

DATE : 28-01-2018

 ALL INDIA OPEN TEST (AIOT)
 (JEE MAIN PATTERN)

TARGET : JEE (MAIN+ADVANCED) 2018

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HINTS & SOLUTIONS
PART-A : PHYSICS

1. A beaker

Sol. Real depth = r.i. (apparent depth)

In the first case,

$$\text{Real depth, } h_1 = \mu(b - a)$$

In the second case,

$$\text{Real depth, } h_2 = \mu(d - c)$$

 Since, $h_2 > h_1$, the difference of real depth

$$h_2 - h_1 = \mu[d - c - b + a]$$

Since, liquid is added

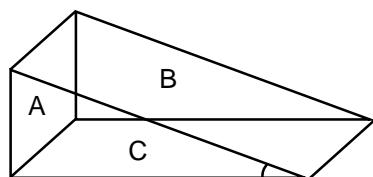
$$\therefore h_2 - h_1 = d - b$$

$$\therefore d - b = \mu(d - c - b + a)$$

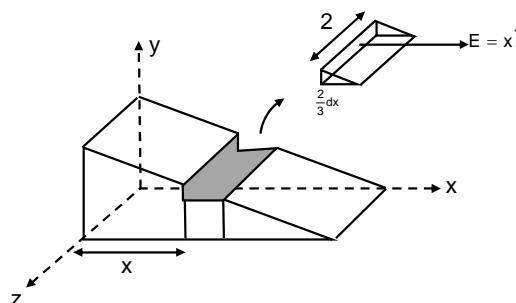
$$\therefore \mu = \frac{d - b}{d - c - b + a}$$

2. Electric field

Sol. Flux through A in zero as area is \perp to \vec{E} total flux due to

 B and C due to y component of E.F. is zero. Net flux is
 only due to x-component of E.F. due to B.


$$d\phi = x \times 2 \times \frac{2}{3} dx = \frac{4x}{3} dx$$



$$\therefore \phi = \int_0^3 \frac{4x}{3} dx = \frac{4x^2}{3} \Big|_0^3 = 6$$

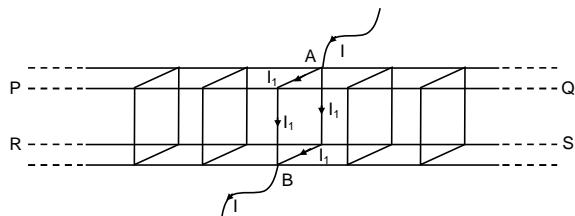
$$\phi = \frac{q}{\epsilon_0} = 6$$

$$Q = 6\epsilon_0$$

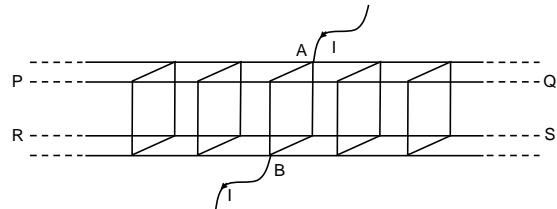
4. The vernier

$$\text{Sol. Least count} = \left(1 - \frac{49}{50}\right) 0.5^\circ = \frac{1^\circ}{100} = 0.6'$$

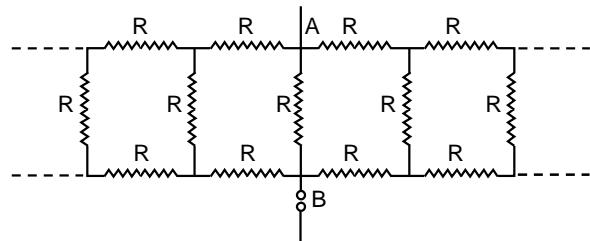
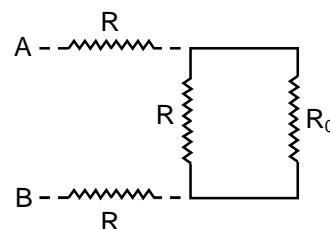
5. The network.....

Sol.


No current flows in line PQ and RS.



So, the given network is equivalent to


 Let equivalent resistance on either side of AB = R_0


$$2R + \frac{RR_0}{R + R_0} = R_0$$

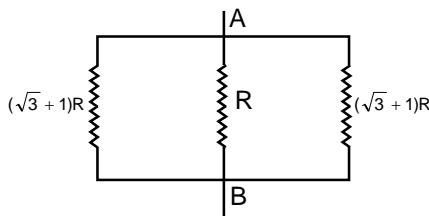
$$= R^2 + 2RR_0 + RR_0 = RR_0 + R_0^2$$

$$R_0^2 - 2RR_0 - 2R^2 = 0$$

$$R_0 = \frac{2R + \sqrt{4R^2 + 8R^2}}{2}$$

$$R_0 = \frac{2R + 2\sqrt{3}R}{2} = (\sqrt{3} + 1)R$$

So, the given network is equivalent to



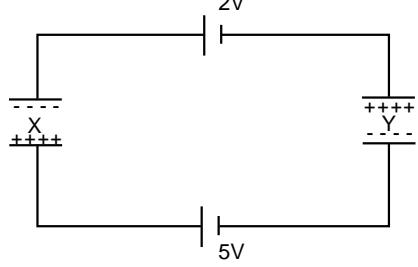
$$\therefore R_{AB} = \frac{R \times \frac{(\sqrt{3} + 1)R}{2}}{\left(1 + \frac{\sqrt{3} + 1}{2}\right)R} = \frac{(\sqrt{3} + 1)R}{(\sqrt{3} + 3)} = \frac{R}{\sqrt{3}}$$

6. Four metallic

Sol. Charge on each capacitor is

$$q = \frac{3C}{2}$$

$$\text{Electric field between plates} = \frac{q}{A\epsilon_0}$$



$$V_x + \frac{Ed}{2} - 5 + \frac{Ed}{2} = V_y$$

$$V_y - V_x = Ed - 5$$

$$= \frac{q}{\epsilon_0 A} d - 5 = \frac{q}{C} d - 5 = \frac{3C/2}{C} d - 5 = \frac{3}{2} d - 5$$

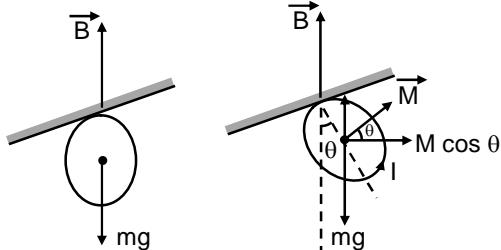
$$= -3.5 \text{ V}$$

$$W_{el} = q(V_y - V_x) = (-e)(-3.5)$$

7. A uniform

Sol. torque due to magnetic field = $MB \cos \theta$

Torque due to weight = $mgR \sin \theta$



No current

after current

In equilibrium

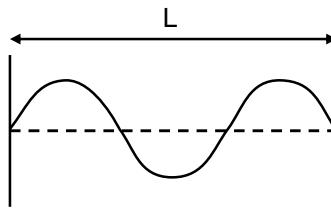
$$mgR \sin \theta = MB \cos \theta = I\pi R^2 \cos \theta B$$

$$\tan \theta = \frac{\pi IRB}{mg} \Rightarrow \theta = \tan^{-1} \left(\frac{\pi IRB}{mg} \right)$$

$$\therefore \text{maximum angular deflection} = 2 \tan^{-1} \left(\frac{\pi IRB}{mg} \right)$$

