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# 11. WAVES ON A STRING

## 1. INTRODUCTION

We come across numerous phenomena in nature based on the properties of wave motion. This chapter describes the equations and properties of wave motion. The study of waves on a string forms the basis of understanding the phenomena associated with sound waves and other mechanical and non-mechanical waves. Wave transmits both energy and momentum from one region to other. Mechanical waves require a medium to travel, whereas non-mechanical waves don't. Wave on a string is a mechanical wave but the properties and concepts studied here will be useful in studying non-mechanical waves as well.

## 2. WAVE MOTION

A wave is a disturbance or variation traveling through space and matter. It is the undulating movement of energy from one point to another. The medium through which the wave passes may experience some oscillations, but the particles in the medium do not travel with the wave. The wave equation, which is a differential equation, expresses the properties of motion in waves. Waves come in all shapes and sizes, and accordingly, the mathematical expression of the wave equation also varies.

### 2.1 Types of Waves

Waves can, broadly, be classified into two types:

**(a)** **Mechanical waves**: Waves that require a medium/matter for their propagation are called mechanical waves. These waves are generated due a disturbance in the medium (particles in the matter) and while the wave travels through the medium, the movement of the medium (particles) is minimal. Therefore mechanical waves propagate only energy, not matter. Both the wave and the energy propagate in the same direction. All waves (mechanical or electromagnetic) have a certain energy. Only a medium possessing elasticity and inertia can propagate a mechanical wave.

**(b)** **Non-mechanical waves/Electromagnetic waves**: Waves that do not require a medium/matter for their propagation are called electromagnetic waves. These waves are formed by the coupling of electric and magnetic fields due to acceleration of electric charge and can travel through vacuum. Depending on the wavelength of the electromagnetic wave, they are classified as radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays, and gamma rays.

## 3. WAVE PULSE ON A STRING

A wave pulse is a single, sudden, and short-duration disturbance that moves from point A to point B through a medium, e.g., a string. We know that waves originate when a disturbance at the source point moves through one particle to its adjacent particles from one end of the medium to the other. Now, when a disturbance-producing source active for a short time, a wave pulse passes through the medium. Conversely, when the source remains active for an extended time, creating a series of motions, it results in a wave train or a wave packet. Thus, a wave train is a group of waves traveling in the same direction.

For example, if the person in figure decides to move his hand up and down 10 times and then stop, a wave train consisting of 10 loops will move on the string.

## 4. EQUATION OF A TRAVELING WAVE

In the figure, let us assume that the man starts moving his hand at t = 0 and finished his job at t = Δt. The vertical displacement of the left end of the string, denoted as y, is a function of time. It is zero for $t<0$ and $t>\Delta t$. This function can be represented by f (t). Let us take the left end of the string as the source of the wave and take the X axis along the string toward right. The function f (t) represents the displacement y of the particle at x=0 as a function of time: y(x=0, t) =f (t).

The disturbance on the string travels towards right at a constant speed. Thus, the displacement produced at the left end at time t reaches the point x at time $t+\left(\frac{x}{v}\right)$ Similarly, the displacement of the particle at point x at time t was generated at the left end at the time t-x/. But the displacement of the left end at time t-x/v is f (t-x/v). Hence, y (x, t) = y(x=0, t-x/v) = f(t-x/v).

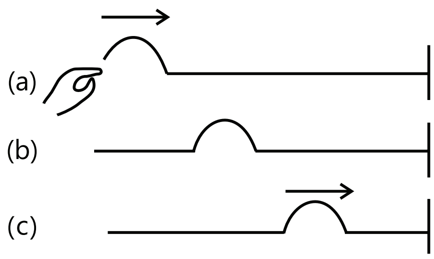


**Figure 11.1**

The displacement of the particle at x at time t, i.e., y(x, t) is generally abbreviated as y and the wave equation is written as y = f (t-x/v). ... (i)

Equation (i) represents a wave traveling in the positive direction x at a constant speed. Such a wave is called a traveling wave or a progressive wave. The function f is dependent on the movement of the source, and therefore, arbitrary. The time t and the position x must be represented in the wave equation in the form t-x/ only. For example,

$y = A\sin\frac{(t-x/v)}{T}$ , and $y=Ae^{-\frac{(t-x/v)}{T}}$ are valid wave equations.

Both these equations represent the movement of the wave in the positive direction x at constant speed v.

In contrast, the equation $y = A\sin\frac{(x^2-v^2t^2)}{L^2}$ does not represent the movement of the wave in the direction x at a constant speed . If a wave travels in the negative direction at a speed , its general equation may be written as

y = f(t + x/v) ... (ii)

Equation (i) can also be written as $y = f\left(\frac{vt-x}{v}\right)$ or $y=g(x-vt)$, ....(iii)

where g is some other function having the following meaning: Let us assume that t = 0 in the wave equation. Then, we get the displacement of various particle at t = 0, i.e., $y=(x,t=0)=g(x)$. Thus, the function g(x) represents the shape of the string at t = 0. Assuming that the displacement of the different particles at t = 0 is represented by the function g(x), the displacement of the particle at x at time t will be y = g(x - vt). Similarly, if the wave is traveling along the negative direction x and the displacement of a different particle at t = 0 is g(x), the displacement of the particle at x at time t will be y = g(x + t) ...(iv)

**Illustration 1:** The wave equation of a wave propagating on a stretched string along its length taken as the positive x axis is given as $y=y_0\exp\left(-\left(\frac{t}{T}-\frac{x}{\lambda}\right)^2\right)$ where $y_0$ = 4 mm, T = 1.0 s and λ = 4 cm.

(a) Find the velocity of the wave.

(b) Find the function f(t) giving the displacement of the particle at x = 0.

(c) Find the function g(x) giving the shape of the string at t = 0.

(d) Plot the shape g(x) of the sting at t = 0.

(e) Plot the shape of the string at t = 5s. **(JEE MAIN)**

**Sol:** The wave moves having natural frequency of $\nu$ and wavelength $\lambda$ has velocity $V = \nu\lambda$.

As the frequency is $\nu = \frac{1}{T}$ the velocity of the wave is then $V = \frac{\lambda}{T}$.

(a) The wave equation can be written as $y = y_0 e^{-\frac{1}{T^2}\left(t-\frac{x}{\lambda/T}\right)^2}$

On comparison with the general equation y = f (t – x/), we can infer that, $\upsilon = \frac{\lambda}{T} = \frac{4\,cm}{1.0\,s} = 4\,cms^{-1}$

(b) Putting x =0 in the given equation $f(t) = y_0\, e^{-(t/T)^2}$ ... (i)

(c) Putting t = 0 in the given equation $g(t) = y_0\, e^{-(x/\lambda)^2}$ ... (ii)

(d)


(e)

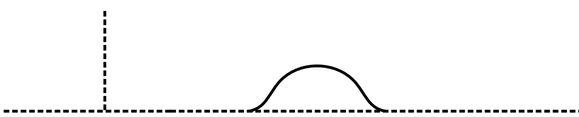



**Figure 11.2**

## 4.1 Sine Wave Traveling on a String

Consider the scenario where the person in the Fig. 11.3 keeps moving his hand up and down continuously. As energy is being constantly supplied by the person, the wave generated at the source keeps oscillating the any part of the string through which it passes. Thus energy passes from the left (the source) to the right continuously till the person gets tired. The nature of the vibration of any particle in the string is similar to that of the left end (the source), the only difference is that there is an interval of x/ between two motions.

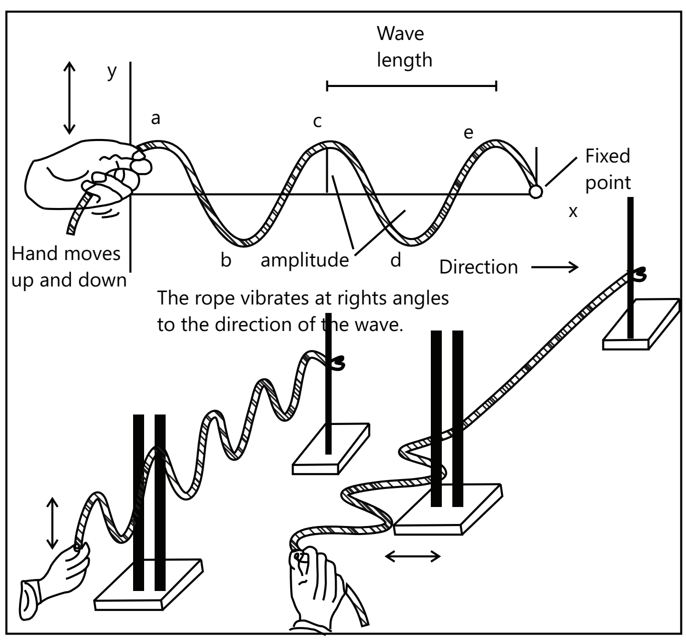



**Figure 11.3**

When the person in the Fig 11.3 oscillates the left end x = 0 in a simple harmonic motion, the equation of motion of this end may be written as f(t) = Asin $\omega$ t ... (i)

where A is the amplitude and $\omega$ is the angular frequency. The time period of oscillation is given by $T = 2\pi/\omega$ and the frequency of oscillation is

$\nu = 1/T = \omega/2\pi$. The wave produced by such an oscillation source is called a sine wave or sinusoidal wave.

The displacement of the particle at x at time t will be

$$y = f(t - x/\upsilon) \quad \text{or} \quad y = A\sin w(t - x/\upsilon) \qquad \text{... (ii)}$$

The velocity of the particle at x at time t is given by $\frac{\partial y}{\partial t} = A\omega\cos(t - x/\upsilon)$ ... (iii)

## PLANCESS CONCEPTS

- While differentiating with respect to t, we should treat x as constant – it is the same particle whose displacement should be considered as a function of time. Therefore, the symbol $\frac{\partial}{\partial t}$ is used in place of $\frac{d}{dt}$.
- In the event that the waves travel along negative x direction, the direction of $V_p$ will change.

  Particle velocity is the same as wave velocity. The two are totally different. While the wave moves on the string at a constant velocity along the x axis, the particle moves up and down with velocity $\frac{\partial}{\partial t}y$, which changes with x and t.

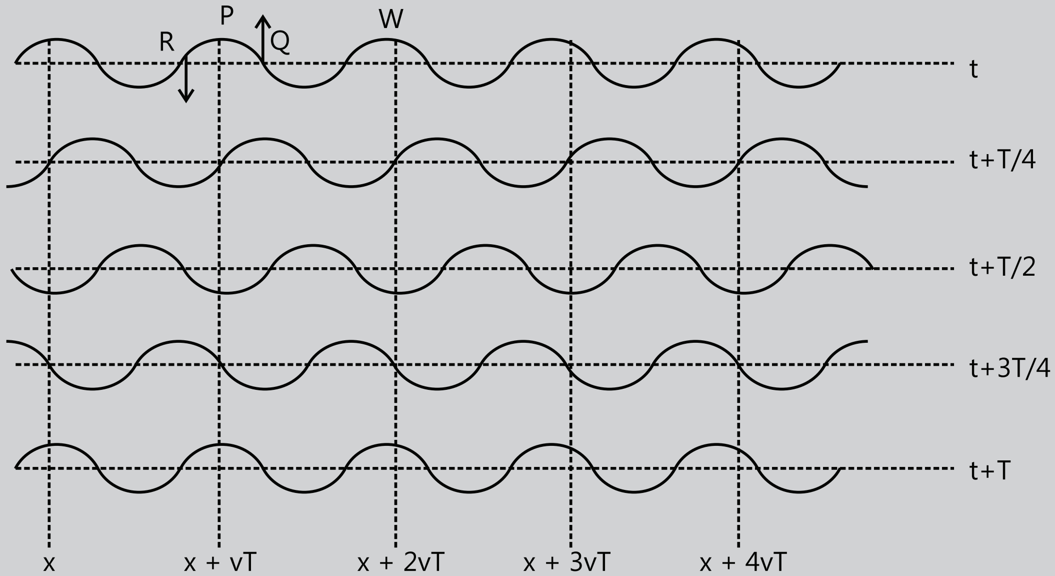



**Figure 11.4**

**Above figure shows change in shape of string with time**

**Vaibhav Krishan (JEE 2009 AIR 22)**

### 4.1.1 Some Important Terms

**(a) Amplitude:** In a wave, the crest represents highest point the wave rises to and equilibrium represents the default position from which a wave arises. Therefore, the distance between the crest and the equilibrium point in a single wave cycle is referred to as the equilibrium.

**(b) Wavelength:** The distance between any two points with the same phase, such as between crests or troughs is referred to as the wavelength $\lambda$. It is generally measured in meters.

**(c) Wave Number:** Wave number is a measurement of a certain number of wavelengths for some given distance. In a sense, the wave number is like a spatial analogue of frequency. Typically, wave number is taken to be $2\pi$ times the number of wavelengths per unit of distance, which is the number of radians for each unit of distance as well. $k = \frac{2\pi}{\lambda}$

**(d) Time Period:** A period T is the time needed for one complete cycle of vibration of a wave to pass a given point.

**(e) Frequency:** Frequency describes the number of waves that pass a fixed place in a given amount of time and is typically measured in hertz. These are related by $f = \frac{1}{T}$

**(f) Angular Frequency:** The angular frequency $\omega$ gives the frequency with which phase changes. It is expressed in radians per second. It is related to the frequency or period by $\omega = 2\pi f = \frac{2\pi}{T}$ ....(i)

**Illustration 2:** Consider the wave y = (5 mm) sin [(1 cm $^{-1}$) x – (60s $^{-1}$) t]. Find (a) the amplitude, (b) the wave number, (c) the wavelength, (d) the frequency, (e) the time period and (f) the wave velocity. **(JEE MAIN)**

**Sol:** Comparing the given equation with $y = A\sin(kx - \omega t)$ we get the values of wave number k, amplitude A and angular frequency. The frequency $\omega = 2\pi\upsilon = 2\pi/T$. The velocity of wave is $v = \nu\lambda$ and the wave number of wave is $K = \frac{2\pi}{\lambda}$.

On comparing the given equation with standard equation of a traveling wave, we find

(a) Amplitude A = 5mm, (b) wave number k = 1 cm$^{-1}$, (c) wavelength $\lambda = \frac{2\pi}{k} = 2\pi\,\text{cm}$

(d) Frequency $\nu = \frac{\omega}{2\pi} = \frac{60}{2\pi}\text{Hz} = \frac{30}{\pi}\text{Hz}$ (e) Time period $T = \frac{1}{\nu} = \frac{\pi}{30}\text{s}$

(f) Wave velocity $v = \nu\lambda = 60\ \text{cms}^{-1}$

## 4.2 Velocity of Waves on a String

The wave speed depends on the properties of the medium. For a string, the speed of a transverse wave traveling along a vibrating string (v) is directly proportional to the square root of the tension of the string (T) over the linear mass density ($\mu$):

$v = \sqrt{\frac{T}{\mu}}$, where the linear density $\mu$ is the mass per unit length of the string ....(i)

## 4.3 Phase Difference

The amount by which two cyclical motions of the same frequency, are out of step with each other. It can be measured in degrees from 0° to 360°, radians from 0 to $2\pi$, or seconds of time.. If two oscillators have the same frequency and no phase difference, they are said to be in phase. Conversely, if they have the same frequency and different phases, then they have a phase difference and they are said to be out of phase with each other. If the phase difference is 180° ($\pi$ radians), then the two oscillators are said to be in anti-phase.

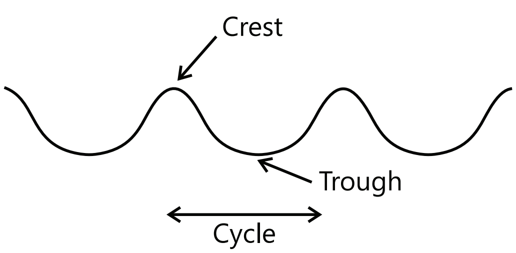



Figure 11.5

### 4.4 Crest and Trough

In a wave, the crest represents highest point the wave rises within a cycle. A trough is the opposite of a crest, hence the minimum or lowest point in a cycle.

## 5. ALTERNATIVE FORMS OF WAVE EQUATION

As seen earlier, the wave equation of a wave traveling in x direction is $y = A\sin\omega(t - x/\upsilon)$,

This can also be written in several other forms such as $y = A\sin(\omega t - kx)$, ... (i)

$$y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad \text{... (ii)}$$

$$y = A\sin[k\ (vt - x)] \quad \text{... (iii)}$$

Please bear in mind our choice of t = 0 in writing equation (v) from which the wave equation has been derived. Also, the point at which the left end x = 0 crosses its mean position y = 0 and goes up has been chosen as the origin of time. For a general choice of the origin of time, a phase constant will have to be added to give the equation

$$y = A\ \sin\ [\omega\ (t - x/v) + \phi] \quad \text{... (iv)}$$

The constant $\varphi$ will be $\pi/2$ If we choose t = 0 at an instant when the left end reaches its extreme position y=A, then the constant $\varphi$ will be $\pi/2$. The equation will then be

$$y = A\cos\omega(t - x/v), \quad \text{... (v)}$$

If on the other hand, t = 0 is taken at the point when the left end is crossing the mean position from an upward to downward direction, $\varphi$ will be $\pi$ and the equation will be

$$y = A\sin\omega\left(\frac{x}{v} - t\right) \text{ or } y = A\ \sin\ (kx - \omega t) \quad \text{... (vi)}$$

**PLANCESS CONCEPTS**

Both $\sin(kx - \omega t)$ and $\sin(\omega t - kx)$ differ just by a phase of "$\pi$". If a particle at t = 0, x = 0 in its mean position is moving upwards (in first wave), then the same particle would be in mean position and the particle would be moving down!

**B Rajiv Reddy (JEE 2012, AIR 11)**

**Illustration 3:** Fig 11.6 shows a string of linear mass density 1.0 Kg $m^{-1}$ and a length of 50 cm. Find the time taken by a wave pulse to travel through the length of the string. Take g = 10 $ms^{-2}$. **(JEE MAIN)**

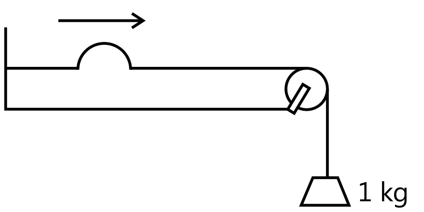


**Figure 11.6**

**Sol:** The wave velocity on stretched string under tension F = mg is given by

$v = \sqrt{\frac{F}{\mu}}$ where $\mu$ is mass per unit length of the string.

The tension in the string is F = mg = 10N. Given that the mass per unit length is $\mu$ = 1.0 Kg $m^{-1}$, the wave velocity is, $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{10N}{0.1kgm^{-1}}} = 10ms^{-1}$.

Therefore, to travel through 50 cm, the wave pulse will take 0.05 s.

**Illustration 4:** A rubber tube that is 12.0 m long and that has a total mass of 0.9 kg is fastened to fixed base. At the other end of the tube, a cord is attached that passes over a pulley and supports an object with a mass of 5.0 kg. If the tube is struck at one end, find the time required for the transverse pulse to reach the other end. ($g = 9.8\ m/s^2$) **(JEE MAIN)**

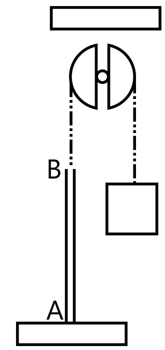



**Figure 11.7**

**Sol:** For the string under the tension T =mg where m is mass of the block. When the rod is struck at lower rod, the wave thus originated travels at speed $v = \sqrt{\frac{T}{\mu}}$ where μ is the mass per unit length of the string.

Tension in the rubber tube AB, T = mg or T = (5.0) (9.8) = 49 N

Mass per unit length of rubber tube, $\mu = \frac{0.9}{12} = 0.075\,kg/m$

∴ Speed of wave on the tube, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{49}{0.075}} = 25.56\,m/s$

∴ The required time is $t = \frac{AB}{v} = \frac{12}{25.56} = 0.47\,s$.

**Illustration 5:** Prove that the equation y = a sin (ωt – kx) satisfies the wave equation $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$ and find speed of the wave and the direction in which it is traveling. **(JEE ADVANCED)**

**Sol:** To prove the above relation, we need to take the ratio of second order time derivative of wave equation and second order displacement derivative of wave equation.

$\frac{\partial^2 y}{\partial t^2} = -\omega^2 a\sin(\omega t - kx)$ and $\frac{\partial^2 y}{\partial x^2} = k^2 a\sin(\omega t - kx)$. We can write these two equation as,

$\frac{\partial^2 y}{\partial t^2} = \frac{\omega^2}{k^2} \cdot \frac{\partial^2 y}{\partial x^2}$. Comparing this with, $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$

We get, wave speed $v = \frac{\omega}{k}$

The negative sign between ωt and kx implies that wave is traveling along positive direction.

**Illustration 6:** The Fig 11.8 shows a snapshot of a sinusoidal traveling wave which was taken at t = 0.3 s. The wavelength is 7.5 cm and the amplitude is 2 cm. Assuming the crest was at x = 0 at t = 0, write the equation of traveling wave. **(JEE ADVANCED)**

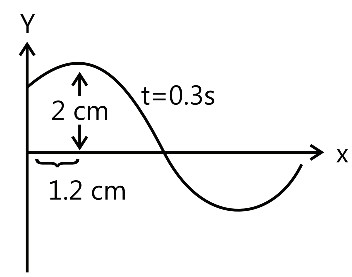



**Figure 11.8**

**Sol:** The equation of travelling wave is $y = A\sin(kx - \omega t)$. The wave number is given by $k = \frac{2\pi}{\lambda}$ and angular frequency of wave is $\omega = vk$.

Given, A = 2 cm, λ = 7.5 cm ∴ $k = \frac{2\pi}{\lambda} = 0.84\,cm^{-1}$

The wave has traveled a distance of 1.2 cm in 0.3 s. Hence, speed of the wave $v = \frac{1.2}{0.3} = 4\,cm/s$

∴ Angular frequency ω = (v) (k) = 3.36 rad/s

Since the wave is traveling along the positive direction x, and crest (maximum displacement) is at x = 0 at t = 0, we can write the wave equation as

Y(x, t) = A cos (kx - ωt) or y(x, t) = A cos (ωt - kx) as cos(-θ) = cos θ

Therefore, the equation of the traveling wave is

y(x, t) = (2 cm) cos [(0.84 cm-1)x – (3.36 rad/s)t]

**Illustration 7:** The mass and length of a rope hanging from the ceiling are 0.1 kg and 2.45 m, respectively. The rope has a uniform width.

(a) Determine the speed of transverse wave in the rope at a point 0.5 m away from the lower end.

(b) Also, calculate the time taken by the wave to travel the full length of the rope.

**(JEE ADVANCED)**

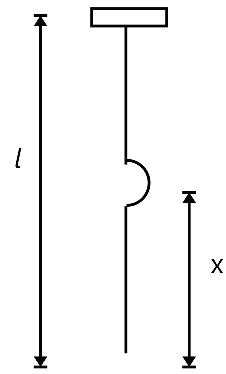



**Figure 11.9**

**Sol:** As the rope hangs under it's own weight, the tension in string at a distance x from hanging end is $T = mg\frac{x}{\ell}$ where $\ell$ is the length of the string and m is mass of the string.

When a transverse waves are generated to travel along length of rope, they travel with speed $v = \sqrt{\frac{T}{\mu}}$ where μ is mass per unit length of string.

The tension in the string will be different at different points owing to the mass of the string and the fact that it is suspended vertically from a ceiling.. The tension at a point which is at a distance x free end will be due to the weight of the string below it. Given that m is the mass of string of length l, the mass of length x of the string will be $\left(\frac{x}{\ell}\right)m$

$$\mu = \frac{0.1}{2.45} = 0.04\text{kg/m; Tension} = mg\left(\frac{x}{\ell}\right) = mg\left(\frac{0.5}{2.45}\right) = 0.20\text{N} \Rightarrow v = \sqrt{\frac{T}{\mu}} = 2.236\text{m/s}$$

(b) From the above equation, we see that velocity of the wave is different at different points. Therefore, if at point x the wave travels a distance dx in time dt, then $dt = \frac{dx}{v} = \frac{dx}{\sqrt{gx}}$

$$\therefore \quad \int_0^1 dt = \int_0^1 \frac{dx}{\sqrt{gx}}; \quad t = 2\sqrt{\frac{\ell}{g}} = 2\sqrt{\frac{2.45}{9.8}} = 1.0\text{s}$$

**Illustration 8:** The mass and length of a rope hanging vertically from a rigid support are 12 m and 6 kg, respectively. A stone of mass 2 kg is attached to the free end of the rope. The rope has a uniform width. If a transverse pulse of wavelength 0.06 m is produced at the lower end of the rope, what will be the wavelength of the pulse when it reaches the top of the rope?

**(JEE ADVANCED)**

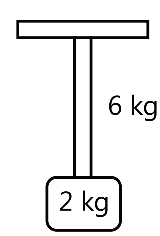



**Figure 11.10**

**Sol:** The wave velocity will be $V = \nu\lambda = \sqrt{\frac{F}{\mu}}$ where F is the tension in rope at a point and μ is mass per unit length of the string. As F is varying along the length of the rope so the velocity will vary along the length of the rope. As source frequency is constant λ will vary.

Owing to the fact that a stone is attached to the lower end of the rope, the tension in the rope will be different at the different points. The tension at the lower end will be 20 N and at the upper end it will be 80 N.

We have, $V = \nu\lambda$ or, $\sqrt{\frac{F}{\mu}} = \nu\lambda$ or, $\frac{\sqrt{F}}{\lambda} = \nu\sqrt{\mu}.$

The frequency of the wave pulse is affected only by the frequency of the source, and hence the wave pulse frequency will be the same across the length of the rope as it depends only on the frequency of the source. As the rope has a uniform width, the mass per unit length will also be consistent across the length of the rope.

Thus, by (i) $\frac{\sqrt{F}}{\lambda}$ is constant.

Hence, $\frac{\sqrt{(2kg)g}}{0.06m} = \frac{\sqrt{(8kg)g}}{\lambda_1}$ where $\lambda_1$ is the wavelength at the top of the rope. This gives $\lambda_1$ = 0.12m

## 6. POWER TRANSMITTED ALONG THE STRING BY A SINE WAVE

The direction of a traveling wave on a string and the direction of the energy transmitted by it is the same. Consider a sine wave traveling along a stretched string in the direction x. The equation for the displacement in the direction y is $y = A \sin \omega (t - x/v)$ ... (i)

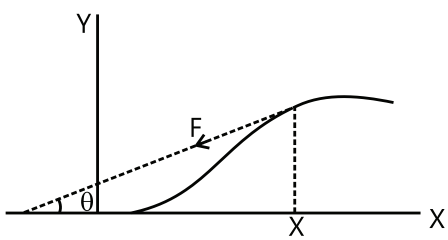


**Figure 11.11**

The Fig 11.11 the portion of the string to the left of the point x exerts a force F on the portion of the string to the right of the point x at time t. The direction of this force is along the tangent to the string at position x. The component of the force along the axis y is $F_y = -F\sin\theta \approx -F\tan\theta = -F\frac{\partial y}{\partial x}$

The power delivered by the force F to the string on the right of position x is, therefore, $P = \left(-F\frac{\partial y}{\partial x}\right)\frac{\partial y}{\partial t}$

By (i), it is $-F\left[\left(-\frac{\omega}{v}\right)A\cos\omega(t-x/v)\right][\omega A\cos\omega(t-x/v)] = \frac{\omega^2 A^2 F}{v}\cos^2\omega(t-x/v)$

This is the rate at which energy is being transferred from left portion of the string to the right portion across the point at x. The $\cos^2$ term oscillates between 0 and 1 during cycle and its average value is 1/2, therefore, the average power transmitted across any point is

$$P_{av} = \frac{1}{2}\frac{\omega^2 A^2 F}{v} = 2\pi^2 \mu c A^2 v^2 \quad \text{... (ii)}$$

The power passing along the length of the string is proportional to the square of the amplitude and square of the frequency of the wave.

