MAJOR TEST-4 (MT-4)
(JEE MAIN PATTERN)
TARGET : JEE (MAIN+ADVANCED) 2018

DATE : 11-01-2018 **COURSE : VIJETA (01JP)**

HINTS & SOLUTIONS

PART-A : PHYSICS

1. The time ....

Sol. $S = \frac{1}{2}at^2$

$$2R\cos\theta = \frac{1}{2}(g\cos\theta)t^2$$

$$t = \sqrt{\frac{4R}{g}} = \text{same}$$

2. The flow of ....

Sol. $t = \frac{4m}{2m/sec} = 2sec.$

$$V = \sqrt{z} = \sqrt{2t} = \sqrt{2}\times\sqrt{t}$$

$$\frac{dx}{dt} = \sqrt{2}\sqrt{t}$$

$$\int dx = \sqrt{2}\int\sqrt{t}dt$$

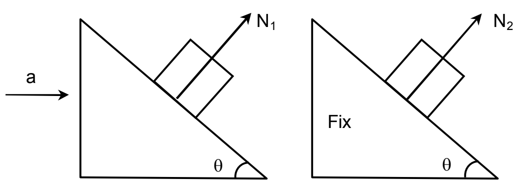
$$x = \sqrt{2}\frac{t^{3/2}}{\frac{3}{2}}$$

$$x = \frac{2}{3}\sqrt{2}\left[2^{3/2}\right]$$

$$x = \frac{2}{3}\sqrt{2}\left[2^1\times\sqrt{2}\right] = \frac{8}{3} \text{ meter}$$

3. A block is kept ....

Sol.



$N_1\cos\theta = mg$ $N_2 = mg\cos\theta$

$$\frac{N_2}{N_1} = \cos^2\theta$$

4. The switch in ....

Sol. During time 't_2' capacitor is discharging with the help of resistor 'R'

$\therefore q = q_0e^{-t/RC}$ $[\because Q = CV]$

$V = V_0e^{-t/RC}$

As $V_0 = \frac{2V}{3}; V = \frac{V}{3}$

$t_2 = RC\ \ell n2$

During time 't_1' capacitor is charging with the help of battery.

$\therefore q = q_0(1 - e^{-t/RC})$ or $V = V_0(1 - e^{-t/RC})$

as $V_0 = \frac{2V}{3}; V = \frac{V}{3}$

$t_1 = RC\ \ell n2$

$T = t_1 + t_2 = 2RC\ \ell n2$

5. In young's ....

Sol. $\frac{9D\lambda}{d} - \frac{3}{2}\frac{D\lambda}{d} = 7.5mm$

$$\frac{D\lambda}{d}[9 - 1.5] = 7.5mm$$

$$\frac{D\lambda}{d} = 1mm$$

$$\frac{1\times\lambda}{0.5\times10^{-3}} = 1\times10^{-3}$$

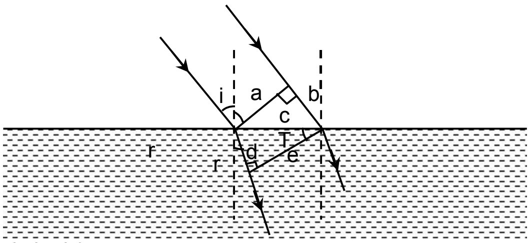
$$\lambda = 0.5\times10^{-6}$$

$$\lambda = 5\times10^{-7}$$

$\lambda = 5000$Å

6. Figure shows ....

Sol.



sin i = b/c

sin r = d/c

sin i = μsinr

μ = b/d

7. The direction ....

Sol. $\tan\theta = \frac{E_{\parallel}}{E_{\perp}} = \frac{\cos\theta_2 - \cos\theta_1}{\sin\theta_2 + \sin\theta_1}$

$= \frac{1-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

θ = 30°

8. An AC ....

Sol. We have, $Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$, thus $\omega\uparrow$, $Z\downarrow$

$\Rightarrow$ i increases

9. A rod of ....

Sol. The maximum emf will be at mean position of oscillation

$\therefore \frac{mg\ell}{3}(1-\cos\alpha) = \frac{1}{2}\left(\frac{mg\ell^2}{3}\right)\omega^2$ and $\varepsilon = \frac{1}{2}B\omega\ell^2$

10. A flexible ....

Sol. E = $\frac{d\phi}{dt}$, $\phi = B\pi r^2$ and $\frac{dr}{dt}$ constant so E is constant

11. An approximate ....

Sol. $\frac{(365\times 24\times 60\times 60)-(\pi\times 10^7)}{365\times 24\times 60\times 60}\times 100\% \cong 0.43\%$

12. In a Searle's ....

Sol. $Y = \frac{\frac{T}{A}}{\frac{x}{\ell}} = \frac{mg}{A}\left[\frac{\ell}{x}\right] = \frac{4\times 50\times 10\times 3}{10^{-5}\times 10^{-3}} = 6\times 10^{11} N/m^2$

13. A system of ....

Sol. $\mu_1\sin\theta_1 = \mu_2\sin\theta_2 \Rightarrow$ at origin x = 0, y = 0 $\Rightarrow \mu = 2$ & $\theta = 60°$

15. A particle of ....

Sol. $W_f = \Delta K$

$-\mu mg2\pi r = \frac{1}{2}m\left(\frac{v_0}{2}\right)^2 - \frac{1}{2}mv_0^2$

16. A body is ....

Sol. Torque about origin experienced by body is

$\vec{\tau} = \vec{r}\times\vec{F} = (2\hat{i}-3\hat{j})\times(3\hat{i}+2\hat{j}+6\hat{k})N-m = -18\hat{i}-12\hat{j}+13\hat{k}$

But as the body is allowed to rotated only along y-axis, the y-component of torque only causes the angular acceleration of the body.