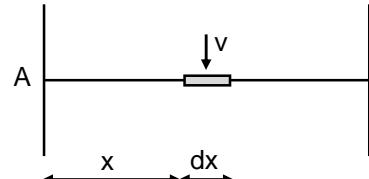
8. A standing

Sol. In 3rd harmonic



$$y = 2A \sin \left(\frac{3\pi x}{L} \right) \cos(\omega t)$$

Max. emf will be when string will become horizontal.



$$\text{Amp at position } x = 2A \sin \left(\frac{3\pi x}{L} \right)$$

$$\text{Velocity at position } x = 2A \sin \left(\frac{3\pi x}{L} \right) \omega$$

$$\therefore d\varepsilon = v B dx$$

$$= B \cdot 2\pi\omega \cdot \sin \left(\frac{3\pi x}{L} \right) dx$$

$$\text{Total emf } \varepsilon = 2A\omega B \int_0^L \sin \left(\frac{3\pi x}{L} \right) dx = \frac{4A\omega B}{k}$$

9. An A.C.

Sol. If in A.C. circuit there are only L and C, phase difference between applied voltage can be either zero or π . So, 'a' can't be the answer.

10. There is

Sol. Induced E.F. outside magnetic region

$$E = \frac{R^2}{2r} \frac{dB}{dt}$$

$$r = R_1, \frac{dB}{dt} = \beta$$

$$= \frac{R^2}{2R_1} \beta$$

$$dV = -\vec{E} \cdot d\vec{l}$$

$$\therefore \Delta V = \varepsilon = \int \varepsilon_1 \cdot d\vec{l} = \frac{R^2}{2R_1} \beta \cdot R_1 \theta$$

$$= \frac{R^2}{2} \theta \beta$$

11. When light

$$\text{Sol. } 2\mu t = \frac{(2n-1)\lambda}{2} \quad \text{for max.}$$

$$2\mu t = n\lambda \quad \text{for min.}$$

$$\text{For min. } 2 \times 1.33 \times t = (4500 \text{ \AA})^{\circ} n \\ \dots \text{(i)}$$

$$\text{For max. } 2 \times 1.33 \times t = \left(\frac{6000}{2} \text{ \AA}^{\circ} \right) (2n-1) \dots \text{(ii)}$$

$$\text{Solving } n = 2$$

$$\text{Put } n = 2 \text{ in 1}$$

$$T = 3383 \text{ \AA}^{\circ}$$

12. Magnetic

Sol. Frequency of rev = v

$$\therefore I = ev$$

$$\therefore B = \frac{\mu_0 I}{2r}$$

$$B = \frac{\mu_0 e v}{2r}$$

$$= \left(\frac{\mu e}{2} \right) \frac{v}{2\pi r} \left(\frac{1}{r} \right) = \left(\frac{\mu e}{4\pi} \right) \frac{e.v}{r^2}$$

In hydrogen atom

$$v = \frac{n\hbar}{2\pi mr}$$

$$\therefore B = \frac{\mu_0}{4\pi} \frac{e h.n}{(4\pi m)r^3}$$

$$\text{and } r = \frac{\epsilon_0 h^2 n^2}{\pi m e^2}$$

$$\therefore B = \frac{\mu_0 \pi m^2 e^7}{8\epsilon_0 h^5 n^5}$$

$$\therefore B \propto e^7$$

$$B \propto \frac{1}{n^5}$$

13. The mean

$$\text{Sol. } \lambda_{(\alpha+\beta)} = \lambda_\alpha + \lambda_\beta$$

$$\Rightarrow \frac{1}{T_{\frac{1}{2}(\alpha+\beta)}} = \frac{1}{T_{\frac{1}{2}(\alpha)}} + \frac{1}{T_{\frac{1}{2}(\beta)}}$$

$$\Rightarrow \frac{1}{T_{\frac{1}{2}(\alpha+\beta)}} = \frac{1}{30} + \frac{1}{60} = \frac{1}{20}$$

$$\therefore T_{\frac{1}{2}(\alpha+\beta)} = 20 \text{ years}$$

\therefore One-fourth of sample will remain after 2 half life = 40

n2 years.

14. In a common

$$\text{Sol. } A_V = \beta \frac{R_{\text{out}}}{R_{\text{in}}} \Rightarrow G = 25 \frac{R_{\text{out}}}{R_1} \dots \text{(i)}$$

$$G_m = \frac{\beta}{R_1} \Rightarrow R_1 = \frac{\beta}{G_m} = \frac{25}{0.03}$$

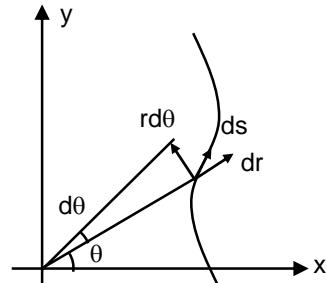
$$G = 25 \frac{R_{\text{out}}}{25} \times 0.03 \dots \text{(i)}$$

$$G' = 20 \frac{R_{\text{out}}}{20} \times 0.02 \dots \text{(ii)}$$

$$G' = \frac{2}{3} G$$

15. A particle is

$$\text{Sol. } ds^2 = (dr)^2 + (rd\theta)^2$$



$$ds^2 = (d\theta)^2 + (\theta \cdot d\theta)^2 = (1 + \theta^2) d\theta^2$$

$$\therefore ds = \sqrt{1 + \theta^2} d\theta$$

$$\therefore s = \int_0^{2\pi} \sqrt{1 + \theta^2} d\theta$$

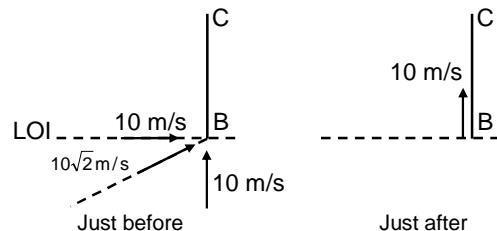
$$= \frac{2\pi \sqrt{1 + 4\pi^2} + \ln |2\pi + \sqrt{1 + 4\pi^2}|}{2}$$

16. A particle is

Sol. Velocity of particle just before hitting BC;

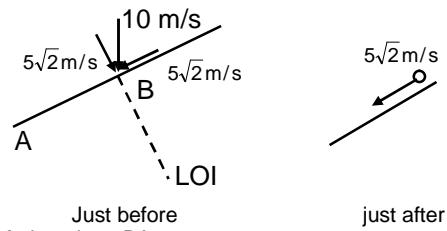
$$v = \sqrt{20^2 - 2 \times 10 \times 10} = 10\sqrt{2} \text{ m/s}$$

Collision with BC



Since BC is smooth particle will hit BA with velocity 10 m/s.