**Illustration 9:** For a sine wave with an amplitude of 2.0 mm, the average power transmitted through a given point on a string is 0.20 W. What will be the power that will be transmitted through this point were the amplitude to be increased to 3.0 mm?. **(JEE ADVANCED)**

**Sol:** The power transmitted by the sine wave is $P \propto A^2$ where A is the amplitude of the wave.

Other things being equal, the power transmitted is proportional to the square of the amplitude.

Thus, $\frac{P_2}{P_1} = \frac{A_2^2}{A_1^2}$ or $\frac{P_2}{0.20W} = \frac{9}{4} = 2.25$ $P_2 = 2.25 \times 0.20\ W = 0.45\ W$

# 7. ENERGY IN WAVE MOTION

Every wave motion involves transfer of energy and momentum.. Waves are produced when force is applied to a portion of the wave medium. When force is applied to a portion of the wave medium, the disturbance thus caused in that portion of the medium generates a wave that exerts a force on the adjoining portions. This, in turn, disturbs those portions, thereby propagating the wave further to the adjacent portions. In this way, a wave can transport energy from one region of space to other.

The energy in wave motion is manifested in three forms, namely, energy density (u), power (P), and intensity (I). We shall discuss them one by one.

## 7.1 Energy Density ($\mu$)

The energy density of a progressive wave is the total mechanical energy (kinetic + potential) per unit volume of the medium through which the wave is propagated. This can be illustrated through an example. Let us imagine a string attached to a tuning fork. When the tuning fork is struck, the vibration transmits energy to the segment of the string attached to it, or in other words, as the vibrating fork moves through its equilibrium position, it stretches a segment of the string, increasing its potential energy, while also imparting transverse speed to the segment, increasing its kinetic energy. Thus, as the wave moves along the string, energy is transferred to the other segments of the string.

## 7.2 Kinetic Energy Per Unit Volume

The kinetic energy of a unit volume of the string can be calculated from the wave function. Mass of unit volume is the density $\rho$. Its displacement from equilibrium is the wave function

$y = A \sin (k x - \omega t)$.

Its speed is $\frac{dy}{dt}$, where x is considered to be fixed. The kinetic energy of unit volume $\Delta K$ is then

$\Delta K = \frac{1}{2}(\Delta m)v_y^2 = \frac{1}{2}\rho\left(\frac{dy}{dt}\right)^2$; Using $y = A \sin (k x - w t)$, we obtain $\frac{dy}{dt} = -wA\cos(kx - wt)$

So the kinetic energy per unit volume is $\Delta K = \frac{1}{2}\rho^2 \omega^2 A^2 \cos^2 (kx - \omega t)$ ... (i)

## 7.3 Potential Energy Per Unit Volume

The work done by the vibrating fork by stretching the segment of the string is the potential energy of the segment. It depends on the slope $\frac{dy}{dx}$. The potential energy per unit volume of the string is related to the slope and tension T and is given by (for small slopes)

$$\Delta U = \frac{1}{2}\rho v^2\left(\frac{dy}{dx}\right)^2 \quad \text{... (ii)}$$

where v = wave speed = $\frac{\omega}{k}$

Using $\frac{dy}{dx} = k A \cos (k x - \omega t)$, we obtain for the potential energy $\Delta U = \frac{1}{2}\rho\omega^2 A^2 \cos^2 (kx - \omega t)$ ... (iii)

which is the same as the kinetic energy. The total energy per unit volume is $\Delta E = \Delta K + \Delta U = \rho\omega^2 A^2 \cos^2 (kx - \omega t)$ ... (iv)

The total energy per unit volume ($\Delta E$) varies with time. As the average value of $\cos^2 (kx - \omega t)$ at any point is $\frac{1}{2}$, the average energy per unit volume (also called the energy density $\mu$) is $\mu = \frac{1}{2}\rho\omega^2 A^2$ ... (v)

## PLANCESS CONCEPTS

In the case of a spring with mass $\rho$ attached to it and oscillating in a simple harmonic wave, the energy density is the same as in equation (v). However, its potential energy is maximum when the displacement is maximum. In the case of a string segment, it is the slope of the spring that determines the potential energy and it is maximum when the slope is maximum, which is at the equilibrium position of the segment – the same position for which the kinetic energy is maximum.

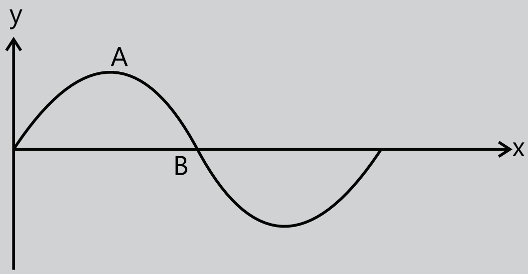



**Figure 11.12**

In the Fig 11.12, the kinetic energy and potential energy both are zero at point A, whereas at point B, both the kinetic energy and potential energy are maximum.

**Aman Gour (JEE 2012, AIR 230)**

## 7.4 Intensity (I)

The intensity of a wave is defined as the flow of energy per unit area of a cross-section of the string in unit time.

Thus, $I = \frac{\text{power}}{\text{area of cross} - \text{section}} = \frac{P}{s}$ or $I = \frac{1}{2}\rho\omega^2 A^2 V$

This is, however, the average intensity transmitted through the string. The instantaneous intensity $\rho\omega^2 A^2 v\sin^2(kx-\omega t)$ or $\rho\omega^2 A^2 v\cos^2(kx-\omega t)$ depends on x and t.

## PLANCESS CONCEPTS

- The relation for power and intensity discussed above are for transverse waves on a string. However, they hold good for other waves also.
- Intensity due to a point source: Assuming that waves are propagated uniformly in all directions, the energy at a distance r from a point source is distributed uniformly on a spherical surface of radius r and area S= $4\pi r^2$. If P is the power per unit area that is incident perpendicular to the direction of propagation, then intensity $I = \frac{P}{4\pi r^2}$ or $I \propto \frac{P}{r^2}$

  Since amplitude $A \propto \sqrt{I}$, a spherical harmonic wave emanating from a point source can therefore, be written as $y(r,t) = \frac{A}{r}\sin(kr - \omega t)$

**T P Varun (JEE 2012, AIR 64)**

**Illustration 10:** An oscillator attached to stretched string with a diameter of 4 mm transmits transverse waves through the length of the string. The amplitude and frequency of the oscillation are $10^{-4}$ m and 10 Hz, respectively. Tension in the string is 100 N, mass density of wire is $4.2 \times 10^3$ kg/m$^3$.

Find: (a) The wave equation along the string

(b) The energy per unit volume of the wave

(c) The average energy flow per unit time across any section of the string

(d) The power required to drive the oscillator. **(JEE ADVANCED)**

**Sol:** The wave equation of string is $y = A\sin(kx - \omega t)$ where the wave number $k = \frac{2\pi}{\lambda}$, the angular frequency $\omega = 2\pi\nu = \frac{2\pi}{T}$ . $\lambda$ is the wavelength and T is the time period of wave. As the string is under tension of 100 N, the wave velocity on string is given by $V = \nu\lambda = \sqrt{\frac{T}{\mu}}$. Use the formula for wave energy in the string .

(a) Speed of transverse wave on the string is, $V = \sqrt{\frac{T}{\rho S}}$ $(\because \mu = \rho S)$

Substituting the values, we have

$$v = \sqrt{\frac{100}{(4.2\times10^3)\left(\frac{\pi}{4}\right)(4.0\times10^{-3})^2}} = 43.53\text{ m/s}; \omega = 2\pi f = 20\pi\frac{\text{rad}}{\text{s}} = 62.83\frac{\text{rad}}{\text{s}}$$

Wave number is $k = \frac{\omega}{V} = 1.44\text{ m}^{-1}$

$\therefore$ The wave equation is $y(x,t) = A\sin(kx - \omega t) = (10^{-4}\text{m})\sin\left[(1.44\text{m}^{-1})x - \left(62.83\frac{\text{rad}}{\text{s}}\right)t\right]$

(b) Energy per unit volume of the string, u = energy density = $\frac{1}{2}\rho\,\omega^2\,A^2$

Substituting the values, we have $u = \left(\frac{1}{2}\right)(4.2\times10^3)(62.83)^2(10^{-4})^2 = 8.29\times10^{-2}\text{ J/m}^3$

(c) Average energy flow per unit time, $P = \text{ power } = \left(\frac{1}{2}\rho\omega^2A^2\right)(sv) = (u)(sv)$

Substituting the values, we have $P = (8.29\times10^{-2})\left(\frac{\pi}{4}\right)(4.0\times10^{-3})^2(43.53) = 4.53\times10^{-5}\text{ J/s}$

(d) Power required to drive the oscillator is obviously $4.53 \times 10^{-5}$ W.

## 8. INTERFERENCE

Interference is a phenomenon that occurs when two waves superimpose while traveling in the same medium. This results in the formation of a wave of greater or lower amplitude. Interference happens with waves that emerge from the same source or have the similar frequencies.

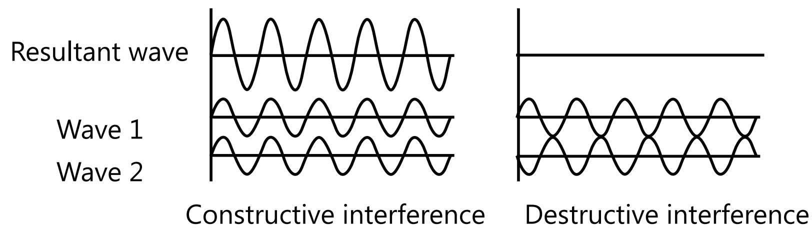



**Figure 11.13**

### 8.1 Principle of Superposition

The principle of superposition of waves states that when two or more waves of same type come together at a single point in space, the total displacement at that point is equal to the sum of the displacements of the individual waves. Constructive interference is the meeting of two waves of equal frequency and phase, i.e., if the crest of a wave meets a crest of another wave of the same frequency at the same point, then the total displacement is the sum of the individual displacements. Destructive interference is the meeting of two waves of equal frequency and opposite phase, i.e., if the crest of one wave meets a trough of another wave then the total displacement is equal to the difference in the individual displacements.

In constructive interference, the phase difference between the waves is a multiple of $2\pi$, whereas in a destructive interference the difference is an odd multiple of $\pi$. If the phase difference is between these two extremes, then the total displacement of the summed waves lies between the minimum and maximum values. If the first wave alone were traveling, the displacement of particles may be written as $y_1=f_1(t-x/v)$. If the second wave alone were traveling, the displacement may be written as $y_2=f_2(t+x/v)$

If both the waves are traveling on the string, the displacement of its different particles will be given by

$y = y1 + y2 = f_1(t-x/v) + f_2(t+x/v)$.

If the two individual displacements are in opposite directions, the magnitude of the resulting displacement may be smaller than the magnitudes of the individual displacements. In a nutshell, when two or more waves pass through a point at the same time, the disturbance at the point is the sum of the disturbances each wave would produce in absence of the other wave(s).

## 8.2 Interference of Wave Going in Same Direction

Let us assume that two identical sources send sinusoidal waves of same angular frequency $\omega$ in the positive direction x. It is also assumed that the wave velocity and consequentially, the wave number k is same for the two waves. One source may send the wave a little later than the other or the two sources may be located at different points. Here, the phases of the two waves at the point of interference will be different. If we assume the amplitudes of the two waves to be $A_1$ and $A_2$ and the phase difference of the two waves to be an angle $\delta$, their equations may be written as

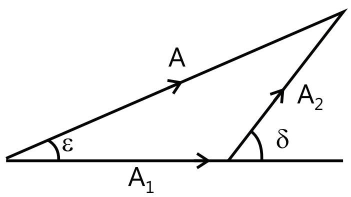



**Figure 11.14**

$y_1 = A_1 \sin(kx-\omega t)$ And $y_2 = A_2\sin(kx-\omega t+\delta)$

According to the principle of superposition, the resultant wave is represented by

$y = y_1 + y_2 = A_1\sin(kx-\omega t) + A_2\sin(kx-\omega t+\delta)$

$= A_1\sin(kx-\omega t) + A_2\sin(kx-\omega t)\cos\delta + A_2\cos(kx-\omega t)\sin\delta$

$= \sin(kx-\omega t)(A_1 + A_2\cos\delta) + \cos(kx-\omega t)(A_2\sin\delta)$

We can evaluate it using the method to combine two simple harmonic motions. If we write

$A_1 + A_2\cos\delta = A\cos\varepsilon$ ... (i)

And $A_2\sin\delta = A\sin\varepsilon$ ... (ii)

We get, $y = A[\sin(kx-\omega t)\cos\varepsilon + \cos(kx-\omega t)\sin\varepsilon] = A\sin(kx-\omega t+\varepsilon)$

Thus, the resultant is indeed a sine wave of amplitude A with a phase difference $\varepsilon$ with the first wave. By (i) and (ii),

$A^2 = A^2\cos^2\varepsilon + A^2\sin^2\varepsilon = (A_2 + A_2\cos\delta)^2 + (A_2\sin\delta)^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\delta$

Or $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\delta}$ ... (iii)

Also $\tan\varepsilon = \dfrac{A\sin\varepsilon}{A\cos\varepsilon} = \dfrac{A_2\sin\delta}{A_1 + A_2\cos\delta}$ ... (iv)

These relations may be remembered by using the following geometrical model can be used to remember these relations: draw a vector of length $A_1$ to represent $y_1 = A_1\sin(kx - \omega\tau)$ and another vector of length $A_2$ at an angle $\delta$ with the first one to represent $y_2 = A_2\sin(kx - \omega\tau + \delta)$. The resultant vector then represents the resultant wave $y = A\sin(kx - \omega\tau + \varepsilon)$. The given Fig 11.14 shows the construction.

**Illustration 11:** The equations of two waves passing simultaneously through a string are given by $y_1 = A_1\sin k(x - vt)$ and $y_2 = A_2\sin k(x - vt + x_0)$, where the wave number $k = 6.28\ cm^{-1}$ and $x_0 = 1.50$ cm. The amplitudes for $A_1$ and $A_2$ are 5.0 mm and 4.0 mm, respectively. Find the phase difference between the waves and the amplitude of the resulting wave. **(JEE ADVANCED)**

**Sol:** As there are two waves passing through the string simultaneously, the phase difference between the two waves will be $\delta = kx_0$. And the resulting amplitude of the waves will be $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\delta}$ .

The phase of the first wave is k (x -vt) and of the second is k (x- vt + $x_0$).

The phase difference is, therefore, $\delta = kx_0 = (6.28\text{cm}^{-1})\ (1.50\ \text{cm}) = 2\pi \times 1.5 = 3\pi$

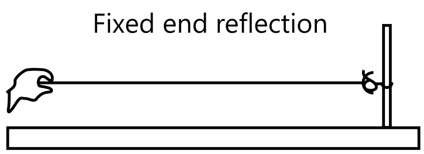
We can thus infer that this is a destructive interference. The amplitude of the resulting wave is given by $|A_1 - A_2|$ = (5.0 − 4.0) mm = 1.0mm.

# 9. BOUNDARY BEHAVIOUR

When a propagating wave reaches the end of the medium it encounters an obstacle or, maybe, another medium through which it could travel. Here, the interface of the two media is referred to as the boundary and the behavior of a wave/pulse at that boundary is described as its boundary behavior.

## 9.1 Fixed End Reflection

Let us consider an elastic string which is attached at one end to a pole on a lab bench while the other end is will be held in the hand and stretched in order to introduce pulses into the medium. The end of the elastic string that is attached to the pole is immovable when a wave or pulse reaches it. If a pulse is introduced at the hand-held end of the rope, it will travel through the string towards the fixed immovable end of the medium. This is called the incident pulse since it is incident (i.e., approaching toward) the boundary with the pole. With the incident pulse reaches the boundary, two things occur:




An elastic rope security to a can be used to study the behavior waves at a fixed end.

**Figure 11.15**

**(a)** Some of the energy transmitted by the pulse is reflected back towards the hand-held end of the rope. This is known as the reflected pulse.

**(b)** That part of the energy that is transmitted to the pole causes the pole to vibrate.

As the vibrations of the pole are not obvious, the energy transmitted to it is not typically discussed. The emphasis here will be on the reflected pulse. What are the characteristics/properties of its motion?

When seen from the fixed immovable end, the reflected pulse is a mirror image of incident pulse. That is, an upward displaced pulse will be reflected and returned as a downward displacement pulse and vice-versa.

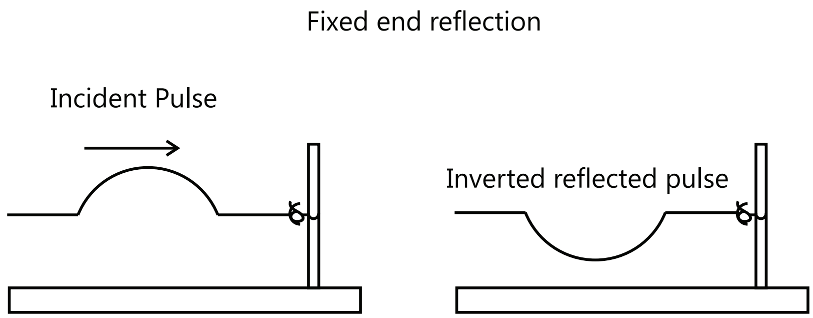
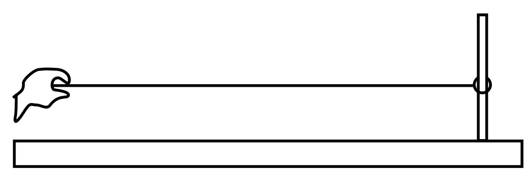



**Figure 11.16**

## 9.2 Free End Reflection

Continuing with the above example, let us consider the situation where instead of being securely attached to a lab pole, the elastic string is attached to a ring that is fixed loosely around the pole. Since the string is no longer attached firmly to the pole, the last particle of the rope will be able to move when a pulse reaches it.

Now, if a pulse is introduced at the hand-held end of the string, it will travel through the string towards the pole at the right end of the medium. However, the string is no longer fixed tightly to



if the end of an elastic roe not fastened to the pole then it will be free move up and down. This provides for the study of waves behavior at free ends.

**Figure 11.17**

the pole and, therefore, the string and the pole will slide past each other. There will be no interaction between the string particle and the pole particle. In other words, when the last particle in the string is displaced upwards, there will be no adjoining pole particle to pull it down. As a result, the upward displacement of the incident pulse is not reversed the in the reflected pulse. Similarly, if the incident pulse has a downward displacement the reflected pulse will also demonstrate a downward displacement. . Inversion is not observed in free end reflection.

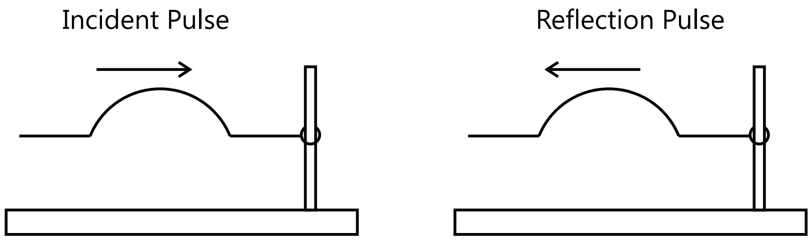



**Figure 11.18**

# 10. REFLECTION AND TRANSMISSION OF A PULSE ACROSS A BOUNDARY

## 10.1 Reflection and Transmission of a Pulse across a Boundary from Less to More Dense

A pulse exhibits two behaviors upon reaching the boundary.

**(a)** A part of the energy transmitted by the incident pulse is reflected and returns towards the hand-held end of a thin string. The pulse that returns to the hand-held end after bouncing off the boundary is known as the reflected pulse.

**(b)** A part of the energy transmitted by the incident pulse is transmitted into the thick string. The disturbance that continues moving to the right is known as the transmitted pulse.

In a wave traveling from a less dense to a denser medium a part of the incident pulse will be reflected off the boundary of the less dense string while another part will be transmitted across the boundary of the thin string into the new medium (thick string). The pulse that moves into the new medium is the transmitted pulse and is not inverted. The pulse that is reflected off the boundary of the thinner string is called the reflected pulse is inverted.

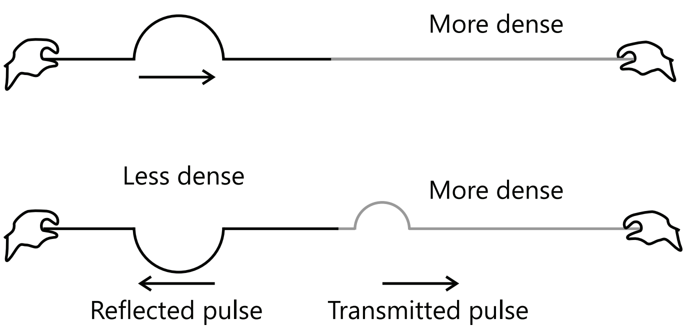



**Figure 11.19**

## 10.2 Reflection and Transmission of a Pulse across a Boundary from More to Less Dense

Here, the transmitted pulse moves through the less dense string/medium, while the reflected pulse travels through the denser string/medium. The transmitted pulse travels faster and has larger wavelength than the reflected pulse. However, the speed and wavelength of the reflected pulse are same as the that of the incident pulse.

Here, a part of the incident pulse will be reflected off the boundary of the denser string/medium and part will transmitted across the boundary of the denser string/medium into the less dense string/medium. There is no inversion, whatsoever.

A wave travelling from a more dense to a less dense medium

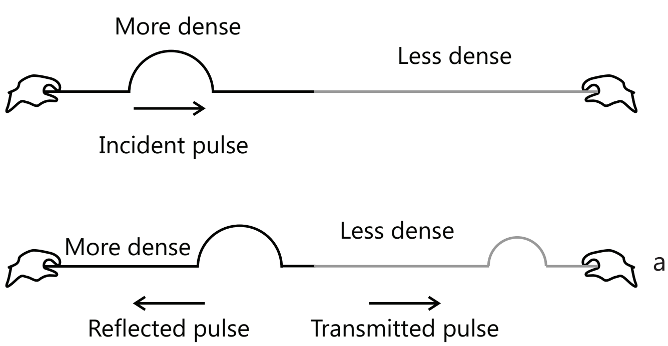



**Figure 11.20**

## PLANCESS CONCEPTS

- The wave speed and the wavelength are always greatest in the least dense string/medium.
- The wave frequency remains constant even when crosses the boundary.
- When moving from less dense string/medium to denser string/medium, the reflected pulse gets inverted.
- The amplitude of the incident pulse is always greater than that of the reflected pulse.

**Anand K (JEE 2011, AIR 47)**

## 11. STANDING WAVES

Standing wave, also called a stationary wave, is combination of two waves moving in opposite directions, each having the same amplitude and frequency. . The manner of this interference makes it appear as if some points along the medium are standing still. For this reason, this wave pattern is referred to as the standing wave pattern. Let us assume that two waves of equal amplitude and frequency propagate towards each other along a string. The equation of two waves are given by $y_1 = A\sin(\omega t - kx)$ and $y_2 = A\sin(\omega t + kx + \delta)$.

To understand these waves, let us discuss the special case when $\delta = 0$.

The displacements of the particles of the string consequent to the interference are given by the principle of superposition as $y = y_1 + y_2 = A\ [\sin(\omega t\text{-}kx) + \sin(\omega t+kx) = 2A \sin\omega t \cos kx$ or $y = (2A \cos kx) \sin \omega t$ ... (xix)

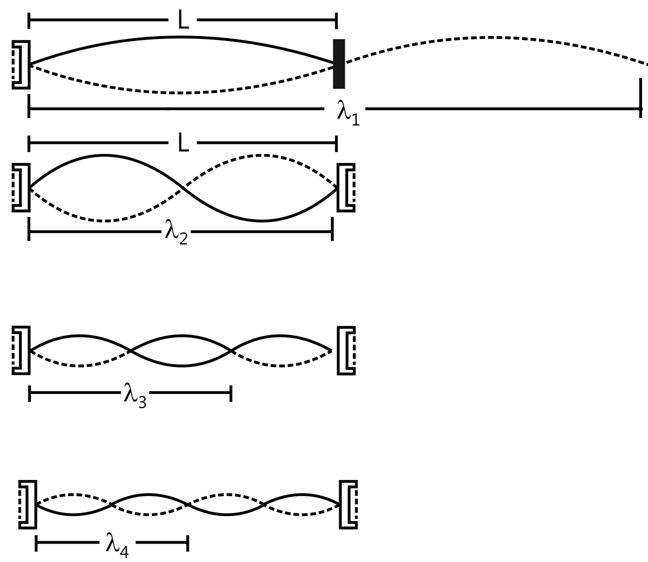



**Figure 11.21**

### 11.1 Nodes and Antinodes

As discussed earlier, the manner of interference of standing wave patterns is such that there are points along the medium that appear to be stationary. These points are referred to as nodes or points of no displacement. There are other points along the medium that undergo v the maximum displacement during each vibrational cycle of the standing wave. These points along the medium are called antinodes, as they represent the other extreme in the standing wave pattern. A standing wave pattern always has nodes and antinodes appearing alternatively in them

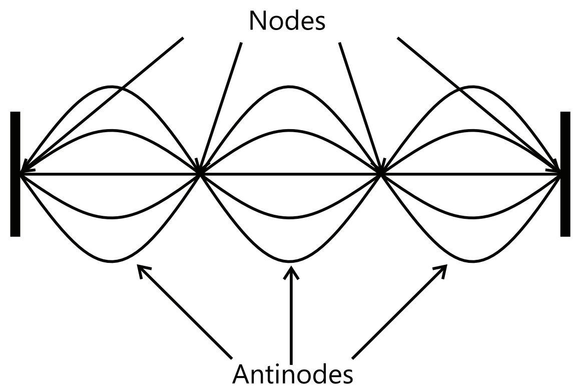



**Figure 11.22**

**PLANCESS CONCEPTS**

Nodes and antinodes are quite different from crests and troughs. In a traveling wave, there points of large upward and downward displacements, referred to as the crest and trough of the wave. However, an antinode refers to a point of the string that remains stationary or appear to be stationary.

