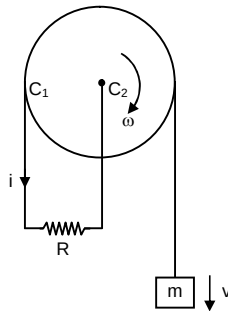


HINTS & SOLUTIONS

PAPER-1

PART-I (PHYSICS)

1. Consider a perfectly
Sol. Let at any time t , velocity of block is v , angular velocity of disc is ω and current from resistance is i .



Writing energy equation

$$mgv = mv \frac{dv}{dt} + I \frac{\omega d\omega}{dt} + i^2 R$$

$$v = \omega a, \quad I = \frac{ma^2}{2}$$

$$\frac{dv}{dt} = a \frac{d\omega}{dt}$$

$$i = \frac{B\omega a^2}{2R} = \frac{Bva}{2R}$$

$$mgv = mv \frac{dv}{dt} + \frac{ma^2}{2} \frac{v}{a} \frac{1}{a} \frac{dv}{dt} + \frac{B^2 a^2}{4R} v^2$$

$$\frac{3m}{2} \frac{dv}{dt} = mg - \frac{B^2 a^2}{4R} v$$

$$\frac{dv}{dt} = \frac{2g}{3} - \frac{B^2 a^2}{6mR} v$$

$$\text{Let } \frac{2g}{3} = \alpha$$

$$\frac{B^2 a^2}{6mR} = \beta ; \quad \frac{dv}{dt} = \alpha - \beta v$$

$$\int_0^v \frac{dv}{\alpha - \beta v} = \int_0^t dt \Rightarrow v = \frac{\alpha}{\beta} (1 - e^{-\beta t})$$

$$= \frac{4mgR}{B^2 a^2} \left(1 - e^{-\left(\frac{B^2 a^2}{6mR}\right)t} \right)$$

$$\frac{dv}{dt} = \alpha e^{-\beta t} \Rightarrow \frac{dv}{dt} = \frac{2g}{3} e^{-\beta t}$$

2. A uniform wire

$$\text{Sol. } \Delta T_1 = T_0 - T_1 \quad \Delta T_2 = T_0 - T_2 \quad \Delta T_3 = T_0 - T_3$$

$$f = \frac{1}{2\ell_0} \sqrt{\frac{y\alpha\Delta T_1}{\mu}} = \frac{2}{2\ell_0} \sqrt{\frac{y\alpha\Delta T_2}{\mu}} = \frac{3}{2\ell_0} \sqrt{\frac{y\alpha\Delta T_3}{\mu}}$$

$$\sqrt{\Delta T_1} = 2\sqrt{\Delta T_2} = 3\sqrt{\Delta T_3}$$

$$\Delta T_1 = 4\Delta T_2 = 9\Delta T_3 \Rightarrow T_1 < T_2 < T_3$$

$$T_0 - T_1 = 4(T_0 - T_2) \Rightarrow T_0 - T_1 = 4T_0 - 4T_2$$

$$4T_2 - T_1 = 3T_0$$

$$4(T_0 - T_2) = 9(T_0 - T_3) \Rightarrow 9T_3 - 4T_2 = 5T_0$$

3. A particle moving

Sol. Let initial velocity is u and constant acceleration is a . given

$$S_{6-7} = S_7 - S_6 = \left(7u + \frac{1}{2}a(49) \right) - \left(6u + \frac{1}{2}a(36) \right)$$

$$= u + \frac{13}{2}a = 20 \Rightarrow 2u + 13a = 40$$

$$S_{8-9} = S_9 - S_8 = \left(9u + \frac{1}{2}a(81) \right) - \left(8u + \frac{1}{2}a(64) \right)$$

$$= u + \frac{17}{2}a = 24 \Rightarrow 2u + 17a = 48$$

$$\Rightarrow a = 2\text{m/s}^2 \text{ and } u = 7\text{m/s}$$

$$S_{9-10} = S_{10} - S_9 = \left(10u + \frac{1}{2}a(100) \right) - \left(9u + \frac{1}{2}a(81) \right)$$

$$= u + \frac{19}{2}a = 26\text{m}$$

4. Consider the

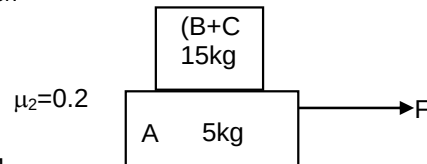
Sol. $(f_{AG})_{\max} = 15\text{N}$

$(f_{AB})_{\max} = 20\text{N}$

$(f_{BC})_{\max} = 15\text{N}$

(A & B)

If force is applied on block A B and C will always move together.



Maximum value of F for which there is no slipping b/w the block is

$$F_{\max} = 15(2) + 15 = 45\text{N}$$

maximum possible friction b/w B and C = 10 N

(C & D)

If force is applied on block C, A and B will never move.

Maximum value of F for which there is no slipping b/w blocks is

$$F_{\max} = 15\text{N}$$

Maximum possible value of friction force between A and B is 15N.

5. Imagine a light

Sol. $\frac{mv^2}{r} = \frac{k}{r^3} \Rightarrow v \propto \frac{1}{r}$

$T = \frac{2\pi r}{v} \Rightarrow T \propto \frac{r}{v} \Rightarrow T \propto r^2$

6. Which of the

Sol. (A & D)

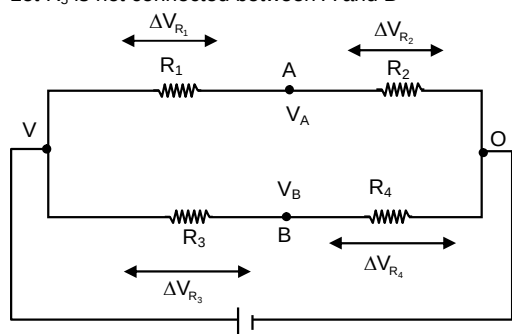
For a force to be conservative work done along any two path (If initial & final position are same) must be same.

(C) Electric field just outside the conductor must be perpendicular to the conductor surface for this charge may or may not be distributed uniformly.

(D) When a conducting plate is oscillated between the magnetic pole pieces, eddy currents are produced due to which plate is damped. This phenomenon is called electro magnetic damping.

7. Consider the given

Sol. Let R_5 is not connected between A and B



$V - V_A = \Delta V_{R_1} \Rightarrow V_A = V - \Delta V_{R_1} = V - \left(\frac{V}{R_1 + R_2} \right) R_1$

$= V \left(\frac{R_1 + R_2 - R_1}{R_1 + R_2} \right) = \frac{VR_2}{R_1 + R_2} = \frac{V}{\left(\frac{R_1}{R_2} \right) + 1}$

$V - V_B = \Delta V_{R_3} \Rightarrow V_B = V - \Delta V_{R_3}$

$= V - \left(\frac{V}{R_3 + R_4} \right) R_3 = V \left(\frac{R_3 + R_4 - R_3}{R_3 + R_4} \right)$

$= \frac{VR_4}{R_3 + R_4} = \frac{V}{\left(\frac{R_3}{R_4} \right) + 1}$

Now if R_5 is connected between A and B
Current will flow from A to B if $V_A > V_B$

$\frac{V}{\left(\frac{R_1}{R_2} \right) + 1} > \frac{V}{\left(\frac{R_3}{R_4} \right) + 1} \Rightarrow \frac{R_3}{R_4} > \frac{R_1}{R_2}$

Current will flow from B to A if $V_A < V_B$

$\frac{V}{\left(\frac{R_3}{R_2} \right) + 1} > \frac{V}{\left(\frac{R_1}{R_2} \right) + 1} \Rightarrow \frac{R_1}{R_2} > \frac{R_3}{R_4}$

8. The equation of

Sol. $Y = A \sin(\omega t - \vec{k} \cdot \vec{r})$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{k} \cdot \vec{r} = (3x + 2y - z)\pi$

$\vec{k} = (3\hat{i} + 2\hat{j} - \hat{k})\pi$

$|\vec{k}| = \pi\sqrt{9 + 4 + 1} = \pi\sqrt{14}$

$|\vec{k}| = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{|\vec{k}|} = \frac{2\pi}{\pi\sqrt{14}} = \sqrt{\frac{2}{7}} \text{ m}$

unit vector in the direction of wave propagation = $\frac{3\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{14}}$

9. A converging lens

Sol. $m = \frac{f}{f + u} = \frac{200}{200 - 150} = 4$

$\frac{x_i}{x_o} = \frac{y_i}{y_o} = 4$

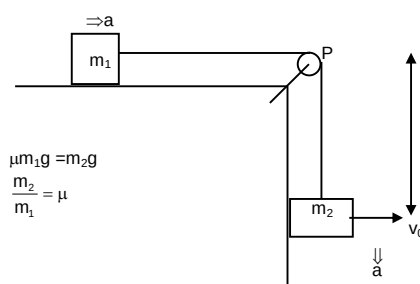
$x_o = 600 \text{ cm}$

$z_i = -450 \text{ cm}$

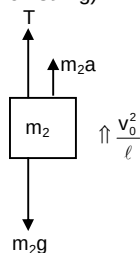
Area of similar triangles is proportional to square of corresponding sides.

10. In the arrangement

Sol.



Lets observe the motion of m_2 from an observer fixed at P (point on string)



$T + m_2a - m_2g = \frac{m_2v_0^2}{\ell} \dots\dots\dots(1)$

for m_1

$T - \mu m_1g = m_1a \dots\dots\dots(2)$

from (1) and $m_1a + \mu m_1g - m_2a - m_2g = \frac{m_2v_0^2}{\ell}$

$a = \frac{m_2 \frac{v_0^2}{\ell}}{(m_1 + m_2)} = \left(\frac{\mu}{1 + \mu} \right) \frac{v_0^2}{\ell}$

ROC of m_2

$T - m_2g = \mu m_1g + \left(\frac{\mu m_1}{1 + \mu} \right) \frac{v_0^2}{\ell} - m_2g$

$= \left(\frac{\mu m_1}{1 + \mu} \right) \frac{v_0^2}{\ell} = m_2 \frac{v_0^2}{R}$

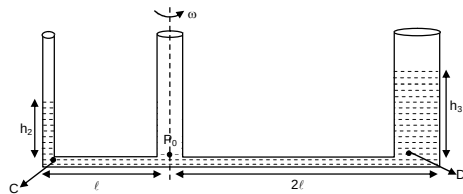
$R = \left(\frac{m_2}{\mu m_1} \right) (1 + \mu) \ell = \ell (1 + \mu)$

11. With a rise of

Sol. With the increase in temperature surface tension of water decreases, Viscosity of water decreases and Viscosity of air increases.