Collision with BA



Just before Motion along BA Final velocity at A

$$v^2 = (5\sqrt{2})^2 + 2 \times 10 \times 10 = 250$$

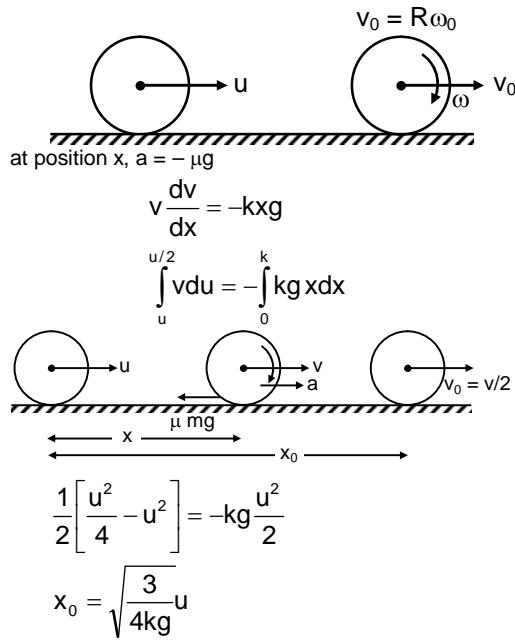
$$v = 5\sqrt{10} \text{ m/s}$$

17. Centre of mass

Sol. Let v_0 = velocity when pure rolling starts

Conservation of angular momentum about lowest point,

$$\mu u R = mv_0 R + mR^2 \left(\frac{v_0}{R} \right) = 2mv_0 R \Rightarrow v_0 = \frac{u}{2}$$



18. Water enters

$$\text{Sol. } \tau = 2(\rho av^2)R = \frac{2\rho(av)^2}{a}R$$

19. Two waves

Sol. Resultant wave

$$y = y_1 + y_2 \\ = \frac{5}{(3x - 4t)^2 + 2} - \frac{5}{(3x + 4t - 6)^2 + 2} \quad \dots(i)$$

$$y = 0 \text{ when } 3x - 4t = 3x + 4t - 6 \Rightarrow t = \frac{6}{8} = \frac{3}{4} \text{ s}$$

y can also be written as

$$y = \frac{5}{(4t - 3x)^2 + 2} - \frac{5}{(3x + 4t - 6)^2 + 2} \quad \dots(ii)$$

Again

$$y = 0 \text{ when } 4t - 3x = 3x + 4t - 6$$

$$x = 1 \text{ m}$$

20. In the system

Sol. Compression in the spring

$$X = \frac{\mu mg}{K}$$

Applying work energy theorem on piston

$$Wg = -W_{sp} - W_{atm} = \frac{1}{2} Kx^2 + P_0 \cdot x \cdot A$$

$$= \frac{\mu^2 m^2 g^2}{2K} + \frac{P_0 \mu mg}{K}$$

$$Q = W_g + \Delta U$$

$$= Wg$$

$$= \frac{\mu_0 mg}{K} \left[AP_0 + \frac{\mu mg}{2} \right]$$

21. S_1 and S_2

Sol. Let initial phase of $S_1 = \theta_1$ and t of $S_2 = \theta_2$

$$\therefore \Delta\theta = \theta_1 - \theta_2 = \frac{\pi}{2}$$

$$\text{At } P_1 \quad \Phi_1 = k \Delta P - \Delta\theta = 0 - \frac{\pi}{2}$$

$$\text{At } P_2 \quad \Phi_2 = k \Delta P - \Delta\theta \\ = 2\pi \times \frac{\lambda}{2} - \frac{\pi}{2} = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

\therefore Resultant intensity at P_2 is same that of P_1

22. A particle

$$\text{Sol. } a = A \sin \omega t_0$$

$$b = A \sin 2\omega t_0$$

$$c = A \sin 3\omega t_0$$

$$a + c = 2A \sin 2\omega t_0 \cos \omega t_0$$

$$\frac{a+c}{2b} = \cos \omega t_0$$

$$w = \frac{1}{t_0} \cos^{-1} \left(\frac{a+c}{2b} \right)$$

$$f = \frac{1}{2\pi t_0} \cos^{-1} \left(\frac{a+c}{2b} \right)$$

23. The rubber

$$\text{Sol. Elastic energy } E = \frac{1}{2} Y (\text{strain})^2 (\text{volume})$$

$$E = \frac{1}{2} \times 6 \times 10^8 \left(\frac{0.05}{0.20} \right)^2 (2 \times 10^{-6} \times 0.25) = 9.375 \text{ J}$$

The elastic energy is converted into kinetic energy.

$$E = \frac{1}{2} mv^2$$

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 9.375}{15 \times 10^{-3}}} = 35.3 \text{ m/s}$$

24. A body emits

$$\text{Sol. } \lambda_m T = b - 3 \times 10^{-3} \text{ m K (Wien's Law)}$$

$$\frac{dE}{dt} = \sigma T^4 \text{ (Boltzmann law)}$$

$$\lambda'_m = 2\lambda_m = 2 \times 480 = 960 \text{ nm.}$$

25. A sinusoidal

$$\text{Sol. } E_c = \frac{110 + 90}{2} = 100$$

$$E_m = \frac{110 - 90}{2} = 10$$

$$\text{modulating index } m = \frac{E_m}{E_c} = 0.1$$

$$\text{amplitude of side band } m \times \frac{E_c}{2} = 5 \text{ V}$$

26. A wire bent

Sol. Since the wire is continuous, tension in the parts AB and BC will be identical. Equating the horizontal and vertical components of forces separately

