HINTS & SOLUTIONS

PART-A : PHYSICS

1. If $\tan\theta > \mu$ .....

Sol. $F\cos\phi + \mu N = mg\sin\theta$

$N = mg\cos\theta + F\sin\phi$

$F = \dfrac{mg\sin\theta - \mu mg\cos\theta}{\mu\sin\phi + \cos\phi}$

2. An uniform .....

Sol. $I_c = \dfrac{2}{3}mR^2$

$I_{cm} = \dfrac{5}{12}mR^2$

When ω is maximum

$mg\dfrac{R}{2} = \dfrac{1}{2}\left(\dfrac{5}{12}mR^2\right)\omega^2$

$\omega^2\dfrac{R}{2} = \dfrac{6g}{5}$

$N_{max} = mg + m.\ \dfrac{6g}{5}$

$N_{max} = \dfrac{11mg}{5}$

3. A steel ball .....

Sol. $D_S - D_H = 0.05$ mm

Where D_S is diameter of ball and D_H is diameter of ball hole.

As we increase temperature

$D_H > D_S$ for ball to enter hole

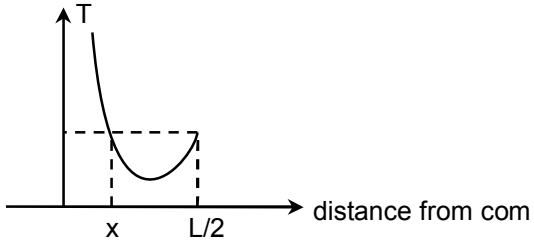
$D_H(1 + 12\times10^{-6}\Delta T) - D_S\left(1 + \dfrac{3.2}{3}\times10^{-6}\Delta T\right) > 0$

$\left(12\times10^{-4} - \dfrac{3.2}{3}\times10^{-4}\right)\Delta T > 0.05$

Gives $\Delta T > 46°$

4. A rod of .....

Sol. From graph of T of rod before x there will be only two points on the either side of centre of mass about which T is same



$x = \dfrac{K^2}{L/2} = \dfrac{L^2/12}{L/2} = \dfrac{L}{6}$

distance from one end should be lie between $\dfrac{L}{3}$ & $\dfrac{2L}{3}$

5. A student .....

Sol. For damped oscillation $\omega = \sqrt{\dfrac{K}{m} - \dfrac{b_1^2}{4m^2}}$

$\omega = \sqrt{\dfrac{4500}{90} - \dfrac{180\times180}{4\times90\times90}} = \sqrt{50-1} = 7$

Time period = $\dfrac{2\pi}{7}$ sec.

6. A sonometer .....

Sol. Frequency of sonometer wire $f = \dfrac{2}{2\times31.25\times10^{-2}}\sqrt{\dfrac{256}{10^{-2}}}$

$= \dfrac{2\times16\times1000}{2\times31.25} = 512$Hz

So possible frequency of tuning for k is 502 and 522Hz.

When frequency of wire is increased by increasing tension, beats are not heard.

Which may be due to

→ Both frequency becomes equal (f = 522 Hz)

→ Beat frequency is more than the value which can be heard (16 beats) (f = 502 Hz)

So both answers are possible. Only one is given in options.

7. A travelling wave .....

Sol. $y = 2\sin\pi\left(t - \dfrac{x}{2} + \dfrac{1}{3}\right)$

$\dfrac{dy}{dx} = 2\cos\left(\pi t - \dfrac{\pi x}{2} + \dfrac{\pi}{3}\right)\times\left(-\dfrac{\pi}{2}\right)$

Equation of wave can be given as

$y = 0.2 \sin\left(\pi t - \frac{\pi}{2}x + \frac{\pi}{3}\right)$

Slope = $\frac{\partial y}{\partial x} = -0.1 \times \pi \cos\left(\pi t - \frac{\pi}{2}x + \frac{\pi}{3}\right)$

$\frac{\partial y}{\partial x} = -\frac{\pi}{10}\cos\left[\pi - \frac{3\pi}{2} + \frac{\pi}{3}\right]$

$= -\frac{\sqrt{3}\pi}{20}$

8. A man can .....

Sol. Speed of river from bank to mid stream

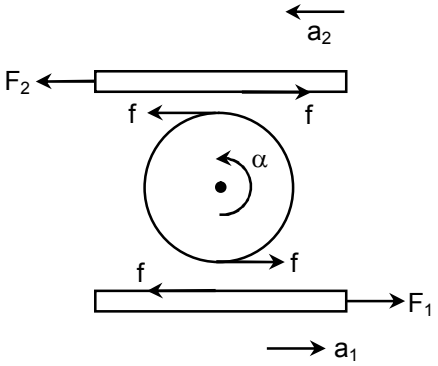
$v = \frac{v_0}{d/2}x = \frac{2v_0x}{d}$

$\int d(\text{drift}) = 2\int_0^{d/2} \frac{dx}{u} \times \frac{2V_0x}{d}$

drift = $\frac{V_0 d}{2u}$

9. A uniform .....

Sol. Since sphere is at rest both friction at top and bottom will be equal as acceleration of sphere is zero. Acceleration of both planks



$a_1 = a_2 = \alpha R = a$ ∴ No slipping

$F_1 - f = ma$ .....(1)

$F_2 - f = 2ma$ .....(2)

$2fR = \frac{2}{5}mR^2\alpha$

$\Rightarrow \quad 2f = \frac{2}{5}$ ma .....(3)

adding equation (1), (2) and (3)