12. Length of arms

Sol.



$$P_C = P_0 + \frac{1}{2} \rho \omega^2 l^2 = P_0 + P g h_2 \Rightarrow \omega^2 l^2 = 2 g h_2$$

$$P_D = P_0 + \frac{1}{2} \rho \omega^2 (4l^2) = P_0 + P g h_3 \Rightarrow 2 \rho \omega^2 l^2 = g h_3$$

$$\Rightarrow \frac{h_3}{h_2} = 4 \Rightarrow h_3 = 4 h_2$$

Volume conservation

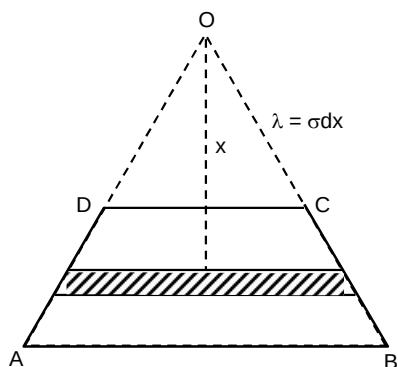
$$A h + 2 A h + 3 A h = A h_2 + 3 A h_3$$

$$h_2 + 3 h_3 = 6 h$$

$$\Rightarrow h_2 = \frac{6h}{13} \quad h_3 = \frac{24h}{13}$$

13. Consider a uniformly

Sol.



$$dE = 2 \left(\frac{1}{4\pi\epsilon_0} \right) \frac{\sigma dx}{x} \left(\frac{1}{2} \right)$$

$$\frac{\sigma}{4\pi\epsilon_0} \frac{dx}{x}$$

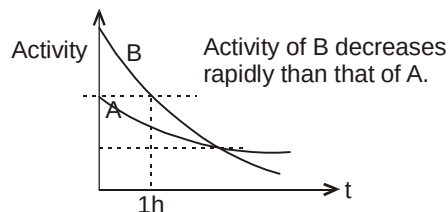
$$E = \frac{\sigma}{4\pi\epsilon_0} \int_{y_0}^{2y_0} \frac{dx}{x}$$

$$= \frac{7\sigma}{44\epsilon_0} \ln \sqrt{2}$$

14. At $t = 0$, a sample

Sol. As $\frac{1}{\lambda} = \frac{t_{1/2}}{\ell n 2} \Rightarrow t_{(1/2) A} > t_{(1/2) B}$

Activity curves are $(T_{1/2} (A) > T_{1/2} (B))$



15. The emissive power of

Sol. Since, $e = a = 0.2$ (Since, $a = (1 - r - t) = 0.2$ for the body B)

$$E = (100) (0.2) = 20 \text{ W/m}^2$$

$$\text{Power emitted} = e.A = 20 \times 10 = 200 \text{ Watt}$$

16. An ideal monatomic

Sol. $TV^{a-1} = \text{const.}$

$$a - 1 = \frac{1}{2}$$

$$a = \frac{3}{2}$$

$$C = C_v - \frac{R}{a-1}$$

$$C = C_v - 2R$$

As V increases, temperature decreases hence internal energy decreases.

17. A car moves

Sol. Frequency of horn directly heard by observer $\frac{v + v_0}{v + v_c} f$

$$\text{Frequency of echo} = \frac{v}{v + v_c} f$$

Frequency of echo of horn as heard by observer.

$$\frac{v}{v - v_c} f \cdot \left(\frac{v + v_0}{v} \right)$$

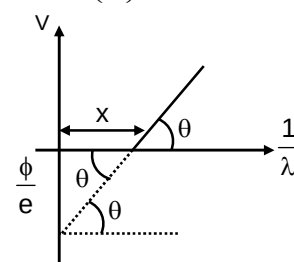
Frequency of Beats :

$$= (v + v_0) f \left\{ \frac{1}{v - v_c} - \frac{1}{v + v_c} \right\} = \frac{2v_c(v + v_0)}{(v^2 - v_c^2)} f$$

18. The graph between

$$\text{Sol. } eV = \frac{hc}{\lambda} - \phi$$

$$v = \frac{hc}{e} \left(\frac{1}{\lambda} \right) - \frac{\phi}{e}$$



$$\tan \theta = \frac{hc}{e}$$

$$\tan \theta = \frac{\phi}{ex}$$

$$\phi = ex \tan \theta$$

$$\phi_1 : \phi_2 : \phi_3 = 3 : 2 : 1$$

19. The electron in.....

Sol. Time period $T_n = \frac{2\pi r_n}{V_n}$

$$T \propto \frac{n^2}{1/n} \text{ i.e., } T \propto n^3$$

Hence, $n_1 = 2n_2$
Choice (b) and (c) are wrong.

20. Select correct

Sol. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$
f is positive
 $m = \frac{f}{f - u}$

PART - II (CHEMISTRY)

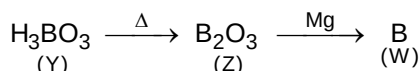
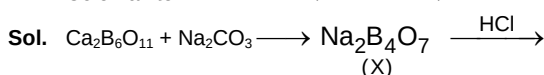
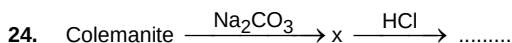
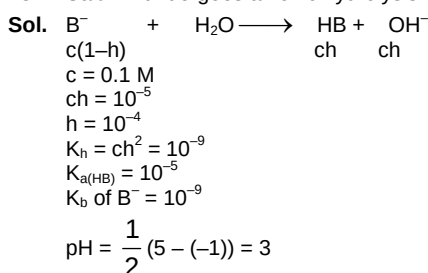
21. If $\left(\frac{\partial Z}{\partial P}\right)_T = \frac{1}{RT} \left(b - \frac{a}{RT}\right) + \frac{2a}{(RT)^3} \left(2b - \frac{a}{RT}\right) P + \dots$

Sol. Given equation is derivative of Virial equation (in terms of pressure) with respect to pressure.

22. Which of following represents equilibrium

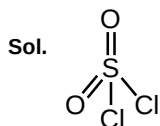
Sol. P and T condition decides equilibrium for such equilibria (not amount).

23. Salt AB undergoes anionic hydrolysis



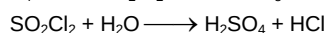
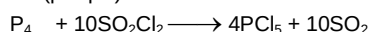
Decahydrated Borax is $Na_2[B_4O_5(OH)_4] \cdot 8H_2O$
So it has 20 O-H bonds.

25. Which is/are correct about SO_2Cl_2



2 ($p\pi-d\pi$) bonds

No ($p\pi-p\pi$) bonds



26. Alternate tetrahedral void in FCC

Sol. In ZnS, S^{2-} occupy FCC lattice points and Zn^{2+} alternate tetrahedral voids.

27. Which of the following is/are correct

Sol. Silver solution is an example of Lyophobic sol.

28. For the cell reaction

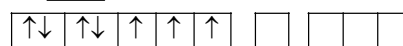
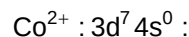
Sol. $E_{cell}^0 = E_{RP(RHS)}^0 - E_{RP(LHS)}^0$
 $= -0.76 - (-1.36) = 0.6$
 $\Delta_r G^0 = -RT \ln K_{eq};$

$$\text{or } \log K_{eq} = \frac{nFE^0}{RT \times 2.303} = \frac{2 \times 0.6}{0.06} = 20$$

$$\Rightarrow \frac{2 \times 0.6}{0.06} \Rightarrow 20; K_f = 10^{20}$$

29. Select the correct option(s)

Sol. (A) $\ln [Co(SCN)_4]$, Co^{2+} is present



3d⁷ 4s 4p

Hybridization is sp^3

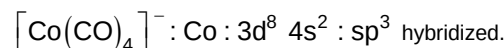
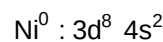
as SCN^- is weak field ligand.

no. of unpaired electrons = 3

$$\mu = \sqrt{n(n+2)} \text{ B. M. (spin only)}$$

$$= \sqrt{15} \text{ B.M.}$$

(C) $Ni(CO)_4 : sp^3$ hybridized CO is strong field



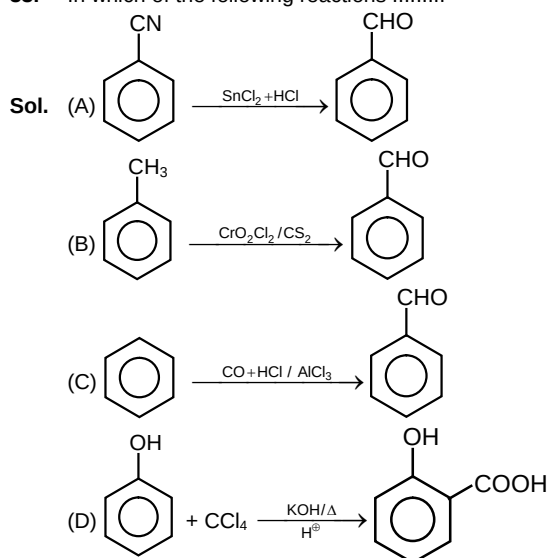
30. Following metal cation gives

Sol. Al, Cr and Zn oxide are Amphoteric in nature.

31. Consider the given reactions

Sol. Oxidation state of N in HNO_3 is +5. It is reduced to NO_2 , N_2O , NO_2 and NO respectively in the given reactions. n-factors are 1, 4, 1, 3 respectively.

33. In which of the following reactions



44. If E_1 and E_2 are

Sol. $P(E_1) = \frac{1}{4}$

$$P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{1}{2}$$

$$P(E_1 \cap E_2) = \frac{1}{8}$$

$$P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{1}{4}$$

$$P(E_2) = \frac{1}{2}$$

$$P(E_1) \cdot P(E_2) = P(E_1 \cap E_2)$$

45. $\lim_{n \rightarrow \infty} \frac{n}{3} \left\{ \left(\frac{3}{n} + \frac{9}{n^2} \right)^2 + \left(\frac{3}{n} + \frac{18}{n^2} \right)^2 \dots \dots \dots \right\}$

Sol. $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{n}{3} \left(\frac{3}{n} + \frac{9r}{n^2} \right)^2$

$$\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{n}{3} \cdot \frac{9}{n^2} \left(1 + \frac{3r}{n} \right)^2$$

$$\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{3}{n} \left(1 + \frac{3r}{n} \right)^2$$

$$3 \int_0^1 (1+3x)^2 dx$$

$$\left(\frac{1+3x^3}{3 \cdot 3} \right)_0^1$$

$$\frac{64}{3} - \frac{1}{3} = 21$$

46. If the roots of the

$$p-1$$

Sol. $x^3 + ax^2 + bx + c = 0 \leftarrow p$

$$p+1$$

$$a = -3p$$

$$b = p(p-1) + p(p+1) + (p-1)(p+1)$$

$$c = -p(p-1)(p+1)$$

$$\frac{a^2}{b+1} = \frac{9p^2}{p^2 - p + p^2 + p + p^2 - 1 + 1} = 3$$

47. $\sum_{n=1}^{\infty} \tan^{-1} \left(\frac{4n}{n^4 + 5} \right) = \dots \dots \dots$

