HINTS & SOLUTIONS

PART : I MATHEMATICS

1. α, β are real roots...................

Sol. $(\alpha + \beta)^2 - 2 < [\alpha(\alpha + \beta)] + [\beta(\alpha + \beta)] \le (\alpha + \beta)^2$

$\Rightarrow \alpha$ and β are integers

2. The interval in which ....

Sol. ${}^{2n}C_{(n-1)}x^{n-1} < {}^{2n}C_n x^n$ and ${}^{2n}C_{(n+1)}x^{n+1} < {}^{2n}C_n x^n$

$\therefore \frac{{}^{2n}C_{(n-1)}}{{}^{2n}C_n} < x < \frac{{}^{2n}C_n}{{}^{2n}C_{(n+1)}}$

3. If the number of terms...................

Sol. $2n + 1 = 53 \Rightarrow n = 26$

so exponent of 5 in (26)! is

$= \left[\frac{26}{5}\right] + \left[\frac{26}{5^2}\right] + ... = 6$

so $p = 7$

4. If largest constant...................

Sol. Using AM $\ge$ GM inequality

$(a + b)^2 + (a + b + 4c)^2 = (a + b)^2 + (a + 2c + b + 2c)^2 \ge$

$(2\sqrt{ab})^2 + \left(2\sqrt{2ac} + 2\sqrt{2bc}\right)^2$

$= 4ab + 8ac + 8bc + 16c\sqrt{ab}$

$= \frac{(a+b)^2 + (a+b+4c)^2}{abc}(a + b + c) \ge$

$\frac{4ab + 8ac + 8bc + 16c\sqrt{ab}}{abc}(a + b + c)$

$= \left[\frac{4}{c} + \frac{8}{b} + \frac{8}{a} + \frac{16}{\sqrt{ab}}\right](a + b + c),$

$\left(\frac{8}{2c} + \frac{8}{a} + \frac{8}{b} + \frac{8}{\sqrt{ab}} + \frac{8}{\sqrt{ab}}\right)\left(\frac{a}{2} + \frac{a}{2} + \frac{b}{2} + \frac{b}{2} + c\right)$

$\ge \left(5\sqrt[5]{\frac{8^5}{2a^2b^2c}}\right)\left(5\sqrt[5]{\frac{a^2b^2c}{2^4}}\right) = 100$

7. b and c are arithmetic ....

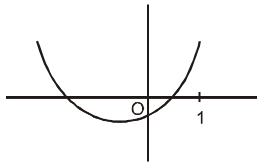
Sol. Let $a - d = 3t$, then $bc = (d + 2t)(d + t) = d^2 + 3td + 2t^2$ and

$\frac{a}{d} = 1 + \frac{3t}{d}$,

so $hk = d^2\left[1 + \frac{3t}{d}\right] = d^2 + 3td < bc \Rightarrow bc > hk$

8. If 6, 8 and 12 are ....

Sol. Let a be the first term and d be the common difference of A.P.



Now $a + (p - 1)d = 6$, $a + (q - 1)d = 8$, $a + (r - 1)d = 12$

$\Rightarrow (q - p)d = 2$ and $(r - q)d = 4$

$\Rightarrow \frac{q-p}{r-q} = \frac{1}{2} \Rightarrow 2q - 2p = r - q \Rightarrow 2p + r = 3q$ .... (1)

given $f(x) = rx^2 + 2px - 2q$

$f(0) = -2q$ and $f(1) = r + 2p - 2q = q$

Now $f(0)f(1) = -2q^2 < 0$

$\Rightarrow f(x) = 0$ has one root between 0 and 1, product of roots = negative

9. An A.P., a G.P., a ....

Sol. t_{n+2} of A.P. $= a + (n + 1)(b - a)$

$= b + (b - a)n$ ……….. (i)

t_{n+2} of G.P. $= a\left(\frac{b}{a}\right)^{n+1} = b\left(\frac{b}{a}\right)^n$ ………. (ii)

t_{n+2} of H.P. $= \frac{1}{\frac{1}{a} + (n+1)\left(\frac{a-b}{ab}\right)} = \frac{ab}{a + (a-b)n}$ …….. (iii)

these terms are in G.P. so

$b^2\left(\frac{b}{a}\right)^{2n} = [b + (b - a)n]\left[\frac{ab}{a + (a-b)n}\right]$

by solving $\frac{b^{2n+2} - a^{2n+2}}{ab(b^{2n} - a^{2n})} = \frac{n+1}{n}$

10. If $f(k) = \sum_{r=1}^{k}\frac{1}{r}$ and ....

Sol. $\because f(k) = \sum_{r=1}^{k}\frac{1}{r} = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{k}$

$\therefore f(1) = 1$

$f(2) = 1 + \frac{1}{2}$

$f(3) = 1 + \frac{1}{2} + \frac{1}{3}$

:

$f(2013) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} .... + \frac{1}{2013}$

Adding $\sum_{r=1}^{2013} f(r)$

$= 1 \times 2013 + \frac{1}{2} \times 2012 + \frac{1}{3} \times 2011 + ... + \frac{1}{2013} \times 1$

$= 1 \times 2013 + \frac{1}{2} \times (2013 - 1) + \frac{1}{3} \times (2013 - 2) + ... + \frac{1}{2013} \times (2013 - 2012)$

$= 2013\left(1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{2013}\right) - \left(\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + ... \frac{2012}{2013}\right)$

$= 2013\ f(2013) - \left\{\left(1 - \frac{1}{2}\right) + \left(1 - \frac{1}{3}\right) + \left(1 - \frac{1}{4}\right) + ... + \left(1 - \frac{1}{2013}\right)\right\}$

$= 2013\ f(2013) - 1 \times 2013 + \left(1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{2013}\right)$

$= 2013\ f(2013) - 2013 + f(2013) = 2014\ f(2013) - 2013$

$\therefore a = 2014, b = 2013, c = -2013$

11. If p, q, r are three ....

Sol. $\because \frac{p+q+r}{3} \geq \sqrt[3]{pqr} \Rightarrow (p+q+r)^3 \geq 27pqr$

but given that $27pqr \geq (p+q+r)^3$

so that $(p + q + r)^3 = 27pqr$ and equality holds when $p = q = r$.

