BIOMOLECULES-1

INTRODUCTION :

- Living matter of cell is called protoplasm.
- **Protoplasm** (Gr. Protos-first+ Plasma-organization) is a living organized substance which is the place for all physical and chemical transformations, as a characteristics of life.
- Protoplasm also includes membranes, cytoplasmic contents and nucleus.
- Protoplasm is **"physical basis of life".**
- Protoplasm is the medium & source of all biological activities.
- Protoplasm basically differentiates a nonliving from living.

HISTORY :

- **Corti** (1772) observed protoplasm first time.
- **Felix Dujardin** (1835) studied the jelly-like substance in protozoa and called it **sarcode**. i.e. flesh of cell.
- **J.E. Purkinje** (1840) coined the term **protoplasm** for the living substance.
- **Hugo von Mohl** (1846) applied this name (protoplasm) for the contents of the embryonic cells of the plants.
- **Max Schultze** (1863) stated that protoplasm is the **physical basis of life**.
- **Hanstein** (1880) proposed the term protoplast for such organized mass of protoplasm, **without cell wall in plants.**
- **O. Hertwig** (1892) propound the protoplasmic theory according to which all living matter, out of which animals and plants are formed, is protoplasm.
- **Fisher, Hardy** and **Wilson** (1916) proposed **colloidal theory of protoplasm**.

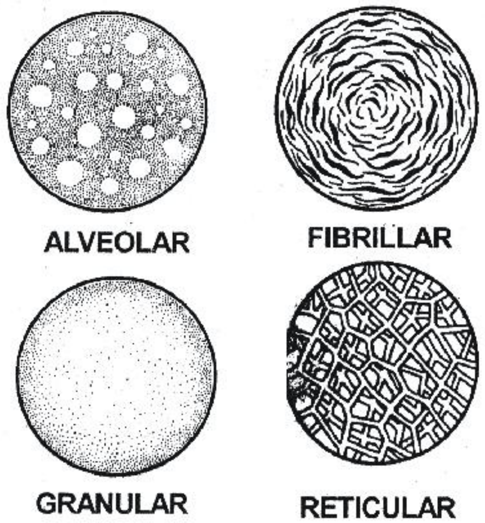
PHYSICAL NATURE OF PROTOPLASM

- Protoplasm is a **greyish, translucent, jelly-like, odourless** and **viscous** substance.
- Protoplasm is **heavier than water**.
- Protoplasm behaves as a **moderate conductor of electricity**.
- Size of colloidal particles is 0.001 to 0.1 μm.
- Mineral ions and smaller inorganic and organic molecules (sugar, salts, simple acids, bases etc.) occurs as **crystalloid solutes** in the protoplasm, while larger, organic molecules (proteins, polysac charides and nucleic acid) occur as **colloidal solutes**. Thus protoplasm is **crystallo-colloidal mixture**.
- Scientists call it a **"mixture of mixtures"**, or **"supermixture"**.
- Protoplasm is a **reversible coloidal system** i.e. it can change from a thicker, highly viscous **"gel" state** to a comparatively more fluid and less viscous **"sol" state** and vice-versa.
- Being a liquid mixture, the protoplasm has a **surface tension**.
- Viscosity = 2 – 20 centipoises.
- Refractive index = 1.4

Physical Appearance of Protoplasm :

Old theories : Protoplasm appears differently in different phases.

- **Alveolar theory-Butschli (1892) :** Protoplasm consists of bubbles or alveoli of a fluid of lesser density distributed in a fluid of greater density.
- **Granular theory : Altmann(1893) :** The protoplasm consists of numerous **fine granules** dispersed uniformly in homogeneous fluid medium.
- **Fibriller theory-Fisher (1894) and Flemming (1897)** This theory maintains that protoplasm consists of numerous **minute fibrils** or thread-like structures dispersed in a fluid medium.



* **Reticular theory-Hanstein, Klein and Carnoy :** According to this theory protoplasm consists of numerous minute fibrils which forms a **network or reticulum** in a fluid medium.

MODERN COLLOIDAL THEORY :

• This was first suggested by **R.A. Fisher (1894**) and latter by **Hardy (1899)** and **Wilson (1916).**
• It consists of a fluid **matrix** or ground substance **(liquid phase)** dispersion phase and **dispersed phase** of granules and globules.
• The solid and semisolid particles range in diameter from 0.001μ to 0.1μ (1μ or micron = 1/1000 mm.)
• The particle are **too big** to form a suspension and are too small to form a **true solution**, they thus remain in the matrix forming a **colloidal system**.
• The liquid phase or dispersion phase of protoplasmic colloids consists mainly of water having dis solved inorganic ions, salts and small molecules.
• The **dispersed phase** comprises mainly of large molecules of **proteins, lipids** and **carbohydrate**.
• **Phase reversal** in protoplasm is due to its colloidal nature. Non living colloid are irreversible in phase change. (Gel $\rightleftharpoons$ Sol)
• Protoplasm shows **elasticity**.
• Elasticity, viscosity, contractility, rigidity are physical properties due to colloidal nature.
• Protoplasm shows **viscosity**.
• Colloidal particles show scattering of light, when a beam of light is passed through colloid it be comes visible. It is called **Tyndall effect**.
• Colloidal particles show zig-zag or irregular movement. The movements are called as **Brownian movement**. It occurs during sol state.
• Protoplasm responds to the stimulus of chemicals, light and heat or mechanical and thus it shows **irritability**.
• Protoplasm shows **conductivity of impulse** produced by stimulus.
• All colloidal solution shows ageing. After some time (age) various activities stops.

MOVEMENTS OF PROTOPLASM

• It can show following types of movement:
Amoeboid movement :
• Naked mass of protoplasm shows creeping movement to form pseudopodia. e.g. Amoeba, Slime moulds, Neutrophils.
Cyclosis :
• Streaming movement of cytoplasm is called cyclosis. It is of two types.
(a) circulation (b) Rotation

CHEMICAL NATURE OF PROTOPLASM

Elements :

• About 36 elements are known to occur in protoplasm.
• About 13 elements are found in more quantity. These universal elements are C, H, O, N, Cl, Ca, P,Na, K, S, Mg, I, Fe.
• Carbon, hydrogen, nitrogen and oxygen are found greatly in the protoplasm and make up 95% of all living material. Thus they are called **major element**.
• Remaining 23 elements are called trace element. They are in very small amount (about 0.376%). they are copper, Cobalt, magnese, zinc, chromium molybdenum, boron, silicon, vanadium, nickle, lflorine, selenium etc

Elements	% amount
Oxygen	62
Carbon	20
Hydrogen	10
Nitrogen	3
Calcium	2.5
Phosphorus	1.14
Chlorine	0.16
Sulphur	0.14
Potassium	0.11
Sodium	0.10
Magnesium	0.07
Iron	0.01
Iodine	0.014

A comparison of elements present in non-living and living matter		
Element	% Weight of	
	Earth's crust	Human body
Hydrogen (H)	0.14	0.5
Carbon (C)	0.03	18.5
Oxygen (O)	46.6	65.0
Nitrogen (N)	Very little	3.3
Sulphur (S)	0.03	0.3
Sodium (Na)	2.8	0.2
Calcium (Ca)	3.6	1.5
Magnesium (Mg)	2.1	0.1
Silicon (Si)	27.7	negligible

COMPOUNDS OF PROTOPLASM :

Although some elements occur in protoplasm **as free ions** but mostly they are found in the form of different kinds of **compounds**.

Average composition of cells

S.No.	[A] Inorganic Compounds	
1	Water	70-90%
2	Ions (Salts, acids, bases, gases)	1%
	[B] Organic	
1	Proteins	10-15%
2	Lipids	2%
3	Carbohydrates	3%
4	Nucleic acids	5-7 %

Do you know Biomicromolecules & Biomacromolecules	
1	**Acid soluble pool contain those compounds which have molecular weights ranging from 18 to around 800 daltons (Da) approximately.**
2	**The acid insoluble fraction, has only four types of organic compounds i.e., proteins, nucleic acids, polysaccharides and lipids. These classes of compounds with the exception of lipids, have molecular weights in the range of ten thousand daltons and above**
3	**Those compounds which have molecular weights less than one thousand dalton usually referred as biomicromolecules or simply biomolecules.**
4	**The acid soluble pool represents roughly the cytoplasmic composition. The macromolecules from cytoplasm and organelles become the acid insoluble fraction. Together they represent the entire chemical composition of living tissues or organisms.**
5	**Lipids , because of ltheir associantion with membranes separate in the macromolecular fraction. Lipids are not strictly macromolecules.**

[A] INORGANIC COMPOUNDS :

1. Water :

- Most aboudant substance of all living organism.
- It forms about 70-90% of the cell.
- Water is transparent, colourless and odourless.
- It has high surface tension, high specific heat, high heat of vaporization.
- pH value of water is 7.
- The density of water is maximum at 4^0C.
- It freezes at 0^0C and boils at 100^0C.
- The molecular of water is composed of 2 atoms of hydrogen and one atom of oxygen linked by covalent bonds. Bond angle 104^0.
- Water exists in two forms-

(i) Free water - about 95%

(ii) Bound water - 5%

FUNCTION OF WATER :

- Water is an important structural component.
- Water gives an ideal medium for chemical reactions. It is a **universal solvent**.
- A number of waste products are eliminated in solution form mainly as urine with the help of water.
- Water keeps the surface of tissues and organs moist.
- In living organism, water gives a medium for distribution or transportation of living matter.
- Water, works as a dispersion phase in the colloidal system of protoplasm.
- Its high specific heat prevents temperature hazards due to sudden change.

Water content in different organism and body parts		
1	Body of jelly fish	99%
2	Dormant seed	6%
3	bone cells	25-40%
4	Skeleton muscles	75%
5	Embryo	90-95%
6	Dentine of tooth	10%
7	Enamel of tooth	5%
8	Brain cell	90%
9	Nerve cells	80%

2. Mineral salts :

- Salts in protoplasm occur in ionised form
- Salt responsible for conducitivity & irritibility
- Some importent minerals and their functions given in below table

Minerals	Important Funcations
Calcium	Importent mineral for formation of bones and teeth.
	Essential for blood clotting, muscle contraction, membrane permebility and nerve impulse conduction.
Phosphorus	Phosphorus is an imporant component of the phospholipids of biomembranes and of the nucleotides of nucleic acids (DNA and RNA).
Sulphur	It is a component of two amino acids which participate in polymerization of some structure proteins.
Iron	Combinationof iron with porphyrin pigment yields heme which is a component of haemoglobin of RBC and myoglobin of muscle.
Magnesium	Combination of magnesium with prophyrin yields the green pigment, chlorophyll, of plants.
Iodine	It is component of thyroxine hormone of thyroid gland.
Copper	It is component of haemocyanin which serves as respiratory pigment in certain higher invertebrates.
Manganase	It is a cofactor in metalloenzymes.
Cobalt	It is component of vitamin B12. It's deficiency may cause anaemia.
Sodium, potassium and Chloride	These ions are mainly responsible for maintenance of osmolarity of cytosol and extracellular fluids.
Chlorine, sodium and potassium	These participate in maintenance of acid base equilibrium.

3. Acids and Bases :

- They form buffer system & maintaing pH of protoplasm (carbonic acid -bicarbonate buffer)
- Phosphoric acid (H_3PO_4) - found in nucleic acids.
- Hydrochloric acid (HCl) - found in gastric juice.

4. Gases :

- There are four major gases in protoplasm, which remains dissolved in its free water.
- These four gases are $CO_2 > O_2 > N_2 > H_2$ (Solubility order)
- CO_2 is used in synthesis of urea and dissociation of oxyhaemoglobin.
- O_2 is used in the oxidation of the substance.
- H_2 is used for ETS.

[B] ORGANIC COMPOUND :

- Four organic compound are found in acid-insoluble pool
- There are three main organic compounds in the protoplasm.
- In organic compounds, the main bonds exist between C–C and C–H.
- Normally, the organic compounds remain suspended or dissolved in the water of the protoplasm.

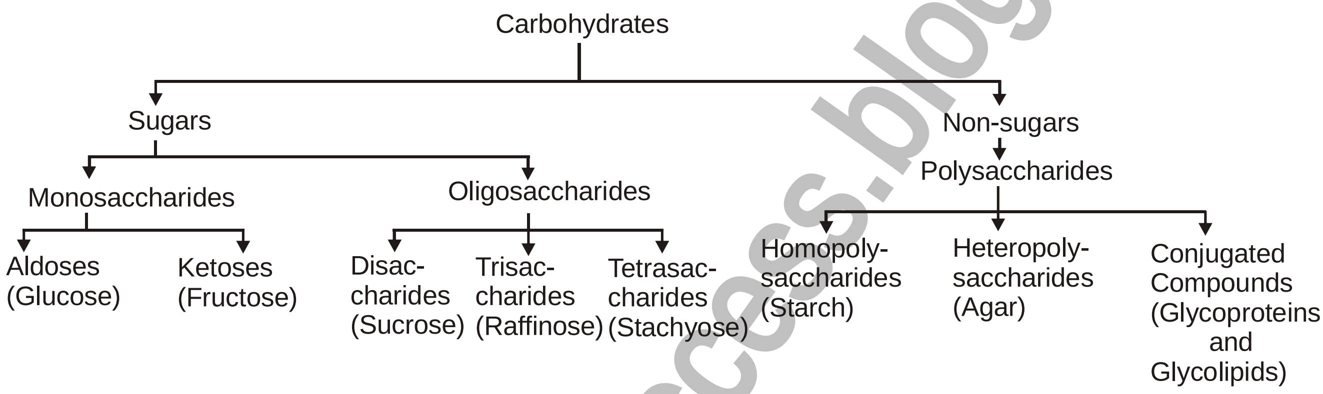
TYPES OF ORGANIC COMPOUNDS :

There are following types of organic compounds in protoplasm.

- Carbohydrates
- Amino acids & Proteins
- Lipids
- Nucleic acid
- Enzyme
- Vitamins

Do you know
1. Hardest material in animal kingdom is enamel.
2. Hardest material in plant kingdom is sporopollenin.

CARBOHYDRATE



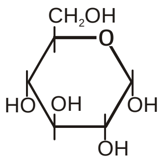
- These compounds are made up of carbon, hydrogen and oxygen. Ratio of H & O is 2 : 1.
- Carbohydrate are aldehyde or keton derivatives of polyhydroxy alcohol.
- A carbohydrate may contain aldehyde or ketone group and is called aldose or ketose, respectively.
- Certain carbohydrates contain some nitrogen, phosphorous, or sulphur.
- Carbohydrate are produced by green plants during photosynthesis. About 80 percent of the dry weight of the plant is made up of carohydrates.
- Carbohydrate are also called saccharides or sugar.
- **Carbohydrate provides 55-65% of total energy required.**
- **Caloric value - 4.1 Kcal/gm**
- **Storage site - mostly liver and muscles**
- **Daily requirement - 500gm approx.**

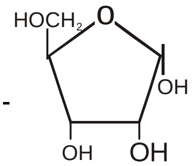
Types of carbohydrate : Carbohydrates are of three types.

[A] Monosaccharides **[B] Oligosaccharides** **[C] Polysaccharides**

(A) Monosaccharides :

- These are simple sugars.
- Monosaccharides are the simplest and smallest carbohydrates.
- These are colourless or white, mostly sweet and crystalline solids which are freely soluble in water.
- Monosaccharide molecules have the general formula, $C_nH_{2n}O_n$ or $C_n(H_2O)_n$.
- They do not undergo hydrolysis but oxidises to CO_2 & H_2O.
- They contain 3 to 7 carbons.
- **Suffix - Ose is used in their nomenclature.**

- Pyranose -  Six member ring sugar e.g. Glucose ($C_6H_{12}O_6$).

- Furanose -  Five member ring sugar e.g. Ribose ($C_5H_{10}O_5$).

- **On the basis of nature of functional group they are classified in two groups.**

(a) Aldoses : They have aldehyde group (–CHO).

(b) Ketoses : They have ketone group (–C=O).

- **Classification based on number of Carbon atoms.**

(i) Trioses : They have 3 carbon atoms, $C_3H_6O_3$ e.g., dihydroxyacetone and glyceraldehyde.

(ii) Tetroses : Tetroses contain 4 carbon atoms, $C_4H_8O_4$ e.g. erythrose, erythrulose.

(iii) Pentoses : They contain 5 Carbon atoms, $C_5H_{10}O_5$ e.g. ribose, deoxyribose, xylose, ribulose, arabinose (Gum arabic).

(iv) Hexoses : They have 6 carbon atoms, $C_6H_{12}O_6$ e.g. glucose, fructose, galactose and mannose.

(v) Heptoses : They have 7 carbon atoms $C_7H_{14}O_7$ e.g. sedoheptulose.

Few Important Monosaccharide :

1. Glucose :

- Glucose is dextrorotatory so it is called "dextrose".
- **Glucose is aldose sugar having –CHO group.**
- **Glucose is found in grapes in abundant quantity so it is also known as "Grape sugar".**
- Glucose is the main - respiratory substrate in the body.
- Other types of hexose are converted into glucose in liver.

2. Fructose :

- Fructose is Levorotatory so it is called "Levulose".
- **Fructose is a ketose sugar having C=O group.**
- **Fructose is found in honey (32-40%) and sweet fruits so it is called as "Fruit Sugar".**
- Fructose is the sweetest natural sugar.
- Fructose is source of energy for sperms and semen.

3. Galactose :

- Galactose is not found in free state- In mammalian body galactose in milk sugar lactose.
- Galactose is also found as a component of galactolipids. i.e. In cerebroside & ganglioside.

4. Mannose :

- Mannose is not found in free state.
- Mannose occurs in albumin of egg and in wood as component of hemicellulose.

5. Ribose and Deoxyribose :

- They are found in nucleic acid, ATP, NAD, NADP, FAD, FMN, Vitamin B_2.

(B) Oligosaccharide :

- Oligosaccharides are formed by condensation of 2 to 10 molecules of monosaccharide.
- Oligosaccharides on hydrolysis yields 2 to 10 monosaccharide units (monomers).
- In oligosaccharides, monosaccharides are linked together by glycosidic bonds.
- Aldehyde or ketone group of one monosaccharide reacts with alcoholic group of another monosaccharide to from glycosidic bond.
- One molecule of H_2O eliminates during glycosidic bond formation.

Types of Oligosaccharides :

Disaccharide :

- Theyare formed by condensation of two molecules of monosaccharides e.g. sucrose, maltose, lactose.

Trisaccharides :

- They are formed by 3 monosaccharide molecules e.g. raffinose, rhaminose and gentianose.

Tetrasaccharides :

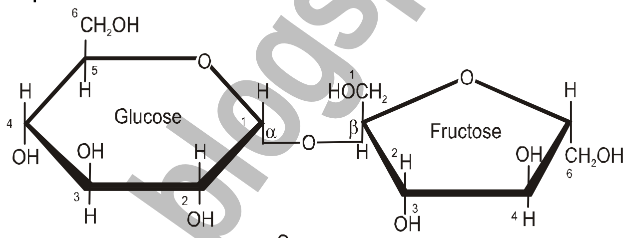
- They are formed by condensation of four molecules of monosaccharides e.g., scorodose and stachyose.
- Pentasaccharide e.g. verbinose, barbacose (Galactose + Galactose + Glucose + Glucose + Fructose)

Important Disaccharides :

(1) Sucrose (cane sugar) :

- Sucrose is obtained from sugar cane and sugar beet, called cane sugar.
- Sucrose is a commercial sugar.
- It is the storage product of photosynthesis in these plants.
- Sucrose is formed by the condensation of one molecule each of glucose and fructose with the removal of one molecule of water. (α-D-Glucose + β-D fructose)
- 1, 2 glycosidic bond is formed between glucose and fructose.

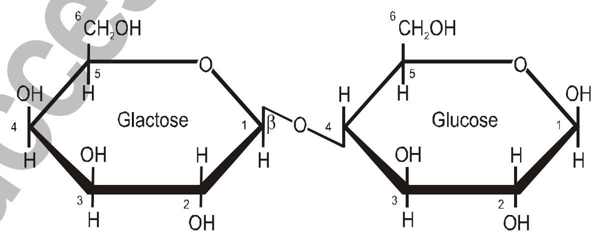
$C_6H_{12}O_6 + C_6H_{12}O_6 \rightarrow C_{12}H_{22}O_{11} + H_2O$
Glucose Fructose Sucrose



Sucrose

(2) Lactose (Milk Sugar) :

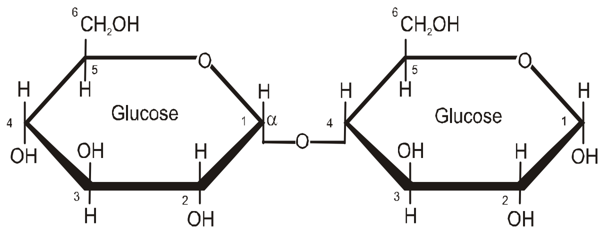
- Lactose or Milk sugar is found naturally in mammalian **milk**.
- Lactose is a reducing sugar.
- Lactose is formed inside mammary glands by condensation of one molecules of each of glucose and galactose (α-D glucose + β-D-galactose)
- Lactose have β-1- 4 glycosidic linkage between glucose and galactose.



Lactose

(3) Maltose (Malt sugar) :

- Maltose or malt sugar is found in germinating starchy seeds.Maltose is formed by condensation of two molecules of α-D glucose and β-D glucose by a 1, 4 glycosidic bond is formed between two molecules of glucose. It is reducing sugar.



Maltose

Functions of Simple Carbohydrate :

- Glucose acts as most common respiratory substrate.
- Glucose is the blood sugar of many animals.
- Mammary glands synthesis lactose of milk from glucose and galactose.
- Glucose is utilised in the synthesising fats and amino acids.
- Trioses, tetroses, pentoses and heptoses are intermediates in the path way of photosynthesis.

(C) Polysaccharides :

- These are complex carbohydrates.
- They are formed by polymerisation of 11 to thousands of monosaccharide monomers.
- $n(C_6H_{12}O_6) \longrightarrow (C_6H_{12}O_6)_n + nH_2O$
  Monosaccharide Polysaccharide
- General formula $(C_6H_{10}O_5)_n$
- **They are tasteless and soluble in water.**
- Suffix-ans is used in nomenclature. Normally they are called as glycans. (Polymers of glucose)

- In a polysaccharide chain (like glycogen), the right end is called the reducing end and the left end is called non-reducing end.
- Depending upon the composition, polysaccharides are of two types.

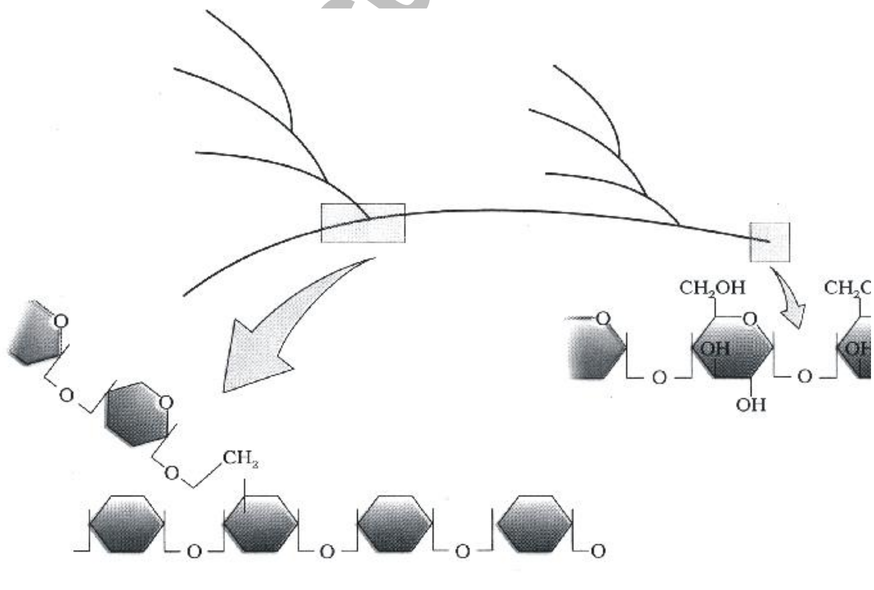
(a) Homopolysaccharides or Homoglycans
- Made up of only one type of monosaccharide monomers. For example - starch, cellulose and glycogen, inulin

(b) Heteropolysaccharides or Heteroglycans
- These are formed by condensation of either monosaccharide derivatives or more than one type of monosacharide monomers. e.g., **chitin, agar, hemicellulose**, **arabagalactans**, **arabaxylans** etc.
- Based on function, polysaccharide are grouped into **two categories**: **Storage** and **structural**.

(i) Food storage polysaccharides :
- They are those polysaccharides, which act as reverse food in the living organisms.
- The main storage polysaccharides are **starch**, **glycogen** and **inulin**.

(1) Starch :
- Starch is the reserve food in plants.
- It is also called amylum.
- It is found in abundance in cereal grains (wheat, rice, maize), legumes (pea, gram, beans), potato, banana etc.
- **Starch consists of two components: amylose and amylopectin (both glucose polymers).**
- **Amylose is more soluble in water but amylopectin is insoluble.**
- **Amylose** have 250-300 glucose units are arranged in an unbranched chain by α 1-4 linkage.
- Amylopectin is a branched chain molecule. Approximately 30 glucose units are linked in one branchby α-1,4 and α-1,6 linkage.
- **Starch can hold I_2 molecules in the helical portion and starch-I_2 is blue in colour.**
- Starch present in potato contains 20% amylose and 80% amylopectin.

(2) Glycogen
- It is a main reserve food in animals, bacteria and fungi.
- It is also known as **animal starch**.
- Glycogen is stored in **muscle cells** and **liver cells**.
- Liver of an adult human may store upto 0.91 kg of glycogen.
- **Glycogen gives red colour with idodine.**



Diagrammatic representation of a portion of glycogen

(3) Inulin
- Linear polymer of fructose units linked with β-1,4 bonds, Inulin is found in **roots of Dahalia and Artichoke**.
- It is water soluble polysaccharide.
- It is not metabolised in human body and **filtered throuh kidney**. It is therefore, used in **testing of kidney function**, **(glomerular filtration).**

(ii) Structural polysaccharides :

(1) Cellulose :

- It is a homopolysaccharide of β-D glucose, having β 1, 4 glycosidic bonds. The chains are unbranched.
- It is the main structural component of **cell wall of plants**, some **fungi** and **protists**.
- **Tuncin** of tunicate is related to cellulose (also called as **animal cellulose**)
- Cellulose is the most abundant organic substance of the plants & also on earth.
- Cotton fibres have about 90% of cellulose & Wood contain 25-50% of cellulose.
- Cellulose is fibrous polysaccharide with a high tensile strength.
- A cellulose molecule consists of an unbrached and linear chain of 6000 or more glucose residues with molecular weight between 0.5 to 2.5 millions.
- Cellulose form the **bulk of human food** and most of the herbivorous.
- It **can not digested** by the human beings, because they lack the enzyme **cellulase**, required to digest cellulose.
- Cellulose is an important **constituent of diet** of ruminats such as cows and buffaloes.
- Cellulose rich wood is used in **production of paper** and as **fuel**.
- Cellulose containing fibres of **cotton**, **linen** and **jute** are used for textile and ropes.
- Cellulose nitrate is used in **propellant explosives**.
- **Cellulose does not contain complex helices and hence cannot hold** f_2**.**

(2) Chitin :

- Chitin is the **second most abundant** organic compound in nature.
- It is present in the **exoskeleton of arthropods** and in the **cell walls of fungi**.
- **In fungal walls, the chitin is often known as fungus cellulose.**
- The monomer of chitin is not glucose but nitrogen containing glucose derivative known as N-acetyl glucosamine.

(3) Pectin :

- It is a heteropolysaccharide composed of sugars arabinose and galactose.
- Pectin found in cell wall where it binds with cellulose fibrils in bundles.
- Salts of pectin i.e. Ca and Mg-pectates form middle lamella in plants.

(4) Hemicellulose :

- It is mixture of polysaccharides xylans, galactans, arabagalactans and glucomannans.
- It is found in the cell wall. It is hardest known carbohydrate.

(5) Hyaluronic acid (Animal cement) :

- It is found in skin, vitreous humour of the eye, the umbilical cord, synovial fluid.
- It is a linear polymer of disaccharide repeat units linked with the help of β-1,4 glycosidic bonds.
- Disaccharids repeat units are D-glucoronic acid and N-acetyl -D- glucosamine.

(6) Chondriotin sulphates :

- They are found in cornea, cartilage, tendons, skin, heart, saliva etc.
- The repeating unit is a disaccharids consisting of glucoronic acid and sulphate ester of N-acetyl galactosomines, linked through a β-1,3 bond. The disaccharides are linked with each other by β-1, 4 bonds.

(7) Heparin :

- It is secreted by mast cells in the intestinal mucosa, liver, lung, spleen and kindey.
- It is an anticoagulant.

(8) Agar-Agar :

- It is a polysaccharide found in **few red algae such as gracillaria, gelidium etc**. It is made up of sulfated galactose. It is important as tissue culture medium.

(9) Tunicin :

- It is cellulose like glycon. It costitutes exoskeleton like covering test on Urochordata.

Mucopolysaccharides :

- They contain acidic or aminated polysaccharide formed from galactose, mannose, sugar derivatives and uronic acids.
- These are jelly like compounds important in packing and connection.
- Mucopolysaccharides are found in the **cell walls of bacteria** and in the **connective tissues** of animals as well as in **body fluids**.
- **Hyaluronic acid, chondriotin sulphate and heparin** are comon mucopolysaccharides.

Function of Carbohydrate :

- They are the primary **source of energy**.
- Normally body obtains 58 to 65% of its required energy from the carbohydrate.
- **Ribose** & **Deoxyribose** are used in the fromation of RNA, DNA, ATP, NAD, FAD etc.
- **Starch** in plants & **glycogen** in animals are the stored form of food.
- Cellulose is used in the formation of cell wall.
- **Chitin** forms exoskeleton in arthropods and few fungal cell wall.
- **Heparin, Mosquitin & herudin** act as anti-coagulants.
- **Chondriotin sulphate** is a component of cartilage, tendons & bones, heart valves.

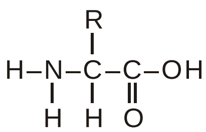
PROTEIN

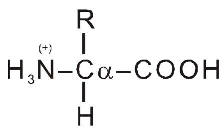
- Protein name is derived from a greek word proteioses which means **"holding first place" (Berzelius and Mulder).**
- Essential elements in protein are **C, H, O, N**, (sulphur is also present in less amount)
- After water, proteins are most abundant compounds in protoplasm. (10-15%) Amount of proteins is different in differetn sources -20% in **muscle**, 7% in **blood plasma**, 3.5% in **cow milk**, 12% in **cereal grain**, 20% in **nuts** and **pulses**, 11-13% in **white part of egg.** 15-17 % in **yellow part of egg**.
- Different elements in simple proteins have following proportion-
- **Carbon**- 50-55%, **Hydrogen**- 7%, **Oxygen**- 20-24%, **Nitrogen**-14-18%, **Sulphur**- 0.3-0.5%.
- **Caloric value - 5.6 kcal/gm**
- **Daily requirement - 70 - 100gm**
- **Proteins are polymer of amino acid (Fisher and Hofmeister).**

Do You Know	
1	**Protein is a heteropolymer and not a homopolymer.**
2	**Collagen is the most abundant protein in animal world.**
3	**Ribulose bisphosphate carboxylase-Oxygenase (RUBISCO) is the most abundant protein in the whole of the biosphere.**

Amino Acid :

- Amino acids are **organic acids**.
- All amino acids contain carboxylic acid group and amino acid group, both linked to a single carbon atom called α carbon.





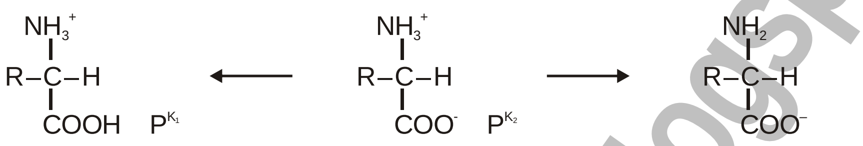
L-α amino acid

General structure of a α - amino acid

- Number of known amino acids are more than 100.
- **Only 20 amino acids are commonly found in proteins of organism. (Magic 20)**
- These 20 amino acids can be classified into two types.

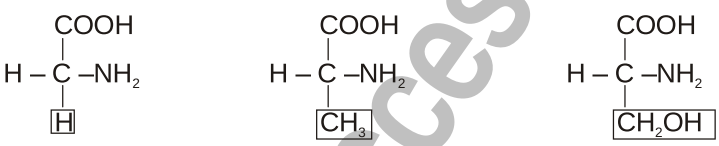
Essential amino acids	Nonessential amino acids
They are not synthesized in the animal cells. Hence, their presence in the food is essential.	**Are those amino acids which are synthesized in animal cells. Hence their presence in the food is not essential.**
They are Threonin, Leucine, Lysin, Tryptophane, Phenylalanine, valine, Isoleucine, Methionine	**They are glycine, serine, aspartic acid, asperagine, prolin, alanine, cysteine, glutamic acid, tyrosine, hydroxyprolin.**

- Semi essential amino acids- Are those amino acids which are **synthesized** in animal cells but not in sufficient quantity, thus there presence in food is essential. They are - **Arginine and Histidine**.
- Amino acids are amphoteric compound which exhibits both the acidic (due to –COOH group) and basic (due to $–NH_2$ group) properties. It is also called as **dipolar or zwitter ions**.



Cation in acid solution | **Zwitterion at isoelectric pH** | **anion in alkaline solution**

- When R group of such proteinaceous amino acids could be a hydrogen, then amino acid is called glycine, if methyl group then alanine amino acid and if hydroxy methyl in place of R group then amino acid is serine and their structures are



Glycine | **Alanine** | **Serine**

Classification of Amino acids :

Amino acids can be classified into three groups depending on their NH_2 or –COOH group number.

(1) Neutral

(2) Acidic and

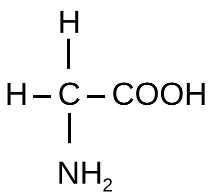
(3) Basic

Neutral amino acids :

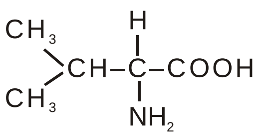
This comprises the largest group and can be further subdivided into aliphatic, aromatic, heterocyclic and sulphur containing amino acids. They have all of them one amino and one carboxylic group, each.

