CHAPTER 1 SOLUTION FOR PROBLEM 1

Use the given conversion factors.

(a) The distance d in rods is

$$d = 4.0\,\text{furlongs} = \frac{(4.0\,\text{furlongs})(201.168\,\text{m/furlong})}{5.0292\,\text{m/rod}} = 160\,\text{rods}\,.$$

(b) The distance in chains is

$$d = 4.0\,\text{furlongs} = \frac{(4.0\,\text{furlongs})(201.168\,\text{m/furlong})}{20.17\,\text{m/chain}} = 40\,\text{chains}\,.$$

(a) Let ρ be the mass per unit volume of iron. It is the same for a single atom and a large chunk. If M is the mass and V is the volume of an atom, then $\rho = M/V$, or $V = M/\rho$. To obtain the volume in m^3, first convert ρ to kg/m^3: $\rho = (7.87\,\mathrm{g/cm^3})(10^{-3}\,\mathrm{kg/g})(10^6\,\mathrm{cm^3/m^3}) = 7.87 \times 10^3\,\mathrm{kg/m^3}$. Then

$$V = \frac{M}{\rho} = \frac{9.27 \times 10^{-26}\,\mathrm{kg}}{7.87 \times 10^3\,\mathrm{kg/m^3}} = 1.18 \times 10^{-29}\,\mathrm{m^3}\,.$$

(b) Set $V = 4\pi R^3/3$, where R is the radius of an atom, and solve for R:

$$R = \left(\frac{3V}{4\pi}\right)^{1/3} = \left[\frac{3(1.18 \times 10^{-29}\,\mathrm{m^3})}{4\pi}\right]^{1/3} = 1.41 \times 10^{-10}\,\mathrm{m}\,.$$

The center-to-center distance between atoms is twice the radius or 2.82×10^{-10} m.

CHAPTER 1 HINT FOR PROBLEM 3

(a) Multiply 1.0 km by the number of meters per kilometer (1.0×10^3 m/km) and the number of microns per meter ($1.0 \times 10^6\ \mu$/m).

(b) Multiply 1.0 km by the number of meters per kilometer, by the number of feet per meter (3.218), and by the number of yards per foot (0.333).

[ans: (a) $10^9\ \mu$m; (b) 10^{-4}; (c) $9.1 \times 10^5\ \mu$m]

CHAPTER 1 HINT FOR PROBLEM 9

The volume of water is the area of the land times the depth of the water if it does not seep into the ground. You must convert $26\,\text{km}^2$ to acres and 2.0 in. to feet. For the first conversion use $1\,\text{km}^2 = 1 \times 10^6\,\text{m}^2$, $1\,\text{m}^2 = (3.281\,\text{ft})^2$, and $1\,\text{acre} = 43\,560\,\text{ft}^2$. See Appendix D.

[ans: 1.1×10^3 acre-fteet]

(a) The ratio of the interval on clock B to the interval on clock A (600 s) is the same as the ratio of the interval between events 2 and 4 on clock B to the same interval on clock A.
(b) The ratio of the interval on clock C to the interval on clock B is the same as the ratio of the interval between events 1 and 3 on clock C to the same interval on clock B.
(c) Measure the time of the event (when clock A reads 400 s) from the time of event 2 on the diagram. The interval is 400 s − 312 s on clock A. Calculate the interval on clock B and add 125 s.
(d) Measure the time of the event (when clock C reads 15.0 s) from the time of event 1 on the diagram. The interval is 15.0 s − 92 s on clock C. Calculate the interval on clock B and add 25 s.

[ans: (a) 495 s; (b) 141 s; (c) 198 s; (d) −245 s]

CHAPTER 2 SOLUTION FOR PROBLEM 3

(a) The average velocity during any time interval is the displacement during that interval divided by the interval: $v_{avg} = \Delta x/\Delta t$, where Δx is the displacement and Δt is the time interval. In this case the interval is divided into two parts. During the first part the displacement is $\Delta x_1 = 40\,\mathrm{km}$ and the time interval is

$$\Delta t_1 = \frac{(40\,\mathrm{km})}{(30\,\mathrm{km/h})} = 1.33\,\mathrm{h}\,.$$

During the second part the displacement is $\Delta x_2 = 40\,\mathrm{km}$ and the time interval is

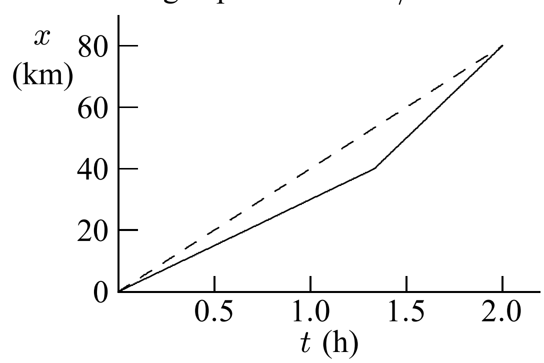
$$\Delta t_2 = \frac{(40\,\mathrm{km})}{(60\,\mathrm{km/h})} = 0.67\,\mathrm{h}\,.$$

Both displacements are in the same direction, so the total displacement is $\Delta x = \Delta x_1 + \Delta x_2 = 40\,\mathrm{km} + 40\,\mathrm{km} = 80\,\mathrm{km}$. The total time interval is $\Delta t = \Delta t_1 + \Delta t_2 = 1.33\,\mathrm{h} + 0.67\,\mathrm{h} = 2.00\,\mathrm{h}$. The average velocity is

$$v_{avg} = \frac{(80\,\mathrm{km})}{(2.0\,\mathrm{h})} = 40\,\mathrm{km/h}\,.$$

(b) The average speed is the total distance traveled divided by the time. In this case the total distance is the magnitude of the total displacement, so the average speed is $40\,\mathrm{km/h}$.

(c) Assume the automobile passes the origin at time $t = 0$. Then its coordinate as a function of time is as shown as the solid lines on the graph to the right. The average velocity is the slope of the dotted line.



(a) Since the unit of ct^2 is that of length and the unit of t is that of time, the unit of c must be that of (length)/(time)^2, or m/s^2.

(b) Since bt^3 has a unit of length, b must have a unit of (length)/(time)^3, or m/s^3.

(c) When the particle reaches its maximum (or its minimum) coordinate its velocity is zero. Since the velocity is given by $v = \mathrm{d}x/\mathrm{d}t = 2ct - 3bt^2$, $v = 0$ occurs for $t = 0$ and for

$$t = \frac{2c}{3b} = \frac{2(3.0\,\mathrm{m/s^2})}{3(2.0\,\mathrm{m/s^3})} = 1.0\,\mathrm{s}\,.$$

For $t = 0$, $x = 0$ and for $t = 1.0\,\mathrm{s}$, $x = 1.0\,\mathrm{m}$. Reject the first solution and accept the second.

(d) In the first 4.0 s the particle moves from the origin to $x = 1.0\,\mathrm{m}$, turns around, and goes back to $x(4\,\mathrm{s}) = (3.0\,\mathrm{m/s^2})(4.0\,\mathrm{s})^2 - (2.0\,\mathrm{m/s^3})(4.0\,\mathrm{s})^3 = -80\,\mathrm{m}$. The total path length it travels is $1.0\,\mathrm{m} + 1.0\,\mathrm{m} + 80\,\mathrm{m} = 82\,\mathrm{m}$.

(e) Its displacement is given by $\Delta x = x_2 - x_1$, where $x_1 = 0$ and $x_2 = -80\,\mathrm{m}$. Thus $\Delta x = -80\,\mathrm{m}$.

The velocity is given by $v = 2ct - 3bt^2 = (6.0\,\mathrm{m/s^2})t - (6.0\,\mathrm{m/s^3})t^2$. Thus

$$\begin{aligned}
&\text{(f)} \quad v(1\,\mathrm{s}) = (6.0\,\mathrm{m/s^2})(1.0\,\mathrm{s}) - (6.0\,\mathrm{m/s^3})(1.0\,\mathrm{s})^2 = 0\\
&\text{(g)} \quad v(2\,\mathrm{s}) = (6.0\,\mathrm{m/s^2})(2.0\,\mathrm{s}) - (6.0\,\mathrm{m/s^3})(2.0\,\mathrm{s})^2 = -12\,\mathrm{m/s}\\
&\text{(h)} \quad v(3\,\mathrm{s}) = (6.0\,\mathrm{m/s^2})(3.0\,\mathrm{s}) - (6.0\,\mathrm{m/s^3})(3.0\,\mathrm{s})^2 = -36.0\,\mathrm{m/s}\\
&\text{(i)} \quad v(4\,\mathrm{s}) = (6.0\,\mathrm{m/s^2})(4.0\,\mathrm{s}) - (6.0\,\mathrm{m/s^3})(4.0\,\mathrm{s})^2 = -72\,\mathrm{m/s}\,.
\end{aligned}$$

The acceleration is given by $a = \mathrm{d}v/\mathrm{d}t = 2c - 6b = 6.0\,\mathrm{m/s^2} - (12.0\,\mathrm{m/s^3})t$. Thus

$$\begin{aligned}
&\text{(j)} \quad a(1\,\mathrm{s}) = 6.0\,\mathrm{m/s^2} - (12.0\,\mathrm{m/s^3})(1.0\,\mathrm{s}) = -6.0\,\mathrm{m/s^2}\\
&\text{(k)} \quad a(2\,\mathrm{s}) = 6.0\,\mathrm{m/s^2} - (12.0\,\mathrm{m/s^3})(2.0\,\mathrm{s}) = -18\,\mathrm{m/s^2}\\
&\text{(l)} \quad a(3\,\mathrm{s}) = 6.0\,\mathrm{m/s^2} - (12.0\,\mathrm{m/s^3})(3.0\,\mathrm{s}) = -30\,\mathrm{m/s^2}\\
&\text{(m)} \quad a(4\,\mathrm{s}) = 6.0\,\mathrm{m/s^2} - (12.0\,\mathrm{m/s^3})(4.0\,\mathrm{s}) = -42\,\mathrm{m/s^2}\,.
\end{aligned}$$

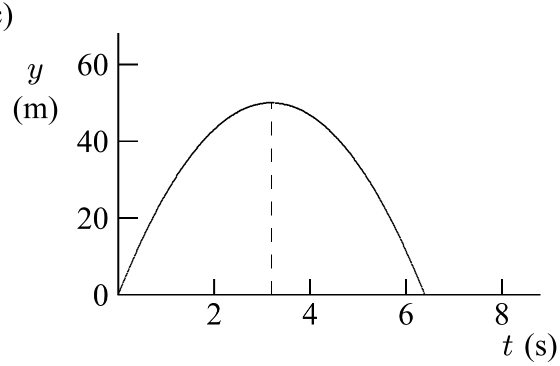
(a) At the highest point the velocity of the ball is instantaneously zero. Take the y axis to be upward, set $v = 0$ in $v^2 = v_0^2 - 2gy$, and solve for v_0: $v_0 = \sqrt{2gy}$. Substitute $g = 9.8\,\mathrm{m/s^2}$ and $y = 50\,\mathrm{m}$ to get

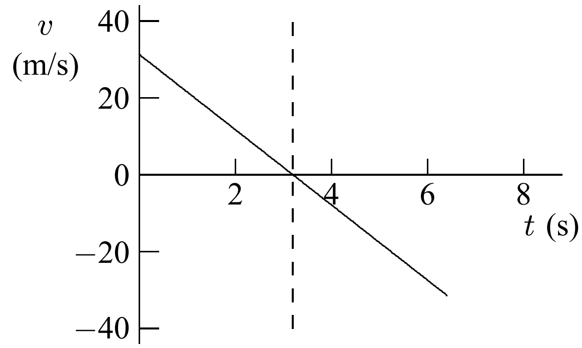
$$v_0 = \sqrt{2(9.8\,\mathrm{m/s^2})(50\,\mathrm{m})} = 31\,\mathrm{m/s}\,.$$

(b) It will be in the air until $y = 0$ again. Solve $y = v_0 t - \frac{1}{2}gt^2$ for t. Since $y = 0$ the two solutions are $t = 0$ and $t = 2v_0/g$. Reject the first and accept the second:

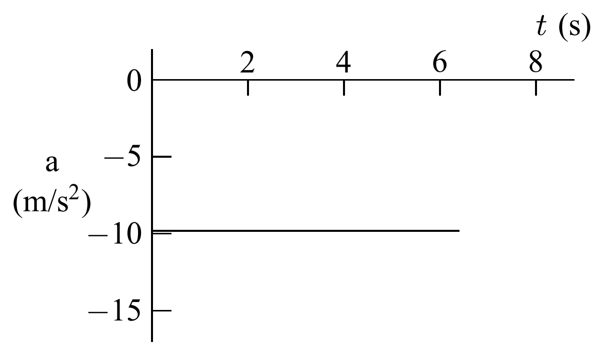
$$t = \frac{2v_0}{g} = \frac{2(31\,\mathrm{m/s})}{9.8\,\mathrm{m/s^2}} = 6.4\,\mathrm{s}\,.$$

(c)





The acceleration is constant while the ball is in flight: $a = -9.8\,\mathrm{m/s^2}$. Its graph is as shown on the right.



CHAPTER 2 HINT FOR PROBLEM 13

(a) Use $v_{\text{avg}} = \Delta x/\Delta t$. To calculate the particle's coordinate at the beginning of the interval substitute $t = 2.00\,\text{s}$ into the equation for x and to calculate its coordinate at the end of the interval substitute $t = 3.00\,\text{s}$. Δx, of course, is the difference and Δt is $1.00\,\text{s}$
(b) Differentiate the expression for the coordinate with respect to time and evaluate the result for $t = 2.00\,\text{s}$.
(c) Do the same for $t = 3.00\,\text{s}$.
(d) Do the same for $t = 2.5\,\text{s}$.
(e) You must find the time when the particle is midway between the two positions. Find the midway point x_m by taking the average of the two coordinates found in part (a), then solve $x_m = 9.75 + 1.50t^3$ for t. Finally, substitute this value for t into the expression for the velocity as a function of time (the derivative of the coordinate).

[ans: (a) 28.5 cm/s; (b) 18.0 cm/s; (c) 40.5 cm/s; (d) 28.1 cm/s; (e) 30.3 cm/s]

CHAPTER 2 HINT FOR PROBLEM 21

Use $v = v_0 + at$, where v is the electron's velocity at any time t, v_0 is its velocity at time $t = 0$, and a is its acceleration. Let $t = 0$ at the instant the electron's velocity is $+9.6\,\mathrm{m/s}$ and evaluate the expression for (a) $t = -2.5\,\mathrm{s}$ and for $t = +2.5\,\mathrm{s}$.

$\Big[$ ans: (a) $+1.6\,\mathrm{m/s}$; (b) $+18\,\mathrm{m/s}$ $\Big]$

Put the origin of an x axis at the position of train A when it starts slowing and suppose the train has velocity v_{A0} (= +40 m/s from the graph) at that time. Its velocity as a function of time is given by $v_{A0} + a_A t$, where a_A is its acceleration. This is the slope of the upper line on the graph and is negative. Solve for the time when train A stops and use $x_A = v_{A0}t + \frac{1}{2}a_A t^2$ to find its position when it stops.

The velocity of train B is given by $v_B = v_{B0} + a_B t$, where v_{B0} is its velocity at $t = 0$ (-30 m/s from the graph), and a_B is its acceleration. This is the slope of the lower line on the graph and is positive. Solve for the time when train B stops and use $x_B = x_{B0} + v_{B0}t + \frac{1}{2}a_B t^2$ to find its position when it stops. Here $x_{B0} = 200$ m. The separation of the trains when both have stopped is the difference of the coordinates you have found.

[ans: 40 m]

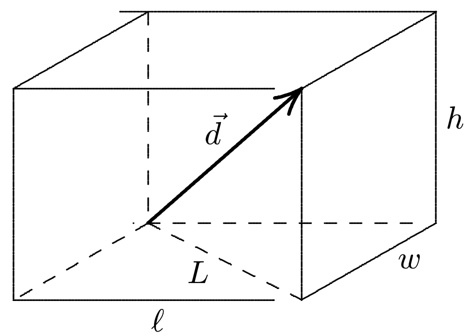
Divide the falling of the ball into two segments: from the top of the building to the top of the window and from the top of the window to the sidewalk. You need to find the lengths of each of these segments.

. You need to know the velocity of the ball as it passes the top of the window going down. Take the origin of a coordinate system to be at the top of the window and suppose the downward direction is positive. Suppose further that the velocity of the ball is v_0 when falls past that point. If h_w is the top-to-bottom dimension of the window, then $h = v_0 t + \frac{1}{2} g t^2$, where t is the time to pass the window. Solve for v_0.

To find the length of the first segment solve for the distance the ball must fall to achieve a velocity of v_0. To find the length of the second segment solve for the distance the ball falls in 1.125 s, starting with a downward velocity of v_0. The time here is the time for the ball to pass the window plus the time for it to fall from the bottom of the window to the sidewalk.

[ans: 20.4 m]

(a) The magnitude of the displacement is the distance from one corner to the diametrically opposite corner: $d = \sqrt{(3.00\,\text{m})^2 + (3.70\,\text{m})^2 + (4.30\,\text{m})^2} = 6.42\,\text{m}$. To see this, look at the diagram of the room, with the displacement vector shown. The length of the diagonal across the floor, under the displacement vector, is given by the Pythagorean theorem: $L = \sqrt{\ell^2 + w^2}$, where ℓ is the length and w is the width of the room. Now this diagonal and the room height form a right triangle with the displacement vector as the hypotenuse, so the length of the displacement vector is given by

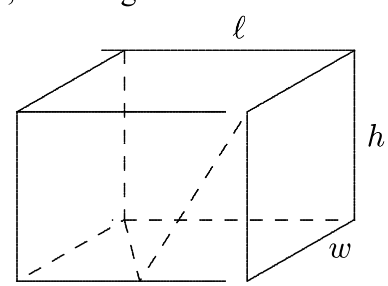


$$d = \sqrt{L^2 + h^2} = \sqrt{\ell^2 + w^2 + h^2}\,.$$

(b), (c), and (d) The displacement vector is along the straight line from the beginning to the end point of the trip. Since a straight line is the shortest distance between two points the length of the path cannot be less than the magnitude of the displacement. It can be greater, however. The fly might, for example, crawl along the edges of the room. Its displacement would be the same but the path length would be $\ell + w + h$. The path length is the same as the magnitude of the displacement if the fly flies along the displacement vector.

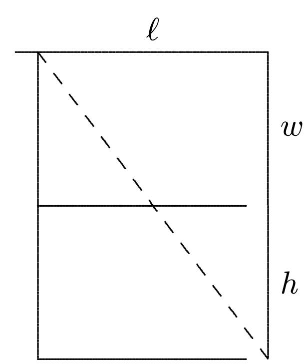
(e) Take the x axis to be out of the page, the y axis to be to the right, and the z axis to be upward. Then the x component of the displacement is $w = 3.70\,\text{m}$, the y component of the displacement is $4.30\,\text{m}$, and the z component is $3.00\,\text{m}$. Thus $\vec{d} = (3.70\,\text{m})\,\hat{\imath} + (4.30\,\text{m})\,\hat{\jmath} + (3.00\,\text{m})\,\hat{k}$. You may write an equally correct answer by interchanging the length, width, and height.

(f) Suppose the path of the fly is as shown by the dotted lines on the upper diagram. Pretend there is a hinge where the front wall of the room joins the floor and lay the wall down as shown on the lower diagram. The shortest walking distance between the lower left back of the room and the upper right front corner is the dotted straight line shown on the diagram. Its length is



$$L_{\text{min}} = \sqrt{(w + h)^2 + \ell^2}$$

$$= \sqrt{(3.70\,\text{m} + 3.00\,\text{m})^2 + (4.30\,\text{m})^2} = 7.96\,\text{m}\,.$$



(a) and (b) The vector $\vec{a}$ has a magnitude 10.0 m and makes the angle 30° with the positive x axis, so its components are $a_x = (10.0\,\text{m})\cos 30° = 8.67\,\text{m}$ and $a_y = (10.0\,\text{m})\sin 30° = 5.00\,\text{m}$. The vector $\vec{b}$ has a magnitude of 10.0 m and makes an angle of 135° with the positive x axis, so its components are $b_x = (10.0\,\text{m})\cos 135° = -7.07\,\text{m}$ and $b_y = (10.0\,\text{m})\sin 135° = 7.07\,\text{m}$. The components of the sum are $r_x = a_x + b_x = 8.67\,\text{m} - 7.07\,\text{m} = 1.60\,\text{m}$ and $r_y = a_y + b_y = 5.0\,\text{m} + 7.07\,\text{m} = 12.1\,\text{m}$.

(c) The magnitude of $\vec{r}$ is $r = \sqrt{r_x^2 + r_y^2} = \sqrt{(1.60\,\text{m})^2 + (12.1\,\text{m})^2} = 12.2\,\text{m}$.

(d) The tangent of the angle θ between $\vec{r}$ and the positive x axis is given by $\tan\theta = r_y/r_x = (12.1\,\text{m})/(1.60\,\text{m}) = 7.56$. θ is either 82.5° or 262.5°. The first angle has a positive cosine and a positive sine and so is the correct answer.

Since $ab\cos\phi = a_x b_x + a_y b_y + a_z b_z$,

$$\cos\phi = \frac{a_x b_x + a_y b_y + a_z b_z}{ab}\,.$$

The magnitudes of the vectors given in the problem are $a = \sqrt{(3.0)^2 + (3.0)^2 + (3.0)^2} = 5.2$ and $b = \sqrt{(2.0)^2 + (1.0)^2 + (3.0)^2} = 3.7$. The angle between them is found from

$$\cos\phi = \frac{(3.0)(2.0) + (3.0)(1.0) + (3.0)(3.0)}{(5.2)(3.7)} = 0.926$$

and the angle is $\phi = 22^\circ$.

Use a coordinate system with its origin at the original position of the ship, its x axis positive to the east, and its y axis positive to the north. Let $\vec{d}_g$ (= (120 km) $\hat{\jmath}$) be the vector from the ship to its goal, $\vec{d}_w$ (= (100 km) $\hat{\imath}$) be the vector to the point where the wind blows the ship, and $\vec{d}$ be the vector describing the required displacement. The vector equation is $\vec{d}_g = \vec{d}_w + \vec{d}$. Solve for $\vec{d}$. The distance it must sail is the magnitude of $\vec{d}$ and you might describe the direction it must sail by calculating the angle $\vec{d}$ makes with north or with east.

[ans: (a) 156 km; (b) 39.8° west of north]

CHAPTER 3 HINT FOR PROBLEM 21

In each case vectorially add the displacements for the three moves, then calculate the magnitude and direction of the result. You might place an x axis parallel to the forward direction and a y axis left to right. Each displacement is then 1 m or 2 m along one of these axes. Once you have the total displacement calculate its magnitude by taking the square root of the sum of the squares of its components. If you use the coordinate system suggested above, the tangent of the angle it makes with the forward direction is its y component divided by its x component. Sketch the displacement to be sure you get the correct value when you evaluate the inverse tangent.

[ans: All displacements have a magnitude of 2.24 m. The angles are: (1) 26.6° right of forward; (2) 63.4° left of forward; (3) 26.6° left of forward]

(a) Use $\vec{a} \cdot \vec{b} = ab\cos\phi$. Where the angle ϕ between them is the difference of the angles they make with the positive x axis.

(b) The magnitude of the vector product is $ab\sin\phi$. Use the right-hand rule to find the direction.

[ans: (a) -18.8; (b) 26.9, in the positive z direction]

CHAPTER 3 HINT FOR PROBLEM 33

Use $\vec{a} \cdot \vec{b} = ab\cos\phi$, where ϕ is the angle between the vectors when they are drawn with their tails at the same point. Solve for $\cos\phi$, then take the inverse.

$\left[\text{ans: } 70.5^\circ\right]$

(a) Take the y axis to be upward and the x axis to be horizontal. Place the origin at the point where the diver leaves the platform. The components of the diver's initial velocity are $v_{0x} = 3.00\,\mathrm{m/s}$ and $v_{0y} = 0$. At $t = 0.800\,\mathrm{s}$ the horizontal distance of the diver from the platform is $x = v_{0x}t = (2.00\,\mathrm{m/s})(0.800\,\mathrm{s}) = 1.60\,\mathrm{m}$.

(b) The driver's y coordinate is $y = -\frac{1}{2}gt^2 = -\frac{1}{2}(9.8\,\mathrm{m/s^2})(0.800\,\mathrm{s})^2 = -3.13\,\mathrm{m}$. The distance above the water surface is $10.0\,\mathrm{m} - 3.13\,\mathrm{m} = 6.86\,\mathrm{m}$.

(c) The driver strikes the water when $y = -10.0\,\mathrm{m}$. The time he strikes is

$$t = \sqrt{-\frac{2y}{g}} = \sqrt{-\frac{2(-10.0\,\mathrm{m})}{9.8\,\mathrm{m/s^2}}} = 1.43\,\mathrm{s}$$

and the horizontal distance from the platform is $x = v_{0x}t = (2.00\,\mathrm{m/s})(1.43\,\mathrm{s}) = 2.86\,\mathrm{m}$.

You want to know how high the ball is from the ground when its horizontal distance from home plate is 97.5 m. To calculate this quantity you need to know the components of the initial velocity of the ball. Use the range information. Put the origin at the point where the ball is hit, take the y axis to be upward and the x axis to be horizontal. If x (= 107 m) and y (= 0) are the coordinates of the ball when it lands, then $x = v_{0x}t$ and $0 = v_{0y}t - \frac{1}{2}gt^2$, where t is the time of flight of the ball. The second equation gives $t = 2v_{0y}/g$ and this is substituted into the first equation. Use $v_{0x} = v_{0y}$, which is true since the initial angle is $\theta_0 = 45°$. The result is $x = 2v_{0y}^2/g$. Thus

$$v_{0y} = \sqrt{\frac{gx}{2}} = \sqrt{\frac{(9.8\,\mathrm{m/s^2})(107\,\mathrm{m})}{2}} = 22.9\,\mathrm{m/s}\,.$$

Now take x and y to be the coordinates when the ball is at the fence. Again $x = v_{0x}t$ and $y = v_{0y}t - \frac{1}{2}gt^2$. The time to reach the fence is given by $t = x/v_{0x} = (97.5\,\mathrm{m})/(22.9\,\mathrm{m/s}) = 4.26\,\mathrm{s}$. When this is substituted into the second equation the result is

$$y = v_{0y}t - \frac{1}{2}gt^2 = (22.9\,\mathrm{m/s})(4.26\,\mathrm{s}) - \frac{1}{2}(9.8\,\mathrm{m/s^2})(4.26\,\mathrm{s})^2 = 8.63\,\mathrm{m}\,.$$

Since the ball started 1.22 m above the ground, it is 8.63 m + 1.22 m = 9.85 m above the ground when it gets to the fence and it is 9.85 m − 7.32 m = 2.53 m above the top of the fence. It goes over the fence.

To calculate the centripetal acceleration of the stone you need to know its speed while it is being whirled around. This the same as its initial speed when it flies off. Use the kinematic equations of projectile motion to find that speed. Take the y axis to be upward and the x axis to be horizontal. Place the origin at the point where the stone leaves its circular orbit and take the time to be zero when this occurs. Then the coordinates of the stone when it is a projectile are given by $x = v_0 t$ and $y = -\frac{1}{2}gt^2$. It hits the ground when $x = 10\,\text{m}$ and $y = -2.0\,\text{m}$. Note that the initial velocity is horizontal. Solve the second equation for the time: $t = \sqrt{-2y/g}$. Substitute this expression into the first equation and solve for v_0:

$$v_0 = x\sqrt{-\frac{g}{2y}} = (10\,\text{m})\sqrt{-\frac{9.8\,\text{m/s}^2}{2(-2.0\,\text{m})}} = 15.7\,\text{m/s}\,.$$

The magnitude of the centripetal acceleration is $a = v^2/r = (15.7\,\text{m/s})^2/(1.5\,\text{m}) = 160\,\text{m/s}^2$.

CHAPTER 4 HINT FOR PROBLEM 13

When the cart reaches its greatest y coordinate the y component of its velocity is zero. Since the acceleration is constant you may write $v_y = v_{0y} + a_y t$, where t is the time. Solve for the time when $v_y = 0$, then substitute this value into the expression for the x component of the velocity: $v_x = v_{0x} + a_x t$.

[ans: (32 m/s $\hat{\imath}$]

Put the origin of a coordinate system at the point where the decoy is released and take the time to zero when it is released. The initial velocity of the decoy is the same as the velocity of the plane.

(a) Solve $x = v_{0x}t$ for the flight time of the decoy.

(b) Evaluate $y = v_{0y}t - \frac{1}{2}gt^2$ for the y coordinate of the landing point. Its magnitude is the height of the plane above the ground when the decoy is released.

[ans: (a) 10.0 s; (b) 897 m]

(a) The ball travels a horizontal distance of 50.0 m in 4.00 s. Calculate the horizontal component of its velocity. This is constant throughout the motion. Before it reaches the top of wall it travels horizontally for 1.00 s. Calculate the horizontal distance it travels from when it is hit to when it reaches the top of the wall. Since the motion is symmetric, the horizontal distance it travels from when it passes the top of the wall on the way down to when it is caught is the same. The total horizontal distance traveled is the sum of these three distances.

(b) and (c) The ball is in flight for 6.00 s. Use $y = y_0 + v_{oy}t - \frac{1}{2}gt^2$ to compute the y component of its initial velocity. Its initial speed is the square root of the sum of the squares of the velocity components and the tangent of the launch angle is the y component divided by the x component.

(d) Use $y = v_{0y}t - \frac{1}{2}gt^2$ to compute the height of the ball 1.00 s after it is hit.

[ans: (a) 75.0 m; (b) 31.9 m/s; (c) 66.9°; (d) 25.5 m]

Let v_r be the man's running speed and v_w be the speed of the moving walk. When running in the direction of travel of the walk the speed of the man relative to the building is $v_r + v_w$ and when running in the opposite direction his speed relative to the building is $v_r - v_w$. The distance run in the first case is given by $t_1(v_r + v_w)$ and in the second by $t_2(v_r - v_w)$, where $t_1 = 2.5\,\mathrm{s}$ and $t_2 = 10.0\,\mathrm{s}$. The two distances are the same. Solve for v_r/v_w.

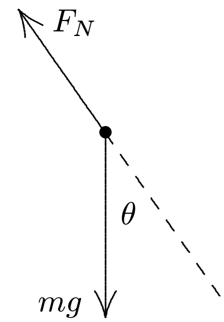
[ans: 5/3]

CHAPTER 5 SOLUTION FOR PROBLEM 13

(a) The free-body diagram is shown in Fig. 5–18 of the text. Since the acceleration of the block is zero, the components of the Newton's second law equation yield $T - mg\sin\theta = 0$ and $F_N - mg\cos\theta = 0$. Solve the first equation for the tension force of the string: $T = mg\sin\theta = (8.5\,\text{kg})(9.8\,\text{m/s}^2)\sin 30^\circ = 42\,\text{N}$.

(b) Solve the second equation for F_N: $F_N = mg\cos\theta = (8.5\,\text{kg})(9.8\,\text{m/s}^2)\cos 30^\circ = 72\,\text{N}$.

(c) When the string is cut it no longer exerts a force on the block and the block accelerates. The x component of the second law becomes $-mg\sin\theta = ma$, so $a = -g\sin\theta = -(9.8\,\text{m/s}^2)\sin 30^\circ = -4.9\,\text{m/s}^2$. The negative sign indicates the acceleration is down the plane.



The free-body diagram is shown at the right. $\vec{F}_N$ is the normal force of the plane on the block and $m\vec{g}$ is the force of gravity on the block. Take the positive x axis to be down the plane, in the direction of the acceleration, and the positive y axis to be in the direction of the normal force. The x component of Newton's second law is then $mg\sin\theta = ma$, so the acceleration is $a = g\sin\theta$.

(a) Place the origin at the bottom of the plane. The equations for motion along the x axis are $x = v_0 t + \frac{1}{2}at^2$ and $v = v_0 + at$. The block stops when $v = 0$.

According to the second equation, this is at the time $t = -v_0/a$. The coordinate when it stops is

$$x = v_0\left(\frac{-v_0}{a}\right) + \frac{1}{2}a\left(\frac{-v_0}{a}\right)^2 = -\frac{1}{2}\frac{v_0^2}{a} = -\frac{1}{2}\frac{v_0^2}{g\sin\theta}$$
$$= -\frac{1}{2}\left[\frac{(-3.50\,\mathrm{m/s})^2}{(9.8\,\mathrm{m/s^2})\sin 32.0^\circ}\right] = -1.18\,\mathrm{m}\,.$$

(b) The time is

$$t = -\frac{v_0}{a} = -\frac{v_0}{g\sin\theta} = -\frac{-3.50\,\mathrm{m/s}}{(9.8\,\mathrm{m/s^2})\sin 32.0^\circ} = 0.674\,\mathrm{s}\,.$$

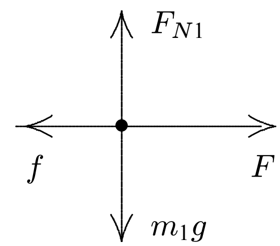
(c) Now set $x = 0$ and solve $x = v_0 t + \frac{1}{2}at^2$ for t. The result is

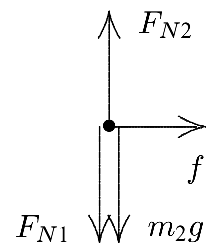
$$t = -\frac{2v_0}{a} = -\frac{2v_0}{g\sin\theta} = -\frac{2(-3.50\,\mathrm{m/s})}{(9.8\,\mathrm{m/s^2})\sin 32.0^\circ} = 1.35\,\mathrm{s}\,.$$

The velocity is

$$v = v_0 + at = v_0 + gt\sin\theta = -3.50\,\mathrm{m/s} + (9.8\,\mathrm{m/s^2})(1.35\,\mathrm{s})\sin 32^\circ = 3.50\,\mathrm{m/s}\,,$$

as expected since there is no friction. The velocity is down the plane.





(a) The free-body diagrams are shown to the right. $\vec{F}$ is the applied force and $\vec{f}$ is the force of block 1 on block 2. Note that $\vec{F}$ is applied only to block 1 and that block 2 exerts the force $-\vec{f}$ on block 1. Newton's third law has thereby been taken into account.

Newton's second law for block 1 is $F - f = m_1 a$, where a is the acceleration. The second law for block 2 is $f = m_2 a$. Since the blocks move together they have the same acceleration and the same symbol is used in both equations. Use the second equation to obtain an expression for a: $a = f/m_2$. Substitute into the first equation to get $F - f = m_1 f/m_2$. Solve for f:

$$f = \frac{Fm_2}{m_1 + m_2} = \frac{(3.2\,\text{N})(1.2\,\text{kg})}{2.3\,\text{kg} + 1.2\,\text{kg}} = 1.1\,\text{N}\,.$$

(b) If $\vec{F}$ is applied to block 2 instead of block 1, the force of contact is

$$f = \frac{Fm_1}{m_1 + m_2} = \frac{(3.2\,\text{N})(2.3\,\text{kg})}{2.3\,\text{kg} + 1.2\,\text{kg}} = 2.1\,\text{N}\,.$$

(c) The acceleration of the blocks is the same in the two cases. Since the contact force f is the only horizontal force on one of the blocks it must be just right to give that block the same acceleration as the block to which $\vec{F}$ is applied. In the second case the contact force accelerates a more massive block than in the first, so it must be larger.

CHAPTER 5 HINT FOR PROBLEM 15

Use Newton's second law in component form. The x component of the acceleration is the slope of the left-hand graph and the y component is the slope of the right-hand graph. Multiply each of these components by the mass of the package to obtain the components of the force. The magnitude is the square root of the sum of the squares of the components and the tangent of the angle that the force makes with the positive x axis is the y component divided by the x component.

$\left[\text{ans: (a) } 11.7\,\text{N; (b) } -59.0^\circ\right]$

Draw a free-body diagram for Tarzan and put in the axes. Remember that the vine pulls, not pushes, on him. The x component of the force of the vine on Tarzan is given by $T\sin\theta$ and the y component is given by $T\cos\theta$, where T is the tension in the vine and θ is the angle the vine makes with the vertical. The net force is the vector sum of the tension force and the gravitational force of Earth on Tarzan (his weight). According to Newton's second law his acceleration is the net force divided by his mass.

[ans: (a) $(285\,\mathrm{N})\,\hat{\mathrm{i}} + (705\,\mathrm{N})\,\hat{\mathrm{j}}$; (b) $(285\,\mathrm{N})\,\hat{\mathrm{i}} - (115\,\mathrm{N})\,\hat{\mathrm{j}}$; (c) $307\,\mathrm{N}$; (d) -22.0°; (e) $3.67\,\mathrm{m/s^2}$; (f) -22.0°]

CHAPTER 5 HINT FOR PROBLEM 37

Draw a free-body diagram for the bundle. The forces on it are the upward tension force of the cable and the downward gravitational force of Earth. Take the tension to have its maximum value and solve the Newton's second law equation for the acceleration. Use $v^2 = 2ah$ to find the speed v of the bundle when it hits the ground. Here h is the starting height of the bundle above the ground.

