Solutions

of

# Waves & Thermodynamics

## Lesson $14^{th}$ to $19^{th}$

By DC Pandey

# 14. Wave Motion

## Introductory Exercise 14.1

**1.** A function, $f$ can represent wave equation, if it satisfy

$$\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}$$

For, $y = a \sin \omega t$,

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 a \sin \omega t = -\omega^2 y$$

but, $\frac{\partial^2 y}{\partial x^2} = 0$

So, $y$ do not represent wave equation.

**2.** $y\,(x,\,t) = ae^{-(bx - et)^2} = ae^{-(kx - \omega t)^2}$

$\Rightarrow\ k = b$ and $\omega = e \Rightarrow v = \frac{\omega}{k} = \frac{c}{b}$

**3.** $y\,(x,\,t) = \frac{1}{1 + (4x + \omega t)^2}$ represent the given pulse, where,

$$y(x,\,0) = \frac{1}{1 + k^2 x^2} = \frac{1}{1 + x^2}$$

$\Rightarrow\quad k = 1$

and $y(x,\,z) = \frac{1}{1 + (x - 2\omega)^2} = \frac{1}{1 + (x - 1)^2}$

$\Rightarrow\qquad \omega = \frac{1}{2}$

$\therefore\qquad v = \frac{\omega}{k} = \frac{1/2}{1} = 0.5\,\text{m/s}$

**4.** $y = \frac{10}{5 + (x + 2t)^2} = \frac{a}{b + (kx + \omega t)^2}$

Amplitude, $y_{\max} = \frac{a}{b} = \frac{10}{5} = 2\,\text{m}$

and $\qquad k = 1;\ \omega = 2$

$v = \frac{\omega}{k} = 2\,\text{m/s}$ and is travelling in (–) $x$ direction.

**5.** $y = \frac{10}{(kx - \omega t)^2 + 2}$

$$y(x,\,0) = \frac{10}{k^2 x^2 + 2} = \frac{10}{x^2 + 2} \Rightarrow k = 1$$

$$\omega = vk = 2\,\text{m/s} \times 1\,\text{m}^{-1} = 2\,\text{rad/s}$$

$\Rightarrow\qquad y = \frac{10}{(x - 2t)^2 + 2}$

## Introductory Exercise 14.2

**1.** $y(x,\,t) = 0.02\,\sin\left(\frac{x}{0.05} + \frac{t}{0.01}\right)\text{m}$

$= A \sin\,(kx + \omega t)\,\text{m}$

$\Rightarrow\ A = 0.02\,\text{m},\ k = \frac{1}{0.05}\,\text{m}^{-1},\ \omega = \frac{1}{0.01}\,\text{s}^{-1}$

(a) $v = \frac{\omega}{k} = \frac{0.05}{0.01}\,\text{m/s} = 5\,\text{m/s}$

(b) $v_p = \frac{\partial y}{\partial t} = A\omega \cos\,(kx + \omega t)$

$v_p(0.2,\,0.3) = 0.02 \times \frac{1}{0.01} \cos\left(\frac{0.2}{0.5} + \frac{0.3}{0.01}\right)$

$= 2\cos\,(4 + 30)$

$= 2\cos\,34$

$= 2(-0.85)$

$= -1.7\,\text{m/s}$

**2.** Yes, $(v_p)_{\max} = A\omega = Ak \cdot \frac{\omega}{k} = (Ak)v$

**3.** $\lambda = 4$ cm, $v = 40$ cm/s (given)

(a) $\nu = \frac{v}{\lambda} = \frac{40\,\text{cm/s}}{4\,\text{cm}} = 10\,\text{Hz}$

(b) $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

$= \frac{2\pi}{4\text{ cm}} \times 2.5\text{ cm} = \frac{5\pi}{4}\text{ rad}$

(c) $\Delta t = \frac{T}{2\pi}\Delta\phi = \frac{1}{2\pi\nu}\Delta\phi$

$= \frac{1}{2\pi \times 10} \times \frac{\pi}{3}$

$= \frac{1}{60}\text{ s}$

(d) $v_p = (v_p)_{max}$

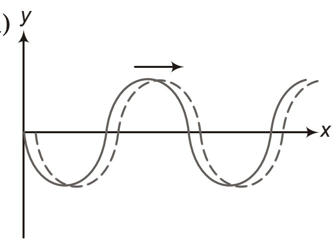
$= -A\omega = -2\pi A\nu$

$= -2\pi \times 2\text{ cm} \times 10\text{ s}^{-1}$

$= -40\pi\text{ cm/s}$

$= -1.26\text{ cm/s}$

**4.** (a)




$y = A\sin(\omega t - kx)$

$= A\sin\left(v \cdot \frac{2\pi}{\lambda}t - \frac{2\pi}{\lambda}x\right)$

$= 0.05\sin\left(12 \times \frac{2\pi}{0.4}t - \frac{2\pi}{0.4}x\right)$

$= 0.05\sin(60\pi t - 5\pi x)$

(b) $y(0.25, 0.15)$

$= 0.05\sin(60\pi \times 0.15 - 5\pi \times 0.25)$

$= 0.05\sin(9\pi - 1.25\pi)$

$= 0.05\sin(7.75\pi) = 0.05\sin(1.75\pi)$

$= -0.0354\text{ m} = -3.54\text{ cm}$

(c) $\Delta t = \frac{T}{2\pi}\Delta\phi = \frac{\Delta\phi}{\omega} = \frac{0.25\pi}{60\pi}$

$= \frac{1}{240}\text{ s} = 4.2\text{ ms}$

# Introductory Exercise 14.3

**1.** $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{m/l}} = \sqrt{\frac{Tl}{m}}$

$= \sqrt{\frac{500 \times 2}{0.06}} = \frac{100\sqrt{5}}{\sqrt{3}} = 129.1\text{ m/s}$

**2.** $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho \cdot A}}$

$= \sqrt{\frac{0.98}{9.8 \times 10^3 \times 10^{-6}}} = 10\text{ m/s}$

# Introductory Exercise 14.4

**1.** $I = \frac{P}{4\pi r^2} = \frac{1\text{ W}}{4\pi \times (1\text{ m})^2} = \frac{1}{4\pi}\text{ W/m}^2$

**2.** For line source, $I = \frac{\rho}{2\pi rl}$

$\Rightarrow I \propto \frac{1}{r}$ and as $I \propto A^2$

$\Rightarrow A \propto \frac{1}{\sqrt{r}}$

# AIEEE Corner

## Subjective Questions (Level 1)

**1.** $y(x, t) = 6.50 \text{ mm} \cos 2\pi \left(\frac{\pi}{28.0 \text{ cm}} - \frac{t}{0.0360\text{s}}\right)$

$$= A\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

$\Rightarrow \quad A = 6.50 \text{ mm}, \ \lambda = 28.0 \text{ cm},$

$$\nu = \frac{1}{T} = \frac{1}{0.036}\text{s}^{-1} = 27.78 \text{ Hz}$$

$$v = \nu\lambda = 28.0 \text{ cm} \times 27.78 \text{ s}^{-1} = 778 \text{ cm/s}$$

$$= 7.78 \text{ m/s}$$

The wave is travelling along (+)ve $x$-axis.

**2.** $y = 5\sin 30\pi\left(t - \frac{x}{240}\right)$

$$= 5\sin\left(30\pi t - \frac{\pi}{8}x\right) = A\sin(\omega t - kx)$$

(a) $y(2, 0) = 5\sin\left(3\pi \times 0 - \frac{\pi}{8} \times 2\right)$

$$= -5\sin\frac{\pi}{4} = -\frac{5}{\sqrt{2}} = -3.535 \text{ cm}$$

(b) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi/8} = 16 \text{ cm}$

(c) $v = \frac{\omega}{k} = \frac{30\pi}{\pi/8} = 240 \text{ cm/s}$

(d) $\nu = \frac{\omega}{2\pi} = \frac{30\pi}{2\pi} = 15 \text{ Hz}$

**3.** $y = 3 \text{ cm} \sin(3.14 \text{ cm}^{-1}x - 314 \text{ s}^{-1} t)$

$$= 3 \text{ cm} \sin(\pi \text{ cm}^{-1}x = 100\pi\text{s}^{-1}\text{t})$$

$$= A\sin(kx - \omega t)$$

(a) $(v_p)_{\max} = A\omega = 3 \text{ cm} \times 100\pi \text{ s}^{-1}$

$$= 300\pi \text{ cm/s} = 3\pi \text{ m/s} = 9.4 \text{ m/s}$$

(b) $a = -\omega^2 y = -(100\pi\text{s}^{-1})^2 \times 3 \text{ cm} \sin(6\pi - 111\pi)$

$$= -300\pi \sin(-105\pi) = 0$$

**4.** (a) $\Delta x = \frac{\lambda}{2\pi}\Delta\phi = \frac{v/\nu}{2\pi}\Delta\phi = \frac{350}{500 \times 2\pi} \times \frac{\pi}{3}$

$$= \frac{7\pi}{60\pi} = \frac{7}{50}\text{m} = 0.166 \text{ m}$$

(b) $\Delta\phi = \frac{2\pi}{T}\Delta t = 2\pi\nu\,\Delta t = 2\pi \times 500 \times 10^{-3}$

$$= \pi = 180°$$

**5.** $y(x, t) = \frac{6}{(kx + \omega t)^2 + 3}$

$$y(x, 0) = \frac{6}{k^2x^2 + 3} = \frac{6}{x^2 + 3}$$

$\Rightarrow \quad k = 1 \text{ m}^{-1}$

$\Rightarrow \omega = vk = 4.5 \text{ m/s} \times 1 \text{ m}^{-1} = 4.5 \text{ rad/s}$

$\Rightarrow y(x, t) = \frac{6}{(x - 4.5t)^2 + 3}$

**6.** $y = 1.0 \sin \pi\left(\frac{x}{2.0} - \frac{t}{0.01}\right)$

$$= 1.0 \sin 2\pi\left(\frac{x}{4.0} - \frac{t}{0.02}\right)$$

$$= A\sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

(a) $A = 1.0 \text{ mm}, \lambda = 4.0 \text{ cm}, T = 0.02\text{s}$

(b) $v_p = \frac{\partial y}{\partial t} = -\omega A\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$

$$= -\frac{2\pi A}{T}\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

$$= -\frac{2\pi \times 1.0 \text{ mm}}{0.02\text{s}}\cos 2\pi\left(\frac{x}{4.0} - \frac{t}{0.02\text{s}}\right)$$

$$= -\frac{\pi}{10}\text{m/s} \cos \pi\left(\frac{x}{2.0 \text{ cm}} - \frac{t}{0.01\text{s}}\right)$$

$v_p(1.0 \text{ cm}, 0.01\text{s}) =$

$$-\frac{\pi}{10}\text{m/s} \cos \pi\left(\frac{1}{2} - \frac{0.01}{0.01}\right)$$

$$= -\frac{\pi}{10}\text{m/s} \cos\frac{\pi}{2} = 0 \text{ m/s}$$

(c) $v_p(3.0, 0.01)$

$$= -\frac{\pi}{10}\cos \pi\left(\frac{3}{2} - 1\right) = 0 \text{ m/s}$$

$v_p\,(5.0\text{ cm}, 0.01\text{s}) = -\dfrac{\pi}{10}\text{ms}\cos\pi\left(\dfrac{5}{2}-1\right)$

$= 0\text{ m/s}$

$v_p(7.0\text{ cm}, 0.01\text{s}) = -\dfrac{\pi}{10}\text{m/s}\cos\pi\left(\dfrac{7}{2}-1\right)$

$= 0\text{ m/s}$

(d) $v_p(1.0\text{ cm}, 0.011\text{s})$

$= -\dfrac{\pi}{10}\text{m/s}$

$\cos\pi\left(\dfrac{1}{2}-\dfrac{0.011}{0.01}\right)$

$= -\dfrac{\pi}{10}\cos\pi\left(\dfrac{1}{12}-1.1\right)$

$= -\dfrac{\pi}{10}\cos 0.6\pi = -\dfrac{\pi}{10}\cos\dfrac{3\pi}{5} = 9.7\text{ cm/s}$

$v_p(1.0\text{ cm}, 0.012\text{s})$

$= -\dfrac{\pi}{10}\text{m/s}\cos\left(\dfrac{1}{2}-\dfrac{0.012}{0.01}\right)$

$= -\dfrac{\pi}{10}\cos\pi(0.5-1.2)$

$= -\dfrac{\pi}{10}\cos 0.7\pi = 18.5\text{ cm/s}$

$v_p(1.0\text{ cm}, 0.013\text{ s}) = -\dfrac{\pi}{10}\text{m/s}$

$\cos\pi\left(\dfrac{1}{2}-\dfrac{0.013}{0.01}\right) = -\dfrac{\pi}{10}\cos 0.8\pi$

$= 25.4\text{ cm/s}$

**7.** (a) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{40\text{ cm}} = \dfrac{\pi}{20}\text{cm}^{-1}$

$= 0.157\text{ rad/cm}$

$T = \dfrac{1}{\nu} = \dfrac{1}{8}\text{s} = 0.125\text{s}$

$\omega = 2\pi\nu = 16\pi\text{ rad/s} = 50.26\text{ rad/s}$

$v = \nu\lambda = 8\text{s}^{-1}\times 40\text{ cm} = 320\text{ cm/s}$

(b) $y(x, t) = A\cos(kx-\omega t)$

$= 15.0\text{ cm}\cos(0.157x - 50.3\,t)$

**8.** $A = 0.06\text{m}$ and $2.5\lambda = 20\text{ cm}$

$\Rightarrow \quad \lambda = \dfrac{20}{2.5}\text{cm} = 8\text{ cm}$

$\nu = \dfrac{v}{\lambda} = \dfrac{300\text{ m/s}}{8\text{ cm}} = 3750\text{ Hz}$

$y = A\sin(kx-\omega t) = 0.06\text{m}$

$\sin\left(\dfrac{2\pi}{0.08}x - 2\pi\times 3750\,t\right)$

$= 0.06\text{m}\sin(78.5\text{ m}^{-1}x - 23561.9\text{ s}^{-1}t)$

**9.** (a) $\nu = \dfrac{v}{\lambda} = \dfrac{8.00\text{ m/s}}{0.32\text{ m}} = 25\text{ Hz}$

$T = \dfrac{1}{\nu} = \dfrac{1}{15}\text{s} = 0.043\text{ Hz}$

$k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{0.32\text{m}} = 19.63\text{ rad/m}$

(b) $y = A\cos(kx+\omega t) = A\cos 2\pi\left(\dfrac{x}{\lambda}+\dfrac{t}{T}\right)$

$= 0.07\text{m}\cos 2\pi\left(\dfrac{x}{0.32\text{m}}+\dfrac{t}{0.04\text{ s}}\right)$

(c) $y = 0.07\text{m}\cos 2\pi\left(\dfrac{0.36}{0.32}+\dfrac{0.15}{0.04}\right)$

$= 0.07\text{ m}\cos 2\pi\left(\dfrac{9}{8}+\dfrac{30}{8}\right)$

$= 0.07\text{ m}\cos\dfrac{39}{4}\pi$

$= 0.07\text{ m}\cos\left(10\pi - \dfrac{\pi}{4}\right)$

$= 0.07\text{ m}\cos\dfrac{\pi}{4} = 0.0495\text{ m}$

(d) $\Delta t = \dfrac{T}{2\pi}\Delta\phi = \dfrac{\Delta\phi}{2\pi\nu} = \dfrac{\pi+\pi/4}{2\pi\times 25}$

$= \dfrac{3}{200}\text{s} = 0.015\text{s}$

**10.** $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{T}{\rho A}} = \sqrt{\dfrac{Mg}{\rho A}}$

$= \sqrt{\dfrac{2\times 9.8}{8920\times 3.14\times(1.2\times 10^{-3})^2}}$

$= \sqrt{\dfrac{2\times 9.8\times 10^4}{89.2\times 3.14\times 1.44}} = 22\text{ m/s}$

**11.** $\lambda \propto \nu \propto \sqrt{T} \propto \sqrt{M}$

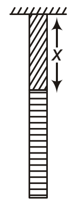
$\Rightarrow \quad \dfrac{\lambda_2}{\lambda_1} = \sqrt{\dfrac{M_2}{M_1}}$

$= \sqrt{\dfrac{8}{2}} = \sqrt{4} = 2.$

$\Rightarrow \quad \lambda_2 = 2\lambda_1$

$= 0.12\text{ m.}$

**12.** $T(x) = \mu(L - x)g,\ v(x) = \sqrt{\dfrac{T(x)}{\mu}}$




$$= \sqrt{g(L-x)}$$

$$\frac{dx}{\sqrt{g(L-x)}} = dt\ ;$$

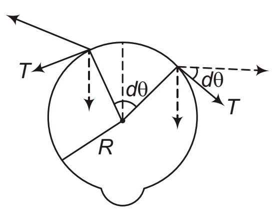
Let, $L - x = y$

$$dx = -dy$$

$$\int_L^0 \frac{-dy}{\sqrt{g}\sqrt{y}} = t$$

$$\therefore \quad t = \frac{1}{\sqrt{g}} \left. \frac{-\sqrt{y}}{1/2} \right|_1^0 = 2\sqrt{\frac{L}{g}}$$

**13.** (a) $dm\ \omega^2 R = 2T \sin d\theta$




$$\mu R\, 2d\theta\ \omega^2 R = 2T\, d\theta$$

$$\Rightarrow \quad \omega^2 R^2 = \frac{T}{\mu}$$

$\therefore$ Wave speed, $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\omega^2 R^2} = R\omega$

(b) Kink remains stationary when rope and kink moves in opposite sence *i.e.*, if rope is rotating anticlockwise then kink has to move clockwise.

**14.** $x$ is being measured from lover end of the string

$$\therefore \quad m(x) = \int dm = \int_0^x \mu_0\, x\, dx = \frac{1}{2}\mu_0 x^2$$

$$\therefore \quad v(x) = \sqrt{\frac{T(x)}{\mu}} = \sqrt{\frac{m(x)g}{\mu}}$$

$$= \sqrt{\frac{\frac{1}{2}\mu_0 x^2 g}{\mu_0 x}} = \sqrt{\frac{1}{2}gx}$$

$$\Rightarrow \quad \int_0^l \frac{dx}{\sqrt{\frac{1}{2}gx}} = \int_0^t dt$$

$$\Rightarrow \quad t = \sqrt{\frac{2}{g}} \times 2\sqrt{l_0}$$

$$\therefore \quad t = \sqrt{\frac{8l_0}{g}}$$

**15.** $\mu = \dfrac{dm}{dx} = kx$

$$\Rightarrow M = \int dm = \int_0^2 kx\, dx = \frac{1}{2}kL^2$$

$$\Rightarrow \quad k = \frac{2M}{L^2}$$

$$v(x) = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{kx}} = \sqrt{\frac{TL^2}{2Mx}} = \frac{dx}{dt}$$

$$\therefore \quad t = \int dt = \int_0^L \sqrt{\frac{2Mx}{TL^2}}\, dx = \sqrt{\frac{2M}{TL^2}}\ \frac{L^{\frac{1}{2}+1}}{\frac{1}{2}+1}$$

$$= \frac{2}{3}\sqrt{\frac{2ML^3}{TL^2}} = \frac{2}{3}\sqrt{\frac{2ML}{T}}$$

**16.** (a) $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{Mg}{\mu}}$

$$= \sqrt{\frac{1.5 \times 9.8}{0.055}} = 16.3 \text{ m/s}$$

(b) $\lambda = \dfrac{v}{\nu} = \dfrac{16.3 \text{ m/s}}{120/\text{s}} = 0.136 \text{ m}$

(c) $\lambda \propto v \propto \sqrt{T} \propto \sqrt{M}$ *i.e.*, if $M$ is doubled both speed and wavelength increases by a factor of $\sqrt{2}$.

**17.** $E = I\,At = 2\pi^2\nu^2 a^2 \rho\, vAt$

$$= 2\pi^2\nu^2 a^2 (\rho A)\,(v.t)$$

$$= 2\pi^2\nu^2 a^2 \mu.l$$

$$= 2\pi^2\nu^2 a^2 m$$

$$= 2 \times (3.14)^2 \times (120)^2 \times (0.16 \times 10^{-3})^2 \times 80 \times 10^{-3}$$

$$= 582 \times 10^{-6}\ \text{J} = 582\ \mu\text{J} = 0.58\ \text{mJ}$$

**18.** $P = \dfrac{E}{t} = IA = 2\pi^2\nu^2 a^2 \rho\nu A = 2\pi^2\nu^2 a^2 \mu v$

$$= 2\pi^2\nu^2 a^2 \sqrt{T\mu}$$

$$= 2 \times (3.14)^2 \times (60)^2 \times (6 \times 10^{-2})^2 \sqrt{80 \times 5 \times 10^{-2}}$$

$$= 4(3.14 \times 60 \times 0.06)^2 = 511.6 \text{ W}$$

**19.** $P = IA = 2\pi^2 \nu^2 a^2 \sqrt{T\mu}$

$= 2 \times (3.14)^2 \times (200)^2 \times 10^{-6} \sqrt{60 \times 6 \times 10^{-3}}$

$= 8 \times (3.14)^2 \times 10^{-2} \times 6 \times 10^{-1}$ W

$= 0.474$ W

$E = Pt = P \cdot \dfrac{l}{v}$

$= \dfrac{0.474 \times 2}{\sqrt{\dfrac{60}{6 \times 10^{-3}}}} = \dfrac{0.474 \times 2}{100}$ J $= 9.48$ mJ

**20.** $P = 2\pi^2 v^2 a^2 \rho v A = 2\pi^2 v^2 a^2 \mu v \,;\, v = \sqrt{\dfrac{T}{\mu}}$

$\mu = \dfrac{T}{v^2}$

$= 2\pi^2 \nu^2 a^2 \dfrac{T}{v^2} \cdot v = 2\pi^2 \nu^2 a^2 \dfrac{T}{v}$

$= \dfrac{2 \times (3.14)^2 \times (100)^2 (0.5 \times 10^{-3})^2 \times 100}{100}$

$= 2 \times (3.14)^2 \times 10^4 \times 0.25 \times 10^{-6}$

$= 4.93 \times 10^{-2}$ W $= 49$ mW

## ■ Objective Questions (Level 1)

**1.** $\omega = \dfrac{150 \times 2\pi}{60} = 5\pi$ rad/s, $A = 0.04$ m and $\theta = \dfrac{\pi}{4}$

$\therefore\ y = A \sin(\omega t + \theta) = 0.04 \sin\left(5\pi t + \dfrac{\pi}{4}\right)$

**2.** $\omega = 600\pi,\ v = 300 \Rightarrow k = \dfrac{\omega}{v} = 2\pi$

$\Rightarrow\ y = A \sin(\omega t - kx)$

$= 0.04 \sin(600\pi t - 2\pi x)$

$y(0.75, 0.01) = 0.04 \sin\left(600\pi \times 0.01 - 2\pi \times \dfrac{3}{4}\right)$

$= 0.04 \sin\left(6\pi - \dfrac{3\pi}{2}\right)$

$= 0.04 \sin\left(4\pi + \dfrac{\pi}{2}\right) = 0.04$ m

**3.** $y(x, t) = \dfrac{1}{2 + 3(kx - \omega t)^2}$

$y(x, 0) = \dfrac{1}{2 + 3k^2x^2} = \dfrac{1}{2 + 3x^2}$

$\Rightarrow\ k = 1$

$y(x, 2) = \dfrac{1}{2 + 3(x - 2\omega)^2} = \dfrac{1}{2 + 3(x - 2)^2}$

$\Rightarrow\ \omega = 1\ \therefore v = \dfrac{\omega}{k} = 1$ m/s

**4.** $y = A \sin(\omega t - kx) = \dfrac{A}{2}$

$\Rightarrow\ \omega t - kx = \dfrac{\pi}{6}$

$\therefore\ \dfrac{2\pi}{T} \cdot \dfrac{T}{6} - \dfrac{\pi}{6} = kx$

$\Rightarrow\ \dfrac{\pi}{6} = \dfrac{2\pi}{\lambda} \times 0.04$

$\therefore\ \lambda = 12 \times 0.04 = 0.48$ m.

**5.** $\lambda = \dfrac{v}{\nu} = \dfrac{300 \text{ m/s}}{25 \text{ Hz}} = 12$m

$\Delta\phi = \dfrac{2\pi}{\lambda} \Delta x = \dfrac{2\pi}{12\text{m}} (16 - 10)\text{m} = \pi$

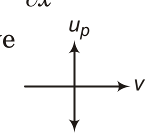
**6.** $y = 0.02 \sin(x + 30t) = A \sin(kx + \omega t)$

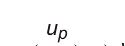
$\Rightarrow\ k = 1,\ \omega = 30$

$v = \dfrac{\omega}{k} = 30 \text{ m/s} = \sqrt{\dfrac{T}{\mu}}$

$\Rightarrow T = \mu v^2 = 1.3 \times 10^{-4} \times 900 = 0.117$ N

**7.** $v_p = \dfrac{\partial y}{\partial t} = \dfrac{\partial y}{\partial x} \cdot \dfrac{\partial x}{\partial t} = v \dfrac{\partial y}{\partial x} = \text{slope} \times v$

In transverse wave 

they are perpendicular *i.e.*, $\dfrac{\pi}{2}$. In longitudinal wave 

, they are either at 0 or $\pi$ so, 0, $\dfrac{\pi}{2}$ and $\pi$ are the possible angles between $v_p$ and $v$.

**8.** $\omega = 2\pi \nu = 200\pi$ rad/s,

$k = \dfrac{\omega}{v} = \omega \sqrt{\dfrac{\mu}{T}} = 200\pi \sqrt{\dfrac{3.5 \times 10^{-3}}{35}}$

$= 2\pi$ rad/m

$y = A\cos(\omega t - kx) = A\cos(200\pi t - 2\pi x)$

$\frac{\partial y}{\partial x} = 2\pi A \sin(200\pi t - 2\pi x)$

When, $y = 0$

$\Rightarrow \quad \sin(200\pi t - 2\pi x) = 0$

$\Rightarrow \quad \sin(200\pi t - 2\pi x) = 1$

$\therefore \quad 2\pi A = \frac{\pi}{20} \Rightarrow A = \frac{1}{40} = 0.025\,\text{m}$

$\therefore \quad y = 0.025\cos(200\pi t - 2\pi x)$

**9.** $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.25} = 8\pi$ rad/s;

$k = \frac{\omega}{v} = \frac{8\pi}{48} = \frac{\pi}{6}$ rad/cm

$y = A\sin(\omega t - kx)$

$= A\sin\left(8\pi \times 1 - \frac{\pi}{6} \times 67\right)$

$= A\sin\frac{\pi}{6} = A\sin 30° = \frac{A}{2} = 3$ cm

$\Rightarrow A = 6$ cm

**10.** $\frac{v_A}{v_B} = \sqrt{\frac{T_A}{\rho A_A} \cdot \frac{\rho A_B}{T_B}} = \sqrt{\frac{T_A \pi d_B^2}{T_B \pi d_A^2}}$

$= \frac{d_B}{d_A}\sqrt{\frac{T_A}{T_B}} = \frac{d_B}{d_B/2}\sqrt{\frac{T_B/2}{T_B}}$

$= 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$

**11.** $E \propto A^2\nu^2$ for $E$ to constant, $A\nu =$ constant

$A_A\nu_A = A_B\nu_B \Rightarrow A_A 4\nu_B = A_B\nu_B$

$\Rightarrow A_B = 4A_A$

**12.** $k = 1$ rad/m; $v = 4$ m/s

$\Rightarrow \quad \omega = vk = 4$ rad/s

$\therefore \quad y = \frac{6}{(kx - \omega t)^2 + 3} = \frac{6}{(x - 4t)^2 + 3}$

**13.** $v_l = \sqrt{\frac{Y}{\rho}}$ and $v_t = \sqrt{\frac{Y\frac{\Delta l}{l}}{\rho}} = v_l\sqrt{\frac{\Delta l}{l}}$

$\Rightarrow \quad \sqrt{\frac{l}{\Delta l}} = \frac{v_l}{v_t} = 10 \quad \therefore \quad \frac{\Delta l}{l} = \frac{1}{100}$

Stress $= Y\frac{\Delta l}{l} = E\frac{\Delta l}{l} = \frac{E}{100}$

**14.** $A = 4$ m, $\omega = \frac{\pi}{5}$, $k = \frac{\pi}{9}$, $\theta = \frac{\pi}{6}$

$\therefore \quad v = \frac{\omega}{k} = \frac{\pi/5}{\pi/9} = \frac{9}{5}$ m/s

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi/9} = 18$ m

$\nu = \frac{\omega}{2\pi} = \frac{\pi/5}{2\pi} = \frac{1}{10}$ Hz

**15.** $\omega = 10\pi$ and $k = 0.1\pi$

$\Rightarrow \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.1\pi} = 20$ m

$\therefore \quad \Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{20} \times 10 = \pi$

**16.** $y = \frac{2}{(2x - 6.2t)^2 + 20}$

$\Rightarrow \quad A = \frac{2}{20} = 0.1$ m, $k = 2$ rad/m

and $\quad \omega = 6.2$ rad/s

$\therefore \quad v = \frac{\omega}{k} = \frac{6.2}{2} = 3.1$ m/s

$\nu = \frac{\omega}{2\pi} = \frac{6.2}{2 \times 3.1} = 1$ Hz

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2} = \pi$ m

**17.** $I = 2\pi^2\nu^2 A^2\rho v = \frac{1}{2}\omega^2 A^2 \rho v$

$u = \frac{E}{V} = \frac{IST}{V} = \frac{2\pi^2\nu^2 A^2 \rho v\, St}{V}$

$= 2\pi^2\nu^2 A^2 \rho = \frac{1}{2}\rho\omega^2 A^2$

$P = \frac{E}{t} = I.S = 2\pi^2\nu^2 A^2 \rho v \cdot S$

$= \frac{1}{2}\rho\omega^2 A^2 v \cdot S$

$E = Pt \Rightarrow P = \frac{E}{t} = IS \Rightarrow I = \frac{P}{S}$

**18.** $y = A\sin(\pi x + \pi t)$

$y(x, 0) = A\sin(\pi x) \Rightarrow y = 0$ for $x = 0$ and 1

$a = -\omega^2 y = -\omega^2 A\sin(\pi x)$

$\Rightarrow a = \pm\omega^2 A$ at $x = \frac{1}{2}$ and $\frac{3}{2}$

$v_P = \pi A \cos(\pi x) \Rightarrow v_p = 0$ for $x = \frac{1}{2}$ and $\frac{3}{2}$

So all the above options are correct.

**19.** $y = A \sin \frac{2\pi}{a}(x - bt) = A \sin(kx - \omega t)$

$$k = \frac{2\pi}{a},\ \omega = \frac{2\pi b}{a}$$

$$\Rightarrow v = \frac{\omega}{k} = \frac{2\pi b/a}{2\pi/a} = b$$

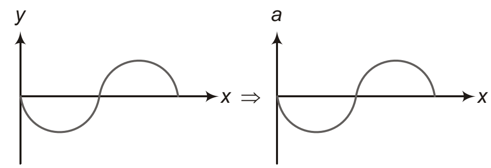
$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2\pi/a} = a$$

**20.** $y = A \sin 2\pi\left(\frac{x}{a} - \frac{t}{b}\right) = A \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$

$\Rightarrow \qquad \lambda = a,\ T = b$

$\Rightarrow \qquad v = \nu\lambda = \frac{\lambda}{T} = \frac{a}{b}$

**21.**



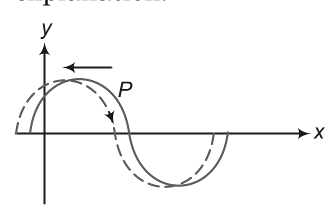

as $\qquad a = -\omega^2 y$

# JEE Corner

## Assertion and Reason

1. For propagation of transverse waves medium require tension which is possible due to modulus of rigidity. And in gases there is no such Young's modulus or surface tension. So the reason given is correct explanation.
2. Surface tension of water plays the role of modulus of rigidity and that is why transverse waves can travel on liquid surface.
3. Both the waves are travelling in same direction with a phase difference of $\pi$. So reason is false.
4. $v = f\lambda$ is constant for a particular medium so if frequency is doubled wavelength becomes half, and speed remains constant. Thus assertion is false.
5. Sound is mechanical wave which requires material medium for propagation and as on moon there is no atmosphere, sound cannot travel.
6. Angular wave number, $\bar{\nu} = \frac{2\pi}{\lambda}$ while wave number, $k = \frac{1}{\lambda}$ which is defined as the number of waves per unit length.
7. Electromagnetic wave are non-mechanical, they travel depending upon electric and magnetic properties of medium. They can travel in medium as well as an vacuum. So reason is false.
8. As speed, $v = \sqrt{\frac{T}{\mu}} \Rightarrow v \propto \frac{1}{\sqrt{\mu}}$ in second string $\mu$ is more (by looking) so $v$ will be less. Thus reason is true explanation of assertion.
9. At point $A$ both $v_p$ and $\Delta l$ is zero *ie*, K.E. and P.E. are minimum while at $B$ both $v_p$ and $\Delta l$ are maximum *i.e.*, both K.E. and P.E. are maximum. Thus both assertion and reason are true but not correct explanation.

**12.**




If $P$ is moving downword then it shows that the wave is travelling in (–) ve $x$ direction. So assertion is false.

**11.** $A = 2a \cos \frac{\Delta\phi}{2}$, for $A = a$

$\Rightarrow \quad \cos \frac{\Delta\phi}{2} = \frac{1}{2} = \cos \frac{\pi}{3}$

$\therefore \quad \Delta\phi = \frac{2\pi}{3} = \frac{360°}{3} = 120°$

Assertion is true but the reason is false.