A. Aliphatic amino acids :

- Glycine (Gly) G
  (Aminoacetic acid)

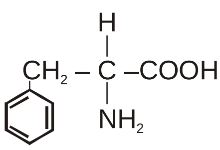


- Valine (Val) V
  (α-aminoisovaleric acid)

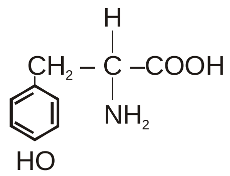


B. Aromatic Amino acids

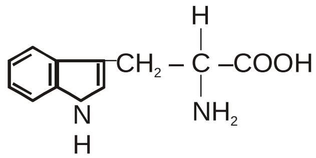
- Phenylalanine (Phe) F
  (α-amino, β-phenyl propionic acid)



- Tyrosine (Tyr) Y
  (p-hydroxyphenylalanine)

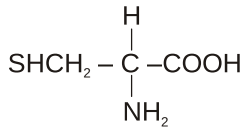


- Tryptophan (Trp) W
  (α-amino, β-indole propionic acid)



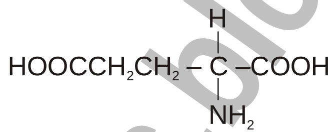
C. Heterocyclic amino acid

- Cysteine (Cys) C
  (a-amino, b-mercapto propionic acid)



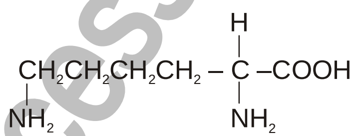
II. Acidic amino acids :

- Glutamic acid (Glu) E
  (α-aminoglutaric acid)



III. Basic Amino Acids :

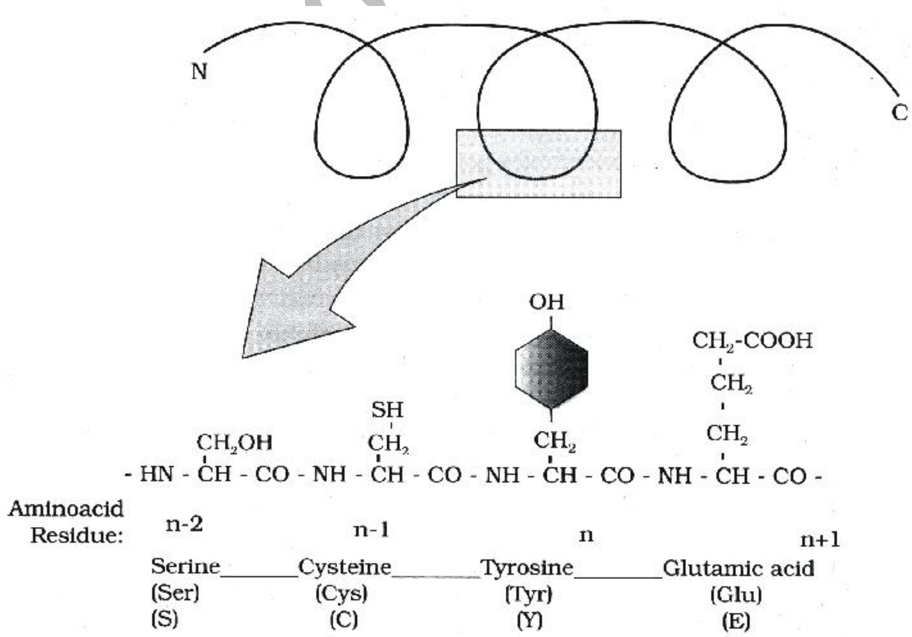
- Lysine (Lys) K
  (α, ε-diaminocaproic acid)



Structure of proteins :

(1) Primary structure :

- A **straight chain** of amino acids linked by **peptide bond** form primary structure of proteins.
- This structure of proteins is most unstable.
- **Newly formed proteins on ribosomes** have primary structure.



Primary structure of a portion of a hypothetical protein. N and C refer to the two termini of every protein. Single letter codes and

(2) Secondary structure :

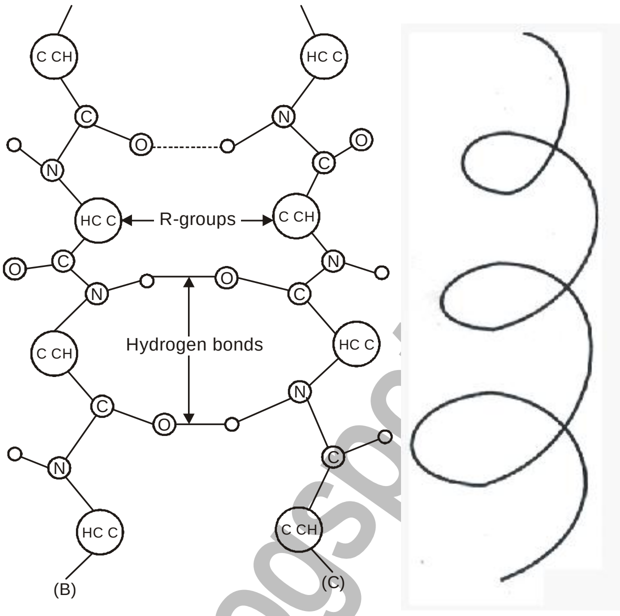
- Protein molecules are **spirally coiled**.
- In addition to **peptide bond**, amino acid are linked by **hydrogen bond** formed between oxygen of carboxylic group and hydrogen of amide group. This structure is of two types-

(i) ~-Helix :

- Right handed rotation of spirally coiled chain with approximately 3½ amino acid in each trun.
- This structure have intramolecular hydrogen bonding i.e. between two amino acid of same chain e.g. **Keratin, Myosin, Tropomyosin**

(ii) -Helix or -pleated structure :

- Protein molecule have zig-zag structure.
- Two or more protein molecules are held together by intermolecular hydrogen bonding, e.g. fibroin (silk).



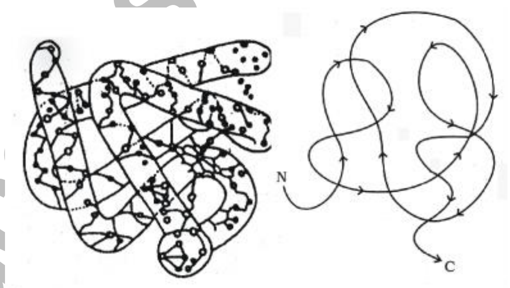
SECONDARY STRUCTURE OF PROTEIN

- Proteins of sec. structure are insoluble in water and fibrous in appearance. Keratin is a fibrous, tough resistant to digestion, sclero protein. Hard ness of keratin is due to abundance of cysteinamino acid in its structure.

(3) Tertiary structure-

- Proteins of tertiary structure are **highly folded** to give a globular appearance. Mostly are water soluble.
- This structure of protein have following bonds-

(i) Peptide bonds
(ii) Hydrogen bonds
(iii) Disulphide bond
(iv) Hydrophobic bond
(v) Ionic bond

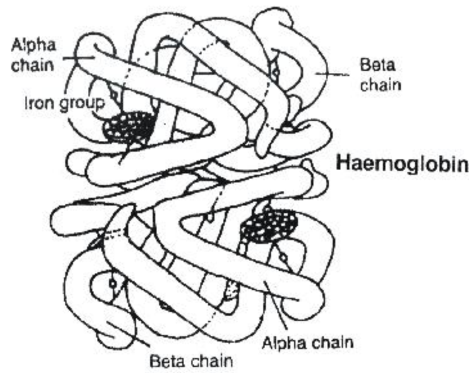


TERTIARY STRUCTURE OF PROTEIN

- The tertiary structure of the protein molecules are so arranged as to hide non-polar sides inside and expose the polar side chains.
- Majority of proteins and enzymes in protoplasm exhibit tertiary structure.

(4) Quaternary structure:

- Two or more poly peptide chains of tertiary structure unite by different types of bond to form quaternary structure of protein.
- Different polypeptide chains may be similar (lacticdehydrogenase) or disimilar types (Haemoglobin, insulin).
- Quaternary structure is most stable structure of protein.
- Adult human haemoglobin consists of 4 subunits two of these are identical to each other. Hence, two subunits of α type and two subunits of β type.



QUATERNARY STRUCTURE OF PROTEIN

Properties of proteins :

- Proteins are large sized molecules.
- Many protein form colloidal solutions.
- A protein may bind as well as react with a variety of chemicals.
- Proteins can not pass through cell membranes.
- The distruption of bond of tertiary proteins structure is called denaturation.

Types of Proteins :

• Protein are of three types-

(i) Simple proteins (ii) Conjugated proteins (iii) Derived proteins

(i) Simple Proteins:

• They are made up of only amino acids.
• They are of two types - Fibrous and globular proteins.

1. Fibrous proteins :

• They are thread like structural proteins.
• Fibrous protein generally contain secondary structure and are insoluble in water e.g., **collagen of connective tissue, actin and myosin of muscles, keratin of scales, feathers, hairs, claws, nails, horns and hoofs, silk of spider web**.

2. Globular proteins :

• They are **spherical** non-contractile proteins which may be enzymatic or non-enzymatic.
• They have **tertiary** or **quaternary** structure.
• Smaller globular proteins are usually soluble in water and are not coagulated by heat e.g., histones.
• Egg albumin, serum globulins and glutelins are examples of large globular proteins, which get coagulated by heat.

(a) Albumins :

• They are water soluble and occur as serum albumin in blood plasma and loctoalbumin in milk.

(b) Globulins :

• They are soluble in weak acids and alkalies but insoluble in water e.g. **lactoglobulin** in milk.

(c) Histones :

• They are water soluble and are rich in lysine and arginine (amino acids) they occur in **eukaryotic nucleus**.

(d) Protamines :

• These water soluble arginine rich proteins occur in DNA of spermatozoa of some fishes e.g. salmine in Salmon.

(e) Gluteins :

• They occur only in plants like wheat e.g., **glutenin**.

(f) Prolamines :

• They occur only in plants e.g., **zein** in corn and **gliadin** in wheat.

(ii) Conjugated proteins :

• These are formed by the binding of a simple protein with a non-protein called the prosthetic group.
• These are of following types-

(a) Chromoproteins :

• These proteins contain pigment (coloured) as prosthetic group e.g., haemoglobin, haemocyanin, cytochrome, flavoprotien and rhodopsin.

(b) Glycoproteins :

• Contain protein + carbohydrate less than 4% eg., plasma glycoprotein secreted from liver and immunoglobulin produced by lymphocyetes.

(c) Mucoprotein : They have carbohydrate more than 4% e.g., Muerins of bacteria cell wall ossomucoid in bones, tendenomucoid in tendons, chondromucoid in cartilage. Various mucoproteins are found in vitrous humor synovial fluid.

(d) Nucleoproteins :

• Contain protein + nucleic acid. e.g., histone and non-histone protein join with DNA to make chromosomes.

(e) Metalloproteins :

• Contain protein + metal ions e.g., Arginase (Mn & Mg), Carbonic anhydrase (Zn), Tyrosinase (Cu), Xanthine oxidase (Mo) etc.

(f) Phosphoproteins :

• Contain protein + phosphate e.g., casein in milk and ovo-vitellin in eggs.

(g) Lipoproteins :

• Contain protein + lipids e.g., high density lipoprotein (HDL), low density lipoprotein (LDL) and very low density lipoprotein (VLDL). Mostly important in membranes.

(iii) Derived proteins: These are denatured or hydrolysed protein.
- **Primary derived protein :** Denatured product e.g. Fibrin, Myosan
- **Secondary derived protein :** Digestion product of proteins eg. Peptones, Proteose di & tripetide.

Function of proteins :

1. Structural proteins :
- Many proteins serve as building material of cells and tissues.
- Some proteins form supporting structures e.g., **elastin of ligaments, collagen of tendons, cartilages, cartilage bone** and **connective tissue**.

2. Protective structure :
- Fibrous protein keratin is the major constituent of external protective structure of animals like hair, feather, horny layer of skin, nails, claws, hoofs etc.

Some Proteins and their Functions	
Protein	**Functions**
Collagen	Intercellular ground substance
Trypsin	Enzyme
Insulin	Hormone
Antibody	Fights infectious agents
Receptor	Sensory reception (smell, taste, hormone, etc.)
GLUT-4	Enables glucose transport into cells

3. Enzymes :
- Many proteins function as enzymes to catalyse biochemical reactions.

4. Carrier proteins :
- Some proteins act as carriers which bind and transport specific molecules across a membrane or in a body fluid.
- Haemoglobin transports oxygen in the body.
- α-globulin of blood carries thyroxine and bilirubin & β-globulin transport vitamin A, D and K.

5. Receptor proteins :
- A number of proteins present on the **external surface of cell membrane** act as receptor molecules.

6. Hormones :
- Some hormones are proteinaceous e.g., **insulin**

7. Contractile proteins :
- **Myosin** and **actin** make the muscle fibres contractile to bring about movements and locomotion.

8. Defence :
- Some proteins act as **antibodies** that participate in the defence mechanism of the body.

9. Storage proteins :
- These occur in milk, eggs and seeds to nourish the young ones.
- Iron storing protein commonly found in animal tissue is **ferritin**.

10. Protein Buffers :
- Proteins also help in maintaining a **balance of acidity** and **alkalanity** by combining with excess acids and bases.

11. Visual pigments :
- **Rhodopsin** and **iodopsin** are protein pigments.

12. Toxins :
- Many toxins of microbes, plants and animals are proteins.

13. Blood clotting proteins :
- The proteins **fibrinogen** and **thrombin** help in blood clotting to check bleeding from injuries.

14. Sweetest substance :
- Monellin, a protein derived from an African berry in 2000 times sweeter than sucrose.

15. Repressor :
- Most of the repressor that regulate gene (operon concept) are protein in nature.

LIPIDS

- **SIMPLE LIPIDS or HOMOLIPIDS**
  - **Fats and oils** (Triglycerides) Mixed triglycerides Symmetrical triglycerides
  - **Waxes** Sperm whale wax Bees wax Carnauba wax
- **COMPOUND LIPIDS or HETEROLIPIDS**
  - **Phospholipids** (Phosphatides) Phosphoglycerides Lecithins Cephalins Plasmalogens Phosphoinositides Linositols Phosphosphingsides (Sphingomyelins)
  - **Glycolipids** (Galactolipids) Kerasin Phrenosin Nervon Oxynervon
- **DERIVED LIPIDS**
  - **Steroids** C_{29},$C_{28}$$C_{27}$ steroids C_{24} steroids C_{12},C_{19} C_{18}steroids
  - **terpenes** Monoterpenes, Sesquiterpenes, Diterpenes, Triterpenes, Polyterpenes
  - **Carotenoids** Lycopene, Carotenes Xanthophylls, Tetraterpenes

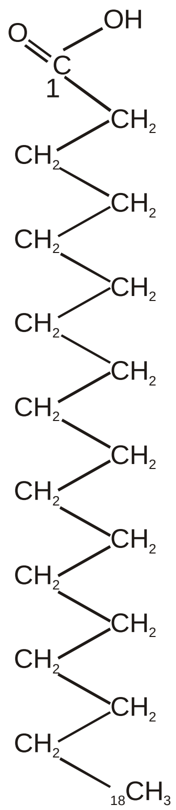
- They are made of carbon, hydrogen and oxygen. hydrogen and Oxygen ratio is never same as H_2O.
- The number of oxygen atoms in a lipid molecule is always less than number of hydrogen and carbon atoms.
- Sometimes small amount of phosphorus, nitrogen and sulphur are also present.
- Lipids are esters of fatty acids and related substances.
- They include substance like cooking oil, butter, ghee, natural rubber, cholesterol etc.
- Lipids exhibit a variety of structures but have certain common characteristics.
- Lipids are insoluble in water. But soluble in organic solvents like chloroform, benzene and acetone.
- The basic components of all lipids are fatty acids and many lipids have both glycerol (tri hydroxy propane) and fatty acids.
- **Structure of Glycerol**

CH_2–OH
|
CH–OH
|
CH_2–OH

- **Caloric value – 9.45Kcal/gm**
- **Storage site – subcutaneous fat, adipose cells**
- **Daily requirement – 50gm**
- Lipids in protoplasm are about 2%. These are present as small globules in protoplasm.
- They do not undergo further polymerisation.

Fatty acids :

- **Fatty acids are organic acids with a hydrocarbon chain ending in carboxylic group (–COOH).**
- The hydrocarbon chains of fatty acid may possess straight or ring structure.
- **Most fatty acids have an even number of carbon atoms between 14 and 22, mostly 16 or 18.**
- Plants and few animals can synthesis all types of fatty acids.
- Some animals including man cannot synthesis few fatty acids eg. linoleic



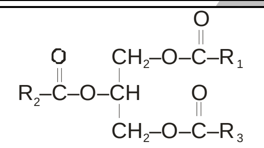
Stearic acid

acid, linolenic acid and arachionic acids. These fatty acid are called essential fatty acids (EFA). They obtain these fatty acids through edible oils eg. sunflower, groundnut cotton seed, coconut oils etc.
- Fatty acids are of two types:

(i) Saturated fatty acids
(ii) Unsaturated fatty acids

(i) Saturated Fatty acids :
- **They do not have double bonds in their carbon chains.**
- They have higher melting points and are solid at normal temperature.
- Saturated fatty acids have general formula $C_2H_{2n}O_2$.
- The most common fatty acids are e.g., palmitic acid ($C_{16}H_{32}O_2$) or $CH_3(CH_2)_{14}COOH$ and stearic acid ($C_{18}H_{32}O_2$) or $CH_3(CH_2)_{16}COOH$
- Animals that living in warm climate have large quantity of saturated fatty acids.

(ii) Unsaturated fatty acids :
- **They have one or more double or triple bonds in their carbon chains.**
- Most common unsaturated fatty acids are e.g., Oleic acid ($C_{18}H_{34}O_2$), linoleic acid ($C_{18}H_{32}O_2$) and arachidonic acid ($C_{20}H_{32}O_2$). (most essential fatty acids)
- **The unsaturated fatty acids have lower melting points and are liquid at normal temperature.**
- **More unsaturated fatty acids are found in higher plants and in animals that live at low temperature.**
- In hydrogenation unsaturated fatty acids are changed to saturated and the oil becomes a solid fat.

Do You Know	
1	**Oils have lower melting (e.g., gingely oil) and hence remain as oil in winters.**
2	**The fatty acids are found esterified with glycerol. They can be then monoglycerides, diglycerides and triglycerides.**
3	 Structure of Triglyceride (R_1,R_2 and R_3 are fatty acids)

Classification of lipids :
- Bloor (1943) classified lipids into three types-simple, compound and derived.

(i) Simple lipids :
- Simple lipids are formed of fatty acids and trihydric alcohol (glycerol) only.
- Simple lipids are of following types-

1. Neutral or true fats- (Glycerides and triglycerides) :
- **Fats are esters of fatty acid and glycerol.**
- Each molecule of glycerol can react withthree molecules of fatty acids.
- **Fatty acid (3 molecules) + glycerol (one molecule) $\rightarrow$ Lipid (one molecule) + water (three molecule)**
- **Triglycerides are the most common fats in cells.**
- Mono-and di-glycerides occur as intermediates in certain biosynthetic reaction.
- The three fatty acids are similar only in few fats. They are called pure fats.
- Most fats have dissimilar or two of the three fatty acids are similar. They are called mixed fats, (e.g., butter)
- **Depending on the physical nature, fats are differentiated into oils and hard fats.**
- **(A)** Oils are generally liquid at room temperature.
- Oils are rich in unsaturated fatty acids and short chain fatty acids. e.g., groundnut oil, rape seed oil, mustard oil.
- The oils have a tendency to solidify, are called drying oils.
- **(B)** Hard fats are solid at room temperature.
- Hard fats contain long chain saturated fatty acids and have high melting point e.g., animal fat.

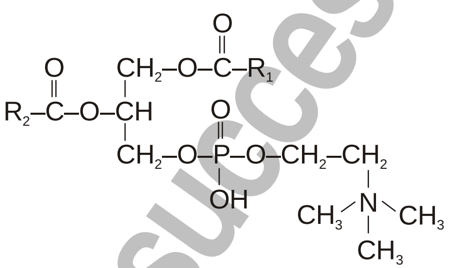
2. Waxes :
- Waxes are highly insoluble esters of long-chain. Monohydroxy alcohol.
- **Waxes are esters of fatty acids of high molecular weight with alcohol except glycerol.**
- Waxes have a higher melting point than neutral fats.

- Sebaceous glands of mammalian skin secrete waxy sebum which acts as a lubricant to keep the hair and skin soft.
- The sebum secreted by skin glands of wooly mammals is commonly called lanolin or "wool oil".
- Ceruminous and sebaceous glands of the skin that covers the external ear passage in mammals secrete ear wax.
- Spermaceti, a wax found in skull of whale, dolphin.
- Honey bees constructed their beehives with bees wax (myricyl palmitate) secreted by their abdominal glands.
- **Paraffin wax is obtained from petroleum. Candles are made of paraffin wax and stearic acid.**

(ii) Compound lipids :
- The compound lipids contain fatty acid, alcohols and other compound as phosphorus, aminonitrogen, carbohydrates.
- Compound lipids are of following types-

1. Phospholipids (Fatty acids + Glycerol + H_3PO_4 + Other compound) :
- These are triglyceride lipids in which one fatty acid is replaced by a phosphate group.
- Some phospholipids also have a nitrogenous compound such as choline (in lecithin), ethanolamine (in cephalin).
- **Phospholipids are amphipathic carrying both hydrophillic (water attracting) polar and hydrophobic (water repellant) non-polar groups because of this property they form bilayers.**
- In aqueous medium, phospholipid molecules arrange in a double layered membrane or lipid bilayer.
- Few examples of phospholipids are-

(a) Lecithin : It is formed by one molecule of glycerol, two molecules of fatty acids and one molecule of phosphoric acid. Choline is attached with phosphoric acid. Lecithin is found in yolk, brain, soyabean membrane. It also acts as lipid carrier in blood.



Phospholipid (Lecithin)

(b) Cephalin : It is like lecithine but choline is replaced by amino ethyl alcohol(Ethanol amine)
- Cephalin is found in yolk, blood platelets and Nerve tissues.

(c) Plasmalogens : Occur in vertebrate cardiac muscles, ciliate protists and certian cells of invertebrates.
- The plasmalogens is platelet-activating factor (PAF) which is released from basophils (WBC, in Vertebrates) to stimulate the blood platelets.

(d) Sphingolipid : It is similar to lecithin but have sphingosin in place of glycerol. Sphingomylins are important as myelin sheath.

2. Glycolipids :
- Glycolipids contain fatty acids, and amino alcohol and one or more simple sugars.
- The glycolipids are components of cell membranes, myelin sheath of nerve fibres and membrane of chloroplasts.
- Animal cells contain cerebrosides and gangliosides.

(a) Cerebrosides : are important lipids of white matter of cells of brain and myelin sheath of the nerve fibres.
(b) Gangliosides : are found in grey matter of the brain, membraneof RBCs, spleen. They also have neuraminic acid.

3. Lipoproteins :
- Lipoprotein are composed of lipids (mainly phospholipids) and proteins. They are present in the blood, milk and egg yolk.

4. Cutin and suberin :
- Cutin is a complex lipid found in plant cell walls and cuticle. It binds epidermal cells and reduces transpiration.

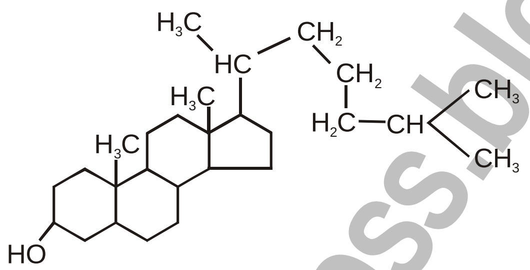
- Suberin is a mixture of fatty substances. It is present in the wall of cork cells and endodermal cells. It makes cell wall strong and impermeable to water.

(iii) Derived lipids :

- These lipids are obtained by the hydrolysis of simple or compound lipids.
- Although these are the products of hydrolysis of lipids, but even then they have some properties of lipids.
- Derived lipids are following types.

1. Steroids (sterols) :

- Important steroids which have ketone group are called as sterone eg. Testosterone, sterols have – OH group eg. cholesterol.
- Sterols are lipids of high molecular weight e.g., cholesterol, ergasterol, stigmasterol, campesterol etc.
- Cholesterol ($C_{27}H_{45}OH$) is the common sterol found in many animals, human beings and some plants e.g, Potato.
- **Cholesterol and its esters are insoluble in water.**
- Some steroides are hormones like progesterone, estrogen, testosterone and carticosterone.
- **Cholesterol forms vitamin D on exposure to ultraviolet rays.**
- Cholesterol in an sessential comonent of animal cell membrane and the cell membrane of mycoplasmas.
- **Ergosterol and stigmosterol and found in plants, fungi.**



STRUCTURE OF CHOLESTEROL

3. Terpenes :

- Terpenes are lipid like hydrocarbons formed of isoprene (C_5H_8) units.
- They are major components of 'essential oils' produced by certain plants.
- The terpenes include certain fat soluble vitamins like A, E, K, carotenoids and certain coenzymes like coenzyme Q.

4. Prostaglandins :

- Prostaglandins are hormone like compound derived from 20 carbon polysaturated fatty acids such as arachidonic acid.
- They are present in human seminal fluid, testis, kidney, placenta, uterus, stomach, lung, brain and heart.
- The main function of prostaglandins is binding of hormones to membranes of target cells. Regulation of B.P. gonadial peristalisis and osmoregulation.

Function of lipids :

1. Food material :

- Lipids provide food, highly rich in calorific value. One gram lipid produces 9.3 kilocalories of heat.

2. Food reserve :

- Lipids are insoluble in aqueous solutions, and therefore can be stored readily in the body as a food reserve.

3. Structural components :

- They make an impotant consitituent of the cell membrane.

4. Heat insulation :

- The fats are characterised for their high insulating capacity. Great quantities of fat are deposited in the sub-cutaneous layers in aquatic mammals such as whale and in animals living in cold climates.

5. Fatty acid absorption :

- Phospholipids play in important role in the absorption and transportation of fatty acids & fat soluble vitamins.

6. Hormone synthesis :

- The sex hormones, adrenocorticoids, cholic acids and also vitamin D are all synthesised from cholesterol, a steroidal lipid.

7. Vitamin carriers :

- Lipids act as carriers of natural fat-soluble vitamin such as vatamin A, D and E.

EXERCISE

1. Cellulose form a major portion of food grazing cattle, it is -
(A) Digested by bacteria of almentary canal
(B) Digested by animal itself.
(C) Digested partly by the animal and partly by bacteria
(D) Passed out undigested

2. Glycoprotein are known to play important role in cell recognition, the specificity of this recognition is provided largely by-
(A) Protein portion of the glycoproteins
(B) Carbohydrate portion of the glycoproteins
(C) Both carbohydrate and protein portion of glycoproteins
(D) Lipid portion of glycoprotein

3. The first amino acids taking part in protein synthesis -
(A) Met (B) Val (C) Arg (D) Tryp.

4. Dipetide is-
(A) Structure of two peptide bonds
(B) Two amino acids linked by one peptide bond
(C) Bond between one amino acid and one peptide
(D) None

5. Decreasing order of amount of organic compound in animal body -
(A) Carbohydrate, Protein, Fat and Nucleic acid
(B) Protein, Fats, Nucleic acid and Carbohydrate and nucleic acid
(C) Protein, Fats, Carbohydrate and Nucleic acid
(D) Carbohydrate, Fats, Proteins and Nucleic acid

6. Characteristic feature of haemoglobin -
(A) Reversible union with oxygen
(B) Red colour
(C) Presence of Cu
(D) Presence of globulin protein

7. Correct about glycogen is that it is -
(A) A disaccharide stored in liver and can react with NH_3 to form proteins
(B) Synthesized in liver and takes part in formation of bile and lipase besides being a source of energy
(C) A polysaccharide synthesized and stored in liver cells
(D) Synthesized in blood and stored in liver and muscles to provide glucose in times to need

8. Cotton fibres are made of -
(A) Protiens (B) Starch (C) Lignin (D) Cellulose

9. Most diverse macromolecules found in cells, both physically and chemically are -
(A) Lipids (B) Carbohydrates (C) Proteins (D) Nucleic acid

10. Cellulose is made up of -
(A) Unbranched chain of glucose molecules linked by β 1-4 glycosidic bonds
(B) Branched chains of glucose molecules lined by α 1-6glycosidic bond in straight chains and α 1-6 glycosidic bonds at branch points
(C) Unbranched chains of glucose molecules lined by α 1-6 glycosidic bonds
(D) Branched chains of glucose molecules linked by α 1-6 glycosidic bonds at branch points

11. Lactose is composed of -
(A) Glucose + galactose (B) Glucose + glucose
(C) glucose + fructose (D) fructose + fructose

12. Enormous diversity of protein molecules is mainly due to diversity of-
(A) Peptide bonds (B) Amino groups of amino acids
(C) R groups of amino acids (D) Amino acid sequences within the protien molecules

13. The antibodies are -
(A) γ (Gamma) - globulins (B) Albumines
(C) Vitamins (D) Sugar

14. Which is not an essential macro-element for growth of plants-
(A) Na (B) Ca (C) Zn (D) K

15. Most abundant molecule of Nature is-
(A) Glucose (B) Cellulose (C) Starch (D) Ribose

16. Identify the protein which does not contain any metal -
(A) Phytochrome (B) Cytochrome (C) Glycoprotein (D) Ferritin

17. Which one is storage protein -
(A) Ferroprotein (B) Chromoprotein (C) Mucoprotein (D) Glutelin

18. Prokaryotic genetic material contain -
(A) neither DNA nor histones (B) DNA but no histones
(C) Either DNA or histones (D) Both DNA and histones

19. Which protein found in maximum amount-
(A) Catalase (B) Zinc carbonic anhydrase
(C) Transferase (D) RUBISCO

20. Living substance of cells was called "sarcode" by -
(A) Robert Brown (B) Dujardin (C) Robert Hooke (D) Purkinje

21. Saline solution is given to patients of cholera because -
(A) Na^+ prevents water loss from body
(B) NaCl function as regulatory material
(C) NaCl produces energy
(D) NaCl is antibacterial

22. A nutrient which is used to build protoplasm is-
(A) Fat (B) Protein (C) Carbohydrate (D) Calcium

23. Two elements needed for building healthy teeth and bones are :
(A) iron and calcium (B) phosphorous and iron
(C) calcium and phosphorous (D) CO_2 and H_2O

24. If all the peptide bonds of protein are broken, then the remaining part is :
(A) amide (B) oligosaccharide (C) polypeptide (D) amino acid

25. Which of the following is a disaccharide?
(A) Glucose (B) Fructose (C) Sucrose (D) Galactose

26. The chemical formula of starch is :
(A) $(C_6H_{10}O_5)n$ (B) $(C_6H_{12}O_6)n$ (C) $C_{12}H_{12}O_{11}$ (D) CH_3COOH

27. Which elements is located at the centre of the porphyrin ring in chlorophyll-
(A) Calcium (B) Magnesium (C) Potassium (D) Mangnese

28. The major role of minor elements inside living organisms is to acts as-
(A) Co-factors of enzymes (B) Builiding blocks of important amino acids
(C) Constituent of hormones (D) Binder of cell structure

29. In protoplasm fat store is in the form of-
(A) Polypeptide (B) Triglyceride
(C) Polysaccharide (D) Nucleosides

30. Antibody is formed by-
(A) Protein (B) Carbohydrate
(C) Nucleic acid (D) Lipid

31. Which sugar is present in nucleic acid-
(A) Pentose (B) Hexose (C) Fructose (D) Glucose

32. Major cause of anaemia-
(A) Deficiency of Ca (B) Deficiency of Fe
(C) Deficiency of Na (D) Deficiency of Mg

33. Deficiency of copper causes-
(A) Pellagra (B) Anemia and damage to CNS
(C) Influenza (D) Xerophthalmia

34. Cytochromes are-
(A) Riboflavin nucleotides (B) Pyrimidine nucleotides
(C) Iron porphyrin proteins (D) Flavoproteins

35. Starch and cellulose are the compounds made up of many units of
(A) Simple sugar (B) Fatty acid (C) Glycerol (D) Amino acid

36. Which of the following is the characteristic of plants
(A) Glucose and cellulose (B) Pyruvic acid and glucose
(C) Cellulose and starch (D) Starch and pyruvic acid

37. Inulin found in plant cell is a
(A) Lipid (B) Protein (C) Polysaccharide (D) Vitamin

38. Which of the following structure is not common in all protein
(A) Primary structure (B) Secondary structure (C) Tertiary structure (D) Quarternary structure

39. Oval shaped and eccentric starch particles are found in
(A) Wheat (B) Maize (C) Potato (D) Rice

40. What are the most diversed molecules in the cell
(A) Lipids (B) Proteins (C) Carbohydrates (D) Mineral salts

41. No cell could live without
(A) Phytochrome (B) Enzymes (C) Chloroplasts (D) Protein

42. Lipids are insoluble in water, because lipids molecules are
(A) Neutral (B) Zwitter ions (C) Hydrophobic (D) Hydrophilic

43. α-helical model of protein was discovered by
(A) Pauling and Correy (B) Watson (C) Morgan (D) Berzelus

44. Aleurone grains are
(A) Enzymes (B) Carbohydrates (C) Protein (D) Fat

45. High content of lysine is present in
(A) Wheat (B) Apple (C) Maize (D) Banana

46. Arachidonic acid is
(A) Non-essential fatty acid (B) Essential fatty acid
(C) Polyunsaturated fatty acid (D) Both (B) and (C)

47. Maltose consists of which one of the following
(A) β - glucose and β - galactose (B) α - glucose and α - fructose
(C) α - sucrose nad β - glucose (D) Glucose and glucose

48. Paraffin wax is
(A) Ester (B) Acid (C) Monohydric alcohol (D) Cholesterol

49. Match the items in column I with items in column II and choose the correct answer

Column I		Column II	
A	Triglyceride	1	Animal hormones
B	Membrane lipid	2	feathers and leaves
C	Steroid	3	phospolipids
D	Wax	4	fat stored in form of droplets

(A) A - 4, B - 3, C - 1, D - 2 (B) A - 2, B - 3, C - 4, D - 1
(C) A - 3, B - 4, C - 1, D - 2 (D) A - 4, B - 1, C - 2, D - 3

50. In which one of the following groups, all the three are examples of polysaccharides
(A) Starch, glycogen, cellulose (B) Sucrose, maltose, glucose
(C) Glucose, fructose, lactose (D) Galactose, starch, sucrose

51. Given below is the chemical formula of

$$CH_3(CH_2)_{14}-\overset{\overset{\large O}{\|}}{C}-OH$$

(A) Palmitic acid (B) Stearic acid (C) Glycerol (D) Galactose

52. Select the wrong statement
(A) The building blocks of lipids are amino acids
(B) Majority of enzymes contain a non-protein part called the prosthetic group
(C) The thylakoids are arranged one above the other likea stack of coins forming a granum
(D) Crossing-over occurs at pachytene stage of meiosis I

53. What does the following equation denote? Amino acid + ATP $\rightarrow$ Aminoacyl AMP + PP
(A) Elongation of chain (B) chain termination
(C) Activation of amino acid (D) None of these

54. Which of the following fats is least harmful for heart
(A) Saturated fat (B) Cholesterol (C) Polyunsaturated fat (D) Oils

55. Quarternary structure of protein
(A) Consists of four subunits
(B) May be either α or β
(C) Is unrelated to two function of the protein
(D) Is dictated by the primary structures of the individual subunits

56. The "repeating unit" of glycogen is
(A) Fructose (B) Mannose (C) Glucose (D) Galactose

57. Cellulose, the most important constituent of plant cell wall is made of
(A) Unbranched chain of glucose molecules linked by α 1, 4 glycosidic bond
(B) Branched chain of glucose molecules linked by β 1, 4 glycosidic bond in straight chain and α 1, 6 glycosidic bond at the site of branching
(C) Unbranched chain of glucose molecules linked by β 1, 4 glycosidic bond
(D) Branched chain of glucose molecules linked by α 1, 6 glycosidic bond at the site of branching

58. Conjugated proteins containing carbohydrates as prosthetic group are known as
(A) Chromoproteins (B) Glycoproteins (C) Lipoproteins (D) Nucleoproteins

59. Which is an essential amino acid
(A) Serine (B) Aspartic acid (C) Glycine (D) Phenylalanine

60. Most abundant organic compound on earth is
(A) Protein (B) Cellulose (C) Lipids (D) Steroids

61. Spoilage of oil can be detected by which fatty acid
(A) Oleic acid (B) Linolenic acid (C) Linoleic acid (D) Erucic acid

62. Cytochrome is
(A) Metallo flavoprotein
(B) Fe containing porphyrin pigment
(C) Glycoprotein
(D) Lipid

63. Element necessary for the middle lamella
(A) Ca (B) Zn (C) K (D) Cu

64. In plants, inulin and pectin are
(A) Reserve materials (B) wastes (C) Excretory material (D) Insect-attracting material

65. Collagen is
(A) Fibrous protein (B) Globular protein (C) Lipid (D) Carbohydrate

66. Lipids are insoluble in water because lipid molecules are
(A) Hydrophilic (B) Hydrophobic (C) Neutral (D) Zwitter ions

67. Which of the following is a reducing sugar
(A) Galactose (B) Gluconic acid (C) β-methyl galactoside (D) Sucrose

68. Which steroid is used for transformation
(A) Cortisol (B) Cholesterol (C) Testosterone (D) Progesterone

69. The major portion of the dry weight of plants comprises of
(A) Carbon, nitrogen and hydrogen
(B) Carbon, hydrogen and oxygen
(C) Nitrogen, phosphorus and potassium
(D) Calcium, magnesium and sulphur

70. During anaerobic digestion of organic waste, such as in producing biogas, which one of the following is left undergraded
(A) Hemicellulose (B) Lipids (C) Cellulose (D) Lignin

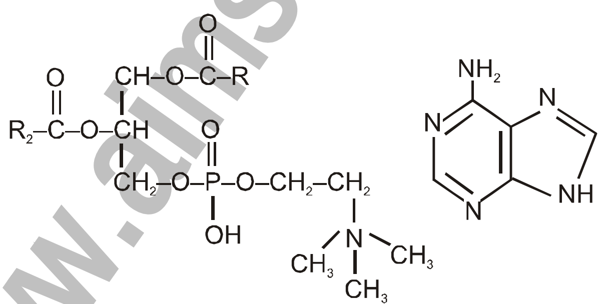
71. Which of the following is the simplest amino acid
(A) Alanine (B) Asparagine (C) Glycine (D) Tyrosine

72. Antibodies in our body are complex
(A) Steroids (B) prostaglandins (C) Glycoproteins (D) Lipoproteins

73. Which one of the following is not a constituent of cell membrane
(A) Cholesterol (B) Glycolipids (C) Proline (D) Phospholipids

74. About 98 percent of the mass of every living organism is composed of just six elements including carbone, hydrogen, nitrogen, oxygen and.
(A) Phosphorus and sulphur
(B) Sulphur and magnesium
(C) magnesium and sodium
(D) Calcium and phosphorus

75. Which one of the following structural formulae of two organic compounds is correctly identified along with its related function ?