$F_1 + F_2 = \left(3 + \frac{2}{5}\right)$ ma = $\frac{17}{5}$ ma

x = 7 **Ans.**

10. Air offers .....

Sol. Maximum height = $\frac{u^2}{2(g+a)} = \frac{96 \times 96}{2 \times 16} = 288\text{m}$

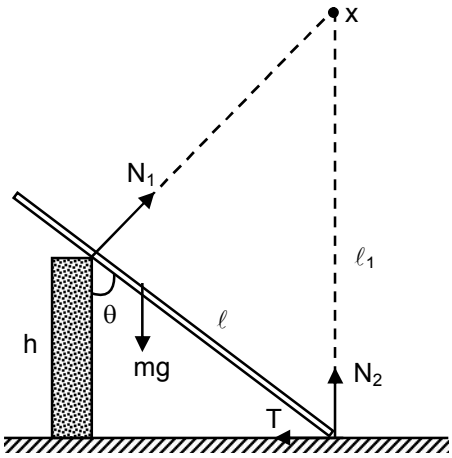
Time of flight = $\sqrt{\frac{2H}{g+a}} + \sqrt{\frac{2H}{g-a}}$

= $\sqrt{\frac{2 \times 288}{16}} + \sqrt{\frac{2 \times 288}{4}}$

= 6 + 12 = 18s

11. A uniform .....

Sol.



Torque about X will be zero

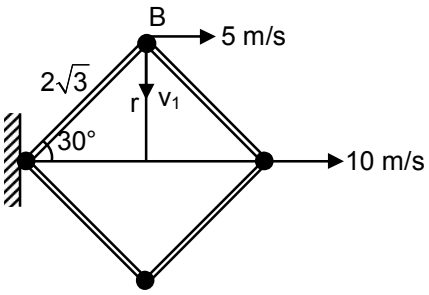
T × ℓ_1 = mg $\frac{\ell}{2}$ sinθ

T × $\frac{h}{\cos^2\theta} = mg\frac{\ell}{2}\sin\theta$

T = $\frac{mg\ell\sin\theta\cos^2\theta}{2h}$

12. A hinged .....

Sol.



speed of D = $\sqrt{5^2 + v_1^2 + (\omega r)^2}$

$v_1 = 5\sqrt{3}$ [Constraint motion]

$\omega r = 4\sqrt{2} \times \sqrt{3} = 4\sqrt{6}$

speed = 14 m/s

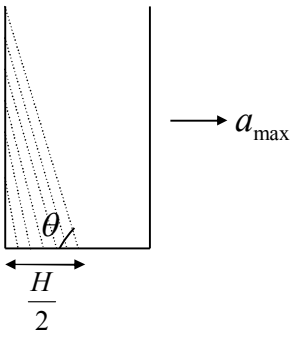
13. In adiabatic .....

Sol. $\frac{4}{28}\left(\frac{7R}{2}\right)(3T_0 - T) = \frac{(0.8)}{4}\left(\frac{5R}{2}\right)(T - T_0)$

$T = 2T_0$

14. An open .....

Sol. Volume of liquid inside container is $\frac{H^3}{4}$ .when container is moved with maximum possible acceleration.



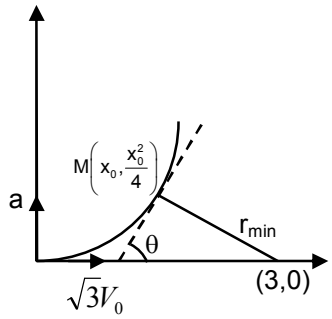
$$\tan\theta = \frac{a}{g} = 2 \qquad \Rightarrow a = 2g$$

Force due to left wall is

$$\frac{(0+\rho gh)}{2}.H^2 = \frac{\rho g H^3}{2} = 2mg$$

15. Two particles .....

Sol. motion of P with respect to Q is parabolic



x = $\sqrt{3}V_0 t$

y = $\frac{5}{2}t^2$

y = $\frac{x^2}{4}$

M is foot of perpendicular drawn on path of P with Q.

Let M is $\left(x_0, \frac{x_0^2}{4}\right)$

$$\frac{-2}{x_0} = \frac{\frac{x_0^2}{4} - 0}{(x_0 - 3)}$$

$$x_0 = 2$$

So M is (2, 1)

So QM is $\sqrt{2}$ meter

$$2 = \sqrt{3} \times \sqrt{\frac{10}{3}}t$$

t = $\frac{2}{\sqrt{10}}$sec.

$V_x = \sqrt{3}V_0 = \sqrt{10}$ m/s

$V_y = 5t \Rightarrow$ At t = 2s.

$V_y = \sqrt{10}$ m/s

Speed = $\sqrt{10+10} = \sqrt{20}$ m/s

16. A uniform rod.....

Sol. $\Delta\ell = 0$

17. A transverse .....

Sol. Wave speed, $V_0 = \sqrt{\frac{T}{\mu}} = 10 m/s$

Particle speed, $V = V_0.\frac{\partial y}{\partial x} = V_0 \tan 45^\circ = 10 m/s$