## ■ Match the Columns

**1.** $y = a \sin (bt - cx) = A \sin (\omega t - kx)$

| | | |
|---|---|---|
| (a) | $v = \frac{\omega}{k} = \frac{b}{c}$ | $r$ |
| (b) | $(v_p)_{max} = A\omega = ab$ | $s$ |
| (c) | $\nu = \frac{\omega}{2\pi} = \frac{b}{2\pi}$ | $p$ |
| (d) | $\lambda = \frac{2\pi}{k} = \frac{2\pi}{c}$ | $s$ |

**2.** $y = 4 \text{ cm} \sin (\pi t + 2\pi x)$

$v_p = 4\pi \text{ cm/s} \cos (\pi t + 2\pi x)$

$a = -4\pi^2 \text{ cm/s}^2 \sin (\pi t + 2\pi x)$

| | | |
|---|---|---|
| (a) | $v_p (0, t) = 4\pi \text{ cm s} \cos \pi t$<br>$= \pm 4\pi$ for $\cos \pi t = \pm 1$<br>or $\pi t = n\pi \Rightarrow t = n = 0, 1, 2, 3, \rightarrow$ | $q, r$ |
| (b) | $a(0, t) = -4\pi^2 \text{ cm/s}^2 \sin \pi t$<br>$= \pm 4\pi^2$ for $\sin \pi t = \pm 1$<br>or $\pi t = (2n + 1)\frac{\pi}{2}$<br>$\Rightarrow t = n + \frac{1}{2} = 0.5, 2.5 \rightarrow$ | $p, s$ |
| (c) | $v_p (0.5, t) = 4\pi \text{ cm/s} \cos (\pi t + \pi)$<br>$= \pm 4\pi$ for $\pi t + \pi = n\pi$<br>or $t = n - 1 = 0, 1, 2, 3, \rightarrow$ | $q, r$ |
| (d) | $a (0.5, t) = -4\pi^2 \text{ cm/s}^2$<br>$= \pm 4\pi^2 \Rightarrow \pi t + \pi = (2n + 1)\frac{\pi}{2}$<br>or $t = n - \frac{1}{2} = 0.5, 1.5, 2.5 \rightarrow$ | $p$ |

**3.** $y = A \sin (\omega t \pm kx)$ at $t = 0$

$\Rightarrow y = \pm A \sin kx$

$v_p = \pm \omega A \cos kx$ and $a = -\omega^2 y$

| | | |
|---|---|---|
| (a) | $v_p = \pm \omega A \cos kx \rightarrow$ | $s$ |
| (b) | $a_A = (+) v_e$ as $y_A$ is negative $\rightarrow$ | $p$ |
| (c) | $v_B = \pm A\omega \rightarrow$ | $s$ |
| (d) | $a_B = 0$ ve $y_0 = 0 \rightarrow$ | $r$ |

**4.** (a) $u = \frac{E}{V} = \frac{IST}{V} = \frac{2\pi^2 \nu^2 A^2 \rho v s t}{V}$

$= 2\pi^2 \nu^2 A^2 \rho = \frac{1}{2} \rho \omega^2 A^2$

$[u] = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}] \rightarrow s$

(b) $P = \frac{E}{t} = \frac{ISt}{t} = IS = 2\pi^2 \nu^2 A^2 \rho v s$

$= \frac{1}{2} \omega^2 A^2 \rho v s = \frac{1}{2} \rho \omega^2 A^2 s\, v \rightarrow q$

$[P] = \frac{E}{t} = \frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}] \rightarrow p$

(c) $I = \frac{E}{St} = \frac{[ML^2T^{-2}]}{[L^2T]}$

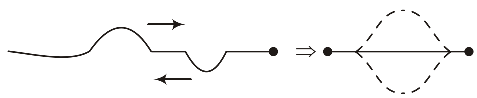
$= [MT^{-3}] = [ML^0T^{-3}] \rightarrow s$

(d) $\frac{1}{\lambda} = [L^{-1}] = [M^0L^{-1}T^0] \rightarrow s$

| | | | |
|---|---|---|---|
| **5.** | (a) | $y = A \sin (\omega t - kx) \rightarrow$ | $p$ |
| | | $v_p = \omega A \cos (\omega t - kx) \rightarrow$ | $r$ |
| | | $a = -\omega^2 A \sin (\omega t - kx)$ | |
| | (b) | $y = A \sin (kx - \omega t) \rightarrow$ | $p$ |
| | | $v_p = -\omega A \cos (kx - \omega t)$ | |
| | | $a = -\omega^2 A \sin (kx - \omega t)$ | |
| | (c) | $y = -A \cos (\omega t + kx) \rightarrow$ | $q$ |
| | | $v_p = \omega A \sin (\omega t + kx)$ | |
| | | $a = \omega^2 A \cos (\omega t + kx) \rightarrow$ | $s$ |
| | (d) | $y = -A \cos (kx - \omega t) \rightarrow$ | $p$ |
| | | $v_p = -A\, \omega \sin (kx - \omega t)$ | |
| | | $a = \omega^2 A \cos (kx - \omega t) \rightarrow$ | d $s$ |

# 15. Superposition of Waves

## Introductory Exercise 15.1

**1.** 

When displacement of all the particles is momentarily zero, then there is no elastic potential energy stored in the string and as the speed is maximum at mean position, so entire energy is purely kinetic.

**2.** (a) $v = \sqrt{\frac{T}{\mu}}$

$\Rightarrow \quad \frac{v_2}{v_1} = \sqrt{\frac{\mu_1}{\mu_2}}$

$= \sqrt{\frac{\mu_1}{0.25\,\mu_1}} = \frac{1}{\sqrt{0.25}}$

$= \frac{1}{0.5} = 2$

$\Rightarrow \quad v_2 = 2v_1 = 20 \text{ cm/s}$

(b) $a_t = \frac{2v_2}{v_1 + v_2} a_i$

$= \frac{2 \times 20}{10 + 20} a_i = \frac{4}{3} a_i$

and $a_r = \frac{v_2 - v_1}{v_2 + v_1} a_i$

$= \frac{20 - 10}{20 + 10} a_i = \frac{1}{3} a_i$

**3.** (a) For fixed end, a phase change of $\pi$ takes place in reflected wave and direction becomes opposite.

as $\quad Y_i = 0.3 \cos (2x - 40t)$

$\Rightarrow \quad Y_r = 0.3 \cos (2x + 40t + \pi)$

(b) For free end, there is no change in phase for reflected wave and direction becomes opposite.

as $\quad Y_i = 0.3 \cos (2x - 40t)$

$\Rightarrow \quad Y_r = 0.3 \cos (2x + 40t)$

**4.** $v_1 = \frac{\omega}{k_1} = \frac{50}{2} = 25 \text{ m/s}$ and $v_2 = 50 \text{ m/s}$

$\Rightarrow \quad a_t = \frac{2v_2}{v_1 + v_2} a_i = \frac{2 \times 50}{25 + 50} a_i$

$= \frac{4}{3} \times 2 \times 10^{-3} \text{ m} = \frac{8}{3} \text{ mm.}$

$a_r = \frac{v_2 - v_1}{v_2 + v_1} a_i = \frac{50 - 25}{50 + 25} a_i$

$= \frac{1}{3} \times 2 \times 10^{-3} \text{ m} = \frac{2}{3} \text{ mm.}$

as $v_2 > v_1 \Rightarrow$ the boundary is rearer and there is no phase change.

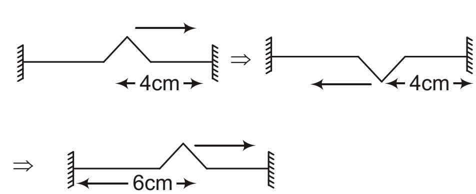
$k_2 = \frac{\omega}{v_2}$

$= \frac{50\,\pi}{50} = \pi$

$\therefore \quad y_r = \frac{2}{3} \times 10^{-3} \cos \pi (0.2\,x + 50t)$

and $y_t = \frac{8}{3} \times 10^{-3} \cos \pi (1.0\,x - 50t)$

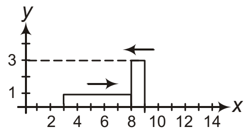
**5.** $t_1 = \frac{2 \times 40 \text{ cm}}{1 \text{ cm/s}} = 8$ s, inverted



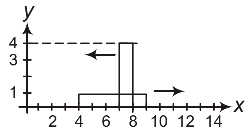
$$t_2 = \frac{4+10+6}{1}$$

$= 20\,\text{s}$ upright

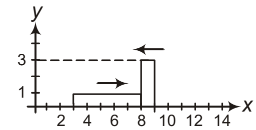
**6.**



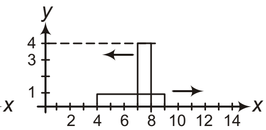

$t$ = 2s



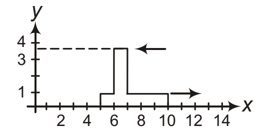

$t$ = 3s



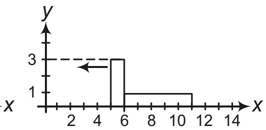

$t$ = 2s




$t$ = 3s




$t$ = 2s




$t$ = 5s

# Introductory Exercise 15.2

**1.** $y = 5 \sin \frac{\pi x}{3} \cos 40\pi t = 2a \sin kx \cos \omega t$

$a = \frac{5}{2} = 2.5\,\text{cm},\ k = \frac{\pi}{3}\,\text{cm}^{-1},\ \omega = 40\,\pi\text{s}^{-1}$

$$v = \frac{\omega}{k} = \frac{40\pi}{\pi/3} = 120\,\text{cm/s}$$

$$\Delta x = \frac{\lambda}{2} = \frac{1}{2}\cdot\frac{2\pi}{k} = \frac{\pi}{k} = \frac{\pi}{\pi/9}\,\text{cm} = 3\,\text{cm}$$

$$v_P = \frac{dy}{dt} = -200\,\pi \sin \frac{\pi x}{3} \sin 40\pi t$$

$$v_P\left(1.5, \frac{9}{8}\right) = -200\,\pi \sin\left(\frac{\pi}{3}\cdot\frac{3}{2}\right) \sin\left(40\,\pi \times \frac{9}{8}\right)$$

$$= -200\,\pi \sin\left(\frac{\pi}{2}\right) \sin(45\,\pi)$$

$$= -200\,\pi \times 1 \times 0$$

$$= 0\,\text{cm/s}$$

**2.** Two waves with different amplitudes can produce partial stationary waves with amplitude of antinodes being $a_1 + a_2$ and amplitude of nodes being $a_1 \sim a_2$. As here node is not stationary that is why energy is also transported through nodes.

**3.** (a) $\frac{\lambda}{2} = 2\,\text{m} \Rightarrow \lambda = 4\,\text{m}$,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{100}{4\times10^{-2}}} = \frac{10^2}{2} = 50\,\text{m/s}$$

$$\nu = \frac{v}{\lambda} = \frac{50}{4} = 12.5\,\text{Hz}$$

and is fundamental tone or first harmonic.

$$y = 0.1 \sin \frac{2\pi}{\lambda} x \sin 2\pi\nu t$$

$$= 0.1 \sin \frac{2\pi}{4} x \sin 2\pi \times 12.5\, t$$

$$= 0.1 \sin \frac{\pi}{2} x \sin 25\,\pi t$$

(b) $3\,\frac{\lambda}{2} = 2\,\text{m} \Rightarrow \lambda = \frac{4}{3}\,\text{m}$ and $v = 50\,\text{m/s}$

$\nu = \frac{v}{\lambda} = \frac{50}{4/3}\,\text{Hz} = 37.5\,\text{Hz}$ and is 2nd overtone or 3rd harmonic.

$$y = 0.04 \sin \frac{2\pi}{4/3} x \sin 2\pi \times 37.5\, t$$

$$= 0.04 \sin \frac{3\pi}{2} x` \sin 75\,\pi t$$

**4.** $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{Fl}{m}} = \sqrt{\frac{400\times 4}{160\times10^{-3}}}$

$$= \sqrt{\frac{1600}{16\times10^{-2}}} = 10^2 = 100\,\text{m/s}$$

(a) $\frac{\lambda_0}{4} = l \Rightarrow \lambda_0 = 4l = 16\,\text{m}$

$$\frac{3\lambda_1}{4} = l \Rightarrow \lambda_1 = \frac{4l}{3} = \frac{16}{3}\text{ m} = 5.33\text{ m}$$

and $\frac{5\lambda_2}{4} = l \Rightarrow \lambda_2 = \frac{4l}{5} = \frac{16}{5}\text{ m} = 3.2\text{ m}$

(b) $\nu_0 = \frac{v}{\lambda_0} = \frac{100}{16} = 6.25\text{ Hz}$

$$\nu_1 = \frac{v}{\lambda_1} = \frac{100}{16/3} = 18.75\text{ Hz}$$

$$\nu_3 = \frac{v}{\lambda_3} = \frac{100}{16/5} = 31.25\text{ Hz}$$

**5.** $l = n\frac{\lambda}{2} = \frac{0.54}{2}n = 0.27\,n$

and $l = (n+1)\frac{\lambda'}{2} = \frac{0.48}{2}(n+1)$

$$= 0.24\,(n+1)$$

$\Rightarrow 0.27\,n = 0.24\,n + 0.24$

$\Rightarrow 0.03\,n = 0.24 \Rightarrow n = 8$

(a) These are 8th and 9th harmonic

(b) $l = 0.27\,n = 0.27 \times 8 = 2.16\text{ m}$

(c) $\frac{\lambda_0}{2} = l \Rightarrow \lambda_0 = 2l = 4.32\text{ m}$

**6.** $5\nu_0 - 2\nu_0 = 54\text{ Hz}$

$\Rightarrow 3\nu_0 = 54\text{ Hz} \Rightarrow \nu_0 = 18\text{ Hz}$

**7.** $\nu_0 = \frac{1}{2l}\sqrt{\frac{F}{\mu}}$

$$\Rightarrow \frac{\nu_2}{\nu_1} = \sqrt{\frac{F_2}{F_1}}$$

$$= \sqrt{\frac{M+2.2}{2.2}} = \frac{260}{220} = \frac{13}{11}$$

$$\Rightarrow \frac{M+2.2}{2.2} = \frac{169}{121} = 1 + \frac{48}{121}$$

$$= 1 + \frac{M}{2.2}$$

$$\Rightarrow M = \frac{48 \times 2.2}{121} = \frac{9.6}{11} = 0.873\text{ kg}$$

**8.** $n\nu_0 = 250\text{ Hz}$ and $(n+1)\nu_0 = 300\text{ Hz}$

$\Rightarrow \nu_0 = 50\text{ Hz}$

and $n = 5 \Rightarrow$ So these are 5th and 6th harmonics.

$$\nu_0 = \frac{1}{2l}\sqrt{\frac{F}{\mu}}$$

$$\Rightarrow F = 4l^2 v_0^2 \mu = 4 \times 50^2 \times \frac{36 \times 10^{-3}}{1} = 360\text{ N}$$

# AIEEE Corner

## ■ Subjective Question (Level 1)

**1.** $A = \sqrt{A_1^2 + A_1^2 + 2A_1\,A_1\cos 90^\circ}$

$$= A_1\sqrt{2} = 4\sqrt{2}\text{ cm} = 5.66\text{ cm}$$

**2.** $v_2 = 2\,v_1$

$$A_r = \frac{v_2 - v_1}{v_2 + v_1}A = \frac{v_1}{3v_1}A = \frac{1}{3}A$$

$$A_t = \frac{2v_2}{v_2 + v_1}A = \frac{4v_1}{3v_1}A = \frac{4}{3}A$$

$$\frac{I_r}{I_i} = \left(\frac{A_r}{A}\right)^2 = \frac{1}{9}$$

and $\frac{I_t}{I_i} = 1 - \frac{1}{9} = \frac{8}{9}$

$$A = \sqrt{10^2 + 20^2 + 2 \times 10 \times 20\cos\frac{\pi}{3}}$$

$$= \sqrt{100 + 400 + 200} = \sqrt{700} = 10\sqrt{7}$$

$$= 26.46\text{ units}$$

$$\tan\theta = \frac{20\sin\frac{\pi}{3}}{10 + 20\cos\frac{\pi}{3}}$$

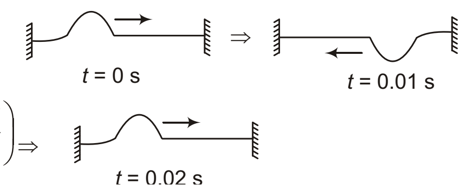
$$= \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right) = 0.714\text{ rad}$$

$\therefore$ Phase $= 5x + 25t + 0.714$ rad.

**4.** $y_1 = 1\text{ cm}\sin(\pi\text{ cm}^{-1}x - 50\,\pi\text{s}^{-1}t)$

$$y_2 = 1.5 \text{ cm} \sin\left(\frac{\pi}{2} \text{cm}^{-1} x - 100\,\pi \text{s}^{-1} t\right)$$




$\Rightarrow$

$$y_1\,(4.5, 5 \times 10^{-3}) = 1 \text{ cm} \sin\left(4.5\,\pi - \frac{250\,\pi}{1000}\right)$$

$$= 1 \sin\left(\frac{9}{2}\pi - \frac{\pi}{4}\right)$$

$$= 1 \sin\left(\frac{17\pi}{4}\right)$$

$$= 1 \text{ cm} \sin\left(4\pi + \frac{\pi}{4}\right)$$

$$= 1 \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \text{ cm and}$$

$$y_2\,(4.5, 5 \times 10^{-3}) = 1.5 \text{ cm} \sin\left(\frac{9\pi}{4} - \frac{500\,\pi}{1000}\right)$$

$$= 1.5 \text{ cm} \sin\left(\frac{9\pi}{4} - \frac{\pi}{2}\right)$$

$$= 1.5 \sin\left(\frac{5\pi}{4}\right)$$

$$= 1.5 \sin\left(\pi + \frac{\pi}{4}\right)$$

$$= -1.5 \sin\left(\frac{\pi}{4}\right)$$

$$= -\frac{1.5}{\sqrt{2}} \text{ cm}$$

$$\therefore \quad y = y_1 + y_2 = \frac{1}{\sqrt{2}} - \frac{1.5}{\sqrt{2}}$$

$$= -\frac{0.5}{\sqrt{2}} = -\frac{1}{2\sqrt{2}} \text{ cm}$$

**5.** $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{16\text{N}}{0.4 \times 10^{-3} \times 10^2 \text{ kg/N}}}$

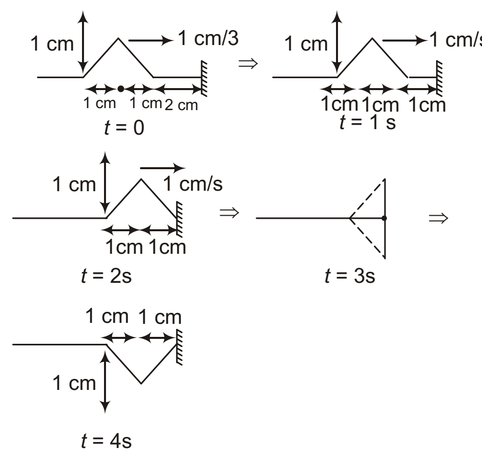
$$= \sqrt{\frac{16 \times 10^2}{4}} = 20 \text{ m/s}$$

(a) For same shape, time,

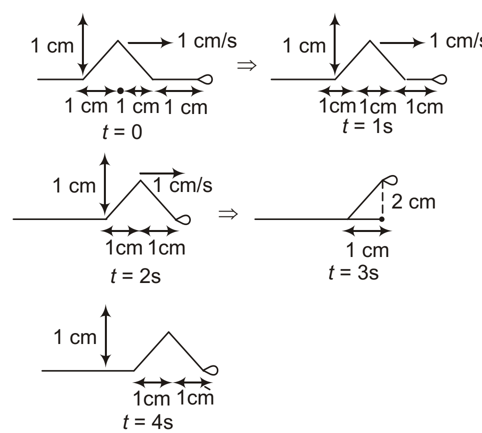
$$t = \frac{2l}{v} = \frac{2 \times 0.2}{20} \text{ s} = 0.02 \text{ s}$$

(b)

**6.** (a)




(b)




**7.** $y = 1.5 \sin(0.4\,x) \cos(200t)$

$$= 2A \sin kx \cos \omega t$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.4} = 5\pi \text{ m} = 15.7 \text{ m}$$

$$\nu = \frac{\omega}{2\pi} = \frac{200}{2\pi} = \frac{100}{\pi} \text{ Hz} = 31.8 \text{ Hz}$$

$$v = \frac{\omega}{k} = \frac{200}{0.4} = 500 \text{ m/s}$$

**8.** $y = y_1 + y_2 = 3\text{ cm}\sin(\pi x + 0.6\,\pi t) + 3\text{ cm}\sin(\pi x - 0.6\,\pi t)$

$= 6\text{ cm}\sin\pi x\cos 0.6\,\pi t = R\cos 0.6\,\pi t$

where, $R = 6\text{ cm}\sin\pi x.$

(a) $R\,(0.25) = 6\text{ cm}\sin\pi\times\frac{1}{4}$

$= \frac{6}{\sqrt{2}} = 3\sqrt{2}\text{ cm} = 4.24\text{ cm}$

(b) $R\,(0.50) = 6\text{ cm}\sin\pi\times\frac{1}{2} = 6\text{ cm}$

(c) $R\,(1.50) = 6\text{ cm}\sin\frac{3\pi}{2} = -6\text{ cm}$

$\Rightarrow\quad |R| = 6\text{ cm}$

(d) For antinodes, $R = \pm 6\text{ cm}$

$\Rightarrow\quad \sin\pi x = \pm 1 \Rightarrow \pi x = (2n+1)\frac{\pi}{2}$

or $x = n + \frac{1}{2} = 0.5\text{ cm}, 1.5\text{ cm}, 2.5\text{ cm}$

**9.** $\lambda = \frac{2\pi}{4} = \frac{2\pi}{\pi/2} = 4\text{ cm}$

(a) Distance between successive antinodes $= \frac{\lambda}{2} = 2\text{ cm}$

(b) $R(x) = 2A\sin kx$

$= 2\times\pi\text{ cm}\sin\frac{\pi}{2}\times 0.5$

$= 2\pi\sin\frac{\pi}{4}$

$= \frac{2\pi}{\sqrt{2}} = \sqrt{2}\,\pi\text{ cm}$

**10.** $\nu_n = \frac{n+1}{2l}\sqrt{\frac{T}{\mu}} = \frac{n+1}{2\times 20}\sqrt{\frac{20}{9\times 10^{-3}}}$

$= \frac{n+1}{60}\times\frac{100\sqrt{2}}{3} = \frac{5\sqrt{2}}{9} = \frac{5\sqrt{2}}{9}(n+1)$

$= 0.786\,(n+1)$

$= 0.786\text{ Hz}$

1.57 Hz, 2.36 Hz, 3.14 Hz

**11.** (a) $T = \mu v^2 = \mu\nu^2\lambda^2$

$= \frac{1.2\times 10^{-3}}{0.7}\times(220)^2\times(1.4)^2$

$= 162.6\text{ N}$

(b) $\nu_2 = 3\,\nu_0 = 3\times 220\text{ Hz} = 660\text{ Hz}$

**12.** $\nu_n = \frac{n+1}{2l}\sqrt{\frac{T}{\mu}} = \frac{n+1}{2\times 0.6}\sqrt{\frac{50}{0.01}}$

$= \frac{50\sqrt{2}}{1.2}(n+1) = 58.93\,(n+1)\text{ Hz}$

$\nu_n \le 20{,}000\text{ Hz} \Rightarrow n = 338$

$\therefore \nu_{338} = 339\times 58.93 = 19975.8\text{ Hz}$

$= 19.976\text{ kHz}$

**13.** $n\nu_0 = 420\text{ Hz}$ and $(n+1)\,\nu_0 = 490\text{ Hz}$

$\Rightarrow \nu_0 = 70\text{ Hz}$ and $n = 6$

$\therefore \nu_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \Rightarrow l = \frac{1}{2\nu_0}\sqrt{\frac{T}{\mu}} = \sqrt{\frac{\frac{450}{0.005}}{2\times 70}}$

$= \frac{300}{140} = 2.143\text{ m}$

**14.** $\lambda = \frac{v}{\nu} = \frac{400\text{ m/s}}{800\text{ Hz}} = \frac{1}{2}\text{ m},$

$l = 4\frac{\lambda}{2} = 2\lambda = 1\text{ m}$

(a) $4\nu_0 = 400\text{ Hz} \Rightarrow \nu_0 = 100\text{ Hz}$

(b) $7\nu_0 = 700\text{Hz}$

**16.** $\nu_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

$\Rightarrow\quad \nu_0 \propto \frac{1}{l}$

$\nu_1 : \nu_2 : \nu_3 = 1 : 2 : 3$

$= \frac{1}{l_1} : \frac{1}{l_2} : \frac{1}{l_3}$

$\Rightarrow\quad l_1 : l_2 : l_3 = \frac{1}{1} : \frac{1}{2} : \frac{1}{3}$

$= 6 : 3 : 2 = 6x : 3x : 2x$

$6x + 3x + 2x = 1\text{ m}$

$\Rightarrow\quad x = \frac{1}{11}\text{ m}$

$\therefore$position of first bridge $= 6x = \frac{6}{11}\text{ m}$

and position of second bridge

$= 6x + 3x = 9x = \frac{9}{11}\text{ m}$

From the same end or $1 - \frac{9}{11} = \frac{2}{11}\text{ m}$

from other end .

**17.** $\nu_0 = \dfrac{v}{2l}$

$\Rightarrow \quad \nu'_0 = \dfrac{v}{2l'}$

$\Rightarrow \quad l' = \dfrac{\nu_0}{\nu'_0}\, l = \dfrac{124}{186} \times 90 \text{ cm} = 60 \text{ cm}$

Thus length of the vibrating string has to be 60 cm.

**18.** $\dfrac{\lambda}{2} = 15 \text{ cm} \Rightarrow \lambda = 30 \text{ cm},$

$R_{\max} = 2A = 0.85 \text{ cm},$

$T = 0.075 \text{ s}$

(a) $y = 2A \sin kx \sin \omega t$

$= 0.85 \text{ cm} \sin\left(\dfrac{2\pi}{0.3 \text{ m}}\, x\right) \sin\left(\dfrac{2\pi}{0.075 \text{ s}}\, t\right)$

(b) $v = \dfrac{\omega}{k} = \dfrac{2\pi/0.075}{2\pi/0.3} = \dfrac{0.3}{0.075} = 4 \text{ m/s}$

(c) $\dfrac{\lambda}{4} = \dfrac{30}{4} = 7.5 \text{ cm}$

$\therefore \quad R\,(7.5 + 3) = 2A \sin kx$

$= 0.85 \sin \dfrac{2\pi}{30} \times 10.5$

$\therefore \quad R(11.5 \text{ cm}) = 0.85 \sin\left(\dfrac{21\pi}{30}\right)$

$= 0.85 \sin\,(0.7\,\pi)$

$= 0.85\,(126°) = 0.688 \text{ cm}$

**19.** $\nu_0 = \dfrac{v}{2l} = \dfrac{48}{2 \times 1.5} = 16 \text{ Hz}$

and $\quad \lambda_0 = 2l = 3 \text{ m}$

$\nu_2 = 3\nu_0 = 48 \text{ Hz and } \lambda_2 = \dfrac{v}{\nu_2} = \dfrac{48}{48} = 1 \text{ m}$

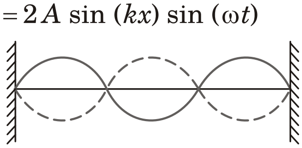
$\nu_3 = 4\,\nu_0 = 64 \text{ Hz}$

and $\lambda_3 = \dfrac{v}{\nu_3} = \dfrac{48}{64} = \dfrac{3}{4} = 0.75 \text{ m}$

**20.** $y = 5.60 \text{ cm} \sin\,(0.340 \text{ rad/cm } x)$

$\sin\,(50.0 \text{ rad/s } t)$

$= 2A \sin\,(kx) \sin\,(\omega t)$

(a) 

(b) $2A = 5.60 \text{ cm} \Rightarrow A = 2.80 \text{ cm}$

(c) $l = 3\,\dfrac{\lambda}{2} = \dfrac{3}{2} \cdot \dfrac{2\pi}{k} = \dfrac{3\pi}{k}$

$= \dfrac{3\pi}{0.0340} \text{ cm} = 277.2 \text{ cm}$

(d) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{0.0340} \text{ cm} = 184.8 \text{ cm}$

$\nu = \dfrac{\omega}{2\pi} = \dfrac{50}{2\pi} = 7.96 \text{ Hz}$

$T = \dfrac{1}{\nu} = \dfrac{1}{7.96} \text{ s} = 0.216 \text{ s}$

$v = \nu\lambda = 7.96 \text{ Hz} \times 184.8 \text{ cm}$

$= 1470 \text{ cm/s}$

(e) $(v_p)_{\max} = R_{\max}\ \omega = 2A\omega$

$= 5.60 \text{ cm} \times 50 \text{ rad/s}$

$= 280 \text{ cm/s}$

(f) for eight harmonic,

$8\,\dfrac{\lambda}{2} = l \Rightarrow \lambda' = \dfrac{l}{4} = \dfrac{277.2}{4} = 69.3 \text{ cm}$

$k = \dfrac{2\pi}{\lambda'} = \dfrac{2 \times 3.14}{69.3} = 0.0907 \text{ rad/cm}$

$v' = 8\, v_0 = \dfrac{8}{3} \cdot v = \dfrac{8}{3} \times 7.96 \text{ Hz}$

$= 21.22 \text{ Hz}$

$\omega' = 2\pi\nu = 133.4 \text{ rad/s}$

$\Rightarrow \ y = 5.60 \text{ cm} \sin\,(0.0907 \text{ rad/s} \cdot x)$

$\sin \cdot (133 \text{ rad/s} \cdot t)$

**21.** (a) $v = \nu\lambda = \nu_0 \cdot 2l = 60 \times 2 \times 0.8$

$= 96 \text{ m/s}$

(b) $T = \mu v^2 = \dfrac{40 \times 10^{-3}}{80 \times 10^{-2}} \times (96)^2$

$= \dfrac{96^2}{20} = 460.8 \text{ N}$

(c) $(v_p)_{\max} = R_{\max}\ \omega$

$= 0.3 \text{ cm} \times 2\pi$

$\times 60 \text{ rad/s} = 113 \text{ cm/s} = 1.13 \text{ m/s}$

$a_{\max} = \omega^2 R_{\max} = (120\,\pi)^2 \times 0.3 \text{ cm/s}^2$

$= 426.4 \text{ m/s}^2$

## Objective Questions (Level 1)

**1.** $\dfrac{\nu_2}{\nu_1} = \sqrt{\dfrac{T_2}{T_1}} \Rightarrow \dfrac{3}{2} = \sqrt{\dfrac{T+2.5}{T}}$

$\Rightarrow \quad 9T = 4\,(T + 2.5)$

$\Rightarrow \quad 5T = 10 \Rightarrow T = 2\text{N}$

**2.** $\nu = \dfrac{n+1}{2l}\sqrt{\dfrac{T}{\mu}} = \dfrac{n+1}{2l}\sqrt{\dfrac{T}{\pi r^2 \rho}}$

$= \dfrac{n+1}{2lr}\sqrt{\dfrac{T}{\pi\rho}} = \dfrac{n+1}{ld}\sqrt{\dfrac{T}{\pi\rho}} = \text{constant.}$

$n + 1 \propto \dfrac{ld}{\sqrt{T}}$

$\dfrac{n_1+1}{n_2+1} = \dfrac{l_1}{l_2}\cdot\dfrac{d_1}{d_2}\sqrt{\dfrac{T_2}{T_1}}$

$= \dfrac{1}{2} \times \dfrac{1}{3} \times \sqrt{2}$

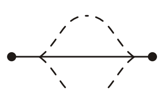
$= \dfrac{1}{3\sqrt{2}} = 1 : 3\sqrt{2}$

or $\dfrac{n_2+1}{n_1+1} = 3\sqrt{2}$

**3.** $f \propto \dfrac{1}{l}; l = l_1 + l_2 + l_3$

$\Rightarrow \dfrac{1}{f_0} = \dfrac{1}{f_1} + \dfrac{1}{f_2} + \dfrac{1}{f_3}$

**4.** During overlapping the displacement of particles is zero while velocity is maximum. So the entire energy is purely kinetic.



**5.** $y\,(x, y) = y_1 + y_2 = a\cos(kx + \omega t) + y_2$

$= -2a \sin kx \sin \omega t$ is necessary for a node at $x = 0$. Thus,

$y_2 = 2a \sin kx \sin \omega t - a\cos(kx + \omega t)$

$= -2a \sin kx \sin \omega t - a \cos kx \cos \omega t + a \sin kx \sin \omega t$

$= -a[\cos kx \cos \omega t + \sin kx \sin \omega t]$

$= -a\cos(kx - \omega t)$

**6.** In transverse stationary wave, longitudinal strain is maximum at node. While in longitudinal stationary wave at displacement node pressure and density are maximum. So all are correct.

**7.** In stationary wave all particles errors the mean position simultaneously and are at their maximum displacement simultaneously at different instant at this time all of them are at rest. So all are correct.

**8.** Maximum displacement

$y_{max} = 3A - A + 2A = 4A$

$\dfrac{v_t}{v_l} = \dfrac{v_t}{v_l} = \dfrac{\sqrt{\dfrac{\frac{Y\,\Delta l}{l}}{\rho}}}{\sqrt{\dfrac{Y}{\rho}}} = \sqrt{\dfrac{\Delta l}{l}}$

$= \sqrt{\dfrac{\frac{1}{\eta}\,l}{l}} = \dfrac{1}{\sqrt{\eta}}$

**10.** $f_n = \dfrac{n+1}{2 \times 1}\sqrt{\dfrac{100}{0.01}} = 50\,(n+1)$

$= 50\text{ Hz}, 100\text{ Hz}, 150\text{ Hz}$

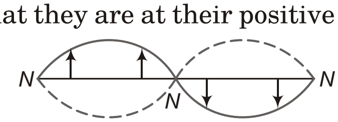
$n_n = \dfrac{2n+1}{4 \times 1}\sqrt{\dfrac{100}{0.01}} = 25\,(2m+1)$

$= 25\text{ Hz}, 75\text{Hz}, 125\text{ Hz}$

$\therefore \; n_2 = 75\text{ Hz} = \dfrac{f_1 + f_2}{2}$

$= \dfrac{50\text{ Hz} + 100\text{Hz}}{2} = 75\text{ Hz}$

**11.** In stationary waves all particles perform SHM such that they are at their positive and negative extremes one time each in a time period, where they come to rest. Particles between two successive nodes are in phase while beside node are in opposite phase. So all the particles cannot be at positive extreme simultaneously.




**12.** The question is wrong, string has to be fix at one end and free at other. Then

$(2n+1)\,\nu_0 = 90\text{ Hz}, (2n+3)\,\nu_0 = 50\text{ Hz}$

and $(2n+5)\,\nu_0 = 210\text{ Hz}$

$\Rightarrow 2\nu_0 = 60$ Hz or $\nu_0 = 30$ Hz and $n = 1$

*i.e.*, vibrations are 3rd, 5th and 7th harmonic.

$$\lambda_0 = 2l = 1.6\,\text{m}$$

$$\therefore \quad v = \nu_0\lambda_0 = 30\,\text{Hz} \times 1.6\,\text{m} = 48\,\text{m/s}$$

**13.** $y = y_1 + y_2 + y_3 = 12\sin\left(\theta - \frac{\pi}{2}\right) + 6\sin(\theta + 0) + 4\sin\left(\theta + \frac{\pi}{2}\right)$

$= 6\sin\theta - 12\cos\theta + 4\cos\theta$

$= 6\sin\theta - 8\cos\theta$

$\Rightarrow R = \sqrt{6^2 + 8^2 + 2\times 6\times 8\cos\frac{\pi}{2}}$

$= \sqrt{100} = 10\,\text{mm}$

**14.** $(2n+1)\,\nu_0 = 105$ Hz

and $(2n+3)\,\nu_0 = 175$ Hz

$\Rightarrow \quad 2\nu_0 = 70$ Hz

$\Rightarrow \quad \nu_0 = 35$ Hz

**15.** $l_1 : l_2 : l_3 = \frac{1}{\nu_1} : \frac{1}{\nu_2} : \frac{1}{\nu_3} = \frac{1}{1} : \frac{1}{3} : \frac{1}{4}$

$= 12 : 4 : 3$

$\therefore\ 12x + 4x + 3x = 114$ cm

$\Rightarrow \quad x = \frac{114}{19}$ cm $= 6$ cm

$\therefore\ l_1 = 12x = 72$ cm, $l_2 = 4x = 24$ cm, $l_3 = 3x = 18$ cm

**16.** $f \propto \sqrt{T}$

$$\frac{f/2}{f} = \sqrt{\frac{V\rho g - V\sigma_1 g}{V\rho g}} = \sqrt{1 - \frac{\sigma_2}{\rho}} = \frac{1}{2}$$

$\Rightarrow 1 - \frac{\sigma_1}{\rho} = \frac{1}{4} \Rightarrow \frac{\sigma_1}{\rho} = \frac{3}{4}$

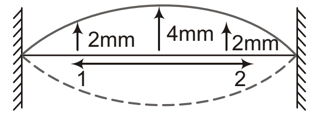
$$\frac{f/3}{f} = \sqrt{\frac{V\rho g - V\sigma_2 g}{V\rho g}} = \sqrt{1 - \frac{\sigma_2}{\rho}} = \frac{1}{3}$$

$\Rightarrow \quad 1 - \frac{\sigma_2}{\rho} = \frac{1}{q} \Rightarrow \frac{\sigma_2}{\rho} = \frac{8}{9}$

$\therefore \quad \frac{\sigma_1}{\sigma_2} = \frac{3/4}{8/9} = \frac{27}{32} \Rightarrow \frac{\sigma_2}{\sigma_1} = \frac{32}{27} = 1.18$

where $\sigma_1$ is density of water and $\sigma_2$ is density of the other liquid.

