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15. CALORIMETRY AND THERMAL EXPANSION

1. INTRODUCTION

Have not been we dealing with the temperature and thermal energy in our daily life? Such as, we store our perishable food in refrigerator, switch on the heater of the car if we ever feel cold, and always handle hot utensils with thermal glove. To make a cup of cold coffee, ice cubes are used by our mother and how can of coke kept out of refrigerate comes to the room temperature.

2. DEFINITION OF HEAT

Heat is energy in transient. Heat energy flows from one body to another body due to their temperature difference. It is measured in units of calories. The SI unit is Joule. 1 calorie = 4.2J

Illustration 1: What is the difference between heat and temperature? **(JEE MAIN)**

Sol: Temperature is associated with kinetic energy of atoms/molecule while heat is energy in transit. Temperature is a measure of the motion of the molecules or atoms within a substance; more specifically, it is the measure of the average kinetic energy of the molecules or atoms in a substance. Heat is the flow of energy from one body to another as a result of a temperature difference. It is important to point out that matter does not contain heat; it contains molecular kinetic energy and not heat. Heat flows and it is the energy that is being transferred. Once heat has been transferred to an object, it ceases to be heat. It becomes internal energy.

3. DEFINITION OF CALORIE

The amount of heat needed to increase the temperature of 1 g of water from 14.5°C to 15.5°C at a pressure of 1 atm is called 1 calorie.

1 kilo calorie =10^3 calories; 1 calorie = 4.186 Joule

If the temperature of a body a mass m is raised through a temperature ΔT, then the heat, ΔQ, given to the body is $\Delta Q = m.s.\Delta T$ where s is the specific heat of the body which is defined as the amount of the heat required to raise the temperature of a unit mass of the body through 1°C. Its unit is cal/gm/°C or J/kg/K.

Thermal capacity of a body is the quantity of heat required to raise its temperature through 1°C and is equal to the product of mass and specific heat of the body. $Q=m\int_{T1}^{T_2} s\,dt$ (be careful about unit of temperature, use units according to the given units of s)

PLANCESS CONCEPTS

Historically, first calorie was defined and hence such a weird unit conversion is used between calorie and Joule.

Chinmay Spurandare (JEE 2012, AIR 698)

4. PRINCIPLE OF CALORIMETRY

When two bodies at different temperatures are mixed, heat will pass from the body at a higher temperature to the body at a lower temperature until the temperature of the mixture becomes constant. The principle of calorimetry implies that heat lost by the body at a higher temperature is equal to the heat gained by the other body at a lower temperature assuming that there is no loss of heat in the surroundings.

5. TEMPERATURE SCALES

5.1 Kelvin Temperature Scale

Kelvin is a temperature scale designed such that zero K is defined as absolute zero (at absolute zero, a hypothetical temperature, all molecular movement stops- all actual temperatures are above absolute zero) and the size of one unit is the same as the size of one degree Celsius. Water freezes at 273.15K; water boils at 373. 15K. [K=C+273.15°, F = (9/5) C+32°] . For calculation purposes, we take 0^oC = 273K.

5.2 Celsius Temperature Scale

Celsius Temperature Scale - Temperature Scale according to which the temperature difference between the reference temperature of the freezing and boiling of water is divided into 100 degrees. The freezing point is taken as zero degree Celsius and the boiling point as 100 degrees Celsius. The Celsius scale is widely known as the centigrade scale because it is divided into 100 degrees.

5.3 Fahrenheit Scale

Fahrenheit temperature scale is a scale based on 32 for the freezing point of water and 212 for the boiling point of water, the interval between the two being divided into 180 parts.

The 18^{th} –century German physicist Daniel Gabriel Fahrenheit originally took as the zero of his scale the temperature of an ice- salt mixture and selected the value of 30 and 90 for the point of water and normal body temperature, respectively; these later were revised to 32 and 96, but the final scale required an adjustment to 98.6 for the latter value

	Kelvin	Celsius	Fahrenheit
Water boils	373.16K	100^0C	212^0F
Water freezes	273.16K	0^0	32^0F
Absolute zero	0 k	-273.16^0C	-459.7^0F

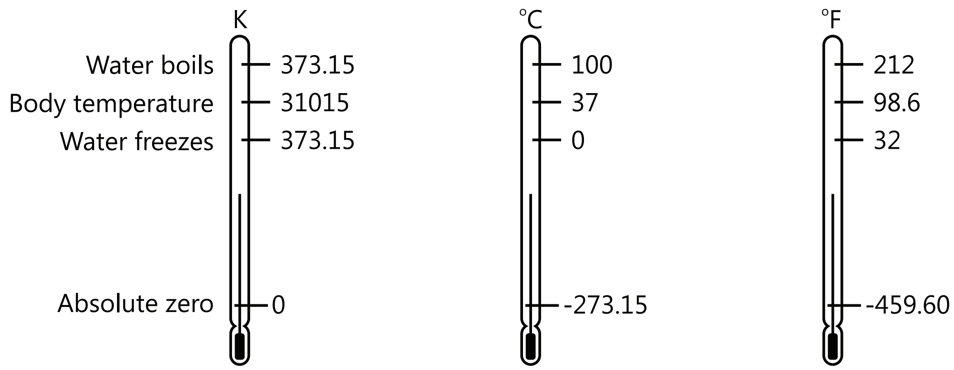


Figure 15.1

PLANCESS CONCEPTS

For easy conversion of temperature units, remember the following equation

$$\left(\frac{C-0}{100-0}\right) = \left(\frac{F-32}{212-32}\right) = \left(\frac{K-273}{373-273}\right)$$

Where C, F and K are respectively temperatures in Celsius, Fahrenheit and Kelvin scale. Note the values used in denominator, are actually the boiling and melting points of water in respective scales, so quite easy to remember.

Nitin Chandrol (JEE 2012, AIR 134)

Illustration 2: Express a temperature of 60° F in degree Celsius and in Kelvin. **(JEE MAIN)**

(i) $T_C = T - 273.15$ (ii) $T_F = 32 + \frac{9}{5}T_C$

Sol: (Using above formulas) Find the temp in Celsius first, then in Kelvin as kelvin and Celsius have more simple relation. Substituting T_F =60°C in Eq. (ii); $T_C = \frac{5}{9}\left(T_F - 32\right) = \frac{5}{9}\left(60°C - 32°C\right) = 15.55°C$

From Eq. (i) T=Tc +273.15=15.55°C+273.15=288.7K

Illustration 3: Calculate the temperature which has the same value on (i) the Celsius and Fahrenheit (ii) Fahrenheit and Kelvin scales. **(JEE ADVANCED)**

Sol: The value of temp which shows same reading on the Celsius as well as on the Fahrenheit (i part) and on the Kelvin and Fahrenheit (ii part).

(i) Let the required temperature be x°, now $T_F = \frac{9}{5}T_C + 32$

or $5T_F = 9T_C + 160$ or 5X=9X+160 $\therefore X = \frac{160}{-4} = -40°$ $\Rightarrow$ -40°C =-40°F

(ii) Let the required temperature be x° $\frac{T_F - 32}{180} = \frac{T_k - 273.15}{100}$

$$\therefore \frac{X-32}{180} = \frac{T_k - 273.15}{100}$$

On solving we get, X = 574.6

5.4 Triple Point of Water

The triple point of water is that unique temperature and pressure at which water can coexist in equilibrium between the solid, liquid and gaseous states. The pressure at the triple point of water is 4.58 mm of Hg and the temperature is 273.16K (or 0.01°C). The absolute or Kelvin temperature T at any point is then defined, using a constant volume gas thermometer for an ideal gas as: $T = 273.16 \times \frac{p}{p_{tp}}$ [ideal gas; constant volume]

In this relation, p_{tp} is the pressure in the thermometer at the triple point temperature of water and P is the pressure in the thermometer when it is at the point where T is being measured. Note that if we let $P=P_{tp}$ in this relation, T=273.16 K as it must.

Illustration 4: When in thermal equilibrium at the triple point of water, the pressure of Hg in a constant volume gas thermometer is 1020 Pa. The pressure of He is 288 pa when the thermometer is in thermal equilibrium with liquid nitrogen at its normal boiling point. What is the normal boiling point of nitrogen as measured using this thermometer? **(JEE MAIN)**

Sol: As we consider volume of the fluid to be constant, and hence T/P ratio remains constant, Normal boiling point of nitrogen is $T = 273.16 \times \frac{p}{p_{tp}}$; Here P=288 Pa; p_{tp} =1020 Pa

$\therefore\ T = 273.16 \times \frac{288}{1020} = 77.1\text{K}$

6. HEAT CAPACITY

The heat capacity of a body is defined as the amount of heat required to raise its temperature by 1°C. It is also known as the thermal capacity of the body. Suppose a body has mass m and specific heat c. Heat capacity = Heat required to raise the temperature of the body by 1°C = mc × 1 =mc

$\therefore$ Heat capacity =mc

Hence heat capacity of a body (solid or liquid) is equal to the product of its mass and specific heat. Clearly, the SI unit of heat capacity is J/°C or J/K. The greater the mass of a body, the greater is its heat capacity.

7. SPECIFIC HEAT CAPACITY

When we supply heat to a solid substance (or liquid), its temperature increases. It is found that the amount of heat Q absorbed by the solid substance (or liquid), is

(i) Directly proportional to the mass (m) of the substance i.e., $Q \propto m$

(ii) Directly proportional to the rise in temperature (ΔT) i.e.. $Q \propto \Delta T$

Combining the two factors, we have, $Q \propto m\,\Delta T$ .... (i)

or $Q = cm\,\Delta T$

Where C is constant of proportionality and is called specific heat capacity or simply specific heat of the substance.

From eq. (i), we have $c = \frac{Q}{m\Delta T}$ .... (ii)

If m = 1 kg and ΔT = 1°C, then c = Q.

Hence the specific heat of a solid (or liquid) may be defined as the amount of heat required to raise the temperature of 1kg of solid (or liquid) through 1°C (or 1K). It is clear from eq. (ii) that SI unit of specific heat is J kg^{-1} °C^{-1} or J kg^{-1} K^{-1}.

PLANCESS CONCEPTS

Don't get confused here with the terminology of heat capacity and specific heat capacity. Always remember that Specific heat capacity is the property of material and heat capacity is property of a given body.

B Rajiv Reddy (JEE 2012, AIR 11)

Illustration 5: A geyser heats water flowing at the rate of 3.0 liters per minute from 27°C to 77°C. If the geyser operates on a gas burner, what is the rate of combustion of the fuel if its heat of combustion is 4.0×10^4 J g^{-1}? **(JEE ADVANCED)**

Sol: The total heat required to increase the temperature of the water is equal to the heat supplied by the combustion of gas per minute. Mass of 3 liters of water =3kg $\therefore$ Mass of water flowing per minute, m= 3 kg=3000 g min^{-1}

Rise of temperature, $\Delta\theta$=77-27=50°C; Heat absorbed by water per minute= $mc\Delta\theta$ $=3000\times 1\times 50$cal

$$= 3000\times 1\times 50\times 4.2 \text{ J min}^{-1} = 630000 \text{ J min}^{-1}$$

$\therefore$ Heat supply by gas burner= 630000 J min^{-1} and heat of combustion of fuel = 4.0×10^4 J g^{-1}

$\therefore$ Rate of combustion of fuel = $\dfrac{630000}{4.0\times 10^4} = 15.75$ g min^{-1}

Illustration 6: A copper block of mass 60 g is heated till its temperature is increased by 20°C. Find the heat supplied to the block. Specific heat capacity of copper= 0.09 cal g^{-1} $°C^{-1}$. **(JEE MAIN)**

Sol: Here the heat is utilized to increase the temperature of the block only.

The heat supplied is $Q=ms\Delta\theta$ = (60 g) (0.09 cal g^{-1} $°C^{-1}$) (20°C) = 108 cal.

The quantity ms is called the heat capacity of the body. Its unit is J K^{-1}. The mass of water having the same heat capacity as given body is called the water equivalent of the body.

8. MOLAR SPECIFIC HEAT CAPACITY FOR SOLIDS OR LIQUIDS

The molar specific heat of a solid (or liquid) is defined as the amount of heat required to raise the temperature of 1 mole of the solid (or liquid) through 1°C (or 1K). It is denoted by the symbol C. Therefore, the amount of heat Q required to raise the temperature of n moles of a solid (or liquid) through a temperature change ΔT is given by; $Q = nC\Delta T$

It is clear that SI unit of C is J mol^{-1} K^{-1}. For any material of mass m and molecular mass M, the number of moles n=m/M. $\therefore$ $Q = \frac{m}{M}C\Delta T$ also $\therefore$ $Q = mC\Delta T$ $\therefore \frac{m}{M}C\Delta T = mc\ \Delta T$ or $C = Mc$ ...(i)

Eq. (i) gives the relation between molar specific heat C and the ordinary specific heat.

9. MOLAR SPECIFIC HEAT CAPACITY FOR THE GASES

The amount of heat required to increase the temperature of 1 mole of a gas through 1°C is called molar heat capacity.

The number of moles, n, in mass m of the gas is given by $n = \dfrac{\text{Mass of the gas}}{\text{Molecular weight}}$

9.1 Molar Specific Heat at Constant Volume, C_v:

If $(\Delta Q)_V$ is the heat required to raise the temperature of mass m gm or n moles of gas of molecular weight M at constant volume through temperature ΔT, $(\Delta Q)_V = mc_v\Delta T = nMc_v\,\Delta T = nC_v\,\Delta T$

Where C_v molar specific is heat at constant volume and is equal to Mc_V.

9.2 Molar specific heat at Constant Pressure, C_p:

If $(\Delta Q)_p$ is the heat required to raise the temperature of mass m gm or n moles of gas of molecular weight M at constant pressure through temperature ΔT, $(\Delta Q)_p = mc_p\Delta T = nMc_p\,\Delta T = nC_p\,\Delta T$

Where C_p molar specific is heat at constant volume and is equal to Mc_p.

For monatomic gases, $C_p = \frac{5R}{2}, C_V = \frac{3R}{2}; \gamma = \frac{C_p}{C_v} = \frac{5}{3} = 1.67$; for diatomic gases, $C_p = \frac{7R}{2}, C_V = \frac{5R}{2}\ \gamma = \frac{C_p}{C_v} = \frac{7}{5} = 1.4$;

Mayer's relation gives, $C_p - C_V = R$; where $C_V = \frac{R}{\gamma - 1}, C_p = \frac{\gamma R}{\gamma - 1}$

Illustration 7: How much heat is required to raise the temperature of an ideal monoatomic gas by 10 K if the gas is maintained at constant pressure? **(JEE MAIN)**

Sol: The process is at constant pressure here. Formula for heat capacity of gas at constant pressure is used.

The heat required is given by $Q = n\ C_p\,\Delta T$ Here n=1 & $\Delta T = 10$ K;

$C_p = \frac{5}{2}R = \frac{5}{2} \times 8.3\ \text{J mol}^{-1}\ \text{K}^{-1}$; $\therefore\ Q = 1 \times \frac{5}{2} \times 8.3 \times 10 = 207.5$ J

PLANCESS CONCEPTS

Without calculation, one can tell that C_p is always greater that C_v. Think of a situation in which we need to raise the temperature of same amount of gas in constant pressure conditions and constant volume conditions. It is quite obvious that in constant volume conditions all the heat will be used up to raise internal energy of gas. We see that the rise in internal energy of gas is same in both cases as increase in temperature is same. However, we see that for constant pressure conditions, more heat is required as some of it will also be used to expand the volume. This condition requires that C_p must be greater than C_v.

Anand K (JEE 2011, AIR 47)

Illustration 8: Calculate the amount of heat necessary to raise the temperature of 2 moles of He gas from 20°C to 50°C using (i) constant –volume process and (ii) constant-pressure process.

For He $C_V = 1.5$ R and $C_p = 2.49$ R. **(JEE ADVANCED)**

Sol: Heat capacity at constant volume and constant pressure are applicable here.

(i) The amount of heat required for constant –volume process is $Q_v = C_v\Delta T$; Here n=2 moles;

$C_V = 1.5\ R = 1.5 \times 8.314\text{J mol}^{-1}\ {}^0\text{C}^{-1}$; $\Delta T = 50\text{-}20 = 30°\text{C}$; $Q_v = 2 \times (1.5 \times 8.314) \times 30 = 748$ (ii) The amount of heat required for constant –pressure process is $Q_p = nC_p\Delta T$

Here n=2 moles; $C_p = 2.49\ R = 2.49 \times 8.314\text{J mol}^{-1}\ {}^0\text{C}^{-1}$; $\Delta T = 30°\text{C}$

$\therefore\ Q_p = (2.49 \times 8.314) \times 30 = 1242\text{J}$

Since the temperature rise is the same in the two cases, the change in internal energy is same i.e, 748 J. however, in constant-pressure excess heat supplied =1242-748=494 J. This extra heat of 494 J went into the work of expansion of the gas.

10. LATENT HEAT

The amount of heat required to change a unit mass of a substance completely from one state to another at constant temperature is called the latent heat of the substance.

If a substance of mass m required heat Q to change completely from one state to another at constant temperature, then, the latent heat $L = \frac{Q}{m}$. The SI unit of latent heat of a substance is J kg^{-1}. There are two types of latent heats viz. latent heat of fusion and latent heat of vaporization.

(a) Latent heat of fusion. We know that a solid changes into liquid at a constant temperature which is called the melting point.

The amount of heat required to change the unit mass of solid mass into its liquid state at constant temperature is called the latent heat of fusion of the solid.

For example, the latent heat of fusion of ice is 334 J/kg. It means to change 1 kg of ice at 0°C into liquid water at 0°C, we must supply 334 KJ of heat.

(b) Latent heat of vaporization. We know that a liquid changes into gaseous state at a constant temperature which called the boiling point. The amount of heat required to change the unit mass of a liquid into its gaseous state at constant temperature is called latent heat of vaporization of the liquid.

Illustration 9: A piece of ice of mass 100 g and at temperature 0°C is put in 200 g of water at 25°C. How much ice will melt as the temperature of the water reaches 0°C? The specific heat capacity of water =4200 JK^{-1} and the specific latent heat of fusion of ice= 3.4×10^5 JK^{-1}. **(JEE MAIN)**

Sol: Total heat lost by the water equal to the total heat gained by the ice.

The heat released as the water cools down from 25°C to 0°C is

$Q = ms\Delta\theta = (0.2\ kg)(4200\ Jk^{-1}\ K^{-1})(25K) = 21000\,J.$

The amount of ice melted by this much heat is given by $m = \frac{Q}{L} = \frac{21000\,J}{3.4 \times 10^5\ Jkg^{-1}} = 62g$

11. WATER EQUIVALENT

The water equivalent of a body is defined as the mass of water that will absorb or lose the same amount of heat as the body for the same rise or fall in temperature. The water equivalent of a body is measured in kg in SI unit and in g in C.G.S. units. Suppose the water equivalent of a body is 10 kg. It means that if the body is heated through, say 10°C, it will absorb the same amount of heat as absorbed by 10 Kg of water when heated through 10°C. Consider a body of mass m and specific heat required to raise the temperature of the body through ΔT is $Q = cm\Delta T$ ...(i)

Suppose w is water equivalent of this body. Then, by definition, Q is given by:

$Q = w\ \Delta T$ From eqs. (ii) And (ii), we have, $w\ \Delta T = cm\ \Delta T$ or w= m c

Thus the water equivalent of a body is numerically equal to the product of the mass of the body and its specific heat. Note that mc is the heat capacity of the body. Therefore, we may conclude that water equivalent and heat capacity of a body are numerically equal.

Illustration 10: A calorimeter of water equivalent 15g contains 165 g of water at 25°C. Steam at 100° is passed through the water for some time. The temperature is increased to 30°C and the mass of calorimeter and its contents are increased by 1.5 g. Calculate the specific latent heat of vaporization of water. Specific heat capacity of water is 1 cal g^{-1} $°C^{-1}$. **(JEE ADVANCED)**

Sol: The change in mass of the content of calorimeter is due to formation of more water from condensation of steam and all comes to the same temperature.

let L be the specific latent heat of vaporization of water. The mass of the steam condensed is 1.5 g. Heat lost in condensation of steam is $Q_1 = (1.5g)L$. The condensed water cools from 100°C to 30 °C. Heat lost in the process is

$$Q_2 = (1.5g)\left(1\ \text{cal}g^{-1}\,{}^0C^{-1}\right)(70°C) = 105\,\text{cal}.$$

Heat supplied to the calorimeter and to the cold water during the rise in temperature from 25°C to 30°C is

$$Q_3 = (15g + 165g)\left(1\ \text{cal}g^{-1}\ {}^0C^{-1}\right)(5°C) = 900\,\text{cal}.$$

If no heat is lost to the surrounding.

$(1.5g)L + 105\,\text{cal} = 900\,\text{cal}$ or L= 530 cal g^{-1}

Illustration 11: The water equivalent of a body is 10 kg. What does it mean? **(JEE MAIN)**

Sol: It means that if a body is heated through say 5°C, it will absorb the same amount of heat as absorbed by 10 kg of water when heated through 5°C.

12. MECHANICAL EQUIVALENT OF HEAT

In early days, heat was not recognized as a form of energy. Heat was supposed to be something needed to raise the temperature of a body or to change its phase. Calorie was defined as the unit of heat. A number of experiments were performed to show that the temperature may also be increased by doing mechanical work on the system. These experiments established that heat is equivalent to mechanical energy and measured how much mechanical energy is equivalent to a calorie. If mechanical work W produces the same temperature change as heat H, we write, W=JH. Where J is called mechanical equivalent of heat. It is clear that if W and H are both measured in the same unit then J=1. If W is measured in joule (work done by a force of 1 N in displacing an object by 1 m in its direction) and H in calorie (heat required to raise the temperature of 1 g of water by 1°C) then J is expressed in joule per calorie. The value of J gives how many joules of mechanical work is needed to raise the temperature of 1 g of water by 1°C.

Illustration 12: Assuming that the density of air at N.T.P=0.0013 g/cc, C_p = 0.239 cal $g^{-1}K^{-1}$ and the ratio C_p/C_V = 1.40, calculate the mechanical equivalent of heat. **(JEE MAIN)**

Sol: Compare the value of Gas Constant (R) by calculating in different unit (Calorie and Joule). $R = C_p - C_V$. And R=PV/T, then find the ratio (in joule/in calorie).

Now, C_p = 0.239 cal $g^{-1}K^{-1}$; C_p/C_V =1.40; $\therefore\ C_V = \frac{C_p}{1.40} = \frac{0.239}{1.40} = 0.171\,\text{cal}\ g^{-1}K^{-1}$

Volume of 1 g of air at N.T.P. = $\frac{1}{0.0013}\text{cc} = \frac{10^{-6}}{0.0013}\text{m}^3$

Volume of 1 kg $(=1000g)$ of air at N.T.P., $V = \frac{10^{-6}}{0.0013} \times 1000 \frac{10^{-3}}{0.0013}\text{m}^3$

Normal pressure, p = hρg = $0.76 \times 13600 \times 9.8$ = 101292.8 Nm^{-2}

Normal temperature, T = 273 K

Gas constant r for 1 kg of air is given by; $R = \frac{PV}{T} = 101292.8 \times \frac{10^{-3}}{0.0013} \times \frac{1}{273} = 285.4\ Jkg^{-1}\ K^{-1}$

Note that $R = C_p - C_V$. It means that if 1 kg of air is heated through 1°C (or 1K) first at constant pressure and then at constant volume, then extra heat needed for constant-pressure process to do this work of 285.4 J i.e., W = 285.4 J

Heat supplied to do work is $Q = 1000g \times C_p \times 1K - 1000g \times C_V \times 1K$

$$= 1000g \times 0.239 \times 1K - 1000g \times 0.171 \times 1K$$

$$= 239 - 171 = 68\ cal;\ \text{Now}\ W = JQ;\qquad \therefore\ J = \frac{W}{Q} = \frac{285}{68} = 4.2J/cal$$

13. LAW OF HEAT EXCHANGE

When a hot body is mixed or kept in contact with a cold body, the hot body loses heat and its temperature falls. On the other hand, the cold body gains heat and its temperature rises. The final temperature of the mixture will lie between the original temperatures of the hot body and the cold body. If a system is completely isolated, no energy can flow into and out of the system. Therefore according to the law of conservation of energy, the heat lost by one body is equal to the heat gained by other body i.e. Heat lost = heat gained

This is known as law of heat exchange.