**GV Abhinav (JEE 2012, AIR 329)**

## 11.2 Differences Between Standing Waves and Traveling Waves

| Standing Wave | Traveling Wave |
|---|---|
| The disturbance produced in a region appears stationary. | The disturbance produced in a region is transmitted with a definite velocity |
| Different particles move with different amplitudes | The motion of all particles are similar in nature |
| The particles at node always remain at rest | There is no particle which always remains at rest |
| All particles cross their mean positions together | At no point all the particles are at mean positions together |
| All the particles between two successive nodes reach their extreme positions together, thus moving in phase. | The phases of nearby particles are always different |
| The energy of one region is always confined to that region | Energy is transmitted from one region of space to other |

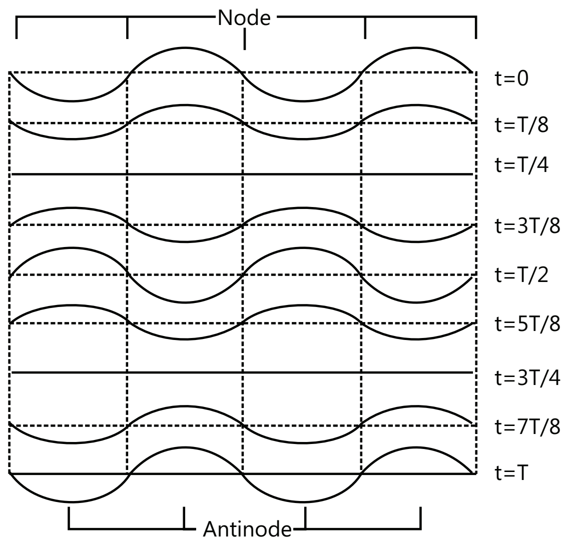



**Figure 11.23**

**Illustration 12:** The interference of two waves with equal amplitudes and frequencies traveling in opposite directions produces a standing wave having the equation Y = A cos kx sin $\omega$t in which

A = 1.0 mm, k = 1.57 $cm^{-1}$ and $\omega$ = 78.5 $s^{-1}$

(a) Find the velocity of the component traveling waves.

(b) Find the node closest to the origin in the region x > 0.

(c) Find the antinode closest to the origin in the region x > 0.

(d) Find the amplitude of the particle at x = 2.33 cm. **(JEE ADVANCED)**

**Sol:** Here the two waves of same amplitude and frequency interfere with each other to form the standing waves, the velocity of the resultant wave will be $v = \frac{\omega}{k}$ where $\omega$ is the angular frequency of the wave and k is the wave number. The distance of the node from the origin is given by $kx = \frac{n\pi}{2}$. And distance of antinode from origin is given by $kx = n\pi$.

(a) The standing wave is formed by the superposition of the waves

$y_1 \frac{A}{2}\sin(\omega t - kx)$ and $y_2 \frac{A}{2}\sin(\omega t + kx)$. The wave velocity (magnitude) of the waves is $v = \frac{\omega}{k} = \frac{78.5\ s^{-1}}{1.57\ cm^{-1}} = 50\ cms^{-1}$

(b) For a node, cos kx = 0

The smallest position x satisfying this relation is given by $kx = \frac{\pi}{2}$ or, $x = \frac{\pi}{2k} = 1cm$

(c) For an antinode, |cos kx| = 1 or cos kx = $\pm 1$

$Kx = 0,\ \pi, 2\pi, \ldots\ldots n\pi \Rightarrow x \neq 0, x_{min} = \frac{\pi}{k} = \frac{3.14}{1.57} = 2cm$

(d) The amplitude of vibration of the particle at x is given by |A cos kx|

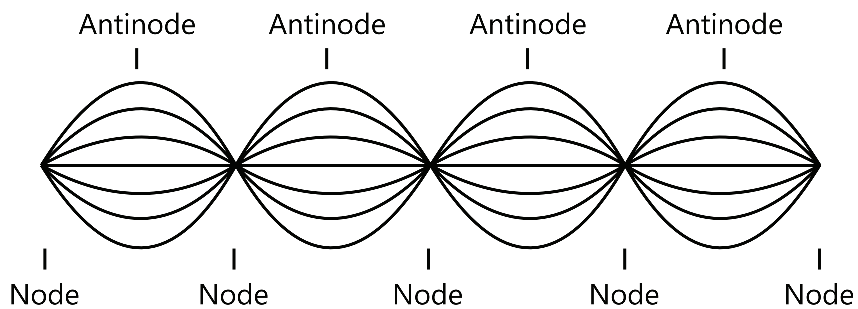



**Figure 11.24**

For the given point, $kx = \left(1.57cm^{-1}\right)\left(2.33cm\right) = \frac{7}{6}\pi = \pi + \frac{\pi}{6}$

Thus, the amplitude will be $\left(1.0\ mm\right)\ |\ \cos(\pi + \pi/6)\,| = \frac{\sqrt{3}}{2}mm = 0.86mm$

## 11.3 Standing Waves on a String Fixed at Both Ends (Qualitative Discussion)

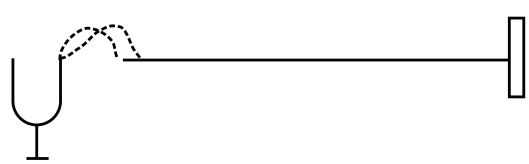


**Figure 11.25**

Let us take the example of string fixed at both the ends –- one end to a wall and the other end tied to a tuning fork. The tuning fork vibrates longitudinally with a small amplitude producing sine waves of amplitude A which travel along the string towards the wall. The said wave then gets reflected and travels toward the fork. This wave, being reflected from a fixed end, will be an inverted wave.. These waves are again hit the fork back and as the fork is heavy and vibrates longitudinally with a small amplitude, it acts like a fixed end and the waves reflected from the fork get inverted again. Therefore, the wave produced directly by the fork initially and the twice-reflected wave have same shape, though the twice-reflected wave has already travelled a length 2L.

Let us assume that the length of the string is $2L=\lambda$. The wave moving from the tuning fork to the wall and the wave reflected back from the wall to the tuning fork interfere constructively and the resultant wave that proceeds towards the wall has an amplitude 2A. This wave of amplitude 2A is again reflected back by the wall and then again reflected by the fork. Now, this twice-reflected wave again interfaces constructively with the new incident wave and a wave of amplitude 3A is produced. Thus, the amplitude keeps progressing. The string gets energy from the vibrating and the amplitude builds up. Same arguments hold if 2L is any integral multiple of $\lambda$ that is $L=n\lambda/2$, where n is an integer.

However, in the above discussion, we have not factored in any loss of energy due to air viscosity or due to the inflexibility of the string. In the steady state, waves of invariable amplitude will be present on the string from left to right as well as from right to left. These opposing waves will produce standing waves on the string. Nodes and antinodes will be formed along the string and there will be large amplitudes of vibration at the antinodes. We can then say that the string is in resonance with the fork. The condition, $L=n\lambda/2$, for such a resonance may be stated in a different way. We have from equation (9), $\upsilon = \nu\lambda$ or $\lambda = \upsilon/\nu$

The condition for resonance is, therefore,

$$L = n\frac{\lambda}{2} \quad \text{or} \quad L = \frac{n\upsilon}{2\nu} \quad \text{or} \quad \nu = \frac{n\upsilon}{2L} = \frac{n}{2L}\sqrt{F/\mu} \qquad \text{... (i)}$$

The lowest frequency with which a standing wave can be set up in a string fixed at both the ends is thus

$$\nu_o = \frac{1}{2L}\sqrt{F/\mu} \qquad \text{... (ii)}$$

This is called the fundamental frequency of the string. All other possible frequencies of standing waves are integral multiples of this fundamental frequency. Equation (xx) gives the natural frequencies, normal frequencies, or resonant frequencies.

**Illustration 13:** Shown in the Fig 11.26 is a wire with a length of 50 cm and a mass of 20 g. It supports a mass of 1.6 kg. Find the fundamental frequency of the portion of the string between the wall and the pulley.

Take $g=10ms^{-2}$. **(JEE ADVANCED)**

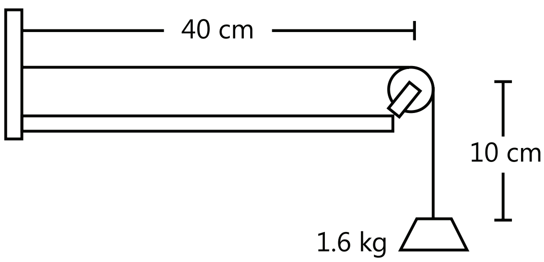



**Figure 11.26**

**Sol:** The string is subjected to uniform tension due to weight of the block of mass 1.6 kg. The fundamental frequency of the string between the fixed support and pulley is given by

$\nu_o = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$ where $\mu$ is the mass per unit length of string.

The tension in the string is F = (1.6kg) $(10ms^{-2})$ = 16N.

The linear mass density is $\mu = \frac{20\,g}{50\,cm} = 0.04\,kgm^{-1}$

The fundamental frequency is $\nu_o = \frac{1}{2L}\sqrt{\frac{F}{\mu}} = \frac{1}{2\times(0.4m)}\sqrt{\frac{16N}{0.04\,kgm^{-1}}}$ = 25 Hz

## 11.4 Analytical Treatment of Vibration of a String Fixed at Both Ends

Let us assume a string of length L which is kept fixed at the ends x= 0 and x= L. For certain wave frequencies, standing waves are set up in the string. Due to the repeated reflection of the wave at the ends and the damping effects, waves going in the positive direction x interfere to give a resultant wave $y_1 = A\sin(kx-\omega t)$. Similarly, the waves going in the negative direction x interfere to give the resultant wave $y_2 = A\sin(kx+\omega t+\delta)$. As a result, the displacement of the particle of the string at position x and at time t is given by the principle of superposition as $y = y_1 + y_2 = A\sin(kx-\omega t) + \sin(kx+\omega t+\delta)$

$$= 2A\sin(kx - \frac{\delta}{2})\cos(\omega t + \frac{\delta}{2}) \quad \text{... (i)}$$

If standing waves are formed, the ends x = 0 and x = L must be nodes because they are kept fixed. Thus, we have the boundary conditions y= 0 at x=0 for all t and y=0 at x=l for all t.

The first boundary condition is satisfied by equation (i) if $\sin\frac{\delta}{2} = 0$, or $\delta = 0$.

Equation (i) then becomes $y = 2A\sin kx\cos\omega t$ ... (ii)

The second boundary condition will be satisfied if

$\sin kL = 0$ or $kL = n\pi$, where n=1, 2, 3, 4, 5,.......

or $\frac{2\pi L}{\lambda} = n\pi$ or $L = \frac{n\lambda}{2}$ ... (iii)

If the length of the string is an integral multiple of $\lambda/2$, standing waves are produced.

Again writing $\lambda = \upsilon T = \frac{\upsilon}{v}$, equation (xxv) becomes $v = \frac{n\upsilon}{2L} = \frac{n}{2L}\sqrt{F/\mu}$

Which is same as equation (xx). The lowest possible frequency is $v_o = \frac{\upsilon}{2L} = \frac{1}{2L}\sqrt{F/\mu}$ ... (iv)

This is the fundamental frequency of the string. The other natural frequencies with which standing wave can be formed on the string are

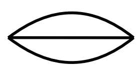
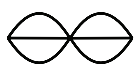
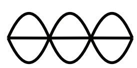
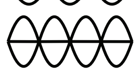
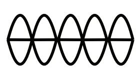
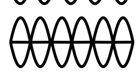
| Harmonic | Pattern | No. of Loops | Length-Wavelength relationship |
|---|---|---|---|
| 1st |  | 1 | L = 1/2 • λ |
| 2nd |  | 2 | L = 2/2 • λ |
| 3rd |  | 3 | L = 3/2 • λ |
| 4rd |  | 4 | L = 4/2 • λ |
| 5th |  | 5 | L = 5/2 • λ |
| 6th |  | 6 | L = 6/2 • λ |

**Figure 11.27**

$v_1 = 2v_0 = \frac{2}{2L}\sqrt{F/\mu}$ 1st overtone, or 2rd harmonic,

$v_2 = 3v_0 = \frac{3}{2L}\sqrt{F/\mu}$ 2nd overtone, or 3rd harmonic,

$v_3 = 4v_0 = \frac{4}{2L}\sqrt{F/\mu}$ 3rd overtone, or 4rd harmonic, etc.

In general, any integral multiple of the fundamental frequency is a valid frequency. These higher frequencies are called overtones. Thus, $v_1 = 2v_0$ is the first overtone, $v_2 = 3v_0$ is the second overtone, etc. An integral multiple of a frequency is called its harmonic. Thus, for a string fixed at both the ends, all the overtones are harmonics of the fundamental frequency and vice-versa.

## 11.5 Vibration of a String Fixed at One End

If a string is set up in such a way that one end of it remains fixed while the other end is free to move in a transverse direction, standing waves can be produced. The free end can be created by connecting the string to a very light thread. If the vibrations of the "correct" frequency are produced by the source, standing waves are produced. Assuming end x=0 is fixed and x = L is free, the equation is again given by $y = 2A\sin kx\cos\omega t$ which is the same as equation (xxii), with the boundary condition that x = L is an antinode. The boundary condition that x = 0 is a node is automatically satisfied by the above equation as it is fixed. For x = L to be an antinode, $\text{Sin } kL = \pm 1$

$$\text{or} \quad kL = \left(n+\frac{1}{2}\right)\pi \quad \text{or} \quad \frac{2\pi L}{\lambda} = \left(n+\frac{1}{2}\right)\pi \quad \text{or} \quad \frac{2Lv}{\upsilon} = n+\frac{1}{2} \quad \text{or} \quad v = \left(n+\frac{1}{2}\right)\frac{\upsilon}{2L} = \frac{n+\frac{1}{2}}{2L}\sqrt{F/\mu} \qquad \text{... (i)}$$

These are the normal frequencies of vibration. The fundamental frequency is obtained when n = 0,

i. e., $\nu_o = \upsilon / 4L$

The overtone frequency are $\nu_1 = \frac{3\upsilon}{4L} = 3\nu_o$

$\nu_2 = \frac{5\upsilon}{4L} = 5\nu_o$, $\nu_3 = \frac{7\upsilon}{4L} = 7\nu_o$, etc

It can be seen that all the harmonics of the fundamental frequency are not the valid frequencies for the standing waves. Only the odd harmonics are the overtones. The string shapes for some of the normal modes are shown in Fig 11.28.

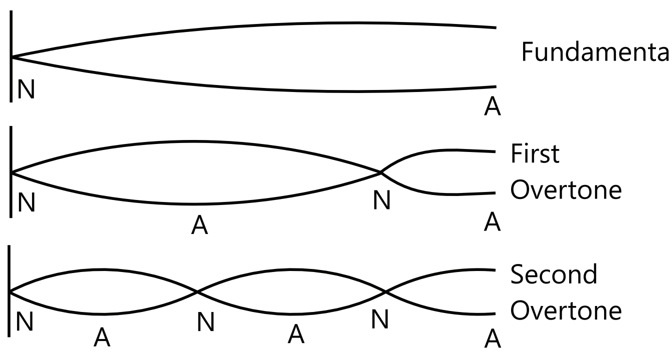



**Figure 11.28**

**Illustration 14:** A string is vibrating up and down as the fifth harmonic and completes 21 vibrational cycles in 5 seconds. The length of the string is 8.2 meters. Determine the frequency, period, wavelength and speed for this wave. **(JEE MAIN)**

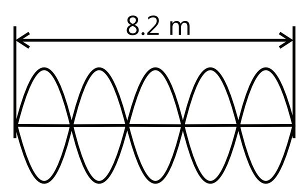



**Figure 11.29**

**Sol:** The frequency of the wave is $f = \frac{\text{number of cycles produced}}{\text{total time}}$. The time period of wave $T = \frac{1}{f}$. When string is vibrating in fifth harmonics, then 2L = 5λ. The wave velocity is v = f λ.

Given: L = 8.2 m and 21 cycles in 5 seconds. The frequency here refers to the number of back-and-forth movements of a point on the string and is measured as the number of cycles per unit of time. In this case, it is f = (21 cycles)/(5 seconds) = 4.2 Hz

The period is the reciprocal of the frequency. T = 1/ (4.2 Hz) = 0.238 s.

The wavelength of the wave is correlated to the length of the rope. For the fifth harmonic as shown in the picture, the length of the rope is equivalent to five halves of a wavelength. That is, $L = \frac{5}{2}\lambda$ where λ is the wavelength. Rearranging and substituting the equation gives the following results:

$\lambda = (2/5) \times L = (2/5) \times (8.2\text{ m}) = 3.28\text{ m}$

The wavelength and frequency wave can be used to calculate the speed of a wave using the wave equation

$V = f\lambda = (4.2\text{Hz}).(3.28\text{m}) = 13.8\text{m/s}$

# 12. LAWS OF TRANSVERSE VIBRATIONS OF A STRING

For a string fixed at both ends, the fundamental frequency of vibration is given by equation (ix). The statements known as "Laws of transverse vibrations of a strings" can be derived from equation (ix).

## 12.1 Law of Length

Tension and mass per unit length of the string remaining the same, the fundamental frequency of vibration of a string (fixed at both ends) is inversely proportional to the length of the string.

$\nu \propto 1/L$ if F and $\mu$ are constants.

## 12.2 Law of Tension

The length and the mass per unit length of the string remaining the same, the fundamental frequency of a string is proportional to the square root of its tension. $\nu \propto \sqrt{F}$ L if F and $\mu$ are constants.

## 12.3 Law of mass

The length and the tension remain the same, the fundamental frequency of a string is inversely proportional to the square root of the linear mass density, i.e., mass per unit length.

$\nu \propto \frac{1}{\sqrt{\mu}}$ if L and F are constants.

These above laws can be experimentally studied with an apparatus called sonometer.

## 12.4 Sonometer

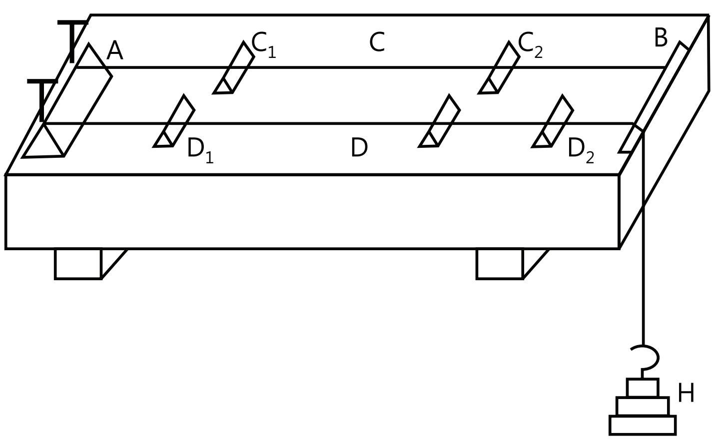



Figure 11.30

A sonometer is an apparatus that is used to study the transverse vibrations of strings. It is also called the monochord because it often has only one string. It consists of a rectangular wooden box with two fixed bridges near the ends, with a pulley fixed at one end. A string is fixed at one end, which is then run over the bridges and the pulley, and then attached to a weight holder hanging below the pulley. Additional weights can be added to the holder to increase the tension in the wire. A third, movable bridge, can be placed under the string to change the length of the vibrating section of the string. This device demonstrates the relationship between the frequency of the sound produced when a string is plucked and the tension, length, and mass per unit length of the string. These relationships are referred to as Mersenne's law after Marin Mersenne (1588–1648), who studied and formulated them. For small amplitude vibration, the frequency is proportional to:

**(a)** The square root of the tension of the string

**(b)** The reciprocal of the square root of the linear density of the string,

**(c)** The reciprocal of the length of wire of sonometer

**Illustration 15:** Resonance is obtained in a sonometer experiment when the experimental wire with a length of 21 cm between the bridges is excited by a tuning fork of frequency 256 Hz. If a tuning fork of frequency 384 Hz is used, what should be the length of the experimental wire to get the resonance? **(JEE MAIN)**

**Sol:** For sonometer wire the ratio of lengths of vibrating string is $\frac{\ell_1}{\ell_2} = \frac{\nu_2}{\nu_1}$.

By the law of length, $\ell_1\nu_1 = \ell_2\nu_2$ or $\ell_2 = \frac{\nu_1}{\nu_2}\ell_1 = \frac{256}{384}\times 21\text{cm} = 14\,\text{cm}$

## 13. TRANSVERSE AND LONGITUDINAL WAVES

When there is a disturbance at the source in a string, it causes displacement of the particles of the string. The direction of such displacements is perpendicular to the direction of the propagation of the wave. Such waves are called transverse waves. The wave on a string is a transverse wave.. Light waves are also an example of transverse waves. Here, the value of the electric field changes with space and time and the changes are propagated in space. The direction of the electric field is perpendicular to the direction of propagation of light when light travels in free space.

Sound waves are an example of non-transverse waves. The particles of the medium are carried along the direction of propagation of sound. We shall study in some detail the mechanism of sound waves in the next chapter. If the displacement produced by the passing wave is along the direction of the wave propagation, the wave is called a longitudinal wave. Sound waves are longitudinal.

### 13.1 Compression and Rarefaction

A longitudinal wave consists of compressions and rarefactions. Those regions in a longitudinal wave where particles are clustered together are compressions. Conversely, those regions where the particles are furthest apart are called rarefactions.

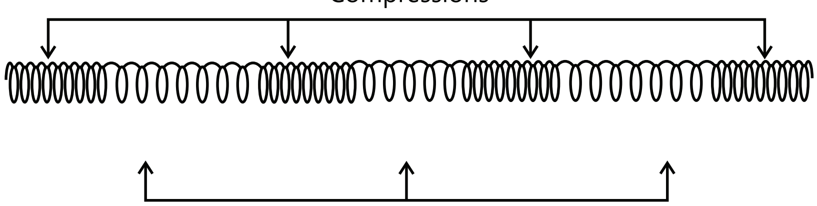



**Figure 11.31**

**Illustration 16:** A sonometer wire has a length of 100 cm and a fundamental frequency of 330 Hz. Find

(a) The velocity of propagation of transverse waves along the wire and

(b) The wavelength of the resulting sound in air if velocity of sound in air is 330 m/s. **(JEE ADVANCED)**

**Sol:** When sonometer wire is set to vibrate in its fundamental frequency, then wavelength is $\lambda$=2L, the wave velocity is $v = f\lambda$ where f is the frequency of oscillation.

(a) In case of transverse vibration of string for fundamental mode:

$L = (\lambda / 2)$, i.e., $\lambda = 2L = 2\times I = 2m$

i.e., the wavelength of transverse wave propagation on string is 2 m. Since the frequency of the wire is given to be 330 Hz, so from $\upsilon = f\lambda$, the velocity of transverse waves along the wire will be

$V_{wire} = 330\times 2 = 660\,m/s$

i.e., for transverse mechanical waves propagation along the wire, Hz,m and m/s

(b) Here vibration wire will act as source and produce sound, i.e., longitudinal waves in air. Now as frequency does not change with change in medium so Hz and as velocity in air is given to be = 330 m/s so from $\upsilon = f\lambda$;

$\lambda_{air} = (V_{air} / f) = (330 / 330) = 1m$

i. e., for sound (longitudinal mechanical waves) in air produced by vibration of wire (body),

f = 330Hz, $\lambda$=2m and v =330m/s

## 14. POLARIZATION OF WAVES

Let us assume that we have a cardboard with a slit in it through which a stretched string is passed such that the card is placed in a perpendicular position to the string. (See Fig 11.32). If we take the string as the X axis, the cardboard will be in Y-Z plane. Now we generate a wave along the X axis such that the particles of the string are displaced in direction Y as the wave passes. If the slit in the cardboard is also aligned along the Y axis, the portion of the string in the slit can vibrate freely in the slit and the wave will pass through the slit. Now, if we turn the cardboard by $90^0$ in its plane, the slit will be aligned along the Z axis. As the wave reaches the slit, the portion of the string in the slit tries to move along the Y axis but the narrow slit on the cardboard becomes an obstruction. Consequentially, the wave is not able to pass through the slit. However, if the slit is inclined to the Y axis at a different other angle, only a part of the wave is transmitted and in the transmitted wave the disturbance is produced parallel to the slit. The same experiment can be conducted with two chairs as shown in the Fig 11.33. If the displacement produced is always along a fixed direction, then the wave is said to be linearly polarized in that direction. The examples considered in this chapter are linearly polarized in y direction. By the same token, a wave that produces a displacement along the z direction, is a linearly polarized wave, polarized in z-direction. Its equation is given by $z = A\sin\omega(t - x / \upsilon)$ .Linearly polarized waves are referred to as plane polarized. In the event that each particle of a string moves in a small circle when the wave is propagated, the wave is called circularly polarized. If each particle goes in ellipse, the wave is called polarized. If the particles are move randomly in the plane perpendicular to the direction of propagation, the wave is called un-polarized.

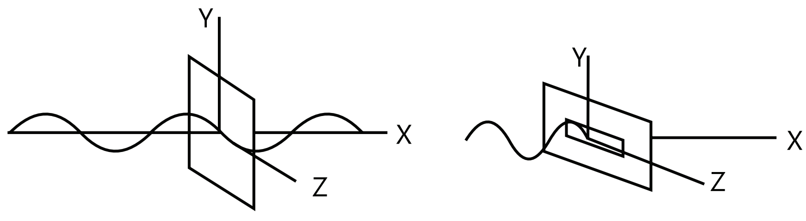



**Figure 11.32**

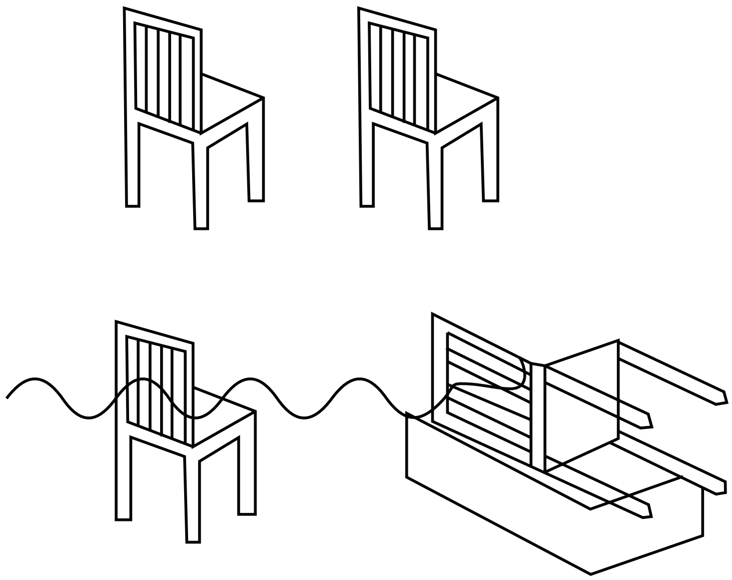


**Figure 11.33**

# PROBLEM-SOLVING TACTICS

**1.** Understanding and remembering all formulae is the key to solving problems in these sections. If the relation between the given quantities and the questions asked is known, it will be easy to solve most of the problems. All the quantities discussed in this topic are in some sense related to each other.

**2.** The concept of reflection (of waves) can be encapsulated in a single point: "Inversion- Reflected wave will invert only when it encounters a denser medium. And transmitted wave will never invert." If this much is clear, one can easily identify the case in every question.

**3.** Waves must always be understood in the context of transfer of energy rather than as just some function of x and t for better understanding of physics.

**4.** For questions pertaining to the derivation of the wave equation, one can begin easily with only the x part and subsequently add or subtract vt from x depending on the direction of velocity.

**5.** Most questions related to velocity and energy appear complicated due to the introduction of the usual Newton mechanics. This should, however, be treated just as some additional information to calculate tension in the string (e.g., Pulley systems).