**17.** $R = 2A\sin Kx = 4\,\text{mm}\sin\frac{2\pi}{\lambda}x$




$= 4\,\text{mm}\sin\frac{2\pi x}{3\,\text{m}}$

$2\,\text{mm} = 4\,\text{mm}\sin\frac{2\pi x}{3}$

$\Rightarrow \quad \frac{2\pi x}{3} = \frac{\pi}{3}$

$\Rightarrow \quad x = 0.5\,\text{m}$

Thus points 1 and 2 are at 0.5 m from their nearest boundary. So separation between them is

$1.5\,\text{m} - 2\times 0.5\,\text{m} = 0.5\,\text{m} = 50\,\text{m}$

**18.** $y = -A\sin(\omega t - kx)$

$= -A\sin\left(2\pi\nu t - \frac{2\pi\nu}{v}x\right)$

$= -A\sin(6\pi t - 2\pi x)$

$y(3, t) = +A = -A\sin(6\pi t - 6\pi)$

$= A\sin(6\pi - 6\pi t)$

$\therefore \quad 6\pi - 6\pi t = \frac{\pi}{2} \Rightarrow \frac{11}{2} = 6t$

$\Rightarrow \quad t = \frac{11}{12}$ s

**19.** $\Delta\phi_1 = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\cdot\frac{3}{2}\lambda = 3\pi$

and $\Delta\phi_2 = -\frac{\pi}{2} \Rightarrow \Delta\phi = 3\pi - \frac{\pi}{2} = \frac{5\pi}{2}$

**20.** $n\nu_0 = 400$ Hz, $(n+1)\nu = 450$ Hz

$\Rightarrow \nu_0 = 50$ Hz and $n = 8$

$\nu_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \Rightarrow l = \frac{1}{2\nu_0}\sqrt{\frac{T}{\mu}}$

$= \frac{1}{2\times 50}\sqrt{\frac{490}{0.1}} = \frac{70}{100} = 0.7$ m

**21.** $3\cdot\frac{\lambda}{2} = 1\,\text{m}, \lambda = \frac{2}{3}$ m

$v = \nu\lambda = 300\,\text{Hz}\times\frac{2}{3}\,\text{m} = 200\,\text{m/s}$

**22.** $l = \frac{\lambda_1}{2}, \frac{2\lambda_2}{2}, \frac{3\lambda_3}{2}$

$\Rightarrow \quad \lambda_1 = 2l, \lambda_2 = \frac{2l}{2}, \lambda_3 = \frac{2l}{3}$

$\therefore \quad \lambda_1 : \lambda_2 : \lambda_3 = 2l : \frac{2l}{2} : \frac{2l}{3}$

$= 1 : \frac{1}{2} : \frac{1}{3}$

**23.** $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

$= \frac{2\pi}{vT}\Delta x = \frac{2\pi}{300 \times 0.04} \times (16 - 10)$

$= \frac{2\pi}{12} \times 6 = \pi$

**24.** $v = \frac{\omega}{k} = \frac{30}{1} = 30 \text{ m/s}$

$= \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}}$

$\therefore T = \rho A v^2 = 8000 \times 10^{-6} \times 900 = 7.2 \text{ N}$

**25.** $5\nu_0 = 480 \text{ Hz}, 2\nu_0 = \frac{2}{5} \times 480 \text{ Hz}$

$= 192 \text{ Hz}$

**26.** $\frac{I_r}{I_i} = 0.64 = \left(\frac{A_r}{A_i}\right)^2$

$\Rightarrow \quad \frac{A_r}{A_i} = 0.8 \Rightarrow A_r = 0.8 A_i$

$A_r = \frac{v_2 - v_1}{v_2 + v_1} A_i = 0.8 A_i = \frac{4}{5} A_i$

$\Rightarrow \quad 5v_2 - 5v_1 = \pm(4v_2 + 4v_1)$

$\Rightarrow \quad v_2 = 9v_1, \frac{1}{9}v_1$

For, $v_2 > v_1$ the boundary is rarer and there will not be any change in phase of reflected wave and for $v_2 < v_1$ a phase change of 180° takes place.

$\therefore Y_r = 0.8 A \sin(kx + \omega t + 30° + 180°)$

**27.** $\nu_A = \frac{1}{2l}\sqrt{\frac{T}{\frac{\rho\pi d^2}{4}}} = \frac{1}{ld}\sqrt{\frac{T}{\pi\rho}}$

$\nu_B = \frac{1}{2 \times 2l}\sqrt{\frac{2T}{2\rho\pi\frac{4d^2}{4}}}$

$= \frac{1}{4ld}\sqrt{\frac{T}{\pi\rho}} = \frac{1}{4}\nu_A$

$\therefore$Third overtone of $\nu_B = 4\nu_B = \nu_A$

## Passage (Q 28 to 30)

$I_r = (100\% - 36\%) I_i = 64\% I_i = 0.64 I_i$

$\therefore \quad \frac{A_r}{A_i} = \sqrt{\frac{I_r}{I_i}} = \sqrt{0.64} = 0.8 = \pm\frac{v_2 - v_1}{v_2 + v_1}$

$\Rightarrow \quad 0.8 v_2 + 0.8 v_1 = \pm(v_2 - v_1)$

$\Rightarrow \quad -0.2 v_2 = 1.8 v_1$

$\Rightarrow \quad v_2 = 9 v_1$

for rarer boundary

or $1.8 v_2 = 0.8 v_1 \Rightarrow v_2 = \frac{1}{9} v_1$

for danser boundary

**28.** $A_r = 0.8 A$

**29.** $Y = A \sin\left(ax + bt + \frac{\pi}{2}\right) + 0.8$

$A \sin\left(ax - bt + \frac{\pi}{2} + \pi\right)$

$= A \sin\left(ax + bt + \frac{\pi}{2}\right) - 0.8$

$A \sin\left(ax - bt + \frac{\pi}{2}\right)$

$= A \cos(ax + bt) - 0.8 A \cos(ax - bt)$

$= A \cos ax \cos bt - A \sin ax \sin bt$

$- 0.8 A \cos ax \cos bt$

$- 0.8 A \sin ax \sin bt$

$= 0.2 A \cos ax \cos bt - 1.8 A \sin ax \sin bt$

$= 0.2 A \cos ax \cos bt - 0.2 A \sin ax \sin bt$

$- 1.6 A \sin ax \sin bt$

$= 0.2 A \cos(ax + bt)$

$- 1.6 A \sin(ax) \sin(bt)$

$= cA \cos(ax + bt) - 1.6 A \sin ax \sin bt$

$\Rightarrow e = 0.2$

**30.** For antinodes, $\sin ax = \pm 1$

$\Rightarrow \quad ax = (2n + 1)\frac{\pi}{2}$

$x = (2n + 1)\frac{\pi}{2a} = \frac{\pi}{2a}, \frac{3\pi}{2a}, \frac{5\pi}{2a}$

So for second antinode, $x = \frac{3\pi}{2a}$

**31.** $\frac{\nu_0 + 15}{\nu_0} = \sqrt{\frac{1 + 0.21}{1}} = 1 \cdot 1$

$\Rightarrow \quad 15 = 0.1\,\nu_0 \Rightarrow \nu_0 = 150\text{ Hz}$

$\frac{\nu_2}{\nu_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{1.21}{1}} = 1.1$

$\Rightarrow \quad \nu_2 = 110\%$ of $v_1$

$\lambda_0 = 2l$ which do not change

So, (a), (c) and (d) are correct.

**32.** For interference, sources must be coherent there frequency has to be equal and phase different has to be constant. So, (a) and (d) are correct.

**33.** Stationary waves are formed due to superposition **(here use of the term 'interference' is literary and not scientific because interference is a different phenomenon than stationary waves)** of waves having some amplitude, same frequency and travelling opposite direction. Here nodes are the points who always remain at rest. Total energy is always conserved.

**34.** A medium is said to be rarer if speed of wave in it is higher. And as frequency is constant, wavelength increases while frequency is constant, wavelength increases while phase do not change during change in medium.

**35.** $Y = A \sin kx \cos \omega t = 2a \sin kx \cos \omega t$

$a = \frac{A}{2}$, third overtone means fourth harmonic and wire oscillate with four loops.

$l = 4\frac{\lambda}{2} = 2\lambda = 2 \cdot \frac{2\pi}{k} = \frac{4\pi}{k}$

and stationary wave do not propagate.

**36.** For stationary waves, frequency and amplitude has to be same and direction has to be opposite with constant phase difference.

It is satisfied in (b) and (d) only.

**37.** $y = y_1 + y_2 = 2A \cos kx \sin \omega t$

$= R \sin \omega t$

$R = 2A \cos kx$ so at $x = 0$ there is antinode.

$\therefore \quad \cos kx = \pm 1$

$\Rightarrow \quad kx = n\pi, x = \frac{n\pi}{k} = 0, \frac{\pi}{4}, \frac{2\pi}{x},$

are antinodes.

# JEE Corner

## Assertion and Reason

**1.** $y_1 + y_2 = A \sin(\omega t + kx) + A \cos(\omega t - kx)$

$= A \sin(\omega t + kx) + A \sin\left(\frac{\pi}{2} - \omega t + kx\right)$

$= 2A \sin \frac{\omega t + kx + \frac{\pi}{2} - \omega t + kx}{2} \cos \frac{\omega t + kx - \frac{\pi}{2} + \cot - kx}{2}$

$= 2A \sin\left(kx + \frac{\pi}{4}\right) \cos\left(\omega t - \frac{\pi}{4}\right)$

$= R \cos\left(\omega t - \frac{\pi}{4}\right)$

where, $R = 2A \sin\left(kx + \frac{\pi}{4}\right)$;

$R(0) = 2A \sin\frac{\pi}{4} = A\sqrt{2}$

So, at $x = 0$, node is not present, *i.e.*, Assertion is false.

**2.** In stationary waves only nodes are at rest and not other particles. It is so called as energy is not transmitted, thus assertion is false.

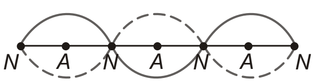
**3.** In rarer medium speed of wave is higher and as

$$A_t = \frac{2v_2}{v_1 + v_2} A_i$$

$\Rightarrow \quad A_t > A_i$

so reason is correct explanation to assertion.

**4.** In second overtone or third harmonic there are three loops or three antinodes or four nodes. And length of the string, $l = 3\frac{\lambda}{2}$ so, assertion and reason are both true.




**5.** As speed of wave is constant in stretched wire, and $v = f\lambda$, so with increase in frequency, wavelength decreases. So reason is correct explanation of assertion.

**6.** In stationary waves, amplitude of nodes is zero and it is possible only when superposing waves has same amplitude. But it is not the only condition, there has to be same frequency, opposite direction of propagation and constant phase difference. So assertion is not completely true.

**7.** Energy lying between conservative node and antinode is constant where it moves to and fro between node and antinode.

**8.** $$\frac{I_{\max}}{I_{\min}} = \frac{25}{1} = \left(\frac{5}{1}\right)^2 = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2$$

$\Rightarrow \quad 5(A_1 - A_2) = A_1 + A_2$

$\Rightarrow \quad 4A_1 = 6A_2 \Rightarrow A_1 : A_2 = 3:2.$

Thus reason is the correct explanation of assertion.

**9.** $$y = A\sin\left(\theta - \frac{\pi}{2}\right) + A\sin\theta + A\sin\left(\theta + \frac{\pi}{2}\right)$$

$$= -A\cos\theta + A\sin\theta + A\cos\theta$$

$$= A\sin\theta$$

$\therefore \quad R = A \Rightarrow I_f = I_i$

Assertion and reason are both true but reason do no explain assertion.

**10.** For two coherent sources phase difference has to be constant and that constant be same at all points as $\Delta\phi \neq \Delta\phi(t)$. Different light sources can never be coherent. So phase difference must be same, thus assertion is false.

## ■ Match the Columns

**1.** $v_1 = \sqrt{\frac{T}{\mu}}$ and $v_2 = \sqrt{\frac{T}{9\mu}} = \frac{v_1}{3}$

$\Rightarrow \quad \frac{v_1}{v_2} = 3$

$$A_r = \frac{v_1 - v_1/3}{v_1 + v_1/3} A_i = \frac{2/3}{4/3} A_i = \frac{1}{2} A_i$$

and $$A_t = \frac{2v_2}{v_1 + v_2} A_i = \frac{2\,v_1/3}{v_1 + v_1/3} A_i = \frac{1}{2} A_i$$

(a) $\frac{A_1}{A_2} = \frac{A_r}{A_t} = \frac{1/2\,A_i}{1/2\,A_i} = 1 \rightarrow$ q

(b) $\frac{v_1}{v_2} = 3 \rightarrow$ r

(c) $\frac{I_r}{I_i} = \left(\frac{A_r}{A_i}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ and

$$\frac{I_t}{I_i} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{I_r}{I_t} = \frac{I_r/I_i}{I_t/I_i} = \frac{1/4}{3/4} = \frac{1}{3} \rightarrow \text{s}$$

(d) $P = IS = 2\pi^2 \nu^2 A^2 \rho v \rho$

$$= \frac{1}{2}\omega^2 A^2 \mu \sqrt{\frac{T}{\mu}}$$

$$= \frac{1}{2}\omega^2 A^2 \sqrt{T\mu}$$

$$\frac{P_1}{P_2} = \frac{\frac{1}{2}\omega^2 A_1^2 \sqrt{T\mu_1}}{\frac{1}{2}\omega^2 A_2^2 \sqrt{T\mu_2}}$$

$$= \frac{A_1^2}{A_2^2}\sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{\mu_1}{9\mu_1}}$$

$$= \frac{1}{3} \to s$$

**2.** (a) $\frac{\nu_2}{\nu_4} = \frac{\frac{3}{2l}v}{\frac{5}{2l}v} = \frac{3}{5} \to r$

(b) Number of nodes in 3rd harmonic is 4 and in Fifth harmonic 6,

so, $\frac{4}{6} = \frac{2}{3} \to p$

(c) Number of antnodes in 3rd harmonic is 3 and in fifth harmonic 5, 50, $\frac{3}{5} \to r$

(d) $\frac{\lambda_2}{\lambda_4} = \frac{\nu_4}{\nu_2} = \frac{5}{3} \to s$

**3.** In danser medium speed of wave is lesser and in rarer medium it is greater.

(a) When wave goes from denser to rarer medium its speed increases $\to p$

(b) As frequency do not change with change in medium then with increase in speed wavelength increases $\to p$

(c) As $v_t > v_i$ then $A_t > A_i \to p$

(d) Frequency remains unchanged $\to r$

**4.** $R = \sqrt{A^2 + A^2 + 2 \cdot A \cdot A \cdot \cos\theta}$

$= 2A \cos\frac{\theta}{2}$

(a) $R(60°) = 2A \cos\frac{60°}{2}$

$= 2A \cos 30° = 2A \cdot \frac{\sqrt{3}}{2} = A\sqrt{3} \to s$

(b) $R(120°) = 2A \cos 120°/2 = 2A \cos 60°$

$= 2A \cdot \frac{1}{2} = A \to s$

(c) $R(90°) = 2A \cos 90°/2$

$= 2A \cos 45° = A\sqrt{2}$

$\Rightarrow I_R = 2A^2 = 2I_i \to p$

(d) $R(0°) = 2A \cos (0°/2) = 2A$

$\Rightarrow I_R = 4A^2 = 4I_i \to r$

**5.** $\nu_2 = 3\nu_0 = 210 \text{ Hz} \Rightarrow \nu_0 = 70 \text{ Hz}$

(a) $\nu_0 = 70 \text{ Hz} \to s$

(b) $\nu_2 = 3\nu_0 = 210 \text{ Hz} \to p$

(c) $\nu_3 = 4\nu_0 = 4 \times 70 \text{ Hz} = 280 \text{ Hz} \to r$

(d) $\nu_1 = 2\nu_0 = 140 \text{ Hz} \to s$

# 16. Sound Waves

## Introductory Exercise 16.1

**1.** $P_0 = S_0 kB$

$$\Rightarrow \quad B = \frac{P_0}{S_0 k} = \frac{P_0 \lambda}{2\pi S_0}$$

$$= \frac{14 \times 0.35}{2 \times 3.14 \times 5.5 \times 10^{-6}}$$

$$= 1.4 \times 10^5 \text{ N/m}^2$$

**2.** $\lambda = \frac{v}{\nu} \Rightarrow \lambda_{max} = \frac{1450 \text{ m/s}}{20 \text{ Hz}} = 72.5 \text{ m},$

$$\lambda_{min} = \frac{1450 \text{ m/s}}{20000 \text{ Hz}} = 7.25 \text{ cm}$$

**3.** Pressure wave and displacement wave has a phase difference of $\frac{\pi}{2}$, so,

(a) When pressure is maximum, displacement is minimum *i.e.*, zero.

(b) $S_0 = \frac{p_0}{kB} = \frac{p_0}{\frac{2\pi}{\lambda} \cdot \rho v^2}$

$$= \frac{p_0 \lambda}{2\pi \rho v^2} = \frac{p_0 \; v\lambda}{2\pi \nu \rho v^2} = \frac{p_0}{2\pi \nu \rho v}$$

$$= \frac{10}{2 \times 3.14 \times 10^3 \times 1.29 \times 340} \text{ m}$$

$$= 3.63 \times 10^{-6} \text{ m}$$

**4.** $S_0 = \frac{P_0}{kB} = \frac{P_0}{2\pi \nu \rho v} = \frac{P_0}{\omega \rho v}$

$$= \frac{P_0 k}{\rho \omega^2}$$

$$= \frac{12 \times 8.18}{1.29 \times (2700)^2}$$

$$= 1.04 \times 10^{-5} \text{ m}$$

## Introductory Exercise 16.2

**1.** $\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = 2 \Rightarrow T_2 = 4T_1 = 4 \times 273 \text{ K}$

$$= 3 \times 273°\text{C} = 819°\text{C}$$

**2.** $v_t = v_0 \left(1 + \frac{t}{273}\right)^{1/2} = v_0 \left(1 + \frac{t}{546}\right)$

$$v_{30} - v_3 = v_0 \left[1 + \frac{30}{546} - 1 + \frac{3}{546}\right]$$

$$= v_0 \left(\frac{33}{546}\right)$$

$$= 332 \times \frac{33}{546} = 20.06 \text{ m/s}$$

**3.** $v = \nu\lambda = 250 \times 8 = 2000 \text{ m/s}$

$$B = \rho v^2 = 900 \times (2000)^2$$

$$= 36 \times 10^8 \text{ N/m}$$

$$= 3.6 \times 10^9 \text{ Pa}$$

**4.** $v = \sqrt{\frac{\gamma R t}{M}} = \sqrt{\frac{\frac{7}{5} \times 8.314 \times 273}{32 \times 10^{-3}}}$

$$= 315 \text{ m/s}$$

# Introductory Exercise 16.3

1. $P_0 = S_0 kB = 2\pi \nu\rho v\, S_0$

$= 2 \times 3.14 \times 300 \times 1.2 \times 344 \times 6 \times 10^{-6}$

$= 4.67$ Pa

$$I = \frac{P_0^2}{2\rho v} = \frac{(4.67)^2}{2 \times 1.2 \times 344}$$

$= 2.64 \times 10^{-2}\ \text{W/m}^2$

$$L = 10 \log \frac{I}{I_0} = 10 \log \frac{2.64 \times 10^{-2}}{10^{-12}}$$

$= 104$ dB

2. $$2L - L = 10 \log \frac{\eta I}{I_0} - 10 \log \frac{I}{I_0}$$

$= 10 \log (\eta) = 9$ dB

$\Rightarrow \log \eta = 0.9, \quad \eta = 10^{0.9} = 7.9$

3. $$I \propto \frac{1}{r^2} \Rightarrow I = \frac{k}{r^2}$$

$$L_F - L_M = 10 \log \frac{I_F}{I_M}$$

$$= 10 \log \left(\frac{r_M}{r_F}\right)^2$$

$$= 20 \log \left(\frac{3}{0.3}\right) = 20 \text{ dB}$$

4. (a) $$I = \frac{P_0^2}{2\rho v}; I_{max} = \frac{(28)^2}{2 \times 1.29 \times 345}$$

$= 0.881\ \text{W/m}^2$

$$L_{max} = 10 \log \frac{0.881}{10^{-12}} = 119.45 \text{ dB}$$

$$I_{min} = \frac{(2 \times 10^{-5})^2}{2 \times 1.29 \times 345}$$

$= 4.49 \times 10^{-13}\ \text{W/m}^2$

$$L_{min} = 10 \log \frac{4.49 \times 10^{-13}}{10^{-12}} \text{ dB}$$

$= -3.48$ dB

(b) $$S_0 = \frac{\text{Po}}{kB} = \frac{\text{Po}}{2\pi \nu\rho v}$$

$$(S_0)_{max} = \frac{28}{2 \times 3.14 \times 500 \times 1.29 \times 345}$$

$= 2 \times 10^{-5}$ m

$$(S_0)_{min} = \frac{2 \times 10^{-5}}{2 \times 3.14 \times 500 \times 1.29 \times 345}$$

$= 1.43 \times 10^{11}$ m

# Introductory Exercise 16.4

1. $$(2n - 1)\frac{\lambda}{2} = 12 \text{ cm}$$

and $$(2n + 1)\frac{\lambda}{2} = 36 \text{ cm}$$

$\Rightarrow \lambda = 36 - 12 = 24$ cm

$$\nu = \frac{v}{\lambda} = \frac{330 \text{ m/s}}{0.24 \text{ m}} = 1375 \text{Hz}$$

2. $$\Delta x = \frac{\lambda}{2\pi}\Delta\phi = \frac{\lambda}{2\pi}\cdot\frac{\pi}{3} = \frac{\lambda}{6}$$

$$= \frac{v}{6\nu} = \frac{350}{6 \times 500} = 0.117 \text{ m} = 11.7 \text{ cm}$$

$$\Delta\phi = \frac{2\pi}{T}\Delta t = 2\pi\nu\Delta t = 2\pi \times 500 \times 10^{-3}$$

$= \pi$ rad $= 180°$

3. $$\Delta x_1 = 2\sqrt{H^2 + \frac{d^2}{4}} - d = n\lambda$$

and

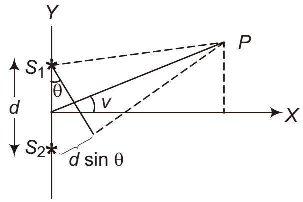
$$\Delta x_2 = 2\sqrt{(H + h)^2 + \frac{d^2}{4}} - d = \left(n + \frac{1}{2}\right)\lambda$$

$$\Rightarrow \frac{\lambda}{2} = 2\sqrt{(H + h)^2 + \frac{d^2}{4}} - 2\sqrt{H^2 + \frac{d^2}{4}}$$

or $$\lambda = 4\sqrt{(H + h)^2 + \frac{d^2}{4}} - 4\sqrt{H^2 + \frac{d^2}{4}}$$

$$\lambda = 2\sqrt{4(H + h)^2 + d^2} - 2\sqrt{4H^2 + d^2}$$

4. $\Delta x_p = d \sin\theta = \left(n + \frac{1}{2}\right)\lambda$ for minima




(a) $\therefore\ d\sin\theta = \frac{\lambda}{2}$ for first minima

$$\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right) = \sin^{-1}\left(\frac{v}{2\nu d}\right)$$

$$= \sin^{-1}\left(\frac{340}{2\times 600\times 2}\right)$$

$$= \sin^{-1}(0.142) = 0.142\,\text{rad}$$

$$= 8.14°$$

(b) For, first maxima $d\sin\theta = \lambda$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{340}{1200}\right)$$

$$= 16.46°$$

(c) $\Delta x_{max} \le d \Rightarrow n\lambda \le d,\ n \le \frac{d}{\lambda}$

$$= \frac{2\times 600}{340} = 3.53$$

$$\Rightarrow \quad n = 3 \text{ maxima.}$$

**5.** (a) For coherent speakers in phase,

$$I_R = 4I_0 \cos^2\frac{\theta}{2}$$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{2} = \pi = \theta$$

$$\Rightarrow \quad I_R = 4I_0\cos\frac{\pi}{2} = 0$$

(b) For incoherent sources,

$$I_R = I_1 + I_2 = I_0 + I_0 = 2I_0$$

(c) For coherent speakers with a phase difference 180°.

$$\Delta\phi' = 180° + \Delta\phi = \pi + \pi = 2\pi$$

$$\Rightarrow \quad I'_R = 4I_0\cos^2\frac{2\pi}{2} = 4I_0$$

**6.** $60\text{ dB} = 10\log\frac{I_0}{10^{-12}}$

$$\Rightarrow \quad 10^6 \times 10^{-12} = I_0$$

$$\Rightarrow \quad I_0 = 10^{-6}\ \text{W/m}^2$$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi\nu}{v}\Delta x$$

$$= \frac{2\pi\times 170}{340}\times(11-8) = 3\pi = \theta$$

(a) $\therefore I_R = 4I_0\cos^2\frac{\theta}{2} = 4I_0\cos^2\frac{3\pi}{2} = 0$

(b) $\Delta\phi' = 3\pi + \pi = 4\pi$

$$\Rightarrow \quad I'_R = 4I_0\cos\frac{4\pi}{2} = 4I_0$$

$$= 4\times 10^{-6}\ \text{W/m}^2$$

$$L'_R = 10\log\frac{4\times 10^{-6}}{10^{-12}}$$

$$= 10\log 10^6\text{ dB} + 10\log 4$$

$$= 60\text{ dB} + 2\log 2\text{ dB}$$

$$= 60\text{ dB} + 6\text{ dB} = 66\text{ dB}$$

(e) $\Delta\phi'' = \frac{2\pi\nu}{v}\cdot\Delta x = \frac{2\pi\times 85}{340}\times(11-8)$

$$= \frac{3\pi}{2} = \theta$$

$$I_R'' = 4I_0\cos^2\frac{3\pi}{4}$$

$$= 4I_0\cos^2\left(\pi - \frac{\pi}{4}\right) = 2I_0$$

$$\Rightarrow L_R'' = 10\log\frac{2\times 10^{-6}}{10^{-12}} = 63\text{ dB}$$

**7.** (a) $I_1 = \frac{10^{-3}}{4\pi\times 2^2} = \frac{10^{-3}}{16\pi}$

$$= 19.9\times 10^{-6}\ \text{W/m}^2$$

$$= 19.9\ \mu\text{W/m}^2$$

$$I_2 = \frac{10^{-3}}{4\pi\times 3^2} = \frac{10^{-3}}{36\pi}$$

$$= 8.84\times 10^{-6}\ \text{W/m}^2$$

$$= 8.84\ \mu\text{W/m}^2$$

(b) $(I_P)_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$

$$= (4.46 + 2.97)^2$$

$$= 55.27\ \mu\text{W/m}^2$$

(c) $(I_P)_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

$$= (4.46 - 2.97)^2$$

$$= 2.22\ \mu\text{W/m}^2$$

(d) $I_P = I_1 + I_2 = 28.7\ \mu\text{W/m}^2$

# Introductory Exercise 16.5

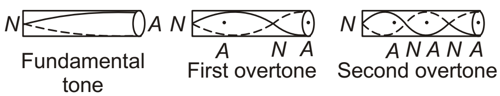
**1.** (a) $\nu_0 = \dfrac{v}{4l_c} \Rightarrow l_c = \dfrac{v}{4\nu_0} = \dfrac{345\ \text{m/s}}{4 \times 220\ \text{Hz}}$

$= 0.392\ \text{m}$

(b) $\dfrac{3v}{2l_0} = 5\nu_0$

$\Rightarrow \quad l_0 = \dfrac{3v}{10\nu_0} = \dfrac{3 \times 345}{10 \times 220} = 0.470\ \text{m}$

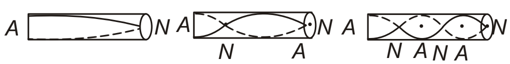
**2.** (a)




$d_A = l = 0.8$ m $\quad d_A = \dfrac{l}{3}, l = \dfrac{0.8}{3}$ m, $\quad d_A = \dfrac{l}{5}, \dfrac{3l}{5}, l$

$= \dfrac{0.8}{5}$ m, $\dfrac{2.4}{4}$ m, 0.8 m

(b)




$d_A = 0_n \qquad d_A = 0, \dfrac{2l}{3} \qquad d_A = 0, \dfrac{2l}{5}, \dfrac{4l}{5}$

$= 0, 0.533$ m $\qquad = 0$ m, 0.32 m, 0.64 m

**3.**

| | |
|---|---|
| 2 0 | 400, 560 |
| 4 | 20, 28 |

$\Rightarrow$ HCF of the two shows, 80 and the values, 400 Hz and 560 Hz are odd multiples of 80. These conservative harmonics are odd, which can be seen in closed organ pipe only.

(b) These are 5th and 7th harmonic.

(c) $\nu_0 = \dfrac{v}{4l_c}$

$\Rightarrow \quad l_c = \dfrac{v}{4\nu_0} = \dfrac{344}{4 \times 80} = 1.075\ \text{m}$

**4.** $v = \nu\lambda = 1000 \times 2 \times 6.77 \times 10^{-2}$ m/s

$= 135.4$ m/s

$v = \sqrt{\dfrac{\gamma RT}{M}} \Rightarrow r = \dfrac{Mv^2}{RT}$

$= \dfrac{n \times 127 \times 10^{-3} \times (135.4)^2}{8.314 \times 400} = 0.7\ n$

As $1 < r < 2$

$\Rightarrow n = 2 \Rightarrow r = 0.7 \times 2 = 1.4 = \dfrac{7}{5}$ diatonic

**5.** $\nu = \dfrac{(2n+1)v}{4l_1} = \dfrac{(2n+3)v}{4l_2}$

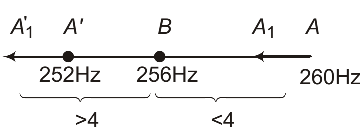
$\dfrac{2n+3}{2n+1} = \dfrac{l_2}{l_1} = \dfrac{100}{60} = \dfrac{5}{3}$

$\Rightarrow \qquad n = 1$

$\therefore v = \dfrac{4l_1\ \nu}{2n+1} = \dfrac{4 \times 0.6 \times 440}{3} = 352\ \text{m/s}$

# Introductory Exercise 16.6

**1.**




$\Rightarrow \qquad \nu_A = 252$ Hz

$\nu_A = (256 \pm 4)$ Hz

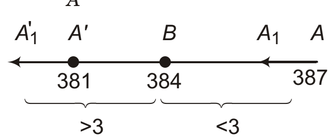
and $\quad \nu_A - n = (256 \pm 6)$ Hz

$\therefore \quad 256 \pm 4 - n = 256 \pm 6$

$\pm 4 \mp 6 = n \Rightarrow n = -4 + 6 = 2$

$\therefore \quad \nu_A = 256 - 4 = 252$ Hz

**2.**




$\Rightarrow \qquad \nu_A = 387$ Hz

$\nu_A = (384 \pm 3)$ Hz

and $\quad \nu_A - n = 384 \pm \text{m}, \text{m} < 3$

$\therefore \quad 384 \pm 3 - n = 385 \pm \text{m}$

$$\Rightarrow \quad \pm 3 - n = \pm m$$
$$\Rightarrow \quad \pm 3 \mp m = n = (+) \text{ ve}$$
$$\Rightarrow \quad n = +3 - m$$
$$\Rightarrow \quad \nu_A = 384 + 3 = 387 \text{ Hz}$$
$$6 \text{ Hz} = 600 \text{ Hz} = \frac{1}{2l}\sqrt{\frac{T_A}{\mu}}$$
and
$$600 \text{ Hz} = \frac{1}{2l}\sqrt{\frac{T_B}{\mu}}$$
$$\Rightarrow \quad \frac{606}{600} = \sqrt{\frac{T_A}{T_B}} = 1.01$$
$$\Rightarrow \quad \frac{T_A}{T_B} = 1.02$$

**4.** $$256 \pm 4 = \frac{v}{2 \times 0.25}$$
and
$$256 = \frac{v}{2 \times (0.25 - x)}$$
$$\frac{256}{252} = \frac{2 \times 0.25}{2\,(0.25 - x)} = \frac{1}{1 - 4x}$$
$$256 - 4 \times 256\, x = 252$$
$$\Rightarrow \quad 4 = 4 \times 256\, x$$
$$x = \frac{1}{256} \text{ m} = \frac{100}{256} \text{ cm} = 0.4 \text{ cm}$$

# Introductory Exercise 16.7

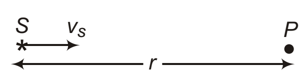
**1.** When source is moving,
$$\nu_s' = \frac{v}{v + v_s}\nu = \frac{1}{1 \mp \frac{v_s}{v}}\nu$$
$$= \left(1 \mp \frac{v_s}{v}\right)^{-1}\nu$$
$$= \left(1 \pm \frac{v_s}{v}\right)\nu = \left(1 \pm \frac{u}{v}\right)\nu$$
When observer is moving, $\nu_0 = \dfrac{v \pm v_0}{v}\nu$
$$= \left(1 \pm \frac{v_0}{v}\right)\nu = \left(1 \pm \frac{v}{v}\right)\nu$$
So, it can be seen that, $\nu_0$ and $\nu_s$ are equal if $u << v$.

**2.** $$\lambda = \frac{340}{200} = 1.7 \text{ m}$$
(a) $$\lambda' = \lambda - uT = 1.7 \text{ m} - \frac{80}{200} = 1.7 \text{ m}$$
$$- 0.4 \text{ m} = 1.3 \text{ m}$$
(b) $$\nu' = \frac{v}{\lambda'} = \frac{340 \text{ m/s}}{1.3 \text{ m}}$$
$$= 262 \text{ Hz}$$

**3.** For doppler effect there has to be relative motion between source and receiver, but as they are at rest relative to each other that's why there is no shift in wavelength and frequency.