$\left[\right.$ans: (a) $1.4\,\mathrm{m/s^2}$; (b) $4.1\,\mathrm{m/s}$ $\left.\right]$

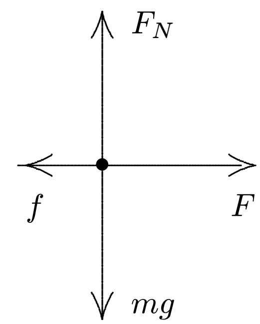
CHAPTER 5 HINT FOR PROBLEM 47

Draw a free-body diagram for each of the blocks. The forces on the left-hand block are the force of gravity $m_1 g$, down, and the tension force of the cord T, up. The forces on the right-hand block are the force of gravity $m_2 g$ and the tension force T, up. Let a_1 be the acceleration of block 1 and a_2 be the acceleration of block 2, then write a Newton's second law equation for each block. Note that the tension force on block 1 has the same magnitude as the tension force on block 2 and that the accelerations are actually vertical components.

The magnitudes of the accelerations are the same since the blocks are connected by the cord. Their signs, however, depend on the coordinate system used. If, for example, you take the upward direction to be positive for both blocks then $a_1 = -a_2$. Substitute $a_1 = a$ and $a_2 = -a$ into the second law equations and solve them simultaneously for a and the cord tension.

[ans: (a) $3.6\,\mathrm{m/s^2}$; (b) $17\,\mathrm{N}$]

(a) The free-body diagram for the bureau is shown to the right. $\vec{F}$ is the applied force, $\vec{f}$ is the force of friction, $\vec{F}_N$ is the normal force of the floor, and $m\vec{g}$ is the force of gravity. Take the x axis to be horizontal and the y axis to be vertical. Assume the bureau does not move and write the Newton's second law equations. The x component is $F - f = 0$ and the y component is $F_N - mg = 0$. The force of friction is then equal in magnitude to the applied force: $f = F$. The normal force is equal in magnitude to the force of gravity: $F_N = mg$. As F increases, f increases until $f = \mu_s F_N$. Then the bureau starts to move. The minimum force that must be applied to start the bureau moving is

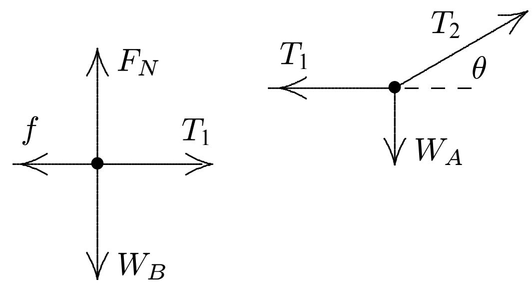


$$F = \mu_s F_N = \mu_s mg = (0.45)(45\,\text{kg})(9.8\,\text{m/s}^2) = 2.0 \times 10^2\,\text{N}\,.$$

(b) The equation for F is the same but the mass is now $45\,\text{kg} - 17\,\text{kg} = 28\,\text{kg}$. Thus

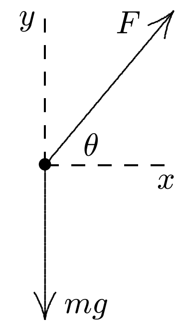
$$F = \mu_s mg = (0.45)(28\,\text{kg})(9.8\,\text{m/s}^2) = 1.2 \times 10^2\,\text{N}\,.$$

The free-body diagrams for block B and for the knot just above block A are shown to the right. T_1 is the magnitude of the tension force of the rope pulling on block B, T_2 is the magnitude of the tension force of the other rope, f is the magnitude of the force of friction exerted by the horizontal surface on block B, F_N is the magnitude of the normal force exerted by the surface on block B, W_A is the weight of block A, and W_B is the weight of block B. θ (= 30°) is the angle between the second rope and the horizontal.



For each object take the x axis to be horizontal and the y axis to be vertical. The x component of Newton's second law for block B is then $T_1 - f = 0$ and the y component is $F_N - W_B = 0$. The x component of Newton's second law for the knot is $T_2 \cos\theta - T_1 = 0$ and the y component is $T_2 \sin\theta - W_A = 0$. Eliminate the tension forces and find expressions for f and F_N in terms of W_A and W_B, then select W_A so $f = \mu_s F_N$. The second Newton's law equation gives $F_N = W_B$ immediately. The third gives $T_2 = T_1/\cos\theta$. Substitute this expression into the fourth equation to obtain $T_1 = W_A/\tan\theta$. Substitute $W_A/\tan\theta$ for T_1 in the first equation to obtain $f = W_A/\tan\theta$. For the blocks to remain stationary f must be less than $\mu_s F_N$ or $W_A/\tan\theta < \mu_s W_B$. The greatest that W_A can be is the value for which $W_A/\tan\theta = \mu_s W_B$. Solve for W_A:

$$W_A = \mu_s W_B \tan\theta = (0.25)(711\,\text{N})\tan 30^\circ = 1.0 \times 10^2\,\text{N}\,.$$



The free-body diagram for the plane is shown to the right. F is the magnitude of the lift on the wings and m is the mass of the plane. Since the wings are tilted by 40° to the horizontal and the lift force is perpendicular to the wings, the angle θ is 50°. The center of the circular orbit is to the right of the plane, the dashed line along x being a portion of the radius. Take the x axis to be to the right and the y axis to be upward. Then the x component of Newton's second law is $F\cos\theta = mv^2/R$ and the y component is $F\sin\theta - mg = 0$, where R is the radius of the orbit. The first equation gives $F = mv^2/R\cos\theta$ and when this is substituted into the second, $(mv^2/R)\tan\theta = mg$ results. Solve for R:

$$R = \frac{v^2}{g}\tan\theta\,.$$

The speed of the plane is $v = 480\,\text{km/h} = 133\,\text{m/s}$, so

$$R = \frac{(133\,\text{m/s})^2}{9.8\,\text{m/s}^2}\tan 50^\circ = 2.2\times 10^3\,\text{m}\,.$$

In each case you must decide if the block moves or not. If it moves the frictional force is kinetic in nature; if it does not move the frictional force is static in nature. Assume the block does not move and find the frictional force that is need to hold it stationary. Draw a free-body diagram for the block. The forces on it are the gravitational force, the normal force of the plane, the frictional force of the plane, and the applied force $\vec{P}$. Take the x axis to be parallel to the plane and the y axis to perpendicular to the plane, then write the Newton's second law equations in component form, with the acceleration equal to zero. Calculate the frictional and normal forces and compare the magnitude of the frictional force with the product of the coefficient of static friction and the magnitude of the normal force. If it is less the block does not move and the frictional force you computed is the actual frictional force. If the frictional force is greater the block does move and the magnitude of the frictional force is the product of the coefficient of kinetic friction and the magnitude of the normal force.

[ans: (a) $(17\,\mathrm{N})\,\hat{\mathrm{i}}$; (b) $(20\,\mathrm{N})\,\hat{\mathrm{i}}$; (c) $(15\,\mathrm{N})\,\hat{\mathrm{i}}$ndans

CHAPTER 6 HINT FOR PROBLEM 29

If the smaller block does not slide the frictional force of the larger block on it must have magnitude mg and this must be less than the product of the coefficient of static friction between the blocks and the magnitude of the normal force of the blocks on each other. Write the horizontal component of the Newton's second law equation for the smaller block. The horizontal forces on it are the applied force and the normal force of the larger block. Do the same for the larger block. The only horizontal force on it is the normal force of the smaller block. According to Newton's third law the two normal forces have the same magnitude. These equations can be solved for the magnitude of the normal force and then the value of the maximum static frictional force can be computed.

[ans: 4.9×10^2 N]

CHAPTER 6 HINT FOR PROBLEM 37

To round the curve without sliding the frictional force must be equal to mv^2/r, where m is the mass of the bicycle and rider, v is their speed, and r is the radius of the curve, and it must be less than $\mu_s F_N$, where μ_s is the coefficient of kinetic friction between the tires and the road and F_N is the normal force of the road on the tires.

[ans: 21 m]

(a) Let F be the magnitude of the force exerted by the cable on the astronaut. The force of the cable is upward and the force of gravity is mg is downward. Furthermore, the acceleration of the astronaut is $g/10$, upward. According to Newton's second law, $F - mg = mg/10$, so $F = 11mg/10$. Since the force $\vec{F}$ and the displacement $\vec{d}$ are in the same direction the work done by $\vec{F}$ is

$$W_F = Fd = \frac{11mgd}{10} = \frac{11(72\,\text{kg})(9.8\,\text{m/s}^2)(15\,\text{m})}{10} = 1.16 \times 10^4\,\text{J}\,.$$

(b) The force of gravity has magnitude mg and is opposite in direction to the displacement. Since $\cos 180° = -1$, it does work

$$W_g = -mgd = -(72\,\text{kg})(9.8\,\text{m/s}^2)(15\,\text{m}) = -1.06 \times 10^4\,\text{J}\,.$$

(c) The total work done is $W = 1.16 \times 10^4\,\text{J} - 1.06 \times 10^4\,\text{J} = 1.1 \times 10^3\,\text{J}$. Since the astronaut started from rest the work-kinetic energy theorem tells us that this must be her final kinetic energy.

(d) Since $K = \frac{1}{2}mv^2$ her final speed is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.1 \times 10^3\,\text{J})}{72\,\text{kg}}} = 5.3\,\text{m/s}\,.$$

(a) As the body moves along the x axis from $x_i = 3.0\,\text{m}$ to $x_f = 4.0\,\text{m}$ the work done by the force is

$$W = \int_{x_i}^{x_f} F_x\,dx = \int_{x_i}^{x_f} -6x\,dx = -3x^2\Big|_{x_i}^{x_f} = -3(x_f^2 - x_i^2)$$
$$= -3\left[(4.0)^2 - (3.0)^2\right] = -21\,\text{J}\,.$$

According to the work-kinetic energy theorem, this is the change in the kinetic energy:

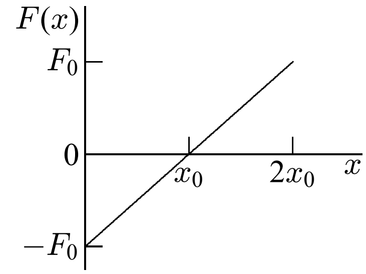
$$W = \Delta K = \tfrac{1}{2}m(v_f^2 - v_i^2)\,,$$

where v_i is the initial velocity (at x_i) and v_f is the final velocity (at x_f). The theorem yields

$$v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2(-21\,\text{J})}{2.0\,\text{kg}} + (8.0\,\text{m/s})^2} = 6.6\,\text{m/s}\,.$$

(b) The velocity of the particle is $v_f = 5.0\,\text{m/s}$ when it is at $x = x_f$. Solve the work-kinetic energy theorem for x_f. The net work done on the particle is $W = -3(x_f^2 - x_i^2)$, so the work-kinetic energy theorem yields $-3(x_f^2 - x_i^2) = \frac{1}{2}m(v_f^2 - v_i^2)$. Thus

$$x_f = \sqrt{-\frac{m}{6}(v_f^2 - v_i^2) + x_i^2} = \sqrt{-\frac{2.0\,\text{kg}}{6\,\text{N/m}}\left[(5.0\,\text{m/s})^2 - (8.0\,\text{m/s})^2\right] + (3.0\,\text{m})^2} = 4.7\,\text{m}\,.$$



(a) The graph shows F as a function of x if x_0 is positive. The work is negative as the object moves from $x = 0$ to $x = x_0$ and positive as it moves from $x = x_0$ to $x = 2x_0$. Since the area of a triangle is $\frac{1}{2}$(base)(altitude), the work done from $x = 0$ to $x = x_0$ is $-\frac{1}{2}(x_0)(F_0)$ and the work done from $x = x_0$ to $x = 2x_0$ is $\frac{1}{2}(2x_0 - x_0)(F_0) = \frac{1}{2}(x_0)(F_0)$. The total work is the sum, which is zero.

(b) The integral for the work is

$$W = \int_0^{2x_0} F_0\left(\frac{x}{x_0} - 1\right)\, dx = F_0\left(\frac{x^2}{2x_0} - x\right)\Bigg|_0^{2x_0} = 0\,.$$

The magnitude of the force is given by Newton's second law: $F = ma$, where m is the mass of the luge and rider and a is the magnitude of their acceleration. Since the force is constant and directed oppositely to the displacement, the work it does is $W = -Fd$, where d is the distance traveled while stopping. According to the work-kinetic energy theorem this must be the change in kinetic energy of the luge and rider and since the luge stops, $-\frac{1}{2}mv^2 = -Fd$, where v is the initial speed of the luge. Solve for d. Use $W = -\frac{1}{2}mv^2$ or $W = -Fd$ to calculate the work done by the force.

[ans: (a) 1.7×10^2 N; (b) 3.4×10^2 m; (c) -5.8×10^4 J; (d) 3.4×10^2 N; (e) 1.7×10^2 N; (f) -5.8×10^4 J]

The work done by the cable is given by $W = Td$, where T is the tension force of the cable and d is the distance the elevator cab travels (d_1 in part (a) and d_2 in part (b)). According to Newton's second law the acceleration of the cheese (and also of the elevator) is $a = F_N/m_c$, where m_c is the mass of the cheese, and the tension force of the cable is $T = m_e a$, where m_e is the mass of the elevator cab. (Strictly, it should be the mass of the cab and cheese together, but the mass of the cheese is so small it may be neglected here.)

(a) Put d equal to d_1 (= 2.40 m) and F_N = 3.00 N, then solve for W.

(b) Put $d = d_2$ (= 10.5 m) and $W = 92.61 \times 10^3$ J, then solve for F_N.

[ans: (a) 25.9 kJ; (b) 2.45 N]

CHAPTER 7 HINT FOR PROBLEM 35

Use the work-kinetic energy theorem: the work done by the force is equal to the change in the kinetic energy of the object. Find the speed of the object at the beginning and end of the interval by differentiating its coordinate with respect to time.

[ans: 5.3×10^2 J]

(a) The work done by the force is given by $W = \vec{F} \cdot \Delta\vec{d}$, where $\Delta\vec{d} = \vec{d}_f - \vec{d}_i$. Use the component equation for the value of the scalar product: $\vec{F} \cdot \Delta\vec{d} = F_x(\Delta\vec{d})_x + F_y(\Delta\vec{d})_y + F_z(\Delta\vec{d})_z$.
(b) The average power is the work done by the machine's force divided by the time.

[ans: (a) 1.0×10^2 J; (b) 8.4 W

(a) The only force that does work as the flake falls is the force of gravity and it is a conservative force. If K_i is the kinetic energy of the flake at the edge of the bowl, K_f is its kinetic energy at the bottom, U_i is the gravitational potential energy of the flake-Earth system with the flake at the top, and U_f is the gravitational potential energy with it at the bottom, then $K_f + U_f = K_i + U_i$. Take the potential energy to be zero at the bottom of the bowl. Then the potential energy at the top is $U_i = mgr$, where r is the radius of the bowl and m is the mass of the flake. $K_i = 0$ since the flake starts from rest. Since the problem asks for the speed at the bottom, write $\frac{1}{2}mv^2$ for K_f. The energy conservation equation becomes $mgr = \frac{1}{2}mv^2$, so

$$v = \sqrt{2gr} = \sqrt{2(9.8\,\mathrm{m/s^2})(0.220\,\mathrm{m})} = 2.08\,\mathrm{m/s}\,.$$

(b) Note that the expression for the speed ($v = \sqrt{2gr}$) does not contain the mass of the flake. The speed would be the same, 2.08 m/s, regardless of the mass of the flake.

(c) The final kinetic energy is given by $K_f = K_i + U_i - U_f$. Since K_i is greater than before, K_f is greater. This means the final speed of the flake is greater.

Information given in the second sentence allows us to compute the spring constant. Solve $F = kx$ for k:

$$k = \frac{F}{x} = \frac{270\,\mathrm{N}}{0.02\,\mathrm{m}} = 1.35 \times 10^4\,\mathrm{N/m}\,.$$

(a) Now consider the block sliding down the incline. If it starts from rest at a height h above the point where it momentarily comes to rest, its initial kinetic energy is zero and the initial gravitational potential energy of the block-Earth system is mgh, where m is the mass of the block. We have taken the zero of gravitational potential energy to be at the point where the block comes to rest. We also take the initial potential energy stored in the spring to be zero. Suppose the block compresses the spring a distance x before coming momentarily to rest. Then the final kinetic energy is zero, the final gravitational potential energy is zero, and final spring potential energy is $\frac{1}{2}kx^2$. The incline is frictionless and the normal force it exerts on the block does no work, so mechanical energy is conserved. This means $mgh = \frac{1}{2}kx^2$, so

$$h = \frac{kx^2}{2mg} = \frac{(1.35 \times 10^4\,\mathrm{N/m})(0.055\,\mathrm{m})^2}{2(12\,\mathrm{kg})(9.8\,\mathrm{m/s^2})} = 0.174\,\mathrm{m}\,.$$

If the block traveled down a length of incline equal to ℓ, then $\ell \sin 30^\circ = h$, so $\ell = h/\sin 30^\circ = (0.174\,\mathrm{m})/\sin 30^\circ = 0.35\,\mathrm{m}$.

(b) Just before it touches the spring it is 0.055 m away from the place where it comes to rest and so is a vertical distance $h' = (0.055\,\mathrm{m})\sin 30^\circ = 0.0275\,\mathrm{m}$ above its final position. The gravitational potential energy is then $mgh' = (12\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(0.0275\,\mathrm{m}) = 3.23\,\mathrm{J}$. On the other hand, its initial potential energy is $mgh = (12\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(0.174\,\mathrm{m}) = 20.5\,\mathrm{J}$. The difference is its final kinetic energy: $K_f = 20.5\,\mathrm{J} - 3.23\,\mathrm{J} = 17.2\,\mathrm{J}$. Its final speed is

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(17.2\,\mathrm{J})}{12\,\mathrm{kg}}} = 1.7\,\mathrm{m/s}\,.$$

(a) The magnitude of the force of friction is $f = \mu_k N$, where μ_k is the coefficient of kinetic friction and N is the normal force of the surface on the block. The only vertical forces acting on the block are the normal force, upward, and the force of gravity, downward. Since the vertical component of the block's acceleration is zero, Newton's second law tells us that $N = mg$, where m is the mass of the block. Thus $f = \mu_k mg$. The increase in thermal energy is given by $\Delta E_{\text{th}} = f\ell = \mu_k mg\ell$, where ℓ is the distance the block moves before coming to rest. Its value is $\Delta E_{\text{th}} = (0.25)(3.5\,\text{kg})(9.8\,\text{m/s}^2)(7.8\,\text{m}) = 67\,\text{J}$.

(b) The block had its maximum kinetic energy just as it left the spring and entered the part of the surface where friction acts. The maximum kinetic energy equals the increase in thermal energy, 67 J.

(c) The energy that appears as kinetic energy is originally stored as the potential energy of the compressed spring. Thus $\Delta E = \frac{1}{2}kx^2$, where k is the spring constant and x is the compression. Solve for x:

$$x = \sqrt{\frac{2\Delta E}{k}} = \sqrt{\frac{2(67\,\text{J})}{640\,\text{N/m}}} = 0.46\,\text{m}\,.$$

Use conservation of mechanical energy. When the car is a distance y above ground level and is traveling with speed v the mechanical energy is given by $\frac{1}{2}mv^2 + mgy$, where m is the mass of the car. Write this expression for the initial values and for the values when the car is at another point (A, B, C, or the point where it stops on the last hill). Equate the two expressions to each other and solve for the unknown (either the speed or the height above the ground). Notice that the mass of the car cancels from the conservation of energy equation.

[ans: (a) $17.0\,\mathrm{m/s}$; (b) $26.5\,\mathrm{m/s}$; (c) $33.4\,\mathrm{m/s}$; (d) $56.7\,\mathrm{m}$; (e) all the same]

(a) Use $U = -\int F\,dx$ and select the constant of integration so that $U = 27\,\mathrm{J}$ for $x = 0$.
(b) The force is zero at the value of x for which the potential energy is maximum.
(c) and (d) Set the expression for U equal to zero and solve for x.

[ans: (a) $U = 27 + 12x - 3x^2$; (b) 39 J; (c) $-1.6\,\mathrm{m}$; (d) $5.6\,\mathrm{m}$]

Take the gravitational potential energy to be zero when the whole cord is stuck to the ceiling. Now compute the potential energy when the cord is hanging by one end. Consider an infinitesimal segment of cord with mass dm a distance y from the ceiling. The potential energy associated with this segment is $dU = -gy\,dm$. If the length of the segment is dy, then $dm = (dy/L)M$, where L is the length of the cord and M is its mass. The total potential energy is the sum over segments:

$$U = -\int_0^L gy\frac{M}{L}\,dy\,.$$

Evaluate the integral.

[ans: $-18\,\mathrm{mJ}$]

(a) The work done by the spring force is given by $W_s = -\frac{1}{2}kd^2$, where d is the spring compression and k is the spring constant.

(b) The change in the thermal energy is given by $\Delta E_{th} = fd$, where f is the magnitude of the frictional force. This is the product of the magnitude of the normal force of the floor on the block and the coefficient of kinetic friction. Use Newton's second law to obtain the magnitude of the normal force.

(c) Use the energy equation $W = \Delta E_{mec} + \Delta E_{th}$, where W is the work done by external forces and ΔE_{mec} is the change in the mechanical energy. Take the system to be composed of the block and the spring. Then no external forces do work and the mechanical energy is the sum of the kinetic energy of the block and the potential energy stored in the spring. Solve for the initial kinetic energy and then the initial speed.

[ans: (a) -0.90 J; (b) 0.46 J; (c) 1.0 m/s]

Take the x axis to be to the right in the figure, with the origin at the shore. Let m_b be the mass of the boat and x_{bi} its initial coordinate. Let m_d be the mass of the dog and x_{di} his initial coordinate. The coordinate of the center of mass is

$$x_{\text{com}} = \frac{m_b x_{bi} + m_d x_{di}}{m_b + m_d} \,.$$

Now the dog walks a distance d to the left on the boat. The new coordinates x_{bf} and x_{df} are related by $x_{bf} = x_{df} + d$, so the coordinate of the center of mass can be written

$$x_{\text{com}} = \frac{m_b x_{bf} + m_d x_{df}}{m_b + m_d} = \frac{m_b x_{df} + m_b d + m_d x_{df}}{m_b + m_d} \,.$$

Since the net external force on the boat-dog system is zero the velocity of the center of mass does not change. Since the boat and dog were initially at rest the velocity of the center of mass is zero. The center of mass remains at the same place and the two expressions we have written for x_{com} must equal each other. This means $m_b x_{bi} + m_d x_{di} = m_b x_{df} + m_b d + m_d x_{df}$. Solve for x_{df}:

$$\begin{aligned} x_{df} &= \frac{m_b x_{bi} + m_d x_{di} - m_b d}{m_b + m_d} \\ &= \frac{(18\,\text{kg})(6.1\,\text{m}) + (4.5\,\text{kg})(6.1\,\text{m}) - (18\,\text{kg})(2.4\,\text{m})}{18\,\text{kg} + 4.5\,\text{kg}} = 4.2\,\text{m} \,. \end{aligned}$$

(a) Let m be the mass and $v_i\,\hat{\imath}$ be the velocity of the body before the explosion. Let m_1, m_2, and m_3 be the masses of the fragments. (The mass of the third fragment is 6.00 kg.) Write $v_1\,\hat{\jmath}$ for the velocity of fragment 1, $-v_2\,\hat{\imath}$ for the velocity of fragment 2, and $v_{3x}\,\hat{\imath} + v_{3y}\,\hat{\jmath}$ for the velocity of fragment 3. Since the original body and two of the fragments all move in the xy plane the third fragment must also move in that plane. Conservation of linear momentum leads to $mv_i\,\hat{\imath} = m_1v_1\,\hat{\jmath} - m_2v_2\,\hat{\imath} + m_3v_{3x}\,\hat{\imath} + m_3v_{3y}\,\hat{\jmath}$, or $(mv_i + m_2v_2 - m_3v_{3x})\,\hat{\imath} - (m_1v_1 + m_3v_{3y})\,\hat{\jmath} = 0$. The x component of this equation gives

$$v_{3x} = \frac{mv_i + m_2v_2}{m_3} = \frac{(20.0\,\mathrm{kg})(200\,\mathrm{m/s}) + (4.00\,\mathrm{kg})(500\,\mathrm{m/s})}{6.0\,\mathrm{kg}} = 1.00 \times 10^3\,\mathrm{m/s}\,.$$

The y component gives

$$v_{3y} = -\frac{m_1v_1}{m_3} = -\frac{(10.0\,\mathrm{kg})(100\,\mathrm{m/s})}{6.0\,\mathrm{kg}} = -167\,\mathrm{m/s}\,.$$

Thus $\vec{v}_3 = (1.00 \times 10^3\,\mathrm{m/s})\,\hat{\imath} - (167\,\mathrm{m/s})\,\hat{\jmath}$. The velocity has a magnitude of $1.01 \times 10^3\,\mathrm{m/s}$ and is $9.48°$ below the x axis.

(b) The initial kinetic energy is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(20.0\,\mathrm{kg})(200\,\mathrm{m/s})^2 = 4.00 \times 10^5\,\mathrm{J}\,.$$

The final kinetic energy is

$$\begin{aligned} K_f &= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 \\ &= \frac{1}{2}\left[(10.0\,\mathrm{kg})(100\,\mathrm{m/s})^2 + (4.00\,\mathrm{kg})(500\,\mathrm{m/s})^2 + (6.00\,\mathrm{kg})(1014\,\mathrm{m/s})^2\right] \\ &= 3.63 \times 10^6\,\mathrm{J}\,. \end{aligned}$$

The energy released in the explosion is $3.63 \times 10^6\,\mathrm{J} - 4.00 \times 10^5\,\mathrm{J} = 3.23 \times 10^6\,\mathrm{J}$.

(a) Use conservation of mechanical energy to find the speed of either ball after it has fallen a distance h. The initial kinetic energy is zero, the initial gravitational potential energy is Mgh, the final kinetic energy is $\frac{1}{2}Mv^2$, and the final potential energy is zero. Thus $Mgh = \frac{1}{2}Mv^2$ and $v = \sqrt{2gh}$. The collision of the ball of M with the floor is a collision of a light object with a stationary massive object. The velocity of the light object reverses direction without change in magnitude. After the collision, the ball is traveling upward with a speed of $\sqrt{2gh}$. The ball of mass m is traveling downward with the same speed. Use Eq. 9–75 to find an expression for the velocity of the ball of mass M after the collision:

$$\begin{aligned} v_{Mf} &= \frac{M-m}{M+m}v_{Mi} + \frac{2m}{M+m}v_{mi} = \frac{M-m}{M+m}\sqrt{2gh} - \frac{2m}{M+m}\sqrt{2gh} \\ &= \frac{M-3m}{M+m}\sqrt{2gh}\,. \end{aligned}$$

For this to be zero, $m = M/3 = (0.63\,\text{kg})/3 = 0.21\,\text{kg}$.

(b) Use the same equation to find the velocity of the ball of mass m after the collision:

$$v_{mf} = -\frac{m-M}{M+m}\sqrt{2gh} + \frac{2M}{M+m}\sqrt{2gh} = \frac{3M-m}{M+m}\sqrt{2gh}\,.$$

Substitute $M = 3m$ to obtain $v_{mf} = 2\sqrt{2gh}$. Now use conservation of mechanical energy to find the height h' to which the ball rises. The initial kinetic energy is $\frac{1}{2}mv_{mf}^2$, the initial potential energy is zero, the final kinetic energy is zero, and the final potential energy is mgh'. Thus $\frac{1}{2}mv_{mf}^2 = mgh'$, so

$$h' = \frac{v_{mf}^2}{2g} = \frac{8gh}{2g} = 4h = 4(1.8\,\text{m} = 7.2\,\text{m}\,,$$

where $2\sqrt{2gh}$ was substituted for v_{mf}.

(a) Use $\vec{a}_{\rm com} = (m_1\vec{a}_1 + m_2\vec{a}_2)/(m_1+m_2)$, where $\vec{a}_1$ is the acceleration of block 1 and $\vec{a}_2$ is the acceleration of block 2. First use Newton's second law to find the magnitude of the acceleration of the blocks. If T is the tension force of the cord, block 1 obeys $T = m_1 a$ and block 2 obeys $m_2 g - T = m_2 a$. Eliminate T and solve for a. Then $\vec{a}_1 = a\,\hat{\mathrm{i}}$ and $\vec{a}_2 = -a\,\hat{\mathrm{j}}$.

(b) Since the acceleration is constant and the system is released from rest the velocity of the center of mass as a function of time t is $\vec{v}_{\rm com} = \vec{a}_{\rm com} t$.

(d) The center of mass follows a straight line. If θ is the angle between the path and the x axis, then $\tan\theta = a_{{\rm com}\,y}/a_{{\rm com}\,y}$.

$\big[$ans: (a) $(2.35\,\mathrm{m/s^2})\,\hat{\mathrm{i}} - (1.57\,\mathrm{m/s^2})\,\hat{\mathrm{j}}$; (b) $\left[(2.35\,\mathrm{m/s^2})\,\hat{\mathrm{i}} - (1.57\,\mathrm{m/s^2})\,\hat{\mathrm{j}}\right] t$; (d) straight, at a downward angle of 34° $\big]$

CHAPTER 9 HINT FOR PROBLEM 31

(a) The impulse is given by the integral $J = \int F\,dt$ over the duration of a single impact. This is just the area enclosed by one of the triangles on the graph. The area, of course, is half the product of the base (= 10 ms) and the altitude (= 200 N).
(b) The average force is given by $F_{avg} = J/\Delta t$, where Δt is the duration of the impact.
(c) Use $F_{avg} = \Delta p/\Delta t$, where Δp is the change in the momentum of the snowball stream in time Δt, a time interval that includes many impacts. Since the snowballs stick to the wall, the momentum of each snowball changes by mv, where v is its speed. If N snowballs hit per unit time, the change in the total momentum is $\Delta p = mvN\,\Delta t$.

[ans: (a) $1.00\,\mathrm{N\cdot s}$; (b) $100\,\mathrm{N}$; (c) $20\,\mathrm{N}$]

The momentum of the bullet-bock 1 system is conserved in the encounter of the bullet with that block. Let m be the mass of the bullet and v_0 be its initial velocity. Let $v_{\rm int}$ be the velocity of the bullet after it emerges from block 1 but before it strikes block 2. Let M_1 be the mass of block 1 and let V_1 be its velocity after the bullet emerges from it. Then $mv_0 = mv_{\rm int} + M_1V_1$.

Momentum is also conserved in the encounter of the bullet with block 2. Let M_2 be the mass of that block and let V_2 be its velocity after the bullet becomes embedded in it. Then $mv_{\rm int} = (m + M_2)V_2$. Solve the second equation for $v_{\rm int}$ and then the first for v_0.

[ans: (a) 721 m/s; (b) 937 m/s]

(a) Use 1 rev = 2π rad and 1 min = 60 s to obtain

$$\omega = \frac{200\,\text{rev}}{1\,\text{min}} = \frac{(200\,\text{rev})(2\pi\,\text{rad/rev})}{(1\,\text{min})(60\,\text{s/min})} = 20.9\,\text{rad/s}\,.$$

(b) The speed of a point on the rim is given by $v = \omega r$, where r is the radius of the flywheel and ω must be in radians per second. Thus $v = (20.9\,\text{rad/s})(0.60\,\text{m} = 12.5\,\text{m/s}$.

(c) If ω is the angular velocity at time t, ω_0 is the angular velcoty at $t = 0$, and α is the angular acceleration, then since the anagular acceleration is constant $\omega = \omega_0 + \alpha t$ and

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{(1000\,\text{rev/min}) - (200\,\text{rev/min})}{1.0\,\text{min}} = 800\,\text{rev/min}^2\,.$$

(d) The flywheel turns through the angle θ, which is

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (200\,\text{rev/min})(1.0\,\text{min}) + \frac{1}{2}(800\,\text{rev/min}^2)(1.0\,\text{min})^2 = 600\,\text{rev}\,.$$

CHAPTER 10 SOLUTION FOR PROBLEM 41

Use the parallel-axis theorem. According to Table 10–2, the rotational inertia of a uniform slab about an axis through the center and perpendicular to the large faces is given by

$$I_{\text{com}} = \frac{M}{12}(a^2 + b^2)\,.$$

A parallel axis through a corner is a distance $h = \sqrt{(a/2)^2 + (b/2)^2}$ from the center, so

$$\begin{aligned} I &= I_{\text{com}} + Mh^2 = \frac{M}{12}(a^2 + b^2) + \frac{M}{4}(a^2 + b^2) = \frac{M}{3}(a^2 + b^2) \\ &= \frac{0.172\,\text{kg}}{3}\left[(0.035\,\text{m})^2 + (0.084\,\text{m})^2\right] = 4.7 \times 10^{-4}\,\text{kg}\cdot\text{m}^2\,. \end{aligned}$$

(a) Use constant acceleration kinematics. If down is taken to be positive and a is the acceleration of the heavier block, then its coordinate is given by $y = \frac{1}{2}at^2$, so

$$a = \frac{2y}{t^2} = \frac{2(0.750\,\mathrm{m})}{(5.00\,\mathrm{s})^2} = 6.00 \times 10^{-2}\,\mathrm{m/s^2}\,.$$

The lighter block has an acceleration of $6.00 \times 10^{-2}\,\mathrm{m/s^2}$, upward.

(b) Newton's second law for the heavier block is $m_h g - T_h = m_h a$, where m_h is its mass and T_h is the tension force on the block. Thus

$$T_h = m_h(g - a) = (0.500\,\mathrm{kg})(9.8\,\mathrm{m/s^2} - 6.00 \times 10^{-2}\,\mathrm{m/s^2}) = 4.87\,\mathrm{N}\,.$$

(c) Newton's second law for the lighter block is $m_l g - T_l = -m_l a$, where T_l is the tension force on the block. Thus

$$T_l = m_l(g + a) = (0.460\,\mathrm{kg})(9.8\,\mathrm{m/s^2} + 6.00 \times 10^{-2}\,\mathrm{m/s^2}) = 4.54\,\mathrm{N}\,.$$

(d) Since the cord does not slip on the pulley, the tangential acceleration of a point on the rim of the pulley must be the same as the acceleration of the blocks, so

$$\alpha = \frac{a}{R} = \frac{6.00 \times 10^{-2}\,\mathrm{m/s^2}}{5.00 \times 10^{-2}\,\mathrm{m}} = 1.20\,\mathrm{rad/s^2}\,.$$

(e) The net torque acting on the pulley is $\tau = (T_h - T_l)R$. Equate this to $I\alpha$ and solve for I:

$$I = \frac{(T_h - T_l)R}{\alpha} = \frac{(4.87\,\mathrm{N} - 4.54\,\mathrm{N})(5.00 \times 10^{-2}\,\mathrm{m})}{1.20\,\mathrm{rad/s^2}} = 1.38 \times 10^{-2}\,\mathrm{kg \cdot m^2}\,.$$

CHAPTER 10 HINT FOR PROBLEM 11

Use the equation for constant-angular acceleration rotation: $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, where θ is the angular position at time t (measured from the angular position at $t = 0$), ω_0 is the angular velocity at $t = 0$, and α is the angular acceleration. Solve for α.

(b) the average angular velocity is the angular displacement divided by the time interval.

(c) Evaluate the constant-acceleration equation $\omega = \omega_0 + \alpha t$.

(d) Use $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$. You might take the time to be zero at the end of the first 5 seconds and take ω_0 to be the answer to part (c).

[ans: (a) $2.0\,\mathrm{rad/s^2}$; (b) $5.0\,\mathrm{rad/s}$; (c) $10\,\mathrm{rad/s}$; (d) $75\,\mathrm{rad}$]

CHAPTER 10 HINT FOR PROBLEM 25

The tangential component of the acceleration is given by $a_t = \alpha r$ and the radial component is given by $a_r = \omega^2 r$, where α is the angular acceleration, ω is the angular velocity, and r is the distance from the rotation axis to the point. Use $\omega = d\theta/dt$ to find ω and $\alpha = d\omega/dt$ to calculate α.

[ans: (a) 6.4 cm/s^2; (b) 2.6 cm/s^2]

CHAPTER 10 HINT FOR PROBLEM 39

(a) The rotational inertia of a rod of length d and mass M, rotating about an axis through its center and perpendicular to it, is $\frac{1}{12}Md^2$ (see Table 10–2). Use the parallel-axis theorem to find the rotational inertias of the rods in this problem. For one rod the rotation axis is $d/2$ from its center and for the other it is $3d/2$ from its center. The rotational inertia of either of the particles is the product of its mass and the square of the its distance from the rotation axis. Sum the rotational inertias to obtain the total rotational inertia.

(b) The rotational kinetic energy is given by $K = \frac{1}{2}I\omega^2$, where I is the total rotational inertia.

[ans: (a) $0.023\,\mathrm{kg\cdot m^2}$; (b) $11\,\mathrm{mJ}$]

Use $\tau_{\text{net}} = I\alpha$, where τ_{net} is the net torque on the cylinder, I is its rotational inertia, and α is its angular acceleration. The net torque is the sum of the individual torques. The magnitude of the torque associated with $\vec{F}_1$ is F_1R; the magnitude of the torque associated with $\vec{F}_2$ is F_2R; the magnitude of the torque associated with $\vec{F}_3$ is F_3r; the magnitude of the torque associated with $\vec{F}_4$ is 0. If the torque, acting alone, tends to produce an angular acceleration in the counterclockwise direction it enters the sum as a positive value; if it tends to produce an angular acceleration in the clockwise direction it enters the sum as a negative value. The rotational inertia is $I = \frac{1}{2}MR^2$, where M is the mass of the cylinder. Solve for α. If the result is positive the angular acceleration is in the counterclockwise direction; if it is negative the angular acceleration is in the clockwise direction.