# FORMULAE SHEET

| S. No | Term | Description |
|---|---|---|
| 1 | Wave | It is a disturbance or variation traveling through a medium due to the repeated undulating motion of particles of the medium through their equilibrium position. Examples are sound waves travelling through an intervening medium, water waves etc. |
| 2 | Mechanical waves | Waves that are propagated through a material medium are called MECHANICAL WAVES. These are governed by Newton's Law of Motion. Sound waves are mechanical waves propagated through the atmosphere from a source to the listener and it requires a medium for its propagation. |
| 3 | Non mechanical waves | Waves which are not propagated through a material medium. Eg: light waves, EM waves. |
| 4 | Transverse wave | These are waves in which the displacements or oscillations are perpendicular to the direction of propagation of the wave. |
| 5 | Longitudinal wave | Longitudinal wave waves in which the displacement or oscillations in medium are parallel to the direction of propagation of wave. Example: sound waves |
| 6 | Equation of harmonic wave | At any time t, displacement y of the particle from its equilibrium position as a function of the coordinate x of the particle is $y(x, t) = A\sin(\omega t - kx)$ where,<br>A is the amplitude of the wave, K- is the wave number<br>$\omega$ is angular frequency of the wave and ($\omega$t-kx) is the phase |
| 7 | Wave number | Wavelength $\lambda$ and wave number k are related by the relation $k = 2\pi/\lambda$ |

| | | |
|---|---|---|
| 8 | Frequency | Time period T and frequency f of the wave are related to $\omega$ by $\omega/2\pi = f = 1/T$ |
| 9 | Speed of wave | Speed of the wave is given by $v = \omega/k = \lambda/T = \lambda f$ |
| 10 | Speed of a transverse wave | The tension and the linear mass density of a stretched string, and not the frequency, determines the speed of a transverse wave i.e., $v=\sqrt{\frac{T}{\mu}}$<br>T = Tension in the string<br>$\mu$ = Linear mass density of the string. |
| 11 | Speed of longitudinal waves | Speed of longitudinal waves in a medium is given by<br>$v = \sqrt{\frac{B}{\rho}}$; B = bulk modulus; $\rho$ = Density of the medium speed of longitudinal waves in ideal gas is<br>$v = \sqrt{\frac{\gamma P}{\rho}}$ P = Pressure of the gas,<br>$\rho$ = Density of the gas and $\gamma = C_p / C_V$ |
| 12 | Principle of superposition | It states that when two or more waves of same type come together at a single point in space, the total displacement at that point is equal to the sum of the displacements of the individual waves. It is given by $y=\sum y_i(x,t)$ |
| 13 | Interference of waves | Two sinusoidal waves traveling in the same direction interfere to produce a resultant sinusoidal wave traveling in that direction if they have the same amplitude and frequency, with resultant wave given by the relation<br>$y'(x,t)=[2A_m\cos(u/2)]\sin(\omega t-kx+u/2)$ where u is the phase difference between two waves.<br>If u = 0, then interference would be fully constructive.<br>If u = $\pi$, then waves would be out of phase and the interference would be destructive. |
| 14 | Reflection of waves | An incident wave encountering a boundary gets reflected. If an incident wave is represented by<br>$y_i(x,t) = A\sin(\omega t-kx)$ then reflected wave at rigid boundary is<br>$y_r(x,t) = A\sin(\omega t+kx+\pi)=-A\sin(\omega t+kx)$<br>And for reflections at open boundary, the reflected wave is given by $y_r(x,t) = A\sin(\omega t+kx)$ |
| 15 | Standing waves | When two identical waves moving in opposite directions meet, the interference produces standing waves. The particle displacement in standing wave is given by $y(x,t) = [2A\sin(kx)]\sin(\omega t)$. The amplitude of standing waves is different at different point i.e., at nodes amplitude is zero and at antinodes amplitude is maximum or equal to sum of amplitudes of constituting waves. |

| | | |
|---|---|---|
| 16 | Normal modes of stretched string | Frequency of transverse waves in a stretched string of length L and fixed at both the ends is given by<br>$f = nv/2L$ where n = 1, 2, 3........<br>The above relation gives a set of frequencies called normal modes of oscillation of the system. Mode n=1 is called the fundamental mode with frequency $f_1 = v/2L$. Second harmonic is the oscillation mode with n = 2 and so on.<br>Thus the string has infinite number of possible frequency of vibration which are harmonics of fundamental frequency $f_1$ such that $f_n = nf_1$. |

# Solved Examples

## JEE Main/Boards

**Example 1:** The length of a wave propagated on a long stretched string is taken as the positive x axis. The wave equation is given by

$y = y_0 e^{-\left(\frac{t}{T}-\frac{x}{\lambda}\right)^2}$ where $y_0 = 4\,\text{mm}$, $T = 1.0\,\text{s}$ and $\lambda = 4\text{cm}$.

(a) Find the velocity of the wave.

(b) Find the function finding the displacement of the particle at x = 0.

(c) Find the function giving the shape of the string at = 0.

(d) Plot the shape of the string at t =0.

(e) Plot the shape of the string at t=5s.

**Sol:** The wave moves having natural frequency of $\nu$ and wavelength $\lambda$ has velocity $V = \nu\lambda$. As the frequency is $\nu = \frac{1}{T}$ the velocity of the wave is then $V = \frac{\lambda}{T}$.

(a) The wave equation may be written as

$$y = y_0 e^{-\frac{1}{T}\left(t-\frac{x}{\lambda/t}\right)^2}$$

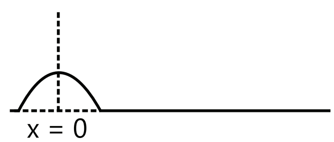
Comparing with the general equation we see that

$$\nu = \frac{\lambda}{1.0\,\text{s}} = 4\,\text{cm} = 4\text{cm/sec}$$

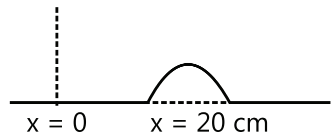
(b) Putting x = 0 in the given equation $f(t) = y_0\, e^{-(t/T)^2}$ … (i)

(c) Putting t = 0 in the given equation $g(t) = y_0\, e^{-(x/\lambda)^2}$ ... (ii)

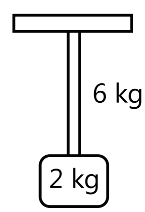
(d)




(e)




**Example 2:** The dimensions of a uniform rope are as follows: length 12 m, mass 6 kg. The rope hangs vertically from a rigid support with a slab of a mass of 2 kg is attached to the free end of the rope. If a transverse pulse of wavelength 0.06 m is transmitted from the free end of the rope, what is the wavelength of the pulse when it reaches the top of the rope?




**Sol:** The wave velocity will be $V = \nu\lambda = \sqrt{\frac{F}{\mu}}$ where F is the tension in string at a point and $\mu$ is mass per unit length of the string. As F is varying along the length of the rope so the velocity will vary along the length of the rope. As source frequency is constant $\lambda$ will vary.

We have, $V = \nu\lambda$

Or, $\sqrt{\frac{F}{\mu}} = \nu\lambda$ or $\frac{\sqrt{F}}{\lambda} = \nu\sqrt{\mu}$

Since the frequency of the wave pulse is dependent only on the frequency of the source, it will be consistent across the length of the rope. The mass per unit length will also be consistent for the entire rope as the rope is uniform. Thus,

By (i) $\frac{\sqrt{F}}{\lambda}$ is constant.

Hence, $\frac{\sqrt{(2kg)g}}{0.06} = \frac{\sqrt{(8kg)g}}{\lambda_1}$

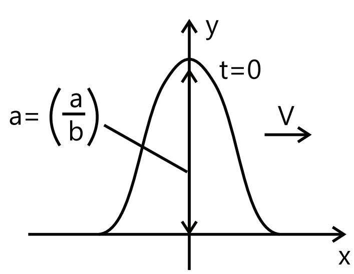
where $\lambda_1$ is the wavelength at the top of the rope. This gives $\lambda_1$ = 0.12m.

**Example 3:** A traveling wave pulse is given by $y = \frac{10}{5+(x+2t)^2}$. What is the direction, velocity and amplitude of the pulse?

**Sol:** The wave equation given above is of form $y = \frac{a}{b+(x \mp \upsilon t)^2}$ where 'a' is the amplitude of the disturbance.

A wave pulse is a disturbance confined to only in a small part of the medium at a given instant [see figure] and its shape does not change during propagation. It is usually expressed by the form $y = \frac{a}{b+(x \mp \upsilon t)^2}$

Comparing the above with the given pulse we find that $f(x \mp \upsilon t) = (x+2t)^2$




i.e, the pulse is traveling along negative x axis with velocity 2 m/s.

Further, amplitude is the maximum value of wave function which will be when $(x+2t)^2 = 0$

So, $A = y_{max} = \frac{10}{5} = 2$

**Example 4:** Consider a tube that is closed at one end and has a vibrating diaphragm at the other end. The diaphragm, which may be assumed to be the displacement node, produces a stationary wave pattern at the frequency of 2000 Hz, in which the distance between adjacent nodes is 8 cm. When the frequency is gradually reduced, the stationary wave pattern disappears but another stationary wave pattern reappears at a frequency of 1600 Hz. Calculate

(i) The speed of sound in air.

(ii) The distance between adjacent nodes at a frequency of 1600 Hz,

(iii) The distance between diaphragm and the closed end

(iv) The next lower frequencies at which stationary wave patterns will be obtained.

**Sol:** The standing waves generated inside the tube closed at one end, have the wavelength $n\lambda = 2L$ where L is length of the tube. The velocity of the wave in air is given by v = f$\lambda$, where n is the frequency of the sound wave.

Since the node-to node distance is $\lambda/2$, $\lambda/2 = 0.08$ or $\lambda$ = 0.16m

(i) c = n $\lambda$; $\therefore$ c= 2000 × 0.16 = 320ms$^{-1}$

(ii) 320 = 1600 × $\lambda/2$ or $\lambda = 0.2$m

$\therefore$ Distance between nodes = 0.2/2 = 0.1 m = 10cm.

(iii) Since there are nodes at the ends, the distance between the closed end and the membrane must be exact integrals of $\lambda/2$.

$\therefore$ 0.4 = n $\lambda/2 = \nu' \times 0.2/2 \Rightarrow \frac{n}{n'} = \frac{5}{4}$

When n = 5, n' = 4 l = n × $0.16/2 = 0.4\mu = 40$cm

(iv) For the next lower frequency n = 3, 2, 1

$\therefore$ 0.4 = 3 $\lambda$ / 2 or $\lambda$ = 0.8/3

Since c=n $\lambda$, $n = \frac{320}{0.8/3} = 1200$Hz

Again 0.4 = 1. $\lambda$ / 2 or $\lambda$ = 0.4M

$\therefore$ n = 320/0.4 = 800 Hz

Again 0.4 = 1. $\lambda$ / 2 or $\lambda$ = 0.8M

$\therefore$ n = 320/0.8 = 400 Hz

**Example 5:** Consider a tuning fork of frequency 256 Hz and an open organ pipe of slightly lower frequency. Both are at 17°C temperature. When sounded together, they produce 4 beats per second. When the temperature of air in the pipe is altered, the number of beats per second first diminishes to zero and then increases again to 4. Determine the quantum of temperature change in the pipe? Also, in what direction has the temperature of the air in the pipe been altered?

**Sol:** In an open organ pipe the frequency of the wave is $n = \frac{V_t}{\lambda}$ where $V_t$ is the velocity of wave at temperature t and $\lambda$=2L is the wavelength of the vibrating wave. If temperature of air inside the organ pipe changes, the velocity of wave also changes, since $V \propto \sqrt{T}$.

$n=\frac{V_{17}}{2l}$ where l = length of the pipe;

$\therefore \quad 256-\frac{V_{17}}{2l}=4 \quad$ or $\quad \frac{V_{17}}{2l}=252$

Since beats decreases first and then increases to 4, the frequency of the pipe increases. This can happen only if the temperature increases.

Let t be the final temperature, in Celsius,

Now $\frac{V_t}{2l}$ =256 + 4 or $\frac{V_t}{2l}$ = 260

Dividing $\frac{V_t}{V_{17}}=\frac{260}{252}$ or $\sqrt{\frac{273+t}{273+17}}=\frac{260}{252}$

$(\therefore V<\sqrt{T})$ or $t=308.7 - 273 = 35.7^0 C.$

$\therefore$ Rise in temperature =35.7 – 17 = 18.7 $^0$C.

**Example 6:** Determine the fundamental frequency and the first four overtones of a 15 cm pipe

(a) If the pipe is closed at one ends,

(b) If the pipe is open at both ends

(c) How many overtones are within the human auditory range in each of the above cases? Velocity of sound in air = 330 $ms^{-1}$.

**Sol:** For the organ pipe closed at one end, the fundamental frequency of the wave of wavelength $\lambda$ is given by, $n_0 = \frac{v}{4L}$.The frequency of $i^{th}$ over tone is given by $n_i = (i+1) \times n_0$ where i=1,2,3.... etc.

(a) $n_0 = \frac{V}{4l}$

Wher $n_0$ = frequency of the fundamental node

$\Rightarrow n_0 = \frac{330}{4 \times 0.15} = 550Hz$

The first four overtones are $3n_0$, $5n_0$, $7n_0$ and $9n_0$

$\therefore$ So, the required frequencies are 550, 1650, 2750, 3850, and 4950 Hz.

(b) $n_0 = \frac{V}{2l} = \frac{330}{2 \times 0.15} = 1100Hz$

The first overtones are $2n_0$, $3n_0$, $4n_0$ and $5n_0$

So, the required frequency are 1100, 2200, 3300, 4400, and 5500 Hz

The frequency of the $n^{th}$ overtone is (n +1) $n_0$.

$\therefore$ (n +1) $n_0$ =20000 or (n+1)100 = 20000

Or n = 17.18

The acceptable value is 17.

**Example 7:** The displacement of a particle of a string carrying a traveling wave is given by

$y=(3.0\,cm)\sin 6.28(0.50x-50t)$,

where x is in centimeter and t in second. Find (a) the amplitude, (b) the wavelength, (c) the frequency and (d) the speed of the wave.

**Sol:** In a open organ pipe the frequency of the wave is $n = \frac{V_t}{\lambda}$ where $V_t$ is the velocity of wave at temperature t and $\lambda$=2L is the wavelength of the vibrating wave. If temperature of air inside the organ pipe changes, the velocity of wave also changes, since $V \propto \sqrt{T}$.

On comparing with the standard wave equation

$y=A\sin(kx-\omega t) = A\sin 2\pi(\frac{x}{\lambda}-\frac{t}{T})$

we see that, Amplitude = A = 3.0 cm,

Wavelength = $\lambda = \frac{1}{0.50}$ cm = 2.0 cm, and the frequency

$= \nu = \frac{1}{T} = 50Hz$

The speed of the wave is $V=\nu\lambda$

= (50 $s^{-1}$) (2.0cm) = 100 cm $s^{-1}$

**Example 8:** The equation for a wave traveling in the direction x on a string is

y = (3.0 cm) sin [(3.14 $cm^{-1}$) x – (314 $s^{-1}$) t].

(a) Find the maximum velocity of a particle of the string.

(b) Find the acceleration of a particle at x = 6.0 cm at time t = 0.11 s

**Sol:** The maximum velocity is $v=\frac{\partial y}{\partial t}$ While the acceleration $a=\frac{\partial v}{\partial t}$

(a) The velocity of the particle at x at time t is $v=\frac{\partial y}{\partial t}$

$$=(3.0\text{ cm})(-314s^{-1})\cos[(3.14cm^{-1})x-(314s^{-1})t]$$

$$=(-9.4ms^{-1})\cos[((3.14cm^{-1})x-(314s^{-1})t]$$

The maximum velocity of a particle will be v = 9.4 $ms^{-1}$.

(b) The acceleration of the particle at x at time t is

$$a=\frac{\partial v}{\partial t}=-(9.4ms^{-1})\,(314s^{-1})\sin[(3.14cm^{-1})\,x-(314\,s^{-1})\,t]$$

$$=-(2952ms^{-2})\sin[(3.14\,ms^{-1})\,x-(3.14\,s^{-1})].$$

The acceleration of the particle at x = 6.0 cm at time t = 0.11 s = 1(2952ms-$^{2}$) sin [6π - 11 π] = 0.

**Example 9:** One end of a long string is attached to an oscillator moving in transverse direction at a frequency of 20 Hz. The string has a cross-section area of 0.80 $mm^2$ and a density of 12.5 g $cm^{-3}$ . It is subjected to a tension of 64 N along the X axis. At t= 0, the source is at a maximum displacement y = 1.0 cm. (a) Find the speed of the wave traveling on the string. (b) Write the equation for the wave. (c) What is the displacement of the particle of the string at x = 50 cm at time t = 0.05 s? (d) What is the velocity of this particle at this instant?

**Sol:** As the wave is under tension F, the maximum wave velocity of wave is $v=\sqrt{\frac{F}{\mu}}$ where μ is the mass per unit length of the string. The wave equation is y=A cos (ωt) where ωis angular frequency and A is amplitude of wave.

(a) The mass of 1 m long part of the string is m= (0.80mm$^{2}$) × (1 m) × (12.5gcm$^{-3}$)

= (0.80×$10^{-6}$ $m^3$) × (12.5×$10^3$kg $m^{-3}$) = 0.01 kg

The linear mass density is μ 0.01 kg $m^{-1}$. The wave speed is $v=\sqrt{\frac{F}{\mu}}=\sqrt{\frac{64N}{0.01kgm^{-1}}}=80ms^{-1}$

(b) The amplitude of the source is A = 1.0cm and the frequency is =20 Hz. The angular frequency is $\omega=2\pi v=40\,\pi\,s^{-1}$. Also at t =0, the displacement is equal to its amplitude, i.e., at t=0, x = A. The equation of motion of the source is, therefore, y = (1.0cm) cos[(40 π $s^{-1}$)t] ... (i)

The equation of the wave traveling on the string along the position X – axis is obtained by replacing t with $t-x/v$ in equation (i). It is, therefore,

$$y=(1.0cm)\cos\left[(40\pi s^{-1})\left(t-\frac{x}{v}\right)\right]$$

$$=(1.0\,cm)\cos\left[(40\pi s^{-1})t-\left(\frac{x}{2}m^{-1}\right)\right] \quad \text{... (ii)}$$

Where the value of v has been put from part (a).

(c) The displacement of the particle at x = 50 cm at time t = 0.05 s is by equation (ii),

$$y=(1.0\,cm)\cos[(40\pi\,s^{-1})(0.05s)-\left(\frac{\pi}{2}m^{-1}\right)(0.5m)]$$

$$=(1.0\,cm)\cos\left[2\pi-\frac{\pi}{4}\right]=\frac{1.0}{\sqrt{2}}=0.71cm$$

(d) The velocity of the particle at position x at time t is, by equation (ii),

$$\upsilon=\frac{\partial y}{\partial t}=-(1.0\,cm)(40\pi s^{-1})\sin\left[(40\,\pi\,s^{-1})t-\left(\frac{\pi}{2}m^{-1}\right)x\right]$$

Putting the values of x and t,

$$\upsilon=-(40\pi\,cms^{-1})\sin\left(2\pi-\frac{\pi}{2}\right)=\frac{40\pi}{\sqrt{2}}cms^{-1}\approx 89cms^{-1}$$

**Example 10:** The speed of a transverse wave traveling through a wire is 80 m $s^{-1}$. The length of the wire is 50 cm, the mass is 5.0 g, the area of cross-section of the wire is 1.0 $mm^2$, and its Young modulus is 16 ×$10^{11}$ $Nm^{-2}$. Find the extension of the wire over its natural length.

**Sol:** The maximum velocity of the wave is $v=\sqrt{F/\mu}$ where F is the tension in the string and μ is mass per unit length of string. And the Young's modulus of the string is $Y=\frac{F/A}{\Delta L/L}$.

The linear mass density is

$$\mu=\frac{5\times10^{-3}kg}{50\times10^{-2}m}=1.0\times10^{-2}kgm^{-1}$$

The wave speed is $\upsilon=\sqrt{F/\mu}$ .

Thus, the tension is

$F=\mu v^2=(1.0\times10^{-2}\ kgm^{-1})\times6400\ m^2s^{-2}=64\ N$

The Young modulus is given by $Y=\frac{F/A}{\Delta L/L}$

The extension is, therefore,

$$\Delta L=\frac{FL}{AY}=\frac{(64N)(0.50m)}{(1.0\times10^{-6}m^2)\times(16\times10^{11}Nm^{-2})}=0.02mm$$

# JEE Advanced/Boards

**Example 1:** The interference of two traveling waves of equal amplitude and frequency moving in opposite directions along a string produces a standing wave having the equation

$y = A\cos kx \sin\omega t$ in which A = 1.0 mm, $k = 1.57cm^{-1}$ and $\omega = 78.5s^{-1}$.

(a) Find the velocity of the component traveling waves.

(b) Find s the node closest to the origin in the region x > 0.

(c) Find the antinode closest to the origin in the region x>0

(d) Find the amplitude of the particle at x= 2.33 cm.

**Sol:** Here the two waves of same amplitude and frequency interfere with each other to form the standing waves, the velocity of the resultant wave will be $V = \frac{\omega}{K}$ where ω is the angular frequency of the wave and K is the wave number. At the node the waves are 90° opposite in phase, so that the amplitude of resulting wave is zero at the node.

(a) The standing wave is formed by the superposition of the waves

$$y_1 = \frac{A}{2}\sin(\omega t - kt) \text{ and } y_2 = \frac{A}{2}\sin(\omega t + kt)$$

The wave velocity (magnitude) of either of the waves is

$$v = \frac{\omega}{k} = \frac{78.5s^{-1}}{1.57cm^{-1}} = 50cm/s.$$

(b) For a node, cos kx = 0

The smallest position value of x satisfying this relation is given by $kx = \frac{\pi}{2}$

$$\text{Or} \quad x = \frac{\pi}{2k} = \frac{3.14}{2\times 1.57cm^{-1}} = 1cm$$

(c) For an antinode, |coskx| = 1

The smallest positive x satisfying this relation is given by $kx = \pi$ or $x = \frac{\pi}{k} = 2cm$

(d) The amplitude of vibration of the particle at x is given by |Acoskx|. For the given point,

$$kx = (1.57\,cm^{-1})(2.33cm) = \frac{7}{6}\pi = \pi + \frac{\pi}{6}$$

Thus, the amplitude will be (1.0mm)|

$$\cos(\pi + \pi/6)| = \frac{\sqrt{3}}{2}mm = 0.86mm$$

**Example 2:** A tuning fork of frequency 500 Hz is used to generate a transverse harmonic wave of amplitude 0.01 m at one end (x = 0) of a long, horizontal string. At a given instant of time the displacement of the particle at x = 0.1 m is –0.005m and that of the particle at x = 0.2 m is +0.005m. Calculate the wavelength and wave velocity. Assuming that the wave is traveling along the positive direction x and that the end x = 0 is at equilibrium position at t = 0, obtain the equation of the wave.

**Sol:** The fork is the source to generate the transverse wave on string whose frequency is also 500 Hz. The equation of this wave is given by $y = A\sin(kx - \omega t)$ where k is the wave number and x is the displacement of particle. The wave velocity is given by $V = \nu\lambda$ where υ is the frequency of source

Since the wave is traveling along positive direction x and the displacement of the end x = 0 is at time t = 0, the general equation of this wave is

$$y(x,t) = A\sin\left\{\frac{2\pi}{\lambda}(\upsilon t - x)\right\} \quad \text{... (i)}$$

Where A = 0.01 m. When x = 0.1m, Y = -0.005m

$$\therefore \quad -0.005 = 0.01\sin\left\{\frac{2\pi}{\lambda}(\upsilon t - x_1)\right\}$$

$$\text{Where} \quad x_1 = 0.1m \text{ or } \sin\left\{\frac{2\pi}{\lambda}(\upsilon t - x_1)\right\} = -\frac{1}{2}$$

$$\therefore \text{Phase } \sin\phi_1 = \frac{2\pi}{\lambda}(\upsilon t - x_1) = \frac{7\pi}{6} \quad \text{... (ii)}$$

When x = 0.2m y= + 0.005. Therefore, we have +0.005

$$= 0.01 \sin\left\{\frac{2\pi}{\lambda}(Vt - x_2)\right\}$$

Where $x_2$ = 0.2 m

$$\therefore \phi_2 = \frac{2\pi}{\lambda}(Vt - x_1) = \frac{2\pi}{6} \quad \text{... (iii)}$$

From eqs. (ii) and (iii)

$$\therefore \quad \Delta\phi = \phi_1 - \phi_2 = \pi$$

$$\text{Now,} \quad \Delta\phi = -\frac{2\pi}{\lambda}\Delta x \quad \text{thus,}$$

$$\pi = -\frac{2\pi}{\lambda}(x_1 - x_2) = \frac{2\pi}{\lambda}(0.1 - 0.2) \quad \text{or } \lambda = 0.2m$$

Now, frequency n of the wave = frequency of the tuning fork = 500 Hz. Hence, wave velocity

$$\upsilon = n\lambda = 500\times 0.2 = 100ms^{-1}$$

Substituting for A, λ, and v in equation. (i) We get $y(x,t)=0.01\sin\{10\pi(100t-x)\}$

This is the equation of the wave where y and x are in meters and t in seconds.

**Example 3:** Two tuning forks A and B sounded together produce 6 beats per second. With the introduction of an air resonance tube closed at one end, the two forks give resonance when the two air columns are 24 cm and 25 cm, respectively. Calculate the frequencies of forks.

**Sol:** Beats are produced when the two waves of similar amplitude but different frequency are interacting with each other. For vibrating air column $\frac{\ell_1}{\ell_2}=\frac{f_2}{f_1}$ where $\ell$ is length of vibrating air column and f is frequency of tuning fork.

Let the frequency of the first fork be $f_1$ and that of second be $f_2$.

We then have, $f_1=\frac{v}{4\times 24}$ and $f_2=\frac{v}{4\times 25}$

We also see that $f_1 > f_2$

$\therefore \quad f_1 - f_2 = 6$ ... (i)

And $\frac{f_1}{f_2}=\frac{25}{24}$ ... (ii)

Solving (i) and (ii), we get $f_1=150$ Hz and $f_2=144$ Hz

**Example 4:** The oscillation of a string of length 60 cm fixed at both ends is represented by the equation: $y=4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$ where x and y are in cm and *t* is in seconds.

(a) What is the maximum displacement of a point at x = 5 cm?

(b) Where are the nodes located along the string?

(c) What is the velocity of a particle at x = 7.5 cm & at t = 0.25 sec.

(d) Write down the components waves which give the above wave on superposition.

**Sol:** The wave equation oscillating string is written in form of $y=2a\cos\left(\frac{2\pi x}{\lambda}\right)\sin\left(\frac{2\pi vt}{\lambda}\right)$ where x is displacement and v is velocity of wave. The maximum velocity of wave is $v=\frac{\partial y}{\partial t}$ and the distance of nodes from any fixed end of string found using relation $kx=\frac{\pi}{2}$

Comparing given equation with equation of standing wave, $y=2a\cos\left(\frac{2\pi x}{\lambda}\right)\sin\left(\frac{2\pi vt}{\lambda}\right)$

$\frac{2\pi}{\lambda}=\frac{\pi}{15,}$; $\lambda=30$ cm; $a=2$ cm

$\frac{2\pi v}{\lambda}=96\pi \quad \Rightarrow v=1440\text{ cm/sec}$

(a) $x = 5$ cm, $y_{max}=4\sin\left(\frac{\pi\times 5}{15}\right)=\frac{4\sqrt{3}}{2}=2\sqrt{3}$ cm

(b) As $\lambda=30$ cm; nodes are at 0, 15, 30, 45, 60 cm

(c) $\frac{\partial y}{\partial t}=-4\sin\left(\frac{\pi x}{15}\right)\sin(96\pi r)\times 96\pi$

x = constant For x = 7.5 cm, t = 0.25 sec.

