Solutions of

Electricity & Magnetism

Lesson 20th to 25th

By DC Pandey

20 Current Electricity

Introductory Exercise 20.1

1. $i = \frac{q}{t}$, here $q = e$, $t = \frac{2\pi r}{v}$

$\therefore \quad i = \frac{ev}{2\pi r}$

$$= \frac{1.6 \times 10^{-19} \times 2.2 \times 10^{6}}{2 \times 3.14 \times 5 \times 10^{-11}}$$

$= 1.12 \times 10^{-3}$ A

$= 1.12$ mA

2. No. of atoms in 63.45 g of Cu $= 6.023 \times 10^{23}$

$\therefore$ No. of atoms in 1 cm^3 (8.89 g) of Cu

$$= \frac{6.023 \times 10^{23}}{63.54} \times 8.89$$

$= 8.43 \times 10^{22}$

As one conduction electron is present per atoms,

$n = 8.43 \times 10^{22}\ cm^{-3}$ or $8.43 \times 10^{28}\ m^{-3}$

As $i = neAv_d$

$\Rightarrow \quad v_d = \frac{i}{neA}$

$$= \frac{2.0}{8.43 \times 10^{28} \times 1.6 \times 10^{-19} \times 3.14 \times (0.5 \times 10^{-3})^2}$$

$= 1.88 \times 10^{-6}\ ms^{-1}$

3. Yes.

As current always flows in the direction of electric field.

4. False.

In the absence of potential difference, electrons passes random motion.

5. Current due to both positive and negative ions is from left to right, hence, there is a net current from left to right.

6. $i = 10 + 4t \Rightarrow \frac{dq}{dt} = 10 + 4t$

$\Rightarrow \quad \int_0^q dq = \int_0^{10} (10 + 4t)\, dt$

$\Rightarrow \quad q = [10t + 2t^2]_0^{10} = 300$ C

Introductory Exercise 20.2

1. $R = \frac{\rho L}{A}$

$$= 1.72 \times 10^{-8} \times \frac{35}{3.14 \times \left(\frac{2.05}{2} \times 10^{-3}\right)^2}$$

$= 0.57\ \Omega$

2. (a) $\quad J = \frac{E}{\rho}$

$\Rightarrow \quad i = JA = \frac{EA}{\rho}$

$$= \frac{0.49 \times 3.14 \times (0.42 \times 10^{-3})^2}{2.75 \times 10^{-8}}$$

$= 9.87$ A

(b) $V = EL = 0.49 \times 12 = 5.88$ V

(c) $R = \frac{V}{i} = \frac{5.88}{9.87} = 0.6\ \Omega$

3. Let us consider the conductor to be made up of a number of elementary discs. The conductor is supposed to be extended to form a complete cone and the vertex O of the cone is taken as origin with the conductor placed along x-axis with its two ends at $x = r$ and $x = l + r$. Let θ be the semi-vertical angle of the cone.

Consider an elementary disc of thickness dx at a distance x from origin.

Resistance of this disc,

$$dR = \rho \frac{dx}{A}$$

If y be the radius of this disc, then

$$A = \pi y^2$$

But $y = x \tan\theta$

$$dR = \rho \frac{dx}{\pi x^2 \tan^2\theta}$$

$\therefore$ Resistance of conductor

$$R = \int dR = \int_r^{l+r} \frac{\rho dx}{\pi x^2 \tan^2\theta}$$

$$R = \frac{\rho}{\pi \tan^2\theta}\left[-\frac{1}{x}\right]_r^{l+r}$$

$$= \frac{\rho}{\pi \tan^2\theta}\left[\frac{1}{r} - \frac{1}{l+r}\right]$$

$$R = \frac{\rho l}{\pi r(l+r)\tan^2\theta}$$

But, $r\tan\theta = a$

$(r + l)\tan\theta = b$

$$R = \frac{\rho l}{\pi ab}$$

4. True. $\rho = \frac{1}{\sigma}$

$\therefore$ $\rho \times \sigma = \frac{1}{\sigma} \times \sigma = 1$

5. $R_{Cu} = R_{Fe}$

$$4.1(1 + \alpha_{Cu}\Delta T) = 3.9(1 + \alpha_{Fe}\Delta T)$$

$$4.1[1 + 4.0 \times 10^{-3}(T - 20)] = 3.9[1 + 5.0 \times 10^{-3}(T - 20)]$$

$$4.1 + 16.4 \times 10^{-3}(T - 20) = 3.9 + 19.5 \times 10^{-3}(T - 20)$$

$$3.1 \times 10^{-3}(T - 20) = 0.2$$

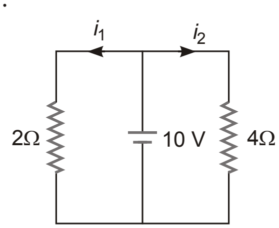
$$T - 20 = \frac{0.2}{3.1 \times 10^{-3}}$$

$$= 64.5°C$$

$$T = 84.5°C$$

Introductory Exercise 20.3

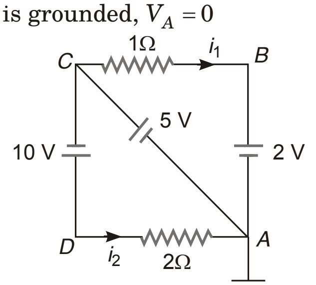
1. Potential difference across both the resistors is 10 V.



Hence, $i_1 = \frac{10}{2} = 5$ A

and $i_2 = \frac{10}{4} = 2.5$ A

2. As A is grounded, $V_A = 0$



$$V_B = V_A + 2 = 2 \text{ V}$$

$$V_C = V_A + 5 = 5 \text{ V}$$

$$V_D = V_C + 10 = 15 \text{ V}$$

$\therefore$ $i_1 = \frac{V_C - V_B}{1} = 3$ A

and $i_2 = \frac{V_D - V_A}{2} = \frac{15}{2} = 7.5$ A

3. Current in the given loop is

$$i = \frac{E + 15}{8}$$

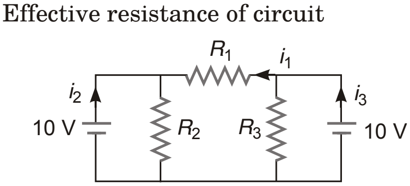
$$V_{AB} = E - 2i = E - 2 \cdot \left(\frac{E + 15}{8}\right) = 0$$

$\Rightarrow$ $E = 5$ V

4. Effective emf,

$$E = 8 \times 1 - 2 \times 1 = 6 \text{ V}$$

Effective resistance of circuit



$$R = R_{\text{external}} + 10r = 2 + 10 \times 1 = 12\,\Omega$$

$$\therefore \quad i = \frac{E}{R} = \frac{6}{12} = 0.5 \text{ A}$$

5. As $R_2 = R_3$ and $V_1 = V_2$

Potential difference across R_1 is zero.

Hence, current through $R_1 \Rightarrow i_1 = 0$

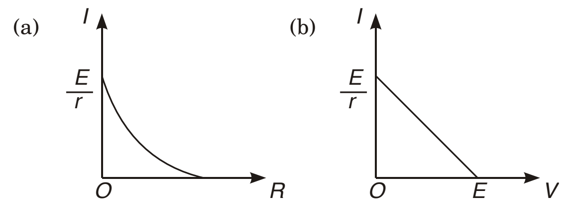
and current through R_2

$$\Rightarrow \quad i_2 = \frac{V_1}{R_2} = \frac{10}{10} = 1 \text{ A}$$

6. $i = \dfrac{E}{R + r}$

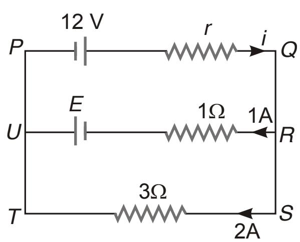
Also, $V = E - ir$

$$i = \frac{E - V}{r}$$



Introductory Exercise 20.4

1.



Applying KCL at junction R

$$i = 1 + 2 = 3 \text{ A}$$

$$V_{ST} = V_{RU} = V_{QP}$$

Taking $V_{ST} = V_{RU}$

$$6 = 1 - E$$

$$E = -5 \text{ V}$$

And from

$$V_{ST} = V_{QP}$$

$$6 = -ir + 12$$

$$r = \frac{12 - 6}{i} = \frac{6}{3} = 2\,\Omega$$

2. Power delivered by the 12 V power supply,

$$P_1 = Vi = 12 \times 3 = 36 \text{ W}$$

and power dissipated in 3 Ω resister,

$$P_3 = i_3^2 R_3 = 2^2 \times 3 = 12 \text{ W}$$

Introductory Exercise 20.5

1. $E = \dfrac{\dfrac{E_1}{r_1} + \dfrac{E_2}{r_2} + \dfrac{E_3}{r_3}}{\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3}} = \dfrac{\dfrac{10}{1} + \dfrac{4}{2} + \dfrac{6}{2}}{\dfrac{1}{1} + \dfrac{1}{2} + \dfrac{1}{2}}$

$$= \frac{10 + 2 + 3}{2}$$

$$= 7.5 \text{ V}$$

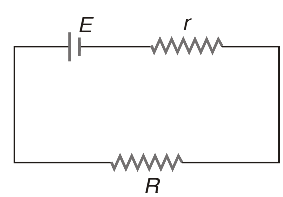
and $\quad \dfrac{1}{r} = \dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3}$

$$= \frac{1}{1} + \frac{1}{2} + \frac{1}{2} = 2$$

$$\Rightarrow \quad r = \frac{1}{2}$$

$$= 0.5\,\Omega$$

2. $i = \dfrac{E}{R + r}$



Rate of dissipation of energy

$$P = i^2 R = \frac{E^2 R}{(R + r)^2}$$

For maximum or minimum power

$$\frac{dP}{dR} = 0$$

$$\Rightarrow \quad E^2 \left[\frac{(R + r)^2 - 2R(R + r)}{(R + r)^4} \right] = 0$$

$$\Rightarrow \qquad E^2 \frac{(R+r)(r-R)}{(R+r)^4} = 0$$

$$\Rightarrow \qquad \frac{E^2(r-R)}{(R+r)^3} = 0$$

$$\Rightarrow \qquad R = r$$

$$\frac{d^2P}{dR^2} = E^2\left[\frac{(R+r)^3(-1) - 3(r-R)(R+r)^2}{(R+r)^6}\right]$$

$$= \frac{-E^2(4r-2R)}{(R+r)^4}$$

Clearly $\frac{d^2P}{dR^2}$ is negative at $R = r$.

Hence, P is maximum at $R = r$

and $P_{\max} = \frac{E^2 r}{(r+r)^2} = \frac{E^2}{4r}$

3. When the batteries are connected in series

$$E_{\text{eff}} = 2E = 4\text{V}, \; r_{\text{eff}} = 2r = 2\,\Omega$$

For maximum power

$$R = r_{\text{eff}} = 2\Omega$$

and $P_{\max} = \frac{E_{\text{eff}}^2}{4r_{\text{eff}}} = \frac{(4)^2}{4 \times 2} = 2$ W

4. $I_g = 5$ mA, $G = 1\,\Omega$, $V = 5$ V

$$R = \frac{V}{I_g} - G = \frac{5}{5 \times 10^{-3}} - 1$$

$$= 999\,\Omega$$

A 999 Ω resistance must be connected in series with the galvanometer.

5. $G = 100\,\Omega$, $i_g = 50\,\mu$A, $i = 5$ mA

$$\therefore \qquad S = \frac{i_g G}{i - i_g} = \frac{50 \times 10^{-6} \times 100}{5 \times 10^{-3} - 50 \times 10^{-6}}$$

$$= \frac{1}{1 - 0.01} = \frac{1}{0.99}$$

$$= \frac{100}{99}\,\Omega$$

By connecting a shunt resistance of $\frac{100}{99}\,\Omega$.

6. $i_g = \frac{V}{G}$

and $R = \frac{nV}{i_g} - G = (n-1)G$

7. $V_{AB} = \frac{15}{16}E$

Potential gradient

$$k = \frac{V_{AB}}{L} = \frac{15E}{16 \times 600}$$

$$= \frac{E}{640}\text{ V/cm}$$

(a) $\frac{E}{2} = kL \Rightarrow L = \frac{E}{2k} = 320$ cm

(b) $V = kl = \frac{E}{640} \times 560 = \frac{7E}{8}$

Also, $V = E - ir$

$$\therefore \qquad E - ir = \frac{7E}{8}$$

$$i = \frac{E}{8r}$$

AIEEE Corner

Subjective Questions (Level 1)

1. $i = \frac{q}{t} = \frac{ne}{t}$

Given,

$$i = 0.7,\; t = 1\text{ s},\; e = 1.6 \times 10^{-19}\text{ C}$$

$$\therefore \qquad n = \frac{it}{e} = \frac{0.7 \times 1}{1.6 \times 10^{-19}}$$

$$= 4.375 \times 10^8$$

2. $q = it = 3.6 \times 3 \times 3600$

$$= 38880\text{ C}$$

3. (a) $q = it = 7.5 \times 45 = 337.5$ C

(b) $q = ne \Rightarrow n = \frac{q}{e}$

$$= \frac{337.5}{1.6 \times 10^{-19}} = 2.11 \times 10^{21}$$

4. $T = \frac{2\pi r}{v} \Rightarrow f = \frac{1}{T} = \frac{v}{2\pi r}$

$$= \frac{2.2 \times 10^6}{2 \times 3.14 \times 5.3 \times 10^{-11}}$$

$$= 6.6 \times 10^{19}\text{ s}^{-1}$$

$$I = \frac{q}{T} = ef$$

$$= 1.6 \times 10^{-19} \times 6.6 \times 10^{19}$$

$$= 10.56\text{ A}$$

5. (a) $I = 55 - 0.65t^2$

$$I = \frac{dq}{dt}$$

$$\Rightarrow \quad dq = Idt$$

$$\Rightarrow \quad q = \int I\,dt$$

$$\therefore \quad q = \int_0^8 Idt = \int_0^8 (55 - 0.65t^2)\,dt$$

$$= 55[t]_0^8 - 0.65\left[\frac{t^2}{2}\right]_0^8$$

$$= 440 - 20.8 = 419.2\text{ C}$$

(b) If current is constant

$$I = \frac{q}{t} = \frac{419.2}{8} = 52.4\text{ A}$$

6. $i \propto v_d$

$$\therefore \quad \frac{v_{d_2}}{v_{d_1}} = \frac{i_2}{i_1}$$

$$\Rightarrow \quad v_{d_2} = \frac{i_2}{i_1} v_{d_1} = \frac{6.00}{1.20} \times 1.20 \times 10^{-4}$$

$$= 6.00 \times 10^{-4}\text{ ms}^{-1}$$

7. $$v_d = \frac{i}{neA}$$

$$= \frac{1}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 1 \times 10^{-4}}$$

$$= 0.735 \times 10^{-6}\text{ ms}^{-1}$$

$$= 0.735\,\mu\text{m/s}$$

$$t = \frac{l}{v_d} = \frac{10^3}{0.735 \times 10^{-6}}$$

$$= 1.36 \times 10^9\text{ s} = 43\text{ yr}$$

8. Distance covered by one electron in 1 s

$$= 1 \times 0.05 = 0.05\text{ cm}$$

Number of electrons in 1 cm of wire

$$= 2 \times 10^{21}$$

$\therefore$ Number of electrons crossing a given area per second

= Number of electrons in 0.05 cm of wire

$$= 0.05 \times 2 \times 10^{21} = 10^{20}$$

$$i = \frac{q}{t} = \frac{ne}{t}$$

$$= \frac{10^{20} \times 1.6 \times 10^{-19}}{1} = 1.6 \times 10 = 16\text{ A}$$

9. $R = \rho \dfrac{L}{A}$

Given,

$$\rho = 0.017\,\mu\Omega\text{-m}$$

$$= 1.7 \times 10^{-8}\,\Omega\text{-m}$$

$$l = 24.0\text{ m}$$

$$A = \pi\left(\frac{d}{2}\right)^2 = 3.14 \times \left(\frac{2.05}{2} \times 10^{-3}\right)^2$$

$$= 3.29 \times 10^{-6}\text{ m}^2$$

$$R = 1.7 \times 10^{-8} \times \frac{24.0}{3.29 \times 10^{-6}}$$

$$= 0.12\,\Omega$$

10. $$R = \rho\frac{L}{A}$$

$$A = \frac{\rho L}{R}$$

If D is density, then

$$m = DV = DAL = \frac{D\rho L^2}{R}$$

$$= \frac{8.9 \times 10^3 \times 1.72 \times 10^{-8} \times (3.5)^2}{0.125}$$

$$= 1.5 \times 10^{-2}\text{ kg} = 15\text{ g}$$

11. At 20°C,

$R_1 = 600\,\Omega$, $R_2 = 300\,\Omega$

At 50°C,

$$R_1' = R_1(1 + \alpha_1 \Delta t)$$

$$= 600(1 + 0.001 \times 30) = 600 \times 1.03$$

$$= 618\,\Omega$$

$$R_2' = R_2(1 + \alpha_2 \Delta t)$$

$$= 300(1 + 0.004 \times 30) = 336$$

$$\therefore \quad R' = R_1' + R_2' = 618 + 336$$

$$= 954\,\Omega$$

$$\alpha = \frac{R' - R}{R \times \Delta t} = \frac{954 - 900}{900 \times 30}$$

$$R = 600 + 300 = 900\,\Omega$$

$$= 0.002\,°\text{C}^{-1}$$

12. As both the wires are connected in parallel,

$$V_{\text{Al}} = V_{\text{Cu}}$$

$$i_{\text{Al}} R_{\text{Al}} = i_{\text{Cu}} R_{\text{Cu}}$$

$$i_{\text{Al}}\rho_{\text{Al}} \frac{L_{\text{Al}}}{\pi d_{\text{Al}}^2} = i_{\text{Cu}}\rho_{\text{Cu}} \frac{L_{\text{Cu}}}{\pi d_{\text{Cu}}^2}$$

$$\Rightarrow \quad d_{\text{Cu}} = d_{\text{Al}} \sqrt{\frac{i_{\text{Cu}}\,\rho_{\text{Cu}}\,L_{\text{Cu}}}{i_{\text{Al}}\rho_{\text{Al}}L_{\text{Al}}}}$$

$$= 1 \times 10^{-3} \sqrt{\frac{2 \times 0.017 \times 6}{3 \times 0.028 \times 7.5}}$$

$$= 0.569 \times 10^{-3}\text{ m}$$

$$= 0.569\text{ mm.}$$

13. (a) $E = \frac{V}{L} = \frac{0.938}{75 \times 10^{-2}} = 1.25 \text{ V/m}$

(b) $J = \frac{E}{\rho} \Rightarrow \rho = \frac{1.25}{4.4 \times 10^{7}}$

$$\rho = 2.84 \times 10^{-8} \ \Omega\text{-m}$$

14. (a) $J = \frac{E}{\rho} = \frac{V}{\rho L}$

Current density is maximum when L is minimum, *ie*, $L = d$, potential difference should be applied to faces with dimensions $2d \times 3d$.

$$J_{\text{min.}} = \frac{V}{\rho d}.$$

(b) $i = \frac{V}{R} = \frac{VA}{\rho L}$

Current is maximum when L is minimum and A is maximum.

Hence, in this case also, V should be applied to faces with dimensions $2d \times 3d$

and $i_{\max} = \frac{V(2d \times 3d)}{\rho(d)} = \frac{6Vd}{\rho}.$

15. (a) $R = \rho \frac{L}{A}$

$\rho = \frac{RA}{L}$

$[r = \frac{d}{2} = 1.25 \text{ mm} = 1.25 \times 10^{-3} \text{ m}]$

$= \frac{0.104 \times 3.14 \times (1.25 \times 10^{-3})^2}{14.0}$

$= 3.64 \times 10^{-7} \ \Omega\text{-m}$

(b) $i = \frac{V}{R} = \frac{EL}{R} = \frac{1.28 \times 14}{0.104} = 172.3 \text{ A}$

(c) $i = neAv_d$

$$v_d = \frac{i}{neA}$$

$= \frac{172.3}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 3.14 \times (1.25 \times 10^{-3})^2}$

$= 2.58 \times 10^{-3} \text{ ms}^{-1}$

16. For zero thermal coefficient of resistance, $\Delta R = 0$

$$R_C \alpha_C \Delta T + R_{Fe} \alpha_{Fe} \Delta T = 0$$

$$\frac{R_1}{R_2} = \frac{-\alpha_{Fe}}{\alpha_C} = \frac{-5.0 \times 10^{-3}}{-0.5 \times 10^{-3}} = 10$$

$$R_1 = 10 R_2$$

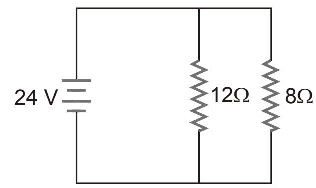
Also, $R_1 + R_2 = 20$

$\Rightarrow \quad 10R_2 + R_2 = 20$

$\Rightarrow \quad R_2 = \frac{20}{11} \Omega = 1.82 \ \Omega$

and $\quad R_1 = 20 - R_2 = 20 - 1.82 \ \Omega = 18.2 \ \Omega$

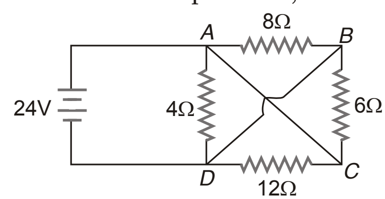
17. The circuit can be redrawn as



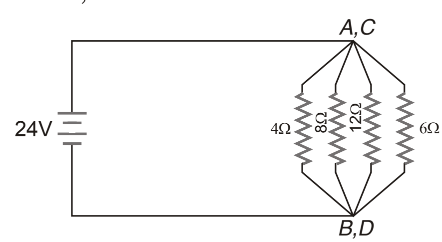
$$R_{\text{eff}} = \frac{8 \times 12}{12 + 8} = 4.8 \ \Omega$$

$$I = \frac{V}{R_{\text{eff}}} = \frac{24}{4.8} = 5 \text{ A}$$

18. Here, A and C are at same potential and B and D are at same potential,



Hence, the circuit can be redrawn as



$\therefore \quad \frac{1}{R} = \frac{1}{4} + \frac{1}{8} + \frac{1}{12} + \frac{1}{6}$

$= \frac{6 + 3 + 2 + 4}{24}$

$= \frac{15}{24} = \frac{5}{8}$

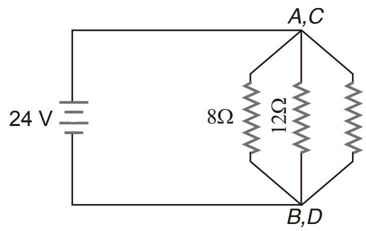
$R = \frac{8}{5} \Omega$

$= 1.6 \ \Omega$

$i = \frac{V}{R} = \frac{24}{1.6}$

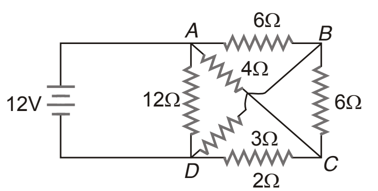
$= 15 \text{ A}$

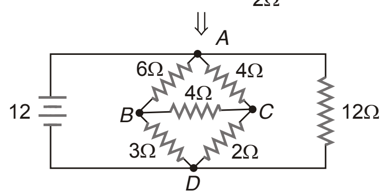
19. Given circuit is similar to that in previous question but $4 \ \Omega$ resistor is removed. So the effective circuit is given by



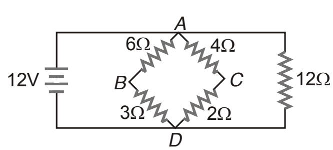
$$\frac{1}{R} = \frac{1}{8} + \frac{1}{12} + \frac{1}{6}$$
$$\frac{1}{R} = \frac{3+2+4}{24} = \frac{9}{24} = \frac{3}{8}$$
$$R = \frac{8}{3}\,\Omega = 2.67\,\Omega$$
$$i = \frac{V}{R} = \frac{24}{2.67} = 9\text{ A}$$

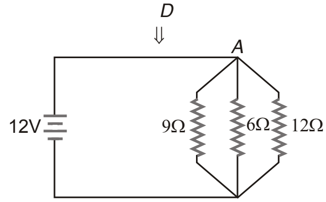
20.

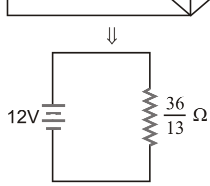




Wheatstone bridge is balanced, hence $4\,\Omega$ resistance connected between B and C be removed and the effective circuit becomes

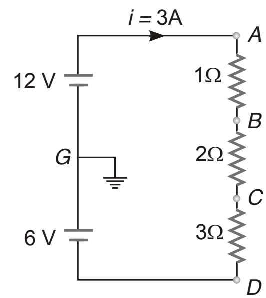






$$i = \frac{V}{R} = \frac{12}{36/13}$$
$$= \frac{13}{3}\text{ A}$$

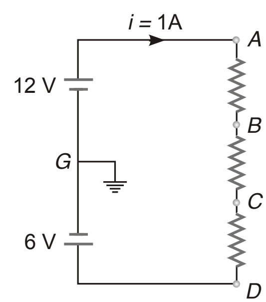
21. (a) $i = \dfrac{12+6}{1+2+3} = 3\text{ A}$



$$V_G = 0$$
$$V_A = V_G + 12 = 12\text{ V}$$
$$V_A - V_B = 3\text{ V}$$
$$V_B = 12 - 3 = 9\text{ V}$$
$$V_B - V_C = 6\text{ V}$$
$$V_C = 9 - 6 = 3\text{ V}$$
$$V_G - V_D = 6\text{ V},\ V_D = -6\text{ V}$$

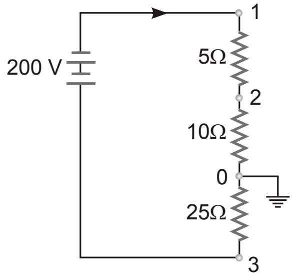
(b) If 6 V battery is reversed

$$i = \frac{12-6}{1+2+3} = 1\text{ A}$$



$$V_G = 0,$$
$$V_A - v_G = 12\text{ V},\ \ V_A = 12\text{ V}$$
$$V_A - V_B = 1\text{ V}$$
$$\Rightarrow \quad V_B = 11\text{ V}$$
$$V_B - V_C = 2\text{ V}$$
$$\Rightarrow \quad V_C = 9\text{ V}$$
$$V_D - V_G = 6\text{ V}$$
$$\Rightarrow \quad V_D = 6\text{ V}$$

22. $i = \dfrac{200}{5 + 10 + 25} = 5\text{ A}$



(i) $V_3 - V_0 = 5 \times 25$

$\Rightarrow \quad V_3 = 125\text{ V}$

(ii) $V_0 - V_2 = 5 \times 10$

$V_2 = -50\text{ V}$

(iii) $V_2 - V_1 = 5 \times 5$

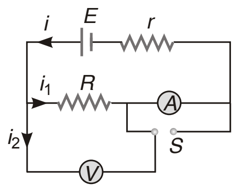
$V_1 = -75\text{ V}$

(iv) $V_{3-2} = 5 \times 35 = 175\text{ V}$

(v) $V_{1-2} = -5 \times 5 = -25\text{ V}$

(vi) $V_{1-3} = -200\text{ V}$

23. (a)



$$R_{\text{eff}} = R \,||\, R_v + R_a + r$$

$$= \frac{50 \times 200}{50 + 200} + 2 + 1$$

$$= 43\ \Omega$$

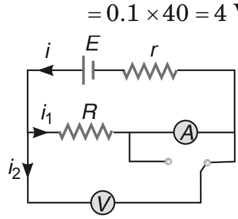
$$i = \frac{E}{R_{\text{eff}}} = \frac{4.3}{43} = 0.1\text{ A}$$

$\therefore$ Reading of ammeter, $i = 0.1$ A

and reading of voltmeter $= i(R \,||\, R_v)$

$= 0.1 \times 40 = 4$ V

(b)



$$R_{\text{eff}} = (R_a + R) \,||\, R_v + r$$

$$= \frac{52 \times 200}{52 + 200} + 1$$

$$= 42.26\ \Omega$$

$$i = \frac{E}{R_{\text{eff}}} = 0.102\ \Omega$$

Reading of voltmeter

$$V = E - ir$$

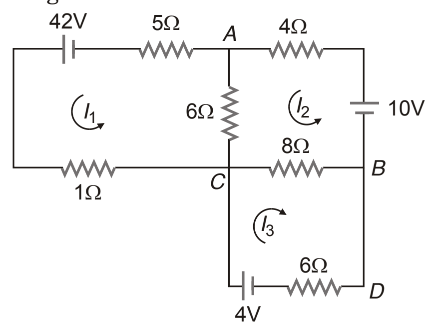
$$= 4.3 - 0.102 \times 1$$

$$\approx 4.2\ \Omega$$

Reading of Ammeter,

$$i_1 = \frac{V}{R + R_a} = \frac{4.2}{42} = 0.08\text{ A}$$

24. Consider the directions of current as shown in figure.



Applying KVL in loop 1, 2 and 3, we respectively get,

$$I_1 + 6(I_1 - I_2) + 5I_1 = 42$$

$$\Rightarrow \quad 12I_1 - 6I_2 = 42$$

$$\Rightarrow \quad 2I_1 - I_2 = 7 \qquad \ldots\text{(i)}$$

$$4I_2 + 6(I_2 - I_1) + 8(I_2 + I_3) = 10$$

$$\Rightarrow \quad 9I_2 - 3I_1 + 4I_3 = 5 \qquad \ldots\text{(ii)}$$

$$8(I_2 + I_3) + 16I_3 = 4$$

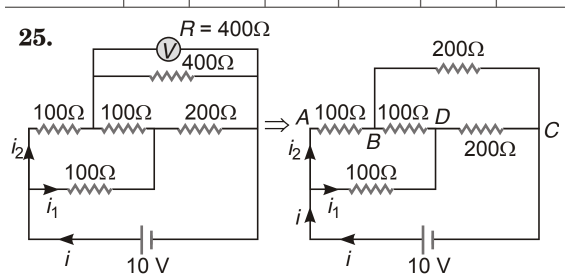
$$2I_2 + 6I_3 = 1 \qquad \ldots\text{(iii)}$$

On solving, we get,

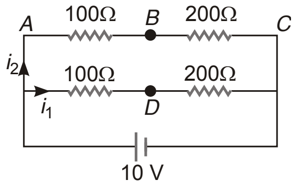
$I_1 = 4.7$ A, $I_2 = 2.4$ A, $I_3 = 0.5$ A

Resistor	5Ω	1Ω	4Ω	6Ω	8Ω	16Ω
Current	4.7 A	4.7 A	2.4 A	2.3 A	2.9 A	0.5 A

25.



As Wheatstone bridge is balanced, 100Ω resistance between B and D can be removed, *ie*,



$$\therefore \qquad i_1 = i_2 = \frac{10}{300} = \frac{1}{30}\text{ A}$$

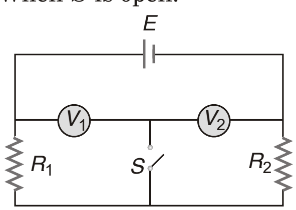
Hence, reading of voltmeter

= Potential difference between B and C

$$= 200 \times i_2 = \frac{20}{3}\text{ V}$$

$$= 6.67\text{ V}$$

26. (a) (i) When S is open.



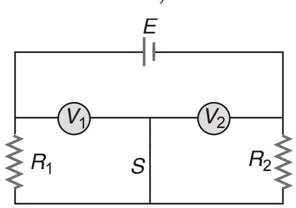
$$V_1 = \frac{R_1}{R_{V_1} + R_{V_2}} E = \frac{3000}{5000} \times 200$$

$$= 120\text{ V}$$

$$V_2 = \frac{R_{V_2}}{R_{V_1} + R_{V_2}} E = \frac{2000}{5000} \times 200$$

$$= 80\text{ V}$$

(ii) When S is closed,

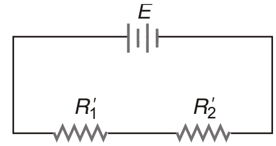


Now, R_1 and V_1 are in parallel and their effective resistance

$$R_1' = \frac{R_1 R_{V_1}}{R_1 + R_{V_1}} = \frac{6000}{5} = 1200\,\Omega$$

Similarly,

R_2 and V_2 are in parallel with their effective resistance,



$$R_2' = \frac{R_2 R_{V_2}}{R_2 + R_{V_2}} = \frac{6000}{5} = 1200\,\Omega$$

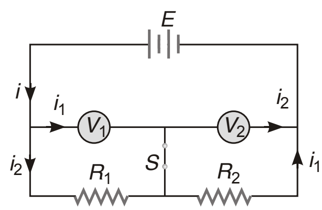
As $\qquad R_1' = R_2'$

Hence,

reading of V_1 = reading of V_2

$$= \frac{1200}{1200 + 1200} \times 200 = 100\text{ V}$$

(b) Current distribution is shown in figure



$$i = \frac{E}{R_1' + R_2'}$$

$$i = \frac{200}{2400} = \frac{1}{12}\text{ A}$$

$$i_1 = \frac{R_{V_1}}{R_1 + R_{V_1}} i = \frac{3000}{5000} \times \frac{1}{12}$$

$$= \frac{1}{20}\text{ A}$$

$$i_2 = \frac{R_1}{R_1 + R_{V_1}} i = \frac{2000}{5000} \times \frac{1}{12}$$

$$= \frac{1}{30}\text{ A}$$

$\therefore$ Current flowing through

$$S = i_1 - i_2 = \frac{1}{20} - \frac{1}{30}$$

$$= \frac{1}{60}\text{ A}$$

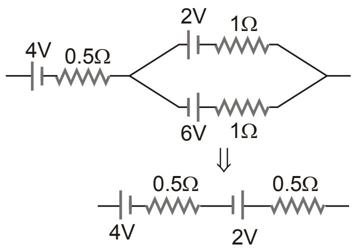
27. Effective emf of 2 V and 6 V batteries connected in parallel

$$E' = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} = \frac{2 \times 1 - 6 \times 1}{1 + 1}$$

$$= -2\text{ V}$$

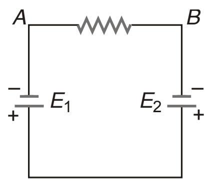
and $\qquad r' = \dfrac{r_1 r_2}{r_1 + r_2} = \dfrac{1}{2}\,\Omega$

$$= 0.5\,\Omega$$



Net emf, $E = 4 - 2 = 2$ V

28. (a)



As $E_1 > E_2$

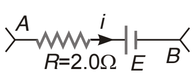
Current will flow from B to A.

(b) E_1 is doing positive work

(c) As current flows from B to A through resistor, B is at higher potential.

29. $i^2R = 2\text{ W} < 5\text{ W}$

Clearly X is doing negative work.

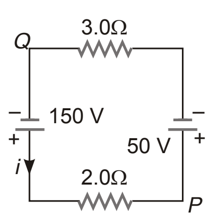


(a) $P = Vi \Rightarrow V = \dfrac{P}{i} = \dfrac{0.5}{1.0} = 5.0$ V

(b) $E = V - iR = 5 - 2 = 3$ V

(c) It is clear from figure that positive terminal of X is towards left.

30. $i = \dfrac{150 - 50}{3 + 2} = 20$ A



$$V_P - V_Q = 50 + 3.0\,i$$
$$V_Q = 100 - (50 + 60)$$
$$= -10\text{ V}$$

31. (a) As voltmeter is ideal, it has infinite resistance, therefore current is zero.

(b) $V = E - ir \Rightarrow E = 5.0$ V

(c) Reading of voltmeter $\Rightarrow V = 5.0$ V

32. $V_1 = E - i_1 r \Rightarrow E - 1.5\,r = 8.4$ ...(i)

$V_2 = E + i_2 r \Rightarrow E + 3.5\,r = 9.4$ ...(ii)

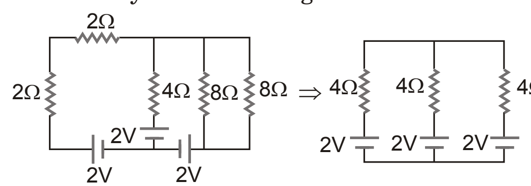
On solving, we get

$$r = 0.2\,\Omega$$
$$E = 8.7\text{ V}$$

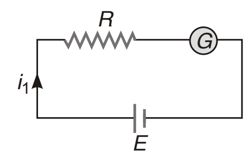
33. In case of charging

$V = E + i\,r = 2 + 5 \times 0.1 = 2.5$ V

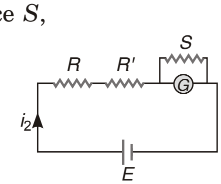
34. Clearly current through each branch is zero.



35. $i_1 = \dfrac{E}{R + G}$



On shunting the galvanometer with resistance S,



$$i_2 = \frac{E}{R + R' + \dfrac{GS}{S + G}}$$

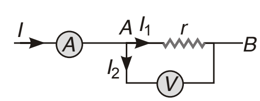
As $i_1 = i_2$

$$\frac{E}{R + G} = \frac{E}{R + R' + \dfrac{GS}{G + S}}$$

$$\Rightarrow \quad R + R' + \frac{GS}{G + S} = R + G$$

$$R' = \frac{G^2}{G + S}$$

36.



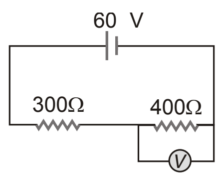
$$I_2 = \frac{r}{R + r} I = \frac{V}{R}$$

$$\Rightarrow \qquad \frac{r}{R+r} = \frac{V}{IR}$$

$$\Rightarrow \qquad \frac{R}{r} = \frac{IR - V}{V} = \frac{5 \times 2500 - 100}{100}$$

$$r = \frac{100}{12400} \times 2500 = 20.16\,\Omega$$

37.

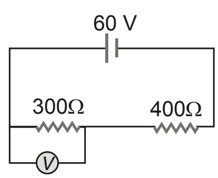


Let R be the resistance of voltmeter

As reading of voltmeter is 30 V,

$$\frac{1}{R} + \frac{1}{400} = \frac{1}{300} \Rightarrow R = 1200\,\Omega$$

If voltmeter is connected across $300\,\Omega$ resistor,



Effective resistance of $300\,\Omega$ resistor and voltmeter

$$R' = \frac{300 \times 1200}{300 + 1200} = 240\,\Omega$$

$$i = \frac{60}{400 + 240}$$

$$= \frac{60}{640}\,\text{A}$$

$$= \frac{3}{32}\,\text{A}$$

$\therefore$ Reading of voltmeter,

$$V = iR' = \frac{3}{32} \times 240$$

$$= 22.5\,\text{V}$$

38. $V_2 = \dfrac{R'}{R_1 + R_2{}'} V,$

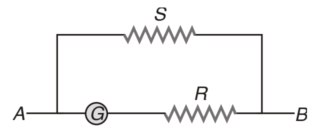
$$R_2{}' = \frac{rR_2}{r + R_2} = \frac{120}{3}$$

$$= 40\,\Omega$$

$$V_2 = \frac{40}{60 + 40} 120$$

$$= 48\,\text{V}$$

39. $S = \dfrac{i_g}{i - i_g}(G + R)$



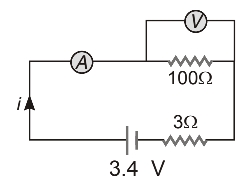
$$R = \frac{i - i_g}{i_g} S - G$$

$$= \frac{20 - 10^{-3}}{10^{-3}} \times 0.005 - 20$$

$$= 79.995\,\Omega$$

40. $r = \dfrac{L_1 - L_2}{L_2} R = \dfrac{0.52 - 0.4}{0.4} \times 5 = 1.5\,\Omega$

41. Let R be the resistance of voltmeter



$$R_e = 3 + 2 + \frac{100R}{100 + R}$$

$$= 5 + \frac{100R}{100 + R}$$

$$i = \frac{3.4}{5 + \dfrac{100R}{100 + R}} = 0.04$$

$$\Rightarrow \qquad 0.2 + \frac{4R}{100 + R} = 3.4$$

$$\Rightarrow \qquad R = 400\,\Omega$$

Reading of voltmeter,

$$V = i \times \frac{100R}{100 + R} = 0.04 \times \frac{100 \times 400}{100 + 400}$$

$$= 3.2\,\text{V}$$

If the voltmeter had been ideal,

Reading of voltmeter

$$= \frac{100}{105} \times 3.4 = 3.24\,\text{V}$$

42. $\dfrac{L_1}{L_2} = \dfrac{R_1}{R_2}$

$$\Rightarrow \qquad \frac{L_1}{40 - L_1} = \frac{8}{12} \qquad (L_1 + L_2 = 40\,\text{cm})$$

$$\Rightarrow \qquad L_1 = 16\,\text{cm} \quad \text{from } A.$$

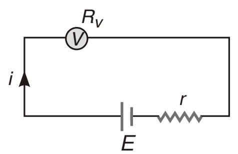
43. $S = \dfrac{i_g}{i - i_g}(G + R)$

$\Rightarrow \quad R = \dfrac{i - i_g}{i_g} S - G$

$= \dfrac{20 - 0.0224}{0.0244} \times 0.0250 - 9.36$

$= 12.94\,\Omega$

44. (a) $i = \dfrac{E}{R_V + r}$



$V = iR_V = \dfrac{E}{R_V + r} R_V$

(b) $\dfrac{r}{R_V + r} = 1\% = \dfrac{1}{100}$

$R_V = 99r = 99 \times 0.45$

$= 44.55\,\Omega$

(c) $\dfrac{V}{E} = \dfrac{R_V}{R_V + r}$

As R_V decreases, V decreases, decreasing accuracy of voltmeter.

45. (a) When ammeter is connected

$I_A = \dfrac{E}{R_A + R + r}$

When ammeter is removed

$I = \dfrac{E}{R + r} = \dfrac{R_A + R + r}{R + r} I_A$

(b) $\dfrac{I_A}{I} = 99\%$

$\dfrac{R + r}{R_A + R + r} = \dfrac{99}{100}$

$\Rightarrow \quad R_A = \dfrac{1}{99}(R + r) = \dfrac{1}{99}(3.8 + 0.45)$

$R_A = 0.043\,\Omega$

(c) As $\dfrac{I_A}{I} = \dfrac{R + r}{R_A + R + r}$, as R_A increases, I_A decreases, decreasing the accuracy of ammeter.

46. $I_{max} = \sqrt{\dfrac{\rho_{max}}{R}} = \sqrt{\dfrac{36}{2.4}} = \sqrt{15}$ A

For the given circuit

$R_e = \dfrac{1}{2} R + R = \dfrac{3}{2} R$

Maximum power dissipated by the circuit

$P'_{max} = I_{max}^2 R_e$

$= 15 \times \dfrac{3}{2} \times 2.4 = 54$ W

47. Total power of the circuit, $P = P_1 + P_2 + P_3$

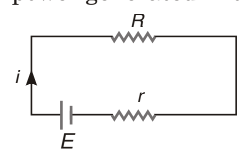
$= 40 + 60 + 75$

$= 175$ W

As $P = \dfrac{V^2}{R} \Rightarrow R = \dfrac{V^2}{P}$

$= \dfrac{(120)^2}{175} = 82.3\,\Omega$

48. Thermal power generated in the battery



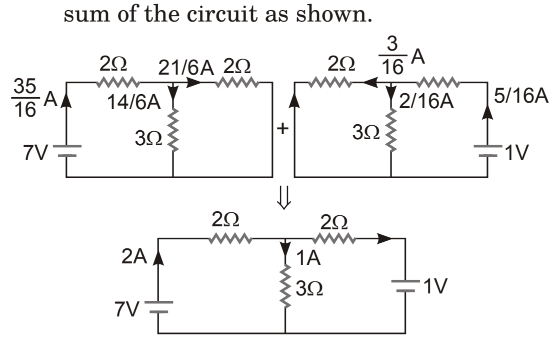
$P_1 = i^2 r = i(E - V)$

$= 0.6$ W

Power development in the battery by electric forces

$P_2 = IE = 2.6$ W

49. The given circuit can be considered as the sum of the circuit as shown.



$\therefore \quad P_1 = 7 \times 2 = 14$ W,

$P_2 = -1 \times 1 = 1$ W

50. (a) $i = \dfrac{E_1 - E_2}{R_1 + R_2} = \dfrac{12 - 6}{4 + 8} = 0.5$ A

(b) Power dissipated in $R_1 = I^2 R_1 = 1$ W

and power dissipated in $R = I^2 R_2 = 2$ W

(c) Power of battery $E_1 = E_1 I$

$= 12 \times 0.5 = 6$ W (supplied)

Power of battery $E_2 = E_2 I$

$= -6 \times 0.6 = -3$ W (absorbed)

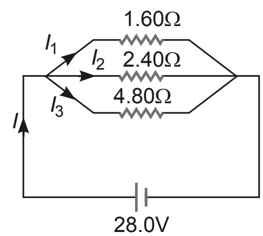
51. $I = \dfrac{E}{R+r} = \dfrac{12}{5+1} = 2\text{ A}$

(a) $P = EI = 12 \times 2 = 24\text{ W}$

(b) $P_1 = I^2R = 2^2 \times 5 = 20\text{ W}$

(c) $P_2 = I^2r = 2^2 \times 1 = 4\text{ W}$

52. (a)



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R} = \frac{1}{1.60} + \frac{1}{2.40} + \frac{1}{4.80}$$

$$R = 0.80\ \Omega$$

(b) $I_1 = \dfrac{V}{R_1} = \dfrac{28.0}{1.60} = 17.5\text{ A}$

$$I_2 = \frac{V}{R_2} = \frac{28.0}{2.40} = 11.67\text{ A}$$

$$I_3 = \frac{V}{R_3} = \frac{28.0}{4.80} = 5.83\text{ A}$$

(c) $I = I_1 + I_2 + I_3 = 35.0\text{ A}$

(d) As all the resistance connected in parallel, voltage across each resistor is 28.0 V.

(e) $P_1 = \dfrac{V^2}{R_1} = \dfrac{(28)^2}{1.6} = 490\text{ W}$

$$P_2 = \frac{V^2}{R_2} = \frac{(28)^2}{2.4} = 326.7\text{ W}$$

$$P_3 = \frac{V^2}{R_3} = \frac{(28)^2}{4.8} = 163.3\text{ W}$$

(f) As, $P = \dfrac{V^2}{R}$

Resistor with least resistance will dissipate maximum power.

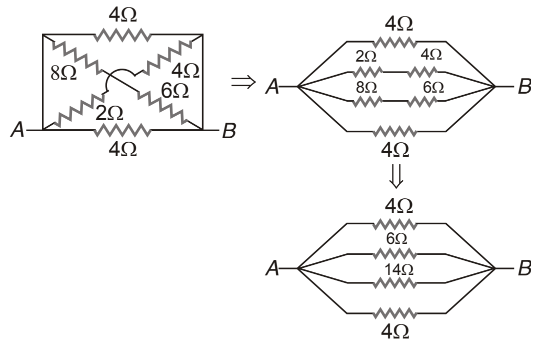
53. (a) $P = \dfrac{V^2}{R} \Rightarrow V = \sqrt{PR}$

$$= \sqrt{5 \times 15 \times 10^3} = 2.74 \times 10^2$$

$$= 274\text{ V}$$

(b) $P = \dfrac{V^2}{R} = \dfrac{(120)^2}{9 \times 10^3} = 1.6\text{ W}$

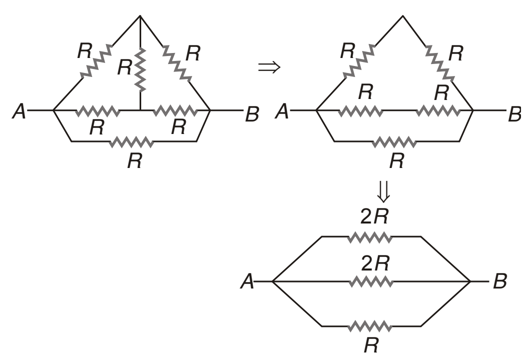
54. (a) $\dfrac{1}{R} = \dfrac{1}{4} + \dfrac{1}{6} + \dfrac{1}{14} + \dfrac{1}{4}$



$$\frac{1}{R} = \frac{31}{42}$$

$$R = \frac{42}{31}$$

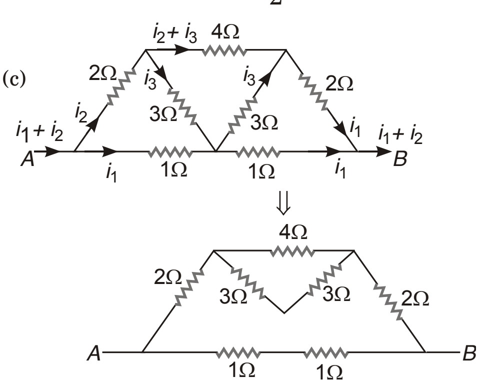
(b) $\dfrac{1}{R_e} = \dfrac{1}{2R} + \dfrac{1}{2R} + \dfrac{1}{R}$

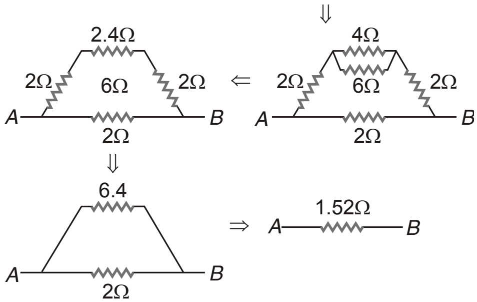


Wheatstone bridge is balanced

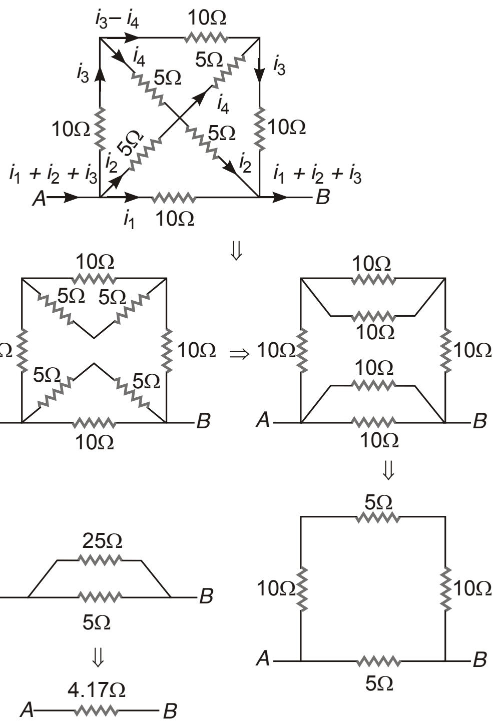
$\Rightarrow$ $R_e = \dfrac{R}{2}$

(c)

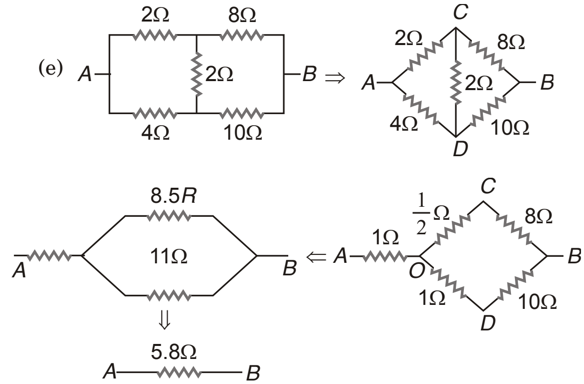




(d)



(e)



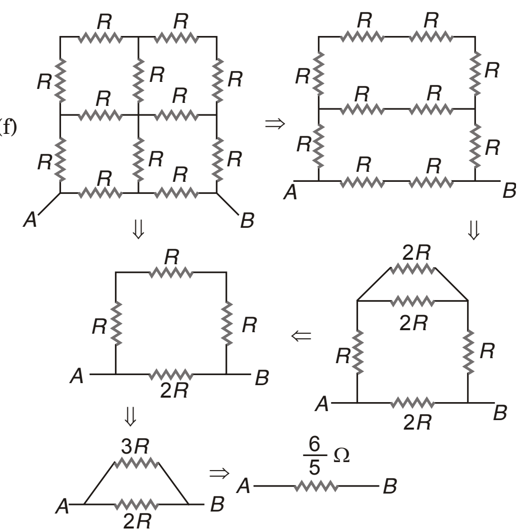
By Star-Delta Method

$$R_A = \frac{R_2R_3}{R_1 + R_2 + R_3}$$

$$R_B = \frac{R_1R_3}{R_1 + R_2 + R_3}$$

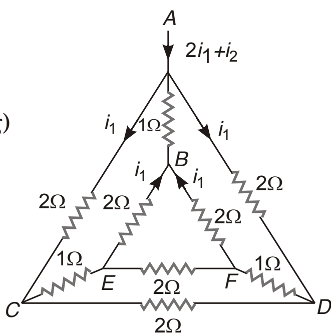
$$R_C = \frac{R_2R_3}{R_1 + R_2 + R_3}$$

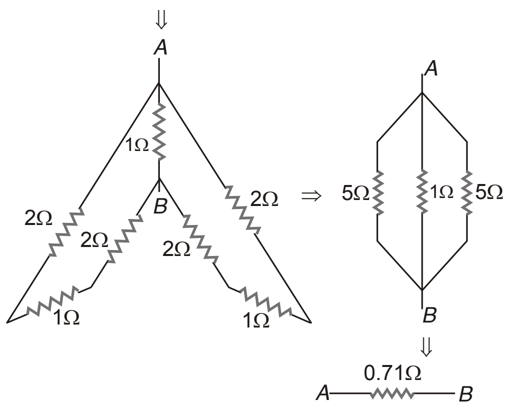
(f)



As circuit is symmetrical about perpendicular bisector of AB, lying on it are at same potential.

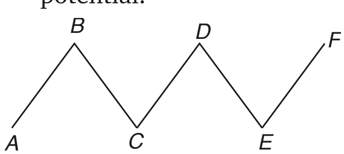
(g)





Clearly C and D, E and F are at same potential.

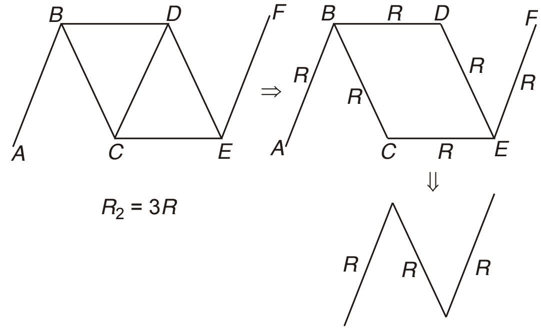
55.



Let R be the resistance of each conductor, and R_1 be the effective resistance between A and F in first case then,

$\therefore \qquad R_1 = 5R$

If R_2 be effective resistance between A and F in second case then,

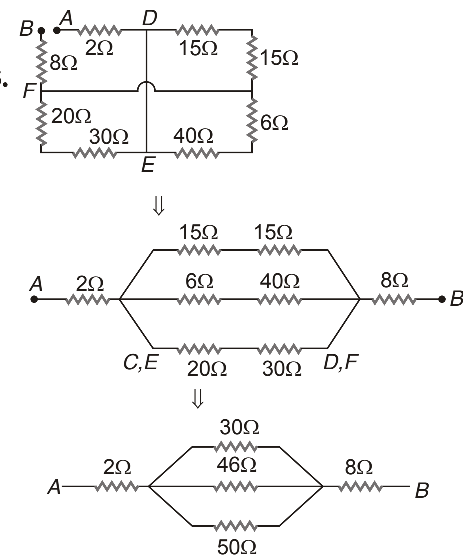


$R_2 = 3R$

$$\therefore \qquad \frac{R_2}{R_1} = \frac{3R}{5R} = 0.6$$

$$\Rightarrow \qquad R_2 = 0.6R_1$$

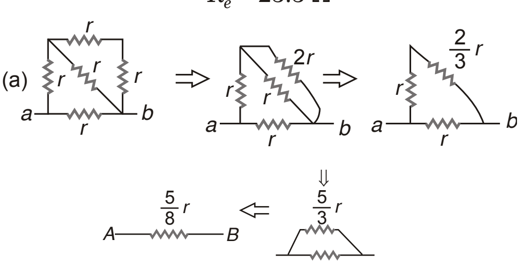
56.



Here, C and E , D and F are at same potential.

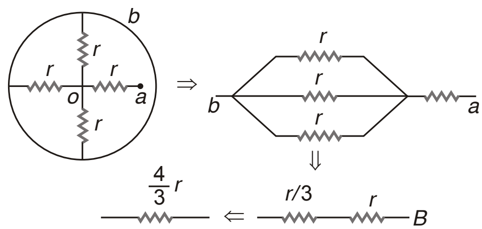
$$R_e = 23.3\ \Omega$$

57. (a)

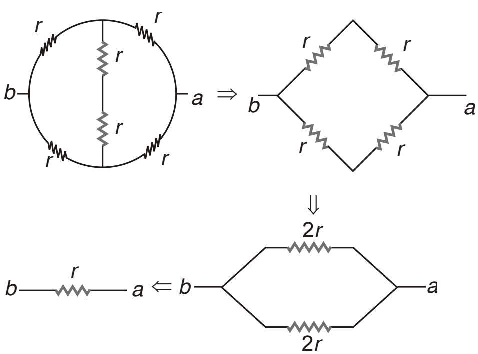


17

(b)

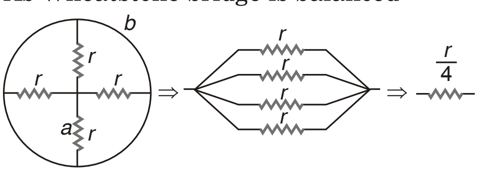


(c)

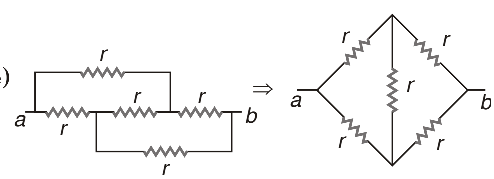


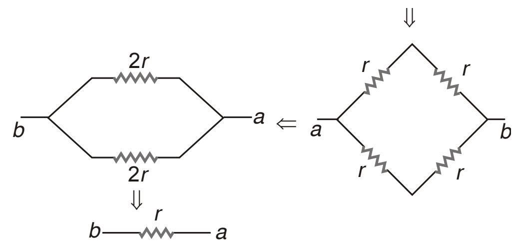
As Wheatstone bridge is balanced

(d)



(e)

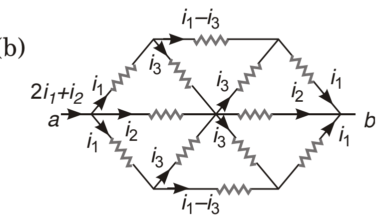


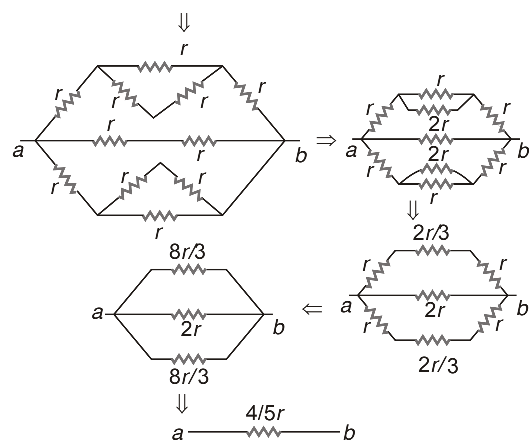


As Wheatstone bridge is balanced.

58. $R_e = \dfrac{r}{2}$

(b)





Objective Questions (Level-1)

1. When ammeter is connected in series

$$R_e = R + R_A$$

Hence, net current decreases. So R_A should be very low.

2. Amount of charge entering per second from one face is equal to the amount of charge leaving per second at the other, hence I is constant.

Again,

$$v_d = \frac{I}{neA} = \text{not constant.}$$

As $\qquad v_d = \dfrac{eF}{m}\tau$

$\Rightarrow \qquad E = \dfrac{mv_d}{e\tau} = \text{not constant}$

3. $R = \dfrac{V}{I}$

$\Rightarrow \qquad [R] = \dfrac{[V]}{[I]} = \dfrac{[ML^2T^{-3}I^{-1}]}{[I]}$

$$= [ML^2T^{-3}I^{-2}]$$

4. $\sigma = \dfrac{1}{\rho}$

As unit of resistivity is ohm-m and unit of σ is ohm^{-1}- m^{-1}.

5. Fact.

6. $E = I(R + r)$

Case I

$$E = 0.5(3.75 + r)$$

Case II

$$E = 0.4(4.75 + r)$$

On solving

$$r = 0.25\,\Omega,\ E = 2\text{ V}$$

7. $\frac{I}{I_g} = \frac{50}{20} \Rightarrow I = \frac{5}{2} I_g$

$$S = \frac{I_g}{I - I_g} G \Rightarrow G = \frac{I - I_g}{I_g} S$$

$$= \frac{3}{2} \times 12$$

$$= 18\,\Omega$$

8. $I_g = 2\% I = \frac{1}{50} I$

$$S = \frac{I_g}{I - I_g} G = \frac{G}{49}$$

9. $P = \frac{V^2}{R}$

$$P + \Delta P = \frac{V^2}{R + \Delta R}$$

As $R \propto l$

$$\Delta R = -10\% R$$

$$\Delta P = \frac{V^2}{0.9R} - \frac{V^2}{R} = \left(\frac{1}{0.9} - 1\right) P$$

$$= \frac{10}{9} P \approx 11\% P$$

10. Potential difference between any two points is zero.

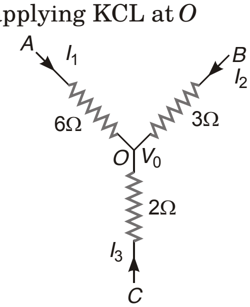
11.

$$r = \frac{l_1 - l_2}{l_2} R$$

$$= \frac{75 - 60}{60} \times 10$$

$$= 2.5\,\Omega$$

12. (b) By applying KCL at O



$$I_1 + I_2 + I_3 = 0$$

$$\frac{6 - V_0}{6} + \frac{3 - V_0}{3} + \frac{2 - V_0}{2} = 0$$

$$\Rightarrow 6 - V_0 + 2(3 - V_0) + 3(2 - V_0) = 0$$

$$V_0 = 3\text{ V}$$

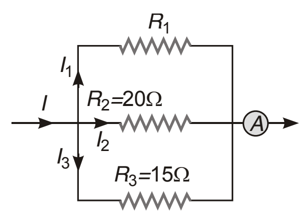
13. $v_d = \frac{I}{neA} = \frac{I}{ne\,\pi r^2}$

$$v_d' = \frac{2I}{ne\pi(2r)^2} = \frac{v_d}{2} = \frac{v}{2}$$

14. Voltmeter has higher resistance than ammeter.

Again higher the range of voltmeter, higher will be its resistance.