[ans: (a) $9.7\,\text{rad/s}^2$; (b) counterclockwise]Use $\tau_{\text{net}} = I\alpha$, where τ_{net} is the net torque on the cylinder, I is it rotational inertia, and α is its angular acceleration. The net torque is the sum of the individual torques. The magnitude of the torque associated with $\vec{F}_1$ is F_1R; the magnitude of the torque associated with $\vec{F}_2$ is F_2R; the magnitude of the torque associated with $\vec{F}_3$ is F_3r; the magnitude of the torque associated with $\vec{F}_4$ is 0. If the torque, acting alone, tends to produce an angular acceleration in the counterclockwise direction it enters the sum as a positive value; it tends to produce an angular acceleration in the clockwise direction it enter the sum as a negative value. The rotational inertia is $I = \frac{1}{2}MR^2$, where M is the mass of the cylinder. Solve for α. If the result is positive it is in the counterclockwise direction; if it is negative the acceleration is in the clockwise direction.

[ans: (a) $9.7\,\text{rad/s}^2$; (b) counterclockwise]

(a) The angular momentum is given by the vector product $\vec{\ell} = m\vec{r} \times \vec{v}$, where $\vec{r}$ is the position vector of the particle and $\vec{v}$ is its velocity. Since the position and velocity vectors are in the xy plane we may write $\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath}$ and $\vec{v} = v_x\,\hat{\imath} + v_y\,\hat{\jmath}$. Thus

$$\vec{r} \times \vec{v} = (x\,\hat{\imath} + y\,\hat{\jmath}) \times (v_x\,\hat{\imath} + v_y\,\hat{\jmath}) = xv_x\,\hat{\imath} \times \hat{\imath} + xv_y\,\hat{\imath} \times \hat{\jmath} + yv_x\,\hat{\jmath} \times \hat{\imath} + yv_y\,\hat{\jmath} \times \hat{\jmath}\,.$$

Use $\hat{\imath} \times \hat{\imath} = 0$, $\hat{\imath} \times \hat{\jmath} = \hat{k}$, $\hat{\jmath} \times \hat{\imath} = -\hat{k}$, and $\hat{\jmath} \times \hat{\jmath} = 0$ to obtain

$$\vec{r} \times \vec{v} = (xv_y - yv_x)\,\hat{k}\,.$$

Thus

$$\begin{aligned}\vec{\ell} &= m(xv_y - yv_x)\,\hat{k} \\ &= (3.0\,\mathrm{kg})\left[(3.0\,\mathrm{m})(-6.0\,\mathrm{m/s}) - (8.0\,\mathrm{m})(5.0\,\mathrm{m/s})\right]\,\hat{k} = (-1.7 \times 10^2\,\mathrm{kg}\cdot\mathrm{m^2/s})\,\hat{k}\,.\end{aligned}$$

(b) The torque is given by $\vec{\tau} = \vec{r} \times \vec{F}$. Since the force has only an x component we may write $\vec{F} = F_x\,\hat{\imath}$ and

$$\vec{\tau} = (x\,\hat{\imath} + y\,\hat{\jmath}) \times (F_x\,\hat{\imath}) = -yF_x\,\hat{k} = -(8.0\,\mathrm{m})(-7.0\,\mathrm{N})\,\hat{k} = (56\,\mathrm{N}\cdot\mathrm{m})\,\hat{k}\,.$$

(c) According to Newton's second law, $\vec{\tau} = \mathrm{d}\vec{\ell}/\mathrm{d}t$, so the time rate of change of the angular momentum is $56\,\mathrm{kg}\cdot\mathrm{m^2/s^2}$, in the positive z direction.

(a) No external torques act on the system consisting of the man, bricks, and platform, so the total angular momentum of that system is conserved. Let I_i be the initial rotational inertia of the system and let I_f be the final rotational inertia. If ω_i is the initial angular velocity and ω_f is the final angular velocity, then $I_i\omega_i = I_f\omega_f$ and

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{6.0\,\mathrm{kg\cdot m^2}}{2.0\,\mathrm{kg\cdot m^2}}\right)(1.2\,\mathrm{rev/s}) = 3.6\,\mathrm{rev/s}\,.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}I_i\omega_i^2$, the final kinetic energy is $K_f = \frac{1}{2}I_f\omega_f^2$, and their ratio is

$$\frac{K_f}{K_i} = \frac{I_f\omega_f^2}{I_i\omega_i^2} = \frac{(2.0\,\mathrm{kg\cdot m^2})(3.6\,\mathrm{rev/s})^2}{(6.0\,\mathrm{kg\cdot m^2})(1.2\,\mathrm{rev/s})^2} = 3.0\,.$$

(c) The man did work in decreasing the rotational inertia by pulling the bricks closer to his body. This energy came from the man's store of internal energy.

(a) If we consider a short time interval from just before the wad hits to just after it hits and sticks, we may use the principle of conservation of angular momentum. The initial angular momentum is the angular momentum of the falling putty wad. The wad initially moves along a line that is $d/2$ distant from the axis of rotation, where d is the length of the rod. The angular momentum of the wad is $mvd/2$. After the wad sticks, the rod has angular velocity ω and angular momentum $I\omega$, where I is the rotational inertia of the system consisting of the rod with the two balls and the wad at its end. Conservation of angular momentum yields $mvd/2 = I\omega$. If M is the mass of one of the balls, $I = (2M + m)(d/2)^2$. When $mvd/2 = (2M + m)(d/2)^2\omega$ is solved for ω, the result is

$$\omega = \frac{2mv}{(2M + m)d} = \frac{2(0.0500\,\text{kg})(3.00\,\text{m/s})}{[2(2.00\,\text{kg}) + 0.0500\,\text{kg}](0.500\,\text{m})} = 0.148\,\text{rad/s}\,.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}mv^2$, the final kinetic energy is $K_f = \frac{1}{2}I\omega^2$, and their ratio is $K_f/K_i = I\omega^2/mv^2$. When $I = (2M + m)d^2/4$ and $\omega = 2mv/(2M + m)d$ are substituted, this becomes

$$\frac{K_f}{K_i} = \frac{m}{2M + m} = \frac{0.0500\,\text{kg}}{2(2.00\,\text{kg}) + 0.0500\,\text{kg}} = 0.0123\,.$$

(c) As the rod rotates the sum of its kinetic and potential energies is conserved. If one of the balls is lowered a distance h, the other is raised the same distance and the sum of the potential energies of the balls does not change. We need consider only the potential energy of the putty wad. It moves through a 90° arc to reach the lowest point on its path, gaining kinetic energy and losing gravitational potential energy as it goes. It then swings up through an angle θ, losing kinetic energy and gaining potential energy, until it momentarily comes to rest. Take the lowest point on the path to be the zero of potential energy. It starts a distance $d/2$ above this point, so its initial potential energy is $U_i = mgd/2$. If it swings through the angle θ, measured from its lowest point, then its final position is $(d/2)(1 - \cos\theta)$ above the lowest point and its final potential energy is $U_f = mg(d/2)(1 - \cos\theta)$. The initial kinetic energy is the sum of the kinetic energies of the balls and wad: $K_i = \frac{1}{2}I\omega^2 = \frac{1}{2}(2M + m)(d/2)^2\omega^2$. At its final position the rod is instantaneously stopped, so the final kinetic energy is $K_f = 0$. Conservation of energy yields $mgd/2 + \frac{1}{2}(2M + m)(d/2)^2\omega^2 = mg(d/2)(1 - \cos\theta)$. When this equation is solved for $\cos\theta$, the result is

$$\begin{aligned}\cos\theta &= -\frac{1}{2}\left(\frac{2M + m}{mg}\right)\left(\frac{d}{2}\right)\omega^2 \\ &= -\frac{1}{2}\left[\frac{2(2.00\,\text{kg}) + 0.0500\,\text{kg}}{(0.0500\,\text{kg})(9.8\,\text{m/s}^2)}\right]\left(\frac{0.500\,\text{m}}{2}\right)(0.148\,\text{rad/s})^2 = -0.0226\,.\end{aligned}$$

The result for θ is 91.3°. The total angle of the swing is $90^\circ + 91.3^\circ = 181^\circ$.

CHAPTER 11 HINT FOR PROBLEM 13

At the loop bottom the vector sum of the gravitational and normal forces must equal the product of the mass of the ball and the centripetal acceleration. That is, $F_N - Mg = Mv^2/r$, where v is the speed of the ball and r is the radius of the loop. Use conservation of energy to find an expression for v^2. Take the potential energy to be zero at the bottom of the loop. Then the initial potential energy is Mgh. The initial kinetic energy is zero and the final kinetic energy is $\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$, where ω is the angular speed of the ball. Since the ball does not slide the speed and angular speed are related by $v = \omega R$. Use this relationship to eliminate ω from the energy equation, then solve for v^2. Substitute this expression and $F_N = 2.00Mg$ into $F_N - Mg = Mv^2/r$ and solve for I/MR^2.

[ans: 0.50]

CHAPTER 11   HINT FOR PROBLEM 29

(a) Use Newton's second law to find the acceleration: $\vec{a} = \vec{F}/m$, where m is the mass of the object.

(b) The angular momentum is $\vec{\ell} = m\vec{d} \times \vec{v}$. Use the component form of the vector product:

$$\vec{d} \times \vec{v} = (yv_z - zv_y)\,\hat{\imath} + (zv_x - xv_z)\,\hat{\jmath} + (xv_y - yv_x)\,\hat{k}\,.$$

(c) The torque is $\vec{\tau} = m\vec{d} \times \vec{F}$. Use the component form of the vector product:

$$\vec{d} \times \vec{F} = (yF_z - zF_y)\,\hat{\imath} + (zF_x - xF_z)\,\hat{\jmath} + (xF_y - yF_x)\,\hat{k}\,.$$

(d) Equate the two forms of the scalar product, $\vec{v} \cdot \vec{F} = vF \cos\phi$ and $\vec{v} \cdot \vec{F} = v_xF_x + v_yF_y + v_zF_z$, then solve for ϕ, the angle between $\vec{v}$ and $\vec{F}$.

[ans: (a) $(3.00\,\mathrm{m/s^2})\,\hat{\imath} - (4.00\,\mathrm{m/s^2})\,\hat{\jmath} + (2.00\,\mathrm{m/s^2})\,\hat{k}$; (b) $(42.0\,\mathrm{kg \cdot m^2/s})\,\hat{\imath} + (24.0\,\mathrm{kg \cdot m^2/s})\,\hat{\jmath} + (60.0\,\mathrm{kg \cdot m^2/s})\,\hat{k}$; (c) $(-8.00\,\mathrm{N \cdot m})\,\hat{\imath} - (26.0\,\mathrm{N \cdot m})\,\hat{\jmath} - (40.0\,\mathrm{N \cdot m})\,\hat{k}$; (d) 127°](a) Use Newton's second law to find the acceleration: $\vec{a} = \vec{F}/m$, where m is the mass of the object.

(b) The angular momentum is $\vec{\ell} = m\vec{d} \times \vec{v}$, where m is the mass of the object. Use the component form of the vector product:

$$\vec{d} \times \vec{v} = (yv_z - zv_y)\,\hat{\imath} + (zv_x - xv_z)\,\hat{\jmath} + (xv_y - yv_x)\,\hat{k}\,.$$

(c) The torque is $\vec{\tau} = m\vec{d} \times \vec{F}$. Use the component form of the vector product:

$$\vec{d} \times \vec{F} = (yF_z - zF_y)\,\hat{\imath} + (zF_x - xF_z)\,\hat{\jmath} + (xF_y - yF_x)\,\hat{k}\,.$$

(d) Equate the two forms of the scalar product, $\vec{v} \cdot \vec{F} = vF \cos\phi$ and $\vec{v} \cdot \vec{F} = v_xF_x + v_yF_y + v_zF_z$ and solve for ϕ, the angle between $\vec{v}$ and $\vec{F}$.

[ans: (a) $(3.00\,\mathrm{m/s^2})\,\hat{\imath} - (4.00\,\mathrm{m/s^2})\,\hat{\jmath} + (2.00\,\mathrm{m/s^2})\,\hat{k}$; (b) $(42.0\,\mathrm{kg \cdot m^2/s})\,\hat{\imath} + (24.0\,\mathrm{kg \cdot m^2/s})\,\hat{\jmath} + (60.0\,\mathrm{kg \cdot m^2/s})\,\hat{k}$; (c) $(-8.00\,\mathrm{N \cdot m})\,\hat{\imath} - (26.0\,\mathrm{N \cdot m})\,\hat{\jmath} - (40.0\,\mathrm{N \cdot m})\,\hat{k}$; (d) 127°]

(a) The skaters must remain equidistance from the center of the circle and they are 3.0 m apart.

(b) The total angular momentum of the skaters is conserved. While the skaters are skating along straight lines, each has an angular momentum of $mvd/2$ about the center of the circle. Here d is the separation of the skaters. After they start skating around the circle each has an angular momentum of $m(d/2)^2\omega$, where ω is their angular speed. Equate the two expression for the total angular momentum and solve for ω.

(c) The kinetic energy is given by $K = \frac{1}{2}I\omega^2$, where the rotational inertia of the two-skater system is $2m(d/2)^2$.

(d) The angular momentum of the system is conserved as the skaters pull along the pole. Thus $I_{\text{new}}\omega_{\text{new}} = I_{\text{old}}\omega_{\text{old}}$. Solve for ω_{new}.

(e) The kinetic energy is now $K = \frac{1}{2}I\omega_{\text{new}}^2$.

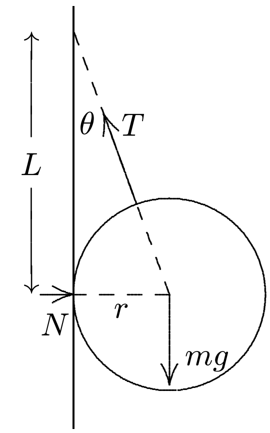
(f) The total energy of the system is the sum of the kinetic, potential, and internal energies and remains constant. The kinetic energy changed but the potential energy did not.

[ans: (a) 1.5 m; (b) 0.93 rad/s; (c) 98 J; (d) 8.4 rad/s; (e) 8.8×10^2 J; (f) internal energy of the skaters]

(a) The angular momentum of the two-disk system is conserved as the small disk slides. Let I_1 be the rotational inertia of the large disk, I_2 be the rotational inertia of the small disk in its initial location, and I_2' be the rotational inertia of the small disk in its final location, all about the axis through the center of the large disk. If ω_i is the initial angular velocity of the disks and ω_f is their final angular velocity, then conservation of angular momentum yields $(I_1 + I_2)\omega_i = (I_1 + I_2')\omega_f$. The center of the small disk moved a distance of $2r$. The parallel axis theorem tells us that $I_2' = I_2 + m(2r)^2$. According to Table 10–2, $I_1 = \frac{1}{2}(10m)(3r)^2$ and $I_2 = \frac{1}{2}mr^2$. You can now solve for ω_f.

(b) The old kinetic energy is $K_0 = \frac{1}{2}(I_1 + I_2)\omega_i^2$ and the new kinetic energy is $K = \frac{1}{2}(I_1 + I_2')\omega_f^2$.

[ans: (a) 18 rad/s; (b) 0.92]



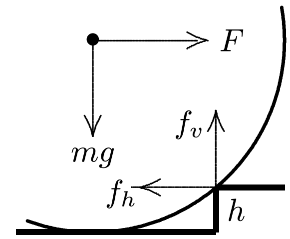
Three forces act on the sphere: the tension force $\vec{T}$ of the rope (acting along the rope), the force of the wall $\vec{N}$ (acting horizontally away from the wall), and the force of gravity $m\vec{g}$ (acting downward). Since the sphere is in equilibrium they sum to zero. Let θ be the angle between the rope and the vertical. Then, the vertical component of Newton's second is $T\cos\theta - mg = 0$. The horizontal component is $N - T\sin\theta = 0$.

(a) Solve the first equation for T: $T = mg/\cos\theta$. Substitute $\cos\theta = L/\sqrt{L^2+r^2}$ to obtain

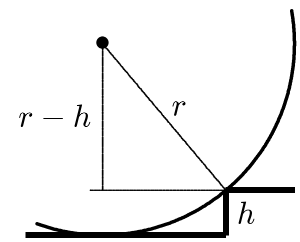
$$T = \frac{mg\sqrt{L^2+r^2}}{L} = \frac{(0.85\,\text{kg})(9.8\,\text{m/s}^2)\sqrt{(0.080\,\text{m})^2+(0.042\,\text{m})^2}}{0.080\,\text{m}}$$
$$= 9.7\,\text{N}\,.$$

(b) Solve the second equation for N: $N = T\sin\theta$. Use $\sin\theta = r/\sqrt{L^2+r^2}$ to obtain

$$N = \frac{Tr}{\sqrt{L^2+r^2}} = \frac{mg\sqrt{L^2+r^2}}{L}\,\frac{r}{\sqrt{L^2+r^2}} = \frac{mgr}{L} = \frac{(0.85\,\text{N})(9.8\,\text{m/s}^2)(0.042\,\text{m})}{0.080\,\text{m}} = 4.4\,\text{N}\,.$$



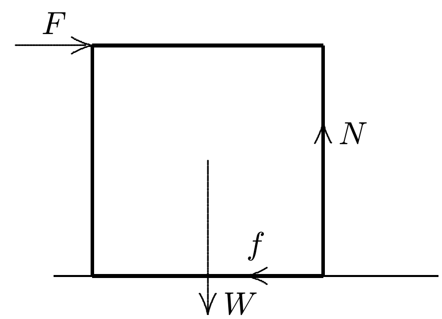
Consider the wheel as it leaves the lower floor. The floor no longer exerts a force on the wheel, and the only forces acting are the force F applied horizontally at the axle, the force of gravity mg acting vertically at the center of the wheel, and the force of the step corner, shown as the two components f_h and f_v. If the minimum force is applied the wheel does not accelerate, so both the total force and the total torque acting on it are zero.



Calculate the torque around the step corner. Look at the second diagram to see that the distance from the line of F to the corner is $r - h$, where r is the radius of the wheel and h is the height of the step. The distance from the line of mg to the corner is $\sqrt{r^2 + (r-h)^2} = \sqrt{2rh - h^2}$. Thus $F(r-h) - mg\sqrt{2rh - h^2} = 0$. The solution for F is

$$F = \frac{\sqrt{2rh - h^2}}{r - h}mg = \frac{\sqrt{2(0.0600\,\text{m})(0.0300\,\text{m}) - (0.0300\,\text{m})^2}}{0.0600\,\text{m} - 0.0300\,\text{m}}(0.800\,\text{kg})(9.8\,\text{m/s}^2) = 13.6\,\text{N}$$

(a) Examine the box when it is about to tip. Since it will rotate about the lower right edge, that is where the normal force of the floor is exerted. This force is labeled N on the diagram to the right. The force of friction is denoted by f, the applied force by F, and the force of gravity by W. Note that the force of gravity is applied at the center of the box. When the minimum force is applied the box does not accelerate, so the sum of the horizontal force components vanishes:



$$F - f = 0\,,$$

the sum of the vertical force components vanishes:

$$N - W = 0\,,$$

and the sum of the torques vanishes:

$$FL - \frac{WL}{2} = 0\,.$$

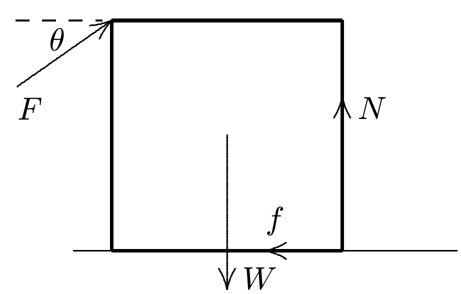
Here L is the length of a side of the box and the origin was chosen to be at the lower right edge.Solve the torque equation for F:

$$F = \frac{W}{2} = \frac{890\,\text{N}}{2} = 445\,\text{N}\,.$$

(b) The coefficient of static friction must be large enough that the box does not slip. The box is on the verge of slipping if $\mu_s = f/N$. According to the equations of equilibrium $N = W = 890\,\text{N}$ and $f = F = 445\,\text{N}$, so $\mu_s = (445\,\text{N})/(890\,\text{N}) = 0.50$.

(c) The box can be rolled with a smaller applied force if the force points upward as well as to the right. Let θ be the angle the force makes with the horizontal. The torque equation then becomes $FL\cos\theta + FL\sin\theta - WL/2 = 0$, with the solution

$$F = \frac{W}{2(\cos\theta + \sin\theta)}\,.$$



You want $\cos\theta + \sin\theta$ to have the largest possible value. This occurs if $\theta = 45°$, a result you can prove by setting the derivative of $\cos\theta + \sin\theta$ equal to zero and solving for θ. The minimum force needed is

$$F = \frac{W}{4\cos 45^\circ} = \frac{890\,\text{N}}{4\cos 45^\circ} = 315\,\text{N}\,.$$

CHAPTER 12 HINT FOR PROBLEM 3

The rope is along straight lines from the supports to the center. Suppose these lines make the angle θ with the horizontal. Then $\tan\theta = 2h/L$, where h is the sag of the rope and L is the distance between supports. The factor 2 arises because the center of the rope is a distance $L/2$ from each support. The rope pulls up on the object with a force of $2T\sin\theta$, where T is the tension in the rope. This must equal the weight of the suspended object. Solve for T

$\left[\text{ans: } 7.92\,\text{kN}\right]$

CHAPTER 12 HINT FOR PROBLEM 15

The floor pushes up on the foot at point P with a force that is equal in magnitude to the weight of the person. To find the force of the calf muscle write an expression for the net torque about B and to find the force of the lower leg bones write an expression for the net torque about A. In each case the net torque must vanish and you can solve the resulting equation for one of the forces.

[ans: (a) 2.7 kN; (b) up; (c) 3.6 kN; (d) down]

CHAPTER 12 HINT FOR PROBLEM 23

(a) Consider the net torque on the strut about the hinge. If M is the mass of the concrete block and L is the length of the strut, the magnitude of the torque produced by the block is $LMg\cos\theta$. If m is the mass of the strut the magnitude of the torque associated with the gravitational force on the strut is $(mgL/2)\cos\theta$. The factor 2 arises because the force of gravity may be taken to act at the center of the strut. A little trigonometry shows that the angle between the strut and the cable is $\theta - \phi$, so if T is the tension in the cable the magnitude of the torque it produces is $TL\sin(\theta - \phi$. The first two torques are clockwise and the last is counterclockwise.l The three torques must sum to zero since the system is in equilibrium. Solve for T.

(b) and (c) The horizontal and vertical components of the net force on the strut are both zero. Use these conditions to solve for the horizontal and vertical components of the force of the hinge.

[ans: (a) 6.63 kN; (b) 5.74 kN; (c) 5.96 kN]

At the point where the forces balance $GM_e m/r_1^2 = GM_s m/r_2^2$, where M_e is the mass of Earth, M_s is the mass of the Sun, m is the mass of the space probe, r_1 is the distance from the center of Earth to the probe, and r_2 is the distance from the center of the Sun to the probe. Substitute $r_2 = d - r_1$, where d is the distance from the center of Earth to the center of the Sun, to find

$$\frac{M_e}{r_1^2} = \frac{M_s}{(d - r_1)^2}\,.$$

Take the positive square root of both sides, then solve for r_1. A little algebra yields

$$r_1 = \frac{d\sqrt{M_e}}{\sqrt{M_s} + \sqrt{M_e}} = \frac{(150 \times 10^9\,\mathrm{m})\sqrt{5.98 \times 10^{24}\,\mathrm{kg}}}{\sqrt{1.99 \times 10^{30}\,\mathrm{kg}} + \sqrt{5.98 \times 10^{24}\,\mathrm{kg}}} = 2.6 \times 10^8\,\mathrm{m}\,.$$

Values for M_e, M_s, and d can be found in Appendix C.

(a) The magnitude of the force on a particle with mass m at the surface of Earth is given by $F = GMm/R^2$, where M is the total mass of Earth and R is Earth's radius. The acceleration due to gravity is

$$a_g = \frac{F}{m} = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11}\,\mathrm{m^3/s^2 \cdot kg})(5.98 \times 10^{24}\,\mathrm{kg})}{(6.37 \times 10^6\,\mathrm{m})^2} = 9.83\,\mathrm{m/s^2}\,.$$

(b) Now $a_g = GM/R^2$, where M is the total mass contained in the core and mantle together and R is the outer radius of the mantle (6.345×10^6 m, according to Fig. 13–36). The total mass is $M = 1.93 \times 10^{24}\,\mathrm{kg} + 4.01 \times 10^{24}\,\mathrm{kg} = 5.94 \times 10^{24}\,\mathrm{kg}$. The first term is the mass of the core and the second is the mass of the mantle. Thus

$$a_g = \frac{(6.67 \times 10^{-11}\,\mathrm{m^3/s^2 \cdot kg})(5.94 \times 10^{24}\,\mathrm{kg})}{(6.345 \times 10^6\,\mathrm{m})^2} = 9.84\,\mathrm{m/s^2}\,.$$

(c) A point 25.0 km below the surface is at the mantle-crust interface and is on the surface of a sphere with a radius of $R = 6.345 \times 10^6$ m. Since the mass is now assumed to be uniformly distributed the mass within this sphere can be found by multiplying the mass per unit volume by the volume of the sphere: $M = (R^3/R_e^3)M_e$, where M_e is the total mass of Earth and R_e is the radius of Earth. Thus

$$M = \left[\frac{6.345 \times 10^6\,\mathrm{m}}{6.37 \times 10^6\,\mathrm{m}}\right]^3 (5.98 \times 10^{24}\,\mathrm{kg}) = 5.91 \times 10^{24}\,\mathrm{kg}\,.$$

The acceleration due to gravity is

$$a_g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11}\,\mathrm{m^3/s^2 \cdot kg})(5.91 \times 10^{24}\,\mathrm{kg})}{(6.345 \times 10^6\,\mathrm{m})^2} = 9.79\,\mathrm{m/s^2}\,.$$

Let N be the number of stars in the galaxy, M be the mass of the Sun, and r be the radius of the galaxy. The total mass in the galaxy is NM and the magnitude of the gravitational force acting on the Sun is $F = GNM^2/r^2$. The force points toward the galactic center. The magnitude of the Sun's acceleration is $a = v^2/R$, where v is its speed. If T is the period of the Sun's motion around the galactic center then $v = 2\pi R/T$ and $a = 4\pi^2 R/T^2$. Newton's second law yields $GNM^2/R^2 = 4\pi^2 MR/T^2$. The solution for N is

$$N = \frac{4\pi^2 R^3}{GT^2 M}\,.$$

The period is 2.5×10^8 y, which is 7.88×10^{15} s, so

$$N = \frac{4\pi^2 (2.2 \times 10^{20}\,\mathrm{m})^3}{(6.67 \times 10^{-11}\,\mathrm{m^3/s^2 \cdot kg})(7.88 \times 10^{15}\,\mathrm{s})^2 (2.0 \times 10^{30}\,\mathrm{kg})} = 5.1 \times 10^{10}\,.$$

(a) Use the law of periods: $T^2 = (4\pi^2/GM)r^3$, where M is the mass of the Sun (1.99×10^{30} kg) and r is the radius of the orbit. The radius of the orbit is twice the radius of Earth's orbit: $r = 2r_e = 2(150 \times 10^9\,\text{m}) = 300 \times 10^9\,\text{m}$. Thus

$$\begin{aligned} T &= \sqrt{\frac{4\pi^2 r^3}{GM}} \\ &= \sqrt{\frac{4\pi^2(300 \times 10^9\,\text{m})^3}{(6.67 \times 10^{-11}\,\text{m}^3/\text{s}^2 \cdot \text{kg})(1.99 \times 10^{30}\,\text{kg})}} = 8.96 \times 10^7\,\text{s}\,. \end{aligned}$$

Divide by $(365\,\text{d/y})(24\,\text{h/d})(60\,\text{min/h})(60\,\text{s/min})$ to obtain $T = 2.8\,\text{y}$.

(b) The kinetic energy of any asteroid or planet in a circular orbit of radius r is given by $K = GMm/2r$, where m is the mass of the asteroid or planet. Notice that it is proportional to m and inversely proportional to r. The ratio of the kinetic energy of the asteroid to the kinetic energy of Earth is $K/K_e = (m/m_e)(r_e/r)$. Substitute $m = 2.0 \times 10^{-4} m_e$ and $r = 2r_e$ to obtain $K/K_e = 1.0 \times 10^{-4}$.

Symmetry tells us that the horizontal component of the net force on the central sphere is zero. A little geometry shows that the distance from a vertex to the center of the triangle is $\sqrt{3}L/4$, where L is the length of a triangle side. Furthermore, the angle between a side and the line from a vertex to the center is 30° and the sine of this angle is $1/2$. Thus vertical component of the force on the central sphere is

$$F_{\text{net, }y} = \frac{GMm_4}{(\sqrt{3}L/4)^2} - \frac{2Gmm_4(1/2)}{(\sqrt{3}L/4)^2}\,.$$

Set this expression equal to zero and solve for M. Notice that the algebraic result for M does not depend on the value of m_4.

[ans: (a) m; (b) 0]

The gravitational force on a particle that is located at the equator has magnitude $F = GmM/R^2$, where M is the mass of the star and R is its radius. For the particle to remain on the surface this must equal the centripetal force $m\omega^2 R$ required to keep the particle on its circular path. Here ω is the angular speed of the star. Set these two expression equal and solve for M.

$\big[$ans: 5×10^{24} kg $\big]$

CHAPTER 13 HINT FOR PROBLEM 29

If $U(R_s)$ is the gravitational potential energy when the projectile when it is on the surface of the planet and K is its initial kinetic energy, then to escape the planet the mechanical energy must be at least zero. Set the mechanical energy equal to zero and solve for K.

$\left[\text{ans: } 5.0 \times 10^9\ \text{J}\ \right]$

CHAPTER 13 HINT FOR PROBLEM 39

The period T and orbit radius r are related by the law of periods: $T^2 = (4\pi^2/GM)r^3$, where M is the mass of Mars. Convert the given period to seconds and solve for M.

$\left[\text{ans: } 6.5 \times 10^{23}\text{ kg}\right]$

CHAPTER 13 HINT FOR PROBLEM 47

(a) The semimajor axis is given by

$$a = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3},$$

where M is the mass of the Sun (which can be found in Appendix C), and T is the period of the motion. Be sure to substitute the period in seconds.

(b) The eccentricity e of the orbit is related to the aphelion distance R_a by

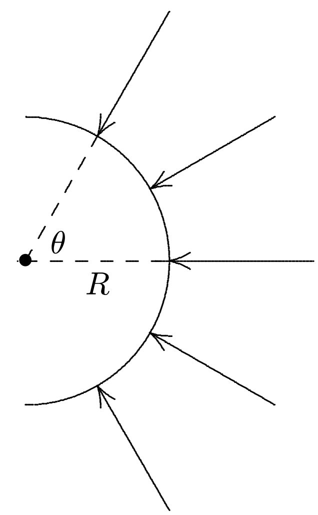
$$e = 1 - \frac{R_a}{a}.$$

The mean orbital radius of Pluto can also be found in Appendix C.

[ans: (a) 1.9×10^{13} m; (b) $3.6R_p$]

The period T is given by the Kepler's law of periods: $T^2 = (4\pi^2/GM)r^3$. The speed is given by $v = 2\pi r/T$, the kinetic energy by $K = \frac{1}{2}Mv^2$, and the magnitude of the angular momentum by mrv.

[ans: (a) $r^{3/2}$; (b) $1/r$; (c) $\sqrt{r}$; (d) $1/\sqrt{r}$]



(a) At every point on the surface there is a net inward force, normal to the surface, due to the difference in pressure between the air inside and outside the sphere. The diagram to the right shows half the sphere and some of the force vectors. We suppose a team of horses is pulling to the right. To pull the sphere apart it must exert a force at least as great as the horizontal component of the net force of the air.

Consider the force acting at the angle θ shown. Its horizontal component is $\Delta p \cos\theta \mathrm{d}A$, where $\mathrm{d}A$ is an infinitesimal area element at the point where the force is applied. We take the area to be that of a ring of constant θ on the surface. The radius of the ring is $R\sin\theta$, where R is the radius of the sphere. If the angular width of the ring is $\mathrm{d}\theta$, in radians, then its width is $R\,\mathrm{d}\theta$ and its area is $\mathrm{d}A = 2\pi R^2 \sin\theta\,\mathrm{d}\theta$. Thus the net horizontal component of the force of the air is given by

$$F_h = 2\pi R^2\,\Delta p \int_0^{\pi/2} \sin\theta\cos\theta\,\mathrm{d}\theta$$
$$= \pi R^2\,\Delta p \sin^2\theta\,\Big|_0^{\pi/2} = \pi R^2\,\Delta p\,.$$

This is the force that must be exerted by each team of horses to pull the sphere apart.

(b) Use $1\,\mathrm{atm} = 1.00 \times 10^5\,\mathrm{Pa}$ to show that $\Delta p = 0.90\,\mathrm{atm} = 9.00 \times 10^4\,\mathrm{Pa}$ The sphere radius is $0.30\,\mathrm{m}$, so $F_h = \pi(0.30\,\mathrm{m})^2(9.00 \times 10^4\,\mathrm{Pa}) = 2.5 \times 10^4\,\mathrm{N}$.

(c) One team of horses could be used if one half of the sphere is attached to a sturdy wall. The force of the wall on the sphere would balance the force of the horses. Two teams were probably used to heighten the dramatic effect.

(a) At depth y the gauge pressure of the water is $p = \rho g y$, where ρ is the density of the water. Consider a strip of water with width W and thickness $\mathrm{d}y$, across the dam. Its area is $\mathrm{d}A = W\,\mathrm{d}y$ and the force it exerts on the dam is $\mathrm{d}F = p\,\mathrm{d}A = \rho g W y\,\mathrm{d}y$. The total force of the water on the dam is

$$\begin{aligned} F &= \int_0^D \rho g W y\,\mathrm{d}y = \tfrac{1}{2}\rho g W D^2 \\ &= \tfrac{1}{2}(0.998 \times 10^3\,\mathrm{kg/m^3})(9.8\,\mathrm{m/s^2})(314\,\mathrm{m})(35.0\,\mathrm{m})^2 = 1.80 \times 10^9\,\mathrm{N}\,. \end{aligned}$$

(b) Again consider the strip of water at depth y. Its moment arm for the torque it exerts about O is $D - y$ so the torque it exerts is $\mathrm{d}\tau = \mathrm{d}F(D - y) = \rho g W y(D - y)\,\mathrm{d}y$ and the total torque of the water is

$$\begin{aligned} \tau &= \int_0^D \rho g W y(D - y)\,\mathrm{d}y = \rho g W\left(\frac{1}{2}D^3 - \frac{1}{3}D^3\right) = \tfrac{1}{6}\rho g W D^3 \\ &= \tfrac{1}{6}(0.998\,\mathrm{kg/m^3})(9.8\,\mathrm{m/s^2})(314\,\mathrm{m})(35.0\,\mathrm{m})^3 = 2.19 \times 10^{10}\,\mathrm{m}\,. \end{aligned}$$

(c) Write $\tau = rF$, where r is the effective moment arm. Then,

$$r = \frac{\tau}{F} = \frac{\frac{1}{6}\rho g W D^3}{\frac{1}{2}\rho g W D^2} = \frac{D}{3} = \frac{35.0\,\mathrm{m}}{3} = 11.7\,\mathrm{m}\,.$$

(a) The force of gravity mg is balanced by the buoyant force of the liquid $\rho g V_s$: $mg = \rho g V_s$. Here m is the mass of the sphere, ρ is the density of the liquid, and V_s is the submerged volume. Thus $m = \rho V_s$. The submerged volume is half the volume enclosed by the outer surface of the sphere, or $V_s = \frac{1}{2}(4\pi/3)r_o^3$, where r_o is the outer radius. This means

$$m = \frac{4\pi}{6}\rho r_o^3 = \left(\frac{4\pi}{6}\right)(800\,\text{kg/m}^3)(0.090\,\text{m})^3 = 1.22\,\text{kg}\,.$$

Air in the hollow sphere, if any, has been neglected.

(b) The density ρ_m of the material, assumed to be uniform, is given by $\rho_m = m/V$, where m is the mass of the sphere and V is its volume. If r_i is the inner radius, the volume is

$$V = \frac{4\pi}{3}\left(r_o^3 - r_i^3\right) = \frac{4\pi}{3}\left[(0.090\,\text{m})^3 - (0.080\,\text{m})^3\right] = 9.09 \times 10^{-4}\,\text{m}^3\,.$$

The density is

$$\rho = \frac{1.22\,\text{kg}}{9.09 \times 10^{-4}\,\text{m}^3} = 1.3 \times 10^3\,\text{kg/m}^3\,.$$

(a) The continuity equation yields $Av = aV$ and Bernoulli's equation yields $\frac{1}{2}\rho v^2 = \Delta p + \frac{1}{2}\rho V^2$, where $\Delta p = p_2 - p_1$. The first equation gives $V = (A/a)v$. Use this to substitute for V in the second equation. You should obtain $\frac{1}{2}\rho v^2 = \Delta p + \frac{1}{2}\rho(A/a)^2 v^2$. Solve for v. The result is

$$v = \sqrt{\frac{2\,\Delta p}{\rho\left(1 - \frac{A^2}{a^2}\right)}} = \sqrt{\frac{2a^2\,\Delta p}{\rho(a^2 - A^2)}}\,.$$

(b) Substitute values to obtain

$$v = \sqrt{\frac{2(32 \times 10^{-4}\,\mathrm{m}^2)^2(41 \times 10^3\,\mathrm{Pa} - 55 \times 10^3\,\mathrm{Pa})}{(998\,\mathrm{kg/m^3})\left[(32 \times 10^{-4}\,\mathrm{m}^2)^2 - (64 \times 10^{-4}\,\mathrm{m}^2)^2\right]}} = 3.06\,\mathrm{m/s}\,.$$

The density of water was obtained from Table 14–1 of the text. The flow rate is $Av = (64 \times 10^{-4}\,\mathrm{m}^2)(3.06\,\mathrm{m/s}) = 2.0 \times 10^{-2}\,\mathrm{m^3/s}$.