THERMAL EXPANSION

1. DEFINITION OF THERMAL EXPANSION

It is the expansion due to increase in temperature. Most substances expand when they are heated. Thermal expansion is a consequence of the change in average separation between the constituent atoms of an object. Atoms of an object can be imagined to be connected to one another by stiff springs as shown in figure. At ordinary temperatures, the atoms in a solid oscillate about their equilibrium positions with an amplitude of approximately 10^{-11} m. The average spacing between the atoms is about $10^{-1°}$m. As the temperature of the solid increases, the atoms oscillate with greater amplitudes, as a result the average separation between them increases, and consequently the object expands. More precisely, thermal expansion arises from the asymmetrical nature of the potential energy curve.

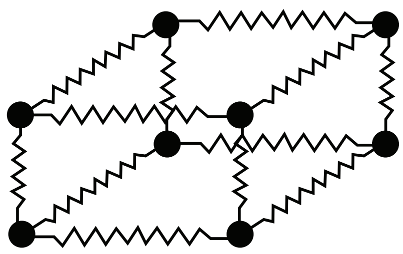


Figure 15.2

2. THERMAL EXPANSION OF SOLIDS

2.1 Linear Expansion

When a solid substance is heated, most of them generally expand. If a solid has a length L_0 and has a very small area of cross-section, at a temperature T_0, its length increases to L_T when its temperature is increased by ΔT. The increase in length, ΔL, is then given by,

$\Delta L = L_T - L_0 = L_0 \times \alpha \times \Delta T$ Where α is the coefficient of linear expansion which is given by

$$\alpha = \frac{L_T - L_0}{L_0 \Delta T};\ \ L_T = L_0(1 + \alpha\,\Delta T)$$

The coefficient of linear expansion is equal to the increase in length per unit length per degree rise of temperature.

The SI unit of α is /°C or /K. Its value is different for different solid materials. For example α for aluminum is $2.4\times10^{-5}\ /^{0}C$ whereas for brass, its value is $2.0\times10^{-5}/^{o}C$. Note that the change in temperature ΔT will be the same whether it is measured in Celsius scale or on the Kelvin scale: $\Delta T\ ^{0}C = \Delta T\ K$.

2.2 Superficial Expansion

If a solid plate of area A_0 and of very small thickness is heated through a temperature ΔT so that its area increases to A_T, then the increase in area ΔA is given by

$$\Delta A = A_T - A_0 = \beta A_0 \Delta T \qquad \text{or} \qquad \beta = \frac{A_T - A_0}{A_0 \Delta T}$$

Where β is called the coefficient of superficial expansion. $\beta = 2\alpha$

Hence the coefficient of superficial expansion of a solid may be defined as the fractional change in surface area $(\Delta S / S)$ per degree change in temperature. Its SI unit is also $/^{0}C$ or /K.

Note that exchange in temperature ΔT will be the same whether it is measured on the Celsius scale or on the Kelvin scale.

2.3 Volume Expansion

If a solid of initial volume V_0 at any temperature is heated so that its volume is increased to V_T with increase of temperature ΔT, the increase in volume, ΔV, is given by

$$\Delta V = V_T - V_0 = \gamma V_0 \Delta T\ ;\ \gamma = \frac{V_T - V_0}{V_0 \Delta T} = \frac{\Delta V}{V_0 \Delta T}$$

Where γ is called the coefficient of volume or cubical expansion. $\gamma = 3\alpha$

As the temperature of solid increases, the amplitude of oscillation of atoms increases which results in an increase of average distance between atoms with increase of temperature due to which the volume increases. If ρ_0 is the density of a solid at 0°C and ρ_T is its density T°C, then for a constant mass m of the solid,

$$\rho_0 = \frac{m}{V_0} \text{ And } \rho_T = \frac{m}{V_T} \text{ where } V_0 \text{ and } V_T \text{ are its respective volume at 0°C \& T°C}$$

$$\therefore \frac{\rho_0}{\rho_T} = \frac{V_T}{V_0} = \frac{V_0(1+\gamma T)}{V_0} = 1+\gamma T \quad \therefore \rho_T = \frac{\rho_0}{1+\gamma T}$$

Hence coefficient of cubical expansion of a solid may be defined as the fractional change in volume $(\Delta V / V)$ per degree change in temperature. Its SI unit is / °C or /K.

PLANCESS CONCEPTS

For anisotropic solids $\beta = \alpha_1 + \alpha_2$ and $\gamma = \alpha_1 + \alpha_2 + \alpha_3$.

Here α_1, α_2 and α_3 are coefficients of linear expansion in X, Y and Z directions.

For solid value of are generally small so we can write density, $d = d_0(1-\gamma\Delta T)$ (Using binomial expansion). γ is not always positive. It can have a negative value.

E.g. For water, density increases from 0 to 4°C so γ is -ve (0 to 4°C) and for 4°C to higher temperature γ is –ve. At 4°C density is maximum. Coefficients of thermal expansion are generally not independent of temperature. But for JEE purpose you are supposed to assume it as a constant if not mentioned.

If α is not constant

(i) (α varies with distance) Let α = ax +b; Total expansion= $\int$ expansion of length dx = $\int_0^1 (ax+b)\,dx\Delta t$

PLANCESS CONCEPTS

(ii) (α Varies with temperature) ; Let $\alpha = f(T)$; $\Delta\ell = \int_{T_1}^{T_2} \alpha\ell_0 dT$

Caution: If α is in°C,

Then put T_1 and T_2 in °C. Similarly if α is in K then put T_1 and T_2 in K.

If you have a difficulty in remembering the definition of different capacity then just look at the units given heat capacity and figure out whether it's per unit mass/mole/ or for entire mass.

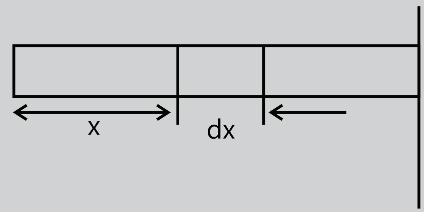


Figure 15.3

Yashwanth Sandupatla (JEE 2012, AIR 821)

3. PRACTICAL APPLICATION OF THERMAL EXPANSION OF SOLIDS

There are a large number of important practical applications of thermal expansion of solids. However, we shall brief only a few of them by way of illustration.

(a) While laying the railway tracks, a small gap is left between the successive lengths of the rails. This gap is provided to allow for the expansion of the rails during summer. If no gap is left, these expansions cause the rails to buckle.

(b) When the iron tyre of a wheel to be put on the wheel, the tyre is made slightly smaller in diameter than that of wheel. The iron tyre is first heated uniformly till its diameter becomes more than that of the wheel and is then slipped over the wheel. On cooling the tyre contracts and makes a tight fit on the wheel.

(c) In bridges, one end is rigidly fastened to its abutment while the other rests on rollers. This provision allows the expansion and contraction to take place during changes in temperature.

(d) The fact that a solid expands on heating and contracts on cooling is utilized in riveting e.g., riveting two metal plates together, joining steel girders etc. For joining two steel plates, holes are drilled between them. The rivets (small rods) are made red hot and inserted in the holes in the plates. The ends of the rivets are hammered into the head. After some time, the rivets contract on being and hold the plates very tightly.

(e) The concrete roads and floor are always made in sections and enough space is provided between the sections. This provision allows expansion and contraction to take place due to change in temperature.

PLANCESS CONCEPTS

- If a solid object has a hole in it, what happens to size of the hole, when the temperature of the object increases? A common misconception is that if the object expands, the hole will shrink because material expands into the hole. But the, truth is that if the Object expands the hole will expand too, because every linear dimension of an object change in the same way when the temperature changes.

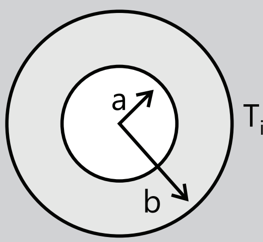


Figure 15.4

- Effect of temperature on the time period of a pendulum:

  The time Period of a simple pendulum is given by.

$$T = 2\pi\sqrt{\frac{\ell}{g}}; \qquad \text{or} \qquad T \propto \sqrt{\ell}$$

PLANCESS CONCEPTS

As the temperature is increased length of the pendulum and hence, time period gets increased or a pendulum clock becomes slow and it's loses the time. Time lost in time t(by a pendulum clock whose actual time period is T and changed time period at some higher temperature is T') is $\Delta t = \left(\frac{\Delta T}{T'}\right) t$.

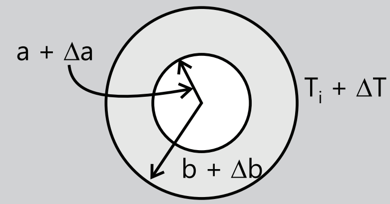


Figure 15.5

Similarly, if the temperature is decreased the length and hence, the time period gets decreased. A pendulum clock on this case runs fast and it gains the time. Time gained in time t is the same, i.e., $\Delta t = \left(\frac{\Delta T}{T'}\right) t$.

Gv Abhinav (JEE 2012, AIR 329)

Illustration 13: A steel ruler exactly 20 cm long is graduated to give correct measurements at 20°C.

(a) What happens to the reading if the temperature decreases below 20°C?

(b) What is the actual length of the ruler at 40°C? **(JEE MAIN)**

Sol: Lowering the temperature, shorten the scale from 1 m of original length. It'll show length of 1m lengthier than its length. And hence will show 1m to be more than 1m. It will now measure more. And reverse, in case of increasing the temperature.

(a) If the temperature decreases, the length of the ruler also decreases through thermal contraction. Below 20°C, each centimeter division is actually somewhat shorter than 1.0 cm, so the steel ruler gives reading that are too long.

(b) At 40°C, the increase in length of the ruler is

$$\Delta\ell = \ell\alpha\Delta T \quad = (20)(1.2\times10^{-5})(40^0 - 20^0) = 0.48\times 10^{-2} \text{ cm}$$

$\therefore$ The actual length of the ruler is, $\ell' = \ell + \Delta\ell = 20.0048$ cm

Illustration 14: A second pendulum clock has a steel wire. The clock is calibrated at 20°C. How much time does the clock lose or gain in one week when the temperature is increased to 30° C?

$\alpha_{steel} = 1.2\times10^{-5}\left({}^0C\right)^{-1}$ **(JEE ADVANCED)**

Sol: Increment in length increase the time period of oscillation.

The time period of second's pendulum is 2 seconds. As the temperature increases, length and hence, time period increases, clock becomes slow and it loses the time. The change in time period is $\Delta T = \frac{1}{2}T\alpha\Delta\theta$ = $\left(\frac{1}{2}\right)(2)(1.2\times10^{-5})(30^0 - 20^0) = 1.2\times10^{-4}$ s $\therefore$ New time period is, $T' = T + \Delta T = (2 + 1.2\times10^{-4}) = 2.00012$s

$\therefore$ Time lost in one week $\Delta t = \left(\frac{\Delta T}{T'}\right)t = \left(\frac{1.2\times10^{-4}}{2.0012}\right)(7\times24\times3600) = 36.28$s

4. THERMAL EXPANSION OF LIQUIDS

As a liquid in a vessel acquires the shape of the vessel, its heating increases the volume of the vessel initially due to expansion of the vessel which decreases the level of the liquid initially. When the temperature of the liquid is increased further, it increased the volume of the liquid. Thus the observed or apparent expansion of the liquid is lesser than the real level of the liquid when the temperature increases. Thus the apparent expansion of the liquid is lesser than the real expansion of the liquid which gives a value of coefficient of real expansion more than that for the coefficient of apparent expansion.

The coefficient of real expansion, g_r, of a liquid is defined as the real increase in volume per degree rise of temperature per unit original volume of the liquid.

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Orignal volume} \times \text{rise in temperature}}$$

The coefficient of apparent expansion, γ_a of a liquid is defined as the ratio of the observed increase in volume of the liquid with respect to the original level before heating per degree rise of temperature to the original volume of the liquid.

$$\gamma_a = \frac{\text{Observed increased in volume}}{\text{Orignal volume} \times \text{rise in temperature}}$$

If γ_g is the coefficient of cubical expansion of the material of the vessel, then $\gamma_r = \gamma_a + \gamma_g$; $\gamma_g = 3\alpha$

Then density of the liquid ρ_T at temperature T is related to density ρ_o at °C as $\rho_T = \frac{\rho_0}{1+\gamma T}$

Where γ is the coefficient of real expansion of the liquid and T is the increase in temperature.

It is clear that $\gamma_r > \gamma_a$ and both are measured unit $°C^{-1}$. It can be shown that: $\gamma_r = \gamma_a + \gamma_g$

Where γ_g is the coefficient of cubical expansion of glass (or material of the container).

Illustration 15: Find the coefficient of volume expansion for an ideal gas at constant pressure. **(JEE MAIN)**

Sol: Recall the formula for coefficient of volume expansion for ideal gas.

For an ideal gas $PV = nRT$

As P is constant, we have $P.dV = nRdt$ $\therefore \frac{dV}{dT} = \frac{nR}{P}$ or $\gamma = \frac{1}{v}.\frac{dV}{dT} = \frac{nR}{PV} = \frac{nR}{nRT} = \frac{1}{T}$ $\therefore \gamma = \frac{1}{T}$

5. THERMAL EXPANSION OF GASES

The molecules in an ideal gas have only kinetic energy due to their motion but do not possess any potential energy. The thermodynamic state of any gas is defined in terms of its pressure, volume and temperature denoted as P, V and T respectively. A change in one of these quantities produces a corresponding change in the other quantities depending upon the condition under which the transformation take place. Such changes are governed by the following gas laws:

5.1 Boyle's Law

The pressure of given mass of a gas is inversely proportional to its volume if temperature T remains constant

$P \propto \frac{1}{V}$ or $PV = \text{constant}$;

If the pressure P_1 and volume V_1 changes to the respective values P_2, V_2 when the temperature remains

Constant, then $P_1 V_1 = P_2 V_2$.

5.2 Charles's Law of Volume

The volume V of a given mass of a gas is directly proportional to its absolute temperature, T, when its pressure remains constant. $V \propto T \quad \frac{V}{T} = \text{constant}$

If the volume V_1 and temperature T_1 are respectively changed to V_2, T_2 at constant pressure, then $\frac{V_1}{T_1} = \frac{V_2}{T_2}$. Where temperatures T_1 and T_2 are in Kelvin scale.

If V_0 and V_t are volume of the gas at 0°C and t°C respectively,

$$\frac{V_t}{273+t} = \frac{V_0}{273};\ V_t = V_0\left[1+\frac{t}{273}\right]V_0 = \left[1+\alpha_v t\right]$$

Where α_V is the volume coefficient of a gas and is equal to 1/273.

5.3. Gay Lussac's Law of Pressure

The pressure of a given mass of a gas is directly proportional to its absolute temperature provided the volume of the gas is kept constant. $P \alpha T$ or $\frac{P}{T}$constant

If the pressure and temperature P_1, T_1 is change respectively to P_2, T_2 at constant volume, then

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} = \text{constant.}$$

If P_t and P_0 are pressure of the gas at t °C & 0^0 respectively, then $P_t = P_0\left(1+\alpha_p t\right) = P_0\left(1+\frac{t}{273}\right)$

Where α_p is equal to the pressures coefficient of the gas which also equal to 1/273.

5.4 Gas Equation

If the above mentioned three laws are combined, then $\frac{PV}{T}$ =constant; $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \text{constant}$.

The value of the constant depends on the mass of the gas.

If the gas has n moles, PV=nRT which is called the equation of the state of an ideal gas. R is called the universal or molar gas constant and its value in S.I. units is 8.314J. mol^{-1} K^{-1}.

6. RELATION BETWEEN COEFFICIENTS OF EXPANSION

We shall now show that for solid, the approximate relations between α, β and γ are:

$\beta = 2\alpha$ and $\gamma = 3\alpha$;

(a) Relation between β and α. Consider a square plate of side ℓ_0 at °C and ℓ_1 at t °C.

$\ell_1 = \ell_0\left(1+\alpha t\right)$;

Area of plate at 0°C, $A_0 = \ell_0^2$;

Area of plate at t°C, $A_1 = \ell_1^2 = \ell_0^2\left(1+\alpha t\right)^2 = A_0\left(1+\alpha t\right)^2$

Also Area of plate at t°C, $A_1 = A_0\left(1+\beta t\right)$

$\therefore\ A_0\left(1+\alpha t\right)^2 = A_0\left(1+\beta t\right)$ or $\therefore 1+\alpha^2t^2+2\alpha t = 1+\beta t$

Since the value of α is small, the term α^2t^2 may be neglected. $\therefore\ \beta = 2\alpha$

The result is altogether general because any flat surface can be regarded as a collection of small squares.

(b) Relation between γ and α. Consider a cube of side ℓ_0 at °C and ℓ_1 at t °C.

$\therefore\ \ell_1 = \ell_0\left(1+\alpha t\right)$; Volume of cube at 0C, $V_0 = \ell_0^3$;

Volume of cube at t °C, $V_1 = \ell_1^3 = \ell_0^3\left(1+\alpha t\right)^3 = V_0\left(1+\alpha t\right)$

Also Volume of cube at t °C, $V_1 = V_0\left(1+\gamma t\right)$; $\therefore\ V_0\left(1+\alpha t\right)^3 = V_0\left(1+\gamma t\right)$

Or $1+3\alpha t+3\alpha^2t^2+\alpha^3t^3 = 1+\gamma t$

Since the value of α is small, we can neglect the higher power of α.

$\therefore\ 3\alpha t = \gamma t$ or $\gamma = 3\alpha$

Again, result is general because any solid can be regarded as a collection of small cubes.

7. VARIATION OF DENSITY WITH TEMPERATURE

Variation of Density with temperature: Most substances expand when they are heated, i.e.. Volume of a given mass of a substance increases on heating, so the density should decrease (as $\rho \propto \frac{1}{V}$)

Let us see how the density (ρ) varies with increase in temperature. $\rho = \frac{m}{v}$ or $\rho \propto \frac{1}{V}$ (for a given mass)

$$\therefore \quad \frac{\rho'}{\rho} = \frac{V}{V'} = \frac{V}{V+\Delta V} = \frac{1}{1+\gamma\Delta T}; \qquad \therefore\ \rho' = \frac{\rho}{1+\gamma\Delta T}$$

This expression can also be written as, $\rho' = \rho\left(1+\gamma\Delta T\right)^{-1}$

As γ is small. $\left(1+\gamma\Delta T\right)^{-1} \neq 1-\gamma\Delta T$ $\therefore$ $\rho' \simeq \rho\left(1-\gamma\Delta T\right)$

Illustration 16: A glass flask of volume 200 cm^3 is just filled with mercury at 20° C. How much mercury will over flow when the temperature of the system is raised to 100°C? The coefficient of volume expansion of glass is 1.2×10^{-5}/°C and that of mercury is 18×10^{-5}/°C. **(JEE MAIN)**

Sol: Increase in temperature, increase the volume of both, mercury as well as flask but mercury expands more than flask because the coefficient of volume expansion of mercury is more than of flask.

The increase in the volume of the flask is $\Delta V = \gamma_R V\Delta T = \left(12\times10^{-5}\right)\times\left(200\right)\times\left(100-20\right) = 0.19cm^3$

The increase in the volume of the mercury is $\Delta V' = \gamma_m V\Delta T = \left(18\times10^{-5}\right)\times\left(200\right)\times\left(100-20\right) = 2.88cm^3$

$\therefore$ The volume of the mercury that will overflow $\Delta V' - \Delta V = 2.88 - 0.19 = 2.69cm^3$

Illustration 17: A sheet of brass is 40 cm long and 8 cm broad at 0 °C. If the surface area at 100°C is 320.1 cm^2, find the coefficient of linear expansion of brass. **(JEE MAIN)**

Sol: Calculate the coefficient of area expansion, coefficient of linear exp. Equal to half of coeff. of area expansion.

Surface area of sheet at 0°C, $A_0 = 40\times 8 = 320cm^2$

Surface area of sheet at 100°C, A_{100}=320.1cm^2

Rise in temperature, $\Delta T = 100-0 = 100°C$

Increase in surface are $\Delta A = A_{100} - A_0 = 320.1 - 320 = 0.1\,cm^2$

Coefficient of surface expansion β is given by; $\beta = \frac{\Delta A}{A_0\times\Delta T} = \frac{0.1}{320\times100} = 31\times10^{-7}$ / °C

$\therefore$ Coefficient of linear expansion, $\alpha = \frac{\beta}{2} = \frac{31\times10^{-7}}{2} = 15.5\times10^{-7}\ /\ ^\circ C$

8. THERMAL STRESS

If the ends of rods of length L_0 are rigidly fixed and it is heated, its length L_0 tends to increase due to increase in temperature ΔT, but it is prevented from expansion. It results in setting up compressive or tensile stress in the rod which is called the thermal stress.

As Y= $\frac{\text{Stress}}{\text{Strain}}$, Stress=Y× Strain = $\frac{Y\Delta L}{L_0} = \frac{Y\alpha L_0 \Delta T}{L_0} = Y\alpha\Delta T$ The force, F, on rigid support is given by.

Where A is area of cross-section of the rod.

If ΔT represent a decrease in temperature, then F/A and F are tensile stress and tensile force respectively.

Note: When the temperature of a gas enclosed in a vessel of rigid material is increased, then thermal stress is equal to the increase in pressure (ΔP) and is given by: $\Delta P = K\gamma\Delta T$

Where K= bulk modulus of gas; γ =coefficient of cubical expansion; ΔT =increase in temperature

Proof. $V = V(1+\gamma\Delta T)$ or $V - V = V\gamma\Delta T$ or $\Delta V = V\gamma\Delta T$ now $K = \frac{V\Delta P}{\Delta V} = \frac{V\Delta P}{\gamma V\Delta T}$ $\therefore$ $\Delta P = \gamma K\Delta T$

Illustration 18: A steel wire of 2.0mm cross-section is held straight (but under no tension) by attaching it firmly to two rigid walls at a distance 1.50 m apart, at 30° C. If the temperature now decreases to -10°C, and if the end points remain fixed, what will be the tension in the wire? For steel, $Y = 200000 M\ Pa$ **(JEE MAIN)**

Sol: Here the concept of strain is applicable with linear expansion. Decreased temp. tends to decrease the length of wire but strain keep it intact.

Conceptualize: If free to do so, the wire would contract but since we have tied its ends, it will not contract and maintain its original length.

Classify: Until now we have seen when the length of a wire is changed, it produces strain and hence stress. This situation is different as strain will be produced because of wire maintaining its length. At a lower temperature the wire would have an unstrained length smaller than the original length. However since its ends are tied, it will maintain its length but develop strain. Or in other words it has longer length than what it would have had at this temperature if not tied at its ends.

Compute: If free to do so, the wire would contract a distance ΔL as it cooled, where

$\Delta L = \alpha L, \Delta T = (1.2\times10^{-5}\ ^0C^{-1})(1.5m)(40^0C) = 7.2\times10^{-4}$ m

But the ends are fixed. As a result, forces at the ends must, in effect, stretch the wire this same length, ΔL. Therefore,

from Y= $(F/A)/(\Delta L/L)$, we have tension F= $\frac{YA\Delta L}{L} = \frac{(2\times10^{11}\ N/m^2)(2\times10^{-6}\ m^2)(7.2\times10^{-4}\ m)}{1.5m}$ = 192 N

Conclude: Strictly, we should have substituted $(1.5\pm7.2\times10^{-4})$ m for L in the expression of tension. However. The error incurred in not doing so, is negligible.