Sol. $\sum_{n=1}^{\infty} \tan^{-1} \left(\frac{4n}{n^4 + 5} \right) = \sum_{n=1}^{\infty} \tan^{-1} \left(\frac{4n}{1 + (n^2 + 2)^2 - 4n^2} \right)$

$$= \sum_{n=1}^{\infty} \tan^{-1} \left(\frac{4n}{1 + (n^2 + 2n + 2)(n^2 - 2n + 2)} \right)$$

$$\begin{aligned} & \sum_{n=1}^{\infty} \left[\tan^{-1}(n^2 + 2n + 2) - \tan^{-1}(n^2 - 2n + 2) \right] \\ &= 2 \tan^{-1}(\infty) - \tan^{-1}1 - \tan^{-1}2 = \frac{3\pi}{4} - \tan^{-1}2 \\ &= \frac{\pi}{4} + \tan^{-1} \frac{1}{2} \end{aligned}$$

48. If A^5 is null square

Sol. $(I + A + A^2 + \dots + A^n)(I - A) = I$

$$\text{fi } I - A^{n+1} = I \quad \text{fi } A^{n+1} = 0$$

as A^5 is null matrix fi $n+1$ can be any number greater than or equal to 5. fi $n = 4$

49. If $x = \frac{1^2}{1} + \frac{2^2}{3} + \frac{3^2}{5} + \dots \dots \dots$

Sol. $x - y = 1^2 \left(\frac{1}{1} - \frac{1}{3} \right) + 2^2 \left(\frac{1}{3} - \frac{1}{5} \right) \dots \dots \dots 1001^2 \left(\frac{1}{2001} - \frac{1}{2003} \right)$

$$= \frac{1^2 \cdot 2}{1 \cdot 3} + \frac{2^2 \cdot 2}{3 \cdot 5} \dots \dots \dots + \frac{2 \cdot 1001^2}{2001 \cdot 2003}$$

$$\frac{x - y}{2} = \left(\frac{1^2}{1 \cdot 3} + \frac{2^2}{3 \cdot 5} \dots \dots \dots \frac{1001^2}{2001 \cdot 2003} \right)$$

$$T_r = \frac{r^2}{(2r-1)(2r+1)}$$

$$= \frac{1}{4} \left(\frac{4r^2 + 1 - 1}{(2r-1)(2r+1)} \right)$$

$$= \frac{1}{4} \left(1 + \frac{1}{(2r-1)(2r+1)} \right)$$

$$= \frac{1}{4} \left(1 + \left(\frac{1}{(2r-1)} - \frac{1}{(2r+1)} \right) \right) \frac{1}{8}$$

$$\sum_{r=1}^{1001} T_r = \frac{1}{4} (1001) + \frac{1}{8} \left(1 - \frac{1}{2003} \right)$$

$$\frac{x - y}{2} = \frac{1001}{4} + \frac{2002}{2003 \cdot 8}$$

$$x - y = \frac{1001}{2} + \frac{1001}{2 \cdot 2003}$$

$$[x - y] = 500$$

50. Let a differentiable.....

Sol. We have $2|f(x) - f(y)| \leq |x - y|$

$$\Rightarrow \left| \frac{f(x) - f(y)}{x - y} \right| \leq \frac{1}{2} \Rightarrow |f'(x)| \leq \frac{1}{2}$$

But $f'(x) \geq \frac{1}{2}$. So $f'(x) = \frac{1}{2}$ so the curve is $y = \frac{x}{2} + c$

51. The function $f(x)$

Sol. $f(x) = 2 + \frac{3}{x-2}$ is bijective

$$f^{-1}(x) = 2 + \frac{3}{x-2}$$

$$\lim_{x \rightarrow 0^+} f(e^{1/x}) = 2$$

52. If r_1 and r_2 are

Sol. Let any point $(r \cos \theta, r \sin \theta)$ in xy plane

We have to maximize & minimize r

$$5r^2 \cos^2 \theta + 5r^2 \sin^2 \theta + 6r^2 \sin \theta \cos \theta - 8 = 0$$

$$5r^2 + 3r^2 \sin 2\theta - 8 = 0$$

$$r^2 = \frac{8}{5 + 3 \sin 2\theta}$$

$$r_{\max} = 2$$

$$r_{\min} = 1$$

$$r_1 + r_2 = 3$$

53. $\int \frac{dx}{ax^2 + bx + c} = \dots\dots\dots$

Sol.

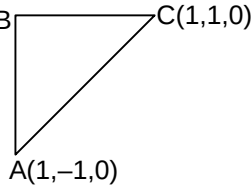
$$\int \frac{dx}{ax^2 + bx + c} = \frac{1}{a} \int \frac{dx}{\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}}$$

$$= k_1 \tan^{-1} \frac{x + A}{B} + C \text{ if } \frac{b^2 - 4ac}{4a^2} < 0$$

54. A plane cuts the

एक समतल, आयताकार

Sol. $(0,0,0)B \quad C(1,1,0)$



$z = 0$ is one of the plane perpendicular to

$$x + y = 0, x - y = 0, x = 1$$

Now the triangle formed is take one vertex on z-axis i.e., $(0, 0, k)$, then $1^2 + 1^2 + k^2 = 2^2$

$$\Rightarrow k = \pm \sqrt{2}$$

Now the direction ratio of plane having point $(0, 0, \sqrt{2})$, $(1, 1, 0)$ and $(1, -1, 0)$ is $(\sqrt{2}, 0, 1)$

and the direction ratio of plane having point $(0, 0, -\sqrt{2})$, $(1, 1, 0)$ and $(1, -1, 0)$ is $(-\sqrt{2}, 0, 1)$

55. Consider the circle

Sol. $x^2 + (ax^2 - b)^2 = 1 \Rightarrow a^2 x^4 + (1 - 2ab)x^2 + (b^2 - 1) = 0$

$$\Rightarrow a^2 t^2 + (1 - 2ab)t + (b^2 - 1) = 0$$

$$\Rightarrow f(t) = 0$$

$$D = 4a^2 - 4ab + 1$$

$$a > b > 1 \Rightarrow D > 0, f(0) > 0 \text{ and } \frac{2ab - 1}{2a^2} > 0$$

$$\Rightarrow t_1 > 0, t_2 > 0 \Rightarrow \text{four distinct real values of } x$$

$$b < -1 \Rightarrow D > 0, f(0) > 0 \text{ and } \frac{2ab - 1}{2a^2} < 0$$

$$\Rightarrow t_1 < 0, t_2 < 0 \Rightarrow \text{no real value of } x$$

$$-1 < b < 1 \Rightarrow f(0) < 0 \Rightarrow t_1 > 0, t_2 < 0$$

$$\Rightarrow \text{two distinct real values of } x$$

56. If $\sqrt{\alpha_1 - 1} + 2\sqrt{\alpha_2 - 4} + \dots\dots\dots$

Sol. $2\sqrt{\alpha_1 - 1} + 4\sqrt{\alpha_2 - 4} + 6\sqrt{\alpha_3 - 9} + 8\sqrt{\alpha_4 - 16}$

$$= (\alpha_1 - 1) + 1 + (\alpha_2 - 4) + 4 + (\alpha_3 - 9) + 9 + (\alpha_4 - 16) + 16$$

$$\Rightarrow (\sqrt{\alpha_1 - 1} - 1)^2 + (\sqrt{\alpha_2 - 4} - 2)^2 + (\sqrt{\alpha_3 - 9} - 3)^2$$

$$+ (\sqrt{\alpha_4 - 16} - 4)^2 = 0$$

$$\Rightarrow \sqrt{\alpha_1 - 1} = 1, \sqrt{\alpha_2 - 4} = 2, \sqrt{\alpha_3 - 9} = 3,$$

$$\sqrt{\alpha_4 - 16} = 4$$

$$\Rightarrow \alpha_1 = 2, \alpha_2 = 8, \alpha_3 = 18, \alpha_4 = 32$$

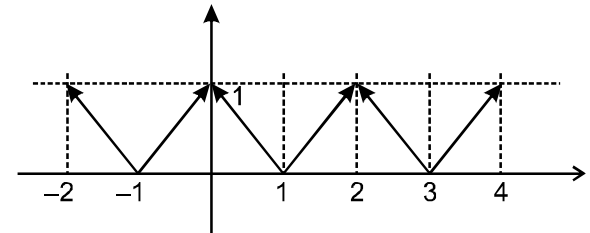
57. Which of the following

Sol. $\sin 82\frac{1}{2}^\circ = \cos 7\frac{1}{2}^\circ$

$$\sin 127\frac{1}{2}^\circ = \cos 37\frac{1}{2}^\circ \quad \sin 97\frac{1}{2}^\circ = \cos 7\frac{1}{2}^\circ$$

58. Let $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) \dots\dots\dots$

Sol.



$$x - [x] = \{x\}$$

$$x - [x + 1] = \{x\} - 1$$

$$\int_{-2}^4 f(x) dx = 6 \cdot \frac{1}{2} (1.1) = 3$$

59. Let $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} = \dots\dots\dots$

Sol. Since it is an identity the value of L.H.S and R.H.S are equal for all values of x

$$\text{put } x = 0 \Rightarrow \alpha_1 \alpha_2 \alpha_3 \alpha_4 = 6$$

60. If a chord of the circle

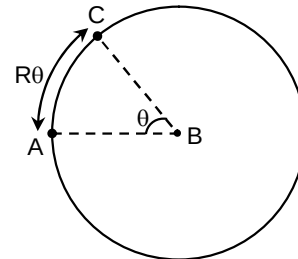
Sol. Let chord be $P = (5, 5)$

$$Q = (-2, -2)$$

PAPER-2 PART- I (PHYSICS)

1. Two men A

Sol. Let they meet at C.

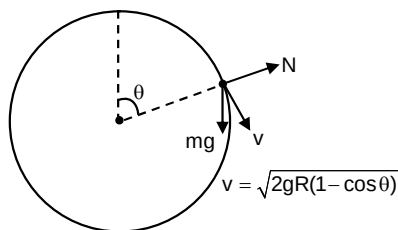


$$\frac{R}{v} = \sqrt{\frac{2R\theta}{a}}$$

$$v = R \sqrt{\frac{a}{2R\theta}} = \sqrt{\frac{aR}{2\theta}}$$

2. Two beads each

Sol.