(A) B : Adenine - a nucleotide that makes up nucleic acids
(B) A : Triglyceride - major source of energy
(C) B : Uracil - a component of DNA
(D) A : Lecithin - a component of cell membrane

ANSWERS

1.	(A)	**2.**	(B)	**3.**	(A)	**4.**	(B)	**5.**	(C)	**6.**	(A)	**7.**	(C)
8.	(D)	**9.**	(C)	**10.**	(A)	**11.**	(A)	**12.**	(D)	**13.**	(A)	**14.**	(A)
15.	(B)	**16.**	(C)	**17.**	(D)	**18.**	(B)	**19.**	(D)	**20.**	(B)	**21.**	(A)
22.	(B)	**23.**	(C)	**24.**	(D)	**25.**	(C)	**26.**	(A)	**27.**	(B)	**28.**	(A)
29.	(B)	**30.**	(A)	**31.**	(A)	**32.**	(B)	**33.**	(B)	**34.**	(C)	**35.**	(A)
36.	(C)	**37.**	(C)	**38.**	(D)	**39.**	(C)	**40.**	(B)	**41.**	(D)	**42.**	(C)
43.	(A)	**44.**	(C)	**45.**	(A)	**46.**	(B)	**47.**	(D)	**48.**	(A)	**49.**	(A)
50.	(A)	**51.**	(A)	**52.**	(A)	**53.**	(C)	**54.**	(C)	**55.**	(D)	**56.**	(C)
57.	(C)	**58.**	(B)	**59.**	(D)	**60.**	(B)	**61.**	(D)	**62.**	(B)	**63.**	(A)
64.	(A)	**65.**	(A)	**66.**	(B)	**67.**	(A)	**68.**	(B)	**69.**	(B)	**70.**	(D)
71.	(C)	**72.**	(C)	**73.**	(C)	**74.**	(A)	**75.**	(D)				

1. FAMILY SOLANACEAE

Classification

Kingdom	**–**	**Plantae**
Sub-Kingdom	**–**	**Phanerogamia**
Division	**–**	**Angiospermae**
Class	**–**	**Dicotyledonae**
Sub-Class	**–**	**Gamopetalae**
Series	**–**	**Bicarpellatae**
Order	**–**	**Polymoniales**
Family	**–**	**Solanaceae**

Distribution :
90 genera and 2000 species include in this family. Plants are mostly found in Tropical and temperate region.

Habit :
Plants are mostly **Annual or perennial herbs. Ex : Nicotiana tabacum, solanum nigrum or shrubs Ex: Cestrum nocturnum, trees and climbers are rare.**

Root :
Tap root system.

Stem :
Herbaceous or woody, Erect, **hairy or Prickly Ex: Solanum xanthocarpum. In some plants it is modified into underground tubers Ex: Solanum tuberosum.**

Leaf :
Simple, petiolated, exstipulate, Alternate, entire margin or dessected margin, unicostate reticulate venation.
Exception : In **tomato (Lycopersicum) leaves are pinnately compound** in which the tip of the main axis terminates in a flower.

Inflorescence :
(a) Monochasial scorpioid Cyme - Ex: Atropa belladona

(b) Monochasial Helicoid Cyme - Ex: Solanum nigrum

Exceptions :
(i) Solitary terminal - Ex: Datura

(ii) Solitary axillary - Ex: Physalis

Flower :
Pedicellate, Ebracteate, bisexual, Actinomorphic, complete, hypogynous, Pentamerous.
Exception : Flower Zygomorphic - Ex: Hyocymous, schizanthus and salpiglossis.

Calyx :
5, Gamosepalous aestivation valvate, **Persistent- (a) Accrescent- enlarged baloon like on present fruit Ex: physalis, Withania, (b) Marcescent-dry & hard calyx present on fruit Ex: S.melongena,** odd sepal posterior.
Exception : Spiny : Ex: S. xanthocarpum.

Corolla :
5, Gamopetalous, valvate or imbricate aestivation, **bell shaped or infundibulum.**
Exception Bilipped in schizanthus.

Androecium :
5, Polyandrous, **Epipetalous**, Anther dithecous, basifixed, introse.
Exception :
(i) 4 stamens are found in Salpiglossis in which two long and two short (didynamous stage) staamensare faound.

(ii) 2 stamens are present in schizanthus.

Gynoecium :
Bicarpellary, syncarpous. Bilocular, **Axile placentation. Placenta is swollen, ovary situated on the thalamus obliquely. It is multilocular in datura and tomate due to the formation of false septum.**

Fruit :
Berry : Single fleshy and non-dehiscent **Ex : Tomato, Brinjal, Chillies, physalis.**

Seed :
Endospermic, embryo erect or curved.

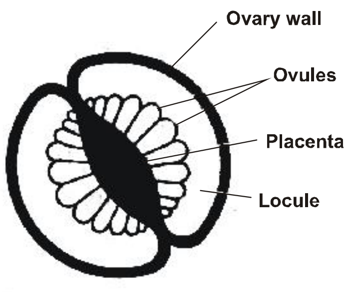
Pollination :
Normally Entomophily

Exception :
(i) Solanum tuberosum-self pollination

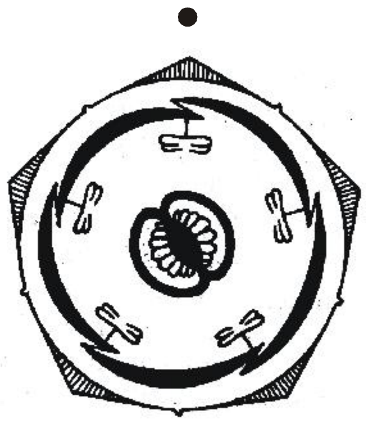
(ii) Salpiglossis-cleistogamous pollination.

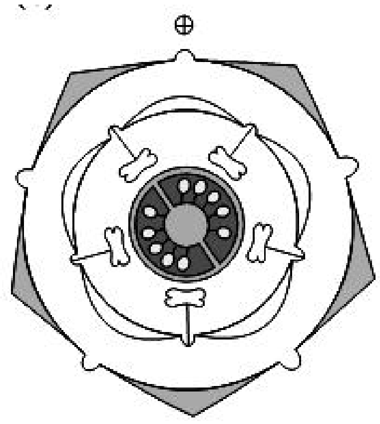
Floral formula : Br. $\oplus$ ⚥ $K_{(5)}\ \overset{\frown}{C_{(5)}\ A_5}\ \underline{G}_{(2)}$

Floral diagr



T.S. of ovary


Floral diagram



Note : Above floral diagram of NCERT text book is not completely correct because in the calyx part - odd sepal anterior condition has been indicated but in Solanaceae the odd sepal is posterior that lies near the mother axis.

Economic Importance :

(1) Food :

(i) Potato (Solanum tuberosum) : Edible part is starchy tuber

(ii) Tomato (Lycopersicum esculantum) : Frurits are useful for manufacturing chutany & sauce

(iii) Brinjal or Eggplant (Solanum melongena) : Fruits are used as vegetable Edible part is pericarp and placenta.

(iv) Chillies (Capsicum annum) : Fruits used as condiments

(v) Capsicum frutescence : Fruits used as Vegetables

(vi) Gooseberry : (Physalis peruvianum) : Fruits are edible

2. Medicines :

(i) Aswagandha (Withania somnifera) : Roots yield a nerve tonic. It is used in manufacturing aurvedic tonic aswagandharishtra. Aswagandha is also used in the treatment of rheumatism, hiccap, cough, fever etc.

(ii) Atropa balladona : Medicine Belladona is obtained from roots. It is used in pain, as a sedative Atropine is used for the expansion of eyepupil before eye test. It is also used as opium antidote.

(iii) Henbane (Hyocymus niger) : Medicine Henbane obtains from dried leaves and flowers which contain scopolamine and Hyocyamine alkaloid It is used in treatment of asthma, whooping cough, and as pain killer.

(iv) Datura (Datura stramonium, D. metal or thorn apple) : Stramonium medicine obtains from leaves and flowers which contain atropine, Hyocyamine and scopolamine alkaloids. stramonium is used in the treatment of parkinsonia and asthma while scopolamine is used as sedative and pain killer.

(v) Tobacco (Nicotiana tabacum) : Nicotine and anabasin alkaloids obtain from its leaves. It is used as sedative, Antispasmodic and vermifuge. Tobacco is poisonous. Former affects CNS (central nervous system) Tobacco alkaloid anabasine & nicotine are narcotic, cause cancer of mouth, lungs and heart disease.

3. Ornamental Plants :

(i) Lady of the night (Night jasmine) - Cestrum nocturnum

(ii) Day jasmine - Cestrum diurnum

(iii) Pitunia hybrida, P. alba

(iv) Yesterday - Today tomorrow - Brunfelsia hopeana

(v) Butterfly flower - Schizanthus sps

(vi) Jasmine - Solanum Jasminoides

(vii) Salpiglossis

2. FAMILY - LILIACEAE

Classification

kingdom	**–**	**Plantae**
Sub-kingdom	**–**	**phanerogamia**
Divison	**–**	**Angiospermae**
Class	**–**	**Monocotyledonae**
Series	**–**	**Coronarieae**
Family	**–**	**Liliaceae**

Distribution :

250 genera and 4000 species, cosmopolitan distribution.

Habit :

Normal **Parennial herbs Ex: Asphodelus** some are **shrubs Ex: Dracaena, some are climbers Ex: Smilax, Gloriosa, some are trees Ex: Yucca.**

Root :

Usually adventitous roots. **Fasciculated or tuberous roots are found in Asparagus.**

Stem :

Aerial or under ground. Food stores in the underground stem like.

1. Bulb : Ex: Onion and Garlic

2. Corm : Ex: Colchicum automnale

Modification of aerial stem
(i) Cladode : Ex: Asparagus
Exception : Abnormal secondary growth occur in Dracaena and Yucca.

Leaves :

Cauline, Radical (leaves locate on under ground stem) **Ex: Asphodelus,** Stipulate sessile, various types of phyllotaxy (alternate, opposite or whorled), parallel venation.

Exception :
(i) Reticulate venation - Ex : Smilax.

(ii) In Gloriosa leaf tips are modified in to tendrils while in smilax stipules are converted in to tendrils.

(iii) In Ruscus, leaves are modified into scales.

(iv) In Asparagus - leaves are converted into spines.

Inflorescence :

Usually racemose or **solitary axillary or terminal, penicle in Dracaena, Yucca, Spadix in Aloe** While **scapigerous umbel** in **onion (Allium sepa)** in which Inflorescence come out from under ground stem and clusters of flowers develop on the tip as umbel but it is actually scorpioid cyme instead of umbel.

Flower :

Complete, Bracteate, **Trimerous**, Actinomorphic, Hypogynous.

Exception :

(i) Zygomorphic flower - Ex: Lilium.

(ii) Unisexual and Incomplete flower - Ex: Smilax and Ruscus.

(iii) Tetramerous flower - Ex: Paris, Aspidiastra.

Parianth :

6 Tepals, in two whorls **–3 + 3 Polyphyllous**, Imbricate, odd tepal anterior.

Androecium : 6 Stamens arrange in two whorls **3 + 3**, Polyandrous, **Epiphyllous**, Anther Dithecous, Basifixed or Versatile, Introse.

Exception :

(i) In paris 8 Stamens in two whorl, 4 + 4.

(ii) In Ruscus only 3 stamens of inner whorl are present.

Gynoecium :

Tricarpellary, Syncarpous, **Axile placentation**, Trilocular, Stigma trifid. **In onion gynobasic style is present.**

Fruit : Berry - Ex: Lilly or Capsule Ex: Onion.

Seed : Endospermic.

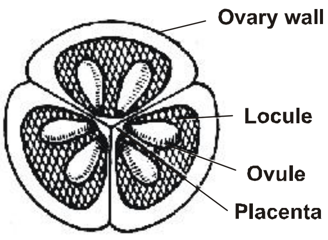
Pollination :

Entomophilous, **Pollination through a specific insect Ex : By Pronuba yuccasela in Yucca**.

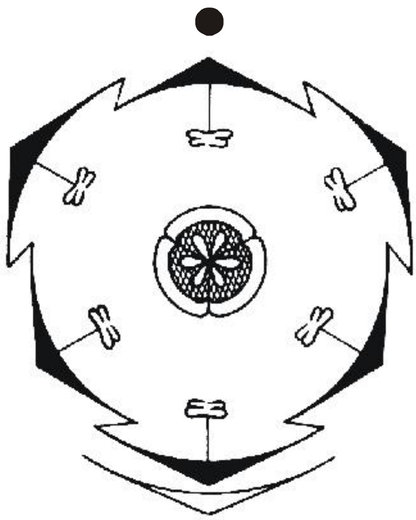
Floral - formula : Br. $\oplus$ ⚥ $\widehat{P_{(3+3)}A_{3+3}}\ \underline{G}_{(3)}$

Note : In NCERT text book, in the floral formula (Br $\oplus$ ⚥ **$P_{(3+3)}$ A_{3+3} $\underline{G}_{(3)}$) epiphyllous condition ($\widehat{P_{(3+3)}A_{3+3}}$) is not mentioned although it is mentioned in floral diagram**

Floral diagram :



T.S. of ovary


floral diagram

Economic Importance :

(1) Food :

(i) Onion - (Allium cepa) : Edible part is bulbs and fleshy leaves.

(ii) Garlic - (Allium sativum) : Bulbs.

(iii) Asparagus officinalis : Fasciculated roots.

(2) Medicines :

(i) Smilax zeylanica : Roots yield sarsperilla like drug for purifying blood, piles, leprosy, gonorrhoea.

(ii) Allium sativum : Useful in Heart disease and rheumatism.

(iii) Asphodelus tenuifolius : Its seed are useful in the treatment of ulcer and swelling.

(iv) Crinum asiaticum : Extract of leaves is used in rheumatism and ear pain.

(v) Fritillaria cirrhosa : Drieds bulbs useful in Tuberculosis and Asthma.

(3) Other uses :

(i) Colchicum luteum and C. automnale : Colchicine obtain from roots which is used in experiment of induce polyploidy.

(ii) Indian bow string hemp (Sansvieria roxburghiana) : Fibres obtain from leaves which are useful for making rops, nets, etc.

(iii) Dragon's blood plant (Dracaena draca) : Red coloured resin obtain from its stem. It is also called Dragon blood. Metals are polished by this resin.

(iv) Phormium tenax : Its oil is used in making paints & Varnish.

(4) Ornamental plants :

(i) Mother's in Law's tongue	–	**Sansivieria trifasciata**
(ii) Drager plant	–	**Yucca alolifolia**
(iii) Glory lily	–	**Gloriosa superba**
(iv) Asparagus fern	–	**Asparagus plumosus**
(v) Tulips	–	**Tulipa gesneriana**
(iv) Lily	–	**Lilium bulbiferum**
(vii) Butcher's boom	–	**Ruscus aculeatus**
(viii) Satavar	–	**Asparagus officinale**

3. FAMILY LEGUMINOSAE

Classification :

Kingdom	**–**	**Plantae**
Sub - Kingdom	**–**	**Phenerogamia**
Division	**–**	**Angiospermae**
Class	**–**	**Dicotyledonae**
Sub - class	**–**	**Polypetalae**
Series	**–**	**Calyciflorae**
Order	**–**	**Rosales**
Family	**–**	**Leguminosae**

It is also called Legume family this is the second largest family of Dicots. **Leguminosae is divided into three sub-families on the basis of variations in coralla, Androecium and other parts**. These sub families are as follows.

(I) Papilionatae **(II) Caesalpinoideae** **(III) Mimosoideae**

(I) PAPILIONATAE (FABACEAE) :

Distribution :

It includes about **400 genera and 12000 species, cosmopolitan distribution.** According to ICBN (International Code of Botanical Nomenclature) its new name is **'Lotoideae'.** Most of the plants of this sub family are found in Tropical and sub-tropical areas.

Habit :

Usually Annual or parennial herbs, shrubs, some are **Tendril climbers like Pisum sativum, Lathyrus odorotus,** some are **Twiners like Clitoria** and some are **trees like Delbergia sisso**.

Roots :

Tap root system, Many plants have nodules on secondary roots. **Nitrogen fixing bacteria-Rhizobium lie in the root nodules in the symbiotic form**.

Stem :

Erect, Herbaceous or woody, cylindrical, branched, solid, some are **twiners like Dolichus lab lab**.

Leaf :

Stipulate, Alternate, Unipinnately compound and imparipinnate, **Pulvinous leaf base**, ReticulateVenation.
In Pisum sativum and Lathyrus odoratus, upper leaf lets are modified into tendrils.
Exception : Palmately compound leaves, **Ex: Trifolium and Melilotus**.

Inflorescence : Usually Raceme or Solitary axillary, **Ex: Lathyrus aphaca.**

Flower :

Bracteate, bracteolate, Pedicellate, bisexual, Zygomorphic, Pentamerous, **Perigynous**.

Calyx :

5, Gamosepalous, Valvate or imbrcate aestivation, odd sepal anterior.

Corolla :

5, Polypetalous, **Descending imbricate aestivation** in which the posterior large bilobed petal called **vexillum or standard** overlaps the two smaller lateral petals named **wings or alae**. The latter overlaps the two small anterior petals which are fused lightly by the upper anterior margins called **keel or carina**. This type of coralla is also called **Papilionaceous corolla**.
Exception : Petals absent in Lespedeza.

Androecium :

10 stamens, **Diadelphous** in which filaments of 9 stamens are fused while one stamen is free, Anther dithecous, Dorsifixed, Introse, Inserted.

Exception :
(i) 9 Monadelphous stamens are found in Arachis hypogea and Delbergia sisso.

(ii) 10 free or polyandrous stamens present in Sophora.

(iii) 10 monadelphous stamens present in Crotolaria and Pongamia.

Gynoecium :

Monocarpellary, Unilocular, **Marginal Placentation**, style one.

Fruit :

Legume or pod which is single, dry, dehiscent fruit.
Exception : Lomentum in Arachis and Desmodium.

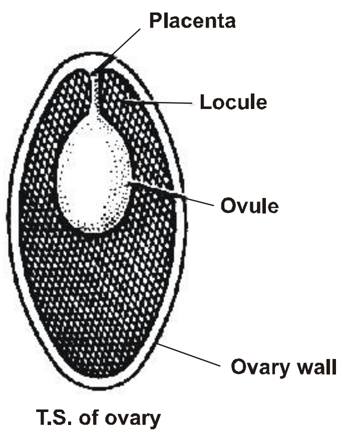
Seed : **Non-endospermic**.

Pollination :

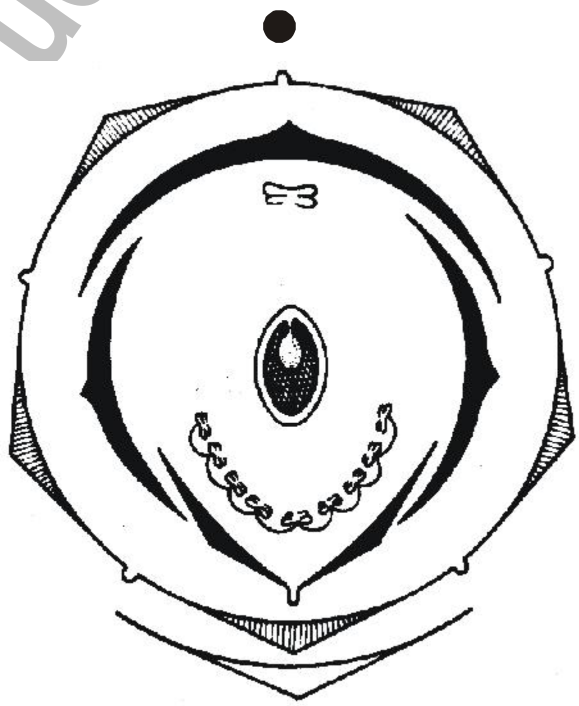
Entamophilly but self pollination occurs in Pisum sativum.

Floral formula : **Br. % ⚥** $K_{(5)}\ C_{1+2+(2)}\ A_{1+(9)}\ G_{\underline{1}}$

Floral diagram :



T.S. of ovary


floral diagram

Economic Importance :

(1) Pulses :

(i)	**Green gram**	**–**	**Phaseolus radiatus syn. vigna radiata**
(ii)	**Gram**	**–**	**Cicer aurietinum**
(iii)	**Pea**	**–**	**Pisum sativum**
(iv)	**Lentils**	**–**	**Lens culinaris**
(v)	**Pegeon pea**	**–**	**Cajanus cajan**
(vi)	**Cow pea**	**–**	**Vigna unguiculata**
(vii)	**soyabean**	**–**	**Glycin max**
(viii)	**French bean**	**–**	**Phaseolus vulgaris**
(ix)	**Sew gram**	**–**	**Vigna aconitifolia**
(x)	**Black gram**	**–**	**Phaseolus mungo syn Vigna munga**

Point of Remember

Khaseri dal (Lathyrus sativus) : The use of this pulse causes 'lathyrism disease'.

(2) Vegetables :

(i)	**Fenugreek**	**–**	**Trigonella foenum-graceum**
(ii)	**Lablab**	**–**	**Dolicus lablab**
(iii)	**Cluster bean**	**–**	**Cyamopsis tetragonoloba**
(iv)	**Cow pea**	**–**	**Vigna unguiculata**
(v)	**Bakala**	**–**	**Vicia Laba**

(3) Oils :

(i) Ground nut (Arachis hypogea) : Oil is obtained from seeds and used for cooking and manufacture of vegetable ghee. Oil cake is used for cattle and as bio-fertilizer.

(ii) Soyabean (Glycin max) : Oil is obtained from seeds, used for cooking, and manufacturing for ink of printing, Paints, Insectides. Oil cake contain 32-42% proteins which are called soya nagates.

(iii) Pongamia pinnatta : Oil from seeds used for manufacturing soap and lubricants.

(4) Fibres :

(i) Sun hemp (Crotolaria juncea) : Bast fibres obtain from stem which are useful in manufacturing Rops, cannvas, sacs.

(ii) Dhaincha (Iesbania cannabinus) :

(5) Fodder :

(i) Egyptean clover or Barseem - Trifolium alexandrium.

(ii) Alfa alfa - (Medicago sativa).

(iii) Indian Clover or senji - Melilotus alba.

(iv) Sweet clover or van methi - M.indica.

(v) Subabool - Leucaena leucocephala.

(6) Dyes :

(i) Indigo or neel (Indigophora tinctoria) : Blue dye obtains from leaves and young branches.

(ii) Dak (Butea monosperma) : Yellow orange dye obtains from petals of flowers. It is also called **"Flame of Forest".**

(iii) Red sandel (Pterocarpous santalinus) : Red dye extracted from heart wood.

(7) Timber :

(i) Shisham - Dalbergia sisso
(ii) Indian rose wood (kala shisham)- Dalbergia latifolia
(iii) Indian Kino tree - Pterocarpus marsupium.
(iv) African black wood - Dalbergia melanoxylon.

(8) Gum :

(i) Tragacanth (Astragalus gummifer) : Tragacanth gum is useful in confectionary, textile industry and cosmetics.

(ii) Cluster bean (Cyamopsis tetragonoloba) : Guargum obtains from seeds, used in textiles, paper industry and cosmetics.

(iii) Bengal kino (Butea) : Gum has medicinal use.

(9) Insecticides :

(i) Deris elliptica : "Rotenone" Insecticide is obtained.

(10) Ornamental plants :

(i) Phoolmantar or sweet pea - Lathyrus odoratus

(ii) Indian telegraph plant - Desmodium gyrans

(iii) Japanees pagoda tree - Sophora japonica

(iv) Indian coral tree - Erythrima undica

(v) Butterfly pea - Clitoria termata

(vi) Flame of Forest - Butea monosperma

(11) Medicines :

(i) Jeweller's weight 'Ratti'- (Abrus prictorious) : Oinment prepared from leaves used in leucoderma seeds were used by jewellers in weighing ornaments.

(ii) Liquorice (Glycyrrhiza glabra) : Roots are useful in cough.

(II) SUB-FAMILY CAESALPINOIDEAE

Distribution :
It includes 150 genera and 2800 species these are usually found in Tropical and sub-tropical areas.

Habit :
Mostly trees like - **Gulmohar (Delonix regea), Amaltas (Cassia fistula), Tamarindus indica, Kachnar (Bauhinia variegata).**

Roots :
Tap root system.

Stem :
Woody, Erect, branched, cylindrical, solid.

Leaf :
Cauline & Ramal, Petiolate, Pulvinous leaf base, Alternate, Unipinnately compound and paripinnate, Reticulate Venation.

Exception : Bipinnately compound leaves are found in Parkinsonia and gulmohar. **Phyllode present in Parkinsonia**.

Inflorescence : Raceme or panicle.

Flower :
Pedicellate, Bracteate, Bisexual, Zygomorphic, Perigynous, Pentamerous.

Calyx :
5 sepals, polysepalous, odd sepal anterior, Imbricate astivation.

Corolla :
5 petals, polypetalous, **Ascending imbricate aestivation.** In Tamarindus indica two anterior petals are found as a scale.
Exception : Petals absent in Saraca indica.

Androecium :
10 stamens in two whorls 5+ 5, Polyandrous, usually three posterior stamens are sterile and resting 7 stamens are fertile. **The 3 sterile stamens are known as staminode**.

Exception :
(i) Tamarindus - 7 stamens, monadelphous, in which 4 staminode.

(ii) Bauhinia variegata - 5 Normal stamens.

(iii) Parkinsonia - All the 10 stamens are fertile.

Gynoecium : As Papilionatae.

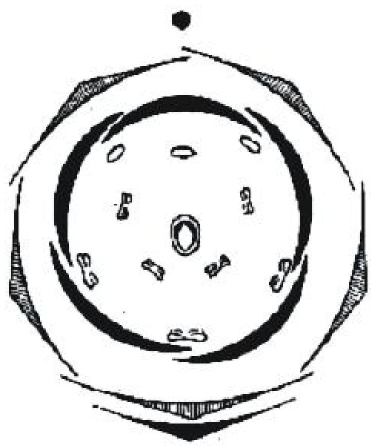
Fruit :
Legume. Exception Lomentum Ex: Tamarindus.

Seed :
Non-endospermic or Endospermic.

Pollination : Entamophily.

Floral formula : Br. % ⚥ $K_5\ C_5\ A_{5+5 \text{ or } 7+3 \text{ (Staminodes)}}\ G_{\underline{1}}$

Floral diagram :



Floral diagram

Economic Importance :

(1) Food :

(i) Bauhinia Variegata : Floral buds are used as vegetable.

(ii) Tamarind (Tamarindus indica) : Leaves and unripe fruits used for preparing chutney.

(2) Dyes :

(i) Caesalpinia sappan ('Gulal') : Red-orange dye obtains from heart wood that is used in textile industry.

(ii) Logwood (Haematoxylon compechianum) : Haematoxylin stain is obtained from heart wood which is used in the staining of Nucleus.

(3) Medicines :

(i) Cassia fistula : Immature fruits are laxative.

(ii) Cassia sophora : Leaves are useful in Ringworm.

(iii) Cassia glauca : Bark and leaves are used in Diabetes and Gonorrhoea.

(iv) Saraca indica : Bark is useful in menstrual disorder. Aurvedic medicine 'Asokarishtra' is prepared by its bark.

(4) Tanning :

(i) Bauhinia purpurea - Bark is used

(ii) Bauhinia malabarica - Bark is used

(iii) Caesalpinia digyna - Bark is used

(5) Gum :

It is obtained from the stem of **Bauhinia variegata and B. Vahalii (camel'sfoot tree)**.

(6) Other uses :

(i) Anjan (Hardwikia binata) : Tiers and ploughs are manufactured by its wood.

(ii) Phanera Vahalii : Gum obtains for commercial purpose and Bark fibres are useful in making Ropes and Baskets.

(7) Ornamental Plants :

(i) Gulmohar or Royal poinciana - Delonix regea.

(ii) Kachnar - Bauhinia Variegata.

(iii) Ashok - Saraca indica.

(iv) Peacock Flower - Caesalpinia pulcherima.

(v) Amaltas - Cassia fistula.

(vi) Jerusalem thorn - Parkinsonia aculeata (Vilaiti Kikar).

(III) SUB-FAMILY MIMOSOIDEAE

Distribution : 56 genera & 2800 species, these are usually found in the tropical and sub-tropical areas.

Habit :

Mostly trees **Ex: Acacia, Some are shrubs Ex: Dicrostaychis, rarely woody climber Ex: Entada.** Some plants are xerophyte **Ex: Acacia, some are floating hydrophytes Ex: Neptunia oleracia.**

Root :
Tap root system.

Stem :
Erect, solid, cylindrical, branched, woody.

Leaf :
Petiolate, Alternate, Stipulate, pinnately compound and paripinnate, Reticulate Venation, Pulvinus leaf base.

Exception :

(i) Phyllode Ex: Australian acacia.
(ii) Stipules convert into spines in Acacia nilotica.

Inflorescence :

(i) **Racemose head :** The tip of floral axis is swollen and sessile flowers are basipetally arranged on the former. **Ex: Acacia.**

(ii) **Spike : Ex: Prosopis.**

Flower :
Bracteate, Actinomorphic, bisexual, Perigynous, Sessile, Tetramerous or pentamerous.