CHAPTER 14 HINT FOR PROBLEM 9

The pressure p in a fluid at depth h below the surface is $p = p_0 + \rho g h$, where p_0 is the pressure at the surface and ρ is the density of the fluid.

[ans: 1.90×10^4 Pa]

The force on the output piston has magnitude $F_o = kx$, where k is the spring constant and x is the distance the spring is compressed. The force on the input piston has magnitude $F_i = mg$, where m is the mass of sand in the container. The forces obey Pascal's principle: $F_o/A_0 = F_i/A_i$, where A_o is the cross-sectional area of the output piston and A_i is the cross-sectional area of the input piston. Replace F_o with kx and F_i with mg, then solve for m.

[ans: 8.50 kg]

(a) Let V be the volume of water displaced by the car and ρ_w $(= 0.998 \times 10^3\,\mathrm{kg/m^3})$ be the density of water. Then the magnitude of the buoyant force on the car is $\rho_w g V$. Since the car is essentially in equilibrium this must equal the weight mg of the car, where m is its mass. solve $mg = \rho_w g V$ for V.

(b) Let V_w be the volume of water in the car. The total weight of the car and the water in it is $mg + \rho_w g V_w$. The magnitude of the buoyant force is now $\rho_w g V_{\text{total}}$, where V_{total} is the total volume of the car $(= 5.00\,\mathrm{m^3} + 0.75\,\mathrm{m^3} + 0.800\,\mathrm{m^3} = 6.55\,\mathrm{m^3})$. Solve $mg + \rho_w g V_w = \rho_w g V_{\text{total}}$ for V_w.

[ans: $4.75\,\mathrm{m^3}$]

(a) Use the equation of continuity: $V_b A_b = v_2 A_2$, where v_b is the water speed and A_b is the cross-sectional ares of the pipe at the basement and v_2 is the speed and A_2 is the cross-sectional area of the pipe at the second floor. If d is the diameter of the pipe its cross-sectional area is $\pi d^2/4$. Solve for v_2.

(b) Use the Bernoulli equation: $p_b + \frac{1}{2}\rho v_b^2 = p_2 + \frac{1}{2}v_2^2 + h$, where p_b is the pressure at the basement, p_2 is the pressure at the second floor, h is the height of the second floor above the basement, and ρ is the density of water.

[ans: (a) $3.9\,\mathrm{m/s}$; (b) $88\,\mathrm{kPa}$]

Use Bernoulli's principle and the projectile motion equations to develop an expression for x as a function of h. The pressure inside the tank at the hole is $p_0 + \rho g h$, where p_0 is atmospheric pressure and ρ is the density of water. The pressure outside the tank at the hole is p_0. Let v_0 be the speed of the water as it leaves the hole. The speed of the water inside the tank is quite small and may be taken to be zero. Then the principle yields $p_0 + \rho g h = p_0 + \frac{1}{2}\rho v_0^2$. Algebraically solve this equation for v_0. If it takes time Δt for water to fall from the hole to the ground, a distance $H - h$, then $H - h = \frac{1}{2}g(\Delta t)^2$. Solve for Δt. The distance from the tank to the point on the ground where the water hits is $x = v_0\,\Delta t$.

(a) Evaluate the expression you developed for x.

(b) Solve the expression for h. There are two solutions. One is 10 cm. You want the other one.

(c) Set the derivative of x with respect to h equal to zero and solve for h.

[ans: (a) 35 cm; (b) 30 cm; (c) 20 cm]

CHAPTER 14 HINT FOR PROBLEM 61

Bernoulli's principle gives $p_A = p_B + \frac{1}{2}\rho_{\text{air}}v^2$. The pressure difference is $p_A - p_B = \rho g h$. Use this to substitute for $p_A - p_B$ in the Bernoulli equation, then solve for v.

$\left[\text{ans: (b) } 63.3\,\text{m/s}\right]$

The maximum force that can be exerted by the surface must be less than $\mu_s N$ or else the block will not follow the surface in its motion. Here, μ_s is the coefficient of static friction and N is the normal force exerted by the surface on the block. Since the block does not accelerate vertically, you know that $N = mg$, where m is the mass of the block. If the block follows the table and moves in simple harmonic motion, the magnitude of the maximum force exerted on it is given by $F = ma_m = m\omega^2 x_m = m(2\pi f)^2 x_m$, where a_m is the magnitude of the maximum acceleration, ω is the angular frequency, and f is the frequency. The relationship $\omega = 2\pi f$ was used to obtain the last form.

Substitute $F = m(2\pi f)^2 x_m$ and $N = mg$ into $F < \mu_s N$ to obtain $m(2\pi f)^2 x_m < \mu_s mg$. The largest amplitude for which the block does not slip is

$$x_m = \frac{\mu_s g}{(2\pi f)^2} = \frac{(0.50)(9.8\,\mathrm{m/s^2})}{(2\pi \times 2.0\,\mathrm{Hz})^2} = 0.031\,\mathrm{m}\,.$$

A larger amplitude requires a larger force at the end points of the motion. The surface cannot supply the larger force and the block slips.

(a) Take the angular displacement of the wheel to be $\theta = \theta_m \cos(2\pi t/T)$, where θ_m is the amplitude and T is the period. Differentiate with respect to time to find the angular velocity: $\Omega = -(2\pi/T)\theta_m \sin(2\pi t/T)$. The symbol Ω is used for the angular velocity of the wheel so it is not confused with the angular frequency. The maximum angular velocity is

$$\Omega_m = \frac{2\pi\theta_m}{T} = \frac{(2\pi)(\pi\,\text{rad})}{0.500\,\text{s}} = 39.5\,\text{rad/s}\,.$$

(b) When $\theta = \pi/2$, then $\theta/\theta_m = 1/2$, $\cos(2\pi t/T) = 1/2$, and

$$\sin(2\pi t/T) = \sqrt{1 - \cos^2(2\pi t/T)} = \sqrt{1 - (1/2)^2} = \sqrt{3}/2\,,$$

where the trigonometric identity $\cos^2 A + \sin^2 A = 1$ was used. Thus

$$\Omega = -\frac{2\pi}{T}\,\theta_m \sin\left(\frac{2\pi t}{T}\right) = -\left(\frac{2\pi}{0.500\,\text{s}}\right)(\pi\,\text{rad})\left(\frac{\sqrt{3}}{2}\right) = -34.2\,\text{rad/s}\,.$$

The minus sign is not significant. During another portion of the cycle its angular speed is $+34.2\,\text{rad/s}$ when its angular displacement is $\pi/2$ rad.

(c) The angular acceleration is

$$\alpha = \frac{d^2\theta}{dt^2} = -\left(\frac{2\pi}{T}\right)^2 \theta_m \cos(2\pi t/T) = -\left(\frac{2\pi}{T}\right)^2 \theta\,.$$

When $\theta = \pi/4$,

$$\alpha = -\left(\frac{2\pi}{0.500\,\text{s}}\right)^2 \left(\frac{\pi}{4}\right) = -124\,\text{rad/s}^2\,.$$

Again the minus sign is not significant.

(a) The period of the pendulum is given by $T = 2\pi\sqrt{I/mgd}$, where I is its rotational inertia, m is its mass, and d is the distance from the center of mass to the pivot point. The rotational inertia of a rod pivoted at its center is $mL^2/12$ and, according to the parallel-axis theorem, its rotational inertia when it is pivoted a distance d from the center is $I = mL^2/12 + md^2$. Thus

$$T = 2\pi\sqrt{\frac{m(L^2/12 + d^2)}{mgd}} = 2\pi\sqrt{\frac{L^2 + 12d^2}{12gd}} .$$

The square of the period is $4\pi^2(L^2 + 12d^2)/12gd)$ and the derivative of this with respect to d is

$$\frac{dT^2}{dd} = \frac{4\pi^2}{12g}\left[24 - \frac{L^2 + 12d^2}{d^2}\right] .$$

Set this equal to zero and solve for d. The result is $d = L/\sqrt{12}$. Substitute this back into the expression for T and obtain

$$T = 2\pi\sqrt{\frac{L}{\sqrt{3}g}} = 2\pi\sqrt{\frac{2.20\,\mathrm{m}}{\sqrt{3}(9.8\,\mathrm{m/s^2})}} = 2.26\,\mathrm{s} .$$

(b) According to the expression obtained in part (a) for the minimum period this period is proportional to $\sqrt{L}$, so it increases as L increases.

(c) According to the expression obtained in part (a) for the minimum period this period is independent of m. T does not change when m increases.

(a) The displacement x and acceleration a at any instant of time are related by $x = -\omega^2 a$, where ω is the angular frequency. The frequency is $f = \omega/2\pi$.

(b) The angular frequency, mass m, and spring constant k are related by $\omega^2 = k/m$.

(c) If x_m is the amplitude then we may take the displacement to be $x = x_m \cos(\omega t)$. The velocity is $v = -\omega x_m \sin(\omega t)$. Use the trigonometric identity $\sin^2(\omega t)+\cos^2(\omega t) = 1$ to find an expression for x_m.

[ans: (a) 5.58 Hz; (b) 0.325 kg; (c) 0.400 m]

CHAPTER 15 HINT FOR PROBLEM 25

Suppose the smaller block is on the verge of slipping when the acceleration of the blocks has its maximum value. This occurs when the displacement of the blocks is equal to the amplitude x_m of their oscillation and the value is $a_m = \omega^2 x_m$, where ω is the angular frequency of the oscillation. The magnitude of the force of friction is $f = ma_m = m\omega^2 x_m$. Since the block is on the verge of slipping $f = \mu_s N = \mu_s mg$, where N $(= mg)$ is the magnitude of the normal force of either block on the other. Solve for x_m. The angular frequency is given by $\omega = \sqrt{k/(m + M)}$.

[ans: 22 cm]

CHAPTER 15 HINT FOR PROBLEM 33

(a) Use $f = (1/2\pi)\sqrt{k/m}$, where k is the spring constant and m is the mass of the object.
(b) The potential energy is given by $U = \frac{1}{2}kx^2$.
(c) The kinetic energy is given by $K = \frac{1}{2}mv^2$, where v is the speed of the object.
(d) The mechanical energy, which is the sum of the kinetic and potential energies, is $\frac{1}{2}kx_m^2$, where x_m is the amplitude of the oscillation.

[ans: (a) 2.25 Hz; (b) 1125 J; (c) 250 J; (d) 86.6 cm]

CHAPTER 15 HINT FOR PROBLEM 41

The period of a physical pendulum with rotational inertia I and mass m, suspended from a point that is a distance h from its center of mass, is $T = 2\pi\sqrt{I/mgh}$. The distance between the point of suspension and the center of oscillation is the same as the length of a simple pendulum with the same period. The period of a simple pendulum of length L_0 is $T = 2\pi\sqrt{L_0/g}$. Set these two expressions for the period equal to each other and solve for L_0.

At time $t = 0$ the angle is $\theta_0 = \theta_m \cos(\phi)$ and its rate of change is $(d\theta/dt)_0 = -\omega\theta_m \sin(\phi)$, where ω (= 4.44 rad/s) is the angular frequency of oscillation. Solve the first equation for $\cos(\phi)$ and the second for $\sin(\phi)$. Use the results to find an expression for $\tan(\phi)$ in terms of θ_0, $(d\theta/dt)_0$, and ω. Then find θ itself. Be sure you obtain the correct result for ϕ. Both the sine and cosine should be positive. Now add the expressions for the square of the sine and the square of the cosine to find an expression for θ_m.

[ans: (a) 0.845 rad; (b) 0.0602 rad]

(a) In the expression for y, the quantity y_m is the amplitude and so is $0.12\,\text{mm}$.

(b) The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string, so the wavelength is $\lambda = v/f = \sqrt{\tau/\mu}/f$ and the angular wave number is

$$k = \frac{2\pi}{\lambda} = 2\pi f\sqrt{\frac{\mu}{\tau}} = 2\pi(100\,\text{Hz})\sqrt{\frac{0.50\,\text{kg/m}}{10\,\text{N}}} = 141\,\text{m}^{-1}\,.$$

(c) The frequency is $f = 100\,\text{Hz}$, so the angular frequency is $\omega = 2\pi f = 2\pi(100\,\text{Hz}) = 628\,\text{rad/s}$.

(d) The positive sign is used since the wave is traveling in the negative x direction.

(a) The wave speed at any point on the rope is given by $v = \sqrt{\tau/\mu}$, where τ is the tension at that point and μ is the linear mass density. Because the rope is hanging the tension varies from point to point. Consider a point on the rope a distance y from the bottom end. The forces acting on it are the weight of the rope below it, pulling down, and the tension, pulling up. Since the rope is in equilibrium these balance. The weight of the rope below is given by $\mu g y$, so the tension is $\tau = \mu g y$. The wave speed is $v = \sqrt{\mu g y/\mu} = \sqrt{gy}$.

(b) The time dt for the wave to move past a length dy, a distance y from the bottom end, is $dt = dy/v = dy/\sqrt{gy}$ and the total time for the wave to move the entire length of the rope is

$$t = \int_0^L \frac{dy}{\sqrt{gy}} = 2\sqrt{\frac{y}{g}}\,\Bigg|_0^L = 2\sqrt{\frac{L}{g}}\,.$$

Possible wavelengths are given by $\lambda = 2L/n$, where L is the length of the wire and n is an integer. The corresponding frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu} = \sqrt{\tau L/M}$, where τ is the tension in the wire, μ is the linear mass density of the wire, and M is the mass of the wire. $\mu = M/L$ was used to obtain the last form. Thus

$$f = \frac{n}{2L}\sqrt{\frac{\tau L}{M}} = \frac{n}{2}\sqrt{\frac{\tau}{LM}} = \frac{n}{2}\sqrt{\frac{250\,\mathrm{N}}{(10.0\,\mathrm{m})(0.100\,\mathrm{kg})}} = n(7.91\,\mathrm{Hz})\,.$$

(a) For $n = 1$, $f = 7.91\,\mathrm{Hz}$.

(b) For $n = 2$, $f = 15.8\,\mathrm{Hz}$.

(c) For $n = 3$, $f = 23.7\,\mathrm{Hz}$.

CHAPTER 16 SOLUTION FOR PROBLEM 49

The waves have the same amplitude, the same angular frequency, and the same angular wave number, but they travel in opposite directions.

(a) The amplitude of each of the constituent waves is half the amplitude of the standing wave or 0.50 cm.

(b) Since the standing wave has three loops the string is three half-wavelengths long. If L is the length of the string and λ is the wavelength, then $L = 3\lambda/2$, or $\lambda = 2L/3 = 2(3.0\,\text{m})/3 = 2.0\,\text{m}$. The angular wave number is $k = 2\pi/\lambda = 2\pi/(2.0\,\text{m}) = 3.1\,\text{m}^{-1}$.

(c) If v is the wave speed, then the frequency is

$$f = \frac{v}{\lambda} = \frac{3v}{2L} = \frac{3(100\,\text{m/s})}{2(3.0\,\text{m})} = 50\,\text{Hz}\,.$$

The angular frequency is $\omega = 2\pi f = 2\pi(50\,\text{Hz}) = 3.1 \times 10^2\,\text{rad/s}$.

(d) Since the first wave travels in the negative x direction, the second wave must travel in the positive x direction and the sign in front of ω must be the negative sign.

CHAPTER 16 HINT FOR PROBLEM 11

Determine the sign in front of ω from the direction of travel of the wave. At $t = 0$ the displacement at $x = 0$ is $y_m \sin(\phi)$. Solve this for ϕ and, of the possible solutions, choose the value that makes $\sin(-\omega t + \phi)$ a positive sine function. Then the displacement for $t = 0$ is $y_m \sin(kx + \phi)$. At $x = 0$ the wave is given by $y = y_m \sin(\pm\omega t + \phi)$, where you should use the appropriate sign in front of ω.

The amplitude is the maximum displacement of the string from $y = 0$. The angular wave number is $k = 2\pi/\lambda$, where λ is the wavelength. The angular frequency is $\omega = 2\pi/T$, where T is period, which can be read from the graph. The wave speed is given by $v = \omega/k$. The transverse particle velocity is the derivative of y with respect to t.

[ans: (a) negative sine function; (b) 4.0 cm; (c) $0.31\,\mathrm{cm}^{-1}$; (d) $0.63\,\mathrm{s}^{-1}$; (e) π rad; (f) negative sign; (g) 2.0 cm/s; (h) -2.5 cm/s]

The rate with which kinetic energy passes a point on the cord is given by

$$\frac{dK}{dt} = \tfrac{1}{2}\mu v\omega^2 y_m^2 \cos^2(kx - \omega t)\,,$$

where μ is the linear mass density of the cord, v is the wave speed, ω is the angular frequency, and y_m is the amplitude. A maximum on the graph has the value $\frac{1}{2}\mu v\omega^2 y_m^2$. Read this value from either graph. The linear mass density of the cord is given. The period T can be read from the second graph and $\omega = 2\pi/T$ can be used to compute ω. The wavelength λ can be read from the first graph and the wave speed can be computed using $v = \lambda/T$. You can now compute y_m.

[ans: 3.2 mm]

The amplitude y_m is given in the problem statement. The angular wave number is given by $k = 2\pi/\lambda$, where λ is the wavelength, which is the same for both waves and for the resultant. It can therefore be read from the graph. The angular frequency is given by $\omega = v/k$, where v is the wave speed, which you can easily calculate since you know that the wave travels 57.0 cm in 8.0 ms. The resultant amplitude, which can be read from the graph, is related to the phase constant ϕ_2 by $y_{\text{result},\,m} = 2y_m \cos(\phi_2/2)$. Solve for ϕ_2. The sign in front of ω is determined by the direction of travel of the wave.

[ans: (a) 9.0 mm; (b) $16\,\text{m}^{-1}$; (c) $1.1 \times 10^3\,\text{s}^{-1}$; (d) 2.7 rad; (e) positive sign]

CHAPTER 16 HINT FOR PROBLEM 43

(a) The wave speed is determined by the linear mass density of the string and the tension in it.

(b) The wavelength is twice the distance between nodes of the standing wave pattern. In this case one and one-half wavelengths fit into the distance between the string ends.

(c) The frequency f is given by $f = v/\lambda$, where v is the wave speed and λ is the wavelength.

[ans: (a) $144\,\mathrm{m/s}$; (b) $60.0\,\mathrm{cm}$; (c) $241\,\mathrm{Hz}$]

Let t_f be the time for the stone to fall to the water and t_s be the time for the sound of the splash to travel from the water to the top of the well. Then, the total time elapsed from dropping the stone to hearing the splash is $t = t_f + t_s$. If d is the depth of the well, then the kinematics of free fall gives $d = \frac{1}{2}gt_f^2$, or $t_f = \sqrt{2d/g}$. The sound travels at a constant speed v_s, so $d = v_s t_s$, or $t_s = d/v_s$. Thus the total time is

$$t = \sqrt{\frac{2d}{g}} + \frac{d}{v_s}\,.$$

This equation is to be solved for d. Rewrite it as

$$\sqrt{\frac{2d}{g}} = t - \frac{d}{v_s}$$

and square both sides to obtain

$$\frac{2d}{g} = t^2 - 2\frac{t}{v_s}d + \frac{1}{v_s^2}d^2\,.$$

Now multiply by gv_s^2 and rearrange to get

$$gd^2 - 2v_s(gt + v_s)d + gv_s^2t^2 = 0\,.$$

This is a quadratic equation for d. Its solutions are

$$d = \frac{2v_s(gt + v_s) \pm \sqrt{4v_s^2(gt + v_s)^2 - 4g^2v_s^2t^2}}{2g}\,.$$

The physical solution must yield $d = 0$ for $t = 0$, so we take the solution with the negative sign in front of the square root. Once values are substituted the result $d = 40.7\,\mathrm{m}$ is obtained.

CHAPTER 17 SOLUTION FOR PROBLEM 25

(a) Let I_1 be the original intensity and I_2 be the final intensity. The original sound level is $\beta_1 = (10\,\text{dB})\log(I_1/I_0)$ and the final sound level is $\beta_2 = (10\,\text{dB})\log(I_2/I_0)$, where I_0 is the reference intensity. Since $\beta_2 = \beta_1 + 30\,\text{dB}$,

$$(10\,\text{dB})\log(I_2/I_0) = (10\,\text{dB})\log(I_1/I_0) + 30\,\text{dB}\,,$$

or

$$(10\,\text{dB})\log(I_2/I_0) - (10\,\text{dB})\log(I_1/I_0) = 30\,\text{dB}\,.$$

Divide by 10 dB and use $\log(I_2/I_0) - \log(I_1/I_0) = \log(I_2/I_1)$ to obtain $\log(I_2/I_1) = 3$. Now use each side as an exponent of 10 and recognize that

$$10^{\log(I_2/I_1)} = I_2/I_1\,.$$

The result is $I_2/I_1 = 10^3$. The intensity is increased by a factor of 1.0×10^3.

(b) The pressure amplitude is proportional to the square root of the intensity so it is increased by a factor of $\sqrt{1000} = 32$.

(a) The expression for the Doppler shifted frequency is

$$f' = f\,\frac{v \pm v_D}{v \mp v_S}\,,$$

where f is the unshifted frequency, v is the speed of sound, v_D is the speed of the detector (the uncle), and v_S is the speed of the source (the locomotive). All speeds are relative to the air. The uncle is at rest with respect to the air, so $v_D = 0$. The speed of the source is $v_S = 10\,\mathrm{m/s}$. Since the locomotive is moving away from the uncle the frequency decreases and we use the plus sign in the denominator. Thus

$$f' = f\,\frac{v}{v + v_S} = (500.0\,\mathrm{Hz})\left(\frac{343\,\mathrm{m/s}}{343\,\mathrm{m/s} + 10.00\,\mathrm{m/s}}\right) = 485.8\,\mathrm{Hz}\,.$$

(b) The girl is now the detector. Relative to the air she is moving with speed $v_D = 10.00\,\mathrm{m/s}$ toward the source. This tends to increase the frequency and we use the plus sign in the numerator. The source is moving at $v_S = 10.00\,\mathrm{m/s}$ away from the girl. This tends to decrease the frequency and we use the plus sign in the denominator. Thus $(v + v_D) = (v + v_S)$ and $f' = f = 500.0\,\mathrm{Hz}$.

(c) Relative to the air the locomotive is moving at $v_S = 20.00\,\mathrm{m/s}$ away from the uncle. Use the plus sign in the denominator. Relative to the air the uncle is moving at $v_D = 10.00\,\mathrm{m/s}$ toward the locomotive. Use the plus sign in the numerator. Thus

$$f' = f\,\frac{v + v_D}{v + v_S} = (500.0\,\mathrm{Hz})\left(\frac{343\,\mathrm{m/s} + 10.00\,\mathrm{m/s}}{343\,\mathrm{m/s} + 20.00\,\mathrm{m/s}}\right) = 486.2\,\mathrm{Hz}\,.$$

(d) Relative to the air the locomotive is moving at $v_S = 20.00\,\mathrm{m/s}$ away from the girl and the girl is moving at $v_D = 20.00\,\mathrm{m/s}$ toward the locomotive. Use the plus signs in both the numerator and the denominator. Thus $(v + v_D) = (v + v_S)$ and $f' = f = 500.0\,\mathrm{Hz}$.

CHAPTER 17 HINT FOR PROBLEM 27

The sound level in decibels is given by $\beta = (10\,\mathrm{dB})\log(I/I_0)$, where I is the intensity and I_0 is the reference intensity ($1 \times 10^{-12}\,\mathrm{W/m^2}$). Solve for I. The intensity is related to the sound displacement amplitude s_m by $I = \frac{1}{2}\rho v \omega^2 s_m^2$, where ρ is the density of air ($1.21\,\mathrm{kg/m^3}$), ω is the angular frequency (2π times the frequency), and v is the speed of sound ($343\,\mathrm{m/s}$).

[ans: (a) $10\,\mu\mathrm{W/m^2}$; (b) $0.10\,\mu\mathrm{W/m^2}$; (c) 70 nm; (d) 7.0 nm]

(a) The rate of energy transport is given by $\frac{1}{2}\rho v\omega^2 s_m^2 A$, where ρ is the density of air ($1.21\,\mathrm{kg/m^3}$), s_m is the displacement amplitude ($12.0\,\mathrm{nm}$), ω is the angular frequency, v is the speed of sound ($343\,\mathrm{m/s}$), and A is the cross-sectional area of the tube. Use $A = \pi R^2$, where R is the internal radius of the tube, to compute A.

(b) The two waves together carry twice as much energy as one wave alone.

(c), (d), and (e) Since the waves interfere the amplitude is now $2s_m \cos(\phi/2)$, where ϕ is the phase difference.

[ans: (a) 0.34 nW; (b) 0.68 nW; (c) 1.4 nW; (d) 0.88 nW; (e) 0]

CHAPTER 17 HINT FOR PROBLEM 41

The top of the water is a displacement node and the top of the well is a displacement antinode. At the lowest resonant frequency exactly one-fourth of a wavelength fits into the depth of the well. If d is the depth and λ is the wavelength then $\lambda = 4d$. The frequency is $f = v/\lambda = v/4d$, where v is the speed of sound. The speed of sound is given by $v = \sqrt{B/\rho}$, where B is the bulk modulus and ρ is the density of air in the well.

$\left[\text{ans: } 12.4\,\text{m}\right]$

CHAPTER 17 HINT FOR PROBLEM 55

Use the Doppler shift equation. In the reflector frame the speed of the source is $29.9\,\mathrm{m/s} + 65.8\,\mathrm{m/s} = 95.7\,\mathrm{m/s}$, the speed of the detector (the reflector) is zero, and the speed of sound is $329\,\mathrm{m/s} + 65.8\,\mathrm{m/s} = 394.8\,\mathrm{m/s}$. The source is moving toward the detector. In the source frame the speed of the source is zero, the speed of the detector is $95.7\,\mathrm{m/s}$ and the speed of sound is $329\,\mathrm{m/s} + 29.9\,\mathrm{m/s} = 358.9\,\mathrm{m/s}$. The detector is moving toward the source. In each case the wavelength is the speed of sound divided by the frequency.

[ans: (a) 1.58 kHz; (b) 0.208 m; (c) 2.16 kHz; (d) 0.152 m]

If V_c is the original volume of the cup, α_a is the coefficient of linear expansion of aluminum, and ΔT is the temperature increase, then the change in the volume of the cup is $\Delta V_c = 3\alpha_a V_c \, \Delta T$. See Eq. 18–11. If β is the coefficient of volume expansion for glycerin then the change in the volume of glycerin is $\Delta V_g = \beta V_c \, \Delta T$. Note that the original volume of glycerin is the same as the original volume of the cup. The volume of glycerin that spills is

$$\begin{aligned}\Delta V_g - \Delta V_c &= (\beta - 3\alpha_a) V_c \, \Delta T \\ &= \left[(5.1 \times 10^{-4}\,/\mathrm{C}^\circ) - 3(23 \times 10^{-6}\,/\mathrm{C}^\circ)\right](100\,\mathrm{cm}^3)(6\,\mathrm{C}^\circ) = 0.26\,\mathrm{cm}^3 \,.\end{aligned}$$

(a) There are three possibilities:

1. None of the ice melts and the water-ice system reaches thermal equilibrium at a temperature that is at or below the melting point of ice.

2. The system reaches thermal equilibrium at the melting point of ice, with some of the ice melted.

3. All of the ice melts and the system reaches thermal equilibrium at a temperature at or above the melting point of ice.

First suppose that no ice melts. The temperature of the water decreases from T_{Wi} (= 25°C) to some final temperature T_f and the temperature of the ice increases from T_{Ii} (= −15°C) to T_f. If m_W is the mass of the water and c_W is its specific heat then the water rejects heat

$$Q = c_W m_W (T_{Wi} - T_f)\,.$$

If m_I is the mass of the ice and c_I is its specific heat then the ice absorbs heat

$$Q = c_I m_I (T_f - T_{Ii})\,.$$

Since no energy is lost these two heats must be the same and

$$c_W m_W (T_{Wi} - T_f) = c_I m_I (T_f - T_{Ii})\,.$$

The solution for the final temperature is

$$\begin{aligned} T_f &= \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii}}{c_W m_W + c_I m_I} \\ &= \frac{(4190\,\mathrm{J/kg\cdot K})(0.200\,\mathrm{kg})(25^\circ\mathrm{C}) + (2220\,\mathrm{J/kg\cdot K})(0.100\,\mathrm{kg})(-15^\circ\mathrm{C})}{(4190\,\mathrm{J/kg\cdot K})(0.200\,\mathrm{kg}) + (2220\,\mathrm{J/kg\cdot K})(0.100\,\mathrm{kg})} \\ &= 16.6^\circ\mathrm{C}\,. \end{aligned}$$

This is above the melting point of ice, so at least some of the ice must have melted. The calculation just completed does not take into account the melting of the ice and is in error.

Now assume the water and ice reach thermal equilibrium at $T_f = 0^\circ\mathrm{C}$, with mass m ($< m_I$) of the ice melted. The magnitude of the heat rejected by the water is

$$Q = c_W m_W T_{Wi}\,,$$

and the heat absorbed by the ice is

$$Q = c_I m_I (0 - T_{Ii}) + m L_F\,,$$

where L_F is the heat of fusion for water. The first term is the energy required to warm all the ice from its initial temperature to 0°C and the second term is the energy required to melt mass m of the ice. The two heats are equal, so

$$c_W m_W T_{Wi} = -c_I m_I T_{Ii} + m L_F \,.$$

This equation can be solved for the mass m of ice melted:

$$\begin{aligned} m &= \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii}}{L_F} \\ &= \frac{(4190\,\mathrm{J/kg\cdot K})(0.200\,\mathrm{kg})(25^\circ\mathrm{C}) + (2220\,\mathrm{J/kg\cdot K})(0.100\,\mathrm{kg})(-15^\circ\mathrm{C})}{333 \times 10^3\,\mathrm{J/kg}} \\ &= 5.3 \times 10^{-2}\,\mathrm{kg} = 53\,\mathrm{g}\,. \end{aligned}$$

Since the total mass of ice present initially was 100 g, there is enough ice to bring the water temperature down to 0°C. This is the solution: the ice and water reach thermal equilibrium at a temperature of 0°C with 53 g of ice melted.

(b) Now there is less than 53 g of ice present initially. All the ice melts and the final temperature is above the melting point of ice. The heat rejected by the water is

$$Q = c_W m_W (T_{Wi} - T_f)$$

and the heat absorbed by the ice and the water it becomes when it melts is

$$Q = c_I m_I (0 - T_{Ii}) + c_W m_I (T_f - 0) + m_I L_F \,.$$

The first term is the energy required to raise the temperature of the ice to 0°C, the second term is the energy required to raise the temperature of the melted ice from 0°C to T_f, and the third term is the energy required to melt all the ice. Since the two heats are equal,

$$c_W m_W (T_{Wi} - T_f) = c_I m_I (-T_{Ii}) + c_W m_I T_f + m_I L_F \,.$$

The solution for T_f is

$$T_f = \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii} - m_I L_F}{c_W (m_W + m_I)} \,.$$

Substitute given values to obtain $T_f = 2.5^\circ\mathrm{C}$.

(a) The change in internal energy ΔE_{int} is the same for path iaf and path ibf. According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$, where Q is the heat absorbed and W is the work done by the system. Along iaf $\Delta E_{\text{int}} = Q - W = 50\,\text{cal} - 20\,\text{cal} = 30\,\text{cal}$. Along ibf $W = Q - \Delta E_{\text{int}} = 36\,\text{cal} - 30\,\text{cal} = 6\,\text{cal}$.

(b) Since the curved path is traversed from f to i the change in internal energy is $-30\,\text{cal}$ and $Q = \Delta E_{\text{int}} + W = -30\,\text{cal} - 13\,\text{cal} = -43\,\text{cal}$.

(c) Let $\Delta E_{\text{int}} = E_{\text{int},\,f} - E_{\text{int},\,i}$. Then, $E_{\text{int},\,f} = \Delta E_{\text{int}} + E_{\text{int},\,i} = 30\,\text{cal} + 10\,\text{cal} = 40\,\text{cal}$.

(d) and (e) The work W_{bf} for the path bf is zero, so $Q_{bf} = E_{\text{int},\,f} - E_{\text{int},\,b} = 40\,\text{cal} - 22\,\text{cal} = 18\,\text{cal}$. For the path ibf $Q = 36\,\text{cal}$ so $Q_{ib} = Q - Q_{bf} = 36\,\text{cal} - 18\,\text{cal} = 18\,\text{cal}$.

CHAPTER 18 HINT FOR PROBLEM 13

The change in the surface area is $\Delta A = 2A\alpha\,\Delta T$, where A is the original surface area, ΔT is the change in temperature (on the Kelvin or Celsius scale), and α is the coefficient of linear expansion for brass. See Table 18–2 for the value of α. A cube has 6 faces, each with an area equal to the square of an edge length.

$\left[\text{ans: } 11\,\text{cm}^2\right]$

Use $V = V_0 + V_0\beta_{\text{liquid}}\,\Delta T$ to compute the new volume V of the liquid and $A = A_0 + 2A\alpha_{\text{glass}}\Delta T$ to compute the new cross-sectional area of the tube. Here V_0 is the original volume of fluid and A_0 is the original cross-sectional area of the tube. The height h of the fluid after the temperature increase can be found from $Ah = V$. You will need to use $V_0 = A_0L/2$, where L is the length of the tube.

[ans: 0.13 mm]

The steam is converted to water, giving up energy $m_S L_V$, where m_S is the mass of steam and L_V is the heat of vaporization of water. The temperature of the water is then reduced from T_{S0} (= 100°C) to T (= 50°C), giving up energy $m_S c(T_0 - T)$, where c is the specific heat of water. The ice melts, absorbing energy $m_I L_F$, where m_I is the mass of ice and L_F is the heat of fusion of water. Then the temperature of the resulting water is raised from T_{I0} (= 0°C) to T, absorbing energy $m_I c(T - T_{I0}$. The energy given up by the substance that originally was steam must equal the energy absorbed by the substance that was originally ice. Solve for m_S.

[ans: 33 g]

CHAPTER 18 HINT FOR PROBLEM 47

According to the first law of thermodynamics the change in the internal energy of the gas as it goes from c to a via d is $\Delta E_{\text{int, } c\to a} = Q_{c\to d} + Q_{d\to a} - W_{c\to d} - W_{d\to a}$. Since the internal energy depends only on the state of the gas $\Delta E_{\text{int, } c\to a} = -\Delta E_{\text{int, } a\to c}$. You should recognize that $W_{d\to a} = 0$ since the volume of the gas does not change over this portion of the path. All other quantities are given.

[ans: 60 J]

When the rods are welded together end to end the rate of energy conduction is

$$P_{\text{cond}} = \frac{kA}{2L}\,\Delta T\,,$$

where k is the thermal conductivity, A is the cross-sectional area of each rod, L is the length of each rod, and ΔT is the difference in temperature of the ends of the composite. When they are welded together side by side the length is L and the cross-sectional area is $2A$, so the rate of energy conduction is

$$P_{\text{cond}} = \frac{2kA}{L}\,\Delta T\,.$$

Note that this is 4 times as large as previously.

[ans: 0.50 min]

According to Eq. 18–37 the rate of energy conduction through the composite is given by

$$P_{\text{cond}} = \frac{A\Delta T}{\sum L_i/k_i},$$

where A is the cross-sectional area, ΔT is the temperature difference of the ends, k_i is the thermal conductivity of layer i, and L_i is the thickness of layer i. The rate of energy conduction is the same throughout the composite. Through layer 4 it is

$$P_{\text{cond}} = \frac{k_4 A}{L_4}\Delta T_4,$$

where ΔT_4 is the energy difference of the ends of layer 4. Equate the two expressions for the rate of energy conduction and solve for ΔT_4. Note that the area cancels from the equation. Since you know the temperature at one end of layer 4 you can calculate the temperature at the other end.

[ans: -4.2°C]

Suppose the gas expands from volume V_i to volume V_f during the isothermal portion of the process. The work it does is

$$W = \int_{V_i}^{V_f} p \, \mathrm{d}V = nRT \int_{V_i}^{V_f} \frac{\mathrm{d}V}{V} = nRT \ln \frac{V_f}{V_i} \,,$$

where the ideal gas law $pV = nRT$ was used to replace p with nRT/V. Now $V_i = nRT/p_i$ and $V_f = nRT/p_f$, so $V_f/V_i = p_i/p_f$. Also replace nRT with p_iV_i to obtain

$$W = p_i V_i \ln \frac{p_i}{p_f} \,.$$

Since the initial gauge pressure is 1.03×10^5 Pa, $p_i = 1.03 \times 10^5$ Pa$+1.013 \times 10^5$ Pa $= 2.04 \times 10^5$ Pa. The final pressure is atmospheric pressure: $p_f = 1.013 \times 10^5$ Pa. Thus

$$W = (2.04 \times 10^5\,\mathrm{Pa})(0.140\,\mathrm{m}^3) \ln \frac{2.04 \times 10^5\,\mathrm{Pa}}{1.013 \times 10^5\,\mathrm{Pa}} = 2.00 \times 10^4\,\mathrm{J} \,.$$

During the constant pressure portion of the process the work done by the gas is $W = p_f(V_i - V_f)$. Notice that the gas starts in a state with pressure p_f, so this is the pressure throughout this portion of the process. Also note that the volume decreases from V_f to V_i. Now $V_f = p_iV_i/p_f$, so

$$\begin{aligned} W &= p_f \left(V_i - \frac{p_i V_i}{p_f} \right) = (p_f - p_i)V_i \\ &= (1.013 \times 10^5\,\mathrm{Pa} - 2.04 \times 10^5\,\mathrm{Pa})(0.140\,\mathrm{m}^3) = -1.44 \times 10^4\,\mathrm{J} \,. \end{aligned}$$

The total work done by the gas over the entire process is $W = 2.00 \times 10^4\,\mathrm{J} - 1.44 \times 10^4\,\mathrm{J} = 5.60 \times 10^3\,\mathrm{J}$.