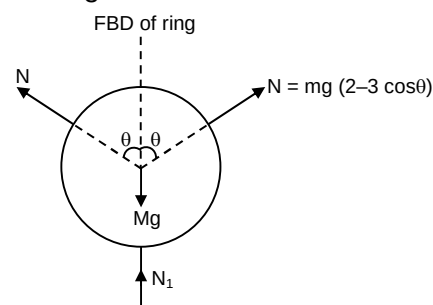


$$mg \cos \theta - N = \frac{mv^2}{R} = 2mg(1 - \cos \theta)$$

$$N = 3mg \cos \theta - 2mg \Rightarrow mg(3 \cos \theta - 2)$$

$$\text{for } \cos \theta > \frac{2}{3} \quad N = \text{Positive}$$

$$\cos \theta < \frac{2}{3} \quad N = \text{Negative}$$



$$N_1 + 2N \cos \theta = Mg$$

$$N_1 = Mg - 2mg \cos \theta (2 - 3 \cos \theta)$$

Ring will loose contact if

$$2mg(2 \cos \theta - 3 \cos^2 \theta) = Mg$$

$$4mg \cos \theta - 6mg \cos^2 \theta = Mg$$

$$6mg \cos^2 \theta - 4mg \cos \theta + Mg = 0$$

$$\cos \theta = \frac{4mg \pm \sqrt{16m^2g^2 - 24mMg^2}}{12mg} = \frac{2m \pm \sqrt{4m^2 - 6mM}}{6m}$$

for the situation to occur

$$4m^2 - 6mM \geq 0$$

$$2m \geq 3M$$

$$\frac{m}{M} \geq \frac{3}{2}$$

3. Two cells of emf.....

$$\text{Sol. } \varepsilon_1 = 300 \alpha \quad \dots\dots\dots(i)$$

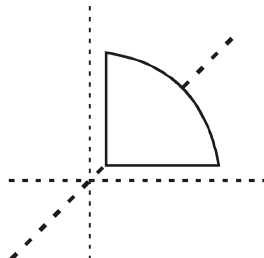
$$-\varepsilon_2 + \varepsilon_1 = 100 \alpha \quad \dots\dots\dots(ii)$$

where, α is the potential gradient

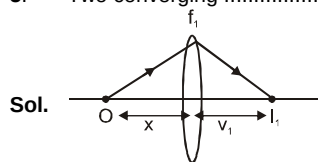
$$\therefore \frac{\varepsilon_2}{\varepsilon_1} = \frac{2}{3}$$

4. A convex lens

Sol. Each part will have different principal axis therefore number of images formed will be 4.



5. Two converging



Sol.

Image -1

$$u_1 = -x$$

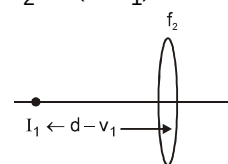
$$\frac{1}{v_1} - \frac{1}{-x} = \frac{1}{f_1}$$

$$v_1 = \frac{x f_1}{x - f_1}$$

$$m_1 = \frac{v_1}{u_1} = \frac{v_1}{-x} = -\left(\frac{f_1}{x - f_1}\right)$$

Image -2

$$u_2 = -(d - v_1)$$



$$\frac{1}{v_2} - \frac{1}{-(d - v_1)} = \frac{1}{f_2} \quad ; \quad v_2 = \frac{(d - v_1)f_2}{d - v_1 - f_2}$$

$$m_2 = \frac{v_2}{u_2} = -\left(\frac{f_2}{d - v_1 - f_2}\right)$$

$$m_1$$

$$m_2 = \left(\frac{f_1}{x - f_1}\right) \left(\frac{f_2}{d - \frac{x f_1}{x - f_1} - f_2}\right) = \frac{f_1 f_2}{x(d - f_1 - f_2) - df_1 + f_1 f_2}$$

Since m is independent of x

$$\Rightarrow (d - f_1 - f_2) = 0 \Rightarrow d = f_1 + f_2$$

$$\Rightarrow m = -\frac{f_2}{f_1}$$

6. Magnetic field at

$$\text{Sol. } \frac{\mu_0 I_1}{2\pi \times 10\text{cm}} = \frac{\mu_0(2)}{2 \times (5\text{cm})}$$

$$\Rightarrow \frac{I_1}{2\pi} = 2$$

$$I_1 = 4\pi$$

7. In the AC circuit

$$\text{Sol. } \tan \phi = \frac{X_L - X_C}{R}$$

8. A uniform cylinder

$$\text{Sol. } E = -mgX + \frac{1}{2}mv^2 \left(1 + \frac{1}{2}\right) + \frac{K}{2}(\ell_0 + 2x)^2$$

$$\frac{dE}{dX} = -mg + \frac{m}{2} \cdot 2V \frac{dV}{dX} \cdot \frac{3}{2} + \frac{K}{2} 2(\ell_0 + 2x) \cdot 2 = 0$$

$$\frac{d^2 X}{dt^2} = -\frac{8K}{3m} X$$

Alternative Sol.

$$\frac{1}{2} k(2r\theta)^2 - mgr\theta + \frac{1}{2} \left(\frac{mr^2}{2} + mr^2 \right) \omega^2 = C$$

$$\frac{1}{2} k4r^2 2\theta \dot{\theta} - mgr \dot{\theta} + \frac{3mr^2}{4} 2\omega \alpha = 0$$

$$\alpha = -\frac{8k}{3m} \theta + \text{constant}$$

$$\therefore T = 2\pi \sqrt{\frac{3m}{8k}}$$

9. A photon strikes

Sol. Energy required to just remove the electron = 13.6 eV

$\therefore$ Energy required = 13.6 + 16.4 = 30 eV

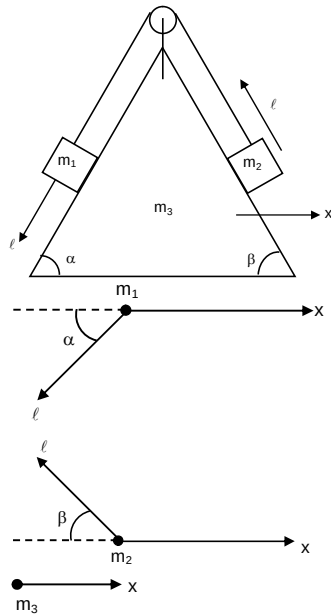
If E be the photon energy 25%

E = 30 eV, E = 120 eV = 24 × 5 eV.

X = 5 **Ans.**

10. In the arrangement

Sol.



$$m_1(x - l \cos \alpha) + m_2(x - l \cos \beta) + m_3 x = 0$$

$$x(m_1 + m_2 + m_3) = (m_1 \cos \alpha + m_2 \cos \beta) l$$

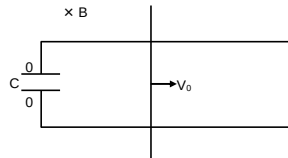
$$x = \frac{(m_1 \cos \alpha + m_2 \cos \beta) l}{m_1 + m_2 + m_3}$$

11. Final velocity

12. Total energy

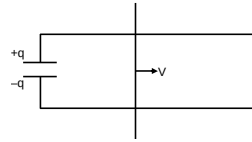
Just before rod is kept on rail

Sol.



after time dt

Just after rod is kept on rail



$$q = CVB\ell \quad i = \frac{q}{dt} = \frac{CVB\ell}{dt}$$

Force on rod $f = i\ell B$

Impulse of force on rod $= fdt = CVB^2\ell^2$

Using impulse momentum theorem for rod

$$mV_0 - B^2\ell^2 C V = mV$$

$$V = \frac{mV_0}{m + B^2\ell^2 C} = \frac{mV_0}{m + 2m} = \frac{V_0}{3}$$

$$\text{Initially energy } \varepsilon_i = \frac{1}{2} mV_0^2$$

$$\text{Final energy } \varepsilon_f = \frac{1}{2} mV^2 + \frac{C^2 B^2 \ell^2}{2C} V^2$$

$$= \frac{1}{2} m \left(\frac{V_0}{3} \right)^2 + \frac{1}{2} (2m) \left(\frac{V_0}{3} \right)^2$$

$$= \frac{1}{18} mV_0^2 + \frac{2}{18} mV_0^2$$

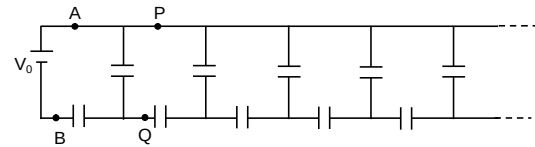
$$= \frac{1}{6} mV_0^2$$

$$\text{Heat loss} = \frac{1}{2} mV_0^2 - \frac{1}{6} mV_0^2 = \frac{1}{3} mV_0^2$$

13. The equivalent

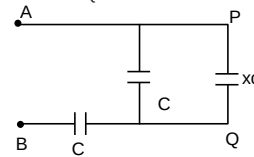
14. Total heat loss

Sol.



Lets find equivalent capacitance b/w A and B. Let $C_{AB} = xC$

then $C_{PQ} = xC$



$$C_{AB} = \frac{C(x+1)C}{C + (x+1)C} = xC$$

$$\frac{x+1}{x+2} = x \Rightarrow x+1 = x^2 + 2x$$

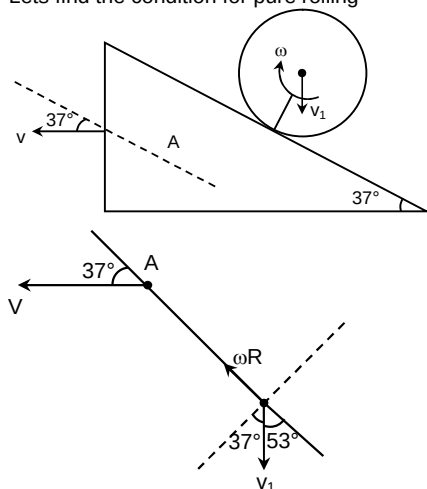
$$x^2 + x - 1 \Rightarrow x = \frac{-1 \pm \sqrt{1+4}}{2}$$

$$x = \left(\frac{\sqrt{5}-1}{2} \right) \quad C_{AB} = \left(\frac{\sqrt{5}-1}{2} \right) C$$

$$\text{Heat produced } H_1 = \frac{1}{2} C_{AB} V_0^2 = \left(\frac{\sqrt{5}-1}{4} \right) C V_0^2$$

15. If at certain instant

Sol. Lets find the condition for pure rolling



$$v \sin 37^\circ = v_1 \cos 37^\circ$$

$$v_1 = v \tan 37^\circ = \frac{3}{4} v$$

$$\omega R - v_1 \sin 37^\circ = v \cos 37^\circ$$

$$\omega R - \left(\frac{3}{4}v\right)\left(\frac{3}{5}\right) = \frac{4}{5}v$$

$$\omega R - \frac{9}{20}v = \frac{16}{20}v$$

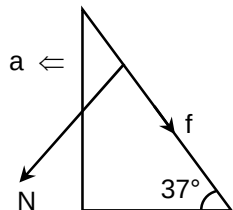
$$\Rightarrow \omega R = \frac{25}{20}v = \frac{5}{4}v$$

$$\omega = \frac{5v}{4R}$$

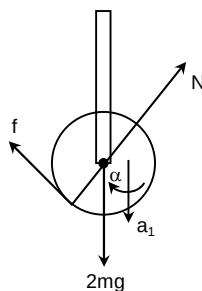
$$\begin{aligned} K &= \frac{1}{2}mv^2 + \frac{1}{2}(2m)v_1^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\omega^2 \\ &= \frac{1}{2}mv^2 + m\left(\frac{9}{16}v^2\right) + \frac{1}{4}mR^2\left(\frac{25v^2}{16R^2}\right) \\ &= \frac{1}{2}mv^2 + \frac{9mv^2}{16} + \frac{25mv^2}{64} = \left(\frac{32+36+25}{64}\right)mv^2 \\ &= \frac{93}{64}mv^2 \end{aligned}$$