So, $\alpha = \frac{\tau_y}{I} = \frac{-12}{10}\hat{j} = -1.2\hat{j}$ rad/s^2

17. Consider the ....

Sol. $\frac{n(n-1)}{2}$ = 15 $\Rightarrow$ n = 6

Possible transition

for z = 2 (4 $\rightarrow$ 2)

for z = 3 (6 $\rightarrow$ 3)

Element should be hydrogen like atoms so ion will be He^+

Binding energy =

$\frac{(13.6)Z^2}{n^2} = \frac{(13.6)(4)}{36} = \frac{13.6}{9} = 1.6eV$

18. The position ....

Sol. Released energy = 2 × 4 × 8 – 2 × 1 – 7 × 7 = 13 MeV

19. A coin moves ....

Sol. Friction is supportive here.

20. Three waves......................

Sol. A is transmitted via ground wave, B via sky wave and C via space wave.

21. A particle is ....

Sol. Where m = $\frac{f}{f-u}=\frac{-f}{-f+\left(\frac{f}{2}-\frac{gt^2}{2}\right)}=\frac{2f}{f+gt^2}$

$\Rightarrow V_1 = -\left(\frac{2f}{f+gt^2}\right)^2(gt)=\frac{-4f^2gt}{\left(f+gt^2\right)^2}$

For maximum speed $\frac{dV_1}{dt}=0$

$\Rightarrow t = \sqrt{\frac{f}{3g}} \Rightarrow V_{Imax} = \frac{3}{4}\sqrt{3fg}$

22. Suppose that ....

Sol. $\eta_1 = \frac{Q_1-Q_2}{Q_1}=1-\frac{Q_2}{Q_1}$

$Q_1=\frac{Q_2}{1-\eta_1}$

$\eta_2 = \frac{Q_2-Q_3}{Q_2}=1-\frac{Q_3}{Q_2}$

$Q_3 = Q_2(1-\eta_2)$

Now $\eta = \frac{Q_1-Q_3}{Q_1}=1-\frac{Q_3}{Q_1}$

$= 1-\frac{Q_2(1-\eta_2)}{Q_2}(1-\eta_1)$

23. This value of ....

Sol. $\frac{1}{\chi}=\frac{T-T_C}{C}$

$20=\frac{1160-T_C}{C}$

$10=\frac{1160-T_C}{C}$

$10=\frac{720-T_C}{C}\Rightarrow 10=\frac{440}{C}$

$\Rightarrow C = 44$

24. When a glass ....

Sol. $\frac{6}{\cos\theta}=\frac{4}{\cos 60^\circ}$

$\cos\theta=\frac{3}{4}$

25. A rod of ....

Sol. Temperature as a function of time $T=\frac{1}{10}t^\circ C$

$\frac{dQ}{dt}=kA\frac{dT}{dx}$

$\Rightarrow \int_0^Q dQ=\int_0^{(10\times60)}\left(\frac{kA}{0.4}\right)\times0.1tdt=\left.\frac{100\times1\times10^{-4}\times0.1t^2}{0.8}\right|_0^{600}=450$

26. Two tuning ....

Sol. $f_A=\left[\frac{(V-V_w)}{(V-V_w)+V_s}\right]f=\frac{330}{340}f$

$f_B=\left[\frac{(V+V_w)}{(V+V_w)-V_s}\right]f=\frac{350}{340}f$

$|f_B-f_A|=\left|\frac{20}{340}\times85\right|=5Hz$

27. Assume that ....

Sol. $m\propto v^xd^yg^z$

$[M] = [LT^{-1}]^x[ML^{-3}]^y[LT^{-2}]^z$

$\Rightarrow$ x = 6 (value of k)

28. A ring rotates ....

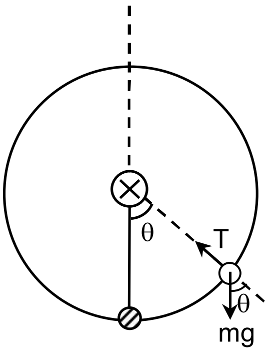
Sol. Tangential acceleration α = 6,

Centripetal accleration = ω^2R = 9 $\Rightarrow \omega$ = 3 rad/s

$\Rightarrow \frac{\alpha}{\omega}=\frac{6}{3}=2$

29. A 3kg ball is ....

Sol.



$T - mg\cos60° = \frac{mv^2}{R}$

$30 - 15 = \frac{3(v^2)}{R} \Rightarrow \frac{v^2}{R} = 5$

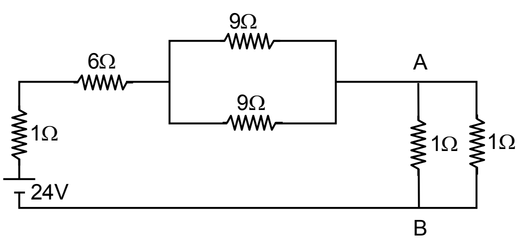
$a_t = g\sin60° = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$

$a = \sqrt{a_t^2 + a_c^2} = \sqrt{75+25} = 10 m/s^2$

$\frac{a}{2}$ **= 5 m/s² Ans.**

30. If the switches......................

Sol.