$$\frac{mv^2}{r} = T \sin 30^\circ + T \sin 60^\circ \quad (\text{i})$$

$$mg = T \cos 30^\circ + T \cos 60^\circ \quad (\text{ii})$$

As the right-hand sides of (i) and (ii) are identical

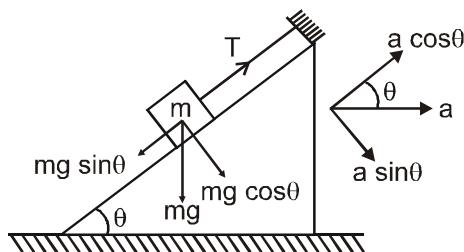
$$\frac{mv^2}{r} = mg$$

$$\text{or } v = \sqrt{gr}$$

27. In the given

$$\text{Sol. } v = \frac{t^2}{2}$$

$$a = t$$



At the instant when the normal is just zero

$$mg \cos \theta = ma \sin \theta$$

$$a = g \cot \theta$$

$$\text{since } \theta = 45^\circ$$

$$a = t = 10$$

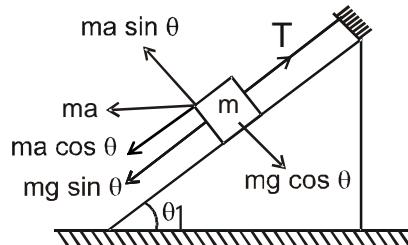
$$\Rightarrow t = 10 \text{ sec.}$$

Alternative solutions

$$v = \frac{t^2}{2}$$

$$a = t$$

Normal will be zero at the instant $ma \sin \theta = mg \cos \theta$



$$a = g \cot \theta$$

$$\text{since } \theta = 45^\circ$$

$$a = t = 10$$

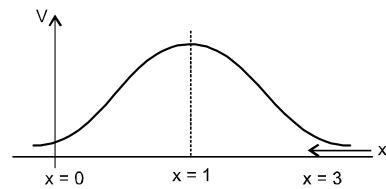
$$\Rightarrow t = 10 \text{ sec.}$$

28. The electromagnetic

Sol. In electromagnetic radiations the rays with increasing energy or decreasing wavelength are,
 $\text{RW} > \mu\text{W} > \text{IR} > \text{visible light} > \text{UV} > \text{X-rays} > \gamma\text{-rays}$

29. A particle of

Sol.



to send the particle from $x = 3$ to $x = 0$, the particle has to be sent to the point ($x = 1$) where force changes the direction

So by applying energy conservation between $x = 3$ to $x = 1$
 $K_i + U_i = K_f + U_f$

$$\frac{1}{2} mu^2 + \frac{10}{4 + (3-1)^2} = 0 + \frac{10}{4 + (1-1)^2}$$

$$\text{Solving } u = \frac{\sqrt{5}}{2} \text{ m/s}$$

30. Plane surface

Sol. Focal length of silvered mirror

$$\frac{1}{f} = \frac{1}{f_m} - \frac{2}{f_\ell}$$

$$\frac{1}{f} = \frac{1}{\infty} - \frac{2}{f_\ell}$$

$$f = -\frac{f_\ell}{2}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-60} + \frac{1}{-30} = \frac{1}{f}$$

$$f = -20 \text{ cm}$$

Focal length of unsilvered lens will be 40 cm

$$\text{So, } \frac{1}{v} + \frac{1}{30} = \frac{1}{40}$$

$$\frac{1}{v} = \frac{1}{40} - \frac{1}{60} = \frac{3-2}{120} = +\frac{1}{120}$$

$$\Rightarrow v = 120 \text{ cm.}$$

PART-B : CHEMISTRY

31. The rate constant of

Sol. $R = K [A]$

$$R = 4 \times 10^{-3} \times 0.02 = 8 \times 10^{-5} \text{ M sec}^{-1}$$

32. The volume-temperature

Sol. The correct order of pressure is

$$p_1 > p_3 > p_2 \text{ (For same volume & moles } p \propto T\text{).}$$

35. What is the value of

$$\text{Sol. } \alpha = \frac{7.8}{390} = 2 \times 10^{-2}$$

$$K_a = c\alpha^2 = 16 \times 10^{-6}$$

$$\text{or } pK_a = 4.8$$

37. 1 mol CH_3COOH is

Sol. $\Delta T_b = i K_b \cdot m$

$$\text{Given molality} = \frac{1 \times 1000}{250} = 4 \text{m, 6.4}$$

$$= i \times 2 \times 4 \text{ or } i = 0.8$$

For dimerisation

$$i = 1 - \frac{\beta}{2}$$

$$\Rightarrow 0.8 - 1 = \frac{-\beta}{2}$$

$$\text{or } \beta = 0.4 \Rightarrow 40\%$$

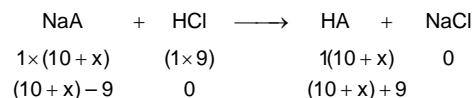
38. Potassium crystallizes

$$\text{Sol. distance between nearest neighbours} = 2R = \frac{5.2 \times \sqrt{3}}{2} = 4.5 \text{ \AA}$$

39. An acidic buffer solution

Sol. For given buffer solution $\text{pH}_1 = \text{pKa}$

Now 9 mL of 1 M HCl is added to $(10 + x)$ mL of this solution



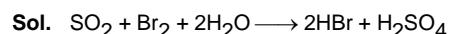
$$\text{pH}_1 - \text{pH}_2 = 1$$

$$\log \frac{(10+x)-9}{(10+x)+9} = -1$$

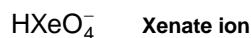
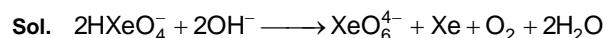
$$\frac{10+x-9}{10+x+9} = \frac{1}{10}$$

$$x = 1 \text{ mL.}$$

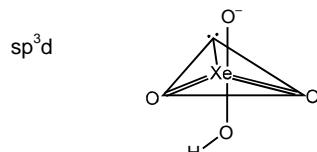
41. Bromine water reacts



43. The xenate ion undergoes

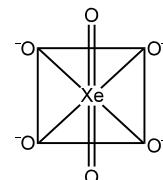


S.N. = 5



(No peroxide group)

Structure of $[\text{XeO}_6]^{4-}$



44. Select the incorrect

Sol. Distance between two nearest tetrahedral void is $\frac{a}{2}$

45. Which one of the following

Sol. (3) Carbonate ores are calcined in absence of air to obtain the metal oxides.

46. The gas escaping from

| | O_2 |
|---|--------------|
| $2\text{CO} + \text{O}_2 \longrightarrow 2\text{CO}_2$ | 14 |
| $2\text{H}_2 + \text{O}_2 \longrightarrow 2\text{H}_2\text{O}$ | 1.5 |
| $\text{CH}_4 + 2\text{O}_2 \longrightarrow \text{CO}_2 + 2\text{H}_2\text{O}$ | 1.2 |
| $\text{C}_2\text{H}_4 + 3\text{O}_2 \longrightarrow 2\text{CO}_2 + 2\text{H}_2\text{O}$ | 0.6 |

$$\frac{17.3 \text{ Parts } \times 5}{86.5} = 86.5 \text{ part of the air}$$

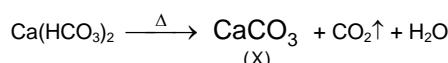
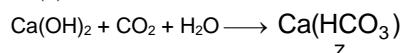
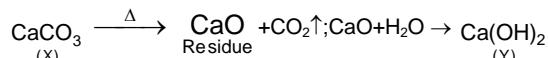
$$200 \text{ m}^3 \text{ of the gas } 2 \times 86.5 = 173 \text{ m}^3 \text{ Ans.}$$

47. Find total number of

Sol. Two geometrical isomer (cis and trans) and two linkage isomer ($-\text{SCN}$ and $-\text{CNS}$).