$\frac{\partial y}{\partial t}=-4\sin\left(\frac{\pi\times 7.5}{15}\right)\sin(96\pi\times 0.25)\times 96\pi=0$

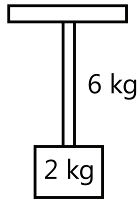
(d) Component waves $y=-4\sin\left(\frac{\pi x}{15}\right)\sin(96\pi t)$

$=2\sin\left(\frac{\pi x}{15}+96\pi t\right)+2\sin\left(\frac{\pi x}{15}-96\pi t\right)$

$\Rightarrow$ Component waves are $y_1=2\sin\left(\frac{\pi x}{15}+96\pi t\right)$;

$y_2=2\sin\left(\frac{\pi x}{15}-96\pi t\right)$

**Example 5:** A uniform rope hangs vertically from a rigid support with a slab of mass 2 kg attached to the free end of the rope. The rope has a length of 12 m and a mass of 6 kg. A transverse pulse of wavelength 0.06 m is produced at the free end of the rope. Determine the wavelength of the pulse when it reaches the top of the rope?




**Sol:** The wave velocity will be $V=\nu\lambda=\sqrt{\frac{F}{\mu}}$ where F is the tension in rope at a point and μ is mass per unit length of the string. As F is varying along the length of the rope so the velocity will vary along the length of the rope. As source frequency is constant $\lambda$ will vary.

As the rope is stretched using a slab, its tension will be different at different points along the length of the rope. The tension at the free end will be (2 kg) g while at the upper end it will be (8kg) g.

We have, $v = \nu\lambda \Rightarrow \sqrt{F/\mu} = \nu\lambda$ or $\sqrt{F}/\lambda = \nu\sqrt{\mu}$ ... (i)

Since the frequency of the wave pulse is dependent only on the frequency of the source, it will be consistent across the length of the rope. The mass per unit length will also be consistent for the entire rope as the rope is uniform. Thus, by

(i) $\frac{\sqrt{F}}{\lambda}$ is constant . Hence, $\frac{\sqrt{(2kg)g}}{0.06m} = \frac{\sqrt{(8kg)g}}{\lambda_1}$,

Where $\lambda_1$ is the wavelength at the top of the rope, this gives $\lambda_1 = 0.12m$.

**Example 6:** Two waves passing through a region are represented by

$y = (1.0\,cm)\sin[(3.14\,cm^{-1})x - (157s^{-1})t]$

and $y = (1.5\,cm)\sin[(1.57\,cm^{-1})x - (314\,s^{-1})t]$

Find the displacement of the particle at x = 4.5 cm at time t = 5.0 ms.

**Sol:** As the waves are superimposed on each other, the resultant displacement is $Y = y_1 + y_2$.

According to the principle of superposition, each wave produces its own disturbance and the resultant disturbance is equal to the vector sum of the individual disturbances. The displacements of the particle at x = 4.5cm at time t = 5.0 due to the two waves are,

$$y_1 = (1.0\,cm)\sin[(3.14\,cm^{-1})(4.5\,cm) - (157s^{-1})](5.0\times10^{-3}s)$$

$$= (1.0\,cm)\sin\left(4.5\pi - \frac{\pi}{4}\right) = (1.0\,cm)\sin[4\pi + \pi/4]$$

$$= \frac{1.0\,cm}{\sqrt{2}} \text{ and}$$

$$y_2 = (1.5\,cm)\sin[(1.57\,cm^{-1})(4.5\,cm) - (314s^{-1})] \times (5.0\times10^{-3}s)$$

$$= (1.5\,cm)\sin\left(2.25\pi - \frac{\pi}{2}\right) = (1.5\,cm)\sin[2\pi + \pi/4]$$

$$= -(1.5\,cm)\sin\frac{\pi}{4} = -\frac{1.5\,cm}{\sqrt{2}}$$

The net displacement is

$$y = y_1 + y_2 = \frac{-0.5cm}{\sqrt{2}} = -0.35\,cm.$$

**Example 7:** The vibrations of a string fixed at both ends are described by the equation

$y = (5.00mm)\sin[(1.57cm^{-1})x]\sin[(314\,s^{-1})t]$

(a) What is the maximum displacement of the particle at x =5.66cm?

(b) What are the wavelengths and the wave speeds of the two transverse waves that combine to give the above vibration?

(c) What is the velocity of the particle at x = 5.66 cm at time t = 2.00s?

(d) If the length of the string is 10.0 cm, locate the nodes and the antinodes. How many loops are formed in the vibration?

**Sol:** The transverse velocity of particle of string is $u = \frac{\partial y}{\partial t}$. The wave velocity is $V = \nu\lambda$. Comparing wave equation with y=A sin kx sin $\omega$t, we get the amplitude A and angular frequency of the wave.

(a) The amplitude of the vibration of the particle at position x is

$A = |(5.00\text{ mm})\sin[(1.57cm^{-1})x]$

For $x = 5.66\,cm$,

$$A = \left|(5.00\text{ mm})\sin\left[\frac{\pi}{2}\times 5.66\right]\right|$$

$$= \left|(5.00\text{ mm})\sin\left(2.5\pi + \frac{\pi}{3}\right)\right|$$

$$= \left|(5.00\text{ mm})\cos\frac{\pi}{3}\right| = 2.50\text{ mm}$$

(b) From the given equation, the wave number $k = 1.57\text{ cm}^{-1}$ and the angular frequency $\omega$ = 314 s-1. Thus, the wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\times3.14}{1.57\text{ cm}^{-1}} = 4.00\text{ cm}$$

and Frequency is $\nu = \frac{\omega}{2\pi} = \frac{314s^{-1}}{2\times3.14} = 50\text{ s}^{-1}$

The wave speed is

$\upsilon = \nu\lambda = (50s^{-1})\,(4.00\text{ cm}) = 2.00ms^{-1}$.

(c) The velocity of the particle at position x at time t is given by

$$\upsilon = \frac{\partial y}{\partial t} = (5.00\text{ mm})\sin[(1.57\text{ cm}^{-1})x]\,[314s^{-1} \times\cos(314s^{-1})t]$$

$= (157\ m\ s^{-1})$ in $(1.57\ cm^{-1})\ x \cos(314\ s^{-1})t$

(d) The nodes occur where the amplitude is zero, i.e., sin (1.57cm-1) x = 0 or $\left(\frac{\pi}{2}cm^{-1}\right)x = n\pi$

Where n is an integer. Thus, x = 2n cm.

The nodes, therefore, occur at x = 0, 2 cm, 4 cm, 6 cm, 8cm and 10 cm. Antinodes occur in between them, i.e., at x = 1 cm, 3 cm, 5cm, 7 cm and 9 cm. The string vibrates in 5 loops.

**Example 8:** A guitar of 90 cm length has a fundamental frequency of 124 Hz. Where should it be pressed to produce a fundamental frequency of 186 Hz?

**Sol:** As wires of guitar resemble the sonometer wire, thus the fundamental frequency of the guitar wire fixed at both ends is $\nu = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$. And for two vibrating strings, the ratio of their vibrating lengths is $\frac{\ell_1}{\ell_2} = \frac{\nu_2}{\nu_2}$.

The fundamental frequency of a string fixed at both ends is given by $\nu = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$

As F and $\mu$ are fixed,

$\frac{\nu_1}{\nu_2} = \frac{L_2}{L_1}$ or $L_2 = \frac{\nu_1}{\nu_2}L_1$

$= \frac{124\,Hz}{186\ Hz}(90\ cm) = 60\ cm$

Thus, the string should be pressed at 60 cm from an end.

**Example 9:** The total length of a sonometer wire is 1 m between the fixed ends. Where the two bridges should be placed in the sonometer so that the three segments of the wire have their fundamental frequencies in the ratio 1:2:3?

**Sol:** For sonometer the ratio of length of wires is $L \propto \frac{1}{\nu}$

where v is the frequency of the wave and L is length of vibrating string.

Suppose the lengths of the three segments are $L_1$, $L_2$, and $L_3$, respectively. The fundamental frequencies are

$\nu_1 = \frac{1}{2L_1}\sqrt{F/\mu}$

$\nu_2 = \frac{1}{2L_2}\sqrt{F/\mu}$ ; $\nu_3 = \frac{1}{2L_3}\sqrt{F/\mu}$

So that $\nu_1 L_1 = \nu_2 L_2 = \nu_3 L_3$. ... (i)

As $\nu_1 : \nu_2 : \nu_3 = 1:2:3$ we have

$\nu_2 = 2\nu_1$ and $\nu_3 = 3\nu_1$ so that by (i)

$L_2 = \frac{\nu_1}{\nu_2}L_1 = \frac{L_1}{2}$ and $\nu_3 = \frac{\nu_1}{\nu_3}L_1 = \frac{L_1}{3}$ and

$L_1 + L_2 + L_3 = 1m$

We get $L_1\left(1 + \frac{1}{2} + \frac{1}{3}\right) = 1m$

$L_1 = \frac{6}{11}m$ Thus, $L_2 = \frac{L_1}{2} = \frac{6}{11}m$

$L_3 = \frac{\nu_1}{3} = \frac{2}{11}m$

One bridge should be placed at $\frac{6}{11}m$ from one end and the other should be placed at $\frac{2}{11}m$ from other end.

**Example 10:** A wire having a linear mass density 5.0 $\times 10^{-3}$ kg $m^{-1}$ resonates at a frequency of 420 Hz when it is stretched between two rigid supports with a tension of 450 N.. The next higher frequency at which the same wire resonates is 490 Hz. Find the length of the wire.

**Sol:** For vibrating string the $n^{th}$ harmonic of fundamental frequency is $f = \frac{n}{2L}\sqrt{\frac{F}{\mu}}$. Here L is the length of vibrating string and F is the tension in the string. The two given frequencies correspond to two consecutive values n and (n+1).

Suppose the wire vibrates at 420 Hz in its nth harmonic and at 490 Hz in its $(n + 1)^{th}$ harmonic.

$420\,s^{-1} = \frac{n}{2L}\sqrt{F/\mu}$ ... (i)

and $490\,s^{-1} = \frac{(n+1)}{2L}\sqrt{F/\mu}$ ... (ii)

This gives $\frac{490}{420} = \frac{(n+1)}{n}$ or n = 6

Putting the value in (i),

$$420 s^{-1} = \frac{6}{2L}\sqrt{\frac{450N}{5.0 \times 10^{-3}\,kgm^{-1}}} = \frac{900}{L}\,ms^{-1}$$

Or $L = \frac{900}{420}m = 2.1m$

# JEE Main/Boards

## Exercise 1

**Q.1** Audible frequencies have a range 40 hertz to 30,000 hertz. Explain this range in terms of

(i) Period T

(ii) Wavelength $\lambda$ in air, and

(iii) Angular frequency

Give velocity of sound in air is 350 $ms^{-1}$

**Q.2** From a radio station, the frequency of waves is 15 Mega cycle/sec. Calculate their wavelength.

**Q.3** The velocity of sound in air at N.T.P is 331 $ms^{-1}$. Find its velocity when the temperature rises to $91^0$C and its pressure is doubled.

**Q.4** A displacement wave is represented by = 0.25 × $10^{-3}$ sin (500 t – 0.025 x). Deduce (i) amplitude (ii) period (iii) angular frequency (iv) wavelength (v) amplitude of particle velocity (vi) amplitude of particle acceleration. , the t and x are in cm, sec and meter respectively.

**Q.5** The length of a sonometer wire between two fixed ends is 110 cm. Where should be two bridges be placed so as to divide the wire into three segments, whose fundamental frequencies are in the ratio 1: 2: 3?

**Q.6** Calculate the velocity of sound in a gas, in which two wave lengths 2.04 m and 2.08 m produce 20 beats in 6 seconds.

**Q.7** A tuning fork of unknown frequency gives 6 beats per second with a tuning fork of frequency 256. It gives same number of beats/ sec when loaded with wax. Find the unknown frequency.

**Q.8** Is it possible to have longitudinal waves on a string? A transverse wave in a steel rod?

**Q.9** What type of mechanical waves do you expect to exist in (a) vacuum (b) air (c) inside the water (d) rock (e) on the surface of water?

**Q.10** What will be the speed of sound in a perfect rigid rod?

**Q.11** What is the distance between compression and its nearest rarefaction in a longitudinal wave?

**Q.12** What is the distance between a node and an adjoining antinode in a stationary wave?

**Q.13** Explain why waves on strings are always transverse.

**Q.14** What is a wave function? Give general form of wave function. What is a periodic function?

**Q.15** Distinguish between harmonics and overtones.

**Q.16** A stone is dropped into a well in which water is 78.4 m deep. After how long will the sound of splash be heard at the top? Take velocity of sound in air = 332 $ms^{-1}$

**Q. 17** From a cloud at an angle of $30^0$ to the horizontal, we hear the thunder clap 8s after seeing the lightening flash. What is the height of the cloud above the ground if the velocity of sound in air is 330 m/s?

**Q.18** A steel wire 0.72m long has a mass of $5.0 \times 10^{-3}$ kg. If the wire is under a tension of 60 N, what is the speed of transverse wave on the wire?

**Q.19** For a metal, bulk modulus of elasticity is $7.5 \times 10^{10}$ $Nm^{-2}$, and density is $2.5 \times 10^3$ $m^{-3}$. Deduce the velocity of longitudinal waves.

**Q.20** A steel wire 70 cm long has mass of 7g. If the wire is under a tension of 100 N, what is the speed of transverse waves in the wire?

**Q.21** Two waves of angular frequencies 50 and 5000 rad $s^{-1}$ have the same displacement amplitude, $3 \times 10^{-5}$ cm. Deduce the acceleration amplitude for them.

**Q.22** The equation of a wave traveling in x- direction on a string is y = (3.0 cm) sin [(3.14$cm^{-1}$) x – (314 $s^{-1}$)t]

(a) Find the max. Velocity of a particle of the string.

(b) Find the acceleration of a particle at x = 6.0 cm and at time t = 00.11 s.

**Q.23** A fork of frequency 250 Hz is held over and maximum sound is obtained when the column of air is 31 cm or 97 cm. Determine (i) velocity of sound (ii) the end correction (iii) the radius of the tube.

**Q.24** In an experiment, it was found that a tuning fork and a sonometer gave 5 beats/sec, both when length of wire was 1 m and 1.05m. Calculate the frequency of the fork.

## Exercise 2

### Single Correct Choice Type

**Q.1** A wave is propagating along x–axis. The displacement of particle of the medium in z – direction at t = 0 is given by: z = exp [- (x +2)$^2$], where 'x' is in meters. At t = 1s, the same wave disturbance is given by: z = exp [-2 ( 2-x ) $^2$]. Then, the wave propagation velocity is

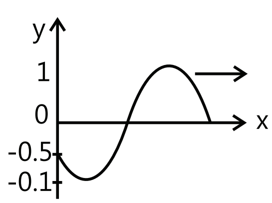
(A) 4 m/s in + x direction

(B) 4 m/s in - x direction

(C) 2 m/s in + x direction

(D) 2 m/s in - x direction

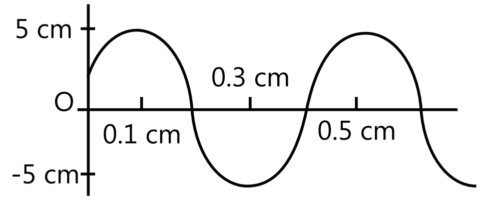
**Q.2** The equation of a wave traveling along the positive x – axis, as shown in figure at t = 0 is given by




(A) $\sin\left(kx-\omega t+\frac{\pi}{6}\right)$ (B) $\sin\left(kx-\omega t-\frac{\pi}{6}\right)$

(C) $\sin\left(\omega t-kx+\frac{\pi}{6}\right)$ (D) $\sin\left(\omega t-kx-\frac{\pi}{6}\right)$

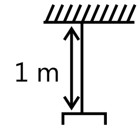
**Q.3** In the figure shown the shape of part of a long string in which transverse wave are produced by attaching one end of the string to tuning fork of frequency 250 Hz. What is the velocity of the waves?




(A) 1.0 ms$^{-1}$ (B) 1.5 ms$^{-1}$

(C) 2.0 ms$^{-1}$ (D) 2.5 ms$^{-1}$

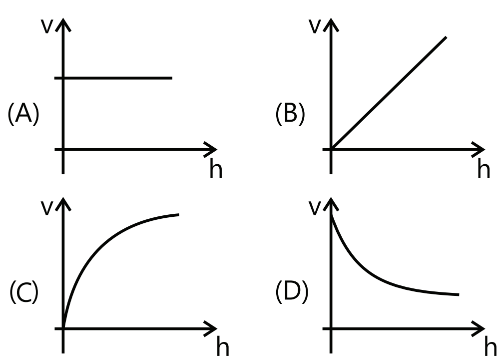
**Q.4** A block of mass 1 kg is hanging vertically from a string of length 1 m and mass/ length = 0.001 Kg/m. A small pulse is generated at its lower end. The pulse reaches the top end in approximately




(A) 0.2 sec (B) 0.1 sec

(C) 0.02 sec (D) 0.01 sec

**Q. 5** A uniform rope having some mass hangs vertically from a rigid support. A transvers wave pulse is produced at the lower end. The speed (v) of the wave pulse varies with height (h) from the lower end as:




**Q. 6** A wire of 10$^{-2}$ kgm$^{-1}$ passes over a frictionless light pulley fixed on the top of a frictionless inclined plane which makes an angle of 30$^0$ with the horizontal. Masses m and M are tied at two ends of wire such that m rests on the plane and M hangs freely vertically downwards. The entire system is in equilibrium and a transverse wave propagates along the wire with a velocity of 100 ms$^{-1}$. Then,

(A) M = 5kg (B) $\frac{m}{M}=\frac{1}{4}$

(C) m=20kg (D) $\frac{m}{M}=4$

**Q.7** Consider a function $y = 10 \sin^2 (100\pi t + 5 \pi z)$ where y, z are in cm and t is in second.

(A) The function represents a traveling, periodic wave propagating in (-z) direction with speed 20m/s.

(B) The function does not represent a traveling wave.

(C) The amplitude of the wave is 5 cm.

(D) The amplitude of the wave is 10 cm.

**Q. 8** The displacement from the position of equilibrium of a point 4 cm from a source of sinusoidal oscillations is half the amplitude at the moment t = T/ 6 (T is the time period). Assume that the source was at mean position at t = 0. The wavelength of the running wave is

(A) 0.96m (B) 0.48m (C) 0.24m (D) 0.12m

**Q. 9** The period of oscillations of a point is 0.04 sec. and the velocity of propagation of oscillation is 300m/sec. The difference of phases between the oscillations of two points at distance 10 and 16m respectively from the source of oscillations is

(A) $2\pi$ (B) $\pi/2$ (C) $\pi/4$ (D) $\pi$

**Q.10** A motion is described by $y = \dfrac{3}{a^2 + (x+3t)^2}$ where y, x are in meter and t is in second.

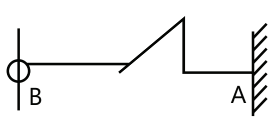
(A) This represents equation of progressive wave propagation along – x direction with 3 $ms^{-1}$.

(B) This represents equation of progressive wave propagation along + x direction with 3 $ms^{-1}$.

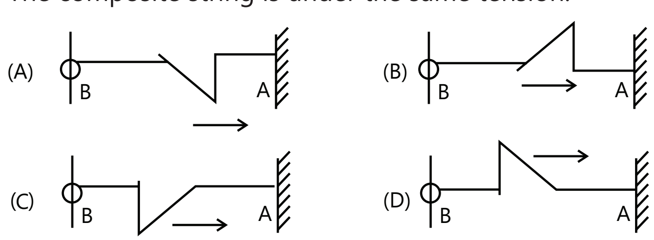
(C) This does not represent a progressive wave equation.

(D) Data is insufficient to arrive at any conclusion.

**Q.11** A pulse shown here is reflected from the rigid wall A and then from free end B. The shape of the string after these 2 reflection will be




**Q.12** A composition string is made up by joining two strings of different masses per unit length →μ and 4μ. The composite string is under the same tension.




A transverse wave pulse: Y = (6 mm) sin (5t + 40x), where 't' is in seconds and 'x' in meters, is sent along the lighter string towards the joint. The joint is at x = 0. The equation of the wave pulse reflected from the joint is

(A) (2 mm) sin (5t – 40x)

(B) (4 mm) sin (40t – 5x)

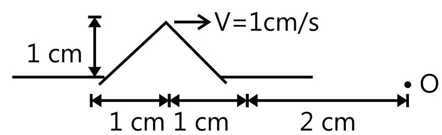
(C) - (2 mm) sin (5t – 40x)

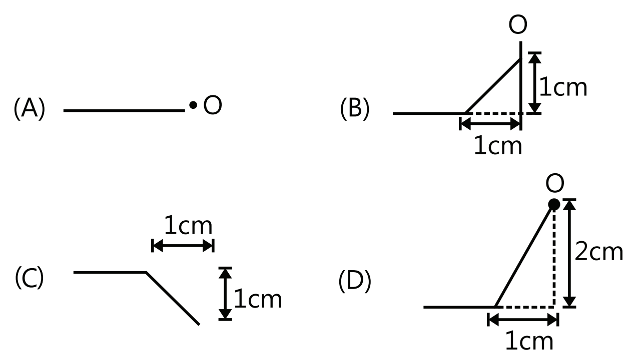
(D) (2 mm) sin (5t – 10x)

**Q. 13** In the previous question, the percentage of power transmitted to the heavier string through the joint is approximately

(A) 33% (B) 89% (C) 67% (D) 75%

**Q.14** A wave pulse on a string has the dimension shown in figure. The waves speed is V= 1 cm/s. If point O is a free end. The shape of wave at time t = 3 s is:




**Q.15** A string 1 m long is drawn by a 300 Hz vibrator attached to its end. The string vibrates in 3 segments. The speed of transverse waves in the string is equal to

(A) 100 m/s (B) 200 m/s

(C) 300 m/s (D) 400 m/s

**Q.16** The resultant amplitude due to superposition of two waves $y_1 = 5\sin(\omega t - kx)$ and $y_2 = -5\cos(\omega t - kx - 150^0)$

(A) 5 (B) $5\sqrt{3}$

(C) $5\sqrt{2-\sqrt{3}}$ (D) $5\sqrt{2+\sqrt{3}}$

**Q.17** A wave represented by the equation $y = A\cos(kx - \omega t)$ is superimposed with another wave to from a stationary wave such that the point x = 0 is a node. The equation of the other wave is:

(A) $-A\sin(kx + \omega t)$ (B) $-A\cos(kx + \omega t)$

(C) $A\sin(kx + \omega t)$ (D) $A\cos(kx + \omega t)$

**Q.18** A taut string at both ends vibrates in its $n^{th}$ overtone. The distance between adjacent Node and Antinode is found to be 'd'. If the length of the string is L, then

(A) $L = 2d(n+1)$ (B) $L = d(n+1)$

(C) $L = 2dn$ (D) $L = 2d(n-1)$

**Q.19** A metallic wire of length L is fixed between two rigid supports. If the wire is cooled through a temperature difference $\Delta T$ (Y = young's modulus, $\rho$ = density, $\alpha$ = coefficient of linear expansion) then the frequency of transverse vibration is proportional to:

(A) $\frac{\alpha}{\sqrt{\rho Y}}$ (B) $\frac{\sqrt{Y\alpha}}{\rho}$ (C) $\frac{\rho}{\sqrt{Y\alpha}}$ (D) $\sqrt{\frac{\rho\alpha}{\sqrt{Y}}}$

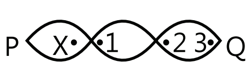
**Q.20** A standing wave $Y = A\sin\left(\frac{20}{3}\pi x\right)\cos(1000\pi t)$ is maintained in a taut string where y and x are expressed in meters. The distance between the successive points oscillating with the amplitude A/2 across a node is equal to

(A) 25 cm (B) 2.5 cm (C) 5 cm (D) 10 cm

**Q.21** A string of length 0.4m & mass $10^{-2}$kg is tightly clamped at its ends. The tension in the string is 1.6 N. Identical wave pulses are produced at one end at equal intervals of time, $\Delta t$. The minimum value of $\Delta t$ which allows constructive interference between successive pulses is:

(A) 0.05s (B) 0.10s (C) 0.20s (D) 0.40s

**Q. 22** Fig 11.46, show a stationary wave between two fixed points P and Q. which points (s) of 1, 2 and 3 are in phase with the point X?