12. If the fraction ....

Sol. It can be reduce in $\frac{\text{linear}}{\text{linear}}$ only when both have 2 roots common so

common roots are

$\{x^3 + (a-10)x^2 - x + a - 6\} - \{x^3 + (a-6)x^2 - x + a - 10\} = 0$

$-4x^2 + 4 = 0$

$x^2 = 1 \Rightarrow x = \pm 1$

so roots of $x^3 + (a-10)x^2 - x + a - 6 = 0$ are $1, -1, \alpha$

$\Sigma\alpha = -(a-10)$

$-1 + 1 + \alpha = -a + 10$

$\alpha = 10 - a$ ...... (i)

$\Sigma\alpha\beta\gamma = -(a-6)$

$-\alpha = -(a-6)$

$\alpha = a - 6$

$10 - a = a - 6$

$2a = 16$

$a = 8$

13. Let $(1 + x^2)^2 (1 + x)^n$ ....

Sol. $2\,{}^nC_1 = 1 + (2 + {}^nC_2)$

$n^2 - 5n + 6 = 0 \Rightarrow n = 2, 3$

15. (P) Let P and Q be ....

Sol. (P) $Q(P(Q(x)))$ has degree $n \times 7n = 7n^2$

$\therefore 7n^2 = 7 \Rightarrow n = 1$ Ans.

(Q) $\frac{10^{2013} - 1}{10^{33} - 1} = \frac{(10^{33})^{61} - 1}{10^{33} - 1}$

$= (10^{33\times 60} + 10^{33\times 59} + 10^{33\times 58} + ... + 1)$

$\Rightarrow$ unit digit $= 1$

(R) Real and distinct

$\Rightarrow D > 0 \Rightarrow m < 1 \quad m \in [-1, 1)$

$2\alpha\beta - 4 = 2(m^2 - 3m + 3) - 4 = 2(m^2 - 3m + 1)$

$\therefore$ max value = 10

16. (P) The number ....

Sol. (P) From given equation

$$\left(\frac{\sqrt{2}}{\sqrt{5+2\sqrt{2}}}\right)^x + \left(\frac{\sqrt{2}+1}{\sqrt{5+2\sqrt{2}}}\right)^x = 1$$

which is of the form $(\sin\theta)^x + (\cos\theta)^x = 1$

$\Rightarrow$ x = 2 is only the real solution.

$\therefore$ Number of solutions = only one.

(Q) As $x \in [0, 3] \Rightarrow x = 0, 1, 2, 3$

Required intervals [0, 1), [1, 2), [2, 3)

For $x \in [0, 1)$ the equation $x^2 - 3x + [x] = 0$

reduces to $x^2 - 3x = 0$, $x = 0$ $x = 3$, but $x = 3 \notin [0, 1)$

$\Rightarrow$ only one solution in this case

For $x \in [1, 2)$ equation reduces to

$x^2 - 3x + 1 = 0$, in this no value of $x \in [1, 2)$

For $x \in [2, 3)$ equation $x^2 - 3x + [x] = 0$ reduces to

$x^2 - 3x + 2 = 0$

$\Rightarrow$ x = 2 and x = 1, but $x = 1 \notin [2, 3)$

$\therefore$ only x = 2 is the solution in this case.

$\therefore$ Total number of solutions of $x^2 - 3x + [x] = 0$ is two.

(R) $a^x + a^{-x} \geq 2$ (using AM $\geq$ GM)

and $0 \leq \cos^2 x \leq 1$

$\therefore 3^x + 3^{-x} \geq 2$

and $0 \leq 2\cos^2\left(\frac{x^2 + x}{4}\right) \leq 2$

$3^x + 3^{-x} = 2$ & $2\cos^2\left(\frac{x^2 + x}{4}\right) = 2$

which is possible only if x = 0

$\therefore$ Number of solution is only one.

(S) solving given equations we get x = 28/a which is the common root of the given equations

$\Rightarrow \frac{(28)^2}{a^2} - 28 - 21 = 0 \Rightarrow a = \pm 4$

But $a > 0 \Rightarrow a = 4$

17. If $(1 + 3 + 5 + ...p) +$ ....

Sol. Note that p, q, r are positive odd integers. Let p = 2k - 1, where k is a positive integer.

