2480 nm is incident on it. The band gap in (eV) for the semiconductor is
(a) 0.5 eV (b) 0.7 eV
(c) 1.1 eV (d) 2.5 eV
(2005)

15. In a common base amplifier, the phase difference between the input signal voltage and output voltage is
(a) 0 (b) $\pi/2$
(c) $\pi/4$ (d) π
(2005)

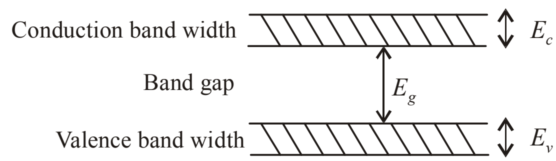
16. In a full wave rectifier circuit operating from 50 Hz mains frequency, the fundamental frequency in the ripple would be
(a) 100 Hz (b) 70.7 Hz
(c) 50 Hz (d) 25 Hz
(2005)

17. A solid which is transparent to visible light and whose conductivity increases with temperature is formed by
(a) metallic binding
(b) ionic binding
(c) covalent binding
(d) van der Waals binding
(2006)

18. In the ratio of the concentration of electrons that of holes in a semiconductor is 7/5 and the ratio of currents is 7/4 then what is the ratio of their drift velocities?
(a) 4/7 (b) 5/8 (c) 4/5 (d) 5/4.
(2006)

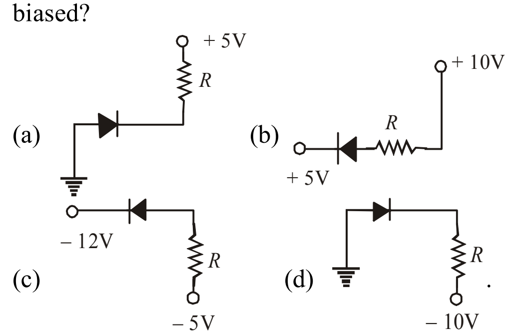
19. In common base mode of a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of the base current amplification factor (β) will be
(a) 48 (b) 49 (c) 50 (d) 51.
(2006)

20. If the lattice constant of this semiconductor is decreased, then which of the following is correct?



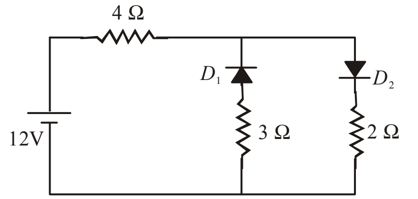
(a) all E_c, E_g, E_v decrease
(b) all E_c, E_g, E_v increase
(c) E_c, and E_v increase, but E_g decreases
(d) E_c, and E_v decrease, but E_g increases.
(2006)

21. In the following, which one of the diodes is reverse biased?



(2006)

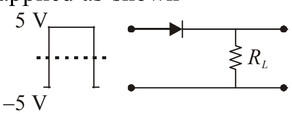
22. The circuit has two oppositely connect ideal diodes in parallel. What is the current following in the circuit?



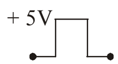
(a) 1.33 A (b) 1.71 A
(c) 2.00 A (d) 2.31 A.

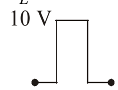
(2006)

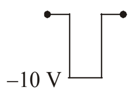
23. If in a p - n junction diode, a square input signal of 10 V is applied as shown



Then the output signal across R_L will be

(a) 

(b) 

(c) 

(d) 

(2007)

24. Carbon, silicon and germanium have four valence electrons each. At room temperature which one of the following statements is most appropriate ?

(a) The number of free electrons for conduction is significant only in Si and Ge but small in C.
(b) The number of free conduction electrons is significant in C but small in Si and Ge.
(c) The number of free conduction electrons is negligibly small in all the three.
(d) The number of free electrons for conduction is significant in all the three.

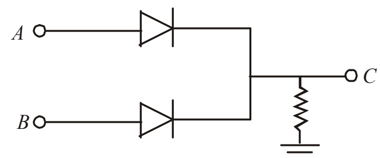
(2007)

25. A working transistor with its three legs marked P, Q and R is tested using a multimeter. No conduction is found between P and Q. By connecting the common (negative) terminal of the multimeter to R and the other (positive) terminal to P or Q, some resistance is seen on the multimeter. Which of the following is true for the transistor?

(a) It is an *npn* transistor with R as collector.
(b) It is an *npn* transistor with R as base.
(c) It is a *pnp* transistor with R as collector.
(d) It is a *pnp* transistor with R as emitter.

(2008)

26. In the circuit below, A and B represent two inputs and C represents the output. The circuit represents



(a) OR gate (b) NOR gate
(c) AND gate (d) NAND gate.

(2008)

Answer Key

1.	(c)	**2.**	(c)	**3.**	(c)	**4.**	(a)	**5.**	(a)	**6.**	(c)
7.	(b)	**8.**	(a)	**9.**	(a)	**10.**	(d)	**11.**	(c)	**12.**	(b)
13	(c)	**14.**	(a)	**15.**	(a)	**16.**	(a)	**17.**	(c)	**18.**	(d)
19.	(b)	**20.**	(d)	**21.**	(a)	**22.**	(c)	**23.**	(a)	**24.**	(a)
25.	(a)	**26.**	(a)								

EXPLANATIONS

1. **(c) :** Semiconductors, like Si, Ge, act as insulators at low temperature.
2. **(c) :** For conductor, ρ increases as temperature rises. For semiconductor, ρ decreases as temperature rises.
3. **(c) :** The energy band gap is maximum in insulators.
4. **(a) :** The emitter is most heavily doped.
5. **(a) :** Wave nature of electron and covalent bonds are correlated.
6. **(c) :** Copper is conductor and germanium is semiconductor. When cooled, the resistance of copper strip decreases and that of germanium increases.
7. **(b) :** Variation of number of charge carriers with temperature is responsible for variation of resistance in a metal and a semiconductor.
8. **(a) :** Electric field is zero in the middle of the depletion layer of a reverse baised *p-n* junction.
9. **(a) :** Electrons of *n*-type emitter move from emitter to base and then base to collector when *npn* transistor is used as an amplifier.
10. **(d) :** In common emitter configuration, current gain is

$$A_i = \frac{-(h_{fe})}{1+(h_{oe})(R_L)} = \frac{-50}{1+(25\times10^{-6})\times(1\times10^3)}$$

$$= -\frac{50}{1+0.025} = \frac{-50}{1.025} = -48.78.$$

11. **(c) :** Copper is a conductor
Germanium is a semiconductor
When cooled, the resistance of copper decreases and that of germanium increases.
12. **(b) :** Pauli's exclusion principle explains band structure of solids.
13. **(c) :** When *p-n* junction diode is forward biased, both the depletion region and barrier height are reduced.
14. **(a) :** Band gap = Energy of photon of λ = 2480 nm

$$\therefore \quad \text{Energy} = \frac{hc}{\lambda}\text{ J} = \frac{hc}{\lambda e}(\text{eV})$$

$$\therefore \quad \text{Band gap} = \frac{(6.63\times10^{-34})\times(3\times10^8)}{(2480\times10^{-9})\times(1.6\times10^{-19})}\text{ eV}$$

$= 0.5$ eV.

15. **(a) :** In a common base amplifier, the phase difference between the input signal and output voltage is zero.
16. **(a) :** Frequency of full wave rectifier
$= 2 \times$ input frequency $= 2 \times 50 = 100$ Hz.
17. **(c) :** Covalent binding.
18. **(d) :** Drift velocity $v_d = \dfrac{I}{nAe}$

$$\frac{(v_d)_{\text{electron}}}{(v_d)_{\text{hole}}} = \left(\frac{I_e}{I_h}\right)\left(\frac{n_h}{n_e}\right) = \frac{7}{4}\times\frac{5}{7} = \frac{5}{4}.$$

19. **(b) :** $\beta = \dfrac{I_c}{I_b} = \dfrac{I_c}{I_e - I_c} = \dfrac{5.488}{5.60-5.488} = \dfrac{5.488}{0.112} = 49.$
20. **(d) :** E_c and E_v decrease but E_g increases if the lattice constant of the semiconductor is decreased.
21. **(a) :** Figure (a) represent a reverse biased diode.
22. **(c) :** Since diode D_1 is reverse biased, therefore it will act like an open circuit.
Effective resistance of the circuit is $R = 4 + 2 = 6\ \Omega$.
Current in the circuit is $I = E/R = 12/6 = 2$ A.
23. **(a) :** The current will flow through R_L when diode is forward biased.
24. **(a) :** C, Si and Ge have the same lattice structure and their valence electrons are 4. For C, these electrons are in the second orbit, for Si it is third and germanium it is the fourth orbit. In solid state, higher the orbit, greater the possibility of overlapping of energy bands. Ionization energies are also less therefore Ge has more conductivity compared to Si. Both are semiconductors.Carbon is an insulator.
25. **(a) :** It is *npn* transistor with *R* as collector. If it is connected to base, it will be in forward bias.
26. **(a) :** It is OR gate. When either of them conducts, the gate conducts.

CHAPTER

25

Practical Physics

1. An experiment is performed to find the refractive index of glass using a travelling microscope. In this experiment distances are measured by
   (a) a screw gauge provided on the microscope
   (b) a vernier scale provided on the microscope
   (c) a standard laboratory scale
   (d) a meter scale provided on the microscope.
   (2008)

2. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, she measures the column length to be x cm for the second resonance. Then
   (a) $36 > x > 18$ (b) $18 > x$
   (c) $x > 54$ (d) $54 > x > 36$.
   (2008)

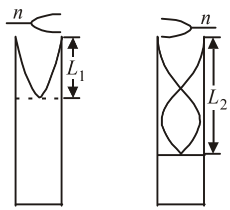
3. Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of – 0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is
   (a) 3.38 mm   (b) 3.32 mm
   (c) 3.73 mm   (d) 3.67 mm.
   (2008)

Answer Key

1.	(b)	**2.**	(c)	**3.**	(a)

1. **(b) :** A travelling microscope moves horizontally on a main scale provided with a vernier scale, provided with the microscope.

2. **(c) :** $v_1 = \sqrt{\frac{\gamma RT}{M}}$ assuming M is the average molar mass of the air (*i.e.*, nitrogen) and γ is also for nitrogen.



1[st] resonance 2[nd] resonance

$v_1 = \sqrt{\frac{\gamma RT_1}{M}}$; $v_2 = \sqrt{\frac{\gamma RT_2}{M}}$ where T_1 and T_2 stand for winter and summer temperatures.

$L_1 = \frac{v_1}{n} = \frac{\lambda}{4} = 18\text{ cm}$. At temperature T_1

At T_2, summer, $v_2 > v_1$.

$L_2 = \frac{v_2}{n} = \frac{3\lambda}{4} > 3 \times 18.$

$\therefore$ $L_2 > 54$ cm.

3. **(a) :** Least count of the screw gauge

$$= \frac{0.5\text{ mm}}{50} = 0.01\text{ mm}$$

Main scale reading = 3 mm.
Vernier scale reading = 35
$\therefore$ Observed reading = 3 + 0.35 = 3.35
zero error = –0.03
$\therefore$ actual diameter of the wire = 3.35 – (–0.03)
= 3.38 mm.

Solved Paper - 2009

1. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

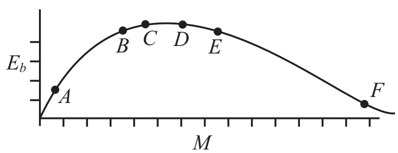
Statement-1: For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement-2: The net work done by a conservative force on an object moving along a closed loop is zero.

(a) Statement-1 is true, Statement-2 is false
(b) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true.

[Electrostatics]

2. The above is a plot of binding energy per nucleon E_b, against the nuclear mass M; A, B, C, D, E, F correspond to different nuclei. Consider four reactions:



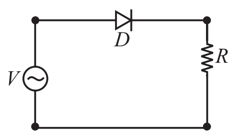
(i) $A + B \rightarrow C + \varepsilon$ (ii) $C \rightarrow A + B + \varepsilon$
(iii) $D + E \rightarrow F + \varepsilon$ (iv) $F \rightarrow D + E + \varepsilon$

where ε is the energy released? In which reactions is ε positive?

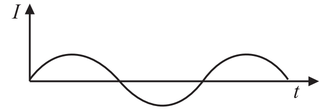
(a) (i) and (iv)   (b) (i) and (iii)
(c) (ii) and (iv)   (d) (ii) and (iii)

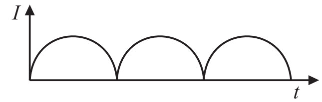
[Atoms, Molecules and Nuclei]

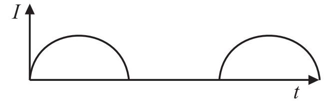
3. A p-n junction (D) shown in the figure can act as a rectifier.

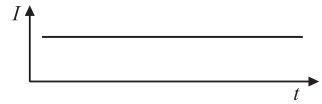


An alternating current source (V) is connected in the circuit. The current(I) in the resistor(R) can be shown by

(a)


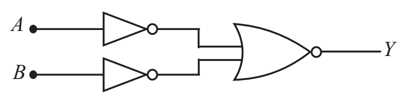
(b)


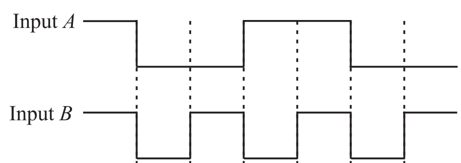
(c)


(d)


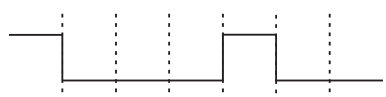
[Solids and Semiconductor Devices]

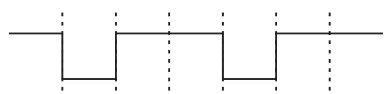
4. The logic circuit shown below has the input waveforms 'A' and 'B' as shown. Pick out the correct output waveform.

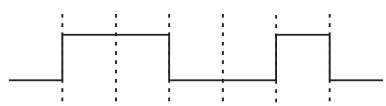


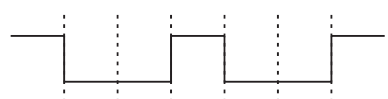


Output is

(a)


(b)


(c)


(d)


[Solids and Semiconductor Devices]

5. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time?

(a) $a^2T^2 + 4\pi^2v^2$ (b) aT/x
(c) $aT + 2\pi v$ (d) aT/v

[Oscillations]

6. In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance u and the image distance v, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the x-axis meets the experimental curve at P. The coordinates of P will be

(a) $(2f, 2f)$ (b) $(f/2, f/2)$
(c) (f, f) (d) $(4f, 4f)$

[Practical Physics]

7. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of

(a) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (b) $\frac{1}{6}\frac{l\omega}{g}$
(c) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (d) $\frac{1}{6}\frac{l^2\omega^2}{g}$

[Rotational Motion and Moment of Inertia]

8. Let $P(r) = \frac{Q}{\pi R^4}r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point 'p' inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is

(a) 0 (b) $\frac{Q}{4\pi\varepsilon_0 r_1^2}$
(c) $\frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$ (d) $\frac{Qr_1^2}{3\pi\varepsilon_0 R^4}$

[Electrostatics]

9. The transition from the state $n = 4$ to $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from

(a) $2 \to 1$ (b) $3 \to 2$
(c) $4 \to 2$ (d) $5 \to 4$

[Atoms, Molecules and Nuclei]

10. One kg of a diatomic gas is at a pressure of 8×10^4 N/m^2. The density of the gas is 4 kg/m^3. What is the energy of the gas due to its thermal motion?

(a) 3×10^4 J (b) 5×10^4 J
(c) 6×10^4 J (d) 7×10^4 J

[Heat and Thermodynamics]

11. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1: The temperature dependence of resistance is usually given as $R = R_0(1 + \alpha\Delta t)$. The resistance of a wire changes from 100 Ω to 150 Ω when its temperature is increased from 27°C to 227°C. This implies that $\alpha = 2.5 \times 10^{-3}$/°C

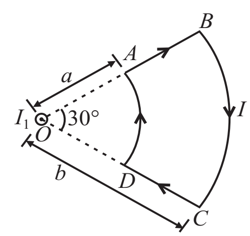
Statement-2: $R = R_0(1 + \alpha\Delta t)$ is valid only when the change in the temperature ΔT is small and $\Delta R = (R - R_0) << R_0$.

(a) Statement-1 is true, Statement-2 is false
(b) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true

[Current Electricity]

Directions : Question numbers 12 and 13 are based on the following paragraph.

A current loop $ABCD$ is held fixed on the plane of the paper as shown in the figure. The arcs BC (radius = b) and DA (radius = a) of the loop are joined by two straight wires AB and CD. A steady current I is flowing in the loop. Angle made by AB and CD at the origin O is 30°. Another straight thin wire with steady current I_1 flowing out of the plane of the paper is kept at the origin.



12. The magnitude of the magnetic field (B) due to loop $ABCD$ at the origin (O) is

(a) zero (b) $\frac{\mu_0 I(b-a)}{24ab}$
(c) $\frac{\mu_0 I}{4\pi}\left[\frac{b-a}{ab}\right]$
(d) $\frac{\mu_0 I}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$

13. Due to the presence of the current I_1 at the origin

(a) the forces on AB and DC are zero

(b) the forces on AD and BC are zero

(c) the magnitude of the net force on the loop is given by $\frac{I_1 I}{4\pi}\mu_0\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$

(d) the magnitude of the net force on the loop is given by $\frac{\mu_0 I I_1}{24ab}(b-a)$.

[Magnetic Effect of Current]

14. A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4^{th} bright fringe of the unknown light. From this data, the wavelength of the unknown light is

(a) 393.4 nm (b) 885.0 nm

(c) 442.5 nm (d) 776.8 nm

[Wave Optics]

15. Two points P and Q are maintained at the potentials of 10 V and –4 V respectively. The work done in moving 100 electrons from P to Q is

(a) -9.60×10^{-17} J (b) 9.60×10^{-17} J

(c) -2.24×10^{-16} J (d) 2.24×10^{-16} J

[Electrostatics]

16. The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is (hc = 1240 eV nm)

(a) 3.09 eV (b) 1.41 eV

(c) 1.51 eV (d) 1.68 eV

[Electrons and Photons]

17. A particle has an initial velocity $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is

(a) 10 units (b) $7\sqrt{2}$ units

(c) 7 units (d) 8.5 units

[Description of Motion in 2 and 3 Dimension]

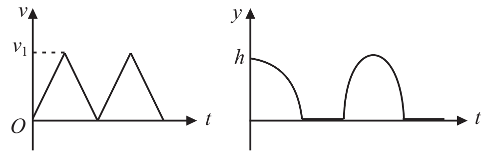
18. A motor cycle starts from rest and accelerates along a straight path at 2 m/s^2. At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest?
(Speed of sound = 330 ms^{-1}).

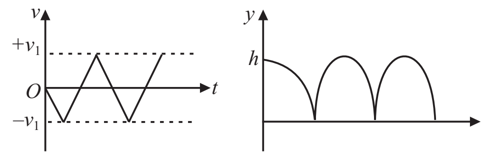
(a) 49 m (b) 98 m

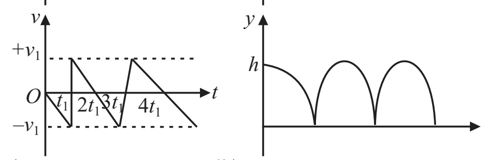
(c) 147 m (d) 196 m

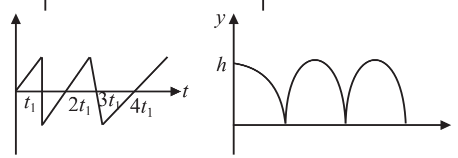
[Waves]

19. Consider a rubber ball freely falling from a height h = 4.9 m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time and the height as function of time will be

(a) 

(b) 

(c) 

(d) 

[Work, Energy and Power]

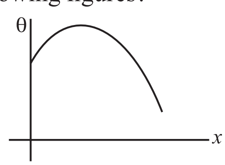
20. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then the Q/q equals

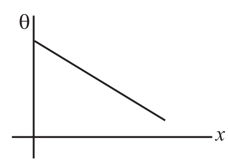
(a) $-2\sqrt{2}$ (b) –1

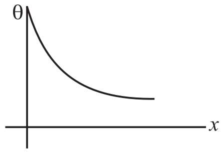
(c) 1 (d) $-\frac{1}{\sqrt{2}}$

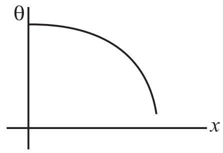
[Electrostatics]

21. A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figures?

(a) 

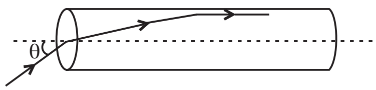
(b) 

(c) 

(d) 

[Transfer of Heat]

22. A transparent solid cylindrical rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the figure.



The incident angle θ for which the light ray grazes along the wall of the rod is

(a) $\sin^{-1}\left(\frac{1}{2}\right)$ (b) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(c) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

[Ray Optics]

23. Three sound waves of equal amplitudes have frequencies (υ – 1), υ, (υ + 1). They superpose to give beats. The number of beats produced per second will be

(a) 4 (b) 3
(c) 2 (d) 1

[Waves]

24. The height at which the acceleration due to gravity becomes $g/9$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth is

(a) $2R$ (b) $\frac{R}{\sqrt{2}}$

(c) $R/2$ (d) $\sqrt{2}R$

[Gravitation]

25. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount?

(a) F (b) $4F$
(c) $6F$ (d) $9F$

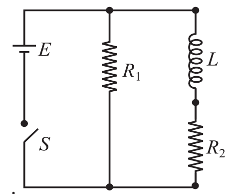
[Properties of Matter]

26. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (= 0.5°), then the least count of the instrument is

(a) one minute (b) half minute
(c) one degree (d) half degree

[Practical Physics]

27. An inductor of inductance L = 400 mH and resistors of resistances R_1 = 2 Ω and R_2 = 2 Ω are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at t = 0. The potential drop across L as a function of time is



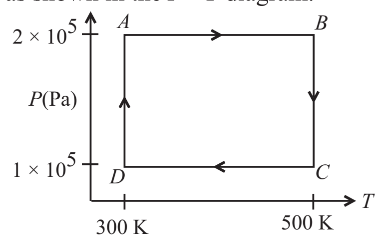
(a) $6e^{-5t}$ V (b) $\frac{12}{t}e^{-3t}$ V

(c) $6(1-e^{-t/0.2})$ V (d) $12e^{-5t}$ V

[Electromagnetic Induction and Alternating Current]

Directions: Question numbers 28, 29 and 30 are based on the following paragraph.

Two moles of helium gas are taken over the cycle *ABCDA*, as shown in the *P* - *T* diagram.



28. Assuming the gas to be ideal the work done on the gas in taking it from A to B is

(a) $200R$ (b) $300R$
(c) $400R$ (d) $500R$

29. The work done on the gas in taking it from D to A is

(a) $-414R$ (b) $+414R$
(c) $-690R$ (d) $+690R$

30. The net work done on the gas in the cycle *ABCDA* is

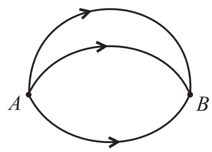
(a) zero (b) $276R$
(c) $1076R$ (d) $1904R$

[Heat and Thermodynamics]

EXPLANATIONS

1. **(c) :** Work done = potential difference $(V_B - V_A) \times q$, the charge.

V_A and V_B only depend on the initial and final positions and not on the path. Electrostatic force is a conservative force.



If the loop is completed, $V_A - V_A = 0$. No net work is done as the initial and final potentials are the same.

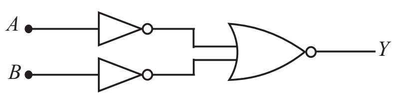
Both the statements are true but statement-2 is not the reason for statement-1.

2. **(a) :** When two nucleons combine to form a third one, and energy is released, one has fusion reaction. If a single nucleus splits into two, one has fission. The possibility of fusion is more for light elements and fission takes place for heavy elements.

Out of the choices given for fusion, only A and B are light elements and D and E are heavy elements. Therefore $A + B \rightarrow C + \varepsilon$ is correct. In the possibility of fission is only for F and not C. Therefore $F \rightarrow D + E + \varepsilon$ is the correct choice.

3. **(c) :** (a) is original wave (b) is a full-wave rectified (c) is the correct choice. The negative waves are cut off when the diode is connected in reverse bias (d) is not the diagram for alternating current.

4. **(a) :**



By de Morgan's theorem, $\left(\overline{\overline{A} + \overline{B}}\right) = A \cdot B$.

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} + \overline{B}$	$\overline{\overline{A} + \overline{B}}$	Verify $A \cdot B$
1	1	0	0	0	1	1
0	0	1	1	1	0	0
0	1	1	0	1	0	0
1	0	0	1	1	0	0

This is the same as AND Gate of A and B.

5. **(b) :** For a simple harmonic motion,

acceleration, $a = -\omega^2 x$ where ω is a constant $= \dfrac{2\pi}{T}$.

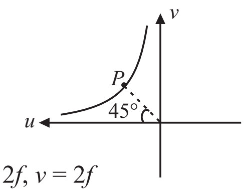
$$a = -\frac{4\pi^2}{T^2} \cdot x \;\Rightarrow\; \frac{aT}{x} = -\frac{4\pi^2}{T}.$$

The period of oscillation T is a constant.

$\therefore\ \dfrac{aT}{x}$ is a constant.

6. **(a) :** According to New Cartesian coordinate system used in our 12th classes, for a convex lens, as u is negative, the lens equation is $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$.

One has to take that u is negative again for calculation, it effectively comes to $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$.



If u = radius of curvature, $2f$, $v = 2f$

i.e., $\dfrac{1}{2f} + \dfrac{1}{2f} = \dfrac{1}{f}$.

v and u are have the same value when the object is at the centre of curvature. The solution is (a).

According the real and virtual system, u is +ve and v is also +ve as both are real. If $u = v$, $u = 2f$ = radius of curvature.

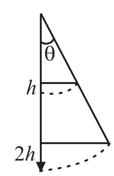
$$\therefore\ \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \;\Rightarrow\; \frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}.$$

The answer is the same (a).

(The figure given is according to New Cartesian system).

7. **(d) :** The uniform rod of length l and mass m is swinging about an axis passing through the end.

When the centre of mass is raised through h, the increase in potential energy is mgh. This is equal to the kinetic energy $= \dfrac{1}{2} I\omega^2$.

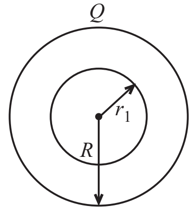


$$\Rightarrow\ mgh = \frac{1}{2}\left(m\frac{l^2}{3}\right) \cdot \omega^2 \qquad \therefore\ h = \frac{l^2 \cdot \omega^2}{6g}.$$

8. **(c) :** If the charge density, $\rho = \dfrac{Q}{\pi R^4} r$,

The electric field at the point p distant r_1 from the centre, according to Gauss's theorem is

$E \cdot 4\pi r_1^2$ = charge enclosed/ε_0

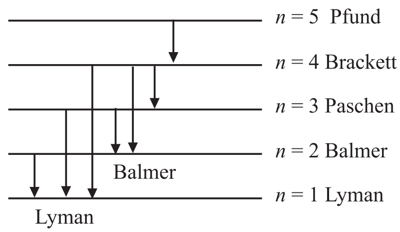


$$E \cdot 4\pi r_1^2 = \frac{1}{\varepsilon_0}\int \rho dV$$

$$\Rightarrow \quad E \cdot 4\pi r_1^2 = \frac{1}{\varepsilon_0}\int_0^{r_1} \frac{Qr}{\pi R^4} \cdot 4\pi r^2 dr$$

$$\Rightarrow \quad E = \frac{Qr_1^2}{4\pi\varepsilon_0 R^4}.$$

9. **(d) :**



Transition 4 → 3 is in Paschen series. This is not in the ultraviolet region but this is in infrared region. Transition 5 → 4 will also be in infrared region (Brackett).

10. **(b) :** The thermal energy or internal energy is

$U = \frac{5}{2}\mu RT$ for diatomic gases. (5 is the degrees of freedom as the gas is diatomic)

But $PV = \mu RT$

$$V = \frac{\text{mass}}{\text{density}} = \frac{1\text{ kg}}{4\text{ kg/m}^3} = \frac{1}{4}\text{ m}^3$$

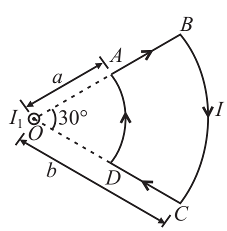
$P = 8 \times 10^4$ N/m^2.

$$\therefore \quad U = \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} = 5 \times 10^4 \text{ J}$$

11. **(a) :** From the statement given, $\alpha = 2.5 \times 10^{-3}$/°C. The resistance of a wire change from 100 Ω to 150 Ω when the temperature is increased from 27°C to 227°C.

It is true that α is small. But (150 – 100) Ω or 50 Ω is not very much less than 100 Ω *i.e.*, $R - R_0 << R_0$ is not true.

12. **(b) :** O is along the line CD and AB. They do not contribute to the magnetic induction at O. The field due to DA is positive or out of the paper and that due to BC is into the paper or negative.



The total magnetic field due to loop $ABCD$ at O is
$B = B_{AB} + B_{BC} + B_{CD} + B_{DA}$

$$\Rightarrow \quad B = 0 - \frac{\mu_0 I}{4\pi b} \times \frac{\pi}{6} + 0 + \frac{\mu_0 I}{4\pi a} \times \frac{\pi}{6}$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{24ab}(b-a), \text{ out of the paper or positive.}$$

13. **(b) :** The straight wire is perpendicular to the segments and the fields are parallel. There will be no force. Due to parts AB and CD, their fields are equal and opposite and their effects also cancel each other.

14. **(c) :** For interference, by Young's double slits, the path difference $\frac{xd}{D} = n\lambda$ for bright fringes and $\frac{xd}{D} = (2n+1)\frac{\lambda}{2}$ for getting dark fringes.

The central fringes when $x = 0$, coincide for all wavelengths.

The third fringe of $\lambda_1 = 590$ nm coincides with the fourth bright fringe of unknown wavelength λ.

$$\therefore \quad \frac{xd}{D} = 3 \times 590 \text{ nm} = 4 \times \lambda \text{ nm}$$

$$\therefore \quad \lambda = \frac{3 \times 590}{4} = 442.5 \text{ nm}.$$

15. **(d) :** + 10 V P ●·········● Q – 4 V

Work done in moving 100e^- from P to Q,
(Work done in moving 100 negative charges from the positive to the negative potential).

$W = (100e^-)(V_Q - V_P)$
$= (-100 \times 1.6 \times 10^{-19})(-14 \text{ V}) = 2.24 \times 10^{-16}$ J.

16. **(b) :** The wavelength of light illuminating the photoelectric surface = 400 nm.

$$i.e., \; h\upsilon = \frac{1240 \text{ eV nm}}{400 \text{ nm}} = 3.1 \text{ eV}.$$

Max. kinetic energy of the electrons = 1.68 eV.

$h\upsilon = W_\phi$ + kinetic energy

$\therefore$ W_ϕ, the work function = $h\upsilon$ – kinetic energy
= 3.1 – 1.68 eV = 1.42 eV.

17. **(b) :** $v = u + at$

$\vec{v} = (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j}) \times 10$

$\vec{v} = (3 + 4)\hat{i} + (4 + 3)\hat{j}$

$\Rightarrow \; |\vec{v}| = \sqrt{49 + 49} = \sqrt{98} = 7\sqrt{2}$ units

(This value is about 9.9 units close to 10 units. If (a) is given that is also not wrong).

18. **(b) :** The source is at rest, the observer is moving away from the source.

$$\therefore \quad f' = f\frac{(v_{\text{sound}} - v_{\text{obs}})}{v_{\text{sound}}}$$

$$\Rightarrow \quad \frac{f'}{f} \times v_{\text{sound}} = v_{\text{sound}} - v_{\text{obs}}$$

$$\Rightarrow \frac{f'}{f} \times v_{\text{sound}} - v_{\text{sound}} = -v_{\text{obs}}$$

$$v_{\text{sound}}\left(\frac{f'}{f} - 1\right) = -v_{\text{obs}}$$

$330(0.94 - 1) = -v_{\text{obs}}$

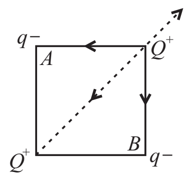
$\Rightarrow \; v_{\text{obs}} = 330 \times 0.06 = 19.80 \text{ ms}^{-1}$.

$$\therefore \; s = \frac{v^2 - u^2}{2a} = \frac{(19.80)^2}{2 \times 2} = 98 \text{ m}.$$

19. **(c) :** $v = u + gt$. As the ball is dropped, $v = gt$ when coming down. v increases, makes collision, the value of v becomes +ve, decreases, comes to zero and increases. The change from $+v$ to $-v$ is almost instantaneous. Using –ve signs when coming down, (c) is correct.

Further $h = \frac{1}{2}gt^2$ is a parabola. Therefore (c).

20. **(a) :** The force of repulsion by Q is cancelled by the resultant attracting force due to q^- and q^- at A and B.



Force of repulsion,

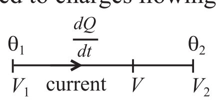
$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(a^2 + a^2)} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{2a^2}$$

Total force of attraction along the diagonal (taking $\cos\theta$ components)

$$= \frac{1}{4\pi\varepsilon_0}\left\{\frac{Qq}{a^2}\cdot\frac{1}{\sqrt{2}} + \frac{Qq}{a^2\sqrt{2}}\right\} = \frac{1}{4\pi\varepsilon_0}\left\{\frac{Qq\sqrt{2}}{a^2}\right\}$$

$$\Rightarrow \frac{Q^2}{2a^2} = \frac{Qq\sqrt{2}}{a^2} \;\; \Rightarrow \frac{Q^2}{Qq^-} = -2\sqrt{2}.$$

21. **(b) :** Heat flow can be compared to charges flowing in a conductor.

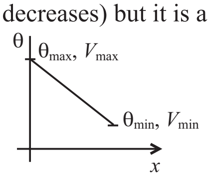


Current is the same.

The potential difference $V_1 - V$ at any point $= I \times$ Resistance $= I \times \frac{\rho l}{A}$

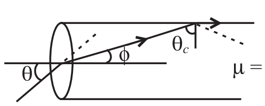
Potential difference is $\propto l$ but negative.

As l increases, potential decreases (temperature decreases) but it is a straight line function.



Potential difference is proportional to resistance (thermal as well as electric).

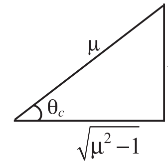
22. **(d) :**



μ = ref. index of the rod

If θ_c has to be the critical angle, $\theta_c = \sin^{-1}\frac{1}{\mu}$

But $\theta_c = 90° - \phi$, $\theta_i = \theta$.



$$\frac{\sin\theta_i}{\sin\phi} = \mu = \frac{2}{\sqrt{3}} \;\; \Rightarrow \frac{\sin\theta}{\cos\theta_c} = \mu.$$

But

$$\cos\theta_c = \frac{\sqrt{\mu^2 - 1}}{\mu} \;\; \therefore \;\; \sin\theta = \mu\frac{\sqrt{\mu^2 - 1}}{\mu} = \sqrt{\mu^2 - 1}.$$

$$\therefore \; \theta = \sin^{-1}\sqrt{\frac{4}{3} - 1} = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

So that θ_c is making total internal reflection.

23. **(a) :** The given sources of sound produce frequencies, $(\upsilon - 1)$, υ and $(\upsilon + 1)$.

For two sources of frequencies υ_1 and υ_2,

$y_1 = A \cos 2\pi\upsilon_1 t$

$y_2 = A \cos 2\pi\upsilon_2 t$

Superposing, one gets

$$y = 2A\cos 2\pi\left(\frac{\upsilon_1 - \upsilon_2}{2}\right)t\cos 2\pi\left(\frac{\upsilon_1 + \upsilon_2}{2}\right)t.$$

The resultant frequency obtained is $\frac{\upsilon_1 + \upsilon_2}{2}$ and this wave is modulated by a wave of frequency $\frac{\upsilon_1 - \upsilon_2}{2}$ (rather the difference of frequencies/2).

The intensity waxes and wanes. For a cosine curve (or sine curve), the number of beats $= \upsilon_1 \sim \upsilon_2$.

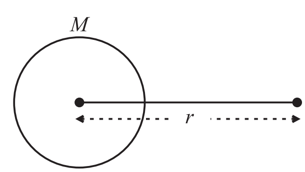
Frequencies	Mean	Beats
$\upsilon + 1$ and υ	$(\upsilon + 0.5)$ Hz	1
υ and $\upsilon - 1$	$\upsilon - 0.5$	1
$(\upsilon + 1)$ and $(\upsilon - 1)$	υ	2

Total number of beats = 4.

One should detect three frequencies, υ, $\upsilon + 0.5$ and $\upsilon - 0.5$ and each frequency will show 2 beats, 1 beat and 1 beat per second, respectively.

Total number of beats = 4

24. **(a) :** The acceleration due to gravity at a height h from the ground is given as $g/9$.



$$\frac{GM}{r^2} = \frac{GM}{R^2}\cdot\frac{1}{9}$$

$\therefore \;\; r = 3R$

The height above the ground is $2R$.

25. **(d) :** For the same material, Young's modulus is the same and it is given that the volume is the same and the area of cross-section for the wire l_1 is A and that of l_2 is $3A$.

$V = V_1 = V_2$

$V = A \times l_1 = 3A \times l_2 \;\Rightarrow\; l_2 = l_1/3$

$$Y = \frac{F / A}{\Delta l / l} \;\Rightarrow\; F_1 = YA\frac{\Delta l_1}{l_1}$$

$$F_2 = Y3A\frac{\Delta l_2}{l_2}$$

Given $\Delta l_1 = \Delta l_2 = \Delta x$ (for the same extension)

$$\therefore\; F_2 = Y \cdot 3A \cdot \frac{\Delta x}{l_1 / 3} = 9 \cdot \left(\frac{YA\Delta x}{l_1}\right) = 9F_1 \text{ or } 9F.$$

26. (a) : Least count $= \dfrac{\text{value of 1 main scale division}}{\text{The number of divisions on the vernier scale}}$

as shown below.

Here n vernier scale divisions = $(n - 1)$ M.S.D.

$$\therefore\; 1\,\text{V.S.D.} = \frac{n-1}{n}\ \text{M.S.D}$$

$$\text{L.C.} = 1\,\text{M.S.D.} - 1\,\text{V.S.D}$$

$$= 1\,\text{M.S.D} - \frac{(n-1)}{n}\,\text{M.S.D.}$$

$$\Rightarrow\; \text{L.C.} = 0.5^\circ - \frac{29}{30} \times 0.5^\circ$$

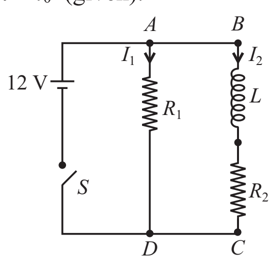
$$\Rightarrow\; \text{L.C.} = \frac{0.5}{30} = \frac{1}{30} \times \frac{1}{2} = \frac{1}{60}^\circ = 1\,\text{min}.$$

27. (d) : For the given R, L circuit the potential difference across $AD = V_{BC}$ as they are parallel.

$I_1 = E/R_1$.

$I_2 = I_0(1 - e^{-t/\tau})$ where τ = mean life or L/R.

$\tau = t_0$ (given).



$$E\,(\text{across } BC) = L\frac{dI_2}{dt} + R_2 I_2$$

$I_2 = I_0(1 - e^{-t/t_0})$.

$$\text{But } I_0 = \frac{E}{R_2} = \frac{12}{2} = 6\ \text{A}.$$

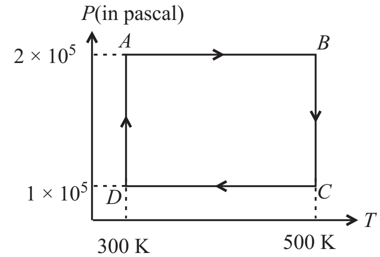
$$\tau = t_0 = \frac{L}{R} = \frac{400 \times 10^{-3}\,\text{H}}{2\,\Omega} = 0.2\,\text{s}$$

$\therefore\; I_2 = 6(1 - e^{-t/0.2})$

Potential drop across $L = E - R_2 I_2$

$= 12 - 2 \times 6(1 - e^{-t/0.2}) = 12e^{-t/0.2} = 12e^{-5t}$ V.

28. (c) :



Path AB, P is the same, ΔT is 200 K.

$PV = nRT$ for all process

$\therefore\; P\Delta V = nR\Delta T = 2R\,200 = 400R$.

Work done on the gas from A to $B = 400R$.

29. (b) : D to A, temperature remains the same.

$$\therefore\; \text{Work done by the gas} = W = nRT\ln\frac{V_2}{V_1}$$

$$= nRT\ln\frac{P_1}{P_2}$$

$$\therefore\; W = 2 \times R(300)\ln\left(\frac{10^5}{2 \times 10^5}\right)$$

$\Rightarrow\; W = -600R\,(0.693) = -415.8R$.

This is the work done by the gas

$\therefore\;$ Work done on the gas = $+415.8R$.

Nearest to (b).

30. (b) : Total work done on the gas when taking from A to $B = 400R$, from C to D is equal and opposite. They cancel each other.

For taking from D to A, work done on the gas $= +414R$.

Work done on the gas in taking it from B to C, pressure is decreased, temperature remain the same, volume increases.

$\Rightarrow\; W_{BC} + W_{DA} = 2\ln 2(500R - 300R)$.

$\Rightarrow\; W_{BC+DA} = (2\ln 2) \times (200R)$

$= 400R \times 0.693 = 277R$.

$\therefore\;$ Work done along AB and CD cancel each other because pressure changes but temperature is the same.

Net work done on the gas of 2 moles of helium through the whole network = $277R$ per cycle or nearest to the answer (b).

Solved Paper - 2010

Directions : Questions number 1-3 are based on the following paragraph.

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

1. The initial shape of the wavefront of the beam is
(a) planar (b) convex
(c) concave
(d) convex near the axis and concave near the periphery

2. The speed of light in the medium is
(a) maximum on the axis of the beam
(b) minimum on the axis of the beam
(c) the same everywhere in the beam
(d) directly proportional to the intensity I

3. As the beam enters the medium, it will
(a) travel as a cylindrical beam
(b) diverge
(c) converge
(d) diverge near the axis and converge near the periphery

[Wave Optics]

Directions : Questions number 4-5 are based on the following paragraph.

A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c.

4. The speed of daughter nuclei is
(a) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$ (b) $c\frac{\Delta m}{M+\Delta m}$
(c) $c\sqrt{\frac{2\Delta m}{M}}$ (d) $c\sqrt{\frac{\Delta m}{M}}$

5. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then
(a) $E_1 = 2E_2$ (b) $E_2 = 2E_1$
(c) $E_1 > E_2$ (d) $E_2 > E_1$

[Atoms, Molecules and Nuclei]

Directions : Questions number 6 - 7 contain Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

6. **Statement-1 :** When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.

Statement-2 : Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.
(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true.

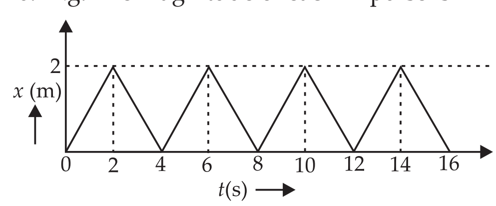
[Electrons and Photons]

7. **Statement-1 :** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.
(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true.

[Work, Energy and Power]

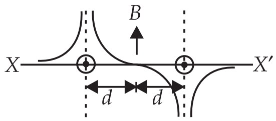
8. The figure shows the position - time (x-t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is

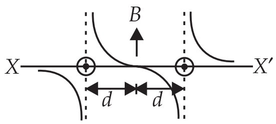


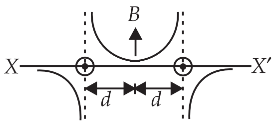
(a) 0.2 N s (b) 0.4 N s
(c) 0.8 N s (d) 1.6 N s

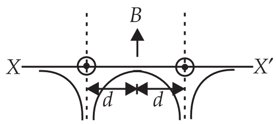
[Laws of Motion]

9. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX' is given by

(a) 

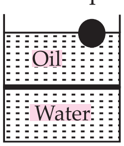
(b) 

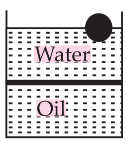
(c) 

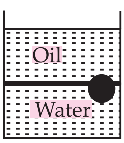
(d) 

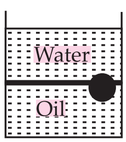
[Magnetic Effect of Current]

10. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position?

(a) 

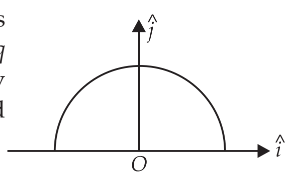
(b) 

(c) 

(d) 

[Properties of Matter]

11. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is



(a) $\dfrac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (b) $\dfrac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$

(c) $-\dfrac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (d) $-\dfrac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

[Electrostatics]

12. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to $32V$, the efficiency of the engine is

(a) 0.25 (b) 0.5
(c) 0.75 (d) 0.99

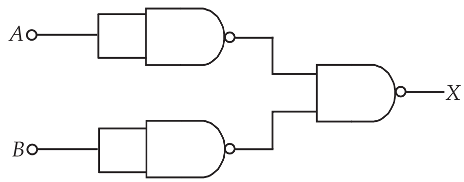
[Heat and Thermodynamics]

13. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are

(a) 4, 4, 2 (b) 5, 1, 2
(c) 5, 1, 5 (d) 5, 5, 2

[Units and Measurements]

14. The combination of gates shown below yields



(a) NAND gate (b) OR gate
(c) NOT gate (d) XOR gate

[Solids and Semiconductor Devices]

15. If a source of power 4 kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called

(a) γ-rays (b) X-rays
(c) ultraviolet rays (d) microwaves

[Electrons and Photons]

16. A radioactive nucleus (initial mass number A and atomic number Z) emits 3α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

(a) $\dfrac{A-Z-4}{Z-2}$ (b) $\dfrac{A-Z-8}{Z-4}$

(c) $\dfrac{A-Z-4}{Z-8}$ (d) $\dfrac{A-Z-12}{Z-4}$

[Atoms, Molecules and Nuclei]

17. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\dfrac{5}{4} - \dfrac{r}{R}\right)$ upto $r = R$, and $\rho(r) = 0$ for

$r > R$, where r is the distance from the origin. The electric field at a distance r ($r < R$) from the origin is given by

(a) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$ (b) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$

(c) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$ (d) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$

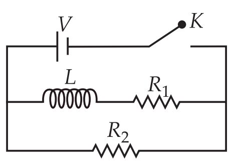
[Electrostatics]

18. In a series LCR circuit $R = 200\ \Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is

(a) 242 W (b) 305 W
(c) 210 W (d) zero W

[Electromagnetic Induction and Alternating Current]

19. In the circuit shown below, the key K is closed at $t = 0$. The current through the battery is



(a) $\frac{V(R_1+R_2)}{R_1R_2}$ at $t=0$ and $\frac{V}{R_2}$ at $t=\infty$

(b) $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at $t=0$ and $\frac{V}{R_2}$ at $t=\infty$

(c) $\frac{V}{R_2}$ at $t=0$ and $\frac{V(R_1+R_2)}{R_1R_2}$ at $t=\infty$

(d) $\frac{V}{R_2}$ at $t=0$ and $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at $t=\infty$

[Electromagnetic Induction and Alternating Current]

20. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is

(a) $y^2 = x^2 +$ constant (b) $y = x^2 +$ constant
(c) $y^2 = x +$ constant (d) $xy =$ constant

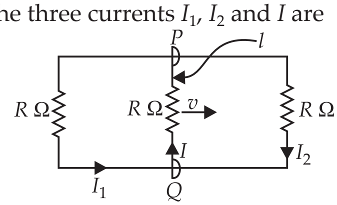
[Description of Motion in 2 and 3 Dimensions]

21. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

[Electromagnetic Induction and Alternating Current]

22. A rectangular loop has a sliding connector PQ of length l and resistance $R\ \Omega$ and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



(a) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$

(b) $I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$

(c) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$

(d) $I_1 = I_2 = I = \frac{Blv}{R}$

[Electromagnetic Induction and Alternating Current]

23. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

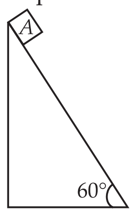
$$y = 0.02\,(\text{m})\sin\left[2\pi\left(\frac{t}{0.04(\text{s})} - \frac{x}{0.50(\text{m})}\right)\right].$$

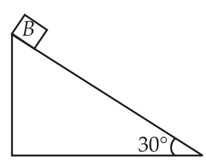
The tension in the string is

(a) 6.25 N (b) 4.0 N (c) 12.5 N (d) 0.5 N

[Waves]

24. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?





(a) $4.9\ ms^{-2}$ in vertical direction
(b) $4.9\ ms^{-2}$ in horizontal direction
(c) $9.8\ ms^{-2}$ in vertical direction
(d) zero

[Laws of Motion]

25. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point $P(R, \theta)$ on the circle of radius R is (Here θ is measured from the x-axis)

(a) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

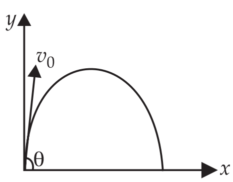
(b) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$

(c) $-\frac{v^2}{R}\sin\theta\,\hat{i} + \frac{v^2}{R}\cos\theta\,\hat{j}$

(d) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$

[Description of Motion in 2 and 3 Dimensions]

26. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is



(a) $\frac{1}{2} mgv_0t^2 \cos\theta\,\hat{i}$

(b) $-mgv_0t^2 \cos\theta\,\hat{j}$

(c) $mgv_0t \cos\theta\,\hat{k}$

(d) $-\frac{1}{2} mg\, v_0t^2 \cos\theta\,\hat{k}$

where $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively.

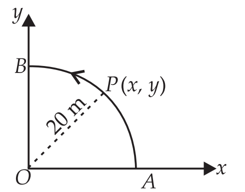
[Description of Motion in One Dimension]

27. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density $0.8\ g\ cm^{-3}$, the angle remains the same. If density of the material of the sphere is $1.6\ g\ cm^{-3}$, the dielectric constant of the liquid is

(a) 1 (b) 4 (c) 3 (d) 2

[Electrostatics]

28. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of P when $t = 2$ s is nearly



(a) $14\ m\ s^{-2}$

(b) $13\ m\ s^{-2}$

(c) $12\ m\ s^{-2}$

(d) $7.2\ m\ s^{-2}$

[Description of Motion in 2 and 3 Dimensions]

29. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is

(a) $\frac{b^2}{6a}$ (b) $\frac{b^2}{2a}$ (c) $\frac{b^2}{12a}$ (d) $\frac{b^2}{4a}$

[Properties of Matter]

30. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly

(a) $\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + \alpha_2}{2}$ (b) $\frac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$

(c) $\alpha_1 + \alpha_2, \frac{\alpha_1 + \alpha_2}{2}$ (d) $\alpha_1 + \alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2}$

[Current Electricity]

EXPLANATIONS

1. **(a) :** As the beam is initially parallel, the shape of wavefront is planar.

2. **(b) :** Given $\mu = \mu_0 + \mu_2 I$

As $\quad \mu = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$

$\mu = \frac{c}{v}$ or $v = \frac{c}{\mu} = \frac{c}{\mu_0 + \mu_2 I}$

As the intensity is maximum on the axis of the beam, therefore v is minimum on the axis of the beam.

3. **(c)**

4. **(c) :** Mass defect, $\Delta M = \left[(M+\Delta m) - \left(\frac{M}{2}+\frac{M}{2}\right)\right]$

$= [M + \Delta m - M] = \Delta m$

Energy released, $Q = \Delta Mc^2 = \Delta mc^2$ ...(i)

According to law of conservation of momentum, we get

$(M+\Delta m)\times 0 = \frac{M}{2}\times v_1 - \frac{M}{2}\times v_2$ or $v_1 = v_2$

Also, $Q = \frac{1}{2}\left(\frac{M}{2}\right)v_1^2 + \frac{1}{2}\left(\frac{M}{2}\right)v_2^2 - \frac{1}{2}(M+\Delta m)\times(0)^2$

$= \frac{M}{2}v_1^2 \quad (\because \quad v_1 = v_2)$ ...(ii)

Equating equations (i) and (ii), we get

$\left(\frac{M}{2}\right)v_1^2 = \Delta mc^2$

$v_1^2 = \frac{2\Delta mc^2}{M}$

$v_1 = c\sqrt{\frac{2\Delta m}{M}}$

5. **(d) :** After decay, the daughter nuclei will be more stable, hence binding energy per nucleon of daughter nuclei is more than that of their parent nucleus.

Hence, $E_2 > E_1$

6. **(a) :** According to Einstein's photoelectric equation

$K_{max} = h\upsilon - \phi_0$

where,

υ = frequency of incident light

ϕ_0 = work function of the metal

Since $K_{max} = eV_0$

$V_0 = \frac{h\upsilon}{e} - \frac{\phi_0}{e}$

As $\upsilon_{X\text{-rays}} > \upsilon_{\text{Ultraviolet}}$

Therefore, both K_{max} and V_0 increase when ultraviolet light is replaced by X-rays.

Statement-2 is false.

7. **(b)**

8. **(c) :** Here, mass of the body, $m = 0.4$ kg

Since position-time (x-t) graph is a straight line, so motion is uniform. Because of impulse direction of velocity changes as can be seen from the slopes of the graph.

From graph,

Initial velocity, $u = \frac{(2-0)}{(2-0)} = 1 \text{ m s}^{-1}$

Final velocity, $v = \frac{(0-2)}{(4-2)} = -1 \text{ m s}^{-1}$

Initial momentum, $p_i = mu = 0.4 \times 1 = 0.4$ N s

Final momentum, $p_f = mv = 0.4 \times (-1) = -0.4$ N s

Impulse = Change in momentum = $p_f - p_i$

$= -0.4 - (0.4)$ N s $= -0.8$ N s

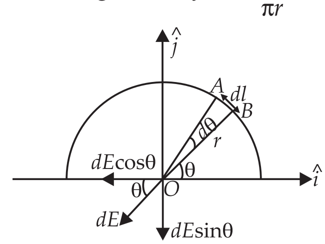
$|\text{Impulse}| = 0.8$ N s

9. **(b)**

10. **(c) :** As $\rho_{oil} < \rho_{water}$, so oil should be over the water. As $\rho > \rho_{oil}$, so the ball will sink in the oil but $\rho < \rho_{water}$ so it will float in the water.

Hence option (c) is correct.

11. **(d) :** Linear charge density, $\lambda = \frac{q}{\pi r}$



Consider a small element AB of length dl subtending an angle $d\theta$ at the centre O as shown in the figure.

$\therefore$ Charge on the element, $dq = \lambda dl$

$= \lambda r d\theta \left(\because \quad d\theta = \frac{dl}{r}\right)$

The electric field at the centre O due to the charge element is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{\lambda r d\theta}{4\pi\varepsilon_0 r^2}$$

Resolve dE into two rectangular components

By symmetry, $\int dE\cos\theta = 0$

The net electric field at O is

$$\vec{E} = \int_0^{\pi} dE\sin\theta(-\hat{j}) = \int_0^{\pi}\frac{\lambda r d\theta}{4\pi\varepsilon_0 r^2}\sin\theta(-\hat{j})$$

$$= -\int_0^{\pi}\frac{qr\sin\theta d\theta}{4\pi^2\varepsilon_0 r^3}\hat{j} \qquad \left(\because\ \lambda = \frac{q}{\pi r}\right)$$

$$= -\int_0^{\pi}\frac{q\sin\theta d\theta}{4\pi^2\varepsilon_0 r^2}\hat{j} = -\frac{q}{4\pi^2\varepsilon_0 r^2}[-\cos\theta]_0^{\pi}\hat{j}$$

$$= -\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$$

12. (c) : For an adiabatic process

$TV^{\gamma-1}$ = constant

$\therefore\ T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1} = T_2\left(\frac{32V}{V}\right)^{\gamma-1} = T_2(32)^{\gamma-1}$$

For diatomic gas, $\gamma = \frac{7}{5}$

$\therefore\ T_1 = T_2(32)^{\frac{7}{5}-1} = T_2(32)^{2/5} = T_2(2^5)^{2/5} = 4T_2$

Efficiency of the engine, $\eta = 1 - \frac{T_2}{T_1} = \left(1 - \frac{1}{4}\right)$

$$\eta = \frac{3}{4} = 0.75$$

13. (b) : (i) All the non-zero digits are significant.

(ii) All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all.

(iii) If the number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant.

(iv) The power of 10 is irrelevant to the determination of significant figures.

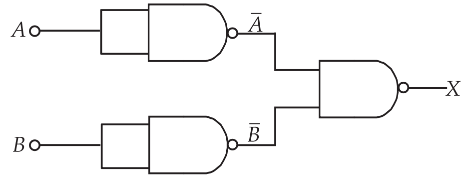
According to the above rules,

23.023 has 5 significant figures.

0.0003 has 1 significant figures.

2.1×10^{-3} has 2 significant figures.

14. (b) :



The Boolean expression of the given circuit is

$X = \overline{\overline{A}\cdot\overline{B}}$

$= \overline{\overline{A}} + \overline{\overline{B}}$ (Using De Morgan theorem)

$= A + B$ (Using Boolean identity)

This is same as the Boolean expression of OR gate.

Alternative method

The truth table of the given circuit is as shown in the table

A	B	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$X = \overline{\overline{A}\cdot\overline{B}}$
0	0	1	1	1	0
0	1	1	0	0	1
1	0	0	1	0	1
1	1	0	0	0	1

This is same as that of OR gate.

15. (b) : Here,

Power of a source, $P = 4$ kW $= 4 \times 10^3$ W

Number of photons emitted per second, $N = 10^{20}$

Energy of photon, $E = h\upsilon = \frac{hc}{\lambda}$

$\because\ E = \frac{P}{N}$

$\frac{hc}{\lambda} = \frac{P}{N}$

or $\lambda = \frac{Nhc}{P} = \frac{10^{20}\times 6.63\times 10^{-34}\times 3\times 10^8}{4\times 10^3}$

$= 4.972 \times 10^{-9}$ m $= 49.72$ Å

It lies in the X-ray region.

16. (c) : When a radioactive nucleus emits an alpha particle, its mass number decreases by 4 while the atomic number decreases by 2.

When a radioactive nucleus, emits a β^+ particle (or positron (e^+)) its mass number remains unchanged while the atomic number decreases by 1.

$\therefore\ {}^{A}_{Z}X \xrightarrow{3\alpha} {}^{A-12}_{Z-6}Y \xrightarrow{2e^+} {}^{A-12}_{Z-8}W$

In the final nucleus,

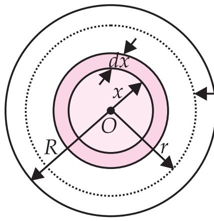
Number of protons, $N_p = Z - 8$

Number of neutrons, $N_n = A - 12 - (Z - 8)$

$= A - Z - 4$

$\therefore \quad \frac{N_n}{N_p} = \frac{A - Z - 4}{Z - 8}$

17. **(c) :** Consider a thin spherical shell of radius x and thickness dx as shown in the figure.



Volume of the shell, $dV = 4\pi x^2 dx$

Let us draw a Gaussian surface of radius $r(r < R)$ as shown in the figure above.

Total charge enclosed inside the Gaussian surface is

$$Q_{in} = \int_0^r \rho dV = \int_0^r \rho_0 \left(\frac{5}{4} - \frac{x}{R}\right) 4\pi x^2 dx$$

$$= 4\pi\rho_0 \int_0^r \left(\frac{5}{4}x^2 - \frac{x^3}{R}\right) dx$$

$$= 4\pi\rho_0 \left[\frac{5}{12}x^3 - \frac{x^4}{4R}\right]_0^r = 4\pi\rho_0 \left[\frac{5}{12}r^3 - \frac{r^4}{4R}\right]$$

$$= \frac{4\pi\rho_0}{4}\left[\frac{5}{3}r^3 - \frac{r^4}{R}\right] = \pi\rho_0 \left[\frac{5}{3}r^3 - \frac{r^4}{R}\right]$$

According to Gauss's law

$$E 4\pi r^2 = \frac{Q_{in}}{\varepsilon_0}$$

$$E 4\pi r^2 = \frac{\pi\rho_0}{\varepsilon_0}\left[\frac{5}{3}r^3 - \frac{r^4}{R}\right]$$

$$E = \frac{\pi\rho_0 r^3}{4\pi r^2 \varepsilon_0}\left[\frac{5}{3} - \frac{r}{R}\right]$$

$$E = \frac{\rho_0 r}{4\varepsilon_0}\left[\frac{5}{3} - \frac{r}{R}\right]$$

18. **(a) :** Here, $R = 200\ \Omega$, $V_{rms} = 220$ V, $\upsilon = 50$ Hz

When only the capacitance is removed, the phase difference between the current and voltage is

$$\tan\phi = \frac{X_L}{R}$$

$$\tan 30° = \frac{X_L}{R} \text{ or } X_L = \frac{1}{\sqrt{3}}R$$

When only the inductance is removed, the phase difference between current and voltage is

$$\tan\phi' = \frac{X_C}{R}$$

$$\tan 30° = \frac{X_C}{R} \text{ or } X_C = \frac{1}{\sqrt{3}}R$$

As $X_L = X_C$, therefore the given series LCR is in resonance.

$\therefore$ Impedance of the circuit is $Z = R = 200\ \Omega$

The power dissipated in the circuit is

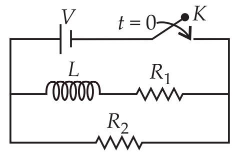
$$P = V_{rms} I_{rms} \cos\phi$$

$$= \frac{V_{rms}^2}{Z}\cos\phi \qquad \left(\because I_{rms} = \frac{V_{rms}}{Z}\right)$$

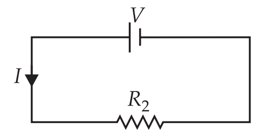
At resonance, power factor $\cos\phi = 1$

$$\therefore P = \frac{V_{rms}^2}{Z} = \frac{(220\text{ V})^2}{(200\ \Omega)} = 242\text{ W}$$

19. **(c) :**



At time $t = 0$, the inductor acts as an open circuit. The corresponding equivalent circuit diagram is as shown in the figure (i).

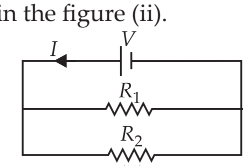


(i)

The current through battery is

$$I = \frac{V}{R_2}$$

At time $t = \infty$, the inductor acts as a short circuit. The corresponding equivalent circuit diagram is as shown in the figure (ii).



(ii)

$\therefore$ The current through the battery is

$$I = \frac{V}{R_{eq}} = \frac{V}{\frac{R_1 R_2}{R_1 + R_2}} \qquad (\because R_1 \text{ and } R_2 \text{ are in parallel})$$

$$= \frac{V(R_1 + R_2)}{R_1 R_2}$$

20. **(a) :** Here, $\vec{v} = K(y\hat{i} + x\hat{j})$

$$\vec{v} = Ky\hat{i} + Kx\hat{j} \qquad ...(i)$$

$$\because \vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} \qquad ...(ii)$$

Equating equations (i) and (ii), we get

$$\frac{dx}{dt} = Ky; \frac{dy}{dt} = Kx$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{Kx}{Ky} = \frac{x}{y} \quad \text{...(iii)}$$

Integrating both sides of the above equation, we get

$$\int y dy = \int x dx$$

$$y^2 = x^2 + \text{constant}$$

21. (d) : During discharging of capacitor through a resistor,

$$q = q_0 e^{-t/RC} \quad \text{...(i)}$$

The energy stored in the capacitor at any instant of time t is

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{(q_0 e^{-t/RC})^2}{C} \quad \text{(Using(i))}$$

$$= \frac{1}{2}\frac{q_0^2}{C} e^{-2t/RC} = U_0 e^{-2t/RC} \quad \text{...(ii)}$$

where $U_0 = \frac{1}{2}\frac{q_0^2}{C}$, the maximum energy stored in the capacitor.

According to given problem

$$\frac{U_0}{2} = U_0 e^{-2t_1/RC} \quad \text{(Using (ii))} \quad \text{...(iii)}$$

and $$\frac{q_0}{4} = q_0 e^{-t_2/RC} \quad \text{(Using (i))} \quad \text{...(iv)}$$

From equation (iii), we get

$$\frac{1}{2} = e^{-2t_1/RC}$$

Taking natural logarithms of both sides, we get

$$\ln 1 - \ln 2 = -\frac{2t_1}{RC} \quad \text{or} \quad t_1 = \frac{RC \ln 2}{2} \quad (\because \ln 1 = 0)$$

From equation (iv), we get

$$\frac{1}{4} = e^{-t_2/RC}$$

Taking natural logarithms of both sides of the above equation, we get

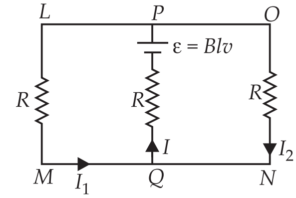
$$\ln 1 - \ln 4 = -\frac{t_2}{RC}$$

$$t_2 = RC \ln 4 = 2RC \ln 2 \quad (\because \ln 4 = 2\ln 2)$$

$$\therefore \frac{t_1}{t_2} = \frac{RC \ln 2}{2} \times \frac{1}{2RC \ln 2} = \frac{1}{4}$$

22. (c) : Emf induced across PQ is $\varepsilon = Blv$.

The equivalent circuit diagram is as shown in the figure.



Applying Kirchhoff's first law at junction Q, we get

$$I = I_1 + I_2 \quad \text{...(i)}$$

Applying Kirchhoff's second law for the closed loop $PLMQP$, we get

$$-I_1R - IR + \varepsilon = 0$$

$$I_1R + IR = Blv \quad \text{...(ii)}$$

Again, applying Kirchhoff's second law for the closed loop $PONQP$, we get

$$-I_2R - IR + \varepsilon = 0$$

$$I_2R + IR = Blv \quad \text{...(iii)}$$

Adding equations (ii) and (iii), we get

$$2IR + I_1R + I_2R = 2Blv$$

$$2IR + R(I_1 + I_2) = 2Blv$$

$$2IR + IR = 2Blv \quad \text{(Using (i))}$$

$$3IR = 2Blv$$

$$I = \frac{2Blv}{3R} \quad \text{...(iv)}$$

Substituting this value of I in equation (ii), we get

$$I_1 = \frac{Blv}{3R}$$

Substituting the value of I in equation (iii), we get

$$I_2 = \frac{Blv}{3R}$$

Hence, $I_1 = I_2 = \frac{Blv}{3R}$, $I = \frac{2Blv}{3R}$

23. (a) : Here, linear mass density $\mu = 0.04 \text{ kg m}^{-1}$

The given equation of a wave is

$$y = 0.02 \sin\left[2\pi\left(\frac{t}{0.04} - \frac{x}{0.50}\right)\right]$$

Compare it with the standard wave equation

$$y = A\sin(\omega t - kx)$$

we get,

$$\omega = \frac{2\pi}{0.04} \text{ rad s}^{-1}; \; k = \frac{2\pi}{0.5} \text{ rad m}^{-1}$$

Wave velocity, $$v = \frac{\omega}{k} = \frac{(2\pi/0.04)}{(2\pi/0.5)} \text{ m s}^{-1} \quad \text{...(i)}$$

Also $$v = \sqrt{\frac{T}{\mu}} \quad \text{...(ii)}$$

where T is the tension in the string and μ is the linear mass density

Equating equations (i) and (ii), we get

$$\frac{\omega}{k}=\sqrt{\frac{T}{\mu}} \quad \text{or} \quad T=\frac{\mu\omega^2}{k^2}$$

$$T=\frac{0.04\times\left(\frac{2\pi}{0.04}\right)^2}{\left(\frac{2\pi}{0.05}\right)^2}=6.25\text{ N}$$

24. **(a) :** The acceleration of the body down the smooth inclined plane is $a = g\sin\theta$

It is along the inclined plane.

where θ is the angle of inclination

$\therefore$ The vertical component of acceleration a is

$$a_{(\text{along vertical})} = (g\sin\theta)\sin\theta = g\sin^2\theta$$

For block A

$$a_{A(\text{along vertical})} = g\sin^2 60°$$

For block B

$$a_{B(\text{along vertical})} = g\sin^2 30°$$

The relative vertical acceleration of A with respect to B is

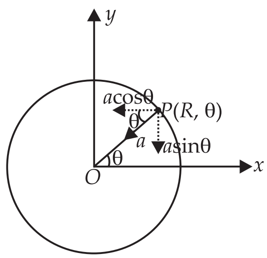
$$a_{AB(\text{along vertical})} = a_{A(\text{along vertical})} - a_{B(\text{along vertical})}$$

$$= g\sin^2 60° - g\sin^2 30°$$

$$= g\left(\left(\frac{\sqrt{3}}{2}\right)^2-\left(\frac{1}{2}\right)^2\right)$$

$$= \frac{g}{2} = 4.9\text{ m s}^{-2}\text{ in vertical direction.}$$

25. **(d) :**



For a particle in uniform circular motion,

Acceleration, $a=\frac{v^2}{R}$ towards the centre

From figure,

$$\vec{a}=-a\cos\theta\hat{i}-a\sin\theta\hat{j} = -\frac{v^2}{R}\cos\theta\hat{i}-\frac{v^2}{R}\sin\theta\hat{j}$$

26. **(d) :** The position vector of the particle from the origin at any time t is

$$\vec{r}=v_0\cos\theta t\,\hat{i}+(v_0\sin\theta t-\frac{1}{2}gt^2)\hat{j}$$

$\therefore$ Velocity vector, $\vec{v}=\frac{d\vec{r}}{dt}$

$$\vec{v}=\frac{d}{dt}(v_0\cos\theta t\;\hat{i}+(v_0\sin\theta t-\frac{1}{2}gt^2)\hat{j})$$

$$= v_0\cos\theta\,\hat{i}+(v_0\sin\theta-gt)\,\hat{j}$$

The angular momentum of the particle about the origin is

$$\vec{L}=\vec{r}\times m\vec{v}$$

$$\vec{L}=m(\vec{r}\times\vec{v})$$

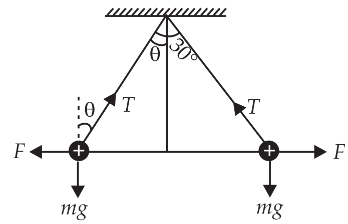
$$= m\left[(v_0\cos\theta t\,\hat{i}+(v_0\sin\theta t-\frac{1}{2}gt^2)\hat{j})\times(v_0\cos\theta\;\hat{i}+(v_0\sin\theta-gt)\hat{j})\right]$$

$$= m\left[(v_0^2\cos\theta\sin\theta t-v_0gt^2\cos\theta)\hat{k}+(v_0^2\sin\theta\cos\theta t-\frac{1}{2}gt^2v_0\cos\theta)(-\hat{k})\right]$$

$$= m\left[v_0^2\sin\theta\cos\theta t\hat{k}-v_0gt^2\cos\theta\hat{k}-v_0^2\sin\theta\cos\theta t\hat{k}+\frac{1}{2}v_0gt^2\cos\theta\hat{k}\right]$$

$$= m\left[-\frac{1}{2}v_0gt^2\cos\theta\hat{k}\right]=-\frac{1}{2}mgv_0t^2\cos\theta\hat{k}$$

27. **(d) :**



Initially, the forces acting on each ball are

(i) Tension T

(ii) Weight mg

(iii) Electrostatic force of repulsion F

For its equilibrium along vertical,

$$T\cos\theta = mg \quad \text{...(i)}$$

and along horizontal,

$$T\sin\theta = F \quad \text{...(ii)}$$

Dividing equation (ii) by (i), we get

$$\tan\theta=\frac{F}{mg} \quad \text{...(iii)}$$

When the balls are suspended in a liquid of density σ and dielectric constant K, the electrostatic force will become $(1/K)$ times, *i.e.* $F' = (F/K)$ while weight

$$mg' = mg - \text{Upthrust}$$

$$= mg - V\sigma g \qquad [\text{As Upthrust} = V\sigma g]$$

$$mg' = mg\left[1-\frac{\sigma}{\rho}\right] \qquad \left[\text{As } V=\frac{m}{\rho}\right]$$

For equilibrium of balls,

$$\tan\theta'=\frac{F'}{mg'}=\frac{F}{Kmg[1-(\sigma/\rho)]} \quad \text{...(iv)}$$

According to given problem, $\theta' = \theta$

From equations (iv) and (iii), we get

$$K=\frac{1}{\left(1-\frac{\sigma}{\rho}\right)}$$

$$K=\frac{\rho}{(\rho-\sigma)}=\frac{1.6}{(1.6-0.8)}=2$$

28. **(a) :** $s = t^3 + 3$

$$\therefore\ v=\frac{ds}{dt}=\frac{d}{dt}(t^3+3)=3t^2$$

Tangential acceleration, $a_t=\frac{dv}{dt}=\frac{d}{dt}(3t^2)=6t$

At $t = 2$ s,

$v = 3(2)^2 = 12$ m/s, $a_t = 6(2) = 12$ m/s^2

Centripetal acceleration,

$$a_c=\frac{v^2}{R}=\frac{(12)^2}{20}=\frac{144}{20}=7.2\text{ m/s}^2$$

Net acceleration,

$$a=\sqrt{(a_c)^2+(a_t)^2}=\sqrt{(7.2)^2+(12)^2}\approx 14\text{ m/s}^2$$

29. **(d) :** $U=\frac{a}{x^{12}}-\frac{b}{x^6}$

$$\text{Force, }F=-\frac{dU}{dx}=-\frac{d}{dx}\left(\frac{a}{x^{12}}-\frac{b}{x^6}\right)$$

$$=-\left[\frac{-12a}{x^{13}}+\frac{6b}{x^7}\right]=\left[\frac{12a}{x^{13}}-\frac{6b}{x^7}\right]$$

At equilibrium $F = 0$

$$\therefore\ \frac{12a}{x^{13}}-\frac{6b}{x^7}=0\ \text{ or }\ x^6=\frac{2a}{b}$$

$$U_{\text{at equilibrium}}=\frac{a}{\left(\frac{2a}{b}\right)^2}-\frac{b}{\left(\frac{2a}{b}\right)}$$

$$=\frac{ab^2}{4a^2}-\frac{b^2}{2a}=\frac{b^2}{4a}-\frac{b^2}{2a}=-\frac{b^2}{4a}$$

$U(x = \infty) = 0$

$D = [U(x=\infty) - U_{\text{at equilibrium}}]$

$$=\left[0-\left(-\frac{b^2}{4a}\right)\right]=\frac{b^2}{4a}$$

30. **(a) :** Let R_0 be the resistance of both conductors at 0°C.

Let R_1 and R_2 be their resistance at t°C. Then

$R_1 = R_0(1 + \alpha_1 t)$

$R_2 = R_0(1 + \alpha_2 t)$

Let R_s is the resistance of the series combination of two conductors at t°C. Then

$R_s = R_1 + R_2$

$R_{s0}(1 + \alpha_s t) = R_0(1 + \alpha_1 t) + R_0(1 + \alpha_2 t)$

where, $R_{s0} = R_0 + R_0 = 2R_0$

$\therefore\ 2R_0(1 + \alpha_s t) = 2R_0 + R_0 t(\alpha_1 + \alpha_2)$

$2R_0 + 2R_0\alpha_s t = 2R_0 + R_0 t(\alpha_1 + \alpha_2)$

$$\therefore\ \alpha_s=\frac{\alpha_1+\alpha_2}{2}$$

Let R_p is the resistance of the parallel combination of two conductors at t°C. Then

$$R_p=\frac{R_1R_2}{R_1+R_2}$$

$$R_{p0}(1+\alpha_p t)=\frac{R_0(1+\alpha_1 t)\,R_0(1+\alpha_2 t)}{R_0(1+\alpha_1 t)+R_0(1+\alpha_2 t)}$$

$$\text{where, }R_{p0}=\frac{R_0R_0}{R_0+R_0}=\frac{R_0}{2}$$

$$\therefore\ \frac{R_0}{2}(1+\alpha_p t)=\frac{R_0^2(1+\alpha_1 t)(1+\alpha_2 t)}{2R_0+R_0(\alpha_1+\alpha_2)t}$$

$$\frac{R_0}{2}(1+\alpha_p t)=\frac{R_0^2(1+\alpha_1 t+\alpha_2 t+\alpha_1\alpha_2 t^2)}{R_0(2+(\alpha_1+\alpha_2)t)}$$

$$\frac{1}{2}(1+\alpha_p t)=\frac{(1+\alpha_1 t+\alpha_2 t+\alpha_1\alpha_2 t^2)}{(2+(\alpha_1+\alpha_2)t)}$$

As α_1 and α_2 are small quantities

$\therefore\ \alpha_1\alpha_2$ is negligible

$$\therefore\ \frac{1}{2}(1+\alpha_p t)=\frac{1+(\alpha_1+\alpha_2)t}{2+(\alpha_1+\alpha_2)t}=\frac{1+(\alpha_1+\alpha_2)t}{2\left[1+\frac{(\alpha_1+\alpha_2)t}{2}\right]}$$

$$=\frac{1}{2}[1+(\alpha_1+\alpha_2)t]\left[1+\frac{(\alpha_1+\alpha_2)t}{2}\right]^{-1}$$

$$=\frac{1}{2}[1+(\alpha_1+\alpha_2)t]\left[1-\frac{(\alpha_1+\alpha_2)t}{2}\right]$$

[By binomial expansion]

$$=\frac{1}{2}\left[1-\frac{(\alpha_1+\alpha_2)t}{2}+(\alpha_1+\alpha_2)t-\frac{(\alpha_1+\alpha_2)^2t^2}{2}\right]$$

As $(\alpha_1 + \alpha_2)^2$ is negligible

$$\therefore\ \frac{1}{2}(1+\alpha_p t)=\frac{1}{2}\left[1+\frac{1}{2}(\alpha_1+\alpha_2)t\right]$$

$$\alpha_p t=\frac{(\alpha_1+\alpha_2)}{2}t$$

$$\alpha_p=\frac{\alpha_1+\alpha_2}{2}$$

Solved Paper - 2011

1. 100 g of water is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J kg^{-1} K^{-1})

(a) 4.2 kJ (b) 8.4 kJ
(c) 84 kJ (d) 2.1 kJ

[Heat and Thermodynamics]

2. The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 when $\frac{1}{3}$ of it had decayed is

(a) 7 min (b) 14 min
(c) 20 min (d) 28 min

[Atoms, Molecules and Nuclei]

3. A mass M, attached to a horizontal spring, executes SHM with a amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is

(a) $\frac{M}{M+m}$ (b) $\frac{M+m}{M}$
(c) $\left(\frac{M}{M+m}\right)^{1/2}$ (d) $\left(\frac{M+m}{M}\right)^{1/2}$

[Oscillations]

4. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is

(a) 12.1 eV (b) 36.3 eV
(c) 108.8 eV (d) 122.4 eV

[Atoms, Molecules and Nuclei]

5. The transverse displacement $y(x,t)$ of a wave on a string is given by

$$y(x,t) = e^{-(ax^2+bt^2+2\sqrt{ab}xt)}$$

This represents a

(a) wave moving in $+x$-direction with speed $\sqrt{\frac{a}{b}}$
(b) wave moving in $-x$-direction with speed $\sqrt{\frac{b}{a}}$
(c) standing wave of frequency $\sqrt{b}$
(d) standing wave of frequency $\frac{1}{\sqrt{b}}$

[Waves]

6. A resistor R and 2 μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed. ($\log_{10}2.5 = 0.4$)

(a) $1.3 \times 10^4\ \Omega$ (b) $1.7 \times 10^5\ \Omega$
(c) $2.7 \times 10^6\ \Omega$ (d) $3.3 \times 10^7\ \Omega$

[Electromagnetic Induction and Alternating Current]

7. A current I flows in an infinitely long wire with cross-section in the form of a semicircular ring of radius R. The magnitude of the magnetic induction along its axis is

(a) $\frac{\mu_0 I}{\pi^2 R}$ (b) $\frac{\mu_0 I}{2\pi^2 R}$
(c) $\frac{\mu_0 I}{2\pi R}$ (d) $\frac{\mu_0 I}{4\pi R}$

[Magnetic Effect of Current]

8. A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively

(a) 372 K and 310 K (b) 372 K and 330 K
(c) 330 K and 268 K (d) 310 K and 248 K

[Heat and Thermodynamics]

9. An object moving with a speed of 6.25 m s^{-1}, is decelerated at a rate given by $\frac{dv}{dt} = -2.5\sqrt{v}$ where v is the instantaneous speed. The time taken by the object, to come to rest, would be

(a) 1 s (b) 2 s
(c) 4 s (d) 8 s

[Description of Motion in One Dimension]

10. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre; a, b are constants. Then the charge density inside the ball is

(a) $-24\pi a\varepsilon_0 r$ (b) $-6a\varepsilon_0 r$
(c) $-24\pi a\varepsilon_0$ (d) $-6a\varepsilon_0$

[Electrostatics]

11. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m s^{-1}. The speed of the image of the second car as seen in the mirror of the first one is

(a) $\frac{1}{10}$ m s^{-1} (b) $\frac{1}{15}$ m s^{-1}

(c) 10 m s^{-1} (d) 15 m s^{-1}

[Ray Optics]

12. If a wire is stretched to make it 0.1% longer, its resistance will

(a) increase by 0.05% (b) increase by 0.2%

(c) decrease by 0.2% (d) decrease by 0.05%

[Current Electricity]

13. Three perfect gases at absolute temperatures T_1,T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is

(a) $\frac{(T_1+T_2+T_3)}{3}$ (b) $\frac{n_1T_1+n_2T_2+n_3T_3}{n_1+n_2+n_3}$

(c) $\frac{n_1T_1^2+n_2T_2^2+n_3T_3^2}{n_1T_1+n_2T_2+n_3T_3}$ (d) $\frac{n_1^2T_1^2+n_2^2T_2^2+n_3^2T_3^2}{n_1T_1+n_2T_2+n_3T_3}$

[Heat and Thermodynamics]

14. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d(d << l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them

(a) $v \propto x^{-1/2}$ (b) $v \propto x^{-1}$

(c) $v \propto x^{1/2}$ (d) $v \propto x$

[Electrostatics]

15. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (Surface tension of soap solution = 0.03 N m^{-1})

(a) 4π mJ (b) 0.2π mJ

(c) 2π mJ (d) 0.4π mJ

[Properties of Matter]

16. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is

(a) $\pi\sqrt{LC}$ (b) $\frac{\pi}{4}\sqrt{LC}$

(c) $2\pi\sqrt{LC}$ (d) $\sqrt{LC}$

[Electromagnetic Induction and Alternating Current]

17. Two bodies of masses m and $4m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is

(a) zero (b) $-\frac{4Gm}{r}$

(c) $-\frac{6Gm}{r}$ (d) $-\frac{9Gm}{r}$

[Gravitation]

18. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc

(a) remains unchanged

(b) continuously decreases

(c) continuously increases

(d) first increases and then decreases

[Rotational Motion and Moment of Inertia]

19. Let the x–z plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is

(a) 30° (b) 45°

(c) 60° (d) 75°

[Ray Optics]

20. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance X_0 ($X_0 > A$). If the maximum separation between them is ($X_0 + A$), the phase difference between their motion is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

[Oscillations]

21. **Direction :** The question has a paragraph followed by two statements, Statement-1 and Statement-2. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement-1 : When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π.

Statement-2 : The centre of the interference pattern is dark.

(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true.

[Wave Optics]

22. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by

(a) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2$ K (b) $\frac{(\gamma-1)}{2\gamma R}Mv^2$ K
(c) $\frac{\gamma Mv^2}{2R}$ K (d) $\frac{(\gamma-1)}{2R}Mv^2$ K

[Heat and Thermodynamics]

23. A screw gauge gives the following reading when used to measure the diameter of a wire.

Main scale reading : 0 mm
Circular scale reading : 52 divisions

Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.

The diameter of wire from the above data is :

(a) 0.52 cm (b) 0.052 cm
(c) 0.026 cm (d) 0.005 cm

[Practical Physics]

24. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} N A^{-1}m^{-1} due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 m s^{-1}, the magnitude of the induced emf in the wire of aerial is

(a) 1 mV (b) 0.75 mV
(c) 0.50 mV (d) 0.15 mV

[Electromagnetic Induction and Alternating Current]

25. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1 : Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

Statement-2 : The state of ionosphere varies from hour to hour, day to day and season to season.

(a) Statement-1 is true, statement-2 is false.
(b) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true.

[Electromagnetic Waves]

26. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is

(a) $\frac{3}{2}g$ (b) g
(c) $\frac{2}{3}g$ (d) $\frac{g}{3}$

[Rotational Motion and Moment of Inertia]

27. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is

(a) $\pi\frac{v^2}{g}$ (b) $\pi\frac{v^4}{g^2}$
(c) $\frac{\pi}{2}\frac{v^4}{g^2}$ (d) $\pi\frac{v^2}{g^2}$

[Description of Motion in 2 and 3 Dimensions]

28. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements :

Statement-1 : A metallic surface is irradiated by a monochromatic light of frequency $\upsilon > \upsilon_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement-2 : The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

(a) Statement-1 is true, statement-2 is false.
(b) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true.

[Electrons and Photons]

29. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion if reversed, is

(a) less than 3
(b) more than 3 but less than 6
(c) more than 6 but less than 9
(d) more than 9

[Rotational Motion and Moment of Inerita]

30. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 m s^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to

(a) 5.0×10^{-3} m
(b) 7.5×10^{-3} m
(c) 9.6×10^{-3} m
(d) 3.6×10^{-3} m

[Properties of Matter]

EXPLANATIONS

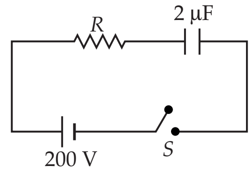
1. **(b) :** $\Delta Q = ms\Delta T$
Here, $m = 100$ g $= 100 \times 10^{-3}$ kg
$s = 4184$ J kg^{-1} K^{-1} and $\Delta T = (50 - 30) = 20°$C
$\therefore \quad \Delta Q = 100 \times 10^{-3} \times 4184 \times 20 = 8.4 \times 10^3$ J
As $\Delta Q = \Delta U + \Delta W$
$\therefore$ Change in internal energy
$\Delta U = \Delta Q = 8.4 \times 10^3$ J $= 8.4$ kJ $\qquad (\because \Delta W = 0)$

2. **(c) :** Number of undecayed atoms after time t_2,
$$\frac{N_0}{3} = N_0 e^{-\lambda t_2} \qquad \text{...(i)}$$
Number of undecayed atoms after time t_1,
$$\frac{2}{3} N_0 = N_0 e^{-\lambda t_1} \qquad \text{...(ii)}$$
Dividing (ii) by (i), we get
$2 = e^{\lambda(t_2 - t_1)}$ or $\ln 2 = \lambda(t_2 - t_1)$
or $(t_2 - t_1) = \dfrac{\ln 2}{\lambda}$
As per question, $t_{1/2}$ = half life time = 20 min
$\therefore \quad t_2 - t_1 = 20$ min $\qquad \left[\because \; t_{1/2} = \dfrac{\ln 2}{\lambda}\right]$

3. **(d) :** $T_1 = 2\pi\sqrt{\dfrac{M}{k}}$ $\qquad$ ...(i)
When a mass m is placed on mass M, the new system is of mass $= (M + m)$ attached to the spring. New time period of oscillation
$$T_2 = 2\pi\sqrt{\frac{(m+M)}{k}} \qquad \text{...(ii)}$$
Consider v_1 is the velocity of mass M passing through mean position and v_2 velocity of mass $(m + M)$ passing through mean position.
Using, law of conservation of linear momentum
$Mv_1 = (m + M)v_2$
$M(A_1\omega_1) = (m + M)(A_2\omega_2)$
$(\because \; v_1 = A_1\omega_1$ and $v_2 = A_2\omega_2)$
or $\dfrac{A_1}{A_2} = \dfrac{(m+M)}{M}\dfrac{\omega_2}{\omega_1}$
$$= \left(\frac{m+M}{M}\right) \times \frac{T_1}{T_2}\left(\because \; \omega_1 = \frac{2\pi}{T_1} \text{ and } \omega_2 = \frac{2\pi}{T_2}\right)$$
$$\frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}} \qquad \text{(Using (i) and (ii))}$$

4. **(c) :** Using, $E_n = -\dfrac{13.6 Z^2}{n^2}$ eV
Here, $Z = 3$ (For Li^{++})
$$\therefore \quad E_1 = -\frac{13.6(3)^2}{(1)^2} \text{ eV}$$
$E_1 = -122.4$ eV
and $E_3 = \dfrac{-13.6 \times (3)^2}{(3)^2} = -13.6$ eV
$\Delta E = E_3 - E_1 = -13.6 + 122.4 = 108.8$ eV

5. **(b) :** $y(x, t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}\,xt)}$
$= e^{-(\sqrt{a}x + \sqrt{b}t)^2}$
Comparing equation (i) with standard equation
$y(x, t) = f(ax + bt)$
As there is positive sign between x and t terms, hence wave travel in $-x$ direction.
$$\text{Wave speed} = \frac{\text{Coefficient of } t}{\text{Coefficient of } x} = \sqrt{\frac{b}{a}}$$

6. **(c) :**


In case charging of capacitor through the resistance is
$V = V_0(1 - e^{-t/RC})$
Here, $V = 120$ V, $V_0 = 200$ V, $R = ?$
$C = 2$ μF and $t = 5$ s.
$\therefore \quad 120 = 200(1 - e^{-5/R \times 2 \times 10^{-6}})$
or $e^{-5/R \times 2 \times 10^{-6}} = \dfrac{80}{200}$
Taking the natural logarithm on both sides, we get
$$\frac{-5}{R \times 2 \times 10^{-6}} = \ln(0.4) = -0.916$$
$\Rightarrow \quad R = 2.7 \times 10^6\ \Omega$

7. **(a)**

8. **(a) :** The efficiency of Carnot engine,
$$\eta = \left(1 - \frac{T_2}{T_1}\right)$$
$$\therefore \quad \frac{1}{6} = \left(1 - \frac{T_2}{T_1}\right) \qquad \left(\text{Given, } \eta = \frac{1}{6}\right)$$

$$\frac{T_2}{T_1}=\frac{5}{6} \quad \Rightarrow \quad T_1=\frac{6T_2}{5} \qquad \text{...(i)}$$

As per question, when T_2 is lowered by 62 K, then its efficiency becomes $\frac{1}{3}$

$$\therefore \quad \frac{1}{3}=\left(1-\frac{T_2-62}{T_1}\right)$$

$$\frac{T_2-62}{T_1}=1-\frac{1}{3}$$

$$\frac{T_2-62}{\frac{6}{5}T_2}=\frac{2}{3} \qquad \text{(Using (i))}$$

$$\frac{5(T_2-62)}{6T_2}=\frac{2}{3}$$

$5T_2-310=4T_2 \Rightarrow T_2=310$ K

From equation (i),

$$T_1=\frac{6\times 310}{5}=372 \text{ K}$$

9. (b) : $\frac{dv}{dt}=-2.5\sqrt{v}$ or $\frac{1}{\sqrt{v}}dv=-2.5\,dt$

On integrating, within limit (v_1 = 6.25 m s^{-1} to $v_2=0$)

$$\therefore \int_{v_1=6.25\text{ m s}^{-1}}^{v_2=0} v^{-1/2}dv=-2.5\int_0^t dt$$

$$2\times\left[v^{1/2}\right]_{6.25}^{0}=-(2.5)t \Rightarrow t=\frac{-2\times(6.25)^{1/2}}{-2.5}=2 \text{ s}$$

10. (d) : $\phi=ar^2+b$

Electric field, $E=\frac{-d\phi}{dr}=-2ar$ ...(i)

According to Gauss's theorem,

$$\oint \vec{E}\cdot d\vec{S}=\frac{q_{\text{inside}}}{\varepsilon_0}$$

or $-2ar4\pi r^2=\frac{q_{\text{inside}}}{\varepsilon_0}$ (Using (i))

$$q_{\text{inside}}=-8\varepsilon_0 a\pi r^3$$

Charge density inside the ball is

$$\rho_{\text{inside}}=\frac{q_{\text{inside}}}{\frac{4}{3}\pi r^3}$$

$$\therefore \quad \rho_{\text{inside}}=\frac{-8\varepsilon_0 a\pi r^3}{\frac{4}{3}\pi r^3}$$

$$\rho_{\text{inside}}=-6a\varepsilon_0$$

11. (b)

12. (b) : Resistance of wire

$$R=\frac{\rho l}{A} \qquad \text{...(i)}$$

On stretching, volume (V) remains constant.

So $V=Al$ or $A=\frac{V}{l}$

$$\therefore \quad R=\frac{\rho l^2}{V} \qquad \text{(Using (i))}$$

Taking logarithm on both sides and differentiating we get,

$$\frac{\Delta R}{R}=\frac{2\Delta l}{l} \qquad (\because V \text{ and } \rho \text{ are constants})$$

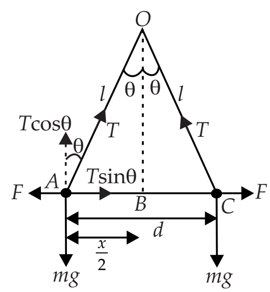
or $\frac{\Delta R}{R}\%=\frac{2\Delta l}{l}\%$

Hence, when wire is stretched by 0.1% its resistance will increase by 0.2%.

13. (b) : The final temperature of the mixture is

$$T_{\text{mixture}}=\frac{T_1n_1+n_2T_2+n_3T_3}{n_1+n_2+n_3}$$

14. (a) : Figure shows equilibrium positions of the two sphere.



$$\therefore \quad T\cos\theta=mg$$

and $T\sin\theta=F=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}$

$$\therefore \quad \tan\theta=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2 mg}$$

When charge begins to leak from both the spheres at a constant rate, then

$$\tan\theta=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2 mg}$$

$$\frac{x}{2l}=\frac{1q^2}{4\pi\varepsilon_0 x^2 mg} \qquad \left(\because \tan\theta=\frac{x}{2l}\right)$$

or $\frac{x}{2l}\propto\frac{q^2}{x^2}$

or $q^2\propto x^3 \Rightarrow q\propto x^{3/2}$

$$\frac{dq}{dt} \propto \frac{3}{2} x^{1/2} \frac{dx}{dt}$$

or $\quad v \propto x^{-1/2} \qquad \left(\because \frac{dq}{dt} = \text{constant}\right)$

15. (d) : Here, surface tension, $S = 0.03$ N m^{-1}
$r_1 = 3$ cm $= 3 \times 10^{-2}$ m, $r_2 = 5$ cm $= 5 \times 10^{-2}$ m
Since bubble has two surfaces,
Initial surface area of the bubble

$$= 2 \times 4\pi r_1^2 = 2 \times 4\pi \times (3 \times 10^{-2})^2$$
$$= 72\pi \times 10^{-4}\ \text{m}^2$$

Final surface area of the bubble

$$= 2 \times 4\pi r_2^2 = 2 \times 4\pi (5 \times 10^{-2})^2 = 200\pi \times 10^{-4}\ \text{m}^2$$

Increase in surface energy

$$= 200\pi \times 10^{-4} - 72\pi \times 10^{-4} = 128\pi \times 10^{-4}$$

$\therefore$ Work done $= S \times$ increase in surface energy

$$= 0.03 \times 128 \times \pi \times 10^{-4} = 3.84\pi \times 10^{-4}$$
$$= 4\pi \times 10^{-4}\ \text{J} = 0.4\pi\ \text{mJ}$$

16. (b) : Charge on the capacitor at any instant t is

$$q = q_0 \cos\omega t \qquad \text{...(i)}$$

Equal sharing of energy means

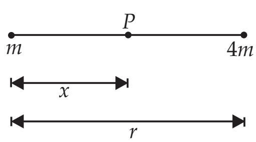
Energy of a capacitor $= \frac{1}{2}$ Total energy

$$\frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\left(\frac{1}{2}\frac{q_0^2}{C}\right) \Rightarrow q = \frac{q_0}{\sqrt{2}}$$

From equation (i)

$$\frac{q_0}{\sqrt{2}} = q_0 \cos\omega t$$
$$\cos\omega t = \frac{1}{\sqrt{2}}$$
$$\omega t = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$
$$t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC} \qquad \left(\because \omega = \frac{1}{\sqrt{LC}}\right)$$

17. (d) : Let x be the distance of the point P from the mass m where gravitational field is zero.



$$\therefore \frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2} \quad \text{or} \quad \left(\frac{x}{(r-x)}\right)^2 = \frac{1}{4}$$

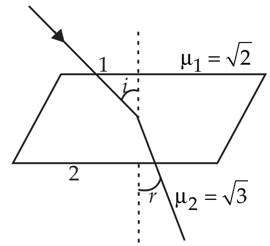
or $\quad x = \frac{r}{3} \qquad$ ...(i)

Gravitational potential at a point P is

$$= -\frac{Gm}{x} - \frac{G(4m)}{(r-x)}$$
$$= -\frac{Gm}{\left(\frac{r}{3}\right)} - \frac{G(4m)}{\left(r - \frac{r}{3}\right)} \qquad \text{(Using (i))}$$
$$= -\frac{3Gm}{r} - \frac{3G(4m)}{2r} = -9\frac{Gm}{r}$$

18. (d)

19. (b)



Here, $\vec{A} = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\,\hat{k}$

$$\cos i = \frac{10}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + (-10)^2}} = \frac{10}{20}$$

$$\cos i = \frac{1}{2} \quad \text{or} \quad i = \cos^{-1}\left(\frac{1}{2}\right) = 60°$$

Using Snell's law, $\mu_1 \sin i = \mu_2 \sin r$

$$\sqrt{2}\sin 60° = \sqrt{3}\sin r \Rightarrow r = 45°$$

20. (b) **21. (c)**

22. (d) : Kinetic energy of vessel $= \frac{1}{2}mv^2$
Increase in internal energy

$$\Delta U = nC_V\Delta T$$

where n is the number of moles of the gas in vessel.
As the vessel is stopped suddenly, its kientic energy is used to increase the temperature of the gas

$$\therefore \quad \frac{1}{2}mv^2 = \Delta U$$
$$\frac{1}{2}mv^2 = nC_V\Delta T$$
$$\frac{1}{2}mv^2 = \frac{m}{M}C_V\Delta T \qquad \left(\because n = \frac{m}{M}\right)$$
$$\Delta T = \frac{Mv^2}{2C_V}$$

or $\quad \Delta T = \frac{Mv^2(\gamma-1)}{2R}\ \text{K} \qquad \left(\because C_V = \frac{R}{(\gamma-1)}\right)$

23. (b) : Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$$

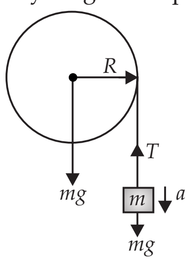
$$= \frac{1}{100} \text{ mm} = 0.01 \text{ mm}$$

Diameter of wire = Main scale reading
+ circular scale reading × Least count
= 0 + 52 × 0.01 = 0.52 mm = 0.052 cm

24. (d) : Here, $B_H = 5.0 \times 10^{-5}$ N A^{-1} m^{-1}
$l = 2$ m and $v = 1.5$ m s^{-1}
Induced emf, $\varepsilon = B_H vl = 5 \times 10^{-5} \times 1.50 \times 2$
$= 15 \times 10^{-5}$ V $= 0.15$ mV

25. (b)

26. (c) : The free body diagram of pulley and mass



$mg - T = ma$

$$\therefore \quad a = \frac{mg - T}{m} \qquad \text{...(i)}$$

As per question, pulley to be consider as a circular disc.

$\therefore$ Angular acceleration of disc

$$\alpha = \frac{\tau}{I} \qquad \text{...(ii)}$$

Here, $\tau = T \times R$

and $I = \frac{1}{2}mR^2$ (For circular disc)

$$\therefore \quad T = \frac{mR\alpha}{2} \qquad \text{(Using (ii))}$$

$$\text{Therefore, } a = \frac{mg - \frac{mR\alpha}{2}}{m} \qquad \text{(Using (i))}$$

$$ma = mg - \frac{ma}{2} \qquad \left(\because \ \alpha = \frac{a}{R}\right)$$

$$\therefore \quad a = \frac{2g}{3}$$

27. (b) : $R_{max} = \frac{v^2 \sin 90^\circ}{g} = \frac{v^2}{g}$

$$\text{Area} = \pi(R_{max})^2 = \frac{\pi v^4}{g^2}$$

28. (d) : The maximum kinetic energy of the electron
$K_{max} = h\upsilon - h\upsilon_0$
Here, υ_0 is threshold frequency.
The stopping potential is
$eV_0 = K_{max} = h\upsilon - h\upsilon_0$
Therefore, if υ is doubled K_{max} and V_0 is not doubled.

29. (b) : Torque exerted on pulley $\tau = FR$

$$\text{or} \quad \alpha = \frac{FR}{I} \qquad \left(\because \ \alpha = \frac{\tau}{I}\right)$$

Here, $F = (20t - 5t^2)$, $R = 2$ m, $I = 10$ kg m^2

$$\therefore \quad \alpha = \frac{(20t - 5t^2) \times 2}{10}$$

$$\alpha = (4t - t^2) \quad \text{or} \quad \frac{d\omega}{dt} = (4t - t^2)dt$$

$$d\omega = (4t - t^2)dt$$

On integrating, $\omega = 2t^2 - \frac{t^3}{3}$

At $t = 6$ s, $\omega = 0$

$$\omega = \frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3}$$

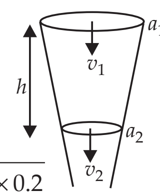
$$d\theta = \left(2t^2 - \frac{t^3}{3}\right)dt$$

On integration, $\theta = \frac{2t^3}{3} - \frac{t^4}{12}$

At, $t = 6$ s, $\theta = 36$ rad

$$2\pi n = 36 \Rightarrow n = \frac{36}{2\pi} < 6$$

30. (d) : Here, $d_1 = 8 \times 10^{-3}$ m
$v_1 = 0.4$ m s^{-1}
$h = 0.2$ m



According to equation of motion,

$$v_2 = \sqrt{v_1^2 + 2gh} = \sqrt{(0.4)^2 + 2 \times 10 \times 0.2}$$

≈ 2 m s^{-1}

$\therefore$ According to equation of continuity

$$a_1 v_1 = a_2 v_2$$

$$\pi \times \left(\frac{8 \times 10^{-3}}{2}\right)^2 \times 0.4 = \pi \times \left(\frac{d_2}{2}\right)^2 \times 2$$

$$d_2 = 3.6 \times 10^{-3} \text{ m}$$

CHAPTER 1

Atoms, Molecules and Chemical Arithmetic

1. Number of atoms in 558.5 gram Fe (at. wt. of Fe = 55.85 g mol^{-1}) is
   (a) twice that in 60 g carbon
   (b) 6.023×10^{22}
   (c) half that in 8 g He
   (d) $558.5 \times 6.023 \times 10^{23}$.

   (2002)

2. With increase of temperature, which of these changes?
   (a) Molality
   (b) Weight fraction of solute
   (c) Fraction of solute present in water
   (d) Mole fraction.

   (2002)

3. What volume of hydrogen gas, at 273 K and 1 atm. pressure will be consumed in obtaining 21.6 g of elemental boron (atomic mass = 10.8) from the reduction of boron trichloride by hydrogen?
   (a) 89.6 L
   (b) 67.2 L
   (c) 44.8 L
   (d) 22.4 L.

   (2003)

4. If we consider that 1/6, in place of 1/12, mass of carbon atom is taken to be the relative atomic mass unit, the mass of one mole of a substance will
   (a) decrease twice
   (b) increase two fold
   (c) remain unchanged
   (d) be a function of the molecular mass of the substance.

   (2005)

5. How many moles of magnesium phosphate, $Mg_3(PO_4)_2$ will contain 0.25 mole of oxygen atoms?
   (a) 0.02
   (b) 3.125×10^{-2}
   (c) 1.25×10^{-2}
   (d) 2.5×10^{-2}.

   (2006)

6. In the reaction,
   $2Al_{(s)} + 6HCl_{(aq)} \rightarrow 2Al^{3+}{}_{(aq)} + 6Cl^{-}{}_{(aq)} + 3H_{2(g)}$
   (a) 11.2 L $H_{2(g)}$ at STP is produced for every mole $HCl_{(aq)}$ consumed
   (b) 6 L $HCl_{(aq)}$ is consumed for every 3 L $H_{2(g)}$ produced
   (c) 33.6 L $H_{2(g)}$ is produced regardless of temperature and pressure for every mole Al that reacts
   (d) 67.2 L $H_{2(g)}$ at STP is produced for every mole Al that reacts.

   (2007)

Answer Key

1.	(a)	2.	(c)	3.	(b)	4.	(a)	5.	(b)	6.	(c)

1. **(a) :** Fe (no. of moles) = $\frac{558.5}{55.85} = 10$ moles
C (no. of moles) = 60/12 = 5 moles.
(atomic weight of carbon = 12)

2. **(c) :** Volume increases with rise in temperature.

3. **(b) :** $2BCl_3 + 3H_2 \rightarrow 6HCl + 2B$
or $BCl_3 + \frac{3}{2}H_2 \rightarrow 3HCl + B$
10.8 g boron requires hydrogen = $\frac{3}{2} \times 22.4$ L
21.6 g boron will require hydrogen
$$= \frac{3}{2} \times \frac{22.4}{10.8} \times 21.6 = 67.2 \text{ L.}$$

4. **(a) :** 1 atomic mass unit on the scale of 1/6 of C-12 = 2 amu on the scale of 1/12 of C-12.
Now, atomic mass of an element
$$= \frac{\text{Mass of one atom of the element}}{\text{1 amu (Here on the scale of } \frac{1}{6} \text{ of C–12)}}$$
$$= \frac{\text{Mass of one atom of the element}}{\text{2 amu (Here on the scale of } \frac{1}{12} \text{ of C–12)}}$$
$\therefore$ Numerically the mass of a substance will become half of the normal scale.

5. **(b) :** 1 mole of $Mg_3(PO_4)_2$
$\Rightarrow$ 3 moles of Mg atom + 2 moles of P atom + 8 moles of O atom
8 moles of oxygen atoms are present in = 1 mole of $Mg_3(PO_4)_2$
0.25 mole of oxygen atoms are present in = $\frac{1 \times 0.25}{8}$
= 3.125×10^{-2} moles of $Mg_3(PO_4)_2$.

6. **(c) :**
$$2Al_{(s)} + 6HCl_{(aq)} \longrightarrow 2Al^{3+}_{(aq)} + 6Cl^-_{(aq)} + 3H_{2(g)}$$
At STP, 6 moles of HCl produces 3 moles
or 3×22.4 lit of HCl
$\therefore$ 1 mole of HCl produces
$\frac{3 \times 22.4}{6}$ = 11.2 lit of HCl
Again at STP, 2 moles of Al produces 3 moles
or 3×22.4 lit of HCl
or 1 mole of Al produces $\frac{3 \times 22.4}{2}$
= 33.6 lit of HCl

CHAPTER
2

States of Matter

1. Value of gas constant R is
(a) 0.082 litre atm (b) 0.987 cal mol^{-1} K^{-1}
(c) 8.3 J mol^{-1} K^{-1} (d) 83 erg mol^{-1} K^{-1}.
(2002)

2. Kinetic theory of gases proves
(a) only Boyle's law
(b) only Charles' law
(c) only Avogadro's law
(d) all of these.
(2002)

3. For an ideal gas, number of moles per litre in terms of its pressure P, gas constant R and temperature T is
(a) PT/R (b) PRT
(c) P/RT (d) RT/P.
(2002)

4. Na and Mg crystallize in *bcc* and *fcc* type crystals respectively, then the number of atoms of Na and Mg present in the unit cell of their respective crystal is
(a) 4 and 2 (b) 9 and 14
(c) 14 and 9 (d) 2 and 4.
(2002)

5. How many unit cells are present in a cube-shaped ideal crystal of NaCl of mass 1.00 g?
[Atomic masses : Na = 23, Cl = 35.5]
(a) 2.57×10^{21} (b) 5.14×10^{21}
(c) 1.28×10^{21} (d) 1.71×10^{21}.
(2003)

6. According to the kinetic theory of gases, in an ideal gas, between two successive collisions a gas molecule travels
(a) in a circular path
(b) in a wavy path
(c) in a straight line path
(d) with an accelerated velocity.
(2003)

7. A pressure cooker reduces cooking time for food because
(a) heat is more evenly distributed in the cooking space
(b) boiling point of water involved in cooking is increased
(c) the higher pressure inside the cooker crushes the food material
(d) cooking involves chemical changes helped by a rise in temperature.
(2003)

8. As the temperature is raised from 20°C to 40°C, the average kinetic energy of neon atoms changes by a factor of which of the following?
(a) 1/2 (b) $\sqrt{313/293}$
(c) 313/293 (d) 2.
(2004)

9. In van der Waals equation of state of the gas law, the constant b is a measure of
(a) intermolecular repulsions
(b) intermolecular attraction
(c) volume occupied by the molecules
(d) intermolecular collisions per unit volume.
(2004)

10. What type of crystal defect is indicated in the diagram below?
Na^+ Cl^- Na^+ Cl^- Na^+ Cl^-

Cl^- $\square$ Cl^- Na^+ $\square$ Na^+
Na^+ Cl^- $\square$ Cl^- Na^+ Cl^-
Cl^- Na^+ Cl^- Na^+ $\square$ Na^+.

(a) Frenkel defect
(b) Schottky defect
(c) interstitial defect
(d) Frenkel and Schottky defects.

(2004)

11. An ionic compound has a unit cell consisting of A ions at the corners of a cube and B ions on the centres of the faces of the cube. The empirical formula for this compound would be

(a) AB (b) A_2B
(c) AB_3 (d) A_3B

(2005)

12. Which one of the following statements is not true about the effect of an increase in temperature on the distribution of molecular speeds in a gas?

(a) The most probable speed increases.
(b) The fraction of the molecules with the most probable speed increases.
(c) The distribution becomes broader.
(d) The area under the distribution curve remains the same as under the lower temperature.

(2005)

13. Total volume of atoms present in a face-centred cubic unit cell of a metal is (r is atomic radius)

(a) $\frac{20}{3}\pi r^3$ (b) $\frac{24}{3}\pi r^3$
(c) $\frac{12}{3}\pi r^3$ (d) $\frac{16}{3}\pi r^3$

(2006)

14. Equal masses of methane and oxygen are mixed in an empty container at 25°C. The fraction of the total pressure exerted by oxygen is

(a) 1/2 (b) 2/3
(c) $\frac{1}{3}\times\frac{273}{298}$ (d) 1/3.

(2007)

15. In a compound, atoms of element Y form *ccp* lattice and those of element X occupy 2/3rd of tetrahedral voids. The formula of the compound will be

(a) $X_3 Y_4$ (b) $X_4 Y_3$
(c) $X_2 Y_3$ (d) $X_2 Y$

(2008)

Answer Key

1.	(c)	**2.**	(d)	**3.**	(c)	**4.**	(d)	**5.**	(a)	**6.**	(c)
7.	(b)	**8.**	(c)	**9.**	(c)	**10.**	(b)	**11.**	(c)	**12.**	(b)
13.	(d)	**14.**	(d)	**15.**	(b)						

1. **(c) :** Units of R
   (i) in lit atm $\Rightarrow$ 0.082 lit atm mol^{-1} k^{-1}
   (ii) in C.G.S. system $\Rightarrow 8.314 \times 10^7$ erg mol^{-1} k^{-1}
   (iii) in M.K.S. system $\Rightarrow$ 8.314 J mol^{-1} k^{-1}
   (iv) in calories $\Rightarrow$ 1.987 cal mol^{-1} k^{-1}

2. **(d) : Explanation of the Gas Laws on the basis of Kinetic Molecular Model**
   One of the postulates of kinetic theory of gases is Average K.E. $\propto T$

   or, $\frac{1}{2}mnC_{rms}^2 \propto T$ or, $\frac{1}{2}mnC_{rms}^2 = kT$

   Now, $PV = \frac{1}{3}mnC_{rms}^2 = \frac{2}{3}\times\frac{1}{2}mnC_{rms}^2 = \frac{2}{3}kT$

   (i) Boyle's Law :
   - Constant temperature means that the average kinetic energy of the gas molecules remains constant.
   - This means that the rms velocity of the molecules, C_{rms} remains unchanged.
   - If the rms velocity remains unchanged, but the volume increases, this means that there will be fewer collisions with the container walls over a given time.
   - Therefore, the pressure will decrease

     i.e. $P \propto \frac{1}{V}$

     or PV = constant.

   (ii) Charles' Law :
   - An increase in temperature means an increase in the average kinetic energy of the gas molecules, thus an increase in C_{rms}.
   - There will be more collisions per unit time, furthermore, the momentum of each collision increases (molecules strike the wall harder).
   - Therefore, there will be an increase in pressure.
   - If we allow the volume to change to maintain constant pressure, the volume will increase with increasing temperature (Charles law).

   (iii) Avogadro's Law
   It states that under similar conditions of pressure and temperature, equal volume of all gases contain equal number of molecules. Considering two gases, we have

   $P_1V_1 = \frac{2}{3}kT_1$ and $P_2V_2 = \frac{2}{3}kT_2$

   Since $P_1 = P_2$ and $T_1 = T_2$, therefore

   $\frac{P_1V_1}{P_2V_2} = \frac{(2/3)kT_1}{(2/3)kT_2} \Rightarrow \frac{V_1}{V_2} = \frac{n_1}{n_2}$

   If volumes are identical, obviously $n_1 = n_2$.

3. **(c) :** From ideal gas equation, $PV = nRT$
   $\therefore$ $n/V = P/RT$ (number of moles $= n/V$)

4. **(d) :** *bcc* - Points are at corners and one in the centre of the unit cell.

   Number of atoms per unit cell $= 8\times\frac{1}{8}+1 = 2$.

   fcc - Points are at the corners and also centre of the six faces of each cell.

   Number of atoms per unit cell $= 8\times\frac{1}{8}+6\times\frac{1}{2} = 4$.

5. **(a) :** mass (m) = density × volume = 1.00 g
   Mol. wt. (M) of NaCl = 23 + 35.5 = 58.5
   Number of unit cell present in a cube shaped crystal of NaCl of mass 1.00 g $= \frac{\rho\times a^3\times N_A}{M\times Z} = \frac{m\times N_A}{M\times Z}$

   $= \frac{1\times 6.023\times 10^{23}}{58.5\times 4}$.

   (In NaCl each unit cell has 4 NaCl units. Hence $Z = 4$).

   $\therefore$ Number of unit cells $= 0.02573 \times 10^{23}$
   $= 2.57 \times 10^{21}$ unit cells.

6. **(c) :** According to the kinetic theory of gases, gas molecules are always in rapid random motion colliding with each other and with the wall of the container and between two successive collisions a gas molecule travels in a straight line path.

7. **(b) :** According to Gay Lussac's law, at constant pressure of a given mass of a gas is directly proportional to the absolute temperature of the gas. Hence, on increasing pressure, the temperature is also increased. Thus in pressure cooker due to increase in pressure the boiling point of water involved in cooking is also increased.

8. **(c) :** $K_b = 3/2\ RT$

$$\frac{K_{40}}{K_{20}} = \frac{T_{40}}{T_{20}} = \frac{273+40}{273+20} = \frac{313}{293}.$$

9. **(c) :** van der Waals constant for volume correction b is the measure of the effective volume occupied by the gas molecules.

10. **(b) :** When an atom or ion is missing from its normal lattice site, a lattice vacancy is created. This defect is known as Schottky defect.
Here equal number of Na^+ and Cl^- ions are missing from their regular lattice position in the crystal. So it is Schottky defect.

11. **(c) :** Number of A ions per unit cell = $\frac{1}{8} \times 8 = 1$

Number of B ions per unit cell = $\frac{1}{2} \times 6 = 3$

Empirical formula = AB_3.

12. **(b) :** Most probable velocity is defined as the speed possessed by maximum number of molecules of a gas at a given temperature. According to Maxwell's distribution curves, as temperature increases, most probable velocity increases and fraction of molecule possessing most probable velocity decreases.

13. **(d) :** In case of a face-centred cubic structure, since four atoms are present in a unit cell, hence volume

$$V = 4\left(\frac{4}{3}\pi r^3\right) = \frac{16}{3}\pi r^3.$$

14. **(d) :** Let the mass of methane and oxygen be m gm.
Mole fraction of oxygen, x_{O_2}

$$= \frac{\frac{m}{32}}{\frac{m}{32}+\frac{m}{16}} = \frac{m}{32} \times \frac{32}{3m} = \frac{1}{3}$$

Let the total pressure be P.
$\therefore$ Partial pressure of O_2, $p_{O_2} = P \times x_{O_2}$

$$= P \times \frac{1}{3} = \frac{1}{3}P.$$

15. **(b) :** Number of Y atoms per unit cell in ccp lattice $(N) = 4$
number of tetrahedral voids = $2N = 2 \times 4 = 8$
number of tetrahedral voids occupied by X
= $2/3^{\text{rd}}$ of the tetrahedral void = $2/3 \times 8 = 16/3$
Hence the formula of the compound will be
$X_{16/3}Y_4 = X_4Y_3$

CHAPTER
3

Atomic Structure, Chemical Bonding and Molecular Structure

1. In which of the following species the interatomic bond angle is 109° 28′?
(a) NH_3, $(BF_4)^{-1}$ (b) $(NH_4)^+$, BF_3
(c) NH_3, BF_3 (d) $(NH_2)^{-1}$, BF_3.
(2002)

2. In which of the following species is the underlined carbon having sp^3 hybridisation?
(a) $CH_3\underline{C}OOH$ (b) $CH_3\,\underline{C}H_2\,OH$
(c) $CH_3\underline{C}OCH_3$ (d) $CH_2 = \underline{C}H - CH_3$.
(2002)

3. Number of sigma bonds in P_4O_{10} is
(a) 6 (b) 7
(c) 17 (d) 16.
(2002)

4. In a hydrogen atom, if energy of an electron in ground state is 13.6 eV, then that in the 2nd excited state is
(a) 1.51 eV (b) 3.4 eV
(c) 6.04 eV (d) 13.6 eV.
(2002)

5. A square planar complex is formed by hybridisation of which atomic orbitals?
(a) s, p_x, p_y, d_{yz} (b) $s, p_x, p_y, d_{x^2-y^2}$
(c) s, p_x, p_y, d_{z^2} (d) s, p_y, p_z, d_{xy}.
(2002)

6. Uncertainty in position of a minute particle of mass 25 g in space is 10^{-5} m. What is the uncertainty in its velocity (in ms^{-1})? ($h = 6.6 \times 10^{-34}$ Js)
(a) 2.1×10^{-34} (b) 0.5×10^{-34}
(c) 2.1×10^{-28} (d) 0.5×10^{-23}.
(2002)

7. Which of the following are arranged in an increasing order of their bond strengths?
(a) $O_2^- < O_2 < O_2^+ < O_2^{2-}$
(b) $O_2^{2-} < O_2^- < O_2 < O_2^+$
(c) $O_2^- < O_2^{2-} < O_2 < O_2^+$
(d) $O_2^+ < O_2 < O_2^- < O_2^{2-}$.
(2002)

8. In Bohr series of lines of hydrogen spectrum, the third line from the red end corresponds to which one of the following inter-orbit jumps of the electron for Bohr orbits in an atom of hydrogen?
(a) $3 \to 2$ (b) $5 \to 2$
(c) $4 \to 1$ (d) $2 \to 5$.
(2003)

9. The de Broglie wavelength of a tennis ball of mass 60 g moving with a velocity of 10 metres per second is approximately
(Planck's constant, $h = 6.63 \times 10^{-34}$ Js)
(a) 10^{-33} metres (b) 10^{-31} metres
(c) 10^{-16} metres (d) 10^{-25} metres.
(2003)

10. The orbital angular momentum for an electron revolving in an orbit is given by $\sqrt{l(l+1)}\cdot\frac{h}{2\pi}$. This momentum for an s-electron will be given by
(a) $+\frac{1}{2}\cdot\frac{h}{2\pi}$ (b) zero
(c) $\frac{h}{2\pi}$ (d) $\sqrt{2}\cdot\frac{h}{2\pi}$.
(2003)

11. Which one of the following compounds has the smallest bond angle in its molecule?

(a) SO_2 (b) OH_2
(c) SH_2 (d) NH_3.

(2003)

12. The pair of species having identical shapes for molecules of both species is
(a) CF_4, SF_4 (b) XeF_2, CO_2
(c) BF_3, PCl_3 (d) PF_5, IF_5.

(2003)

13. Which of the following sets of quantum numbers is correct for an electron in 4*f* orbital?

(a) $n = 4, l = 3, m = +4, s = +\frac{1}{2}$

(b) $n = 4, l = 4, m = -4, s = -\frac{1}{2}$

(c) $n = 4, l = 3, m = +1, s = +\frac{1}{2}$

(d) $n = 3, l = 2, m = -2, s = +\frac{1}{2}$.

(2004)

14. Consider the ground state of Cr atom ($Z = 24$). The numbers of electrons with the azimuthal quantum numbers, $l = 1$ and 2 are, respectively
(a) 12 and 4 (b) 12 and 5
(c) 16 and 4 (d) 16 and 5.

(2004)

15. The wavelength of the radiation emitted, when in a hydrogen atom electron falls from infinity to stationary state 1, would be (Rydberg constant $= 1.097 \times 10^7$ m^{-1})
(a) 91 nm (b) 192 nm
(c) 406 nm (d) 9.1×10^{-8} nm.

(2004)

16. The correct order of bond angles (smallest first) in H_2S, NH_3, BF_3 and SiH_4 is
(a) $H_2S < SiH_4 < NH_3 < BF_3$
(b) $NH_3 < H_2S < SiH_4 < BF_3$
(c) $H_2S < NH_3 < SiH_4 < BF_3$
(d) $H_2S < NH_3 < BF_3 < SiH_4$.

(2004)

17. The bond order in NO is 2.5 while that in NO^+ is 3. Which of the following statements is true for these two species?
(a) Bond length in NO^+ is greater than in NO
(b) Bond length in NO is greater than in NO^+
(c) Bond length in NO^+ is equal to that in NO
(d) Bond length is unpredictable.

(2004)

18. Which one of the following has the regular tetrahedral structure?
(a) XeF_4 (b) SF_4
(c) BF_4^- (d) $[Ni(CN)_4]^{2-}$.
(Atomic nos.: B = 5, S = 16, Ni = 28, Xe = 54)

(2004)

19. The maximum number of 90° angles between bond pair-bond pair of electrons is observed in
(a) dsp^3 hybridisation
(b) sp^3d hybridisation
(c) dsp^2 hybridisation
(d) sp^3d^2 hybridisation.

(2004)

20. Which one of the following species is diamagnetic in nature?
(a) He_2^+ (b) H_2
(c) H_2^+ (d) H_2^-.

(2005)

21. In a multi-electron atom, which of the following orbitals described by the three quantum numbers will have the same energy in the absence of magnetic and electric fields?
(i) $n = 1, l = 0, m = 0$
(ii) $n = 2, l = 0, m = 0$
(iii) $n = 2, l = 1, m = 1$
(iv) $n = 3, l = 2, m = 1$
(v) $n = 3, l = 2, m = 0$
(a) (i) and (ii) (b) (ii) and (iii)
(c) (iii) and (iv) (d) (iv) and (v)

(2005)

22. Of the following sets which one does NOT contain isoelectronic species?
(a) PO_4^{3-}, SO_4^{2-}, ClO_4^-
(b) CN^-, N_2, C_2^{2-}
(c) SO_3^{2-}, CO_3^{2-}, NO_3^-
(d) BO_3^{3-}, CO_3^{2-}, NO_3^-

(2005)

23. Which of the following statements in relation to the hydrogen atom is correct?

(a) $3s$ orbital is lower in energy than $3p$ orbital
(b) $3p$ orbital is lower in energy than $3d$ orbital
(c) $3s$ and $3p$ orbitals are of lower energy than $3d$ orbital
(d) $3s$, $3p$ and $3d$ orbitals all have the same energy

(2005)

24. According to Bohr's theory, the angular momentum of an electron in 5^{th} orbit is

(a) $25\frac{h}{\pi}$ (b) $1.0\frac{h}{\pi}$
(c) $10\frac{h}{\pi}$ (d) $2.5\frac{h}{\pi}$.