PROBLEM-SOLVING TACTICS

While solving a problem of heat transfer in these cases, do look for state changes because that's where students generally make a mistake. State changes cause some of the energy to be used up as latent heat and hence must be taken care of always.

FORMULAE SHEET

1. Type of thermal expansion

		Coefficient of expansion	For temperature change Δt change in
(i)	Linear	$\alpha = \lim_{\Delta t \to 0} \frac{1}{\ell_0}\frac{\Delta \ell}{\Delta t}$	Length $\Delta \ell = \ell_0 \alpha \Delta t$
(ii)	Superficial	$\beta = \lim_{\Delta t \to 0} \frac{1}{A_0}\frac{\Delta A}{\Delta t}$	Area $\Delta A = A_0 \beta \Delta t$
(iii)	Volume	$\gamma = \lim_{\Delta t \to 0} \frac{1}{V_0}\frac{\Delta V}{\Delta t}$	Volume $\Delta V = V_0 \gamma \Delta t$

- For isotropic solids $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$(let) so $\beta = 2\alpha$ and $\gamma = 3\alpha$
- For anisotropic solids $\beta = \alpha_1 + \alpha_2$ and $\gamma = \alpha_1 + \alpha_2 + \alpha_3$ Here α_1, α_2 and α_3 are coefficient of linear expansion in X, Y, and Z directions.

Variation in density: With increase of temperature volume increases so density decreases and vice-versa.

$$\rho = \frac{\rho_0}{(1+\gamma \Delta t)} \approx \rho_0 (1 - r\, \Delta T)$$

Thermal Stress: A rod of length ℓ_0 is clamped between two fixed walls with distance ℓ_0.

If temperature is changed by amount Δt then stress= $\frac{F}{A}$ (area assumed to be constant)

Strain= $\frac{\Delta \ell}{\ell_0}$; so, $Y = \frac{F/A}{\Delta \ell / \ell_0} = \frac{F\ell_0}{A\Delta \ell} - \frac{F}{A\alpha\Delta t}$ or F=YA $\alpha \Delta t$

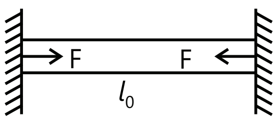


Figure 15.6

- $\Delta Q = mc\Delta T$ where c: Specific heat capacity
- $\Delta Q = nC\Delta T$ C: Molar heat capacity
- Heat transfer in phase change : $\Delta Q = mL$ L: latent heat of substance
- 1 Calorie= 4.18 joules of mechanical work
- Law of Calorimetry: heat released by one of the substances = Heat absorbed by other substances.

Solved Examples

JEE Main/Boards

Example 1: Calculate the amount of heat required to convert 1.00kg of ice at -10°C into steam at 100°C at normal pressure. Specific heat capacity of ice = 2100 Jk^{-1} K^{-1}, latent heat of fusion of ice=3.36×10^5 $JKg^{-1}K^{-1}$, specific heat capacity of water= 4200 $JKg^{-1}K^{-1}$ and latent heat of vaporization of water =2.25×10^6 JKg^{-1}.

Sol: Here the temperature of ice and water changes along with change in phases. i. e. ice to water and then water to steam.

Heat required to take the ice from -10 °C to 0^0C = $(1kg)(2100\ JKg^{-1}K^{-1})(10K) = 21000$ J.

Heat required to melt the ice at 0 °C to water = $(1kg)(3.36 \times 10^5\ JKg^{-1}) = 336000$ J.

Heat required to take 1 kg of water from 0 °C to 100 $= (1kg)(4200 JKg^{-1}K^{-1})(100K) = 420000$J.

Heat required to convert 1kg of water at 100°C into steam $= (1kg)(2.25 \times 10^6\ JKg^{-1}) = 2.25 \times 10^6$ J.

Total heat required = 3.03×10^6 J.

Example 2: A 5 g piece of ice at-20°C is put into 10g of water at 30°C. Assuming that heat is exchanged only between the ice and the water, find the final temperature of the mixture. Specific heat capacity of ice. = 2100 JKg^{-1} $°C^{-1}$ specific heat capacity of water = 4200 Jkg^{-1} $°C^{-1}$ and latent heat of fusion of ice = 3.36×10^5 JKg^{-1}.

Sol: Always proceed in similar questions assuming the final temperature to be the temperature of phase change (i.e. 0 here)

The heat given by the water when it cools down from 30°C to 0°C is

$$(0.0kg)\left(4200JKg^{-1}\ {}^{\circ}C^{-1}\right)(30°C) = 1260\,J$$

The heat required to bring the ice to 0°C is

$$(0.005kg)\left(2100\,JKg^{-1}\ {}^{\circ}C^{-1}\right)\left(20^{\circ}C\right) = 210\,J.$$

The heat required to melt 5 g of ice is

$$(0.005kg)\left(3.36 \times 10^5\ JKg^{-1}\ °C^{-1}\right) = 1680\,J\cdot$$

We see that whole of the ice cannot be melted as the required amount of heat is not provided by the water. Also, the heat is enough to bring the ice to 0°C. Thus the final temperature of the mixture is 0°C with some of the ice is melted.

Example 3: A thermally isolated vessel contains 100g of water at 0°C. When air above the water is pumped out, some of the water freezes and some evaporates at 0°C itself. Calculate the mass of ice formed if no water is left in the vessel. Latent heat of vaporization of water at 0°C= 2.10×10^6 JKg^{-1} and latent heat of fusion= 3.36×10^5 JKg^{-1}.

Sol: Some water evaporates and Heat of vaporization comes from water itself and hence remaining water freezes by giving the heat for vaporization.

Total mass of water=M=100g. Latent heat of vaporization of water at 0°C=L_1=$21.0 \times 10^5 Jkg^{-1}$ latent heat of fusion of ice= L_2= 3.36×10^5 JKg^{-1}. Suppose, the mass of the ice formed = m. Then the mass of water evaporated = M – m. Heat taken by the water to evaporate = (M – m)L_1 and heat given by the water in freezing=mL_2. Thus, mL_2= (M-m) L_1

$$\text{or, } m = \frac{ML_1}{L_1 + L_2} = \frac{(100g)\left(2.10 \times 10^6\ JKg^{-1}\right)}{(21.0 + 3.36)10^5\ JKg^{-1}} = 86g.$$

Example 4: A lead bullet penetrates into a solid object and melts. Assuming that 50% of its kinetic energy was used to heat it, calculate the initial speed of the bullet. The initial temperature of the bullet is 27°C and its melting point is 327°C. Latent heat of fusion of lead = 2.25×10^4 JKg^{-1} and specific heat capacity of lead = 125JKg^{-1} K^{-1}

Sol: Kinetic energy of bullet spatially converted into heat and melt it.

Let the mass of bullet = m.

Heat required to take the bullet from 27°C to 327°C =

$$m \times \left(125JKg^{-1}\ K^{-1}\right)(300K)$$

$$= m \times \left(3.75 \times 10^4\ JKg^{-1}\right)$$

Heat required to melt the bullet

$$= m \times \left(2.10 \times 10^6\ JKg^{-1}\right).$$

If the initial speed be v, the kinetic energy is $\frac{1}{2}mv^2$ and hence the heat developed is $\frac{1}{2}\left(\frac{1}{2}mv^2\right) = \frac{1}{4}mv^2$. thus,

$\frac{1}{4}mv^2 = m(3.75 + 2.5) \times 10^4 JKg^{-1}$ or v= 500 ms^{-1}

Example 5: An aluminum vessel of mass 0.5 kg contains 0.2 kg of water at 20°C. A block iron of mass 0.2 kg at 100°C is gently put onto the water. Find the equilibrium temperature of the mixture. Specific heat capacities of aluminum, iron and water are 910 $JKg^{-1}K^{-1}$, 470 $JKg^{-1}K^{-1}$ and 4200 $JKg^{-1}K^{-1}$ respectively.

Sol: Heat lost by the iron block increase the temperature of vessel and water.

Mass aluminum = 0.5kg,

Mass of water = 0.2kg;

Mass of iron = 0.2kg

Temp. of aluminum and water = 20°C = 293K

Temperature of iron = 100°C = 373K

Specific heat aluminum = 910J/kg-K

Specific heat of iron = 470J/kg-K

Specific heat of water = 4200J/kg-K

Heat gain = Heat lost;

$$\Rightarrow (T - 293)(0.5 \times 910 + 0.2 \times 4200) = 0.2 \times 470 \times (373 - T)$$

$$\Rightarrow (T - 293)(455 + 8400) = 49(373 - T);$$

$$\Rightarrow (T-293)\left(\frac{1295}{94}\right) = (373-T);$$

$$\Rightarrow (T-293)\times 14 = 373-T$$

$$\Rightarrow T = \frac{4475}{15} = 298K \quad \therefore \quad T = 298-273 = 25^0 C$$

The final temp= $25°C$.

Example 6: A Piece of iron of mass 100 g is kept inside a furnace for long time and then put in a calorimeter of water equivalent 10 g containing 240 g of water 20°C. The mixture attains an equilibrium temperature of 60°C.

Find the temperature of furnace. Specific heat capacity of iron=$470 JKg^{-1} °C^{-1}$.

Sol: This can be calculated in reverse manner, Heat lost by the iron piece is equal to heat required to increase the temperature of water and calorimeter.

Mass of iron = 100g

Water Eq of calorimeter = 10g;

Mass of water = 240g

Let the Temp. of surface = 0°C

S_{iron} = 470J/kg °C

Total heat gained = Total heat lost.

So, $\frac{100}{1000}\times 470\times(\theta-60) = \frac{250}{1000}\times 4200\times(60-20)$

$$\Rightarrow 470 - 47\times 60 = 25\times 42\times 40$$

$$\Rightarrow \theta = 4200 + \frac{2820}{47} = \frac{44820}{47} = 953.61^0 C$$

Example 7: The temperature of equal masses of three different liquids A, B and C are 120°C, 19°C and 280°C respectively. The temperature when A and B are mixed is 160°C, and when B and C are mixed, it is 23°C what will be the temperature when A and C are mixed?

Sol: All liquids have same mass. The heat lost by one equals to heat gain by other, so we can try to solve for the ratio of their heat capacities.

The temp. of A $= 12°$ C

The temp. of B = 19 °C

The temp. of C = 28 °C

The temp of A + B = $16°$ C

The temp. of B + C = $23°$ C

In accordance with the principle of calorimetry when A& B are mixed

$$M_{CA}(16-12) = M_{CB}(19-16)$$

$$\Rightarrow M_{CA}4 = M_{CB}3 \Rightarrow M_{CA} = \frac{3}{4}M_{CB} \qquad ...(i)$$

And when B and C are mixed;

$$M_{CB}(23-19) = M_{CC}(28-23)$$

$$\Rightarrow 4M_{CB} = 5M_{CC} \Rightarrow M_{CC} = \frac{4}{5}M_{CB} \qquad ...(ii)$$

When A& C are mixed, if T is the common temperature of mixture

$$M_{CA}(T-12) = M_{CC}(28-T) = \left(\frac{3}{4}\right)CB(T-12)$$

$$\Rightarrow \left(\frac{4}{4}\right)M_{CB}(28-T)$$

$$= 15T - 180 = 488 - 16T \Rightarrow T = \frac{628}{31} = 20.258 = 20.3°C$$

Example 8: A glass cylinder can contain m_0 =100g of mercury at a temperature of T_0=0°C. When T_1=20°C, the cylinder can contain m_1 =99.7g of mercury. In both cases the temperature of the mercury is assumed to be equal to that of the cylinder. Use this data of find the coefficient of linear expansion of glass α, bearing in mind that the coefficient of volume expansion of mercury $\gamma_1 = 18\times 10^{-5}\,deg^{-1}$

Sol: Get the γ of glass with the information of mercury. Find the relation between the densities at different temperature, and then get coefficient of linear expansion of glass cylinder.

When the cylinder is heated, its volume increases according to the same Law as that of the glass: $V_1 = V_0(1+\gamma T_1)$ where γ is the coefficient of volume expansion of glass. If the densities of mercury at the temperature T_0 and T_1 are denoted by ρ_0 and ρ_1. We can write that $m_0 = V_0\rho_0$ and $m_1 = V_1\rho_1$, where $\rho_1 = \frac{\rho_0}{1+\gamma T_1}$. This system of equations will give the following expression for γ ;

$$\gamma = \frac{m_1(1+\gamma_1 T_1)}{m_0 T_0} \approx 3\times 10^{-5}\,deg^{-1}$$

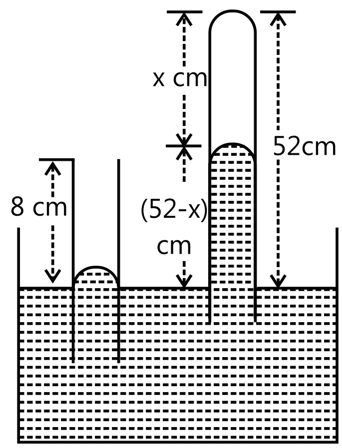
The coefficient of linear expansion, $\alpha = \frac{\gamma}{3} = 10^{-5}\,deg^{-1}$

JEE Advanced/Boards

Example 1: An open glass tube is immersed in mercury in such a way that a length of 8cm extends above the mercury level. The open end of the tube is then closed and raised further by 44 cm. What will be the length of the air column above mercury in the tube? Atmospheric pressure= 76 cm of mercury.

Sol: Air column will get trapped and follow PV=constant.

Let A be the area of cross section of the tube.



Initial Atmospheric pressure of air in the tube outside the mercury surface=P_1=76 cm of Hg

Initial volume of air, V_1=8A

New pressure of air in the tube

$P_2 = 76-(52-x)=(42-x)\text{cm of Hg}$

New volume of air, $V = xA$

As $P_1V_1 = P_2V_2$; $\quad 76\times 8A = (42+x)xA$

or $608 = x^2 + 24x$

or $x^2 + 24x - 608$

$= 0, x = \dfrac{-24 \pm \sqrt{(24)^2 - 4\times 608}}{2}$

$\therefore\ x = 15.2\,\text{cm}$ or $x = -39.4\,\text{cm}$

Which is negative

$\therefore$ The length of air column =15.4cm

Example 2: An air bubble starts rising from bottom of a lake. Its diameter is 3.6 mm at the bottom and 4 mm at the surface. The depth of the lake is 250 cm and the temperature at the surface is 40°C. What is the temperature at the bottom of the lake? Given atmospheric pressure=76 cm of Hg and g=980cm/sec^2.

Sol: Here the amount air remains while P, V and T all parameters changes. Hence PV/T =constant.

Volume of the bubble of lake

$= V_1 = \frac{4}{3}\pi r_1^3 = \frac{4}{3}\pi(0.18)^3\ cm^3$

Pressure on the bubble P_1

= Atmospheric pressure + Pressure due to a column of 250 cm of water

$= 76\times 13.6\times 980 + 250\times 1\times 980$

$= (76\times 13.6 + 250)980\text{dyne}/cm^2;\ T_1 = ?$

Volume of the bubble at the surface of lake

$V_2 = \frac{4}{3}\pi r_2^3 = \frac{4}{3}\pi(0.2)^3\ cm^3$

Pressure on the bubble P_2

=Atmospheric pressure = $76\times 13.6\times 980\ \text{dyne}/cm^2$

T_2= 273+40°C = 313°K As

$\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$ or $\dfrac{(76\times 13.6+250)980\times 4\pi(0.18)^3}{T_1\times 3}$

$= \dfrac{(76\times 13.6\times 980)4\pi(0.2)^3}{313\times 3};$

or $\dfrac{1283\times(0.18)^3}{T_1} = \dfrac{1033.6(0.2)^3}{313}$

$T_1 = \dfrac{1283\times(0.18)^3\times 313}{(1033.6)(0.2)^3} = 1283.35^0K$

$\therefore\ T_1 = 1283.35-273 = 10.35°C$

Example 3: A mixture of 250 gm of water and 200 gm of ice at 0°C is kept in a calorimeter which has a water equivalent of 50 gm. If 200 gm of a steam at 100°C is passed through this mixture, calculate the final temperature and weight of the content of the calorimeter. Latent heat of fusion of ice=80 Cal/gm. latent heat of vaporization of water of steam=540cal/gm., Specific heat of water=1cal/gm./°C.

Sol: Latent heat of vaporization of water is approx. 7 times of latent heat of fusion. So 1g steam can melt about 7g of ice. The mass of steam equals to mass of ice, so part of steam is condensed to melt the ice.

Heat lost by 200 gm. of steam before it is condensed to water at 100°C

$= 200 \times 540 = 108000 cal$ ... (i)

Heat gained by 200 gm. of ice at 0°C

$= mL + m \times s \times \Delta T = 200 \times 80 \times 1 \times (100 - 0)$

$= 36000$ cal

Heat gained by 250gm of water and 50 gm of water equivalent of calorimeter at 100°C to 0°C

$= 200 \times 80 \times (100 - 0) + 50 \times (100 - 0)$

$= 300 \times 100 = 30000 cal$

Total heat gained

$30000 cal + 36000 = 66000 cal$ ...(ii)

Amount of heat lost by the system (i) is greater than heat gained by ice. This shows that only a part of the steam will condense to water at 100°C which will be sufficient for melting ice.

Let M be mass of steam which will be sufficient for melting ice,

$\therefore$ Mass M of steam required is given by.

Or $M = 66000 / 540 = \frac{1100}{9} = 122.2 gm$

Final temperature of system= 100°C

Weight contents

= Weight of ice +Water+ Steam condensed

=250+200+122.2=572.2gm

Example 4: A copper calorimeter of mass 100 gm contains 200g of a mixture of ice and water; Steam at 100°C under normal pressure is passed into the calorimeter and the temperature of the mixture is allowed to rise to 50°C. If the mass of the calorimeter and its contents is now 330gm, what was the ratio of ice and water in the beginning? Neglect heat losses. Given that:

Specific heat of copper= $0.42 \times 10^3 J / kg K$.

Specific heat of water = $4.2 \times 10^3 J / kg K$.

Latent heat of fusion of ice = $3.36 \times 10^5 J / kg K$.

Latent heat of condensation of steam

$= 22.5 \times 10^5 J / kg K$.

Sol: Total amount of heat lost by the steam will bring the water and calorimeter to 50 degree temp. remaining heat would have been used to melt the ice.

Heat is lost by steam in getting condensed and heat is gained by the water, ice and the calorimeter. Let the calorimeter originally contains x gm of ice and (200-x) gm. of water.

Heat gained by calorimeter

$= \frac{100}{1000} \times 0.42 \times 10^3 \times (50 - 0) = 2100 J$

Heat gained by ice

$= \frac{x}{1000} \times \left[3.36 \times 10^5 + \left(4.2 \times 10^3 \times 50\right)\right]$

= x [336 + 210] = x × 546 J

Heat gained by water

$= \left[\frac{200 - x}{1000}\right]\left[4.2 \times 10^3 \times 50\right] = 42000 - 210x J$

Heat lost by steam

$\left[\frac{330 - 200 - 100}{1000}\right]\left[22.5 \times 10^3 \times 4.2 \times 10^3 \times 50\right]$

$= 30\left[2250 + 210\right] = 30 \times 2460 = 73800 J$

Heat gained = heat lost;

2100+546x+42000-210x = 73800;

336x = 73800-44100 = 29700

Mass of ice = $x = \frac{29700}{336} = 88.39 gm$

Mass of water = 111.61 gm

Ratio of ice to water = 88.39:111.6=1:1.263 $\approx$ 0.79

Example 5: A one liter flask contains some mercury. It is found that at different temperatures the volume of air inside the flask remains the same. What is the volume of mercury in the flask? Given coefficient of linear expansion of glass= 3×10^{-6} per degree Celsius.

Coefficient of volume expansion of Hg

$= 1.8 \times 10^{-4}$ per degree Celsius.

Sol: Volume of air in the flask is independent of temperature.

Let x be the volume of mercury in the flask

Volume of air = Volume of flask – Volume of Hg.

= 1000 cm^3 – x cm^3

At any Temperature 'T' –

Volume of flask = 1000 + 1000 x 3 α_g ΔT. ...(i)

and Volume of Hg $= x + x \times \gamma_m \times \Delta T$ ...(ii)

Hence volume of air = Volume of flask – Volume of Hg

$= 1000 - x + (1000 \times 3\alpha_g - x \times \gamma_m) \times \Delta T$

Given: Volume of air remains constant at all temperatures

Hence, coefficient of ΔT

i.e. $(1000 \times 3\alpha_g - x \times \gamma_m) = 0$

$$\Rightarrow x = \frac{3 \times 1000 \times \alpha_g}{\gamma_m} = \frac{9 \times 1000\ cm^3 \times 10^{-6}\ /^{o}C}{1.8 \times 10^{-4}\ /\ ^{o}C}$$

$= 50\ cm^3$

Example 6: A piece of metal weights 46 gm in air. When it is immersed in a liquid of specific gravity 1.24 at 37°C, it weighs 30 gm. When the temperature of the liquid is raised to 42°C, the metal piece weights 30.5 gm. The specific gravity of the liquid at 42°C, is 1.20. Calculate the coefficient of linear expansion of the metal.

Sol: Applying Archimedes' principle, i.e. lose in wt= wt of liquid displaced. We can get volume of metal at two temp. so we can have coefficient of volume expansion. Weight of the piece of metal in air=46gm. weight of the piece of metal in liquid at 27°C =30gm

$\therefore$ Loss of the weight of the piece of metal in liquid = 46-30 = 16gm = Weight of liquid displaced

Volume of liquid Displaced

$$= \frac{\text{Weight of liquid displaced}}{\text{Density}} = \frac{16}{124}\text{c.c}$$

The volume of metal piece at 27°C, is $\therefore\ V_{27} = \frac{16}{124}$c.c

Weight of the piece of metal in air= 46 gm

Weight of the piece of metal in liquid at = 42°C = 30.5gm

Loss of the weight of the piece of metal in liquid = 46 - 30.5 = 15.5gm

The Volume of the liquid Displaced

$$= \frac{\text{Weight of liquid displaced}}{\text{Density}} = \frac{15.5}{1.20}\text{c.c}$$

The volume of piece of metal at 42°C

$$= V_{42} = \frac{15.5}{1.20}\text{c.c};\ V_{42} = V_{27}\left(1 + \gamma T\right)$$

$$\therefore \frac{15.5}{1.20} = \frac{16}{124}\left(1 + \gamma \times 15\right)\left(\because T = 42 - 27 = 15\right)$$

$$1 + \gamma 15 = \frac{15.5}{1.20} \times \frac{16}{124}$$

$$\text{Or } \gamma = \frac{1}{15} = \left[\frac{15.5}{1.20} \times \frac{1.24}{16} - 1\right] = \frac{1}{15} \times \frac{1}{960}$$

$$\therefore \alpha = \frac{1}{3 \times 15 \times 960} = 2.31 \times 10^{-5}\ /^{0}C$$

Example 7: A composite rod is made by joining a copper rod end to end with a second rod of different material but of the same cross-section. At 25°C, the composite rod is 1 m in length of which of the copper rod is 30 cm. At 125°C, the length of the composite rod increases by 1.91 mm. When the composite rod is not allowed to expand by holding it between two rigid walls, it is found that the lengths of the two constituents do not change with the rise of temperature. Find the Young's modulus and the coefficient of the linear expansion of the second rod. Given Young's modulus copper $= 1.3 \times 10^{11} N/m^2$, coefficient of the linear expansion of copper=.$1.7 \times 10^{-5}\ /^{0}C$

Sol: First part, α_2 can be calculated and then same compressive force applied by the wall, this will give the Young's modulus of the material.