Calyx :
4-5 sepals, Polysepalous or Gamosepalous, aestivation Valvate.

Corolla :
4-5 petals, sometime Gamopetalous, Valvate.

Androecium :
Indefinate stamens, Polyandrous, filament long, Anther dithecous, Dorsifixed, Introse, **Exserted.**

Exception :
(i) Prosopis - 10 stamens, polyandrous

(ii) Mimosa - 4 stamens, polyandrous

(iii) Acrocarpus - 5 stamens, polyandrous

(iv) Albizia - stamens Monadelphous

Gynoecium :
As papilionatae.

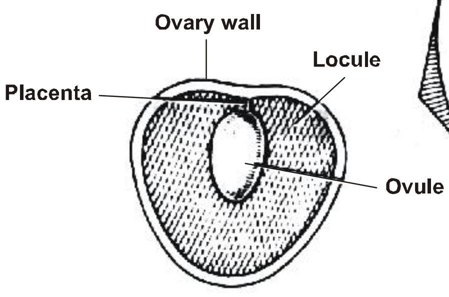
Fruit :
Mostly lomentum or legume.

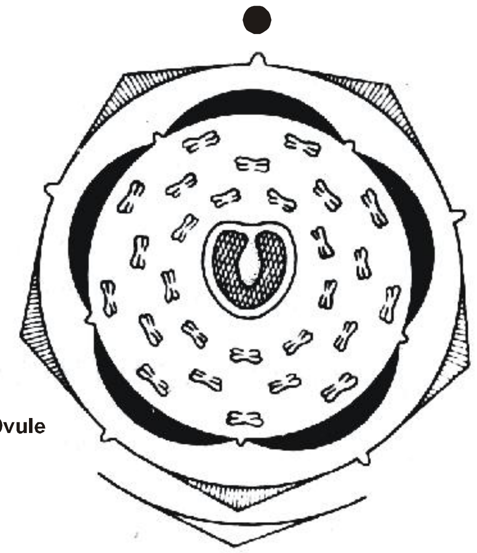
Seed :
Non-endospermic.

Pollination :
Entomophilly.

Floral formula : Br. $\oplus$ ⚥ $K_{4-5 \text{ or } (4-5)}$ $C_{4-5 \text{ or } (4-5)}$ A_{∞} $G_{\underline{1}}$

Floral diagram :





floral diagram

Economic Importance :

(1) Timber and fuel :

(i) Desi babool - Acacia arabica.

(ii) Khazari - Prosopis Cinerarifolia.

(iii) Jumbo - Xylia Xylocarpa.

(iv) Cerrish - Albizzia lebake.

(v) Indian iron wood - Xylia dolabini formis.

(2) Food & Fodder :

(i) Albizzia lebac : Leaves are used as food and fodder.

(ii) Pithecolobium dulce : Fruit and aril are Red coloured and sweet.

(iii) Neptunia oleracia : Fruits are edible.

(3) Gum :

It is mostly obtained from Acacia species. The gum obtain from Acacia sengal is called **"Gum arabic"**. It is used in medicines, confectionery & textile.

(4) Ornamental Plants :

(i) Chui-mui (Mimosa pudica) - Touch me not.
(ii) Lazwanti - Neptunia oleracia.
(iii) Australian acacia - Acacia melaxylon.
(iv) Hedge plant - Pithecolobium dulce.

(5) Other uses :

(i) Soappod (Acacia concina) : 'Shikakai' Pod contains saponin used for washing hair.
(ii) Acacia catechu : Katha is obtained from Heart wood.
(iii) Prosopis specigera : Grown as wind breaker in Rajasthan.
(iv) Acacia farmesiana : 'Cassic' perfume obtain from its flowers.

EXERCISE

1. Colchicum autumnale is a member of
(A) Brassicaceae (B) Liliaceae (C) Poaceae (D) Fabaceae

2. Axile placentation occurs in
(A) Asteraceae and Fabaceae (B) Brassicaceae and Solanaceae
(C) Solanaceae and Liliaceae (D) Brassicaceae and Solanaceae

3. $\oplus$ ⚥ $\overset{\frown}{P_{(3+3)}\,A_{3+3}}\ \underline{G}_{(3)}$ is floral formula of
(A) Liliaceae (B) Brassicaceae (C) Asteraceae (D) Poaceae

4. Red Pepper is
(A) Capsicum anuum (B) Solanum nigrum
(C) Lycopersicum esculentum (D) Physalis peruviana

5. Oil yielding legume is
(A) Carthamus (B) Glycine max (C) Ricinus (D) Vigna sinesis

6. Lycopersicum esculentum belongs to family
(A) Brassicaceae (B) Solanaceae (C) Liliaceae (D) Poaceae

7. Belladona is obtained from
(A) Atropa (B) Hyoscyamus (C) Calendula (D) Aconitum

8. Name the plant from seeds of which oil is obtained
(A) Cicer arietinum (B) Saccharum officinarum
(C) Saccharum munja (D) Arachis hypogea

9. Name the family having (9) + 1 arrangement of stamens
(A) Solanaceae (B) Asteraceae (C) Liliaceae (D) Fabaceae.

10. Largest family of flowering plants is
(A) Fabaceae (B) Liliaceae (C) Poaceae (D) Asteraceae.

11. Epipetalous stamen, obliquely placed placenta and fruit berry or capsule are diagnostic features of family :
(A) Cruciferae (B) Solanaceae (C) Malvaceae (D) Labiatae

12. Seeds which are used as Jweller's weight
(A) Xanthium (B) Abrus precatorius (C) Calotropis (D) Thespasia

13. Cladode is the modification of
(A) Leaf (B) Root (C) Petiole (D) Stem

14. The fruit of orange is
(A) Pepo (B) Pome (C) Hesperidium (D) Drupe

15. Edible part of cabbage is
(A) Flower (B) Inflorescence (C) Stem (D) Bud

16. Diadelphous condition occurs in
(A) Solanaceae (B) Fabaceae (C) Asteraceae (D) Liliaceae

17. Flower of Fabaceae is
(A) Complete, zygomorphic, pentamerous (B) Complete, actinomorphic, trimerous
(C) Incomplete, zygomorphic, trimerous (D) Incomplete, actinomorphic, pentamerous

18. Family Liliaceae is characterised by
(A) Trimerous flower (B) Tetramerous flower
(C) Pentamerous flower (D) Zygomorphic flower

19. In sweet pea tendrils are modified
(A) Stipule (B) Stem (C) Leaf (D) Leaflet

20. In which of the following aestivation of sepals & petals one margin covers the other & its margin is covered by previous one
(A) Valvate (B) Twisted (C) Imbricate (D) Quincuncial

21. Swollen placentae, oblique septum and epipetalous stgamen conniving are characteristics of family
(A) Brassicaceae (B) Asteraceae (C) Poaceae (D) Solanaceae

22. Plants which are used as green manure in crop fields and in sandy soil
(A) Crotalaria juncea and Alhagi camelorum (B) Calotropis procera and Phyllanthus niruri
(C) Saccharum munja and Lantana camara (D) Dichanthium annulatum and Acacia nilotica

23. Epipetalous stamens and axile placentation are found in
(A) Cruciferae (B) Leguminosae (C) Malvaceae (D) Liliaceae

24. Colchicum plant which gives colchicine alkaloid belongs to which family
(A) Leguminosae (B) Malvaceae (C) Liliaceae (D) Cruciferae

25. Subfamilies of Leguminosae family are differentiated on the basis of
(A) Gynoecium (B) Corolla & Androecium
(C) Nature of plant (D) Nature of fruit

26. Floral diagram represents
(A) Position of Flower (B) Number and arrangement of floral parts
(C) Structure of Flower (D) Nature of plant

27. Green Gram is
(A) Vigna radiata (B) Vigna mungo (C) Phaseolus vulgaris (D) Phaseolus coccineus

28. Ornamental plant 'Tulip' belongs to which family
(A) Asterceae (B) Brassicaceae (C) Solanaceae (D) Liliaceae

29. Trimerous flower, superior ovary with axile placentation are characteristic of
(A) Liliaceae (B) Cucurbitaceae (C) Solanaceae (D) Asteraceae

30. Which of the following members of family Solanaceae is rich in vitamin C
(A) Guava (B) Gooseberry (C) Strawberry (D) Tomato

31. Three crops that contribute maximum to global food production are
(A) Wheat, Rice and Maize (B) Wheat, Rice and Barley
(C) Wheat, Maize and Sorghum (D) Rice, Maize and Sorghum

32. In Solanaceae the fruit is
(A) Drupe (B) Berry or Capsule (C) Siliqua (D) Pod or achene

33. Perianth occurs in family
(A) Solanaceae (B) Fabaceae (C) Brassicaceae (D) Liliaceae

34. Colour of Bougainvillea is due to
(A) Coloured sepals (B) Coloured bracts (C) Coloured petals (D) None

35. What type of placentation is seen in Sweet Pea ?
(A) Marginal (B) Basal (C) Axile (D) Free central

36. Pentamerous actinomorphic flowers, bicarpellary ovary with oblique septum and a fruit of capsule or berry, belong to family
(A) Liliaceae (B) Asteraceae (C) Brassicaceae (D) Solanaceae

37. Aloe used in medicine belongs to family
(A) Solanaceae (B) Liliaceae (C) Asteraceae (D) Malvaceae

38. Soyabean belongs to
(A) Fabaceae (B) Poaceae (C) Solanaceae (D) Asteraceae

39. Name the most advanced family of monocots
(A) Arecaceae (B) Orchidaceae (C) Poaceae (D) None of the above

40. Leguminous plant used for prevention of Parkinson's syndrome is
(A) Acacia catechu (B) Acacia arabica (C) Abrus precatorius (D) Arabidopsis

41. Monocarpellary ovary, diadelphous androecium and marginal placentation occur in
(A) Brassicaceae (B) Asteraceae (C) Liliaceae (D) Papilionaceae / Fabaceae

42. The floral formula $\oplus$ ⚥ $K_{(5)}\ \overset{\frown}{C_{(5)}\ A_5}\ \underline{G}_{(2)}$ is that of
(A) Tobacco (B) Tulip (C) Soybean (D) Sunnhemp

43. Keel is characteristic of the flowers of
(A) Cassia (B) Calotropis (C) Bean (D) Gulmohur

44. Consider the following four statement A, B, C, and D select the right option for two **correct** statements.

Statements

(a) In vexillary aestivation the large posterior petal is called - standard, two lateral ones are wings and two small anterior petals are termed keel.

(b) The floral formula for Liliaceae is $\oplus$ ⚥ P_{3+3} A_{3+3} $G_{\underline{3}}$

(c) In pea flower the stamens are monadelphous

(d) The floral formula for Solanaceae is $\oplus$ ⚥ $K_{(3)}$ $C_{(3)}$ $A_{(4)}$ $G_{(\underline{2})}$

The correct statements are

(A) (c) and (d) (B) (a) and (c) (C) (a) and (b) (D) (b) and (c)

45. The term "Keel" is used for special type of

(A) Sepals (B) Petals (C) Stamens (D) Carpels

46. Polyadelphous stamens are found in

(A) Cotton (B) Sunflower (C) Grain (D) Lemon

47. Replum is the characteristic feature of the

(A) Asteraceae (B) Brassicaceae (C) Malvaceae (D) Liliaceae

48. ....ane Quinine are obtained from the of

(A) Leguminosae (B) Asteraceae (C) Rubiaceae (D) Poaceae

49. Which of the following includes largest number of genera and species of plants?

(A) Brassicaceae (B) Liliaceae (C) Malvaceae (D) Asteraceae

50. Flowers are zygomorphic in

(A) Mustard (B) Radish (C) Lily (D) Candytuft

ANSWERS

1.	(B)	**2.**	(C)	**3.**	(A)	**4.**	(A)	**5.**	(B)	**6.**	(B)	**7.**	(A)
8.	(D)	**9.**	(D)	**10.**	(D)	**11.**	(B)	**12.**	(B)	**13.**	(D)	**14.**	(C)
15.	(D)	**16.**	(B)	**17.**	(A)	**18.**	(A)	**19.**	(D)	**20.**	(B)	**21.**	(D)
22.	(A)	**23.**	(C)	**24.**	(C)	**25.**	(B)	**26.**	(B)	**27.**	(A)	**28.**	(D)
29.	(A)	**30.**	(D)	**31.**	(A)	**32.**	(B)	**33.**	(D)	**34.**	(B)	**35.**	(A)
36.	(D)	**37.**	(B)	**38.**	(A)	**39.**	(B)	**40.**	(C)	**41.**	(D)	**42.**	(A)
43.	(C)	**44.**	(C)	**45.**	(B)	**46.**	(D)	**47.**	(B)	**48.**	(C)	**49.**	(D)
50.	(D)												

Mole Concept-1

Introduction :

There are a large number of objects around us which we can see and feel.
Anything that occupies space and has mass is called matter.

Ancient Indian and Greek Philosopher's beleived that the wide variety of object around us are made from combination of five basic elements : Earth, Fire, Water, Air and Sky.
The Indian Philosopher kanad (600 BC) was of the view that matter was composed of very small, indivisible particle called *"parmanus"*.
Ancient Greek Philosophers also believed that all matter was composed of tiny building blocks which were hard and indivisible.
The Greek philosopher Democritus named these building blocks as atoms, meaning indivisible.
All these people have their philosophical view about matter, they were never put to experimental tests, nor ever explain any scientific truth.
It was ***John Dalton*** who firstly developed a theory on the structure of matter, later on which is known as ***Dalton's atomic theory.***

DALTON'S ATOMIC THEORY :

- Matter is made up of very small indivisible particles called atoms.
- All the atoms of a given element are identical in all respect i.e. mass, shape, size, etc.
- Atoms cannot be created or destroyed by any chemical process.
- Atoms of different elements are different in nature.

Classification of matter

- **on the basis of physical behaviour**
  - Solids
  - Liquids
  - Gases
- **on the basis of chemical behaviour**
  - Pure substances
    - Element
    - Compound
  - Mixtures

Basic Definitions :

Relative atomic mass :

One of the most important concept come out from Dalton's atomic theory was that of relative atomic mass or relative atomic weight. This is done by expressing mass of one atom with respect to a fixed standard. Dalton used hydrogen as the standard (H = 1). Later on oxygen (O = 16) replaced hydrogen as the reference. Therefore relative atomic mass is given as

On hydrogen scale :

$$\text{Relative atomic mass (R.A.M)} = \frac{\text{Mass of one atom of an element}}{\text{mass of one hydrogen atom}}$$

On oxygen scale :

$$\text{Relative atomic mass (R.A.M)} = \frac{\text{Mass of one atom of an element}}{\frac{1}{16} \times \text{mass of one oxygen atom}}$$

- The present standard unit which was adopted internationally in 1961, is based on the mass of one carbon-12 atom.

$$\text{Relative atomic mass (R.A.M)} = \frac{\text{Mass of one atom of an element}}{\frac{1}{12}\times \text{mass of one C}-12\text{ atom}}$$

Atomic mass unit (or amu) :

The atomic mass unit (amu) is equal to $\left(\frac{1}{12}\right)^{th}$ mass of one atom of carbon-12 isotope.

$\therefore$ 1 amu $= \frac{1}{12}$ × mass of one C-12 atom

$\simeq$ mass of one nucleon in C-12 atom.

$= 1.66 \times 10^{-24}$ gm or 1.66×10^{-27} kg

- **one amu is also called one Dalton (Da).**
- **Today, amu has been replaced by 'u' which is known as unified mass**

Atomic & molecular mass :

It is the mass of 1 atom of a substance it is expressed in amu.

- Atomic mass = R.A.M × 1 amu

$$\text{Relative molecular mass} = \frac{\text{mass of one molecule of the substance}}{\frac{1}{12}\times \text{mass of one}-\text{C}-12\text{ atom}}$$

- Molecular mass = Relative molecular mass × 1 amu

Note : Relative atomic mass is nothing but the number of nucleons present in the atom.

Solved Examples

Example-1 Find the relative atomic mass of 'O' atom and its atomic mass.

Solution The number of nucleons present in 'O' atom is 16.

$\therefore$ relative atomic mass of 'O' atom = 16.

Atomic mass = R.A.M × 1 amu = 16 × 1 amu = 16 amu

Mole : The Mass / Number Relationship

Mole is a chemical counting SI unit and defined as follows :

A mole is the amount of a substance that contains as many entities (atoms, molecules or other particles) as there are atoms in exactly 0.012 kg (or 12 gm) of the carbon-12 isotope.

From mass spectrometer we found that there are 6.023×10^{23} atoms present in 12 gm of C-12 isotope.

The number of entities in 1 mol is so important that it is given a separate name and symbol known as Avogadro constant denoted by N_A.

i.e. on the whole we can say that 1 mole is the collection of 6.02×10^{23} entities. Here entities may represent atoms, ions, molecules or even pens, chair, paper etc also include in this but as this number (N_A) is very large therefore it is used only for very small things.

HOW BIG IS A MOLE ?

Amount of water in world's oceans (litres) | Age of earth (seconds) | Population of earth

Avogadro's number 602,200,000,000,000,000,000,000

Distance from earth to sun (centimeters)

❍ **Note :** **In modern practice gram-atom and gram-molecule are termed as mole.**

Gram Atomic Mass :

The atomic mass of an element expressed in gram is called gram atomic mass of the element.

or

It is also defined as mass of 6.02×10^{23} atoms.

or

It is also defined as the mass of one mole atoms.

For example for oxygen atom :

Atomic mass of 'O' atom = mass of one 'O' atom = 16 amu

gram atomic mass = mass of 6.02×10^{23} 'O' atoms

$= 16 \text{ amu} \times 6.02 \times 10^{23}$

$= 16 \times 1.66 \times 10^{-24} \text{ g} \times 6.02 \times 10^{23} = 16 \text{ g}$

$(\because \quad 1.66 \times 10^{-24} \times 6.02 \times 10^{23} \simeq 1)$

Solved Examples

Example-2 How many atoms of oxygen are their in 16 g oxygen.

Solution Let x atoms of oxygen are present

So, $16 \times 1.66 \times 10^{-24} \times x = 16$ g

$$x = \frac{1}{1.66 \times 10^{-24}} = N_A$$

Gram molecular mass :

The molecular mass of a substance expressed in gram is called the gram-molecular mass of the substance.

or

It is also defined as mass of 6.02×10^{23} molecules

or

It is also defined as the mass of one mole molecules.

For example for 'O_2' molecule :

Molecular mass of 'O_2' molecule = mass of one 'O_2' molecule

= 2 × mass of one 'O' atom

= 2 × 16 amu

= 32 amu

gram molecular mass = mass of 6.02×10^{23} 'O_2' molecules $= 32 \text{ amu} \times 6.02 \times 10^{23}$

$= 32 \times 1.66 \times 10^{-24} \text{ gm} \times 6.02 \times 10^{23} = 32 \text{ gm}$

Solved Examples

Example-3 The molecular mass of H_2SO_4 is 98 amu. Calculate the number of moles of each element in 294 g of H_2SO_4.

Solution Gram molecular mass of H_2SO_4 = 98 gm

moles of $H_2SO_4 = \frac{294}{98}$ = 3 moles

$\mathbf{H_2SO_4}$	**H**	**S**	**O**
One molecule	2 atom	one atom	4 atom
$1 \times N_A$	$2 \times N_A$ atoms	$1 \times N_A$ atoms	$4 \times N_A$ atoms
$\therefore$ one mole	2 mole	one mole	4 mole
$\therefore$ **3 mole**	**6 mole**	**3 mole**	**12 mole**

Gay-Lussac's Law of Combining Volume :

According to him elements combine in a simple ratio of atoms, gases combine in a simple ratio of their volumes provided all measurements should be done at the same temperature and pressure

$$H_2(g) \quad + \quad Cl_2(g) \longrightarrow 2HCl$$
$$1 \text{ vol} \qquad 1 \text{ vol} \qquad\quad 2 \text{ vol}$$

Avogadro's hypothesis :

Equal volume of all gases have equal number of molecules (not atoms) at same temperature and pressure condition.

S.T.P. (Standard Temperature and Pressure)

At S.T.P. condition : temperature = 0°C or 273 K
pressure = 1 atm = 760 mm of Hg

and volume of one mole of gas at STP is found to be experimentally equal to 22.4 litres which is known as molar volume.

Note : **Measuring the volume is equivalent to counting the number of molecules of the gas.**

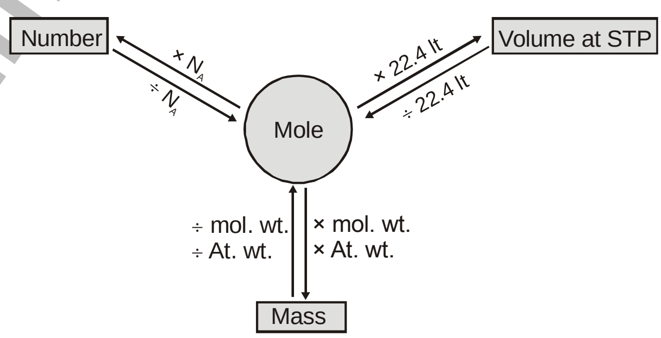
Solved Examples

Example-4 Calculate the volume in litres of 20 g hydrogen gas at STP.

Solution No. of moles of hydrogen gas = $\frac{\text{Mass}}{\text{Molecular mass}} = \frac{20 \text{ gm}}{2 \text{ gm}}$ = 10 mol

volume of hydrogen gas at STP = 10 × 22.4 lt.

Y-map : Interconversion of mole - volume, mass and number of particles :



The laws of chemical combination :

Atoine Lavoisier, ***John Dalton*** and other scientists formulate certain law concerning the composition of matter and chemical reactions. These laws are known as the law of chemical combination.

(i) The law of conservation of mass :

In a chemical change total mass remains conserved.
i.e. mass before reaction is always equal to mass after reaction.

Solved Examples

Example-5 $H_2(g) + \frac{1}{2} O_2(g) \longrightarrow H_2O(l)$

Solution $H_2(g) + \frac{1}{2} O_2(g) \longrightarrow H_2O(l)$

	$H_2(g)$	$\frac{1}{2} O_2(g)$	$H_2O(l)$
Before reaction initially	1 mole	$\frac{1}{2}$ mole	0
After the reaction	0	0	1 mole

mass before reaction = mass of 1 mole $H_2(g)$ + $\frac{1}{2}$ mole $O_2(g)$

= 2 + 16 = 18 gm

mass after reaction = mass of 1 mole water = 18 gm

(ii) Law of constant or Definite proportion :

All chemical compounds are found to have constant composition irrespective of their method of preparation or sources.

Example :

In water (H_2O), Hydrogen and Oxygen combine in 2 : 1 molar ratio, this ratio remains constant whether it is tap water, river water or sea water or produced by any chemical reaction.

Solved Examples

Example-6 1.80 g of a certain metal burnt in oxygen gave 3.0 g of its oxide. 1.50 g of the same metal heated in steam gave 2.50 g of its oxide. Show that these results illustrate the law of constant proportion.

Solution In the first sample of the oxide,

Wt. of metal = 1.80 g,

Wt. of oxygen = (3.0 – 1.80) g = 1.2 g

$\therefore \quad \frac{\text{wt. of metal}}{\text{wt. of oxygen}} = \frac{1.80g}{1.2g} = 1.5$

In the second sample of the oxide,

Wt. of metal = 1.50 g,

Wt. of oxygen = (2.50 – 1.50) g = 1 g.

$\therefore \quad \frac{\text{wt. of metal}}{\text{wt. of oxygen}} = \frac{1.50\,g}{1g} = 1.5$

Thus, in both samples of the oxide the proportions of the weights of the metal and oxygen a fixed. Hence, the results follow the law of constant proportion.

(iii) The law of multiple proportion :

When one element combines with the other element to form two or more different compounds, the mass of one element, which combines with a constant mass of the other, bear a simple ratio to one another.

Note : Simple ratio here means the ratio between small natural numbers, such as 1 : 1, 1 : 2, 1 : 3, later on this simple ratio becomes the valency and then oxidation state of the element.
See oxidation number of carbon also have same ratio 1 : 2 in both the oxide.

Solved Examples

Example-7 Carbon is found to form two oxides, which contain 42.9% and 27.3% of carbon respectively. Show that these figures illustrate the law of multiple proportions.

Solution **Step-1**

To calculate the percentage composition of carbon and oxygen in each of the two oxides.

	First oxide	Second oxide	
Carbon	42.9 %	27.3 %	(Given)
Oxygen (by difference)	57.1%	72.7 %	

Step-2

To calculate the masses of carbon which combine with a fixed mass i.e., one part by mass of oxygen in each of the two oxides.

In the first oxide, 57.1 parts by mass of oxygen combine with carbon = 42.9 parts.

∴ 1 part by mass of oxygen will combine with carbon = $\frac{42.9}{57.1}$ = 0.751.

In the second oxide. 72.7 parts by mass of oxygen combine with carbon = 27.3 parts.

∴ 1 part by mass of oxygen will combine with carbon = $\frac{27.3}{72.7}$ = 0.376

Step-3.

To compare the masses of carbon which combine with the same mass of oxygen in both the oxides.

The ratio of the masses of carbon that combine with the same mass of oxygen (1 part) is .

0.751 : 0.376 or 2 : 1

Since this is simple whole number ratio, so the above data illustrate the law of multiple proportions.

Percentage Composition :

Here we are going to find out the percentage of each element in the compound by knowing the molecular formula of compound.

We know that according to law of definite proportions any sample of a pure compound always possess constant ratio with their combining elements.

Solved Examples

Example-8 Every molecule of ammonia always has formula NH_3 irrespective of method of preparation or sources. i.e. 1 mole of ammonia always contains 1 mol of N and 3 mole of H. In other words 17 gm of NH_3 always contains 14 gm of N and 3 gm of H. Now find out % of each element in the compound.

Solution **Mass % of N in NH_3** = $\frac{\text{Mass of N in 1 mol } NH_3}{\text{Mass of 1 mol of } NH_3} \times 100$ = $\frac{14 \text{ gm}}{17}$ × 100 = 82.35 %

Mass % of H in NH_3 = $\frac{\text{Mass of H is 1 mol } NH_3}{\text{Mass of 1 mole of } NH_3} \times 100$ = $\frac{3}{17}$ × 100 = 17.65 %

Empirical and molecular formula :

We have just seen that knowing the molecular formula of the compound we can calculate percentage composition of the elements. Conversely if we know the percentage composition of the elements initially, we can calculate the relative number of atoms of each element in the molecules of the compound. This gives us the empirical formula of the compound. Further if the molecular mass is known then the molecular formula can easily be determined.

The empirical formula of a compound is a chemical formula showing the relative number of atoms in the simplest ratio. An empirical formula represents the simplest whole number ratio of various atoms present in a compound.

The molecular formula gives the actual number of atoms of each element in a molecule. The molecular formula shows the exact number of different types of atoms present in a molecule of a compound.

The molecular formula is an integral multiple of the empirical formula.

i.e. molecular formula = empirical formula × n

where $n = \dfrac{\text{molecular formula mass}}{\text{empirical formula mass}}$

Solved Examples

Example-9 Acetylene and benzene both have the empirical formula CH. The molecular masses of acetylene and benzene are 26 and 78 respectively. Deduce their molecular formulae.

Solution $\because$ Empirical Formula is CH

Step-1 The empirical formula of the compound is CH

$\therefore$ Empirical formula mass = (1 × 12) + 1 = 13.

Molecular mass = 26

Step-2 To calculate the value of 'n'

$$n = \frac{\text{Molecular mass}}{\text{Empirical formula mass}} = \frac{26}{13} = 2$$

Step-3 To calculate the molecular formula of the compound.

Molecular formula = n × (Empirical formula of the compound)

= 2 × CH = C_2H_2

Thus the molecular formula is **C_2H_2**

Similarly for benzene

To calculate the value of 'n'

$$n = \frac{\text{Molecular mass}}{\text{Empirical formula mass}} = \frac{78}{13} = 6$$

thus the molecular formula is 6 × CH = **C_6H_6**

Example-10 An organic substance containing carbon, hydrogen and oxygen gave the following percentage composition.

C = 40.684% ; H = 5.085% and O = 54.228%

The molecular weight of the compound is 118 gm. Calculate the molecular formula of the compound.

Solution **Step-1**

To calculate the empirical formula of the compound.

Element	Symbol	Percentage of element	At. mass of element	Relative no. of atoms $= \frac{\text{Percentage}}{\text{At. mass}}$	Simplest atomic ratio	Simplest whole no. atomic ratio
Carbon	C	40.687	12	$\frac{40.687}{12} = 3.390$	$\frac{3.390}{3.389} = 1$	2
Hydrogen	H	5.085	1	$\frac{5.085}{1} = 5.085$	$\frac{5.085}{3.389} = 1.5$	3
Oxygen	O	54.228	16	$\frac{54.228}{16} = 3.389$	$\frac{3.389}{3.389} = 1$	2

$\therefore$ Empirical Formula is $C_2H_3O_2$

Step-2 To calculate the empirical formula mass.

The empirical formula of the compound is $C_2H_3O_2$.

$\therefore$ Empirical formula mass = (2 × 12) + (3 × 1) + (2 × 16) = 59.

Step-3 To calculate the value of 'n'

$$n = \frac{\text{Molecular mass}}{\text{Empirical formula mass}} = \frac{118}{59} = 2$$

Step-4 To calculate the molecular formula of the salt.

Molecular formula = n × (Empirical formula) = 2 × $C_2H_3O_2$ = $C_4H_6O_4$

Thus the molecular formula is $C_4H_6O_4$.

DENSITY :

It is of two type.

- Absolute density
- Relative density

For Liquid and Solids

- Absolute density = $\frac{\text{mass}}{\text{volume}}$
- Relative density or specific gravity = $\frac{\text{density of the substance}}{\text{density of water at } 4°C}$

We know that density of water at 4ºC = 1 g/ml.

For Gases :

- Absolute density (mass/volume) = $\frac{\text{Molar mass}}{\text{Molar volume}}$

Relative density or Vapour density :

Vapour density is defined as the density of the gas with respect to hydrogen gas at the same temperature and pressure.

Vapour density = $\frac{d_{gas}}{d_{H_2}}$

$$V.D. = \frac{M_{gas}}{M_{H_2}} = \frac{M_{gas}}{2}$$

$\mathbf{M_{gas} = 2\ V.D.}$

Relative density can be calculated w.r.t. to other gases also.

Solved Examples

Example-11 What is the V.D. of SO_2 with respect to CH_4

Solution $V.D. = \frac{M.W.\ SO_2}{M.W.\ CH_4}$

$V.D = \frac{64}{16} = 4$

Example-12 7.5 litre of the particular gas at S.T.P. weighs 16 gram. What is the V.D. of gas

Solution 7.5 litre = 16 gram

moles = $\frac{7.5}{22.4} = \frac{16}{M}$

M = 48 gram V.D = $\frac{48}{2} = 24$

Chemical Reaction :

It is the process in which two or more than two substances interact with each other where old bonds are broken and new bonds are formed.

Chemical Equation :

All chemical reaction are represented by chemical equations by using chemical formula of reactants and products. Qualitatively a chemical equation simply describes what the reactants and products are. However, a balanced chemical equation gives us a lot of quantitative information. Mainly the molar ratio in which reactants combine and the molar ratio in which products are formed.

Attributes of a balanced chemical equation:

(a) It contains an equal number of atoms of each element on both sides of equation.(POAC)
(b) It should follow law of charge conservation on either side.
(c) Physical states of all the reagents should be included in brackets.
(d) All reagents should be written in their standard molecular forms (not as atoms)
(e) The coefficients give the relative molar ratios of each reagent.

Solved Examples

Example-13 Write a balance chemical equation for following reaction :
When potassium chlorate ($KClO_3$) is heated it gives potassium chloride (KCl) and oxygen (O_2).

Solution $KClO_3$ (s) $\xrightarrow{\Delta}$ KCl (s) + O_2 (g) (unbalanced chemical equation)

$2KClO_3$ (s) $\xrightarrow{\Delta}$ 2 KCl (s) + 3 O_2 (g) (balanced chemical equation)

Remember a balanced chemical equation is one which contains an equal number of atoms of each element on both sides of equation.

Interpretation of balanced chemical equations :

Once we get a balanced chemical equation then we can interpret a chemical equation by following ways

- Mass - mass analysis
- Mass - volume analysis
- Mole - mole analysis
- Vol - Vol analysis (separately discussed as **eudiometry or gas analysis**)

Now you can understand the above analysis by following example

- **Mass-mass analysis :**

Consider the reaction

$2KClO_3 \longrightarrow 2KCl + 3O_2$ According to stoichiometry of the reaction

mass-mass ratio: 2 × 122.5 : 2 × 74.5 : 3 × 32

or $\dfrac{\text{Mass of } KClO_3}{\text{Mass of KCl}} = \dfrac{2\times122.5}{2\times74.5}$

$\dfrac{\text{Mass of } KClO_3}{\text{Mass of } O_2} = \dfrac{2\times122.5}{3\times32}$

Solved Examples

Example-14 367.5 gram $KClO_3$ (M = 122.5) when heated. How many gram KCl and oxygen is produced.

Solution Balance chemical equation for heating of $KClO_3$ is

$$2KClO_3 \longrightarrow 2KCl + 3O_2$$

mass-mass ratio : 2 × 122.5 gm : 2 × 74.5 gm : 3 × 32 gm

$$\frac{\text{mass of } KClO_3}{\text{mass of KCl}} = \frac{2\times122.5}{2\times74.5} \Rightarrow \frac{367.5}{W} = \frac{122.5}{74.5}$$

W = 3 × 74.5 = 223.5 gm

$$\frac{\text{Mass of } KClO_3}{\text{Mass of } O_2} = \frac{2\times122.5}{3\times32} \Rightarrow \frac{367.5}{W} = \frac{2\times122.5}{3\times32}$$

W = 144 gm

- **Mass - volume analysis :**

Now again consider decomposition of $KClO_3$

$$2KClO_3 \longrightarrow 2KCl + 3O_2$$

mass volume ratio : 2 × 122.5 gm : 2 × 74.5 gm : 3 × 22.4 lt. at STP

we can use two relation for volume of oxygen

$$\frac{\text{Mass of } KClO_3}{\text{volume of } O_2 \text{ at STP}} = \frac{2\times122.5}{3\times22.4 \text{ lt}} \qquad ...(i)$$

and $$\frac{\text{Mass of KCl}}{\text{volume of } O_2 \text{ at STP}} = \frac{2\times74.5}{3\times22.4 \text{ lt}} \qquad ...(ii)$$

Solved Examples

Example-15 367.5 gm $KClO_3$ (M = 122.5) when heated, how many litre of oxygen gas is produced at STP.