(2006)

25. Which of the following molecules/ions does not contain unpaired electrons?
(a) O_2^{2-} (b) B_2
(c) N_2^+ (d) O_2.

(2006)

26. Uncertainty in the position of an electron (mass = 9.1×10^{-31} kg) moving with a velocity 300 ms^{-1}, accurate upto 0.001% will be ($h = 6.6 \times 10^{-34}$ Js)
(a) 19.2×10^{-2} m (b) 5.76×10^{-2} m
(c) 1.92×10^{-2} m (d) 3.84×10^{-2} m.

(2006)

27. Among the following mixtures, dipole-dipole as the major interaction, is present in
(a) benzene and ethanol
(b) acetonitrile and acetone
(c) KCl and water
(d) benzene and carbon tetrachloride.

(2006)

28. In which of the following molecules/ions are all the bonds not equal?
(a) SF_4 (b) SiF_4
(c) XeF_4 (d) BF_4^-.

(2006)

29. Which of the following species exhibits the diamagnetic behaviour?
(a) NO (b) O_2^{2-}
(c) O_2^+ (d) O_2.

(2007)

30. The charge/size ratio of a cation determines its polarizing power. Which one of the following sequences represents the increasing order of the polarizing power of the cationic species, K^+, Ca^{2+}, Mg^{2+}, Be^{2+}?
(a) $Ca^{2+} < Mg^{2+} < Be^+ < K^+$
(b) $Mg^{2+} < Be^{2+} < K^+ < Ca^{2+}$
(c) $Be^{2+} < K^+ < Ca^{2+} < Mg^{2+}$
(d) $K^+ < Ca^{2+} < Mg^{2+} < Be^{2+}$.

(2007)

31. In which of the following ionization processes, the bond order has increased and the magnetic behaviour has changed?
(a) $N_2 \rightarrow N_2^+$ (b) $C_2 \rightarrow C_2^+$
(c) $NO \rightarrow NO^+$ (d) $O_2 \rightarrow O_2^+$.

(2007)

32. Which of the following sets of quantum numbers represents the highest energy of an atom?
(a) $n = 3, l = 0, m = 0, s = +\frac{1}{2}$
(b) $n = 3, l = 1, m = 1, s = +\frac{1}{2}$
(c) $n = 3, l = 2, m = 1, s = +\frac{1}{2}$
(d) $n = 4, l = 0, m = 0, s = +\frac{1}{2}$.

(2007)

33. Which of the following hydrogen bonds is the strongest?
(a) O — H - - - F
(b) O — H - - - H
(c) F — H - - - F
(d) O — H - - - O.

(2007)

34. Which one of the following pairs of species have the same bond order?
(a) NO^+ and CN^+ (b) CN^- and NO^+
(c) CN^- and CN^+ (d) O_2^- and CN^-

(2008)

35. The ionization enthalpy of hydrogen atom is 1.312×10^6 J mol^{-1}. The energy required to excite the electron in the atom from $n = 1$ to $n = 2$ is
(a) 9.84×10^5 J mol^{-1}
(b) 8.51×10^5 J mol^{-1}

(c) 6.56×10^5 J mol^{-1}
(d) 7.56×10^5 J mol^{-1}

(2008)

36. Which one of the following constitutes a group of the isoelectronic species?

(a) N_2, O_2^-, NO^+, CO
(b) C_2^{2-}, O_2^-, CO, NO
(c) NO^+, C_2^{2-}, CN^-, N_2
(d) CN^-, N_2, O_2^{2-}, C_2^{2-}

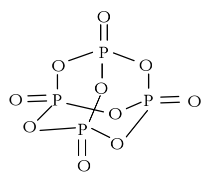
(2008)

Answer Key

1.	(a)	**2.**	(b)	**3.**	(d)	**4.**	(a)	**5.**	(b)	**6.**	(c)
7.	(b)	**8.**	(b)	**9.**	(a)	**10.**	(b)	**11.**	(c)	**12.**	(b)
13.	(c)	**14.**	(b)	**15.**	(a)	**16.**	(a)	**17.**	(b)	**18.**	(c)
19.	(d)	**20.**	(b)	**21.**	(d)	**22.**	(c)	**23.**	(d)	**24.**	(d)
25.	(a)	**26.**	(c)	**27.**	(b)	**28.**	(a)	**29.**	(b)	**30.**	(d)
31.	(c)	**32.**	(c)	**33.**	(c)	**34.**	(b)	**35.**	(a)	**36.**	(c)

1. **(a) :** Both undergoes sp^3 hybridization. The expected bond angle should be 109º28′ but actual bond angle is less than 109º28′ because of the repulsion between lone pair and bonded pairs due to which contraction occurs.

2. **(b) :** In molecules (a) $\left(CH_3-\overset{\overset{\displaystyle O}{||}}{C}-OH\right)$, (c) $\left(CH_3-\overset{\overset{\displaystyle O}{||}}{C}-CH_3\right)$ and (d) $(CH_2=CH-CH_3)$, the carbon atom has a multiple bond, only (b) has sp^3 hybridization.

3. **(d) :** 

No. of σ bonds = 16
No. of π bonds = 4

4. **(a) :** 2[nd] excited state will be the 3rd energy level.

$$E_n = \frac{13.6}{n^2}\ \text{eV} \quad \text{or} \quad E = \frac{13.6}{9} = 1.51\ \text{eV}.$$

5. **(b) :** dsp^2 hybridisation gives square planar structure with s, p_x, p_y and $d_{x^2-y^2}$ orbitals with bond angles of 90°.

6. **(c) :** According to Heisenberg uncertainty principle,

$$\Delta x \cdot m\Delta v = \frac{h}{4\pi}$$

$$\Delta v = \frac{6.6\times 10^{-34}}{4\times 3.14\times 25\times 10^{-5}}$$

$\therefore\ \Delta v = 2.1\times 10^{-28}\ \text{ms}^{-1}$.

7. **(b) :** Molecular orbital configuration of

$O_2 \Rightarrow \sigma(1s)^2\sigma^*(1s)^2\sigma(2s)^2\sigma^*(2s)^2\sigma(2p_z)^2\pi(2p_x)^2$
$\pi(2p_y)^2\pi^*(2p_x)^1\pi^*(2p_y)^1$; $\text{B.O.} = \frac{10-6}{2} = 2$

$O_2^+ \Rightarrow \sigma(1s)^2\sigma^*(1s)^2\sigma(2s)^2\sigma^*(2s)^2\sigma(2p_z)^2\pi(2p_x)^2$
$\pi(2p_y)^2\pi^*(2p_x)^1$; $\text{B.O.} = \frac{10-5}{2} = 2.5$

$O_2^- \Rightarrow \sigma(1s)^2\sigma^*(1s)^2\sigma(2s)^2\sigma^*(2s)^2\sigma(2p_z)^2\pi(2p_x)^2$
$\pi(2p_y)^2\pi^*(2p_x)^2\pi^*(2p_y)^1$; $\text{B.O.} = \frac{10-7}{2} = 1.5$

$O_2^{2-} \Rightarrow \sigma(1s)^2\sigma^*(1s)^2\sigma(2s)^2\sigma^*(2s)^2\sigma(2p_z)^2\pi(2p_x)^2$
$\pi(2p_y)^2\pi^*(2p_x)^2\pi^*(2p_y)^2$; $\text{B.O.} = \frac{10-8}{2} = 1$

Hence increasing order of bond order is
$O_2^{2-} < O_2^- < O_2 < O_2^+$

8. **(b) :** The electron has minimum energy in the first orbit and its energy increases as n increases. Here n represents number of orbit, *i.e.* 1[st], 2[nd], 3[rd] .... The third line from the red end corresponds to yellow region *i.e.* 5. In order to obtain less energy electron tends to come in 1[st] or 2[nd] orbit. So jump may be involved either $5\rightarrow 1$ or $5\rightarrow 2$. Thus option (b) is correct here.

9. **(a) :** $\lambda = \frac{h}{mv} = \frac{6.63\times 10^{-34}\times 1000}{60\times 10}$ m
$= 11.05\times 10^{-34}$ m $= 1.105\times 10^{-33}$ metres.

10. **(b) :** The value of l (azimuthal quantum number) for s -electron is equal to zero.

Orbital angular momentum $= \sqrt{l(l+1)}\cdot\frac{h}{2\pi}$

Substituting the value of l for s-electron

$= \sqrt{0(0+1)}\cdot\frac{h}{2\pi} = 0.$

11. **(c) :**

	SO_2	OH_2	SH_2	NH_3
Bond angle :	119.5°	104.5°	92.5°	106.5°

12. **(b) :** Central atom in each being sp hybridised shows linear shape.
$F-Xe-F$ $\qquad O=C=O$

13. **(c) :** For $4f$ orbital electrons, $n = 4$
$l = 3$ (because $\overset{s}{0}\ \overset{p}{1}\ \overset{d}{2}\ \overset{f}{3}$)
$m = +3, +2, +1, 0, -1, -2, -3$
$s = \pm 1/2$.

14. **(b) :** $_{24}Cr \rightarrow 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^5\ 4s^1$
we know for p, $l = 1$ and for d, $l = 2$.
For $l = 1$, total number of electrons = 12 [$2p^6$ and $3p^6$]
For $l = 2$, total number of electrons = 5 [$3d^5$]

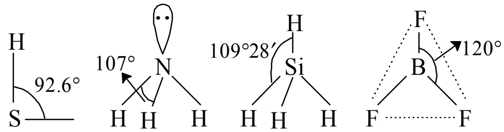
15. **(a) :** $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \text{ m}^{-1}\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$$

$\therefore \quad \lambda = 91 \times 10^{-9}$ m = 91 nm.

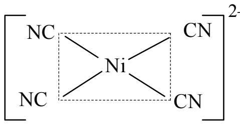
16. **(a) :** The correct order of bond angle (smallest first) is $H_2S < NH_3 < SiH_4 < BF_3$

$92.6° < 107° < 109°28' < 120°$

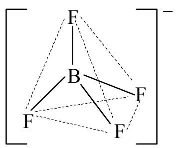


17. **(b) :** Higher the bond order, shorter will be the bond length. Thus NO^+ is having higher bond order than that of NO so NO^+ has shorter bond length.

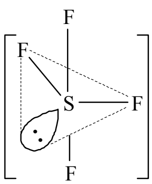
18. **(c) :**



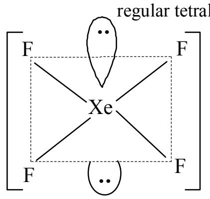
square planar



regular tetrahedral

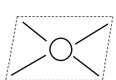


see-saw shaped

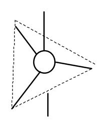


square planar

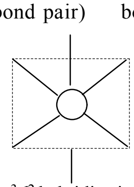
19. **(d) :**



dsp^2 hybridisation (four 90° angles between bond pair and bond pair)



sp^3*d* or *dsp*^{3} hybridisation (six 90° angles between bond pair and bond pair)



sp^3*d*^{2} hybridisation (twelve 90° angle between bond pair and bond pair)

20. **(b) :** $He_2^+ \rightarrow \sigma(1s)^2\, \sigma^*(1s)^1$, one unpaired electron
$H_2 \rightarrow \sigma(1s)^2\, \sigma^*(1s)^0$, no unpaired electron
$H_2^+ \rightarrow \sigma(1s)^1\, \sigma^*(1s)^0$, one unpaired electron
$H_2^- \rightarrow \sigma(1s)^2\, \sigma^*(1s)^1$, one unpaired electron.
Due to absence of unpaired electrons, H_2 will be diamagnetic.

21. **(d) :** Orbitals having same $(n + l)$ value in the absence of electric and magnetic field will have same energy.

22. **(c) :** Number of electrons in SO_3^{2-}
$= 16 + 8 \times 3 + 2 = 42$
Number of electrons in $CO_3^{2-} = 6 + 8 \times 3 + 2 = 32$
Number of electrons in $NO_3^- = 7 + 8 \times 3 + 1 = 32$
These are not isoelectronic species as number of electrons are not same.

23. **(d) :** For hydrogen the energy order of orbital is
$1s < 2s = 2p < 3s = 3p = 3d < 4s = 4p = 4d = 4f$

24. **(d) :** Angular momentum of the electron, $mvr = \frac{nh}{2\pi}$
when $n = 5$ (given)

$\therefore \quad$ Angular momentum $= \frac{5h}{2\pi} = 2.5\frac{h}{\pi}$.

25. **(a) :** The molecular orbital configuration of O_2^{--} ion is
KK $\sigma(2s)^2\, \sigma^*(2s)^2\, \sigma(2p_z)^2\, \pi(2p_x)^2\, \pi(2p_y)^2\, \pi^*(2p_x)^2\, \pi^*(2p_y)^2$
Here KK represents non-bonding molecular orbital of 1*s* orbital.
O_2^{2-} contains no unpaired electrons.
The molecular orbital configuration of B_2 molecule is
KK $\sigma(2s)^2\, \sigma^*(2s)^2\, \pi(2p_x)^1\, \pi(2p_y)^1$
It contains 2 unpaired electrons.
The molecular orbital configuration of N_2^+ ion is
KK $\sigma(2s)^2\, \sigma^*(2s)^2\, \sigma(2p_z)^2\, \pi(2p_x)^2\, \pi(2p_y)^2\, \sigma(2p_z)^1$
It contains one unpaired electron.
The molecular orbital configuration of O_2 molecule is
KK $\sigma(2s)^2\, \sigma^*(2s)^2\, \pi(2p_x)^2\, \pi(2p_y)^2\, \sigma(2p_z)^2\, \pi^*(2p_x)^1\, \pi^*(2p_y)^1$
It contains 2 unpaired electrons.

26. **(c) :** According to Heisenberg's uncertainty principle

$$\Delta x \times \Delta p = \frac{h}{4\pi}.$$

$$\Delta x \cdot (m \cdot \Delta v) = \frac{h}{4\pi} \quad \Rightarrow \quad \Delta x = \frac{h}{4\pi m \cdot \Delta v}$$

Here $\Delta v = \frac{0.001}{100} \times 300 = 3 \times 10^{-3}$ ms^{-1}.

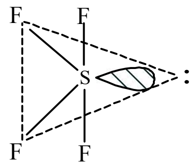
$$\therefore \quad \Delta x = \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 3 \times 10^{-3}}$$

$$= 1.92 \times 10^{-2} \text{ m}.$$

27. **(b) :** Dipole-dipole interactions occur among the polar molecules. Polar molecules have permanent

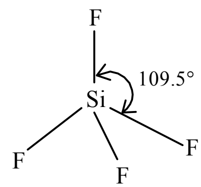
dipoles. The positive pole of one molecule is thus attracted by the negative pole of the other molecule. The magnitude of dipole-dipole forces in different polar molecules is predicted on the basis of the polarity of the molecules, which in turn depends upon the electronegativities of the atoms present in the molecule and the geometry of the molecule (in case of polyatomic molecules, containing more than two atoms in a molecule).

28. **(a) :**

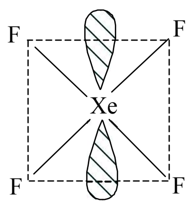


SF_4 molecule shows sp^3d hybridisation but its expected trigonal bipyramidal geometry gets distorted due to presence of a lone pair of electrons and it becomes distorted tetrahedral or see-saw with the bond angles equal to 89° and 177° instead of the expected angles of 90° and 180° respectively.

SiF_4 : sp^3 hybridisation and tetrahedral geometry.



XeF_4 : sp^3d^2 hybridisation, shape is square planar instead of octahedral due to presence of two lone pair of electrons on Xe atom.



BF_4^- : sp^3 hybridisation and tetrahedral geometry.

29. **(b) :** Molecular orbital configuration is

NO $\Rightarrow \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^2 \pi 2p_y^2 \pi^* 2p_x^1$

$\Rightarrow$ paramagnetic

$O_2 \Rightarrow \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^2 \pi 2p_y^2 \pi^* 2p_x^1 \pi^* 2p_y^1$

$\Rightarrow$ paramagnetic

$O_2^{2-} \Rightarrow \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^2 \pi 2p_y^2 \pi^* 2p_x^2 \pi^* 2p_y^2$

$\Rightarrow$ diamagnetic

$O_2^+ \Rightarrow \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^2 \pi 2p_y^2 \pi^* 2p_x^1$

$\Rightarrow$ paramagnetic

30. **(d) :** High charge and small size of the cations increases polarisation.

As the size of the given cations decreases as

$K^+ > Ca^{2+} > Mg^{2+} > Be^{2+}$

Hence, polarising power decreases as

$K^+ < Ca^{2+} < Mg^{2+} < Be^{2+}$

31. **(c) :** Molecular orbital configuration of

$O_2 \Rightarrow \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^2 \pi 2p_y^2 \pi^* 2p_x^1 \pi^* 2p_y^1$

$\Rightarrow$ paramagnetic

Bond order $= \frac{10-6}{2} = 2$

$O_2^+ \Rightarrow \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^2 \pi 2p_y^2 \pi^* 2p_x^1$

$\Rightarrow$ paramagnetic

Bond order $= \frac{10-5}{2} = 2.5$

$N_2 \Rightarrow \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \pi 2p_x^2 \pi 2p_y^2 \pi 2p_Z^2$

$\Rightarrow$ paramagnetic

Bond order $= \frac{10-4}{2} = 3$

$N_2^+ \Rightarrow \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \pi 2p_x^2 \pi 2p_y^2 \pi 2p_z^1$

$\Rightarrow$ paramagnetic

Bond order $= \frac{9-4}{2} = 2.5$

$C_2 \Rightarrow \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \pi 2p_x^2 \pi 2p_y^2$

$\Rightarrow$ diamagnetic

Bond order $= \frac{8-4}{2} = 2$

$C_2^+ \Rightarrow \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \pi 2p_x^2 \pi 2p_y^1$

$\Rightarrow$ paramagnetic

Bond order $= \frac{7-4}{2} = 1.5$

NO $\Rightarrow \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^2 \pi 2p_y^2 \pi^* 2p_x^1$

$\Rightarrow$ paramagnetic

Bond order $= \frac{10-5}{2} = 2.5$

$NO^+ \Rightarrow \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^2 \pi 2p_y^2$

$\Rightarrow$ diamagnetic.

Bond order $= \frac{10-4}{2} = 3$.

32. **(c) :** $n = 3, l = 0$ represents $3s$ orbital

$n = 3, l = 1$ represents $3p$ orbital

$n = 3, l = 2$ represents $3d$ orbital

$n = 4, l = 0$ represents $4s$ orbital

The order of increasing energy of the orbitals is

$3s < 3p < 4s < 3d.$

33. **(c) :** Because of highest electronegativity of F, hydrogen bonding in F — H - - - F is strongest.

34. **(b) :** In the given pair of species number of electron in NO^+ = number of electron in CN^- = 14 electrons. So they are isoelectronic in nature.

Hence bond order of these two species will be also similar which is shown below.

$NO^+ \rightarrow \sigma 1s^2\ \sigma^*1s^2\ \sigma 2s^2\ \sigma^*2s^2\ \sigma 2p_z^2\ \pi 2p_x^2\ \pi 2p_y^2$

B.O $= 1/2\ [N_b - N_a] = 1/2\ [10 - 4]$

or B.O $= 3$

$CN^- \rightarrow \sigma 1s^2\ \sigma^*1s^2\ \sigma 2s^2\ \sigma^*2s^2\ \pi 2p_x^2\ \pi 2p_y^2\ \sigma 2p_z^2$

B.O $=1/2\ [10 - 4]$ or B.O $= 3$

35. **(a) :** The ionisation of H-atom is the energy absorbed when the electron in an atom gets excited from first shell (E_1) to infinity (*i.e.*, E_∞)

I.E $= E_\infty - E_1$

$1.312 \times 10^6 = 0 - E_1$

$E_1 = -1.312 \times 10^6$ J mol^{-1}

$$E_2 = -\frac{1.312\times10^6}{(2)^2} = -\frac{1.312\times10^6}{4}$$

Energy of electron in second orbit ($n = 2$)

$\therefore$ Energy required when an electron makes transition from $n = 1$ to $n = 2$

$$\Delta E = E_2 - E_1 = -\frac{1.312\times10^6}{4} - (-1.312\times10^6)$$

$$= \frac{-1.312\times10^6 + 5.248\times10^6}{4} = 0.984 \times 10^6$$

$\Delta E = 9.84 \times 10^5$ J mol^{-1}

36. **(c) :** Number of electrons in each species are given below

$N_2 = 14$ $CN^- = 14$

$O_2^- = 17$ $C_2^{2-} = 14$

$NO^+ = 14$ $O_2^{2-} = 18$

$CO = 14$ $NO = 15$

It is quite evident from the above that NO^+, C_2^{2-}, CN^-, N_2 and CO are isoelectronic in nature. Hence option (c) is correct.

CHAPTER

4

Solutions

1. Freezing point of an aqueous solution is (–0.186)°C. Elevation of boiling point of the same solution is K_b = 0.512°C, K_f = 1.86°C, find the increase in boiling point.

(a) 0.186°C (b) 0.0512°C

(c) 0.092°C (d) 0.2372°C.

(2002)

2. In mixture *A* and *B* components show –ve deviation as

(a) $\Delta V_{\text{mix}} > 0$

(b) $\Delta H_{\text{mix}} < 0$

(c) $A - B$ interaction is weaker than $A - A$ and $B - B$ interaction

(d) $A - B$ interaction is stronger than $A - A$ and $B - B$ interaction.

(2002)

3. In a 0.2 molal aqueous solution of a weak acid H*X*, the degree of ionization is 0.3. Taking K_f for water as 1.85, the freezing point of the solution will be nearest to

(a) – 0.480°C (b) – 0.360 °C

(c) – 0.260 °C (d) + 0.480 °C.

(2003)

4. 25 ml of a solution of barium hydroxide on titration with a 0.1 molar solution of hydrochloric acid gave a litre value of 35 ml. The molarity of barium hydroxide solution was

(a) 0.07 (b) 0.14

(c) 0.28 (d) 0.35.

(2003)

5. If liquids *A* and *B* form an ideal solution, the

(a) enthalpy of mixing is zero

(b) entropy of mixing is zero

(c) free energy of mixing is zero

(d) free energy as well as the entropy of mixing are each zero.

(2003)

6. Which one of the following aqueous solutions will exhibit highest boiling point?

(a) 0.01 M Na_2SO_4

(b) 0.01 M KNO_3

(c) 0.015 M urea

(d) 0.015 M glucose.

(2004)

7. 6.02×10^{20} molecules of urea are present in 100 ml of its solution. The concentration of urea solution is

(a) 0.001 M (b) 0.01 M

(c) 0.02 M (d) 0.1 M.

(2004)

8. To neutralise completely 20 mL of 0.1 M aqueous solution of phosphorous acid (H_3PO_3), the volume of 0.1 M aqueous KOH solution required is

(a) 10 mL (b) 20 mL

(c) 40 mL (d) 60 mL.

(2004)

9. Which of the following liquid pairs shows a positive deviation from Raoult's law?

(a) Water - hydrochloric acid

(b) Benzene - methanol

(c) Water - nitric acid

(d) Acetone - chloroform.

(2004)

10. Which one of the following statements is false?

(a) Raoult's law states that the vapour pressure of a component over a solution is proportional to its mole fraction.
(b) The osmotic pressure (π) of a solution is given by the equation ($\pi = MRT$, where M is the molarity of the solution.
(c) The correct order of osmotic pressure for 0.01 M aqueous solution of each compound is $BaCl_2 > KCl > CH_3COOH >$ sucrose.
(d) Two sucrose solutions of same molality prepared in different solvents will have the same freezing point depression.

(2004)

11. If α is the degree of dissociation of Na_2SO_4, the vant Hoff's factor (i) used for calculating the molecular mass is
(a) $1 + \alpha$ (b) $1 - \alpha$
(c) $1 + 2\alpha$ (d) $1 - 2\alpha$

(2005)

12. Benzene and toluene form nearly ideal solutions. At 20°C, the vapour pressure of benzene is 75 torr and that of toluene is 22 torr. The partial vapour pressure of benzene at 20°C for a solution containing 78 g of benzene and 46 g of toluene in torr is
(a) 50 (b) 25
(c) 37.5 (d) 53.5

(2005)

13. Two solutions of a substance (non electrolyte) are mixed in the following manner. 480 ml of 1.5 M first solution + 520 mL of 1.2 M second solution. What is the molarity of the final mixture?
(a) 1.20 M (b) 1.50 M
(c) 1.344 M (d) 2.70 M

(2005)

14. Equimolal solutions in the same solvent have
(a) same boiling point but different freezing point
(b) same freezing point but different boiling point
(c) same boiling and same freezing points
(d) different boiling and different freezing points

(2005)

15. Density of a 2.05 M solution of acetic acid in water is 1.02 g/mL. The molality of the solution is
(a) 1.14 mol kg^{-1} (b) 3.28 mol kg^{-1}
(c) 2.28 mol kg^{-1} (d) 0.44 mol kg^{-1}.

(2006)

16. 18 g of glucose ($C_6H_{12}O_6$) is added to 178.2 g of water. The vapour pressure of water for this aqueous solution at 100°C is
(a) 759.00 torr (b) 7.60 torr
(c) 76.00 torr (d) 752.40 torr.

(2006)

17. The density (in g mL^{-1}) of a 3.60 M sulphuric acid solution that is 29% H_2SO_4 (molar mass = 98 g mol^{-1}) by mass will be
(a) 1.45 (b) 1.64
(c) 1.88 (d) 1.22

(2007)

18. A mixture of ethyl alcohol and propyl alcohol has a vapour pressure of 290 mm at 300 K. The vapour pressure of propyl alcohol is 200 nm. If the mole fraction of ethyl alcohol is 0.6, its vapour pressure (in mm) at the same temperature will be
(a) 360 (b) 350
(c) 300 (d) 700.

(2007)

19. A 5.25% solution of a substance is isotonic with a 1.5% solution of urea (molar mass = 60 g mol^{-1}) in the same solvent. If the densities of both the solutions are assumed to be equal to 1.0 g cm^{-3}, molar mass of the substance will be
(a) 210.0 g mol^{-1} (b) 90.0 g mol^{-1}
(c) 115.0 g mol^{-1} (d) 105.0 g mol^{-1}.

(2007)

20. At 80°C, the vapour pressure of pure liquid A is 520 mm of Hg and that of pure liquid B is 1000 mm of Hg. If a mixture solution of A and B boils at 80°C and 1 atm pressure, the amount of A in the mixture is (1 atm = 760 mm of Hg)
(a) 50 mol percent (b) 52 mol percent
(c) 34 mol percent (d) 48 mol percent

(2008)

21. The vapour pressure of water at 20°C is 17.5 mm Hg. If 18 g of glucose ($C_6H_{12}O_6$) is added to 178.2 g of water at 20°C, the vapour pressure of the resulting solution will be

(a) 17.325 mm Hg (b) 17.675 mm Hg

(c) 15.750 mm Hg (d) 16.500 mm Hg

(2008)

22. Gold numbers of protective colloids A, B, C and D are 0.50, 0.01, 0.10 and 0.005, respectively. The correct order of their protective powers is

(a) $B < D < A < C$

(b) $D < A < C < B$

(c) $C < B < D < A$

(d) $A < C < B < D$

(2008)

Answer Key

1.	(b)	**2.**	(b), (d)	**3.**	(a)	**4.**	(b)	**5.**	(a)	**6.**	(a)
7.	(b)	**8.**	(c)	**9.**	(b)	**10.**	(d)	**11.**	(c)	**12.**	(a)
13.	(c)	**14.**	(c)	**15.**	(c)	**16.**	(d)	**17.**	(d)	**18.**	(b)
19.	(a)	**20.**	(a)	**21.**	(a)	**22.**	(d)				

1. **(b) :** $\Delta T_b = K_b \dfrac{W_B}{M_B \times W_A} \times 1000$

$$\Delta T_f = K_f \frac{W_B}{M_B \times W_A} \times 1000$$

$\dfrac{\Delta T_b}{\Delta T_f} = \dfrac{K_b}{K_f}$ or $\dfrac{\Delta T_b}{0.186} = \dfrac{0.512}{1.86}$ or, $\Delta T_b = 0.0512°C$.

2. **(b, d) :** For negative deviation, from Raoult's law, $\Delta V_{mix} < 0$ and $\Delta H_{mix} < 0$. Here $A - B$ attractive force is greater than $A - A$ and $B - B$ attractive forces.

3. **(a) :** $HX \rightleftharpoons H^+ + X^-$

$$\begin{array}{ccc} 1 & 0 & 0 \\ 1 - 0.3 & 0.3 & 0.3 \end{array}$$

Total number of moles after dissociation
$= 1 - 0.3 + 0.3 + 0.3 = 1.3$.

$$\frac{K_f \text{ (observed)}}{K_f \text{ (experimental)}} = \frac{\text{no. of moles after dissociation}}{\text{no. of moles before dissociation}}$$

or, $\dfrac{K_f \text{ (observed)}}{1.85} = \dfrac{1.3}{1}$

or, K_f (observed) $= 1.85 \times 1.3 = 2.405$.
$\Delta T_f = K_f \times$ molality $= 2.405 \times 0.2 = 0.4810$.
Freezing point of solution $= 0 - 0.481 = -0.481°C$.

4. **(b) :** $Ba(OH)_2$ $\quad$ HCl
$M_1V_1 = M_2V_2$
$M_1 \times 25 = 0.1 \times 35$

or, $M_1 = \dfrac{0.1 \times 35}{25} = 0.14$.

5. **(a) :** For ideal solutions, $\Delta H_{mix} = 0$, neither heat is evolved nor absorbed during dissolution.

6. **(a) :** Elevation in boiling point is a colligative property which depends upon the number of solute particles.
Greater the number of solute particles in a solution, higher the extent of elevation in boiling point.

$$Na_2SO_4 \rightarrow 2Na^+ + SO_4^{2-}$$

7. **(b) :** Moles of urea $= \dfrac{6.02 \times 10^{20}}{6.02 \times 10^{23}} = 10^{-3}$ moles

Concentration (molarity) of solution

$= \dfrac{\text{no. of moles of solute}}{\text{no. of litres of solution}} = \dfrac{10^{-3}}{100} \times 1000 = 0.01$ M.

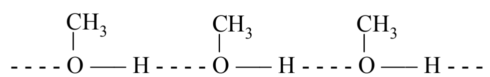
8. **(c) :** H_3PO_3 is a dibasic acid.
N_1V_1 (acid) $= N_2V_2$ (base)
$0.1 \times 2 \times 20 = 0.1 \times 1 \times V_2$

$\therefore\ V_2 = \dfrac{0.1 \times 2 \times 20}{0.1 \times 1} = 40$ mL.

9. **(b) :** In solutions showing positive deviation, the observed vapour pressure of each component and total vapour pressure are greater than predicted by Raoult's law, *i.e.*

$$p_A > p_A^o x_A;\ p_B > p_B^o x_B;\ p > p_A + p_B$$

In solution of methanol and benzene, methanol molecules are held together due to hydrogen bonding as shown below:



On adding benzene, the benzene molecules get in between the molecules of methanol, thus breaking the hydrogen bonds. As the resulting solution has weaker intermolecular attractions, the escaping tendency of alcohol and benzene molecules from the solution increases. Consequently the vapour pressure of the solution is greater than the vapour pressure as expected from Raoult's law.

10. **(d) :** The extent of depression in freezing point varies with the number of solute particles for a fixed solvent only and it's a characteristic feature of the nature of solvent also. $\Delta T_f = k_f \times m$
For different solvents, value of k_f is also different. So, for two different solvents the extent of depression may vary even if number of solute particles be dissolved in them.

11. (c) : $Na_2SO_4 \rightleftharpoons 2Na^+ + SO_4^{2-}$

$$\begin{array}{ccc} 1 & 0 & 0 \\ 1-\alpha & 2\alpha & \alpha \end{array}$$

Vant Hoff factor (i) = $\dfrac{1-\alpha+2\alpha+\alpha}{1} = 1 + 2\alpha.$

12. (a) : According to Raoult's law, $P_B = P^\circ_B X_B$

$P^\circ_B = 75$ torr

$$X_B = \frac{78/78}{(78/78)+(46/92)} = \frac{1}{1+0.5} = \frac{1}{1.5}$$

$$P_B = 75 \times \frac{1}{1.5} = 50 \text{ torr.}$$

13. (c) : Total millimoles of solute
= 480 × 1.5 + 520 × 1.2 = 720 + 624 = 1344.
Total volume = 480 + 520 = 1000.

Molarity of the final mixture = $\dfrac{1344}{1000} = 1.344$ M.

14. (c) : According to Raoult's law equimolal solutions of all the substances in the same solvent will show equal elevation in boiling points as well as equal depression in freezing point.

15. (c) : Molality, $m = \dfrac{M}{1000d - MM_2} \times 1000$

where M = molarity, d = density, M_2 = molecular mass

$$m = \frac{2.05}{1000 \times 1.02 - 2.05 \times 60} = \frac{2.05}{897}$$

$= 2.28 \times 10^{-3}$ mol g^{-1} = 2.28 mol kg^{-1}.

16. (d) : $\dfrac{p^0 - p_s}{p_s} = \dfrac{n}{N}$

$$\frac{760 - p_s}{p_s} = \frac{18/180}{178.2/18} = \frac{1/10}{9.9}$$

$$\Rightarrow \quad 760 - p_s = \frac{1}{99} p_s \quad \Rightarrow \quad 760 \times 99 - 99 p_s = p_s$$

$$\Rightarrow 100 p_s = 760 \times 99 \Rightarrow p_s = \frac{760 \times 99}{100} = 752.4 \text{ torr.}$$

17. (d) : 3.6 M solution means 3.6 mole of H_2SO_4 is present in 1000 ml of solution.

∴ Mass of 3.6 moles of H_2SO_4 = 3.6 × 98 g
= 352.8 g

∴ Mass of H_2SO_4 in 1000 ml of solution
= 352.8 g

Given, 29 g of H_2SO_4 is present in 100 g of solution

∴ 352.8 g of H_2SO_4 is present in

$$\frac{100}{29} \times 352.8 = 1216 \text{ g of solution}$$

Now, density = $\dfrac{\text{Mass}}{\text{Volume}} = \dfrac{1216}{1000} = 1.216$ g/mL
= 1.22 g/mL

18. (b) : According to Raoult's law,

$$P = P_A + P_B = P_A^0 x_A + P_B^0 x_B$$

or $\quad 290 = P_A^0 \times (0.6) + 200 \times (1 - 0.6)$

or $\quad 290 = 0.6 \times P_A^0 + 0.4 \times 200$

or $\quad P_A^0 = 350$ mm.

19. (a) : Isotonic solutions have same osmotic pressure.

$\pi_1 = C_1RT, \quad \pi_2 = C_2RT$

For isotonic solution, $\pi_1 = \pi_2$

∴ $\quad C_1 = C_2$.

or, $\quad \dfrac{1.5/60}{V} = \dfrac{5.25/M}{V}$

[Where M = molecular weight of the substance]

or, $\dfrac{1.5}{60} = \dfrac{5.25}{M} \quad$ or $\quad M = 210.$

20. (a) : We have, P_A^0 = 520 mm Hg

and P_B^0 = 1000 mm Hg

Let mole fraction of A in solution = X_A
and mole fraction of B in solution = X_B
Then, at 1 atm pressure *i.e.* at 760 mm Hg

$P_A^0 X_A + P_B^0 X_B = 760$ mm Hg

$P_A^0 X_A + P_B^0 (1 - X_A) = 760$ mm Hg

⇒ $520 X_A + 1000 - 1000 X_A = 760$ mm Hg

⇒ $X_A = \dfrac{1}{2}$ or 50 mol percent.

21. (a) : In solution containing non-volatile solute, pressure is directly proportional to its mole fraction.

P_{solution} = vapour pressure of its pure component × mole fraction in solution

$\therefore \quad P_{sol} = P^\circ X_{solvent}$

let A be the solute and B the solvent

$$\therefore \quad X_B = \frac{n_B}{n_A + n_B} = \frac{\frac{178.2}{18}}{\frac{18}{180} + \frac{178.2}{18}}$$

$$X_B = \frac{9.9}{9.94} = 0.99$$

Now $P_{solution} = P^\circ X_{solvent} = 17.5 \times 0.99$

$P_{solution} = 17.32$

22. (d) : The different protecting colloids differ in their protecting powers. Zsigmondy introduced a term called Gold number to describe the protective power of different colloids. Smaller the value of gold number greater will be protecting power of the protective colloid. Thus

$$\text{protective power of colloid} \propto \frac{1}{\text{Gold number}}$$

CHAPTER
5

Energetics

1. For the reactions,
$C + O_2 \rightarrow CO_2$; $\Delta H = -393$ J
$2Zn + O_2 \rightarrow 2ZnO$; $\Delta H = -412$ J
(a) carbon can oxidise Zn
(b) oxidation of carbon is not feasible
(c) oxidation of Zn is not feasible
(d) Zn can oxidise carbon.
(2002)

2. If an endothermic reaction is non-spontaneous at freezing point of water and becomes feasible at its boiling point, then
(a) ΔH is –ve, ΔS is +ve
(b) ΔH and ΔS both are +ve
(c) ΔH and ΔS both are –ve
(d) ΔH is +ve, ΔS is –ve
(2002)

3. A heat engine absorbs heat Q_1 at temperature T_1 and heat Q_2 at temperature T_2. Work done by the engine is J ($Q_1 + Q_2$). This data
(a) violates 1st law of thermodynamics
(b) violates 1st law of thermodynamics if Q_1 is –ve
(c) violates 1st law of thermodynamics if Q_2 is –ve
(d) does not violate 1st law of thermodynamics.
(2002)

4. If at 298 K the bond energies of C – H, C – C, C $=$ C and H – H bonds are respectively 414, 347, 615 and 435 kJ mol^{-1}, the value of enthalpy change for the reaction
$H_2C = CH_{2(g)} + H_{2(g)} \rightarrow H_3C - CH_{3(g)}$
at 298 K will be
(a) +250 kJ (b) –250 kJ
(c) +125 kJ (d) –125 kJ.
(2003)

5. The enthalpy change for a reaction does not depend upon the
(a) physical states of reactants and products
(b) use of different reactants for the same product
(c) nature of intermediate reaction steps
(d) difference in initial or final temperatures of involved substances.
(2003)

6. In an irreversible process taking place at constant T and P and in which only pressure-volume work is being done, the change in Gibbs free energy (dG) and change in entropy (dS), satisfy the criteria
(a) $(dS)_{V, E} < 0$, $(dG)_{T, P} < 0$
(b) $(dS)_{V, E} > 0$, $(dG)_{T, P} < 0$
(c) $(dS)_{V, E} = 0$, $(dG)_{T, P} = 0$
(d) $(dS)_{V, E} = 0$, $(dG)_{T, P} > 0$.
(2003)

7. The internal energy change when a system goes from state A to B is 40 kJ/mole. If the system goes from A to B by a reversible path and returns to state A by an irreversible path what would be the net change in internal energy?
(a) 40 kJ (b) > 40 kJ
(c) < 40 kJ (d) zero.
(2003)

8. An ideal gas expands in volume from 1×10^{-3} m^3 to 1×10^{-2} m^3 at 300 K against a constant pressure of 1×10^5 Nm^{-2}. The work done is
(a) – 900 J (b) – 900 kJ

(c) 270 kJ (d) 900 kJ.

(2004)

9. The enthalpies of combustion of carbon and carbon monoxide are –393.5 and –283 kJ mol^{-1} respectively. The enthalpy of formation of carbon monoxide per mole is

(a) 110.5 kJ (b) 676.5 kJ

(c) –676.5 kJ (d) –110.5 kJ.

(2004)

10. For a spontaneous reaction the ΔG, equilibrium constant (K) and E°_{cell} will be respectively

(a) –ve, >1, +ve (b) +ve, >1, –ve

(c) –ve, <1, –ve (d) –ve, >1, –ve

(2005)

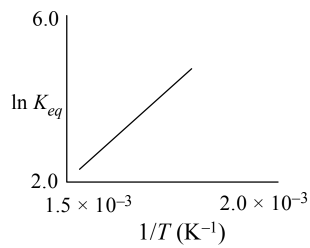
11. Consider the reaction: $N_2 + 3H_2 \rightarrow 2NH_3$ carried out at constant temperature and pressure. If ΔH and ΔU are the enthalpy and internal energy changes for the reaction, which of the following expressions is true?

(a) $\Delta H = 0$ (b) $\Delta H = \Delta U$

(c) $\Delta H < \Delta U$ (d) $\Delta H > \Delta U$

(2005)

12. A schematic plot of ln K_{eq} versus inverse of temperature for a reaction is shown in the figure. The reaction must be



(a) exothermic

(b) endothermic

(c) one with negligible enthalpy change

(d) highly spontaneous at ordinary temperature

(2005)

13. If the bond dissociation energies of XY, X_2 and Y_2 (all diatomic molecules) are in the ratio of 1 : 1 : 0.5 and ΔH_f for the formation of XY is –200 kJ mol^{-1}. The bond dissociation energy of X_2 will be

(a) 100 kJ mol^{-1} (b) 200 kJ mol^{-1}

(c) 800 kJ mol^{-1} (d) 400 kJ mol^{-1}

(2005)

14. The standard enthalpy of formation (ΔH_f°) at 298 K for methane, $CH_{4(g)}$ is –74.8 kJ mol^{-1}. The additional information required to determine the average energy for C – H bond formation would be

(a) the dissociation energy of H_2 and enthalpy of sublimation of carbon

(b) latent heat of vaporisation of methane

(c) the first four ionisation energies of carbon and electron gain enthalpy of hydrogen

(d) the dissociation energy of hydrogen molecule, H_2.

(2006)

15. An ideal gas is allowed to expand both reversibly and irreversibly in an isolated system. If T_i is the initial temperature and T_f is the final temperature, which of the following statements is correct?

(a) $(T_f)_{irrev} > (T_f)_{rev}$

(b) $T_f > T_i$ for reversible process but $T_f = T_i$ for irreversible process

(c) $(T_f)_{rev} = (T_f)_{irrev}$

(d) $T_f = T_i$ for both reversible and irreversible processes.

(2006)

16. The enthalpy changes for the following processes are listed below:

$Cl_{2(g)} = 2Cl_{(g)}$, 242.3 kJ mol^{-1}

$I_{2(g)} = 2I_{(g)}$, 151.0 kJ mol^{-1}

$ICl_{(g)} = I_{(g)} + Cl_{(g)}$, 211.3 kJ mol^{-1}

$I_{2(s)} = I_{2(g)}$, 62.76 kJ mol^{-1}

Given that the standard states for iodine and chlorine are $I_{2(s)}$ and $Cl_{2(g)}$, the standard enthalpy of formation for $ICl_{(g)}$ is

(a) – 14.6 kJ mol^{-1} (b) –16.8 kJ mol^{-1}

(c) +16.8 kJ mol^{-1} (d) +244.8 kJ mol^{-1}.

(2006)

17. ($\Delta H - \Delta U$) for the formation of carbon monoxide (CO) from its elements at 298 K is

(R = 8.314 JK^{-1} mol^{-1})

(a) –1238.78 J mol^{-1} (b) 1238.78 J mol^{-1}

(c) –2477.57 J mol^{-1} (d) 2477.57 J mol^{-1}.

(2006)

18. In conversion of limestone to lime,

$$CaCO_{3(s)} \rightarrow CaO_{(s)} + CO_{2(g)}$$

the values of ΔH° and ΔS° are +179.1 kJ mol^{-1} and 160.2 J/K respectively at 298 K and 1 bar. Assuming that ΔH° and ΔS° do not change with temperature,

temperature above which conversion of limestone to lime will be spontaneous is

(a) 1118 K (b) 1008 K

(c) 1200 K (d) 845 K.

(2007)

19. Assuming that water vapour is an ideal gas, the internal energy change (ΔU) when 1 mol of water is vapourised at 1 bar pressure and 100°C, (given : molar enthalpy of vapourisation of water at 1 bar and 373 K = 41 kJ mol^{-1} and R = 8.3 J mol^{-1} K^{-1}) will be

(a) 41.00 kJ mol^{-1} (b) 4.100 kJ mol^{-1}

(c) 3.7904 kJ mol^{-1}

(d) 37.904 kJ mol^{-1}.

(2007)

20. Identify the correct statement regarding a spontaneous process:

(a) Lowering of energy in the reaction process is the only criterion for spontaneity.

(b) For a spontaneous process in an isolated system, the change in entropy is positive.

(c) Endothermic processes are never spontaneous.

(d) Exothermic processes are always spontaneous.

(2007)

21. Oxidising power of chlorine in aqueous solution can be determined by the parameters indicated below:

$$1/2\ Cl_2(g) \xrightarrow{1/2\ \Delta_{diss}H^{\ominus}} Cl(g) \xrightarrow{\Delta_{eg}H^{\ominus}} Cl^-(g) \xrightarrow{\Delta_{hyd}H^{\ominus}} Cl^-(aq)$$

The energy involved in the conversion of $1/2\ Cl_{2(g)}$ to $Cl^-_{(aq)}$ (using data, $\Delta_{diss}H^{\ominus}_{Cl_2} = 240$ kJ mol^{-1}, $\Delta_{eg}H^{\ominus}_{Cl} = -349$ kJ mol^{-1}, $\Delta_{hyd}H^{\ominus}_{Cl^-} = -381$ kJ mol^{-1}) will be

(a) +120 kJ mol^{-1} (b) +152 kJ mol^{-1}

(c) –610 kJ mol^{-1} (d) –850 kJ mol^{-1}

(2008)

22. Standard entropy of X_2, Y_2 and XY_3 are 60, 40 and 50 J K^{-1} mol^{-1}, respectively. For the reaction, $1/2\ X_2 + 3/2\ Y_2 \rightarrow XY_3$, $\Delta H = -30$ kJ, to be at equilibrium, the temperature will be

(a) 1000 K (b) 1250 K

(c) 500 K (d) 750 K

(2008)

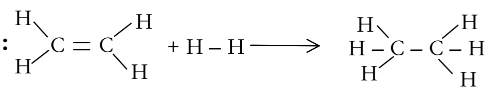
Answer Key

1.	(d)	**2.**	(b)	**3.**	(d)	**4.**	(d)	**5.**	(c)	**6.**	(b)
7.	(d)	**8.**	(a)	**9.**	(d)	**10.**	(a)	**11.**	(c)	**12.**	(a)
13.	(c)	**14.**	(a)	**15.**	(a)	**16.**	(c)	**17.**	(a)	**18.**	(a)
19.	(d)	**20.**	(b)	**21.**	(c)	**22.**	(d)				

1. **(d) :** ΔH = negative shows that the reaction is spontaneous. Higher value for ΔH shows that the reaction is more feasible.

2. **(b) :** For endothermic reaction, ΔH = +ve
Now, $\Delta G = \Delta H - T\Delta S$
For non-spontaneous reaction, ΔG should be positive
Now ΔG is positive at low temperature if ΔH is positive.
ΔG is negative at high temperature if ΔS is positive.

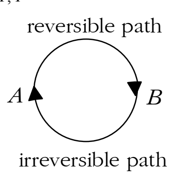
3. **(d) :** It does not violate first law of thermodynamics but violates second law of thermodynamics.

4. **(d) :** 

$\Delta H_{\text{Reaction}} = \Sigma\, BE_{\text{reactant}} - \Sigma\, BE_{\text{product}}$
$= 4 \times 414 + 615 + 435 - (6 \times 414 + 347)$
$= 2706 - 2831$
$= -125$ kJ

5. **(c) :** This is according to Hess's law.

6. **(b) :** For spontaneity, change in entropy (dS) must be positive, means it should be greater than zero. Change in Gibbs free energy (dG) must be negative means that it should be lesser than zero. $(dS)_{V,E} > 0$, $(dG)_{T,P} < 0$.

7. **(d) :** 

We know that for a cyclic process the net change in internal energy is equal to zero and change in the internal energy does not depend on the path by which the final state is reached.

8. **(a) :** $W = -P\Delta V$
$= -1 \times 10^5(1 \times 10^{-2} - 1 \times 10^{-3})$
$= -1 \times 10^5 \times 9 \times 10^{-3} = -900$ J.

9. **(d) :** $C_{(s)} + O_{2(g)} \rightarrow CO_{2(g)}$;
$\Delta H = -393.5$ kJ mol^{-1} ... (i)
$CO_{(g)} + \frac{1}{2}O_{2(g)} \rightarrow CO_{2(g)}$;
$\Delta H = -283$ kJ mol^{-1} ... (ii)
On subtraction equation (ii) from equation (i), we get
$C_{(s)} + O_{2(g)} \rightarrow CO_{(g)}$; $\Delta H = -110.5$ kJ mol^{-1}.
The enthalpy of formation of carbon monoxide per mole
$= -110.5$ kJ mol^{-1}.

10. **(a) :** For spontaneous process, ΔG = –ve
Now $\Delta G = -RT \ln K$
When $K > 1$, ΔG = –ve
Again $\Delta G^\circ = -nFE^\circ$
When E° = +ve, ΔG° = –ve

11. **(c) :** $N_2 + 3H_2 \rightarrow 2NH_3$
$\Delta n = 2 - 4 = -2$
$\Delta H = \Delta U + \Delta nRT = \Delta U - 2RT$
$\therefore\ \Delta H < \Delta U$.

12. **(a) :** $\ln\dfrac{K_2}{K_1} = \dfrac{\Delta H}{R}\left[\dfrac{1}{T_1} - \dfrac{1}{T_2}\right]$

$$\ln\frac{6}{2} = \frac{\Delta H}{R}[1.5\times10^{-3} - 2\times10^{-3}]$$

or, $\ln 3 = \dfrac{\Delta H}{R}\times(-0.5\times10^{-3})$

ΔH of reaction comes out to be negative. Hence reaction is exothermic.

13. **(c) :** Let the bond dissociation energy of XY, X_2 and Y_2 be x kJ mol^{-1}, x kJ mol^{-1} and $0.5x$ kJ mol^{-1} respectively.

$$\frac{1}{2}X_2 + \frac{1}{2}Y_2 \rightarrow XY;\ \Delta H_f = -200 \text{ kJ mol}^{-1}$$

$\Delta H_{\text{reaction}}$ = [(sum of bond dissociation energy of all reactants) – (sum of bond dissociation energy of product)]

$$= \left[\frac{1}{2}\Delta H_{X_2} + \frac{1}{2}\Delta H_{Y_2} - \Delta H_{XY}\right]$$

$$= \frac{x}{2} + \frac{0.5x}{2} - x = -200$$

$$\therefore \quad x = \frac{200}{0.25} = 800 \text{ kJ mol}^{-1}.$$

14. (a) : $C + 2H_2 \rightarrow CH_4$; $\Delta H^\circ = -74.8$ kJ mol^{-1}

In order to calculate average energy for C – H bond formation we should know the following data.

$C_{(graphite)} \rightarrow C_{(g)}$; ΔH_f° = enthalpy of sublimation of carbon

$H_{2(g)} \rightarrow 2H_{(g)}$; ΔH° = bond dissociation energy of H_2

15. (a) : If a gas was to expand by a certain volume reversibly, then it would do a certain amount of work on the surroundings. If it was to expand irreversibly it would have to do the same amount of work on the surroundings to expand in volume, but it would also have to do work against frictional forces. Therefore the amount of work have greater modulus but –ve sign.

$W_{\text{irrev.}} > W_{\text{reve.}}$; $T_{f\,\text{irrev.}} > T_{f\,\text{rev.}}$

16. (c) : $\frac{1}{2} I_{2\,(s)} + \frac{1}{2} Cl_{2\,(g)} \rightarrow ICl_{\,(g)}$

$$\Delta H_{ICl_{(g)}} = \left[\frac{1}{2}\Delta H_{I_2(s)\rightarrow I_2(g)} + \frac{1}{2}\Delta H_{I-I} + \frac{1}{2}\Delta H_{Cl-Cl}\right] - [\Delta H_{I-Cl}]$$

$$= \left[\frac{1}{2}\times 62.76 + \frac{1}{2}\times 151.0 + \frac{1}{2}\times 242.3\right] - [211.3]$$

= [31.38 + 75.5 + 121.15] – 211.3

= 228.03 – 211.3 = 16.73 kJ/mol.

17. (a) : $\Delta H - \Delta U = \Delta n_g RT$

$$C + \frac{1}{2} O_2 \rightarrow CO$$

$$\Delta n_g = 1 - \left(1 + \frac{1}{2}\right) = -\frac{1}{2}.$$

$$\Delta H - \Delta U = -\frac{1}{2}\times 8.314 \times 298 = -1238.78 \text{ J mol}^{-1}.$$

18. (a) : For $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

For a spontaneous process $\Delta G^\circ < 0$

i.e. $\Delta H^\circ - T\Delta S^\circ < 0$

or $\Delta H^\circ < T\Delta S^\circ$ or, $T\Delta S^\circ > \Delta H^\circ$

or $T > \frac{\Delta H^\circ}{\Delta S^\circ}$ *i.e.* $T > \frac{179.1 \times 1000}{160.2}$

or $T > 1117.9$ K $\approx$ 1118 K.

19. (d) : $\Delta U = \Delta H - \Delta nRT$

= 41000 – 1 × 8.314 × 373

= 41000 – 3101.122

= 37898.878 J mol^{-1} = 37.9 kJ mol^{-1}.

20. (b) : In an isolated system, there is neither exchange of energy nor matter between the system and surrounding. For a spontaneous process in an isolated system, the change in entropy is positive, *i.e.* $\Delta S > 0$.

Most of the spontaneous chemical reactions are exothermic. A number of endothermic reactions are spontaneous *e.g.* melting of ice (an endothermic process) is a spontaneous reaction.

The two factors which are responsible for the spontaneity of a process are

(i) tendency to acquire minimum energy

(ii) tendency to acquire maximum randomness.

21. (c) : $\frac{1}{2} Cl_{2(g)} \rightarrow Cl_{(g)}$;

$$\Delta H_1 = \frac{1}{2}\Delta_{diss} H^\ominus_{Cl_2} = \frac{240}{2} = 120 \text{ kJmol}^{-1}$$

$Cl_{(g)} \rightarrow Cl^-_{(g)}$; $\Delta H_2 = \Delta_{eg} H^\ominus_{Cl} = -349$ kJmol^{-1}

$Cl^-_{(g)} + aq \rightarrow Cl^-_{(aq)}$;

$\Delta H_3 = \Delta_{hyd} H^\ominus = -381$ kJmol^{-1}

The required reaction is $\frac{1}{2} Cl_{2(g)} \rightarrow Cl^-_{(aq)}$; ΔH

Then $\Delta H = \frac{1}{2}\Delta_{diss} H^\ominus + \Delta_{eg} H^\ominus + \Delta_{hyd} H^\ominus$

$= 120 + (-349) + (-381) = -610$ kJ mol^{-1}

22. (d) : $\frac{1}{2}X_2 + \frac{3}{2}Y_2 \rightarrow XY_3$

$$\Delta S^0_{\text{reaction}} = \Delta S^0_{\text{products}} - \Delta S^0_{\text{reactants}}$$

$$\therefore\ \Delta S^0_{\text{reaction}} = \Delta S^0_{XY_3} - \frac{1}{2}\Delta S^0_{X_2} - \frac{3}{2}\ \Delta S^0_{Y_2}$$

$$= 50 - \frac{1}{2} \times 60 - \frac{3}{2} \times 40 = -40 \text{ J K}^{-1}\ \text{mol}^{-1}$$

Using equation, $\Delta G = \Delta H - \text{T}\Delta S$

We have $\Delta H = -30$ kJ, $\Delta S = -40$ J K^{-1} mol^{-1}

and at equilibrium $\Delta G = 0$. Therefore

$$T = \frac{\Delta H}{\Delta S} = \frac{-30 \times 1000}{-40} = 750 \text{ K}$$

CHAPTER
6

Equilibrium

1. 1 M NaCl and 1 M HCl are present in an aqueous solution. The solution is
(a) not a buffer solution with pH < 7
(b) not a buffer solution with pH > 7
(c) a buffer solution with pH < 7
(d) a buffer solution with pH > 7.
(2002)

2. Species acting as both Bronsted acid and base is
(a) $(HSO_4)^{-1}$ (b) Na_2CO_3
(c) NH_3 (d) OH^{-1}.
(2002)

3. Let the solubility of an aqueous solution of $Mg(OH)_2$ be x then its K_{sp} is
(a) $4x^3$ (b) $108x^5$
(c) $27x^4$ (d) $9x$.
(2002)

4. For the reaction
$CO_{(g)} + (1/2)\ O_{2(g)} = CO_{2(g)}$, K_p/K_c is
(a) RT (b) $(RT)^{-1}$
(c) $(RT)^{-1/2}$ (d) $(RT)^{1/2}$.
(2002)

5. Change in volume of the system does not alter the number of moles in which of the following equilibria?
(a) $N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$
(b) $PCl_{5(g)} \rightleftharpoons PCl_{3(g)} + Cl_{2(g)}$
(c) $N_{2(g)} + 3H_{2(g)} \rightleftharpoons 2NH_{3(g)}$
(d) $SO_2Cl_{2(g)} \rightleftharpoons SO_{2(g)} + Cl_{2(g)}$.
(2002)

6. In which of the following reactions, increase in the volume at constant temperature does not affect the number of moles at equilibrium ?
(a) $2NH_3 \rightarrow N_2 + 3H_2$
(b) $C_{(g)} + (1/2)\ O_{2(g)} \rightarrow CO_{(g)}$
(c) $H_{2(g)} + O_{2(g)} \rightarrow H_2O_{2(g)}$
(d) none of these.
(2002)

7. Consider the reaction equilibrium:
$2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}$; $\Delta H° = -198$ kJ.
On the basis of Le Chatelier's principle, the condition favourable for the forward reaction is
(a) lowering of temperature as well as pressure
(b) increasing temperature as well as pressure
(c) lowering the temperature and increasing the pressure
(d) any value of temperature and pressure.
(2003)

8. For the reaction equilibrium,
$N_2O_{4(g)} \rightleftharpoons 2NO_{2(g)}$
the concentrations of N_2O_4 and NO_2 at equilibrium are 4.8×10^{-2} and 1.2×10^{-2} mol L^{-1} respectively. The value of K_c for the reaction is
(a) 3.3×10^2 mol L^{-1}
(b) 3×10^{-1} mol L^{-1}
(c) 3×10^{-3} mol L^{-1}
(d) 3×10^3 mol L^{-1}.
(2003)

9. The solubility in water of a sparingly soluble salt AB_2 is 1.0×10^{-5} mol L^{-1}. Its solubility product will be
(a) 4×10^{-15} (b) 4×10^{-10}
(c) 1×10^{-15} (d) 1×10^{-10}.
(2003)

10. The correct relationship between free energy change in a reaction and the corresponding equilibrium constant K_c is

(a) $\Delta G = RT \ln K_c$ (b) $-\Delta G = RT \ln K_c$
(c) $\Delta G^\circ = RT \ln K_c$ (d) $-\Delta G^\circ = RT \ln K_c$.

(2003)

11. Which one of the following statements is not true?

(a) The conjugate base of $H_2PO_4^-$ is HPO_4^{2-}.
(b) pH + pOH = 14 for all aqueous solutions.
(c) The pH of 1×10^{-8} M HCl is 8.
(d) 96,500 coulombs of electricity when passed through a $CuSO_4$ solution deposits 1 gram equivalent of copper at the cathode.

(2003)

12. When rain is accompanied by a thunderstorm, the collected rain water will have a pH value

(a) slightly lower than that of rain water without thunderstorm
(b) slightly higher than that when the thunderstorm is not there
(c) uninfluenced by occurrence of thunderstorm
(d) which depends on the amount of dust in air.

(2003)

13. The conjugate base of $H_2PO_4^-$ is

(a) PO_4^{3-} (b) P_2O_5
(c) H_3PO_4 (d) HPO_4^{2-}.

(2004)

14. What is the equilibrium expression for the reaction $P_{4\,(s)} + 5O_{2\,(g)} \rightleftharpoons P_4O_{10\,(s)}$?

(a) $K_c = \dfrac{[P_4O_{10}]}{[P_4][O_2]^5}$ (b) $K_c = \dfrac{[P_4O_{10}]}{5[P_4][O_2]}$

(c) $K_c = [O_2]^5$ (d) $K_c = \dfrac{1}{[O_2]^5}$.

(2004)

15. For the reaction, $CO_{(g)} + Cl_{2(g)} \rightleftharpoons COCl_{2(g)}$, the K_p/K_c is equal to

(a) $1/RT$ (b) RT
(c) $\sqrt{RT}$ (d) 1.0

(2004)

16. The equilibrium constant for the reaction
$N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$
at temperature T is 4×10^{-4}. The value of K_c for the reaction : $NO_{(g)} \rightleftharpoons \frac{1}{2}N_{2(g)} + \frac{1}{2}O_{2(g)}$ at the same temperature is

(a) 2.5×10^2 (b) 50
(c) 4×10^{-4} (d) 0.02

(2004)

17. The molar solubility (in mol L^{-1}) of a sparingly soluble salt MX_4 is s. The corresponding solubility product is K_{sp}. s is given in terms of K_{sp} by the relation

(a) $s = (K_{sp}/128)^{1/4}$ (b) $s = (128K_{sp})^{1/4}$
(c) $s = (256K_{sp})^{1/5}$ (d) $s = (K_{sp}/256)^{1/5}$.

(2004)

18. Consider an endothermic reaction $X \rightarrow Y$ with the activation energies E_b and E_f for the backward and forward reactions, respectively. In general

(a) $E_b < E_f$
(b) $E_b > E_f$
(c) $E_b = E_f$
(d) there is no definite relation between E_b and E_f

(2005)

19. The solubility product of a salt having general formula MX_2, in water is 4×10^{-12}. The concentration of M^{2+} ions in the aqueous solution of the salt is

(a) 2.0×10^{-6} M (b) 1.0×10^{-4} M
(c) 1.6×10^{-4} M (d) 4.0×10^{-10} M.

(2005)

20. The exothermic formation of ClF_3 is represented by the equation:
$Cl_{2(g)} + 3F_{2(g)} \rightleftharpoons 2ClF_{3(g)}$; $\Delta H = -329$ kJ
Which of the following will increase the quantity of ClF_3 in an equilibrium mixture of Cl_2, F_2 and ClF_3?

(a) Increasing the temperature
(b) Removing Cl_2
(c) Increasing the volume of the container
(d) Adding F_2

(2005)

21. For the reaction
$2NO_{2(g)} \rightleftharpoons 2NO_{(g)} + O_{2(g)}$
($K_c = 1.8 \times 10^{-6}$ at 184°C)
($R = 0.0831$ kJ/(mol.K)). When K_p and K_c are compared at 184°C it is found that

(a) K_p is greater than K_c
(b) K_p is less than K_c
(c) $K_p = K_c$
(d) whether K_p is greater than, less than or equal to K_c depends upon the total gas pressure

(2005)

22. Hydrogen ion concentration in mol/L in a solution of pH = 5.4 will be
(a) 3.98×10^8 (b) 3.88×10^6
(c) 3.68×10^{-6} (d) 3.98×10^{-6}
(2005)

23. What is the conjugate base of OH^-?
(a) O_2 (b) H_2O
(c) O^- (d) O^{2-}
(2005)

24. Among the following acids which has the lowest pK_a value?
(a) CH_3COOH (b) $(CH_3)_2CH - COOH$
(c) $HCOOH$ (d) CH_3CH_2COOH
(2005)

25. An amount of solid NH_4HS is placed in a flask already containing ammonia gas at a certain temperature and 0.50 atm. pressure. Ammonium hydrogen sulphide decomposes to yield NH_3 and H_2S gases in the flask. When the decomposition reaction reaches equilibrium, the total pressure in the flask rises to 0.84 atm. The equilibrium constant for NH_4HS decomposition at this temperature is
(a) 0.30 (b) 0.18
(c) 0.17 (d) 0.11
(2005)

26. Phosphorus pentachloride dissociates as follows in a closed reaction vessel,
$$PCl_{5(g)} \rightleftharpoons PCl_{3(g)} + Cl_{2(g)}$$
If total pressure at equilibrium of the reaction mixture is P and degree of dissociation of PCl_5 is x, the partial pressure of PCl_3 will be
(a) $\left(\frac{x}{x+1}\right)P$ (b) $\left(\frac{2x}{1-x}\right)P$
(c) $\left(\frac{x}{x-1}\right)P$ (d) $\left(\frac{x}{1-x}\right)P$
(2006)

27. The equilibrium constant for the reaction
$$SO_{3(g)} \rightleftharpoons SO_{2(g)} + \frac{1}{2}O_{2(g)}$$
is $K_c = 4.9 \times 10^{-2}$. The value of K_c for the reaction
$2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}$ will be
(a) 416 (b) 2.40×10^{-3}
(c) 9.8×10^{-2} (d) 4.9×10^{-2}.
(2006)

28. Given the data at 25°C,
$Ag + I^- \rightarrow AgI + e^-$; $E° = 0.152$ V
$Ag \rightarrow Ag^+ + e^-$; $E° = -0.800$ V
What is the value of log K_{sp} for AgI?
$$\left(2.303\frac{RT}{F} = 0.059 \text{ V}\right)$$
(a) -8.12 (b) $+8.612$
(c) -37.83 (d) -16.13
(2006)

29. The first and second dissociation constants of an acid H_2A are 1.0×10^{-5} and 5.0×10^{-10} respectively. The overall dissociation constant of the acid will be
(a) 0.2×10^5 (b) 5.0×10^{-5}
(c) 5.0×10^{15} (d) 5.0×10^{-15}.
(2007)

30. The pK_a of a weak acid (HA) is 4.5. The pOH of an aqueous bufferd solution of HA in which 50% of the acid is ionized is
(a) 7.0 (b) 4.5
(c) 2.5 (d) 9.5
(2007)

31. In a saturated solution of the sparingly soluble strong electrolyte $AgIO_3$ (molecular mass = 283) the equilibrium which sets in is
$AgIO_{3(s)} \rightleftharpoons Ag^+_{(aq)} + IO^-_{3\,(aq)}$.
If the solubility product constant K_{sp} of $AgIO_3$ at a given temperature is 1.0×10^{-8}, what is the mass of $AgIO_3$ contained in 100 ml of its saturated solution?
(a) 1.0×10^{-4} g (b) 28.3×10^{-2} g
(c) 2.83×10^{-3} g (d) 1.0×10^{-7} g.
(2007)

32. The equilibrium constants K_{p1} and K_{P2} for the reactions $X \rightleftharpoons 2Y$ and $Z \rightleftharpoons P + Q$, respectively are in the ratio of 1 : 9. If degree of dissociation of X and Z be equal then the ratio of total pressures at these equilibria is
(a) 1 : 9 (b) 1 : 36
(c) 1 : 1 (d) 1 : 3
(2008)

33. For the following three reactions (i), (ii) and (iii), equilibrium constants are given

(i) $CO_{(g)} + H_2O_{(g)} \rightleftharpoons CO_{2(g)} + H_{2(g)}; K_1$
(ii) $CH_{4(g)} + H_2O_{(g)} \rightleftharpoons CO_{(g)} + 3H_{2(g)}; K_2$
(iii) $CH_{4(g)} + 2H_2O_{(g)} \rightleftharpoons CO_{2(g)} + 4H_{2(g)}; K_3$
Which of the following relation is correct ?

(a) $K_3 \cdot K_2^3 = K_1^2$ (b) $K_1\sqrt{K_2} = K_3$
(c) $K_2 K_3 = K_1$ (d) $K_3 = K_1K_2$

(2008)

34. The pK_a of a weak acid, HA, is 4.80. The pK_b of a weak base, BOH is 4.78. The pH of an aqueous solution of the corresponding salt, BA, will be
(a) 9.22 (b) 9.58
(c) 4.79 (d) 7.01

(2008)

35. Four species are listed below :
(i) HCO_3^- (ii) H_3O^+
(iii) HSO_4^- (iv) HSO_3F
Which one of the following is the correct sequence of their acid strength?
(a) iii < i < iv < ii
(b) iv < ii < iii < i
(c) ii < iii < i < iv
(d) i < iii < ii < iv

(2008)

Answer Key

1.	(a)	**2.**	(a)	**3.**	(a)	**4.**	(c)	**5.**	(a)	**6.**	(d)
7.	(c)	**8.**	(c)	**9.**	(c)	**10.**	(d)	**11.**	(c)	**12.**	(a)
13.	(d)	**14.**	(d)	**15.**	(a)	**16.**	(b)	**17.**	(d)	**18.**	(a)
19.	(b)	**20.**	(d)	**21.**	(a)	**22.**	(d)	**23.**	(d)	**24.**	(b)
25.	(d)	**26.**	(a)	**27.**	(a)	**28.**	(d)	**29.**	(d)	**30.**	(d)
31.	(c)	**32.**	(b)	**33.**	(d)	**34.**	(d)	**35.**	(d)		

1. **(a) :** HCl is a strong acid and its salt do not form buffer solution. As the resultant solution is acidic, hence pH is less than 7.

2. **(a) :** According to Bronsted-Lowry concept, a Bronsted acid is a substance which can donate a proton to any other substance and a Bronsted base is a substance which can accept a proton from any other substance.
$(HSO_4)^-$ can accept and donate a proton.
$(HSO_4)^- + H^+ \rightarrow H_2SO_4$
$(HSO_4)^- - H^+ \rightarrow SO_4^{2-}$.

3. **(a) :** $Mg(OH)_2 \rightarrow \underset{x}{[Mg^{2+}]} + \underset{2x}{2[OH^-]}$
$K_{sp} = [Mg^{2+}]\,[OH^-]^2$
or, $K_{sp} = (x) \times (2x)^2 = x \times 4x^2 = 4x^3$.

4. **(c) :** $K_p = K_c(RT)^{\Delta n}$; $\Delta n = 1 - \left(1 + \frac{1}{2}\right) = 1 - \frac{3}{2} = -\frac{1}{2}$.
$\therefore \quad \frac{K_p}{K_c} = (RT)^{-1/2}$.

5. **(a) :** In this reaction the ratio of number of moles of reactants to products is same *i.e.* 2 : 2, hence change in volume will not alter the number of moles.

6. **(d) :** For those reactions, where $\Delta n = 0$, increase in volume at constant temperature does not affect the number of moles at equilibrium.

7. **(c) :** The conversion of SO_2 to SO_3 is an exothermic reaction, hence decrease the temperature will favour the forward reaction. There is also a decrease in volume or moles in product side. Thus the reaction is favoured by low temperature and high pressure. (Le-Chatelier's principle).

8. **(c) :** $[N_2O_4] = 4.8 \times 10^{-2}$ mol L^{-1}
$[NO_2] = 1.2 \times 10^{-2}$ mol L^{-1}
$$K_c = \frac{[NO_2]^2}{[N_2O_4]} = \frac{1.2 \times 10^{-2} \times 1.2 \times 10^{-2}}{4.8 \times 10^{-2}}$$
$= 0.3 \times 10^{-2} = 3 \times 10^{-3}$ mol L^{-1}.

9. **(c) :** $AB_2 \rightleftharpoons A^{2+} + 2B^-$
$S = 1.0 \times 10^{-5}$ mol L^{-1}
$K_{sp} = [A^{2+}]\,[B^-]^2 = 1.0 \times 10^{-5} \times (1.0 \times 10^{-5})^2$
$= 1.0 \times 10^{-15}$.

10. **(d) :** $\Delta G = \Delta G^\circ + 2.303\, RT\log K_c$
At equilibrium, $\Delta G = 0$
$\Delta G^\circ = -2.303\, RT\log K_c$

11. **(c) :** pH of an acid cannot exceed 7. Here we should also consider $[H^+]$ that comes from H_2O.
Now $[H^+] = [H^+]_{\text{from HCl}} + [H^+]_{\text{from } H_2O}$
$= 10^{-8} + 10^{-7} = 10^{-8} + 10 \times 10^{-8} = 11 \times 10^{-8}$
$\therefore \quad pH = -\log(11 \times 10^{-8}) = 6.9587$

12. **(a) :** Due to thunderstorm, temperature increases. As temperature increases, $[H^+]$ also increases, hence pH decreases.

13. **(d) :** Conjugate base is formed by the removal of H^+ from acid.
$H_2PO_4^- \rightarrow HPO_4^{2-} + H^+$

14. **(d) :** $P_{4\,(s)} + 5O_{2\,(g)} \rightleftharpoons P_4O_{10\,(s)}$
$$K_c = \frac{[P_4O_{10\,(s)}]}{[P_{4\,(s)}][O_{2(g)}]^5}$$
We know that concentration of a solid component is always taken as unity.
$$K_c = \frac{1}{[O_2]^5}.$$

15. **(a) :** $CO_{(g)} + Cl_{2(g)} \rightleftharpoons COCl_{2(g)}$
$\Delta n = 1 - 2 = -1$
$K_p = K_c(RT)^{\Delta n}. \quad \therefore \quad \frac{K_p}{K_c} = (RT)^{-1} = \frac{1}{RT}$.

16. **(b) :** $N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$
$$K_c = \frac{[NO]^2}{[N_2]\,[O_2]} = 4 \times 10^{-4}$$
$NO_{(g)} \rightleftharpoons \frac{1}{2}N_{2(g)} + \frac{1}{2}O_{2(g)}$
$$K_c' = \frac{[N_2]^{1/2}\,[O_2]^{1/2}}{[NO]} = \frac{1}{\sqrt{K_c}} = \frac{1}{\sqrt{4 \times 10^{-4}}}$$
$$= \frac{1}{2 \times 10^{-2}} = \frac{100}{2} = 50.$$

17. **(d) :** MX_4 (solid) $\rightleftharpoons \underset{s}{M^{4+}_{(aq)}} + \underset{4s}{4X^-_{(aq)}}$
Solubility product, $K_{sp} = s \times (4s)^4 = 256\, s^5$.
$$\therefore \quad s = \sqrt[5]{\frac{K_{sp}}{256}} = \left(\frac{K_{sp}}{256}\right)^{1/5}.$$

18. (a) : For endothermic reaction, $\Delta H = +ve$
$\Delta H = E_f - E_b$, it means $E_b < E_f$.

19. (b) : $MX_{2(s)} \rightleftharpoons M^{2+}_{(aq)} + 2X^-_{(aq)}$
$\qquad\qquad s \qquad 2s$
$K_{sp} = s \cdot (2s)^2 = 4s^3$
$4 \times 10^{-12} = 4s^3$ or, $s^3 = 1 \times 10^{-12}$
or, $s = 1 \times 10^{-4}$ M $\Rightarrow$ $[M^{2+}] = 1 \times 10^{-4}$ M.

20. (d) : $Cl_{2(g)} + 3F_{2(g)} \rightleftharpoons 2ClF_{3(g)}$; $\Delta H = -329$ kJ.
Favourable conditions:
(i) As the reaction is exothermic, hence decrease in temperature will favour the forward reaction.
(ii) Addition of reactants or removal of product will farour the forward reaction.
(iii) Here $\Delta n = 2 - 4 = -2$ (*i.e.*, –ve) hence decrease in volume or increase in pressure will favour the forward reaction..

21. (a) : $K_p = K_c (RT)^{\Delta n}$
$\Delta n = 3 - 2 = 1$.
$K_p = K_c (0.0831 \times 457)^1$. $\therefore$ $K_p > K_c$.

22. (d) : $pH = -\log[H^+]$
$[H^+]$ = antilog (–pH) = antilog(–5.4) = 3.98×10^{-6}.

23. (d) : Conjugate base of OH^- is O^{2-}.
$OH^- \rightleftharpoons O^{2-} + H^+$

24. (b) : Higher the pK_a value, weaker is the acid. Hence, strongest acid has lowest pK_a value.

25. (d) :

	$NH_4HS_{(s)}$	$\rightleftharpoons NH_{3(g)}$	$+ H_2S_{(g)}$
Initial pressure	0	0.5	0
At equi.	0	0.5 + x	x

Total pressure = 0.5 + 2x = 0.84 $\therefore$ x = 0.17 atm.
$K_p = p_{NH_3} \times p_{H_2S} = (0.5 + 0.17)(0.17) = 0.11$ atm^2.

26. (a) : Given $PCl_{5\,(g)} \rightleftharpoons PCl_{3\,(g)} + Cl_{2\,(g)}$

$t = 0$	1	0	0
t_{eq}	$1 - x$	x	x

Total number of moles = $1 - x + x + x = 1 + x$

Thus partial pressure of $PCl_3 = \left(\frac{x}{1+x}\right)P$.

27. (a) : $SO_{3(g)} \rightleftharpoons SO_{2(g)} + 1/2\ O_{2(g)}$
$$\frac{[SO_2][O_2]^{1/2}}{[SO_3]} = K_c = 4.9 \times 10^{-2} \quad \text{... (i)}$$
$SO_{3(g)} + 1/2\ O_{2(g)} \rightleftharpoons SO_{3(g)}$
$$\frac{[SO_3]}{[SO_2][O_2]^{1/2}} = K_c' = \frac{1}{4.9 \times 10^{-2}} \quad \text{... (ii)}$$
For $2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}$
$$\frac{[SO_3]^2}{[SO_2]^2[O_2]} = K_c'^2 = \frac{1}{4.9 \times 4.9 \times 10^{-4}}$$
$$= \frac{10000}{24.01} = 416.49$$

28. (d) : $AgI_{(s)} + e^- \rightarrow Ag_{(s)} + I^-$, $E° = -0.152$ V
$Ag_{(s)} \rightarrow Ag^+ + e^-$, $E° = -0.800$ V
$AgI_{(s)} \rightarrow Ag^+ + I^-$, $E° = -0.952$ V
$$E°_{cell} = \frac{0.059}{n}\log K \quad i.e. \quad -0.952 = \frac{0.059}{1}\log K_{sp}$$
$$\text{or,}\quad \log K_{sp} = -\frac{0.952}{0.059} = -16.135$$

29. (d) : $H_2A \rightleftharpoons H^+ + HA^-$;
$$K_1 = \frac{[H^+][HA^-]}{[H_2A]} = 1 \times 10^{-5}.$$
$$HA^- \rightleftharpoons H^+ + A^{2-};\quad K_2 = 5 \times 10^{-10} = \frac{[H^+][A^{2-}]}{[HA^-]}$$
$$K = \frac{[H^+]^2[A^{2-}]}{[H_2A]} = K_1 \times K_2 = 1 \times 10^{-5} \times 5 \times 10^{-10}$$
$$= 5 \times 10^{-15}.$$

30. (d) : For acidic buffer, $pH = pK_a + \log\frac{[A^-]}{[HA]}$
When the acid is 50% ionised, $[A^-] = [HA]$
or $pH = pK_a + \log 1$ or $pH = pK_a$
Given pK_a = 4.5 $\therefore$ pH = 4.5
$\therefore$ pOH = 14 – 4.5 = 9.5

31. (c) : $AgIO_3 \rightleftharpoons Ag^+ + IO_3^-$ [S = Solubility]
$\qquad\qquad\qquad S \qquad S$
$K_{sp} = S^2$
or, $S^2 = 1.0 \times 10^{-8}$ or, $S = 1.0 \times 10^{-4}$ mol/lit
$= 1.0 \times 10^{-4} \times 283$ g/lit
$$= \frac{1.0 \times 10^{-4} \times 283}{1000} \text{ gm/ml}$$
$$= \frac{1.0 \times 10^{-4} \times 283 \times 100}{1000} \text{ gm/100ml}$$
$= 28.3 \times 10^{-4}$ gm/100 ml
$= 2.83 \times 10^{-3}$ gm/100 ml.

32. (b) :

	$X \rightleftharpoons$	$2Y$;	$Z \rightleftharpoons$	P	$+ Q$
Initial mol.	1	0	1	0	0
At equilibrium	$1 - a$	$2a$	$1 - a$	a	a

$$K_{P_1} = \frac{P_Y^2}{P_X} = \frac{\left(\frac{2\alpha}{1+\alpha}P_1\right)^2}{\left(\frac{1-\alpha}{1+\alpha}P_1\right)}$$

$$K_{P_2} = \frac{P_P P_Q}{P_Z} = \frac{\left(\frac{\alpha}{1+\alpha} P_2\right)\left(\frac{\alpha}{1+\alpha} P_2\right)}{\left(\frac{1-\alpha}{1+\alpha} P_2\right)}$$

$$\Rightarrow K_{P_1} = \frac{4\alpha^2 P_1}{1-\alpha^2} \quad \text{... }(i)$$

$$\Rightarrow K_{P_1} = \frac{4\alpha^2 P_1}{1-\alpha^2} \quad \text{...}(ii)$$

Given is $\frac{K_{P_1}}{K_{P_2}} = \frac{1}{9}$ ...(*iii*)

Substituting values of from equation (*i*) & (*ii*) into (*iii*), we get

$$\frac{\frac{4\alpha^2 P_1}{1-\alpha^2}}{\frac{\alpha^2 P_2}{1-\alpha^2}} = \frac{1}{9} \Rightarrow \frac{4P_1}{P_2} = \frac{1}{9} \Rightarrow \frac{P_1}{P_2} = \frac{1}{36}$$

33. **(d) :** $CO_{(g)} + H_2O_{(g)}$ $CO_{2(g)} + H_{2(g)}$

$$K_1 = \frac{[CO_2][H_2]}{[CO][H_2O]} \quad \text{.... (i)}$$

$CH_{4(g)} + H_2O_{(g)}$ $CO_{(g)} + 3H_{2(g)}$

$$K_2 = \frac{[CO][H_2]^3}{[CH_4][H_2O]} \quad \text{.... (ii)}$$

$CH_{4(g)} + 2H_2O_{(g)}$ $CO_{2(g)} + 4H_{2(g)}$

$$K_3 = \frac{[CO_2][H_2]^4}{[CH_4][H_2O]^2} \quad \text{.... (iv)}$$

From equations (i), (ii) and (iii) ; $K_3 = K_1 \times K_2$.

34. **(d) :** Given that pK_a = 4.8 and pK_b = 4.78

$\therefore$ pH = 7 + 1/2 (pK_a – pK_b)

= 7 + 1/2 (4.80 – 4.78) = 7.01

35. **(d) :** HSO_3F is the super acid. Its acidic strength is greater than any given species. The pK_a value of other species are given below :

HCO_3^- $\rightarrow$ 10.25

H_3O^+ $\rightarrow$ –1.74

HSO_4^- $\rightarrow$ 1.92

Lesser the pK_a value, higher will be its acidic strength. Hence sequence of acidic strength will be

$HSO_3F > H_3O^+ > HSO_4^- > HCO_3^-$

CHAPTER 7

Redox Reactions & Electrochemistry

1. EMF of a cell in terms of reduction potential of its left and right electrodes is

(a) $E = E_{left} - E_{right}$ (b) $E = E_{left} + E_{right}$

(c) $E = E_{right} - E_{left}$ (d) $E = -(E_{right} + E_{left})$.

(2002)

2. When $KMnO_4$ acts as an oxidising agent and ultimately forms $[MnO_4]^{-1}$, MnO_2, Mn_2O_3, Mn^{2+} then the number of electrons transferred in each case respectively is

(a) 4, 3, 1, 5 (b) 1, 5, 3, 7

(c) 1, 3, 4, 5 (d) 3, 5, 7, 1.

(2002)

3. Which of the following is a redox reaction?

(a) $NaCl + KNO_3 \rightarrow NaNO_3 + KCl$

(b) $CaC_2O_4 + 2HCl \rightarrow CaCl_2 + H_2C_2O_4$

(c) $Mg(OH)_2 + 2NH_4Cl \rightarrow MgCl_2 + 2NH_4OH$

(d) $Zn + 2AgCN \rightarrow 2Ag + Zn(CN)_2$.

(2002)

4. Conductivity (unit Siemen's S) is directly proportional to area of the vessel and the concentration of the solution in it and is inversely proportional to the length of the vessel then the unit of the constant of proportionality is

(a) $Sm\ mol^{-1}$ (b) $Sm^2\ mol^{-1}$

(c) $S^{-2}m^2\ mol$ (d) $S^2m^2\ mol^{-2}$.

(2002)

5. If ϕ denotes reduction potential, then which is true?

(a) $E°_{cell} = \phi_{right} - \phi_{left}$

(b) $E°_{cell} = \phi_{left} + \phi_{right}$

(c) $E°_{cell} = \phi_{left} - \phi_{right}$

(d) $E°_{cell} = -(\phi_{left} + \phi_{right})$.

(2002)

6. What will be the emf for the given cell

$Pt\,|\,H_2\,(P_1)\,|\,H^+_{(aq)}\,||\,H_2\,(P_2)\,|\,Pt$

(a) $\frac{RT}{F}\log\frac{P_1}{P_2}$ (b) $\frac{RT}{2F}\log\frac{P_1}{P_2}$

(c) $\frac{RT}{F}\log\frac{P_2}{P_1}$ (d) none of these.

(2002)

7. Which of the following reaction is possible at anode?

(a) $2\,Cr^{3+} + 7H_2O \rightarrow Cr_2O_7^{2-} + 14H^+$

(b) $F_2 \rightarrow 2F^-$

(c) $(1/2)\,O_2 + 2H^+ \rightarrow H_2O$

(d) none of these.

(2002)

8. The heat required to raise the temperature of body by 1ºC is called

(a) specific heat (b) thermal capacity

(c) water equivalent (d) none of these.

(2002)

9. When during electrolysis of a solution of $AgNO_3$, 9650 coulombs of charge pass through the electroplating bath, the mass of silver deposited on the cathode will be

(a) 1.08 g (b) 10.8 g

(c) 21.6 g (d) 108 g.

(2003)

10. For the redox reaction:

$Zn_{(s)} + Cu^{2+}\,(0.1\ M) \rightarrow Zn^+\,(1M) + Cu_{(s)}$

taking place in a cell, $E°_{cell}$ is 1.10 volt. E_{cell} for the cell will be $\left(2.303\frac{RT}{F} = 0.0591\right)$

(a) 2.14 V (b) 1.80 V

(c) 1.07 V (d) 0.82 V.

(2003)

11. For a cell reaction involving a two-electron change, the standard e.m.f. of the cell is found to be 0.295 V at 25°C. The equilibrium constant of the reaction at 25°C will be

(a) 1×10^{-10} (b) 29.5×10^{-2}
(c) 10 (d) 1×10^{10}.

(2003)

12. Standard reduction electrode potentials of three metals *A*, *B* and *C* are +0.5 V, –3.0 V and –1.2 V respectively. The reducing power of these metals are

(a) $B > C > A$ (b) $A > B > C$
(c) $C > B > A$ (d) $A > C > B$.

(2003)

13. Consider the following $E°$ values.

$E°_{Fe^{3+}/Fe^{2+}} = +0.77\text{ V}$; $E°_{Sn^{2+}/Sn} = -0.14\text{ V}$

Under standard conditions the potential for the reaction

$Sn_{(s)} + 2Fe^{3+}_{(aq)} \rightarrow 2Fe^{2+}_{(aq)} + Sn^{2+}_{(aq)}$ is

(a) 1.68 V (b) 1.40 V
(c) 0.91 V (d) 0.63 V.

(2004)

14. The standard e.m.f. of a cell, involving one electron change is found to be 0.591 V at 25°C. The equilibrium constant of the reaction is (F = 96,500 C mol^{-1}, R = 8.314 JK^{-1}mol^{-1})

(a) 1.0×10^{1} (b) 1.0×10^{5}
(c) 1.0×10^{10} (d) 1.0×10^{30}.

(2004)

15. The limiting molar conductivities $\Lambda°$ for NaCl, KBr and KCl are 126, 152 and 150 S cm^2 mol^{-1} respectively. The $\Lambda°$ for NaBr is

(a) 128 S cm^2 mol^{-1}
(b) 176 S cm^2 mol^{-1}
(c) 278 S cm^2 mol^{-1}
(d) 302 S cm^2 mol^{-1}.

(2004)

16. In a cell that utilizes the reaction

$Zn_{(s)} + 2H^{+}_{(aq)} \rightarrow Zn^{2+}_{(aq)} + H_{2(g)}$

addition of H_2SO_4 to cathode compartment, will

(a) lower the *E* and shift equilibrium to the left
(b) lower the *E* and shift the equilibrium to the right
(c) increase the *E* and shift the equilibrium to the right
(d) increase the *E* and shift the equilibrium to the left.

(2004)

17. The $E°_{M^{3+}/M^{2+}}$ values for Cr, Mn, Fe and Co are – 0.41, +1.57, 0.77 and +1.97 V respectively. For which one of these metals the change in oxidation state from +2 to +3 is easiest?

(a) Cr (b) Mn
(c) Fe (d) Co.

(2004)

18. The highest electrical conductivity of the following aqueous solutions is of

(a) 0.1 M acetic acid
(b) 0.1 M chloroacetic acid
(c) 0.1 M fluoroacetic acid
(d) 0.1 M difluoroacetic acid

(2005)

19. Aluminium oxide may be electrolysed at 1000°C to furnish aluminium metal (At. Mass = 27 amu; 1 Faraday = 96,500 Coulombs). The cathode reaction is

$$Al^{3+} + 3e^{-} \rightarrow Al^{0}$$

To prepare 5.12 kg of aluminium metal by this method would require

(a) 5.49×10^{7} C of electricity
(b) 1.83×10^{7} C of electricity
(c) 5.49×10^{4} C of electricity
(d) 5.49×10^{10} C of electricity

(2005)

20.

Electrolyte	KCl	KNO_3	HCl	NaOAc	NaCl
(S cm^2 mol^{-1})	149.9	145.0	426.2	91.0	126.5

Calculate molar conductance of acetic acid using appropriate molar conductances of the electrolytes listed above at infinite dilution in H_2O at 25°C.

(a) 517.2 (b) 552.7
(c) 390.7 (d) 217.5

(2005)

21. Which of the following chemical reactions depicts the oxidising behaviour of H_2SO_4?

(a) $2HI + H_2SO_4 \rightarrow I_2 + SO_2 + 2H_2O$
(b) $Ca(OH)_2 + H_2SO_4 \rightarrow CaSO_4 + 2H_2O$
(c) $NaCl + H_2SO_4 \rightarrow NaHSO_4 + HCl$
(d) $2PCl_5 + H_2SO_4 \rightarrow 2POCl_3 + 2HCl + SO_2Cl_2$.

(2006)

22. The molar conductivities $\Lambda°_{NaOAc}$ and $\Lambda°_{HCl}$ at infinite dilution in water at 25°C are 91.0 and 426.2 S cm^2/mol respectively. To calculate $\Lambda°_{HOAc}$, the additional value required is

(a) $\Lambda^\circ_{H_2O}$ (b) Λ°_{KCl}

(c) Λ°_{NaOH} (d) Λ°_{NaCl}.

(2006)

23. Resistance of a conductivity cell filled with a solution of an electrolyte of concentration 0.1 M is 100 Ω. The conductivity of this solution is 1.29 Sm^{-1}. Resistance of the same cell when filled with 0.2 M of the same solution is 520 Ω. The molar conductivity of 0.02 M solution of the electrolyte will be

(a) 124×10^{-4} S m^2 mol^{-1}

(b) 1240×10^{-4} S m^2 mol^{-1}

(c) 1.24×10^{-4} S m^2 mol^{-1}

(d) 12.4×10^{-4} S m^2 mol^{-1}.

(2006)

24. The equivalent conductances of two strong electrolytes at infinite dilution in H_2O (where ions move freely through a solution) at 25°C are given below:

$\Lambda^\circ_{CH_3COONa} = 91.0$ S cm^2/equiv.

$\Lambda^\circ_{HCl} = 426.2$ Scm^2/equiv.

What additional information/quantity one needs to calculate Λ° of an aqueous solution of acetic acid?

(a) Λ° of chloroacetic acid ($ClCH_2COOH$)

(b) Λ° of NaCl

(c) Λ° of CH_3COOK

(d) the limiting equivalent conductance of H^+ ($\lambda^\circ_{H^+}$).

(2007)

25. The cell,

$Zn \mid Zn^{2+}$ (1 M) $\|$ Cu^{2+} (1 M) $\mid$ Cu ($E^\circ_{cell} = 1.10$ V)

was allowed to be completely discharged at 298 K. The relative concentration of Zn^{2+} to Cu^{2+}

$\left(\frac{[Zn^{2+}]}{[Cu^{2+}]}\right)$ is

(a) 9.65×10^4 (b) antilog(24.08)

(c) 37.3 (d) $10^{37.3}$.

(2007)

26. Given $E^\circ_{Cr^{3+}/Cr} = -0.72$ V, $E^\circ_{Fe^{2+}/Fe} = -0.42$ V. The potential for the cell

$Cr \mid Cr^{3+}(0.1\ M) \mid\mid Fe^{2+}(0.01\ M) \mid Fe$ is

(a) −0.26 V (b) 0.26 V

(c) 0.339 V (d) −0.339 V

(2008)

27. Amount of oxalic acid present in a solution can be determined by its titration with $KMnO_4$ solution in the presence of H_2SO_4. The titration gives unsatisfactory result when carried out in the presence of HCl, because HCl

(a) oxidises oxalic acid to carbon dioxide and water

(b) gets oxidised by oxalic acid to chlorine

(c) furnishes H^+ ions in addition to those from oxalic acid

(d) reduces permanganate to Mn^{2+}

(2008)

Answer Key

1.	(c)	**2.**	(c)	**3.**	(d)	**4.**	(b)	**5.**	(a)	**6.**	(b)
7.	(a)	**8.**	(b)	**9.**	(b)	**10.**	(c)	**11.**	(d)	**12.**	(a)
13.	(c)	**14.**	(c)	**15.**	(a)	**16.**	(c)	**17.**	(a)	**18.**	(d)
19.	(a)	**20.**	(c)	**21.**	(a)	**22.**	(d)	**23.**	(a)	**24.**	(b)
25.	(d)	**26.**	(b)	**27.**	(d)						

1. **(c) :** E_{cell} = Reduction potential of cathode (right) – reduction potential of anode (left)
$= E_{right} - E_{left}$.

2. **(c) :**
$$\overset{+3}{Mn_2O_3} \xleftarrow{-4e^-} \overset{+7}{[KMnO_4]} \xrightarrow{-e^-} [MnO_4]^{-1}$$
$$[KMnO_4] \xrightarrow{-5e^-} Mn^{2+} \qquad [KMnO_4] \xrightarrow{-3e^-} \overset{+4}{MnO_2}$$

3. **(d) :** The oxidation states show a change only in reaction (d).
$$\overset{0}{Zn} + 2\overset{+1}{Ag}CN \longrightarrow 2\overset{0}{Ag} + \overset{+2}{Zn}(CN)_2$$
$$\text{Zn: } -2e^- \qquad \text{Ag: } +2e^-$$

4. **(b) :** $S \propto A$ (A = area)
$S \propto C$ (C = concentration)
$S \propto \frac{1}{L}$ (L = length)
Combining we get, $S \propto \frac{AC}{L}$
or $\quad S = K\frac{AC}{L}$ [K = constant of proportionality]
or $K = \frac{SL}{AC}$
$$\therefore \quad \text{Unit of } K = \frac{\text{S} \times \text{m}}{\text{m}^2 \times \frac{\text{mol}}{\text{m}^3}} = \frac{\text{S} \times \text{m} \times \text{m}^3}{\text{m}^2 \times \text{mol}}$$
$$= \text{Sm}^2\ \text{mol}^{-1}$$

5. **(a) :** $E_{cell} = E_{right\ (cathode)} - E_{left\ (anode)}$.

6. **(b) :** $2H^+ + 2e^- \longrightarrow H_2(P_2)$
$H_2(P_1) \longrightarrow 2H^+ + 2e^-$
Overall reaction : $H_2(P_1) \longrightarrow H_2(P_2)$
$$E = E° - \frac{RT}{nF}\log\frac{P_2}{P_1} = 0 - \frac{RT}{nF} og\frac{P_2}{P_1} = \frac{RT}{nF}\log\frac{P_1}{P_2}$$

7. **(a) :** Here Cr^{3+} is oxidised to $Cr_2O_7^{2-}$.

8. **(b) :** It is also known as heat capacity.

9. **(b) :** The mass of silver deposited on the cathode
$$= \frac{108 \times 9650}{96500} = 10.8 \text{ g.}$$

10. **(c) :** $E_{cell} = E°_{cell} - \frac{0.0591}{n}\log\frac{1}{0.1}$
Here $n = 2$, $E°_{cell} = 1.10$ V
$$E_{cell} = 1.10 - \frac{0.0591}{2}\log 10$$
$E_{cell} = 1.10 - 0.0295 = 1.0705$ V.

11. **(d) :** $E°_{cell} = \frac{0.0591}{n}\log K_c$
$$0.295 = \frac{0.0591}{2}\log K_c$$
or, $\quad 0.295 = 0.0295 \log K_c$
or, $\quad K_c$ = antilog 10 $\quad$ or $\quad K_c = 1 \times 10^{10}$.

12. **(a) :**

	A	B	C
$E°_{red}$	+0.5 V	–3.0 V	–1.2 V

More is the value of reduction potential, more is the tendency to get reduced, *i.e.* less is the reducing power.
The reducing power follows the following order:
$$B > C > A.$$

13. **(c) :** $E°_{cell} = E°_{Sn/Sn^{2+}} + E°_{Fe^{3+}/Fe^{2+}}$
$= 0.14 + 0.77 = 0.91$ V.

14. **(c) :** $E_{cell} = E°_{cell} - \frac{0.0591}{n}\log K_c$
$$0 = 0.591 - \frac{0.0591}{1}\log K_c$$
$\Rightarrow \quad -0.591 = -0.0591 \log K_c$
$$\Rightarrow \quad \log K_c = \frac{0.591}{0.0591} = 10$$
$\therefore \quad K_c$ = antilog 10 = 1×10^{10}.

15. **(a) :** $\Lambda°_{NaCl} = \Lambda°_{Na^+} + \Lambda°_{Cl^-}$ ... (i)
$\Lambda°_{KBr} = \Lambda°_{K^+} + \Lambda°_{Br^-}$ ... (ii)
$\Lambda°_{KCl} = \Lambda°_{K^+} + \Lambda°_{Cl^-}$ ... (iii)
Equation (i) + (ii) – (iii)
$\Lambda°_{NaBr} = \Lambda°_{Na^+} + \Lambda°_{Br^-}$
$= 126 + 152 - 150 = 128$ S cm^2 mol^{-1}.

16. **(c) :** $Zn_{(s)} + 2H^+_{(aq)} \rightleftharpoons Zn^{2+}_{(aq)} + H_{2\,(g)}$
$$E_{cell} = E°_{cell} - \frac{0.059}{2}\log\frac{[Zn^{2+}] \times p_{H_2}}{[H^+]^2}$$
On adding H_2SO_4 the $[H^+]$ will increase therefore

E_{cell} will also increase and the equilibrium will shift towards the right.

17. (a) : $Cr^{2+} \mid Cr^{3+} = +0.41$ V
$Mn^{2+} \mid Mn^{3+} = -1.57$ V
$Fe^{2+} \mid Fe^{3+} = -0.77$ V
$Co^{2+} \mid Co^{3+} = -1.97$ V
More is the value of oxidation potential more is the tendency to get oxidised.
As Cr will have maximum oxidation potential value, therefore its oxidation will be easiest.

18. (d) : Higher the acidity, higher will be the tendency to release protons and hence lighter will be the electrical conductivity. Difluoroacetic acid will be strongest acid due to electron withdrawing effect of two fluorine atoms so as it will show maximum electrical conductivity.

19. (a) : From Faraday's 1st law,
$W = Z \times Q$ [W = weight, Z = electrochemical equivalent, Q = quantity of electricity]
Now $E = Z \times F$
[E = equivalent weight , F = faraday]

or $\quad W = \dfrac{E}{F} \times Q$

or $\quad Q = \dfrac{W \times F}{E}$

or $\quad Q = \dfrac{W \times F}{\frac{A}{n}}$

[A = Atomic weight , n = valency of ion]

or $\quad Q = \dfrac{n \times w \times f}{A}$

$= \dfrac{3 \times 5.12 \times 10^3 \times 96500}{27} = 5.49 \times 10^7$C

20. (c) : $\Lambda^{\infty}_{AcOH} = \Lambda^{\infty}_{AcONa} + \Lambda^{\infty}_{HCl} - \Lambda^{\infty}_{NaCl}$
$= 91.0 + 426.2 - 126.5$
$= 390.7$ S cm^2 mol^{-1}.

21. (a) : In the reaction,

$$\underset{+1\,-1}{2HI} + \underset{+1\,+6\,-2}{H_2SO_4} \rightarrow \underset{0}{I_2} + \underset{+4}{SO_2} + \underset{+1\,-2}{2H_2O}$$

Ist half reaction : $\underset{-1}{2HI} \rightarrow \underset{0}{I_2}$

In this reaction oxidation number of I increases by one, thus this is an oxidation reaction and HI behaves as a reducing agent.

IInd half reaction : $\underset{+6}{H_2SO_4} \rightarrow \underset{+4}{SO_2}$

In this reaction oxidation number of S decreases by two, thus this is a reduction reaction and H_2SO_4 behaves as oxidising agent.

22. (d) : $CH_3COONa + HCl \rightarrow CH_3COOH + NaCl$
From the reaction,

$$\Lambda^\circ_{CH_3COONa} + \Lambda^\circ_{HCl} = \Lambda^\circ_{CH_3COOH} + \Lambda^\circ_{NaCl}$$

or, $\quad \Lambda^\circ_{CH_3COOH} = \Lambda^\circ_{CH_3COONa} + \Lambda^\circ_{HCl} - \Lambda^\circ_{NaCl}$

Thus to calculate the value of $\Lambda^\circ_{CH_3COOH}$ one should know the value of Λ°_{NaCl} along with $\Lambda^\circ_{CH_3COONa}$ and Λ°_{HCl}.

23. (a) : $\kappa = \dfrac{1}{R}\left(\dfrac{l}{a}\right)$ *i.e.*, $1.29 = \dfrac{1}{100}\left(\dfrac{l}{a}\right)$

$l/a = 129$ m^{-1}.
$R = 520\ \Omega$ for 0.2 M, $C = 0.02$ M

$$\lambda_m = \kappa \times \frac{1000}{\text{molarity}} = \frac{1 \times 129}{520} \times \frac{1000}{0.02} \times 10^{-6}\ \text{m}^3$$

$= 124 \times 10^{-4}$ S m^2 mol^{-1}.

24. (b) : According to Kohlrausch's law, the molar conductivity at infinite dilution (Λ°) for weak electrolyte, CH_3COOH is

$$\Lambda^\circ_{CH_3COOH} = \Lambda^\circ_{CH_3COONa} + \Lambda^\circ_{HCl} - \Lambda^\circ_{NaCl}$$

So, for calculating the value of $\Lambda^\circ_{CH_3COOH}$, value of Λ°_{NaCl} should also be known.

25. (d) : $Zn + Cu^{2+} \longrightarrow Zn^{2+} + Cu$

$$E_{cell} = E^\circ_{cell} - \frac{0.059}{2} \log \frac{[Zn^{2+}]}{[Cu^{2+}]}$$

When the cell is completely discharged, $E_{cell} = 0$

$$0 = 1.1 - \frac{0.059}{2} \log \frac{[Zn^{2+}]}{[Cu^{2+}]}$$

or $\quad \log \dfrac{[Zn^{2+}]}{[Cu^{2+}]} = \dfrac{2 \times 1.1}{0.059}$ or, $\log \dfrac{Zn^{2+}}{Cu^{2+}} = 37.3$

or $\quad \dfrac{Zn^{2+}}{Cu^{2+}} = 10^{37.3}$.

26. (b) : $Cr \rightarrow Cr^{3+} + 3e^-$ $\quad E^\circ_{red} = -0.72$ V
$Fe^{2+} + 2e^- \rightarrow Fe$ $\quad E^\circ_{red} = -0.42$ V

$2Cr + 3Fe^{2+} \rightarrow 2Cr^{3+} + 3Fe$
$E^\circ_{cell} = E^\circ_{cathode} - E^\circ_{anode} = -0.42 - (-0.72)$
$E^\circ_{cell} = 0.3$

According to Nernst equation

$$E_{\text{cell}} = E^{\circ}_{\text{cell}} - \frac{0.059}{n_{\text{cell}}}\log_{10}\frac{[Cr^{3+}]^2}{[Fe^{2+}]^3}$$

$$E_{\text{cell}} = 0.3 - \frac{0.059}{6}\log_{10}\frac{(0.1)^2}{(0.01)^3}$$

$$E_{\text{cell}} = 0.3 - \frac{0.059}{6}\log_{10}10^4$$

$E_{\text{cell}} = 0.3 - 0.039$

$\therefore \quad E_{\text{cell}} = 0.261$ V

27. (d) : Oxalic acid present in a solution can be determined by its titration with $KMnO_4$ solution in the presence of H_2SO_4.

$$2KMnO_4 + 3H_2SO_4 + 5\begin{array}{c}COOH\\ |\\ COOH\end{array} \longrightarrow$$

$$K_2SO_4 + 2MnSO_4 + 10CO_2 + 5H_2O$$

Titration cannot be done in the presence of HCl because $KMnO_4$ being a strong oxidizing agent oxidises HCl to Cl_2 and get itself reduced to Mn^{2+}. So actual amount of oxalic acid in solution cannot be determined.

CHAPTER

8

Kinetics

1. Units of rate constant of first and zero order reactions in terms of molarity M unit are respectively
(a) sec^{-1}, $Msec^{-1}$ (b) sec^{-1}, M
(c) $Msec^{-1}$, sec^{-1} (d) M, sec^{-1}.
(2002)

2. For the reaction $A + 2B \rightarrow C$, rate is given by $R = [A]\,[B]^2$ then the order of the reaction is
(a) 3 (b) 6
(c) 5 (d) 7.
(2002)

3. The differential rate law for the reaction
$H_2 + I_2 \rightarrow 2HI$ is
(a) $-\frac{d[H_2]}{dt} = -\frac{d[I_2]}{dt} = -\frac{d[HI]}{dt}$
(b) $\frac{d[H_2]}{dt} = \frac{d[I_2]}{dt} = \frac{1}{2}\frac{d[HI]}{dt}$
(c) $\frac{1}{2}\frac{d[H_2]}{dt} = \frac{1}{2}\frac{d[I_2]}{dt} = -\frac{d[HI]}{dt}$
(d) $-2\frac{d[H_2]}{dt} = -2\frac{d[I_2]}{dt} = \frac{d[HI]}{dt}$.
(2002)

4. The formation of gas at the surface of tungsten due to adsorption is the reaction of order
(a) 0 (b) 1
(c) 2 (d) insufficient data.
(2002)

5. The rate law for a reaction between the substances A and B is given by rate = $k\,[A]^n\,[B]^m$. On doubling the concentration of A and halving the concentration of B, the ratio of the new rate to the earlier rate of the reaction will be as
(a) $\frac{1}{2^{m+n}}$ (b) $(m + n)$
(c) $(n - m)$ (d) $2^{(n-m)}$.
(2003)

6. For the reaction system:
$$2NO_{(g)} + O_{2(g)} \rightarrow 2NO_{2(g)},$$
volume is suddenly reduced to half its value by increasing the pressure on it. If the reaction is of first order with respect to O_2 and second order with respect to NO_2, the rate of reaction will
(a) diminish to one-fourth of its initial value
(b) diminish to one-eighth of its initial value
(c) increase to eight times of its initial value
(d) increase to four times of its initial value.
(2003)

7. In the respect of the equation $k = Ae^{-E_a/RT}$ in chemical kinetics, which one of the following statements is correct?
(a) k is equilibrium constant
(b) A is adsorption factor
(c) E_a is energy of activation
(d) R is Rydberg constant.
(2003)

8. In a first order reaction, the concentration of the reactant, decreases from 0.8 M to 0.4 M in 15 minutes. The time taken for the concentration to change from 0.1 M to 0.025 M is
(a) 30 minutes (b) 15 minutes
(c) 7.5 minutes (d) 60 minutes.
(2004)

9. The rate equation for the reaction $2A + B \rightarrow C$ is found to be : rate = $k[A]\,[B]$. The correct statement in relation to this reaction is that the
(a) unit of k must be s^{-1}
(b) $t_{1/2}$ is a constant

(c) rate of formation of C is twice the rate of disappearance of A

(d) value of k is independent of the initial concentrations of A and B.

(2004)

10. A reaction involving two different reactants can never be

(a) unimolecular reaction

(b) first order reaction

(c) second order reaction

(d) bimolecular reaction

(2005)

11. $t_{1/4}$ can be taken as the time taken for the concentration of a reactant to drop to 3/4 of its initial value. If the rate constant for a first order reaction is k, the $t_{1/4}$ can be written as

(a) $0.10/k$ (b) $0.29/k$

(c) $0.69/k$ (d) $0.75/k$

(2005)

12. A reaction was found to be second order with respect to the concentration of carbon monoxide. If the concentration of carbon monoxide is doubled, with everything else kept the same, the rate of reaction will be

(a) remain unchanged

(b) tripled

(c) increased by a factor of 4

(d) doubled.

(2006)

13. Rate of a reaction can be expressed by Arrhenius equation as : $k = Ae^{-E/RT}$. In this equation, E represents

(a) the energy above which all the colliding molecules will react

(b) the energy below which colliding molecules will not react

(c) the total energy of the reacting molecules at a temperature, T

(d) the fraction of molecules with energy greater than the activation energy of the reaction.

(2006)

14. The following mechanism has been proposed for the reaction of NO with Br_2 to form NOBr.

$NO_{(g)} + Br_{2(g)} \rightleftharpoons NOBr_{2(g)}$

$NOBr_{2(g)} + NO_{(g)} \rightarrow 2NOBr_{(g)}$

If the second step is the rate determining step, the order of the reaction with respect to $NO_{(g)}$ is

(a) 1 (b) 0

(c) 3 (d) 2.

(2006)

15. The energies of activation for forward and reverse reactions for $A_2 + B_2 \rightleftharpoons 2AB$ are 180 kJ mol^{-1} and 200 kJ mol^{-1} respectively. The presence of a catalyst lowers the activation energy of both (forward and reverse) reactions by 100 kJ mol^{-1}. The enthalpy change of the reaction ($A_2 + B_2 \rightarrow 2AB$) in the presence of a catalyst will be (in kJ mol^{-1})

(a) 20 (b) 300

(c) 120 (d) 280.

(2007)

16. Consider the reaction, $2A + B \rightarrow$ products. When concentration of B alone was doubled, the half-life did not change. When the concentration of A alone was doubled, the rate increased by two times. The unit of rate constant for this reaction is

(a) s^{-1} (b) L mol^{-1} s^{-1}

(c) no unit (d) mol L^{-1} s^{-1}.

(2007)

17. For a reaction $\frac{1}{2}A \rightarrow 2B$ rate of disappearance of A is related to the rate of appearance of B by the expression

(a) $-\frac{d[A]}{dt} = 4\frac{d[B]}{dt}$

(b) $-\frac{d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt}$

(c) $-\frac{d[A]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$

(d) $-\frac{d[A]}{dt} = \frac{d[B]}{dt}$

(2008)

Answer Key

1. (a)	**2.** (a)	**3.** (d)	**4.** (a)	**5.** (d)	**6.** (c)
7. (c)	**8.** (a)	**9.** (d)	**10.** (a)	**11.** (b)	**12.** (c)
13. (b)	**14.** (d)	**15.** (a)	**16.** (b)	**17.** (c)	

1. **(a) :** Unit of $K = (\text{mol L}^{-1})^{1-n}\ \text{sec}^{-1}$,
where n = order of reaction
$n = 0 \Rightarrow$ zero order reaction
$n = 1 \Rightarrow$ first order reaction

2. **(a) :** Order is the sum of the power of the concentrations terms in rate law expression.
$R = [A] \cdot [B]^2$
Thus, order of reaction = 1 + 2 = 3.

3. **(d) :** $H_2 + I_2 \rightarrow 2HI$
When 1 mole of H_2 and 1 mole of I_2 reacts, 2 moles of HI are formed in the same time interval.
Thus the rate may be expressed as
$$\frac{-d[H_2]}{dt} = \frac{-d[I_2]}{dt} = \frac{1}{2}\frac{d[HI]}{dt}$$
The negative sign signifies a decrease in concentration of the reactant with increase of time.

4. **(a)**

5. **(d) :** $\text{Rate}_1 = k\,[A]^n\,[B]^m$
On doubling the concentration of A and halving the concentration of B
$\text{Rate}_2 = k\,[2A]^n\,[B/2]^m$
Ratio between new and earlier rate
$$= \frac{k[2A]^n\,[B/2]^m}{k[A]^n\,[B]^m} = 2^n \times \left(\frac{1}{2}\right)^m = 2^{n-m}.$$

6. **(c) :** $\text{Rate}_1 = k\,[NO]^2\,[O_2]$
When volume is reduced to 1/2, concentration becomes two times.
$\text{Rate}_2 = k\,[2NO]^2\,[2O_2]$
$$\frac{\text{Rate}_1}{\text{Rate}_2} = \frac{k[NO]^2\,[O_2]}{k[2NO]^2\,[2O_2]} \quad \text{or} \quad \frac{\text{Rate}_1}{\text{Rate}_2} = \frac{1}{8}$$
$\therefore\ \text{Rate}_2 = 8\ \text{Rate}_1$.

7. **(c) :** In Arrhenius equation, $k = Ae^{-E_a/RT}$
k = rate constant, A = frequency factor
T = temperature, R = gas constant, E_a = energy of activation.
This equation can be used for calculation of energy of activation.

8. **(a) :** The concentration of the reactant decreases from 0.8 M to 0.4 M in 15 minutes,
i.e. $t_{1/2}$ = 15 minute.
Therefore, the concentration of reactant will fall from 0.1 M to 0.025 M in two half lives.
i.e. $2t_{1/2} = 2 \times 15 = 30$ minutes.

9. **(d) :** $2A + B \rightarrow C$
rate = $k\,[A]\,[B]$
The value of k (velocity constant) is always independent of the concentration of reactant and it is a function of temperature only.
For a second order reaction, unit of rate constant, k is L mol^{-1} sec^{-1} for a second order reaction,
$$t_{1/2} = \frac{1}{ka}$$
i.e. $t_{1/2}$ is inversely proportional to initial concentration.
$$2A + B \rightarrow C$$
$$\text{Rate} = -\frac{1}{2}\frac{d[A]}{dt} = -\frac{d[B]}{dt} = \frac{d[C]}{dt}$$
i.e. rate of formation of C is half the rate of disappearance of B.

10. **(a) :** Generally, molecularity of simple reactions is equal to the sum of the number of molecules of reactants involved in the balanced stoichiometric equation. Thus, a reaction involving two different reactants can never be unimolecular.
But a reaction involving two different reactants can a first order reaction. For example, for the following reaction
$$RCl + H_2O \rightarrow ROH + HCl$$
Expected rate law :
Rate = $k[RCl][H_2O]$ expected order = 1 + 1 = 2
But actual rate law :
Rate = $k'[RCl]$ actual order = 1
Here water is taken in excess, hence its concentration may be taken constant.
Here the molecularity of the reaction = 2 and the order of the reaction = 1.

11. **(b) :** $t_{1/4} = \dfrac{2.303}{k}\log\dfrac{4}{3} = \dfrac{0.29}{k}$.

12. **(c) :** Given $r_1 = \frac{dx}{dt} = k[CO]^2$

$r_2 = k[2CO]^2 = 4k[CO]^2$

Thus, according to the rate law expression doubling the concentration of CO increases the rate by a factor of 4.

13. **(b) :** $k = Ae^{-E/RT}$

where E = activation energy, *i.e.* the minimum amount of energy required by reactant molecules to participate in a reaction.

14. **(d) :** $NO_{(g)} + Br_{2(g)} \rightleftharpoons NOBr_{2(g)}$

$NOBr_{2(g)} + NO_{(g)} \rightarrow 2NOBr_{(g)}$ [rate determining step]

Rate of the reaction (r) = $K\,[NOBr_2]\,[NO]$

where $[NOBr_2] = K_C\,[NO][Br_2]$

$r = K \cdot K_C \cdot [NO][Br_2][NO]$

$r = K'\,[NO]^2\,[Br_2]$.

The order of the reaction with respect to $NO_{(g)} = 2$.

15. **(a) :** $\Delta H_R = E_f - E_b = 180 - 200 = -20$ kJ mol^{-1}.

The correct answer for this question should be -20 kJ mol^{-1}. But no option given is correct. Hence we can ignore sign and select option (a).

16. **(b) :** Rate = $K\,[A]^x\,[B]^y$

When $[B]$ is doubled, keeping $[A]$ constant half-life of the reaction does not change.

Now, for a first order reaction $t_{1/2} = \frac{0.693}{K}$

i.e. $t_{1/2}$ is independent of the concentration of the reactant. Hence the reaction is first order with respect to B. Now when $[A]$ is doubled, keeping $[B]$ constant, the rate also doubles. Hence the reaction is first order with respect to A.

$\therefore$ Rate = $[A]^1\,[B]^1$ $\therefore$ order = 2.

Now for a nth order reaction, unit of rate constant is (lit)$^{n-1}$ (mol)$^{1-n}$ sec^{-1} when $n = 2$, unit of rate constant is lit mol^{-1} sec^{-1}.

17. **(c) :** For this reaction,

$$\text{Rate} = -\frac{1}{1/2}\frac{d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt}$$

$$\Rightarrow \quad -\frac{d[A]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$$

CHAPTER
9

Nuclear Chemistry

1. If half-life of a substance is 5 yrs, then the total amount of substance left after 15 years, when initial amount is 64 grams is
(a) 16 grams
(b) 2 grams
(c) 32 grams
(d) 8 grams.
(2002)

2. β-particle is emitted in radioactivity by
(a) conversion of proton to neutron
(b) form outermost orbit
(c) conversion of neutron to proton
(d) β-particle is not emitted.
(2002)

3. The radionucleide $^{234}_{90}\text{Th}$ undergoes two successive β-decays followed by one α-decay. The atomic number and the mass number respectively of the resulting radionucleide are
(a) 92 and 234
(b) 94 and 230
(c) 90 and 230
(d) 92 and 230.
(2003)

4. The half-life of a radioactive isotope is three hours. If the initial mass of the iosotope were 256 g, the mass of it remaining undecayed after 18 hours would be
(a) 4.0 g
(b) 8.0 g
(c) 12.0 g
(d) 16.0 g.
(2003)

5. Consider the following nuclear reactions:
$^{238}_{92}M \rightarrow {}^{X}_{Y}N + 2\,{}^{4}_{2}\text{He}$; ${}^{X}_{Y}N \rightarrow {}^{A}_{B}L + 2\beta^{+}$
The number of neutrons in the element L is
(a) 142
(b) 144
(c) 140
(d) 146.
(2004)

6. The half-life of a radioisotope is four hours. If the initial mass of the isotope was 200 g, the mass remaining after 24 hours undecayed is
(a) 1.042 g
(b) 2.084 g
(c) 3.125 g
(d) 4.167 g.
(2004)

7. Hydrogen bomb is based on the principle of
(a) nuclear fission
(b) natural radioactivity
(c) nuclear fusion
(d) artificial radioactivity
(2005)

8. A photon of hard gamma radiation knocks a proton out of $^{24}_{12}\text{Mg}$ nucleus to form
(a) the isotope of parent nucleus
(b) the isobar of parent nucleus
(c) the nuclide $^{23}_{11}\text{Na}$
(d) the isobar of $^{23}_{11}\text{Na}$
(2005)

9. In the transformation of $^{238}_{92}\text{U}$ to $^{234}_{92}\text{U}$, if one emission is an α-particle, what should be the other

emission(s)?
(a) two β^-
(b) two β^- and one β^+
(c) one β^- and one γ
(d) one β^+ and one β^-.

(2006)

10. A radioactive element gets spilled over the floor of a room. Its half-life period is 30 days. If the initial velocity is ten times the permissible value, after how many days will it be safe to enter the room?
(a) 100 days
(b) 1000 days
(c) 300 days
(d) 10 days.

(2007)

11. Which of the following nuclear reactions will generate an isotope?
(a) β-particle emission
(b) Neutron particle emission
(c) Positron emission
(d) α-particle emission.

(2007)

Answer Key

1.	(d)	**2.**	(c)	**3.**	(c)	**4.**	(a)	**5.**	(b)	**6.**	(c)
7.	(c)	**8.**	(c)	**9.**	(a)	**10.**	(a)	**11.**	(b)		

1. **(d) :** $t_{1/2} = 5$ years, $n = \dfrac{T}{t_{1/2}} = \dfrac{15}{5} = 3$

$$n = N_0\left(\frac{1}{2}\right)^n = 64 \times \left(\frac{1}{2}\right)^3 = 8 \text{ gm}$$

2. **(c) :** Since the nucleus does not contain β-particles, it is produced by the conversion of a neutron to a proton at the moment of emission.

$$_0n^1 \rightarrow {}_{+1}p^1 + {}_{-1}e^0$$

3. **(c) :** $_{90}Th^{234} \xrightarrow{-2\beta} {}_{92}X^{234} \xrightarrow{-\alpha} {}_{90}Th^{230}$
Elimination of 1α and 2β particles give isotope.

4. **(a) :** $t_{1/2} = 3$ hours, $n = T/t_{1/2} = 18/3 = 6$.

$$N = N_0\left(\frac{1}{2}\right)^n \Rightarrow N = 256\left(\frac{1}{2}\right)^6$$
$\Rightarrow$ $N = 4.0$ gm.

5. **(b) :** ${}^{238}_{92}M \rightarrow {}^{230}_{88}N + 2\,{}^{4}_{2}He$

${}^{230}_{88}N \rightarrow {}^{230}_{86}L + 2\beta^+$

Therefore, number of neutrons in element L
$= 230 - 86 = 144$.

6. **(c) :** $t_{1/2} = 4$ hours

$$n = \frac{T}{t_{1/2}} = \frac{24}{4} = 6 \ ; \ N = N_0\left(\frac{1}{2}\right)^n$$

or, $N = 200 \times \left(\frac{1}{2}\right)^6 = 3.125$ g.

7. **(c) :** Hydrogen bomb is based on the principal of nuclear fusion. In hydrogen bomb, a mixture of deuterium oxide and tritium oxide is enclosed in a space surrounding an ordinary atomic bomb. The temperature produced by the explosion of the atomic bomb initiates the fusion reaction between ${}^3_1H$ and ${}^2_1H$ releasing huge amount of energy.

8. **(c) :** ${}^{24}_{12}Mg + \gamma \rightarrow {}^{23}_{11}Na + {}^{1}_{1}p$

9. **(a) :** $_{92}U^{238} \xrightarrow{-\alpha} {}_{90}A^{234} \xrightarrow{-\beta} {}_{91}B^{234} \xrightarrow{-\beta} {}_{92}U^{234}$
Thus in order to get $_{92}U^{234}$ as end product 1α and 2β particles should be emitted.

10. **(a) :** Let A be the activity for safe working.
Given $A_0 = 10\,A$
$A_0 \times N_0$ and $A \times N$

$$t = \frac{2.303}{\lambda}\log\frac{N_0}{N} = \frac{2.303}{\lambda}\log\frac{A_0}{A}$$

$$= \frac{2.303}{0.693/30}\log\frac{10A}{A} = \frac{2.303 \times 30}{0.693}\log 10$$

$$= \frac{2.303 \times 30}{0.693} = 99.69 \text{ days} \approx 100 \text{ days.}$$

11. **(b) :** The atoms of the some elements having same atomic number but different mass numbers are called isotopes.

$${}^{A}_{Z}X \xrightarrow{-\alpha} {}^{A-4}_{Z-2}Y; \quad {}^{A}_{Z}X \xrightarrow{-\beta} {}^{A}_{Z+1}Y$$

$${}^{A}_{Z}X \xrightarrow{-{}^{1}_{0}n} {}^{A-1}_{Z}X \qquad {}^{A}_{Z}X \xrightarrow{+\beta^+} {}^{A}_{Z-1}Y$$

CHAPTER

10

Surface Chemistry

1. Which one of the following characteristics is not correct for physical adsorption?
   (a) Adsorption on solids is reversible
   (b) Adsorption increases with increase in temperature
   (c) Adsorption is spontaneous
   (d) Both enthalpy and entropy of adsorption are negative.

(2003)

2. The volume of a colloidal particle, V_c as compared to the volume of a solute particle in a true solution V_s could be
   (a) ~ 1
   (b) $\sim 10^{23}$
   (c) $\sim 10^{-3}$
   (d) $\sim 10^{3}$

(2005)

3. The disperse phase in colloidal iron (III) hydroxide and colloidal gold is positively and negatively charged, respectively. Which of the following statements is NOT correct?
   (a) Magnesium chloride solution coagulates, the gold sol more readily than the iron (III) hydroxide sol
   (b) Sodium sulphate solution causes coagulation in both sols
   (c) Mixing of the sols has no effect
   (d) Coagulation in both sols can be brought about by electrophoresis

(2005)

4. In Langmuir's model of adsorption of a gas on a solid surface
   (a) the rate of dissociation of adsorbed molecules from the surface does not depend on the surface covered
   (b) the adsorption at a single site on the surface may involve multiple molecules at the same time
   (c) the mass of gas striking a given area of surface is proportional to the pressure of the gas
   (d) the mass of gas striking a given area of surface is independent of the pressure of the gas.