15. $I_2 = \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} I$



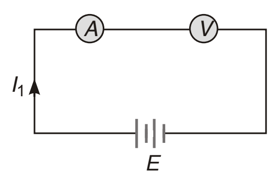
$$\Rightarrow \qquad \frac{1}{R_1} = \frac{I}{I_2 R_2} - \left(\frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$= \frac{0.8}{0.3 \times 20} - \left(\frac{1}{20} + \frac{1}{15}\right)$$

$$= \frac{1}{60}$$

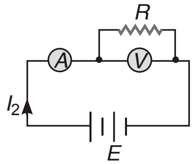
$$R_1 = 60\,\Omega$$

16. (d) $I_1 = \frac{E}{R_A + R_V},\ V_1 = I_1 R_V$



$$= E - I_1 R_A$$

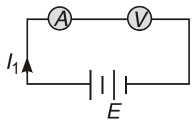
If resistance is connected in parallel with voltmeter,



$$I_2 = \frac{E}{R_A + \frac{RR_V}{R + R_V}} > I_1$$

and $\quad V_2 = E - I_2 R_A < V_1$

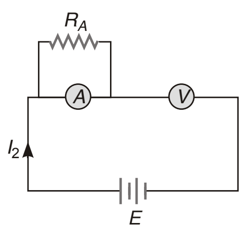
17. Before connectivity resistance is parallel with ammeter



$$I_1 = \frac{E}{R_A + R_V},\ V_1 = I_1 R_V$$

$$= E - I_1 R_A$$

After connecting resistance in parallel to the ammeter.



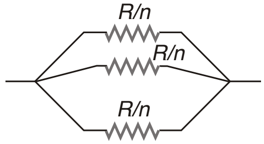
$$I_2 = \frac{E}{\frac{R_A}{2} + R_V},$$

Reading of ammeter $= \frac{1}{2} I_2$

$$= \frac{E}{R_A + 2R_V} > \frac{1}{2} I_1$$

$$V = I_2 R_V = \frac{2E}{R_A + 2R_V} < 2V_1$$

18. $R_e = \frac{R}{n^2}$



$$P = \frac{V^2}{R}$$

$$P_e = \frac{V^2}{R_e} = n^2 P$$

19. As bulb A is in series with entire circuit.

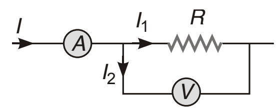
20. $I = \frac{E_1 + E_2}{R + r_1 + r_2} = \frac{18}{R + 3}$

$$V_{ab} = E_2 - Ir_2 = 0$$

$$3 - \frac{18}{R+3} \times 1 = 0$$

$\Rightarrow \qquad R = 3\ \Omega$

21. $I_2 = \frac{R}{R + R_V} I$



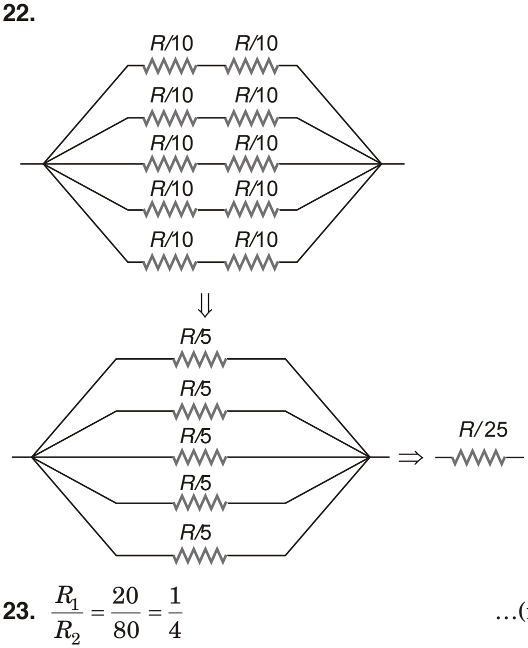
$$\frac{V}{R_V} = \frac{R}{R + R_V} I$$

$$\frac{100}{2500} = \frac{R}{R + 2500} 5$$

$$R + 2500 = 125\ R$$

$$R = \frac{2500}{24} \Omega \approx 100\ \Omega$$

22.



23. $\frac{R_1}{R_2} = \frac{20}{80} = \frac{1}{4}$ ...(i)

$$\frac{R_1 + 15}{R_2} = \frac{40}{60} = \frac{2}{3}$$

$$\frac{R_1}{R_2} + \frac{15}{R_2} = \frac{2}{3}$$

$$\frac{15}{R_2} = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$$

$$\Rightarrow \qquad R_2 = 36\,\Omega,$$

$$R_1 = \frac{R_2}{4} = 9\Omega$$

24. (b) As $V_1 = \frac{V}{2}$, $R_1 = R_2$

$$\frac{R_V \times 100}{100 + R_V} = 50$$

$$R_V = 100\,\Omega$$

25. (d) $\frac{E}{R_{PB} + r} = \frac{2}{4+1} = 0.4\,\Omega$

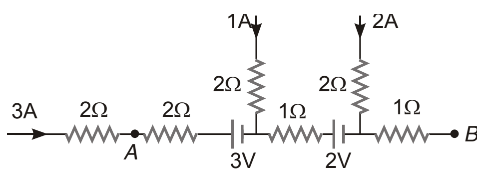
$$V_{AB} = IR_{AB} = 1.6\,\Omega$$

$$K = \frac{V_{PB}}{L} = \frac{1.6}{100} = 0.016 \text{ V/cm}$$

$$L = \frac{E_1}{K} = \frac{1.2}{0.016} = 75 \text{ cm}$$

26. (d) $V_{AB} = 3 \times 2 + 3 + 1 \times 4 - 2 + 6 \times 1$

$= 17$ V



27. (c) $E_e = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} = 2$ V

$$r_e = \frac{r_1 r_2}{r_1 + r_2} = 0.5\,\Omega$$

For maximum power $R = r_e$

and $\quad P_{max} = \frac{E_e^2}{4r_e} = \frac{(2)^2}{4 \times 0.5} = 2$ W

28. (a) $V = \frac{R}{R+r} E$

$$r = \left(\frac{E}{V} - 1\right) R = \left(\frac{2.2}{1.8} - 1\right) 5$$

$$= \frac{10}{9}\,\Omega$$

29. (d) $I = \frac{E_1 - E_2}{R_1 + R_2 + r_1 + r_2}$

$$= \frac{10 - 5}{25 + 15 + 2.5 + 2.5} = \frac{1}{9}\text{ A}$$

$$V_{AB} = -I(25 + 15)$$

$$= -\frac{1}{9} \times 40 \approx -4\text{ V}$$

30. (a) $V_{AB} = kL = 0.2 \times 100 = 20$ mV

$$V_{AB} = \frac{R_{AB}}{R_{AB} + R} E$$

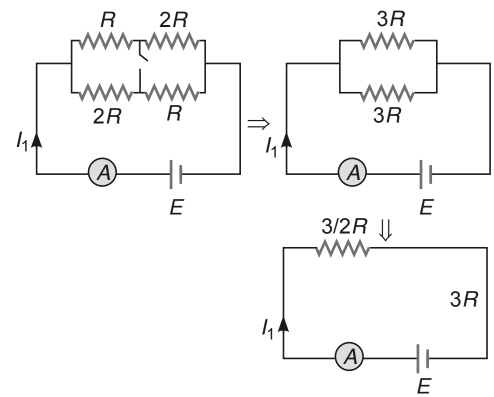
$$\Rightarrow \qquad 0.02 = \frac{R_{AB}}{R_{AB} + 490} \times 2$$

$$\Rightarrow \qquad R_{AB} + 490 = 100 R_{AB}$$

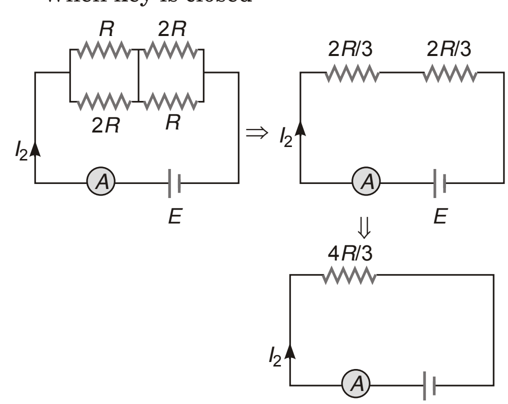
$$R_{AB} = \frac{490}{99} \approx 4.9\,\Omega$$

31. (c) When key is open,

$$I_1 = \frac{2E}{3R}$$



When key is closed



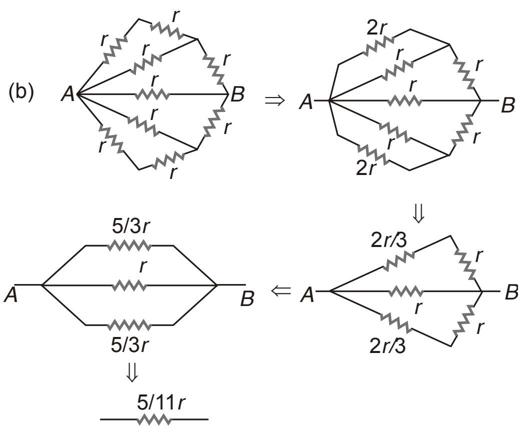
$$I_2 = \frac{3E}{4R}$$

$$\therefore \qquad \frac{I_1}{I_2} = \frac{8}{9}$$

32. (b) $S = \frac{I_g}{I - I_g} G = \frac{\frac{1}{34} I}{\frac{33}{34} I} \times 3663$

$= 111\,\Omega$

33. (b)

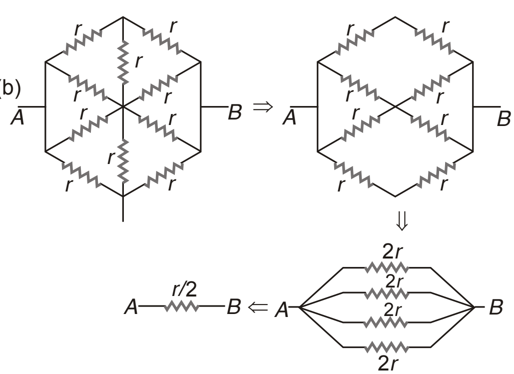


$R_e = \frac{5}{11} r$

$r = \frac{11}{5} \times 1.5$

$= 3.3\ \Omega$

34. (b)

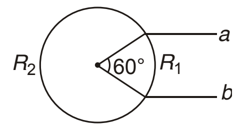


As the circuit is symmetrical about perpendicular bisector of *AB*, all points lying on it are at same potential.

35. (c) $R_1 = \frac{L_1}{L_1 + L_2} R$

$\Rightarrow \quad R_1 = \frac{R}{6} = 3\ \Omega$

$R_2 = \frac{l_2}{l_1 + l_2} \Rightarrow R_2 = 15\ \Omega$

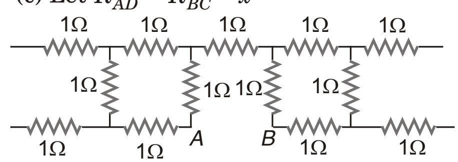


Hence R_1 and R_2 are in parallel

$R_e = \frac{R_1 R_2}{R_1 + R_2}$

$= 2.5\Omega$

36. (c) Let $R_{AD} = R_{BC} = x$



Clearly $x < 1$ as $1\,\Omega$ resistor is in parallel with some combination.

Now $\quad R_{AB} = x + 1 + x$

$= 2x + 1$

As $x < 1$

$1 < R_{AB} < 3$

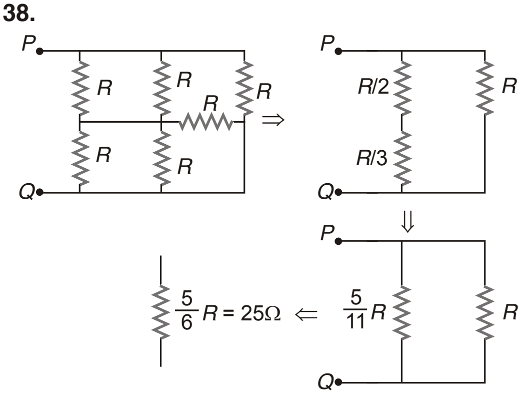
37. (d) $R_{AB} = R + \frac{R(R + R_0)}{2R + R_0} = R_0$

$\Rightarrow \quad 2R^2 + RR_0 + R^2 + RR_0 = 2RR_0 + R_0^2$

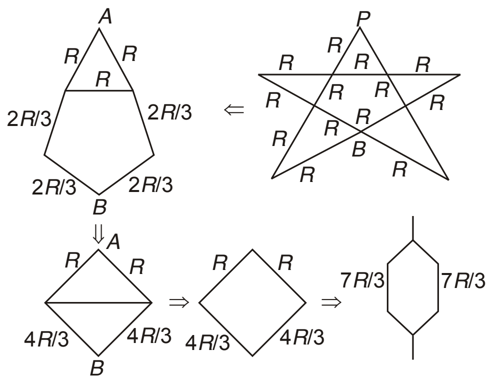
$\Rightarrow \quad 3R^2 = R_0^2$

$\Rightarrow \quad R = \frac{R_0}{\sqrt{3}}$

38.



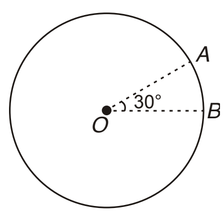
39. Wheatstone bridge is balanced.



$\Rightarrow \quad R_e = \frac{7}{6}R$

40. (d) $R_1 = \frac{L_1}{L_1 + L_2} R = \frac{1}{12} R$

$= 3\ \Omega$



$R_2 = \frac{L_2}{L_1 + L_2} = \frac{11}{12} R = 33\ \Omega$

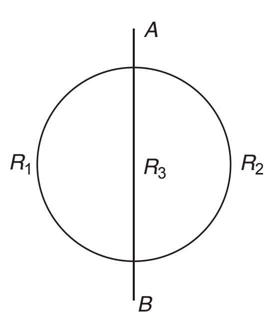
R_1 and R_2 are in parallel,

$R_e = \frac{R_1 R_2}{R_1 + R_2} = \frac{3 \times 33}{3 + 33}$

$= 2.75\ \Omega$

41. (a) Resistance per unit length of wire

$= \frac{4}{2\pi r}$



$R_1 = \frac{4}{2\pi r} \times \pi r = 2 = R_2$

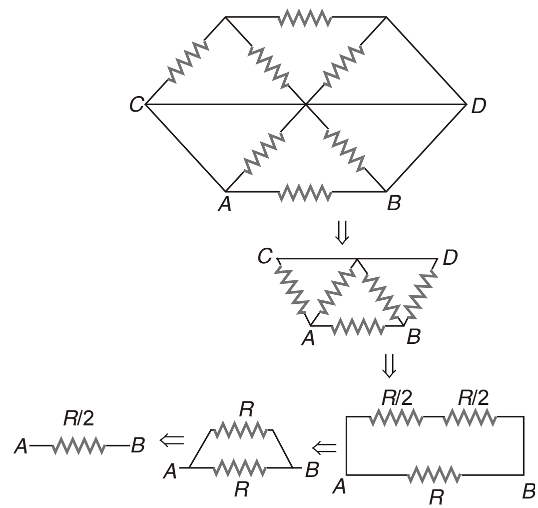
$R_3 = \frac{4}{2\pi r} \times 2r = \frac{4}{\pi}$

$\therefore \quad \frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

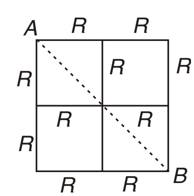
$= \frac{1}{2} + \frac{1}{2} + \frac{\pi}{4} = \frac{4+\pi}{4}$

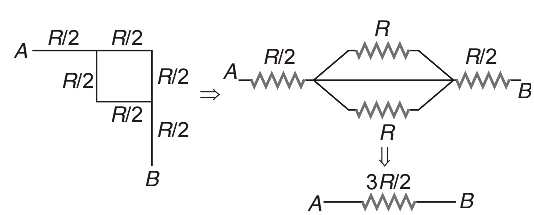
$R_e = \frac{4}{4+\pi}\ \Omega$

42. (d) Points C and D are shorted hence the portion above line CD can be removed.



43. (b) As AB is line of symmetry, we can fold the network about AB.





JEE Corner

Assertion and Reason

1. (d) $V = IR$, If $V = 0$ either $I = 0$ or $R = 0$
2. (b) As all the resistors are in parallel potential difference is same, hence
   $P = \frac{V^2}{R}$ is maximum if R is minimum.
3. (b) $dH = I^2 dRt = \frac{I^2 t \rho}{A} dH$
   I is same everywhere, hence portion having less area is more heated.
   Again $J = \frac{I}{A}$
   $\therefore \quad J_A > J_B$.
   Reason is also correct but does not explain assertion.
4. (b) Both assertion and reason are correct but reason does not explain the cause of decrease in voltmeter reading.
5. (b) As $R_A < R_V$, more current passes through ammeter when positions of ammeter and voltmeter are interchanged and potential difference across voltmeter becomes less that emf of cell.
6. (c) During charging current inside the battery flows from positive terminal to negative terminal. Reason is false while assertion is true.
7. (d) $I = \frac{E}{R + r}$ is maximum when R is zero hence reason is false.
   $P = \frac{E^2 R}{(R + r)^2}$ is maximum at $R = r$.
8. (c) $I = \frac{V}{R}$, $P = \frac{V^2}{R}$ both I and P are inversly proportional to R hence both decrease with increase in R which increases with temperature.
   According to Ohm's law $V \propto I$ not $V = IR$.
   As R can be variable also.
9. (d) Drift velocity is average velocity of all the electrons but velocity of all electrons is not constant.
10. (a) $R = \rho \frac{L}{A}$
    $$\rho = \frac{m}{ne^2 \tau}$$
    with increase in temperature, electron collide more frequently, *i.e.*, τ decreases, increasing ρ and hence R.
11. (d) $E = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} \quad \therefore \; E_1 < E < E_2$
    If $E_1 < E_2$
    $r = \frac{r_1 r_2}{r_1 + r_2}, r < r_1, r < r_2$
12. (d) $\frac{R_1}{R_2} = \frac{L_1}{L_2}$
    Hence there is no effect of one while measuring using meter bridge.

Objective Questions (Level-2)

1. (b) $I = \frac{E_2 - E_1}{r_1 + r_2} = \frac{1.5 - 1.3}{r_1 + r_2}$
   $$= \frac{0.2}{r_1 + r_2} \qquad \ldots(\text{i})$$
   $V = E_1 + Ir_1$
   $$\Rightarrow \qquad 1.45 = 1.3 + \frac{0.2}{r_1 + r_2} r_1$$
   $$\frac{r_1}{r_1 + r_2} = \frac{0.15}{0.2} \Rightarrow 0.2 r_1 = 0.15 r_1 + 0.15 r_2$$
   $$0.05 r_1 = 0.15\, r_2 \quad r_1 = 3 r_2$$
2. (c) Let R = Resistance of voltmeter,
   $$V_1 = \frac{ER}{R_1 + R} = 198\text{ V} \qquad \ldots(\text{i})$$
   $$V_2 = \frac{ER}{R_2 + R} = \frac{ER}{2R_1 + R} = 180\text{ V} \qquad \ldots(\text{ii})$$
   $$\frac{2R_1 + R}{R_1 + R} = \frac{198}{180} = \frac{11}{10}$$
   $$20R_1 + 10R = 11R_1 + 11R$$
   $$9R_1 = R$$

From Eq. (i),

$$\frac{ER}{R_1 + R} = 198$$

$$E = 198 \times \frac{10R_1}{9R_1} = 220\text{ V}$$

3. (b) $P = I^2R$

As R is same for all bulbs and maximum current passes through bulb A, it will glow most brightly.

4. (c) $R + R_A = \frac{V}{I} = 5\,\Omega$

$$R = 5 - R_A < 5\,\Omega$$

5. (a) $r = \frac{L_1 - L_2}{L_2} R = \frac{10}{60} \times 132.40$

$$\approx 22.1\,\Omega$$

6. (b) Current through R when S is open.

$$I_1 = \frac{E_1 + E_2}{R + r_1 + r_2}$$

Current through R when S is closed

$$I_2 = \frac{E_1}{R + r_1}$$

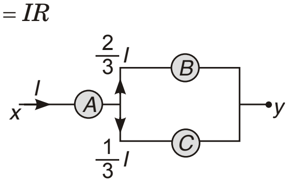
$$\Delta I = I_2 - I_1$$

$$= \frac{E_1}{R + r_1} - \frac{E_1 + E_2}{R + r_1 + r_2}$$

$$= \frac{E_1 r_2 - E_2(R + r_1)}{(R + r_1)(R + r_1 + r_2)}$$

$\Delta I = +$ ve if $E_1 r_2 > E_2(R + r_1)$

7. (a) $V_A = IR$



$$V_B = \frac{2}{3} I \times 1.5R = IR$$

$$V_C = \frac{1}{3} I \times 3R = IR$$

$\therefore$ $$V_A = V_B = V_C$$

8. (d) Current through $15\,\Omega$ resistor

$$= \frac{30}{15} = 2\text{ A}$$

$$V_{BC} = (2 + 5) \times 5 = 35\text{ V}$$

Voltage drop across $R = 100 - (30 + 55)$

$$= 35\text{ V}$$

$\therefore$ Required ratio $= \frac{35}{35} = 1$

9. (a) $r = \frac{L_1 - L_2}{L_2} R = \frac{x - y}{y} R$

10. (d) $\frac{R - 20}{40 - 20} = \frac{t - 10}{30 - 20}$

$\Rightarrow$ $$R = t + 10$$

$$I = \frac{E}{R} = \frac{10}{t + 10}$$

$$\frac{dq}{dt} = \frac{10}{t + 10}$$

$$q = \int_{10}^{30} \frac{10}{t + 10} dt = 10\left[\log_e(t + 10)\right]_{10}^{30}$$

$$= 10 \log_e 2$$

11. (b) Let l_1 length is kept fixed and l_2 is stretched,

$$R_1 = \rho \frac{l_1}{A},\ R_2 = \rho \frac{l_2}{A}$$

Initial resistance,

$$R = R_1 + R_2 \qquad \ldots\text{(i)}$$

Now full is stretched $\frac{3}{2}$ times, *ie*,

$$l_2' = \frac{3}{2}(l_1 + l_2) - l_1$$

$$= \frac{1}{2}(l_1 + 3l_2)$$

$$A_2' = \frac{A_2' l_2'}{l_2} = \frac{2Al_2}{l_1 + 3l_2}$$

$$R_2' = \rho \frac{\frac{1}{2}(l_1 + 3l_2)^2}{2Al_2}$$

$$R_2' = \rho \frac{(l_1 + 3l_2)^2}{4Al_2}$$

Now, $$R' = 4R$$

$$R_1 + R_2' = 4(R_1 + R)$$

$$l_1 + \frac{(l_1 + 3l_2)^2}{4l_2} = 4(l_1 + l_2)$$

$\Rightarrow$ $$\frac{l_2}{l_1} = \frac{1}{7}$$

$\Rightarrow$ $$\frac{l_2}{l_1 + L_2} = \frac{1}{8}$$

12. (b) $\frac{X}{R} = \frac{l_1}{100 - l_1} \Rightarrow l_1 = 40\text{ cm}$

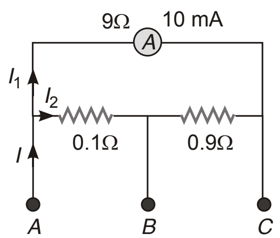
If $R' = 8\,\Omega$

$$\frac{X}{R'} = \frac{l_1'}{100 - l_1'}$$

$$\Rightarrow \qquad l_1' = 60 \text{ cm}$$

$$l_1' - l_1 = 20 \text{ cm}$$

13. (c) $I_1 = \dfrac{0.1}{0.1 + 9.9} I$



But $I_1 = 10$ mA

$$I = \frac{10}{0.1} \times 10 \text{ mA} = 1000 \text{ mA}$$

$$= 1 \text{ kA}$$

14. (d) Effective emf of two cells

$$E = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} = \frac{2 \times 6 + 4 \times 2}{2 + 6}$$

$$= \frac{20}{8} = 2.5 \text{ V}$$

$$R_{AB} = 4 \times 4 = 16\,\Omega$$

$$V_{AB} = \frac{R_{AB}}{R + R_{AB}} E_0 = \frac{16}{4 + 16} \times 12$$

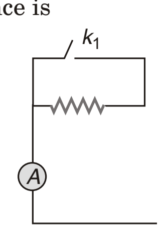
$$= 9.6 \text{ V}$$

$$k = \frac{V_{AB}}{L} = 2.4 \text{ V/m}$$

Now, $E = kl$

$$\Rightarrow \qquad L = \frac{E}{k} = \frac{2.5}{2.4} = \frac{25}{24}$$

15. When k_1 and k_2 both are closed, the resistance R_1 is short circuited. Therefore net resistance is



$$R_{net} = r + \frac{100 \times 100}{100 + 100} = r + 50$$

$$\therefore \qquad I_0 = \frac{E}{r + 50} \qquad \text{...(i)}$$

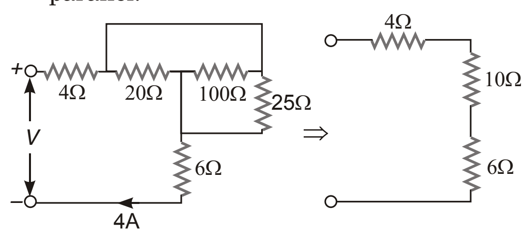
when, k_1 is open and k_2 is closed, net resistance is

$$R_{net} = r + R_1 + \frac{100 \times 100}{100 + 100} = (r + R_1 + 50)$$

$$\therefore \qquad \frac{I_0}{2} = \frac{E}{r + R_1 + 50} \qquad \text{...(ii)}$$

The above two equations are satisfied if $r = 0$ and $R_1 = 50\,\Omega$.

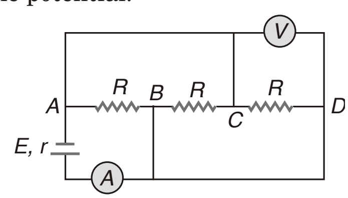
16. (b) $20\,\Omega$, $100\,\Omega$ and $25\,\Omega$ resistors are in parallel.



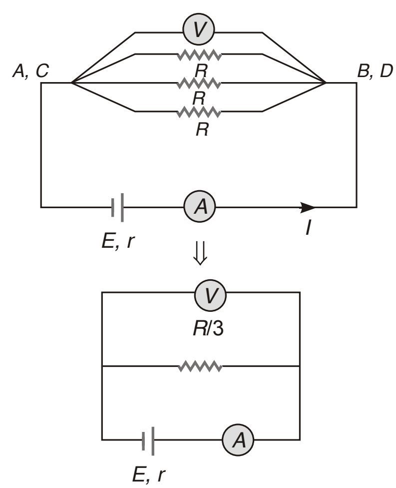
$$R = 20\,\Omega$$

$$V = IR = 80 \text{ V}$$

17. (a) Hence, points A and C, B and D are at same potential.



The equivalent circuit is given by



$$I = \frac{E}{\frac{R}{3} + r} = 1\text{ A}$$

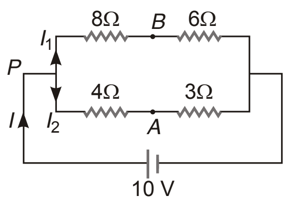
$$V = I \times \frac{R}{3} = 3\text{ V}$$

18. (c) $S = \frac{I_g}{I - I_g} G, \quad G = r, S = \frac{r}{4}$

$$I_g = \frac{1}{4}(I - I_g) \Rightarrow I_g = \frac{1}{5} I$$

$$= 0.006\text{ A}$$

19. (d) $I_1 = \frac{10}{14} = \frac{5}{7}$ A



$$I_2 = \frac{10}{7}\text{ A}$$

$$V_P - V_A = I_2 \times 4$$

$$= \frac{40}{7}\text{ A}$$

$$V_P - V_B = I_1 \times 8 = \frac{40}{7}\text{ A}$$

$$V_A - A_B = 0$$

Another method

As, $\frac{R_1}{R_2} = \frac{R_3}{R_4}, V_B = V_A$

20. (b) For series connection

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{3}{2}$$

Now, $R_1 = \rho \frac{L_1}{A_1} = \frac{\rho L_1}{\pi r_1^2}$,

$$R_2 = \delta \frac{L_2}{A_2} = \frac{\rho L_2}{\pi r_2^2}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{L_1}{L_2} \times \left(\frac{r_2}{r_1}\right)^2$$

$$\frac{r_1}{r_2} = \sqrt{\frac{R_2 L_1}{R_1 L_2}} = \sqrt{\frac{2 \times 6}{3 \times 1}} = 2$$

$$\frac{r_2}{r_1} = \frac{1}{2}$$

21. (a) Voltage sensitivity of voltmeter

$$\propto \frac{1}{\text{Resistance of voltmeter}}$$

$$\therefore \quad \frac{V_{s_1}}{V_{s_2}} = \frac{R_2 + G}{R_1 + G}$$

$$\frac{30}{20} = \frac{R_2 + 50}{2950 + 50}$$

$$R_2 + G = \frac{30 \times 3000}{20} = 4500$$

$$\Rightarrow \quad R_2 = 4450\ \Omega$$

22. (b) For $x = 0$

$$V_{AB} = E$$

$$k_1 = \frac{E}{L}$$

$$E_0 = k_1 L_1 = \frac{EL_1}{L} \quad \ldots\text{(i)}$$

For $x = x$ (say)

$$V_{AB} = \frac{R_{AB}}{R_{AB} + x} E$$

$$k_2 = \frac{R_{AB} E}{(R_{AB} + x) L}$$

$$E_0 = k_2 L_2 = \frac{R_{AB} E L_2}{(R_{AB} + x) L} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii),

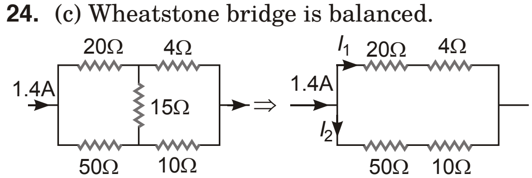
$$L_1 = \frac{R_{AB} + L_2}{(R_{AB} + x)}$$

$$\Rightarrow \quad 20 = \frac{10 \times 30}{10 + x}$$

$$\Rightarrow \quad x = 5\ \Omega$$

23. (d) To obtain null point similar terminal of both the batteries should be connected.

24. (c) Wheatstone bridge is balanced.

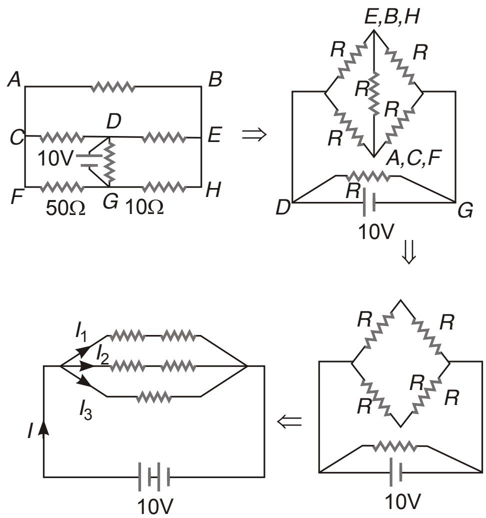


$$\therefore \quad I_1 = \frac{R_2}{R_1 + R_2} \times I$$

$$= \frac{60}{84} \times 1.4$$

$$= 1\text{ A}$$

25. (b)



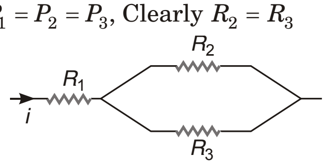
$$I_1 = \frac{10}{2 \times 2} = 2.5\,\text{A}$$

26. (b) Effective resistance of voltmeter and 3 kΩ resistor,

$$R_1 = \frac{3 \times 6}{2 + 6} = 2\,\text{k}\Omega$$

$$V_1 = \frac{R_1}{R_1 + R_2} E = \frac{2}{4} \times 10 = 5\,\text{V}$$

27. (d) $P_1 = P_2 = P_3$, Clearly $R_2 = R_3$



$\therefore$ $$i_2 = i_3 = \frac{i}{2}$$

$$P_1 = i^2 R_1,\ P_2 = \left(\frac{i}{2}\right)^2 \times R_2 = \frac{1}{4} i^2 R_2$$

$$P_3 = \frac{1}{4} i^2 R_3$$

$$R_2 = 4R_1,\ R_3 = 4R_1$$

$\therefore$ $$R_1 : R_2 : R_3 = 1 : 4 : 4$$

28. As $E = kL_1 \Rightarrow k = \dfrac{E}{L_1} = \dfrac{2}{500} = 250$

$$= \frac{1}{250}\,\text{V/cm}$$

$$V = kL_2 = \frac{1}{250} \times 490\,\text{cm}$$

$$= 1.96\,\text{V}$$

29. (c) $$r = \frac{L_1 - L_2}{L_2} R$$

$\Rightarrow$ $$R = \frac{L_2 r}{L_1 - L_2}$$

$$= \frac{490 \times 10}{10} = 490\,\Omega$$

More than One Correct Options

30. $$H = P_1 t_1 = P_2 t_2$$

$\Rightarrow$ $$t_1 = \frac{H}{P_1},\ t_2 = \frac{H}{P_2}$$

If connected in series

$$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}$$

$\Rightarrow$ $$t = t_1 + t_2$$

If connected in parallel

$$P = P_1 + P_2$$

$\Rightarrow$ $$t = \frac{t_1 t_2}{t_1 + t_2}$$

31. $$E = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} = \frac{6 \times 3 + 5 \times 2}{2 + 3}$$

$$= 5.6\,\text{V}$$

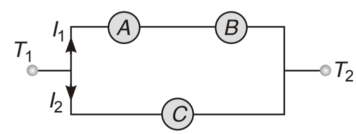
As there is no load.

$$V = E = 5.6\,\text{V}$$

If $$E_1 = E_2,\ I = 0$$

$$I = \frac{E_1 - E_2}{r_1 + r_2} = \frac{6 - 5}{2 + 3} = 0.2\,\text{A}$$

32. Let V = Potential difference between T_1 and T_2.



$$I_1 = \frac{V}{R_A + R_B}$$

$$I_2 = \frac{V}{R_C}$$

Now,. $$I_A = I_B = I_1$$

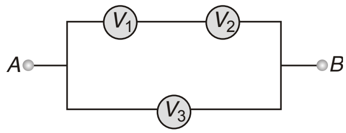
$$I_C = I_2$$

Also, $V = I_C R_C = I_1 (R_A + R_B)$

$$= I_A R_A + I_D R_B$$

$$\frac{I_B}{I_C} = \frac{I_1}{I_2} = \frac{R_C}{R_A + R_B}$$

33. As $R_1 \neq R_2$



$$V_1 \neq V_2$$

Also, $$V_3 = V_1 + V_2$$

34. As $R_1 = R_2$

$$V_1 = V_2$$

$$R = \rho \frac{L}{A}$$

But

$$L_2 = 2L_1$$

and $$R_1 = R_2$$

$\therefore$ $$A_2 = 2\,A_1$$

Also, $v_d \propto \dfrac{1}{A}$ (For constant current)

$$v_{d_2} = \frac{1}{2} v_{d1} \Rightarrow v_{d_1} = 2\,v_{d_2}$$

Again, $v_d \propto E$

$\therefore$ $$E_1 = 2\,E_2$$

35. If $E > 18$ V current will flow from B to A and *vice-versa*.

36. $V = kl$

If Jockey is shifted towards right, I and hence k will decreases as $k \propto I$.

Hence L will increase.

If E_1 is increased, k will increase, hence L will decrease.

If E_2 is increased L will increase as V will increase.

If ρ is closed V will decrease hence L will decrease.

37. $I_e = \dfrac{E}{R_e + r_e}$, Initially, $I = \dfrac{E}{R + r}$

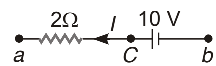
If S_1 is closed

$$I_e = \frac{E}{\frac{R}{2} + r} > I$$

If S_2 is closed

$$I_e = \frac{E}{R + \frac{r}{2}} > I$$

38. $V_b - V_a = -10 + 2I = 2\,\text{V}$

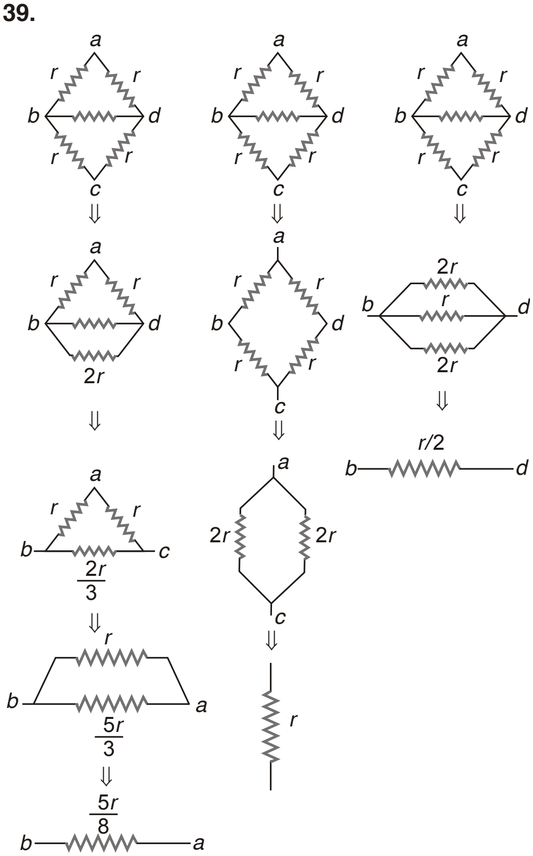


$$I = 6\,\text{A}$$

From b to a.

$$V_c - V_a = 2 \times 6 = 12\,\text{V}$$

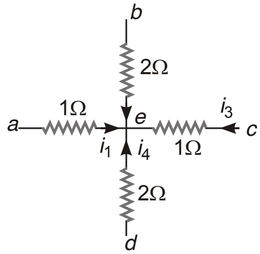
39.



Match the Columns

1. By applying KCL at e

$$i_1 + i_2 + i_3 + i_4 = 0$$



$$\frac{2 - V_e}{1} + \frac{4 - V_e}{2} + \frac{6 - V_e}{1} + \frac{4 - V_e}{2} = 0$$

$$V_e = 4\,\text{V},\ I_1 = -2\,\text{A},\ i_2 = 0,\ i_3 = 2\,\text{A},\ i_4 = 0$$

2. Current is same at every point and $A_1 < A_2$

$$J = \frac{i}{A} \Rightarrow J_1 > J_2$$

$$v_d = \frac{i}{neA} \Rightarrow v_{d_1} > v_{d_2}$$

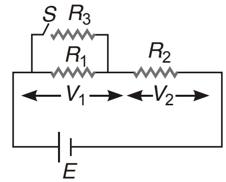
$$r = \frac{R}{L} = \frac{\rho}{A} \Rightarrow r_1 > r_2$$

$$k = \frac{V}{L} \Rightarrow k_1 > k_2$$

3. When switch S is closed

V_1 decreases, V_2 increases,

$\therefore$ Current through R_1 decreases and through R_2 increases.



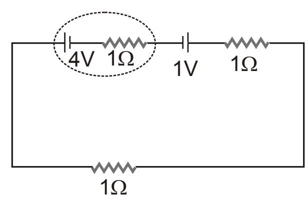
4. $$[R] = \frac{[V]}{[I]} = \frac{[ML^2T^{-3}A^{-1}]}{[A]} = [ML^2T^{-3}A^{-2}]$$

$$[V] = \frac{[W]}{[q]} = \frac{[ML^2T^{-2}]}{[AT]} = [ML^2T^{-3}A^{-1}]$$

$$[\sigma] = \frac{[R][A]}{[L]} = \frac{[ML^2T^{-3}A^{-2}][L^2]}{[L]} = [ML^3T^{-3}A^{-2}]$$

$$[\sigma] = \frac{1}{[\rho]} = [M^{-1}L^{-3}T^3A^2]$$

5. $$I = \frac{E_A - E_B}{R + r_A + r_A} = 1\,\text{A}$$



$$V_A = E_A - Ir_A = 3\text{ V}$$

$$V_B = E_B + Ir_B = 2\text{ V}$$

$$P_A = IV_A = 3\text{ W}$$

$$P_B = IV_B = 2\text{ W}$$

21 Electrostatics

Introductory Exercise 21.1

1. No, because charged body can attract an uncharged by inducing charge on it.
2. Yes.
3. On clearing, a phonograph record becomes charged by friction.
4. No. of electrons in 3 g mole of hydrogen atom

$$= 3 \times 6.022 \times 10^{23}$$

$$\therefore \quad q = ne = 3 \times 6.022 \times 10^{23} \times 1.6 \times 10^{-19}$$

$$= 2.9 \times 10^{5} \text{ C}$$

Introductory Exercise 21.2

1. $$F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2}$$

$$F_g = \frac{G m_1 m_2}{r^2}$$

$$\frac{F_e}{F_g} = \frac{e^2}{4\pi\varepsilon_0 \cdot G m_1 m_2}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times 9.11 \times 10^{-31} \times 1.67 \times 10^{-27}}$$

$$= 2.27 \times 10^{39}$$

2. $$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$\varepsilon_0 = \frac{q_1 q_2}{4\pi F r^2}$$

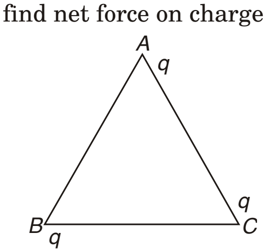
$$[\varepsilon_0] = \frac{[q_e]^2}{[F][r]^2}$$

$$= \frac{[\text{IT}]^2}{[\text{MLT}^{-2}][\text{L}]^2}$$

$$= [\text{M}^{-1}\text{L}^{-3}\text{T}^{4}\text{I}^{2}]$$

SI units of $\varepsilon_0 = \text{C}^2 \text{N}^{-1} \text{m}^{-2}$.

3. Let us find net force on charge at A.

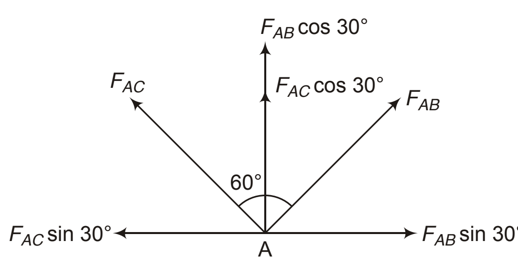


$$F_{AB} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{a^2} \quad F_{AC} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{a^2}$$

Net force on charge at A

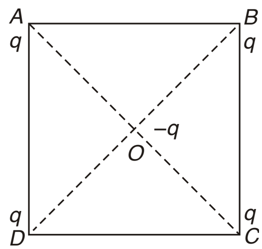
$$F_A = F_{AB} \cos 30° + F_{AC} \cos 30°$$

$$= \frac{\sqrt{3} q^2}{4\pi\varepsilon_0 \cdot a^2}$$



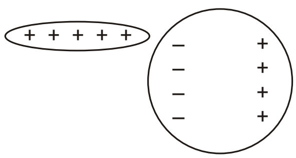
4. $\vec{\mathbf{F}}_{OA} = \vec{\mathbf{F}}_{OC}$

and $\vec{\mathbf{F}}_{OB} = -\vec{\mathbf{F}}_{OD}$



Hence, net force on charge at centre is zero.

5. No. In case of induction while charge comes closer and like charge moves further from the source.



The cause of attraction is more attractive force due to small distance. But if electrostatic force becomes independent of distance, attractive force will become equal to repulsive force, hence net force becomes zero.

6. When the charged glass rod is brought near the metal sphere, negative induces on the portion of sphere near the charge, hence it get attracted. But when the sphere touches the rod it becomes positively charged due to conduction and gets repelled by the rod.

7. Yes as $q_{\min} = e$

$$F_{\min} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2}$$

8. No. Electrostatic force is independent of presence or absence of other charges.

9. $\vec{F}_{21} = -\vec{F}_{12} = (-4\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\,\text{N}$.

Introductory Exercise 21.3

1. False. $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$

2. $V_A > V_B$ as electric lines of force move from higher potential to lower potential.

3. False. Positively charged particle moves in the direction of electric field while negatively charged particle moves opposite to the direction of electric field.

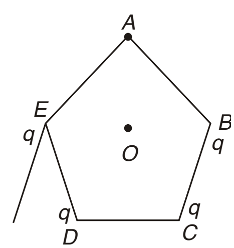
4. False. Direction of motion can be different from direction of force.

5. $E = \frac{\sigma}{\varepsilon_0} \Rightarrow \sigma = E\varepsilon_0 = 3.0 \times 8.85 \times 10^{-12}$

$$= 2.655 \times 10^{-11}\ \text{C/m}^2$$

6. q_1 and q_3 are positively charged as lines of force are directed away from q_1 and q_3. q_2 is negatively charged because electric field lines are towards q_2.

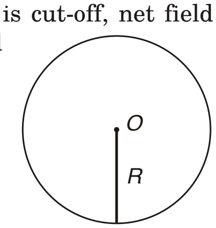
7. If a charge q is placed at A also net field at centre will be zero.



Hence net field at O is same as produced by A done but in opposite direction,02 *i.e.*,

$$E = \frac{1}{4\pi\varepsilon} \cdot \frac{q}{a^2}$$

8. Net field at the centre (O) of wire is zero. If a small length of the wire is cut-off, net field will be equal to the field due to cut-off portion, *i.e.*,



$$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{R^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{\frac{q}{2\pi R}\,dl}{R^2}$$

$$= \frac{q\,dl}{8\pi^2\varepsilon_0 R^3}$$

9. $$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^3} \vec{\mathbf{r}}$$
$$= -\frac{9 \times 10^9 \times 2 \times 10^{-6}}{(3^2 + 4^2)^{3/2}} (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) = -144 (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \text{ N/C}$$

Introductory Exercise 21.4

1. Gain in KE = loss of PE
$$\frac{1}{2} mv^2 = \frac{1}{4\pi\varepsilon_0} \cdot q_1 q_2 \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$
$$\frac{1}{2} \times 10^{-4} v^2$$
$$= -1 \times 10^{-6} \times 2 \times 10^{-6} \times 9 \times 10^9 \left(\frac{1}{1} - \frac{1}{0.5}\right)$$
$$v^2 = 360$$
$$v = 6\sqrt{10} \text{ ms}^{-1}$$

2. $W = q(V_A - V_B)$
$$= 2 \times 10^{-6} \left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{-1 \times 10^{-6}}{1} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{-1 \times 10^{-6}}{2}\right)$$
$$= -9 \times 10^{-3} \text{ J}$$
$$= -9 \text{ m J}$$

3. Whenever work is done by electric force, potential energy is decreased.
$$W = -\Delta U$$
$$U_2 = U_1 - W = -8.6 \times 10^{-8} \text{ J}$$

4. No. As $U = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r}$

If there are three particles
$$U = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}}\right]$$

Here U may be zero.

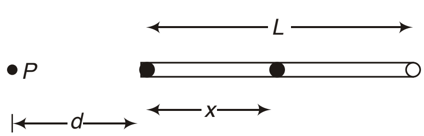
In case of more than two particles PE of systems may same as if they were separated by infinite distance but not in case of two particles.

Introductory Exercise 21.5

1. $$V_{ba} = \frac{W_{a \to b}}{q} = 12 \times 10^2 = 1200 \text{ V}$$

2. $\lambda = \alpha x$

(a) SI Units of λ = C/m
$$\alpha = \frac{\lambda}{x}$$
Hence SI unit of $\alpha = \dfrac{\text{C/m}}{\text{m}} = \text{C/m}^2$.

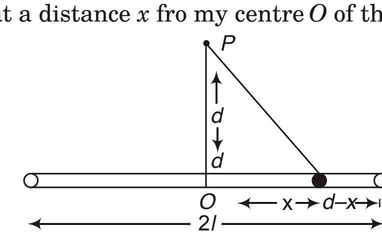


(b) Consider an elementary portion of rod at a distance x from origin having length dx. Electric potential at P due to this element.
$$dV = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda\, dx}{x + d}$$

Net electric potential at P
$$V = \int_0^L \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda\, dx}{x + d}$$

$$\Rightarrow \quad \frac{\alpha}{4\pi\varepsilon_0} \cdot \int_0^L \frac{x\, dx}{x + d}$$
$$= \frac{\alpha}{4\pi\varepsilon_0} \cdot \left[\int_0^L dx - d\int_0^L \frac{dx}{x + d}\right]$$
$$= \frac{\alpha}{4\pi\varepsilon_0} \cdot [[x]_0^L - d\,[\ln (x + d)]_0^L]$$
$$= \frac{\alpha}{4\pi\varepsilon_0} \cdot \left[L - d \ln \frac{L + d}{d}\right]$$

3. Consider an elementary portion of length dx at a distance x fro my centre O of the rod.



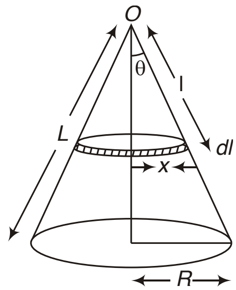
Electric potential at P due to this element,
$$dV = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda\, dx}{\sqrt{d^2 + x^2}}$$
$$V = \frac{\lambda}{4\pi\varepsilon_0} \cdot \int_{-l}^{l} \frac{dx}{\sqrt{d^2 + x^2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \cdot \left[\sin^{-1}\frac{x}{d}\right]_{-l}^{l}$$

$$= \frac{q}{4\pi\varepsilon_0 \cdot 2l} \times 2\sin^{-1}\frac{x}{d}$$

$$V = \frac{q}{4\pi\varepsilon_0\, l}\sin^{-1}\frac{x}{d}$$

4. Consider the cone to be made up of large number of elementary rings.



Consider one such ring of radius x and thickness dl. Let θ be the semi-vertical angle of cone and R be the radius of cone.

Charge on the elementary ring;

$$dQ = \sigma\, dA = \frac{Q}{\pi RL} \cdot 2\pi x\, dl$$

or $$dQ = \frac{2Ql\sin\theta}{RL}\, dl$$

Potential at O due to this ring

$$dV = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dQ}{l}$$

$$= \frac{Q\sin\theta}{2\pi\,\varepsilon_0 RL}\, dl$$

Total potential at O

$$V = \frac{Q\sin\theta}{2\pi\varepsilon_0\, RL}\int_0^L dl = \frac{QL\sin\theta}{2\pi\varepsilon_0\, RL}$$

$$= \frac{Q}{2\pi\varepsilon_0\, L} \quad [L\sin\theta = R]$$

$$U = qV$$

$$= \frac{Qq}{2\pi\varepsilon_0\, L}$$

Introductory Exercise 21.6

1. (a) $V = a(x^2 - y^2)$

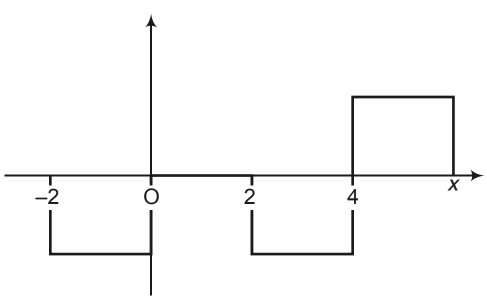
$$E = -\left(\frac{\partial v}{\partial x}\hat{\mathbf{i}} = \frac{\partial v}{\partial y}\hat{\mathbf{j}}\right) = -2ax\,\hat{\mathbf{i}} + 2y\,\hat{\mathbf{j}}$$

(b) $V = axy$

$$E = -\left(\frac{\partial v}{\partial x}\hat{\mathbf{i}} = \frac{\partial v}{\partial y}\hat{\mathbf{j}}\right) = -a(y\,\hat{\mathbf{i}} + x\,\hat{\mathbf{j}})$$

2. From $x = -2$ to $x = 0$ & $x = 2$ to $x = 4$

V is increasing uniformly.



Hence, E is uniform and negative

From $x = 0$ to $x = 2$

V is constant hence E is zero.

For $x > 4$

V is decreasing at constant rate, hence E is positive.

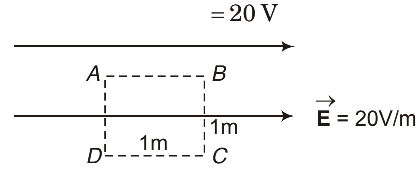
3. $E = -\dfrac{dV}{dr} = -\dfrac{(50 - 100)}{5 - 0} = 10$ V/m

True.

4. (a) $V_P - V_D = \vec{\mathbf{E}} \cdot \vec{\mathbf{l}} = 0$

(b) $V_P - V_C = \vec{\mathbf{E}} \cdot \vec{\mathbf{l}} = 20 \times 1 \times \cos 0°$

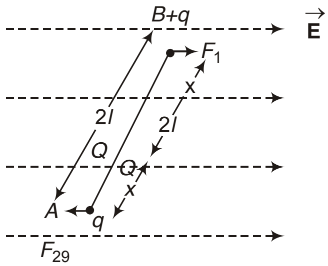
$= 20$ V



(c) $V_B - V_D = -20 \times 1 = -20$ V

(d) $V_C - V_D = -20 \times 1 = -20$ V

Introductory Exercise 21.7



1. $F_1 = qE$ towards right

   $F_2 = qE$ towards left

   Net torque about θ,

   $$\tau = qE(2l - x)\sin\theta + qEx\sin\theta$$
   $$= q(2l)E\sin\theta = pE\sin\theta$$
   $$\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$$

2. $$E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(\sqrt{y^2+a^2})^2}$$
   $$E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(\sqrt{y^2+a^2})^2}$$
   $$E_3 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2q}{y^2}$$

   Net field at P

$$\vec{\mathbf{E}} = -(E_3 - E_1\cos\theta - E_2\cos\theta)\hat{\mathbf{j}}$$
$$= \frac{-q}{4\pi\varepsilon_0}\left[\frac{2}{y^2} - \frac{\cos\theta}{y^2+a^2} - \frac{\cos\theta}{y^2+a^2}\right]\hat{\mathbf{j}}$$
$$= -\frac{2q}{4\pi\varepsilon_0}\left[\frac{1}{y^2} - \frac{y}{(y^2+a^2)^{3/2}}\right]\hat{\mathbf{j}}$$
$$= -\frac{2q}{4\pi\varepsilon_0}\left[\frac{(y^2+a^2)^{3/2} - y^3}{y^2(y^2+a^2)^{3/2}}\right]\hat{\mathbf{j}}$$
$$= -\frac{2q}{4\pi\varepsilon_0}\left[\frac{y^3\left(1+\frac{q^2}{y^2}\right)^{3/2} - y^3}{y^2(y^2+a^2)^{3/2}}\right]\hat{\mathbf{j}}$$

As $y >> a$

$$E = -\frac{2q}{4\pi\varepsilon_0}\cdot\left[\frac{y^3\left(1+\frac{3q^2}{2y^2}\right) - y^3}{y^5}\right]\hat{\mathbf{j}}$$
$$E = -\frac{3qa^2}{4\pi\varepsilon_0 y^4}\hat{\mathbf{j}}$$

Introductory Exercise 21.8

1. (a) Charge q is completely the hemisphere hence flux through hemisphere is zero.

   (b) Charge inside the sphere is q hence flux through hemisphere

   $$\phi = \frac{q}{\varepsilon_0}$$

   (c) As charge q is at the surface, net flux through hemisphere

   $$\phi = \frac{q}{2\,\varepsilon_0}$$

2. When charge is at any of the vertex, net flux through the cube,

   $$\phi = \frac{q}{8\varepsilon_0}$$

   If charge q is at D,

   flux through three faces containing D is zero and the flux ϕ is divided equal among other three faces, hence

   $$\phi_{\text{EFGH}} = \frac{1}{\phi} = \frac{q}{2\pi\varepsilon_0}$$

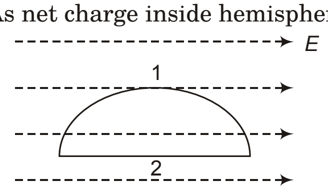
   and $\phi_{\text{AEHD}} = 0$

3. True. As electric field is uniform, flux entering the cube will be equal to flux leaving it.

   $\therefore$ $\phi_{\text{net}} = 0 \Rightarrow \phi_{\text{net}} = \frac{q}{\varepsilon_0}$

   $$\Rightarrow q = 0$$

4. (a) As net charge inside hemisphere is zero,



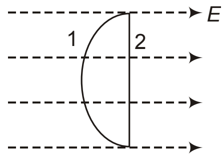
$$\phi_1 + \phi_2 = 0$$

But E is parallel to surface 2.

$\therefore \qquad \phi_2 = 0$

Hence, $\phi_1 = 0$

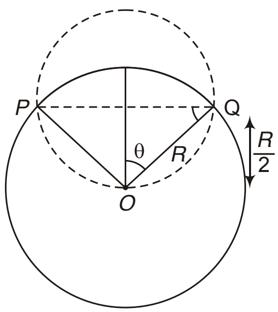
(b) Again, $\phi_1 + \phi_2 = 0$



$$\phi_2 = E \times \pi R^2 = \pi R^2 E$$

$$\therefore \qquad \phi_1 = -\phi_2 = -\pi R^2 E$$

5. $\cos\theta = \frac{R/2}{R} = \frac{1}{2}, \theta = 60°$



$\angle POQ = 2\theta = 120°$

$\therefore$ Length of arc $PQ = \frac{2\pi}{3} R$

Charge inside sphere,

$$q = \frac{q_0}{2\pi R} \times \frac{2\pi}{3} R \Rightarrow \frac{q_0}{3}$$

$\therefore$ Flux through the sphere

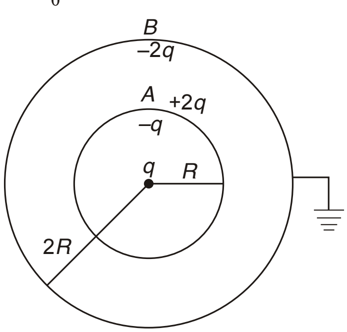
$$\phi = \frac{q}{\varepsilon_0} = \frac{q_0}{3\,\varepsilon_0}$$

6. Net charge inside the cube = 0.

$\therefore$ Net flux through the cube = 0.

Introductory Exercise 21.9

1. $V_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q + q + q_B}{2R} = 0$



$$q_B = -2q$$

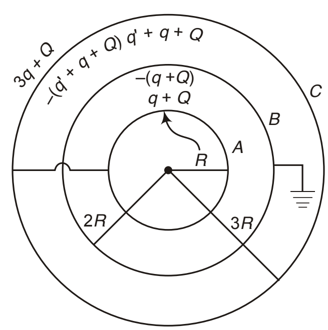
Total charge inside a conducting sphere appears on its outer surface,

$\therefore$ Charge on outer surface of $A = 2q$

and charge on outer surface of B

$$= 2q - 2q = 0$$

2. Let q' = charge on sphere B and charge ϕ flows from sphere C to A.



$$V_B = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q' + q + Q}{2R} + \frac{2q - Q}{3R} \right) = 0$$

$$\Rightarrow \qquad 3q' + q + \phi = 0 \qquad \text{...(i)}$$

Again, $V_P = V_C$

$$\frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{q+Q}{R} + \frac{q}{2R} + \frac{2q - Q}{3R} \right] = \frac{1}{4\pi\varepsilon_0} \cdot \frac{3q + q'}{3R}$$

$$6(q + Q) + 3q' + 2(2q - Q) = 2(3q + q')$$

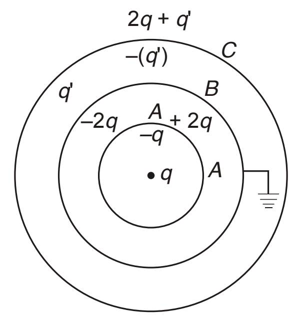
$$4q + 4q + q' = 0$$

On solving

$$Q = -\frac{5}{11}q, \; q' = -\frac{24}{11}q$$

	A	B	C
Charge on inner surface	0	$-(q+Q) = -\frac{6}{11}q$	$-(q'+q+Q) = \frac{18}{11}q$
Charge on outer surface	$q+Q = \frac{6}{11}$	$q'+q+Q = -\frac{18}{11}q$	$3q+q = \frac{9}{11}q$

3.



	A	B	C
Charge on inner surface	$-q$	$-2q$	$+\frac{4}{3}q$
Charge on outer surface	$+2q$	$-\frac{4q}{3}$	$+\frac{2}{3}q$

AIEEE Corner

Subjective Questions (Level-1)

1. $F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q(Q-q)}{r^2}$

For maximum force

$$\frac{dF}{dq} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q-2q}{r^2} = 0$$

$$q = \frac{Q}{2}$$

$$\frac{d^2F}{dq^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-2}{r^2} < 0$$

Hence F is maximum at $q = \frac{Q}{2}$.

2. Minimum possible charge on a particle $= e$.

$$\therefore F_{\min} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r^2} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{(1\times10^{-2})^2}$$

$$= 2.3\times10^{-24}\text{ N}$$

3. $$F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2} \quad \ldots\text{(i)}$$

$$F_g = \frac{Gm_1m_2}{r^2} \quad \ldots\text{(ii)}$$

$$\frac{F_e}{F_g} = \frac{q_1q_2}{4\pi\varepsilon_0 Gm_1m_2}$$

$$= \frac{(3.2\times10^{-19})^2\times9\times10^9}{6.67\times10^{-11}\times(6.64\times10^{-27})^2}$$

$$= 3.1\times10^{35}$$

4. $$F_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2} \quad \ldots\text{(i)}$$

$$F_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r^2} \quad \ldots\text{(ii)}$$

[As both the spheres are identical, find charge on both the spheres will be equal]

$$q = \frac{q_1 - q_2}{2}$$

$\Rightarrow \quad q_1 - q_2 = 2q$

From Eq. (ii),

$$q^2 = 4\pi\varepsilon_0 r^2F_2$$

$$= \frac{(50\times10^{-2})^2\times0.036}{9\times10^9} = 10^{-12}$$

$$q = 10^{-6}\text{ C} = 1\,\mu\text{C}$$

From Eq. (i),

$$q_1q_2 = 4\pi\varepsilon_0 r^2F_1 = \frac{(50\times10^{-2})^2\times0.108}{9\times10^9}$$

$$= 3\times10^{-12}$$

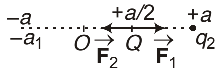
Also, $q_1 + q_2 = 2q = 2\times10^{-6}$

On solving

$$q_1 = \pm 3\,\mu\text{C}$$

and $q_2 = \mp 1\,\mu\text{C}$

5. (a) $F_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 Q}{(3a/2)^2}$



$$F_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4q_1 Q}{9a^2} \quad \ldots\text{(i)}$$

$$F_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2 Q}{(a/2)^2}$$

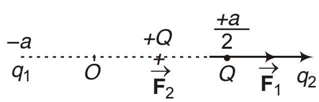
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{4q_2 Q}{a^2} \quad \ldots\text{(ii)}$$

For net force on Q to be zero

$$F_1 = F_2$$

or $$q_1 = 9q_2$$

(b) $F_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4q_1 Q}{25a^2}$



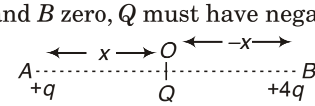
$$F_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4q_2 Q}{9a^2}$$

For net force on Q to be zero.

$$F_1 + F_2 = 0$$

$$\Rightarrow \quad \frac{q_1}{q_2} = \frac{25}{9}$$

6. (a) In order to make net force on charge at A and B zero, Q must have negative sign.