Solution You can use here equation (1)

$$\frac{\text{mass of } KClO_3}{\text{volume of } O_2 \text{ at STP}} = \frac{2\times122.5}{3\times22.4 \text{ lt}} \Rightarrow \frac{367.5}{V} = \frac{2\times122.5}{3\times22.4 \text{ lt}}$$

V = 3 × 3 × 11.2 ⇒ **V = 100.8 lt**

- **Mole-mole analysis :**

This analysis is very much important for quantitative analysis point of view. Students are advised to clearly understand this analysis.

Now consider again the decomposition of $KClO_3$.

$$2KClO_3 \longrightarrow 2KCl + 3O_2$$

In very first step of mole-mole analysis you should read the balanced chemical equation like ***2 moles $KClO_3$ on decomposition gives you 2 moles KCl and 3 moles O_2.*** and from the stoichiometry of reaction we can write

$$\frac{\text{Moles of } KClO_3}{2} = \frac{\text{Moles of KCl}}{2} = \frac{\text{Moles of } O_2}{3}$$

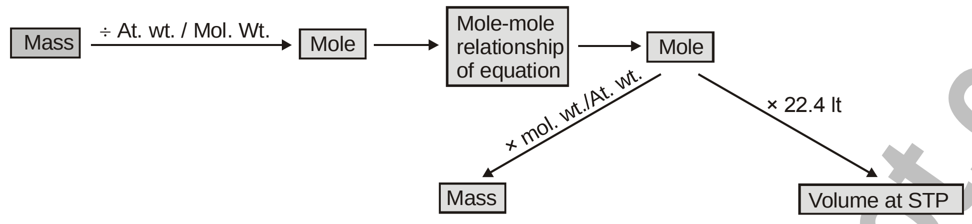
Now for any general balance chemical equation like

$$aA + bB \longrightarrow cC + dD$$

you can write.

$$\frac{\text{Moles of A reacted}}{a} = \frac{\text{moles of B reacted}}{b} = \frac{\text{moles of C produced}}{c} = \frac{\text{moles of D produced}}{d}$$

Note : In fact mass-mass and mass-vol analysis are also interpreted in terms of mole-mole analysis you can use following chart also.



Limiting reagent :

The reactant which is consumed first and limits the amount of product formed in the reaction, and is therefore, called limiting reagent.

Limiting reagent is present in least stoichiometric amount and therefore, controls amount of product.

The remaining or left out reactant is called the excess reagent.

When you are dealing with balance chemical equation then if number of moles of reactants are not in the ratio of stoichiometric coefficient of balanced chemical equation, then there should be one reactant which is limiting reactant.

Solved Examples

Example-16 Three mole of Na_2CO_3 is reacted with 6 moles of HCl solution. Find the volume of CO_2 gas produced at STP. The reaction is

$Na_2CO_3 + 2HCl \longrightarrow 2\,NaCl + CO_2 + H_2O$

Solution

From the reaction :	$Na_2CO_3 + 2HCl \longrightarrow 2\,NaCl + CO_2 + H_2O$
given moles	3 mol 6 mol
given mole ratio	1 : 2
Stoichiometric coefficient ratio	1 : 2

See here given moles of reactant are in stoichiometric coefficient ratio therefore none reactant left over.

Now use Mole-mole analysis to calculate volume of CO_2 produced at STP

$$\frac{\text{Moles of } Na_2CO_3}{1} = \frac{\text{Mole of } CO_2 \text{ Produced}}{1}$$

Moles of CO_2 produced = 3

volume of CO_2 produced at STP = 3 × 22.4 L = 67.2 L

Example-17 6 moles of Na_2CO_3 is reacted with 4 moles of HCl solution. Find the volume of CO_2 gas produced at STP. The reaction is

$Na_2CO_3 + 2HCl \longrightarrow 2\,NaCl + CO_2 + H_2O$

Solution

From the reaction :	$Na_2CO_3 + 2HCl \longrightarrow 2\,NaCl + CO_2 + H_2O$
given mole of reactant	6 : 4
given molar ratio	3 : 2
Stoichiometric coefficient ratio	1 : 2

See here given number of moles of reactants are not in stoichiometric coefficient ratio. Therefore there should be one reactant which consumed first and becomes limiting reagent.

But the question is how to find which reactant is limiting, it is not very difficult you can easily find it. According to the following method.

How to find limiting reagent :

Step : I Divide the given moles of reactant by the respective stoichiometric coefficient of that reactant.

Step : II See for which reactant this division come out to be minimum. The reactant having minimum value is limiting reagent for you.

Step : III Now once you find limiting reagent then your focus should be on limiting reagent

From Step I & II $\quad Na_2CO_3 \quad HCl$

$$\frac{6}{1} = 6 \qquad \frac{4}{2} = 2 \text{ (division is minimum)}$$

$\therefore$ **HCl is limiting reagent**

From Step III

$$\text{From } \frac{\text{Mole of HCl}}{2} = \frac{\text{Moles of } CO_2 \text{ produced}}{1}$$

$\therefore$ mole of CO_2 produced = 2 moles

$\therefore$ volume of CO_2 produced at S.T.P. = 2 × 22.4 = 44.8 lt.

Principle of Atom Conservation (POAC) :

POAC is conservation of mass. Atoms are conserved, moles of atoms shall also be conserved in a chemical reaction (but not in nuclear reactions.)

This principle is fruitful for the students when they don't get the idea of balanced chemical equation in the problem.

The strategy here will be around a particular atom. We focus on a atom and conserve it in that reaction. This principle can be understand by the following example.

Consider the decomposition of $KClO_3$ (s) $\rightarrow$ KCl (s) + O_2 (g) (unbalanced chemical reaction)

Apply the principle of atom conservation (POAC) for K atoms.

Moles of K atoms in reactant = moles of K atoms in products

or moles of K atoms in $KClO_3$ = moles of K atoms in KCl.

Now, since 1 molecule of $KClO_3$ contains 1 atom of K

or 1 mole of $KClO_3$ contains 1 mole of K, similarly,1 mole of KCl contains 1 mole of K.

Thus, moles of K atoms in $KClO_3$ = 1 × moles of $KClO_3$

and moles of K atoms in KCl = 1 × moles of KCl.

$\therefore$ moles of $KClO_3$ = moles of KCl

$$\text{or} \quad \frac{\textbf{wt. of } \mathbf{KClO_3} \textbf{ in g}}{\textbf{mol. wt. of } \mathbf{KClO_3}} = \frac{\textbf{wt. of KCl in g}}{\textbf{mol. wt. of KCl}}$$

- The above equation gives the mass-mass relationship between $KClO_3$ and KCl which is important in stoichiometric calculations.

  Again, applying the principle of atom conservation for O atoms,

  moles of O in $KClO_3$ = 3 × moles of $KClO_3$

  moles of O in O_2 = 2 × moles of O_2

  $\therefore$ 3 × moles of $KClO_3$ = 2 × moles of O_2

  $$\text{or} \quad 3 \times \frac{\text{wt. of } KClO_3}{\text{mol. wt. of } KClO_3} = 2 \times \frac{\text{vol. of } O_2 \text{ at NTP}}{\text{standard molar vol. (22.4 lt.)}}$$

- The above equations thus gives the mass-volume relationship of reactants and products.

Solved Examples

Example-18 27.6 g K_2CO_3 was treated by a series of reagents so as to convert all of its carbon to $K_2Zn_3[Fe(CN)_6]_2$. Calculate the weight of the product.
[mol. wt. of K_2CO_3 = 138 and mol. wt. of $K_2Zn_3[Fe(CN)_6]_2$ = 698]

Solution Here we have not knowledge about series of chemical reactions but we know about initial reactant and final product accordingly

$$K_2CO_3 \xrightarrow[\text{Steps}]{\text{Several}} K_2Zn_3[Fe(CN)_6]_2$$

Since C atoms are conserved, applying POAC for C atoms,
moles of C in K_2CO_3 = moles of C in $K_2Zn_3[Fe(CN)_6]_2$
1 × moles of K_2CO_3 = 12 × moles of $K_2Zn_3[Fe(CN)_6]_2$
($\because$ 1 mole of K_2CO_3 contains 1 moles of C)

$$\frac{\text{wt. of } K_2CO_3}{\text{mol. wt.of } K_2CO_3} = 12 \times \frac{\text{wt. of the product}}{\text{mol. wt.of product}}$$

wt. of $K_2Zn_3[Fe(CN)_6]_2$ = $\frac{27.6}{138} \times \frac{698}{12}$ = 11.6 g

Solutions :

A mixture of two or more substances can be a solution. We can also say that "a solution is a homogeneous mixture of two or more substances,'' 'Homogeneous' means 'uniform throughout'. Thus a homogeneous mixture, i.e., a solution, will have uniform composition throughout.

Properties of a solution :

- A solution is clear and transparent. For example, a solution of sodium chloride in water is clear and tranparent.
- The solute in a solution does not settle down even after the solution is kept undisturbed for some time.
- In a solution, the solute particle cannot be distinguished from the solvent particles or molecules even under a microscope. In a true solution, the particles of the solute disappear into the space between the solvent molecules.
- The components of a solution cannot be separated by filtration.

Concentration terms :

The following concentration terms are used to expressed the concentration of a solution. These are

- Molarity (M)
- Molality (m)
- Mole fraction (x)
- % calculation
- Normality (N)
- ppm

❍ Remember that all of these concentration terms are related to one another. By knowing one concentration term you can also find the other concentration terms. Let us discuss all of them one by one.

Molarity (M) :

The number of moles of a solute dissolved in 1 L (1000 ml) of the solution is known as the molarity of the solution.

i.e., $\text{Molarity of solution} = \frac{\text{number of moles of solute}}{\text{volume of solution in litre}}$

Let a solution is prepared by dissolving w gm of solute of mol.wt. M in V ml water.

$\therefore$ Number of moles of solute dissolved $= \frac{w}{M}$

$\therefore$ V ml water have $\frac{w}{M}$ mole of solute

$\therefore$ 1000 ml water have $\frac{w \times 1000}{M \times V_{ml}}$ $\therefore$ Molarity (M) = $\frac{w \times 1000}{(\text{Mol. wt of solute}) \times V_{ml}}$

Some other relations may also useful.

Number of millimoles = $\frac{\text{mass of solute}}{(\text{Mol. wt. of solute})} \times 1000$ = (Molarity of solution × V_{ml})

- Molarity of solution may also given as :

  $\frac{\text{Number of millimole of solute}}{\text{Total volume of solution in ml}}$

- Molarity is a unit that depends upon temperature. It varies inversely with temperature .

  Mathematically : Molarity decreases as temperature increases.

  Molarity $\propto \frac{1}{\text{temperature}} \propto \frac{1}{\text{volume}}$

- If a particular solution having volume V_1 and molarity = M_1 is diluted upto volume V_2 mL than

  $M_1V_1 = M_2V_2$

  M_2 : Resultant molarity

- If a solution having volume V_1 and molarity M_1 is mixed with another solution of same solute having volume V_2 mL & molarity M_2

  then $M_1V_1 + M_2V_2 = M_R (V_1 + V_2)$

  M_R = Resultant molarity = $\frac{M_1V_1 + M_2V_2}{V_1 + V_2}$

Solved Examples

Example-19 149 gm of potassium chloride (KCl) is dissolved in 10 Lt of an aqueous solution. Determine the molarity of the solution (K = 39, Cl = 35.5)

Solution Molecular mass of KCl = 39 + 35.5 = 74.5 gm

$\therefore$ Moles of KCl = $\frac{149 \text{ gm}}{74.5 \text{ gm}}$ = 2

$\therefore$ Molarity of the solution = $\frac{2}{10}$ = 0.2 M

Molality (m) :

The number of moles of solute dissolved in1000 gm (1 kg) of a solvent is known as the molality of the solution.

i.e., molality = $\frac{\text{number of moles of solute}}{\text{mass of solvent in gram}} \times 1000$

Let Y gm of a solute is dissolved in X gm of a solvent. The molecular mass of the solute is M_0. Then Y/M_0 mole of the solute are dissolved in X gm of the solvent. Hence

Molality = $\frac{Y}{M_0 \times X} \times 1000$

- ***Molality is independent of temperature changes.***

Mole fraction (x) :

The ratio of number of moles of the solute or solvent present in the solution and the total number of moles present in the solution is known as the mole fraction of substances concerned.

Let number of moles of solute in solution = n

Number of moles of solvent in solution = N

$\therefore$ Mole fraction of solute (x_1) = $\frac{n}{n+N}$

$\therefore$ Mole fraction of solvent (x_2) = $\frac{N}{n+N}$

also $x_1 + x_2 = 1$

❍ Mole fraction is a pure number. It will remain independent of temperature changes.

% calculation :

The concentration of a solution may also expressed in terms of percentage in the following way.

- **% weight by weight (w/w) :** It is given as mass of solute present in per 100 gm of solution.

i.e. % w/w = $\frac{\text{mass of solute in gm}}{\text{mass of solution in gm}} \times 100$

- **% weight by volume (w/v) : It is given as mass of solute present in per 100 ml of solution.**

i.e., % w/v = $\frac{\text{mass of solute in gm}}{\text{volume of solution in ml}} \times 100$

- **% volume by volume (v/v) : It is given as volume of solute present in per 100 ml solution.**

i.e., % v/v = $\frac{\text{volume of solute in ml}}{\text{volume of solution in ml}} \times 100$

Miscellaneous :

- **AVERAGE/ MEAN ATOMIC MASS :**

The weighted average of the isotopic masses of the element's naturally occuring isotopes.

Mathematically, average atomic mass of X (A_x) = $\frac{a_1x_1 + a_2x_2 + ... + a_nx_n}{100}$

Where :

a_1, a_2, a_3 .... atomic mass of isotopes.

and x_1, x_2, x_3 .... mole % of isotopes.

- **MEAN MOLAR MASS OR MOLECULAR MASS:**

The average molar mass of the different substance present in the container = $\frac{n_1M_1 + n_2M_2 + ...n_nM_n}{n_1 + n_2 + ...n_n}$.

Where :

M_1, M_2, M_3 .... are molar masses.

n_1, n_2, n_3 .... moles of substances.

KVPY PROBLEMS (PREVIOUS YEARS)

1. 3.01×10^{23} molecules of elemental Sulphur will react with 0.5 mole of oxygen gas completely to produce **[KVPY_2008_SA]**

(A) 6.02×10^{23} molecules of SO_3 (B) 6.02×10^{23} molecules of SO_2
(C) 3.01×10^{23} molecules of SO_3 (D) 3.01×10^{23} molecules of SO_2

Sol.

S	+	O_2	$\longrightarrow$	SO_2
1 mole		1 mole		1 mole
$\frac{1}{2}$ mole		$\frac{1}{2}$ mole		$\frac{1}{2}$ mole
3.01×10^{23}		0.5 mole		?

$\therefore$ 3.01×10^{23} molecules of SO_2 will be formed.

2. The density of a salt solution is1.13 g cm^{-3} and it contains 18% of NaCl by weight. The volume of the solution containing 36.0 g of the salt will be : **[KVPY_2008_SA]**

(A) 200 cm^3 (B) 217 cm^3 (C) 177 cm^3 (D) 157cm^3

Sol. Consider the volume of the solution = x cm^3
Then the mass of the solution will be = 1.13x
(mass = density × volume)
The solution contains 18% of NaCl by weight

$\therefore \frac{18}{100} \times 1.13x = 36$

$x = \frac{3600}{18 \times 1.13} = 177\ cm^3$

3. 10 g of a crystalline metal sulphate salt when heated generates approximately 6.4 g of an anhydrous salt of the same metal. The molecular weight of the anhydrous salt is 160 g. The number of water molecules present in the crystal is : **[KVPY_2008_SA]**

(A) 1 (B) 2 (C) 3 (D) 5

Sol. Consider that the salt contains x molecules of water .
Molecular weight of anhydrous salt = 160 g
so molecular weight of hydrated salt will be = 160 + 18x g

Then, no. of moles of water present in 10x gm of hydrated salt = $\frac{10}{160+18x} \times x$

and weight of water present in 10 gm of hydrated salt = $\frac{10x}{160+18x} \times 18$

Hydrated salt	$\longrightarrow$	Anhydrous salt +	Water
10g		6.4 g	3.6g

$\frac{180x}{160+18x} = 3.6$

180x = 576 + 64.8 x

x = 5

4. One mole of nitrogen gas on reaction with 3.01 x 10^{23} molecules of hydrogen gas produces - **[KVPY_2009_SA]**

(A) one mole of ammonia (B) 2.0 x 10^{23} molecules of ammonia
(C) 2 moles of ammonia (D) 3.01×10^{23} molecules of ammonia

Sol.

	N_2	+	$3H_2$	$\rightleftharpoons$	$2NH_3$
Initial no. of moles	1		0.5		0
After reaction	1 – 0.167		0.5 – 0.5		0.334 mole

0.334 moles of NH_3 = 2.0×10^{23} molecules

5. 10 ml of an aqeuous solution containing 222 mg of calcium chloride (mol. wt. = 111) is diluted to 100 ml. The concentration of chloride ion in the resulting solution is - **[KVPY_2009_SA]**

(A) 0.02 mol/lit. (B) 0.01 mol/lit. (C) 0.04 mol/lit (D) 2.0 mol/lit.

Sol. Initially concentration of salt in solution $= \dfrac{222 \times 10^{-3}}{111 \times 10 \times 10^{-3}} = 0.2M$

On dilution the final concentration of $CaCl_2$ will be

$M_1V_1 = M_2V_2$

$0.2 \times 10 = M_2 \times 100$

$M_2 = 0.02$ M

$CaCl_2 \longrightarrow Ca^{2+} + 2Cl^-$

0.02 M   0.02 M   2 × 0.02

$[Cl^-] = 0.04M = 0.04$ mole/L

6. Aluminium reduces maganese dioxide to manganese at high temperature. The amount of aluminium required to reduce one gram mole of manganese dioxide is - **[KVPY_2009_SA]**

(A) 1/2 gram mole (B) 1 gram mole (C) 3/4 gram mole (D) 4/3 gram mole

Sol. $4Al + 3MnO_2 \longrightarrow 3Mn + 2Al_2O_3$

To reduce 3 moles of MnO_2 required moles of Al = 4

So, for one mole of MnO_2 required moles of Al will be = 4/3

7. The molar mass of $CaCO_3$ is 100 g. The maximum amount of carbon dioxide that can be liberated on heating 25 g of $CaCO_3$ is : **[KVPY_2010_SA]**

(A) 11 g (B) 5.5 g (C) 22 g (D) 2.2 g

Sol. $CaCO_3 (s) \xrightarrow{\Delta} CaO (s) + CO_2 (g)$

Number of mole $\left(\dfrac{25}{100}\right)$ $\left(\dfrac{25}{100}\right)$

Amount of $CO_2 = \left(\dfrac{25}{100}\right) \times 44 = 11$ gram.

8. Mass of a liquid is weighed correct to three decimal place and its volume is measured correct to one decimal place. The density of the liquid calculated from the above data will be correct to **[KVPY_2007_SB]**

(A) three decimal place (B) two decimal place (C) one decimal place (D) four decimal place

Ans. **(C)**

9. When the size of a spherical nanoparticle decreases from 30 nm to 10 nm, the ratio surface area/volume becomes **[KVPY_2011_SB]**

(A) 1/3 of the original (B) 3 times the original (C) 1/9 of the original (D) 9 times the original

Sol. **(B)**

$r_1 = 30$ nm $r_2 = 10$ nm

$$\left(\frac{\text{surface area}}{\text{volume}}\right) = \frac{4\pi r^2}{\frac{4\pi r^3}{3}} = \frac{3}{r} \Rightarrow \frac{(3/r_2)}{(3/r_1)} = \frac{30}{10} = 3$$

10. Emulsification of 10 ml of oil in water produces 2.4×10^{18} droplets. If the surfaces tension at the oil-water interface is 0.03 Jm^{-2} and the area of each droplet is 12.5×10^{-16} m^2, the energy spent in the formation of oil droplets is : **[KVPY_2011_SB]**

(A) 90 J (B) 30 J (C) 900 J (D) 10 J

Sol. **(A)**

Total area of droplets $= 2.4 \times 10^{18} \times 12.5 \times 10^{-16} = 3000$ m^2

Energy liberated = 3000 × 0.03 = 90 J

Exercise

1. If the atomic mass of Sodium is 23, the number of moles in 46 g of sodium is :
(A) 1 (B) 2 (C) 2.3 (D) 4.6

2. The charge on 1 gram ions of Al^{3+} is : (N_A = Avogadro number, e = charge on one electron)
(A) $\frac{1}{27} N_A e$ coulomb (B) $\frac{1}{3} \times N_A e$ coulomb (C) $\frac{1}{9} \times N_A e$ coulomb (D) $3 \times N_A e$ coulomb

3. Which of the following contains the greatest number of atoms ?
(A) 1.0 g of butane (C_4H_{10}) (B) 1.0 g of nitrogen (N_2)
(C) 1.0 g of silver (Ag) (D) 1.0 g of water (H_2O)

4. A gaseous mixture contains CO_2(g) and N_2O(g) in 2 : 5 ratio by mass. The ratio of the number of molecules of CO_2(g) and N_2O(g) is **:**
(A) 5 :2 (B) 2 : 5 (C) 1 : 2 (D) 5 : 4

5. A sample of aluminium has a mass of 54.0 g. What is the mass of the same number of magnesium atoms? (At. wt. Al = 27, Mg=24)
(A) 12 g (B) 24 g (C) 48 g (D) 96 g.

6. The weight of a molecule of the compound $C_{60}H_{22}$ is :
(A) 1.09×10^{-21} g (B) 1.24×10^{-21} g (C) 5.025×10^{-23} g (D) 16.023×10^{-23} g

7. Density of ozone relative to methane under the same temperature & pressure conditions is :
(A) 1 (B) 3 (C) 1.5 (D) 2.5

8. Vapour density of a gas if its density is 0.178 g/L at NTP is :
(A) 0.178 (B) 2 (C) 4 (D) 0.089

9. A nugget of gold and quartz was found to contain x g of gold and y g of quartz and has density d. If the densities of gold and quartz are d_1 and d_2 respectively then the correct relation is :
(A) $\frac{x}{d_1} + \frac{y}{d_2} = \frac{x+y}{d}$ (B) $xd_1 + yd_2 = (x + y)\, d$
(C) $\frac{x}{d_2} + \frac{y}{d_1} = \frac{x+y}{d}$ (D) $\frac{x+y}{d} + \frac{x}{d_1} + \frac{x}{d_2} = 0$

10. The atomic weights of two elements A and B are 40 and 80 respectively. If x g of A contains y atoms, how many atoms are present in 2x g of B?
(A) $\frac{y}{2}$ (B) $\frac{y}{4}$ (C) y (D) 2y

11. The empirical formula of a compound of molecular mass 120 is CH_2O. The molecular formula of the compound is :
(A) $C_2H_4O_2$ (B) $C_4H_8O_4$ (C) $C_3H_6O_3$ (D) all of these

12. Calculate the molecular formula of compound which contains 20% Ca and 80% Br (by wt.) if molecular weight of compound is 200. (Atomic wt. Ca = 40, Br = 80)
(A) $Ca_{1/2}Br$ (B) $CaBr_2$ (C) CaBr (D) Ca_2Br

13. What weight of $CaCO_3$ must be decomposed to produce the sufficient quantity of carbon dioxide to convert 21.2 kg of Na_2CO_3 completely in to $NaHCO_3$. [Atomic mass Na = 23, Ca = 40]
$CaCO_3 \longrightarrow CaO + CO_2$
$Na_2CO_3 + CO_2 + H_2O \longrightarrow 2NaHCO_3$
(A) 100 Kg (B) 20 Kg (C) 120 Kg (D) 30 Kg

14. 12 g of alkaline earth metal gives 14.8 g of its nitride. Atomic weight of metal is
(A) 12 (B) 20 (C) 40 (D) 14.8

15. Calculate the amount of Ni needed in the Mond's process given below

$$Ni + 4CO \longrightarrow Ni(CO)_4$$

If CO used in this process is obtained through a process, in which 6 g of carbon is mixed with 44 g CO_2.
(A) 14.675 g (B) 29 g (C) 58 g (D) 28 g

16. For the reaction $2P + Q \rightarrow R$, 8 mol of P and 5 mol of Q will produce
(A) 8 mol of R (B) 5 mol of R (C) 4 mol of R (D) 13 mol of R

17. How many mole of $Zn(FeS_2)$ can be made from 2 mole zinc, 3 mole iron and 5 mole sulphur.
(A) 2 mole (B) 3 mole (C) 4 mole (D) 5 mole

18. Equal weight of 'X' (At. wt. = 36) and 'Y' (At. wt. = 24) are reacted to form the compound X_2Y_3. Then :
(A) X is the limiting reagent
(B) Y is the limiting reagent
(C) No reactant is left over and mass of X_2Y_3 formed is double the mass of 'X' taken
(D) none of these

19. 25.4 g of iodine and 14.2g of chlorine are made to react completely to yield a mixture of ICl and ICl_3. Calculate the number of moles of ICl and ICl_3 formed.
(A) 0.1 mole, 0.1 mole (B) 0.1 mole, 0.2 mole (C) 0.5 mole, 0.5 mole (D) 0.2 mole, 0.2 mole

20. What weights of P_4O_6 and P_4O_{10} will be produced by the combustion of 31g of P_4 in 32g of oxygen leaving no P_4 and O_2.
(A) 2.75g, 219.5g (B) 27.5g, 35.5g (C) 55g, 71g (D) 17.5g, 190.5g

21. If 500 ml of 1 M solution of glucose is mixed with 500 ml of 1 M solution of glucose final molarity of solution will be :
(A) 1 M (B) 0.5 M (C) 2 M (D) 1.5 M

22. The volume of water that must be added to a mixture of 250 ml of 0.6 M HCl and 750 ml of 0.2 M HCl to obtain 0.25 M solution of HCl is :
(A) 750 ml (B) 100 ml (C) 200 mℓ (D) 300 mℓ

23. What volume of 0.10 M H_2SO_4 must be added to 50 mL of a 0.10 M NaOH solution to make a solution in which the molarity of the H_2SO_4 is 0.050 M ?
(A) 400 mL (B) 50 mL (C) 100 mL (D) 150 mL

24. What approximate volume of 0.40 M $Ba(OH)_2$ must be added to 50.0 mL of 0.30 M NaOH to get a solution in which the molarity of the OH^- ions is 0.50 M?
(A) 33 mL (B) 66 mL (C) 133 mL (D) 100 mL

25. Mole fraction of A in H_2O is 0.2. The molality of A in H_2O is :
(A) 13.9 (B) 15.5 (C) 14.5 (D) 16.8

26. What is the molarity of H_2SO_4 solution that has a density of 1.84 g/cc and contains 98% by mass of H_2SO_4? (Given atomic mass of S = 32)
(A) 4.18 M (B) 8.14 M (C) 18.4 M (D) 18 M

27. The molarity of the solution containing 2.8%(mass / volume) solution of KOH is : (Given atomic mass of K = 39) is :
(A) 0.1 M (B) 0.5 M (C) 0.2 M (D) 1 M

28. Four 1-1 litre flasks are separately filled with the gases H_2, He, O_2 and O_3 at the same temperature and pressure. The ratio of total number of atoms of these gases present in different flask would be :
(A) 1 : 1 : 1 : 1 (B) 1 : 2 : 2 : 3 (C) 2 : 1 : 2 : 3 (D) 3 : 2 : 2 : 1

29. Which of the following expressions is correct (n = no. of moles of the gas, N_A = Avogadro constant, m = mass of 1 molecule of the gas, N = no. of molecules of the gas)?
(A) $n = m N_A$ (B) $m = N_A$ (C) $N = nN_A$ (D) $m = mn/N_A$

30. The volume of 1 mol of a gas at standard temperature and pressure is
(A) 11.2 litres (B) 22.4 litres (C) 100 litres (D) None of these

31. 2 moles of nitrogen atoms at NTP occupy a volume of :
(A) 11.2 L (B) 44.8 L (C) 22.4 L (D) 5.6 L

32. Under the same conditions, two gases have the same number of molecules. They must
(A) be noble gases (B) have equal volumes
(C) have a volume of 22.4 dm^3 each (D) have an equal number of atoms

33. 3g of a hydrocarbon on combustion in excess of oxygen produces 8.8 g of CO_2 and 5.4 g of H_2O. The data illustrates the law of :
(A) conservation of mass (B) multiple proportions
(C) constant proportions (D) none of these

34. The vapour density of a gas A is twice that of a gas B. If the molecular weight of B is M, the molecular weight of A will be :
(A) M (B) 2M (C) 3M (D) M / 2

35. The relative density of a gas A with respect to another gas B is 2. The vapour density of the gas B is 20, the vapour density of the gas A is :
(A) 30 (B) 40 (C) 50 (D) 60

36. Which is incorrect statement about 1.7 g of NH_3
(A) It contain 0.3 mol H – atom (B) it contain 2.408×10^{23} atoms
(C) Mass % of hydrogen is 17.65% (D) vapour density of NH_3 is 17

37. NX is produced by the following step of reactions

$$M + X_2 \longrightarrow MX_2$$
$$3MX_2 + X_2 \longrightarrow M_3X_8$$
$$M_3X_8 + N_2CO_3 \longrightarrow NX + CO_2 + M_3O_4$$

How much M (metal) is consumed to produce 206 gm of NX. (Take at wt of M = 56, N=23, X = 80)

(A) 42 gm (B) 56 gm (C) $\frac{14}{3}$ gm (D) $\frac{7}{4}$ gm

38. $A + B \rightarrow A_3B_2$ (unbalanced)
$A_3B_2 + C \rightarrow A_3B_2C_2$ (unbalanced)
Above two reactions are carried out by taking 3 moles each of A and B and one mole of C. Then which option is/are correct ?
(A) 1 mole of $A_3B_2C_2$ is formed (B) $1/2$ mole of $A_3B_2C_2$ is formed
(C) 1/2 mole of A_3B_2 is formed (D) $1/2$ mole of A_3B_2 is left finally

39. If 27 g of Carbon is mixed with 88 g of Oxygen and is allowed to burn to produce CO_2 , then which is incorrect:
(A) Oxygen is the limiting reagent. (B) Volume of CO_2 gas produced at NTP is 50.4 L.
(C) C and O combine in mass ratio 3 : 8. (D) (A) & (C) both.

40. In a certain operation 358 g of $TiCl_4$ is reacted with 96 g of Mg. Calculate % yield of Ti if 32 g of Ti is actually obtained [At. wt. Ti = 48, Mg = 24]
(A) 35.38 % (B) 66.6 % (C) 100 % (D) 60 %

41. Find out % of O & H in H_2O compound.

42. Acetylene & butene have empirical formula CH & CH_2 respectively. The molecular mass of acetylene and butene are 26 & 56 respectively deduce their molecular formula.

43. An oxide of nitrogen gave the following percentage composition :

N = 25.94 and O = 74.06

Calculate the empirical formula of the compound.

44. Find the density of CO_2(g) with respect to N_2O(g).

45. Formation of polyethene from calcium carbide takes place as follows :

$$CaC_2 + H_2O \rightarrow Ca(OH)_2 + C_2H_2 \rightarrow C_2H_4;$$
$$n(C_2H_4) \rightarrow (-CH_2-CH_2-)_n.$$

Determine the amount of polyethylene possibly obtainable from 64.0 kg $CaCl_2$ can be.

46. The molality of a sulphuric acid solution is 0.2. Calculate the total weight of the solution having 1000 gm of solvent.

47. When 170 g NH_3 (M =17) decomposes how many grams of N_2 & H_2 is produced.

48. 340 g NH_3 (M = 17) when decompose how many litres of nitrogen gas is produced at STP.

49. 4 mole of $MgCO_3$ is reacted with 6 moles of HCl solution. Find the volume of CO_2 gas produced at STP, the reaction is

$$MgCO_3 + 2HCl \rightarrow MgCl_2 + CO_2 + H_2O.$$

50. 117 gm NaCl is dissolved in 500 ml aqueous solution. Find the molarity of the solution.

Answers

1.	(B)	**2.**	(D)	**3.**	(A)	**4.**	(B)	**5.**	(C)	**6.**	(B)	**7.**	(B)
8.	(B)	**9.**	(A)	**10.**	(C)	**11.**	(B)	**12.**	(B)	**13.**	(B)	**14.**	(C)
15.	(A)	**16.**	(C)	**17.**	(A)	**18.**	(C)	**19.**	(A)	**20.**	(B)	**21.**	(A)
22.	(C)	**23.**	(C)	**24.**	(A)	**25.**	(A)	**26.**	(C)	**27.**	(B)	**28.**	(C)
29.	(C)	**30.**	(B)	**31.**	(C)	**32.**	(B)	**33.**	(A)	**34.**	(B)	**35.**	(B)
36.	(D)	**37.**	(A)	**38.**	(B)	**39.**	(A)	**40.**	(A)	**41.**	88.89%, 11.11%		
42.	C_2H_2 and C_4H_8.			**43.**	N_2O_5	**44.**	1	**45.**	28 Kg.	**46.**	1019.6 g.		
47.	140 g, 30 g.			**48.**	224 lit.	**49.**	67.2 lit.	**50.**	4 M.				

FUNDAMENTALS OF MATHEMATICS

NUMBER SYSTEM

CLASSIFICATION OF NUMBERS

(i) Natural numbers :
Counting numbers are known as **natural numbers**.
N = { 1, 2, 3, 4, ... }.

(ii) Whole numbers :
All natural numbers together with 0 form the collection of all **whole numbers**.
W = { 0, 1, 2, 3, 4, ... }.

(iii) Integers :
All natural numbers, 0 and negative of natural numbers form the collection of all **integers**.
I or **Z** = { ..., – 3, – 2, – 1, 0, 1, 2, 3, ... }.

(iv) Rational numbers :

These are real numbers which can be expressed in the form of $\frac{p}{q}$, where **p** and **q** are integers and $q \neq 0$.

e.g. 2/3, 37/15, -17/19.

