P10-17

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ADVANCED PATTERN PART TEST-4(APT-4)

TARGET : JEE (MAIN+ADVANCED) 2018

SUBJECT : MATHEMATICS COURSE : VIJAY (01JR)

Date : 14-01-2018 Time: 2 Hours Maximum Marks : 168

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

GENERAL :

1. The sealed booklet is your Question Paper. Do not break the seal till you are instructed to do so.
2. The question paper CODE is printed on the right hand top corner of this sheet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the question.
4. Blank spaces are provided within this booklet for rough work.
5. Write your Name and Roll Number in the space provided on the below cover.
6. After the open booklet, verify that the booklet contains all the **40** questions along with the options are legible.

QUESTION PAPER FORMAT AND MARKING SCHEME :

7. This questions paper consists of **four sections.**
8. Each section as detailed in the following table :

Section	Question Type	Number of Questions	Category-wise Marks for Each Question				Maximum Marks of the Section
			Full Marks	Partial Marks	Zero Marks	Negative Marks	
1	One or More Correct Option(s)	14	+4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened	+1 For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened	0 If none of the bubbles is darkened	–2 In all other cases	56
2	Comprehension (One or More Correct Option(s))	6	+4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened	+1 For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened	0 If none of the bubbles is darkened	–2 In all other cases	24
3	Match the Column	2	For each entry in Column-I +2 If only the bubble(s) corresponding to all the correct match(es) is(are) darkened	–	0 if not attempted	–1 In all other cases	16
4	Single digit Integer (0-9)	18	+4 If only the bubbles corresponding to the correct answer is darkened	–	0 if not attempted	–1 In all other cases	72

OPTICAL RESPONSE SHEET :

9. Darken the appropriate bubbles on the original by applying sufficient pressure.
10. The original is machine-gradable and will be collected by the invigilator at the end of the examination.
11. Don not tamper with or mutilate the ORS.
12. Write your name, roll number and the name of the examination centre and sign with pen in the space provided for this purpose on the original.
    Do not write any of these details anywhere else. Darken the appropriate bubble under each digit of your roll number.

DARKENING THE BUBBLES ON THE ORS :

13. Use a **BLACK BALL POINT** to darken the bubbles in the upper sheet.
14. Darken the bubble **COMPLETELY.**
15. Darken the bubble **ONLY** if you are sure of the answer.
16. The correct way of darkening a bubble is as shown here : ●
17. There is **NO** way to erase or "un-darkened bubble.
18. The marking scheme given at the beginning of each section gives details of how darkened and **not darkened** bubbles are evaluated.

DO NOT BREAK THE SEAL WITHOUT BEING INSTRUCTED TO DO SO BY THE INVIGILATOR

NAME OF THE CANDIDATE : ……

ROLL NO. : ……

I have read all the instructions and shall abide by them

- -

Signature of the Candidate

I have verified the identity, name and roll number of the candidate.

- -

Signature of the Invigilator

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MATHEMATICS

SECTION – 1 : (Maximum Marks : 56)

- This section contains **FOURTEEN** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories :

Full Marks	:	+4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding to **each correct option,** provided NO incorrect option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	–2	In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks ; darkening only (A) and (D) will result in +2 marks and darkening (A) and (B) will result in –2 marks, as a wrong option is also darkened.

1. The equation of normal to the curve $x + y = x^y$, where it cuts the x-axis is equal to

(A) $x + y - 1 = 0$ (B) $y = 3x - 3$ (C) $y = x - 1$ (D) $2y = x - 1$

2. $\int \frac{\cos x - \sin x}{7 - 9\sin 2x} dx =$ (c being an arbitrary constant)

(A) $\frac{1}{12} \ell n \left| \frac{4 + 3(\sin x + \cos x)}{4 - 3(\sin x + \cos x)} \right| + C$ (B) $\frac{1}{24} \ell n \left| \frac{4 + 3(\sin x + \cos x)}{4 - 3(\sin x + \cos x)} \right| + C$