$= 1 + 3 + 5 + ... + p = 1 + 3 + 5 + ... + (2k - 1)$

= Sum of first k odd positive integers

$= k^2 = \left(\frac{p+1}{2}\right)^2 \quad \left(\because 2k - 1 = p, \therefore k = \frac{p+1}{2}\right)$

Similarly $1 + 3 + 5 + ... + q = \left(\frac{q+1}{2}\right)^2$

and $1 + 3 + 5 + ... + r = \left(\frac{r+1}{2}\right)^2$

So, we are given that $\left(\frac{p+1}{2}\right)^2 + \left(\frac{q+1}{2}\right)^2 = \left(\frac{r+1}{2}\right)^2$

$\Rightarrow (p + 1)^2 + (q + 1)^2 = (r + 1)^2$

As we know that $3^2 + 4^2 = 5^2$, $6^2 + 8^2 = 10^2$ and p > 6, therefore, smallest possible values of p + q + r is given by

p + 1 = 8, q + 1 = 6, r + 1 = 10

$\Rightarrow$ p = 7, q = 5, r = 9

18. If x, y, z are ....

Sol. A.M. ≥ G.M. $\left\{(x-3),\ y+\frac{1}{2},\ z+\frac{5}{3}\right\}$

$$\frac{x-3+y+\frac{1}{2}+z+\frac{5}{3}}{3} \geq \sqrt[3]{(x-3)\left(y+\frac{1}{2}\right)\left(z+\frac{5}{3}\right)}$$

$$\frac{2+\frac{13}{6}}{3} \geq \sqrt[3]{\frac{(x-3)\,(2y+1)(3z+5)}{6}}$$

$$\left(\frac{25}{18}\right)^3 \times 6 \geq (x-3)\,(2y+1)(3z+5)$$

19. If S is the sum....................

Sol. Here , S = (10 + 12 + 14 + 16 + ... + 98) – (10 + 20 + 30 + ... + 98)
= 2{(1 + 2 + 3 + ... + 49) - (1 + 2 + 3 + 4)} - 10{1 + 2 + 3 + ... + 9}

$$= 2\left\{\frac{49\times(49+1)}{2}\right\} - 2\times 10 - 10\times\frac{9\times(9+1)}{2}$$

$$= 49\times 50 - 20 - 450 = 1980$$

20. Number of terms....................

Sol. $\left(1+\left(x^3+\frac{1}{x^3}\right)\right)^{\lambda}$ let $\lambda = \Sigma n = \frac{n(n+1)}{2}$

$$= 1 + {}^{\lambda}C_1\left(x^3+\frac{1}{x^3}\right) + {}^{\lambda}C_2\left(x^3+\frac{1}{x^3}\right)^2 + {}^{\lambda}C_3\left(x^3+\frac{1}{x^3}\right)^3 ....$$

Now in each term 2 extra dissimilar terms are added

eg $\left(x^3+\frac{1}{x^3}\right)^3 = x^9 + \frac{1}{x^9} + 3\left(x^3+\frac{1}{x^3}\right)$

no of terms $= 1+2+2+2....(\lambda+1)$ terms

1 + (2 + 2 ...... λ terms)

$$= 1 + 2\left(\frac{n}{2}\right)(n+1)$$

PART : II PHYSICS

21. A non–viscous ....

Sol. Applying Bernoulli is equation from section–(1) and (2)

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P_1 + \frac{1}{2}\rho V^2 + 0 = P_2 + \frac{1}{2}\rho(2v)^2 + \rho g h$$

and $P_1 - P_2 = \rho g(2h)$

Solving we get,

$$V = \sqrt{\frac{2gh}{3}}$$

(C) Work done by gravitation force per unit value W_{gr} = decrease in gravitational $PT_{value} = \rho g h_1 - \rho g h_2$

$W_{gr} = 0 - \rho g h$

(D) Work done by elastic force volume,
W_e = decrease in elastic $P.E._{vol}$ = decrease in pressure

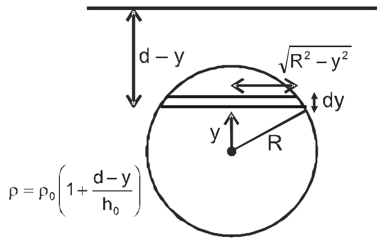
$= P_1 - P_2 = \rho g(2h)$.

22. A uniform solid ....

Sol. $dB = \pi(R^2 - y^2)dy \quad \rho_0\left(1+\frac{d-y}{h_0}\right)g$

$$dB = \frac{\pi\rho_0 g}{h_0}\ [(R^2 - y^2)\,(h_0 + d - y)]\ dy$$

$$= \frac{\pi\rho_0 g}{h_0}\,[R^2(h_0 + d)dy - R^2 y dy - (h_0 + d)y^2 dy + y^3 dy]$$



$$B = \int_{y=-R}^{+R} dB = \frac{\pi\rho_0 g}{h_0}$$

$$\left(R^2(h_0+d)y - \frac{R^2y^2}{2} - (h_0+d)\frac{y^3}{3} + \frac{y^4}{4}\right)_{-R}^{+R}$$

$$B = \frac{\pi\rho_0 g}{h_0}\left[(h_0+d)R^2(2R) - \frac{(h_0+d)}{3}\quad(2R^3)\right] =$$

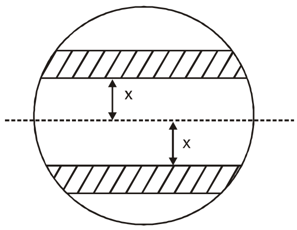
$$\frac{\pi\rho_0 g}{h_0}\left[\frac{4}{3}(h_0+d)R^3\right]$$

$$= \frac{4}{3}\pi R^3 g \quad \frac{\rho_0}{h_0}(h_0+d) = \frac{4}{3}\pi R^3 g\sigma$$

$$\Rightarrow \quad \sigma = \frac{\rho_0}{h_0}(h_0+d)$$

$$\sigma = \rho_0\left(1+\frac{d}{h_0}\right)$$

Alternate solution



$$\sigma v g = \int\left[\rho_0\left(1+\frac{d-x}{h_0}\right)dvg + \rho_0\left(1+\frac{d+x}{h_0}\right)dvg\right]$$