18. Two disks .....

Sol. If impulse of friction till slipping is I then

$-IR_A = I_A\omega_A = I_A \times 5$

$+IR_B = I_B\omega_B - 0$

For no slipping $\omega_A R_A = \omega_B R_B$

Solving this we get ω_B = 8 rad/s

19. In a resonance .....

Sol. $e = \frac{\ell_3 - 3\ell_1}{2}$ = 2.0 cm

de = $\frac{d\ell_2 + 3d\ell_1}{2}$

% error = $\frac{d\ell_2 + 3d\ell_1}{2 \times e} \times 100$

= $\frac{0.1 + 3 \times 0.1}{2 \times 2.0} \times 100$ = 10%

20. Two particle .....

Sol. $\frac{R_A}{R_B} = \frac{\cos(\theta - \phi)}{\cos(\theta + \phi)} = \frac{\frac{4}{5} \times \frac{\sqrt{3}}{2} + \frac{3}{5} \times \frac{1}{2}}{\frac{4}{5} \times \frac{\sqrt{3}}{2} - \frac{3}{5} \times \frac{1}{2}} = \frac{4\sqrt{3}+3}{4\sqrt{3}-3}$

= $\frac{4+\sqrt{3}}{4-\sqrt{3}}$

21. In an refrigerator.....

Sol. Efficiency of ideal engine

$\eta = 1 - \frac{270}{300} = \frac{1}{10}$

Efficiency of refrigerator = $\frac{1}{20}$

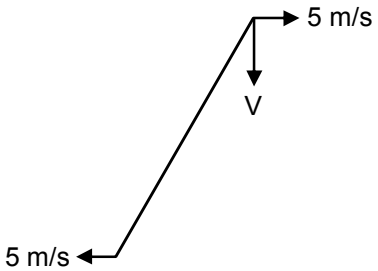
$\frac{1}{20} = \frac{1\,kJ}{Q} \Rightarrow Q_1 = 20$ kJ

22. Equation motion .....

Sol. Value of function is not repeated in a fixed intervel.

23. Point B of .....

Sol.



Constrained motion along rod

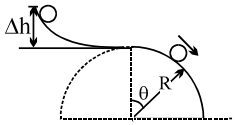
$\frac{4v}{5} - 3 = 3 \Rightarrow V = \frac{15}{2}$ m/s

Speed = $\sqrt{\left(\frac{15}{2}\right)^2 + (5)^2}$

24. A skier plans.....

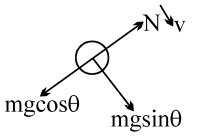
Sol. $\Delta h = \frac{R}{4} + R(1 - \cos\theta)$

$\frac{1}{2}mv^2 = mg\Delta h = \frac{mgR}{4}\{1 + 4(1 - \cos\theta)\}$



$\therefore \quad \frac{mv^2}{R} = \frac{mg}{2}(5 - 4\cos\theta)$

$mg\cos\theta - N = \frac{mv^2}{R}$



$mg\cos\theta = \frac{mg}{2}(5 - 4\cos\theta)$

$\cos\theta = 5/6$

25. The work .....

Sol. $(10 \times 11 - 10 \times 6) \times 10^{-4} \times 2T = \Delta w$

$50 \times 10^{-4} \times 2T = 3 \times 10^{-4}$

$\Rightarrow \quad T = \frac{3}{100} = 3 \times 10^{-2}$ N/m

26. A trolley is.....

Sol. Velocity of bullet with respect to ground

$= 40\hat{i} + 40\hat{j} + 50\hat{k}$

Time of fight = $\frac{2 \times 50}{10}$ = 10 sec.

Range of centre of mass

$= \frac{2 \times 40\sqrt{2} \times 50}{10} = 400\sqrt{2}$ m

coordinate of centre of mass after 10 sec = (500, 400, 0)

one of the particle will fall on trolley

Position of trolley at the end of 10 sec.

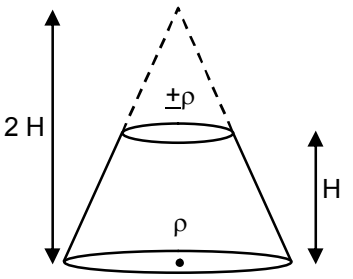
= (200, 0, 0)

Position of other particle = (800, 800, 0)

distance = $800\sqrt{2}$ m $\Rightarrow$ M = 8

27. A frustum of.....

Sol.



$y_{cm} = \frac{\rho \times \frac{1}{3}\pi(R)^2 \times 2H \times \frac{2H}{4} - \rho \times \frac{1}{3}\pi\frac{R^2}{4}H\frac{5H}{4}}{\rho \times \left(\frac{1}{3}\pi(R)^2 \times 2H - \frac{1}{3}\pi\frac{R^2}{4}H\right)}$

$= \frac{11H}{28} \Rightarrow x = 7$

28. A car starts .....

Sol. Acc = $\frac{1}{3}m/s^2 = \frac{60 \times 60}{3}m/min^2$ = 1800 m/min^2

initially velocity 18 km/h = $\frac{18000}{60} = 300m/min$

$4800 = 300 + \frac{1800}{2}(2n - 1)$

$\frac{2 \times 4500}{1800} = 2n - 1$

$\Rightarrow n = 3$

29. A composite .....

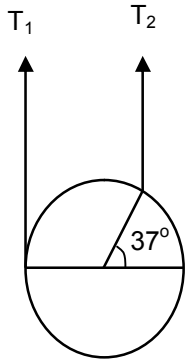
Sol. $y_{cm} = \dfrac{\frac{L}{2}\cdot A\times\frac{3\rho}{2}\cdot\frac{L}{4}+\frac{L}{2}A\cdot\frac{\rho}{4}\cdot\frac{3L}{4}}{\frac{L}{2}A\cdot\frac{3\rho}{2}+\frac{L}{2}A\cdot\frac{\rho}{4}} = \frac{9L}{28}$

$\Rightarrow$ height immersed = $2\times\frac{9L}{28}=\frac{9L}{14}$

$\Rightarrow$ x = 9

30. An uniform .....

Sol.