$i = \frac{24}{1+6+4.5+0.5} = \frac{24}{12} = 2A$

$V_{AB} = 2 \times 0.5 = 1V$

PART-B : CHEMISTRY

31. 0.1 molar aqueous solution …………

Sol. $\pi = iCRT$

$= 2 \times 0.1 \times \frac{1}{12} \times 300$

$= 5$ atm

$P_{ext} <$ O.P.

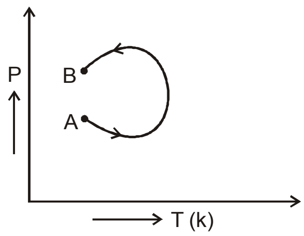
Hence net flow of solvent molecules will be from (solvent to solution)

32. The solubility of MX, MX_2 …………

Sol.

Salt	K_{sp}
MX	$S^2 = 10^{-8}$
MX_2	$4S^3 = 4 \times 10^{-12}$
MX_3	$27S^4 = 27 \times 10^{-16}$

33. …………

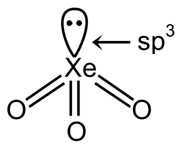


Sol. $T_A = T_B$ $P_B > P_A$ $\therefore$ $V_A > V_B$

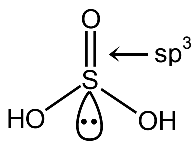
$\Rightarrow$ Volume first increases as pressure firstly decreases and temperature increases.

34. Which of the following is …………

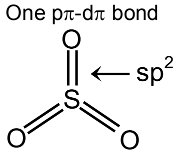
Sol.



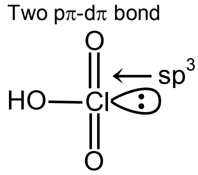
Three pπ-dπ bonds



One pπ-dπ bond



Two pπ-dπ bond



Two pπ-dπ bonds

35. The value of K_p for the …………

Sol. $$CO_2\,(g) + C(s) \rightleftharpoons 2CO(g)$$

t = 0	0.48	—	0
t = equi^m	0.48 - x	—	2x

$K_P = \dfrac{(2x)^2}{(0.48 - x)} = 3$

x = 0.33 bar

38. Select correct statement …………

Sol. $[AlCl(H_2O)_5]Cl_2$

Hybridisation $\rightarrow sp^3d^2$

Oxidation Number $\rightarrow$ +3

39. For a real gas under low …………

Sol. At low pressure, Vander waal's equation for a real gas is given as :

$Z = 1 - \dfrac{a}{RTV_m}$

intercept = 1

slope = –ve

40. Find out standard enthalpy…………

Sol. $CH_3OH + \dfrac{3}{2}O_2 \longrightarrow CO_2\,(g) + 2H_2O\,(g)\ \ \Delta H^o = -726$ kJ/mole

$\Delta H_{rxn.} = -726 = [-393 + (2 \times -286)] - [\Delta H_f\,(CH_3OH)]$

43. **Statement-1 :** In the extraction…………

Sol. Statement-2 is the correct explanation of the Statement-1.

44. In the concentration …………

Sol. $$E_{cell} = -\frac{RT}{nF}\ell n\frac{[H^+]_{anode}}{[H^+]_{cathode}}$$

$$= -\frac{RT}{nF}\ell n\frac{\dfrac{K_a[HA]_{anode}}{[NaA]_{anode}}}{\dfrac{K_a[HA]_{cathode}}{[NaA]_{cathode}}}$$

45. When H_2O_2 is added to an …………

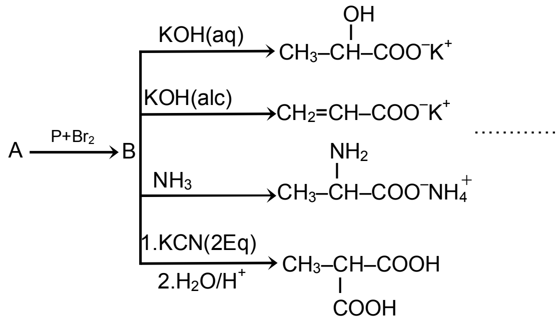
Sol. $Cr_2O_7^{2-} + 2H^+ + 4H_2O_2 \longrightarrow 2CrO(O_2)_2$ (blue coloured) + $5H_2O$.

46. Final product of following …………

Sol. Product is $CH_3 - CH = CH - CH_2 - \underset{\underset{O}{\|}}{C} - H$

47. Which of the following …………

Sol. The presence of group makes the diazo group a strong electrophile

48. 