(A) 1, 2 and 3 (B) 1 and 2 only

(C) 2 and 3 only (D) 3 only

**Q.23** A wave travels uniformly in all directions from a point source in an isotropic medium. The displacement of the medium at any point at a distance r from the source may be represented by (A is a constant representing strength of source)

(A) $[A/\sqrt{r}]\sin(kr - \omega t)$ (B) $[A/r]\sin(kr - \omega t)$

(C) $[Ar]\sin(kr - \omega t)$ (D) $[A/r^2]\sin(kr - \omega t)$

**Q.24** A sinusoidal progressive wave is generated in a string. Its equation is given by $Y = (2mm)\sin(2\pi x - 100\pi t + \pi/3)$. The time when particle at x = 4 m first passes through mean position, will be

(A) $\frac{1}{150}$ sec (B) $\frac{1}{12}$ sec

(C) $\frac{1}{300}$ sec (D) $\frac{1}{100}$ sec

**Q.25** A transverse wave is described by the equation $Y = A\sin[2\pi x(ft - x/\lambda)]$. The maximum particle velocity is equal to four times the wave velocity if:

(A) $\lambda = \pi A/4$ (B) $\lambda = \pi A/2$

(C) $\lambda = \pi A$ (D) $\lambda = 2\pi A$

## Previous Years' Questions

**Q. 1** A transverse wave is described by the equation $y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$. The maximum particle velocity is equal to four time the wave velocity if ***(1984)***

(A) $\lambda = \pi$ (B) $\lambda = \pi y_0/2$

(C) $\lambda = 2\pi$ (D) $\lambda = 2\pi y_0$

**Q.2** A wave represented by the equation $y = a\cos(kx - \omega t)$ is superimposed with another wave to from a stationary wave such that point x = 0 is a node. The equation for the other wave is ***(1988)***

**Q.3** The displacement y of a particle executing periodic motion is given by $y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000t)$. This expression may be considered to be a result of the superposition of ...................... Independent harmonic motions. ***(1992)***

(A) Two (B) Three (C) Four (D) Five

**Q.4** The extension in a string, obeying Hooke's law, is x. The speed of transverse wave in the stretched string is. If the extension in the string is increased to 1.5 x, the speed of transverse wave will be ***(1996)***

(A) 1.22 (B) 0.61 (C) 1.50 (D) 0.75

**Q. 5** A traveling wave in a stretched string is described by the equation; $Y = A \sin(kx - \omega t)$

The maximum particle velocity is ***(1997)***

(A) $A\omega$ (B) $\omega/\kappa$ (C) $d\omega/d\kappa$ (D) $x/\omega$

**Q.6** Two vibrating strings of the same material but of lengths L and 2L have radii 2r and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes. The one of length L with frequency $V_1$ and the other with frequency $V_2$. The ratio $V_1/V_2$ is given by ***(2000)***

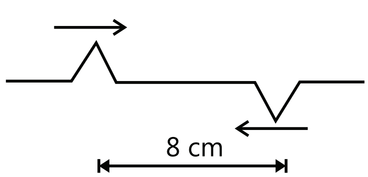
(A) 2 (B) 4 (C) 8 (D) 1

**Q.7** The ends of a stretched wire of length L are fixed at x = 0 and x = L. In one experiment the displacement of the wire is $y_1 = A\sin\left(\frac{\pi x}{L}\right)\sin\omega t$ and energy is $E_1$ and in other experiment its displacement is $y_2 = A\sin\left(\frac{2\pi x}{L}\right)\sin 2\omega t$ and energy is $E_2$. Then ***(2011)***

(A) $E_2 = E_1$ (B) $E_2 = 2E_1$

(C) $E_2 = 4E_1$ (D) $E_2 = 16E_1$

**Q.8** Two pulses in a stretched string, whose centers are initially 8 cm apart, are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2 s the total energy of the pulses will be ***(2001)***




(A) Zero

(B) Purely kinetic

(C) Purely potential

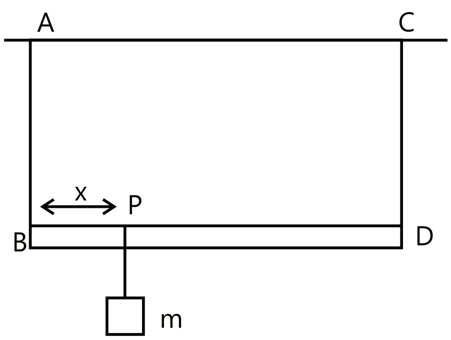
(D) Partly kinetic and partly potential

**Q.9** A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by mass M. The wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is ***(2002)***

(A) 25 kg (B) 5kg

(C) 12.5 kg (D) 1/25 kg

**Q.10** A massless rod BD is suspended by two identical massless strings AB and CD of equal lengths. A block of mass m is suspended from point P such that BP is equal to x. If the fundamental frequency of the left wire is twice the fundamental frequency of right wire, then the value of x is ***(2006)***




(A) l/5 (B) l/4 (C) 4l/5 (D) 3l/4

**Q.11** A hollow pipe of length 0.8 m is closed at one end. At its open end, a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is $320 ms^{-1}$, the mass of the string is ***(2010)***

(A) 5 kg (B) 10kg (C) 20 kg (D) 40 kg

**Q.12** The displacement of particles in a string stretched in the x – direction is represented by y. Among the following expressions for y, those describing wave motion is (are) ***(1987)***

(A) $\cos kx \sin\omega t$ (B) $k^2x^2 - \omega^2t^2$

(C) $\cos^2(kx + \omega t)$ (D) $\cos\left(k^2x^2 - \omega^2t^2\right)$

**Q.13** A wave is represented by the equation;
$y = A \sin\left(10\pi x + 15\pi t + \pi/3\right)$

Where x is in meter and t is in second. The expression represents ***(1990)***

(A) A wave traveling in the position x – direction with a velocity 1.5 m/s

(B) A wave traveling in the negative x – direction with a velocity 1.5 m/s

(C) A wave traveling in the negative x – direction with a wavelength 0.2 m

(D) A wave traveling in the position x – direction with a wavelength 0.2 m

**Q.14** Two identical straight wires are stretched so as to produce 6 beats/ s when vibrating simultaneously. On changing the tension slightly in one of them, the beat frequency remains unchanged. Denoting by $T_1$, $T_2$ the higher and the lower initial tension in the strings, then it could be said that while making the above changes in tension ***(1991)***

(A) $T_2$ was decreased (B) $T_2$ was increased

(C) $T_1$ was decreased (D) $T_1$ was increased

**Q. 15** A wave disturbance in a medium is described by

$$y(x,\ t) = 0.02\cos\left(50\pi t+\frac{\pi}{2}\right)\cos(10\pi x),$$

Where x and y are in meter and t is in second. ***(1995)***

(A) A node occurs at x = 0.15m

(B) An antinode occurs at x = 0.3 m

(C) The speed of wave is 5 $ms^{-1}$

(D) The wavelength of wave is 0.2 m

**Q.16** The (x, y) coordinates of the corners of a square plate are (0, 0), (L, 0), (L, L) and (0, L). The edges of the plates are clamped and transverse standing waves are set-up in it. If $u(x,\ y)$ denotes the displacement of the plate at the point (x, y) at some instant of time, the possible expression (s) for u is (are) (a = positive constant) ***(1998)***

(A) $a\cos(\pi x/2L)\cos(\pi y/2L)$

(B) $a\sin(\pi x/L)\sin(\pi y/L)$

(C) $a\sin(\pi x/L)\sin(2\pi y/L)$

(D) $a\cos(2\pi x/L)\cos(\pi y/L)$

**Q.17** A transverse sinusoidal wave of amplitude a, wavelength $\lambda$ and frequency is traveling on a stretched string. The maximum speed of any point on the string is $\nu/10$, where is the speed of propagation of the wave. If a = $10^{-3}$m and $\nu$ = 10m/s, then $\lambda$ and f are given by ***(1998)***

(A) $\lambda=2\pi\times10^{-2}$ m (B) $\lambda=10^{-3}$ m

(C) $f=\frac{10^{-3}}{2\pi}$Hz (D) f = $10^4$Hz

**Q.18** In a wave motion, y = a sin(kx – wt), y can represent ***(1999)***

(A) Electric field (B) Magnetic field

(C) Displacement (D) Pressure

**Q.19** Standing waves can be produced ***(1999)***

(A) On a string clamped at both ends

(B) On a string clamped at one end and free at the other

(C) When incident wave gets reflected from a wall

(D) When two identical waves with a phase difference of $\pi$ are moving in the same direction

**Q.20**. A wave travelling along the x-axis is described by the equation y(x, t) = 0.005 cos ($\alpha x - \beta t$). If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then $\alpha$ and $\beta$ in appropriate units are ***(2008)***

(A) $\alpha$ = 25.00 $\pi$, $\beta = \pi$ (B) $\alpha=\frac{0.08}{\pi},\frac{2.0}{\pi}$

(C) $\alpha=\frac{0.04}{\pi},\beta=\frac{1.0}{\pi}$ (D) $\alpha=12.50\pi,\beta=\frac{\pi}{2.0}$

**Q.21** The equation of a wave on a string of linear mass density 0.04 kg $m^{-1}$ is given by $y=0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(t)}-\frac{x}{0.50(m)}\right)\right]$. The tension in the string is ***(2010)***

(A) 4.0 N (B) 12.5 N

(C) 0.5 N (D) 6.25 N

**Q.22** The transverse displacement y (x,t) of a wave on a string is given by $y(a,t)=e^{-(ax^2+bt^2+2\sqrt{ab}xt)}$.This represents a ***(2011)***

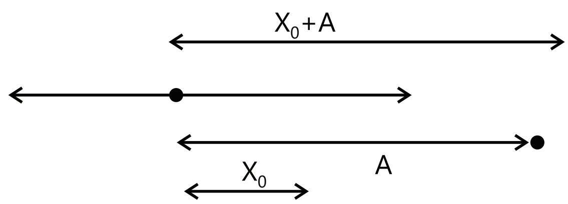
(A) Wave moving in – x direction with speed $\sqrt{\frac{b}{a}}$

(B) Standing wave of frequency $\sqrt{b}$

(C) Standing wave of frequency $\frac{1}{\sqrt{b}}$

(D) Wave moving in + x direction with $\sqrt{\frac{a}{b}}$

**Q.23** Two particles are executing simple harmonic motion of the same amplitude A and frequency $\omega$ along the x-axis. Their mean position is separated by distance $X_0$ ($X_0$ > A). If the maximum separation between them is ($X_0$ + A), the phase difference between their motion is: ***(2011)***




(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

**Q.24** A mass M, attached to a horizontal spring, executes S.H.M. with amplitude $A_1$. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude $A_2$. The ratio of $\left(\frac{A_1}{A_2}\right)$ is: ***(2011)***

(A) $\frac{M+m}{M}$ (B) $\left(\frac{M+m}{M}\right)^{1/2}$

(C) $\left(\frac{M}{M+m}\right)^{1/2}$ (D) $\frac{M}{M+m}$

**Q.25** A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%.What is the fundamental frequency of steel if density and elasticity of steel are $7.7 \times 10^3$ kg/m$^3$ and $2.2 \times 10^{11}$ N/m$^2$ respectively? ***(2013)***

(A) 188.5 Hz (B) 178.2 Hz

(C) 200.5 Hz (D) 770 Hz

**Q.26** The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90s using a wrist watch of 1s resolution. The accuracy in the determination of g is: ***(2015)***

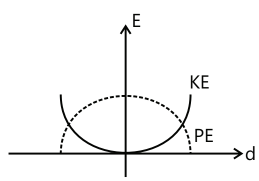
(A) 2% (B) 3% (C) 1% (D) 5%

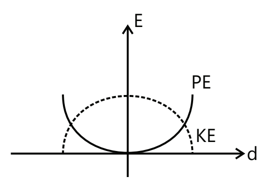
**Q.27** A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to $T_M$. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to : (g = gravitational acceleration) ***(2015)***

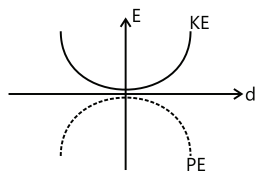
(A) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$ (B) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{A}$

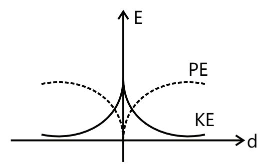
(C) $\left[1 - \left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (D) $\left[1 - \left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

**Q.28** For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (Graphs are schematic and not drawn to scale) ***(2015)***

(A)



(B)



(C)



(D)



**Q.29** A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is: (take g = 10 ms$^{-2}$) ***(2016)***

(A) 2s (B) $2\sqrt{2}$ s (C) $\sqrt{2}$ s (D) $2\pi\sqrt{2}$ s

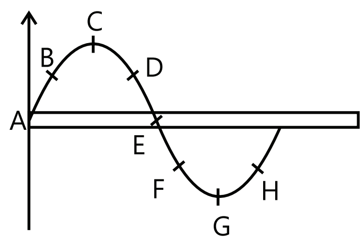
**Q.30** A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is. ***(2016)***

(A) 3A (B) $A\sqrt{3}$ (C) $\frac{7A}{3}$ (D) $\frac{A}{3}\sqrt{41}$

# JEE Advanced/Boards

## Exercise 1

**Q.1** A transverse wave is traveling along a string from left to right. The figure. represents the shape of the string (snap - shot) at a given instant. At this instant (a) which points have an upward velocity (b) which points will have downward velocity (c) which points have zero velocity (d) which points have maximum magnitude of velocity?

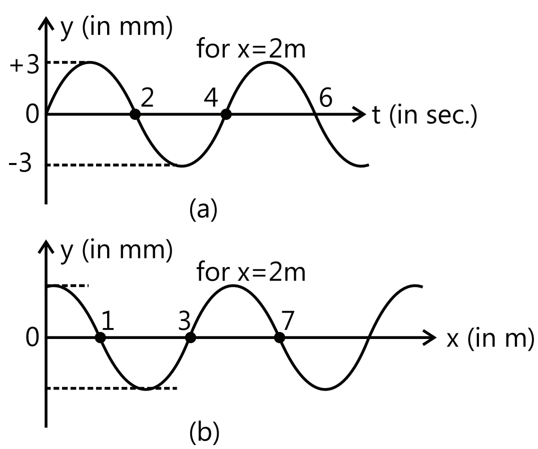



**Q.2** A sinusoidal wave propagates along a string. In figure (a) and (b). 'y' represents displacement of particle from the mean position. 'x' & 't' have usual meanings. Find:

(a) Wavelength, frequency and speed of the wave.

(b) Maximum velocity and maximum acceleration of the particles

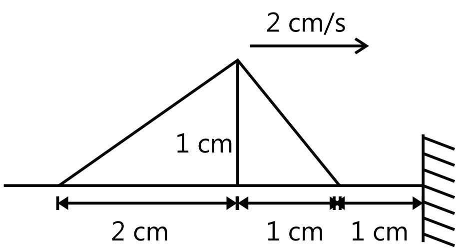
(c) The magnitude of slope of the string at x = 2 at t = 4 sec.




**Q.3** The extension in a string, obeying Hook's low is x. the speed of wave in the stretched string is v. If the extension in the string increased to 1.5x find the new the speed of wave.

**Q.4** A steel wire has a mass of 5gm and is under tension 450N. Find the maximum average power that can be carried by the transverse wave in the wire if the amplitude is not to exceed 20% of the wavelength.

**Q.5** The figure shown a triangle pulse on a rope at t = 0. It is approaching a fixed end at 2 cm/s




(a) Draw the pulse at t = 2sec.

(b) The particle speed on the leading edge at the instant depicted is______.

**Q.6** Two strings A and B with μ =2 kg/m and μ = 8 kg/m respectively are joined in series and kept on a horizontal table with both the ends fixed. The tension in the string is 200 N. If a pulse of amplitude 1 cm travels in A towards the junction, then find the amplitude of reflected and transmitted pulse.

**Q.7** A parabolic pulse given by equation y (in cm) = $0.3 - 0.1\,(x - 5t)^2$ ($y \geq 0$) x in meter and t in second traveling in a uniform string. The pulse passes through a boundary beyond which its velocity becomes 2.5 m/s. What will be the amplitude of pulse in this medium after transmission?

**Q.8** A 40 cm long wire having a mass 3.2 gm and area of c.s 1 $mm^2$ is stretched between the support 40.05 cm apart. In its fundamental mode, it vibrates with a frequency 1000/64 Hz. Find the young's modulus of the wire.

**Q.9** A string of mass 0.2 kg/m and length L = 0.6m is fixed at both ends and stretched such that is has a tension of 80 N. The string is vibrating in its third normal mode, has an amplitude of 0.5 cm. What is the frequency of oscillation? What is the maximum transverse velocity amplitude?

**Q.10** A rope**,** under tension of 200N and fixed at both ends, oscillates in a second – harmonic standing wave pattern. The displacement of the rope is given by:

$$Y = (0.10m)\ (\sin \pi x / 2) \sin 12\pi t$$

Where x = 0 at one end of the rope, x is in meters and t is in seconds. What are

(a) The length of the rope

(b) The speed of the progressive waves on the rope, and

(c) The mass of the rope

(d) If the rope oscillates in a third – harmonic standing wave pattern, what will be the period of oscillation?

**Q.11** A stretched uniform wire of a sonometer between two fixed knife edges, when vibrates in its second harmonic gives 1 beat per second with a vibrating tuning fork of frequency 200 Hz. Find the percentage change in the tension of the wire to be in unison with the tuning fork.

**Q.12** A string fixed at both ends has consecutive standing wave modes for which the distances between adjacent nodes are 18 cm and 16 cm respectively.

(a) What is the length of the string?

(b) If the tension is 10 N and the linear mass density is 4kg/m, what is the fundamental frequency?

**Q.13** In a mixture of gases, the average number of degree of freedom per molecules is 6. The rms speed of the molecules of the gas is c. find the velocity of sound in the gas.

## Exercise 2

### Single Correct Choice Type

**Q.1** A wave is represented by the equation $Y = 10\sin 2\pi(100t - 0.02x) + 10\sin 2\pi(100t + 0.02x)$ The maximum amplitude and loop length are respectively

(A) 20 units and 30 units

(B) 20 units and 25 units

(C) 30 units and 20 units

(D) 25 units and 20 units

**Q.2** A string of length 1 m and linear mass density 0.01 $kgm^{-1}$ is stretched to a tension of 100N. When both ends of the string are fixed, the three lowest frequencies for standing wave are $f_1$, $f_2$ and $f_3$. When only one end of the string is fixed, the three lowest frequencies for standing wave are $n_1$, $n_2$ and $n_3$. Then

(A) $n_3 = 5n_1 = f_3 = 125$ Hz

(B) $f_3 = 5f_1 = n_3 = 125$ Hz

(C) $f_3 = n_2 = 3f_1 = 150$ Hz

(D) $n_2 = \frac{f_1 + f_2}{2} = 75Hz$

**Q.3** A chord attached to a vibrating string from divides it into 6 loops, when its tension is 36N. the tension at which it will vibrate in 4 loops is

(A) 24N (B) 36N (C) 64N (D) 81N

**Q.4** A wave equation is given as y = cos (500t – 70x), where y in mm and t is in sec.

(A) The wave is not a transverse propagating wave.

(B) The speed of wave is 50/7 m/s

(C) The frequency of oscillation 1000$\pi$ Hz

(D) Two closest points which are in same phase have separation 45 $\pi$/7 cm.

**Q.5** A wave pulse passing on a string with a speed of 40 cm $s^{-1}$ in the negative x – direction has its maximum at x = 0 at t = 0. Where will this maximum be located at t = 5s?

(A) 2 m (B) 3 m (C) 1 m (D) 2.5 m

**Q.6** A steel wire of length 64 cm weights 5 g. If it is stretched by a force of 8 N, what would be the speed of a transverse wave passing on it?

(A) 10 m/s (B) 38 m/s (C) 32 m/s (D) 22 m/s

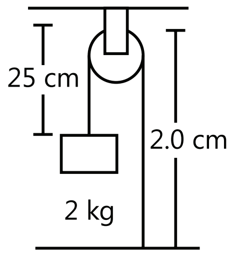
**Q.7** Two blocks each having a mass of 3.2 kg are connected by a wire CD and the system is suspended from the ceiling by another wire AB. The linear mess density of the wire AB is 10 g $m^{-1}$ and that of CD is 8 $gm^{-1}$. Find the speed of a transverse wave pulse produced in AB and in CD.

(A) 80 m/s, 63 m/s (B) 75 m/s, 54 m/s

(C) 82 m/s, 33 m/s (D) 87 m/s , 60 m/s

**Q.8** In the arrangement shown in figure, the string has a mass of 4.5 g. How much time will it take for a transverse disturbance produced at the floor to reach the pulley?

(Take g = 10 ms$^{-2}$)




(A) 0.03 s (B) 0.02 s (C) 0.01 s (D) 0.04 s

**Assertion Reasoning Type**

**Q.9 Statement-I:** In a sinusoidal traveling wave on a string potential energy of deformation of string element at extreme position is maximum

**Statement-II:** The particle in sinusoidal traveling wave perform SHM.

(A) Statement-I is true, statement-II is true, statement-II is a correct explanation for statement-I

(B) Statement-I is true, statement-II is true, statement-II is NOT correct explanation for statement-I

(C) Statement-I is true, statement-II is false

(D) Statement-I is false, statement-II is true

**Q. 10 Statement-I:** When a pulse on string reflects from free end, the resultant pulse is formed in such a way that slope of string at free end is zero.

**Statement-II:** Zero resultant slope ensures that there is no force components perpendicular to string.

(A) Statement-I is true, statement-II is true, statement-II is a correct explanation for Statement-I

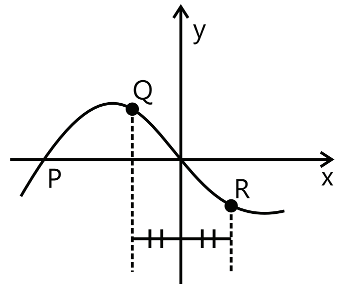
(B) Statement-I is true, statement-II is true, statement-II is NOT correct explanation for Statement-I

(C) Statement-I is true, statement-II is false

(D) Statement-I is false, statement-II is true

**Multiple Correct Choice Type**

**Q.11** At a certain moment, the photograph of a string on which a harmonic wave is traveling to the right is shown. Then, which of the following is true regarding the velocities of the points P, Q and R on the string.

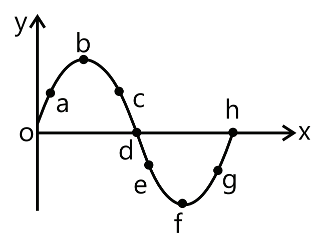



(A) $V_P$ is upwards (B) $V_Q = -V_R$

(C) $|V_P| > |V_Q| = |V_R|$ (D) $V_Q = V_R$

**Comprehension Type**

The figure represents the instantaneous picture of a transverse harmonic wave traveling along the negative X – axis. Choose the correct alternative (s) related to the movement of the mine points shown in the figure.




**Q.12** The point/s moving upward is/are

(A) a (B) c (C) f (D) g

**Q.13** The point/s moving downwards is/are

(A) o (B) b (C) d (D) h

**Q.14** The stationary points is/ are

(A) o (B) b (C) f (D) h

**Q.15** The point/s moving with maximum velocity is/are

(A) b (B) c (C) d (D) h

## Previous Years' Questions

**Q. 1** An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water, so that one half of its volume is submerged. The new fundamental frequency (in Hz) is ***(1995)***

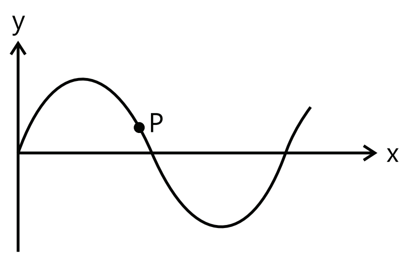
(A) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ (B) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$

(C) $300\left(\frac{2\rho}{2\rho-1}\right)$ (D) $300\left(\frac{2\rho-1}{2\rho}\right)$

**Q.2** A string of length 0.4m and mass $10^{-2}$ kg is tightly clamped at its ends. The tension in the string is 1.6N. Identical wave pulses are produced at one end at equal intervals of time $\Delta t$, The minimum value of $\Delta t$, which allows constructive interference between successive pulses, is ***(1998)***

(A) 0.05 s (B) 0.10 s (C) 0.20 s (D) 0.40s

**Q. 3** A transverse sinusoidal wave moves along a string in the positive x – direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap – shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is ***(2008)***




(A) $\frac{\sqrt{3}\pi}{50}\hat{j}\,m/s$ (B) $-\frac{\sqrt{3}\pi}{50}\hat{j}\,m/s$

(C) $\frac{\sqrt{3}\pi}{50}\hat{i}\,m/s$ (D) $-\frac{\sqrt{3}\pi}{50}\hat{i}\,m/s$

**Q.4** A vibrating string of certain length l under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats/s when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased the number of beats reduces to 2 per second. Assuming the velocity of sound in air to be 340 m/s, the frequency n of the tuning fork in Hz is ***(2008)***

(A) 344 (B) 336 (C) 117.3 (D) 109.3

**Paragraph 1:**

Two plane harmonic sound waves are expressed by the equations.

$y_1(x,t) = A\cos(\pi x - 100\pi t)$

and $y_2(x,t) = A\cos(0.4\pi x - 92\pi t)$

(All parameters are in MKS) ***(2006)***

**Q.5** How many times does an observer hear maximum intensity in one second?

(A) 4 (B) 10 (C) 6 (D) 8

**Q.6** What is the speed of sound?

(A) 200 m/s (B) 180 m/s (C) 192 m/s (D) 96 m/s

**Q.7** At x = 0 how many times the amplitude of $y_1 + y_2$ is zero in one second?

(A) 192 (B) 48 (C) 100 (D) 96

**Q.8** A wave equation which gives the displacement along the y – direction is given by; $y = 10^{-4}\sin(60t + 2x)$. Where x and y are in meter and t is time in second. This represents a wave ***(1981)***

(A) Traveling with a velocity of 30 m/s in the negative x – direction

(B) Of wavelength $\pi$ m

(C) Of frequency $30/\pi$ Hz

(D) Of amplitude $10^{-4}$ m

**Q.9** As a wave propagates ***(1999)***

(A) The wave intensity remains constant for a plane wave

(B) The wave intensity decreases as the inverse of the distance from the source for a spherical wave

(C) The wave intensity decreases as the inverse square of the distance from the source for a spherical wave

(D) Total intensity of the spherical wave over the spherical surface centered at the source remains constant at all times

**Q.10** $Y(x,t) = \frac{0.8}{[(4x+5t)^2+5]}$ represents a moving pulse where x and y are in meter and t is in second. Then, ***(1999)***

(A) Pulse is moving in positive x – direction

(B) In 2 s it will travel a distance of 2.5 m

(C) Its maximum displacement is 0.16 m

(D) It is a symmetric pulse

**Q.11** A copper wire is held at the two ends by rigid supports. At $30^0$C, the wire is just taut, with negligible tension. Find the speed of transverse waves in this wire at $10^0$C. ***(1998)***

Given: Young modulus of copper = $1.3 \times 10^{11}$ N/m$^2$

Coefficient of linear expansion of copper = $1.7 \times 10^{-5}$ $^oC^{-1}$

Density of copper = $9 \times 10^3$ kg/m$^2$

**Q.12** A string 25 cm long and having a mass of 2.5 g is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats/ s are heard. It is observed that decreasing the tension in the string decreases the beat frequency. If the speed of sound in air is 320 m/s find the tension in the string. ***(1999)***

**Q.13** A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope? ***(1984)***

**Q.14** A steel wire of length 1 m, mass 0.1 kg and uniform cross-sectional area $10^{-6}$ m$^2$ is rigidly fixed at both ends. The temperature of the wire is lowered by $20^0$C. If transverse waves are set – up by plucking the string in the middle, calculate the frequency of the fundamental mode of vibration. ***(2009)***

Given : $Y_{steel} = 2\times10^{11}\, N/m^2$ and

$\alpha_{steel} = 1.21\times10^{-5} / ^oC.$

**Q.15** When two progressive waves $y_1$ = 4 sin (2x - 6t) and $y_2$ = 3 sin $\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is: ***(2010)***

**Q.16** A horizontal stretched string fixed at two ends, is vibrating in its fifth harmonic according to the equation y(x, t) = 0.01m sin [(62.8m$^{-1}$)x] cos[(628s$^{-1}$)t]. Assuming $\pi$ = 3.14, the correct statement(s) is (are): ***(2013)***

(A) The number of nodes is 5.

(B) The length of the string is 0.25 m.

(C) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01m.

(D) The fundamental frequency is 100 Hz.

