HINTS & SOLUTIONS

PART : I MATHEMATICS

1. $\int \frac{x^3+5x^2-3x+4}{\sqrt{x^2+x+1}}dx$ is..................

Sol. Here $\varphi(x) = x^3+5x^2-3x+4$ is a polynomial of third degree.

We write

$$\int \frac{x^3+5x^2-3x+4}{\sqrt{x^2+x+1}}dx$$

$$=\left(Ax^2+Bx+c\right)\sqrt{x^2+x+1}+D\int\frac{1}{\sqrt{x^2+x+1}}dx.$$

Differentiating and multiplying by $\sqrt{x^2+x+1}$, we get

$$x^3+5x^2-3x+4$$

$$=(2Ax+B)\left(x^2+x+1\right)+\frac{1}{2}\left(Ax^2+Bx+C\right)(2x+1)+D.$$

Equating the coefficients of like powers of x, we get

$1=3A,$

$5=\frac{5}{2}A+2B,$

$-3=2A+\frac{3}{2}B+C$

$4=B+\frac{1}{2}C+D.$

From these, we obtain

$A=1/3$; $B=25/12$; $C=-163/24$; $D=85/16.$

2. If f(x) be a twice ....

Sol. $t^2f(x) - 2t f'(x) + f''(x) = 0$ has equal roots

$D = 0 \Rightarrow \frac{f''(x)}{f'(x)} = \frac{f'(x)}{f(x)}$

$\Rightarrow \ln(f'(x)) = \ln(f(x)) - \ell n\, c$

$\Rightarrow f(x) = cf'(x) \Rightarrow f(0) = cf'(0) \Rightarrow c = \frac{1}{2}$

$\therefore \frac{f'(x)}{f(x)} = 2 \Rightarrow \ln(f(x)) = 2x + k \Rightarrow \ln(f(0)) = k \Rightarrow k = 0$

$\Rightarrow \ln(f(x)) = 2x \Rightarrow f(x) = e^{2x}$

$t^2.e^{2x} - 4te^{2x} + 4e^{2x} = 0$

$\Rightarrow t = 2$

$$\lim_{x\to0}\left(\frac{f(x)-1}{x}-\frac{t}{2}\right) = \lim_{x\to0}\left(\frac{e^{2x}-1}{2x}\times2-\frac{2}{2}\right)$$

$= 2 - 1 = 1$

3. Find the area enclose ....

Sol. To describe the first arc OPA of the cycloid, θ varies from 0 to 2π. The coordinates of A are $(2a\pi,0)$.

The required area $=\int_0^{2\pi a} y\,dx.$

We have $dx = a(1-\cos\theta)d\theta$

$\therefore$ The required area $=\int_0^{2\pi} a(1-\cos\theta)\cdot a(1-\cos\theta)d\theta$

$$=a^2\int_0^{2\pi}\left(1-2\cos\theta+\cos^2\theta\right)d\theta$$

$$=a^2\int_0^{2\pi}\left(1-2\cos\theta+\frac{1+\cos2\theta}{2}\right)d\theta$$

$$=a^2\left|\theta-2\sin\theta+\frac{\theta}{2}+\frac{\sin2\theta}{4}\right|_0^{2\pi}$$

$=3a^2\pi.$

Note. The integral

$$\int_0^{2\pi} a^2(1-\cos\theta)^2 d\theta$$

Could also be evaluated differently as follows. We have

$$\int_0^{2\pi} a^2(1-\cos\theta)^2 d\theta = 2a^2\int_0^{\pi}(1-\cos\theta)^2 d\theta$$

$$=8a^2\int_0^{\pi}\sin^4\frac{\theta}{2}d\theta$$

$$=16a^2\int_0^{\pi/2}\sin^4\phi\, d\phi, \text{ where } \frac{\theta}{2}=\phi.$$

$$=16a^2\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}=3a^2\pi.$$

4. Solve ....

Sol. We have, $\frac{dy}{dx}+\frac{x^2+1}{x\left(x^2-1\right)}y=\frac{1}{x^2},$

so that it is linear.

Here, $P=\frac{x^2+1}{x\left(x^2-1\right)},$

$$\int P\,dx = \int \frac{x^2+1}{x\left(x^2-1\right)}dx$$

$$= \int\left(\frac{1}{x+1}+\frac{1}{x-1}-\frac{1}{x}\right)dx = \log\frac{x^2-1}{x}.$$

Thus, the integrating factor is

$$e^{\int P\,dx} = e^{\log\frac{x^2-1}{x}} = \frac{x^2-1}{x}.$$

Multiplying by $\left(x^2-1\right)/x$, we obtain

$$\frac{x^2-1}{x}\left[\frac{dy}{dx}+\frac{x^2+1}{x\left(x^2-1\right)}y\right] = \frac{1}{x^2}\cdot\frac{x^2-1}{x}.$$

Thus, the solution is

$$y\frac{x^2-1}{x} = \int\frac{x^2-1}{x^3}dx + c = \log x + \frac{1}{2x^2} + c.$$

5. For $\int_0^\infty e^{-\alpha x}\frac{\sin\beta x}{x}dx$, ...

Sol. We write

$$\varphi(\alpha,\beta) = \int_0^\infty e^{-\alpha x}\frac{\sin\beta x}{x}dx \quad .....(1)$$

Assuming the validity of differentiation under integral sign, we have, on differentiating w.r.t. β

$$\varphi_\beta(\alpha,\beta) = \int_0^\infty e^{-\alpha x}\cos\beta x\,dx$$

$\frac{\alpha}{\alpha^2+\beta^2}$, if $\alpha > 0$

Integrating w.r.t. β, we get

$$\varphi(\alpha,\beta) = \tan^{-1}\frac{\beta}{\alpha} + c, \quad ......(2)$$

were, c, is a constant. From (1), we have,

$$\varphi(\alpha,\beta) = 0 \quad ......(3)$$

so that putting $\beta = 0$ in (2), we obtain c = 0. Thus

$$\varphi(\alpha,\beta) = \tan^{-1}\frac{\beta}{\alpha}, \quad .....(4)$$

where $\alpha > 0$.