(2006)

Answer Key

1.	(b)	**2.**	(d)	**3.**	(c)	**4.**	(c)

1. **(b) :** During adsorption, there is always decrease in surface energy which appears as heat. Therefore adsorption always takes place with evolution of heat, *i.e.* it is an exothermic process and since the adsorption process is exothermic, the physical adsorption occurs readily at low temperature and decreases with increasing temperature. (Le Chatelier's principle).

2. **(d) :** For true solution the diameter range is 1 to 10 Å and for colloidal solution diameter range is 10 to 1000 Å.

$$\frac{V_c}{V_s} = \frac{(4/3)\,\pi r_c^{\,3}}{(4/3)\pi r_s^{\,3}} = \left(\frac{r_c}{r_s}\right)^3.$$

Ratio of diameters = $(10/1)^3 = 10^3$

$V_c/V_s \simeq 10^3$.

3. **(c) :** Opposite charges attract each other. Hence on mixing coagulation of two sols may be take place.

4. **(c) :** Assuming the formation of a monolayer of the adsorbate on the surface of the adsorbent, it was derived by Langmuir that the mass of the gas adsorbed per gram of the adsorbent is related to the equilibrium pressure according to the equation:

$$\frac{x}{m} = \frac{aP}{1+bP}$$

where x is the mass of the gas adsorbed on m gram of the adsorbent, P is the pressure and a, b are constants.

CHAPTER

11

Metallurgy

1. Aluminium is extracted by the electrolysis of
   (a) bauxite
   (b) alumina
   (c) alumina mixed with molten cryolite
   (d) molten cryolite.

   (2002)

2. The metal extracted by leaching with a cyanide is
   (a) Mg (b) Ag
   (c) Cu (d) Na.

   (2002)

3. Cyanide process is used for the extraction of
   (a) barium (b) aluminium
   (c) boron (d) silver.

   (2002)

4. When the sample of copper wth zinc impurity is to be purified by electrolysis, the appropriate electrodes are

	cathode	**anode**
(a)	pure zinc	pure copper
(b)	impure sample	pure copper
(c)	impure zinc	impure sample
(d)	pure copper	impure sample.

   (2002)

5. Which one of the following ores is best concentrated by froth-flotation method?
   (a) Magnetite (b) Cassiterite
   (c) Galena (d) Malachite.

   (2004)

6. During the process of electrolytic refining of copper, some metals present as impurity settle as 'anode mud'. These are
   (a) Sn and Ag (b) Pb and Zn
   (c) Ag and Au (d) Fe and Ni.

   (2005)

7. Which of the following factors is of no significance for roasting sulphide ores to the oxides and not subjecting the sulphide ores to carbon reduction directly?
   (a) CO_2 is more volatile than CS_2
   (b) Metal sulphides are thermodynamically more stable than CS_2
   (c) CO_2 is thermodynamically more stable than CS_2
   (d) Metal sulphides are less stable than the corresponding oxides

   (2008)

Answer Key

1.	(c)	**2.**	(b)	**3.**	(d)	**4.**	(d)	**5.**	(c)	**6.**	(c)
7.	(a)										

1. **(c) :** Aluminium is obtained by the electrolysis of the pure alumina (20 parts) dissolved in a bath of fused cryolite (60 parts) and fluorspar (20 parts).

2. **(b) :** Silver ore forms a soluble complex with NaCN from which silver is precipitated using scrap zinc.

$$Ag_2S + 2NaCN \rightarrow \underset{\text{(soluble)}}{\underset{\text{sod. argentocyanide}}{Na[Ag(CN)_2]}} \xrightarrow{Zn} Na_2[Zn(CN)_4] + Ag\downarrow$$

3. **(d) :** Gold and silver are extracted from their native ores by Mac-Arthur forrest cyanide process.

4. **(d) :** The impure metal is made anode while a thin sheet of pure metal acts as cathode. On passing the current, the pure metal is deposited on the cathode and equivalent amount of the metal gets dissolved from the anode.

5. **(c) :** Froth-flotation method is used for the concentration of sulphide ores. The method is based on the preferential wetting properties with the frothing agent and water. Here galena (PbS) is the only sulphide ore.

6. **(c) :** In the electrolytic refining of copper the more electropositive impurities like Fe, Zn, Ni, Co, etc. dissolve in the solution and less electropositive impurities such as Ag, Au and Pt collect below the anode in the form of anodic mud.

7. **(a) :** Oxidising roasting is a very common type of roasting in metallurgy and is carried out to remove sulphur and arsenic in the form of their volatile oxides. CS_2 is more volatile than CO_2. So option (a) is of no significance for roasting sulphide ores to their oxides. The reduction process is on the thermodynamic stability of the products and not on their volatility.

CHAPTER

12

Periodic Properties

1. Which is the correct order of atomic sizes?
   (a) Ce > Sn > Yb > Lu
   (b) Sn > Ce > Lu > Yb
   (c) Lu > Yb > Sn > Ce
   (d) Sn > Yb > Ce > Lu.
   (At. Nos. : Ce = 58, Sn = 50, Yb = 70 and Lu = 71)
   (2002)

2. According to the periodic law of elements, the variation in properties of elements is related to their
   (a) atomic masses
   (b) nuclear masses
   (c) atomic numbers
   (d) nuclear neutron-proton number ratios.
   (2003)

3. Which one of the following groupings represents a collection of isoelectronic species?
   (At. nos.: Cs-55, Br-35)
   (a) Na^+, Ca^{2+}, Mg^{2+} (b) N^{3-}, F^-, Na^+
   (c) Be, Al^{3+}, Cl^- (d) Ca^{2+}, Cs^+, Br.
   (2003)

4. Which one of the following ions has the highest value of ionic radius?
   (a) Li^+ (b) B^{3+}
   (c) O^{2-} (d) F^-.
   (2004)

5. Which one of the following sets of ions represents the collection of isoelectronic species?
   (a) K^+, Ca^{2+}, Sc^{3+}, Cl^-
   (b) Na^+, Ca^{2+}, Sc^{3+}, F^-
   (c) K^+, Cl^-, Mg^{2+}, Sc^{3+}
   (d) Na^+, Mg^{2+}, Al^{3+}, Cl^-.
   (Atomic nos.: F = 9, Cl = 17, Na = 11, Mg = 12, Al = 13, K = 19, Ca = 20, Sc = 21)
   (2004)

6. The formation of the oxide ion $O^{2-}_{(g)}$ requires first an exothermic and then an endothermic step as shown below.
   $O_{(g)} + e^- = O^-_{(g)}$; $\Delta H^\circ = -142$ kJmol^{-1}
   $O^-_{(g)} + e^- = O^{2-}_{(g)}$; $\Delta H^\circ = 844$ kJmol^{-1}
   This is because
   (a) oxygen is more electronegative
   (b) oxygen has high electron affinity
   (c) O^- ion will tend to resist the addition of another electron
   (d) O^- ion has comparatively larger size than oxygen atom.
   (2004)

7. Which among the following factors is the most important in making fluorine the strongest oxidising agent?
   (a) Electron affinity
   (b) Ionization energy
   (c) Hydration enthalpy
   (d) Bond dissociation energy.
   (2004)

8. Lattice energy of an ionic compound depends upon
   (a) charge on the ion only
   (b) size of the ion only
   (c) packing of the ion only
   (d) charge and size of the ion.
   (2005)

9. Based on lattice energy and other considerations which one of the following alkali metal chlorides is expected to have the highest melting point?
   (a) LiCl (b) NaCl
   (c) KCl (d) RbCl
   (2005)

10. In which of the following arrangements the order is NOT according to the property indicated against it?

(a) $Al^{3+} < Mg^{2+} < Na^+ < F^-$ - increasing ionic size

(b) B < C < N < O - increasing first ionisation enthalpy

(c) I < Br < F < Cl - increasing electron gain enthalpy (with negative sign)

(d) Li < Na < K < Rb - increasing metallic radius

(2005)

11. Which one of the following sets of ions represents a collection of isoelectronic species?

(a) $K^+, Cl^-, Ca^{2+}, Sc^{3+}$

(b) $Ba^{2+}, Sr^{2+}, K^+, S^{2-}$

(c) $N^{3-}, O^{2-}, F^-, S^{2-}$

(d) $Li^+, Na^+, Mg^{2+}, Ca^{2+}$.

(2006)

12. The increasing order of the first ionisation enthalpies of the elements B, P, S and F (lowest first) is

(a) F < S < P < B (b) P < S < B < F

(c) B < P < S < F (d) B < S < P < F.

(2006)

13. The decreasing values of bond angles from NH_3 (106°) to SbH_3 (101°) down group-15 of the periodic table is due to

(a) increasing bond-bond pair repulsion

(b) increasing *p*-orbital character in sp^3

(c) decreasing lone pair-bond pair repulsion

(d) decreasing electronegativity.

(2006)

14. Following statements regarding the periodic trends of chemical reactivity of the alkali metals and the halogens are given. Which of these statements gives the correct picture?

(a) The reactivity decreases in the alkali metals but increases in the halogens with increase in atomic number down the group

(b) In both the alkali metals and the halogens the chemical reactivity decreases with increase in atomic number down the group

(c) Chemical reactivity increases with increase in atomic number down the group in both the alkali metals and halogens

(d) In alkali metals the reactivity increases but in the halogens it decreases with increase in atomic number down the group.

(2006)

Answer Key

1.	(a)	**2.**	(c)	**3.**	(b)	**4.**	(c)	**5.**	(a)	**6.**	(c)
7.	(d)	**8.**	(d)	**9.**	(b)	**10.**	(b)	**11.**	(a)	**12.**	(d)
13.	(c)	**14.**	(d)								

1. **(a) :** Generally as we move from left to right in a period, there is regular decrease in atomic radii and in a group as the atomic number increases the atomic radii also increases. Thus the atomic radius of Sn should be less than lanthanides. La > Sn. But due to lanthanide contraction, in case of lanthanides there is a continuous decrease in size with increase in atomic number. Hence the atomic radius follow the given trend :
Ce > Sn > Yb > Lu.

2. **(c) :** According to modified modern periodic law, the properties of elements are periodic functions of their atomic numbers.

3. **(b) :** Isoelectronic species are the neutral atoms, cations or anions of different elements which have the same number of electrons but different nuclear charge. Number of electrons in $N^{3-} = 7 + 3 = 10$.

   Number of electrons in $F^- = 9 + 1 = 10$

   Number of electrons in $Na^+ = 11 - 1 = 10$.

4. **(c) :** This can be explained on the basis of z/e $\left\{\frac{\text{nuclear charge}}{\text{no. of electrons}}\right\}$, whereas z/e ratio increases, the size decreases and when z/e ratio decreases the size increases.

   For Li^+, $\frac{z}{e} = \frac{3}{2} = 1.5$

   For B^{3+}, $\frac{z}{e} = \frac{5}{2} = 2.5$

   For O^{2-}, $\frac{z}{e} = \frac{8}{10} = 0.8$

   For F^-, $\frac{z}{e} = \frac{9}{10} = 0.9$

   Hence, O^{2-} has highest value of ionic radius.

5. **(a) :** Isoelectronic species are those which have same number of electrons.
$K^+ = 19 - 1 = 18$; $Ca^{2+} = 20 - 2 = 18$
$Sc^{3+} = 21 - 3 = 18$; $Cl^- = 17 + 1 = 18$
Thus all these ions have 18 electrons in them.

6. **(c) :** The addition of second electron in an atom or ion is always endothermic.

7. **(d) :** The bond dissociation energy of F – F bond is very low. The weak F – F bond makes fluorine the strongest oxidising halogen.

8. **(d) :** The value of lattice energy depends on the charges present on the two ions and the distance between them.

9. **(b) :** In case of halides of alkali metals, melting point decreases going down the group because lattice enthalpies decreases as size of alkali metal increases. But LiCl has lower melting point in comparison to NaCl due to covalent nature. Thus, NaCl is expected to have the highest melting point among given halides.

10. **(b) :** As we move from left to right across a period, ionisation enthalpy increases with increasing atomic number. So the order of increasing ionisation enthalpy should be B < C < N < O.
But $N(1s^2 2s^2 2p^3)$ has a stable half filled electronic configuration. So, ionization enthalpy of nitrogen is greater than oxygen.
So, the correct order of increasing the first ionization enthalpy is B < C < O < N.

11. **(a) :** $K^+ = 19 - 1 = 18\ e^-$
$Cl^- = 17 + 1 = 18\ e^-$
$Ca^{2+} = 20 - 2 = 18\ e^-$
$Sc^{3+} = 21 - 3 = 18\ e^-$
Thus all the species are isoelectronic.

12. **(d) :**

Element:	B	S	P	F
I.E. (eV):	8.3	10.4	11.0	17.4

In general as we move from left to right in a period, the ionisation enthalpy increases with increasing atomic number. The ionisation enthalpy decreases as we move down a group. $P(1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^3)$ has a stable half filled electronic configuration than $S\ (1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^4)$. For this reason, ionisation enthalpy of P is higher than S.

13. (c) :

	NH_3	PH_3	AsH_3	SbH_3
Bond angle	106.5°	93.5°	91.5°	91.3°

The bond angle in ammonia is less than 109° 28′ due to repulsion between lone pairs present on nitrogen atom and bonded pairs of electrons. As we move down the group, the bond angles gradually decrease due to decrease in bond pair lone pair repulsion.

14. (d) : All the alkali metals are highly reactive elements since they have a strong tendency to lose the single valence *s*-electron to form unipositive ions having inert gas configuration. This reactivity arises due to their low ionisation enthalpies and high negative values of their standard electrode potentials.

However, the reactivity of halogens decreases with increase in atomic number due to following reasons:

(a) As the size increases, the attraction for an additional electron by the nucleus becomes less.

(b) Due to decrease in electronegativity from F to I, the bond between halogen and other elements becomes weaker and weaker.

CHAPTER

13

Chemistry of Non-metals

1. In XeF_2, XeF_4, XeF_6 the number of lone pairs on Xe are respectively
(a) 2, 3, 1 (b) 1, 2, 3
(c) 4, 1, 2 (d) 3, 2, 1.
(2002)

2. Which of the following statements is true?
(a) HF is less polar than HBr
(b) Absolutely pure water does not contain any ions
(c) Chemical bond formation takes place when forces of attraction overcome the forces of repulsion
(d) In covalency transference of electron takes place.
(2002)

3. When H_2S is passed through Hg_2S we get
(a) HgS (b) $HgS + Hg_2S$
(c) $Hg_2S + Hg$ (d) Hg_2S.
(2002)

4. Glass is a
(a) micro-crystalline solid
(b) super-cooled liquid
(c) gel
(d) polymeric mixture.
(2003)

5. Graphite is a soft solid lubricant extremely difficult to melt. The reason for this anomalous behaviour is that graphite
(a) is a non-crystalline substance
(b) is an allotropic form of diamond
(c) has molecules of variable molecular masses like polymers
(d) has carbon atoms arranged in large plates of rings of strongly bound carbon atoms with weak interplate bonds.
(2003)

6. Which one of the following processes will produce hard water?
(a) Saturation of water with $CaCO_3$
(b) Saturation of water with $MgCO_3$
(c) Saturation of water with $CaSO_4$
(d) Addition of Na_2SO_4 to water.
(2003)

7. Which one of the following pairs of molecules will have permanent dipole moments for both members?
(a) SiF_4 and NO_2 (b) NO_2 and CO_2
(c) NO_2 and O_3 (d) SiF_4 and CO_2.
(2003)

8. Which one of the following substances has the highest proton affinity?
(a) H_2O (b) H_2S
(c) NH_3 (d) PH_3.
(2003)

9. Which one of the following is an amphoteric oxide?
(a) ZnO (b) Na_2O
(c) SO_2 (d) B_2O_3.
(2003)

10. Concentrated hydrochloric acid when kept in open air sometimes produces a cloud of white fumes. The explanation for it is that
(a) concentrated hydrochloric acid emits strongly smelling HCl gas all the time
(b) oxygen in air reacts with the emitted HCl gas to form a cloud of chlorine gas

(c) strong affinity of HCl gas for moisture in air results in forming of droplets of liquid solution which appears like a cloudy smoke
(d) due to strong affinity for water, concentrated hydrochloric acid pulls moisture of air towards itself. This moisture forms droplets of water and hence the cloud.

(2003)

11. What may be expected to happen when phosphine gas is mixed with chlorine gas?
(a) The mixture only cools down
(b) PCl_3 and HCl are formed and the mixture warms up
(c) PCl_5 and HCl are formed and the mixture cools down
(d) $PH_3 \cdot Cl_2$ is formed with warming up.

(2003)

12. The states of hybridisation of boron and oxygen atoms in boric acid (H_3BO_3) are respectively
(a) sp^2 and sp^2 (b) sp^2 and sp^3
(c) sp^3 and sp^2 (d) sp^3 and sp^3.

(2004)

13. In a hydrogen-oxygen fuel cell, combustion of hydrogen occurs to
(a) generate heat
(b) create potential difference between the two electrodes
(c) produce high purity water
(d) remove adsorbed oxygen from electrode surface.

(2004)

14. Which one of the following statements regarding helium is incorrect?
(a) It is used to fill gas in balloons instead of hydrogen because it is lighter and non-inflammable.
(b) It is used as a cryogenic agent for carrying out experiments at low temperatures.
(c) It is used to produce and sustain powerful superconducting magnets.
(d) It is used in gas-cooled nuclear reactors.

(2004)

15. The molecular shapes of SF_4, CF_4 and XeF_4 are
(a) the same with 2, 0 and 1 lone pairs of electrons on the central atom respectively.
(b) the same with 1, 1 and 1 lone pair of electrons on the central atoms respectively.
(c) different with 0, 1 and 2 lone pairs of electrons on the central atom respectively.
(d) different with 1, 0 and 2 lone pairs of electrons on the central atom respectively.

(2005)

16. The number of hydrogen atom(s) attached to phosphorus atom in hypophosphorous acid is
(a) zero (b) two
(c) one (d) three

(2005)

17. The correct order of the thermal stability of hydrogen halides ($H - X$) is
(a) HI > HBr > HCl > HF
(b) HF > HCl > HBr > HI
(c) HCl < HF > HBr < HI
(d) HI > HCl < HF > HBr

(2005)

18. In silicon dioxide
(a) each silicon atom is surrounded by four oxygen atoms and each oxygen atom is bonded to two silicon atoms
(b) each silicon atom is surrounded by two oxygen atoms and each oxygen atom is bonded to two silicon atoms
(c) silicon atom is bonded to two oxygen atoms
(d) there are double bonds between silicon and oxygen atoms

(2005)

19. Which of the following statements is true?
(a) H_3PO_3 is a stronger acid than H_2SO_3
(b) In aqueous medium HF is a stronger acid than HCl
(c) $HClO_4$ is a weaker acid than $HClO_3$
(d) HNO_3 is a stronger acid than HNO_2.

(2006)

20. The stability of dihalides of Si, Ge, Sn and Pb increases steadily in the sequence
(a) $PbX_2 \ll SnX_2 \ll GeX_2 \ll SiX_2$

(b) $GeX_2 \ll SiX_2 \ll SnX_2 \ll PbX_2$
(c) $SiX_2 \ll GeX_2 \ll PbX_2 \ll SnX_2$
(d) $SiX_2 \ll GeX_2 \ll SnX_2 \ll PbX_2$.

(2007)

21. Identify the incorrect statement among the following.
(a) Br_2 reacts with hot and strong NaOH solution to give NaBr and H_2O.
(b) Ozone reacts with SO_2 to give SO_3.
(c) Silicon reacts with $NaOH_{(aq)}$ in the presence of air to give Na_2SiO_3 and H_2O.
(d) Cl_2 reacts with excess of NH_3 to give N_2 and HCl.

(2007)

22. Regular use of the following fertilizers increases the acidity of soil?
(a) Ammonium sulphate
(b) Potassium nitrate
(c) Urea
(d) Superphosphate of lime.

(2007)

23. In context with the industrial preparation of hydrogen from water gas ($CO + H_2$), which of the following is the correct statement?
(a) CO is oxidised to CO_2 with steam in the presence of a catalyst followed by absorption of CO_2 in alkali
(b) CO and H_2 are fractionally separated using differences in their densities
(c) CO is removed by absorption in aqueous Cu_2Cl_2 solution
(d) H_2 is removed through occlusion with Pd

(2008)

24. Among the following substituted silanes the one which will give rise to cross linked silicone polymer on hydrolysis is
(a) R_3SiCl (b) R_4Si
(c) $RSiCl_3$ (d) R_2SiCl_2

(2008)

Answer Key

1.	(d)	**2.**	(c)	**3.**	(c)	**4.**	(b)	**5.**	(d)	**6.**	(c)
7.	(c)	**8.**	(c)	**9.**	(a)	**10.**	(c)	**11.**	(c)	**12.**	(b)
13.	(b)	**14.**	(a)	**15.**	(d)	**16.**	(b)	**17.**	(b)	**18.**	(a)
19.	(d)	**20.**	(c)	**21.**	(d)	**22.**	(a)	**23.**	(a)	**24.**	(c)

1. **(d) :** XeF_2 sp^3d 3 lone pairs
XeF_4 sp^3d^2 2 lone pairs
XeF_6 sp^3d^3 1 lone pair

2. **(c) :** Due to the higher electronegativity of F, HF is more polar than HBr pure water contains H^+ and OH^- ions. In covalency, sharing of electrons between two non-metal atoms takes place.

3. **(c)**

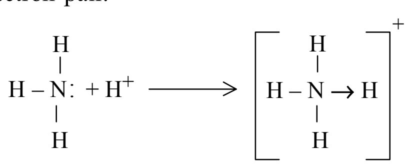
4. **(b) :** Glass is a transparent or translucent amorphous supercooled solid solution (supercooled liquid) of silicates and borates, having a general formula $R_2O{\cdot}MO{\cdot}6SiO_2$ where R = Na or K and M = Ca, Ba, Zn or Pb.

5. **(d) :** Graphite has a two-dimensional sheet like structure and each carbon atom makes a use of sp^2 hybridisation.
The above layer structure of graphite is less compact than that of diamond. Further, since the bonding between the layers involving only weak van der Waal's forces, these layers can slide over each other. This gives softness, greasiness and lubricating character of graphite.

6. **(c) :** Permanent hardness is introduced when water passes over rocks containing the sulphates or chlorides of both of calcium and magnesium.

7. **(c) :** NO_2 and O_3 both have unsymmetrical structures, so they have permanent dipole moment.

8. **(c) :** Ammonia is a Lewis base, accepting proton to form ammonium ion as it has tendency to donate an electron pair.

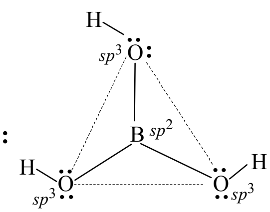


9. **(a) :** ZnO is an amphoteric oxide and dissolves readily in acids forming corresponding zinc salts and alkalies forming zincates.
$$ZnO + H_2SO_4 \rightarrow \underset{\text{zinc sulphate}}{ZnSO_4} + H_2O$$
$$ZnO + 2NaOH \rightarrow \underset{\text{sodium zincate}}{Na_2ZnO_2} + H_2O$$

10. **(c) :** HCl gas in presence of moisture in air forms droplets of liquid solution in the form of cloudy smoke.

11. **(c) :** Phosphine burns in the atmosphere of chlorine and forms phosphorus pentachloride.
$$PH_3 + 4Cl_2 \rightarrow PCl_5 + 3HCl$$

12. **(b) :**

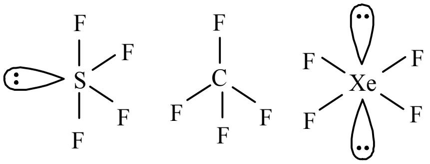


13. **(b) :** Direct conversion of chemical energy to electric energy can be made considerably more efficient (*i.e.* upto 75%) than the 40% maximum now obtainable through burning of fuel and using the heat to form steam for driving turbines. Furthermore, the water obtained as a byproduct may be used for drinking by the astronauts.
At anode : $2H_{2(g)} + 4OH^-_{(aq)} \rightarrow 4H_2O_{(l)} + 4e^-$
At cathode : $O_{2(g)} + 2H_2O_{(l)} + 4e^- \rightarrow 4OH^-_{(aq)}$
$$2H_{2(g)} + O_{2(g)} \rightarrow 2H_2O_{(l)}$$

14. **(a) :** Helium is twice as heavy as hydrogen, its lifting power is 92 percent of that of hydrogen.
Helium has the lowest melting and boiling points of any element which makes liquid helium an ideal coolant for many extremely low-temperature applications such as superconducting magnets, and cryogenic research where temperatures close to absolute zero are needed.

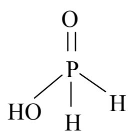
15. **(d) :** SF_4 (sp^3d, trigonal bipyramidal with one equatorial position occupied by 1 lone pair), CF_4

(sp^3, tetrahedral, no lone pair), XeF_4 (sp^3d^2, square planar, two lone pairs).



(SF_4) (CF_4) (XeF_4)

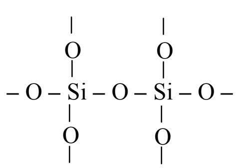
16. **(b) :** Hypophosphorous acid



Number of hydrogen atom(s) attached to phosphorus atom = 2.

17. **(b) :** As the size of the halogen atom increases from F to I, H – X bond length in HX molecules also increases from H – F to H – I (H – F < H – Cl < H – Br < H – I).
The increase in H – X bond length decreases the strength of H – X bond from H – F to H – I (H – F > H – Cl > H – Br > H – I). The decrease in the strength of H – X bond is evident from the fact that H – X bond dissociation energies decrease from H – F to H – I. Due to successive decrease in the strength of H – X bond from H – F to H – I, thermal stability of HX molecules also decreases from HF to HI (HF > HCl > HBr > HI).

18. **(a) :** Silicon dioxide exhibits polymorphism. It is a network solid in which each Si atom is surrounded tetrahedrally by four oxygen atoms.



19. **(d) :** Higher is the oxidation state of the central atom, greater is the acidity.
Hence, $HClO_4$ is a stronger acid than $HClO_3$.
HNO_3 is a stronger acid than HNO_2.
Now, greater is the electronegativity and higher is the oxidation state of the central atom, greater is the acidity. Hence H_2SO_3 is a stronger acid than H_3PO_3.
Due to higher dissociation energy of H – F bond and molecular association due to hydrogen bonding in HF, HF is a weaker acid than HCl.

20. **(c) :** Due to the inert pair effect (the reluctance of ns^2 electrons of outermost shell to participate in bonding) the stability of M^{2+} ions (of group IV elements) increases as we go down the group.

21. **(d) :** $3Br_2 + 6NaOH \rightarrow 5NaBr + NaBrO_3 + 3H_2O$

$O_3 + SO_2 \rightarrow O_2 + SO_3$

$Si + 2NaOH + O_2 \rightarrow Na_2SiO_3 + H_2O$

Cl_2 reacts with excess of ammonia to produce ammonium chloride and nitrogen.

$$2NH_3 + 3Cl_2 \longrightarrow N_2 + 6HCl$$
$$6NH_3 + 6HCl \longrightarrow 6NH_4Cl$$
$$\overline{8NH_3 + 3Cl_2 \longrightarrow N_2 + 6NH_4Cl}$$

22. **(a) :** $$(NH_4)_2SO_4 + 2H_2O \longrightarrow \underset{\text{Strong acid}}{(2H^+ + SO_4^{2-})} + \underset{\text{Weak base}}{2NH_4OH}$$

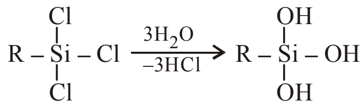
$(NH_4)_2SO_4$ on hydrolysis produces strong acid H_2SO_4, which increases the acidity of the soil.

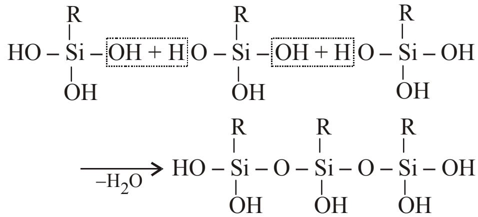
23. **(a) :** Carbon monoxide is oxidised to carbon dioxide by passing the gases and steam over an iron oxide or cobalt oxide or chromium oxide catalyst at 673 K resulting in the production of more H_2.

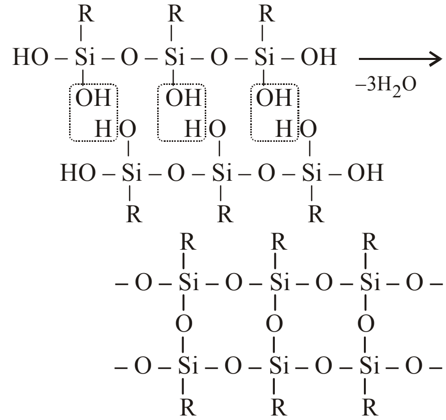
$$CO + H_2O \xrightarrow[673\text{ K}]{Fe_2O_3,\ CoO} CO_2 + H_2$$
$$CO_2 \xrightarrow[\text{alkali}]{NaOH} Na_2CO_3$$

CO_2 is absorbed in alkali (NaOH).
The entire reaction is called water gas shift reaction.

24. **(c) :** $RSiCl_3$ on hydrolysis gives a cross linked silicone. The formation can be explained in three steps

(i) 

(ii) 

(iii) 

Cross linked silicone

CHAPTER 14

Chemistry of Metals

1. KO_2 (potassium super oxide) is used in oxygen cylinders in space and submarines because it
   (a) absorbs CO_2 and increases O_2 content
   (b) eliminates moisture
   (c) absorbs CO_2
   (d) produces ozone.
   (2002)

2. Alum helps in purifying water by
   (a) forming Si complex with clay particles
   (b) sulphate part which combines with the dirt and removes it
   (c) coagulating the mud particles
   (d) making mud water soluble.
   (2002)

3. A metal M readily forms its sulphate MSO_4 which is water-soluble. It forms its oxide MO which becomes inert on heating. It forms an insoluble hydroxide $M(OH)_2$ which is soluble in NaOH solution. Then M is
   (a) Mg (b) Ba
   (c) Ca (d) Be.
   (2002)

4. A red solid is insoluble in water. However it becomes soluble if some KI is added to water. Heating the red solid in a test tube results in liberation of some violet coloured fumes and droplets of a metal appear on the cooler parts of the test tube. The red solid is
   (a) $(NH_4)_2Cr_2O_7$ (b) HgI_2
   (c) HgO (d) Pb_3O_4.
   (2003)

5. Which one of the following nitrates will leave behind a metal on strong heating?
   (a) Ferric nitrate
   (b) Copper nitrate
   (c) Manganese nitrate
   (d) Silver nitrate.
   (2003)

6. For making good quality mirrors, plates of float glass are used. These are obtained by floating molten glass over a liquid metal which does not solidify before glass. The metal used can be
   (a) mercury (b) tin
   (c) sodium (d) magnesium.
   (2003)

7. One mole of magnesium nitride on the reaction with an excess of water gives
   (a) one mole of ammonia
   (b) one mole of nitric acid
   (c) two moles of ammonia
   (d) two moles of nitric acid.
   (2004)

8. Beryllium and aluminium exhibit many properties which are similar. But, the two elements differ in
   (a) exhibiting maximum covalency in compounds
   (b) forming polymeric hydrides
   (c) forming covalent halides
   (d) exhibiting amphoteric nature in their oxides.
   (2004)

9. Aluminium chloride exists as dimer, Al_2Cl_6 in solid state as well as in solution of non-polar solvents such as benzene. When dissolved in water, it gives
   (a) $Al^{3+} + 3Cl^-$
   (b) $[Al(H_2O)_6]^{3+} + 3Cl^-$
   (c) $[Al(OH)_6]^{3-} + 3HCl$
   (d) $Al_2O_3 + 6HCl$.
   (2004)

10. The soldiers of Napolean army while at Alps during freezing winter suffered a serious problem as regards to the tin buttons of their uniforms. White metallic tin buttons got converted to grey powder. This transformation is related to

(a) an interaction with nitrogen of the air at very low temperatures
(b) a change in the crystalline structure of tin
(c) a change in the partial pressure of oxygen in the air
(d) an interaction with water vapour contained in the humid air.

(2004)

11. Among the properties (*A*) reducing (*B*) oxidising (*C*) complexing, the set of properties shown by CN^- ion towards metal species is

(a) *A, B* (b) *B, C*
(c) *C, A* (d) *A, B, C.*

(2004)

12. The number and type of bonds between two carbon atoms in calcium carbide are

(a) one sigma, one pi
(b) one sigma, two pi
(c) two sigma, one pi
(d) two sigma, two pi

(2005)

13. Heating an aqueous solution of aluminium chloride to dryness will give

(a) $AlCl_3$ (b) Al_2Cl_6
(c) Al_2O_3 (d) $Al(OH)Cl_2$

(2005)

14. A metal, *M* forms chlorides in +2 and +4 oxidation states. Which of the following statements about these chlorides is correct?

(a) MCl_2 is more volatile than MCl_4
(b) MCl_2 is more soluble in anhydrous ethanol than MCl_4
(c) MCl_2 is more ionic than MCl_4
(d) MCl_2 is more easily hydrolysed than MCl_4.

(2006)

Answer Key

1. (a)	**2.** (c)	**3.** (d)	**4.** (b)	**5.** (d)	**6.** (a)
7. (c)	**8.** (a)	**9.** (b)	**10.** (b)	**11.** (c)	**12.** (b)
13. (b)	**14.** (c)				

1. **(a) :** $4KO_2 + 2CO_2 \rightarrow 2K_2CO_3 + 3O_2$
2. **(c) :** The negatively charged colloidal particles of impurities get neutralised by the Al^{3+} ions and settle down and pure water can be decanted off.
3. **(d) :** Be forms water soluble $BeSO_4$, water insoluble $Be(OH)_2$ and BeO. $Be(OH)_2$ is insoluble in NaOH giving sodium beryllate Na_2BeO_2.
4. **(b) :** The precipitate of mercuric iodide dissolves in excess of potassium iodide forming a complex, K_2HgI_4.
$$HgI_2 + 2KI \rightarrow K_2HgI_4$$
HgI_2 on heating liberates I_2 gas.
$$HgI_2 \rightleftharpoons Hg + \underset{\text{violet vapours}}{I_2}$$
5. **(d) :** When heated at red heat, $AgNO_3$ decomposes to metallic silver.
$$2AgNO_3 \rightarrow 2Ag + 2NO_2 + O_2$$
6. **(a) :** Mercury is such a metal which exists as liquid at room temperature.
7. **(c) :** $Mg_3N_2 + 6H_2O \rightarrow 3Mg(OH)_2 + 2NH_3$
8. **(a) :** Beryllium has the valency +2 while aluminium exhibits its valency as +3.
9. **(b) :** $Al_2Cl_6 + 12H_2O \rightleftharpoons 2[Al(H_2O)_6]^{3+} + 6Cl^-$
10. **(b) :** Grey tin is very brittle and easily crumbles down to a powder in very cold climates.
$$\underset{\text{(cubic)}}{\text{Grey tin}} \rightleftharpoons \underset{\text{(tetragonal)}}{\text{White tin}}$$
The change of white tin to grey tin is accompanied by increase in volume. This is called tin disease or tin plague.
11. **(c) :** CN^- ions act both as reducing agent as well as good complexing agent.
12. **(b) :** Calcium carbide is ionic carbide having $[:C \equiv C:]^{2-}$.
$$Ca^{2+}\,[:C \underset{2\,\pi}{\overset{1\,\sigma}{\equiv}} C:]^{2-}$$
13. **(b) :** Aluminium chloride in aqueous solution exists as ion pair.
$2AlCl_3 + aq. \rightarrow [AlCl_2(H_2O)_4]^+_{(aq)} + [AlCl_4(H_2O)_2]^-_{(aq)}$
The crystallisation of $AlCl_3$ from aqueous solution, therefore, yields an ionic solid of composition $[AlCl_2(H_2O)_4]^+\ [AlCl_4(H_2O)_2]^- \cdot xH_2O$. This compound decomposes at about 190°C to give the non-ionic dimer Al_2Cl_6.
$$[AlCl_2(H_2O)_4]^+\ [AlCl_4(H_2O)_2]^- \cdot xH_2O \xrightarrow[190°C]{\text{heat}} Al_2Cl_6 + H_2O$$
14. **(c) :** The elements of group 14 show an oxidation state of +4 and +2. The compounds showing an oxidation state of +4 are covalent compound and have tetrahedral structures. *e.g.* $SnCl_4$, $PbCl_4$, $SiCl_4$, etc. whereas those which show +2 oxidation state are ionic in nature and behave as reducing agent. *e.g.* $SnCl_2$, $PbCl_2$, etc.
Further as we move down the group, the tendency of the element to form covalent compound decreases but the tendency to form ionic compound increases.

CHAPTER
15

Chemistry of Representative Elements

1. In case of nitrogen, NCl_3 is possible but not NCl_5 while in case of phosphorus, PCl_3 as well as PCl_5 are possible. It is due to
(a) availability of vacant *d* orbitals in P but not in N
(b) lower electronegativity of P than N
(c) lower tendency of H-bond formation in P than N
(d) occurrence of P in solid while N in gaseous state at room temperature.
(2002)

2. Which one of the following statements is correct?
(a) Manganese salts give a violet borax test in the reducing flame.
(b) From a mixed precipitate of AgCl and AgI, ammonia solution dissolves only AgCl.
(c) Ferric ions give a deep green precipitate on adding potassium ferrocyanide solution.
(d) On boiling a solution having K^+, Ca^{2+} and HCO_3^- ions we get a precipitate of $K_2Ca(CO_3)_2$.
(2003)

3. Several blocks of magnesium are fixed to the bottom of a ship to
(a) keep away the sharks
(b) make the ship lighter
(c) prevent action of water and salt
(d) prevent puncturing by under-sea rocks.
(2003)

4. In curing cement plasters water is sprinkled from time to time. This helps in
(a) keeping it cool
(b) developing interlocking needle-like crystals of hydrated silicates
(c) hydrating sand and gravel mixed with cement
(d) converting sand into silicic acid.
(2003)

5. The solubilities of carbonates decrease down the magnesium group due to a decrease in
(a) lattice energies of solids
(b) hydration energies of cations
(c) inter-ionic attraction
(d) entropy of solution formation.
(2003)

6. The substance not likely to contain $CaCO_3$ is
(a) a marble statue
(b) calcined gypsum
(c) sea shells
(d) dolomite.
(2003)

7. Among Al_2O_3, SiO_2, P_2O_3 and SO_2 the correct order of acid strength is
(a) $SO_2 < P_2O_3 < SiO_2 < Al_2O_3$
(b) $SiO_2 < SO_2 < Al_2O_3 < P_2O_3$
(c) $Al_2O_3 < SiO_2 < SO_2 < P_2O_3$
(d) $Al_2O_3 < SiO_2 < P_2O_3 < SO_2$.
(2004)

8. Of the following outer electronic configurations of atoms, the highest oxidation state is achieved by which one of them?
(a) $(n-1)d^8ns^2$ (b) $(n-1)d^5ns^1$
(c) $(n-1)d^3ns^2$ (d) $(n-1)d^5ns^2$.
(2004)

9. Which of the following oxides is amphoteric in character?
(a) CaO (b) CO_2
(c) SiO_2 (d) SnO_2
(2005)

10. The structure of diborane (B_2H_6) contains
(a) four $2c$-$2e$ bonds and two $3c$-$2e$ bonds
(b) two $2c$-$2e$ bonds and four $3c$-$2e$ bonds
(c) two $2c$-$2e$ bonds and two $3c$-$3e$ bonds
(d) four $2c$-$2e$ bonds and four $3c$-$2e$ bonds
(2005)

11. What products are expected from the disproportionation reaction of hypochlorous acid?
(a) $HClO_3$ and Cl_2O
(b) $HClO_2$ and $HClO_4$
(c) HCl and Cl_2O
(d) HCl and $HClO_3$.
(2006)

12. The ionic mobility of alkali metal ions in aqueous solution is maximum for
(a) K^+ (b) Rb^+
(c) Li^+ (d) Na^+.
(2006)

13. Identify the wrong statement in the following.
(a) Acid rain is mostly because of oxides of nitrogen and sulphur
(b) Chlorofluorocarbons are responsible for ozone layer depletion
(c) Greenhouse effect is responsible for global warming
(d) Ozone layer does not permit infrared radiation from the sun to reach the earth
(2008)

14. Which one of the following is the correct statement?
(a) $B_2H_6 \cdot 2NH_3$ is known as 'inorganic benzene'.
(b) Boric acid is a protonic acid.
(c) Beryllium exhibits coordination number of six.
(d) Chlorides of both beryllium and aluminium have bridged chloride structures in solid phase.
(2008)

Answer Key

1.	(a)	**2.**	(b)	**3.**	(b)	**4.**	(b)	**5.**	(b)	**6.**	(b)
7.	(d)	**8.**	(b)	**9.**	(d)	**10.**	(a)	**11.**	(d)	**12.**	(b)
13.	(d)	**14.**	(d)								

1. **(a) :** $_7N = 1s^2\ 2s^2\ 3p^3$
$_{15}P = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^3$
In phosphorus the $3d$-orbitals are available.

2. **(b) :** The solubility product of AgCl, AgBr and AgI at the room temperature are 2.8×10^{-10}, 5.0×10^{-13} and 8.5×10^{-17} respectively. Thus, AgI is the least soluble silver halide.
The lattice energies of AgBr and AgI are even higher because of greater number of electrons in their anions. Consequently, they are even less soluble than AgCl. Due to greater solubility of AgCl than AgI, ammonia solution dissolves only AgCl and forms a complex.
$$AgCl + 2NH_4OH \rightarrow \underset{\text{Diammine silver chloride}}{[Ag(NH_3)_2]Cl} + 2H_2O$$

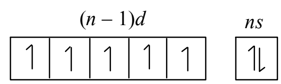
3. **(b) :** Magnesium, on account of its lightness, great affinity for oxygen and toughness is used in ship. Being a lighter element, magnesium makes the ship lighter when it is fixed to the bottom of the ship.

4. **(b) :** Water develops interlocking needle-like crystals of hydrated silicates. The reactions involved are the hydration of calcium aluminates and calcium silicates which change into their colloidal gels. At the same time, some calcium hydroxide and aluminium hydroxides are formed as precipitates due to hydrolysis. Calcium hydroxide binds the particles of calcium silicate together while aluminium hydroxide fills the interstices rendering the mass impervious.

5. **(b) :** The stability of the carbonates of the alkaline earth metals increases on moving down the group. The solubility of carbonate of metals in water is generally low. However they dissolve in water containing CO_2 yielding bicarbonates, and this solubility decreases on going down in a group with the increase in stability of carbonates of metals, and decrease in hydration energy of the cations.

6. **(b) :** The composition of gypsum is $CaSO_4 \cdot 2H_2O$. It does not have $CaCO_3$.

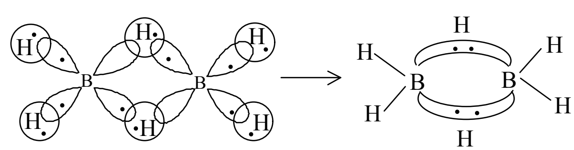
7. **(d) :** Acidity of the oxides of non metals increases with the electronegativity and oxidation number of the element.
$$Al_2O_3 < SiO_2 < P_2O_3 < SO_2$$
Al_2O_3 is amphoteric. SiO_2 is slightly acidic whereas P_2O_3 and SO_2 are the anhydrides of the acids H_3PO_3 and H_2SO_3.

8. **(b) :** 

$(n-1)d^5\ ns^2$ can achieve the maximum oxidation state of +7.

9. **(d) :** CaO-basic, CO_2 and SiO_2-acidic, SnO_2-amphoteric, as it reacts both with acids and bases.
$SnO_2 + 4HCl \rightarrow SnCl_4 + 2H_2O$
$SnO_2 + 2NaOH \rightarrow Na_2SnO_3 + H_2O$

10. **(a) :** According to molecular orbital theory, each of the two boron atoms is in sp^3 hybrid state. Of the four hybrid orbitals, three have one electron each while the fourth is empty. Two of the four orbitals of each of the boron atom overlap with two terminal hydrogen atoms forming two normal B – H σ-bonds. One of the remaining hybrid orbital (either filled or empty) of one of the boron atoms, $1s$ orbital of hydrogen atoms (bridge atom) and one of hybrid orbitals of the other boron atom overlap to form a delocalised orbital covering the three nuclei with a pair of electrons. Such a bond is known as three centre two electron ($3c - 2e$) bonds.



Structure of diborane

11. **(d) :** $3HClO_{4(aq)} \rightarrow HClO_{3(aq)} + 2HCl_{(aq)}$
It is a disproportionation reaction of hypochlorous

acid where the oxidation number of Cl changes from +1 (in ClO^-) to +5 (in ClO_3^-) and –1 (in Cl^-).

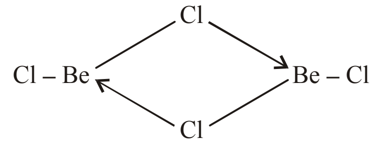
12. **(b) :** The alkali metal ion exist as hydrated ions $M^+(H_2O)_n$ in the aqueous solution. The degree of hydration, decreases with ionic size as we go down the group. Hence Li^+ ion is mostly hydrated *e.g.* $[Li(H_2O)_6]^+$. Since the mobility of ions is inversely proportional to the size of the their hydrated ions, hence the increasing order of ionic mobility is Li^+ ($Na^+ < K^+ < Rb^+$)

13. **(d) :** The thick layer of ozone called ozoneplanket which is effective in absorbing harmful ultraviolet rays given out by the sun acts as a protective shield. It does not permit the ultra violet rays from sun to reach the earth.

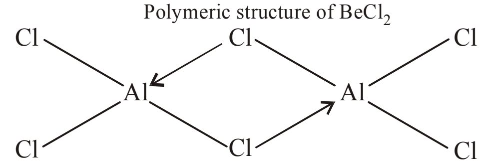
14. **(d) :** Boric acid is a weak monobasic acid ($K_a = 1.0 \times 10^{-9}$). It is a notable part that boric acid does not act as a protonic acid (*i.e.*, proton donor) but behaves as a Lewis acid by accepting a pair of electrons from OH^- ions.

$$B(OH)_3 + 2H_2O \rightarrow [B(OH)_4]^- + H_3O^+$$

$BeCl_2$ like Al_2Cl_6 has a bridged polymeric structure in solid phase generally as shown below.



Polymeric structure of $BeCl_2$



Polymeric structure of Al_2Cl_6

Beryllium exhibits coordination number of four as it has only four available orbitals in its valency shell.

Also,

$$3B_2H_6 + 6NH_3 \longrightarrow 3[BH_2(NH_3)_2]^+BH_4^-$$

or

$$B_2H_6 \cdot 2NH_3$$

or

$$B_2H_6 \cdot 2NH_3 \xrightarrow{\text{Heat, 450 K}} \underset{\text{Borazine}}{2B_3N_3H_6} + 12H_2$$

Borazine has structure similar to benzene and therefore,

it is called inorganic benzene. Hence option (d) is correct.

CHAPTER

16

Transition Metals including Lanthanides

1. Which of the following ions has the maximum magnetic moment?
   (a) Mn^{2+}
   (b) Fe^{2+}
   (c) Ti^{2+}
   (d) Cr^{2+}.

   (2002)

2. Most common oxidation states of Ce (cerium) are
   (a) +2, +3
   (b) +2, +4
   (c) +3, +4
   (d) +3, +5.

   (2002)

3. Arrange Ce^{3+}, La^{3+}, Pm^{3+} and Yb^{3+} in increasing order of their ionic radii.
   (a) $Yb^{3+} < Pm^{3+} < Ce^{3+} < La^{3+}$
   (b) $Ce^{3+} < Yb^{3+} < Pm^{3+} < La^{3+}$
   (c) $Yb^{3+} < Pm^{3+} < La^{3+} < Ce^{3+}$
   (d) $Pm^{3+} < La^{3+} < Ce^{3+} < Yb^{3+}$.

   (2002)

4. The most stable ion is
   (a) $[Fe(OH)_3]^{3-}$
   (b) $[Fe(Cl)_6]^{3-}$
   (c) $[Fe(CN)_6]^{3-}$
   (d) $[Fe(H_2O)_6]^{3+}$.

   (2002)

5. How do we differentiate between Fe^{3+} and Cr^{3+} in group III?
   (a) By taking excess of NH_4OH solution
   (b) By increasing NH_4^+ ion concentration
   (c) By decreasing OH^- ion concentration
   (d) Both (b) and (c).

   (2002)

6. A reduction in atomic size with increase in atomic number is a characteristic of elements of
   (a) high atomic masses
   (b) d-block
   (c) f-block
   (d) radioactive series.

   (2003)

7. The atomic numbers of vanadium (V), chromium (Cr), manganese (Mn) and iron (Fe) are respectively 23, 24, 25 and 26. Which one of these may be expected to have the highest second ionisation enthalpy?
   (a) V (b) Cr
   (c) Mn (d) Fe.

   (2003)

8. The number of d-electrons retained in Fe^{2+} (At. no. Fe = 26) ions is
   (a) 3 (b) 4
   (c) 5 (d) 6.

   (2003)

9. What would happen when a solution of potassium chromate is treated with an excess of dilute nitric acid?
   (a) Cr^{3+} and $Cr_2O_7^{2-}$ are formed
   (b) $Cr_2O_7^{2-}$ and H_2O are formed
   (c) CrO_4^{2-} is reduced to +3 state of Cr
   (d) CrO_4^{2-} is oxidised to +7 state of Cr.

   (2003)

10. The radius of La^{3+} (Atomic number of La = 57) is 1.06 Å. Which one of the following given values will be closest to the radius of Lu^{3+} (Atomic number of Lu = 71)?
    (a) 1.60 Å
    (b) 1.40 Å

(c) 1.06 Å
(d) 0.85 Å.

(2003)

11. Excess of KI reacts with $CuSO_4$ solution and then $Na_2S_2O_3$ solution is added to it. Which of the statements is incorrect for this reaction?
(a) Cu_2I_2 is formed
(b) CuI_2 is formed
(c) $Na_2S_2O_3$ is oxidised
(d) Evolved I_2 is reduced.

(2004)

12. Cerium (Z = 58) is an important member of the lanthanoids. Which of the following statements about cerium is incorrect?
(a) The common oxidation states of cerium are +3 and +4.
(b) The +3 oxidation state of cerium is more stable than +4 oxidation state.
(c) The +4 oxidation state of cerium is not known in solutions.
(d) Cerium (IV) acts as an oxidising agent.

(2004)

13. The correct order of magnetic moments (spin only values in B.M.) among is
(a) $[MnCl_4]^{2-} > [CoCl_4]^{2-} > [Fe(CN)_6]^{4-}$
(b) $[MnCl_4]^{2-} > [Fe(CN)_6]^{4-} > [CoCl_4]^{2-}$
(c) $[Fe(CN)_6]^{4-} > [MnCl_4]^{2-} > [CoCl_4]^{2-}$
(d) $[Fe(CN)_6]^{4-} > [CoCl_4]^{2-} > [MnCl_4]^{2-}$.
(Atomic nos.: Mn = 25, Fe = 26, Co = 27)

(2004)

14. Heating mixture of Cu_2O and Cu_2S will give
(a) $Cu + SO_2$
(b) $Cu + SO_3$
(c) $CuO + CuS$
(d) Cu_2SO_3

(2005)

15. The oxidation state of chromium in the final product formed by the reaction between KI and acidified potassium dichromate solution is
(a) +4
(b) +6
(c) +2
(d) +3

(2005)

16. Calomel (Hg_2Cl_2) on reaction with ammonium hydroxide gives
(a) $HgNH_2Cl$
(b) $NH_2 - Hg - Hg - Cl$
(c) Hg_2O
(d) HgO

(2005)

17. The lanthanide contraction is responsible for the fact that
(a) Zr and Y have about the same radius
(b) Zr and Nb have similar oxidation state
(c) Zr and Hf have about the same radius
(d) Zr and Zn have the same oxidation state

(2005)

18. Which of the following factors may be regarded as the main cause of lanthanide contraction?
(a) Poor shielding of one of 4*f* electron by another in the subshell
(b) Effective shielding of one of 4*f* electrons by another in the subshell
(c) Poorer shielding of 5*d* electrons by 4*f* electrons
(d) Greater shielding of 5*d* electrons by 4*f* electrons

(2006)

19. Nickel (Z = 28) combines with a uninegative monodentate ligand X^- to form a paramagnetic complex $[NiX_4]^{2-}$. The number of unpaired electron(s) in the nickel and geometry of this complex ion are, respectively
(a) one, tetrahedral
(b) two, tetrahedral
(c) one, square planar
(d) two, square planar.

(2006)

20. The "spin-only" magnetic moment [in units of Bohr magneton, (μ_B)] of Ni^{2+} in aqueous solution would be (atomic number of Ni = 28)
(a) 2.84 (b) 4.90
(c) 0 (d) 1.73

(2006)

21. Identify the incorrect statement among the following:
(a) 4*f* and 5*f* orbitals are equally shielded.
(b) *d*-Block elements show irregular and erratic chemical properties among themselves.
(c) La and Lu have partially filled *d*-orbitals and no other partially filled orbitals.

(d) The chemistry of various lanthanoids is very similar.

(2007)

22. The actinoids exhibit more number of oxidation states in general than the lanthanoids. This is because

(a) the $5f$ orbitals extend further from the nucleus than the $4f$ orbitals

(b) the $5f$ orbitals are more buried than the $4f$ orbitals

(c) there is a similarity between $4f$ and $5f$ orbitals in their angular part of the wave function

(d) the actinoids are more reactive than the lanthanoids.

(2007)

23. Larger number of oxidation states are exhibited by the actinoids than those by the lanthanoids, the main reason being

(a) More reactive nature of the actinoids than the lanthanoids

(b) $4f$ orbitals more diffused than the $5f$ orbitals

(c) Lesser energy difference between $5f$ and $6d$ than between $4f$ and $5d$ orbitals

(d) More energy difference between $5f$ and $6d$ than between $4f$ and $5d$ orbitals

(2008)

24. In which of the following octahedral complexes of Co (At. no. 27), will the magnitude of Δ_{oct} be the highest?

(a) $[Co(NH_3)_6]^{3+}$ (b) $[Co(CN)_6]^{3-}$

(c) $[Co(C_2O_4)_3]^{3-}$ (d) $[Co(H_2O)_6]^{3+}$

(2008)

Answer Key

1.	(a)	**2.**	(c)	**3.**	(a)	**4.**	(b)	**5.**	(d)	**6.**	(c)
7.	(b)	**8.**	(d)	**9.**	(b)	**10.**	(d)	**11.**	(b)	**12.**	(c)
13.	(a)	**14.**	(a)	**15.**	(d)	**16.**	(a)	**17.**	(c)	**18.**	(a)
19.	(b)	**20.**	(a)	**21.**	(a)	**22.**	(a)	**23.**	(c)	**24.**	(b)

1. **(a) :** Mn^{2+} ($3s^2 3p^6 3d^5$) has the maximum number of unpaired electrons (5) and therefore has maximum moment.

2. **(c) :** The common stable oxidation state of all the lanthanides is +3. The oxidation states of +2 and +4 are also exhibited and these oxidation states are only stable in those cases where stable $4f^0$, $4f^7$ or $4f^{14}$ configurations are achieved. Ce^{4+} is stable due to $4f^0$ configuration.

3. **(a) :** According to their positions in the periods, these values are in the order:

	Yb^{3+} <	Pm^{3+} <	Ce^{3+}	< La^{3+}
At. Nos.	70	61	58	57
Ionic radii (pm)	86	98	103	106

Ionic size decreases from La^{3+} to Lu^{3+} due to lanthanide contraction.

4. **(b) :** A more basic ligand forms stable bond with metal ion, Cl^- is most basic amongst all.

5. **(d) :** NH_4^+ ions are increased to suppress release of OH^- ions, hence solubility product of $Fe(OH)_3$ is attained. Colour of precipitate is different.

6. **(c) :** With increase in atomic number *i.e.* in moving down a group, the number of the principal shell increases and therefore, the size of the atom increases. But in case of *f* -block elements there is a steady decrease in atomic size with increase in atomic number due to lanthanide contraction.
As we move through the lanthanide series, 4*f* electrons are being added one at each step. The mutual shielding effect of *f* electrons is very little. This is due to the shape of the *f* -orbitals. The nuclear charge, however increases by one at each step. Hence, the inward pull experienced by the 4*f* electrons increases. This causes a reduction in the size of the entire $4f^n$ shell.

7. **(b) :** The second ionisation potential values of Cu and Cr are sufficiently higher than those of neighbouring elements. This is because of the electronic configuration of Cu^+ which is $3d^{10}$ (completely filled) and of Cr^+ which is $3d^5$ (half-filled), *i.e.*, for the second ionisation potentials, the electron is to be removed from very stable configurations.

8. **(d) :** ${}_{26}Fe = 1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p^6\, 3d^6\, 4s^2$
$Fe^{++} = 1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p^6\, 3d^6$
The number of *d* -electrons retained in Fe^{2+} = 6.

9. **(b) :** Dilute nitric acid converts chromate into dichromate and H_2O.

$$2K_2CrO_4 + 2HNO_3 \rightarrow K_2Cr_2O_7 + 2KNO_3 + H_2O$$

or, $$\underset{\text{yellow}}{2CrO_4^{2-}} \overset{H^+}{\rightleftharpoons} \underset{\text{orange}}{Cr_2O_7^{2-}} + H_2O$$

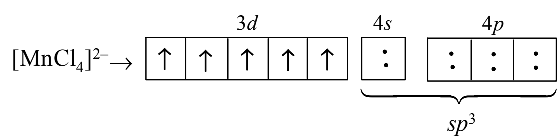
10. **(d) :** Due to lanthanide contraction, the ionic radii of Ln^{3+} (lanthanide ions) decreases from La^{3+} to Lu^{3+}. Thus the lowest value (here 0.85 Å) is the ionic radius of Lu^{3+}.

11. **(b) :** $$4K\overset{-1}{I} + 2CuSO_4 \longrightarrow \overset{0}{I_2} + Cu_2I_2 + 2K_2SO_4$$

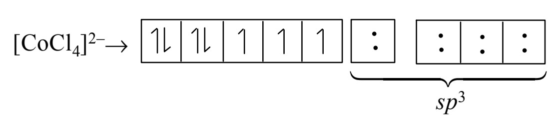
$$\overset{0}{I_2} + 2Na_2\overset{+2}{S_2}O_3 \longrightarrow Na_2\overset{+2.5}{S_4}O_6 + 2Na\overset{-1}{I}$$

12. **(c) :** +4 oxidation state of cerium is also known in solutions.

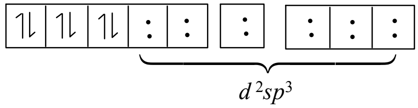
13. **(a) :**



Number of unpaired electrons = 5



Number of unpaired electrons = 3

$[Fe(CN)_6]^{4-} \rightarrow$ 

Number of unpaired electrons = 0

Magnetic moment = $n\sqrt{n+2}$

where n = number of unpaired electrons.

i.e. greater the number of unpaired electrons, greater will be the paramagnetic character.

14. **(a) :** $Cu_2S + 2Cu_2O \rightarrow 6Cu + SO_2$

This is an example of auto-reduction.

15. (d) :

$$\overset{+6}{Cr_2}O_7^{2-} + 14H^+ + 6I^- \longrightarrow 2\overset{+3}{Cr^{3+}} + 7H_2O + 3I_2$$

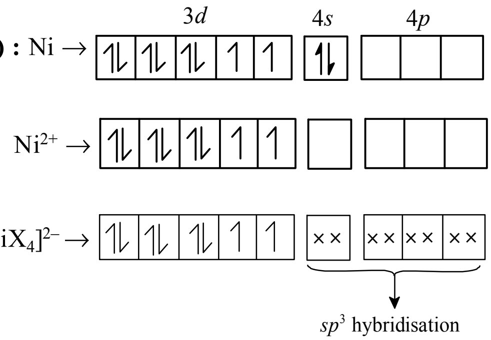
16. **(a) :** Calomel on reaction with ammonium hydroxide turns black. The black substance is a mixture of mercury and mercuric amino chloride.

$Hg_2Cl_2 + 2NH_4OH \rightarrow$

$$\underbrace{Hg\begin{matrix} \diagup NH_2 \\ \diagdown Cl \end{matrix}}_{\text{Black}} + Hg + NH_4Cl + 2H_2O$$

17. **(c) :** In each vertical column of transition elements, the elements of second and third transition series resemble each other more closely than the elements of first and second transition series on account of lanthanide contraction. The pairs of elements such as Zr-Hf, Mo-W, Nb-Ta etc; possess almost the same properties.

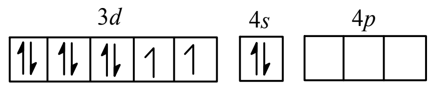
18. **(a) :** As we proceed from one element to the next element in the lanthanide series, the nuclear charge, *i.e.* atomic number increases by one unit and the addition of one electron occurs at the same time in 4*f*-energy shell. On account of the very diffused shapes of *f*-orbitals, the 4*f*-electrons shield each other quite poorly from the nuclear charge. Thus, the effect of nuclear charge increase is somewhat more than the changed shielding effect. This brings the valence shell nearer to the nucleus and hence the size of atom or ion goes on decreasing as we move in the series. The sum of the successive reactions is equal to the total lanthanide contraction.

19. **(b) :** 

Number of unpaired electrons = 2

Geometry = tetrahedral.

20. **(a) :** $_{28}Ni \rightarrow [Ar]\ 3d^8\ 4s^2$



Number of unpaired electrons (n) = 2

$\mu = \sqrt{n(n+2)} = \sqrt{2(2+2)} = \sqrt{8} \approx 2.84$

21. **(a) :** The decrease in the force of attraction exerted by the nucleus on the valency electrons due to presence of electrons in the inner shells is called shielding effect. An 4*f* orbital is nearer to the nucleus than 5*f* orbitals. Hence shielding of 4*f* is more than 5*f*.

22. **(a) :** As the distance between the nucleus and 5*f* orbitals (actinides) is more than the distance between the nucleus and 4*f* orbitals (lanthanides) hence the hold of nucleus on valence electron decreases in actinides. For this reason the actinoides exhibit more number of oxidation states in general.

23. **(c) :** Actinoids show different oxidation states such as +2, +3, +4, +5, +6 and +7. However +3 oxidation state is most common among all the actinoids. The wide range of oxidation states of actinoids is attributed to the fact that the 5*f*, 6*d* and 7*s* energy levels are of comparable energies. Therefore all these three subshells can participate.

24. **(b) :** Strong field ligand such as CN^-, usually produce low spin complexes and large crystal field splittings. H_2O is a weaker field ligand than NH_3 and $C_2O_4^{2-}$ therefore

$\Delta_{oct}\,[Co(H_2O)_6]^{3+} < \Delta_{oct}[Co(C_2O_4)]^{3-} < [Co(NH_3)_6]^{3+}$

Common ligands in order of increasing crystal field strength are given below :

$I^- < Br^- < Cl^- < F^- < OH^- < C_2O_4^{2-} < H_2O < NH_3 < en < NO_2^- < CN^-$

CHAPTER

17

Coordination Compounds & Organometallics

1. $CH_3 - Mg - Br$ is an organometallic compound due to
(a) Mg – Br bond (b) C – Mg bond
(c) C – Br bond (d) C – H bond.
(2002)

2. The type of isomerism present in nitropentamine chromium (III) chloride is
(a) optical (b) linkage
(c) ionization (d) polymerisation.
(2002)

3. In the coordination compound, $K_4[Ni(CN)_4]$, the oxidation state of nickel is
(a) –1 (b) 0
(c) +1 (d) +2.
(2003)

4. Ammonia forms the complex ion $[Cu(NH_3)_4]^{2+}$ with copper ions in alkaline solutions but not in acidic solutions. What is the reason for it?
(a) In acidic solutions hydration protects copper ions.
(b) In acidic solutions protons coordinate with ammonia molecules forming NH_4^+ ions and NH_3 molecules are not available.
(c) In alkaline solutions insoluble $Cu(OH)_2$ is precipitated which is soluble in excess of any alkali.
(d) Copper hydroxide is an amphoteric substance.
(2003)

5. One mole of the complex compound $Co(NH_3)_5Cl_3$, gives 3 moles of ions on dissolution in water. One mole of the same complex reacts with two moles of $AgNO_3$ solution to yield two moles of AgCl (*s*). The structure of the complex is
(a) $[Co(NH_3)_5Cl]Cl_2$
(b) $[Co(NH_3)_3Cl_2]\cdot 2NH_3$
(c) $[Co(NH_3)_4Cl_2]Cl\cdot NH_3$
(d) $[Co(NH_3)_4Cl]Cl_2\cdot NH_3$.
(2003)

6. The coordination number of a central metal atom in a complex is determined by
(a) the number of ligands around a metal ion bonded by sigma bonds
(b) the number of ligands around a metal ion bonded by pi-bonds
(c) the number of ligands around a metal ion bonded by sigma and pi-bonds both
(d) the number of only anionic ligands bonded to the metal ion.
(2004)

7. Which one of the following complexes is an outer orbital complex?
(a) $[Fe(CN)_6]^{4-}$ (b) $[Mn(CN)_6]^{4-}$
(c) $[Co(NH_3)_6]^{3+}$ (d) $[Ni(NH_3)_6]^{2+}$.
[Atomic nos.: Mn = 25, Fe = 26, Co = 27, Ni = 28]
(2004)

8. Coordination compounds have great importance in biological systems. In this context which of the following statements is incorrect?
(a) Chlorophylls are green pigments in plants and contain calcium.
(b) Haemoglobin is the red pigment of blood and contains iron.
(c) Cyanocobalamine is B_{12} and contains cobalt.
(d) Carboxypeptidase-A is an enzyme and contains zinc.
(2004)

9. Which one of the following has largest number of isomers?
(a) $[Ru(NH_3)_4Cl_2]^+$ (b) $[Co(NH_3)_5Cl]^{2+}$
(c) $[Ir(PR_3)_2H(CO)]^{2+}$ (d) $[Co(en)_2Cl_2]^+$
(R = alkyl group, en = ethylenediamine)
(2004)

10. The oxidation state of Cr in $[Cr(NH_3)_4Cl_2]^+$ is
(a) +3 (b) +2
(c) +1 (d) 0
(2005)

11. The IUPAC name of the coordination compound $K_3[Fe(CN)_6]$ is
(a) potassium hexacyanoferrate (II)
(b) potassium hexacyanoferrate (III)
(c) potassium hexacyanoiron (II)
(d) tripotassium hexacyanoiron (II)
(2005)

12. Which of the following compounds shows optical isomerism?
(a) $[Cu(NH_3)_4]^{2+}$ (b) $[ZnCl_4]^{2-}$
(c) $[Cr(C_2O_4)_3]^{3-}$ (d) $[Co(CN)_6]^{3-}$
(2005)

13. Which one of the following cyano complexes would exhibit the lowest value of paramagnetic behaviour?
(a) $[Cr(CN)_6]^{3-}$ (b) $[Mn(CN)_6]^{3-}$
(c) $[Fe(CN)_6]^{3-}$ (d) $[Co(CN)_6]^{3-}$
(2005)

14. The value of the 'spin only' magnetic moment for one of the following configurations is 2.84 BM. The correct one is
(a) d^4 (in strong ligand field)
(b) d^4 (in weak ligand field)
(c) d^3 (in weak as well as in strong fields)
(d) d^5 (in strong ligand field)
(2005)

15. The IUPAC name for the complex $[Co(NO_2)(NH_3)_5]Cl_2$ is
(a) nitrito-N-pentaamminecobalt(III) chloride
(b) nitrito-N-pentaamminecobalt(II) chloride
(c) pentaammine nitrito-N-cobalt(II) chloride
(d) pentaammine nitrito-N-cobalt(III) chloride.
(2006)

16. In $Fe(CO)_5$, the Fe – C bond possesses
(a) π-character only (b) both σ and π characters
(c) ionic character (d) σ-character only.
(2006)

17. How many EDTA (ethylenediaminetetraacetic acid) molecules are required to make an octahedral complex with a Ca^{2+} ion?
(a) Six (b) Three
(c) One (d) Two.
(2006)

18. Which of the following has a square planar geometry?
(a) $[PtCl_4]^{2-}$ (b) $[CoCl_4]^{2-}$
(c) $[FeCl_4]^{2-}$ (d) $[NiCl_4]^{2-}$.
(At. nos.: Fe = 26, Co = 27, Ni = 28, Pt = 78)
(2007)

19. The coordination number and the oxidation state of the element E in the complex $[E(en)_2(C_2O_4)]NO_2$ (where (en) is ethylene diamine) are, respectively
(a) 6 and 3 (b) 6 and 2
(c) 4 and 2 (d) 4 and 3
(2008)

Answer Key

1.	(b)	2.	(b)	3.	(b)	4.	(b)	5.	(a)	6.	(a)
7.	(d)	8.	(a)	9.	(d)	10.	(a)	11.	(b)	12.	(c)
13.	(d)	14.	(a)	15.	(d)	16.	(b)	17.	(c)	18.	(a)
19.	(a)										

1. **(b) :** Compounds that contain at least one carbon-metal bond are called organometallic compounds.

2. **(b) :** The nitro group can attach to metal through nitrogen as ($-NO_2$) or through oxygen as nitrito ($-ONO$).

3. **(b) :** Let the oxidation number of
Ni in $K_4[Ni(CN)_4] = x$
$1 \times 4 + x \times (-1) \times 4 = 0 \Rightarrow 4 + x - 4 = 0 \Rightarrow x = 0.$

4. **(b) :** In acidic solution, NH_3 forms a bond with H^+ to give NH_4^+ ion which does not have a lone pair on N atom. Hence it cannot act as a ligand.

5. **(a) :** Given reactions can be explained as follows:
$[Co(NH_3)_5Cl]Cl_2 \rightleftharpoons [Co(NH_3)_5Cl]^{2+} + 2Cl^- \Rightarrow 3$ ions.
$[Co(NH_3)_5Cl]Cl_2 + 2AgNO_3 \rightarrow [Co(NH_3)_5Cl](NO_3)_2 + 2AgCl$

6. **(a) :** The number of atoms of the ligands that are directly bound to the central metal atom or ion by coordinate bonds is known as the coordination number of the metal atom or ion.
Coordination number of metal = number of σ bonds formed by metal with ligands.

7. **(d) :**

Complex ion	Hybridization of central ion
$[Fe(CN)_6]^{4-}$	d^2sp^3 (inner)
$[Mn(CN)_6]^{4-}$	d^2sp^3 (inner)
$[Co(NH_3)_6]^{3+}$	d^2sp^3 (inner)
$[Ni(NH_3)_6]^{2+}$	sp^3d^2 (outer)

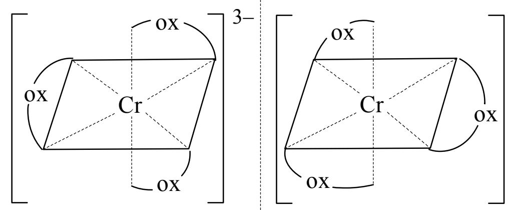
8. **(a) :** Chlorophyll are green pigments in plants and contains magnesium instead of calcium.

9. **(d) :** $[Co(en)_2Cl_2]^+$ shows geometrical as well as optical isomerism.

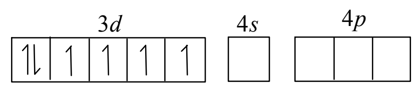
10. **(a) :** Let the oxidation state of Cr in $[Cr(NH_3)_4Cl_2]^+ = x$
$x + 4(0) + 2(-1) = +1$
$x - 2 = +1$ or, $x = +1 + 2 = +3.$

11. **(b) :** $K_3[Fe(CN)_6]$
Potassium hexacyanoferrate(III)

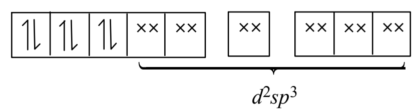
12. **(c) :** Optical isomers rarely occur in square planar complexes on account of the presence of axis of symmetry. Optical isomerism is very common in octahedral complexes having general formulae: $[Ma_2b_2c_2]^{n\pm}$, $[Mabcdef]^{n\pm}$, $[M(AA)_3]^{n\pm}$, $[M(AA)_2a_2]^{n\pm}$, $[M(AA)_2ab]^{n\pm}$ and $[M(AB)_3]^{n\pm}$ (where AA = symmetrical bidentate ligand and AB = unsymmetrical bidentate ligand).



13. **(d) :** $[Co(CN)_6]^{3-}$
$Co \rightarrow [Ar]\ 3d^7\ 4s^2$
$Co^{3+} \rightarrow [Ar]\ 3d^6\ 4s^0$



In presence of strong field ligand CN^- pairing of electrons takes place.



There is no unpaired electron, so the lowest value of paramagnetic behaviour is observed.

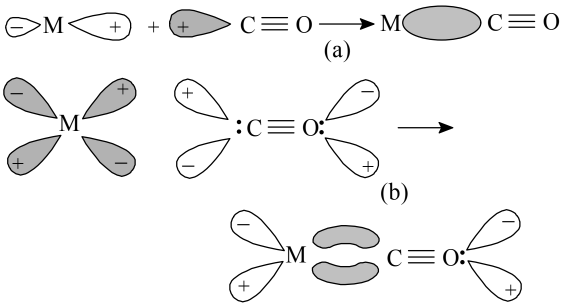
14. **(a) :** Spin only magnetic moment $= \sqrt{n(n+2)}$ B.M.
Where n = no. of unpaired electron.
Given, $\sqrt{n(n+2)} = 2.84$
or, $n(n+2) = 8.0656$
or, $n = 2$
In an octahedral complex, for a d^4 configuration in a strong field ligand, number of unpaired electrons = 2

15. **(d) :** $[Co(NO_2)(NH_3)_5]Cl_2$
pentaaminenitrito-N-cobalt(III) chloride

16. **(b) :** In a metal carbonyl, the metal carbon bond possesses both the σ- and π-character. A σ-bond between metal and carbon atom is formed when a vacant hybrid bond of the metal atom overlaps with

an orbital of C atom of carbon monoxide containing a lone pair of electrons.

Formation of π-bond is caused when a filled orbital of the metal atom overlaps with a vacant antibonding π^* orbital of C atom of CO. This overlap is also called back donation of electrons by metal atom to carbon.



(a) The formation of the metal ← carbon σ-bond using an unshared pair of the C atom. (b) The formation of the metal → carbon π-bond.

The π-overlap is perpendicular to the nodal plane of σ-bond.

17. **(c) :** EDTA, which has four donor oxygen atoms and two donor nitrogen atoms in each molecule forms complex with Ca^{2+} ion. The free acid H_4EDTA is insoluble and the disodium salt Na_2H_2EDTA is the most used reagent.

$Ca^{2+} + [H_2EDTA]^{2-} \rightarrow [Ca(EDTA)]^{2-} + 2H^+$

18. **(a) :** In 4-coordinate complexes Pt, the four ligands are arranged about the central 2-valent platinum ion in a square planar configuration.

19. **(a) :** In the given complex $[E(en)_2(C_2O_4)]^+NO_2^-$ ethylene diamine is a bidentate ligand and $(C_2O_4^{2-})$ oxalate ion is also bidentate ligand. Therefore co-ordination number of the complex is 6 *i.e.*, it is an octahedral complex.

Oxidation number of E in the given complex is

$x + 2 \times 0 + 1 \times (-2) = +1$

$\therefore\ x = 3$

CHAPTER 18

General Organic Chemistry

1. Arrangement of $(CH_3)_3C–$, $(CH_3)_2CH–$, CH_3CH_2 – when attached to benzyl or an unsaturated group in increasing order of inductive effect is

(a) $(CH_3)_3C - < (CH_3)_2CH - < CH_3CH_2 -$
(b) $CH_3CH_2 - < (CH_3)_2CH - < (CH_3)_3C -$
(c) $(CH_3)_2CH - < (CH_3)_3C - < CH_3CH_2 -$
(d) $(CH_3)_3C - < CH_3CH_2 - < (CH_3)_2CH -$.

(2002)

2. Racemic mixture is formed by mixing two

(a) isomeric compounds
(b) chiral compounds
(c) meso compounds
(d) optical isomers.

(2002)

3. A similarity between optical and geometrical isomerism is that

(a) each forms equal number of isomers for a given compound
(b) if in a compound one is present then so is the other
(c) both are included in stereoisomerism
(d) they have no similarity.

(2002)

4. Which of the following does not show geometrical isomerism?

(a) 1,2-dichloro-1-pentene
(b) 1,3-dichloro-2-pentene
(c) 1,1-dichloro-1-pentene
(d) 1,4-dichloro-2-pentene.

(2002)

5. The reaction :

$$(CH_3)_3C - Br \xrightarrow{H_2O} (CH_3)_3C - OH$$

(a) elimination reaction
(b) substitution reaction
(c) free radical reaction
(d) displacement reaction.

(2002)

6. In a compound C, H and N atoms are present in 9 : 1 : 3.5 by weight. Molecular weight of compound is 108. Molecular formula of compound is

(a) $C_2H_6N_2$ (b) C_3H_4N
(c) $C_6H_8N_2$ (d) $C_9H_{12}N_3$.

(2002)

7. Among the following four structures I to IV,

$C_2H_5 - \overset{CH_3}{\overset{|}{CH}} - C_3H_7$ (I)

$CH_3 - \overset{O}{\overset{||}{C}} - \overset{CH_3}{\overset{|}{CH}} - C_2H_5$ (II)

$H - \overset{H}{\overset{|}{\underset{\underset{H}{|}}{C}}}{}^{\oplus}$ (III)

$C_2H_5 - \overset{CH_3}{\overset{|}{CH}} - C_2H_5$ (IV)

it is true that

(a) all four are chiral compounds
(b) only I and II are chiral compounds
(c) only III is a chiral compound
(d) only II and IV are chiral compounds.

(2003)

8. The compound formed in the positive test for nitrogen with the Lassaigne solution of an organic compound is

(a) $Fe_4[Fe(CN)_6]_3$ (b) $Na_3[Fe(CN)_6]$
(c) $Fe(CN)_3$ (d) $Na_4[Fe(CN)_5NOS]$.

(2004)

9. The ammonia evolved from the treatment of 0.30 g of an organic compound for the estimation of nitrogen was passed in 100 mL of 0.1 M sulphuric acid. The excess of acid required 20 mL of 0.5 M sodium hydroxide solution for complete neutralization. The organic compound is

(a) acetamide (b) benzamide
(c) urea (d) thiourea.

(2004)

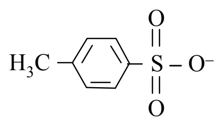
10. Due to the presence of an unpaired electron, free radicals are

(a) chemically reactive
(b) chemically inactive
(c) anions
(d) cations

(2005)

11. The decreasing order of nucleophilicity among the nucleophiles is

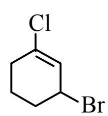
(1) $CH_3C(=O)-O^-$ (2) CH_3O^-

(3) CN^- (4) 

(a) 1, 2, 3, 4 (b) 4, 3, 2, 1
(c) 2, 3, 1, 4 (d) 3, 2, 1, 4.

(2005)

12. The IUPAC name of the compound shown below is



(a) 2-bromo-6-chlorocyclohex-1-ene
(b) 6-bromo-2-chlorocyclohexene
(c) 3-bromo-1-chlorocyclohexene
(d) 1-bromo-3-chlorocyclohexene.

(2006)

13. $CH_3Br + Nu^- \rightarrow CH_3 - Nu + Br^-$
The decreasing order of the rate of the above reaction with nucleophiles (Nu^-) *A* to *D* is
[Nu^- = (*A*) PhO^-, (*B*) AcO^-, (*C*) HO^-,
(*D*) CH_3O^-]

(a) $D > C > A > B$ (b) $D > C > B > A$
(c) $A > B > C > D$ (d) $B > D > C > A$.

(2006)

14. The increasing order of stability of the following free radicals is

(a) $(CH_3)_2\dot{C}H < (CH_3)_3\dot{C} < (C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C}$
(b) $(C_6H_5)_3\dot{C} < (C_6H_5)_2\dot{C}H < (CH_3)_3\dot{C} < (CH_3)_2\dot{C}H$
(c) $(C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C} < (CH_3)_3\dot{C} < (CH_3)_2\dot{C}H$
(d) $(CH_3)_2\dot{C}H < (CH_3)_3\dot{C} < (C_6H_5)_3\dot{C} < (C_6H_5)_2\dot{C}H$

(2006)

15. Increasing order of stability among the three main conformations (*i.e.* eclipse, anti, gauche) of 2-fluoroethanol is

(a) eclipse, gauche, anti
(b) gauche, eclipse, anti
(c) eclipse, anti, gauche
(d) anti, gauche, eclipse.

(2006)

16. Which of the following is the correct order of decreasing S_N2 reactivity?

(a) $R_2CHX > R_3CX > RCH_2X$
(b) $RCHX > R_3CX > R_2CHX$
(c) $RCH_2X > R_2CHX > R_3CX$
(d) $R_3CX > R_2CHX > RCH_2X$.
(*X* is a halogen)

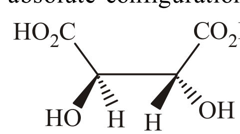
(2007)

17. Which one of the following conformations of cyclohexane is chiral?

(a) Boat (b) Twist boat
(c) Rigid (d) Chair.

(2007)

18. The absolute configuration of

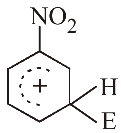
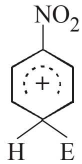


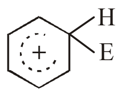
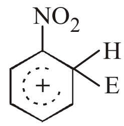
is

(a) *S*, *R* (b) *S*, *S*
(c) *R*, *R* (d) *R*, *S*

(2008)

19. The electrophile, $E^{\oplus}$ attacks the benzene ring to generate the intermediate σ-complex. Of the following, which σ-complex is of lowest energy?

(a)  (b) 

(c)  (d) 

(2008)

20. The correct decreasing order of priority for the functional groups of organic compounds in the IUPAC system of nomenclature is

(a) $-CONH_2$, $-CHO$, $-SO_3H$, $-COOH$

(b) $-COOH$, $-SO_3H$, $-CONH_2$, $-CHO$

(c) $-SO_3H$, $-COOH$, $-CONH_2$, $-CHO$

(d) $-CHO$, $-COOH$, $-SO_3H$, $-CONH_2$

(2008)

Answer Key

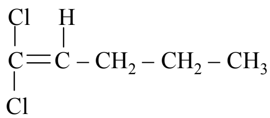
1.	(b)	**2.**	(d)	**3.**	(c)	**4.**	(c)	**5.**	(b)	**6.**	(c)
7.	(b)	**8.**	(a)	**9.**	(c)	**10.**	(a)	**11.**	(d)	**12.**	(c)
13.	(a)	**14.**	(a)	**15.**	(a)	**16.**	(c)	**17.**	(b)	**18.**	(c)
19.	(c)	**20.**	(c)								

1. **(b) :** $-CH_3$ group has +I effect, as number of $-CH_3$ group increases the inductive effect increases.

2. **(d) :** An equimolar mixture of two *i.e.* dextro and laevorotatory optical isomers is termed as racemic mixture or *dl* form or (±) mixture.

3. **(c) :** Both involves compounds having the same molecular and structural formulae, but different spatial arrangement of atoms or groups.

4. **(c) :**



Condition for geometrical isomerism is presence of two different atoms of groups attached to each carbon atom containing double bond.
Identical groups (Cl) on C - 1 will give only one compound. Hence it does not show geometrical isomerism.

5. **(b) :** This is an example of nucleophilic substitution reaction.

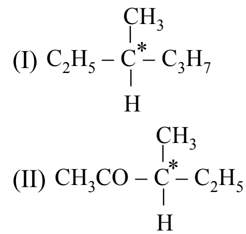
$$\underset{\text{Substrate}}{(CH_3)_3C-Br} + \underset{\text{nucleophile}}{OH^-} \longrightarrow (CH_3)_3C-OH + \underset{\text{Leaving group}}{Br^-}$$

6. **(c) :**

C		H		N
9	:	1	:	3.5
$\frac{9}{12}$	:	$\frac{1}{1}$	:	$\frac{3.5}{14}$
$\frac{3}{4}$	:	$\frac{1}{1}$	:	$\frac{1}{4}$
3	:	4	:	1

Empirical formula = C_3H_4N
$(C_3H_4N)_n = 108$
$(12 \times 3 + 1 \times 4 + 14)_n = 108$
$54n = 108 \Rightarrow n = 108/54 = 2$
Molecular formula = $C_6H_8N_2$.

7. **(b) :** A chiral object or compound can be defined as the one that is not superimposable on its mirror image, or we can say that all the four groups attached to a carbon atom must be different.
Only I and II are chiral compounds.



8. **(a) :** $3Na_4[Fe(CN)_6] + 4Fe^{3+} \rightarrow \underset{\text{Prussian blue}}{Fe_4[Fe(CN)_6]_3} + 12Na^+$

9. **(c) :** Equivalents of NH_3 evolved

$$= \frac{100 \times 0.1 \times 2}{1000} - \frac{20 \times 0.5}{1000} = \frac{1}{100}$$

Percent of nitrogen in the unknown organic compound $= \frac{1}{100} \times \frac{14}{0.3} \times 100 = 46.6$

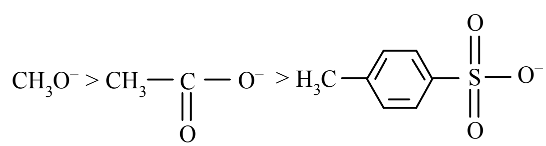
Percentage of nitrogen in urea $(NH_2)_2CO$

$$= \frac{14 \times 2}{60} \times 100 = 46.6$$

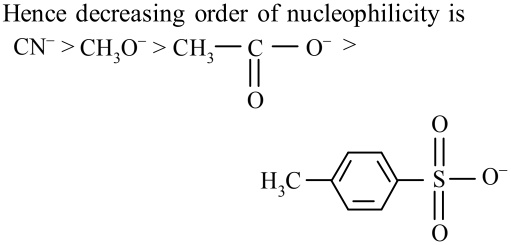
∴ The compound must be urea.

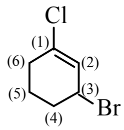
10. **(a) :** Free radicals are highly reactive due to presence of an unpaired electron. They readily try to pair-up the odd electrons.

11. **(d) :** Strong bases are generally good nucleophile. If the nucleophilic atom or the centre is the same, nucleophilicity parallels basicity, *i.e.*, more basic the species, stronger is the nucleophile. Hence basicity as well as nucleophilicity order is



Now CN^- is a better nucleophile than CH_3O^-. Hence decreasing order of nucleophilicity is



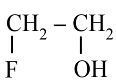
12. **(c) :**  3-bromo-1-chlorocyclohexene

13. **(a) :** If the nucleophilic atom or the centre is same, nucleophilicity parallels basicity, *i.e.* more basic the species stronger is the nucleophile.

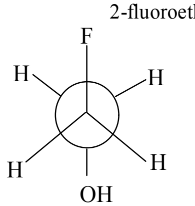
$$CH_3O^- > HO^- > PhO^- > AcO^-$$

Here, the nucleophilic atom *i.e.* O is the same in all these species. This order can be easily explained on the general concept that a weaker acid has a stronger conjugate base.

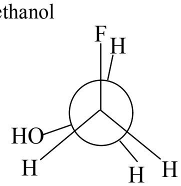
14. **(a) :** On the basis of hyperconjugation effect of the alkyl groups, the order of stability of free radical is as follows: tertiary > secondary > primary.
Benzyl free radicals are stabilised by resonance and hence are more stable than alkyl free radicals. Further as the number of phenyl group attached to the carbon atom holding the odd electron increases, the stability of a free radical increases accordingly.
i.e. $(CH_3)_2\dot{C}H < (CH_3)_3\dot{C} < (C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C}$

15. **(a) :** 

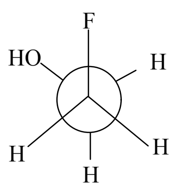
2-fluoroethanol



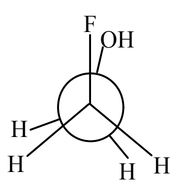
anti or staggered



eclipsed



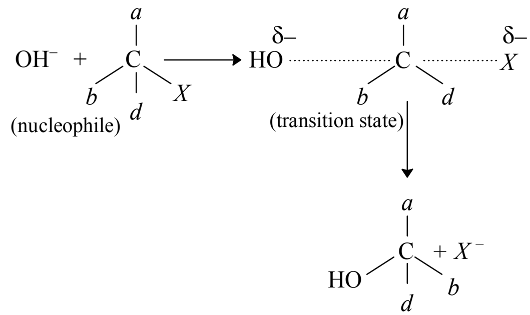
skew or gauche



fully eclipsed

The anti conformation is most stable in which F and OH groups are far apart as possible and minimum repulsion between two groups occurs.
In fully eclipsed conformation F and OH groups are so close that the steric strain is maximum, hence this conformation is most unstable. The order of stability of these conformations is
anti > gauche > partially eclipsed > fully eclipsed

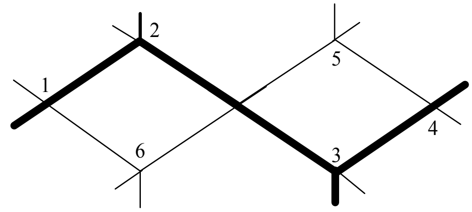
16. **(c) :** S_N2 mechanism occurs as



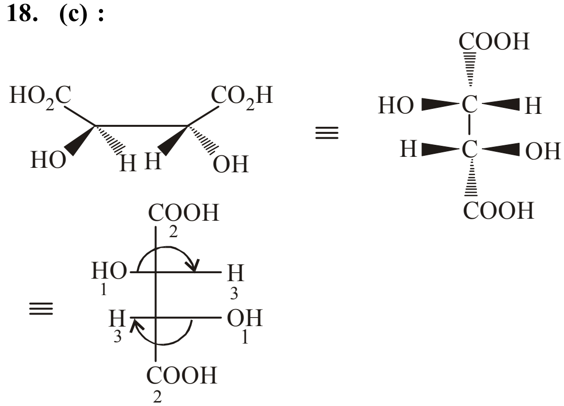
In S_N2 reaction, in the transition state there will be five groups attached to the carbon atom at which reaction occurs. Thus there will be crowding in the transition state, and the bulkier the group, the more the reaction will be hindered sterically. Hence S_N2 reaction is favoured by small groups on the carbon atom attached to halogens. So the decreasing order of reactivity of halide is

$$\underset{\text{(primary)}}{RCH_2X} > \underset{\text{(secondary)}}{R_2CHX} > \underset{\text{(tertiary)}}{R_3CX}$$

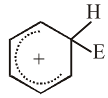
17. **(b) :** The twist boat conformation of cyclohexane is optically active as it does not have any plane of symmetry.



18. **(c) :**



(R, R)

19. **(c) :** 

This structure will be of lowest energy due to resonance stabilisation of +ve charge. In all other three structures, the presence of electron-withdrawing NO_2 group will destabilize the +ve charge and hence they will have greater energy.

20. **(c) :** The order of preference of functional groups is as follows:

$-SO_3H > -COOH > -COOR > -COX >$
$-COCl > -CONH_2 > -CHO > -CN > >C=O$
$> -OH > -SH > -NH_2 > >C=C< > -C \equiv C- >$
$-NO_2 > -NO > -X$

Hydrocarbons

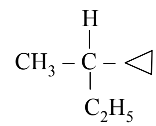
1. Which of these will not react with acetylene?
(a) NaOH (b) ammonical $AgNO_3$
(c) Na (d) HCl.
(2002)

2. What is the product when acetylene reacts with hypochlorous acid?
(a) CH_3COCl (b) $ClCH_2CHO$
(c) Cl_2CHCHO (d) ClCHCOOH.
(2002)

3. Butene-1 may be converted to butane by reaction with
(a) Zn - HCl (b) Sn - HCl
(c) Zn - Hg (d) Pd/H_2.
(2003)

4. On mixing a certain alkane with chlorine and irradiating it with ultraviolet light, it forms only one monochloroalkane. This alkane could be
(a) propane (b) pentane
(c) isopentane (d) neopentane.
(2003)

5. Which one of the following has the minimum boiling point?
(a) *n*-butane (b) 1-butyne
(c) 1-butene (d) isobutene.
(2004)

6. Amongst the following compounds, the optically active alkane having lowest molecular mass is
(a) $CH_3 - CH_2 - CH_2 - CH_3$
(b) $CH_3 - CH_2 - \underset{}{\overset{\overset{\large CH_3}{|}}{CH}} - CH_3$
(c) 
(d) $CH_3 - CH_2 - C \equiv CH$.
(2004)

7. Which one of the following is reduced with zinc and hydrochloric acid to give the corresponding hydrocarbon?
(a) Ethyl acetate (b) Acetic acid
(c) Acetamide (d) Butan-2-one.
(2004)

8. 2-Methylbutane on reacting with bromine in the presence of sunlight gives mainly
(a) 1-bromo-2-methylbutane
(b) 2-bromo-2-methylbutane
(c) 2-bromo-3-methylbutane
(d) 1-bromo-3-methylbutane
(2005)

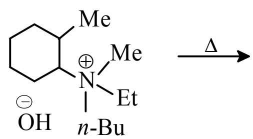
9. Reaction of one molecule of HBr with one molecule of 1, 3-butadiene at 40°C gives predominantly
(a) 3-bromobutene under kinetically controlled conditions
(b) 1-bromo-2-butene under thermodynamically controlled conditions
(c) 3-bromobutene under thermodynamically controlled conditions
(d) 1-bromo-2-butene under kinetically controlled conditions.
(2005)

10. Of the five isomeric hexanes, the isomer which can give two monochlorinated compounds is
(a) n-hexane
(b) 2,3-dimethylbutane
(c) 2,2-dimethylbutane
(d) 2-methylpentane

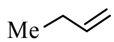
(2005)

11. Acid catalyzed hydration of alkenes except ethene leads to the formation of
(a) primary alcohol
(b) secondary or tertiary alcohol
(c) mixture of primary and secondary alcohols
(d) mixture of secondary and tertiary alcohols

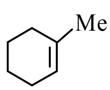
(2005)

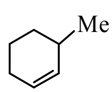
12. 

The alkene formed as a major product in the above elimination reaction is

(a) 

(b) $CH_2 = CH_2$

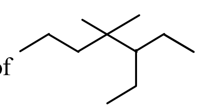
(c) 

(d) 

(2006)

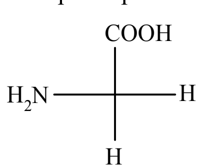
13. The compound formed as a result of oxidation of ethyl benzene by $KMnO_4$ is
(a) benzyl alcohol (b) benzophenone
(c) acetophenone (d) benzoic acid.

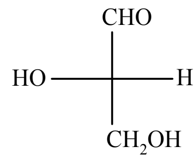
(2007)

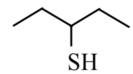
14. The IUPAC name of  is
(a) 3-ethyl-4-4-dimethylheptane
(b) 1,1-diethyl-2,2-dimethylpentane
(c) 4,4-dimethyl-5,5-diethylpentane
(d) 5,5-diethyl-4,4-dimethylpentane.

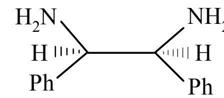
(2007)

15. Which of the following molecules is expected to rotate the plane-polarised light?

(a) 

(b) 

(c) 

(d) 

(2007)

16. Which of the following reactions will yield 2,2-dibromopropane?
(a) $CH_3 - CH = CH_2 + HBr \rightarrow$
(b) $CH_3 - C \equiv CH + 2HBr \rightarrow$
(c) $CH_3CH = CHBr + HBr \rightarrow$
(d) $CH \equiv CH + 2HBr \rightarrow$

(2007)

17. The reaction of toluene with Cl_2 in presence of $FeCl_3$ gives predominantly
(a) m-chlorobenzene
(b) benzoyl chloride
(c) benzyl chloride
(d) o- and p-chlorotoluene.

(2007)

18. Presence of a nitro group in a benzene ring
(a) deactivates the ring towards electrophilic substitution
(b) activates the ring towards electrophilic substitution
(c) renders the ring basic
(d) deactivates the ring towards nucleophilic substitution.

(2007)

19. Toluene is nitrated and the resulting product is reduced with tin and hydrochloric acid. The product so obtained is diazotised and then heated with cuprous bromide. The reaction mixture so formed contains
(a) Mixture of o- and m-bromotoluenes
(b) Mixture of o- and p-bromotoluenes
(c) Mixture of o- and p-dibromobenzenes
(d) Mixture of o- and p-bromoanilines

(2008)

20. In the following sequence of reactions, the alkene affords the compound B

$CH_3CH{=}CHCH_3 \xrightarrow{O_3} A \xrightarrow[Zn]{H_2O} B.$
The compound B is
(a) CH_3CHO (b) CH_3CH_2CHO
(c) CH_3COCH_3 (d) $CH_3CH_2COCH_3$
(2008)

21. The hydrocarbon which can react with sodium in liquid ammonia is
(a) $CH_3CH_2C \equiv CCH_2CH_3$
(b) $CH_3CH_2CH_2C \equiv CCH_2CH_2CH_3$
(c) $CH_3CH_2C \equiv CH$
(d) $CH_3CH = CHCH_3$
(2008)

22. The treatment of CH_3MgX with $CH_3C \equiv C - H$ produces
(a) CH_4
(b) $CH_3 - CH = CH_2$
(c) $CH_3C \equiv C - CH_3$
(d) $CH_3 - \overset{\overset{\displaystyle H}{|}}{C} = \overset{\overset{\displaystyle H}{|}}{C} - CH_3$
(2008)

Answer Key

1.	(a)	**2.**	(c)	**3.**	(d)	**4.**	(d)	**5.**	(d)	**6.**	(c)
7.	(d)	**8.**	(b)	**9.**	(b)	**10.**	(b)	**11.**	(b)	**12.**	(d)
13.	(d)	**14.**	(a)	**15.**	(b)	**16.**	(b)	**17.**	(d)	**18.**	(a)
19.	(b)	**20.**	(a)	**21.**	(c)	**22.**	(a)				

1. **(a) :** Acetylene does not react with NaOH because product would be the stronger acid H_2O and the stronger base $(CH_3-C\equiv\bar{C})$. Acetylene reacts with the other three as:

$$\underset{\text{CNa}}{\overset{\text{CH}}{|||}} \xleftarrow[NH_{3(l)}]{Na} \underset{\text{CH}}{\overset{\text{CH}}{|||}} \xrightarrow{HCl} \underset{\text{CHCl}}{\overset{CH_2}{||}} \xrightarrow{HCl} \underset{CHCl_2}{\overset{CH_3}{|}}$$

$$\underset{\text{CH}}{\overset{\text{CH}}{|||}} \xrightarrow{[AgNO_3 + NH_4OH]} \underset{\text{CAg}}{\overset{\text{CAg}}{|||}} + NH_4NO_3$$
white ppt.

2. **(c) :**

$$CH\equiv CH \xrightarrow{HOCl} \underset{\text{CHCl}}{\overset{\text{CHOH}}{||}} \xrightarrow{HOCl} \underset{CHCl_2}{\overset{CH(OH)_2}{|}} \xrightarrow{-H_2O} \underset{CHCl_2}{\overset{\text{CHO}}{|}}$$

3. **(d) :** $\underset{\text{Butene - 1}}{H_3C-CH_2-CH=CH_2} \xrightarrow{Pd/H_2} \underset{\text{Butane}}{H_3C-CH_2-CH_2-CH_3}$

4. **(d) :** The number of monohalogenation products obtained from any alkene depends upon the number of different types of hydrogen it contains.
Compound containing only one type of hydrogen gives only one monohalogenation product.

$\underset{\text{propane}}{CH_3CH_2CH_3}$ — two types of hydrogen (two monohalogenation product)

$\underset{\text{pentane}}{CH_3CH_2CH_2CH_2CH_3}$ — two types of hydrogen (two monohalogenation product)

$\underset{\text{isopentane}}{CH_3-\overset{\overset{CH_3}{|}}{C}H-CH_2-CH_3}$ — three types of hydrogen (three monohalogenation product)

$\underset{\text{neopentane}}{H_3C-\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}-CH_3}$ — one type of hydrogen (one monohalogenation product)

Thus the given alkane should be neopentane.

5. **(d) :** Among the isomeric alkanes, the normal isomer has a higher boiling point than the branched chain isomer. The greater the branching of the chain, the lower is the boiling point.
The *n*-alkanes have larger surface area in comparison to branched chain isomers (as the shape approaches that of a sphere in the branched chain isomers). Thus, intermolecular forces are weaker in branched chain isomers, therefore, they have lower boiling points in comparison to straight chain isomers.

6. **(c) :** $CH_3-\underset{\underset{C_2H_5}{|}}{\overset{\overset{H}{|}}{C^*}}-\triangleleft$

Optically active due to presence of chiral carbon atom.

7. **(d) :** Butan-2-one will get reduced into butane when treated with zinc and hydrochloric acid following Clemmensen reaction whereas Zn/HCl do not reduce ester, acid and amide.

$$\underset{\text{Butan-2-one}}{CH_3-CH_2-\overset{\overset{O}{||}}{C}-CH_3} \xrightarrow[HCl]{Zn-Hg} \underset{\text{Butane}}{CH_3-CH_2CH_2-CH_3}$$

8. **(b) :** The reactivity order of abstraction of H atoms towards bromination of alkane is $3°H > 2°H > 1°H$.

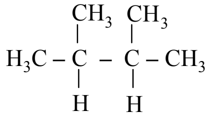
So, $\underset{\text{2-methylbutane}}{(CH_3)_2CHCH_2CH_3} \xrightarrow[127°C]{Br_2/h\upsilon} \underset{\text{2-bromo-2-methylbutane}}{(CH_3)_2C(Br)CH_2CH_3}$

9. **(b) :** 1,2-addition product is kinetically controlled product while 1,4-addition product is thermodynamically controlled product and formed at comparatively higher temperature.

$$CH_2=CH-CH=CH_2 \xrightarrow[40°]{HBr} \underset{\text{1,4-addition}}{CH_2(Br)CH=CHCH_3} + \underset{\text{1,2-addition}}{CH_3CH(Br)-CH=CH_2}$$

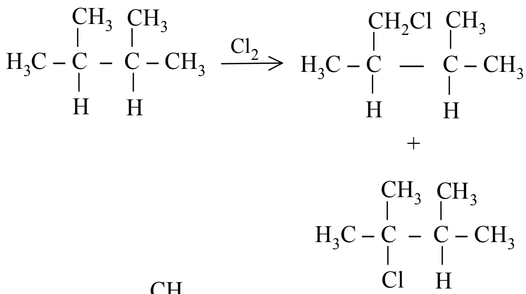
Therefore, 1-bromo-2-butene will be the main product under thermodynamically controlled conditions.

10. **(b) :** The number of monohalogenation products obtained from any alkane depends upon the number of different types of hydrogen it contains.

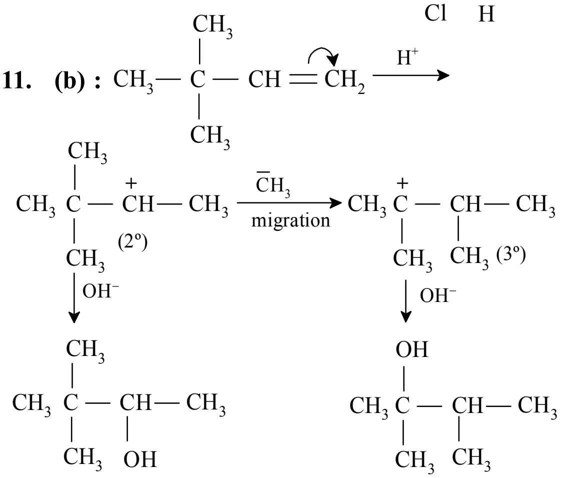


2,3-dimethylbutane

2,3-dimethylbutane has two types of hydrogen atoms so on monochlorination gives only two monochlorinated compounds.

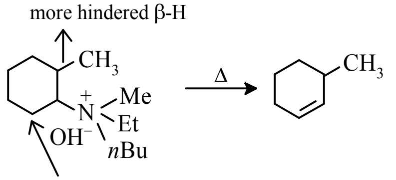


11. (b) :



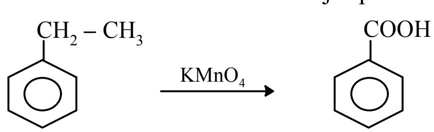
(secondary alcohol) (tertiary alcohol)

12. (d) :



In Hofmann elimination reaction, it is the less sterically hindered β-hydrogen that is removed and hence less substituted alkene is the major product.

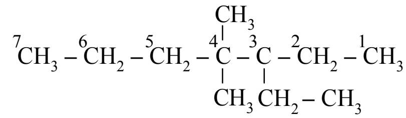
13. (d) :



Ethyl benzene Benzoic acid

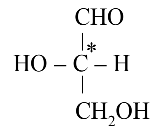
When oxidises with alkaline $KMnO_4$ or acidic $Na_2Cr_2O_7$, the entire side chain (in benzene homologues) with atleast one H at α-carbon, regardless of length is oxidised to –COOH.

14. (a) :



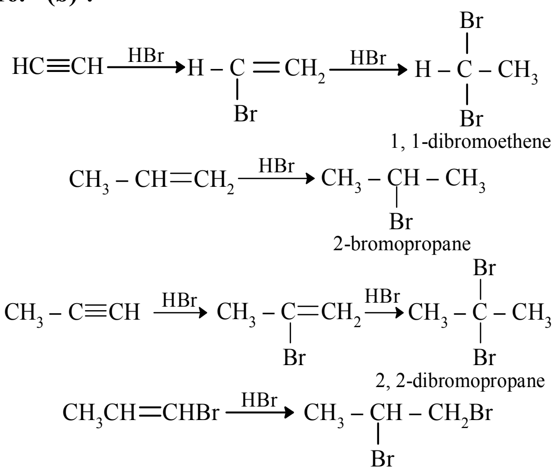
3-ethyl-4, 4-dimethylheptane

15. (b) :



Due to the presence of chiral carbon atom, it is optically active, hence it is expected to rotate plane of polarized light.

16. (b) :



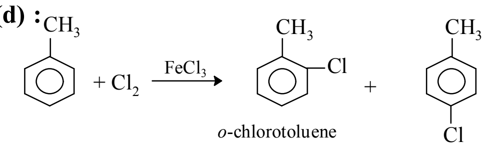
1, 1-dibromoethene

2-bromopropane

2, 2-dibromopropane

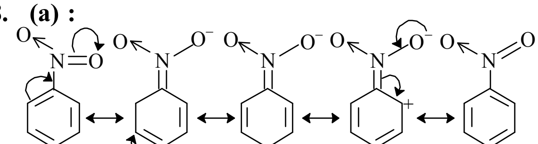
1, 2-dibromopropane

17. (d) :



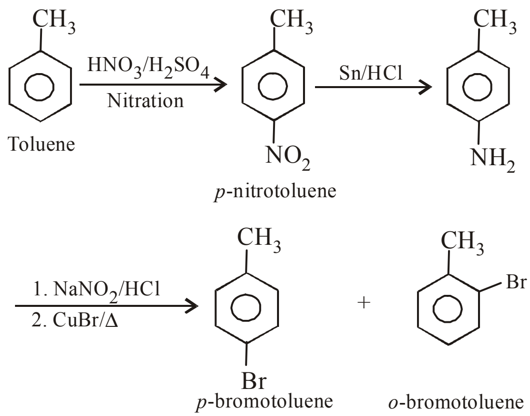
o-chlorotoluene *p*-chlorotoluene

18. (a) :

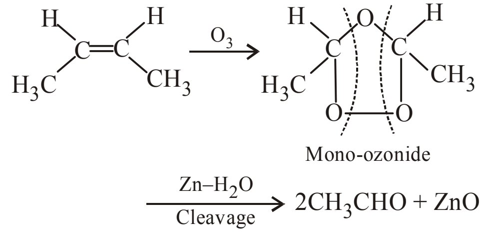


From the resonating structures of it can be seen that the nitrogroup withdrawn electrons from the rings and hence it deactivates the benzene ring for further electophilic substitution.

19. (b) : The reaction sequence is as follows :



20. (a) : The complete reaction sequence is as follows



21. (c) : Terminal alkynes react with sodium in liquid ammonia to yield ionic compounds *i.e.* sodium alkylides.

22. (a) : Grignard reagent reacts with compounds having active or acidic hydrogen atom to give alkane.

$CH_3MgX + CH_3C \equiv C\text{–}H \rightarrow$

$CH_4 + CH_3C \equiv CMgX$

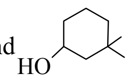
Halides, Hydroxy Compounds & Ethers

1. An ether is more volatile than an alcohol having the same molecular formula. This is due to
   (a) dipolar character of ethers
   (b) alcohols having resonance structures
   (c) inter-molecular hydrogen bonding in ethers
   (d) inter-molecular hydrogen bonding in alcohols.
   (2003)

2. During dehydration of alcohols to alkenes by heating with concentrated H_2SO_4 the initiation step is
   (a) protonation of alcohol molecule
   (b) formation of carbocation
   (c) elimination of water
   (d) formation of an ester.
   (2003)

3. Bottles containing C_6H_5I and $C_6H_5CH_2I$ lost their original labels. They were labelled *A* and *B* for testing. *A* and *B* were separately taken in a test tube and boiled with NaOH solution. The end solution in each tube was made acidic with dilute HNO_3 and then some $AgNO_3$ solution was added. Substance *B* gave a yellow precipitate. Which one of the following statements is true for this experiment?
   (a) *A* was C_6H_5I
   (b) *A* was $C_6H_5CH_2I$
   (c) *B* was C_6H_5I
   (d) Addition of HNO_3 was unnecessary.
   (2003)

4. For which of the following parameters the structural isomers C_2H_5OH and CH_3OCH_3 would be expected to have the same values? (Assume ideal behaviour)
   (a) Heat of vaporisation
   (b) Vapour pressure at the same temperature
   (c) Boiling points
   (d) Gaseous densities at the same temperature and pressure.
   (2004)

5. The IUPAC name of the compound  is

   (a) 3,3-dimethyl-1-hydroxy cyclohexane
   (b) 1,1-dimethyl-3-hydroxy cyclohexane
   (c) 3,3-dimethyl-1-cyclohexanol
   (d) 1,1-dimethyl-3-cyclohexanol.
   (2004)

6. Which of the following will have a meso-isomer also?
   (a) 2-chlorobutane
   (b) 2,3-dichlorobutane
   (c) 2,3-dichloropentane
   (d) 2-hydroxypropanoic acid.
   (2004)

7. Acetyl bromide reacts with excess of CH_3MgI followed by treatment with a saturated solution of NH_4Cl gives
   (a) acetone
   (b) acetamide
   (c) 2-methyl-2-propanol
   (d) acetyl iodide.
   (2004)

8. Among the following compounds which can be dehydrated very easily ?
   (a) $CH_3CH_2CH_2CH_2CH_2OH$
   (b) $CH_3CH_2CH_2\underset{}{\overset{\overset{\displaystyle OH}{|}}{C}}HCH_3$

(c) $CH_3CH_2\underset{\displaystyle OH}{\overset{\displaystyle CH_3}{C}}CH_2CH_3$

(d) $CH_3CH_2\underset{\displaystyle CH_3}{CH}CH_2CH_2OH$

(2004)

9. Which of the following compounds is not chiral?
(a) 1-chloropentane
(b) 2-chloropentane
(c) 1-chloro-2-methylpentane
(d) 3-chloro-2-methylpentane.

(2004)

10. The best reagent to convert pent-3-en-2-ol into pent-3-en-2-one is
(a) acidic permanganate
(b) acidic dichromate
(c) chromic anhydride in glacial acetic acid
(d) pyridinium chloro-chromate

(2005)

11. Tertiary alkyl halides are practically inert to substitution by S_N2 mechanism because of
(a) insolubility (b) instability
(c) inductive effect (d) steric hindrance

(2005)

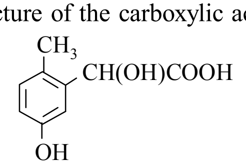
12. Alkyl halides react with dialkyl copper reagents to give
(a) alkenes (b) alkyl copper halides
(c) alkanes (d) alkenyl halides

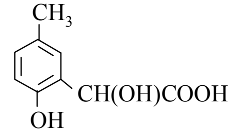
(2005)

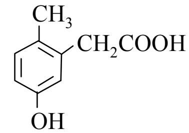
13. Elimination of bromine from 2-bromobutane results in the formation of
(a) equimolar mixture of 1 and 2-butene
(b) predominantly 2-butene
(c) predominantly 1-butene
(d) predominantly 2-butyne

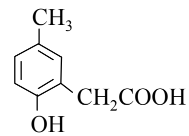
(2005)

14. *p*-cresol reacts with chloroform in alkaline medium to give the compound *A* which adds hydrogen cyanide to form the compound *B*. The latter on acidic hydrolysis gives chiral carboxylic acid. The structure of the carboxylic acid is

(a) 

(b) 

(c) 

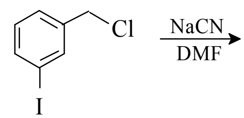
(d) 

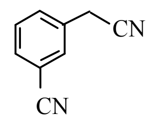
(2005)

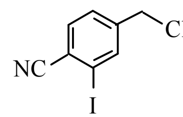
15. Phenyl magnesium bromide reacts with methanol to give
(a) a mixture of anisole and Mg(OH)Br
(b) a mixture of benzene and Mg(OMe)Br
(c) a mixture of toluene and Mg(OH)Br
(d) a mixture of phenol and Mg(Me)Br.

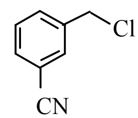
(2006)

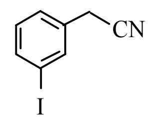
16. The structure of the major product formed in the following reaction is



(a) 

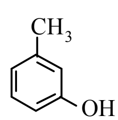
(b) 

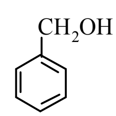
(c) 

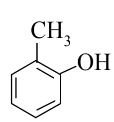
(d) 

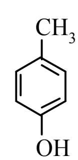
(2006)

17. The structure of the compound that gives a tribromoderivative on treatment with bromine water is

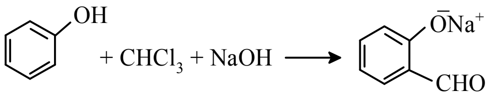
(a) 

(b) 

(c) 

(d) 

(2006)

18. 

The electrophile involved in the above reaction is

(a) dichloromethyl cation $(\overset{\oplus}{C}HCl_2)$

(b) dichlorocarbene $(:CCl_2)$

(c) trichloromethyl anion $(\overset{\ominus}{C}Cl_3)$

(d) formyl cation $(\overset{\oplus}{C}HO)$.

(2006)

19. Fluorobenzene (C_6H_5F) can be synthesised in the laboratory

(a) by heating phenol with HF and KF

(b) from aniline by diazotization followed by heating the diazonium salt with HBF_4

(c) by direct fluorination of benzene with F_2 gas

(d) by reacting bromobenzene with NaF solution.

(2006)

20. Reaction of *trans*-2-phenyl-1-bromocyclopentane on reaction with alcoholic KOH produces

(a) 4-phenylcyclopentene

(b) 2-phenylcyclopentene

(c) 1-phenylcyclopentene

(d) 3-phenylcyclopentene.

(2006)

21. HBr reacts with $CH_2 = CH - OCH_3$ under anhydrous conditions at room temperature to give

(a) CH_3CHO and CH_3Br

(b) $BrCH_2CHO$ and CH_3OH

(c) $BrCH_2 - CH_2 - OCH_3$

(d) $H_3C - CHBr - OCH_3$.

(2006)

22. In the following sequence of reactions,

$$CH_3CH_2OH \xrightarrow{P + I_2} A \xrightarrow[\text{ether}]{Mg} B \xrightarrow{HCHO} C \xrightarrow{H_2O} D$$

the compound *D* is

(a) propanal (b) butanal

(c) *n*-butyl alcohol (d) *n*-propyl alcohol.

(2007)

23. The organic chloro compound, which shows complete stereochemical inversion during a S_N2 reaction, is

(a) CH_3Cl (b) $(C_2H_5)_2CHCl$

(c) $(CH_3)_3CCl$ (d) $(CH_3)_2CHCl$

(2008)

24. Phenol, when it first reacts with concentrated sulphuric acid and then with concentrated nitric acid, gives

(a) nitrobenzene

(b) 2, 4, 6-trinitrobenzene

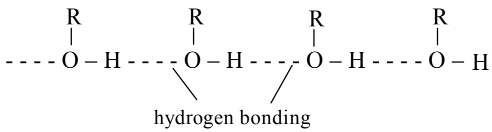
(c) *o*-nitrophenol (d) *p*-nitrophenol

(2008)

Answer Key

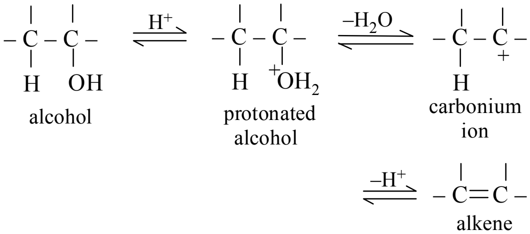
1.	(d)	**2.**	(a)	**3.**	(a)	**4.**	(d)	**5.**	(c)	**6.**	(b)
7.	(c)	**8.**	(c)	**9.**	(a)	**10.**	(d)	**11.**	(d)	**12.**	(c)
13.	(b)	**14.**	(b)	**15.**	(b)	**16.**	(d)	**17.**	(a)	**18.**	(b)
19.	(b)	**20.**	(d)	**21.**	(d)	**22.**	(d)	**23.**	(a)	**24.**	none

1. (d) : The reason for the lesser volatility of alcohols than ethers is the intermolecular association of a large number of molecules due to hydrogen bonding as – OH group is highly polarised.



No such hydrogen bonding is present in ethers.

2. (a) : Dehydration of alcohol to alkene in presence of concentrated H_2SO_4 involves following steps :



Thus, the initiation step is protonation of alcohol.

3. (a) :

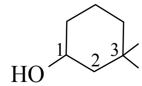
A: $C_6H_5I \xrightarrow{NaOH} C_6H_5ONa \xrightarrow{HNO_3/H^+} C_6H_5OH \xrightarrow{AgNO_3}$ No yellow precipitate

B: $C_6H_5CH_2I \xrightarrow{NaOH} C_6H_5CH_2ONa \xrightarrow{HNO_3/H^+} C_6H_5CH_2OH \xrightarrow{AgNO_3}$ Yellow precipitate

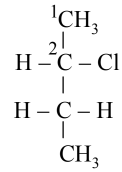
Thus *A* must be C_6H_5I.

4. (d) : Vapour density $= \dfrac{\text{Molecular weight}}{2}$

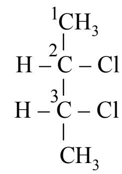
As both the compounds have same molecular weights, both will have the same vapour density. Hence, gaseous density of both ethanol and dimethyl ether would be same under identical conditions of temperature and pressure. The rest of these three properties; vapour pressure, boiling point and heat of vaporization will differ as ethanol has hydrogen bonding whereas ether does not.

5. (c) :  → 3,3-dimethyl-1-cyclohexanol

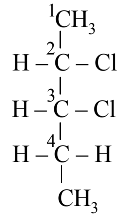
6. (b) :



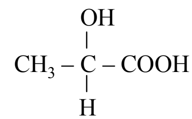
2-chlorobutane



2,3-dichlorobutane



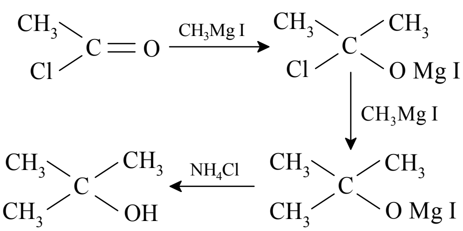
2,3-dichloropentane



2-hydroxypropanoic acid

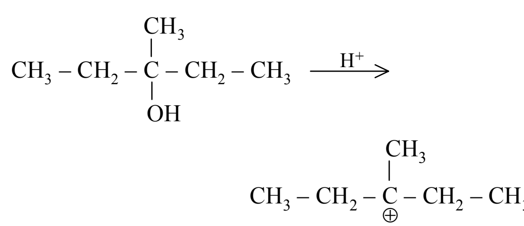
2,3-dichlorobutane have meso isomer due to the presence of plane of symmetry.

7. (c) :



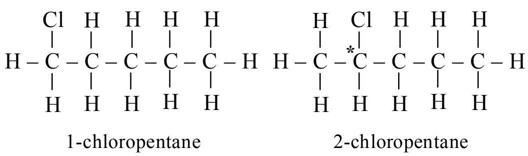
2-methyl-2-propanol

8. (c) : The ease of dehydration of alcohols is tertiary > secondary > primary according to the order of stability of the carbocations.



The more stable carbocation is generated thus more easily it will be dehydrated.

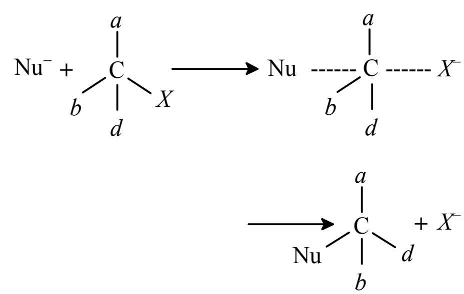
9. **(a) :** To be optically active the compound or structure should possess chiral or asymmetric centre.



1-chloropentane, 2-chloropentane, 1-chloro-2-methylpentane, 3-chloro-2-methylpentane

10. **(d) :** Pyridinium chlorochromate oxidises an alcoholic group selectively in the presence of carbon-carbon double bond.

11. **(d) :**



In an S_N2 reaction, in the transition state, there will be five groups attached to the carbon atom at which reaction occurs.
Thus there will be crowding in the transition state, and presence of bulky groups make the reaction sterically hindered.

12. **(c) :** In Corey House synthesis of alkane, alkyl halide reacts with lithium dialkyl cuprate.

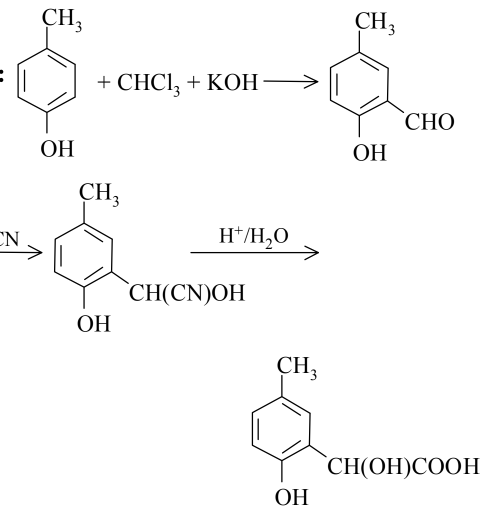
$$R_2CuLi + R'X \rightarrow RR' + RCu + LiX$$

13. **(b) :** $CH_3CH_2CH(Br)CH_3 \xrightarrow{\text{KOH (alc.)}} CH_3CH{=}CHCH_3 + CH_3CH_2CH{=}CH_2$

2-bromobutane → 2-butene (80%) + 1-butene (20%)

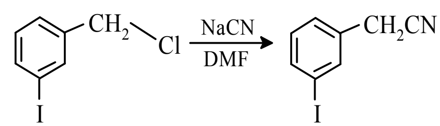
In elimination reaction of alkyl halide major product is obtained according to Saytzeff's rule, which states that when two alkenes may be formed, the alkene which is most substituted one predominates.