**Q.17** One end of a taut string of length 3m along the x axis is fixed at x = 0. The speed of the waves in the string is 100 ms$^{-1}$. The other end of the string is vibrating in the y direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is (are): ***(2014)***

(A) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$

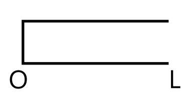
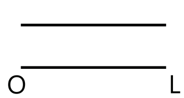
(B) $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(C) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

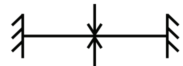
(D) $y(t) = A\sin\frac{5\pi x}{6}\cos 250\pi t$

**Q.18** A metal rod AB of length 10x has its one end A in ice at 0°C and the other end B in water at 100°C. If a point P on the rod is maintained at 400°C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of $\lambda x$ from the ice end A, find the value of $\lambda$. [Neglect any heat loss to the surrounding.] ***(2009)***

**Q.19.** Column I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as $\lambda_f$. Match each system with statements given in column II describing the nature and wavelength of the standing waves. ***(2011)***

| **Column I** | **Column II** |
|---|---|
| (A) Pipe closed at one end  | (p) Longitudinal waves |
| (B) Pipe open at both ends  | (q) Transverse waves |

| Column I | Column II |
|---|---|
| (C) Stretched wire clamped at both ends   | (r) $\lambda_f = L$ |

| Column I | Column II |
|---|---|
| (D) Stretched wire clamped at both ends and at mid-point   | (s) $\lambda_f = 2L$ |
| | (t) $\lambda_f = 4L$ |

# PlancEssential Questions

## JEE Main/Boards

### Exercise 1

Q. 5 Q. 17 Q. 22

### Exercise 2

Q.1 Q.4 Q.5

Q.6 Q.10 Q.14

Q.21

## JEE Advanced/Boards

### Exercise 1

Q. 2 Q.10 Q.12

### Exercise 2

Q.1 Q.2 Q.4

Q.9 Q.10

# Answer Key

## JEE Main/Boards

### Exercise 1

**Q. 2** 20 m

**Q. 3** 382.2 $ms^{-1}$

**Q. 4** (i) $0.25 \times 10^{-3}$ cm

(ii) $\pi/250$ sec (iii) 500 rad/sec

(iv) 80 $\pi$ meters

(v) 0.125 cm/s (vi) 62.5 cm/sec$^2$

**Q. 6** 353.6 $ms^{-1}$

**Q. 7** 250 Hz

**Q. 16** 4.23 s

**Q. 17** 1.320 km

**Q. 18** 92.95 $ms^{-1}$

**Q. 19** $5.27 \times 10^3$ $ms^{-1}$

**Q. 20** 100 $ms^{-1}$

**Q. 21** $7.5 \times 10^{-2}$ $cms^{-2}$; $7.5 \times 10^{2}$ $cms^{-2}$

**Q. 22** (a) 9.42 m/s (b) zero

**Q. 23** 330 $ms^{-1}$, 0.02 m, 0.033 m

**Q. 24** 205 Hz

## Exercise 2

### Single Correct Choice Type

| | | | | | |
|---|---|---|---|---|---|
| **Q.1** A | **Q.2** D | **Q. 3** A | **Q. 4** D | **Q. 5** C | **Q. 6** C |
| **Q.7** C | **Q.8** B | **Q. 9** D | **Q. 10** A | **Q. 11** A | **Q. 12** C |
| **Q.13** B | **Q.14** D | **Q.15** B | **Q.16** A | **Q. 17** B | **Q. 18** B |
| **Q.19** B | **Q.20** C | **Q.21** B | **Q.22** C | **Q. 23** B | **Q.24** C |
| **Q.25** B | | | | | |

## Previous Years' Questions

| | | | | | |
|---|---|---|---|---|---|
| **Q. 1** B | **Q. 2** C | **Q. 3** B | **Q. 4** A | **Q. 5** A | **Q. 6** D |
| **Q. 7** C | **Q. 8** B | **Q. 9** A | **Q. 10** A | **Q. 11** B | **Q. 12** A, C |
| **Q. 13** B, C | **Q. 14** D | **Q. 15** A, B, C, D | **Q. 16** B,C | **Q. 17** A | **Q. 18** A,B |
| **Q. 19** A | **Q. 20** A | **Q. 21** D | **Q. 22** A | **Q. 23** D | **Q. 24** C |
| **Q. 25** B | **Q. 26** B | **Q. 27** A | **Q. 28** B | **Q. 29** B | **Q. 30** C |

# JEE Advanced/Boards

## Exercise 1

**Q. 1** (a) D,E,F, (b) A,B,H, (c) C,G, (d) A,E

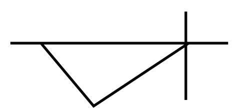
(b) $\frac{3\pi}{2}$ mm / s, $\frac{3\pi^2}{4}$ mm / $s^2$, (c) $\frac{3\pi}{2}$

**Q. 2** (a) $\lambda$=4m, $f=\frac{1}{4}$Hz, 1 m/s

**Q. 3** 1.22 v

**Q. 4** 106.59 kW

**Q. 5** (a)



(b) 2 cm/s

**Q.6** $A_1=-\frac{1}{3}$cm, $A_1=\frac{2}{3}$cm

**Q. 7** 0.2 cm

**Q. 8** 1 ×$10^9$ $Nm^{-2}$

**Q. 9** 50Hz, 50$\pi$ cm/sec

**Q. 10** 4 m, 24 m/s, 25/18 kg, 1/9 sec

**Q. 11** 1.007%

**Q. 12** (a) 144 cm; (b) 17.36 Hz

**Q. 13** 2/3c

## Exercise 2

### Single Correct choice type

| | | | | | |
|---|---|---|---|---|---|
| **Q. 1** B | **Q. 2** D | **Q. 3** D | **Q. 4** B | **Q. 5** A | **Q. 6** C |
| **Q. 7** A | **Q. 8** B | | | | |

### Assertion Reasoning Type

**Q. 9** D **Q. 10** A

**Multiple Correct Choice Type**

**Q.11** C, D

**Comprehension Type**

**Q. 12** A, D **Q.13** C **Q.14** B, C **Q.15** C, D

## Previous Years' Questions

| | | | | | |
|---|---|---|---|---|---|
| **Q.1** A | **Q.2** B | **Q.3** A | **Q.4** A | **Q.5** A | **Q.6** A |
| **Q.7** C | **Q.8** A, B, C, D | **Q.9** A | **Q.10** A | **Q.11** 70.1 m/s | **Q.12** 27.04 N |
| **Q.13** 0.12 m | **Q.14** 11 Hz | **Q.15** 5 | **Q. 16** B, C | **Q. 17** A, C, D | **Q. 18** 9 |

**Q. 19** A $\rightarrow$ p, t; B $\rightarrow$ p, s; C $\rightarrow$ q, s; D $\rightarrow$ q, r

# Solutions

## JEE Main/Boards

### Exercise 1

**Sol 1:** (i) $\frac{1}{30,000}\,s \le T \le \frac{1}{40}$

(ii) $\frac{350}{30000}\,m/s \le \lambda \le \frac{350}{40}\,m/s$

(iii) $80\pi$ rads$^{-1} \le \omega \le 60000\pi$ rads$^{-1}$

**Sol 2:** f = 15 × $10^6$ Hz

$\lambda = \frac{V}{f} = \frac{3\times10^8}{15\times10^6} = 20$ m

**Sol 3:** $V' = \sqrt{\frac{4}{3}}\,V = \sqrt{\frac{4}{3}} \times 331 = 382.2$ ms$^{-1}$

**Sol 4:** (i) A = 0.25 × $10^{-3}$ cm

(ii) $T = \frac{2\pi}{500} = \frac{\pi}{250}$ s

(iii) $\omega$ = 500 rad/s

(iv) $\lambda = \frac{2\pi}{0.025}$ m = $80\pi$ m

(v) $V_{max}$ = 0.25 × $10^{-3}$ × 500 cm s$^{-1}$

$V_{max.}$ = 0.125 cms$^{-1}$

(vi) $a_{max.} = V_{max}\,\omega$ = 0.125 × 500

$a_{max.}$ = 6.25 cms$^{-2}$

**Sol 5:**

$f \propto \frac{1}{\ell}$

$f \rightarrow 1 : 2 : 3$

$\ell \rightarrow 1 : \frac{1}{2} : \frac{1}{3}$

$\ell \rightarrow 6 : 3 : 2$

Bridges must be placed at 60 cm from one end and 20 cm from another end

**Sol 6:** $\frac{V}{2.04} - \frac{V}{2.08} = \frac{20}{6}$; $V\left(\frac{0.04}{2.04\times2.05}\right) = \frac{20}{6}$

V = 353. 6 ms$^{-1}$

**Sol 7:** On loading with wax frequency decreases

f – 256 = ± 6

f = 256 ± 6 Hz

f = 262 Hz

**Sol 8:** No because string is not stretchable yes transverse waves are possible in a steel rod.

**Sol 9:** (a) No wave possible as there is no particle.

(b) Longitudinal waves (direction of motion of particles parallel to direction of propagation of wave)

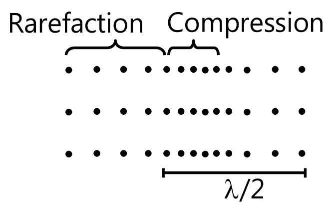
(c) Longitudinal

(d) Both are possible

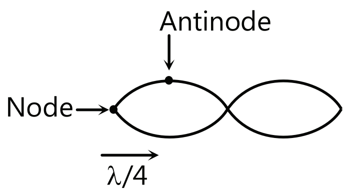
(e) Combined longitudinal & transverse (ripples)

**Sol 10:** Infinite as young's modulus of a rigid body is infinite

**Sol 11:** Half the wavelength ($\lambda/2$)




**Sol 12:**



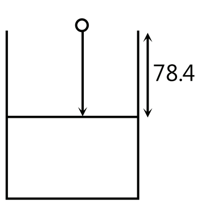

**Sol 13:** Strings cannot be compressed or extended hence there won't be regions of compression and rarefaction. Strings have elasticity of shape. Hence wave on strings are transverse.

**Sol 14:** Refer theory

**Sol 15:** A harmonic of a wave is a component frequency of the signal that is an integer multiple of the fundamental frequency. If f is the fundamental frequency the harmonics have frequency 2f, 3f, 4f ..... etc.

An overtone is any frequency higher than the fundamental frequency of a sound.

**Sol 16:**

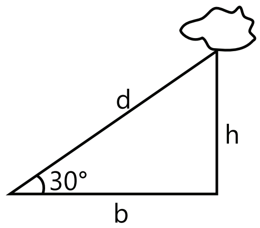



Time to reach water = $\sqrt{\dfrac{2\times 78.4}{9.8}}$ = 45

Time for sound to reach top = $\dfrac{78.4}{332}$ = 0.23 s

Total time = 4.23 s

**Sol 17:**




$$d\left(\frac{1}{330}-\frac{1}{3\times 10^8}\right) = 8 \times 330 \times 3 \times 10^8$$

d $(3\times 10^8 - 330)$

d $\cong$ 8 × 330

d = 264 m

height of cloud = 1320 m = 1.32 Km

**Sol 18:** $m = \dfrac{5\times 10^{-3}}{0.72} = \dfrac{1}{144}$

T = 60 N

$$V = \sqrt{\frac{t}{\mu}} = \sqrt{\frac{60}{1/144}} = 24\sqrt{15} = 92.95\ ms^{-1}$$

**Sol 19:** $B = 7.5 \times 10^{10}\ N\ m^{-2}$

$E = 2.7 \times 10^3\ kg\ m^{-3}$

$$V = \sqrt{\frac{B}{e}} = \sqrt{\frac{7.5\times 10^{10}}{2.7\times 10^3}} = 5270.46\ ms^{-1}$$

$V = 5.27 \times 10^3\ ms^{-1}$

**Sol 20:** $\mu = \dfrac{7\times 10^{-3}}{0.7} = 10^{-2}$ kg/m

T = 100 N

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{100}{10^{-2}}} = 100\ ms^{-1}$$

**Sol 21:** $\omega$, = 50 $rads^{-1}$ $CO_2$ = 5000 $rads^{-1}$

$a = 3 \times 10^{-5} \times (50)^2\ cms^{-2}$

$a_1 = 7.5 \times 10^{-2}\ cms^{-2}$

$a_2 = 3 \times 10^{-5} \times (5000)^2\ cms^{-2}$

$a_2 = 750\ cms^{-2}$

$a_2 = 7.5\ ms^{-2}$

**Sol 22:** y = (3.0 cm) sin [($3.14\ cm^{-1}$)x – ($314\ s^{-1}$)t]

(a) $V_{max.} = 3.0\ cm \times 314\ s^{-1} = 942\ cm\ s^{-1}$

$= 9.42\ ms^{-1}$

(b) $a = -3 \times (314)^2 \sin(\pi \times 6 - 100\pi \times 0.11)$

$a = -3 \times (314)^2 \sin(-5\pi)$

$a = 0$

**Sol 23:** f = 250 Hz

$(31 + h) = \frac{\lambda}{2}$

$(97 + h) = \frac{3\lambda}{4} \Rightarrow 66 = \frac{\lambda}{2}$

$\lambda = 132$ cm

$V = f\lambda = 250 \times 1.32\ ms^{-1} \Rightarrow V = 330\ ms^{-1}$

$H = \frac{132}{4} - 31 = 2$ cm = 0.02 m

Radius of tube = $\frac{\text{End Cross section}}{0.6}$

$= \frac{0.02}{0.6} = \frac{0.2}{6} = \frac{0.1}{3} = 0.033$ m

**Sol 24:** $\frac{V}{2} - F = 5$

$F - \frac{V}{2.1} = 5$

$\frac{V}{2} - \frac{V}{2.1} = 10$

$V = 420\ ms^{-1}$

$F = 5 + \frac{420}{2.1} = 205$ Hz

## Exercise 2

### Single Correct Choice Type

**Sol 1: (A)** $z = e^{-(x-vt+c)^2}$

$c - V \times 0 = 2$

$c - V \times 1 = -2$

$V = +4$ m/s

**Sol 2: (D)** $y = -\sin\left(kx - \omega t + \frac{\pi}{6}\right)$

$\Rightarrow y = \sin\left(\omega t - kx - \frac{\pi}{6}\right)$

**Sol 3: (A)** $V = f\lambda \Rightarrow V = 250 \times \frac{0.4}{100} = 1\ ms^{-1}$

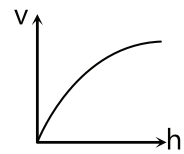
**Sol 4: (D)** $V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{10}{0.001}} = 100\ ms^{-1}$

$T = \frac{1\,m}{100 ms^{-1}} = 0.01$ sec

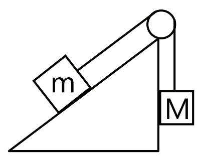
**Sol 5: (C)** $V \propto T^{1/2}$

T decreases linearly with height

$\therefore$ Parabolic curve




**Sol 6: (C)**




$T = V^2M = 100$ N; $T = mg \sin\theta$

$Mg = 100$ N ; $100 = m \times 10 \times \frac{1}{2}$

M = 10 kg, m = 20 kg

$\frac{m}{M} = 2$

**Sol 7: (C)** $y = 5(1 - \cos(200\pi t + 10\pi z))$

Amplitude = 5 cm

**Sol 8: (B)** $y = A \sin(kx + wt)$

$A \sin\left(4k + \frac{\pi}{3}\right) = \frac{A}{2} \Rightarrow 4k + \frac{\pi}{3} = \frac{\pi}{6}$

$\Rightarrow K = \frac{2\pi}{\lambda} = \frac{\pi}{24}$

$\lambda = 48$ cm ; $\lambda = 0.48$ m

**Sol 9: (D)** T = 0.04 sec; $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.04} = 50\pi$

V = 300 m/s, $k = \frac{\omega}{V} = \frac{50\pi}{300} = \frac{\pi}{6}$

$\Delta\phi = \frac{\pi}{6} \times 6 = \pi$

**Sol 10: (A)** $y = \dfrac{3}{a^2 + (x-3t)^2}$; $\quad y = \dfrac{3}{a^2 + (x+vt)^2}$

V = – 3 m/s

**Sol 11: (A)** Phase change of $\pi$ due to reflection from rigid wall.

**Sol 12: (C)** $V_1 \to$ speed in light string

$V_2 \to$ speed in heavy string

$V_2 = \dfrac{V_1}{2}$

$A_r = \dfrac{V_2}{V_1 + V_2} A = \dfrac{\frac{1}{2}}{1+\frac{1}{2}}$ 6 mm = 2 mm

y= (2mm) sin (kx – wt)

y = (2mm) sin(40x – 5t)

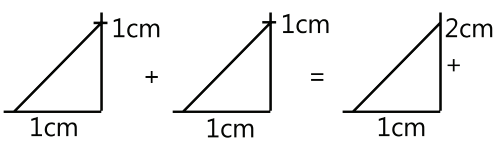
**Sol 13: (B)** For a given string Power $\propto A^2$

Power reflected = $\left(\dfrac{1}{3}\right)^2 P = P/9$

Power transmitted = $\dfrac{8P}{9}$

$\therefore$ 89% power transmitted

**Sol 14: (D)** By superposition




**Sol 15: (B)** $\ell$ = 1m f = 300 Hz

$\dfrac{3}{2}\lambda = \ell$; $f = \dfrac{v}{\ell}$; $\lambda = \dfrac{2\ell}{3}$; $v = f\lambda$

$v = 300 \times \dfrac{2}{3} \times 1 = 200$ m/s

**Sol 16: (A)** $y_1 = 5\sin(\omega t - kx)$

$y_2 = -5\cos(\omega t - kx - 150°)$

$y_1 + y_2 = 5(\sin(\omega t - kx) - \sin(\omega t - kx - 60°))$

$= 10(\sin 30° \cos(\omega t - kx - 30°))$

$= 5\cos(\omega t - kx - 30°)$

**Sol 17: (B)** $y_1 = A\cos(kx - \omega t)$

$y_2 = A\cos(kx + \omega t + \phi)$

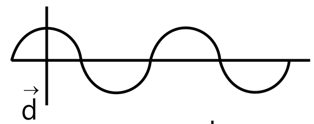
$y_1 + y_2 = 2A\cos 20\left(kx + \dfrac{\phi}{2}\right)\cos\left(\omega t + \dfrac{\phi}{2}\right)$

$\cos\dfrac{\phi}{2} = 0$ ; $\phi = \pi$

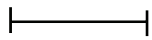
$y_2 = -A\cos(kx + \omega t)$

**Sol 18: (B)** $(n+1) = \dfrac{L}{2d}$

L = (n + 1)d




No. of loops $= \dfrac{L}{2d}$

**Sol 19: (B)** 

$T = Y\alpha\Delta TA$

$f \propto \dfrac{v}{\ell}$; $\quad \dfrac{v}{\ell} = \dfrac{1}{2\ell^2}\sqrt{\dfrac{T}{\mu}}$

$f \propto \dfrac{1}{r^2}\left(\dfrac{T}{\mu}\right)^{1/2}$

$\propto \dfrac{1}{\ell^2}\left(\dfrac{y\alpha\Delta T_A}{\frac{m}{A\ell}}\right)^{1/2}$ ; $\propto \dfrac{1}{\ell^2}\left(\dfrac{y\alpha\Delta T}{\rho}\right)^{1/2}$

**Sol 20: (C)** $y = A\sin\left(\dfrac{20}{3}\pi x\right)\cos(1000\,\pi t)$

$\sin\left(\dfrac{20\pi}{3}x\right) = \dfrac{1}{2}$

$\dfrac{20x}{3} = \dfrac{\pi}{6}$; $\quad x = \dfrac{1}{40}$ m

Distance = $2x = \dfrac{1}{20}$ m = 5 cm

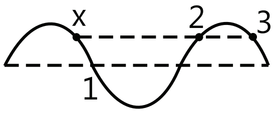
**Sol 21: (B)** $\ell$ = 0.4 m, m = $10^{-2}$ kg, T = 1.6 N

$\mu = \dfrac{10^{-2}}{0.4} = 2.5 \times 10^{-2}$ kg/m

$f = \dfrac{1}{2 \times 0.4}\sqrt{\dfrac{1.6}{2.5 \times 10^{-2}}} = \dfrac{1}{0.8} \times \dfrac{4}{0.5} = 10$ Hz

T = 0.1 s

**Sol 22: (C)** $\sin\phi = \sin(\pi - \phi)$




**Sol 23: (B)** Refer theory $y = \frac{A}{r}\sin(kr - \omega t)$

**Sol 24: (C)** At x = 4

$\Rightarrow y = (2mm)\sin\left(8\pi + \frac{\pi}{3} - 100\pi t\right)$

$\Rightarrow y = (2mm)\sin\left(\frac{\pi}{3} - 100\pi t\right)$

$\frac{\pi}{3} = 100\pi t$ ; $t = \frac{1}{300}$ sec

**Sol 25: (B)** $V_{pmax} = 2\pi fA$

$V_{wave} = \frac{2\pi f}{2\pi/\lambda} = f\lambda$

$2\pi fA = 4f\lambda$

$\lambda = \frac{\pi A}{2}$

## Previous Years' Questions

**Sol 1: (B)** We velocity $v = \frac{\text{coefficient of t}}{\text{coefficient of x}} = \frac{2\pi f}{2\pi/\lambda}$

$= \lambda f$

Maximum particle velocity $v_{pm} = \omega A = 2\pi f y_0$

Given, $V_{pm} = 4V$ or $2\pi f\, y_0 = 4\lambda f$

$\therefore \lambda = \frac{\pi y_0}{2}$

**Sol 2: (C)** For a stationary wave to form, two indentical waves should travel in opposite direction. Further at x = 0, resultant y (from both the waves) should be zero at all instant.

**Sol 3: (B)** The given equation can be written as

$y = 2(2\cos^2\frac{t}{2})\sin(1000\, t)$

$y = 2(\cos t + 1)\sin(1000\, t)$

$= 2\cos t \sin 1000\, t + 2\sin(1000\, t)$

$= \sin(1001t) + \sin(999t) + 2\sin(1000t)$

i.e., the given expression is a result of superposition of three independent harmonic motions of angular frequencies 999, 1000 and 1001 rad/s.

**Sol 4: (A)** From Hooke's law

Tension in a string (T) $\propto$ extension (x) and speed of sound in string $v = \sqrt{T/\mu}$ or $v \propto \sqrt{T}$

Therefore, $v \propto \sqrt{x}$

x is increased to 1.5 times i.e., speed will increase by $\sqrt{1.5}$ times of 1.22 times. Therefore speed of sound in new position will be 1.22 v.

**Sol 5: (A)** This is an equation of a travelling wave in which particles of the medium are in SHM and maximum particle velocity in SHM is $A\omega$, where A is the amplitude and $\omega$ the angular velocity.

**Sol 6: (D)** Fundamental frequency is given by

$V = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$ (with both the ends fixed)

$\therefore$ Fundamental frequency

$V \propto \frac{1}{\ell\sqrt{\mu}}$ (for same tension in both strings)

Where $\mu$ = mass per unit length of wire

$= \rho.A$ ($\rho$ = density)

$= \rho(\pi r^2)$ or $\sqrt{\mu} \propto r \therefore v \propto \frac{1}{r\ell}$

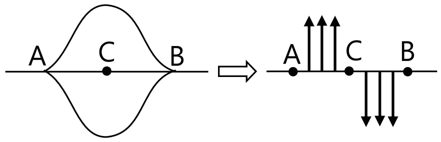
$\therefore \frac{V_1}{V_2} = \left(\frac{r_2}{r_1}\right)\left(\frac{\ell_2}{\ell_1}\right) = \left(\frac{r}{2r}\right)\left(\frac{2L}{L}\right) = 1$

**Sol 7: (C)** Energy $E \propto (\text{amplitude})^2 (\text{frequency})^2$

Amplitude (A) is same in both the cases, but frequency $2\omega$ in the second case is two times the frequency ($\omega$) in the first case

Therefore, $E_2 = 4E_1$

**Sol 8: (B)** After two seconds both the pulses will move 4 cm towards each other. So by their superposition, the resultant displacement at every point will be zero. Therefore total energy will be purely in the form of kinetic. Half of the particles will be moving upwards and half downwards.




**Sol 9: (A)** Let $f_0$ = frequency of tuning fork

Then, $f_0 = \frac{5}{2\ell}\sqrt{\frac{9g}{\mu}} = \frac{3}{2\ell}\sqrt{\frac{Mg}{\mu}}$

($\mu$ = mass per unit length of wire)

Solving this, we get M = 25 kg

In the first case, frequency corresponds to fifth harmonic while in the second case it corresponds to third harmonic.

**Sol 10: (A)** $f \propto v \propto \sqrt{T}$

$f_{AB} = 2f_{CD}$

$\therefore T_{AB} = 4T_{CD}$ ...(i)

Further $\Sigma\tau_p = 0$

$\therefore T_{AB}(x) = T_{CD}(l - x)$ or $4x = l - x$

$(T_{AB} = 4T_{CD})$

or $x = l/5$

**Sol 11: (B)** The fundamental mode in a pipe closed at one end and the second harmonic in a string are shown in figure. It can be seen that $\lambda_p / 4 = Lp$ and $\lambda_s = L_s$.

For the pipe closed at one end,

$$v_p = \frac{\nu_p}{\lambda_p} = \frac{\nu_p}{4L_p} = \frac{320}{4(0.8)} = 100 \text{ Hz}$$

Where $\nu_p$ =320m/s is the velocity of sound in the pipe and $L_p$=0.8m is length of the pipe. For string of mass m, length $L_s$ and having tension T, velocity of the string is given by,

$$v_s = \frac{\nu_s}{\lambda_s} = \sqrt{\frac{T/(m/L_S)}{L_s}} = \sqrt{\frac{T}{mL_s}} = \sqrt{\frac{50}{m(0.5}} = \frac{10}{\sqrt{m}}$$

At resonance $\nu_p = \nu_s$ substitute $\nu_p$ and $\nu_s$ from first and second equation to get m= 0.01kg=10 gram.

**Sol 12: (A, C)** options satisfy the condition;

$$\frac{\partial^2 y}{\partial x^2} = \text{(constant)} \frac{\partial^2 y}{\partial t^2}$$

**Sol 13: (B, C)** $\omega = 15\pi$, k = 10 p

Speed of wave, $v = \frac{\omega}{k} = 1.5$ m/s

Wavelength of wave $\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = 0.2$ m

$10\pi$ x and $15\pi$t have the same sign. Therefore, wave is traveling in negative x-direction.

**Sol 14: (D)** $T_1 > T_2$

$\therefore v_1 > v_2$

or $f_1 > f_2$

and $f_1 - f_2 = 6$ Hz

Now, if $T_1$ is increased, $f_1$ will increase or $f_1 - f_2$ will increase. Therefore, (d) option is wrong.

If $T_1$ is decreased, $f_1$ will decrease and it may be possible that now $f_2 - f_1$ become 6 Hz. Therefore, (C) option is correct. Similarly, when $T_2$ is increased, $f_2$ will increase and again $f_2$-$f_1$ may become equal to 6 Hz. So, (B) is also correct. But (A) is wrong.

**Sol 15: (A, B, C, D)** It is given that

$y(x, t) = 0.02\cos(50\pi t + \pi/2)\cos(10\pi x)$

$\cong A\cos(\omega t + \pi/2)\cos kx$

Node occurs when $kx = \frac{\pi}{2}, \frac{3\pi}{2}$ etc.

$10\pi x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\Rightarrow$ x = 0.05m, 0.15 m option (a)

Antinode occurs when $kx = \pi, 2\pi, 3\pi$ etc.

$10\pi x = \pi, 2\pi, 3\pi$ etc.

$\Rightarrow$ x = 0.1m, 0.2, 0.3 m option (b)

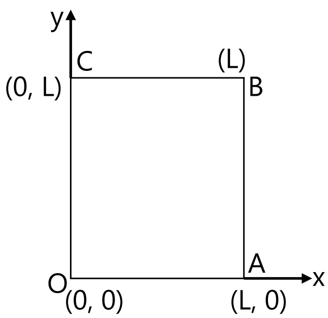
Speed of the wave is given by,

$v = \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5$ m/s option (c)

Wavelength is given by,

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = \left(\frac{1}{5}\right)$ m = 0.2 m

**Sol 16: (B, C)** Since, the edges are clamped, displacement of the edges u(x, y) = 0 for




Line, OA i.e., y = 0; $0 \le x \le L$

AB i.e., x = L; $0 \le y \le L$

BC i.e., y = L; $0 \le x \le L$

OC i.e., x = 0; $0 \le y \le L$

The above conditions are satisfied only in alternatives (B) and (C).

Note that u (x, y) = 0, for all four values eg, in alternative (D), u(x, y) = 0 for y = 0, y = L but it is not zero for x = 0 or x = L. Similarly, in option (A) u (x, y) = 0 at x = L, y = L but it is not zero for x = 0 or y = 0 while in option (B) and (C), u (x, y) = 0 for x = 0, y = 0, x = L and y = L.

**Sol 17: (A)** Maximum speed of any point on the string = aω = a(2πf)

$\therefore = \frac{v}{10} = \frac{10}{10} = 1$ (Given: v = 10 m/s)

$\therefore 2\pi af = 1$ ;  $f = \frac{1}{2\pi a}$

a = $10^{-3}$ m (Given)

$\therefore f = \frac{1}{2\pi\times 10^{-3}} = \frac{10^3}{2\pi}$ Hz

Speed of wave v = f l

$\therefore (10 \text{ m/s}) = \left(\frac{10^3}{2\pi}S^{-1}\right)\lambda$ ; $\lambda = 2\pi \times 10^{-2}$m

**Sol 18: (A, B)** In case of sound wave, y can represent pressure and displacement, while in case of an electromagnetic wave it represents electric and magnetic fields.