Let the charge Q is planed at a distance x from A (+ Q charge)

$$F_{OA} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ}{x^2}$$

$$F_{OB} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4qQ}{(x-x)^2}$$

For net force on Q to be zero.

$$F_{OA} = F_{OB}$$

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ}{x^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4qQ}{(L-x)^2}$$

$$(L-x)^2 = (2x)^2$$

$$x = \frac{L}{3}$$

Force on A,

$$F_{AB} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4q^2}{L^2}$$

$$F_{AO} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ}{x^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{qqQ}{L^2}$$

For net force on Q to be zero.

$$F_{AB} = F_{AO}$$

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{4q^2}{L^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qqQ}{L^2}$$

$$\Rightarrow \quad q = \frac{9}{4}Q$$

$$\Rightarrow \quad Q = \frac{4}{9}q$$

As Q is negative $\Rightarrow q = \frac{-4}{9}q$

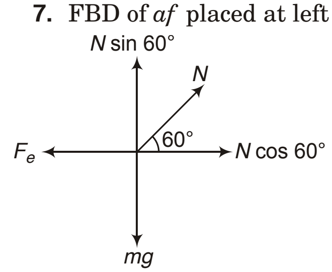
(b) PE of the system

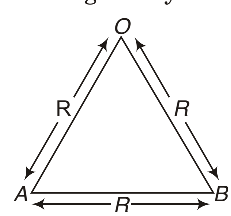
$$U = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{4q^2}{L} + \frac{qQ}{x} + \frac{4qQ}{L-x}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{4q^2}{L} - \frac{4qQ}{3L} - \frac{8qQ}{3L}\right] = 0$$

Hence, equilibrium is unstable.

7. FBD of af placed at left can be given by





ΔABD is equilateral

As beads are in equilibrium

$$mg = N\sin 60°$$

$$F_e = N\cos 60°$$

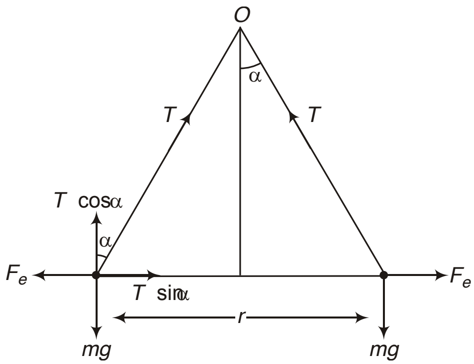
$$\frac{F_e}{mg} = \cot 60°$$

$$q^2 = 4\pi\varepsilon_0 R^2\, mg \cot 60°$$

$$q = \sqrt{\frac{4\pi\varepsilon_0 R^2\, mg}{3}}$$

$$= 2R\sqrt{\frac{6\pi\varepsilon_0 mg}{\sqrt{3}}}$$

8. As ball are in equilibrium



$$F_e = T \sin\alpha$$
$$mg = T\cos\alpha$$
$$F_e = mg\tan\alpha$$
$$q^2 = 4\pi\varepsilon_0 r^2 \tan\alpha$$

Here, $r = 2l\sin\alpha$

$$q^2 = 16\pi\varepsilon_0\, l^2 \sin^2\alpha\tan\alpha$$
$$q = 3.3\times10^{-8}\ \text{C.}$$

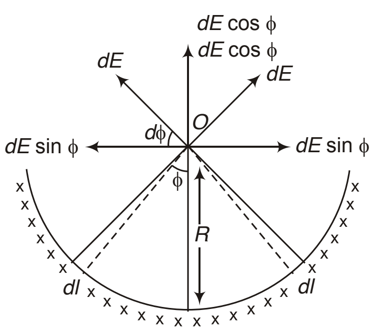
9. Same as Q.7. Introductory Exercise 21.3.

10. See Q.7. Introductory Exercise 21.3.

11. $$E = \frac{1}{4\pi\varepsilon_0}\cdot 1\,\frac{q}{r^3}\,\vec{\mathbf{r}}$$
$$= \frac{9\times10^9\times(-8.0\times10^{-9})}{((1.2)^2+(1.6)^2)^{3/2}}(1.2\,\hat{\mathbf{i}} - 1.6\,\hat{\mathbf{j}})$$
$$= -18\sqrt{2}\,(1.2\,\hat{\mathbf{i}} - 1.6\,\hat{\mathbf{j}})\ \text{N/C.}$$

12. Consider an elementary portion on the ring of length dl subtending angle $d\phi$ at centre 'O' of the ring.

Charge on this portion,



$$dq = \lambda\, dl = \lambda\, R d\phi$$

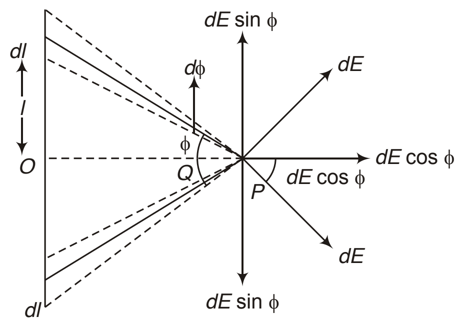
$\therefore$ $$dE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{R^2} = \frac{1}{4\pi\varepsilon_0}\,\frac{\lambda\, d\phi}{R}$$

Here, $dE\sin\phi$ components of field will cancel each other.

Hence, Net field at O

$$E = \int dE\cos\phi = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda}{R}\int_{-\pi/2}^{\pi/2}\cos\phi\, d\phi$$
$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\lambda}{R}$$

13. Consider elementary portion of the rod of length dl at a distance l from the centre O of the rod.



Charge on this portion

$$dq = \lambda\, dl = \frac{Q}{L}dl$$

$\therefore$ $$dE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{(a\sec\phi)^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\,dl}{La^2\sec^2\phi}$$

Now,

$$l = a\tan\phi$$

$\Rightarrow$ $$dl = a\sec^2\phi\, d\phi$$

$\therefore$ $$dE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\,d\phi}{La}$$

Net Electric field at P.

$$E = \int dE\cos\phi$$

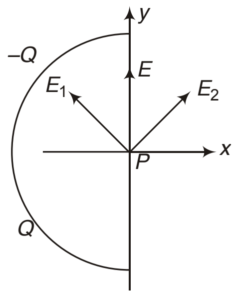
[$dE\sin\phi$ components will cancel each other as rod in symmetrical about P.]

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{La}\int_{-\theta}^{\theta}\cos\phi\, d\phi$$
$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Q\sin\theta}{La}$$

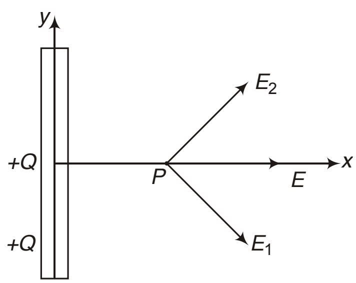
But $$\sin\theta = \frac{L}{2\sqrt{a^2+\left(\frac{L}{2}\right)^2}} = \frac{L}{\sqrt{4a^2+L^2}}$$

$\therefore$ $$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Q}{a\sqrt{4a^2+L^2}}$$

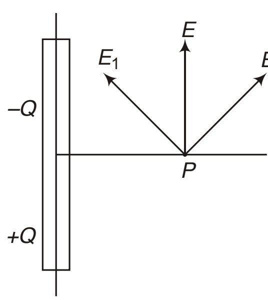
14. (a) As shown in figure, direction of electric field at P will be along + ve y-axis.



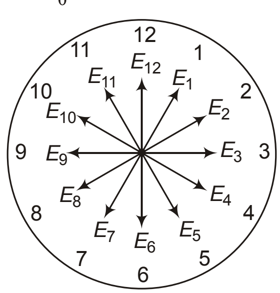
(b) Positive x-axis.



(c) Positive y-axis.

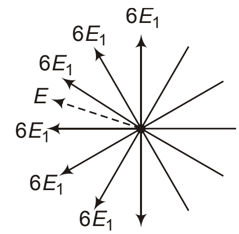


15. Let $E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}$



Resultant fields of two opposite charges can be shown as given in figure.

Clearly resultant field is along angle bisector of field towards 9 and 10.



Hence time shown by clock in the direction of electric field is 9 : 30.

16. (a) $a = \frac{F}{m} = \frac{-eE}{m}$

$$= \frac{-1.6 \times 10^{-19} \times 1 \times 10^3}{9.1 \times 10^{-31}}$$

$$= -1.76 \times 10^{14} \text{ ms}^{-2}$$

$$u = 5.00 \times 10^8 \text{ cm/s} = 5 \times 10^6 \text{ ms}^{-1}$$

$$v = 0$$

$$v^2 - u^2 = 2as$$

$$s = \frac{(5 \times 10^6)^2}{2 \times 1.7 \times 10^{14}} = 1.4 \times 10^{-2} = 1.4 \text{ cm}$$

(b) $v = u + at$

$$t = \frac{5 \times 10^6}{1.76 \times 10^{14}} = 2.8 \times 10^{-8} = 28 \text{ ns.}$$

(c) Δk = work done by electric field.

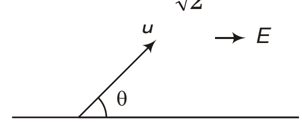
$$= F \cdot x = -eEx$$

$$= -1.6 \times 10^{-19} \times 1 \times 10^3 \times 8 \times 10^{-3}$$

$$= -1.28 \times 10^{-18} \text{ J}$$

Loss of KE $= 1.28 \times 10^{-18}$ J

17. Here, $u_x = u \cos 45° = \frac{25}{\sqrt{2}} \text{ ms}^{-1}$



$$u_y = u \sin 45° = \frac{25}{\sqrt{2}} \text{ ms}^{-1}$$

$$a_x = qE = 2 \times 10^{-6} \times 2 \times 10^7$$

$$= 40 \text{ ms}^{-1}$$

$$a_y = -10 \text{ ms}^{-1}$$

$$y = u_{yt} + \frac{1}{a_y} t^2$$

$$y = \frac{25}{\sqrt{2}} t - 5t^2$$

at the end of motion,

$$t = T \text{ and } y = 0$$

$$\therefore \quad T = \frac{5}{\sqrt{2}} \text{ s}$$

Also at the end of motion,

$$x = R$$

$$\therefore \quad x = u_x t + \frac{1}{2} a_x t^2$$

$$R = \frac{25}{\sqrt{2}} \times \frac{5}{\sqrt{2}} + 20 \times \left(\frac{5}{\sqrt{2}}\right)^2$$

$$= 312.5 \text{ m}$$

18. (a) $$R = \frac{\mu^2 \sin 2\theta}{qE}$$

$$\sin 2\theta = \frac{qER}{mu^2}$$

$$= \frac{1.6 \times 10^{-19} \times 720 \times 1.27 \times 10^{-3}}{1.67 \times 10^{-27} \times (9.55 \times 10^3)^2}$$

$$= 0.96$$

$$2\theta = 88° \text{ or } 92°$$

$$\theta = 44° \text{ or } 46°$$

$$T = \frac{2mh \sin\theta}{2E}$$

$$= \frac{2 \times 9.55 \times 10^3 \times \frac{1}{\sqrt{2}} \times 1.67 \times 10^{-31}}{1.6 \times 10^{-19} \times 720}$$

$$= 1.95 \times 10^{-11} \text{ s}$$

19. (a) $$\vec{\mathbf{a}} = -\frac{e\vec{\mathbf{E}}}{m} = -\frac{1.6 \times 10^{-19} \times 120}{9.1 \times 10^{-31}} \hat{\mathbf{j}}$$

$$= -2.1 \times 10^{13} \hat{\mathbf{i}} \text{ m/s}$$

(b) $$t = \frac{\Delta x}{u_x} = \frac{2 \times 10^{-2}}{1.5 \times 10^5} = \frac{4}{3} \times 10^{-7} \text{ s}$$

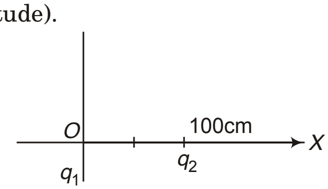
$$v_y = u_y + a_y t$$

$$= 3.0 \times 10^6 \times 2.1 \times 10^{13} \times \frac{4}{3} \times 10^{-7}$$

$$= 0.2 \times 10^6 \text{ m/s}$$

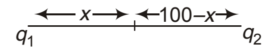
$$\vec{\mathbf{v}} = (1.5 \times 10^5)\hat{\mathbf{i}} + (0.2 \times 10^6)\hat{\mathbf{j}}$$

20. Absolute potential can be zero at two points on the x-axis. One in between the charges and other on the left of charge a_1 (smaller in magnitude).



Case I.

In between two charges : let potential is zero at a distance x from q_1 towards q_2.



$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{x} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{100 - x} = 0$$

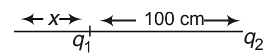
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2 \times 10^{-6}}{x} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{3 \times 10^{-6}}{100 - x} = 0$$

$$\Rightarrow \quad 200 - 2x = 3x$$

$$x = 20 \text{ cm}$$

Case II.

Consider the potential is zero at a distance x from charge q, on its left.



$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{x} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{100 + x} = 0$$

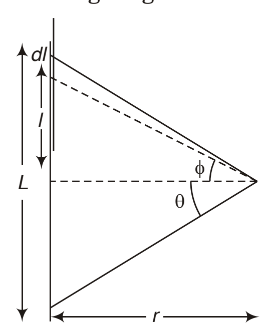
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2 \times 10^{-6}}{x} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{-3 \times 10^{-6}}{100 + x} = 0$$

$$200 + 2x = 3x$$

$$x = 200 \text{ cm}$$

21. Let us first find the potential at a point on the perpendicular bisector of a line charge.

Consider a line of carrying a line charge density λ having length L.



Consider an elementary portion of length dl on the rod.

Charge on this portion

$$dq = \lambda \, dl$$

$$\therefore \quad dV = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\lambda \, dl}{r \sec\phi}$$

Now, $$l = r \tan\phi$$

$$dl = r \sec^2\phi \, d\phi$$

$$\therefore \quad dV = \frac{\lambda \sec\phi\, d\phi}{4\pi\varepsilon_0}$$

$$\therefore \quad V = \int dV = \frac{\lambda}{4\pi\varepsilon_0}\cdot\int_{-\theta}^{\theta} \sec^2\phi\, d\phi$$

$$= \frac{\lambda}{4\pi\varepsilon_0}[\ln|\sec\theta + \tan\theta|]_{-\theta}^{\theta}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left|\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta}\right|\right]$$

$$= \frac{2\lambda}{4\pi\varepsilon_0}\ln|\sec\theta + \tan\theta|$$

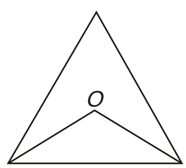
In the given condition

$$\theta = 60°$$

Potential due to one side

$$V_1 = V_2 = V_3 = \frac{2\lambda}{4\pi\varepsilon_0}\cdot\ln|\sec 60° + \tan 60°|$$

$$= \frac{2\lambda}{4\pi\varepsilon_0}\cdot\ln|2+\sqrt{3}|$$



Total potential at O

$$V = 3V_1 = \frac{6\lambda}{4\pi\varepsilon_0}\cdot\ln|2+\sqrt{3}|$$

$$= \frac{Q}{2\pi\varepsilon_0\, a}\cdot\ln|2+\sqrt{3}|$$

22. (a) $V_2 - V_1 = -\vec{\mathbf{E}}\cdot\vec{\mathbf{d}} = -250\times 20\times 10^{-2}$

$$= -50\text{ V}$$

$$W = \Delta V = q(V_2 - V_1)$$

$$= 12\times 10^{-6}\times -50 = -0.6\text{ mJ}$$

(b) $V_2 - V_1 = -50$ V

23. By work energy theorem

$$W = \Delta K$$

$$q(V_1 - V_2) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$-5\times 10^{-6}(20 - 800)$$

$$= \frac{1}{2}\times 2\times 10^{-4}(V_2^2 - (5)^2)$$

$$v_2^2 = 55$$

$$v_2 = \sqrt{55} = 7.42\text{ ms}^{-1}$$

When a particle is released in electric field it moves in such a way that, it decreases its PE and increases KE

Hence, particle at B is faster than that at A.

24. Centre of circle is equidistant from every point on its periphery,

Hence, $V_0 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$,

where $q = Q_1 + Q_2 = -SQ$

$$\therefore \quad V_0 = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{5Q}{R}$$

Similarly, $V_p = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\sqrt{R^2+Z^2}}$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot\frac{SQ}{\sqrt{R^2+Z^2}}$$

25. Initial PE

$$U_i = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r_1}$$

$$U_f = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r_2}$$

Work done by electric force

$$W = -\Delta U = -(U_f - U_i)$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot q_1q_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$\Rightarrow W = -9\times 10^9\times 2.4\times 10^{-6}\times(-4.3\times 10^{-6})\left(\frac{1}{0.25\sqrt{2}} - \frac{1}{0.15}\right)$$

$$W = -0.356\text{ mJ}$$

26. (a) $U = \frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_1}{r_{31}}\right]$

$$= 9\times 10^9\left[\frac{4\times 10^{-9}\times(-3\times 10^{-9})}{0.2} + \frac{(-3\times 10^{-9})\times(2\times 10^{-9})}{0.1} + \frac{4\times 10^{-9}\times 2\times 10^{-9}}{0.1}\right]$$

$$U = 9\times 10^{-8}[-6 - 6 + 8] = -360\text{ nJ}$$

(b) Let the distance of q_3 from q_1 is x cm. Then

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{q_1q_2}{0.2} + \frac{q_2q_3}{0.2 - x} + \frac{q_3q_1}{x}\right] = 0$$

$$\Rightarrow 9\times 10^9\left[\frac{4\times 10^{-9}\times(-3\times 10^{-9})}{20\times 10^{0-2}} + \frac{(-3\times 10^{-9})\times 2\times 10^{-9}}{(20 - x)\times 10^{-2}}\right.$$

$$+\frac{2\times10^{-9}\times4\times10^{-9}}{x\times10^{-2}}\Bigg] = 0$$

$$\Rightarrow \quad -\frac{6}{10}-\frac{6}{20-x}+\frac{8}{x} = 0$$

$$\Rightarrow \quad x = 6.43 \text{ cm}$$

27. Let Q be the third charge

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{q^2}{d}+\frac{qQ}{d}+\frac{qQ}{d}\right] = 0$$

$$Q = -\frac{q}{2}$$

28. $V = -\vec{\mathbf{E}}\cdot\vec{\mathbf{r}}$

(a) $\vec{\mathbf{r}} = 5\,\hat{\mathbf{k}}$

$$V = -(5\,\hat{\mathbf{i}}-3\,\hat{\mathbf{j}})-(5\,\hat{\mathbf{k}}) = 0$$

(b) $\vec{\mathbf{r}} = 4\,\hat{\mathbf{i}}+3\,\hat{\mathbf{k}}$

$$V = -(5\,\hat{\mathbf{i}}-3\,\hat{\mathbf{j}})-(4\,\hat{\mathbf{i}}+3\,\hat{\mathbf{j}})$$

$$= -20 \text{ kV}$$

29. $\vec{\mathbf{E}} = 400\,\hat{\mathbf{j}}$ V/m

(a) $\vec{\mathbf{r}} = 20\,\hat{\mathbf{j}}$ cm $= (0.2\,\hat{\mathbf{j}})$ m

$$V = -\vec{\mathbf{E}}\cdot\vec{\mathbf{r}} = -80 \text{ V}$$

(b) $\vec{\mathbf{r}} = (-0.3\,\hat{\mathbf{j}})$ m

$$V = -\vec{\mathbf{E}}\cdot\vec{\mathbf{r}} = 120 \text{ V}$$

(c) $\vec{\mathbf{r}} = (0.15\,\hat{\mathbf{k}})$

$$V = 0$$

30. $\vec{\mathbf{E}} = 20\,\hat{\mathbf{i}}$ N/C

(a) $\vec{\mathbf{r}} = (4\,\hat{\mathbf{i}}+2\,\hat{\mathbf{j}})$ m

$$V = -\vec{\mathbf{E}}\cdot\vec{\mathbf{r}} = -80 \text{ V}$$

(b) $\vec{\mathbf{r}} = (2\,\hat{\mathbf{i}}+3\,\hat{\mathbf{j}})$ m

$$V = -\vec{\mathbf{E}}\cdot\vec{\mathbf{r}} = -40 \text{ V}$$

31. (a) $[A] = \dfrac{[V]}{[xy+yz+zx]} = \dfrac{[\mathrm{ML^2T^{-3}I^{-1}}]}{[\mathrm{L^2}]}$

$$= [\mathrm{ML^0T^{-3}I^{-1}}]$$

(b) $E = -\vec{\nabla}V = -\left(\dfrac{\partial v}{\partial x}\hat{\mathbf{i}}+\dfrac{\partial v}{\partial y}\hat{\mathbf{j}}+\dfrac{\partial v}{\partial z}\hat{\mathbf{k}}\right)$

$$= -A\,[(y+z)\,\hat{\mathbf{i}}+(z+x)\,\hat{\mathbf{j}}+(x+y)\,\hat{\mathbf{k}}]$$

(c) at (1m, 1m, 1m)

$$E = -10(2\,\hat{\mathbf{i}}+2\,\hat{\mathbf{j}}+2\,\hat{\mathbf{k}})$$

$$= -20(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$$

32. $V_B - V_0 = -\vec{\mathbf{E}}\cdot\vec{\mathbf{r}}$

$$\Rightarrow \quad V - 0 = -(40+60)$$

$$\Rightarrow \quad V = -100$$

33. (a) $E_x = -\dfrac{\partial v}{\partial x} = -(Ay-2Bx)$

$$E_y - \frac{\partial V}{\partial y} = -(Ax+C)$$

$$E_z = -\frac{\partial V}{\partial Z} = 0$$

(b) For $E = 0$

$$E_x = 0 \text{ and } E_y = 0$$

Hence, $\quad E_y = 0$

$$Ax + C = 0$$

$$x = -\frac{C}{A}$$

$E_x = 0$

$$Ay - 2B\left(-\frac{C}{A}\right) = 0$$

$$y = -\frac{2BC}{A^2}$$

Hence, E is zero at $\left(-\dfrac{C}{A}, -\dfrac{2BC}{A^2}\right)$.

34. $\phi = \dfrac{q}{\varepsilon_0}$

$$q = \varepsilon_0\phi = 8.8\times10^{-12}\times360$$

$$= 3.18\times10^{-9} \text{ C}$$

$$= 3.186 \text{ nC}$$

36. (a) $\phi = \dfrac{q}{\varepsilon_0} = -\dfrac{3.60\times10^{-6}}{8.85\times10^{-12}}$

$$= 4.07\times10^{5} \text{ V-m.}$$

(b) $\phi = \dfrac{q}{\varepsilon_0} \Rightarrow q = \varepsilon_0\phi$

$$= 8.85\times10^{-12}\times780 = 6.903\times10^{-9}$$

$q = 6.903$ nC

(c) No.
Net flux through a closed surface does not depend on position of charge.

36. $\vec{\mathbf{E}} = \left(\dfrac{3}{5}E_0\,\hat{\mathbf{i}}+\dfrac{4}{5}E_0\,\hat{\mathbf{j}}\right)$

$$\vec{\mathbf{S}} = 0.2\,\hat{\mathbf{j}}\,\mathrm{m}^2 = \frac{1}{5}\hat{\mathbf{j}}\,\mathrm{m}^2$$

$$\therefore \quad \phi = \vec{\mathbf{E}}\cdot\vec{\mathbf{S}} = \frac{4}{25}\,\mathrm{Nm^2/C}$$

$$= \frac{4}{25} \times 2.0 \times 10^3 \text{ N-m}^2/\text{C}$$
$$= 320 \text{ N-m}^2/\text{C}$$

37. $\vec{\mathbf{E}} = \frac{E_0 x}{l} \hat{\mathbf{i}}\; x_1 = 0$

$$\vec{\mathbf{E}}_1 = 0$$
$$x_2 = a$$
$$\vec{\mathbf{E}}_2 = \frac{E_0 a}{l} \hat{\mathbf{i}}$$

Flux entering the surface

$$\phi_1 = 0$$

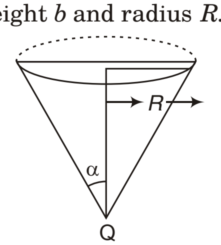
Flux leaving the surface

$$\phi_2 = E_2 a^2 = \frac{E_0 a^3}{l}$$
$$= \frac{5 \times 10^3 \times (1 \times 10^{-2})^3}{2 \times 10^{-2}}$$
$$= 0.25 \text{ N-m}^2/\text{C}$$

Net flux, $\phi = \frac{q}{\varepsilon_0}$

$$\phi_2 - \phi_1 = \frac{q}{\varepsilon_0}$$
$$q = \varepsilon_0 (\phi_2 - \phi_1)$$
$$= 8.85 \times 10^{-12} \times 0.25$$
$$= 2.21 \times 10^{-12} \text{ C} = 2.21 \text{ pC}$$

38. Consider the charge is placed at vertex of the cone of height b and radius R.



Let α be the semi-vertical angle of the cone, then solid angle subtended by the cone.

$$\Omega = 2\pi(1 - \cos\alpha)$$

Flux passing through cone

$$\phi = \frac{\Omega}{4\pi} \cdot \phi_{\text{total}}$$

But $\qquad \phi = \frac{1}{4} \phi_{\text{total}} \qquad$ (Given)

$$\Omega = \pi$$
$$2\pi(1 - \cos\alpha) = \pi$$
$$1 - \cos\alpha = \frac{1}{2}$$
$$\cos\alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$$
$$R = b \tan\alpha = \sqrt{3}\, b$$

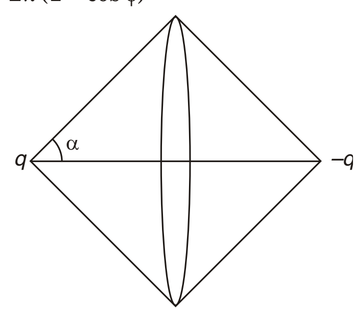
Hence proved

39. $\vec{\mathbf{E}} = -B\hat{\mathbf{i}} + C\hat{\mathbf{j}} - D\hat{\mathbf{k}},\; \vec{\mathbf{S}}_1 = -L^2\hat{\mathbf{i}},\; \vec{\mathbf{S}}_2 = -L^2\hat{\mathbf{j}},$
$\vec{\mathbf{S}}_3 = -L^2\hat{\mathbf{i}}, \quad \vec{\mathbf{S}}_4 = -L^2\hat{\mathbf{j}}, \quad \vec{\mathbf{S}}_4 = -L^2\hat{\mathbf{k}},$
$\vec{\mathbf{S}}_6 = -L^2\hat{\mathbf{k}}$

$$\therefore \quad \phi_1 = \vec{\mathbf{E}} \cdot \vec{\mathbf{S}}_1 = BL^2,\; \phi_2 = \vec{\mathbf{E}} \cdot \vec{\mathbf{S}}_2 = CL^2,$$
$$\phi_3 = \vec{\mathbf{E}} \cdot \vec{\mathbf{S}}_3 = -BL^2,\; \phi_4 = \vec{\mathbf{E}} \cdot \vec{\mathbf{S}}_4 = -CL^2,$$
$$\phi_5 = \vec{\mathbf{E}} \cdot \vec{\mathbf{S}}_5 = -DL^2,\; \phi_6 = \vec{\mathbf{E}} \cdot \vec{\mathbf{S}}_6 = DL^2$$

(b) $\phi = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6 = 0$

40. $\Omega = 2\pi(1 - \cos\phi)$

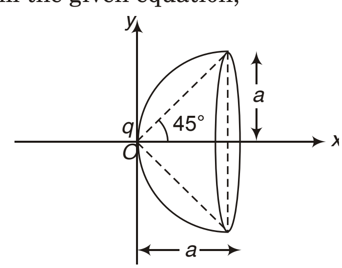


$$\phi_1 = \phi_2 = \frac{\Omega}{4\pi} \cot \phi_{\text{total}} = \frac{2\pi(1 - \cos\alpha)}{4\pi} \cdot \frac{q}{\varepsilon_0}$$
$$= \frac{1}{2\varepsilon_0}(1 - \cos\alpha)$$

Total flux through the ring

$$\phi = \phi_1 + \phi_2$$
$$= \frac{q}{\varepsilon_0}(1 - \cos\alpha) = \frac{q}{\varepsilon_0}\left(1 - \frac{l}{\sqrt{R^2 + l^2}}\right)$$

41. From the given equation,



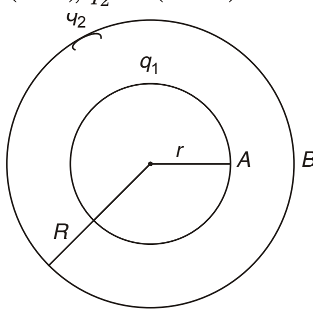
radius of hemisphere $= a$
and its centre is at $(a, 0, 0)$

$$\Omega = 2\pi(1-\cos\alpha) = 2\pi\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\phi = \frac{\Omega}{4\pi}\phi_{total} = \frac{2\pi\left(1-\frac{1}{\sqrt{2}}\right)}{4\pi}\cdot\frac{q}{\varepsilon_0}$$

$$\phi = \frac{q}{2\,\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$$

42. $q_1 = \sigma(4\pi r^2),\ q_2 = \sigma(4\pi R^2)$



But, $q_1 + q_2 = Q \Rightarrow \sigma = \dfrac{Q}{4\pi(r^2+R^2)}$

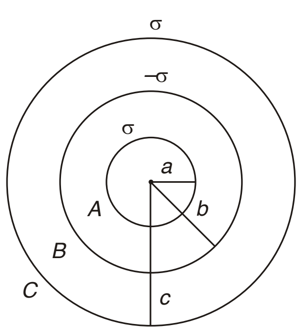
$$V_A = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_1}{r}+\frac{q_2}{R}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{\sigma(4\pi r^2)}{r}+\frac{\sigma(4\pi R^2)}{R}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q(r+R)}{r^2+R^2}$$

$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_1+q_2}{R}\right) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$$

43. $q_A = \sigma(4\pi a^2),\ q_B = -\sigma(4\pi b^2)$



$$q_C = \sigma(4\pi c^2)$$

$$V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A}{a}+\frac{q_B}{b}+\frac{q_C}{c}\right)$$

$$= \frac{\sigma}{\varepsilon_0}(a+b+c)$$

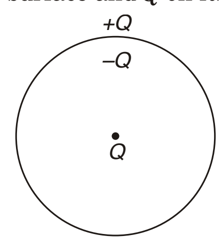
$$V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A+q_B}{b}+\frac{q_C}{c}\right)$$

$$= \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{b}-b+c\right)$$

$$V_C = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_A+q_B+q_C}{c}\right)$$

$$= \frac{\sigma}{\varepsilon_0}\left(\frac{a^2-b^2+c^2}{c}\right)$$

44. (a) As charge Q is placed at the centre of the sphere, charge $-Q$ will appear on the inner surface and Q on its outer surface.



Hence, $\sigma_{in} = \dfrac{-Q}{4\pi a^2}$

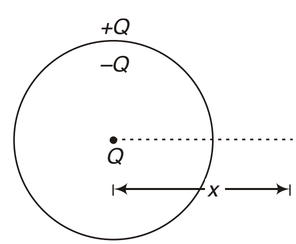
and $\sigma_{out} = \dfrac{Q}{4\pi a^2}$

(b) Entire charge inside the sphere appears on its outer surface, hence

$$\sigma_{in} = -\frac{Q}{4\pi a^2} \text{ and } \sigma_{out} = \frac{Q+q}{4\pi a^2}$$

(c) In case (a)

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{x^2}$$



In case (b)

$E = E_1 + E_2$

E_1 = Field due to charge Q.

E_2 = Field due to charge on shell.

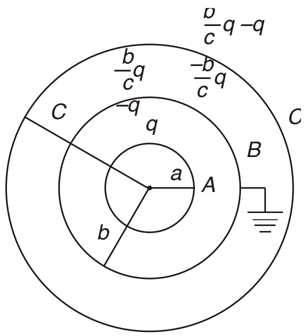
$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{x^2}$$

for $x < a$

As field due to shell is zero for $x < a$.

and $\quad E = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{Q+q}{x^2}$, for $x > a$

45. Let Q be the charge on the shell B,

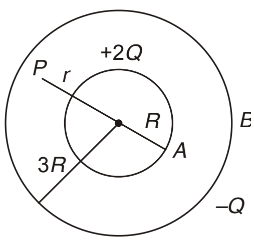


$$V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q+Q}{b} + \frac{-q}{c}\right] = 0$$

$$Q = q\left(\frac{b-c}{c}\right)$$

Charge distribution on different surfaces is shown in figure.

46. (a) Let E_1 and E_2 be the electric field at P due to inner shell and outer shell respectively.



Now, $E_1 = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{2Q}{r}$ and $E_2 = 0$

$$\therefore \quad E = E_1 + E_2 = E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{r}$$

(b) $V_A = \dfrac{1}{4\pi\varepsilon_0} \cdot \left[\dfrac{2Q}{R} - \dfrac{Q}{3R}\right]$

$$V_B = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{2Q-Q}{3R}\right]$$

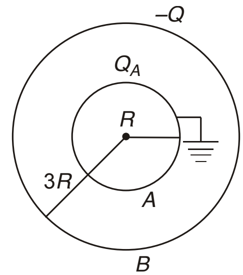
$$V_A - V_B = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{2Q}{R} - \frac{2Q}{3R}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{4Q}{3R}$$

(c) Whenever two concentric conducting spheres are joined by a conducting wire entire charge flows to the outer sphere.

$$\therefore \quad Q_A = 0, Q_B = 0$$

(d) Let Q_A be the charge on inner sphere.



$$V_A = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{Q_A}{R} - \frac{Q}{3R}\right] = 0$$

$$Q_A = \frac{Q}{3}$$

47. (a) At $r = R$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{Q}{R} + \frac{-2Q}{2R} + \frac{3Q}{3R}\right]$$

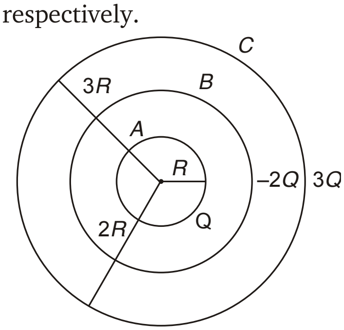
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R}$$

At $r = 3R$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{Q - 2Q + 3Q}{3R}\right]$$

$$= \frac{1}{4\pi\varepsilon_9} \cdot \frac{2Q}{3R}$$

(b) Let E_1, E_2 and E_3 be the electric fields at $r = \dfrac{5}{2}R$ due to shells A, B and C respectively.



$$\therefore \quad E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{\left(\frac{5}{2}R\right)^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{4Q}{25R} \quad \text{(outwards)}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{\left(\frac{5}{2}R\right)^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{8Q}{25R} \quad \text{(inward)}$$

$$E_3 = 0$$

Net field at $r = \frac{5}{2}R$

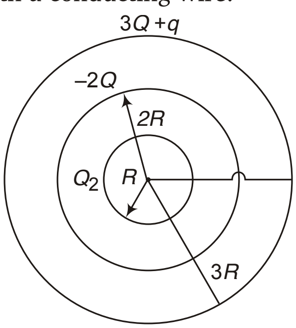
$$E = E_2 + E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4Q}{25R} \quad \text{(inward)}$$

(c) Total electrostatic energy of system is the sum of self-energy of three shell and the energy of all possible pairs *i.e.*,

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{Q^2}{2R} + \frac{(-2Q)^2}{2 \times 2R} + \frac{(3Q)^2}{2 \times 3R} + \frac{Q(-2Q)}{2R} + \frac{(-2Q) \times 3Q}{3R} + \frac{Q \times 3Q}{3R} \right]$$

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R}$$

(d) Let q charge flows from innermost shell to outermost shell on connecting them with a conducting wire.



$$V_A = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{Q - q}{R} + \frac{-2Q}{2R} + \frac{3Q + q}{3R} \right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{3Q - 2q}{3R}$$

$$V_B = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{Q - q - 2Q + 3Q + q}{3R} \right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{3R}$$

But $V_A = V_B$

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{3Q - 2q}{3R} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{3R}$$

$$\Rightarrow \quad q = \frac{Q}{2}$$

$\therefore$ Charge on innermost shell $= Q - q = \frac{Q}{2}$

and charge on outermost shell $= 3Q + q = \frac{7Q}{2}$

and
$$V_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{3Q - 2q}{3R}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{3R}$$

(c) In this case

$$E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{2R\left(\frac{5}{2}R\right)^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{25R} \quad \text{(outward)}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Q}{\left(\frac{5}{2}R\right)^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{8Q}{25R} \quad \text{(inward)}$$

Net electric field at $r = \frac{5}{2}R$

$$E = E_2 - E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{6Q}{25R} \quad \text{(inward)}$$

Objective Questions (Level-1)

1. $\phi = E \cdot A$

Units of $\phi = \text{N/C} \times \text{m}^2 = \text{N-m}^2/\text{C}$

or $\text{V/m} \times \text{m}^2 = \text{V-m}$

2. Net force

$$F' = mg - qE$$

$$g' = g - \frac{qE}{m}$$

$$T' = 2\pi\sqrt{\frac{l}{g_1}} > T$$

3. Electric lines of force terminate at negative charge.

4. $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{l^2}$

Initial PE

$$U_i = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q^2}{l} + \frac{q^2}{l} + \frac{q^2}{l} \right) = 3Fl$$

Find PE

$$U_f = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q^2}{2l}+\frac{q^2}{2l}+\frac{q^2}{2l}\right) = \frac{3}{2}Fl$$

$$W = U_f - U_i = -\frac{3}{2}Fl$$

5. KE $= qV$

$$\frac{1}{2}mv^2 = qV$$

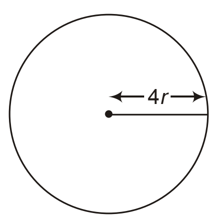
$$v = \sqrt{\frac{2qV}{m}}$$

$$\Rightarrow \quad V_1 : V_2 : V_3 = \sqrt{\frac{q_1V_1}{m_1}} : \sqrt{\frac{q_2V_2}{m_2}} : \sqrt{\frac{q_3V_3}{m_3}}$$

$$\Rightarrow \quad V_1 : V_2 : V_3 = \sqrt{\frac{e\times 1}{m}} : \sqrt{\frac{e\times 2}{2m}} : \sqrt{\frac{2e\times 4}{4m}}$$

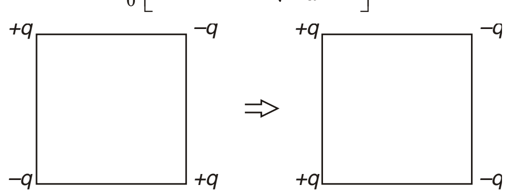
$$V_1 : V : V_3 = 1 : 1 : \sqrt{2}$$

6. $V = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{r}$



$$V' = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2q}{4r} = \frac{V}{2}$$

7. $U_i = \dfrac{1}{4\pi\varepsilon_0}\left[-\dfrac{q^2}{a}\times 4 + \dfrac{q^2}{\sqrt{2}\,a}\times 2\right]$



$$= \frac{-q^2}{4\pi\varepsilon_0\,a}[4-\sqrt{2}]$$

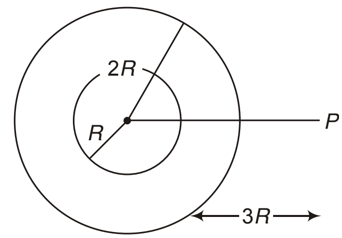
$$U_f = \frac{1}{4\pi\varepsilon_0}\cdot\left[-\frac{q^2}{a}\times 2 + \frac{q^2}{a}\times 2 - \frac{q^2}{\sqrt{2}}\times 2\right]$$

$$= -\frac{\sqrt{2}\,q^2}{4\pi\varepsilon_0\,a}$$

$$W = U_f - U_i = -\frac{\sqrt{2}\,q^2}{4\pi\varepsilon_0\,a} + \frac{q^2}{4\pi\varepsilon_0\,a}[4-\sqrt{2}]$$

$$= \frac{q^2}{4\pi\varepsilon_0 a}[4-2\sqrt{2}]\,\text{J}$$

8. Potential at point P



$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1+q_2}{3R}$$

$$\Rightarrow \quad \frac{9\times 10^9\times 3\times 10^{-6}}{3R} = 9000$$

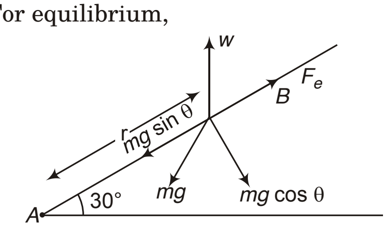
$$R = 1\text{ m}$$

9. As distance of every point of ring from axis is same.

$$V = \frac{kq}{\sqrt{R^2+x^2}},\ \text{But } x = 2\sqrt{R}$$

$$= \frac{kq}{3R}$$

10. For equilibrium,



$$mg\sin 30^\circ = F_e$$

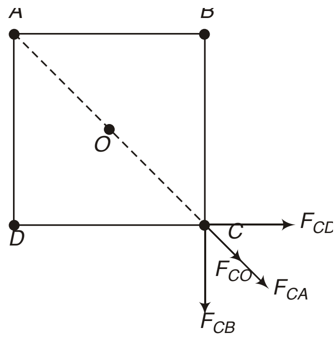
$$mg\sin 30^\circ = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r^2}$$

$$r = q\sqrt{\frac{1}{4\pi\varepsilon_0\,mg\sin 30^\circ}}$$

$$= 2.0\times 10^{-6}\sqrt{\frac{91\times 10^9}{0.1\times 10\times\frac{1}{2}}}$$

$$\approx 20\text{ cm}$$

11. Net force on $C = 0$



$$F_{CB} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(2\sqrt{2}-1)^2 Q^2}{a^2}$$

$$F_{CD} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(2\sqrt{2}-1)^2 Q^2}{a^2}$$

$$F_{CA} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(2\sqrt{2}-1)^2 Q^2}{2a^2}$$

$$F_{CO} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2(2\sqrt{2}-1)Q^2}{a^2}$$

Net force on C

$$F = F_{CA} + F_{CO} + F_{CB}\cos 45^\circ + F_{CO}\cos 45^\circ$$

$$= \frac{(2\sqrt{2}-1)Q}{a^2}\left[\frac{(2\sqrt{2}-1)Q}{2} + \frac{(2\sqrt{2}-1)Q}{\sqrt{2}} + \frac{(2\sqrt{2}-1)Q}{\sqrt{2}} + \frac{2q}{1}\right] = 0$$

$$q = -\frac{7Q}{4}$$

12. $E = \dfrac{\sigma}{\varepsilon_0}$

$$F = eF = \frac{e\sigma}{\varepsilon_0}$$

Acceleration of proton

$$a = \frac{F}{m} = \frac{\sigma e}{m\varepsilon_0}$$

$$s = ut + \frac{1}{a}t^2$$

$$u = 0$$

$$t = \sqrt{\frac{25}{a}} = \sqrt{\frac{25\, m\varepsilon_0}{\sigma e}}$$

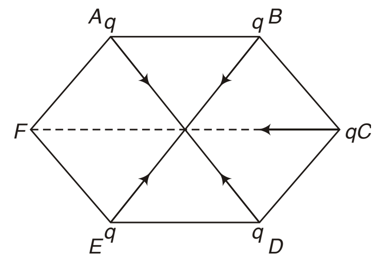
$$= \sqrt{\frac{2 \times 0.1 \times 1.67 \times 10^{-27} \times 8.8 \times 10^{-12}}{2.21 \times 10^{-9} \times 1.6 \times 10^{-19}}}$$

$$= 2\sqrt{2}\ \mu\text{s}$$

13. Data is not sufficient.

14. If the charges have opposite sign, electric field is zero on the left of smaller charge.

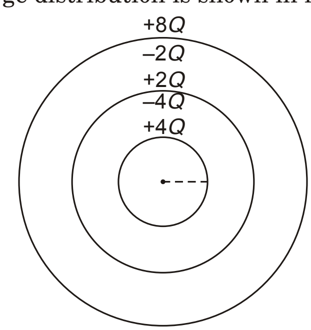
15. Net field is only due to charge on C.



$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(2a)^2} = \frac{q}{16\pi\varepsilon_0 a^2}$$

16. On touching two spheres, equal charge will appear on both the spheres and for a given total charge, force between two spheres is maximum if charges on them are equal.

17. Charge distribution is shown in figure.



18. $V = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{r}$

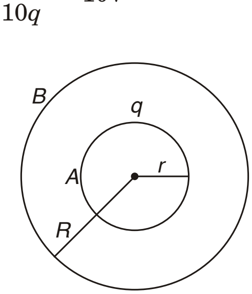
If drops coalesce, total volume remains conserved,

$$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$$

$R = 10r$

$$V' = \frac{1}{4\pi\varepsilon_0} \cdot \frac{1000q}{10q} = 10\text{V}$$

19. $V_A = \dfrac{1}{4\pi\varepsilon_0} \cdot \left[\dfrac{q}{r} + \dfrac{Q}{R}\right]$



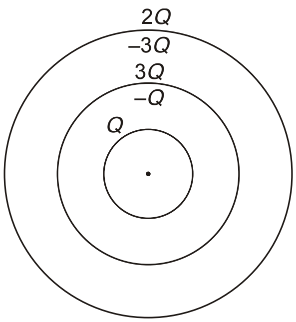
$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{q+Q}{R}\right]$$

$$V_A - V_B = \frac{q}{4\pi\varepsilon_0}\cdot\left[\frac{1}{r}-\frac{1}{R}\right]$$

$\therefore \qquad V_A - V_B \propto q$

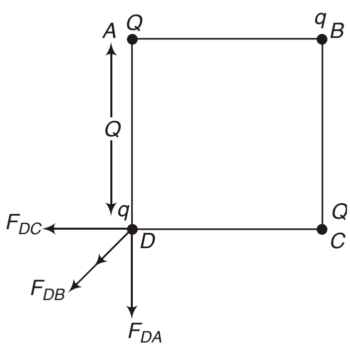
If q is doubled, $V_A - V_B$ will become double.

20. Charge distribution is shown in figure.



21. $\phi = \vec{\mathbf{E}}\cdot\vec{\mathbf{S}} = (5\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}})\cdot(\hat{\mathbf{i}}) = 5$ V-m.

22. $F_{DA} = F_{DC} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{a^2}$



$$F_{DB} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{2a^2}$$

Net force on charge at D

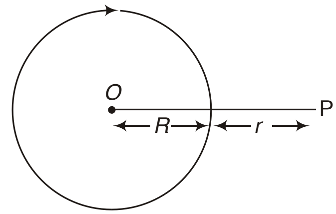
$$\vec{\mathbf{F}}_0 = F_{DB} + F_{DA}\cos 45° + F_{DB}\cos 45° = 0$$

$$\Rightarrow \qquad \frac{1}{4}4\pi\varepsilon_0\cdot\frac{q}{a^2}\left[\frac{q}{2}+\frac{Q}{\sqrt{2}}+\frac{Q}{\sqrt{2}}\right] = 0$$

$$q = -2\sqrt{2}\,Q$$

23. As $V_B = 0$, Total charge inside B must be zero and hence charge on its outer surface is zero and on its inner surface is $-q$.

24. $V_p = \frac{1}{V_0}$



$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R+r} = \frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{3q}{2R}$$

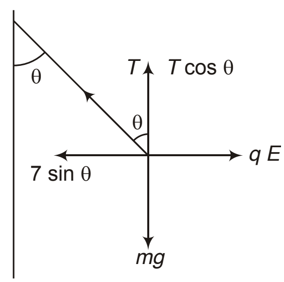
$$\frac{1}{R+r} = \frac{3°}{4R}$$

$$4R = 3R + 3r$$

$$r = \frac{R}{3}$$

25. Net charge on any dipole is zero.

26. For net force to be zero.



$$T\cos\theta = mg \Rightarrow T = \frac{mg}{\cos\theta}$$

$$\text{or } T\sin\theta = qE \Rightarrow T = \frac{qE}{\sin\theta}$$

27. $E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a} = \frac{V_1}{a}$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{b^2} = \frac{V_2}{b}$$

But $\qquad E_1 = E_2$

$$\frac{V_1}{a} = \frac{V_2}{b}$$

$$\Rightarrow \qquad \frac{V_1}{V_2} = \frac{a}{b}$$

28. Electric field on equatorial lines of dipole is opposite to dipole moment.

29. Potential difference between two concentric spheres is independent of charge on outer sphere.

30. $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = Er$$

$$r = \left|\frac{V}{E}\right| = \frac{3000}{500} = 6 \text{ m}$$

$$q = 4\pi\varepsilon_0 rV = \frac{6 \times (-3000)}{9 \times 10^9} = -2\,\mu\text{C}$$

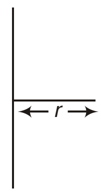
31. $F_1 = F_2$

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_1^2} = \frac{1}{4\pi K\varepsilon_0} \cdot \frac{q_1 q_2}{r_2^2}$$

$$r_2 = \frac{r_1}{\sqrt{K}} = \frac{50}{\sqrt{5}} = 10\sqrt{5} \text{ m}$$

$$\approx 22.3 \text{ m}$$

32. Electric field at a distance r from infinite line charge



$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$dV = -E\,dr$$

$$\int_{V_1}^{V_2} dV = -\int_a^b E\,dr$$

$$\Rightarrow \quad V_2 - V_1 = \frac{\lambda}{2\pi\varepsilon_0} \cdot \ln\frac{1}{2}$$

$$W = q(V_2 - V_1) = \frac{q\lambda}{2\pi\varepsilon_0} \ln\frac{1}{2}$$

33. As negative charge is at less distance from the line charge, it is attracted towards the line charge.

34. $r = \sqrt{(4-1)^2 + (2-2)^2 + (0-4)^2} = 5 \text{ m}$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = \frac{9 \times 10^9 \times 2 \times 10^{-8}}{5} = 36 \text{ V}$$

(b) and (c) are wrong.

35. $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$

At a distance r from the centre,

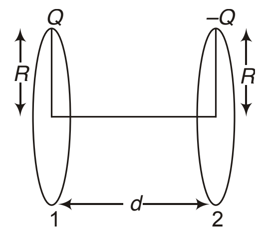
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} = \frac{VR}{r^2}$$

36. When outer sphere is earthed field between the region of two spheres in non-zero and is zero in all other regions.

37. $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{s}} = qEs\cos\theta$

$$E = \frac{W}{qs\cos\theta} = \frac{4}{0.2 \times 2 \times \cos 60^\circ} = 20 \text{ N/C}$$

38. $V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{Q}{R} - \frac{Q}{\sqrt{d^2 + R^2}}\right]$



$$V_2 = \frac{1}{4\pi\varepsilon_0}\left[-\frac{Q}{R} + \frac{Q}{\sqrt{d^2 + R^2}}\right]$$

$$V_1 - V_2 = \frac{1}{4\pi\varepsilon_0}\left[-\frac{2Q}{R} - \frac{2Q}{\sqrt{d^2 + R^2}}\right]$$

$$V_1 - V_2 = \frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{R} + \frac{1}{\sqrt{d^2 + R^2}}\right]$$

39. Electric field inside a hollow sphere is always zero.

40. $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{r}} = q\,\vec{\mathbf{E}} \cdot \vec{\mathbf{r}}$

$$= q(E_1\,\hat{\mathbf{i}} + E_2\,\hat{\mathbf{j}}) - (a\,\hat{\mathbf{i}} + b\,\hat{\mathbf{j}})$$

$$= q(aE_1 + bE_2)$$

JEE Corner

Assertion and Reason

1. Negative charge always moved towards increasing potential.

On moving from A to B potential energy of negative charge decreases hence its KE increases.

2. $U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$

If q_1 and q_2 have opposite sign, U decreases with decrease in r.

$F = -\frac{dU}{dr} \Rightarrow$ work done by conservative force always decreases PE.

3. $E = -\frac{dV}{dr} = -(10) = 10$ V/m along x-axis.

4. $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$

Inside the solid sphere.

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qr}{R^3}$$

at $r = \frac{R}{2}$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{2R^2} = \frac{V}{2R}$$

Assertion is correct.

Reason is false as electric field inside the sphere is directly proportional to distance from centre but not outside it.

5. Gauss theorem is valid only for closed surface but electric flux can be obtained for any surface.

6. Let V_0 = Potential at origin,

$$V_A = -(4\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \cdot (4\hat{\mathbf{i}}) = -16\text{ V}$$

$$V_B = -(4\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \cdot (4\hat{\mathbf{i}}) = -16\text{ V}$$

$$V_A = V_B$$

Hence, Assertion is false.

7. In the line going A and B, the energy of third charge is minimum at centre.

8. Dipole has both negative and positive charges hence work done is not positive.

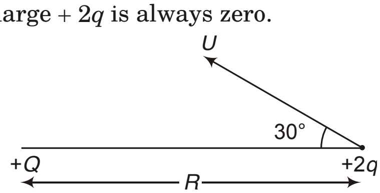
9. Charge outside a closed surface can produce electric field but cannot produce flux.

10. $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qx}{(x^2 + a^2)^{3/2}}$ is maximum at $x = \frac{a}{\sqrt{2}}$

But $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\sqrt{a^2 + x^2}}$ is maximum at $x = 0$.

Objective Questions (Level-2)

1. Electrostatic force always acts along the line joining the two charges, hence net torque on charge $+2q$ is always zero.

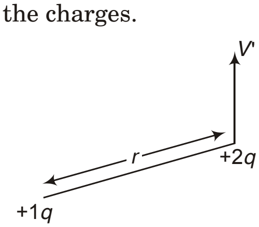


As net torque is zero angular momentum of charge remains conserved.

Initial angular momentum

$$L_i = m(V \sin 30°)R$$

When the separation between the charges become minimum, direction of motion of charge $+2q$ become perpendicular to the line joining the charges.



∴ find angular momentum

$$L_f = mv'r = \frac{mvr}{\sqrt{2}}$$

By conservation of angular momentum

$$L_i = L_f \Rightarrow r = \frac{\sqrt{3}}{2} R$$

2. $\vec{\mathbf{v}}_1 = v\hat{\mathbf{j}}, \vec{\mathbf{v}}_2 = 2v\cos 30°\,\hat{\mathbf{i}} + 2v\sin 30°\,\hat{\mathbf{j}}$

$$= \sqrt{3}\,\hat{\mathbf{i}} + v\hat{\mathbf{j}}$$

As velocity along y-axis is unchanged, electric field along x-axis is zero.

For motion along x-axis,

$$v_x^2 - u_x^2 = 2a_x(x - x_0)$$

$$a_x = \frac{(\sqrt{3}v)^2 - 0}{2a} = \frac{3v^2}{2a}$$

$$F_x = ma_x = \frac{3mv^2}{2a}$$

$$\Rightarrow \quad \vec{\mathbf{F}} = \frac{3mv^2}{2a}\hat{\mathbf{i}}$$

Also, $\vec{\mathbf{F}} = -e\vec{\mathbf{E}}$

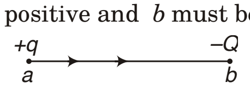
$$\vec{\mathbf{E}} = -\frac{3mv^2}{2ea}\hat{\mathbf{i}}$$

Rate of work done by electric field at B

$$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} = \left(\frac{3mv}{2a}\hat{\mathbf{i}}\right) \cdot (\sqrt{3}v\,\hat{\mathbf{i}} + v\hat{\mathbf{j}})$$

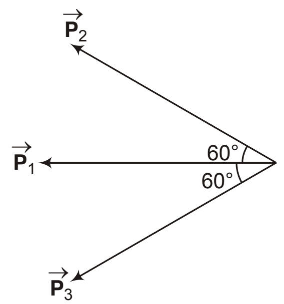
$$= \frac{3\sqrt{3}\,mv^3}{2a}$$

3. Electric field is always possible, hence a must be positive and b must be negative.



4. The system can be assumed as a combination of three identical dipoles as shown in figure.

Here, $P_1 = P_2 = P_3 = Q(2a)$



Net dipole moment of the system

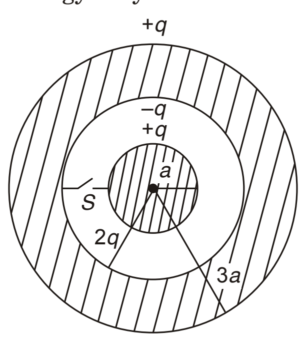
$$P = P_1 + P_2 \cos 60^\circ + P_3 \cos 60^\circ$$
$$= 2p = 4Qa$$

Electric field on equatorial lines of short dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{P}{x^3}$$
$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{4Qa}{x^3} = \frac{Q\,a}{\pi\varepsilon_0 x^3}$$

5. Potential at centre will be same as potential at the surface of inner shell *i.e.*, 10 V.

6. Initial charge distribution is shown in figure, Initial energy of system



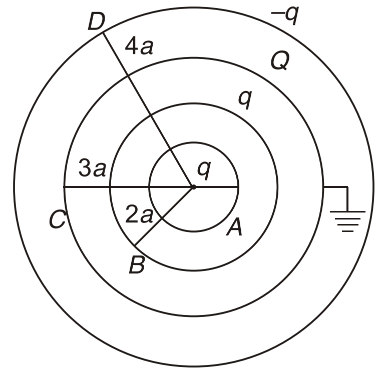
$$U_i = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{q^2}{2a} + \frac{(-q)^2}{2 \times 2a} + \frac{q^2}{2 \times 3a} + \frac{q(-q)}{2a} + \frac{q(q)}{3a} + \frac{q(-q)}{3a} \right]$$
$$= \frac{5a^2}{48\pi\varepsilon_0\, a}$$

When switch S is closed, entire charge flows to the outer surface of outer shell,

$$U_f = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{2 \times 3a} = \frac{q^2}{24\pi\varepsilon_0\, a}$$

$$\text{Heat produced} = U_i = U_f = \frac{q^2}{8\pi\varepsilon_0 a}$$
$$= \frac{kq^2}{2a}$$

7. Let Q charge flows to C



$$V_C = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{q + Q}{3a} + \frac{(-q)}{4a} \right] = 0$$
$$Q = -\frac{q}{4}$$
$$\therefore \quad V_A = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{q}{2a} + \frac{Q}{3a} + \frac{-q}{4a} \right]$$
$$= \frac{1}{4\pi\varepsilon_0} \left[\frac{q}{\sigma a} \right]$$
$$V_A - V_C = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\sigma a}$$
$$= \frac{kq}{\sigma a}$$

8. $V_S = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{R}$

and $\quad V_C = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{3q}{2R}$

$$\therefore \; V_C - V_S = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{2R}$$
$$= \frac{1}{4p\varepsilon_0} \cdot \frac{\frac{4}{3}\pi\,\delta R^3}{2R} = \frac{\delta\, R^2}{\sigma E_0}$$

9. As particle comes to rest, force must be repulsive, hence it is positively charged.

Again on moving down its KE first increases than decreases, PE will first decrease than increase.

10. (1) is correct as the points having zero potential are close to Q_2, $|Q_2| < |Q_1|$.

Again as potential near Q_1 is positive, Q_1 is positive, hence (2) is correct.

At point A and B potential is zero not field, hence they may or may not be equilibrium point.

Hence (3) is wrong.

At point C potential is minimum, Q positive charge placed at this point will have unstable equilibrium but a negative charge will be in stable equilibrium at this position.

Hence, (4) is wrong.

11. V_1 is always negative and V_2 is always positive.

12. Electric field between the two points is positive near q_1 and negative near q_2, hence q_1 is positive and q_2 is negative.

Again neutral point is closer to q_1, hence $q_1 < q_2$.

13. Electric field due to a conductor does not depend on position of charge inside it.

14. $\vec{\mathbf{E}} = 400\cos 45^\circ\,\hat{\mathbf{i}} + 4000\sin 45^\circ\,\hat{\mathbf{j}}$

$$= 200\sqrt{2}\,(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

$$\vec{\mathbf{V}}_A - \vec{\mathbf{V}}_B = -\,\vec{\mathbf{E}}\cdot\vec{\mathbf{r}}_{AB}$$

$$= -\,200\sqrt{2}\,(\hat{\mathbf{i}} + \hat{\mathbf{j}})\cdot(2\,\hat{\mathbf{j}} - 3\,\hat{\mathbf{i}}) \times 10^{-2}$$

$$= 2\sqrt{2}\text{ V} \approx 2.8\text{ V}$$

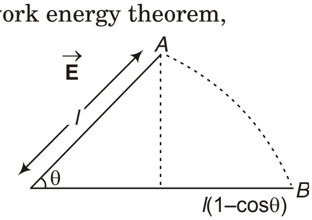
15. Potential difference between two concentric spherical shells does not depend on charge of outer sphere. Hence,

$$V_A{}' - V_B{}' = V_A - V_B$$

But $V_B{}' = 0$

$\therefore \quad V_A{}' = V_A - V_B.$

16. By work energy theorem,



Work done by electric field

= charge is KE

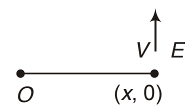
$$qE\,l(1 - \cos\theta) = \frac{1}{2}mv^2 - 0$$

$$v = \sqrt{\frac{qE\,l}{m}}$$

At point B

$$T = qE + \frac{mv^2}{r} = 2\,qE$$

17. Velocity of particle at any instant



$$V = at = \frac{qE}{m}t$$

$\therefore \quad L = mvr = qE\,x_0 t$

Hence, angular momentum of the particle increases with time.

18. By work energy theorem

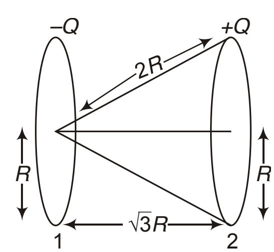
$$W = \Delta K$$

$$\Rightarrow \quad q\,(V_S - V_C) = 0 - \frac{1}{2}mv^2$$

$$\Rightarrow \quad q\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R} - \frac{1}{4\pi\varepsilon_0}\cdot\frac{3Q}{2R}\right) = -\frac{1}{2}mv^2$$

$$\Rightarrow \quad u = \sqrt{\frac{Q\,q}{4\pi\varepsilon_0 mR}}$$

19. Potential at the centre of negatively charged ring



$$V_1 = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{-Q}{R} + \frac{Q}{2R}\right)$$

$$= -\frac{Q}{8\pi\varepsilon_0 R}$$

Potential at the centre of positively charged ring

$$V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{Q}{R} - \frac{Q}{2R}\right)$$

$$= \frac{Q}{8\pi\varepsilon_0 R}$$

Kinetic energy required = Work done required

$$= q\,(V_2 - V_1) = \frac{Q}{4\pi\varepsilon_0 R}$$

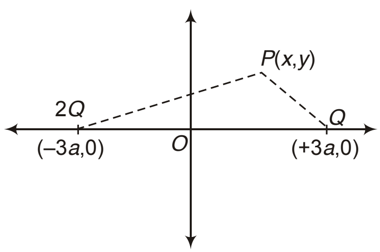
20. $E_x = -\frac{V_{x_2} - V_{x_1}}{x_2 - x_1} = -\frac{16-4}{-2-2} = 3$ V/m

$E_y = -\frac{V_{y_3} - V_{y_1}}{y_3 - y_1} = -\frac{12-4}{4-2} = -4$ V/m

$\therefore E = E_x \hat{\mathbf{i}} + E_y \hat{\mathbf{j}} = (3\hat{\mathbf{i}} - 4\hat{\mathbf{j}})$ V/m.

21. Consider a point $P(x, y)$ where potential is zero.

Now, V_P



$$V_P = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{\sqrt{(3a-x)^2+y^2}} + \frac{-2Q}{\sqrt{(x+3a)^2+y^2}}\right) = 0$$

$\Rightarrow \quad (x+3a)^2 + y^2 = 4[(3a-x)^2 + y^2]$

$\Rightarrow \quad 3x^2 + 3y^2 - 30ax + 27a^2 = 0$

$\Rightarrow \quad x^2 + y^2 - 10ax + 9a^2 = 0$

The equation represents a circle with radius

$$= \sqrt{\left(\frac{10a}{2}\right)^2 - 9a^2} = 4a$$

and centre at $\left(\frac{10}{2}a, 0\right) = (5a, 0)$

Clearly points $x = a$ and $x = 9a$ lie on this circle.