16. Acceleration of

Sol.



$$N \sin 37^\circ - f \cos 37^\circ = ma$$



.....(1)

$$2mg - N \cos 37^\circ - f \sin 37^\circ = 2ma_1 = 2m\left(\frac{3a}{4}\right) \quad \text{.....(2)}$$

$$f \times R = I\alpha = \left(\frac{mR^2}{2}\right) \times \left(\frac{5a}{4R}\right) \quad \text{.....(3)}$$

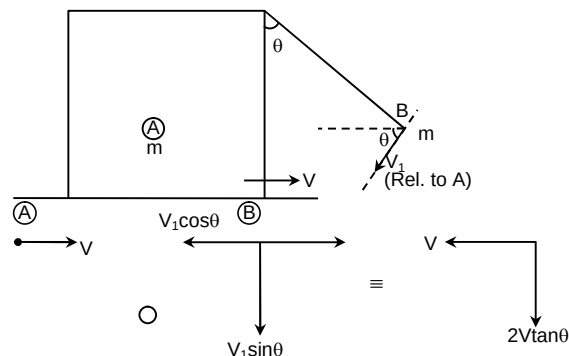
After solving

$$f = \frac{5ma}{8}, N = \frac{5ma}{2}, a = \frac{16g}{31}$$

17. Speed of the block

18. Normal reaction offered

Sol.



Applying conservation of linear momentum.

$$mV = m(V_1 \cos \theta - V) \Rightarrow V_1 = \frac{2V}{\cos \theta} \quad \left[V_1 = \frac{2V}{\cos \theta} \right]$$

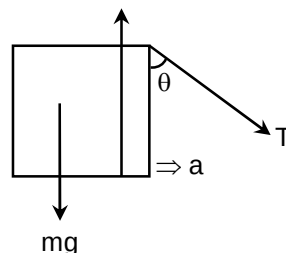
Applying conservation of mechanical energy

$$\begin{aligned} mg \ell \cos \theta &= \frac{1}{2}mV^2 + \frac{1}{2}m(V^2 + 4V^2 \tan^2 \theta) \\ &= \frac{1}{2}mV^2(2 + 4 \tan^2 \theta) = mV^2(1 + 2 \tan^2 \theta) \end{aligned}$$

$$V = \sqrt{\frac{g \ell \cos \theta}{1 + 2 \tan^2 \theta}}$$

$$V_1 = 2 \sqrt{\frac{g \ell}{\cos \theta (1 + 2 \tan^2 \theta)}} = 2 \sqrt{\frac{g \ell \cos \theta}{1 + \sin^2 \theta}}$$

FBD of A

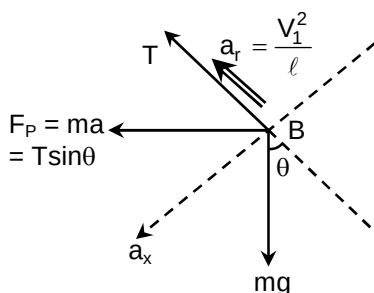


$$\begin{aligned} N &= T \cos \theta + mg \\ T \sin \theta &= ma \end{aligned}$$

$$N = T \cos \theta + mg$$

$$T \sin \theta = ma$$

Motion of B w.r.t. A



$$T + T \sin^2 \theta - mg \cos \theta = \frac{mV_1^2}{l}$$

$$T(1 + \sin^2 \theta) = mg \cos \theta + \frac{m}{l} \left(4 \cdot \frac{g l \cos \theta}{1 + \sin^2 \theta} \right)$$

$$T(1 + \sin^2 \theta) = mg \left(\cos \theta + \frac{4 \cos \theta}{1 + \sin^2 \theta} \right)$$

$$T(1 + \sin^2 \theta) = mg \cos \theta \left(\frac{5 + \sin^2 \theta}{1 + \sin^2 \theta} \right)$$

$$T = mg \cos \theta \left(\frac{5 + \sin^2 \theta}{(1 + \sin^2 \theta)^2} \right)$$

$$N = T \cos \theta + mg = mg \left(1 + \frac{\cos^2 \theta (5 + \sin^2 \theta)}{(1 + \sin^2 \theta)^2} \right)$$

19. Maximum percentage

Sol. For maximum error in g :

$$g = \frac{AB}{A+B} \Rightarrow \frac{1}{g} = \frac{1}{A} + \frac{1}{B} \Rightarrow \frac{dg}{g^2} = \frac{dA}{A^2} + \frac{dB}{B^2}$$

$$\frac{dg}{g} = g \left(\frac{dA}{A^2} + \frac{dB}{B^2} \right)$$

$$\left(\frac{dg}{g} \right)_{\max} = 2 \left(\frac{0.12}{(6)^2} + \frac{0.15}{(3)^2} \right) = 4\%$$

20. Maximum percentage

Sol. $f = \frac{AB}{C+D}$

$$\left(\frac{df}{f} \right) = \frac{dA}{A} + \frac{dB}{B} + \frac{d(C+D)}{C+D} = \frac{0.12}{6} +$$

$$\frac{0.15}{3} + \frac{0.04}{2+4} + \frac{0.12}{2+4}$$

$$\left(\frac{df}{f} \right)_{\max} = 9.67\%$$

PART- II (CHEMISTRY)

21. Total number of electrons in ${}_{24}\text{Cr}^{3+}$

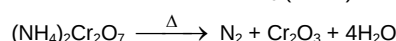
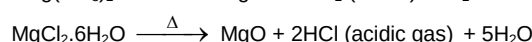
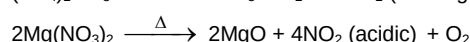
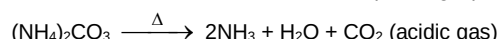
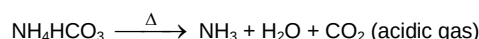
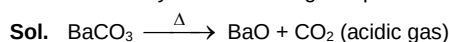
Sol.

1s ²	2s ²	2p ⁶	3s ²	3p ⁶	3d ³
↑↓	↑↓	↑↓ ↑↓ ↑↓	↑↓	↑↓ ↑↓ ↑↓	↑ ↑ ↑
or					
↑	↑	↑			

22. One mole of a substance is cooled at the rate

Sol. $\Delta S_{\text{fusion}} = \frac{\Delta H}{T} = \frac{[0.4 \times 10^3] \times 30}{400} = 30 \text{ J mole}^{-1} \text{ K}^{-1}$

23. How many of the following compounds



24. A first order reaction is completed

Sol. $K = \frac{2.303}{2} \log \frac{100}{80} \Rightarrow K = \frac{2.303}{2} \log \frac{5}{4} \text{ min}^{-1}$

$$K = \frac{2.303}{2} \times 0.1 \text{ min}^{-1}$$

$$t_{1/2} = \frac{0.693}{K} = 6 \text{ min}$$

25. Suppose that at time t the state

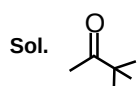
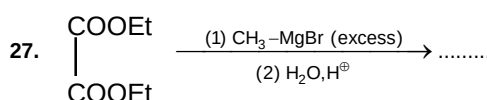
Sol. $\Psi = (2/\pi C^2)^{3/4} e^{-(x^2+y^2+z^2)/C^2}$

$$|\Psi|^2 dx dy dz = (2/\pi C^2)^{3/2} e^{-2(x^2+y^2+z^2)/C^2} dx dy dz$$

$$[2/(4\pi \text{nm}^2)]^{3/2} e^{-2[(1.2)^2 + (-1)^2 + (0)^2]/4} (0.004 \text{ nm})^3$$

$$= 1.200 \times 10^{-9}$$

Ans. $y = 9$

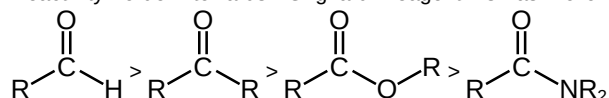


28. How many of the following compound

Sol. Acetal and sucrose do not give any precipitation with tollen's reagent.

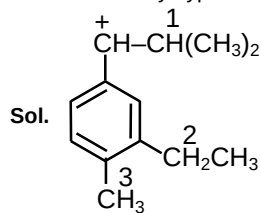
29. How many of the following favours

Sol. Reactivity order towards Grignard reagent is as follows



Only (i), (iv), (vi) and (vii) undergoes nucleophilic addition reaction.

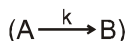
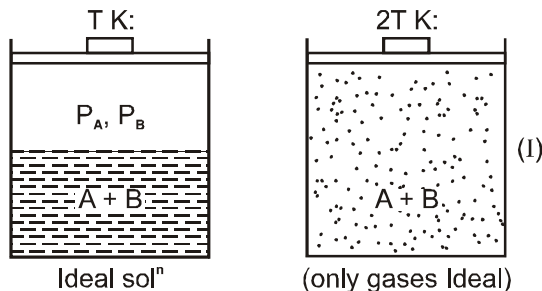
30. How many hyperconjugable H-atoms



Total hyperconjugable H-atoms = 6

31. If partial vapour pressure of A is twice

Sol. (A)



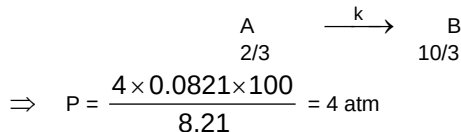
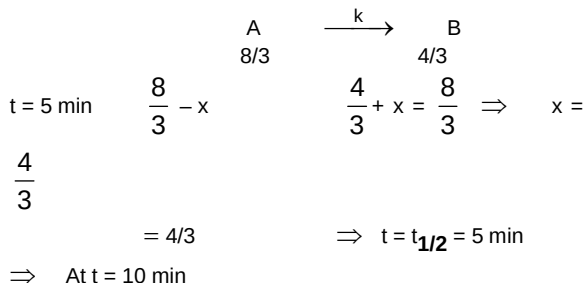
$$\frac{n_A}{n_B} = \frac{P_A}{P_B} = 2; \text{ Total V.P.} = 2 \text{ atm};$$

$$PV = nRT \Rightarrow 2 \times 8.21 = (n_A + n_B) \times 0.0821 \times 50$$

$$\Rightarrow n_A + n_B = 4$$

32. Vapours of A and B are passed into a

Sol. (B) $\frac{n_A}{n_B} = \frac{2}{1}$



33. $\text{KOH} + \text{O}_3 \longrightarrow [\text{X}] + \text{O}_2 + \text{H}_2\text{O}$

Sol. $2\text{KOH} + 5\text{O}_3 \longrightarrow 2\text{KO}_3$ (orange coloured) + $5\text{O}_2 + \text{H}_2\text{O}$.