**4.** $$\lambda = \frac{v}{\nu} = \frac{344}{500} = 0.688 \text{ m}$$
(a) $$\lambda_{\text{front}} = \lambda - uT = 0.688 - \frac{30}{500}$$
$$= 0.688 - 0.060 = 0.628 \text{ m}$$
(b) $$\lambda_{\text{behind}} = \lambda + uT = 0.688 + 0.060$$
$$= 0.748 \text{ m}$$
(c) $$\nu_{\text{front}} = \frac{344}{0.628} = 547.8 \text{ Hz}$$
(d) $$\nu_{\text{behind}} = \frac{344}{0.748} = 459.9 \text{ Hz}$$

**5.** $$\nu' = \frac{v - w - v_0}{v - w + v_s}\nu = \frac{340 - 5 - 20}{340 - 5 + 10} \times 300 \text{ Hz}$$
$$= \frac{315}{345} \times 300 \text{ Hz} = 273.9 \text{ Hz}$$

**6.**


# AIEEE Corner

## Subjective Questions (Level 1)

**1.** $d = d_1 + d_2 = v\dfrac{t_1}{2} + v\dfrac{t_2}{2}$

$= \dfrac{v}{2}(t_1 + t_2) = \dfrac{332}{2}\left(\dfrac{3}{2} + \dfrac{5}{2}\right)$

$= 332 \times 2 = 664$ m

The time for third eco is,

$t = t_1 + t_2 = \dfrac{3}{2} + \dfrac{5}{2} = 4\,\text{s}$

**2.** $v = \sqrt{\dfrac{\gamma RT}{M}} = \sqrt{\dfrac{\frac{7}{5} \times 8.314 \times 300}{2 \times 10^{-3}}}$

$= \sqrt{21 \times 8.314 \times 10^4} = 1321$ m/s

**3.** $v = \sqrt{\dfrac{\gamma p}{\rho}}$

$= \sqrt{\dfrac{\frac{5}{3} \times 76 \times 10^{-2} \times 13.6 \times 10^3 \times 9.8}{0.179}}$

$= \sqrt{\dfrac{5 \times 76 \times 136 \times 9.8}{3 \times 0.179}} = 971$ m/s

**4.** (a) $B = \rho v^2 = \rho \nu^2 \lambda^2$

$= 1300 \times 16 \times 10^4 \times 64$

$= 1.33 \times 10^{10}$ N/m$^2$

(b) $Y = \rho v^2 = \dfrac{\rho\, l^2}{t^2} = \dfrac{6400 \times (1.5)^2}{(3.9 \times 10^{-4})^2}$

$= 9.47 \times 10^{10}$ Pa

**5.** $v_t = \sqrt{\dfrac{\Delta l}{l}} v_l \Rightarrow \dfrac{\Delta l}{l} = \left(\dfrac{v_t}{v_l}\right)^2; \dfrac{F}{A} = Y\dfrac{\Delta l}{l}$

$= Y\left(\dfrac{v_t}{t_l}\right)^2 = Y\left(\dfrac{1}{30}\right)^2 = \dfrac{Y}{900}$

**6.** $M_{\text{mix}} = \dfrac{2 \times 2 + 1 \times 14}{2 + 1} = 6$ m/mole

$\dfrac{v_{\text{mix}}}{v_{H_2}} = \sqrt{\dfrac{M_{H_2}}{M_{\text{mix}}}} = \sqrt{\dfrac{2}{6}} = \dfrac{1}{\sqrt{3}}$

$v_{\text{mix}} = \dfrac{1}{\sqrt{3}} v_{H_2} = \dfrac{1}{\sqrt{3}} v_0 \sqrt{\dfrac{T_2}{T_1}} = \dfrac{v_0}{\sqrt{3}}\sqrt{\dfrac{300}{273}}$

$= \dfrac{v_0}{\sqrt{2.73}} = \dfrac{1300}{\sqrt{2.73}} = 787$ m/s

**7.** $L_1 = 10 \log \dfrac{10^{-6}}{10^{-12}} = 60 \log 10 = 60$ dB

$L_2 = 10 \log \dfrac{10^{-9}}{10^{-12}} = 30 \log 10 = 30$ dB

$\Rightarrow L_1 = 2L_2$

**8.** $100\text{ dB} = 10 \log \dfrac{I}{I_0}$ dB

$\Rightarrow \quad I = 10^{10} I_0 = 10^{-2}$ W/m$^2$

$P = 4\pi r^2 I = 4\pi \times (40)^2 \times 10^{-2}$

$= 64\,\pi\,\text{W} = 201$ W

**9.** (a) $60\text{ dB} = 10 \log \dfrac{I}{I_0}$ dB

$\Rightarrow \quad I = 10^6 I_0 = 10^{-6}$ W/m$^2$

(b) $P = AI = 120 \times 10^{-4} \times 10^{-6}$ W

$= 1.2 \times 10^{-8}$ watt

**10.** (a) $\Delta L = 13\text{ dB} = 10 \log \dfrac{I_2}{I_1}$ dB

$\Rightarrow \quad I_2 = 10^{1.3} I_1 = 20\, I_1$

(b) As with doubling the intensity, loudness increases by 3 dB irrespective of the initial intensity.

**11.** $I = \dfrac{P}{4\pi r^2} = \dfrac{5}{4\pi(20)^2} = \dfrac{5}{4\pi \times 400}$

$= \dfrac{1}{320\pi}$ W/m$^2$ $= 9.95 \times 10^{-4}$ W/m$^2$

(b) $I = 2\pi^2 \nu^2 a^2 \rho v \Rightarrow a = \dfrac{1}{\pi\nu}\sqrt{\dfrac{I}{2\rho v}}$

$= \dfrac{1}{300\,\pi}\sqrt{\dfrac{1}{320\,\pi \times 2 \times 1.29 \times 330}}$

$= \dfrac{1}{300\,\pi \times 10^{12}}\sqrt{\dfrac{1}{85.5}}$

$= 1.15 \times 10^{-6}$ m

**12.** $60 \text{ dB} = 10 \log \dfrac{I}{10^{-12}} \text{ dB}$

$\Rightarrow I = 10^{-6} \text{ W/m}^2$ and $a = \dfrac{1}{\pi\nu}\sqrt{\dfrac{T}{2\rho v}}$

$= \dfrac{1}{800\,\pi}\sqrt{\dfrac{10^{-6}}{2 \times 1.29 \times 330}} = 13.6 \times 10^{-9} \text{ m}$

**13.** $102 \text{ dB} = 10 \log \dfrac{I}{I_0} \text{ dB}$

$\Rightarrow \quad I = 10^{10.2}\, I_0 = 10^{10.2-12}$

$= 10^{-1.8} \text{ W/m}^2$

$P = 4\pi r^2 I$

$= 4 \times 3.14 \times (20)^2 \times 10^{-1.8}$

$= 80 \text{ W}$

**14.** $I = 2\pi^2 v^2 a^2 \rho v$

$= 2 \times (3.14)^2 \times (300)^2$

$\times (0.2 \times 10^{-3})^2 \times 1.29 \times 330 \text{ W/m}^2$

$= 30.25 \text{ W/m}^2$

$L = 10 \log \dfrac{I}{I_0} \text{ dB} = 10 \log \dfrac{30.25}{10^{-12}} \text{ dB}$

$= 134.8 \text{ dB}$

**15.** (a) $v_w = \sqrt{\dfrac{B}{\rho}} = \sqrt{\dfrac{2.18 \times 10^9}{10^3}}$

$= 1.48 \times 10^3 \text{ m/s}$

$A_w = \sqrt{\dfrac{I}{2\pi^2 \nu^2 \rho v}} = \dfrac{1}{\pi\nu}\sqrt{\dfrac{I}{2\rho v}}$

$= \dfrac{1}{3400\,\pi}\sqrt{\dfrac{3 \times 10^{-6}}{2 \times 10^3 \times 1.48 \times 10^3}}$

$= 9.44 \times 10^{-11} \text{ m}$

$\lambda_w = \dfrac{v_N}{\nu} = \dfrac{1.48 \times 10^3}{3400} = 0.43 \text{ m}$

(b) $v_a = \sqrt{\dfrac{\gamma p}{\rho}} = \sqrt{\dfrac{1.4 \times 10^5}{1.2}} = 341.6 \text{ m/s}$

$A_a = \dfrac{1}{3400\,\pi}\sqrt{\dfrac{3 \times 10^{-6}}{2 \times 1.2 \times 341.6}}$

$= 5.66 \times 10^{-9} \text{ m}$

$\lambda_a = \dfrac{341.6}{3400} = 0.1 \text{ m}$

(c) $A_a > A_w$; $\dfrac{A_a}{A_w} = \dfrac{5.66 \times 10^{-9}}{9.44 \times 10^{-11}} = 60$

As bulk modulus of water is much larger than air, such that displacement of particles of medium becomes less.

**16.** $I = \dfrac{p_0^2}{2\rho v} = \dfrac{(6 \times 10^{-5})^2}{2 \times 1.29 \times 343} \text{ W/m}^2$

$= 4 \times 10^{-12} \text{ W}$

$\therefore \; L = 10 \log \dfrac{I}{I_0} = 10 \log \dfrac{4 \times 10^{-12}}{10^{-12}}$

$= 20 \log 2 = 6 \text{ dB}$

**17.** $\nu_o = \dfrac{v}{2l} = 594 \text{ Hz};$

$\nu_c = \dfrac{v}{4l} = \dfrac{\nu_0}{2} = \dfrac{594}{2} \text{ Hz} = 297 \text{ Hz}$

**18.** $\nu_0 = \dfrac{(n+1)v}{2l} = (n+1)\dfrac{344}{2 \times 0.45}$

$= (n+1) \times 382.2 \text{ Hz}$

$= 382.2 \text{ Hz}, 764.4 \text{ Hz}, 1146.7 \text{ Hz},$

$\nu_t = \dfrac{(2n+1)v}{2l}$

$= (2n+1)\dfrac{344}{2 \times 0.45}$

$(2n+1) \times 191.1 \text{ Hz}$

$= 191.1 \text{ Hz}, 573.3 \text{ Hz}, 955.6 \text{ Hz}$

**19.** $\nu_c = \dfrac{v}{4l}$

$\Rightarrow \; v = 4l\,\nu_c = 4 \times 0.15 \times 500 = 300 \text{ m/s}$

$\nu_o = \dfrac{v}{2l_o} = \dfrac{300}{2 \times 0.6} = 250 \text{ Hz}$

**20.** $y = A \cos kx \cos \omega t$

$= A \cos \dfrac{2\pi}{1.6} x \cos 2\pi \dfrac{330}{1.6} t$

$= A \cos 3.93x \cos 1296\, t$

**21.** $\nu = \dfrac{2n+1}{4 \times 0.5} v = \dfrac{2n+3}{4 \times 0.84} v$

$\Rightarrow \quad \dfrac{2n+3}{2n+1} = \dfrac{84}{50} = 1.68$

$\Rightarrow \quad 3 - 1.68 = 2n \times 0.68$

$\therefore \; n = 0.97 = 1$ as $n$ is an integer

$$v = \frac{4lv}{2n+1} = \frac{4 \times 0.5 \times 512}{3} \text{ m/s}$$

$= 341.3$ m/s

$$\nu = \frac{2n+5}{4l} \nu$$

$$\Rightarrow \; l = \frac{2n+5}{4\nu} v = \frac{7}{4 \times 512} \times 341.3$$

$= 1.167 \text{ m} = 116.7 \text{ cm}$

**22.** $\nu_c = \frac{v}{4l} = \frac{340}{4 \times 1} = 85 \text{ Hz}$

$$\nu_s = \frac{v}{\lambda} = \frac{1}{0.4}\sqrt{\frac{F}{\mu}} = 85$$

$$\Rightarrow F' = (85 \times 0.4)^2 \; \mu = (34)^2 \times \frac{4 \times 10^{-3}}{0.4}$$

$= 11.65$ N

**23.** $\nu_c = \frac{v}{4(l+e)} \Rightarrow v = 4\nu\,(l + l)$

$= 4\nu(l + 0.3d)$

$= 4 \times 480\,(0.16 + 0.3 \times 0.05) = 336$ m/s

**24.** (a) $\nu_e = \frac{(2n+1)\,\nu}{4l}$

$$\Rightarrow \quad 440 = \frac{5 \times 330}{4l}$$

$$\Rightarrow \quad l = \frac{5 \times 330}{4 \times 440} = \frac{15}{16} \text{ m}$$

(b) 

$\frac{5\lambda}{4} = l = \frac{15}{16}$

$$\Rightarrow \quad \lambda = \frac{15}{16} \times \frac{4}{5} = \frac{3}{4} \text{ m}$$

$$\Delta p = \Delta p_0 \cos kx = \Delta p_0 \cos \frac{2\pi}{3/4} \times \frac{15}{32}$$

$$= \Delta p_0 \cos \frac{15\pi}{12} = \Delta p_0 \cos \frac{5\pi}{4} = \frac{\Delta p_0}{\sqrt{2}}$$

(c) At open end there is pressure node, so, $p_{\max} = \Delta p_{\min} = \Delta p_0$

(d) At closed end there is pressure antinode, such that, $p_{\max} = p_0 + \Delta p_0$ and $p_{\min} = p_0 - \Delta p_0$

**25.** (a) $\nu_c = \frac{v}{4l_c}$

$$\Rightarrow l_c = \frac{v}{4\nu_c} = \frac{345}{4 \times 220} = 0.392 \text{ m}$$

(b) $l_0 = \frac{5\lambda}{4}, \; l_0 = \frac{3\lambda}{2} = \frac{3}{2} \cdot \frac{4}{5}\, l_c$

$$= \frac{6}{5}\, l_c = \frac{6}{5} \times 0.392 \text{ m} = 0.47 \text{ m}$$

**26.** $\nu_s = \nu_c \Rightarrow \frac{v_s}{2 \times 0.8\, l_c} = \frac{v_s}{4\, l_c}$

$$\Rightarrow \quad \frac{v_s}{v_a} = \frac{1.6}{4} = 0.4$$

**27.** (a) $\lambda_s = \frac{v}{\nu} = \frac{340}{300} = \frac{17}{15} \text{ m} = 1.13 \text{ m}$

(b) $\lambda_a = \lambda - v_s T \Rightarrow \frac{v - v_s}{\nu}$

$$= \frac{340 - 30}{300} = \frac{31}{30} = 1.03 \text{ m}$$

$$\lambda_b = \lambda + v_s T = \frac{v + v_s}{\nu}$$

$$= \frac{340 + 30}{300} = \frac{37}{30} = \frac{37}{30} = 1.23 \text{ m}$$

**28.** $\nu = \frac{1}{2l}\sqrt{\frac{F}{\mu}} \Rightarrow \frac{\Delta\nu}{\nu} = \frac{1}{2}\frac{\Delta F}{F}$

$$\Rightarrow \frac{\Delta F}{F} = 2\frac{\Delta \nu}{\nu} = 2 \times \frac{1.5}{440} = \frac{3}{440} = \pm\, 0.68\%$$

$\nu' = \nu + \Delta\nu = 440 \pm 1.5$

$= 438.5$ Hz or $441.5$ Hz

**29.** $v = 0.32$ m/s;

$\lambda = vT = 0.32 \times 1.6 \text{ m} = 0.512$ m.

$$\lambda_a' = \lambda - v_s T \Rightarrow v_s = \frac{\lambda - \lambda'}{T} = v - \frac{\lambda'}{T}$$

$$= 0.32 - \frac{0.12}{1.6} = 0.245 \text{ m/s}$$

$\lambda_b'' = \lambda + v_s T = 0.512 \text{ m} + 0.245 \times 1.6$

$= 0.512 + 0.392 = 0.904$ m.

**30.** (a) $\nu_a = \frac{v - v_0}{v - v_s}\nu = \frac{340 + 18}{240 - 30} \times 262 \text{ Hz}$

$$= \frac{358}{310} \times 262 \text{ Hz} = 302.5 \text{ Hz}$$

(b) $\nu_r = \frac{v - v_0}{v + v_s}\nu = \frac{340 - 18}{340 + 30} \times 262$

$$= \frac{322}{370} \times 262 \text{ Hz} = 228 \text{ Hz}$$

**31.** $\Delta\nu = \frac{v}{v - v_s}\nu - \frac{v}{v + v_s}\nu$

$$= \frac{2vv_s\nu}{v^2 - v_s^2} \simeq \frac{2v_s\nu}{v} \Rightarrow v = \frac{v\Delta\nu}{2v_s}$$

$$\therefore \quad \nu = \frac{340 \times 4}{2 \times 1} = 680 \text{ Hz}$$

**32.** $\nu_0 = \nu_c \pm \Delta\nu = 110 \pm 2.2; \nu_c = \frac{v}{4l_c}$

$$\Rightarrow \quad l_c = \frac{v}{4\nu_c} = \frac{330}{4 \times 110}$$

$$\therefore \quad l_c = \frac{3}{4}\text{m}; \nu_0 = \frac{2v}{2l_0}$$

$$\Rightarrow \quad l_0 = \frac{2v}{2\nu_0} = \frac{2 \times 330}{2(330 \pm 2.2)}$$

$$= 0.993 \text{ m or } 1.007 \text{ m}$$

**33.** $\nu_p = \nu_Q \pm \frac{7}{2}$ and $\nu_P < \nu_Q$ as beat frequency increases waxing of $P$.

$$\nu_Q + 5 = \frac{v}{v - v_s}\nu_Q = \frac{332}{332 - 5}\nu_Q$$

$$= \frac{332}{327}\nu_Q \Rightarrow 5 = \frac{5}{327}\nu_Q$$

$$\Rightarrow \quad \nu_Q = 327 \text{ Hz and}$$

$$\nu_P = 327 - \frac{7}{2} = 323.5 \text{ Hz}$$

When $Q$ gives 5 beats with its own echo.

OR

$$\nu_P = \nu_Q - \frac{7}{2} = \nu'_q - 5 = \frac{332}{327}\nu_Q - 5$$

$$\Rightarrow \quad 5 - \frac{7}{2} = \frac{5}{327}\nu_Q$$

$$\Rightarrow \quad \nu_Q = \frac{327 \times 1.5}{5} = 98.1 \text{ Hz}$$

$$\Rightarrow \quad \nu_P = 98.1 - 2.5 = 94.6 \text{ Hz}$$

When $P$ gives 5 beats with the echo of $Q$.

**34.** $\Delta\nu = \frac{v}{v - v_s}\nu - \frac{v}{v + v_s}\nu \Rightarrow \frac{2vv_s\nu}{v^2 - v_s^2} \simeq \frac{2v_s\nu}{v}$

$$\Rightarrow v_s = \frac{v\nu}{2\nu} = \frac{340 \times 2}{2 \times 680}$$

$$\therefore \quad v_s = \frac{1}{2}\text{m/s}$$

**35.** $(2n + 1)\frac{\lambda}{2} = 11.5 \text{ cm}$

$$(2n + 3)\frac{\lambda}{2} = 34.5 \text{ cm}$$

$$\Rightarrow \frac{2n + 3}{2n + 1} = \frac{34.5}{11.5} = 3 \Rightarrow 4n = 0 \Rightarrow n = 0$$

$$\therefore \quad \frac{\lambda}{2} = 11.5 \text{ cm} \Rightarrow \lambda = 23 \text{ cm}$$

$$\nu = \frac{v}{\lambda} = \frac{331.2 \text{ m/s}}{0.23 \text{ m}} = 1440 \text{ Hz}$$

**36.** $\lambda = \frac{v}{\nu} = \frac{330}{220} = 1.5 \text{ m}$

$$\Delta x = S_2P - S_1P = 3 - \frac{3}{4} = \frac{9}{4}\text{m}$$

$$= \frac{3}{2} \cdot \frac{3}{2} = \frac{3}{2}\lambda = (2n + 1)\frac{\lambda}{2}$$

Here, $S_1P = \frac{3}{4} = \frac{1}{2}\lambda$

$$\Rightarrow \quad \phi_1 = \frac{2\pi}{\lambda}S_1P = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

and $\quad S_2P = 3 = 2 \cdot \frac{3}{2} = 2\lambda$

$$\Rightarrow \quad \phi_2 = \frac{2\pi}{\lambda} \cdot 2\lambda = 4\pi$$

Destructive interference will take place at $P$.

$$\therefore \quad P_P = P_{\min} = (\sqrt{P_1} - \sqrt{P_2})^2$$

$$= (\sqrt{1.8 \times 10^{-3}} - \sqrt{1.2 \times 10^{-3}})^2$$

$$= 0.6 \times 10^{-3}(\sqrt{3} - \sqrt{2})^2$$

$$= 0.6 \times 10^{-3} \times 0.1 = 6 \times 10^{-5} \text{ W}$$

**37.** $\Delta x = 2\sqrt{2^2 + \left(\frac{x}{2}\right)^2} - x = n\lambda = 1 \times \lambda$

$$= \frac{360 \text{ m/s}}{360 \text{ Hz}} = 1 \text{ m}$$

$$2\sqrt{4 + \frac{x^2}{4}} = 1 + x$$

or $\quad 4\left(4 + \frac{x^2}{4}\right) = 1 + 2x + x^2$

$$16 - 1 = 2x$$

$$x = 7.5 \text{ m}$$

## Objective Questions (Level 1)

1. Sound cannot travel in vacuum, as it is mechanical wave.

2. Longitudinal waves can travel through all mechanical mediums.

3. $\sqrt{\frac{\gamma RT}{32}} = \sqrt{\frac{\gamma R \times 288}{28}}$

$T = \frac{32}{28} \times 288 \text{ K} = \frac{8}{7} \times 288 \text{K} = 56°\text{C}$

4. Third overtone is 7th harmonic *ie,* there 4 nodes and 4 antinodes.


5. $\nu \propto \frac{v}{l} \Rightarrow \nu \propto \sqrt{T}$ so with increase in temperature, frequency increases.

6. For sound water is rarer medium and air is densor medium so, it bends towards normal while going from water to air.

7. $\nu_c = \frac{v}{4l_c} = \nu_o = \frac{v}{2l_o} \Rightarrow \frac{l_c}{l_o} = \frac{2}{4} = 1:2$

8. $\frac{\nu_2}{\nu_1} = \sqrt{\frac{F_2}{F_1}}$

$\Rightarrow F_2 = \left(\frac{\nu_2}{\nu_1}\right)^2 F_1$

$\Rightarrow M_2 = \left(\frac{\nu_2}{\nu_1}\right)^2 M_1 = \left(\frac{256}{320}\right)^2 \times 10 \text{ kg}$

$= 6.4 \text{ kg}$

$\therefore\ OM = M_2 - M_1 = 6.4 - 10 = -3.6 \text{ kg}$

*i.e.,* Mass has to be decreased by 3.6 kg

9. $\nu_{\text{direct}} = \frac{v}{v - v_s}\nu$ and $\nu_{\text{reflected}} = \frac{v}{u - v_s}\nu$

as $\nu_D = \nu_R$ so there will be no beats *i.e.,* beat frequency will be zero.

10. $\Delta\nu = \nu_2 - \nu_1 = \frac{v}{\lambda_2} - \frac{v}{\lambda_1} = v\frac{(\lambda_1 - \lambda_2)}{\lambda_1\lambda_2}$

$\Rightarrow\ v = \frac{\lambda_1\lambda_2\Delta\nu}{\Delta\lambda}$

$\therefore\ v = \frac{1 \times 1.01 \times \frac{10}{3}}{0.01} = 337 \text{ m/s}$

11. $\nu' = \frac{v}{v+v}\nu = \frac{1}{2}\nu = n\nu \Rightarrow n = \frac{1}{2} = 0.5$

12. $I_{\max} = (\sqrt{I} + \sqrt{I})^2 = 4I = NI \Rightarrow N = 4$

13. $\nu = \frac{v}{4(l_1 + e)} = \frac{3v}{4(l_2 + e)}$

$\Rightarrow\ l_2 + e = 3l_1 + 3e$

$\Rightarrow\ e = \frac{l_2 - 3l_1}{2}$

$\therefore\ e = \frac{42 - 3 \times 17}{2} \text{cm} = 0.5 \text{ cm}$

$v = 4\nu(l_1 + e) = 4 \times 500(17 + 8.5) \times 10^{-2}$

$= 20 \times 17.5 = 350 \text{ m/s}$

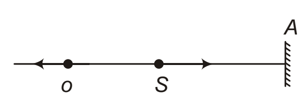
14. At the moment when velocity of source is perpendicular to the line joining source and observer then there is no Doppler effect *i.e.,* $n + n_1 = n \Rightarrow n_1 = 0$

15. $\nu = \frac{(n+1)v}{4l} = (2n+1)\frac{340}{4 \times 1} = 85(2n+1)$

$= 85, 255, 425, 595, 765, 935$

$\therefore$ 6 frequencies below 1 kHz.

16. $\Delta\nu = \frac{v - v_0}{v - v_s}\nu - \frac{v - v_0}{v + v_s}\nu = \nu\left(1 - \frac{v + v_0}{v + v_s}\right)$

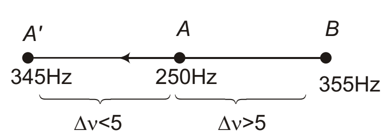



$= \nu \cdot \frac{v_s + v_0}{v + v_s} = \frac{10}{360} \times 180 = 5 \text{ Hz}$

17. $\Delta\nu = n = \nu_1 - \nu_2 = \frac{v}{\lambda_1} - \frac{v}{\lambda_2}$

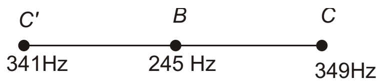
$= \frac{v(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2} \Rightarrow v = \frac{n\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$

18.



As beat frequency between $A$ and $B$ decreases on loading $A$.

*i.e.*, $\nu_B < \nu_A \Rightarrow \nu_B = 345$ Hz




After loading $A$, $\nu_A' = 345 + 2 = 247$ Hz

and $\nu_A' - \nu_c = \pm 6 \Rightarrow \nu_c = \lambda_A' \mp 6$

$= 347 \mp 6$

$= 341$ or $353$ Hz.

As possible frequency of C are 341 Hz and 249 Hz then only 341 Hz is justified.

**19.** $e = \dfrac{l_2 - 3l_1}{2} = \dfrac{122 - 3 \times 40}{2}$ cm $= 1$ cm

So, $\dfrac{v}{l(l_1 + e)} = \dfrac{5v}{4(l_1 + e)}$

$\Rightarrow \quad l_3 = 5l_1 + 4l$

$= 5 \times 40 + 4 \times 1 = 204$ cm

**20.** $\dfrac{\Delta\nu}{\nu} = \dfrac{1}{2}\dfrac{\Delta F}{F} \Rightarrow \Delta\nu = \dfrac{1}{2}\nu\dfrac{\Delta F}{F}$

$= \dfrac{1}{2} \times 200 \times \dfrac{1}{100} = 1$ Hz

**21.** $\nu = \dfrac{2n+1}{4l}v \quad \Rightarrow \quad l = \dfrac{2n+1}{4\nu}v$

$= (2n + 1)\dfrac{340}{4 \times 340} = \dfrac{2n+1}{4}$ m

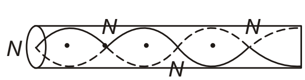
$= \dfrac{1}{4}$ m, $\dfrac{3}{4}$ m, $\dfrac{5}{4}$ m.

As, $l_{max} = 120$ cm $\Rightarrow l = 25$ cm 75 cm.

$\therefore$Height of water column

$= 120$ cm $- 75$ cm $= 45$ cm

**22.** $7\dfrac{\lambda}{4} = 105$ cm $\Rightarrow \lambda = \dfrac{105 \times 4}{7} = 60$ cm




$\Rightarrow \quad \dfrac{\lambda}{4} = \dfrac{60}{4} = 15$ cm

So, nodes are at, $\dfrac{\lambda}{4}, 3\dfrac{\lambda}{4}, 5\dfrac{\lambda}{4}$ and $7\dfrac{\lambda}{4}$ from closed end *i.e.*, they are at, 15 cm, 45 cm, 75 cm and 105 cm.

**23.** $\nu_c = \dfrac{v}{4l} = 512$ Hz, $\nu_o = \dfrac{v}{2l} = 2\dfrac{v}{4l}$

$= 2\nu_c = 2 \times 512$ Hz $= 1024$ Hz

**24.** $M_{min} = \dfrac{1 \times 32 + 1 \times 2}{1 + 1} = 17$

$\dfrac{v_{min}}{v_{H_2}} = \sqrt{\dfrac{M_{H_2}}{M_{min}}} = \sqrt{\dfrac{2}{17}}$

**25.** $\nu_a > f$ and $\nu_r < f$ but $\nu_a =$ constant and $\nu_r =$ cosntant.

So, curve in (b) represents correctly.

**26.** $u = \dfrac{(2n+1)v}{4l} - \dfrac{(m+1)v}{2l}$

How, $\dfrac{(2n+1)v}{4 \times 2l} - \dfrac{(m+1)v}{4 \times 2l}$

$= \dfrac{4}{2} = 2$ beat/s

**27.** $\Delta\nu = \nu_a - \nu_r = \dfrac{v}{v - v_1}\nu - \dfrac{v}{v + v_1}\nu$

$= \dfrac{2vv_1\nu}{(v - v_1)(v + v_1)}$

$= \dfrac{2 \times 320 \times 4 \times 243}{316 \times 324} = 6$ Hz

**28.** $\nu_c = \dfrac{(2n+1)v}{4l_c} = \dfrac{320}{4 \times 1}(2n + 1)$

$= (2n + 1) \times 80$ Hz $= 80$ Hz, 240 Hz, 400 Hz,...

$\nu_0 = \dfrac{(n+1)v}{2l_0} = \dfrac{320}{2 \times 1.6}(n + 1)$

$= (n + 1) \times 100$ Hz $= 100$ Hz, 200 Hz, 300 Hz, 400 Hz,...

$\therefore \ \nu_c = \nu_o = 400$ Hz

**29.** $I_{max} = 4I_0$

and $\quad I'_{max} = 4I_{max} = 16I_0$

$L' = 10\text{dB} + 10\log 16$

$= 10$ dB $+ 40\log 2$ dB $= 22$ dB

**30.** $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{\pi/2} = 4$ m

$l = 5\dfrac{\lambda}{4} = 5 \times \dfrac{4}{4}$ m $= 5$ m

**31.** $d = (2n+1)\frac{\lambda}{4} = \frac{(2n+1)}{4}\cdot\frac{330}{660}$ m

$= \frac{330}{24}(2n+1)$ cm $= (2n+1)\times 13.75$ cm.

$= 13.75$ cm, $41.25$ cm, $68.75$ cm, $96.25$ cm etc.

**32.** $\Delta\nu = \frac{v}{\lambda_1} - \frac{v}{\lambda_2} = \frac{v(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}$

$= \frac{332\times 1\times 10^{-2}}{0.49\times 0.5} = 13.15$ Hz

**33.** $\Delta\nu_B = \nu_A - \nu_B = \frac{300}{300-30}\times 300 - 300$

$= 33.33$ Hz and $\nu'_A \neq \nu'_B$

So both (a) and (b) options are wrong.

**34.** $f_a = \frac{v+v_0}{v} f = \left(1+\frac{v_0}{v}\right) f$ and

$f_r = \left(1-\frac{v_0}{v}\right) f$

$$\frac{f_a}{f_r} = \frac{v+v_0}{v-v_0}$$

$\Rightarrow \quad (f_a - f_r)v = (f_a + f_r)v_0$

$\Rightarrow \quad \frac{v}{v_0} = \frac{f_a + f_r}{f_a - f_r}.$

and

$f_a - f_r = \frac{2v_0}{v} f = 2\left(\frac{f_a - f_r}{f_a + f_r}\right) f$

$\Rightarrow \quad f = \frac{f_a + f_r}{2}.$

# JEE Corner

## Assertion and Reason

**1.** $\nu_c = (2n+1)\frac{v}{4l} = \frac{v}{4l}, \frac{3v}{4l}, \frac{5v}{4l}, \ldots$

while, $\nu_0 = \frac{(n+1)v}{2l} = \frac{v}{2l}, \frac{2v}{2l}, \frac{3v}{2l}, \frac{4v}{2l}, \ldots$

it can be seen that $\nu_c \neq \nu_o$ at all situation and $\nu_c = \frac{1}{2}\nu_o$ so assertion is true but reason is false.

**2.** Apparent frequency is constant for constant relative velocity so assertion is false.

**3.** At a point of minimum displacement pressure amplitude is maximum *i.e.*, pressure difference is maximum not pressure. So assertion is false.

**4.** The deriver receiver two sounds one direct, $\nu_0 = \nu$ and other $\nu_R = \frac{v+u}{v-u}\nu$ such that be detects beats. So reason is true explanation of assertion.

**5.** With increase in intensity sound level increases in lograthmic order so assertion is false.

**6.** Speed of sound $v = \sqrt{\frac{\gamma p}{\rho}}$, with increase in only pressure density increases such that $p/\rho$ remains constant. Again $v = \sqrt{\frac{\gamma RT}{M}}$ so both assertion and reason are true but reason is not correct explanation of assertion.

**7.** $\nu_A = \nu_B + 4$ when $A$ is loaded with little wax then $\nu_A$ sightly decreases and then beat frequency decreases, but if it is heavily loaded with wax then its frequency goes much below $\nu_B$ such that beat frequency increases. So, assertion and reason are both true but reason is not correct explanation of assertion.

**8.** $150 \,|\, 450, 750, \ldots$ ; $3, \; 5$

The frequencies are odd harmonics then the pipe is closed and fundamental frequency is also 150 Hz. So assertion and reason are both true but reason is not correct explanation of assertion.

**9.** $\nu \propto \dfrac{1}{l+e}$ with increase in diameter end correction, $e$, increases and $\nu$ decreases. So reason is correct explanation of assertion.

**10.** With increasing length of air column, number of overtone increases and not the wavelength so assertion is false.

## ■ Objective Questions (Level 2)

**1.** At the boundary between two mediums, one part of incident wave gets reflected and other part gets transmitted or refracted.

**2.** $\dfrac{3\lambda}{2} = 3.9\,\pi \Rightarrow \lambda = \dfrac{3.9\,\pi}{1.5}$

$\Rightarrow k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{\dfrac{3.9\,\pi}{1.5}} = \dfrac{3}{3.9}\,\rho_0 = S_0\,kB$

$\Rightarrow \quad S_0 = \dfrac{\rho_0}{kB} = \dfrac{10^{-2} \times 10^5}{\dfrac{3}{3.9} \times 1.3 \times (200)^2}$

$= \dfrac{3.9 \times 10^{-1}}{12 \times 1.3} = 0.025 \text{ m } = 2.5 \text{ cm}$

**3.** $\dfrac{A_t}{A_i} = \dfrac{2v_2}{v_1 + v_2} = \dfrac{2 \times 100}{200 + 100} = \dfrac{2}{3}$

**4.** $\Delta\nu = \dfrac{v}{v + v_2}\nu - \dfrac{v}{v + v_1}\nu$

$= \dfrac{v\nu(v_1 - v_2)}{(v + v_2)(v + v_1)} \simeq \dfrac{\nu(v_1 - v_2)}{v}$

$\therefore \; v_1 - v_2 = \dfrac{v\Delta\nu}{\nu} = \dfrac{340 \times 10}{1700} = 2 \text{ m/s}$

**5.** $v_s = gt = 10 \text{ m/s}$

$\Delta\nu = \dfrac{v + v_0}{v - v_s}\nu - \dfrac{v - v_0}{v + v_s}\nu$

$= \left(\dfrac{300 + 2}{300 - 10} - \dfrac{300 - 2}{300 + 10}\right) \times 150 \text{ Hz}$

$= \left(\dfrac{302}{290} - \dfrac{298}{310}\right) \times 150 = 12 \text{ Hz}$

**6.** $7\dfrac{\lambda}{2} = L \Rightarrow \lambda = \dfrac{2L}{7}$

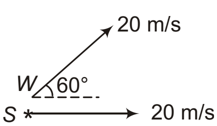
$A = a\cos kx = a\cos\dfrac{2\pi}{2\dfrac{L}{7}} \cdot \dfrac{L}{7} = a\cos\pi = -a$

**7.** For maxima, $n\lambda = 3$

$\Rightarrow \lambda = \dfrac{3}{n}; \; \nu = \dfrac{v}{\lambda} = \dfrac{nv}{3} = 110\,n.$

$\therefore \nu = 110, 220, 330$ Hz, ..etc. maxima will be formed so maximum will not be formed at 120 Hz and 100 Hz.