(a) The distribution function gives the fraction of particles with speeds between v and $v + \mathrm{d}v$, so its integral over all speeds is unity: $\int P(v)\,\mathrm{d}v = 1$. Evaluate the integral by calculating the area under the curve in Fig. 19–22. The area of the triangular portion is half the product of the base and altitude, or $\frac{1}{2}av_0$. The area of the rectangular portion is the product of the sides, or av_0. Thus $\int P(v)\,\mathrm{d}v = \frac{1}{2}av_0 + av_0 = \frac{3}{2}av_0$, so $\frac{3}{2}av_0 = 1$ and $av_0 = 2/3$.
(b) The average speed is given by

$$v_{\text{avg}} = \int vP(v)\,\mathrm{d}v\,.$$

For the triangular portion of the distribution $P(v) = av/v_0$ and the contribution of this portion is

$$\frac{a}{v_0}\int_0^{v_0} v^2\,\mathrm{d}v = \frac{a}{3v_0}\,v_0^3 = \frac{av_0^2}{3} = \frac{2}{9}v_0\,,$$

where $2/3v_0$ was substituted for a. $P(v) = a$ in the rectangular portion and the contribution of this portion is

$$a\int_{v_0}^{2v_0} v\,\mathrm{d}v = \frac{a}{2}\,(4v_0^2 - v_0^2) = \frac{3a}{2}v_0^2 = v_0\,.$$

Thus $v_{\text{avg}} = \frac{2}{9}v_0 + v_0 = 1.2v_0$ and $v_{\text{avg}}/v_0 = 1.2$.
(c) The mean-square speed is given by

$$v_{\text{rms}}^2 = \int v^2P(v)\,\mathrm{d}v\,.$$

The contribution of the triangular section is

$$\frac{a}{v_0}\int_0^{v_0} v^3\,\mathrm{d}v = \frac{a}{4v_0}\,v_0^4 = \frac{1}{6}\,v_0^2\,.$$

The contribution of the rectangular portion is

$$a\int_{v_0}^{2v_0} v^2\,\mathrm{d}v = \frac{a}{3}\,(8v_0^3 - v_0^3) = \frac{7a}{3}v_0^3 = \frac{14}{9}\,v_0^2\,.$$

Thus $v_{\text{rms}} = \sqrt{\frac{1}{6}v_0^2 + \frac{14}{9}v_0^2} = 1.31v_0$ and $v_{\text{rms}}/v_0 = 1.3$. (d) The number of particles with speeds between $1.5v_0$ and $2v_0$ is given by $N\int_{1.5v_0}^{2v_0} P(v)\,\mathrm{d}v$. The integral is easy to evaluate since $P(v) = a$ throughout the range of integration. Thus the number of particles with speeds in the given range is $Na(2.0v_0 - 1.5v_0) = 0.5Nav_0 = N/3$, where $2/3v_0$ was substituted for a. The fraction of particles in the given range is 0.33.

(a) Since the process is at constant pressure energy transferred as heat to the gas is given by $Q = nC_p\,\Delta T$, where n is the number of moles in the gas, C_p is the molar specific heat at constant pressure, and ΔT is the increase in temperature. For a diatomic ideal gas $C_p = \frac{7}{2}R$. Thus

$$Q = \frac{7}{2}nR\,\Delta T = \frac{7}{2}(4.00\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(60.0\,\text{K}) = 6.98\times 10^3\,\text{J}\,.$$

(b) The change in the internal energy is given by $\Delta E_{\text{int}} = nC_V\,\Delta T$, where C_V is the specific heat at constant volume. For a diatomic ideal gas $C_V = \frac{5}{2}R$, so

$$\Delta E_{\text{int}} = \frac{5}{2}nR\,\Delta T = \frac{5}{2}(4.00\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(60.0\,\text{K}) = 4.99\times 10^3\,\text{J}\,.$$

(c) According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$, so

$$W = Q - \Delta E_{\text{int}} = 6.98\times 10^3\,\text{J} - 4.99\times 10^3\,\text{J} = 1.99\times 10^3\,\text{J}\,.$$

(d) The change in the total translational kinetic energy is

$$\Delta K = \frac{3}{2}nR\,\Delta T = \frac{3}{2}(4.00\,\text{mol})(8.314\,\text{J/mol}\cdot\text{K})(60.0\,\text{K}) = 2.99\times 10^3\,\text{J}\,.$$

CHAPTER 19 HINT FOR PROBLEM 9

Use the ideal gas law. The partial pressure for gas 1 is $p_1 = n_1RT/V$ and the partial pressure for gas 2 is $p_2 = n_2RT/V$. Here n_1 is the number of moles of gas 1, n_2 is the number of moles of gas 2, T is the temperature, and V is the volume of the container. The total pressure is $p = p_1 + p_2$. You want to calculate $p_2/(p_1 + p_2)$.

[ans: 0.2]

CHAPTER 19 HINT FOR PROBLEM 25

(a) Use $\epsilon = L_V/N$, where L_V is the heat of vaporization and N is the number of molecules per gram. Divide the molar mass of water by Avogadro's number to obtain the mass of an atom. The reciprocal of the mass in grams is the number of molecules per gram.
(b) The average translational kinetic energy is $K_{\text{avg}} = \frac{3}{2}kT$.

$\left[\text{ans: (a) } 1.61 \times 10^{-20}\text{ cal } (6.76 \times 10^{-20}\text{ J}); \text{ (b) } 10.7\right]$

(a) The rms speed is given by $v_{\text{rms}} = \sqrt{3RT/M}$. The molar mass of hydrogen is $2.02 \times 10^{-3}\,\text{kg/mol}$.
(b) When the surfaces of the spheres that represent an H_2 molecule and an Ar atom are touching, the distance between their centers is the sum of their radii.
(c) Since the argon atoms are essentially at rest, in time t the hydrogen atom collides with all the argon atoms in a cylinder of radius d and length vt, where v is its speed. That is, the number of collisions is $\pi d^2 vtN/V$, where N/V is the concentration of argon atoms.

[ans: (a) $7.0\,\text{km/s}$; (b) $2.0 \times 10^{-8}\,\text{cm}$; (c) $3.5 \times 10^{10}\,\text{collisions/s}$]

(a) The work is given by the integral $W = \int p\,dV$, where p is the pressure and dV is a volume element. According to the ideal gas law $p = nRT/V$, so $W = (nRT/V)\,dV$. Take the volume to be $V = V_0 + \alpha t$ and the temperature to be $T = T_0 + \beta t$, where t is the time. Find the values of V_0, T_0, α, and β so that the expressions for V and T give correct values for $t = 0$ (when the process begins) and for $t = 2.00\,\text{h}$. Substitute the expressions for V, T, and $dV = \alpha dt$ into the equation for the work to obtain

$$W = \int_0^{2.00\,\text{h}} \frac{nR(T_0 + \beta t)}{V_0 + \alpha t} \alpha\, dt\,.$$

Evaluate the integral.

(b) According to the first law of thermodynamics the energy absorbed as heat is $Q = \Delta E_{\text{int}} + W$, where ΔE_{int} is the change in the internal energy, which may be computed using $\Delta E_{\text{int}} = nC_V\,\Delta T$, where C_V is the molar specific heat for constant volume processes and ΔT is the change in temperature.

(c) The molar specific heat for the process is $Q/n\Delta T$.

(d), (e), and (f) The work done by the gas during the isothermal portion of the process is $W = nRT\ln(V_f/V_i)$, where V_i is the initial volume and V_f is the final volume. The temperature here is the initial temperature. The work done during the constant-volume portion is zero. The energy absorbed as heat and the molar specific heat can be computed in the same manner as in parts (b) and (c).

[ans: (a) 7.72×10^4 J; (b) 5.46×10^4 J; (c) $5.17\,\text{J/mol}\cdot\text{K}$; (d) 4.32×10^4 J; (e) $8,86 \times 10^4$ J; (f) $8.38\,\text{J/mol}\cdot\text{K}$]

CHAPTER 19 HINT FOR PROBLEM 57

The change in the internal energy is the same for the two paths. In particular it is $Q - W$, where Q is the net energy input as heat and W is the net work done by the gas. Add up the heat input and the work for the segments of path 1, then calculate $Q - W$. Along an isotherm the work done is equal to the heat input and along an adiabat the heat input is zero.

$\left[\text{ans: } -20\,\text{J}\right]$

(a) The energy that leaves the aluminum as heat has magnitude $Q = m_a c_a (T_{ai} - T_f)$, where m_a is the mass of the aluminum, c_a is the specific heat of aluminum, T_{ai} is the initial temperature of the aluminum, and T_f is the final temperature of the aluminum-water system. The energy that enters the water as heat has magnitude $Q = m_w c_w (T_f - T_{wi})$, where m_w is the mass of the water, c_w is the specific heat of water, and T_{wi} is the initial temperature of the water. The two energies are the same in magnitude since no energy is lost. Thus $m_a c_a (T_{ai} - T_f) = m_w c_w (T_f - T_{wi})$ and

$$T_f = \frac{m_a c_a T_{ai} + m_w c_w T_{wi}}{m_a c_a + m_w c_w} .$$

The specific heat of aluminum is $900\,\mathrm{J/kg\cdot K}$ and the specific heat of water is $4190\,\mathrm{J/kg\cdot K}$. Thus,

$$\begin{aligned} T_f &= \frac{(0.200\,\mathrm{kg})(900\,\mathrm{J/kg\cdot K})(100^\circ\,\mathrm{C}) + (0.0500\,\mathrm{kg})(4190\,\mathrm{J/kg\cdot K})(20^\circ\,\mathrm{C})}{(0.200\,\mathrm{kg})(900\,\mathrm{J/kg\cdot K}) + (0.0500\,\mathrm{kg})(4190\,\mathrm{J/kg\cdot K})} \\ &= 57.0^\circ\,\mathrm{C} . \end{aligned}$$

This is equivalent to 330 K.

(b) Now temperatures must be given in kelvins: $T_{ai} = 393\,\mathrm{K}$, $T_{wi} = 293\,\mathrm{K}$, and $T_f = 330\,\mathrm{K}$. For the aluminum, $\mathrm{d}Q = m_a c_a\,\mathrm{d}T$ and the change in entropy is

$$\begin{aligned} \Delta S_a &= \int \frac{\mathrm{d}Q}{T} = m_a c_a \int_{T_{ai}}^{T_f} \frac{\mathrm{d}T}{T} = m_a c_a \ln \frac{T_f}{T_{ai}} \\ &= (0.200\,\mathrm{kg})(900\,\mathrm{J/kg\cdot K}) \ln \frac{330\,\mathrm{K}}{373\,\mathrm{K}} = -22.1\,\mathrm{J/K} . \end{aligned}$$

(c) The entropy change for the water is

$$\begin{aligned} \Delta S_w &= \int \frac{\mathrm{d}Q}{T} = m_w c_w \int_{T_{wi}}^{T_f} \frac{\mathrm{d}T}{T} = m_w c_w \ln \frac{T_f}{T_{wi}} \\ &= (0.0500\,\mathrm{kg})(4190\,\mathrm{J/kg\cdot K}) \ln \frac{330\,\mathrm{K}}{293\,\mathrm{K}} = +24.9\,\mathrm{J/K} . \end{aligned}$$

(d) The change in the total entropy of the aluminum-water system is $\Delta S = \Delta S_a + \Delta S_w = -22.1\,\mathrm{J/K} + 24.9\,\mathrm{J/K} = +2.8\,\mathrm{J/K}$.

(a) The efficiency is

$$\varepsilon = \frac{T_H - T_C}{T_H} = \frac{(235 - 115)\,\mathrm{K}}{(235 + 273)\,\mathrm{K}} = 0.236\,.$$

Note that a temperature difference has the same value on the Kelvin and Celsius scales. Since the temperatures in the equation must be in kelvins, the temperature in the denominator was converted to the Kelvin scale.

(b) Since the efficiency is given by $\varepsilon = |W|/|Q_H|$, the work done is given by $|W| = \varepsilon|Q_H| = 0.236(6.30 \times 10^4\,\mathrm{J}) = 1.49 \times 10^4\,\mathrm{J}$.

(a) Suppose there are n_L molecules in the left third of the box, n_C molecules in the center third, and n_R molecules in the right third. There are $N!$ arrangements of the N molecules, but $n_L!$ are simply rearrangements of the n_L molecules in the right third, $n_C!$ are rearrangements of the n_C molecules in the center third, and $n_R!$ are rearrangements of the n_R molecules in the right third. These rearrangements do not produce a new configuration. Thus, the multiplicity is

$$W = \frac{N!}{n_L!\,n_C!\,n_R!}\,.$$

(b) If half the molecules are in the right half of the box and the other half are in the left half of the box, then the multiplicity is

$$W_B = \frac{N!}{(N/2)!\,(N/2)!}\,.$$

If one-third of the molecules are in each third of the box, then the multiplicity is

$$W_A = \frac{N!}{(N/3)!\,(N/3)!\,(N/3)!}\,.$$

The ratio is

$$\frac{W_A}{W_B} = \frac{(N/2)!\,(N/2)!}{(N/3)!\,(N/3)!\,(N/3)!}\,.$$

(c) For $N = 100$,

$$\frac{W_A}{W_B} = \frac{50!\,50!}{33!\,33!\,34!} = 4.16 \times 10^{16}\,.$$

The change in entropy of a block is $\Delta S = \int (mc/T)\, dT = mc \ln(T_f/T_i)$, where m is the mass of the block, c is the specific heat, T_i is the initial temperature, and T_f is the final temperature. Since the blocks are identical the final temperature is halfway between the two initial temperatures. To find the value of mc, consider the irreversible process of Fig. 20–5. The energy absorbed as heat is related to the change in temperature by $Q = mc\,\Delta T$. Solve for mc.

Now go back to the reversible process of Fig. 20–6. Since the block and its reservoir are isolated and the process is reversible the change in the entropy of the reservoir is the negative of the change in the entropy of the block.

[ans: (a) $-710\,\mathrm{mJ/K}$; (b) $+710\,\mathrm{mJ/K}$; (c) $+723\,\mathrm{mJ/K}$; (d) $-723\,\mathrm{mJ/K}$; (e) $+13\,\mathrm{mJ/K}$; (f) 0]

CHAPTER 20 HINT FOR PROBLEM 13

(a) The energy absorbed by the lower temperature block equals the energy emitted by the higher temperature block. Thus $m_C c_C(T_C - T) = m_L c_L(T - T_L)$, where m_C is the mass of the copper block, T_C is its initial temperature, c_C is the specific heat of copper, m_L is the mass of the lead block, T_L is its initial temperature, c_L is the specific heat of lead, and T is the final temperature of the blocks. Solve for T.
(b) The two blocks form a system that does no work on its environment and absorbs no energy as heat from its environment.
(c) Consider a constant-volume process that takes a block from its initial temperature to its final temperature. The change in entropy is $\Delta S = \int (mc/T)\, dT$.

[ans: (a) 320 K; (b) 0; (c) +1.72 J/K]

CHAPTER 20 HINT FOR PROBLEM 23

The efficiency of a Carnot engine is given by $\epsilon_C = 1 - (T_L/T_H)$, where T_L is the temperature of the low-temperature reservoir and T_H is the temperature of the high-temperature reservoir, both on the Kelvin scale. Solve for T_H for each of the given efficiencies.

[ans: 97 K]

CHAPTER 20 HINT FOR PROBLEM 43

The possible configurations in the form (n_1, n_2) are: $(1, 7)$, $(2, 6)$, $(3, 5)$, $(4, 4)$, $(3, 5)$, $(2, 6)$, and $(1, 7)$. The corresponding multiplicities are calculated using $W = 8!/(n_1!n_2!)$ and entropies are calculated using $S = k \ln W$, where k is the Boltzmann constant.

Assume the spheres are far apart. Then the charge distribution on each of them is spherically symmetric and Coulomb's law can be used. Let q_1 and q_2 be the original charges and choose the coordinate system so the force on q_2 is positive if it is repelled by q_1. Take the distance between the charges to be r. Then the force on q_2 is

$$F_a = -\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} .$$

The negative sign indicates that the spheres attract each other.

After the wire is connected, the spheres, being identical, have the same charge. Since charge is conserved, the total charge is the same as it was originally. This means the charge on each sphere is $(q_1 + q_2)/2$. The force is now one of repulsion and is given by

$$F_b = \frac{1}{4\pi\epsilon_0}\frac{(q_1 + q_2)^2}{4r^2} .$$

Solve the two force equations simultaneously for q_1 and q_2. The first gives

$$q_1 q_2 = -4\pi\epsilon_0 r^2 F_a = -\frac{(0.500\,\mathrm{m})^2(0.108\,\mathrm{N})}{8.99\times 10^9\,\mathrm{N\cdot m^2/C^2}} = -3.00\times 10^{-12}\,\mathrm{C}^2$$

and the second gives

$$q_1 + q_2 = 2r\sqrt{4\pi\epsilon_0 F_b} = 2(0.500\,\mathrm{m})\sqrt{\frac{0.0360\,\mathrm{N}}{8.99\times 10^9\,\mathrm{N\cdot m^2/C^2}}} = 2.00\times 10^{-6}\,\mathrm{C} .$$

Thus

$$q_2 = \frac{-(3.00\times 10^{-12}\,\mathrm{C}^2)}{q_1}$$

and

$$q_1 - \frac{3.00\times 10^{-12}\,\mathrm{C}^2}{q_1} = 2.00\times 10^{-6}\,\mathrm{C} .$$

Multiply by q_1 to obtain the quadratic equation

$$q_1^2 - (2.00\times 10^{-6}\,\mathrm{C})q_1 - 3.00\times 10^{-12}\,\mathrm{C}^2 = 0 .$$

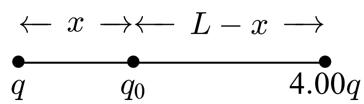
The solutions are

$$q_1 = \frac{2.00\times 10^{-6}\,\mathrm{C} \pm \sqrt{(-2.00\times 10^{-6}\,\mathrm{C})^2 + 4(3.00\times 10^{-12}\,\mathrm{C}^2)}}{2} .$$

If the positive sign is used, $q_1 = 3.00\times 10^{-6}\,\mathrm{C}$ and if the negative sign is used, $q_1 = -1.00\times 10^{-6}\,\mathrm{C}$. Use $q_2 = (-3.00\times 10^{-12})/q_1$ to calculate q_2. If $q_1 = 3.00\times 10^{-6}\,\mathrm{C}$, then $q_2 = -1.00\times 10^{-6}\,\mathrm{C}$ and if $q_1 = -1.00\times 10^{-6}\,\mathrm{C}$, then $q_2 = 3.00\times 10^{-6}\,\mathrm{C}$. Since the spheres are identical, the solutions are essentially the same: one sphere originally had charge $-1.00\times 10^{-6}\,\mathrm{C}$ and the other had charge $+3.00\times 10^{-6}\,\mathrm{C}$.

CHAPTER 21 SOLUTION FOR PROBLEM 17

If the system of three particles is to be in equilibrium, the force on each particle must be zero. Let the charge on the third particle be q_0. The third particle must lie on the x axis since otherwise the two forces on it would not be along the same line and could not sum to zero. Thus the y coordinate of the particle must be zero. The third particle must lie between the other two since otherwise the forces acting on it would be in the same direction and would not sum to zero. Suppose the third particle is a distance x from the particle with charge q, as shown on the diagram to the right. The force acting on it is then given by



$$F_0 = \frac{1}{4\pi\epsilon_0}\left[\frac{qq_0}{x^2} - \frac{4.00qq_0}{(L-x)^2}\right] = 0\,,$$

where the positive direction was taken to be toward the right. Solve this equation for x. Canceling common factors yields $1/x^2 = 4.00/(L-x)^2$ and taking the square root yields $1/x = 2.00/(L-x)$. The solution is $x = 0.333L$.

The force on q is

$$F_q = \frac{1}{4\pi\epsilon_0}\left[\frac{qq_0}{x^2} + \frac{4.00q^2}{L^2}\right] = 0\,.$$

Solve for q_0: $q_0 = -4.00qx^2/L^2 = -0.444q$, where $x = 0.333L$ was used.

The force on the particle with charge $4.00q$ is

$$\begin{aligned} F_{4q} &= \frac{1}{4\pi\epsilon_0}\left[\frac{4.00q^2}{L^2} + \frac{4.00qq_0}{(L-x)^2}\right] = \frac{1}{4\pi\epsilon_0}\left[\frac{4.00q^2}{L^2} + \frac{4.00(0.444)q^2}{(0.444)L^2}\right] \\ &= \frac{1}{4\pi\epsilon_0}\left[\frac{4.00q^2}{L^2} - \frac{4.00q^2}{L^2}\right] = 0\,. \end{aligned}$$

With $q_0 = -0.444q$ and $x = 0.333L$, all three charges are in equilibrium.

Calculate the x and y components of the forces of particles 1, 2, and 4 on particle 3. Add the x components to find the x component of the net force and add the y components to find the y component of the net force. Use Coulomb's law to calculate the magnitude of each force. The charges of particles 1 and 3 have the same sign so the force of 1 is in the negative y direction. The charges of particles 2 and 3 have opposite signs so the force of 2 has positive x and y components. The distance between these particles is $\sqrt{2}a$ and the force makes an angle of 45° with the positive x direction. The charges of particles 3 and 4 have opposite signs so the force of 4 is in the negative x direction.

[ans: (a) $0.17\,\mathrm{N}$; (b) $-0.046\,\mathrm{N}$]

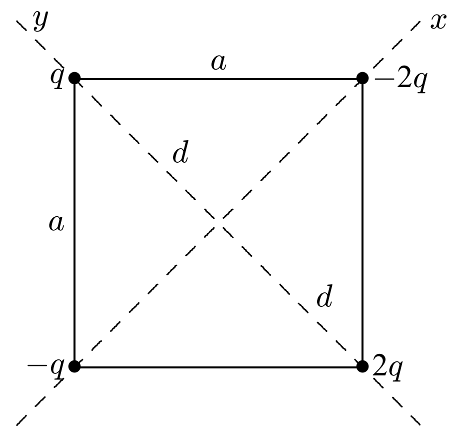
Divide the shell into concentric shells of infinitesimal thickness dr. The shell with radius r has volume $dV = 4\pi r^2\,dr$ and contains charge $dq = \rho\,dV = 4\pi r^2 \rho\,dr$. Integrate this expression. The lower limit is the inner radius of the original shell and the upper limit is its outer radius.

[ans: 3.8×10^{-8} C]

CHAPTER 21 HINT FOR PROBLEM 25

The current is the charge that is intercepted by Earth's surface per unit time. If N is the number of protons that hit each square meter of the surface per second, then the current is $i = NAe$, where A is the area of the surface, given by $A = 4\pi R^2$. Look up the radius R of Earth in Appendix C.

$\left[\text{ans: } 122\,\text{mA}\right]$



Choose the coordinate axes as shown on the diagram to the right. At the center of the square, the electric fields produced by the particles at the lower left and upper right corners are both along the x axis and each points away from the center and toward the particle that produces it. Since each particle is a distance $d = \sqrt{2}a/2 = a/\sqrt{2}$ away from the center, the net field due to these two particles is

$$E_x = \frac{1}{4\pi\epsilon_0}\left[\frac{2q}{a^2/2} - \frac{q}{a^2/2}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{a^2/2} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(1.0\times10^{-8}\,\mathrm{C})}{(0.050\,\mathrm{m})^2/2} = 7.19\times10^4\,\mathrm{N/C}.$$

At the center of the square, the field produced by the particles at the upper left and lower right corners are both along the y axis and each points away from the particle that produces it. The net field produced at the center by these particles is

$$E_y = \frac{1}{4\pi\epsilon_0}\left[\frac{2q}{a^2/2} - \frac{q}{a^2/2}\right] = \frac{1}{4\pi\epsilon_0}\frac{q}{a^2/2} = 7.19\times10^4\,\mathrm{N/C}\,.$$

The magnitude of the net field is

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{2(7.19\times10^4\,\mathrm{N/C})^2} = 1.02\times10^5\,\mathrm{N/C}$$

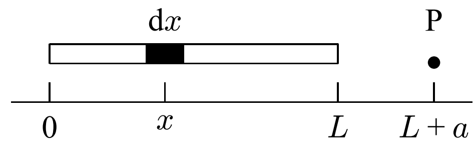
and the angle it makes with the x axis is

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}(1) = 45^\circ\,.$$

It is upward in the diagram, from the center of the square toward the center of the upper side.

(a) The linear charge density λ is the charge per unit length of rod. Since the charge is uniformly distributed on the rod, $\lambda = -q/L = -(4.23 \times 10^{-15}\,\mathrm{C})/(0.0815\,\mathrm{m}) = -5.19 \times 10^{-14}\,\mathrm{C/m}$.

(b) and (c) Position the origin at the left end of the rod, as shown in the diagram. Let $\mathrm{d}x$ be an infinitesimal length of rod at x. The charge in this segment is $\mathrm{d}q = \lambda\,\mathrm{d}x$. The charge $\mathrm{d}q$ may be taken to be a point charge. The electric field it produces at point P has only an x component and this component is given by



$$\mathrm{d}E_x = \frac{1}{4\pi\epsilon_0}\,\frac{\lambda\,\mathrm{d}x}{(L+a-x)^2}\,.$$

The total electric field produced at P by the whole rod is the integral

$$\begin{aligned} E_x &= \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{\mathrm{d}x}{(L+a-x)^2} = \frac{\lambda}{4\pi\epsilon_0}\,\frac{1}{L+a-x}\bigg|_0^L \\ &= \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{a}-\frac{1}{L+a}\right] = \frac{\lambda}{4\pi\epsilon_0}\,\frac{L}{a(L+a)}\,. \end{aligned}$$

When $-q/L$ is substituted for λ the result is

$$E_x = -\frac{1}{4\pi\epsilon_0}\,\frac{q}{a(L+a)} = -\frac{(8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2})(4.23 \times 10^{-15}\,\mathrm{C})}{(0.120\,\mathrm{m})(0.0815\,\mathrm{m}+0.120\,\mathrm{m})} = -1.57 \times 10^{-3}\,\mathrm{N/C}\,.$$

The negative sign indicates that the field is toward the rod and makes an angle of 180° with the positive x direction.

(d) Now

$$E_x = -\frac{1}{4\pi\epsilon_0}\,\frac{q}{a(L+a)} = -\frac{(8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2})(4.23 \times 10^{-15}\,\mathrm{C})}{(50\,\mathrm{m})(0.0815\,\mathrm{m}+50\,\mathrm{m})} = -1.52 \times 10^{-8}\,\mathrm{N/C}\,.$$

(e) The field of a point particle at the origin is

$$E_x = -\frac{q}{4\pi\epsilon_0 a^2} = -\frac{(8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2})(4.23 \times 10^{-15}\,\mathrm{C})}{(50\,\mathrm{m})^2} = -1.52 \times 10^{-8}\,\mathrm{N/C}\,.$$

At a point on the axis of a uniformly charged disk a distance z above the center of the disk, the magnitude of the electric field is

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right],$$

where R is the radius of the disk and σ is the surface charge density on the disk. See Eq. 22–26. The magnitude of the field at the center of the disk ($z = 0$) is $E_c = \sigma/2\epsilon_0$. You want to solve for the value of z such that $E/E_c = 1/2$. This means

$$\frac{E}{E_c} = 1 - \frac{z}{\sqrt{z^2 + R^2}} = \frac{1}{2}$$

or

$$\frac{z}{\sqrt{z^2 + R^2}} = \frac{1}{2}.$$

Square both sides, then multiply them by $z^2 + R^2$ to obtain $z^2 = (z^2/4) + (R^2/4)$. Thus, $z^2 = R^2/3$ and $z = R/\sqrt{3} = (0.600\,\mathrm{m})/\sqrt{3} = 0.346\,\mathrm{m}$.

The particles are equidistance from P and their charges have the same magnitude. The y components of their fields sum to zero and their x components are the same, so you need calculate only the x component of one of the fields, then double it. The x component of the field of either particle is given by $E_x = -(1/4\pi\epsilon_0)qx/r^3$, where x is the coordinate of the particle and r is its distance from P.

[ans: (a) $1.38 \times 10^{-10}\,\mathrm{N/C}$; (b) negative x direction]

At P the electric fields of the two charged particles have the same magnitude, which is given by $E = (1/4\pi\epsilon_0)q/(r^2 + d^2/4)$. Their x components sum to zero and their y components have the same value: $E_y = -E \sin\theta$, where θ is the angle between the x axis and the line from either particle to P. Use $\sin\theta = (d/2)/\sqrt{r^2 + d^2/4}$ to substitute for $\sin\theta$. Double the result to take both particles into account, then use the binomial theorem to find the expression for the net field for $r >> d$.

[ans: (a) $qd/4\pi\epsilon_0 r^3$; (b) negative y direction]

Symmetry tells us that the horizontal component of the net electric field at P is zero. Divide the rod into sections of infinitesimal width dx and treat each section as a point particle with charge $dq = \lambda\, dx$, where λ ($= q/L$) is the linear charge density of the rod. Put the origin at the center of the rod. The magnitude of the field produced at P by the section at x is $dE = (1/4\pi\epsilon_0)(\lambda\, dx)/r^2$ and its vertical component is $E \sin\theta$. Here r is the distance from the section to P and θ is the angle between the line from the section to P and the positive x direction. Substitute $r = \sqrt{x^2 + R^2}$ and $\sin\theta = R/r = R/\sqrt{x^2 + R^2}$ and integrate over the length of the rod.

[ans: a) 12.4 N/C; (b) positive y direction]

CHAPTER 22 HINT FOR PROBLEM 53

The potential energy of an electric dipole is given by $U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$, where $\vec{p}$ is the dipole moment, $\vec{E}$ is the electric field, and θ is the angle between the dipole moment and the electric field. The work required of an external agent is the change in the potential energy.

$\left[\text{ans: } 1.22 \times 10^{-23}\,\text{J}\ \right]$

Assume the charge density of both the conducting rod and the shell are uniform. Neglect fringing. Symmetry can be used to show that the electric field is radial, both between the rod and the shell and outside the shell. It is zero, of course, inside the rod and inside the shell since they are conductors.

(a) and (b) Take the Gaussian surface to be a cylinder of length L and radius r, concentric with the conducting rod and shell and with its curved surface outside the shell. The area of the curved surface is $2\pi rL$. The field is normal to the curved portion of the surface and has uniform magnitude over it, so the flux through this portion of the surface is $\Phi = 2\pi rLE$, where E is the magnitude of the field at the Gaussian surface. The flux through the ends is zero. The charge enclosed by the Gaussian surface is $Q_1 - 2.00Q_1 = -Q_1$. Gauss' law yields $2\pi r\epsilon_0 LE = -Q_1$, so

$$E = -\frac{Q_1}{2\pi\epsilon_0 Lr} = -\frac{(8.99 \times 10^9\,N \cdot \mathrm{m}^2/\mathrm{C}^2)(3.40 \times 10^{-12}\,\mathrm{C})}{(11.00\,\mathrm{m})(26.0 \times 10^{-3}\,\mathrm{m})} = -0.214\,\mathrm{N/C}\,.$$

The magnitude of the field is $0.214\,\mathrm{N/C}$. The negative sign indicates that the field points inward.

(c) and (d) Take the Gaussian surface to be a cylinder of length L and radius r, concentric with the conducting rod and shell and with its curved surface between the conducting rod and the shell. As in (a), the flux through the curved portion of the surface is $\Phi = 2\pi rLE$, where E is the magnitude of the field at the Gaussian surface, and the flux through the ends is zero. The charge enclosed by the Gaussian surface is only the charge Q_1 on the conducting rod. Gauss' law yields $2\pi\epsilon_0 rLE = Q_1$, so

$$E = \frac{Q_1}{2\pi\epsilon_0 Lr} = \frac{2(8.99 \times 10^9\,\mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2)(3.40 \times 10^{-12}\,\mathrm{C})}{(11.00\,\mathrm{m})(6.50 \times 10^{-3}\,\mathrm{m})} = +0.855\,\mathrm{N/C}\,.$$

The positive sign indicates that the field points outward.

(e) Consider a Gaussian surface in the form of a cylinder of length L with the curved portion of its surface completely within the shell. The electric field is zero at all points on the curved surface and is parallel to the ends, so the total electric flux through the Gaussian surface is zero and the net charge within it is zero. Since the conducting rod, which is inside the Gaussian cylinder, has charge Q_1, the inner surface of the shell must have charge $-Q_1 = -3.450 \times 10^{-12}\,\mathrm{C}$.

(f) Since the shell has total charge $-2.00Q_1$ and has charge $-Q_1$ on its inner surface, it must have charge $-Q_1 = -3.40 \times 10^{-12}\,\mathrm{C}$ on its outer surface.

(a) To calculate the electric field at a point very close to the center of a large, uniformly charged conducting plate, we may replace the finite plate with an infinite plate with the same area charge density and take the magnitude of the field to be $E = \sigma/\epsilon_0$, where σ is the area charge density for the surface just under the point. The charge is distributed uniformly over both sides of the original plate, with half being on the side near the field point. Thus

$$\sigma = \frac{q}{2A} = \frac{6.0 \times 10^{-6}\,\mathrm{C}}{2(0.080\,\mathrm{m})^2} = 4.69 \times 10^{-4}\,\mathrm{C/m^2}\,.$$

The magnitude of the field is

$$E = \frac{\sigma}{\epsilon_0} = \frac{4.69 \times 10^{-4}\,\mathrm{C/m^2}}{8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}} = 5.3 \times 10^{7}\,\mathrm{N/C}\,.$$

The field is normal to the plate and since the charge on the plate is positive, it points away from the plate.

(b) At a point far away from the plate, the electric field is nearly that of a point particle with charge equal to the total charge on the plate. The magnitude of the field is $E = q/4\pi\epsilon_0 r^2$, where r is the distance from the plate. Thus

$$E = \frac{(8.99 \times 10^{9}\,\mathrm{N \cdot m^2/C^2})(6.0 \times 10^{-6}\,\mathrm{C})}{(30\,\mathrm{m})^2} = 60\,\mathrm{N/C}\,.$$

To find an expression for the electric field inside the shell in terms of A and the distance from the center of the shell, select A so the field does not depend on the distance.

Use a Gaussian surface in the form of a sphere with radius r_g, concentric with the spherical shell and within it ($a < r_g < b$). Gauss' law will be used to find the magnitude of the electric field a distance r_g from the shell center.

The charge that is both in the shell and within the Gaussian sphere is given by the integral $q_s = \int \rho \, dV$ over the portion of the shell within the Gaussian surface. Since the charge distribution has spherical symmetry, we may take dV to be the volume of a spherical shell with radius r and infinitesimal thickness dr: $dV = 4\pi r^2 \, dr$. Thus,

$$q_s = 4\pi \int_a^{r_g} \rho r^2 \, dr = 4\pi \int_a^{r_g} \frac{A}{r} r^2 \, dr = 4\pi A \int_a^{r_g} r \, dr = 2\pi A(r_g^2 - a^2) \, .$$

The total charge inside the Gaussian surface is $q + q_s = q + 2\pi A(r_g^2 - a^2)$.

The electric field is radial, so the flux through the Gaussian surface is $\Phi = 4\pi r_g^2 E$, where E is the magnitude of the field. Gauss' law yields

$$4\pi\epsilon_0 E r_g^2 = q + 2\pi A(r_g^2 - a^2) \, .$$

Solve for E:

$$E = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{r_g^2} + 2\pi A - \frac{2\pi A a^2}{r_g^2} \right] \, .$$

For the field to be uniform, the first and last terms in the brackets must cancel. They do if $q - 2\pi A a^2 = 0$ or $A = q/2\pi a^2 = (45.0 \times 10^{-15}\,\mathrm{C})/2\pi(2.00 \times 10^{-2}\,\mathrm{m})^2 = 1.79 \times 10^{-11}\,\mathrm{C/m^2}$.

CHAPTER 23 HINT FOR PROBLEM 7

Think of the proton as being at the center of a cube with edge length d. According to Gauss's law the net electric flux through the surface of the cube is e/ϵ_0. Since the proton is at the center one-sixth of the net flux is through each face of the cube.

$\left[\text{ans: } 3.01\,\text{nN}\cdot\text{m}^2/\text{C}\ \right]$

(a) The charge on the drum is the product of its surface charge density and the surface area of its curved surface: $q = \sigma 2\pi RL$, where σ is the charge density ($2.0\,\mu\mathrm{C/m^2}$ from Problem 16), R is its radius, and L is its length.

(b) The magnitude of the electric field at the drum surface is given by $E = \lambda/2\pi\epsilon_0 R$, where $lambda$ is the linear charge density. Since $\lambda = q/L$, $E = q/2\pi\epsilon_0 RL$. Thus $q_2/R_2L_2 = q_1/R_1L_1$, where the subscripts 1 refer to the old values and the subscripts 2 refer to the new values. Solve for q_2.