$T_1\ \ell = T_2\left(\frac{4\ell}{5}\right)$

$5\ T_1 = 4\ T_2$

$K = \frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}}\cdot\frac{\ell_2}{\ell_1}$

$= \sqrt{\frac{4}{5}}\cdot\frac{7}{10}$

$= \frac{7}{5\sqrt{5}}$

PART-B : CHEMISTRY

31. The Poisson's ratio .....

Sol. $\frac{C_P}{C_V} = 1.4 = \gamma$

$C_P - C_V = R$

$\gamma C_V - C_V = R$

or $C_V = \frac{R}{(\gamma-1)}$ and $C_P - \frac{C_P}{\gamma} = R$ or $C_P = \frac{R\gamma}{\gamma-1}$

$C_V = \frac{2}{0.4} = 5$ cal/mol K

Also c_V or $s_V = \frac{2}{0.4\times32} = 0.15$ cal/mol K

32. The electrons identified .....

Sol. (a) 5 p (b) 5 s (c) 4 d (d) 4 p

Acc. to (n + ℓ) rule, increasing order of energy

(d) < (b) < (c) < (a)

33. Dissolving 180 g of glucose.....

Sol. Mole = $\frac{180}{180}$ = 1 mol

mass of solution = 1180 g

$V = \frac{1180}{1.15\times1000} = \frac{118}{115}$ L

$M = \frac{1\times115}{118} = 0.97$ M

34. How many litres of water.....

Sol. pH = 1 $\Rightarrow$ $[H^+] = 10^{-1} = 0.1$ M

pH = 2 $\Rightarrow$ $[H^+] = 10^{-2} = 0.01$ M

for dilution of HCl $M_1V_1 = M_2V_2$

$0.1\times2 = 0.01\times V_2$

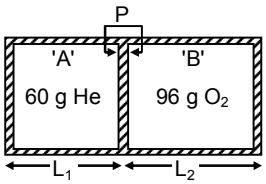
$V_2 = 20$ L

Volume of water added = 20 – 2 = 18 litre

35. Molar volume of an.....

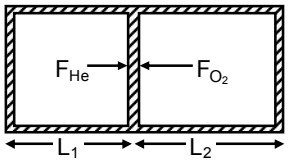
Sol. $Z = \frac{0.9}{0.45} = 2$ Z > 1

36.



.....

Sol.



At equilibrium, $F_{He} = F_{O_2}$

$P_{He}\times A = P_{O_2}\times A$

$\frac{n_{He}RT}{V_{He}}\times A = \frac{n_{O_2}RT\times A}{V_{O_2}}$

$\frac{n_{He}}{L_1} = \frac{n_{O_2}}{L_2}$

$\therefore\ \frac{L_1}{L_2} = \frac{n_{He}}{n_{O_2}} = \frac{60/4}{96/32} = \frac{15}{3} = 5$

37. For a real gas …….

Sol. A real gas behaves ideally at Boyle's temperature in low pressure range.

$$Z = \frac{PV_M}{RT} = 1$$

$$P = \frac{RT}{V_M} = \frac{R\frac{a}{Rb}}{V_M} = \frac{a}{V_M.b}$$

38. When 1 mole of strong……

Sol. $H_2A + 2NaOH \longrightarrow 2NaA + 2H_2O \quad \Delta H = E$
(SA) (SB)

$$\underset{(WA)}{HB} + NaOH \longrightarrow NaB + H_2O \quad \Delta H < \frac{E}{2}$$

HB is monoprotic & weak acid, so few amount of energy is required for its dissociation.

39. For a reversible reaction …….

Sol. For reaction $\Delta n < 0$ so high pressure is favoured for forward reaction.
Reaction is endothermic so high temperature favours forward reaction.

40. Enthalpy of atomisation…….

Sol.  in producing 1 mole C(g), $\frac{1}{2} \times 4 = 2$ C–C bonds are broken.

$\Rightarrow 2 \times \in_{c-c} = 600$; $\in_{c-c} = 300$ kJ/mol.

41. $A(s) \rightleftharpoons 2B(g) + C(g)$ …….

Sol.

	$A(s) \rightleftharpoons 2B(g) + C(g)$	
eq.1	2p	p
eq.2	p	p'

$P_{T_1} = 3p$

$K_p = 4p^3$

$(p)^2 p' = 4p^3$

$p' = 4p$

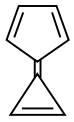
$P_{T_2} = p + 4p = 5p$

$$\frac{P_{T_2}}{P_{T_1}} = \frac{5p}{3p} = 5/3$$

42. Determine the pH.....

Sol. $pH = 4.7 + \log \frac{0.2}{0.1} = 5$

43. Which of the following ……

Sol.  has aromatic character.

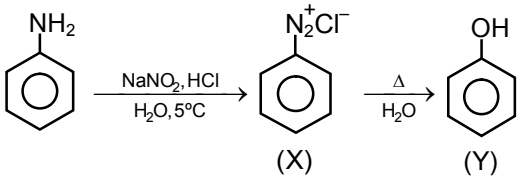
44. When borax is …….

Sol. $[B_4O_5(OH)_4]^{2-} + 5H_2O \rightleftharpoons 2B(OH)_3 + 2[B(OH)_4]^-$
or
$[B_4O_7]^{2-} + 7H_2O \longrightarrow 2B(OH)_3 + 2[B(OH)_4]^-$

45. Correct order of bond…….

Sol. Bond order of $O_2^{++} = 3$
Bond order of $O_2^{+} = 2.5$
Bond order of $O_2 = 2$
Order of bond strength = $O_2^{++} > O_2^{+} > O_2$

46. Identify the X and Y…….

Sol. 