34. Which of the following statement is

Sol. (A) Correct statement (B) $2\text{NO}_2 + \text{O}_3 \longrightarrow \text{N}_2\text{O}_5 + \text{O}_2$

(C) O_3^- is paramagnetic.

(D) $2\text{I}_2 + 9[\text{O}_3] \longrightarrow \text{I}_4\text{O}_9$ (yellow solid) + 9O_2

35. Salt (A) contains

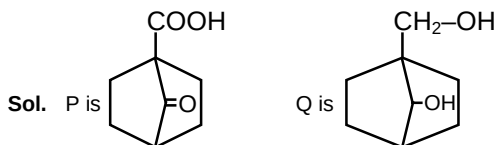
36. The orange solution is

Sol. (35, 36)

(A) = Bi^{3+} , (B) = BiI_3 (black), (C) = BiOI (orange), (D) = $[\text{BiI}_4]^-$,
(E) = $\text{Bi}(\text{C}_6\text{H}_3\text{O}_3)$ (yellow)

BiI_4^- is orange complex.

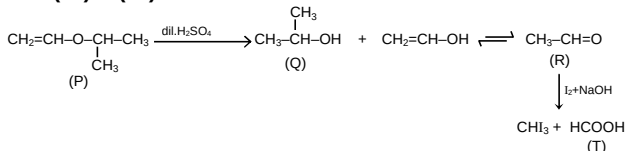
38. Choose the correct option.....



39. P will be

40. Which amongs the following

Sol. (39) & (40)



PART- III (MATHEMATICS)

41. If $(1 + x + x^2)^{3n+1}$
Sol. $(1 + x + x^2)^{3n+1} = a_0 + a_1x + a_2x^2 + \dots + a_{6n}x^{6n} + a_{6n+1}x^{6n+1} + a_{6n+2}x^{6n+2}$

putting $x = \omega$ and ω^2 , we get

$$(1 + \omega + \omega^2)^{3n+1} = 0 = a_0 + a_1\omega + a_2\omega^2 + a_3 + \dots + a_{6n} + a_{6n+1}\omega + a_{6n+2}\omega^2 \dots (1)$$

$$(1 + \omega^2 + \omega^4)^{3n+1} = 0 = a_0 + a_1\omega^2 + a_2\omega^4 + a_3 + \dots + a_{6n} + a_{6n+1}\omega^2 + a_{6n+2}\omega^4 \dots (2)$$

Adding (1) and (2), we get

$$\sum_{r=0}^{2n} (2a_{3r} - (a_{3r+1} + a_{3r+2})) = 0.$$

42. $f: \mathbb{R} \rightarrow \mathbb{R}$ be a twice.....

Sol. Given inequality can be written as :

$$f''(x) - 2f'(x) \geq 3 \text{ (if } f'(x) - 2f(x))$$

$$\text{Let } f'(x) - 2f(x) = g(x)$$

$$\Rightarrow g'(x) - 3g(x) \geq 0 \text{ multiply } e^{-3x}$$

$$\Rightarrow \frac{d}{dx} (g(x)e^{-3x}) \geq 0 \Rightarrow g(x)e^{-3x} \text{ is non-decreasing.}$$

$$\text{Now } g(0) = f'(0) - 2f(0) = -2$$

$$f'(x) - 2f(x) \geq -2e^{3x}, \forall x \geq 0, e^{-2x}$$

$$\Rightarrow \frac{d}{dx} (f(x)e^{-2x}) \geq -2e^x, \forall x \geq 0$$

$$\Rightarrow \frac{d}{dx} (f(x)e^{-2x} + 2e^x) \geq 0$$

$$\Rightarrow f(x)e^{-2x} + 2e^x \geq 3$$

$$\Rightarrow f(x) \geq 3e^{2x} - 2e^{3x}, \forall x \geq 0$$

comparing $ah(bx) - bh(ax)$ with $3e^{2x} - 2e^{3x}$ we get

$$h(x) = e^x, a = 3, b = 2$$

$$\Rightarrow (a + b)h(0) = 5$$

43. If number of values

Sol. $(z^3) = ((\bar{\omega}^7)) \Rightarrow |z|^3 = |\bar{\omega}|^7 = |\omega|^7 \text{ or } |z|^{15} = |\omega|^{35}$

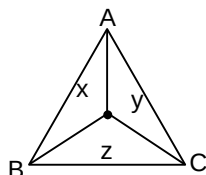
Again $z^5 \cdot \omega^{11} = 1 \Rightarrow |z|^5 \cdot |\omega|^{11} = 1 \text{ or } |z|^{15} \cdot |\omega|^{33} = 1 \dots (ii)$

From (i) and (ii) $\Rightarrow |z| = |\omega| = 1$

Again $-(\bar{\omega})^{35} = \frac{1}{\omega^{33}} \Rightarrow (\bar{\omega})^2 = -1 = i^2 \Rightarrow \omega = i \text{ or } -i$

44. Let O be an interior.....

Sol.



$$\vec{OA} + 2\vec{OB} + 3\vec{OC} = \vec{0} \quad \dots\dots(i)$$

Let x, y, z be the area of triangle OAB, OBC and OCA respectively and area $(\Delta ABC) = A$

$$\Rightarrow x + y + z = A \quad \dots\dots(ii)$$

Taking cross product with $\vec{OA}$ on the both sides (i) and then take modulus on both sides, we get

$$\frac{x}{y} = \frac{3}{2} \Rightarrow x = \frac{3}{2}y$$

$$\frac{x}{y} = \frac{3}{2} \Rightarrow x = \frac{3}{2}y$$

$$\text{similarly, } \frac{x}{z} = 3 \text{ and } \frac{y}{z} = 2 \Rightarrow z = \frac{y}{2}$$

$$\therefore \text{ from (ii) , } y\left(\frac{3}{2} + 1 + \frac{1}{2}\right) = A \Rightarrow \frac{A}{y} = 3$$

45. In ΔABC , if

$$\text{Sol. } \frac{c+a}{12} = \frac{a+b}{14} = \frac{b+c}{18} = \frac{s}{11}$$

$$b+c = \frac{18}{11}s$$

$$\Rightarrow 2s - a = \frac{18}{11}s$$

$$\Rightarrow s - a = \frac{7}{11}s$$

$$r_1 = \frac{\Delta}{s-a} = \frac{11\Delta}{7s} = \frac{11}{7}r$$

$$a+c = \frac{12s}{11}$$

$$2s - b = \frac{12s}{11}$$

$$s - b = \frac{s}{11}$$

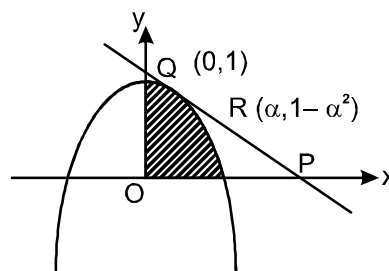
$$r_2 = \frac{\Delta}{s-b} = \frac{11\Delta}{s} = 11r$$

$$\Rightarrow \frac{7r_1 - r_2}{11} = \frac{11r - 11r}{11r} = 0$$

46. If the tangent to

Sol. A_1 = area under the curve and axes

$$= \int_0^1 (1-x^2)dx = \left[x - \frac{x^3}{3} \right]_0^1 = \frac{2}{3} \quad \dots(i)$$



$$\text{now } y = 1 - x^2$$

$$\therefore y' = -2x$$

$$\therefore y'(\alpha, 1 - \alpha^2) = -2\alpha$$

Equation of tangent to the curve $y = 1 - x^2$

$$\Rightarrow (y - (1 - \alpha^2)) = -2\alpha(x - \alpha)$$

$$\Rightarrow 2\alpha x + y = \alpha^2 + 1$$

$$\therefore P \equiv \left(\frac{\alpha^2 + 1}{2\alpha}, 0 \right); Q \equiv (0, \alpha^2 + 1)$$

$$A = \text{Area of triangle POQ} = \frac{1}{2} (OP) (OQ) = \frac{1}{4} \frac{(\alpha^2 + 1)^2}{\alpha}$$

$\therefore$

$$\frac{dA}{d\alpha} = \frac{1}{4} \frac{2\alpha(\alpha^2 + 1)2\alpha - (\alpha^2 + 1)^2}{\alpha^2} = \frac{1}{4} \frac{3\alpha^4 + 2\alpha^2 - 1}{\alpha^2}$$

$$\text{For maximum / minimum } \frac{dA}{d\alpha} = 0$$

$$\Rightarrow 3\alpha^4 + 2\alpha^2 - 1 = 0 \quad \alpha^2 = \frac{1}{3} \quad \Rightarrow \alpha = \pm \frac{1}{\sqrt{3}}$$

$$\text{Since } 0 < \alpha < 1 \Rightarrow \alpha = \frac{1}{\sqrt{3}}, \frac{d^2A}{d\alpha^2} > 0$$

Hence A is minimum.

$$A = \frac{1}{4} \frac{\left(\frac{1}{3} + 1\right)^2}{\frac{1}{\sqrt{3}}} = \frac{4}{3\sqrt{3}} \quad \dots(ii)$$

Since $A = kA_1$ (given)

$$\Rightarrow \frac{4}{3\sqrt{3}} = \frac{2}{3}k \Rightarrow k = \frac{2}{\sqrt{3}}$$

47. If the differential

Sol. $\frac{dy}{dx} (\cos y - \sin y) + (\cos y + \sin y) e^{-x} = e^{e^{-x}}$

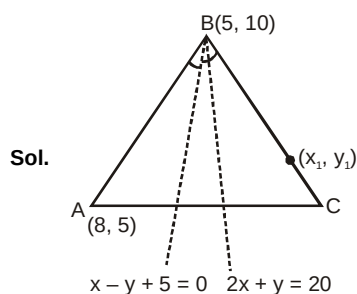
$$\cos y + \sin y = u$$

$$\Rightarrow \frac{du}{dx} + e^{-x}u = e^{e^{-x}} = x$$

$$\Rightarrow e^{-e^{-t}} = t$$

$$\Rightarrow t \cdot e^{-t} = 1$$

48. The vertex A of triangle



Point of the intersection of $x - y + 5 = 0$ and $2x + y = 20$

gives the co-ordinate of vertex B i.e. (5,10)

Image of point A with respect to $x - y + 5 = 0$ lies on the line

BC.