Length of copper rod at 25°C, l_1=30m

Length of second rod at 25°C, l_2=70cm

If α_1 and α_2 are respective linear expansion coefficients, the total expansion of the composite rod when the temperature rises by

Δt is $\left(l_1\alpha_1 + l_2\alpha_2\right)\Delta t$.

$$\therefore \left(30 \times 1.7 \times 10^{-5} + 70\alpha_2\right) \times 100 = 0.191$$

$$\alpha_2 = 2 \times 10^{-5}\ /^{0}C$$

If the two rods do not change in length of heating, The compressions of the two rods due to thermal stress must be $l_1\alpha_1\Delta t$ and $l_2\alpha_2\Delta t$ respectively. If A area of cross-section of each rod, then

Tension developed in copper rod, $F_1 = Y_1 A \alpha_1 \Delta t$

Tension developed in second rod, $F_2 = Y_2 A \alpha_2 \Delta t$

F_1 And F_2 should be equal and opposite

Since the composite rod is in equilibrium,

$$\therefore Y_1\alpha_1 = Y_2\alpha_2;\ Y_2 = Y_1\alpha_1/\alpha_2$$

$$= \frac{13 \times 10^{11} \times 1.7 \times 10^{-5}}{2 \times 10^{-5}} = 1.1 \times 10^{11}\ N/m^2$$

JEE Main/Boards

Exercise 1

Q.1 What are the S.I and c.g.s. unit of heat? How are they related?

Q.2 What is the specific heat of water in SI units? Does it vary with temperature?

Q.3 What is the specific heat of gas in an isothermal process?

Q.4 What is principle behind calorimeter?

Q.5 Briefly explain the concept of heat and concept of temperature?

Q.6 Explain what is meant by specific heats of a substance. What are its units? How is molar specific heat different from specific heat?

Q.7 Define the two principle specific heat of gas. Which is greater and why?

Q.8 What do you understand by change of state? What change occurs with temperature, when heat is given to a solid body?

Q.9 A faulty thermometer has its fixed point marked at 5 and 95. The temperature of a body as measured by faulty thermometer is 59. Find the correct temperature of the body on Celsius scale.

Q.10 A blacksmith fixed iron ring on the rim of the wooden wheel of a bullock cart. The diameter of the rim and the ring is 5.243 m and 5.231 m respectively at 27°C. To what temperature should the ring be heated so as to fit rim of the wheel? Coefficient of linear expansion of iron=$1.20\times10^{-5}\,K^{-1}$.

Q.11 A sheet of brass is 50 cm long and 10 cm broad at 0°C. The area of the surface increases by 1.9 cm^2 at 100°C. Find the coefficient of linear expansion of brass?

Q.12 A sphere of aluminum of 0.047 kg is placed for sufficient time in a vessel containing boiling water, so that the sphere is at 100°C. It is then immediately transferred to 0.14 kg copper calorimeter containing 0.25kg of water of 20°C. The temperature of water rises and attains a steady state at 23°C. Calculate the specific heat capacity of aluminum. Specific heat capacity of copper = $0.386 \times 10^3 Jkg^{-1}K^{-1}$. Specific heat capacity of water= $4.18\times10^3\,K^{-1}$

Q13 How many grams of ice at-14°C are needed to cool 200 grams of water from 25°C to 10°C. Take sp. Heat of ice=0.5cal/g ^0C and latent heat of ice=80 cal/g

Q.14 A tank of volume 0.2m^3 contains Helium gas at a temp. of 300K and pressure 10^5 N/m^2. Find the amount of heat required to raise the temp. to 500K. The molar heat capacity of helium at constant volume is 3.0 ca/mole-K. Neglect any expansion in the volume of the tank. Take r=8.31J/mole-K.

Q.15 5 moles of oxygen is heated at constant volume from 10°C to 20°C. Calculate the amount of heat required, if C_p=8 cal/mole °C and R=8.36 joule/ mole°C.

Exercise 2

Single Correct Choice Type

Q.1 Overall change in volume and radii of a uniform cylindrical steel wire are 0.2% and 0.002% respectively when subjected to some suitable force. Longitudinal tensile stress acting on the wire is $\left(Y = 2.0\times10^{11}Nm^{-2}\right)$

(A) $3.2\times10^9 Nm^{-2}$ (B) $3.2\times10^7 Nm^{-2}$

(C) $3.6\times10^7 Nm^{-2}$ (D) $4.08\times10^8 Nm^{-2}$

Q.2 A solid sphere of radius R made of material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass m is placed on the piston to compress the liquid, the fractional change in the radius of the sphere $\delta R / R$ is

(A) Kmg /A (B) Kmg /3A

(C) mg /A (D) mg /3AR

Q.3 A cylindrical wire of radius 1 mm, length 1 m, Young's modulus= $2\times10^{11}Nm^2$, Poisson's ratio $\mu=\pi/10$ is stretched by a force of 100 N. Its radius will become

(A) 0.99998mm (B) 0.99999mm

(C) 0.99997mm (D) 0.99995mm

Q.4 A block of mass 2.5kg is heated to a temperature of 500°C and placed a large ice block. What is the maximum amount of ice that can melt (approx.)? Specific heat for the body=0.1 cal/gm°C.

(A) 1kg (B) 1.5 kg (C) 2kg (D) 2.5kg

Q.5 1 kg of ice at- 10°C is mixed with 4.4kg of water at 30°C. The final temperature of mixture is: (specific heat of ice is 2100 J/kg/k)

(A) 2.3°C (B) 4.4°C (C) 5.3°C (D) 8.7°C

Q.6 Steam at 100°C is added slowly to 1400 gm of water at 16°C, until the temperature of water is raised to 80°C. The mass of steam required to do this is $\left(L_V=540\,cal/gm\right)$:

(A) 165gm (B) 125gm

(C) 250gm (D) 320gm

Q.7 Ice at 0°C is added to 200g of water initially at 70°C in a vacuum flask. When 50g of ice has been added and has all melted the temperature of the flask and contents is 40°C. When a further 80g of ice has been added and has all melted, the temperature of the whole is 10°C. Calculate the specific latent heat of fusion of ice. $\left[\text{Take}\,s_w=1cal/gm^\circ C\right]$

(A) $3.8\times10^5\,J/kg$ (B) $1.2\times10^5\,J/kg$

(C) $2.4\times10^5\,J/kg$ (D) $3.0\times10^5\,J/kg$

Q.8 A continuous flow water heater (geyser) has an electrical power rating=2 KW and efficiency of conversion of electrical power into heat=80%. If water is flowing through the device at the rate of 100 cc/sec, and the inlet temperature is 10°C, the outlet temperature will be.

(A) 12.2°C (B) 13.8°C (C) 20°C (D) 16°C

Q.9 A rod of length 2m rests on smooth horizontal floor. If the rod is heated from 0°C to 20°C, find the longitudinal strain developed. $\left(\alpha=5\times10^{-5}\,/^0\,C\right)$

(A) 10^{-3} (B) 2×10^{-3} (C) Zero (D) None

Q.10 A steel tape gives correct measurement at 20°C. A piece of wood is being measured with the steel tape gives correct measurement at 20°C. A piece of wood is being measured with the steel tape at 0°C. The reading is 25 cm on the tape, The real length of the given piece of wood must be:

(A) 25cm (B) <25cm

(C) >25cm (D) Cannot say

Q.11 A metallic rod 1 cm long with a square cross-section is heated through t°C. If Young's modulus of elasticity of the metal is E and the mean coefficient of linear expansion is α per degree Celsius, then the compressional force required to prevent the rod from expanding along its length is: (Neglect the change of cross-sectional area)

(A) $EA\alpha t$ (B) $EA\alpha t(1+\alpha t)$

(C) $EA\alpha t(1-\alpha t)$ (D) $E/\alpha t$

Q 12. A solid ball is completely immersed in a liquid. The coefficient of volume expansion of the ball and liquid are 3×10^{-6} and 8×10^{-6} per°C respectively. The percentage change in upthrust when the temperature is increased by 100°C is.

(A) 0.5% (B) 0.11% (C) 1.1% (D) 0.05%

Previous Years' Questions

Q.1. 70 cal of heat are required to raise the temperature of 2 mole of an ideal diatomic gas at constant pressure from 35°C. The amount of heat required (in calorie) to raise the temperature of the same gas through the same range (30°C to 35°C) at constant volume is. ***(1985)***

(A) 30 (B) 50 (C) 70 (D) 90

Q.2 Steam at 100°C is passed into 1.1kg of water contained in a calorimeter of water equivalent 0.02kg at 15°C. Till the temperature calorimeter and its constants rises to 80°C. The mass of steam condensed in kg is ***(1986)***

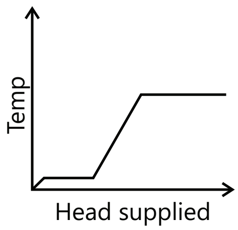
(A) 0.130 (B) 0.065

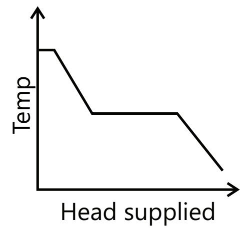
(C) 0.260 (D) 0.135

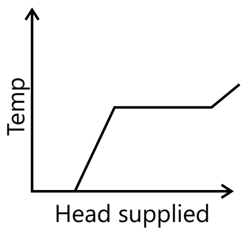
Q.3 Two cylinders A and B fitted with piston contain equal amount of an ideal diatomic gas at 300K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of the gas in B is ***(1988)***

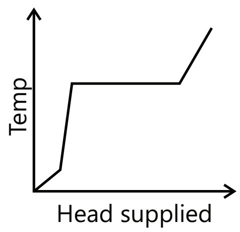
(a) 30 K (b) 18 K (c) 50 K (d) 42 K

Q.4 A block of ice at-10°C is slowly heated and converted to steam at 100°C. Which of the following curves represent the phenomena qualitatively? ***(2000)***

(A)


(B)


(C)


(D)


Q.5 Two rods, one made of the aluminum and the other made of steel, having initial length l_1 and l_2 are connected together to form a single rod of length $l_1 + l_2$. The coefficients of linear expansion for aluminum and steel are α_a and α_s respectively. If the length of each rod increases by the same amount when their temperature are raised by t°C, then find the ratio $\frac{l_1}{l_1 + l_2}$ ***(2003)***

(A) $\frac{\alpha_s}{\alpha_a}$ (B) $\frac{\alpha_a}{\alpha_s}$

(C) $\frac{\alpha_s}{(\alpha_a + \alpha_s)}$ (D) $\frac{\alpha_a}{(\alpha_a + \alpha_s)}$

Q.6 2 kg ice at-20°C is mixed 5 kg of water at 20°C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 kcal/kg/°C and 0.5 Kcal/kg/°C while the latent heat of fusion of ice is 80 kcal/kg ***(2003)***

(A) 7 kg (B) 6kg (C) 4kg (D) 2kg

Q.7 Two identical conducting rods are first connected independently to two vessels, one containing water at 100°C and the other containing ice at 0°C. In the second case, the rods are joined end to end and connected to the same vessels. Let q_1 and q_2 gram per second be the rate of melting of ice in the two cases respectively. The ratio $\frac{q_1}{q_2}$ is ***(2004)***

(A) $\frac{1}{2}$ (B) $\frac{2}{1}$ (C) $\frac{4}{1}$ (D) $\frac{1}{4}$

Q.8 Calorie is defined as the amount of heat required to raise temperature of 1 g of water by 1^0C and it is defined under which of the following conditions? ***(2005)***

(A) From 14.5°C to 15.5°C at 760 mm of Hg

(B) From 98.5°C to°C 99.5°C at 760 mm of Hg

(C) From 13.5°C to 14.5°C at 76 mm of Hg

(D) From 3.5°C to 4.5°C at 76 mm of Hg

Q.9 This question contains Statement-I and Statement-II. Of the four choices given after the statements, choose the one that best describes the two statements. ***(2009)***

Statement-I: The temperature dependence of resistance is usually given as $R = R_o(1 + \alpha\Delta t)$. The resistance of a wire changes from 100 Ω to 150 Ω when its temperature is increased from 27°C to 227°C. This implies that $\alpha = 2.5 \times 10^{-3}$ /°C.

Statement-II: $R = R_i (1 + \alpha\Delta T)$ is valid only when the change in the temperature ΔT is small and $\Delta R = (R - R_o) << R_o$.

(A) Statement-I is true, statement-II is false

(B) Statement-I is true, statement-II is true; statement-II is the correct explanation of statement-I.

(C) Statement-I is true, statement-II is true; statement-II is not the correct explanation of statement-I.

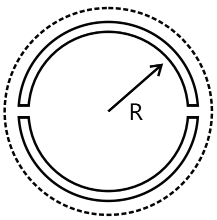
(D) Statement-I is false, statement-II is true

Q.10 Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly ***(2010)***

(A) $\frac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$ (B) $\alpha_1 + \alpha_2, \frac{\alpha_1 + \alpha_2}{2}$

(C) $\alpha_1 + \alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2}$ (D) $\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + \alpha_2}{2}$

Q.11 A wooden wheel of radius R is made of two semicircular parts (see figure); The two parts are held together by a ring made of a metal strip of cross sectional area S and length L. L is slightly less than 2pR. To fit the ring on the wheel, it is heated so that its temperature rises by DT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is a, and its Youngs' modulus is Y, the force that one part of the wheel applies on the other part is: ***(2012)***



(A) $2\pi SY\alpha\Delta T$ (B) $SY\alpha\Delta T$

(C) $\pi SY\alpha\Delta T$ (D) $2SY\alpha\Delta T$

Q.12 Three rods of copper, brass and steel are welded together to form a Y-shaped structure. Area of cross-section of each rod = 4 cm^2. End of copper rod is maintained at 100°C whereas ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cm respectively. The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is **(2014)**

(A) 1.2 cal/s (B) 2.4 cal/s

(C) 4.8 cal/s (D) 6.0 cal/s

Q.13 A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up ? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take g = 9.8 ms^{-2} ***(2016)***

(A) 6.45×10^{-3} kg (B) 9.89×10^{-3} kg

(C) 12.89×10^{-3} kg (D) 2.45×10^{-3} kg

Q.14 A pendulum clock lose 12 s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively: ***(2016)***

(A) 60°C ; $\alpha = 1.85 \times 10^{-4}$/°C

(B) 30°C ; $\alpha = 1.85 \times 10^{-3}$/°C

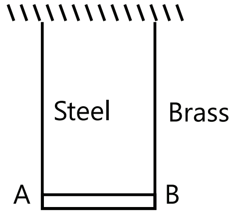
(C) 55°C ; $\alpha = 1.85 \times 10^{-2}$/°C

(D) 25°C ; $\alpha = 1.85 \times 10^{-5}$/°C

JEE Advanced/Boards

Exercise 1

Q.1 A light rigid bar is suspended horizontally from two vertical wires, one of steel and one of steel brass, as shown in figure. Each wire is 2.00 m long. The diameter of the steel wire is 0.60 mm and the length of the bar AB is 0.20 m. When a mass of 10kg is suspended from the center of AB, bar remains horizontal.



(i) What is the tension in each wire?

(ii) Calculate the extension of the steel wire and the energy stored in it.

(iii) Calculate the diameter of the brass wire.

(iv) If the brass wire is replaced by another brass wire of diameter 1 mm, where should the mass be suspended so that AB would remain horizontal? The Young's modulus for steel=2.0×10^{11} Pa, the Young's modulus for brass=1.0×10^{11} Pa.

Q.2 A steel rope has length L, area of cross-section A, Young's modulus Y. [Density=d]

(a) It is pulled on a horizontal frictionless floor with a constant horizontal force F = [dALg]/2 applied at one end. Find the strain at the midpoint.

(b) If the steel rope is vertical and moving with the force acting vertically upward at the upper, end. Find the strain at a point L/3 from lower end.

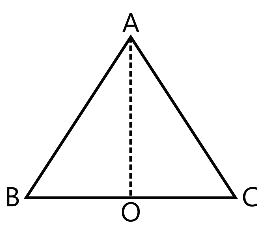
Q.3 An aluminum container of mass 100 gm contains 200 gm of ice at-20 ^0C. Heat is added to system at the rate of 100 cal/s. Find the temperature of the system after 4 minutes (specific heat of ice=0.5 and L=80 cal /gm, specific heat Al=2.0.cal/gm/°C)

Q.4 A volume of 120 ml of drink (half alcohol + half water by mass) originally at a temperature of 25°C is cooled by adding 20gm ice at 0°C. If all the ice melts, find the final temperature of drink. (Density of drink=0.833 gm/cc, specific heat of alcohol = 0.6 cal/gm/°C)

Q.5 A hot liquid contained in a container of negligible heat capacity loses temperature at the rate of 3K/min, just before it begins to solidify. The temp remains constant for 30 min.

Find the ratio of specific heat capacity of liquid to specific latent heat of fusion. (Given that of losing heat is constant).

Q.6 Three aluminum rods of equal length form an equilateral triangle ABC. Taking O (midpoint of rid BC) as the origin, find the increase in Y-coordinate of the center of mass per unit change in temperature of the system. Assume the length of each rod is 2m, and $\alpha_{al} = 4\sqrt{3} \times 10^{-6} /^0 C$.



Q.7 A thermostat chamber at a small height h above earth's surface maintained at 30°C has a clock fitted in it with an uncompensated pendulum. The clock designer correctly designs it for height h, but for temperature of 20°C. If this chamber is taken to earth's surface, the clock in it would click correct time. Find the coefficient of linear expansion of material of pendulum. (Earth's radius is R)

Q.8 A metal rod A of length 25 cm expands by 0.050cm. When its temperature is raised from 0°C to 100°C, another rod B of a different metal of length 40cm expands by 0.040cm for the same rise in temperature. A third rod C of 50cm length is made up of pieces of rods A and B placed end to end expands by 0.03 cm on heating from 0°C to 50°C. Find the length of each portion of the composite rod.

Q.9 A wire of cross-section area $4 \times 10^{-4}\ m^2$ has modulus of elasticity $2 \times 10^{11}\ N/m^2$ and length 1 m is stretched between two vertical rigid poles. A mass of 1 kg is suspended at its middle. Calculate the angle it makes with horizontal.

Q.10 A copper calorimeter of mass 100 gm contains 200 gm of a mixture of ice and water. Steam at 100°C under normal pressure is passed into the calorimeter and the temperature of the mixture is allowed to rise to 50°C. If the mass of the calorimeter and its contents is now 330 gm, what was the ratio of ice and water in the beginning? Neglect heat losses.

Given: specific heat capacity of copper
$= 0.42 \times 10^3\ Jkg^{-1}\ K^{-1}$.

Specific heat capacity of water

$= 4.2 \times 10^3\ Jkg^{-1}\ K^{-1}$.

Specific heat of fusion of ice $= 3.36 \times 10^5\ Jkg^{-1}$.

Latent heat of condensation of steam

$= 22.5 \times 10^5\ Jkg^{-1}$.

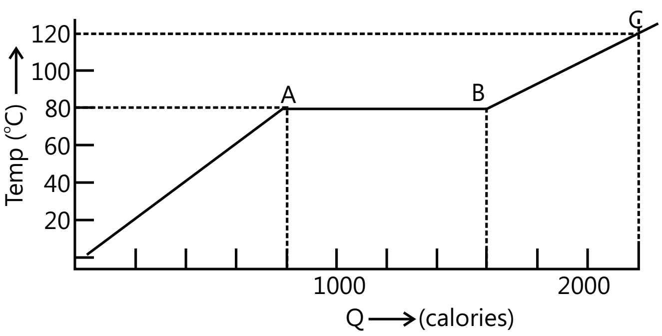
Q.11 Two 50 cm ice cubes are dropped into 250gm of water into a glass. If the water is initially at a temperature of 25°C and the temperature of ice is 15°C. Find the final temperature of water. (Specific heat of ice=0.5cal/gm/°C and L=80 cal/gm). Find the final amount of water and ice.

Q.12 A flow calorimeter is used is measure the specific heat of liquid. Heat is added at a known rate to a steam of the liquid as it passes through the calorimeter at a known rate. Then a measurement of the resulting temperature difference between the inflow and the outflow point of the liquid steam enables us to compute the specific heat of the liquid. A liquid of density 0.2g/cm^3 flows through a calorimeter at the rate of cm^3/s. Heat is added by means of a 250-W electric heating oil, and a temperature difference of 25°C is established in steady-state condition between the inflow and the outflow points. Find the specific heat of the liquid.

Q.13 Two identical calorimeters A and B contain equal quantity of water at 20°C. A 5 gm piece of metal X of specific heat 0.2 cal g^{-1} $(C°)^{-1}$ is dropped into A and 5 gm piece of metal Y into B. The equilibrium temperature in A is 22°C and in B 23°C. The initial temperature of both the metal is 40°C. Find the specific heat of metal Y in cal g^{-1} $(C°)^{-1}$.

Q.14 The temperature of 100 gm of water is to be raised from 24°C to 90°C by adding steam to it. Calculate the mass of the steam required for this purpose.

Q.15 A substance is in a solid form at 0°C. The amount of heat added to substance and its temperature are plotted in the following graph. If the relative specific heat capacity of the solid substance is 0.5, find from the graph.



(i) The mass of substance;

(ii) The specific latent heat of the melting process, and

(iii) The specific heat of the substance in the liquid state.

Q.16 A solid receives heat by radiation over its surface at the rate of 4kW. The heat convection rate from the surface of solid to the surrounding is 5.2 kW, and heat is generated at a rate of 1.7KW over the volume of the solid. The rate of change of the temperature of the solid is 0.5°Cs^{-1}. Find the heat capacity of the solid.

Q.17 Water is heated from 10°C to 90°C in a residential hot water heater at a rate of 70 liter per minute. Natural gas with a density of 1.2kg/m^3 is used in the heater, which has a transfer efficiency of 32%. Find the gas consumption rate in cubic meters per hour. (Heat combustion for natural gas is 8400kcal/kg)

Q.18 If two rods of length L and 2 L having coefficients of linear expansion α and 2α respectively are connected so that length becomes 3 L, determine the average coefficient of linear expansion of the composite rod.

Q.19 A clock pendulum made of invar has a period of 0.5 sec at 20°C. If the clock is used in a climate where average temperature is 30°C, approximately. How much faster or slower will the clock run in 10^6 sec. (α_{invar} = 1×10^{-6}/°C)

Q.20 A U-tube filled with a liquid of volumetric coefficient of 10^{-5}/°C lies in a vertical plane. The height of liquid column in the left vertical limb is 100 cm. The liquid in the left vertical limb is maintained at a temperature=0°C while the liquid in the right limb is maintained at a temperature=100°C. Find the difference in levels in the two limbs.

Q.21 An iron bar (young's modulus $=10^{11}\ N/m^2$, $\alpha=10^{-6}\ /^0C$) 1 m long and $10^{-3}\ m^2$ in area is heated from 0°C to 100°C without being allowed to bend or expand. Find the compressive force developed inside the bar.

Q.22 An isosceles triangle is formed with a rod of length l_1 and coefficient of linear expansion α_1 for the base and two thin rods each of length l_2 and coefficient of linear expansion α_2 for the two pieces, if the difference between the apex and the midpoint of the base remain unchanged as the temperatures varied show that

$$\frac{l_1}{l_2}=2\sqrt{\frac{\alpha_2}{\alpha_1}}$$

Q.23 A steel drill making 180 rpm is used to drill a hole in a block of steel. The mass of the steel block and the drill is 180gm. If the entire mechanical work is used up in producing heat and the rate of rise in temperature of the block and the drill is 0.5°C/s. find

(i) the rate of working of the drill in watts, and

(ii) the torque required to drive the drill.