…………

Sol: $\underset{(A)}{CH_3CH_2COOH} \xrightarrow[\text{HVZ reaction}]{P/Br_2} \underset{(B)}{CH_3-\underset{\underset{Br}{|}}{CH}-COOH}$

(B) will give above reactions

49. $CH_3 - CH_2 - CH_3 \xrightarrow[(2)\ y\ \ (3)\ z]{(1)\ x}$ …………

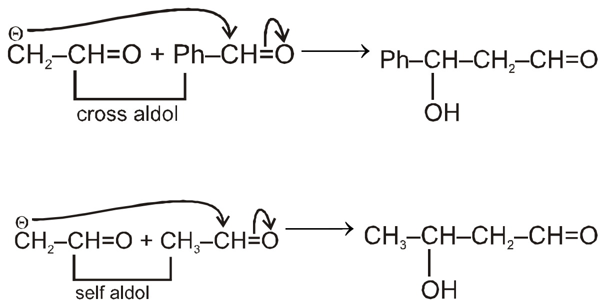
Sol. $CH_3-CH_2-CH_3 \longrightarrow CH_3 - \underset{\underset{Cl}{|}}{CH} - CH_3$

$\xrightarrow[\text{alc. KOH}/\Delta]{-HCl} CH_3-CH=CH_2$

$\xrightarrow{HBr,\ R_2O_2(AMK)} CH_3-CH_2-CH_2-Br.$

50. Identify the option which …………

Sol. $Ph-CH=O + CH_3CH=O \xrightarrow{\text{Dil NaOH}}$

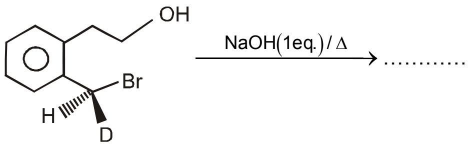


51 The labels of two bottles............

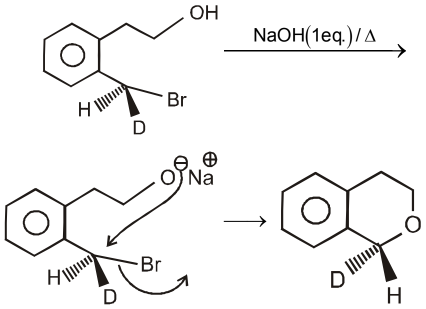
Sol. Neutral $FeCl_3$ gives positive test with I and II so I and II cannot distinguish with $FeCl_3$ solution.

$CHCl_3$ + NaOH react with both compound so the cannot distinguish.

52.

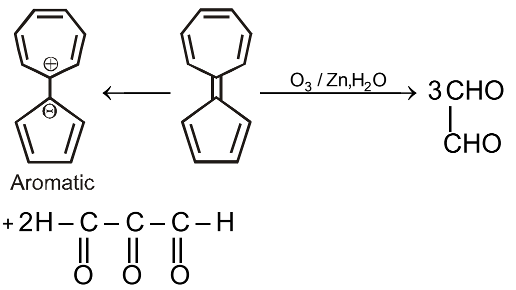


Sol.



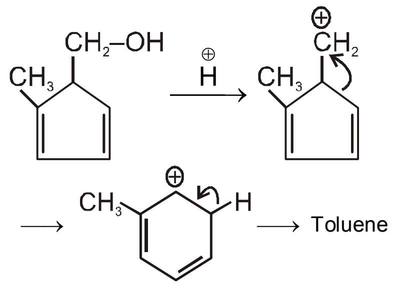
53. $C_{12}H_{10}$ is an aromatic compound ....

Sol.



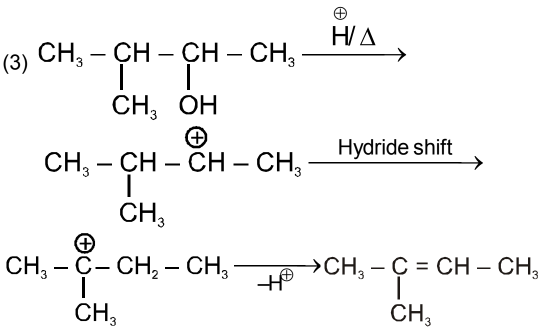
54. Which of the following reaction............

Sol. (1)



(2) It is E–2 reaction.

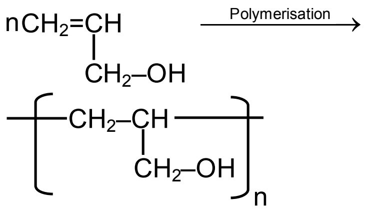
(3)



(4) No β-H so reaction is not possible.

55. Which of the following represents............

Sol.



56. 100 mL, 0.2 M CH_3COOH ....

Sol. $CH_3COOH + NaOH \longrightarrow CH_3COONa + H_2O$

20	10	
10	–	10

$$pH = pK_a + \log \frac{10}{10}$$

$$pH = pK_a$$

57. The following data were obtained ....

Sol. $SO_2Cl_2(g) \longrightarrow SO_2(g) + Cl_2(g)$

0.5	–	–
0.5-x	x	x

$P_T = 0.5 + x = 0.6$

x = 0.1

$$K = \frac{2.303}{100}\log\frac{0.5}{0.4} = \frac{2.303}{1000}$$

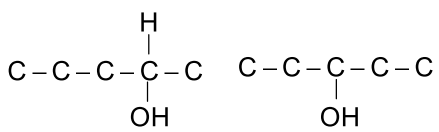
When $P_T = 0.65$

x = 0.15

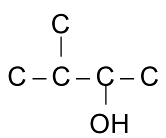
$$\text{Rate} = K\,[P_{SO_2Cl_2}] = \frac{2.303}{1000} \times 0.35 \qquad = 8.06 \times 10^{-4}$$

59. How many isomeric alcohol ....

Sol.