Also we assume that $\varphi(\alpha,\beta)$ is a continuous function of α for $\alpha > 0$..

We have, from (1),

$$\varphi(0,\beta) = \int_0^\infty \frac{\sin\beta x}{x}dx,$$

and from (4)

$$\lim_{\alpha\to0}\varphi(0,\beta) = \lim_{\alpha\to0}\left(\tan^{-1}\frac{\beta}{\alpha}\right) = \begin{cases}\pi/2, & \text{if } \beta > 0,\\ 0, & \text{if } \beta = 0,\\ -\pi/2 & \text{if } \beta < 0.\end{cases}$$

Also, because of continuity,

$$\lim_{\alpha\to0}\varphi(\alpha,\beta) = (0,\beta).$$

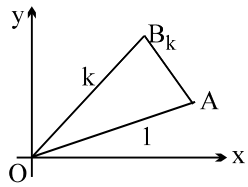
$$\therefore \int_0^\infty \frac{\sin\beta x}{x}dx = \begin{cases}\pi/2, & \text{if } \beta > 0,\\ 0, & \text{if } \beta = 0,\\ -\pi/2 & \text{if } \beta < 0.\end{cases}$$

In particular, we have,

$$\int_0^\alpha \frac{\sin x}{x}dx = \frac{\pi}{2}.$$

6. For positive integers ....

Sol. $OB_k = k$



$\angle AOB_k = \frac{k\pi}{2n}$

$S_k = \frac{1}{2}k\sin\frac{k\pi}{2n}$ (using $\Delta = \frac{1}{2}ab\sin\theta$)

$$\therefore L = \frac{k}{2n^2}\sum_{k=1}^{n}\sin\frac{k\pi}{2n} = \frac{1}{2n}\sum_{k=1}^{n}\frac{k}{n}\sin\frac{k\pi}{2n} = \frac{1}{2}\int_0^1 x\cdot\sin\frac{\pi x}{2}dx$$

$$= \frac{1}{2}\left[\underbrace{\frac{-2}{\pi}x\cos\frac{\pi x}{2}\Big|_0^1}_{\text{zero}} + \frac{2}{\pi}\int_0^1\cos\frac{\pi x}{2}dx\right]$$

$$= \frac{1}{2}\left[0+\frac{2}{\pi}\cdot\frac{2}{\pi}\left(\sin\frac{\pi x}{2}\right)_0^1\right] = \frac{2}{\pi^2}$$

7. Through any point ....

Sol. Given $A_1 = \frac{A_2}{2}$

Also $A_1 + A_2 = xy$

$$3A_1 = xy$$

$$A_1 = \frac{xy}{3}$$

$$A_1 = \int_x^0 y\,dx$$

Differentiate both sides

$$3y = y + \frac{x\,dy}{dx}$$

$$2y = \frac{x\,dy}{dx}$$

$$2\frac{dx}{x} = \frac{dy}{y}$$

$$2\ln x = \ln y + \ln c$$

$$\ln x^2 = \ln yc$$

$$x^2 = yc$$

Parabola

8. The value of..................

Sol. $1 < \ell n\, x < \dfrac{x}{e}$

$\left(\dfrac{e}{x}\right)^{\frac{1}{3}} < \dfrac{1}{\sqrt[3]{\ell n\, x}} < 1$

$\therefore\ e^{\frac{1}{3}} \int \dfrac{dx}{x^{1/3}} < I < 1$

$\Rightarrow e^{\frac{1}{3}} \cdot \dfrac{3}{2}\left(16^{\frac{1}{3}} - 9^{\frac{1}{3}}\right) < I < 1$

9. Let 'e' be the ....

Sol. $\dfrac{1}{e^2} + \dfrac{1}{(f(e))^2} = 1 \Rightarrow f(e) = \dfrac{e}{\sqrt{e^2-1}}$

$f(f(e)) = e$

$f(f(f(e))) = \dfrac{e}{\sqrt{e^2-1}}$

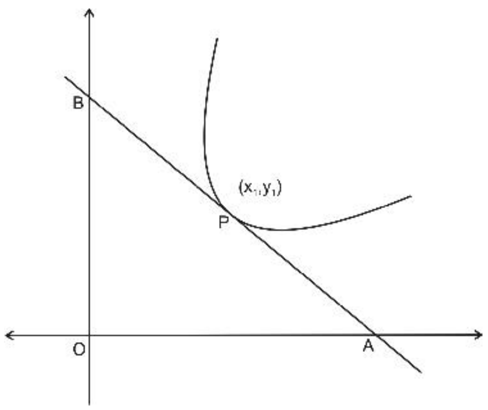
$G(e) = f(f(f...f(e))) = \begin{cases} e, \text{if n is even} \\ \dfrac{e}{\sqrt{e^2-1}}, \text{if n is odd} \end{cases}$

$\therefore\ \int_1^3 G(e)\cdot de = \left.\dfrac{e^2}{2}\right|_1^3$ (n is even) = 4

and $= \int_1^3 \dfrac{e}{\sqrt{e^2-1}} de = 2\sqrt{2}$

10. A curve passes ....

Sol.