22. Work done $= qEy =$ Charge in KE

Hence, $\quad K_f = \frac{1}{2}mv^2 + qEy$

All other statements are correct.

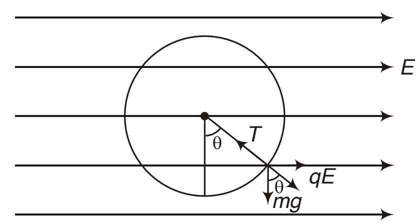
23. Electrostatic force of attraction provides necessary centripetal force.

ie, $\quad \frac{mv^2}{r} = \frac{\lambda q}{2\pi\varepsilon_0 r}$

$\Rightarrow \quad V = \sqrt{\frac{\lambda q}{2\pi\varepsilon_0 m}}$

$$T = \frac{2\pi r}{V} = 2\pi r\sqrt{\frac{2\pi\varepsilon_0 m}{\lambda q}} = 2\pi r\sqrt{\frac{m}{2K\lambda q}}$$

24. $T - mg\cos\theta - qE\sin\theta = \frac{mv^2}{l}$



Tension will be minimum when velocity is minimum.

Minimum possible in the string is zero.

ie, $\quad \frac{mv^2}{l} = -(mg\cos\theta + qE\sin\theta)$

Diff. both sides w.r.t. θ

$$\frac{2mv}{l}\frac{dv}{d\theta} = mg\sin\theta - qE\sin\theta \qquad \text{...(i)}$$

For minima or maxima

$$\frac{dv}{d\theta} = 0 \Rightarrow \theta = \tan^{-1}\frac{qE}{mg}$$

or $\quad \pi + \tan^{-1}\frac{qE}{mg}$

Differentiating Eq. (i) again,

$$\frac{2mv}{l}\frac{d^2v}{d\theta^2} + \frac{2m}{l}\cdot\left(\frac{dv}{d\theta}\right)^2 = mg\sin\theta + qE\sin\theta$$

$\therefore \frac{d^2v}{d\theta^2} = +\text{ve for } \theta = \tan^{-1}\frac{qE}{mg}$

and –ve for $\theta = \pi + \tan^{-1}\frac{qE}{mg}$

25. $q_A = \sigma(4\pi a^2)$, $q_B = -\sigma(4\pi b^2)$ and $q_C = -\sigma(4\pi c^2)$

$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_A + q_B}{b} + \frac{q_C}{c}\right) = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b^2} - b + c\right]$$

26. $U_i = \frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{q^2}{a} - \frac{-q^2}{a} + \frac{q^2}{a} - \frac{q^2}{a} - \frac{q^2}{\sqrt{2}a}\times 2\right]$

$$= \frac{-\sqrt{2}q^2}{4\pi\varepsilon_0 a}$$

$U_f = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{a}$

$W = U_f - U_i = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{a}(\sqrt{2}+1)$

27. $q_A = \sigma(4\pi a^2)$, $q_B = -\sigma(4\pi b^2)$, $q_C = \sigma(4\pi c^2)$

Given, $V_A = V_C$

$$\frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_A}{a}+\frac{q_B}{b}+\frac{q_C}{c}\right)=\frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_A+q_B+q_C}{c}\right)$$

$$\Rightarrow \quad a-b+c=\frac{a^2-b^2}{c}+c$$

$$\Rightarrow \quad a+b=c$$

28. Potential at minimum at mid-point in the region between two charges, and is always positive.

29. $U_i = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q^2}{r} = U$

$$U_f = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r}\times 3 = 3U$$

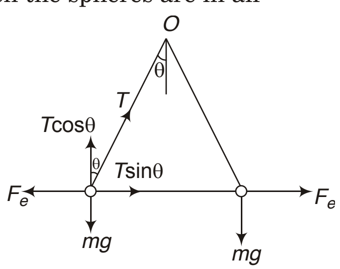
$$\therefore W = U_f - U_i = 2U$$

30. Loss of KE = Gain in PE

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{r}$$

$$r \propto \frac{1}{v^2}$$

31. When the spheres are in air

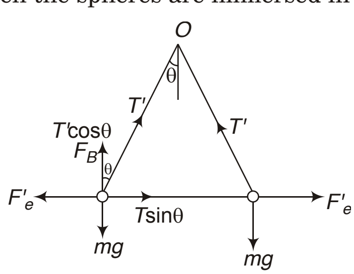


$$T\cos\theta = mg$$

$$T\sin\theta = F_e$$

$$\therefore \quad F_e = mg\tan\theta \quad \ldots\text{(i)}$$

When the spheres are immersed in liquid



$$F_1{}' = T'\sin\theta$$

$$g - F_B = T'\cos\theta$$

$$\therefore \quad F_e{}' = (mg - F_B)\tan\theta \quad \ldots\text{(ii)}$$

On dividing Eq. (ii) by Eq. (i),

$$\frac{F_e{}'}{F_e} = \frac{mg - F_B}{mg}$$

$$\frac{1}{K} = 1 - \frac{F_B}{mg} = 1 - \frac{0.8}{1.6} = \frac{1}{2}$$

$$K = 2$$

32. $V_P = \dfrac{1}{4\pi\varepsilon_0}\cdot\left[\dfrac{q}{a}\times 2 - \dfrac{q}{\sqrt{a^2+b^2}}\times 2\right]$

$$= \frac{2q}{4\pi\varepsilon_0}\cdot\left[\frac{\sqrt{a^2+b^2}-a}{a\sqrt{a^2+b^2}}\right]$$

$$= \frac{2q}{4\pi\varepsilon_0}\left[\frac{a\left(1+\dfrac{b^2}{a^2}\right)^{1/2} - a}{a\sqrt{a^2+b^2}}\right] = \frac{2q}{4\pi\varepsilon_0}\cdot\frac{b^2}{a^3}$$

[As $b << a$]

33. In any case electric field at origin is $\dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{5q}{r^2}$ along x-axis and $\dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{5q}{r^2}$ along y-axis.

34. $\langle u \rangle = \dfrac{1}{2}\varepsilon_0 E^2 = \dfrac{1}{2}\varepsilon_0\left(\dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{R^2}\right)^2$

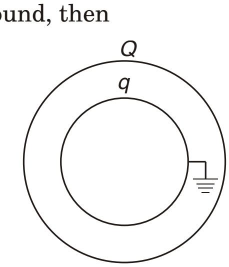
$$= \frac{1}{2}\varepsilon_0\left[\frac{9\times 10^9 \times \frac{1}{9}\times 10^{-9}}{12}\right]$$

$$= \frac{\varepsilon_0}{2}\ \text{J/m}^3$$

35. If Q is initial charge on B

then, $V_A - V_B = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{Q}{b} = V$

Now, if A is earthed, let charge q moves on A from ground, then



$$V_A = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q}{a}+\frac{Q}{b}\right) = 0$$

$$q = -\frac{a}{b}Q$$

$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q+Q}{b}$$
$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{b}\left(1-\frac{a}{b}\right) = V\left(1-\frac{a}{b}\right)$$

36. $\vec{\mathbf{E}} = -\left(\frac{\partial v}{\partial x}\hat{\mathbf{i}} + \frac{\partial v}{\partial y}\hat{\mathbf{j}} + \frac{\partial v}{\partial z}\hat{\mathbf{k}}\right)$

$$= -\left(\frac{-2}{1}\hat{\mathbf{i}} + \frac{-2}{1}\hat{\mathbf{j}} + \frac{-2}{1}\hat{\mathbf{k}}\right)$$
$$= 2(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})\ \text{N/C}$$

If V_P is potential at P, then

$$V_P - V_0 = -\vec{\mathbf{E}}\cdot\vec{\mathbf{r}}$$
$$V_P - 10 = -2(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = -6$$
$$V_P = 4\ \text{V}$$

37. On touching two spheres, charge is equally divided among them, then due to induction a charge $\left(-\frac{q}{2}\right)$ appears on the earthed sphere.

38. Negative charge will induce on the conductor near P.

39. $\begin{cases} 0 & \text{for } r < r_A \\ \frac{kQ_P}{r} & \text{for } r_P < r < r_B \\ \frac{k(Q_A - Q_B)}{r} & \text{for } r > r_B \end{cases}$

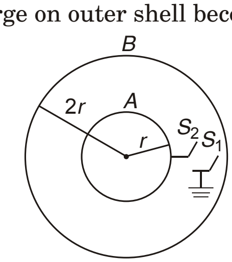
As $|Q_B| > |Q_A|$

E is –ve for $r > r_B$.

40. $\vec{\mathbf{E}} = -\left(\frac{\partial v}{\partial x}\hat{\mathbf{i}} + \frac{\partial v}{\partial y}\hat{\mathbf{j}}\right)$

$$= k(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$$
$$|\vec{\mathbf{E}}| = k\sqrt{y^2 + x^2} = kr$$

41. Let charge on outer shell becomes q.



$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{Q+q}{2r}\right) = 0$$
$$q = -Q$$

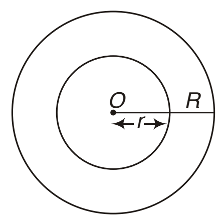
42. Let charge q' flows through the switch to the ground, then

$$\frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{Q-q'}{r} - \frac{Q}{2r}\right] = 0$$
$$q' = \frac{1}{2}Q$$

43. After n steps

$$q' = \frac{1}{2^n}Q \text{ and } q = \frac{-1}{2^{n-1}}Q$$
$$\therefore\quad V_A = \frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q'}{r} + \frac{q}{2r}\right) = 0$$
$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q'+q}{2r}$$
$$= \frac{1}{2^{n+1}}\left[\frac{Q}{4\pi\varepsilon_0 r}\right]$$

44. Consider a spherical Gaussian surface of radius $r\,(< R)$ and concentric with the sphere,



Charge on a small sphere of radius r

$$dq = \delta\, dV = 4\pi r^2 \delta\, dr$$
$$= r\,\pi\delta_0\left(r^2 - \frac{r^3}{R}\right)dr$$

Total charge inside the Gaussian surface,

$$q = 4\pi\,\delta_0\int_0^r\left(r^2 - \frac{r^3}{R}\right)dr$$
$$= 4\pi\delta_0\left[\frac{r^3}{3} - \frac{r^4}{4R}\right]$$
$$\therefore E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2} = \frac{\delta_0}{\varepsilon_0}\left[\frac{r}{3} - \frac{r^2}{4R}\right]$$

45. Total charge inside the surface.

$$Q = 4\pi\delta_0\left[\frac{R^3}{3} - \frac{R^3}{r}\right] = \frac{1}{3}\pi\delta_0 R^3$$
$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2} = \frac{\delta_0 R^3}{12\varepsilon_0 r^2}$$

46. $E = \frac{\delta_0}{\varepsilon_0}\left[\frac{r}{3} - \frac{r^2}{4R}\right]$

For maximum intensity of electric field

$$\frac{dE}{dr}=\frac{\delta_0}{\varepsilon_0}\left[\frac{1}{3}-\frac{r}{2R}\right]=0$$

$$\Rightarrow \qquad r=\frac{2}{3}R$$

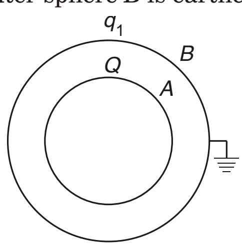
$$\frac{d^2E}{dr^2}=-\frac{\delta_0}{2R\varepsilon_0}=-\text{ve},$$

hence E is maximum at $r=\frac{2}{3}R$.

47. $$E_{\max}=\frac{\rho_0}{\varepsilon_0}\left[\frac{\frac{2\lambda}{3}}{3}-\frac{\left(\frac{2R}{3}\right)^2}{4R}\right]=\frac{\rho_0 R}{q\varepsilon_0}$$

48. Potential difference between two concentric spheres do not depend on the charge on outer sphere.

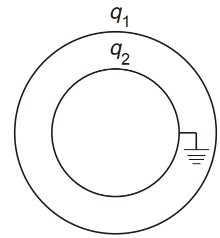
49. When outer sphere B is earthed



$$V_B=\frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{Q+q_1}{b}\right)=0$$

$$q_1=-Q$$

Now, if A is earthed



$$q_A=\frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_2}{a}+\frac{q_1}{b}\right)=0$$

$$q_2=-\frac{a}{b}q_1=\frac{a}{b}Q$$

50. When connected by conducting wires, entire charge from inner sphere flows to the outer sphere, *ie*,

$$q_3=q_1+q_2\ \left(\frac{a}{b}-1\right)Q$$

$$=\frac{a-b}{b}Q$$

More than One Correct Options

1. Before earthing the surface B,

$$V_A=\frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_A}{R}+\frac{q_B}{2R}\right)=2\text{ V}$$

$$V_B=\frac{1}{4\pi\varepsilon_0}\cdot\left(\frac{q_A+q_B}{2R}\right)=\frac{3}{2}\text{ V}$$

$$\Rightarrow \qquad \frac{q_A}{q_B}=\frac{1}{2}$$

On earthing the sphere B,

$$q_A'=q_A$$

$$V_B=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_A'+q_B'}{2R}=0$$

$$q_B'=-q_A'$$

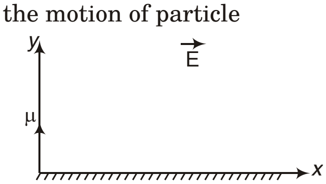
$$\Rightarrow \qquad \frac{q_A'}{q_B'}=-1$$

As potential difference does not depend on charge on outer sphere,

$$V_A'-V_B'=V_A-V_B=\frac{V}{2}$$

$$V_A'=\frac{1}{2}V$$

2. For the motion of particle



$$u_x=0, v_x=v, a_x=\frac{qE}{m}, a_y=-g,$$

$$x_0=0,\ y_0=0$$

$$x=x_0+u_xt+\frac{1}{2}a_xt^2$$

$$x=\frac{qE}{2m}t^2 \qquad \text{...(i)}$$

$$y=y_0+u_yt+\frac{1}{2}a_yt^2$$

$$=ut-\frac{1}{2}gt^2 \qquad \text{...(ii)}$$

At the end of motion

$$t=T,\ y=0,\ x=R$$

From Eq. (ii),

$$0 = \left(u - \frac{1}{2}gT\right)T$$

$$T = \frac{2u}{g} = \frac{2 \times 10}{10} = 2\text{ s}$$

From Eq. (i),

$$R = \frac{qE}{2m}T^2$$

$$= \frac{1 \times 10^{-3} \times 10^4}{2 \times g} \times 4 = 10\text{ m}$$

Now, $v_y^2 - u_y^2 = 2a_y(y - y_0)$

At highest point (*i.e.*, $y = H$), $v_y = 0$

$$0 - (10)^2 = -2 \times 10(H - 0)$$

$$H = 5\text{ m}$$

3. Let R be the radius of the sphere

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R + r_1}$$

$$\Rightarrow \quad \frac{q \times 10^9 \times q}{(R + S) \times 10^{-2}} = 100 \quad \text{...(i)}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R + r_2}$$

$$\Rightarrow \quad \frac{9 \times 10^9 \times q}{(R + 10) \times 10^{-2}} = 75 \quad \text{...(ii)}$$

On solving,

$R = 10$ cm,

$$q = \frac{5}{3} \times 10^{-9}\text{ C} = \frac{50}{3} \times 10^{-10}\text{ C}$$

Electric field on surface,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2} = \frac{9 \times 10^9 \times \frac{5}{3} \times 10^{-9}}{(10 \times 10^{-2})}$$

$$= 1500\text{ V/m}$$

Potential at surface,

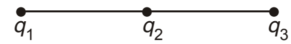
$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R} \quad \frac{9 \times 10^9 \times \frac{5}{3} \times 10^{-9}}{10 \times 10^{-2}}$$

$$= 150\text{ V}$$

Potential at Centre

$$V_C = \frac{3}{2}V_S = 225\text{ V}$$

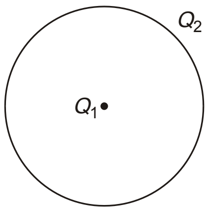
4. For all charges to be in equilibrium, force experienced by either charge must be zero *ie*., force due to other two charges must be equal and opposite.



Hence all the charges must be collinear, charges q_1, and q_3 must have same sign and q_2 must have opposite sign, q_2 must have maximum magnitude.

Such on equilibrium is always unstable.

5. Flux through any closed surface depends only on charge inside the surface but electric field at any point on the surface depends on charges inside as well as outside the surface.



6. As net charge on an electric dipole is zero, net flux through the sphere is zero.

But electric field at any point due to a dipole cannot be zero.

7. Gauss's law gives total electric field and flux due to all charges.

8. If two concentric spheres carry equal and opposite charges, Electric field is **non-zero** only in the region between two sphere and potential is is *zero* only outside both the spheres.

9. As force on the rod due to electric field is towards right, force on the rod due to hinge must be left.

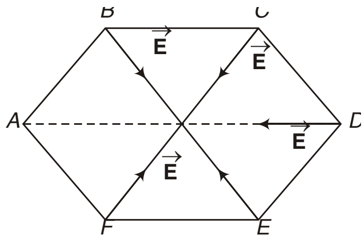
The equilibrium is clearly neutral.

10. If moved along perpendicular bisector, for all identical charges, electrostatic potential energy is maximum at mid point and if moved along the line joining the particles, electrostatic potential energy is minimum at the mid-point.

Match the Columns

1. (a → s), (b → q), (c → r), (d → p).

If charge at B is removed



$$E_{net} = E_D \cos 30° + E_E \cos 30° = \sqrt{3}E$$

If charge at C is removed

$$E_{net} = E_D \cos 60° + E_f \cos 60° = E$$

If charge at D is removed

$$\vec{E}_{net} = 0 \text{ and } \vec{E}_B = -\vec{E}_E$$

and $$\vec{E}_E = -\vec{E}_E$$

If charge at B and C both are removed,

$$E_{net} = E_E + E_D \cos 60° + E_F \cos 60° = 2E$$

2. (a → q), (b → p), (c → s), (d → r).

$$V = -\vec{E} \cdot \vec{r}$$

If $\vec{r} = 4\hat{i}$, $V = -8\,V$,

If $\vec{r} = -4\hat{i}$, $V = 8\,V$

If $\vec{r} = 4\hat{j}$, $V = -16\,V$,

If $\vec{r} = -4\hat{j}$, $V = 16\,V$

3. For a solid sphere

$$V_{in} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{2R^3}(3R^2 - r^2)$$

at $r = \frac{R}{2}$

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{2R^3}\left(3R^2 - \frac{R^2}{4}\right) = \frac{11}{8}V$$

$$V_{out} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

at $r = 2R$

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{2R} = \frac{V}{2}$$

$$E_{in} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qr}{R^3}$$

at $r = \frac{R}{2}$

$$E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{2R^2} = \frac{V}{2R} = \frac{V}{2} \qquad (\because R = 1\text{ m})$$

$$E_{out} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

at $r = 2R$

$$E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(2R)^2} = \frac{V}{4R} = \frac{V}{4}$$

$\therefore (a \to s), (b \to q), (c \to q), (d \to p)$.

4. $(a \to r), (b \to q), (c \to q), (d \to s)$

5. $(a \to p), (b \to q), (c \to r), (d \to s)$

For a spherical shell,

$$E = \begin{cases} 0 & \text{for } r < R \\ \dfrac{Kq}{r^2} & \text{for } r \geq R \end{cases}$$

$$V = \begin{cases} \dfrac{Kq}{R} & \text{for } r \leq R \\ \dfrac{Kq}{r} & \text{for } r \geq R \end{cases}$$

For a solid sphere,

$$E = \begin{cases} \dfrac{Kqr}{R^3} & \text{for } r \leq R \\ \dfrac{Kq}{r^2} & \text{for } r \geq R \end{cases}$$

$$V = \begin{cases} \dfrac{Kq}{2R^2}(2R^2 - r^2) & \text{for } r \leq R \\ \dfrac{Kq}{r^2} & \text{for } r \geq R \end{cases}$$

22 Capacitors

Introductory Exercise 22.1

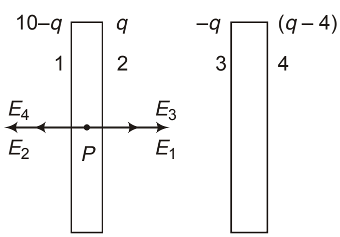
1. $C = \frac{q}{V} = [C] = \frac{[\text{AT}]}{[\text{ML}^2\text{T}^{-3}\text{A}^{-1}]}$

$$= [\text{M}^{-1}\text{L}^{-2}\text{T}^4\text{A}^2]$$

2. False.

Charge will flow if there is potential difference between the conductors. It does not depend on amount of charge present.

3. Consider the charge distribution shown in figure.



Electric field at point P

$$E_P = E_1 + E_3 - E_2 - E_4$$

$$= \frac{10-q}{2\varepsilon_0 A} - \frac{q}{2\varepsilon_0 A} + \frac{q}{2\varepsilon_0 A} - \frac{q-4}{2\varepsilon_0 A}$$

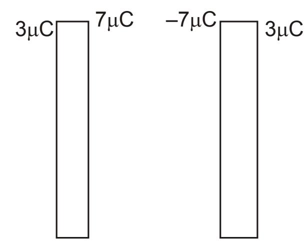
But P lies inside conductor

$\therefore E_P = 0$

$\Rightarrow \quad 10 - q - q + q - q + 4 = 0$

$\Rightarrow \quad q = 7\ \mu\text{C}$

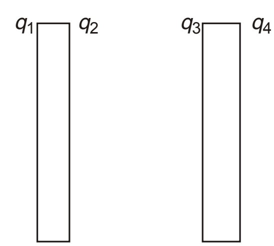
Hence, the charge distribution is shown in figure.



Sort-cut Method

Entire charge resides on outer surface of conductor and will be divided equally on two outer surfaces.

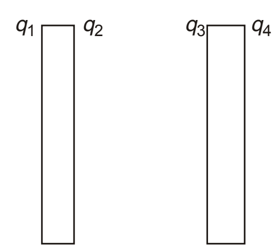
Hence, if q_1 and q_2 be charge on two plates then.



$$q_1 = q_4 = \frac{q_1 + q_2}{2} = 3\ \mu\text{C}$$

$$q_2 = \frac{q_1 - q_2}{2} = 7\ \mu\text{C}$$

$$q_3 = \frac{q_2 - q_1}{2} = -7\ \mu\text{C}$$



4. Charge distribution is shown in figure.

$$q_1 = q_4 = \frac{q_1 + q_2}{2} = -\frac{q}{2}$$

$$q_2 = \frac{q_1 - q_2}{2} = \frac{5q}{2}$$

$$q_3 = \frac{q_2 - q_1}{2} = -\frac{5q}{2}$$

$\therefore$ Charge on capacitor = Charge on inner side of positive plate.

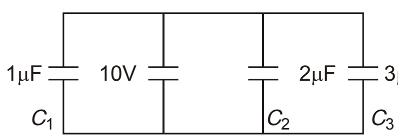
$$q = \frac{5q}{2}$$

and $$C = \frac{\varepsilon_0 A}{d}$$

$\therefore$ $$V = \frac{q}{C} = \frac{5q\,d}{2\varepsilon_0 A}$$

Introductory Exercise 22.2

1. All the capacitors are in parallel



$$q_1 = C_1 V = 1 \times 10 = 10\ \mu\text{C}$$

$$q_2 = C_2 V = 2 \times 10 = 20\ \mu\text{C}$$

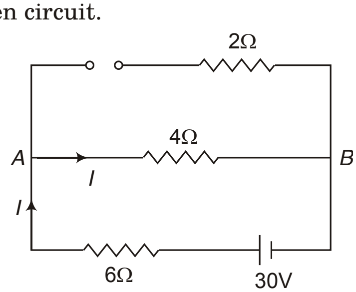
$$q_3 = C_3 V = 3 \times 10 = 30\ \mu\text{C}$$

2. Potential difference across the plates of capacitor

$$V = 10\ \text{V}$$

$$q = CV = 4 \times 10 = 40\ \mu\text{C}$$

3. In the steady state capacitor behaves as open circuit.



$$I = \frac{30}{6+4} = 3\ \text{A}$$

Potential difference across the capacitor,

$$V_{AB} = 4 \times I = 4 \times 3 = 12\ \text{V}$$

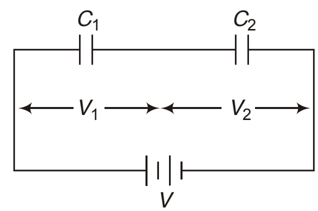
$\therefore$ Charge on capacitor

$$q = CV_{AB} = 2 \times 12 = 24\ \mu\text{C}$$

4. (a) $$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1} + \frac{1}{2}$$

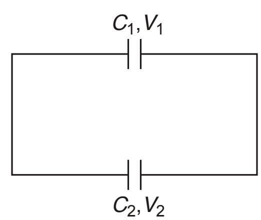
$$C_e = \frac{2}{3}\ \mu\text{C}$$

$$q = C_e V = \frac{2}{3} \times 1200 = 800\ \mu\text{C}$$



(b) $$V_1 = \frac{q}{C_1} = \frac{800}{1} = 800\ \text{V}$$

$$V_2 = \frac{q}{C_2} = \frac{800}{2} = 400\ \text{V}$$



Now, if they are connected in parallel,

Common potential, $\left(V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)$

$$= \frac{1 \times 800 \times 2 \times 400}{1+2} = \frac{1600}{3}\ \text{V}$$

$$q_1 = C_1 V = \frac{1600}{3}\ \mu\text{C},\ q_2 = C_2 V = \frac{3200}{3}\ \mu\text{C}$$

5. Common potential

$$V = \frac{C_1V_2 + C_2V_2}{C_1 + C_2}$$

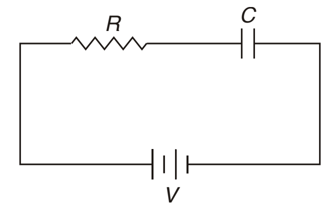
But $V = 20$, $V_2 = 0$, $V_1 = 100\ \text{V}$, $C_1 = 100\ \mu\text{C}$

$\therefore$ $$\frac{100 \times 100 + C_2 \times 0}{400 + C_2} = 20$$

$\Rightarrow$ $$C_2 = 400\ \mu\text{C}$$

Introductory Exercise 22.3

1. Let q be the final charge on the capacitor, work done by battery



$$W = qV$$

Energy stored in the capacitor

$$U = \frac{1}{2} qV$$

$\therefore$ Energy dissipated as heat

$$H = U - W = \frac{1}{2} qV = U$$

2. We have

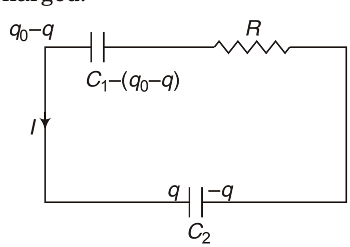
$$I = I_0 e^{-t/\tau}$$

$$\frac{I_0}{2} = I_0 e^{-t/\tau} \Rightarrow e^{-t/\tau} = \frac{1}{2}$$

$$t = \tau \ln 2 = 0.693\ \tau$$

$t = 0.693$ time constant.

3. Let capacitor C_1 is initially charged and C_2 is uncharged.



At any instant, let charge on C_2 be q, charge on C_1 at that instant $= q_0 - q$

By Kirchhoff's voltage law,

$$\frac{(q_0 - q)}{C} - IR - \frac{q}{C} = 0$$

$$\Rightarrow \quad \frac{dq}{dt} = \frac{q_0 - 2q}{RC}$$

$$\Rightarrow \quad \int_0^q \frac{dq}{q_0 - 2q} = \int_0^t \frac{dt}{RC}$$

$$\Rightarrow \quad \frac{[\ln (q_0 - 2q)]_0^q}{-2} = \frac{1}{RC} [t]_0^t$$

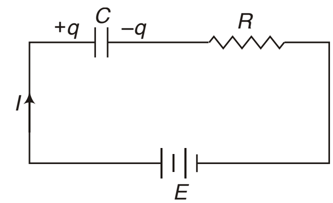
$$\Rightarrow \quad q = \frac{q_0}{2}(1 - e^{-t/\tau})$$

$\therefore$ At time t,

Charge on $C_1 = q = \frac{q_0}{2}(1 - e^{-t/\tau})$

Charge on $C_2 = q_0 - q = \frac{q_0}{2}(1 + e^{-t/\tau})$

4. Let q be the charge on capacitor at any instant t



By Kirchhoff's voltage law

$$\frac{q}{C} + IR = E$$

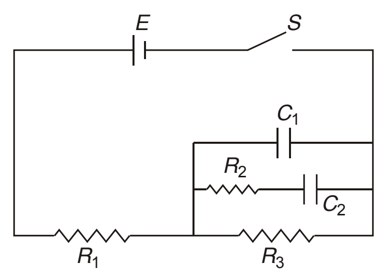
$$\frac{dq}{dt} = \frac{CE - q}{RC}$$

$$\int_{q_0}^{q} \frac{dq}{CE - q} = \int_0^t \frac{dt}{RC}$$

$$\Rightarrow \quad q_0 = CE(1 - e^{-t/\tau}) + q_0 e^{-t/\tau}$$

where, $\tau = RC$

5. (a) When the switch is just closed, Capacitors behave like short circuit.



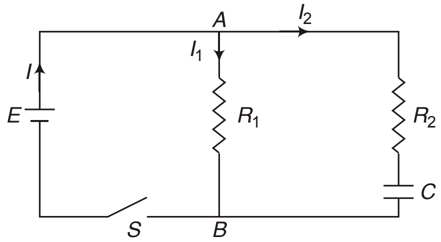
$\therefore$ Initial current

$$I_i = \frac{E}{R_1}$$

(b) After a long time, *i.e.*, in steady state, both the capacitors behaves open circuit,

$$I_f = \frac{E}{R_1 + R_3}$$

6. (a) Immediately after closing the switch, capacitor behaves as short circuit,



$\therefore \quad I_1 = \dfrac{E}{R_1}$ and $I_2 = \dfrac{E}{R_2}$

(b) In the steady state, capacitor behaves as open circuit,

$\therefore \quad I_1 = \dfrac{E}{R_1}, I_2 = 0$

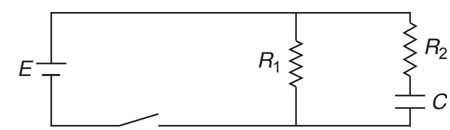
(c) Potential difference across the capacitors in the steady state,

$$V = E$$

$\therefore$ Energy stored in the capacitor

$$U = \frac{1}{2} CE^2$$

(d) After the switch is open



$$R_e = R_1 + R_2$$
$$\tau = R_3 C = (R_1 + R_2) C$$

AIEEE Corner

Subjective Questions (Level-1)

1. $C = \dfrac{\varepsilon_0 A}{d} \Rightarrow A = \dfrac{Cd}{\varepsilon_0} = \dfrac{1 \times 1 \times 10^{-3}}{8.85 \times 10^{-12}}$

$$= 1.13 \times 10^8 \text{ m}^2$$

2. $C_1 = \dfrac{\varepsilon_0 A_1}{d}$ and $C_2 = \dfrac{\varepsilon_0 A_2}{d}$

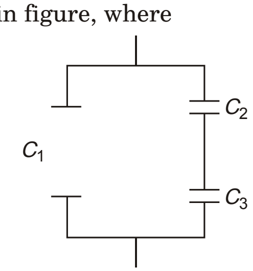
If connected in parallel

$$C = C_1 + C_2 = \frac{\varepsilon_0 A_1}{d} + \frac{\varepsilon_2 A_2}{d}$$
$$= \frac{\varepsilon_0 (A_1 + A_2)}{d} = \frac{\varepsilon_0 A}{d}$$

where, $A = A_1 + A_2 =$ effective area.

Hence proved.

3. The arrangement can be considered as the combination of three different capacitors as shown in figure, where

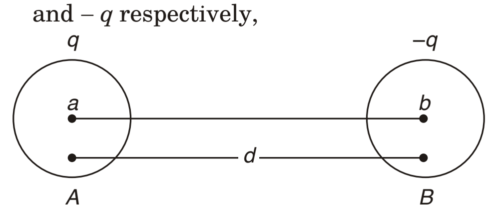


$$C_1 = \frac{k_1 \varepsilon_0 \frac{A}{2}}{2d} = \frac{k_1 \varepsilon_0 A}{4d}$$
$$C_2 = \frac{k_2 \varepsilon_0 \frac{A}{2}}{2d/2} = \frac{k_2 \varepsilon_0 A}{2d}$$
$$C_3 = \frac{k_3 \varepsilon_0 \frac{A}{2}}{2d/2} = \frac{k_3 \varepsilon_0 A}{2d}$$

Therefore, the effective capacitance,

$$C = C_1 + \frac{C_2 C_3}{C_2 + C_3}$$
$$= \frac{\varepsilon_0 A}{2d} \left(\frac{k_1}{2} + \frac{k_1 k_3}{k_2 + k_3} \right)$$

4. (a) Let the spheres A and B carry charges q and $-q$ respectively,



$\therefore \quad V_A = \dfrac{1}{4\pi\varepsilon_0} \cdot \left[\dfrac{q}{a} - \dfrac{q}{d} \right]$

$$V_B = \frac{1}{4\pi\varepsilon_0} \cdot \left[-\frac{q}{b} + \frac{q}{d} \right]$$

Potential difference between the spheres,

$$V = V_A - V_B = \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{a} + \frac{1}{b} - \frac{2}{d} \right]$$
$$C = \frac{q}{V} = \frac{4\pi\varepsilon_0}{\frac{1}{a} + \frac{1}{b} - \frac{2}{d}}$$

Hence proved.

(b) If $d \to \infty$

$$C = \frac{4\pi\varepsilon_0}{\frac{1}{a} + \frac{1}{b}} = \frac{4\pi\varepsilon_0 \cdot ab}{a + b}$$

If two isolated spheres of radii a and b are connected in series,

then,

$$C' = \frac{C_1 C_2}{C_1 + C_2}$$

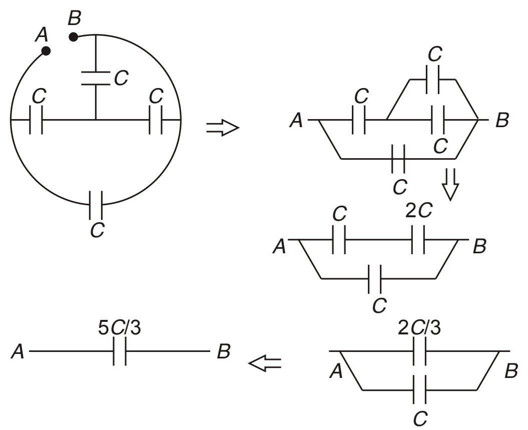
where, $C_1 = 4\pi\varepsilon_0 a$, $C_2 = 4\pi\varepsilon_0 b$

$$\therefore \qquad C' = \frac{4\pi\varepsilon_0 \cdot ab}{a + b}$$

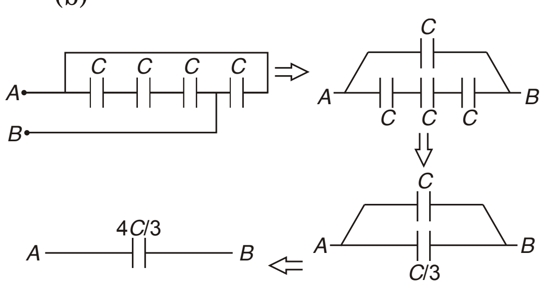
$$\therefore \qquad C' = C$$

Hence proved.

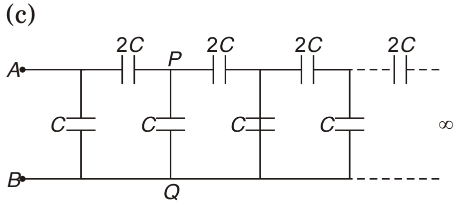
5. (a)



(b)



(c)



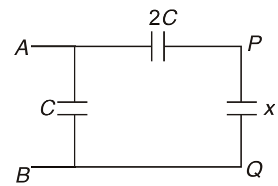
Let effective capacitance between A and B

$$C_{AB} = x$$

As the network is infinite,

$$C_{PQ} = C_{AB} = x$$

Equivalent circuit is shown in figure,



$$R_{AB} = C + \frac{2Cx}{2C + x} = x$$

$$\Rightarrow \qquad 2C^2 + Cx + 2Cx = 2Cx + x^2$$

$$\Rightarrow \qquad x^2 - Cx - 2C^2 = 0$$

On solving, $x = 2C$ or $-C$

But x cannot be negative,

Hence, $x = 2C$

6. $q = CV = 7.28 \times 25 = 182\ \mu\text{C}$

7. (a) $V = \dfrac{q}{C} = \dfrac{0.148 \times 10^{-6}}{245 \times 10^{-12}} = 604\ \text{V}$

(b) $C = \dfrac{\varepsilon_0 A}{d} \Rightarrow A = \dfrac{Cd}{\varepsilon_0}$

$$= \frac{245 \times 10^{-12} \times 0.328 \times 10^{-3}}{8.85 \times 10^{-12}}$$

$$= 9.08 \times 10^{-3}\ \text{m}^2$$

$$= 90.8\ \text{cm}^2$$

(c) $\sigma = \dfrac{q}{A} = \dfrac{0.148 \times 10^{-6}}{9.08 \times 10^{-3}} = 16.3\ \mu\text{C/m}^2$

8. (a) $E_0 = 3.20 \times 10^5$ V/m

$E = 2.50 \times 10^5$ V/m

$$k = \frac{E_0}{E} = \frac{3.20 \times 10^5}{2.50 \times 10^5} = 1.28$$

(b) Electric field between the plates of capacitor is given by

$$E = \frac{\sigma}{\varepsilon_0}$$

$$\Rightarrow \qquad \sigma = \varepsilon_0 E = 8.8 \times 10^{-12} \times 3.20 \times 10^5$$

$$= 2.832 \times 10^{-6}\ \text{C/m}^2$$

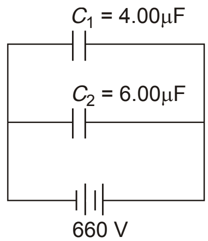
$$= 2.832\ \mu\text{C/m}^2$$

9. (a) $q_1 = C_1 V = 4 \times 660 = 2640\ \mu\text{C}$

$q_2 = C_2 V = 6 \times 660 = 3960\ \mu\text{C}$

As C_1 and C_2 are connected in parallel,

$$V_1 = V_2 = V = 660 \text{ V}$$



(b) When unlike plates of capacitors are connected to each other,

Common potential

$$V = \frac{C_2V_2 - C_1V_1}{C_1 + C_2} = \frac{6 \times 660 - 4 \times 660}{6}$$

$$= 220 \text{ V}$$

$$q_1 = C_1V = 4 \times 220 = 880 \ \mu\text{C}$$

$$q_2 = C_2V = 6 \times 220 = 1320 \ \mu\text{C}$$

10. $E = \dfrac{V}{d} = \dfrac{400}{5 \times 10^{-3}} = 8 \times 10^4$ V/m

Energy density,

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2} \times 8.85 \times 10^{-12} \times (8 \times 10^{-4})^2$$

$$= 2.03 \times 10^{-2} \text{ J/m}^3$$

$$= 20.3 \text{ mJ/m}^3$$

11. Dielectric strength = maximum possible electric field

$$E = \frac{V}{d} \Rightarrow d = \frac{V}{E}$$

$$= \frac{5500}{1.6 \times 10^7} = 3.4 \times 10^{-4} \text{ m}$$

$$C = \frac{k\varepsilon_0 A}{d} \Rightarrow A = \frac{Cd}{k\varepsilon_0}$$

$$= \frac{1.25 \times 10^{-9} \times 3.4 \times 10^{-4}}{3.6 \times 8.85 \times 10^{-12}}$$

$$= 1.3 \times 10^{-2} \text{ m}^2$$

$$= 0.013 \text{ m}^2$$

12. Let C_P and C_S be the effective capacitance of parallel and series combination respectively.

For parallel combination,

$$U_P = 0.19 \text{ J}$$

$$U_P = \frac{1}{2}C_P V^2$$

$$\Rightarrow \quad C_P = \frac{2U_P}{V^2} = \frac{2 \times 0.1}{(2)^2} = 0.05 \text{ F}$$

$$= 50 \text{ mF}$$

For series combination,

$$U_S = 1.6 \times 10^{-2} \text{ J} = 0.016 \text{ J}$$

$$U_S = \frac{1}{2}C_S V^2$$

$$\Rightarrow \quad C_S = \frac{2U_S}{V^2} = \frac{2 \times 0.016}{(2)^2} = 0.008 \text{ F}$$

$$= 8 \text{ mF}$$

Now, $C_P = C_1 + C_2 = 5$ mF

or $\quad C_2 = (5 - C_1)$ mF

and $\dfrac{1}{C_S} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{1}{8}$

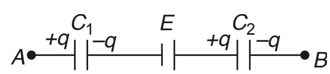
$$\Rightarrow \quad \frac{1}{C_1} + \frac{1}{S - C_1} = \frac{1}{8}$$

On solving,

$C_1 = 40$ mF, $C_2 = 10$ mF or *vice-versa*.

13. In the given circuit,

$$V_A - V_B = \frac{q}{C_1} - E + \frac{q}{C_2} = 5$$



$$\Rightarrow \quad \frac{q}{10^{-6}} - 10 + \frac{q}{2 \times 10^{-6}} = 5$$

$$\Rightarrow \quad q = 10 \times 10^{-6} \text{ C} = 10 \ \mu\text{C}$$

$$\therefore \quad V_1 = \frac{q}{C_1} = 10 \text{ V}, V_2 = \frac{q}{C_2} = 5 \text{ V}$$

14. (a) In order to increase voltage range n times, n-capacitors must be connected in series.

Hence, to increase voltage range to 500V, 5 capacitors must be connected in series.

Now, effective capacitance of series combination,

$$C_S = C_n = \frac{10}{5} = 2 \text{ pF}$$

Hence, no parallel grouping of such units is required.

Hence, a series grouping of 5 such capacitors will have effective capacitance 2 pF and can withstand 500 V.

(b) If n capacitors are connected in series and m such units are connected in parallel,

$$V_e = nV$$

$$C_e = \frac{mC}{n}$$

Here, $V = 100$ V

$$V_e = 300 \text{ V}$$

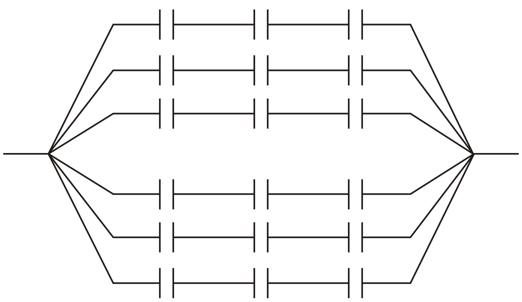
$$\therefore \quad n = \frac{V_e}{V} = 3$$

$C = 10\text{ pF}$

$C_e = 20\text{ pF}$

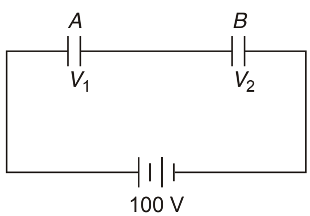
$$m = \frac{nC_e}{C} = \frac{3 \times 20}{10} = 6$$

Hence, the required arrangement is shown in figure.



15. Case I.

$$V_1 = \frac{C_2}{C_1 + C_2} V = 60\text{ V}$$



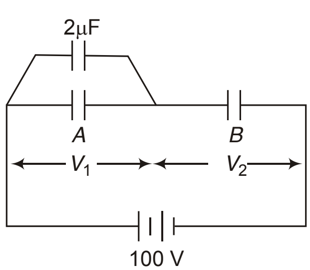
$$V_2 = \frac{C_1}{C_1 + C_2} V = 40\text{ V}$$

$$\Rightarrow \quad \frac{C_1}{C_2} = \frac{2}{3}$$

$$\Rightarrow \quad C_2 = \frac{3}{2} C_1$$

Case II.

$$V_1 = \frac{C_2}{C_1 + C_2 + 2} = 10\text{ V}$$



$$V_2 = \frac{C_1 + 2}{C_1 + C_2 + C} = 90\text{ V}$$

$$\Rightarrow \quad \frac{C_1 + 2}{C_2} = \frac{90}{10} = 9$$

$$\Rightarrow \quad C_1 + 2 = 9C_2$$

$$\Rightarrow \quad C_1 + 2 = 9 \times \frac{3}{2} C_1$$

$$\Rightarrow \quad \frac{25}{2} C_1 = 2 \Rightarrow C_1 = \frac{4}{25}\ \mu\text{F}$$

$$= 0.16\ \mu\text{F}$$

$$C_2 = \frac{3}{2} C_1 = 0.24\ \mu\text{F}$$

16. (a) $q = CV = 10 \times 12 = 120\ \mu\text{C}$

(b) $C = \dfrac{\varepsilon_0 A}{d}$

If separation is doubled, capacitance will become half. *i.e.,*

$$C' = \frac{C}{2}$$

$$q' = E' V = \frac{C}{2} V = 60\ \mu\text{C}$$

(c) $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{\pi \varepsilon_0 r^2}{d}$

If r is doubled, C will become four times, *i.e.*,

$$C' = 4C$$

$$q' = C' V = 480\ \mu\text{C}$$

17. Heat produced = Energy stored in the capacitor

$$H = \frac{1}{2} CV^2 = \frac{1}{2} \times 450 \times 10^{-6} \times (295)^2$$

$$= 19.58\text{ J}$$

18. (a) $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{8.85 \times 10^{-12} \times 2}{5 \times 10^{-3}}$

$$= 3.54 \times 10^{-6}\text{ F}$$

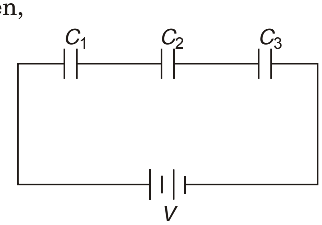
$$= 3.54\ \mu\text{F}$$

(b) $q = CV = 3.54 \times 10^{-9} \times 10000$

$$= 35.4 \times 10^{-6} = 35.4\ \mu\text{C}$$

(c) $E = \dfrac{V}{d} = \dfrac{10000}{5 \times 10^{-3}} = 2 \times 10^6\text{ V/m}$

19. Given,



$$C_1 = 8.4\ \mu\text{F},\ C_2 = 8.2\ \mu\text{F}$$

$$C_3 = 4.2\ \mu\text{F},\ V = 36\text{ V}$$

(a) Effective capacitance,

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{8.4} + \frac{1}{8.2} + \frac{1}{4.2} \Rightarrow C_e = 2.09\ \mu\text{F}$$

$$q = C_e V = 2.09 \times 36 = 75.2\ \mu\text{C}$$

As combination is series, charge on each capacitor is same, *i.e.*, 75.2 μC.

(b) $U = \frac{1}{2} qV = \frac{1}{2} \times 75.2 \times 36 \times 10^{-6}$

$$= 1.35 \times 10^{-3}\ \text{J} = 1.35\ \text{mJ}$$

(c) Common potential,

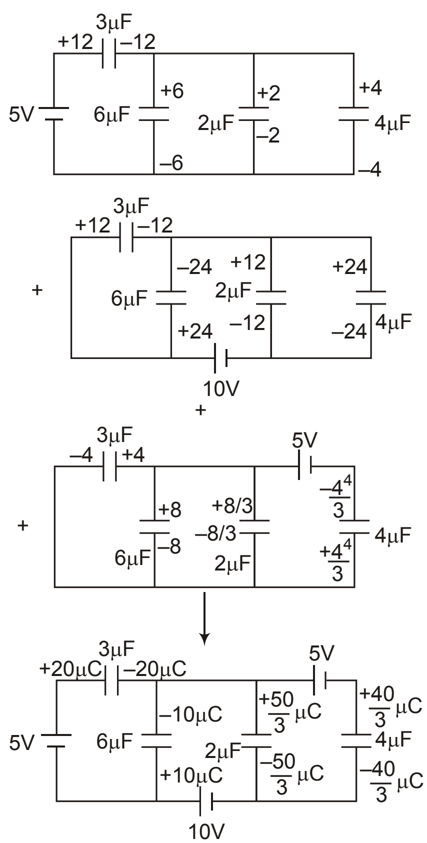
$$V = \frac{C_1V_1 + C_2V_2 + C_3V_3}{C_1 + C_2 + C_3} = 10.85\ \text{V}$$

(d) $U' = \frac{1}{2}(C_1 + C_2 + C_3)V^2$

$$= \frac{1}{2} \times (8.4 + 8.2 + 4.2) \times (10.85)^2 \times 10^{-6}$$

$$= 1.22 \times 10^{-3}\ \text{J} = 1.22\ \text{mJ}$$

20. The Given circuit can be considered as the sum of three circuits as shown

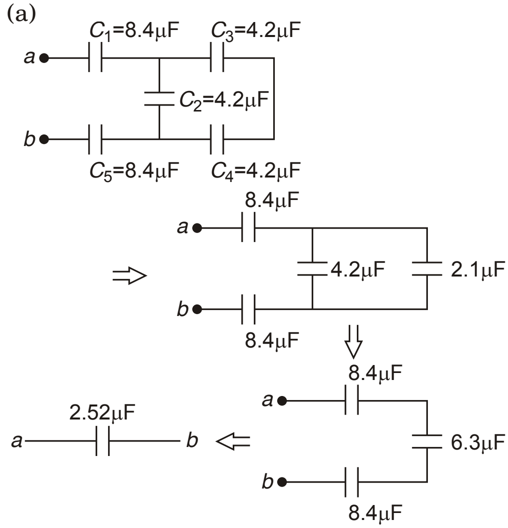


(Charge is shown in μC).

Hence, charge on 6 μF capacitor = 10 μC

and Charge on 4 μF capacitor = $\frac{40}{3}$ μC

21. (a)



(b) Charge supplied by the source of emf

$$q = CV = 2.52 \times 10^{-6} \times 220$$

$$= 554.4\ \mu\text{C}$$

$$q_1 = q_5 = q = 554.4\ \mu\text{C}$$

$$q_2 = \frac{4.2}{4.2 + 2.1} q$$

$$= \frac{4.2}{6.3} \times 554.4\ \mu\text{C} = 369.6\ \mu\text{C}$$

and $q_3 = q_4 = \frac{2.1}{4.2 + 2.1} q = \frac{2.1}{6.3} \times 554.4\ \mu\text{C}$

$$= 184.8\ \mu\text{C}$$

$$V_1 = \frac{q_1}{C_1} = \frac{554.4}{8.4} = 66\ \text{V} = V_5$$

$$V_2 = \frac{q_2}{C_2} = \frac{369.6}{4.2} = 88\ \text{V}$$

$$V_3 = V_4 = \frac{q_3}{C_3} = \frac{184.8}{4.2} = 44\ \text{V}$$

22. Let C_1 and C_2 be the capacitances of A and B respectively.

$$\therefore \quad C_1 = \frac{k_1\varepsilon_0 A_1}{d_1}, C_2 = \frac{k_1\varepsilon_0 A_2}{d_2}$$

Now, $\quad V_1 = \frac{C_2}{C_1 + C_2} V$

$$\Rightarrow \frac{C_2}{C_1 + C_2} = \frac{130}{230} = \frac{13}{23} \quad \text{...(i)}$$

$$V_2 = \frac{C_1}{C_1 + C_2} V$$

$$\Rightarrow \quad \frac{C_1}{C_1 + C_2} = \frac{10}{23} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$\frac{C_1}{C_2} = \frac{10}{13}$$

If dielectric slab of C_1 is replaced by one for which $k = 5$ then,

$$C_1' = \frac{5\varepsilon_0 A_1}{d_1} = \frac{5}{2} C_1$$

$$\therefore \quad \frac{V_2'}{V_1'} = \frac{C_1'}{C_2} = \frac{5C_1}{2C_2} = \frac{50}{26}$$

$$V_2' + \frac{50}{26} V_1' = 230$$

Also,

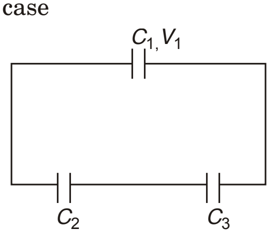
$$V_1' + V_2' = 230$$

$$V_1' = \frac{50}{26} V_1'$$

$$V_1' = 78.68 \text{ V}$$

and $\quad V_2 = 151.32$ V

23. In this case



Common potential,

$$V = \frac{C_1 V_1}{C_1 + \frac{C_2 C_3}{C_2 + C_3}}$$

$$V = \frac{1 \times 110}{1 + 1.2}$$

$$= \frac{110}{2.2}$$

$$= 50 \text{ V}$$

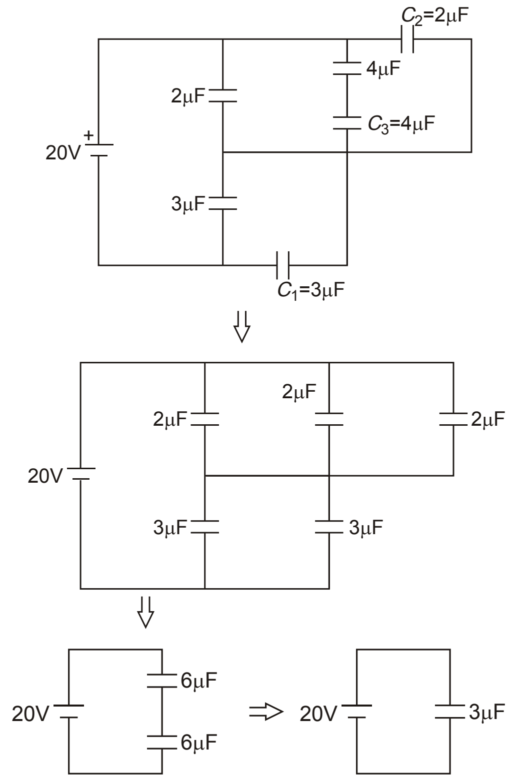
Charge flown through connecting wires,

$$= \frac{C_2 C_3}{C_2 + C_3} V$$

$$= 1.2 \times 50$$

$$= 60 \ \mu\text{C}$$

24. (a) Hence, effective capacitance across the battery is 3 μF.



(b) $q = CV = 3 \times 20 = 60 \ \mu\text{C}$

(c) Potential difference across C_1

$$V_1 = \frac{6}{6+6} \times 20 = 10 \text{ V}$$

$$q_1 = C_1 V_1 = 3 \times 10 = 30 \ \mu\text{C}$$

(d) Potential difference across C_2

$$V_2 = \frac{6}{6+6} \times 20 = 10 \text{ V}$$

$$q_2 = C_2 V_2 = 2 \times 10 = 20 \ \mu\text{C}$$

(e) Potential difference across C_3

$$V_3 = \frac{4}{4+4} \times V_2 = 5 \text{ V}$$

$$q_3 = C_3 V_3 = 4 \times 2 = 20 \ \mu\text{C}$$

25. (a) When switch S_2 is open, C_1 and C_3 are in series, C_2 and C_4 are in series their effective capacitances are in parallel with each other.

Hence,

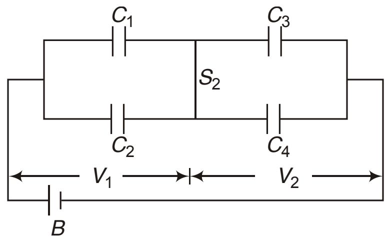
$$q_1 = q_3 = \frac{C_1 C_3}{C_1 + C_3} V$$

$$= \frac{1 \times 3}{1 + 3} \times 12 = 9\,\mu\text{C}$$

$$q_2 = q_4 = \frac{C_2 C_4}{C_2 + C_4}$$

$$= \frac{2 \times 4}{2 + 4} \times 12 = 16\,\mu\text{C}$$

(b) When S_2 is closed, C_1 is in parallel with C_2 and C_3 is in parallel with C_4.



Therefore,

$$V_1 = V_2 = \frac{C_3 + C_4}{C_1 + C_2 + C_3 + C_4}\,\text{V}$$

$$= \frac{7}{10} \times 12 = 8.4\,\text{V}$$

$$V_3 = V_4 = \frac{C_1 + C_2}{C_1 + C_2 + C_3 + C_4}\,\text{V}$$

$$= \frac{3}{10} \times 12 = 3.6\,\text{V}$$

$$q_1 = C_1 V_1 = 1 \times 8.4 = 8.4\,\mu\text{C}$$

$$q_2 = C_2 q_2 = 2 \times 8.4 = 16.8\,\mu\text{C}$$

$$q_3 = C_3 V_3 = 3 \times 3.6 = 10.8\,\mu\text{C}$$

$$q_4 = C_4 V_4 = 4 \times 3.6 = 14.4\,\mu\text{C}$$

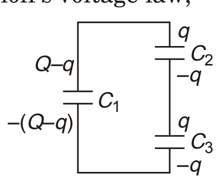
26. Initial charge on C_1

$$Q = C_1 V_0$$

Now, if switch S is thrown to right.

Let charge q flows from C_1 to C_2 and C_3.

By Kirchhoff's voltage law,



$$\frac{q}{C_2} + \frac{q}{C_3} - \frac{Q - q}{C_1} = 0$$

$$q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) = \frac{Q}{C_1}$$

$$q = \frac{C_2 C_3 Q}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$

$$= \frac{C_1 C_2 C_3 V}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$

$$\therefore \quad q_1 = Q - q = \frac{C_1^2 (C_2 + C_3) V}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$

$$q_2 = q_3 = q = \frac{C_1 C_2 C_3 V}{C_1 C_2 + C_2 C_3 + C_3 C_1}$$

27. $C = \dfrac{\varepsilon_0 A}{d}$, $q = CV = \dfrac{\varepsilon_0 A V}{d}$

(a) $C' = \dfrac{\varepsilon_0 A}{2d}$, $q' = q$

(As battery is disconnected)

$$V' = \frac{q'}{C'} = 2\,\text{V}$$

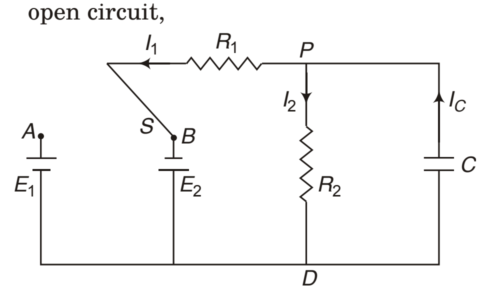
(b) $V_i = \dfrac{1}{2C} V^2 = \dfrac{\varepsilon_0 A V^2}{2d}$

$$U_f = \frac{1}{2} C' V'^2 = \frac{1}{2} \cdot \frac{\varepsilon_0 A}{2d} (2V)^2$$

$$= \frac{\varepsilon_0 A V^2}{d}$$

(c) $W = U_f - U_i = \dfrac{\varepsilon_0 A V^2}{2d}$

28. In the steady state, capacitor behaves as open circuit,



$$I_1 = I_2 = \frac{E_1}{R_1 + R_2} = 1\text{ mA and } I_C = 0$$

$$V_{PD} = I_2 R_2 = \frac{E_1 R_2}{R_1 + R_2}$$

When switch is shifted to B,

At this instant,

$$V_{PD} = \frac{E_1 R_2}{R_1 + R_2}$$

$$I_2 = \frac{V_{PD}}{R_2} = \frac{E_1}{R_1 + R_2} = 1\text{ mA}$$

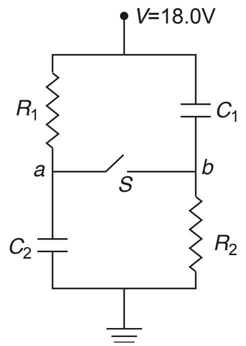
$$I_1 = \frac{E_2 + V_{PD}}{R_1} = \frac{E_2 + \dfrac{E_1R_2}{R_1 + R_2}}{R_1}$$

$$= \frac{(R_1 + R_2)E_2 + E_1R_2}{R_1}$$

$$= 2 \text{ mA}$$

$$I_C = I_1 + I_2 = 1 + 2 = 3 \text{ mA}$$

29. (a) When switch S is open, no current pass through the circuit,



Hence,

$$V_b - 0 = 0$$

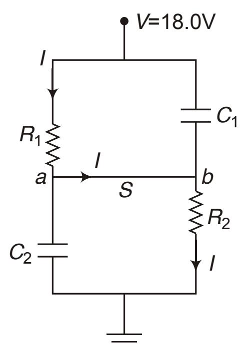
$$V_b = 0$$

$$18 - V_a = 0 \Rightarrow V_a = 18 \text{ V}$$

$$\Rightarrow \quad V_a - V_b = 18 \text{ V}$$

(b) a is at higher potential.

(c) When switch S is closed,



$$I = \frac{V}{R_1 + R_2} = 2 \text{ A}$$

$$V_b - 0 = IR_2 = 2 \times 3$$

$$V_b = 6 \text{ V}$$

(d) $q_1 = C_1V = 6 \times 18 = 108\ \mu\text{C}$

$$q_2 = C_2V = 3 \times 18 = 54\ \mu\text{C}$$

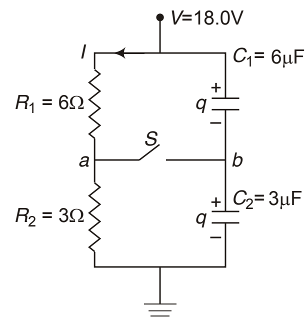
After closing the switch,

$$q_1' = C_1V_1 = 6 \times 12 = 72\ \mu\text{C}$$

$$q_2' = C_2V_2 = 3 \times 6 = 18\ \mu\text{C}$$

$\Delta q_1 = 18\ \mu\text{C}$, $\Delta q_2 = -36\ \mu\text{C}$

30. (a) $I = \dfrac{V}{R_1 + R_2} = \dfrac{18}{9} = 2 \text{ A}$



$$q = \frac{C_1C_2}{C_1 + C_2}V = 2 \times 18 = 36\ \mu\text{C}$$

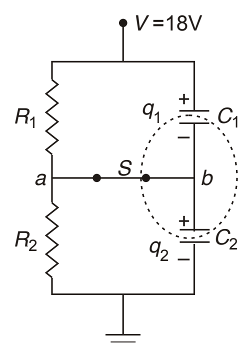
Now, $V_a - 0 = IR_2 \Rightarrow V_a = 6 \text{ V}$

and $V_b - 0 = \dfrac{q}{C_2} = \dfrac{36}{3} = 12 \text{ V}$

$$V_a - V_b = -6 \text{ V}$$

(b) b is at higher potential.

(c) When switch S is closed, in steady state,



$$V_a - v_b = 6 \text{ V}$$

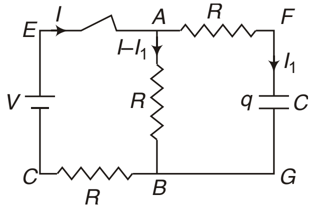
$$q_1 = C_1V_1 = 6 \times 12 = 72\ \mu\text{C}$$

$$q_2 = C_2V_2 = 3 \times 6 = 18\ \mu\text{C}$$

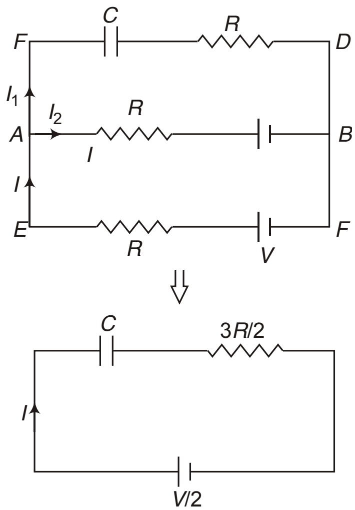
Charge flown through S

$$= q_1 - q_2 = 72 - 18 = 54\ \mu\text{C}$$

31.