(C) $C - \frac{1}{24} \ell n \left| \frac{4 - 3(\sin x + \cos x)}{4 + 3(\sin x + \cos x)} \right|$ (D) $C - \frac{1}{12} \ell n \left| \frac{4 - 3(\sin x + \cos x)}{4 + 3(\sin x + \cos x)} \right|$

Space for Rough Work

3. $I_1 = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$, $I_2 = \int_0^{\pi} \frac{x^3 \sin x}{(\pi^2 - 3\pi x + 3x^2)(1+\cos^2 x)} dx$, then

(A) $I_1 = \frac{\pi^2}{8}$ (B) $I_1 = \frac{\pi^2}{4}$ (C) $I_1 = I_2$ (D) $I_1 > I_2$

4. If $\vec{a}$ and $\vec{b}$ are two vectors and angle between $\vec{a}$ and $\vec{b}$ is θ, then

(A) $\left|\vec{a} \times \vec{b}\right|^2 + \left(\vec{a}.\vec{b}\right)^2 = |\vec{a}|^2 |\vec{b}|^2$

(B) $\left|\vec{a} \times \vec{b}\right| = \left(\vec{a}.\vec{b}\right)$, if $\theta = \frac{\pi}{4}$

(C) $\vec{a} \times \vec{b} = \left(\vec{a}.\vec{b}\right)\hat{n}$, ($\hat{n}$ is normal unit vector), if $\theta = \frac{\pi}{4}$

(D) $\vec{a} \times \vec{b}.\left(\vec{a}+\vec{b}\right) = 0$

5. Solution of the differential equation $\frac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = \frac{x \sin^2(x^2+y^2)}{y^3}$

(A) $-\cot(x^2+y^2) = \left(\frac{x}{y}\right)^2 + c$ (B) $\frac{y^2}{x^2+y^2c} = -\tan^2(x^2+y^2)$

(C) $-\cot(x^2+y^2) = \left(\frac{y}{x}\right)^2 + c$ (D) $\frac{y^2+x^2c}{x^2} = -\tan^2(x^2+y^2)$

Space for Rough Work

6. Let f(x) be differentiable function on the interval $(-\infty, \infty)$ such that f(1) = 5 and $\lim_{a \to x} \frac{af(x) - xf(a)}{a - x} = 2$, $\forall\ x \in R$. Then which of the following alternative(s) is/are correct ?

(A) f(x) has an inflection point

(B) $f'(x) = 3\ \forall\ x \in R$

(C) $\int_0^2 f(x)\,dx = 10$

(D) Area bounded by f(x) with co-ordinate axes is $\frac{2}{3}$

7. If $\int \frac{(x-1)\,dx}{x^2\sqrt{2x^2 - 2x + 1}} = \frac{\sqrt{f(x)}}{g(x)} + C$, then

(A) $f(x) = 2x^2 - 2x + 1$ (B) $g(x) = x + 1$

(C) $g(x) = x$ (D) $f(x) = \sqrt{2x^2 - 2x}$

8. Let $f(x) = \begin{cases} x\sin\left(\frac{\pi}{x}\right) & \text{at } x \neq 0 \\ 0 & \text{at } x = 0 \end{cases}$

(A) $f'(x)$ vanishes atleast once in $\left[\frac{1}{3}, \frac{1}{2}\right]$

(B) $f'(x)$ vanishes atleast once in $\left[\frac{1}{4}, \frac{1}{2}\right]$

(C) $f'(x)$ vanishes atleast once in $\left[\frac{1}{k+1}, \frac{1}{k}\right]$ for every $K \in N$

(D) $f'(x)$ satisfies Rolle's theorem on [0 , 1]

Space for Rough Work

9. For the function $f(x) = ||x|^2 - 2|x| - 3|$

(A) The number of points where f(x) is not differentiable is 3

(B) The number of points where f(x) attains local minimum values is 3

(C) The number of points where f(x) attains local maximum values is 3

(D) The number of points where f(x) attains global minimum values is 2

10. Let $I = \int \frac{e^x}{e^{4x}+1} dx$ and $J = \int \frac{e^{-x}}{e^{-4x}+1} dx$ which of the following are correct ?