Eq. of tangent at $p : y - y_1 = m(x - x_1)\left[m = \dfrac{dy}{dx}\right)$

Area of $\Delta OAB = 2$

$\dfrac{1}{2}\left(x_1 - \dfrac{y_1}{m}\right)(y_1 - mx_1) = 2$

$4m + (y_1 - mx_1)^2 = 0$

$y_1 - mx_1 = \pm 2\sqrt{-m}$ ....(1)

diff.,

$m - m - xm' = \pm 2 \cdot \dfrac{1}{2\sqrt{-m}}(-1)\cdot m'$

$\therefore\ m' = 0$

$\Rightarrow m = c$

or $m = \dfrac{-1}{x^2}$

putting in (1)

$y - cx = \pm 2\sqrt{-c}$

As it passes through (1, 1)

$1 - c = \pm 2\sqrt{-c} \quad \Rightarrow \quad c = -1$

$x + y = 2$ (in I quad)

and, $y - x\left(\dfrac{-1}{x^2}\right) = \pm\dfrac{2}{x}$

$xy = 1$

11. Let $f(x) = \int_0^x \dfrac{dt}{\sqrt{1+t^3}}$, ...

Sol. $f(x) = \int_0^x \dfrac{dt}{\sqrt{1+t^3}}$

$f(g(x)) = \int_0^{g(x)} \dfrac{dt}{\sqrt{1+t^3}}$, as g is inverse of f.

$\therefore\ f(g(x)) = x$

$x = \int_0^{g(x)} \dfrac{dt}{\sqrt{1+t^3}}$

Differentiating

$I = \dfrac{g'(x)}{\sqrt{1+g^3}} \quad \Rightarrow (g')^2 = 1 + g^3$

diff. again,

$2g'g'' = 3g^2g^1 \quad \Rightarrow \quad 2g'' = 3g^2$

12. Let $\ell_1 = \lim\limits_{x\to\infty} \sqrt{\dfrac{x - \cos^2 x}{x + \sin x}}$ ....

Sol. $\ell_1 = \lim\limits_{x\to\infty} \sqrt{\dfrac{1 - \dfrac{\cos^2 x}{x}}{1 + \dfrac{\sin x}{x}}} = 1$

$\ell_2 = \lim\limits_{h\to 0^+} 2\int_0^1 \dfrac{h\,dx}{h^2 + x^2}$

$= \lim\limits_{h\to 0^+} \left[2\dfrac{h}{h}\tan^{-1}\dfrac{x}{h}\right]_{0^+}^1 = \pi$ **Ans.**

Note:

$\dfrac{22}{7} > \pi \quad \left[\dfrac{22}{7} = 3.1428571 \text{ and } \pi \approx 3.1415929\right]$

13. Let f(x) be a ....

Sol. Let $\dfrac{f(x)dx}{x(x+1)^2(x-2)^3}$

$$= \frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} + \frac{D}{(x-2)} + \frac{E}{(x-2)^2} + \frac{F}{(x-2)^3}$$

now $\int \frac{f(x)\,dx}{x(x+1)^2(x-2)^3}$ dx will be logarithmic function if

C = E = F = 0

$\therefore$ f(x) = (x + 1)(x – 2)^2

$\therefore$ f(1) = 2 and f ′(2) = 0

$$\therefore \int_0^4 f(x)\,dx = \int_0^4 (x+1)(x-2)^2\,dx \quad \text{put } x-2 = t$$

$$= \int_{-2}^{2} (t+3)t^2\,dt = 2\int_0^2 (3t^2)\,dt = \frac{6}{3}(8) = 16$$

14. If the function f(x) is ....

Sol. Differentiate

$$2f(x).f'(x) = \frac{f(x)\sin x}{2+\cos x}$$

Integrate

$$2f(x) = C - \ln(2+\cos x)$$

x = 0 ; f(0) = 0 $\Rightarrow$ C = ln 3

15. Let I_n =..................

Sol. We have

$$I_n = 2\int_0^1 x\left(1+\frac{x^2}{2}+\frac{x^4}{4}+\frac{x^6}{6}+......+\frac{x^{2n}}{2n}\right)dx$$

$$\left(\int_{-1}^{1}(\text{odd})\,dx = 0\right)$$

$$= 2\left[\frac{x^2}{1\cdot 2}+\frac{x^4}{2\cdot 4}+\frac{x^6}{4\cdot 6}+.......+\frac{x^{2n+2}}{2n(2n+2)}\right]_0^1$$

$$= 2\left[\frac{1}{1\cdot 2}+\frac{1}{2\cdot 4}+\frac{1}{4\cdot 6}+.......+\frac{1}{2n(2n+2)}\right]$$

$$= 1+\frac{1}{2}\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)++\left(\frac{1}{3}-\frac{1}{4}\right)+.....+\left(\frac{1}{n}-\frac{1}{n+1}\right)\right]$$

Hence $\lim\limits_{n\to\infty}\left[1+\frac{1}{2}\left(1-\frac{1}{n+1}\right)\right] = \frac{3}{2}$

$\therefore$ p = 3 ; q = 2

16. The figure shows..................

Sol. We have $A(t) = \int_0^t \sin x^2\,dx$; $B(t) = \frac{t\sin t^2}{2}$

$$\therefore \lim_{t\to 0}\frac{A(t)}{B(t)} = \lim_{t\to 0}\frac{2\int_0^t \sin x^2\,dx}{t\sin t^2}$$

$$= \lim_{t\to 0}\frac{2\int_0^t \sin x^2\,dx}{t^3\,\frac{\sin t^2}{t^2}} = \lim_{t\to 0}\frac{2\int_0^t \sin x^2\,dx}{t^3}$$

Hence $\lim\limits_{t\to 0}\frac{A(t)}{B(t)} = \lim\limits_{t\to 0}\frac{2\sin t^2}{3t^2}$ = **Ans.**]

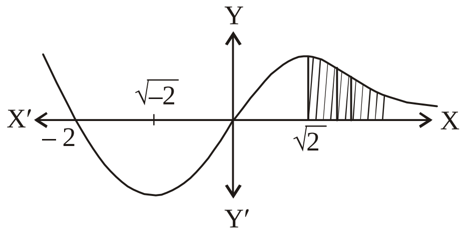
17. The order of the ....