**8.**




$\nu' = \dfrac{v + \omega\cos 60°}{v + w\cos 60° - v_s}$

$= \dfrac{300 + 10}{300 + 10 - 20} \times 500 \text{ Hz}$

$= \dfrac{310}{290} \times 500 = 534 \text{ Hz}$

**9.** $\Delta\nu = \nu_R - \nu_0 = \left[\dfrac{v + 20}{v - 10} - \dfrac{v + 20}{v + 10}\right] \times 500$

$= \left(\dfrac{360}{300} - \dfrac{360}{350}\right) \times 500 \text{ Hz} = 31 \text{ Hz}$

**10.** $\Delta\nu = \dfrac{404\pi}{2\pi} - \dfrac{400\pi}{2\pi} = 202 - 200 = 2 \text{ Hz}$

$\dfrac{I_{max}}{I_{min}} = \left(\dfrac{2 + 1}{2 - 1}\right)^2 = 9 : 1$

**11.** $3\frac{\lambda}{4} = 34 \text{ cm} \Rightarrow \lambda = \frac{4}{3} \times 34 \text{ cm}$

$\Rightarrow \nu = \frac{v}{\lambda} \Rightarrow v_{51} = \nu\lambda = \frac{136}{3}\nu$

$\frac{v_{16}}{v_{51}} = \sqrt{\frac{273+16}{273+51}} = \sqrt{\frac{289}{324}} = \frac{1}{\sqrt{1.121}} = \frac{1}{1.1}$

$\Rightarrow \nu\lambda_{16} = \frac{\nu\lambda_{51}}{1.1}$

$\therefore \quad \lambda_{16} = \frac{136}{3 \times 1.1} = 41.21 \text{ cm}$

**12.** $176 \times \frac{v-v}{v-22} = 165 \times \frac{v+v}{v}$

$\Rightarrow \quad 176\, v(v-v) = 165(v+v)(v-22)$

$\therefore$

$176 \times 330(330-v) = 165(330+v)(330-22)$

or $\quad 1.143\,(330 - v) = 330 + v$

or $\quad 0.143 \times 330 = 2.143\,v \Rightarrow v = 22 \text{ m/s}$

**13.** $M_{\min} = \frac{2 \times 32 + 3 \times 48}{2+3} = 41.6$

$\frac{\nu_2}{\nu_1} = \frac{v_2}{v_1} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{32}{41.6}} = \sqrt{0.77}$

$= 0.875 = 175 \text{ Hz}$

**14.** $v_0 = gt = 30 \text{ m/s}$

$1100 = \frac{v+30}{v} \times 1000, \; 1.1v = v + 30$

$0.1\,v = 30 \Rightarrow v = 300 \text{ m/s}$

**Passage (Q 5 to 17)**

$v_m + v_p = 8 \text{ m/s}, \; 50v_m = 150v_p$

$\Rightarrow \quad v_m = 3v_p, \; 4v_p = 8 \text{ m/s}$

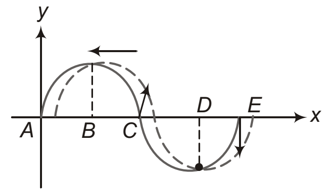
$v_p = 2 \text{ m/s and } v_m = 6 \text{ m/s}$

**15.** $\nu' = \frac{v+2}{v-6} f_0 = \frac{332}{324} f_0 = \text{constant}$

**16.** $\nu'' = \frac{v-2}{v+6} f_0 = \frac{328}{336} f_0 = \text{constant}$

**17.** $\nu'' < f_0 < \nu'$ and graph is (a)

**18.**




Both (a) and b are correct.

**More Than One**

**19.** $\nu = \frac{(2n+1)v}{4l}$

$\Rightarrow l = \frac{v}{4\nu}(2n+1) = \frac{330}{4 \times 264}(2n+1) \text{ m}$

$= (2n+1) \times 31.25 \text{ cm}$

$= 31.25 \text{ cm}, 93.75 \text{ cm}, 156.25 \text{ cm}$

**20.** (a) $v \propto p^0$, (b) $v \propto \sqrt{T} \Rightarrow v^2 \propto T$,

where $T$ is absolute temperature.

(c) $v \propto \sqrt{F}$ (d) $\nu \propto \frac{1}{l}$

$\therefore$(c) and (d) are correct.

**21.** $P_0 = BAk; \; B = -\frac{\Delta p}{\frac{\Delta V}{V}} - \frac{\Delta p}{\frac{\Delta p}{p}}$

$\therefore \quad \Delta\rho = \frac{\Delta p \rho}{B} = BAk\frac{\rho}{B} = \rho A k$

$\rho \propto p;$

Pressure and density equations are in opposite phase *i.e.*, $\Delta\phi = \frac{\pi}{2}$ and not $\pi$.

So, (a), (b) and (c) are correct.

**22.** $\frac{5v}{4l_c} = \frac{3v}{2l_o} \Rightarrow \frac{1.25}{l_c} = \frac{2}{l_o} \Rightarrow \frac{l_o}{l_c} = \frac{2}{1.25} = \frac{8}{5}.$

(a) $\nu_c = \frac{v}{4l_c} = \frac{v}{4 \times \frac{5}{8} l_o} = \frac{2v}{5l_o}$

$= \frac{4}{5} \cdot \frac{v}{2l_o} = \frac{4}{5}\nu_o \Rightarrow \nu_c < \nu_o$

(b) $\nu_c = \frac{3v}{4l_c} = \frac{3v}{4 \times \frac{5}{8} l_0} = \frac{12}{5} \cdot \frac{v}{2l_0}$

$= \frac{6}{5} \cdot \frac{2v}{2l_o} = \frac{6}{5} \cdot \nu_o \Rightarrow \nu_c > \nu_o.$

(c) $\nu_c = \frac{15v}{4l_c} = \frac{15v}{4 \times \frac{5}{8} l_o} = \frac{6v}{l_o} = 12\frac{v}{2l_o}$

$= 12v_0$ twelbth harmonic.

(d) Closed organ pipe cannot have tenth harmonic it only has odd harmonics.

**23.** $f = \frac{v}{4(l+e)} = \frac{1}{4(l+e)}\sqrt{\frac{\gamma RT}{M}}$

(a) increase in $r \Rightarrow$ increase in $e \Rightarrow$ decrease in $f$

(b) increase in $T \Rightarrow$ increase in $v \Rightarrow$ increase in $f$

(c) increase in $M \Rightarrow$ decrease in $v \Rightarrow$ decrease in $f$

(d) increase in $P \Rightarrow$ increase in $\rho \Rightarrow$ no change in $v \Rightarrow$ no change in $f$

**24.** $f_a = \frac{v}{v - v_s} f$ and $f_r = \frac{v}{v + v_s} f$ are constants during approach and received.

## Match the Columns

**1.** $\nu_o = \frac{v}{2l} = f$

(a) $\nu_c = \frac{v}{4 \times 2l} = \frac{f}{4} = 0.25f \to s$

(b) $\nu_{c2} = \frac{5v}{4 \times 2l} = \frac{5}{4} \cdot f = 1.25f \to p$

(c) $\nu_{c1} = \frac{3v}{4 \times 2l} = \frac{3}{4} f = 0.75f \to r$

(d) $\nu_{c1} = \frac{3v}{4 \times 2l} = 0.75f \to r$

**2.** $\Delta\nu_1 = \left(\frac{v}{v - v_s} - \frac{v}{v + v_s}\right) f = \frac{2vv_f}{v^2 - v_s^2} f$

$= \frac{2v\frac{v}{4}}{v^2 - \frac{v^2}{16}} f = \frac{16}{15} \times \frac{1}{2} f = \frac{8}{15} f$

$\Delta\nu_2 = \left(\frac{v + v_s}{v - v_s} - 1\right) f = \frac{2v_s}{v - v_s} f$

$= \frac{2v/4}{v - v/4} f = \frac{2}{3} f$

$\Delta\nu_3 = \left(\frac{v}{v - v_5} - \frac{v}{v - v_5}\right) f = 0$

(a) $\to \Delta\nu_1 = \frac{8}{15} f \to q$

(b) $\to \Delta\nu_2 = \frac{2}{3} f \to p$

(c) $\Delta\nu_3 = 0 \to s$

(d) $\Delta\nu_3' = 0 \to s$

**3.** $f = f_T - f_S$

(a) If tuning fork is loaded $f_T$ decreases such that beat frequency may increase or decrease depending upon amount of wax $\to r, s$

(b) If prongs are filed, beat frequency must increase $\to p$

(c) If tension is increased beat frequency may increase or decrease depending upon the amount of change in tension. $\to r, s$

(d) If tension is decreased, beat frequency must increase $\to p$

**4.** (a) For point source, $I \propto \frac{1}{r}$, and $A \propto \frac{1}{r} \to r$

(b) $\to q$

(c) For line source, $I \propto \frac{1}{r}$ and

$$A \propto \frac{1}{\sqrt{r}} \to q$$

(d) $\to p$

**5.** $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi} = 2$ m

$\lambda = 5\frac{\lambda}{4} = \frac{5}{2}$ m $= 2.5$ m

(a) $l = 2.5$ m $\to s$

(b) $\lambda = 2$ m $\to r$

(c) $\frac{\lambda}{2}, \frac{2\lambda}{2} = 1$m, 2m $\to p, r$

(d) $\frac{\lambda}{4}, 3\frac{\lambda}{4} = 0.5$ m, 1.5 m $\to q$

# 17. Thermometry, Thermal Expansion & Kinetic Theory of Gases

## Introductory Exercise 17.1

**1.** (a) $\dfrac{C}{5} = \dfrac{F-32}{9}$ for $F = 0, C = -\dfrac{5}{9} \times 32$

$= -17.8°\text{C}$

(b) $\dfrac{K-273.15}{5} = \dfrac{F-32}{9}$ for $K = 0,$

$F = -\dfrac{9}{5} \times 273.15 + 32 = -459.67°\text{F}$

**2.** (a) $\dfrac{x}{5} = \dfrac{2x-32}{9} \Rightarrow x = \dfrac{10x}{9} - 17.8$

$\Rightarrow \quad 17.8 = \left(\dfrac{10}{9} - 1\right)x$

$\Rightarrow \quad x = 17.8 \times 9 = 160.2\ °\text{C}$

(b) $\dfrac{x}{5} = \dfrac{x/2 - 32}{9} \Rightarrow x = \dfrac{5}{18}x - 17.8$

$\Rightarrow 17.8 = -\dfrac{13}{18}x \Rightarrow x = -24.65\ °\text{C}$

**3.** $\dfrac{C-5}{99-5} = \dfrac{F-32}{212-32}$

$\Rightarrow \quad \dfrac{C-5}{94} = \dfrac{F-32}{180}$

$\Rightarrow \quad \dfrac{52-5}{94} = \dfrac{F-32}{180}$

$\Rightarrow \quad F = 32 + \dfrac{180}{94} \times 47 = 122\ °\text{F}$

**4.** $\dfrac{K-273.15}{5} = \dfrac{F-32}{9}$

$\Rightarrow \quad x - 273.15 = \dfrac{5}{9}x - 17.8$

$\Rightarrow \quad \dfrac{4}{9}x = 255.35 \Rightarrow x = 574.54$

**5.** $\dfrac{C}{5} = \dfrac{F-32}{9} \quad \Rightarrow \dfrac{9}{5}x = x - 32$

$\Rightarrow \quad \dfrac{4}{5}x = -32$

$\Rightarrow \quad x = -\dfrac{5}{4} \times 32 = -40\ °\text{C}$

**6.** $\Delta t = \dfrac{1}{2}\alpha t\ \Delta\theta$

$= \dfrac{1}{2} \times 1.2 \times 10^{-5} \times 86400 \times 30$

$= 1.5 \times 1.2 \times 8.64\ \text{s} = 15.55\ \text{s}$ given.

**7.** As from 0°C to 4°C, density of water increases so the volume of wooden block above water level increases and as from 4°C to 10°C density of water decreases so the volume of block above water decreases.

**8.** $V_1\rho_1 g = V_2'\sigma_1 g$ and $V_2\rho_2 g = V_2'\sigma_2 g$

$\Rightarrow \quad \dfrac{\Delta V_1}{V_1} = 1 - \dfrac{V_1'}{V_1} = -1 - \dfrac{\sigma_1}{\rho_1}$

and $\dfrac{\Delta V_2}{V_2} = 1 - \dfrac{\sigma_2}{\rho_2}$

$\therefore \quad \dfrac{\Delta V_2}{V_2} - \dfrac{\Delta V_1}{V_1}$

$= \left(1 - \dfrac{\sigma_2}{\rho_2}\right) - \left(1 - \dfrac{\sigma_1}{\rho_1}\right) = \dfrac{\sigma_1}{\rho_1} - \dfrac{\sigma_2}{\rho_2}$

$= \dfrac{\sigma_1}{\rho_1} - \dfrac{\sigma_1(1-\gamma_2\Delta T)}{\rho_1(1-\gamma_1 T)}$

$$= \frac{\sigma_1}{\rho_1}\left(\frac{\gamma_2 - \gamma_1}{1 - \gamma_1}\right)_T^{\Delta T}$$

$$= \frac{\sigma_1}{\rho_1} - \frac{\sigma_1(1 - \gamma_2 \Delta T)}{\rho_1(1 - \gamma_1 T)}$$

$$= \frac{\sigma_1}{\rho_1}\left(\frac{\gamma_2 - \gamma_1}{1 - \gamma_1}\right)_T^{\Delta T}$$

**9.** On cooling brass contracts more than iron $(\alpha_B > \alpha_{Fe})$ such that brass disk gets loosen from hole of iron.

**10.** $V \propto T \Rightarrow V = kT \Rightarrow \ln V = \ln k + \ln T$

$$\Rightarrow \frac{\Delta V}{V} = \frac{\Delta T}{T} \Rightarrow \frac{\Delta V}{V \Delta T} = \frac{1}{T} = \gamma$$

# Introductory Exercise 17.2

**1.** For ideal gases, $pV = nRT$

$$\Rightarrow \quad T = \frac{V}{nR}\, p = \frac{VM}{mR}\, p$$

$$\text{Slope} = \frac{VM}{mR}$$

As slope $\propto \frac{1}{m} \Rightarrow m_2 < m_1$

**2.** $pV = nRT \Rightarrow \frac{p_2}{p_1} = \frac{T_2}{T_1}$

$$= \frac{360}{300} = \frac{6}{5}$$

$$\Rightarrow p_2 = \frac{6}{5}\, p_1 = \frac{6}{5} \times 10 \text{ atm} = 12 \text{ atm}$$

**3.** $M_{\text{mix}} = \frac{\frac{1}{4} \times 28 + \frac{1}{4} \times 44}{\frac{1}{4} + \frac{1}{4}} = \frac{7 + 11}{\frac{1}{2}} = 36$

$$pV = nRT = \frac{m}{M} RT$$

$$\Rightarrow pM = \frac{m}{V} RT = \rho RT \Rightarrow \rho = \frac{pM}{RT}$$

$$\therefore \quad \rho = \frac{1.01 \times 10^5 \times 36 \times 10^{-3}}{8.31 \times 290}$$

$$= \frac{1.01 \times 36}{8.31 \times 2.9} = 1.5 \text{ kg/m}^3$$

**4.** $pV = nRT = \frac{N}{N_A} RT$

$$\Rightarrow N = \frac{pVN_A}{RT}$$

$$= \frac{10^{-6} \times 13.6 \times 10^3 \times 10 \times 250 \times 10^{-6} \times 6.02 \times 10^{23}}{8.31 \times 300}$$

$$= \frac{13.6 \times 5 \times 6.02}{8.31 \times 6} \times 10^{15} = 8.21 \times 10^{15}$$

**5.** $pV = nRT$

$$\Rightarrow \quad V = \frac{nR}{p} \cdot T$$

$$\text{Slope} \propto \frac{1}{\rho}$$

$$\Rightarrow \quad p_1 > p_2$$

**6.** $pV = nRT \Rightarrow p = (nRT)\, \frac{1}{V}$

$\Rightarrow y = mx$ is a straight line passing through origin.

# Introductory Exercise 17.3

**1.** Average velocity depends on the direction of motion of gas molecules and as container do not move such that their net effect becomes zero, due to the reason that some molecules are moving in one direction while other are moving in opposite direction. But in case of average speed only magnitudes are in use which do not cancel each other.

**2.** $\text{KE} = \frac{3}{2} kT = \frac{3}{2} \times \frac{8.31}{6 \times 10^{23}} \times 300 \text{ J}$

$= \frac{3}{4} \times 8.31 \times 10^{-21} \text{ J}$

$= 6.21 \times 10^{-21} \text{ J}$

**3.** $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$,

$v_{\text{He}} = \sqrt{\frac{3 \times 8.31 \times 300}{4 \times 10^{-3}}}$

$= 1.37 \times 10^3 \text{ m/s}$

$v_{\text{Ne}} = \sqrt{\frac{3 \times 8.31 \times 300}{20.2 \times 10^{-3}}} = 608.5 \text{ m/s}$

$\text{KE} = \frac{3}{2} kT = 6.21 \times 10^{-21} \text{ J}$

**4.** $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$\Rightarrow T = \frac{Mv_{\text{rms}}^2}{3R} = \frac{4 \times 10^{-3} \times 10^6}{3 \times 8.31}$

$= 160.45 \text{ K}$

**5.** $\rho = \frac{n_1\rho_1 + n_2\rho_2}{n_1 + n_2} = \frac{(1 - n_2)\rho_1 + n_2\rho_2}{1 - n_2 + n_2}$

$= \rho_1 + n_2 (\rho_2 - \rho_1)$

$\Rightarrow n_2 = \frac{\rho - \rho_1}{\rho_2 - \rho_1} = \frac{1.293 - 1.429}{1.251 - 1.429}$

$= \frac{136}{178} = 0.764 = 76.4\%$ by mass

**6.** $\frac{V_2}{V_1} = \frac{p_1 T_2}{p_2 T_1} = \frac{(p_0 + h\rho g)}{p_0 \times 277}$

$= \frac{(1.01 \times 10^5 + 40 \times 10^3 \times 10) \times 293}{1.01 \times 10^5 \times 277}$

$= \frac{5.01 \times 293}{1.01 \times 277} = 5.25$

$\Rightarrow V_2 = 5.25 V_1 = 105 \text{ cm}^3$

**7.** $N = nN_A = \frac{1}{18} \times 6 \times 10^{23}$

$= \frac{1}{3} \times 10^{23}$;

$S = 4\pi R^2 = 4 \times 3.14 \times (6400 \times 10^3 \times 10^2)^2$

$= 5.14 \times 10^{18} \text{ cm}^2$

$\therefore \quad \frac{N}{S} = \frac{10^{23}}{3 \times 5.14 \times 10^{18}}$

$= 6.5 \times 10^3$ molecules/cm$^2$

(a) $nC_V = \frac{3}{2} nR = 35 \text{ J/K}$

$\Rightarrow \quad n = \frac{70}{3R} = 2.8$ mole

(b) $U = \frac{3}{2} nRT = 35 \text{ J/K} \times 273 \text{ K} = 9555 \text{ J}$

(c) $C_p = C_V + R = \frac{5}{2} R = 20.8$ J/K mole

**8.** (a) $n(C_p - C_V) = nR = 29.1 \text{ J/K}$

$\Rightarrow n = \frac{29.1}{8.314}$ mole $= 3.5$ mole

(b) $C_V = nc_V = n\frac{3}{2}R = 3.5 \times 1.5 \times 8.314$

$= 43.65 \text{ J/K}$

$C_p = nc_p = n\frac{5}{2} R = C_V + nR$

$= 43.65 + 3.5 \times 8.314$

$= 72.75 \text{ J/K}$

(c) $C_V' = nc_V = n \times \frac{5}{3} R = 72.75 \text{ J/K}$

$C_p' = nc_p = n \times \frac{7}{2} R$

$= 72.75 + 3.5 \times 8.314$

$= 101.85 \text{ J/K}$

**10.** $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ and $v_{\text{av}} = \sqrt{\frac{8RT}{\pi M}}$

Here $3 > \frac{8}{\pi} \Rightarrow v_{\text{rms}} > v_{\text{av}}$,

*i.e.*, the statement is true.

# AIEEE Corner

## Subjective Questions (Level 1)

**1.** $\frac{C'}{5} = \frac{68-32}{9} = \frac{36}{9} = 4$

$\Rightarrow C' = 20°\text{C}\,;\ \frac{K'-273}{5} = \frac{68-32}{9} = 4$

$\Rightarrow K' = 293\ \text{K}$

$\frac{C'}{5} = \frac{5-32}{9} = -\frac{27}{9} = -3$

$\Rightarrow C' = -15°\text{C};\ \frac{K'-273}{5} = \frac{5-32}{9} = -3$

$\Rightarrow K' = 258\ \text{K}$

$\frac{C'}{5} = \frac{176-32}{9} = \frac{144}{9} = 16$

$\Rightarrow C' = 80°\text{C};\ \frac{K'-273}{5} = 16$

$\Rightarrow K' = 353\ \text{K}$

**2.** $\frac{30}{5} = \frac{F'-32}{9} \Rightarrow F' = 54 + 32 = 86°\text{F}$

$= 546°\text{R}$

$\frac{5}{5} = \frac{F'-32}{9} \Rightarrow F' = 9 + 32 = 41°\text{F} = 501°\text{R}$

$-\frac{20}{5} = \frac{F'-32}{9}$

$\Rightarrow \quad F' = -36 + 32 = -41°\text{F}$

$= 456°\text{R}$

**3.** $\frac{x}{5} = \frac{x-32}{9} \Rightarrow 32 = x - \frac{9}{5}x = -\frac{4}{5}x$

$\Rightarrow \quad x = -\frac{5}{4} \times 32 = -40°$

$\Rightarrow \quad -40°\text{C} = -40°\text{F}$

**4.** $\frac{\Delta C}{5} = \frac{\Delta F}{9} \Rightarrow \Delta F = \frac{9}{5}\Delta C = \frac{9}{5} \times 40 = 72°$

$\therefore\ F_2 = F_1 + 72° = 140.2°\text{F}$

**5.** $\frac{32-20}{80-20} = \frac{C'-0}{100-0}$

$\Rightarrow \frac{12}{60} = \frac{C'}{100} \Rightarrow C' = \frac{12 \times 100}{60} = 20°\text{C}$

**6.** $\frac{T_2}{T_1} = \frac{p_2}{p_1} = \frac{160}{80} = 2 \ \Rightarrow T_2 = 2T_1$

$\therefore\ T_2 = 2 \times 273.15\ \text{K} = 546.30\ \text{K}$

**7.** $R_t = R_0(1 + \alpha\,\Delta\theta)$

$\Rightarrow 3.50 = 2.50\,(1 + 100\alpha) \Rightarrow 1 = 250\,\text{K}$

or $\alpha = \frac{1.0}{250} = 4 \times 10^{-3}/°\text{C}$

$\therefore\ 6.50 = 2.50\,(1 + 4 \times 10^{-3}\,\Delta\theta)$

$\Rightarrow \quad 4 = 10^{-2} \times \Delta\theta$

$\Rightarrow \ \Delta\theta = 400 \ \Rightarrow \ \theta_2 = 400°\text{C}$

$\Rightarrow \ \Delta\theta = 400 \ \Rightarrow \ \theta_2 = 400°\text{C}$

*i.e.*, boiling point of sulphur is 400°C.

**8.** $\frac{T_2}{T_1} = \frac{p_2}{p_1} = \frac{75 + 45}{75 + 5} = \frac{120}{80} = \frac{3}{2}$

$T_2 = \frac{3}{2}\,T_1 = \frac{3}{2} \times 300.15\ \text{K}$

$= 450.225\ \text{K} = 177.08°\text{C}$

**9.** $\Delta\gamma = \gamma\,(\alpha_{Br} - \alpha_{Fe})\,\Delta\theta$

$\Rightarrow \quad \Delta\theta = \frac{\Delta\gamma}{\gamma} \cdot \frac{1}{\alpha_{Br} - \alpha_{Fe}}$

$= \frac{0.01 \times 10^{-3}}{6 \times 10^{-2}} \cdot \frac{1}{\alpha_{Br} - \alpha_{Fe}}$

$= \frac{10^{-3}}{6\,(\alpha_{Br} - \alpha_{Fe})}$

$\therefore \quad \theta_2 = \theta_1 + \frac{10^{-3}}{6\,(\alpha_{Br} - \alpha_{Fe})}$

$= 30°\text{C} + \frac{10^{-3}}{6\,(\alpha_{Br} - \alpha_{Fe})} = 30°\text{C} + \frac{100}{6 \times 0.63}$

$= 57.78°\text{C}.$

**10.** (a) $\Delta l = l\ \alpha\Delta\theta \simeq 88.42 \times 2.4 \times 10^{-5} \times 30$

$= 0.064\ \text{cm}$

(b) $\Delta l = l\,(\alpha_{Al} - \alpha_{St})\,\Delta\theta$

$= 88.42\,(2.4 - 1.2) \times 10^{-5} \times 30$

$= 0.032\ \text{cm}$

$l_S = l + \Delta l = 88.42 + 0.032\ \text{cm}$

$= 88.45\ \text{cm}$

**11.** $\frac{\Delta l}{l} \times 100\% = \alpha \Delta \theta \times 100\%$

$= -1.2 \times 10^{-5} \times 35 \times 100\%$

$= -0.042\%$

**12.** $F = YA \frac{\Delta l}{l} = YA \alpha \Delta\theta$

$= 2 \times 10^{11} \times 2 \times 10^{-6} \times 1.2 \times 10^{-5} \times 40$

$= 4 \times 1.2 \times 40 \text{ N} = 160 \times 1.2 \text{ N} = 192 \text{ N}$

**13.** $V\sigma g = (50 - 45) \times 10^{-3}$ kg

$= 5 \times 10^{-3}$ kg

$V' \sigma' g = (50 - 45.1) \times 10^{-3}$ kg

$= 4.9 \times 10^{-3}$ kg

$$V(1 + \gamma_s \Delta\theta) \frac{\sigma}{1 + \gamma_l \Delta\theta} g = 4.9 \times 10^{-3}$$

$$\frac{1 + \gamma_s \Delta\theta}{1 + \gamma_l \Delta\theta} = \frac{4.9}{5}$$

$$\Rightarrow 5 + 5\gamma_s \Delta\theta = 4.9 + 4.9 \gamma_e \Delta\theta$$

$$\gamma_l = \frac{0.1 + 5\gamma_s \Delta\theta}{4.9 \Delta\theta} = \frac{1}{49\Delta\theta} + \frac{5}{4.9}\gamma_s$$

$$\gamma_s = \frac{1}{49 \times 75} + \frac{5}{4.9} \times 12 \times 10^{-6}$$

$= 272.1 \times 10^{-6} + 12.2 \times 10^{-6}$

$= 2.84 \times 10^{-4}$ °C

**14.** $M = 14 + 3 = 17$ g/mole

$= 17 \times 10^{-3}$ kg/mole

$$\Rightarrow M = \frac{17 \times 10^{-3}}{6.033 \times 10^{-23}} \text{ kg/molecule}$$

$= 2.82 \times 10^{-26}$ kg/molecule

**15.** $n = \frac{pV}{RT} = \frac{1.52 \times 10^6 \times 10^{-2}}{8.314 \times 298.15} = 6.13$

$$\rho = \frac{m}{V} = \frac{nM}{V} = \frac{6.13 \times 2 \times 10^{-3}}{10^{-2}}$$

$= 1.23$ kg/m$^3$

$$\rho' = \frac{m'}{V} = \frac{nM'}{V} = \frac{16 \, nM}{V} = 16\rho$$

$= 19.62$ kg/m$^3$

**16.** $p_2 = p_1 \frac{V_1}{V_2} = 1 \text{ atm} \times \frac{76}{6}$

$= 12.7$ atm

**17.** $V_2 = \frac{p_1 V_1}{T_1} \cdot \frac{T_2}{p_2} = \frac{p_1}{p_2} \cdot \frac{T_2}{T_1} \cdot V_1$

$$= \frac{1}{0.5} \times \frac{270}{300} \times 500 \text{ m}^3 = 900 \text{ m}^3$$

**18.** $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$

$$\Rightarrow \frac{\left(\frac{mg}{A} + p_0\right) A \cdot h_i}{293} = \frac{\left(\frac{mg}{A} + p_0\right) A h_f}{273}$$

$$\Rightarrow h_f = \frac{373}{293} h_i = \frac{373}{293} \times 4 \text{ cm} = 50.9 \text{ cm}$$

**19.** $p_1 = p_2 \Rightarrow \frac{n_1}{V_1} = \frac{n_2}{V_2} = \frac{25/28}{L_1 A} = \frac{40/4}{L_2 A}$

$$\Rightarrow \quad \frac{L_1}{L_2} = \frac{25}{28} \times \frac{1}{10} = \frac{5}{56} = 0.089$$

$$\frac{n_1}{n_2} = \frac{25/28}{40/4} = \frac{25}{280} = \frac{5}{56} = 0.089$$

**20.** $n = n_1 + n_2$

$$\Rightarrow p(V_1 + V_2) = p_1 V_1 + p_2 V_2$$

$$\Rightarrow p = \frac{p_1 V_1 + p_2 V_2}{V_1 + V_2}$$

$$\therefore p = \frac{1.38 \times 0.11 + 0.69 \times 0.16}{0.11 + 0.16} \text{ MP}_a$$

$$= \frac{0.1518 + 0.1104}{0.27} = \frac{0.2622}{0.27} = 0.97 \text{ MP}_a$$

**21.** $\frac{pV_1}{T} + \frac{pV_2}{T} = \frac{p_1 V_1}{T_1} + \frac{p_1 V_2}{T_2}$

$$\frac{1 \text{ atm}}{293 \text{ K}} \times 600 \text{ cm}^3$$

$$= p_1 \left(\frac{400 \text{ cm}^3}{373 \text{ K}} + \frac{200 \text{ cm}^3}{273 \text{ K}}\right)$$

$$\Rightarrow p_1 = \frac{600/293}{\frac{400}{373} + \frac{200}{273}} \text{ atm}$$

$$\therefore \quad p_1 = \frac{3}{293\left(\frac{2}{373} + \frac{1}{273}\right)} \text{ atm}$$

$$= \frac{3}{1.57 + 1.07} = \frac{3}{2.64} \text{ atm}$$

$= 1.136$ atm

**22.** $V = \frac{nRT}{p} = \frac{1 \times 8.314 \times 273.15}{1.013 \times \times 10^5} \text{ m}^3$

$= 0.02242 \text{ m}^3 = 22.42 \text{ litre}$

**23.** $p_2 = \frac{p_1 V_1}{T_1} \cdot \frac{T_2}{V_2} = p_1 \frac{V_1}{V_2} \cdot \frac{T_2}{T_1}$

$= 1.5 \times 10^5 \times \frac{0.75}{0.48} \times \frac{430}{300}$

$= 3.36 \times 10^5 \text{ Pa}$

**24.** $\rho = p_1 + p_1' + p_2$

$= \frac{n_1 RT}{V} + \frac{n_1' RT}{V} + \frac{n_2 RT}{V}$

$= \left[0.7 \times \frac{1.4}{28} + 0.3 \times \frac{1.4}{14} + \frac{0.4}{4}\right] \frac{RT}{V}$

$= \left[\frac{0.7}{20} + \frac{0.3}{10} + \frac{1}{10}\right] \times \frac{8.314 \times 1500}{5 \times 10^{-3}}$

$= \frac{3.3}{20} \times 8.314 \times 3 \times 10^5 \text{ Pa}$

$= 4.11 \times 10^5 \text{ Pa}$

**25.** $\text{RKE} = 2 \times \frac{1}{2} kT = \frac{1}{2} I\omega^2$

$\Rightarrow \omega = \sqrt{\frac{2kT}{I}} = \sqrt{\frac{2 \times 1.38 \times 10^{-23} \times 300}{8.28 \times 10^{-38} \times 10^{-7}}}$

$\therefore \omega = 10^{12} \sqrt{\frac{6 \times 1.38}{8.28}} = 10^{12} \text{ rad/s}$

**26.** $v = \sqrt{\frac{\gamma p}{\rho}}$

$\Rightarrow \gamma = \frac{\rho v^2}{p} = \frac{1.3 \times (330)^2}{1.013 \times 10^5} = 1.398$

$= \frac{f+2}{f} \Rightarrow f = \frac{2}{0.398} \simeq 5$

**27.** $\gamma = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{V_1} + n_2 C_{V_2}} = \frac{3 \times \frac{5}{2} + 2 \times \frac{7}{2}}{3 \times \frac{3}{2} + 2 \times \frac{5}{2}}$

$= \frac{15 + 14}{9 + 10} = \frac{29}{19} = 1.53$

**28.** $K = \frac{3}{2} pV$

$\Rightarrow \frac{K_2}{K_1} = \frac{\frac{3}{2} p_2 V_2}{\frac{3}{2} p_1 V_1} = \frac{3}{2} \cdot \frac{15}{5} = 4.5$

$\Rightarrow K_2 = 4.5 \text{ K}$

**29.** $C_p = \frac{f+2}{2} R = 29 \Rightarrow f = \frac{58}{R} - 2 = 5$

$pT = p \cdot \frac{pV}{nR}$

$\Rightarrow p^2 V = \text{constant}$

$\Rightarrow pV^{1/2} = \text{constant} \Rightarrow a = \frac{1}{2},$

$c = \frac{f}{2} R + \frac{R}{1 - \frac{1}{2}} = \frac{f+4}{2} R = 29$

$f = \frac{58}{R} - 4 = 3$

**30.** $\text{TKE} = \frac{3}{5}$ of total energy and $\text{RKE} = \frac{2}{5}$ of total energy, so the gas is diatomic.

$\text{TKE} = \frac{3}{2} kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 \text{ J}$

$= 6.21 \times 10^{-21} \text{ J/molecule}$

$\Delta Q = nC_V \;\Delta T = 1 \cdot \frac{5}{2} \times 8.314 \times 1 = 20.8 \text{ J}$

**31.** $C_p = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2}$

$= \frac{2.5\,R + 3.5\,R}{1+1} = 3R$

$C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$

$= \frac{1.5\,R + 2.5\,R}{1+1} = 2R$

$\gamma = \frac{C_P}{C_V} = \frac{3R}{2R} = 1.5$

**32.** $\gamma = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{V_1} + n_2 C_{V_2}} = \frac{(n_1 + n_2) C_{p_1}}{(n_1 + n_2) C_{V_1}}$

$= \frac{C_{p_1}}{C_{V_1}} = \gamma$

**33.** $p = aV^b \Rightarrow pV^{-b} = \text{constant}$

$C = \frac{\Delta Q}{n \Delta T} = 0$ for adiabatic process for which $pV^{\gamma} = $ constant comparing, we get, $b = -\gamma$

**34.** $p = kV \Rightarrow pV^{-1} = \text{constant}$

$\Rightarrow pV = a$ constant $\Rightarrow a = -1$

$C = C_V + \dfrac{R}{1-a} = C_V + \dfrac{R}{1+1} = C_V + \dfrac{R}{2}$

**35.** $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}} = \sqrt{\dfrac{3 \times 8.314 \times 373.15}{2 \times 10^{-3}}}$

$= 2.16 \times 10^3$ m/s

$= 2.16$ km/s

**36.** $v_{\text{rms}} = \sqrt{\dfrac{(500)^2 + (600)^2 + (700)^2 + (800)^2 + (900)^2}{5}}$

$= \dfrac{100}{\sqrt{5}} \sqrt{25 + 36 + 49 + 64 + 81}$

$= 714$ m/s

$v_{\text{av}} = \dfrac{500 + 600 + 700 + 800 + 900}{5}$

$= 20\,(5 + 6 + 7 + 8 + 9) = 700$ m/s

$v_{\text{rms}} \neq v_{\text{av}}$

**37.** $\text{KE} = \dfrac{3}{2}\, pV$

$\Rightarrow N \times 6 \times 10^{-26} = 1.5 \times 2 \times 10^5 \times 100 \times 10^{-3} \times 10^{-3}$

$\therefore \quad N = \dfrac{3 \times 10}{6 \times 10^{-26}} = 5 \times 10^{26}$

$= \dfrac{5000}{6.023} \times 6.023 \times 10^{23} = 830.15\,\text{Na}$

$\therefore \quad n = 8300.15$ moles

**38.** Frequency of collision, $\nu = \dfrac{v}{2\sqrt{3}\, l} = \dfrac{v}{2\sqrt{3} \cdot V}$

$= \dfrac{1}{2\sqrt{3}V} \sqrt{\dfrac{3RT}{M}}$

$\therefore \nu = \sqrt{\dfrac{RT}{4VM}} = \sqrt{\dfrac{RT}{4 \cdot \dfrac{nRT}{p} \cdot M}} = \sqrt{\dfrac{p}{4nM}}$

$= \sqrt{\dfrac{2 \times 10^{+5}}{4 \times 1 \times 46 \times 10^{-3}}}$

$= 41.04 \times 10^3$/s

**39.** $\text{KE} = \dfrac{3}{2}\, pV = \dfrac{3}{2} \times 10^5 \times 2 \times 10^{-6} = 0.3$ J

$N = \dfrac{m}{m_1} = \dfrac{50 \times 10^{-6}}{8 \times 10^{-26}} = 6.25 \times 10^{20}$

$\therefore K_1 = \dfrac{K}{N} = \dfrac{0.3}{6.25 \times 10^{20}} = \dfrac{30}{6.25} \times 10^{-22}$ J

$= 4.8 \times 10^{-22}$ J

**40.** $v_0 = \sqrt{\dfrac{3RT_0}{M_0}}$

(a) $\dfrac{v}{v_0} = \sqrt{\dfrac{T}{T_0}} = \sqrt{\dfrac{573}{293}} = 1.4 \Rightarrow v = 1.4\, v_0$

(b) $v = v_0$ as RMS speed changes with temperature and not with pressure.