14. **(b) :**

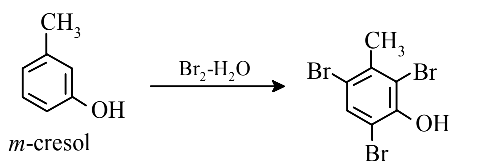


15. **(b) :** $CH_3OH + C_6H_5MgBr \rightarrow C_6H_6 + Mg(OCH_3)Br$

16. **(d) :**



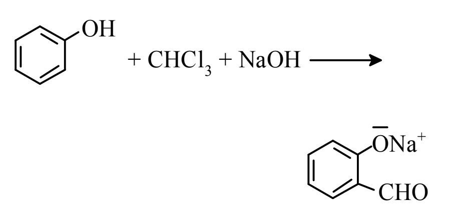
17. **(a) :** Since the compound on treatment with Br_2-water gives a tribromoderivative, therefore it must be *m*-cresol, because it has two *ortho* and one *para* position free with respect to OH group and hence can give tribromoderivative.



m-cresol

2,4,6-tribromo-3-methylphenol

18. **(b) :**

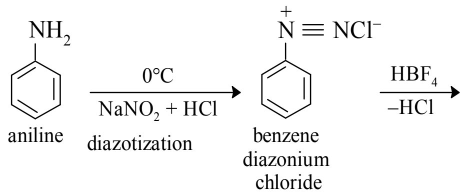


(Reimer-Tiemann reaction)

The electrophile is dichlorocarbene, $:CCl_2$ generated from chloroform by the action of a base.

$$OH^- + CHCl_3 \rightleftharpoons HOH + :CCl_3^- \rightarrow Cl^- + :CCl_2$$

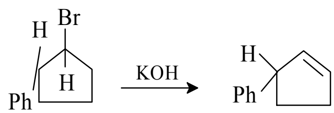
19. (b) :



$$C_6H_5\overset{+}{N}_2BF_4^- \xrightarrow{\Delta} C_6H_5F + BF_3 + N_2$$

benzene diazonium tetra fluoroborate — fluorobenzene

20. (d) :



3-phenylcyclopentene

It follows E2 mechanism.

Hughes and Ingold proposed that bimolecular elimination reactions take place when the two groups to be eliminated are *trans* and lie in one plane with the two carbon atoms to which they are attached *i.e.* E2 reactions are stereoselectively *trans*.

21. (d) : Methyl vinyl ether is a very reactive gas. It is hydrolysed rapidly by dilute acids at room temperature to give methanol and aldehyde.

However, under anhydrous conditions at room temperature, it undergoes many addition reactions at the double bond.

$$H_2C{=}CH-OCH_3 \xrightarrow{H^+} H_2\underset{|}{\underset{H}{C}}-\overset{\oplus}{C}H-OCH_3$$

$$\xrightarrow{Br^-} H_3C-\overset{Br}{\overset{|}{C}}H-OCH_3$$

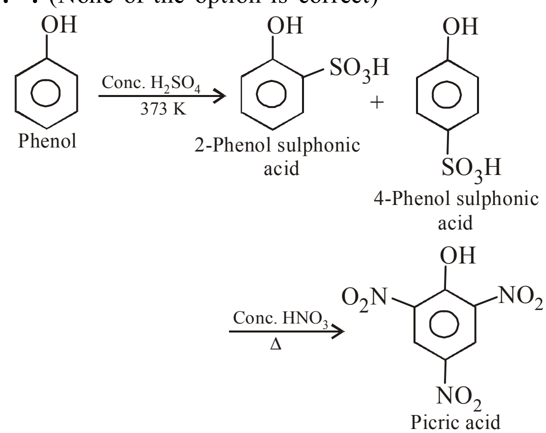
22. (d) :

$$CH_3CH_2OH \xrightarrow[I_2]{P} CH_3CH_2I \xrightarrow{Mg} CH_3CH_2MgI \xrightarrow{HCHO} H-\underset{CH_2CH_3}{\overset{H}{C}}-OMgI \xrightarrow{H_2O} H-\underset{CH_2CH_3}{\overset{H}{C}}-OH$$

n-propyl alcohol (*D*)

23. (a) : In S_N2 reactions, the nucleophile attacks from back side resulting in the inversion of molecule. Also, as we move from 1° alkyl halide to 3° alkyl halide, the crowding increases and +I effect increases which makes the carbon bearing halogen less positively polarised and hence less readily attacked by the nucleophile.

24. : (None of the option is correct)



CHAPTER **21**

Aldehydes, Ketones, Carboxylic Acids & Their Derivatives

1. Which of the following compounds has wrong IUPAC name?

(a) $CH_3 - CH_2 - CH_2 - COO - CH_2CH_3 \rightarrow$ ethyl butanoate

(b) $CH_3 - \underset{\displaystyle CH_3}{\underset{|}{CH}} - CH_2 - CHO \rightarrow$ 3-methylbutanal

(c) $CH_3 - \underset{\displaystyle OH}{\underset{|}{CH}} - \underset{\displaystyle CH_3}{\underset{|}{CH}} - CH_3 \rightarrow$ 2-methyl-3-butanol

(d) $CH_3 - \underset{\displaystyle CH_3}{\underset{|}{CH}} - \overset{\displaystyle O}{\overset{||}{C}} - CH_2 - CH_3 \rightarrow$ 2-methyl-3-pentanone

(2002)

2. $CH_3CH_2COOH \xrightarrow[\text{red P}]{Cl_2} A \xrightarrow{\text{alc. KOH}} B.$

What is B ?

(a) CH_3CH_2COCl

(b) CH_3CH_2CHO

(c) $CH_2 = CHCOOH$

(d) $ClCH_2CH_2COOH$.

(2002)

3. On vigorous oxidation by permanganate solution, $(CH_3)_2C = CH - CH_2 - CHO$ gives

(a) $CH_3 - \underset{\displaystyle CH_3}{\underset{|}{\overset{\displaystyle OH}{\overset{|}{C}}}} - \overset{\displaystyle OH}{\overset{|}{CH}} - CH_2CH_3$

(b) $\begin{matrix} CH_3 \\ CH_3 \end{matrix}\!\!>\! COOH + CH_3CH_2COOH$

(c) $\begin{matrix} CH_3 \\ CH_3 \end{matrix}\!\!>\! CH - OH + CH_3CH_2CH_2OH$

(d) $\begin{matrix} CH_3 \\ CH_3 \end{matrix}\!\!>\! C = O + CH_3CH_2CHO.$

(2002)

4. The IUPAC name of $CH_3COCH(CH_3)_2$ is

(a) isopropylmethyl ketone

(b) 2-methyl-3-butanone

(c) 4-methylisopropyl ketone

(d) 3-methyl-2-butanone.

(2003)

5. When $CH_2 = CH - COOH$ is reduced with $LiAlH_4$, the compound obtained will be

(a) $CH_3 - CH_2 - COOH$

(b) $CH_2 = CH - CH_2OH$

(c) $CH_3 - CH_2 - CH_2OH$

(d) $CH_3 - CH_2 - CHO$.

(2003)

6. The general formula $C_nH_{2n}O_2$ could be for open chain

(a) diketones (b) carboxylic acids

(c) diols (d) dialdehydes.

(2003)

7. In the anion $HCOO^-$ the two carbon-oxygen bonds are found to be of equal length. What is the reason for it?

(a) Electronic orbitals of carbon atom are hybridised.

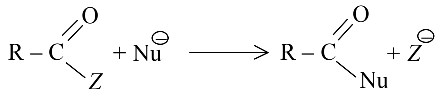
(b) The $C = O$ bond is weaker than the $C - O$ bond.

(c) The anion $HCOO^-$ has two resonating structures.

(d) The anion is obtained by removal of a proton from the acid molecule.

(2003)

8. Rate of the reaction



is fastest when Z is

(a) Cl (b) NH_2

(c) OC_2H_5 (d) $OCOCH_3$.

(2004)

9. Consider the acidity of the carboxylic acids:

(i) PhCOOH

(ii) o-$NO_2C_6H_4COOH$

(iii) p-$NO_2C_6H_4COOH$

(iv) m-$NO_2C_6H_4COOH$

Which of the following order is correct?

(a) i > ii > iii > iv (b) ii > iv > iii > i

(c) ii > iv > i > iii (d) ii > iii > iv > i.

(2004)

10. On mixing ethyl acetate with aqueous sodium chloride, the composition of the resultant solution is

(a) $CH_3COOC_2H_5 + NaCl$

(b) $CH_3COONa + C_2H_5OH$

(c) $CH_3COCl + C_2H_5OH + NaOH$

(d) $CH_3Cl + C_2H_5COONa$.

(2004)

11. Which one of the following undergoes reaction with 50% sodium hydroxide solution to give the corresponding alcohol and acid?

(a) Phenol (b) Benzaldehyde

(c) Butanol (d) Benzoic acid.

(2004)

12. The increasing order of the rate of HCN addition to compounds A - D is

A. HCHO B. CH_3COCH_3

C. $PhCOCH_3$ D. PhCOPh.

(a) $A < B < C < D$ (b) $D < B < C < A$

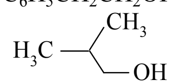
(c) $D < C < B < A$ (d) $C < D < B < A$.

(2006)

13. Among the following the one that gives positive iodoform test upon reaction with I_2 and NaOH is

(a) $CH_3CH_2CH(OH)CH_2CH_3$

(b) $C_6H_5CH_2CH_2OH$

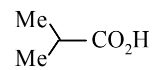
(c) 

(d) $PhCHOHCH_3$.

(2006)

14. The correct order of increasing acid strength of the compounds

(A) CH_3CO_2H (B) $MeOCH_2CO_2H$

(C) CF_3CO_2H (D)  is

(a) $B < D < A < C$

(b) $D < A < C < B$

(c) $D < A < B < C$

(d) $A < D < C < B$.

(2006)

15. The compound formed as a result of oxidation of ethyl benzene by $KMnO_4$ is

(a) benzyl alcohol (b) benzophenone

(c) acetophenone (d) benzoic acid.

(2007)

Answer Key

1. (c)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (b)	**6.** (b)
7. (c)	**8.** (a)	**9.** (d)	**10.** (a)	**11.** (b)	**12.** (c)
13. (d)	**14.** (c)	**15.** (d)			

1. **(c) :** $\overset{1}{C}H_3-\underset{OH}{\overset{2}{C}H}-\underset{CH_3}{\overset{3}{C}H}-\overset{4}{C}H_3$

3-methyl-2-butanol

2. **(c) :** $CH_3CH_2COOH \xrightarrow[\text{red P}\\ -HCl]{Cl_2} CH_3CHClCOOH$

$\xrightarrow[-HCl]{\text{alc. KOH}} CH_2 = CHCOOH$

Acrylic acid

3. **(b) :** Aldehydic group gets oxidised to carboxylic group.
Double bond breaks and carbon gets oxidised to carboxylic group.

4. **(d) :** $H-\underset{H}{\overset{H}{\overset{|}{C}}}{}_1-\overset{O}{\overset{||}{C}}{}_2-\underset{CH_3}{\overset{H}{\overset{|}{C}}}{}_3-\underset{H}{\overset{H}{\overset{|}{C}}}{}_4-H$

3-methyl-2-butanone

5. **(b) :** $LiAlH_4$ is a strong reducing agent, it reduces carboxylic group into primary alcoholic group without affecting the basic skeleton of compound.

$CH_2 = CH-COOH \xrightarrow[[H^+]]{LiAlH_4} CH_2 = CH-CH_2OH$

6. **(b) :** Diketones - $C_nH_{2n-2}O_2$, Carboxylic acid - $C_nH_{2n}O_2$
Diols - $C_nH_{3n}O_2$, Dialdehydes - $C_nH_nO_2$.

7. **(c) :** $HCOO^-$ exists as

$$H-\overset{O}{\overset{||}{C}}-O^- \longleftrightarrow H-\overset{O^-}{\overset{|}{C}}=O$$

So, the two carbon-oxygen bonds are found to be of equal length.

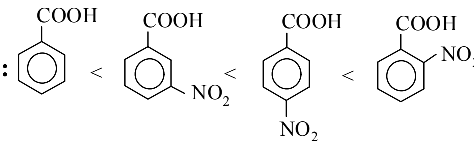
8. **(a) :** $R-C(=O)-Z + Nu^{\ominus} \longrightarrow R-C(=O)-Nu + Z^{\ominus}$

Reactivity of the acid derivatives decreases as the basicity of the leaving group increases. The basicity of the leaving group increases as

$Cl^- < RCOO^- < RO^- < NH_2^-$

Secondly least stabilization by resonance due to ineffective overlapping between the $3p$ orbital of Cl and $2p$ orbital of carbon.

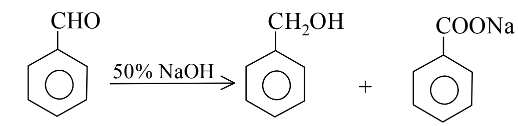
9. **(d) :**



Electron withdrawing group increases the acidity of benzoic acid, *o*-isomer will have higher acidity then corresponding *m* and *p* isomer due to ortho-effect. As *M* group (*i.e.* NO_2) at *p*-position have more pronounced electron withdrawing effect than as $-NO_2$ group at *m*-position (–I effect)
$\therefore$ Correct order of acidity is ii > iii > iv > i.

10. **(a) :** $CH_3COOC_2H_5 + NaCl_{(aq)} \rightarrow$ no reaction
i.e., the resultant solution contains ethyl acetate and sodium chloride.

11. **(b) :** Benzaldehyde will undergo Cannizzaro reaction on treatment with 50% NaOH to produce benzyl alcohol and benzoic acid as it does not contain α-hydrogen.



benzaldehyde benzyl alcohol sodium benzoate

12. **(c) :** Addition of HCN to carbonyl compounds is a characteristic nucleophilic addition reaction of carbonyl compounds.
Order of reactivity:

$$(H)(H)C=O > (R)(H)C=O > (R)(R)C=O$$

The lower reactivity of ketones over aldehydes is due to +I-effect of the alkyl (R) group and steric hindrance. As the size of the alkyl group increases, the reactivity of the ketones further decreases.

$$(CH_3)(CH_3)C=O > ((CH_3)_2CH)((CH_3)_2CH)C=O > ((CH_3)_3)((CH_3)_3)C=O$$

The aromatic aldehydes and ketones are less reactive than their aliphatic analogous. This is due to the +R-effect of the benzene ring.

$$PhCHO > PhCOCH_3 > PhCOPh$$

From the above information, it is clear that increasing order of the rate of HCN addition to compounds HCHO, CH_3COCH_3, $PhCOCH_3$ and PhCOPh is

$PhCOPh < PhCOCH_3 < CH_3COCH_3 < HCHO$.

13. **(d) :** Iodoform test is given by only the compounds containing CH_3CO – or CH_3CHOH – group.

$$PhCHOHCH_3 + 4I_2 + 6NaOH \xrightarrow{\Delta} CHI_3 + PhCOONa + 5NaI + 5H_2O$$

14. **(c) :** Effect of substituent on the acid strength of aliphatic acids:

(i) Acidity decreases as the +I-effect of the alkyl group increases.

$$HCOOH > CH_3COOH > (CH_3)_2CHCOOH > (CH_3)_3CCOOH$$

(ii) Acidity decreases as the –I-effect as well as number of halogen atoms decreases.

$$FCH_2COOH > ClCH_2COOH > BrCH_2COOH > ICH_2COOH > CH_3COOH$$

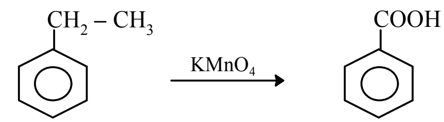
$$F_3CCOOH > F_2CHCOOH > FCH_2COOH > CH_3COOH$$

(iii) Electron donating substituents like – R, – OH, – NH_2 etc. tend to decrease while electron withdrawing substituents like –NO_2, –CHO etc. tend to increase the acid strength of substituted acid.

On the basis of given information the relative order of increasing acid strength of the given compounds is

$$(CH_3)_2COOH < CH_3COOH < CH_3OCH_2COOH < CF_3COOH$$

15. **(d) :**



Ethyl benzene Benzoic acid

When oxidises with alkaline $KMnO_4$ or acidic $Na_2Cr_2O_7$, the entire side chain (in benzene homologues) with atleast one H at α-carbon, regardless of length is oxidised to – COOH.

CHAPTER 22

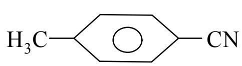
Nitrogen Containing Compounds

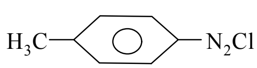
1. When primary amine reacts with chloroform in ethanolic KOH then the product is
(a) an isocyanide (b) an aldehyde
(c) a cyanide (d) an alcohol.
(2002)

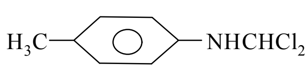
2. The correct order of increasing basic nature for the bases NH_3, CH_3NH_2 and $(CH_3)_2NH$ is
(a) $CH_3NH_2 < NH_3 < (CH_3)_2NH$
(b) $(CH_3)_2NH < NH_3 < CH_3NH_2$
(c) $NH_3 < CH_3NH_2 < (CH_3)_2NH$
(d) $CH_3NH_2 < (CH_3)_2NH < NH_3$.
(2003)

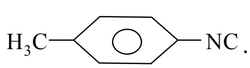
3. Ethyl isocyanide on hydrolysis in acidic medium generates
(a) ethylamine salt and methanoic acid
(b) propanoic acid and ammonium salt
(c) ethanoic acid and ammonium salt
(d) methylamine salt and ethanoic acid.
(2003)

4. The reaction of chloroform with alcoholic KOH and *p* -toluidine forms

(a) 

(b) 

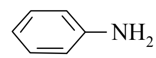
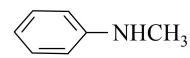
(c) 

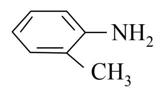
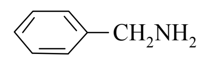
(d) 

(2003)

5. Which one of the following does not have sp^2 hybridized carbon?
(a) Acetone (b) Acetic acid
(c) Acetonitrile (d) Acetamide.
(2004)

6. Which of the following is the strongest base?

(a)  (b) 

(c)  (d) 

(2004)

7. Which one of the following methods is neither meant for the synthesis nor for separation of amines?
(a) Hinsberg method
(b) Hofmann method
(c) Wurtz reaction
(d) Curtius reaction
(2005)

8. Amongst the following the most basic compound is
(a) benzylamine (b) aniline
(c) acetanilide (d) *p*-nitroaniline
(2005)

9. Reaction of cyclohexanone with dimethylamine in the presence of catalytic amount of an acid forms a compound if water during the reaction is continuously removed. The compound formed is generally known as
(a) a Schiff's base (b) an enamine
(c) an imine (d) an amine
(2005)

10. An organic compound having molecular mass 60 is found to contain C = 20%, H = 6.67% and N = 46.67% while rest is oxygen. On heating it gives NH_3 alongwith a solid residue. The solid residue gives violet colour with alkaline copper sulphate solution. The compound is

(a) CH_3NCO (b) CH_3CONH_2
(c) $(NH_2)_2CO$ (d) $CH_3CH_2CONH_2$

(2005)

11. Which one of the following is the strongest base in aqueous solution?

(a) Methylamine (b) Trimethylamine
(c) Aniline (d) Dimethylamine.

(2007)

12. In the chemical reaction,

$CH_3CH_2NH_2 + CHCl_3 + 3KOH \rightarrow (A) + (B) + 3H_2O$,

the compounds (A) and (B) are respectively

(a) C_2H_5NC and 3 KCl
(b) C_2H_5CN and 3KCl
(c) $CH_3CH_2CONH_2$ and 3KCl
(d) C_2H_5NC and K_2CO_3.

(2007)

Answer Key

1.	(a)	**2.**	(c)	**3.**	(a)	**4.**	(d)	**5.**	(c)	**6.**	(d)
7.	(c)	**8.**	(a)	**9.**	(b)	**10.**	(c)	**11.**	(d)	**12.**	(a)

1. **(a) :** When a primary amine reacts with chloroform with ethanolic KOH, then a bad smell compound isocyanide is formed. This is called carbylamine reaction and this reaction is used as a test of primary amines.

$$\underset{\text{primary amine}}{RNH_2} + CHCl_3 + 3KOH \xrightarrow{\Delta} \underset{\text{isocyanide}}{RNC} + 3KCl + 3H_2O$$

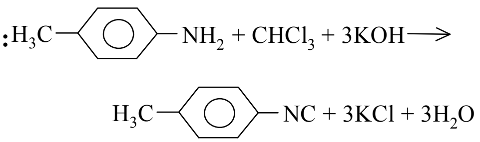
2. **(c) :** Except the amines containing tertiary butyl group, all lower aliphatic amines are stronger bases than ammonia because of +I (inductive) effect. The alkyl groups, which are electron releasing groups, increase the electron density around the nitrogen thereby increasing the availability of the lone pair of electrons to proton or Lewis acids and making the amine more basic. The observed order in the case of lower members is found to be as secondary > primary > tertiary. This anomalous behaviour of tertiary amines is due to steric factors *i.e.* crowding of alkyl groups cover nitrogen atom from all sides and thus makes it unable for protonation.
Thus the relative strength is in order

$$(CH_3)_2NH > CH_3NH_2 > NH_3.$$

3. **(a) :** Alkyl isocyanides are hydrolysed by dilute mineral acids to form primary amines.

$$\underset{\text{ethylisocyanide}}{C_2H_5N \overset{\rightarrow}{\equiv} C} + 2H_2O \xrightarrow{H^+} \underset{\text{ethylamine}}{C_2H_5NH_2} + \underset{\text{methanoic acid}}{HCOOH}$$

4. **(d) :**



The above reaction is known as carbylamine reaction and is generally used to convert primary amine into isocyanide.

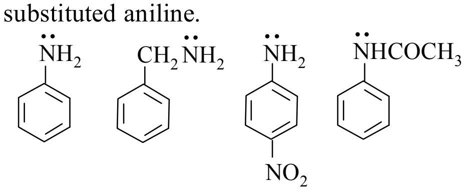
5. **(c) :** $\overset{sp^3}{CH_3} - \overset{sp^2}{CO} - \overset{sp^3}{CH_3}$ (Acetone) ; $\overset{sp^3}{CH_3} - \overset{sp^2}{COOH}$ (Acetic acid)

$\overset{sp^3}{CH_3} - \overset{sp}{C} \equiv N$ (Acetonitrile) ; $\overset{sp^3}{CH_3} - \overset{sp^2}{CONH_2}$ (Acetamide)

6. **(d) :** In this compound, the non-bonding electron pair of nitrogen does not take part in resonance. In other three compounds, the non-bonding electron pair of nitrogen is delocalized into benzene ring by resonance, as a result the electron density on the N atom decreases, due to which basicity decreases.

7. **(c) :** In Wurtz reaction alkyl halide reacts with sodium metal in the presence of dry ether to give alkane.

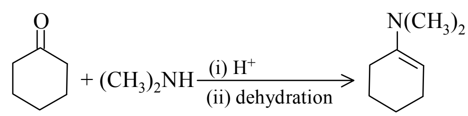
8. **(a) :** Due to resonance of electron pair in aniline, basic strength decreases. In benzylamine electron pair is not involved in resonance. Further the presence of electron donating groups in the benzene ring increase the basic strength while electron withdrawing group decrease the basic strength of substituted aniline.



I II III IV

Decreasing order of basic strength is II > I > IV > III.

9. **(b) :**



enamine

10. **(c) :**

Element	Simplest ratio	Percentage	Relative no. of atom
C	20.00	1.67	1
H	6.67	6.67	4
N	46.67	3.33	2
O	26.66	1.67	1

The molecular formula is CH_4N_2O.

So, the compound is H_2NCONH_2.

$$2NH_2CONH_2 \xrightarrow{\Delta} \underset{\text{biuret}}{NH_2CONHCONH_2} + NH_3\uparrow$$

Biuret gives violet colour with alkaline copper sulphate solution.

11. **(d) :** The increasing order of basicity of the given compounds is

$(CH_3)_2NH > CH_3NH_2 > (CH_3)_3N > C_6H_5NH_2$

Due to the +I effect of alkyl groups, the electron density on nitrogen increases and thus the availability of the lone pair of electrons to proton increases and hence the basicity of amines also increases. So aliphatic amines are more basic than aniline.

In case of tertiary anine $(CH_3)_3N$, the covering of alkyl groups over nitrogen atom from all sides makes the approach and bonding by a proton relatively difficult, hence the basicity decreases. Electron withdrawing (C_6H_5 –) groups decreases electron density on nitrogen atom and thereby decreasing basicity.

12. **(a) :** $CH_3CH_2NH_2 + CHCl_3 + 3KOH \longrightarrow C_2H_5NC + 3KCl + 3H_2O$

This is called carbylamine reaction.

CHAPTER 23

Polymers

1. Polymer formation from monomers starts by
   (a) condensation reaction between monomers
   (b) coordinate reaction between monomers
   (c) conversion of monomer to monomer ions by protons
   (d) hydrolysis of monomers.
   (2002)

2. Nylon threads are made of
   (a) polyvinyl polymer
   (b) polyester polymer
   (c) polyamide polymer
   (d) polyethylene polymer.
   (2003)

3. Which of the following is a polyamide?
   (a) Teflon (b) Nylon-66
   (c) Terylene (d) Bakelite
   (2005)

4. Which of the following is fully fluorinated polymer?
   (a) Neoprene (b) Teflon
   (c) Thiokol (d) PVC
   (2005)

5. Bakelite is obtained from phenol by reaction with
   (a) $HCHO$ (b) $(CH_2OH)_2$
   (c) CH_3CHO (d) CH_3COCH_3
   (2008)

Answer Key

1.	(a)	**2.**	(c)	**3.**	(b)	**4.**	(b)	**5.**	(a)

1. **(a) :** Polymerisation takes place either by condensation or addition reactions.
2. **(c) :** Nylon threads are polyamides. They are the condensation polymers of diamines and dibasic acids.

$$\underset{\text{adipic acid}}{n\ HOOC(CH_2)_4COOH} + \underset{\text{hexamethylene diamine}}{n\ H_2N(CH_2)_6NH_2} \xrightarrow[\text{high pressure}]{280°C} \underset{\text{nylon (polyamide)}}{HO\left[OC-(CH_2)_4-CONH(CH_2)_6NH\right]_n}$$

3. **(b) :** Polymers having amide linkages (– CONH) are known as polyamides.

$$\underset{\text{hexamethylene diamine}}{n(H_2N(CH_2)_6NH_2)} + \underset{\text{adipic acid}}{n(HOOC(CH_2)_4COOH)}$$

$$\downarrow$$

$$\underset{\text{Nylon-66}}{-\!\!\left(HN-(CH_2)_6-\underbrace{\boxed{NHCO}}_{\text{amide linkage}}-(CH_2)_4-CO\right)_n\!\!-}$$

4. **(b) :** Neoprene : $\left[CH_2-CH=\underset{}{\overset{Cl}{\overset{|}{C}}}-CH_2\right]_n$

Teflon : $\left[CF_2-CF_2\right]_n$

Thiokol :

$$-CH_2\left[CH_2-S-S-CH_2\right]_nCH_2-S-S-CH_2CH_2-$$

PVC : $\left[CH_2-\underset{Cl}{\underset{|}{CH}}\right]_n$

5. **(a) :** Bakelite is a thermosetting polymer which is made by reaction between phenol and HCHO.

CHAPTER 24

Biomolecules

1. RNA is different from DNA because RNA contains
(a) ribose sugar and thymine
(b) ribose sugar and uracil
(c) deoxyribose sugar and thymine
(d) deoxyribose sugar and uracil.
(2002)

2. The functional group, which is found in amino acid is
(a) – COOH group
(b) – NH_2 group
(c) – CH_3 group
(d) both (a) and (b).
(2002)

3. Complete hydrolysis of cellulose gives
(a) D-fructose
(b) D-ribose
(c) D-glucose
(d) L-glucose.
(2003)

4. The reason for double helical structure of DNA is operation of
(a) van der Waal's forces
(b) dipole-dipole interaction
(c) hydrogen bonding
(d) electrostatic attractions.
(2003)

5. Identify the correct statement regarding enzymes.
(a) Enzymes are specific biological catalysts that can normally function at very high temperatures ($T \sim 1000$ K).
(b) Enzymes are normally heterogeneous catalysts that are very specific in action.
(c) Enzymes are specific biological catalysts that cannot be poisoned.
(d) Enzymes are specific biological catalysts that possess well-defined active sites.
(2004)

6. Which base is present in RNA but not in DNA?
(a) Uracil
(b) Cytosine
(c) Guanine
(d) Thymine.
(2004)

7. Insulin production and its action in human body are responsible for the level of diabetes. This compound belongs to which of the following categories?
(a) A co-enzyme
(b) A hormone
(c) An enzyme
(d) An antibiotic.
(2004)

8. In both DNA and RNA, heterocyclic base and phosphate ester linkages are at
(a) C_5' and C_2' respectively of the sugar molecule
(b) C_2' and C_5' respectively of the sugar molecule
(c) C_1' and C_5' respectively of the sugar molecule
(d) C_5' and C_1' respectively of the sugar molecule
(2005)

9. The term anomers of glucose refers to
(a) isomers of glucose that differ in configurations at carbons one and four (C-1 and C-4)
(b) a mixture of (D)-glucose and (L)-glucose
(c) enantiomers of glucose
(d) isomers of glucose that differ in configuration at carbon one (C-1).
(2006)

10. The pyrimidine bases present in DNA are
(a) cytosine and adenine
(b) cytosine and guanine
(c) cytosine and thymine
(d) cytosine and uracil.
(2006)

11. The secondary structure of a protein refers to
(a) fixed configuration of the polypeptide backbone
(b) α-helical backbone
(c) hydrophobic interactions
(d) sequence of α-amino acids.
(2007)

12. α-*D*-(+)-glucose and β-*D*-(+)-glucose are
(a) enantiomers (b) conformers
(c) epimers (d) anomers
(2008)

Answer Key

1.	(b)	**2.**	(d)	**3.**	(c)	**4.**	(c)	**5.**	(d)	**6.**	(a)
7.	(b)	**8.**	(c)	**9.**	(d)	**10.**	(c)	**11.**	(b)	**12.**	(d)

1. **(b) :**

		DNA	RNA
(a)	Pyrimidine derivatives	Cytosine Thymine	Cytosine Uracil
(b)	Purine derivatives	Adenine Guanine	Adenine Guanine
(c)	Sugar	Deoxyribose	Ribose

2. **(d) :** An amino acid is a bifunctional organic molecule that contains both a carboxyl group, –COOH, as well as an amino group, $-NH_2$.

3. **(c) :** $\underset{\text{Cellulose}}{(C_6H_{10}O_5)_n} + nH_2O \xrightarrow{H^+} \underset{\text{D-glucose}}{nC_6H_{12}O_6}$

Cellulose is a straight chain polysaccharide composed of D-glucose units which are joined by β-glycosidic linkages. Hence cellulose on hydrolysis produces only D-glucose units.

4. **(c) :** The two polynucleotide chains or strands of DNA are linked up by hydrogen bonding between the nitrogenous base molecules of their nucleotide monomers.

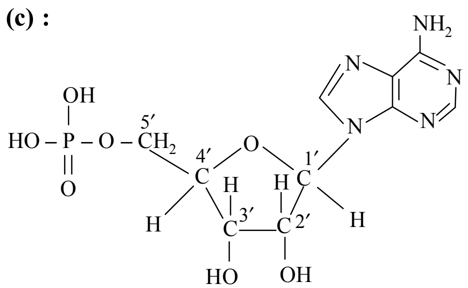
Adenine ═ Thymine (two hydrogen bonds) Cytosine ≡ Guanine (three hydrogen bonds)

5. **(d) :** Enzymes are shape selective specific biological catalysts which normally functions effectively at body temperature.

6. **(a) :** RNA contains cytosine and uracil as pyrimidine bases while DNA has cytosine and thymine. Both have the same purine bases *i.e.* guanine and adenine.

7. **(b) :** Insulin is a proteinaceous hormone secreted by β-cells by islet of Langerhans of pancreas in our body.

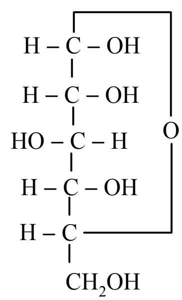
8. **(c) :**



9. **(d) :** Due to cyclic hemiacetal or cyclic hemiketal structures, all the pentoses and hexoses exist in two stereoisomeric forms *i.e.* α form in which the OH at C_1 in aldoses and C_2 in ketoses lies towards the right and β form in which it lies towards left. Thus glucose, fructose, ribose, etc., all exist in α and β form. Glucose exists in two forms α-D-glucose and β-D glucose.

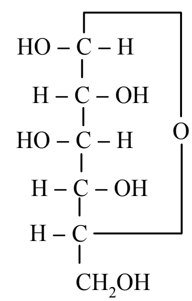
α-D-(+) glucose ⇌ equilibrium mixture
⇌ β-(D)- (+) glucose

As a result of cyclization the anomeric (C-1) becomes asymmetric and the newly formed – OH group may be either on left or on right in Fischer projection thus resulting in the formation of two isomers (anomers). The isomers having – OH group to the left of the C-1 is designated β-D-glucose and other having – OH group on the right as α-D-glucose.



α-D-glucose

and



β-D-glucose

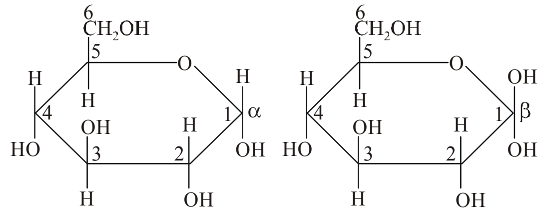
10. **(c) :** DNA contains cytosine and thymine as pyrimidine bases and guanine and adenine as purine bases.

11. **(b) :** Secondary structure of proteins is mainly of two types.

(i) α-helix : This structure is formed when the chain of α-amino acid coils as a right handed screw (called α-helix) because of the formation of hydrogen bonds between amide groups of the same peptide chain.

(ii) β-plated sheet : In this structure the chains are held together by a very large number of hydrogen bonds between $C{=}O$ and NH of different chains.

12. (d) : Structures of α-*D*-(+)-glucose and β-*D*-(+)-glucose are :

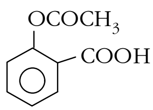


α-D-(+)-glucopyranose β-D-(+)-glucopyranose

A pair of stereoisomers which differ in configuration at C-1 are known as anomers.

CHAPTER
25

Chemistry in Action

1. The compound 

is used as

(a) antiseptic (b) antibiotic
(c) analgesic (d) pesticide.

(2002)

2. Which of the following could act as a propellant for rockets?

(a) Liquid hydrogen + liquid nitrogen
(b) Liquid oxygen + liquid argon
(c) Liquid hydrogen + liquid oxygen
(d) Liquid nitrogen + liquid oxygen.

(2003)

3. The compound formed on heating chlorobenzene with chloral in the presence of concentrated sulphuric acid is

(a) gammexene (b) DDT
(c) freon (d) hexachloroethane.

(2004)

4. The smog is essentially caused by the presence of

(a) O_2 and O_3
(b) O_2 and N_2
(c) oxides of sulphur and nitrogen
(d) O_3 and N_2.

(2004)

5. Which one of the following types of drugs reduces fever?

(a) Analgesic (b) Antipyretic
(c) Antibiotic (d) Tranquiliser

(2005)

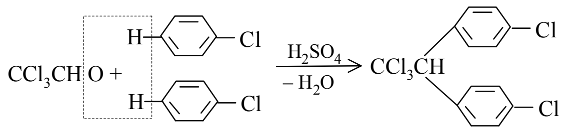
Answer Key

1. (c) **2.** (c) **3.** (b) **4.** (c) **5.** (b)

1. **(c) :** The compound is acetyl salicylic acid (Aspirin). Drugs which relieve or decrease pain are termed analgesics.

2. **(c) :** Liquid hydrogen (because of its low mass and high enthalpy of combustion) and liquid oxygen (as it is a strong supporter of combustion) are used as an excellent fuel for rockets.

3. **(b) :** DDT is prepared by heating chlorobenzene and chloral with concentrated sulphuric acid.



1,1,1-trichloro-2,2 bis (*p*-chlorophenyl) ethane or DDT

4. **(c) :** Photochemical smog is caused by oxides of sulphur and nitrogen.

5. **(b) :** An antipyretic is a drug which is responsible for lowering temperature of the feverish organism to normal but has no effect on normal temperature states.

Solved Paper - 2009

1. Knowing that the chemistry of lanthanoids (Ln) is dominated by its +3 oxidation state, which of the following statements is incorrect ?

(a) Because of the large size of the Ln(III) ions the bonding in its compounds is predominantly ionic in character.
(b) The ionic sizes of Ln(III) decrease in general with increasing atomic number.
(c) Ln(III) compounds are generally colourless.
(d) Ln(III) hydroxides are mainly basic in character.

[Transition Metals Including Lanthanoids]

2. A liquid was mixed with ethanol and a drop of concentrated H_2SO_4 was added. A compound with a fruity smell was formed. The liquid was

(a) CH_3OH (b) HCHO
(c) CH_3COCH_3 (d) CH_3COOH

[Aldehydes, Ketones, Carboxylic Acids and Their Derivatives]

3. Arrange the carbanions,
$(CH_3)_3\bar{C}$, $\bar{C}Cl_3$, $(CH_3)_2\bar{C}H$, $C_6H_5\bar{C}H_2$
in order of their decreasing stability

(a) $C_6H_5\bar{C}H_2 > \bar{C}Cl_3 > (CH_3)_3\bar{C} > (CH_3)_2\bar{C}H$
(b) $(CH_3)_2\bar{C}H > \bar{C}Cl_3 > C_6H_5\bar{C}H_2 > (CH_3)_3\bar{C}$
(c) $\bar{C}Cl_3 > C_6H_5\bar{C}H_2 > (CH_3)_2\bar{C}H > (CH_3)_3\bar{C}$
(d) $(CH_3)_3\bar{C} > (CH_3)_2\bar{C}H > C_6H_5\bar{C}H_2 > \bar{C}Cl_3$

[General Organic Chemistry]

4. The alkene that exhibits geometrical isomerism is

(a) propene (b) 2-methylpropene
(c) 2-butene (d) 2-methyl-2-butene

[General Organic Chemistry]

5. In which of the following arrangements, the sequence is not strictly according to the property written against it ?

(a) $CO_2 < SiO_2 < SnO_2 < PbO_2$: increasing oxidising power
(b) HF < HCl < HBr < HI : increasing acid strength
(c) $NH_3 < PH_3 < AsH_3 < SbH_3$: increasing basic strength
(d) B < C < O < N : increasing first ionization enthalpy

[Chemistry of Representative Elements]

6. The major product obtained on interaction of phenol with sodium hydroxide and carbon dioxide is

(a) benzoic acid (b) salicylaldehyde
(c) salicylic acid (d) phthalic acid

[Halides, Hydroxy Compounds and Ethers]

7. Which of the following statements is incorrect regarding physisorption ?

(a) It occurs because of van der Waals forces.
(b) More easily liquefiable gases are adsorbed readily.
(c) Under high pressure it results into multi molecular layer on adsorbent surface.
(d) Enthalpy of adsorption ($\Delta H_{\text{adsorption}}$) is low and positive.

[Surface Chemistry]

8. Which of the following on heating with aqueous KOH produces acetaldehyde ?

(a) CH_3COCl (b) CH_3CH_2Cl
(c) CH_2ClCH_2Cl (d) CH_3CHCl_2

[Halides, Hydroxy Compounds and Ethers]

9. In an atom, an electron is moving with a speed of 600 m/s with an accuracy of 0.005%. Certainty with which the position of the electron can be located is ($h = 6.6 \times 10^{-34}$ kg m^2 s^{-1}, mass of electron, $e_m = 9.1 \times 10^{-31}$ kg)

(a) 1.52×10^{-4} m (b) 5.10×10^{-3} m
(c) 1.92×10^{-3} m (d) 3.84×10^{-3} m

[Atomic Structure, Chemical Bonding & Molecular Structure]

10. In a fuel cell methanol is used as fuel and oxygen gas is used as an oxidizer. The reaction is

$$CH_3OH_{(l)} + \frac{3}{2}O_{2(g)} \longrightarrow CO_{2(g)} + 2H_2O_{(l)}.$$

At 298 K standard Gibb's energies of formation for $CH_3OH_{(l)}$, $H_2O_{(l)}$ and $CO_{2(g)}$ are –166.2, –237.2 and –394.4 kJ mol^{-1} respectively. If standard enthalpy of combustion of methanol is –726 kJ mol^{-1}, efficiency of the fuel cell will be

(a) 80 % (b) 87%
(c) 90% (d) 97%

[Energetics]

11. Two liquids X and Y form an ideal solution. At 300 K, vapour pressure of the solution containing 1 mol of X and 3 mol of Y is 550 mm Hg. At the same temperature, if 1 mol of Y is further added to this solution, vapour pressure of the solution increases by 10 mm Hg. Vapour pressure (in mm Hg) of X and Y in their pure states will be, respectively

(a) 200 and 300 (b) 300 and 400
(c) 400 and 600 (d) 500 and 600

[Solutions]

12. The half-life period of a first order chemical reaction is 6.93 minutes. The time required for the completion of 99% of the chemical reaction will be (log 2 = 0.301)

(a) 230.3 minutes (b) 23.03 minutes
(c) 46.06 minutes (d) 460.6 minutes

[Kinetics]

13. Given : $E^{\circ}_{Fe^{3+}/Fe} = -0.036$ V, $E^{\circ}_{Fe^{2+}/Fe} = -0.439$ V. The value of standard electrode potential for the change, $Fe^{3+}_{(aq)} + e^- \rightarrow Fe^{2+}_{(aq)}$ will be

(a) – 0.072 V (b) 0.385 V
(c) 0.770 V (d) – 0.270 V

[Redox Reactions and Electrochemistry]

14. On the basis of the following thermochemical data : ($\Delta_f G^{\circ}\ H^{+}_{(aq)} = 0$).

$$H_2O_{(l)} \rightarrow H^+_{(aq)} + OH^-_{(aq)}\ ;\ \Delta H = 57.32 \text{ kJ}$$

$$H_{2(g)} + \frac{1}{2}O_{2(g)} \rightarrow H_2O_{(l)};\ \Delta H = -286.2 \text{ kJ}$$

The value of enthalpy of formation of OH^- ion at 25°C is

(a) – 22.88 kJ (b) – 228.88 kJ
(c) + 228.88 kJ (d) – 343.52 kJ

[Energetics]

15. Copper crystallizes in *fcc* with a unit cell length of 361 pm. What is the radius of copper atom ?

(a) 108 pm (b) 127 pm
(c) 157 pm (d) 181 pm

[States of Matter]

16. Which of the following has an optical isomer ?

(a) $[Co(NH_3)_3Cl]^+$ (b) $[Co(en)(NH_3)_2]^{2+}$
(c) $[Co(H_2O)_4(en)]^{3+}$ (d) $[Co(en)_2(NH_3)_2]^{3+}$

[Coordination Compounds and Organometallics]

17. Solid $Ba(NO_3)_2$ is gradually dissolved in a 1.0×10^{-4} M Na_2CO_3 solution. At what concentration of Ba^{2+} will a precipitate begin to form ?(K_{sp} for $BaCO_3 = 5.1 \times 10^{-9}$)

(a) 4.1×10^{-5} M (b) 5.1×10^{-5} M
(c) 8.1×10^{-8} M (d) 8.1×10^{-7} M

[Equilibrium]

18. Which one of the following reactions of xenon compounds is not feasible ?

(a) $XeO_3 + 6HF \rightarrow XeF_6 + 3H_2O$
(b) $3XeF_4 + 6H_2O \rightarrow 2Xe + XeO_3 + 12HF + 1.5\ O_2$
(c) $2XeF_2 + 2H_2O \rightarrow 2Xe + 4HF + O_2$
(d) $XeF_6 + RbF \rightarrow Rb[XeF_7]$

[Chemistry of Representative Elements]

19. Using MO theory predict which of the following species has the shortest bond length ?

(a) O_2^{2+} (b) O_2^{+}
(c) O_2^{-} (d) O_2^{2-}

[Atomic Structure, Chemical Bonding & Molecular Structure]

20. In context with the transition elements, which of the following statements is incorrect ?

(a) In addition to the normal oxidation states, the zero oxidation state is also shown by these elements in complexes.
(b) In the highest oxidation states, the transition metals show basic character and form cationic complexes.
(c) In the highest oxidation states of the first five transition elements (Sc to Mn), all the 4*s* and 3*d* electrons are used for bonding.
(d) Once the d^5 configuration is exceeded, the tendency to involve all the 3*d* electrons in bonding decreases.

[Transition Metals Including Lanthanoids]

21. Calculate the wavelength (in nanometre) associated with a proton moving at 1.0×10^3 ms^{-1}.
(Mass of proton = 1.67×10^{-27} kg and $h = 6.63 \times 10^{-34}$ Js)

(a) 0.032 nm (b) 0.40 nm
(c) 2.5 nm (d) 14.0 nm

[Atomic Structure, Chemical Bonding & Molecular Structure]

22. A binary liquid solution is prepared by mixing *n*-heptane and ethanol. Which one of the following statements is correct regarding the behaviour of the solution ?

(a) The solution formed is an ideal solution
(b) The solution is non-ideal, showing +ve deviation from Raoult's law.
(c) The solution is non-ideal, showing –ve deviation from Raoult's law.
(d) *n*-heptane shows +ve deviation while ethanol shows –ve deviation from Raoult's law.

[Solutions]

23. The number of stereoisomers possible for a compound of the molecular formula
$CH_3 - CH = CH - CH(OH) - Me$ is
(a) 3 (b) 2
(c) 4 (d) 6

[General Organic Chemistry]

24. The IUPAC name of *neo*-pentane is
(a) 2-methylbutane (b) 2,2-dimethylpropane
(c) 2-methylpropane (d) 2,2-dimethylbutane

[General Organic Chemistry]

25. The set representing the correct order of ionic radius is
(a) $Li^+ > Be^{2+} > Na^+ > Mg^{2+}$
(b) $Na^+ > Li^+ > Mg^{2+} > Be^{2+}$
(c) $Li^+ > Na^+ > Mg^{2+} > Be^{2+}$
(d) $Mg^{2+} > Be^{2+} > Li^+ > Na^+$

[Chemistry of Metals]

26. The two functional groups present in a typical carbohydrate are
(a) –OH and –COOH (b) –CHO and –COOH
(c) $>C=O$ and –OH (d) –OH and –CHO

[Biomolecules]

27. The bond dissociation energy of B – F in BF_3 is 646 kJ mol^{-1} whereas that of C – F in CF_4 is 515 kJ mol^{-1}. The correct reason for higher B – F bond dissociation energy as compared to that of C – F is
(a) smaller size of B-atom as compared to that of C-atom
(b) stronger σ bond between B and F in BF_3 as compared to that between C and F in CF_4
(c) significant $p\pi$-$p\pi$ interaction between B and F in BF_3 whereas there is no possibility of such interaction between C and F in CF_4.
(d) lower degree of $p\pi$-$p\pi$ interaction between B and F in BF_3 than that between C and F in CF_4.

[Chemistry of Representative Elements]

28. In Cannizzaro reaction given below

$$2\,PhCHO \xrightarrow{:OH^-} PhCH_2OH + PhCOO^-$$

the slowest step is
(a) the attack of $:OH^-$ at the carboxyl group
(b) the transfer of hydride to the carbonyl group
(c) the abstraction of proton from the carboxylic group
(d) the deprotonation of $PhCH_2OH$

[Aldehydes, Ketones, Carboxylic Acids and Their Derivatives]

29. Which of the following pairs represents linkage isomers ?
(a) $[Cu(NH_3)_4][PtCl_4]$ and $[Pt(NH_3)_4][CuCl_4]$
(b) $[Pd(PPh_3)_2(NCS)_2]$ and $[Pd(PPh_3)_2(SCN)_2]$
(c) $[Co(NH_3)_5(NO_3)]SO_4$ and $[Co(NH_3)_5(SO_4)]NO_3$
(d) $[PtCl_2(NH_3)_4]Br_2$ and $[PtBr_2(NH_3)_4]Cl_2$

[Coordination Compounds and Organometallics]

30. Buna-N synthetic rubber is a co-polymer of
(a) $H_2C = CH - C(Cl) = CH_2$ and $H_2C = CH - CH = CH_2$
(b) $H_2C = CH - CH = CH_2$ and $H_5C_6 - CH = CH_2$
(c) $H_2C = CH - CN$ and $H_2C = CH - CH = CH_2$
(d) $H_2C = CH - CN$ and $H_2C = CH - C(Cl) = CH_2$

[Chemitry in Action]

EXPLANATIONS

1. **(c) :** Ln^{3+} compounds are generally coloured in the solid state as well as in aqueous solution. Colour appears due to presence of unpaired *f*-electrons which undergo *f*-*f* transition.

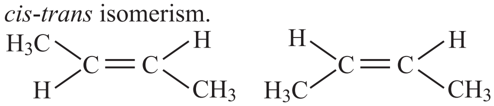
2. **(d) :** Since the compound formed has a fruity smell, it is an ester, thus the liquid to which ethanol and conc. H_2SO_4 are added must be an acid.

$$\therefore\ CH_3COOH + C_2H_5OH \xrightarrow{\text{conc.}H_2SO_4} CH_3COOC_2H_5 + H_2O.$$

3. **(c) :** The groups having $+I$ effect decrease the stability while groups having $-I$ effect increase the stability of carbanions. Benzyl carbanion is stabilized due to resonance. Also, out of 2° and 3° carbanions, 2° carbanions are more stable, thus the decreasing order of stability is :

$$\bar{C}Cl_3 > C_6H_5\bar{C}H_2 > (CH_3)_2\bar{C}H > (CH_3)_3\bar{C}.$$

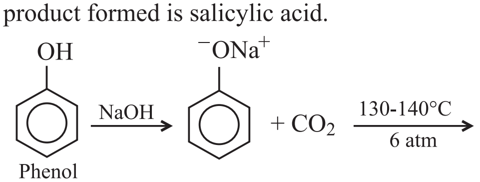
4. **(c) :** When two groups attached to a double bonded carbon atom are same, the compound does not exhibit geometrical isomerism.
Compounds in which the two groups attached to a double bonded carbon are different, exhibit geometrical isomerism, thus, only 2-butene exhibits *cis-trans* isomerism.

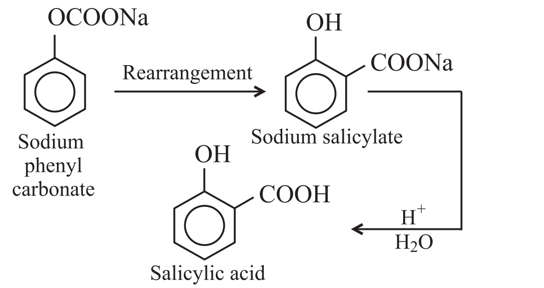


trans-2-butene　　*cis*-2-butene

5. **(c) :** In group 15 hydrides, the basic character decreases on going down the group due to decrease in the availability of the lone pair of electrons because of the increase in size of elements from N to Bi. Thus, correct order of basicity is
$NH_3 > PH_3 > AsH_3 > SbH_3$.

6. **(c) :** The reaction of phenol with NaOH and CO_2 is known as Kolbe-Schmidt or Kolbe's reaction. The product formed is salicylic acid.





7. **(d) :** Physical adsorption is an exothermic process (*i.e.*, $\Delta H = -$ve) but its value is quite low because the attraction of gas molecules and solid surface is weak van der Waals forces.

8. **(d) :** $\underset{\text{Gem-dihalide}}{CH_3CHCl_2} \xrightarrow{\text{aq. KOH}} \underset{\text{unstable}}{CH_3CH(OH)_2} \xrightarrow{-H_2O} \underset{\text{acetaldehyde}}{CH_3CHO}$

9. **(c) :** Given, velocity of e^-, $v = 600\ ms^{-1}$
Accuracy of velocity = 0.005%

$$\therefore\ \Delta v = \frac{600 \times 0.005}{100} = 0.03$$

According to Heisenberg's uncertainty principle,

$$\Delta x \cdot m\Delta v \geq \frac{h}{4\pi}$$

$$\Rightarrow\ \Delta x = \frac{6.6 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 0.03}$$

$= 1.92 \times 10^{-3}$ m.

10. **(d) :** For the given reaction,

$$CH_3OH_{(l)} + \frac{3}{2}O_{2(g)} \longrightarrow CO_{2(g)} + 2H_2O_{(l)},$$

$$\Delta H = -726\ kJ\ mol^{-1}$$

also, $\Delta G_f^\circ[CH_3OH_{(l)}] = -166.2\ kJ\ mol^{-1}$
$\Delta G_f^\circ[H_2O_{(l)}] = -237.2\ kJ\ mol^{-1}$
and $\Delta G_f^\circ[CO_{2(g)}] = -394.4\ kJ\ mol^{-1}$
Now,

$$\Delta G^\circ_{\text{reaction}} = \sum \Delta G^\circ_{f\ \text{products}} - \sum \Delta G^\circ_{f\ \text{reactants}}$$

$= [-394.4 + 2 \times (-237.2)] - (-166.2)$
$= -702.6\ kJ\ mol^{-1}$

$$\% \text{ Efficiency} = \frac{\Delta G}{\Delta H} \times 100 = \frac{-702.6}{-726} \times 100$$
$$= 96.77\%$$

$\therefore$ Efficiency $\approx 97\%$.

11. (c) : $P_T = p_X^\circ x_X + p_Y^\circ x_Y$

where, P_T = Total presure

p_X° = Vapour pressure of X in pure state

p_Y° = Vapour pressure of Y in pure state

x_X = Mole fraction of $X = 1/4$

x_Y = Mole fraction of $Y = 3/4$.

(i) When $T = 300$ K, $P_T = 550$ mm Hg

$$\therefore \ 550 = p_X^\circ\left(\frac{1}{4}\right) + p_Y^\circ\left(\frac{3}{4}\right)$$

$$\Rightarrow \quad p_X^\circ + 3p_Y^\circ = 2200 \qquad .....(1)$$

(ii) When at $T = 300$ K, 1 mole of Y is added,

$P_T = (550 + 10)$ mm Hg

$\therefore$ $x_X = 1/5$ and $x_Y = 4/5$

$$\Rightarrow \ 560 = p_X^\circ\left(\frac{1}{5}\right) + p_Y^\circ\left(\frac{4}{5}\right)$$

$$\text{or} \ \ p_X^\circ + 4p_Y^\circ = 2800 \qquad ....(2)$$

On solving equations (1) and (2), we get

p_Y° = 600 mm Hg and p_X° = 400 mm Hg.

12. (c) : Given, $t_{1/2} = 6.93$ min

$$\lambda = \frac{0.693}{t_{1/2}} \qquad (\text{for } 1^{\text{st}} \text{ order reaction})$$
$$= \frac{0.693}{6.93}$$

Since reaction follows 1st order kinetics,

$$t = \frac{2.303}{\lambda}\log\frac{[A_0]}{[A]}$$

where $[A_0]$ = initial concentration

and $[A]$ = concentration of A at time t.

$\because$ Reaction is 99% complete, $\therefore \ \dfrac{[A_0]}{[A]} = \dfrac{100}{1}$.

$$\text{or} \quad t = \frac{2.303 \times 6.93}{0.693} \times \log(100)$$

$= 23.03 \times 2 \log(10) = 46.06$ minutes.

13. (c) : Given,

$Fe^{3+} + 3e^- \longrightarrow Fe \quad ; \ E_1^\circ = -0.036$ V.

$Fe^{2+} + 2e^- \longrightarrow Fe \quad ; \ E_2^\circ = -0.439$ V.

Required equation is

$Fe^{3+} + e^- \longrightarrow Fe^{2+} \quad ; \ E_3^\circ = ?$

Applying $\Delta G^\circ = -nFE^\circ$

$\therefore \quad \Delta G_3^\circ = \Delta G_1^\circ - \Delta G_2^\circ$

$(-n_3FE_3^\circ) = (-n_1FE_1^\circ) - (-n_2FE_2^\circ)$

$E_3^\circ = 3E_1^\circ - 2E_2^\circ = 3 \times (-0.036) - 2 \times (-0.439)$

$E_3^\circ = -0.108 + 0.878 = 0.77$ V.

14. (b) : The reaction for the formation of $OH^-_{(aq)}$ is

$$H_{2(g)} + \frac{1}{2}O_{2(g)} \longrightarrow H^+_{(aq)} + OH^-_{(aq)}$$

This is obtained by adding the two given equations.

$\therefore \quad \Delta H$ for the above reaction = 57.32 + (–286.2)
= –228.88 kJ.

15. (b) : Since Cu crystallizes in *fcc* lattice,

$\therefore$ radius of Cu atom,

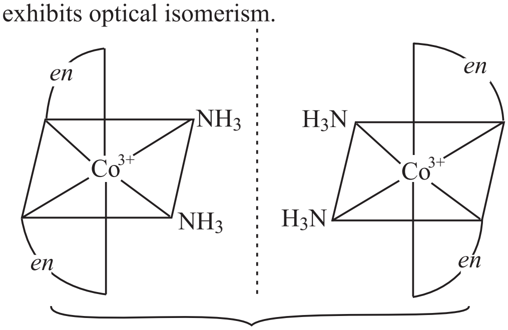
$$r = \frac{a}{2\sqrt{2}} \quad (a = \text{edge length})$$

Given, a = 361 pm

$$\therefore \ r = \frac{361}{2\sqrt{2}} \approx 127 \text{ pm}$$

16. (d) : Optical isomerism is usually exhibited by octahedral compounds of the type $[M(AA)_2B_2]$, where (AA) is a symmetrical bidentate ligand. Square planar complexes rarely show optical isomerism on accout of presence of axis of symmetry.

Thus among the given options, $[Co(en)_2(NH_3)_2]^{3+}$ exhibits optical isomerism.



Non-superimposable
mirror images

17. (b) : K_{sp} for $BaCO_3 = [Ba^{2+}][CO_3^{2-}]$

given, $[CO_3^{2-}] = 1 \times 10^{-4}$ M (from Na_2CO_3)

$K_{sp} = 5.1 \times 10^{-9}$

$\therefore \quad 5.1 \times 10^{-9} = [Ba^{2+}] \times [10^{-4}]$.

$\Rightarrow \quad [Ba^{2+}] = 5.1 \times 10^{-5}$ M.

Thus, when $[Ba^{2+}] = 5.1 \times 10^{-5}$ M, $BaCO_3$ precipitate will begin to form.

18. (a) : The reaction is not feasible because XeF_6 formed will further produce XeO_3 by getting

hydrolysed.
$XeF_6 + 3H_2O \longrightarrow XeO_3 + 3H_2F_2$.

19. **(a) :** According to MOT, the molecular orbital electronic configuration of

$O_2^{2+}: (\sigma 1s)^2(\sigma^* 1s)^2(\sigma 2s)^2(\sigma^* 2s)^2(\sigma 2p_z)^2(\pi 2p_x)^2 = (\pi 2p_y)^2$

$\therefore$ B.O. $= \frac{10-4}{2} = 3$.

$O_2^{+}: (\sigma 1s)^2(\sigma^* 1s)^2(\sigma 2s)^2(\sigma^* 2s)^2(\sigma 2p_z)^2(\pi 2p_x)^2 = (\pi 2p_y)^2(\pi^* 2p_x)^1$

$\therefore$ B.O. $= \frac{10-5}{2} = 2.5$.

$O_2^{-}: (\sigma 1s)^2(\sigma^* 1s)^2(\sigma 2s)^2(\sigma^* 2s)^2(\sigma 2p_z)^2(\pi 2p_x)^2 = (\pi 2p_y)^2(\pi^* 2p_x)^2 = (\pi^* 2p_y)^1$

$\therefore$ B.O. $= \frac{10-7}{2} = 1.5$.

$O_2^{2-}: (\sigma 1s)^2(\sigma^* 1s)^2(\sigma 2s)^2(\sigma^* 2s)^2(\sigma 2p_z)^2(\pi 2p_x)^2 = (\pi 2p_y)^2(\pi^* 2p_x)^2 = (\pi^* 2p_y)^2$

$\therefore$ B.O. $= \frac{10-8}{2} = 1$.

$\because$ B.O. $\propto \frac{1}{\text{Bond length}}$,

$\therefore$ O_2^{2+} has the shortest bond length.

20. **(b) :** When the transition metals are in their highest oxidation state, they no longer have tendency to give away electrons, thus they are not basic but show acidic character and form anionic complexes.

21. **(b) :** According to de-Broglie's equation,

$\lambda = \frac{h}{mv}$

Given, $v = 1.0 \times 10^3$ ms^{-1}

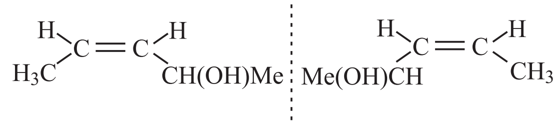
$\therefore$ $\lambda = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1.0 \times 10^3} = 3.9 \times 10^{-10}$ m.

or $\lambda \approx 0.4$ nm.

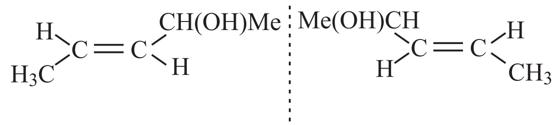
22. **(b) :** The solution containing *n*-heptane and ethanol shows non-ideal behaviour with positive deviation from Raoult's law. This is because the ethanol molecules are held together by strong H-bonds, however the forces between *n*-heptane and ethanol are not very strong, as a result they easily vapourise showing higher vapour presure than expected.

23. **(c) :** The given compound has a C $=$ C group and one chiral (*) carbon,

$H_3C - HC{=}CH - \overset{*}{C}H(OH) - Me$

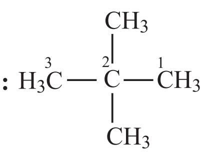


d, l isomers of *cis*-form



d, l isomers of *trans*-form

$\therefore$ Total stereoisomers = 4.

24. **(b) :** 

neo-pentane or 2,2-dimethylpropane

25. **(b) :** Moving from left to right in a period, the ionic radii decrease due to increase in effective nuclear charge as the additional electrons are added to the same shell, however from top to bottom the ionic radii increase with increasing atomic number and presence of additional shells. Also Li and Mg are diagonally related and hence the order is
$Na^+ > Li^+ > Mg^{2+} > Be^{2+}$.

26. **(c) :** Carbohydrates are essentially polyhydroxy aldehydes and polyhydroxy ketones. Thus the two functional groups present are $>C{=}O$ (aldehyde or ketone) and –OH.

27. **(c) :** In BF_3, B is sp^2 hybridised and has a vacant 2*p*-orbital which overlaps laterally with a filled 2*p*-orbital of F forming strong *p*π-*p*π bond. However in CF_4, C does not have any vacant *p*-orbitals to undergo π-bonding.
Thus $B.E._{B-F} > B.E._{C-F}$.

28. **(b) :** Rate determining step is always the slowest step. In case of Cannizzaro reaction, H-transfer to the carbonyl group is the rate determining step and hence the slowest.
Mechanism :

$$C_6H_5-\underset{\substack{|\\ H}}{C}=O + OH^- \xrightarrow{\text{Fast}}$$

$$C_6H_5-\overset{\substack{H\\ |}}{\underset{\substack{|\\ OH}}{C}}-O^- + O=\overset{\substack{H\\ |}}{C}-C_6H_5 \xrightarrow[\text{Hydride transfer}]{\text{Slow,}}$$

$$C_6H_5-\underset{\substack{|\\ OH}}{C}=O + {}^-O-\overset{\substack{H\\ |}}{\underset{\substack{|\\ H}}{C}}-C_6H_5 \xrightarrow{\text{Rearrangement}}$$

$$C_6H_5-CH_2-OH + C_6H_5COO^-$$

29. (b) : Linkage isomerism is exhibited by compounds containing ambidentate ligand.
In $[Pd(PPh_3)_2(NCS)_2]$, the linkage of NCS and Pd is through N.
In $[Pd(PPh_3)_2(SCN)_2]$, the linkage of SCN and Pd is through S.

30. (c) : Buna-N is a co-polymer of butadiene and acrylonitrile.

$$\underset{\text{1,3-Butadiene}}{nCH_2=CH-CH=CH_2} + \underset{\text{Acrylonitrile}}{nCH_2=\overset{\substack{CN\\ |}}{CH}} \longrightarrow$$

$$\underset{\text{Buna-N}}{\left[CH_2-CH=CH-CH_2-CH_2-\overset{\substack{CN\\ |}}{CH}\right]_n}$$

Solved Paper - 2010

1. Ionisation energy of He^+ is 19.6×10^{-18} J atom^{-1}. The energy of the first stationary state (n = 1) of Li^{2+} is

(a) 8.82×10^{-17} J atom^{-1}
(b) 4.41×10^{-16} J atom^{-1}
(c) -4.41×10^{-17} J atom^{-1}
(d) -2.2×10^{-15} J atom^{-1}

[Atomic Structure, Chemical Bonding & Molecular Structure]

2. 29.5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method and the evolved ammonia was absorbed in 20 mL of 0.1 M HCl solution. The excess of the acid required 15 mL of 0.1 M NaOH solution for complete neutralization. The percentage of nitrogen in the compound is

(a) 29.5 (b) 59.0
(c) 47.4 (d) 23.7

[General Organic Chemistry]

3. The energy required to break one mole of Cl—Cl bonds in Cl_2 is 242 kJ mol^{-1}. The longest wavelength of light capable of breaking a single Cl—Cl bond is
($c = 3 \times 10^8$ m s^{-1} and $N_A = 6.02 \times 10^{23}$ mol^{-1})

(a) 494 nm (b) 594 nm
(c) 640 nm (d) 700 nm

[Atomic Structure, Chemical Bonding & Molecular Structure]

4. Percentages of free space in cubic close packed structure and in body centred packed structure are respectively

(a) 48% and 26% (b) 30% and 26%
(c) 26% and 32% (d) 32% and 48%

[States of Matter]

5. From amongst the following alcohols the one that would react fastest with conc.HCl and anhydrous $ZnCl_2$, is

(a) 1-Butanol (b) 2-Butanol
(c) 2-Methylpropan-2-ol
(d) 2-Methylpropanol

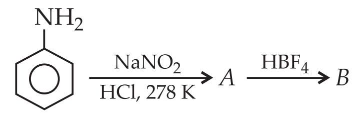
[Halides, Hydroxy Compounds & Ethers]

6. If 10^{-4} dm^3 of water is introduced into a 1.0 dm^3 flask at 300 K, how many moles of water are in the vapour phase when equilibrium is established? (Given : Vapour pressure of H_2O at 300 K is 3170 Pa; R = 8.314 J K^{-1} mol^{-1})

(a) 1.27×10^{-3} mol (b) 5.56×10^{-3} mol
(c) 1.53×10^{-2} mol (d) 4.46×10^{-2} mol

[States of Matter]

7. In the chemical reaction,



the compounds A and B respectively are

(a) nitrobenzene and chlorobenzene
(b) nitrobenzene and fluorobenzene
(c) phenol and benzene
(d) benzene diazonium chloride and fluorobenzene

[Nitrogen Containing Compounds]

8. For a particular reversible reaction at temperature T, ΔH and ΔS were found to be both +ve. If T_e is the temperature at equilibrium, the reaction would be spontaneous when

(a) $T = T_e$ (b) $T_e > T$
(c) $T > T_e$ (d) T_e is 5 times T

[Energetics]

9. Out of the following, the alkene that exhibits optical isomerism is

(a) 2-methyl-2-pentene (b) 3-methyl-2-pentene
(c) 4-methyl-1-pentene (d) 3-methyl-1-pentene

[General Organic Chemistry]

10. If sodium sulphate is considered to be completely dissociated into cations and anions in aqueous solution, the change in freezing point of water (ΔT_f), when 0.01 mol of sodium sulphate is dissolved in 1 kg of water, is (K_f = 1.86 K kg mol^{-1})

(a) 0.0186 K (b) 0.0372 K
(c) 0.0558 K (d) 0.0744 K

[Solutions]

11. The edge length of a face centred cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is

(a) 144 pm (b) 288 pm
(c) 398 pm (d) 618 pm

[States of Matter]

12. The correct order of increasing basicity of the given conjugate bases ($R = CH_3$) is
(a) $RCOO^- < HC \equiv C^- < NH_2^- < R^-$
(b) $RCOO^- < HC \equiv C^- < R^- < NH_2^-$
(c) $R^- < HC \equiv C^- < RCOO^- < NH_2^-$
(d) $RCOO^- < NH_2^- < HC \equiv C^- < R^-$
[Equilibrium]

13. In aqueous solution the ionisation constants for carbonic acid are
$$K_1 = 4.2 \times 10^{-7} \text{ and } K_2 = 4.8 \times 10^{-11}$$
Select the correct statement for a saturated 0.034 M solution of the carbonic acid.
(a) The concentration of H^+ is double that of CO_3^{2-}.
(b) The concentration of CO_3^{2-} is 0.034 M.
(c) The concentration of CO_3^{2-} is greater than that of HCO_3^-.
(d) The concentration of H^+ and HCO_3^- are approximately equal.
[Equilibrium]

14. The Gibb's energy for the decomposition of Al_2O_3 at 500°C is as follows :
$2/3Al_2O_3 \longrightarrow 4/3Al + O_2$, $\Delta_rG = +966$ kJ mol^{-1}.
The potential difference needed for electrolytic reduction of Al_2O_3 at 500°C is at least
(a) 5.0 V (b) 4.5 V
(c) 3.0 V (d) 2.5 V
[Redox Reactions & Electrochemistry]

15. The correct sequence which shows decreasing order of the ionic radii of the element is
(a) $O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$
(b) $Al^{3+} > Mg^{2+} > Na^+ > F^- > O^{2-}$
(c) $Na^+ > Mg^{2+} > Al^{3+} > O^{2-} > F^-$
(d) $Na^+ > F^- > Mg^{2+} > O^{2-} > Al^{3+}$
[Periodic Properties]

16. Solubility product of silver bromide is 5.0×10^{-13}. The quantity of potassium bromide (molar mass taken as 120 g mol^{-1}) to be added to 1 litre of 0.05 M solution of silver nitrate to start the precipitation of AgBr is
(a) 5.0×10^{-8} g (b) 1.2×10^{-10} g
(c) 1.2×10^{-9} g (d) 6.2×10^{-5} g
[Equilibrium]

17. The standard enthalpy of formation of NH_3 is -46 kJ mol^{-1}. If the enthalpy of formation of H_2 from its atoms is -436 kJ mol^{-1} and that of N_2 is -712 kJ mol^{-1}, the average bond enthalpy of N—H bond in NH_3 is
(a) -1102 kJ mol^{-1} (b) -964 kJ mol^{-1}
(c) $+352$ kJ mol^{-1} (d) $+1056$ kJ mol^{-1}
[Energetics]

18. A solution contains 2.675 g of $CoCl_3 \cdot 6NH_3$ (molar mass = 267.5 g mol^{-1}) is passed through a cation exchanger. The chloride ions obtained in solution were treated with excess of $AgNO_3$ to give 4.78 g of AgCl (molar mass = 143.5 g mol^{-1}). The formula of the complex is (At. mass of Ag = 108 u)
(a) $[CoCl(NH_3)_5]Cl_2$ (b) $[Co(NH_3)_6]Cl_3$
(c) $[CoCl_2(NH_3)_4]Cl$ (d) $[CoCl_3(NH_3)_3]$
[Coordination Compounds and Organometallics]

19. The time for half life period of a certain reaction $A \longrightarrow$ Products is 1 hour. When the initial concentration of the reactant A is 2.0 mol L^{-1}, how much time does it take for its concentration to come from 0.50 to 0.25 mol L^{-1} if it is a zero order reaction ?
(a) 1 h (b) 4 h
(c) 0.5 h (d) 0.25 h
[Kinetics]

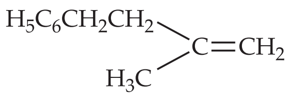
20. Consider the reaction :
$Cl_{2(aq)} + H_2S_{(aq)} \longrightarrow S_{(s)} + 2H^+_{(aq)} + 2Cl^-_{(aq)}$
The rate of reaction for this reaction is
rate = $k[Cl_2][H_2S]$
Which of these mechanism is/are consistent with this rate equation ?
A. $Cl_2 + H_2S \longrightarrow H^+ + Cl^- + Cl^+ + HS^-$ (slow)
$Cl^+ + HS^- \longrightarrow H^+ + Cl^- + S$ (fast)
B. $H_2S \Leftrightarrow H^+ + HS^-$ (fast equilibrium)
$Cl_2 + HS^- \longrightarrow 2Cl^- + H^+ + S$ (slow)
(a) A only (b) B only
(c) Both A and B (d) Neither A nor B
[Kinetics]

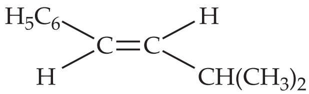
21. Three reactions involving $H_2PO_4^-$ are given below:
(i) $H_3PO_4 + H_2O \longrightarrow H_3O^+ + H_2PO_4^-$
(ii) $H_2PO_4^- + H_2O \longrightarrow HPO_4^{2-} + H_3O^+$
(iii) $H_2PO_4^- + OH^- \longrightarrow H_3PO_4 + O^{2-}$
In which of the above does $H_2PO_4^-$ act as an acid ?
(a) (i) only (b) (ii) only
(c) (i) and (ii) (d) (iii) only
[Equilibrium]

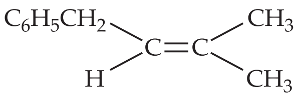
22. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of 44 u. The alkene is
(a) ethene (b) propene
(c) 1-butene (d) 2-butene
[Hydrocarbons]

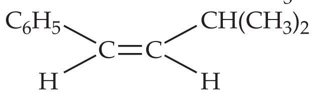
23. The main product of the following reaction is

$$C_6H_5CH_2CH(OH)CH(CH_3)_2 \xrightarrow{\text{Conc.}H_2SO_4}$$

(a) 

(b) 

(c) 

(d) 

[Halides, Hydroxy Compounds & Ethers]

24. Biuret test is not given by

(a) proteins (b) carbohydrates
(c) polypeptide (d) urea

[Practical Chemistry]

25. The correct order of $E^\circ_{M^{2+}/M}$ values with negative sign for the four successive elements Cr, Mn, Fe and Co is

(a) Cr > Mn > Fe > Co (b) Mn > Cr > Fe > Co
(c) Cr > Fe > Mn > Co (d) Fe > Mn > Cr > Co

[Transition Metals Including Lanthanides]

26. Which one of the following has an optical isomer ?

(a) $[Zn(en)_2]^{2+}$ (b) $[Zn(en)(NH_3)_2]^{2+}$
(c) $[Co(en)_3]^{3+}$ (d) $[Co(H_2O)_4(en)]^{3+}$

[Coordination Compounds and Organometallics]

27. The polymer containing strong intermolecular forces *e.g.*, hydrogen bonding is

(a) natural rubber (b) teflon
(c) nylon-6,6 (d) polystyrene

[Polymers]

28. At 25°C, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions start precipitating in the form of $Mg(OH)_2$ from a solution of 0.001 M Mg^{2+} ions ?

(a) 8 (b) 9 (c) 10 (d) 11

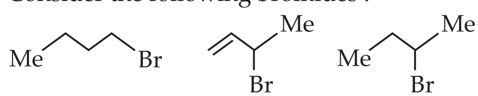
[Equilibrium]

29. On mixing, heptane and octane form an ideal solution. At 373 K, the vapour pressure of the two liquid components (heptane and octane) are 105 kPa and 45 kPa respectively. Vapour pressure of the solution obtained by mixing 25.0 g of heptane and 35 g of octane will be (molar mass of heptane = 100 g mol^{-1} and of octane = 114 g mol^{-1})

(a) 144.5 kPa (b) 72.0 kPa
(c) 36.1 kPa (d) 96.2 kPa

[Solutions]

30. Consider the following bromides :



(*A*) (*B*) (*C*)

The correct order of S_N1 reactivity is

(a) $A > B > C$ (b) $B > C > A$
(c) $B > A > C$ (d) $C > B > A$

[Halides, Hydroxy Compounds & Ethers]

EXPLANATIONS

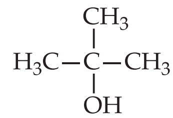
1. **(c) :** $I.E.(He^+) = 19.6 \times 10^{-18}$ J atom^{-1}
E_1 (for H) $\times Z^2 = I.E.$
$E_1 \times 4 = -19.6 \times 10^{-18}$
E_1 (for Li^{2+}) $= E_1$ (for H) $\times 9$
$$= \frac{-19.6 \times 10^{-18} \times 9}{4} = -44.1 \times 10^{-18} \text{ J atom}^{-1}$$

2. **(d) :** The % of N according to Kjeldahl's method
$$= \frac{1.4 \times N_1 \times V}{w}$$
N_1 = Normality of the standard acid = 0.1 N
w = Mass of the organic compound taken
= 29.5 mg = 29.5×10^{-3} g
V = Volume of N_1 acid neutralised by ammonia
= (20 – 15) = 5 ml.
$$\Rightarrow \quad \%N = \frac{1.4 \times 0.1 \times 5}{29.5 \times 10^{-3}} = 23.7$$

3. **(a) :** Energy required to break 1 mol of bonds
= 242 kJ mol^{-1}
$\therefore$ Energy required to break 1 bond
$$= \frac{242 \times 10^3}{6.02 \times 10^{23}} \text{ J}$$
We know that, $E = \frac{hc}{\lambda}$
Given, $c = 3 \times 10^8$ m s^{-1}.
$$\Rightarrow \frac{242 \times 10^3}{6.02 \times 10^{23}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda}$$
$$\therefore \quad \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8 \times 6.02 \times 10^{23}}{242 \times 10^3}$$
$= 0.494 \times 10^{-6}$ m = 494 nm.

4. **(c) :** The packing efficiency in a ccp structure
= 74%
$\therefore$ Percentage free space = 100 – 74 = 26%
Packing efficiency in a body centred structure
= 68%
$\therefore$ Percentage free space = 100 – 68 = 32%

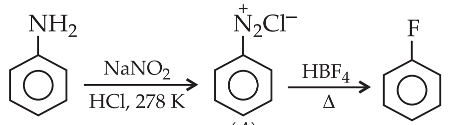
5. **(c) :** The reagent, conc.HCl and anhydrous $ZnCl_2$ is Lucas reagent, which is used to distinguish between 1°, 2° and 3° alcohols.
3° alcohol + Lucas reagent $\longrightarrow$ Immediate turbidity.
2° alcohol + Lucas reagent $\longrightarrow$ Turbidity after 5 mins.
1° alcohol + Lucas reagent $\longrightarrow$ No reaction.
Thus, the required alcohol is 2-methylpropan-2-ol, *i.e.*,



(3° alcohol)

6. **(a) :** The volume occupied by water molecules in vapour phase is $(1 - 10^{-4})$ dm^3, *i.e.*, approximately 1 dm^3.
$PV = nRT$
$3170 \times 1 \times 10^{-3} = n_{H_2O} \times 8.314 \times 300$
$$n_{H_2O} = \frac{3170 \times 10^{-3}}{8.314 \times 300} = 1.27 \times 10^{-3} \text{ mol}$$

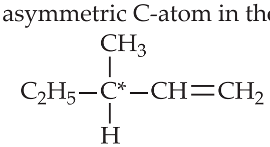
7. **(d) :**



(A) Benzenediazonium chloride
(B) Fluorobenzene

8. **(c) :** According to Gibb's formula,
$$\Delta G = \Delta H - T\Delta S$$
Since ΔH and ΔS, both are +ve, for $\Delta G < 0$, the value of $T > T_e$.

9. **(d) :** 3-Methyl-1-pentene exhibits optical isomerism as it has an asymmetric C-atom in the molecule.



3-methyl-1-pentene

10. **(c) :** Depression in freezing point, $\Delta T_f = i \times K_f \times m$
For sodium sulphate, $i = 3$
$$m = \frac{0.01}{1 \text{ kg}} = 0.01 \text{ m}.$$
Given, $K_f = 1.86$ K kg mol^{-1}.
$\therefore \quad \Delta T_f = 3 \times 1.86 \times 0.01 = 0.0558$ K.

11. **(a) :** In fcc lattice, $r_c + r_a = \frac{a}{2}$
Given, $a = 508$ pm
$r_c = 110$ pm

$\therefore \quad 110 + r_a = \frac{508}{2} \Rightarrow r_a = 144$ pm.

12. (a) : The order of acidity can be explained on the basis of the acidity of the acids of the given conjugate base. Stronger is the acid, weaker is the conjugate base. Since RCOOH is the strongest acid amongst all, $RCOO^-$ is the weakest base. Due to sp hybridised carbon, acetylene is also acidic and hence a weak base but stronger than $RCOO^-$. As sp^3 carbon is less electronegative than sp^3 nitrogen, R^- is more basic than NH_2^-.

13. (d) : $H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$; $K_1 = 4.2 \times 10^{-7}$

$HCO_3^- \rightleftharpoons H^+ + CO_3^{2-}$; $K_2 = 4.8 \times 10^{-11}$

$\because$ $K_1 >> K_2$, so H_2CO_3 ionises more than HCO_3^- and hence, contribution of H^+ is mostly due to ionisation of carbonic acid, thus the concentrations of H^+ and HCO_3^- are approximately equal.

14. (d) : The ionic reactions are :

$2/3Al_2^{3+} + 4e^- \longrightarrow 4/3Al$

$2/3O_3^{2-} \longrightarrow O_2 + 4e^-$

Thus, no. of electrons transferred = 4 = n.

$\Delta G = -nFE = -4 \times 96500 \times E$

or $966 \times 10^3 = -4 \times 96500 \times E$

$$\Rightarrow E = -\frac{966 \times 10^3}{4 \times 96500} = -2.5 \text{ V}.$$

15. (a) : All the given species are isoelectronic. Among isoelectronic species, anions generally have greater size than cations.
Also greater, the nuclear charge (Z) of the ion, smaller the size. Thus the order is :

$$O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$$

16. (c) : Given, $(K_{sp})_{AgBr} = 5.0 \times 10^{-13}$

The required equation is,

$KBr + AgNO_3 \longrightarrow AgBr + KNO_3$

Given, $[AgNO_3] = 0.05$ M

$\Rightarrow [Ag^+] = [NO_3^-] = 0.05$ M

$[Ag^+][Br^-] = (K_{sp})_{AgBr}$

$\Rightarrow 0.05 \times [Br^-] = 5 \times 10^{-13}$

$$\Rightarrow [Br^-] = \frac{5 \times 10^{-13}}{5 \times 10^{-2}} = 1 \times 10^{-11} \text{ M}$$

$\because \quad [K^+] = [Br^-] = [KBr]$

$\therefore \quad [KBr] = 1 \times 10^{-11}$ M.

$$\text{Molarity} = \frac{n_{KBr}}{V_{Solution}(L)}$$

$$1 \times 10^{-11} = \frac{w_{KBr}/120}{1} \quad (\text{Mol. wt. of KBr} = 120)$$

$\Rightarrow \quad w_{KBr} = 1 \times 10^{-11} \times 120 = 120 \times 10^{-11}$

$w_{KBr} = 1.2 \times 10^{-9}$ g

17. (c) : $1/2N_2 + 3/2H_2 \longrightarrow NH_3$

$B.E.$ 712 436

$$\therefore \quad (\Delta H_f^\circ)_{NH_3} = \left[\frac{1}{2}B.E._{N_2} + \frac{3}{2}B.E._{H_2} - 3B.E._{N-H}\right]$$

$$-46 = \left[\frac{1}{2} \times 712 + \frac{3}{2} \times 436 - 3B.E._{N-H}\right]$$

$-46 = 356 + 654 - 3B.E._{N-H}$

$3B.E._{N-H} = 1056$

$$B.E._{N-H} = \frac{1056}{3} = 352 \text{ kJ mol}^{-1}$$

18. (b) : No. of moles of $CoCl_3 \cdot 6NH_3 = \frac{2.675}{267.5} = 0.01$.

No. of moles of AgCl $= \frac{4.78}{143.5} = 0.03$.

Since 0.01 moles of the complex $CoCl_3 \cdot 6NH_3$ gives 0.03 moles of AgCl on treatment with $AgNO_3$, it implies that 3 chloride ions are ionisable, in the complex. Thus, the formula of the complex is $[Co(NH_3)_6]Cl_3$.

19. (d) : For a zero order reaction, $t_{1/2}$ is given as

$$t_{1/2} = \frac{[A_0]}{2k} \quad \text{or} \quad k = \frac{[A_0]}{2t_{1/2}}$$

Given, $t_{1/2} = 1$hr, $[A_0] = 2$M

$$\therefore \quad k = \frac{2}{2 \times 1} = 1 \text{ mol L}^{-1} \text{ hr}^{-1}$$

Integrated rate law for zero order reaction is

$[A] = -kt + [A_0]$

Here, $[A_0] = 0.5$ M and $[A] = 0.25$ M

$\Rightarrow \quad 0.25 = -t + 0.5 \qquad \Rightarrow t = 0.25$ hours.

20. (a) : The rate equation depends upon the rate determining step. The given rate equation is only consistent with the mechanism A.

21. (b) : In equation (ii), $H_2PO_4^-$ acts as a proton donor and thus, acts as an acid.

22. (d) : $\underset{\text{(Symm. alkene)}}{RCH{=}CHR} \xrightarrow[\text{(ii) Zn, } H_2O]{\text{(i) } O_3} 2RCHO$

Molecular mass of RCHO = 44

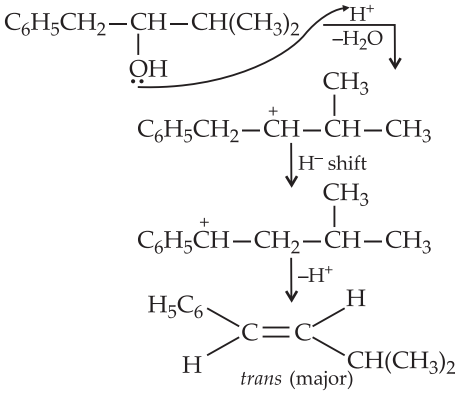
$\Rightarrow \quad R + 12 + 1 + 16 = 44$

Mol. mass of $R = 44 - 29 = 15$

This is possible, only when R is $-CH_3$ group.

$\therefore$ The aldehyde is CH_3CHO and the symmetrical alkene is $CH_3HC{=}CHCH_3$.

$$\underset{\text{2-Butene}}{CH_3CH{=}CHCH_3} \xrightarrow[\text{(ii) Zn, } H_2O]{\text{(i) } O_3} \underset{\text{Acetaldehyde}}{2CH_3CHO}$$

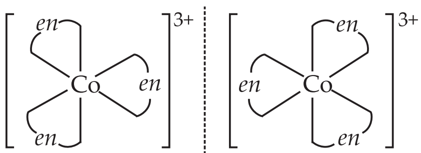
23. **(b) :** 

The preferential formation of this compound is due to conjugation in the compound.

24. **(b) :** Biuret test is used to characterise the presence of —CONH group in a compound.

25. **(b) :** $E^\circ_{Mn^{2+}/Mn} = -1.18$ V
$E^\circ_{Cr^{2+}/Cr} = -0.91$ V
$E^\circ_{Fe^{2+}/Fe} = -0.44$ V
$E^\circ_{Co^{2+}/Co} = -0.28$ V

26. **(c) :** Optical isomers rarely occur in square planar complexes due to the presence of axis of symmetry.
Optical isomerism is common in octahedral complexes of the general formula, $[Ma_2b_2c_2]^{n\pm}$, $[Mabcdef]^{n\pm}$, $[M(AA)_3]^{n\pm}$, $[M(AA)_2a_2]^{n\pm}$, $[M(AA)_2ab]^{n\pm}$ and $[M(AB)_3]^{n\pm}$. Thus, among the given options, only $[Co(en)_3]^{3+}$ shows optical isomerism.



27. **(c) :** Nylon-6,6 involves amide (CONH) linkage therefore, it will also have very strong inter molecular hydrogen bonding between $\rangle$NH·······OC$\langle$ group of two polyamide chains.

28. **(c) :** $(K_{sp})_{Mg(OH)_2} = [Mg^{2+}][OH^-]^2$
$1 \times 10^{-11} = [0.001][OH^-]^2$
$\Rightarrow [OH^-]^2 = \dfrac{10^{-11}}{10^{-3}} = 10^{-8}$
$\Rightarrow [OH^-] = 10^{-4}$
pOH = 4
Thus, pH = 14 – 4 = 10

29. **(b) :** Given, $p^\circ_{heptane} = 105$ kPa
$p^\circ_{octane} = 45$ kPa
$w_{heptane} = 25$ g
$w_{octane} = 35$ g.

$n_{heptane} = \dfrac{25}{100} = 0.25$

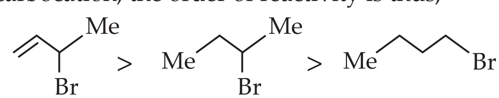
$n_{octane} = \dfrac{35}{114} = 0.30$

$x_{heptane} = \dfrac{0.25}{0.25 + 0.30} = 0.45$

$x_{octane} = \dfrac{0.30}{0.25 + 0.30} = 0.54$

$p_{Total} = x_{heptane}p^\circ_{heptane} + x_{octane}p^\circ_{octane}$
$= 0.45 \times 105 + 0.54 \times 45$
$= 47.25 + 24.3 = 71.55 \approx 72$ kPa.

30. **(b) :** S_N1 reaction rate depends upon the stability of the carbocation, as carbocation formation is the rate determining step. Compound (*B*), forms a 2° allylic carbocation which is the most stable, the next stable carbocation is formed from (*C*), it is a 2° carbocation, (*A*) forms the least stable 1° carbocation, the order of reactivity is thus,



(*B*) > (*C*) > (*A*)

Solved Paper - 2011

1. The presence or absence of hydroxy group on which carbon atom of sugar differentiates RNA and DNA.
(a) 1^{st} (b) 2^{nd}
(c) 3^{rd} (d) 4^{th}
[Biomolecules]

2. Among the following the maximum covalent character is shown by the compound
(a) $FeCl_2$ (b) $SnCl_2$
(c) $AlCl_3$ (d) $MgCl_2$
[Atomic Structure, Chemical Bonding and Molecular Structure]

3. Which of the following statement is wrong ?
(a) The stability of hydrides increases from NH_3 to BiH_3 in group 15 of the periodic table.
(b) Nitrogen cannot form $d\pi$-$p\pi$ bond.
(c) Single N — N bond is weaker than the single P – P bond.
(d) N_2O_4 has two resonance structure.
[Chemistry of Representative Elements]

4. Phenol is heated with a solution of mixture of KBr and $KBrO_3$. The major product obtained in the above reaction is
(a) 2-bromophenol (b) 3-bromophenol
(c) 4-bromophenol
(d) 2, 4, 6-tribromophenol
[Halide, Hydroxy Compounds and Ethers]

5. A 5.2 molal aqueous solution of methyl alcohol, CH_3OH, is supplied. What is the mole fraction of methyl alcohol in the solution ?
(a) 0.100 (b) 0.190
(c) 0.086 (d) 0.050
[Solutions]

6. The hybridisation of orbitals of N atom in NO_3^-, NO_2^+ and NH_4^+ are respectively
(a) sp, sp^2, sp^3 (b) sp^2, sp, sp^3
(c) sp, sp^3, sp^2 (d) sp^2, sp^3, sp
[Atomic Structure, Chemical Bonding and Molecular Structure]

7. Ethylene glycol is used as an antifreeze in a cold climate. Mass of ethylene glycol which should be added to 4 kg of water to prevent it from freezing at – 6°C will be : (K_f for water = 1.86 K kg mol^{-1}, and molar mass of ethylene glycol = 62 g mol^{-1})
(a) 804.32 g (b) 204.30 g
(c) 400.00 g (d) 304.60 g
[Solutions]

8. The reduction potential of hydrogen half-cell will be negative if
(a) $p(H_2)$ = 1 atm and $[H^+]$ = 2.0 M
(b) $p(H_2)$ = 1 atm and $[H^+]$ = 1.0 M
(c) $p(H_2)$ = 2 atm and $[H^+]$ = 1.0 M
(d) $p(H_2)$ = 2 atm and $[H^+]$ = 2.0 M
[Redox Reactions and Electrochemistry]

9. Which of the following reagents may be used to distinguish between phenol and benzoic acid ?
(a) Aqueous NaOH (b) Tollen's reagent
(c) Molisch reagent (d) Neutral $FeCl_3$
[Practical Chemistry]

10. Trichloroacetaldehyde was subjected to Cannizzaro's reaction by using NaOH. The mixture of the products contains sodium trichloroacetate ion and another compound. The other compound is
(a) 2,2,2-trichloroethanol
(b) trichloromethanol
(c) 2,2,2-trichloropropanol
(d) chloroform
[Aldehydes, Ketones, Carboxylic Acids and Their Derivatives]

11. Which one of the following orders presents the correct sequence of the increasing basic nature of the given oxides ?
(a) $Al_2O_3 < MgO < Na_2O < K_2O$
(b) $MgO < K_2O < Al_2O_3 < Na_2O$
(c) $Na_2O < K_2O < MgO < Al_2O_3$
(d) $K_2O < Na_2O < Al_2O_3 < MgO$
[Periodic Properties]

12. A gas absorbs a photon of 355 nm and emits at two wavelengths. If one of the emission is at 680 nm, the other is at
(a) 1035 nm (b) 325 nm
(c) 743 nm (d) 518 nm
[Atomic Structure, Chemical Bonding and Molecular Structure]

13. Which of the following statements regarding sulphur is incorrect ?

(a) S_2 molecule is paramagnetic.
(b) The vapour at 200°C consists mostly of S_8 rings.
(c) At 600°C the gas mainly consists of S_2 molecules.
(d) The oxidation state of sulphur is never less than +4 in its compounds.

[Chemistry of Non-metals]

14. The entropy change involved in the isothermal reversible expansion of 2 moles of an ideal gas from a volume of 10 dm^3 to a volume of 100 dm^3 at 27°C is

(a) 38.3 J mol^{-1} K^{-1} (b) 35.8 J mol^{-1} K^{-1}
(c) 32.3 J mol^{-1} K^{-1} (d) 42.3 J mol^{-1} K^{-1}

[Energetics]

15. Which of the following facts about the complex $[Cr(NH_3)_6]Cl_3$ is wrong ?

(a) The complex involves d^2sp^3 hybridisation and is octahedral in shape.
(b) The complex is paramagnetic.
(c) The complex is an outer orbital complex.
(d) The complex gives white precipitate with silver nitrate solution.

[Coordination Compounds and Organometallics]

16. The structure of IF_7 is

(a) square pyramid (b) trigonal bipyramid
(c) octahedral
(d) pentagonal bipyramid

[Atomic Structure, Chemical Bonding and Molecular Structure]

17. The rate of a chemical reaction doubles for every 10°C rise of temperature. If the temperature is raised by 50°C, the rate of the reaction increases by about

(a) 10 times (b) 24 times
(c) 32 times (d) 64 times

[Kinetics]

18. The strongest acid amongst the following compounds is

(a) CH_3COOH (b) HCOOH
(c) $CH_3CH_2CH(Cl)CO_2H$
(d) $ClCH_2CH_2CH_2COOH$

[Aldehydes, Ketones, Carboxylic Acids and Their Derivatives]

19. Identify the compound that exhibits tautomerism.

(a) 2-Butene (b) Lactic acid
(c) 2-Pentanone (d) Phenol

[General Organic Chemistry]

20. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm. Some of the CO_2 is converted into CO on the addition of graphite. If the total pressure at equilibrium is 0.8 atm, the value of K is

(a) 1.8 atm (b) 3 atm
(c) 0.3 atm (d) 0.18 atm

[Equilibrium]

21. In context of the lanthanoids, which of the following statement is not correct ?

(a) There is a gradual decrease in the radii of the members with increasing atomic number in the series.
(b) All the members exhibit +3 oxidation state.
(c) Because of similar properties the separation of lanthanoids is not easy.
(d) Availability of 4f electrons results in the formation of compounds in +4 state for all the members of the series.

[Transition Metals Including Lanthanides]

22. 'a' and 'b' are van der Waals' constants for gases. Chlorine is more easily liquefied than ethane because

(a) a and b for Cl_2 > a and b for C_2H_6
(b) a and b for Cl_2 < a and b for C_2H_6
(c) a for Cl_2 < a for C_2H_6 but b for Cl_2 > b for C_2H_6
(d) a for Cl_2 > a for C_2H_6 but b for Cl_2 < b for C_2H_6

[States of Matter]

23. The magnetic moment (spin only) of $[NiCl_4]^{2-}$ is

(a) 1.82 BM (b) 5.46 BM
(c) 2.82 BM (d) 1.41 BM

[Coordination Compounds and Organometallics]

24. In a face centred cubic lattice, atom A occupies the corner positions and atom B occupies the face centre positions. If one atom of B is missing from one of the face centred points, the formula of the compound is

(a) A_2B (b) AB_2
(c) A_2B_3 (d) A_2B_5

[States of Matter]

25. The outer electronic configuration of Gd (Atomic No : 64) is

(a) $4f^3\, 5d^5\, 6s^2$ (b) $4f^8\, 5d^0\, 6s^2$
(c) $4f^4\, 5d^4\, 6s^2$ (d) $4f^7\, 5d^1\, 6s^2$

[Transition Metals Including Lanthanides]

26. Boron cannot form which one of the following anions ?

(a) BF_6^{3-} (b) BH_4^-

(c) $B(OH)_4^-$ (d) BO_2^-

[Chemistry of Representative Elements]

27. Ozonolysis of an organic compound gives formaldehyde as one of the products. This confirms the presence of

(a) two ethylenic double bonds
(b) a vinyl group
(c) an isopropyl group
(d) an acetylenic triple bond

[Aldehydes, Ketones, Carboxylic Acids and Their Derivatives]

28. Sodium ethoxide has reacted with ethanoyl chloride. The compound that is produced in this reaction is

(a) diethyl ether (b) 2-butanone
(c) ethyl chloride (d) ethyl ethanoate

[Halides, Hydroxy Compounds and Ethers]

29. The degree of dissociation (α) of a weak electrolyte, A^xB^y is related to van't Hoff factor (i) by the expression

(a) $\alpha = \dfrac{i-1}{(x+y-1)}$ (b) $\alpha = \dfrac{i-1}{(x+y+1)}$

(c) $\alpha = \dfrac{(x+y-1)}{i-1}$ (d) $\alpha = \dfrac{(x+y+1)}{i-1}$

[Solutions]

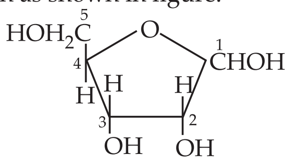
30. Silver mirror test is given by which one of the following compounds ?

(a) Acetaldehyde (b) Acetone
(c) Formaldehyde (d) Benzophenone

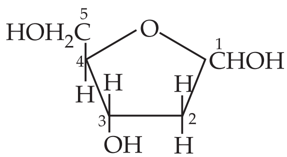
[Aldehydes, Ketones, Carboxylic Acids and Their Derivatives]

EXPLANATIONS

1. **(b) :** The sugar molecule found in RNA is D-ribose while the sugar in DNA is D-2-deoxyribose. The sugar D-2-deoxyribose differs from ribose only in the substitution of hydrogen for an – OH group at 2-position as shown in figure.



D-ribose (RNA)



D-2-deoxyribose (DNA)

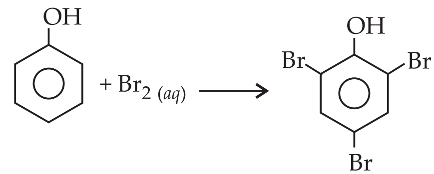
2. **(c) :** We know that, extent of polarisation $\propto$ covalent character in ionic bond.
Fajan's rule states that
(i) the polarising power of cation increases, with increase in magnitude of positive charge on the cation
$\therefore$ polarising power $\propto$ charge of cation
(ii) the polarising power of cation increases with the decrease in the size of a cation.
$\therefore$ polarising power $\propto \dfrac{1}{\text{size of cation}}$
Here the $AlCl_3$ is satisfying the above two conditions *i.e.*, Al is in +3 oxidation state and also has small size. So it has more covalent character.

3. **(a) :** Thermal stability decreases gradually from NH_3 to BiH_3. So the stability also decreases.

	NH_3	PH_3	AsH_3	SbH_3	BiH_3
Decomposition temperature	1300°C	440°C	280°C	150°C	room temp.

The size of the central atom increases from N to Bi therefore, the tendency to form a stable covalent bond with small atom like hydrogen decreases and therefore, stability decreases.

4. **(d) :** $KBr_{(aq)} + KBrO_{3\,(aq)} \longrightarrow Br_{2\,(aq)}$
This bromine reacts with phenol gives 2,4,6-tribromophenol.



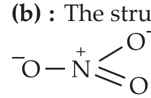
2,4,6-tribromophenol

5. **(c) :** Mole fraction of solute $= \dfrac{n}{N+n}$
n = number of moles of solute
N = number of moles of solvent
Here solute is methyl alcohol, solvent is water.
Given $n = 5.2$, $N = \dfrac{1000}{18}$

$$\therefore \quad \text{Mole fraction} = \frac{5.2}{5.2+\frac{1000}{18}}$$

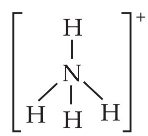
$$= \frac{5.2\times 18}{93.6+1000} = \frac{93.6}{1093.6} = 0.0855 \approx 0.086$$

6. **(b) :** The structures of NO_3^-, NO_2^+ and NH_4^+ is



sp^2 hybridisation

$O=\overset{+}{N}=O$ sp hybridisation



sp^3 hybridisation

7. **(a) :** $\Delta T_f = K_f \times m = K_f \times \dfrac{w_2 \times 1000}{w_1 \times m_2}$
w_1 and w_2 = wt. of solvent and solute
m_2 = molecular wt. of solute
$\Delta T_f = 0 - (-6) = 6$

$$\therefore \quad 6 = \frac{1.86\times w_2 \times 1000}{4000\times 62}$$

$$w_2 = \frac{6\times 62\times 4000}{1000\times 1.86} = 800 \text{ g.}$$

8. **(c) :** $2H^+_{(aq)} + 2e^- \longrightarrow H_{2\,(g)}$

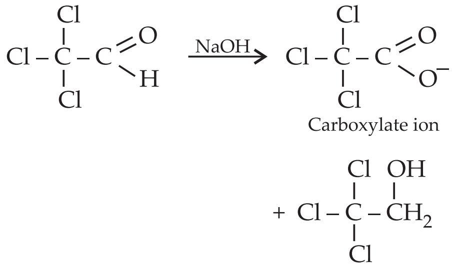
$$E_{\text{red}} = E^\circ_{\text{red}} - \frac{0.0591}{n}\log\frac{p_{H_2}}{[H^+]^2}$$

$$E_{\text{red}} = 0 - \frac{0.0591}{2}\log\frac{2}{(1)^2}$$

E_{red} will only be negative when p_{H_2} > [H^+]. So option (c) is correct.

9. **(d) :** Phenol gives violet colouration with neutral ferric chloride solution.
Benzoic acid gives buff coloured (pale dull yellow) precipitate with neutral ferric chloride solution.

10. **(a) :** In Cannizzaro's reaction one molecule is oxidised to carboxylate ion and the other is reduced to alcohol.



Carboxylate ion

2,2,2-trichloroethanol

11. **(a) :** While moving from left to right in periodic table basic character of oxide of elements will decrease.

$\therefore \quad \xrightarrow[\text{Increasing basic strength}]{Al_2O_3 < MgO < Na_2O}$

And while descending in the group basic character of corresponding oxides increases.

$\therefore \quad \xrightarrow[\text{Increasing basic strength}]{Na_2O < K_2O}$

$\therefore$ Correct order is
$Al_2O_3 < MgO < Na_2O < K_2O$

12. **(c) :** We know that
$E = h\upsilon = hc/\lambda$

$E = E_1 + E_2 \quad \text{or} \quad \frac{hc}{\lambda} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$

$\Rightarrow \quad \frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \quad \Rightarrow \quad \frac{1}{355} = \frac{1}{680} + \frac{1}{\lambda_2}$

$\therefore \quad \lambda_2 = \frac{355 \times 680}{680 - 355} = 742.769 \text{ nm} \approx 743 \text{ nm}.$

13. **(d) :** Sulphur exhibits –2, +2, +4, +6 oxidation states but +4 and +6 are more common.

14. **(a) :** Entropy change for an isothermal process is

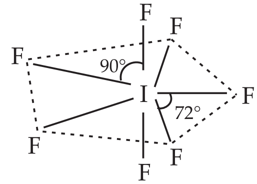
$\Delta S = 2.303 nR \log\left(\frac{V_2}{V_1}\right)$

$\Delta S = 2.303 \times 2 \times 8.314 \times \log\left(\frac{100}{10}\right)$

$= 38.294 \text{ J mol}^{-1} \text{ K}^{-1} \approx 38.3 \text{ J mol}^{-1} \text{ K}^{-1}.$

15. **(c) :** The complex $[Cr(NH_3)_6]Cl_3$ involves d^2sp^3 hybridization as it involves $(n-1)d$ orbitals for hybridization. It is an inner orbital complex.

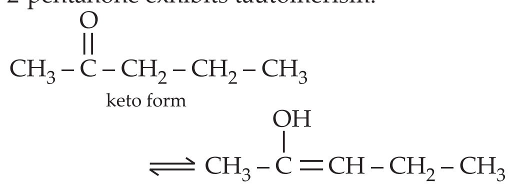
16. **(d) :** The structure is pentagonal bipyramidal having sp^3d^3 hybridisation as given below:



17. **(c) :** $\frac{\text{Rate at } 50°\text{C}}{\text{Rate at } T_1°\text{C}} = 2^{\frac{50}{10}} = 2^5 = 32$ times.

18. **(c) :** $CH_3CH_2CH(Cl)COOH$ is the strongest acid due to –I effect of Cl.

19. **(c) :** The type of isomerism in which a substance exists in two readily interconvertible different structures leading to a dynamic equilibrium is known as tautomerism.
2-pentanone exhibits tautomerism.



keto form

enol form

20. **(a) :** $CO_{2\,(g)} + C_{\,(s)} \rightleftharpoons 2CO_{\,(g)}$

0.5 atm

$0.5 - P \qquad\qquad 2P$

Total pressure $= 0.5 - P + 2P = 0.8$

$P = 0.3$

$K_P = \frac{P_{CO}^2}{P_{CO_2}} = \frac{(2P)^2}{(0.5-P)} = \frac{(0.6)^2}{(0.5-0.3)} = 1.8$ atm

21. **(d) :** Availability of 4*f* electrons does not result in the formation of compounds in +4 state for all the members of the series.

22. **(d) :**

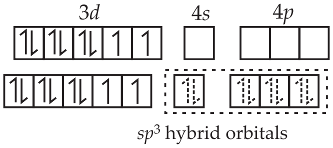
	a (dm^3 atm mol^{-2})	b(dm^3 mol^{-1})
Cl_2	6.49	0.0562
C_2H_6	5.49	0.0638

From the above values, a for Cl_2 > a for ethane
b for ethane > b for Cl_2.

23. **(c) :** In the paramagnetic and tetrahedral complex $[NiCl_4]^{2-}$, the nickel is in +2 oxidation state and the ion has the electronic configuration $3d^8$. The hybridisation scheme is as shown in figure.

Ni^{2+}, [Ar]$3d^8$:

[NiCl$_4$]$^{2-}$:



$\mu_{B.M.} = \sqrt{n(n+2)} = \sqrt{2(2+2)} = \sqrt{8} = 2.82$ BM

24. **(d) :** A $\quad$ B

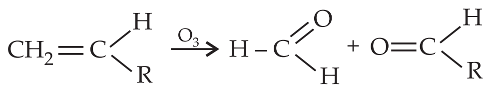
$8 \times \frac{1}{8}$ $\quad$ $5 \times \frac{1}{2}$

Formula of the compound is A_2B_5.

25. **(d) :** The electronic configuration of $_{64}$Gd = [Xe]^{54} $4f^7\ 5d^1\ 6s^2$

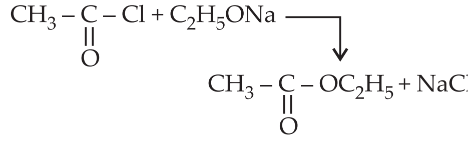
26. **(a) :** Due to non-availability of d-orbitals, boron is unable to expand its octet. Therefore, the maximum covalency of boron cannot exceed 4.

27. **(b) :**



Vinyl group ($CH_2 = CH -$) on ozonolysis gives formaldehyde.

28. **(d) :**



Ethyl ethanoate

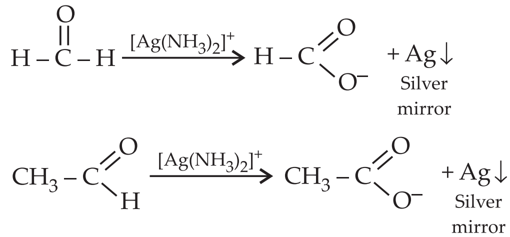
29. **(a) :** $A^xB^y \longrightarrow xA^{y+} + yB^{x-}$

$1-\alpha \qquad x\alpha \qquad y\alpha$

$i = 1 - \alpha + x\alpha + y\alpha = 1 + \alpha(x + y - 1)$

$\therefore \quad \alpha = \frac{i-1}{(x+y-1)}$

30. **(a, c):** Formaldehyde and acetaldehyde can be oxidised by Tollen's reagent to give silver mirror.



CHAPTER 1

Set, Relations and Functions

1. Which one is not periodic?

(a) $|\sin 3x| + \sin^2 x$ (b) $\cos\sqrt{x} + \cos^2 x$

(c) $\cos 4x + \tan^2 x$ (d) $\cos 2x + \sin x$.

(2002)

2. The period of $\sin^2\theta$ is

(a) π^2 (b) π (c) π^3 (d) $\pi/2$.

(2002)

3. The domain of $\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$ is

(a) [1, 9] (b) [–1, 9]

(c) [–9, 1] (d) [–9, –1].

(2002)

4. The function $f(x) = \log(x + \sqrt{x^2+1})$ is

(a) an odd function

(b) a periodic function

(c) neither an even nor an odd function

(d) an even function.

(2003)

5. A function f from the set of natural numbers to integers defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\frac{n}{2}, & \text{when } n \text{ is even} \end{cases} \quad \text{is}$$

(a) onto but not one-one

(b) one-one and onto both

(c) neither one-one nor onto

(d) one-one but not onto.

(2003)

6. Domain of definition of the function

$$f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x), \text{ is}$$

(a) $(-1, 0) \cup (1, 2)$ (b) $(1, 2) \cup (2, \infty)$

(c) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

(d) $(1, 2)$.

(2003)

7. If $f: R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is

(a) $\frac{7(n+1)}{2}$ (b) $7n(n + 1)$

(c) $\frac{7n(n+1)}{2}$ (d) $\frac{7n}{2}$.

(2003)

8. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is

(a) not symmetric (b) transitive

(c) a function (d) reflexive.

(2004)

9. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is

(a) {1, 2, 3, 4} (b) {1, 2, 3, 4, 5, 6}

(c) {1, 2, 3} (d) {1, 2, 3, 4, 5}.

(2004)

10. If $f: R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is

(a) [0, 1] (b) [–1, 1]

(c) [0, 3] (d) [–1, 3].

(2004)

11. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then

(a) $f(x) = f(-x)$ (b) $f(2 + x) = f(2 - x)$

(c) $f(x + 2) = f(x - 2)$ (d) $f(x) = -f(-x)$.

(2004)

12. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

(a) [1, 2] (b) [2, 3) (c) [2, 3] (d) [1, 2).

(2004)

13. Let $R = \{(3, 3)\ (6, 6)\ (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$ be a relation on the set $A = \{3, 6, 9, 12\}$. The relation is

(a) reflexive and symmetric only
(b) an equivalence relation
(c) reflexive only
(d) reflexive and transitive only. *(2005)*

14. Let $f : (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then f is both one-one and onto when B is the interval

(a) $\left[0, \frac{\pi}{2}\right)$ (b) $\left(0, \frac{\pi}{2}\right)$
(c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. *(2005)*

15. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

Interval	**Function**
(a) $[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(b) $(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$
(c) $(-\infty, -4]$	$x^3 + 6x^2 + 6$
(d) $\left(-\infty, \frac{1}{3}\right]$	$3x^2 - 2x + 1$

(2005)

16. A real valued function $f(x)$ satisfies the functional equation $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$ where a is a given constant and $f(0) = 1$. $f(2a - x)$ is equal to

(a) $f(x)$ (b) $-f(x)$
(c) $f(-x)$ (d) $f(a) + f(a - x)$. *(2005)*

17. Let W denote the words in the English dictionary. Define the relation R by :
$R = \{(x, y) \in W \times W \mid$ the words x and y have at least one letter in common$\}$.
Then R is

(a) not reflexive, symmetric and transitive
(b) reflexive, symmetric and not transitive
(c) reflexive, symmetric and transitive
(d) reflexive, not symmetric and transitive. *(2006)*

18. The set $S = \{1, 2, 3, ..., 12\}$ is to be partitioned into three sets A, B, C of equal size. Thus $A \cup B \cup C = S, A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is

(a) $\frac{12!}{(4!)^3}$ (b) $\frac{12!}{(4!)^4}$
(c) $\frac{12!}{3!(4!)^3}$ (d) $\frac{12!}{3!(4!)^4}$. *(2007)*

19. Let R be the real line. Consider the following subsets of the plane $R \times R$:
$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$
$T = \{(x, y) : x - y \text{ is an integer}\}$.
Which one of the following is true?

(a) T is an equivalence relation on R but S is not
(b) Neither S nor T is an equivalence relation on R
(c) Both S and T are equivalence relations on R
(d) S is an equivalence relation on R but T is not *(2008)*

20. Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$ where
$Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$.
Show that f is invertible and its inverse is

(a) $g(y) = \frac{y-3}{4}$ (b) $g(y) = \frac{3y+4}{3}$
(c) $g(y) = 4 + \frac{y+3}{4}$ (d) $g(y) = \frac{y+3}{4}$ *(2008)*

Answer Key

1.	(b)	**2.**	(b)	**3.**	(a)	**4.**	(a)	**5.**	(b)	**6.**	(c)
7.	(c)	**8.**	(a)	**9.**	(c)	**10.**	(d)	**11.**	(c)	**12.**	(b)
13.	(d)	**14.**	(c)	**15.**	(d)	**16.**	(b)	**17.**	(b)	**18.**	(a)
19.	(a)	**20.**	(a)								

1. **(b) :** Period of $|\sin 3x|$ is $\frac{\pi}{3}$ and period of $\sin^2 x$ is π

(a) Same as the period of $|\sin x|$ or $\frac{1-\cos 2x}{2}$ whose period is π

Now period of $|\sin 3x| + \sin^2 x$ is the L.C.M of their periods

$\therefore$ L.C.M of $\left(\frac{\pi}{3}, \pi\right) = \frac{\text{LCM }(\pi,\pi)}{\text{HCF }(3,1)} = \pi$

(c, d) Similarly we can say that $\cos 4x + \tan^2 x$ and $\cos 2x + \sin x$ are periodic function.

(b) Now $\cos^2 x$ is periodic with period π and for period of $\cos\sqrt{x}$ let us take.

$f(x) = \cos\sqrt{x}$

Let $f(x + T) = f(x)$

$\Rightarrow \cos\sqrt{T+x} = \cos\sqrt{x}$

$\Rightarrow \sqrt{T+x} = 2n\pi \pm \sqrt{x}$

which gives no value of T independent of x

$\therefore f(x)$ cannot be periodic

now say $g(x) = \cos^2 x + \cos\sqrt{x}$ which is sum of a periodic and non periodic function and such function have no period.

So, $\cos\sqrt{x} + \cos^2 x$ is non periodic function.

2. **(b) :** Let $f(\theta) = \sin^2\theta = |\sin\theta|$

Period of $|\sin\theta|$ is π (well known fact so keep the result in memory)

3. **(a) :** If $y = \sin^{-1}x$, then $-1 \le x \le 1$

$\therefore -1 \le \log_3\left(\frac{x}{3}\right) \le 1 \quad \left[\text{as } y = \sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]\right]$

$\Rightarrow \frac{1}{3} \le \frac{x}{3} \le 3^1$

$\Rightarrow 1 \le x \le 9$

4 **(a) :** $f(x) = \log\left[\sqrt{x^2+1}+x\right]$

$\therefore f(-x) = \log\left[\sqrt{1+x^2}-x\right]$

$= -\log\left[\frac{1}{\sqrt{1+x^2}-x}\right] = -\log\left[\frac{\sqrt{1+x^2}+x}{1}\right]$

$= -f(x) \Rightarrow f(x) + f(-x) = 0$

$\Rightarrow f(x)$ is an odd function

5. **(b) :** If n is odd, let $n = 2k - 1$

Let $f(2k_1 - 1) = f(2k_2 - 1)$

$\Rightarrow \frac{2k_1-1-1}{2} = \frac{2k_2-1-1}{2}$

$\Rightarrow k_1 = k_2$

$\Rightarrow f(n)$ is one-one functions if n is odd

Again, If $n = 2k$ (i.e. n is even)

Let $f(2k_1) = f(2k_2)$

$\Rightarrow -\frac{2k_1}{2} = -\frac{2k_2}{2}$

$\Rightarrow k_1 = k_2$

$\Rightarrow f(n)$ is one-one if n is even

Again $f(n) = \frac{n-1}{2}$

$f'(n) = \frac{1}{2} > 0\ \forall n \in N$ if n is odd

and $f'(n) = \frac{-1}{2} < 0\ \forall n \in N$ if n is even

Now all such function which are either increasing or decreasing in the stated domain are said to be onto function. Finally $f(n)$ is one-one onto function.

6. **(c) :** Let $g(x) = \frac{3}{4-x^2}$ $\quad\therefore x \ne \pm 2$

$\therefore D_f\, g(x) = R - \{-2, 2\}$

$f(x) = \log_{10}(x^3 - x)$ $\quad\therefore x^3 - x > 0$

$x(x + 1)(x - 1) > 0$

$$\begin{array}{cccc} - & + & - & + \\ & -1 & 0 & 1 \end{array}$$

$\therefore x \in (-1, 0) \cup (1, \infty)$

$\therefore$ Domain of $f(x)$ is $(-1, 0) \cup (1, 2) \cup (2, \infty)$

7. **(c) :** Let $x = 0 = y$ $\quad\therefore f(0) = 0$

and $x = 1, y = 0$ $\quad\therefore f(1 + 0) = f(1) + f(0) = 7$ (given)

$x = 1, y = 1$ $\quad\therefore f(1 + 1) = 2f(1) = 2(7)$

$\therefore f(2) = 2(7)$

$x = 1, y = 2$ $\quad\therefore f(3) = f(1) + f(2)$

$= 7 + 2(7)$

$= 3(7)$

and so on.

$\therefore \sum_{r=1}^{n} f(r) = f(1) + f(2) + f(3) + ... + f(n)$

$= 7(1 + 2 + 3 + ...+ n) = \dfrac{7n(n+1)}{2}$

8. **(a) :** R is a function as $A = \{1, 2, 3, 4\}$ and $(2, 4) \in R$ and $(2, 3) \in R$

R is not reflexive as $(1, 1) \notin R$

R is not symmetric as $(2, 3) \in R$ but $(3, 2) \notin R$

R is not transitive as $(1, 3) \in R$ and $(3, 1) \in R$ but $(1, 1) \notin R$.

9. **(c) :** $F(x)$ to be defined for $x \in N$.

(i) $\therefore 7 - x > 0 \quad \Rightarrow x < 7$

(ii) $x - 3 \geq 0 \quad \Rightarrow x \geq 3$

(iii) $x - 3 \leq 7 - x \quad \Rightarrow x \leq 5$

$\therefore$ from (i), (ii), (iii)

$x = 3, 4, 5$

$\therefore F(3) = {}^4P_0,\ F(4) = {}^3P_1,\ F(5) = {}^2P_2$

$\therefore \{1, 2, 3\}$ is required range

10. **(d):** Let $f(x) = g(x) + 1$

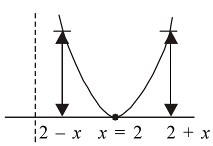
where $g(x) = 2\left[\dfrac{1}{2}\sin x - \dfrac{\sqrt{3}}{2}\cos x\right]$

$= 2 \sin (x - 60°)$

$\therefore \quad -2 \leq 2 \sin (x - 60°) \leq 2$

$-1 \leq 2 \sin (x - 60°) + 1 \leq 3$

11. **(c):** If $y = f(x)$ is symmetrical about the line $x = \alpha$ then $f(x + \alpha) = f(x - \alpha)$



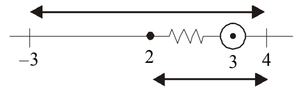
$\therefore f(x + 2) = f(x - 2)$

12. **(b) :** $f(x) = \dfrac{p(x)}{q(x)}$ (say)

then Domain of $f(x)$ is $D_f\, p(x) \cap D_f\, q(x),\ q(x) \neq 0$

now D_f of $p(x)$ is $-\dfrac{\pi}{2} \leq \sin^{-1}(x - 3) \leq \dfrac{\pi}{2}$

$\Rightarrow -\sin\dfrac{\pi}{2} \leq x - 3 \leq \sin\dfrac{\pi}{2}$



$\Rightarrow \ 2 \leq x \leq 4$ ...(i)

Again $9 - x^2 > 0 \Rightarrow x^2 < 9$

$|x| < 3$

i.e. $-3 < x < 3$ ...(ii)

from (i) and (ii) we have

$\therefore\ 2 \leq x < 3$ is correct Domain

13. **(d) :** For $(3, 9) \in R,\quad (9, 3) \notin R$

$\therefore$ relation is not symmetric which means our choice (a) and (b) are out of court. We need to prove reflexivity and transitivity.

For reflexivity $a \in R, (a, a) \in R$ which is hold i.e. R is reflexive. Again,

for transitivity of $(a, b) \in R$, $(b, c) \in R$

$\Rightarrow (a, c) \in R$

which is also true in $R = \{(3, 3)(6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$.

14. **(c) :** For $x \in (-1, 1)$ we have

$f(x) = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$

$\therefore f(\tan\theta) = \tan^{-1}\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)$ (By $x = \tan\theta$)

$= \tan^{-1}\tan 2\theta = 2\tan^{-1}x$

$\Rightarrow -\dfrac{\pi}{2} < \tan^{-1}\left(\dfrac{2x}{1-x^2}\right) < \dfrac{\pi}{2}$.

15. **(d) :** $f(x) = x^3 + 6x^2 + 6$

$f'(x) = 3x^2 + 12x = 3x(x+4)$

$f'(x) > 0 \Rightarrow x < -4 \cup x > 0$

the interval $x < -4$ *i.e* $(-\infty, -4]$ matched correctly and after checking others we find that $f(x) = 3x^2 - 2x + 1$

$\Rightarrow f'(x) > 0$ for $x > 1/3$ which is not given in the choice.

16. **(b) :** Given $f(x-y) = f(x)f(y) - f(a-x)f(a+y)$ (*)

let $x = 0 = y$

$f(0) = (f(0))^2 - (f(a))^2$

$1 = 1 - (f(a))^2 \Rightarrow f(a) = 0$

$\therefore f(2a - x) = f(a - (x - a))$

$= f(a)f(x-a) - f(a+x-a)f(0)$

By using (*)

$= 0 - f(x)(1) = -f(x)$

$(\because\ f(a) = 0,\ f(0) = 1)$

17. (b) : Given relation R such that

$R = \{(x, y) \in W \times W \mid$ the word x and y have atleast one letter in common$\}$

where W denotes set of words in English dictionary

Clearly $(x, x) \in R\ \forall\ x \in W$

$\therefore$ (x, x) has every letter common $\therefore$ R is reflexive

Let $(x, y) \in R$ then $(y, x) \in R$ as x and y have atleast one letter in common. $\Rightarrow$ R is symmetric.

But R is not transitive

$\therefore$ Let x = DON, y = NEST, z = SHE

then $(x, y) \in R$ and $(y, z) \in R$. But $(x, z) \notin R$.

$\therefore$ R is reflexive, symmetric but not transitive.

18. (a) : Number of way $= {}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4 = \dfrac{12!}{(4!)^3}$.

19. (a) : To be an equivalence relation the relation must be all – reflexive, symmetric and transitive.

$T = \{(x, y) : x - y \in Z\}$ is

reflexive – for $(x, x) \in Z$ i.e. $x - x = 0 \in Z$

symmetric – for $(x, y) \in Z \Rightarrow x - y \in Z$

$\Rightarrow y - x \in Z$ i.e. $(y, x) \in Z$

transitive – for $(x, y) \in Z$ and $(y, w) \in Z$

$\Rightarrow x - y \in Z$ and $y - w \in Z$, giving

$x - w \in Z$ i.e. $(x, w) \in Z$.

$\therefore$ T is an equivalence relation on R.