(X) (Y)

47. Which of the following…….

Sol. Fact

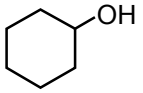
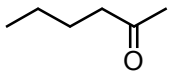
48. Which of the following…….

Sol. (1) The double chain silicates can be drawn in which two simple chains are joined together by shared oxygen. Such compounds are also known as amphiboles.
(2) If two oxygen atoms per tetrahedron are shared to form closed rings such that the structure with general formula $(SiO_3^{2-})_n$ or $(SiO_3)_n^{2n-}$ is obtained, the silicates containing these anions are called cyclic silicates.
(3) Orthosilicates contain discrete $[SiO_4]^{4-}$ units i.e., there is no sharing of corners with one another.

49. What is the product of …….

Sol. $CH_3–\underset{\underset{CH_3}{|}}{C}=CH_2 \xrightarrow[\text{peroxide}]{HBr} CH_3–\underset{\underset{CH_3}{|}}{CH}–CH_2–Br$
(Anti Markovnikoff's product)

51. Which of the following …….

Sol.  and  functional isomers.

52. A solid compound 'X' on…….

Sol. The given compound X must be $CaCO_3$. It can be explained by following reactions,

$$\underset{(X)}{CaCO_3} \xrightarrow{\Delta} \underset{\text{Residue}}{CaO} + CO_2\uparrow ; CaO + H_2O \longrightarrow \underset{(Y)}{Ca(OH)_2}$$

$$Ca(OH)_2 + CO_2 + H_2O \longrightarrow \underset{Z}{Ca(HCO_3)}$$

$$Ca(HCO_3)_2 \xrightarrow{\Delta} \underset{(X)}{CaCO_3} + CO_2\uparrow + H_2O$$

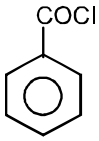
53. The d-orbitals which are …….

Sol. In ICl_4^-, I has sp^3d hybridisation.

54. Which alcohol gives …….

Sol. 3° alcohol gives instant turbidity with Lucas reagent.

55. The IUPAC name of …….

Sol.  is known as benzene carbonyl chloride..

56. The degencracy of 1st …….

Sol. For H, n = 2 is first excited state. we have 2s, $2p_x$, $2p_y$ and $2p_z$.

57. How many of the …….

Sol. False statements are 1,7 and 8, remaining are true.

58. What is the ratio of …….

Sol. $Mg(OH)_2 \rightleftharpoons Mg^{2+} + 2OH^-$

x 2x + 3y

$Al(OH)_3 \rightleftharpoons Al^{3+} + 3OH^-$

y 3y + 2x

Since K_{sp} of $Mg(OH)_2$ > K_{sp} of $Al(OH)_3$

$\therefore$ x >> y $\therefore$ 2x + 3y $\simeq$ 2x

$4 \times 10^{-12} = [Mg^{2+}][OH^-]^2$

$= x \times (2x)^2$

$\therefore$ $x = 10^{-4}$

Similarly $1 \times 10^{-33} = [Al^{3+}][OH^-]^3$

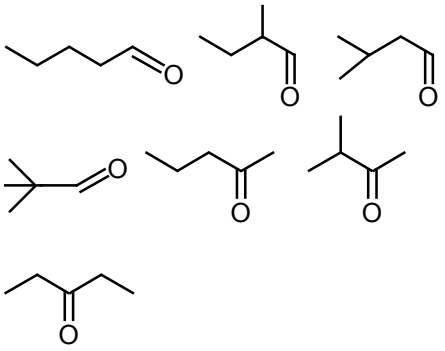
$1 \times 10^{-33} = y \times (2x)^3$

$\therefore$ $y = \frac{10^{-21}}{8}$

Thus $\frac{x}{y} = 8 \times 10^{17}$

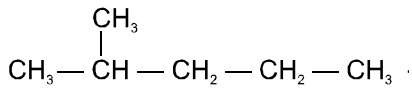
$\therefore$ Ans. $= 8 \times 10^{+17} \times 10^{-17} = 8$

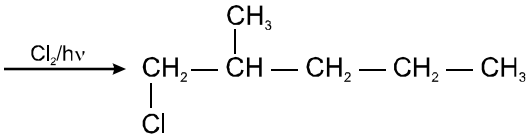
59. How many carbonyls …….

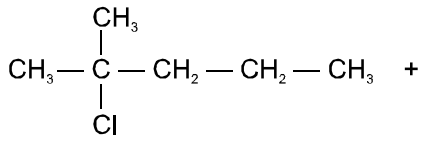
Sol. 

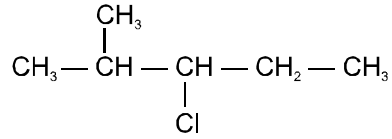
60. How many monochloro…….

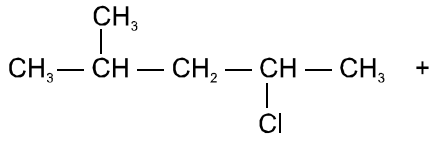
Sol.