$$\sigma v = 2\rho_0\left(1+\frac{d}{h_0}\right)\int_0^{v/2} dV = \rho_0 v\left(1+\frac{d}{h_0}\right)$$

$$\Rightarrow \quad \sigma = \rho_0\left(1+\frac{d}{h_0}\right)$$

23. A large open ....

Sol. $-A\dfrac{dy}{dt} = a\sqrt{2gy}$

$$\frac{2A}{a\sqrt{2g}}\left(\sqrt{H}-\sqrt{\frac{H}{n}}\right) = T_1$$

$$\frac{2A}{a\sqrt{2g}}\left(\sqrt{\frac{H}{n}}-0\right) = T_2$$

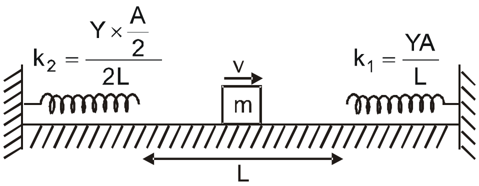
$T_1 = T_2$

n = 4.

24. In the given ....

Sol. Rod behaves as spring of spring constant = $\dfrac{YA}{\ell}$

Equivalent system is:



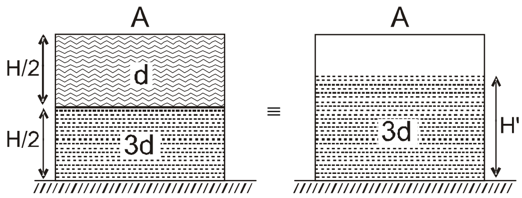
The time period of oscillations of block is

$$T = \frac{2L}{V} + \frac{1}{2}\left(2\pi\sqrt{\frac{mL}{YA}}\right) + \frac{1}{2}\left(2\pi\sqrt{\frac{m\times 2L}{Y\times A/2}}\right)$$

$$= \frac{2L}{V} + 3\pi\sqrt{\frac{m\,L}{A\,Y}}$$

25. A container ....

Sol. A, B, D



$$\frac{H}{2}\times d + \frac{H}{2}\times 3d = H'\times 3d$$

$\Rightarrow H' = \dfrac{2H}{3}$

$V_{efflux} = \sqrt{2g(H'-h)}$

V_{efflux} is maximum when h = H'/2

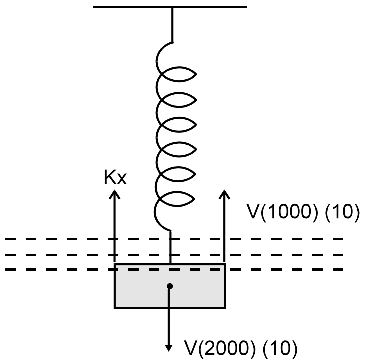
$\therefore \quad V_{max} = \sqrt{\dfrac{2gH}{3}}$

Range $\quad R = V_{efflux} \times \sqrt{\dfrac{2(H'-h)}{g}}$

$R_{max} = \dfrac{2H}{3}$

26. A block of density ....

Sol. Kx = V(2000) (10) – V (1000) (10)



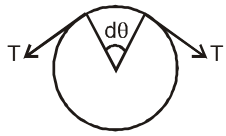
$= \dfrac{10}{2000}$ [1000 × 10]

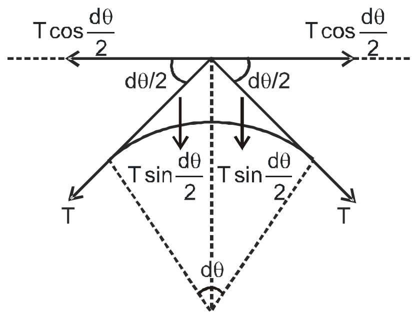
Kx = 50 N ... (b)

$U_{stored} = \dfrac{1}{2} \times (100)\left(\dfrac{50}{100}\right)^2 = \dfrac{1}{2}\times\dfrac{2500}{100} = 12.5\text{ J}$

27. A uniform ring ....

Sol. Let T be the tension in the string.





$$2T\sin\frac{d\theta}{2} = \frac{m}{2\pi R}\times R\omega^2\cdot Rd\theta.$$

$$T = \frac{mR\omega^2}{2\pi}$$

$$Y = \frac{T/A}{\Delta\ell/\ell}$$

$$\frac{\Delta\ell}{\ell} = \frac{T}{Y.A} \Rightarrow \Delta\ell = \frac{T}{Y.A}\times\ell = \frac{mR\omega^2}{2\pi}\times\frac{1}{Y.A}\times 2\pi R = \frac{mR^2\omega^2}{Y.A}$$

$$\frac{\Delta\ell}{\ell} = \frac{\Delta R}{R} = \frac{T}{Y.A} = \frac{m.R\omega^2}{2\pi A.Y}$$

$$\Rightarrow \Delta R = \frac{mR^2\omega^2}{2\pi A.Y}$$

$$V = \frac{1}{2}K.X^2 = \frac{1}{2}\left(\frac{Y.A}{\ell}\right) \times (\Delta\ell)^2$$

$$= \frac{1}{2}\frac{Y.A}{2\pi R} \times \left(\frac{mR^2\omega^2}{Y.A}\right)^2 = \frac{1}{4\pi}\left(\frac{m^2R^3\omega^4}{Y.A}\right)$$

28. If ρ is the density ....

Sol. The stress is maximum at the uppermost point and is equal to the weight of the rod divided by the area. At the highest point,

breaking stress = weight of rod/area of cross section of the rod

$$\sigma = \frac{AL\rho g}{A} = L\rho g$$

$$L = \frac{\sigma}{\rho g}.$$

The stress decreases linearly to zero at the lowest end.