(c) $\dfrac{v}{v_0} = \sqrt{\dfrac{M_0}{M}} = \sqrt{\dfrac{M_0}{3M_0}} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad v = \dfrac{v_0}{\sqrt{3}} = 0.58\, v_0$

**41.** $\sqrt{\dfrac{\gamma RT}{M_{H_2}}} = \sqrt{\dfrac{\gamma RT'}{M_{O_2}}} \Rightarrow T = \dfrac{M_{H_2}}{M_{O_2}} \cdot T'$

$= \dfrac{2}{32} \times 320 = 20\text{ K} = -253°\text{C}$

**42.** $\dfrac{1}{2}\, m v_e^2 = \dfrac{GMm}{R_e} = g_e R_e m$

$\Rightarrow v_e = \sqrt{2 g_e R_e} = v_{H_2} = \sqrt{\dfrac{3RT}{M}}$

$\Rightarrow T_e = \dfrac{2 g_e R_e M}{3R}$

$= \dfrac{2 \times 9.8 \times 6.367 \times 10^6 \times 2 \times 10^{-3}}{3 \times 8.314}$

$= 10007$ K

and $T_m = \dfrac{2 g_m R_m M}{3R}$

$= \dfrac{2 \times 1.6 \times 1.75 \times 10^6 \times 2 \times 10^{-3}}{3 \times 8.314}$

$= 449$ K

**43.** (a) $\text{KE} = \dfrac{3}{2}\, kT = \dfrac{3}{2} \times 1.38 \times 10^{-23} \times 300$ J

$= 6.21 \times 10^{-21}$ J

(b) $\text{KE}' = \dfrac{3}{2}\, kT \cdot N_a = 6.023 \times 10^{23} \times 6.21 \times 10^{-21}$ J

$= 3740$ J

(c) $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.314 \times 300}{32 \times 10^{-3}}}$

$= 483.6$ m/s

## Objective Questions (Level 1)

**1.** $v_{av} = \sqrt{\frac{8RT}{\pi M}}$

**2.** $v_{rms} = \sqrt{\frac{1^2 + 0^2 + 2^2 + 3^2}{4}}$

$= \sqrt{\frac{14}{4}} = \sqrt{3.5}$ m/s

**3.** $\frac{\Delta l}{l} = -\alpha\Delta\theta = -12 \times 10^{-6} \times 50$

$= -600 \times 10^{-6} = -6 \times 10^{-4}$

**4.** $V \propto T \Rightarrow \frac{V_2}{V_1} = \frac{T_2}{T_1} = 2;$

$\frac{\Delta V}{V} = \frac{V_2 - V_1}{V_1} = 2 - 1 = 1 = 100\%$

**5.** $\text{KE} \propto T \Rightarrow \frac{K_2}{K_1} = \frac{T_2}{T_1} = \frac{2E}{E} = 2$

$\Rightarrow T_2 = 2T_1 = 2 \times 283$ K

$\therefore T_2 = 566 \text{ K} = 293°\text{C}$

**6.** $\text{TE} = \frac{f}{2} kT = \frac{n}{2} kT.$

**7.** $\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{1200}{300}} = 2 \Rightarrow v_2 = 2v_1$

**8.** (a) $p_{av} = m_1 v$ is different for different $m_1$

(b) $(\text{KE})_{\text{molecule}} = \frac{3}{2} kT$ is same for any gas.

(c) $(\text{KE})/V = \frac{3}{2}\frac{pV}{V} = \frac{3}{2} p$ is different as $p$ is different for different.

(d) $(\text{KE})_m = \frac{3}{2}\frac{pV}{m} = \frac{3}{2}\frac{p}{\rho}$ is different as $\frac{p}{\rho}$ is different.

**9.** $\frac{p_1V_1}{RT_1} + \frac{p_2V_2}{RT_2} = \frac{pV_1}{RT} + \frac{pV_2}{RT} = \frac{p(V_1 + V_2)}{RT}$

$$T = \frac{p(V_1 + V_2)}{\frac{p_1V_1}{T_1} + \frac{p_2V_2}{T_2}} = \frac{T_1T_2\; p(V_1 + V_2)}{p_1V_1T_2 + p_2V_2T_1}$$

**10.** $\alpha = \frac{\Delta l}{l\,\Delta\theta} = \frac{0.08 \times 10^{-3}}{10 \times 10^{-2} \times 100}$

$= 8 \times 10^{-6} /°\text{C}$

$\Delta V = V\gamma\Delta\theta = 3V\alpha\Delta\theta = 3 \times 100 \text{ cc} \times 8 \times 10^{-6} \times 100$

$= 0.24$ cc

$\Rightarrow V' = 100 \text{ cc} + 0.24 \text{ cc} = 100.24$ cc

**11.** $T = T_0 + \tan 45° V = T_0 + V$

$pV = nRT = nR(T_0 + V) = nRT_0 + nRV$

or $p = nR + \frac{nRT_0}{V} \Rightarrow p = a + \frac{b}{V}$

*ie*, $p$ *versus* $V$ graph will be hyperbola.

**12.** $p^2V =$ constant

$\Rightarrow \left(\frac{nRT}{V}\right)^2 V = \text{constant.}$

$\Rightarrow T^2 \propto V$

$\Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{V_2}{V_1}} = \sqrt{\frac{3V_0}{V_0}} = \sqrt{3}$

$\Rightarrow T_2 = \sqrt{3}\, T_1 = \sqrt{3}\, T_0$

**13.** $p = \frac{1}{3}\rho v_{rms}^2 = \frac{1}{3}\frac{m}{V} v_{rms}^2$

$\Rightarrow mT = \text{constant}$

$\frac{m_1}{m_2} = \frac{T_2}{T_1} = \frac{310}{280} = 1.1$

**14.** As temperature of vessels $A$ and $B$ are some so is average velocity of $O_2$, *i.e.*, $u$.

**15.** $N = nN_a = \frac{pV}{RT} N_a$

$= \frac{10^{-13} \times 10^{-6}}{8.314 \times 300} \times 6.023 \times 10^{+23}$

$$= \frac{602.3}{8.314 \times 3} = 24$$

**16.** $\frac{1}{2} mv^2 = \frac{GMm}{R} - \frac{GMm}{R+h} = mgh$

$\Rightarrow \quad h \simeq \frac{v^2}{2g} = \frac{3RT}{2gM}$

$\therefore \quad h = \frac{3 \times 8.314 \times 273}{2 \times 10 \times 28 \times 10^{-3}}$ m

$= 12.16 \times 10^3$ m $\approx 12$ km

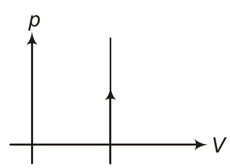
**17.** $\frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1} = \frac{19}{11}$ and $l_1 - l_2 = 30$ cm

$\Rightarrow \quad l_1 = \frac{11}{19} l_1 = 30$ cm

$\Rightarrow \quad l_1 = \frac{19}{8} \times 30$ cm $= 71.25$ cm

and $l_2 = \frac{11}{19} l_1 = 41.25$ cm

**18.** $p \propto V \propto T \Rightarrow V =$ constant




**19.** $V = V_0 + \tan\theta \cdot T,\ pV = nRT = \frac{m}{M} RT$

$\therefore \quad p(V_0 + \tan\theta \times T) = \frac{m}{M} RT$

$\Rightarrow \quad \tan\theta = \frac{1}{pT}\left[\frac{m}{M} RT - pV_0\right]$

or $\quad \tan\theta = \frac{mR}{p_M} - \frac{V_0}{T}$

$\therefore \tan\theta$ remains same when $m \to 2m$ and $p \to 2p$

**20.** $n_1 = \frac{p_1 V}{RT_1}$ and $n_2 = \frac{p_2 V}{RT_2}$

$\frac{n_1}{n_2} = \frac{p_1}{T_1} \cdot \frac{T_2}{p_2} = \frac{10 \times 300}{5 \times 330} = \frac{600}{330} = \frac{20}{11}$

$\therefore\ n_2 = \frac{11}{20} n_1$

$\therefore \Delta m = m_1 - m_2 = m_1 = \frac{11}{20} m_1$

$= \frac{9}{20} \times 28$ g $= \frac{63}{5}$ g

**21.** $\rho = \frac{m}{V} = \frac{m}{(n_1 + n_2)\frac{RT}{p}} = \frac{mp}{(n_1 + n_2)RT}$

$= \frac{12 \times 1.01 \times 10^5 \times 10^{-3}}{(2+2) \times 8.314 \times 300} = 0.12$ kg/m$^3$

**22.** $p = k\rho = \frac{kM}{V}$

$\Rightarrow pV =$ constant is for isothermal process, *i.e.*, $T =$ constant

**23.** $\frac{p^2}{\rho} =$ constant

$\Rightarrow \quad p^2 V$constant $\Rightarrow \quad pT =$ constant

$\frac{p_2}{p_1} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{\rho/2}{\rho}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad p_2 = \frac{p_1}{\sqrt{2}}$

$\frac{T_2}{T_1} = \frac{p_1}{p_2} = \sqrt{2}$

$\Rightarrow \quad T_2 = \sqrt{2}T_1 = \sqrt{2}T$

as $pT =$ constant $\Rightarrow p \propto \frac{1}{T}$

*i.e.*, $p$ - $T$ graph is hyperbola.

**24.** $p^2 V =$ constant

$\Rightarrow PT =$ constant and $T^2 V^{-1} =$ constant.

$\frac{p_2}{p_1} = \sqrt{\frac{V_1}{V_2}} = \sqrt{\frac{V}{4V}} = \frac{1}{2}$

$\Rightarrow \quad p_2 = \frac{p_1}{2} = \frac{p}{2}$

$\frac{T_2}{T_1} = \sqrt{\frac{V_2}{V_1}} = \sqrt{\frac{4V}{V}} = 2$

$\Rightarrow \quad T_2 = 2T_1 = 2T$

as $p \propto \frac{1}{T} \Rightarrow$ p-$T$ graph is hyperbola.

**25.** $\frac{C - 0}{100 - 0} = \frac{F - 32}{212 - 32} = \frac{F - \text{MP}}{\text{BP} - \text{MP}}$

$\Rightarrow$ ice point $= 32°$F and steam point $= 212°$F

**26.** $p \propto V$ at 'a' $p = p_0$ and $V = V_0$ and at 'b', $p = 2p_0$ and $V = 2V_0$, $\rho = \frac{m}{V}$

$\Rightarrow \quad \frac{\rho_b}{\rho_a} = \frac{V_a}{V_b} = \frac{V_0}{2V_0} = \frac{1}{2}$

$\Rightarrow \quad \rho_b = \frac{1}{2}\,\text{Pa}$

$\frac{T_b}{T_a} = \frac{P_b V_b}{P_a V_a} = \frac{2p_0 \cdot 2V_0}{P_0 V_0} = 4$

$\Rightarrow \quad T_b = 4T_a$

$U \propto T \Rightarrow U_b = 4U_a$

as $\quad P \propto V \Rightarrow T \propto V^2$

$\Rightarrow$ Parabola passing through origin

**27.** (a) TKE $= \frac{3}{2} nRT$ is independent of type of gas $\rightarrow$ true.

(b) In one degree of freedom for one mole of gas, $V = \frac{1}{2} RT$

(c) false

(d) false

**28.** $V \propto T \Rightarrow V = \tan\theta \cdot T$

$pV = nRT = \frac{m}{M} RT$

$\Rightarrow p \cdot T \tan\theta = \frac{mRT}{M} \Rightarrow \tan\theta + \frac{mR}{MP}$

$\tan\theta_1 > \tan\theta_2 \Rightarrow \frac{m_1}{p_1} > \frac{m_2}{p_2}$

$\Rightarrow$ all $a$, $b$, $c$ and $d$ are possible.

**29.** $pV = nRT$

$\Rightarrow p = \frac{n}{V} RT = \frac{N_a m/V}{M} RT = \frac{m}{M} RT$

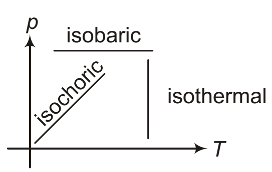
$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kN_a T}{M}} = \sqrt{\frac{3kT}{M}}$

$\Rightarrow$ (a) and (d) are correct.

# JEE Corner

## Assertion & Reason

**1.** Assertion is false.




**2.** Assertion and reason are both true but reason is not correct explanation of assertion. As at low temperature atoms in molecules are tightly bound such that they cannot oscillate.

**3.** $pV = nRT = \frac{2}{3} \cdot \text{KE}$

$\Rightarrow p = \frac{2}{3}\frac{\text{KE}}{V} \Rightarrow p = \frac{2}{3} E.$

Assertion and reason are both true but reason cannot explain assertion.

**4.** Internal energy remains same in train frame of reference, so temperature do not change, but KE of gas molecules in ground frame increases.

**5.** According to equipartition theory, energy is equally distributed for each degree of freedom, so assertion is false.

**6.** At high temperature and low pressure intermolecular distance is much larger than size of the molecules and intermolecular forces can be neglected. So, assertion and reason are both true but not correct explanation.

**7.** At 4°C, volume is minimum or density is maximum *i.e.*, liquid will overflow on increasing or decreasing temperature This reason is false.

**8.** Temperature remains constant as pressure is double and volume is halfed, so internal energy remains constant. So reason partially explains assertion.

**9.** Assertion and reason are both true but not correct explanation.

**10.** $V = \frac{nR}{\rho} T = \left(\frac{MR}{Mp}\right) T \Rightarrow$ slope $\propto m$; reason is correct explanation.

## ■ Match The Columns

**1.**

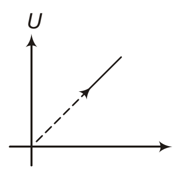
(a) TKE $= \frac{3}{2} nRT = \frac{3}{2} \cdot 2RT = 3RT \rightarrow$ $r$

(b) RKE $= \frac{2}{2} nRT = \frac{2}{2} \cdot 2RT = 2RT \rightarrow$ $p$

(c) PE $= \rightarrow$ $s$

(d) TKE $= \frac{5}{2} nRT = 5RT \rightarrow$ $s$

**2.**




$U = p \Rightarrow T \propto \frac{1}{V} \Rightarrow VT = \text{constant}$

$\Rightarrow pT^2 = \text{constant}$ and $pV^2 = \text{constant}$

(a) $U$ increases $\Rightarrow T$ increases

$\Rightarrow P$ decreases $\rightarrow r$

(b) $p$ increase $\Rightarrow V$ decreases $\rightarrow r$

(c) $U$ increases $\Rightarrow T$ increases $\rightarrow q$

(d) $\frac{T}{V} = \frac{TV}{V^2} = \frac{\text{constant}}{V^2} =$ increase as $V$ decreases $\rightarrow q$

**3.** $x_1 = 3$, $x_2 = \frac{8}{\pi}$, $x_3 = 2$ and $x_4 = r$

(a) $\rightarrow r$, (b) $\rightarrow s$, (c) $\rightarrow q$, (d) $\rightarrow s$

**4.**

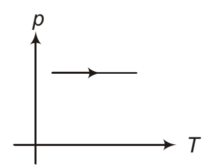
(a) density of water is $\rightarrow$ maximum of 4°C — $s$

(b) depends of change in $\rightarrow$ density of solid and liquid — $s$

(c) depends of change in $\rightarrow$ density of solid and liquid — $s$

(d) $\Delta T = \frac{1}{2} \alpha T \Delta\theta$ increases with $\rightarrow$ increasing temperature — $p$

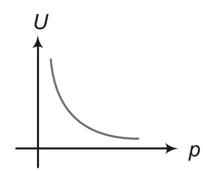
**5.** (a) $p =$ constant $\rightarrow q$




$\longrightarrow q$

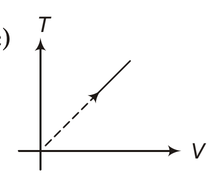
(b) $V \propto T$

$\Rightarrow \frac{1}{\rho} \propto U$



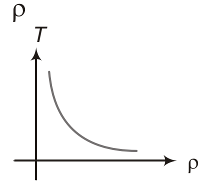

$\longrightarrow R$

(c)




$\longrightarrow p$

(d) $V \propto T \Rightarrow \frac{1}{\rho} \propto T$




$\longrightarrow r$

# 18. First Law of Thermodynamics

## Introductory Exercise 18.1

**1.** (a) $\Delta W = p\Delta V = -1.7 \times 10^5 (1.2 - 0.8)$ J

$= -6.8 \times 10^4$ J

(b) $\Delta V = 1.1 \times 10^5$ J

$\Rightarrow \Delta Q = \Delta U + \Delta W = -17.8 \times 10^4$ J

*i.e.,* $1.78 \times 10^5$ J of heat has flown out of the gas.

(c) No, it is independent of the type of the gas.

**2.** (a) In $p$ - $V$ graph of cyclic process, clockwise rotation gives positive work and anticlockwise gives negative work. And as loop 1 has greater area than loop 2, that is why total work done by the system is positive.

(b) As in cyclic process change in internal energy is zero, that's why for positive work done by the system, heat flows into the system.

(c) In loop '1' work done is positive so, heat flows into the system and in loop '2' work done is negative so heat flows out of the system.

**3.** As the box is insulated *i.e.*, no heat exchange takes place with surrounding and as the gas expands against vacuum *i.e.*, zero pressure that's why no work has been done and there is no change in internal energy. Thus, temperature do not change, internal energy and gas does not do any work.

**4.** $U = \frac{f}{2} nRT = \frac{3}{2} nRT$

$$\Rightarrow \quad n = \frac{2U}{3RT} = \frac{2 \times 100}{3 \times 8.314 \times 300}$$

$= 0.0267$ mole.

**5.** $\Delta Q = ms\,\Delta\theta = 1 \times 387 \times 30\,\text{J} = 11610\,\text{J}$

$$\Delta V = V\gamma\,\Delta\theta = \frac{m}{\rho} \times 3\,\alpha\,\Delta\theta$$

$$= \frac{1}{8.92 \times 10^3} \times 3 \times 7 \times 10^{-6} \times 30$$

$= 7.1 \times 10^{-8}\ \text{m}^{-3}$

$\Delta W = p\,\Delta V = 1.01 \times 10^5 \times 7.06 \times 10^{-8}$

$= 7.13 \times 10^{-3}$ J

$\Delta U = \Delta Q - \Delta W = 11609.99$ J

## Introductory Exercise 18.2

1. (a) At constant volume,

$\Delta U = 0 \Rightarrow \Delta W = 0$

$\Delta Q = nC_V\ \Delta T$

$$\Rightarrow \quad \Delta = \frac{\Delta Q}{nC_V} = \frac{200}{1 \times \frac{3}{2} \times 8.314} = 16.04\ \text{K}$$

$\therefore T_f = T_i + \Delta T = 300 + 16.04 = 316.04$ K

(b) At constant pressure,

$$\Delta T = \frac{\Delta Q}{nC_p} = \frac{200}{1 \times \frac{5}{2} \times 8.314} = 9.62\ \text{K}$$

$\Rightarrow T_f = 300 + 9.62\,\text{K} = 309.62\,\text{K}$

**2.** For adiabatic process,

$pV\gamma = \text{constant} = c$ (say)

$$\therefore \int_{V_i}^{V_f} p\,dV = \int_{V_i}^{V_f} \frac{c}{V^\gamma}\,dV = c\int_{V_i}^{V_f} V^{-\gamma}\,dV$$

$$= c\left.\frac{V^{-\gamma+1}}{-\gamma+1}\right|_{V_i}^{V_f} = c\,\frac{V_f^{-\gamma+1} - V_i^{-\gamma+1}}{1-\gamma}$$

$$= \frac{p_f V_f \cdot V_f^{-\gamma+1} - p_i V_i^{\gamma} \cdot V_f^{-\gamma+1}}{1-\gamma}$$

$$= \frac{p_f V_f - p_i V_i}{1-\gamma} = \frac{p_i V_i - p_f V_f}{\gamma - 1}$$ **(Proved)**

**3.** $\Delta W_{AB} = +500\text{ J}, \Delta Q_{AB} = +250\text{ J}$

$\Rightarrow \Delta U_{AB} = -250\text{ J}$

$\Delta W_{AC} = +700\text{ J}, \Delta Q_{AC} = +300\text{ J}$

$\Rightarrow \Delta U_{AC} = -400\text{ J}$

(a) Path $BC$ is isochoric process, *i.e.*,

$\Delta W_{BC} = 0$

$\therefore \quad \Delta Q_{BC} = \Delta U_{BC} = \Delta U_{AC} - \Delta U_{AB}$

$= -400\text{ J} - (-250\text{ J}) = -150\text{ J}$

(b) $\Delta W_{CDA} = \Delta W_{CD} + \Delta W_{DA}$

$= -800\text{J} + 0 = -800\text{ J}$

(Work is negative as volume is decreasing)

$\Delta U_{CDA} = \Delta U_{AC} = -\Delta U_{AC} = 400\text{ J}$

$\Rightarrow \Delta Q_{CDA} = \Delta W_{CDA} + \Delta U_{CDA}$

$= -800\text{ J} + 400\text{ J} = -400\text{ J}$

**4.** (a) $$T = \frac{pV}{nR} = \frac{1\times 10^{-2} \times 2\times 10^{5}}{1\times 8.314}$$

$= 240.6\text{ K}$

(b) $$\Delta W = \frac{p\Delta V}{\gamma - 1} = \frac{2\times 10^{5} \times 5 \times 10^{-3}}{\frac{5}{3} - 1}$$

$$= \frac{10^3}{2/3}\text{ J}$$

**5.** (a)

$$\Delta K = \frac{p^2}{2m_i} - \frac{p^2}{2m_f} = \frac{p^2}{2}\left(\frac{1}{m_i} - \frac{1}{m_f}\right)$$

$$= \frac{(10\times 10^{-3} \times 200)^2}{2}\left[\frac{1}{10\times 10^{-3}} - \frac{1}{2.01}\right]$$

$$= 2\left[100 - \frac{1}{2}\right] = 199\text{ J}$$

(b) $$\Delta Q = nC_V\,\Delta T \Rightarrow \Delta T = \frac{\Delta Q}{nC_V} = \frac{\Delta Q}{\frac{m}{M}C_V}$$

$$= \frac{M\,\Delta Q}{m\times 3R} = \frac{200\times 199}{2010\times 3\times 8.314}$$

$= 0.8\ °\text{C}$

**6.** $\Delta W = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CD} + \Delta W_{DA}$

$$= nRT_1 m\left(\frac{p_1}{p_2}\right) + p_2(V_C - V_{BC})$$

$$+ nRTm\left(\frac{p_2}{p_1}\right) + p_1(V_1 - V_2)$$

$$= nR(T_2 - T_1)\ln\left(\frac{p_2}{p_1}\right) + p_1V_2 - p_1V_1$$

$$+ p_1V_1 - p_1V_2$$

$$= (p_2V_2 - p_1V_1)\ln\left(\frac{p_2}{p_1}\right)$$

**7.** $\Delta W_{ABCA} = (+)\text{ve} \Rightarrow \Delta W_{AB} = (+)\text{ve}$,

$\Delta W_{BC} = 0, \Delta W_{CA} = (-)\text{ve}$

For $BC$, $\Delta Q = (-)\text{ve} \Rightarrow \Delta U_{BC} = (-)\text{ve}$ and

$\Delta W_{BC} = 0$

For $CA$, $\Delta U = (-)\text{ve} \Rightarrow \Delta Q_{CA} = (-)\text{ve}$ as $\Delta W_{CA} = (-)\text{ve}$.

| | $\Delta U$ | $\Delta W$ | $\Delta Q$ |
|---|---|---|---|
| *AB* | + | + | + |
| *BC* | – | 0 | – |
| *CA* | – | – | – |
| Total | 0 | + | + |

For $AB$, as $\Delta U_{ABCA} = 0$ and $\Delta U_{BC} = (-)\text{ve}$,

$\Delta U_{CA} = (-)\text{ ve}$

$\Rightarrow \quad \Delta U_{AB} = (-)\text{ ve}$

As $\quad \Delta Q_{ABCA} = \Delta W_{ABCA} = (+)\text{ve}$ and

$\Delta Q_{BC} = (-)\text{ve}$

$\Delta Q_{CA} = (-)\text{ve} \Rightarrow \Delta Q_{AB} = (+)\text{ve}$

In isobaric process, $\Delta W = p\Delta V = nR\,\Delta T$

$= 0.2 \times 8.314 \times (300 - 200) = 166.3\text{ J}$

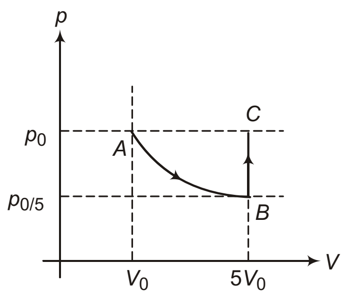
**9.** $$\Delta W = \int p\,dV = \int \alpha V^2\,dV = \frac{1}{3}\alpha V^3$$

$$= \frac{1}{3}\times 5 \times 1.01 \times 10^5 \times (2^3 - 1^3)$$

$= 1.18 \times 10^6\text{ J}$

# Introductory Exercise 18.3

**1.**




$$\Delta W_{BB} = nRT \ln\left(\frac{V_B}{V_A}\right) = 3R \times 273 \ln 5$$

$$= 10959 \text{ J}$$

$$\Delta W_{BC} = 0$$

$$\Delta Q = \Delta U + \Delta W$$

$$\Delta U = \Delta Q - \Delta W$$

$$= 80000 - 10959$$

$$= 69041$$

$$T_f = 5T_i = 5 \times 273 \text{ K} = 1365 \text{ K}$$

$$\Delta Q_{ABC} = \Delta Q_{AB} + \Delta Q_{BC}$$

$$= \Delta W_{BC} + 0 + 0 + \Delta U_{BC}$$

$$\Delta Q_{BC} = nC_V \Delta T \Rightarrow C_V = \frac{\Delta Q_{BC}}{n\Delta T}$$

$$= \frac{69041}{3 \times 4 \times 273} = 21.07$$

$$C_p = C_V + R = 29.39$$

$$\Rightarrow \quad \gamma = \frac{C_p}{C_V} = \frac{29.39}{21.07} = 1.4$$

**2.** $\Delta Q = \Delta U + \Delta W; \Delta Q = nC_p \Delta T$

$$\Rightarrow \quad 1600 = 1 \cdot C_p \cdot 72$$

$$\Rightarrow \quad C_p = 22.22$$

$$C_V = C_p - R = 13.9 \Rightarrow \gamma = \frac{C_p}{C_V} = 1.6$$

$$\Delta W = \Delta Q - \Delta U = 1600 - nC_V \Delta T$$

$$= 1600 - 1 \times 13.9 \times 72$$

$$= 1600 - 1000.8 \text{ J}$$

$$= 599.2 \text{ J}$$

and $\Delta U = nC_V \Delta T = 1 \times 13.9 \times 72$

$$= 1001$$

$$= 1 \text{ kJ}$$

**3.** $\Delta W = \frac{1}{2} \Delta p \Delta V$

$$= \frac{1}{2} \times 20 \times 1.01 \times 10^5 \times 1 \times 10^{-3}$$

$$= 10 \times 101 = 1010 \text{ J}$$

$$\therefore p = \frac{n\Delta W}{\Delta t} = \frac{100 \times 1010 \text{ J}}{60 \text{s}}$$

$$= 1.68 \text{ kW}$$

# AIEEE Corner

## ■ Subjectve Questions (Level 1)

**1.** $\Delta U = \Delta Q - \Delta W = 254 \text{ J} + 73 \text{ J}$

$$= 327 \text{ J}$$

**2.** (a) $\Delta T = \frac{\Delta Q}{nC_V} = \frac{2\Delta Q}{3nR} = \frac{2 \times 200}{2 \times 1 \times 8.314}$

$$= 16 \text{ K}$$

$$\Rightarrow \quad T_f = T_i + \Delta T = 316 \text{ K}$$

(b) $\Delta T' = \frac{\Delta Q}{nC_p} = \frac{2\Delta Q}{5nR} = \frac{2 \times 200}{5 \times 1 \times 8.314}$

$$= 9.6 \text{ K}$$

$$\Rightarrow T'_f = T_i + \Delta T' = 309.6 \text{ K}$$

**3.** $\Delta U = nC_V \Delta T$, in adiabatic process,

$\Delta Q = 0$ and $\Delta U = -\Delta W$

where, $\Delta W = \dfrac{nR\,\Delta T}{1-\gamma}$

$\Rightarrow \quad \Delta U = \dfrac{nR\Delta T}{\gamma - 1}$ for all process.

**4.** $\Delta V = 0 \Rightarrow \Delta W = 0$

$\therefore \Delta Q = \Delta U = nC_V\,\Delta T = n\cdot\frac{5}{2}R\,\Delta T$

$= \frac{5}{2}(p_f V_f - p_i V_i) = \frac{5}{2}(p_f - V_i)V$

$= \frac{5}{2}(5\times 10^5 - 10^5)\times 10\times 10^{-3}$

$= \frac{5}{2}\times 4\times 10^5 \times 10^{-2} = 10^4$ J

**5.** $\Delta Q_1 = \Delta U_1 = nC_V\Delta T = n\cdot\frac{5}{2}R\,(3T_i - T_i)$

$= 5\,nRT_i$

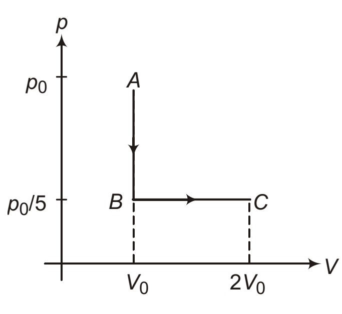
$\Delta Q_2 = nC_p\Delta T = n\cdot\frac{5}{2}R\,(6T_i - 3T_i)$

$= 7.5\,nRT_i$

$\therefore c = \dfrac{\Delta Q}{n\,\Delta T} = \dfrac{12.5\,nRT_i}{n\,(6T_i - T_i)} = \dfrac{12.5R}{5} = 2.5R$

**6.** $\Delta W_{AB} = 0, \Delta W_{BC} = \frac{p_0}{2}\times V_0 = \frac{1}{2}p_0V_0$

$= \frac{1}{2}\cdot nRT_0 = 300\,R$




$\Delta Q = (\Delta U + \Delta W)_{AB} + (\Delta U + \Delta W)_{BC}$

$= \Delta U_{AB} + \Delta U_{BC} + \Delta W_{BC}$

$= 0 + 300\,R$

(As $T_A = T_C$)

$= 2.49\times 10^3$ J $= 2.49$ kJ

**7.** $\Delta U = \Delta Q - \Delta W = 1200$ J $- 2100$ J

$= 900$ J

$\Delta T = \dfrac{\Delta U}{nC_V} = \dfrac{900}{5\times\frac{3}{2}\times 8.314} = 14.43$

$\Rightarrow T_f = T_i - \Delta T = 127°\text{C} - 14.43°\text{C}$

$= 112.6°\text{C}$

**8.** When gas expands it does positive work on the surrounding and for this purpose heat has to be supplied into the system.

**9.** $\Delta W = \rho\Delta V = \rho(V_f - V_i)$

$= \rho m\left(\dfrac{1}{\rho_f} - \dfrac{1}{\rho_i}\right) = \rho m\left(\dfrac{1}{1000} - \dfrac{1}{999.9}\right)$

$= -\dfrac{10^5\times 2\times 0.1}{1000\times 999.9} = -0.02$ J

(work done is negative as volume decreases)

$\Delta Q = ms\,\Delta\theta = 2\times 4200\times 4$

$= 33600$ J

$\Delta U = \Delta Q - \Delta W = 33600.02$ J

**10.** $\Delta W = p\Delta V \approx pV_f = p\dfrac{m}{\rho}$

$= \dfrac{10^5\times 10\times 10^{-3}}{0.6} = 1666.67$ J

$\Delta Q = ms\Delta\theta + mL$

$= 10^{-2}\times 4200\times 100 + 10^{-2}\times 2.5\times 10^6$

$= 4200$ J $+ 25000$ J $= 29200$ J

$\Delta U = \Delta Q - \Delta W = 29200$ J $- 1666.67$ J

$= 27533.33$ J $= 2.75\times 10^4$ J

**11.** $\Delta W = p\,\Delta V$

$= 1.013\times 10^5\times 1670\times 10^{-6}$

$= 1.013\times 167$ J $= 169.2$ J

$\Delta Q = mL = 10^{-3}\times 2.256\times 10^6$ J

$= 2256$ J

$\therefore \Delta U = \Delta Q - \Delta W = (2256 - 169.2)$ J

$= 2086.8$ J $\approx 2087$ J

**12.** $\Delta W = p\Delta V = -2.3\times 10^5\times 0.5$

$= -1.15\times 10^5$ J

$\Delta U = -1.4\times 10^5$ J

$\Rightarrow \Delta Q = \Delta U + \Delta W$

$= -(1.4 + 1.15) \times 10^5 \text{ J}$
$= -2.55 \times 10^5 \text{ J}$

Thus, $2.55 \times 10^5$ J of heat flows out of the system and it is independent of the type of the gas.

**13.** In a cyclic process, $U = 0 \Rightarrow \Delta Q = \Delta W$

(a) $\therefore W_\eta = (Q_1 + Q_2 + Q_3 + Q_4) - (W_1 + W_2 + W_3)$

$= (5960 - 5585 - 2980 + 3645) - (2200 - 825 - 1100)$

$= 1040 - 275 = 765 \text{ J}$

(b) $\eta = \dfrac{\text{work done}}{\text{heat supplied}} = \dfrac{1040}{9605} = 10.83\%$

**14.** (a) $\Delta W = \dfrac{1}{2}\overrightarrow{\mathbf{AB}} \times \overrightarrow{\mathbf{AC}} = \dfrac{1}{2} \times 2p_0 \times V_0$

$= p_0 V_0$

(b) $T_C = \dfrac{2\,p_0 V_0}{nR}$ and $T_A = \dfrac{p_0 V_0}{nR}$

$\Rightarrow \Delta Q_{CA} = nC_p \Delta T = -nC_p \cdot \dfrac{p_0 V_0}{nR}$

$= -\dfrac{5}{2} R \cdot \dfrac{p_0 V_0}{R} = -\dfrac{5}{2} p_0 V_0$

$T_B = \dfrac{3 p_0 V_0}{nR}$, $\Delta Q_{AB} = nC_V \Delta T$

$= n \times \dfrac{3}{2} R \left( \dfrac{3 p_0 V_0}{nR} - \dfrac{p_0 V_0}{nR} \right)$

$= \dfrac{3}{2} \times 2\, p_0 V_0 = 3 p_0 V_0$

(c) $\Delta Q_{AB} + \Delta Q_{BC} + \Delta Q_{CA} = \Delta W$

$= 3 p_0 V_0 + \Delta Q_{BC} - \dfrac{5}{2} p_0 V_0 = p_0 V_0$

$\Rightarrow \quad \Delta Q_{BC} = \dfrac{p_0 V_0}{2}$

(d) Temperature is maximum at a point $D$ lying somewhere between $B$ and $C$ where the product $pV$ is maximum.