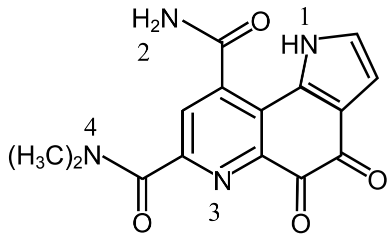
$(d + \ell)$



$(d + \ell)$

60. Write the number of that …………

Sol.



Lone pair of III is localised therefore it is most basic.

PART-C : MATHEMATICS

61. Which of the ....

Sol. (2) is not true as it contradict

De morgan's laws

62. Let R be the ....

Sol. If x = 1.2, y = 1

(1.2, 1) $\in$ S but (1, 1.2) $\notin$ S so

S is not equivalence

Clearly T is an equivalence relation as sum or difference of two integers is an integer.

63. The complete ....

Sol. $\because$ |sin x| = $\begin{cases} \sin x & , \quad x \in (0, \pi] \\ -\sin x & , \quad x \in (\pi, 2\pi) \end{cases}$

$\therefore$ If x $\in$ (0, π], sin x – sin x < 1 is always true

If x $\in$ (π, 2π), then – sin x – sin x < 1

$\sin x > \frac{-1}{2} \Rightarrow x \in \left(\frac{11\pi}{6}, 2\pi\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

Hence $x \in \left(0, \frac{7\pi}{6}\right) \cup \left(\frac{11\pi}{6}, 2\pi\right)$

64. If ax + by = 1 ....

Sol. ax + by = 1 touching $x^2 + y^2 = p^2$

$\left|\frac{0+0-1}{\sqrt{a^2+b^2}}\right| = p \Rightarrow a^2 + b^2 = \frac{1}{p^2}$

Locus of (a, b) is $x^2 + y^2 = \frac{1}{p^2}$ which is a circle of radius $\frac{1}{p}$

65. If $\int \{x(1-x^2)\}^{1/3}$ ....

Sol. $I = \int \frac{(x-x^3)^{1/3}}{x^4} dx = \int \left(\frac{1}{x^2} - 1\right)^{1/3} . \frac{1}{x^3} dx$

put $\frac{1}{x^2} - 1 = t \Rightarrow \frac{1}{x^3} dx = -\frac{dt}{2}$

$= -\frac{1}{2}\int t^{1/3}.dt = -\frac{1}{2} . \frac{t^{4/3}}{\left(\frac{4}{3}\right)} + K$

$= -\frac{3}{8}\left(\frac{1}{x^2} - 1\right)^{4/3} \Rightarrow A = -\frac{3}{8}$ B = – 2, C = $\frac{4}{3}$

66. Equation of circle......................

Sol. $\therefore$ xy – 2x – y + 2 = 0

$\Rightarrow$ x(y – 2) – (y – 2) = 0

$\therefore$ centre is (1, 2)

$\therefore$ Required circle is orthogonal to $x^2 + y^2 + 2x + 4y - 4 = 0$

$\therefore$ radius = length of tangent drawn from (1, 2) to the given circle

$= \sqrt{1+4+2+8-4} = \sqrt{11}$

$\therefore$ required circle is $(x-1)^2 + (y-2)^2 = (\sqrt{11})^2$

$\Rightarrow x^2 + y^2 - 2x - 4y - 6 = 0$

67. There are 12......................

Sol. ${}^7C_4 + {}^7C_3 \times {}^5C_1 + {}^7C_2 \times {}^5C_2 = 420$

$2. {}^7P_3 = 2 \times 7 \times 6 \times 5 = 420$

Alter: ${}^{12}C_4 - ({}^5C_4 + {}^5C_3 . {}^7C_1) = 495 - (5 + 70) = 420$

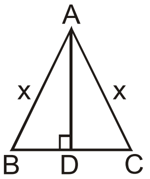
68. The first term ....

Sol. $\left|\frac{2}{1-r} - 2\right| < \frac{1}{10} \Rightarrow \left|\frac{1}{1-r} - 1\right| < \frac{1}{20} \Rightarrow$

$\left|\frac{1-1+r}{1-r}\right| < \frac{1}{20} \Rightarrow \left|\frac{r}{1-r}\right| < \frac{1}{20}$

69. The two equal ....

Sol. Let ABC is an isosceles triangle with AB = AC = x cm



given $\frac{dx}{dt}$ = 3 cm/sec also BC = 6 cm

AD $\perp$ BC so BD = DC = 3

AD = $\sqrt{x^2 - 9}$

$\Delta = \frac{1}{2}.6.\sqrt{x^2-9}$

$\frac{d\Delta}{dt} = 3\ \frac{1}{2\sqrt{x^2-9}}.2x\frac{dx}{dt}$

at x = 6

$= \frac{3}{2}.2.6.\ \frac{1}{3\sqrt{3}}.3$

$= 6\sqrt{3}$

70. In a complex ....

Sol. z lies on the circle with centre $\left(-\frac{1}{3}, 2\right)$ and radius = $\frac{10}{3}$

so maximum area of ΔPAB = $\frac{1}{2}\ .\ \frac{10}{3}.5 = \frac{25}{3}$

z_1, z_2 and centre of circle are collinear here maximum height of triangle is radius of circle.