29. Lower end of ....

Sol. S = 0.5 N/m $r = 10^{-3}$ m $\theta_C = 120°$ $\rho = 5 \times 10^3$ kg/m^3

$$h_{,max} = \frac{2S\cos\theta_c}{r\rho g} = \frac{(2)\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{(10^{-3})(5\times10^3)(10)} = 10^{-2} \text{ m} = 1\text{cm}$$

If $h = \frac{h_{max}}{2}$

$$\frac{2S\cos\theta}{r\rho g} = \frac{1}{2}\frac{2S\cos\theta_c}{r\rho g}$$

$$\Rightarrow \cos\theta = -\frac{1}{4}$$

$$\theta = \cos^{-1}\left(-\frac{1}{4}\right)$$

If $h = \frac{h_{max}}{3}$

$$\frac{2S\cos\theta}{r\rho g} = \frac{1}{3}\frac{2S\cos\theta_c}{r\rho g}$$

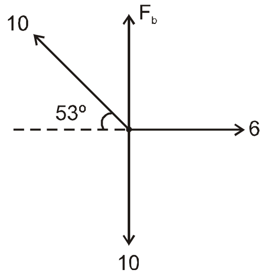
$$\Rightarrow \cos\theta = -\frac{1}{6}$$

$$\theta = \cos^{-1}\left(-\frac{1}{6}\right)$$

30. An external ....

Sol. $F_{drag} = 6\pi\eta RV$

$$= 6\pi \frac{20}{6\pi} \times 0.1 \times 5 = 10 \text{ N}$$

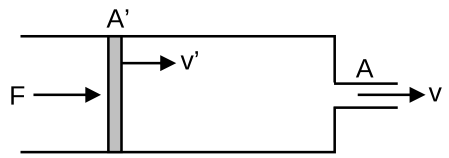


$F_b + 8 = 10$

$F_b = 2$

31. In a certain ....

Sol.



$$\frac{F}{A} + \frac{1}{2}\rho v'^2 = \frac{1}{2}\rho v^2 \qquad \text{(i)}$$

$A'v' = Av$ (ii)

$F \propto v^2$ (A)

$P = F \cdot v'$

(B)

$$Av = \text{volume flow rate} = \frac{\text{volume}}{t}$$

$$\therefore t \propto \frac{1}{v} \qquad \text{(C)}$$

W.D. = ΔK $\Rightarrow$ (D)

32. A light wire of ....

Sol. Here stretching force is same but stress will be different for wires.

For upper wire

Stress = Y × strain

$$\Rightarrow \frac{mg}{\pi r^2} = Y \times \frac{\Delta\ell}{\ell}$$

$$\Rightarrow \Delta\ell_1 = \frac{mg}{\pi r^2}\frac{\ell}{Y}$$

for lower wire

$$\Delta\ell_2 = \frac{mg}{\pi(2r)^2} \times \frac{2\ell}{Y} = \frac{mg\ell}{2\pi r^2 Y}$$

total elongation = $\frac{3}{2}\frac{mg\ell}{\pi r^2 Y}$

Energy stored per unit volume = $\frac{(\text{stress})^2}{2Y}$

$$\frac{\text{force}}{\text{area of cross section}} = \frac{F}{\pi r^2}$$

$$\frac{\text{बल}}{\text{अनुप्रस्थकाट क्षैत्राफल}} = \frac{F}{\pi r^2}$$

Hence ratio will be = 16 : 1

33. A copper wire ....

Sol. σ (breaking stress)

$$= \frac{F}{A} = \frac{mL\omega_f^2}{A} = 10^{10} N/m^2$$

$$Y = \frac{FL}{A\Delta L}$$

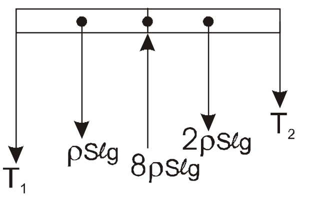
$$Y = \frac{\sigma_i L}{\Delta L} = \frac{(20)^2 \times 1}{10^{-6} \times 10^{-3}} = 4 \times 10^{11} N/m^2$$

34. When a capillary ....

Sol $h = \frac{2T\cos\theta}{\rho g r}$

35. A rod is formed ....

Sol. Consider the FBD shown in the figure.



Force balance

$$\Rightarrow \quad T_1 + T_2 = 5\rho S\ell g$$

Torque balance about point P

$$\Rightarrow T_1 \times 2\ell + \rho S\ell g \times \frac{3}{2}\ell - 8\rho S\ell g \times \ell + \rho S\ell g \times \ell = 0$$

$$T_1 = \frac{11\rho S\ell g}{4}$$

$$\Rightarrow T_2 = \frac{9\rho S\ell g}{4}$$

36. A cubical block ....

Sol. $F = mA = F_0 - F_V = F_0 - \frac{\eta A v}{d}$

$$A = \frac{F_0}{m} - \frac{\eta A}{md} v = a - bv$$

$$\frac{dv}{dt} = a - bv \quad \Rightarrow \quad \int_0^v \frac{dv}{a-bv} = \int_0^t dt$$

$$\Rightarrow \left(-\frac{1}{b}\right) \ell n \left(\frac{a-bv}{a}\right) = t \Rightarrow V = \frac{a}{b}(1 - e^{-bt})$$

$$\frac{dx}{dt} = \frac{a}{b}(1 - e^{-bt}) \Rightarrow \int_0^x dx = \frac{a}{b}\int_0^t (1 - e^{-bt})dt$$

$$x = \frac{a}{b}t - \frac{a}{b^2} + \frac{a}{b^2} e^{-bt}$$

$$A = ae^{-bt}$$

$$k = \frac{1}{2}mv^2$$

$$\frac{dk}{dt} = mv\frac{dv}{dt} = \frac{ma}{b}(1 - e^{-bt})(ae^{-bt})$$

$$= \frac{ma^2}{b}(e^{-bt} - e^{-2bt})$$

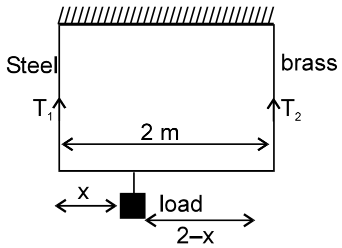
37. In the middle ....