❖ All natural numbers, whole numbers and integers are rational.

❖ Rational numbers include all Integers (without any decimal part to it), terminating fractions (fractions in which the decimal parts are terminating e.g. 0.75, – 0.02 etc.) and also non-terminating but recurring decimals e.g. 0.666....., – 2.333...., etc.

Fractions :

(a) Common fraction **:** Fractions whose denominator is not 10.

(b) Decimal fraction **:** Fractions whose denominator is 10 or any power of 10.

(c) Proper fraction **:** Numerator < Denominator i.e. $\frac{3}{5}$.

(d) Improper fraction **:** Numerator > Denominator i.e. $\frac{5}{3}$.

(e) Mixed fraction **:** Consists of integral as well as fractional part i.e. $3\frac{2}{7}$.

(f) Compound fraction **:** Fraction whose numerator and denominator themselves are fractions. i.e. $\frac{2/3}{5/7}$.

❖ Improper fraction can be written in the form of mixed fraction.

(v) Irrational Numbers :

All real number which are not rational are **irrational numbers**. These are non-recurring as well as non-terminating type of decimal numbers.

For Ex. : $\sqrt{2}$, $\sqrt[3]{4}$, $2+\sqrt{3}$, $\sqrt{2+\sqrt{3}}$, $\sqrt[4]{\sqrt[7]{3}}$ etc.

(vi) Real numbers : Numbers which can represent actual physical quantities in a meaningful way are known as **real numbers**. These can be represented on the number line. Number line is geometrical straight line with arbitrarily defined zero (origin).

(vii) Prime numbers : All natural numbers that have one and itself only as their factors are called **prime numbers** i.e. prime numbers are exactly divisible by 1 and themselves. e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23,...etc. If P is the set of prime number then **P** = {2, 3, 5, 7,...}.

(viii) Composite numbers : All natural numbers, which are not prime are **composite numbers**. If C is the set of composite number then **C** = {4, 6, 8, 9, 10, 12,...}.

- 1 is neither prime nor composite number.

(ix) Co-prime Numbers : If the H.C.F. of the given numbers (not necessarily prime) is 1 then they are known as **co-prime numbers**. e.g. 4, 9 are co-prime as H.C.F. of (4, 9) = 1.

- Any two consecutive numbers will always be co-prime.

(x) Even Numbers : All integers which are divisible by 2 are called **even numbers**. Even numbers are denoted by the expression 2n, where n is any integer. So, if E is a set of even numbers, then **E** = { ..., – 4, –2, 0, 2, 4,...}.

(xi) Odd Numbers : All integers which are not divisible by 2 are called **odd numbers**. Odd numbers are denoted by the general expression 2n –1 where n is any integer. If O is a set of odd numbers, then **O** = {..., –5, –3, –1, 1, 3, 5,...}.

(xii) Imaginary Numbers : All the numbers whose square is negative are called **imaginary numbers**.

e.g. 3i, -4i, i, ... ; where **i =** $\sqrt{-1}$.

(xiii) Complex Numbers : The combined form of real and imaginary numbers is known as **complex numbers**. It is denoted by **Z = A + iB** where **A** is real part and **B** is imaginary part of Z and **A, B** $\in$ **R**.

- The set of complex number is the super set of all the sets of numbers.

SQUARES AND SQUARE ROOT

Squares : When a number is multiplied by itself then the product is called the **square** of that number.

Perfect Square : A natural number is called a **perfect square** if it is the square of any other natural number e.g. 1, 4, 9,... are the squares of 1, 2, 3,... respectively.

Square roots : The square root of a number **x** is that number which when multiplied by itself gives **x** as the product. As we say square of 3 is 9, then we can also say that square root of 9 is 3.

The symbol use to indicate the square root of a number is '$\sqrt{\ }$', i.e. $\sqrt{81}$ = 9, $\sqrt{225}$ = 15 ...etc.

- We can calculate the square root of positive numbers only. However the square root of a positive number may be a positive or a negative number.
  e.g. $\sqrt{25}$ = + 5 or – 5.

Properties of Square Roots :

(i) If the unit digit of a number is 2, 3, 7 or 8, then it does not have a square root in N.

(ii) If a number ends in an odd number of zeros, then it does not have a square root in N.

(iii) The square root of an even number is even and square root of an odd number is odd. e.g. $\sqrt{81}$ = 9, $\sqrt{256}$ = 16, $\sqrt{324}$ = 18 ...etc.

(iv) Negative numbers have no square root in set of real numbers.

CUBES AND CUBE ROOT

Cube : If any number is multiplied by itself three times then the result is called the **cube** of that number.

Perfect cube : A natural number is said to be a **perfect cube** if it is the cube of any other natural number.

SURDS AND EXPONENTS (INDICES)

Any irrational number of the form $\sqrt[n]{a}$ is given a special name **Surd**. Where **'a'** is called radicand, rational. Also the symbol $\sqrt[n]{\ }$ is called the **radical** sign and the **index n** is called **order of the surd**.

$\sqrt[n]{a}$ is read as **nth root of 'a'** and can also be written as $a^{\frac{1}{n}}$.

Identification of Surds :

(i) $\sqrt[3]{4}$ is a surd as radicand is a rational number.

Similar examples : $\sqrt[3]{5}, \sqrt[4]{12}, \sqrt[5]{7}, \sqrt{12}, ...$

(ii) $2 + \sqrt{3}$ is a surd (as surd + rational number will give a surd)

Similar examples : $3-\sqrt{2}, \sqrt{3}+1, \sqrt[3]{3}+1, ...$

(iii) $\sqrt{7-4\sqrt{3}}$ is a surd as $7 - 4\sqrt{3}$ is a perfect square of $(2-\sqrt{3})$.

Similar examples : $\sqrt{7+4\sqrt{3}}, \sqrt{9-4\sqrt{5}}, \sqrt{9+4\sqrt{5}}, ...$

(iv) $\sqrt[3]{\sqrt{3}}$ is a surd as $\sqrt[3]{\sqrt{3}} = \left(3^{\frac{1}{2}}\right)^{\frac{1}{3}} = 3^{\frac{1}{6}} = \sqrt[6]{3}$

Similar examples : $\sqrt[3]{\sqrt[3]{5}}, \sqrt[4]{\sqrt[5]{6}}, ...$

(v) These are not a surds :

(A) $\sqrt[3]{8}$, because $\sqrt[3]{8} = \sqrt[3]{2^3}$ which is a rational number.

(B) $\sqrt{2+\sqrt{3}}$, because $2 + \sqrt{3}$ is not a perfect square.

(C) $\sqrt[3]{1+\sqrt{3}}$, because radicand is an irrational number.

Laws of Surds :

(i) $\left(\sqrt[n]{a}\right)^n = \sqrt[n]{a^n} = a$

(ii) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ [Here order should be same]

(iii) $\sqrt[n]{a} \div \sqrt[n]{b} = \sqrt[n]{\frac{a}{b}}$

(iv) $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a} = \sqrt[m]{\sqrt[n]{a}}$

(v) $\sqrt[n]{a} = \sqrt[n\times p]{a^p}$ or, $\sqrt[n]{a^m} = \sqrt[n\times p]{a^{m\times p}}$ [Important for changing order of surds]

Comparison of Surds :

It is clear that if **x > y > 0** and **n > 1** is a **(+ve)** integer then $\sqrt[n]{x} > \sqrt[n]{y}$. e.g. $\sqrt[3]{16} > \sqrt[3]{12}$, $\sqrt[5]{36} > \sqrt[5]{25}$ and so on.

Conjugate Surds :

- R.F. of $\sqrt{a}-\sqrt{b}$ and $\sqrt{a}+\sqrt{b}$ type surds are called **conjugate surds** .
- Sometimes conjugate surd and reciprocals are same.

FACTORS AND MULTIPLES

Factors : 'a' is a factor of **'b'** if there exists a relation such that **a × n = b**, where **'n'** is any natural number.

- 1 is a factor of all numbers as 1 × b = b.
- Factor of a number cannot be greater than the number (infact the largest factor will be the number itself). Thus factors of any number will lie between 1 and the number itself (both inclusive) and they are limited.

Multiples : 'a' is a multiple of **'b'** if there exists a relation of the type **b × n = a**. Thus the multiples of 6 are 6 × 1 = 6, 6 × 2 = 12, 6 × 3 = 18, 6 × 4 = 24, and so on.

- The smallest multiple will be the number itself and the number of multiples would be infinite.
- **NOTE :**

To understand what multiples are, let's just take an example of multiples of 3. The multiples are 3, 6, 9, 12,.... so on. We find that every successive multiples appears as the third number after the previous.

So if one wishes to find the number of multiples of 6 less than 255, we could arrive at the number through $\frac{255}{6}$ = 42 (and the remainder 3). The remainder is of no consequence to us. So in all there are 42 multiples. If one wishes to find the multiples of 36, find $\frac{255}{36}$ = 7 (and the remainder is 3). Hence, there are 7 multiples of 36.

Factorization : It is the process of splitting any number into form where it is expressed only in terms of the most basic prime factors.
For example, $36 = 2^2 \times 3^2$. 36 is expressed in the factorized from in terms of its basic prime factors.

Number of factors : For any composite number C, which can be expressed as $C = a^p \times b^q \times c^r \times .....$, where a, b, c ..... are all prime factors and p, q, r are positive integers, then the number of factors is equal to $(p + 1) \times (q + 1) \times (r + 1)$....
e.g. $36 = 2^2 \times 3^2$. So the factors of $36 = (2 + 1) \times (2 + 1) = 3 \times 3 = 9$.

HCF AND LCM

LCM (least Common Multiple) : The LCM of given numbers, as the name suggests is the smallest positive number which is a multiple of each of the given numbers.

HCF (Highest Common factor) : The HCF of given numbers, as the name suggests is the largest factor of the given set of numbers.
Consider the numbers 12, 20 and 30. The factors and the multiples are :

Factors	Given numbers	Multiples
1, 2, 3, 4, 6, 12	12	12, 24, 36, 48, 60, 72, 84, 96, 108, 120....
1, 2, 4, 5, 10, 20	20	20, 40, 60, 80, 100, 120.....
1, 2, 3, 5, 6, 10, 15, 30	30	30, 60, 90, 120....

The common factors are 1 and 2 and the common multiples are 60, 120...
Thus the highest common factor is 2 and the least common multiple meaning of HCF is that the HCF is the largest number that divides all the given numbers.
Also since a number divides its multiple, the meaning of LCM is that it is smallest number which can be divided by the given numbers.
❖ HCF will be lesser than or equal to the least of the numbers and LCM will be greater than or equal to the greatest of the numbers.

❖ For any two numbers x and y :

$x \times y = HCF(x, y) \times LCM(x, y)$.

HCF and LCM of fractions :

$$\text{LCM of fractions} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

$$\text{HCF of fractions} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$$

Make sure the fractions are in the most reducible form.

DIVISIBILITY

Division Algorithm : General representation of result is,

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

Dividend = (Divisor × Quotient) + Remainder

NOTE :
(i) $(x^n - a^n)$ is divisible by $(x - a)$ for all the values of n.
(ii) $(x^n - a^n)$ is divisible by $(x + a)$ and $(x - a)$ for all the even values of n.
(iii) $(x^n + a^n)$ is divisible by $(x + a)$ for all the odd values of n.

Test of Divisibility :

No.	Divisiblity Test
2	Unit digit should be 0 or even
3	The sum of digits of no. should be divisible by 3
4	The no formed by last 2 digits of given no. should be divisible by 4.
5	Unit digit should be 0 or 5.
6	No should be divisible by 2 & 3 both
8	The number formed by last 3 digits of given no. should be divisible by 8.
9	Sum of digits of given no. should be divisible by 9
11	The difference between sums of the digits at even & at odd places should be zero or multiple of 11.
25	Last 2 digits of the number should be 00, 25, 50 or 75.

Rule for 7 : Double the last digit of given number and subtract from remaining number the result should be zero or divisible by 7.

REMAINDERS

The method of finding the remainder without actually performing the process of division is termed as **remainder theorem**.

❖ Remainder should always be positive. For example if we divide –22 by 7, generally we get –3 as quotient and –1 as remainder. But this is wrong because remainder is never be negative hence the quotient should be –4 and remainder is +6. We can also get remainder 6 by adding –1 to divisor 7 (7–1 = 6).

CYCLICITY

We are having 10 digits in our number systems and some of them shows special characteristics like they, repeat their unit digit after a cycle, for example 1 repeat its unit digit after every consecutive power. So, its cyclicity is 1 on the other hand digit 2 repeat its unit digit after every four power, hence the cyclicity of 2 is four. The cyclicity of digits are as follows:

Digit	Cyclicity
0, 1, 5 and 6	1
4 and 9	2
2, 3, 7 and 8	4

So, if we want to find the last digit of 2^{45}, divide 45 by 4. The remainder is 1 so the last digit of 2^{45} would be same as the last digit of 2^1 which is 2.

To Find the Unit Digit in Exponential Expressions :

(i) When there is 2 in unit's place of any number.
Since, in 2^1 unit digit is 2, in 2^2 unit digit is 4, in 2^3 unit digit is 8, in 2^4 unit digit is 6, after that the unit's digit repeats. e.g. unit digit$(12)^{12}$ is equal to the unit digit of, 2^4 i.e.6

(ii) When there is 3 in unit's place of any number.
Since, in 3^1 unit digit is 3, in 3^2 unit digit is 9, in 3^3 unit digit is 7, in 3^4 unit digit is 1, after that the unit's digit repeats.

(iii) When there is 4 in unit's place of any number.
Since, in 4^1 unit digit is 4, in 4^2 unit digit is 6, after that the unit's digit repeats.

(iv) When there is 5 in unit's place of any number.
Since, in 5^1 unit digit is 5, in 5^2 unit digit is 5 and so on.

(v) When there is 6 in unit's place of any number.
Since, in 6^1 unit digit is 6, in 6^2 unit digit is 6 & so on.

(vi) When there is 7 in unit's place of any number.
Since, in 7^1 unit digit is 7, in 7^2 unit digit is 9, in 7^3 unit digit is 3, in 7^4 unit digit is 1, after that the unit's digit repeats.

(vii) When there is 8 in unit's place of any number.
Since, in 8^1 unit digit is 8, in 8^2 unit digit is 4, in 8^3 unit digit is 2, in 8^4 unit digit is 6, after that unit's digit repeats after a group of 4.

(viii) When there is 9 in unit's place of any number.
Since, in 9^1 unit's digit is 9, in 9^2 unit's digit is 1, after that unit's digit repeats after a group of 2.

(ix) When there is zero in unit's place of any number.
There will always be zero in unit's place.

To Find the Last Two Digits in Exponential Expressions :

❖ We know that the binomial theorem : $(a + b)^n = a^n + \frac{n}{1!} a^{n-1}b + \frac{n(n-1)}{2!} a^{n-2}b^2 + ... + b^n$.

(i) Last two digits of numbers ending in 1 :
Let's start with some example.

Ex. What are the last two digits of 31^{786}?

Sol. $31^{786} = (30 + 1)^{786} = 30^{786} + 786 \times 30^{785} \times 1 + \frac{786\,(786-1)}{2!} \times 30^{784} \times 1^2 + ... + 1^{786}$.

Note that all the terms excluding last two terms will end in two or more zeroes. The last two terms are $786 \times 30 \times 1^{785}$ and 1^{786}. Now, the second last term will end with one zero and the tens digit of the second last term will be the product of 786 and 3 i.e. 8. Therefore, the last two digits of the second last term will be 80. The last digit of the last term is 1. So the last two digits of 31^{786} are 81.

(ii) Last two digits of numbers ending in 3, 7 or 9 :

Ex. Find the last two digits of 19^{266}.

Sol. $19^{266} = (19^2)^{133}$. Now, 19^2 ends in 61 ($19^2 = 361$) therefore, we need to find the last two digits of $(61)^{133}$.
Once the number is ending in 1 we can straight away get the last two digits with the help of the previous method. The last two digits are 81 (6 × 3 = 18, so the tenth digit will be 8 and last digit will be 1).

(iii) Last two digits of numbers ending in 2, 4, 6 or 8 :

There is only one even two-digit number which always ends in itself (last two digits) - 76 i.e. 76 raised to any power gives the last two digits as 76. Therefore, our purpose is to get 76 as last two digits for even numbers. We know that 24^2 ends in 76 and 2^{10} ends in 24. Also, 24 raised to an even power always ends with 76 and 24 raised to an odd power always ends with 24. Therefore, 24^{34} will end in 76 and 24^{53} will end in 24.

Ex. Find the last two digits of 2^{543}.

Sol. $2^{543} = (2^{10})^{54} \times 2^3 = (24)^{54}$ (24 raised to an even power). $2^3 = 76 \times 8 = 08$.

HIGHEST POWER DIVIDING A FACTORIAL

Factorial n : Product of **n** consecutive natural numbers is known as **'factorial n'** it is denoted by **'n!'**.

So, n! = n.(n – 1).(n – 2).............3.2.1.

e.g. 5! = 5 × 4 × 3 × 2 × 1 = 120.

❖ The value of factorial zero is equal to the value of factorial one. Hence **0! = 1 = 1!**

The approach to finding the highest power of **x** dividing **y!** is $\left[\frac{y}{x}\right] + \left[\frac{y}{x^2}\right] + \left[\frac{y}{x^3}\right]$......., where **[ ]** represents just the integral part of the answer and ignoring the fractional part.

PERCENTAGE

The word **'percentage'** literally means **'per hundred'** or **'for every hundred.'** Therefore, whenever we calculate something as a part of 100, that part is numerically termed as percentage.
In other words, percentage is a ratio whose second term is equal to 100. i.e. 1 : 4 can be written as 25 : 100 or 25%, 3 : 8 can be written as 37.5 : 100 or 37.5%, 3 : 2 can be written as 150 : 100 or 150%, and so on.

❖ **To express a% as a fraction divide it by 100.**
i.e. a% = a/100

❖ **To express a fraction (x/y) as a percent multiply it by 100.**
i.e. x/y = (x/y x 100)%

Basic Formula of Percentage :

❖ **p% of a number N is = N ×** $\frac{p}{100}$.

❖ **To increase or decrease a number by x% , multiply the number by** $\frac{[100 \pm x]}{100}$.

Where, (+) $\Rightarrow$ Increase, (–) $\Rightarrow$ Decrease.

REMARK :
To solve these type of problems calculate x% of given number & add or subtract the value from given number for increase or decrease respectively.

❖ **To calculate what percentage of a is b, use the formula : Percentage =** $\frac{b}{a} \times 100$.

❖ **Percentage increase/decrease when a quantity 'a' is increased/decreased to become another quantity 'b'.**

$$\text{Percentage Increase/Decrease} = \frac{\text{Increase/Decrease}}{\text{Initial Value}} \times 100 = \begin{bmatrix} \frac{b-a}{a} \times 100, \text{ when } b > a \text{ ;(increase)} \\ \frac{a-b}{a} \times 100, \text{ when } b < a \text{ ;(decrease)} \end{bmatrix}$$

$$\text{Therefore new quantity b} = \begin{bmatrix} a \times \left(1 + \frac{\text{percentage increase}}{100}\right) \\ a \times \left(1 - \frac{\text{percentage decrease}}{100}\right) \end{bmatrix}$$

❖ **If one quantity A is x% more or less than another quantity B, than B is less or more than A by :** $\left(\frac{x}{100 \pm x}\right) \times 100$

IMPORTANT CONCEPTS ASSOCIATED WITH PERCENTAGE

Conversion of Fractions into Percentages :

Knowing conversion of common fractions into percentages helps your convert many fractions into percentage immediately, For example, knowing that $\frac{1}{8}$ = 12.5% will help you convert fractions like $\frac{3}{8}$ or $\frac{5}{8}$ into percentages immediately.

❖ **Given below are the fractions converted into percentage.**

Fraction	Percentage	Fraction	Percentage	Fraction	Percentage
$\frac{1}{2}$	50%	$\frac{1}{10}$	10%	$\frac{1}{18}$	5.55%
$\frac{1}{3}$	33.33%	$\frac{1}{11}$	9.09%	$\frac{1}{19}$	5.26%
$\frac{1}{4}$	25%	$\frac{1}{12}$	8.33%	$\frac{1}{20}$	5%
$\frac{1}{5}$	20%	$\frac{1}{13}$	7.69%	$\frac{1}{21}$	4.76%
$\frac{1}{6}$	16.66%	$\frac{1}{14}$	7.14%	$\frac{1}{22}$	4.54%
$\frac{1}{7}$	14.28%	$\frac{1}{15}$	6.66%	$\frac{1}{23}$	4.34%
$\frac{1}{8}$	12.50%	$\frac{1}{16}$	6.25%	$\frac{1}{24}$	4.16%
$\frac{1}{9}$	11.11%	$\frac{1}{17}$	5.88%	$\frac{1}{25}$	4%

SUCCESSIVE CHANGES IN PERCENTAGE

If a quantity **x** is increased or decreased successively by A%, B%, C% then the final value of **x** will be

$$= x\left(1 \pm \frac{A}{100}\right)\left(1 \pm \frac{B}{100}\right)\left(1 \pm \frac{C}{100}\right).$$

❖ Let the present population of town be **P** and let there be an increase or decrease of **R%** per annum.

Then Population after **n** years = $P\left(1 \pm \frac{R}{100}\right)^n$

❖ If length & breadth of a rectangle is changed by **a % & b%** respectively, than **% change** in area will be

$= \left\{a \pm b \pm \frac{(a \times b)}{100}\right\}\%$ (use +ve for increase & -ve for decrease)

DEFINITIONS

(i) Cost price (C.P.) : The amount for which an article is bought is called its cost price, abbreviated to CP.

(ii) Selling price (S.P.) : The amount for which an article is sold is called its selling price, abbreviated to SP.

(iii) Gain : When S.P. > C.P. then there is a gain.
Gain = S.P. – C.P.

(iv) Loss : When S.P. < C.P. then there is a loss.
Loss = C.P. – S.P.

REMARK
The gain or loss is always reckoned on the cost price.

SOME IMPORTANT FORMULAE

(i) Gain = S.P. – C.P.

(ii) Loss = C.P. – S.P.

(iii) Gain% = $\left(\frac{\text{Gain}}{\text{C.P.}} \times 100\right)\%$

(iv) Loss% = $\left(\frac{\text{Loss}}{\text{C.P.}} \times 100\right)\%$

(v) To find S.P. when C.P. and gain% or loss% are given.

(a) S.P. = $\frac{(100 + \text{Gain}\,\%)}{100}$ × C.P.

(b) S.P. = $\frac{(100 - \text{Loss}\,\%)}{100} \times$ C.P.

(vi) To find C.P. when S.P. and gain% or loss% are given

(a) C.P. = $\frac{100}{100 + \text{Gain}\%} \times$ S.P.

(b) C.P. = $\frac{100}{100 - \text{Loss}\%} \times$ S.P.

DISCOUNT

(i) Marked price : In big shops and department stores, every article is tagged with a card and its price is written on it. This is called the **marked price** of that article, abbreviated to MP. For books, the printed price is the marked price.

(ii) List price : Items which are manufactured in a factory are marked with a price according to the list supplied by the factory, at which the retailer is supposed to sell them. This price is known as the **list price** of the article.

(iii) Discount : In order to increase the sale or clear the old stock, sometimes the shopkeepers offer a certain percentage of rebate on the marked price. This rebate is known as **discount**.

❖ **An important fact :** The discount is always reckoned on the marked price.
Clearly, **Selling Price = Marked Price – Discount**

RATIO AND PROPORTION

Ratio :

The comparison of two quantities a and b of similar kind is represented as a : b is called a ratio also it can be represented as $\frac{a}{b}$.

In the ratio a : b, we call **a** as the **first term** or **antecedent** and **b**, the **second term** or **consequent.**

e.g. The ratio 5 : 9 represents $\frac{5}{9}$, with antecedent = 5 and consequent = 9.

❖ The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

e.g. 4 : 5 = 8 : 10 = 12 : 15 etc. Also, 4 : 6 = 2 : 3.

Proportion :

The equality of two ratios is called proportion.
If a : b = c : d, we write, a : b : : c : d and we say that a, b, c, d are in proportion.
where, **a** is called first proportional, **b** is called second proportional, **c** is called third proportional and **d** is called fourth proportional.

❖ **Law of Proportion :**
Product of means = Product of extremes
Thus, if a : b : : c : d $\Rightarrow$ (b × c) = (a × d),
Here a and d are called **extremes,** while b and c are called **mean terms.**

❖ Mean proportional of two given numbers **a** and **b** is $\sqrt{ab}$.

Some other ratios :

Compounded Ratio : The compounded ratio of the ratios (a : b), (c : d), (e : f) is (ace : bdf).

Duplicate ratio : The duplicate ratio of (a : b) is ($a^2 : b^2$).

Sub-duplicate ratio : The sub-duplicate ratio of (a : b) is ($\sqrt{a}$: $\sqrt{b}$).

Triplicate ratio : The triplicate ratio of (a : b) is ($a^3 : b^3$).

Sub-triplicate ratio : The sub-triplicate ratio of (a : b) is $\left(a^{\frac{1}{3}} : b^{\frac{1}{3}}\right)$.

Componendo : If $\frac{a}{b} = \frac{c}{d}$ then, the componendo is $\frac{a+b}{b} = \frac{c+d}{d}$.

Dividendo : If $\frac{a}{b} = \frac{c}{d}$ then, the dividendo is $\frac{a-b}{b} = \frac{c-d}{d}$.

Componendo and Dividendo : If $\frac{a}{b} = \frac{c}{d}$, then the componendo-dividendo is $\frac{a+b}{a-b} = \frac{c+d}{c-d}$.

VARIATION :

(i) We say that **x** is directly proportional to **y**, if **x = ky** for some constant **k** and we write, $\mathbf{x \propto y}$.

(ii) We say that **x** is inversely proportional to **y**, if **xy = k** for some constant **k** and we write, $\mathbf{x \propto \frac{1}{y}}$.

MIXTURE AND ALLIGATION

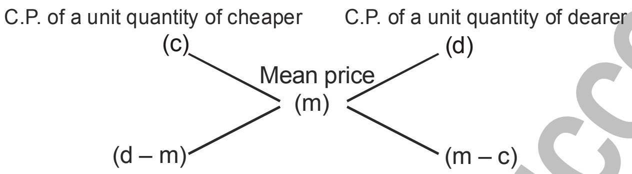
Alligation : It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.

Mean Price : The cost price of a unit quantity of mixture is called the **mean price**.

Rule of Alligation : If two ingredients are mixed, then,

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{(\text{C.P. of dearer}) - (\text{Mean price})}{(\text{Mean price}) - (\text{C.P. of cheaper})}$$

We can also represent this thing as under



∴ Suppose a container contains x units of liquid from which y units are taken out and replaced by water. After n operations, the quantity of pure liquid

$$= \left[x\left(1 - \frac{y}{x}\right)^n\right] \text{units.}$$

WORK AND TIME

Work is defined as the amount of job assigned or the amount of job actually done.
Work is always considered as a whole or 1.
Units of work : Work is measured by many units i.e. men-days, men-hours, men-minutes, machine-hours or in general person-time, machine-time.

❖ **If A and B can do a piece of work in x and y days respectively while working alone, then they will take $\left(\frac{xy}{x+y}\right)$ days to complete the work if both are working together.**

Proof : A's one day work = $\frac{1}{x}$

B's one day work = $\frac{1}{y}$

and (A + B)'s one day work = $\frac{1}{x} + \frac{1}{y}$

(A + B)'s one day work = $\frac{x+y}{xy}$

∴ Time taken by both A and B (working together) to complete the work = $\frac{xy}{x+y}$.

❖ **If A, B, C can do a piece of work in x, y, z days respectively while working alone, then will together take $\frac{1}{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}$ days to complete the work.**

❖ **A can finish a work in x days and B is k times as efficient as A (i.e. B will complete the work in $\frac{x}{k}$ days) Then time taken by both A & B working together to finish the job will be $\frac{x}{k+1}$.**

❖ **If A is k times as good as B and takes x days less than B to finish the work. Then the amount of time required by A and B working together is $\frac{kx}{k^2-1}$ days.**

PIPES AND CISTERNS

Here the work done is in terms of filling or emptying a cistern.

Inlet pipe : It is the pipe connected to cistern which fill the cistern (time taken is in +ve).
Outlet pipe : It is the pipe connected to cistern which empties the cistern. (time taken is –ve).

❖ If an inlet pipe fills a cistern in 'a' hours, then $\left(\frac{1}{a}\right)^{th}$ part is filled in 1 hr.

❖ If two inlet pipes A & B can fill a cistern in 'm' & 'n' hours respectively then together they will take $\left(\frac{mn}{m+n}\right)$ hrs. to fill the cistern.

❖ If an inlet pipe fills a cistern in 'm' hours and an outlet pipe empties it in 'n' hours, then the net part filled in 1 hr. when both the pipes are opened is $\left(\frac{1}{m}-\frac{1}{n}\right)$ hours and the cistern will get filled in $\frac{mn}{n-m}$ hours, for cistern to get filled, m < n.

❖ If m > n, the cistern will never get filled, in this case a completely filled cistern gets emptied in $\left(\frac{mn}{m-n}\right)$ hours.

❖ If an inlet pipe fills a cistern in m hrs. and takes n hrs. longer to fill the cistern due to leak in the cistern, then the time in which the leak will empty the cistern in $m \times \left(1+\frac{m}{n}\right)$.

TIME, SPEED AND DISTANCE

❖ Speed = $\frac{\text{distance}}{\text{time}}$

❖ time = $\frac{\text{distance}}{\text{speed}}$

❖ Distance = Speed × time

❖ If a certain distance (from A to B) is covered at **u km/hr** and the same distance (from B to A) is covered at **v km/hr**. then the average speed during the whole journey is = $\frac{2uv}{u+v}$ km/hr.

Average speed :
If a body travels d_1, d_2, d_3,........, d_n distances with speeds S_1,S_2,.......,S_n,........ respectively, then the average speed of the body through the total distance is given by :

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{d_1+d_2+d_3+..........+d_n}{t_1+t_2+t_3+.......+t_n}$$

Where, $t_1 = \frac{d_1}{s_1}$, $t_2 = \frac{d_2}{s_2}$ ...

DEFINITIONS

(i) **Principle :** The money borrowed or lent out is called principle.

(ii) **Interest :** The additional money paid by the borrower is called the interest.

(iii) **Amount :** The total money (interest + principal) paid by the borrower is called the amount. $[\therefore A = P + I]$

(iv) **Rate of interest :** If the borrower paid interest of Rs. x on Rs.100 for 1 year, then the rate of interest is x percent per annum.

(v) **Time :** The period for which the sum is borrowed is called the time.

(vi) **Conversion Period :** The fixed interval of time at the end of which the interest is calculated and added to the principle at the beginning of the interval is called the conversion period.

(vii) **Simple Interest :** If the principal remains the same throughout the loan period, then the interest paid by the borrower is called simple interest.

$$S.I. = \frac{P \times R \times T}{100}$$

(viii) **Compound Interest :** If the borrower and the lender agree to fix up a certain interval of time (Say, a year or a half year or a quarter of year etc.) so that the Amount (= Principle + Interest) at the end of an interval becomes the principle for the next interval, then the total interest over all the intervals calculated in this way is called the compound interest and is abbreviated as C.I.

Note : S.I. and C.I. are equal for I^{st} year.

SIMPLE INTEREST

To find simple interest and the amount when rate of interest is given as percent per year :

Ex.1 Find the simple interest and the amount on Rs. 2400 for 3 years 5 months and 15 days at the rate of 9%.

Sol. **Given :** Principal (P) = Rs. 2400, Rate (R) = 9%.

Time (T) = 3 years 5 months and 15 days = $\frac{83}{24}$ years.

To find : Simple interest and the amount

Simple interest = $\frac{PRT}{100}$

= Rs. 2400 × $\frac{9}{100} \times \frac{83}{24}$

= Rs. 747

And the amount = Rs. 2400 + Rs. 747

= Rs. 3147.

COMPOUND INTEREST

Computation of Compound Interest when Interest is compounded Annually.

Ex.2 Find the compound interest on Rs. 8000 for 3 year at 5% per annum.

Sol. Principal for the first year = Rs. 8000, Rate = 5% per annum, T = 1 year.

Interest for the first year = $\frac{P \times R \times T}{100}$ = Rs. $\left[\frac{8000 \times 5 \times 1}{100}\right]$

= Rs. 400

$\therefore$ Amount at the end of the first year = Rs. (8000 + 400) = Rs. 8400

Now, principal for the second year = Rs. 8400

Interest for the second year = $\frac{P \times R \times T}{100}$

= Rs. $\left[\frac{8400 \times 5 \times 1}{100}\right]$ = Rs. 420

$\therefore$ Amount at the end of the second year
= Rs. (8400 + 420) = Rs. 8820

Interest for the third year = $\frac{P \times R \times T}{100}$

$$= \text{Rs.} \frac{8820 \times 5 \times 1}{100} = \text{Rs. } 441$$

$\therefore$ Amount at the end of the third year
= Rs. (8820 + 441) = Rs. 9261

Now, we know that total C.I. = Amount – Principal
= Rs. (9261 – 8000) = Rs. 1261

We can also find the C.I. as follows

Total C.I. = Interest for the first year + Interest for the second year + Interest for third year
= Rs. (400 + 420 + 441) = Rs. 1261

Computation of Compound Interest When Interest is compounded Half yearly.

Ex.3 Find the compound interest on Rs. 8000 for $1\frac{1}{2}$ years at 10% per annum, interest being payable half yearly.

Sol. We have Rate of interest = 10% per annum = 5% per half year, Time = $1\frac{1}{2}$ years = 3 half year.

Original principal = Rs. 8000

Interest for the first half year = Rs. $\left[\frac{8000 \times 5 \times 1}{100}\right]$

= Rs. 400

Amount at the end of the first half year = Rs. 8000 + Rs. 400 = Rs. 8400.