$S = \{(x, y) : y = x + 1, 0 < x < 2\}$ is not reflexive for $(x, x) \in S$ would imply $x = x + 1$

$\Rightarrow 0 = 1$ (impossible)

Thus S is not an equivalence relation

20. (a) : Let $f(x_1) = f(x_2),\ x_1, x_2 \in N$

$\Rightarrow 4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$

Thus $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. Hence the function is one-one. Let $y \in Y$ be a number of the form $y = 4k + 3$, for some $k \in N$, then $y = f(x)$

$\Rightarrow 4k + 3 = 4x + 3 \Rightarrow x = k \in N$

Thus corresponding to any $y \in Y$ we have $x \in N$. The function then is onto.

The function, being both one-one and onto is invertible.

$y = 4x + 3 \Rightarrow x = \dfrac{y-3}{4} \quad \therefore\ f^{-1}(x) = \dfrac{x-3}{4}$

or $g(y) = \dfrac{y-3}{4}$ is the inverse of the function.

CHAPTER 2

Complex Numbers

1. z and ω are two nonzero complex number such that $|z| = |\omega|$ and Arg z + Arg $\omega = \pi$ then z equals
(a) $\bar{\omega}$ (b) $-\bar{\omega}$ (c) ω (d) $-\omega$
(2002)

2. If $|z-4| < |z-2|$, its solution is given by
(a) Re(z) > 0 (b) Re(z) < 0
(c) Re(z) > 3 (d) Re(z) > 2.
(2002)

3. The locus of the centre of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally (z, z_1 & z_2 are complex numbers) will be
(a) an ellipse (b) a hyperbola
(c) a circle (d) none of these
(2002)

4. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then
(a) $x = 2n$, where n is any positive integer
(b) $x = 4n + 1$, where n is any positive integer
(c) $x = 2n + 1$, where n is any positive integer
(d) $x = 4n$, where n is any positive integer.
(2003)

5. If z and ω are two non-zero complex numbers such that $|z\omega| = 1$, and Arg(z) – Arg(ω) = $\pi/2$, then $\bar{z}\omega$ is equal to
(a) –1 (b) i
(c) $-i$ (d) 1.
(2003)

6. Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex further, assume that the origin, z_1 and z_2 form an equilateral triangle, then
(a) $a^2 = 2b$ (b) $a^2 = 3b$
(c) $a^2 = 4b$ (d) $a^2 = b$.
(2003)

7. Let z, ω be complex numbers such that $z + i\bar{\omega} = 0$ and $z\omega = \pi$. Then arg z equals
(a) $3\pi/4$ (b) $\pi/2$ (c) $\pi/4$ (d) $5\pi/4$.
(2004)

8. If $z = x - iy$ and $z^{1/3} = p + iq$, then $\dfrac{\left(\frac{x}{p}+\frac{y}{q}\right)}{(p^2+q^2)}$ is equal to
(a) 2 (b) –1 (c) 1 (d) –2.
(2004)

9. If $|z^2 - 1| = |z|^2 + 1$, then z lies on
(a) a circle (b) the imaginary axis
(c) the real axis (d) an ellipse.
(2004)

10. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then arg$z_1$ – argz_2 is equal to
(a) $-\pi$ (b) $\pi/2$ (c) $-\pi/2$ (d) 0.
(2005)

11. If $\omega = \dfrac{z}{z-(1/3)i}$ and $|\omega| = 1$, then z lies on
(a) a circle (b) an ellipse
(c) a parabola (d) a straight line.
(2005)

12. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$ is
(a) i (b) 1 (c) –1 (d) $-i$.
(2006)

13. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of

$$\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+\ldots+\left(z^6+\frac{1}{z^6}\right)^2 \text{ is}$$

(a) 18 (b) 54 (c) 6 (d) 12.

(2006)

14. If $|z + 4| \leq 3$, then the maximum value of $|z + 1|$ is

(a) 6 (b) 0 (c) 4 (d) 10.

(2007)

15. The conjugate of a complex number is $\frac{1}{i-1}$. Then that complex number is

(a) $\frac{1}{i-1}$ (b) $\frac{-1}{i-1}$ (c) $\frac{1}{i+1}$ (d) $\frac{-1}{i+1}$

(2008)

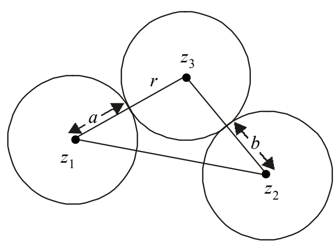
Answer Key

1.	(b)	**2.**	(c)	**3.**	(b)	**4.**	(d)	**5.**	(c)	**6.**	(b)
7.	(a)	**8.**	(d)	**9.**	(b)	**10.**	(d)	**11.**	(d)	**12.**	(d)
13.	(d)	**14.**	(a)	**15.**	(d)						

1. **(b) :** Let $|z| = |\omega| = r$
$\therefore\ z = re^{i\alpha}$ and $\omega = re^{i\beta}$
where $\alpha + \beta = \pi$ (given)
Now $\bar{z} = re^{i\alpha} = re^{i(\pi - \beta)}$
$= re^{i\pi} \cdot e^{-i\beta}$
$= -re^{-i\beta}$
$= -\bar{\omega}$

2. **(c) :** $|z - 4| < |z - 2|$
or $|a - 4 + ib| < |(a - 2) + ib|$ by taking $z = a+ib$
$\Rightarrow \quad (a - 4)^2 + b^2 < (a - 2)^2 + b^2$
$\Rightarrow \quad -8a + 4a < -16 + 4$
$4a > 12$
$a > 3$
$\Rightarrow \quad \text{Re}(z) > 3$

3. **(b) :**

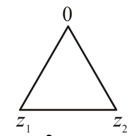


$z_1z_3 - z_3\ z_2 = (a + r) - (b + r)$
$= a - b =$ a constant, which represent a hyperbola
Since, A hyperbola is the locus of a point which moves in such a way that the difference of its distances from two fixed points (foci) is always constant.

4. **(d) :** Given $\left(\dfrac{1+i}{1-i}\right)^x = 1$
$\Rightarrow \left(\dfrac{2i}{2}\right)^x = 1$
$\Rightarrow i^x = 1$
$\Rightarrow i^x = (i)^{4n}$
$\Rightarrow x = 4n,\ n \in I^+$

5. **(c) :** $|z\omega| = 1 \Rightarrow |z||\omega| = 1$ So $|z| = \dfrac{1}{|\omega|}$ ... (1)
Again $Arg(z) - Arg(\omega) = \dfrac{\pi}{2}$
$\therefore \quad \dfrac{z}{\omega} = \left|\dfrac{z}{\omega}\right| i = |z|^2 i$ from (1)
$\therefore \quad \dfrac{z}{\omega} = z\bar{z}\,i \Rightarrow \bar{z}\,\omega = \dfrac{1}{i} = -i.$

6. **(b) :** As z_1, z_2 are roots of $z^2 + az + b$
$\therefore \quad z_1 + z_2 = -a,\ z_1z_2 = b$
Again $0, z_1, z_2$ are vertices of an equilateral triangle

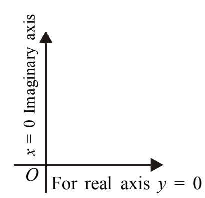


$\therefore \quad 0^2 + z_1^2 + z_2^2 = 0z_1 + z_1z_2 + z_20 = 0$
$z_1^2 + z_2^2 = z_1z_2$
$\Rightarrow \quad (z_1 + z_2)^2 = 3z_1z_2$
$a^2 = 3b$

7 **(a) :** $\bar{z} + i\bar{\omega} = 0$
$\Rightarrow \quad \bar{z} = -i\bar{\omega}$
$\Rightarrow \quad z = i\omega$
$\Rightarrow \quad \omega = -iz$
$\therefore \quad \arg(-iz^2) = \pi$
$\Rightarrow \quad \arg(-i) + 2\arg(z) = \pi$
$\Rightarrow \quad 2\arg(z) = \pi + \pi/2 = 3\pi/2$
$\arg(z) = 3\pi/4$

8. **(d) :** $z^{1/3} = p + iq$
$\Rightarrow \quad x - iy = (p + iq)^3$
$\Rightarrow \quad x - iy = p^3 - 3pq^2 + i(3p^2q - q^3)$
$\Rightarrow \quad x = p^3 - 3pq^2$ and $y = -(3p^2q - q^3)$
$\dfrac{x}{p} = p^2 - 3q^2$ and $\dfrac{y}{q} = -(3p^2 - q^2)$ (*)
adding the equations of (*) we get
$\dfrac{x}{p} + \dfrac{y}{q} = -2(p^2 + q^2)$

9 **(b) :** $|z^2 - 1| = |z|^2 + 1$
$\Rightarrow \quad$ Let $z = x + iy$
$\Rightarrow \quad (x - 1)^2 + y^2$
$= (x^2 + y^2) + 1$
$\Rightarrow \quad 2x = 0$
$x = 0$
$\Rightarrow \quad z$ lies on imaginary axis.



10. (d) : *Method I :* Let $z_1 = \cos\theta_1 + i\sin\theta_1$,
$z_2 = \cos\theta_2 + i\sin\theta_2$

$\therefore\quad z_1 + z_2 = (\cos\theta_1 + \cos\theta_2) + i(\sin\theta_1 + \sin\theta_2)$

Now $|z_1 + z_2| = |z_1| + |z_2|$

$\Rightarrow \sqrt{(\cos\theta_1 + \cos\theta_2)^2 + (\sin\theta_1 + \sin\theta_2)^2} = 1 + 1$

$\Rightarrow 2(1 + \cos(\theta_1 - \theta_2)) = 4$ (by squaring)

$\Rightarrow \cos(\theta_1 - \theta_2) = 1 \Rightarrow \theta_1 - \theta_2 = 0 \quad (\because \cos 0° = 1)$

$\Rightarrow \text{Arg } z_1 - \text{Arg } z_2 = 0.$

11. (d) : Given $\omega = \dfrac{3z}{3z - i} \quad \therefore \ |\omega| = \dfrac{3|z|}{|3z - i|}$

$\Rightarrow |3z - i| = 3|z|$

$\Rightarrow |3(x) + i(3y - 1)| = |3(x + iy)| \ (z = x + iy)$

$\Rightarrow (3x)^2 + (3y - 1)^2 = 9(x^2 + y^2) \Rightarrow 6y - 1 = 0$ which is straight line.

12. (d) : $\sum_{k=1}^{n}\left(\sin\dfrac{2k\pi}{n+1} + i\cos\dfrac{2k\pi}{n+1}\right)$

$\therefore\quad = \sum_{k=1}^{10}\left(\sin\dfrac{2k\pi}{11} + i\cos\dfrac{2k\pi}{11}\right) = -i$

13. (d) : $z^2 + z + 1 = 0$

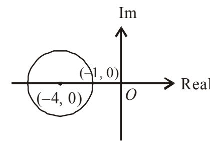
$\Rightarrow\quad z = \omega, \omega^2$

$\therefore\quad \left(z + \dfrac{1}{z}\right)^2 + \left(z^2 + \dfrac{1}{z^2}\right)^2 + \ldots..\left(z^6 + \dfrac{1}{z^6}\right)^2$

$= 4(\omega + \omega^2)^2 + 2(\omega^3 + \omega^3)^2$

$= 4(-1)^2 + 2(2^2) = 4 + 8 = 12$

14. (a) : z lies on or inside the circle with centre $(-4, 0)$ and radius 3 units.
Hence maximum distance of z from $(-1, 0)$ is 6 units.



15. (d) : $\bar{z} = \dfrac{1}{i - 1}$

We have $z = \overline{(\bar{z})}$ giving $z = \dfrac{1}{\bar{i} - 1} = \dfrac{1}{-i - 1} = \dfrac{-1}{i + 1}$

CHAPTER 3

Matrices and Determinants

1. If $a > 0$ and discriminant of $ax^2 + 2bx + c$ is –ve, then $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is

(a) +ve

(b) $(ac - b^2)(ax^2 + 2bx + c)$

(c) –ve (d) 0. *(2002)*

2. If l, m, n are the p^{th}, q^{th} and r^{th} term of a GP, all positive, then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals

(a) –1 (b) 2 (c) 1 (d) 0. *(2002)*

3. If 1, ω, ω^2 are the cube roots of unity, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to

(a) 1 (b) ω (c) ω^2 (d) 0. *(2003)*

4. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

(a) $\alpha = a^2 + b^2$, $\beta = 2ab$

(b) $\alpha = a^2 + b^2$, $\beta = a^2 - b^2$

(c) $\alpha = 2ab$, $\beta = a^2 + b^2$

(d) $\alpha = a^2 + b^2$, $\beta = ab$. *(2003)*

5. Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. The only correct statement about the matrix A is

(a) A^{-1} does not exist

(b) $A = (-1)I$, where I is a unit matrix

(c) A is a zero matrix (d) $A^2 = I$. *(2004)*

6. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$ and $10(B) = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$. If B is the inverse of matrix A, then α is

(a) 2 (b) –1 (c) –2 (d) 5. *(2004)*

7. If $a_1, a_2, a_3, \ldots, a_n, \ldots$ are G.P., then the value of the determinant $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$, is

(a) 2 (b) 1 (c) 0 (d) –2. *(2004)*

8. If $A^2 - A + I = 0$, then the inverse of A is

(a) A (b) $A + I$ (c) $I - A$ (d) $A - I$. *(2005)*

9. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathematical induction

(a) $A^n = 2^{n-1}A - (n-1)I$

(b) $A^n = nA - (n-1)I$

(c) $A^n = 2^{n-1}A + (n-1)I$

(d) $A^n = nA + (n-1)I$. (2005)

10. If $a^2 + b^2 + c^2 = -2$ and

$$f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

then $f(x)$ is a polynomial of degree

(a) 0 (b) 1 (c) 2 (d) 3. (2005)

11. The system of equations $\alpha x + y + z = \alpha - 1$, $x + \alpha y + z = \alpha - 1$, $x + y + \alpha z = \alpha - 1$ has no solutions, if α is

(a) either -2 or 1 (b) -2
(c) 1 (d) not -2. (2005)

12. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true?

(a) $A = B$ (b) $AB = BA$
(c) either A or B is a zero matrix
(d) either A or B is an identity matrix. (2006)

13. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a, b \in N$. Then

(a) there cannot exist any B such that $AB = BA$
(b) there exist more than one but finite number B's such that $AB = BA$
(c) there exists exactly one B such that $AB = BA$
(d) there exist infinitely many B's such that $AB = BA$. (2006)

14. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$ then D is

(a) divisible by x but not y
(b) divisible by y but not x
(c) divisible by neither x nor y
(d) divisible by both x and y. (2007)

15. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals

(a) 1/5 (b) 5 (c) 5^2 (d) 1. (2007)

Directions : Question number 16 is Assertion-Reason type. This question contains two statements : Statement-1 (Assertion) and Statement-2 (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

(a) Statement-1 is true, Statement-2 is false
(b) Statemen-1 is false, Statement-2 is true
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

16. Let A be a 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by $tr(A)$, the sum of diagonal entries of A. Assume that $A^2 = I$.
Statement-1 : If $A \neq I$ and $A \neq -I$, then $\det A = -1$.
Statement-2 : If $A \neq I$ and $A \neq -I$, then $tr(A) \neq 0$. (2008)

17. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$ and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to

(a) 1 (b) 2 (c) -1 (d) 0 (2008)

18. Let A be a square matrix all of whose entries are integers. Then which one of the following is true?

(a) If $\det A = \pm 1$, then A^{-1} need not exist
(b) If $\det A = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers
(c) If $\det A \neq \pm 1$, then A^{-1} exists and all its entries are non-integers
(d) If $\det A = \pm 1$, then A^{-1} exists and all its entries are integers (2008)

Answer Key

1. (c)	**2.** (d)	**3.** (d)	**4.** (a)	**5.** (d)	**6.** (d)
7. (c)	**8.** (c)	**9.** (b)	**10.** (c)	**11.** (b)	**12.** (b)
13. (d)	**14.** (d)	**15.** (a)	**16.** (a)	**17.** (a)	**18.** (d)

1. **(c) :** $C_1 \to xC_1 + C_2 - C_3$

$$= \frac{1}{x}\begin{vmatrix} 0 & b & ax+b \\ 0 & c & bx+c \\ ax^2+2bx+c & bx+c & 0 \end{vmatrix}$$

$$= \frac{(ax^2+2bx+c)}{x}\,[b^2x + bc - acx - bc]$$

$= (b^2 - ac)(ax^2 + 2bx + c)$

$= (+\text{ve})(-\text{ve}) < 0$

2. **(d) :** Let A be the first term and R be the common ratio of G. P.

$\therefore \quad l = t_p = AR^{p-1}$

$\Rightarrow \quad \log l = \log A + (p-1)\log R$

Similarly, $\log m = \log A + (q-1)\log R$

and $\log n = \log A + (r-1)\log R$

$$\therefore \begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix}$$

$$= \underset{c_1 \propto c_3}{\begin{vmatrix} \log A - \log R & p & 1 \\ \log A - \log R & q & 1 \\ \log A - \log R & r & 1 \end{vmatrix}} + \underset{c_1 \propto c_2}{\begin{vmatrix} p\log R & p & 1 \\ q\log R & q & 1 \\ r\log R & r & 1 \end{vmatrix}}$$

$= 0 + 0$

$= 0$

3. **(d) :** As ω is cube root of unity $\therefore \omega^3 = \omega^{3n} = 1$

$$\therefore \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$$

$= (\omega^{3n} - 1) - \omega^n(\omega^{2n} - \omega^{2n}) + \omega^{2n}(\omega^n - \omega^n)$

$= 0$

4. **(a) :** $A^2 = AA = \begin{pmatrix} a & b \\ b & a \end{pmatrix}\begin{pmatrix} a & b \\ b & a \end{pmatrix}$

$$= \begin{pmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix}$$

5. **(d) :** (i) $|A| = 1 \therefore A^{-1}$ does not exist is wrong statement

(ii) $(-1)\,I = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \neq A \Rightarrow$ (b) is false

(iii) A is clearly a non zero matrix $\therefore$ (c) is false

We left with (d) only.

6. **(d) :** Given $A^{-1} = B = 10\,A^{-1} = 10\,B$

$$\Rightarrow \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix} = 10\,A^{-1}.$$

$$\Rightarrow \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}(A) = 10I$$

$$\Rightarrow \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{pmatrix} \quad ...(*)$$

$\Rightarrow -5 + \alpha = 0$

(equating A_{21} entry both sides of (*))

$\Rightarrow \alpha = 5$

7. **(c) :** $\dfrac{a_2}{a_1} = \dfrac{a_3}{a_2} = ... \dfrac{a_n}{a_{n-1}} = r$

which means $a_n, a_{n+1}, a_{n+2} \in$ G.P.

$\Rightarrow a_{n+1}^2 = a_n\,a_{n+2}$

$\Rightarrow 2\log a_{n+1} - \log a_n - \log a_{n+2} = 0 \quad ...(i)$

Similarly $2\log a_{n+4} - \log a_{n+3} - \log a_{n+5} = 0 \quad ...(ii)$

and $2\log a_{n+7} - \log a_{n+6} - \log a_{n+8} = 0 \quad ...(iii)$

Using $C_1 \to C_1 + C_3 - 2C_2$

we get $\Delta = 0$

8. **(c) :** $A^2 - A + I = 0 \Rightarrow I = A - A \cdot A$

$IA^{-1} = AA^{-1} - A(AA^{-1})$, $A^{-1} = I - A$.

9. **(b) :** $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

$\therefore\ A^2 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix},\ A^3 = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ so $A^n = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$

and $nA - (n-1)I = \begin{pmatrix} n & 0 \\ n & n \end{pmatrix} - \begin{pmatrix} n-1 & 0 \\ 0 & n-1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix} = A^n.$

10. (c) : Applying $C_2 \to C_2 + C_3 + C_1$

$f(x) = 1 + 2x + x(a^2 + b^2 + c^2)$

$$\begin{vmatrix} 1+a^2x & 1 & (1+c^2)x \\ (1+a^2)x & 1 & (1+c^2)x \\ (1+a^2)x & 1 & 1+c^2x \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$ and using $a^2 + b^2 + c^2 = -2$ we have

$$(1+2x-2x)\begin{vmatrix} 1-x & 0 & 0 \\ 0 & 0 & x-1 \\ (1+a^2)x & 1 & 1+c^2x \end{vmatrix} = (1-x)^2$$

$= x^2 - 2x + 1\ \therefore$ degree of $f(x)$ is 2.

11. (b) : For no solution $|A| = 0$ and $(\text{adj } A)(B) \neq 0$

Now $|A| = 0 \Rightarrow \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$

$\Rightarrow \alpha^3 - 3\alpha + 2 = 0 \Rightarrow (\alpha - 1)^2(\alpha + 2) = 0$

$\Rightarrow \alpha = 1, -2.$

But for $\alpha = 1$, $|A| = 0$ and $(\text{adj } A)(B) = 0$

$\Rightarrow$ for $\alpha = 1$ there exist infintiely many solution.

Also the each equation becomes

$x + y + z = 0$ again for $\alpha = -2$

$|A| = 0$ but $(\text{adj } A)(B) \neq 0\ \Rightarrow \exists$ no solution.

12. (b) : Give $A^2 - B^2 = (A + B)(A - B)$

$\Rightarrow\quad 0 = BA - AB$

$\Rightarrow\quad BA = AB$

13. (d) : $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ *and* $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$

Now $AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} a & 2b \\ 3a & 4b \end{pmatrix}$ ... (i)

and $BA = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} a & 2a \\ 3b & 4b \end{pmatrix}$ ... (ii)

As $AB = BA \Rightarrow 2a = 2b \Rightarrow a = b$

$\therefore\quad B = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} = aI_2 \Rightarrow \exists$ infinite value of $a = b \in N$

14. (d) : $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

(Apply $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$)

$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix} = 1(xy - 0) = xy$

Hence D is divisible by both x and y.

15. (a) : $A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 25 & 25\alpha + 5\alpha^2 & 5\alpha + 5\alpha + 25\alpha^2 \\ 0 & \alpha^2 & 25\alpha + 5\alpha^2 \\ 0 & 0 & 25 \end{bmatrix}$$

Given $|A^2| = 25$, $625\alpha^2 = 25 \Rightarrow |\alpha| = \frac{1}{5}.$

16. (a) : Let $A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$. We have

$A^2 = \begin{bmatrix} \alpha^2 + \beta\gamma & \beta(\alpha+\delta) \\ \gamma(\alpha+\delta) & \delta^2 + \beta\gamma \end{bmatrix}$

$A^2 = I = \begin{bmatrix} \alpha^2 + \beta\gamma & \beta(\alpha+\delta) \\ \gamma(\alpha+\delta) & \delta^2 + \beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

giving $\alpha^2 + \beta\gamma = 1 = \delta^2 + \beta\gamma$

and $\gamma(\alpha + \delta) = \beta(\alpha + \delta) = 0$

As $A \neq I$, $A \neq -I$, we have $\alpha = -\delta$

$\det A = \begin{vmatrix} \sqrt{1-\beta\gamma} & \beta \\ \gamma & -\sqrt{1-\beta\gamma} \end{vmatrix} = -1 + \beta\gamma - \beta\gamma = -1$

Statement-1 is therefore true.

tr $(A) = \alpha + \delta = 0\quad \{\alpha = -\delta\}$

Statement-2 is false because tr $(A) = 0$

17. (a) : System of equations

$x - cy - bz = 0$

$cx - y + az = 0$

$bx + ay - z = 0$

has non trivial solution if the determinant of coefficient matrix is zero

$$\Rightarrow \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$\Rightarrow 1(1 - a^2) + c(-c - ab) - b(ca + b) = 0$

$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

18. (d) : Each entry of A is an integer, so the cofactor of every entry is an integer. And then each entry of adjoint is integer.

Also det $A = \pm 1$ and we know that

$$A^{-1} = \frac{1}{\det A}(adj\ A)$$

This means all entries in A^{-1} are integers.

CHAPTER
4

Quadratic Equations

1. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation whose roots are α/β and β/α is
(a) $3x^2 - 25x + 3 = 0$
(b) $x^2 + 5x - 3 = 0$
(c) $x^2 - 5x + 3 = 0$
(d) $3x^2 - 19x + 3 = 0$.
(2002)

2. Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then
(a) $a + b + 4 = 0$
(b) $a + b - 4 = 0$
(c) $a - b - 4 = 0$
(d) $a - b + 4 = 0$.
(2002)

3. Product of real roots of the equation
$$x^2 + |x| + 9 = 0$$
(a) is always positive
(b) is always negative
(c) does not exist
(d) none of these.
(2002)

4. If p and q are the roots of the equation $x^2 + px + q = 0$, then
(a) $p = 1, q = -2$ (b) $p = 0, q = 1$
(c) $p = -2, q = 0$ (d) $p = -2, q = 1$.
(2002)

5. If a, b, c are distinct +ve real numbers and $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ is
(a) less than 1 (b) equal to 1
(c) greater than 1 (d) any real no.
(2002)

6. The value of a for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is
(a) –2/3 (b) 1/3 (c) –1/3 (d) 2/3.
(2003)

7. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in
(a) geometric progression
(b) harmonic progression
(c) arithmetic-geometric progression
(d) arithmetic progression.
(2003)

8. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is
(a) 4 (b) 1 (c) 3 (d) 2.
(2003)

9. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation
(a) $x^2 + 18x - 16 = 0$ (b) $x^2 - 18x + 16 = 0$
(c) $x^2 + 18x + 16 = 0$ (d) $x^2 - 18x - 16 = 0$.
(2004)

10. If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$ then its roots are
(a) 0, –1 (b) –1, 1 (c) 0, 1 (d) –1, 2.
(2004)

11. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of q is
(a) 3 (b) 12 (c) 49/4 (d) 4.
(2004)

12. If the cube roots of unity are 1, ω, ω^2 then the roots of the equation $(x-1)^3 + 8 = 0$, are
(a) $-1, -1, -1$
(b) $-1, -1 + 2\omega, -1 - 2\omega^2$
(c) $-1, 1 + 2\omega, 1 + 2\omega^2$
(d) $-1, 1 - 2\omega, 1 - 2\omega^2$.
(2005)

13. The value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assume the least value is
(a) 0 (b) 1 (c) 2 (d) 3.
(2005)

14. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals
(a) 3 (b) -2 (c) 1 (d) 2.
(2005)

15. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval
(a) $(6, \infty)$ (b) $(5, 6]$
(c) $[4, 5]$ (d) $(-\infty, 4)$.
(2005)

16. If the equation $a_nx^n + a_{n-1}x^{n-1} + ... + a_1x = 0$, $a_1 \neq 0$, $n \geq 2$, has a positive root $x = \alpha$, then the equation $na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + ... + a_1 = 0$ has a positive root, which is
(a) smaller than α
(b) greater than α
(c) equal to α
(d) greater than or equal to α.
(2005)

17. If the roots of the quadratic equation $x^2 + px + q = 0$ are tan30° and tan15°, respectively then the value of $2 + q - p$ is
(a) 2 (b) 3 (c) 0 (d) 1.
(2006)

18. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4, lie in the interval
(a) $-2 < m < 0$ (b) $m > 3$
(c) $-1 < m < 3$ (d) $1 < m < 4$.
(2006)

19. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is
(a) $(3, \infty)$ (b) $(-\infty, -3)$
(c) $(-3, 3)$ (d) $(-3, \infty)$.
(2007)

20. The quadratic equations
$x^2 - 6x + a = 0$
and $x^2 - cx + 6 = 0$
have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is
(a) 2 (b) 1 (c) 4 (d) 3
(2008)

Answer Key

1.	(d)	**2.**	(a)	**3.**	(c)	**4.**	(a)	**5.**	(a)	**6.**	(d)
7.	(b)	**8.**	(a)	**9.**	(b)	**10.**	(a)	**11.**	(c)	**12.**	(d)
13.	(b)	**14.**	(c)	**15.**	(d)	**16.**	(a)	**17.**	(b)	**18.**	(c)
19.	(c)	**20.**	(a)								

1. **(d) :** We need the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ which are reciprocal of each other, which means product of roots is $\frac{\alpha}{\beta}\frac{\beta}{\alpha} = 1$. In our choice (a) and (d) have product of roots 1, so choices (b) and (d) are out of court. In the problem choice, None of these is not given. If out of four choices only one choice satisfies that product of root is 1 then you select that choice for correct answer. Now for proper choice we proceed as,

$\alpha \neq \beta$, but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$,

changing α, β by x

$\therefore$ α, β are roots of $x^2 - 5x + 3 = 0$

$\Rightarrow \alpha + \beta = 5, \alpha\beta = 3$

now, $S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{19}{3}$ and product $\frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha} = 1$

$\therefore$ Required equation,

$x^2 -$ (sum of roots) $x +$ product of roots $= 0$

$\Rightarrow x^2 - \frac{19}{3}x + 1 = 0$

$\Rightarrow 3x^2 - 19x + 3 = 0$ is correct answer.

2 **(a) :** Let α, β are roots of $x^2 + bx + a = 0$

$\therefore$ $\alpha + \beta = -b$ and $\alpha\beta = a$

again let γ, δ are roots of $x^2 + ax + b = 0$

$\therefore$ $\gamma + \delta = -a$ and $\gamma\delta = b$

Now given

$\alpha - \beta = \gamma - \delta$

$\Rightarrow (\alpha - \beta)^2 = (\gamma - \delta)^2$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$

$\Rightarrow b^2 - 4a = a^2 - 4b$

$\Rightarrow b^2 - a^2 = -4(b - a)$

$\Rightarrow (b - a)(b + a + 4) = 0$

$\Rightarrow b + a + 4 = 0$ as $(a \neq b)$

3. **(c) :** $x^2 + |x| + 9 = 0$

$\Rightarrow |x|^2 + |x| + 9 = 0$

$\Rightarrow$ \ no real roots $\quad (^3\ D < 0)$

4. **(a) :** Given $S = p + q = -p$ and product $pq = q$

$\Rightarrow q(p - 1) = 0$

$\Rightarrow q = 0, p = 1$

Now If $q = 0$ then $p = 0 \Rightarrow p = q$

If $p = 1$, then $p + q = -p$

$q = -2p$

$q = -2(1)$

$q = -2$

$\Rightarrow p = 1$ and $q = -2$

5. **(a) :** In such type of problem if sum of the squares of number is known and we needed product of numbers taken two at a time or needed range of the product of numbers taken two at a time. We start square of the sum of the numbers like

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow 2(ab + bc + ca) = (a + b + c)^2 - (a^2 + b^2 + c^2)$

$\Rightarrow ab + bc + ca = \frac{(a+b+c)^2 - 1}{2} < 1$

6. **(d) :** Let $\alpha, 2\alpha$ are roots of the given equation

$\therefore$ sum of the roots

$\alpha + 2\alpha = 3\alpha = \frac{1 - 3a}{a^2 - 5a + 3}$ ...(i)

and product of roots

$\alpha(2\alpha) = 2\alpha^2 = \frac{2}{a^2 - 5a + 3}$ ...(ii)

By (i) and (ii) we have

$\frac{9\alpha^2}{2\alpha^2} = \frac{(1-3a)^2}{(a^2 - 5a + 3)^2} \times \frac{a^2 - 5a + 3}{2}$

$\Rightarrow 9(a^2 - 5a + 3) = (1 - 3a)^2$

$\Rightarrow a = \frac{2}{3}$

7. **(b) :** Given $\alpha + \beta$

$= \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$

$\Rightarrow 2a^2c = bc^2 + ab^2$

$\Rightarrow \frac{2a}{b} = \frac{c}{a} + \frac{b}{c}$

$\Rightarrow \frac{c}{a}, \frac{a}{b}, \frac{b}{c} \in$ A.P

$\Rightarrow$ reciprocals are in H.P

8. **(a) :** Given $x^2 - 3|x| + 2 = 0$

If $x \geq 0$ *i.e.* $|x| = x$

$\therefore$ The given equation can be written as

$x^2 - 3x + 2 = 0$

$\Rightarrow (x - 1)(x - 2) = 0$

$\Rightarrow x = 1, 2$

Similarly for $x < 0$, $x^2 - 3|x| + 2 = 0$

$\Rightarrow x^2 + 3x + 2 = 0$

$\Rightarrow x = -1, -2$

Hence 1, –1, 2, –2 are four solutions of the given equation.

9. **(b) :** Let the two number be α, β

$\therefore \frac{\alpha+\beta}{2} = 9$ and $\sqrt{\alpha\beta} = 4$

$\therefore$ Required equation

$x^2 - 2(\text{Average value of } \alpha, \beta)x + \sqrt{G.M}^2 = 0$

$x^2 - 2(9)x + 16 = 0$

10. **(a) :** As $1 - p$ is root of $x^2 + px + 1 - p = 0$

$\Rightarrow (1 - p)^2 + p(1 - p) + (1 - p) = 0$

$(1 - p)[1 - p + p + 1] = 0$

$\Rightarrow p = 1$

$\therefore$ Given equation becomes $x^2 + x = 0$

$\Rightarrow x = 0, -1$

11. **(c) :** As $x^2 + px + q = 0$ has equal roots $\therefore p^2 = 4q$ and one root of $x^2 + px + 12 = 0$ is 4.

$\therefore 16 + 4p + 12 = 0 \therefore p = -7$

$\therefore p^2 = 4q \Rightarrow q = \frac{49}{4}$

12. **(d) :** ***Method (1) :*** (By making the equation from the given roots)

Let us consider $x = -1, -1, -1$

$\therefore$ Required equation from given roots is $(x + 1)(x + 1)(x + 1) = 0$

$(x + 1)^3 = 0$ which does not match with the given equation

$(x - 1)^3 + 8 = 0$ so $x = -1, -1, -1$ cannot be the proper choice.

Again consider $x = -1, -1 + 2\omega, -1 -2\omega^2$

$\therefore$ Required equation from given roots is

$\Rightarrow (x + 1)(x + 1 - 2\omega)(x + 1 + 2\omega^2) = 0$

$\Rightarrow (x + 1)[(x + 1)^2 + (x + 1)(2\omega^2 - 2\omega) - 4\omega^3] = 0$

$\Rightarrow (x + 1)[(x + 1)^2 + 2(x + 1)(\omega^2 - \omega) - 4] = 0$

$\Rightarrow (x + 1)^3 + 2(x + 1)^2(\omega^2 - \omega) - 4(x + 1) = 0$

which cannot be expressed in the form of given equation $(x - 1)^3 + 8 = 0$. Now consider the roots $x_i = -1, 1 - 2\omega, 1 - 2\omega^2$ $(i = 1, 2, 3)$

and the equation with these roots is given by

x^3 – (sum of the roots)x^2 + x(Product of roots taken two at a time) – Product of roots taken all at a time = 0

Now sum of roots $x_1 + x_2 + x_3$

$= -1 + 1 - 2\omega + 1 - 2\omega^2 = 3$

Product of roots taken two at a time

$= -1 + 2\omega - 1 + 2\omega^2 + 1 + 2(\omega^2 + \omega) + 4\omega^3 = 3$

Product of roots taken all at a time

$= (-1)[(1 - 2\omega)(1 - 2\omega^2)] = -7$

$\therefore$ Required equation is $x^3 - 3x^2 + 3x + 7 = 0$

$\Rightarrow x^3 - 3x^2 + 3x - 1 + 8 = 0 \Rightarrow (x - 1)^3 + 8 = 0$

which matched with given equation.

Method 2 (by taking cross checking)

As $(x - 1)^3 + 8 = 0$ ...(*)

and $x = -1$ satisfies $(x - 1)^3 + 8 = 0$

i.e. $(-2)^3 + 8 = 0 \quad\Rightarrow\quad 0 = 0$

Similarly for $1 - 2\omega$ we have $(x - 1)^3 + 8 = 0$

$\Rightarrow (1 - 2\omega - 1)^3 + 8 = 0$

$\Rightarrow (-2\omega)^3 + 8 = 0$

$\Rightarrow -8 + 8 = 0$

and for $1 - 2\omega^2$

we have $(1 - 2\omega^2 - 1)^3 + 8 = 0$

$\Rightarrow \omega^6(-8) + (8) = 0 \Rightarrow 0 = 0$

$\therefore -1, 1 - 2\omega, 1 - 2\omega^2$ are roots of $(x - 1)^3 + 8 = 0$ but on the other hand the other roots does not satisfies the equation $(x - 1)^3 + 8 = 0$.

13. **(b) :** Let $f(a) = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (a - 2)^2 + 2(a + 1)$

$\therefore f'(a) = 2(a - 2) + 2$

For Maxima | Minima $f'(a) = 0$

$\Rightarrow 2[a - 2 + 1] = 0 \Rightarrow a = 1$

Again $f''(a) = 2$,

$f''(1) = 2 > 0 \Rightarrow$ at $a = 1$, $f(a)$ will be least.

14. **(c) :** Let α, $\alpha + 1$ are consecutive integer

$\therefore (x + \alpha)(x + \alpha + 1) = x^2 - bx + c$

comparing both sides we get $\Rightarrow -b = 2\alpha + 1$

$c = \alpha^2 + \alpha$

$\therefore\ b^2 - 4c = (2\alpha + 1)^2 - 4(\alpha^2 + \alpha) = 1.$

15. **(d) :** Given $x^2 - 2kx + k^2 + k - 5 = 0$

Roots are less than 5 $\Rightarrow D \geq 0$

$\Rightarrow (-2k)^2 \geq 4(k^2 + k - 5) \Rightarrow k \leq 5$ ...(A)

Again $f(5) > 0$

$\Rightarrow 25 - 10k + k^2 + k - 5 > 0$

$\Rightarrow k^2 - 9k + 20 > 0 \Rightarrow (k-4)(k-5) > 0$

$\Rightarrow k < 4 \cup k > 5$ ...(B)

Also $\frac{\text{sum of roots}}{2} < 5 \Rightarrow k < 5$ ...(C)

from (A), (B), (C) we have

$k \in (-\infty, 4)$ as the choice gives number $k < 5$ is (d).

16. **(a) :** If possible say

$f(x) = a_0x^n + a_1x^{n-1} + \ldots + a_nx \quad \therefore f(0) = 0$

Now $f(\alpha) = 0$ ($\therefore x = \alpha$ is root of given equation)

$\therefore\ f'(x) = na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots + a_1 = 0$

has at least one root in $]0, \alpha[$

$\Rightarrow na_n x^{n-1} + (n-1)\, a_{n-1}\, x^{n-2} + \ldots + a_1 = 0$

has a +ve root smaller than α.

17. **(b) :** $\alpha = \tan 30°$, $\beta = \tan 15°$ are roots of the equation $x^2 + px + q = 0$

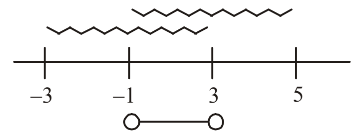
$\therefore \tan\alpha + \tan\beta = -p$ and $\tan\alpha \cdot \tan\beta = q$

using $\tan\alpha + \tan\beta = \tan(\alpha + \beta)$
$(1 - \tan\alpha\tan\beta)$

$\Rightarrow -p = 1 - q \Rightarrow q - p = 1 \Rightarrow 2 + q - p = 3$

18. **(c) :** Let α, β are roots of the equation

$(x^2 - 2mx + m^2) = 1$



$\Rightarrow \quad x = m \pm 1 = m + 1,\ m - 1$

Now $-2 < m + 1 < 4$ ...... (i)

and $-2 < m - 1 < 4$ .... (ii)

$\left\{\begin{array}{l}\Rightarrow \quad -3 < m < 3 \quad \text{......(A)} \\ \text{and} \quad -1 < m < 5 \quad \text{.......(B)}\end{array}\right.$

By (A) & (B) we get $-1 < m < 3$ as shown by the number line.

19. **(c) :** $x^2 + ax + 1 = 0$

Let roots be α and β, then $\alpha + \beta = -a$ and $\alpha\beta = 1$

$|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$, $|\alpha - \beta| = \sqrt{a^2 - 4}$

Since, $|\alpha - \beta| < \sqrt{5} \Rightarrow \sqrt{a^2 - 4} < \sqrt{5}$

$\Rightarrow \quad a^2 - 4 < 5 \Rightarrow a^2 < 9 \Rightarrow -3 < a < 3.$

20. **(a) :** Let α and 4β be the root of

$$x^2 - 6x + a = 0$$

and α and 3β be those of the equation

$$x^2 - cx + 6 = 0$$

From the relation between roots and coefficients

$\alpha + 4\beta = 6$ and $4\alpha\beta = a$

$\alpha + 3\beta = c$ and $3\alpha\beta = 6$

we obtain $\alpha\beta = 2$ giving $a = 8$

The first equation is $x^2 - 6x + 8 = 0 \quad \Rightarrow x = 2, 4$

For $\alpha = 2$, $4\beta = 4 \quad \Rightarrow \quad 3\beta = 3$

For $\alpha = 4$, $4\beta = 2 \quad \Rightarrow \quad 3\beta = 3/2$ (not an integer)

So the common root is $\alpha = 2$.

CHAPTER

5

Permutation and Combination

1. Number greater than 1000 but less than 4000 is formed using the digits 0, 2, 3, 4 repetition allowed is

(a) 125 (b) 105
(c) 128 (d) 625.

(2002)

2. Five digit number divisible by 3 is formed using 0, 1, 2, 3, 4, 6 and 7 without repetition. Total number of such numbers are

(a) 312 (b) 3125
(c) 120 (d) 216.

(2002)

3. The sum of integers from 1 to 100 that are divisible by 2 or 5 is

(a) 3000 (b) 3050
(c) 3600 (d) 3250.

(2002)

4. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 are

(a) 216 (b) 375
(c) 400 (d) 720.

(2002)

5. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by

(a) 30 (b) $5! \times 4!$
(c) $7! \times 5!$ (d) $6! \times 5!$.

(2003)

6. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is

(a) 196 (b) 280
(c) 346 (d) 140.

(2003)

7. If ${}^nC_r$ denotes the number of combinations of n things taken r at a time, then the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals

(a) ${}^{n+2}C_{r+1}$ (b) ${}^{n+1}C_r$
(c) ${}^{n+1}C_{r+1}$ (d) ${}^{n+2}C_r$.

(2003)

8. How many ways are there to arrange the letters in the word GARDEN with the vowels in alphabetical order?

(a) 360 (b) 240
(c) 120 (d) 480.

(2004)

9. Then number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is

(a) 3^8 (b) 21
(c) 5 (d) ${}^8C_3$.

(2004)

10. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number

(a) 602 (b) 603
(c) 600 (d) 601.

(2005)

11. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is

(a) ${}^{56}C_4$ (b) ${}^{56}C_3$

(c) $^{55}C_3$ (d) $^{55}C_4$.

(2005)

12. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote is

(a) 5040 (b) 6210
(c) 385 (d) 1110.

(2006)

13. The sum of the series
$^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - ...... + {}^{20}C_{10}$ is

(a) 0 (b) $^{20}C_{10}$
(c) $-{}^{20}C_{10}$ (d) $\frac{1}{2}\,{}^{20}C_{10}$.

(2007)

Directions : Question number 14 is Assertion-Reason type. The question contains two statements : Statement-1 (Assertion) and Statement-2 (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

(a) Statement-1 is true, Statement-2 is false
(b) Statemen-1 is false, Statement-2 is true
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

14. In a shop there are five types of ice-creams available. A child buys six ice-creams.

Statement-1 : The number of different ways the child can buy the six ice-creams is $^{10}C_5$.

Statement-2 : The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row.

(2008)

15. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?

(a) $7 \cdot {}^6C_4 \cdot {}^8C_4$ (b) $8 \cdot {}^6C_4 \cdot {}^7C_4$
(c) $6 \cdot 7 \cdot {}^8C_4$ (d) $6 \cdot 8 \cdot {}^7C_4$

(2008)

Answer Key

1. (c)	**2.** (d)	**3.** (b)	**4.** (d)	**5.** (d)	**6.** (a)
7. (a)	**8.** (a)	**9.** (b)	**10.** (d)	**11.** (a)	**12.** (a)
13. (d)	**14.** (b)	**15.** (a)			

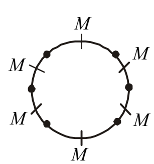
1. **(c) :** Let number of digits formed x.
$\therefore$ $1000 < x < 4000$, which means left extreme digit will be either 2 or 3.
$\therefore$ Required numbers $= {}^2C_1 \times H\ T\ U$
where H = Hundred place
$= {}^2C_1 \times 4 \times 4 \times 4$ $\quad$ T = Ten's place
$= 128$ $\quad$ U = Unit place

2. **(d) :**

3. **(b) :** Set of numbers divisible by 2 are 2, 4, 6, ...100
Set of numbers divisible by 5 are 5, 10, 15, ...100
Set of numbers divisible by 10 are 10, 20, 30, ...100
Now sum of numbers divisible by 2 is given by
$S_{50} = \frac{50}{2}[2 + 100]$ using $S_n = \frac{n}{2}[a + l]$
$S_{50} = 25[102]$
Similarly, $S_{20} = \frac{20}{2}[5 + 100] = 10 \times 105 = 1050$
and $S_{10} = \frac{10}{2}[10 + 100] = 5 \times 110$
$\therefore$ Required sum $= 25 \times 102 + 1050 - 550$
$= 25[102 + 42 - 22]$
$= 25 \times 122$
$= 3050$

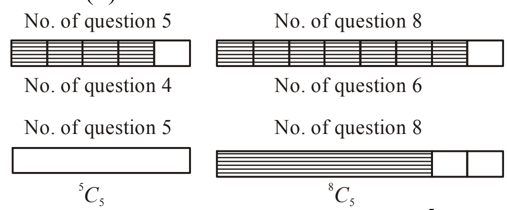
4. **(d) :** Odd numbers are 1, 3, 5, 7
We have to fill up four places like $TH\ H\ T\ U$
(Case: If repetition is allow)
${}^5C_1\ 6^2\ {}^4C_1 = 5 \times 6^2 \times 4$
$= 5 \times 36 \times 4$
$= 720$

5. **(d) :** Number of women 5
Number of men 6



Number of ways of 6 men at a round Table is $n - 1! = (6 - 1)! = 5!$
Now we left with six places between the men and there are 5 women, these 5 women can be arranged themselves by ${}^6P_5$ way.
$\therefore$ Required number of ways $= 5! \times {}^6P_5 = 5! \times 6!$

6. **(a) :** Case (i) :
Required ways for first case $= {}^5C_4 \times {}^8C_6 = 140$
Case (ii):



$\therefore$ Required ways for case (ii) $= {}^5C_5 \times {}^8C_5$
$= \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$
Total number of ways $= 140 + 56$
$= 196$

7. **(a) :** Consider ${}^nC_{r-1} + 2\,{}^nC_r + {}^nC_{r+1}$
$= ({}^nC_{r-1} + {}^nC_r) + ({}^nC_r + {}^nC_{r+1})$
$= {}^{n+1}C_r + {}^{n+1}C_{r+1}$
$= {}^{n+2}C_{r+1}$

8. **(a) :** Number of letters = 6
Number of vowels = 2 namely A.E these alphabets can be arrange themselves by 2! ways
$\therefore$ Number of words $= \frac{6!}{2!} = 360$

9. **(b) : (i)** Each box must contain at least one ball since no box remains empty so we have the following cases

Box	Number of balls				
I	1,	1,	1,	2,	2,
I	1,	2,	3,	3,	2,
III	6,	5,	4,	3,	4,

$\therefore$ Number of ways
$3 \times \frac{1 \times 3!}{2!} + 3! \times 2$
$= 9 + 6 \times 2 = 21$

As $\boxed{1, 1, 6}$ $\boxed{2, 3, 4}$ $\boxed{2, 2, 4}$ have case ways and $\boxed{1, 2, 5}$ $\boxed{1, 3, 4}$ have equal number of ways of arranging the balls in the different boxes.

(ii): Let the number of balls in the boxes are x, y, z respectively then $x + y + z = 8$ and no box is empty so each $x, y, z \geq 1$

$\Rightarrow l + m + n + 3 = 8$ where $l = x - 1$, $m = y - 1$, $n = z - 1$

i.e. $(l + 1) + (m + 1) + (n + 1) = 8$ are non negative integers

$\therefore$ Required number of ways $= {}^{n+r-1}C_r$

$= {}^{3+5-1}C_5 = {}^7C_5 = {}^7C_2$

10. (d) : $\underset{\text{Fixed}}{\boxed{S}}$ A C H I N

No. of word start with $A = 5!$

No. of word start with $C = 5!$

No. of word start with $H = 5!$

No. of word start with $I = 5!$

No. of word start with $N = 5!$

Total words $= 5! + 5! + 5! + 5! + 5! = 5(5!) = 600$

Now add the rank of SACHIN so required rank of SACHIN $= 600 + 1 = 601$.

11. (a) ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$

Putting $r = 6, 5, 4, 3, 2, 1$ we get

$${}^{50}C_4 + {}^{50}C_3 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$$

$$\left(\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}\right)$$

$= {}^{51}C_4 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

$= {}^{52}C_4 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

$= {}^{53}C_4 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3 = {}^{54}C_4 + {}^{54}C_3 + {}^{55}C_3$

$= {}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4$

12. (a) : A voter can vote one candidate or two or three or four candidates

$\therefore$ Required number of ways

$$= {}^{10}c_1 + {}^{10}c_2 + {}^{10}c_3 + {}^{10}c_4 = 385$$

13. (d) : $\because$ ${}^{20}C_0 + {}^{20}C_1x + \ldots\ldots + {}^{20}C_{10}x^{10} +$

$$\ldots\ldots + {}^{20}C_{20}x^{20} = (1+x)^{20}$$

After putting $x = -1$, we get

$${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots$$

$$+ {}^{20}C_{10} - {}^{20}C_{11} - {}^{20}C_{12} + \ldots + {}^{20}C_{20} = 0$$

$$2({}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots - {}^{20}C_9) + {}^{20}C_{10} = 0$$

$${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots - {}^{20}C_9 + {}^{20}C_{10} = \frac{1}{2}\,{}^{20}C_{10}$$

14. (b) : We have to find the number of integral solutions if $x_1 + x_2 + x_3 + x_4 + x_5 = 6$

and that equals ${}^{5+6-1}C_{5-1} = {}^{10}C_4$

Thus Statement-1 is false.

Number of different ways of arranging $6A$'s and $4B$'s in a row

$= \dfrac{\lfloor 10}{\lfloor 6 \times \lfloor 4} = {}^{10}C_4 =$ Number of different ways the child can buy the six ice-creams.

$\therefore$ Statement-2 is true

So, Statement-1 is false, Statement-2 is true.

15. (a) : Leaving S, we have 7 letters M, I, I, I, P, P, I.

way of arranging them $= \dfrac{\lfloor 7}{\lfloor 2 \lfloor 4} = 7 \cdot 5 \cdot 3$

And four S can be put in 8 places in ${}^8C_4$ ways.

The required number of ways

$$= 7 \cdot 5 \cdot 3 \cdot {}^8C_4 = 7 \cdot {}^6C_4 \cdot {}^8C_4.$$

CHAPTER

6

Mathematical Induction and its Application

1. If $a_n = \sqrt{7+\sqrt{7+\sqrt{7+....}}}$ having n radical signs then by methods of mathematical induction which is true

(a) $a_n > 7,\ \forall\ n \geq 1$ (b) $a_n > 3,\ \forall\ n \geq 1$

(c) $a_n < 4,\ \forall\ n \geq 1$ (d) $a_n < 3,\ \forall\ n \geq 1$.

(2002)

2. Let $S(k) = 1 + 3 + 5 + ... + (2k - 1) = 3 + k^2$. Then which of the following is true?

(a) $S(k) \Rightarrow S(k-1)$

(b) $S(k) \Rightarrow S(k+1)$

(c) $S(1)$ is correct

(d) principle of mathematical induction can be used to prove the formula.

(2004)

Directions : Question number 3 is Assertion-Reason type. This question contains two statements : Statement-1 (Assertion) and Statement-2 (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

(a) Statement-1 is true, Statement-2 is false

(b) Statemen-1 is false, Statement-2 is true

(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

3. **Statement-1 :** For every natural number $n \geq 2$,

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+...+\frac{1}{\sqrt{n}} > \sqrt{n}.$$

Statement-2 : For every natural number $n \geq 2$,

$$\sqrt{n(n+1)} < n+1.$$

(2008)

Answer Key

1.	(b)	**2.**	(b)	**3.**	(d)

1. **(b) :** $a_n = \sqrt{7 + a_n}$

$\Rightarrow a_n^2 - a_n - 7 = 0$

$\therefore\ a_n = \dfrac{1 \pm \sqrt{1+28}}{2}$

$= \dfrac{1 \pm \sqrt{29}}{2}$

> 3

2. **(b) :** $S(k) = 1 + 3 + ... + (2k - 1) = 3 + k^2$ ...(i)

When $k = 1$, L.H.S of $S(k) \neq$ R.H.S of $S(k)$

So $S(1)$ is not true.

Now $S(k + 1)$; $1 + 3 + 5 + ... + (2k - 1) + (2k + 1)$

$= 3 + (k + 1)^2$ ...(ii)

Let $S(k)$ is true $\therefore\ 1 + 3 + 5 + ... + (2k - 1) = k^2 + 3$

$\Rightarrow 1 + 3 + 5 + ... + (2k - 1) + (2k + 1)$

$= 3 + k^2 + 2k + 1 = (k + 1)^2 + 3$

$\Rightarrow S(k + 1)$ true $\quad \therefore\ S(k) \Rightarrow S(k + 1)$

3. **(d) :** Statement-1

Let $P(n): \dfrac{1}{\sqrt{1}} + \dfrac{1}{\sqrt{2}} + ... + \dfrac{1}{\sqrt{n}} > \sqrt{n}$

Step 1 : For $n = 2$, $P(2): \dfrac{1}{\sqrt{1}} + \dfrac{1}{\sqrt{2}} > \sqrt{2}$ is true

Step 2 : Assume $P(n)$ is true for $n = k$, i.e.

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{k}} > \sqrt{k} \quad ...(i)$$

Step 3 : For $n = k + 1$, we have to show that

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1} \quad ...(ii)$$

By Assumption step, we get

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{k}} > \sqrt{k}$$

Adding $\dfrac{1}{\sqrt{k+1}}$ on both sides, we get

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \sqrt{k} + \frac{1}{\sqrt{k+1}} \quad ...(iii)$$

Statement-2

For $n = k$

$\sqrt{k(k+1)} < k + 1$

$\Rightarrow \sqrt{k}\sqrt{k+1} < \sqrt{k+1}\sqrt{k+1} \Rightarrow \sqrt{k} < \sqrt{k+1}$

$\because\ \sqrt{k+1} > \sqrt{k}$ For $k \geq 2$

$\Rightarrow 1 > \dfrac{\sqrt{k}}{\sqrt{k+1}} \Rightarrow \sqrt{k} > \dfrac{k}{\sqrt{k+1}}$

multiplyingby $\sqrt{k}$

$\Rightarrow \sqrt{k} > \dfrac{(k+1)-1}{\sqrt{k+1}} \Rightarrow \sqrt{k} > \sqrt{k+1} - \dfrac{1}{\sqrt{k+1}}$

$$\Rightarrow \sqrt{k} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1} \quad ...(iv)$$

From (iii) & (iv)

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$$

hence (ii) is true for $n = k + 1$

hence $P(n)$ is true for $n \geq 2$

So, Statement-1 and Statement-2 are correct but Statement-2 is not explanation of Statement-1

CHAPTER

7

Binomial Theorem

1. The positive integer just greater than $(1 + .0001)^{10000}$ is

(a) 4 (b) 5 (c) 2 (d) 3.

(2002)

2. r and n are positive integers $r > 1$, $n > 2$ and coefficient of $(r + 2)^{\text{th}}$ term and $3r^{\text{th}}$ term in the expansion of $(1 + x)^{2n}$ are equal, then n equals

(a) $3r$ (b) $3r + 1$ (c) $2r$ (d) $2r + 1$.

(2002)

3. The coefficients of x^p and x^q in the expansion of $(1 + x)^{p+q}$ are

(a) equal

(b) equal with opposite signs

(c) reciprocals of each other

(d) none of these.

(2002)

4. If the sum of the coefficients in the expansion of $(a + b)^n$ is 4096, then the greatest coefficient in the expansion is

(a) 1594 (b) 792 (c) 924 (d) 2924.

(2002)

5. If x is positive, the first negative term in the expansion of $(1 + x)^{27/5}$ is

(a) 5^{th} term (b) 8^{th} term

(c) 6^{th} term (d) 7^{th} term.

(2003)

6. The number of integral terms in the expansion of $(\sqrt{3}+\sqrt[8]{5})^{256}$ is

(a) 33 (b) 34 (c) 35 (d) 32.

(2003)

7. The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same if α equals

(a) $-3/10$ (b) $10/3$

(c) $-5/3$ (d) $3/5$.

(2004)

8. The coefficient of x^n in expansion of $(1 + x)(1 - x)^n$ is

(a) $(-1)^{n-1}(n - 1)^2$ (b) $(-1)^n(1 - n)$

(c) $(n - 1)$ (d) $(-1)^{n-1} n$.

(2004)

9. If $s_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$, then $\frac{t_n}{s_n}$ is equal to

(a) $n - 1$ (b) $\frac{1}{2}n - 1$

(c) $\frac{1}{2}n$ (d) $\frac{2n-1}{2}$.

(2004)

10. If the coefficient of x^7 in $\left[ax^2 + \left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation

(a) $a + b = 1$ (b) $a - b = 1$

(c) $ab = 1$ (d) $\frac{a}{b} = 1$.

(2005)

11. If x is so small that x^3 and higher powers of x may be neglected, then $\dfrac{(1+x)^{3/2}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$ may be approximated as

(a) $3x+\frac{3}{8}x^2$ (b) $1-\frac{3}{8}x^2$

(c) $\frac{x}{2}-\frac{3}{8}x^2$ (d) $-\frac{3}{8}x^2$.

(2005)

12. The sum of the series

$1+\frac{1}{4\cdot 2!}+\frac{1}{16\cdot 4!}+\frac{1}{64\cdot 6!}+ .... \infty$ is

(a) $\frac{e+1}{\sqrt{e}}$ (b) $\frac{e-1}{\sqrt{e}}$ (c) $\frac{e+1}{2\sqrt{e}}$ (d) $\frac{e-1}{2\sqrt{e}}$.

(2005)

13. If the expansion in powers of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 + ...$, then a_n is

(a) $\frac{b^n-a^n}{b-a}$ (b) $\frac{a^n-b^n}{b-a}$

(c) $\frac{a^{n+1}-b^{n+1}}{b-a}$ (d) $\frac{b^{n+1}-a^{n+1}}{b-a}$.

(2006)

14. For natural numbers m, n if $(1 - y)^m (1 + y)^n = 1 + a_1y + a_2y^2 + ...$, and $a_1 = a_2 = 10$, then (m, n) is

(a) (20, 45) (b) (35, 20)

(c) (45, 35) (d) (35, 45).

(2006)

15. In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of 5th and 6th terms is zero, then a/b equals

(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$

(c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$.

(2007)

Directions : Question number 16 is Assertion-Reason type. This question contains two statements : Statement-1 (Assertion) and Statement-2 (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

(a) Statement-1 is true, Statement-2 is false

(b) Statemen-1 is false, Statement-2 is true

(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

16. Statement-1 : $\sum_{r=0}^{n}(r+1)\,{}^nC_r = (n+2)2^{n-1}$

Statement-2 : $\sum_{r=0}^{n}(r+1)\,{}^nC_r\, x^r = (1+x)^n + nx(1+x)^{n-1}$

(2008)

Answer Key

1.	(c)	**2.**	(c)	**3.**	(a)	**4.**	(c)	**5.**	(d)	**6.**	(a)
7.	(a)	**8.**	(b)	**9.**	(c)	**10.**	(c)	**11.**	(d)	**12.**	(c)
13.	(d)	**14.**	(d)	**15.**	(b)	**16.**	(c)				

1. **(c) :** Let $R = \left(1 + \frac{1}{10^4}\right)^{1000}$

$= 1 + 1000\left(\frac{1}{10^4}\right)^1 + 1000\frac{999}{2}\left(\frac{1}{10^4}\right)^2 + ...$

$+ \left(\frac{1}{10^4}\right)^{10^3}$

$< 1 + \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + ...\infty = \frac{10}{9}$

$\therefore R < \frac{10}{9}$

$\therefore$ The number just greater than $\frac{10}{9}$ is 2.

2. **(c) :** Given $r > 1$, $n > 2$ and
Coefficient of
T_{r+2} = Coefficient of T_{3r} in $(1 + x)^{2n}$
$\Rightarrow {}^{2n}C_{r+1} = {}^{2n}C_{3r-1}$
$\Rightarrow 3r - 1 = r + 1$ and $\left\{\begin{array}{l} 2n - 3r + 1 = r + 1 \\ \Rightarrow 2n = 4r \\ n = 2r \end{array}\right.$

$2r = 2$
$r = 1$
$\left\{\begin{array}{l} \because {}^nC_x = {}^nC_y \\ \Rightarrow x + y = n \\ \text{or } x = y \end{array}\right.$

3. **(a) :** In the expansion of $(1 + x)^{p+q}$
$T_{r+1} = {}^{p+q}C_r x^r$
$\therefore$ Coefficient of $x^p = {}^{p+q}C_p$

$= \frac{(p+q)!}{p!(p+q-p)!} = \frac{(p+q)!}{p!q!}$ ...(i)

Also coefficient of x^q in $(1 + x)^{p+q}$ is
$= {}^{p+q}C_q$

$= \frac{(p+q)!}{q!(p+q-q)!}$

$= \frac{(p+q)!}{q!p!}$ ....(ii)

$\therefore$ By (i) and (ii)

Coefficient of x^p in $(1 + x)^{p+q}$ = Coefficient of x^q in $(1 + x)^{p+q}$

4 **(c) :** Consider $(a + b)^n = C_0 a^n + C_1 a^{n-1}b + C_2 a^{n-2}b^2 + ... + C_n b^n$
Putting $a = b = 1$
$\therefore \quad 2^n = C_0 + C_1 + C_2 + ... + C_n$
$2^n = 4096 = 2^{12}$
$\Rightarrow \quad n = 12$ (even)
Now $(a + b)^n = (a + b)^{12}$
as $n = 12$ is even so coefficient of greatest term is

${}^nC_{\frac{n}{2}} = {}^{12}C_{\frac{12}{2}} = {}^{12}C_6$

$= \frac{12}{6} \times \frac{11}{5}\cdot\frac{10}{4}\frac{9}{3}\cdot\frac{8}{2}\cdot\frac{7}{1}$

$= \frac{11 \times 9 \cdot 8 \cdot 7}{3 \cdot 2 \cdot 1}$

$= 11 \times 3 \cdot 4 \cdot 7 = 924$

5. **(d) :** General term in the expansion of $(1 + x)^{\frac{27}{5}}$

$T_{r+1} = \frac{n(n-1)\ .....\ (n-r+1)}{r!}x^r$

$\therefore n - r + 1 < 0$

$\Rightarrow \frac{27}{5} + 1 < r$

$\Rightarrow r > \frac{32}{5}$

$r > 6$

6. **(a) :** $(3^{1/2} + 5^{1/8})^{256}$

$T_{r+1} = {}^{256}C_r\ (\sqrt{3})^{256-r} 5^{\frac{r}{8}}$

For integral terms $\frac{256-r}{2}, \frac{r}{8}$ are both positive integer

$\therefore r = 0, 8, 16, ...256$

$\therefore 256 = 0 + (n - 1)8$ using $t_n = a + (n - 1)d$

$\therefore \frac{256}{8} = n - 1$

$\therefore n = \frac{256}{8} + 1$

$n = 32 + 1 \Rightarrow n = 33$

7. **(a) :** Coefficient of middle term in $(1 + \alpha x)^4$ = coefficient of middle term in $(1 - \alpha x)^6$

$\therefore\ {}^4C_2\alpha^2 = {}^6C_3(-\alpha)^3$

$\Rightarrow \alpha = -\dfrac{3}{10}$

8. **(b):** $(1 + x)(1 - x)^n = (1 - x)^n + x(1 - x)^n$

$\therefore$ Coefficient of x^n is $= (-1)^n + (-1)^{n-1}\,{}^nC_1$

$= (-1)^n [1 - n]$

9. **(c):** $t_n = \displaystyle\sum_{r=0}^{n} \frac{r}{{}^nC_r}$

$$t_n = \sum_{r=0}^{n} \frac{n-(n-r)}{{}^nC_{n-r}}$$

$$t_n = n\sum_{r=0}^{n} \frac{1}{{}^nC_r} - \sum_{r=0}^{n} \frac{n-r}{{}^nC_{n-r}}$$

$$t_n = n\sum \frac{1}{{}^nC_r} - \sum_{r=0}^{n} \frac{r}{{}^nC_r} \quad \text{replacing } n-r \text{ by } r$$

$t_n = ns_n - t_n$

$\therefore\ \dfrac{t_n}{s_n} = \dfrac{n}{2}$

10. **(c) :** T_{r+1} of $\left(ax^2 + \dfrac{1}{bx}\right)^{11} = {}^{11}C_r(ax^2)^r\left(\dfrac{1}{bx}\right)^{11-r}$

T_{r+1} of $\left(ax - \dfrac{1}{bx^2}\right)^{11} = {}^{11}C_r(ax)^r\left(-\dfrac{1}{bx^2}\right)^{11-r}$

$\therefore$ Coeff. of x^7 in $\left(ax^2 + \dfrac{1}{bx}\right)^{11} = {}^{11}C_5\dfrac{a^6}{b^5}$

and coefficient of x^{-7} in $\left(ax - \dfrac{1}{bx^2}\right)^{11} = {}^{11}C_6\dfrac{a^5}{b^6}$

Now ${}^{11}C_5\dfrac{a^6}{b^5} = {}^{11}C_6\dfrac{a^5}{b^6} \quad \therefore\ ab = 1.$

11. **(d) :** $\dfrac{(1+x)^{3/2} - \left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$

$$= \frac{\left(1+\frac{3}{2}x+\frac{3}{2}\frac{1}{2}\frac{1}{2!}x^2+...\right)-\left(1+3\cdot\frac{1}{2}x+\frac{3\cdot 2}{2!}\cdot\frac{1}{4}x^2+....\right)}{(1-x)^{1/2}}$$

$$= -\frac{3}{8}x^2(1-x)^{-1/2} = -\frac{3}{8}x^2\left[1+\frac{1}{2}x+\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{1}{2!}x^2+...\right]$$

$= -\dfrac{3}{8}x^2 +$ higher powers of x^2.

12. **(c) :** $1 + \dfrac{1}{4(2!)} + \dfrac{1}{16(4!)} + \dfrac{1}{64(6!)} + ...\infty$

$$= 1 + \frac{1}{2^2\,2!} + \frac{1}{2^4(4!)} + \frac{1}{2^6(6!)} + ...\infty$$

$$= \frac{1}{2}\left[2\left(1 + \frac{1}{2^2\,2!} + \frac{1}{2^4(4!)} + \frac{1}{2^6(6!)} + ...\infty\right)\right]$$

$$= \frac{1}{2}\left[2\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + ...\infty\right)\right]$$

$= \dfrac{1}{2}\left[e^x + e^{-x}\right]$ where $x = 1/2$

$$= \frac{1}{2}\left[e^{1/2} + e^{-1/2}\right] = \frac{e+1}{2\sqrt{e}}$$

13. **(d) :** From given

$$\frac{1}{(1-ax)(1-bx)} = (1-ax)^{-1}(1-bx)^{-1}$$

$= (a_0 + a_1x + ... + a_nx^n + ...)\,(1 - bx)^{-1}$

$= (1 + ax + a^2x^2 + ... + a^{n-1}x^{n-1} + a^nx^n + ...)\,(1 + bx + b^2x^2\ .... + b^nx^n + ...)$

$\Rightarrow\ (a_0 + a_1x + ... + a_nx^n + ...)$

$= 1 + x(a + b) + x^2(a^2 + ab + b^2) + x^3(a^3 + a^2b + ab^2 + b^3) + ... + ... + ... + x^n(a^n + a^{n-1}b + a^{n-2}b^2 + ... + ab^{n-1} + b^n) + ...$

On comparing the coefficient of x^n both sides we have

$a^n = a^n + a^{n-1}b + a^{n-2}b^2 + ... + a\,b^{n-1} + b^n$

$$= \frac{(a^n + a^{n-1}b + a^{n-2}b^2 + ... + ab^{n-1} + b^n)(b-a)}{b-a}$$

(Multiplying and dividing by $b - a$)

$$= \frac{b^{n+1} - a^{n+1}}{b-a}.$$

14. **(d) :** $(1 - y)^m (1 + y)^n$

$= 1 + a_1y + a_2y_2 + a_3y^3 + ... + \quad ...... (*)$

Differentiating w.r. to y both sides of (*) we have

$-m(1 - y)^{m-1}(1 + y)^n + (1 - y)^m\,n(1 + y)^{n-1}$

$= a_1 + 2a_2y + 3a_3y^2 + 4a_4y^3 : ....$

$\Rightarrow \quad n(1+y)^{n-1}(1-y)^m - m(1-y)^{m-1}(1+y)^n$
$= a_1 + 2a_2y + 3a_2y^2 + 4a_4y^3 + ...$ .....(**)
Again differentiating (**) with respect to y we have
$[n(n-1)(1+y)^{n-2}(1-y)^m + n(1+y)^{n-1}(-m)$
$(1-y)^{m-1}]$
$-[m(1+y)^n(m-1)(1-y)^{m-2}(1-y)^{m-1}n(1+y)+n-1]$
$= 2a_2 + 6a_3y + ...$ .... (***)
Now putting $y = 0$ in (**) and (***) we get
$n - m = a_1 = 10$ (A)
and $m^2 + n^2 - (m+n) - 2\,mn = 2a_2 = 20$ ....(B)
Solving (A) and (B)
$n = 45,\ m = 35$
$\therefore \quad (m, n) = (35, 45)$

15. (b) : ${}^nC_4\,a^{n-4}(-b)^4 = -\left({}^nC_5\,a^{n-5}(-b)^5\right)$
$\Rightarrow \dfrac{a}{b} = \dfrac{n-4}{5}.$

16. (c) : $\sum_{r=0}^{n}(r+1)\,{}^nC_r = \sum_{r=0}^{n} r\cdot{}^nC_r + \sum_{r=0}^{n}{}^nC_r$
$= \sum_{r=0}^{n} r\cdot\frac{n}{r}\cdot{}^{n-1}C_{r-1} + \sum_{r=0}^{n}{}^nC_r$
$= n\cdot 2^{n-1} + 2^n = 2^{n-1}(n+2)$
Thus Statement-1 is true.
Again $\sum_{r=0}^{n}(r+1)\,{}^nC_r x^r = \sum_{r=0}^{n} r\cdot{}^nC_r x^r + \sum{}^nC_r x^r$
$= n\sum_{r=0}^{n}{}^{n-1}C_{r-1}x^r + \sum_{r=0}^{n}{}^nC_r x^r$
$= nx(1+x)^{n-1} + (1+x)^n$
Substitute $x = 1$ in the above identity to get
$\sum(r+1)\,{}^nC_r = n\cdot 2^{n-1} + 2^n$
Statement-2 is also true & explains Statement-1 also.

Sequences and Series

1. If 1, $\log_9(3^{1-x}+2)$, $\log_3[4\cdot 3^x - 1]$ are in AP. then x equals
(a) $\log_3 4$ (b) $1-\log_3 4$
(c) $1-\log_4 3$ (d) $\log_4 3$. *(2002)*

2. $1^3-2^3+3^3-4^3+\ldots+9^3=$
(a) 425 (b) -425 (c) 475 (d) -475. *(2002)*

3. Sum of infinite number of terms in GP is 20 and sum of their square is 100. The common ratio of GP is
(a) 5 (b) 3/5 (c) 8/5 (d) 1/5. *(2002)*

4. The value of $2^{1/4}\cdot 4^{1/8}\cdot 8^{1/6}\ldots\infty$ is
(a) 1 (b) 2 (c) 3/2 (d) 4. *(2002)*

5. Fifth term of an GP is 2, then the product of its 9 terms is
(a) 256 (b) 512 (c) 1024 (d) none. *(2002)*

6. If the system of linear equations $x+2ay+az=0$, $x+3by+bz=0$, $x+4cy+cz=0$ has a non-zero solution, then a, b, c
(a) are in G.P.
(b) are in H.P.
(c) satisfy $a+2b+3c=0$
(d) are in A.P. *(2003)*

7. Let $f(x)$ be a polynomial function of second degree. If $f(1)=f(-1)$ and a, b, c are in A.P., then $f'(a)$, $f'(b)$ and $f'(c)$ are in
(a) G.P.
(b) H.P.
(c) Arithmetic-Geometric Progression
(d) A.P. *(2003)*

8. The sum of the series
$$\frac{1}{1\cdot 2}-\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}-\ldots \text{ upto } \infty$$
is equal to
(a) $\log_e 2-1$ (b) $\log_e 2$
(c) $\log_e(4/e)$ (d) $2\log_e 2$. *(2003)*

9. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3)
(a) lie on an ellipse
(b) lie on a circle
(c) are vertices of a triangle
(d) lie on a straight line. *(2003)*

10. Let R_1 and R_2 respectively be the maximum ranges up and down on an inclined plane and R be the maximum range on the horizontal plane. Then, R_1, R, R_2 are in
(a) A.P.
(b) G.P.
(c) H.P.
(d) Arithmetic-Geometric Progression (A.G.P.). *(2003)*

11. Let T_r be the r^{th} term of an A.P. whose first term is a and common difference is d. If for some positive integers $m, n, m\neq n$, $T_m=\frac{1}{n}$, and $T_n=\frac{1}{m}$, then $a-d$ equals
(a) $1/mn$ (b) 1
(c) 0 (d) $\frac{1}{m}+\frac{1}{n}$. *(2004)*

12. The sum of first n terms of the series $1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + ...$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is

(a) $\frac{n(n+1)^2}{4}$ (b) $\frac{n^2(n+1)}{2}$

(c) $\frac{3n(n+1)}{2}$ (d) $\left[\frac{n(n+1)}{2}\right]^2$. *(2004)*

13. The sum of series $\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + ...$ is

(a) $\frac{(e-1)^2}{2e}$ (b) $\frac{(e^2-1)}{2e}$

(c) $\frac{(e^2-1)}{2}$ (d) $\frac{(e^2-2)}{e}$. *(2004)*

14. If the coefficients of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms in the binomial expansion of $(1 + y)^m$ are in A.P., then m and r satisfy the equation

(a) $m^2 - m(4r - 1) + 4r^2 + 2 = 0$

(b) $m^2 - m(4r + 1) + 4r^2 - 2 = 0$

(c) $m^2 - m(4r + 1) + 4r^2 + 2 = 0$

(d) $m^2 - m(4r - 1) + 4r^2 - 2 = 0$. *(2005)*

15. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$ then x, y, z are in

(a) HP

(b) Arithmetic-Geometric progression

(c) AP (d) GP. *(2005)*

16. If $a_1, a_2, a_3, ..., a_n, ...$ are in G.P., then the determinant

$$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix},$$ is equal to

(a) 0 (b) 1

(c) 2 (d) 4. *(2005)*

17. Let $a_1, a_2, a_3, ...$ be terms of an A.P. If $\frac{a_1 + a_2 + ... + a_p}{a_1 + a_2 + ... + a_q} = \frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals

(a) 41/11 (b) 7/2

(c) 2/7 (d) 11/41. *(2006)*

18. If $a_1, a_2, ..., a_n$ are in H.P., then the expression $a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$ is equal to

(a) $n(a_1 - a_n)$ (b) $(n - 1)(a_1 - a_n)$

(c) na_1a_n (d) $(n - 1)a_1a_n$. *(2006)*

19. The sum of the series $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - ....$ upto infinity is

(a) $e^{-\frac{1}{2}}$ (b) $e^{+\frac{1}{2}}$

(c) e^{-2} (d) e^{-1}. *(2007)*

20. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression is equals

(a) $\sqrt{5}$ (b) $\frac{1}{2}(\sqrt{5}-1)$

(c) $\frac{1}{2}(1-\sqrt{5})$ (d) $\frac{1}{2}\sqrt{5}$. *(2007)*

21. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is

(a) 4 (b) – 4

(c) – 12 (d) 12

(2008)

Answer Key

1.	(c)	**2.**	(a)	**3.**	(b)	**4.**	(b)	**5.**	(b)	**6.**	(b)
7.	(d)	**8.**	(c)	**9.**	(d)	**10.**	(c)	**11.**	(c)	**12.**	(b)
13.	(a)	**14.**	(b)	**15.**	(a)	**16.**	(a)	**17.**	(d)	**18.**	(d)
19.	(d)	**20.**	(b)	**21.**	(c)						

1. **(c) :** As $1, \frac{1}{2}\log_3(3^{1-x}+2), \log_3(4\cdot3^x-1) \in AP$

$\Rightarrow \log_3(3^{1-x}+2) = \log_3(4\cdot 3^x - 1) + 1$

$\Rightarrow 3^{1-x}+2 = (4\cdot3^x - 1)\times 3 \because \log_3 3 = 1.$

$\Rightarrow 3^{1-x}+2 = 12\cdot3^x - 3$

$\Rightarrow 3^x[(3^{1-x})+2] = 12\cdot3^{2x} - 3\cdot3^x$

(multiplying 3^x both side)

$\Rightarrow 12t^2 - 5t - 3 = 0$ where $t = 3^x$

$\Rightarrow (3t+1)(4t-3) = 0$

$\Rightarrow t = -1/3, t = 3/4$

$\Rightarrow 3^x = -1/3$ which is not possible

and $t = \frac{3}{4}$

$\Rightarrow 3^x = \frac{3}{4}$

$\Rightarrow x\log_3 3 = \log_3 3 - \log_3 4$

(By taking logarithm at the base 3 both sides)

$\Rightarrow x = 1 - \log_3 4$

2. **(a) :** $(1^3+3^3+5^3+.....+9^3) - (2^3+4^3+6^3+8^3)$

$= (1^3+3^3+5^3+....+9^3) - 2^3(1^3+2^3+3^3+4^3)$

$= [1^3+3^3+.....+(2n-1)^3]_{n=\text{odd}=5} - 2^3[1^3 + 2^3 + ... + n^3]_{n=\text{even}=4}$

$= [2n(n+1)(n+2)(n+3) - 12n(n+1)(n+2) + 13n(n+1) - n]_{n=5\,(\text{odd})} - 2^3\left[\frac{n^2(n+1)^2}{4}\right]_{\substack{n=4\\(\text{even})}}$

(Remember this result)

$= [2\times5\times6\times7\times8 - 12\times5\times6\times7 + 13 \times 5\times 6 - 5] - 2^3\left(\frac{16\times25}{4}\right)$

$= [3750 - 5(505)] - 2\times16\times25$

$= 1225 - 800 = 425$

3. **(b) :** Let terms of G.P are $a, ar, ar^2, ...$

$\therefore S_\infty = \frac{a}{1-r}$ where a = first term,

r = common ratio

$s_\infty = 20$

According to question $\frac{a}{1-r} = 20$

$\Rightarrow a = 20(1-r)$ ...(i)

also $\frac{a^2}{1-r^2} = 100$

$\Rightarrow \frac{a}{1-r}\cdot\frac{a}{1+r} = 100$

$\Rightarrow a = 5(1+r)$ ...(ii)

Solving (i) and (ii) we have $r = 3/5$

4. **(b) :** $S_\infty = 2^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+...\infty} = 2^\lambda(\text{say})$ ...(*)

Where $\lambda = \frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32} + ...\infty$ ... (*A*)

$\frac{\lambda}{2} = 0 + \frac{1}{8}+\frac{2}{16}+\frac{3}{32}+\frac{4}{64} + ...\infty$ ... (*B*)

Now $(B)-(A) \Rightarrow \frac{\lambda}{2} = \frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+...$

$\frac{\lambda}{2} = \frac{a}{1-r} = \frac{1}{4}\times\frac{2}{1} \therefore \lambda = 1$

so $S_\infty = 2^1$

5. **(b) :** Let first term of an G.P is a and common ratio r

$\therefore t_5 = ar^4 = 2$

$\therefore \prod_{i=1}^{9} a_i = a\cdot ar\, ar^2 .....ar^8$

$= a^9 r^{\frac{8\times9}{2}}$

$= a^9r^{36}$

$= (ar^4)^9$

$= 2^9 = 512$

6. **(b) :** For non trivial solution the determinant of the coefficient of various term vanish

i.e. $\begin{vmatrix}1 & 2a & a\\ 1 & 3b & b\\ 1 & 4c & c\end{vmatrix} = 0$

$\Rightarrow (3bc - 4bc) - 2a(c-b) + a(4c-3b) = 0$

$\Rightarrow \frac{2ac}{a+c} = b$

$\Rightarrow a, b, c \in$ H.P

7. **(d) :** Let the polynomial be $f(x) = ax^2 + bx + c$
given $f(1) = f(-1) \Rightarrow b = 0$
$\therefore f(x) = ax^2 + c$
now $f'(x) = 2ax$
$\therefore f'(a) = 2a^2, f'(b) = 2ab, f'(c) = 2ac$
as $a, b, c \in$ A.P
$\Rightarrow a^2, ab, ac \in$ A.P
$\Rightarrow 2a^2, 2ab, 2ac \in$ A.P
$\Rightarrow f'(a), f'(b), f'(c) \in$ A.P

8. **(c):** $s = \frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - ....\infty$

Let $s_1 = \frac{1}{1\cdot 2} + \frac{1}{3\cdot 4} + \frac{1}{5\cdot 6} + ...\infty$

$\therefore t_n = \frac{1}{(2n-1)(2n)} = \frac{1}{2n-1} - \frac{1}{2n}$

$\therefore s_n = \Sigma t_n = \sum\left(\frac{1}{2n-1} - \frac{1}{2n}\right)$

$= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}$ .....∞

$= \log_e 2$ ...(A)

Again $s_2 = \frac{1}{2\cdot 3} + \frac{1}{4\cdot 5} + \frac{1}{6\cdot 7} + ...\infty$

$t'_n = \frac{1}{(2n)(2n+1)}$

$s_2 = \sum t_n' =$

$\sum \frac{1}{(2n)(2n+1)} = \sum\left(\frac{1}{2n} - \frac{1}{2n+1}\right)$

$= \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + ...\infty$

$= -\left[-\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + ...\infty\right]$

$= -[\log_e 2 - 1]$
$= 1 - \log_e 2$ ...(B)
Now $s = s_1 - s_2 = (A) - (B)$
$= \log_e 2 - 1 + \log_e 2 = \log(4/e)$

9. **(d) :** Let $x_1 = a \therefore x_2 = ar, x_3 = ar^2$
and $y_1 = b \therefore y_2 = br, y_3 = br^2$
Now $A(a, b), B(ar, br), C(ar^2, br^2)$

Now slope of $AB = \frac{b(1-r)}{a(1-r)} = \frac{b}{a}$ and

slope of $BC = \frac{br(1-r)}{ar(1-r)} = \frac{b}{a}$

as slope of AB = slope of BC
$\therefore AB \parallel BC$, but point B is common so
A, B, C are collinear.

10. **(c) :** Let θ be the angle of inclination of plane to horizontal and u be the velocity of projection of the projectile

$\therefore \quad R_1 = \frac{u^2}{g(1+\sin\theta)}, R_2 = \frac{u^2}{g(1-\sin\theta)}$

$\therefore \quad \frac{1}{R_1} + \frac{1}{R_2} = \frac{2g}{u^2} = \frac{2}{R}$

$\Rightarrow \quad R_1, R_1, R_2 \in H.P.$

11. **(c) :** $T_m = a + (m-1)d = \frac{1}{n}$ ...(i)

$T_n = a + (n-1)d = \frac{1}{m}$ ...(ii)

Now $T_m - T_n = \frac{1}{n} - \frac{1}{m} = (m-n)d$

$\Rightarrow d = \frac{1}{mn}$ and $a = \frac{1}{mn}$

$\therefore a - d = 0$

12. **(b):** As S_n is needed for n is odd let $n = 2k + 1$
$\therefore S_n = S_{2k+1}$
= Sum up to $2k$ terms + $(2k+1)^{\text{th}}$ term

$= \frac{2k(2k+1)^2}{2}$ + last term

$= \frac{(n-1)n^2}{2} + n^2$ as $n = 2k + 1$

$= \frac{n^2(n+1)}{2}$

13. **(a) :** $e^x + e^{-x} = 2\left[1 + \frac{x^2}{2!} + \frac{x^4}{4!} + ...\infty\right]$

$\frac{e+e^{-1}}{2} - 1 = \frac{1}{2!} + \frac{1}{4!} + ...\infty$

$\frac{(e-1)^2}{2e} = \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + ...\infty$

14. **(b) :** $T_{r+1} = {}^mC_r\, y^r$. $\therefore {}^mC_{r-1} + {}^mC_{r+1} = 2 \times {}^mC_r$

$\Rightarrow \frac{m!}{(r-1)!(m-r+1)!} + \frac{m!}{(r+1)!(m-r-1)!}$
$= 2\frac{m!}{r!(m-r)!}$

$$\Rightarrow \frac{r(r+1)}{(r+1)!(m-r+1)!}+\frac{(m-r+1)(m-r)}{(r+1)!(m-r+1)!}$$

$$=\frac{2(r+1)(m-r+1)}{(r+1)!(m-r+1)!}$$

$\Rightarrow \quad r(r+1)+(m-r+1)(m-r)$

$= 2(r+1)(m-r+1)$

$\Rightarrow r(r+1)+(m-r)^2+m-r-2(r+1)$

$(m-(r-1))=0$

$\Rightarrow r(r+1)+m^2+r^2-2mr+m-r+2$

$(r^2-1)-2m(r+1)=$

0

$\Rightarrow m^2 \; - \; m(4r+1)+4r^2-2=0.$

15. (a) : Given $|a|<1$, $|b|<1$, $|c|<1$,

$a, b, c \in$ AP

and

$$\sum_{n=0}^{\infty} a^n=\frac{1}{1-a},\ \sum_{n=0}^{\infty} b^n=\frac{1}{1-b},\ \sum_{n=0}^{\infty} c^n=\frac{1}{1-c}$$

$$\therefore\ x=\frac{1}{1-a},\ y=\frac{1}{1-b},\ z=\frac{1}{1-c}$$

$$\Rightarrow a=\frac{x-1}{x},\ b=\frac{y-1}{y},\ c=\frac{z-1}{z}$$

as $a, b, c \in$ AP $\therefore\ 2b=a+c$

$$2\left(\frac{y-1}{y}\right)=\frac{x-1}{x}+\frac{z-1}{z} \Rightarrow \frac{2}{y}=\frac{1}{x}+\frac{1}{z}$$

$\Rightarrow x, y, z \in$ H.P.

16. (a) : Let t_r denote the r^{th} term of G.P. with first term b and common ratio R

$$\therefore\ t_r=bR^{r-1}.\therefore\ \log r=\log b+(r-1)\log R$$

Now from given determinant we have

$$\begin{vmatrix} \log b+(r-1)\log R & \log b+r\log R & \log b+(r+1)\log R \\ \log b+(r+2)\log R & \log b+(r+3)\log R & \log b+(r+4)\log R \\ \log b+(r+5)\log R & \log b+(r+6)\log R & \log b+(r+7)\log R \end{vmatrix}$$

$\Rightarrow$ using (applying $C_2 \to 2C_2 - \ (C_1+C_3)$)

$$=\frac{1}{2}\begin{vmatrix} \log b+(r-1)\log R & 0 & \log b+\log R(r+1) \\ \log b+(r+2)\log R & 0 & \log b+(r+4)\log R \\ \log b+(r+5)\log R & 0 & \log b+(r+7)\log R \end{vmatrix}$$

$$=\frac{1}{2}\times 0=0.$$

17. (d) : Given $a_1, a_2, a_3, \ldots$ be terms of A.P.

$$\frac{a_1+a_2+\ldots.a_p}{a_1+a_2+\ldots a_q}=\frac{p^2}{q^2}$$

$$\Rightarrow \frac{\frac{p}{2}[2a_1+(p-1)d]}{\frac{q}{2}[2a_1+(q-1)d]}=\frac{p^2}{q^2}$$

$$\Rightarrow \frac{2a_1+(p-1)d}{2a_1+(q-1)d}=\frac{p}{q}$$

$\Rightarrow \quad [2a_1+(p-1)d]q=p[2a_1+(q-1)d]$

$\Rightarrow \quad 2a_1(q-p)=d[(q-1)p-(p-1)q]$

$\Rightarrow \quad 2a_1(q-p)=d(q-p) \Rightarrow 2a_1=d$

$$\Rightarrow \frac{a_6}{a_{21}}=\frac{a_1+5d}{a_1+20d}=\frac{a_1+10a_1}{a_1+40a_1}$$

$$\Rightarrow \frac{a_6}{a_{21}}=\frac{11}{41}$$

18. (d) : Given $a_1, a_2, \ldots a_n$ are in H.P.

$$\Rightarrow \frac{1}{a_1}, \frac{1}{a_2} \ldots.. \frac{1}{a_n} \in \text{A.P.}$$

$$\Rightarrow \frac{1}{a_2}-\frac{1}{a_1}=d$$

$$\Rightarrow a_1a_2=\frac{a_1-a_2}{d}=\frac{a_1}{d}-\frac{a_2}{d} \qquad \ldots \text{(i)}$$

$$a_2a_3=\frac{a_2}{d}-\frac{a_3}{d} \qquad \ldots \text{(ii)}$$

$\vdots$

$$a_{n-1}a_n=\frac{a_{n-1}}{d}-\frac{a_n}{d} \qquad \ldots (n)$$

Adding (i), (ii) .... (n) equations we get

$$a_1a_2+a_2a_3+a_3a_4+\ldots.\ a_{n-1}a_n=\frac{a_1}{d}-\frac{a_n}{d}$$

Also $\dfrac{1}{a_n}=\dfrac{1}{a_1}+(n-1)d$

$$\Rightarrow \frac{a_1-a_n}{d}=(n-1)a_1a_n$$

$\therefore \quad a_1a_2+a_2a_3+\ldots a_{n-1}a_n=(n-1)a_1a_n.$

19. (d) : $\because\ e^{-x}=1-x+\dfrac{x^2}{2!}-\dfrac{x^3}{3!}+\dfrac{x^4}{4!}-\ldots\ldots..$

upto infinity

Then put $x = 1$, we get

$$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \text{........ upto infinity.}$$

20. (b) : Given, $a = ar + ar^2 \Rightarrow r^2 + r - 1 = 0$

$$\Rightarrow r = \frac{-1+\sqrt{5}}{2}.$$

21. **(c) :** Let the GP be $a, ar, ar^2, ar^3, \ldots$

we have $a + ar = 12$ ...(1)

$ar^2 + ar^3 = 48$ ...(2)

on division we have

$$\frac{ar^2(1+r)}{a(1+r)} = \frac{48}{12} \Rightarrow r^2 = 4$$

$\therefore \quad r = \pm 2$

But the terms are alternately positive and negative,

$\therefore r = -2$

Now $a = \frac{12}{1+r} = \frac{12}{1-2} = \frac{12}{-1} = -12$ From (1)

CHAPTER
9

Differential Calculus

1. $\underset{x\to 0}{Lim}\dfrac{\log x^n-[x]}{[x]}$, $n \in N$, ([x] denotes greatest integer less than or equal to x)
(a) has value – 1 (b) has value 0
(c) has value 1 (d) does not exist
(2002)

2. f is defined in [–5, 5] as
$$f(x)=\begin{cases}x, \text{ if } x \text{ is rational and}\\ -x, \text{ if } x \text{ is rational. Then}\end{cases}$$
(a) $f(x)$ is continuous at every x, except $x = 0$
(b) $f(x)$ is discontinuous at every x, except $x = 0$
(c) $f(x)$ is continuous everywhere
(d) $f(x)$ is discontinuous everywhere.
(2002)

3. $f(x)$ and $g(x)$ are two differentiable function on [0, 2] such that $f''(x)-g''(x)=0$, $f'(1)=2g'(1)=4$, $f(2)=3g(2)=9$ then $f(x)-g(x)$ at $x = 3/2$ is
(a) 0 (b) 2 (c) 10 (d) 5.
(2002)

4. If $f(1) = 1$, $f'(1) = 2$, then $\underset{x\to 1}{\text{Lt}}\dfrac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ is
(a) 2 (b) 4 (c) 1 (d) 1/2.
(2002)

5. The maximum distance from origin of a point on the curve $x = a\sin t - b\sin\left(\dfrac{at}{b}\right)$
$y = a\cos t - b\cos\left(\dfrac{at}{b}\right)$, both $a, b > 0$ is
(a) $a - b$ (b) $a + b$
(c) $\sqrt{a^2+b^2}$ (d) $\sqrt{a^2-b^2}$.
(2002)

6. $\lim\limits_{x\to 0}\dfrac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is
(a) 1 (b) –1
(c) 0 (d) does not exist.
(2002)

7. If $f(x + y) = f(x)\cdot f(y)\ \forall\ x, y$ and $f(5) = 2$, $f'(0) = 3$, then $f'(5)$ is
(a) 0 (b) 1 (c) 6 (d) 2.
(2002)

8. $\lim\limits_{x\to\infty}\left(\dfrac{x^2+5x+3}{x^2+x+3}\right)^{\frac{1}{x}}$
(a) e^4 (b) e^2
(c) e^3 (d) 1.
(2002)

9. Let $f(2) = 4$ and $f'(2) = 4$ then
$\lim\limits_{x\to 2}\dfrac{x f(2)-2f(x)}{x-2}$ equals
(a) 2 (b) – 2
(c) – 4 (d) 3.
(2002)

10. If $2a + 3b + 6c = 0$ ($a, b, c \in R$) then the quadratic equation $ax^2 + bx + c = 0$ has
(a) At least one in (0, 1)
(b) At least one root in [2, 3]
(c) At least one root in [4, 5]
(d) none of these *(2002)*

11. If $\lim\limits_{x\to 0}\dfrac{\log(3+x)-\log(3-x)}{x}=k$, the value of k is
(a) –1/3 (b) 2/3 (c) –2/3 (d) 0.
(2003)

12. If $f(x) = x^n$, then the value of

$f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + ... + \frac{(-1)^n f^n(1)}{n!}$ is

(a) 2^{n-1} (b) 0 (c) 1 (d) 2^n.

(2003)

13. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its maximum and minumum at p and q respectively such that $p^2 = q$, then a equals

(a) 1 (b) 2 (c) 1/2 (d) 3.

(2003)

14. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then $f(x)$ is

(a) continuous for all x, but not differentiable at $x = 0$

(b) neither differentiable not continuous at $x = 0$

(c) discontinuous everywhere

(d) continuous as well as differentiable for all x.

(2003)

15. The real number x when added to its inverse gives the minimum value of the sum at x equal to

(a) 1 (b) –1 (c) –2 (d) 2.

(2003)

16. The value of

$$\lim_{n\to\infty} \frac{1+2^4+3^4+....+n^4}{n^5} - \lim_{n\to\infty} \frac{1+2^3+3^3+....+n^3}{n^5}$$

is

(a) zero (b) 1/4 (c) 1/5 (d) 1/30.

(2003)

17. $\lim_{x\to\pi/2} \frac{[1-\tan(x/2)][1-\sin x]}{[1+\tan(x/2)][\pi-2x]^3}$ is

(a) 0 (b) 1/32 (c) ∞ (d) 1/8.

(2003)

18. Let $f(a) = g(a) = k$ and their n^{th} derivatives $f^n(a)$, $g^n(a)$ exist and are not equal for some n. Further if

$$\lim_{x\to a} \frac{f(a)g(x) - f(a) - g(a)f(x) + g(a)}{g(x) - f(x)} = 4,$$

then the value of k is

(a) 2 (b) 1

(c) 0 (d) 4.

(2003)

19. If $\lim_{x\to\infty}\left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b, are

(a) $a \in \mathbf{R}$, $b = 2$ (b) $a = 1$, $b \in \mathbf{R}$

(c) $a \in \mathbf{R}$, $b \in \mathbf{R}$ (d) $a = 1$ and $b = 2$.

(2004)

20. Let $f(x) = \frac{1-\tan x}{4x-\pi}$, $x \neq \frac{\pi}{4}$, $x \in \left[0, \frac{\pi}{2}\right]$.

$f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) 1 (d) –1.

(2004)

21. A function $y = f(x)$ has a second order derivative $f''(x) = 6(x - 1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is

(a) $(x + 1)^3$ (b) $(x - 1)^3$

(c) $(x - 1)^2$ (d) $(x + 1)^2$.

(2004)

22. If $2a + 3b + 6c = 0$, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval

(a) (2, 3) (b) (1, 2) (c) (0, 1) (d) (1, 3).

(2004)

23. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h\to 0} \frac{1}{h} f(1+h) = 5$, then $f'(1)$ equals

(a) 4 (b) 3 (c) 6 (d) 5.

(2005)

24. Let f be the differentiable for $\forall\ x$. If $f(1) = -2$ and $f'(x) \geq 2$ for [1, 6], then

(a) $f(6) < 8$ (b) $f(6) \geq 8$

(c) $f(6) = 5$ (d) $f(6) < 5$.

(2005)

25. If f is a real-valued differentiable function satisfying $|f(x) - f(y)| \leq (x-y)^2$, $x, y \in R$ and $f(0) = 0$, then $f(1)$ equals

(a) 1 (b) 2 (c) 0 (d) – 1.

(2005)

26. The normal to the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ at any point θ is such that

(a) it makes angle $\frac{\pi}{2}+\theta$ with x-axis
(b) it passes through the origin
(c) it is at a constant distance from the origin
(d) it passes through $\left(a\frac{\pi}{2}, -a\right)$.

(2005)

27. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then $\lim_{x \to \alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to

(a) 0 (b) $\frac{a^2}{2}(\alpha-\beta)^2$
(c) $\frac{1}{2}(\alpha-\beta)^2$ (d) $\frac{-a^2}{2}(\alpha-\beta)^2$.

(2005)

28. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is

(a) $\frac{1}{18\pi}$ cm/min (b) $\frac{1}{36\pi}$ cm/min
(c) $\frac{5}{6\pi}$ cm/min (d) $\frac{1}{54\pi}$ cm/min.

(2005)

29. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is

(a) 1/4 (b) 41 (c) 1 (d) 17/7.

(2006)

30. The function $g(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at

(a) $x = 2$ (b) $x = -2$
(c) $x = 0$ (d) $x = 1$.

(2006)

31. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2, 0) and (3, 0) is

(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/6$ (c) $\pi/4$.

(2006)

32. The set of points where $f(x) = \frac{x}{1+|x|}$ is differentiable, is

(a) $(-\infty, 0) \cup (0, \infty)$
(b) $(-\infty, -1) \cup (-1, \infty)$
(c) $(-\infty, \infty)$
(d) $(0, \infty)$.

(2006)

33. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is

(a) $\frac{3}{2}x^2$ (b) $\sqrt{\frac{x^3}{8}}$
(c) $\frac{1}{2}x^2$ (d) πx^2.

(2006)

34. If $x^m \cdot y^n = (x + y)^{m+n}$, then dy/dx is

(a) $\frac{y}{x}$ (b) $\frac{x+y}{xy}$
(c) xy (d) $\frac{x}{y}$.

(2006)

35. A value of c for which conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3] is

(a) $\log_3 e$ (b) $\log_e 3$
(c) $2 \log_3 e$ (d) $\frac{1}{2}\log_e 3$.

(2007)

36. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(c) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$.

(2007)

37. Let $f : R \to R$ be a function defined by $f(x) = \min\{x + 1, |x| + 1\}$. Then which of the following is true ?

(a) $f(x)$ is differentiable everywhere
(b) $f(x)$ is not differentiable at $x = 0$
(c) $f(x) \geq 1$ for all $x \in R$
(d) $f(x)$ is not differentiable at $x = 1$.

(2007)

38. The function $f : R / \{0\} \to R$ given by

$$f(x)=\frac{1}{x}-\frac{2}{e^{2x}-1}$$

can be made continuous at $x = 0$ by defining $f(0)$ as

(a) 0 (b) 1

(c) 2 (d) -1.

(2007)

39. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p + q)$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{2}$ (d) 2.

(2007)

40. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have?

(a) 5 (b) 7

(c) 1 (d) 3 *(2008)*

41. Let $f(x)=\begin{cases}(x-1)\sin\frac{1}{x-1} & \text{if } x\neq 1\\ 0 & \text{if } x=1\end{cases}$

Then which one of the following is true?

(a) f is differentiable at $x = 1$ but not at $x = 0$

(b) f is neither differentiable at $x = 0$ nor at $x = 1$

(c) f is differentiable at $x = 0$ and at $x = 1$

(d) f is differentiable at $x = 0$ but not at $x = 1$

(2008)

42. Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$. Then which one of the following holds?

(a) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

(b) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$

(c) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$

(d) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

(2008)

Answer Key

1.	(d)	**2.**	(b)	**3.**	(d)	**4.**	(a)	**5.**	(a)	**6.**	(a)
7.	(c)	**8.**	(d)	**9.**	(c)	**10.**	(a)	**11.**	(b)	**12.**	(b)
13.	(b)	**14.**	(a)	**15.**	(a)	**16.**	(c)	**17.**	(b)	**18.**	(d)
19.	(b)	**20.**	(a)	**21.**	(b)	**22.**	(c)	**23.**	(d)	**24.**	(b)
25.	(c)	**26.**	(a), (c)	**27.**	(b)	**28.**	(a)	**29.**	(b)	**30.**	(a)
31.	(a)	**32.**	(c)	**33.**	(c)	**34.**	(a)	**35.**	(c)	**36.**	(d)
37.	(a)	**38.**	(b)	**39.**	(c)	**40.**	(c)	**41.**	(b)	**42.**	(b)

1. **(d) :** $\underset{x\to 0}{\text{Lt}} \dfrac{\log x^n - [x]}{[x]} = \underset{x\to 0}{\text{Lt}} \dfrac{n\log x}{[x]} - 1$

which does not exist as $\underset{x\to 0}{\text{Lt}} \dfrac{\log x}{[x]}$ does not exist

2. **(b)**

3. **(d) :** As $f''(x) - g''(x) = 0$

$\Rightarrow \quad f'(x) - g'(x) = k$

$f'(1) - g'(1) = k$

$\therefore \quad k = 2$

so $\quad f'(x) - g'(x) = 2$

$\Rightarrow \quad f(x) - g(x) = 2x + k_1$

$f(2) - g(2) = 4 + k_1$

$k_1 = 2$

so $\quad f(x) - g(x) = 2x + 2$

$\therefore \quad [f(x) - g(x)]_{x=\frac{3}{2}} = \dfrac{2\times 3}{2} + 2 = 5$

4. **(a) :** $\underset{x\to 1}{\text{Lt}} \dfrac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ (0/0 form)

$\underset{x\to 1}{\text{Lt}} \dfrac{1}{2\sqrt{f(x)}} \times \dfrac{2\sqrt{x}}{1} \times f'(x)$

$= \dfrac{2\times 1\times 2}{2} = 2$

5. **(a) :**

Let $A(0,0)$, $B(x, y) = \begin{cases} a\sin t - b\sin\dfrac{at}{b} = x \\ a\cos t - b\cos\dfrac{at}{b} = y \end{cases}$

$\therefore \sqrt{x^2 + y^2} = AB$

$= \sqrt{a^2(\sin^2 t + \cos^2 t) + b^2\left(\sin^2\left(\dfrac{at}{b}\right) + \cos^2\left(\dfrac{at}{b}\right)\right) - 2ab\cos\left(t - \dfrac{at}{b}\right)}$

$= \sqrt{a^2 + b^2 - 2ab\cos\alpha}$ (since $|\cos\alpha| \le 1$)

$\le \sqrt{a^2 + b^2 - 2ab} = a - b.$

6. **(a) :** $\underset{x\to 0}{Lt} \dfrac{\sqrt{2}\sqrt{\sin^2 x}}{x\sqrt{2}} = \underset{x\to 0}{Lt} \dfrac{\sin x}{x} = 1.$

7. **(c) :** Given $f(x + y) = f(x) f(y)$

$\therefore \; f(0 + 0) = (f(0))^2$

$\Rightarrow \quad f(0) = 0$ or $f(0) = 1$ but $f(0) \ne 0$

Now $f'(x) = \underset{h\to 0}{\text{Lt}} \dfrac{f(x+h) - f(x)}{h}$

$= \underset{h\to 0}{\text{Lt}} \dfrac{f(x)f(h) - f(x)}{h}$

$f'(x) = f(x) \underset{h\to 0}{\text{Lt}} \dfrac{f(h)-1}{h}$

$\therefore \quad f'(0) = f(0) \underset{h\to 0}{\text{Lt}} \dfrac{f(h)-1}{h}$

$3 = \underset{h\to 0}{\text{Lt}} \dfrac{f(h)-1}{h}$ ($\because f(0) = 1$)

Now $f'(x) = f(x) \underset{h\to 0}{\text{Lt}} \dfrac{f(h)-1}{h}$

$\therefore \quad f'(5) = f(5) \times 3$

$= 2 \times 3 = 6$

8. **(d) :** We have $\lim\limits_{x\to\infty} \left(\dfrac{x^2+5x+3}{x^2+x+3}\right)^{1/x}$

$= \lim\limits_{x\to\infty} \left(\dfrac{1+\dfrac{5}{x}+\dfrac{3}{x^2}}{1+\dfrac{1}{x}+\dfrac{3}{x^2}}\right)^{1/x} = 1^0 = 1$

9. **(c) :** $\underset{x\to 2}{\text{Lt}} \dfrac{xf(2) - 2f(x) + 2f(2) - 2f(2)}{x-2}$

$\underset{x\to 2}{\text{Lt}} \dfrac{(x-2)f(2) - 2[f(x) - f(2)]}{x-2}$

$= \lim\limits_{x\to 2} [f(2) - 2f'(x)]$

$= 4 - 2 \times 4$

$= -4$

10. **(a) :** Let us consider $f(x) = \dfrac{ax^3}{3} + \dfrac{bx^2}{2} + cx$

$\therefore f(0) = 0$ and $f(1) = \dfrac{a}{3} + \dfrac{b}{2} + c$

$= \dfrac{2a + 3b + 6c}{6}$

$= 0$ given.

As $f(0) = f(1) = 0$ and $f(x)$ is continuous and differentiable also in [0, 1].

$\therefore$ By Rolle's theorem $f'(x) = 0$

$\Rightarrow ax^2 + bx + c = 0$ has at least one root in the interval (0, 1).

11. (b) : $$\underset{x\to 0}{\text{Lt}} \frac{\log(3+x)-\log(3-x)}{x} = k$$

$$\therefore \quad k = \underset{x\to 0}{\text{Lt}} \frac{\log\left(1+\frac{x}{3}\right)-\log\left(1-\frac{x}{3}\right)}{x}$$

$$k = \underset{x\to 0}{\text{Lt}} \frac{\log\left(1+\frac{x}{3}\right)}{\frac{x}{3}\times 3} + \underset{x\to 0}{\text{Lt}} \frac{\log\left(1-\frac{x}{3}\right)}{-\frac{x}{3}\times 3}$$

$$k = \frac{1}{3}+\frac{1}{3} = \frac{2}{3}$$

12. (b) : $f(x) = x^n \therefore f(1) = 1 = {}^nC_0$

$\therefore \quad f'(x) = nx^{n-1}$ so $-f'(1) = -n = -{}^nC_1$

$f''(x) = n(n-1)x^{n-2}$ so $\dfrac{f''(1)}{2!} = \dfrac{n(n-1)}{2!} = {}^nC_2$

$f^n(x) = n(n-1)\ ...1 \therefore \dfrac{f^n(1)(-1)^n}{n!} = (-1)^n\ {}^nC_n$

$$\therefore \quad f(1) - \frac{f'(1)}{1!}+\frac{f''(1)}{2!}-\frac{f'''(1)}{3!} + \ldots + \frac{(-1)^n f^n(1)}{n!} = {}^nC_0 - {}^nC_1 + {}^nC_2 \ ....+ (-1)^n C_n$$

now $(1 + x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$ ...(*)

Putting $x = -1$ in both side of * we get

$0 = C_0 - C_1 + C_2 - C_3 + ...$

13. (b) : For maximum and minima $f'(x) = 0$

$\Rightarrow \quad 6x^2 - 18ax + 12a^2 = 0$ and $f''(x) = 12x - 18a$

$f'(x) = 0$

$\Rightarrow \quad x = a, 2a$ and $f''(a) < 0$ and $f''(2a) > 0$

Now $p = a$ and $q = 2a$

and $p^2 = q$

$\Rightarrow \quad a^2 = 2a$

$\Rightarrow \quad a^2 - 2a = 0$

$\Rightarrow \quad a(a - 2) = 0$

$\Rightarrow \quad a = 0, a = 2$

14. (a) : Given $f(x) = \begin{cases} x\, e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$$\underset{x\to 0^+}{\text{Lt}} f(x) = \underset{x\to 0^+}{\text{Lt}} xe^{-2/x} = 0 \qquad ...(A)$$

$$\text{and } \underset{x\to 0^-}{\text{Lt}} f(x) = \underset{x\to 0^-}{\text{Lt}} e^{\left[-\frac{1}{x}+\frac{1}{x}\right]} = 0 \qquad ...(B)$$

As $\underset{x\to 0}{\text{LHL}} = \underset{x\to 0}{\text{RHL}}$ $\therefore f(x)$ is continuous at $x = 0$

Again RHD at $x = 0$ is

$$\underset{x\to 0^+}{\text{Lt}} \frac{(0+h)e^{-\left(\frac{1}{h}+\frac{1}{h}\right)}-0}{h} = 0$$

also we have L.H.D at $x = 0$

is $\dfrac{(0-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)}-0}{-h} = 1$

so L.H.D $\neq$ R.H.D at $x = 0$

$\therefore f(x)$ is non differentiable at $x = 0$

15. (a) : $f(x) = x + 1/x$

$f'(x) = 1 - 1/x^2$ and $f''(x) = \dfrac{2}{x^3}$

now $f'(x) = 0$

$\Rightarrow x = \pm 1$

$\therefore f''(1) > 0$

$\Rightarrow x = 1$ is point of minima.

16. (c) : $$\underset{n\to\infty}{\text{Lt}} \frac{1^4+2^4+3^4+\ldots+n^4}{n^5} - \underset{n\to\infty}{\text{Lt}} \frac{1^3+2^3+\ldots+n^3}{n^5}$$

$$= \underset{n\to\infty}{\text{Lt}} \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30\cdot n^5} - \underset{n\to\infty}{\text{Lt}} \frac{n^2(n+1)^2}{4\times n^5}$$

$$= \frac{6}{30} - 0 \left[\text{Using } 1^4 + 2^4 + \ldots. + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{n(n+1)(6n^3+9n^2+n-1)}{30}\right]$$

$$= \frac{1}{5}$$

17. (b) : $\underset{x\to\pi/2}{\text{Lt}} \dfrac{\tan\left(\dfrac{\pi}{4}-\dfrac{x}{2}\right)(1-\sin x)}{4.\left(\dfrac{\pi-2x}{4}\right)(\pi-2x)^2}$

$$\underset{x\to\pi/2}{\text{Lt}} \frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{4.\left(\frac{\pi}{4}-\frac{x}{2}\right)} \frac{1-\cos\left(\frac{\pi}{2}-x\right)}{(\pi-2x)^2}$$

$$\underset{x\to\pi/2}{\text{Lt}} \frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{4.\left(\frac{\pi}{4}-\frac{x}{2}\right)} \frac{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{4^2\left(\frac{\pi-2x}{4}\right)^2}$$

$$= \frac{1}{4}\times\frac{2}{16} = \frac{1}{32}$$

18. (d) :

$$\underset{x\to a}{\text{Lt}} \frac{f(a)\,g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)} = 4$$

$$\Rightarrow \underset{x\to a}{\text{Lt}} \frac{f(a)[g(x)-f(x)]}{g(x)-f(x)} = 4$$

$$\Rightarrow \underset{x\to a}{\text{Lt}}\ f(a) = 4$$

$k = 4$

19. (b) : $e^2 = \left(1+\dfrac{a}{x}+\dfrac{b}{x^2}\right)^{2x}$ (1^∞ form)

$$e^2 = e^{\underset{x\to\infty}{\text{Lt}}\left[1+\frac{a}{x}+\frac{b}{x^2}-1\right](2x)}$$

$e^2 = e^{2a}$

$\Rightarrow 2a = 2 \therefore a = 1$ and $b \in R$

20. (a) : $\underset{x\to\pi/4}{\text{Lt}} \dfrac{1-\tan x}{4x-\pi}$ putting $4x - \pi = t$

$$\therefore \underset{x\to\pi/4}{\text{Lt}} \frac{(1-\tan x)\times(1+\tan x)}{(1+\tan x)\left[-4\left(\frac{\pi}{4}-x\right)\right]}$$

$$\underset{x\to\pi/4}{\text{Lt}} -\frac{\tan\left(\frac{\pi}{4}-x\right)\times(1+\tan x)}{4\left(\frac{\pi}{4}-x\right)} = -1/2$$

21. (b) : Given $f''(x) = 6(x-1)$

$\Rightarrow f'(x) = \dfrac{6(x-1)^2}{2} + c$

$\Rightarrow 3 = 3 + c$ $\left[\begin{array}{l}\because f(x)=y=3x+5\\ f'(x)=3\ \forall\ x\in R\end{array}\right]$

$\Rightarrow c = 0$

so $f'(x) = 3(x-1)^2$

$\Rightarrow f(x) = (x-1)^3 + c_1$ as curve passes through $(2, 1)$

$\Rightarrow 1 = (2-1)^3 + c_1$

$\Rightarrow c_1 = 0$

$\therefore f(x) = (x-1)^3$

22. (c) : Let $f(x) = \dfrac{ax^3}{3}+\dfrac{bx^2}{2}+cx$

Note : In such type of problems we always consider $f(x)$ as the integration of L.H.S of the given equation without constant. Here integration of $ax^2 + bx + c$ is $\dfrac{ax^3}{3}+\dfrac{bx^2}{2}+cx$ called it by $f(x)$. Now use the intervals in $f(x)$ if $f(x)$ satisfies the given condition then at least one root of the equation $ax^2 + bx + c = 0$ must lies in that interval.

Now $f(x) = \dfrac{ax^3}{3}+\dfrac{bx^2}{2}+cx$

$f(0) = 0$ and $f(1) = \dfrac{a}{3}+\dfrac{b}{2}+c$

$= \dfrac{2a+3b+6c}{6}$

$= 0$ given $2a + 3b + 6c = 0$

$\therefore x = 0$ and $x = 1$ are roots of

$f(x) = \dfrac{ax^3}{3}+\dfrac{bx^2}{2}+cx = 0$

$\therefore$ at least one root of the equation $ax^2 + bx + c = 0$ lies in $(0, 1)$

23. (d) : As $f(x)$ is differenatiable at $x = 1$

$5 = \lim\limits_{h\to0} \dfrac{f(1+h)}{h}$ assumes 0/0 form

$5 = \lim\limits_{h\to0} \dfrac{f'(1)}{1}$ $\therefore f'(1) = 5.$

24. (b) : Let if possible $f'(x) = 2$ for

$\Rightarrow f(x) = 2x + c$ (Integrating both side w.r.t. x)

$\therefore f(1) = 2 + c, -2 = 2 + c$

$\Rightarrow c = -4 \quad \therefore f(x) = 2x - 4$

$\therefore f(6) = 2 \times 6 - 4 = 8 \quad \therefore f(6) \geq 8.$

25. **(c) :** Given $|f(x)-f(y)| \le (x-y)^2$

$$\lim_{x\to y}\left|\frac{f(x)-f(y)}{x-y}\right| \le \lim_{x\to y}|x-y|$$

$\Rightarrow |f'(x)| \le 0,\ f(x) = 0$

$(|f'(x)|<0,\ \text{not possible})$

$\Rightarrow f(x) = k$ (by integration)

$\Rightarrow f(x) = 0 \qquad \because f(0) = 0$

$\Rightarrow f(x)\ (\forall\ x\in R) = 0 \quad \therefore f(1) = 0.$

26. **(a), (c) :**

$\dfrac{dy}{dx} = \dfrac{dy}{d\theta}\cdot\dfrac{d\theta}{dx} = \tan\theta$ = slope of tangent

$\therefore$ Slope of normal to the curve $= -\cot\theta$

$= \tan(90+\theta).$

Now equation of normal to the curve

$$[y-a(\sin\theta-\theta\cos\theta)] = -\frac{\cos\theta}{\sin\theta}(x-a(\cos\theta+a\sin\theta))$$

$\Rightarrow x\cos\theta+y\sin\theta = a(1)$

Now distance from (0, 0) to $x\cos\theta + y\sin\theta = a$ is

distance $(d) = \dfrac{(0+0-a)}{1}$

$\therefore$ distance is constant $= a$.

27. **(b) :** As α is root of $ax^2 + bx + c = 0$

$\therefore a\alpha^2 + b\alpha + c = 0.$ Now

$$\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$$

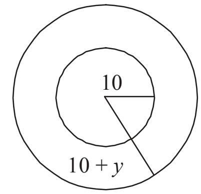
$$= \lim_{x\to\alpha}\frac{2\sin^2\left(\dfrac{ax^2+bx+c}{2}\right)}{(x-\alpha)^2}$$

$$= \lim_{x\to\alpha}\frac{2\sin^2\left[\dfrac{a(x-\alpha)(x-\beta)}{2}\right]}{a^2\left[\dfrac{(x-\alpha)^2(x-\beta)^2}{4}\right]}\times\frac{a^2(x-\beta)^2}{4}$$

$$= \lim_{x\to\alpha}\left[\frac{\sin\left(\dfrac{a(x-\alpha)(x-\beta)}{2}\right)}{\dfrac{a(x-\alpha)(x-\beta)}{2}}\right]^2\times\frac{a^2(x-\beta)^2}{2}$$

$$= 1\times\frac{a^2}{2}(\alpha-\beta)^2.$$

28. **(a) :** $v = \dfrac{4}{3}\pi(y+10)^3$ where y is thickness of ice



$\Rightarrow \dfrac{dv}{dt} = 4\pi(10+y)^2\dfrac{dy}{dt}$

$\left(\dfrac{dy}{dt}\right)_{\text{at } y=5} = \dfrac{50}{4\pi(15)^2}$

$\left(\text{as } \dfrac{dv}{dt} = 50\ \text{cm}^3/\text{min.}\right)$

$= \dfrac{1}{18\pi}$ cm/min.

29. **(b) :** For the range of the expression

$\dfrac{3x^2+9x+17}{3x^2+9x+7} = y = \dfrac{ax^2+bx+c}{px^2+qx+r}$,

[find the solution of the inequality

$A y^2 + B y + K \ge 0$]

where $A = q^2 - 4pr = -3$, $B = 4ar + 4PC - 2bq$

$= 126$

$K = b^2 - 4ac = -123$

i.e., solve $-3y^2 + 126 + y - 123 \ge 0$

$\Rightarrow \quad 3y^2 - 126y + 123 \le 0$

$\Rightarrow \quad y^2 - 42y + 41 \le 0$

$\Rightarrow \quad (y-1)(y-42) \le 0$

$\Rightarrow \quad 1 \le y \le 42$

$\Rightarrow$ maximum value of y is 42

30. **(a) :** Let $g(x) = \dfrac{x}{2}+\dfrac{2}{x}$

$\therefore \quad g'(x) = \dfrac{1}{2}-\dfrac{2}{x^2}$

for maxima and minima $g'(x) = 0 \Rightarrow x = \pm 2$

Again $g''(x) = \dfrac{4}{x^3} > 0$ for $x = 2$

< 0 for $x = -2 \quad \therefore x = 2$ is point of minima

31. **(a) :** Given equation $y = x^2 - 5x + 6$, given points (2, 0), (3, 0)

$\therefore \dfrac{dy}{dx} = 2x-5$

say $m_1 = \left(\dfrac{dy}{dx}\right)_{\substack{x=2\\y=0}} = 4-5 = -1$

and $m_2 = \left(\frac{dy}{dx}\right) at_{\substack{x=3\\y=0}} = 6-5 = 1$

since $m_1 m_2 = -1$

$\Rightarrow$ tangents are at right angle i.e. $\frac{\pi}{2}$

32. **(c) :** Given $f(x) = \frac{x}{1+|x|}$

$$\Rightarrow f(x) = \begin{cases} \frac{x}{1-x}, & x < 0 \\ \frac{x}{1+x}, & x \geq 0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} \frac{1}{(1-x)^2}, & x < 0 \\ \frac{1}{(1+x)^2}, & x \geq 0 \end{cases}$$

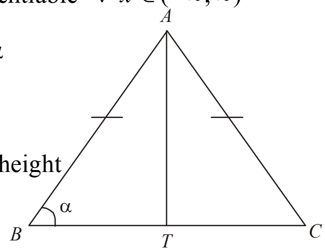
$f'(x)$ is finite quantity $\forall\ x \in R$

$\therefore\ f'(x)$ is differentiable $\forall\ x \in (-\infty, \infty)$

33. **(c) :** $AT = x \sin \alpha$

$BT = x \cos \alpha$

Area of triangle



ABC $= \frac{1}{2}$ base $\times$ height

$= \frac{1}{2}(2BT)(AT)$

$= \frac{1}{2}(2x^2 \cos\alpha \sin\alpha)$

$= \frac{1}{2}x^2 \sin 2\alpha \leq \frac{1}{2}x^2$ as $-1 \leq \sin 2\alpha \leq 1$

$\therefore$ Maximum are of $\Delta ABC = \frac{1}{2}x^2$

34. **(a) :** $x^m \times y^n = (x+y)^{m+n}$

Taking log both sides we get

$m \log x + n \log y = (m+n) \log(x+y)$

Differentiating w.r.t. x we get

$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{m+n}{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{n}{y} - \frac{m+n}{x+y}\right) = \frac{m+n}{x+y} - \frac{m}{x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{nx+ny-my-ny}{y(x+y)}\right) = \frac{mx+nx-mx-my}{x(x+y)}$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{nx-my}{nx-my}\right)\frac{y}{x} = \frac{y}{x} \Rightarrow \frac{dy}{dx} = \frac{y}{x}.$$

35. **(c) :** By LMVT,

$$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{f(3)-f(1)}{3-1}$$

$$f'(c) = \frac{\log_e 3 - \log_e 1}{2} = \frac{1}{2}\log_e 3$$

$\Rightarrow \frac{1}{c} = \frac{1}{2}\log_e 3 = \frac{1}{2\log_3 e}$ $\quad \therefore\ c = 2 \log_3 e$.

36. **(d) :** $f'(x) = \frac{1}{1+(\sin x + \cos x)^2}.(\cos x - \sin x)$

$$f'(x) = \frac{\cos x - \sin x}{2 + \sin 2x}$$

If $f'(x) > 0$ then $f(x)$ is increasing function

For $-\frac{\pi}{2} < x < \frac{\pi}{4}, \cos x > \sin x$

Hence $y = f(x)$ is increasing in $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$.

37. **(a) :** $f(x) = \min\{x+1, |x|+1\}$

$\Rightarrow f(x) = x + 1, x \in R$

Hence $f(x)$ is differentiable for all $x \in R$.

38. **(b) :** $f(0) = \lim\limits_{x\to 0}\left[\frac{1}{x} - \frac{2}{e^{2x}-1}\right]$

$$= \lim_{x\to 0}\frac{e^{2x}-1-2x}{x(e^{2x}-1)} \left(\frac{0}{0}\text{form}\right)$$

By using L' Hospital rule

$$f(0) = \lim_{x\to 0}\frac{2e^{2x}-2}{(e^{2x}-1)+2xe^{2x}} \left(\frac{0}{0}\text{ form}\right)$$

Again use L' Hospital rule

$$f(0) = \lim_{x\to 0}\frac{4e^{2x}}{4e^{2x}+4xe^{2x}} = 1.$$

39. **(c) :** Let $p = \cos\theta$, $q = \sin\theta$

$0 \leq \theta \leq \pi/2$

$p + q = \cos\theta + \sin\theta$

$\Rightarrow$ maximum value of $(p+q) = \sqrt{2}$

Second method

By using A.M $\geq$ G.M., $\frac{p^2+q^2}{2} \geq pq \Rightarrow pq \leq \frac{1}{2}$

$(p+q)^2 = p^2 + q^2 + 2pq \Rightarrow (p+q) \leq \sqrt{2}$.