71. For the system.......................

Sol. $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc - a^3 - b^3 - c^3$

$= -\frac{1}{2}$ (a + b + c)(a - b)^2 + (b – c)^2 + (c – a)^2

Now when p = q = r, then system is homogenous hence consistent. If a = b = c and p,q,r are distinct then system represent three parallel planes hence inconsistent

72. If $(x^2 + y^2)dy$ ....

Sol. $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$

put y = vx

are get $\frac{1+v^2}{v^3}$ dv = $-\ \frac{1}{x}$dx

$\Rightarrow \int \frac{1+v^2}{v^3}dv = -\int \frac{1}{x}dx$

$\Rightarrow -\frac{1}{2v^2} + \ell nv = -\ \ell nx + c$

$\Rightarrow -\frac{x^2}{2y^2} + \ell n\left(\frac{y}{x}\right) = \ell nx + c$

Now y(1) = 1 $\Rightarrow$ c = $-\ \frac{1}{2}$

Its given $y(x_0) = e$

$\Rightarrow\ \ x_0 = \sqrt{3}\,e$

73. If $f(x) = (e^{(x-1)})^2$ ....

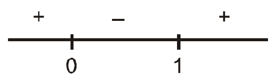
Sol. $f(x) = e^{2(x-1)}\ .\ (x-1)^2$

$f'(x) = e^{2(x-1)}\ .\ 2(x-1) + (x-1)^2 .\ e^{2(x-1)}\ .\ 2$

$\Rightarrow e^{2(x-1)}\ (\ 2x - 2 + 2x^2 + 2 - 4x\ ) = 0$

$\Rightarrow 2x^2 - 2x = 0$

x = 0, 1



here $f''(1) > 0$

Hence at x = 1 f(x) is minimum

74. The mean and ....

Sol. $\frac{1}{n}\sum_{i=1}^{n} x_i = 5\ ,\ \frac{1}{n}\sum x_i^2 - \bar{x}^2 = 0$

$\Rightarrow \frac{1}{n}\sum x_i^2 = \bar{x}^2 = 25 \Rightarrow$ 25n = 400 $\Rightarrow$ n = 16

75. If f(x) = $\int_1^x \frac{\ell n t}{1+t}dt$ ....

Sol. $f\left(\frac{1}{x}\right) = \int_1^{1/x} \frac{\ln t}{(1+t)}dt = \int_1^x \frac{-\ln t}{\frac{1+u}{u}} \times \frac{-1}{u^2}du =$

$\int_1^x \frac{\ln t}{t(1+t)}dt$

Now, $f(x) + f\left(\frac{1}{x}\right) = \int_1^x \frac{\ln t}{(1+t)}\left(1+\frac{1}{t}\right)dt = \int_1^x \frac{\ln t}{t}\,dt$

$= \left[\frac{(\ln t)^2}{2}\right]_1^x = \frac{(\ln t)^2}{2}$

76. If $(1 + x)^n = C_0$ ....

Sol. Obvious sum

$= {}^{2n}C_{n-2} = \frac{(2n)!}{(2n-n+2)!(n-2)!} = \frac{(2n)!}{(n-2)!(n+2)!}$

77. If $\vec{x}, \vec{y}, \vec{z}$ are ....

Sol. $|\vec{x} + \vec{y} + \vec{z}|^2 = 4$

$\Rightarrow \vec{x}.\vec{y} + \vec{y}.\vec{z} + \vec{z}.\vec{x} = \frac{1}{2}$ ....(1)

now $\vec{a}.\vec{x} = \frac{3}{2} \Rightarrow 1 + \vec{x}.\vec{y} + \vec{x}.\vec{z} = \frac{3}{2}$

$\vec{x}.\vec{y} + \vec{x}.\vec{z} = \frac{1}{2}$ ....(2)

Similarly $\vec{a}.\vec{y} = \frac{7}{4} \Rightarrow \vec{x}.\vec{y} + \vec{z}.\vec{y} = \frac{3}{4}$ .....(3)

from (1),(2) & (3) $\vec{x}.\vec{y} = \frac{3}{4}$, $\vec{y}.\vec{z} = 0$, $\vec{x}.\vec{z} = -\frac{1}{4}$

Hence (1), (2), (4) options are correct

Now, $\vec{z} \times (\vec{x} \times \vec{y}) = -\vec{b}$

$\Rightarrow (\vec{z} \,.\, \vec{y})\ \vec{x} - (\vec{x}.\vec{z})\ \vec{y} = -\vec{b}$

$\Rightarrow 0 + \frac{1}{4}\vec{y} = -\vec{b} \Rightarrow \vec{y} \,||\, \vec{b}$

78. $\int_0^4 \ell n(\sqrt{(x-2)^2+4} + (x-2))dx$ ....