$$\frac{x_1 - 8}{1} = \frac{y_1 - 5}{-1} = -2 \left(\frac{8 - 5 + 5}{1 + 1} \right)$$

$$x_1 = 0, \quad y_1 = 13$$

$\therefore$ co-ordinate of image (0, 13)

$$\text{equation of line BC } y - 10 = \frac{13 - 10}{0 - 5} (x - 5)$$

$$-5y + 50 = 3x - 15$$

$$3x + 5y = 65$$

Let co-ordinates of point C is $\left(t, \frac{65 - 3t}{5}\right)$, mid-point of A and

C is $\left(\frac{8+t}{2}, \frac{90-3t}{10}\right)$ lies on the line

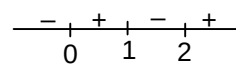
$$2x + y = 20, \text{ hence satisfies the line } 2\left(\frac{8+t}{2}\right) + \frac{90-3t}{10} = 20$$

$$7t = 30 \Rightarrow t = \frac{30}{7}$$

Hence co-ordinates of point C is $\left(\frac{30}{7}, \frac{73}{7}\right)$

49. Let $f(x)$ be a function.....

Sol. $f'(x) = x(x^2 - 3x + 2) = x(x-1)(x-2)$ the sign scheme for $f'(x)$ is as below



$\therefore f'(x) \leq 0$ in $1 \leq x \leq 2$ and $f'(x) \geq 0$ in $2 \leq x \leq 5$

$\therefore f(x)$ is decreasing in $[1, 2]$ and increasing in $[2, 3]$

maximum $f(x)$ = the greatest among $\{f(1), f(3)\}$

$$f(1) = \int_1^1 x(x^2 - 3x + 2) dx = 0, \quad f(3) = \int_1^3 x(x^2 - 3x + 2) dx = 2$$

$$= \int_1^3 x(x^2 - 3x + 2) dx = 2$$

$\therefore$ maximum $f(x) = 2$

50. Three numbers

$\{1, 2, \dots, 10\}$ से

Sol. Let E_1 be the event getting minimum number 3 and E_2 be the event getting maximum number

7 Then $P(E_1) = P(\text{getting number 3 and other two from number 4 to 10})$

$$= \frac{{}^1C_1 \times {}^7C_2}{{}^{10}C_3} = \frac{7}{40}$$

$P(E_2) = P(\text{getting one number 7 and other two from number}$

$$1 \text{ to } 6) = \frac{{}^1C_1 \times {}^6C_2}{{}^{10}C_3} = \frac{1}{8}$$

$P(E_1 \cap E_2) = P(\text{getting one number 3, second number 7 and third from 4 to 6})$

$$= \frac{{}^1C_1 \times {}^1C_1 \times {}^3C_1}{{}^{10}C_3} = \frac{1}{40}$$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = \frac{11}{40}$$

So $k = 6$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = \frac{11}{40}$$

So $k = 6$

51. The value of
Sol. line are in same plane so

$$\begin{vmatrix} 1 & -1 & -1 \\ 2 & 3 & \lambda \\ 3 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 1(9 - 2\lambda) + 1(6 - 3\lambda) - 1(-5) = 0$$

$$20 - 5\lambda = 0 \Rightarrow \lambda = 4$$

$$\text{So } \sin^{-1} \sin 4 = \sin^{-1} \sin(\pi - 4) = \pi - 4$$

52. Angle between
Sol. normal vector to plane containing lines is

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 3 & 2 & 3 \end{vmatrix} = \hat{i} + 6\hat{j} - 5\hat{k}$$

So angle between planes

$$\cos \theta = \frac{4 + 6 - 10}{\sqrt{21}\sqrt{62}} = 0 \Rightarrow \theta = \frac{\pi}{2}$$

53. Let $F(x) = \int_{\sin x}^{\cos x} e^{(1+\sin^{-1}t)^2} dt$

Sol. $F'(x) = e^{(1+\sin^{-1}(\cos x))^2} \cdot (-\sin x) - e^{(1+\sin^{-1}(\sin x))^2} \cdot \cos x$

$$F'(0) = -e = F' \left(\frac{\pi}{2} \right)$$

Since $F'(x)$ is continuous and differentiable and

$$F'(0) = F' \left(\frac{\pi}{2} \right)$$

Hence Rolle's theorem is applicable for $y = F'(x)$ in $\left[0, \frac{\pi}{2} \right]$

$$\therefore \exists \text{ some } c \in \left(0, \frac{\pi}{2} \right) \text{ s.t. } F''(c) = 0$$

54. If a function $y = f(x)$

Sol. Differentiating the equation w.r. to x , we get

$$2f(x)f'(x) = (f(x))^2 + (f'(x))^2$$

$$\Rightarrow (f(x) - f'(x))^2 = 0 \Rightarrow f'(x) = f(x)$$

$$\Rightarrow \frac{f'(x)}{f(x)} = 1 \Rightarrow \ln f(x) = x + c$$

$$\text{but } x = 0, f(0) = \sqrt{m^2} \quad m \in \mathbb{R}^+$$

$$\therefore c = \ln m$$

$$\therefore \ln f(x) = x + \ln m \Rightarrow \ln \left(\frac{f(x)}{m} \right) = x$$

$$\Rightarrow f(x) = me^x, m \in \mathbb{R}^+$$

$$f'(x) > 0 \quad \forall x \in \mathbb{R}$$

55. The probability of

56. If white ball is

Sol. $P(i)\alpha i$

$$P(i) = Ki$$

$$P(1) = K, P(2) = 2K, \dots, P(6) = 6K$$

$$K + 2K + \dots + 6K = 1$$

$$K(1 + 2 + \dots + 6) = 1$$

$$K = \frac{1}{21}$$

$$P(1) = \frac{1}{21}, P(2) = \frac{2}{21}, \dots, P(6) = \frac{6}{21}$$

$$P(\text{Prime number}) = \frac{2}{21} + \frac{3}{21} + \frac{5}{21} = \frac{10}{21}$$

$$P(\text{not prime}) = \frac{11}{21}$$

$$P(B) = \frac{10}{21} \times \frac{3}{5} + \frac{11}{21} \times \frac{2}{5} = \frac{52}{105}$$

$$P(w) = \frac{10}{21} \times \frac{2}{5} + \frac{11}{21} \times \frac{3}{5} = \frac{53}{105}$$

$$P\left(\frac{\text{urnB}}{w}\right) = \frac{\frac{33}{105}}{\frac{53}{105}} = \frac{33}{53}$$

57. If $|P| = 1, |Q| = 1$

$$\begin{aligned} \text{Sol. } & (\text{adj } P^{-1}) \cdot \text{adj } B (\text{adj } Q^{-1}) \\ & = (P^{-1})^{-1} \text{adj } B (Q^{-1})^{-1} = PAQ \end{aligned}$$

58. If A and P are

Sol. $(PAP)^T = P^T A^T P^T$
 $= (-P) (-A) (-P) = -P A P$

59. f(x) is equal

Sol. $f'(x) = f(x) + \int_0^2 f(x) dx$

let $\int_0^2 f(x) dx = k \Rightarrow f'(x) - f(x) = k$

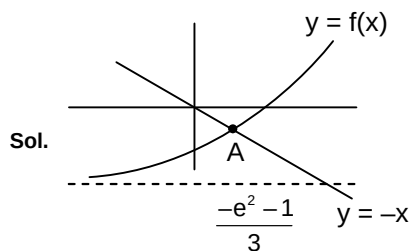
$\Rightarrow e^{-x} f(x) = k \int e^{-x} dx \Rightarrow f(x) = ce^x - k$

Since $\int_0^2 f(x) dx = k \Rightarrow k = \frac{c(e^2 - 1)}{3}$

& $f(0) = \frac{4 - e^2}{3} \Rightarrow c = 1, k = \frac{e^2 - 1}{3}$

$f(x) = e^x - \left(\frac{e^2 - 1}{3} \right)$

60. The number of



Number of solutions of $f(x) + x = 0$

$\Rightarrow f(x) = -x$

point A is only solution

DATE : 14-05-2017
ANSWER KEY
CODE-0
PAPER-1
PART- I (PHYSICS)

- | | | | | | | |
|-----------|-----------|-----------|-----------|-----------|----------|----------|
| 1. (ABC) | 2. (BCD) | 3. (AC) | 4. (BC) | 5. (AD) | 6. (AD) | 7. (BC) |
| 8. (AC) | 9. (BCD) | 10. (AC) | 11. (ABD) | 12. (ABC) | 13. (BC) | 14. (AD) |
| 15. (ACD) | 16. (ABC) | 17. (BCD) | 18. (AC) | 19. (AD) | 20. (AC) | |

PART- II (CHEMISTRY)

- | | | | | | | |
|------------|------------|------------|------------|------------|-----------|-----------|
| 21. (ABC) | 22. (ABCD) | 23. (ABCD) | 24. (BCD) | 25. (ABCD) | 26. (ACD) | 27. (ACD) |
| 28. (BC) | 29. (AC) | 30. (ABD) | 31. (AB) | 32. (AB) | 33. (ABC) | 34. (AB) |
| 35. (ABCD) | 36. (ABCD) | 37. (AB) | 38. (ABCD) | 39. (AC) | 40. (AB) | |

PART- III (MATHEMATICS)

- | | | | | | | |
|------------|------------|-----------|-----------|------------|-----------|-----------|
| 41. (ABCD) | 42. (ABCD) | 43. (ABC) | 44. (ACD) | 45. (ABD) | 46. (BCD) | 47. (BCD) |
| 48. (ACD) | 49. (AB) | 50. (ABD) | 51. (ABD) | 52. (ABCD) | 53. (CD) | 54. (BC) |
| 55. (ABC) | 56. (ABD) | 57. (ABC) | 58. (ABC) | 59. (C) | 60. (AB) | |

PAPER-2
PART- I (PHYSICS)

- | | | | | | | |
|-----------|------------|-----------|---------|---------|---------|-----------|
| 1. (2) | 2. (3,6,9) | 3. (4, 8) | 4. (4) | 5. (3) | 6. (4) | 7. (1, 4) |
| 8. (4, 8) | 9. (5) | 10. (7) | 11. (C) | 12. (C) | 13. (A) | 14. (C) |
| 15. (A) | 16. (B) | 17. (C) | 18. (B) | 19. (B) | 20. (C) | |

PART- II(CHEMISTRY)

- | | | | | | | |
|-----------|---------|---------|---------------|---------|---------|---------|
| 21. (3,4) | 22. (3) | 23. (8) | 24. (1,2,3,4) | 25. (9) | 26. (5) | 27. (1) |
| 28. (7) | 29. (4) | 30. (6) | 31. (A) | 32. (B) | 33. (B) | 34. (D) |
| 35. (A) | 36. (A) | 37. (A) | 38. (B) | 39. (C) | 40. (B) | |

PART- III (MATHEMATICS)