Sol. $y = C_1\cos 2x + \frac{C_2}{2}(1+\cos 2x) + \frac{C_3}{2}(1-\cos 2x) + C_4$

$$= \left(C_1+\frac{C_2}{2}-\frac{C_3}{2}\right)\cos 2x + \left(\frac{C_2}{2}+\frac{C_3}{2}+C_4\right) = A\cos 2x + B$$

18. Area enclosed by..................

Sol.



Required area $A = \int_0^\infty \underset{(II)}{e^{-x}}\underset{(I)}{(x^2+2x)}\,dx = 4$ **Ans.**

(I. B. P.)

19. If $S = \sum_{r=1}^{\infty}\left(\frac{1}{r^2}\right)$, then..................

Sol. $I = x\,\ell nx\cdot\ell n(1-x)\Big|_0^1 - \int_0^1 x\left(\frac{\ell n(1-x)}{x}-\frac{\ell n\,x}{(1-x)}\right)dx$

$$= 0 - \int_0^1 \ell n(1-x)\,dx - \int_0^1\left(\frac{1-x-1}{1-x}\right)\ell n x\,dx$$

$$= -2\int_0^1 \ell n x\,dx + \int_0^1\frac{\ell n x}{(1-x)}dx$$

$$= -2\left(x\,\ell n x - x\right)_0^1 + \int_0^1\frac{\ell n(1-x)}{x}dx$$

$$= 2 - \int_0^1\left(1+\frac{x}{2}+\frac{x^2}{3}+.....\right)dx$$

$$= 2 - S$$

20. If $\int\frac{3x^2+2x}{x^6+2x^5+x^4+2x^3+2x^2+5}dx$ ....

Sol. $\int \frac{3x^2+2x}{x^6+2x^5+x^4+2x^3+2x^2+5}dx$

$= \int \frac{3x^2+2x}{(x^3+x^2+1)^2+4}dx = \frac{1}{2}\tan^{-1}\left(\frac{x^3+x^2+1}{2}\right)+C$

$$F(x)=\frac{1}{2}\tan^{-1}\left(\frac{x^3+x^2+1}{2}\right)+C$$

$$\therefore F(1)-F(0)=\frac{1}{2}\left(\tan^{-1}\frac{3}{2}-\tan^{-1}\frac{1}{2}\right)=\frac{1}{2}\tan^{-1}\frac{4}{7}$$

$$\therefore 0<F(1)-F(0)<\frac{1}{2}.\frac{\pi}{2}<1$$

$\therefore$ [F(1) – F(0)] = 0

PART : II PHYSICS

21. An open organ ....

Sol. f = $\frac{v}{2(\ell+2e)}$ where e = end correction = 0.6 r

$\therefore$ f = $\frac{v}{2(\ell+2\times0.6r)}$ = $\frac{v}{2(\ell+1.2\ \ r)}$

$\therefore$ $\frac{\Delta f}{f}$ = $\frac{\Delta v}{v}$ – $\frac{\Delta(\ell+1.2\ \ r)}{\ell+1.2\ \ r}$

= $\frac{\Delta v}{v}$ – $\frac{\Delta\ell+1.2\ \ \Delta\ \ r}{\ell+1.2\ \ r}$

here $\frac{\Delta v}{v}$ = 0 (given)

$\frac{\Delta f}{f}$ × 100 = – $\frac{\Delta\ell+1.2\ \ \Delta r}{\ell+1.2\ \ r}\times100$

for maximum % error : $\Delta\ell$ = 0.1, Δr = 0.05

$\left(\frac{\Delta f}{f}\times100\right)_{max}$ = $\frac{0.1+1.2\times0.05}{94+1.2\times5}$ × 100 = **0.16%**

Ans.

22. A wire of length ....
1 x 10^{-3} kg/m ....

Sol. $420=\frac{P}{2L}\sqrt{\frac{T}{m}}$

$$490=\frac{P+1}{2L}\sqrt{\frac{T}{m}}$$

$$\frac{490}{420}=\frac{P+1}{P}$$

P = 6

substituting P = 6 in above equation

$420=\frac{P}{2L}\sqrt{\frac{T}{m}}$ $\Rightarrow$ $L=\frac{10}{7}$ m

23. A sonometer ....

Sol. f = $\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$ = $\frac{1}{2\times0.1}\sqrt{\frac{64}{10^{-2}}}$ = 400Hz

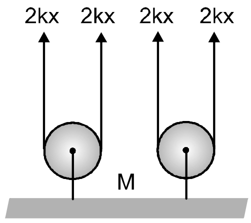
Now f′ = f$\left(\frac{v}{v+v_s}\right)$

399 = 400 × $\frac{300}{300+v_s}$

$\Rightarrow$ v_s = 0.75 m/s

24. The natural ....

Sol. If the mass M is displaced by x from its mean position each spring further stretched by 2x.