Sol. Let I = $\int_0^4 \ell n(\sqrt{(x-2)^2+4} + (x-2))dx$

Then I = $\int_0^4 \ell n(\sqrt{(4-x-2)^2+4} + (4-x-2))dx$

$= \int_0^4 \ell n\sqrt{(x-2)^2+4} - (x-2)\,dx$

$= \int_0^4 \ell n\left(\frac{4}{\sqrt{(x-2)^2+4} + (x-2)}\right)dx$

$\Rightarrow$ I = 4ℓn 4 – I

$\therefore$ I = 2ℓn 4 = 4 ℓn 2

79. The value of 'a' for ....

Sol. $\alpha + \beta = 4a - 6$, $\alpha\beta = -4a - 4$, where α,β are roots

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

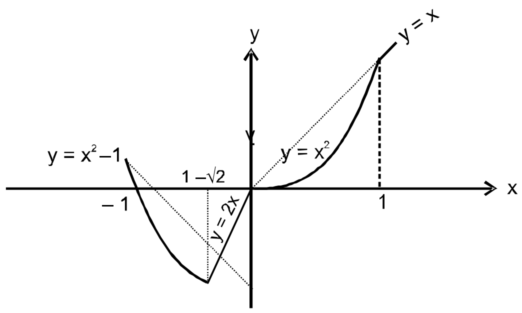
$= (4a - 6)^2 + 2(4a + 4)$

$= 16a^2 - 40a + 44$

For least value $a = \frac{-(-40)}{2(16)} = \frac{5}{4}$

80. Let f(x) ....

Sol.



From the graph

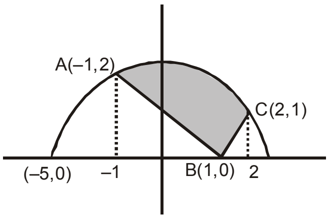
f(x) is continuous everywhere & not differentiable at

$1-\sqrt{2}$, 0, 1

i.e. exactly at three point

81. The area bounded ....

Sol. Shaded region is required area



Hence area $= \int_{-1}^{2} \sqrt{5-x^2}\,dx - \frac{1}{2}.2.2 - \frac{1}{2}.1.1$

82. A fair coin is ....

Sol. $P(H_i) = \frac{1}{2}$, $P(A_m) = \frac{{}^{10}C_m}{2^{10}}$

$P(H_i \cap A_m) = \frac{{}^{9}C_{m-1}}{2^{10}}$ for H_i and A_m to be independent

$\frac{{}^{9}C_{m-1}}{2^{10}} = \frac{1}{2} \cdot \frac{{}^{10}C_m}{2^{10}}$

m = 5

83. Let f : $\left[-1 , \; -\frac{1}{2}\right]$.......................

Sol. Let $\cos^{-1} x = \theta$

where $-1 \le x \le -\frac{1}{2}$

i.e $\frac{2\pi}{3} \le \theta \le \pi$

Then $y = 4x^2 - 3x = \cos 3\theta$

where $2\pi \le 3\theta \le 3\pi$

i.e. $y = \cos(3\theta - 2\pi)$

where $0 \le 3\theta \le -2\pi \le \pi$

$\therefore 3\theta - 2\pi = \cos^{-1} y$

i.e. $3\cos^{-1}x - 2\pi = \cos^{-1}y$

$3\cos^{-1}x = 2\pi + \cos^{-1}y$

$\cos^{-1}x = \frac{2\pi}{3} + \frac{1}{3}\cos^{-1}y$

$x = \cos\left(\frac{2\pi}{3} + \frac{1}{3}\cos^{-1} y\right)$

$\therefore f^{-1}(x) = \cos\left(\frac{2\pi}{3} + \frac{1}{3}\cos^{-1} x\right)$

84. If f(x) is a ....

Sol. Here $f(x) = \frac{x^3 - 2x^2 - x + 2}{x^2 - 4}$, $x \ne \pm 2$

$\because$ f(x) is continuous at x = 2

$\Rightarrow f(2) = \lim_{x \to 2} f(x)$

$= \lim_{x \to 2} \frac{(x^2-1)\;(x-2)}{(x+2)\;(x-2)} = \frac{3}{4}$

85. Let f : R $\to$ R ....

Sol. $f(x).f(y) - f(xy) = x + y$ ...(1)

put x = y = 1

$\Rightarrow f^2(1) - f(1) = 2$

$f^2(1) - f(1) - 2 = 0$

f(1) = 2 or –1

$\Rightarrow$ f(1) = 2 ($\because$ f(1) > 0)

Now put y = 1 in eq^n. (1)

$\Rightarrow f(1).f(x) - f(x) = x + 1$

$\Rightarrow f(x) = x + 1$

$\Rightarrow y = x + 1$

$\Rightarrow x = y - 1$

$\Rightarrow f^{-1}(x) = x - 1$

$\Rightarrow f(x).\, f^{-1}(x) = x^2 - 1 = h(x)$

$h(\sin x + \cos x) = (\sin x + \cos x)^2 - 1 = \sin 2x$

Required interval length is $\frac{\pi}{2}$.

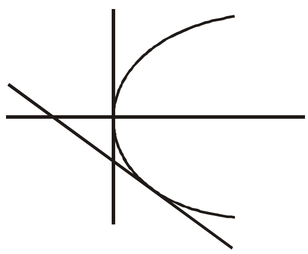
86. If x + y + 1 = 0 is ....