Sol. Pressure in the air inside the column of mercury is equal to the weight of mercury over the air divided by the internal cross sectional area of the tube. When the temperature increases, the weight of the upper part of the mercury column does not change. That is why the pressure in the air is also constant. For the isobaric process, the change in volume is proportional to the change in temperature. The same is true for the lengths of the air column.

$$\frac{\ell}{\ell_0} = \frac{T}{T_0} \Rightarrow \ell = \frac{\ell_0 T}{T_0} = 11$$

38. A light rod of ....

Sol.



$$\text{Stress} = \frac{T}{A} = \frac{T_1}{A_1} \Rightarrow \frac{T_1}{T_2} = \frac{1}{2}$$

& for equilibrium,

$mg = T_1 + T_2 = \frac{3T_2}{2}$ (After balance)

& $mgx - T_2.(2) = 0$ (Target balance) $\frac{3T_2}{2}x - 2T_2 = 0$ $x = \frac{4}{3}$ m

39. A thin rod of ....

Sol. Strain (ε) = $\frac{\Delta \ell}{\ell} = \propto \Delta T = (10^{-5})(200) = 2 \times 10^{-3}$

Stress = Y (strain)

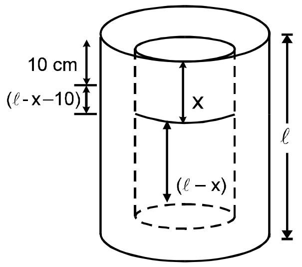
Stress = $10^{11} \times 2 \times 10^{-3} = 2 \times 10^{8}$ N/m^2

$\Rightarrow$ Required force = stress × Area = $(2 \times 10^8)(2 \times 10^{-6}) = 4 \times 10^2 = 400$ N

$\therefore$ Mass to be attached = $\frac{400}{g} = 40$ kg Ans.

40. A tube with both ....

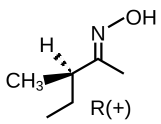
Sol. After oil is filled up, pressure at the depth of lower end should equate if measured from inside and outside the tube. Suppose depth of oil is x cm then :



$1000.g. [(\ell - 10)cm] = 800\ g. (x\ cm) + 1000\ g[(\ell - x)cm]$

$\Rightarrow$ x = 50 cm

PART : III CHEMISTRY

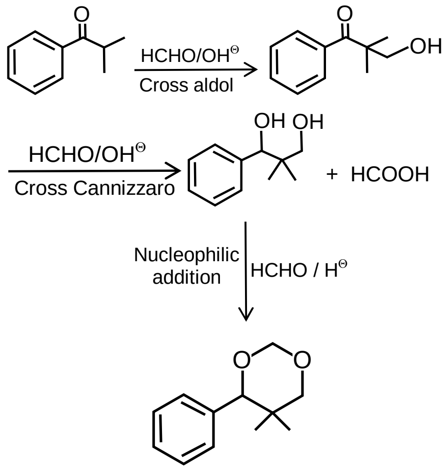
41. 

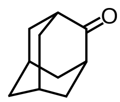
....

Sol. One step front side substitution occurs when the incoming group is attached to the leaving group and thus held to the front side in a three membered ring T.S. such is the case in rearrangements including Beckmann, Hofmann, Curtius, Schmidt Wolff, Lossen and Baeyer-villiger which give retention.

Above example is Beckmann rearrangement.

42. The major product ....

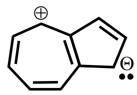
Sol. 

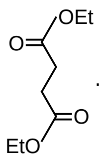
44. 

....

Sol. Ketones may be oxidized to esters by peracids or hydrogen peroxide a process known as Baeyer-villiger oxidation.

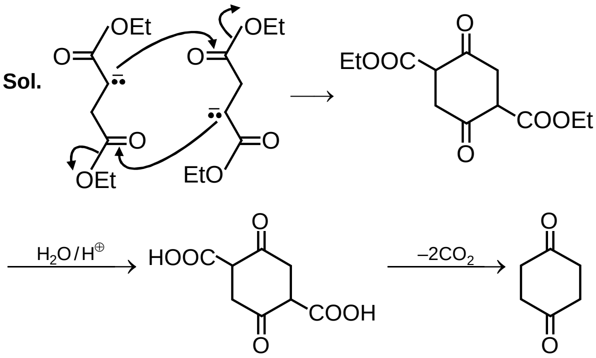
46. Which of the following ....

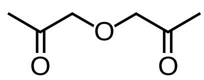
Sol. 

47. 

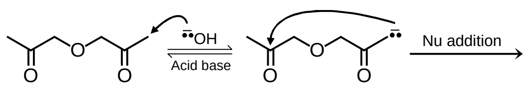
....