(a) Consider the circuit as combination of two cells of emf E and OV.



$$E_e = \frac{E_1R_2 + E_2R_1}{R_1 + R_2} = \frac{V}{2}$$

$$R_e = R + \frac{R}{2} = \frac{3R}{2}$$

$$q = q_0(1 - e^{-t/\tau})$$

$$q_0 = \frac{CE}{2}$$

$$\tau = \frac{3RC}{2}$$

$$\therefore \quad q = \frac{CE}{2}(1 - e^{-2t/3RC})$$

(b) $I_1 = \dfrac{dq}{dt} = \dfrac{E}{3R}e^{-2t/3RC}$

In loop $EDBA$ $\dfrac{q}{C} + I_1R - I_2R = 0$

$$I_2 = \frac{q}{RC} + I_1$$

$$= \frac{E}{2R}(1 - e^{-2t/3RC}) + \frac{E}{3R}e^{-2t/3RC}$$

$$= \frac{E}{6R}(3 - e^{-2t/3RC})$$

Objective Questions (Level 1)

1. $F = \dfrac{Q^2}{2\varepsilon_0 A}$ is independent of d.

2. $C = \dfrac{q}{V}$

On connecting the plates V becomes zero.

3. The system can assumed to a parallel combination of two spherical conductors.

$$C = C_1 + C_2 = 4\pi\varepsilon_0 a + 4\pi\varepsilon_0 b$$
$$= 4\pi\varepsilon_0(a + b)$$

4. $V = \dfrac{q}{C}$

On connecting in series

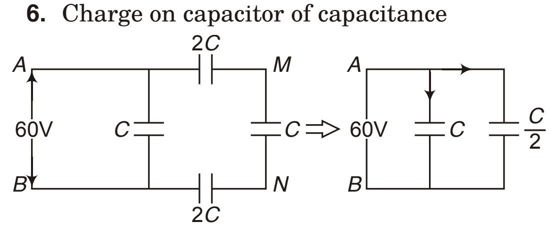
$q' = q =$ Charge on any capacitor

$$C' = \frac{C}{n}$$

$$\therefore \quad V' = \frac{nq}{C} = nV$$

5. Incorrect diagram.

6. Charge on capacitor of capacitance



$$\frac{C}{2} = \frac{C}{2}V = 30\text{ C}$$

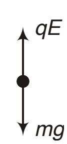
$$V_{MN} = \frac{q}{C} = 30\text{ V}$$

7. For equilibrium,

$$qE = mg$$

$$q\frac{V}{d} = \frac{4}{3}\pi r^3 \rho g$$

$$V \propto \frac{r^3}{q}$$

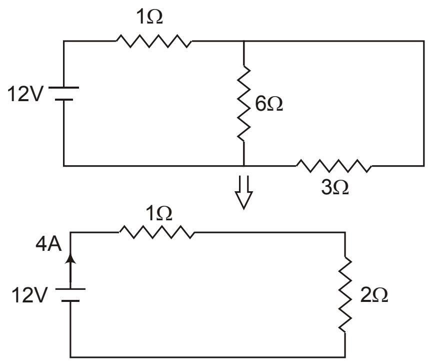


$$\frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^3 \times \frac{q_1}{q_2}$$

$$\Rightarrow \quad V_2 = 4\ \text{V}$$

8. Electric field between the plates is uniform but in all other regions it is zero.

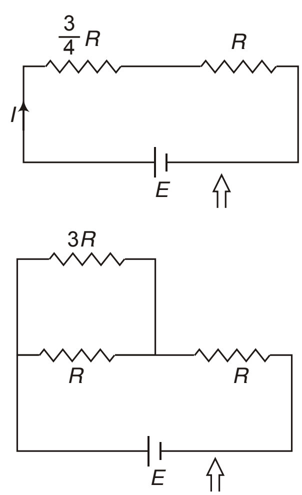
9. Initially the capacitor offers zero resistance.

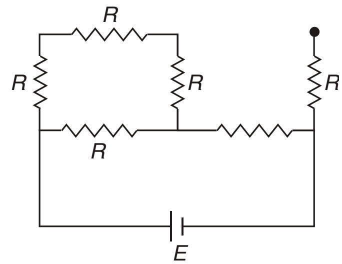


$$i = \frac{12}{1+2} = 4\ \text{A}$$

10. $q = CV = CE$

11. In the steady state, capacitor behaves as open circuit. the equivalent diagram is given by





$$I = \frac{E}{R + \frac{3}{4}R} = \frac{4E}{7R}$$

But potential difference across capacitor,

$$V = IR$$

$$\Rightarrow \quad 10 = \frac{4E}{7R} R$$

$$\Rightarrow \quad E = 17.5\ \text{V}$$

12. As all the capacitors are connected in series potential difference across each capacitor is

$$V = \frac{E}{4} = \frac{10}{4} = 2.5\ \text{V}$$

$$V_A - V_N = 3\text{V} = 7.5\ \text{V}$$

$$V_A = 7.5\ \text{V}$$

$$V_N - V_B = 2.5\ \text{V}$$

$$V_B = -2.5\ \text{V}$$

13. Heat produced = Loss of energy

$$= \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

$$= \frac{2 \times 10^{-6} \times 2 \times 10^{-6}}{2(2+2) \times 10^{-6}}(100 - 0)$$

$$= 5 \times 10^{-3}\ \text{J} = 5\ \text{mJ}$$

14.

$$q = q_0 e^{-t/\eta}$$

$$I = I_0 e^{-t/\eta}$$

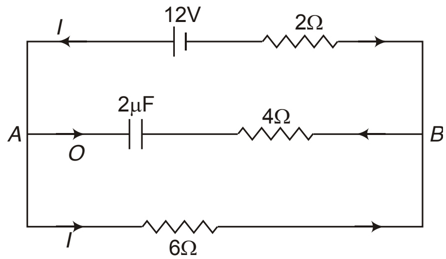
$$P = I^2 R = I_0^2 e^{-2t/\eta} R = P_0 e^{-2t/\eta}$$

$$\Rightarrow \quad \eta' = \frac{\eta}{2}$$

15. Common potential $= \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2} = \dfrac{E}{2}$

16. $V_A - V_B = 6 + 3 \times 2 - \dfrac{9}{1} + 3 \times 3 = 12\ \text{V}$

17. In the steady state, current through battery



$$I = \frac{12}{6+2} = \frac{3}{2}\ \text{A}$$

Potential difference across the capacitor,

$$V_{AB} = 6 \times \frac{3}{2} = 9\ \text{N}$$

$\therefore \quad q = CV_{AB} = 2 \times 9 = 18\ \mu\text{C}$

18. C_2 and C_3 are in parallel

Hence, $V_2 = V_3$

Again Kirchhoff's junction rule

$$-q_1 + q_2 + q_3 = 0$$

$\Rightarrow \quad q_1 = q_2 + q_3$

19. For the motion of electron

$$R = \frac{mu^2 \sin 2\theta}{eE} = l \qquad \text{...(i)}$$

and $$H = \frac{mu^2 \sin^2 \theta}{2eE} = d \qquad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i),

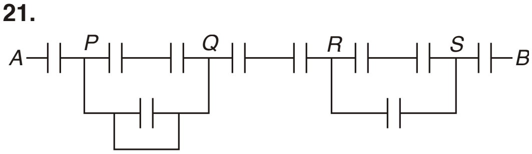
$$\tan\theta = \frac{4d}{l}$$

20. $V = Ed \Rightarrow d = \dfrac{V}{E} = \dfrac{2V\varepsilon_0}{6}$

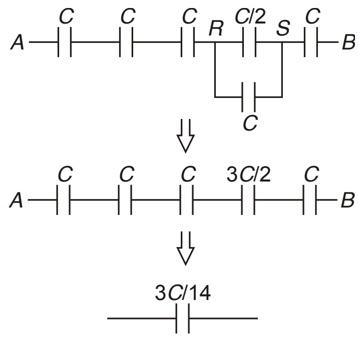
$$= \frac{2 \times 5 \times 8.85 \times 10^{-12}}{10^{-7}}$$

$$= 8.85 \times 10^{-4} = 0.88\ \text{mm}$$

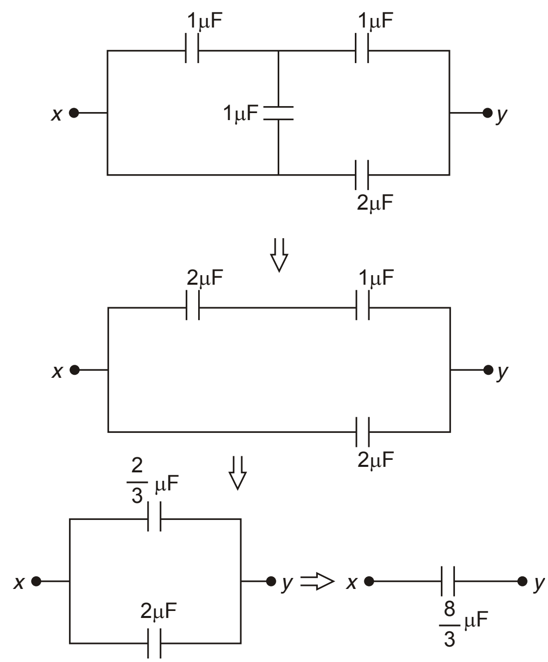
21.



P and Q are at same potential, hence capacitor connected between them have no effect on equivalent capacitance.



22.



23. $C_1 = \dfrac{k_1\varepsilon_0 A}{2d} + \dfrac{k_2\varepsilon_0 A}{2d}$

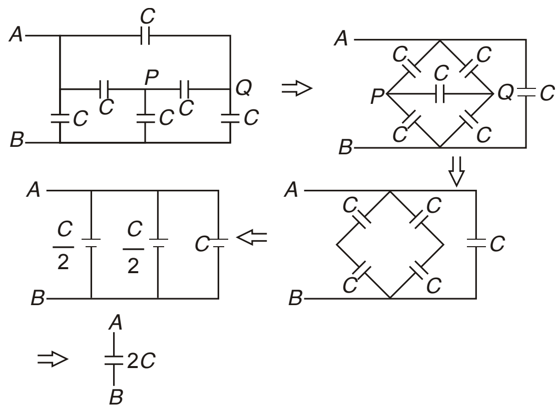
$$= \frac{(k_1 + k_2)\varepsilon_0 A}{2d} \qquad \text{(Parallel grouping)}$$

$$\frac{1}{C_2} = \frac{d}{2k_1\varepsilon_0 A} + \frac{d}{2k_2\varepsilon_0 A} \text{(Series grouping)}$$

$$C_2 = \frac{2k_1k_2}{k_1 + k_2}\,\frac{\varepsilon_0 A}{d}$$

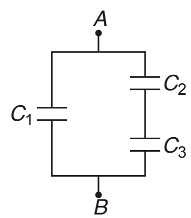
$$\frac{C_1}{C_2} = \frac{(k_1 + k_2)^2}{4k_1k_2} = \frac{(2+3)^2}{4 \times 2 \times 3} = \frac{25}{24}$$

24.



25. Cases (a), (b) and (c) are balanced Wheatstone bridge.

26. The given arrangement can be considered as the combination of three capacitors as shown in figure.



Hence,
$$C_1 = \frac{k_1\varepsilon_0 A}{2d}$$

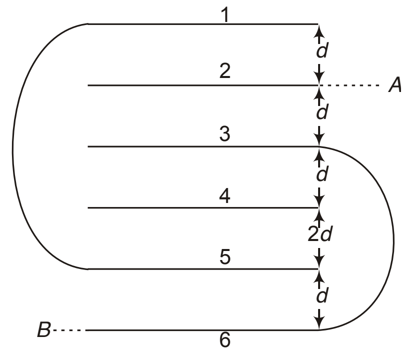
$$C_2 = \frac{k_2\varepsilon_0 \frac{A}{2}}{d/2} = \frac{k_2\varepsilon_0 A}{d}$$

$$C_3 = \frac{k_3\varepsilon_0 \frac{A}{2}}{d/2} = \frac{k_3\varepsilon_0 A}{d}$$

Effective capacitance,

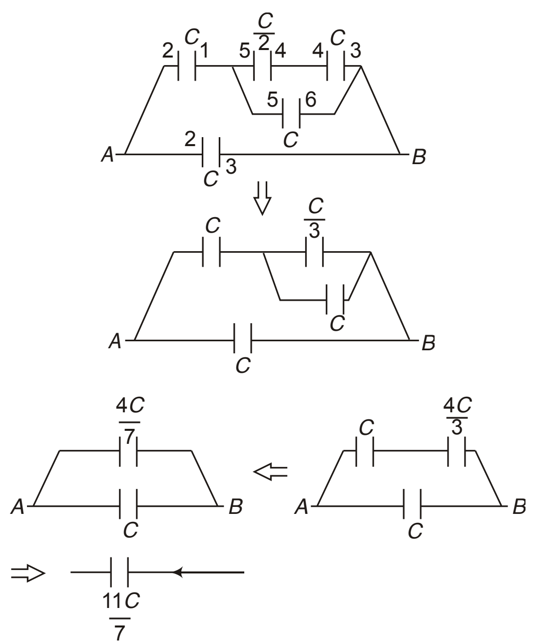
$$C = C_1 + \frac{C_2C_3}{C_1 + C_2} = \frac{\varepsilon_0 A}{d}\left[\frac{k_1}{2} + \frac{k_2k_3}{k_2 + k_3}\right]$$

27. Here, plate 1 is connected to plate 5 and plate 3 is connected to plate 6.



Capacitance of all other capacitance is same, i.e., $C = \frac{\varepsilon_0 A}{d}$ but that of formed by plates 4 and 5 is $\frac{C}{2}$ as distance between these two plates is $2d$.

The equivalent circuit is shown in figure.



$$\therefore C_{eq} = \frac{11}{7}C = \frac{11\varepsilon_0 A}{7d} = \frac{11}{7} \times 7\ \mu\text{F} = 11\ \mu\text{F}$$

JEE Corner

Assertion and Reason

1. Capacitance $=\frac{q}{V}$ is constant for a given capacitor.
2. Reason correctly explains the assertion.
3. $U=\frac{1}{2}qV, W=qV$
4. For discharging of capacitor

$$q=q_0e^{-t/\tau}$$
$$\frac{dq}{dt}=-\frac{q_0}{\tau}e^{-t/\tau}$$
$$=-\frac{q_0}{RC}e^{-t/\tau}$$

Hence, more is the resistance, less will be the slope.

5. Charge on two capacitors will be same only if both the capacitors are initially uncharged.
6. As potential difference across both the capacitors is same, charge will not flow through the switch.
7. C and R_2 are shorted.
8. Time constant for the circuit,

$$\tau=RC$$

9. In series, charge remains same

and $$U=\frac{q^2}{2C}\Rightarrow U\propto\frac{1}{C}$$

10. In series charge remains same

$\therefore$ $$V_1=\frac{q}{C_1}, V_2=\frac{q}{C_2}$$

On inserting dielectric slab between the plates of the capacitor, C_2 increases and hence, V_2 decreases. So more charge flows to C_2.

Objective Questions (Level 2)

1. $$E=\begin{cases}-\frac{4Q}{\varepsilon_0 A}\hat{\mathbf{i}} & \text{for } x<d\\ -\frac{2Q}{\varepsilon_0 A}\hat{\mathbf{i}} & \text{for } d<x<2d\\ \frac{4Q}{\varepsilon_0 A}\hat{\mathbf{i}} & \text{for } 2d<x<3d\end{cases}$$

2. Let E_0 = external electric field

and E = electric field due to sheet

$\therefore$ $$E_1=E_0-E=8$$
$$E_2=E_0+E=12$$
$\Rightarrow$ $$E=2\text{ V/m}\Rightarrow\frac{\sigma}{2\varepsilon_0}=2$$
$$\sigma=4\varepsilon_0$$

3. When the switch is just closed, capacitors behave like short circuit, no current pass through either 6 Ω or 5 Ω resistor.
4. For charging of capacitor

$$I=I_0e^{-t/\tau}$$
$$\ln I=\log I_0-\frac{t}{\tau}$$
$$\ln I=\ln\frac{V}{R}-\frac{t}{RC}$$

But, $I_{01}=I_{02}$

$\Rightarrow$ $$\frac{V_1}{R_1}=\frac{V_2}{R_2}$$

Also, $\frac{1}{R_1C_1}>\frac{1}{R_2C_2}$

$\Rightarrow$ $$R_2C_2>R_1C_1$$

As only two parameters can be different,

$$C_1=C_2$$
$$R_2>R_1$$
and $$V_2>V_1$$

5. Charge on capacitor at the given instant.

$$q=\frac{q_0}{2}=\frac{CE}{2}$$

Heat produced = Energy stored in capacitor

$$=\frac{q^2}{2C}=\frac{CE^2}{8}$$

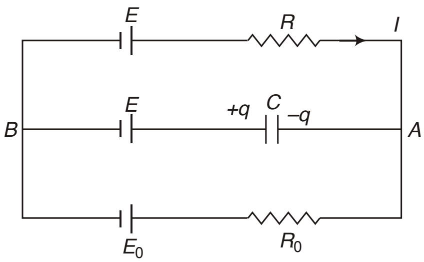
Heat liberated inside the battery,

$$=\frac{r}{r+2r}\times\text{Total heat produced}$$
$$=\frac{CE^2}{24}$$

6. Capacitor is not inside any loop.

7. $I = \dfrac{E - E_0}{R + R_0}$

$$V_{BA} = \frac{q}{C} - E$$



$$-E + IR = \frac{q}{C} - E$$

$$q = IRC + \frac{(E - E_0)RC}{R + R_0}$$

8. $C = \dfrac{C_1 C_2}{C_1 + C_2}$

$$C_1 = C_2 = \frac{\varepsilon_0 A}{d}$$

$$C = \frac{\varepsilon_0 A}{2d}$$

$$C_1' = \frac{2\varepsilon_0 A}{d}, C_2' = \frac{\varepsilon_0 A}{2d}$$

$$C' = \frac{C_1' C_2'}{C_1' + C_2'} = \frac{2\varepsilon_0 A}{5d} < C$$

9. $$R_e = \frac{R}{3}$$

$$\tau = R_e C = \frac{RC}{3}$$

$$q = q_0(1 - e^{-t/\tau})$$

$$= CV(1 - e^{-3t/RC})$$

10. Energy loss $= \dfrac{C_1 C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$

$$= \frac{2 \times 4}{2 \times (2 - 14)}(100 - 50)^2 \times 10^{-6}$$

$$= 1.7 \times 10^{-3} \text{ J}$$

11. $q = q_0 e^{-t/RC}$

$$I = -\frac{dq}{dt} = \frac{q_0}{RC} e^{-t/RC}$$

at $t = 0$

$$I = \frac{q_0}{RC} = 10$$

$$V_0 = \frac{q_0}{C} = 10 \times R = 10 \times 10 = 100 \text{ V}$$

12. $V_A = -V_B$

ie, $V_A - V_P = V_P - V_B$ $\quad [V_P = 0]$

$$\Rightarrow \quad \frac{q}{C_{123}} = \frac{q}{C_n}$$

$$\Rightarrow \quad C_n = C_{123}$$

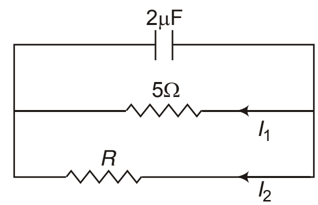
$$\Rightarrow \quad \frac{1}{C_n} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

13. When connected with reverse polarity

$$H = \frac{C_1 C_2}{2(C_1 + C_2)}(V_1 + V_2)^2$$

$$= \frac{C \times 2C}{2(C + 2C)} \times (V + 4V)^2 = \frac{25}{3} CV^2$$

14. $\dfrac{H_1}{H_2} = \dfrac{R_2}{R_1} = \dfrac{R}{S}$



Also, $H_1 + H_2 = \dfrac{1}{2} CV^2 = \dfrac{1}{2} \times 2 \times 10^{-6} \times (5)^2$

$$H_1 + H_2 = 25 \text{ μJ}$$

$$H_2 = 25 - 10 = 15 \text{ μJ}$$

$$\frac{10}{15} = \frac{R}{5} \Rightarrow R = \frac{10}{3} \Omega$$

15. When current in the resistor is 1 A.

$$IR + \frac{q}{C} = E$$

$$1 \times 5 + \frac{q}{2} = 10$$

$$\Rightarrow \quad q = 10 \text{ μC}$$

When the switch is shifted to position 2. In steady state, charge on capacitor

$$q' = 5 \times 2 = 10 \text{ μC}$$

but with opposite polarity.

$\therefore$ Total charge flown through 5 V battery,

$$= q + q' = 20 \text{ μC}$$

Work done by the battery $= 20 \times 5$

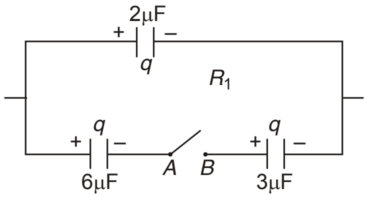
$$= 100 \text{ μJ}$$

Heat produced $= W - \Delta U$

But, $\quad \Delta U = 0$

$\therefore \quad H = W = 100$ μJ

16. $V_A - V_B = -\frac{q}{6} + \frac{q}{2} - \frac{q}{3} = 0$



Hence, no charge will flow from A to B.

17. As potential difference across both the capacitors is same, they are in parallel.

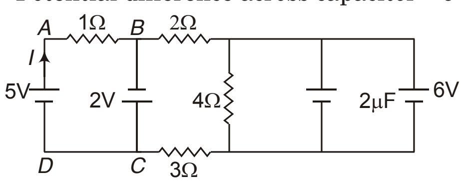
Hence, effective capacitance,

$$C = \frac{2\varepsilon_0 A}{d}$$

$$U = \frac{1}{2}CV^2 = \frac{\varepsilon_0 A}{d}V^2$$

18. Rate of charging decreases as it just charged.

19. Potential difference across capacitor = 6 V



$$q = CV = 2 \times 6 = 12\ \mu C$$

In loop $ABCD$,

$$I \times 1 - 2 - 5 = 0 \Rightarrow I = 7\ A$$

20. While charging

$$R_e = R \Rightarrow \tau = RC$$

While discharging

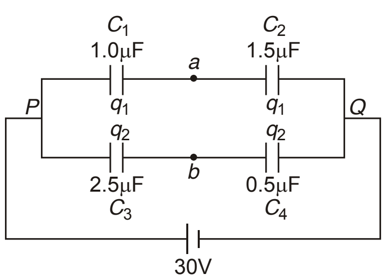
$$R_e = 2R \Rightarrow \tau = 2RC$$

21. Common potential,

$$V = \frac{C_2V_2 - C_1V_1}{C_1 + C_2} = \frac{3 \times 100 - 1 \times 100}{1 + 3}$$

$$= 25\ V$$

22. $q_1 = \frac{1 \times 1.5}{1 + 1.5} \times 30 = 18\ \mu C$



$$q_2 = \frac{2.5 \times 0.5}{2.5 + 0.5} \times 30$$

$$= 12.5\ \mu C$$

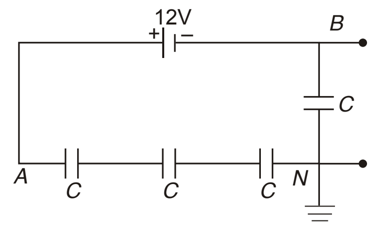
$$\therefore\ V_p - V_a = \frac{q_1}{C_1} = 18\ V$$

$$V_p - V_b = \frac{q_2}{C_3}$$

$$= \frac{12.5}{2.5} = 5\ V$$

$$V_b - V_a = 13\ V$$

23. As all the capacitors are identical, potential difference across each capacitor,



$$V = \frac{E}{4} = 3\ V$$

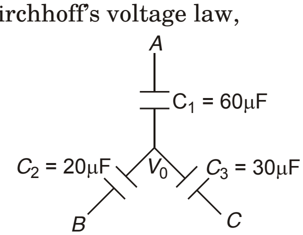
$$V_N - V_B = 3\ V$$

$$V_B = -3\ V$$

$$V_A - V_B = 12\ V$$

$$\Rightarrow\ V_A = 9\ V$$

24. By Kirchhoff's voltage law,



$$q_1 + q_2 + q_3 = 0$$

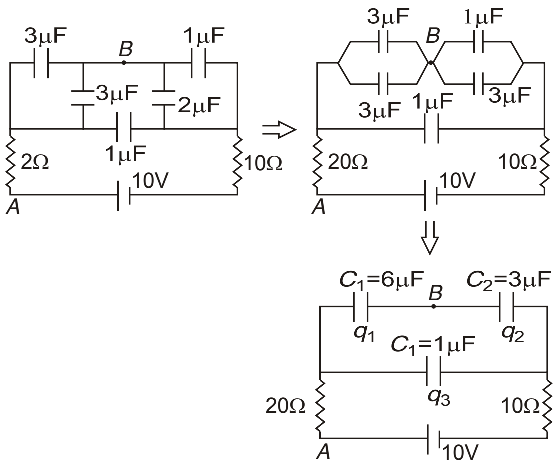
$$C_1(C_A - V_0) + C_2(V_B - V_0) + C_3(V_C - V_0) = 0$$

$$V_0 = \frac{C_1V_A + C_2V_B + C_3V_C}{C_1 + C_2 + C_3}$$

$$V_0 = \frac{60 \times 6 + 2 \times 20 + 3 \times 30}{60 + 20 + 30}$$

$$= \frac{49}{11}\ V$$

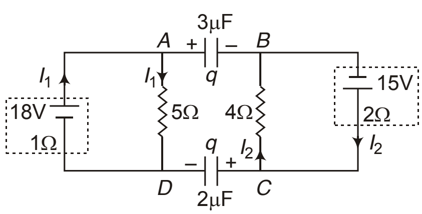
25. In the steady state, there will be no current in the circuit.



$$\therefore \quad V_1 = \frac{C_2}{C_1 + C_2} V = \frac{3}{3+6} 10 = \frac{10}{3} \text{ V}$$

26. $I_1 = \frac{E_1}{R_1 + r_1} = \frac{18}{5+1} = 3 \text{ A}$

$$I_2 = \frac{E_2}{R_2 + r_2} = \frac{15}{4+2} = 2.5 \text{ A}$$



In loop *ABCD*,

$$\frac{q}{3} - I_2 R_2 + \frac{q}{2} - I_1 R_1 = 0$$

$$\Rightarrow \quad \frac{5q}{6} = 3 \times 5 + 2.5 \times 4 \Rightarrow q = 30 \, \mu\text{C}$$

27. During discharging

$$q = q_0 \, e^{-t/\tau}$$

$$q_0 = CE = 10 \, \mu\text{C}$$

at $t = 12$ s,

$$q = 10e^{-12/6} = 10e^{-2}$$

$$= (0.37)^2 10 \, \mu\text{C}$$

28. $q = \frac{C_1 C_2}{C_1 + C_2} (E_1 - E_2)$

$$V_{ap} = \frac{q}{C_2} = \frac{C_1}{C_1 + C_2} (E_1 - E_2)$$

$$= \left(\frac{E_1 - E_2}{C_1 + C_2} \right) C_1$$

29. $H = \frac{C_1 C_2}{2(C_1 + C_2)} (V_1 - V_2)^2$ is independent of resistance.

30. Immediately after switch is closed, capacitor behaves like short circuit.

31. $i_1 = \frac{V}{2R} e^{-t/RC}, \; i_2 = \frac{V}{R} e^{-t/RC}$

$$\frac{i_1}{i_2} = \frac{1}{2} e^{\frac{5t}{6RC}}$$

Increases with time.

32. $R = \frac{\rho d}{A} = \frac{d}{\sigma A}$

$$C = \frac{k\varepsilon_0 A}{d}$$

$$\tau = RC = \frac{d}{\sigma A} \times \frac{k\varepsilon_0 A}{d}$$

$$= \frac{\varepsilon_0}{6} = \frac{8.85 \times 10^{-12}}{7.4 \times 10^{-12}} = 6 \text{ s}$$

33. $i = i_0 e^{-t/\tau}$

$$\Rightarrow \quad \frac{i_0}{2} = i_0 e^{\frac{-\ln 4}{RC}}$$

$$\Rightarrow \quad \frac{\ln 4}{RC} = \ln 2$$

$$\Rightarrow \quad \ln 4 = \ln 2^{RC}$$

$$\Rightarrow \quad RC = 2$$

$$\Rightarrow \quad R = \frac{2}{C} = \frac{2}{0.5} = 4 \, \Omega$$

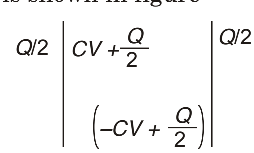
34. Potential difference across each capacitor is equal, hence they are in parallel, charge on each capacitor

$$q = C_e V = 2 \times 10 = 20 \, \mu\text{C}$$

As plate *C* contributed to two capacitors, charge on plate,

$$C = 2q = +40 \, \mu\text{C}$$

35. Charge distribution on the plates of the capacitor is shown in figure

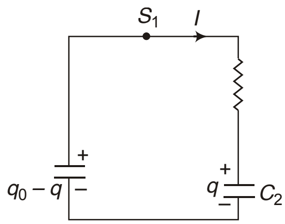


$$\therefore \quad V' = \frac{Q'}{C} = \frac{CV + \frac{Q}{2}}{C}$$

$$= V + \frac{Q}{2C}$$

36. Let q be the charge on C_2 (or charge flown through the switches at any instant of time)

By Kirchhoff's law



$$IR + \frac{q}{C_2} - \frac{q_0 - q}{C_1} = 0$$

$$\frac{dq}{dt} = \frac{C_2 q_0 - (C_1 + C_2) q}{C_1 C_2 R}$$

$$\int_0^q \frac{dq}{C_2 q_0 - (C_1 + C_2) q} = \int_0^t \frac{dt}{C_1 C_2 R}$$

$$= \frac{1}{C_1 + C_2} \left[\ln |C_2 q_0 - (C_1 + C_2) q\right]_0^q$$

$$= \frac{1}{C_1 C_2 R} t$$

$$q = \frac{C_2 q_0}{C_1 + C_2}\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\tau = \frac{C_1 + C_2}{C_1 C_2 R}$$

or $$q = \frac{C}{C_1} q_0 \left(1 - e^{-\frac{t}{RC}}\right)$$

where, $$C = \frac{C_1 C_2}{C_1 + C_2}$$

37. $$H = \frac{C_1 C_2}{2(C_1 + C_2)} (V_1 - V_2)^2$$

$$= \frac{C_1 C_2}{2(C_1 + C_2)} \left(\frac{q_0}{C_1}\right)^2$$

$$= \frac{C_2 q_0^2}{2C_1 (C_1 + C_2)} = \frac{C\, q_0^2}{2C_1^2}$$

38. Electric field in the gap will remain same.

39. Electric field inside the dielectric slab

$$E' = \frac{E}{k} = \frac{V}{kd}.$$

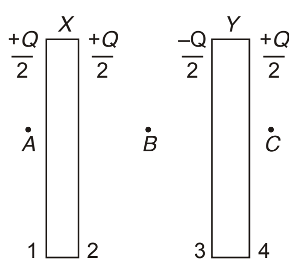
More Than One Correct Options

1. Charge distribution is shown in figure

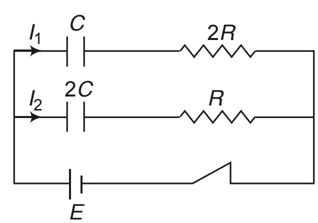
$$E_B = E_1 + E_2 + E_3 + E_4$$

$$= \frac{Q}{4\varepsilon_0 A} + \frac{Q}{4\varepsilon_0 A} + \frac{Q}{4\varepsilon_0 A} - \frac{Q}{4\varepsilon_0 A}$$

$$= \frac{Q}{2\varepsilon_0 A}$$



$|E_A| = |E_C| = |E_1 + E_2 - E_3 + E_4|$ but E_A and E_C have opposite direction.



$$q_2 = 2CE\left(1 - e^{-\frac{t}{RC}}\right)$$

$$\frac{dq_1}{dt} = \frac{E}{R} e^{-\frac{t}{RC}}$$

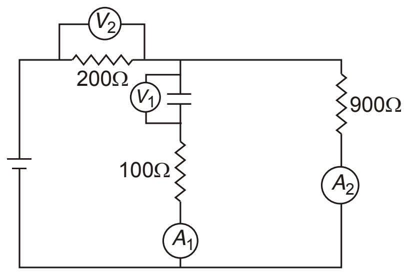
$$\frac{dq_2}{dt} = \frac{2E}{R} e^{-\frac{t}{2RC}}$$

$$\frac{q_1}{q_2} = \frac{1}{2} \Rightarrow \frac{q_{0_1}}{q_{0_2}} = \frac{1}{2}$$

$$\tau_1 = \tau_2 = RC$$

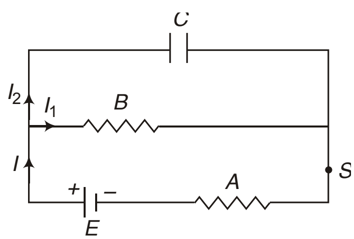
2. $$q_1 = CE\left(1 - e^{-\frac{t}{RC}}\right)$$

3. $$V_1 = \frac{q}{C} = \frac{4 \times 10^{-3}}{100 \times 10^{-6}} = 40\text{V}$$



$$I_1 = 0$$
$$I_2 = \frac{V_1}{900} = \frac{40}{900} = \frac{2}{45}\text{ A}$$
$$V_2 = I_2 \times 200 = \frac{2}{45} \times 200 = \frac{80}{9}\text{ V}$$
$$E = V_1 + V_2 = 40 + \frac{80}{9}$$
$$= \frac{440}{9}\text{ V}$$

4. Initially $I_1 = 0, I_2 = I = \frac{E}{R}$



As the capacitor starts charging,
I_2 decreases and I_1 increases,
In the steady state

$$I_1 = I = \frac{E}{R}, I_2 = 0$$

At any instant

$$P_1 = I_1^2 R, P_2 = I_2^2 R$$

Steady state potential difference across the capacitor,

$$V = \frac{E}{2}$$
$$U = \frac{1}{2}CV^2 = \frac{CE^2}{8}$$

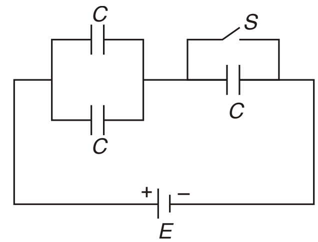
5. $F = \frac{Q^2}{2\varepsilon_0 A}$ independent of d.

$E = \frac{Q}{\varepsilon_0 A}$ independent of d.

$$U = \frac{Q^2}{2C} = \frac{Q^2 d}{2\varepsilon_0 A}$$

$$\Rightarrow \quad U \propto d$$
$$V = \frac{Q}{C} = \frac{Qd}{\varepsilon_0 A}$$
$$\Rightarrow \quad V \propto d$$

6. When switch S is open



$$C_e = \frac{C \times 2C}{C + 2C} = \frac{2}{3}C$$
$$q_1 = \frac{2}{3}CE$$

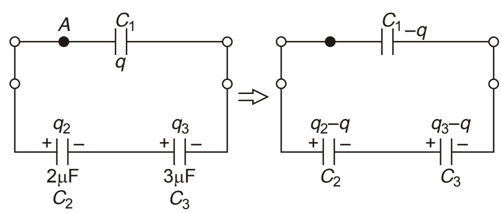
When switch S is closed

$$C_e' = 2C$$
$$q_2 = 2CE$$

Charge flown through the battery

$$\Delta q = q_2 - q_1 = \frac{4}{3}CE = \text{ positive}$$

7. Let charge q flows to C_1 at it falls to the free end of the wire.



By Kirchhoff's voltage law,

$$\frac{q_2 - q}{C_2} + \frac{q_3 - q}{C_3} - \frac{q}{C_1} = 0$$
$$q = \frac{\frac{q_2}{C_2} + \frac{q_3}{C_3}}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{q}{C_3}}$$
$$= \frac{V_2 + V_3}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$
$$q = \frac{150 + 120}{\frac{1}{2} + \frac{1}{3} + \frac{1}{1.5}} = 180\ \mu\text{C}$$
$$q_2' = q_2 - q = 150 \times 2 - 180$$
$$= 120\ \mu\text{C}$$

8. $C = \dfrac{\varepsilon_0 A}{d}$

$$U = \frac{Q^2}{2C} = \frac{Q^2 d}{2\varepsilon_0 A} \Rightarrow U \propto d$$

$$V = \frac{Q}{C} = \frac{Qd}{\varepsilon_0 A} \Rightarrow V \propto d$$

$$C = \frac{\varepsilon_0 A}{d} \Rightarrow C \propto \frac{1}{d}$$

$E = \dfrac{Q}{\varepsilon_0 A} \Rightarrow E$ is independent of d.

9. $R = 1 + 2 = 3\ \Omega, C = 2\text{ F}$

$$q_0 = CV_0 = 2 \times 6 = 12\text{C}$$

At any instant

$$q = q_0 \left(e^{\frac{-t}{RC}} \right)$$

$$I = \frac{dq}{dt} = \frac{q_0}{RC} e^{\frac{-t}{RC}}$$

at $t = 0$

$$I = \frac{q_0}{RC} = \frac{12}{3 \times 2} = 2\text{ A}$$

at $t = 6 \ln 2$

$$I = \frac{q_0}{RC} e^{\frac{-6 \ln 2}{6}} = \frac{12}{6} \times \frac{-1}{2}$$

$$= 2 \times \frac{1}{2} = 1\text{ A}$$

Potential difference across $1\ \Omega$ resistor

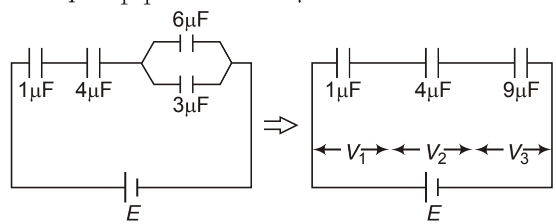
$\Rightarrow \quad 1 \times 1 = 1\text{ V}$

Potential difference across $2\ \Omega$ resistor

$\Rightarrow \quad 1 \times 2 = 2\text{ V}$

$\therefore$ By Kirchhoff's voltage law, potential difference across capacitors $= 1 + 2 = 3\text{ V}$.

10. $q = C_1 V_1 = 1 \times 10 = 10\ \mu\text{F}$



$$V_2 = \frac{q}{C_2} = \frac{10}{4} = 2.5\text{ V}$$

$$V_3 = \frac{q}{C_3} = \frac{10}{9}\text{ V}$$

Match the Columns

1. $C_1' = kC_1 = 8\ \mu\text{F}$,

$$C_2' = \frac{C_2}{k} = 2\ \mu\text{F}$$

$$q' = \frac{C_1' C_2'}{C_1' + C_2'} V = (1.6V)\ \mu\text{C}$$

$$q = (2V)\ \mu\text{C}$$

$\therefore q' < q$

$$U_2' = \frac{q'^2}{C_2'} = \frac{(1.6\,V)^2}{2 \times 2} = 0.64V^2,$$

$$U_2 = \frac{q^2}{2C_2} = \frac{(2)^2}{2 \times 2} = (1\,V)\ \mu\text{C}$$

$$U_2' < U_2$$

$$V_2' = \frac{q'}{C_2'} = \frac{1.6V}{8} = 0.2V,$$

$$V_2 = \frac{q}{C_2} = \frac{(2V)}{4} = 0.5V$$

$$V_2' < V$$

$$E_2' = \frac{q'}{\varepsilon_0 A} = \frac{1.6V}{\varepsilon_0 A},$$

$$E_2 = \frac{q}{k\varepsilon_0 A} = \frac{2V}{2 \times \varepsilon_0 A} = \frac{V}{\varepsilon_0 A}$$

$$E_2' > E_2$$

(a → q), (b → q), (c → q), (s → p).

2. Before switch S is closed, charge distribution is shown in figure (1).

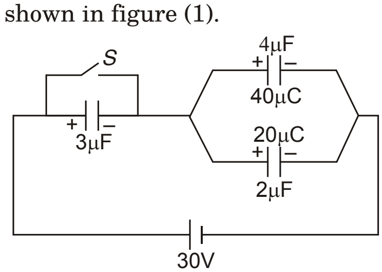


Fig. 1

After switch S is closed, charge distribution is shown in figure (2).

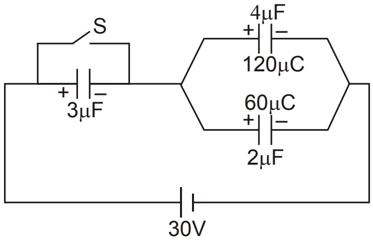
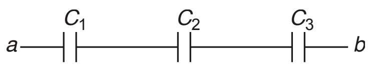


Fig. 2

(a → s), (b → p), (c → q), (d → s).

3. (a → q), (b → p, r), (c → q), (d → p, π)

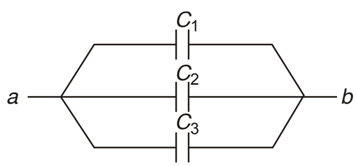
(p)



$$V_1 = V_2 = V_3 = \frac{V}{3}$$

$$q_1 = q_2 = q_3 = \frac{CV}{3}$$

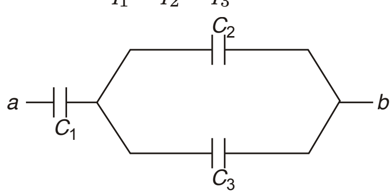
(q)



$$V_1 = V_2 = V_3 = V$$

$$q_1 = q_2 = q_3 = CV$$

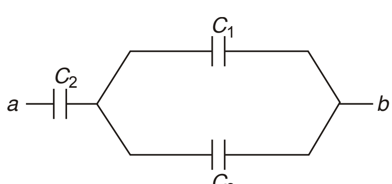
(r)



$$V_1 = \frac{2V}{3},\ V_2 = V_3 = \frac{V}{3}$$

$$q_1 = \frac{2CV}{3},\ q_2 = q_3 + \frac{CV}{3}$$

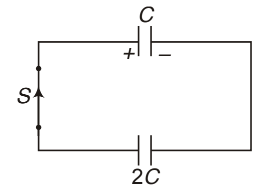
(s)



$$V_2 = \frac{2V}{3},\ V_1 = V_2 = \frac{V}{3}$$

$$q_2 = \frac{2CV}{3},\ q_1 = q_3 = \frac{CV}{3}$$

4. Common potential



$$V' = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{V}{3}$$

$$U_1 = \frac{1}{2}C_1V'^2 = \frac{1}{18}CV^2$$

$$U_2 = \frac{1}{2}C_2V'^2 = \frac{1}{9}CV^2$$

$$\Delta U = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

$$= \frac{C \times C}{2(C_1 + C_2)}(V - 0)^2$$

$$= \frac{1}{6}CV^2$$

(a → r), (b → p), (c → q).

5.
$$C_1 = \frac{k\varepsilon_0 A}{2d} + \frac{\varepsilon_0 A}{2d}$$

$$= (k + 1)\frac{\varepsilon_0 A}{2d} = \frac{3\,\varepsilon_0 A}{2d}$$

$$= \frac{1}{C_2} = \frac{d}{2k\varepsilon_0 A} + \frac{d}{2\varepsilon_0 A}$$

$$= \frac{d}{2\varepsilon_0 A}\left(\frac{1 + k}{k}\right)$$

$$\Rightarrow \quad C_2 = \frac{2k\varepsilon_0 A}{d\,(1 + k)} = \frac{4\varepsilon_0 A}{3d}$$

$$\frac{C_1}{C_2} = \frac{9}{8}$$

As combination is series, $q_1 = q_2$

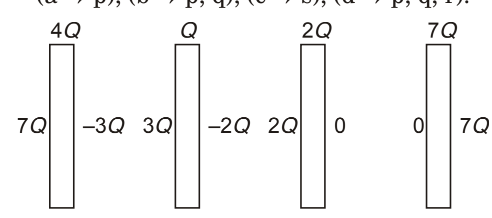
$$\Rightarrow \quad \frac{q_1}{q_2} = 1$$

$$\frac{U_1}{U_2} = \frac{C_2}{C_1} = \frac{8}{9}$$

(a → s), (b → s), (c → s).

6. Charge distribution is shown in figure.

(a → p), (b → p, q), (c → s), (d → p, q, r).



23 Magnetics

Introductory Exercise 23.1

1. $[F_e] = [F_m]$

$$[qE] = [qvB]$$

$$\Rightarrow \quad \left[\frac{E}{B}\right] = [v] = [\mathrm{L\,T^{-1}}]$$

2. $\vec{\mathbf{F}} = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$

$$\therefore \quad \vec{\mathbf{F}} \perp \vec{\mathbf{v}} \text{ and } \vec{\mathbf{F}} \perp \vec{\mathbf{B}}$$

Because cross product of any two vectors is always perpendicular to both the vectors.

3. No. As $\vec{\mathrm{F}}_m = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$

$$\Rightarrow \quad |\vec{\mathbf{F}}_m| = qvB\sin\theta$$

If $F_m = 0$, either $B = 0$ or $\sin\theta = 0$,
i.e., $\theta = 0$

4. $\vec{\mathbf{F}} = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$

$$= -4 \times 10^{-6} \times 10^{-6} \times 10^{-2}[(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (2\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 3\hat{\mathbf{k}})]$$

$$= -4 \times 10^{-2}(4\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 16\hat{\mathbf{k}})$$

$$= -16(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \times 10^{-2}\ \mathrm{N}$$

Introductory Exercise 23.2

1. As magnetic field can exert force on charged particle, it can be accelerated in magnetic field but its speed cannot increases as magnetic force is always perpendicular to the direction of motion of charged particle.

2. $\vec{\mathbf{F}}_m = -e(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$

By Fleming's left hand rule, $\vec{\mathbf{B}}$ must be along positive z-axis.

3. As magnetic force provides necessary centripetal force to the particle to describe a circle.

$$qvB = \frac{mv^2}{r}$$

$$\Rightarrow \quad r = \frac{mv}{qB}$$

(a) $r = \dfrac{mv}{qB}$

$$\Rightarrow \quad r \propto m$$

Hence, electron will describe smaller circle.

(b) $T = \dfrac{2\pi r}{v} = \dfrac{2\pi m}{qB}$

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

$$\Rightarrow \quad f \propto \frac{1}{m}$$

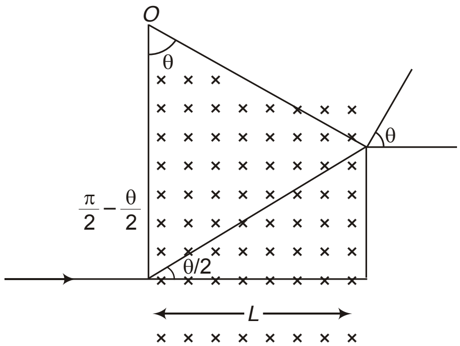
$\therefore$ electron have greater frequency.

4. Electrons are refocused on x-axis at a distance equal to pitch, *i.e.,*

$$n = p = v_{\|}T$$

$$= \frac{2\pi mv\cos\theta}{eB}$$

5. (a) If $L \geq r = \frac{mv}{qB}$,



(b) The particle will describe a semi-circle.

Hence, $\theta = \pi$

(c) $$\frac{L}{l} = \cos\frac{\theta}{2}$$

$$\Rightarrow \quad \frac{L}{2R\sin\frac{\theta}{2}} = \cos\frac{\theta}{2}$$

$$\frac{L}{R} = \sin\theta \Rightarrow \sin\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{6}$$

6. $r = \frac{mv}{eB} = \frac{\sqrt{2mk}}{eB}$

$$= \frac{\sqrt{2m\,eV}}{eB} = \frac{1}{B}\sqrt{\frac{2mV}{e}}$$

For electron,

$$r = \frac{1}{0.2}\sqrt{\frac{2 \times 9.1 \times 10^{-31} \times 100}{1.6 \times 10^{-19}}}$$

$$= 1.67 \times 10^{-4}\text{ m} = 0.0167\text{ cm}$$

For proton

$$r = \frac{1}{0.2}\sqrt{\frac{2 \times 1.67 \times 10^{-27} \times 100}{1.6 \times 10^{-19}}}$$

$$= 7 \times 10^{-3}\text{ m} = 0.7\text{ cm}$$

7. $r = \frac{mv}{qB} = \frac{\sqrt{2mk}}{qB}$

$$\therefore \quad r \propto \frac{\sqrt{m}}{q}$$

$$\therefore \quad r_p : r_d : r_\alpha = \frac{\sqrt{1}}{1} : \frac{\sqrt{2}}{1} : \frac{\sqrt{4}}{2}$$

$$= 1 : \sqrt{2} : 1$$

Introductory Exercise 23.3

1. Let at any instant

$$\vec{\mathbf{V}} = V_x\,\hat{\mathbf{i}} + V_y\,\hat{\mathbf{j}} + V_z\,\hat{\mathbf{k}}$$

Now, $V_x^2 + V_y^2 = V_0^2 = \text{constant}$

and $$V_2 = V_0 - \frac{qE}{m}f$$

$\vec{\mathbf{V}}$ is minimum when $V_2 = 0$

at $$f = \frac{mv_0}{qE}$$

and $$V_{\min} = V_0$$

2. After one revolution, $y = 0$,

$x = p =$ pitch of heating

$$= \frac{2\pi\,mv\sin\theta}{qB}$$

Hence, coordination of the particle,

$$= (x, b) = \left(0, \frac{2\pi\,mv\sin\theta}{qB}\right)$$

3. $\vec{\mathbf{F}} = i(\vec{\mathbf{l}} \times \vec{\mathbf{B}}) = ilB\,[\hat{\mathbf{i}} \times (\hat{\mathbf{j}} + \hat{\mathbf{k}})]$

$$(\vec{\mathbf{F}}) = \sqrt{2}\,ilB$$

4. No. as $\hat{\mathbf{i}} \times (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = \hat{\mathbf{i}} \times \hat{\mathbf{j}} + \hat{\mathbf{i}} \times (\hat{\mathbf{j}} \times \hat{\mathbf{k}})$

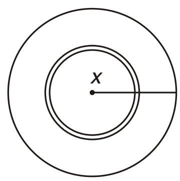
But $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = 0$

$\therefore\ \hat{\mathbf{i}} \times (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = \hat{\mathbf{i}} \times (\hat{\mathbf{j}} \times \hat{\mathbf{k}})$

Introductory Exercise 23.4

1. Consider the disc to be made up of large number of elementary concentric rings. Consider one such ring of radius x and thickness dx.

Charge on this ring



$$dq = \frac{q}{\pi R^2} \cdot dA + \frac{q}{\pi R^2} \times 2\pi x\, dx$$

$$dq = \frac{2qx\, dx}{R^2}$$

Current in this ring,

$$di = \frac{dq}{T} + \frac{\omega\, dq}{2\pi} = \frac{\omega q x\, dx}{\pi R^2}$$

$\therefore$ Magnetic moment of this ring,

$$dM = di \times A = \frac{\omega q x\, dx}{\pi R^2} \times \pi x^2$$

$$= \frac{\omega q}{R^2} x^3 dx$$

$\therefore$ Magnetic moment of entire disc,

$$M = \int dM = \frac{\omega q}{R^2} \int_0^R x^3 dx$$

$$= \frac{\omega q}{R^2} \left[\frac{R^4}{4} \right] = \frac{1}{4} \omega q R^2$$

2. $\vec{\mathbf{M}} = i \times [(\vec{\mathbf{OA}} \times \vec{\mathbf{AB}})]$

$\vec{\mathbf{OA}} = OA \cos\theta\, \hat{\mathbf{j}} + OA \sin\theta\, \hat{\mathbf{k}}$

$\vec{\mathbf{AB}} = AB\, \hat{\mathbf{i}}$

$\therefore$ $\vec{\mathbf{M}} = i \times OA \cdot AB\, [(\cos\theta\, \hat{\mathbf{j}} + \sin\theta\, \hat{\mathbf{k}}) \times \hat{\mathbf{i}}]$

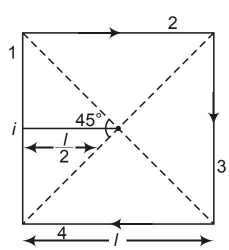
$$= 4 \times 0.2 \times 0.1 \left[\left(\frac{\sqrt{3}}{2} \hat{\mathbf{j}} + \frac{1}{2} \hat{\mathbf{k}} \right) \times \hat{\mathbf{i}} \right]$$

$$= (0.04\, \hat{\mathbf{j}} - 0.07\, \hat{\mathbf{k}}) \text{ A-m}^2$$

Introductory Exercise 23.5

1. (a) $B_1 = B_2 = B_3 = B_4$

$$= \frac{\mu_0}{4\pi} \cdot \frac{i}{l/2} [\sin 45° + \sin 45°]$$



$$= \frac{\mu_0}{4\pi} \cdot \frac{2\sqrt{2} i}{l}$$

Net magnetic field at the centre of the square,

$$B = B_1 + B_2 + B_3 + B_4 = \frac{\mu_0}{4\pi} \cdot \frac{8\sqrt{2} i}{l}$$

$$= \frac{2\sqrt{2}\, \mu_0 i}{\pi l} = 28.3\ \mu\text{T (inward)}$$

(b) If the conductor is converted into a circular loop, then

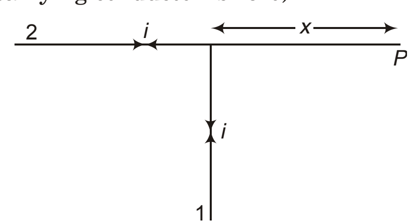
$$2\pi r = 4l \Rightarrow r = \frac{2l}{\pi}$$

$$B = \frac{\mu_0 i}{2} r = \frac{\pi \mu_0 i}{4l} = 24.7\ \mu\text{T (inward)}$$

2. $B = \dfrac{\mu_0}{4\pi} \cdot \dfrac{i}{x}$

(As P is lying near one end of conductor 1)

$B_2 = 0$ (Magnetic field on the axis of a current carrying conductor is zero)

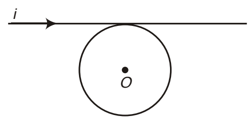


$$B = B_1$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{i}{x}$$

By right hand thumb rule, direction of magnetic field at P is inward.

3. Magnetic field due to straight conductor at O



$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2i}{R}$$

Magnetic field at O due to circular loop

$$B_2 = \frac{\mu_0 i}{2R}$$

By right hand thumb rule, both the filds are acting inward.

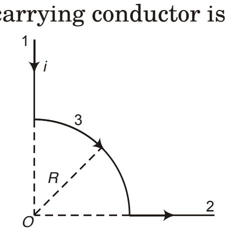
Hence,

$$B = B_1 + B_2 = \frac{\mu_0 i}{2\pi R}(1+\pi)$$

$$= \frac{4\pi \times 10^{-7} \times 7}{2\pi \times 10 \times 10^{-2}}\left(1 + \frac{22}{7}\right)$$

$$= 58 \times 10^{-6}\ \text{T} = 58\ \mu\text{T (inward)}.$$

4. $B_1 = B_2 = 0$ (Magnetic field on the axis of current carrying conductor is zero)



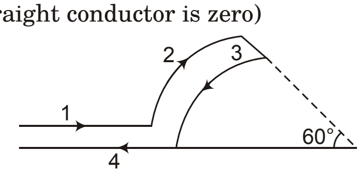
$$B_3 = \frac{1}{4} \cdot \frac{\mu_0 i}{2R} = \frac{\mu_0 i}{8R}$$

$$= \frac{4\pi \times 10^{-7} \times 5}{8 \times 3 \times 10^{-2}}$$

$$= 2.62 \times 10^{-5}\ \text{T}$$

$$= 26.2\ \mu\text{T (inward)}.$$

5. $B_1 = B_2 = 0$ (Magnetic field on the axis of straight conductor is zero)



$$B_2 = \frac{60°}{360°} \cdot \frac{\mu_0 i}{2b} = \frac{\mu_0 i}{12b}\ \text{(inward)}$$

$$B_3 = \frac{60°}{360°} = \frac{\mu_0 i}{2a} = \frac{\mu_0}{12a}\ \text{(outward)}$$

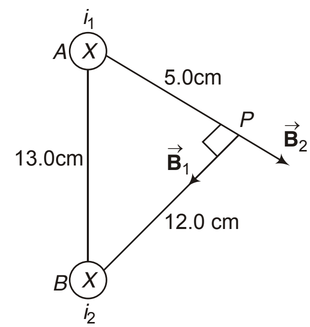
As $B_3 > B_2$,

Net magnetic field at P,

$$B = B_3 - B_2$$

$$= \frac{\mu_0 i}{12}\left[\frac{1}{a} - \frac{1}{b}\right]$$

6. AB, AP and BP from Pythagorus triplet, hence $\angle APB = 90°$



$$\therefore \quad \vec{\mathbf{B}}_1 = \frac{\mu_0}{4\pi} \cdot \frac{2i_1}{r_1}\hat{\mathbf{PB}}$$

$$\vec{\mathbf{B}}_2 = \frac{\mu_0}{4\pi} \cdot \frac{2i_2}{r_2}\hat{\mathbf{AP}}$$

$$B = \sqrt{B_1^2 + B_2^2}$$

$$= \frac{\mu_0}{2\pi}\sqrt{\left(\frac{i_1}{r_1}\right)^2 + \left(\frac{i_2}{r_2}\right)^2}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \times \sqrt{\left(\frac{3}{0.05}\right)^2 + \left(\frac{3}{0.12}\right)^2}$$

$$= 1.3 \times 10^{-5}\ \text{T}$$

$$= 13\ \mu\text{T}$$

7. $\tau = NIAB\cos\theta$

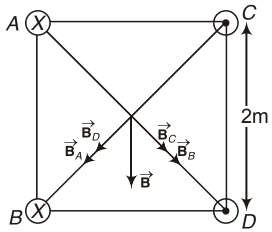
$$= 100 \times 1.2 \times 0.4 \times 0.3 \times 0.8 \times \cos 30°$$

$$= 9.98\ \text{N-m}$$

Rotation will be clockwise as seen from above.

Introductory Exercise 23.6

1. By right hand thumb rule, direction of magnetic field due to conductor A, B, C and D are as shown in figure.



$$B_A = B_B = B_C = B_D = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r}$$

Here, $I = 5$ A

$$r = \frac{a}{\sqrt{2}} = \frac{0.2}{\sqrt{2}} = 0.14$$

$\therefore$ Net magnetic field at P

$$B = \sqrt{(B_A + B_D)^2 + (B_B + B_C)^2}$$

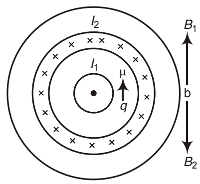
$$= \frac{\mu_0}{4\pi} \cdot \frac{4\sqrt{2}\, I}{r}$$

$$= \frac{10^{-7} \times 4\sqrt{2} \times 5}{0.2/\sqrt{2}} = 20 \times 10^{-6} \text{ T}$$

$$= 20\ \mu\text{T}$$

Clearly resultant magnetic field is downward.

2. At point A



$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{I_1}{r_1}$$

$B_2 = 0$ (Magnetic field inside a current carrying hollow cylinder is zero)

$$\therefore\ B_a = B_1 + B_2 = \frac{\mu_0}{4\pi} \cdot \frac{I_1}{r_1}$$

$$= \frac{10^{-7} \times 1}{1 \times 10^{-3}} - 10^{-4} \text{ T}$$

$$= 100\ \mu\text{T (upward)}$$

At point B

$$B_1 = \frac{\mu_0}{4\pi\varepsilon_0} \cdot \frac{I_1}{r_2} \uparrow,\ B_2 = \frac{\mu_0}{4\pi} \cdot \frac{I_2}{r_2} \downarrow$$

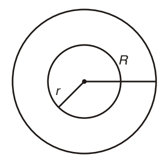
Net field at B

$$B = B_2 - B_1 = \frac{\mu_0}{4\pi} \cdot \frac{1}{r_2}(I_2 - I_1)$$

$$= \frac{10^{-7}}{3 \times 10^{-3}} \times (3 - 2) = 0.67 \times 10^{-4} \text{ T}$$

$$= 67\ \mu\text{T}$$

3. Consider the cylinder to be made up of large number of elementary hollow cylinders.



Consider one such cylinder of radius r and thickness dr.

Current passing through this hollow cylinder,

$$di = j dA = j(2\pi r\, dr) 2\pi\, br^2 dr$$

(a) Total current inside the portion of radius r_1,

$$I_1 = \int di = 2\pi b \int_0^{r_1} r^2 dr$$

$$= 2\pi b \left[\frac{r_1^3}{3}\right]_0^{r_1}$$

$$= \frac{2}{3}\pi\, b r_1^3$$

By ampere's circuital law,

$$\oint_{2\pi r_1} B \cdot dl = \mu_0 i_1$$

$$B_1 \times 2\pi r_1 = \mu_0 \left(\frac{2}{3}\pi\, b r_1^3\right)$$

$$\Rightarrow \qquad B_1 = \frac{\mu_0\, b r_1^2}{3}$$

(b) Total current inside the cylinder

$$i = 2\pi b \int_0^R r^2 dr$$

$$= \frac{2}{3}\pi\, bR^3$$

$$B_2 = \frac{\mu_0}{4\pi} \frac{2i}{r_2} = \frac{\mu_0\, bR^3}{3r_2}$$

AIEEE Corner

Subjective Questions (Level-1)

1. Positive. By Flemings left hand rule.

2. $F_m = evB\sin\theta$

$\Rightarrow \quad v = \dfrac{F_e}{eB\sin\theta}$

$= \dfrac{4.6\times10^{-15}}{1.6\times10^{-19}\times3.5\times10^{-3}\times\sin 60^\circ}$

$= 9.46\times10^{6}$ m/s

3. $F_m = qvB\sin\theta$

$= (2\times1.6\times10^{-19})\times10^{5}\times0.8\times1$

$= 2.56\times10^{-14}$ N

4. (a) $\vec{\mathbf{F}}_m = e(\vec{\mathbf{v}}\times\vec{\mathbf{B}})$

$= -1.6\times10^{-19}[(2.0\times10^{6})\hat{\mathbf{i}} + (3.0\times10^{6})\hat{\mathbf{j}}]$

$\times(0.03\,\hat{\mathbf{i}} + 0.15\,\hat{\mathbf{j}})$

$= -(6.24\times10^{-4}\text{ N})\hat{\mathbf{k}}$

(b) $= \vec{\mathbf{F}}_m\, e(\vec{\mathbf{v}}\times\vec{\mathbf{B}}) = -(6.24\times10^{-4}\text{N})\hat{\mathbf{k}}$

5. $\vec{\mathbf{F}}_m = e(\vec{\mathbf{v}}\times\vec{\mathbf{B}})$

$(6.4\times10^{-19})\hat{\mathbf{k}} = -1.6\times10^{-19}[(2\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}})$

$\times(B_x\,\hat{\mathbf{i}} + 3B_x\,\hat{\mathbf{j}})]$

$6.4\times10^{-19}\,\hat{\mathbf{k}} = -1.6\times10^{-19}[2B_x\,\hat{\mathbf{k}}]$

$B_x = \dfrac{6.4\times10^{-19}}{-3.2\times10^{-19}} = -2.0$ T

6. (a) As magnetic force always acts perpendicular to magnetic field, magnetic field must be along x-axis.