Specific heat of steel=0.1 and J=4.2 J/cal. Use: $P=\tau\omega$

Q.24 Ice at-20°C is filled up to height h=10cm in a uniform cylindrical vessel. Water at temperature θ°C is filled in another identical vessel up to the same height h= 10 cm. Now water from second vessel is poured into first vessel and it is found that level of upper surface falls through $\Delta h=0.5$ cm when thermal equilibrium is reached. Neglecting thermal capacity of vessels, change in density of water due to change in temperature and loss of heat due to radiation, calculate initial temperature θ of water.

Given, density of water, $\rho_W=1gm\,cm^{-3}$

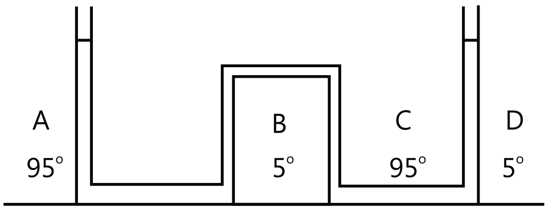
Density of ice, $\rho_i=0.9gm\,cm^3$

Specific of water, .. $s_W = 1 cal/gm\,^0C$

Specific heat of ice, Specific $s_i = 0.5 cal/gm\,^0C$

Latent heat of ice, $L = 80 cal/gm$

Q.25 The apparatus shown in the figure consists of four glass columns connected by horizontal sections. The height of central columns B & C are 49 cm each. The two outer columns A & D are open to the atmosphere. A & C are maintained at a temperature of 95° C while the column B & D are maintained at 5° C. the height of the liquid in A & D measured from the base line are 52.8 cm & 51 cm respectively. Determine the coefficient of thermal expansion of the liquid.

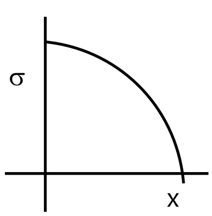


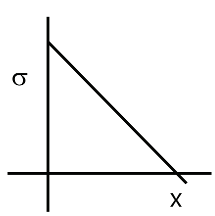
Q.26 Toluene liquid of volume 300 cm^3 at 0°C is contained in a beaker and another quantity of toluene of volume 110 cm^3 at 100°C is in another beaker. (The combined volume Is 410 cm^3). Determine the total Volume of the mixture of the toluene liquid when they are mixed together. Given the coefficient of volume expansion $\gamma = 0.001/C$ and all forms of heat losses can be ignored. Also find the final temperature of the mixture.

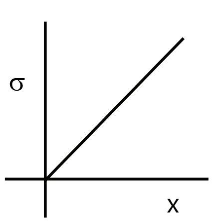
Exercise 2

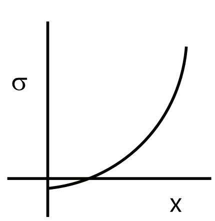
Single Correct Choice Type

Q.1 A uniform rod is rotating in gravity free region with constant angular velocity. The variation of tensile stress with distance X from axis of rotation is best represented by which of the following graphs.

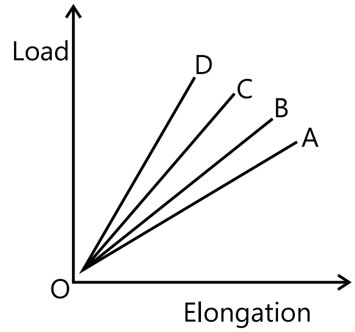
(A) 

(B) 

(C) 

(D) 

Q.2 The load versus strain graph for four wires of the same material is shown in the figure. The thickest wire is represented by the line.



(A) OB (B) OA

(C) OD (D) OC

Q.3 10 gm of ice at 0°C is kept in a calorimeter of water equivalent 10 gm. How much heat should be supplied to the apparatus to evaporate the water thus formed? (Neglect loss of heat)

(A) 6200 cal (B) 7200 cal

(C) 13600 cal (D) 8200 cal

Q.4 Heat is being supplied at a constant rate to a sphere of ice which is melting at the rate 0.1 gm/sec. It melts completely in 100sec. Assumed no loss of heat. The rate of rise of temperature thereafter will be

(A) 0.8°C/sec (B) 5.40°C/sec

(C) 3.6°C/sec (D) will change with time

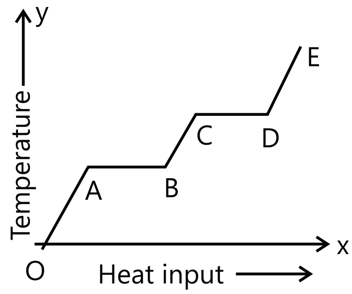
Q.5 Ice at 0°C is added to 200g of water initially at 70°C in a vacuum flask. When 50g of ice has been added and has all melted.

The temperature of the flask and contents is 40°C. When a further 80g of ice has been added and has all melted, the temperature of the whole is 10°C. Calculate the specific latent heat of fusion of ice. $\left[\text{Take } s_w = 1 cal/gm\,^0C\right]$

(A) $3.8 \times 10^5 J/kg$ (B) $1.2 \times 10^5 J/kg$

(C) $2.4 \times 10^5 J/kg$ (D) $3.0 \times 10^5 J/kg$

Q.6 A solid material is supplied with heat at a constant rate. The temperature of material is changing with heat input as shown in the figure. What does slope DE represent



(A) Latent heat of liquid

(B) Latent heat of vapour

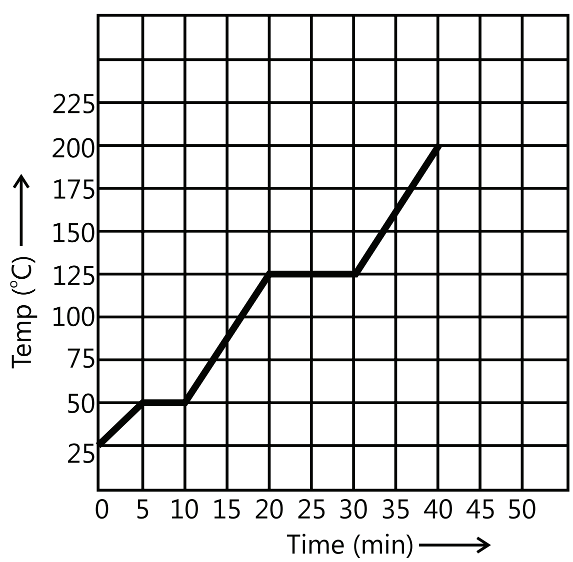
(C) Heat capacity of vapour

(D) Inverse of heat capacity of vapour

Q.7 A block of ice with mass m falls into a lake. After impact, a mass of ice m/5 melts. Both the block of ice and the lake have a temperature of 0°C. If L represents heat of fusion, the minimum distance the ice fell before striking the surface is

(A) $\frac{L}{5g}$ (B) $\frac{5L}{g}$ (C) $\frac{gL}{5m}$ (D) $\frac{mL}{5g}$

Q.8 The graph shown in the figure represents change in the temperature of 5 kg of a substance as it absorbs heat at a constant rate of 42 kJ min^{-1}. The latent heat of vaporization of the substance is:



(A) $630 KJkg^{-1}$ (B) $126 KJkg^{-1}$

(C) $84 KJkg^{-1}$ (D) $12.6 KJkg^{-1}$

Q.9 The density of material A is 1500kg/ m^3 and that of another material B is 2000 kg/m^3. It is found that the heat capacity of 8 volumes of A is equal to heat capacity of 12 volume of B. The ratio of specific heats of A and B will be

(A) 1:2 (B) 3:1 (C) 3:2 (D) 2:1

Q.10 Find the amount of heat supplied to decrease the volume of an ice water mixture by 1 cm^3 without any change in temperature. $\left(\rho_{ice} = 0.9\rho_{water}, L_{ice} = 80\ cal/gm\right)$.

(A) 360 cal (B) 500 cal

(C) 720 cal (D) None of these

Q.11 Some steam at 100 °C is passed into 1.1kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C so that the temperature of the calorimeter and its contents rises to 80 °C. What is the mass of steam condensing? (In kg)

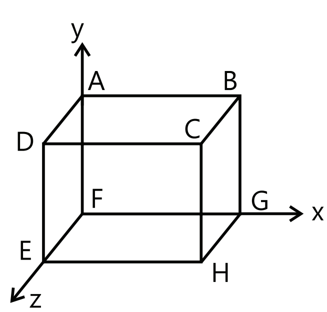
(A) 0.130 (B) 0.065

(C) 0.260 (D) 0.135

Q.12 A thin copper wire of length L increases in length by 1% when heated from temperature T_1 to T_2. What is the percentage change in area when a thin copper plate having dimensions $2L \times L$ is heated from T_1 to T_2?

(A) 1% (B) 2% (C) 3% (D) 4%

Q.13 The coefficients thermal expansion of steel and metal X are respectively 12×10^{-6} and 12×10^{-6} per °C. At 40°C, the side of a cube of metal X was measured using steel vernier callipers. The reading was 100 mm. assuming that the calibration of the vernier was done at 0°C, then the actual length of the side cube at 0°C will be



(A) >100mm

(B) <100 mm

(C) =100mm

(D) Data insufficient to conclude

Q.14 A cuboid ABCDEFGH is anisotropic with

$\alpha_x = 1\times10^{-5}/°C$ $\alpha_y = 2\times10^{-5}/°C$ $\alpha_z = 3\times10^{-5}/°C$.

Coefficient of superficial. Expansion of faces can be

(A) $\beta_{ABCD} = 5\times10^{-5}/°C$

(B) $\beta_{BCGH} = 4\times10^{-5}/°C$

(C) $\beta_{CDEH} = 3\times10^{-5}/°C$

(D) $\beta_{EFGH} = 2\times10^{-5}/°C$

Q.15 The coefficient of apparent expansion of a liquid in a copper vessel is C and in a silver vessel is S. The coefficient of volume expansion of copper is γ_C. What is the coefficient of linear expansion of silver?

(A) $\frac{(C+\gamma_C+S)}{3}$ (B) $\frac{(C-\gamma_C+S)}{3}$

(C) $\frac{(C+\gamma_C-S)}{3}$ (D) $\frac{(C-\gamma_C-S)}{3}$

Q.16 A sphere of diameter 7 cm and mass 266.5gm floats in a bath of a liquid. As the temperature is raised, the sphere just begins to sink at a temperature 35°C. If the density of a liquid at 0°C is 1.527 gm/cc, then neglecting the expansion of the sphere, the coefficient of cubical expansion of the liquid is f:

(A) 8.486×10^{-4} per °C (B) 8.486×10^{-5} per °C

(C) 8.486×10^{-6} per °C (D) 8.486×10^{-3} per °C

Q. 17 The volume of the bulb of a mercury thermometer at 0°C is V_0 and cross section of the capillary is A_0.

The coefficient of linear expansion of glass is α_g per °C and the cubical expansion of mercury γ_m per °C.

If the mercury just fills the bulb at 0°C, what is the length of mercury column in capillary at T°C?

(a) $\frac{V_0T(\gamma_m+3\alpha_g)}{A_0(1+2\alpha_gT)}$ (B) $\frac{V_0T(\gamma_m-3\alpha_g)}{A_0(1+2\alpha_gT)}$

(c) $\frac{V_0T(\gamma_m+3\alpha_g)}{A_0(1+3\alpha_gT)}$ (D) $\frac{V_0T(\gamma_m-2\alpha_g)}{A_0(1+3\alpha_gT)}$

Q.18 A thin walled cylindrical metal vessel of linear coefficient of expansion $10^{-3}°C^{-1}$ contains benzene of volume expansion coefficient $10^{-3}°C^{-1}$. If the vessel and its contents are now heated by 10°C, the pressure due to the liquid at the bottom.

(A) Increases by 2% (B) decreases by 1%

(C) decreases by 2% (D) remains unchanged

Q.19 A rod of length 20 cm is made of metal. It expands by 0.075 cm when its temperature is raised from 0°C to 100 °C. Another rod of a different metal B having the same length expands by 0.045cm for the same change in temperature, A third rod of the same length is composed of two parts one of metal A and the other of metal B. This rod expands by 0.06 cm for the same change in temperature. The portion made of metal A has the length:

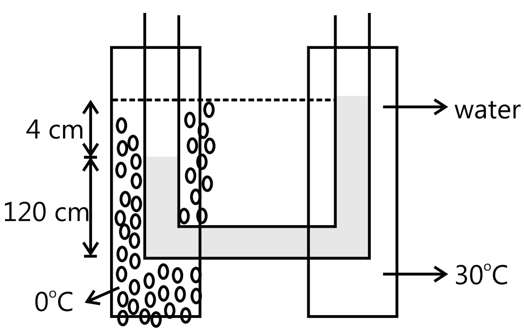
(A) 20cm (B) 10cm (C) 15cm (D) 18cm

Q.20 A glass flask contains some mercury at the room temperature. It is found that at different temperatures the volume of air inside the flask remains the same. If the volume of mercury in the flask is 300cm^3, then volume of the flask is (given the coefficient of volume expansion of mercury and coefficient of linear expansion of glass are $1.8\times10^{-4}\,(^0C)^{-1}$ respectively)

(A) 4500 cm^3 (B) 450 cm^3

(C) 2000 cm^3 (D) 6000 cm^3

Q.21 Two vertical glass tubes filled with a liquid are connected by a capillary tube as shown in the figure. The tube on the left is put in an ice bath at 0°C while the tube on the right is kept at 30°C in a water bath. The difference in the levels of the liquid in the tube is 4cm while the height of the liquid column at 0°C is 120 cm. The coefficient of volume expansion of liquid is (ignore expansion of glass tube)



(A) 22 × 10^{-3} / °C (B) 1.1 × 10^{-3} / °C

(C) 11 × 10^{-3} / °C (D) 2.2 × 10^{-3} / °C

Multiple Correct Choice Type

Q.22 A composite rod consists of a steel rod of length 25cm and area 2A and a copper rod of length 50cm and area A. The composite rod is subjected to an axial load F. If the Young's modulus of steel and copper are in the ratio2:1.

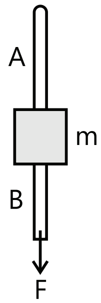
(A) The extension produced in copper rod will be more.

(B) The extension in copper and steel part will be in the ratio 2:1.

(C) The stress applied to the copper rod will be more.

(D) No extension will be produced in the steel rod.

Q.23 The wires A and B shown in the figure are mode of the same material and have radii r_A and r_B respectively. The block between them has a mass m. When the force F is mg/3, one of the wires breaks.



(A) A breaks if $r_A = r_B$

(B) A breaks if $r_A < 2r_B$

(C) Either A or B may break if $r_A = 2r_B$

(D) The length of A and B must be known to predict which wire will break

Q.24 Four rods A, B, C, D of same length and material but of different radii $r, r\sqrt{2}, r\sqrt{3}$ and 2r respectively are held between two rigid walls. The temperature of all rods is increased by same amount. If the rods do not bend, then

(A) The stress in the rods are in the ratio 1: 2: 3: 4.

(B) The force on the rod exerted by the wall in the ratio 1: 2: 3: 4.

(C) The energy stored in the rods due to elasticity is in the ratio 1: 2: 3: 4.

(D) The strain produced in the rod are in the ratio 1: 2: 3: 4.

Q. 25 A body of mass M is attached to the lower end of a metal wire, whole upper end is fixed. The elongation of the wire is l.

(A) Loss in gravitation potential energy of M is Mgl

(B) The elastic potential energy stored in the wires is Mgl

(C) The elastic potential energy stored in the wires is 1/2 Mgl

(D) Heat produced is 1/2 Mgl.

Q.26 An experiment is performed to measure the specific heat of copper. A lump of copper is heated in an oven, and then dropped into a beaker of water. To calculate the specific heat of copper, the experimenter must know or measure the value of all the quantities below EXCEPT the

(A) Heat capacity of water and beaker

(B) Original temperature of the copper and the water

(C) Final (equilibrium) temperature of the copper and the water

(D) Time taken to achieve equilibrium after the copper is dropped into the water

Q.27 When the temperature of a copper coin is raised by 80°C, its diameter increases by 0.2%

(A) Percentage rise in the area of a face is 0.4%

(B) Percentage rise in the thickness is 0.4%

(C) Percentage rise in the volume is 0.6%

(D) Coefficient of linear expansion of copper is $0.25 \times 10^{-4}\ {}^0C^{-1}$

Comprehension Type

(Questions 28 -31)

Solids and liquids both expand on heating. The density of substance decreases on expanding according to the relation

$$\rho_2 = \frac{\rho_1}{1+\gamma(T_2 - T_1)}$$

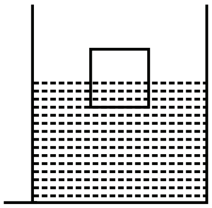
Where, $\rho_1 \rightarrow$ density at T_1; $\rho_2 \rightarrow$ density at T_2

$\gamma \rightarrow$ coeff. of volume expansion of substances

When a solid is submerged in a liquid, liquid exerts an upward force on solid which is equal to the weight of liquid displaced by submerged part of solid.

Solid will float or sink depending on relative densities of solid and liquid.

A cubical block of solid floats in a liquid with half of its volume submerged in the liquid as shown in figure (at temperature T)



$\alpha_s \rightarrow$ coeff. of linear expansion of solid

$\gamma_\ell \rightarrow$ coeff. of volume expansion of solid;

$\rho_s \rightarrow$ Density of solid at temp.T ;

$\rho_\ell \rightarrow$ Density of liquid at temp.T

Q.28 The relation between density of solid and liquid at temperature T is

(A) $\rho_s = 2\rho_\ell$ (B) $\rho_s = (1/2)\rho_\ell$

(C) $\rho_s = \rho_\ell$ (D) $\rho_s = (1/4)\rho_\ell$

Q.29 If temperature of system increases, then fraction of solid submerged in liquid

(A) Increases (B) decreases

(C) Remain the same (D) Inadequate information

Q.30 Imagine fraction submerged does not change on increasing temperature.

The relation between γ_l and α_s is

(A) $\gamma_\ell = 3\alpha_s$ (B) $\gamma_\ell = 2\alpha_s$

(C) $\gamma_\ell = 4\alpha_s$ (D) $\gamma_\ell = (3/2)\alpha$

Q.31 Imagine the depth of the block submerged in the liquid does not change on increasing temperature then

(A) $\gamma_\ell = 2\alpha$ (B) $\gamma_\ell = 3\alpha$

(C) $\gamma_\ell = (3/2)\alpha$ (D) $\gamma_\ell = (4/3)\alpha$

Assertion Reasoning Type

Q.32 Statement-I: The coefficient of volume expansion has dimension K^{-1}.

Statement-II: The coefficient of volume expansion is defined as the change in volume per unit volume per unit change in temperature.

(A) Statement-I is true, statement-II is true and Statement-II is correct explanation of statement-I

(B) Statement-I is true, statement-II is true and statement-II is not correct explanation of statement-I

(C) Statement-I is true, statement-II is false

(D) Statement-I is false, statement-II is true

Q.33 Statement-I: Water kept in an open vessel will quickly evaporate on the surface of the moon.

Statement-II: the temperature at the surface of the moon is much higher than saturation point of the water.

Q.34 Statement-I: When a solid iron ball is heated, percentage increase in its volume is largest.

Statement-II: Coefficient of superficial expansion is twice that of linear expansion whereas coefficient of volume expansion is three time of linear expansion.

Q.35 Statement-I: A beaker is completely filed with water at 4°C. It will over flow, both when heated or cooled.

Statement-II: There is expansion of water below and above 4°C.

Q.36 Statement-I: latent heat of fusion of ice is 336000 Jkg^{-1}

Statement-II: latent heat refers to change of state without any change in temperature.

Q.37 Statement-I: Specific heat of a body is always greater than its thermal capacity.

Statement-II: Thermal capacity is the heat required for raising temperature of unit mass of the body through unit degree.

Previous Year's Questions

Q.1 A bimetallic strip is formed out of two identical strips-one of copper and other of brass. The coefficient of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. then, R is ***(1999)***

(A) Proportional to ΔT

(B) Inversely proportional to ΔT

(C) Proportional to $|\alpha_B - \alpha_C|$

(D) Inversely proportional to $|\alpha_B - \alpha_C|$

Q.2 300g of water at 25°C is added to 100g of ice at 0°C. The final temperature of the mixture is ...°C ***(1989)***

Q.3 A substance of mass M kg required a power input of P watts to remain in the molten states at its melting point. When the power source is turned off, the sample completely solidifies in time t seconds. The latent heat of fusion of the substance is .... ***(1992)***

Q.4 At given temperature, the specific heat of a gas at a constant pressure is always greater than its specific heat at constant volume. **(True or False)** ***(1987)***

Q.5 The temperature of 100g of water is to be raised from 24°C to 90°C by adding steam to it. Calculate the mass of the steam required for this purpose, ***(1996)***

Q.6 A cube of coefficient of linear expansion α_s is floating in a bath containing a liquid of coefficient of volume expansion γ_1. When the temperature is raised by ΔT, the depth up to which the cube is submerged in the liquid remain the same. Find the relation between γ_1 and α_s showing all the steps**.** ***(2004)***

Q.7 In an insulated vessel, 0.05 kg steam at 373 K and 0.45kg of ice at 253 K are mixed. Find the final temperature of the mixture (in Kelvin). ***(2006)***

Given, L_{fusion} = 80 cal/g=336 J/g,

$L_{vaporization}$ = 540 cal/g = 2268 J/g,

S_{ice} = 2100 J/kg, K=0.5 cal/g-K and

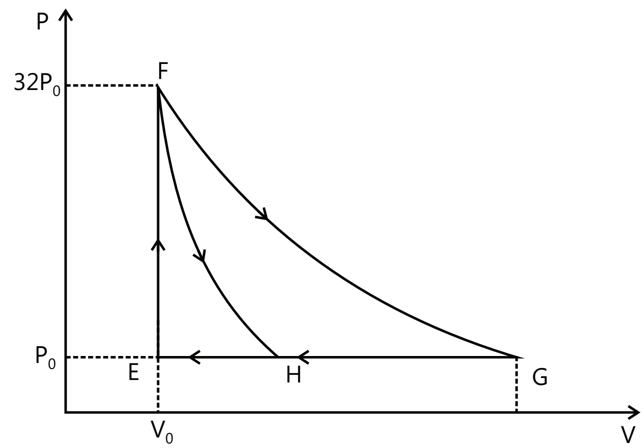
S_{water} = 4200 J/kg, K = 1cal/g-K

Q.8 A piece of ice (heat capacity =2100Jkg^{-1}°C^{-1} and latent heat = $3.36\times10^5 Jkg^{-1}$) of mass m gram is at -5°C at atmospheric pressure. It is given 420J of heat so that the ice starts melting. Finally when the ice- water mixture is in equilibrium, it is found that 1 g of ice has melted. Assuming there is no other heat exchange in the process, the value of m is? ***(2010)***

Q.9 A metal rod AB of length 10x has its one end A in ice at 0°C and the other end B in water at 100°C. If a point P on the rod is maintained at 400°C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of λx from the ice end A, find the value of λ. [Neglect any heat loss to the surrounding.] ***(2009)***

Q.10 Steel wire of length 'L' at 40°C is suspended from the ceiling and then a mass 'm' is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length 'L'. The coefficient of linear thermal expansion of the steel is 10^{-5}/°C, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that L >> diameter of the wire. Then the value of 'm' in kg is nearly: ***(2011)***

Q.11 Parallel rays of light of intensity I = 912 Wm^{-2} are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan-Boltzmann constant $\sigma = 5.7\times10^{-8}\ Wm^{-2}\ K^{-4}$ and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to ***(2014)***



(A) 330 K (B) 660 K (C) 990 K (D) 1550 K

Q.12 An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true? ***(2016)***

(A) The temperature distribution over the filament is uniform

(B) The resistance over small sections of the filament decreases with time

(C) The filament emits more light at higher band of frequencies before it breaks up

(D) The filament consumes less electrical power towards the end of the life of the bulb

Q.13 The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1 m at 10°C. Now the end P is maintained at 10°C, while the end S is heated and maintained at 400°C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is $1.2\times10^{-5}\ K^{-1}$, the change in length of the wire PQ is ***(2016)***

(A) 0.78 mm (B) 0.90 mm (C) 1.56 mm (D) 2.34 mm

PlancEssential Questions

JEE Main/Boards

Exercise 1

Q. 10 Q.14

Exercise 2

Q.8 Q.10

JEE Advanced/Boards

Exercise 1

Q.1 Q.6 Q.8
Q.13 Q.15 Q.25

Exercise 2

Q. 1 Q.2 Q.19
Q.23 Q.24

Answer Key

JEE Main/Boards

EXERCISE 1

Q.1 1 joule=10^{-7} ergs

Q.2 4180 J kg^{-1} K^{-1}, yes

Q.3 $c = \frac{\Delta Q}{m\Delta T}$

Q.4 Heat gained=Heat lost; i.e. mass of the body sp. heat$\times$ rise its temperature=mass of the other body $\times$ sp. Heat $\times$ fall in its temperature

Q.9 60°C

Q.10 217.73°C

Q.11 1.9×10^{-5} K^{-1}

Q.12 911 J kg^{-1} K^{-1}

Q.13 31g

Q.14 4813.2cal.