Principal for the second half year = Rs. 8400

Interest for the second half year = Rs. $\left(\frac{8400 \times 5 \times 1}{100}\right)$ = Rs. 420

Amount at the end of the second half year
= Rs. 8400 + Rs. 420 = Rs. 8820

Principal for the third half year = Rs. 8820

Interest for the third half year

$$= \text{Rs.} \left(\frac{8820 \times 5 \times 1}{100}\right) = \text{Rs. } 441$$

Amount at the end of third half year = Rs. 8820 + Rs. 441 = Rs. 9261

$\therefore$ Compound interest = Rs. 9261 – Rs. 8000
= Rs. 1261

Computation of compound Interest when Interest is Compounded Quarterly :

Ex.4 Find the compound interest on Rs. 10,000 for 1 year at 20% per annum interest being payable quarterly.

Sol. We have Rate of interest = 20% per annum

= $\frac{20}{4}$ = 5% per quarter

Time = 1 year = 4 quarters.

Principal for the first quarter = Rs. 10000

Interest for the first quarter = Rs. $\left(\frac{10000 \times 5 \times 1}{100}\right)$ = Rs. 500

Amount at the end of first quarter = Rs. 10000 + Rs. 500 = Rs. 10500

Principal for the second quarter = Rs. 10500

Interest for the second quarter = Rs. $\left(\frac{10500 \times 5 \times 1}{100}\right)$ = Rs. 525

Amount at the end of second quarter

= Rs. 10500 + Rs. 525 = Rs. 11025

Principal for the third quarter = Rs. 11025

Interest for the third quarter = Rs. $\frac{11025 \times 5 \times 1}{100}$

= Rs. 551.25

Amount at the end of the third quarter

= Rs. 11025 + Rs. 551.25 = Rs. 11576.25

Principal for the fourth quarter = Rs. 11576.25

Interest for the fourth quarter

= Rs. $\left(\frac{11576.25 \times 5 \times 1}{100}\right)$ = Rs. 578.8125

Amount at the end of the fourth quarter

= Rs. 11576.25 + Rs. 578.8125

= Rs. 12155.0625

Compound interest = Rs. 12155.0625 – Rs. 10000

= Rs. 2155.0625

Computation of compound interest by using formulae :

(i) Let P be the principal and the rate of interest be R% per annum. If the interest is compounded annually then the amount A and the compound interest C.I. at the end of n years.

Given by, $A = P\left(1+\frac{R}{100}\right)^n$

and C.I. = A – P = $P\left\{\left(1+\frac{R}{100}\right)^n - 1\right\}$ respectively

(ii) Let P be the principal and the rate of interest be R% per annum. If the interest is compounded k times in a year annually, then the amount A and the compound interest. C.I. at the end of n years is given by

$$A = P\left(1+\frac{R}{100k}\right)^{nk}$$

and C.I. = A – P = $P\left\{\left(1+\frac{R}{100k}\right)^{nk} - 1\right\}$ respectively

(iii) Let P be the principal and the rate of interest be R_1% for first year, R_2% for second year, R_3% for third year and so on and in the last R_n% for the n^{th} year. Then, the amount A and the compound interest C.I. at the end of n years are given by

$$A = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)..........\left(1+\frac{R_n}{100}\right)$$

and C.I. = A – P respectively

(iv) Let P be the principal and the rate of interest be R% per annum. If the interest is compounded annually but time is the fraction of a year, say $5\frac{1}{4}$ year, then amount A is given by

$$A = P\left(1+\frac{R}{100}\right)^5\left(1+\frac{R/4}{100}\right)$$

and C.I. = A – P

INEQUATIONS

A statement involving variable (s) and the sign of inequality viz, >,< ≥ or ≤ is called an **inequation**.

An inequation may contain one or more variables Also, it may be linear or quadratic or cubic etc.

(i) $3x - 2 < 0$ (ii) $2x^2 + 3x + 4 > 0$ (iii) $2x + 5y \geq 4$

(a) Properties of inequalities

(i) If '**a**' is a positive no. i.e. **a > 0** then for $x < y$

$$\Rightarrow \quad \frac{x}{a} < \frac{y}{a} \text{ \& } ax < ay.$$

(ii) If '**a**' is –ve i.e. **a < 0** then for $x < y$

$$\Rightarrow \quad \frac{x}{a} > \frac{y}{a} \text{ \& } ax > ay.$$

(iii) If '**a**' is a +ve no. i.e. **a > 0** then for $x > y$

$$\Rightarrow \quad \frac{x}{a} > \frac{y}{a} \text{ \& } ax > ay.$$

(iv) If '**a**' is a –ve no. i.e. **a < 0** then for $x > y$

$$\Rightarrow \quad \frac{x}{a} < \frac{y}{a} \text{ \& } ax < ay.$$

TYPES OF INTERVALS

(i) Closed interval : Let **a** and **b** be two given real numbers such that $a < b$. Then the set of all real numbers x such that $a \leq x \leq b$ is called **closed interval** and is denoted by [a, b] may be graphed as :

(ii) Open interval : If **a** and **b** are two real numbers such that $a < b$, then the set of all real numbers x satisfying $a < x < b$ is called an **open interval** and is denoted by (a, b) or]a, b[and may be graphed as :

(iii) Semi-closed or semi-open interval : If **a** and **b** are two real numbers such that $a < b$, then the sets $(a, b] = \{x \in R : a < x \leq b\}$ and $[a, b) = \{x \in R : a \leq x < b\}$ are known as **semi-open** or **semi-closed intervals**. (a, b] and [a, b) are also denoted by]a, b] and [a, b[respectively.

LINEAR INEQUATION IN ONE VARIABLE

Let **a** be a non-zero real number and **x** be a variable. Then inequations of the form $ax + b < 0$, $ax + b \leq 0$, $ax + b > 0$ and $ax + b \geq 0$ are known as linear inequations in one variable **x**.

For example, $9x - 15 > 0$, $5x - 4 \geq 0$, $3x + 2 < 0$ and $2x - 3 \leq 0$ are linear inequations in one variable.

(a) Solving linear inequations in one variable

Rule 1: Same number may be added to (or subtracted from) both side of an inequation without changing the sign of inequality.

Rule 2 : Both sides of an inequation can be multiplied (or divided) by the same positive real number without changing the sign of inequality. However, the sign of inequality is reversed when both sides of an inequation are multiplied or divided by a negative number.

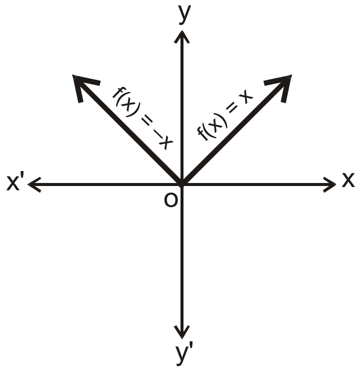
Rule 3 : Any term of an inequation may be taken to the other side with its sign changed without affecting the sign of inequality.

ABSOLUTE VALUE OF A REAL NUMBER

The function f(x) defined by

$$f(x) = |x| = \begin{cases} x, \text{ when } x \geq 0 \\ -x, \text{ when } x < 0 \end{cases}$$

is called the modulus function. It is also called the absolute value function.



The distance between two real numbers x and y is defined as $|x - y|$.

(a) Inequations involving absolute value

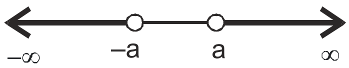
Result 1. If a is a positive real number, then

(i) $|x| < a \Leftrightarrow -a < x < a$ i.e. $x \in (-a, a)$

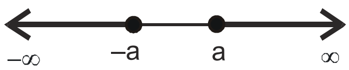
(ii) $|x| \le a \Leftrightarrow -a \le x \le a$ i.e. $x \in [-a, a]$

Result 2. If a is a positive real number, then

(i) $|x| > a \Leftrightarrow x < -a$ or $x > a$



(ii) $|x| \ge a \Leftrightarrow x \le -a$ or $x \ge a$



Result 3. Let r be a positive real number and **a** be a fixed real number. Then,

(i) $|x - a| < r \Leftrightarrow a - r < x < a + r$ i.e. $x \in (a - r, a + r)$

(ii) $|x - a| \le r \Leftrightarrow a - r \le x \le a + r$ i.e. $x \in [a - r, a + r]$

(iii) $|x - a| > r \Leftrightarrow x < a - r$, or $x > a + r$

(iv) $|x - a| \ge r \Leftrightarrow x \le a - r$, or $x \ge a + r$

LOGARITHM OF A NUMBER

The logarithm of the number N to the base ' a ' is the exponent indicating the power to which the base ' a ' must be raised to obtain the number N. This number is designated as $\log_a N$. Hence:

$$\log_a N = x \Leftrightarrow a^x = N, \ a > 0, \ a \ne 1 \ \& \ N > 0$$

If a = 10, then we write log b rather than $\log_{10} b$.
If a = e, we write $\ell n b$ rather than $\log_e b$. Here ' e ' is called as Napiers base & has numerical value equal to 2.7182.

REMEMBER

$\log_{10} 2 = 0.3010$; $\log_{10} 3 = 0.4771$;
$\ell n\ 2 = 0.693$; $\ell n\ 10 = 2.303$

REMARK :

The existence and uniqueness of the number $\log_a N$ can be determined with the help of set of conditions, a > 0 and a ≠ 1 and N > 0.

Fundamental Logarithmic Identity :

$$a^{\log_a N} = N, \ a > 0, \ a \ne 1 \ \& \ N > 0$$

THE PRINCIPAL PROPERTIES OF LOGARITHM

Let M & N are arbitrary positive numbers, $a > 0$, $a \neq 1$, $b > 0$, $b \neq 1$ and α is any real number then ;

(i) $\log_a (MN) = \log_a M + \log_a N$; in general $\log_a(x_1 x_2 ......x_n) = \log_a x_1 + \log_a x_2 + ... + \log_a x_n$

(ii) $\log_a (M/N) = \log_a M - \log_a N$

(iii) $\log_a M^{\alpha} = \alpha \log_a M$

(iv) $\log_{a^{\beta}} M = \dfrac{1}{\beta} \log_a M$

(v) changing of base $\log_b M = \dfrac{\log_a M}{\log_a b}$

REMARKS :

(i) $\log_a 1 = 0$ (ii) $\log_a a = 1$

(iii) $\log_{1/a} a = -1$ (iv) $\log_b a = \dfrac{1}{\log_a b}$

(v) $a^x = e^{x \ell n a}$

LOGARITHMIC EQUATION

The equality $\log_a x = \log_a y$ is possible if and only if $x = y$ i.e. $\mathbf{\log_a x = \log_a y \Leftrightarrow x = y}$

Always check that the solutions should satisfy $x > 0$, $y > 0$, $a > 0$, $a \neq 1$.

SETS

A well defined collection of objects is known as **sets**. If **a** is an element of a set **A**, then we write **a** $\in$ **A** and say **a** belongs to **A**. If **a** does not belong to **A**, then **a** $\notin$ **A** is written.

For example : The collection of all states in the Indian union is a set but collection of good cricket players of India is not a set, since the term "good player is vague and it is not well defined.

Some letters are reserved for the sets as listed below :

N : For the set of Natural numbers.

Z : For the set of Integers.

Z⁺ : For the set of all positive Integers.

Q : For the set of all Rational numbers.

Q⁺ : For the set of all positive Rational numbers.

R : For the set of all Real numbers.

R⁺ : For the set of all Positive real numbers.

C : For the set of all Complex numbers.

TYPE OF SETS

(a) Empty Set :

A set is said to be **empty** or **null** or **void** set if it has no element and it is denoted by ϕ **or { }**.

(b) Singleton Set :

A set consisting of a single element is called a **singleton set**.

(c) Finite Set :

A set is called a **finite set** if it is either void set or its element can be listed (counted labelled) by natural numbers **1, 2, 3 ....** and the process of listing terminates at a certain natural number **n** (say).

For example : Set of all persons on the earth is a finite set.

(d) Infinite Set :

A set whose elements cannot be listed by natural numbers **1, 2, 3,......** for any natural number **n** is called an **infinite set**.
For example : Set of all points in a plane is an infinite set.

(e) Cardinal Number of a Finite Set :

The number **n** in the above definition is called the **cardinal number** or **order** of a finite set **A** and is denoted by **n(A)**.

(f) Equivalents Set :

Two finite sets **A** and **B** are **equivalent** if their cardinal numbers are same. i.e. **n(A) = n(B)**.
For example : A = {1,2,3} and B = {a,b,c} are equivalent sets.

(g) Equal Set :

Two sets **A** and **B** are said to be equal if every element of **A** is a member of **B**, and every element of **B** is member of **A**.

NOTE :
Equal sets are equivalents but equivalent sets need not be equal.

(h) Subset :

Let **A** and **B** be two sets. If every element of **A** is an element of **B**, then **A** is called a subset of **B**. If **A** is a subset of **B**, we write $A \subseteq B$, which is read as **"A is a subset of B"** or **"A is contained in B"**. Thus, $A \subseteq B$ if $a \in A \Rightarrow a \in B$. The symbol " $\Rightarrow$ " stands for **"implies".** If **A** is not a subset of **B**, we write $A \not\subset B$.

NOTE :
Every set is a subset of itself and the empty set is subset of every set. These two subsets are called **improper subsets**. A subsets **A** of a set **B** is called a **proper subset** of B if $A \neq B$ and we write $A \subset B$.

SOME RESULTS ON SUBSET :
(i) Every set is a subset of itself
(ii) The empty set is a subset of every set.
(iii) The total number of subsets of a finite set containing n element is 2^n.

(i) Universal Set :

A set that contains all sets in a given context is called the **Universal Set**.

(j) Power Set :

Let **A** be a set. Then the collection or family of all subsets of **A** is called the **power set** of **A** and is denoted by **P(A)**.

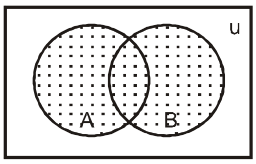
VENN DIAGRAM

Diagram drawn to represent sets are called **Venn-Euler diagram** or simply **Venn diagram**. In Venn-diagram the universal set U is represented by points within a rectangle and its subsets are represented by points in closed curves (usually circles) within the rectangle.

OPERATION ON SETS

(a) Union of Sets :

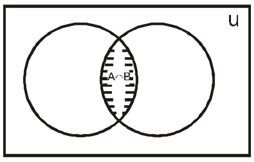
Let **A** and **B** be two sets. The **union** of **A** and **B** is the set of all those elements which belong **either to A or to B or to both A and B**.



Thus, $A \cup B = \{x : x \in A \text{ or } x \in B\}$.

(b) Intersection of Sets :

Let **A** and **B** be two sets. The **intersection** of **A** and **B** is the set of all those elements that belong to **both A and B**.



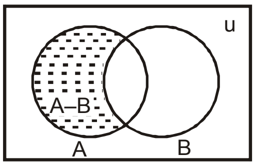
Thus, **$A \cap B = \{x : x \in A \text{ and } x \in B\}$.**

(c) Disjoint Sets :

Two sets **A** and **B** are said to be **disjoint**, if **$A \cap B = \phi$**. If **$A \cap B \neq \phi$**, then A and B are said to be intersecting or overlapping sets.

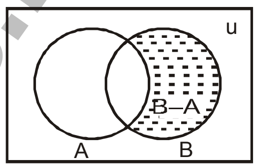
(d) Difference of Sets :

Let **A** and **B** be two sets. The **difference** of **A and B**, written as **A – B**, is the set of all those elements of **A which do not belong to B**.

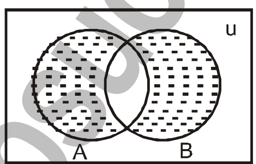


Thus, **$A - B = \{x : x \in A \text{ and } x \notin B\}$**
or, **$A - B = \{x \in A : x \notin B\}$**.
Clearly, $x \in A - B \Leftrightarrow x \in A$ and $x \notin B$.
Similarly, the difference **B – A** is the set of all those elements of **B** that do not belong to **A**

i.e. **$B - A = \{x \in B : x \notin A\}$**.

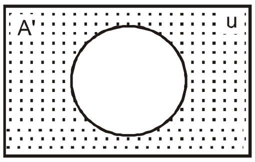


(e) Symmetric Difference of Two Sets :
Let **A** and **B** be two sets. The **symmetric difference** of sets **A** and **B** is the set **$(A - B) \cup (B - A)$** and is denoted by **$A \Delta B$**.



(f) Complement of a Set :

Let **U** be the universal set and let **A** be a set such that **$A \subset U$**. Then, the **complement of A** with respect to **U** is denoted by **A'** or **A^c** or **U– A** and is defined the set of all those elements of **U** which are not in **A**.



Thus, **$A' = \{x \in U : x \notin A\}$**.

Clearly, **$x \in A' \Leftrightarrow x \notin A$**.

MORE RESULTS ON OPERATIONS ON SETS

If **A**, **B** and **C** are **finite sets** and **U** be the **finite universal set**, then
(i) $A - B = A \cap B' = A - (A \cap B)$
(ii) $A - B = A \Leftrightarrow A \cap B = \phi$
(iii) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$
(vi) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
(vii) $n(A \cup B) = n(A) + n(B) \Leftrightarrow$ A, B are disjoint non-void sets.
(viii) $n(A - B) = n(A) - n(A \cap B)$
(ix) $n(A \Delta B)$ = No. of elements which belong to exactly one of A or B
$= n((A - B) \cup (B - A)) = n(A - B) + n(B - A)$ [$\because$ (A – B) and (B – A) are disjoint]
$= n(A) - n(A \cap B) + n(B) - n(A \cap B)$
$= n(A) + n(B) - 2n(A \cap B)$

(x) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(xi) Number of elements in exactly two of the sets A, B, C = $n(A \cap B) + n(B \cap C) + n(C \cap A) - 3\,n(A \cap B \cap C)$

(xii) Number of elements in exactly one of the sets A, B, C

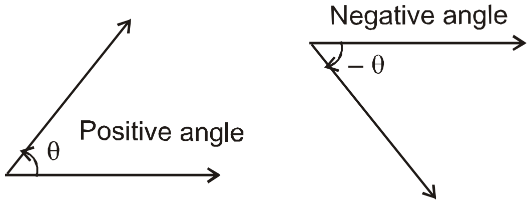
$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

(xiii) $n(A' \cup B') = n((A \cap B)') = n(u) - n(A \cap B)$

(xiv) $n(A' \cap B') = n((A \cup B)') = n(u) - n(A \cup B)$.

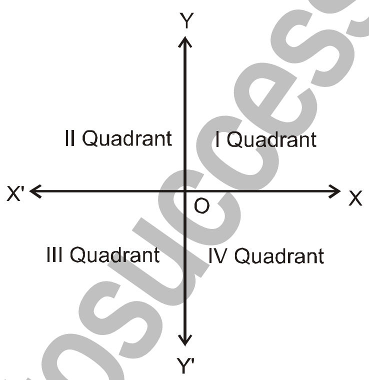
ANGLE

An angle is the amount of rotation of a revolving line with respect to a fixed line. If the rotation is in **anticlock-wise sense**, then the angle measured is **positive** and if the rotation is in **clock-wise sense**, then the angle measured is **negative**.



QUADRANTS

Let **X'OX** and **YOY'** be two lines at right angles in a plane. These lines divide the plane into four equal parts are known as **quadrants**. The lines **X'OX** and **YOY'** are known as **X-axis** and **Y-axis** respectively. These two lines taken together are known as the **co-ordinate axe**s. The regions **XOY, YOX', X'OY'** and **Y'OX** are known as **first**, **second**, **third** and **fourth quadrants** respectively.



(a) Systems of measurement of angles :

(i) Sexagesimal system

(ii) Centesimal system

(iii) Circular system

(i) Sexagesimal system : In this system a **right angle** is divided into **90** equal parts called **degrees**. Each **degree** is divided into **60** equal parts called **minutes** and each **minute** is divided into **60** equal parts called **seconds**.

Thus, **1 right angle = 90 degrees** (90^o)

1^o = 60 minutes (**60'**)

1' = 60 seconds (**60"**)

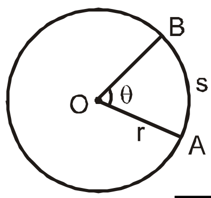
(ii) Centesimal system : In this system a **right angle** is divided into **100** equal parts, called **grades**. Each **grade** is sub divided into **100 minutes**, and each **minute** into **100 seconds**.

Thus, **1 right angle** = **100 grades (100^g)**

1 grade = 100 minutes (100')

1 minute = 100 seconds (100")

(iii) Circular system : In this system the unit of measurement is radian. **One radian**, written as **1^c**, is the measure of an angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.



The number of radians in an angle subtended by an arc of a circle at the centre is equal to $\frac{\text{length of arc}}{\text{radius}}$.

$\therefore \quad \theta = \frac{s}{r}$

Where, θ = angle in radian, s = arc length and r = radius.

(b) Relation Between Three System of Measurement of Angles : $\frac{D}{90} = \frac{G}{100} = \frac{2R}{\pi}$

Where, D = number of degrees,

G = number of grades,

and R = number of radians.

NOTE :

(i) The angle between two consecutive digits in a clock = **30º = (π/6 radians)**.

(ii) The **hour hand** rotates through an angle of **30º** in **one hour**, i.e. **(1/2)º** in **one minute**.

(iii) The **minute hand** rotates through an angle of **6º** in **one minute**.

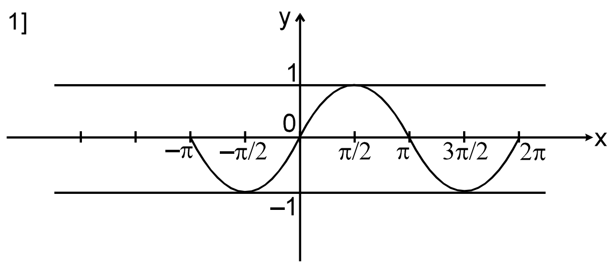
TRIGONOMETRIC RATIOS OF ALLIED ANGLES

Two angles are said to **allied** when their sum or difference is either **zero** or a **multiple of 90º**. If θ is any angle, then – **θ, 90 ± θ, 180 ± θ, 270 ± θ, 360 ± θ** etc. are called **allied angles.**

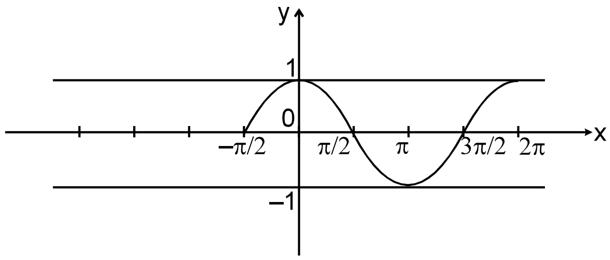
sin (– θ) = – sin θ	cos (– θ) = cos θ
sin (90 + θ) = cos θ	cos (90 + θ) = – sin θ
tan (90 + θ) = – cot θ	cot (90 + θ) = – tan θ
sec (90 + θ) = – cosec θ	cosec (90 + θ) = sec θ
sin (180 – θ) = + sin θ	sin (180 + θ) = – sin θ
cos (180 – θ) = – cos θ	cos (180 + θ) = – cos θ
tan (180 – θ) = – tan θ	tan (180 + θ) = tan θ
cot (180 – θ) = – cot θ	cot (180 + θ) = cot θ
sec (180 – θ) = – sec θ	sec (180 + θ) = – sec θ
cosec (180 – θ) = cosec θ	cosec (180 + θ) = – cosec θ
sin (270 – θ) = – cos θ	sin (270+ θ) = – cos θ
cos (270 – θ) = – sin θ	cos (270 + θ) = sin θ
tan (270 – θ) = cot θ	tan (270 + θ) = – cot θ
cot (270 – θ) = tan θ	cot (270 + θ) = – tan θ
sec (270 – θ) = – cosec θ	sec (270 + θ) = cosec θ
cosec (270 – θ) = – sec θ	cosec (270 + θ) = – sec θ
sin (360 – θ) = – sin θ	sin (360 + θ) = sin θ
cos (360 – θ) = cos θ	cos (360 + θ) = cos θ
tan (360 – θ) = – tan θ	tan (360 + θ) = tan θ
cot (360 – θ) = – cot θ	cot (360 + θ) = cot θ
sec (360 – θ) = sec θ	sec (360 + θ) = sec θ
cosec (360 – θ) = – cosec θ	cosec (360 + θ) = cosec θ

GRAPHS OF TRIGONOMETRIC FUNCTIONS

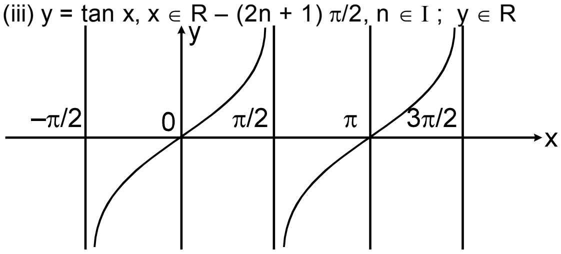
(i) y = sin x x ∈ R; y ∈ [–1, 1]



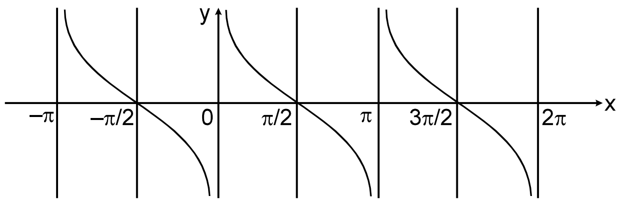
(ii) $y = \cos x$ $x \in R$; $y \in [-1, 1]$



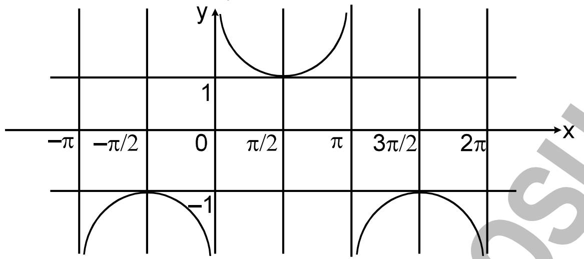
(iii) $y = \tan x$, $x \in R - (2n + 1)\,\pi/2$, $n \in I$; $y \in R$



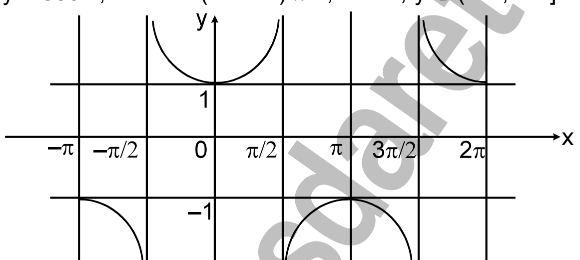
(iv) $y = \cot x$, $x \in R - n\pi$, $n \in I$; $y \in R$



(v) $y = \text{cosec } x$,
$x \in R - n\pi$, $n \in I$; $y \in (-\infty, -1] \cup [1, \infty)$



(vi) $y = \sec x$, $x \in R - (2n + 1)\,\pi/2$, $n \in I$; $y \in (-\infty, -1] \cup$ $[1, \infty)$



TRIGONOMETRIC FUNCTIONS OF SUM OR DIFFERENCE OF TWO ANGLES

(i) $\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$

(ii) $\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

(iii) $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$
$= \sin (A+B).\ \sin (A - B)$

(iv) $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$
$= \cos (A+B).\ \cos (A - B)$

(v) $\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

(vi) $\cot(A \pm B) = \dfrac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$

(vii) $\tan(A + B + C)$

$= \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$.

TRANSFORMATION FORMULAE

(i) $\sin(A + B) + \sin(A - B) = 2 \sin A \cos B$ (ii) $\sin(A+B) - \sin(A - B) = 2 \cos A \sin B$

(iii) $\cos(A+B) + \cos(A - B) = 2 \cos A \cos B$ (iv) $\cos(A - B) - \cos(A+B) = 2 \sin A \sin B$

(v) $\sin C + \sin D = 2 \sin \dfrac{C+D}{2} \cos \dfrac{C-D}{2}$

(vi) $\sin C - \sin D = 2 \cos \dfrac{C+D}{2} \sin \dfrac{C-D}{2}$

(vii) $\cos C + \cos D = 2 \cos \dfrac{C+D}{2} \cos \dfrac{C-D}{2}$ (viii) $\cos C - \cos D = -2 \sin \dfrac{C+D}{2} \sin \dfrac{C-D}{2}$

MULTIPLE AND SUB-MULTIPLE ANGLES

(i) $\sin 2A = 2 \sin A \cos A$; $\sin A = 2 \sin \dfrac{A}{2} \cos \dfrac{A}{2}$

(ii) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2 \sin^2 A$; $2 \cos^2 \dfrac{A}{2} = 1 + \cos A$, $2 \sin^2 \dfrac{A}{2} = 1 - \cos A$.

(iii) $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$, $\tan \theta = \dfrac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$

(iv) $\sin 2A = \dfrac{2 \tan A}{1 + \tan^2 A}$, $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

(v) $\sin 3A = 3 \sin A - 4 \sin^3 A$

(vi) $\cos 3A = 4 \cos^3 A - 3 \cos A$

(vii) $\tan 3A = \dfrac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$

(viii) $\sin 15°$ or $\sin \dfrac{\pi}{12} = \dfrac{\sqrt{3} - 1}{2\sqrt{2}} = \cos 75°$ or $\cos \dfrac{5\pi}{12}$

(ix) $\cos 15°$ or $\cos \dfrac{\pi}{12} = \dfrac{\sqrt{3} + 1}{2\sqrt{2}} = \sin 75°$ or $\sin \dfrac{5\pi}{12}$

(x) $\tan 15° = \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3} = \cot 75°$; $\tan 75° = \dfrac{\sqrt{3} + 1}{\sqrt{3} - 1} = 2 + \sqrt{3} = \cot 15°$

(xi) $\sin \dfrac{\pi}{10}$ or $\sin 18° = \dfrac{\sqrt{5} - 1}{4}$ & $\cos 36°$ or $\cos \dfrac{\pi}{5} = \dfrac{\sqrt{5} + 1}{4}$.

RANGE OF TRIGONOMETRIC EXPRESSION

$E = a \sin \theta + b \cos \theta$

$E = \sqrt{a^2 + b^2} \sin(\theta + \alpha)$, where $\tan \alpha = \dfrac{b}{a}$

$= \sqrt{a^2+b^2}\ \cos(\theta - \beta)$, where $\tan\beta = \dfrac{a}{b}$

Hence for any real value of θ,

$-\sqrt{a^2+b^2} \le E \le \sqrt{a^2+b^2}$

So, the Maximum value $= \sqrt{a^2+b^2}$

And Minimum value $= -\sqrt{a^2+b^2}$

Trigonometric Equation

A man can do all things if he but wills them. Albertus

Euclid taught me that without assumptions there is no proof. Therefore, in any argument, examine the assumptions. Eric Temple Bell

Trigonometric Equation :

An equation involving one or more trigonometric ratios of an unknown angle is called a trigonometric equation.

Solution of Trigonometric Equation :

A solution of trigonometric equation is the value of the unknown angle that satisfies the equation.

e.g. if $\sin\theta = \dfrac{1}{\sqrt{2}} \Rightarrow \theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{9\pi}{4}, \dfrac{11\pi}{4}, \ldots\ldots\ldots$

Thus, the trigonometric equation may have infinite number of solutions (because of their periodic nature) and can be classified as :

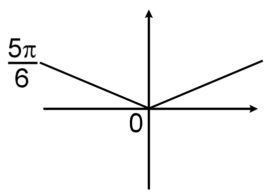
(i) Principal solution (ii) General solution.

Principal solutions :

The solutions of a trigonometric equation which lie in the interval $[0, 2\pi)$ are called Principal solutions.

e.g. Find the Principal solutions of the equation $\sin x = \dfrac{1}{2}$.

Solution :



$\because$ $\sin x = \dfrac{1}{2}$

$\because$ there exists two values

i.e. $\dfrac{\pi}{6}$ and $\dfrac{5\pi}{6}$ which lie in $[0, 2\pi)$ and whose sine is $\dfrac{1}{2}$

$\therefore$ Principal solutions of the equation $\sin x = \dfrac{1}{2}$ are $\dfrac{\pi}{6}, \dfrac{5\pi}{6}$

General Solution :

The expression involving an integer 'n' which gives all solutions of a trigonometric equation is called General solution.

General solution of some standard trigonometric equations are given below.

General Solution of Some Standard Trigonometric Equations :

(i) If $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n \alpha$ where $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $n \in I$.

(ii) If $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$ where $\alpha \in [0, \pi]$, $n \in I$.

(iii) If $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$ where $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $n \in I$.

(iv) If $\sin^2\theta = \sin^2\alpha \Rightarrow \theta = n\pi \pm \alpha$, $n \in I$.

(v) If $\cos^2\theta = \cos^2\alpha \Rightarrow \theta = n\pi \pm \alpha$, $n \in I$.

(vi) If $\tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha$, $n \in I$. [**Note:** α is called the principal angle]

Some Important deductions :

(i) $\sin\theta = 0 \Rightarrow \theta = n\pi$, $n \in I$

(ii) $\sin\theta = 1 \Rightarrow \theta = (4n + 1)\frac{\pi}{2}$, $n \in I$

(iii) $\sin\theta = -1 \Rightarrow \theta = (4n - 1)\frac{\pi}{2}$, $n \in I$

(iv) $\cos\theta = 0 \Rightarrow \theta = (2n + 1)\frac{\pi}{2}$, $n \in I$

(v) $\cos\theta = 1 \Rightarrow \theta = 2n\pi$, $n \in I$

(vi) $\cos\theta = -1 \Rightarrow \theta = (2n + 1)\pi$, $n \in I$

(vii) $\tan\theta = 0 \Rightarrow \theta = n\pi$, $n \in I$

Way of solving of Trigonometric Equations :

1 Trigonometric equations which can be solved by use of factorization.

2 Trigonometric equations which can be solved by reducing them in quadratic equations.

3 Trigonometric equations which can be solved by transforming a sum or difference of trigonometric ratios into their product.

4 Trigonometric equations which can be solved by transforming a product of trigonometric ratios into their sum or difference.

5 Trigonometric Equations of the form a sinx + b cosx = c, where a, b, c $\in$ R, can be solved by dividing both sides of the equation by $\sqrt{a^2 + b^2}$.

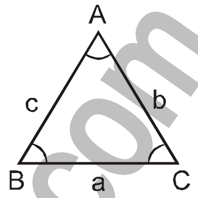
6 Trigonometric equations of the form P(sinx ± cosx, sinx cosx) = 0, where p(y, z) is a polynomial, can be solved by using the substitution sinx ± cosx = t.