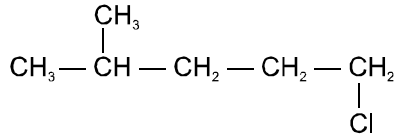












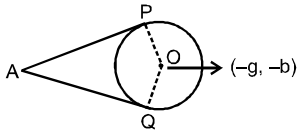
PART-C : MATHEMATICS

61. Sum of the series.....

Sol. $\sum(-1)^r\ {}^{10}C_r\left(\frac{1}{3}\right)^r + \sum(-1)^r\ {}^{10}C_r\left(\frac{8}{9}\right)^r$

$= \left(1-\frac{1}{3}\right)^n + \left(1-\frac{8}{9}\right)^n = \left(\frac{2}{3}\right)^n + \left(\frac{1}{9}\right)^n = \frac{6^n+1}{3^{2n}}$

62. A point A(2, 1) is .....

Sol. 

AO will be diameter of circle (x – 2)(x + g)+(y – 1)(y + f) = 0

63. The range of.....

Sol. Point (α, α) will lie is shaded regin

it $\alpha^2 + \alpha^2 - 1 < 0$

$\Rightarrow 2a^2 - 1 < 0$

$\Rightarrow -\frac{1}{2} < \alpha < \frac{1}{\sqrt{2}}$ ... (1)

and $\alpha + \alpha - 1 \Rightarrow \alpha > \frac{1}{2}$ ... (2)

$\therefore$ common solution of (1) and (2) is:

$$\frac{1}{2} < \alpha < \frac{1}{\sqrt{2}}$$

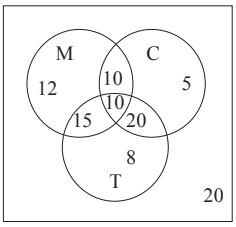
64. In a cricket .....

Sol. All the eleven players can be arranged in 11! ways.
But as from the order of three players is fixed

$\therefore$ number of ways = $\frac{11!}{3!} = {}^{11}C_3 \cdot 8!$

65. An Investigator.....

Sol.



66. The joint equation.....

Sol : For joint equation

$$x^2 - 3y^2 - 4x + 6\sqrt{3}y - 5 = 0$$

$$\left(x - \sqrt{3}y + C_1\right)\left(x + \sqrt{3}y + C_2\right) = 0$$

So equations of straight line are

$$y = \frac{x}{\sqrt{3}} + \frac{1}{\sqrt{3}} \text{ \& } y = -\frac{x}{\sqrt{3}} + \frac{5}{\sqrt{3}}$$

Then altitude will also make angle $\frac{\pi}{3}$ with each line so

equation is $x - 2 = 0$

67. Equation of the .....

Sol. Let equation of hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\frac{9}{a^2} - \frac{1}{b^2} = 1$$

$a^2 e^2 = a^2 + b^2$

$3a^2 = a^2 + b^2$

Solving

$a^2 = \frac{17}{2}$, $b^2 = 17$

68. If $\Delta = a^2 - (b-c)^2$, ...

Sol. $\Delta = a^2 - (b-c)^2 \Rightarrow b^2 + c^2 - a^2 = 2bc - \Delta$

$2bc \cos A = 2bc - \Delta \quad \left(\because \cos A = \frac{b^2 + c^2 - a^2}{2bc}\right)$

$= 2bc - \frac{1}{2} bc \sin A$

$\Rightarrow 4 \cos A + \sin A = 4 \Rightarrow \tan \frac{A}{2} = \frac{1}{4} \quad \therefore \tan A = \frac{8}{15}$

69. If the expansion .....

Sol. $2\,{}^{m}C_{r+1} = {}^{m}C_r + {}^{m}C_{r+2}$

$\Rightarrow 4\,{}^{m}C_{r+1} = {}^{m+1}C_{r+1} + {}^{m+1}C_{r+2} = {}^{m+2}C_{r+2}$

$$4\frac{m!}{(r+1)!(m-r-1)!} = \frac{(m+2)!}{(r+2)!(m-r)!}$$

$$\Rightarrow 4 = \frac{(m+2)(m+1)}{(r+2)(m-r)}$$

$4r^2 + r(8 - 4m) + m^2 - 5m + 2 = 0$

$$r = \frac{(m-2) \pm \sqrt{m+2}}{2}$$

$\Rightarrow m = 7$ gives $r = 4$ and $m = 14$ gives $r = 8$

70. Number of four .....

Sol. Number are of form

$\therefore \quad x + y + z = 7 \; ; \; 0 \le x, y\, z \le 7$

Number of ways = ${}^{9}C_2$ (9 identical balls in 3 different boxes, empty allowed)

= 36

71. If $ax^2 + bx + c = 0$.....

Sol. $5x^2 + 6x + 12 = 0$

$D = 36 - 4 \times 60 = -204 < 0$

$\Rightarrow$ roots of $5x^2 + 6x + 12 = 0$ are imaginary

$\therefore$ both the roots are common

$\therefore \frac{a}{5} = \frac{b}{6} = \frac{c}{12} = k$

$\Rightarrow a = 5k, b = 6k, c = 12k$

since $a + b < c$

$\therefore$ triangle is not formed

72. If $(x-5)^{x^2-5x+6} = 1$, ...

Sol. $(x-5)^{x^2-5x+6} = 1 \Rightarrow (x-5)(x-3)(x-2) = 1$

$\therefore$ x = 2, 3 are the solutions

$x - 5 = 1 \Rightarrow \quad x = 6$

$x - 5 = -1 \Rightarrow \quad x = 4$

$\therefore$ all the solutions are 2, 3, 4, 6

73. If $a > b > 0$ are .....

Sol. Let

$$y = \sqrt{a\,b + (a-b)\sqrt{a\,b + (a-b)\sqrt{a\,b + (a-b)\sqrt{a\,b + ...}}}}$$