40. (c) : Let $f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560$

$\therefore f'(x) = 7x^6 + 70x^4 + 48x^2 + 30$

$\Rightarrow f'(x) > 0 \ \forall \ x \in R$

i.e. $f(x)$ is an strictly increasing function.

so it can have at the most one solution. It can be shown that it has exactly one solution.

41. (b) : By definition

$$f'(1) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h},\ \text{if the limit exists.}$$

$$\therefore \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0}\frac{(1+h-1)\sin\dfrac{1}{(1+h-1)}-0}{h} = \lim_{h\to 0}\sin\frac{1}{h}$$

As the limit dosen't exist,

$\therefore$ it is not diffentiable at $x = 1$

Again $f'(0) = \lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{h}$, if the limit exists

$$\therefore \lim_{h\to 0}\frac{f(h)-f(0)}{h} = \lim_{h\to 0}\frac{(h-1)\sin\dfrac{1}{h-1}-\sin 1}{h}$$

But this limit dosen't exist. Hence it is not differentiable at $x = 0$.

42. (b) : Denote $x^3 - px + q$ by $f(x)$

i.e. $f(x) = x^3 - px + q$

Now for expression, $f'(x) = 0$, i.e. $3x^2 - p = 0$

$$x = -\sqrt{\frac{p}{3}}, \sqrt{\frac{p}{3}}$$

$f''(x) = 6x$

$$f''\left(-\sqrt{\frac{p}{3}}\right) < 0$$

$$f''\left(\sqrt{\frac{p}{3}}\right) > 0$$

Thus maxima at $-\sqrt{\dfrac{p}{3}}$ and minima at $\sqrt{\dfrac{p}{3}}$.

CHAPTER 10

Integral Calculus

1. $\int_0^{\sqrt{2}} [x^2]\,dx$ is
(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$
(c) $\sqrt{2}-1$ (d) $\sqrt{2}-2$. *(2002)*

2. $I_n = \int_0^{\pi/4} \tan^n x\,dx$, then $\lim_{n\to\infty} n[I_n + I_{n-2}]$ equals
(a) 1/2 (b) 1 (c) ∞ (d) 0. *(2002)*

3. $\int_{\pi}^{10\pi} |\sin x|\,dx$ is
(a) 20 (b) 8 (c) 10 (d) 18. *(2002)*

4. If $y = f(x)$ makes +ve intercept of 2 and 0 unit x and y and encloses an area of 3/4 square unit with the axes then $\int_0^2 x\, f'(x)\,dx$ is
(a) 3/2 (b) 1 (c) 5/4 (d) –3/4. *(2002)*

5. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\,dx$ is
(a) $\pi^2/4$ (b) π^2 (c) 0 (d) $\pi/2$. *(2002)*

6. The area bounded by the curves $y = \ln x$, $y = \ln|x|$, $y = |\ln x|$ and $y = |\ln|x||$ is
(a) 4 sq. units (b) 6 sq. units
(c) 10 sq. units (d) none of these *(2002)*

7. $\lim_{n\to\infty} \frac{1^p + 2^p + 3^p + \ldots + n^p}{n^{p+1}}$ is
(a) $\frac{1}{p+1}$ (b) $\frac{1}{1-p}$
(c) $\frac{1}{p} - \frac{1}{p-1}$ (d) $\frac{1}{p+2}$. *(2002)*

8. The area of the region bounded by the curves $y = |x-1|$ and $y = 3 - |x|$ is
(a) 3 sq. units (b) 4 sq. units
(c) 6 sq. units (d) 2 sq. units. *(2003)*

9. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)g(x)\,dx$ is
(a) $e + \frac{e^2}{2} - \frac{3}{2}$ (b) $e - \frac{e^2}{2} - \frac{3}{2}$
(c) $e + \frac{e^2}{2} + \frac{5}{2}$ (d) $e - \frac{e^2}{2} - \frac{5}{2}$. *(2003)*

10. If $f(y) = e^y$, $g(y) = y$; $y > 0$ and
$$F(t) = \int_0^t f(t-y)g(y)\,dy, \text{ then}$$
(a) $F(t) = e^t - (1+t)$ (b) $F(t) = t\,e^t$
(c) $F(t) = te^{-t}$
(d) $F(t) = 1 - e^t(1+t)$. *(2003)*

11. If $f(a+b-x) = f(x)$, then $\int_a^b x\,f(x)\,dx$ is equal to
(a) $\frac{a+b}{2}\int_a^b f(x)\,dx$ (b) $\frac{b-a}{2}\int_a^b f(x)\,dx$
(c) $\frac{a+b}{2}\int_a^b f(a+b-x)\,dx$
(d) $\frac{a+b}{2}\int_a^b f(b-x)\,dx$. *(2003)*

12. The value of the integral $I = \int_0^1 x(1-x)^n\,dx$ is
(a) $\frac{1}{n+2}$ (b) $\frac{1}{n+1} - \frac{1}{n+2}$

(c) $\frac{1}{n+1}+\frac{1}{n+2}$ (d) $\frac{1}{n+1}$. *(2003)*

13. The value of $\lim\limits_{x\to 0}\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}$ is

(a) 2 (b) 1 (c) 0 (d) 3. *(2003)*

14. Let $\frac{d}{dx}F(x)=\left(\frac{e^{\sin x}}{x}\right), x>0$.

If $\int_1^4 \frac{3}{x}e^{\sin x^3}\,dx = F(k)-F(1)$, then one of the possible values of k is

(a) 16 (b) 63 (c) 64 (d) 15. *(2003)*

15. $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{n}e^{r/n}$ is

(a) $1-e$ (b) $e-1$ (c) e (d) $e+1$. *(2004)*

16. If $\int\frac{\sin x}{\sin(x-\alpha)}dx = Ax+B\log\sin(x-\alpha)+C$, then value of (A, B) is

(a) $(-\sin\alpha, \cos\alpha)$ (b) $(\cos\alpha, \sin\alpha)$
(c) $(\sin\alpha, \cos\alpha)$ (d) $(-\cos\alpha, \sin\alpha)$ *(2004)*

17. $\int\frac{dx}{\cos x-\sin x}$ is equal to

(a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|+C$

(b) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right|+C$

(c) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+C$

(d) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+C$. *(2004)*

18. The value of $\int_{-2}^{3}|1-x^2|\,dx$ is

(a) 7/3 (b) 14/3 (c) 28/3 (d) 1/3. *(2004)*

19. The value of $I = \int_0^{\pi/2}\frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}dx$ is

(a) 2 (b) 1 (c) 0 (d) 3. *(2004)*

20. If $\int_0^{\pi} x\,f(\sin x)dx = A\int_0^{\pi/2} f(\sin x)dx$, then A is

(a) $\pi/4$ (b) π (c) 0 (d) 2π. *(2004)*

21. If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}\,dx$ and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}\,dx$, then the value of $\frac{I_2}{I_1}$ is

(a) -1 (b) -3 (c) 2 (d) 1. *(2004)*

22. The area of the region bounded by the curves $y = |x-2|$, $x = 1$, $x = 3$ and the x-axis is

(a) 3 (b) 2 (c) 1 (d) 4. *(2004)*

23. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ is

(a) ab (b) $2ab$ (c) a/b (d) $\sqrt{ab}$ *(2005)*

24. $\int\left\{\frac{(\log x-1)}{1+(\log x)^2}\right\}^2 dx$ is equal to

(a) $\frac{x}{x^2+1}+C$ (b) $\frac{\log x}{(\log x)^2+1}+C$

(c) $\frac{x}{(\log x)^2+1}+C$ (d) $\frac{xe^x}{1+x^2}+C$. *(2005)*

25. Let $F : R\to R$ be a differentiable function having $f(2) = 6$, $f'(2) = \left(\frac{1}{48}\right)$. Then $\lim\limits_{x\to 2}\int_6^{f(x)}\frac{4t^3}{x-2}dt$ equals

(a) 36 (b) 24 (c) 18 (d) 12. *(2005)*

26. Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\left(\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is

(a) $\left(\frac{\pi}{4}-\sqrt{2}+1\right)$ (b) $\left(\frac{\pi}{4}+\sqrt{2}-1\right)$

(c) $\left(1-\frac{\pi}{4}+\sqrt{2}\right)$ (d) $\left(1-\frac{\pi}{4}-\sqrt{2}\right)$. *(2005)*

27. If $I_1 = \int_0^1 2^{x^2}\,dx$, $I_2 = \int_0^1 2^{x^3}\,dx$, $I_3 = \int_1^2 2^{x^2}\,dx$ and $I_4 = \int_1^2 2^{x^3}\,dx$ then

(a) $I_1 > I_2$ (b) $I_2 > I_1$ (c) $I_3 > I_4$ (d) $I_3 = I_4$.

(2005)

28. The area enclosed between the curve $y = \log_e(x + e)$ and the coordinate axes is

(a) 2 (b) 1 (c) 4 (d) 3.

(2005)

29. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is

(a) 1 : 2 : 3 (b) 1 : 2 : 1

(c) 1 : 1 : 1 (d) 2 : 1 : 2. *(2005)*

30. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}\,dx$, $a > 0$, is

(a) $\pi/2$ (b) $a\pi$ (c) 2π (d) π/a

(2005)

31. $\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + ... + \frac{1}{n}\sec^2 1\right]$ equals

(a) $\frac{1}{2}\operatorname{cosec}1$ (b) $\frac{1}{2}\sec 1$

(c) $\frac{1}{2}\tan 1$ (d) $\tan 1$.

(2005)

32. The value of the integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}\,dx$ is

(a) 1/2 (b) 3/2 (c) 2 (d) 1.

(2006)

33. $\int_0^{\pi} x f(\sin x)dx$ is equal to

(a) $\pi\int_0^{\pi} f(\cos x)dx$ (b) $\pi\int_0^{\pi} f(\sin x)dx$

(c) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)dx$ (d) $\pi\int_0^{\pi/2} f(\cos x)dx$.

(2006)

34. $\int_{-3\pi/2}^{-\pi/2} [(x+\pi)^3 + \cos^2(x+3\pi)]dx$ is equal to

(a) $\frac{\pi^4}{32}$ (b) $\frac{\pi^4}{32}+\frac{\pi}{2}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{2}-1$.

(2006)

35. The value of $\int_1^a [x]\, f'(x)\,dx$, $a > 1$, where $[x]$ denotes the greatest integer not exceeding x is

(a) $a f(a) - \{f(1) + f(2) + ... + f([a])\}$

(b) $[a] f(a) - \{f(1) + f(2) + ... + f([a])\}$

(c) $[a] f([a]) - \{f(1) + f(2) + ... + f(a)\}$

(d) $a f([a]) - \{f(1) + f(2) + ... + f(a)\}$.

(2006)

36. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{\log t}{1+t}dt$, Then $F(e)$ equals

(a) 1 (b) 2 (c) 1/2 (d) 0.

(2007)

37. The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $2\sqrt{2}$ (c) 2 (d) π.

(2007)

38. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$ equals

(a) $\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

(b) $\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

(c) $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

(d) $\frac{1}{2}\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$. *(2007)*

39. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is

(a) 1/6 (b) 1/3 (c) 2/3 (d) 1.

(2007)

40. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}}dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}}dx$.

Then which one of the following is true?

(a) $I > \frac{2}{3}$ and $J < 2$ (b) $I > \frac{2}{3}$ and $J > 2$

(c) $I < \frac{2}{3}$ and $J < 2$ (d) $I < \frac{2}{3}$ and $J > 2$

(2008)

41. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to

(a) $\frac{4}{3}$ (b) $\frac{5}{3}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(2008)

42. The value of is $\sqrt{2}\int \frac{\sin x\, dx}{\sin\left(x-\frac{\pi}{4}\right)}$

(a) $x-\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

(b) $x+\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

(c) $x-\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

(d) $x+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

(2008)

Answer Key

1.	(c)	**2.**	(b)	**3.**	(d)	**4.**	(d)	**5.**	(b)	**6.**	(a)
7.	(a)	**8.**	(b)	**9.**	(b)	**10.**	(a)	**11.**	(a), (c)	**12.**	(b)
13.	(b)	**14.**	(c)	**15.**	(b)	**16.**	(b)	**17.**	(d)	**18.**	(c)
19.	(a)	**20.**	(b)	**21.**	(c)	**22.**	(c)	**23.**	(b)	**24.**	(c)
25.	(c)	**26.**	(c)	**27.**	(a)	**28.**	(b)	**29.**	(c)	**30.**	(a)
31.	(c)	**32.**	(b)	**33.**	(d)	**34.**	(c)	**35.**	(b)	**36.**	(c)
38.	(c)	**39.**	(a)	**40.**	(c)	**41.**	(a)	**42.**	(d)		

1. **(c) :** $\int_0^{\sqrt{2}} [x^2]dx = \int_0^1 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx$

$= 0 + \int_1^{\sqrt{2}} 1\, dx = \sqrt{2} - 1$

2. **(b) :** $I_n = \int_0^{\pi/4} \tan^n x\, dx$

$I_{n-2} = \int_0^{\pi/4} \tan^{n-2} x\, dx$

$\therefore\ I_n + I_{n-2} =$

$\int_0^{\pi/4} \tan^n x\, dx + \int_0^{\pi/4} \tan^{n-2} x\, dx$

$= \int_0^{\pi/4} \tan^{n-2} x \times (\sec^2 x - 1)\, dx + \int_0^{\pi/4} \tan^{n-2} x\, dx$

$= \int_0^{\pi/4} \tan^{n-2} x \sec^2 x\, dx$

$I_n + I_{n-2} = \dfrac{1}{n+1}$

$\therefore\ n(I_n + I_{n-2}) = \dfrac{1}{1 + 1/n}$

$\therefore\ \underset{n \to \infty}{\text{Lt}}\ n(I_n + I_{n-2}) = 1$

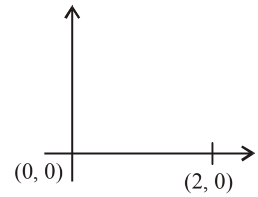
3. **(d) :** $\int_{\pi}^{10\pi} |\sin x|\, dx$

$= \int_0^{10\pi} |\sin x|\, dx - \int_0^{\pi} |\sin x|\, dx$

$= 10 \times 2 - 1 \times 2$

$= 18$ (Using period of $|\sin x| = \pi$)

4. **(d) :** Given $\int_0^2 f(x)\, dx = 3/4$



$\therefore\ \int_0^2 x f'(x)\, dx = x\int_0^2 f'(x)\, dx - \int_0^2 f(x)\, dx$

$= (x \cdot f(x))_0^2 - 3/4$

$= 2f(2) - \dfrac{3}{4}$

$= 0 - \dfrac{3}{4}$ [$\because f(2) = 0$, curve having intercept 2 units on x-axis.]

$= -3/4$

5. **(b) :** $2\int_{-\pi}^{\pi} \dfrac{x}{1+\cos^2 x}\, dx + 2\int_{-\pi}^{\pi} \dfrac{x \sin x}{1+\cos^2 x}\, dx$

$= 0 + 2\int_{-\pi}^{\pi} \dfrac{x \sin x}{1+\cos^2 x}\, dx$

$= 2 \cdot 2\int_0^{\pi} \dfrac{x \sin x}{1+\cos^2 x}$

$= 4\int_0^{\pi} \dfrac{x \sin x}{1+\cos^2 x}\, dx$

$= 4 \times \dfrac{\pi}{2}\int_0^{\pi} \dfrac{\sin x}{1+\cos^2 x}\, dx$

$\left(\text{by using } \int_0^{\pi} x f(\sin x)\, dx = \dfrac{\pi}{2}\int_0^{\pi} f(\sin x)\, dx\right)$

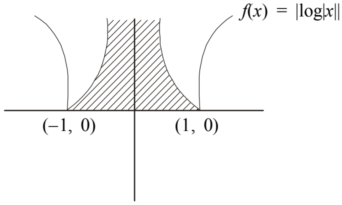
$= 4\dfrac{\pi}{2} \times 2 \times \int_0^{\pi/2} \dfrac{\sin x}{1+\cos^2 x}\, dx$

$= 4\pi\ (\tan^{-1} \cos x)_{\frac{\pi}{2}}^{0}$ (By putting $\cos x = t$)

$= 4\pi \times \left(\dfrac{\pi}{4} - 0\right)$

$= \pi^2$

6. **(a) :** Required Area



$= 2\int_0^1 \left|\log|x|\right| dx$

$= 2\left[\left(x\left|\log|x|\right|\right)\right]_0^1 - \int_0^1 \left(-\frac{1}{x}\right)\cdot x\, dx$

$= 2[(1 - 0) + (x)_0^1] = 4$ sq. units.

7. **(a) :** $\underset{n\to\infty}{\text{Lt}} \frac{1^p + 2^p + ... + n^p}{n^p} \times \frac{1}{n}$

$= \underset{n\to\infty}{\text{Lt}} \frac{1}{n}\left[\left(\frac{1}{n}\right)^p + \left(\frac{2}{n}\right)^p + ... + \left(\frac{n}{n}\right)^p\right]$

$= \underset{n\to\infty}{\text{Lt}} \sum_{r=1}^{r=n} \frac{1}{n}\left(\frac{r}{n}\right)^p$

$= \int_0^1 x^p dx = \frac{1}{p+1}$

8. **(b) :** Required area

$= \int_{-1}^{0} (3+x) - (-x+1) + \int_0^1 (3-x) - (-x+1) dx$

$+ \int_1^2 (3-x) - (x-1)\, dx$

$= \int_{-1}^{0} 2(1+x)\, dx + \int_0^1 2\, dx + \int_1^2 (4-2x)\, dx$

$= 4$ sq. units

9. **(b) :** As $f(x) = f'(x)$ and $f(0) = 1$

$\Rightarrow \quad \frac{f'(x)}{f(x)} = 1$

$\Rightarrow \quad \log(f(x)) = x$

$\Rightarrow \quad f(x) = e^x + k$

$\Rightarrow \quad f(x) = e^x$ as $f(0) = 1$

Now $g(x) = x^2 - e^x$

$\therefore \int_0^1 f(x)\, g(x)\, dx = \int_0^1 e^x(x^2 - e^x)\, dx$

$= \int_0^1 x^2 e^x dx - \int_0^1 e^{2x} dx$

$= [(x^2 - 2x + 2)e^x]_0^1 - \left(\frac{e^{2x}}{2}\right)_0^1$

$= (e - 2) - \left(\frac{e^2 - 1}{2}\right)$

$= e - \frac{e^2}{2} - \frac{3}{2}$

Using $f^n(x)e^x\, dx = e^x[f^n(x) - f_1{}^n(x) + f_2{}^n(x) + ... + (-1)^n f_n(x)]$

where $f_1, f_2, ... f_n$ are derivatives of first, second ...n^{th} order.

10. **(a) :** From given $F(t) = \int_0^t f(t-y)\, g(y)\, dy$

$= \int_0^t e^{t-y} y\, dy$ (By replacing $y \to t - y$ in $f(y)$)

$F(t) = -\int_t^0 (t-\theta)e^\theta d\theta = \int_0^t (t-\theta)e^\theta d\theta$

$= (t\, e^\theta)_0^t - [(\theta - 1)\, e^\theta]_0^t$

$= t(e^t - 1) - (t - 1)e^t - 1$

$= e^t(t - t + 1) - t - 1$

$= e^t - (t + 1)$

11. **(a), (c) :** Let $I = \int_a^b x\, f(x)\, dx$

$I = \int_a^b (a+b-x)\, f(a+b-x)\, dx$

$I = \int_a^b (a+b)\, f(a+b-x)\, dx - \int_a^b x\, f(a+b-x)\, dx$

$I = \int_a^b (a+b) f(x) dx - \int_a^b x f(x) dx$

$\therefore\ I = \frac{a+b}{2}\int_a^b f(x)\, dx = \frac{a+b}{2}\int_a^b f(a+b-x) dx$

12. (b) : $\int_0^1 x(1-x)^n\,dx$

Putting $x = \sin^2\theta$

$dx = 2\sin\theta\cos\theta\,d\theta$

and $x = 0, \theta = 0$

$x = 1, \theta = \pi/2$

$\therefore \int_0^1 x(1-x)^n dx = \int_0^{\pi/2} \sin^2\theta\cos^{2n}\theta\,(2\sin\theta\cos\theta)d\theta$

$= 2\int_0^{\frac{\pi}{2}} \sin^3\theta\cos^{2n+1}\theta\,d\theta$

$\left[\text{Using} \int_0^{\pi/2} \sin^{2n+1}\theta\cos^{2n+1}\theta\,d\theta = \frac{[(2n)(2n-2)...2][(2n)(2n-2)...2]}{(4n+2)(4n)(4n-2)....2}\right]$

$\therefore 2\int_0^{\pi/2} \sin^3\theta\cos^{2n+1}\theta\,d\theta$

$= \frac{2[2\times(2n)(2n-2)(2n-4)\,...4.2]}{(2n+4)(2n+2)(2n)(2n-2)...\,4.2}$

$= \frac{2\times2\times1}{(2n+4)(2n+2)}$

$= \frac{1}{(n+2)(n+1)}$

$= \frac{1}{n+1} - \frac{1}{n+2}$ (by partial fraction)

13. (b) : $\lim_{x\to0} \frac{(\tan t)_0^{x^2}}{x\sin x}$

$= \lim_{x\to0} \frac{\tan x^2}{x\sin x}$

$= \lim_{x\to0} \frac{\tan x^2}{x^2\,\frac{\sin x}{x}}$

$= \lim_{x\to0} \frac{\tan x^2}{x^2} \frac{1}{\lim_{x\to0}\frac{\sin x}{x}} = 1\times1 = 1$

14. (c): Given $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = F(k) - F(1)$

$\Rightarrow \int_1^4 \frac{3x^2}{x^3} e^{\sin x^3} dx = F(k) - F(1)$

$\Rightarrow \int_1^{64} \frac{e^{\sin z}}{z} dz = F(k) - F(1)$ where $(x^3 = z)$

$\Rightarrow [F(z)]_1^{64} = F(k) - F(1)$

$\Rightarrow F(64) - F(1) = F(k) - F(1)$

$\Rightarrow k = 64$

15. (b) : $\lim_{n\to\infty} \sum_{r=1}^{r=n} \frac{1}{n} e^{\frac{r}{n}}$

$= \int_0^1 e^x dx = e - 1$

16. (b) : $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B\log\sin(x-\alpha) + C$

$\Rightarrow$ Differentiating *w.r.* to x both sides

$\Rightarrow \frac{\sin x}{\sin(x-\alpha)} = A + \frac{B\cos(x-\alpha)}{\sin(x-\alpha)}$

$\Rightarrow \sin x = A\sin(x-\alpha) + B\cos(x-\alpha)$

$\sin x = A(\sin x\cos\alpha - \cos x\sin\alpha) + B(\cos x\cos\alpha + \sin x\sin\alpha)$

$\sin x = \sin x(A\cos\alpha + B\sin\alpha) + \cos x(B\cos\alpha - A\sin\alpha)$

Now solving $A\cos\alpha + B\sin\alpha = 1$

and $B\cos\alpha - A\sin\alpha = 0$

$(A, B) = (\cos\alpha, \sin\alpha)$

17. (d) : $\int \frac{1}{a\cos x - b\sin x} dx$ where $a = b = 1$

let $a = r\cos\theta = 1$

$b = r\sin\theta = 1$

$\therefore r = \sqrt{2}$

$\theta = \tan^{-1}(b/a)$

$= \frac{1}{\sqrt{2}} \int \frac{1}{\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x} dx$

$= \frac{1}{\sqrt{2}} \int \frac{1}{\cos(x+\pi/4)} dx$

$= \frac{1}{\sqrt{2}} \int \frac{1}{\sin\left(\frac{\pi}{2} + x + \frac{\pi}{4}\right)} dx$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{2\sin\left(\frac{x}{2}+\frac{3\pi}{8}\right)\cos\left(\frac{x}{2}+\frac{3\pi}{8}\right)} dx$$

$$= \frac{1}{2\sqrt{2}} \int \frac{\sec^2\left(3\frac{\pi}{8}+\frac{x}{2}\right)}{\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)} dx$$

$$= \frac{1}{2\sqrt{2}} \times 2\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right| + c$$

$$= \frac{1}{\sqrt{2}} \log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right| + c$$

18. (c) : $\int_{-2}^{3} |1-x^2|\,dx = \int_{-2}^{3} |(1-x)(1+x)|\,dx$

Putting $1 - x^2 = 0 \therefore x = \pm 1$

Points $-2, -1, 1, 3$

$\therefore |1 - x^2| = \begin{cases} 1-x^2 & \text{if } |x|<1 \\ (1-(1-x^2)) & \text{if } x<-1 \text{ and } x \ge 1 \end{cases}$

$$\therefore \int_{-2}^{3} |(1-x^2)|\,dx$$

$$= \int_{-2}^{-1} (x^2-1)dx + \int_{-1}^{1} (1-x^2)dx + \int_{1}^{3} (x^2-1)dx$$

$$= \frac{4}{3} + 2\left(\frac{2}{3}\right) + \frac{20}{3} = \frac{28}{3}$$

19. (a) : $\int_{0}^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{(\sin x + \cos x)^2}} dx$

$$= \int_{0}^{\pi/2} (\sin x + \cos x)dx$$

$$= \left(\frac{\cos x}{-1} + \sin x\right)_0^{\frac{\pi}{2}}$$

$= 1 - (-1) = 2$

20. (b) : $\int_{0}^{\pi} x f(\sin x)dx = A\int_{0}^{\frac{\pi}{2}} f(\sin x)dx$

or $A\int_{0}^{\frac{\pi}{2}} f(\sin x)dx = \frac{\pi}{2}\int_{0}^{\pi} f(\sin x)dx = \int_{0}^{\pi} xf(\sin x)dx$

$$\Rightarrow A\int_{0}^{\frac{\pi}{2}} f(\sin x)dx = \frac{\pi}{2} \times 2\int_{0}^{\frac{\pi}{2}} f(\sin x)dx$$

$$\Rightarrow A\int_{0}^{\frac{\pi}{2}} f(\sin x) = \pi\int_{0}^{\frac{\pi}{2}} f(\sin x)dx$$

$\Rightarrow A = \pi$

21. (c) : As $f(x) = \frac{e^x}{1+e^x}$

$\therefore \quad f(a) = \frac{e^a}{1+e^a}$ and $f(-a) = \frac{e^{-a}}{1+e^{-a}}$

$\therefore \quad f(-a) + f(a) = 1$

Now $\int_{f(-a)}^{f(a)} x\,g\{x(1-x)\}\,dx =$

$$\int_{f(-a)}^{f(a)} (1-x)g\{(1-x)(x)\}\,dx$$

using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$$\Rightarrow \quad 2\int_{f(-a)}^{f(a)} x\,g\{x(1-x)\}\,dx$$

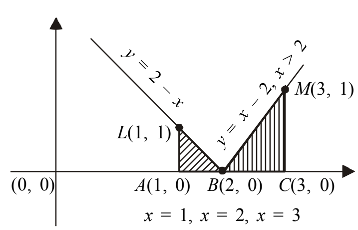
$$= \int_{f(-a)}^{f(a)} g\{(1-x)x\}\,dx$$

$\Rightarrow \quad 2I_1 = I_2$

$\therefore \quad \frac{I_2}{I_1} = \frac{2}{1}$

22. (c) : $y = \begin{cases} x-2 & \text{if } x>2 \\ 0 & \text{if } x=0 \\ 2-x & \text{if } x<2 \end{cases}$

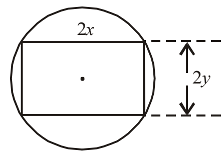
Required area = Area of ΔLAB + Area of ΔMBC



$= \frac{1}{2}[AL \times AB + BC \times CM]$

$= \frac{1}{2}\ [1 \times 1 + 1 \times 1] = 1$

23. (b) : Any point on the ellipse



$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$

is $(a\cos\theta, b\sin\theta)$ so the area of rectangle inscribed in the ellipse is given by

$A = (2a\cos\theta)\ (2b\sin\theta)$

$\therefore\ A = 2ab\sin 2\theta \Rightarrow \frac{dA}{d\theta} = 4ab\cos 2\theta$

Now for maximum area

$\frac{dA}{d\theta} = 0 \Rightarrow \theta = \frac{\pi}{4}$ and $\left(\frac{d^2A}{d\theta^2}\right)_{\theta=\pi/4} = -8ab\sin 2\theta$

as $\frac{d^2A}{d\theta^2} < 0$. $\therefore$ Area is maximum for $\theta = \pi/4$.

$\therefore$ sides of rectangle are $\frac{2a}{\sqrt{2}}, \frac{2b}{\sqrt{2}}$

Required area $= 2ab$.

24. (c) : Method by cross check

Consider $f(x) = \frac{x}{(\log x)^2 + 1}$

$\therefore\ f'(x) = \frac{1+(\log x)^2 - \frac{2x\log x}{x}}{(1+(\log x)^2)^2}$

$\therefore\ f'(x) = \frac{1+(\log x)^2 - 2\log x}{(1+\log^2 x)^2} = \left(\frac{(1-\log x)}{(1+\log x)^2}\right)^2$

$\therefore\ \int\left(\frac{(1-\log x)^2}{1+(\log x)^2}\right)dx = \int f'(x)\,dx = f(x)$

$\therefore\ \int\left(\frac{1-\log x}{1+(\log x)^2}\right)^2 dx = \frac{x}{1+(\log x)^2}$

Hence (c) is correct answer and we can check the other choices by the similar argument.

25. (c) : $\lim\limits_{x\to 2} \int\limits_6^{f(x)} \frac{4t^3}{x-2}\,dt$ (0/0) form,

$= \lim\limits_{x\to 2} \frac{f'(x) \times 4(f(x))^3}{1}$

$= 4f'(2) \times (f(2))^3 = \frac{1}{48} \times 4 \times 6 \times 6 \times 6 = 18.$

26. (c) : According to question

$\int\limits_{\pi/4}^{B} f(x)dx = \int\limits_{\pi/4}^{B(>\pi/4)} \left(B\sin B + \frac{\pi}{4}\cos B + B\sqrt{2}\right)$

$f(\beta) = \sin B + B\cos B - \frac{\pi}{4}\sin B + \sqrt{2}$

$\therefore\ f\left(\frac{\pi}{2}\right) = 1 - \frac{\pi}{4} + \sqrt{2}.$

27. (a) : For $0 < x < 1$, $x^2 > x^3$ $\therefore$ $\boxed{2^{x^2} > 2^{x^3}}$

and for $1 < x < 2$, $x^3 > x^2$ $\therefore$ $2^{x^3} > 2^{x^2}$

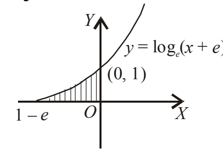
i.e. $\boxed{2^{x^2} < 2^{x^3}} \Rightarrow I_3 < I_4$

as $2^{x^2} > 2^{x^3}$

$\therefore\ \int\limits_0^1 2^{x^2}dx > \int\limits_0^1 2^{x^3}dx$

$\therefore\ I_1 > I_2.$

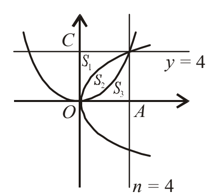
28. (b) : Required area $= \int\limits_{1-e}^{0} \log_e(x+e)dx$



$= \int\limits_1^e \log z\,dz$

$= \left[z(\log_e z - 1)\right]_1^e = 1.$

29. (c) : Total area $= 4 \times 4 = 16$ sq. units



Area of $S_3 = \int\limits_0^4 \frac{x^2}{4}dx = \frac{16}{3} = S_1$

$\therefore\ S_2 = 16 - \frac{16}{3} \times 2 = \frac{16}{3}.$

$\therefore\ S_1 : S_2 : S_3$ is $1 : 1 : 1$.

30. (a) : Let $f(x) = \int\limits_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}dx \quad (a > 0)$ ...(1)

$\therefore\ f(x) = \int\limits_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^{-x}}dx$

$\therefore \int_a^b f(x)dx = \int_a^b f(a+b-x)$

$\therefore \quad f(x) = \int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1+a^x} dx$ ... (2)

$2f(x) = \int_{-\pi}^{\pi} \cos^2 x\, dx = 2\int_0^{\pi} \cos^2 x\, dx$

$= 2\times 2\int_0^{\pi/2} \cos^2 x\, dx$, $2f(x) = 4 \times \frac{1}{2} \times \frac{\pi}{2}$

$\left[\begin{array}{l}\text{By using } \int_0^{\pi/2} \sin^n x\, dx \\ = \frac{n-1}{n}\cdot\frac{n-3}{n-2}.....\frac{1}{2}\times\frac{\pi}{2} \text{ if } n \text{ is even}\end{array}\right]$

$f(x) = \frac{\pi}{2}$

31. (c) :

$\lim_{n\to\infty} \frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\left(\frac{4}{n^2}\right) + ... + \frac{1}{n}\sec^2 1$

$= \lim_{n\to\infty} \frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\left(\frac{4}{n^2}\right) + ... + \frac{n}{n^2}\sec^2\left(\frac{n^2}{n^2}\right)$

$= \lim_{n\to\infty} \sum_{r=1}^{r=n} \left(\frac{r}{n^2}\right)\sec^2\left(\frac{r}{n}\right)^2$

$= \lim_{n\to\infty} \sum_{r=0}^{r=n} \frac{1}{n}\left(\frac{r}{n}\right)\sec^2\left(\frac{r}{n}\right)^2$

$= \int_0^1 x\sec^2(x^2)dx = \frac{1}{2}\tan 1.$

32. (b) : Using fact

$\int_a^b \frac{f(x)}{f(a+b+x)+f(x)} = \int_a^b f(x)dx = \frac{b-a}{2}$

$\therefore \quad \int_3^6 \frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}dx = \frac{6-3}{2} = \frac{3}{2}$

33. (d) : Let $I = \int_0^{\pi} xf(\sin x)dx$ ..... (i)

$I = \int_0^{\pi} (\pi - x) f(\sin x)dx$ ..... (ii)

using $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

By (i) & (ii) on adding

$\therefore \quad I = \frac{\pi}{2}\int_0^{\pi} f(\sin x)dx = 2\frac{\pi}{2}\int_0^{\frac{\pi}{2}} f(\sin x)dx$

[using $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$ if $f(2a-x) = f(x)$]

$= \pi\int_0^{\frac{\pi}{2}} f\left(\sin(\frac{\pi}{2}-x)\right)dx = \pi\int_0^{\frac{\pi}{2}} f(\cos x)dx$

34. (c) : Let $I = \int_{\frac{-3\pi}{2}}^{\frac{-\pi}{2}} \left[(x+\pi)^3 + \cos^2(x+3\pi)\right]dx$

Putting $x + \pi = z$

also $x = \frac{-\pi}{2} \Rightarrow z = \frac{\pi}{2}$ and $x = \frac{-3\pi}{2} \Rightarrow z = \frac{-\pi}{2}$

$\therefore \quad dx = dz$

and $\quad x + 3\pi = z + 2\pi$

$\therefore \quad l = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} [z^3 + \cos^2(2\pi + z)]dz$

$= \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} z^3 dz + \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos^2 z\, dz$

$= 0$ (an odd function) $+2\int_0^{\frac{\pi}{2}} \cos^2 z\, dz$

$= 0 + 2\frac{1}{2}\times\frac{\pi}{2}$

$\left\{\text{using fact } \int_0^{\frac{\pi}{2}} \sin^n x\, dx\right.$

$= \left\{\begin{array}{ll} \frac{n-1}{n}\cdot\frac{n-3}{n-2}......\frac{1}{2}\times\frac{\pi}{2} & \text{if } \quad n = 2m \\ \frac{n-1}{n}\frac{n-3}{n-2}......\frac{2}{2} & \text{if } n = 2m+1 \end{array}\right\}$

$= \frac{\pi}{2}$

35. (b) : $\int_2^a [x]f'(x)dx$, say $[a] = K$ such that $a > 1$

$= \int_1^2 1f'(x)dx + \int_2^3 2f'(x)dx + ... + \int_{K-1}^{K} (K-1)f'(x)dx + \int_K^a Kf'(x)dx$

$= f(2) - f(1) + 2[f(3) - f(2)] + 3[f(4) - f(3)] + ...$
$(K-1)\,[f(K) - f(K-1)] + K[f(a) - f(K)]$

$= -\,[f(1) + f(2) + ... + f(K)] + K\,f(a)$

$= [a]\,f(a) - [f(1) + f(2) + ... + f([a])]$

36. (c) : $F(x)=\int_1^x \frac{\ln t}{1+t}dt+\int_1^{1/x}\frac{\ln t}{1+t}dt$

$$F(x)=\int_1^x\left(\frac{\ln t}{1+t}+\frac{\ln t}{(1+t)t}\right)dt=\int_1^x\frac{\ln t}{t}dt=\frac{1}{2}(\ln x)^2$$

$F(e) = 1/2$.

37. $\left[\sec^{-1}t\right]_{\sqrt{2}}^{x}=\frac{\pi}{2}$

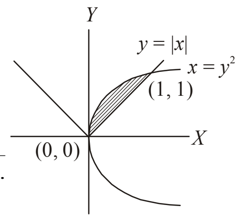
$\sec^{-1}x-\sec^{-1}\sqrt{2}=\frac{\pi}{2}\Rightarrow \sec^{-1}x=\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$

$x=-\sqrt{2}$. There is no correct option.

38. (c) : $\frac{1}{2}\int\frac{dx}{\sin\left(x+\frac{\pi}{6}\right)}=\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+c.$

39. (a) : Required area

$=\int_0^1(\sqrt{x}-x)dx$



$=\left[\frac{2}{3}x^{3/2}-\frac{x^2}{2}\right]_0^1=\frac{2}{3}-\frac{1}{2}=\frac{1}{6}.$

40. (c) : In the interval of integration $\sin x < x$

$$I=\int_0^1\frac{\sin x}{\sqrt{x}}dx<\int_0^1\frac{x}{\sqrt{x}}dx=\int_0^1\sqrt{x}dx=\left[\frac{2}{3}x^{3/2}\right]_0^1=\frac{2}{3}$$

$\therefore\ I<\frac{2}{3}$

Also $J=\int_0^1\frac{\cos x}{\sqrt{x}}dx<\int_0^1\frac{1}{\sqrt{x}}dx=[2\sqrt{x}]_0^1=2$

$\therefore\ J<2$

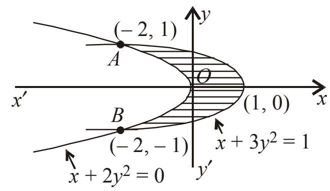
41. (a) : Solution $x + 2y^2 = 0$ and $x + 3y^2 = 1$ we have

$1-3y^2=-2y^2 \Rightarrow y^2=1 \quad \therefore \quad y=\pm 1$

$y=-1 \Rightarrow x=-2$

$y=1 \Rightarrow x=-2$

The bounded region is as under



The desired area $=2\int_0^1[(1-3y^2)-(-2y^2)]dy$

$=2\int_0^1(1-y^2)dy=2\left[y-\frac{y^3}{3}\right]_0^1$

$=2\times\frac{2}{3}=\frac{4}{3}$ sq. units

42. (d) : $\sqrt{2}\int\frac{\sin x}{\sin\left(x-\frac{\pi}{4}\right)}dx$

$$=\sqrt{2}\int\frac{\sin\left(x-\frac{\pi}{4}+\frac{\pi}{4}\right)}{\sin\left(x-\frac{\pi}{4}\right)}dx$$

$$=\sqrt{2}\int\left[\cos\frac{\pi}{4}+\cot\left(x-\frac{\pi}{4}\right)\sin\frac{\pi}{4}\right]dx$$

$$=\sqrt{2}\cdot\frac{1}{\sqrt{2}}x+\sqrt{2}\cdot\frac{1}{\sqrt{2}}\ln\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$$

$$=x+\ln\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$$

c being a constant of integration.

CHAPTER 11

Differential Equations

1. If $y = (x + \sqrt{1+x^2})^n$, then

$$(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} \text{ is}$$

(a) n^2y (b) $-n^2y$
(c) $-y$ (d) $2x^2y$.

(2002)

2. The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$

(a) $\frac{1}{4}e^{-2x}$ (b) $\frac{1}{4}e^{-2x} + cx + d$
(c) $\frac{1}{4}e^{-2x} + cx^2 + d$ (d) $\frac{1}{4}e^{-2x} + c + d$.

(2002)

3. The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{\frac{2}{3}} = 4\frac{d^3y}{dx^3}$ are

(a) $1, \frac{2}{3}$ (b) 3, 1
(c) 3, 3 (d) 1, 2.

(2002)

4. The degree and order of the differential equation of the family of all parabolas whose axis is x-axis, are respectively

(a) 1, 2 (b) 3, 2
(c) 2, 3 (d) 2, 1.

(2003)

5. The solution of the differential equation

$$(1+y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0 \text{ is}$$

(a) $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$
(b) $xe^{\tan^{-1} y} = \tan^{-1} y + k$
(c) $xe^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$
(d) $(x-2) = ke^{-\tan^{-1} y}$.

(2003)

6. If $x = e^{y + e^{y + \ldots \text{ to } \infty}}$, $x > 0$ then $\frac{dy}{dx}$ is

(a) $\frac{1-x}{x}$ (b) $\frac{1}{x}$
(c) $\frac{x}{1+x}$ (d) $\frac{1+x}{x}$.

(2004)

7. The differential equation for the family of curves $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant is

(a) $(x^2 - y^2)y' = 2xy$ (b) $2(x^2 + y^2)y' = xy$
(c) $2(x^2 - y^2)y' = xy$ (d) $(x^2 + y^2)y' = 2xy$.

(2004)

8. The solution of the differential equation

$$ydx + (x + x^2y)dy = 0 \text{ is}$$

(a) $\frac{1}{xy} + \log y = C$ (b) $-\frac{1}{xy} + \log y = C$
(c) $-\frac{1}{xy} = C$ (d) $\log y = Cx$.

(2004)

9. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where $c > 0$, is a parameter, is of order and degree as follows

(a) order 1, degree 1 (b) order 1, degree 2
(c) order 2, degree 2 (d) order 1, degree 3.

(2005)

10. If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is

(a) $x\log\left(\frac{y}{x}\right) = cy$ (b) $y\log\left(\frac{x}{y}\right) = cx$

(c) $\log\left(\frac{x}{y}\right) = cy$ (d) $\log\left(\frac{y}{x}\right) = cx$.

(2005)

11. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants is of

(a) second order and second degree

(b) first order and second degree

(c) first order and first degree

(d) second order and first degree.

(2006)

12. The differential equation of all circles passing through the origin and having their centres on the x-axis is

(a) $y^2 = x^2 + 2xy\frac{dy}{dx}$ (b) $y^2 = x^2 - 2xy\frac{dy}{dx}$

(c) $x^2 = y^2 + xy\frac{dy}{dx}$ (d) $x^2 = y^2 + 3xy\frac{dy}{dx}$.

(2007)

13. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is

(a) $(x-2)^2 y'^2 = 25 - (y-2)^2$

(b) $(x-2) y'^2 = 25 - (y-2)^2$

(c) $(y-2) y'^2 = 25 - (y-2)^2$

(d) $(y-2)^2 y'^2 = 25 - (y-2)^2$

(2008)

14. The solution of the differential equation

$$\frac{dy}{dx} = \frac{x+y}{x}$$

satisfying the condition $y(1) = 1$ is

(a) $y = x \ln x + x$ (b) $y = \ln x + x$

(c) $y = x \ln x + x^2$ (d) $y = x e^{(x-1)}$

(2008)

Answer Key

1.	(a)	**2.**	(b)	**3.**	(c)	**4.**	(a)	**5.**	(a)	**6.**	(a)
7.	(a)	**8.**	(b)	**9.**	(d)	**10.**	(d)	**11.**	(d)	**12.**	(a)
13.	(d)	**14.**	(a)								

1. **(a) :** $y_1 = n\left[x+\sqrt{1+x^2}\right]^{n-1}\left[1+\dfrac{x}{\sqrt{1+x^2}}\right]$

$y_1 = n\left[x+\sqrt{1+x^2}\right]^{n}\cdot\dfrac{1}{\sqrt{1+x^2}}$

$y_1 = \dfrac{ny}{\sqrt{1+x^2}}\quad\left(y_1=\dfrac{dy}{dx}\right)$

$\Rightarrow y_1^2(1+x^2) = n^2y^2$

$\Rightarrow y_1^2(2x) + (1+x^2)(2y_1y_2) = 2yy_1n^2$

$\Rightarrow y_2(1+x^2) + xy_1 = n^2y$

2. **(b) :** Given $\dfrac{d^2y}{dx^2} = e^{-2x}$

$\therefore\quad \dfrac{dy}{dx} = \dfrac{e^{-2x}}{-2} + c$

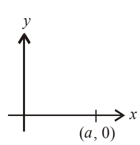
$\therefore\quad y = \dfrac{e^{-2x}}{4} + cx + d$

3. **(c) :** $\left(1+3\dfrac{dy}{dx}\right)^{\frac{2}{3}} = 4\left(\dfrac{d^3y}{dx^3}\right)$

$\Rightarrow\quad \left(1+3\dfrac{dy}{dx}\right)^{2} = \left[4\dfrac{d^3y}{dx^3}\right]^3$

$\therefore$ highest order is 3 whose exponent is also 3.

4. **(a):** As axis of parabola is x-axis which means focus lies on x-axis. Equation of such parabola's is given by



$y^2 = 4a(x-k)$ ... (i)

$\Rightarrow\quad 2yy_1 = 4a$ (by differentiating (i) w.r. to x)

$\Rightarrow\quad y\dfrac{dy}{dx} = 2a$ ... (ii)

$\Rightarrow\quad \dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2 = 0$

(by differentiating (ii) w.r. to x)

$\Rightarrow$ order 2 and degree 1 (Concept: Exponent of highest order derivative is called degree and order of that derivative is called order of the differential equation.)

5. **(a) :** From the given equation

$(1+y^2)\dfrac{dx}{dy} + 1x = e^{\tan^{-1}y}$

$\Rightarrow\quad \dfrac{dx}{dy} + \dfrac{1}{1+y^2}x = \dfrac{e^{\tan^{-1}y}}{1+y^2}$

$\Rightarrow x\cdot\text{I.F} = \int y\cdot\text{I.F}\,dy$

where $\text{I.F} = e^{\int\frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

$\Rightarrow\quad x\,e^{\tan^{-1}y} = \dfrac{e^{2\tan^{-1}y}}{2} + c$

$\Rightarrow\quad 2x\,e^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$

6. **(a) :** $x = e^{y+e^{y+e^{y\ldots}}}\quad\Rightarrow x = e^{y+x}$

Differentiate *w.r.* to x after taking logarithm both sides

$\therefore\quad \dfrac{1}{x} = 1 + \dfrac{dy}{dx}$

$\Rightarrow\quad \dfrac{dy}{dx} = \dfrac{1-x}{x}$

7. **(a) :** Given family of curve is

$x^2 + y^2 - 2ay = 0$ ...(1)

$\Rightarrow\quad 2a = \dfrac{x^2+y^2}{y}$

Also from (1) $2x + 2yy' - 2a\,y' = 0$

$\Rightarrow\quad 2x + 2yy' - \left(\dfrac{x^2+y^2}{y}\right)y' = 0$

$\Rightarrow\quad 2xy + y'(2y^2 - x^2 - y^2) = 0$

$\Rightarrow\quad y'(x^2 - y^2) = 2xy$

8. **(b) :** $y\,dx = -(x^2y + x)\,dy$

$\Rightarrow\quad ydx + xdy = -x^2y\,dy$

$$\Rightarrow \quad \frac{ydx + xdy}{(xy)^2} = \frac{-dy}{y}$$

$$\Rightarrow \quad \frac{\frac{d}{dx}(xy)}{(xy)^2} = -\frac{dy}{y}$$

$$-\frac{1}{xy} = -\log y + k$$

$$\Rightarrow \quad -\frac{1}{xy} + \log y = c$$

9. **(d) :** $y^2 = 2c\left(x+\sqrt{c}\right)$ ...(*)

$\therefore\ 2yy_1 = 2c$ $\qquad \therefore\ yy_1 = c$

Now putting $c = yy_1$ in (*) we get

$$y^2 = 2\cdot yy_1\left(x+\sqrt{yy_1}\right) \Rightarrow (y^2-2xyy_1)^2 = 4(yy_1)^3$$

$\Rightarrow (y^2 - 2xyy_1)^2 = 4y^3y_1^3 \ \Rightarrow$ order 1, degree 3.

10. **(d) :** $x\frac{dy}{dx} = y(\log y - \log x + 1)$

$\therefore\ \frac{dy}{dx} = \frac{y}{x}\left(\log\left(\frac{y}{x}\right)+1\right)$ Now put $\frac{y}{x} = v$

$\therefore\ v\log v\, dx = x\, dv$

$$\Rightarrow \frac{dv}{v\log v} = \frac{dx}{x} \Rightarrow \log\left(\frac{y}{x}\right) = cx.$$

11. **(d) :** Given $A x^2 + B y^2 = 1$

As solution having two constants, $\therefore$ order of differential equation is 2 so our choices (b) & (c) are discarded from the list, only choices (a) and (b) are possible

Again $A x^2 + B y^2 = 1$ .... (*)

$$\Rightarrow \quad -\frac{A}{B} = \frac{y}{x}\frac{dy}{dx} \qquad \text{.... (i)}$$

differentiating (*) w.r. to x

Again on differentiating

$$-\frac{A}{B} = y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 \qquad \text{......(ii)}$$

By (i) and (ii) we get

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = y\left(\frac{dy}{dx}\right)$$

$\Rightarrow$ order 2 degree 1.

12. **(a) :** General equation of all such circles is $(x-h)^2 + (y-0)^2 = h^2$ .... (i) where h is parameter

$\Rightarrow (x - h)^2 + y^2 = h^2$

Differentiating, we get $2(x-h) + 2y\frac{dy}{dx} = 0$

$h = x + y\frac{dy}{dx}$ to eliminate h, putting value of h in equation .... (i) ,

$\therefore$ we get $y^2 = x^2 + 2xy\frac{dy}{dx}$.

13. **(d) :** The equation to circle is

$(x - \alpha)^2 + (y - 2)^2 = 25$ ...(1)

Differentiation w.r.t. x

$$(x-\alpha) + (y-2)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad x-\alpha = -(y-2)\frac{dy}{dx} \qquad \text{...(2)}$$

From (1) and (2) on eliminate 'α'

$$(y-2)^2\left(\frac{dy}{dx}\right)^2 + (y-2)^2 = 25$$

$\Rightarrow \quad (y - 2)^2(y')^2 = 25 - (y - 2)^2$

14. **(a) : 1st Method (Homogeneous equation):**

Let $y = vx$, so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$

we have $v + x\frac{dv}{dx} = \frac{x+vx}{x} = 1 + v$

$$\Rightarrow \quad x\frac{dv}{dx} = 1 \Rightarrow \quad dv = \frac{dx}{x}$$

$$\Rightarrow \quad v = \ln x + \ln k$$

As $v = y/x$ we have $y = x \ln x + (\ln k)x$

At $x = 1$, $y = 1$ giving

$1 = 0 + (\ln k) \quad \therefore\ \ln k = 1$, Then $y = x \ln x + x$

2nd Method (Inspection) :

Rewriting the equation

$\frac{dy}{dx} = \frac{x+y}{x}$ as

$xdy - ydx = xdx$

we have $\frac{xdy - ydx}{x^2} = \frac{dx}{x}$

$$d\left(\frac{y}{x}\right) = \frac{dx}{x}$$

on integration $\frac{y}{x} = \ln x + k$

$\Rightarrow \quad y = x \ln x + kx$

As before, evaluating constant, $y = x \ln x + x$

CHAPTER
12

Two Dimensional Geometry

1. Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are
(a) $x = \pm(y + 2a)$ (b) $y = \pm(x + 2a)$
(c) $x = \pm(y + a)$ (d) $y = \pm(x + a)$.
(2002)

2. If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subtends an angle of measure 45° at the major segment of the circle then value of m is
(a) $2 \pm \sqrt{2}$ (b) $-2 \pm \sqrt{2}$
(c) $-1 \pm \sqrt{2}$ (d) none of these
(2002)

3. If the pair of lines
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
intersect on the y-axis then
(a) $2fgh = bg^2 + ch^2$ (b) $bg^2 \neq ch^2$
(c) $abc = 2fgh$ (d) none of these
(2002)

4. The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is
(a) $4 \le x^2 + y^2 \le 64$ (b) $x^2 + y^2 \le 25$
(c) $x^2 + y^2 \ge 25$ (d) $3 \le x^2 + y^2 \le 9$
(2002)

5. The point of lines represented by
$3ax^2 + 5xy + (a^2 - 2)y^2 = 0$
and $\perp$ to each other for
(a) two values of a (b) $\forall\ a$
(c) for one value of a (d) for no values of a.
(2002)

6. Locus of mid point of the portion between the axes of $x \cos\alpha + y \sin\alpha = p$ where p is constant is
(a) $x^2 + y^2 = \dfrac{4}{p^2}$ (b) $x^2 + y^2 = 4p^2$
(c) $\dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{2}{p^2}$ (d) $\dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{4}{p^2}$.
(2002)

7. The centre of the circle passing through (0, 0) and (1, 0) and touching the circle $x^2 + y^2 = 9$ is
(a) $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ (b) $\left(\dfrac{1}{2}, -\sqrt{2}\right)$
(c) $\left(\dfrac{3}{2}, \dfrac{1}{2}\right)$ (d) $\left(\dfrac{1}{2}, \dfrac{3}{2}\right)$
(2002)

8. The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length $3a$ is
(a) $x^2 + y^2 = 9a^2$ (b) $x^2 + y^2 = 16a^2$
(c) $x^2 + y^2 = 4a^2$ (d) $x^2 + y^2 = a^2$.
(2002)

9. A triangle with vertices (4, 0), (–1, –1), (3, 5) is
(a) isosceles and right angled
(b) isosceles but not right angled
(c) right angled but not isosceles
(d) neither right angled nor isosceles
(2002)

10. The foci of the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{b^2} = 1$ and the hyperbola $\dfrac{x^2}{144} - \dfrac{y^2}{81} = \dfrac{1}{25}$ coincide. Then the value of b^2 is
(a) 5 (b) 7 (c) 9 (d) 1.
(2003)

11. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then

(a) $t_2 = -t_1 + \frac{2}{t_1}$ (b) $t_2 = t_1 - \frac{2}{t_1}$

(c) $t_2 = t_1 + \frac{2}{t_1}$ (d) $t_2 = -t_1 - \frac{2}{t_1}$.

(2003)

12. If the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then

(a) $r < 2$ (b) $r = 2$

(c) $r > 2$ (d) $2 < r < 8$.

(2003)

13. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq. units. Then the equation of the circle is

(a) $x^2 + y^2 + 2x - 2y = 47$

(b) $x^2 + y^2 - 2x + 2y = 47$

(c) $x^2 + y^2 - 2x + 2y = 62$

(d) $x^2 + y^2 + 2x - 2y = 62$.

(2003)

14. A square of side a lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle α $(0 < \alpha < \pi/4)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is

(a) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$

(b) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$

(c) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

(d) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$.

(2003)

15. If the pairs of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then

(a) $p = -q$ (b) $pq = 1$

(c) $pq = -1$ (d) $p = q$.

(2003)

16. Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t)$, $(b\sin t, -b\cos t)$ and $(1, 0)$, where t is a parameter, is

(a) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

(b) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$

(c) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$

(d) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$.

(2003)

17. If the equation of the locus of point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then $c =$

(a) $a_1^2 - a_2^2 + b_1^2 - b_2^2$

(b) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$

(c) $\sqrt{(a_1^2 + b_1^2 - a_2^2 - b_2^2)}$

(d) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$.

(2003)

18. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

(a) $\left(\frac{-9}{8}, \frac{9}{2}\right)$ (b) $(2, -4)$

(c) $(2, 4)$ (d) $\left(\frac{9}{8}, \frac{9}{2}\right)$

(2004)

19. The normal to the curve $x = a(1 + \cos\theta)$, $y = a\sin\theta$ at θ always passes through the fixed point

(a) $(0, 0)$ (b) $(0, a)$ (c) $(a, 0)$ (d) (a, a).

(2004)

20. Let $A(2, -3)$ and $B(-2, 1)$ be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line

(a) $3x + 2y = 5$ (b) $2x - 3y = 7$

(c) $2x + 3y = 9$ (d) $3x - 2y = 3$.

(2004)

21. The equation of the straight line passing through the point $(4, 3)$ and making intercepts on the co-ordinate axes whose sum is -1 is

(a) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$

(b) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = 1$

(c) $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

(d) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$.

(2004)

22. If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product, then c has the value

(a) 2 (b) -1 (c) 1 (d) -2.

(2004)

23. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then c equals

(a) 3 (b) -1 (c) 1 (d) -3.

(2004)

24. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is

(a) $2ax - 2by + (a^2 + b^2 + 4) = 0$
(b) $2ax + 2by - (a^2 + b^2 + 4) = 0$
(c) $2ax + 2by + (a^2 + b^2 + 4) = 0$
(d) $2ax - 2by - (a^2 + b^2 + 4) = 0$.

(2004)

25. A variable circle passes through the fixed point $A(p, q)$ and touches x-axis. The locus of the other end of the diameter through A is

(a) $(y - p)^2 = 4qx$ (b) $(x - q)^2 = 4py$
(c) $(x - p)^2 = 4qy$ (d) $(y - q)^2 = 4px$.

(2004)

26. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameters of a circle of circumference 10π, then the equation of the circle is

(a) $x^2 + y^2 + 2x + 2y - 23 = 0$
(b) $x^2 + y^2 - 2x - 2y - 23 = 0$
(c) $x^2 + y^2 - 2x + 2y - 23 = 0$
(d) $x^2 + y^2 + 2x - 2y - 23 = 0$.

(2004)

27. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is

(a) $x^2 + y^2 + x + y = 0$
(b) $x^2 + y^2 - x + y = 0$
(c) $x^2 + y^2 - x - y = 0$
(d) $x^2 + y^2 + x - y = 0$.

(2004)

28. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then

(a) $d^2 + (2b - 3c)^2 = 0$
(b) $d^2 + (3b + 2c)^2 = 0$
(c) $d^2 + (2b + 3c)^2 = 0$
(d) $d^2 + (3b - 2c)^2 = 0$.

(2004)

29. The eccentricity of an ellipse, with its centre at the origin, is 1/2. If one of the directrices is $x = 4$, then the equation of the ellipse is :

(a) $4x^2 + 3y^2 = 12$ (b) $3x^2 + 4y^2 = 12$
(c) $3x^2 + 4y^2 = 1$ (d) $4x^2 + 3y^2 = 1$.

(2004)

30. If the straight lines $x = 1 + s$, $y = -3 - \lambda s$, $z = 1 + \lambda s$ and $x = \frac{t}{2}$, $y = 1 + t$, $z = 2 - t$, with parameters s and t respectively, are co-planar, then λ equals

(a) $-1/2$ (b) -1 (c) -2 (d) 0.

(2004)

31. Let P be the point $(1, 0)$ and Q a point on the locus $y^2 = 8x$. The locus of mid point of PQ is

(a) $x^2 - 4y + 2 = 0$ (b) $x^2 + 4y + 2 = 0$
(c) $y^2 + 4x + 2 = 0$ (d) $y^2 - 4x + 2 = 0$.

(2005)

32. The line parallel to the x-axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is

(a) below the x-axis at a distance of 2/3 from it
(b) below the x-axis at a distance of 3/2 from it
(c) above the x-axis at a distance of 2/3 from it
(d) above the x-axis at a distance of 3/2 from it.

(2005)

33. If non-zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ always passes through a fixed point. That point is

(a) $(-1, -2)$ (b) $(-1, 2)$
(c) $\left(1, -\frac{1}{2}\right)$ (d) $(1, -2)$.

(2005)

34. If a vertex of a triangle is $(1, 1)$ and the mid points of two sides through this vertex are $(-1, 2)$ and $(3, 2)$, then the centroid of the triangle is

(a) $\left(\frac{-1}{3}, \frac{7}{3}\right)$ (b) $\left(-1, \frac{7}{3}\right)$
(c) $\left(\frac{1}{3}, \frac{7}{3}\right)$ (d) $\left(1, \frac{7}{3}\right)$.

(2005)

35. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q then the line $5x + by - a = 0$ passes through P and Q for

(a) no value of a

(b) exactly one value of a

(c) exactly two values of a

(d) infinitely many values of a. *(2005)*

36. A circle touches the x-axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is

(a) a circle (b) an ellipse

(c) a parabola (d) a hyperbola. *(2005)*

37. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is

(a) $2ax + 2by - (a^2 - b^2 + p^2) = 0$

(b) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$

(c) $2ax + 2by - (a^2 + b^2 + p^2) = 0$

(d) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$. *(2005)*

38. An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{4}$. *(2005)*

39. The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

(a) a circle (b) an ellipse

(c) a hyperbola (d) a parabola. *(2005)*

40. If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then

(a) $3a^2 - 2ab + 3b^2 = 0$

(b) $3a^2 - 10ab + 3b^2 = 0$

(c) $3a^2 + 2ab + 3b^2 = 0$

(d) $3a^2 + 10ab + 3b^2 = 0$. *(2005)*

41. A straight line through the point $A(3, 4)$ is such that its intercept between the axes is bisected at A. Its equation is,

(a) $x + y = 7$ (b) $3x - 4y + 7 = 0$

(c) $4x + 3y = 24$ (d) $3x + 4y = 25$. *(2006)*

42. The locus of the vertices of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is

(a) $xy = \frac{105}{64}$ (b) $xy = \frac{3}{4}$

(c) $xy = \frac{35}{16}$ (d) $xy = \frac{64}{105}$. *(2006)*

43. In an ellipse, the distance between its focii is 6 and minor axis is 8. Then its eccentricity is,

(a) 3/5 (b) 1/2

(c) 4/5 (d) $1/\sqrt{5}$. *(2006)*

44. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, then the equation of the circle is

(a) $x^2 + y^2 + 2x - 2y - 47 = 0$

(b) $x^2 + y^2 + 2x - 2y - 62 = 0$

(c) $x^2 + y^2 - 2x + 2y - 62 = 0$

(d) $x^2 + y^2 - 2x + 2y - 47 = 0$. *(2006)*

45. Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of chord of the circle C that subtend an angle of $2\pi/3$ at its centre is

(a) $x^2 + y^2 = \frac{3}{2}$ (b) $x^2 + y^2 = 1$

(c) $x^2 + y^2 = \frac{27}{4}$ (d) $x^2 + y^2 = \frac{9}{4}$. *(2006)*

46. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}$, $x > 0$ and $y = 3x$, $x > 0$, then a belongs to

(a) $\left(0, \frac{1}{2}\right)$ (b) $(3, \infty)$

(c) $\left(\frac{1}{2}, 3\right)$ (d) $\left(-3, -\frac{1}{2}\right)$. *(2006)*

47. For the Hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant when α varies ?

(a) abscissae of vertices

(b) abscissae of foci

(c) eccentricity (d) directrix. *(2007)*

48. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is

(a) (2, 4) (b) (–2, 0)
(c) (–1, 1) (d) (0, 2). *(2007)*

49. Let $A(h, k)$, $B(1, 1)$ and $C(2, 1)$ be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1 square unit, then the set of values which 'k' can take is given by

(a) {–1, 3} (b) {–3, –2}
(c) {1, 3} (d) {0, 2}. *(2007)*

50. Let $P = (-1, 0)$, $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR is

(a) $\frac{\sqrt{3}}{2}x + y = 0$ (b) $x + \sqrt{3y} = 0$
(c) $\sqrt{3}x + y = 0$ (d) $x + \frac{\sqrt{3}}{2}y = 0.$

(2007)

51. If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines $xy = 0$, then m is

(a) 1 (b) 2 (c) –1/2 (d) –2.
(2007)

52. Consider a family of circles which are passing through the point (–1, 1) and are tangent to x-axis. If (h, k) are the coordinate of the centre of the circles, then the set of values of k is given by the interval

(a) $-\frac{1}{2} \le k \le \frac{1}{2}$ (b) $k \le \frac{1}{2}$
(c) $0 \le k \le \frac{1}{2}$ (d) $k \ge \frac{1}{2}$. *(2007)*

53. The normal to a curve at $P(x, y)$ meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a

(a) circle (b) hyperbola
(c) ellipse (d) parabola. *(2007)*

54. The perpendicular bisector of the line segment joining $P(1, 4)$ and $Q(k, 3)$ has y-intercept – 4. Then a possible value of k is

(a) – 4 (b) 1 (c) 2 (d) – 2
(2008)

55. A parabola has the origin as its focus and the line $x = 2$ as the directrix. Then the vertex of the parabola is at

(a) (2, 0) (b) (0, 2) (c) (1, 0) (d) (0, 1)
(2008)

56. The point diametrically opposite to the point P(1, 0) on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is

(a) (3, 4) (b) (3, – 4)
(c) (– 3, 4) (d) (– 3, – 4)
(2008)

57. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is

(a) $\frac{5}{3}$ (b) $\frac{8}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$
(2008)

Answer Key

1.	(b)	**2.**	(c)	**3.**	(a)	**4.**	(a)	**5.**	(a)	**6.**	(d)
7.	(b)	**8.**	(c)	**9.**	(a)	**10.**	(b)	**11.**	(d)	**12.**	(d)
13.	(b)	**14.**	(c)	**15.**	(c)	**16.**	(a)	**17.**	(d)	**18.**	(d)
19.	(c)	**20.**	(c)	**21.**	(d)	**22.**	(a)	**23.**	(d)	**24.**	(b)
25.	(c)	**26.**	(c)	**27.**	(c)	**28.**	(c)	**29.**	(b)	**30.**	(c)
31.	(d)	**32.**	(b)	**33.**	(d)	**34.**	(d)	**35.**	(a)	**36.**	(c)
37.	(c)	**38.**	(b)	**39.**	(c)	**40.**	(c)	**41.**	(c)	**42.**	(a)
43.	(a)	**44.**	(d)	**45.**	(d)	**46.**	(c)	**47.**	(b)	**48.**	(b)
49.	(a)	**50.**	(c)	**51.**	(a)	**52.**	(d)	**53.**	(b), (c)	**54.**	(a)
55.	(c)	**56.**	(d)	**57.**	(b)						

1 **(b) :** Let common tangent to the curves be $y = mx + c$ ...(1)

$= mx + \frac{a}{m}$

and $\therefore$ $y^2 = 8ax = 4(2a)x$

$\therefore$ Equation of tangent to parabola

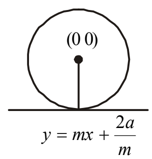
$y = mx + \frac{2a}{m}$ ...(2)

which is also tangent to the circle

$$x^2 + y^2 = 2a^2 = (\sqrt{2}a)^2$$

Now Distance from (0, 0) to the tangent line = Radius of circle

$\therefore$ $\sqrt{2}\,a = \pm \frac{2a}{m} \times \frac{1}{\sqrt{1 + m^2}}$



$\Rightarrow$ $m^2(1 + m^2) - 2 = 0$

$\Rightarrow$ $(m^2 - 1)(m^2 + 2) = 0$

$\Rightarrow$ $m = \pm 1$

Required equation of tangent $y = mx + \frac{2a}{m}$

$= \pm (x + 2a)$

2. **(c) :** Equation of chord $y = mx + 1$

Equation of circle $x^2 + y^2 = 1$

Joint equation of the curve through the intersection of line and circle be given by $x^2 + y^2 = (y - mx)^2$, (By homorizing the equation)

$\Rightarrow$ $x^2(1 - m^2) + 2mxy = 0$

Now $\tan\theta = \pm \frac{2\sqrt{h^2 - ab}}{a + b}$ where $\begin{cases} a = 1 - m^2 \\ h = m, b = 0 \end{cases}$

$$\tan 45° = \pm \frac{2\sqrt{m^2 - 0}}{1 - m^2}$$

$\Rightarrow$ $1(1 - m^2) = \pm 2m$

$\Rightarrow$ $m^2 \pm 2m - 1 = 0$

$\Rightarrow$ $m = \pm 1 \pm \sqrt{2}$

$\Rightarrow$ $m = 1 \pm \sqrt{2}$ and $-1 \pm \sqrt{2}$

3. **(a) :** As $s = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a pair of line

$\therefore$ $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$

or $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ ...(1)

Now say point of intersection on y axis be $(0, y_1)$ and point of intersection of pair of line be obtained by solving the equations $\frac{\partial s}{\partial x} = 0 = \frac{\partial s}{\partial y}$

$\therefore$ $\frac{\partial s}{\partial x} = 0 \Rightarrow ax + by + g = 0 \Rightarrow$

and $\frac{\partial s}{\partial y} = 0 \Rightarrow hx + by + f = 0 \Rightarrow$

$\begin{cases} hy_1 + g = 0 \\ by_1 + f = 0 \end{cases}$ (*)

On comparing the equation given in (*) we get $bg = fh$ and $bg^2 = fgh$ ...(2)

Again $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ meet at y-axis $\therefore$ $x = 0$

$\Rightarrow$ $by^2 + 2fy + c = 0$ whose roots must be equal

$\therefore$ $f^2 = bc$

$af^2 = abc$ ...(3)

Now using (2) and (3) in equation (1) we have

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow$ $(abc - af^2) + (fgh - bg^2) + fgh - ch^2 = 0$

$\Rightarrow$ $0 + 0 + fgh - ch^2 = 0$

$\therefore$ $ch^2 = fgh$ ...(4)

Now adding (2) and (4)

$2fgh = ch^2 + bg^2$

4. **(a) :** Let (α, β) is the centre of the circle whose radius is 3.

$\therefore$ Equation of such circle be

$(x - \alpha)^2 + (y - \beta)^2 = 3^2$

$\Rightarrow$ $\alpha^2 + \beta^2 - 2\alpha x - 2\beta y + 25 = 9$

$\Rightarrow$ $x^2 + y^2 - 2x^2 - 2y^2 + 25 = 9$

$\Rightarrow$ $x^2 + y^2 = 25 - 9$

$\Rightarrow$ $x^2 + y^2 = 16$ and $x^2 + y^2 = 25$

$\Rightarrow$ $4 \le x^2 + y^2 \le 64$

5. **(a) :** Using fact : Pair of lines $Ax^2 + 2hxy + By^2 = 0$ are b to each other if $A + B = 0$

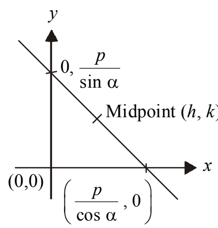
$\Rightarrow \quad 3a + a^2 - 2 = 0$

$\Rightarrow \quad a^2 + 3a - 2 = 0$

$\Rightarrow$ There exist two value of a as $D > 0$

$\therefore \; a = \dfrac{-3 \pm \sqrt{17}}{2}$

6. **(d) :** $\therefore$ (h, k) is $\left(\dfrac{p}{2\cos\alpha}, \dfrac{p}{2\sin\alpha}\right)$



$\therefore \quad \cos\alpha = \dfrac{p}{2h},$

$\sin\alpha = \dfrac{p}{2k}$

$\Rightarrow \quad \sin^2\alpha + \cos^2\alpha$

$= \dfrac{p^2}{4h^2} + \dfrac{p^2}{4k^2}$

$= \dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{4}{p^2}$

7. **(b) :** Let equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$

As it passes through (0, 0) so $c = 0$

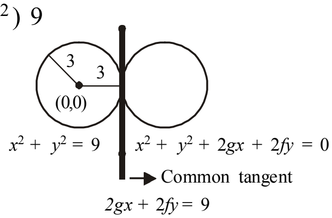
and as it passes (1, 0) so $-g = \dfrac{1}{2}$.

Now $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 = 9$ touches each other.

$\therefore$ equation of common tangent is $2gx + 2fy - 9 = 0$ and distance from the centre of circle $x^2 + y^2 = 9$ to the common tangent is equal to the radius of the circle $x^2 + y^2 = 9$

$\therefore \quad \dfrac{|0 + 0 - 9|}{\sqrt{4g^2 + 4f^2}} = 3$

$\Rightarrow 3^2 = 4(g^2 + f^2)\ 9$



$= 4\left(\dfrac{1}{4} + f^2\right)$

$9 = 1 + 4f^2$

$\therefore \quad f^2 = 2$

$f = \pm\sqrt{2}$

$\therefore \quad -f = \pm\sqrt{2}$

$\therefore$ Centre of the required circle be

$\left(\dfrac{1}{2}, \sqrt{2}\right), \left(\dfrac{1}{2}, -\sqrt{2}\right)$

8. **(c) :** Given median of the equilateral triangle is $3a$.

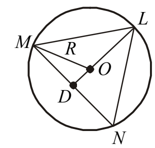
In $\Delta\, LMD$, $(LM)^2 = (LD)^2 + (MD)^2$

$(LM)^2 = 9a^2 + \left(\dfrac{LM}{2}\right)^2$

$\Rightarrow \dfrac{3}{4}(LM)^2 = 9a^2 \qquad \therefore\ (LM)^2 = 12a^2$

Again in triangle OMD, $(OM)^2 = (OD)^2 + (MD)^2$

$R^2 = (3a - R)^2 + \left(\dfrac{LM}{2}\right)^2$



$\Rightarrow R^2 = 9a^2 + R^2 - 6aR + 3a^2$

$\Rightarrow 6aR = 12a^2$

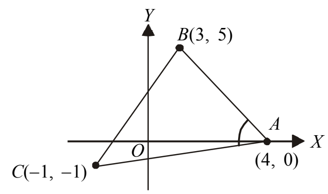
$R = 2a$

So equation of circle be

$(x - 0)^2 + (y - 0)^2 = R^2 = (2a)^2$

$\Rightarrow x^2 + y^2 = 4a^2$

9. **(a) :** $AB = \sqrt{26}$, $AC = \sqrt{26}$



$\therefore \quad ABC$ is isosceles

Again product of the slope of $\overline{AC}$ and $\overline{AB}$

$= \dfrac{1}{5} \times (-5) = -1$

$\Rightarrow \quad AC$ b AB

$\Rightarrow$ right at A

10. **(b) :** Eccentricity for $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

is $b^2 = a^2(1 - e^2)$

and eccentricity for $\dfrac{x^2}{\frac{144}{25}} - \dfrac{y^2}{\frac{81}{25}} = 1$ is

$$\left\{\begin{array}{l} e_1 = \dfrac{a_1^2 + b_1^2}{a_1^2} \\ \therefore e_1 = \sqrt{1 + \dfrac{81}{144}} = \dfrac{15}{12} \end{array}\right.$$

Again foci $= a_1 e_1 = \dfrac{12}{5} \times \dfrac{15}{12} = 3$

$\therefore$ focus of hyperbola is $(3, 0) = (ae, 0)$

so focus of ellipse $(ae, 0) = (4e, 0)$

As their foci are same $\therefore\ 4e = 3 \qquad \therefore\ e = 3/4$

$\therefore\ e^2 = 1 - \left(\dfrac{b}{a}\right)^2$

$= 1 - \dfrac{b^2}{16}$

or $\dfrac{b^2}{16} = 1 - e^2 = 1 - \dfrac{9}{16}$

$b^2 = 7$

11. **(d) :** Since the normal at $(bt_1^2, 2bt_1)$, on parabola $y^2 = 4bx$ meet the parabola again at $(bt_2^2, 2bt_2)$ therefore $t_1x + y = 2bt_1 + bt_1^3$ passes through $(bt_2^2, 2bt_2)$

$\Rightarrow\quad t_1bt_2^2 + 2bt_2 = 2bt_1 + bt_1^3$

$\Rightarrow\quad t_1(t_2^2 - t_1^2) = 2(t_1 - t_2)$

$\Rightarrow\quad t_1(t_2 + t_1) = -2$

$\Rightarrow\quad t_2 + t_1 = -\dfrac{2}{t_1}$

$\Rightarrow\quad t_2 = -\dfrac{2}{t_1} - t_1$

12. **(d) :** $(x - 1)^2 + (y - 3)^2 = r^2\ \therefore\ C_1(1, 3)$ and $r_1 = t_1 = r$

$(x - 4)^2 + 1(y + 1)^2 = 9\ \therefore\ C_2(4, -1)$ and $r_2 = t_2 = 3$

so $C_1C_2 = \sqrt{(4-1)^2 + (3+1)^2} = 5$

Now for intersecting circles

$r_1 + r_2 > C_1C_2$ and $|r_1 - r_2| < C_1C_2$

$\therefore \Rightarrow r + 3 > 5$ and $|r - 3| < 5$

$\Rightarrow\quad r > 2$ and $-5 < r - 3 < 5$

$\Rightarrow\quad r > 2$ and $-2 < r < 8$

$\Rightarrow\quad r \in (2, 8)$

13. **(b) :** Co-ordinate of centre may be $(1, -1)$ or $(-1, 1)$ but $1, -1$ satisfies the given equations of diameter, so choices (a) and (d) are out of court.

Again $\pi R^2 = 154,\ \ R^2 = 49\ \therefore\ R = 7$

$\therefore$ Required equation of circle be

$(x - 1)^2 + (y + 1)^2 = 49$

$\Rightarrow x^2 + y^2 - 2x + 2y = 47$

14. **(c) :** According to the problem square lies above x-axis

Now equation of AB using two point form. We get

$y - y_1 = m(x - x_1)$

$(y - a\sin\alpha) = -\dfrac{\cancel{a}(\cos\alpha - \sin\alpha)}{\cancel{a}(\cos\alpha + \sin\alpha)}\ [x - a\cos\alpha]$

$\Rightarrow y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha)$

$= a\sin\alpha(\cos\alpha + \sin\alpha) + a\cos\alpha\,(\cos\alpha - \sin\alpha)$

$= a(\sin^2\alpha + \cos^2\alpha)$

$= a(1)$

15. **(c) :** Given equations are

$x^2 - 2qxy - y^2 = 0 \qquad ...(1)$

$x^2 - 2pxy - y^2 = 0 \qquad ...(2)$

Joint equation of angle bisector of the line (i) and (ii) are same

$\therefore\quad \dfrac{x^2 - y^2}{1+1} = \dfrac{xy}{-q}$

$\Rightarrow\quad qx^2 + 2xy - qy^2 = 0 \qquad ...(3)$

Now (2) and (3) are same, taking ratio of their coefficients

$\therefore\quad \dfrac{1}{q} = \dfrac{-p}{1}$

$\Rightarrow\quad pq = -1$

16. **(a) :** Let (h, k) be the co-ordinate of centroid

$\therefore\quad h = \dfrac{a\cos t + b\sin t + 1}{3}$,

$k = \dfrac{a\sin t - b\cos t + 0}{3}$

$\Rightarrow\quad 3h - 1 = a\cos t + b\sin t \qquad ...(i)$

$3k = a\sin t - b\cos t \qquad ...(ii)$

By squaring (i) and (ii) then adding we get

$(3h - 1)^2 + (3k)^2$

$= a^2(\cos^2 t + \sin^2 t) + b^2(\cos^2 t + \sin^2 t)$

replacing (h, k) by (x, y) we get choice (a) is correct.

17. **(d) :** Let α, β is the point of locus, equidistant from (a_1, b_1) and (a_2, b_2) is given by

$(\alpha - a_1)^2 + (\beta - b_1)^2 = (\alpha - a_2)^2 + (\beta - b_2)^2$

$\Rightarrow a_1^2 + b_1^2 - 2a_1\alpha - 2b_1\beta - a_2^2 - b_2^2 + 2a_2\alpha + 2b_2\beta = 0$

$\Rightarrow 2(a_2 - a_1)\alpha + 2(b_2 - b_1)\beta + a_1^2 + b_1^2 - b_2^2 - a_2^2 = 0$

$\Rightarrow (a_2 - a_1)x + (b_2 - b_1)y + \dfrac{1}{2}(a_1^2 + b_1^2 - a_2^2 - b_2^2) = 0$

$\Rightarrow c = -\dfrac{1}{2}[a_1^2 + b_1^2 - a_2^2 - b_2^2]$

18. **(d) :** Given $y^2 = 18x$ and $\frac{dy}{dt} = 2\frac{dx}{dt}$

$\therefore \quad 2y\frac{dy}{dt} = 18\frac{dx}{dt}$

$\Rightarrow 2y\frac{dy}{dt} = \frac{18}{2}\frac{dy}{dt}$

$\Rightarrow y = 9/2 \quad \therefore x = \frac{y^2}{18} = \frac{81}{72} = \frac{9}{8}$

So the required point is $x = \frac{9}{8}$, $y = 9/2$

19. **(c) :** The equation of normal at θ is

$y - y_1 = -\frac{1}{\frac{dy}{dx}}(x - x_1)$

$\Rightarrow \quad y - a\sin\theta = \frac{\sin\theta}{\cos\theta}(x - a(1 - \cos\theta))$

which passes through $(a, 0)$

20. **(c) :** Let locus of point $C(h, k)$ and centroid (α, β)

As (α, β) lies on $2x + 3y = 1 \therefore 2\alpha + 3\beta = 1$

Now centroid of ABC is

$\left(\frac{2+(-2)+h}{3} \frac{-3+1+k}{3}\right)$ or $\left(\frac{h}{3}, \frac{k-2}{3}\right)$

$\therefore \quad 2\left(\frac{h}{3}\right) + \frac{3(k-2)}{3} = 1$

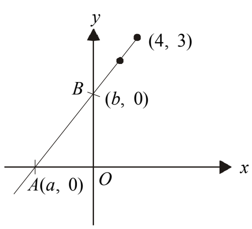
$\Rightarrow \quad 2h + 3k = 9$

$\Rightarrow \quad 2x + 3y = 9$

21. **(d) :** Given $OA + OB = -1$

i.e. $\quad a + b = -1$

$\therefore$ equation of the line be



$\frac{x}{a} - \frac{y}{1+a} = 1$

$\Rightarrow \quad \frac{4}{a} - \frac{3}{1+a} = 1$

$\Rightarrow \quad a = \pm 2$ (as $a = 2$ $b = -3$ and $a = -2$ $b = 1$)

so equation are $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$

22. **(a) :** If m_1 and m_2 are slope of the lines then by given condition $m_1 + m_2 = 4m_1m_2$

$\Rightarrow \quad -\frac{2c}{7} = -\frac{4}{7}$

$\Rightarrow \quad c = 2$

By using $ax^2 + 2hxy + by^2 = 0$

$\Rightarrow \quad m_1 + m_2 = \frac{-2h}{b}$ and $m_1m_2 = \frac{a}{b}$

23. **(d) :** The equation $ax^2 + 2hxy + by^2 = 0$

$= (y - m_1x)(y - m_2x)$

$\Rightarrow \quad m_1 + m_2 = -\frac{2h}{b} = \frac{1}{4c} \quad \text{...(*)}$

$m_1m_2 = \frac{3}{2}c$

and $3x + 4y = 0 \Rightarrow m_1 = -3/4$

$\therefore \quad m_2 = -\frac{2}{c}$

now by (*) we have

$-\left(\frac{3}{4} + \frac{2}{c}\right) = \frac{1}{4c}$

$\Rightarrow \quad -\frac{3}{4} = \frac{1}{4c} + \frac{2}{c}$

$\Rightarrow \quad -\frac{3}{4} = \frac{1}{4c} + \frac{8}{4c}$

$\Rightarrow \quad -\frac{3}{4} = \frac{9}{4c}$

$\therefore \quad c = -3$

24. **(b) :** Let the equation of circle cuts orthogonally the circle $x^2 + y^2 = 4$ is

$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \text{...(i)}$

$\therefore 2g_1g_2 + 2f_1f_2 = c_1c_2$ (where $(-g, -f)$ are point of locus)

$\Rightarrow c = -4$

Again circle (i) passes through (a, b), so

$a^2 + b^2 + 2ga + 2fb + 4 = 0$

Now replacing g, f by x, y respectively

$\therefore 2ax + 2by - (a^2 + b^2 + 4) = 0$

25. **(c) :** Equation of circle AB as diameter is given by

$(x - p)(x - \alpha) + (y - q)(y - \beta) = 0$

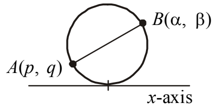
$\Rightarrow x^2 + y^2 - x(p + \alpha) - y(q + \beta) + p\alpha + q\beta = 0$ ...(1)

Now (1) touches axis of x so put $y = 0$ in (1) we have

$x^2 - x(p + \alpha) + p\alpha + q\beta = 0$ ...(2)

and $D = 0$ in equation (2)

$\therefore\ (p + \alpha)^2 = 4[p\alpha + q\beta]$



$\Rightarrow (p - \alpha)^2 = 4q\beta$

Now $\alpha \to x$, $\beta \to y$

$\therefore\ (p - x)^2 = 4q(y)$ which required locus of one end point of the diameter.

26. **(c) :** As per given condition centre of the circle is the point of intersection of the $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$

$\therefore$ centre is $(1, -1)$

Also circumference of the circle be given $2\pi r = 10\pi$

$\therefore\ r = 5$

$\therefore$ Required equation of circle is

$(x - 1)^2 + (y + 1)^2 = 5^2$

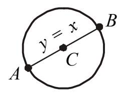
or $x^2 + y^2 - 2x + 2y - 23 = 0$

27. **(c) :** Given circle $x^2 + y^2 - 2x = 0$ ... (1)

and line be $y = x$ ... (2)

Solving (1) and (2) we get

$x = 0, 1\ \therefore\ y = 0, 1$



$\therefore\ A(0, 0)$, $B(1, 1)$ and equation of circle in the diameter form is $(x - 0)(x - 1) + (y - 0)(y - 1) = 0$

$\Rightarrow x^2 + y^2 - (x + y) = 0$

28. **(c) :** The point of intersection of parabola's $y^2 = 4ax$ and $x^2 = 4ay$ are $A(0, 0)$, $B(4a, 4a)$ as the line $2bx + 3cy + 4d = 0$ passes through these points

$\therefore\ d = 0$ and $2b(4a) + 3c(4a) = 0$

$\Rightarrow 2b + 3c = 0$

$\Rightarrow (2b + 3c)^2 + d^2 = 0$

29. **(b) :** Equation of directrix $x = 4$ which is parallel to y-axis so axis of the ellipse is x-axis. Let equation of ellipse be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1\ (a > b)$

Again $e = 1/2$ and $e^2 = 1 - \dfrac{b^2}{a^2}$

$\Rightarrow \left(\dfrac{b}{a}\right)^2 = 1 - 1/4 = 3/4$ ...(*)

Also the equation of one directrix is $x = 4$

$\therefore$ equation of directrix $x = \dfrac{a}{e}$

$\therefore\ 4 = \dfrac{a}{e}$

$\Rightarrow\ a = 2$ ($\because\ e = 1/2$)

Further $b^2 = \dfrac{a^2 \times 3}{4}$ by (*)

$b^2 = \dfrac{4 \times 3}{4} = 3$

Hence equation of ellipse is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$\Rightarrow \dfrac{x^2}{4} + \dfrac{y^2}{3} = 1$ or $3x^2 + 4y^2 = 12$

30. **(c) :** From the given lines we have

$\dfrac{x - 1}{1} = \dfrac{y + 3}{\lambda} = \dfrac{z - 1}{\lambda} = s$ ... (A)

and $\dfrac{x - 0}{1} = \dfrac{y - 1}{2} = \dfrac{z - 2}{-2}$ ...(B)

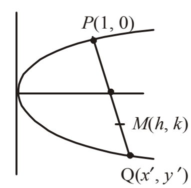
As lines (A) and (B) are coplanar

$$\therefore \begin{vmatrix} 1 & -4 & -1 \\ 1 & -\lambda & \lambda \\ 1 & 2 & -2 \end{vmatrix} = 0$$

$\Rightarrow\ (2\lambda - 2\lambda) + 4(-2 - \lambda) - 1(2 + \lambda) = 0$

$\Rightarrow\ 5\lambda = -10\ \therefore\ \lambda = -2$

31. **(d) :** Let $M(x', y')$ be point of locus mid point of PQ.



$\Rightarrow \dfrac{x' + 1}{2} = h,\ \dfrac{y' + 0}{2} = k,$

$\therefore\ x' = 2h - 1,\ y' = 2k$

Now (x', y') lies on $y^2 = 8x$

$\Rightarrow\ (2k)^2 = 8(2h - 1)$

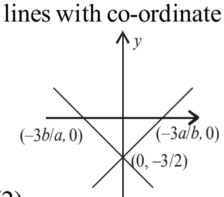
$\Rightarrow\ y^2 = 2(2x - 1) \Rightarrow y^2 - 4x + 2 = 0.$

32. **(b) :** Intercepts made by the lines with co-ordinate axis is

$(-3b/a\ , 0)$, $(0, -3/2)$

and $(0, -3/2)$

and $(3a/b, 0)$ and



common intercept is $(0, -3/2)$.

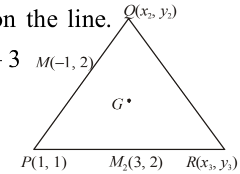
33. **(d) :** Let us take the two set of values of $a = 1$, $b = 1/2$, $c = 1/3$

and $a = 1/2$, $b = 1/3$, $c = 1/4$

Putting these value in the given equation we get

$x + 2y + 3 = 0$ and $2x + 3y + 4 = 0$ } *
solving the equations of * we have $x = 1$, $y = -2$
$(1, -2)$ is required point on the line.

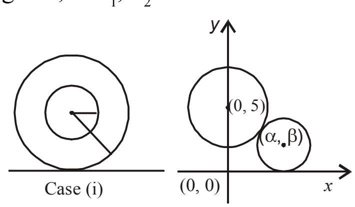
34. (d) : $\therefore\ x_2 = 2(-1) - 1 = -3$
$y_2 = 2 \times 2 - 1 = 3$
$x_3 = 3 \times 2 - 1 = 5$
$y_3 = 2 \times 2 - 1 = 3$



$$\therefore \text{ Centroid } G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$
$$= \left(\frac{1-3+5}{3}, \frac{1+3+3}{3}\right) = \left(1, \frac{7}{3}\right).$$

35. (a) : As the line passes through P and Q which are the point of intersection of two circles. It means given line is the equation of common chord and the equation of common chord of two intersecting circle is
$S_1 - S_2 = 0$
$= 5ax + (c - d)y + a + 1 = 0.$
Now $5ax + (c - d)y + a + 1 = 0$ and $5x + by - a = 0$ represent same equation.
$$\therefore \frac{5a}{5} = \frac{c-d}{b} = \frac{a+1}{-a}$$
$\Rightarrow a^2 + a + 1 = 0$ and $\dfrac{c-d}{b} + 1 = -\dfrac{1}{a}$
$\Rightarrow \left(a + \dfrac{1}{2}\right)^2 + \dfrac{3}{4} = 0$ and $-(c - d + b) = b/a$
$d - b - c = +b/a$ has no solution. $\therefore$ No value of a exist.

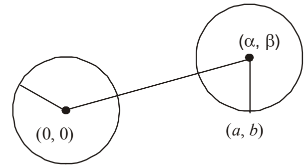
36. (c) : Let locus of centre be α, β then according to given, if r_1, r_2 are radii of circles then



Internal touch. This case does not exist as centre of circle is (0, 3) and radius is 2.

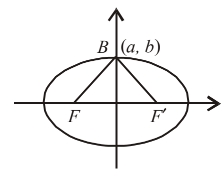
$C_1C_2 = r_2 \pm r_1$
$\Rightarrow\ (\alpha - 0)^2 + (\beta - 3)^2$
$= |\beta \pm 2|$
$\Rightarrow\ \alpha^2 + \beta^2 - 6\beta + 9 = \beta^2 + 4 + 4\beta$
and $\alpha^2 + \beta^2 - 6\beta + 9 = \beta^2 - 4\beta + 4$
$\Rightarrow\ \alpha^2 - 10\beta + 5 = 0$ and $x^2 = 2\beta + 5$
$\Rightarrow\ x^2 = 10y - 5$ and $x^2 = 2y - 5$
both are parabolas but $x^2 = 2y - 5$ does not exist.

37. (c) : Let locus of the centre of circle be (α, β).



If C_1, C_2 are centres of the circles with radii r_1, r_2 respectively then $(C_1C_2)^2 = r_1^2 + r_2^2$
$\Rightarrow\ \alpha^2 + \beta^2 = p^2 + (\alpha - a)^2 + (\beta - b)^2$
$\Rightarrow\ p^2 + a^2 + b^2 - 2a\alpha - 2b\beta = 0$
$\Rightarrow\ 2ax + 2by - (a^2 + b^2 + p^2) = 0.$

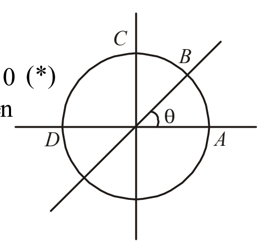
38. (b) : $F'(-ae, 0)$, $F(ae, 0)$
Slope of $BF = \dfrac{b}{ae} = m_1$ (say)
slope of $BF' = \dfrac{b}{-ae} = m_2$ (say)
now $m_1 \times m_2 = -1 \ \Rightarrow \dfrac{b}{ae} \times \dfrac{b}{-ae} = -1$
$\Rightarrow b^2 = a^2e^2 \ \Rightarrow a^2 - a^2e^2 = a^2e^2$
$$\left(\because\ e^2 = 1 - \frac{b^2}{a^2}\right)$$



$\Rightarrow 1 - e^2 = e^2,\ 2e^2 = 1,\ e = \pm\dfrac{1}{\sqrt{2}}.$

39. (c) : Given $y = \alpha x + \beta$ and $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$
$\therefore\ b^2x^2 - a^2y^2 = a^2b^2$
$\Rightarrow b^2x^2 - a^2(\alpha x + \beta)^2 = a^2b^2$
by using $y = a\alpha + \beta$
$\Rightarrow x^2(b^2 - a^2\alpha^2) - 2a^2\alpha\beta x + (-\beta^2a^2 - a^2b^2) = 0$ ....(*)
Now the line $y = \alpha x + \beta$ will be tangent to circle if both roots if (*) are equal
$\therefore$ keeping $D = 0$ in (*) we have
$4\alpha^2a^4\beta^2 = 4(b^2 - a^2\alpha^2)(-\beta^2a^2 - a^2b^2)$
$\Rightarrow\ \alpha^2a^2\beta^2 = (b^2 - a^2\alpha^2)(-\beta^2 - b^2)$
$\Rightarrow\ \alpha^2a^2\beta^2 = -b^2\beta^2 + \beta^2a^2\alpha^2 - b^4 + a^2\alpha^2b^2$
$\Rightarrow\ a^2\alpha^2b^2 = b^2(b^2 + \beta^2) \Rightarrow a^2\alpha^2 = b^2 + \beta^2$
$\Rightarrow a^2x^2 - y^2 = b^2.$

40. (c) :
$ax^2 + 2(a+b)xy + by^2 = 0$ (*)
Let θ be the angle between the lines represent by *



$$\therefore\ \tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$\tan\theta = \left|\dfrac{2\sqrt{(a+b)^2 - ab}}{a+b}\right|$

$\{3 \text{ area } \widehat{OAB} = \text{ Area of } \widehat{DBC}\}$

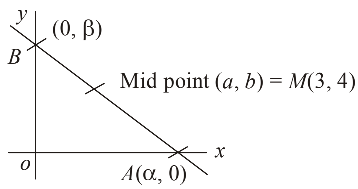
Now $\theta = 45$

$\{\because$ area of one sector = 3 time the area of another sector$\}$

$\therefore \quad \tan 45° = \left|\dfrac{2\sqrt{(a+b)^2 - ab}}{a+b}\right|$

$\Rightarrow 3a^2 + 2ab + 3b^2 = 0.$

41. (c) : Now the equation of line which meet the x-axis and y-axis



at $A(\alpha, 0)$ B $(0, \beta)$ is given by

$\dfrac{x}{\alpha} + \dfrac{y}{\beta} = 1$

where $\alpha = 2a = 6$

and $\beta = 2b = 8$

$\therefore \quad$ required equation be $\dfrac{x}{6} + \dfrac{y}{8} = y$

$\Rightarrow \quad 4x + 3y = 24$

42. (a) : Let h, k be the locus of the vertex of family of parabola

$$y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$$

$$\therefore \quad k = \frac{a^3h^2}{3} + \frac{a^2h}{2} = 2a$$

$$\Rightarrow \quad \frac{3k}{a^3} = h^2 + \frac{3h}{2a} - \frac{6}{a^2}$$

$$\Rightarrow \quad \frac{3}{a^3}\left(k + \frac{35a}{16}\right) = \left(h + \frac{3}{4a}\right)^2$$

i.e., $\left\{x^2 = \dfrac{3}{a^3}y, \text{ where } x = h + \dfrac{3}{4a}, y = k + \dfrac{35a}{16}\right\}$

$\Rightarrow \quad$ vertex is $\left(\dfrac{-3}{4a}, \dfrac{-35a}{16}\right)^2$

$$\therefore \quad hk = \left(\frac{-3}{4a}\right)\left(\frac{-35a}{16}\right)$$

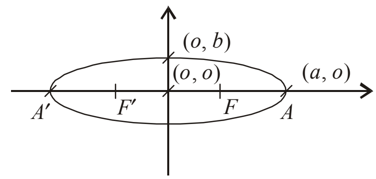
$$\Rightarrow \quad hk = \frac{105}{64}$$

$\Rightarrow \quad xy = \dfrac{105}{64}$ (taking $h = x$, $k = y$)

43. (a) : Let $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ $(a > b)$

Given $2b = 8$ ... (i)

and $2ae = 6$ ... (ii)



By (i) and (ii) we have $\dfrac{b}{ae} = \dfrac{4}{3}$

$$\Rightarrow \quad \frac{b^2}{a^2} = \frac{16}{9}e^2$$

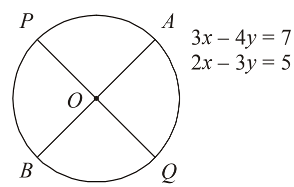
$$\Rightarrow \quad 1 - e^2 = \frac{16}{9}e^2$$

$(\because\ b^2 = a^2(1 - e^2)$ as $a > b)$

$$\Rightarrow \quad e = \frac{3}{5}$$

44. (d) : Let $OA = r$

Given area = 49π



$\Rightarrow \quad \pi r^2 = 49\pi$

$r = 7$

Point of intersection of AB and PQ is $(1, -1)$

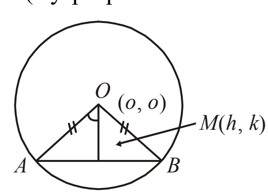
$\therefore \quad$ equation of circle is $(x-1)^2 + (y+1)^2 = 7^2$

$\Rightarrow \quad x^2 + y^2 - 2x + 2y - 47 = 0$

45. (d) : Let AB is chord of circle and $M(h, k)$ be mid point of AB $\angle AOM = 60°$

Now $OA = OB = 3$ and

$OM \perp AB$ (By properties of circle)



Now $OA = \sqrt{h^2+k^2}$, $OM = r\cos\theta$

$$\sqrt{h^2+k^2} = 3\cos 60°$$

$$\sqrt{h^2+k^2} = \frac{3}{2}$$

$$\Rightarrow \quad h^2+k^2 = \frac{9}{4}$$

$$\Rightarrow \quad x^2+y^2 = \frac{9}{4}$$

46. **(c) :** Given lines are $y = \frac{x}{2} (x > 0)$ and $y = 3x$ $(x > 0)$ using (a, a^2) in these lines

$$a^2 - \frac{a}{2} > 0 \quad \text{... (i)}$$

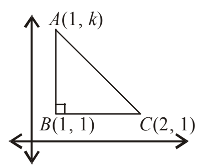
and $a^2 - 3a < 0$ ... (ii)

Solving (i) and (ii) we get $\frac{1}{2} < a < 3$

47. **(b) :** $\because$ $b^2 = a^2(e^2 - 1) \Rightarrow \sin^2\alpha = \cos^2\alpha\,(e^2 - 1)$
$\Rightarrow \tan^2\alpha + 1 = e^2 \Rightarrow e^2 = \sec^2\alpha$
Vertices $\equiv (\pm\cos\alpha, 0)$
Coordinate of focii are $(\pm ae, 0) \equiv (\pm 1, 0)$
$\Rightarrow$ if α varies then the abscissa of foci remain constant.

48. **(b) :** Let P is the required point, then P lies on directrix $x = -2$ of $y^2 = 8x$
Hence $P \equiv (-2, 0)$.

49. **(a) :** $\frac{1}{2} \times |k-1| \times 1 = 1$
$k = -1, 3$.



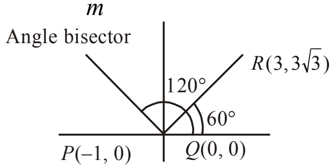
50. **(c) :** Slope of the required angle bisector is $\tan 120° = -\sqrt{3}$
Hence equation of the angle bisector is
$y = -\sqrt{3}(x - 0)$
$\Rightarrow \sqrt{3}x + y = 0$

51. **(a) :** Sum of the slopes $= -\dfrac{\text{co-efficient of } xy}{\text{co-efficient of } y^2}$

$\therefore$ Sum of slopes $= -\dfrac{(1-m^2)}{m} = 0$

$\Rightarrow m = \pm 1$.

Second method
Equation of bisectors of lines



$xy = 0$ are $y = \pm x$
Put $y = \pm x$ in
$my^2 + (1 - m^2)xy - mx^2 = 0$, we get
$(1 - m^2)x^2 = 0 \Rightarrow m = \pm 1$.

52. **(d) :** Equation of circle $(x - h)^2 + (y - k)^2 = k^2$
It is passing through $(-1, 1)$ then
$(-1 - h)^2 + (1 - k)^2 = k^2 \Rightarrow h^2 + 2h - 2k + 2 = 0$, $D \geq 0$
$2k - 1 \geq 0 \Rightarrow k \geq \frac{1}{2}$.

53. **(b, c) :** Equation of normal at $P(x, y)$ is

$$Y - y = -\frac{dx}{dy}(X - x) \quad \Rightarrow G \equiv \left(x + y\cdot\frac{dy}{dx}, 0\right)$$

$$\left|x + y\frac{dy}{dx}\right| = |2x| \;\Rightarrow\; y\frac{dy}{dx} = x \;\text{ or }\; y\frac{dy}{dx} = -3x$$

$ydy = xdx$ or $ydy = -3xdx$
After integrating, we get

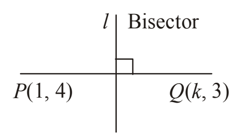
$$\frac{y^2}{2} = \frac{x^2}{2} + c \text{ or } \frac{y^2}{2} = -\frac{3x^2}{2} + c \;\Rightarrow x^2 - y^2 = -2c$$

or $3x^2 + y^2 = 2c \Rightarrow x^2 - y^2 = c_1$ or $3x^2 + y^2 = c_2$.

54. **(a) :** The slope of

$$l = -\frac{1}{\text{the slope of the original line } PQ}$$

$$= -\frac{1}{\frac{3-4}{k-1}} = (k-1)$$



The midpoint $= \left(\dfrac{k+1}{2}, \dfrac{7}{2}\right)$

The equation to the bisector l is

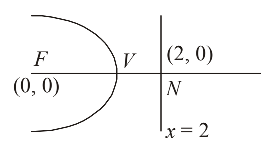
$$\left(y - \frac{7}{2}\right) = (k-1)\left(x - \frac{k+1}{2}\right)$$

As $x = 0$, $y = -4$ satisfies it, we have

$$\left(-4 - \frac{7}{2}\right) = (k-1)\left(0 - \frac{k+1}{2}\right) \;\Rightarrow\; -\frac{15}{2} = -\frac{k^2-1}{2}$$

$\Rightarrow k^2 - 1 = 15 \;\Rightarrow\; k^2 = 16 \;\therefore\; k = \pm 4$.

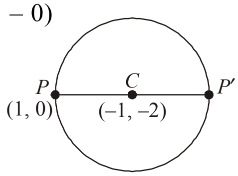
55. **(c) :** The vertex is the mid point of FN, that is, vertex $= (1, 0)$



56. **(d) :** The centre C of the circle is seen to be $(-1, -2)$. As C is the mid point P and P', the coordinate of P' is given by

$P' \equiv (2 \times -1 - 1, 2 \times -2 - 0)$

$\equiv (-3, -4)$



Remark : If P be (α, β) and $C(h, k)$ then

$P' \equiv (2h - \alpha, 2k - \beta)$

57. **(b) :** Obviously the major axis is along the x-axis The distance between the focus and the corresponding directrix $= \left|\frac{a}{e} - ae\right| = 4$

$\Rightarrow \frac{a}{e} - ae = 4$ (note that $\frac{a}{e} > ae$)

$\Rightarrow a\left(\frac{1}{e} - e\right) = 4 \Rightarrow a\left(2 - \frac{1}{2}\right) = 4$

$\Rightarrow a \cdot \frac{3}{2} = 4 \therefore a = \frac{8}{3}$

Remark : The question should have read "*The corresponding directrix*" in place of "*the directrix*".

CHAPTER 13

Three Dimensional Geometry

1. The d.r. of normal to the plane through (1, 0, 0), (0, 1, 0) which makes an angle $\pi/4$ with plane $x + y = 3$ are
   (a) $1, \sqrt{2}, 1$ (b) $1, 1, \sqrt{2}$
   (c) 1, 1, 2 (d) $\sqrt{2}, 1, 1$.
   (2002)

2. Two systems of rectangular axes have the same origin. If a plane cuts them at distance a, b, c and a', b', c' from the origin, then
   (a) $\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}-\frac{1}{c'^2}=0$
   (b) $\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$
   (c) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$
   (d) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$.
   (2003)

3. The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is
   (a) $11\frac{3}{4}$ (b) 13
   (c) 39 (d) 26.
   (2003)

4. The radius of the circle in which the sphere
   $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$
   is cut by the plane $x + 2y + 2z + 7 = 0$ is
   (a) 2 (b) 3 (c) 4 (d) 1.
   (2003)

5. A tetrahedron has vertices at O (0, 0, 0), A (1, 2, 1), B (2, 1, 3) and C (–1, 1, 2). Then the angle between the faces OAB and ABC will be
   (a) $\cos^{-1}(17/31)$ (b) 30°
   (c) 90° (d) $\cos^{-1}(19/35)$.
   (2003)

6. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $(1, \vec{a}, \vec{a}^2)$, $(1, \vec{b}, \vec{b}^2)$ and $(1, \vec{c}, \vec{c}^2)$ are non-coplanar, then the product abc equals
   (a) –1 (b) 1
   (c) 0 (d) 2.
   (2003)

7. The two lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d$ will be perpendicular, if and only if
   (a) $aa' + bb' + cc' = 0$
   (b) $(a + a')(b + b') + (c + c') = 0$
   (c) $aa' + cc' + 1 = 0$
   (d) $aa' + bb' + cc' + 1 = 0$.
   (2003)

8. The lines
   $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$
   are coplanar if
   (a) $k = 1$ or -1 (b) $k = 0$ or -3
   (c) $k = 3$ or -3 (d) $k = 0$ or -1.
   (2003)

9. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with

y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals:

(a) 3/5 (b) 1/5

(c) 2/3 (d) 2/5.

(2004)

10. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is

(a) 7/2 (b) 5/2

(c) 3/2 (d) 9/2

(2004)

11. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The co-ordinates of each of the points of intersection are given by

(a) $(3a, 2a, 3a), (a, a, 2a)$

(b) $(3a, 2a, 3a), (a, a, a)$

(c) $(3a, 3a, 3a), (a, a, a)$

(d) $(2a, 3a, 3a), (2a, a, a)$.

(2004)

12. The intersection of the spheres $x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and $x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the sphere and the plane

(a) $x - y - 2z = 1$ (b) $x - 2y - z = 1$

(c) $x - y - z = 1$ (d) $2x - y - z = 1$.

(2004)

13. If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the midpoint of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ then a equals

(a) 1 (b) –1

(c) 2 (d) –2.

(2005)

14. The distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and the plane $\vec{r}\cdot(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is

(a) $\frac{10}{3\sqrt{3}}$ (b) $\frac{10}{9}$

(c) $\frac{10}{3}$ (d) $\frac{3}{10}$.

(2005)

15. If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$, the value of λ is

(a) $-\frac{3}{5}$ (b) $\frac{5}{3}$

(c) $\frac{-4}{3}$ (d) $\frac{3}{4}$.

(2005)

16. The plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius

(a) 1 (b) 3

(c) $\sqrt{2}$ (d) 2.

(2005)

17. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(a) 90° (b) 0°

(c) 30° (d) 45°.

(2005)

18. The two lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are perpendicular to each other if

(a) $aa' + cc' = -1$ (b) $aa' + cc' = 1$

(c) $\frac{a}{a'} + \frac{c}{c'} = -1$ (d) $\frac{a}{a'} + \frac{c}{c'} = 1$.

(2006)

19. The image of the point (–1, 3, 4) in the 3 plane $x - 2y = 0$ is

(a) $\left(-\frac{17}{3}, -\frac{19}{3}, 4\right)$ (b) (15, 11, 4)

(c) $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$ (d) $\left(\frac{9}{5}, -\frac{13}{5}, 4\right)$.

(2006)

20. If a line makes an angle of $\pi/4$ with the positive directions of each of x-axis and y-axis, then the angle that the line makes with the positive direction of the z-axis is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$.

(2007)

21. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the coordinates of the other end of the diameter are

(a) (4, 3, 5) (b) (4, 3, –3)

(c) (4, 9, –3) (d) (4, –3, 3).

(2007)

22. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vectors $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals

(a) -4 (b) -2 (c) 0 (d) 1.

(2007)

23. Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x-axis, then $\cos\alpha$ equals

(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$.

(2007)

24. The line passing through the points $(5, 1, a)$ and $(3, b, 1)$ crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then

(a) $a = 8, b = 2$ (b) $a = 2, b = 8$
(c) $a = 4, b = 6$ (d) $a = 6, b = 4$

(2008)

25. If the straight lines

$$\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3} \text{ and } \frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$$

intersect at a point, then the integer k is equal to

(a) -2 (b) -5 (c) 5 (d) 2

(2008)

Answer Key

1.	(b)	**2.**	(c)	**3.**	(b)	**4.**	(b)	**5.**	(d)	**6.**	(a)
7.	(c)	**8.**	(b)	**9.**	(a)	**10.**	(a)	**11.**	(b)	**12.**	(d)
13.	(d)	**14.**	(a)	**15.**	(b)	**16.**	(a)	**17.**	(a)	**18.**	(a)
19.	(d)	**20.**	(b)	**21.**	(c)	**22.**	(b)	**23.**	(c)	**24.**	(d)
25.	(b)										

1. **(b) :** Let DR's of normal to plane are a, b, c

$\therefore \quad a(x-1)+b(y)+c(z)=0 \qquad ...(*)$

$\Rightarrow \quad a(0-1)+b(1)+c(0)=0$

(by using (0, 1, 0) in (*))

$\Rightarrow \quad -a+b=0 \Rightarrow a=b$

Also angle between (*) and $x+y+0z=3$ is $\pi/4$

$$\therefore \quad \cos\frac{\pi}{4}=$$

$$\frac{a+a}{\sqrt{1^2+1^2}\sqrt{a^2+b^2+c^2}}=\frac{2a}{\sqrt{2}\sqrt{2a^2+c^2}}$$

$\Rightarrow \quad 2a^2+c^2=4a^2$

$\Rightarrow \quad c=\pm\sqrt{2}\,a$

$\therefore \quad DR's\ a,\ b,\ c\ i.e.\ a,\ a,\ \pm\sqrt{2}a$

$\therefore$ Required DR's are $1, 1, \sqrt{2}$ or $1, 1, -\sqrt{2}$

Hence $1, 1, \sqrt{2}$ match with choice (b)

2. **(c) :** Now equation of the plane through $(a, 0, 0)$ $(0, b, 0)$ $(0, 0, c)$ is

$$\Rightarrow \frac{x}{x-\text{Intercept}}+\frac{y}{y-\text{Intercept}}+\frac{z}{z-\text{Intercept}}$$

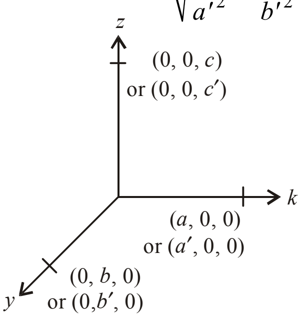
$= 1 \qquad ...(*)$

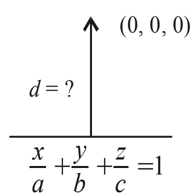
$$\Rightarrow \quad \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

So the distance from (0, 0, 0) to this plane to the plane (*) is given by

$$d_1=\frac{|0+0+0-1|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$$

$$\text{Similarly, } d_2=\frac{1}{\sqrt{\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}}}$$



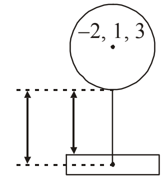


Now $d_1=d_2$ given (as origin is same)

$$\Rightarrow \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=\frac{1}{\sqrt{\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}}}$$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$$

3. **(b) :** In order to determine the shortest distance between the plane and sphere, we find the distance from the centre of sphere to the plane- Radius of sphere



$12x+4y+3z-327=0$

$\therefore$ Centre of sphere is $(-2, 1, 3)$

Required distance is

$$\frac{|-24+4+9-327|}{\sqrt{12^2+4^2+3^2}}-\sqrt{(2)^2+1^2+3^2+155}$$

$= 26 - 13$

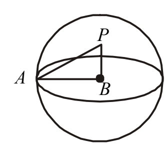
$= 13$ units.

4. **(b) :** The radius and centre of sphere $x^2+y^2+z^2+2x-2y-4z-19=0$ is

$\sqrt{1^2+1^2+4+19}=5$ and centre $(-1, 1, 2)$

PB $\perp$ from centre to the plane

$$\frac{|-1+2+4+7|}{\sqrt{1+2^2+2^2}}=4$$



Now $(AB)^2=AP^2-PB^2$

$= 25 - 16$

$= 9$

$\therefore AB = 3$

5. **(d) :** Concept using angle between the faces is equal to the angle between their normals.

$\therefore$ vector b to the face OAB is $\overrightarrow{OA}\times\overrightarrow{OB}$

$= 5i - j - 3k$ and vector b to the face ABC is $\overrightarrow{AB}\times\overrightarrow{AC}=i-5j-3k$

$\therefore$ Let θ be the angle between the faces OAB and ABC

$$\therefore \cos\theta = \frac{(5i - j - 3k)\cdot(i - 5j - 3k)}{|5i - j - 3k|\,|i - 5j - 3k|}$$

$$\cos\theta = \frac{19}{35} \quad \therefore \theta = \cos^{-1}\left(\frac{19}{35}\right)$$

6. **(a) :** As vectors $(1, \vec{a}, \vec{a}^2)$, $(1, \vec{b}, \vec{b}^2)$, $(1, \vec{c}, \vec{c}^2)$ are non coplanar.

$$\therefore \quad \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0 \quad \text{... (A)}$$

$$\text{now} \quad \begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix} = 0$$

On solving, we get

$$\Rightarrow (1 + abc)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$\Rightarrow (1 + abc) = 0$ by using (A)

7. **(c) :** Given lines can be written as

$$\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c} \quad \text{and}$$

$$\frac{x-b'}{a'} = \frac{y-0}{1} = \frac{z-d}{c'}$$

$\therefore$ Required condition of perpendicularity is $aa' + cc' + 1 = 0$

8. **(b) :** Using fact, two line

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \quad \text{and}$$

$$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2} \quad \text{are coplanar if}$$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad k^2 + 3k = 0$

$k = 0$ or $k = -3$

9. **(a) :** If a line makes the angle α, β, γ with x, y, z axis respectively then

$l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad 2l^2 + m^2 = 1$ or $2n^2 + m^2 = 1$

$\Rightarrow \quad 2\cos^2\theta = 1 - \cos^2\beta$ $(\alpha = \gamma = \theta)$

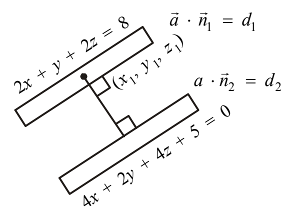
$2\cos^2\theta = \sin^2\beta$

$\Rightarrow \quad 2\cos^2\theta = 3\sin^2\theta$ (given $\sin^2\beta = 3\sin^2\theta$)

$\Rightarrow \quad 5\cos^2\theta = 3$

10. **(a) :** Let x_1, y_1, z_1 be any point on the plane $2x + y + 2z - 8 = 0$

$\therefore 2x_1 + y_1 + 2z_1 - 8 = 0$



$$\therefore d = \frac{|2(2x + y + 2z - 8) + 21|}{\sqrt{4^2 + 2^2 + 4^2}} = \frac{21}{6} = \frac{7}{2}$$

11. **(b) :** Given $AB = \frac{x}{1} = \frac{y+a}{1} = \frac{z}{1}$

$CD : \frac{x+a}{2} = \frac{y}{1} = \frac{z}{1}$

Let $P \equiv (r, r - a, r)$ and $Q = (2\lambda - a, \lambda, \lambda)$

Direction ratios of PQ are $r - 2\lambda + a, r - \lambda - a, r - \lambda$

According to question direction ratios of PQ are (2, 1, 2)

$$\therefore \underset{(i)}{\frac{r - 2\lambda + a}{2}} = \underset{(ii)}{\frac{r - \lambda - a}{1}} = \underset{(iii)}{\frac{r - \lambda}{2}}$$

(ii) and (iii) $\Rightarrow r - \lambda = 2a$ ...(1)

(i) and (iii) $\Rightarrow \lambda = a$ $\quad r = 3a, \ \lambda = a$

$\therefore p \equiv (3a, 2a, 3a)$ and $Q \equiv (a, a, a)$.

12. **(d) :** Equation of the plane of intersection of two spheres $S_1 = 0 = S_2$ is given by $S_1 - S_2 = 0$

$\Rightarrow \quad 10x - 5y - 5z = 5$

$\Rightarrow \quad 2x - y - z = 1$

13. **(d) :** Centre of spheres are (–3, 4, 1) and (5, –2, 1)

M (1, 1, 1)

C_1 (–3, 4, 1) $\qquad$ C_2 (5, –2, 1) using mid point in the equation

$2ax - 3ay + 4az + 6 = 0$

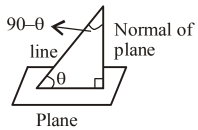
$\Rightarrow 2a - 3a + 4a + 6 = 0 \Rightarrow a = -2.$

14. (a) : $d = \left|\dfrac{\vec{a}\cdot\vec{n} - d}{\sqrt{n}}\right|$

$\therefore\ d = \left|\dfrac{(2i - 2j + 3k)\cdot(i + 5j + k) - (-5)}{\sqrt{1^2 + 5^2 + 1^2}}\right|,$

$d = \dfrac{10}{3\sqrt{3}}.$

15. (b) : Angle between the line and plane is same as the angle between the line and normal to the plane



$\therefore\ \cos(90 - \theta) = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

$\Rightarrow \dfrac{1}{3}\ \dfrac{(1\times 2 + 2\times(-1)) + 2\sqrt{\lambda}}{\sqrt{1^2 + 2^2 + 2^2}\sqrt{2^2 + 1^2 + \lambda}}$

$\Rightarrow \lambda = \dfrac{5}{3}.$

16. (a) : Centre of sphere is 1/2, 0, –1/2

R = Radius of sphere is $\sqrt{g^2 + f^2 + w^2 - c}$

$= \sqrt{\dfrac{1}{4} + \dfrac{1}{4} + 2}\ \ \therefore\ R = \sqrt{\dfrac{5}{2}}$

$d = \perp$ distance from centre to the plane is equal to

$d = \left|\dfrac{\frac{1}{2} + 0 + \frac{1}{2} - 4}{\sqrt{1^2 + 2^2 + 1^2}}\right|,\ d = \dfrac{3}{\sqrt{6}}.$

$\therefore$ Radius of the circle

= Radius of sphere – perpendicular distance from centre of sphere to plane

$= \sqrt{\left(\sqrt{\dfrac{5}{2}}\right)^2 - \left(\dfrac{9}{6}\right)} = \sqrt{\dfrac{15}{6} - \dfrac{9}{6}} = 1.$

17. (a) : From given line

$\dfrac{x}{3} = \dfrac{y}{2} = \dfrac{z}{-6}$ and $\dfrac{x}{2} = \dfrac{y}{-12} = \dfrac{z}{-3}$

$\cos\theta = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

$\cos\theta = \dfrac{6 - 24 + 18}{\sqrt{3^2 + 2^2 + (-6)^2}\sqrt{2^2 + (-12)^2 + (-3)^2}} = 0$

$\therefore\ \theta = 90^\circ.$

18. (a) : Fact : Two lines $\dfrac{x - x_1}{a_1} = \dfrac{x - y_1}{b_1} = \dfrac{z - z_1}{c_1}$

and $\dfrac{x - x_2}{a_2} = \dfrac{y - y_2}{b_2} = \dfrac{z - z_2}{c_2}$

are $\perp$ if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Given lines can be written as $\dfrac{x - b}{a} = \dfrac{y}{1} = \dfrac{z - d}{c}$ (i)

and $\dfrac{x - b'}{a'} = \dfrac{y}{1} = \dfrac{z - d'}{c'}$ ...(ii)

As lines are perpendicular

$\therefore\ \ a\,a' + 1 + c\,c' = 0$

$\Rightarrow\ \ a\,a' + c\,c' = -1$

19. (d) : Image of point (x', y', z') in $ax + by + cz + d = 0$ is

$\dfrac{x - x'}{a} = \dfrac{y - y'}{b} = \dfrac{z - z'}{c} = \dfrac{-2(ax' + by' + cz' + d)}{a^2 + b^2 + c^2}$

$\Rightarrow\ \ \dfrac{x + 1}{1} = \dfrac{y - 3}{-2} = \dfrac{z - 4}{0} = \dfrac{-2(-1 - 6)}{5}$

$\therefore\ \ x = \dfrac{9}{5}, y = \dfrac{-13}{5}, z = 4$

20. (b) : Let required angle is θ

$\because\ \ l = \cos\dfrac{\pi}{4}, m = \cos\dfrac{\pi}{4}$ then $n = \cos\theta$

We know that $l^2 + m^2 + n^2 = 1$

$\Rightarrow\ \ \cos^2\dfrac{\pi}{4} + \cos^2\dfrac{\pi}{4} + \cos^2\theta = 1$

$\Rightarrow\ \ \dfrac{1}{2} + \dfrac{1}{2} + \cos^2\theta = 1$

$\Rightarrow\ \ \cos^2\theta = 0 \Rightarrow \theta = \pi/2$

Thus required angle is $\pi/2$.

21. (c) : Centre of sphere $\equiv (3, 6, 1)$

Let the other end of diameter is (α, β, γ)

$3 = \dfrac{\alpha + 2}{2} \Rightarrow \alpha = 4\ ,\ 6 = \dfrac{\beta + 3}{2} \Rightarrow \beta = 9$

$1 = \dfrac{\gamma + 5}{2} \Rightarrow \gamma = -3.$

22. (b) : $\vec{a} = \hat{i} + \hat{j} + \hat{k},\ \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and

$\vec{c} = x\hat{i} + (x - 2)\hat{j} - \hat{k}$

$[\vec{a}\ \vec{b}\ \vec{c}] = 0$

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x - 2 & -1 \end{vmatrix} = 0$

$\Rightarrow 1(1 - 2x + 4) - 1(-1 - 2x) + 1(x - 2 + x) = 0$

$\Rightarrow 5 - 2x + 1 + 2x + 2x - 2 = 0$

$\Rightarrow x = -2.$

23. **(c) :** Direction of the line, $L = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 3 & 2 \end{vmatrix}$

$= 3\hat{i} - 3\hat{j} + 3\hat{k}$.

Then $\cos\alpha = \frac{3}{\sqrt{9+9+9}} = \frac{1}{\sqrt{3}}$.

Second method

If direction cosines of L be l, m, n, then

$2l + 3m + n = 0$, $l + 3m + 2n = 0$

After solving, we get, $\frac{l}{3} = \frac{m}{-3} = \frac{n}{3}$

$\therefore\ l : m : n = \frac{1}{\sqrt{3}} : -\frac{1}{\sqrt{3}} : \frac{1}{\sqrt{3}} \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$.

24. **(d) :** The equation of the line passing through $(3, b, 1)$ and $(5, 1, a)$ is

$\frac{x-5}{2} = \frac{y-1}{1-b} = \frac{z-a}{a-1} = \mu$ (say)

The line crosses the yz plane where $x = 0$, i.e.