48. A solid compound 'X'

Sol. The given compound X must be CaCO_3 . It can be explained by following reactions,

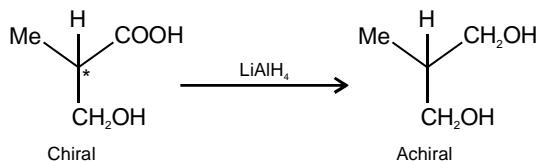


49. In which of the following

Sol. Maltose has hemi acetal linkage so it can reduce tollen's reagent and all mono saccharides (e.g. Fructose, Glucose, Ribose, Mannose) give tollen's test but polysaccharides (Cellulose, Starch, Amylopectin.....) do not give tollen's test.

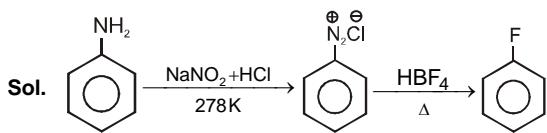
50. Compound (A) ($C_4H_8O_3$)

Sol. $NaHCO_3$ test shows the presence of the ($-COOH$) group, and from the structures given in the problem, only the compound in (a) on reduction with $LiAlH_4$ gives achiral product.



The compounds (2), (3) and (4) with $LiAlH_4$ will give chiral products. So the answer is (1).

51. In the chemical reactions



52. Which reagent is not

Sol. Fehling solution does not oxidise aromatic aldehydes.

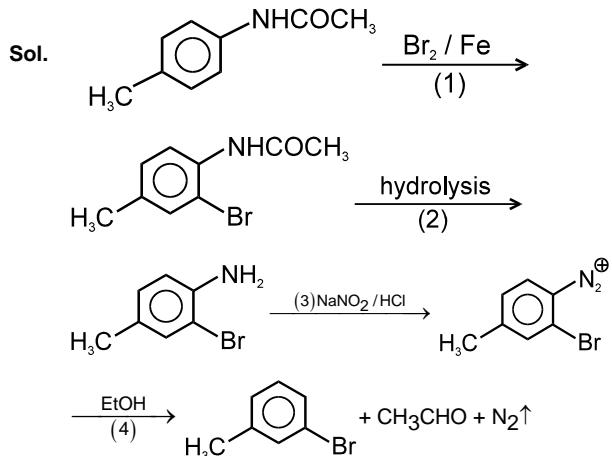
53. Which of the following

Sol. $\xrightarrow{AgClO_4}$ After removal of Cl-atom, formed intermediate is anti-aromatic so this reaction is not possible.

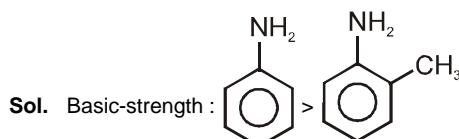
54. How many structures

Sol. Primary amines give carbyl amine test. Total 4 primary amines structures are possible with $C_4H_{11}N$.

55. The end product of following

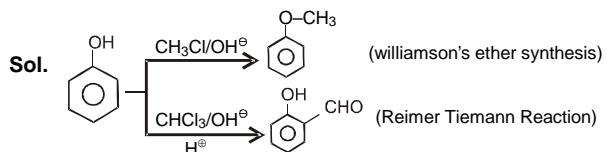


56. Select the incorrect

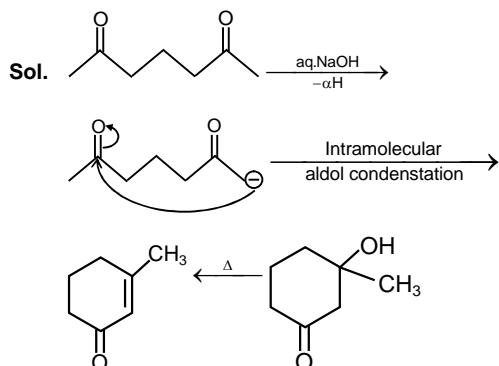


(due to ortho effect)

57. In the following



58. Which of the following



60. Which of the following

Sol. Ascorbic acid is one form of vitamin C

PART-C : MATHEMATICS

61. Let p : Sindhu

$$\begin{aligned} \text{Sol. } \sim(\sim q \wedge p) &= (\sim(\sim q)) \vee (\sim p) \\ &= \sim p \vee q \\ &= p \rightarrow q \\ \therefore \text{ contrapositive is } \sim q &\rightarrow \sim p \end{aligned}$$

62. The mean marks.....

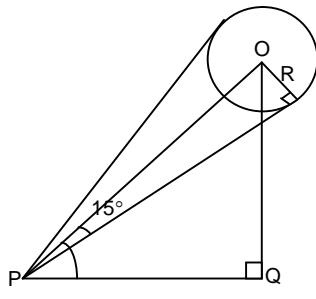
$$\begin{aligned} \text{Sol. } n + m &= 120 \quad \dots(1) \\ 56 &= \frac{50n + 60m}{120} \quad \dots(2) \\ (1) \& (2) \rightarrow n = 48 \& m = 72 \\ \therefore \frac{9n}{m} &= 6 \end{aligned}$$

63. U_1, U_2, \dots, U_{15}

$$\begin{aligned} \text{Sol. Let } |U_1 \cup \dots \cup U_{15}| &= m \\ \text{Then } \sum_{i=1}^{15} |U_i| &= 3m \\ \Rightarrow 30 &= 3m \Rightarrow m = 10 \\ 4 \times 10 &= 10 \times n \Rightarrow n = 4 \end{aligned}$$

64. A spherical

Sol.



$$OP = R \operatorname{cosec}(15^\circ)$$

$$OP \sin(60^\circ) = 6$$

$$R \operatorname{cosec}(15^\circ) \sin(60^\circ) = 6$$

65. X is a binomial

Sol. $\operatorname{Var}(X) = npq$

$$pq \max = \frac{1}{4}$$

67. Let α, β be

$$\begin{aligned} \text{Sol. Let } y &= (\alpha + \beta) - \sqrt{\alpha^2 + \beta^2} \\ y &= -\sqrt{2} - \sqrt{4} \\ y + \sqrt{2} &= -2 \\ y^2 + 2\sqrt{2}y - 2 &= 0 \end{aligned}$$

$$68. \frac{3 + \cot(76^\circ) \cdot \cot(16^\circ)}{\cot(76^\circ) + \cot(16^\circ)}$$

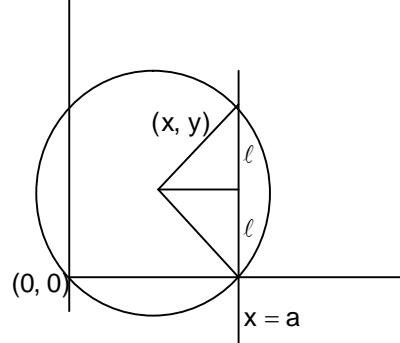
Sol. Changing into sines & cosines, the expression simplifies to $\cot(44^\circ)$.