Sol. Solving $x^2 = \lambda y$ & $x + y + 1 = 0$

$x^2 = \lambda(-x - 1) = 0 \Rightarrow x^2 + \lambda x + \lambda = 0$

$D = 0 \Rightarrow \lambda^2 - 4\lambda = 0 \Rightarrow \lambda = 0, 4$

at $\lambda = 0 \to y^2 = 4x$, $x = 0$, $y = -x - 1$



line intersect y axis so can't be tangent

So $\lambda = 4$ only

87. For some non......................

Sol. $\vec{x}.\vec{a} = \vec{x}.\vec{b} = \vec{x}.\vec{c} = 0$

$\Rightarrow$ $\vec{x}$ is perpendicular to $\vec{a}, \vec{b}, \vec{c}$ vectors and hence $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

$[\vec{a}\ \vec{b}\ \vec{c}] = 0$

88. A circular sector......................

Sol. Let radius of sector = r and angle = θ

$\theta = \frac{\ell - 2r}{r}$

Area = $\frac{1}{2} r^2\theta = \frac{1}{2} r^2 \left(\frac{\ell - 2r}{r}\right)$

$A = \frac{1}{2}(r - 2r^2)$

$\frac{dA}{dr} = \frac{\ell - 4r}{2} = 0 \Rightarrow r = \frac{\ell}{4}$

$\frac{d^2A}{dr^2} = -2 < 0$ maxima

$\therefore A_{max} = \frac{r}{2}(4r - 2r) = r^2 = \frac{\ell^2}{16}$

Maximum area of rectangle $A'_{max} = \left(\frac{\ell}{4}\right)^2 = \frac{\ell^2}{16}$

ratio = 1

89. The value of ....

Sol. $L = \lim_{n\to\infty}\left(\frac{2}{2-\frac{1}{n^2}}\right)\frac{1}{n}.\cos\left(\frac{1+\frac{1}{n}}{2-\frac{1}{n}}\right) - \frac{1}{\left(\frac{1}{n}-2\right)} \times \frac{(-1)^n}{\left(1+\frac{1}{n^2}\right)}.\frac{1}{n}$

$$= \lim_{n\to\infty}\frac{1}{n}\left[\frac{2}{2-\frac{1}{n^2}}\cos\left(\frac{1+\frac{1}{n}}{2-\frac{1}{n}}\right) - \frac{1}{\left(\frac{1}{n}-2\right)} \times \frac{(-1)^n}{1+\frac{1}{n^2}}\right]$$

$$= 0 \times \left[\frac{2}{2}\cos\frac{1}{2} \pm \frac{1}{2} \times 1\right] = 0$$

90. The shortest ....

Sol. A(–3, 6, 0) is a point on first line

B(–2, 0, 7) is a point on second line

direction ratios of the line of shortest distances are

$$\frac{a}{3-2} = \frac{b}{-8+4} = \frac{c}{-4+12}$$

i.e. < 1, – 4 , 8 >

$\therefore$ shortest distance

$$= \frac{(-2+3) \quad . \quad 1 + (0-6)(-4) + (7-0) \quad 8}{\sqrt{1+16+64}}$$

$$= \frac{81}{9} = 9$$

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MAJOR TEST-4 (MT-4)
(JEE MAIN PATTERN)
TARGET : JEE (MAIN+ADVANCED) 2018

DATE : 11-01-2018 **COURSE : VIJETA (01JP)**

ANSWER KEY

CODE-0

PHYSICS

1.	(4)	**2.**	(2)	**3.**	(3)	**4.**	(2)	**5.**	(3)	**6.**	(3)	**7.**	(1)
8.	(3)	**9.**	(2)	**10.**	(4)	**11.**	(1)	**12.**	(1)	**13.**	(1)	**14.**	(2)
15.	(4)	**16.**	(3)	**17.**	(4)	**18.**	(2)	**19.**	(3)	**20.**	(2)	**21.**	(2)
22.	(3)	**23.**	(1)	**24.**	(1)	**25.**	(3)	**26.**	(5)	**27.**	(6)	**28.**	(2)
29.	(5)	**30.**	(1)										

CHEMISTRY

31.	(2)	**32.**	(2)	**33.**	(1)	**34.**	(3)	**35.**	(2)	**36.**	(4)	**37.**	(1)
38.	(4)	**39.**	(1)	**40.**	(1)	**41.**	(4)	**42.**	(4)	**43.**	(1)	**44.**	(1)
45.	(3)	**46.**	(4)	**47.**	(4)	**48.**	(2)	**49.**	(2)	**50.**	(1)	**51**	(2)
52.	(2)	**53.**	(2)	**54.**	(4)	**55.**	(2)	**56.**	(5)	**57.**	(8)	**58.**	(4)
59.	(5)	**60.**	(3)										

MATHEMATICS

61.	(2)	**62.**	(1)	**63.**	(3)	**64.**	(4)	**65.**	(4)	**66.**	(2)	**67.**	(4)
68.	(4)	**69.**	(4)	**70.**	(2)	**71.**	(2)	**72.**	(2)	**73.**	(2)	**74.**	(2)
75.	(2)	**76.**	(2)	**77.**	(2)	**78.**	(3)	**79.**	(2)	**80.**	(2)	**81.**	(3)
82.	(3)	**83.**	(4)	**84.**	(2)	**85.**	(2)	**86.**	(4)	**87.**	(0)	**88.**	(1)
89.	(0)	**90.**	(9)										