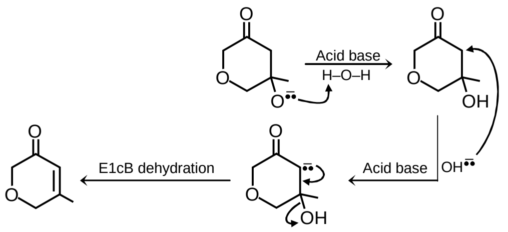
(2 moles)

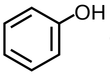
Sol. 

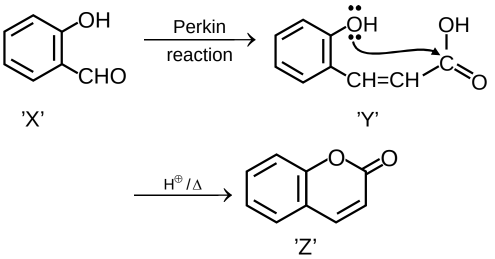
48. 

....

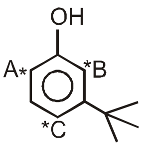
Sol. 



49.  ....

Sol. 

50. The reactivity of ....

Sol. 

–OH group is strongly activating & O, P–directing due to its powerful +M effect.

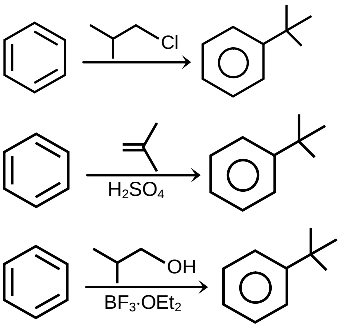
With I_2 only A is substituted, since –I is large, steric inhibition by large $–CMe_3$ group forbids substitution at B or C.

–Br and –Cl become progressively more reactive, due to –

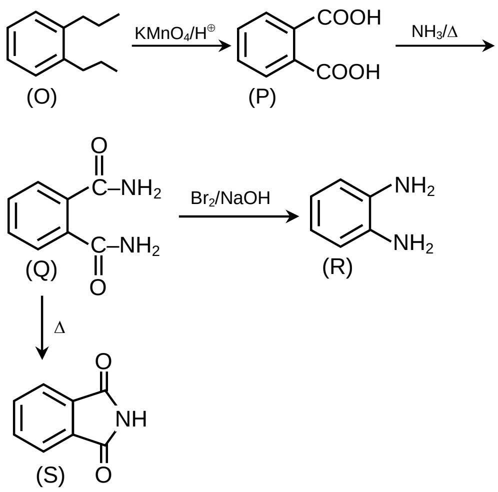
(a) increasing electrophilic nature of $X^{\oplus}$ (not mentioned is any option).

(b) Smaller size most sterically hindered loaction is B which is substituted only by –Cl.

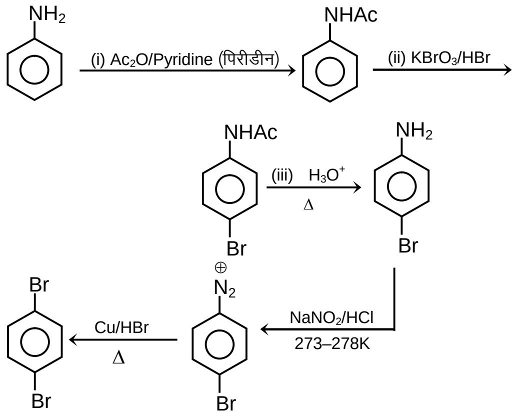
51. Among the following.........

Sol. 

53. Treatment of compound **O** ....

Sol. 

54. The product(s) of the ....

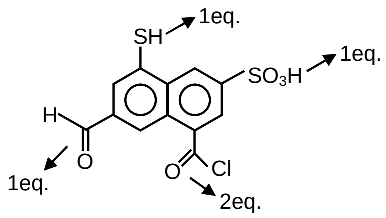
Sol. 

59. How many of the ....

Sol. Incorrect product (v), (vi) & (x).

60. How many equivalent ....

Sol. Value of x is 5



ADVANCED PATTERN PART TEST-4 (APT-4)

TARGET : JEE (MAIN+ADVANCED) 2017

DATE : 03-05-2017 REVISION PLAN-2 COURSE : VIJETA (ADP), VIJAY (ADR)

ANSWER KEY

CODE-0

PART : I MATHEMATICS

1.	(B)	**2.**	(B)	**3.**	(D)	**4.**	(A)	**5.**	(BC)	**6.**	(AC)	**7.**	(A)
8.	(AD)	**9.**	(AC)	**10.**	(BCD)	**11.**	(ABD)	**12.**	(ACD)	**13.**	(AB)	**14.**	(AC)
15.	(A)	**16.**	(B)	**17.**	(21)	**18.**	(25)	**19.**	(20)	**20.**	(21)		

PART : II PHYSICS

21.	(B)	**22.**	(D)	**23.**	(C)	**24.**	(A)	**25.**	(ABD)	**26.**	(BC)	**27.**	(ACD)
28.	(AB)	**29.**	(ACD)	**30.**	(ABCD)	**31.**	(ABCD)	**32.**	(AB)	**33.**	(AB)	**34.**	(BD)
35.	(A)	**36.**	(B)	**37.**	(11)	**38.**	(30)	**39.**	(40)	**40.**	(50)		

PART : III CHEMISTRY

41.	(D)	**42.**	(A)	**43.**	(A)	**44.**	(C)	**45.**	(A)	**46.**	(ABC)	**47.**	(C)
48.	(ABCD)	**49.**	(C)	**50.**	(D)	**51.**	(BCD)	**52.**	(ABC)	**53.**	(ABD)	**54.**	(B)
55.	(C)	**56.**	(B)	**57.**	(23)	**58.**	(41)	**59.**	(73)	**60.**	(15)		