Q.15 300 cal

Exercise 2

Single Correct Choice Type

Q.1 D	**Q.2** B	**Q.3** D	**Q.4** B	**Q.5** D	**Q.6** A
Q.7 A	**Q.8** B	**Q.9** C	**Q.10** B	**Q.11** B	**Q.12** D

Previous Year's Questions

Q.1 B	**Q.2** D	**Q.3** D	**Q.4** A	**Q.5** C	**Q.6** B
Q.7 C	**Q.8** A	**Q.9** A	**Q. 10** D	**Q.11** D	**Q.12** C
Q.13 C	**Q.14** D				

JEE Advanced/Boards

Exercise 1

Q.1 (i) 50N, (ii) 1.77 cm, 0.045 J (iii) 8.48×10^{-4} m (iv) x=0.12m **Q.2** (a) (dgL)/4Y, (b) (dgL)/6Y

Q.3 25.5°C **Q.4** 4°C **Q.5** 1/90 **Q.6** 4×10^{-6} m / °C **Q.7** h/5R

Q.8 10cm, 40cm **Q.9** 1/200 rad **Q.10** 1: 1.26 **Q.11** 0 °C,125 / 4 g ice, 1275/4g water

Q.12 5000 J/°C Kg **Q.13** 27/85 **Q.14** 12gm **Q.15** (i) 0.02kg (ii) 40,000calkg^{-1} K^{1} (iii) 750 cal/ kg °C

Q.16 1000 J (C°)$^{-1}$ **Q.17** 104.16 M^3 λ^{-1} **Q.18** $5\alpha / 3$ **Q.19** 5sec slow **Q.20** 0.1cm

Q.21 10,000N **Q.23** (a) 37.8J/s (watts,), (b) 2.005 N-m **Q.24** 45°C

Q.25 2×10^{-4} °C **Q.26** Decrease by 0.75cm^3, 25°C

Exercise 2

Single Correct Choice Type

Q.1 A	**Q.2** C	**Q.3** D	**Q.4** A	**Q.5** A	**Q.6** D
Q.7 A	**Q.8** C	**Q.9** D	**Q.10** C	**Q.11** A	**Q.12** B
Q.13 A	**Q.14** C	**Q.15** C	**Q.16** A	**Q.17** B	**Q.18** C
Q.19 B	**Q.20** D	**Q.21** C			

Multiple Correct Choice Type

Q.22 A, C	**Q.23** A, B, C	**Q.24** B, C	**Q.25** A, C, D	**Q.26** D	**Q.27** A, C, D

Comprehension Type

Q.28 B	**Q.29** D	**Q.30** A	**Q.31** A

Assertion Reasoning Type

Q.32 A	**Q.33** A	**Q.34** A	**Q.35** B	**Q.36** B	**Q.37** D

Previous Year's Questions

Q.1 B, D	**Q.2** 6.25 grams	**Q.3** $L = \frac{Pt}{M}$	**Q.4** true	**Q.5** 12g	**Q.6** $\gamma_l = 2\alpha_s$
Q.7 273 K	**Q.8** 8	**Q.9** 9	**Q.10** 3	**Q.11** A	**Q.12** A, D
Q.13 A					

Solutions

JEE Main/Boards

Exercise 1

Sol 1: S.I unit of heat = joules

C.g.s unit of heat = erg.

and of 1 joule = 1 newton × 1m

= 10^5 Dyne × 10^2 cm = 10^7 Dyne × cm

1 Joule = 10^{-7} Erg

Sol 2: Specific heat of water at approximately room temp is 4180 J Kg^{-1} K^{-1}.

Yes, the specific heat of water varies with the temp.

Sol 3: Now for isothermal process $\Delta T = 0$, but if heat is not zero $\Rightarrow$ specific heat $\rightarrow \infty$

$$C = \frac{\Delta Q}{m\Delta T} \text{ as } \Delta T \rightarrow 0$$

$C \rightarrow \infty$.

Sol 4: When two bodies at different temp are mixed, the heat will pass from a body at higher temp to a lower temp body until the temp of the mixture becomes constant. The principle of calorimetry implies that heat lost by a body at a higher temperature is equal to heat gained by another body at a lower temperature assuming that there is no loss of heat to the surroundings.

Sol 5: Heat $\Rightarrow$ Heat is the energy that is transferred from one body to another because of temperature difference.

Temperature $\Rightarrow$ Temperature of a body is basically a measure of the energy that the particles of that body have. [Vibrational energy]

Sol 6: Specific heat is amount of energy required to increase the temperature of 1 kg of a substance by 1°C so its units: J. $kg^{-1}K^{-1}$ Molar specific heat is energy required to increase the temperature of 1 mole of a substance by 1°C.

Sol 7: The principal specific heat capacities of a gas:

(a) The specific heat capacity at constant value (C_v) is defined as the quantity of heat required to raise the temperature of 1 kg of gas by 1 K, if the volume of gas remains constant.

(b) The specific heat capacity at constant pressure (C_p) is defined as the quantity of heat required to raise the temperature of 1 kg of gas by 1K, if the pressure of gas is constant. C_p is always greater than C_v, since if the volume of the gas increases, work must be done by the gas to push back the surroundings.

Sol 8: Change of state occurs because of the weakening of the intermolecular forces between the molecules of the substance, once heat is given to the body. As temp increases, the molecular vibrations increases and the intermolecular forces weaken.

Sol 9: Correct thermometer = 0°C and 100°C

$$\text{So ratio} = \frac{95-5}{0+100-0} = 0.9$$

So 0.9 scale of faulty = 1 scale of correct

$\Rightarrow 5 + 0.9\,x = 59$

$$\Rightarrow x = \frac{54}{0.9} = 60$$

so x = 60°C

Sol 10: We have $L = L_0(1 + \alpha\Delta T)$

$\Rightarrow L/L_0 = 1 + \alpha\Delta T$

$$\Rightarrow \Delta T = \frac{(1-L/L_0)}{\alpha} = \left(\frac{1-5.231/5.243}{1.2\times10^{-5}}\right)$$

$\Rightarrow \Delta T = 190.73\text{ K} \Rightarrow T - 27 = 190.73$

$\Rightarrow T = 217.73°C$

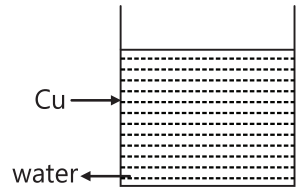
Sol 11: $A_0 = 500\text{ cm}^2$. $\Delta A = 1.9\text{ cm}^2$

Now, $\Delta A = 2\alpha.A_0(\Delta T)$

$\Rightarrow 1.9\text{ cm}^2 = 2 \times \alpha(500\text{ cm}^2)(100K)$

$\Rightarrow \alpha = 1.9 \times 10^{-5}\text{ K}^{-1}$

Sol 12:



Now, amount of heat lost by the aluminium ball = amount of heat gained by

(Container + water)

$\Rightarrow M_A . S_A . \Delta T_A$

$= M_C . S_C . \Delta T_C + M_W . S_W . \Delta T_W$.

$$\Rightarrow S_A = \frac{M_C.S_C.\Delta T_C + M_W.S_W.\Delta T_W}{M_A.\Delta T_A}$$

$$= \frac{0.14\times0.386\times10^3\times(23-20)+0.25\times4.13\times10^3\times(23-20)}{0.047\times(100-23)}$$

$= 0.911 \times 10^3 = 911\text{ J Kg}^{-1}\text{K}^{-1}$

Sol 13: Let the mass of ice be m (in grams) then head gained by ice

$= m . S_i . \Delta T + m . L + m . S_W . \Delta T$

$= m . (0.5) \times (14) + m . (80) + m . 1 \times 10$

= 17m + 80 m = 97m

And heat lost by water $= m . S_W . \Delta T$

$= 200 \times 1 \times (25 - 10) = 200 \times 15$

So assuming no heat loss to surroundings

200 × 15 = 97 m

$\Rightarrow m = \frac{200 \times 15}{97} = 30.93 \text{ gm} \approx 31 \text{ gm.}$

Sol 14: We have PV = nRT

$\Rightarrow n = \frac{PV}{RT} = \frac{10^5 \times 0.2}{8.31 \times 300}$

$= \frac{2 \times 100}{8.31 \times 3} = 8.022$ moles

Now volume = Const.

So heat supplied = n . Cv . ΔT

$8.022 \text{ mol} \times 3.0 \frac{\text{cal}}{\text{mole.K}} \times (500 - 300)\text{K}$

= 4813.2 cal

Sol 15: We have $C_P = C_V + R$

$R = 8.36 \text{ joules/mole °C} = \frac{8.36}{4.18} \text{ cal/mole°C}$

R= 2 cal/mole °C

so $C_P = C_V + R \Rightarrow C_v = C_P - R$

= (8 – 2) cal/mole°C = 6 cal/mole°C

So $Q = nC_v\Delta T$

= 5 × 6 × (20 – 10) = 300 cal.

Exercise 2

Single Correct Choice Type

Sol 1: (D) V = A × λ

$\Rightarrow \frac{dV}{V} = \frac{dA}{A} + \frac{d\ell}{\ell}$

$\Rightarrow \frac{0.2}{100} = -2 \times \frac{dr}{r} + \frac{d\ell}{\ell}$

$\Rightarrow \frac{d\ell}{\ell} = \frac{0.2}{100} - \left(\frac{-2 \times 0.002}{100}\right)$

$\frac{d\ell}{\ell} = \frac{0.2 + 0.004}{100} \Rightarrow \frac{0.204}{100}$

So stress = Y × strain

$= 2 \times 10^{11} \times \frac{0.204}{100} = 4.08 \times 10^8 \text{ N/m}^2.$

Sol 2: (B) $\frac{1}{K} = \frac{-1.\partial V}{V\partial p}$ and $\Delta P = \frac{mg}{A}$ So

$\frac{\Delta P}{K} = \frac{dV}{V} = \frac{4\pi R^2.dR}{\frac{4\pi}{3}R^3} = \frac{3dR}{R}$

$= \frac{mg}{3AK} = \frac{dR}{R}$

After correction

Sol 3: (D) Stress = Y × Strain

⇒ Strain = Stress/Y

$= \frac{100}{\pi r^2} \times \frac{1}{2 \times 10^{11}} = \frac{1}{2 \times 10^9 \times 3.14 \times 10^{-6}}$

$= \frac{1}{6.28 \times 10^3} = \frac{10}{6.28} \times 10^{-4} = 1.6 \times 10^{-4}$

Now, $\mu . \frac{d\ell}{\ell} = \frac{-dr}{r} \Rightarrow dr = -1 \text{ mm} \times \mu \times \frac{d\ell}{\ell}$

$= \frac{-1.6 \times 10^{-4} \times 3.14}{10}, \ r - r_0 = -5.024 \times 10^{-5}$

Sol 4: (B) 2.5×10^3 gm. (0.1 cal/gm°C) . (500 – 0)

= m × L

$\Rightarrow \frac{2.5 \times 100 \times 0.1 \times 500}{80} = m.$

$= 1.5625 \times 10^3$ g

Sol 5: (D) 1 cal = 4.2 J

⇒ Specific heat of ice $= \frac{21000}{42}$ cal/Kg. K

= 500 cal/kg. K = 0.5 cal/gm . K

So suppose the mixture is at temperature T, then

$m_i . S_i . (\Delta T) + m_i L = + m_i . S_w (T - 0)$

$= m_w . S_w (30 - T)$

⇒ 1000 × 0.5 × 10 + 80 × 1000 + 1000 × 1 × T

= 4400 × 1 × (30 – T)

⇒5000 + 80000 + 1000T

= 4400 × 30 – 4400T

⇒5400T = 47000

⇒ T = 8.7°C

Sol 6: (A) $ms\,\Delta T = m_s . L_v$

⇒ 1400 × 1 × 64 = m × 540

Sol 7: (A) Let the heat capacity of the flask be M

Then L = latent heat of fusion

Then 50 L + 50(40 – 0) = 200 × (70 – 40) + (70 – 40) × M

$\Rightarrow$50 L + 2000 = 6000 + 30 M

$\Rightarrow$ 5L = 3M + 400 ...(i)

And 80L + 80 (10 – 0)=250 × (40 – 10) + M (30)

$\Rightarrow$80L + 800 = 7500 + 30 M

$\Rightarrow$8L = 3M + 670

$\Rightarrow$8L = 5L – 400 + 670

$\Rightarrow$L = 90 cal/gm

$\Rightarrow 90 \times 4.2 \times 10^3$ J/kg = 3.8×10^5 J/Kg

Sol 8: (B) $\frac{dV}{dt}$ = 100 cm^3/sec

$\Rightarrow \frac{dm}{dt} = \rho \cdot \frac{dV}{dt}$ = 100 gm/sec.

Now, power used in heating = 2000 × 0.8 = 1600 W.

Now, power = $\frac{d(ms\Delta T)}{dt} = s.\Delta T \cdot \frac{dm}{dt}$

Now assuming ΔT is same

So $1600 = \frac{100}{1000}\frac{kg}{sec} \times 4200 \frac{J}{kg.K} \times \Delta T$

$\Rightarrow \frac{1600 \times 10}{4200} = \Delta T = 3.8°C.$

$\Rightarrow$ T – 10° = 3.8° $\Rightarrow$ T = 13.8°C (B)

Sol 10: (B) Refer theory

Sol 11: (B) Actual length = $L_0(L + \alpha\Delta T) = L_0(1 + \alpha t)$

Change in length = $L_0(1 + \alpha t) - L_0 = L_0\alpha t$

So strain = $\frac{L_0\alpha t}{L_0(1+\alpha t)} = \frac{\alpha t}{(1+\alpha t)}$

$\Rightarrow$ Stress = E × strain = $\frac{E\alpha t}{(1+\alpha t)}$

$\Rightarrow$ Force = A × stress = $\frac{EA\alpha t}{1+\alpha t}$

Sol 12: (D) $\rho_\ell(t) \cdot V_s(t) \cdot g = \frac{\rho_\ell}{(1+8\times10^{-6}\times10^2)}$

$\times v_s(1 + 3 \times 10^{-6} \times 10^2) \times g$

$= \rho_\ell.v_s.g.(1 + 3 \times 10^{-4})(1 - 8 \times 10^{-4})$

so difference = $\rho_\ell.v_s.g(-5 \times 10^{-4})$

Previous Years' Questions

Sol 1: (B) $Q_1 = nC_p\,\Delta T$, $Q_2 = nC_v\,\Delta T$,

$\frac{Q_2}{Q_1} = \frac{C_v}{C_p} = \frac{1}{\gamma}$ or $Q_2 = \frac{Q_1}{\gamma} = \frac{70}{1.4}$ = 50 cal

Sol 2: (D) Heat required

$Q = (1.1 + 0.02) \times 10^3 \times 1 \times (80 - 15)$

= 72800 cal.

Therefore, mass of steam condensed (in kg)

$m = \frac{Q}{L} = \frac{72800}{540} \times 10^{-3} = 0.135$ kg

Sol 3: (D) A is free to move, therefore, heat will be supplied at constant pressure

$\therefore\quad dQ_A = nC_p dT_A$ ...(i)

B is held fixed, therefore, heat will be supplied at constant volume.

$\therefore\quad dQ_B = nC_v dT_B$ ...(ii)

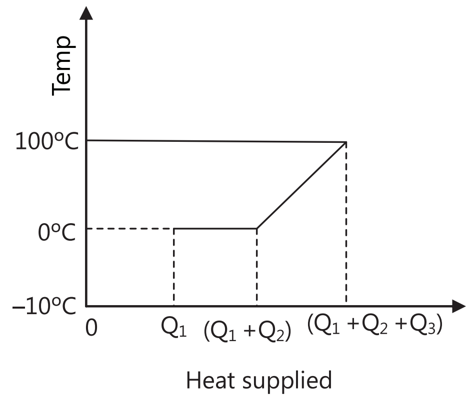
But $dQ_A = dQ_B$ (given)

$\therefore\quad nC_p dT_A = nC_v dT_B \therefore dT_B = \left(\frac{C_p}{C_v}\right) dT_A$

$= \gamma(dT_A)$ [γ =1.4 (diatomic)]

$(dT_A = 30\text{ K}) = (1.4)(30\text{ K})$; $\therefore\quad dT_B = 42$ K

Sol 4: (A) The temperature of ice will first increase from – 10°C to 0°C. Heat supplied in this process will be



$Q_1 = ms_i(10)$

where, m = mass of ice

s_i = specific heat of ice

Then, ice starts melting. Temperature during melting will remain constant (0°C).

Heat supplied in this process will be

Q_2 = mL, L = latent heat of melting.

Now the temperature of water will increase from 0ºC to 100ºC. Heat supplied will be

$Q_3 = ms_w (100)$

where, s_w = Specific heat of water.

Finally, water at 100ºC will be converted into steam at 100ºC and during this process temperature again remains constant. Temperature versus heat supplied graph will be as shown in figure.

Sol 5: (C) Given $\Delta\ell_1 = \Delta\ell_2$ or $\ell_1\alpha_a t = \ell_2\alpha_5 t$

$$\therefore \frac{\ell_1}{\ell_2} = \frac{\alpha_s}{\alpha_a} \text{ or } \frac{\ell_1}{\ell_2+\ell_2} = \frac{\alpha_s}{\alpha_a+\alpha_s}$$

Sol 6: (B) Heat released by 5 kg of water when its temperature falls from 20ºC to 0ºC is,

$Q_1 = mC\Delta\theta = (5)(10^3)(20\text{-}0) = 10^5$ cal

when 2 kg ice at – 20ºC comes to a temperature of 0ºC, it takes an energy

$Q_2 = mC\Delta\theta = (2)(500)(20) = 0.2 \times 10^5$ cal

The remaining heat

$Q = Q_1 - Q_2 = 0.8 \times 10^5$ cal will melt a mass m of the ice,

Where, $m = \frac{Q}{L} = \frac{0.8\times10^5}{80\times10^3} = 1$ kg

So, the temperature of the mixture will be 0ºC, mass of water in it is 5 + 1 = 6 kg and mass of ice is 2 – 1 = 1 kg.

Sol 7: (C) $\frac{dQ}{dt} = L\left(\frac{dm}{dt}\right)$

or $\frac{\text{Temperaute difference}}{\text{Thermal resistance}} = L\left(\frac{dm}{dt}\right)$

or $\frac{dm}{dt} \propto \frac{1}{\text{Thermal resistance}}$; $q \propto \frac{1}{R}$

In the first case rods are in parallel and thermal resistance is $\frac{R}{2}$ while in second case rods are in series and thermal resistance is 2R.

$$\frac{q_1}{q_2} = \frac{2R}{R/2} = \frac{4}{1}$$

Sol 8: (A) 1 Calorie is the heat required to raise the temperature of 1 g of water from 14.5ºC to 15.5ºC at 760 mm of Hg.

Sol 9: (A) A current loop ABCD is held fixed on the plane of the paper as shown in the figure. The arcs BC (radius = b) and DA (radius = a) of the loop are joined by two straight wires AB and CD. A steady current I is flowing in the loop. Angle made by AB and CD at the origin O is 30°. Another straight thin wire with steady current I_1 flowing out of the plane of the paper is kept at the origin.

Sol 10: (D) Let R_0 be the initial resistance of both conductors

∴ At temperature q their resistance will be,

$R_1 = R_0(1+\alpha_1\theta)$ and $R_2 = R_0(1+\alpha_2\theta)$

for, series combination, $R_s = R_1 + R_2$

$R_{s0}(1+\alpha_s\theta) = R_0(1+\alpha_1\theta) + R_0(1+\alpha_2\theta)$

where $R_{s0} = R_0 + R_0 = 2R_0$

$\therefore 2R_0(1+\alpha_s\theta) = 2R_0 + R_0\theta(\alpha_1+\alpha_2)$

or $\alpha_s = \frac{\alpha_1+\alpha_2}{2}$

For parallel combination, $R_p = \frac{R_1R_2}{R_1+R_2}$

$$R_{p0}(1+\alpha_p\theta) = \frac{R_0(1+\alpha_1\theta)R_0(1+\alpha_2\theta)}{R_0(1+\alpha_1\theta)+R_0(1+\alpha_2\theta)}$$

Where, $R_{p0} = \frac{R_0R_0}{R_0+R_0} = \frac{R_0}{2}$

$$\therefore \frac{R_0}{2}(1+\alpha_p\theta) = \frac{R_0^2(1+\alpha_1\theta+\alpha_2\theta+\alpha_1\alpha_2\theta)}{R_0(2+\alpha_1\theta+\alpha_2\theta)}$$

As α_1 and α_2 are small quantities

∴ $\alpha_1 \alpha_2$ is negligible

or $\alpha_p = \frac{\alpha_1+\alpha_2}{2+(\alpha_1+\alpha_2)\theta} = \frac{\alpha_1+\alpha_2}{2}\left[1-(\alpha_1+\alpha_2)\theta\right]$

as $(\alpha_1+\alpha_2)^2$ is negligible

∴ $\alpha_p = \frac{\alpha_1+\alpha_2}{2}$

Sol 11: (D) If temperature increases by ΔT,

Increase in length L, ΔL = LαΔT

$\therefore \frac{\Delta L}{L} = \alpha\Delta T$

Let tension developed in the ring is T.

$\therefore \frac{T}{S} = Y\frac{\Delta L}{L} = Y\alpha\Delta T$

$\therefore T = SY\alpha\Delta T$

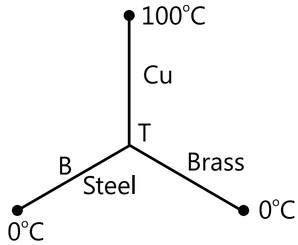
From FBD of one part of the wheel,

F = 2T

Where, F is the force that one part of the wheel applies on the other part.

$\therefore$ F = 2SYαΔT

Sol 12: (C) $Q = Q_1 + Q_2$



$$\frac{0.92\times 4(100-T)}{46}=\frac{0.26\times 4\times (T-0)}{13}+\frac{0.12\times 4\times T}{12}$$

$\Rightarrow$ 200 – 2T = 2T + T

$\Rightarrow$ T = 40°C

$$\Rightarrow Q=\frac{0.92\times 4\times 60}{46}=4.8\,\text{cal/s}$$

Sol 13: (C) Let m mass of fat is used.