[ans: (a) $0.32\,\mu$C; (b) $0.14\,\mu$C]

The electric field at a distance r from the cylindrical axis is given by $E = \lambda/2\pi r$, where λ is the linear charge density inside the Gaussian cylinder of radius r. For part (a) the point is outside both the rod and the cylindrical shell, so $\lambda = (Q_1 + Q_2)/L$. For part (b) the point is outside the rod but inside the shell, so $\lambda = Q_1/L$. The field is radially outward if E is positive and radially inward if E is negative. There can be no net charge inside a Gaussian cylinder that is completely within the shell, so the charge on the rod and the interior surface of the shell must sum to zero. The charge on the interior and exterior surfaces of the shell must sum to Q_2.

[ans: (a) $0.214\,\mathrm{N/C}$; (b) inward; (c) $0.855\,\mathrm{N/C}$; (d) outward; (e) $-3.4 \times 10^{-12}\,\mathrm{C}$; (f) $-3.40 \times 10^{-12}\,\mathrm{C}$]

CHAPTER 23 HINT FOR PROBLEM 33

The magnitude of the electric field of a large plate with surface charge density σ is $E = \sigma/2\epsilon_0$. If σ is positive the field points away from the plate and if σ is negative it points toward the plate. Thus the x component of the field is $\sigma/2\epsilon_0$ to the right of the plate and $-\sigma/2\epsilon_0$ to the left. Write equations for the x component of the field between plates 1 and 2, between plates 2 and 3, and outside plate 3, then solve these for σ_2/σ_2.

$\left[\text{ans: } -1.5\ \right]$

CHAPTER 23 HINT FOR PROBLEM 45

Use Gauss' law with a spherical Gaussian surface in the form of a sphere with radius r. The electric flux through the surface is $4\pi r^2 E$, where E is the magnitude of the electric field. For part (a) only the charge on the smaller shell is enclosed by the Gaussian surface and part (b) the charge on both shells is enclosed.

$\Big[$ ans: (a) $2.50 \times 10^4\,\mathrm{N/C}$; (b) $1.35 \times 10^4\,\mathrm{N/C}$ $\Big]$

Use a spherical Gaussian surface with radius r, concentric with the charge distribution. The electric field is radially outward, normal to the surface, and has a uniform magnitude over the surface, so the electric flux through the surface is $4\pi r^2 E$, where E is the magnitude of the electric field a distance r from the center of the charge distribution. The charge enclosed by the Gaussian surface is

$$q_{\text{enc}} = \int_0^r 4\pi(r')^2 \rho \, dr'$$

Thus

$$4\pi r^2 \epsilon_0 E = \int_0^r 4\pi(r')^2 \rho \, dr' \,.$$

Differentiate with respect to r and solve for ρ.

$\left[\text{ans: } 6\epsilon_0 K r^3 \right]$

The disk is uniformly charged. This means that when the full disk is present each quadrant contributes equally to the electric potential at P, so the potential at P due to a single quadrant is one-fourth the potential due to the entire disk. First find an expression for the potential at P due to the entire disk.

Consider a ring of charge with radius r and width dr. Its area is $2\pi r\,dr$ and it contains charge $dq = 2\pi\sigma r\,dr$. All the charge in it is a distance $\sqrt{r^2 + D^2}$ from P, so the potential it produces at P is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{2\pi\sigma r\,dr}{\sqrt{r^2 + D^2}} = \frac{\sigma r\,dr}{2\epsilon_0\sqrt{r^2 + D^2}}\,.$$

The total potential at P is

$$V = \frac{\sigma}{2\epsilon_0}\int_0^R \frac{r\,dr}{\sqrt{r^2 + D^2}} = \frac{\sigma}{2\epsilon_0}\sqrt{r^2 + D^2}\bigg|_0^R = \frac{\sigma}{2\epsilon_0}\left[\sqrt{R^2 + D^2} - D\right]\,.$$

The potential V_{sq} at P due to a single quadrant is

$$\begin{aligned} V_{\text{sq}} &= \frac{V}{4} = \frac{\sigma}{8\epsilon_0}\left[\sqrt{R^2 + D^2} - D\right] \\ &= \frac{7.73 \times 10^{-15}\,\text{C/m}^2}{8(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}\left[\sqrt{(0.640\,\text{m})^2 + (0.259\,\text{m})^2} - 0.259\,\text{m}\right] \\ &= 4.71 \times 10^{-5}\,\text{V}\,. \end{aligned}$$

The work required is equal to the potential energy of the system, relative to a potential energy of zero for infinite separation. Number the particles 1, 2, 3, and 4, in clockwise order starting with the particle in the upper left corner of the arrangement. The potential energy of the interaction of particles 1 and 2 is

$$\begin{aligned} U_{12} &= \frac{q_1 q_2}{4\pi\epsilon_0 a} = \frac{(8.99\times 10^9\,\mathrm{N\cdot m^2/C^2})(2.30\times 10^{-12}\,\mathrm{C})(-2.30\times 10^{-12}\,\mathrm{C})}{0.640\,\mathrm{m}} \\ &= -7.43\times 10^{-14}\,\mathrm{J}\,. \end{aligned}$$

The distance between particles 1 and 3 is $\sqrt{2}a$ and both these particles are positively charged, so the potential energy of the interaction between particles 1 and 3 is $U_{13} = -U_{12}/\sqrt{2} = +5.25\times 10^{-14}\,\mathrm{J}$. The potential energy of the interaction between particles 1 and 4 is $U_{14} = U_{12} = -7.43\times 10^{-14}\,\mathrm{J}$. The potential energy of the interaction between particles 2 and 3 is $U_{23} = U_{12} = -7.43\times 10^{-14}\,\mathrm{J}$. The potential energy of the interaction between particles 2 and 4 is $U_{24} = U_{13} = 5.25\times 10^{-14}\,\mathrm{J}$. The potential energy of the interaction between particles 3 and 4 is $U_{34} = U_{12} = -7.43\times 10^{-14}\,\mathrm{J}$.

The total potential energy of the system is

$$\begin{aligned} U &= U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34} \\ &= -7.43\times 10^{-14}\,\mathrm{J} + 5.25\times 10^{-14}\,\mathrm{J} - 7.43\times 10^{-14}\,\mathrm{J} - 7.43\times 10^{-14}\,\mathrm{J} \\ &\quad - 7.43\times 10^{-14}\,\mathrm{J} + 5.25\times 10^{-14}\,\mathrm{J} = -1.92\times 10^{-13}\,\mathrm{J}\,. \end{aligned}$$

This is equal to the work that must be done to assemble the system from infinite separation.

(a) The electric potential is the sum of the contributions of the individual spheres. Let q_1 be the charge on one, q_2 be the charge on the other, and d be their separation. The point halfway between them is the same distance $d/2$ (= 1.0 m) from the center of each sphere, so the potential at the halfway point is

$$V = \frac{q_1 + q_2}{4\pi\epsilon_0 d/2} = \frac{(8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2})(1.0 \times 10^{-8}\,\mathrm{C} - 3.0 \times 10^{-8}\,\mathrm{C})}{1.0\,\mathrm{m}} = -1.80 \times 10^2\,\mathrm{V}\,.$$

(b) The distance from the center of one sphere to the surface of the other is $d - R$, where R is the radius of either sphere. The potential of either one of the spheres is due to the charge on that sphere and the charge on the other sphere. The potential at the surface of sphere 1 is

$$\begin{aligned} V_1 &= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{R} + \frac{q_2}{d-R}\right] \\ &= (8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2})\left[\frac{1.0 \times 10^{-8}\,\mathrm{C}}{0.030\,\mathrm{m}} - \frac{3.0 \times 10^{-8}\,\mathrm{C}}{2.0\,\mathrm{m} - 0.030\,\mathrm{m}}\right] \\ &= 2.9 \times 10^3\,\mathrm{V}\,. \end{aligned}$$

(c) The potential at the surface of sphere 2 is

$$\begin{aligned} V_2 &= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{d-R} + \frac{q_2}{R}\right] \\ &= (8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2})\left[\frac{1.0 \times 10^{-8}\,\mathrm{C}}{2.0\,\mathrm{m} - 0.030\,\mathrm{m}} - \frac{3.0 \times 10^{-8}\,\mathrm{C}}{0.030\,\mathrm{m}}\right] \\ &= -8.9 \times 10^3\,\mathrm{V}\,. \end{aligned}$$

The magnitude of the electric field a distance r from the sphere center, inside the sphere, is $E = qr/4\pi\epsilon_0 R^3$. Integrate the negative of this from $r = 0$ to $r = 1.45\,\text{cm}$ in part (a) and to $r = R$ in part (b).

$\left[\text{ans: (a) } -0.268\,\text{mV; (b) } -0.681\,\text{mV}\right]$

CHAPTER 24 HINT FOR PROBLEM 19

The electric potential due to an electric dipole is given by $V = (1/4\pi\epsilon_0)(p/r)\cos\theta$, where p is the magnitude of the dipole moment and the angle θ is measured from the dipole axis. Here $\theta = 0$.

[ans: 16.3 μV]

CHAPTER 24 HINT FOR PROBLEM 27

All the charge on any of the rods is equidistant from the origin so the electric potential produced at the origin by any rod is $V = (1/4\pi\epsilon_0)q/R$, where Q is the charge on the rod and R is the radius of the rod. Add the contributions of the three rods.

[ans: 13 kV]

The force on the electron is $\vec{F} = -e\vec{E}$, where $\vec{E}$ is the electric field at the position of the electron. The electric field components are $E_x = -\partial V/\partial x$, $E_y = -\partial V/\partial y$, and $E_z = \partial V/\partial z$. The first two partial derivatives are the slopes of the graphs.

$\left[\text{ans: } (-4.0 \times 10^{-16}\,\text{N})\,\hat{\imath} + (1.6 \times 10^{-16}\,\text{N})\,\hat{\jmath}\ \right]$

Use conservation of mechanical energy. The potential energy when the positron is at some coordinate x is $U = eV$, where values of V can be read from the graph. When the positron is at $x = 0$ the potential energy is zero, so the mechanical energy is the same as the positron's kinetic energy there: it is $\frac{1}{2}mv^2$, where v is the speed of the positron as it enters the field. If the mechanical energy is greater than the potential energy when the positron is at $x = 50\,\text{cm}$, then the positron will continue in the positive x direction and will exit the field at $x = 50\,\text{cm}$. If the mechanical energy is less than the potential energy with the positron at $x = 50\,\text{cm}$ then the direction of motion of the positron will reverse at some point before $x = 50\,\text{cm}$ and the positron will exit the field at $x = 0$.

[ans: (a) 0; (b) $1.0 \times 10^7\,\text{m/s}$]]

(a) After the switches are closed, the potential differences across the capacitors are the same and the two capacitors are in parallel. The potential difference from a to b is given by $V_{ab} = Q/C_{\text{eq}}$, where Q is the net charge on the combination and C_{eq} is the equivalent capacitance.

The equivalent capacitance is $C_{\text{eq}} = C_1 + C_2 = 4.0 \times 10^{-6}$ F. The total charge on the combination is the net charge on either pair of connected plates. The charge on capacitor 1 is

$$q_1 = C_1 V = (1.0 \times 10^{-6}\,\text{F})(100\,\text{V}) = 1.0 \times 10^{-4}\,\text{C}$$

and the charge on capacitor 2 is

$$q_2 = C_2 V = (3.0 \times 10^{-6}\,\text{F})(100\,\text{V}) = 3.0 \times 10^{-4}\,\text{C}\,,$$

so the net charge on the combination is $3.0 \times 10^{-4}\,\text{C} - 1.0 \times 10^{-4}\,\text{C} = 2.0 \times 10^{-4}\,\text{C}$. The potential difference is

$$V_{ab} = \frac{2.0 \times 10^{-4}\,\text{C}}{4.0 \times 10^{-6}\,\text{F}} = 50\,\text{V}\,.$$

(b) The charge on capacitor 1 is now $q_1 = C_1 V_{ab} = (1.0 \times 10^{-6}\,\text{F})(50\,\text{V}) = 5.0 \times 10^{-5}\,\text{C}$.

(c) The charge on capacitor 2 is now $q_2 = C_2 V_{ab} = (3.0 \times 10^{-6}\,\text{F})(50\,\text{V}) = 1.5 \times 10^{-4}\,\text{C}$.

(a) Let q be the charge on the positive plate. Since the capacitance of a parallel-plate capacitor is given by $\epsilon_0 A/d$, the charge is $q = CV = \epsilon_0 AV/d$. After the plates are pulled apart, their separation is d' and the potential difference is V'. Then $q = \epsilon_0 AV'/d'$ and

$$V' = \frac{d'}{\epsilon_0 A} q = \frac{d'}{\epsilon_0 A} \frac{\epsilon_0 A}{d} V = \frac{d'}{d} V = \frac{8.00\,\text{mm}}{3.00\,\text{mm}}(6.00\,\text{V}) = 16.0\,\text{V}\,.$$

(b) The initial energy stored in the capacitor is

$$U_i = \frac{1}{2}CV^2 = \frac{\epsilon_0 AV^2}{2d} = \frac{(8.85 \times 10^{-12}\,\text{F/m})(8.50 \times 10^{-4}\,\text{m}^2)(6.00\,\text{V})}{2(3.00 \times 10^{-3}\,\text{mm})} = 4.51 \times 10^{-11}\,\text{J}$$

and the final energy stored is

$$U_f = \frac{1}{2}C'(V')^2 = \frac{1}{2}\frac{\epsilon_0 A}{d'}(V')^2 = \frac{(8.85 \times 10^{-12}\,\text{F/m})(8.50 \times 10^{-4}\,\text{m}^2)(16.0\,\text{V})}{2(8.00 \times 10^{-3}\,\text{mm})} = 1.20 \times 10^{-10}\,\text{J}\,.$$

(c) The work done to pull the plates apart is the difference in the energy: $W = U_f - U_i = 1.20 \times 10^{-10}\,\text{J} - 4.51 \times 10^{-11}\,\text{J} = 7.49 \times 10^{-11}\,\text{J}$.

(a) The electric field in the region between the plates is given by $E = V/d$, where V is the potential difference between the plates and d is the plate separation. The capacitance is given by $C = \kappa\epsilon_0 A/d$, where A is the plate area and κ is the dielectric constant, so $d = \kappa\epsilon_0 A/C$ and

$$E = \frac{VC}{\kappa\epsilon_0 A} = \frac{(50\,\mathrm{V})(100 \times 10^{-12}\,\mathrm{F})}{5.4(8.85 \times 10^{-12}\,\mathrm{F/m})(100 \times 10^{-4}\,\mathrm{m}^2)} = 1.0 \times 10^4\,\mathrm{V/m}\,.$$

(b) The free charge on the plates is $q_f = CV = (100 \times 10^{-12}\,\mathrm{F})(50\,\mathrm{V}) = 5.0 \times 10^{-9}\,\mathrm{C}$.

(c) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is $q/2\epsilon_0 A$, the field between the plates is

$$E = \frac{q_f}{2\epsilon_0 A} + \frac{q_f}{2\epsilon_0 A} - \frac{q_i}{2\epsilon_0 A} - \frac{q_i}{2\epsilon_0 A}\,,$$

where the first term is due to the positive free charge on one plate, the second is due to the negative free charge on the other plate, the third is due to the positive induced charge on one dielectric surface, and the fourth is due to the negative induced charge on the other dielectric surface. Note that the field due to the induced charge is opposite the field due to the free charge, so the fields tend to cancel. The induced charge is therefore

$$\begin{aligned} q_i &= q_f - \epsilon_0 AE \\ &= 5.0 \times 10^{-9}\,\mathrm{C} - (8.85 \times 10^{-12}\,\mathrm{F/m})(100 \times 10^{-4}\,\mathrm{m}^2)(1.0 \times 10^4\,\mathrm{V/m}) \\ &= 4.1 \times 10^{-9}\,\mathrm{C} = 4.1\,\mathrm{nC}\,. \end{aligned}$$

CHAPTER 25 HINT FOR PROBLEM 9

C_1 and C_2 are in parallel and the combination is in series with C_3. First find the equivalent capacitance C_{12} of C_1 and C_2, then the equivalent capacitance of C_{12} and C_3.

[ans: $3.16\,\mu$F]

C_3 and C_5 are in series and this combination is in parallel with C_2 and C_4. C_1 and C_6 are in parallel and this combination is in series with the combination of C_3, C_5, C_2, and C_4. The charge stored by the equivalent capacitor is $C_{\text{eq}}V$. Find the charge and potential difference of any of the capacitors by remembering that the potential differences are the same across two capacitors in parallel and that the charges are the same for two capacitors in series. In addition, the potential difference for a parallel connection is the same as the potential difference across the equivalent capacitor and the charge on each capacitor of a series connection is the same as the charge for the equivalent capacitor.

[ans: (a) $3.00\,\mu$F; (b) $60\,\mu$C; (c) 10 V; (d) $30.0\,\mu$C; (e) 10 V; (f) $20.0\,\mu$C; (g) 5.00 V; (h) $20.0\,\mu$C]

CHAPTER 25 HINT FOR PROBLEM 25

The energy stored in a capacitor is $\frac{1}{2}CV^2$, where C is its capacitance and V is the potential difference across its plates. You need to convert $10\,\mathrm{kW\cdot h}$ to joules.

$\Big[$ ans: $72\,\mathrm{F}$ $\Big]$

CHAPTER 25 HINT FOR PROBLEM 33

The three capacitors each carry the same charge, so the one with the smallest capacitance has the greatest potential difference. Take this to be 100 V. Calculate the charge on this capacitor (and hence on each of the others) and then the potential differences across the other capacitors. The potential difference between points A and B is the sum of the potential differences across the capacitors.

[ans: (a) 190 V; (b) 95 mJ]

The capacitance of the capacitor before the slab is inserted is $C = \epsilon_0 A/d$, where A is a plate area and d is the plate separation. You may think of the capacitor after the slab is inserted as two capacitors in series. One has capacitance $\epsilon_0 A/(d-b)$ and the other has capacitance $\kappa\epsilon_0 A/b$, where b is the thickness of the slab. The charge on the capacitor before the slab is inserted is $q = CV$, where V is the potential difference across the battery and, since the battery is then disconnected, no charge leaves the capacitor as the slab is inserted. The electric field between a plate and the dielectric is given by $E = q/\epsilon_0$ and the electric field in the dielectric is given by E/κ. The potential difference across the plates is the charge on a plate divided by the capacitance. The work done in inserting the slab is the change in the energy stored by the capacitor, calculated using $U = \frac{1}{2}q^2/C$.

[ans: (a) 89 pF; (b) 0.12 nF; (c) 11 nC; (d) 11 nC; (e) 10 kV/m; (f) 2.1 kV/m; (g) 88 V; (h) $-0.17\,\mu$J]

(a) The magnitude of the current density is given by $J = nqv_d$, where n is the number of particles per unit volume, q is the charge on each particle, and v_d is the drift speed of the particles. The particle concentration is $n = 2.0 \times 10^8\,\text{cm}^{-3} = 2.0 \times 10^{14}\,\text{m}^{-3}$, the charge is $q = 2e = 2(1.60 \times 10^{-19}\,\text{C}) = 3.20 \times 10^{-19}\,\text{C}$, and the drift speed is $1.0 \times 10^5\,\text{m/s}$. Thus,

$$J = (2 \times 10^{14}\,\text{m}^{-3})(3.2 \times 10^{-19}\,\text{C})(1.0 \times 10^5\,\text{m/s}) = 6.4\,\text{A/m}^2\,.$$

(b) Since the particles are positively charged, the current density is in the same direction as their motion, to the north.

(c) The current cannot be calculated unless the cross-sectional area of the beam is known. Then $i = JA$ can be used.

The resistance of conductor A is given by

$$R_A = \frac{\rho L}{\pi r_A^2}\,,$$

where r_A is the radius of the conductor. If r_o is the outside radius of conductor B and r_i is its inside radius, then its cross-sectional area is $\pi(r_o^2 - r_i^2)$ and its resistance is

$$R_B = \frac{\rho L}{\pi(r_o^2 - r_i^2)}\,.$$

The ratio is

$$\frac{R_A}{R_B} = \frac{r_o^2 - r_i^2}{r_A^2} = \frac{(1.0\,\text{mm})^2 - (0.50\,\text{mm})^2}{(0.50\,\text{mm})^2} = 3.0\,.$$

(a) and (b) Calculate the electrical resistances of the wires. Let ρ_C be the resistivity of wire C, r_C be its radius, and L_C be its length. Then the resistance of this wire is

$$R_C = \rho_C \frac{L_C}{\pi r_C^2} = (2.0 \times 10^{-6}\,\Omega\cdot\text{m})\frac{1.0\,\text{m}}{\pi(0.50\times 10^{-3}\,\text{m})^2} = 2.54\,\Omega\,.$$

Let ρ_D be the resistivity of wire D, r_D be its radius, and L_D be its length. Then the resistance of this wire is

$$R_D = \rho_D \frac{L_D}{\pi r_D^2} = (1.0 \times 10^{-6}\,\Omega\cdot\text{m})\frac{1.0\,\text{m}}{\pi(0.25\times 10^{-3}\,\text{m})^2} = 5.09\,\Omega\,.$$

If i is the current in the wire, the potential difference between points 1 and 2 is

$$\Delta V_{12} = iR_C = (2.0\,\text{A})(2.54\,\Omega) = 5.1\,\text{V}$$

and the potential difference between points 2 and 3 is

$$\Delta V_{23} = iR_D = (2.0\,\text{A})(5.09\,\Omega) = 10\,\text{V}\,.$$

(c) and (d) The rate of energy dissipation between points 1 and 2 is

$$P_{12} = i^2 R_C = (2.0\,\text{A})^2(2.54\,\Omega) = 10\,\text{W}$$

and the rate of energy dissipation between points 2 and 3 is

$$P_{23} = i^2 R_D = (2.0\,\text{A})^2(5.09\,\Omega) = 20\,\text{W}\,.$$

CHAPTER 26 HINT FOR PROBLEM 7

The cross-sectional area of wire is given by $A = \pi r^2$, where r is its radius. The magnitude of the current density is $J = i/A$. Solve for r and double the result to obtain the diameter.

$\left[\text{ans: } 0.38\,\text{mm}\ \right]$

CHAPTER 26 HINT FOR PROBLEM 13

The conductivity is given by $\sigma = J/E$, where J is the magnitude of the current density and E is the magnitude of the electric field in the wire. The magnitude of the current density is the current divided by the cross-sectional area and the magnitude of the electric field is the potential difference divided by the length.

$\left[\text{ans: } 2.0 \times 10^6\,(\Omega \cdot \text{m})^{-1}\right]$

CHAPTER 26 HINT FOR PROBLEM 29

The current is the potential difference divided by the resistance. The magnitude of the current density is the current divided by the cross-sectional area. To calculate the drift speed v_d use $J = nev_d$, where J is the magnitude of the current density and n is the free-electron concentration. The magnitude of the electric field is the potential difference divided by the front-to-rear length.

[ans: (a) 38.3 mA; (b) 109 A/m^2; (c) 1.28 cm/s; (d) 227 V/m]

The rate of energy dissipation is given by $P = V^2/R$, where V is the potential difference across the resistor and R its resistance. For the same resistor connected to two different batteries the ratio of the dissipation rates is $P_1/P_2 = V_1^2/V_2^2$.

[ans: 0.135 W]

CHAPTER 26 HINT FOR PROBLEM 43

Multiply the power of the bulb by the duration of a month in hours and divide by 1000 to get the energy dissipated in kilowatt·hours. Finally, multiply by the cost of a kilowatt·hour to get the total cost. Use $P = V^2/R$, where P is the power of the bulb, V is the potential difference, and R is the resistance, to calculate the resistance of the bulb and $V = iR$ to calculate the current i.

[ans: (a) \$4.46 US; (b) $144\,\Omega$; (c) 0.833 A]

CHAPTER 27 SOLUTION FOR PROBLEM 5

(a) Let i be the current in the circuit and take it to be positive if it is to the left in R_1. Use Kirchhoff's loop rule: $\mathcal{E}_1 - iR_2 - iR_1 - \mathcal{E}_2 = 0$. Solve for i:

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{12\,\mathrm{V} - 6.0\,\mathrm{V}}{4.0\,\Omega + 8.0\,\Omega} = 0.50\,\mathrm{A}\,.$$

A positive value was obtained, so the current is counterclockwise around the circuit.

(b) and (c) If i is the current in a resistor R, then the power dissipated by that resistor is given by $P = i^2R$. For R_1, the power dissipated is

$$P_1 = (0.50\,\mathrm{A})^2(4.0\,\Omega) = 1.0\,\mathrm{W}$$

and for R_2, the power dissipated is

$$P_2 = (0.50\,\mathrm{A})^2(8.0\,\Omega) = 2.0\,\mathrm{W}\,.$$

(d) and (e) If i is the current in a battery with emf $\mathcal{E}$, then the battery supplies energy at the rate $P = i\mathcal{E}$ provided the current and emf are in the same direction. The battery absorbs energy at the rate $P = i\mathcal{E}$ if the current and emf are in opposite directions. For battery 1 the power is

$$P_1 = (0.50\,\mathrm{A})(12\,\mathrm{V}) = 6.0\,\mathrm{W}$$

and for battery 2 it is

$$P_2 = (0.50\,\mathrm{A})(6.0\,\mathrm{V}) = 3.0\,\mathrm{W}\,.$$

(f) and (g) In battery 1, the current is in the same direction as the emf so this battery supplies energy to the circuit. The battery is discharging. The current in battery 2 is opposite the direction of the emf, so this battery absorbs energy from the circuit. It is charging.

(a) First find the currents. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is to the left. Let i_3 be the current in R_3 and take it to be positive if it is to downward. The junction rule produces

$$i_1 + i_2 - i_3 = 0\,.$$

The loop rule applied to the left-hand loop produces

$$\mathcal{E}_1 - i_1 R_1 - i_3 R_3 = 0$$

and applied to the right-hand loop produces

$$\mathcal{E}_2 - i_2 R_2 - i_3 R_3 = 0\,.$$

Substitute $i_3 = -i_1 + i_2$, from the first equation, into the other two to obtain

$$\mathcal{E}_1 - i_1 R_1 - i_1 R_3 - i_2 R_3 = 0$$

and

$$\mathcal{E}_2 - i_2 R_2 - i_1 R_3 - i_2 R_3 = 0\,.$$

The first of these yields

$$i_1 = \frac{\mathcal{E}_1 - i_2 R_3}{R_1 + R_3}\,.$$

Substitute this into the second equation and solve for i_2. You should obtain

$$\begin{aligned} i_2 &= \frac{\mathcal{E}_2(R_1 + R_3) - \mathcal{E}_1 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3} \\ &= \frac{(1.00\,\mathrm{V})(4.00\,\Omega + 5.00\,\Omega) - (3.00\,\mathrm{V})(5.00\,\Omega)}{(4.00\,\Omega)(2.00\,\Omega) + (2.00\,\Omega)(5.00\,\Omega) + (4.00\,\Omega)(5.00\,\Omega)} = -0.158\,\mathrm{A}\,. \end{aligned}$$

Substitute this into the expression for i_1 to obtain

$$i_1 = \frac{\mathcal{E}_1 - i_2 R_3}{R_1 + R_3} = \frac{3.00\,\mathrm{V} - (-0.158\,\mathrm{A})(5.00\,\Omega)}{4.00\,\Omega + 5.00\,\Omega} = 0.421\,\mathrm{A}\,.$$

Finally,

$$i_3 = i_1 + i_2 = (0.421\,\mathrm{A}) + (-0.158\,\mathrm{A}) = 0.263\,\mathrm{A}\,.$$

Note that the current in R_2 is actually to the right. The current in R_1 is to the right and the current in R_3 is downward.

(a), (b), and (c) The rate with which energy is dissipated in R_1 is

$$P_1 = i_1^2 R_1 = (0.421\ \text{A})^2(4.00\ \Omega) = 0.709\ \text{W}\,.$$

The rate with which energy is dissipated in R_2 is

$$P_2 = i_2^2 R_2 = (-0.158\ \text{A})^2(2.00\ \Omega) = 0.0499\ \text{W}\,,$$

and the rate with which energy is dissipated in R_3 is

$$P_3 = i_3^2 R_3 = (0.263\ \text{A})^2(5.00\ \Omega) = 0.346\ \text{W}\,,$$

(d) and (e) The power of battery 1 is

$$i_1\mathcal{E}_1 = (0.421\ \text{A})(3.00\ \text{V}) = 1.26\ \text{W}$$

and the power of battery 2 is

$$i_2\mathcal{E}_2 = (-0.158\ \text{A})(1.00\ \text{V}) = -0.158\ \text{W}\,.$$

The negative sign indicates that battery 2 is actually absorbing energy from the circuit.

(a), (b), and (c) At $t = 0$, the capacitor is completely uncharged and the current in the capacitor branch is as it would be if the capacitor were replaced by a wire. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is downward. Let i_3 be the current in R_3 and take it to be positive if it is downward. The junction rule produces $i_1 = i_2 + i_3$, the loop rule applied to the left-hand loop produces $\mathcal{E} - i_1 R_1 - i_2 R_2 = 0$, and the loop rule applied to the right-hand loop produces $i_2 R_2 - i_3 R_3 = 0$. Since the resistances are all the same, you can simplify the mathematics by replacing R_1, R_2, and R_3 with R. The solution to the three simultaneous equations is

$$i_1 = \frac{2\mathcal{E}}{3R} = \frac{2(1.2 \times 10^3\,\mathrm{V})}{3(0.73 \times 10^6\,\Omega)} = 1.1 \times 10^{-3}\,\mathrm{A}$$

and

$$i_2 = i_3 = \frac{\mathcal{E}}{3R} = \frac{1.2 \times 10^3\,\mathrm{V}}{3(0.73 \times 10^6\,\Omega)} = 5.5 \times 10^{-4}\,\mathrm{A}\,.$$

(d), (e), and (f) At $t = \infty$, the capacitor is fully charged and the current in the capacitor branch is zero. Then $i_1 = i_2$ and the loop rule yields

$$\mathcal{E} - i_1 R_1 - i_1 R_2 = 0\,.$$

The solution is

$$i_1 = i_2 = \frac{\mathcal{E}}{2R} = \frac{1.2 \times 10^3\,\mathrm{V}}{2(0.73 \times 10^6\,\Omega)} = 8.2 \times 10^{-4}\,\mathrm{A}\,.$$

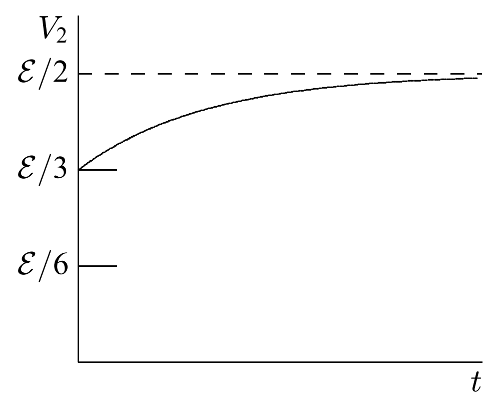
(g) and (h) The potential difference across resistor 2 is $V_2 = i_2 R_2$. At $t = 0$ it is

$$V_2 = (5.5 \times 10^{-4}\,\mathrm{A})(0.73 \times 10^6\,\Omega) = 4.0 \times 10^2\,\mathrm{V}$$

and at $t = \infty$ it is

$$V_2 = (8.2 \times 10^{-4}\,\mathrm{A})(0.73 \times 10^6\,\Omega) = 6.0 \times 10^2\,\mathrm{V}\,.$$

(i) The graph of V_2 versus t is shown to the right.



Since the battery is being charged the potential difference across its terminals is given by $\mathcal{E} + ir$, where $\mathcal{E}$ is its emf and r is its internal resistance. Energy is dissipated at a rate that is given by i^2r and is being converted to chemical energy at a rate of $\mathcal{E}i$. When the battery is being discharged the potential difference across its terminals is given by $\mathcal{E} - ir$.

[ans: (a) 14 V; (b) 1.0×10^2 W; (c) 6.0×10^2 W; (d) 10 V, 1.0×10^2 W]

R_1 and R_2 are in parallel and their equivalent resistor is in series with R_3.

$\left[\text{ans: } 4.50\,\Omega\ \right]$

The dissipation rate in R_3 is given by $i_3^2 R_3$, where i_3 is the current in R_3. Use the loop and junction rules to find an expression for i_3 in terms of R_3, then set its derivative with respect to R_3 equal to zero and solve for R_3.

[ans: $1.43\,\Omega$]

CHAPTER 27 HINT FOR PROBLEM 37

Write one junction and two loop equation, then solve them simultaneously for the current in R_3. Assume the ammeter has zero resistance.

[ans: 0.45 A]

CHAPTER 27 HINT FOR PROBLEM 43

Use the loop equation for the loop containing R_1, R_2, R_s, and R_x. Write an expression for the potential difference $V_b - V_a$ and set it equal to zero. Solve these two equations for R_x.

The energy initially stored by the capacitor is given by $q_0^2/2C$, where q_0 is the initial charge. Solve for q_0. At any time t the charge on the capacitor is given by $q = q_0 e^{-t/\tau}$, and the current is given by $i = (q_0/\tau)e^{-t/\tau}$, where τ (= RC) is the capacitive time constant. The potential difference across the capacitor is q/C, the potential difference across the resistor is iR, and the rate of thermal energy production in the resistor is given by i^2R.

[ans: (a) 1.0×10^{-3} C; (b) 1.0×10^{-3} A; (c) $(1.0 \times 10^3\,\mathrm{V})\,e^{-t}$; (d) $(1.0 \times 10^3\,\mathrm{V})\,e^{-t}$; (e) $P = e^{-2t}$ W]

For the particle to be undeflected the only component of the net force that does not vanish is the component that is parallel to the particle velocity. The electric field is the smallest possible if all components of the net force vanish. Thus $e(\vec{E} + \vec{v} \times \vec{B}) = 0$. The electric field $\vec{E}$ must be perpendicular to both the particle velocity $\vec{v}$ and the magnetic field $\vec{B}$. The magnetic field is perpendicular to the velocity, so $\vec{v} \times \vec{B}$ has magnitude vB and the magnitude of the electric field is given by $E = vB$. Since the particle has charge e and is accelerated through a potential difference V, $\frac{1}{2}mv^2 = eV$ and $v = \sqrt{2eV/m}$. Thus

$$E = B\sqrt{\frac{2eV}{m}} = (1.2\,\mathrm{T})\sqrt{\frac{2(1.60 \times 10^{-19}\,\mathrm{C})(10 \times 10^3\,\mathrm{V})}{9.99 \times 10^{-27}\,\mathrm{kg}}} = 6.8 \times 10^5\,\mathrm{V/m}\,.$$

(a) If v is the speed of the positron, then $v \sin\phi$ is the component of its velocity in the plane that is perpendicular to the magnetic field. Here ϕ is the angle between the velocity and the field (89°). Newton's second law yields $eBv \sin\phi = m(v \sin\phi)^2/r$, where r is the radius of the orbit. Thus $r = (mv/eB)\sin\phi$. The period is given by

$$T = \frac{2\pi r}{v \sin\phi} = \frac{2\pi m}{eB} = \frac{2\pi(9.11 \times 10^{-31}\,\mathrm{kg})}{(1.60 \times 10^{-19}\,\mathrm{C})(0.100\,\mathrm{T})} = 3.58 \times 10^{-10}\,\mathrm{s}\,.$$

The expression for r was substituted to obtain the second expression for T.

(b) The pitch p is the distance traveled along the line of the magnetic field in a time interval of one period. Thus $p = vT\cos\phi$. Use the kinetic energy to find the speed: $K = \frac{1}{2}mv^2$ means

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(2.0 \times 10^3\,\mathrm{eV})(1.60 \times 10^{-19}\,\mathrm{J/eV})}{9.11 \times 10^{-31}\,\mathrm{kg}}} = 2.651 \times 10^7\,\mathrm{m/s}\,.$$

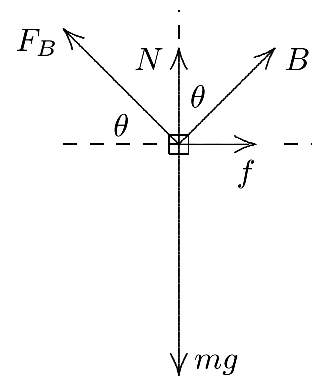
Thus

$$p = (2.651 \times 10^7\,\mathrm{m/s})(3.58 \times 10^{-10}\,\mathrm{s})\cos 89.0^\circ = 1.66 \times 10^{-4}\,\mathrm{m}\,.$$

(c) The orbit radius is

$$r = \frac{mv \sin\phi}{eB} = \frac{(9.11 \times 10^{-31}\,\mathrm{kg})(2.651 \times 10^7\,\mathrm{m/s})\sin 89.0^\circ}{(1.60 \times 10^{-19}\,\mathrm{C})(0.100\,\mathrm{T})} = 1.51 \times 10^{-3}\,\mathrm{m}\,.$$

The magnetic force must push horizontally on the rod to overcome the force of friction. But it can be oriented so it also pulls up on the rod and thereby reduces both the normal force and the maximum possible force of static friction.



Suppose the magnetic field makes the angle θ with the vertical. The diagram to the right shows the view from the end of the sliding rod. The forces are also shown: F_B is the force of the magnetic field if the current is out of the page, mg is the force of gravity, N is the normal force of the stationary rails on the rod, and f is the force of friction. Notice that the magnetic force makes the angle θ with the *horizontal*. When the rod is on the verge of sliding, the net force acting on it is zero and the magnitude of the frictional force is given by $f = \mu_s N$, where μ_s is the coefficient of static friction. The magnetic field is perpendicular to the wire so the magnitude of the magnetic force is given by $F_B = iLB$, where i is the current in the rod and L is the length of the rod.