ADVANCED PATTERN PART TEST-4 (APT-4)

TARGET : JEE (MAIN+ADVANCED) 2017

DATE : 03-05-2017 REVISION PLAN-2 COURSE : VIJETA (ADP), VIJAY (ADR)

ANSWER KEY

CODE-1

PART : I MATHEMATICS

1.	(A)	**2.**	(C)	**3.**	(C)	**4.**	(B)	**5.**	(AC)	**6.**	(AD)	**7.**	(B)
8.	(AC)	**9.**	(AD)	**10.**	(ACD)	**11.**	(ACD)	**12.**	(BCD)	**13.**	(AC)	**14.**	(BC)
15.	(A)	**16.**	(B)	**17.**	(21)	**18.**	(25)	**19.**	(20)	**20.**	(21)		

PART : II PHYSICS

21.	(A)	**22.**	(B)	**23.**	(A)	**24.**	(D)	**25.**	(BCD)	**26.**	(AB)	**27.**	(ABC)
28.	(CD)	**29.**	(ABD)	**30.**	(ABCD)	**31.**	(ABCD)	**32.**	(CD)	**33.**	(CD)	**34.**	(AD)
35.	(C)	**36.**	(A)	**37.**	(11)	**38.**	(30)	**39.**	(40)	**40.**	(50)		

PART : III CHEMISTRY

41.	(C)	**42.**	(B)	**43.**	(B)	**44.**	(D)	**45.**	(B)	**46.**	(ABD)	**47.**	(D)
48.	(ABCD)	**49.**	(D)	**50.**	(C)	**51.**	(ACD)	**52.**	(ABD)	**53.**	(ABC)	**54.**	(A)
55.	(D)	**56.**	(A)	**57.**	(23)	**58.**	(41)	**59.**	(73)	**60.**	(15)		

ADVANCED PATTERN PART TEST-4 (APT-4)

TARGET : JEE (MAIN+ADVANCED) 2017

DATE : 03-05-2017 REVISION PLAN-2 COURSE : VIJETA (ADP), VIJAY (ADR)

ANSWER KEY

CODE-2

PART : I MATHEMATICS

1.	(B)	**2.**	(B)	**3.**	(D)	**4.**	(A)	**5.**	(BC)	**6.**	(AC)	**7.**	(A)
8.	(AD)	**9.**	(AC)	**10.**	(BCD)	**11.**	(ABD)	**12.**	(ACD)	**13.**	(AB)	**14.**	(AC)
15.	(A)	**16.**	(B)	**17.**	(21)	**18.**	(25)	**19.**	(20)	**20.**	(21)		

PART : II PHYSICS

21.	(B)	**22.**	(D)	**23.**	(C)	**24.**	(A)	**25.**	(ABD)	**26.**	(BC)	**27.**	(ACD)
28.	(AB)	**29.**	(ACD)	**30.**	(ABCD)	**31.**	(ABCD)	**32.**	(AB)	**33.**	(AB)	**34.**	(BD)
35.	(A)	**36.**	(B)	**37.**	(11)	**38.**	(30)	**39.**	(40)	**40.**	(50)		

PART : III CHEMISTRY

41.	(D)	**42.**	(A)	**43.**	(A)	**44.**	(C)	**45.**	(A)	**46.**	(ABC)	**47.**	(C)
48.	(ABCD)	**49.**	(C)	**50.**	(D)	**51.**	(BCD)	**52.**	(ABC)	**53.**	(ABD)	**54.**	(B)
55.	(C)	**56.**	(B)	**57.**	(23)	**58.**	(41)	**59.**	(73)	**60.**	(15)		

ADVANCED PATTERN PART TEST-4 (APT-4)

TARGET : JEE (MAIN+ADVANCED) 2017

DATE : 03-05-2017 REVISION PLAN-2 COURSE : VIJETA (ADP), VIJAY (ADR)

ANSWER KEY

CODE-3

PART : I MATHEMATICS

1.	(A)	**2.**	(C)	**3.**	(C)	**4.**	(B)	**5.**	(AC)	**6.**	(AD)	**7.**	(B)
8.	(AC)	**9.**	(AD)	**10.**	(ACD)	**11.**	(ACD)	**12.**	(BCD)	**13.**	(AC)	**14.**	(BC)
15.	(A)	**16.**	(B)	**17.**	(21)	**18.**	(25)	**19.**	(20)	**20.**	(21)		

PART : II PHYSICS

21.	(A)	**22.**	(B)	**23.**	(A)	**24.**	(D)	**25.**	(BCD)	**26.**	(AB)	**27.**	(ABC)
28.	(CD)	**29.**	(ABD)	**30.**	(ABCD)	**31.**	(ABCD)	**32.**	(CD)	**33.**	(CD)	**34.**	(AD)
35.	(C)	**36.**	(A)	**37.**	(11)	**38.**	(30)	**39.**	(40)	**40.**	(50)		

PART : III CHEMISTRY

41.	(C)	**42.**	(B)	**43.**	(B)	**44.**	(D)	**45.**	(B)	**46.**	(ABD)	**47.**	(D)
48.	(ABCD)	**49.**	(D)	**50.**	(C)	**51.**	(ACD)	**52.**	(ABD)	**53.**	(ABC)	**54.**	(A)
55.	(D)	**56.**	(A)	**57.**	(23)	**58.**	(41)	**59.**	(73)	**60.**	(15)		