$$y = \sqrt{ab + (a-b)y}$$

$y^2 = ab + (a - b)\, y$

$y^2 - (a - b)\, y - ab = 0$

$$y = \frac{(a-b) \pm \sqrt{(a-b)^2 + 4x1xab}}{2}$$

$y = \dfrac{(a-b) \pm (a+b)}{2}$

$y = a, -b$

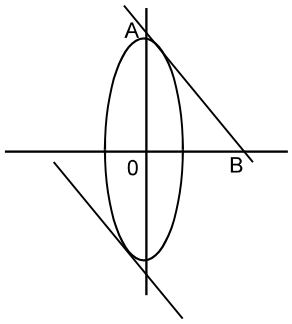
$y = -b$ (not possible)

$\therefore$ $y = a$

$\therefore$ Independent of b

74. Distance between.....

Sol.



$y = mx \pm \sqrt{a^2m^2 + b^2}$

$y = -\dfrac{4}{3}x \pm \sqrt{18 \times \dfrac{16}{9} + 32}$

$y = -\dfrac{4}{3}x \pm 8$

Distance between tangents $= \dfrac{16}{\sqrt{1+\dfrac{16}{9}}} = \dfrac{16 \times 3}{5} = \dfrac{48}{5}$

75. $-1 < \cos\left(2x - \dfrac{\pi}{3}\right) < \dfrac{1}{2}$ .....

Sol. $\dfrac{\pi}{3} < 2x - \dfrac{\pi}{3} < \pi \cup \pi < 2x - \dfrac{\pi}{3} < \dfrac{5\pi}{3}$

$\dfrac{2\pi}{3} < 2x < \dfrac{4\pi}{3} \cup \dfrac{4\pi}{3} < 2x < 2\pi$

$\dfrac{\pi}{3} < x < \dfrac{2\pi}{3} \cup \dfrac{2\pi}{3} < x < \pi$

general solution

$x \in \left(\dfrac{\pi}{3} + n\pi, \dfrac{2\pi}{3} + n\pi\right) \cup \left(\dfrac{2\pi}{3} + n\pi, (n+1)\pi\right), n \in I$

76. The series of .....

Sol. Number of elements in the n^{th} group is $(2n - 1)$

$\therefore$ Number of elements in the 20^{th} group is 39

$\therefore$ $\ell = 39$

77. Eccentricity of .....

Sol. $\dfrac{1}{3} = \dfrac{1}{2}(1 - e^2)$

$e^2 = 1 - \dfrac{2}{3}$; $e = \dfrac{1}{\sqrt{3}}$

78. If $S_n = 1 + \dfrac{1}{2} +$ .....

Sol. $S_n = \dfrac{\left(1 - \left(\dfrac{1}{2}\right)^n\right)}{1 - \dfrac{1}{2}}$

$S_n = 2 - \left(\dfrac{1}{2}\right)^{n-1} \Rightarrow \left(\dfrac{1}{2}\right)^{n-1} < \dfrac{1}{100}$

$\Rightarrow$ $n - 1 \geq 7$ $\Rightarrow$ $n \geq 8$

79. If a, b, c and d .....

Sol. $1 + a \geq 2\sqrt{a}$, $1 + b \geq 2\sqrt{b}$, $1 + c \geq 2\sqrt{c}$, $1 + d \geq 2\sqrt{d}$

$\therefore$ $(1 + a)(1 + b)(1 + c)(1 + d) \geq 16\sqrt{abcd} = 16$

$\therefore$ minimum value of $(1 + a)(1 + b)(1 + c)(1 + d)$ is 16

80. If a,b,c denote, ...

Sol. $c^2 = a^2 + b^2 \Rightarrow \angle C = \dfrac{\pi}{2}$

$\therefore$ $\Delta = \dfrac{1}{2}ab \sin C = \dfrac{1}{2}ab$

$\Rightarrow$ $\sqrt{s(s-a)(s-b)(s-c)} = \dfrac{ab}{2}$

$\Rightarrow$ $4s(s - a)(s - b)(s - c) = a^2b^2$.

81. The mean and .....

Sol. Standard deviation is independent of origin shifting

82. The number of .....

Sol. $\left(\dfrac{x^3 + x + x^4 + 1}{x^2}\right)^{15} = \dfrac{a_0 + a_1x + a_2x^2 + ...+ a_{60}x^{60}}{x^{30}}$

83. If $(1 + x)(1 + x^2)$ .....

Sol. Degree of x on L.H.S is 1 + 2 + 4 +.....+ 128 = 255

84. A ray of light .....

Sol. Focus (0, 2). The point of intersection of curve and y = 4 is (0, 4). From the reflection property of reflected ray passes through focus.