**Note:** In general, y is general physical quantity which is made to oscillate at one place and these oscillations are propagated to other places also.

**Sol 19: (A)** Standing waves can be produced only when two similar type of waves (same frequency and speed, but amplitude may be different) travel in oposite directions.

**Sol 20: (A)** y = 0.005 cos (αx − βt)

Comparing the equation with the standard form,

$$y = A\cos\left[\left(\frac{x}{\lambda}-\frac{t}{T}\right)2\pi\right]$$

2π/λ = α and 2π/T = β

α = 2π/0.08 = 25.00 π

β = π

**Sol 21: (D)** $T = \mu v^2 = \mu\frac{\omega^2}{k^2} = 0.04\frac{(2\pi/0.004)^2}{(2\pi/0.50)^2} = 6.25\text{N}$

**Sol 22: (A)** $y_{(x,t)} = e^{-}(\sqrt{a}x+\sqrt{b}t)^2 V = \sqrt{\frac{b}{a}}$

Wave moving in − ve x –direction.

**Sol 23: (D)** $\phi_1 = 0$ ; $\phi_2 = \frac{\pi}{2}$

**Sol 24: (C)** Energy of simple harmonic oscillator is constant.

$$\Rightarrow \frac{1}{2}M\omega^2A_1^2 = \frac{1}{2}(m+M)\omega^2A_2^2$$

$$\frac{A_1^2}{A_2^2} = \frac{M+m}{M}$$

$$\therefore \frac{A_1}{A_2} = \sqrt{\frac{M+m}{M}}$$

**Sol 25: (B)** $f = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{Ad}}$

Also, $Y = \frac{T\ell}{A\Delta\ell}$  $\Rightarrow \frac{T}{A} = \frac{Y\Delta\ell}{\ell}$  $\Rightarrow f = \frac{1}{2\ell}\sqrt{\frac{y\Delta\ell}{\ell d}}$

$\ell = 1.5\text{m}, \frac{\Delta\ell}{\ell} = 0.01, d = 7.7\times 10^3 \text{ kg/m}^3$

$y = 2.2\times 10^{11}\text{N/m}^2$

After solving

$$f = \sqrt{\frac{2}{7}}\times\frac{10^3}{3}\text{Hz}$$

$f \approx 178.2\text{Hz}$

**Sol 26: (B)** Given $\frac{\Delta L}{L} = \frac{0.1}{20}$

$T = \frac{90}{100}$sec. $\Delta T = \frac{1}{100}$sec.

$\frac{\Delta T}{T} = \frac{1}{90}$

$g = \left(\frac{1}{4\pi^2}\right)\frac{L}{T^2}$

$\Rightarrow \frac{\Delta g}{g} \times 100\% = \frac{\Delta L}{L} \times 100 + \frac{2\Delta T}{T} \times 100$

$\frac{\Delta g}{g} \times 100\% = \left(\frac{0.1}{20}\right)100 + 2\left(\frac{1}{90}\right)100 = 2.72\%$

So, nearest option is 3%.

**Sol 27: (A)**

$T = 2\pi\sqrt{\frac{\ell}{g}}$

$T_M = 2\pi\sqrt{\frac{\ell + \Delta\ell}{g}}$ $\Delta\ell = \frac{Mg\ell}{AY}$

$\frac{T_M}{T} = \sqrt{\frac{\ell + \Delta\ell}{\ell}}$

$\left(\frac{T_M}{T}\right)^2 = 1 + \frac{\Delta\ell}{\ell}$

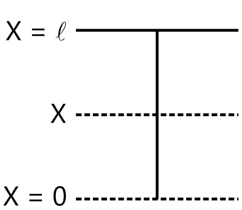
$\left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{AY}$

$\frac{1}{y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$

**Sol 28: (B)** K.E. is maximum at mean position, whereas P.E. is minimum.

At extreme position, K.E. is minimum and P.E. is maximum.

**Sol 29: (B)** Let mass per unit length be $\lambda$




$T = \lambda gx$ $v = \sqrt{\frac{T}{\lambda}} = \sqrt{gx}$

$v^2 = gx$

$a = \frac{vdv}{dx} = \frac{g}{2}$

$\ell = \frac{1}{2}\frac{g}{2}t^2 \Rightarrow t = \sqrt{\frac{4\ell}{g}} = 2\sqrt{2}$ sec

**Sol 30: (C)**

$v = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}$

$v = \sqrt{5}\frac{A\omega}{3}$

$v_{new} = 3v = \sqrt{5}A\omega$

So the new amplitude is given by

$v_{new} = \omega\sqrt{A_{new}{}^2 - x^2} \Rightarrow \sqrt{5}A\omega = \omega\sqrt{A_{new}{}^2 - \left(\frac{2A}{3}\right)^2}$

$A_{new} = \frac{7A}{3}$

# JEE Advanced/Boards

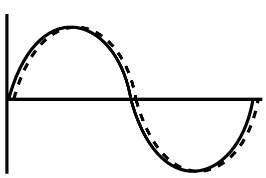
## Exercise 1

**Sol 1:** (a) D, E, F

(b) A, B, H

(c) C, G

(d) A, E



**Sol 2:** (a) $\lambda$ = 4 m

$f = \frac{1}{T} = 0.25$ Hz

$V = f\lambda = 1$ ms$^{-1}$ in –ve n-direction

(b) $V_{max.} = 0.5\ \pi \times 3$ mm s$^{-1}$ = $1.5\pi$ mm s$^{-1}$

$a_{max.} = 0.75\pi^2$ mm $s^{-2}$

(c) $y = (3\text{ mm})\sin\left(\frac{\pi}{2}x - \frac{\pi}{2}t + \pi\right)$

$= (3\text{ mm})\sin\left(\frac{\pi}{2}(x - t + 2)\right)$

$\left.\frac{dy}{dx}\right|_t = \left(\frac{3\pi}{2}\right)\cos\frac{\pi}{2}(x - t + 2)$

Slope at x = 2m & t = 4 sec

$= \frac{3\pi}{2}\cos\frac{\pi}{2}(2 - 4 + 2) = \frac{3\pi}{2}$

**Sol 3:** $V \propto \sqrt{T}$, $V' = \sqrt{1.5}\,V$

**Sol 4:** $\mu = 5 \times 10^{-3}$ kg/m ; $V = \sqrt{\frac{450}{5\times10^{-3}}} = 300$ m/s

T = 450 N

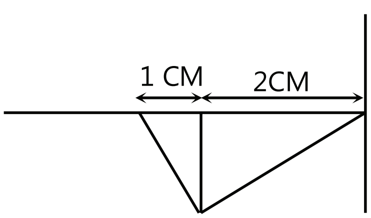
$A \le \frac{\lambda}{5}$

$P_{avg\ max.} = \frac{1}{2}\frac{\omega^2A^2F}{V}$

$= \frac{1}{2} \times 4\pi^2 \frac{f^2\lambda^2}{25} \times \frac{450}{300}$

$= \frac{2\pi^2}{25} \times 450 \times \frac{(300)^2}{300} = 106.59$ kW

**Sol 5:**




$V_p = -\text{slope} \times V_{wave} = -(-1) \times -2$ cm/s

$V_p = -2$ cm/s

–ve sign represents particle moving down

**Sol 6:** $\mu_A = 2$ kg/m$\mu_B = 8$ kg/m

T = 200 N

$V_A = \sqrt{\frac{200}{2}} = 10$m/s $V_B = \sqrt{\frac{200}{8}} = 5$m/s

$A_r = -\frac{V_B}{V_A + V_B}A$; $A_T = \frac{V_A}{V_A + V_B}A$

$A_r = -\frac{1}{3}$ cm; $A_T = \frac{2}{3}$ cm

**Sol 7:** $V_1 = 5$ m/s

$V_2 = 2.5$ m/s

$A_T = \frac{5}{7.5}A = \frac{2}{3} \times 0.3 = 0.2$ cm

**Sol 8:** $\ell = 0.4$ m ; m = 3.2 g ; A = 1 $mm^2$

$m = \frac{3.2}{0.4} \times 10^{-3} = 8 \times 10^{-3}$

$\frac{100}{64} = \frac{1}{2\times0.4}\sqrt{\frac{T}{8\times10^{-3}}}$

$\left(\frac{800}{64}\right)^2 \times 8 \times 10^{-3} = T$

$\frac{10^4}{8\times8} \times 5 \times 10^{-3} = T$

T = 1.25 N

$Y = \frac{T\ell}{A\Delta\ell} = \frac{1.25\times0.4}{10^{-6}\times5\times10^{-4}} = \frac{0.5}{5} \times 10^{10}$ $Nm^{-2}$

$Y = 10^9$ $Nm^{-2}$

**Sol 9:** $\mu = 0.2$ kg/m

L = 0.6 m

T = 80 N

$f = \frac{3}{2\times0.6}\sqrt{\frac{80}{0.2}} = \frac{3}{1.2} \times 20 = 50$ Hz

$V_{max} = 2\pi fA$

$= 2\pi \times 50 \times 0.5$ $cms^{-1} = 50\pi$ $cms^{-1}$

**Sol 10:** T = 200 N

$k = \frac{\pi}{2}\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi/2} = 4$ m

(a) Second harmonic

$\therefore \ell = \lambda = 4$ m

(b) $V = \frac{12\pi}{\pi/2} = 24$ $ms^{-1}$

(c) $\mu = \frac{T}{V^2} = \frac{200}{24\times24}$

mass $= \mu \times \ell = \frac{200}{24\times24} \times 4 = 1.39$ kg

(d) $f = \frac{3}{8} \times 24 = 9$ Hz

$T = \frac{1}{f}; T = \frac{1}{9}$ sec

**Sol 11:** $\sqrt{\frac{T'}{T}}$ f= $\frac{200}{199}$

$\frac{T'}{T} = \left(\frac{200}{199}\right)^2$

T′ = 1.01007

ΔT = T′ – T = 0.01007

% change in tension = 1.007%

**Sol 12:** $\lambda_1$ = 36 cm $\lambda_2$ = 32 cm

$\lambda_1 = \frac{2\ell}{n} = 36; \lambda_2 = \frac{2\ell}{(n+1)} = 32$

$\Rightarrow \frac{n+1}{n} = \frac{36}{32} = \frac{9}{8}$

⇒ 8n + 8 = 9n

⇒ n = 8

$\ell = \frac{36\times 8}{2}$ = 144 cm

$\ell$ = 1.44 m

(b) $f_0 = \frac{1}{2\times 1.44}\sqrt{\frac{10}{4\times 10^{-3}}}$

$\Rightarrow f_0 = \frac{50}{2\times 1.44}$

$f_0$ = 17.36 Hz

**Sol 13:** $V = \sqrt{\frac{\gamma RT}{M}}$

$\sqrt{\frac{RT}{M}} = \frac{c}{\sqrt{3}}$

$\gamma = 1 + \frac{2}{6} = \frac{4}{3}$

$V = \sqrt{\gamma} \times \frac{c}{\sqrt{3}} = \sqrt{\frac{4}{3}} \times \frac{c}{\sqrt{3}}$

$V = \frac{2}{3}c$

# Exercise 2

## Single Correct Choice Type

**Sol 1: (B)** y = 20 sin 2π(100t) cos(2π(0.02x))

$A_{max.}$ = 20 units

$\frac{2\pi}{\lambda} = 2\pi(0.02)$

λ = 50 units

Maximum loop length = $\frac{\lambda}{2} = \frac{50}{2}$ = 25 units

**Sol 2: (D)** M = 0.01 kg $m^{-1}$

T = 100N; $f_1 = \frac{1}{2}\sqrt{\frac{100}{0.01}}$ = 50Hz

$f_1 = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$ ; $f_2 = 2f_1$ ; $f_3 = 3f_1$

$n_1 = \frac{f_1}{2}$ ; $n_2 = \frac{3f_1}{2}$ ; $n_3 = \frac{5f_1}{2}$

**Sol 3: (D)** $\nu = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$

$\nu_1 = \nu_2$

$6 \times \sqrt{36} = 4 \times \sqrt{T}$

$T = \frac{36^2}{4^2}$ = 81 N

**Sol 4: (B)** y = cos (70x – 500 t)

Transverse wave as particle oscillate perpendicular to the direction of motion

$V = \frac{500}{70} = \frac{50}{7}$ m/s

$f = 500 = \frac{250}{\pi}$

$\lambda = \frac{2\pi}{70}$ m $= \frac{20\pi}{7}$ cm

**Sol 5: (A)** $y = A\cos(kx - \omega t)$

For this the maximum occurs at x = 0 and at t = 0

For getting the maximum at t = 5 sec

$kx = 5\omega \Rightarrow x = 5\omega / k$

x = 5 v = 200 cm = 2m

**Sol 6: (C)** The speed of the transverse is given as

$$V = \sqrt{\frac{T}{\mu}}$$

Here, T is tension and is Mass per unit length. Now putting values in above equation, we get

$$V = \sqrt{8 \times 64 \times \frac{1000}{100} \times 5} = 32\,m/s$$

**Sol 7: (A)** $\nu = \sqrt{\frac{T}{\mu}} \Rightarrow \nu_{AB} = \sqrt{\frac{6.4 \times 10}{10 \times 10^{-3}}} = 80ms^{-1}$

$$\nu_{CD} = \sqrt{\frac{3.2 \times 10}{8 \times 10^{-3}}} = 63ms^{-1}$$

**Sol 8: (B)** $\nu = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{20}{2 \times 10^{-3}}} = 100ms^{-1}$

$$\mu = \frac{4.5}{2.25} = 2gm^{-1}$$

$$\Rightarrow s = \mu t - \frac{1}{2}gt^2$$

$$\Rightarrow 2 = 100t - 5t^2$$

$$\Rightarrow t^2 - 20t + 0.4 = 0$$

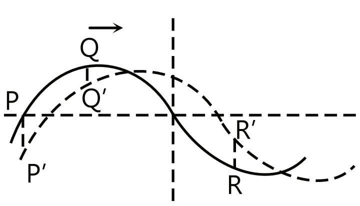
$$\Rightarrow t = \frac{+20 \pm \sqrt{400 - 1.6}}{2} = 0.02s$$

**Assertion Reasoning Type**

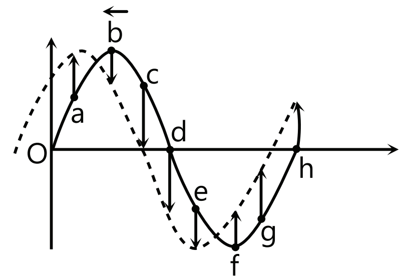
**Sol 9: (D)** Potential energy is maximum at the extremes and particle oscillate in SHM.

**Sol 10: (A)** There cannot be a perpendicular force to a string.

**Sol 11: (C, D)**




**Comprehension Type**




**Sol 12: (A, D)** Upward

a, g , h

**Sol 13: (C)** Downward

c, d, e

**Sol 14: (B, C)** Stationary

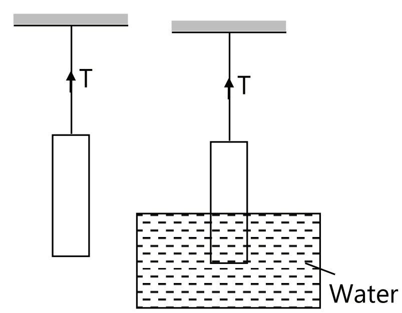
b, f

**Sol 15: (C, D)** Maximum velocity

o, d, h

## Previous Years' Questions

**Sol 1: (A)** The diagramatic representation of the given problem is shown in figures. The expression of fundamental frequency is $V = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$




In air T = mg (Vρ)g

$$\therefore v = \frac{1}{2\ell}\sqrt{\frac{A\rho g}{\mu}} \qquad ...(i)$$

When the object is half immersed in water

$$T' = mg - \text{upthrust} = V\rho g - \left(\frac{V}{2}\right)\rho_w g$$

$$= \left(\frac{V}{2}\right) g\,(2\rho - \rho_w)$$

The new fundamental frequency is

$$V' = \frac{1}{2\ell} \times \sqrt{\frac{T'}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{(Vg/2)(2\rho - \rho_w)}{\mu}} \quad \text{...(ii)}$$

$$\therefore \frac{v'}{v} = \left(\sqrt{\frac{2\rho - \rho_w}{2\rho}}\right)$$

$$\text{or } v' = v\left(\frac{2\rho - \rho_w}{2\rho}\right)^{1/2} = 300\left(\frac{2\rho - 1}{2\rho}\right)^{1/2} \text{ Hz}$$

**Sol 2: (B)** Mass per unit length of the string.

$$m = \frac{10^{-2}}{0.4} = 2.5 \times 10^{-2} \text{ kg/m}$$

$\therefore$ Velocity of wave in the string,

$$V = \sqrt{\frac{T}{m}} = \sqrt{\frac{1.6}{2.5 \times 10^{-2}}}$$

v = 8 m/s

For constructive interference between successive pulses

$$\Delta t_{min} = \frac{2\ell}{v} = \frac{(2)(0.4)}{8} = 0.10\text{s}$$

(After two reflections, the wave pulse is in same phase as it was produced, since in one reflection its phase changes by $\pi$, and if at this moment next identical pulse is produced, then constructive interference will be obtained.)

**Sol 3: (A)** Particle velocity $v_p$ = – v (slope of y-x graph)

Here, v = + ve, as the wave is traveling in positive x-direction.

Slope at P is negative.

$\therefore$ Velocity of particle is in positive y (or $\hat{j}$) direction.

**Sol 4: (A)** With increase in tension, frequency of vibrating string will increase. Since number of beats are decreasing. Therefore, frequency of vibrating string or third harmonic frequency of closed pipe should be less that the frequency tuning fork by 4.

$\therefore$ Frequency of tuning fork

= Third harmonic frequency of closed pipe +4

$$= 3\left(\frac{v}{4\ell}\right) + 4 = 3\left(\frac{340}{4 \times 0.75}\right) + 4 = 344 \text{ Hz}$$

**Sol 5: (A)** In one second number of maximas is called the beat frequency. Hence,

$$f_b = f_1 - f_2 = \frac{100\pi}{2\pi} - \frac{92\pi}{2\pi} = 4\text{Hz}$$

**Sol 6: (A)** Speed of wave $v = \frac{\omega}{k}$

$$\text{or } v = \frac{100\pi}{0.5\pi} \text{ or } \frac{92\pi}{0.46\pi} = 200 \text{ m/s}$$

**Sol 7: (C)** At x = 0, y = $y_1 + y_2$

= 2A cos 96$\pi$t cos 4$\pi$t

Frequency of cos (96 $\pi$t) function is 48 Hz and that of cos (4$\pi$t) function is 2 Hz.

In one second, cos function becomes zero at 2f times, where f is the frequency. Therefore, first function will become zero at 96 times and the second at 4 times. But second will not overlap with first. Hence, net y will become zero 100 times in 1 s.

**Sol 8: (A, B, C, D)** y = $10^{-4}$ sin (60t + 2x)

A = $10^{-4}$ m, $\omega$ = 60 rad/s, k = 2$m^{-1}$

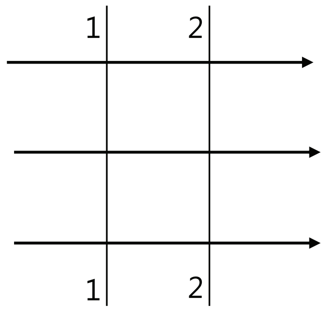
$$\text{Speed of wave, } v = \frac{\omega}{k} = 30 \text{ m/s}$$

$$\text{Frequency, } f = \frac{\omega}{2\pi} = \frac{30}{\pi} \text{ Hz.}$$

$$\text{Wavelength } \lambda = \frac{2\pi}{k} = \pi \text{m}$$

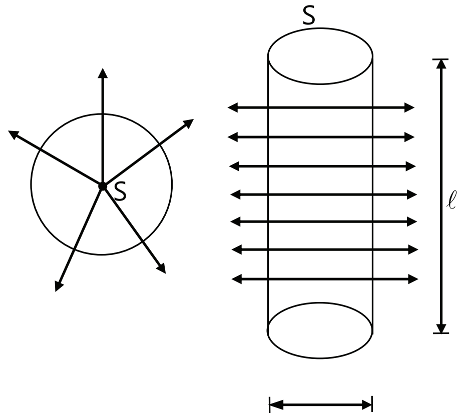
Further, 60 t and 2x are of same sign. Therefore, the wave should travel in negative x-direction.

**Sol 9: (A)** For a plane wave intesity (energy crossing per unit area per unit time) is constant at all points.




But for a spherical wave, intensity at a distance r from a point source of power P (energy transmitted per unit time) is given by

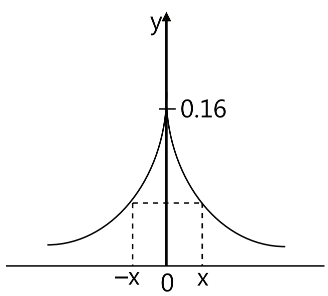
$$I = \frac{P}{4\pi r^2} \text{ or } I \propto \frac{1}{r^2}$$




Note: for a line source $I \propto \frac{1}{r}$

Because, $I = \frac{P}{\pi r \ell}$

**Sol 10: (A)** The shape of pulse at x = 0 t = 0 would be as shown, in figure(a).




t= 0(a)

$$Y(0,0) = \frac{0.8}{5} = 0.16\text{m}$$

From the figure it is clear that $y_{max} = 0.16$ m

Pulse will be symmetric (Symmetry is checked about $y_{max}$) if at t = 0

y(x) = y(–x)

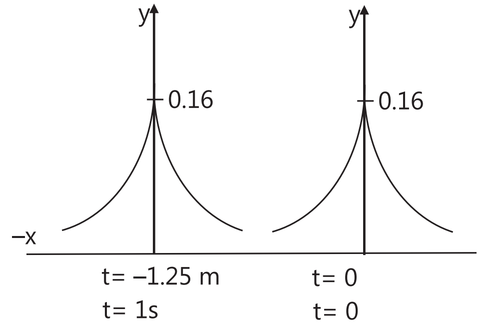
From the given equation

$$\text{And} \left.\begin{aligned} y(x) &= \frac{0.8}{16x^2+5} \\ y(-x) &= \frac{0.8}{16x^2+5} \end{aligned}\right\} \text{at } t = 0$$

or y (x) = y (– x)

Therefore, pulse is symmetric.

Speed of pulse, at t = 1s, and x = – 1.25 m




(a) (b)

Value of y is again 0.16 m, i.e., pulse has traveled a distance of 1.25 m in 1 s in negative x-direction or we can say that the speed of pulse is 1.25 m/s and it is traveling in negative x-direction. Therefore, it will travel a distance of 2.5 m in 2 s. The above statement can be better understood from figure (b)

**Sol 11:** Tension due to thermal stresses,

$T = YA\,\alpha\,.\,\Delta\theta$

$$v = \sqrt{\frac{T}{\mu}}$$

Hence, $\mu$ = mass per unit length = $\rho A$

$$\therefore v = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{YA\alpha.\Delta\theta}{\rho A}} = \sqrt{\frac{Y\alpha\,\Delta\theta}{\rho}}$$

Substituting the values we have,

$$v = \sqrt{\frac{1.3\times10^{11}\times1.7\times10^{-5}\times20}{9\times10^{3}}} = 70.1 \text{ m/s}$$

**Sol 12:** By decreasing the tension in the string beat frequency is decreasing, it means frequency of string was greater than frequency of pipe. Thus,

First overtone frequency of string–Fundamental frequence of closed pipe = 8

$$\therefore 2\left(\frac{v_1}{2\ell_1}\right) - \left(\frac{v_2}{4\ell_2}\right) = 8$$

$$\text{or } v_1 = \ell_1\left[8 + \frac{v_2}{4\ell_2}\right]$$

Substituting the value, we have

$$v_1 = 0.25\left[8 + \frac{320}{4\times0.4}\right] = 52 \text{ m/s}$$

$$\text{Now, } v_1 = \sqrt{\frac{T}{\mu}}$$

$\therefore T = \mu v_1^2 = \left(\frac{m}{\ell}\right) v_1^2 = \left(\frac{2.5\times10^{-3}}{0.25}\right)(52)^2 = 27.04$ N

**Sol 13:** $v = \sqrt{T/\mu}$

$$\frac{v_{top}}{v_{bottom}} = \sqrt{\frac{T_{top}}{T_{bottom}}} = \sqrt{\frac{6+2}{2}} = 2 \qquad \text{...(i)}$$

Frequency will remain unchanged. Therefore, equation (i) can be written as, $\frac{f\lambda_{top}}{f\lambda_{bottom}} = 2$

Or $\lambda_{top} = 2(\lambda_{bottom}) = 2 \times 0.06 = 0.12$ m

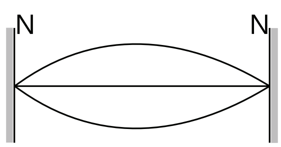
**Sol 14:** The temperature stress is $\sigma = Y\alpha\Delta q$

or tension in the steel wire $T = \sigma A = YA\alpha\Delta q$

Substituting the values, we have

$T = (2 \times 10^{11})(10^{-6})(1.21 \times 10^{-5})(20) = 48.4$ N

Speed of transverse wave on the wire, $v = \sqrt{\frac{T}{\mu}}$




$\lambda/2 = l$

Hence, $\mu$ = mass per unit length of wire = 0.1 kg/m

$\therefore v = \sqrt{\frac{48.4}{0.1}} = 22$ m/s

Fundamental frequency $f_0 = \frac{v}{2\ell} = \frac{22}{2\times1} = 11$ Hz

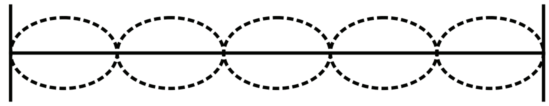
**Sol 15:**

$$A_{eq} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$A_{eq} = \sqrt{4^2 + 3^2 + 2(4)(3)\cos\frac{\pi}{2}}$$

$$A_{eq} = 5$$

**Sol 16: (B, C)** $y = 0.01\text{ m} \sin(20\pi x)\cos 200\pi t$



No. of nodes is 6

$$20\pi = \frac{2\pi}{\lambda}$$

$$\therefore \lambda = \frac{1}{10}\text{m} = 0.1\text{m}$$

Length of the spring $= 0.5 \times \frac{1}{2} = 0.25$

Mid point is the antinode

Frequency at this mode is $f = \frac{200\pi}{2\pi} = 100\text{Hz}$

$\therefore$ Fundamental frequency $= \frac{100}{5} = 20\text{Hz}$

**Sol 17: (A, C, D)** Taking y(t) = A f(x) g(t) & Applying the conditions:

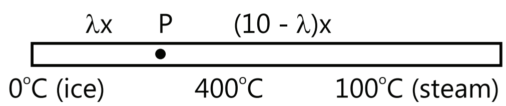
1; here x = 3m is antinode & x = 0 is node

2; possible frequencies are odd multiple of fundamental frequency.

where, $\nu_{fundamental} = \frac{v}{4\ell} = \frac{25}{3}\text{Hz}$

The correct options are A, C, D.

**Sol 18:**




$$\frac{dm_{ice}}{dt} = \frac{dm_{vapour}}{dt}$$

$$\frac{400kS}{\lambda x L_{ice}} = \frac{300kS}{(100-\lambda)xL_{vapour}}$$

$$\lambda = 9$$

**Sol 19:** A $\rightarrow$ p, t; B $\rightarrow$ p, s; C $\rightarrow$ q, s; D $\rightarrow$ q, r

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