$p = -\dfrac{2p_0}{V_0} + 5p_0$

$\Rightarrow \quad pV = \left( \dfrac{-2p_0}{V_0} V + 5p_0 \right) V$

$= -\dfrac{2p_0}{V_0} V^2 + 5p_0 V$

For $pV$ = maximum $\dfrac{d}{dV}(pV) = 0$

$\Rightarrow \quad -2V \cdot \dfrac{2p_0}{V_0} + 5p_0 = 0$

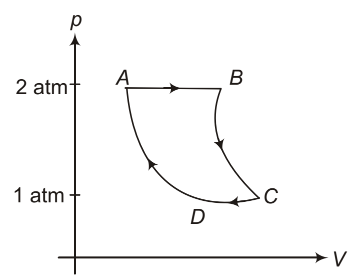
$\Rightarrow \quad V = \dfrac{5V_0}{4}$

$p = -\dfrac{2p_0}{V_0} \cdot \dfrac{5V_0}{4} + 5p_0 = \dfrac{5}{2} p_0$

$\therefore \quad T_{\max} = \dfrac{(pV)_{\max}}{nR}$

$\therefore \quad p = \dfrac{\dfrac{5}{2} p_0 \cdot \dfrac{5}{4} V_0}{1 \cdot R} = \dfrac{25 p_0 V_0}{8R}$

**15.** $V_A = \dfrac{nRT_A}{p_A} = \dfrac{2R \times 300}{2 \times 10^5} = 3 \times 10^{-3} R$




$V_B = \dfrac{2R \times 400}{2 \times 10^5} = 4 \times 10^{-3} R,$

$V_C = \dfrac{2R \times 400}{10^5} = 8 \times 10^{-3} R$

$V_0 = \dfrac{2R \times 300}{10} = 6 \times 10^{-3} R$

$\Delta W = 2 \times 10^5 (4 - 3) \times 10^{-3} R + 2R \times 400 \ln\left(\dfrac{8}{4}\right) + 1 \times 10^5 (6 - 8) \times 10^{-3}$

K

$+ 2R \times 300 \ln\left(\dfrac{3}{6}\right)$

$\therefore \; \Delta W = 200R + 800R \ln 2 - 200R - 600R \ln 2$

$= 2000R \ln 2 = 1153 \text{ J}$

As $\Delta Q = \Delta W = 1153$ J and $\Delta U = 0$ cyclic process.

**16.** $\Delta W = \frac{1}{2}\left(\frac{3V_0}{2} - \frac{V_0}{2}\right)(p_B - p_0)$

$$+ \frac{1}{2}\left(\frac{3V_0}{2} - \frac{V_0}{2}\right)(p_0 - p_0)$$

$$= \frac{1}{2}V_0(p_B - p_0) + \frac{1}{2}V_0(p_0 - p_D)$$

$$= \frac{1}{2}V_0(p_B - p_D)$$

where, $p_B = p_0 + \frac{p_0}{V_0}\cdot\frac{V_0}{2} = \frac{3p_0}{2}$

and $p_D = p_0 - \frac{p_0}{V_0}\cdot\frac{V_0}{2} = \frac{p_0}{2}$

$$\therefore \Delta W = \frac{1}{2}V_0\left(\frac{3}{2}p_0 - \frac{1}{2}p_0\right) = \frac{1}{2}p_0V_0$$

$$\Delta W_{ABC} = p_0\left(\frac{3V_0}{2} - \frac{V_0}{2}\right) + \frac{1}{2}\left(\frac{3V_0}{2} - \frac{V_0}{2}\right)$$

$$(p_B - p_0) = p_0V_0 + \frac{1}{2}V_0\left(\frac{3}{2}p_0 - p_0\right)$$

$$= \frac{5}{4}p_0V_0$$

$$\Delta U_{ABC} = nC_V(T_C - T_A)$$

$$= n\cdot\frac{3}{2}R\left[\frac{p_0\frac{3V_0}{2}}{nR} - \frac{p_0\frac{V_0}{2}}{nR}\right] = \frac{3}{2}p_0V_0$$

$$\therefore \Delta Q_{\text{supplied}} = \frac{5}{4}p_0V_0 = \frac{11}{4}p_0V_0$$

$$\eta = \frac{\frac{1}{2}p_0V_0}{\frac{11}{4}p_0V_0} = \frac{2}{11} = 0.1818 = 18.18\%$$

**17.** (a) As the cyclic process is clockwise *i.e.*, work done is positive, so heat is absorbed by the system.

(b) In cyclic process work done is equal to the net heat absorbed (as change in internal energy is zero) so, work done in one cycle is 7200 J.

(c) In anticlockwise rotation, work done is negative and heat is liberated by the system, and its magnitude is 7200 J.

**18.** (a) As area under clockwise loop is more than that at anticlockwise loop, so network done is positive.

(b) In loop I work done is positive and in loop II work done is negative.

(c) As network done in one cycle is positive so heat flows into the system.

(d) In loop I heat flows into the system and in loop II heat flows out of the system.

**19.** $T_A = \frac{p_AV_A}{nR} = \frac{1.01\times 10^5 \times 22.4}{10^3 \times 8.314}$

$$= 273\text{ K}$$

$$T_B = \frac{p_BV_A}{nR} = \frac{2p_AV_A}{nR} = 2T_A$$

$$= 546\text{ K} = T_c$$

$$V_c = \frac{nRT_c}{p_c} = \frac{nRT_c}{p_A} = \frac{nRT_B}{p_A}$$

$$= \frac{2nRT_A}{p_A} = 2V_A = 44.8\text{ m}^3$$

**20.** (a) $\Delta W = AB \times BC$

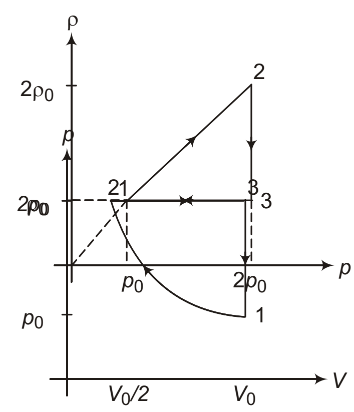
$$= (4 - 1.5)\times 10^{-6} \times (4 - 2)\times 10^5$$

$$= 2.5 \times 0.2 = 0.5\text{ J}$$

(b) $\Delta Q = \Delta W$ as $\Delta U = 0$ in a cycle

$\Rightarrow \Delta Q = 0.5\text{ J}$

**21.** As $\rho \propto \frac{1}{V}$



(a) $\Delta W_{12} = nRT_0 \ln\left(\frac{V_0/2}{V_0}\right)$

$= -p_0V_0 \ln 2 = -p_0 \frac{M}{\rho_0} \ln 2$

$\Delta W_{23} = 2p_0\left(V_0 - \frac{V_0}{2}\right) = p_0V_0$

$= p_0 \frac{M}{\rho_0}; \Delta W_{31} = 0$

(b) $\Delta Q_{231} = \Delta Q_{23} + \Delta Q_{31}$

$= nC_V\Delta T_{23} + \Delta W_{23} + nC_V\Delta T_{31}$

$= n \times \frac{3}{2}R\left(\frac{2p_0V_0}{nR} - \frac{2p_0\frac{V_0}{2}}{nR}\right) + p_0V_0$

$+ n \times \frac{3}{2}R\left(\frac{p_0V_0}{nR} - \frac{2p_0 \times \frac{V_0}{2}}{nR}\right) +$

$p_0V = \frac{3}{2}p_0V_0 + p_0V_0 = \frac{5}{2}p_0V_0$

$\therefore$ Heat rejected $= \Delta Q_{231} - \Delta W$

$= \frac{5}{2}p_0V_0 - p_0V_0 + p_{V_0} \ln 2$

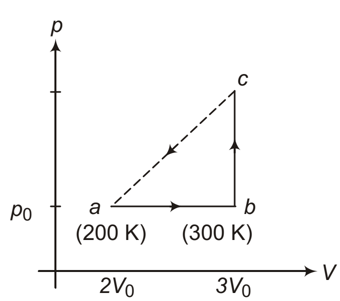
$= \frac{3}{2}p_0V_0 + p_0V_0 \ln 2$

$= p_0V_0\left(\frac{5}{2} + \ln 2\right) - \frac{p_0M}{p_0}\left(\frac{3}{2} - \ln 2\right)$

(c) $\eta = \frac{\text{work done}}{\text{heat supplied}} = \frac{\Delta W}{\Delta Q_{231}}$

$= \frac{p_0V_0 - p_0V_0 \ln 2}{\frac{5}{2}p_0V_0} = \frac{2}{3}(1 - \ln 2)$

**22.** $\Delta W_{AB} = p_0(3V_0 - 2V_0) = p_0V_0;$




$\Delta W_{BC} = 0, \Delta W_{CA} = ?$

$\Delta Q = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$

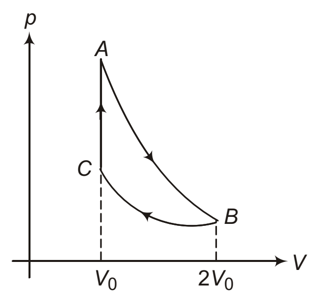
$-800 \text{ J} = P_0V_0 + 0 + \Delta W_{CA}$

$\Rightarrow \Delta W_{CA} = -800 \text{ J} - p_0V_0$

$= -800 \text{ J} - \frac{1}{2}nRT_A$

$\therefore \Delta W_{CA} = -800 \text{ J} - 200R = -2463 \text{ J}$

**23.** $\Delta W_{AB} = \frac{p_BV_B - p_AV_A}{1-\gamma}$




$= \frac{3}{2}(p_AV_A - p_BV_B)$

$= \frac{3}{2}nR(T_A - T_B) = \frac{3}{2}nRT_B\left(\frac{T_A}{T_B} - 1\right); TV^{\gamma - 1}$

$= \frac{3}{2}nRT_B\left(\left(\frac{2}{T}\right)^{\frac{5}{3}-1} - 1\right) = \frac{3}{2}nRT_B(2^{2/3} - 1)$

$\Delta W_{BC} = nRT_B \ln\left(\frac{V_0}{2V_0}\right)$

$= -nRT_B \ln 2$ and $\Delta W_{CA} = 0$

Heat Supplied

$\Delta Q_{CA} = \Delta U_{CA} = \frac{3}{2}nR(T_A - T_C)$

$= \frac{3}{2}nR(T_A - T_B)$

$= \frac{3}{2}nRT_B\left(\frac{T_A}{T_D} - 1\right) = \frac{3}{2}nRT_B(2^{2/3} - 1)$

$\therefore \eta = \frac{\Delta W}{\Delta Q_{CA}}$

$= \frac{\frac{3}{2}nRT_B(2^{2/3} - 1) - nRT_B \ln 2 + 0}{\frac{3}{2}nRT_B(2^{2/3} - 1)}$

$$\eta = 1 - \frac{2}{3} \cdot \frac{\ln 2}{2^{2/3} - 1} = 1 - 0.7867 = 0.213$$

$$= 21.3\%$$

## ■ Objective Questions (Level 1)

**1.** $U = nC_V T = \frac{3}{2} \times 1 \times RT = \frac{3}{2} RT$

$$\Rightarrow \quad T = \frac{2U}{3R}$$

$$T_D = \frac{2V_0}{3R} = 300\,\text{K} \Rightarrow U_0 = 450R,$$

$$T_A = \frac{4V_0}{3R} = 600\,\text{K}$$

$$\Delta W = \Delta W_{AB} + W_{CD} = nRT_A \ln\left(\frac{2V_0}{V_0}\right)$$

$$+ nRT_D \ln\left(\frac{V_0}{2V_0}\right) = nR(T_A - T_D)\ln 2$$

$$= 1 \times R \times (600 - 300)\ln 2$$

$$= 300R \ln 2 = \Delta Q$$

**2.** $\Delta W_{12} = p\Delta V = nR\Delta T$

$$= 2R \times 300 = 600R$$

$\Delta W_{23} = ?;\ \Delta W_{31} = 0.$

As, $\Delta Q = \Delta W_{12} + \Delta W_{23} + \Delta W_{31}$

$$= -300\text{ J} = 600R + \Delta W_{23} + 0$$

$$\Rightarrow \Delta W_{23} = -300\text{ J} - 600R = -5288\text{ J}$$

**3.** $nC_p \Delta T_1 = nC_V \Delta T_2$

$$\Rightarrow \quad \frac{7}{2} \times 30 = \frac{5}{2} \Delta T_2$$

$$\Rightarrow \quad \Delta T_2 = 42\text{ K}$$

**4.** $TV^{n-1} = \text{constant}$

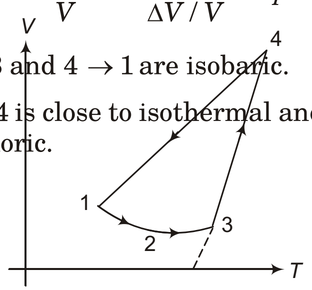
$$\Rightarrow pV \cdot V^{n-1} = pV^n = \text{constant}$$

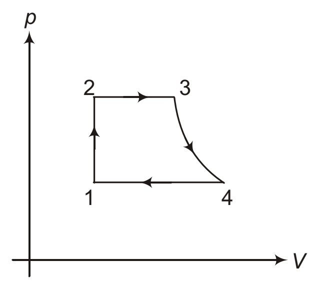
$$\ln p + n \ln V = \ln c$$

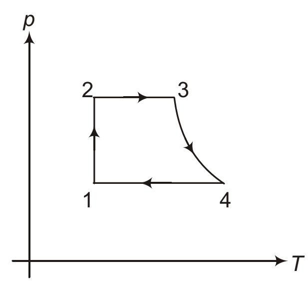
$$\frac{\Delta p}{p} = -n\frac{\Delta V}{V} \Rightarrow -\frac{\Delta p}{\Delta V / V} = np = B$$

**5.** $2 \to 3$ and $4 \to 1$ are isobaric.

$3 \to 4$ is close to isothermal and $1 \to 2$ is isochoric.




**6.** $W = \int p\,dV = \int kV\,dV = \frac{k}{2} V^2$

$$= \frac{1}{2} pV = \frac{1}{2} nR\,(T_2 - T_1) = \frac{R}{2}(T_2 - T_1)$$

**7.** $p \propto V^2,\ W = \int p\,dV = \int kV^2\,dV$

$$= \frac{1}{3} kV^3 = \frac{1}{3} pV$$

$$= \frac{1}{3} nR\,(T_f - T_i) = (+)\,\text{ve}$$

**8.** $\Delta W = -nRT \ln\left(\frac{V_f}{V_i}\right)$

$$= -nRT \ln\left(\frac{1}{2}\right) = nRT \ln 2$$

**9.** $\Delta U = 600\text{ J} - 150\text{ J} = 450\text{ J}$

$$= nC_V \Delta T = \frac{3}{2} R \cdot n\Delta T$$

$$C = \frac{\Delta Q}{n\Delta T} = \frac{600\text{ J}}{\frac{450\text{ J}}{\frac{3}{2}R}} = \frac{3}{2}R \times \frac{600}{450}$$

$$= \frac{3}{2}R \times \frac{4}{3} = 2R$$

**10.** $\Delta W_1 = (+)\text{ ve}, \Delta W_2 = 0, \Delta W_3 = (-)\text{ ve}$

and $\Delta U_1 = \Delta U_2 = \Delta U_3$

as $\Delta Q = \Delta U + \Delta W \Rightarrow Q_1 > Q_2 > Q_3$

**11.** $U = 2p_0 2V_0 - 2p_0 V_0 = 2p_0 V_0$

and $\Delta W = p_0(2V_0 - V_0) = p_0 V_0$

$\therefore\ \Delta Q = \Delta U + \Delta W = 3p_0 V_0$

**12.** In adiabatic compression, temperature of the gas increases and as $pV \propto T$ so, $pV$ increases.

**13.** As $\Delta W_1 < \Delta W_2$ while $\Delta U_1 = \Delta U_2$

$\Rightarrow\ \Delta Q_1 < \Delta Q_2$

$\Rightarrow\ C_1 < C_2$

$\Rightarrow\ \frac{C_1}{C_2} < 1$

**14.** $$\Delta W = nR(4T - T) + \frac{nR(5T - 4T)}{1-\gamma} + nR(3T - 5T) + \frac{nR(T - 3T)}{1-\gamma}$$

$$= 3nRT - 2nRT + \frac{nRT}{1-\gamma} - \frac{2nRT}{1-\gamma}$$

$$= nRT + \frac{nRT}{\gamma - 1}$$

$$= \frac{nRT}{\gamma - 1}(\gamma - 1 + 1) = \frac{\gamma}{\gamma - 1}nRT$$

$$= \frac{5/3}{5/3 - 1}\cdot 1RT = 2.5RT$$

**15.** $Up$ = constant

$$= \frac{3}{2}nRT\cdot\frac{nM}{V} = \frac{3}{2}n^2MR\frac{T}{V}$$

$\Rightarrow T \propto V$ *i.e.*, isobaric process.

$$\frac{\Delta U}{\Delta W} = \frac{\Delta U}{\Delta Q - \Delta U} = \frac{3/2}{\frac{5}{2} - \frac{3}{2}} = \frac{3}{2}$$

$$= \frac{C_V}{C_p - C_V} = \frac{C_V}{R}$$

**16.** $\Delta W = 50 \times (0.4 - 0.1) + \frac{1}{2} \times 50 \times (0.2 - 0.1)$

$= 15 + 2.5 = 27.5\text{ J}$

$\Delta U = 2.5\text{ J}$

$\Rightarrow\ \Delta Q = \Delta U + \Delta W = 20\text{ J}$

**17.** $W_1 = \int_{V_0}^{2V_0} pdV = p(2V_0 - V_0) = pV_0$

$$W_2 = \int_{V_0}^{2V_0} kVdV = \frac{1}{2}kV^2$$

$$= \frac{1}{2}k(4V_0^2 - V_0^2) = \frac{3}{2}kV_0^2 = \frac{3}{2}PV_0$$

$\Rightarrow\ W_1 < W_2$

**18.** $\Delta W = \pi r_1 r_2 = \pi ab$

$$= \pi\frac{r_2 - r_1}{2}\cdot\frac{(p_2 - p_1)}{2}$$

$$= \frac{\pi}{4}(p_2 - p_1)(V_2 - V_1)$$

**19.** $$W = \int PdV = \int\frac{nRT}{V - b}dV = nRT\int\frac{dx}{x}$$

$x = V - b$

$dx = dV$

$$= nRT\ln x = nRT\ln(V - b)\Big|_V^{2V}$$

$$= nRT[\ln(2V - b) - \ln(V - b)]$$

$$= nRT\ln\left|\frac{2V - b}{V - b}\right| = RT\ln\left|\frac{2V - b}{V - b}\right|$$

as $n = 1$ mole

**20.** $AB$ is isochoric process, so, $\Delta W_{AB} = 0$

$BC$ is isothermal process, so,

$$\Delta W_{BC} = nRT_2\ln\left(\frac{V_2}{V_1}\right) = RT_2\ln\left(\frac{V_2}{V_1}\right)$$

$CA$ is close to isobaric process, so,

$$\Delta W_{CA} = nRT = nR(T_1 - T_2) = R(T_1 - T_2)$$

**21.** $\Delta Q = \Delta U + \Delta W = -\Delta Q + \Delta W$

$\Rightarrow \Delta W = 2\Delta Q$

$$\Delta U = nC_V\Delta T = n\frac{f}{2}R\Delta T = \frac{n}{\gamma - 1}R\Delta T;$$

$$\gamma = \frac{f+2}{f} = 1 + \frac{2}{f}$$

$$f = \frac{2}{\gamma - 1}$$

$$\Delta W = \int p\,dV = 2\,\Delta Q - \frac{2n}{\gamma - 1} R\Delta T$$

$$= \frac{2\,nR\Delta T}{1-\gamma} = \frac{nR\Delta T}{1-a} \text{ for polytropic}$$

process with $pV^a$ = constant

$$\therefore \quad \frac{2}{1-\gamma} = \frac{1}{1-a} \Rightarrow 1 - a = \frac{1-\gamma}{2}$$

$$\Rightarrow \quad 1 - \frac{1}{2} + \frac{\gamma}{2} = a$$

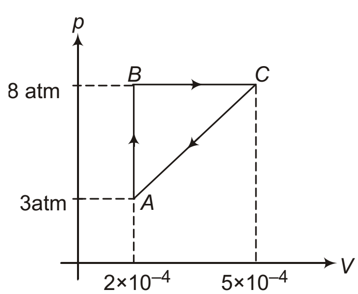
or $\quad a = \frac{1}{2} + \gamma = \frac{1+\gamma}{2}$

$$\therefore \quad pV^a = \text{constant}$$

$$= TV^{a-1} = TV^{\frac{1+\gamma}{2} - 1}$$

$$= TV^{\frac{\gamma - 1}{2}} = \text{constant}$$

**22.** $\Delta W_{AB} = 0, \Delta U_{AB} = 600$ J




$$\Delta W_{BC} = 8 \times 10^5 (5-2) \times 10^{-4} = 240 \text{ J}$$

$$\Delta U_{BC} = Q_{BC} - W_{BC} = 200 - 240$$

$$= -40 \text{ J}$$

$\Delta U = \Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA} = 0$ in cyclic process.

$$\therefore \ \Delta U_{CA} = -\Delta U_{AB} - \Delta U_{BC}$$

$$= -600 \text{ J} + 40 \text{ J} = -560 \text{ J}$$

**23.** Starting and ending points along $x$-axis in graph are not clear, so nothing can be said about the magnitude of work.

It can only be said that work done in $ABC$ is negative and that in $DEF$ is positive. Looking at the graph, area can be assumed to be equal so,

$$W_{DEF} = -W_{ABC}.$$

**24.** $\Delta W_{\text{isobaric}} = p\Delta V = p(2V - V) = pV$

$$\Delta W_{\text{isothermal}} = nRT \ln\left(\frac{2V}{V}\right) = pV \ln 2$$

$$= 0.693\ pV$$

$$\Delta W_{\text{adiabatic}} = \frac{p_f \cdot 2V - p_i V}{1-\gamma}$$

$$= \frac{p_i \left(\frac{V}{2V}\right)^r \cdot 2V - p_i V}{1-\gamma} = \frac{pV(2^{1-r} - 1)}{1-r}$$

$$= pV\left(\frac{1-2^{1-r}}{r-1}\right) = pV\left(\frac{1-4^{-1/3}}{2/3}\right)$$

$$= 0.55\ pV$$

So, work done is minimum in adiabatic process.

**25.** $\Delta Q = \Delta U + \Delta W$

$$\frac{7}{2} RT_0 = 10 \times \frac{5}{2} R\Delta T + 10R\Delta T = 35R\Delta T$$

$$T_0 = 100T = 10\,(T - T_0)$$

$$\Rightarrow \quad 11\,T_0 = 10\,T$$

$$\Rightarrow \quad T = 1.1\,T_0$$

$$\frac{pV_0}{RT_0} = \frac{pV}{R \times 1.1\,T_0}$$

$$\Rightarrow \quad V = \frac{11}{10} V_0 = 1.1\,V_0$$

**26.** $\Delta W = (3\,p_0 - p_0)(2V_0 - V_0) = 2p_0 V_0$

$$\Delta Q_{\text{supplied}} = n\frac{3}{2}R\left(\frac{3\,p_0 V_0}{nR} - \frac{p_0 V_0}{nR}\right)$$

$$+ n \cdot \frac{5}{2} R\left(\frac{3\,p_0 \cdot 2V_0}{nR} - \frac{3\,p_0 V_0}{nR}\right)$$

$$= \frac{3}{2} nR \cdot \frac{2p_0 V_0}{nR} + \frac{5}{2} nR \frac{3\,p_0 V_0}{nR}$$

$$= 3\,p_0 V_0 + \frac{15}{2} p_0 V_0 = \frac{21}{2} p_0 V_0$$

$$\eta = \frac{\Delta W}{\Delta r} = \frac{2p_0 V_0}{\frac{21}{2} p_0 V_0} = \frac{4}{21}$$

**27.** $\Delta W_{12} < \Delta W_{13}$ can be seen from area under the curve, while $\Delta V_1 = \Delta V_2$

$\Rightarrow \quad \Delta Q_{12} < \Delta Q_{13}$

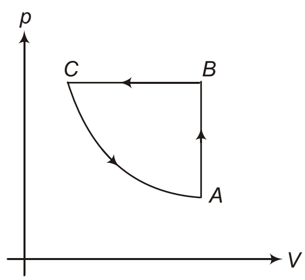
$\Rightarrow \quad Q_2 < Q_1$ or $Q_1 > Q_2$

**28.** $\Delta W_{CA} = p_0(V_0 - 2V_0) = -p_0V_0$

and $\quad \Delta U_{CA} = -\frac{3}{2}p_0V_0$

$\Rightarrow \quad \Delta Q_{CA} = -\frac{5}{2}p_0V_0$

**29.** $\Delta Q_{AB} = 200\,\text{kJ} = nC_V\Delta T;$




$\Delta U_{BC} = -100\,\text{kJ}$ and $\Delta W_{BC} = -50\,\text{kJ}$

$\Delta W_{AB} = 0 \Rightarrow \Delta U_{AB} = 200\,\text{kJ}, \Delta Q_{CA} = 0$

$\Delta U_{ABC} = \Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA} = 0$

or 200 kJ – 100 kJ + $\Delta U_{CA} = 0$

$\Delta U_{CA} = -100$ kJ

$\Delta Q_{AB} + \Delta Q_{BC} + \Delta Q_{CA}$

$= 200$ kJ + (– 100 kJ – 50 kJ) + 0

$= 50\,\text{kJ}$

$\Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$

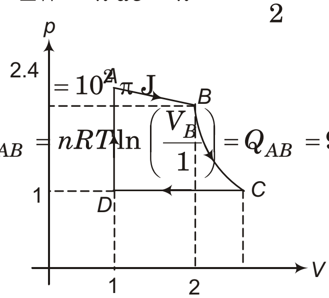
$= 0 + 200$ kJ + $\Delta W_{CA}$

$= \Delta Q_{ABC} = 50$ kJ

$\therefore \Delta W_{CA} = -150$ kJ

**30.** $\Delta Q = \Delta W = \pi\, ab = \pi \times \frac{20 \times 10^{-3}}{2} \times \frac{20 \times 10^{3}}{2}$

$= 10^4\pi$ J




**31.** $\Delta W_{AB} = nRT\ln\left(\frac{V_B}{1}\right) = Q_{AB} = 9 \times 10^4$ J

$\Rightarrow \quad 800\,T\ln V_B = 9 \times 10^4$ J

$\Rightarrow \quad T\ln V_D = \frac{225}{2}$

$\Delta W_{ABCD} = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CD} + \Delta W_{DA}$

$= nRT\ln\left(\frac{V_B}{V_A}\right) + \frac{p_Cp_C - p_Bp_B}{1-\gamma} + p_C(V_D - V_C) + 0$

$= 9 \times 10^4 + \frac{10^5 - nRT_B}{1 - \frac{5}{3}} + 10^5(2-1)$

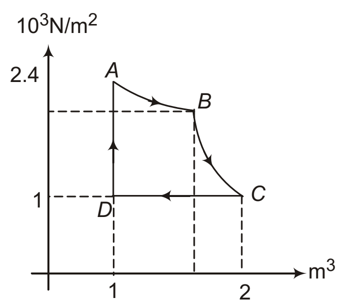
$= 19 \times 10^4 - \frac{3}{2}(10^5 - 800\,T_B)$

$= 4 \times 10^4 + 1200\,T_B$

$= 4 \times 10^4 + 1200 \times \frac{2.4 \times 10^5 \times 1}{100 \times 8}$

$= 4 \times 10^5$ J

**31.** $\Delta W = \Delta W_{AB} + \frac{p_CV_C - p_BV_B}{1-\gamma} + \Delta W_{CD}$




$= 9 \times 10^4 + \frac{2 \times 10^5 - 9 \times 10^4}{1 - 5/3} - 1 \times 10^5$

$= 9 \times 10^4 + \frac{3}{2} \times 11 \times 10^4 - 10 \times 10^4$

$= \left(\frac{33}{2} - 1\right) \times 10^4 = 15.5 \times 10^4$

**32.** $\Delta W = \int p\,dV = \int kV\,dV = \frac{1}{2}kV^2$

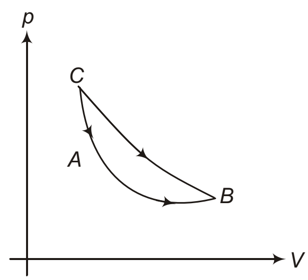
$$= \frac{1}{2} pV = \frac{1}{2} nRT_0 = \frac{1}{2} RT_0$$

$$\Delta U = nC_V \Delta T = 1 \cdot \frac{3}{2} RT_0$$

$$\Rightarrow \Delta Q = \left(\frac{3}{2} + \frac{1}{2}\right) RT_0 = 2RT_0$$

**33.** $pT = \text{constant} = p \dfrac{pV}{nR} = \dfrac{p^2 V}{nR}$

$$\Rightarrow \quad p^2 V = \text{constant}$$




$$\therefore \quad p_0^2 V_0 = \left(\frac{p_0}{2}\right)^2 V \Rightarrow V = 4V_0$$

$$\Rightarrow \quad T = \frac{\frac{p_0}{2} \cdot 4V_0}{nR} = 2 \frac{p_0 V_0}{nR} = 2T_0$$

$$\therefore \quad \Delta U = nC_V \Delta T = 2 \times \frac{3}{2} R\,(2T_0 - T_0)$$

$$= 3R \cdot \frac{p_0 V_0}{2R} = \frac{3}{2} p_0 V_0$$

**35.** $\Delta W_{BC} = nRT_0 \ln \left(\dfrac{V_C}{V_B}\right)$

$$= nRT_0 \ln \left(\frac{p_B}{p_C}\right) = 2 \cdot nRT_0 \ln \left(\frac{V_B}{V_A}\right)$$

$$= 2nRT_0 \ln \left(\frac{p_A}{p_B}\right)$$

$$\therefore \ln \left(\frac{p_B}{p_C}\right) = \ln \left(\frac{p_0}{p_0/2}\right)^2 = \ln 4$$

$$\Rightarrow \quad p_B = 4p_C$$

$$\Rightarrow \quad p_C = \frac{p_B}{4} = \frac{p_0}{8}$$

**36.** As, $\Delta W_a > \Delta W_b \Rightarrow \Delta W_1 > \Delta W_2$

while, $\Delta U_1 = \Delta U_2 \Rightarrow \Delta Q_1 > \Delta Q_2$

**37.** $\eta = 1 - \dfrac{T_{\text{sink}}}{T_{\text{source}}} = 1 - \dfrac{300}{600}$

$$= 1 - \frac{1}{2} = 0.5 = 50\%$$

**38.** As the volume is adiabatically decreased, temperature of the gas increases and as the time elapsed, temperature normalizes *i.e.*, decreases and so pressure also decreases.

**39.** As the compression is quick , the process is adiabatic while leads to heating of the gas.

**40.** $pV^\gamma = \text{constant}$

$$= \frac{nRT}{V} V^\gamma = nRTV^{\gamma - 1}$$

$$\Rightarrow TV^{\gamma - 1} = \text{constant}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma - 1} = \left(\frac{L_2}{L_1}\right)^{\frac{5}{3} - 1} = \left(\frac{L_2}{L_1}\right)^{\frac{2}{3}}$$

**41.** $pV^\gamma = \text{constant} = p\left(\dfrac{n\gamma T}{p}\right)^\gamma$

$$\Rightarrow \quad p^{1-\gamma} T^\gamma = \text{constant}$$

$$\Rightarrow \quad p^{\gamma - 1} \propto T^\gamma$$

$$\Rightarrow \quad p \propto T^{\frac{\gamma}{\gamma - 1}}$$

As $\dfrac{\gamma}{\gamma - 1} = \dfrac{7/5}{\frac{7}{5} - 1} = \dfrac{7}{2}$ for diatom gases.

$$\therefore \quad p \propto T^{3.5} \Rightarrow \alpha = 3.5$$

**42.** $pV^x = \text{constant}$ , $\Delta W = \dfrac{nR\Delta T}{1 - x}$,

$$\Delta U = n \cdot \frac{5}{2} R\Delta T$$

$$C = \frac{\Delta Q}{n\Delta T} = \frac{\frac{nR\Delta T}{1-x} + \frac{5}{2} nR\Delta T}{n\Delta T}$$

$$= \frac{5}{2} R + \frac{R}{1 - x} < 0$$

$$\frac{5}{2} R < \frac{R}{x - 1} \Rightarrow x - 1 < \frac{2}{5}$$

$x < \frac{7}{5} \Rightarrow x < 1.4$ but $x > 1$ as for $x < 1$,

$C$ will become positive.

$\therefore \quad 1 < x < 1.4$

**43.** $C_V = \dfrac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2} = \dfrac{13}{6} R$

(a) $\dfrac{2 \times \frac{5}{2} R + 4 \times \frac{5}{2} R}{2 + 4} = \dfrac{15}{6} R$

(b) $\dfrac{2 \times \frac{5}{2} R + 4 \times \frac{3}{2} R}{2 + 4} = \dfrac{11}{6} R$

(c) $\dfrac{2 \times \frac{3}{2} R + 4 \times \frac{5}{2} R}{2 + 4} = \dfrac{13}{6} R$ and

(d) $\dfrac{2 \times \frac{6}{2} R + 4 \times \frac{3}{2} R}{2 + 4} = \dfrac{12}{6} R$

**Passage 44 & 45**

**44.** $\Delta W_{ABCA} = \frac{1}{2} \times p \times V = \frac{pV}{2} = \Delta Q_{\text{net}}$

**45.** $CA \rightarrow$ isobaric and $BC \rightarrow$ isochoric,

$\therefore \quad \dfrac{C_p}{C_v} = \gamma = \dfrac{5}{3}$

**46.** $pV^{\gamma} = \text{constant} = p\left(\dfrac{nRT}{p}\right)^{\gamma}$

$\Rightarrow \quad p^{1-\gamma} T^{\gamma} = \text{constant}$

$\Rightarrow \quad T \propto p^{\frac{\gamma - 1}{\gamma}}$

$\therefore \quad T \propto p^{\frac{5/3 - 1}{5/3}} \Rightarrow T \propto p^{2/5}$

$\therefore \quad \dfrac{T_B}{T_A} = \left(\dfrac{p_B}{p_A}\right)^{2/5} = \left(\dfrac{2p_c}{3p_c}\right)^{2/5} = 0.85$

$\therefore \quad T_B = 0.85 T_A = 850 \text{ K}$

**47.** $\Delta W_{AB} = \dfrac{nRT}{1 - \gamma} = \dfrac{1 \times \frac{25}{3} \times 150}{\frac{5}{3} - 1}$

$= 75 \times 25 \text{ J} = 1875 \text{ J}$

**48.** $\Delta W_{BC} = 0, \Delta Q_{BC} = \Delta U_{BC}$

$= n \times \frac{3}{2} R (T_C - T_B)$

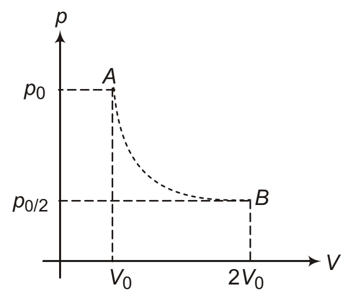
$= n \times \frac{3}{2} R \left(\frac{p_C V}{nR} - \frac{p_B V}{nR}\right)$

$= \frac{3}{2}\left(\frac{1}{3} p_A - \frac{2}{3} p_A\right) V$

$= -\frac{1}{2} p_A V = -\frac{1}{2} \times \frac{3}{2} p_B V = -\frac{3}{4} \cdot nRT_B$

$= -\frac{3}{4} \times 1 \times \frac{25}{3} \times 850 = -5312.5 \text{ J}$

**49.** $\Delta W_{AB} = (+)$ ve, $T_A = T_B$




$$p = -\frac{p_0}{2V_0} V + \frac{3}{2} p_0$$

$\Rightarrow \quad \dfrac{nRT}{V} = -\dfrac{p_0}{2V_0} V + \dfrac{3}{2} p_0$

or $\quad T = -\dfrac{p_0}{2nRV_0} V^2 + \dfrac{3p_0}{2nR} V_0$

$\Rightarrow \quad y = ax^2 + bx$ is parabola .