$F_1 = qv_1B\sin\theta_1$

$\Rightarrow \quad B = \dfrac{F_1}{qv_1B\sin\theta_1} = \dfrac{5\sqrt{2}\times10^{-3}}{1\times10^{-6}\times10^{6}\times\dfrac{1}{\sqrt{2}}}$

$\Rightarrow \quad B = 10^{-3}$ T

or $\quad \vec{\mathbf{B}} = (10^{-3}\text{ T})\hat{\mathbf{i}}$

(b) $F_2 = qv_2 B\sin\theta_2$

$= 1\times10^{-6}\times10^{6}\times10^{-3}\times\sin 90^\circ$

$= 10^{-3}$ N

$F_2 = 1$ mN

7. Let $\vec{\mathbf{B}} = B_x\,\hat{\mathbf{i}} + B_y\,\hat{\mathbf{j}} + B_z\,\hat{\mathbf{k}}$

(a) $\vec{\mathbf{F}} = q(\vec{\mathbf{v}}\times\vec{\mathbf{B}})$

$7.6\times10^{-3}\,\hat{\mathbf{i}} - 5.2\times10^{-3}\,\hat{\mathbf{k}}$

$= -7.8\times10^{-6}\times3.8\times10^{3}(B_z\,\hat{\mathbf{i}} - B_x\,\hat{\mathbf{k}})$

$\Rightarrow \quad B_x = -0.175$ T, $B_z = -0.256$ T

(b) Cannot be determined by this information.

(c) As $\vec{\mathbf{F}} = q(\vec{\mathbf{v}}\times\vec{\mathbf{B}})$

$\vec{\mathbf{F}}\perp\vec{\mathbf{B}}$

Hence, $\vec{\mathbf{B}}\cdot\vec{\mathbf{F}} = 0$

8. $\vec{\mathbf{B}} = B\,\hat{\mathbf{i}}$

(a) $\vec{\mathbf{v}} = v\,\hat{\mathbf{j}}$

$\vec{\mathbf{F}} = q(\vec{\mathbf{v}}\times\vec{\mathbf{B}}) = -qvB\,\hat{\mathbf{k}}$

(b) $\vec{\mathbf{v}} = v\,\hat{\mathbf{j}}$

$\vec{\mathbf{F}} = q(\vec{\mathbf{v}}\times\vec{\mathbf{B}}) = qvB\,\hat{\mathbf{j}}$

(c) $\vec{\mathbf{v}} = -v\,\hat{\mathbf{i}}$

$\vec{\mathbf{F}} = q(\vec{\mathbf{v}}\times\vec{\mathbf{B}}) = 0$

(d) $\vec{\mathbf{v}} = v\cos 45^\circ\,\hat{\mathbf{i}} - v\cos 45^\circ\,\hat{\mathbf{k}}$

$\vec{\mathbf{F}} = q(\vec{\mathbf{v}}\times\vec{\mathbf{B}}) = -\dfrac{qvB}{\sqrt{2}}\,\hat{\mathbf{j}}$

(e) $\vec{\mathbf{v}} = v\cos 45^\circ\,\hat{\mathbf{j}} - v\cos 45^\circ\,\hat{\mathbf{k}}$

$\vec{\mathbf{F}} = q(\vec{\mathbf{v}}\times\vec{\mathbf{B}}) = \dfrac{qvB}{\sqrt{2}}(-\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$= -\dfrac{qvB}{\sqrt{2}}(\hat{\mathbf{j}} + \hat{\mathbf{k}})$

9. $r = \dfrac{mv}{qB} = \dfrac{\sqrt{2mk}}{eB} = \dfrac{\sqrt{2\,meV}}{eB}$

$\Rightarrow \quad B = \sqrt{\dfrac{2mV}{e}}r$

$= \sqrt{\dfrac{2\times9.1\times10^{-31}\times2\times10^{3}}{1.6\times10^{-19}}}\times0.180$

$$= 0.36 \times 10^{-4}\ \text{T}$$

$$B = 3.6 \times 10^{-4}\ \text{T}$$

10. (a) $r = \dfrac{mv}{qB} \Rightarrow v = \dfrac{qBr}{m}$

$$= \frac{1.6 \times 10^{-19} \times 2.5 \times 6.96 \times 10^{-3}}{3.34 \times 10^{-27}}$$

$$= 8.33 \times 10^{5}\ \text{ms}^{-1}$$

(b) $t = \dfrac{T}{2} = \dfrac{\pi m}{qB}$

$$= \frac{3.14 \times 3.34 \times 10^{-27}}{1.6 \times 10^{-19} \times 2.5}$$

$$= 2.62 \times 10^{-8}\ \text{s}$$

(c) $k = eV = \dfrac{1}{2} mv^2$

$$\Rightarrow V = \frac{mv^2}{2e}$$

$$= \frac{3.34 \times 10^{-27} \times (8.33 \times 10^{5})^2}{2 \times 1.6 \times 10^{-19}}$$

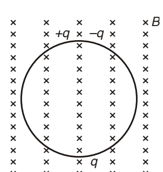
$$= 7.26 \times 10^{3}\ \text{V}$$

$$= 7.26\ \text{kV}$$

11. (a) $-q$. As initially particle is neutral, charge on two particles must be equal and opposite.

(b) The will collide after completing half rotation, *i.e.*,

$$t = \frac{T}{2} = \frac{\pi m}{qB}$$



12. Here, $r = \dfrac{10.0}{2} = 5.0$ cm,

(a) $r = \dfrac{mv}{qB} \Rightarrow B = \dfrac{mv}{qr}$

$$= \frac{9.1 \times 10^{-31} \times 1.41 \times 10^{6}}{1.6 \times 10^{-19} \times 5 \times 10^{-2}}$$

$$= 1.6 \times 10^{-4}\ \text{T}$$

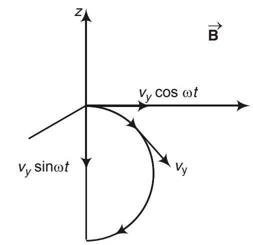
By Fleming's left hand rule, direction of magnetic field must be inward.

(b) $t = \dfrac{T}{2} = \dfrac{\pi m}{qB}$

$$= \frac{3.14 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 1.6 \times 10^{-4}}$$

$$= 1.1 \times 10^{-7}\ \text{s}$$

13. The component of velocity along the magnetic field (*i.e.*, v_x) will remain unchanged and the proton will move in a helical path.



At any instant,

Components of velocity of particle along Y-axis and Z-axis

$$v'_y = v_y \cos\theta = v_y \cos \omega t$$

and $\quad v'_z = -v_z \sin\theta = v_z \sin \omega t$

where, $\quad \omega = \dfrac{qB}{m}$

$$\therefore \vec{\mathbf{v}} = v_x \hat{\mathbf{i}} + v_y \cos \omega t\, \hat{\mathbf{j}} - v_z \sin \omega t\, \hat{\mathbf{k}}$$

14. For the electron to hit the target, distance GS must be multiple of pitch, *i.e.*,

$$GS = np$$

For minimum distance, $n = 1$

$$\Rightarrow \quad GS = p = \frac{2\pi\, mv \cos\theta}{qB}$$

$$\Rightarrow \quad p = \frac{2\pi \sqrt{2\,mk} \cos 60^\circ}{qB} \qquad (mv = \sqrt{2\,mk})$$

$$\Rightarrow \quad B = \frac{2\pi \sqrt{2\,mk} \cos 60^\circ}{qp}$$

$$= \frac{2 \times 3.14 \times \sqrt{2 \times 9.1 \times 10^{-31} \times 2 \times 1.6 \times 10^{-16} \times \frac{1}{2}}}{1.6 \times 10^{-19} \times 0.1}$$

$$\Rightarrow \quad B = 4.73 \times 10^{-4}\ \text{T}$$

15. (a) From Question 5 (c)

Introductory Exercise 23.2

$$\frac{L}{R} = \sin\theta \Rightarrow L = R \sin\theta$$

$$R \sin 60^\circ = \frac{R}{2}$$

$$\Rightarrow \quad L = \frac{mv}{2qB} = \frac{mv_0}{2qB_0}$$

(b) Now, $L' = 2.1\,L = 1.05R$

As $L' > R$,

Particle will describe a semicircle and move out of the magnetic field moving in opposite direction, *i.e.,*

$$v' = -v = -v_0\,\hat{\mathbf{i}}$$

and $$t = \frac{T}{2} = \frac{\pi m}{qB_0}$$

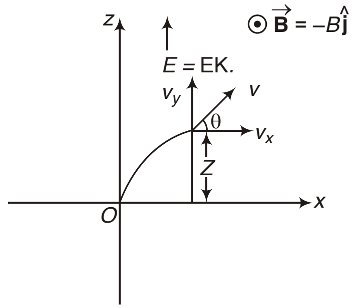
16. $\vec{\mathbf{v}} = (50\ \text{ms}^{-1})\,\hat{\mathbf{i}},\ \vec{\mathbf{B}} = (2.0\ \text{mT})\,\hat{\mathbf{j}}$

As particle move with uniform velocity,

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) = 0$$

$$\Rightarrow \quad \vec{\mathbf{E}} = \vec{\mathbf{B}} \times \vec{\mathbf{v}} = -(0.1\ \text{N/C})\hat{\mathbf{k}}$$

17. If v be the speed of particle at point (0, y, z) then by work-energy theorem,



$$W = \Delta K = \frac{1}{2}mv^2$$

But work done by magnetic force is zero,

hence, network done = work done by electric force

$$= qEZ$$

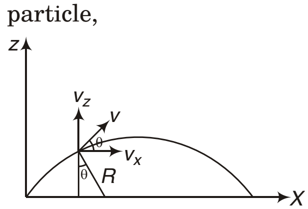
$$\therefore \quad qE_0Z = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2qE_0Z}{m}}$$

As the magnetic field is along Y-axis, particle will move in XZ-plane.

The path of particle will be a cycloid. In this case, instantaneous centre of curvature of the particle will move along X-axis.

As magnetic force provides centripetal force to the particle,



$$qvB_0 = \frac{mv^2}{R}$$

$$v = \frac{qB_0R}{m}$$

$$v_x = v\cos\theta = \frac{qB_0R\cos\theta}{m}$$

$$= \frac{qB_0Z}{m} \quad (\because R\cos\theta = Z)$$

Now, $v_z = \sqrt{v^2 - v_x^2} = \sqrt{\dfrac{2qE_0Z}{m} - \dfrac{q^2B_0^2Z^2}{m^2}}$

18. Given, $\vec{\mathbf{E}} = E\,\hat{\mathbf{j}},\ \vec{\mathbf{B}} = B\,\hat{\mathbf{k}}$,

$$\vec{\mathbf{v}} = v\cos\theta\,\hat{\mathbf{j}} + v\sin\theta\,\hat{\mathbf{k}}$$

As protons are moving undeflected,

$\vec{\mathbf{F}} = 0 \Rightarrow e(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) = 0$

$$\Rightarrow \quad e(E\,\hat{\mathbf{j}} - vB\cos\theta\,\hat{\mathbf{j}}) = 0$$

or $$v = \frac{E}{B\cos\theta}$$

Now, if electric field is switched off

$$p = \frac{2\pi\, mv\sin\theta}{qB} = \frac{2\pi\, mE\tan\theta}{qB^2}$$

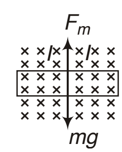
(Component of velocity along magnetic field $= v_z = v\sin\theta$)

19. $F = IlB\sin\theta$

$$I = \frac{F}{lB\sin\theta} = \frac{0.13}{0.2 \times 0.067 \times \sin 90^\circ}$$

$= 9.7$ A

20. For no tension in springs



$$F_m = mg$$

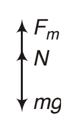
$$\Rightarrow \quad IlB = mg$$

$$I = \frac{mg}{lB} = \frac{13.0 \times 10^{-3} \times 10}{62.0 \times 10^{-2} \times 0.440}$$

$= 0.48$ A

By Fleming left hand rule, for magnetic force to act in upward direction, current in the wire must be towards right.

21. (a) FBD of metal bar is shown in figure, for metal to be in equilibrium,



$$F_m + N = mg$$

$$\Rightarrow \quad F_m = mg - N$$

$$\Rightarrow \quad IlB = m - N$$

$$\Rightarrow \quad \frac{V}{R}lB = mg - N$$

$$\Rightarrow \quad V = \frac{R}{lB}(mg - N)$$

For largest voltage,

$N = 0$

$$V = \frac{Rmg}{lB} = \frac{25 \times 750 \times 10^{-3} \times 9.8}{50.0 \times 10^{-2} \times 0.450}$$

$$= 817.5 \text{ V}$$

(b) If $IlB > mg$

$$IlB - mg = ma$$

$$a = \frac{IlB - mg}{m} = \frac{VlB}{Rm} - g$$

$$= \frac{817.5 \times 50 \times 10^{-2} \times 0.45}{2 \times 750 \times 10^{-3}} - 9.8$$

$$= 112.8 \text{ m/s}^2$$

22. $I = 3.50$ A, $l = -(1.00 \text{ cm})\hat{\mathbf{i}}$

$\Rightarrow \quad l = -(1.00 \times 10^{-2} \text{ m})\hat{\mathbf{i}}$

(a) $\vec{\mathbf{B}} = -(0.65 \text{ T})\hat{\mathbf{j}}$

$$\vec{\mathbf{F}}_m = I(\vec{\mathbf{l}} \times \vec{\mathbf{B}}) = -(0.023 \text{ N})\hat{\mathbf{k}}$$

(b) $\vec{\mathbf{B}} = +(0.56 \text{ T})\hat{\mathbf{k}}$

$$\vec{\mathbf{F}}_m = I(\vec{\mathbf{l}} \times \vec{\mathbf{B}}) = (0.0196 \text{ N})\hat{\mathbf{j}}$$

(c) $\vec{\mathbf{B}} = -(0.33 \text{ T})\hat{\mathbf{i}}$

$$\vec{\mathbf{F}}_m = I(\vec{\mathbf{l}} \times \vec{\mathbf{B}}) = 0$$

(d) $\vec{\mathbf{B}} = (0.33 \text{ T})\hat{\mathbf{i}} - (0.28 \text{ T})\hat{\mathbf{k}}$

$$\vec{\mathbf{F}}_m = I(\vec{\mathbf{l}} \times \vec{\mathbf{B}}) = -(0.0098 \text{ N})\hat{\mathbf{j}}$$

(e) $\vec{\mathbf{B}} = +(0.74 \text{ T})\hat{\mathbf{j}} - (0.36 \text{ T})\hat{\mathbf{k}}$

$$\vec{\mathbf{F}}_m = I(\vec{\mathbf{l}} \times \vec{\mathbf{B}}) = -(0.0259 \text{ N})\hat{\mathbf{k}} + (0.0126 \text{ N})\hat{\mathbf{j}}$$

$$= (0.0126 \text{ N})\hat{\mathbf{j}} - (0.0259 \text{ N})\hat{\mathbf{K}}$$

23. $\vec{\mathbf{B}} = (0.020 \text{ T})\hat{\mathbf{j}}$

$\vec{\mathbf{l}}_1 = \vec{\mathbf{ab}} = -(40.0 \text{ cm})\hat{\mathbf{j}}$

$$= -(40.0 \times 10^{-2} \text{ m})\hat{\mathbf{j}}$$

$\vec{\mathbf{F}}_1 = I(\vec{\mathbf{l}}_1 \times \vec{\mathbf{B}}) = 0$

$\vec{\mathbf{l}}_2 = \vec{\mathbf{bc}} = (40.0 \text{ cm})\hat{\mathbf{k}}$

$$= -(400 \times 10^{-2} \text{ m})\hat{\mathbf{k}}$$

$\vec{\mathbf{F}}_2 = I(\vec{\mathbf{l}}_2 \times \vec{\mathbf{B}}) = (0.04 \text{ N})\hat{\mathbf{i}}$

$\vec{\mathbf{l}}_3 = \vec{\mathbf{cd}} = -(40 \times 10^{-2})\hat{\mathbf{i}} + (40 \times 10^{-2} \text{ m})\hat{\mathbf{j}}$

$\vec{\mathbf{F}}_3 = I(\vec{\mathbf{l}}_3 \times \vec{\mathbf{B}}) = -(0.04 \text{ N})\hat{\mathbf{k}}$

$\vec{\mathbf{l}}_4 = \vec{\mathbf{da}} = (40 \times 10^{-2} \text{ m})\hat{\mathbf{i}} - (40 \times 10^{-2} \text{ m})\hat{\mathbf{k}}$

$\vec{\mathbf{F}}_4 = I(\vec{\mathbf{l}}_4 \times \vec{\mathbf{B}}) = (0.04 \text{ N})\hat{\mathbf{i}} + (0.04 \text{ N})\hat{\mathbf{k}}$

24. $\vec{\mathbf{M}} = IA\hat{\mathbf{M}}$

$$= 0.20 \times \pi(8.0 \times 10^{-2})^2(0.60\,\hat{\mathbf{i}} - 0.80\,\hat{\mathbf{j}})$$

$$= (40.2 \times 10^{-4})(0.60\,\hat{\mathbf{i}} - 0.80\,\hat{\mathbf{j}})\text{A-m}^2$$

$\vec{\mathbf{B}} = (0.25 \text{ T})\hat{\mathbf{i}} + (0.30 \text{ T})\hat{\mathbf{k}}$

(a) $\vec{\tau} = \vec{\mathbf{M}} \times \vec{\mathbf{B}}$

$$= (40.2 \times 10^{-4})(-0.24\,\hat{\mathbf{i}} - 0.18\,\hat{\mathbf{j}} + 0.2\,\hat{\mathbf{k}})$$

$$= (-9.6\,\hat{\mathbf{i}} - 7.2\,\hat{\mathbf{j}} + 8.0\,\hat{\mathbf{k}}) \times 10^{-4} \text{ N-m.}$$

(b) $U = -\vec{\mathbf{M}} \cdot \vec{\mathbf{B}} = -(40.2 \times 10^{-4})(0.15)$ J

$$\approx -6.0 \times 10^{-4} \text{ J}$$

25. Consider the wire is bent in the form of a loop of N turns,

Radius of loop, $\quad r = \frac{L}{2\pi N}$

Magnetic dipole moment associated with the loop

$$M = NiA = Ni \times \pi r^2 = \frac{iL^2}{4\pi N^2}$$

$$\tau = MB \sin 90° = \frac{iL^2 B}{4\pi N}$$

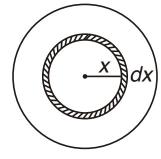
Clearly τ is maximum, when $N = 1$

and the maximum torque is given by

$$\tau_m = \frac{iL^2 B}{4\pi}$$

26. Consider the disc to be made up of large number of elementary rings. Consider on such ring of radius x and thickness dx.

Charge on this ring,



$$dq = \frac{q}{\pi R^2} \times 2\pi x\, dx = \frac{2q}{R^2} x\, dx$$

Current associated with this ring,

$$di = \frac{dq}{T} = \frac{\omega\, dq}{2\pi} = \frac{\omega q}{\pi R^2} x\, dx$$

Magnetic moment of this ring

$$dM = \pi x^2 di = \frac{\omega q}{R^2} x^3 dx$$

Magnetic moment of entire disc,

$$M = \int dM = \frac{\omega q}{R^2}\int_0^R x^3 dx = \frac{1}{4}\omega q R^2 \quad \text{...(i)}$$

Magnetic field at the centre of disc due to the elementary ring under consideration

$$dB = \frac{\mu_0\, di}{2x} = \frac{\mu_0 \omega q^2}{2\pi R^2} dx$$

Net magnetic field at the centre of the disc,

$$B = \int dB = \frac{\mu_0 \omega q}{2\pi R^2}\int_0^R dx = \frac{\mu_0 \omega q}{2\pi R}$$

$$\therefore \quad \frac{M}{B} = \frac{\pi R^3}{2\mu_0}$$

27. (a) By principle of conservation of energy,

Gain in KE = Loss in PE

$$\text{KE} = -\text{PE}\cos\theta + \text{ME}$$

$$f\cos\theta = 1 - \frac{K}{\text{ME}} = 1 - \frac{0.80 \times 10^{-3}}{0.02 \times 52 \times 10^{-3}}$$

$$= \frac{10}{13}$$

$$\theta = \cos^{-1}\frac{10}{13} = 76.7^\circ$$

(b) $\theta = \cos^{-1}\frac{10}{13} = 76.7^\circ$

Entire KE will again get converted into PE

28. $\Delta U = U_2 - U_1 = -MB - (+MB)$

$$= -2MB$$

$$= -2 \times 1.45 \times 0.835 = -2.42 \text{ J}$$

29. (a) $T = \frac{2\pi r}{v} = \frac{2 \times 3.14 \times 5.3 \times 10^{-11}}{2.2 \times 10^6}$

$$= 1.5 \times 10^{-16} \text{ s}$$

(b) $i = \frac{e}{T} = \frac{1.6 \times 10^{-19}}{1.5 \times 10^{-16}} = 1.1 \times 10^{-3}\text{A}$

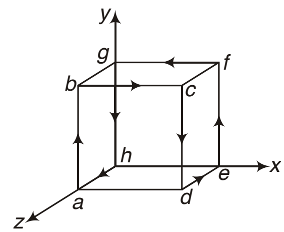
$$= 1.1 \text{ mA}$$

(c) $M = \pi r^2 i$

$$= 3.14 \times (5.3 \times 10^{-11})^2 \times 1.1 \times 10^{-3}$$

$$= 9.3 \times 10^{-24} \text{ A-m}^2$$

30. Suppose equal and opposite currents are flowing in sides ad and eh, so that three complete current carrying loops are formed,



$$\vec{\mathbf{M}}_{abcd} = -i l^2 \hat{\mathbf{k}}$$

$$\vec{\mathbf{M}}_{efgh} = i l^2 \hat{\mathbf{k}}$$

$$\vec{\mathbf{M}}_{adeh} = i l^2 \hat{\mathbf{j}}$$

$\therefore$ Total magnetic moment of the closed path,

$$\vec{\mathbf{M}} = \vec{\mathbf{M}}_{abcd} + \vec{\mathbf{M}}_{efgh} + \vec{\mathbf{M}}_{adeh} = i l^2 \hat{\mathbf{j}}$$

31. Circuit is same as in Q.30

$$\vec{\mathbf{M}} = i l^2 \hat{\mathbf{j}} = \hat{\mathbf{j}}$$

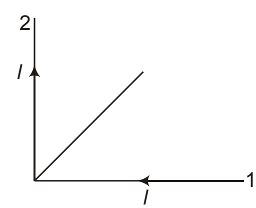
$$\vec{\mathbf{B}} = 2\hat{\mathbf{j}}$$

$$\vec{\tau} = \vec{\mathbf{M}} \times \vec{\mathbf{B}} = 0$$

32. $B_1 = \frac{\mu_0}{4\pi}\cdot\frac{I}{r}$

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{I}{r}$$

Here, B_1 and B_2 are perpendicular to each other, hence,



$$B = \sqrt{B_1^2 + B_2^2}$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{\sqrt{2}I}{r} = \frac{10^{-7} \times \sqrt{2} \times 5}{35 \times 10^{-2}}$$

$$= 2.0 \times 10^{-6} \text{ T}$$

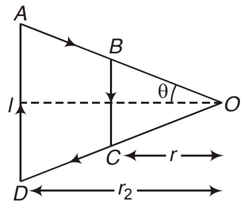
$$= 2.0\ \mu\text{T}$$

33. Clearly $\Delta BOC \sim \Delta AOB$

$$\therefore \quad \frac{r_2}{r_6} = \frac{AD}{BC}$$

$$\Rightarrow \quad r_2 = 2r$$

$$= 100 \text{ mm}$$



and $AD = 2BC = 200$ mm

$$\theta = \cos^{-1} \frac{r}{\frac{BC}{2}} = 45^\circ$$

$$B_{BC} = \frac{\mu_0}{4\pi}\frac{I}{r}[\sin 45^\circ + \sin 45^\circ]\frac{\sqrt{2}\,I}{r}$$

$$= \frac{\mu_0}{4\pi} \quad \text{(outwards)}$$

$$B_{AD} = \frac{\mu_0}{4\pi} \cdot \frac{I}{r_2}(\sin 45^\circ + \sin 45^\circ)$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{\sqrt{2}\,I}{r_2} \quad \text{(inwards)}$$

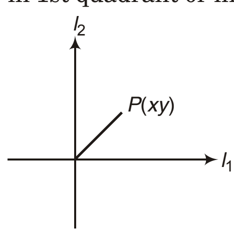
Net magnetic field at O.

$$B = B_{BC} - B_{AD} = \frac{\sqrt{2}\,\mu_0 I}{4\pi}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \sqrt{2} \times 10^{-7} \times \sqrt{2}\left[\frac{1}{50 \times 10^{-3}} - \frac{1}{100 \times 10^{-3}}\right]$$

$$= 2 \times 10^{-6}\ \text{T} = 2\ \mu\text{T} \quad \text{(outwards)}$$

34. Let us consider a point $P(x, y)$ where magnetic field is zero. Clearly the point must lie either in 1st quadrant or in 3rd quadrant.



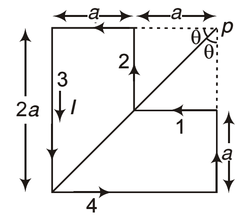
$$B = \frac{\mu_0}{4\pi} \cdot \frac{2I_1}{y} - \frac{\mu_0}{4\pi} \cdot \frac{2I_2}{x} = 0$$

$$\Rightarrow \quad I_1 x = I_2 y$$

$$\Rightarrow \quad y = \left(\frac{I_1}{I_2}\right) x$$

35. $\theta = 45^\circ$

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \cdot \frac{I}{a}(\sin\theta + \sin\theta)$$



$$= \frac{\mu_0}{4\pi} \cdot \frac{I}{a\sqrt{2}} \quad \text{(inwards)}$$

$$B_3 = B_4 = \frac{\mu_0}{4\pi}\frac{I}{\sqrt{2}a}(\sin 0 + \sin 0)$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I}{2a\sqrt{2}} \quad \text{(outwards)}$$

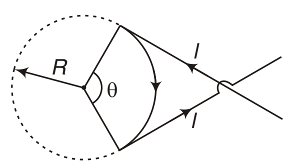
Net magnetic field at P

$$B = B_1 + B_2 - (B_3 + B_4)$$

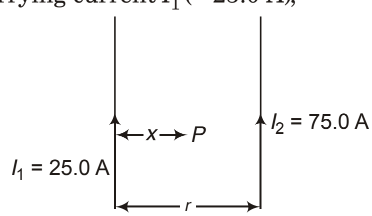
$$= \frac{\mu_0}{4\pi} \cdot \frac{I}{\sqrt{2}a} \quad \text{(inwards)}$$

36. $B = 2 \times \frac{\mu_0}{4\pi} \cdot \frac{I}{R} - \frac{\mu_0 I}{2R} \times \frac{\theta}{2\pi} = 0$

$$\Rightarrow \quad \theta = 2\ \text{rad.}$$



37. (a) Consider a point P in between the two conductors at a distance x from conductor carrying current I_1 (= 25.0 A),



Magnetic field at P

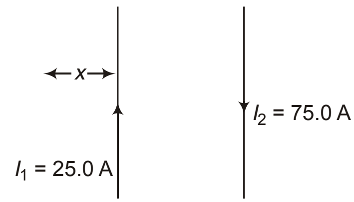
$$B = \frac{\mu_0}{4\pi} \cdot \frac{I_1}{x} - \frac{\mu_0}{4\pi} \cdot \frac{I_2}{r - x} = 0$$

$$\Rightarrow \quad \frac{I_1}{x} = \frac{I_2}{r - x}$$

$$\Rightarrow \quad \frac{r - x}{x} = \frac{I_2}{I_1}$$

$$\Rightarrow \quad x = \frac{I_1}{I_1 + I_2} r = \frac{25.0}{100.0} \times 40 = 10\ \text{cm}$$

(b) Consider a point Q lying on the left of the conductor carrying current I_1 at a distance x from it.



$$B = \frac{\mu_0}{4\pi} \cdot \frac{I_1}{x} - \frac{\mu_0}{4\pi} \cdot \frac{I_2}{r+x} = 0$$

$$\Rightarrow \quad \frac{I_1}{x} = \frac{I_2}{r+x}$$

$$\Rightarrow \quad x = \frac{I_1}{I_2 - I_1} r = \frac{25.0}{50.0} \times 40$$

$$= 20 \text{ cm}$$

38. $B = \dfrac{\mu_0}{4\pi} \cdot \dfrac{2N\pi r^2 I}{(r^2 + x^2)^{3/2}}$

But, $x = R$

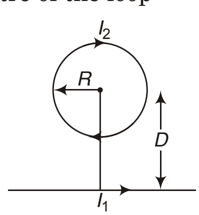
$$B = \frac{\mu_0 NI}{4\sqrt{2}r} \Rightarrow N = \frac{4\sqrt{2}\,Br}{\mu_0 I}$$

$$= \frac{4\sqrt{2} \times 6.39 \times 10^{-4} \times 6 \times 10^{-2}}{4\pi \times 10^{-7} \times 2.5}$$

$\Rightarrow \quad N = 69$

39. For magnetic field at the centre of loop to be zero, magnetic field due to straight conductor at centre of loop must be outward, hence I_1 must be rightwards.

At the centre of the loop



$$B = B_1 - B_2$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2I_1}{D} - \frac{\mu_0 I_2}{2R} = 0$$

$$I_1 = \frac{\pi D}{R} I_2$$

40. (a) $B = \dfrac{\mu_0 NI}{2R} \Rightarrow I = \dfrac{2BR}{\mu_0 N}$

$$= \frac{2 \times 0.0580 \times 2.40 \times 10^{-2}}{4\pi \times 10^{-7} \times 800}$$

$$\Rightarrow \quad I = 2.77 \text{ A}$$

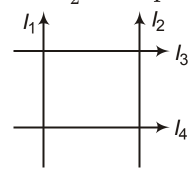
(b) On the axis of coil,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\,NIA}{(r^2 + x^2)^{3/2}}$$

$$\frac{B_C}{B} = \frac{(r^2 + x^2)^{3/2}}{r^3} \Rightarrow \left(\frac{r^2 + x^2}{r^2}\right)^{3/2} = 2$$

$$\Rightarrow \quad x = 0.0184 \text{ m}$$

41. Let the current $I_2 (= I)$ upwards



$$B = -B_1 + B_2 - B_3 + B_4$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2}{r} [-I_1 + I_2 - I_3 + I_4] = 0$$

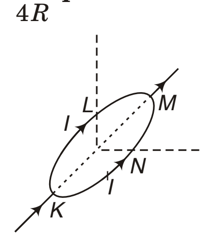
$$I_2 = I_1 + I_3 - I_4$$

$$= 10 + 8 - 20$$

$$= -2 \text{ A}$$

Negative sign indicates that current I is directed downwards.

42. $\vec{\mathbf{B}}_{KLM} = -\dfrac{\mu_0 I}{4R} \hat{\mathbf{i}}$



$$\vec{\mathbf{B}}_{KNM} = \frac{\mu_0 I}{4R} \hat{\mathbf{j}}$$

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_{KLM} + \vec{\mathbf{B}}_{KNM} = \frac{\mu_0 I}{4R}(-\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

(a) $\vec{\mathbf{F}} = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}}) = -\dfrac{\mu_0 Iqv}{4R} \hat{\mathbf{k}}$

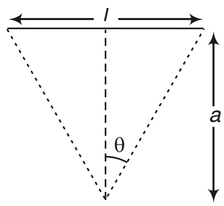
(b) $\vec{\mathbf{l}}_1 = \vec{\mathbf{l}}_2 = -2R\hat{\mathbf{k}}$

$$\vec{\mathbf{F}}_1 = I(\vec{\mathbf{l}}_1 \times \vec{\mathbf{B}}) = 2IRB\,\hat{\mathbf{i}}$$

$$\vec{\mathbf{F}}_2 = I(\vec{\mathbf{l}}_2 \times \vec{\mathbf{B}}) = 2IRB\,\hat{\mathbf{i}}$$

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = 4IRB\,\hat{\mathbf{i}}$$

43. (a) Length of each side



$$l = \frac{2\pi r}{n}$$

$$\theta = \frac{\pi}{n}$$

$$a = \frac{l}{2}\cot\theta = \frac{\pi r}{n}\cot\theta$$

$$B = n \times \frac{\mu_0}{4\pi}\cdot\frac{i}{a}(2\sin\theta)$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{2n^2\sin\theta}{\pi r\cot\theta}$$

$$= \frac{\mu_0 i n^2 \sin^2\frac{\pi}{n}}{2\pi^2 r\cos\frac{\pi}{n}}$$

(b) $$\lim_{n\to\infty} B = \lim_{n\to\infty}\frac{\mu_0 i n^2\sin^2\frac{\pi}{n}}{2\pi^2 r\cos\frac{\pi}{n}}$$

$$\Rightarrow \quad \lim_{n\to 0} = \frac{\mu_0 i}{2r}$$

44. $\oint \vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = 3.83\times 10^{-7}$ T-m

(a) By Ampere's circuital law

$$\oint \vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = \mu_0 I$$

$$\Rightarrow I = \frac{1}{\mu_0}\oint \vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = \frac{1}{4\pi\times 10^{-7}}\times 3.83\times 10^{-7}$$

$$= 0.3\text{A}$$

(b) If we integrate around the curve in the opposite direction, the value of line integral will become negative, *i.e.*,

$$-3.83\times 10^{-7}\text{ T-m.}$$

45. $\oint \vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = \mu_0 I$

As the path is taken counter-clockwise direction, $\oint \vec{\mathbf{B}}\cdot\vec{\mathbf{dl}}$ will be positive if current is outwards and will be negative if current is inwards.

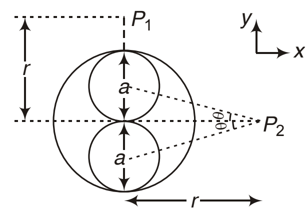
$$\oint_a \vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = 0$$

$$\oint_b \vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = -\mu_0 I_1 = -5.0\times 10^{-6}\text{ T-m}$$

$$\oint_c \vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = \mu_0(I_2 - I_1) = 2.5\times 10^{-6}\text{ T-m}$$

$$\oint_d \vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = \mu_0(I_2 + I_3 - I_1) = 5.0\times 10^{-6}\text{ T-m}$$

46.



Current density

$$J = \frac{I}{\pi a^2 - 2\pi\left(\frac{a}{2}\right)^2} = \frac{2I}{\pi a^2}$$

Let us consider both the cavities are carrying equal and opposite currents with current density J.

Let B_1, B_2 and B_3 be magnetic fields due to complete cylinder, upper and lower cavity respectively.

(a) At point P_1

$$\vec{\mathbf{B}}_1 = -\frac{\mu_0}{4\pi}\cdot\frac{2I_1}{r}\hat{\mathbf{i}} = -\frac{\mu_0}{4\pi}\cdot\frac{2J\times\pi a^2}{r}\hat{\mathbf{i}}$$

$$= -\frac{\mu_0 I}{\pi r}\hat{\mathbf{i}}$$

$$\vec{\mathbf{B}}_2 = \frac{\mu_0}{4\pi}\cdot\frac{2I_2}{r-\frac{a}{2}}\hat{\mathbf{i}} = \frac{\mu_0}{4\pi}\cdot\frac{2J\times\pi\left(\frac{a}{2}\right)^2}{r-\frac{a}{2}}\hat{\mathbf{i}}$$

$$= -\frac{\mu_0 I}{4\pi\left(r-\frac{a}{2}\right)}\hat{\mathbf{i}}$$

$$\vec{\mathbf{B}}_3 = \frac{\mu_0}{4\pi}\cdot\frac{2I_3}{r+\frac{a}{2}}\hat{\mathbf{i}} = \frac{\mu_0}{4\pi\left(r+\frac{a}{2}\right)}\hat{\mathbf{i}}$$

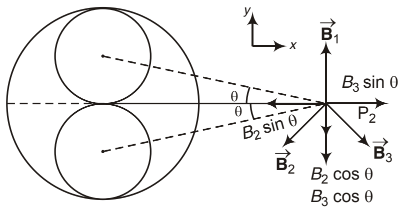
$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 + \vec{\mathbf{B}}_3$$

$$= \frac{\mu_0 I}{4\pi}\left[-\frac{4}{r} + \frac{1}{r-\frac{a}{2}} + \frac{1}{r+\frac{a}{2}}\right]\hat{\mathbf{i}}$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi r}\left[\frac{2r^2 - a^2}{4r^2 - a^2}\right]\hat{\mathbf{i}}$$

$$\therefore (\vec{\mathbf{B}}) = \frac{\mu_0 I}{4\pi r}\left[\frac{2r^2 - a^2}{4r^2 - a^2}\right], \text{ towards left.}$$

(b) At point P_2



$$\vec{\mathbf{B}}_1 = \frac{\mu_0}{4\pi}\cdot\frac{2I_1}{r}\hat{\mathbf{j}} = \frac{\mu_0 I}{\pi r}\hat{\mathbf{j}}$$

$$\vec{\mathbf{B}}_2 = \frac{\mu_0}{4\pi}\cdot\frac{2I_2}{\sqrt{r^2+\frac{a^2}{4}}}[-\sin\theta\,\hat{\mathbf{i}} - \cos\theta\,\hat{\mathbf{j}}]$$

$$= \frac{-\mu_0 I}{2\pi\sqrt{4r^2+a^2}}[\sin\theta\,\hat{\mathbf{i}} + \cos\theta\,\hat{\mathbf{j}}]$$

$$\vec{\mathbf{B}}_3 = \frac{\mu_0}{4\pi}\cdot\frac{2I_3}{\sqrt{r^2+\frac{a^2}{4}}}[\sin\theta\,\hat{\mathbf{i}} - \cos\theta\,\hat{\mathbf{j}}]$$

$$= \frac{\mu_0 I}{2\pi\sqrt{4r^2+a^2}}[\sin\theta\,\hat{\mathbf{i}} - \cos\theta\,\hat{\mathbf{j}}]$$

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 + \vec{\mathbf{B}}_3$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{2}{r} - \frac{2\cos\theta}{\sqrt{4r^2+a^2}}\right]\hat{\mathbf{j}}$$

but, $\cos\theta = \dfrac{r}{\sqrt{r^2+\frac{a^2}{4}}} = \dfrac{2r}{\sqrt{4r^2+a^2}}$

$$\therefore \quad \vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi}\left[\frac{2}{r} - \frac{4r}{4r^2+a^2}\right]\hat{\mathbf{j}}$$

$$= \frac{\mu_0 I}{4\pi r}\left[\frac{2r^2+a^2}{4r^2+a^2}\right]\hat{\mathbf{j}}$$

$$(\vec{\mathbf{B}}) = \frac{\mu_0 I}{4\pi r}\left[\frac{2r^2+a^2}{4r^2+a^2}\right], \text{ upwards.}$$

47. Let us first find magnetic field due a current carrying infinite plate.

Consider a rectangular amperian loop ($WXYZ$) as shown in Fig. 1.

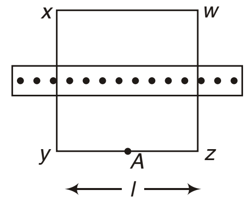


Fig.1

$$\oint_{WXYZ} \vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = \mu_0 \lambda l$$

$$\Rightarrow \int_{W\to X}\vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} + \int_{X\to Y}\vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} + \int_{Y\to Z}\vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} + \int_{Z\to W}\vec{\mathbf{B}}\cdot\vec{\mathbf{dl}} = \mu_0\lambda l$$

$$Bl + 0 + Bl + 0 = \mu_0 \lambda l$$

$$B = \frac{1}{2}\mu_0\lambda$$

In Fig. 2.

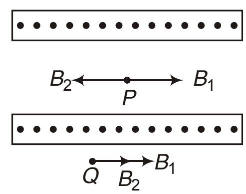


Fig.2

At point P,

$$B_1 = B_2 = \frac{1}{2}\mu_0\lambda$$

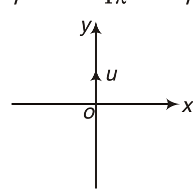
$$B = B_1 - B_2 = 0,$$

At point Q,

$$B_1 = B_2 = \frac{1}{2}\mu_0\lambda$$

$$B = B_1 + B_2 = \mu_0\lambda$$

48. $\vec{\mathbf{B}} = \dfrac{\mu_0}{4\pi}\cdot\dfrac{I\,\vec{\mathbf{dl}}\times\vec{\mathbf{r}}}{r^3} = \dfrac{\mu_0}{4\pi}\cdot\dfrac{q(\vec{\mathbf{v}}\times\vec{\mathbf{r}})}{r^3}$



$$\vec{\mathbf{v}} = (8.00\times10^6\ \text{ms}^{-1})\hat{\mathbf{j}}$$

(a) $\vec{\mathbf{r}} = (0.500\ \text{m})\hat{\mathbf{i}}$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\cdot\frac{q(\vec{\mathbf{v}}\times\vec{\mathbf{r}})}{r^3}$$

$$= \frac{10^{-7} \times 6.00 \times 10^{-6}[(8.00 \times 10^{6}\,\hat{\mathbf{j}}) \times (0.500)\,\hat{\mathbf{i}}\,]}{(0.500)^3}$$

$$\Rightarrow \vec{\mathbf{B}} = -(1.92 \times 10^{-5}\ \mathrm{T})\,\hat{\mathbf{k}}$$

(b) $\vec{\mathbf{r}} = -(0.500\ \mathrm{m})\,\hat{\mathbf{j}}$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \cdot \frac{q(\vec{\mathbf{v}} \times \vec{\mathbf{r}})}{r^3} = 0$$

(c) $\vec{\mathbf{r}} = +(0.500\ \mathrm{m})\,\hat{\mathbf{k}}$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \cdot \frac{q(\vec{\mathbf{v}} \times \vec{\mathbf{r}})}{r^3} = (1.92 \times 10^{-5}\ \mathrm{T})\,\hat{\mathbf{i}}$$

(d) $\vec{\mathbf{r}} = -(0.50\ \mathrm{m})\,\hat{\mathbf{j}} + 0.500\ \mathrm{m}\,\hat{\mathbf{k}}$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \cdot \frac{q(\vec{\mathbf{v}} \times \vec{\mathbf{r}})}{r^3} = (1.92 \times 10^{-5}\ \mathrm{T})\,\hat{\mathbf{i}}$$

49. $q = -4.80\ \mu\mathrm{C} = -4.80 \times 10^{-6}\ \mathrm{C}$

$$\vec{\mathbf{v}} = (6.80 \times 10^{5}\ \mathrm{m/s})\,\hat{\mathbf{i}}$$

(a) $\vec{\mathbf{r}} = (0.500\ \mathrm{m})\,\hat{\mathbf{i}}$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \cdot \frac{q(\vec{\mathbf{v}} \times \vec{\mathbf{r}})}{r^3} = 0$$

(b) $\vec{\mathbf{r}} = (0.500\ \mathrm{m})\,\hat{\mathbf{j}}$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \cdot \frac{q(\vec{\mathbf{v}} \times \vec{\mathbf{r}})}{r^3} = -(1.3 \times 10^{-6}\ \mathrm{T})\,\hat{\mathbf{k}}$$

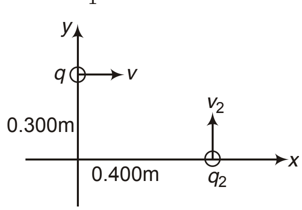
(c) $\vec{\mathbf{r}} = (0.500\ \mathrm{m})\,\hat{\mathbf{i}} + (0.500\ \mathrm{m})\,\hat{\mathbf{j}}$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \cdot \frac{q(\vec{\mathbf{v}} \times \vec{\mathbf{r}})}{r^3} = -(1.31 \times 10^{-6}\ \mathrm{T})\,\hat{\mathbf{k}}$$

(d) $\vec{\mathbf{r}} = (0.500\ \mathrm{m})\,\hat{\mathbf{k}}$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \cdot \frac{q(\vec{\mathbf{v}} \times \vec{\mathbf{r}})}{r^3} = (1.31 \times 10^{-6}\ \mathrm{T})\,\hat{\mathbf{j}}$$

50. $\vec{\mathbf{B}}_1 = \dfrac{\mu_0}{4\pi} \cdot \dfrac{q_1(\vec{\mathbf{v}}_1 \times \vec{\mathbf{r}}_2)}{r_1^3}$



$\Rightarrow$

$$\vec{\mathbf{B}}_1 = \frac{10^{-7} \times 4.00 \times 10^{-6}\,[(2.00 \times 10^{5}\,\hat{\mathbf{i}}) \times (-0.300\,\hat{\mathbf{j}})]}{(0.300)^3}$$

$$= -(8.89 \times 10^{-7}\ \mathrm{T})\,\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}}_2 = \frac{\mu_0}{4\pi} \cdot \frac{q_2(\vec{\mathbf{v}}_2 \times \vec{\mathbf{r}}_2)}{r_2^3}$$

$\Rightarrow$

$$\vec{\mathbf{B}}_2 = \frac{10^{-7} \times (-1.5 \times 10^{-6})[(8.00 \times 10^{5}\,\hat{\mathbf{i}}) \times (-0.400\,\hat{\mathbf{j}})]}{(0.400)^2}$$

$$= -(7.5 \times 10^{-7}\ \mathrm{T})\,\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = -(16.4 \times 10^{-6}\ \mathrm{T})\,\hat{\mathbf{k}}$$

$$= -(1.64 \times 10^{-6}\ \mathrm{T})\,\hat{\mathbf{k}}$$

or $\quad B = 1.64 \times 10^{-6}\ \mathrm{T}$ (inwards)

51. Magnetic force per unit length on the conductor AB,

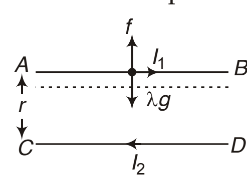
$$f = \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{r}$$

For equilibrium

$$f = \frac{m}{l} g = \lambda g$$

$$\Rightarrow \qquad \lambda g = \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{r} \qquad \ldots\text{(i)}$$

Suppose wire AB is depressed by x,



Net force on unit length of wire AB

$$\lambda a = \lambda g - f'$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{r} - \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{r - x}$$

$$= -\frac{\mu_0}{4\pi} \cdot \frac{2\,I_1I_2 \cdot x}{r(r - x)}$$

If $x << r$

$$\lambda a = -\frac{\mu_0 I_1 I_2}{2r^2} x$$

$$a = -\frac{\mu_0 I_1 I_2}{2\lambda\, r^2} x \qquad \ldots\text{(ii)}$$

General equation of SHM

$$a = -\omega^2 x \qquad \ldots\text{(ii)}$$

Hence, motion of wire AB will be simple harmonic.

From Eqs. (i) and (ii),

$$\omega = \sqrt{\frac{\mu_0 I_1 I_2}{2\lambda r^2}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2\lambda r^2}{\mu_0 I_1 I_2}} = 2\pi\sqrt{\frac{r}{g}}$$

$$= 2 \times 3.14\sqrt{\frac{0.01}{9.8}}$$

$$= 0.2 \text{ s}$$

52. (a) $f = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r}$

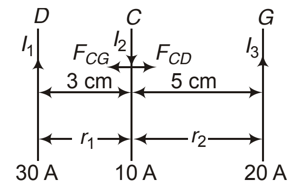
$$I_2 = \frac{f\,r}{\frac{\mu_0}{4\pi} \cdot 2I_1}$$

$$= \frac{4.00 \times 10^{-5} \times 2.50 \times 10^{-2}}{10^{-7} \times 2 \times 0.600}$$

$$= 8.33 \text{ A}$$

(b) As the wires repel each other, current must be in opposite directions.

53. $f_{CD} = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r_1}$



$$f_{CG} = \frac{\mu_0}{4\pi} \cdot \frac{2I_2 I_3}{r_2}$$

$$f = f_{CD} - f_{CG}$$

$$f = f_{CD} - f_{CG} = \frac{\mu_o}{4\pi} \cdot 2I_2\left[\frac{I_1}{r_1} - \frac{I_3}{r_2}\right]$$

$$= 10^{-7} \times 2 \times 10\left[\frac{30}{3 \times 10^{-2}} - \frac{20}{5 \times 10^{-2}}\right]$$

$$f = 12 \times 10^{-4} \text{ N} = 1.2 \times 10^{-3} \text{ N/m}$$

$$F = f\,l = 1.2 \times 10^{-3} \times 25 \times 10^{-2}$$

$$= 3 \times 10^{-4} \text{ N}$$

54. Force per unit length on wire MN

$$f_{MN} = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{a}$$

$$F = f_{MN} \times L = \frac{\mu_0 I_1 I_2 L}{2\pi a}$$

Torque acting on the loop is zero because magnetic field is parallel to the area vector.

Objective Questions (Level 1)

1. Fact

2. $T = \frac{2\pi m}{qB}$ is independent of speed.

3. Outside the wire

$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r}$ where, r is distance from the centre.

4. The path will be parabola if force acting on the particle is constant in magnitude as well as in direction.

5. $B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r}$

$$\mu_0 = \frac{4\pi rB}{2I}$$

$$\text{Units of } \mu_0 = \frac{\text{m} \times \text{Wb/m}^2}{\text{A}}$$

$$= \text{Wbm}^{-1}\text{A}^{-1}$$

6. Fact

7. $\vec{\mathbf{M}} = i\,\vec{\mathbf{A}}$, where $\vec{\mathbf{A}}$ = Area vector.

8. Force acting on a closed current carrying loop is always zero.

9. $M = NIA$

10. $\vec{\mathbf{a}} \perp \vec{\mathbf{B}} \Rightarrow \vec{\mathbf{a}} \cdot \vec{\mathbf{B}} = 0$

$$(x\,\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}) = 2x + 3 - 4 = 0$$

$$\Rightarrow x = 0.5$$

12. A current carrying closed loop never experiences a force magnetic field.

13. $r = \frac{mv}{qB} = \frac{P}{qB}$,

$$P = mv = \text{momentum.}$$

$$\therefore \qquad r \propto \frac{1}{q}$$

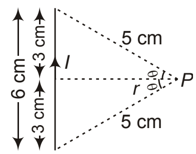
$$\therefore \qquad \frac{r_p}{r_\alpha} = \frac{q_\alpha}{q_p} \Rightarrow r_p : r_\alpha = 2:1$$

14. $W = MB(\cos\theta_1 - \cos\theta_2)$

Here, $\theta_1 = \pi$, $\theta_2 = \pi - \theta$

$$W = MB(\cos\pi - \cos(\pi - \theta))$$
$$= -MB(1 - \cos\theta)$$

15. $B_P = \frac{\mu_0}{4\pi} \cdot \frac{I}{r}(2\sin\theta)$



$$r = \sqrt{5^2 - 3^2} = 4 \text{ cm}$$
$$= 4 \times 10^{-2} \text{ m}$$

$\sin\theta = \frac{3}{5}$

$$\therefore \quad B_P = \frac{10^{-7} \times 50 \times 2 \times \frac{3}{5}}{4 \times 10^{-2}}$$
$$= 1.5 \times 10^{-4} \text{ T}$$
$$= 1.5 \text{ gauss.}$$

16. Magnetic field on the axis of current carrying circular loop,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2M}{(r^2 + x^2)^{3/2}} \quad \text{...(i)}$$

Magnetic field at the centre of current carrying circular loop,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$\frac{B_2}{B_1} = \frac{(r^2 + x^2)^{3/2}}{r^3}$$
$$= \frac{(3^2 + 4^2)^{3/2}}{3^3}$$
$$= \frac{125}{27}$$

$$\Rightarrow \quad B_2 = \frac{125}{27} \times 54 = 150 \,\mu\text{T}$$

17. $\vec{\mathbf{F}} = I(\vec{\mathbf{I}} \times \vec{\mathbf{B}}) = I(\vec{\mathbf{ba}} \times \vec{\mathbf{B}})$

$= -I(\vec{\mathbf{ab}} \times \vec{\mathbf{B}}) = I(\vec{\mathbf{B}} \times \vec{\mathbf{ab}})$

18. Kinetic energy of electron,

$$K = \frac{1}{2}mv^2 = eV$$

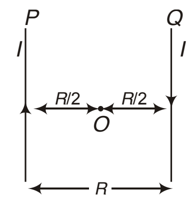
$$\Rightarrow \quad v = \sqrt{\frac{2eV}{m}}$$

Magnetic force,

$$F_m = evB\sin\theta$$
$$F_m \propto v \Rightarrow F_m \propto \sqrt{v}$$

Hence, if potential difference is doubled, force will become $\sqrt{2}$ times.

19. Magnetic field at O due to P,



$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2I}{R/2} = \frac{\mu_0 I}{\pi R} \quad \text{(inwards)}$$

Magnetic field at O due to Q,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I}{R/2} = \frac{\mu_0 I}{\pi R} \quad \text{(inwards)}$$

Net magnetic field at O,

$$B = B_1 + B_2 = \frac{2\mu_0 I}{\pi R}$$

20. As solved in Question 16,

$$\frac{B_2}{B_1} = \left(\frac{x^2 + R^2}{R^2}\right)^{3/2}$$

$$\Rightarrow \quad \left(\frac{x^2 + R^2}{R^2}\right)^{3/2} = 8$$

$$\Rightarrow \quad \frac{x^2 + R^2}{R^2} = 4$$

$$\Rightarrow \quad x = \sqrt{3}\,R$$

21. Component of velocity of particle along magnetic field, *i.e.*,

$$v_y = \frac{qE}{m}t = \alpha E t$$

is not constant, hence pitch is variable.

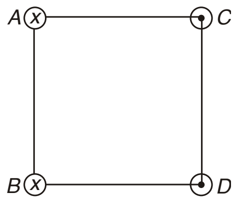
22. $r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$

Now, $R = \frac{\sqrt{2mK}}{eB}$

$$R' = \frac{\sqrt{2m(2K)}}{e(3R)} = \frac{\sqrt{2}}{3}R$$

23. Same as question 1. Introductory exercise 23.6.

Note. Her diagram is wrong correct diagram should be



24. $r = \dfrac{mv}{qB} = \dfrac{\sqrt{2\,mK}}{qB} = \dfrac{\sqrt{2\,mqV}}{qB}$ $[K = qV]$

$\Rightarrow \qquad r = \sqrt{\dfrac{2\,mV}{q}}\left(\dfrac{1}{B}\right)$

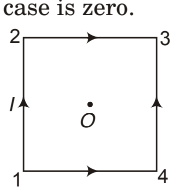
25. Magnetic field due to a conductor of finite length.

$$B = \frac{\mu_0}{4\pi}\cdot\frac{I}{r}(\sin\alpha + \sin\beta)$$

Here, $\alpha = -\theta_2$, $\beta = \theta_1$ and $r = a$

$\therefore \qquad B = \dfrac{\mu_0 I}{2a}(\sin\theta_1 - \sin\theta_2)$

26. In case C, magnetic field of conductor 1-2 and 2-3 at O is inward while those of 3-4 and 4-1 at O is outward, hence net magnetic field at O in this case is zero.

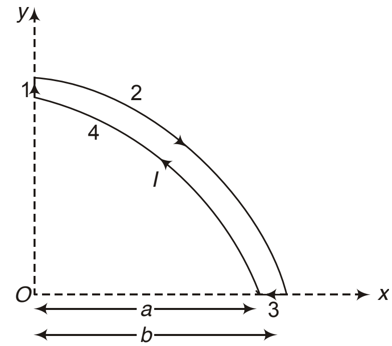


27. $\vec{\mathbf{dF}} = I(\vec{\mathbf{dl}} \times \vec{\mathbf{B}})$

But $\vec{\mathbf{B}} \,||\, \vec{\mathbf{dl}}$ at every point,

hence, $\vec{\mathbf{dF}} = 0$.

28. $B_1 = B_3 = 0$ (Magnetic field on the axis of current carrying straight conductor is zero)



$$\vec{\mathbf{B}}_2 = \frac{-1}{4}\left(\frac{\mu_0 I}{2b}\right)\hat{\mathbf{k}} = -\frac{\mu_0 I}{8b}\hat{\mathbf{k}},$$

$$\vec{\mathbf{B}}_3 = \frac{1}{4}\left(\frac{\mu_0 I}{2a}\right)\hat{\mathbf{k}} = \frac{\mu_0 I}{8a}\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 + \vec{\mathbf{B}}_3 + \vec{\mathbf{B}}_4 = \frac{\mu_0 I}{8}\left[\frac{1}{a} - \frac{1}{b}\right]\hat{\mathbf{k}}$$

29. Current associated with electron,

$$I = \frac{q}{T} = ef$$

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 ef}{2R}$$

30. Same as question 1(a). Introductory Exercise 23.5.

31. At point 1,

Magnetic field due to inner conductor is non-zero, but due to outer conductor is zero.

Hence, $B_1 \neq 0$

At point 2,

Magnetic field due to both the conductors is equal and opposite.

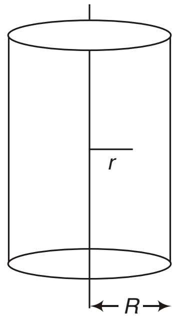
Hence, $B_2 = 0$

32. Apply Fleming's left hand rule or right hand thumb rule.

33. Magnetic field due to straight conductors at O is zero because O lies on axis of both the conductors.

Hence, $B = \dfrac{\phi}{2\pi}\cdot\dfrac{\mu_0 I}{2x} = \dfrac{\mu_0 I\phi}{4\pi x}$

34. Inside a solid cylinder having uniform current density,



$$B = \frac{\mu_0 I r}{2\pi R^2}$$

Here, $r = R - x$

$\therefore$ $$B = \frac{\mu_0 I (R - x)}{2\pi R^2}$$

35. Magnetic force is acting radially outward on the loop.

JEE Corner

Assertion and Reason

1. For parabolic path, acceleration must be constant and should not be parallel or antiparallel to velocity.

2. By Fleming's left hand rule.

3. Magnetic force on upper wire must be in upward direction, hence current should be in a direction opposite to that of wire 1.

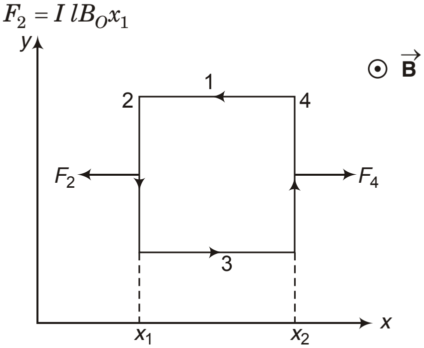
Reason is also correct but does not explain Assertion.

4. $\tau = MB \sin \alpha$

$\alpha = 90°$

$\therefore \tau = MB \neq 0$

5. $F_2 = I\, lB_O x_1$



$$F_4 = I\, lB_0 x_2$$

$\therefore$ $$F_4 > F_2$$

Hence, net force is along X-axis.

6. Radii of both is different because mass of both is different

$$r = \frac{mv}{qB} = \frac{\sqrt{2\,meV}}{e\,B}$$

7. For equilibrium

$\vec{\mathbf{F}}_e + \vec{\mathbf{F}}_m = 0$

$\Rightarrow$ $$q\,\vec{\mathbf{E}} = -q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$$

$\Rightarrow$ $$\vec{\mathbf{E}} = -\vec{\mathbf{v}} \times \vec{\mathbf{B}} = \vec{\mathbf{B}} \times \vec{\mathbf{v}}$$

8. $P_m = \vec{\mathbf{F}}_m \cdot \vec{\mathbf{v}}$

As $\vec{\mathbf{F}}_m$ is always perpendicular to $\vec{\mathbf{v}}$,

$$\vec{\mathbf{P}}_m = 0$$

Again, $P_e = \vec{\mathbf{F}}_e \cdot \vec{\mathbf{v}}$, may or may not be zero.

9. Reason correctly explains Assertion.

10. Magnetic force cannot change speed of particle as it is always perpendicular to the speed of the particle.

11. $a = \dfrac{v^2}{R}$

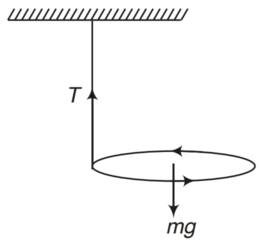
but R also depends on v.

$\therefore$ $$a = \frac{F_m}{m} = \frac{qvB}{m}$$

$\Rightarrow$ $$a \propto v$$

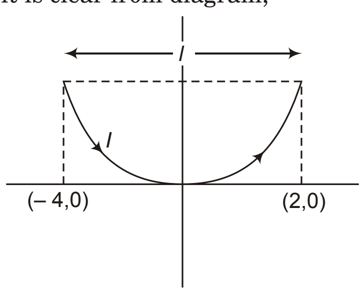
Objective Questions (Level 2)

1. For net torque to be zero.



$$IAB_0 = mgR$$
$$I = \frac{mgR}{AB_0} = \frac{mgR}{\pi R^2 B_0}$$
$$= \frac{mg}{\pi RB_0}$$

2. As it is clear from diagram,



Effective length of wire,

$$\vec{\mathbf{l}} = (4\text{ m})\hat{\mathbf{i}}$$
$$\vec{\mathbf{F}} = I(\vec{\mathbf{l}} \times \vec{\mathbf{B}})$$
$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}}{m} = \frac{I}{m}(\vec{\mathbf{l}} \times \vec{\mathbf{B}})$$
$$= \frac{2}{0.1}(4\hat{\mathbf{i}} \times (-0.02\hat{\mathbf{k}})) = 1.6\hat{\mathbf{j}}\text{ m/s}^2$$

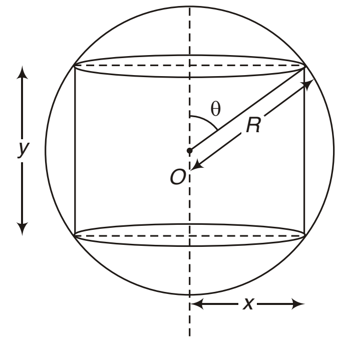
3. Impulse = Change in momentum

$$\int I\,lB\,dt = mv - 0$$
$$lB\int dq = mv$$
$$dq = \frac{mv}{lB} = \frac{m\sqrt{2gh}}{lB}$$

4. Consider the sphere to be made up of large number of hollow, coaxial cylinder of different height and radius. Consider one such cylinder of radius x, height y and thickness.