Net restoring force

F = – 8kx

M.a = –8 kx

f = $\frac{1}{2\pi}\sqrt{\left|\frac{a}{x}\right|}$

= $\frac{1}{2\pi}\sqrt{\frac{8k}{M}}$ = $\frac{1}{\pi}\sqrt{\frac{2k}{M}}$

25. What will be ....

Sol. F = $-\frac{dU}{dx}$ = $\frac{2a}{x^3}$ – $\frac{b}{x^2}$ = $\frac{1}{x^2}\left[\frac{2a}{x}-b\right]$

F = 0 at x = $\frac{2a}{b}$

Let F = $\frac{2a}{\left(\frac{2a}{b}+y\right)^3}$ – $\frac{b}{\left(\frac{2a}{b}+y\right)^2}$ = $\frac{2b^3a}{8a^3}\left(1-\frac{3by}{2a}\right)$

– $\frac{b\ \ .\ \ b^2}{4a^2}\left[1-\frac{yb}{a}\right]$

F = $\frac{b^3}{4a^2}\left[1-\frac{3by}{2a}-1+\frac{yb}{a}\right]$

= $\frac{b^3}{4a^2}\left[\frac{-by}{2a}\right]$ = – $\frac{b^4}{8a^3}$y

T = $2\pi\sqrt{\frac{8a^3m}{b^4}}$ = $\frac{4\pi a}{b^2}\sqrt{2ma}$

Ans. T = $\frac{4\pi a}{b^2}\sqrt{2ma}$

26. A tube of given ....

Sol. Restoring force

F = 2s (3ρ – ρ) gx = 4sρgx

m = Lsρ + Ls3ρ = 4 Lsρ

ω = $\sqrt{\frac{4s\rho g}{4L\rho s}}$ = $\sqrt{\frac{g}{L}}$

27. A block A of ....

Sol. Max compression in spring

$$\frac{1}{2}(2m)\left(\frac{v_0}{2}\right)^2 = \frac{1}{2}KX_1^2 \Rightarrow X_1 = \sqrt{\frac{mv_0^2}{2K}}$$

Max elongation in spring

$$\frac{1}{2}(m)\left(\frac{v_0}{2}\right)^2 = \frac{1}{2}KX_2^2 \Rightarrow X_2 = \frac{1}{2}\sqrt{\frac{mv_0^2}{K}}$$

B makes first half oscillation with A

So $t_1 = \pi\sqrt{\frac{2m}{K}}$

B makes rest half oscillation alone

$t_2 = \pi\sqrt{\frac{m}{K}}$

$t = t_1 + t_2 = \pi\sqrt{\frac{m}{K}}\ (\sqrt{2} + 1)$

28. Three cars ....

Sol. Frequencies observed by A

$f_1 = f_2$ $\quad f_2 = f\left(\frac{320+5}{320+10}\right) = f\frac{325}{330}$

Frequencies observed by B

$f_1 = f\left(\frac{320-10}{320-5}\right) = f\frac{310}{315}$

$f_2 = f\left(\frac{320+10}{320+5}\right) = f\frac{330}{325}$

Frequencies observed by C

$f_1 = f$

$f_2 = f\left(\frac{320-5}{320-10}\right) = f\frac{315}{310}$

29. A pipe closed ....

Sol. $f_{fun.} = \begin{cases} \frac{v}{2\ell} & \text{for open pipe} \\ \frac{v}{4\ell} & \text{for closed pipe} \end{cases}$

$f \propto \sqrt{T}$ but f does not depend on pressure.

for closed pipe $f_{1st\ overtone} = 3f_{fundamental}$.

30. The displacement ....

Sol. $a = -\omega^2 y$

$F = -m\omega^2 y$

$U = \frac{1}{2}ky^2$

31. A source emit ....

Sol. $\lambda' = \frac{V - V_s}{f} = \frac{332-32}{1000} = 0.3$ m

$f' = f\frac{(V+V_0)}{V-V_s} = 1000 \times \frac{332+64}{332-32} = 1320$ Hz

$\lambda'' = \frac{V - V_0}{f'} = 0.2$ m

32. Consider rigid ....

Sol. Energy density ($x_0 = 0$)

= Energy density ($x = x_0$)

= constant = $\frac{du}{dV} = \frac{du}{A_1 vdt} = \frac{1}{2}\rho\omega^2 A_0^2$

Where A_1 = Area of cross section of the rod then

density × (amplitude)^2 = constant

$\rho_0 A_0^2 = \rho_0 (1 + Kx) A^2$

$A = \frac{A_0}{\sqrt{1+Kx}}$

Then

$V = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{Y}{\rho_0(1+Kx)}}$ f = VA

$\lambda = \frac{f}{V} = f_0\sqrt{\frac{\rho_0(1+Kx)}{Y}}$

time taken $\frac{dx}{dt} = \sqrt{\frac{Y}{\rho_0(1+Kx)}}$

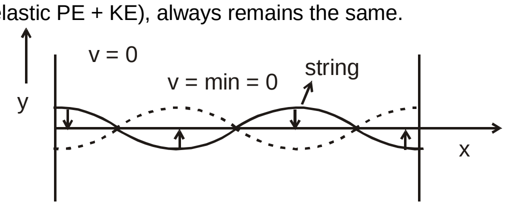
$$\int_0^x \left(\sqrt{1+Kx}\right) dx = \sqrt{\frac{Y}{\rho_0}}\int_0^t dt$$

$$t = \frac{2}{3}\sqrt{\frac{\rho_0}{Y}}\left(\frac{1+Kx}{K}\right)^{3/2}$$

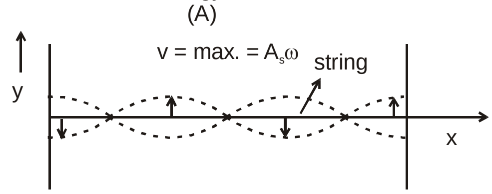
33. In case of ....

Sol. In travelling wave potential energy per unit length = kinetic energy per unit length = $\frac{1}{2}\mu V_P^2$

In standing wave When all the particles are at their extreme positions KE is minimum while elastic PE is maximum (as shown in figure A), and when all the particles (simultaneously) pass through their mean position KE will be maximum while elastic PE minimum (Figure B). The total energy confined in a segment (elastic PE + KE), always remains the same.