7 Trigonometric equations which can be solved by the use of boundness of the trigonometric ratios sinx and cosx.

SOLUTION OF TRIANGLE

Sine Rule :

In any triangle ABC, the sines of the angles are proportional to the opposite sides



i.e. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

Cosine Formula :

In any ΔABC

(i) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ or $a^2 = b^2 + c^2 - 2bc\cos A = b^2 + c^2 + 2bc\cos(B + C)$

(ii) $\cos B = \frac{c^2 + a^2 - b^2}{2ca}$ **(iii)** $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Projection Formula :

In any ΔABC

(i) $a = b\cos C + c\cos B$ **(ii)** $b = c\cos A + a\cos C$

(iii) $c = a\cos B + b\cos A$

Napier's Analogy - tangent rule :

In any ΔABC

(i) $\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$ **(ii)** $\tan\frac{C-A}{2} = \frac{c-a}{c+a}\cot\frac{B}{2}$

(iii) $\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$

Trigonometric Functions of Half Angles :

(i) $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$; $\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$; $\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

(ii) $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$; $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$; $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

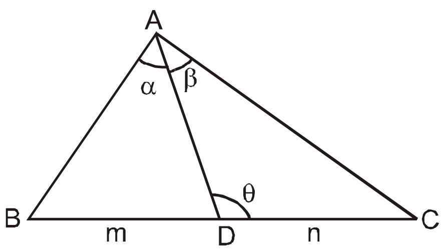
(iii) $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{\Delta}{s(s-a)} = \frac{(s-b)(s-c)}{\Delta}$ where $s = \frac{a+b+c}{2}$ is semi perimeter and Δ is the area of triangle.

(iv) $\sin A = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)} = \frac{2\Delta}{bc}$

Area of Triangle (Δ)

$\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B = \sqrt{s(s-a)(s-b)(s-c)}$

m - n Rule :



$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

$= n\cot B - m\cot C$

Radius of Circumcirlce :

If R be the circumradius of ΔABC, then $R = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C} = \frac{abc}{4\Delta}$

Radius of The Incircle :

If 'r' be the inradius of ΔABC, then

(i) $r = \frac{\Delta}{s}$

(ii) $r = (s-a)\tan\frac{A}{2} = (s-b)\tan\frac{B}{2} = (s-c)\tan\frac{C}{2}$

(iii) $r = \frac{a\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{A}{2}}$ and so on

(iv) $r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Radius of The Ex- Circles :

If r_1, r_2, r_3 are the radii of the ex-circles of ΔABC opposite to the vertex A, B, C respectively, then

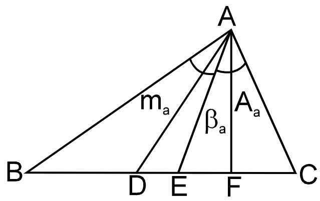
(i) $r_1 = \frac{\Delta}{s-a}$; $r_2 = \frac{\Delta}{s-b}$; $r_3 = \frac{\Delta}{s-c}$

(ii) $r_1 = s\tan\frac{A}{2}$; $r_2 = s\tan\frac{B}{2}$; $r_3 = s\tan\frac{C}{2}$

(iii) $r_1 = \frac{a\cos\frac{B}{2}\cos\frac{C}{2}}{\cos\frac{A}{2}}$ and so on

(iv) $r_1 = 4R\sin\frac{A}{2}\cdot\cos\frac{B}{2}\cdot\cos\frac{C}{2}$

Length of Angle Bisectors, Medians & Altitudes :



(i) Length of an angle bisector from the angle A = $\beta_a = \frac{2bc\cos\frac{A}{2}}{b+c}$;

(ii) Length of median from the angle A = $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

&(iii) Length of altitude from the angle A = $A_a = \frac{2\Delta}{a}$

NOTE : $m_a^2 + m_b^2 + m_c^2 = \frac{3}{4}(a^2 + b^2 + c^2)$

The Distances of The Special Points from Vertices and Sides of Triangle :

(i) Circumcentre (O) : $OA = R$ and $O_a = R\cos A$

(ii) Incentre (I) : $IA = r\,\text{cosec}\,\frac{A}{2}$ and $I_a = r$

(iii) Excentre (I_1) : $I_1A = r_1\,\text{cosec}\,\frac{A}{2}$ and $I_{1a} = r_1$

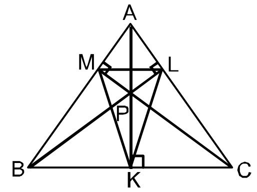
(iv) Orthocentre (H) : $HA = 2R\cos A$ and $H_a = 2R\cos B\cos C$

(v) Centroid (G) : $GA = \frac{1}{3}\sqrt{2b^2 + 2c^2 - a^2}$ and $G_a = \frac{2\Delta}{3a}$

Orthocentre and Pedal Triangle :

The triangle KLM which is formed by joining the feet of the altitudes is called the Pedal Triangle.

(i) Its angles are $\pi - 2A$, $\pi - 2B$ and $\pi - 2C$.



(ii) Its sides are $a \cos A = R \sin 2A$,
$b \cos B = R \sin 2B$ and
$c \cos C = R \sin 2C$

(iii) Circumradii of the triangles PBC, PCA, PAB and ABC are equal.

Excentral Triangle :

The triangle formed by joining the three excentres I_1, I_2 and I_3 of Δ ABC is called the excentral or excentric triangle.

(i) Δ ABC is the pedal triangle of the $\Delta\, I_1 I_2 I_3$.

(ii) Its angles are $\frac{\pi}{2} - \frac{A}{2}$, $\frac{\pi}{2} - \frac{B}{2}$ and $\frac{\pi}{2} - \frac{C}{2}$.

(iii) Its sides are $4R \cos\frac{A}{2}$, $4R \cos\frac{B}{2}$ and $4R \cos\frac{C}{2}$.

(iv) $I I_1 = 4R \sin\frac{A}{2}$; $I I_2 = 4R \sin\frac{B}{2}$; $I I_3 = 4R \sin\frac{C}{2}$.

(v) Incentre I of Δ ABC is the orthocentre of the excentral $\Delta\, I_1 I_2 I_3$.

Distance Between Special Points :

(i) Distance between circumcentre and orthocentre $OH^2 = R^2 (1 - 8 \cos A \cos B \cos C)$

(ii) Distance between circumcentre and incentre $OI^2 = R^2 \left(1 - 8 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}\right) = R^2 - 2Rr$

(iii) Distance between circumcentre and centroid $OG^2 = R^2 - \frac{1}{9}(a^2 + b^2 + c^2)$

PLANE GEOMETRY

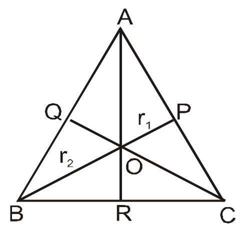
MENSURATION

❖ **Pre-requisite : Before going through this chapter, you should be thorough with the basic concepts of the chapter explained in X NCERT.**

Some important results

AN IMPORTANT PROPERTY

In an equilateral triangle perpendiculars drawn form all the three vertices intersect each other in the ratio of 2 :1 from the vertex
to the base.



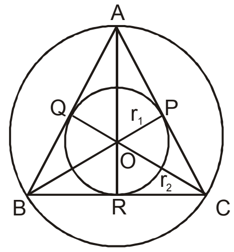
$$\therefore \frac{AO}{OR} = \frac{BO}{OP} = \frac{CO}{OQ} = \frac{2}{1} = \frac{r_2}{r_1}$$

OP = OQ = OR, all are the inradii.
OA = OB = OC, all are the circumradii.

❖ **NOTE :**
Radii means more than one radius.

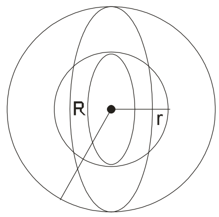
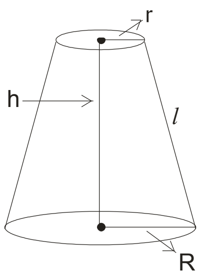
O is the centre of two circles. Circle PQR is called as incircle (touching the sides) and circle ABC is called as circumcircle (touching the vertices)

$\therefore \qquad \text{Inradius} = \dfrac{\text{side}}{2\sqrt{3}}$

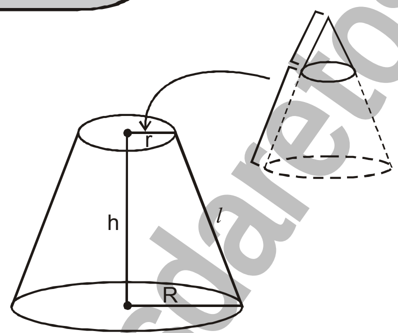


and $\qquad \text{Circumradius} = \dfrac{\text{side}}{\sqrt{3}}$

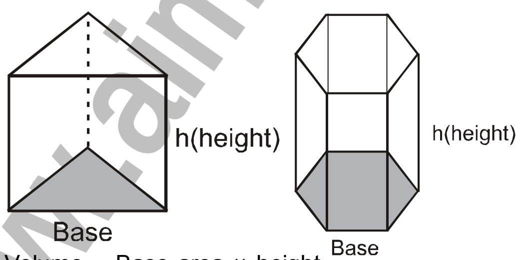
2.

S.No.	Name	Figure	Nomenclature	Volume	Curved/Lateral surface area	Total surface area
1	**Spherical shell**		r → inner radius R → outer radius	$\frac{4}{3}\pi\left[R^3 - r^3\right]$	——	$4\pi[R^2 + r^2]$
2	**Frustum of a cone**			$\frac{\pi}{3}h\left(r^2 + Rr + R^2\right)$	$\pi(r+R)l$	lateral surface area + $\pi[R^2 + r^2]$

FRUSTUM OF A CONE



PRISM



Volume = Base area × height

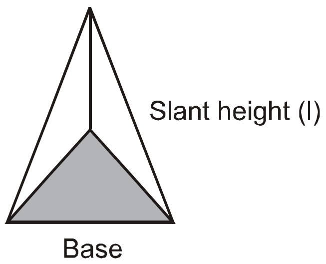
Lateral surface area = perimeter of the base × height

PYRAMID

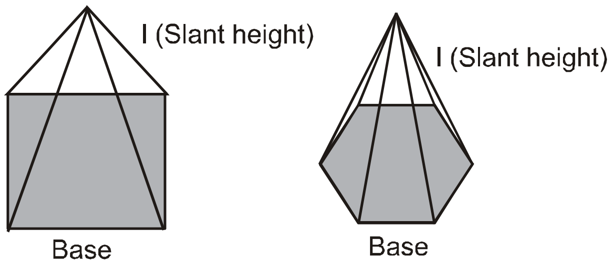
Volume = $\frac{1}{3}$ × base area × height

Lateral surface area

= $\frac{1}{2}$ × perimeter of the base × slant height



Total surface area = lateral surface area + base area



Useful Formulae of Mensuration to Remember :

1. Volume of a cuboid = ℓbh.

2. Surface area of cuboid = $2(\ell b + bh + h\ell)$.

3. Volume of cube = a^3

4. Surface area of cube = $6a^2$

5. Volume of a cone = $\frac{1}{3}\pi r^2 h$.

6. Curved surface area of cone = $\pi r\ell$ (ℓ = slant height)

7. Curved surface area of a cylinder = $2\pi rh$.

8. Total surface area of a cylinder = $2\pi rh + 2\pi r^2$.

9. Volume of a sphere = $\frac{4}{3}\pi r^3$.

10. Surface area of a sphere = $4\pi r^2$.

11. Area of a circular sector = $\frac{1}{2} r^2 \theta$, when θ is in radians.

12. Volume of a prism = (area of the base) × (height).

13. Lateral surface area of a prism = (perimeter of the base) × (height).

14. Total surface area of a prism = (lateral surface area) + 2 (area of the base) (Note that lateral surfaces of a prism are all rectangle).

15. Volume of a pyramid = $\frac{1}{3}$ (area of the base) × (height).

16. Curved surface area of a pyramid = $\frac{1}{2}$ (perimeter of the base) × (slant height). (Note that slant surfaces of a pyramid are triangles).

QUADRATIC EQUATIONS

1. Polynomial :

A function f defined by $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots. + a_1 x + a_0$
where $a_0, a_1, a_2, \ldots\ldots, a_n \in R$ is called a polynomial of degree n with real coefficients ($a_n \neq 0, n \in W$).
If $a_0, a_1, a_2, \ldots.., a_n \in C$, it is called a polynomial with complex coefficients.

2. Quadratic polynomial & Quadratic equation :

A polynomial of degree 2 is known as quadratic polynomial. Any equation f(x) = 0, where f is a quadratic polynomial, is called a quadratic equation. The general form of a quadratic equation is

$ax^2 + bx + c = 0$ ....(i)

Where a, b, c are real numbers, $a \neq 0$.
If a = 0, then equation (i) becomes linear equation.

3. Difference between equation & identity :

If a statement is true for all the values of the variable, such statements are called as identities. If the statement is true for some or no values of the variable, such statements are called as equations.

Example : (i) $(x + 3)^2 = x^2 + 6x + 9$ is an identity
(ii) $(x + 3)^2 = x^2 + 6x + 8$, is an equation having no root.
(iii) $(x + 3)^2 = x^2 + 5x + 8$, is an equation having – 1 as its root.

A quadratic equation has exactly two roots which may be real (equal or unequal) or imaginary.
$ax^2 + bx + c = 0$ is:

★ a quadratic equation if $a \neq 0$ Two Roots

★ a linear equation if $a = 0, b \neq 0$ One Root

★ a contradiction if $a = b = 0, c \neq 0$ No Root

★ an identity if $a = b = c = 0$ Infinite Roots

If $ax^2 + bx + c = 0$ is satisfied by three distinct values of 'x', then it is an identity.

4. Relation Between Roots & Co-efficients:

(i) The solutions of quadratic equation, $ax^2 + bx + c = 0$, ($a \neq 0$) is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The expression, $b^2 - 4ac \equiv D$ is called discriminant of quadratic equation.

(ii) If α, β are the roots of quadratic equation,
$ax^2 + bx + c = 0$ ....(i)
then equation (i) can be written as
$a(x - \alpha)(x - \beta) = 0$
or $ax^2 - a(\alpha + \beta)x + a\alpha\beta = 0$ ......(ii)
equations (i) and (ii) are identical,

$\therefore$ by comparing the coefficients sum of the roots, $\alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

and product of the roots, $\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

(iii) Dividing the equation (i) by a, $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$\Rightarrow \quad x^2 - \left(\dfrac{-b}{a}\right)x + \dfrac{c}{a} = 0 \quad \Rightarrow \quad x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow \quad x^2 -$ (sum of the roots) x + (product of the roots) = 0

Hence we conclude that the quadratic equation whose roots are α & β is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

5. Nature of Roots:

Consider the quadratic equation, $ax^2 + bx + c = 0$ having α, β as its roots;

$$D \equiv b^2 - 4ac$$

- $D = 0$
  Roots are equal i.e. $\alpha = \beta = -b/2a$
  & the quadratic expression can be expressed as a perfect square of a linear polynomial
- $D \neq 0$
  Roots are unequal
  - $a, b, c \in R$ & $D > 0$
    Roots are real
    - $a, b, c \in Q$ &
      D is square of a rational number
      $\Rightarrow$ Roots are rational
      ↓
      $a = 1,\ b, c \in I$ & D is square of an integer
      $\Rightarrow$ Roots are integral.
    - $a, b, c \in Q$ &
      D is not square of a rational number
      $\Rightarrow$ Roots are irrational
      i.e. $\alpha = p + \sqrt{q}$, $\beta = p - \sqrt{q}$
  - $a, b, c \in R$ & $D < 0$
    Roots are imaginary $\alpha = p + iq$, $\beta = p - iq$

6. Common Roots:

Consider two quadratic equations, $a_1x^2 + b_1x + c_1 = 0$ & $a_2x^2 + b_2x + c_2 = 0$.

(i) If two quadratic equations have both roots common, then the equations are identical and their co-efficient are in proportion.

i.e. $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$.

(ii) If only one root is common, then the common root 'α' will be:

$$\alpha = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} = \frac{b_1c_2 - b_2c_1}{c_1a_2 - c_2a_1}$$

Hence the condition for one common root is:

$$\Rightarrow \quad (c_1a_2 - c_2a_1)^2 = (a_1b_2 - a_2b_1)\ (b_1c_2 - b_2c_1)$$

Note : If $f(x) = 0$ & $g(x) = 0$ are two polynomial equation having some common root(s) then those common root(s) is/are also the root(s) of $h(x) \equiv a\,f(x) + bg\,(x) = 0$.

7. Graph of Quadratic Expression:

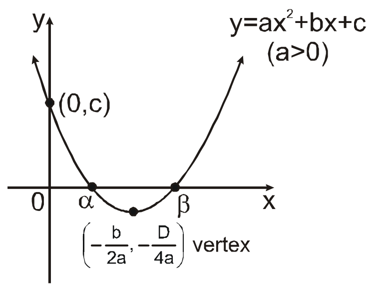
$$y = f(x) = ax^2 + bx + c$$

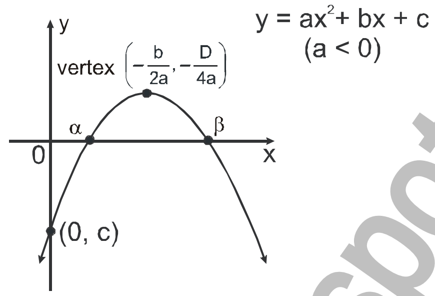
or $\quad \left(y + \dfrac{D}{4a}\right) = a\left(x + \dfrac{b}{2a}\right)^2$

★ the graph between x, y is always a parabola.

★ the co–ordinate of vertex are $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$

★ If a > 0 then the shape of the parabola is concave upwards & if a < 0 then the shape of the parabola is concave downwards.





★ the parabola intersect the y–axis at point (0, c).

★ the x–co–ordinate of point of intersection of parabola with x–axis are the real roots of the quadratic equation f (x) = 0. Hence the parabola may or may not intersect the x–axis.

8. Range of Quadratic Expression $f(x) = ax^2 + bx + c$.

(i) Range :

If a > 0 $\Rightarrow$ $f(x) \in \left[-\frac{D}{4a}, \infty\right)$

If a < 0 $\Rightarrow$ $f(x) \in \left(-\infty, -\frac{D}{4a}\right]$

Hence maximum and minimum values of the expression f (x) is $-\frac{D}{4a}$ in respective cases and it occurs at $x = -\frac{b}{2a}$ (at vertex).

(ii) Range in restricted domain:

Given $x \in [x_1, x_2]$

(a) If $-\frac{b}{2a} \notin [x_1, x_2]$ then,

$$f(x) \in [\min\{f(x_1), f(x_2)\},\ \max\{f(x_1), f(x_2)\}]$$

(b) If $-\frac{b}{2a} \in [x_1, x_2]$ then,

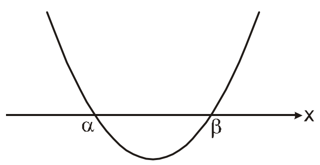
$$f(x) \in \left[\min\left\{f(x_1), f(x_2), -\frac{D}{4a}\right\},\ \max\left\{f(x_1), f(x_2), -\frac{D}{4a}\right\}\right]$$

9. Sign of Quadratic Expressions :

The value of expression $f(x) = ax^2 + bx + c$ at $x = x_0$ is equal to y–co–ordinate of the point on parabola $y = ax^2 + bx + c$ whose x–co–ordinate is x_0. Hence if the point lies above the x–axis for some $x = x_0$, then $f(x_0) > 0$ and vice–versa.

We get six different positions of the graph with respect to x–axis as shown.

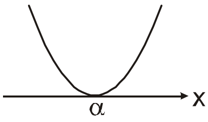
(i)



Conclusions :

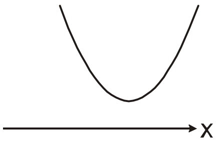
(a) a > 0
(b) D > 0
(c) Roots are real & distinct.
(d) f(x) > 0 in $x \in (-\infty, \alpha) \cup (\beta, \infty)$
(e) f(x) < 0 in $x \in (\alpha, \beta)$

(ii)



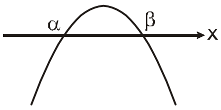
(a) $a > 0$
(b) $D = 0$
(c) Roots are real & equal.
(d) $f(x) > 0$ in $x \in R - \{\alpha\}$

(iii)



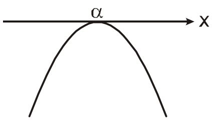
(a) $a > 0$
(b) $D < 0$
(c) Roots are imaginary.
(d) $f(x) > 0 \ \forall \ x \in R$.

(iv)



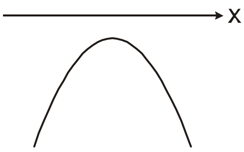
(a) $a < 0$
(b) $D > 0$
(c) Roots are real & distinct.
(d) $f(x) < 0$ in $x \in (-\infty, \alpha) \cup (\beta, \infty)$
(e) $f(x) > 0$ in $x \in (\alpha, \beta)$

(v)



(a) $a < 0$
(b) $D = 0$
(c) Roots are real & equal.
(d) $f(x) < 0$ in $x \in R - \{\alpha\}$

(vi)



(a) $a < 0$
(b) $D < 0$
(c) Roots are imaginary.
(d) $f(x) < 0 \ \forall \ x \in R$.

10. Solution of Quadratic Inequalities:

The values of 'x' satisfying the inequality $ax^2 + bx + c > 0$ $(a \neq 0)$ are:

(i) If $D > 0$, i.e. the equation $ax^2 + bx + c = 0$ has two different roots α & β such that $\alpha < \beta$

Then $a > 0 \Rightarrow x \in (-\infty, \alpha) \cup (\beta, \infty)$

$a < 0 \Rightarrow x \in (\alpha, \beta)$

(ii) If $D = 0$, i.e. roots are equal, i.e. $\alpha = \beta$.

Then $a > 0 \Rightarrow x \in (-\infty, \alpha) \cup (\alpha, \infty)$

$a < 0 \Rightarrow x \in \phi$

(iii) If $D < 0$, i.e. the equation $ax^2 + bx + c = 0$ has no real root.

Then $a > 0 \Rightarrow x \in R$

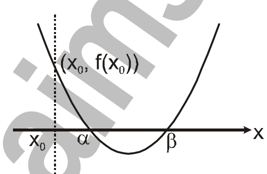
$a < 0 \Rightarrow x \in \phi$

(iv) Inequalities of the form $\frac{P(x)\ Q(x)\ R(x)\ldots\ldots\ldots}{A(x)\ B(x)\ C(x)\ldots\ldots\ldots} \begin{smallmatrix} < \\ = \\ > \end{smallmatrix} 0$ can be quickly solved using the method of intervals, where A, B, C........, P, Q, R......... are linear functions of 'x'.

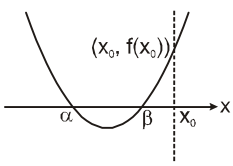
11. Location of Roots :

Let $f(x) = ax^2 + bx + c$, where $a > 0$ & $a, b, c \in R$.

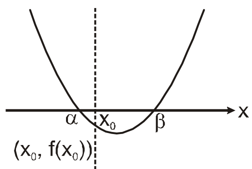
(i)



(ii)



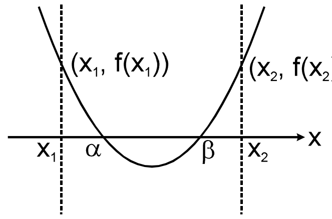
(iii)

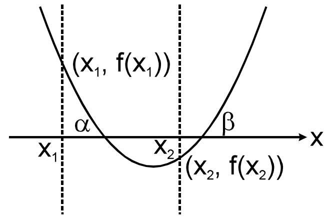


(i) Conditions for both the roots of $f(x) = 0$ to be greater than a specified number 'x0' are $b^2 - 4ac \geq 0$ & $f(x_0) > 0$ & $(-b/2a) > x_0$.

(ii) Conditions for both the roots of $f(x) = 0$ to be smaller than a specified number 'x_0' are $b^2 - 4ac \geq 0$ & $f(x_0) > 0$ & $(-b/2a) < x_0$.

(iii) Conditions for a number 'x_0' to lie between the roots of $f(x) = 0$ is $f(x_0) < 0$.

(iv) 

(v) 

(iv) Conditions that both roots of f (x) = 0 to be confined between the numbers x_1 and x_2, $(x_1 < x_2)$ are $b^2 - 4ac \geq 0$ & $f(x_1) > 0$ & $f(x_2) > 0$ & $x_1 < (-b/2a) < x_2$.

(v) Conditions for exactly one root of f (x) = 0 to lie in the interval (x_1, x_2) i.e. $x_1 < x < x_2$ is $f(x_1).\, f(x_2) < 0$.

12. Theory Of Equations:

If $\alpha_1, \alpha_2, \alpha_3, ...\alpha_n$ are the roots of the equation;

$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n = 0$ where $a_0, a_1, ..., a_n$ are all real & $a_0 \neq 0$ then,

$$\sum \alpha_1 = -\frac{a_1}{a_0},\ \sum \alpha_1\alpha_2 = +\frac{a_2}{a_0},\ \sum \alpha_1\alpha_2\alpha_3 = -\frac{a_3}{a_0},......,\ \alpha_1\,\alpha_2\,\alpha_3.......\alpha_n = (-1)^n\frac{a_n}{a_0}$$

Note :

(i) If α is a root of the equation f(x) = 0, then the polynomial f(x) is exactly divisible by $(x - \alpha)$ or $(x - \alpha)$ is a factor of f(x) and conversely.

(ii) Every equation of n^{th} degree $(n \geq 1)$ has exactly n roots & if the equation has more than n roots, it is an identity.

(iii) If the coefficients of the equation f(x) = 0 are all real and $\alpha + i\beta$ is its root, then $\alpha - i\beta$ is also a root. i.e. imaginary roots occur in conjugate pairs.

(iv) An equation of odd degree will have odd number of real roots and an equation of even degree will have even numbers of real roots.

(v) If the coefficients in the equation are all rational & $\alpha + \sqrt{\beta}$ is one of its roots, then $\alpha - \sqrt{\beta}$ is also a root where $\alpha, \beta \in Q$ & β is not square of a rational number.

(vi) If there be any two real numbers 'a' & 'b' such that f(a) & f(b) are of opposite signs, then f(x) = 0 must have odd number of real roots (also atleast one real root) between 'a' and 'b'.

(vii) Every equation f(x) = 0 of degree odd has atleast one real root of a sign opposite to that of its last term. (If coefficient of highest degree term is positive).

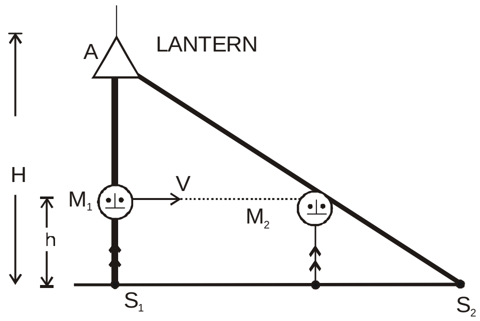
RECTILINEAR MOTION

Exercise 2 ; Part -I

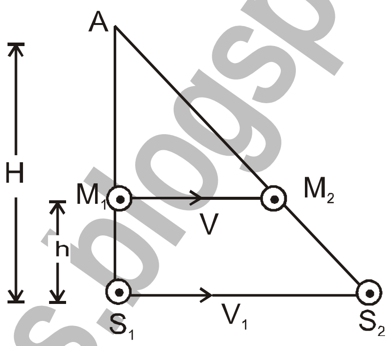
4. A man walking with a speed 'v' constant in magnitude and direction passes under a lantern hanging at a height H above the ground (consider lantern as a point source). Find the velocity with which the edge of the shadow of the man's head moves over the ground, if his height is 'h'.
नियत चाल v (दिशा व परिमाण दोनों में नियत) से चलता हुआ एक व्यक्ति जमीन से H उँचाई पर लटकी हुई लालटेन के पास से गुजरता है (लालटेन को बिन्दु स्त्रोत माने), तो व्यक्ति के सिर की परछाई जमीन पर किस वेग से गति करेगी (यदि व्यक्ति की ऊँचाई 'h' है।)?

Ans : $\left(\frac{H}{H-h}\right)v$

Sol.



⇒



Where V_1 = velocity of shadow
छाया का वेग

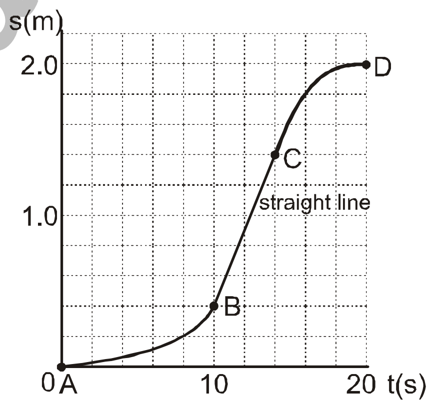
Let माना t = 0, Man is at M_1. व्यक्ति M_1 पर है
and और t = t Man is at M_2. व्यक्ति M_2 पर है

As क्योंकि $\Delta AM_1M_2 \sim \Delta AS_1S_2$.

$$\therefore \quad \frac{AM_1}{AS_1} = \frac{M_1M_2}{S_1S_2} \quad \Rightarrow \quad \frac{H-h}{H} = \frac{Vt}{V_1t}$$

$$\therefore \quad V_1 = \frac{VH}{H-h}$$

10._ A point moves rectilinearly in one direction. Fig. shows **[M.Bank_Irodov._1.4]**
एक कण एक दिशा में सरल रेखीय गति करता है। दिया गया आरेख



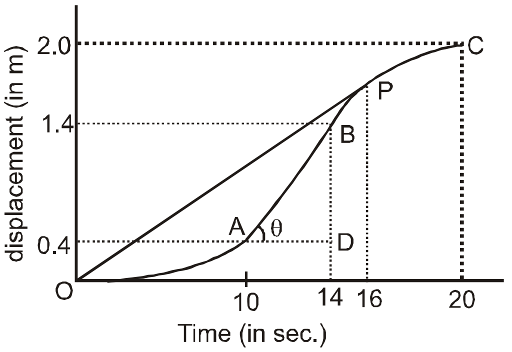
the displacement s traversed by the point as a function of the time t.
कण द्वारा तय किया गया विस्थापन s को, समय t के फलन के रूप में दर्शाता है।
Using the plot find:
आरेख का उपयोग करते हुये बताईये ।

(a) the average velocity of the point during the time of motion;
यात्रा के दौरान कण का औसत वेग
(b) the maximum velocity;
कण का अधिकतम वेग
(c) the time t_0 at which the instantaneous velocity is equal to the mean velocity averaged over the first t_0 seconds.
आरेख में वह क्षण t_0 क्या होगा जब तात्क्षणिक वेग प्रथम t_0 सेकण्ड समय तक, के औसत वेग के बराबर होगा।
Ans : (a) 10 cm/s; (b) 25 cm/s; (c) t_0 = 16s

Sol. the slope of displacement time graph gives velocity.

(a) Average velocity $= \dfrac{\text{total displacement}}{\text{total time}}$

$= \dfrac{2}{20} = \dfrac{1}{10}$ m/s = 10 cm/s



The given graph can be divided into three parts.
(i) OA $\rightarrow$ This part of graph is increasing order graph. So, velocity increases from O to A and acceleration is positive.
(ii) AB $\rightarrow$ This part of graph is constant slope graph (straight line). It means velocity remains constant during AB.
(b) BC $\rightarrow$ This part of graph is decreasing order graph.
So, velocity decreases and acceleration is negative.
From this discussion, it is clear that during AB velocity is constant and maximum.
v_{max} = slope of AB portion of graph

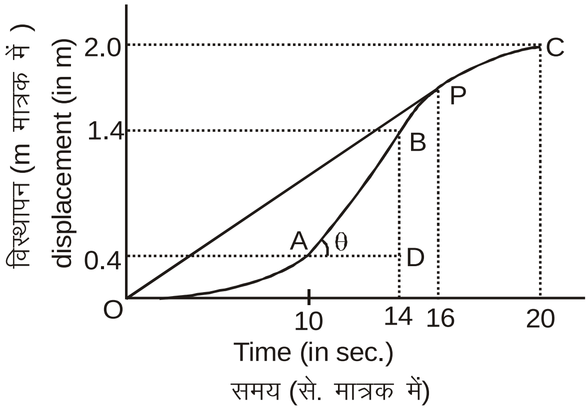
$= \dfrac{BD}{AD} = \dfrac{1}{4}$ m/s = 25 cm/s.

(c) Instantaneous velocity is given by slope of a point on s-t graph.
Since, straight line OP is tangent at the point P in fig.
So, instantaneous velocity at point P is equal to average velocity between O to P. The corresponding times is t_0 = 16 sec.

Sol. विस्थापन–समय वक्र की ढाल वेग के बराबर होती है

(a) औसत वेग $= \dfrac{\text{कुल विस्थापन}}{\text{कुल समय}}$

$= \dfrac{2}{20} = \dfrac{1}{10}$ m/s = 10 cm/s



दिया गया वक्र तीन भागों में विभाजित किया जा सकता है

(i) OA $\rightarrow$ वक्र का यह भाग की ढ़ाल धनात्मक व बढ़ते क्रम में है, अतः O से A तक वेग बढ़ता है व त्वरण धनात्मक है।

(ii) AB $\rightarrow$ वक्र का यह भाग नियत ढाल है। इसका अर्थ है कि AB के दौरान वेग नियत है।

(b) BC $\rightarrow$ वक्र का यह भाग की ढ़ाल घटते क्रम में है।

अतः वेग घट रहा है तथा त्वरण ऋणात्मक है।

इससे यह सिद्ध होता है कि भाग AB के दौरान वेग अधिकतम तथा नियत है।

v_{max} = वक्र के AB भाग की ढाल

$= \frac{BD}{AD} = \frac{1}{4}$ m/s = 25 cm/s.

(c) s-t वक्र के किसी बिन्दु पर तात्क्षणिक वेग का मान उस बिन्दु पर वक्र की ढाल के बराबर है।

चूंकि, सीधी रेखा OP बिन्दु P पर स्पर्श रेखा है।

अतः बिन्दु P पर तात्क्षणिक वेग O से P के बीच औसत वेग के समान है। संगत समय t_0 = 16 sec है