Now, $y = 2R\cos\theta$, $x = R\sin\theta$, $dx = R\cos\theta\,d\theta$

Charge on this cylinder,



$$dq = \frac{q}{\frac{4}{3}\pi R^3}\cdot(2\pi\,yx\,dx)$$
$$= 3q\cos^2\theta\sin\theta\,d\theta$$

Current associated with this cylinder,

$$di = \frac{dq}{T} = \frac{\omega\,dq}{2\pi} = \frac{3\omega q}{2\pi}\cos^2\theta\sin\theta\,d\theta$$

Magnetic moment associated with this cylinder,

$$dM = di\,A = \frac{3q\omega}{2\pi}\cos^2\theta\sin\theta\,d\theta \times \pi x^2$$
$$dM = \frac{3}{2}R^2\omega qA\cos^2\theta\sin^3\theta\,d\theta$$
$$M = \int dM = \frac{3}{2}R^2 q\int_{\pi 2}^{0}\cos^2\theta\sin^3\theta\,d\theta$$
$$= \frac{3}{2}R^2\omega q\int_{\pi/2}^{0}\cos^2\theta(1-\cos^2\theta)\sin\theta\,d\theta$$
$$= \frac{3}{2}R^2\omega q\left[\frac{\cos^3\theta}{3} - \frac{\cos^5\theta}{5}\right]_{\pi/2}^{0}$$
$$= \frac{1}{5}R^2\omega q$$

5. As solved in question 5(c). Introductory Exercise 23.2.

$$\frac{L}{R} = \sin\theta$$

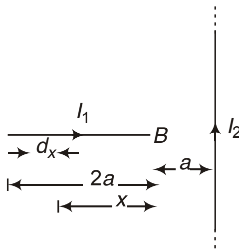
Here, $L = d$, $R = \dfrac{mV}{qB}$

$$\therefore \quad \frac{qBd}{mV} = \sin\theta$$

or
$$\frac{q}{m} = \frac{V\sin\theta}{Bd}$$

6. Force on portion AC will more compared to that on portion CB.

7. Consider an elementary portion of the wire carrying current I_1 of length dx at a distance x from end B.



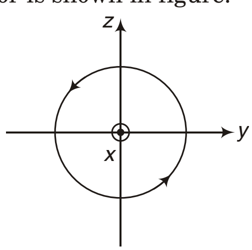
Force on this portion

$$dF = I_1 dx\, B$$
$$= \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{a+x} dx$$

Total force on wire AB

$$F = \int dF = \frac{\mu_0}{4\pi} \cdot 2I_1 I_2 \int_a^{2a} \frac{dx}{a+x}$$
$$= \frac{\mu_0 I_1 I_2}{2\pi} \ln 3$$

8. Magnetic field line due to current carrying conductor is shown in figure.



9.
$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2IA_1}{(x_1^2 + r_1^2)^{3/2}}$$
$$= \frac{\mu_0}{4\pi} \cdot \frac{2I \cdot \pi r_1^2}{(x_1^2 + r_1^2)^{3/2}}$$
$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I \cdot \pi r_2^2}{(x_2^2 + r_2^2)^{3/2}}$$
$$\frac{B_1}{B_2} = \frac{r_1^2 (x_2^2 + r_2^2)^{3/2}}{r_2^2 (x_1^2 + r_1^2)^{3/2}}$$

But, $r_1 = x_1 \tan\theta$

and $r_2 = x_2 \tan\theta$

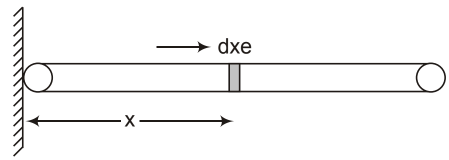
$\therefore$ $\frac{B_1}{B_2} = 2$

10. $b - a$ must be less than or equal to radius of circular path,

i.e., $$b - a \le \frac{mv}{qB}$$

or $$v \ge \frac{qB(b-a)}{m}$$

11. Consider an elementary portion of length dx at a distance x from the pivoted end.



Charge on this portion

$$dq = \frac{q}{l} dx$$

Current associated with this portion

$$di = \frac{dq}{T} = \frac{qf}{l} dx$$

Magnetic moment of this portion

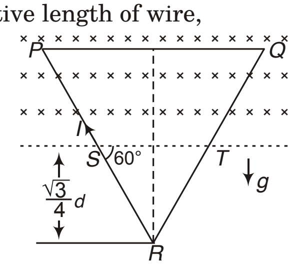
$$dM = \pi x^2 di = \frac{\pi qf}{l} x^2 dx$$
$$M = \frac{\pi qf}{l} \int_0^l x^2 dx = \frac{1}{3} \pi qf l^2$$

12. At $x = 0$, $y = \pm 2$ m

Effective length of wire

$$l = (4 \text{ m})\hat{\mathbf{j}}$$
$$\therefore \quad \vec{\mathbf{F}}_m = I(\vec{\mathbf{l}} \times \vec{\mathbf{B}}) = 3(4\hat{\mathbf{j}} \times 5\hat{\mathbf{k}})$$
$$= 60\,\hat{\mathbf{i}} \text{ N}$$

13. Effective length of wire,



$$l = ST = 2 \times \frac{\sqrt{3}}{4} a \times \cot 60^\circ$$
$$= \frac{a}{2}$$

For equilibrium, $IlB = Mg$

$$\Rightarrow \quad I = \frac{2Mg}{lB}$$

14. For particle not collide with the solenoid, radius of path of particle ≤ half or radius of solenoid.

$$\frac{mv}{qB} \geq \frac{r}{2}$$

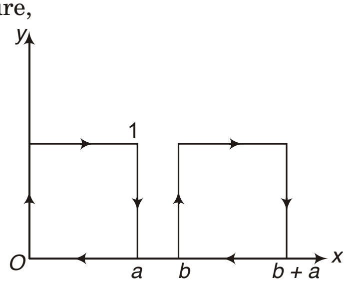
But $B = \mu_0 n\, i$

$$\Rightarrow \qquad v > \frac{rqB}{2m} = \frac{\mu_0 qr\, n\, i}{2m}$$

16. Magnetic force cannot do work on charged particle, hence its energy will remain same, so that θ remains same.

Again, magnetic force is always along the string, it will never produce a torque hence, T will also remain same.

17. Let the x-coordinates of loops be as shown in figure,



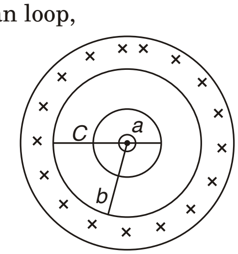
then, $F_1 = Ia\,(B_0 a) - 0 = I\,a^2 B_0$

$F_2 = Ia\,(B_0\,(b + a)) - Ia\,(B_0 b)$

$= I\,a^2 B_0$

$\therefore \qquad F_1 = F_2 \neq 0$

18. Consider an amperian loop of radius $x\,(b < x < c)$, threaded by current the amperian loop,



$$I' = I - \frac{x^2 - b^2}{c^2 - b^2} I$$

$$= \frac{c^2 - x^2}{c^2 - b^2} I$$

$$I = \frac{\mu_0 I'}{2\pi x} = \frac{\mu_0 I\,(c^2 - x^2)}{2\pi x\,(c^2 - b^2)}$$

19. As $\vec{\mathbf{E}} = -\vec{\mathbf{v}} \times \vec{\mathbf{B}}$

Net force on the particle must be zero.

20. Consider an elementary portion of length dy at $y - y$ on the wire.

Force on this portion,

$$dF = I\,(\vec{\mathbf{dy}} \times \vec{\mathbf{B}})$$

Here, $\vec{\mathbf{dy}} = -dy\,\hat{\mathbf{j}}$ (Current is directed along negative y-axis).

$dF = -I\,\{dy\,\hat{\mathbf{j}}(0.3\,y\,\hat{\mathbf{i}} + 0.4y\,\hat{\mathbf{j}})\}$

$= -2 \times 10^{-3}(-0.3\,y\,dy\,\hat{\mathbf{k}})$

Total force on the wire,

$F = \int dF = -2 \times 10^{-3} \int_0^1 (-0.3\,y\,dy\,\hat{\mathbf{k}})$

$F = (3 \times 10^{-4}\,\hat{\mathbf{k}})$ N

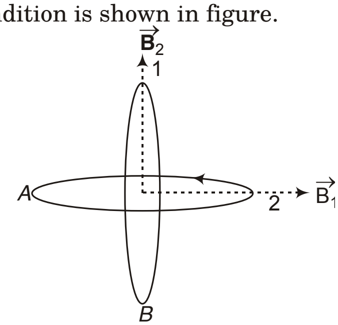
21. $\vec{\mathbf{E}} = -\vec{\mathbf{v}} \times \vec{\mathbf{B}}$

$|\vec{\mathbf{E}}| = vB = \frac{rqB}{m} B$

$= \frac{(5 \times 10^{-2})(20 \times 10^{-6})(0.1)^2}{(20 \times 10^{-9})}$

$\therefore\ \vec{\mathbf{E}} = 0.5$ V/m $\qquad (1\,\mu\text{g} = 10^{-9}\text{kg})$

22. Condition is shown in figure.



$$B_1 = \frac{\mu_0 I_1}{2R_1}$$

$$B_2 = \frac{\mu_0 I_2}{2R_2}$$

$\therefore \qquad B = \sqrt{B_1^2 + B_2^2}$

$$= \frac{\mu_0}{2}\sqrt{\left(\frac{I_1}{R_1}\right)^2 + \left(\frac{I_2}{R_2}\right)^2}$$

$$= \frac{4\pi \times 10^{-7}}{2} \times \sqrt{\left(\frac{5}{\frac{5}{\sqrt{2}} \times 10^{-2}}\right)^2 + \left(\frac{5\sqrt{2}}{5 \times 10^{-2}}\right)^2}$$

$$= \frac{4\pi \times 10^{-7} \times 2}{2 \times 10^{-2}} = 4\pi \times 10^{-5} \text{ T}$$

23. Initially, net force on the particle is zero. Hence,

$$V = \frac{E}{B}$$

Now, if electric field is switched off.

$$r = \frac{mv}{qB} = \frac{E}{SB^2} \qquad \left(\frac{q}{m} = S\right)$$

24. For equilibrium,

$f = \frac{mg}{l}$ [f = magnetic force per unit length on the conductors]

$$\Rightarrow \quad \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{r} = \lambda g$$

$$\Rightarrow \quad r = \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{\lambda g}$$

$$= \frac{10^{-7} \times 2 \times 100 \times 50}{0.01 \times 10}$$

$$= 0.01 \text{ m}$$

Clearly, equilibrium of conductor B is unstable.

25. If $\vec{\mathbf{B}}_1, \vec{\mathbf{B}}_2$ and $\vec{\mathbf{B}}_3$ be magnetic fields at the given point due to the wires along x, y and z axis respectively, then

$$\vec{\mathbf{B}}_1 = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a} \hat{\mathbf{j}}$$

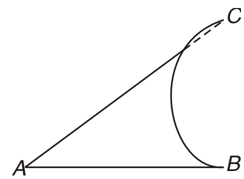
$$\vec{\mathbf{B}}_2 = -\frac{\mu_0}{4\pi} \cdot \frac{2I}{a} \hat{\mathbf{i}}$$

$$\vec{\mathbf{B}}_3 = 0$$

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 + \vec{\mathbf{B}}_3 = \frac{\mu_0 i}{2\pi a} (\hat{\mathbf{j}} - \hat{\mathbf{i}})$$

26. Effective length, $l = AC = \sqrt{4^2 + 3^2}$

$$= 5 \text{ m}$$



$$F = IlB = 2 \times 5 \times 2 = 20 \text{ N}$$

27. At point P,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qx}{(R^2 + x^2)^{3/2}}$$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2iA}{(R^2 + x^2)^{3/2}}$$

Hence, $i = \frac{q}{T} = \frac{qv}{2\pi R}$

and $A = \pi R^2$

$$\therefore \quad \frac{E}{B} = \frac{1}{\mu_0\varepsilon_0} \cdot \frac{1}{v} = \frac{c^2}{v} \qquad \left[c = \frac{1}{\mu_0\varepsilon_0}\right]$$

More than One Correct Options

1. $B_1 = \frac{\mu_0 N_1 I_1}{2R_1} = \frac{4\pi \times 10^{-7} \times 50 \times 2}{2 \times 5 \times 10^{-2}}$

$$= 4\pi \times 10^{-4} \text{ T}$$

$B_2 = \frac{\mu_0 N_2 I_2}{2R_2} = \frac{4\pi \times 10^{-7} \times 100 \times 2}{2 \times 10 \times 10^{-2}}$

$$= 4\pi \times 10^{-4} \text{ T}$$

If current is in same sense,

$$B = B_1 + B_2 = 8\pi \times 10^{-4} \text{ T}$$

And if current is in opposite sense,

$$B = B_1 - B_2 = 0$$

2. $\vec{\mathbf{F}} = \vec{\mathbf{F}}_e + \vec{\mathbf{F}}_m = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$

If $\vec{\mathbf{F}} = 0$

Either, $\vec{\mathbf{E}} = -\vec{\mathbf{v}} \times \vec{\mathbf{B}}$,

$$\vec{\mathbf{E}} \neq 0, \vec{\mathbf{B}} = 0$$

or $\quad \vec{\mathbf{E}} = 0$, or $\vec{\mathbf{v}} \times \vec{\mathbf{B}} = 0$

Again, If $\vec{\mathbf{v}} \times \vec{\mathbf{B}} = 0$

Either $\vec{\mathbf{B}} = 0$

or $\theta = 0°$, *i.e.*, $\vec{\mathbf{v}} \parallel \vec{\mathbf{B}}$.

3. The particle will describe a circle in x-y plane with radius,

$$r = \frac{mv}{qB} = \frac{1 \times \sqrt{8^2 + 6^2}}{1 \times 2} = 5 \text{ m}$$

and $$T = \frac{2\pi m}{qB} = \pi \text{ s} = 3.14 \text{ s}$$

4. $$\vec{\tau} = MB \sin\theta$$

$$U = -pE\cos\theta$$

$$\theta = 80°$$

Hence, $\tau = 0, U = pE =$ maximum.

As PE (U) is maximum, equilibrium is unstable.

5. Fact.

6. Upward and downward components of force will cancel each other while leftward force is more than rightward force, hence net force is leftwards.

7. $$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$$

$$= q\{E_0 \hat{\mathbf{k}} + (v\hat{\mathbf{j}}) \times (B_0 \hat{\mathbf{i}})\}$$

$$= q(E_0 - vB_0)\hat{\mathbf{k}}$$

If $v < \frac{E_0}{B_0}$, particle will deflect towards positive z-axis.

If $v > \frac{E_0}{B_0}$, particle will deflect towards negative z-axis.

If $v = \frac{E_0}{B_0}$, particle will move undeflected and its KE will remain constant.

8. $K = eV \Rightarrow K \propto V$ will become double

$R = \frac{\sqrt{2mK}}{qB} \Rightarrow R \propto \sqrt{K}$ will become $\sqrt{2}$ times.

$\omega = \frac{qB}{2\pi m}$ is independent of kinetic energy.

9. Use right hand thumb rule.

10. For cd to be in equilibrium, force on it must be repulsive while for ab to be in equilibrium, force on it must be attractive.

Equilibrium of cd will be stable while that of ab will be unstable.

Match the Columns

1. (a → r), (b → q), (c → p), (d → r)

$$\vec{\mathbf{F}}_m = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}}) = -e(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$$

and $$\vec{\mathbf{F}}_m = q\vec{\mathbf{E}} = -e\vec{\mathbf{E}}$$

2. (a → r), (b → s), (c → q), (d → p)

As $\vec{\mathbf{F}}_m = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$

By Fleming's left hand rule, positively charged particles deflects towards left and negatively charged particles deflects towards right.

Again, $$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

$$\Rightarrow \quad r \propto \frac{\sqrt{m}}{q}$$

3. (a → p, s), (b → p, q), (c → p, r), (d → p, s)

Whenever a closed current carrying loop is placed in uniform magnetic field, net force experienced by it is zero.

Also $\quad \tau = PE\sin\alpha$

is maximum if $\alpha = 90°$, *i.e.*, in case (b) only.

And $\quad U = -PE\cos\alpha$

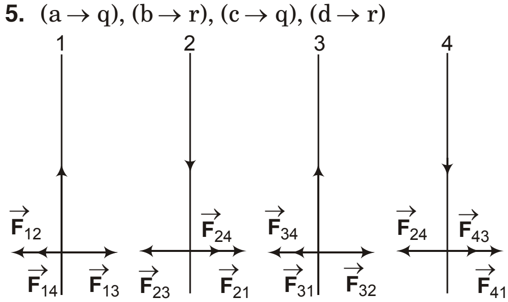
U is positive if α is obtuse, *i.e.*, in cases (a) and (d).

and U is minimum if $\alpha = 0$, *i.e.*, in case (c).

4. (a → q), (b → r), (c → s), (d → s)

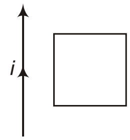
Use right hand thumb rule.

5. (a → q), (b → r), (c → q), (d → r)



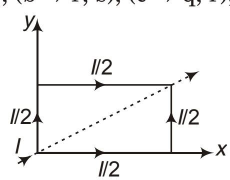
Direction of different forces on different wires is shown in figure.

6. (a → q, s), (b → p, r), (c → p, r), (d → q, s)



When the current is increased or the loop is moved towards the wire, magnetic flux linked with the loop increases. As a result of this, induced current will produce in the loop to decrease the magnetic field. Because initial magnetic flux linked with the loop is inward, induced magnetic flux will be outward and induced current will be anti-clockwise and *vice-versa*.

7. (a → r, s), (b → r, s), (c → q, r), (d → p, r)



Effective lengths of two conductors,

$$l_1 = l_2 = l\,\hat{\mathbf{i}} + l\,\hat{\mathbf{j}}$$

If $\qquad \vec{\mathbf{B}} = B_0\,\hat{\mathbf{i}}$

$$\vec{\mathbf{F}} = \frac{I}{2}(\vec{\mathbf{l}}_1 \times \vec{\mathbf{B}}) + \frac{I}{2}(\vec{\mathbf{l}}_2 \times \vec{\mathbf{B}}) = -B_0 I\,l\,\hat{\mathbf{k}}$$

$\vec{\tau} = 0$, because lines of action of force on the two wires are equal and opposite.

If $\qquad \vec{\mathbf{B}} = B_0\,\hat{\mathbf{j}}$

$$\vec{\mathbf{F}} = B_0 I\,l\,\hat{\mathbf{k}}$$

Again, lines of action of force on the two wires are equal and opposite.

$$\tau = 0$$

If $\quad \vec{\mathbf{B}} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

$$\vec{\mathbf{F}} = 0$$

$$\vec{\tau} = 0$$

If $\quad \vec{\mathbf{B}} = B_0\,\hat{\mathbf{k}}$

$$\vec{\mathbf{F}} = B_0 I\,l(\hat{\mathbf{i}} - \hat{\mathbf{j}})$$

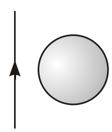
$$\Rightarrow |\vec{\mathbf{F}}| = \sqrt{2}B_0 I\,l$$

$$\tau = 0$$

24 Electromagnetic Induction

Introductory Exercise 24.1

1. Magnetic field inside the loop due to current carrying conductor is inwards.



As the current in the conductor increases, magnetic flux linked with the loop increases as a result of which, induced current will produce in the loop to produce an outward magnetic field, *i.e.*, induced current will be anti-clockwise.

2. No.

Emf is induced if the field is time varying.

3. $\frac{d\phi_B}{dt}$ = induced emf

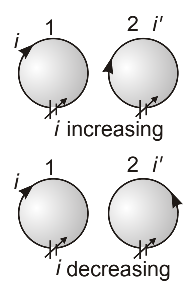
$$\therefore \qquad \left[\frac{d\phi_B}{dt}\right] = [V] = [ML^2 T^{-3} I^{-1}]$$

Introductory Exercise 24.2

1. If the outward magnetic flux increases, induced current will be in such a way that it produces inwards magnetic flux, *i.e.,* it will be clockwise.

2. Magnetic flux linked with the coil will not change, hence induced current will be zero.

3. If the current in coil 1 (clockwise) increases, outward magnetic flux linked with the coil 2 increases and the coil 2 will produce induced current in clockwise direction to oppose the change in magnetic flux linked with it.



Hence, if the current in coil 1 increases, induced current will be in same sense and *vice-versa*.

Introductory Exercise 24.3

1. $\phi_B = BS = B_0 S\, e^{-at}$

$$e = -\frac{d\phi_B}{dt} = a\, B_0 S\, e^{-at}$$

2. No.

As, $\qquad F_m = i\, lB = 0$

Because, $i = 0$ as the circuit is not closed. As net force acting on the bar is zero, no external force is required to move the bar with constant velocity.

3. $|e| = \dfrac{\phi_2 - \phi_1}{t}$

But, $\phi_1 = NB_1A\cos\theta$, $\phi_2 = NB_2A\cos\theta$

$\therefore \quad |e| = \dfrac{NA\cos\theta(B_2 - B_1)}{t}$

$$\Rightarrow A = \frac{|e|t}{N(B_2 - B_1)\cos\theta}$$

$$= \frac{80.0 \times 10^{-3} \times 0.4}{50 \times (600 \times 10^{-6} - 200 \times 10^{-6}) \times \dfrac{\sqrt{3}}{2}}$$

$$= 1.85\ \text{m}^2$$

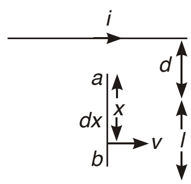
Side of square, $a = \sqrt{A} = 1.36$ m

Total length of wire $= 50 \times 4a$

$$= 50 \times 4 \times 1.36 = 272\ \text{m}$$

4. (a) Consider an elementary portion of length dx of the bar at a distance x from end a.

Magnetic field at this point,



$$B = \frac{\mu_0}{4\pi} \cdot \frac{2i}{a + x}$$

Induced emf in this portion,

$$de = B\,dxv = \frac{\mu_0}{4\pi} \cdot \frac{2vi}{d + x}\,dx$$

5. (a) EMF induced in the bar ab,

$$e = \int de = \frac{\mu_0}{4\pi} \cdot 2vi \int_0^l \frac{dx}{d + x}$$

$$= \frac{\mu_0}{4\pi} 2vi [\ln(d + x)]_0^l$$

$$= \frac{\mu_0 vi}{2\pi} \ln \frac{d + l}{d}$$

$$= \frac{\mu_0 vi}{2\pi} \ln\left(1 + \frac{l}{d}\right)$$

(b) Magnetic field in the region ab is inwards, hence by Fleming's left hand rule, positive charge will move up and a will be at higher potential.

Or

Use Fleming's right hand rule.

(c) No.

As flux linked with the square loop will remain same.

Introductory Exercise 24.4

1. Potential difference across an inductor,

$$V = L\frac{di}{dt} = L\frac{d}{dt}(3t\sin t)$$

$$= 3L[\sin t + t\cos t]$$

Introductory Exercise 24.5

1. (a) Total number of turns on the solenoid,

$$N = \frac{l}{d} = \frac{40 \times 10^{-2}}{0.10 \times 10^{-2}}$$

$$= 400$$

$$L = \frac{\mu_0 N^2 A}{l}$$

$$= \frac{4\pi \times 10^{-7} \times (400)^2 \times 0.90 \times 10^{-4}}{40 \times 10^{-2}}$$

$$= 4.5 \times 10^{-5}\ \text{H}$$

(b) $\quad e = -L\dfrac{di}{dt}$

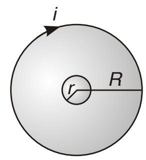
$$= -4.5 \times 10^{-5} \times \frac{0 - 10}{0.10}$$

$$= 4.5 \times 10^{-3}\ \text{V}$$

$$= 4.5\ \text{mV}$$

Introductory Exercise 24.6

1. Consider a current i is flowing in the outer loop.



Magnetic field at the centre of the loop.

$$B = \frac{\mu_0 i}{2R}$$

As $R >> r$, magnetic field inside smaller loop may assumed to be constant.

Hence, magnetic flux linked with the smaller loop,

$$\phi_m = B \times \pi r^2 = \frac{\mu_0 \pi r^2 i}{2R}$$

$$M = \frac{\phi_m}{i} = \frac{\pi \mu_0 r^2}{2R}$$

Introductory Exercise 24.7

1. (a) $V_0 = i_0 R = 36 \times 10^{-3} \times 175 = 6.3$ V

(b) $i = i_0 (1 - e^{-t/\tau})$

where, $\tau = \dfrac{L}{R}$

Now, at $t = 58\ \mu s$

$$i = 4.9 \text{ mA}$$

$\therefore\ 4.9 = 36(1 - e^{-58/\tau})$

$$\Rightarrow \quad e^{-58/\tau} = \frac{31.1}{36}$$

$$\Rightarrow \quad \tau = 397\ \mu s$$

$$\frac{L}{R} = 397\ \mu s$$

$$\Rightarrow \quad L = 175 \times 397 \times 10^{-6}$$

$$= 69 \text{ mH}$$

(c) $\tau = 397\ \mu s$

2.
$$[L] = \frac{[e]}{\left[\frac{di}{dt}\right]} = \frac{[V][t]}{[i]}$$

and
$$[R] = \frac{[V]}{[i]}$$

$$\therefore \quad \left[\frac{L}{R}\right] = \frac{[L]}{[R]} = [\text{T}]$$

3. (a) Initially

$$E = L\frac{di}{dt}$$

$$\Rightarrow \quad \frac{di}{dt} = \frac{E}{L}$$

$$= \frac{12.0}{3.00} = 4 \text{ A/s}$$

(b) $E = V_L + V_R$

$$\Rightarrow \quad E = L\frac{di}{dt} + iR$$

$$\Rightarrow \quad \frac{di}{dt} = \frac{1}{L}[E - iR]$$

$$= \frac{1}{3.00} \times [12 - 1 \times 7]$$

$$\Rightarrow \quad \frac{di}{dt} = \frac{5}{3} = 1.67 \text{ A/s}$$

(c) $\tau = \dfrac{L}{R} = \dfrac{3}{7}$

$$i = i_0(1 - e^{-t/\tau})$$

$$= \frac{E}{R}(1 - e^{-t/\tau}) = \frac{12}{7}(1 - e^{-1.4/3})$$

$\Rightarrow \quad i = 0.639$ A

(d) $i_0 = \dfrac{E}{R} = \dfrac{12}{7} = 1.71$ A

4. (a) $P = Ei = \dfrac{E^2}{R}(1 - e^{-t/\tau})$

$$= \frac{(12)^2}{7}(1 - e^{-7t/3}) = 20.6(1 - e^{-2.33t})\text{ W}$$

(b) Rate of dissipation of energy,

$$P_R = i^2 R = i_0^2 R(1 - e^{-7t/R})^2$$

$$= 20.6(1 - e^{-2.33t})^2 \text{ W}$$

(c) Rate of increase of magnetic energy

$$P_L = ei = L\frac{di}{dt}i$$

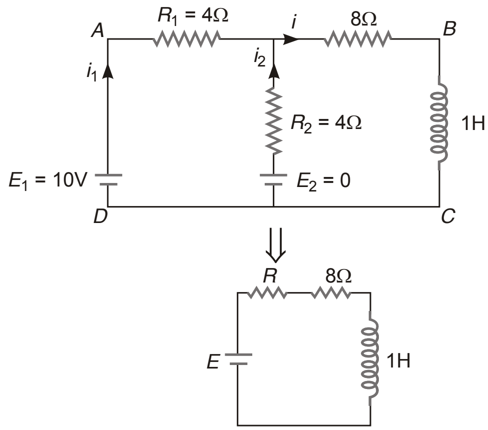
$$= 20.6(e^{-2.33t} - e^{-4.67t})\text{ W}$$

(d) Clearly, $P = P_R + P_L$

5. No.

$E = V_L + V_R$ and V_R cannot be negative in RL circuit.

6. Consider the system as a combination of two batteries ($E_1 = 10$ V and $E_2 = 0$) as shown



$$E = \frac{E_1 R_2 + E_2 R_1}{R_1 + R_2} = 5 \text{ V}$$

$$R = \frac{R_1 R_2}{R_1 + R_2} = 2\Omega$$

$$i_0 = \frac{E}{R + 8} = \frac{5}{10} = 0.5 \text{ A}$$

$$\tau = \frac{L}{R + 8} = \frac{1}{10}$$

$$i = i_0 (1 - e^{-t/\tau})$$

$$i = 0.5(1 - e^{-10t}) \text{ A}$$

$\therefore$ Current through inductor

$$i = 2.5(1 - e^{-10t}) \text{ A}$$

In loop *ABCDA*

$$i_1 R_1 + 8i + L\frac{di}{dt} - E_1 = 0$$

$$i_1 \times 4 + 8 \times 0.5(1 - e^{-10t}) + 1(5e^{-10t}) - 10 = 0$$

$$i_1 = (1.5 - 0.25\, e^{-10t}) \text{ A}$$

Introductory Exercise 24.8

1. $[C] = \frac{[q]}{[V]} = \frac{[i][T]}{[V]}]$

$$[L] = \frac{[e]}{\left[\frac{di}{dt}\right]} = \frac{[V][T]}{[i]}$$

$$\Rightarrow [\sqrt{LC}] = [\sqrt{L}\sqrt{C}] = [\text{T}]$$

2. In *LC* oscillations, magnetic energy is equivalent to kinetic energy in spring block system.

$$i = \frac{dq}{dt} \Rightarrow v = \frac{dx}{dt}$$

Also *L* is equivalent to inertia (*m*) in electricity, hence

Magnetic energy $= \frac{1}{2}Li^2$ is equivalent to kinetic energy $= \frac{1}{2}mv^2$.

3. In *LC* oscillations,

(a) $\frac{di}{dt} = -\frac{1}{LC}q \Rightarrow q = -LC\frac{di}{dt}$

$$\Rightarrow \quad |q| = 18 \times 10^{-6} \times 0.75 \times 3.40$$

$$= 46.5 \times 10^{-6} \text{ C}$$

$$= 46.5\ \mu\text{C}$$

(b) $e = -L\frac{di}{dt} = -L\left(-\frac{1}{LC}q\right)$

$$= \frac{q}{C} = \frac{4.8 \times 10^{-4}}{18 \times 10^{-6}} = 23.3 \text{ V}$$

4. $i_0 = \omega q_0$

where, $\omega = \frac{1}{\sqrt{LC}}$

$$\Rightarrow \quad V_0 = \frac{q_0}{C} = \frac{i_0}{\omega C}$$

$$V_0 = i_0\sqrt{\frac{L}{C}} = 0.1 \times \sqrt{\frac{20 \times 10^{-3}}{0.5 \times 10^{-6}}}$$

$$= 20 \text{ V}$$

Introductory Exercise 24.9

1. (a) $B = \mu_0 ni$

$$\phi_m = NBA = \mu_0 n\, NAi$$

$$e = -\frac{d\phi_m}{dt} = -\mu_0 nNA\frac{di}{dt}$$

$$= -4\pi \times 10^{-7} \times \frac{25}{0.01} \times 10 \times 5.0 \times 10^{-4} \times (-0.2)$$

$$= 3.14 \times 10^{-6} \text{ V}$$
$$= 3.14 \,\mu\text{V}$$

(b) $E = \dfrac{e}{2\pi R} = \dfrac{3.14 \times 10^{-6}}{2 \times 3.14 \times 25 \times 10^{-2} \times 10}$

$$= 2 \times 10^{-7} \text{ V/m}$$

2. $B = (2.00t^3 - 4.00t^2 + 0.8)$ T

$\dfrac{dB}{dt} = (6.00t^2 - 8.00t)$ T/s

From, $t = 0$ to $t = 1.33$ s, $\dfrac{dB}{dt}$ is negative, hence B is decreasing in that interval.

For $t > 1.33$ s, $\dfrac{dB}{dt}$ is positive, hence B is increasing for $t > 1.33$ s.

(a) For point P_2,

induced emf, $V_2 = -\dfrac{d\phi_{m_2}}{dt} = -\pi R^2 \dfrac{dB}{dt}$

Induced electric field at P_2,

$$E = \frac{V_2}{2\pi r_2} = -\frac{R^2}{2r_2} \cdot \frac{dB}{dt}$$
$$= -\frac{R^2}{2r_2}(6.00t^2 - 8.00t)$$
$$F = -eE = \frac{R^2}{2r_2}(6.00t^2 - 8.00t)$$
$$= 8.0 \times 10^{-21} \text{ N}$$

As magnetic field is increasing in this region, induced electric field will be anti-clockwise and hence, electron will experience force in clockwise sense, *i.e.*, downward at P_2.

(b) For point P_1,

Induced emf, $V_1 = -\dfrac{d\phi_{m_1}}{dt} = -\pi r_1^2 \dfrac{dB}{dt}$

Induced electric field at P_1,

$$E = -\frac{V_1}{2\pi r_1} = -\frac{1}{2} r_1 \frac{dB}{dt}$$
$$= -\frac{1}{2} r_1 (6.00t^2 - 8.00t) = 0.36 \text{ V/m}$$

At, $t = 2.00$ s

magnetic field is increasing, hence, induced electric field will be anti-clockwise, *i.e.*, upward at P_1 and perpendicular to r_1.

AIEEE Corner

Subjective Questions (Level 1)

1. $< e > = -\dfrac{\phi_2 - \phi_1}{t} = -\dfrac{B(A_2 - A_1)}{t}$

$$A_1 = \pi r^2 = 3.14 \times (0.1)^2$$
$$= 3.14 \times 10^{-2} = 0.0314$$
$$A_2 = a^2 = \left(\frac{2\pi r}{4}\right)^2$$
$$= \left(\frac{2 \times 3.14 \times 0.1}{4}\right)^2 = 0.025$$
$$\therefore \quad < e > = -\frac{100(0.025 - 0.0314)}{0.1}$$
$$= 6.4 \text{ V}$$

2. $\phi_1 = NBA = 500 \times 0.2 \times 4 \times 10^{-4}$

$$= 0.04 \text{ Wb}$$

$\phi_2 = -NBA = -0.04$ Wb

Average induced emf,

$$< e > = -\frac{(\phi_2 - \phi_1)}{t}$$

Average induced current,

$$< i > = \frac{< e >}{R} = -\frac{(\phi_2 - \phi_1)}{Rt}$$

Charge flowing through the coil

$$q = < i > t$$
$$\Rightarrow q = -\frac{(\phi_2 - \phi_1)}{R} = -\frac{(-0.04 - 0.04)}{50}$$
$$= \frac{0.08}{50} = 1.6 \times 10^{-3} \text{ C}$$
$$= 1.6 \text{ mC} = 1600 \,\mu\text{C}$$

3. $\phi_1 = NBS$, $\phi_2 = -NBS$

Induced emf,

$$< e > = -\frac{(\phi_2 - \phi_1)}{t} = \frac{2NBS}{t}$$

Induced current

$$< i > = \frac{< e >}{R} = \frac{2NBS}{Rt}$$

Charge flowing through the coil,

$$q = < i > t = \frac{2NBS}{R}$$
$$\Rightarrow \quad B = \frac{qR}{2NS} = \frac{4.5 \times 10^{-6} \times 40}{2 \times 60 \times 3 \times 10^{-6}}$$

$$= 0.5\ \text{T}$$

4. $\vec{\mathbf{B}} = (4.0\,\hat{\mathbf{i}} - 1.8\,\hat{\mathbf{k}}) \times 10^{-3}$ T,

$\vec{\mathbf{S}} = (5.0 \times 10^{-4}\,\hat{\mathbf{k}})\ \text{m}^2$

$\phi = \vec{\mathbf{B}} \cdot \vec{\mathbf{S}} = -9.0 \times 10^{-7}$ Wb

5. $e = Blv = 1.1 \times 0.8 \times 5 = 4.4$ V

By Fleming's right hand rule, north end of the wire will be positive.

6. $A = \pi r^2 = 3.14 \times (12 \times 10^{-2})^2 = 0.045\ \text{m}^2$

(a) For $t = 0$ to $t = 2.0$ s

$$\frac{dB}{dt} = \text{slope} = \frac{0.5 - 0}{2.0 - 1} = 0.25\ \text{T/s}$$

$$e = -\frac{d\phi_m}{dt} = -A\frac{dB}{dt}$$

$$= -0.045 \times 0.25 = -0.011\ \text{V}$$

$$|e| = 0.011\ \text{V}$$

(b) For, $t = 2.0$ s to $t = 4.0$ s

$$\frac{dB}{dt} = \text{slope} = 0\ e = 0$$

(c) For, $t = 4.0$ s to $t = 6.0$ s

$$\frac{dB}{dt} = \text{slope} = \frac{0 - 0.5}{6.0 - 4.0} = -0.25$$

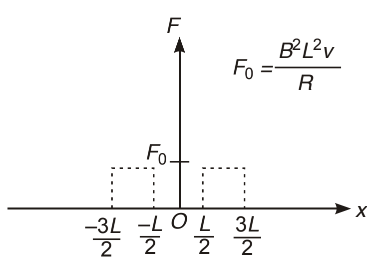
$$e = -\frac{d\phi_m}{dt} = -A\frac{dB}{dt} = 0.11\ \text{V}$$

7. (a) When magnetic flux linked with the coil changes, induced current is produced in it, in such a way that, it opposes the change.

Magnetic flux linked with the coil will change only when coil is entering in (from $x = -\frac{3L}{2}$ to $x = -\frac{L}{2}$) or moving (from $x = \frac{L}{2}$ to $x = \frac{3L}{2}$) of the magnetic field.

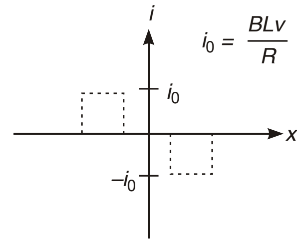
Because, of induced current, an opposing force act on the coil, which is given by

$$F = ilB = \frac{BLv}{R}BL = \frac{B^2L^2v}{R}$$

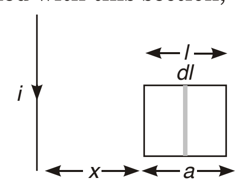


Hence, equal force in direction of motion of coil is required to move the block with uniform speed.

(b) When the coil is entering into the magnetic field, magnetic flux linked with the coil increases and the induced current will produce magnetic flux in opposite direction and will be counter-clockwise and *vice-versa*.



8. Consider an elementary section of length dl of the frame as shown in figure. Magnetic flux linked with this section,



$$d\phi_m = BdB = \frac{\mu_0}{4\pi} \cdot \frac{2i}{x + l}\,adl$$

Total magnetic flux linked with the frame,

$$\phi_m = \int d\phi_m = \frac{\mu_0 ai}{2\pi}\int_0^a \frac{dl}{x + l}$$

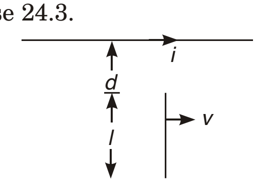
$$= \frac{\mu_0 ai}{2\pi}[\ln(x + a) - \ln x]$$

Induced emf

$$e = \frac{-d\phi_m}{dt} = -\frac{\mu_0 ai}{2\pi} \cdot \left[\frac{1}{x + a} - \frac{1}{x}\right]\frac{dx}{dt}$$

$$= \frac{\mu_0 a^2 i}{2\pi x(x + a)}v = \frac{\mu_0 a^2 iv}{2\pi x(x + a)}$$

9. As solved in Qusetion 4. Introductory Exercise 24.3.



$$e = \frac{\mu_0 iv}{2\pi} \ln\left(1 + \frac{l}{d}\right)$$

Here, $i = 10$ A

$v = 10\ \text{ms}^{-1}$

$l = 10.0\ \text{cm} - 1.0\ \text{cm} = 9.0\ \text{cm}$

$d = 1.0$ cm

$$e = \frac{4\pi \times 10^{-7} \times 10 \times 10}{2\pi} \ln\left(1 + \frac{9.0}{1.0}\right)$$

$$e = (2 \times 10V) \ln(10)\ \text{V}$$

10. Induced current

$$i = \frac{e}{R} = \frac{Blv}{R}$$

Force needed to move the rod with constant speed = Magnetic force acting on the rod

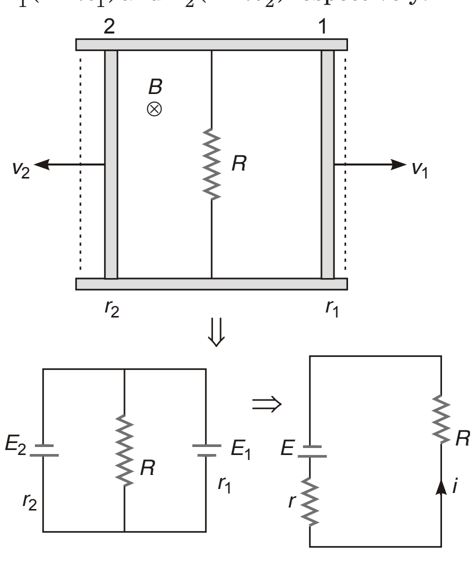
ie., $F = i\,lB = \frac{Blv}{R} lB$

$$= \frac{B^2 l^2 v}{R} = \frac{(0.15)^2 \times (50 \times 10^{-2})^2 \times 2}{3}$$

$\Rightarrow$ $F = 0.00375$

11. Suppose the magnetic field is acting into the plane of paper.

Rods 1 and 2 can be treated as cells of emf $E_1 (= Blv_1)$ and $E_2 (= Blv_2)$ respectively.



Now, $E_1 = Blv_1 = 0.010 \times 10.0 \times 10^{-2} \times 4.00$

$= 0.004$ V

$E_2 = Blv_2 = 0.010 \times 10 \times 0 \times 10^{-2} \times 8.00$

$= 0.008$ V

Effective emf

$$E = \frac{E_2 r_1 - E_1 r_2}{r_1 + r_2}$$

$$= \frac{0.008 \times 15.0 - 0.004 \times 10.0}{15.0 + 10.0}$$

$= 0.0032$ V

$$r = \frac{r_1 r_2}{r_1 + r_2} = \frac{15 \times 10}{25} = 6\,\Omega$$

$$i = \frac{E}{R + r} = \frac{0.0032}{5 + 6} = 0.003\ \text{A} = 0.3\ \text{mA}$$

12. (a) $e = -L\frac{di}{dt} = -0.54 \times (-0.030)$

$= 1.62 \times 10^{-2}$ V

(b) Current flowing from b to a is decreasing, hence, a must be at higher potential.

13. (a) $i = 5 + 16t$, $|e| = 10\text{mV} = 10 \times 10^{-3}$ V

$$|e| = L\frac{di}{dt} \Rightarrow 10 \times 10^{-3} = L\frac{d}{dt}(5 + 16t)$$

$$L = \frac{10 \times 10^{-3}}{16} = 0.625\ \text{mH}$$

(b) at $t = 1$ s

$i = 5 + 16(1) = 21$ A

Energy stored in the inductor,

$$U = \frac{1}{2}Li^2 = \frac{1}{2} \times 0.625 \times 10^{-3} \times (21)^2$$

$= 0.138$ J

$$P = \frac{dU}{dt} = Li\frac{di}{dt} = 0.625 \times 10^{-3} \times 21 \times 16$$

$= 0.21$ W

14. From $t = 0$ to $t = 2.0$ ms

$$\frac{V - 0}{t - 0} = \frac{5.0 - 0}{2.0 \times 10^{-3}} = 0$$

$\Rightarrow$ $V = 2500\,t$

$$L\frac{di}{dt} = 2500\,t$$

$$\int di = \frac{2500}{L}\int t\,dt$$

$\Rightarrow$ $$\int_0^i di = \frac{2500}{L}\int_0^t t\,dt$$

$$i = \frac{1250}{L}t^2$$

at $t = 2.0$ ms

$$i = \frac{1250}{150 \times 10^{-3}} \times (2.0 \times 10^{-3})^2$$

$= 3.33 \times 10^{-2}$ A

From $t = 2.0$ ms to $t = 4.0$ ms

$$\frac{V - 5.0}{t - 2.0 \times 10^{-3}} = \frac{0 - 0.50}{(4.0 - 2.0) \times 10^{-3}}$$

$$V = -2500(t - 2.0 \times 10^{-3}) + 5.0$$
$$= -2500\,t + 10.0$$
$$L\frac{di}{dt} = -2500\,t + 10.0$$
$$di = \frac{1}{L}(-2500\,t + 10.0)\,dt$$
$$i = \frac{1}{L}[-1250\,t^2 + 10.0\,t]$$

at $t = 4$ s

$$i = \frac{1}{150 \times 10^{-3}}[-1250 \times (4.0 \times 10^{-3})^2 + 10.0(4.0 \times 10^{-3})]$$
$$= 3.33 \times 10^{-2} \text{ A}$$

15. (a) $|e| = L\frac{di}{dt} \Rightarrow L = \frac{|e|}{di/dt} = \frac{0.0160}{0.0640}$

$$= 0.250 \text{ H}$$

(b) Flux per turn

$$\phi = \frac{Li}{N} = \frac{0.250 \times 0.720}{400}$$
$$= 4.5 \times 10^{-4} \text{ Wb}$$

16. $|e| = M\frac{di}{dt} = M\frac{i_2 - i_1}{t}$

$$\Rightarrow \quad 50 \times 10^{-3} = M \cdot \frac{12 - 4}{0.5}$$
$$M = \frac{50 \times 10^{-3} \times 0.5}{8} = 3.125 \times 10^{-3} \text{ H}$$
$$= 3.125 \text{ mH}$$

If current changes from 3 A to 9 A in 0.02 s.

$$|e| = M\frac{di}{dt} = M\frac{i_2 - i_1}{t}$$
$$= 3.125 \times 10^{-3} \times \frac{9 - 3}{0.02}$$
$$= 0.9375 \text{ V}$$

17. (a) Magnetic flux linked with secondary coil,

$$\phi_{m_2} = M\,i_1$$
$$M = \frac{\phi_2}{i_1} = \frac{6.0 \times 10^{-3} \times 1000}{3} = 2 \text{ H}$$

(b) $$e = -\frac{d\phi_{m_2}}{dt} = -M\frac{d\,i_1}{dt}$$
$$= -2 \times \frac{0 - 3}{0.2} = 30 \text{ V}$$

(c) $L = \frac{\phi_{m_1}}{i_1} = \frac{600 \times 5 \times 10^{-3}}{3} = 1 \text{ H}$

18. (a) $|e| = M\frac{di}{dt} = 3.25 \times 10^{-4} \times 830$

$$= 0.27 \text{ V}$$

As, $\frac{di}{dt}$ is constant, induced emf is constant.

(b) Coefficient of mutual induction remains same whether current flows in first coil or second.

Hence, $|e| = M_1\frac{di}{dt} = 0.27$ V

19. (a) Magnetic flux linked with the secondary coil,

$$\phi_2 = Mi_1$$
$$M = \frac{\phi_2}{i_1} = \frac{0.0320 \times 400}{6.52}$$
$$= 1.96 \text{ H}$$

(b) $\phi_1 = Mi_2 = 1.96 \times 2.54 = 4.9784$ Wb

Flux per turn through primary coil

$$= \frac{\phi_1}{N_1} = \frac{4.9784}{700}$$
$$= 7.112 \times 10^{-3} \text{ Wb/turn.}$$

20. Same as Question 2. Introductory Exercise 24.4

21.
$$i = i_0(1 - e^{-t/\tau})$$
$$= \frac{E}{R}(1 - e^{-\frac{Rt}{L}})$$
$$\frac{di}{dt} = \frac{E}{L}e^{-\frac{Rt}{L}}$$

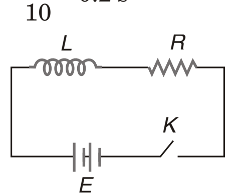
Power supplied by battery,

$$P = Ei = \frac{E^2}{R}(1 - e^{-\frac{Rt}{L}})$$

Rate of storage of magnetic energy

$$P_1 = Li\frac{di}{dt} = \frac{E^2}{R}(1 - e^{-\frac{Rt}{L}})e^{-\frac{Rt}{L}}$$
$$\frac{P_1}{P} = e^{-\frac{Rt}{L}} = e^{-\frac{10 \times 0.1}{1}} = e^{-1} = 0.37$$

22. (a) $\tau = \frac{L}{R} = \frac{2}{10} = 0.2$ s



(b) $i_0 = \frac{E}{R} = \frac{100}{10} = 10\text{ A}$

(c) $i = i_0(1 - e^{-\frac{t}{\tau}})$

$$i = 10(1 - e^{-\frac{1}{0.2}})$$

$$= 10(1 - e^{-5}) = 9.93\text{ A}$$

23. (a) Power delivered by the battery,

$$P = Ei = \frac{E^2}{R}(1 - e^{-\frac{Rt}{L}})$$

$$= \frac{(3.24)^2}{12.8}(1 - e^{\frac{12.8 \times 0.278}{3.56}})$$

$$= 0.82(1 - e^{-1}) = 0.518\text{ W}$$

$$= 518\text{ mW}$$

(b) Rate of dissipation of energy as heat

$$P_2 = i^2R = \frac{E^2}{R}(1 - e^{-\frac{Rt}{L}})^2$$

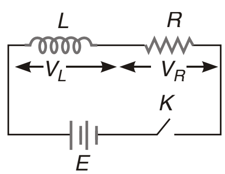
$$= 0.82(1 - e^{-1})^2 = 0.328\text{ W}$$

$$= 328\text{ mW}$$

(c) Rate of storage of magnetic energy

$$P_1 = P - P_2 = 190\text{ mW}$$

24. $E = V_L + V_R = L\frac{di}{dt} + iR$



(a) Initially, $i = 0$

$$\therefore \quad \frac{di}{dt} = \frac{E}{L} = \frac{6.00}{2.50} = 2.40\text{ A/s}$$

(b) When, $i = 0.500$ A

$$\frac{di}{dt} = \frac{E - iR}{L} = \frac{6.00 - 0.500 \times 8.00}{2.50}$$

$$= 0.80\text{ A/s}$$

(c) $i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$

$$= \frac{6.00}{8.00}\left(1 - e^{-\frac{8.00 \times 0.250}{2.5}}\right)$$

$$= 0.750(1 - e^{-0.8}) = 0.413\text{ A}$$

(d) $i_0 = \frac{E}{R} = \frac{6.00}{8.00} = 0.750\text{ A}$

25. (a) $i = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$

But $i = \frac{i_0}{2}$

$$\frac{i_0}{2} = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$e^{-\frac{Rt}{L}} = \frac{1}{2}$$

$$t = \frac{L}{R}\ln 2 = \frac{1.25 \times 10^{-3}}{50.0} \times 0.693$$

$$= 17.3 \times 10^{-6} = 17.3\ \mu\text{s}$$

(b) $U = \frac{1}{2}Li^2 = \frac{1}{2}\left(\frac{1}{2}L\,i_0^2\right)$

$$i = \frac{1}{\sqrt{2}}i_0$$

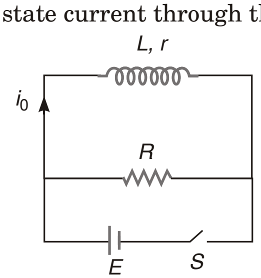
$$i_0\left(1 - e^{-\frac{Rt}{L}}\right) = \frac{i_0}{\sqrt{2}}$$

$$\Rightarrow \quad e^{-\frac{Rt}{L}} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

$$\Rightarrow \quad t = \frac{L}{R}\ln\frac{\sqrt{2}}{\sqrt{2} - 1}$$

$$= 30.7\ \mu\text{s}.$$

26. Steady state current through the inductor



$$i_0 = \frac{E}{r}$$

When the switch S is open

$$\tau = \frac{L}{R + r}$$

(a) $i = i_0\,e^{-t/\tau}$

$$\Rightarrow \quad i = \frac{E}{r}e^{-\left(\frac{(R+r)}{L}\right)}$$

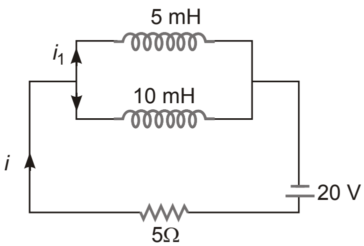
(b) Amount of heat generated in the solenoid

$$H = \int_0^\infty i^2r\,dt = i_0^2r\int_0^\infty e^{-2t/\tau}\,dt$$

$$= \frac{E^2}{r}\left\{-\frac{\tau}{2}[e^{-2t/\tau}]_0^\infty\right\}$$

$$= \frac{(R + r)E^2}{2rL}$$

27. At any instant of time,



$$L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt}$$

$$\Rightarrow \qquad L_1 i_1 = L_2 i_2$$

$$i_1 = 2i_2 \qquad \ldots\text{(i)}$$

In steady state,

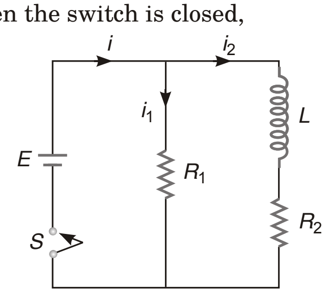
inductors offer zero resistance, hence

$$i = \frac{20}{5} = 4\text{ A}$$

But $\qquad i_1 + i_2 = i$

$$i_2 = \frac{4}{3}\text{A}, \; i_1 = \frac{8}{3}\text{A}$$

28. When the switch is closed,

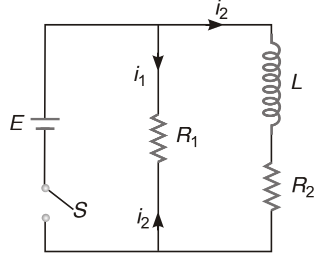


$$i_2 = \frac{E}{R_2}(1 - e^{-R_2 t/L})$$

$$\frac{di_2}{dt} = \frac{E}{L} e^{-R_2 t/L}$$

Potential difference across L

$$V + L\frac{di_2}{dt} = E\, e^{-R_2 t/L} = (12e^{-5t})\text{ V}$$



When the switch S is open, current i_2 flows in the circuit in clockwise direction and is given by

$$i_2 = i_0 e^{-t/\tau}$$

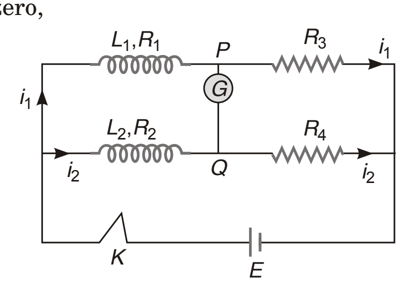
Here, $\qquad i_2 = \frac{E}{R_2}$

$$\tau = \frac{L}{R_1 + R_2}$$

$$i_2 = \frac{E}{R_2} e^{-\left(\frac{R_1 + R_2}{L}\right)t}$$

$$= \frac{12}{2} e^{-10t} = (6\, e^{-10t})\text{ A}$$

29. For current through galvanometer to be zero,



$$V_P = V_Q$$

$$L_1 \frac{di_1}{dt} + i_1 R_1 = L_2 \frac{di_2}{dt} + i_2 R_2 \qquad \ldots\text{(i)}$$

Also, $\qquad i_1 R_3 = i_2 R_4 \qquad \ldots\text{(ii)}$

From Eqs.(i) and (ii),

$$\frac{L_1 \frac{di_1}{dt} + i_1 R_1}{i_1 R_3} = \frac{L_2 \frac{di_2}{dt} + i_2 R_2}{i_2 R_4} \qquad \ldots\text{(iii)}$$

In the steady state,

$$\frac{di_1}{dt} = \frac{di_2}{dt} = 0$$

$$\therefore \qquad \frac{R_1}{R_3} = \frac{R_2}{R_4} \Rightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Again as current through galvanometer is always zero.

$$\frac{i_1}{i_2} = \text{constant}$$

or $$\frac{di_1/dt}{di_2/dt} = \text{constant}$$

or $$\frac{\frac{di_1}{dt}}{\frac{di_2}{dt}} = \frac{i_1}{i_2} \qquad \ldots\text{(iv)}$$

From Eqs. (iii) and (iv),

$$\frac{L_1}{L_2} = \frac{R_3}{R_4} = \frac{R_1}{R_2}$$

30. (a) In LC circuit

Maximum electrical energy = Maximum magnetic energy

$$\Rightarrow \quad \frac{1}{2}CV_0^2 = \frac{1}{2}Li_0^2$$

$$L = C\left(\frac{V_0}{i_0}\right)^2 = 4\times10^{-6}\left(\frac{1.50}{50\times10^{-3}}\right)^2$$

$$= 3.6\times10^{-3}\text{ H}$$

$$\Rightarrow \quad L = 3.6\text{ mH}$$

(b) $f = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\times3.14\sqrt{3.6\times10^{-3}\times4\times10^{-6}}}$$

$$= 1.33\times10^3\text{ Hz}$$

$$= 1.33\text{ kHz}$$

(c) Time taken to rise from zero to maximum value,

$$t = \frac{T}{4} = \frac{1}{4f} = \frac{1}{4\times1.33\times10^3}$$

$$= 3\times10^{-3}\text{ s} = 3\text{ ms.}$$

31. (a) $\omega = 2\pi f = 2\times3.14\times10^3$

$$= 6.28\text{ rad/s}$$

$$T = \frac{1}{f} = \frac{1}{10^3} = 10^{-3}\text{ s} = 1\text{ ms}$$

(b) As initially charge is maximum, (*i.e.., it is extreme position for charge).*

$$q = q_0\cos\omega t$$

$$q_0 = CV_0 = 1\times10^{-6}\times100$$

$$= 10^{-4}$$

$$\therefore \quad q = [10^{-4}\cos(6.28\times10^3)t]\text{ C.}$$

(c) $\omega = \dfrac{1}{\sqrt{LC}}$

$$\Rightarrow L = \frac{1}{\omega^2 C} = \frac{1}{(6.28\times10^3)^2\times10^{-6}}$$

$$= 2.53\times10^{-3}$$

$$\Rightarrow \quad L = 2.53\text{ mH}$$

(d) In one quarter cycle, entire charge of the capacitor flows out.

$$\langle i \rangle = \frac{q}{t} = \frac{4CV}{T}$$

$$= \frac{4\times10^{-6}\times100}{10^{-3}} = 0.4\text{ A}$$

32. (a) $V_0 = \dfrac{q_0}{C} = \dfrac{5.00\times10^{-6}}{4\times10^{-4}}$

$$= 1.25\times10^{-2}\text{ V} = 12.5\text{ mV}$$

(b) Maximum magnetic energy = Maximum electric energy

$$\frac{1}{2}Li_0^2 = \frac{q_0^2}{2C}$$

$$\Rightarrow \quad i_0 = \frac{q_0}{\sqrt{LC}}$$

$$\Rightarrow \quad i_0 = \frac{5.00\times10^{-6}}{\sqrt{0.090\times4\times10^{-4}}} = 8.33\times10^{-4}\text{ A}$$

(c) Maximum energy stored in inductor,

$$= \frac{1}{2}L\,i_0^2$$

$$= \frac{1}{2}\times0.0900\times(8.33\times10^{-4})^2$$

$$= 3.125\times10^{-8}\text{ J}$$

(d) By conservation of energy,

$$\frac{q^2}{2C} + \frac{1}{2}Li^2 = \frac{1}{2}Li_0^2$$

But $i = \dfrac{i_0}{2}$

$$\frac{q^2}{2C} = \frac{3}{8}Li_0^2$$

$$q = \frac{i_0}{2}\sqrt{3LC} = \frac{\sqrt{3}}{2}q_0$$

$$= \frac{1.732}{2}\times5.00\times10^{-6}$$

$$= 4.33\times10^{-6}\text{ C}$$

$$U_m = \frac{1}{2}Li^2 = \frac{1}{4}\left(\frac{1}{2}Li_0^2\right)$$

$$= 7.8\times10^{-9}\text{ J}$$

33. (a) $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{2.0\times10^{-3}\times5.0\times10^{-6}}}$

$$= 10^4\text{ rad/s}$$

$$\left|\frac{di}{dt}\right| = \omega^2 Q$$

$$= (10^4)^2\times100\times10^{-6} = 10^4\text{ A/s}$$

(b) $i = \omega\sqrt{Q_0^2 - Q^2}$

$$= 10^4\sqrt{(200\times10^{-6})^2 - (200\times10^{-6})^2} = 0$$

(c) $i_0 = \omega Q_0 = 10^4\times200\times10^{-6} = 2\text{ A}$

(d) $i = \omega\sqrt{Q_0^2 - Q^2}$

$$\Rightarrow \quad \frac{i_0}{2} = \omega\sqrt{Q_0^2 - Q^2}$$

$$\Rightarrow \quad \frac{\omega Q_0}{2} = \omega\sqrt{Q_0^2 - Q^2}$$

$$Q = \frac{\sqrt{3}}{2}Q_0 = \frac{1.73 \times 200 \times 10^{-6}}{2}$$

$$= 173\ \mu C$$

34. As initially charge is maximum

$$q = q_0 \cos \omega t$$

and $\quad |i| = i_0 \sin \omega t$

where, $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3.3 \times 840 \times 10^{-6}}}$

$$\approx 19 \text{ rad./s}$$

$$i_0 = \omega q_0 = 19 \times 105 \times 10^{-6}$$

$$\approx 2.0 \times 10^{-3} \text{ A} = 2.0 \text{ mA}$$

At $t = 2.00$ ms

(a) $U_e = \frac{q^2}{2C} = \frac{q_0^2}{2C}(\cos^2 \omega t)$

$$= \frac{(105 \times 10^{-6})^2}{2 \times 840 \times 10^{-6}}[\cos^2(38 \text{ rad})]$$

$$U_e = 6.55 \times 10^{-6} \text{ J} = 6.55\ \mu\text{J}$$

(b) $U_m = \frac{1}{2}Li^2 = \frac{1}{2}Li_0^2(\sin \omega t)$

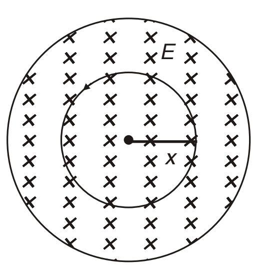
$$= \frac{1}{2} \times 3.3 \times (2 \times 10^{-3})^2 \sin^2(38 \text{ rad})$$

$$= 0.009 \times 10^{-6} \text{ J} = 0.009\ \mu\text{j}$$

(c) $U = \frac{q_0^2}{2C} = \frac{1}{2}Li_0^2$

$$= 6.56 \times 10^{-6} \text{ J} = 6.56\ \mu\text{J}$$

35. As the inward magnetic field is increasing, induced electric field will be anticlockwise.



At a distance x from centre of the region,

Magnetic flux linked with the imaginary loop of radius x

$$\phi_m = \pi x^2 B$$

$$e = \frac{-d\phi_m}{dt} = -\pi x^2 \frac{dB}{dt}$$

Induced electric field,

$$E = \frac{e}{2\pi x} = \frac{1}{2}x\frac{dB}{dt}$$

At a,

$$E = \frac{1}{4}r\frac{dB}{dt}, \text{ towards left.}$$

At b,

$$E = \frac{1}{2}r\frac{dB}{dt}, \text{ upwards.}$$

At c,

$$E = 0$$

36. Inside the solenoid,

$$B = \mu_0 ni$$

$$\frac{dB}{dt} = \mu_0 n\frac{di}{dt}$$

Inside the region of varying magnetic field

$$E = \frac{1}{2}r\frac{dB}{dt} = \frac{1}{2}\mu_0 nr\frac{di}{dt}$$

(a) $r = 0.5 \text{ cm} = 5.0 \times 10^{-3}$ m

$$E = \frac{1}{2}\mu_0 rn\frac{di}{dt}$$

$$= \frac{1}{2} \times 4\pi \times 10^{-7} \times 5.0 \times 10^{-3} \times 900 \times 60$$

$$= 1.7 \times 10^{-4} \text{ V/m}$$

(b) $r = 1.0 \text{ cm} = 1.0 \times 10^{-2}$ m

$$E = \frac{1}{\mu_0}rn\frac{di}{dt}$$

$$= \frac{1}{2} \times 4\pi \times 10^{-3} \times 5.0 \times 10^{-3} \times 900 \times 60$$

$$= 3.4 \times 10^{-4} \text{ V/m}$$

AIEEE Corner

Objective Questions (Level 1)

1. $V = L\dfrac{di}{dt}$

$$[L] = \frac{[V][T]}{[i]} = \frac{[\mathrm{ML^2T^{-3}A^{-1}}][\mathrm{T}]}{[\mathrm{A}]}$$

$$= [\mathrm{ML^2T^{-2}A^{-2}}]$$

2. $M \propto n_1 n_2$

3. Both will tend to oppose the magnetic flux changing with them by increasing current in opposite direction.

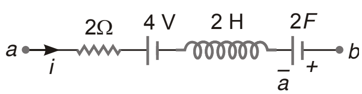
4. Moving charged particle will produced magnetic field parallel to ring, Hence

$$\phi_m = 0$$

Velocity of particle increases continuously due to gravity.