$-5 = 2\mu \quad \therefore \quad \mu = -\frac{5}{2}$

Again $y = \mu(1-b) + 1 = \frac{17}{2}$

$\Rightarrow\ -\frac{5}{2}(1-b) + 1 = \frac{17}{2}$

$\Rightarrow\ -\frac{5}{2}(1-b) = \frac{15}{2}$

$\Rightarrow\ (1-b) = -3 \quad \therefore \quad b = 4$

Again $z = \mu(a-1) + a = -\frac{13}{2}$

$\Rightarrow\ -\frac{5}{2}(a-1) + a = -\frac{13}{2}$

$\Rightarrow\ -\frac{3}{2}a + \frac{5}{2} = -\frac{13}{2}$

$\Rightarrow\ -\frac{3}{2}a = -9 \Rightarrow a = 6$

25. **(b) :** As the lines and intersect,

we have

which on solving given $2k^2 + 5k - 25 = 0$

$\Rightarrow\quad 2k^2 + 10k - 5k - 25 = 0$

$\Rightarrow\quad 2k(k + 5) - 5(k + 5) = 0$

$\Rightarrow\quad (2k - 5)(k + 5) = 0$

$\therefore\ k = -5, \frac{5}{2}$

CHAPTER 14

Vector Algebra

1. If $|\vec{a}| = 4, |\vec{b}| = 2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$ then $(\vec{a}\times\vec{b})^2$ is equal to

(a) 48 (b) 16

(c) $\vec{a}$ (d) none of these

(2002)

2. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $[\vec{a}\ \ \vec{b}\ \ \vec{c}] = 4$ then $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] =$

(a) 16 (b) 64 (c) 4 (d) 8.

(2002)

3. If $\vec{a}, \vec{b}, \vec{c}$ are vectors show that $\vec{a}+\vec{b}+\vec{c} = 0$ and $|\vec{a}| = 7, |\vec{b}| = 5, |\vec{c}| = 3$ then angle between vector $\vec{b}$ and $\vec{c}$ is

(a) 60º (b) 30º (c) 45º (d) 90º.

(2002)

4. If $|a| = 5, |b| = 4, |c| = 3$ thus what will be the value of $|a\cdot b + b\cdot c + c\cdot a|$, given that $\vec{a} + \vec{b} + \vec{c} = 0$

(a) 25 (b) 50 (c) –25 (d) –50.

(2002)

5. $3\lambda\vec{c} + 2\mu(\vec{a}\times\vec{b}) = 0$ then

(a) $3\lambda + 2\mu = 0$ (b) $3\lambda = 2\mu$

(c) $\lambda = \mu$ (d) $\lambda + \mu = 0$.

(2002)

6. $\vec{a} = 3\hat{i} - 5\hat{j}$ and $\vec{b} = 6\hat{i} + 3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c} = \vec{a}\times\vec{b}$ then $|\vec{a}| : |\vec{b}| : |\vec{c}| =$

(a) $\sqrt{34} : \sqrt{45} : \sqrt{39}$

(b) $\sqrt{34} : \sqrt{45} : 39$

(c) 34 : 39 : 45

(d) 39 : 35 : 34.

(2002)

7. If $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$ then $\vec{a} + \vec{b} + \vec{c} =$

(a) abc (b) –1 (c) 0 (d) 2.

(2002)

8. Consider A, B, C and D with position vectors $7\hat{i} - 4\hat{j} + 7\hat{k}$, $\hat{i} - 6\hat{j} + 10\hat{k}$, $-\hat{i} - 3\hat{j} + 4\hat{k}$ and $5\hat{i} - \hat{j} + 5\hat{k}$ respectively. Then $ABCD$ is a

(a) rhombus

(b) rectangle

(c) parallelogram but not a rhombus

(d) square.

(2003)

9. If $\vec{u}, \vec{v}$ and $\vec{w}$ are three non-coplanar vectors, then $(\vec{u}+\vec{v}-\vec{w})\cdot(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})$ equals

(a) $\vec{u}\cdot\vec{v}\times\vec{w}$ (b) $\vec{u}\cdot\vec{w}\times\vec{v}$

(c) $3u\cdot\vec{u}\times\vec{w}$ (d) 0.

(2003)

10. Let $\vec{u} = \hat{i} + \hat{j}$, $\vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}\cdot\hat{n} = 0$ and $\vec{v}\cdot\hat{n} = 0$, then $|\vec{w}\cdot\hat{n}|$ is equal to

(a) 1 (b) 2 (c) 3 (d) 0.

(2003)

11. The vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is

(a) $\sqrt{72}$ (b) $\sqrt{33}$ (c) $\sqrt{288}$ (d) $\sqrt{18}$.

(2003)

12. $\vec{a}, \vec{b}, \vec{c}$ are three vectors, such that $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{b}|=2, |\vec{c}|=3$, then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is equal to

(a) –7 (b) 7 (c) 1 (d) 0.

(2003)

13. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ (λ being some non-zero scalar) then $\vec{a}+2\vec{b}+6\vec{c}$ equals

(a) $\lambda\vec{c}$ (b) $\lambda\vec{b}$ (c) $\lambda\vec{a}$ (d) 0.

(2004)

14. A particle is acted upon by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ which displace it from a point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The work done in standard units by the forces is given by

(a) 25 (b) 30 (c) 40 (d) 15.

(2004)

15. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a}+2\vec{b}+3\vec{c}$, $\lambda\vec{b}+4\vec{c}$ and $(2\lambda-1)\vec{c}$ are non-coplanar for

(a) all except two values of λ
(b) all except one value of λ
(c) all values of λ
(d) no value of λ.

(2004)

16. Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}|=1$, $|\vec{v}|=2$, $|\vec{w}|=3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}, \vec{w}$ are perpendicular to each other then $|\vec{u}-\vec{v}+\vec{w}|$ equals

(a) $\sqrt{14}$ (b) $\sqrt{7}$ (c) 2 (d) 14.

(2004)

17. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non-zero vectors such that $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\,\vec{a}$. If θ is the acute angle between the vectors $\vec{b}$ and $\vec{c}$, then $\sin\theta$ equals

(a) $\frac{2}{3}$ (b) $\frac{\sqrt{2}}{3}$
(c) $\frac{1}{3}$ (d) $\frac{2\sqrt{2}}{3}$.

(2004)

18. If C is the mid point of AB and P is any point outside AB, then

(a) $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=0$
(b) $\overrightarrow{PA}+\overrightarrow{PB}+2\overrightarrow{PC}=\vec{0}$
(c) $\overrightarrow{PA}+\overrightarrow{PB}=\overrightarrow{PC}$
(d) $\overrightarrow{PA}+\overrightarrow{PB}=2\overrightarrow{PC}$

(2005)

19. For any vector $\vec{a}$, the value of $(\vec{a}\times\hat{i})^2+(\vec{a}\times\hat{j})^2+(\vec{a}\times\hat{k})^2$ is equal to

(a) $\vec{a}^2$ (b) $3\vec{a}^2$
(c) $4\vec{a}^2$ (d) $2\vec{a}^2$.

(2005)

20. Let $\vec{a}=\hat{i}-\hat{k}$, $\vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}$ and $\vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$.

Then $[\vec{a}, \vec{b}, \vec{c}]$ depends on

(a) only x (b) only y
(c) neither x nor y (d) both x and y.

(2005)

21. Let a, b and c be distinct non-negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then c is

(a) the arithmetic mean of a and b
(b) the geometric mean of a and b
(c) the harmonic mean of a and b
(d) equal to zero.

(2005)

22. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vector and λ is a real number then $[\lambda(\vec{a}+\vec{b})\lambda^2\vec{b}\;\lambda\vec{c}]=[\vec{a}\;\;\vec{b}+\vec{c}\;\;\vec{b}]$ for

(a) no value of λ
(b) exactly one value of λ
(c) exactly two values of λ
(d) exactly three values of λ.

(2005)

23. If $(\vec{a}\times\vec{b})\times\vec{c}=\vec{a}\times(\vec{b}\times\vec{c})$, where $\vec{a},\vec{b}$ and $\vec{c}$ are any three vectors such that $\vec{a}\cdot\vec{b}\neq 0,\ \vec{b}\cdot\vec{c}\neq 0$, then $\vec{a}$ and $\vec{c}$ are

(a) inclined at an angle of $\pi/3$ between them
(b) inclined at an angle of $\pi/6$ between them
(c) perpendicular
(d) parallel.

(2006)

24. The values of a, for which the points A, B, C with position vectors $2\hat{i}-\hat{j}+\hat{k},\ \hat{i}-3\hat{j}-5\hat{k}$ and $a\hat{i}-3\hat{j}+\hat{k}$ respectively are the vertices of a right-angled triangle at c are

(a) 2 and 1 (b) –2 and –1
(c) –2 and 1 (d) 2 and –1.

(2006)

25. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u}\times 3\hat{v}$ is a unit vector for

(a) no value of θ
(b) exactly one value of θ
(c) exactly two values of θ
(d) more than two values of θ

(2007)

26. The vector $\vec{a}=\alpha\,\hat{i}+2\,\hat{j}+\beta\,\hat{k}$ lies in the plane of the vectors $\vec{b}=\hat{i}+\hat{j}$ and $\vec{c}=\hat{j}+\hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β?

(a) $\alpha = 1,\ \beta = 1$(b) $\alpha = 2,\ \beta = 2$
(c) $\alpha = 1,\ \beta = 2$(d) $\alpha = 2,\ \beta = 1$

(2008)

27. The non-zero vectors $\vec{a},\vec{b}$ and $\vec{c}$ are related by $\vec{a}=8\vec{b}$ and $\vec{c}=-7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is

(a) π (b) 0 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

(2008)

Answer Key

1.	(b)	**2.**	(a)	**3.**	(a)	**4.**	(a)	**5.**	(b)	**6.**	(b)
7.	(c)	**9.**	(a)	**10.**	(c)	**11.**	(b)	**12.**	(a)	**13.**	(d)
14.	(c)	**15.**	(a)	**16.**	(a)	**17.**	(d)	**18.**	(d)	**19.**	(d)
20.	(c)	**21.**	(b)	**22.**	(a)	**23.**	(d)	**24.**	(a)	**25.**	(b)
26.	(a)	**27.**	(a)								

1. **(b) :** Using fact: $(\vec{a} \times \vec{b})^2 = a^2b^2 - (\vec{a} \cdot \vec{b})^2$

$= a^2b^2 - a^2b^2 \cos^2\theta$

$= (4 \times 2)^2 - (4 \times 2)^2 \cos^2 \dfrac{\pi}{6}$

$= 64 \times \sin^2 \dfrac{\pi}{6}$

$= 64 \times \dfrac{1}{4} = 16$

2. **(a) :** Consider $[\vec{a} \times \vec{b}\ \ \vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}]$

$= (\vec{a} \times \vec{b}) \cdot [\vec{k} \times (\vec{c} \times \vec{a})]$ where $k = \vec{b} \times \vec{c}$

$= \vec{a} \times \vec{b} \cdot [(\vec{k} \cdot \vec{a})\vec{c} - (\vec{k} \cdot \vec{c})\vec{a}]$

$= (\vec{a} \times \vec{b}) \cdot \left[[(b \times c) \cdot a]\vec{c} - [(\vec{b} \times \vec{c}) \cdot c]\vec{a}\right]$

$= (\vec{a} \times \vec{b}) \cdot [(\vec{b} \times \vec{c}) \cdot \vec{a}]\vec{c} - 0$

$= \left((\vec{b} \times \vec{c}) \cdot \vec{a}\right)\left[(\vec{a} \times \vec{b}) \cdot \vec{c}\right]$

$= \left[\vec{a} \cdot (\vec{b} \times \vec{c})\right]\left[\vec{c} \cdot (\vec{a} \times \vec{b})\right]$

$= \left[\vec{a} \cdot (\vec{b} \times \vec{c})\right]^2$

$= 16$

3. **(a) :** Given $\vec{a} + \vec{b} + \vec{c} = 0$, we need angle between $\vec{b}$ and $\vec{c}$ so consider $\vec{b} + \vec{c} = -\vec{a}$

$\Rightarrow b^2 + c^2 + 2|b|\ |c| \cos\theta = a^2$

$\Rightarrow \cos\theta = \dfrac{a^2 - b^2 - c^2}{2|b||c|} = \dfrac{49 - 25 - 9}{2 \times 5 \times 3} = \dfrac{1}{2}$

$\therefore \theta = 60°$

4. **(a) :** We have $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow (\vec{a} + \vec{b} + \vec{c})^2 = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\Rightarrow 25 + 16 + 9 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\Rightarrow 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -50$

$\Rightarrow (\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -25$

$\therefore (\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 25$

5. **(b) :** $3\lambda\,\vec{c} = 2\mu\,(\vec{b} \times \vec{a})$

$\Rightarrow$ either $3\lambda = 2\mu$ or $\vec{c} \parallel \vec{b} \times \vec{a}$

but $3\lambda = 2\mu$ is in our court.

6. **(b) :** Given $\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -5 & 0 \\ 6 & 3 & 0 \end{vmatrix}$

$\therefore \quad \vec{c} = 39\,\vec{k}$

Now $|\vec{a}| = \sqrt{34}, |\vec{b}| = \sqrt{45}$ and

$|\vec{c}| = |39\vec{k}| = 39$

$\therefore |\vec{a}| : |\vec{b}| : |\vec{c}| = \sqrt{34} : \sqrt{45} : 39$

7. **(c) :** If possible say $\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \quad \vec{b} + \vec{c} = -\vec{a}$

$\vec{a} \times \left(\vec{b} + \vec{c}\right) = -\vec{a} \times \vec{a}$

$\Rightarrow \quad \vec{a} \times \vec{b} = \vec{c} \times \vec{a}$

similarly $\vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

$\therefore\ \vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

8. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 6\hat{i} + 2\hat{j} - 3\hat{k}$

$\therefore \quad |\overrightarrow{AB}| = \sqrt{49} = 7$

Similarly $\overrightarrow{BC} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$\therefore \quad |\overrightarrow{BC}| = \sqrt{49} = 7$

$\overrightarrow{CD} = -6\hat{i} - 2\hat{j} - \hat{k} \quad |\overrightarrow{CD}| = \sqrt{41}$

$\overrightarrow{DA} = -2\hat{i} + 3\hat{j} - 2\hat{k} \quad |\overrightarrow{DA}| = \sqrt{17}$

9. **(a) :** $(\vec{u} + \vec{v} - \vec{w}) \cdot [\vec{u} \times \vec{v} - \vec{u} \times \vec{w} + \vec{v} \times \vec{w}]$

$\because \vec{v} \times \vec{v} = 0$

$\vec{u} \cdot (\vec{u} \times \vec{v}) - \vec{u} \cdot (\vec{u} \times \vec{w}) + \vec{u} \cdot (\vec{v} \times \vec{w})$

$+ \vec{v} \cdot \vec{u} \times \vec{v} - \vec{v} \cdot (\vec{u} \times \vec{w}) + v \cdot (\vec{v} \times \vec{w})$

$- \vec{w} \cdot (\vec{u} \times \vec{v}) + \vec{w} \cdot (\vec{u} \times \vec{w}) - \vec{w} \cdot (\vec{v} \times \vec{w})$

$= \vec{u} \cdot (\vec{v} \times \vec{w}) + \vec{v} \cdot (\vec{w} \times \vec{u}) - \vec{w} \cdot (\vec{u} \times \vec{v})$

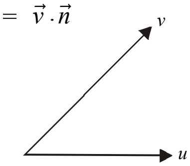
$= \vec{u} \cdot (\vec{v} \times \vec{w}) + \vec{u} \cdot (\vec{v} \times \vec{w}) - \vec{u} \cdot (\vec{v} \times \vec{w})$

$= \vec{u} \cdot (\vec{v} \times \vec{w}) \qquad (^3\ [a\ b\ c] = [b\ c\ a] = [c\ a\ b])$

10. **(c) :** $\hat{n} \parallel \vec{u} \times \vec{v} \ ^3\ \vec{u} \cdot \hat{n} = 0 = \vec{v} \cdot \vec{n}$

now $\hat{n} = \dfrac{\vec{u} \times \vec{v}}{|\vec{u}||\vec{v}|}$

$= \dfrac{1}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}}(-2\hat{k}) = -\hat{k}$



now $\vec{w}\cdot\hat{n} = \left|(i + 2\hat{j} + 3\hat{k})\cdot(-\hat{k})\right|$

$= |-3| = 3$

11. **(b) :** Median through any vertex divide the opposite side into two equal parts

$\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$

$\Rightarrow \overrightarrow{AD} = \frac{1}{2}[\overrightarrow{AB} + \overrightarrow{AC}]$

$= \frac{1}{2}[8i - 2j + 8k]$

$\therefore \left|\overrightarrow{AD}\right| = \sqrt{33}$

12. **(a) :** $\vec{a} + \vec{b} + \vec{c} = 0$

Consider $(a + b + c)^2$

$= a^2 + b^2 + c^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$\Rightarrow (\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = -\frac{(a^2 + b^2 + c^2)}{2}$

$= -\frac{(1^2 + 2^2 + 3^2)}{2}$

$= -7$

13. **(d):** As $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$

$\therefore \vec{a} + 2\vec{b} = P\vec{c}$

and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$ $\therefore \vec{b} + 3\vec{c} = Q\vec{a}$ ...(i)

Now by (i) and (ii) we have

$\vec{a} - 6\vec{c} = P\vec{c} - 2Q\vec{a}$

$\Rightarrow \vec{a}(1 + 2Q) + \vec{c}(-6 - P) = 0$

$\Rightarrow 1 + 2Q = 0$ and $-P - 6 = 0$

$Q = -1/2,\ P = -6$

Putting these value either in (1) or in (ii) we get

$\vec{a} + 2\vec{b} + 6\vec{c} = 0$

14. **(c) :** Total force $\vec{F} = \vec{F}_1 + \vec{F}_2 = 7i + 2j - 4k$ and displacement $\vec{d} = \vec{d}_2 - \vec{d}_1$

$= (5 - 1)i + (4 - 2)j + (k - 3k)$

$\therefore W.D = \vec{F}\cdot\vec{d} = 28 + 4 + 8 = 40$

15. **(a) :** Using the condition of coplanarity of three vectors

$$\therefore \begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & 2\lambda - 1 \end{vmatrix} = 0$$

$\Rightarrow \lambda = 0, \frac{1}{2}.$

16. **(a):** Given $\frac{\vec{v}\cdot\vec{u}}{|u|} = \frac{\vec{u}\cdot\vec{w}}{|u|}$ and $\vec{v}\cdot\vec{w} = 0$

Also $|\vec{u}| = 1,\ |\vec{v}| = 2,\ |\vec{w}| = 3$

Now $|\vec{u} - \vec{v} + \vec{w}|^2$

$= \vec{u}^2 + \vec{v}^2 + \vec{w}^2 - 2\vec{u}\cdot\vec{v} - 2\vec{v}\cdot\vec{w} + 2\vec{u}\cdot\vec{w}$

$= 1 + 4 + 9 + 0 = 14$

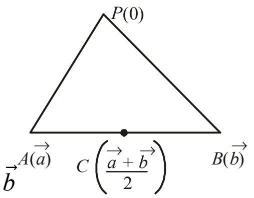
17. **(d) :** $(\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}|b||c|\vec{a}$ (As given)

$\Rightarrow (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow -\vec{b}\cdot\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}| \Rightarrow \cos\theta = -1/3$

$\sin\theta = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

18. **(d) :** Let P is origin



Let $\overrightarrow{PA} = \vec{a},\ \overrightarrow{PB} = \vec{b}$

$\therefore \overrightarrow{PC} = \frac{\vec{a} + \vec{b}}{2}$

Now $\overrightarrow{PA} + \overrightarrow{PB} = \vec{a} + \vec{b}$

$= 2\left(\frac{\vec{a} + \vec{b}}{2}\right) = 2\overrightarrow{PC}.$

19. **(d) :** Let $\vec{a} = a_1 i + b_1 j + c_1 k$

$\therefore \vec{a}^2 = a_1^2 + b_1^2 + c_1^2$ $\qquad \therefore \vec{a}\times\vec{i} = -b_1 k + c_1 j$

$\therefore (\vec{a}\times\vec{i})^2 = b_1^2 + c_1^2$

Similarly $(\vec{a}\times\hat{j})^2 = a_1^2 + c_1^2$

$(\vec{a}\times\hat{k})^2 = a_1^2 + b_1^2$

$\therefore (\vec{a}\times\hat{i})^2 + (\vec{a}\times\hat{j})^2 + (\vec{a}\times\hat{k})^2 = 2(a_1^2 + b_1^2 + c_1^2)$

$= 2\vec{a}^2.$

20. **(c) :** $[a,\ b,\ c] = \vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$$= \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1 - x \\ y & x & 1 + x - y \end{vmatrix} \quad C_3 \to C_3 + C_1$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ y & x & 1 + x \end{vmatrix} = 1(1) = 1$$

which is independent of x and y.

21. (b) : We are given that points lies in the same plane. We know that the vector L, M, N are coplanar if

$$L \cdot (\vec{M} \times \vec{N}) = 0 \Rightarrow \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0 \Rightarrow c = \sqrt{ab}$$

$\therefore$ C is G.M. of a and b.

22. (a) : From given

$\lambda^2(\vec{a}+\vec{b}) \cdot (\lambda\vec{b} \times \lambda\vec{c}) = \vec{a} \cdot (\vec{b}+\vec{c}) \times \vec{b}$

$\Rightarrow \lambda^2 \vec{a} \cdot (\lambda\vec{b} + \lambda\vec{c}) + \lambda^2 \vec{b} \cdot (\lambda\vec{b} \times \vec{c}) = \vec{a} \cdot (\vec{c} \times \vec{b})$

$\Rightarrow \lambda^4 [a \;\; b \;\; c] = -[a \;\; b \;\; c] \Rightarrow \lambda^4 + 1 = 0$

$\Rightarrow (\lambda^2)^2 + 1 = 0 \qquad D < 0$

$\Rightarrow$ No value of λ exist on real axis.

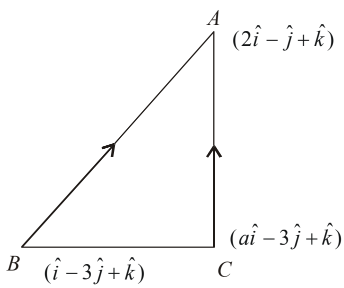
23. (d) : Given $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$

$\Rightarrow \quad (\vec{b} \cdot \vec{c})\vec{a} = (\vec{a} \cdot \vec{b})\vec{c} \qquad \Rightarrow \quad \lambda_1 \vec{a} = \lambda_2 \vec{c}$

$(\lambda_1 = \vec{b} \cdot \vec{c}, \lambda_2 = \vec{a} \cdot \vec{b}$ are scalar quantities)

$\Rightarrow \quad \vec{a} \parallel \vec{c}$

24. (a) : Now $\overrightarrow{CA} \cdot \overrightarrow{CB} = 0$



where $\overrightarrow{CA} = (2-a)\hat{i} - 2\hat{j}$

and $\overrightarrow{CB} = (1-a)\hat{i} - 6\hat{k}$

$\Rightarrow \quad a^2 - 3a + 2 = 0$

$\Rightarrow \quad (a-2)(a-1) = 0$

$\Rightarrow \quad a = 1, 2$

25. (b) : $|2\hat{u} \times 3\hat{v}| = 1 \Rightarrow 6|\hat{u}||\hat{v}||\sin\theta| = 1$

$\Rightarrow \sin\theta = \dfrac{1}{6}$

$2\hat{u} \times 3\hat{v}$ is a unit vector for exactly one value of θ.

26. (a) : $\vec{a}$ lies in the plane of $\vec{b}$ and $\vec{c}$. Also $\vec{a}$ bisects the angle $\vec{b}$ and $\vec{c}$. between Thus

$\vec{a} = \lambda(\vec{b} + \vec{c})$

$$\alpha\hat{i} + 2\hat{j} + \beta\hat{k} = \lambda\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}} + \frac{\hat{j}+\hat{k}}{\sqrt{2}}\right)$$

$$= \lambda\left(\frac{\hat{i}+2\hat{j}+\hat{k}}{\sqrt{2}}\right)$$

on comparison, $\lambda = \sqrt{2}\alpha, \lambda = \sqrt{2}$ and $\lambda = \sqrt{2}\beta$

Thus $\alpha = 1$ and $\beta = 1$

27. (a) : $\vec{a} = 8\vec{b}$

$\vec{c} = -7\vec{b}$

$\vec{a}$ and $\vec{b}$ are parallel and $\vec{b}$ and $\vec{c}$ are antiparallel.

Thus $\vec{a}$ and $\vec{c}$ are antiparallel.

Hence the angle between $\vec{a}$ and $\vec{c}$ is π.

Statistics

1. In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average of the girls?
(a) 73 (b) 65
(c) 68 (d) 74.
(2002)

2. In an experiment with 15 observations on x, the following results were available.
$$\sum x^2 = 2830, \ \sum x = 170$$
One observation that was 20 was found to be wrong and was replaced by the correct value 30. Then the corrected variance is
(a) 188.66
(b) 177.33
(c) 8.33
(d) 78.00.
(2003)

3. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set
(a) is decreased by 2
(b) is two times the original median
(c) remains the same as that of the original set
(d) is increased by 2.
(2003)

4. Consider the following statements :
(1) Mode can be computed from histogram
(2) Median is not independent of change of scale
(3) Variance is independent of change of origin and scale.
Which of these is/are correct?
(a) only (1) and (2)
(b) only (2)
(c) only (1)
(d) (1), (2) and (3).
(2004)

5. In a series of $2n$ observations, half of them equal a and remaining half equal $-a$. If the standard deviation of the observations is 2, then $|a|$ equals
(a) 2 (b) $\sqrt{2}$
(c) $\frac{1}{n}$ (d) $\frac{\sqrt{2}}{n}$.
(2004)

6. In a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is approximately
(a) 20.5 (b) 22.0
(c) 24.0 (d) 25.5.
(2005)

7. Let $x_1, x_2, ..., x_n$ be n observations such that $\sum x_1^2 = 400$ and $\sum x_i = 80$. Then a possible value of n among the following is
(a) 18 (b) 15
(c) 12 (d) 9.
(2005)

8. Suppose a population A has 100 observations 101, 102, ..., 200, and another population B has 100 observations 151, 152, ..., 250. If V_A and V_B represent the variances of the two populations, respectively,

then V_A/V_B is

(a) 1 (b) 9/4

(c) 4/9 (d) 2/3.

(2006)

9. The average marks of boys in class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is

(a) 80 (b) 60

(c) 40 (d) 20.

(2007)

10. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b?

(a) $a = 3, b = 4$ (b) $a = 0, b = 7$

(c) $a = 5, b = 2$ (d) $a = 1, b = 6$

(2008)

Answer Key

1.	(b)	**2.**	(d)	**3.**	(b)	**4.**	(a)	**5.**	(a)	**6.**	(c)
7.	(a)	**8.**	(a)	**9.**	(a)	10.	(a)				

1. **(b) :** Using $\bar{x} = \dfrac{(x_1 + x_2 + ... + x_{100})}{100} = 72$

$\therefore\ x_1 + ... + x_{100} = 7200$ ...(i)

Again $\dfrac{x_1 + x_2 + ... + x_{70}}{70} = 75$

$x_1 + ... + x_{70} = 75 \times 70$ ...(ii)

$\therefore$ Average of 30 girls $= \dfrac{7200 - 5250}{30} = 65$

2. **(d) :** $\sum x = 170$ and $\sum x^2 = 2830$

Increase in $\sum x = 10$ and $\sum x' = 170 + 10 = 180$

Increase in $\sum x^2 = 900 - 400 = 500$ then

$\sum x'^2 = 2830 + 500 = 3330$

$= \dfrac{1}{15} \times 3330 - \left(\dfrac{1}{15} \times 180\right)^2 = 222 - (12)^2 = 78$

3. **(b) :** Total number of observations are 9 which is odd which means median is 5th item now we are increasing 2 in the last four items which does not effect its value. The new median remain unchanged.

4. **(a) :** Mode can be computed by histogram Median will be changed if data's are changed so (2) is correct. Variance depends on change of scale so (3) is not correct.

5. **(a) :** According to problem

X	Value of X	$d =$ value of $X - \bar{X}$	$(X - \bar{X})^2$
x_1	a	a	a^2
x_2	a	a	a^2
–	–	–	–
–	–	–	–
–	–	–	–
–	–	–	–
x_n	a	a	–
x_{n+1}	$-a$	$-a$	a^2
x_{n+2}	$-a$	$-a$	a^2
–	–	–	–
–	–	–	–
–	–	–	–
x_{n+n}	$-a$	$-a$	a_2
$\Sigma X = 0$			$\sum (X - \bar{X})^2 = 2na^2$

$\therefore\ \bar{X} = \dfrac{\Sigma X}{N} = \dfrac{0}{2n} = 0$

Now $SD = \sqrt{\dfrac{\sum (X - \bar{X})^2}{N}} = \sqrt{\dfrac{\sum X^2}{N} - (\bar{X})^2}$

$2 = \sqrt{\dfrac{2na^2}{2n} - 0}$; $2 = \sqrt{a^2}$; $2 = |a|$

6. **(c) :** Using fact, mode = 3 median – 2 mean $= 3 \times 22 - 2 \times 21 = 3(22 - 14) = 3 \times 8 = 24$.

7. **(a) :** Using well known fact that root mean square of number $\geq$ A.M. of the numbers

$\Rightarrow \sqrt{\dfrac{400}{n}} \geq \dfrac{80}{n} \Rightarrow \dfrac{20}{\sqrt{n}} \geq \dfrac{80}{n}$

$\Rightarrow \sqrt{n} \geq 4 \Rightarrow n \geq 16$

$\Rightarrow n = 17$ but not given in choice.

$\therefore\ n = 18$ is correct number.

8. **(a) :** Series $A = 101, 102, 103, ..., 200$

Series $B = 151, 152, 153, ..., 250$

Series B is obtained by adding a fixed quantity to each item of series A, we know that variance is independent of change of origin both series have the same variance so ratio of their variances is 1.

9. **(a) :** Let x and y are number of boys and girls in a class respectively.

$\dfrac{52x + 42y}{x + y} = 50$

$\Rightarrow x = 4y \Rightarrow \dfrac{x}{y} = \dfrac{4}{1}$ and $\dfrac{x}{x + y} = \dfrac{4}{5}$

Required percentage

$= \dfrac{x}{x + y} \times 100 = \dfrac{4}{5} \times 100 = 80\%.$

10. **(a) :** The mean of a, b, 8, 5, 10 is 6

$\Rightarrow \dfrac{a + b + 8 + 5 + 10}{5} = 6$

$\Rightarrow a + b + 23 = 30 \Rightarrow a + b = 7$ ...(1)

Again variance $= \dfrac{\Sigma (x_i - A)^2}{n} = 6.8$

$\Rightarrow \dfrac{(a - 6)^2 + (b - 6)^2 + 4 + 1 + 16}{5} = 6.8$

$\Rightarrow a^2 + b^2 - 12(a + b) + 36 + 21 + 72 = 5 \times 6.8 = 34$

$\Rightarrow a^2 + b^2 - 12 \times 7 + 72 + 21 = 34$

$\therefore\ a^2 + b^2 = 25$ ...(2)

using (1) we have

$a^2 + (7 - a)^2 = 25 \Rightarrow a^2 + 49 - 14a + a^2 = 25$

$\Rightarrow a^2 - 7a + 12 = 0 \quad \therefore\ a = 3, 4$ also $b = 3, 4$

CHAPTER
16

Probability

1. A problem in mathematics is given to three students *A, B, C* and their respective probability of solving the problem is 1/2, 1/3 and 1/4. Probability that the problem is solved is

(a) 3/4 (b) 1/2
(c) 2/3 (d) 1/3.

(2002)

2. A and B are events such that $P(A \cup B) = 3/4$, $P(A \cap B) = 1/4$, $P(\bar{A}) = 2/3$ then $P(\bar{A} \cap B)$ is

(a) 5/12 (b) 3/8
(c) 5/8 (d) 1/4.

(2002)

3. A die is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is

(a) 8/3 (b) 3/8
(c) 4/5 (d) 5/4.

(2002)

4. Events A, B, C are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$ and $P(C) = \frac{1-2x}{2}$. Then set of possible values of x are in the interval

(a) $\left[\frac{1}{3}, \frac{2}{3}\right]$ (b) $\left[\frac{1}{3}, \frac{13}{3}\right]$
(c) $[0, 1]$ (d) $\left[\frac{1}{3}, \frac{1}{2}\right]$.

(2003)

5. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is

(a) 3/5 (b) 1/5
(c) 2/5 (d) 4/5.

(2003)

6. The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively, then $P(X = 1)$ is

(a) 1/16 (b) 1/8 (c) 1/4 (d) 1/32.

(2003)

7. The probability that A speaks truth is 4/5, while this probability for B is 3/4. The probability that they contradict each other when asked to speak on a fact is

(a) 7/20 (b) 1/5
(c) 3/20 (d) 4/5. *(2004)*

8. A random variable X has the probability distribution:

X :	1	2	3	4	5	6	7	8
$P(X)$:	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events $E = \{X \text{ is a prime number}\}$ and $F = \{X < 4\}$, the probability $P(E \cup F)$ is:

(a) 0.35 (b) 0.77
(c) 0.87 (d) 0.50.

(2004)

9. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is:

(a) 128/256 (b) 219/256
(c) 37/256 (d) 28/256.

(2004)

10. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for complement of event A. Then events A and B are

(a) equally likely but not independent
(b) equally likely and mutually exclusive
(c) mutually exclusive and independent
(d) independent but not equally likely.

(2005)

11. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is

(a) 1/9 (b) 2/9
(c) 7/9 (d) 8/9.

(2005)

12. A random variable X has Poisson distribution with mean 2. The $P(X > 1.5)$ equals

(a) 0 (b) $2/e^2$
(c) $3/e^2$ (d) $1-\frac{3}{e^2}$.

(2005)

13. At a telephone enquiry system the number of phone calls regarding relevant enquiry follow Poisson distribution with a average of 5 phone calls during 10-minute time intervals. The probability that there is at the most one phone call during a 10-minute time period is

(a) $\frac{6}{5^e}$ (b) $\frac{5}{6}$ (c) $\frac{6}{55}$ (d) $\frac{6}{e^5}$.

(2006)

14. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is

(a) 0.2 (b) 0.7
(c) 0.06 (d) 0.14.

(2007)

15. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is

(a) 8/729 (b) 8/243
(c) 1/729 (d) 8/9.

(2007)

16. It is given that the events A and B are such that $P(A)=\frac{1}{4}, P(A|B)=\frac{1}{2}$ and $P(B|A)=\frac{2}{3}$. Then $P(B)$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(2008)

17. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) 0 (d) 1

(2008)

Answer Key

1.	(a)	**2.**	(a)	**3.**	(d)	**4.**	(d)	**5.**	(c)	**6.**	(d)
7.	(a)	**8.**	(b)	**9.**	(d)	**10.**	(d)	**11.**	(a)	**12.**	(d)
13.	(d)	**14.**	(d)	**15.**	(b)	**16.**	(c)	**17.**	(d)		

1. **(a) :** Given $P(A) = 1/2$ $\quad \therefore P(\bar{A}) = 1/2$

$P(B) = 1/3 \quad \therefore P(\bar{B}) = \frac{2}{3}$

$P(C) = \frac{1}{4} \quad \therefore P(\bar{C}) = \frac{3}{4}$

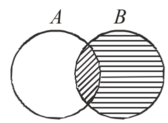
Now problem will be solved if any one of them will solve the problem.

$\therefore$ P(at least one of them solve the problem) = 1 – probability none of them can solve the problem.

or $P(A \cup B \cup C) = 1 - P(\bar{A})\,P(\bar{B})\,P(\bar{C})$

$= 1 - \frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4} = 3/4$

2. **(a) :** Given $P(A \cup B) = 3/4$



$P(A \cap B) = 1/4$

$P(\bar{A}) = 2/3$

$\therefore \quad P(B) = \frac{2}{3} \qquad P(\bar{A} \cap \bar{B}) = P(B) - P(A \cap B)$

By using $P(B) = P(A \cup B) + P(A \cap B) - P(A)$

$\therefore \quad P(\bar{A} \cap B) = \frac{2}{3} - \frac{1}{4} = 5/12$

3. **(d) :** $n = 5$ $p = q = 1/2$ $\quad P(X = r) = {}^5C_r\left(\frac{1}{2}\right)^n$

	x_i	f_i	$f_i x_i$	$f_i x_i^2$
(0)	0	$\left(\frac{1}{2}\right)^5$	0	0
(1)	${}^5C_1\left(\frac{1}{2}\right)^5$	$\frac{1\times 5}{32}$	$\frac{5}{32}$	
(2)	${}^5C_1\left(\frac{1}{2}\right)^5$	$\frac{2\times 10}{32}$	$2^2\,\frac{10}{32}$	
(3)	${}^5C_3\left(\frac{1}{2}\right)^5$	$\frac{3\times 10}{32}$	$3^2\,\frac{10}{32}$	
(4)	${}^5C_4\left(\frac{1}{2}\right)^5$	$\frac{4\times 5}{32}$	$4^2\,\frac{5}{32}$	
(5)	${}^5C_5\left(\frac{1}{2}\right)^5$	$5 \times \frac{1}{32}$	$5^2\,\frac{1}{32}$	
	$\Sigma f_i = 1$	$\Sigma f_i x_i = \frac{80}{32}$	$\Sigma f_i x_i^2 = \frac{240}{32}$	

$$\bar{x} = \text{mean} = \frac{5}{2}$$

$$\text{Now variance} = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$$

$$= \frac{240}{32} - \frac{25}{4}$$

$$= \frac{40}{32}$$

$$= \frac{5}{4}$$

4. **(d) :** A, B, C are mutually exclusive

$\therefore \quad 0 \le P(A) + P(B) + P(C) \le 1 \qquad$ ...(i)

$0 \le P(A), P(B), P(C) \le 1 \qquad$ ...(ii)

now on solving (i) and (ii) we get

$\frac{1}{3} \le x \le \frac{1}{2}$

5. **(c) :** No. of horses = 5

$\therefore$ Probability that A can't win the race

$= \frac{4}{5} \times \frac{3}{4}$

Probability that 'A' must win the race

$= 1 - P(\bar{A})P(\bar{B})$

$= 1 - \frac{12}{20}$

$= \frac{2}{5}$

6. **(d) :** Given mean $np = 4$, $npq = 2$

$\Rightarrow \frac{npq}{np} = \frac{2}{4} \ \therefore q = p = \frac{1}{2}$ and $n = 8$

Now $P(X = r) = {}^8C_r\left(\frac{1}{2}\right)^8$

(Use $p(X = r) = {}^nC_r\, p^r q^{n-r}$

$\therefore p(X = 1) = {}^8C_1\left(\frac{1}{2}\right)^8 = \frac{8}{16\times 16} = \frac{1}{32}$

7. **(a) :** $P(A) = \frac{4}{5}$ $\quad \therefore P(\bar{A}) = \frac{1}{5}$

$P(B) = \frac{3}{4}$ $\quad \therefore P(\bar{B}) = \frac{1}{4}$

Now we needed $P(A)\,P(\bar{B}) + P(B)\,P(\bar{A})$

$= \frac{4}{5}\times\frac{1}{4}+\frac{3}{4}\times\frac{1}{5}=\frac{7}{20}$

8. **(b) :** From the given table prime numbers are 2, 3, 5, 7

'E' denote prime

'F' denote the number < 4

$\therefore$ $P(E) = P(2$ or 3 or 5 or $7)$
(Events 2, 3, 5, 7 are $M.E$)

$= P(2) + P(3) + P(5) + P(7) = .62$

$P(F) = P(1$ or 2 or $3)$ (events 1, 2, 3 are $m.E.$)

$= P(1) + P(2) + P(3) = .50$

$P(E \cap F) = P(2$ or $3) = P(2) + P(3) = .35$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= .62 + .50 - .35 = .77$

9. **(d) :** Given $np = 4$ and $npq = 2$

$q = \frac{npq}{np} = \frac{2}{4} = \frac{1}{2}$ so $p = 1 - 1/2 = \frac{1}{2}$

Now $npq = 2 \therefore n = 8$

$\therefore$ BD is given by $P(X = r) = {}^8C_r\, p^r\, q^{n-r}$

$\therefore$ $P(X = r = 2) = {}^8C_2\left(\frac{1}{2}\right)^8 = \frac{28}{256}$

10. **(d) :** $P(\overline{A\cup B}) = 1-P(A\cup B)$

$= 1-P(A)-P(B)+P(A\cap B)$

$\frac{1}{6} = 1-\frac{3}{4} - P(B)+\frac{1}{4}$

$P(B) = \frac{1}{2}-\frac{1}{6} \Rightarrow P(B) = \frac{4}{12} = \frac{1}{3}$

now $P(A\cap B) = \frac{1}{4} = \frac{1}{3}\times\frac{3}{4} = P(A)P(B)$

so even are independent but not equally likely as $P(A) \neq P(B)$.

11. **(a) :** No. of houses = 3 = No. of favourable cases

No. of applicants = 3,

$\therefore$ Total number of events $= 3^3$

(because each candidate can apply by 3 ways)

Required probability $= \frac{3}{3^3} = \frac{1}{9}$.

12. **(d) :** $P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}$ $(\lambda = \text{mean})$

$\therefore P(X = r > 1.5) = P(2) + P(3) + \ldots \infty$

$= 1 - [P(0) + P(1)]$

$= 1 - \left[e^{-2}+\frac{e^{-2}\times 2^2}{2}\right] = 1-\frac{3}{e^2}$.

13. **(d) :** We know that poission distribution is given by

$P(x=r) = \frac{e^{-\lambda}\lambda^r}{r!}$ where $\lambda = 5$

Now $P(x = r \leq 1) = P(x = 0) + P(x = 1)$

$= \frac{e^{-\lambda}}{0!}+\frac{\lambda_e^{-\lambda}}{1!} = e^{-5}(1+5) = \frac{6}{e^5}$.

14. **(d) :** $P(I) = 0.3, P(\bar{I}) = 1-0.3 = 0.7,$

$P(II) = 0.2, P(\overline{II}) = 1-0.2 = 0.8$

Required probability $= P(\bar{I}\cap II) = P(\bar{I})P(II)$
$= (0.7)(0.2) = 0.14.$

15. **(b) :** Possibility of getting 9 are
(5, 4), (4, 5), (6, 3), (3, 6)

Probability of getting score 9 in
a single throw $= p = \frac{4}{36} = \frac{1}{9}$

Required probability = probability of getting score 9 exactly twice $= {}^3C_2\left(\frac{1}{9}\right)^2\times\left(\frac{8}{9}\right) = \frac{8}{243}$.

16. **(c) :** From the definition of independence of events

$P(A/B) = \frac{P(A\cap B)}{P(B)}$

Then $P(B)\cdot P(A/B) = P(A \cap B)$ ...(1)

Interchanging the role of A and B in (1)

$P(A)P(B/A) = P(B \cap A)$ ...(2)

As $A \cap B = B \cap A$, we have from (1) and (2)

$P(A)P(B/A) = P(B)P(A/B)$

$\Rightarrow \frac{1}{4}\cdot\frac{2}{3} = P(B)\cdot\frac{1}{2} \Rightarrow P(B) = \frac{1}{4}\cdot\frac{2}{3}\cdot 2 = \frac{1}{3}$

17. **(d) :** $A = \{4, 5, 6\}$

Also $B = \{1, 2, 3, 4\}$

We have $A \cup B = \{1, 2, 3, 4, 5, 6\} = S$

Where S is the sample space of the experiment of throwing a die. $P(S) = 1$, for it is a sure event.

Hence $P(A \cup B) = 1$

CHAPTER 17

Trigonometry

1. The number of solution of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is
(a) 2 (b) 3 (c) 0 (d) 1. *(2002)*

2. $\cot^{-1}\left[(\cos\alpha)^{\frac{1}{2}}\right] + \tan^{-1}\left[(\cos\alpha)^{\frac{1}{2}}\right] = x$
then $\sin x =$
(a) 1 (b) $\cot^2(\alpha/2)$
(c) $\tan\alpha$ (d) $\cot(\alpha/2)$ *(2002)*

3. In a triangle with sides a, b, c, $r_1 > r_2 > r_3$ (which are the ex-radii) then
(a) $a > b > c$ (b) $a < b < c$
(c) $a > b$ and $b < c$ (d) $a < b$ and $b > c$. *(2002)*

4. The trigonometric equation $\sin^{-1}x = 2\sin^{-1}a$, has a solution for
(a) all real values (b) $|a| < \frac{1}{2}$
(c) $|a| \geq \frac{1}{\sqrt{2}}$ (d) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$. *(2003)*

5. The sum of the radii of inscribed and circumscribed circles for an n sides regular polygon of side a, is
(a) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ (b) $a\cot\left(\frac{\pi}{2n}\right)$
(c) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$ (d) $a\cot\left(\frac{\pi}{n}\right)$. *(2003)*

6. The upper $3/4^{\text{th}}$ portion of a vertical pole subtends an angle $\tan^{-1}(3/5)$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is
(a) 40 m (b) 60 m
(c) 80 m (d) 20 m. *(2003)*

7. In a triangle ABC, medians AD and BE are drawn. If $AD = 4$, $\angle DAB = \pi/6$ and $\angle ABE = \pi/3$, then the area of the ΔABC is
(a) 16/3 (b) 32/3
(c) 64/3 (d) 8/3. *(2003)*

8. If in a triangle ABC, $a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides a, b and c
(a) are in G.P. (b) are in H.P.
(c) satisfy $a + b = c$ (d) are in A.P. *(2003)*

9. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -21/65$, and $\cos\alpha + \cos\beta = -27/65$, then the value of $\cos\frac{\alpha-\beta}{2}$ is
(a) $\frac{6}{65}$ (b) $\frac{3}{\sqrt{130}}$
(c) $-\frac{3}{\sqrt{130}}$ (d) $\frac{-6}{65}$. *(2004)*

10. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$ then the difference between the maximum and minimum values of u^2 is given by
(a) $(a + b)^2$ (b) $2\sqrt{a^2+b^2}$
(c) $2(a^2 + b^2)$ (d) $(a - b)^2$. *(2004)*

11. The sides of a triangle are $\sin\alpha$, $\cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is
(a) 120° (b) 90°
(c) 60° (d) 150°. *(2004)*

12. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 meters away from the tree the angle of elevation becomes 30°. The breadth of the river is

(a) 40 m (b) 30 m
(c) 20 m (d) 60 m. *(2004)*

13. In a triangle PQR, if $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then
(a) $b = a + c$ (b) $b = c$
(c) $c = a + b$ (d) $a = b + c$ *(2005)*

14. In a triangle ABC, let $\angle C = \frac{\pi}{2}$. If r is the inradius and R is the circumradius of the triangle ABC, then $2(r + R)$ equals
(a) $a + b$ (b) $b + c$
(c) $c + a$ (d) $a + b + c$ *(2005)*

15. If $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$, then $4x^2 - 4xy \cos\alpha + y^2$ is equal to
(a) 4 (b) $2\sin 2\alpha$
(c) $-4\sin^2\alpha$ (d) $4\sin^2\alpha$. *(2005)*

16. If in a ΔABC, the altitudes from the vertices A, B, C on opposite sides are in H.P., then $\sin A$, $\sin B$, $\sin C$ are in
(a) H.P.
(b) Arithmetic-Geometric progression
(c) A.P. (d) G.P. *(2005)*

17. The number of value of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is
(a) 4 (b) 6 (c) 1 (d) 2.
(2006)

18. If $0 < x < \pi$ and $\cos x + \sin x = 1/2$, then $\tan x$ is
(a) $\frac{(1-\sqrt{7})}{4}$ (b) $\frac{(4-\sqrt{7})}{3}$
(c) $-\frac{(4+\sqrt{7})}{3}$ (d) $\frac{(1+\sqrt{7})}{4}$. *(2006)*

19. If $\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the values of x is
(a) 4 (b) 5 (c) 1 (d) 3.
(2007)

20. The largest interval lying in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for which the function,

$$f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x),$$

is defined, is
(a) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (b) $\left[0, \frac{\pi}{2}\right)$
(c) $[0, \pi]$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. *(2007)*

21. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB $(= a)$ subtends an angle of 60° at the foot of the tower, and the angle of elevation of the top of the tower from A or B is 30°. The height of the tower is
(a) $a/\sqrt{3}$ (b) $a\sqrt{3}$
(c) $2a/\sqrt{3}$ (d) $2a\sqrt{3}$. *(2007)*

22. The value of is $\cot\left(\text{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$
(a) $\frac{5}{17}$ (b) $\frac{6}{17}$ (c) $\frac{3}{17}$ (d) $\frac{4}{17}$
(2008)

23. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that $CD = 7$ m. From D the angle of elevation of the point A is 45°. Then the height of the pole is
(a) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}+1}$ m (b) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}$ m
(c) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)$ m (d) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)$ m
(2008)

Answer Key

1.	(b)	**2.**	(a)	**3.**	(a)	**5.**	(a)	**6.**	(a)	**8.**	(d)
9.	(c)	**10.**	(d)	**11.**	(a)	**12.**	(c)	**13.**	(c)	**14.**	(a)
15.	(d)	**16.**	(c)	**17.**	(a)	**18.**	(c)	**19.**	(d)	**20.**	(b)
21.	(a)	**22.**	(b)	**23.**	(c)						

1. **(b) :** $\tan x + \sec x = 2\cos x$

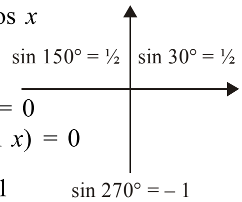
$1 + \sin x = 2\cos^2 x$

$1 + \sin x = 2(1 - \sin^2 x)$

$\Rightarrow\ 2\sin^2 x + \sin x - 1 = 0$

$\Rightarrow\ (2\sin x - 1)(1 + \sin x) = 0$

$\Rightarrow\ \sin x = \dfrac{1}{2}, \sin x = -1$



so there are three solution like $x = 30°, 150°, 270°$

2. **(a) :** Using $\tan^{-1}\theta + \cot^{-1}\theta = \dfrac{\pi}{2} = x$

$\therefore\quad \sin x = \sin\dfrac{\pi}{2} = 1$

3. **(a) :** As $r_1 > r_2 > r_3$

$\Rightarrow\quad \dfrac{\Delta}{s-a} > \dfrac{\Delta}{s-b} > \dfrac{\Delta}{s-c}$

$\Rightarrow\quad \dfrac{s-a}{\Delta} < \dfrac{s-b}{\Delta} < \dfrac{s-c}{\Delta}$

$\Rightarrow\quad a > b > c$

4. $\sin^{-1}x = 2\sin^{-1}a$

$\Rightarrow -\dfrac{\pi}{2} \le 2\sin^{-1}a \le \dfrac{\pi}{2}$

$\Rightarrow -\dfrac{\pi}{4} \le \sin^{-1}a \le \dfrac{\pi}{4}$

$\Rightarrow \sin\left(-\dfrac{\pi}{4}\right) \le a \le \sin\left(\dfrac{\pi}{4}\right)$

$\Rightarrow -\dfrac{1}{\sqrt{2}} \le a \le \dfrac{1}{\sqrt{2}} \quad \left\{\begin{array}{l}\because\ \sin^{-1}x = 2\sin^{-1}a \\ \text{and } -\dfrac{\pi}{2} \le \sin^{-1}x \le \dfrac{\pi}{2}\end{array}\right.$

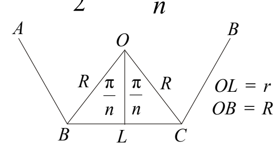
$\Rightarrow |a| \le \dfrac{1}{\sqrt{2}}$ No choice is match.

5. **(a) :** If R be the radius of circum circle of regular polygon of n sides, and r be the radius of inscribed circle the

$R = \dfrac{a}{2}\ \text{cosec}\ \dfrac{\pi}{2n}$ and $r = \dfrac{a}{2}\ \cot\ \dfrac{\pi}{n}$

$\therefore\ R + r =$

$\dfrac{a}{2}\left(\text{cosec}\dfrac{\pi}{n} + \cot\dfrac{\pi}{n}\right)$

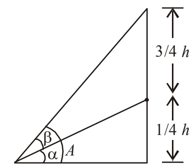


$= \dfrac{a}{2}\left(\dfrac{1+\cos\dfrac{\pi}{n}}{\sin\dfrac{\pi}{n}}\right) = \dfrac{a}{2}\cot\dfrac{\pi}{2n}$

6. **(a) :** $\alpha = A + \beta$

$\therefore\quad \beta = A - \alpha$

$\tan\beta = \dfrac{\tan A - \tan\alpha}{1 - \tan A\tan\alpha}$



$\Rightarrow\quad \dfrac{3}{5} = \dfrac{\dfrac{h}{40} + \left(-\dfrac{h}{160}\right)}{1 - \left(\dfrac{h}{40}\right)\left(-\dfrac{h}{160}\right)}$

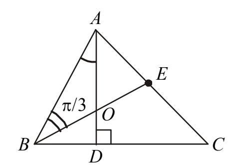
$\Rightarrow\quad h^2 - 200h + 6400 = 0$

$\Rightarrow\quad (h - 40)(h - 160) = 0$

$\Rightarrow\quad h = 40$ or $h = 160$

7. $\dfrac{OB}{AO} = \tan 30°$

$\Rightarrow\quad \overrightarrow{OB} = \dfrac{OA}{\sqrt{3}} = \dfrac{8\sqrt{3}}{9}$



Area of triangle $ADB = \dfrac{1}{2} \times \dfrac{8\sqrt{3}}{9} \times 4 = \dfrac{16\sqrt{3}}{9}$

Area of triangle $ABC = 2 \times \dfrac{16\sqrt{3}}{9} = \dfrac{32\sqrt{3}}{9}$

8. **(d) :** $2a\cos^2\dfrac{C}{2} + 2c\cos^2 A/2 = 3b$

(from given)

$\Rightarrow\quad a(1 + \cos C) + c(1 + \cos A) = 3b$

$\Rightarrow\quad a + c + a\cos C + c\cos A = 3b$

($a\cos C + c\cos A = b$ projection formula)

$\Rightarrow\quad a + c + b = 3b$

$\Rightarrow\quad a + c = 2b$

9. **(c) :** $\sin\alpha + \sin\beta = -\dfrac{21}{65}$

and $\cos\alpha + \cos\beta = -\dfrac{27}{65}$

by squaring and adding we get

$$2(1 + \cos\alpha\cos\beta + \sin\alpha\sin\beta) = \frac{(21)^2 + (27)^2}{(65)^2}$$

$$2[1 + \cos(\alpha - \beta)] = \frac{1170}{(65)^2}$$

$$\cos^2\frac{(\alpha-\beta)}{2} = \frac{1170}{4\times65\times65} = \frac{130\times9}{(130)\times(130)}$$

$$= \frac{9}{130}$$

$$\therefore \cos\frac{\alpha-\beta}{2} = \frac{3}{\sqrt{130}}$$

As $\pi < \alpha - \beta < 3\pi$ then $\cos\left(\dfrac{\alpha-\beta}{2}\right) =$ negative

10. **(d) :** $u^2 = a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) +$

$$2\sqrt{\left(a^4+b^4\right)\sin^2\theta\cos^2\theta + a^2b^2\left(\sin^4\theta+\cos^4\theta\right)}$$

$$= a^2 + b^2 + 2\sqrt{a^2b^2 + \left(a^2-b^2\right)^2\sin^2\theta\cos^2\theta}$$

$$= a^2 + b^2 + 2 + \sqrt{a^2b^2 + \left(\frac{a^2-b^2}{2}\right)\sin 2\theta}$$

$\therefore u^2$ will be maximum or minimum according as $\theta = \pi/4$ or $\theta = 0°$

$\therefore$ Max. $u^2 = 2(a^2 + b^2)$ and

Min. $u^2 = a^2 + b^2 + 2ab = (a + b)^2$

Now Maximum u^2 – Minimum u^2

$= 2(a^2 + b^2) - (a^2 + b^2 + 2ab)$

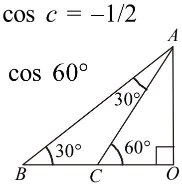
$= a^2 + b^2 - 2ab = (a - b)^2$

11. **(a) :** If $a^2 = \sin^2\alpha$, $b^2 = \cos^2\alpha$, $c^2 = 1 + \sin\alpha\cos\alpha$

then $\cos c = \dfrac{-\sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha}$ $\therefore \cos c = -1/2$

12. **(c) :** Breadth of river $OC = AC\cos 60°$

$= 40\cos 60°$



13. **(c) :** $\angle R = 90° \therefore \angle P + \angle Q = 90°$

$$\therefore \frac{P}{2} = \frac{90}{2} - \frac{Q}{2},\quad \frac{P}{2} = 45 - \frac{Q}{2}$$

$$\Rightarrow \tan\frac{P/2}{1} = \frac{1-\tan Q/2}{1+\tan Q/2}$$

$$\Rightarrow \tan\frac{P}{2} + \tan\frac{Q}{2} = 1 - \tan\frac{P}{2}\cdot\tan\frac{Q}{2}$$

$$\Rightarrow -\frac{b}{a} = 1 - \frac{c}{a}$$

$$\left(\because \tan\frac{P}{2},\ \tan\frac{Q}{2} \text{ are roots of } ax^2+bx+c=0\right)$$

$$\Rightarrow \frac{c-b}{a} = 1 \Rightarrow c = a + b.$$

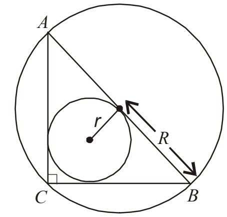
14. **(a) :** $\dfrac{c}{\sin C} = 2R$

$\therefore c = 2R$ ...(A)($\because C = 90°$) and

$$\tan\frac{C}{2} = \frac{r}{s-c}$$

$\therefore r = (s - c)$

$$\left(\tan\frac{C}{2} = \tan 45° = 1\right)$$



$= \dfrac{a+b+c}{2} - c$; $2r = a + b - c$ ...(B)

adding (A) and (B) we get $2(r + R) = a + b$.

15. **(d) :** Using $\cos^{-1}A - \cos^{-1}B$

$$= \cos^{-1}\left(AB + \sqrt{(1-A^2)}\sqrt{(1-B^2)}\right)$$

$$\therefore \cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$$

$$\Rightarrow \frac{xy}{2} + \sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}} = \cos\alpha$$

$$\Rightarrow \left(\cos\alpha - \frac{xy}{2}\right)^2 = (1-x^2)\left(1-\frac{y^2}{4}\right)$$

$$\Rightarrow 4x^2 - 4xy\cos\alpha + y^2 = 4(1-\cos^2\alpha) = 4\sin^2\alpha.$$

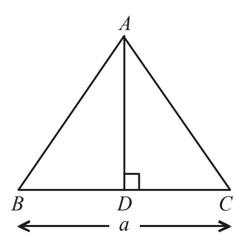
16. **(c) :** Altitude from A to BC is

Area of $\Delta = \dfrac{1}{2}AD\times BC$

$$\therefore \frac{2\cdot\text{ Area of }\Delta}{a} = AD$$

$\therefore$ Altitudes are in H.P.

$$\therefore \frac{2\Delta}{a}, \frac{2\Delta}{b}, \frac{2\Delta}{c} \in \text{HP}$$



$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \in$ HP $\quad \Rightarrow a, b, c \in$ A.P.

17. (a) : $2\sin^2 x + 5 \sin x - 3 = 0$

$\Rightarrow \sin x = \frac{1}{2}, \sin x \neq -3$

there $\sin x = \frac{1}{2}$, we know that each trigonometrical function assumes same value twice in $0 \le x \le 360°$. In our problem $0° \le x \le 540°$. So number of values are 4 like 30°, 150°, 390°, 510°.

18. (c) : $0 < x < \pi$

Given $\cos x + \sin x = \frac{1}{2}$

$\Rightarrow \quad 1 + \sin 2x = \frac{1}{4}$ (By squaring both sides)

$\frac{2\tan x}{1+\tan^2 x} = \frac{-3}{4}$

$\Rightarrow \quad 3\tan^2 x + 8\tan x + 3 = 0$

$\tan x = \frac{-8 \pm \sqrt{64-36}}{6} = \frac{-4 \pm \sqrt{7}}{3}$

$\because \tan x < 0 \Rightarrow \tan x = \frac{-4-\sqrt{7}}{8}$

19. (d) : $\sin^{-1}\left(\frac{x}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right)$

$\frac{x}{5} = \sin\left(\frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right)\right)$

$\frac{x}{5} = \cos\left(\sin^{-1}\frac{4}{5}\right) = \cos\left(\cos^{-1}\frac{3}{5}\right) = \frac{3}{5} \quad \Rightarrow x = 3.$

20. (b) : $f(x)$ is defined if $-1 \le \frac{x}{2} - 1 \le 1$ and $\cos x > 0$

or $0 \le x \le 4$ and $-\frac{\pi}{2} < x < \frac{\pi}{2} \quad \therefore 0 \le x < \frac{\pi}{2}.$

21. (a) : OP = Tower

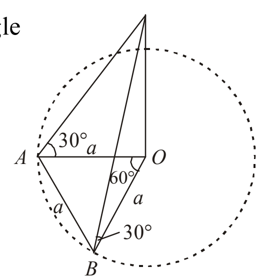
OAB is equilateral triangle

$\therefore OA = OB = AB = a$

In ΔAOP,

$\tan 30° = \frac{OP}{OA}$

$\Rightarrow OP = \frac{a}{\sqrt{3}}$



22. (b) : $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$

$= \cot\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right)$

$= \cot\left(\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right) = \cot\left(\tan^{-1}\frac{17}{6}\right) = \frac{6}{17}$

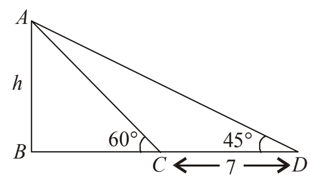
23. (c) : 1st Solution :

Let height of the pole AB be h. Then

$BC = h \cot 60° = h/\sqrt{3}$

$BD = h \cot 45° = h$

As $BD - BC = CD$



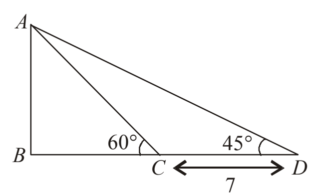
$\Rightarrow h - \frac{h}{\sqrt{3}} = 7 \quad \Rightarrow h(\sqrt{3}-1) = 7\sqrt{3}$

$\Rightarrow h = \frac{7\sqrt{3}}{\sqrt{3}-1} = \frac{7\sqrt{3}(\sqrt{3}+1)}{2} = \frac{7\sqrt{3}}{2}(\sqrt{3}+1)\text{m}$

2nd Solution :

We use the fact that the ratio of distance of B from D and that of B from C i.e. BD to BC is $\sqrt{3}:1$

$\frac{BD}{BC} = \sqrt{3}$, so that $\frac{BD}{CD} = \frac{\sqrt{3}}{\sqrt{3}-1}$



Then $BD = \frac{\sqrt{3}}{\sqrt{3}-1} CD = \frac{\sqrt{3}}{\sqrt{3}-1} \cdot 7 = \frac{7\sqrt{3}}{2}(\sqrt{3}+1)$

As $AB = BD$, the height of the pole

$= \frac{7\sqrt{3}}{2}(\sqrt{3}+1)\text{m}$

Mathematical Logic

Directions : Question number 1 is Assertion-Reason type. This question contains two statements : Statement-1 (Assertion) and Statement-2 (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

(a) Statement-1 is true, Statement-2 is false

(b) Statemen-1 is false, Statement-2 is true

(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

1. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the statement "x is a rational number iff y is a transcendental number".

Statement-1 : r is equivalent to either q or p.

Statement-2 : r is equivalent to $\sim (p \leftrightarrow \sim q)$.

(2008)

6. The statement $p \rightarrow (q \rightarrow p)$ is equivalent to

(a) $p \rightarrow (p \leftrightarrow q)$ (b) $p \rightarrow (p \rightarrow q)$

(c) $p \rightarrow (p \vee q)$ (d) $p \rightarrow (p \wedge q)$

(2008)

Answer Key

1. (a) **2.** (c)

EXPLANATIONS

1. **(a) :** The given statement $r \equiv \sim p \leftrightarrow q$
The Statement-1 is $r_1 \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$
The Statement-2 is
$$r_2 \equiv \sim (p \leftrightarrow \sim q) = (p \wedge q) \vee (\sim q \wedge \sim p)$$
we can establish that $r = r_1$
Thus Statement-1 is true but Statement-2 is false.

6. **(c) :** Let's simplify the statement
$p \to (q \to p) = \sim p \vee (q \to p) = \sim p \vee (\sim q \vee p)$
$= - p \vee p \vee \sim q = p \to (p \vee q)$

Statics*

1. The sum of two forces is 18 N and resultant whose direction is at right angles to the smaller force is 12 N. The magnitude of the two forces are
(a) 13, 5 (b) 12, 6 (c) 14, 4 (d) 11, 7.
(2002)

2. A couple is of moment $\vec{G}$ and the force forming the couple is $\vec{P}$. If $\vec{P}$ is turned through a right angle, the moment of the couple thus formed is $\vec{H}$. If instead, the forces $\vec{P}$ are turned through an angle α, then the moment of couple becomes
(a) $\vec{H}\cos\alpha+\vec{G}\sin\alpha$ (b) $\vec{G}\cos\alpha+\vec{H}\sin\alpha$
(c) $\vec{H}\sin\alpha-\vec{G}\cos\alpha$ (d) $\vec{G}\sin\alpha-\vec{H}\cos\alpha$.
(2003)

3. The resultant of forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If $\vec{Q}$ is doubled, then $\vec{R}$ is doubled. If the direction of $\vec{Q}$ is reversed, then $\vec{R}$ is again doubled. Then $P^2 : Q^2 : R^2$ is
(a) 2 : 3 : 2 (b) 1 : 2 : 3
(c) 2 : 3 : 1 (d) 3 : 1 : 1.
(2003)

4. With two forces acting at a point, the maximum effect is obtained when their resultant is 4 N. If they act at right angles, then their resultant is 3 N. Then the forces are
(a) $\left(2+\frac{1}{2}\sqrt{2}\right)$N and $\left(2-\frac{1}{2}\sqrt{2}\right)$N
(b) $(2+\sqrt{3})$N and $(2-\sqrt{3})$N
(c) $(2+\sqrt{2})$N and $(2-\sqrt{2})$N
(d) $\left(2+\frac{1}{2}\sqrt{3}\right)$N and $\left(2-\frac{1}{2}\sqrt{3}\right)$N.
(2004)

5. In a right angle ΔABC, $\angle A = 90^\circ$ and sides a, b, c are respectively, 5 cm, 4 cm and 3 cm. If a force $\vec{F}$ has moments 0, 9 and 16 in N cm. units respectively about vertices A, B and C, the magnitude of F is
(a) 5 (b) 4 (c) 3 (d) 9.
(2004)

6. Three forces $\vec{P}$, $\vec{Q}$ and $\vec{R}$ acting along IA, IB and IC, where I is the incentre of a ΔABC, are in equilibrium. Then $\vec{P} : \vec{Q} : \vec{R}$ is
(a) $\sec\frac{A}{2} : \sec\frac{B}{2} : \sec\frac{C}{2}$
(b) $\sin\frac{A}{2} : \sin\frac{B}{2} : \sin\frac{C}{2}$
(c) $\cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$
(d) $\operatorname{cosec}\frac{A}{2} : \operatorname{cosec}\frac{B}{2} : \operatorname{cosec}\frac{C}{2}$.
(2004)

7. ABC is a triangle. Forces $\vec{P}, \vec{Q}, \vec{R}$ acting along IA, IB and IC respectively are in equilibrium, where I is the incentre of ΔABC. Then $P : Q : R$ is
(a) $\cos A : \cos B : \cos C$
(b) $\cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$
(c) $\sin\frac{A}{2} : \sin\frac{B}{2} : \sin\frac{C}{2}$
(d) $\sin A : \sin B : \sin C$
(2005)

8. The resultant R of two forces acting on a particle is at right angles to one of them and its magnitude is one third of the other force. The ratio of larger

force to smaller one is

(a) $3 : \sqrt{2}$ (b) $2 : 1$

(c) $3 : 2\sqrt{2}$ (d) $3 : 2.$

(2005)

9. A and B are two like parallel forces. A couple of moment H lies in the plane of A and B and is contained with them. The resultant of A and B after combining is displaced through a distance

(a) $\frac{H}{A+B}$ (b) $\frac{2H}{A-B}$

(c) $\frac{H}{A-B}$ (d) $\frac{H}{2(A+B)}$.

(2005)

10. ABC is a triangle, right angled at A. The resultant of the forces acting along $\overrightarrow{AB}, \overrightarrow{AC}$ with magnitudes $\frac{1}{AB}$ and $\frac{1}{AC}$ respectively is the force along $\overrightarrow{AD}$, where D is the foot of the perpendicular from A to BC. The magnitude of the resultant is

(a) $\frac{AB^2 + AC^2}{(AB)^2(AC)^2}$ (b) $\frac{(AB)(AC)}{AB + AC}$

(c) $\frac{1}{AB} + \frac{1}{AC}$ (d) $\frac{1}{AD}$.

(2006)

11. The resultant of two forces Pn and $3n$ is a force of $7n$. If the direction of $3n$ force were reversed, the resultant would be $\sqrt{19}\ n$. The value of P is

(a) $3\ n$ (b) $4\ n$

(c) $5\ n$ (d) $6\ n$.

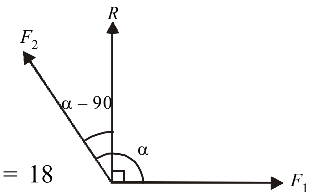
(2007)

Answer Key

1.	(a)	**2.**	(b)	**3.**	(a)	**4.**	(a)	**5.**	(a)	**6.**	(c)
7.	(b)	**8.**	(c)	**9.**	(a)	**10.**	(d)	**11.**	(c)		

1. **(a) :** $F_1 = R\dfrac{\sin(\alpha - 90)}{\sin\alpha}$ and

$F_2 = \dfrac{R\sin 90}{\sin\alpha}$



$F_1 + F_2 = 18$

$\Rightarrow \dfrac{12(1 - \cos\alpha)}{\sin\alpha} = 18$

$\therefore \quad \tan\alpha/2 = \dfrac{3}{2}$ $\quad (\because R = 12)$

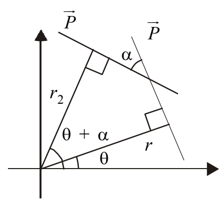
Putting in $F_2 = \dfrac{12}{\sin\alpha} = \dfrac{12}{2\tan\alpha/2}$

$(1 - \tan^2\alpha/2) = 5N$ and $F_1 = 13N$

2. **(b) :** $\vec{G} = \vec{P}\times\vec{r}\ \sin\theta$

$\vec{H} = Pr\cos\theta$

When $\vec{P}$ is turned through an angle α, then



Moment $\vec{M} = \vec{P}\times\vec{r}$

$= Pr\sin(\theta + \alpha)$

$= Pr\sin\theta\cos\alpha + Pr\cos\theta\sin\alpha$

$= \vec{G}\cos\alpha + \vec{H}\sin\alpha$

3. **(a) :** As $\vec{R}$ is resultant

$\therefore \quad \vec{R}^2 = P^2 + Q^2 + 2PQ\cos\theta$ ...(i)

$\therefore \quad 4R^2 = P^2 + 4Q^2 + 4PQ\cos\theta$ ...(ii)

As $\quad 4R^2 = P^2 + Q^2 - 2PQ\cos\theta$ ...(iii)

By (i) and (iii) we get

$5R^2 = 2(P^2 + Q^2)$ ...(iv) *i.e.* $2P^2 + 2Q^2 - 5R^2 = 0$

By (i) and (ii) we have

$2R^2 = 2Q^2 - P^2$...(v) $P^2 - 2Q^2 + 2R^2 = 0$

Now (iv) and (v) yields $\dfrac{P^2}{4-10} = \dfrac{-Q^2}{9} = \dfrac{R^2}{-4-2}$

$\Rightarrow \quad \dfrac{P^2}{-6} = \dfrac{Q^2}{-9} = \dfrac{R^2}{-6}$

$\Rightarrow \quad P^2 : Q^2 : R^2 = 6 : 9 : 6$

4. **(a) :** Let the two forces be $\vec{F}_1$ and $\vec{F}_2$

$\therefore \quad \vec{F}_1 + \vec{F}_2 = 4N$

so $\quad \vec{F}_1^2 + \vec{F}_2^2 + 2\vec{F}_1\cdot F_2 = 4^2 = 16$

$\vec{F}_1^2 + \vec{F}_2^2 = 9$

Now $\quad F_1{}^2 + F_2{}^2 + 2\vec{F}_1\cdot\vec{F}_2 = 16$

$\Rightarrow \quad \vec{F}_1\cdot\vec{F}_2 = \dfrac{7}{2}$

so $\quad \vec{F}_1 - \vec{F}_2 = \sqrt{\left(\vec{F}_1 + \vec{F}_2\right)^2 - 4\vec{F}_1\vec{F}_2}$

$= \sqrt{16-14} = \sqrt{2}\,N$

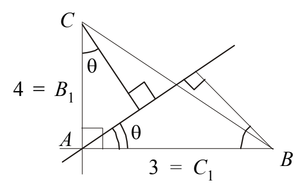
Further $\quad \vec{F}_1 + \vec{F}_2 = 4N$

$\vec{F}_1 - \vec{F}_2 = \sqrt{2}\,N$

$\Rightarrow F_1 = \left(2 + \dfrac{\sqrt{2}}{2}\right)N$ and $F_2 = \left(2 - \dfrac{\sqrt{2}}{2}\right)$

5. **(a) :** Now moment $\vec{F}$ about B say

$B_1 = 3F\sin\theta = 9$



$\Rightarrow \quad F\sin\theta = 3$ ...(i)

Moment $\vec{F}$ about C say

$C_1 = 4F\cos\theta = 16$

$\Rightarrow \quad F\cos\theta = 4$ ...(ii)

No squaring and adding (i) and (ii)

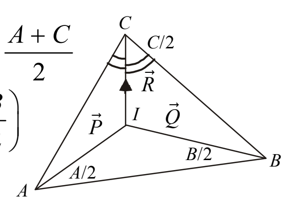
$\vec{F}^2 = 3^2 + 4^2$

$\vec{F} = 5N$

6. **(c) :** $\angle AIC = \pi - \dfrac{A+C}{2}$

$\angle AIC = \pi - \left(\dfrac{\pi}{2} - \dfrac{B}{2}\right)$

$= \dfrac{\pi}{2} + \dfrac{B}{2},$



similarly $\angle BIC = \dfrac{\pi}{2} + \dfrac{A}{2}$

and $\angle AIB = \frac{\pi}{2}+\frac{C}{2}$

Now using Lami's theorem we have

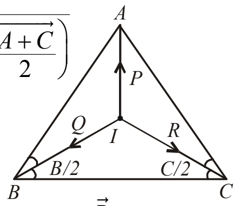
$$\frac{\vec{P}}{\sin\left(\frac{\pi}{2}+\frac{A}{2}\right)} = \frac{\vec{Q}}{\sin\left(\frac{\pi}{2}+\frac{B}{2}\right)} = \frac{\vec{R}}{\sin\left(\frac{\pi}{2}+\frac{C}{2}\right)}$$

Hence $\vec{P}:\vec{Q}:\vec{R}$ is $\cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$

7. (b) : Using Lami's theorem

$$\frac{\vec{P}}{\sin\left(\pi-\frac{\overrightarrow{B+C}}{2}\right)} = \frac{\vec{Q}}{\sin\left(\pi-\frac{\overrightarrow{A+C}}{2}\right)}$$

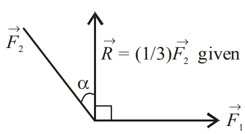
$$= \frac{R}{\sin\left(\pi-\frac{\overrightarrow{B+A}}{2}\right)}$$



$$\Rightarrow \quad \frac{\vec{P}}{\sin\frac{B+C}{2}} = \frac{\vec{Q}}{\sin\frac{C+A}{2}} = \frac{\vec{R}}{\sin\frac{A+B}{2}}$$

$$\Rightarrow \quad \frac{\vec{P}}{\cos\frac{A}{2}} = \frac{\vec{Q}}{\cos\frac{B}{2}} = \frac{\vec{R}}{\cos\frac{C}{2}}.$$

8. (c) : Now $\vec{F_1} = R\,\frac{\sin(90-\alpha)}{\sin\alpha} = \frac{\vec{F_2}}{3}\frac{\cos\alpha}{\sin\alpha}$ ...(1)



and $\vec{F_2} = \frac{R\ \sin 90°}{\sin\alpha} = \frac{1}{3}\frac{\vec{F_2}}{\sin\alpha}$ ...(2)

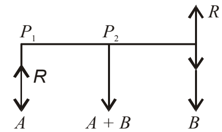
from (1) we have

$$\therefore \quad \frac{\vec{F_1}}{\vec{F_2}} = \frac{1}{3}\frac{\cos\alpha}{\sin\alpha} = \frac{1}{3}\frac{2\sqrt{2}}{3}\times\frac{3}{1}$$

$$\therefore\ \vec{F_2} : \vec{F_1} = 3 : 2\sqrt{2}.$$

9. (a) : Now $AP_1 = BP_2$

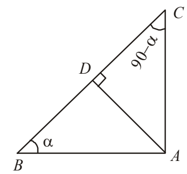
and $\frac{H}{R} = P_1 + P_2$ ..(A)



Let the displacement be y

$$\therefore\ \frac{(A+R)}{B-R} = \frac{P_1-y}{P_2+y} \Rightarrow y = \frac{H}{A+B} \text{ by using (A)}$$

10. (d) : As two forces with magnitudes $\frac{1}{AB}$ and $\frac{1}{AC}$ are at right angle



$\therefore$ magnitude of resultant $=\sqrt{\left(\frac{1}{AB}\right)^2+\left(\frac{1}{AC}\right)^2}$

$$= \frac{\sqrt{(AC)^2+(AB)^2}}{AB\cdot AC} = \frac{BC}{AB\cdot AC} = \frac{BC}{AB\cdot BC} = \frac{1}{AB} = \frac{1}{AD}$$

11. (c) : We know that

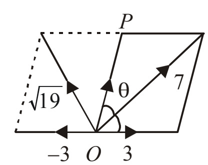
$F^2 = F_2^2 + F_2^2 + 2F_1F_2\cos\theta$, then

$49 = P^2 + 9 + 6P\cos\theta$ .... (i)

$19 = P^2 + 9 - 6P\cos\theta$ .... (ii)

By solving (i) and (ii)

we get $P = 5\ n$.



* *Not in syllabus*

Dynamics*

1. A bead of weight w can slide on smooth circular wire in a vertical plane. The bead is attached by a light thread to the highest point of the wire and in equilibrium, the thread is taut and make an angle θ with the vertical then tension of the thread and reaction of the wire on the bead are
(a) $T = W\cos\theta \quad R = W\tan\theta$
(b) $T = 2W\cos\theta \quad R = W$
(c) $T = W \quad R = W\sin\theta$
(d) $T = W\sin\theta \quad R = W\cot\theta$
(2002)

2. A body travels a distance s in t seconds. It starts from rest and ends at rest. In the first part of the journey, it moves with constant acceleration f and in the second part with constant retardation r. The value of t is given by
(a) $\dfrac{2s}{\frac{1}{f}+\frac{1}{r}}$ (b) $\sqrt{2s(f+r)}$
(c) $\sqrt{2s\left(\frac{1}{f}+\frac{1}{r}\right)}$ (d) $2s\left(\frac{1}{f}+\frac{1}{r}\right)$.
(2003)

3. Two stones are projected from the top of a cliff h metres high, with the same speed u so as to hit the ground at the same spot. If one of the stones is projected horizontally and the other is projected at an angle of θ to the horizontal then $\tan\theta$ equals
(a) $2g\sqrt{\frac{u}{h}}$ (b) $2h\sqrt{\frac{u}{g}}$
(c) $u\sqrt{\frac{2}{gh}}$ (d) $\sqrt{\frac{2u}{gh}}$.
(2003)

4. A particle acted by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The total work done by the forces is
(a) 30 units (b) 40 units
(c) 50 units (d) 20 units.
(2003)

5. Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity $\vec{u}$ and the other from rest with uniform acceleration $\vec{f}$. Let α be the angle between their directions of motion. The relative velocity of the second particle with respect to the first is least after a time
(a) $\dfrac{f\cos\alpha}{u}$ (b) $u\sin\alpha$
(c) $\dfrac{u\cos\alpha}{f}$ (d) $\dfrac{u\sin\alpha}{f}$.
(2003)

6. A particle moves towards east from a point A to a point B at the rate of 4 km/h and then towards north from B to C at the rate of 5 km/h. If $AB = 12$ km and $BC = 5$ km, then its average speed for its journey from A to C and resultant average velocity direct from A to C are respectively
(a) $\frac{17}{9}$ km/h and $\frac{13}{9}$ km/h
(b) $\frac{13}{4}$ km/h and $\frac{17}{4}$ km/h
(c) $\frac{17}{4}$ km/h and $\frac{13}{4}$ km/h

(d) $\frac{13}{9}$ km/h and $\frac{17}{9}$ km/h.

(2004)

7. A velocity $\frac{1}{4}$ m/s is resolved into two components along OA and OB making angles 30º and 45º respectively with the given velocity. Then the component along OB is

(a) $\frac{1}{4}$ m/s (b) $\frac{1}{4}(\sqrt{3}-1)$ m/s

(c) $\frac{1}{8}$ m/s (d) $\frac{1}{8}(\sqrt{6}-\sqrt{2})$ m/s.

(2004)

8. If t_1 and t_2 are the times of flight of two particles having the same initial velocity u and range R on the horizontal, then $t_1^2+t_2^2$ is equal to

(a) $\frac{u^2}{2g}$ (b) $\frac{4u^2}{g^2}$

(c) $\frac{u^2}{g}$ (d) 1.

(2004)

9. Two points A and B move from rest along a straight line with constant acceleration f and f' respectively. If A takes m sec. more than B and describes n units more than B in acquiring the same speed then

(a) $(f+f')m^2 = ff'n$ (b) $(f-f')m^2 = ff'n$

(c) $(f'-f)n = \frac{1}{2}ff'm^2$

(d) $\frac{1}{2}(f+f')m = ff'n^2$.

(2005)

10. A lizard, at an initial distance of 21 cm behind an insect, moves from rest with an acceleration of 2 cm/s^2 and pursues the insect which is crawling uniformly along a straight line at a speed of 20 cm/s. Then the lizard will catch the insect after

(a) 1 s (b) 20 s

(c) 24 s (d) 21 s.

(2005)

11. A particle is projected from a point O with velocity u at an angle of 60º with the horizontal. When it is moving in a direction at right angles to its direction at O, its velocity then is given by

(a) $u/2$ (b) $u/3$

(c) $u/\sqrt{3}$ (d) $2u/3$.

(2005)

12. A particle has two velocities of equal magnitude inclined to each other at an angle θ. If one of them is halved, the angle between the other and the original resultant velocity is bisected by the new resultant. Then θ is

(a) 90° (b) 120°

(c) 45° (d) 60°.

(2006)

13. A body falling from rest under gravity passes a certain point P. It was at a distance of 400 m from P, 4 s prior to passing through P. If g = 10 m/s^2, then the height above the point P from where the body began to fall is

(a) 720 m (b) 900 m

(c) 320 m (d) 680 m.

(2006)

14. A particle just clears a wall of height b at a distance a and strikes the ground at a distance c from the point of projection. The angle of projection is

(a) $\tan^{-1}\frac{bc}{a(c-a)}$ (b) $\tan^{-1}\frac{bc}{a}$

(c) $\tan^{-1}\frac{b}{ac}$ (d) 45°.

(2007)

15. A body weighing 13 kg is suspended by two strings 5 m and 12 m long, their other ends being fastened to the extremities of a rod 13 m long. If the rod be so held that the body hangs immediately below the middle point, Then tensions in the strings are

(a) 5 kg and 12 kg (b) 5 kg and 13 kg

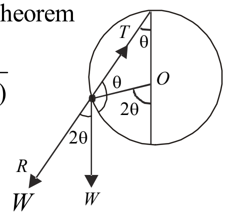
(c) 12 kg and 13 kg (d) 5 kg and 5 kg.

(2007)

Answer Key

1. (b)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (c)
7. (d)	**8.** (b)	**9.** (c)	**10.** (d)	**11.** (c)	**12.** (b)
13. (a)	**14.** (a)	**15.** (a)			

1. **(b) :** According to Lami's theorem



$$\frac{T}{\sin 2\theta} = \frac{R}{\sin(\pi - 2\theta + \theta)} = \frac{W}{\sin(\pi - \theta)}$$

$$\Rightarrow \quad \frac{T}{\sin 2\theta} = \frac{R}{\sin\theta} = \frac{W}{\sin\theta}$$

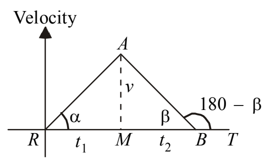
$$\Rightarrow \quad R = W \quad \text{and} \quad T = 2W\cos\theta$$

2. **(c) :** Portions OA, AB of the graph OAB corresponds to motions with acceleration f and retardation r respectively

now area of $\Delta OAB = s$ and $OB = t$

let $OM = t_1$, $MB = t_2$ and $AM = v$

$\therefore$ s = Area of ΔOAB



$\tan(180 - \beta) = -\gamma$

$\therefore \tan\beta = \gamma$

and $\tan\alpha = f$

$$\Rightarrow \quad s = \frac{1}{2}\text{ base} \times \text{height}$$

$$\Rightarrow \quad s = \frac{1}{2} OB \times AM = \frac{tv}{2}$$

$$\therefore \quad v = \frac{2s}{t} \qquad \text{... (i)}$$

Again $f = \dfrac{v}{t_1}$ $\therefore$ $t_1 = \dfrac{v}{f} = \dfrac{2s}{tf}$ by using (i) ... (ii)

and $\quad r = \dfrac{v}{t_2}$ $\therefore$ $t_2 = \dfrac{v}{r} = \dfrac{2s}{tr}$ by using (i) ... (iii)

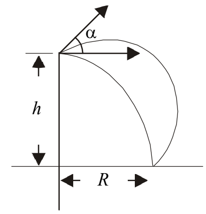
Now total time $t = t_1 + t_2 = \dfrac{2s}{t}\left(\dfrac{1}{f} + \dfrac{1}{r}\right)$

$$\Rightarrow t^2 = 2s\left(\frac{1}{f} + \frac{1}{r}\right)$$

3. **(c) :** $R = tu\cos\alpha = u\sqrt{\dfrac{2h}{g}}$

$$\therefore \ t = \frac{1}{\cos\alpha}\sqrt{\frac{2h}{g}} \qquad \text{... (1)}$$

Again $h = \dfrac{1}{2}gt^2 - (u\sin\alpha) \times t$



$$= \frac{1}{2}\frac{g(2h)}{g\cos^2\alpha} - \frac{u\sin\alpha}{\cos\alpha}\sqrt{\frac{2h}{g}}$$

$$-h\left(\frac{1-\cos^2\alpha}{\cos^2\alpha}\right) = -u\tan\alpha\sqrt{\frac{2h}{g}}$$

$$h\tan^2\alpha = u\tan\alpha\sqrt{\frac{2h}{g}}$$

$$\Rightarrow \tan\alpha = u\sqrt{\frac{2}{gh}}$$

4. **(b) :** Total force $\vec{F_1} + \vec{F_2} = \vec{F} = 7i + 2j - 4k$

and displacement

$$\vec{d} = (5\hat{i} + 4\hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = 4\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\therefore \ W.D. = \vec{F}.\vec{D} = (7\hat{i} + 2\hat{j} - 4\hat{k}).(4\hat{i} + 2\hat{j} - 2\hat{k})$$

$= 28 + 4 + 8$

$= 40$ units

5. **(c) :** Required relative velocity

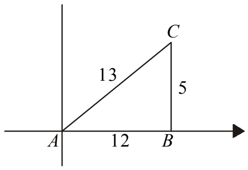
$v^2 = f^2t^2 + u^2 - 2uft\cos\alpha$

$$\therefore \ \frac{d(v)^2}{dt} = 2f^2t - 2uf\cos\alpha$$

$\left(\dfrac{d(v)^2}{dt} = 0\right.$ For maximum and minimum$\left.\right)$ $\Rightarrow t = \dfrac{u\cos\alpha}{f}$

6. **(c) :** According to problem first distance say $S_1 = 12$ and $t_1 = \dfrac{12}{4} = 3$ hr

second distance covered $S_2 = 5$ and $t_2 = \dfrac{5}{5} = 1$ hr



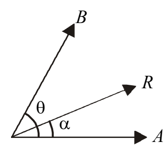
Now average speed $= \dfrac{S_1 + S_2}{t_1 + t_2} = \dfrac{17}{4}$ km/hr

Also average velocity =

$$\frac{\text{Displacement}}{t_2 + t_1} = \frac{\sqrt{12^2 + 5^2}}{4} = \frac{13}{4} \text{ km/hr}$$

7. **(d) :** Given $v = \dfrac{1}{4}$ m/sec

Fact :



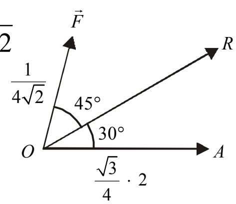
$$A = \frac{R\sin(\theta - \alpha)}{\sin\theta}$$

$$B = \frac{R\sin\alpha}{\sin\theta}$$

$\therefore$ According to problem

$$F = \left(\frac{\frac{1}{4} \times \frac{1}{\sqrt{2}}}{\sqrt{3}+1}\right) 2\sqrt{2}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{8}$$



8. **(b) :** Fact:

(i) $R = \dfrac{u^2 \sin 2\alpha}{g}$ and $t_1 = \dfrac{2u\sin\alpha}{g}$

and $t_2 = \dfrac{2u\sin(90° - \alpha)}{g}$

(ii) For the same velocity of projection and same range there are two direction of projection α and $90 - \alpha$

So $t_1^2 + t_1^2 = \dfrac{4u^2}{g^2}(\sin^2\alpha + \cos^2\alpha) = \dfrac{4u^2}{g^2}$

9. **(c) :** Let B travels x units, $v = u + at$

According to problem $f't = f(t+m)$

$$\Rightarrow \frac{f'-t}{f} = \frac{m}{t} \Rightarrow t^2 = m^2\left(\frac{f}{f'-t}\right)^2 \quad ...(1)$$

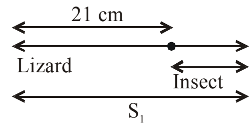
Again $n + x = \dfrac{1}{2}f(t+m)^2$

$$\Rightarrow n + \frac{1}{2}f't^2 = \frac{1}{2}f(t+m)^2 \quad ...(2)$$

Now from (1) and (4) we have

$$\frac{m}{n} = \frac{2(f'-f)}{mff'} \Rightarrow (f'-f)n = \frac{m^2 ff'}{2}.$$

10. **(d) :** $S_1 = ut + \dfrac{1}{2}gt^2$.



Let the lizard catch the insect after t seconds.

$\therefore S = ut + \dfrac{1}{2}gt^2 \Rightarrow t + \dfrac{1}{2} \times 2t^2$

= distance covered by insect

and distance cover by lizard

= 21 cm + distance covered by insect. Now

$\Rightarrow t^2 = 21 + \text{speed of insect} \times \text{time}$

$t^2 = 21 + 20 \times t$

$\Rightarrow t^2 - 20t - 21 = 0 \Rightarrow (t-21)(t+1) = 0$

$\Rightarrow t = 21$, $t = -1$, cannot possible $\therefore t = 21$ sec.

11. **(c) :** $(u)_{\text{along } x\text{-axis}} = v_{\text{along } x\text{-axis}}$

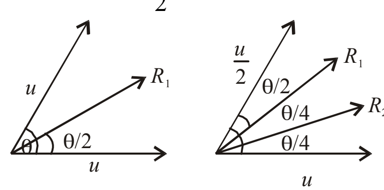
$\Rightarrow \quad u\cos\alpha = v\cos\theta$

$\Rightarrow \quad u\cos 60° = v\cos(90 - 60°)$

$\Rightarrow \quad \dfrac{u}{2} = \dfrac{v \times \sqrt{3}}{2} \Rightarrow v = \dfrac{u}{\sqrt{3}}.$

12. **(b) :** Now $R_1 = 2u\cos\dfrac{\theta}{2}$

Also $2u\cos\dfrac{\pi}{2} = u$



Angle $\dfrac{\theta}{4}$

$\Rightarrow \quad \cos\dfrac{\theta}{2} = \dfrac{1}{2}$

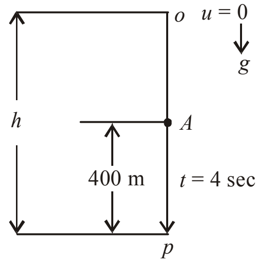
$\Rightarrow \quad \theta = 120°$

13. **(a) :** We know that initial velocity $u = 0$, $a = g$ then, height fallen $h - 400$

Final velocity $v = \sqrt{2g(h-400)}$

Where h = total height up to $P = OP$

In part AP, $s = ut + \frac{1}{2}at^2$ (formula)



$$400 = 4v + \frac{1}{2}g(4)^2$$

$\Rightarrow \quad 400 = 4\sqrt{2g(h-400)} + \frac{1}{2}(160)$

$\Rightarrow \quad 4\sqrt{2g(h-400)} = 400 - 8 \times 10$

$(\because \ a = 10 = g)$

$\Rightarrow \quad 20(h - 400) = (100 - 20)^2$

$\Rightarrow \quad 2(h - 400) = 640$

$\therefore \quad h = 320 + 400 = 720$ m

14. (a) : $a = (u\cos\alpha)t$ and $b = (u\sin\alpha)t - \frac{1}{2}gt^2$

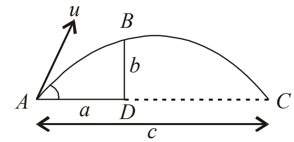
$b = a\tan\alpha - \frac{1}{2}g\frac{a^2}{u^2\cos^2\alpha}$ also, $c = \frac{u^2\sin 2\alpha}{g}$

$\Rightarrow b = a\tan\alpha - \frac{a^2 g}{2}\left(\frac{\sin 2\alpha}{cg}\right)\sec^2\alpha$

$\Rightarrow b = a\tan\alpha - \frac{a^2}{2c}2\tan\alpha$

$\Rightarrow \left(a - \frac{a^2}{c}\right)\tan\alpha = b$

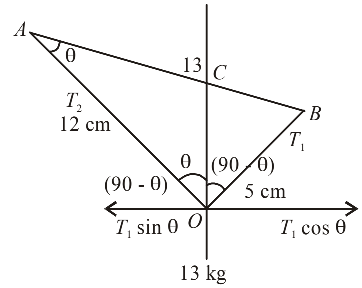
$\tan\alpha = \frac{bc}{a(c-a)}$.



15. (a) : $T_2\cos\left(\frac{\pi}{2} - \theta\right) = T_1\cos\theta \Rightarrow T_2\sin\theta = T_1\cos\theta$

$T_1 \sin\theta + T_2\cos\theta = 13 \quad \because \ OC = CA = CB$

$\Rightarrow \angle AOC = \angle OAC$ and $\angle COB = \angle OBC$



$\therefore \ \sin\theta = \sin A = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$

$\Rightarrow \frac{T_1}{T_2} = \frac{5}{12} \Rightarrow T_1 = \frac{5}{12}T_2$

$\Rightarrow T_2\left(\frac{5}{12} \times \frac{5}{13} + \frac{12}{13}\right) = 13$

$\Rightarrow T_2\left(\frac{169}{12 \times 13}\right) = 13 \quad \therefore \ T_2 = 12$ kgs $\Rightarrow T_1 = 5$ kgs.

* *Not in syllabus*

Solved Paper - 2009

Directions : Questions number 1 to 5 are Assertion-Reason type questions. Each of these questions contains two statements :
Statement - 1 (Assertion) and Statement - 2 (Reason).
Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(b) Statement- 1 is true, Statement-2 is false
(c) Statement-1 is false, Statement-2 is true
(d) Statement-1 is true, Statement-2 is true; Statement-2 is correct explanation for Statement-1

1. **Statement-1 :** $\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.

Statement-2 : $\sim (p \leftrightarrow \sim q)$ is a tautology.

[Mathematical Logic]

2. Let A be a 2×2 matrix

Statement-1 : adj (adj A) = A

Statement-2 : $|\text{adj } A| = |A|$

[Matrices and Determinants]

3. Let $f(x) = (x + 1)^2 - 1, x \geq -1$.

Statement-1 : The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$.

Statement-2 : f is a bijection.

[Sets, Relations and Functions]

4. **Statement-1 :** The variance of first n even natural numbers is $\dfrac{n^2-1}{4}$

Statement-2 : The sum of first n natural numbers is $\dfrac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\dfrac{n(n+1)(2n+1)}{6}$

[Sequences and Series]

5. Let $f(x) = x|x|$ and $g(x) = \sin x$.

Statement-1 : gof is differentiable at $x = 0$ and its derivative is continuous at that point.

Statement-2 : gof is twice differentiable at $x = 0$.

[Differential Calculus]

6. The area of the region bounded by the parabola $(y - 2)^2 = x - 1$, the tangent to the parabola at the point (2, 3) and the x-axis is

(a) 6 (b) 9
(c) 12 (d) 3

[Differential Calculus]

7. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that $x = 0$ is the only real root of $P'(x) = 0$. If $P(-1) < P(1)$, then in the interval $[-1, 1]$:

(a) $P(-1)$ is not minimum but $P(1)$ is the maximum of P
(b) $P(-1)$ is the minimum but $P(1)$ is not the maximum of P
(c) neither $P(-1)$ is the minimum nor $P(1)$ is the maximum of P
(d) $P(-1)$ is the minimum and $P(1)$ is the maximum of P

[Differential Calculus]

8. The shortest distance between the line $y - x = 1$ and the curve $x = y^2$ is

(a) $\dfrac{2\sqrt{3}}{8}$ (b) $\dfrac{3\sqrt{2}}{5}$

(c) $\dfrac{\sqrt{3}}{4}$ (d) $\dfrac{3\sqrt{2}}{8}$

[Two Dimensional Geometry]

9. Let the line $\dfrac{x-2}{3} = \dfrac{y-1}{-5} = \dfrac{z+2}{2}$ lie in the plane $x + 3y - \alpha z + \beta = 0$. Then (α, β) equals

(a) (–6, 7) (b) (5, –15)
(c) (–5, 5) (d) (6, –17)

[Three Dimensional Geometry]

10. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangements is

(a) at least 500 but less than 750
(b) at least 750 but less than 1000
(c) at least 1000 (d) less than 500

[Permutation and Combination]

11. In a binomial distribution $B\left(n,\ p=\frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than

(a) $\frac{1}{\log_{10}4+\log_{10}3}$ (b) $\frac{9}{\log_{10}4-\log_{10}3}$

(c) $\frac{4}{\log_{10}4-\log_{10}3}$ (d) $\frac{1}{\log_{10}4-\log_{10}3}$

[Probability]

12. The lines $p(p^2+1)x-y+q=0$ and $(p^2+1)^2x+(p^2+1)y+2q=0$ are perpendicular to a common line for
(a) exactly one value of p
(b) exactly two values of p
(c) more than two values of p
(d) no value of p

[Two Dimensional Geometry]

13. If A, B and C are three sets such that $A\cap B=A\cap C$ and $A\cup B=A\cup C$, then
(a) $A=C$ (b) $B=C$
(c) $A\cap B=\phi$ (d) $A=B$

[Sets, Relations and Functions]

14. For real x, let $f(x)=x^3+5x+1$, then
(a) f is onto R but not one-one
(b) f is one-one and onto R
(c) f is neither one-one nor onto R
(d) f is one-one but not onto R

[Sets, Relations and Functions]

15. The differential equation which represents the family of curves $y=c_1e^{c_2x}$, where c_1 and c_2 are arbitrary constants, is
(a) $y''=y'y$ (b) $yy''=y'$
(c) $yy''=(y')^2$ (d) $y'=y^2$

[Differential Equations]

16. Let a, b, c be such that $b(a+c)\neq 0$. If

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}+\begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix}=0,$$

then the value of n is
(a) any even integer (b) any odd integer
(c) any integer (d) zero

[Matrices and Determinants]

17. The remainder left out when $8^{2n}-(62)^{2n+1}$ is divided by 9 is
(a) 2 (b) 7
(c) 8 (d) 0

[Binomial Theorem]

18. Let y be an implicit function of x defined by $x^{2x}-2x^x\cot y-1=0$. Then $y'(1)$ equals
(a) 1 (b) log 2
(c) –log 2 (d) –1

[Differential Calculus]

19. If the roots of the equation $bx^2+cx+a=0$ be imaginary, then for all real values of x. The expression $3b^2x^2+6bcx+2c^2$ is
(a) less than $4ab$ (b) greater than $-4ab$
(c) less than $-4ab$ (d) greater than $4ab$

[Quadratic Equations]

20. The sum to infinity of the series

$$1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\ldots\ldots \text{ is}$$

(a) 3 (b) 4
(c) 6 (d) 2

[Sequences and Series]

21. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are

(a) $\frac{6}{5},\frac{-3}{5},\frac{2}{5}$ (b) $\frac{6}{7},\frac{-3}{7},\frac{2}{7}$

(c) $\frac{-6}{7},\frac{-3}{7},\frac{2}{7}$ (d) 6, –3, 2

[Vector Algebra]

22. Let A and B denote the statements
$A:\cos\alpha+\cos\beta+\cos\gamma=0$
$B:\sin\alpha+\sin\beta+\sin\gamma=0$
If $\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)=-\frac{3}{2}$, then
(a) A is false and B is true
(b) both A and B are true
(c) both A and B are false
(d) A is true and B is false

[Trigonometry]

23. One ticket is selected at random from 50 tickets numbered 00, 01, 02, …., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals

(a) $\frac{1}{7}$ (b) $\frac{5}{14}$

(c) $\frac{1}{50}$ (d) $\frac{1}{14}$

[Probability]

24. Three distinct points A, B and C are given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to 1/3. Then the circumcentre of the triangle ABC is at the point

(a) $\left(\frac{5}{4}, 0\right)$ (b) $\left(\frac{5}{2}, 0\right)$

(c) $\left(\frac{5}{3}, 0\right)$ (d) (0, 0)

[Two Dimensional Geometry]

25. If the mean deviation of number 1, 1 + d, 1 + 2d, ..., 1 + 100d from their mean is 255, then the d is equal to

(a) 20.0 (b) 10.1
(c) 20.2 (d) 10.0

[Statistics]

26. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is

(a) $x^2 + 12y^2 = 16$ (b) $4x^2 + 48y^2 = 48$
(c) $4x^2 + 64y^2 = 48$ (d) $x^2 + 16y^2 = 16$

[Two Dimensional Geometry]

27. If $\left|Z - \frac{4}{Z}\right| = 2$,then the maximum value of $|Z|$ is equal to

(a) $\sqrt{5}+1$ (b) 2

(c) $2+\sqrt{2}$ (d) $\sqrt{3}+1$

[Complex Numbers]

28. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p – 5 = 0$ and $x^2 + y^2 + 2x + 2y – p^2 = 0$, then there is a circle passing through P, Q and (1, 1) for

(a) all except one value of p
(b) all except two values of p
(c) exactly one value of p
(d) all values of p

[Two Dimensional Geometry]

29. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality

$$[3\vec{u}\ p\vec{v}\ p\vec{w}] - [p\vec{v}\ \vec{w}\ q\vec{u}] - [2\vec{w}\ q\vec{v}\ q\vec{u}] = 0$$

holds for

(a) exactly two values of (p, q)
(b) more than two but not all values of (p , q)
(c) all values of (p, q)
(d) exactly one value of (p, q)

[Vector Algebra]

30. $\int_0^{\pi}[\cot x]dx$, where [.] denotes the greatest integer function, is equal to

(a) 1 (b) –1
(c) – π/2 (d) π/2

[Integral Calculus]

EXPLANATIONS

1. **(b) :** Let's prepare the truth table

p	q	$\sim q$	$p \leftrightarrow q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$
T	T	F	T	F	T
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	T	F	T

As the column for $\sim(p \leftrightarrow \sim q)$ and $(p \leftrightarrow q)$ is the same, we conclude that $\sim(p \leftrightarrow \sim q)$ is equivalent to $(p \leftrightarrow q)$.

$\sim(p \leftrightarrow \sim q)$ is NOT a tautology because it's statement value is not always true.

2. **(a) :** We have adj(adj A) = $|A|^{n-2}A$

Here $n = 2$, which gives adj(adj A) = A

The statement-1 is true.

Again $|\text{adj } A| = |A|^{n-1}$

Here $n = 2$, which gives $|\text{adj } A| = |A|$

Thus statement-2 is also true. But statement-2 doesn't explain statement-1.

3. **(b) :** The solution of $f(x) = f^{-1}(x)$ are given by $f(x) = x$, which gives $(x+1)^2 - 1 = x$

$\Rightarrow \quad (x+1)^2 - (x+1) = 0 \quad \Rightarrow \quad (x+1)x = 0$

$\therefore \quad x = -1, 0$

But as no co-domain of f is specified, nothing can be said about f being ONTO or not.

4. **(c) :** Sum of first n even natural numbers

$= 2 + 4 + 6 + ... + 2n = 2(1 + 2 + ... + n)$

$$= 2 \cdot \frac{n(n+1)}{2} = n(n+1)$$

$$\text{Mean } (\bar{x}) = \frac{n(n+1)}{n} = n+1$$

$$\text{Variance} = \frac{1}{n}(\sum x_i)^2 - (\bar{x})^2$$

$$= \frac{1}{n}(2^2 + 4^2 + ... + (2n)^2) - (n+1)^2$$

$$= \frac{1}{n} \cdot 2^2(1^2 + 2^2 + ... + n^2) - (n+1)^2$$

$$= \frac{4}{n} \cdot \frac{n(n+1)(2n+1)}{6} - (n+1)^2$$

$$= \frac{2}{3} \cdot (n+1)(2n+1) - (n+1)^2$$

$$= \frac{(n+1)}{3}[2(2n+1) - 3(n+1)]$$

$$= \frac{(n+1)}{3} \cdot (n-1) = \frac{n^2-1}{3}$$

5. **(b) :** $gof(x) = g(f(x)) = \sin(x|x|)$

$$= \begin{cases} -\sin x^2, & x < 0 \\ \sin x^2, & x \ge 0 \end{cases}$$

Let the composite function $gof(x)$ be denoted by $H(x)$.

$$\text{Then } H(x) = \begin{cases} -\sin x^2, & x < 0 \\ \sin x^2, & x \ge 0 \end{cases}$$

$$LH'(0) = \lim_{h \to 0^-} \frac{H(0-h) - H(0)}{-h}$$

$$= \lim_{h \to 0^-} \frac{-\sin h^2}{-h} = \lim_{h \to 0^-} \cdot \frac{\sin h^2}{h^2} \cdot h = 1 \cdot 0 = 0$$

$$RH'(0) = \lim_{h \to 0^+} \frac{H(0+h) - H(0)}{h}$$

$$= \lim_{h \to 0^+} \frac{\sin h^2 - 0}{h} = \lim_{h \to 0^+} \left(\frac{\sin h^2}{h^2}\right) h$$

$= 1 \cdot 0 = 0$

Thus $H(x)$ is differentiable at $x = 0$

$$\text{Also } H'(x) = \begin{cases} -2x\cos x^2 & , x < 0 \\ 0 & , x = 0 \\ 2x\cos x^2 & , x > 0 \end{cases}$$

$H'(x)$ is continuous at $x = 0$ for

$H'(0) = LH'(0) = RH'(0)$

$$\text{Again } H''(x) = \begin{cases} -2\cos x^2 + 4x^2 \sin x^2 & , x < 0 \\ 2\cos x^2 - 4x^2 \sin x^2 & , x \ge 0 \end{cases}$$

$LH''(0) = -2$ and $RH''(0) = 2$

Thus $H(x)$ is NOT twice differentiable at $x = 0$

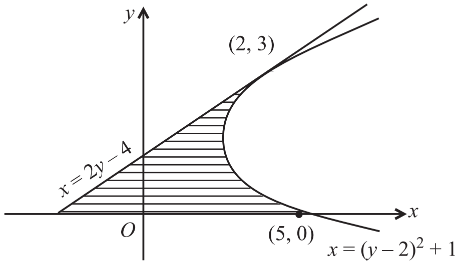
6. **(b) :** $(y-2)^2 = x - 1$

differentiating w.r.t. x, we have $2(y-2)y' = 1$

$$\Rightarrow \quad y' = \frac{1}{2(y-2)} \quad \text{at } (2, 3), \; y' = 1/2$$

The equation of the tangent to the parabola at (2, 3) is

$$y - 3 = \frac{1}{2}(x-2) \quad \Rightarrow \quad x - 2y + 4 = 0$$



The area of the bounded region

$= \int_0^3 [(y-2)^2 + 1 - (2y-4)]\,dy$

$= \int_0^3 (y^2 - 6y + 9)\,dy = \int_0^3 (y-3)^2\,dy$ (Let $3 - y = t$)

$= \int_0^3 (3-y)^2\,dy = \int_0^3 t^2 dt = \left[\frac{t^3}{3}\right]_0^3 = \frac{3^3}{3} = 9$

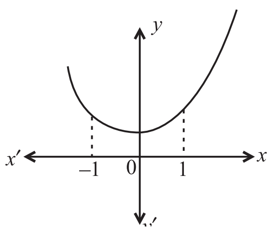
7. **(a) :** $P(x) = x^4 + ax^3 + bx^2 + cx + d$

$P'(x) = 4x^3 + 3ax^2 + 2bx + c$

$P'(0) = 0 \Rightarrow c = 0$

Also $P'(x) = x(4x^2 + 3ax + 2b)$

As $P'(x) = 0$ has no real roots except $x = 0$, we have



D of $4x^2 + 3ax + 2b$ is less than zero

i.e., $(3a)^2 - 4 \cdot 4 \cdot 2b < 0$

then $4x^2 + 3ax + 2b > 0$ $\forall x \in R$

(If $a > 0$, $b^2 - 4ac < 0$ then $ax^2 + bx + c > 0 \ \forall x \in R$)

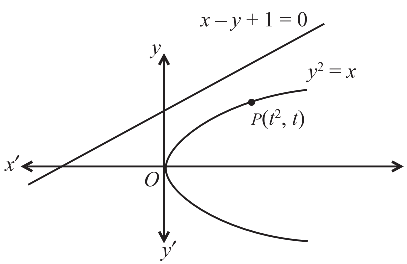
So $P'(x) < 0$ if $x \in [-1, 0)$ *i.e.*, decreasing

and $P'(x) > 0$ if $x \in (0, 1]$ *i.e.*, increasing

Max. of $P(x) = P(1)$

But minimum of $P(x)$ doesn't occur at $x = -1$, *i.e.*, $P(-1)$ is not the minimum.

8. **(d) :**



Let $P(t^2, t)$ be a point on $y^2 = x$

The slope of normal at P

$= -\frac{1}{\text{the slope of tangent at } P} = -\frac{1}{1/2t} = -2t$

The shortest distance between the curves occur along the common normal. Thus $-2t = -1$

$\Rightarrow t = 1/2$

Thus the point P is $(1/4, 1/2)$

The required shortest distance

$= \frac{\left(\frac{1}{4} - \frac{1}{2} + 1\right)}{\sqrt{2}} = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$

9. **(a) :** The line is $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$

The direction ratios of the line are $(3, -5, 2)$.

As the line lies in the plane $x + 3y - \alpha z + \beta = 0$,

we have $(3)(1) + (-5)(3) + 2(-\alpha) = 0$

$\Rightarrow -12 - 2\alpha = 0. \quad \therefore \alpha = -6$

Again $(2, 1, -2)$ lies on the plane

$\Rightarrow 2 + 3 + 2\alpha + \beta = 0$

$\Rightarrow \beta = -2\alpha - 5 = 12 - 5 = 7$

Hence (α, β) is $(-6, 7)$.

10. **(c) :** Out of 6 novels, 4 novels can be selected in ${}^6C_4$ ways.

Also out of 3 dictionaries, 1 dictionary can be selected in ${}^3C_1$ ways.

Since the dictionary is fixed in the middle, we only have to arrange 4 novels which can be done in 4! ways.

Then the number of ways $= {}^6C_4 \cdot {}^3C_1 \cdot 4!$

$= \frac{6 \cdot 5}{2} \cdot 3 \cdot 24 = 1080$

11. **(d) :** Probability of at least one success

$= 1 -$ No success $= 1 - {}^nC_n q^n$

where $q = 1 - p = 3/4$

we want $1 - \left(\frac{3}{4}\right)^n \ge \frac{9}{10}$

$\Rightarrow \frac{1}{10} \ge \left(\frac{3}{4}\right)^n \Rightarrow \left(\frac{3}{4}\right)^n \le \frac{1}{10}$

Taking logarithm on base 10 we have

$n \log_{10}(3/4) \le \log_{10} 10^{-1}$

$\Rightarrow n(\log_{10} 3 - \log_{10} 4) \le -1$

$\Rightarrow n(\log_{10} 4 - \log_{10} 3) \ge 1$

$\Rightarrow n \ge \frac{1}{\log_{10} 4 - \log_{10} 3}$

12. **(a) :** The given lines are perpendicular to a common line, means they are in themselves parallel.

The slope of first line $= \frac{1}{p(p^2+1)}$

The slope of second line

$$= -\frac{p^2+1}{(p^2+1)^2} = -\frac{1}{p^2+1} \quad (\because\ p^2+1 \neq 0)$$

On equating, we have

$$\frac{1}{p(p^2+1)} = -\frac{1}{\left(p^2+1\right)}$$

$\Rightarrow\ p = -1$ (Cancelling $p^2 + 1 \neq 0$)

13. **(b) :** Let $x \in C$

Suppose $x \in A \quad \Rightarrow \quad x \in A \cap C$

$\Rightarrow\ x \in A \cap B \quad (\because\ A \cap C = A \cap B)$

Thus $x \in B$

Again suppose $x \notin A \quad \Rightarrow \quad x \in C \cup A$

$\Rightarrow\ x \in B \cup A \quad \Rightarrow \quad x \in B$

Thus in both cases $x \in C \ \Rightarrow\ x \in B$

Hence $C \subseteq B$ ....(1)

Similarly we can show that $B \subseteq C$ ....(2)

Combining (1) and (2) we get $B = C$.

14. **(b) :** The function is $f : R \to R$

$f(x) = x^3 + 5x + 1$

Let $y \in R$ then $y = x^3 + 5x + 1$

$\Rightarrow\ x^3 + 5x + 1 - y = 0$

As a polynomial of odd degree has always at least one real root, corresponding to any $y \in$ co-domain there $\exists$ some $x \in$ domain such that $f(x) = y$. Hence f is ONTO.

Also f is continuous on R, because it's a polynomial function $f'(x) = 3x^2 + 5 > 0$

$\therefore\ f$ is strictly increasing

Hence f is one-one also.

15. **(c) :** $y = c_1 e^{c_2 x}$

differentiating w.r.t. x, we get

$y' = c_1 c_2 e^{c_2 x} = c_2 y$ ...(i)

Again differentiating w.r.t. x

$y'' = c_2 y'$ ....(ii)

From (i) and (ii) upon division

$$\frac{y'}{y''} = \frac{y}{y'} \quad \Rightarrow \quad y''y = (y')^2$$

Which is the desired differential equation of the family of curves.

16. **(b) :**

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c-1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^n \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c-1 \\ a & -b & c \end{vmatrix} = 0$$

$$\Rightarrow\ D + (-1)^n \begin{vmatrix} a+1 & a-1 & a \\ b+1 & b-1 & -b \\ c+1 & c-1 & c \end{vmatrix} = 0$$

(Changing rows to columns)

$$\Rightarrow\ D + (-1)^n \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c+1 & c-1 \end{vmatrix} = 0$$

(Changing columns in cyclic order doesn't change the determinant)

$\Rightarrow\ D + (-1)^n D = 0 \ \Rightarrow\ \{1 + (-1)^n\}D = 0$

$$\text{Now } D = \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

$$= \begin{vmatrix} a & 2 & a-1 \\ -b & 2 & b-1 \\ c & -2 & c+1 \end{vmatrix} \quad C_2 \to C_2 - C_3$$

$$= \begin{vmatrix} a+c & 0 & a+c \\ -b+c & 0 & b+c \\ c & -2 & c+1 \end{vmatrix} \ R_1 \to R_1 + R_3,\ R_2 \to R_2 + R_3$$

Expanding along 2nd column

$D = 2\{(a + c)(b + c) - (a + c)(c - b)\}$

$= 2(a + c)2b$

$= 4b(a + c) \neq 0$ (By hypothesis)

Now $\{1 + (-1)^n\}D = 0 \ \Rightarrow\ 1 + (-1)^n = 0$

Which mean n = odd integer.

17. **(a) : Ist Solution :** Using Modulo Arithmetic

$8 = -1$ (modulo 9) Also $62 = -1$ (modulo 9)

$\Rightarrow 8^{2n} - (62)^{2n+1} = [(-1)^{2n} - (-1)^{2n+1}]$mod 9

$= (1 + 1)$mod 9 $= 2$ mod9 $\Rightarrow$ Remainder $= 2$

18. **(d) :** $x^{2x} - 2x^x \cot y - 1 = 0$ .....(i)

at $x = 1$ we have

$1 - 2\cot y - 1 = 0$

$\Rightarrow\ \cot y = 0 \quad \therefore\ y = \pi/2$

Differentiating (i) w.r.t. x, we have

$$2x^{2x}(1+\ln x) - 2[x^x(-\text{cosec}^2\, y)\frac{dy}{dx} + \cot y \cdot x^x(1+\ln x)] = 0$$

At $P(1, \pi/2)$ we have

$$2(1+\ln 1) - 2[1(-1)\left(\frac{dy}{dx}\right)_P + 0] = 0$$

$$\Rightarrow\ 2 + 2\left(\frac{dy}{dx}\right)_P = 0 \quad \therefore \quad \left(\frac{dy}{dx}\right)_P = -1$$

19. (b) : The roots of $bx^2 + cx + a = 0$ are imaginary means $c^2 - 4ab < 0 \Rightarrow c^2 < 4ab$

Again the coeff. of x^2 in

$3b^2x^2 + 6bcx + 2c^2$ is +ve, so the minimum value of the expression

$$= -\frac{36b^2c^2 - 4(3b^2)(2c^2)}{4(3b^2)} = -\frac{12b^2c^2}{12b^2} = -c^2$$

As $c^2 < 4ab$ we have $-c^2 > -4ab$

Thus the minimum value is $-4ab$.

20. (a) : Let $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...$

$$\frac{1}{3}S = \quad \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + ...$$

$-$

$$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...$$

$$= \frac{4}{3}\left[1 + \frac{1}{3} + ... \text{to } \infty\right] = \frac{4}{3} \cdot \frac{1}{1 - \frac{1}{3}} = \frac{4}{3} \cdot \frac{1}{2/3} = 2$$

$$\Rightarrow \frac{2}{3}S = 2 \quad \therefore S = 3$$

21. (b) : Let the vector $\overrightarrow{PQ}$ be $(x_1 - x_2, y_1 - y_2, z_1 - z_2)$

we have $x_1 - x_2 = 6$

$y_1 - y_2 = -3$

$z_1 - z_2 = 2$

Length of PQ

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$= \sqrt{6^2 + 3^2 + 2^2} = \sqrt{36 + 9 + 4} = 7$$

The direction cosines of $\overrightarrow{PQ}$ are

$$\left\langle \frac{x_1 - x_2}{PQ}, \frac{y_1 - y_2}{PQ}, \frac{z_1 - z_2}{PQ} \right\rangle$$

i.e., $\left\langle \frac{6}{7}, -\frac{3}{7}, \frac{2}{7} \right\rangle$

22. (b) : $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -3/2$

$\Rightarrow (\cos\beta \cos\gamma + \sin\beta \sin\gamma) + (\cos\gamma \cos\alpha + \sin\gamma \sin\alpha) + (\cos\alpha \cos\beta + \sin\alpha \sin\beta) = -3/2$

$\Rightarrow 2(\cos\beta \cos\gamma + \cos\gamma \cos\alpha + \cos\alpha \cos\beta) + 2(\sin\beta \sin\gamma + \sin\gamma \sin\alpha + \sin\alpha \sin\beta) + 3 = 0$

$\Rightarrow \{\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2(\cos\alpha \cos\beta + \cos\beta \cos\gamma + \cos\gamma \cos\alpha)\} + \{\sin^2\alpha + \sin^2\beta + \sin^2\gamma + 2(\sin\alpha \sin\beta + \sin\beta \sin\gamma + \sin\gamma \sin\alpha)\} = 0$

$\Rightarrow (\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2 = 0$

Which yields simultaneously

$\cos\alpha + \cos\beta + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$

23. (d) : Any number in the set

$S = \{00, 01, 02, ..., 49\}$ is of the form ab where $a \in \{0, 1, 2, 3, 4\}$ and $b \in \{0, 1, 2, ..., 9\}$ for the product of digits to be zero, the number must be of the form either $x0$ which are 5 in numbers, because $x \in \{0, 1, 2, 3, 4\}$

or of the form $0x$ which are 10 in numbers because $x \in \{0, 1, 2, ..., 9\}$

The only number common to both = 00

Thus the number of numbers in S, the product of whose digits is zero = 10 + 5 – 1 = 14

Of these the number whose sum of digits is 8 is just one, i.e. 08

The required probability = 1/14.

24. (a) : Let P be a general point (x, y) such that $\frac{PM}{PN} = \frac{1}{3}$ where $M \equiv (1, 0)$ and $N \equiv (-1, 0)$

we have $\frac{\sqrt{(x-1)^2 + y^2}}{\sqrt{(x+1)^2 + y^2}} = \frac{1}{3}$

$\Rightarrow 9[(x - 1)^2 + y^2] = (x + 1)^2 + y^2$

which reduces to

$8x^2 + 8y^2 - 20x + 8 = 0$

$$\Rightarrow x^2 + y^2 - \frac{10}{4}x + 1 = 0$$

$$\Rightarrow x^2 + y^2 - \frac{5}{2}x + 1 = 0$$

The locus is a circle with centre (5/4, 0)

As points A, B, C lie on this circle, the circumcentre of triangle ABC is (5/4, 0).

25. (b) : The numbers are $1, 1 + d, 1 + 2d, ..., 1 + 100d$.

The numbers are in A.P.

Then mean = 51st term = $1 + 50d = \bar{x}$(says)

Mean deviation (M.D.) $= \frac{1}{n}\sum_{i=1}^{101} |x_i - \bar{x}|$

$$= \frac{1}{101}[50d + 49d + 48d + ... + d + 0 + d + 2d + ... + 50d]$$

$$= \frac{1}{101} \cdot 2d(1 + 2 + ... + 50)$$

$$= \frac{1}{101} \cdot 2d \cdot \frac{50 \cdot 51}{2} = \frac{50 \cdot 51}{101}d$$

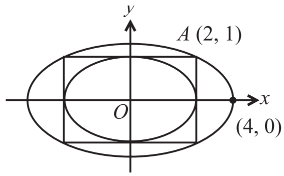
But M.D. = 255 (given)

$$\Rightarrow \frac{50 \cdot 51}{101}d = 255$$

$$\Rightarrow d = \frac{101 \times 255}{50 \times 51} = \frac{101 \times 255}{2550} = 10.1$$

26. (a) : The given ellipse is $\frac{x^2}{4}+\frac{y^2}{1}=1$

i.e., the point A, the corner of the rectangle in 1st quadrant, is (2, 1). Again the ellipse circumscribing the rectangle passess through the point (4, 0), so its equation is



$\frac{x^2}{16}+\frac{y^2}{b^2}=1$

$A(2, 1)$ lies on the above ellipse

$\Rightarrow \frac{4}{16}+\frac{1}{b^2}=1 \quad \Rightarrow \frac{1}{b^2}=1-\frac{1}{4}=\frac{3}{4}$

$\Rightarrow b^2 = 4/3$

Thus the equation to the desired ellipse is

$\frac{x^2}{16}+\frac{3}{4}y^2=1 \Rightarrow x^2+12y^2=16$

27. (a) : We have for any two complex numbers α and β

$||\alpha|-|\beta|| \le |\alpha-\beta|$

Now $\left||Z|-\frac{4}{|z|}\right| \le \left|Z-\frac{4}{Z}\right| \quad \Rightarrow \left||Z|-\frac{4}{|Z|}\right| \le 2$

Set $|z| = r > 0$, then $\left|r-\frac{4}{r}\right| \le 2$

$\Rightarrow -2 \le r-\frac{4}{r} \le 2$

The left inequality gives $r^2 + 2r - 4 \ge 0$

The corresponding roots are

$r=\frac{-2\pm\sqrt{20}}{2}=-1\pm\sqrt{5}$

Thus $r \ge \sqrt{5}-1$ or $r \le -1-\sqrt{5}$

implies that $r \ge \sqrt{5}-1$ (As $r > 0$) ...(i)

Again consider the right inequality

$r-\frac{4}{r} \le 2 \Rightarrow r^2-2r-4 \le 0$

The corresponding roots are

$r=\frac{2\pm\sqrt{20}}{2}=1\pm\sqrt{5}$

Thus $1-\sqrt{5} \le r \le 1+\sqrt{5}$

But $r > 0$, hence $r \le 1+\sqrt{5}$ ...(ii)

(i) and (ii) gives $\sqrt{5}-1 \le r \le \sqrt{5}+1$

So, the greatest value is $\sqrt{5}+1$.