- | | | | | | |
|---------|---------------|-----------|-------------|---------|---------------|
| 41. 0 | 42. 0,1,2,3,4 | 43. 3,5,7 | 44. 5,7,9 | 45. 0 | 46. 0,2,4,6,8 |
| 47. 1 | 48. 1,2,5 | 49. 0,1,2 | 50. 6,7,8,9 | 51. (D) | 52. (B) |
| 53. (B) | 54. (A) | 55. (B) | 56. (D) | 57. (A) | 58. (B) |
| 59. (A) | 60. (B) | | | | |

DATE : 14-05-2017
ANSWER KEY
CODE- 1
PAPER-1
PART- I (PHYSICS)

- | | | | | | | |
|-----------|-----------|-----------|-----------|-----------|----------|----------|
| 1. (ACD) | 2. (ABD) | 3. (AC) | 4. (AD) | 5. (BC) | 6. (BC) | 7. (AD) |
| 8. (AC) | 9. (ABD) | 10. (AC) | 11. (BCD) | 12. (ACD) | 13. (AD) | 14. (BC) |
| 15. (ABC) | 16. (ACD) | 17. (ABD) | 18. (AC) | 19. (BC) | 20. (AC) | |

PART- II (CHEMISTRY)

- | | | | | | | |
|------------|------------|------------|------------|------------|-----------|-----------|
| 21. (ABC) | 22. (ABCD) | 23. (ABCD) | 24. (ACD) | 25. (ABCD) | 26. (ABD) | 27. (BCD) |
| 28. (BD) | 29. (AB) | 30. (ABC) | 31. (CD) | 32. (AD) | 33. (ABD) | 34. (AC) |
| 35. (ABCD) | 36. (ABCD) | 37. (AC) | 38. (ABCD) | 39. (BC) | 40. (AC) | |

PART- III (MATHEMATICS)

- | | | | | | | |
|------------|------------|-----------|-----------|------------|-----------|-----------|
| 41. (ABCD) | 42. (ABCD) | 43. (ABD) | 44. (BCD) | 45. (ABC) | 46. (ACD) | 47. (ACD) |
| 48. (ABD) | 49. (AC) | 50. (BCD) | 51. (ABC) | 52. (ABCD) | 53. (BC) | 54. (BD) |
| 55. (ACD) | 56. (ABC) | 57. (ABC) | 58. (ABD) | 59. (D) | 60. (AC) | |

PAPER-2
PART- I (PHYSICS)

- | | | | | | | |
|-----------|------------|-----------|---------|---------|---------|-----------|
| 1. (2) | 2. (3,6,9) | 3. (4, 8) | 4. (4) | 5. (3) | 6. (4) | 7. (1, 4) |
| 8. (4, 8) | 9. (5) | 10. (7) | 11. (A) | 12. (A) | 13. (C) | 14. (A) |
| 15. (B) | 16. (C) | 17. (A) | 18. (D) | 19. (D) | 20. (A) | |

PART- II (CHEMISTRY)

- | | | | | | | |
|-----------|---------|---------|---------------|---------|---------|---------|
| 21. (3,4) | 22. (3) | 23. (8) | 24. (1,2,3,4) | 25. (9) | 26. (5) | 27. (1) |
| 28. (7) | 29. (4) | 30. (6) | 31. (C) | 32. (C) | 33. (C) | 34. (D) |
| 35. (C) | 36. (D) | 37. (B) | 38. (A) | 39. (D) | 40. (C) | |

PART- III (MATHEMATICS)

- | | | | | | |
|---------|---------------|-----------|-------------|---------|---------------|
| 41. 0 | 42. 0,1,2,3,4 | 43. 3,5,7 | 44. 5,7,9 | 45. 0 | 46. 0,2,4,6,8 |
| 47. 1 | 48. 1,2,5 | 49. 0,1,2 | 50. 6,7,8,9 | 51. (D) | 52. (B) |
| 53. (B) | 54. (A) | 55. (B) | 56. (D) | 57. (A) | 58. (B) |
| 59. (A) | 60. (B) | | | | |

DATE : 14-05-2017
ANSWER KEY
CODE-2
PAPER-1
PART- I (PHYSICS)

- | | | | | | | |
|-----------|-----------|-----------|-----------|-----------|----------|----------|
| 1. (ABC) | 2. (BCD) | 3. (AC) | 4. (BC) | 5. (AD) | 6. (AD) | 7. (BC) |
| 8. (AC) | 9. (BCD) | 10. (AC) | 11. (ABD) | 12. (ABC) | 13. (BC) | 14. (AD) |
| 15. (ACD) | 16. (ABC) | 17. (BCD) | 18. (AC) | 19. (AD) | 20. (AC) | |

PART- II (CHEMISTRY)

- | | | | | | | |
|------------|------------|------------|------------|------------|-----------|-----------|
| 21. (ABC) | 22. (ABCD) | 23. (ABCD) | 24. (BCD) | 25. (ABCD) | 26. (ACD) | 27. (ACD) |
| 28. (BC) | 29. (AC) | 30. (ABD) | 31. (AB) | 32. (AB) | 33. (ABC) | 34. (AB) |
| 35. (ABCD) | 36. (ABCD) | 37. (AB) | 38. (ABCD) | 39. (AC) | 40. (AB) | |

PART- III (MATHEMATICS)

- | | | | | | | |
|------------|------------|-----------|-----------|------------|-----------|-----------|
| 41. (ABCD) | 42. (ABCD) | 43. (ABC) | 44. (ACD) | 45. (ABD) | 46. (BCD) | 47. (BCD) |
| 48. (ACD) | 49. (AB) | 50. (ABD) | 51. (ABD) | 52. (ABCD) | 53. (CD) | 54. (BC) |
| 55. (ABC) | 56. (ABD) | 57. (ABC) | 58. (ABC) | 59. (C) | 60. (AB) | |

PAPER-2
PART- I (PHYSICS)

- | | | | | | | |
|-----------|------------|-----------|---------|---------|---------|-----------|
| 1. (2) | 2. (3,6,9) | 3. (4, 8) | 4. (4) | 5. (3) | 6. (4) | 7. (1, 4) |
| 8. (4, 8) | 9. (5) | 10. (7) | 11. (C) | 12. (C) | 13. (A) | 14. (C) |
| 15. (A) | 16. (B) | 17. (C) | 18. (B) | 19. (B) | 20. (C) | |

PART- II (CHEMISTRY)

- | | | | | | | |
|-----------|---------|---------|---------------|---------|---------|---------|
| 21. (3,4) | 22. (3) | 23. (8) | 24. (1,2,3,4) | 25. (9) | 26. (5) | 27. (1) |
| 28. (7) | 29. (4) | 30. (6) | 31. (A) | 32. (B) | 33. (B) | 34. (D) |
| 35. (A) | 36. (A) | 37. (A) | 38. (B) | 39. (C) | 40. (B) | |

PART- III (MATHEMATICS)

- | | | | | | |
|---------|---------------|-----------|-------------|---------|---------------|
| 41. 0 | 42. 0,1,2,3,4 | 43. 3,5,7 | 44. 5,7,9 | 45. 0 | 46. 0,2,4,6,8 |
| 47. 1 | 48. 1,2,5 | 49. 0,1,2 | 50. 6,7,8,9 | 51. (D) | 52. (B) |
| 53. (B) | 54. (A) | 55. (B) | 56. (D) | 57. (A) | 58. (B) |
| 59. (A) | 60. (B) | | | | |

DATE : 14-05-2017
ANSWER KEY
CODE-3
PAPER-1
PART- I (PHYSICS)

- | | | | | | | |
|-----------|-----------|-----------|-----------|-----------|----------|----------|
| 1. (ACD) | 2. (ABD) | 3. (AC) | 4. (AD) | 5. (BC) | 6. (BC) | 7. (AD) |
| 8. (AC) | 9. (ABD) | 10. (AC) | 11. (BCD) | 12. (ACD) | 13. (AD) | 14. (BC) |
| 15. (ABC) | 16. (ACD) | 17. (ABD) | 18. (AC) | 19. (BC) | 20. (AC) | |

PART- II (CHEMISTRY)

- | | | | | | | |
|------------|------------|------------|------------|------------|-----------|-----------|
| 21. (ABC) | 22. (ABCD) | 23. (ABCD) | 24. (ACD) | 25. (ABCD) | 26. (ABD) | 27. (BCD) |
| 28. (BD) | 29. (AB) | 30. (ABC) | 31. (CD) | 32. (AD) | 33. (ABD) | 34. (AC) |
| 35. (ABCD) | 36. (ABCD) | 37. (AC) | 38. (ABCD) | 39. (BC) | 40. (AC) | |

PART- III (MATHEMATICS)

- | | | | | | | |
|------------|------------|-----------|-----------|------------|-----------|-----------|
| 41. (ABCD) | 42. (ABCD) | 43. (ABD) | 44. (BCD) | 45. (ABC) | 46. (ACD) | 47. (ACD) |
| 48. (ABD) | 49. (AC) | 50. (BCD) | 51. (ABC) | 52. (ABCD) | 53. (BC) | 54. (BD) |
| 55. (ACD) | 56. (ABC) | 57. (ABC) | 58. (ABD) | 59. (D) | 60. (AC) | |

PAPER-2
PART- I (PHYSICS)

- | | | | | | | |
|-----------|------------|-----------|---------|---------|---------|-----------|
| 1. (2) | 2. (3,6,9) | 3. (4, 8) | 4. (4) | 5. (3) | 6. (4) | 7. (1, 4) |
| 8. (4, 8) | 9. (5) | 10. (7) | 11. (A) | 12. (A) | 13. (C) | 14. (A) |
| 15. (B) | 16. (C) | 17. (A) | 18. (D) | 19. (D) | 20. (A) | |

PART- II (CHEMISTRY)

- | | | | | | | |
|-----------|---------|---------|---------------|---------|---------|---------|
| 21. (3,4) | 22. (3) | 23. (8) | 24. (1,2,3,4) | 25. (9) | 26. (5) | 27. (1) |
| 28. (7) | 29. (4) | 30. (6) | 31. (C) | 32. (C) | 33. (C) | 34. (D) |
| 35. (C) | 36. (D) | 37. (B) | 38. (A) | 39. (D) | 40. (C) | |

PART- III (MATHEMATICS)

- | | | | | | |
|---------|---------------|-----------|-------------|---------|---------------|
| 41. 0 | 42. 0,1,2,3,4 | 43. 3,5,7 | 44. 5,7,9 | 45. 0 | 46. 0,2,4,6,8 |
| 47. 1 | 48. 1,2,5 | 49. 0,1,2 | 50. 6,7,8,9 | 51. (D) | 52. (B) |
| 53. (B) | 54. (A) | 55. (B) | 56. (D) | 57. (A) | 58. (B) |
| 59. (A) | 60. (B) | | | | |