69. AD is altitude of.....

$$\begin{aligned} \text{Sol. Let } \angle BAD &= A_1 \& \angle CAD = A_2, \\ \text{Then } \tan A_1 &= 1/3, \tan A_2 = 1/2 \\ A &= A_1 + A_2 = 45^\circ. \end{aligned}$$

70. A circle passes

Sol.



$$(a - x_1)^2 + \ell^2 = x_1^2 + y_1^2$$

$$\text{Locus } y^2 = a^2 + \ell^2 - 2ax$$

71. If $f(x)$ is a 4^{th}

$$\text{Sol. } (x+1)f(x) - x = \lambda x(x-1)(x-2)(x-3)(x-4)$$

$$\text{Put } x = -1, \lambda = -\frac{1}{120}$$

$$\text{Put } x = 5$$

$$6f(5) - 5 = \frac{-1}{120} \times 120$$

$$f(5) = \frac{2}{3}$$

$$73. \text{ Let } f'(x) = \frac{x^2}{1+x^5} \forall x$$

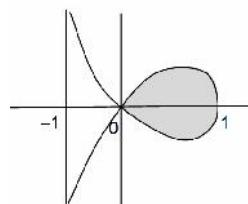
Sol. Applying LMVT in [1, 2]

$$\text{for some } c \text{ in } (1, 2), \text{ we have } \frac{f(2) - f(1)}{2-1} = f'(c)$$

$$\text{Also, } \frac{4}{33} < \frac{c^2}{1+c^5} < \frac{1}{2} \Rightarrow \frac{4}{33} < f(2) < \frac{1}{2}$$

74. Area bounded

Sol.



$$\text{Shaded area} = 2 \int_0^1 x \sqrt{\frac{1-x}{1+x}} dx$$

$$= 2 - \frac{\pi}{2}$$

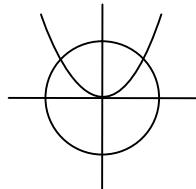
75. A is a 3×3

Sol. By construction

76. Let $R_1 = \{(x, y) : -3 \leq x \leq 3, 0 \leq y \leq 5\}$

Sol. R_1 is interior of circle $x^2 + y^2 = 5^2$

$$R_2 \text{ is interior of parabola } x^2 = \frac{9}{4}y$$



$$R_1 \cap R_2 = \{(x, y) : -3 \leq x \leq 3, 0 \leq y \leq 5\}$$

$$\text{Domain} = [-3, 3] \text{ Range} = [0, 5]$$

$$(0, y) \in R_1 \cap R_2 \text{ where } 0 \leq y \leq 5$$

$$(0, 0) \in R_1 \cap R_2$$

$$(0, 1) \in R_1 \cap R_2$$

$$(0, 3) \in R_1 \cap R_2$$

Hence not a function

77. If $\alpha + \beta + \gamma = \frac{\pi}{2}$

$$\begin{aligned} \text{Sol. } & \begin{vmatrix} \cos(\alpha + \beta + \gamma) & \sin\beta & -\cos\gamma \\ -\sin\beta & \sin(2\alpha + 2\beta + 2\gamma) & \tan\alpha \\ \sin(\alpha + \beta) & -\tan\alpha & \cot(\alpha + \beta + \gamma) \end{vmatrix} \\ &= \begin{vmatrix} 0 & \sin\beta & -\cos\gamma \\ -\sin\beta & 0 & \tan\alpha \\ \cos\gamma & -\tan\alpha & 0 \end{vmatrix} = 0 \end{aligned}$$

78. Let p and q

$$\text{Sol. } p + q = \frac{m-2}{m} \quad pq = \frac{3}{m}$$

$$\frac{p}{q} + \frac{q}{p} = \frac{2}{3} \Rightarrow 3(p^2 + q^2) = 2pq$$

$$\Rightarrow 3((p+q)^2 - 2pq) = \frac{6}{m}$$

$$3\left(\frac{m-2}{m}\right)^2 - \frac{18}{m} = \frac{6}{m}$$

$$\Rightarrow (m-2)^2 = 8m$$

$$\Rightarrow m^2 - 12m + 4 = 0$$

$$m_1 + m_2 = 12 \quad m_1 m_2 = 4$$

$$\frac{m_1}{m_2^2} + \frac{m_2}{m_1^2} = \frac{m_1^3 + m_2^3}{(m_1 m_2)^2}$$

$$= \frac{(m_1 + m_2)^3 - 3m_1 m_2 (m_1 + m_2)}{(m_1 m_2)^2} = 99$$

79. A straight line

$$\text{Sol. } y = x \Rightarrow x = \frac{r}{\sqrt{2}}, y = \frac{r}{\sqrt{2}} \Rightarrow P\left(\frac{r}{\sqrt{2}}, \frac{r}{\sqrt{2}}\right)$$

$$\frac{3r}{\sqrt{2}} - \frac{4r}{\sqrt{2}} = 6 \Rightarrow r = -6\sqrt{2} = OP$$

$$Q\left(\frac{r}{\sqrt{2}}, \frac{r}{\sqrt{2}}\right) \quad \frac{6r}{\sqrt{2}} - \frac{8r}{\sqrt{2}} + c = 0$$

$$r = \frac{c}{\sqrt{2}} \Rightarrow Q$$

$$3(6\sqrt{2}) = 4 \left| \frac{c}{\sqrt{2}} \right| \Rightarrow c = \pm 9$$

80. If m & M are

$$\text{Sol. } M = |2\vec{a}|^2 + |2\vec{b}|^2 + |2\vec{c}|^2 = 12$$

when $\vec{a}, \vec{b}, \vec{c}$ are parallel $|\vec{a} + \vec{b} + \vec{c}|^2 \geq 0$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) \geq 0$$

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} \geq -\frac{3}{2}$$

$$|\vec{a} + \vec{b}|^2 + |\vec{b} + \vec{c}|^2 + |\vec{c} + \vec{a}|^2$$

$$= 2(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2)$$

$$+ \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) \geq 2\left(3 - \frac{3}{2}\right) \geq 3$$

81. Let $a = \cos^{-1}(\cos 20)$

Sol. By graph $a = 20 - 6\pi$,

$$b = 10\pi - 30,$$

$$c = \sin^{-1}(\sin(4\pi - 10)) = 10 - 3\pi$$

$$\Rightarrow a + b + c = \pi$$

$$G.E = \sin(2(\pi)x) + \cos^2((\pi)x)$$

$$= \frac{1}{2} + \frac{1}{2} \cos(2\pi x) + \sin(2\pi x)$$

Maximum value

$$= \frac{1}{2} + \sqrt{\frac{1}{4} + 1} = \frac{1}{2} + \frac{\sqrt{5}}{2} = \frac{\sqrt{5} + 1}{2}$$