28. (a) : The radical axis, which in the case of intersection of the circles is the common chord, of the circles

$S_1 : x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and

$S_2 : x^2 + y^2 + 2x + 2y - p^2 = 0$ is $S_1 - S_2 = 0$

$\Rightarrow x + 5y + 2p - 5 + p^2 = 0$ ...(i)

If there is a circle passing through P, Q and (1, 1) it's necessary and sufficient that (1, 1) doesn't lie on PQ, *i.e.*, $1 + 5 + 2p - 5 + p^2 \neq 0$

$\Rightarrow p^2 + 2p + 1 \neq 0 \Rightarrow (p + 1)^2 \neq 0$

$\therefore p \neq -1$

Thus for all values of p except '–1' there is a circle passing through P, Q and (1, 1).

29. (d) : We have $[l\vec{a}\ m\vec{b}\ n\vec{c}] = lmn[\vec{a}\,\vec{b}\,\vec{c}]$ for scalars l, m, n.

Also $[\vec{a}\,\vec{b}\,\vec{c}]=[\vec{b}\,\vec{c}\,\vec{a}]=[\vec{c}\,\vec{a}\,\vec{b}]$ (cyclic)

And $[\vec{a}\,\vec{b}\,\vec{c}]=-[\vec{a}\,\vec{c}\,\vec{b}]$ (Interchange of any two vectors)

$[3\,\vec{u}\ \ p\vec{v}\ \ p\vec{w}]-[p\vec{v}\ \ \vec{w}\ \ q\vec{u}]-[2\vec{w}\ \ q\vec{v}\ \ q\vec{u}]=0$

$\Rightarrow 3p^2\,[\vec{u}\ \vec{v}\ \vec{w}]-pq[\vec{u}\ \vec{v}\ \vec{w}]+2q^2[\vec{u}\ \vec{v}\ \vec{w}]=0$

$\Rightarrow (3p^2-pq+2q^2)[\vec{u}\ \vec{v}\ \vec{w}]=0$

As $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar, $[\vec{u}\ \vec{v}\ \vec{w}] \neq 0$

Hence $3p^2 - pq + 2q^2 = 0$, $p, q \in R$

As a quadratic in p, roots are real

$\Rightarrow q^2 - 24q^2 \ge 0 \Rightarrow -23q^2 \ge 0$

$\Rightarrow q^2 \le 0 \Rightarrow q = 0$

And thus $p = 0$

Thus $(p, q) \equiv (0, 0)$ is the only possibility.

30. (c) : $I=\int_0^{\pi}[\cot x]dx$

$I=\int_0^{\pi}[\cot(\pi-x)]dx=\int_0^{\pi}[-\cot x]dx$

Adding we have

$2I=\int_0^{\pi}\{[\cot x]+[-\cot x]\}dx$

$2I=\int_0^{\pi}(-1)dx=-\pi \qquad \therefore \quad I=-\pi/2$

Note that $[x] + [-x] = 0, x \in Z = -1, x \notin Z$.

Solved Paper - 2010

1. Consider the following relations:
$R = \{(x, y) \mid x, y$ are real numbers and $x = wy$ for some rational number $w\}$;

$S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) \middle| m, n, p \text{ and } q \text{ are integers such that}\right.$

n, q ≠ 0 and qm = pn}.

Then
(a) R is an equivalence relation but S is not an equivalence relation
(b) neither R nor S is an equivalence relation
(c) S is an equivalence relation but R is not an equivalence relation
(d) R and S both are equivalence relations

[Set, Relations and Functions]

2. The number of complex numbers z such that
$|z - 1| = |z + 1| = |z - i|$ equals
(a) 0 (b) 1 (c) 2 (d) ∞

[Complex Numbers]

3. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$
(a) –2 (b) –1 (c) 1 (d) 2

[Quadratic Equations]

4. Consider the system of linear equations

$$x_1 + 2x_2 + x_3 = 3$$
$$2x_1 + 3x_2 + x_3 = 3$$
$$3x_1 + 5x_2 + 2x_3 = 1$$

The system has
(a) infinite number of solutions
(b) exactly 3 solutions
(c) a unique solution
(d) no solution

[Matrices and Determinants]

5. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is
(a) 3 (b) 36 (c) 66 (d) 108

[Permutations and Combinations]

6. Let $f : (-1, 1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$, $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$
(a) 4 (b) – 4 (c) 0 (d) –2

[Differential Calculus]

7. Let $f : R \to R$ be a positive increasing function with $\lim\limits_{x\to\infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim\limits_{x\to\infty} \frac{f(2x)}{f(x)} =$
(a) 1 (b) 2/3 (c) 3/2 (d) 3

[Differential Calculus]

8. Let $p(x)$ be a function defined on R such that $p'(x) = p'(1 - x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)\, dx$ equals
(a) $\sqrt{41}$ (b) 21 (c) 41 (d) 42

[Integral Calculus]

9. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an A.P. with common difference –2, then the time taken by him to count all notes is
(a) 24 minutes (b) 34 minutes
(c) 125 minutes (d) 135 minutes

[Sequences and Series]

10. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is
(a) $y = 0$ (b) $y = 1$
(c) $y = 2$ (d) $y = 3$

[Differential Calculus]

11. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is
(a) $4\sqrt{2} - 2$ (b) $4\sqrt{2} + 2$
(c) $4\sqrt{2} - 1$ (d) $4\sqrt{2} + 1$

[Integral Calculus]

12. Solution of the differential equation
$\cos x\, dy = y(\sin x - y)\, dx$, $0 < x < \pi/2$ is
(a) $\sec x = (\tan x + c)y$ (b) $y \sec x = \tan x + c$
(c) $y \tan x = \sec x + c$ (d) $\tan x = (\sec x + c)y$

[Differential Equations]

13. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = 0$ and $\vec{a} \cdot \vec{b} = 3$ is

(a) $-\hat{i} + \hat{j} - 2\hat{k}$ (b) $2\hat{i} - \hat{j} + 2\hat{k}$

(c) $\hat{i} - \hat{j} - 2\hat{k}$ (d) $\hat{i} + \hat{j} - 2\hat{k}$

[Vector Algebra]

14. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + 4\hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$

(a) (–3, 2) (b) (2, –3)

(c) (–2, 3) (d) (3, –2)

[Vector Algebra]

15. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

(a) $x = 1$ (b) $2x + 1 = 0$

(c) $x = -1$ (d) $2x - 1 = 0$

[Two Dimensional Geometry]

16. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is

(a) $\frac{23}{\sqrt{15}}$ (b) $\sqrt{17}$

(c) $\frac{17}{\sqrt{15}}$ (d) $\frac{23}{\sqrt{17}}$

[Two Dimensional Geometry]

17. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals

(a) 30° (b) 45° (c) 60° (d) 75°

[Three Dimensional Geometry]

18. Let S be a non-empty subset of R. Consider the following statement:

P : There is a rational number $x \in S$ such that $x > 0$.

Which of the following statements is the negation of the statement P ?

(a) There is a rational number $x \in S$ such that $x \le 0$.

(b) There is no rational number $x \in S$ such that $x \le 0$.

(c) Every rational number $x \in S$ satisfies $x \le 0$.

(d) $x \in S$ and $x \le 0 \Rightarrow x$ is not rational.

[Mathematical Logic]

19. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$

(a) $\frac{25}{16}$ (b) $\frac{56}{33}$ (c) $\frac{19}{12}$ (d) $\frac{20}{17}$

[Trigonometry]

20. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

(a) $-85 < m < -35$ (b) $-35 < m < 15$

(c) $15 < m < 65$ (d) $35 < m < 85$

[Two Dimensional Geometry]

21. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

(a) $\frac{5}{2}$ (b) $\frac{11}{2}$ (c) 6 (d) $\frac{13}{2}$

[Statistics]

22. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is

(a) 1/3 (b) 2/7 (c) 1/21 (d) 2/23

[Probability]

23. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is

(a) there is a regular polygon with $r/R = 1/2$

(b) there is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(c) there is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

(d) there is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

[Trigonometry]

24. The number of 3 × 3 non-singular matrices, with four entries as 1 and all other entries as 0, is

(a) less than 4 (b) 5

(c) 6 (d) at least 7

[Matrices and Determinants]

25. Let $f: R \to R$ be defined by

$$f(x)=\begin{cases}k-2x, & \text{if } x\le -1\\ 2x+3, & \text{if } x>-1\end{cases}$$

If f has a local minimum at $x = -1$, then a possible value of k is

(a) 1 (b) 0

(c) –1/2 (d) –1

[Differential Calculus]

Directions : Question numbers 26 to 30 are Assertion-Reason type questions. Each of these questions contains two statements. Statement-1(Assertion) and Statement-2 (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice:

(a) Statement-1istrue,statement-2istrue;statement-2 is a correct explanation of statement-1.

(b) Statement-1istrue,statement-2istrue;statement-2 is not a correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-1 is true.

26. Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ..., 20}.

Statement-1 : The probability that the chosen numbers when arranged in some order will form an A.P. is $\frac{1}{85}$.

Statement-2 : If the four chosen numbers form an A.P., then the set of all possible values of common difference is {±1, ±2, ±3, ±4, ±5}.

[Probability]

27. Let $S_1=\sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2=\sum_{j=1}^{10} j\,{}^{10}C_j$ and $S_3=\sum_{j=1}^{10} j^2\,{}^{10}C_j$.

Statement-1 : $S_3 = 55 \times 2^9$.

Statement-2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

[Binomial Theorem]

28. **Statement-1 :** The point $A(3, 1, 6)$ is the mirror image of the point B (1, 3, 4) in the plane $x - y + z = 5$.

Statement-2 : The plane $x - y + z = 5$ bisects the line segment joining A (3, 1, 6) and B(1, 3, 4).

[Three Dimensional Geometry]

29. Let $f: R \to R$ be a continuous function defined by

$$f(x)=\frac{1}{e^x+2e^{-x}}.$$

Statement-1 : $f(c) = 1/3$, for some $c \in R$.

Statement-2 : $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all $x \in R$.

[Differential Calculus]

30. Let A be a 2 × 2 matrix with non-zero entries and let $A^2 = I$, where I is 2 × 2 identity matrix. Define $Tr(A)$ = sum of diagonal elements of A and $|A|$ = determinant of matrix A.

Statement-1 : $Tr(A) = 0$.

Statement-2 : $|A| = 1$.

[Matrices and Determinants]

EXPLANATIONS

1. **(c) :** We have $(x, x) \in R$ for $w = 1$ implying that R is reflexive.
For $a \neq 0$, $(a, 0) \notin R$ for any w but $(0, a) \in R$. Thus R is not symmetric.
Hence R is not an equivalence relation.
As $\left(\frac{m}{n}, \frac{m}{n}\right) \in S$ since $mn = mn$, S is reflexive.
$\left(\frac{m}{n}, \frac{p}{q}\right) \in S \Rightarrow qm = pn$.
But this can be written as $np = mq$,
giving $\left(\frac{p}{q}, \frac{m}{n}\right) \in S$. Thus S is symmetric.
Again, $\left(\frac{m}{n}, \frac{p}{q}\right) \in S$ and $\left(\frac{p}{q}, \frac{a}{b}\right) \in S$
means $qm = pn$ and $bp = aq$.
i.e. $\frac{m}{n} = \frac{p}{q}$ and $\frac{p}{q} = \frac{a}{b}$. *i.e.* $\frac{m}{n} = \frac{a}{b}$
Thus $\left(\frac{m}{n}, \frac{a}{b}\right) \in S$
This means S is transitive.

2. **(b) : Ist solution :**
$|z - 1| = |z + 1| = |z - i|$ reads that the distance of desired complex number z is same from three points in the complex plane -1, 1 and i. These points are non-collinear, hence the desired number is the centre of the (unique) circle passing through these three non-collinear points.
2nd solution :
We resort to definition of modulus.
$|z-1| = |z+1| \Rightarrow |z-1|^2 = |z+1|^2$
$\Rightarrow (z-1)(\overline{z}-1) = (z+1)(\overline{z}+1)$
$\Rightarrow z\overline{z} - z - \overline{z} + 1 = z\overline{z} + z + \overline{z} + 1$
$\Rightarrow z + \overline{z} = 0$ (z being purely imaginary)
Thus $x = 0$
Again, $|z-1|^2 = |z-i|^2$
$\Rightarrow (x-1)^2 + y^2 = x^2 + (y-1)^2$
$\Rightarrow 1 + y^2 = (y-1)^2$ (because $x = 0$)
$\Rightarrow 1 + y^2 = y^2 - 2y + 1$
$\therefore y = 0$
Thus, (0, 0) is the desired point.

3. **(b) :** We have $x^2 - x + 1 = 0$ giving $x = \frac{1 \pm i\sqrt{3}}{2}$.
Identifying these roots as ω and ω^2, we have $\alpha = \omega$, $\beta = \omega^2$. We can also take the other way round that would not affect the result.
Now $\alpha^{2009} + \beta^{2009} = \omega^{2009} + \omega^{4018}$
$= \omega^{3k+2} + \omega^{3m+1}$ $(k, m \in N)$
$= \omega^2 + \omega = -1.$ $(\because \omega^{3k} = 1)$

4. **(d) :** $x_1 + 2x_2 + x_3 = 3$
$2x_1 + 3x_2 + x_3 = 3$
$3x_1 + 5x_2 + 2x_3 = 1$
A quick observation tells us that the sum of first two equations yields
$(x_1 + 2x_2 + x_3) + (2x_1 + 3x_2 + x_3) = 3 + 3$
$\Rightarrow 3x_1 + 5x_2 + 2x_3 = 6$
But this contradicts the third equation, *i.e.*,
$3x_1 + 5x_2 + 2x_3 = 1$
As such the system is inconsistent and hence it has no solution.

5. **(d) :** The number of ways $= (^3C_2) \times (^9C_2)$
$= 3 \times \frac{9 \times 8}{2} = 108$

6. **(b) :** $g(x) = \{f(2f(x) + 2\}^2$
We have on differentiation with respect to x,
$g'(x) = 2f(2f(x) + 2) \cdot f'(2f(x) + 2) \cdot 2f'(x)$
Let $x = 0$
$g'(0) = 2f(2f(0) + 2) \cdot f'(2f(0) + 2) \cdot 2f'(0)$
$= 2f(0) \cdot f'(0) \cdot 2f'(0) = (-2)(1)(2) = -4.$

7. **(a) :** As f is a positive increasing function, we have
$f(x) < f(2x) < f(3x)$
Dividing by $f(x)$ leads to $1 < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}$
As $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$, we have by Squeez theorem or Sandwich theorem, $\lim_{x\to\infty} \frac{f(2x)}{f(x)} = 1.$

8. **(b) :** $p'(x) = p'(1 - x)$
On integration,
$p(x) = -p(1 - x) + k$,
k being the constant of integration.
Set $x = 0$ to obtain $p(0) = -p(1) + k$
$\Rightarrow 1 = -41 + k$. $\therefore k = 42$
Now, $I = \int_0^1 p(x)dx = \int_0^1 p(1-x)dx$

On adding we get

$$2I=\int_0^1 p(x)+p(1-x)\,dx=\int_0^1 k dx=\int_0^1 42\,dx=42.$$

Thus $I=21$.

9. **(b) :** We have $a_1+a_2+...+a_n=4500$

$\Rightarrow$ $a_{11}+a_{12}+...+a_n=4500-10\times 150=3000$

$\Rightarrow$ $148+146+....=3000$

$\Rightarrow$ $\frac{n-10}{2}\cdot(2\times 148+(n-10-1)(-21))=3000$

Let $n-10=m$

$\Rightarrow$ $m\times 148-m(m-1)=3000$

$\Rightarrow$ $m^2-149m+3000=0$

$\Rightarrow$ $(m-24)(m-125)=0$

$\therefore$ $m=24, 125,$

giving $n=34, 135$

But for $n=135$, we have

$a_{135}=148+(135-1)(-2)=148-268<0$

But a_{34} is positive.

Hence, $n=34$ is the only answer.

10. **(d) :** $y=x+\frac{4}{x^2}$

On differentiation, $\frac{dy}{dx}=1-\frac{8}{x^3}$

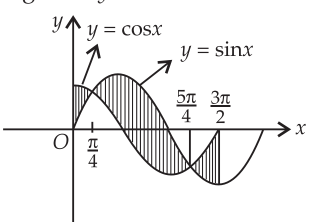
As the tangent is parallel to x-axis, we have

$1-\frac{8}{x^3}=0 \Rightarrow x^3=8. \quad \therefore x=2$

So, $y=2+\frac{4}{2^2}=2+1=3$

Thus (2, 3) is the point of contact and equation of the tangent is $y=3$.

11. **(a) :**



The desired area $=\int_0^{\pi/4}(\cos x-\sin x)dx$

$$+\int_{\pi/4}^{5\pi/4}(\sin x-\cos x)dx+\int_{5\pi/4}^{3\pi/2}(\cos x-\sin x)dx$$

$$=2[\sin x+\cos x]_0^{\pi/4}+[-\cos x-\sin x]_{\pi/4}^{5\pi/4}$$

(As the first and third integrals are equal in magnitude)

$$=2\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-1\right)+\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)$$

$$=\frac{8}{\sqrt{2}}-2=4\sqrt{2}-2$$

12. **(a) : 1st solution:**

$\cos x\,dy=y(\sin x-y)dx$

$\Rightarrow$ $\cos x\,dy=y\sin x\,dx-y^2\,dx$

$\Rightarrow$ $\cos x\,dy-y\sin x\,dx=-y^2\,dx$

$\Rightarrow$ $d(y\cos x)=-y^2\,dx \Rightarrow \frac{d(y\cos x)}{(y\cos x)^2}=-\frac{dx}{\cos^2 x}$

On integration, we have $-\frac{1}{y\cos x}=-\tan x+k$

$\Rightarrow$ $-\sec x=-y\tan x+yk$

$\Rightarrow$ $\sec x=y(\tan x+C)$ where C is a constant

2nd solution:

$\frac{dy}{dx}=\frac{y(\sin x-y)}{\cos x} \Rightarrow \frac{dy}{dx}=y\tan x-y^2\sec x$

$\Rightarrow$ $\frac{dy}{dx}-y\tan x=-y^2\sec x$

$\Rightarrow$ $\frac{1}{y^2}\frac{dy}{dx}-\frac{1}{y}\tan x=-\sec x$

Setting, $-\frac{1}{y}=v$, we have

$\frac{dv}{dx}+(\tan x)v=-\sec x$, which is linear in v.

I.F. $=e^{\int\tan x\,dx}=e^{\ln\sec x}=\sec x$

The solution is

$v\times\sec x=\int-\sec^2 x\,dx+k$

$\Rightarrow$ $v\sec x=-\tan x+k$

$\Rightarrow$ $-\frac{\sec x}{y}=-\tan x-C \Rightarrow \sec x=y(\tan x+C)$

13. **(a) :** We have $\vec{a}\times\vec{b}+\vec{c}=0$

Multiplying vectorially with $\vec{a}$, we have

$\vec{a}\times(\vec{a}\times\vec{b})+\vec{a}\times\vec{c}=0$

$\Rightarrow$ $(\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}+\vec{a}\times\vec{c}=0$

$\vec{a}\times\vec{c}=(\hat{j}-\hat{k})\times(\hat{i}-\hat{j}-\hat{k})=-2\hat{i}-\hat{j}-\hat{k}$

Thus, $3(\hat{j}-\hat{k})-2\vec{b}-2\hat{i}-\hat{j}-\hat{k}=0$

$\therefore$ $\vec{b}=-\hat{i}+\hat{j}-2\hat{k}$

14. **(a) :** $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}+4\hat{k}$,

$\vec{c}=\lambda\hat{i}+\hat{j}+\mu\hat{k}$

$\vec{a}$ and $\vec{c}$ are orthogonal $\Rightarrow \vec{a}\cdot\vec{c}=0$ giving $\lambda-1+2\mu=0$

Also $\vec{b}$ and $\vec{c}$ are orthogonal $\Rightarrow 2\lambda+4+4\mu=0$

Solving the equation we get $\lambda=-3$, $\mu=2$.

15. (c) : From a property of the parabola, the perpendicular tangents intersect at the directrix. The equation of directrix is $x = -1$, hence this is the locus of point P.

16. (d) : As the line passes through (13, 32), we have

$$\frac{13}{5}+\frac{32}{b}=1 \Rightarrow \frac{32}{b}=1-\frac{13}{5}=-\frac{8}{5} \Rightarrow b=-20$$

Thus the line is $\frac{x}{5}-\frac{y}{20}=1$, *i.e.*, $4x-y=20$

The equation of line parallel to $4x - y = 20$ has slope 4.

Thus $-\frac{3}{c}=4. \quad \therefore\ c=-\frac{3}{4}.$

Then the equation to line k is $4x - y = -3$

The distance between lines k and c is

$$\frac{20+3}{\sqrt{4^2+1^2}}=\frac{23}{\sqrt{17}}$$

17. (c) : We have $l=\frac{1}{\sqrt{2}},\quad m=-\frac{1}{2}$

As $l^2 + m^2 + n^2 = 1$, we have $n^2=\frac{1}{4} \Rightarrow n=\frac{1}{2}$

We take positive values, so $n=\frac{1}{2}$

$\Rightarrow \quad \cos\theta=\frac{1}{2}. \quad \therefore\ \theta=60°.$

18. (c) : The given statement is

P : at least one rational $x \in S$ such that $x > 0$.

The negation would be : There is no rational number $x \in S$ such that $x > 0$

which is equivalent to all rational numbers $x \in S$ satisfy $x \le 0$.

19. (b) : $\cos(\alpha + \beta) = 4/5$ giving $\tan(\alpha + \beta) = 3/4$

Also $\sin(\alpha - \beta) = 5/13$ given $\tan(\alpha - \beta) = 5/12$

$\tan(2\alpha)=\tan\{(\alpha+\beta)+(\alpha-\beta)\}$

$$=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}=\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}}$$

$$=\frac{36+20}{48-15}=\frac{56}{33}$$

20. (b) : The circle is $x^2 + y^2 - 4x - 8y - 5 = 0$

$\Rightarrow \quad (x - 2)^2 + (y - 4)^2 = 5^2$

Length of perpendicular from centre (2, 4) on the line $3x - 4y - m = 0$ should be less than radius.

$$\Rightarrow \quad \frac{|6-16-m|}{5}<5 \Rightarrow (10+m)<25$$

$$\Rightarrow \quad -25 < 10 + m < 25 \Rightarrow -35 < m < 15$$

21. (b) : Ist solution:

$$\left.\begin{aligned}\sigma_1^2=4\\ \sigma_2^2=5\end{aligned}\right\} \begin{aligned}\bar{x}=2\\ \bar{y}=4\end{aligned}$$

We have $\frac{\sum x_i}{5}=2 \Rightarrow \sum x_i=10$

Similarly, $\sum y_i=20$

$$\sigma_1^2=\left(\frac{1}{5}\sum x_i^2\right)-\bar{x}^2 \Rightarrow 4=\frac{1}{5}\sum x_i^2-4$$

$$\Rightarrow \frac{1}{5}\sum x_i^2=8. \quad \therefore \sum x_i^2=40.$$

$$\sigma_2^2=\left(\frac{1}{5}\sum y_i^2\right)-\bar{y}^2 \Rightarrow 5=\frac{1}{5}\sum y_i^2-16$$

$$\Rightarrow \frac{1}{5}\sum y_i^2=21. \quad \therefore \sum y_i^2=105$$

$$\sigma^2=\frac{1}{10}\left(\sum x_i^2+\sum y_i^2\right)-\left(\frac{\bar{x}+\bar{y}}{2}\right)^2$$

$$=\frac{1}{10}(40+105)-9=\frac{145-90}{10}=\frac{55}{10}=\frac{11}{2}.$$

2nd solution :

$\sigma_1^2=4,\ n_1=5,\ \bar{x}_1=2$

$\sigma_2^2=5,\ n_2=5,\ \bar{x}_2=4$

$$\bar{x}_{12}=\frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}=\frac{5\times2+5\times4}{10}=3$$

$d_1=(\bar{x}_1-\bar{x}_{12})=-1,\quad d_2=(\bar{x}_2-\bar{x}_{12})=1$

$$\sigma=\sqrt{\frac{n_1\sigma_1^2+n_2\sigma_2^2+n_1d_1^2+n_2d_2^2}{n_1+n_2}}$$

$$=\sqrt{\frac{5.4+5.5+5.1+5.1}{10}}=\sqrt{\frac{55}{10}}=\sqrt{\frac{11}{2}}$$

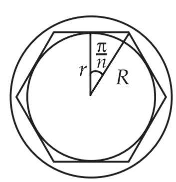
$$\therefore\ \sigma^2=\frac{11}{2}$$

22. (b) : $n(S)={}^9C_3=\frac{9\times8\times7}{6}=84$

$n(E) = {}^3C_1 \cdot {}^4C_1 \cdot {}^2C_1 = 3 \times 4 \times 2 = 24.$

The desired probability $=\frac{24}{84}=\frac{2}{7}.$

23. (c) : We have $\frac{r}{R}=\cos\frac{\pi}{n}$, as is evident from the figure.



Let $\cos\frac{\pi}{n}=\frac{1}{\sqrt{2}}.$

Thus we get $\frac{\pi}{n}=\frac{\pi}{4}$

i.e., $n = 4$, acceptable.

$\cos\frac{\pi}{n}=\frac{1}{2} \Rightarrow \frac{\pi}{n}=\frac{\pi}{3}. \quad \therefore\ n=3,$ acceptable.

$\cos\frac{\pi}{n}=\frac{\sqrt{3}}{2} \Rightarrow \frac{\pi}{n}=\frac{\pi}{6}. \quad \therefore\ n=6,$ acceptable.

But $\cos\frac{\pi}{n}=\frac{2}{3}$ will produce no value of n.

As $\frac{1}{2}<\frac{2}{3}<\frac{1}{\sqrt{2}} \Rightarrow \cos\frac{\pi}{3}<\cos\frac{\pi}{n}<\cos\frac{\pi}{4}$

$\Rightarrow \frac{\pi}{3}>\frac{\pi}{n}>\frac{\pi}{4} \Rightarrow 3<n<4$ (impossible)

24. (d) : $A=\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

Let $A = (a_1b_2c_3 + a_2c_1b_3 + a_3b_1c_2) - (a_1c_2b_3 + a_2b_1c_3 + a_3c_1b_2)$

If any of the terms be non-zero, then det A will be non-zero and all the elements of that term will be 1 each.

Number of non-singular matrices = ${}^6C_1 \times {}^6C_1 = 36$

We can also exhibit more than 6 matrices to pick the right choice.

25. (d) : $\lim_{x\to 1+} f(x)=1$

As $f(-1) = k + 2$

As f has a local miminum at $x = -1$

$f(-1^+) \ge f(-1) \ge f(-1^-) \Rightarrow 1 \ge k+2$

$\Rightarrow k+2\le 1. \quad \therefore\ k\le -1$

Thus $k = -1$ is a possible value.

26. (c) : Number of A.P.'s with common difference 1 = 17

Number of A.P.'s with common difference 2 = 14

Number of A.P.'s with common difference 3 = 11

Number of A.P.'s with common difference 4 = 8

Number of A.P.'s with common difference 5 = 5

Number of A.P.'s with common difference 6 = 2

57

The total number of ways $n(S) = {}^{20}C_4$

The desired probability $=\frac{57}{{}^{20}C_4}$

$=\frac{57\times 24}{20\times 19\times 18\times 17}=\frac{1}{85}$

Now statement-2 is false and statement-1 is true.

27. (c) : $S_1=\sum j(j-1)\ {}^{10}C_j$

$=\sum j(j-1)\cdot\frac{10(10-1)}{(j-1)}\cdot {}^8C_{j-2}$

$=9\times 10\sum_{j=2}^{10} {}^8C_{j-2}=90\times 2^8$

$S_2=\sum_{j=1}^{10} j\cdot {}^{10}C_j=10\sum_{j=1}^{10} {}^9C_{j-1}=10\times 2^9$

$S_3=\sum_{j=1}^{10} j^2\cdot {}^{10}C_j=\sum_{j=1}^{10}(j(j-1)+j)\cdot {}^{10}C_j$

$=\sum_{j=1}^{10} j(j-1)\ {}^{10}C_j+\sum_{j=1}^{10} j\cdot {}^{10}C_j$

$= 90\cdot 2^8 + 10\cdot 2^9 = (45 + 10)2^9 = 55\cdot 2^9.$

Then statement-1 is true and statement-2 is false.

28. (b) : Let the image be (a, b, c)

Thus by image formula, we have

$\frac{a-1}{1}=\frac{b-3}{-1}=\frac{c-4}{1}=-2\left(\frac{1-3+4-5}{3}\right)$

$\Rightarrow \frac{a-1}{1}=\frac{b-3}{-1}=\frac{c-4}{1}=2$

$\therefore\ (a, b, c) = (3, 1, 6)$

Again the midpoint of $A(3, 1, 6)$ and $B(1, 3, 4)$ is $(2, 2, 5)$ & the equation of the plane is $x - y + z = 5$.

As the point lies on the plane, so the plane bisects the segment AB. But it does not explain statement-1.

29. (a) : Using A.M.-G.M. inequality,

$\frac{e^x+2e^{-x}}{2}\ge\sqrt{e^x\cdot 2e^{-x}}$. Thus, $e^x+2e^{-x}\ge 2\sqrt{2}$

Then $\frac{1}{e^x+2e^{-x}}\le\frac{1}{2\sqrt{2}}$

As $\frac{1}{e^x+2e^{-x}}$ is always positive, we have

$0<\frac{1}{e^x+2e^{-x}}\le\frac{1}{2\sqrt{2}}$

Observe that $f(0) = 1/3$. Thus $\exists$ a c such that $f(c) = 1/3$.

Using extreme-value theorem, we can say that as f is continuous, f will attain a value 1/3 at some point. Here we are able to identify the point as well.

30. **(c) :** Let $A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$

$$A^2 = \begin{bmatrix} \alpha^2 + \beta\gamma & \beta(\alpha+\delta) \\ \gamma(\alpha+\delta) & \delta + \beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Which gives $\alpha + \delta = 0$ and $\alpha^2 + \beta\gamma = 1$

So we have $Tr(A) = 0$

det $A = \alpha\delta - \beta\gamma = -\alpha^2 - \beta\gamma = -(\alpha^2 + \beta\gamma) = -1$

Thus statement-1 is true but statement-2 is false.

Solved Paper - 2011

1. If $\vec{a} = \frac{1}{\sqrt{10}}\left(3\hat{i} + \hat{k}\right)$ and $\vec{b} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$, then the value of $\left(2\vec{a} - \vec{b}\right)\cdot\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} + 2\vec{b}\right)\right]$ is

(a) 5 (b) 3
(c) –5 (d) –3

[Vector Algebra]

2. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is

(a) –144 (b) 132
(c) 144 (d) –132

[Binomial Theorem]

3. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re $z = 1$, then it is necessary that

(a) $|\beta| = 1$ (b) $\beta \in (1, \infty)$
(c) $\beta \in (0, 1)$ (d) $\beta \in (-1, 0)$

[Complex Numbers]

4. Consider the following statements
P : Suman is brilliant
Q : Suman is rich
R : Suman is honest
The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as

(a) $\sim Q \leftrightarrow \sim P \wedge R$ (b) $\sim (P \wedge \sim R) \leftrightarrow Q$
(c) $\sim P \wedge (Q \leftrightarrow \sim R)$ (d) $\sim (Q \leftrightarrow (P \wedge \sim R))$

[Mathematical Logic]

5. $\frac{d^2x}{dy^2}$ equals to

(a) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (b) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

(c) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (d) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

[Differential Calculus]

6. **Statement-1 :** The point $A(1, 0, 7)$ is the mirror image of the point $B(1, 6, 3)$ in the line

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$

Statement-2 : The line : $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$.

(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

[Three Dimensional Geometry]

7. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is :

(a) $P(C|D) < P(C)$ (b) $P(C|D) = \frac{P(D)}{P(C)}$
(c) $P(C|D) = P(C)$ (d) $P(C|D) \geq P(C)$

[Probability]

8. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval

(a) $\left[0, \frac{1}{2}\right]$ (b) $\left(\frac{11}{12}, 1\right]$

(c) $\left(\frac{1}{2}, \frac{3}{4}\right]$ (d) $\left(\frac{3}{4}, \frac{11}{12}\right]$

[Probability]

9. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}\,dx$ is

(a) $\frac{\pi}{2}\log 2$ (b) log2
(c) π log2 (d) $\frac{\pi}{8}\log 2$

[Integral Calculus]

10. For $x \in \left(0, \frac{5\pi}{2}\right)$, define $f(x) = \int_0^x \sqrt{t}\sin t\,dt$
Then f has

(a) local minimum at π and local maximum at 2π.
(b) local maximum at π and local minimum at 2π.

(c) local maximum at π and 2π.
(d) local minimum at π and 2π.

[Differential Calculus]

11. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$ and $\vec{a}\cdot\vec{d}=0$. Then the vector $\vec{d}$ is equal to

(a) $\vec{b}+\left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$
(b) $\vec{c}-\left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$
(c) $\vec{b}-\left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$
(d) $\vec{c}+\left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$

[Vector Algebra]

12. Let R be the set of real numbers.

Statement-1 : $A=\{(x,y)\in R\times R : y-x \text{ is an integer}\}$ is an equivalence relation on R.

Statement-2 : $B=\{(x,y)\in R\times R : x=\alpha y$ for some rational number $\alpha\}$ is an equivalence relation on R.

(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

[Set, Relations and Functions]

13. Let A and B be two symmetric matrices of order 3.

Statement-1 : $A(BA)$ and $(AB)A$ are symmetric matrices.

Statement 2 : AB is symmetric matrix if matrix multiplication of A with B is commutative.

(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

[Matrices and Determinants]

14. The two circles $x^2+y^2=ax$ and $x^2+y^2=c^2(c>0)$ touch each other if

(a) $a=2c$
(b) $|a|=2c$
(c) $2|a|=c$
(d) $|a|=c$

[Two Dimensional Geometry]

15. $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$

(a) equals $-\sqrt{2}$
(b) equals $\frac{1}{\sqrt{2}}$
(c) does not exist
(d) equals $\sqrt{2}$

[Differential Calculus]

16. If $A=\sin^2 x+\cos^4 x$, then for all real x

(a) $1\le A\le 2$
(b) $\frac{3}{4}\le A\le\frac{13}{16}$
(c) $\frac{3}{4}\le A\le 1$
(d) $\frac{13}{16}\le A\le 1$

[Trigonometry]

17. The lines $L_1 : y-x=0$ and $L_2 : 2x+y=0$ intersect the line $L_3 : y+2=0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement-1 : The ratio $PR : RQ$ equals $2\sqrt{2}:\sqrt{5}$.

Statement-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

[Two Dimensional Geometry]

18. The domain of the function

$$f(x)=\frac{1}{\sqrt{|x|-x}} \text{ is}$$

(a) $(-\infty, 0)$
(b) $(-\infty,\infty)-\{0\}$
(c) $(-\infty,\infty)$
(d) $(0,\infty)$

[Sets, Relations and Functions]

19. If the angle between the line $x=\frac{y-1}{2}=\frac{z-3}{\lambda}$ and the plane $x+2y+3z=4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals

(a) 2/5
(b) 5/3
(c) 2/3
(d) 3/2

[Three Dimensional Geometry]

20. The shortest distance between line $y-x=1$ and curve $x=y^2$ is

(a) $\frac{8}{3\sqrt{2}}$
(b) $\frac{4}{\sqrt{3}}$

(c) $\frac{\sqrt{3}}{4}$ (d) $\frac{3\sqrt{2}}{8}$

[Two Dimensional Geometry]

21. A man saves ₹ 200 in each of the first three months of his service. In each of the subsequent months his saving increases by ₹ 40 more than the saving of immediately previous month. His total saving from the start of service will be ₹ 11040 after

(a) 20 months (b) 21 months
(c) 18 months (d) 19 months

[Sequences and Series]

22. If the mean deviation about the median of the numbers $a, 2a, \ldots\ldots, 50a$ is 50, then $|a|$ equals

(a) 4 (b) 5
(c) 2 (d) 3

[Statistics]

23. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals

(a) (1, 0) (b) (–1, 1)
(c) (0, 1) (d) (1, 1)

[Complex Numbers]

24. The area of the region enclosed by the curves $y = x$, $x = e$, $y = 1/x$ and the positive x-axis is

(a) 3/2 square units (b) 5/2 square units
(c) 1/2 square units (d) 1 square units

[Integral Calculus]

25. The number of values of k for which the linear equations

$$4x + ky + 2z = 0$$
$$kx + 4y + z = 0$$
$$2x + 2y + z = 0$$

possess a non-zero solution is

(a) 1 (b) zero
(c) 3 (d) 2

[Matrices and Determinants]

26. The values of p and q for which the function

$$f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x} &, x < 0 \\ q &, x = 0 \\ \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}} &, x > 0 \end{cases}$$

is continuous for all x in R, are

(a) $p = -\frac{3}{2}, q = \frac{1}{2}$ (b) $p = \frac{1}{2}, q = \frac{3}{2}$
(c) $p = \frac{1}{2}, q = -\frac{3}{2}$ (d) $p = \frac{5}{2}, q = \frac{1}{2}$

[Differential Calculus]

27. **Statement-1 :** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.

Statement-2 : The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.

(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

[Permutations and Combinations]

28. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is

(a) $3x^2 + 5y^2 - 15 = 0$ (b) $5x^2 + 3y^2 - 32 = 0$
(c) $3x^2 + 5y^2 - 32 = 0$ (d) $5x^2 + 3y^2 - 48 = 0$

[Two Dimensional Geometry]

29. Let I be the purchase value of an equipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T-t)$; where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is

(a) $I - \frac{k(T-t)^2}{2}$ (b) e^{-kT}
(c) $T^2 - \frac{I}{k}$ (d) $I - \frac{kT^2}{2}$

[Differential Equations]

30. If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then $y(\ln 2)$ is equal to

(a) 13 (b) – 2
(c) 7 (d) 5

[Differential Equations]

EXPLANATIONS

12. **(a) :** $y - x$ = integer and $z - y$ = integer
$\Rightarrow$ $z - x$ = integer
$\therefore$ $(x, y) \in A$ and $(y, z) \in A \Rightarrow (x, z)$
$\Rightarrow$ Transitive
Also $(x, x) \in A$ is true $\Rightarrow$ Reflexive
As $(x, y) \in A \Rightarrow (y, x) \Rightarrow$ Symmetric
Hence A is a equivalence relation but B is not.
$(0, y)$ is in B but $(y, 0)$ is not in B.

18. **(a) :** $f(x) = \dfrac{1}{\sqrt{|x| - x}}$

$|x| - x > 0 \Rightarrow |x| > x$
Thus x must be –ve. $\therefore$ $x \in (-\infty, 0)$.

3. **(b) :** Let roots be $1 + ai$, $1 + bi$, then we have, $(a \in R)$
$(1 + ai) + (1 + bi) = -\alpha \Rightarrow 2 + (a + b)i = -\alpha$
$(1 + ai)(1 + bi) = \beta$
Comparing we have, $\alpha = -2$ and $a = -b$
Now $(1 + ai)(1 - ai) = \beta$
$\Rightarrow$ $1 + a^2 = \beta \Rightarrow \beta = 1 + a^2$
As $a^2 \geq 0$ we have $\beta \in (1, \infty)$

23. **(d) :** $(1 + \omega)^7 = (-\omega^2)^7 = -\omega^{14} = -\omega^{12}\,\omega^2$
$= -\omega^2 = 1 + \omega = A + B\omega$ given
Hence on comparison, we have $(A, B) = (1, 1)$.

21. **(b) :** Let it happens after n months.

$$3 \times 200 + \frac{n-3}{2}\{2 \times 240 + (n-4)40\} = 11040$$

$$\Rightarrow \left(\frac{n-3}{2}\right)(480 + 40n - 160) = 11040 - 600$$

$$= 10440$$

$\Rightarrow$ $n^2 + 5n - 546 = 0 \Rightarrow (n + 26)(n - 21) = 0$
$\therefore$ $n = 21$.

27. **(c) :** $x_1 + x_2 + x_3 + x_4 = 10$
The number of positive integral solution is
${}^{6+4-1}C_{4-1} = {}^9C_3$
It is the same as the number of ways of choosing any 3 balls from 9 different places.

2. **(a) :** $(1 - x - x^2 + x^3)^6 = ((1 - x)(1 - x^2))^6$
$= (1 - x)^6\,(1 - x^2)^6$
$= (1 - {}^6C_1x + {}^6C_2x^2 - {}^6C_3x^3 + {}^6C_4x^4 - {}^6C_5x^5 + {}^6C_6x^6)$
$(1 - {}^6C_1x^2 + {}^6C_2x^4 - {}^6C_3x^6 + {}^6C_4x^8 - {}^6C_5x^{10} + {}^6C_6x^{12})$
Coeff. of $x^7 = (-{}^6C_1)(-{}^6C_3) + (-{}^6C_3)({}^6C_2) + (-{}^6C_5)(-{}^6C_1)$
$= 6 \cdot 20 - 20 \cdot 15 + 6 \cdot 6 = 120 - 300 + 36 = -144$

4. **(d) :** The statement can be written as
$P \wedge {\sim}R \Leftrightarrow Q$
Thus the negation is
${\sim}(Q \leftrightarrow P \wedge {\sim}R)$

16. **(c) :** $A = \sin^2 x + \cos^2 x$
We have $\cos^4 x \leq \cos^2 x$
$\sin^2 x = \sin^2 x$
Adding $\sin^2 x + \cos^4 x \leq \sin^2 x + \cos^2 x$
$\therefore$ $A \leq 1$.
Again $A = t + (1 - t)^2 = t^2 - t + 1, t \geq 0$,
where minimum is 3/4
Thus $\dfrac{3}{4} \leq A \leq 1 \cdot$

17. **(a) :** In triangle OPQ, O divides PQ in the ratio of $OP : OQ$ which is $2\sqrt{2} : \sqrt{5}$ but it fails to divide triangle into two similar triangles.

20. **(c) :** Let P be (y^2, y)
Perpendicular distance from P to $x - y + 1 = 0$ is

$$\frac{|y^2 - y + 1|}{\sqrt{2}}$$

As $|y^2 - y + 1| = y^2 - y + 1$ ($y^2 - y + 1 > 0\ \forall\ y \in R$)

Minimum value $= \dfrac{1}{\sqrt{2}} \cdot \dfrac{(4ac - b^2)}{4a}$

$$= \frac{1}{\sqrt{2}} \cdot \frac{4-1}{4 \cdot 1} = \frac{3}{4\sqrt{2}}$$

14. **(d) :** The centres and radii are

$$\left(x - \frac{a}{2}\right)^2 + y^2 = \frac{a^2}{4}, \quad x^2 + y^2 = c^2$$

Centre $\left(\dfrac{a}{2}, 0\right)$ and $(0, 0)$ & radius $= \dfrac{a}{2}$ and c

$$\sqrt{\left(\frac{a}{2}\right)^2 + (0-0)} = \left|\left|\frac{a}{2}\right| \pm c\right| \Rightarrow \left|\frac{a}{2}\right| = \left|\left|\frac{a}{2}\right| \pm c\right|$$

$$\Rightarrow \left|\frac{a}{2}\right| = c - \left|\frac{a}{2}\right|. \ \therefore\ |a| = c.$$

28. **(c) :** Let the ellipse be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$(-3, 1)$ lies on it $\Rightarrow \dfrac{9}{a^2} + \dfrac{1}{b^2} = 1$

Also $b^2 = a^2\left(1-\frac{2}{5}\right) \Rightarrow 5b^2 = 3a^2$

Upon solving we get $a^2 = \frac{32}{7},\ b^2 = \frac{32}{5}$

The equation to ellipse becomes

$3x^2 + 5y^2 = 32$

13. (d) : Let $A(BA) = P$

Then $P^T = (ABA)^T = A^T B^T A^T$ (Transversal rule)

$= ABA = P$

Thus P is symmetric.

Again, $A(BA) = (AB)A$ by associativity.

Also $(AB)^T = B^T A^T = BA = AB$

($\because$ A and B are commutative)

$\Rightarrow$ AB is also symmetric.

25. (d) : For the system to possess non-zero solution,

we have $\begin{vmatrix} 4 & k & 2 \\ k & 4 & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0$

which on expansion gives $k^2 - 6k + 8 = 0$

$\Rightarrow$ $(k-2)(k-4) = 0.$ $\quad\therefore$ $k = 2, 4$

5. (b) : $\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\}$

$= \frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\}\cdot\frac{dx}{dy} = -\left(\frac{dy}{dx}\right)^{-2}\frac{d^2y}{dx^2}\cdot\left(\frac{dy}{dx}\right)^{-1}$

$= -\left(\frac{dy}{dx}\right)^{-3}\frac{d^2y}{dx^2}$

15. (c) : Let $x = 2 + h$

$$\lim_{h\to 0}\frac{\sqrt{1-\cos 2h}}{h} = \lim_{h\to 0}\frac{|\sin h|}{h}$$

RHL = 1, LHL = –1. Thus limit doesn't exist.

26. (a) : $f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$

$\lim_{x\to 0^+} f(x) = \frac{1}{2}$

Again, $\lim_{x\to 0^-} f(x) = \frac{\sin(p+1)+\sin x}{x} = p + 2$

Now, $p + 2 = q = 1/2$

$\therefore$ $p = -3/2,\ q = 1/2.$

10. (b) : $f(x) = \int_0^x \sqrt{t}\sin t\,dt$

$f'(x) = \sqrt{x}\sin x$

$f''(x) = \sqrt{x}\cos x + \frac{1}{2}x^{-1/2}\sin x$

$f''(\pi) = -\sqrt{\pi} < 0$; $f''(2\pi) = \sqrt{2\pi} > 0$

Thus at π maximum and at 2π minimum.

9. (b) : $I = \int_0^1 \frac{8\ln(1+x)}{1+x^2}dx$

Let $J = \int_0^1 \frac{\ln(1+x)}{1+x^2}dx$

Let $x = \tan\theta \Rightarrow J = \int_0^{\pi/4}\ln(1+\tan\theta)d\theta$

Now $J = \int_0^{\pi/4}\ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$

Adding $2J = \int_0^{\pi/4}\ln(1+\tan\theta) + \ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$

$= \int_0^{\pi/4}\ln\left\{(1+\tan\theta)\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)\right\}d\theta$

$2J = \int_0^{\pi/4}(\ln 2)d\theta = \frac{\pi}{4}\ln 2 \Rightarrow 8J = 4\frac{\pi}{4}\ln 2$

$\Rightarrow I = 8J = \pi\ln 2.$

24. (a) : Area $= \frac{1}{2} + \int_1^e \frac{dx}{x} = \frac{1}{2} + \ln x\,\Big|_1^e = \frac{3}{2}$

29. (d) : $\frac{dV}{dt} = -k(T-t)$

On integration, $V = \frac{k(T-t)^2}{2} + \alpha$

At $t = 0$, $V(t) = I \Rightarrow I = \frac{kT^2}{2} + \alpha$

$\therefore$ $\alpha = I - \frac{kT^2}{2}$

As $t = T$, we have $V(T) = \alpha = I - \frac{kT^2}{2}$

30. (c) : $\frac{dy}{dx} = y + 3 \Rightarrow \frac{dy}{y+3} = dx$

$\Rightarrow \ln|y+3| = x + C$

As $y(0) = 2$, we have $\ln 5 = C$
Now $\ln(y + 3) = x + \ln 5$
As $x = \ln 2$ we have
$\ln(y + 3) = \ln 2 + \ln 5 = \ln 10$
$\Rightarrow \quad y + 3 = 10 \quad \Rightarrow \quad y = 7.$

22. (a) : Median is the mean of 25th and 26th observation.

$$M = \frac{25a + 26a}{2} = 25.5a$$

$$MD(M) = \frac{\sum |r_i - M|}{N}$$

$$\Rightarrow \quad 50 = \frac{1}{50}\{2|a| \times (0.5 + 1.5 + ... + 24.5)\}$$

$$\Rightarrow \quad 2500 = 2|a| \cdot \frac{25}{2} \cdot 25. \quad \therefore \quad |a| = 4$$

7. (d) : $P(C|D) = \dfrac{P(C \cap D)}{P(D)}$

as $\quad C \subset D, P(C) \subset P(D). \quad \therefore \quad P(C \cap D) = P(C)$

We have, $\quad P(C|D) = \dfrac{P(C)}{P(D)}$

As $0 < P(D) \le 1$ we have $P(C|D) \ge P(C)$

8. (a) : Probability of at least one failure
$= 1 - P(\text{no failure}) = 1 - p^5$

Now $\quad 1 - p^5 \ge \dfrac{31}{32}$

$\Rightarrow \quad p^5 \le \dfrac{1}{32}$ thus $p \le \dfrac{1}{2} \therefore \; p \in [0, 1/2]$

1. (c) : $(2\vec{a} - \vec{b}) \cdot \{(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})\}$

$= (2\vec{a} - \vec{b}) \cdot \{(\vec{a} \times \vec{b}) \times \vec{a} + 2(\vec{a} \times \vec{b}) \times \vec{b}\}$

$= (2\vec{a} - \vec{b}) \cdot \{(\vec{a} \cdot \vec{a})\,\vec{b} - (\vec{a} \cdot \vec{b})\vec{a} + 2(\vec{a} \cdot \vec{b})\vec{b} - 2(\vec{b} \cdot \vec{b})\vec{a}\}$

$= (2\vec{a} - \vec{b}) \cdot (\vec{b} - 2\vec{a}) = -4\vec{a} \cdot \vec{a} - \vec{b} \cdot \vec{b} = -5$

11. (b) : $\vec{a} \cdot \vec{b} \ne 0 \quad$ (given)

$\vec{a} \cdot \vec{d} = 0$

Now, $\vec{b} \times \vec{c} = \vec{b} \times \vec{d} \Rightarrow \vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b} \times \vec{d})$

$\Rightarrow \quad (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})c = (\vec{a}.\vec{d})\vec{b} - (\vec{a}.\vec{b})\vec{d}$

$\Rightarrow \quad (\vec{a}.\vec{b})\vec{d} = -(\vec{a}.\vec{c})\vec{b} + (\vec{a}.\vec{b})\vec{c}$

$\Rightarrow \quad \vec{d} = -\dfrac{(\vec{a} \cdot \vec{c})\vec{b}}{(\vec{a}.\vec{b})} + \vec{c}$

6. (d) : The direction ratios of the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$ is $(0, 6, -4)$.
The direction ratios of the given line is $(1, 2, 3)$. As $1.0 + 6.2 - 4.3 = 0$ we have the lines as perpendicular
Also the midpoint of AB lies on the given line, so statement 1 and statement 2 are true but statement 2 is not a correct explanation of statement 1. Statement '2' holds even if the line is not perpendicular. This situation is possible.

19. (c) : $\dfrac{x - 0}{1} = \dfrac{y - 1}{2} = \dfrac{z - 3}{\lambda}$

$x + 2y + 3z = 4$

Angle between line and plane (by definition)

$$= \sin^{-1}\left(\frac{1 \cdot 1 + 2 \cdot 2 + \lambda \cdot 3}{\sqrt{1 + 4 + 9}\sqrt{1 + 4 + \lambda^2}}\right) = \sin^{-1}\left(\frac{5 + 3\lambda}{\sqrt{14}\sqrt{5 + \lambda^2}}\right)$$

So, $\dfrac{(5 + 3\lambda)^2}{14(5 + \lambda^2)} + \dfrac{5}{14} = 1 \quad (\sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow \quad \dfrac{(5 + 3\lambda)^2}{5 + \lambda^2} + 5 = 14$

$\Rightarrow \quad (5 + 3\lambda)^2 + 5(5 + \lambda^2) = 14(5 + \lambda^2)$
$\Rightarrow \quad 25 + 30\lambda + 9\lambda^2 + 25 + 5\lambda^2 = 70 + 14\lambda^2$
$\Rightarrow \quad 30\lambda + 50 = 70 \quad \Rightarrow \quad 30\lambda = 20 \quad \therefore \quad \lambda = 2/3$

Solved Paper : AIEEE 2012

PHYSICS

1. Two electric bulbs marked 25 W-220 V and 100 W-220 V are connected in series to a 440 V supply. Which of the bulbs will fuse?

(a) 100 W (b) 25 W
(c) neither (d) both

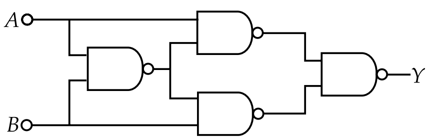
[Thermal and Chemical Effects of Current]

2. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be

(a) 10 m (b) $10\sqrt{2}$ m
(c) 20 m (d) $20\sqrt{2}$ m

[Description of Motion in 2 and 3 Dimensions]

3. Truth table for system of four NAND gates as shown in figure is



(a)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(b)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(c)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(d)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

[Solids and Semiconductor Devices]

4. This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement 1 : Davisson - Germer experiment established the wave nature of electrons.

Statement 2 : If electrons have wave nature, they can interfere and show diffraction.

(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.
(d) Statement 1 is false, Statement 2 is true.

[Electrons and Photons]

5. In Young's double slit experiment, one of the slit is wider than other, so that the amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by

(a) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$ (b) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$

(c) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$ (d) $\frac{I_m}{9}(4+5\cos\phi)$

[Wave Optics]

6. If a simple pendulum has significant amplitude (up to a factor of $1/e$ of original) only in the period between $t = 0$ s to $t = \tau$ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with b as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds

(a) b (b) $\frac{1}{b}$ (c) $\frac{2}{b}$ (d) $\frac{0.693}{b}$

[Oscillations]

7. This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.

If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement 1 : If stretched by the same amount, work done on S_1, will be more than that on S_2.

Statement 2 : $k_1 < k_2$.

(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.

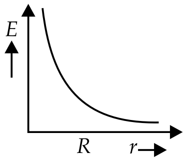
(c) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.
(d) Statement 1 is false, Statement 2 is true.

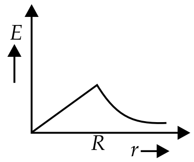
[Work, Energy and Power]

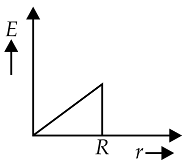
8. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object be shifted to be in sharp focus on film?
(a) 2.4 m (b) 3.2 m
(c) 5.6 m (d) 7.2 m

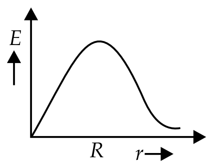
[Ray Optics]

9. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as a function of distance from the centre. The graph which would correspond to the above will be

(a) 

(b) 

(c) 

(d) 

[Electrostatics]

10. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to
(a) induction of electrical charge on the plate.
(b) shielding of magnetic lines of force as aluminium is a paramagnetic material.
(c) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping.
(d) development of air current when the plate is placed.

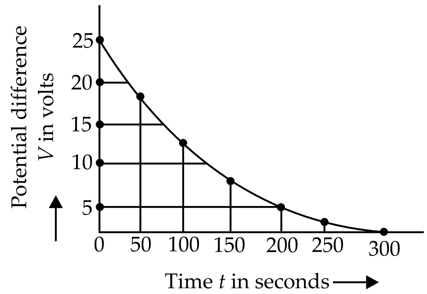
[Magnetic Effect of Current]

11. A spectrometer gives the following reading when used to measure the angle of a prism.
Main scale reading : 58.5 degree
Vernier scale reading : 09 divisions
Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data
(a) 58.77 degree (b) 58.65 degree
(c) 59 degree (d) 58.59 degree

[Practical Physics]

12. A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by (n is an integer)
(a) $\dfrac{n^2h^2}{2(m_1+m_2)r^2}$ (b) $\dfrac{2n^2h^2}{(m_1+m_2)r^2}$
(c) $\dfrac{(m_1+m_2)n^2h^2}{2m_1m_2r^2}$ (d) $\dfrac{(m_1+m_2)^2n^2h^2}{2m_1^2m_2^2r^2}$

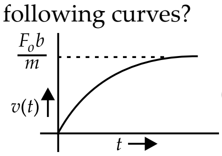
[Atoms, Molecules and Nuclei]

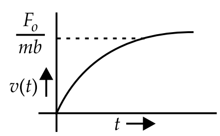
13.


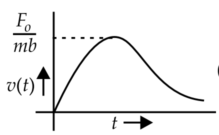
The figure shows an experimental plot for discharging of a capacitor in an R-C circuit. The time constant τ of this circuit lies between
(a) 0 and 50 sec
(b) 50 sec and 100 sec
(c) 100 sec and 150 sec
(d) 150 sec and 200 sec

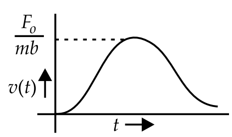
[Electrostatics]

14. A particle of mass m is at rest at the origin at time $t = 0$. It is subjected to a force $F(t) = F_o e^{-bt}$ in the x direction. Its speed $v(t)$ is depicted by which of the following curves?

(a) 

(b) 

(c) 

(d) 

[Laws of Motion]

15. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is

(a) $m_1 : m_2$ (b) $r_1 : r_2$
(c) $1 : 1$ (d) $m_1r_1 : m_2r_2$

[Laws of Motion]

16. A radar has a power of 1 kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance upto which it can detect object located on the surface of the earth (Radius of earth = 6.4×10^6 m) is

(a) 16 km (b) 40 km
(c) 64 km (d) 80 km

[Electromagnetic Waves]

17. Assume that a neutron breaks into a proton and an electron. The energy released during this process is
(Mass of neutron = 1.6725×10^{-27} kg
Mass of proton = 1.6725×10^{-27} kg
Mass of electron = 9×10^{-31} kg)

(a) 7.10 MeV (b) 6.30 MeV
(c) 5.4 MeV (d) 0.73 MeV

[Atoms, Molecules and Nuclei]

18. This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.
An insulating solid sphere of radius R has a uniformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point outside the sphere. The electric potential at infinity is zero.

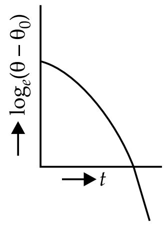
Statement 1 : When a charge q is taken from the centre to the surface of the sphere, its potential energy changes by $\frac{q\rho}{3\varepsilon_0}$.

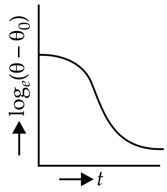
Statement 2 : The electric field at a distance $r (r < R)$ from the centre of the sphere is $\frac{\rho r}{3\varepsilon_0}$.

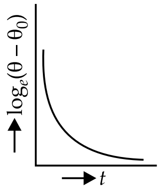
(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is false, Statement 2 is true.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.
(d) Statement 1 is true, Statement 2, is true; Statement 2 is not the correct explanation of Statement 1.

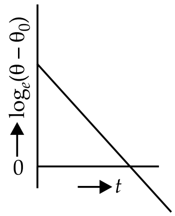
[Electrostatics]

19. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t is

(a)


(b)


(c)


(d)


[Transfer of Heat]

20. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is

(a) zero (b) 1% (c) 3% (d) 6%

[Units and Measurements]

21. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of g and R (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be

(a) 6.4×10^8 Joules (b) 6.4×10^9 Joules
(c) 6.4×10^{10} Joules (d) 6.4×10^{11} Joules

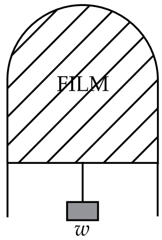
[Gravitation]

22. A cylindrical tube, open at both ends, has a fundamental frequency, f in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now

(a) $\frac{f}{2}$ (b) $\frac{3f}{4}$
(c) $2f$ (d) f

[Waves]

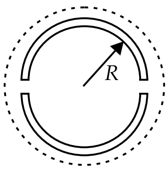
23. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is



(a) 0.1 Nm^{-1} (b) 0.05 Nm^{-1}
(c) 0.025 Nm^{-1} (d) 0.0125 Nm^{-1}

[Properties of Matter]

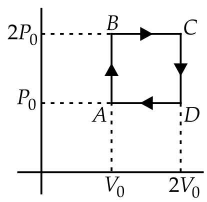
24. A wooden wheel of radius R is made of two semicircular parts (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Young's modulus is Y, the force that one part of the wheel applies on the other part is



(a) $SY\alpha\Delta T$
(b) $\pi SY\alpha\Delta T$
(c) $2SY\alpha\Delta T$
(d) $2\pi SY\alpha\Delta T$

[Properties of Matter]

25. Helium gas goes through a cycle $ABCDA$ (consisting of two isochoric and two isobaric lines) as shown in figure. Efficiency of this cycle is nearly
(Assume the gas to be close to ideal gas)



(a) 9.1%
(b) 10.5%
(c) 12.5% (d) 15.4%

[Heat and Thermodynamics]

26. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be

(a) 3 (b) 5 (c) 6 (d) 2

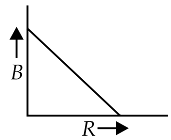
[Atoms, Molecules and Nuclei]

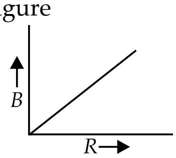
27. Proton, deuteron and alpha particle of the same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are respectively r_p, r_d and r_α. Which one of the following relation is correct?

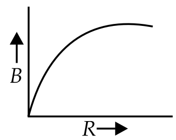
(a) $r_\alpha = r_p < r_d$ (b) $r_\alpha > r_d > r_p$
(c) $r_\alpha = r_d > r_p$ (d) $r_\alpha = r_p = r_d$

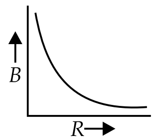
[Magnetic Effect of Current]

28. A charge Q is uniformly distributed over the surface of non-conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure

(a) 

(b) 

(c) 

(d) 

[Electrostatics]

29. An electromagnetic wave in vacuum has the electric and magnetic fields $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then

(a) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$ (b) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$
(c) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$ (d) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$

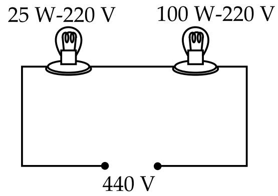
[Electromagnetic Waves]

30. A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be

(a) 1200 K (b) 750 K
(c) 600 K
(d) efficiency of Carnot engine cannot be made larger than 50%

[Heat and Thermodynamics]

1. (b) :



As $R = \dfrac{(\text{Rated voltage})^2}{\text{Rated power}}$

$\therefore$ Resistance of 25 W-220 V bulb is

$$R_1 = \frac{(220)^2}{25}\ \Omega$$

Resistance of 100 W-220 V bulb is

$$R_2 = \frac{(220)^2}{100}\ \Omega$$

When these two bulbs are connected in series, the total resistance is

$$R_s = R_1 + R_2 = (220)^2\left[\frac{1}{25} + \frac{1}{100}\right] = \frac{(220)^2}{20}\ \Omega$$

$$\text{Current, } I = \frac{440}{(220)^2/20} = \frac{2}{11}\ \text{A}$$

Potential difference across 25 W bulb

$$= IR_1 = \frac{2}{11} \times \frac{(220)^2}{25} = 352\ \text{V}$$

Potential difference across 100 W bulb

$$= IR_2 = \frac{2}{11} \times \frac{(220)^2}{100} = 88\ \text{V}$$

Thus the bulb 25 W will be fused, because it can tolerate only 220 V while the voltage across it is 352 V.

2. (c) : Let u be the velocity of projection of the stone.

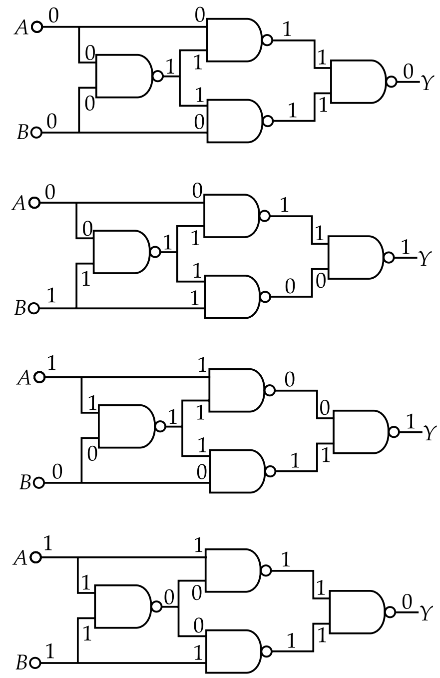
The maximum height a boy can throw a stone is

$$H_{\max} = \frac{u^2}{2g} = 10\ \text{m} \qquad \text{...(i)}$$

The maximum horizontal distance the boy can throw the same stone is

$$R_{\max} = \frac{u^2}{g} = 20\ \text{m} \qquad \text{(Using (i))}$$

3. (d) :



4. (b) : Davisson-Germer experiment showed that electron beams can undergo diffraction when passed through atomic crystals. This shows the wave nature of electrons as waves can exhibit interference and diffraction.

5. (c) : Here, $A_2 = 2A_1$

$\because$ Intensity $\propto$ (Amplitude)^2

$$\therefore \frac{I_2}{I_1} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{2A_1}{A_1}\right)^2 = 4$$

$I_2 = 4I_1$

Maximum intensity, $I_m = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$$= \left(\sqrt{I_1} + \sqrt{4I_1}\right)^2 = \left(3\sqrt{I_1}\right)^2 = 9I_1$$

$$\text{or} \quad I_1 = \frac{I_m}{9} \qquad \text{...(i)}$$

Resultant intensity, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$$= I_1 + 4I_1 + 2\sqrt{I_1(4I_1)}\cos\phi$$

$$= 5I_1 + 4I_1\cos\phi = I_1 + 4I_1 + 4I_1\cos\phi$$

$= I_1 + 4I_1(1 + \cos\phi)$

$= I_1 + 8I_1 \cos^2 \frac{\phi}{2}$ $\qquad \left(\because\ 1 + \cos\phi = 2\cos^2 \frac{\phi}{2}\right)$

$= I_1\left(1 + 8\cos^2 \frac{\phi}{2}\right)$

Putting the value of I_1 from eqn. (i), we get

$$I = \frac{I_m}{9}\left(1 + 8\cos^2 \frac{\phi}{2}\right)$$

6. **(c)**

7. **(d) :** For the same force, $F = k_1x_1 = k_2x_2$ ...(i)

Work done on spring S_1 is

$$W_1 = \frac{1}{2}k_1x_1^2 = \frac{(k_1x_1)^2}{2k_1} = \frac{F^2}{2k_1} \qquad \text{(Using (i))}$$

Work done on spring S_2 is

$$W_2 = \frac{1}{2}k_2x_2^2 = \frac{(k_2x_2)^2}{2k_2} = \frac{F^2}{2k_2} \qquad \text{(Using (i))}$$

$$\therefore \frac{W_1}{W_2} = \frac{k_2}{k_1}$$

As $W_1 > W_2$ $\therefore$ $k_2 > k_1$ or $k_1 < k_2$

Statement 2 is true.

For the same extension, $x_1 = x_2 = x$ ...(ii)

Work done on spring S_1 is

$$W_1 = \frac{1}{2}k_1x_1^2 = \frac{1}{2}k_1x^2 \qquad \text{(Using (ii))}$$

Work done on spring S_2 is

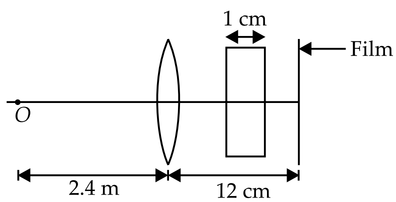
$$W_2 = \frac{1}{2}k_2x_2^2 = \frac{1}{2}k_2x^2 \qquad \text{(Using (ii)}$$

$$\therefore \frac{W_1}{W_2} = \frac{k_1}{k_2}$$

As $k_1 < k_2$ $\therefore$ $W_1 < W_2$

Statement 1 is false.

8. **(c) :**



According to thin lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Here, $u = -2.4$ m $= -240$ cm, $v = 12$ cm

$$\therefore \frac{1}{f} = \frac{1}{12} - \frac{1}{(-240)} = \frac{1}{12} + \frac{1}{240}$$

$$\frac{1}{f} = \frac{21}{240} \text{ or } f = \frac{240}{21} \text{ cm}$$

When a glass plate is interposed between lens and film, so shift produced by it will be

$$\text{Shift} = t\left(1 - \frac{1}{\mu}\right) = 1\left(1 - \frac{1}{1.5}\right) = 1\left(1 - \frac{2}{3}\right) = \frac{1}{3} \text{ cm}$$

To get image at film, lens should form image at distance

$$v' = 12 - \frac{1}{3} = \frac{35}{3} \text{ cm}$$

Again using lens formula

$$\therefore \frac{21}{240} = \frac{3}{35} - \frac{1}{u'} \text{ or } \frac{1}{u'} = \frac{3}{35} - \frac{21}{240} = \frac{1}{5}\left[\frac{3}{7} - \frac{21}{48}\right]$$

$$\frac{1}{u'} = \frac{1}{5}\left[\frac{144 - 147}{336}\right] \text{ or } \frac{1}{u'} = -\frac{3}{1680}$$

$u' = -560$ cm $= -5.6$ m

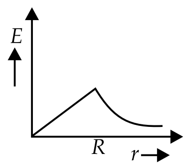
$|u'| = 5.6$ m

9. **(b) :** For uniformly charged sphere

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3} \qquad (\text{For } r < R)$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} \qquad (\text{For } r = R)$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \qquad (\text{For } r > R)$$



The variation of E with distance r from the centre is as shown adjacent figure.

10. **(c)**

11. **(b) :** 30 VSD = 29 MSD

$$1 \text{ VSD} = \frac{29}{30} \text{ MSD}$$

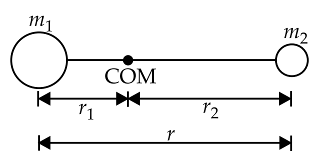
Least count = 1 MSD – 1 VSD

$$= \left(1 - \frac{29}{30}\right)\text{MSD} = \frac{1}{30} \times 0.5°$$

Reading = Main scale reading + Vernier scale reading × least count

$$= 58.5° + 9 \times \frac{0.5°}{30} = 58.5° + 0.15° = 58.65°$$

12. **(c) :** A diatomic molecule consists of two atoms of masses m_1 and m_2 at a distance r apart. Let r_1 and r_2 be the distances of the atoms from the centre of mass.



The moment of inertia of this molecule about an axis passing through its centre of mass and perpendicular to a line joining the atoms is

$I = m_1 r_1^2 + m_2 r_2^2$

As $m_1 r_1 = m_2 r_2$ or $r_1 = \frac{m_2}{m_1} r_2$

$\because \quad r_1 + r_2 = r$

$\therefore \quad r_1 = \frac{m_2}{m_1}(r - r_1)$

On rearranging, we get

$$r_1 = \frac{m_2 r}{m_1 + m_2}$$

Similarly, $r_2 = \frac{m_1 r}{m_1 + m_2}$

Therefore, the moment of inertia can be written as

$$I = m_1\left(\frac{m_2 r}{m_1 + m_2}\right)^2 + m_2\left(\frac{m_1 r}{m_1 + m_2}\right)^2$$

$$= \frac{m_1 m_2}{m_1 + m_2} r^2 \qquad \text{...(i)}$$

According to Bohr's quantisation condition

$$L = \frac{nh}{2\pi}$$

or $L^2 = \frac{n^2 h^2}{4\pi^2}$ ...(ii)

Rotational energy, $E = \frac{L^2}{2I}$

$$E = \frac{n^2 h^2}{8\pi^2 I} \qquad \text{(Using (ii))}$$

$$= \frac{n^2 h^2 (m_1 + m_2)}{8\pi^2 (m_1 m_2) r^2} \qquad \text{(Using (i))}$$

$$= \frac{n^2 \hbar^2 (m_1 + m_2)}{2 m_1 m_2 r^2} \qquad \left(\because \ \hbar = \frac{h}{2\pi}\right)$$

In the question instead of h, $\hbar$ should be given.

13. **(c) :** During discharging of a capacitor

$V = V_0 e^{-t/\tau}$

where τ is the time constant of RC circuit.

At $t = \tau$,

$V = \frac{V_0}{e} = 0.37\, V_0$

From the graph, $t = 0$, $V_0 = 25$ V

$\therefore \quad V = 0.37 \times 25 \text{ V} = 9.25 \text{ V}$

This voltage occurs at time lies between 100 sec and 500 sec. Hence, time constant τ of this circuit lies between 100 sec and 150 sec.

14. **(b) :** $F(t) = F_o e^{-bt}$ (Given)

$ma = F_o e^{-bt}$

$a = \frac{F_o}{m} e^{-bt}$

$\frac{dv}{dt} = \frac{F_o}{m} e^{-bt}$ or $dv = \frac{F_o}{m} e^{-bt} dt$

Integrating both sides, we get

$$\int_0^v dv = \int_0^t \frac{F_o}{m} e^{-bt} dt$$

$$v = \frac{F_o}{m}\left[\frac{e^{-bt}}{-b}\right]_0^t = \frac{F_o}{mb}[1 - e^{-bt}]$$

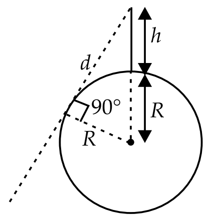
15. **(b) :** Centripetal acceleration, $a_c = \omega^2 r$

$\because \quad \omega = \frac{2\pi}{T}$

As $T_1 = T_2 \Rightarrow \omega_1 = \omega_2$

$\because \quad \frac{a_{c_1}}{a_{c_2}} = \frac{r_1}{r_2}$

16. **(d) :** Maximum distance on earth where object can be detected is d, then



$(h + R)^2 = d^2 + R^2$

$d^2 = h^2 + 2Rh$

$\because \quad h << R$

$\therefore \quad d = \sqrt{2Rh}$

$d = \sqrt{2 \times 6.4 \times 10^6 \times 500} = 8 \times 10^4 \text{ m} = 80 \text{ km}$

17. **(*) :** Mass defect, $\Delta m = m_p + m_e - m_n$

$= (1.6725 \times 10^{-27} + 9 \times 10^{-31} - 1.6725 \times 10^{-27})$ kg

$= 9 \times 10^{-31}$ kg

Energy released = Δmc^2

$= 9 \times 10^{-31} \times (3 \times 10^8)^2$ J

$= \frac{9 \times 10^{-31} \times 9 \times 10^{16}}{1.6 \times 10^{-13}}$ MeV

$= 0.51$ MeV

* None of the given option is correct.

18. **(b) :** Potential at the centre of the sphere,

$$V_C = \frac{R^2 \rho}{2\varepsilon_0}$$

Potential at the surface of the sphere,

$$V_S = \frac{1}{3}\frac{R^2 \rho}{\varepsilon_0}$$

When a charge q is taken from the centre to the surface, the change in potential energy is

$$\Delta U = (V_C - V_S)q = \left(\frac{R^2\rho}{2\varepsilon_0} - \frac{1}{3}\frac{R^2\rho}{\varepsilon_0}\right)q = \frac{1}{6}\frac{R^2 \rho q}{\varepsilon_0}$$

Statement 1 is false.

Statement 2 is true.

19. (d): According to Newton's law of cooling

$$\frac{d\theta}{dt} = -k(\theta - \theta_0) \text{ or } \frac{d\theta}{\theta - \theta_0} = -kdt$$

Integrating both sides, we get

$$\int \frac{d\theta}{\theta - \theta_0} = \int -kdt$$

$\log_e(\theta - \theta_0) = -kt + C$

where C is a constant of integration.

So, the graph between $\log_e(\theta - \theta_0)$ and t is a straight line with a negative slope. Option (d) represents the correct graph.

20. (d): $R = \frac{V}{I} \quad \therefore \quad \frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$

The percentage error in R is

$$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100 = 3\% + 3\% = 6\%$$

21. (c) : Energy required $= \frac{GMm}{R}$

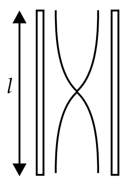
$= gR^2 \times \frac{m}{R} \qquad \left(\because \; g = \frac{GM}{R^2}\right)$

$= mgR$

$= 1000 \times 10 \times 6400 \times 10^3$

$= 64 \times 10^9$ J $= 6.4 \times 10^{10}$ J

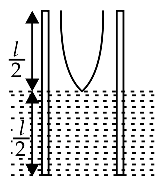
22. (d): When the tube of length l is open at both ends,



$\therefore \quad f = \frac{v}{2l}$ ...(i)

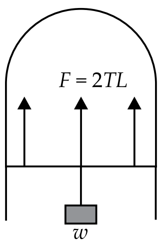
where v is the speed of sound in air.

When the tube is dipped vertically in water and half of it is in water, it behaves closed pipe length $\frac{l}{2}$,



$\therefore \quad f' = \frac{v}{4\left(\frac{l}{2}\right)} = \frac{v}{2l} = f$ (Using(i))

23. (c) : The force due to the surface tension will balance the weight.



$F = w$

$2TL = w$

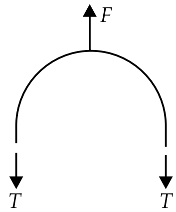
$T = \frac{w}{2L}$

Substituting the given values, we get

$$T = \frac{1.5 \times 10^{-2} \text{ N}}{2 \times 30 \times 10^{-2} \text{ m}} = 0.025 \text{ Nm}^{-1}$$

24. (c) : Increase in length, $\Delta L = L\alpha\Delta T$

$\therefore \quad \frac{\Delta L}{L} = \alpha\Delta T$



The thermal stress developed is

$\frac{T}{S} = Y\frac{\Delta L}{L} = Y\alpha\Delta T$

or $\quad T = SY\alpha\Delta T$

From FBD of one part of the wheel,

or $\quad F = 2T$

Where, F is the force that one part of the wheel applies on the other part.

$\therefore \quad F = 2SY\alpha\Delta T$

25. (d): In case of a cyclic process, work done is equal to the area under the cycle and is taken to be positive if the cycle is clockwise.

$\therefore$ Work done by the gas

W = Area of the rectangle $ABCD = P_0V_0$

Helium gas is a monoatomic gas.

$\therefore \quad C_V = \frac{3}{2}R$ and $C_P = \frac{5}{2}R$

Along the path AB, heat supplied to the gas at constant volume,

$$\therefore \Delta Q_{AB} = nC_V\Delta T = n\frac{3}{2}R\Delta T = \frac{3}{2}V_0\Delta P = \frac{3}{2}P_0V_0$$

Along the path BC, heat supplied to the gas at constant pressure,

$$\therefore \Delta Q_{BC} = nC_P\Delta T = n\frac{5}{2}R\Delta T = \frac{5}{2}(2P_0)\Delta V = 5P_0V_0$$

Along the path CD and DA, heat is rejected by the gas

$$\text{Efficiency, } \eta = \frac{\text{Work done by the gas}}{\text{Heat supplied to the gas}} \times 100$$

$$= \frac{P_0V_0}{\frac{3}{2}P_0V_0 + 5P_0V_0} \times 100 = \frac{200}{13} = 15.4\%$$

26. (c) : Number of spectral lines in the emission spectra,

$$N = \frac{n(n-1)}{2}$$

Here, $n = 4$

$\therefore \quad N = \frac{4(4-1)}{2} = 6$

27. (a) : The radius of the circular path of a charged particle in the magnetic field is given by

$$r = \frac{mv}{Bq}$$

Kinetic energy of a charged particle,

$$K = \frac{1}{2}mv^2 \quad \text{or} \quad v = \sqrt{\frac{2K}{m}}$$

$$\therefore \quad r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2Km}}{qB}$$

As K and B are constants

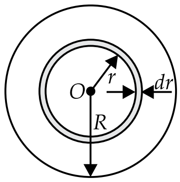
$$\therefore \quad r \propto \frac{\sqrt{m}}{q}$$

$$r_p : r_d : r_\alpha = \frac{\sqrt{m_p}}{q_p} : \frac{\sqrt{m_d}}{q_d} : \frac{\sqrt{m_\alpha}}{q_\alpha}$$

$$= \frac{\sqrt{m}}{e} : \frac{\sqrt{2m}}{e} : \frac{\sqrt{4m}}{2e} = 1 : \sqrt{2} : 1$$

$$\Rightarrow \quad r_\alpha = r_p < r_d$$

28. **(d) :** Consider a elementary ring of radius r and thickness dr of a disc as shown in figure.



Charge on the ring,

$$dq = \frac{Q}{\pi R^2}(2\pi r dr) = \frac{2Qr}{R^2}dr$$

Current due to rotation of charge on ring is

$$I = \frac{dq\omega}{2\pi} = \frac{Qr\omega dr}{\pi R^2}$$

Magnetic field at the centre due to the ring element

$$dB = \frac{\mu_0 I}{2r} = \frac{\mu_0 Qr\omega dr}{2\pi R^2 r} = \frac{\mu_0 Q\omega dr}{2\pi R^2}$$

Magnetic field at the centre due to the whole disc

$$B = \int dB = \frac{\mu_0 Q\omega}{2\pi R^2}\int_0^R dr = \frac{\mu_0 Q\omega R}{2\pi R^2} = \frac{\mu_0 Q\omega}{2\pi R}$$

Since, Q and ω are constants

$$\therefore \quad B \propto \frac{1}{R}$$

Hence variation of B with R should be a rectangular hyperbola as represented in option (d).

29. **(a) :** The direction of polarization is parallel to electric field.

$$\therefore \quad \vec{X} \parallel \vec{E}$$

The direction of wave propagation is parallel to $\vec{E} \times \vec{B}$.

$$\therefore \quad \vec{k} \parallel \vec{E} \times \vec{B}$$

30. **(b) :** Efficiency of Carnot engine,

$$\eta = 1 - \frac{T_2}{T_1}$$

where T_1 is the temperature of the source and T_2 is the temperature of the sink.

For 1st case

$\eta = 40\%$, $T_1 = 500$ K

$$\therefore \quad \frac{40}{100} = 1 - \frac{T_2}{500}$$

$$\frac{T_2}{500} = 1 - \frac{40}{100} = \frac{3}{5}$$

$$T_2 = \frac{3}{5} \times 500 = 300 \text{ K}$$

For 2nd case

$\eta = 60\%$, $T_2 = 300$ K

$$\therefore \quad \frac{60}{100} = 1 - \frac{300}{T_1}$$

$$\frac{300}{T_1} = 1 - \frac{60}{100} = \frac{2}{5}$$

$$T_1 = \frac{5}{2} \times 300 = 750 \text{ K}$$

Solved Paper : AIEEE 2012

CHEMISTRY

1. 2-Hexyne gives *trans*-2-hexene on treatment with
(a) Li/NH_3 (b) $Pd/BaSO_4$
(c) $LiAlH_4$ (d) Pt/H_2 **[Hydrocarbons]**

2. Which of the following on thermal decomposition yields a basic as well as an acidic oxide?
(a) $KClO_3$ (b) $CaCO_3$
(c) NH_4NO_3 (d) $NaNO_3$
[Chemistry of Metals]

3. Which one of the following statements is correct?
(a) All amino acids are optically active.
(b) All amino acids except glycine are optically active.
(c) All amino acids except glutamic acid are optically active.
(d) All amino acids except lysine are optically active.
[Biomolecules]

4. The density of a solution prepared by dissolving 120 g of urea (mol. mass = 60 u) in 1000 g of water is 1.15 g/mL. The molarity of this solution is
(a) 1.78 M (b) 1.02 M
(c) 2.05 M (d) 0.50 M **[Solutions]**

5. The incorrect expression among the following is
(a) in isothermal process,
$$w_{\text{reversible}} = -nRT\ln\frac{V_f}{V_i}$$
(b) $\ln K = \dfrac{\Delta H^\circ - T\Delta S^\circ}{RT}$ (c) $K = e^{-\Delta G^\circ/RT}$
(d) $\dfrac{\Delta G_{\text{system}}}{\Delta S_{\text{total}}} = -T$ **[Energetics]**

6. Which branched chain isomer of the hydrocarbon with molecular mass 72 u gives only one isomer of mono substituted alkyl halide?
(a) Neopentane (b) Isohexane
(c) Neohexane (d) Tertiary butyl chloride
[Hydrocarbons]

7. According to Freundlich adsorption isotherm, which of the following is correct?
(a) $\dfrac{x}{m} \propto p^1$ (b) $\dfrac{x}{m} \propto p^{1/n}$
(c) $\dfrac{x}{m} \propto p^0$
(d) All the above are correct for different ranges of pressure. **[Surface Chemistry]**

8. In which of the following pairs the two species are not isostructural?
(a) PCl_4^+ and $SiCl_4$ (b) PF_5 and BrF_5
(c) AlF_6^{3-} and SF_6 (d) CO_3^{2-} and NO_3^-
[Atomic Structure, Chemical Bonding & Molecular Structure]

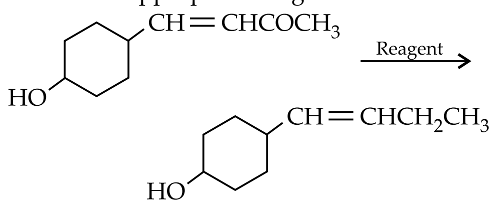
9. How many chiral compounds are possible on monochlorination of 2-methyl butane?
(a) 2 (b) 4
(c) 6 (d) 8
[General Organic Chemistry]

10. The increasing order of the ionic radii of the given isoelectronic species is
(a) $S^{2-}, Cl^-, Ca^{2+}, K^+$ (b) $Ca^{2+}, K^+, Cl^-, S^{2-}$
(c) $K^+, S^{2-}, Ca^{2+}, Cl^-$ (d) $Cl^-, Ca^{2+}, K^+, S^{2-}$
[Periodic Properties]

11. The compressibility factor for a real gas at high pressure is
(a) 1 (b) $1 + Pb/RT$
(c) $1 - Pb/RT$ (d) $1 + RT/Pb$
[States of Matter]

12. Which among the following will be named as dibromidobis (ethylene diamine) chromium(III bromide?
(a) $[Cr(en)_2Br_2]Br$ (b) $[Cr(en)Br_4]^-$
(c) $[Cr(en)Br_2]Br$ (d) $[Cr(en)_3]Br_3$
[Coordination Compounds and Organometallics]

13. In the given transformation, which of the following is the most appropriate reagent?



(a) Zn-Hg/HCl (b) Na, liq. NH_3
(c) $NaBH_4$ (d) NH_2-NH_2, OH^-
[Aldehydes, Ketones, Carboxylic Acids & Their Derivatives]

14. Lithium forms body centred cubic structure. The length of the side of its unit cell is 351 pm. Atomic radius of the lithium will be

(a) 300 pm (b) 240 pm
(c) 152 pm (d) 75 pm
[States of Matter]

15. K_f for water is 1.86 K kg mol^{-1}. If your automobile radiator holds 1.0 kg of water, how many grams of ethylene glycol ($C_2H_6O_2$) must you add to get the freezing point of the solution lowered to –2.8°C?
(a) 93 g (b) 39 g
(c) 27 g (d) 72 g **[Solutions]**

16. The molecule having smallest bond angle is
(a) $AsCl_3$ (b) $SbCl_3$
(c) PCl_3 (d) NCl_3
[Atomic Structure, Chemical Bonding & Molecular Structure]

17. What is DDT among the following?
(a) A fertilizer (b) Biodegradable pollutant
(c) Non-biodegradable pollutant
(d) Greenhouse gas
[Halides, Hydroxy Compounds & Ethers]

18. The pH of a 0.1 molar solution of the acid HQ is 3. The value of the ionization constant, K_a of this acid is
(a) 1×10^{-3} (b) 1×10^{-5}
(c) 1×10^{-7} (d) 3×10^{-1} **[Equilibrium]**

19. Very pure hydrogen (99.9%) can be made by which of the following processes?
(a) Mixing natural hydrocarbons of high molecular weight.
(b) Electrolysis of water.
(c) Reaction of salt like hydrides with water.
(d) Reaction of methane with steam.
[Chemistry of Non-metals]

20. Aspirin in known as
(a) phenyl salicylate (b) acetyl salicylate
(c) methyl salicylic acid
(d) acetyl salicylic acid **[Chemistry in Action]**

21. Which of the following compounds can be detected by Molisch's test?
(a) Sugars (b) Amines
(c) Primary alcohols (d) Nitro compounds
[Biomolecules]

22. The standard reduction potentials for Zn^{2+}/Zn, Ni^{2+}/Ni, and Fe^{2+}/Fe are – 0.76, – 0.23 and – 0.44 V respectively. The reaction $X + Y^{2+} \rightarrow X^{2+} + Y$ will be spontaneous when
(a) X = Ni, Y = Zn (b) X = Fe, Y = Zn
(c) X = Zn, Y = Ni (d) X = Ni, Y = Fe
[Redox Reactions & Electrochemistry]

23. *Ortho*-nitrophenol is less soluble in water than *p*- and *m*-nitrophenols because
(a) *o*-nitrophenol shows intramolecular H-bonding
(b) *o*-nitrophenol shows intermolecular H-bonding
(c) melting point of *o*-nitrophenol is lower than those of *m*- and *p*-isomers
(d) *o*-nitrophenol is more volatile in steam than those of *m*- and *p*-isomers
[Halides, Hydroxy Compounds & Ethers]

24. Iodoform can be prepared from all except
(a) isopropyl alcohol (b) 3-methyl-2-butanone
(c) isobutyl alcohol (d) ethyl methyl ketone
[Halides, Hydroxy Compounds & Ethers]

25. The species which can best serve as an initiator for the cationic polymerization is
(a) HNO_3 (b) $AlCl_3$
(c) BuLi (d) $LiAlH_4$ **[Polymers]**

26. The equilibrium constant (K_c) for the reaction $N_{2(g)} + O_{2(g)} \rightarrow 2NO_{(g)}$ at temperature T is 4×10^{-4}. The value of K_c for the reaction, $NO_{(g)} \rightarrow \frac{1}{2}N_{2(g)} + \frac{1}{2}O_{2(g)}$ at the same temperature is
(a) 2.5×10^2 (b) 4×10^{-4}
(c) 50.0 (d) 0.02 **[Equilibrium]**

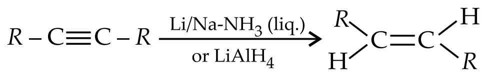
27. For a first order reaction, (A) $\rightarrow$ products, the concentration of A changes from 0.1 M to 0.025 M in 40 minutes. The rate of reaction when the concentration of A is 0.01 M is
(a) 3.47×10^{-4} M/min (b) 3.47×10^{-5} M/min
(c) 1.73×10^{-4} M/min (d) 1.73×10^{-5} M/min
[Kinetics]

28. Which method of purification is represented by the following equation?
$$Ti_{(s)} + 2I_{2(g)} \xrightarrow{523\text{ K}} TiI_{4(g)} \xrightarrow{1700\text{ K}} Ti_{(s)} + 2I_{2(g)}$$
(a) Cupellation (b) Poling
(c) Van Arkel (d) Zone refining
[Metallurgy]

29. Iron exhibits +2 and +3 oxidation states. Which of the following statements about iron is incorrect?
(a) Ferrous compounds are relatively more ionic than the corresponding ferric compounds.
(b) Ferrous compounds are less volatile than the corresponding ferric compounds.
(c) Ferrous compounds are more easily hydrolysed than the corresponding ferric compounds.
(d) Ferrous oxide is more basic in nature than the ferric oxide.
[Transition Metals Including Lanthanides]

30. The electrons identified by quantum numbers n and l:
(1) $n = 4, l = 1$ (2) $n = 4, l = 0$
(3) $n = 3, l = 2$ (4) $n = 3, l = 1$
can be placed in order of increasing energy as
(a) (4) < (2) < (3) < (1) (b) (2) < (4) < (1) < (3)
(c) (1) < (3) < (2) < (4) (d) (3) < (4) < (2) < (1)
[Atomic Structure, Chemical Bonding & Molecular Structure]

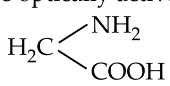
1. **(a, c) :** For *trans* products we should take Na or Li metal in NH_3 or $EtNH_2$ at low temperature or $LiAlH_4$ as reducing agent (anti-addition).



anti-addition product (*trans*-alkene)

2. **(b) :** $CaCO_3 \xrightarrow{\Delta} \underset{\text{metal oxide (basic)}}{CaO} + \underset{\text{non-metal oxide (acidic)}}{CO_2}$

3. **(b) :** Glycine is optically inactive while all other amino acids are optically active.



Glycine (optically inactive)

4. **(c) :** Mass of solute taken = 120 g
Molecular mass of solute = 60 u
Mass of solvent = 1000 g
Density of solution = 1.15 g/mL
Total mass of solution = 1000 + 120 = 1120 g

$$\text{Volume of solution} = \frac{\text{Mass}}{\text{Density}} = \frac{1120}{1.15}\ \text{mL}$$

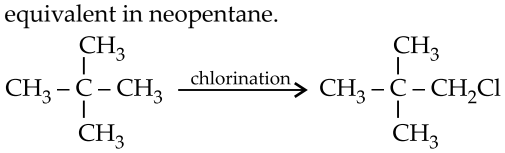
$$\text{Molarity} = \frac{\frac{\text{Mass of solute}}{\text{Molecular mass of solute}}}{\text{Volume of solution}} \times 1000$$

$$= \frac{120/60}{1120/1.15} \times 1000 = \frac{2 \times 1000 \times 1.15}{1120} = 2.05\ \text{M}$$

5. **(b) :** $\Delta G° = -RT \ln K$
$\Delta G° = \Delta H° - T\Delta S°$
$\Delta H° - T\Delta S° = -RT \ln K$

$$\ln K = -\left(\frac{\Delta H° - T\Delta S°}{RT}\right)$$

6. **(a) :** As the molecular mass indicates it should be pentane and neopentane can only form one mono substituted alkyl halide as all the hydrogens are equivalent in neopentane.



Neopentane   Only one mono-chloro derivative possible

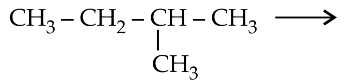
7. **(d) :** According to Freundlich adsorption isotherm

$$\frac{x}{m} = kp^{1/n}$$

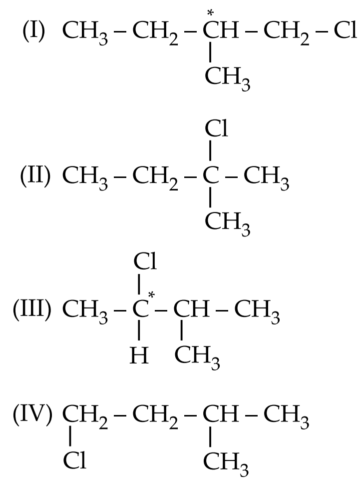
$1/n$ can have values between 0 to 1 over different ranges of pressure.

8. **(b) :** PCl_4^+ and $SiCl_4$ $\Rightarrow$ both tetrahedral
PF_5 $\Rightarrow$ trigonal bipyramidal
BrF_5 $\Rightarrow$ square pyramidal
AlF_6^{3-} and SF_6 both are octahedral, CO_3^{2-} and NO_3^- both are trigonal planar.

9. **(a) :**



2-methyl butane



Possible products on chlorination

Out of four possible isomers only I and III are chiral.

10. **(b) :** For isoelectronic species as effective nuclear charge increases, ionic radii decreases. Nuclear charge is maximum of the specie with maximum protons. Order of nuclear charge:

	Ca^{2+} >	K^+ >	Cl^- >	S^{2-}
Protons :	20	19	17	16
Electrons :	18	18	18	18

Thus increasing order of ionic radii
$Ca^{2+} < K^+ < Cl^- < S^{2-}$

11. **(b) :** For real gases, $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

At high pressure, $P >> a/V^2$

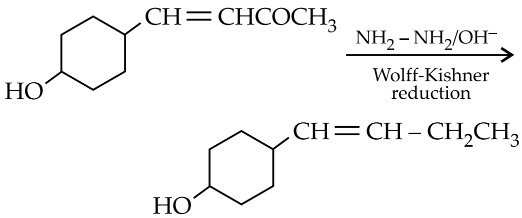
Thus neglecting a/V^2 gives

$$P(V-b) = RT \text{ or } PV = RT + Pb$$

or $\dfrac{PV}{RT} = Z = \dfrac{RT+Pb}{RT} \Rightarrow Z = 1 + Pb/RT$

12. (a)

13. (d) :



– OH group and alkene are acid-sensitive groups so Clemmensen reduction cannot be used and $NaBH_4$ reduces $>C{=}O$ to – CHOH only.

14. (c) : $a = 351$ pm

For *bcc* unit cell, $a\sqrt{3} = 4r$

$$r = \frac{a\sqrt{3}}{4} = \frac{351\times\sqrt{3}}{4} = 152 \text{ pm}$$

15. (a) : $K_f = 1.86$ K kg mol^{-1}

$\Delta T_f = 0 - (-2.8) = 2.8°C$

Mass of solvent = 1.0 kg

Mass of solute = ?

Molecular mass of solute = 62

$\Delta T_f = K_f \times m$

$$m = \frac{\dfrac{\text{Weight of solute}}{\text{Molecular mass of solute}}}{\text{Mass of solvent (g)}} \times 1000$$

$$m = \frac{w/62}{1000} \times 1000 = \frac{w}{62}$$

$\Delta T_f = K_f \times m$

$$2.8 = 1.86 \times \frac{w}{62} \Rightarrow w = \frac{62\times 2.8}{1.86} = 93 \text{ g}$$

16. (b) : As we move down the group the size of atom increases and as size of central atom increases, lone pair-bond pair repulsion also increases. Thus bond angle decreases.

Increasing order of atomic radius :

$$N < P < As < Sb$$

Decreasing order of bond angle :

$$NCl_3 > PCl_3 > AsCl_3 > SbCl_3$$

17. (c)

18. (b) : pH = 3

Molarity = 0.1 M

$[H^+] = \sqrt{K_a C}$

$H^+ = 10^{-pH} = 10^{-3}$

$10^{-3} = \sqrt{K_a \times 0.1}$ or $10^{-6} = K_a \times 0.1$

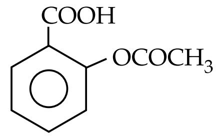
$\therefore \ K_a = 10^{-5}$

19. (b) : Dihydrogen of high purity is usually prepared by the electrolysis of water using platinum electrodes in presence of small amount of acid or alkali.

$$2H_2O_{(l)} \xrightarrow{\text{electrolysis}} 2H_{2(g)} + O_{2(g)}$$

Dihydrogen is collected at cathode.

20. (d) : Aspirin -



Acetyl salicylic acid

21. (a) : Molisch's test is a sensitive chemical test for the presence of carbohydrates, based on the dehydration of carbohydrate by sulphuric acid to produce an aldehyde, which condenses with two molecules of phenol resulting in red or purple coloured compound.

22. (c) : The elements with high negative value of standard reduction potential are good reducing agents and can be easily oxidised.

Thus X should have high negative value of standard potential than Y so that it will be oxidised to X^{2+} by reducing Y^{2+} to Y.

X = Zn, Y = Ni

$Zn + Ni^{2+} \rightarrow Zn^{2+} + Ni$

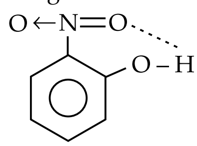
Alternatively, for a spontaneous reaction $E°$ must be positive.

$E° = E°_{\text{reduced species}} - E°_{\text{oxidised species}}$

$= -0.23 - (-0.76)$

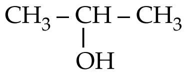
$\Rightarrow E° = +0.53$ V

23. (a) : *o*-Nitrophenol is stable due to intramolecular hydrogen bonding.

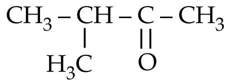


It is difficult to break the H-bonding when dissolved in water thus less soluble.

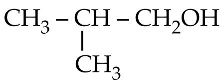
24. (c) : The compounds with $CH_3-\underset{\underset{O}{\|}}{C}-$ or $CH_3-\underset{\underset{OH}{|}}{CH}-$ group form iodoform.



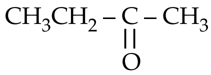
Isopropyl alcohol



3-methyl-2-butanone



Isobutyl alcohol



Ethyl methyl ketone

Thus all the compounds except isobutyl alcohol will form iodoform.

25. **(b) :** Cationic polymerisation is initiated by use of strong Lewis acids such as H_2SO_4, HF, $AlCl_3$, $SnCl_4$ or BF_3 in H_2O.

26. **(c) :** $N_{2(g)} + O_{2(g)} \longrightarrow 2NO_{(g)}$, ... (i)

$K_c = 4 \times 10^{-4}$

By multiplying the equation (i) by $\frac{1}{2}$

$$\frac{1}{2}N_{2(g)} + \frac{1}{2}O_{2(g)} \longrightarrow NO_{(g)} \quad \text{... (ii)}$$

$$K_c' = \sqrt{K_c} = \sqrt{4 \times 10^{-4}} = 2 \times 10^{-2}$$

By reversing the equation (ii), we get

$$NO_{(g)} \longrightarrow \frac{1}{2}N_{2(g)} + \frac{1}{2}O_{2(g)}$$

$$K_c'' = \frac{1}{K_c'} = \frac{1}{2 \times 10^{-2}} = 50.0$$

27. **(a) :** For the first order reaction

$$k = \frac{2.303}{t}\log\frac{a}{a-x}$$

$a = 0.1$ M, $a - x = 0.025$ M, $t = 40$ min

$$k = \frac{2.303}{40}\log\frac{0.1}{0.025} = \frac{2.303}{40}\log 4$$

$$= 0.0347 \text{ min}^{-1}$$

$[A] \longrightarrow$ product

Thus, rate $= k[A]$

rate $= 0.0347 \times 0.01$ M min^{-1}

$= 3.47 \times 10^{-4}$ M min^{-1}.

28. **(c) :** Van Arkel method which is also called as vapour-phase refining is used for preparing ultrapure metals like titanium, zirconium, thorium and uranium.

29. **(c) :** Ferrous oxide is more basic, more ionic, less volatile and less easily hydrolysed than ferric oxide.

30. **(a) :** (1) $n = 4, l = 1 \Rightarrow 4p$

(2) $n = 4, l = 0 \Rightarrow 4s$

(3) $n = 3, l = 2 \Rightarrow 3d$

(4) $n = 3, l = 1 \Rightarrow 3p$

Increasing order of energy is

$3p < 4s < 3d < 4p$

(4) < (2) < (3) < (1)

Alternatively,

for (1) $n + l = 5\ ;\ n = 4$

(2) $n + l = 4\ ;\ n = 4$

(3) $n + l = 5\ ;\ n = 3$

(4) $n + l = 4\ ;\ n = 3$

Lower $n + l$ means less energy and if for two subshells $n + l$ is same than lower n, lower will be the energy.

Thus correct order is (4) < (2) < (3) < (1).

Solved Paper : AIEEE 2012

MATHEMATICS

1. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

[Vector Algebra]

2. If the integral

$\int \frac{5\tan x}{\tan x - 2} dx = x + a\ln|\sin x - 2\cos x| + k$, then a is equal to

(a) 1 (b) 2

(c) – 1 (d) – 2

[Integral Calculus]

3. Consider the function, $f(x) = |x-2| + |x-5|$, $x \in R$

Statement 1 : $f'(4) = 0$

Statement 2 : f is continuous in [2, 5], differentiable in (2, 5) and $f(2) = f(5)$.

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.

[Differential Calculus]

4. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals

(a) 6 (b) 11/5

(c) 29/5 (d) 5

[Two Dimensional Geometry]

5. **Statement 1 :** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$

Statement 2 : If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.

[Two Dimensional Geometry]

6. Three numbers are chosen at random without replacement from {1, 2, 3, ..., 8}. The probability that their minimum is 3, given that their maximum is 6, is

(a) $\frac{1}{4}$ (b) $\frac{2}{5}$

(c) $\frac{3}{8}$ (d) $\frac{1}{5}$

[Probability]

7. Let $ABCD$ be a parallelogram such that $\overrightarrow{AB} = \vec{q}$, $\overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by

(a) $\vec{r} = \vec{q} - \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$ (b) $\vec{r} = -3\vec{q} + \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

(c) $\vec{r} = 3\vec{q} - \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$ (d) $\vec{r} = -\vec{q} + \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$

[Vector Algebra]

8. An equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is

(a) $x - 2y + 2z - 1 = 0$ (b) $x - 2y + 2z + 5 = 0$

(c) $x - 2y + 2z - 3 = 0$ (d) $x - 2y + 2z + 1 = 0$

[Three Dimensional Geometry]

9. In a ΔPQR, if $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to

(a) $\pi/4$ (b) $3\pi/4$

(c) $5\pi/6$ (d) $\pi/6$

[Trigonometry]

10. If $f : R \to R$ is a function defined by $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$, where $[x]$ denotes the greatest integer function, then f is
(a) discontinuous only at non-zero integral values of x.
(b) continuous only at $x = 0$.
(c) continuous for every real x.
(d) discontinuous only at $x = 0$.

[Differential Calculus]

11. **Statement 1 :** The sum of the series $1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400)$ is 8000.

Statement 2 : $\sum_{k=1}^{n} (k^3 - (k-1)^3) = n^3$, for any natural number n.
(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.

[Sequences and Series]

12. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is
(a) 6/5 (b) 5/3
(c) 10/3 (d) 3/5

[Two Dimensional Geometry]

13. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to

(a) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

(c) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$

[Matrices and Determinants]

14. If n is a positive integer, then $(\sqrt{3}+1)^{2n} - (\sqrt{3}-1)^{2n}$ is
(a) an even positive integer.
(b) a rational number other than positive integers.
(c) an irrational number.
(d) an odd positive integer.

[Binomial Theorem]

15. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is
(a) 630 (b) 879
(c) 880 (d) 629

[Permutations and Combinations]

16. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is
(a) $4x^2 + y^2 = 8$ (b) $x^2 + 4y^2 = 16$
(c) $4x^2 + y^2 = 4$ (d) $x^2 + 4y^2 = 8$

[Two Dimensional Geometry]

17. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to
(a) 9/2 (b) 0
(c) – 1 (d) 2/9

[Three Dimensional Geometry]

18. Let $a, b \in R$ be such that the function f given by $f(x) = \ln |x| + bx^2 + ax$, $x \neq 0$ has extreme values at $x = -1$ and $x = 2$.
Statement 1 : f has local maximum at $x = -1$ and at $x = 2$.
Statement 2 : $a = \frac{1}{2}$ and $b = \frac{-1}{4}$
(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.

[Differential Calculus]

19. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies

(a) either on the real axis or on a circle not passing through the origin.
(b) on the imaginary axis.
(c) either on the real axis or on a circle passing through the origin.
(d) on a circle with centre at the origin.

[Complex Numbers]

20. The negation of the statement "If I become a teacher, then I will open a school", is
(a) Neither I will become a teacher nor I will open a school.
(b) I will not become a teacher or I will open a school.
(c) I will become a teacher and I will not open a school.
(d) Either I will not become a teacher or I will not open a school.

[Mathematical Logic]

21. If $g(x) = \int_0^x \cos 4t\, dt$, then $g(x + \pi)$ equals
(a) $g(x) - g(\pi)$ (b) $g(x) \cdot g(\pi)$
(c) $\dfrac{g(x)}{g(\pi)}$ (d) $g(x) + g(\pi)$

[Integral Calculus]

22. A spherical balloon is filled with 4500 π cubic metres of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic metres per minute, then the rate (in metres per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is
(a) 2/9 (b) 9/2
(c) 9/7 (d) 7/9

[Differential Calculus]

23. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has
(a) exactly one real root.
(b) exactly four real roots.
(c) infinite number of real roots.
(d) no real roots.

[Quadratic Equations]

24. Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty is
(a) 2^5 (b) 5^3
(c) 5^2 (d) 3^5

[Set, Relations and Functions]

25. The area bounded between the parabolas $x^2 = \dfrac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is
(a) $\dfrac{20\sqrt{2}}{3}$ (b) $10\sqrt{2}$
(c) $20\sqrt{2}$ (d) $\dfrac{10\sqrt{2}}{3}$

[Integral Calculus]

26. Let P and Q be 3×3 matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to
(a) 0 (b) – 1
(c) – 2 (d) 1

[Matrices and Determinants]

27. Let $x_1, x_2, ..., x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be their variance.
Statement 1 : Variance of $2x_1, 2x_2, ..., 2x_n$ is $4\sigma^2$.
Statement 2 : Arithmetic mean of $2x_1, 2x_2, ..., 2x_n$ is $4\bar{x}$.
(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.

[Statistics]

28. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\dfrac{dp(t)}{dt} = 0.5p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is
(a) $\dfrac{1}{2}\ln 18$ (b) ln 18
(c) 2 ln 18 (d) ln 9

[Differential Equation]

29. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is
(a) – 2 (b) – 1/2
(c) – 1/4 (d) – 4

[Differential Calculus]

30. If 100 times the 100th term of an A.P. with non-zero common difference equals the 50 times its 50th term, then the 150th term of this A.P. is
(a) 150 (b) zero
(c) – 150
(d) 150 times its 50th term

[Sequences and Series]

EXPLANATIONS

1. **(a) :** $\vec{c} = \hat{a} + 2\hat{b},\ \vec{d} = 5\hat{a} - 4\hat{b}$

$\therefore\ \vec{c}\cdot\vec{d} = 0 \Rightarrow\ (\hat{a}+2\hat{b})\cdot(5\hat{a}-4\hat{b}) = 5 - 4\hat{b}\cdot\hat{a} + 10\hat{b}\cdot\hat{a} - 8$

$\Rightarrow\ 6\hat{b}\cdot\hat{a} - 3 = 0\ \Rightarrow\ \hat{b}\cdot\hat{a} = \frac{1}{2}\quad \therefore \theta = \frac{\pi}{3}$

2. **(b) :** $\int \frac{5\tan x}{\tan x - 2}dx = x + a\ln|\sin x - 2\cos x| + k$

Differentiating both sides, we get

$\frac{5\tan x}{\tan x - 2} = 1 + \frac{a(\cos x + 2\sin x)}{\sin x - 2\cos x}$

$\Rightarrow\ \frac{5\sin x}{\sin x - 2\cos x} = \frac{\sin x(1+2a) + \cos x(a-2)}{\sin x - 2\cos x}$

$\Rightarrow\ a = 2$

3. **(a) :** $f(x) = |x-2| + |x-5|$

$$\Rightarrow\ f(x) = \begin{cases} 7-2x, & x < 2 \\ 3, & 2 \le x \le 5 \\ 2x-7, & x > 5 \end{cases}$$

Statement-1 : $f'(4) = 0$. True

Statement-2 : f is continuous in $[2, 5]$, differentiable in $(2, 5)$ and $f(2) = f(5)$. True

But Statement 2 is not a correct explanation for statement 1.

4. **(a) :** $A(1, 1)$; $B(2, 4)$

$P(x_1, y_1)$ divides line segment AB in the ratio 3 : 2

$x_1 = \frac{3(2)+2(1)}{5} = \frac{8}{5}\qquad y_1 = \frac{3(4)+2(1)}{5} = \frac{14}{5}$

$2x + y = k$ passes through $P(x_1, y_1)$

$\therefore\ 2\times\frac{8}{5} + \frac{14}{5} = k\ \Rightarrow\ k = 6$

5. **(d) : Statement 1 :**

$y^2 = 16\sqrt{3}x,\ y = mx + \frac{4\sqrt{3}}{m}$

$\frac{x^2}{2} + \frac{y^2}{4} = 1,\ x = m_1y + \sqrt{4m_1^2 + 2}$

$\Rightarrow\ y = \frac{x}{m_1} - \sqrt{4 + \frac{2}{m_1^2}},\ m = \frac{1}{m_1}$

Now, $\left(\frac{4\sqrt{3}}{m}\right)^2 = \left(-\sqrt{4+\frac{2}{m_1^2}}\right)^2$

$\Rightarrow\ \frac{48}{m^2} = 4 + \frac{2}{m_1^2} = 4 + 2m^2\ \Rightarrow\ \frac{24}{m^2} = 2 + m^2$

$\Rightarrow\ m^4 + 2m^2 - 24 = 0\qquad$ ...(1)

$\Rightarrow\ (m^2+6)(m^2-4) = 0\ \Rightarrow\ m = \pm 2$

Statement 2 : If $y = mx + \frac{4\sqrt{3}}{m}$ is a common tangent to $y^2 = 16\sqrt{3}x$ and ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 - 24 = 0$

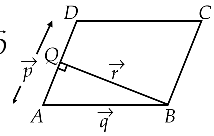
From (1), statement 2 is a correct explanation for statement 1.

6. **(d) :** 3 numbers are chosen from {1, 2, 3, ..., 8} without replacement. Let A be the event that the maximum of chosen numbers is 6.

Let B be the event that the minimum of chosen numbers is 3.

$$P(B/A) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1\cdot1\cdot2}{^8C_3}}{\frac{^5C_2}{^8C_3}} = \frac{2}{10} = \frac{1}{5}$$

7. **(d) :** $\vec{r} = \overrightarrow{BA} + \overrightarrow{AQ}$



$= -\vec{q}$ + projection of $\overrightarrow{BA}$ across $\overrightarrow{AD}$

$= -\vec{q} + \frac{(\vec{p}\cdot\vec{q})\vec{p}}{(\vec{p}\cdot\vec{p})}$

8. **(c) :** Equation of a plane parallel to $x - 2y + 2z - 5 = 0$ and at a unit distance from origin is $x - 2y + 2z + k = 0$

$\Rightarrow\ \frac{|k|}{3} = 1\ \Rightarrow\ |k| = 3$

$\therefore\ x - 2y + 2z - 3 = 0$ or $x - 2y + 2z + 3 = 0$

9. **(d) :** $3\sin P + 4\cos Q = 6$

$4\sin Q + 3\cos P = 1$

$\Rightarrow\ 16 + 9 + 24(\sin(P+Q)) = 37$

$\Rightarrow\ 24(\sin(P+Q)) = 12$

$\Rightarrow\ \sin(P+Q) = \frac{1}{2}\ \Rightarrow\ \sin R = \frac{1}{2}\ \Rightarrow\ R = \frac{5\pi}{6}$ or $\frac{\pi}{6}$

But if $R = \frac{5\pi}{6}$, then $P < \frac{\pi}{6}$ and then $3\sin P < \frac{1}{2}$ and so $3\sin P + 4\cos Q < \frac{1}{2} + 4\ (\neq 6)$. Thus, $R = \frac{\pi}{6}$

10. (c) : $f: R \to R,\ f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$

$$= [x]\cos\left(\pi x - \frac{\pi}{2}\right) = [x]\sin \pi x$$

Let n be an integer.

$\lim_{x \to n^+} f(x) = 0$, $\lim_{x \to n^-} f(x) = 0$

$\therefore\ f(n) = 0$

$\Rightarrow\ f(x)$ is continuous for every real x.

11. (d) : Statement 1 :

$1 + (1 + 2 + 4) + (4 + 6 + 9) + ... + (361 + 380 + 400)$ is 8000

Statement 2 : $\sum_{k=1}^{n} (k^3 - (k-1)^3) = n^3$

Statement 1 : $T_1 = 1$, $T_2 = 7 = 8 - 1$,

$T_3 = 19 = 27 - 8 \Rightarrow T_n = n^3 - (n-1)^3$

$\therefore$ Statement 2 is a correct explanation of statement 1.

12. (c) : Let the equation of the circle is $(x-1)^2 + (y-k)^2 = k^2$

It passes through (2, 3)

$\therefore\ 1 + 9 + k^2 - 6k = k^2$

$\Rightarrow k = \frac{5}{3} \Rightarrow$ diameter $= \frac{10}{3}$

13. (b) : $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix},\ Au_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ Au_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Let $u_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$Au_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\Rightarrow a = 1, 2a + b = 0$

$\Rightarrow\ b = -2, 3a + 2b + c = 0 \Rightarrow c = 1$

Let $u_2 = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$

$Au_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\Rightarrow p = 0, 2p + q = 1 \Rightarrow q = 1,$

$3p + 2q + r = 0 \Rightarrow r = -2$

$$u_1 + u_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

14. (c) : $(\sqrt{3}+1)^{2n} - (\sqrt{3}-1)^{2n}$

$= 2\left[{}^{2n}C_1 \cdot (\sqrt{3})^{2n-1} + {}^{2n}C_3 \cdot (\sqrt{3})^{2n-3} + ...\right]$

irrational number.

15. (b) : Number of ways in which one or more balls can be selected from 10 white, 9 green, 7 black balls is

$= (10 + 1)(9 + 1)(7 + 1) - 1$

$= 880 - 1 = 879$ ways

16. (b) : $(x-1)^2 + y^2 = 1$, $r = 1 \Rightarrow a = 2$

and $x^2 + (y-2)^2 = 4$, $r = 2 \Rightarrow b = 4$

$\Rightarrow\ \frac{x^2}{16} + \frac{y^2}{4} = 1 \Rightarrow x^2 + 4y^2 = 16$

17. (a) : $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = r_1$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1} = r_2$

or $2r_1 + 1 = r_2 + 3$, $3r_1 - 1 = 2r_2 + k$, $4r_1 + 1 = r_2$

$\Rightarrow\ 2r_1 - r_2 = 2$, and $4r_1 - r_2 = -1$

$-2r_1 = 3 \Rightarrow r_1 = \frac{-3}{2}$ and $r_2 = -5$

$\therefore\ -\frac{9}{2} - 1 = -10 + k \Rightarrow k = 10 - \frac{11}{2} = \frac{9}{2}$

18. (a) : $f(x) = \ln|x| + bx^2 + ax, x \neq 0$ has extreme values at $x = -1, x = 2$.

$\Rightarrow\ f'(x) = \frac{1}{x} + 2bx + a$

$f'(-1) = 0$ and $f'(2) = 0$ [Given]

$\Rightarrow\ -1 - 2b + a = 0 \Rightarrow b = -\frac{1}{4}$

and $\frac{1}{2} + 4b + a = 0 \Rightarrow a = \frac{1}{2}$

$f''(x) = -\frac{1}{x^2} + 2b = -\frac{1}{x^2} - \frac{1}{2} = -\left(\frac{1}{x^2} + \frac{1}{2}\right) < 0$

for all $x \in R - \{0\}$

$\Rightarrow\ f$ has a local maximum at $x = -1, x = 2$

$\therefore$ **Statement 1 :** f has local maxima at $x = -1, x = 2$

$\therefore$ **Statement 2 :** $a = \frac{1}{2}, b = -\frac{1}{4}$

19. (c) : $z \neq 1$, $\frac{z^2}{z-1}$ is real.

If z is a real number, then $\frac{z^2}{z-1}$ is real.

Let $z = x + iy$

$\therefore\ \frac{(x^2 - y^2 + 2xiy)((x-1) - iy)}{(x-1)^2 + y^2}$ is real

$\Rightarrow\ -y(x^2 - y^2) + 2xy(x-1) = 0$

$\Rightarrow\ y(x^2 + y^2 - 2x) = 0 \Rightarrow y = 0$ or $x^2 + y^2 - 2x = 0$

$\therefore$ z lies on real axis or on a circle passing through origin.

20. (c) : The given statement is

"If I become a teacher, then I will open a school"

Negation of the given statement is

" I will become a teacher and I will not open a school" $(\because \sim(p \to q) = p \wedge \sim q)$

21. (a, d) : $g(x) = \int_0^x \cos 4t\, dt$

$\Rightarrow g(x) = \left[\frac{\sin 4t}{4}\right]_0^x = \frac{\sin 4x}{4}$

$\Rightarrow g(x+\pi) = \frac{\sin 4(x+\pi)}{4} = \frac{\sin 4x}{4}$

$\Rightarrow g(\pi) = 0 \Rightarrow g(x+\pi) = g(x) + g(\pi)$ or $g(x) - g(\pi)$.

22. (a) : $\frac{dv}{dt} = -72\pi \text{ m}^3/\text{min},\ v_0 = 4500\pi$

$v = \frac{4}{3}\pi r^3 \ \therefore\ \frac{dv}{dt} = \frac{4}{3}\pi \times 3r^2 \times \frac{dr}{dt}$

After 49 min, $v = v_0 + 49 \cdot \frac{dv}{dt} = 4500\pi - 49 \times 72\pi$

$= 4500\pi - 3528\pi = 972\pi$

$\Rightarrow 972\pi = \frac{4}{3}\pi r^3 \Rightarrow r^3 = 243 \times 3 = 3^6 \Rightarrow r = 9$

$\therefore\ -72\pi = 4\pi \times 81 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = -\frac{18}{81} = -\frac{2}{9}$

Thus, radius decreases at a rate of $\frac{2}{9}$ m/min

23. (d) : $e^{\sin x} - e^{-\sin x} - 4 = 0$

$\Rightarrow (e^{\sin x})^2 - 4e^{\sin x} - 1 = 0 \quad \Rightarrow t^2 - 4t - 1 = 0$

$\Rightarrow t = \frac{4 \pm \sqrt{16+4}}{2} = 2 \pm \sqrt{5}$

i.e., $e^{\sin x} = 2 + \sqrt{5}$ or $\underbrace{2 - \sqrt{5}}_{-\text{ve}}$ (neglected)

$\sin x = \ln(2+\sqrt{5}) > 1 \ \therefore$ No real roots.

24. (d) : $X = \{1, 2, 3, 4, 5\};\ Y \subseteq X,\ Z \subseteq X,\ Y \cap Z = \phi$

Number of ways $= 3^5$.

25. (a) : $x^2 = \frac{y}{4}$, $x^2 = 9y$

Area bounded by the parabolas and $y = 2$

$= 2 \times \int_0^2 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right) dy = 5\int_0^2 \sqrt{y}\,dy$

$= 5 \times \frac{(y)^{3/2}}{3/2} = \frac{10}{3} \times 2\sqrt{2} = \frac{20\sqrt{2}}{3}$.

26. (a) : $P^3 = Q^3,\ P^2Q = Q^2P,\ PQ^2 = P^2Q$

$\Rightarrow P(P^2 + Q^2) = (Q^2 + P^2)Q$

$\Rightarrow P(P^2 + Q^2) = (P^2 + Q^2)Q$

$P \neq Q \Rightarrow P^2 + Q^2$ is singular.

Hence, $|P^2 + Q^2| = 0$

27. (b) : $x_1, x_2, x_3, \ldots. x_n$, A.M. $= \bar{x}$, Variance $= \sigma^2$

Statement 2 : A.M. of $2x_1, 2x_2, \ldots., 2x_n$

$= \frac{2(x_1 + x_2 + \ldots.. + x_n)}{n} = 2\bar{x}$

Given A.M. $= 4\bar{x}$ $\therefore$ Statement 2 is false.

28. (c) : $\frac{d(p(t))}{dt} = 0.5p(t) - 450$

$\int_{850}^{p} \frac{2dp}{p-900} = \int_0^t dt \Rightarrow 2\ln\frac{p-900}{-50} = t$

$\Rightarrow p = 900 - 50 \cdot e^{t/2}$

If $p = 0$, then $\frac{900}{50} = e^{t/2} \Rightarrow t = 2\ln 18$

29. (a) : $y = mx + c \Rightarrow 2 = m + c$

Co-ordinates of P & Q : $P(0, c)$, $Q(-c/m, 0)$

$\frac{1}{2} \times |c| \times \left|\frac{c}{m}\right| = A \Rightarrow \frac{c^2}{2m} = A$

$\Rightarrow \frac{(2-m)^2}{2m} = A$

$\Rightarrow \frac{m^2 - 4m + 4}{2m} = A \Rightarrow \frac{m}{2} - 2 + \frac{2}{m} = A$

$\because \frac{dA}{dm} = 0$

$\Rightarrow \frac{1}{2} - \frac{2}{m^2} = 0 \Rightarrow \frac{1}{2} = \frac{2}{m^2} \Rightarrow m^2 = 4 \Rightarrow m = \pm 2$

30. (b) : $100(a + 99d) = 50(a + 49d)$

$\Rightarrow a + 149d = 0$ *i.e.*, $T_{150} = 0$