The vertical component of Newton's second law yields

$$N + iLB\sin\theta - mg = 0$$

and the horizontal component yields

$$iLB\cos\theta - \mu_s N = 0\,.$$

Solve the second equation for N and substitute the resulting expression into the first equation, then solve for B. You should get

$$B = \frac{\mu_s mg}{iL(\cos\theta + \mu_s\sin\theta)}\,.$$

The minimum value of B occurs when $\cos\theta + \mu_s\sin\theta$ is a maximum. Set the derivative of $\cos\theta + \mu_s\sin\theta$ equal to zero and solve for θ. You should get $\theta = \tan^{-1}\mu_s = \tan^{-1}(0.60) = 31°$. Now evaluate the expression for the minimum value of B:

$$B_{\text{min}} = \frac{0.60(1.0\,\text{kg})(9.8\,\text{m/s}^2)}{(50\,\text{A})(1.0\,\text{m})(\cos 31° + 0.60\sin 31°)} = 0.10\,\text{T}\,.$$

CHAPTER 28 HINT FOR PROBLEM 7

Take the magnetic field to be perpendicular to both the electric field and the particle velocity. Since the net force on the electron is zero the magnitudes of the fields are related by $E = vB$, where E is the magnitude of the electric field, B is the magnitude of the magnetic field, and v is the speed of the electron. The magnitude of the electric field is $E = V/d$, where V is the potential difference between the plates and d is the plate separation. The electric field points from the positive plate toward the negative plate. Choose the direction of the magnetic field so that the direction of the magnetic force is opposite to the direction of the electric force.

[ans: $-0.267\,\mathrm{mT}$]

CHAPTER 28 HINT FOR PROBLEM 13

Since the net force on electrons in the solid is zero the electric field is given by $\vec{E} = \vec{v} \times \vec{B}$, where $\vec{v}$ is the velocity of the solid and $\vec{B}$ is the magnetic field. The potential difference across the solid is Ed, where d is the width of the solid along the direction of the electric field.

$\left[\text{ans: (a) } (-600\,\mathrm{mV/m})\,\hat{\mathrm{k}}\text{; (b) } 1.20\,\mathrm{V}\ \right]$

CHAPTER 28 HINT FOR PROBLEM 21

The magnitude of the magnetic force is $F_B = ev_\perp B$, where $v_\perp$ is the component of the particle velocity perpendicular to the magnetic field and B is the magnitude of the field. The pitch is $p = v_\parallel T$, where $v_\parallel$ is the component of the particle velocity along the direction of the field and T is the period of the motion. The period is $T = 2\pi m/eB$.

$\left[\text{ans: } 65.3\,\mathrm{m/s}\right]$

CHAPTER 28 HINT FOR PROBLEM 27

Linear momentum is conserved in the decay and the particles have the same mass, so they move away in opposite directions with the same speed. They collide after they have gone halfway around their circular orbits. This occurs after a time equal to half a period of the motion. The period is given by $T = 2\pi m/eB$.

[ans: (a) 5.06 ns]

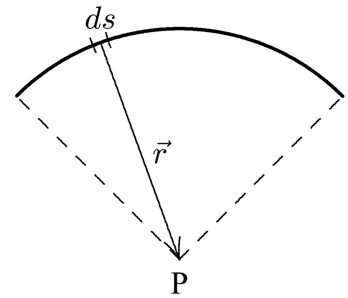
CHAPTER 28 HINT FOR PROBLEM 35

The magnetic force on the wire must be upward and equal in magnitude to the weight of the wire. The magnetic force is given by $i\vec{L} \times \vec{B}$, where i is the current, $\vec{B}$ is the magnetic field, and L is the length of the wire. The vector $\vec{L}$ is in the direction of the current.

[ans: (a) 467 mA; (b) right]

The magnitude of the magnetic dipole moment is the product of the loop area (πR^2, where R is the radius) and the current in the loop. Multiply this by the unit vector in the direction of the dipole moment to find the vector moment. The torque is the vector product of the dipole moment and the magnetic field. The magnetic potential energy is the negative of the scalar product of the dipole moment and the field.

[ans: (a) $(-9.7 \times 10^{-4}\,\mathrm{N \cdot m})\,\hat{\mathrm{i}} - (7.2 \times 10^{-4}\,\mathrm{N \cdot m})\,\hat{\mathrm{j}} + (8.0 \times 10^{-4}\,\mathrm{N \cdot m})\,\hat{\mathrm{k}}$;
(b) $-6.0 \times 10^{-4}\,\mathrm{J}$]



First find the magnetic field of a circular arc at its center. Let $d\vec{s}$ be an infinitesimal segment of the arc and $\vec{r}$ be the vector from the segment to the arc center. $d\vec{s}$ and $\vec{r}$ are perpendicular to each other, so the contribution of the segment to the field at the center has magnitude

$$dB = \frac{\mu_0 i}{4\pi r^2}\, ds\,.$$

The field is into the page if the current is from left to right in the diagram and out of the page if the current is from right to left. All segments contribute magnetic fields in the same direction. Furthermore, r is the same for all of them. Thus the magnitude of the net field at the center is given by

$$B = \frac{\mu_0 i s}{4\pi r^2} = \frac{\mu_0 i\theta}{4\pi r}\,.$$

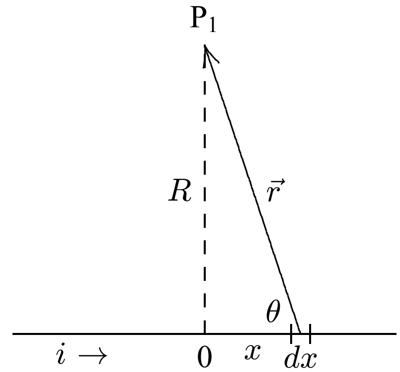
Here s is the arc length and θ is the angle (in radians) subtended by the arc at its center. The second expression was obtained by replacing s with $r\theta$. θ must be in radians for this expression to be valid.

Now consider the circuit of Fig. 29–36. The magnetic field produced by the inner arc has magnitude $\mu_0 i\theta/4\pi b$ and is out of the page. The field produced by the outer arc has magnitude $\mu_0 i\theta/4\pi a$ and is into the page. The two straight segments of the circuit do not produce fields at the center of the arcs because the vector $\vec{r}$ from any point on them to the center is parallel or antiparallel to the current at that point. If the positive direction is out of the page, then the net magnetic field at the center is

$$\begin{aligned}
B &= \frac{\mu_0 i\theta}{4\pi}\left[\frac{1}{b} - \frac{1}{a}\right] \\
&= \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(0.411\,\mathrm{A})(79.0^\circ)(1.745 \times 10^{-2}\,\mathrm{rad/deg})}{4\pi}\left[\frac{1}{0.107\,\mathrm{m}} - \frac{1}{0.135\,\mathrm{m}}\right] \\
&= 1.03 \times 10^{-7}\,\mathrm{T}\,.
\end{aligned}$$

Since $b < a$, the net field is out of the page.

Put the x axis along the wire with the origin at the midpoint and the current in the positive x direction. All segments of the wire produce magnetic fields at P_1 that are into the page so we simply divide the wire into infinitesimal segments and sum the fields due to all the segments. The diagram shows one infinitesimal segment, with length dx. According to the Biot-Savart law, the magnitude of the field it produces at P_1 is given by



$$dB = \frac{\mu_0 i}{4\pi} \frac{\sin\theta}{r^2}\, dx\,.$$

θ and r are functions of x. Replace r with $\sqrt{x^2+R^2}$ and $\sin\theta$ with $R/r = R/\sqrt{x^2+R^2}$, then integrate from $x = -L/2$ to $x = L/2$. The net field is

$$B = \frac{\mu_0 i R}{4\pi} \int_{-L/2}^{L/2} \frac{dx}{(x^2+R^2)^{3/2}} = \frac{\mu_0 i R}{4\pi} \frac{1}{R^2} \frac{x}{(x^2+R^2)^{1/2}}\bigg|_{-L/2}^{L/2} = \frac{\mu_0 i}{2\pi R} \frac{L}{\sqrt{L^2+4R^2}}$$

$$= \frac{4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(0.0582\,\mathrm{A})}{2\pi(0.131\,\mathrm{m})} \frac{0.180\,\mathrm{m}}{\sqrt{(0.180\,\mathrm{m})^2 + 4(0.131\,\mathrm{m})^2}} = 5.03 \times 10^{-8}\,\mathrm{T}\,.$$

(a) Assume that the point is inside the solenoid. The field of the solenoid at the point is parallel to the solenoid axis and the field of the wire is perpendicular to the solenoid axis. The net field makes an angle of 45° with the axis if these two fields have equal magnitudes.

The magnitude of the magnetic field produced by a solenoid at a point inside is given by $B_{sol} = \mu_0 i_{sol} n$, where n is the number of turns per unit length and i_{sol} is the current in the solenoid. The magnitude of the magnetic field produced by a long straight wire at a point a distance r away is given by $B_{wire} = \mu_0 i_{wire}/2\pi r$, where i_{wire} is the current in the wire. We want $\mu_0 n i_{sol} = \mu_0 i_{wire}/2\pi r$. The solution for r is

$$r = \frac{i_{wire}}{2\pi n i_{sol}} = \frac{6.00\,\mathrm{A}}{2\pi(10.0\times10^2\,\mathrm{m}^{-1})(20.0\times10^{-3}\,\mathrm{A})} = 4.77\times10^{-2}\,\mathrm{m} = 4.77\,\mathrm{cm}\,.$$

This distance is less than the radius of the solenoid, so the point is indeed inside as we assumed.

(b) The magnitude of the either field at the point is

$$B_{sol} = B_{wire} = \mu_0 n i_{sol} = (4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(10.0\times10^2\,\mathrm{m}^{-1})(20.0\times10^{-3}\,\mathrm{A}) = 2.51\times10^{-5}\,\mathrm{T}\,.$$

Each of the two fields is a component of the net field, so the magnitude of the net field is the square root of the sum of the squares of the individual fields: $B = \sqrt{2(2.51\times10^{-5}\,\mathrm{T})^2} = 3.55\times10^{-5}\,\mathrm{T}$.

Since the velocity of the proton is perpendicular to the magnetic field, the magnitude of the force on the proton is given by $F = evB$, where v is the speed of the proton and B is the magnitude of the field. The magnitude of the field is given by $B = \mu_0 i/2\pi r$, where i is the current in the wire and r is the distance of the proton from the wire. The direction of the field is given by the fingers of your right hand when your thumb is along the wire in the direction of the current. The direction of the force on the proton is the direction of the vector product $\vec{v} \times \vec{B}$.

$\left[\text{ans: } (-7.76 \times 10^{-23}\,\text{N})\,\hat{\text{i}}\right]$

The magnitude of the magnetic field produced by a straight wire of length L at a point that is a distance R from an end of the wire on a line that is perpendicular to the wire is given by

$$B = \frac{\mu_0 i}{4\pi R} \frac{L}{\sqrt{L^2 + R^2}} .$$

(See problem 19.) To find the direction of the field point the thumb of your right hand along the wire in the direction of the current. Your fingers then curl around the wire in the direction of the field lines. Vectorially add the field of the six wires.

[ans: (a) 20 μT; (b) into]

The magnitude of the force per unit length of a long straight current-carrying wire on a parallel long straight current-carrying wire is given by $F/L = \mu_0 i_a i_b / 2\pi d$, where i_a and i_b are the currents in the wires and d is the separation of the wires. If the currents are in the same direction, the force is one of attraction along a line joining the wires and if the currents are in opposite directions, the force is one of repulsion along such a line. To find the net force on wire 4 sum the forces of the other wires on it.

[ans: $(-125\ \mu\text{N/m})\,\hat{\text{i}} + (41.7\ \mu\text{N/m})\,\hat{\text{j}}$]

se Ampere's law with an Amperian loop that is a circle of radius r, concentric with the wire. The law gives $B2\pi r = \mu_0 i_{\text{enc}}$, where B is the magnitude of the magnetic field a distance r from the central axis of the wire and i_{enc} is the current through the loop. If $r < a$, then $i_{\text{enc}} = (r/a)^2 i$, where i is the total current in the wire. If $r > a$, then $I_{\text{enc}} = i$.

[ans: (a) 0; (b) 0.850 mT; (c) 1.70 mT; (d) 0.850 mT]

CHAPTER 41 HINT FOR PROBLEM T

he magnitude of the magnetic field inside a solenoid is given by $B = \mu_0 n i$, where n is the number of turns per unit length of the solenoid and i is the current in the solenoid.

[ans: 0.30 mT]

The magnitude of the magnetic field inside the solenoid is $B = \mu_0 n i_s$, where n is the number of turns per unit length and i_s is the current. The field is parallel to the solenoid axis, so the flux through a cross section of the solenoid is $\Phi_B = A_s B = \mu_0 \pi r_s^2 n i_s$, where A_s $(= \pi r_s^2)$ is the cross-sectional area of the solenoid. Since the magnetic field is zero outside the solenoid, this is also the flux through the coil. The emf in the coil has magnitude

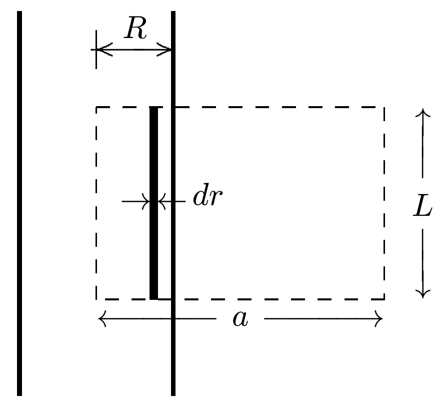
$$\mathcal{E} = \frac{N d\Phi_B}{dt} = \mu_0 \pi r_s^2 N n \frac{di_s}{dt}$$

and the current in the coil is

$$i_c = \frac{\mathcal{E}}{R} = \frac{\mu_0 \pi r_s^2 N n}{R} \frac{di_s}{dt} ,$$

where N is the number of turns in the coil and R is the resistance of the coil. The current changes linearly by 3.0 A in 50 ms, so $di_s/dt = (3.0\,\text{A})/(50 \times 10^{-3}\,\text{s}) = 60\,\text{A/s}$. Thus

$$i_c = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})\pi(0.016\,\text{m})^2(120)(220 \times 10^2\,\text{m}^{-1})}{5.3\,\Omega}(60\,\text{A/s}) = 3.0 \times 10^{-2}\,\text{A} .$$



(a) Suppose each wire has radius R and the distance between their axes is a. Consider a single wire and calculate the flux through a rectangular area with the axis of the wire along one side. Take this side to have length L and the other dimension of the rectangle to be a. The magnetic field is everywhere perpendicular to the rectangle. First consider the part of the rectangle that is inside the wire. The field a distance r from the axis is given by $B = \mu_0 ir/2\pi R^2$ and the flux through the strip of length L and width dr at that distance is $(\mu_0 ir/2\pi R^2)L\,dr$. Thus the flux through the area inside the wire is

$$\Phi_{\text{in}} = \int_0^R \frac{\mu_0 iL}{2\pi R^2}\, r\,dr = \frac{\mu_0 iL}{4\pi}\,.$$

Now consider the region outside the wire. There the field is given by $B = \mu_0 i/2\pi r$ and the flux through an infinitesimally thin strip is $(\mu_0 i/2\pi r)L\,dr$. The flux through the whole region is

$$\Phi_{\text{out}} = \int_R^a \frac{\mu_0 iL}{2\pi}\frac{dr}{r} = \frac{\mu_0 iL}{2\pi}\ln\left(\frac{a}{R}\right)\,.$$

The total flux through the area bounded by the dashed lines is the sum of the two contributions:

$$\Phi = \frac{\mu_0 iL}{4\pi}\left[1 + 2\ln\left(\frac{a}{R}\right)\right]\,.$$

Now include the contribution of the other wire. Since the currents are in the same direction, the two contributions have the same sign. They also have the same magnitude, so

$$\Phi_{\text{total}} = \frac{\mu_0 iL}{2\pi}\left[1 + 2\ln\left(\frac{a}{R}\right)\right]\,.$$

The total flux per unit length is

$$\frac{\Phi_{\text{total}}}{L} = \frac{\mu_0 i}{2\pi}\left[1 + 2\ln\left(\frac{a}{R}\right)\right] = \frac{(4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})(10\,\mathrm{A})}{2\pi}\left[1 + 2\ln\left(\frac{20\,\mathrm{mm}}{1.25\,\mathrm{mm}}\right)\right]$$
$$= 1.31\times 10^{-5}\,\mathrm{Wb/m}\,.$$

(b) Again consider the flux of a single wire. The flux inside the wire itself is again $\Phi_{\text{in}} = \mu_0 iL/4\pi$. The flux inside the region due to the other wire is

$$\Phi_{\text{out}} = \int_{a-R}^a \frac{\mu_0 iL}{2\pi}\frac{dr}{r} = \frac{\mu_0 iL}{2\pi}\ln\left(\frac{a}{a-R}\right)\,.$$

Add Φ_{in} and Φ_{out}, then double the result to include the flux of the other wire and divide by L to obtain the flux per unit length. The total flux per unit length that is inside the wires is

$$\begin{aligned}
\frac{\Phi_{\text{wires}}}{L} &= \frac{\mu_0 i}{2\pi}\left[1+2\ln\left(\frac{a}{a-R}\right)\right] \\
&= \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(10\,\text{A})}{2\pi}\left[1+2\ln\left(\frac{20\,\text{mm}}{20\,\text{mm}-1.25\,\text{mm}}\right)\right] \\
&= 2.26\times10^{-6}\,\text{Wb/m}\,.
\end{aligned}$$

The fraction of the total flux that is inside the wires is

$$\frac{2.26\times10^{-6}\,\text{Wb/m}}{1.31\times10^{-5}\,\text{Wb/m}} = 0.17$$

or 17%.

(c) The contributions of the two wires to the total flux have the same magnitudes but now the currents are in opposite directions, so the contributions have opposite signs. This means $\Phi_{total} = 0$.

(a) Let x be the distance from the right end of the rails to the rod and find an expression for the magnetic flux through the area enclosed by the rod and rails. The magnetic field is not uniform but varies with distance from the long straight wire. The field is normal to the area and has magnitude $B = \mu_0 i/2\pi r$, where r is the distance from the wire and i is the current in the wire. Consider an infinitesimal strip of length x and width dr, parallel to the wire and a distance r from it. The area of this strip is $A = x\,dr$ and the flux through it is $d\Phi_B = (\mu_0 ix/2\pi r)\,dr$. The total flux through the area enclosed by the rod and rails is

$$\Phi_B = \frac{\mu_0 ix}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 ix}{2\pi}\ln\left(\frac{a+L}{a}\right).$$

According to Faraday's law, the emf induced in the loop is

$$\begin{aligned}\mathcal{E} &= \frac{d\Phi}{dt} = \frac{\mu_0 i}{2\pi}\frac{dx}{dt}\ln\left(\frac{a+L}{a}\right) = \frac{\mu_0 iv}{2\pi}\ln\left(\frac{a+L}{a}\right)\\ &= \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(100\,\mathrm{A})(5.00\,\mathrm{m/s})}{2\pi}\ln\left(\frac{1.00\,\mathrm{cm}+10.0\,\mathrm{cm}}{1.00\,\mathrm{cm}}\right)\\ &= 2.40\times10^{-4}\,\mathrm{V}.\end{aligned}$$

(b) If R is the resistance of the rod, then the current in the conducting loop is

$$i_\ell = \frac{\mathcal{E}}{R} = \frac{2.40\times10^{-4}\,\mathrm{V}}{0.400\,\Omega} = 6.00\times10^{-4}\,\mathrm{A}.$$

Since the flux is increasing, the magnetic field produced by the induced current must be into the page in the region enclosed by the rod and rails. This means the current is clockwise.

(c) Thermal energy is generated at the rate

$$P = i_\ell^2 R = (6.00\times10^{-4}\,\mathrm{A})^2(0.400\,\Omega) = 1.44\times10^{-7}\,\mathrm{W}.$$

(d) Since the rod moves with constant velocity, the net force on it is zero. The force of the external agent must have the same magnitude as the magnetic force and must be in the opposite direction. The magnitude of the magnetic force on an infinitesimal segment of the rod, with length dr and a distance r from the long straight wire, is $dF_B = i_\ell B\,dr = (\mu_0 i_\ell i/2\pi r)\,dr$. The total magnetic force on the rod has magnitude

$$\begin{aligned}F_B &= \frac{\mu_0 i_\ell i}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 i_\ell i}{2\pi}\ln\left(\frac{a+L}{a}\right)\\ &= \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(6.00\times10^{-4}\,\mathrm{A})(100\,\mathrm{A})}{2\pi}\ln\left(\frac{1.00\,\mathrm{cm}+10.0\,\mathrm{cm}}{1.00\,\mathrm{cm}}\right)\\ &= 2.87\times10^{-8}\,\mathrm{N}.\end{aligned}$$

Since the field is out of the page and the current in the rod is upward in the diagram, this force is toward the right. The external agent must apply a force of 2.87×10^{-8} N, to the left.

(e) The external agent does work at the rate

$$P = Fv = (2.87 \times 10^{-8}\,\text{N})(5.00\,\text{m/s}) = 1.44 \times 10^{-7}\,\text{W}\,.$$

This is the same as the rate at which thermal energy is generated in the rod. All the energy supplied by the agent is converted to thermal energy.

(a) Assume i is from left to right through the closed switch. Let i_1 be the current in the resistor and take it to be downward. Let i_2 be the current in the inductor and also take it to be downward. The junction rule gives $i = i_1 + i_2$ and the loop rule gives $i_1 R - L(di_2/dt) = 0$. Since $di/dt = 0$, the junction rule yields $(di_1/dt) = -(di_2/dt)$. Substitute into the loop equation to obtain

$$L\frac{di_1}{dt} + i_1 R = 0\,.$$

This equation is similar to Eq. 30–44, and its solution is the function given as Eq. 30–45:

$$i_1 = i_0 e^{-Rt/L}\,,$$

where i_0 is the current through the resistor at $t = 0$, just after the switch is closed. Now, just after the switch is closed, the inductor prevents the rapid build-up of current in its branch, so at that time, $i_2 = 0$ and $i_1 = i$. Thus $i_0 = i$, so

$$i_1 = ie^{-Rt/L}$$

and

$$i_2 = i - i_1 = i\left[1 - e^{-Rt/L}\right]\,.$$

(b) When $i_2 = i_1$,

$$e^{-Rt/L} = 1 - e^{-Rt/L}\,,$$

so

$$e^{-Rt/L} = \frac{1}{2}\,.$$

Take the natural logarithm of both sides and use $\ln(1/2) = -\ln 2$ to obtain $(Rt/L) = \ln 2$ or

$$t = \frac{L}{R}\ln 2\,.$$

CHAPTER 30 HINT FOR PROBLEM 11

The magnetic flux though the coil is the product of the magnitude of the magnetic field, the area of the coil, the number of turns, and the cosine of the angle between the normal to the coil and the magnetic field. This angle, in radians, is given by $2\pi f t$, where f is the frequency of rotation and t is the time. Differentiate the flux with respect to time to find the emf.

$\left[\text{ans: (b) } 0.786\,\text{m}^2\right]$

Use Faraday's law. The area of the circuit is $A_R + A_S \cos(\omega t)$, where A_R is the area of the rectangular portion, A_S is the area of the semicircle, and ω is the angular frequency of rotation. The magnetic flux through the circuit is the product of the area and the magnitude of the magnetic field. Differentiate the flux with respect to time to find the induced emf.

[ans: (a) 40 Hz; (b) 3.2 mV]

Use Faraday's law. The area of the circuit is changing. The magnetic flux through the circuit is Bwx, where B is the magnitude of the magnetic field, w is the width of the circuit, and x is the distance of the rod from the right end of the rails. The rate of change of the flux is Bwv, where v (= dx/dt) is the speed of the rod. The current in the circuit is the induced emf divided by the resistance. The rate of production of thermal energy is i^2R, where i is the current and R is the resistance. Since the rod moves with constant velocity, the force that must be applied is equal in magnitude to the magnetic force on the rod. The rate with which the external force does work is the product of the force and the speed of the rod.

[ans: (a) 0.60 V; (b) up; (c) 1.5 A; (d) clockwise; (e) 0.90 W; (f) 0.18 N; (g) 0.90 W]

CHAPTER 30 HINT FOR PROBLEM 45

The current is the same in the two inductors and the total emf is the sum of their emfs. The equivalent inductance is the total emf divided by the rate of change of the current.

[ans: (b) $L_{\text{eq}} = \sum_{j=1}^{N} L_j$]

The current is given by $i = i_0 e^{-t/\tau_L}$, where i_0 is the current at time $t = 0$ and τ_L is the inductive time constant. Solve for τ_L by taking the natural logarithm of both sides, then use $\tau_L = L/R$ to compute R. Here L is the inductance and R is the resistance.

$\left[\text{ans: } 46\,\Omega\ \right]$

The electric energy density is given by $\frac{1}{2}\epsilon_0 E^2$ and the magnetic energy density is given by $B^2/2\mu_0$. Here E is the magnitude of the electric field and B is the magnitude of the magnetic field. Equate the two energy densities and solve for E.

$\left[\text{ans: } 1.5 \times 10^8\,\mathrm{V/m}\right]$

(a) Since the frequency of oscillation f is related to the inductance L and capacitance C by $f = 1/2\pi\sqrt{LC}$, the smaller value of C gives the larger value of f. Hence, $f_{max} = 1/2\pi\sqrt{LC_{min}}$, $f_{min} = 1/2\pi\sqrt{LC_{max}}$, and

$$\frac{f_{max}}{f_{min}} = \frac{\sqrt{C_{max}}}{\sqrt{C_{min}}} = \frac{\sqrt{365\,\text{pF}}}{\sqrt{10\,\text{pF}}} = 6.0\,.$$

(b) You want to choose the additional capacitance C so the ratio of the frequencies is

$$r = \frac{1.60\,\text{MHz}}{0.54\,\text{MHz}} = 2.96\,.$$

Since the additional capacitor is in parallel with the tuning capacitor, its capacitance adds to that of the tuning capacitor. If C is in picofarads, then

$$\frac{\sqrt{C + 365\,\text{pF}}}{\sqrt{C + 10\,\text{pF}}} = 2.96\,.$$

The solution for C is

$$C = \frac{(365\,\text{pF}) - (2.96)^2(10\,\text{pF})}{(2.96)^2 - 1} = 36\,\text{pF}\,.$$

(c) Solve $f = 1/2\pi\sqrt{LC}$ for L. For the minimum frequency, $C = 365\,\text{pF} + 36\,\text{pF} = 401\,\text{pF}$ and $f = 0.54\,\text{MHz}$. Thus,

$$L = \frac{1}{(2\pi)^2 C f^2} = \frac{1}{(2\pi)^2(401 \times 10^{-12}\,\text{F})(0.54 \times 10^6\,\text{Hz})^2} = 2.2 \times 10^{-4}\,\text{H}\,.$$

(a) For a given amplitude $\mathcal{E}_m$ of the generator emf, the current amplitude is given by

$$I = \frac{\mathcal{E}_m}{Z} = \frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}\,,$$

where R is the resistance, L is the inductance, C is the capacitance, and ω_d is the angular frequency. To find the maximum, set the derivative with respect to ω_d equal to zero and solve for ω_d. The derivative is

$$\frac{dI}{d\omega_d} = -\mathcal{E}_m \left[R^2 + (\omega_d L - 1/\omega_d C)^2\right]^{-3/2} \left[\omega_d L - \frac{1}{\omega_d C}\right] \left[L + \frac{1}{\omega_d^2 C}\right].$$

The only factor that can equal zero is $\omega_d L - (1/\omega_d C)$ and it does for $\omega_d = 1/\sqrt{LC}$. For the given circuit,

$$\omega_d = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})}} = 224\,\mathrm{rad/s}\,.$$

(b) For this value of the angular frequency, the impedance is $Z = R$ and the current amplitude is

$$I = \frac{\mathcal{E}_m}{R} = \frac{30.0\,\mathrm{V}}{5.00\,\Omega} = 6.00\,\mathrm{A}\,.$$

(c) and (d) You want to find the values of ω_d for which $I = \mathcal{E}_m/2R$. This means

$$\frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} = \frac{\mathcal{E}_m}{2R}\,.$$

Cancel the factors $\mathcal{E}_m$ that appear on both sides, square both sides, and set the reciprocals of the two sides equal to each other to obtain

$$R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2 = 4R^2\,.$$

Thus

$$\left(\omega_d L - \frac{1}{\omega_d C}\right)^2 = 3R^2\,.$$

Now take the square root of both sides and multiply by $\omega_d C$ to obtain

$$\omega_d^2 (LC) \pm \omega_d \left(\sqrt{3}CR\right) - 1 = 0\,,$$

where the symbol $\pm$ indicates the two possible signs for the square root. The last equation is a quadratic equation for ω_d. Its solutions are

$$\omega_d = \frac{\pm\sqrt{3}CR \pm \sqrt{3C^2R^2 + 4LC}}{2LC}\,.$$

You want the two positive solutions. The smaller of these is

$$
\begin{aligned}
\omega_2 &= \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \\
&= \frac{-\sqrt{3}(20.0 \times 10^{-6}\,\mathrm{F})(5.00\,\Omega)}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&\qquad + \frac{\sqrt{3(20.0 \times 10^{-6}\,\mathrm{F})^2(5.00\,\Omega)^2 + 4(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})}}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&= 219\,\mathrm{rad/s}
\end{aligned}
$$

and the larger is

$$
\begin{aligned}
\omega_1 &= \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \\
&= \frac{+\sqrt{3}(20.0 \times 10^{-6}\,\mathrm{F})(5.00\,\Omega)}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&\qquad + \frac{\sqrt{3(20.0 \times 10^{-6}\,\mathrm{F})^2(5.00\,\Omega)^2 + 4(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})}}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&= 228\,\mathrm{rad/s}\,.
\end{aligned}
$$

(e) The fractional width is

$$
\frac{\omega_1 - \omega_2}{\omega_0} = \frac{228\,\mathrm{rad/s} - 219\,\mathrm{rad/s}}{224\,\mathrm{rad/s}} = 0.04\,.
$$

(a) The power factor is $\cos\phi$, where ϕ is the phase angle when the current is written $i = I\sin(\omega_d t - \phi)$. Thus $\phi = -42.0°$ and $\cos\phi = \cos(-42.0°) = 0.743$.

(b) Since $\phi < 0$, $\omega_d t - \phi > \omega_d t$ and the current leads the emf.

(c) The phase angle is given by $\tan\phi = (X_L - X_C)/R$, where X_L is the inductive reactance, X_C is the capacitive reactance, and R is the resistance. Now $\tan\phi = \tan(-42.0°) = -0.900$, a negative number. This means $X_L - X_C$ is negative, or $X_C > X_L$. The circuit in the box is predominantly capacitive.

(d) If the circuit is in resonance, X_L is the same as X_C, $\tan\phi$ is zero, and ϕ would be zero. Since ϕ is not zero, we conclude the circuit is not in resonance.

(e), (f), and (g) Since $\tan\phi$ is negative and finite, neither the capacitive reactance nor the resistance is zero. This means the box must contain a capacitor and a resistor. The inductive reactance may be zero, so there need not be an inductor. If there is an inductor, its reactance must be less than that of the capacitor at the operating frequency.

(h) The average power is

$$P_{\text{av}} = \frac{1}{2}\mathcal{E}_m I\cos\phi = \frac{1}{2}(75.0\,\text{V})(1.20\,\text{A})(0.743) = 33.4\,\text{W}\,.$$

(i) The answers above depend on the frequency only through the phase angle ϕ, which is given. If values are given for R, L, and C, then the value of the frequency would also be needed to compute the power factor.

When the capacitor has maximum charge Q the potential difference across it is a maximum. Use $Q = CV$ to compute the maximum charge. Here V is the maximum potential difference. The maximum current is related to the maximum charge by $I = \omega Q$, where ω $(= 1/\sqrt{LC})$ is the angular frequency of oscillation. The maximum energy stored in the inductor is $\frac{1}{2}LI^2$.

[ans: (a) 3.0 nC; (b) 1.7 mA; (c) 4.5 nJ]

The total energy is the sum of the energy stored in the capacitor and the energy stored in the inductor. The total energy is also the energy in the capacitor when it has maximum charge and is the energy in the inductor when the current is maximum. If $q = Q\cos(\omega t + \phi)$ the charge on the capacitor at time $t = 0$ is $q_0 = Q\cos\phi$. There are two solutions for ϕ. To answer part (a) you want the one for which the current is positive and to answer part (b) you want the one for which the current is negative. The current at $t = 0$ is $-\omega Q\sin\phi$.

[ans: (a) 1.98 μJ; (b) 5.56 μC; (c) 12.6 mA; (d) $-46.9°$; (e) $+46.9°$]

CHAPTER 31 HINT FOR PROBLEM 29

The current amplitude is given by $\mathcal{E}_m/X_L$, where X_L ($= \omega L$) is the inductive reactance. The angular frequency ω is related to the frequency f by $\omega = 2\pi f$.

[ans: (a) 95.5 mA; (b) 11.9 mA]

Removing the capacitor is equivalent to setting the capacitance C equal to infinity (think about a parallel-plate capacitor with a plate separation of zero). Set the capacitive reactance equal to zero in the equations for the impedance and phase angle. Use $I = \mathcal{E}_m/Z$, where $\mathcal{E}_m$ is the maximum emf and Z is the impedance, to compute the maximum current I. Use $V_R = IR$ and $V_L = IX_L$ to compute the voltage amplitudes for the resistor and inductor, then draw the phasor diagram. Here X_L (ωL) is the inductive reactance, L is the inductance, ω is the angular frequency, and R is the resistance.

[ans: (a) $218\,\Omega$; (b) $23.4°$; (c) $165\,\text{mA}$]

CHAPTER 31 HINT FOR PROBLEM 41

The phase constant ϕ obeys $\tan\phi = (X_L - X_C)/R$, where X_L $(= \omega L)$ is the inductive reactance, X_C $(= 1/\omega C)$ is the capacitive reactance, R is the resistance, L is the inductance, C is the capacitance, and ω is the angular frequency. Recall that the angular frequency and frequency F are related by $\omega = 2\pi f$.

[ans: $89\,\Omega$]

CHAPTER 31 HINT FOR PROBLEM 49

Replace the three capacitors, which are in parallel, with their equivalent capacitor and the two inductors, which are in series, with their equivalent inductor. The circuit is now a series LC circuit and its resonant frequency is determined by the equivalent capacitance and equivalent inductance.

$\Big[$ ans: (a) 796 Hz; (b) no change; (c) decreased; (d) increased $\Big]$

CHAPTER 31 HINT FOR PROBLEM 61

The power supplied by the generator is the product of the rms voltage V_t and the rms current i_{rms}. The rate of energy dissipation is i^2_{rms}, where R is the total resistance of the two cables.

[ans: (a) 2.4 V; (b) 3.2 mA; (c) 0.16 A]

If the electric field is perpendicular to a region of a plane and has uniform magnitude over the region then the displacement current through the region is related to the rate of change of the electric field in the region by

$$i_d = \epsilon_0 A \frac{dE}{dt}\,,$$

where A is the area of the region. The rate of change of the electric field is the slope of the graph.

For segment a

$$\frac{dE}{dt} = \frac{6.0\times10^5\,\mathrm{N/C} - 4.0\times10^5\,\mathrm{N/C}}{4.0\times10^{-6}\,\mathrm{s}} = 5.0\times10^{10}\,\mathrm{N/C\cdot s}$$

and $i_d = (8.85\times10^{-12}\,\mathrm{F/m})(1.6\,\mathrm{m^2})(5.0\times10^{10}\,\mathrm{N/C\cdot s} = 0.71\,\mathrm{A}$.

For segment b $dE/dt = 0$ and $i_d = 0$.

For segment c

$$\frac{dE}{dt} = \frac{4.0\times10^5\,\mathrm{N/C} - 0}{2.0\times10^{-6}\,\mathrm{s}} = 2.0\times10^{11}\,\mathrm{N/C\cdot s}$$

and $i_d = (8.85\times10^{-12}\,\mathrm{F/m})(1.6\,\mathrm{m^2})(2.0\times10^{11}\,\mathrm{N/C\cdot s} = 2.8\,\mathrm{A}$.

(a) The z component of the orbital angular momentum is given by $L_{\text{orb},\,z} = m_\ell h/2\pi$, where h is the Planck constant. Since $m_\ell = 0$, $L_{\text{orb},\,z} = 0$.

(b) The z component of the orbital contribution to the magnetic dipole moment is given by $\mu_{\text{orb},\,z} = -m_\ell \mu_B$, where μ_B is the Bohr magneton. Since $m_\ell = 0$, $\mu_{\text{orb},\,z} = 0$.

(c) The potential energy associated with the orbital contribution to the magnetic dipole moment is given by $U = -\mu_{\text{orb},\,z} B_{\text{ext}}$, where B_{ext} is the z component of the external magnetic field. Since $\mu_{\text{orb},\,z} = 0$, $U = 0$.

(d) The z component of the spin magnetic dipole moment is either $+\mu_B$ or $-\mu_B$, so the potential energy is either

$$U = -\mu_B B_{\text{ext}} = -(9.27 \times 10^{-24}\ \text{J/T})(35 \times 10^{-3}\ \text{T}) = -3.2 \times 10^{-25}\ \text{J}\,.$$

or $U = +3.2 \times 10^{-25}$ J.