Elastic PE = max = E
Kinetic energy = min = 0
(A)



Elastic PE = min = 0
Kinetic energy = max = E
(B)

34. An organ pipe ....

Sol. Frequency of closed organ pipe are f = $\frac{v}{4\ell}, \frac{3v}{4\ell}, \frac{5v}{4\ell}$,........

f = 100 Hz, 300 Hz, 500 Hz, 700 Hz

$x = \frac{V}{f} = \frac{16}{5}m, \frac{16}{15}m, \frac{16}{25}m$

35. A small block ....

Sol. N = mg

f = ma

As f must be static friction (No slip condition)

$f \le \mu N$

$\Rightarrow$ $ma \le \mu\, mg$ or $ma_0 \le \mu mg$

$\therefore$ $mA\omega^2 \le \mu mg$

$\therefore$ $\omega \le \sqrt{\frac{\mu g}{A}}$

$\omega = \frac{2\pi}{T} \le \sqrt{\frac{\mu g}{A}}$

$\therefore$ $T \ge 2\pi\sqrt{\frac{A}{\mu g}} \Rightarrow \mu \ge \frac{4\pi^2 A}{gT^2}$

36. A disc is hinged ....

Sol. For minimum time period

$x = \frac{R}{\sqrt{2}} \Rightarrow T = 2\pi\sqrt{\frac{\frac{mR^2}{2}+\frac{mR^2}{2}}{\frac{mgR}{\sqrt{2}}}} = 2\pi\sqrt{\frac{\sqrt{2}\ R}{g}}$

37. Sinusoidal waves ....

Sol. (C) $P = \frac{1}{2}\mu\omega^2 A^2 V$

using $V = \sqrt{\frac{T}{\mu}}$

$P = \frac{1}{2}\omega^2 A^2\sqrt{T\mu}$

$\omega = \sqrt{\frac{2P}{A^2\sqrt{T\mu}}}$ $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2P}{A^2\sqrt{T\mu}}}$

using data f = 30 Hz.

38. An ant with ....

Sol. $a_{max} = \omega^2 A = g$

$\omega = 2\pi f = \frac{2\pi v}{\lambda}$

$v = \sqrt{\frac{F}{\mu}}$

$A_{min} = \frac{g}{\omega^2 A}$

$= \frac{g}{A} \times \frac{\mu\lambda^2}{4\pi^2 F} = 2 \times 10^{-3}\ m = 2\ mm$

39. A load of 20 kg ....

Sol. $V_\ell = \sqrt{\frac{Y}{\rho}}$

$V_t = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho.A}}$

$\frac{V_\ell}{V_t} = \sqrt{\frac{Y}{\rho}} \times \sqrt{\frac{\rho.A}{T}}$

$20 = \sqrt{\frac{Y.A}{T}}$

$20 = \sqrt{\frac{19.6 \times 10^{10} \times A}{20 \times 9.8}}$

$\Rightarrow A = 4 \times 10^{-7}\ m^2$

$\Rightarrow A = x \times 10^{-7}\ m^2$

$\Rightarrow x = 4$ **Ans.**

40. Two forks A ....

Sol. $f \propto \frac{1}{L}$

$f_A = \frac{K}{30}\ \&\ f_B = \frac{K}{25}$

$\frac{f_A}{f_B} = \frac{25}{30} = \frac{5}{6}$

$f_A - f_B = 4$

$\Rightarrow f_A = 20\ \&\ f_B = 24$

$f_A + f_B = 44$ **Ans.**

PART : III CHEMISTRY

41. The colourless salt that ....

Sol. K_2SO_4 produce white ppt. with $BaCl_2$ which is insoluble in aqueous HCl.

42. Select the incorrect process ....

Sol. Fe_2O_3 is an oxide ore, froth floatation is not applied.

43. $FeS + H_2SO_4 \longrightarrow B\uparrow$ ....

Sol. $FeS + H_2SO_4 \rightarrow FeSO_4 + \underset{(B)}{H_2S\uparrow}$

$2FeCl_3 + 3H_2S \rightarrow 2FeCl_2 + 2HCl + \underset{\text{(White ppt.)}}{S\downarrow}$

$(CH_3COO)_2Pb + H_2S \rightarrow \underset{\text{(Black ppt.)}}{PbS\downarrow}$

$MnO_4^- + H_2S + H^+ \rightarrow Mn^{2+} + S\downarrow + H_2O$

$H_2S + Na_2[Fe(CN)_5NO] \rightarrow x$

44. When NO_2^- and NO_3^- ....

Sol. $NO_2^- + NH_4^+ \xrightarrow{\Delta} N_2\uparrow + 2H_2O$

$NH_2 - CO - NH_2 + 2HNO_2 \longrightarrow 2N_2\uparrow + CO_2\uparrow + 3H_2O$

$NH_2 - CS - NH_2 + HNO_2 \longrightarrow N_2\uparrow + HSCN + 2H_2O$

$NH_2 - SO_3H + HNO_2 \longrightarrow N_2\uparrow + 2H^+ + SO_4^- + H_2O$

$NO_2^- + N_3^- + 2H^+ \longrightarrow N_2\uparrow + N_2O + H_2O$

45. Match List-I with List-II ....

Sol. $IO_3^- + SO_2 \xrightarrow{H^+} I_2$

$I_2 + \text{starch} \longrightarrow \text{Blue}$

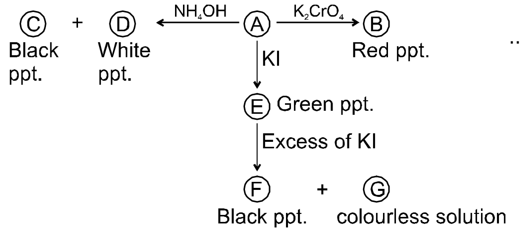
$SO_3^{2-} + MnO_4^- + H^+ \longrightarrow Mn^{2+} + SO_4^{2-} + H_2O$

$SO_3^{2-} + Zn + H^+ \longrightarrow \underset{\text{Rotten egg smell gas}}{H_2S} \uparrow + Zn^{2+} + H_2O$

$CO_2 + Ca(OH)_2 \longrightarrow \underset{\text{white turbidity}}{CaCO_3} \downarrow$

46. Match List-I with List-II...........

Sol. Based on theory

47. 