85. If tan x + tan y = 25.....

Sol. tan x + tan y = 25

cot x + cot y = 30 i.e. $\dfrac{25}{\tan x \tan y} = 30$

$\therefore$ tanx tan y = $\dfrac{25}{30} = \dfrac{5}{6}$

$\therefore$ tan (x + y) = $\dfrac{\tan x + \tan y}{1 - \tan x \tan y} = \dfrac{25}{1 - \dfrac{5}{6}} = 150$

86. If (1 – cot 1º) . .....

Sol. (1 – cot 1º) (1 – cot 44º)

= 1 – cot 1° – cot 44° + cot 1° cot 44° ..... (i)

Also 1 = cot (1° + 44°) = $\frac{\cot 1^\circ \cot 44^\circ - 1}{\cot 1^\circ + \cot 44^\circ}$

$\Rightarrow$ cot 1° + cot 44° = cot 1° cot 44° – 1

$\Rightarrow$ 1 = cot 1° cot 44° – cot 1° – cot 44° ..... (ii)

from (i) and (ii), we get

$\therefore$ (1 – cot 1°) (1– cot 44°) = 2

similarly (1 – cot 2°) (1 – cot 43°) = 2,

(1 – cot 22°) (1 – cot 23°) = 2

$\therefore$ (1 – cot1º)(1 – cot2º)(1 – cot3º) ... (1 – cot44º) = 2^{22}

$\therefore$ n = 22

87. A circle touches .....

Sol. Equation of a circle touching the line y = x at (2, 2) is

$(x-2)^2 + (y-2)^2 + \lambda(x-y) = 0$

i.e. $x^2 + y^2 - (4-\lambda)x - (4+\lambda)y + 8 = 0$

its centre $\left(\frac{4-\lambda}{2}, \frac{4+\lambda}{2}\right)$ lies on y-axis.

$\therefore$ $\frac{4-\lambda}{2} = 0$ i.e. $\lambda = 4$

$\therefore$ the circle is $x^2 + y^2 - 8y + 8 = 0$

$\therefore$ radius = $\sqrt{16-8} = \sqrt{8}$

88. Out of 25 consecutive.....

Sol. $\lambda = {}^{12}C_2 + {}^{13}C_2 = 144$

89. If α, β are the .....

Sol. $4x^2 - 15x + \lambda = 0$

f(1) > 0 and f(2) < 0 and f(3) > 0

f(1) = 4 – 15 + λ > 0

λ > 11

f(2) = 16 – 30 + λ < 0

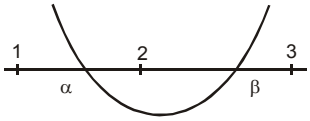
λ < 14

f(3) = 36 – 45 + λ > 0

λ > 9

11 < λ < 14

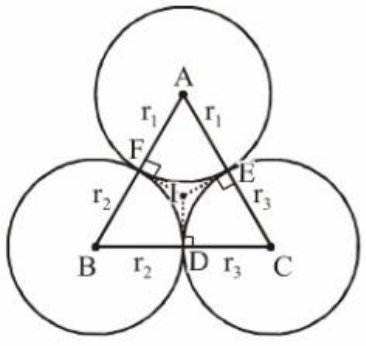
So λ = 12,13 are 2 integral solutions.



90. Three circles .....

Sol: $a = r_2 + r_3$, $b = r_3 + r_1$, $c = r_1 + r_2$

We have given ID = IE = IF = 2



$2 = \frac{\text{Area of } \Delta ABC}{\text{semi perimeter of } \Delta ABC}$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{(r_1+r_2+r_3)r_1r_2r_3}$

$2 = \frac{\sqrt{r_1r_2r_3(r_1+r_2+r_3)}}{(r_1+r_2+r_3)} = \sqrt{\frac{r_1r_2r_3}{r_1+r_2+r_3}}$

$\Rightarrow \frac{r_1r_2r_3}{r_1+r_2+r_3} = 4$

Resonance®
Educating for better tomorrow

MAJOR TEST-3 (MT-3)
(JEE MAIN PATTERN)
TARGET : JEE (MAIN+ADVANCED) 2018

DATE : 05-01-2018

COURSE : VIJETA (01JP)

ANSWER KEY

CODE-0

PHYSICS

1.	(2)	**2.**	(3)	**3.**	(4)	**4.**	(3)	**5.**	(1)	**6.**	(1)	**7.**	(3)
8.	(3)	**9.**	(3)	**10.**	(3)	**11.**	(3)	**12.**	(4)	**13.**	(1)	**14.**	(4)
15.	(3)	**16.**	(1)	**17.**	(1)	**18.**	(1)	**19.**	(4)	**20.**	(4)	**21.**	(4)
22.	(3)	**23.**	(4)	**24.**	(3)	**25.**	(1)	**26.**	(8)	**27.**	(7)	**28.**	(3)
29.	(9)	**30.**	(7)										

CHEMISTRY

31.	(2)	**32.**	(2)	**33.**	(3)	**34.**	(4)	**35.**	(2)	**36.**	(3)	**37.**	(3)
38.	(2)	**39.**	(3)	**40.**	(2)	**41.**	(3)	**42.**	(2)	**43.**	(4)	**44.**	(3)
45.	(1)	**46.**	(3)	**47.**	(3)	**48.**	(4)	**49.**	(1)	**50.**	(2)	**51.**	(1)
52.	(4)	**53.**	(4)	**54.**	(3)	**55.**	(3)	**56.**	(4)	**57.**	(5)	**58.**	(8)
59.	(7)	**60.**	(5)										

MATHEMATICS

61.	(4)	**62.**	(1)	**63.**	(1)	**64.**	(3)	**65.**	(4)	**66.**	(3)	**67.**	(4)
68.	(2)	**69.**	(2)	**70.**	(1)	**71.**	(4)	**72.**	(3)	**73.**	(1)	**74.**	(3)
75.	(4)	**76.**	(3)	**77.**	(2)	**78.**	(1)	**79.**	(3)	**80.**	(4)	**81.**	(2)
82.	(2)	**83.**	(2)	**84.**	(1)	**85.**	(2)	**86.**	(2)	**87.**	(8)	**88.**	(4)
89.	(2)	**90.**	(4)								