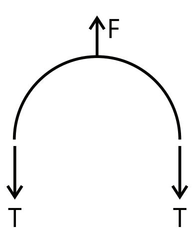
$$m(3.8\times 10^{7})\frac{1}{5}=10(9.8)(1)(1000)$$

$$m=\frac{9.8\times 5}{3.8\times 10^{3}}=12.89\times 10^{-3}\text{kg}$$

Sol 14: (D)

$$\frac{12}{24\times 3600}=\frac{1}{2}\alpha(40-T) \qquad \text{...(i)}$$

$$\frac{-4}{24\times 3600}=\frac{1}{2}\alpha(20-T) \qquad \text{...(ii)}$$



From equation (i) and (ii)

$$-3=\frac{40-T}{20-T}$$

– 60 + 3T = 40 – T

4T = 100

T = 25

From equation (ii)

$$\frac{-4}{24\times 3600}=\frac{1}{2}\alpha(20-25)$$

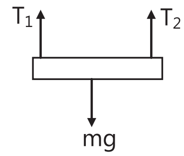
$$\frac{4}{24\times 3600}=\frac{1}{2}\times\alpha\times 5$$

$$\alpha=\frac{8}{24\times 3600\times 5}=1.85\times 10^{-5}\,/^{\circ}\,\text{C}$$

JEE Advanced/Boards

Exercise 1

Sol 1:



(i) 2T = mg = 10 × 10 = 100 (from force and moment balance

$\Rightarrow$ T = 50 N (Tension in each wire)

$T_1 = T_2$ (moment)

2T = mg (Force eq.)

(ii) $\frac{\Delta\ell}{\ell}$ =strain= $\frac{\text{stress}}{Y}$ = $\frac{50/\pi(0.3\times 10^{-3})^2}{2\times 10^{11}}$

$$=\frac{50}{\pi\times(0.3)^2\times 2\times 10^{5}}$$

$$=\frac{50}{\pi\times 2\times(0.3)^2}\times 10^{-5}$$

$= 8.85 \times 10^{-4}$

$\Rightarrow \Delta\ell = 8.85 \times 10^{-4} \times 2$

= 1.77 cm.

Now energy = $\frac{1}{2}$ × stress × strain × $A\ell$

$= \frac{1}{2} \times Y \times (\text{strain})^2 \times A\ell$

$= \frac{1}{2} \times 2 \times 10^{10} \times (8.85 \times 10^{-4})^2 \times \pi \,.\, (0.3 \times 10^{-3})^2$

$= (8.85)^2 \times 10^3 \times \pi \times (0.3)^2 \times 10^{-6} \times 2$

$= (8.85)^2 \times \pi \times (0.3)^2 \times 2 \times 10^{-3}$

= 0.045 J

(iii) $\Delta\ell$ for both has to be same.

$\Rightarrow$ Strains has to be same (ℓ is same for both)

Thus, $\frac{F/A_1}{y_1} = \frac{F/A_2}{y_2}$

$\Rightarrow A_2Y_2 = A_1Y_1 \Rightarrow 2 \times 10^{11} \times \pi\,(0.3 \times 10^{-3})^2$

$= 1 \times 10^{11} \times \pi \times r_1^2$

$\Rightarrow r_1 = \sqrt{2} \times 0.3 \times 10.3$ m.

$\Rightarrow r_1 = 0.424 \times 10^{-3}$m

$\Rightarrow d = 0.848 \times 10^{-3}$ m

$= 0.848 \times 10^{-4}$ m

(iv) For strains to be some (given condition is possible only when the mass is suspended at some distance from centre)

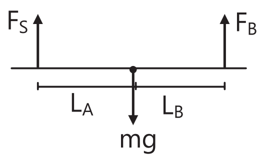
$$\frac{F_s/A_s}{y_s} = \frac{F_B/A_B}{y_B}$$

$$\Rightarrow \frac{F_s}{A_s.y_s} = \frac{F_B}{A_B.y_B}$$

$$\Rightarrow \frac{F_s \times 4}{\pi d_s^2 \times y_s} = \frac{F_B \times 4}{d_B^2 \times y_B \times \pi}$$

$$\frac{F_s}{F_B} = \frac{ds^2 \times y_s}{d_B^2 \times y_B}$$

$$= \frac{(0.6)^2 \times 2 \times 10^{11}}{(1)^2 \times 1 \times 10^{11}}$$



$= 2 \times 0.36 = 0.72$

$\Rightarrow F_S = 0.72F_B$

Torque balance $\Rightarrow F_S.L_A = F_B.L_B$

$\Rightarrow 0.72\ F_B.L_A = F_B.L_B$

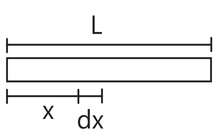
$\Rightarrow L_A .(0.72) = L_B$

Now $L_A + L_B = 0.2$ m

$\Rightarrow L_A\ (1.72) = 0.2$

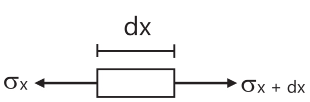
$L_A = 0.116 \approx 0.12$M

Sol 2: (a)



Mass = d(AL) so acceleration = g/2 = $\frac{\text{Force}}{\text{Mass}}$

Now take a small element of length dx.



Then we have $(\sigma_{x+dx} - \sigma_x)\ A = \rho A\ .\ dx(g/2)$

$$\Rightarrow \frac{d\sigma}{dx}(\delta_x) = \frac{\rho g}{2}dx$$

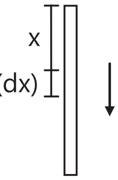
$$\Rightarrow \int_0^{\sigma} d\sigma = \frac{\rho g}{2}\int_0^{x} x$$

$$\sigma = \frac{\rho g x}{2}$$

Now $\varepsilon(x) = \frac{\sigma(x)}{y} = \frac{\rho g x}{2y}$.

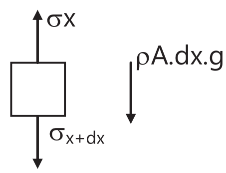
Thus $\varepsilon(L/2) = \frac{\rho g L}{4y}$

(b) (dx)



$$\text{Acceleration} = \frac{(dALg - dALg/2)}{dAL}$$

$= g/2\ m/s^2$



$\boxed{\rho = \text{density}} = d$

Now using force balance

$(\sigma_{x+dx} - \sigma_x)\ A + \rho Agdx = \rho Adx(g/2)$

$$\Rightarrow \left(\frac{\partial \sigma}{\partial x}\right).\ \delta x = \frac{-\rho g}{2}.\delta x$$

$$\Rightarrow \int_{\frac{dLg}{2}}^{\sigma} d\sigma = -\frac{\rho g}{2}\int_0^{x} dx$$

$$\Rightarrow \sigma = \frac{dLg}{2} - \frac{-dgx}{2}$$

$$\sigma(x) = \frac{dg}{2}[L - x]$$

So $\varepsilon(x) = \frac{\sigma(x)}{y} = \frac{dg}{2y}[L - x]$

At x = 2L/3

$$\varepsilon(x) = \frac{dg}{2y}\ .\ [L - 2L/3] = \frac{dgL}{6y}$$

Sol 3: Total heat supplied to the system = 100 cal/s × 240 sec

= 24 × 1000

= 24000 cal

Heat to change temperature from – 20ºC to 0ºC

= 100×0.2×(0 – (–20)]+200×0.5 × (0 + 20)

= 400 + 2000 = 2400 cal

$\Rightarrow$ 24000 – 2400

= 21600 cal are left

Heat used to change the state of all ice

= 80 × 200

= 16000 cal

⇒ Heat left = 21600 – 16000

= 5600 cal

So this much each is used to heat the water produced and the container:

5600 = 100 × 0.2 × (T – 0) + 200 × 1 × (T – 0)

5600 = 20 T + 200T

$\Rightarrow T = \frac{5600}{220} = 25.45°C$

Note: This approach is to be used as we don't know the final state of water.

Sol 4: Mass of drink = 0.833 × 120 = 100 gm

So 50 gm alcohol + 50 gm of water. Let the temperature be T, then

$m_i L_f + m_i . S_w . (T – 0) = m_w . S_w . (25 – T) + m_A . S_A .(25 – T)$

⇒ 20 × 80 + 20 × 1 × T = 50 × 1 × (25 – T) + 50 × 0.6 × (25 – T)

⇒ 1600 + 20 T = 80 × (25 – T)

⇒ 100 T = 80 × 25 – 1600

ΔT = 20 – 16 = 4°C

Sol 5: Let the mass be m, the heat loss rate = R in J/min.

Then $m.S_\ell . \frac{dT}{dt} = R$

$\Rightarrow mS_2.(3) = R$

And also

$m . L_f = 30 R.$

$\Rightarrow m . L_f = 30 \times m S_L . (3)$

$\Rightarrow \frac{1}{90} = \frac{S_2}{L_f} \Rightarrow 1 : 90$

Sol 6: $y_{com} = \frac{y_{BC} m_{BC} + y_{AB}.m_{AB} + y_{AC}.m_{AC}}{m_{BC} + m_{AB} + m_{AC}}$

$$= \frac{m.(0) + \frac{\sqrt{3}a}{4}.m + \frac{\sqrt{3}a}{4}m}{3m}$$

$y_{com} = \frac{a}{2\sqrt{3}}$

Now $\frac{dy_{com}}{dT} = \frac{1}{2\sqrt{3}} \times \frac{da}{dT} = \frac{1}{2\sqrt{3}} \times a_0 . \alpha$

As $= a_0(1 + \alpha(T – T_1)) = \frac{a_0 . 4\sqrt{3} \times 10^{-6}}{2\sqrt{3}}$

$= 4 \times 10^{-6}$ m/°C

$\frac{da}{dT} = a_0 \alpha$

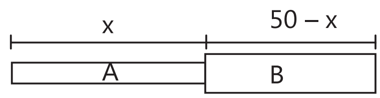
Sol 8: $da = a_0 . \alpha\ \Delta T \Rightarrow 0.05 = 25 . \alpha_A . 100$

$\Rightarrow \alpha_A = 0.2 \times 10^{-4} = 2 \times 10^{-5}$/°C

Similarly $0.04 = 40 . \alpha_B . 100$

$\Rightarrow 10^{-5}$ /°C $= \alpha_B$

Now



Now, $0.03 = x . \alpha_A . (\Delta T) + (50 – x)\alpha_B . \Delta T$

$0.03 = x \times 2 \times 10^{-5} \times 50 + (50 – x)\ 10^{-5} \times 50$

$0.03 = 100x \times 10^{-5} – 50x \times 10^{-5} + 2500 \times 10^{-5}$

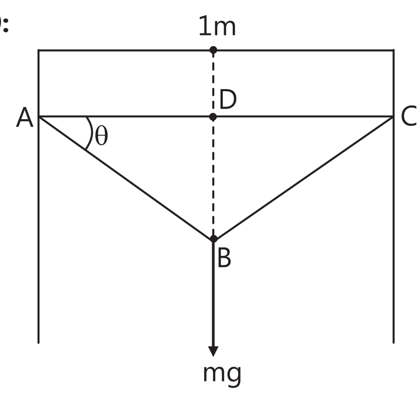
$0.03 = 50x \times 10^{-5} + 0.025$

$\Rightarrow 0.005 = 50x \times 10^{-5}$

⇒ x = 10 cm

So L_A = 10 cm, L_B = 50 – 10 = 40 cm

Sol 9:



We have

2T cos (90° – θ) = mg

⇒ 2T sin θ = mg

⇒ 2T θ ≈ mg [θ is small]

$\Rightarrow T = \frac{mg}{2\theta}$ ...(i)

Now strain = $\frac{AB + BC - AC}{AC} = \frac{2.AB - 2AD}{2AD}$

$= \frac{AB - AD}{AD} = \frac{\sqrt{AD^2 + BD^2} - AD}{AD}$

$= \frac{AD\left[1 + \left(\frac{BD}{AD}\right)^2 \frac{1}{2}\right] - AD}{AD}$

$= \frac{1}{2} \cdot \left(\frac{BD}{AD}\right)^2$, Now as BD = AD tan θ.

Strain = $\frac{1}{2}$. $\tan^2\theta \approx \frac{\theta^2}{2}$ [for small θ]

Now, Stress = Y × Strain

$\Rightarrow \frac{T}{A} = Y \times \frac{\theta^2}{2}$

$\Rightarrow \frac{mg}{2A\theta} = \frac{Y \times \theta^2}{2}$

$\Rightarrow \frac{mg}{AY} = \theta^3 \Rightarrow \theta = 3\sqrt{\frac{mg}{AY}}$

$= 3\sqrt{\frac{1 \times 10}{4 \times 10^{-4} \times 2 \times 10^{11}}}$

$= 3\sqrt{\frac{1}{8 \times 10^6}} = \frac{1}{200}$ rad

Sol 10: Let the mass of ice = m and water = 200 – m So 30 gm (330 – 300) steam is introduced in the system.

Latent heat of condensation for water = $\frac{2250}{4.2}$ cal/gm K.

= 535.71 cal/gm k.

Now, heat lost = 535.71 × 30 + 30× 1 × (100 – 50)

= 17571.3 cal

So heat gained by The mixture = 0.1 × 100 × (50 – 0) + m × 80 + 200 × 1 (50 – 0) = 500 + 10000 + 80 m

⇒17571.3 = 10500 + 80 m

⇒ M_{ice} = 88.4 gm

So m_{water} = 200 – 88.4 = 111.6 gm

So ratio = 88.4 : 111.6 = 1 : 1.26

Sol 11: Heat required to fully melt the ice :

2 × 50 × 0.5 × (15) + 2 × 50 × 80

= 750 + 8000 = 8750 cal

Heat required to convert the water at 0ºC

= 250 × 1 × (25 – 0)

= 250 × 25 = 6250 cal

So the whole water will be converted at 0ºC

Now 6250 – 750 = 5500 cal energy is coming from melting of ice

$\Rightarrow 5500 = mL \Rightarrow m_i = \frac{5500}{80} = 68.75$ gm

So ice melted = 68.75 gm

Ice remained = 100 – 68.75

= 31.25 gm

And water = 250 gm + 68.75 gm = 318.75 gm

Sol 14: Let the mass = m.

So 100 × 1 × (90 – 24)

=mL + m . 1 × (100 – 90)

100 × 66 = 540 m + 10 m.

$\Rightarrow m = \frac{6600}{550} = 12$ gm.

Sol 15: (i) Now, from graph,

800 cal produces 80ºC temperature diff.

⇒ m . S × ΔT = E

⇒ m × 0.5 × 1 cal / gm ºC × 80 = 800

⇒ m = 20 gm = 0.02 Kg

(ii) Heat supplied = 1600 – 800 = 800 cal

⇒ 800 = mL_f = L_f = 40 cal/gm = 40,000 cal/kg.

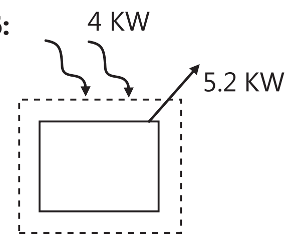
(iii) m . S_ℓ . ΔT = E

⇒ 0.02 × S (120 – 80) = (2200 – 1600)

$0.02 \times S \times 40 = \frac{600}{40}$

$S = \frac{600}{40 \times 0.02} = \frac{15 \times 100}{0.02} = 750$ cal /kg ºC

Sol 16:



Heat stored in system = 4 + 1.7 – 5.2 = 0.5 kW

Now power $P = C_p . \frac{dT}{dt}$

$\Rightarrow 0.5 \times 10^3 = C_p . 0.5$

$\Rightarrow C_p = 10^3$ J/ºC = 1000 J/K.

Sol 17: 70 litre = 70,000 cm^3 = 70,000 gm

= 70 kg. of water

Now heat required per minute

$= m \times S_w . (\Delta T)$

$= 70 \times 1000$ cal/kgºC $\times (90 - 10)$

$= 70 \times 1000 \times 80$

$= 56 \times 10^5$ cal/min

Now, $0.32x = 56 \times 10^5$

$\Rightarrow x = 56 \times 10^5/0.32 = 1.75 \times 10^7$ cal/min

So $mH = 1.75 \times 10^7$.

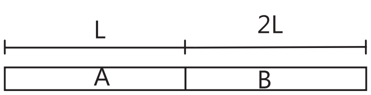
$\Rightarrow m = \frac{1.75\times10^7}{8400\times10^3} = \frac{1.75}{0.84} = 2.08$ kg/min

So for 1 hour = 2.08 kg × 60 = 125 kg/hour.

So volume $= \frac{\text{Mass}}{\text{Density}} = \frac{125\text{kg/hour}}{1.2\text{kg}/\text{m}^3}$

$= 104.16 \times \text{m}^3/\text{hour}$

Sol 18:



$L_T = L_A + L_B$.

$\Rightarrow \frac{dL_T}{dT} = \frac{dL_A}{dT} + \frac{dL_B}{dT}$

$= \alpha . L + (2\alpha) . (2L) = 5\alpha\lambda$.

Now $\frac{1}{L_T} . \frac{dL_T}{dT} = \frac{5\alpha L}{3L} = \frac{5\alpha}{3}$

$\Rightarrow \alpha_T = \frac{5\alpha}{3}$

Sol 19: $T = 2\pi\sqrt{\frac{L}{g}}$

$\frac{dT}{T} = \frac{1}{2} \times \frac{dL}{L}$ [for small changes]

[for quantity

$A = a^m b^n$

$\frac{dA}{A} = m.\frac{da}{a} + n.\frac{db}{b}$]

So $\frac{dL}{L} = (\alpha . \Delta T) = 10^{-6} . (10) = 10^{-5}$

$\Rightarrow \frac{dT}{T} = \frac{1}{2} \times 10^{-5}$

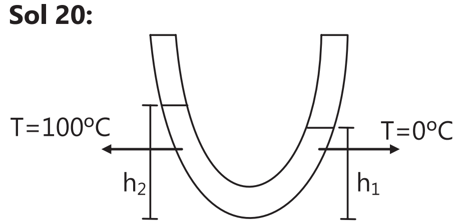
$\Rightarrow dT = 0.5 \times \frac{1}{2} \times 10^{-5}$

[Time period is increasing, so cock has been slowed down]

So in 0.5 sec it loses $\Rightarrow 0.5 \times \frac{1}{2} \times 10^{-5}$ sec

In 10^6 sec it loses $\Rightarrow \frac{0.5\times\frac{1}{2}\times10^{-5}}{0.5} \times 10^6 = 5$ sec

Sol 20:



We have $\rho_1 g . h_1 = \rho_2 . g . h_2$

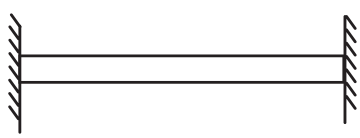
$\rho_0 g h_1 = \frac{\rho_0 g \times h_2}{[1+10^5(\Delta T)]}$

$h_1[1+10^{-3}] = h_2$

$\Rightarrow h_2 = 100 + 0.1 = 100.1$ cm

$\Rightarrow \Delta h = 100.1 - 100 = 0.1$ cm.

Sol 21:



We have $= L_0 (1 + \alpha\Delta T) - L_0 = L_0(\alpha\Delta T)$

And actual length $= L_0 (1 + \alpha\Delta T) = L$

So strain $= \frac{L_0(\alpha\Delta T)}{L_0(1+\alpha\Delta T)} = \frac{\alpha\Delta T}{1+\alpha\Delta T}$

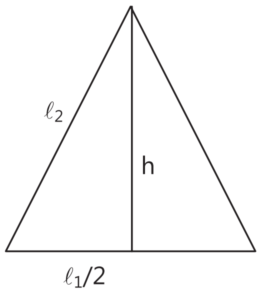
$\Rightarrow$ Stress = Y × Strain $= \frac{Y\alpha\Delta T}{1+\alpha\Delta T}$

So compressions force $= \frac{A\times Y\alpha\Delta T}{1+\alpha\Delta T} = \frac{YA\alpha\Delta T}{1+\alpha\Delta T}$

$= \frac{10^{11}\times10^{-3}\times10^{-6}\times100}{1+10^{-6}\times100}$

$= \frac{10^4}{1+10^{-4}} = 10^4$ N.

Sol 22:



Now using Pythagoras theorem

$$\ell_2^2 = h^2 + \frac{\ell_1^2}{4}$$

Now after increasing temp T,

$$\ell_2^2(1 + \alpha_2.\Delta T)^2 = h^2 + \frac{\ell_1^2}{4}(1 + \alpha_1.\Delta T)^2$$

$$h^2 = \left(\ell_2^2 - \frac{\ell_1^2}{4}\right) + 2.\left[\ell_2^2.\alpha_2 - \frac{\ell_1^2}{4}.\alpha_1\right]\Delta T + \left[\ell_2^2.\alpha_2^2 - \frac{\ell_1^2}{4}.\alpha_1^2\right](\Delta T)^2$$

Now h^2 is independent of ΔT

So, $\ell_2^2.\alpha_2 - \frac{\ell_1^2}{4}.\alpha_1 = 0$ [coeff. of ΔT]

[Now, as $(\Delta T)^2$ has coeff proportional to α^2 and hence negligible]

$$\Rightarrow \ell_2^2.\alpha_2 = \frac{\ell_2^2.\alpha_1}{4} \quad \Rightarrow \boxed{\frac{\ell_1}{\ell_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}}$$

Hence proved.

Sol 23: (i) Entire energy = Heat energy

So power = $m(0.1) \times \frac{dT}{dt} = 180 \times 0.1 \times 0.5$

= 9 cal/s= 9 × 4.2

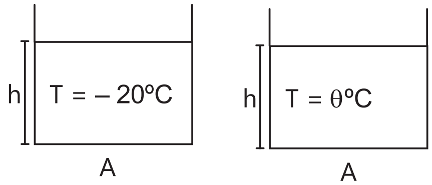
= 37.8 J/s = 37.8 watts

(ii) We have $P = \tau\omega$

So $37.8 = \tau \times \frac{180}{60} \times 2\pi$

$\Rightarrow \tau = \frac{37.8}{6\pi} = 2.005$ Nm.

Sol 24:



Now when both are mixed, 0°C will be the common temperature.

Now, Change in volume

$$= A.(\Delta h) = \frac{m}{\rho_i} - \frac{m}{\rho_w} = \frac{m.[\rho_w - \rho_i]}{\rho_i.\rho_w}$$

Where m = Mass of ice melted,

$$\Rightarrow m = \frac{A(\Delta h)\rho_i\rho_w}{(\rho_w - \rho_i)}$$

So energy gained by this much ice

$$= mL = \frac{A(\Delta h)\rho_i\rho_w \times L}{(\rho_w - \rho_i)}$$

Conservation of energy ⇒

Energy by ice to change temp. + Energy to melt = Energy to convert water temp to 0°C

$$\rho_i.A.h.S_i.(0 + 20) + \frac{A.(\Delta h).\rho_i.\rho_w.L}{(\rho_v - \rho_i)} = \rho_w.A.h.S_w.\theta$$

$$\Rightarrow \rho_i h.S_i.20 + \frac{\Delta h.\rho_i\rho_w.L}{(\rho_v - \rho_i)} = \rho_w.h.S_w.\theta$$

$$\Rightarrow \theta = \frac{\rho_i.S_i \times 20}{\rho_w \times S_w} + \left(\frac{\Delta h}{h}\right)\frac{\rho_i}{(\rho_w - \rho_i)}.\frac{L}{S_w}$$

= 9 + 36 = 45°C

Sol 25: Pressure at the bottom of A is same from both the sides.

$$\rho_A.g.h_A = \rho_0.g.h_0 - \rho_c.g.h_C + \rho_B.g.h_B$$

$$\rho_A h_A = \rho_0.h_0 - \rho_C h_C + \rho_B.h_B \ [\rho_B = \rho_0 = \rho_0]$$

$$\frac{\rho_0}{[1+\alpha.(95-5)]}.h_A = \rho_0.h_D - \frac{\rho_0.h_c}{1+\alpha(95-5)} + \rho_0.h_B$$

$$\frac{h_A}{(1+90\alpha)} = h_0 - \frac{h_C}{(1+90\alpha)} + h_B$$

$$\Rightarrow \frac{(h_A + h_C)}{(1+90\alpha)} = (h_0 + h_B)$$

$$\frac{52.8+49}{1+90\alpha} = (51 + 49)$$

$$\Rightarrow 1 + 90\alpha = \frac{52.8+49}{100}$$

$$1 + 90\alpha = \frac{101.8}{100} \Rightarrow 90\alpha = \frac{1.8}{100}$$

$\Rightarrow \alpha = 0.2 \times 10^{-3}$

$= 2 \times 10^{-4}$/°C

Sol 26: Let the density at 0ºC = ρ_0

Then density at 100ºC = $\dfrac{\rho_0}{1+\gamma.\Delta T}$

= $\dfrac{\rho_0}{1+0.1}$ = $\dfrac{\rho_0}{1.1}$

Density at some temperature

T = $\dfrac{\rho_0}{1+\gamma.(T-0)}$ = $\dfrac{\rho_0}{(1+\gamma T)}$

And from heat transfer.