(c being an arbitrary constant)

(A) $J + I = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{e^{2x}-1}{\sqrt{2}e^x}\right) + c$

(B) $J - I = \frac{1}{2\sqrt{2}} \ell n \left(\frac{e^{2x} - \sqrt{2}e^x + 1}{e^{2x} + \sqrt{2}e^x + 1}\right) + c$

(C) $I = \frac{1}{2\sqrt{2}} \left(\tan^{-1}\left(\frac{e^{2x}-1}{\sqrt{2}e^x}\right) - \frac{1}{2}\ell n\left(\frac{e^{2x} - \sqrt{2}e^x + 1}{e^{2x} + \sqrt{2}e^x + 1}\right)\right) + c$

(D) None of these

11. The value of $\int \frac{\cos^3 x}{\sin^2 x + \sin x} dx$ is equal to :

(A) $\ell n \,|\sin x| + \sin x + C$

(B) $\ell n \,|\sin x| - \sin x + C$

(C) $-\ell n \,|\text{cosec } x| - \sin x + C$

(D) $-\ell n \,|\sin x| + \sin x + C$

Space for Rough Work

12. Let $F(x) = f(x) - f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{\ln t}{1+t+t^2}\,dt$, $\forall\ x \geq 1$. Then

(A) $F(e) = 1$ (B) $F(e) = 0$ (C) $F(\ell nx) = e$ (D) $F(\ell nx) = 0$

13. $\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{\cos^{-1}\left(\frac{2x}{x^2+1}\right)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)}{e^x+1}dx$ is equal to

(A) $\int_{-\pi/6}^{\pi/6} \frac{\sec^2 x}{e^{\tan x}+1}\,dx$ (B) $\int_0^{\pi/6} \sec^2 x\,dx$

(C) $\frac{\pi}{2}\int_{-\pi/6}^{\pi/6} \frac{\sec^2 x\,dx}{e^{\tan x}+1}$ (D) $\frac{\pi}{2\sqrt{3}}$

14. For the function $f(x) = x\cos\frac{1}{x}$, $x \geq 1$,

(A) for at least one x in the interval $[1, \infty)$, $f(x+2) - f(x) < 2$

(B) $\lim_{x\to\infty} f'(x) = 1$

(C) for all x in the interval $[1, \infty)$, $f(x+2) - f(x) > 2$

(D) $f'(x)$ is strictly decreasing in the interval $[1, \infty)$

Space for Rough Work

SECTION – 2 : (Maximum Marks : 24)

- This section contains **THREE** paragraphs
- Based on each paragraph, there will be **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, marks will be awarded in one of the following categories :

Full Marks	:	+4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding to **each correct option,** provided NO incorrect option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	–2	In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks ; darkening only (A) and (D) will result in +2 marks and darkening (A) and (B) will result in –2 marks, as a wrong option is also darkened.

Paragraph for Question Nos. 15 to 16

If f(x) is a differentiable function such that $f(x) = \int_0^x (1+2x\ f(t))\,dt$ & f(1) = e, then

15. Identify the correct statement(s) :

(A) f(x) is an even function
(B) f(x) is an odd function
(C) f(x) is always increasing function
(D) f(x) is always decreasing function

16. $\int_0^1 f(x)\,dx =$

(A) $\frac{e-1}{2}$ (B) is less than 1 (C) $\frac{e+1}{2}$ (D) is greater than 1

Space for Rough Work

Paragraph for Question Nos. 17 to 18

$\int_0^{2a} f(x)\,dx = ka$ if

(i) $f(2a - x) = k - f(x)$ or (ii) $f(a + x) = k - f(x)$

17. Value of integral $I = \int_0^4 \ell n\left(\sqrt{(x-2)^2+4} + (x-2)\right) dx$ is

(A) $\ell n\, 2$ (B) $2\,\ell n\, 2$ (C) $4\,\ell n\, 2$ (D) 0

18. Value of integral $I = \int_0^{\pi} \cos^{-1}(\cos 2x)\, dx$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi^2}{2}$ (C) $\frac{\pi}{2}$ (D) 0