....

Sol. A = Hg_2^{2+}

B = Hg_2CrO_4

C = Hg

D = HgO. $Hg(NH_2)X$

E = Hg_2I_2

F = Hg

G = $[HgI_4]^{2-}$

anion part of 'B' is CrO_4^{2-}

48. Salt A $\xrightarrow{NaOH}$ Gas B + solution C ....

Sol. $NH_4NO_3 + NaOH \longrightarrow NH_3 \uparrow + NaNO_3 + H_2O$

$NH_4NO_2 + NaOH \longrightarrow NH_3 \uparrow + NaNO_2 + H_2O$

$NO_3^- / NO_2^- + Zn + OH^- \longrightarrow NH_3 \uparrow + [Zn(OH)_4]^{2-}$

49. Electrorefining method ....

Sol. Electrorefining method is applicable for purification of Cu, Zn, Sn, Ag, Au, Ni, Pb and Al.

50. Which of the following ....

Sol. $Cl^- + Cr_2O_7^{2-} + H^+ \longrightarrow \underset{\text{Deep red vapours}}{CrO_2Cl_2}$

$CrO_2Cl_2 + OH^- \longrightarrow \underset{\text{yellow colour solution}}{CrO_4^{2-}}$

$CrO_2Cl_2 + H_2O \longrightarrow \underset{\text{yellow colour solution}}{H_2CrO_4}$

Sol. $Cl^- + Cr_2O_7^{2-} + H^+ \longrightarrow CrO_2Cl_2$
गहरी लाल वाष्प

$CrO_2Cl_2 + OH^- \longrightarrow CrO_4^{2-}$
पीले रंग का विलयन

$CrO_2Cl_2 + H_2O \longrightarrow H_2CrO_4$
पीले रंग का विलयन

51. For which of the following ....

Sol. Based on theory

52. Choose the correct ....

Sol. Haematite $\rightarrow$ Fe_2O_3

Cassiterite $\rightarrow$ SnO_2

Cerussite $\rightarrow$ $PbCO_3$

53. $FeC_2O_4 \xrightarrow[\text{in absence of air}]{575°C}$ ....

Sol. $FeC_2O_4 \xrightarrow[\text{in absence of air}]{\Delta} \underset{\text{(Basic oxide)}}{FeO} + \underset{\text{(Netural oxide)}}{CO} + \underset{\text{(Acidic oxide)}}{CO_2}$

A = FeO, B = CO, C = CO_2

CO burns with a blue flame.

CO_2 produce white ppt. with lime water.

55. Find the total number ....

Sol. NO_2^- (NO2), SO_3^{2-} (SO2), HCO_3^- (CO2), $S_2O_3^{2-}$ (SO2)

56. How many of the following ....

Sol. Cu_2O, $Cu(OH)_2$

58. How many of the following ....

Sol. (iv) Bayer's method is used for red bauxite.

59. Among the following find ....

Sol. $Zn^{2+} + NH_4OH \rightarrow Zn(OH)_2 \downarrow$
(white ppt.)

$Zn^{2+} + (NH_4)_2S \rightarrow ZnS \downarrow$
(white ppt.)

$Zn^{2+} + 2OH^- \rightarrow Zn(OH)_2 \downarrow$
(white ppt.)

$Zn^{2+} + [Fe(CN)_6]^{4-} \rightarrow Zn_2[Fe(CN)_6] \downarrow$
(white ppt.)

$ZnCl_2$ is water soluble.

60. Find out total number of ....

Sol. (i) $Mn^{2+} + BiO_3^- \xrightarrow{+H^{\oplus}} MnO_4^- + Bi^{3+} + H_2O$

(ii) $Mn^{2+} + PbO_2 \xrightarrow{+H^{\oplus}} MnO_4^- + Pb^{2+} + H_2O$

(iv) $Mn^{2+} + S_2O_8^{2-} + H_2O \rightarrow MnO_4^- + 2SO_4^{2-} + 2OH^-$

ADVANCED PATTERN PART TEST-5 (APT-5)
TARGET : JEE (MAIN+ADVANCED) 2017

DATE : 10-05-2017 REVISION PLAN-2 COURSE : VIJETA (ADP), VIJAY (ADR)

ANSWER KEY

CODE-0

PART : I MATHEMATICS

1.	(D)	**2.**	(B)	**3.**	(A)	**4.**	(A)	**5.**	(A)	**6.**	(A)	**7.**	(AB)
8.	(ABCD)	**9.**	(BC)	**10.**	(AB)	**11.**	(BD)	**12.**	(ABCD)	**13.**	(ABC)	**14.**	(ACD)
15.	(5)	**16.**	(2)	**17.**	(2)	**18.**	(4)	**19.**	(5)	**20.**	(1)		

PART : II PHYSICS

21.	(A)	**22.**	(A)	**23.**	(D)	**24.**	(A)	**25.**	(C)	**26.**	(D)	**27.**	(AC)
28.	(C)	**29.**	(ACD)	**30.**	(BCD)	**31.**	(ABD)	**32.**	(ABCD)	**33.**	(ACD)	**34.**	(BD)
35.	(4)	**36.**	(2)	**37.**	(9)	**38.**	(2)	**39.**	(4)	**40.**	(4)		