$300\ \rho_o S(T - 0) = \dfrac{110\rho_o}{1.1} S\ (100 - T)$

300 T = 100 (100 - T)

⇒ 400 T = 100 × 100

⇒ T = 25ºC

Now from the expansion and contraction we have

$V = V_{0_1}(1+\alpha\Delta T_1) + V_{o_2}(1+\alpha\Delta T_2)$

$V = V_{0_1} + V_{o_2}$

$\Rightarrow V - V_0 = \Delta V = V_{0_1}\alpha\Delta T_1 + V_{o_2}\alpha\Delta T_2$

= 300 × 0.001 × 25 + 110 × 0.001 × (–75)

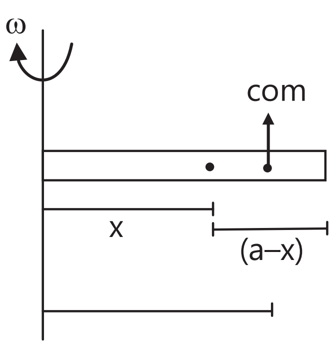
= – 0.75 cm^3

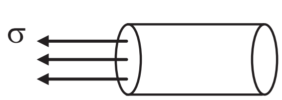
So the volume decreases by 0.75 cm^3

Exercise 2

Single Correct Choice Type

Sol 1: (A)





$r_{com} = \dfrac{a-x}{2} + x = \dfrac{a+x}{2}$

Now σ_x × A = F required for centripetal force

So $\sigma_x \times A = \rho\ .\ (a - x)\ .\ \omega^2\ \dfrac{(a+x)}{2}$

$\Rightarrow \sigma_x = \dfrac{\rho.(a^2-x^2)}{2A}\omega^2$ ⇒ Quadratic

Sol 2: (C) Stress = y × Strain

$\Rightarrow \dfrac{F}{A} = y \times \dfrac{\Delta\ell}{\ell} \Rightarrow \dfrac{F}{\Delta\ell} = \text{Slope} = \dfrac{Ay}{\ell}$

Sol 3: (D) Heat = $mL_1 + (m_w + m_c)\ .\ C_w\ .\ \Delta T + mL_v$

= 10 × 80 + (10 + 10)×1 × 100 + 10 × 540

= 800 + 5400 + 2000 = 8200 cal

Sol 4: (A) ΔH = mL.

$\Rightarrow \dfrac{dH}{dt} = \dfrac{dm}{dt} \times L = 80 \times 0.1$ gm/sec

= 8 cal/sec

So total heat supplied = 800 cal (8 × 100 sec)

So 800 cal = m × L ⇒ m = 10 gm

So $\dfrac{dH}{dt} = mS\dfrac{dT}{dt}$ = 10 × 1 cal/gm K^{-1} × $\dfrac{dT}{dt}$ = 8

$\Rightarrow \dfrac{dT}{dt}$ = 0.8 ºC/sec

Sol 5: (A) Let the heat capacity of the flask be m

Then L = Latent heat of fusion

Then

50 L + 50(40 – 0) =

200 × (70 – 40) + (70 – 40) × m

⇒50 L + 2000 = 6000 + 30 m

⇒ 5L = 3m + 400 ....(i)

And

80L + 80 (10 – 0)=250 × (40 – 10) + m (30)

⇒80L + 800 = 7500 + 30 M

⇒8L = 3M + 670

⇒8L = 5L – 400 + 670

⇒L = 90 cal/gm

⇒90 × 4.2 × 10^3 J/kg = 3.8 × 10^5 J/Kg

Sol 6: (D) Slope = $\dfrac{\Delta T}{H}$ ⇒ Increase of heat capacity

Sol 7: (A) Assuming all potential energy is converted to heat energy

$$mgh = \frac{mL}{5} \Rightarrow h = L/5g$$

Sol 8: (C) Vap. $\Rightarrow$ between 20 – 30 min

$\Rightarrow$ Heat supp = (30 – 20) × 42 KJ = 420 kJ

$\Rightarrow$ mL = 420

$\Rightarrow$ L = 84

Sol 9: (D) 8 volumes of A = 12 volume of B

$\Rightarrow$ 2 volumes of A = 3 volumes of B

So, suppose the volume V,

Then $C_{2V} = C_{3V}$

Thus, $\rho_A . (2V) . S_A = \rho_B . (3V) . S_B$

$\Rightarrow 1500 \times 2\, S_A = 3 \times 2000 \times S_B$

$\Rightarrow S_A/S_B = 2/1$

Sol 12: (B) We have, $\frac{\Delta L}{L} = \alpha\Delta T = 0.01$

So $\frac{dA}{A} = 2\alpha\Delta T = 0.02$

Sol 13: (A) Let the length be = L

Now L (1 + 2 × 10^{-6}× 40)

= 100 mm. (1 + 12 × 10^{-6} × 40)

L is Length at 40ºC

$$\Rightarrow L = 100 \times \frac{(1+12\times10^{-6}\times40)}{\underbrace{(1+2\times10^{-6}\times40)}_{(>1)}}$$

$\Rightarrow$L > 100 mm

Sol 14: (C) $B_{ABCD} = B_{EFGH} = \alpha_x + \alpha_y$

= 3 × 10^{-5}

(A) and (B) are incorrect.

Also $B_{BCGH} = \alpha_y + \alpha_z = (2 + 3) \times 10^{-5}$

= 5 × 10^{-5} $\Rightarrow$ (B) incorrect.

(C) $\Rightarrow \alpha_x + \alpha_y = (2 + 1) \times 10^{-5} = 3 \times 10^{-5}$

(C) is correct.

Sol 15: (C) We have, $x - \gamma_c = C$

And $x - \gamma_s = S$

$\Rightarrow C + \gamma_c - \gamma_s = S$

$\Rightarrow \gamma_s = (C + \gamma_c - S)$

$\Rightarrow \alpha_s = \frac{(C + \gamma_c - S)}{3}$; (C)

Sol 16: (A) Volume of sphere

$$= \frac{4\pi}{3} \times R^3 = \frac{4}{3} \times \frac{22}{3} \times \left(\frac{7}{2}\right)^3 = 179.66(m^3)$$

So density of sphere = 1.4833

Now, the density of sphere = Density of water (for just scenting)

$$\Rightarrow 1.4833 \text{ gm/cm}^3 = \frac{1.527}{1+\gamma.\Delta T} = \frac{1.527}{1+35\gamma}$$

$\Rightarrow \gamma = 8.486 \times 10^{-4}$

Sol 17: (B) Change in volume of Mercury = $V_0\gamma_m.T$

Change in volume of bulb = $V_0 3\alpha_g T$

So excess volume of mercury = $V_0(\gamma_m - 3\alpha_g)T$

And new area of glass = $A_0 (1 + 2\alpha_g T)$

$$\Rightarrow \text{Length} = \frac{V_0.(\gamma_m - 3\alpha_g)T}{A_0(1+2\alpha_g T)}$$

Sol 18: (C) $3\alpha_B = 10^{-3}$ ºC^{-1} and $3\alpha_c = 3 \times 10^{-3}$ ºC^{-1}

So when heated, the ratio of volumes increases by

benzene = $3\alpha_B.\Delta T = 10^{-2}$

(Cylindrical vessel = 3 × 10^{-2})

so new vol : Benzene = $10^{-2}\, V_0 + V_0$

(Cylindrical vessel) = (1 + 3 × 10^{-2})v_0

Change in volume = 2 × $10^{-2}\, V_0$.

So the height will decrease as the volume of cylindrical vessel would be more.

Sol 19: (B) We have $\Delta L = L\alpha\Delta T$

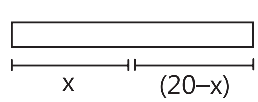
$\Rightarrow 0.075 = 20 \times \alpha_A \times 100$

$3.75 \times 10^{-5} = \alpha_A$

And $0.045 = 20 \times \alpha_B \times 100$

$\Rightarrow 2.25 \times 10^{-5} = \alpha_B$

Now, let the length of A part be x com.



so $\Delta L = x \,.\, \alpha_A \,.\, \Delta T + (20 - x)\, \alpha_B \,.\, \Delta T$

$0.06 = 20\, \alpha_B \,.\, \Delta T + x \,.\, \Delta T\, (\alpha_A - \alpha_B)$

$0.06 = 0.045 + x \times 100 \,.\, [1.5 \times 10^{-5}]$

$\Rightarrow 0.015 = x \times 1.5 - 10^{-3}$

$\Rightarrow x = 10$ cm

Sol 20: (D) Now the volume of air is same

$\Rightarrow \Delta V$ = Same (independent of ΔT)

change in vol. of mercury - change in vol of glass = 0

$\Rightarrow \gamma_m.V_m.\Delta T - \gamma_g.V_g.\Delta T = 0$

$\Rightarrow \boxed{\gamma_m.V_m = \gamma_g.V_g}$

$\Rightarrow 1.8 \times 10^{-4} \times 300 = x \times 9 \times 10^{-6}$

$\Rightarrow x = 20 \times 300 = 6000\text{ cm}^3$

Sol 21: (C) We have

$\rho_1 g h_1 = \rho_2 g h_2$ [pressure is same]

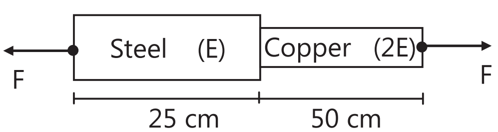
$\Rightarrow \rho_0 \times g \times 120 = \dfrac{124 \times g \times \rho_0}{(1 + \gamma.\Delta T)}$

$\Rightarrow (1 + \gamma.\Delta T) = \dfrac{124}{120}$

$\Rightarrow \gamma.\Delta T = \quad \Rightarrow \gamma = \dfrac{1}{900}$

Multiple Correct Choice Type

Sol 22: (A, C)

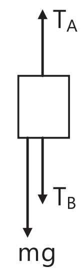


Stress in steel = $\dfrac{F}{2A}$, stress in copper = $\dfrac{F}{A}$

Strain in steel = $\dfrac{F}{2AE}$, strain in copper = $\dfrac{F}{AE}$

Extension = $\dfrac{L_0.F}{2AE}$, extension in copper = $\dfrac{2L_0.F}{AE}$

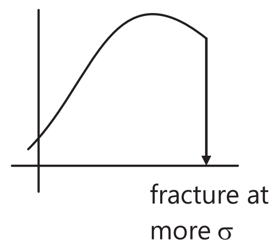
Sol 23: (A, B, C)



Now $T_B = F = mg/3$

And $T_A = mg + T_B = \dfrac{4mg}{3}$

Now, if $r_A = r_B$ then as $T_B < T_A$, the $\sigma_A > \sigma_B$ and hence A will break.

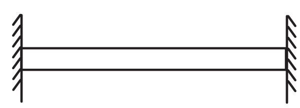


If $r_A > 2r_B \Rightarrow \sigma_A > \sigma_B \Rightarrow$ (B)

If $r_A = 2r_B$

$\Rightarrow \sigma_A = \sigma_B$ and either rope can break.

Sol 24: (B, C)



New length

$= L_0 (1 + \alpha\Delta T)$ and change in length = ΔL

$= L_0 \alpha \Delta T$

So strain in each rod = $\left(\dfrac{\alpha\Delta T}{1 + \alpha\Delta T}\right)$

$\Rightarrow$ Stress = $E\left(\dfrac{\alpha\Delta T}{1 + \alpha\Delta T}\right)$

And Force = $\dfrac{A.E.(\alpha\Delta T)}{(1 + \alpha\Delta T)}$

So, (B) Energy

$= \underbrace{\dfrac{1}{2} \times \text{Stress} \times \text{strain}}_{\text{Same for all}} \times \underbrace{\text{Volume.}}_{A \times \ell}$

Same for all

So Energy α area $\Rightarrow$ (C)

Sol 25: (A, C, D) (A) Stress = $\frac{Mg}{A}$, strain = $\frac{\ell}{L}$

So energy stored = $\frac{1}{2} \times \frac{mg}{A} \times \frac{\ell}{2} \times AL = \frac{mg\ell}{2}$

Sol 26: (D) No kinetics involved

Sol 27: (A, C, D) $\beta = 2\alpha \Rightarrow$ (A)

(C) $\Rightarrow \beta = 3\alpha \Rightarrow$ (C)

$0.002 = \alpha\Delta T = \alpha \times 80$

$\Rightarrow \alpha = \frac{2}{8} \times 10^{-4} \Rightarrow 0.25 \times 10^{-4}$

Sol 28: (B) $(\rho_\ell)(V/2) \times g = (\rho_s) \times V \times g$

$\Rightarrow \rho_\ell = 2\rho_s$.

Sol 29: (D) Let the fraction be f, ΔT = Change in temp

So $\frac{\rho_L}{(1+\gamma_L\Delta T)} \cdot (f \cdot v) \times g = \frac{\rho_s}{(1+\gamma_S\Delta T)} v \times g$

$\Rightarrow f = \frac{\rho_s}{\rho_L} \cdot \frac{(1+\gamma_L\Delta T)}{(1+\gamma_S\Delta T)}$

$f = \frac{1}{2} \times \frac{(1+r_L\Delta T)}{(1+r_S\Delta T)}$

Now the f depends on whether

$\gamma_L > \gamma_s$ or $\gamma_s > \gamma_L$

Sol 30: (A) We have

$\frac{1+\gamma_L T}{1+\gamma_s T} = 1$ for all T.

$\Rightarrow \gamma_L = \gamma_S \Rightarrow \gamma_L = 3\alpha_s$.

Sol 31: (A) We have

$\frac{\rho_\ell}{(1+\gamma_L . \Delta T)} . A_0 (1 + 2\alpha_s\Delta T).h.g = \rho_\ell . A_0 . h \times g$

$\Rightarrow 1 + 2\alpha_s\Delta T = 1 + \gamma_L\Delta T$

$\Rightarrow \boxed{\gamma_L = 2\alpha_s}$

Sol 32: (A) Correct explanation.

Sol 33: (A) Refer theory.

Sol 34: (A) Statement-I may be true statement-II is true.

But statement-I is only possible when $\beta\omega > B_{container}$

Sol 35: (B) Factual

Sol 36: (B) $L_f = 80$ cal/gm $= 80 \times 4.2 \times 10^3$

$= 8 \times 4.2 \times 10^4$ J/kg

= 336000

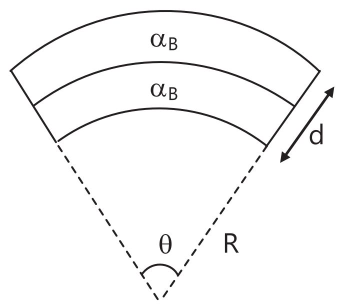
Sol 37: (D) for mass > 1 kg

We have thermal capacity = m.S.

$\Rightarrow$ Thermal cap > S

Previous Years' Questions

Sol 1: (B, D) Let ℓ_0 be the initial length of each strip before heating. Length after heating will be



$\ell_B = \ell_0(1 + \alpha_B\Delta T) = (R + d)\theta$

and $\ell_C = \ell_0(1 + \alpha_C\Delta T) = R\theta$

$\therefore \quad \frac{R+d}{R} = \left(\frac{1+\alpha_B\Delta T}{1+\alpha_C\Delta T}\right)$

$\therefore \quad 1 + \frac{d}{R} = 1 + (\alpha_B - \alpha_c)\Delta T$

[From binomial expansion]

$\therefore \quad R = \frac{d}{(\alpha_B - \alpha_C)\Delta T}$

or $\quad R \propto \frac{1}{\Delta T} \propto \frac{1}{|\alpha_B - \alpha_C|}$

Sol 2: Heat liberated when 300 g water 25ºC goes to water at 0ºC : $Q = ms\Delta\theta = (300)(1)(25) = 7500$ call

From Q = mL, this much heat can melt mass of ice given by

$m = \frac{Q}{L} = \frac{7500}{80} = 93.75$ g

i.e., whole ice will not melt.

Hence, the mixture will be at 0ºC

Mass of water in mixture

= 300 + 93.75 = 393.75 g and

Mass of ice in mixture

= 100 – 93.75 = 6.25 g

Sol 3: Heat lost in time t = Pt = ML

$\therefore \quad L = \frac{Pt}{M}$

Sol 4: $C_P = C_v + R \therefore C_p > C_v$

Sol 5: Let m be the mass of the steam required to raise the temperature of 100 g of water from 24ºC to 90ºC.

Heat lost by steam = Heat gained by

Water $\therefore \quad m(L + s\Delta\theta_1) = 100 s\Delta\theta_2$

or $\quad m = \frac{(100)(s)(\Delta\theta_2)}{L + s(\Delta\theta_1)}$

Here, s = Specific heat of water = 1 cal/g-ºC,

L = Latent heat of vaporization = 540 cal/kg.

$\Delta\theta_1 = (100 - 90) = 10ºC$

and $\Delta\theta_2 = (90 - 24) = 66ºC$

Substituting the values, we have

$m = \frac{(100)(1)(66)}{(540) + (1)(10)} = 12 \text{ g}$

$\therefore \quad m = 12 \text{ g}$

Sol 6: When the temperature is increased, volume of the cube will increase while density of liquid will decrease. The depth upto which the cube is submerged in the liquid remains the same.

Upthrust = Weight. Therefore, upthrust should not change

F = F′

$\therefore V_i \rho_L g = V'_i \rho'_L g$ (V_i=volume immersed)

$\therefore (Ah_i)(\rho_L)(g)$

$= A(1 + 2\alpha_s \Delta T)(h_i)\left(\frac{\rho_L}{1 + \gamma_1 \Delta T}\right) g$

Solving this equation, we get $\gamma_1 = 2\alpha_s$

Sol 7: 0.05 kg steam at 373 K

$\xrightarrow{Q_1}$ 0.05 kg water at 373 K

0.05 kg water at 373 K

$\xrightarrow{Q_2}$ 0.05 kg water at 273 K

0.45 kg ice at 253 K

$\xrightarrow{Q_3}$ 0.45 kg ice at 273 K

0.45 kg ice at 273 K

$\xrightarrow{Q_4}$ 0.45 kg water at 273 K

Q_1 = (50) (540) = 27,000

Q_2 = (50) (1) (100) = 5000

Q_3 = (450) (0.5) (20) = 4500

Q_4 = (450 (80) = 36000

Now since $Q_1 + Q_2 > Q_3$ but $Q_1 + Q_2 < Q_3 + Q_4$ ice will come to 273 K from 253 K, but whole ice will not melt. Therefore, temperature of the mixture is 273K.

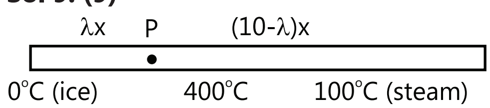
Sol 8: Language of question is slightly wrong. As heat capacity and specific heat are two different physical quantities. Unit of heat capacity is $J\text{-}kg^{-1}$ not $J - kg^{-1} - ºC^{-1}$. The heat capacity given in the question is really the specific heat. Now applying the heat exchange equation.

$420 = (m \times 10^{-3})(2100)(5) + (1 \times 10^{-3})(3.36 \times 10^5)$

Solving this equation we get, m = 8g

$\therefore$ The correct answer is 8.

Sol 9: (9)



$$\frac{dm_{ice}}{dt} = \frac{dm_{vapour}}{dt}$$

$$\frac{400kS}{\lambda x L_{ice}} = \frac{300kS}{(100 - \lambda)x L_{vapour}}$$

$\lambda = 9$

Sol 10: (3) Change in length $\Delta L = \frac{MgL}{YA} = L \propto \Delta T$

$\therefore m \approx 3kg$

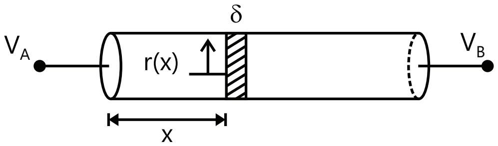
Sol 11: (A) Rate of radiation energy lost by the sphere

= Rate of radiation energy incident on it

$\Rightarrow \sigma \times 4\pi r^2 \left[T^4 - (300)^4\right] = 912 \times \pi r^2$

$\Rightarrow T = \sqrt{11} \times 10^2 \approx 330K$

Sol 12: (A, D) If the temperature distribution was uniform (assuming a uniform cross section for the filament initially) the rate of evaporation from the surface would be same everywhere. But because the filaments break at random locations; it follows that the cross-sections of various filaments are non-uniform.



$$\delta R(x) = \rho \frac{\delta x}{\pi r(x)^2}$$

The temperature of points A and B are decided by ambient temperature are identical. Then the average heat flow through the section S is O. After sufficiently long time, this condition implies that the temperature across the filament will be uniform. If the instantaneous current is i(t) through the filament then by conservation of energy :

$$\frac{(V_B - V_A)^2}{R(t)^2} \times \frac{dx}{\kappa \pi r(x)^2} = e\sigma 2\pi r9x).\delta(x)T^4 + \rho\pi r(x)^2.dxL_v$$

in above κ = Material conductivity

R(t) = Resistance of whole filament as a function of time

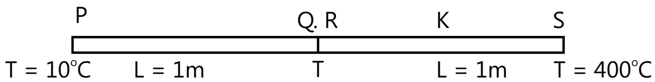
ρ = Material density

L_v = Latent heat of vapourisation for the material at temperature T

Since R(t) increases with time

$$P(t) = \frac{(V_B - V_A)^2}{R(t)} \text{ decreases}$$

Sol 13: (A) Let temperature of junction = T

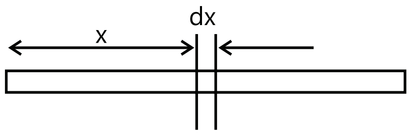


$$\text{Rate of heat transfer } = \frac{dQ}{dt} = \frac{2KA(T-10)}{L} = \frac{KA(400-T)}{L}$$

$\Rightarrow$ 2(T - 10) = 400 - T

3T = 420

T = 140°C



For wire PQ

$$\frac{\Delta T}{\Delta x} = \frac{140-10}{1} = 130$$

Temp. at distance x

T = 10 + 130 x

T - 30 = 130x

Inc. in length of small element

$$\frac{dy}{dx} = \propto \Delta T$$

$$\frac{dy}{dx} = \propto (T-10)$$

$$\frac{dy}{dx} = \propto (130x)$$

$$\int_0^{\Delta L} dy = 130 \propto \int_0^L x\,dx$$

$$\Delta L = \frac{130 \propto x^2}{2}$$

$$\Delta L = \frac{130 \times 1.2 \times 10^{-5} \times 1}{2}$$

$$\Delta L = 78 \times 10^{-5} m = 0.78 mm$$

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