Paragraph for Question Nos. 19 to 20

Let f(x) be real valued continuous function on R defined as $f(x) = x^2 e^{-|x|}$ then

19. f(x) is increasing in

(A) (–2, 0) (B) (2, ∞) (C) (0, 2) (D) (–∞, –2)

20. For which of the following value(s) of k, $y = kx^2$ ($k > 0$) intersect the curve $y = e^{|x|}$ at atleast two points is/are

(A) $\frac{e^2}{4} < k < \infty$ (B) $\frac{e^2}{2}$ (C) $\frac{e^2}{8}$ (D) $0 < k \le \frac{4}{e^2}$

Space for Rough Work

SECTION – 3 : (Maximum Marks : 16)

- This section contains **TWO** questions
- Each question contains two columns, **Column I** and **Column II**
- **Column I** has **four** entries (A),(B), (C) and (D)
- **Column II** has **four** entries (P),(Q), (R) and (S)
- Match the entries in **Column I** with the entries in **Column II**
- One or more entries in **Column I** may match with one or more entries in **Column II**
- The ORS contains a 4 × 4 matrix whose layout will be similar to the one shown below :

(A)	(P)	(Q)	(R)	(S)
(B)	(P)	(Q)	(R)	(S)
(C)	(P)	(Q)	(R)	(S)
(D)	(P)	(Q)	(R)	(S)

- For each entry in **Column I**, darken the bubbles of all the matching entries. For example, if entry (A) in **Column I** matches with entries (P), (Q) and (R), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).

- Marking scheme :

<u>For each entry in **Column I**</u>

+2 If only the bubble(s) corresponding to all the correct match(es) is (are) darkened

0 If none of the bubbles is darkened

–1 In all other cases

Space for Rough Work

21. **Column-f** **Column-ff**

(A) $2\left|\int_0^{\pi/2}\frac{\sin 8x\,.\,\ell n(\cot x)}{\cos 2x}dx\right|$ equal to (P) 0

(B) If $\int_{-1}^{3}\left(\tan^{-1}\left(\frac{x}{x^2+1}\right)+\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)dx$ is $k\pi$, then k = (Q) 2

(C) If $\int_{n-1}^{n+1} f(x)\,dx = n^2$, $\forall\ n \in I$ then the value of $\frac{1}{4}\int_{-3}^{3} f(x)\,dx$ is (R) 1

(D) If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal then $(\mu - \lambda)$ = (S) 5

Space for Rough Work

22. **Column-f** **Column-ff**

(A) If angle between vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\ \vec{c}=\vec{0}$, is $\frac{\lambda\pi}{3}$, then λ is equal to (P) 5

(B) $\int_{-\pi/2}^{\pi/2}(\sin|x|-\cos|x|)dx =$ (Q) 2

(C) Let $\vec{a}=\hat{j}+\sqrt{3}\ \hat{k}$, $\vec{b}=-\hat{j}+\sqrt{3}\ \hat{k}$ and $\vec{c}=2\sqrt{3}\ \hat{k}$ form a triangle. If the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is $\frac{\lambda\pi}{3}$, then λ = (R) 1

(D) If $f(x)=\int_{\pi^2/16}^{x^2}\frac{\cos x\cos\sqrt{\theta}}{1+\sin^2\sqrt{\theta}}d\theta$, then the value of $\frac{5f'(\pi)}{2\pi}$ is (S) 0

Space for Rough Work

SECTION – 4 : (Maximum Marks : 72)

- This section contains **EIGHTEEN** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- Marking scheme :
  - +4 If the bubble corresponding to the answer is darkened
  - 0 If none of the bubbles is darkened
  - –1 In all other cases

23. Let y(x) satisfies the differential equation $y' - y\tan x = 2x\sec x$ and $y(0) = 0$. If $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{\sqrt{k}}$, then find the value of $\frac{k}{64}$.