5. Induced electric field can exist at a point where magnetic field is not present, i.e., outside the region occupying the magnetic field.

6. At, $t = 1$ s



$$q = 4t^2 = 4\,\mathrm{C}$$

$$i = \frac{dq}{dt} = 8t = 8\,\mathrm{A}$$

$$\frac{di}{dt} = 8\,\mathrm{A/s}$$

As, $\dfrac{di}{dt} = \dfrac{d^2q}{dt} = $ Positive

Charge in capacitor is increasing, current i must be towards left.

$$V_{ab} = -2I + 4 - L\frac{di}{dt} - \frac{q}{C}$$

$$= -2\times 8 + 4 - 2\times 8 - \frac{4}{2} = -30\,\mathrm{V}$$

7. $|e| = M\dfrac{di}{dt} = M\dfrac{d}{dt}(i_0 \sin \omega t)$

$$= \omega M i_0 \cos \omega t$$

Maximum induced emf $= \omega M i_0$

$$= 100\pi \times 0.005 \times 10$$

$$= 5\pi$$

8. $\dfrac{1}{2}Li_0^2 = \dfrac{1}{2}CV_0^2 \Rightarrow i_0\sqrt{\dfrac{L}{C}} = 2 \times \sqrt{\dfrac{2}{4\times 10^{-6}}}$

$$= \sqrt{2} \times 10^3\ \mathrm{V}$$

9. $e = \dfrac{1}{2}Bl^2\omega$, is independent of t.

10. $|e| = \dfrac{d\phi}{dt} = \dfrac{\Delta\phi}{t}$

$\Delta\phi = |e|t = iRt$

$$= 10\times 10^{-3} \times 0.5 \times 5$$

$$= 25 \times 10^{-3}\ \mathrm{Wb}$$

$$= 25\ \mathrm{mWb}.$$

11. As inward magnetic field is increasing, induced electric field must be anti-clockwise.

Hence, direction of induced electric field at P will be towards and electron will experience force towards right (opposite to electric field).

12. $\phi = at(\tau - t) = a\tau t - at^2$

$$|e| = \frac{d\phi}{dt} = a\tau - 2at$$

$$i = \frac{|e|}{R} = \frac{a\tau - 2at}{R}$$

$$H = \int_0^\tau i^2 R\,dt = \int_0^\tau \frac{(a\tau - 2at)^2}{R}\,dt$$

$$= \frac{1}{R}\left[\frac{(a\tau - 2at)^3}{3\times(-2a)}\right]_0^\tau$$

$$= \frac{1}{-6Ra}[-a^3\tau^3 - a^3\tau^3]$$

$$= \frac{a^2\tau^3}{3R}$$

13. $E = -L\dfrac{di}{dt}$

14. $V_{BA} = -L\dfrac{di}{dt} + 15 - iR$

$$= -5\times 10^{-3}(-10^3) + 15 - 5\times 1$$

$$= 15\ \mathrm{V}$$

15. $\dfrac{di}{dt} = 10$ A/s, at $t = 0$, $i = 5$A

$$\frac{di}{dt} = 10\ \mathrm{A/s}$$

$$V_A - V_B = iR + L\frac{di}{dt} - E = 0$$

$$= 5 \times 3 + 1 \times 10 - 10 = 15\text{ V}$$

16. $\left(\frac{di}{dt}\right)_{\max} = \left(\frac{d^2q}{dt}\right)_{\max} = \omega^2 q_0 = \frac{q_0}{LC}$

17. $V = L\frac{di}{dt}$

18. $\phi_m = BA\cos\theta$

$$\Rightarrow \quad e = -\frac{d\phi_m}{dt} = BA\sin\theta\frac{d\theta}{dt}$$

$$\Rightarrow \quad iR = BA\sin\theta\frac{d\theta}{dt}$$

$$\Rightarrow \quad \frac{dq}{dt}R = BA\sin\theta\frac{d\theta}{dt}$$

$$\Rightarrow \quad dq = \frac{BA}{R}\sin\theta\, d\theta$$

$$\Rightarrow \quad q = \frac{BA}{R}\int_{\pi/2}^{3\pi/2}\sin\theta\, d\theta = 0$$

19. $\vec{\mathbf{A}} = ab\,\hat{\mathbf{k}}, \vec{\mathbf{B}} = 20t\,\hat{\mathbf{i}} + 10t^2\,\hat{\mathbf{j}} + 50\,\hat{\mathbf{k}}$

$$\phi_m = \vec{\mathbf{B}}\cdot\vec{\mathbf{A}} = 50\,ab$$

$$e = -\frac{d\phi_m}{dt} = 0$$

20. $E = V_b + iR$

$V_b = E - iR = 200 - 20 \times 1.5 = 170\text{ V}$

21. $\frac{V_s}{V_p} = \frac{N_s}{N_p} \Rightarrow V_s = \frac{1}{2} \times 290 = 10\text{ V}$

$$\frac{i_p}{i_s} = \frac{N_s}{N_p}$$

$$\Rightarrow \quad i_s = \frac{N_p}{N_s}i_p = 2 \times 4 = 8\text{ A}$$

22. $V_r = 0$, hence magnetic flux linked with the coil remain same.

$\therefore\ e = \frac{-d\phi}{dt} = 0$

23. $s = \frac{1}{2}at^2$

Due to change in magnetic flux linked with the ring, magnet experiences an upward force, hence,

$$a < g$$

$$s < \frac{1}{2}gt^2 \Rightarrow s < 5\text{ m}$$

24. $V_A - V_B = L\frac{di}{dt} = -\alpha\, t$

25. $i_0 = \frac{E}{R} = \frac{12}{0.3} = 40\text{ A}$

$U_0 = \frac{1}{2}Li_0^2 = \frac{1}{2} \times 50 \times 10^{-3} \times (40)^2$

$= 40\text{ J}$

26. $i = i_0\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$

$$\frac{di}{dt} = \frac{E}{L}e^{-\frac{Rt}{L}}$$

$$V_L = L\frac{di}{dt} = Ee^{-\frac{Rt}{L}}$$

at $t = 0$

$$V_L = E = 20\text{ V}$$

at $t = 20$ ms

$$V_L = Ee^{-\frac{R \times 20 \times 10^{-3}}{L}} \Rightarrow 5 = 20e^{-\frac{R}{50L}}$$

$$\frac{R}{50L} = \ln 4 \Rightarrow R = (100\ln 4)\,\Omega$$

27. $|i| = \frac{|e|}{R} = \frac{1}{R}\cdot\frac{d\phi}{dt} = \frac{1}{R}NA\frac{dB}{dt}$

$$= \frac{10 \times 10 \times 10^{-4}}{20} \times 10^8 \times 10^{-4}$$

$$= 5\text{ A}$$

28. In the steady state, inductor behaves as short circuit, hence entire current flows through it.

29. $\phi_m = AB\cos\theta$

But, $\theta = 90°$

$\therefore \quad \phi_m = 0$

30. $i = \frac{|e|}{R} = -\frac{1}{R}\frac{d\phi_m}{dt}$

$$\Rightarrow \quad \frac{dq}{dt} = \frac{-nBA}{R}\frac{d}{dt}(\cos\theta)$$

$$= \frac{nBA}{R}\sin\theta\frac{d\theta}{dt}$$

$$\Rightarrow \quad dq = \frac{nBA}{R}\sin\theta\, d\theta$$

$$Q_1 = \frac{nBA}{R}\int_0^{\pi}\sin\theta\, d\theta = \frac{2nBA}{R}$$

$$Q_2 = \frac{nBA}{R}\int_0^{2\pi}\sin\theta\, d\theta = 0$$

$$\therefore \quad \frac{Q_2}{Q_1} = 0$$

31. According to Lenz's law, induced current always opposes the cause producing it.

32. $i = i_0\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$

$$= \frac{15}{5}\left(1 - e^{-\frac{5\times 2}{10}}\right) = 3\,(1 - e^{-1})$$

$$= 3\left(1 - \frac{1}{e}\right)\text{A}$$

33. Velocity of AB is parallel to its length.

34. Velocity of rod is parallel to its length.

35. $V_c - V_a = V_c - V_b = BRV$

and $V_a - V_b = 0$

36. Induced current always opposes the cause producing it.

37. $E = -\frac{d\phi}{dt}$

38. Magnetic flux linked with the coil does not change, hence

$$i = \frac{e}{R} = \frac{-1}{R}\cdot\frac{d\phi}{dt} = 0$$

39. $e = Blv\cos\theta = \frac{1}{2}Bl^2\omega\cos\theta$ $\left(\because v = \frac{l}{2}\omega\right)$

As $|\cos\theta|$ varies from 0 to 1

e varies from 0 to $\frac{1}{2}Bl^2\omega$.

JEE Corner

Assertion and Reason

1. Magnetic flux linked with the coil is not changing with time, hence induced current is zero.

2. Both Assertion and Reason are correct but Reason does not explain Assertion.

3. Induced electric field is non-conservative but can exert force on charged particles.

4. $i = 2t - 8$

$\frac{di}{dt} = 2$

$V_a - V_b = L\frac{di}{dt} = 2 \times 2 = 4\text{ V}$

5. $\left(\frac{di}{dt}\right)_{max} = (i_{max})\,\omega = 1 \times 2 = 2\text{ A/s}$

6. $V_a - V_b = V_c - V_a$

$V_c > V_a > V_b$

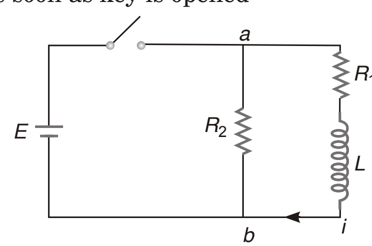
7. Fact.

8. $L = \mu_r\mu_0 n^2 lA$, for ferromagnetic substance,

$$\mu_r > g$$

and L does not depends on i.

9. As soon as key is opened



$$i = i_0 = \frac{E}{R_1}$$

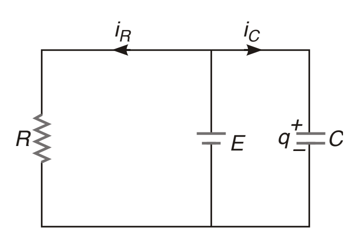
10. Inductors oppose change in current while resistor does not.

Objective Questions (Level 2)

1. By conservation of energy

$$\frac{1}{2}L\,i_0^2 = \frac{1}{2}mv_0^2$$

$$i_0 = \sqrt{\frac{m}{k}}v_0$$



2. Wire AB behaves as a cell of emf, $E = Blv$

$$i_R = \frac{E}{R} = \frac{Blv}{R}$$

$$i_c = 0$$

$$U_c = \frac{1}{2}CE^2 = \frac{1}{2}CB^2l^2v^2$$

3. Apply Fleming's left hand rule.

4. For SHM,

$$v = \omega A \cos \omega t$$

$$e = Blv = Bl\omega A \cos \omega t$$

$$e = \begin{cases} e_0 \cos \omega t & \text{for } nT < t < (2n-1)\dfrac{T}{2} \\ -e_0 \cos \omega t & \text{for } \dfrac{(2n-1)T}{2} > t > nT \end{cases}$$

5. $\phi_m = BA$

At any instant when wires have moved through a distance x,

$$A = (a + 2x)^2$$

$$\phi_m = B(a + 2x)^2$$

$$|e| = \frac{d\phi_m}{dt} = 4B(a + 2x)\frac{dx}{dt}$$

$$= 4B(a + 2x)v_0$$

$$|i| = \frac{|e|}{R} = \frac{4B(a + 2x)v_0}{\lambda \times 4(a + 2x)} = \frac{Bv_0}{\lambda}$$

6. $A = l^2$

$$\frac{dA}{dt} = 2l\frac{dl}{dt} = -2l\alpha \qquad \left(\alpha = -\frac{dl}{dt}\right)$$

$$\phi_m = BA$$

$$e = -\frac{d\phi_m}{dt} = -B\frac{dA}{dt} = 2Bl\alpha$$

at $\quad l = a$

$$e = 2a\,\alpha B$$

7. At this instant, direction of motion of wire PQ is perpendicular to its length.

$\therefore \quad e = Blv$

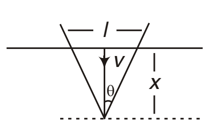
8. $q = CV = CBlv$

$$= 20 \times 10^{-6} \times 0.5 \times 0.1 \times 0.2$$

$$= 0.2\,\mu\text{C}$$

Plate A is positive while plate B is negative.

9. $\phi_m = BA = B\left(\frac{1}{2}lx\right)$



But $l = 2x \tan \theta$

$\therefore \quad \phi_m = B \tan \theta\, x^2$

$$e = -\frac{d\phi_m}{dt} = -2B \tan \theta\, x \frac{dx}{dt}$$

$$= 2B \tan \theta\, vx$$

$$R = r\,l = r(2x \tan \theta)$$

where, r = resistance per unit length of the conductor.

$\therefore \quad i = \frac{e}{R} = \frac{Bv}{r} = \text{constant.}$

10. $\phi_m = BA \cos \theta = BA \cos \omega t$

$$e = -\frac{d\phi_m}{dt} = \omega BA \sin \omega t$$

But $\quad A = b^2$

$\therefore \quad e = b^2 B\omega \sin \omega t$

11. Induced emf

$$e = a^2 \frac{dB}{dt} = (1)^2 \times 2 \times 10^{-3}$$

$$= 2 \times 10^{-3}\text{ V}$$

$$W = qe = 1 \times 10^{-6} \times 2 \times 10^{-3}$$

$$= 2 \times 10^{-9}\text{ J}$$

12. In the steady state, current through capacitor = 0.

$$i_L = \frac{20}{5} = 4\text{ A}$$

$$\phi_1 = 0,\ \phi_2 = i_L L = 4 \times 500 \times 10^{-2}$$

$$= 2\text{ Wb}$$

$$\Delta\phi = \phi_2 - \phi_1 = 2\text{ Wb.}$$

13. $\frac{1}{2}Li^2 = \frac{1}{2}\left(\frac{1}{2}Li_0^2\right)$

$$i = \frac{i_0}{\sqrt{2}}$$

$$i_0\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{i_0}{\sqrt{2}}$$

$$e^{-t/\tau} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

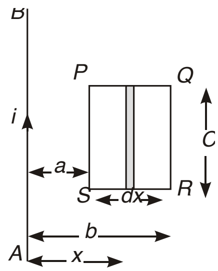
$$t = \tau \ln\left(\frac{\sqrt{2}}{\sqrt{2} - 1}\right)$$

$$= \frac{L}{R}\ln\left(\frac{\sqrt{2}}{\sqrt{2} - 1}\right)$$

14. $B = \frac{\mu_0 i}{2\pi a}$

$$F = qvB = \frac{\mu_0 iqv}{2\pi a}$$

15. Consider an elementary section of loop of width dx at a distance x from wire AB

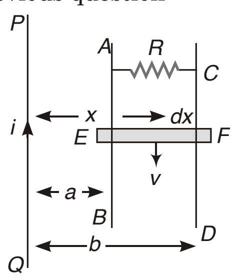


$$d\phi_m = BdA = \frac{\mu_0 i}{2\pi x} C\, dx$$

$$\phi_m = \frac{\mu_0 iC}{2\pi}\int_a^b \frac{dx}{x} = \frac{\mu_0 iC}{2\pi}\ln\frac{b}{a}$$

$$M = \frac{\phi_m}{i} = \frac{\mu_0 C}{2\pi}\ln\frac{b}{a}$$

16. From previous question



$$\phi_m = \frac{\mu_0 iy}{2\pi}\ln\frac{b}{a}$$

$$e = \frac{d\phi_m}{dt} = \frac{\mu_0 i \ln\frac{b}{a}}{2\pi}\frac{dy}{dt}$$

$$e = \frac{\mu_0 iv}{2\pi}\ln\frac{b}{a}$$

$$i = \frac{e}{R} = \frac{\mu_0 iv}{2\pi R}\ln\frac{b}{a}$$

Consider an elementary portion of length dx of the rod at a distance, x from the wire PQ.

Force on this portion,

$$dF = i\,dxB$$

$$= i\frac{\mu_0}{4\pi}\cdot\frac{2i}{x}dx$$

$$F = i\frac{\mu_0}{4\pi}\cdot 2i\int_a^b \frac{dx}{x}$$

$$= \left[\frac{\mu_0 iv}{2\pi R}\ln\frac{b}{a}\right]\frac{\mu_0 i}{2\pi}\ln\frac{b}{a}$$

$$= \frac{1}{vR}\left[\frac{\mu_0 iv}{2\pi}\ln\frac{b}{a}\right]^2$$

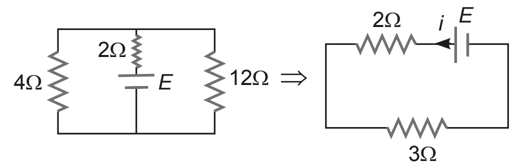
17. $E = \frac{1}{2}r\frac{dB}{dt} \Rightarrow E \propto r$

18. Induced current opposes change in magnetic flux.

19. $V_L = E - iR$

20. The rod can be assumed as a cell of emf $E = Blv$

The equivalent circuit is shown in figure,



$$i = \frac{E}{2+3} = \frac{Blv}{5} = \frac{0.50 \times 0.25 \times 4}{5}$$

$$= 0.1\text{ A}$$

21. Outside the region of magnetic field, induced electric field,

$$E = \frac{r^2}{2R}\frac{dB}{dt} = \frac{Br^2}{2R}$$

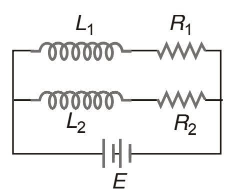
$$F = qE$$

$$\tau = qER = \frac{1}{2}qBr^2$$

22. $V_A - V_0 = B(2R)V$

$\Rightarrow \quad V_A - V_0 = 2BRV$

23. $L_1 = \frac{\eta L}{\eta + 1}, L_2 = \frac{L}{\eta + 1}$



$$R_1 = \frac{\eta R}{\eta + 1}, R_2 = \frac{\eta R}{\eta + 1}$$

$$\frac{1}{L_e} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$= \frac{\eta + 1}{\eta L} + \frac{\eta + 1}{L}$$

$$= \frac{(\eta + 1)}{L}\left[\frac{1}{\eta} + \frac{1}{1}\right]$$

$$L_e = \frac{\eta L}{(\eta + 1)^2}$$

Similarly, $R_e = \frac{\eta R}{(\eta + 1)^2} \quad \therefore \quad \tau = \frac{L_e}{R_e} = \frac{L}{R}$

24. $i = i_0 e^{-t/\tau}$

$$Bi_0 = i_0 e^{-T/\tau}$$

$$\tau = \frac{T}{\ln\left(\frac{1}{B}\right)}$$

25. Given, $i_0^2 R = P$, $\frac{L}{R} = \tau$

when, choke coil is short circuited,

Total heat produced = Magnetic energy stored in the choke coil

$$= \frac{1}{2} L i_0^2 = \frac{1}{2}(R\tau)\left(\frac{P}{R}\right) = \frac{1}{2} P\tau$$

26. $i = i_0 e^{-\frac{Rt}{L}}$

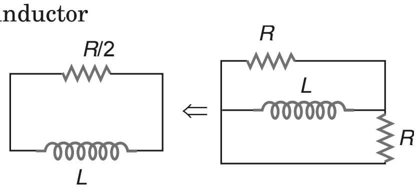
For current to be constant

$$i = i_0$$

$$e^{-\frac{Rt}{L}} = 1$$

$\frac{Rt}{L} = 0 =$ not possible.

27. To final time constant, short the battery and find effective resistance in series with inductor

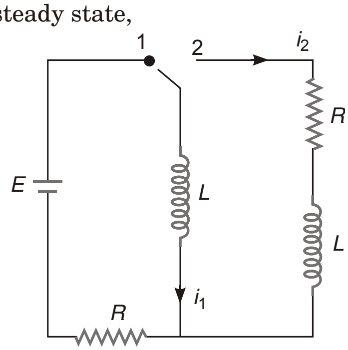


$$R_e = \frac{R}{2}$$

$$\tau = \frac{L}{R_e} = \frac{2L}{R}$$

28. When switch is at position 1.

In steady state,



$$i_1 = \frac{E}{R}$$

$$i_2 = 0$$

When switch is thrown to position 2.

$$i_1 = \frac{E}{R},\ i_2 = -\frac{E}{R}$$

29. $\frac{1}{2} L i^2 = \frac{1}{4}\left(\frac{1}{2} L\, i_0^2\right)$

$$i = \frac{i_0}{2}$$

$$i_0\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{i_0}{2}$$

$$t = \tau \ln 2$$

$\Rightarrow$ $$t = \frac{L}{2}\ln 2$$

30. At the moment when switch is thrown to position 2,

current in capacitor = current in inductor just before throwing the switch to position 2,

$\Rightarrow$ $$i_c = \frac{E}{R}$$

31. Initially, inductor offers infinite resistance, hence,

$$i = 0 \text{ and } \frac{di}{dt} = \text{maximum}$$

$\therefore$ $E = V_L + V_C + V_R$

But $V_C = V_R = 0$

$\Rightarrow$ $V_L = E$

32. Same as Q.12 objective Questions (Level 2).

33. Let V_0 = Potential of metallic rod,

$$V_B - V_0 = B(2R)V = 2BR^2\omega \quad \ldots\text{(i)}$$

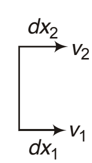
$$V_0 - V_C = B(2R)V = 2BR^2\omega \quad \ldots\text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$$V_B - V_C = 4B\omega R^2$$

34. $e = Blv_c$

$$v_c = \frac{v_1 + v_2}{2}$$



$\therefore$ $$e = \frac{1}{2} Bl(v_1 + v_2)$$

or

$$e = B\frac{dA}{dt}$$

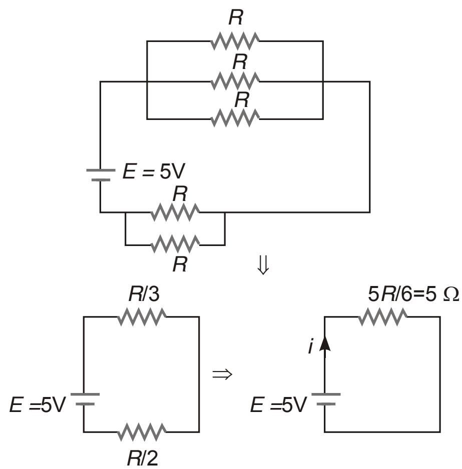
$$dA = \frac{1}{2} l(dx_1 + dx_2)$$

$$e = \frac{1}{2} Bl\left(\frac{dx_1}{dt} + \frac{dx_2}{dt}\right)$$

$$= \frac{1}{2} Bl(v_1 + v_2)$$

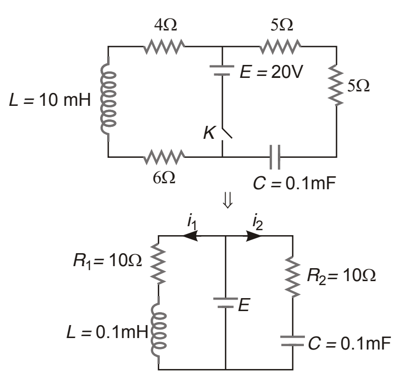
35. Initially, capacitor offer zero resistance and inductor offers infinite resistance.

Effective circuit is given by



$$i = \frac{E}{R} = 1\text{ A}$$

36. $i_1 = \frac{E}{R_1}\left(1 - e^{-\frac{R_1 t}{L}}\right),\ i_2 = \frac{E}{R_2}\left(e^{-\frac{t}{R_2 C}}\right)$



$$i = i_1 + i_2$$
$$= \frac{E}{R_1}\left(1 - e^{-\frac{R_1 t}{L}}\right) + \frac{E}{R_2} e^{-\frac{t}{R_2 C}}$$

at $t = 10^{-3}\ \ln 2$

$$i = \frac{20}{10}\left(1 - e^{-\frac{10 \times 10^{-3}\ \ln 2}{10 \times 10^{-3}}}\right) + \frac{20}{10} e^{-\frac{10^3\ \ln 2}{10 \times 0.1 \times 10^{-3}}}$$

$$= 2\left(1 - \frac{1}{2}\right) + 2\left(\frac{1}{2}\right) = 2\text{ A}$$

37. $|i| = \frac{|e|}{R} = \frac{A}{R}\frac{dB}{dt}$

$$= \frac{B_0 A}{R} = \frac{B_0[(2b)^2 - \pi a^2]}{R}$$
$$= \frac{B_0(4b^2 - \pi a^2)}{R}$$

As inward magnetic field is increasing, net current must be anticlockwise. Hence current in inner circle will be clockwise.

38. From Q. 48 Subjective Questions (Level 1).

$$\phi_m = \frac{\mu_0 ai}{2\pi}\ln\left(1 + \frac{a}{x}\right)$$

Case 1

$$x = b,\ a = a$$
$$\phi_{m_1} = \frac{\mu_0 ai}{2\pi}\ln\left(1 + \frac{a}{b}\right)$$
$$= \frac{\mu_0 ai}{2\pi}\ln\left(\frac{b+a}{b}\right)$$

Case 2

$$x = b - a$$
$$a = a$$
$$\phi_{m_2} = \frac{\mu_0 ai}{2\pi}\ln\left(1 + \frac{a}{b-a}\right)$$
$$= \frac{\mu_0 ai}{2\pi}\ln\left(\frac{b}{b-a}\right)$$
$$< e > = -\frac{\phi_{m_2} - \phi_{m_1}}{t}$$
$$< e > = \frac{< e >}{R} = -\frac{\phi_{m_2} - \phi_{m_1}}{Rt}$$
$$q = < i > t = -\frac{\phi_{m_2} - \phi m_1}{R}$$
$$= -\frac{\mu_0 ai}{2\pi R}\left[\ln\left(\frac{b+a}{b}\right) - \left(\ln\frac{b}{b-a}\right)\right]$$
$$= \frac{-\mu_0 ai}{2\pi R}\ln\left(\frac{b}{b^2 - a^2}\right)$$
$$|q| = \frac{\mu_0 ai}{2\pi R}\ln\left(\frac{b}{b^2 - a^2}\right)$$

39. Magnetic flux linked with the coil.

$$\phi_m = nBA = \frac{\mu_0 n\, iA}{2r}$$
$$|e| = \frac{d\phi_m}{dt}$$
$$\Rightarrow \quad iR = \frac{d\phi_m}{dt}$$

$$\frac{dq}{dt}R = \frac{d\phi_m}{dt} \Rightarrow dq = \frac{1}{R}d\phi_m$$

$$q = \frac{\mu_0 nA}{2rR}\int_0^i di = \frac{\mu_0 n\, iA}{2rR}$$

40. Induced electric field inside the region of varying magnetic fields,

$$E = \frac{1}{2}r\frac{dB}{dt} = \frac{1}{2}r(6t^2 + 2x) = 3r(t^2 + x)\text{ V/m}$$

At, $t = 2.0$ s and $r = \frac{R}{2} = 1.25$ cm

$$= 1.25 \times 10^{-2}\text{ m}$$

$$E = 3 \times 1.25 \times 10^{-2} \times (4 + x)$$

$$= 0.3\text{ V/m}$$

$$F = eE = 1.6 \times 10^{-19} \times 0.3$$

$$= 48 \times 10^{-21}\text{ N}$$

41. $E = \frac{1}{2}r\frac{dB}{dt} \Rightarrow E \propto r$

42. As inward magnetic field is increasing, induced electric field must be anticlockwise.

43. $e = \frac{d\phi_m}{dt} = \pi a^2\frac{dB}{dt} = \pi a^2 B_0$

44. $E = \frac{e}{2\pi a} = \frac{1}{2}aB_0$

45. $\tau = qEa = i\alpha$

$$\alpha = \frac{qEa}{ma^2} = \frac{q \times \frac{1}{2}aB_0 a}{ma^2}$$

$$= \frac{qB_0}{2m}$$

46. $P = \tau\omega = \tau(\alpha t) = i\alpha^2 \cdot t$

$$= ma^2 \times \frac{q^2B_0^2}{\mu\, m^2}t$$

At $t = 1$ s

$$P = \frac{q^2B_0^2a^2}{4m}$$

47. $i = \frac{e}{R} = \frac{A}{R}\cdot\frac{dB}{dt}$

$$\frac{dB}{dt} = 2\text{ T/s},\ A = 0.2 \times 0.4 = 0.08\text{ m}^2$$

$$\therefore\ i = \frac{0.08}{1 \times 1.0} \times 2 = 16\text{ A} \qquad [\because R = r \times (b + 2l)]$$

As outward magnetic field is increasing, induced current must be clockwise.

48. $e = B\frac{dA}{dt} + A\frac{dB}{dt} = Blv + A\frac{dB}{dt}$

At $t = 2$ s,

$$B = 4\text{ T},\ A = 0.2 \times (0.4 - vt) = 0.06\text{ m}^2$$

$$v = 5\text{ cm/s} = 0.05\text{ m/s}$$

$$\therefore\quad e = -4 \times 0.2 \times 0.05 + 0.06 \times 2$$

$$= -0.04 + 0.12 = 0.08\text{ V}$$

49. $F = ilB = \frac{e}{R}lB$

$$= \frac{0.08}{1 \times 0.8} \times 0.2 \times 4$$

$$= 0.008\text{ N}$$

50. When terminal velocity is attained,

power delivered by gravity = power dissipated in two resistors

$$mgv = 0.76 + 1.2$$

$$v = \frac{1.96}{0.2 \times 9.8} = 1\text{ m/s}$$

51. $e = Blv = 0.6 \times 1 \times 1 = 0.6$ V

$$P_1 = \frac{e^2}{R_1}$$

$$\Rightarrow \qquad R_1 = \frac{e^2}{P_1} = \frac{(0.6)^2}{0.76} = 0.47\ \Omega$$

52. $P_2 = \frac{e^2}{R_2}$

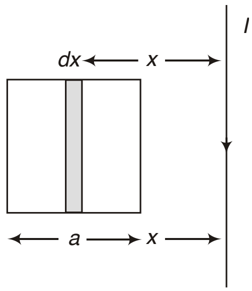
$$\Rightarrow \qquad R_2 = \frac{e^2}{P_2} = \frac{(0.6)^2}{1.2} = 0.3\ \Omega$$

More than One Correct Options

1. $e = B\left(\frac{1}{2}\right)v = \frac{1}{2}BLv$

By Fleming's left hand rule, P must be positive w.r.t. Q.

2. $d\phi_m = BdA = Ba\,dx$



$$= \frac{\mu_0 a\, i}{2\pi x}dx$$

$$\phi_m = \frac{\mu_0 a\, i}{2\pi i}\ln 2$$

$$M = \frac{\phi_m}{i} = \frac{\mu_0 a}{2\pi}\ln 2$$

If the loop is brought close to the wire, upward magnetic flux linked with the loop increases, hence induced current will be clockwise.

3. $\phi = Li =$ Henry-Ampere.

$$L = \frac{V}{di/dt} = \frac{V\,dt}{di} = \frac{\text{Volt-second}}{\text{Ampere}}$$

4. $\tau = \frac{L}{R} = 1\text{ s}$

$$i = i_0(1 - e^{-t/\tau}) = \frac{E}{R}(1 - e^{-t/\tau})$$

$$= 4(1 - e^{-t})$$

At $t = \ln 2$,

$$i = 2A$$

Power supplied by battery, $P = EI = 16$ J/s.

Rate of dissipation of heat in across resistor

$$= i^2R = 8\text{ J/s}$$

$$V_R = iR = 4\text{ V}$$

$$V_a - V_b = E - V_R = 4\text{ V}$$

5. In both the cases, magnetic flux linked with increases, so current i_2 decreases in order to oppose the change.

6. $\phi_1 = BA = 4 \times 2 = 8$ Wb, $\phi_2 = 0$

$$e = \frac{\phi_2 - \phi_1}{t} = \frac{8}{0.1} = 80\text{ V}$$

$$i = \frac{e}{R} = \frac{80}{4} = 20\text{ A}$$

$$q = it = 20 \times 0.1 = 2\text{ C}$$

Current is not given as a function of time, hence heat produced in the coil cannot be determined.

7. In LC oscillations,

$$\omega = \frac{1}{\sqrt{LC}}, f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

$$T = \frac{1}{f} = 2\pi\sqrt{LC}$$

$$i_0 = \omega q_0 = \frac{q_0}{\sqrt{LC}}$$

$$\left(\frac{di}{dt}\right)_{\text{max}} = \omega q_0 = \frac{q_0}{LC}$$

$$(V_L)_{\text{max}} = L\left(\frac{di}{dt}\right)_{\text{max}} = \frac{q_0}{C}$$

8. If magnetic field increases, induced electric field will be anticlockwise and *vice-versa*.

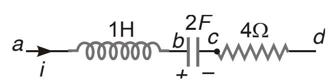
9. $q = 2t^2$

$$i = \frac{dq}{dt} = 4t$$

$$\frac{di}{dt} = 4\text{ A/s}$$

As $\quad \frac{dq}{dt} = \text{Positive}$

Charge on the capacitor is increasing, hence current flows from a to b.



$$t = 1\text{ s}, q = 2\text{C}, \ i = 4\text{ A}$$

$$\frac{di}{dt} = 4\text{ A/s}$$

$$V_a - V_b = L\frac{di}{dt} = 1 \times 4 = 4\text{ V}$$

$$V_b - V_c = \frac{q}{c} = \frac{2}{2} = 1\text{ V}$$

$$V_c - V_d = iR = 4 \times 4 = 16\text{ V}$$

$$V_a - V_d = 4 + 1 + 16 = 21\text{ V}$$

10. $V_a - V_b = \frac{1}{2}Bl^2\omega$

$$V_c - V_b = \frac{1}{2}Bl^2\omega$$

$$V_a - V_c = 0$$

[Direction of velocity of rod a-c is parallel to length a-c]

Match the Columns

1.
$$[B] = \frac{[F]}{[i][l]} = \frac{[\text{MLT}^{-2}]}{[\text{A}][\text{L}]}$$
$$= [\text{ML}^0\text{T}^{-2}\text{A}^{-1}]$$
$$[L] = \frac{[V][dt]}{[di]} = \frac{[\text{ML}^2\text{T}^{-3}][\text{T}]}{[\text{A}]}$$
$$= [\text{ML}^2\text{T}^{-2}\text{A}^{-2}]$$
$$[LC] = [\text{T}^2]$$
$$[\phi_m] = [B][S]$$
$$= [\text{ML}^0\text{T}^{-2}\text{A}^{-1}][\text{L}^2] = [\text{ML}^2\text{T}^{-2}\text{A}^{-1}]$$

2. $i = i_0(1 - e^{-t/\tau})$
$$\tau = \frac{L}{R} = 1\text{ s}$$
$$i_0 = \frac{E}{R} = 5\text{ A}$$
$$V_R = iR = E(1 - e^{-t})$$
$$V_L = E - V_R = Et^{-t}$$

At $t = 0$,
$$V_L = E = 10\text{ V},\ V_R = 0$$

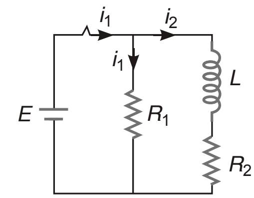
at $t = 1$ s
$$V_L = E(1 - e^{-1}) = \left(1 - \frac{1}{e}\right)10\text{ V}$$
$$V_R = \frac{10}{e}\text{ V}$$

3. In LC oscillations,
$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times \frac{1}{4}}} = 2\text{ rad/s}$$
$$q_0 = 4\text{ C}$$
$$i_0 = \omega q_0 = 8\text{ A}$$
$$\left(\frac{di}{dt}\right)_{\text{max}} = \omega^2 q_0 = 16\text{ A/s.}$$

When, $q = 2$ C
$$V_L = V_C = \frac{q}{C} = 8\text{ V}$$

When, $\left(\frac{di}{dt}\right) = \frac{1}{2}\left(\frac{di}{dt}\right)_{\text{max}} = 8\text{ A/s.}$
$$V_C - V_L = L\frac{di}{dt} = 1 \times 8 = 8\text{ V}$$

4. $i_1 = \frac{E}{R_1} = \frac{9}{6} = 1.6\text{ A}$

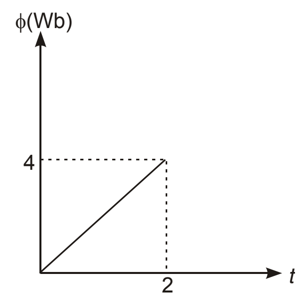


$$i_2 = \frac{E}{R_2}\left(1 - e^{-\frac{R_2 t}{L}}\right) = 3(1 - e^{-t/3})$$

At $t = (\ln 2)$ s
$$V_L = E - i_2R_2 = qe^{-t/3} = \frac{q}{2^{1/3}}$$
$$V_{R_2} = i_2R_2 = q(1 - e^{-t/3})$$
$$= q\left(1 - \frac{1}{2^{1/3}}\right)$$
$$V_{R_1} = i_1R_1 = 9\text{ V}$$
$$V_{bc} = V_L + V_{R_2} = 9\text{ V}$$

(a → s), (b → s), (c → p), (d → p).

5. Induced emf



$|e|$ = slope of ϕ - t graph
$$= \frac{4 - 0}{2 - 0} = 2\text{ V}$$
$$|i| = \frac{|e|}{R} = \frac{2}{2} = 1\text{ A}$$
$$|q| = |i|t = 1 \times 2 = 2\text{ C}$$

As current i is constant
$$H = i^2Rt = (1)^2 \times 2 \times 2 = 4\text{J}$$

25 Alternating Current

Introductory Exercise 25.1

1. $R = \frac{V_{DC}}{I} = \frac{100}{10} = 10\,\Omega$

$Z = \frac{V_{AC}}{I} = \frac{150}{10} = 15\,\Omega$

$X_L = \sqrt{Z^2 - R^2} = \sqrt{(15)^2 - (10)^2}$

$= 5\sqrt{5}\,\Omega$

$L = \frac{X_L}{\omega} = \frac{X_L}{2\pi f} = \frac{5\sqrt{5}}{2 \times 3.14 \times 50}$

$\approx 0.036\,\text{H}$

$\therefore \quad V_L = IX_L = 50\sqrt{5}\,\text{V}$

$= 111.8\,\text{V}$

2. For phase angle to be zero,

$X_L = X_C$

$\Rightarrow \quad \omega L = \frac{1}{\omega C}$

$\Rightarrow \quad L = \frac{1}{\omega^2 C} = \frac{1}{(2\pi f)^2 C}$

$= \frac{1}{(360)^2 \times 10^{-6}}$

$\approx 7.7\,\text{H}$

As $\quad X_L = X_C$

$\therefore \quad Z = R$

$I = \frac{V}{Z} = \frac{120}{20} = 6\,\text{A}$

Introductory Exercise 25.2

1. Resonating frequency,

$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.03 \times 2 \times 10^{-6}}}$

$= \frac{10^4}{\sqrt{6}}$

$f_r = \frac{\omega_r}{2\pi} = \frac{10^4}{2 \times 3.14 \times \sqrt{6}}$

$\approx 1105\,\text{Hz}$

Phase angle at resonance is always 0°.

2. Resistance of arc lamp,

$R = \frac{V_{DC}}{I}$

$= \frac{40}{10} = 4\,\Omega$

Impedance of series combination,

$Z = \frac{V_{AC}}{I} = \frac{200}{10} = 20\,\Omega$

Power factor $= \cos\phi = \frac{R}{Z} = \frac{4}{20} = \frac{1}{5}$

AIEEE Corner

Subjective Questions (Level-1)

1. (a) $X_L = \omega L = 2\pi f L$

$= 2 \times 3.14 \times 50 \times 2$

$= 628\ \Omega$

(b) $X_L = \omega L \Rightarrow L = \dfrac{X_L}{\omega}$

$= \dfrac{X_L}{2\pi f} = \dfrac{2}{2 \times 3.14 \times 50}$

$= 6.37$ mH

(c) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

$= \dfrac{1}{2 \times 3.14 \times 50 \times 2 \times 10^{-6}}$

$= 1592\ \Omega = 1.59\ \text{k}\Omega$

(d) $X_C = \dfrac{1}{\omega C} \Rightarrow C = \dfrac{1}{\omega X_C}$

$= \dfrac{1}{2 \times 3.14 \times 50 \times 2} = 1.59$ mF

2. (a) $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$

$= \sqrt{(300)^2 + \left(400 \times 0.25 - \dfrac{1}{400 \times 8 \times 10^{-6}}\right)^2}$

$= 367.6\ \Omega$

$I_0 = \dfrac{V_0}{Z} = \dfrac{120}{367.6} = 0.326$ A

(b) $\phi = \tan^{-1} \dfrac{X_L - X_C}{R}$

$= \tan^{-1} \dfrac{212.5}{300} \approx -35.3°$

As $X_C > X_L$ voltage will lag behind current by 35.3°.

(c) $V_R = I_0 R = 0.326 \times 300 = 97.8$ V,

$V_L = I_0 X_L = 32.6$ V

$V_C = I_0 X_C = 0.326 \times 312.5$

$= 101.875\ \text{V} \approx 120\ \text{V}$

3. (a) Power factor at resonance is always 1, as $Z = R$, Power factor $= \cos\phi = \dfrac{R}{Z} = 1.$

(b) $P = \dfrac{I_0 E_0 \cos\phi}{2} = \dfrac{E_0^2}{2R}$

$= \dfrac{(150)^2}{3 \times 150} = 75$ W

(c) Because resonance is still maintained, average power consumed will remain same, *i.e.*, 75 W.

4. (a) As voltage is lag behind current, inductor should be added to the circuit to raise the power factor.

(b) Power factor $= \cos\phi = \dfrac{R}{Z}$

$\Rightarrow \quad Z = \dfrac{R}{\cos\phi} = \dfrac{60}{0.720} = \dfrac{250}{3}\ \Omega$

$X_C = \sqrt{Z^2 - R^2}$

$= \sqrt{\left(\dfrac{250}{3}\right)^2 - (60)^2}$

$= 58\ \Omega$

$C = \dfrac{1}{\omega X_C}$

$= \dfrac{1}{2\pi f X_C}$

$= \dfrac{1}{2 \times 3.14 \times 50 \times 58}$

$= 54\ \mu$F

For resonance,

$\omega_r = \dfrac{1}{\sqrt{LC}}$

$\Rightarrow \quad L = \dfrac{1}{\omega_r^2 C}$

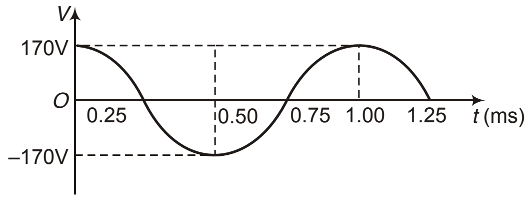
$= \dfrac{1}{(2\pi f)^2 C}$

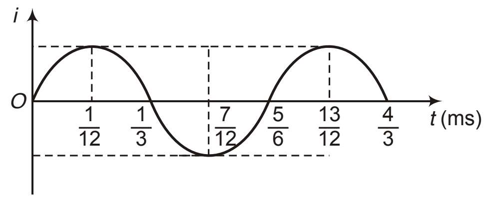
$\Rightarrow \quad L = \dfrac{1}{(2 \times 3.14 \times 50)^2 \times 54 \times 10^{-6}}$

$= 0.185$ H

5. $V(t) = 170 \sin(6280t + \pi/3)$ volt

$i(t) = 8.5 \sin(6280t + \pi/2)$ amp.





(b) $f = \frac{\omega}{2\pi} = \frac{6280}{2 \times 3.14} = 1000\,\text{Hz}$

$= 1\,\text{kHz}$

(c) $\phi = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

$\Rightarrow \quad \cos\phi = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

As phase of i is greater than V, current is leading voltage.

(d) Clearly the circuit is capacitive in nature, we have

$$\cos\phi = \frac{R}{Z}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{R}{Z} \Rightarrow Z = \frac{2}{\sqrt{3}}R$$

Also, $\quad Z = \frac{V_0}{i_0} = \frac{170}{8.5} = 20\,\Omega$

$$R = \frac{\sqrt{3}}{2}Z = 10\sqrt{3}\,\Omega$$

Again, $\quad Z = \sqrt{R^2 + X_C^2} \Rightarrow X_C = \sqrt{Z^2 - R^2}$

$$= \sqrt{400 - 300} = 10\,\Omega$$

$$X_C = \frac{1}{\omega C} \Rightarrow \frac{1}{\omega X_C} = \frac{1}{6280 \times 10}$$

$$= 15.92\,\mu\text{F}$$

6. $I = \frac{V}{X_L} = \frac{V}{\omega L}$

(a) $\omega = 100$ rad/s

$\therefore \quad I = \frac{60}{100 \times 5} = 0.12\,\text{A}$

(b) $\omega = 1000$ rad/s

$\therefore \quad I = \frac{60}{1000 \times 5} = 1.2 \times 10^{-2}\,\text{A}$

(c) $\omega = 10000$ rad/s

$\therefore \quad I = \frac{60}{10000 \times 5} = 1.2 \times 10^{-3}\,\text{A}$

7. $V_R = (2.5\,\text{V}) \cos[(950\,\text{rad/s})\,t]$

(a) $I = \frac{V_R}{R}$

$$= \frac{(2.5\,\text{V}) \cos[(950\,\text{rad/s})\,t]}{300}$$

$$= (8.33\,\text{mA}) \cos[(950\,\text{rad/s})\,t]$$

(b) $X_L = \omega L = 950 \times 0.800$

$= 760\,\Omega$

(c) $V_L = I_0 X_L \cos(\omega t + \pi/2)$

$\Rightarrow \quad V_L = -I_0 X_L \sin\omega t$

$= -6.33 \sin[(950\,\text{rad/s})\,t]\,\text{V}$

8. Given, $L = 0.120$ H, $R = 240\,\Omega$, $C = 7.30\,\mu$F, $I_{rms} = 0.450$ A, $f = 400$ Hz

$$X_L = \omega L = 2\pi f L$$

$$= 2 \times 3.14 \times 400 \times 0.120$$

$$= 301.44\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$= \frac{1}{2 \times 3.14 \times 400 \times 7.3 \times 10^{-6}}$$

$$= 54.43\,\Omega$$

(a) $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$

$$= \frac{240}{\sqrt{(240)^2 + (301.44 - 54.43)^2}}$$

$$= 0.697$$

$$\phi = \cos^{-1}(0.697) \approx 45.8°$$

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{(240)^2 + (301.44 - 54.43)^2} = 344\,\Omega$$

(c) $V_{rms} = I_{rms} Z = 0.450 \times 344$

$= 154.8\,\text{V} \approx 155\,\text{V}$

(d) $P_{av} = V_{rms} I_{rms} \cos\phi$

$= 155 \times 0.450 \times 0.697 = 48.6\,\text{W}$

(e) $P_R = I_{rms}^2 R = (0.450)^2 \times 240 = 48.6\,\text{W}$

(f) and (g) Average power associated with inductor and capacitor is always zero.

Objective Questions (Level-1)

1. In an AC circuit, $\cos\phi$ is called power factor.

2. DC ammeter measures charge flowing in the circuit per unit time, hence it measures average value of current, but average value of AC over a long time is zero.

3. $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$
Hence, for $X_L < X_C$, Z decreases with increase in frequency and for $X_L > X_C$, Z increases with increase in frequency.

4. As voltage leads current and $\phi < \frac{\pi}{2}$, hence either circuit contains inductance and resistance or contains inductance, capacitance and resistance with $X_L > X_C$.

5. RMS value of sine wave AC is $0.707 I_0$, but can be different for different types of AC's.

6. $P = I_v E_v \cos\phi = 0$

7. $Z = \sqrt{R^2 + (X_L - X_C)^2}$

8. $P = \frac{V_0 I_0}{2}$ [V_0 and I_0 are peak voltage and current through resistor only]

9. $$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = 170\text{ V}$$
$$f = \frac{\omega}{2\pi} = \frac{120}{2 \times 3.14} \approx 19\text{ Hz}$$

10. Current is maximum at
$$\omega = \omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5 \times 8 \times 10^{-6}}}$$
$$= 500\text{ rad/s.}$$

11. $$P = \frac{I_0 E_0 \cos\phi}{2}$$
$$= \frac{100 \times 100}{2} \cos\frac{\pi}{3} \times 10^{-3} = 2.5\text{ W}$$

12. $X_C = \frac{1}{\omega C} = \infty$ if $\omega = 0$, *i.e.*, for DC

13. $V = 10\cos 100\pi t$
at $t = \frac{1}{600}$ s,
$$V = 10\cos 100\pi \frac{1}{600}$$
$$= 10\cos\frac{\pi}{6} = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}\text{ V}$$

14. For purely resistive circuit $\phi = 0$.

15. $X_C = \frac{1}{\omega C} \Rightarrow X_C \propto \frac{1}{\omega}$ or $X_C \propto \frac{1}{f}$

16. $$\sin\phi = \frac{X}{Z} = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \quad \phi = \sin^{-1}\left[\frac{1}{\sqrt{3}}\right]$$

17. $\phi = \frac{3\pi}{2}$, $P = \frac{I_0 E_0}{2}\cos\phi = 0$

18. $$R = \frac{V_{\text{DC}}}{I_{\text{DC}}} = 100\,\Omega$$
$$Z = \frac{V_{\text{AC}}}{I_{\text{AC}}} = \frac{100}{0.5} = 200\,\Omega$$
$$X_L = \sqrt{Z^2 - R^2} = 100\sqrt{3}\,\Omega$$
$$L = \frac{X_L}{\omega} = \frac{X_L}{2\pi f} = \frac{100\sqrt{3}}{2\pi \times 50}$$
$$= \left(\frac{\sqrt{3}}{\pi}\right)\text{H}$$

19. $$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \omega C V_{\text{rms}}$$
$$= 100 \times 1 \times 10^{-6} \times \frac{200\sqrt{2}}{\sqrt{2}}$$
$$I_{\text{rms}} = 20\text{ mA}$$

20. $$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(20)^2 + (15)^2}$$
$$= 25\text{ V}, V_0 = 25\sqrt{2}\text{ V}$$

21. $$P = \frac{I_0 V_0 \cos\phi}{2} = 0$$
$$\Rightarrow \quad \cos\phi = 0$$
$$\Rightarrow \quad \phi = 90°$$

22. R is independent of frequency.

23. L is very high so that circuit consumes less power.

24. $\tan\phi = \frac{X_L}{R} \Rightarrow \tan 45° = \frac{X_L}{100}$

$\Rightarrow \quad X_L = 100\,\Omega$

$\omega L = 100\,\Omega$

$L = \frac{100}{2 \times 3.14 \times 10^3} \approx 16\,\text{mH}$

25. The minimum time taken by it in reaching from zero to peak value $= \frac{T}{4}$

$= \frac{1}{4f} = \frac{1}{4 \times 50} = \frac{1}{200} = 5\,\text{ms}$

26. $\phi = 60°$

$$P = \frac{I_0 V_0 \cos\phi}{2} = \frac{4 \times 220 \times \frac{1}{2}}{2}$$

$= 220\,\text{W}$

JEE Corner

Assertion and Reasons

1. X_C and X_L can be greater than Z because

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Hence, $V_C = IX_C$ and $V_L = IX_L$ can be greater than $V = IZ$.

2. At resonance $X_L = X_C$, with further increase in frequency, X_L increases but X_C decreases hence voltage will lead current.

3. $f_r = \frac{1}{2\pi\sqrt{LC}}$, if dielectric slab is inserted between the plates of the capacitor, its capacitance will increase, hence, f_r will decrease.

4. $q =$ Area under graph

$= \frac{1}{2} \times 4 \times (2 + 3) + \frac{1}{2} \times 4 \times (2 + 4)$

$= 22\,\text{C}$

Average current $= \frac{q}{t} = \frac{22}{6} = 3.6\,\text{A}$

5. On inserting ferromagnetic rod inside the inductor, X_L and hence V_L increase. Due to this current will increase if it is lagging and *vice-versa*.

6. $V_R = V_L = V_C \Rightarrow R = X_L = X_C$

Hence, $\phi = 0$ and I is maximum.

as $Z = \sqrt{R^2 + (X_L - X_C)^2}$ is minimum.

7. $I = I_L - I_C = 0$

8. $P = I_{\text{rms}}^2 R = (\sqrt{2})^2 \times 10 = 20\,\text{W}$

9. Inductor coil resists varying current.

10. $I_0 = \frac{E_0}{\sqrt{R^2 + \omega^2 L^2}}$, $\phi = \tan^{-1}\frac{\omega L}{R}$

11. At resonance, current and voltage are in same phase and $I_0 = \frac{V_0}{R}$. Hence, I_0 depends on R.

Objective Questions (Level-2)

Single Correct Options

1. For parallel circuit

$$\phi = \tan^{-1}\left[\frac{1/X_L}{1/R}\right] = \tan^{-1}\frac{4}{3}$$

$= 53°$

2. Current will remain same in series circuit given by

$I = I_0 \sin(\omega t - \phi)$

$= I_0 \sin\left(\omega t - \tan^{-1}\frac{X_L}{R}\right)$

3. $R = R_1 + R_L = 10\,\Omega$

$X_L = \omega L = 10\,\Omega$,

$X_C = \frac{1}{\omega C} = 10\,\Omega$

Reading of ammeter

$$I_{rms} = \frac{V_{rms}}{R} = \frac{10\sqrt{2}}{10}$$

$$= \sqrt{2}\text{ A} = 1.4\text{ A}$$

Reading of voltmeter,

$$V = I_{rms}R_L = 5.6\text{ V}$$

4. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 5 \times 10^3 \times \frac{1}{\pi} \times 10^{-6}}$

$$= 100\,\Omega$$

$$I_R = \frac{V}{R} = \frac{200}{100} = 2\text{ A}$$

$$I_C = \frac{V}{X_C} = \frac{200}{100} = 2\text{ A}$$

[Question is wrong. It should be choose the correct statement].

5. Let $i = i_1 + i_2$

where, $i_1 = 5$ A, $i_2 = 5\sin 100\,\omega t$ A

Average value of $i_1 = 5$ A

Average value of $i_2 = 0$

$\therefore$ Average value of $i = 5$ A

Another method

$$i = 5\left[1 + \cos\left(\frac{\pi}{2} + 100\,\omega t\right)\right]$$

$$= 5\left[2\cos^2\left(\frac{\pi}{4} + 50\,\omega t\right)\right]$$

$$= 10\cos^2\left[\frac{\pi}{4} + 50\,\omega t\right]$$

Average value of $\cos^2\left(\frac{\pi}{4} + 50\,\omega t\right) = \frac{1}{2}$

$\therefore$ average value of $i = \frac{10}{2} = 5$ A.

6. As voltage is leading with current, circuit is inductive, and as $\phi = \frac{\pi}{4}, X_L = R$

or $\quad L = \frac{R}{\omega} = \frac{R}{100}$

7. As $X_C > X_L$ voltage will lag with current.

Again $V = \sqrt{V_R^2 + (V_L - V_C)^2} = 10\text{ V}$

$\therefore \quad V < V_C$

and $\quad \cos\phi = \frac{R}{Z} = \frac{V_R}{V} = \frac{4}{5}$

Hence, a, b and c are wrong.

8. For parallel RLC circuit,

$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\Rightarrow \quad I = \sqrt{\left(\frac{V_0}{R}\right)^2 + \left(\frac{V_0}{X_C} - \frac{V_0}{X_L}\right)^2}$$

$$= V_0\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

9. $V = \sqrt{V_L^2 + V_R^2} = 72.8\text{ V}$

$$\phi = \tan^{-1}\frac{V_L}{V_R} = \tan^{-1}\left(\frac{2}{7}\right)$$

10. Clearly P is capacitor and Q is resistor, as, $V_P = V_Q$, $X_C = R$.

$\therefore$ When connected in series,

$$Z = \sqrt{X_C^2 + R^2} = \sqrt{2}R$$

and $\phi = \frac{\pi}{4}$, leading.

$\therefore I = \frac{1}{4\sqrt{2}}$ A, leading in phase by $\frac{\pi}{4}$.

11. $I = \sqrt{I_R^2 + (I_C - I_L)^2}$

Here, $I_C < I$ or $I_L > I$

12. $I = I_L - I_C = 0.2$ A

13. For a pure inductor voltage leads with current by $\frac{\pi}{2}$.

14. $V_R = IR = 220\text{V}$

Hence it is condition of resonance, i.e.,

$$V_L = V_C = 200\text{ V}$$

15. $\frac{H_1}{H_2} = \frac{I_{DC}^2 R}{I_{rms}^2 R} = \frac{I^2}{(I/\sqrt{2})^2} = 2$

16. $H = I_{rms}^2 R = \frac{I_0^2 R}{2} = \frac{V_0^2 R}{2(R^2 + \omega^2 L^2)}$

17. $V_L = IX_L = I\omega L$

$$V_C = IX_C = \frac{I}{\omega C}$$

If ω is very small,

$$V_L \approx = 0, V_C \approx V_0\,.$$

18. Resistance of coil, $R = \frac{V}{I} = 4\,\Omega$

When connected to battery

$$I = \frac{V}{R + r} = \frac{12}{4 + 4} = 1.5\text{ A}$$

19. $V_R = \sqrt{V^2 - V_C^2} = 6\text{ V}$

$$\phi = \tan^{-1}\frac{V_C}{V_R} = \tan^{-1}\frac{4}{3}$$

20. $V_C = \sqrt{V^2 - V_R^2} = 16\text{ V}$

21. $I = I_0 \sin\left(\frac{\pi}{2}t + \pi\right)$

$I = I_0$ at $\frac{\pi}{2} + \pi = \frac{3\pi}{2}$

$t = 1\text{ s}$

22. $I_0 = \frac{V_0}{\sqrt{2}R}$

$X_C' = \frac{\sqrt{3}}{\omega C} = \sqrt{3}R$

$\Rightarrow \quad I_0 = \frac{V_0}{2R} = \frac{I_0}{\sqrt{2}}$

23. $R = \frac{V_{DC}}{I_{DC}} = \frac{12}{4} = 3\ \Omega$

24. $X_L = \sqrt{Z_1^2 - R^2} = \sqrt{(5)^2 - (3)^2}$

$= 4\ \Omega$

$X_C = \frac{1}{\omega C}$

$= \frac{1}{50 \times 2500 \times 10^{-6}} = 8\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = 5\ \Omega$

Average power $= I_{rms}^2 R = \frac{V_{rms}^2 R}{Z^2}$

$= \frac{(12)^2 \times 3}{(5)^2} = 17.28\text{ W}$

25. Already $X_C > X_L$, with increase in ω, X_C further decrease in ω, X_C increases and X_L decreases, hence, I will decrease.

26. For maximum current

$\omega = \omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 10^{-6} \times 4.9 \times 10^{-3}}}$

$= \frac{10^5}{7}$ rad/s.

27. In resonance,

$Z = \sqrt{R_P^2 + X_C^2} \approx 77\ \Omega$

28. In resonance, $\cos\phi = 1$.

More than One Correct Answers

1. $V_R^2 + V_L^2 = 10000$ ...(i)

$V_L - V_C = 120$ ...(ii)

$V_R^2 + (V_L - V_C)^2 = (130)^2 = 16900$ ...(iii)

On solving

$V_r = 50\text{ V}, V_L = 86.6\text{ V}, V_C = 206.6\text{ V}$

and $\cos\phi = \frac{V_R}{V} = \frac{50}{130} = \frac{5}{15}$

As $V_C > V_L$, circuit is capacitive in nature.

2. $i = 3\sin\omega t + 4\cos\omega t$

$= R\sin(\omega t + \phi)$

$R = 5$ and $\phi = \tan^{-1}\frac{4}{3}$

$i_m = \frac{2i_0}{\pi} = \frac{2R}{\pi} = \frac{10}{\pi}$

If $V = V_m \sin\omega t$

current will lead with the voltage.

If $V = V_m \cos\omega t$

current will lag with voltage.

3. $I = \frac{P}{V} = 1\text{ A}, R = \frac{V}{I} = 60\ \Omega$

For AC,

$Z = \frac{100}{1} = 100\ \Omega$

X_C or $X_L = \sqrt{Z^2 - R^2} = 80\ \Omega$

$L = \frac{X_L}{\omega} = \frac{80}{2\pi \times 50} = \frac{4}{5\pi}\text{ H}$

or $C = \frac{1}{\omega X_C} = \frac{1}{2\pi \times 50 \times 80} = \frac{125}{\pi}\ \mu\text{F}$

or $R' + R = \frac{V}{I}$

$\Rightarrow \quad R' = 100 - 60 = 40\ \Omega$

4. $\cos\phi = \frac{R}{Z} = \begin{cases} 1 & \text{if } R = Z \\ 0 & \text{if } R = 0 \end{cases}$

5. As $X_L > X_C$, voltage will lead with the current.

$Z = \sqrt{R^2 + (X_L - X_C)^2} = 10\sqrt{2}\ \Omega$

$$\phi = \tan^{-1}\frac{X_L - X_C}{R} = \frac{\pi}{4} = 45°$$

$$\cos\phi = \frac{R}{Z} = \frac{1}{\sqrt{2}}$$

6. As $X_L > X_C$, $\omega > \omega_r$

with increase in ω, X_L and hence, Z will increase while with decrease in ω, Z will first decrease and then increase.

7. $X_c = \frac{V_C}{I} = 50\,\Omega$

$$V_R = IR = 80\text{ V}$$

$$V_L = IX_L = 40\text{ V}$$

$$V_{\text{rms}} = \sqrt{V_R^2 + (V_L - V_C)^2} = 100\text{ V},$$

$$V_0 = 100\sqrt{2}\text{ V}$$

8. $I = \dfrac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

with change in L or C I may decrease or increase depending on effect on $\omega L - \frac{1}{\omega C}$.

Match the Columns

1. (a) → (p, r), (b) → (q, r), (c → s), (d) → (p)

Concept based insertion.

2. (a) → (p, s) current and voltage are in same phase so either $X_C = 0$, $X_L = 0$

or $X_C = X_L \neq 0$.

(b) → (q)

$$I = -I_0 \cos\omega t$$

$$= I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\phi = 90° \Rightarrow R = 0$$

(c) → (r, s) current is leading with voltage by $\frac{\pi}{6}$, either $X_L = 0$ or $X_C > X_L$

but X_C and R are non-zero.

(d) → (s) current lags with voltage by $\frac{\pi}{6}$, R and X_L are both non-zero.

3. (a) → (q, s), (b) → (r, s), (c) → (r, s), (d) → (r, s).

$$I = \frac{V}{Z} \text{ and } P = \frac{V^2 r}{Z^2}$$

with increase in L, C or f, Z may increase or decrease, hence power and current.

4. (a → q),

$$R = \frac{V_R}{I} = \frac{40}{2} = 20\,\Omega$$

(b → p)

$$V_C = IX_C = 2 \times 30 = 60\text{ V}$$

(c → r)

$$V_L = IX_C = 2 \times 15 = 30\text{ V}$$

(d → s)

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= 50\text{ V}$$

5. (a → s) R is independent of f.

(b → p) $X_C \propto \frac{1}{f}$

(c → r) $X_L \propto \frac{1}{f}$

(d → q)

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

i.e., first decreases then increases.