82. If C is arbitrary

Sol. Divide numerator and denominator by x^2

$$\int \frac{\frac{2}{x^2} + \frac{\sqrt{x}}{x^2}}{\left(1 + \frac{1}{\sqrt{x}} + \frac{1}{x^2}\right)^2} dx$$

$$\text{put } t = 1 + \frac{1}{\sqrt{x}} + \frac{1}{x} \quad dt = \left(-\frac{1}{2x\sqrt{x}} - \frac{1}{x^2}\right) dx$$

$$= -2 \int \frac{dt}{t^2} = \frac{2}{t} + C = \frac{2x}{x + \sqrt{x} + 1} + C$$

83. Let $\hat{a}, \hat{b}, \hat{c}$ be

Sol. $\hat{a}, \hat{b}, \hat{c}$ will form an equilateral triangle

$$|x\hat{a} + y\hat{b} + z\hat{c}|^2 = x^2 + y^2 + z^2 - xy - yz - zx$$

$$= \frac{1}{2} [(x-y)^2 + (y-z)^2 + (z-x)^2] = \frac{1}{2} [1 + 1 + 4] = 3$$

84. If the number

Sol. Out of numbers $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{1}{3}, \frac{2}{6}, \frac{2}{3}, \frac{4}{6}$ will result in only 3 distinct rational numbers.
 \Rightarrow Total numbers = ${}^6C_2 - 4 = 11$

85. If A, B are events.....

Sol. (A) $P(A \cup B) \geq \max\{P(A), P(B)\} = \frac{2}{3}$

(B) $P(A \cap B) = P(A) + P(B) - P(A \cup B) \geq P(A) + P(B) - 1$
 $= \frac{3}{5} + \frac{2}{3} - 1 = \frac{4}{15}$

$P(A \cap B) \leq \min\{P(A), P(B)\} = \frac{3}{5}$

(C) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$\therefore \frac{4}{15} \leq P(A \cap B) \leq \frac{3}{5}$

$\Rightarrow \frac{4}{15P(B)} \leq \frac{P(A \cap B)}{P(B)} \leq \frac{3}{15P(B)}$

$\Rightarrow \frac{2}{5} P\left(\frac{A}{B}\right) \leq \frac{9}{10}$

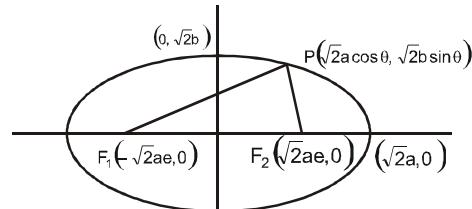
(D) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$\Rightarrow \frac{3}{5} - \frac{3}{5} \leq P(A \cap \bar{B}) \leq \frac{3}{5} - \frac{4}{15}$

$\Rightarrow P(A \cap \bar{B}) \leq \frac{1}{3}$

86. P is a variable

Sol.



area of $\Delta PF_1F_2 = A = \frac{1}{2}(2\sqrt{2}ae) \sqrt{2} b \sin \theta$

$\Rightarrow A = 2abe \sin \theta$

Maximum area $A = 2abe$

$= \frac{2ab}{a} \sqrt{a^2 - b^2} = 2b \sqrt{a^2 - b^2}$

87. Two parabolas

Sol. Parabola open upward and to the right

Let intersection point are P & Q

PA \perp to x-axis to perpendicular

PB \perp to y- axis to perpendicular

PA = PB = PS

\Rightarrow P lies on y = x

88. $\int_2^4 \frac{\sqrt{\ln(81-18x+x^2)}}{2\sqrt{\ln(81-18x+x^2)} + \sqrt{\ln(x^2+6x+9)}} dx \dots\dots$

Sol. Let $I = \int_2^4 \frac{\sqrt{\ln(x-9)^2}}{2\sqrt{\ln(x-9)^2} + \sqrt{\ln(x+3)^2}} dx \dots\dots(i)$

[by property $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$]
 $x \rightarrow 6-x$

$I = \int_2^4 \frac{\sqrt{\ln(x+3)^2}}{2\sqrt{\ln(x+3)^2} + \sqrt{\ln(9-x)^2}} dx \dots\dots(ii)$

Adding (i) & (ii)

$2I = \int_2^4 1 dx = 2 \Rightarrow I = 1$

89. Let $f : (0, \infty) \rightarrow \mathbb{R} \dots\dots$

Sol. $\because F(x^2) = \int_0^{x^2} tf(t) dt = x^4 + x^5$

$\Rightarrow x^2 f(x^2) \cdot 2x = 4x^3 + 5x^4$

$\therefore f(x^2) = \frac{5}{2}x + 2$

$\therefore \sum_{r=1}^{12} f(r^2) = \sum_{r=1}^{12} \left(\frac{5}{2}r + 2 \right)$

$= \frac{5}{2} \times \frac{12 \times 13}{2} + 2 \times 12 = 219$

90. The value of

Sol. $f(x) = \sin x - \cos x - ax + b$

$f'(x) = \cos x + \sin x - a$

$\Rightarrow \left(\sin \left(x + \frac{\pi}{4} \right) \right) - a$

$-1 \leq \sin \left(x + \frac{\pi}{4} \right) \leq 1$

$-\sqrt{2} < \sqrt{2} \sin \left(x + \frac{\pi}{4} \right) \leq \sqrt{2}$

so, to make $f'(x)$ always negative or equal to zero

$a \geq \sqrt{2}$

ANSWER KEY
CODE-0
PHYSICS

| | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1. | (3) | 2. | (4) | 3. | (2) | 4. | (1) | 5. | (2) | 6. | (2) | 7. | (4) |
| 8. | (4) | 9. | (1) | 10. | (2) | 11. | (1) | 12. | (3) | 13. | (4) | 14. | (4) |
| 15. | (1) | 16. | (2) | 17. | (4) | 18. | (1) | 19. | (4) | 20. | (3) | 21. | (4) |
| 22. | (1) | 23. | (3) | 24. | (4) | 25. | (2) | 26. | (3) | 27. | (1) | 28. | (2) |
| 29. | (2) | 30. | (1) | | | | | | | | | | |

CHEMISTRY

| | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 31. | (1) | 32. | (1) | 33. | (3) | 34. | (2) | 35. | (2) | 36. | (3) | 37. | (2) |
| 38. | (1) | 39. | (4) | 40. | (4) | 41. | (3) | 42. | (4) | 43. | (3) | 44. | (3) |
| 45. | (3) | 46. | (2) | 47. | (3) | 48. | (4) | 49. | (2) | 50. | (1) | 51. | (3) |
| 52. | (2) | 53. | (4) | 54. | (1) | 55. | (4) | 56. | (4) | 57. | (2) | 58. | (3) |
| 59. | (1) | 60. | (2) | | | | | | | | | | |