PART : III CHEMISTRY

41.	(A)	**42.**	(C)	**43.**	(D)	**44.**	(A)	**45.**	(C)	**46.**	(B)	**47.**	(ABD)
48.	(CD)	**49.**	(ABCD)	**50.**	(ABD)	**51.**	(BC)	**52.**	(ABC)	**53.**	(ABD)	**54.**	(D)
55.	(4)	**56.**	(2)	**57.**	(2)	**58.**	(6)	**59.**	(4)	**60.**	(3)		

ADVANCED PATTERN PART TEST-5 (APT-5)

TARGET : JEE (MAIN+ADVANCED) 2017

DATE : 10-05-2017 REVISION PLAN-2 COURSE : VIJETA (ADP), VIJAY (ADR)

ANSWER KEY

CODE-1

PART : I MATHEMATICS

1.	(C)	**2.**	(C)	**3.**	(B)	**4.**	(B)	**5.**	(B)	**6.**	(B)	**7.**	(AC)
8.	(ABCD)	**9.**	(BD)	**10.**	(AC)	**11.**	(BC)	**12.**	(ABCD)	**13.**	(ABD)	**14.**	(ABC)
15.	(5)	**16.**	(2)	**17.**	(2)	**18.**	(4)	**19.**	(5)	**20.**	(1)		

PART : II PHYSICS

21.	(C)	**22.**	(C)	**23.**	(B)	**24.**	(D)	**25.**	(A)	**26.**	(B)	**27.**	(BC)
28.	(D)	**29.**	(BCD)	**30.**	(ABD)	**31.**	(BCD)	**32.**	(ABCD)	**33.**	(BCD)	**34.**	(CD)
35.	(4)	**36.**	(2)	**37.**	(9)	**38.**	(2)	**39.**	(4)	**40.**	(4)		

PART : III CHEMISTRY

41.	(C)	**42.**	(C)	**43.**	(C)	**44.**	(B)	**45.**	(B)	**46.**	(A)	**47.**	(ABC)
48.	(BD)	**49.**	(ABCD)	**50.**	(BCD)	**51.**	(BC)	**52.**	(ABD)	**53.**	(ACD)	**54.**	(C)
55.	(4)	**56.**	(2)	**57.**	(2)	**58.**	(6)	**59.**	(4)	**60.**	(3)		

ADVANCED PATTERN PART TEST-5 (APT-5)

TARGET : JEE (MAIN+ADVANCED) 2017

DATE : 10-05-2017 REVISION PLAN-2 COURSE : VIJETA (ADP), VIJAY (ADR)

ANSWER KEY

CODE-2

PART : I MATHEMATICS

1.	(D)	**2.**	(B)	**3.**	(A)	**4.**	(A)	**5.**	(A)	**6.**	(A)	**7.**	(AB)
8.	(ABCD)	**9.**	(BC)	**10.**	(AB)	**11.**	(BD)	**12.**	(ABCD)	**13.**	(ABC)	**14.**	(ACD)
15.	(5)	**16.**	(2)	**17.**	(2)	**18.**	(4)	**19.**	(5)	**20.**	(1)		

PART : II PHYSICS

21.	(A)	**22.**	(A)	**23.**	(D)	**24.**	(A)	**25.**	(C)	**26.**	(D)	**27.**	(AC)
28.	(C)	**29.**	(ACD)	**30.**	(BCD)	**31.**	(ABD)	**32.**	(ABCD)	**33.**	(ACD)	**34.**	(BD)
35.	(4)	**36.**	(2)	**37.**	(9)	**38.**	(2)	**39.**	(4)	**40.**	(4)		

PART : III CHEMISTRY

41.	(A)	**42.**	(C)	**43.**	(D)	**44.**	(A)	**45.**	(C)	**46.**	(B)	**47.**	(ABD)
48.	(CD)	**49.**	(ABCD)	**50.**	(ABD)	**51.**	(BC)	**52.**	(ABC)	**53.**	(ABD)	**54.**	(D)
55.	(4)	**56.**	(2)	**57.**	(2)	**58.**	(6)	**59.**	(4)	**60.**	(3)		

ADVANCED PATTERN PART TEST-5 (APT-5)

TARGET : JEE (MAIN+ADVANCED) 2017

DATE : 10-05-2017 REVISION PLAN-2 COURSE : VIJETA (ADP), VIJAY (ADR)

ANSWER KEY

CODE-3

PART : I MATHEMATICS

1.	(C)	**2.**	(C)	**3.**	(B)	**4.**	(B)	**5.**	(B)	**6.**	(B)	**7.**	(AC)
8.	(ABCD)	**9.**	(BD)	**10.**	(AC)	**11.**	(BC)	**12.**	(ABCD)	**13.**	(ABD)	**14.**	(ABC)
15.	(5)	**16.**	(2)	**17.**	(2)	**18.**	(4)	**19.**	(5)	**20.**	(1)		

PART : II PHYSICS

21.	(C)	**22.**	(C)	**23.**	(B)	**24.**	(D)	**25.**	(A)	**26.**	(B)	**27.**	(BC)
28.	(D)	**29.**	(BCD)	**30.**	(ABD)	**31.**	(BCD)	**32.**	(ABCD)	**33.**	(BCD)	**34.**	(CD)
35.	(4)	**36.**	(2)	**37.**	(9)	**38.**	(2)	**39.**	(4)	**40.**	(4)		

PART : III CHEMISTRY

41.	(C)	**42.**	(C)	**43.**	(C)	**44.**	(B)	**45.**	(B)	**46.**	(A)	**47.**	(ABC)
48.	(BD)	**49.**	(ABCD)	**50.**	(BCD)	**51.**	(BC)	**52.**	(ABD)	**53.**	(ACD)	**54.**	(C)
55.	(4)	**56.**	(2)	**57.**	(2)	**58.**	(6)	**59.**	(4)	**60.**	(3)		