24. $\int_2^4 \left(\log_x 2 - \frac{(\log_x 2)^2}{\ell n 2} \right) dx =$

25. Find the area of the region common to $x^2 + y^2 \geq 4$ and $x^2 + y^2 - 2|x| - 2|y| \leq 0$.

26. A 15 foot ladder is resting against a wall. The bottom is initially 10 ft away and is being pushed towards the wall at $\frac{1}{4}$ ft/sec. If top is moving at a rate of $\frac{n}{4\sqrt{176}}$ ft/sec after 12 sec, then n =

Space for Rough Work

27. The degree and order of the differential equation of the family of all parabolas whose axis is x-axis, are m and n respectively then the value of 3m + 2n is

28. Let $f(x) = \begin{cases} x^2 + 3x + a, & x \le 1 \\ bx + 2 & x > 1 \end{cases}$. If Lagrange's mean value theorem is applicable to f(x) in any finite closed interval then find the value of $\frac{a^2 + b^2}{17}$.

29. The area covered by the curve y = max {2 – x, 2, 1 + x} with x-axis from x = –1 to x = 1 is $\frac{\alpha}{\beta}$, then $\alpha - \beta$ = (Where α, β are coprime to each other)

30. The value of $\pi^2 \int_0^{\pi} \frac{x \sin 2x \sin\left(\frac{\pi}{2}\cos x\right)}{2x - \pi} dx$ is

31. If $f(x) = \int_2^x \frac{dt}{\sqrt{1+t^4}}$ and g be the inverse of f then the value of $\frac{(g'(0))^2}{17}$ is

Space for Rough Work

32. The general solution of the differential equation $(x^2 + y^2)dx = 2xy\,dy$ is $k|(x^m - by^2)| = |x|$, where k is arbitrary constant. The value of m + b is

33. If $\phi(x) = 3f\left(\frac{x^2}{3}\right) + f(3 - x^2)\ \forall\ x \in (-3, 4)$, where $f''(x) > 0\ \forall\ x \in (-3, 4)$, then $\phi(x)$ is increasing in $\left(-\frac{\alpha}{2}, 0\right)$ or $\left(\frac{3}{\beta}, \gamma\right)$, α, β, γ are positive then $\alpha + \beta + \gamma$ is equal to

34. If $I_1 = \int_{-1}^{1}\left(\tan^{-1}x + \tan^{-1}\frac{1}{x}\right)dx$ and $I_2 = \int_{-1}^{1}\left(\cot^{-1}x + \cot^{-1}\frac{1}{x}\right)dx$, then find the value of $\frac{I_2 - I_1}{\pi}$.

35. If f(x) is a polynomial of degree 6, which satisfies $\lim_{x \to 0}\left(1 + \frac{f(x)}{x^3}\right)^{1/x} = e^2$ and has local maximum at x = 1 and local minimum at x = 0 and x = 2, then the value of $\frac{1}{4}\left(\left(\frac{5}{9}\right)^4 f\left(\frac{18}{5}\right)\right)$ is

Space for Rough Work

36. If $\int_0^{x^2} f(t)\,dt = 3x^2 + \int_{x^2}^{1} t\,f(t)\,dt$, then f(2) =

37. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is n, then greatest prime factor of n is

38. For a certain curve y = f(x) satisfying $\frac{d^2y}{dx^2} = 6x - 4$, f(x) has a local minimum value 5 when x = 1. The global maximum value of f(x) if $0 \le x \le 2$, is

39. $\lim_{n\to\infty} 24\left(\sin\frac{\pi}{2n} \cdot \sin\frac{2\pi}{2n} \cdot \sin\frac{3\pi}{2n} \ldots\ldots \sin\frac{(n-1)\pi}{n}\right)^{1/n}$ is equal to :

40. The value of definite integral $\int_{\sqrt{2}-1}^{\sqrt{2}+1} \frac{x^2-1}{(x^2+1)^2}\,dx$ is equal to

Space for Rough Work