MATHEMATICS

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|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 61. | (4) | 62. | (4) | 63. | (4) | 64. | (4) | 65. | (3) | 66. | (3) | 67. | (1) |
| 68. | (1) | 69. | (2) | 70. | (3) | 71. | (4) | 72. | (1) | 73. | (1) | 74. | (3) |
| 75. | (2) | 76. | (4) | 77. | (4) | 78. | (3) | 79. | (4) | 80. | (2) | 81. | (1) |
| 82. | (2) | 83. | (4) | 84. | (1) | 85. | (3) | 86. | (2) | 87. | (1) | 88. | (2) |
| 89. | (4) | 90. | (1) | | | | | | | | | | |

CODE-1
PHYSICS

| | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1. | (1) | 2. | (2) | 3. | (4) | 4. | (3) | 5. | (4) | 6. | (4) | 7. | (2) |
| 8. | (2) | 9. | (3) | 10. | (3) | 11. | (3) | 12. | (1) | 13. | (2) | 14. | (2) |
| 15. | (3) | 16. | (4) | 17. | (2) | 18. | (2) | 19. | (4) | 20. | (1) | 21. | (2) |
| 22. | (3) | 23. | (1) | 24. | (2) | 25. | (4) | 26. | (1) | 27. | (3) | 28. | (4) |
| 29. | (4) | 30. | (3) | | | | | | | | | | |

CHEMISTRY

| | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 31. | (2) | 32. | (2) | 33. | (3) | 34. | (3) | 35. | (3) | 36. | (3) | 37. | (3) |
| 38. | (2) | 39. | (3) | 40. | (4) | 41. | (2) | 42. | (4) | 43. | (4) | 44. | (3) |
| 45. | (4) | 46. | (1) | 47. | (4) | 48. | (4) | 49. | (3) | 50. | (1) | 51. | (4) |
| 52. | (3) | 53. | (4) | 54. | (2) | 55. | (3) | 56. | (4) | 57. | (3) | 58. | (2) |
| 59. | (2) | 60. | (3) | | | | | | | | | | |

MATHEMATICS

| | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 61. | (4) | 62. | (3) | 63. | (3) | 64. | (3) | 65. | (4) | 66. | (4) | 67. | (2) |
| 68. | (2) | 69. | (3) | 70. | (4) | 71. | (3) | 72. | (2) | 73. | (2) | 74. | (4) |
| 75. | (3) | 76. | (3) | 77. | (4) | 78. | (4) | 79. | (3) | 80. | (3) | 81. | (1) |
| 82. | (2) | 83. | (4) | 84. | (3) | 85. | (3) | 86. | (4) | 87. | (3) | 88. | (2) |
| 89. | (4) | 90. | (3) | | | | | | | | | | |

ANSWER KEY
CODE-2
PHYSICS

| | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1. | (3) | 2. | (4) | 3. | (2) | 4. | (1) | 5. | (2) | 6. | (2) | 7. | (4) |
| 8. | (4) | 9. | (1) | 10. | (2) | 11. | (1) | 12. | (3) | 13. | (4) | 14. | (4) |
| 15. | (1) | 16. | (2) | 17. | (4) | 18. | (1) | 19. | (4) | 20. | (3) | 21. | (4) |
| 22. | (1) | 23. | (3) | 24. | (4) | 25. | (2) | 26. | (3) | 27. | (1) | 28. | (2) |
| 29. | (2) | 30. | (1) | | | | | | | | | | |

CHEMISTRY

| | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 31. | (1) | 32. | (1) | 33. | (3) | 34. | (2) | 35. | (2) | 36. | (3) | 37. | (2) |
| 38. | (1) | 39. | (4) | 40. | (4) | 41. | (3) | 42. | (4) | 43. | (3) | 44. | (3) |
| 45. | (3) | 46. | (2) | 47. | (3) | 48. | (4) | 49. | (2) | 50. | (1) | 51. | (3) |
| 52. | (2) | 53. | (4) | 54. | (1) | 55. | (4) | 56. | (4) | 57. | (2) | 58. | (3) |
| 59. | (1) | 60. | (2) | | | | | | | | | | |

MATHEMATICS

| | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 61. | (4) | 62. | (4) | 63. | (4) | 64. | (4) | 65. | (3) | 66. | (3) | 67. | (1) |
| 68. | (1) | 69. | (2) | 70. | (3) | 71. | (4) | 72. | (1) | 73. | (1) | 74. | (3) |
| 75. | (2) | 76. | (4) | 77. | (4) | 78. | (3) | 79. | (4) | 80. | (2) | 81. | (1) |
| 82. | (2) | 83. | (4) | 84. | (1) | 85. | (3) | 86. | (2) | 87. | (1) | 88. | (2) |
| 89. | (4) | 90. | (1) | | | | | | | | | | |

CODE-3
PHYSICS

| | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1. | (1) | 2. | (2) | 3. | (4) | 4. | (3) | 5. | (4) | 6. | (4) | 7. | (2) |
| 8. | (2) | 9. | (3) | 10. | (3) | 11. | (3) | 12. | (1) | 13. | (2) | 14. | (2) |
| 15. | (3) | 16. | (4) | 17. | (2) | 18. | (2) | 19. | (4) | 20. | (1) | 21. | (2) |
| 22. | (3) | 23. | (1) | 24. | (2) | 25. | (4) | 26. | (1) | 27. | (3) | 28. | (4) |
| 29. | (4) | 30. | (3) | | | | | | | | | | |

CHEMISTRY

| | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 31. | (2) | 32. | (2) | 33. | (3) | 34. | (3) | 35. | (3) | 36. | (3) | 37. | (3) |
| 38. | (2) | 39. | (3) | 40. | (4) | 41. | (2) | 42. | (4) | 43. | (4) | 44. | (3) |
| 45. | (4) | 46. | (1) | 47. | (4) | 48. | (4) | 49. | (3) | 50. | (1) | 51. | (4) |
| 52. | (3) | 53. | (4) | 54. | (2) | 55. | (3) | 56. | (4) | 57. | (3) | 58. | (2) |
| 59. | (2) | 60. | (3) | | | | | | | | | | |

MATHEMATICS

| | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 61. | (4) | 62. | (3) | 63. | (3) | 64. | (3) | 65. | (4) | 66. | (4) | 67. | (2) |
| 68. | (2) | 69. | (3) | 70. | (4) | 71. | (3) | 72. | (2) | 73. | (2) | 74. | (4) |
| 75. | (3) | 76. | (3) | 77. | (4) | 78. | (4) | 79. | (3) | 80. | (3) | 81. | (1) |
| 82. | (2) | 83. | (4) | 84. | (3) | 85. | (3) | 86. | (4) | 87. | (3) | 88. | (2) |
| 89. | (4) | 90. | (3) | | | | | | | | | | |