Again, $p = -\dfrac{p}{2V_0} \cdot \dfrac{nRT}{p} + \dfrac{3}{2} p_0$

$\Rightarrow$ is also equation of parabola.

While going from $A$ to $B$ temperature first increases ad than decreases.

**50.** $pV^2 = \text{constant}$

$\Delta W = \int p\, dV = \int \frac{k}{V^2} dV = k\left(-\frac{1}{V}\right)$

$= -pV\Big|_i^f = p_i V_i - p_f V_f$

$= nR(T_i - T_f) = -nR(T_f - T_i) = (-)$ ve

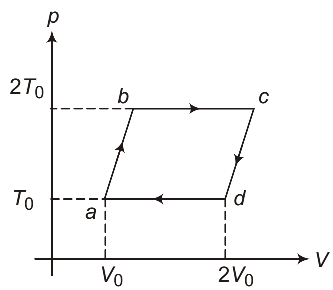
as $T_f > T_i$

as $T_i < T_f \Rightarrow U_i < U_f$

$\Rightarrow \Delta U = (+)$ ve

$\Delta Q = nC_V \Delta T - nR\Delta T = n(C_V - R)\,\Delta T$
$= (+)$ ve as $C_V > R$
*i.e.,* heat is given to the system.

**51.** In cyclic process, $\Delta U = 0$




$$\Delta W = 0 + nR2T_0 \ln\left(\frac{2V_0}{V_0}\right) + 0 + nRT_0 \ln\left(\frac{V_0}{2V_0}\right)$$

$= 2nRT_0 \ln 2 - nRT_0 \ln 2$
$= nRT_0 \ln 2 = (+)$ ve

*i.e.*, $\Delta W > 0$

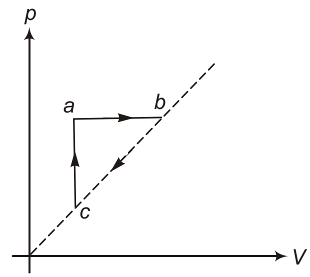
$\Delta Q_{\text{supplied}} = \Delta U_{ab} + \Delta W_{bc}$

$= nC_V(2T_0 - T_0) + nR2T_0 \ln\left(\frac{2V_0}{V_0}\right)$

$= 2 \times \frac{3}{2} RT_0 + 4RT_0 \ln 2$

$= 3RT_0 + 4RT_0 \ln 2$

**52.** $ab \rightarrow$ isochoric, $bc \rightarrow$ isobaric and $ca \rightarrow$ isothermal.




$\Delta W_{ab} = 0, \Delta U_{ca} = 0$

as in $ca$ density is increasing, so volume is decreasing *i.e.,*

$\Delta W_{ca} = (-)$ ve, *i.e.*, $\Delta W_{ca} < 0$

in isochoric process $\Delta Q_{ab}$ is positive for increase in temperature.

**53.** In isochoric process $\Delta W = 0$.

and in adiabatic process

$\Delta Q = 0 \Rightarrow Q_3$ to be minimum

$\Rightarrow \quad Q_2 > Q_1 > Q_3$

# JEE Corner

## ■ Assertion & Reasons

**1.** In adiabatic expression, $\Delta W = (+)$ ve while $\Delta Q = 0$ and as according to first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = -\Delta W$$

*i.e.*, $\Delta U = (-)$ ve this implies decrease in temperature. So, Assertion and reason are both true but not correct explanation.

**2.** Assertion is false, as work done is a path function and not a state function i.e., it depends on the path through which the gas was taken from initial to find state.

**3.** Assertion is false, as first law can be applied for both real and ideal gases.

**4.** During melting of ice its volume decreases, so work done by it is negative and that by atmosphere is positive. So, reason is true explanation of assertion.

**5.** As $\Delta Q = \Delta U + \Delta W \;\Rightarrow\; \Delta U = \Delta Q - \Delta W$, where $\Delta U$ is state function while $\Delta Q$ and $\Delta W$ are path function as for definite initial and final state $\Delta U$ is constant and so is $Q - W$. Thus assertion and reason are both true but not correct explanation.

6. Carnot's engine is ideal heat engine with maximum efficiency but it is not also 100%. So assertion and reason are both true but not correct explanation.

7. $pT = \text{constant} = p \cdot \frac{pV}{R} = \frac{p^2 V}{nR}$

$\Rightarrow p^2 V = \text{constant}$

$\therefore \Delta W = \int p\,dV = \int \sqrt{k}\frac{dV}{\sqrt{V}} = \sqrt{k} \cdot \frac{V^{1/2}}{1/2}$

$= 2\sqrt{k}\sqrt{V} = 2\sqrt{kV} = 2\sqrt{p^2/V}$

$= 2pV = 2nR(T_f - T_i) = 2nRT\,\Delta T$

$\therefore \Delta W = (+)$ ve for $\Delta T = (+)$ ve

and $\frac{nRT}{V} T = \text{constant}.$

$\Rightarrow \quad T^2 \propto V$

or, $\quad V \propto T^2$

Thus assertion is true but reason is false.

8. In adiabatic changes for free expansion, $Q = 0, W = 0$ and $\Delta U = 0$

as in free expansion no work is done against any force.

For ideal gases $pV = \text{constant}$ as $\Delta U = 0$ $\Rightarrow T = \text{constant}$ So, assertion and reason are both true but not correct explanation.

9. Assertion and reason are both true and correct explanation.

10. Assertion and reason are both true and correct explanation.

## ■ Match the Columns

1. (a) $\Delta W = p\int dV = pV = nR(T_f - T_i)$

$= nRT = 2RT \longrightarrow r$

(b) $\Delta U = nC_V T = 2 \times \frac{3}{2} R(2T - t)$

$= 3RT \longrightarrow p$

(c) $\Delta W = \frac{nR(2T - T)}{1 - 5/3} = -\frac{3}{2} \times 2RT$

$= -3RT \longrightarrow s$

(d) $\Delta U = nC_V \Delta T = 3RT \longrightarrow p$

2. (a) In $ab$ slope is more so, pressure is less as $V = \frac{nR}{p} \cdot T$, but is constant and in isobaric process. $\Delta W = p\Delta V = nR\Delta T$ and as $\Delta T$ is same in both process so, $\Delta W$ is same for both $\longrightarrow r$

(b) As $\Delta U = -nC_V \Delta T$ is same for both process $\longrightarrow r$

(c) As $\Delta Q = \Delta U + \Delta W$, it is also same for both process $\longrightarrow s$

(d) Nothing can be said about molar heat capacity $\rightarrow s$

3. (a) $\Delta W = \int pdV$

$= \int \sqrt{\frac{k}{V}} dV = \sqrt{k}\int \frac{dV}{\sqrt{V}}$

$= 2\sqrt{kV} = 2pV = 2nR\Delta T \longrightarrow p$

(b) $\Delta U = nC_V \Delta T = \frac{3}{2} nR\Delta T \longrightarrow s$

(c) $\Delta Q = 2nR\Delta T + \frac{3}{2} nR\Delta T$

$= \frac{7}{2} nR\Delta T \longrightarrow s$

(d) $\longrightarrow s$

4. (a) $\Delta W = p\Delta V = nR\Delta T$ and $\Delta U = nC_V \Delta T$

$\Rightarrow \Delta W < \Delta U \longrightarrow q$

(b) $\Delta W = 0 \Rightarrow \Delta Q = \Delta U, \Delta U = (-)$ ve $\rightarrow p, r$

(c) $\Delta W = (+)$ ve, $\Delta U = (-)$ ve, $\Delta Q = 0 \rightarrow p$

(d) $\Delta W = (+)$ ve, $\Delta U = 0$, $\Delta Q = (+)$ ve $\rightarrow p$

5. (a) $\Delta W_{AB} = p_0 V_0 + \frac{1}{2} p_0 V_0$

$= \frac{3}{2} p_0 V_0 \rightarrow s$

(b) $\Delta U_{AB} = \Delta Q - \Delta W$

$= + 6 p_0 V_0 - \frac{3}{2} p_0 V_0 = + \frac{9}{2} p_0 V_0 \rightarrow s$

(c) $\Delta Q = +6 p_0 V_0$

$= nC \left( \frac{4 p_0 V_0}{nR} - \frac{p_0 V_0}{nR} \right)$

$= \frac{3 p_0 V_0}{R} C$

$\Rightarrow \quad C = 2R \rightarrow p$

(d) $\Delta U$

$= nC_V \left( \frac{4 p_0 V_0}{nR} - \frac{p_0 V_0}{nR} \right)$

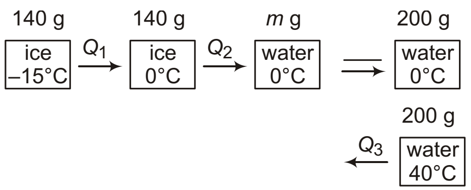
$= 3C_V \frac{p_0 V_0}{R} = \frac{9}{2} p_0 V_0$

$\Rightarrow C_V = \frac{3}{2} R \rightarrow s$

# 19. Calorimetry and Heat Tansfer

## Introductory Exercise 19.1.

**1.**




As Heat gain = Heat loss

$$Q_1 + Q_2 = Q_3$$

$$\Rightarrow 140 \times 0.53 \times 15 + m \times 80 = 200 \times 1 \times 40$$

$$\Rightarrow m = \frac{8000 - 1113}{80} = 86 \text{ g is the mass of}$$

ice melt

$\therefore$ Mass of water

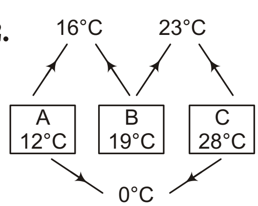
$$= 200 \text{ g} + 86 \text{ g} = 286 \text{ g}$$

and mass of ice

$$= 140 \text{ g} - 86 \text{ g} = 54 \text{ g}$$

while final temperature of mixture is 0°C.

**2.**




$$ms_A \,(16 - 12) = ms_B \,(19 - 16)$$

$$\Rightarrow \quad 4s_A = 3s_B$$

$$ms_B \,(23 - 19) = ms_C \,(28 - 23)$$

$$\Rightarrow \quad 4s_B = 5\, s_C$$

$$\therefore \quad ms_A \,(\theta - 12) = \frac{4}{5}\, ms\,(28 - \theta)$$

or $\quad \frac{3}{4}\, s_B \,(\theta - 12) = \frac{4}{5}\, s_B \,(28 - \theta)$

or $\quad 15\,(\theta - 12) = 16\,(28 - \theta)$

or $\quad 31\theta = 448 + 180$

$$\Rightarrow \quad \theta = 20.26\text{°C}$$

**3.** $mL = ms\Delta\theta$

$$\Rightarrow 80 \text{ cal} = 1 \text{ cal/°C}\,(\theta - 0\text{°C})$$

$$\Rightarrow \theta = 80\text{°C}$$

**4.** As Heat gain = Heat loss

$$\Rightarrow (100 - m) \times 529 = m \times 80$$

$$\therefore \quad 100 \times 529 = 609 \text{ m}$$

$$\Rightarrow m = \frac{100 \times 529}{609} \text{ g} = 86.86 \text{ g of ice will}$$

be formed.

**5.** $P = \frac{d\theta}{dt} = \frac{d}{dt}(ms\Delta\theta) = \frac{dm}{dt}\, s\Delta\theta$

$$\Rightarrow \quad \frac{dm}{dt} = \frac{P}{s\Delta\theta}$$

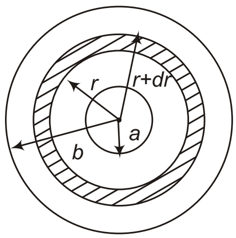
$$\therefore \quad \frac{dm}{dt} = \frac{500 \times 10^6 \text{ J/s}}{4200 \text{ J/kg °C} \times 10\text{°C}}$$

$$= \frac{5}{4.2} \times 10^4 \text{ kg/s} = 1.2 \times 10^4 \text{ kg/s}$$

# Introductory Excersise 19.2

**1.** Rest of the liquid will be heated due to conduction and not convection.

**2.** $\frac{dQ}{dt} = \frac{k \cdot 4\pi r^2 \, (-d\theta)}{dr}$




$\therefore \quad \frac{dQ}{dt} \cdot \frac{dr}{r^2} = -4\pi k \, d\theta$

or $\frac{dQ}{dt} \int_a^b \frac{dr}{r^2} = -4\pi k \int_{T_1}^{T_2} d\theta$

or $\frac{dQ}{dt} \left( \frac{1}{a} - \frac{1}{b} \right) = -4\pi k \, (T_2 - T_1)$

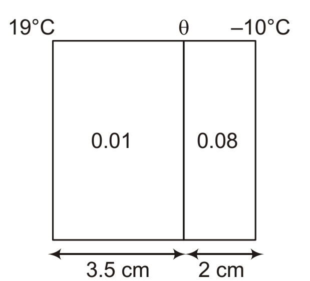
$\Rightarrow \frac{dQ}{dt} = \frac{4\pi k \, (T_1 - T_2)}{\frac{1}{a} - \frac{1}{b}} = 4\pi kab \, \frac{T_1 - T_2}{b - a}$

**3.** $\frac{dQ}{dt} = \frac{kA\Delta\theta}{t}$

$\Rightarrow \quad k = \frac{dQ}{dt} \cdot \frac{t}{A\Delta\theta}$

$\therefore$ Unit of $k = \text{watt} \frac{\text{m}}{\text{m}^2 \text{-} k} = \text{W/m-K}$

**4.** $\frac{K_1 A \, \Delta\theta_1}{l_1} = \frac{K_2 \, A\Delta\theta_2}{l_2}$




$\frac{0.01(19 - \theta)}{3.5} = \frac{0.08 \, (\theta + 10)}{2}$

or $2(19 - \theta) = 28 \, (\theta + 10)$

or $38 - 280 = 30\theta$

or $\theta = -\frac{242}{30} = -8.07°\text{C}$

$\frac{dQ}{dt} = \frac{0.01 \times 1 \times (19 + 8.1)}{3.5 \times 10^{-2}}$

$= 7.74 \text{ W/m}^2$

**5.** $\frac{dQ}{dt} = \frac{dm}{dt} \cdot L = \frac{0.44 \text{ kg}}{300 \text{ s}} \times 2.256 \times 10^6 \text{J/kg}$

$= 3308.8 \text{ J/s}$

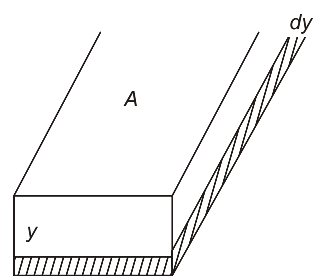
$= \frac{kA \, \theta}{t} = \frac{50.2 \times 0.15 \times (\theta - 100)}{1.2 \times 10^{-2}}$

$= 627.5 \, (\theta - 100)$

$\Rightarrow \theta - 100 = \frac{3308.8}{627.5} = 5.27$

$\Rightarrow \quad \theta = 105.27°\text{C}$

**6.** $\frac{dQ}{dt} = \frac{kA \, [0 - (-\theta)]}{y} = \frac{dm}{dt} \cdot L$




$= \rho \, \frac{dV}{dt} \cdot L = \rho A \, \frac{dy}{dt} \cdot L$

$\Rightarrow \quad \frac{kA\theta}{y} = \rho AL \, \frac{dy}{dt}$

$\Rightarrow \quad \frac{dy}{dt} = \frac{k\theta}{L\rho y}$ (Proved)

**7.** $\frac{dQ}{dt} = e\sigma AT^4$

$= 4 \times 5.67 \times 10^{-8} \times 4\pi \, (4 \times 10^{-2})^2 \times (3000)^4$

$= 0.4 \times 4 \, \pi \times 5.67 \times 4^2 \times 3^4 \text{ J/s}$

$= 3.7 \times 10^4 \text{ watt}$

**8.** $\frac{dQ}{dt} = \frac{\Delta\theta}{R_{th}} \Rightarrow R_{th} = \frac{\Delta\theta}{\frac{d\theta}{dt}} = \frac{K}{W} = \text{KW}^{-1}$

# AIEEE Corner

## Subjective Questions (Level-1)

**1.** $\underset{0°C}{\text{ice}} \xrightarrow{Q_1} \underset{0°C}{\text{Water}} \xrightarrow{Q_2} \underset{100°C}{\text{Water}} \xrightarrow{Q_3} \underset{100°C}{\text{steam}}$

$$Q = Q_1 + Q_2 + Q_3 = mL_f + ms\Delta\theta + mL_v$$
$$= 10\,[80 + 1 \times 100 + 540]$$
$$= 10 \times 720 \text{ cal} = 7200 \text{ cal}$$

**2.** 10 g of water at 40°C do not have sufficient heat energy to melt 15 g of ice at 0°C , so there will be a mixture of ice-water at 0°C. Let the mass of ice left is *mg*.

$$\therefore \quad (15 - m) \times 80 = 10 \times 1 \times 40$$
$$15 - m = 5 \Rightarrow m = 10 \text{ g}$$

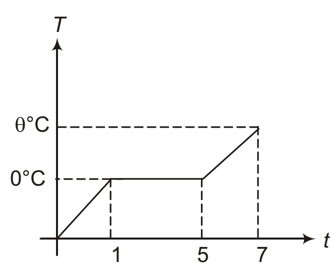
$\therefore$Mass of ice = 10 g

and mass of water = (10 + 5) g = 15 g

**3.** $4 \times s_P\ (60 - 55) = 1 \times s_R \times (55 - 50)$

$$\Rightarrow \quad 4s_P = s_R$$
$$1 \times s_P\ (60 - 55) = 1 \times s_Q\ (55 - 50)$$
$$\Rightarrow \quad s_P = s_Q$$
$$1 \times s_Q\ (60 - \theta) = 1 \times s_R\ (\theta - 50)$$
$$\text{or } s_P\ (60 - \theta) = 4s_P\ (\theta - 50)$$
$$260 = 5\theta \Rightarrow \theta = \frac{260}{5} = 52°C$$

**4.** $\dfrac{dQ}{dt} = \dfrac{m \times 336 \times 10^3 \text{ J/kg}}{4 \times 60 \text{ s}}$




$$= 1400 \text{ J/kg}$$
$$= 1400\ m\text{W/kg}$$
$$= \frac{m \cdot s\Delta\theta}{t} = \frac{m \times 4200\,(\theta - 0)\text{ c}}{2 \times 60 \text{ s}}$$
$$\therefore \quad \frac{1400 \times 2 \times 60}{4200} = \theta$$
$$\Rightarrow \quad \theta = 40°C$$

**5.** $Q = \dfrac{1}{2} \times \dfrac{1}{2} mv^2 = ms\,\Delta\theta + mL$

$$\Rightarrow v = \sqrt{4\,(s\Delta\theta + L)}$$
$$\therefore \quad v = \sqrt{4\,(125 \times 300 + 2.5 \times 10^4)}$$
$$= \sqrt{4 \times (3.75 + 2.5) \times 10^4}$$
$$= \sqrt{4 \times 6.25 \times 10^4} = 500 \text{ m/s}$$

**6.** $\eta\, mg\,\Delta h = ms\Delta\theta$

$$\therefore \quad \Delta\theta = \frac{\eta g \Delta h}{s} = \frac{0.4 \times 10 \times 0.5}{800} = \frac{1}{400}\ °C$$
$$= 2.5 \times 10^{-3}\ °C$$

**7.** $\dfrac{K_1 A\,(\theta - 0)}{l} = \dfrac{K_2 A(100 - \theta)}{l}$

$$\Rightarrow (K_1 + K_2)\,\theta = 100\,K_2$$
$$\therefore \quad \theta = \frac{100\,K_2}{K_1 + K_2} = \frac{100 \times 46}{390 \times 46} = 10.55°C$$

**8.** $i_{CD} = i_{AC} - i_{CB}$

$$\frac{KA(\theta - 25)}{l} = \frac{KA(100 - \theta)}{l/2} - \frac{KA(\theta - 0)}{l/2}$$
$$\text{or} \quad \theta - 25 = 2(100 - \theta) - 2\theta$$
$$\text{or} \quad 5\theta = 225 \Rightarrow \theta = 45°C$$
$$\therefore \quad i_{CD} = \frac{\Delta\theta}{R_{\text{th}}} = \frac{45 - 25}{5} = 4 \text{ W}$$

**9.** $i_A = i_C + i_D$

$$\frac{KA\,(T_1 - \theta)}{l} = \frac{KA(\theta - T_3)}{3l/2} + \frac{KA(\theta - T_2)}{3l/2}$$
$$\Rightarrow T_1 - \theta = \frac{2}{3}(\theta - T_3) + \frac{2}{3}(\theta - T_2)$$
$$\text{or } T_1 + \frac{2}{3}(T_2 + T_3) = \theta\left(1 + \frac{4}{3}\right)$$
$$\Rightarrow \quad \theta = \frac{T_1 + \frac{2}{3}(T_2 + T_3)}{7/3}$$
$$= \frac{3T_1 + 2(T_2 + T_3)}{7}$$

**10.** $\dfrac{KA(200 - \theta_1)}{l} = \dfrac{2KA(\theta_1 - \theta_2)}{l}$

$$= \frac{3KA(\theta_2 - 100)}{l}$$

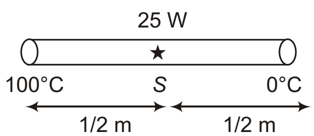
$\therefore \; 200 - \theta_1 = 2(\theta_1 - \theta_2) = 3(\theta_2 - 100)$

$$\Rightarrow \quad \begin{aligned} 3\theta_1 - 2\theta_2 &= 200 \\ \theta_1 + 3\theta_2 &= 500 \\ \hline -11\theta_2 &= -1300 \end{aligned}$$

$$\Rightarrow \quad \theta_2 = \frac{1300}{11} = 118.2°\text{C}$$

$$\theta_1 = \frac{1}{3}[200 + 2\theta_2] = 145.45°\text{C}$$

**11.** $$25 = \frac{400 \times 10^{-4}\,(\theta - 100)}{1/2} + \frac{400 \times 10^{-4}\,(\theta - 0)}{1/2}$$




$$25 = 8 \times 10^{-2}\,[\theta - 100 + \theta]$$

or $$312.5 = 2\theta - 100$$

$$\Rightarrow \quad \theta = \frac{412.5}{2} = 206.25$$

$\therefore \; \Delta\theta_1 = 106.25$ and $\Delta\theta_2 = 206.25$

$$\therefore \; \frac{\Delta\theta_1}{\Delta l} = \frac{106.25°\text{C}}{1/2\text{ m}} = 212.5°\text{C/m}$$

and $$\frac{\Delta\theta_2}{\Delta l} = \frac{206.25\,°\text{C}}{1/2\text{ m}} = 412.5\,°\text{C/m}$$

**12.** $$\frac{dQ}{dt} = e\sigma AT^4 = 0.6 \times 5.67 \times 10^{-8} \times 2 \times (0.1)^2 \times (1073)^4$$

$$= 0.6 \times 5.67 \times (10.73)^4 \times 10^{-2} \times 2$$

$$= 902\text{ W}$$

**13.** $$\left(\frac{dQ}{dt}\right)_1 = e\sigma AT^4 \text{ and } \left(\frac{dQ}{dt}\right)_2 = \sigma AT^4$$

$$\Rightarrow e = \frac{(dQ/dt)_1}{(dQ/dt)_2} = \frac{210}{700} = 0.3$$

**14.** $$\frac{(80 - 50)_c}{5} = \left(\frac{80 + 50}{2} - 20\right)c$$

$$\Rightarrow K = \frac{6}{45};$$

$$\frac{(60 - 30)}{t} = \frac{6}{45}\left(\frac{60 + 30}{2} - 20\right) \Rightarrow t = 9\text{ min}$$

## Objective Questions (Level-1)

**1.** $$\frac{3KA(35 - \theta)}{10} = \frac{KA(\theta - 0)}{20}$$

$$\Rightarrow \quad 6(35 - \theta) = \theta$$

$$\Rightarrow \quad \theta = \frac{6 \times 35}{7} = 30°\text{C}$$

$$\therefore \quad \Delta\theta_A = 35 - 30 = 5°\text{C}$$

**2.** $$\frac{T_S}{T_N} = \frac{\lambda_N}{\lambda_S} = \frac{350}{510} = 0.69$$

According to Wien's law

**3.** $$\left(\frac{dQ}{dt}\right)_2 \Big/ \left(\frac{dQ}{dt}\right)_1 = \frac{\frac{1}{4}\,\frac{K \cdot 4A\,\Delta\theta}{l/2}}{\frac{K\Delta\theta}{l}} = 2$$

$$\Rightarrow \left(\frac{dm}{dt}\right)_2 = 2\left(\frac{dm}{dt}\right)_1 = 0.2\text{ g/s}$$

**4.** $$\frac{dQ}{dt} = \frac{4\pi K(\theta - 0)}{\frac{2a - a}{a \cdot 2a}} = \frac{4\pi\,K(100 - \theta)}{\frac{3a - 2a}{3a \cdot 2a}}$$

$$\Rightarrow \quad 2\theta = 6\,(100 - \theta)$$

$$\Rightarrow \quad \theta = \frac{6}{8} \times 100 = 75°\text{C}$$

**5.** $$\frac{K_1 A(T_2 - T_1)}{d} = \frac{K_2 A(T_3 - T_2)}{3d}$$

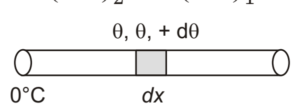
$$\Rightarrow \quad K_1(T_2 - T_1) = \frac{1}{3}\,K_2(T_3 - T_2)$$

$$\Rightarrow \quad K_1 = \frac{1}{3}\,K_2 \Rightarrow K_1 : K_2 = 1 : 3$$

**6.** $$\left(\frac{dQ}{dt}\right)_2 \Big/ \left(\frac{dQ}{dt}\right)_1 = \left[\frac{2K \cdot 2A \cdot \Delta\theta}{2l}\right] \Big/ \left[\frac{KA\,\Delta\theta}{l}\right] = 2$$

$$\Rightarrow \left(\frac{dQ}{dt}\right)_2 = 2\left(\frac{dQ}{dt}\right)_1 = 8\text{ cal/s}$$

**7.**



$$P = \frac{dQ}{dt} = \frac{K \cdot Ad\theta}{dx} = \frac{K_0(1+ax)A\,d\theta}{dx}$$

$$\therefore \int_0^l \frac{dx}{1+ax} = \frac{K_0 A}{P}\int_0^{100} d\theta$$

$$\frac{1}{a}\ln(1+ax)\Big|_0^l = \frac{10^2 \times 10^{-4}}{1}\cdot\theta\Big|_0^{100} = 1$$

$$\Rightarrow \quad \ln(1+al) - \ln 1 = 1$$

$$\Rightarrow \ln(1+al) - \ln 1 = 1$$

$$\Rightarrow \ln(1+al) = 1$$

or $1 + al = e^1$

or $l = \frac{1}{a}(e-1) = e - 1 = 1.7$ m

**8.** $\frac{\lambda_2}{\lambda_1} = \frac{T_1}{T_2} = \frac{2}{3} \Rightarrow \lambda_2 = \frac{2}{3}\lambda_m$

**9.** Heat required to boil 1 g of ice is 180 cal while 1 g of steam can release 540 cal during condenstion. So, temperture of the mixture will be 100°C with 2/3 g steam and 4.3 g water.

**10.** $T_1 < T_2 < T_3$ as temperature of a body decreases in rate of cooling also decreases such that time increases for equal temperature difference.

**11.** Conduction is maximum for which thermal resistance is minimum, as $R_{\text{th}} \propto \frac{l}{r^2}$ then for

(a) 50 (b) 25 (c) 100 (d) 33.33,

So option 'b' has minimum resistance.

**12.** Slope of temperature *versus* heat graph gives increase of specific heat or heat capacity and the portion *DE* is the gaseous state.

**13.** $dQ = m\,sdt = maT^3dT$

$$\Rightarrow \frac{Q}{m} = \frac{a}{4}T^4\Big|_1^2 = \frac{a}{4}(16-1) = \frac{15a}{4}$$

**14.** Resistance becomes 1/4th in parallel of that in series, so times taken will also become 1/4th *ie*, 12/4 = 3 min.

**15.** $ms_1 \times 12 = ms_2 \times 8 \Rightarrow s_1 : s_2 = 2:3$

**16.** $\frac{KA(T - T_c)}{\sqrt{2}\,l} = \frac{KA(T_c - \sqrt{2}\,T)}{l}$

$$\Rightarrow \quad \frac{T}{\sqrt{2}} + \sqrt{2}T = T_c + \frac{T_c}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{3}{\sqrt{2}}T = \frac{1+\sqrt{2}}{\sqrt{2}}T_c$$

$$\Rightarrow \quad T_c = \frac{3}{1+\sqrt{2}}T$$

**17.** $P = (1000 - 160)\text{ W} = 840\text{ W}$

$$= \frac{2 \times 4200 \times 50}{t}$$

$$\therefore \quad t = \frac{42 \times 10^4}{840} = 500\text{ s} = 8\text{ min }20\text{ s}$$

**18.** $\frac{dQ}{dt} = \frac{KA(T_2 - T)}{x} = \frac{2KA(T - T_1)}{4x}$

$$\Rightarrow \quad T_2 - T = \frac{1}{2}T - \frac{1}{2}T_1$$

$$\Rightarrow \quad T_2 + \frac{1}{2}T_1 = \frac{3}{2}T$$

$$\Rightarrow \quad T = \frac{2}{3}\left(T_2 + \frac{1}{2}T_1\right) = \frac{1}{3}(2T_2 + T_1)$$

$$\therefore \quad \frac{dQ}{dt} = \frac{KA}{x}\left[T_2 - \frac{1}{3}(2T_2 + T_1)\right]$$

$$= \frac{KA}{x}[3T_2 - 2T_2 - T_1] \times \frac{1}{3}$$

$$= \frac{KA}{x}(T_2 - T_1) \times \frac{1}{3}$$

$$\Rightarrow \quad f = \frac{1}{3}$$

**19.** $\Delta\theta \propto \frac{1}{K}$

$$\Rightarrow \quad \frac{\Delta\theta_A}{\Delta\theta_B} = \frac{K_B}{K_A} = \frac{1}{2}$$

$$\Rightarrow \quad \Delta\theta_A = \frac{1}{2}\Delta\theta_B = 18°\text{C}$$

## ■ More than One Correct Options

**20.** Amount of heat radiated or absorbed depends upon. Surface type, surface area, surface temperature and temperature of surrounding, so (a) and (b) are correct.

**21.** $\frac{KA(40-\theta)}{l} = \frac{KA(\theta-30)}{l} + \frac{KA(\theta-20)}{l}$

$\Rightarrow \quad 40-\theta = 2\theta - 50$

$\Rightarrow \quad 3\theta = 90°$

or $\quad \theta = 30°C$

So, (b) and (d) are correct.

**22.** $m \times s \times (2\theta - \theta_0) = m \times 2s \times (\theta_0 - \theta)$

$\Rightarrow \quad 4\theta = 3\theta_0 \Rightarrow \theta_0 = \frac{4}{3}\theta$

$c_1 : c_2 = m_1 : s_2 = s_1 : s_2 = 1:2$

So, (b) and (c) are correct.

**23.** In series rate of $R = R_1 + R_2$

$\Rightarrow \frac{1}{q} = \frac{1}{q_1} + \frac{1}{q_2} \Rightarrow q = \frac{q_1 q_2}{q_1 + q_2}$

In parallel $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$\Rightarrow q = q_1 + q_2$ as $q \propto \frac{1}{R}$

So, (b) and (c) are correct.

**24.** (a), (c) and (d) are correct.

# JEE Corner

## Assertion and Reason

1. Assertion is false.
2. According to Wien's law assertion and reason are correct.
3. Assertion and reason are true but not correct explanation.
4. Assertion is true but reason is false as resistance becomes 1/4th.
5. Assertion and reason are both false.
6. Assertion is false as this statement was not given by Newton.
7. Assertion and reason are both true with correct explanation.
8. Both are true but not correct explanation.
9. Assertion is false as temperature at different points become different.
10. As mass of follow sphere is less so cooling will be faster. So, both are true with correct explanation.

## Match the Columns

**1.**

| | | |
|---|---|---|
| (a) | $\sigma = \frac{(dQ/dt)}{AT^4} = \frac{ML^2T^{-2}T^{-1}}{L^2\theta^4}$ $= [MT^{-3}\theta^{-4}]$ | s |
| (b) | $b = \lambda T = L\theta$ | p |
| (c) | $\varepsilon = \frac{E}{At} = \frac{[ML^2T^{-2}]}{[L^2T]} = [MT^{-3}]$ | r |
| (d) | $R\text{th} = \frac{d\theta}{dQ/dt} = \frac{\theta}{[ML^2T^{-2}T^{-1}]}$ $= [M^{-1}L^{-2}T^3\theta]$ | s |

**2.**

| | | |
|---|---|---|
| (a) | Slope of line *ab* | s |
| (b) | Length of line $bc \propto m$ | r |
| (c) | Solid + liquid $\to bc$ | s |
| (d) | Only liquid $\to cd$ | q |

**3.** $\frac{KA(100-\theta_b)}{l} = \frac{KA(\theta_b - \theta_d)}{l} = \frac{KA(\theta_d + 80)}{l}$

$\therefore \; 100 - \theta_b = \theta_b - \theta_d$ and

$100 - \theta_b = \theta_d + 80$

$\therefore \left.\begin{array}{l} \theta_d - 2\theta_b = -100 \\ \theta_d + \theta_b = 20 \end{array}\right\} \Rightarrow -3\theta_b = -120$

$\Rightarrow \quad \theta_b = 40°C \Rightarrow \theta_d = -20°C$

$\theta_c = \theta_f = \frac{40-20}{2} = 10°C$

$\therefore$ (a) $\to q$, (b) $\to p$, (c) $\to p$, (d) $\to r$

**4.** (a) $ms(\theta_1 - \theta) = 2ms(2\theta - \theta_1)$

$\Rightarrow 3\theta_1 = 5\theta \Rightarrow \theta_1 = \frac{5}{3}\theta \to q$

(b) $ms(\theta_2 - \theta) = 3ms(3\theta - \theta_2)$

$\Rightarrow 4\theta_2 = 10\theta \Rightarrow \theta_2 = \frac{5}{2}\theta \rightarrow p$

(c) $2ms(\theta_3 - 2\theta) = 3ms(3\theta - \theta_3)$

$\Rightarrow 5\theta_3 = 13\theta \Rightarrow \theta_3 = \frac{13}{2}\theta \rightarrow s$

(d) $ms(\theta_4 - \theta) + 2ms(\theta_4 - 2\theta)$

$= 3ms(3\theta - \theta_4)$

$\Rightarrow 6\theta_4 = 14\theta \Rightarrow \theta_4 = \frac{7}{3}\theta \rightarrow r$

**5.**

| | | |
|---|---|---|
| (a) | $s = \frac{1}{m}\frac{dQ}{d\theta} = \frac{J}{kg°C} \longrightarrow$ | $q$ |
| (b) | $c = ms = m\frac{dQ}{md\theta} = J°/C \longrightarrow$ | $s$ |
| (c) | $i = \frac{dQ}{dt} = J/s \longrightarrow$ | $r$ |
| (d) | $L = \frac{E}{m} = J/kg \longrightarrow$ | s |