(e) Substitute m_ℓ into the equations given above. The z component of the orbital angular momentum is

$$L_{\text{orb},\,z} = \frac{m_\ell h}{2\pi} = \frac{(-3)(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})}{2\pi} = -3.2 \times 10^{-34}\ \text{J}\cdot\text{s}\,.$$

(f) The z component of the orbital contribution to the magnetic dipole moment is

$$\mu_{\text{orb},\,z} = -m_\ell \mu_B = -(-3)(9.27 \times 10^{-24}\ \text{J/T}) = 2.8 \times 10^{-23}\ \text{J/T}\,.$$

(g) The potential energy associated with the orbital contribution to the magnetic dipole moment is

$$U = -\mu_{\text{orb},\ z} B_{\text{ext}} = -(2.78 \times 10^{-23}\ \text{J/T})(35 \times 10^{-3}\ \text{T}) = -9.7 \times 10^{-25}\ \text{J}\,.$$

(h) The potential energy associated with spin does not depend on m_ℓ. It is $\pm 3.2 \times 10^{-25}$ J.

(a) If the magnetization of the sphere is saturated, the total dipole moment is $\mu_{\text{total}} = N\mu$, where N is the number of iron atoms in the sphere and μ is the dipole moment of an iron atom. We wish to find the radius of an iron sphere with N iron atoms. The mass of such a sphere is Nm, where m is the mass of an iron atom. It is also given by $4\pi\rho R^3/3$, where ρ is the density of iron and R is the radius of the sphere. Thus $Nm = 4\pi\rho R^3/3$ and

$$N = \frac{4\pi\rho R^3}{3m}\,.$$

Substitute this into $\mu_{\text{total}} = N\mu$ to obtain

$$\mu_{\text{total}} = \frac{4\pi\rho R^3\mu}{3m}\,.$$

Solve for R and obtain

$$R = \left[\frac{3m\mu_{\text{total}}}{4\pi\rho\mu}\right]^{1/3}\,.$$

The mass of an iron atom is

$$m = 56\,\text{u} = (56\,\text{u})(1.66\times10^{-27}\,\text{kg/u}) = 9.30\times10^{-26}\,\text{kg}\,.$$

So

$$R = \left[\frac{3(9.30\times10^{-26}\,\text{kg})(8.0\times10^{22}\,\text{J/T})}{4\pi(14\times10^3\,\text{kg/m}^3)(2.1\times10^{-23}\,\text{J/T})}\right]^{1/3} = 1.8\times10^5\,\text{m}\,.$$

(b) The volume of the sphere is

$$V_s = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}(1.82\times10^5\,\text{m})^3 = 2.53\times10^{16}\,\text{m}^3$$

and the volume of Earth is

$$V_e = \frac{4\pi}{3}(6.37\times10^6\,\text{m})^3 = 1.08\times10^{21}\,\text{m}^3\,,$$

so the fraction of Earth's volume that is occupied by the sphere is

$$\frac{2.53\times10^{16}\,\text{m}^3}{1.08\times10^{21}\,\text{m}^3} = 2.3\times10^{-5}\,.$$

The radius of Earth was obtained from Appendix C.

Use the Maxwell law of induction. The magnetic field lines are circles concentric with the boundary of the region and the magnitude of the field is uniform on a line. Thus $\oint \vec{B} \cdot d\vec{s} = 2\pi r B$. Set this equal to $\mu_0 \epsilon_0 \, d\Phi_{E\mathrm{enc}}/dt$ and solve for B. The region of electric flux extends only to $r = R$.

[ans: (a) 3.54×10^{-17} T; (b) 2.13×10^{-17} T]

The magnetic field a distance r from the central axis of a uniform distribution of displacement current is given by $B = \mu_o i_d/2\pi r$, where i_d is the displacement current though a cross-section. For part (a) the point is inside the displacement current distribution so you should take i_d to be $\pi r^2 J_d$. For part (b) the point is outside so you should take it to be $\pi R^2 J_d$.

$\big[$ ans: (a) 75.4 nT; (b) 67.9 nT $\big]$

For each of the original levels there is a new level associated with each possible value of m_ℓ. Thus one value of m_ℓ is associated with level E_1 and three values are associated with level E_2. Use $\mu_{\text{orb}\,z} = m_\ell \mu_B$ and $U = -\mu_{\text{orb}\,z} B_{\text{ext}}$ to compute the difference in energy of the levels for which $m_\ell = 0$ and $m_\ell = 1$, *say*.

[ans: (a) 0; (b) -1, 0, 1; (c) 4.64×10^{-24} J]

The magnitude of the dipole moment is iA where i is the current and A (= πr^2, where r is the orbit radius) is the area bounded by the electron's path. The current is $e/T = ev/2\pi r$, where T is the period of the motion and v is the speed of the electron. The radius of the orbit is $r = mv/eB$ (see Chapter 28). Make substitutions to write the expression for the dipole moment in terms of v, then use $K = \frac{1}{2}mv^2$ to write it in terms of the kinetic energy K. Since the magnetic force must be inward toward the center of the path you can find the direction of travel for a given field direction and hence can find the direction of the dipole moment.To find the magnetization of the gas, vectorially add the dipole moments per unit volume of the electrons and ions.

[ans: (b) K_i/B; (b) $-z$; (c) 0.31 kA/m]

The magnitude of the magnetic field inside a toroid, a distance r from the center, is given by $B_0 = \mu_0 i_p N_p/2\pi r$, where N_p is the number of turns in the primary and i is the current (see Eq. 29–24). Use the average of the inside and outside radii for r and solve for i_p. The total field is $B = B_0 + B_M$ and the magnetic flux through one turn of the secondary coil is $\Phi_B = BA$, where A is the cross-sectional area of the toroid. According to Faraday's law the emf generated in the secondary is $\mathcal{E} = d\Phi_B/dt$, so the current is $i_s = \mathcal{E}/R$, where R is the resistance of the secondary coil. The charge is the integral of the current with respect to time.

[ans: (a) 0.14 A; (b) 79 μC]

(a) Since $c = \lambda f$, where λ is the wavelength and f is the frequency of the wave,

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8\,\text{m/s}}{3.0\,\text{m}} = 1.0 \times 10^8\,\text{Hz}\,.$$

(b) The angular frequency is

$$\omega = 2\pi f = 2\pi(1.0 \times 10^8\,\text{Hz}) = 6.3 \times 10^8\,\text{rad/s}\,.$$

(c) The angular wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3.0\,\text{m}} = 2.1\,\text{rad/m}\,.$$

(d) The magnetic field amplitude is

$$B_m = \frac{E_m}{c} = \frac{300\,\text{V/m}}{3.00 \times 10^8\,\text{m/s}} = 1.00 \times 10^{-6}\,\text{T}\,.$$

(e) $\vec{B}$ must be in the positive z direction when $\vec{E}$ is in the positive y direction in order for $\vec{E} \times \vec{B}$ to be in the positive x direction (the direction of propagation).

(f) The time-averaged rate of energy flow or intensity of the wave is

$$I = \frac{E_m^2}{2\mu_0 c} = \frac{(300\,\text{V/m})^2}{2(4\pi \times 10^{-7}\,\text{H/m})(3.00 \times 10^8\,\text{m/s})} = 1.2 \times 10^2\,\text{W/m}^2\,.$$

(g) Since the sheet is perfectly absorbing, the rate per unit area with which momentum is delivered to it is I/c, so

$$\frac{\text{d}p}{\text{d}t} = \frac{IA}{c} = \frac{(119\,\text{W/m}^2)(2.0\,\text{m}^2)}{3.00 \times 10^8\,\text{m/s}} = 8.0 \times 10^{-7}\,\text{N}\,.$$

(h) The radiation pressure is

$$p_r = \frac{\text{d}p/\text{d}t}{A} = \frac{8.0 \times 10^{-7}\,\text{N}}{2.0\,\text{m}^2} = 4.0 \times 10^{-7}\,\text{Pa}\,.$$

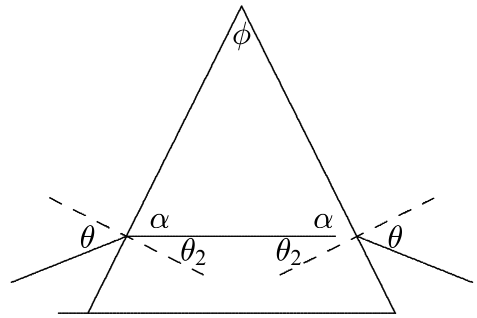
(a) The rotation cannot be done with a single sheet. If a sheet is placed with its polarizing direction at an angle of 90° to the direction of polarization of the incident radiation, no radiation is transmitted.

It can be done with two sheets. Place the first sheet with its polarizing direction at some angle θ, between 0 and 90°, to the direction of polarization of the incident radiation. Place the second sheet with its polarizing direction at 90° to the polarization direction of the incident radiation. The transmitted radiation is then polarized at 90° to the incident polarization direction. The intensity is $I_0 \cos^2\theta \cos^2(90° - \theta) = I_0 \cos^2\theta \sin^2\theta$, where I_0 is the incident radiation. If θ is not 0 or 90°, the transmitted intensity is not zero.

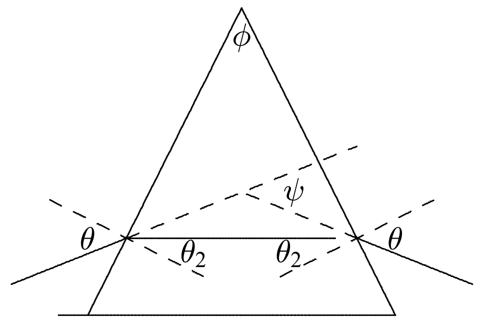
(b) Consider n sheets, with the polarizing direction of the first sheet making an angle of $\theta = 90°/n$ with the direction of polarization of the incident radiation and with the polarizing direction of each successive sheet rotated $90°/n$ in the same direction from the polarizing direction of the previous sheet. The transmitted radiation is polarized with its direction of polarization making an angle of 90° with the direction of polarization of the incident radiation. The intensity is $I = I_0 \cos^{2n}(90°/n)$. You want the smallest integer value of n for which this is greater than $0.60 I_0$.

Start with $n = 2$ and calculate $\cos^{2n}(90°/n)$. If the result is greater than 0.60, you have obtained the solution. If it is less, increase n by 1 and try again. Repeat this process, increasing n by 1 each time, until you have a value for which $\cos^{2n}(90°/n)$ is greater than 0.60. The first one will be $n = 5$.

Look at the diagram on the right. The two angles labeled α have the same value. θ_2 is the angle of refraction. Because the dotted lines are perpendicular to the prism surface $\theta_2 + \alpha = 90°$ and $\alpha = 90° - \theta_2$. Because the interior angles of a triangle sum to $180°$, $180° - 2\theta_2 + \phi = 180°$ and $\theta_2 = \phi/2$.



Now look at the next diagram and consider the triangle formed by the two normals and the ray in the interior. The two equal interior angles each have the value $\theta - \theta_2$. Because the exterior angle of a triangle is equal to the sum of the two opposite interior angles, $\psi = 2(\theta - \theta_2)$ and $\theta = \theta_2 + \psi/2$. Upon substitution for θ_2 this becomes $\theta = (\phi + \psi)/2$.



According to the law of refraction the index of refraction of the prism material is

$$n = \frac{\sin\theta}{\sin\theta_2} = \frac{\sin(\phi + \psi)/2}{\sin\phi/2}\,.$$

CHAPTER 33 HINT FOR PROBLEM 13

The intensity is given by $E_m^2/2c\mu_0$, where E_m is the electric field amplitude. The magnetic and electric field amplitudes are related by $B_m = E_m/c$.

[ans: (a) $1.03\,\mathrm{kV/m}$; (b) $3.43\,\mu\mathrm{T}$]

The radiation pressure on a perfectly absorbing object is given by $p_r = I/c$, where I is the intensity at the object. The intensity a distance r from an isotropically emitting source of power P_s is $I = P_s/4\pi r^2$.

$\left[\text{ans: } 5.9 \times 10^{-8}\,\text{Pa}\right]$

The transmitted intensity is $I = I_0/2$, where I_0 is the incident intensity. The intensity is related to the electric field amplitude E_M by $I = E_m^2/2\mu_0 c$. Since the sheet is absorbing the radiation pressure is $p_r = I_a/c$, where I_a is the intensity of the absorbed light. This is $I_a = I_0 - I$.

[ans: (a) $1.9\,\mathrm{V/m}$; (b) $1.7 \times 10^{-11}\,\mathrm{Pa}$]

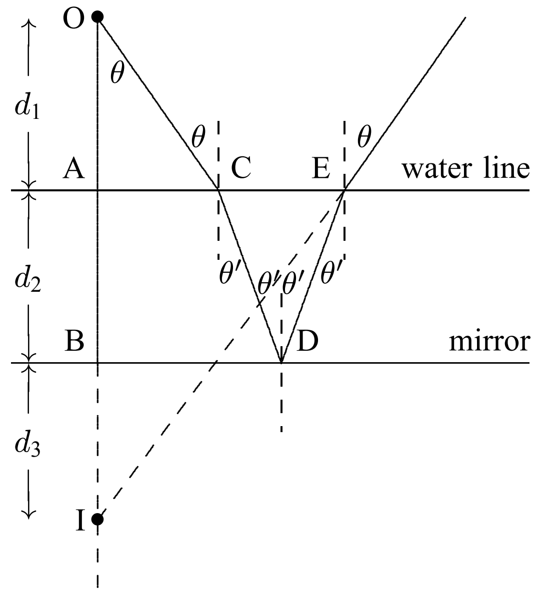
CHAPTER 33 HINT FOR PROBLEM 47

Use the law of refraction. The angle of refraction is 90° and the angle of incidence is given by $tan\theta = L/D$.

[ans: 1.26]

The angle of incidence for the light approaching the boundary between materials 2 and 3 is ϕ. This is the critical angle for total internal reflection, so $n2 \sin\phi = n_3$. Apply the law to the refraction at the interface between materials 1 and 2: $n_1 \sin\theta = n_2 \sin(90^\circ - \phi)$. Solve for θ.

[ans: (a) 1.39 (b) 28.1°; (c) no]



The light bulb is labeled O and its image is labeled I on the digram to the right. Consider the two rays shown on the diagram to the right. One enters the water at A and is reflected from the mirror at B. This ray is perpendicular to the water line and mirror. The second ray leaves the lightbulb at the angle θ, enters the water at C, where it is refracted. It is reflected from the mirror at D and leaves the water at E. At C the angle of incidence is θ and the angle of refraction is θ'. At D the angles of incidence and reflection are both θ'. At E the angle of incidence is θ' and the angle of refraction is θ. The dotted lines that meet at I represent extensions of the emerging rays. Light appears to come from I. We want to compute d_3.

Consideration of the triangle OBE tells us that the distance $d_2 + d_3$ is $L\tan(90^\circ - \theta) = L/\tan\theta$, where L is the distance between A and E. Consideration of the triangle OBC tells us that the distance between A and C is $d_1 \tan\theta$ and consideration of the triangle CDE tells us that the distance between C and E is $2d_2 \tan\theta^{p}rime$, so $L = d_1 \tan\theta + 2d_2 \tan\theta'$, $d_2 + d_3 = (d_1 \tan\theta + 2d_2 \tan\theta')/\tan\theta$, and

$$d_3 = \frac{d_1 \tan\theta + 2d_2 \tan\theta'}{\tan\theta} - d_2 \,.$$

Apply the law of refraction at point C: $\sin\theta = n\sin\theta'$, where n is the index of refraction of water. Since the angles θ and θ' are small we may approximate their sines by their tangents and write $\tan\theta = n\tan\theta'$. Us this to substitute for $\tan\theta$ in the expression for d_3 to obtain

$$d_3 = \frac{nd_1 + 2d_2}{n} - d_2 = \frac{(1.33)(250\,\text{cm}) + 2(200\,\text{cm})}{1.33} - 200\,\text{cm} = 350\,\text{cm}\,,$$

where the index of refraction of water was taken to be 1.33.

Use the lens maker's equation, Eq. 34–10:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right),$$

where f is the focal length, n is the index of refraction, r_1 is the radius of curvature of the first surface encountered by the light and r_2 is the radius of curvature of the second surface. Since one surface has twice the radius of the other and since one surface is convex to the incoming light while the other is concave, set $r_2 = -2r_1$ to obtain

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{2r_1}\right) = \frac{3(n-1)}{2r_1}.$$

Solve for r_1:

$$r_1 = \frac{3(n-1)f}{2} = \frac{3(1.5-1)(60\,\text{mm})}{2} = 45\,\text{mm}.$$

The radii are 45 mm and 90 mm.

Lens 1 is converging and so has a positive focal length. Solve $(1/p_1) + (1/i_1) = (1/f_1)$ for the image distance i_1 associated with the image produced by this lens. The result is

$$i_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(20\,\text{cm})(+9.0\,\text{cm})}{(20\,\text{cm}) - (9.0\,\text{cm})} = 16.4\,\text{cm}\,.$$

This image is the object for lens 2. The object distance is $d - p_2 = (8.0\,\text{cm}) - (16.4\,\text{cm}) = -8.4\,\text{cm}$. The negative sign indicates that the image is behind the second lens. The lens equation is still valid. The second lens has a positive focal length and the image distance for the image it forms is

$$i_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(-8.4\,\text{cm})(5.0\,\text{cm})}{(-8.4\,\text{cm}) - (5.0\,\text{cm})} = +3.1\,\text{cm}\,.$$

The overall magnification is the product of the individual magnifications:

$$m = m_1 m_2 = \left(-\frac{i_1}{p_1}\right)\left(-\frac{i_2}{p_2}\right) = \left(-\frac{16.4\,\text{cm}}{20\,\text{cm}}\right)\left(-\frac{3.1\,\text{cm}}{-8.4\,\text{cm}}\right) = -0.30\,.$$

Since the final image distance is positive the final image is real and on the opposite side of lens 2 from the object. Since the magnification is negative the image is inverted.

The focal length of a concave spherical mirror is positive. Solve $(1/p)+(1/i) = (1/f)$ for i. The magnification is given by $m = -i/p$. If i is positive the image is real; otherwise it is virtual. If m is positive the image is no inverted, otherwise it is. A real image is on the same side of the mirror as the object; a virtual image is on the opposite side.

[ans: (a) -16 cm; (b) -4.4 cm; (c) +0.44; (d) V; (e) NI; (f) opposite]

The type of mirror is found from the sign of the focal length. Solve $(1/p) + (1/i) = (1/f)$ for i. If i is positive the image is real and is on the same side of the mirror as the object; otherwise it is virtual and is on the opposite side. The magnification is given by $m = -i/p$. If m is positive the image is not inverted; otherwise it is.

[ans: (a) concave; (c) +40 cm; (e) +60 cm; (f) -2.0; (g) R; (h) I; (i) same]

Use $(n_1/p) + (n_2/i) = (n_2 - n_1)/r$. Solve for i. If i is positive the image is real and is on the opposite side of the surface from the object; if i is negative the image is virtual and is on the same side of the surface as the object.

[ans: (d) -26 cm; (e) V; (f) same]

he height of the image on the film is $h' = |m|h$, where h is the height of the person and m is the magnification of the lens. Solve $(1/p) + (1/i) = (1/f)$ for i, then use $m = -i/p$ to calculate the magnification. Here p is the object distance, i is the image distance, and f is the focal length of the lens.

[ans: 5.0 mm]

CHAPTER 34 HINT FOR PROBLEM 55

The type of lens tells the sign of the focal length. Solve $(1/p) + (1/i) = (1/f)$ for i and use $m = -i/p$. The sign of i tells if the image is real or virtual and where its position is relative to the lens. The sign of m tells if the image is inverted or not.

$\Big[$ ans: (a) -8.6 cm; (b) +0.39 (c) V; (d) NI; (e) same $\Big]$

Use $m = -i/p$ to obtain i. Solve $(1/p) + (1/i) = (1/f)$ for f. The sign of f tells the type of lens. The sign of i tells if the image is real or virtual and indicates the side of the lens on which the image is formed. The sign of m tells if the image is inverted or not.

[ans: (a) D; (b) -5.3 cm; (d) -4.0 cm; (f) V; (g) NI; (h) same]

The type of lens tells the sign of the focal length. Solve $(1/p_1) + (1/i_1) = (1/f_1)$ for i_1, then calculate $p_2 = d - i_1$ and solve $(1/p_2) + (1/i_2) = (1/f_2)$ for i_2. The sign of i_2 tells if the image is real or virtual and gives the side of lens 2 on which the image is formed. The magnification is the product of the individual magnifications and so is given by $m = (i_1/p_1)(i_2/p_2)$. Its sign tells if the image is inverted or not.

[ans: (a) -4.6 cm; (b) +0.69; (c) V; (d) NI; (e) same]

CHAPTER 34 HINT FOR PROBLEM 97

(a) The image produced by the lens must be 4.00 cm in front of the mirror and therefore 6.00 cm behind the lens. Solve $(1/p)+(1/i)=(1/f)$ for p. (b) The image in the mirror now becomes the object for the lens. The object distance is 14.0 cm. Solve $(1/p)+(1/i)=(1/f)$ for i.

[ans: (a) 3.00 cm; (b) 2.33 cm]

Solve $(1/p)+(1/i) = (1/f)$ for i and explain what happens to i if p is less than f and decreasing. The angle in radians subtended at the eye by the image is $\theta' = h'/|i|$, where h' is the height of the image. Since $h' = |m|h$, and $|m| = |i|/p$, $\theta' = h/p$, where h is the height of the object. The maximum usable angular magnification occurs if the image is at the near point of the eye.

$\left[\text{ans: (b) } P_n\ \right]$

Use $(n_1/p_1) + (n_2/i_1) = (n_2 - n_1)/r$ to find the image distance i_2 for the image formed by the left surface of the sphere. Here medium 1 is air and medium 2 is glass. Set n_1 equal to 1 and replace n_2 with n, the index of refraction for the glass. The incident rays are parallel, so $P_1 = \infty$. The surface is convex so r is positive. The object distance for the right side of the sphere is $p_2 = 2r - i_1$. Solve $(n_1/p_2) + (n_2/i_2) = (n_1 - n_2)/r$ for i_2. Now medium 1 is glass, so $n_1 = n$, and medium 2 is air, so $n_2 = 1$. The surface is concave to the incident light, so r is negative. The sign of i_2 tells if the image is to the right or left of the right side of the sphere.

[ans: (a) (0.5)(2 − n)$x/(n - 1)$; (b) right]

CHAPTER 37 SOLUTION FOR PROBLEM 17

The proper time is not measured by clocks in either frame S or frame S' since a single clock at rest in either frame cannot be present at the origin and at the event. The full Lorentz transformation must be used:

$$x' = \gamma[x - vt]$$

$$t' = \gamma[t - \beta x/c]\,,$$

where $\beta = v/c = 0.950$ and $\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.950)^2} = 3.2026$. Thus,

$$\begin{aligned} x' &= (3.2026)\left[100 \times 10^3\,\text{m} - (0.950)(3.00 \times 10^8\,\text{m/s})(200 \times 10^{-6}\,\text{s}\right] \\ &= 1.38 \times 10^5\,\text{m} = 138\,\text{km} \end{aligned}$$

and

$$t' = (3.2026)\left[200 \times 10^{-6}\,\text{s} - \frac{(0.950)(100 \times 10^3\,\text{m})}{3.00 \times 10^8\,\text{m/s}}\right] = -3.74 \times 10^{-4}\,\text{s} = -374\,\mu\text{s}\,.$$

Calculate the speed of the micrometeorite relative to the spaceship. Let S be the reference frame for which the data is given and attach frame S to the spaceship. Suppose the micrometeorite is going in the positive x direction and the spaceship is going in the negative x direction, both as viewed from S'. Then, in Eq. 38–28, $u' = 0.82c$ and $v = 0.82c$. Notice that v in the equation is the velocity of S' relative to S. Thus, the velocity of the micrometeorite in the frame of the spaceship is

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.82c + 0.82c}{1 + (0.82c)(0.82c)/c^2} = 0.9806c\,.$$

The time for the micrometeorite to pass the spaceship is

$$\Delta t = \frac{L}{u} = \frac{350\,\mathrm{m}}{(0.9806)(3.00 \times 10^8\,\mathrm{m/s})} = 1.19 \times 10^{-6}\,\mathrm{s}\,.$$

Use the two expressions for the total energy: $E = mc^2 + K$ and $E = \gamma mc^2$, where m is the mass of an electron, K is the kinetic energy, and $\gamma = 1/\sqrt{1-\beta^2}$. Thus, $mc^2 + K = \gamma mc^2$ and $\gamma = (mc^2 + K)/mc^2$. This means $\sqrt{1-\beta^2} = (mc^2)/(mc^2 + K)$ and

$$\beta = \sqrt{1 - \left(\frac{mc^2}{mc^2 + K}\right)^2} .$$

Now $mc^2 = 0.511\,\text{MeV}$ so

$$\beta = \sqrt{1 - \left(\frac{0.511\,\text{MeV}}{0.511\,\text{MeV} + 100\,\text{MeV}}\right)^2} = 0.999987 .$$

The speed of the electron is $0.999987c$ or 99.9987% the speed of light.

CHAPTER 37 HINT FOR PROBLEM 5

The time interval in the laboratory frame is $\Delta t = (\Delta x)/v$, where Δx is the length of the track and v is the speed of the particle. A clock traveling with the particle measures the proper time interval.

[ans: 0.446 ps]

CHAPTER 37 HINT FOR PROBLEM 23

Use $\Delta x' = \gamma(\Delta x - v\,\Delta t)$. Since the flashes occur at the same place in S′ set $\Delta x'$ equal to zero and solve for v. You can tell the direction of motion of S′ from the sign of v. To find the time between the flashes as measured in S′, use $\Delta t' = \gamma(\Delta t - v\,\Delta x/c^2)$. The sign of $\Delta t'$ tells which flash occurs first.

[ans: (a) $0.480c$; (b) negative; (c) big flash; (d) $4.39\,\mu$s]

CHAPTER 37 HINT FOR PROBLEM 37

Substitute values into $v = (|\Delta\lambda|/\lambda_0)c$.

$\left[\text{ans: } 0.13c\ \right]$

The page width in the proton's reference frame is $L = L_0/\gamma$, where L_0 is the width in your frame. The time for the trip as measured in your frame is L_0/v and the time in the proton's frame is L/v, where v is the speed of the proton in your frame. Use $E = \gamma mc^2$ to find γ and $\gamma = 1/\sqrt{1-(v/c)^2}$ to find v.

[ans: (a) 0.222 cm; (b) 70.1 ns; (c) 0.740 ns]

Conservation of energy yields $K_\alpha + m_\alpha c^2 + m_N c^2 = K_O + m_O c^2 + K_p + m_p c^2$. Here K_α is the kinetic energy of the alpha particle and m_α is its mass, K_O is the kinetic energy of the oxygen nucleus and m_O is its mass, K_p is the kinetic energy of the proton and m_p is its mass, and m_N is the mass of the nitrogen nucleus. Solve for K_O. The Q of the reaction is $(m_\alpha + m_N - m_O - m_p)c^2$. Use $(1\,\mathrm{u})c^2 = 931.494\,\mathrm{MeV}$ to obtain the requested units.

$\left[\text{ans: (a) } 2.08\,\mathrm{MeV};\ \text{(b) } -1.18\,\mathrm{MeV}\ \right]$

(a) Let R be the rate of photon emission (number of photons emitted per unit time) and let E be the energy of a single photon. Then, the power output of a lamp is given by $P = RE$ if all the power goes into photon production. Now, $E = hf = hc/\lambda$, where h is the Planck constant, f is the frequency of the light emitted, and λ is the wavelength. Thus $P = Rhc/\lambda$ and $R = \lambda P/hc$. The lamp emitting light with the longer wavelength (the 700 nm lamp) emits more photons per unit time. The energy of each photon is less so it must emit photons at a greater rate.

(b) Let R be the rate of photon production for the 700 nm lamp Then,

$$R = \frac{\lambda P}{hc} = \frac{(700 \times 10^{-9}\,\mathrm{m})(400\,\mathrm{J/s})}{(6.626 \times 10^{-34}\,\mathrm{J \cdot s})(3.00 \times 10^{8}\,\mathrm{m/s})} = 1.41 \times 10^{21}\,\mathrm{photon/s}\,.$$

(a) The kinetic energy acquired is $K = qV$, where q is the charge on an ion and V is the accelerating potential. Thus $K = (1.60 \times 10^{-19}\,\text{C})(300\,\text{V}) = 4.80 \times 10^{-17}\,\text{J}$. The mass of a single sodium atom is, from Appendix F, $m = (22.9898\,\text{g/mol})/(6.02 \times 10^{23}\,\text{atom/mol}) = 3.819 \times 10^{-23}\,\text{g} = 3.819 \times 10^{-26}\,\text{kg}$. Thus the momentum of an ion is

$$p = \sqrt{2mK} = \sqrt{2(3.819 \times 10^{-26}\,\text{kg})(4.80 \times 10^{-17}\,\text{J})} = 1.91 \times 10^{-21}\,\text{kg} \cdot \text{m/s}\,.$$

(b) The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}\,\text{J} \cdot \text{s}}{1.91 \times 10^{-21}\,\text{kg} \cdot \text{m/s}} = 3.47 \times 10^{-13}\,\text{m}\,.$$

(a) If m is the mass of the particle and E is its energy, then the transmission coefficient for a barrier of height U and width L is given by

$$T = e^{-2kL}\,,$$

where

$$k = \sqrt{\frac{8\pi^2 m(U-E)}{h^2}}\,.$$

If the change ΔU in U is small (as it is), the change in the transmission coefficient is given by

$$\Delta T = \frac{\mathrm{d}T}{\mathrm{d}U}\,\Delta U = -2LT\,\frac{\mathrm{d}k}{\mathrm{d}U}\,\Delta U\,.$$

Now,

$$\frac{\mathrm{d}k}{\mathrm{d}U} = \frac{1}{2\sqrt{U-E}}\sqrt{\frac{8\pi^2 m}{h^2}} = \frac{1}{2(U-E)}\sqrt{\frac{8\pi^2 m(U-E)}{h^2}} = \frac{k}{2(U-E)}\,.$$

Thus

$$\Delta T = -LTk\,\frac{\Delta U}{U-E}\,.$$

For the data of Sample Problem 38–7, $2kL = 10.0$, so $kL = 5.0$ and

$$\frac{\Delta T}{T} = -kL\frac{\Delta U}{U-E} = -(5.0)\frac{(0.010)(6.8\,\mathrm{eV})}{6.8\,\mathrm{eV} - 5.1\,\mathrm{eV}} = -0.20\,.$$

There is a 20% decrease in the transmission coefficient.

(b) The change in the transmission coefficient is given by

$$\Delta T = \frac{\mathrm{d}T}{\mathrm{d}L}\,\Delta L = -2ke^{-2kL}\,\Delta L = -2kT\,\Delta L$$

and

$$\frac{\Delta T}{T} = -2k\,\Delta L = -2(6.67\times10^{9}\,\mathrm{m}^{-1})(0.010)(750\times10^{-12}\,\mathrm{m}) = -0.10\,.$$

There is a 10% decrease in the transmission coefficient.

(c) The change in the transmission coefficient is given by

$$\Delta T = \frac{\mathrm{d}T}{\mathrm{d}E}\,\Delta E = -2Le^{-2kL}\frac{\mathrm{d}k}{\mathrm{d}E}\,\Delta E = -2LT\frac{\mathrm{d}k}{\mathrm{d}E}\,\Delta E\,.$$

Now, $\mathrm{d}k/\mathrm{d}E = -\mathrm{d}k/\mathrm{d}U = -k/2(U-E)$, so

$$\frac{\Delta T}{T} = kL\frac{\Delta E}{U-E} = (5.0)\frac{(0.010)(5.1\,\mathrm{eV})}{6.8\,\mathrm{eV} - 5.1\,\mathrm{eV}} = 0.15\,.$$

There is a 15% increase in the transmission coefficient.

CHAPTER 38 HINT FOR PROBLEM 15

The kinetic energy of the fastest ejected electron is the difference between the photon energy and the work function.

[ans: 676 km/s]

CHAPTER 38 HINT FOR PROBLEM 21

Write the photoelectric effect equation $K = hf - \Phi$ twice, once for each wavelength and solve the two equations simultaneously for the second wavelength and for the work function.

$\left[\text{ans: (a) } 382\,\text{nm; (b) } 1.82\,\text{eV}\right]$

The fractional change in photon energy is $(f' - f)/f$, where f is the frequency associated with the incident photon and f' is the frequency of the scattered photon. Calculate the wavelength shift and then the wavelength for the scattered photon. Use $f = c/\lambda$ to compute both frequencies.

$\big[$ ans: (a) $-8.1 \times 10^{-9}\,\%$; (b) $-4.9 \times 10^{-4}\,\%$; (c) $-8.8\,\%$; (d) $-66\,\%$ $\big]$

The frequency associated with a photon of energy E is $f = E/h$ and the wavelength is $\lambda = c/f$. The wavelength associated with an electron is $\lambda = h/p$, where p is the magnitude of its momentum. A 1.00 eV electron is nonrelativistic and you can use $K = p^2/2m$ to compute its momentum. A 1.00 GeV electron is relativistic and you should use $(K + mc^2)^2 = (pc)^2 + (mc^2)^2$ but K is so much larger than mc^2 that the expression reduces to $K = pc$. The energies given here are kinetic energies.

[ans: (a) 1.24 μm; (b) 1.22 nm; (c) 1.24 fm; (d) 1.24 fm]

Assume the electron is moving along the x axis and use $\Delta x \cdot \Delta p_x \geq h$, where Δx in the uncertainty in position and Δp_x is the uncertainty in momentum. If Δp_x is to have its least possible value $\Delta x \cdot \Delta P_x = h$. Solve for Δp_x.

$\left[\text{ans: } 2.1 \times 10^{-24}\,\text{kg} \cdot \text{m/s}\ \right]$

The probability that the electron is found in any interval is given by $P = \int |\psi|^2\,dx$, where the integral is over the interval. If the interval width Δx is small, the probability can be approximated by $P = |\psi|^2\,\Delta x$, where the wave function is evaluated for the center of the interval, say. For an electron trapped in an infinite well of width L, the ground state probability density is

$$|\psi|^2 = \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right),$$

so

$$P = \left(\frac{2\,\Delta x}{L}\right)\sin^2\left(\frac{\pi x}{L}\right).$$

(a) Take $L = 100\,\text{pm}$, $x = 25\,\text{pm}$, and $\Delta x = 5.0\,\text{pm}$. Then,

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(25\,\text{pm})}{100\,\text{pm}}\right] = 0.050\,.$$

(b) Take $L = 100\,\text{pm}$, $x = 50\,\text{pm}$, and $\Delta x = 5.0\,\text{pm}$. Then,

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(50\,\text{pm})}{100\,\text{pm}}\right] = 0.10\,.$$

(c) Take $L = 100\,\text{pm}$, $x = 90\,\text{pm}$, and $\Delta x = 5.0\,\text{pm}$. Then,

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(90\,\text{pm})}{100\,\text{pm}}\right] = 0.0095\,.$$

The energy levels are given by

$$E_{n_x\ n_y} = \frac{h^2}{8m}\left[\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right] = \frac{h^2}{8mL^2}\left[n_x^2 + \frac{n_y^2}{4}\right],$$

where the substitutions $L_x = L$ and $L_y = 2L$ were made. In units of $h^2/8mL^2$, the energy levels are given by $n_x^2 + n_y^2/4$. The lowest five levels are $E_{1,1}$ = 1.25, $E_{1,2}$ = 2.00, $E_{1,3}$ = 3.25, $E_{2,1}$ = 4.25, and $E_{2,2} = E_{1,4}$ = 5.00. A little thought should convince you that there are no other possible values for the energy less than 5.

The frequency of the light emitted or absorbed when the electron goes from an initial state i to a final state f is $f = (E_f - E_i)/h$ and in units of $h/8mL^2$ is simply the difference in the values of $n_x^2 + n_y^2/4$ for the two states. The possible frequencies are 0.75 (1,2 $\longrightarrow$ 1,1), 2.00 (1,3 $\longrightarrow$ 1,1), 3.00 (2,1 $\longrightarrow$ 1,1), 3.75 (2,2 $\longrightarrow$ 1,1), 1.25 (1,3 $\longrightarrow$ 1,2), 2.25 (2,1 $\longrightarrow$ 1,2), 3.00 (2,2 $\longrightarrow$ 1,2), 1.00 (2,1 $\longrightarrow$ 1,3), 1.75 (2,2 $\longrightarrow$ 1,3), 0.75 (2,2 $\longrightarrow$ 2,1), all in units of $h/8mL^2$.

There are 8 different frequencies in all. In units of $h/8mL^2$ the lowest is 0.75, the second lowest is 1.00, and the third lowest is 1.25. The highest is 3.75, the second highest is 3.00, and the third highest is 2.25.

The proposed wave function is

$$\psi = \frac{1}{\sqrt{\pi} a^{3/2}} e^{-r/a} \, ,$$

where a is the Bohr radius. Substitute this into the right side of Schrödinger's equation and show that the result is zero. The derivative is

$$\frac{d\psi}{dr} = -\frac{1}{\sqrt{\pi} a^{5/2}} e^{-r/a} \, ,$$

so

$$r^2 \frac{d\psi}{dr} = -\frac{r^2}{\sqrt{\pi} a^{5/2}} e^{-r/a}$$

and

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\psi}{dr} \right) = \frac{1}{\sqrt{\pi} a^{5/2}} \left[-\frac{2}{r} + \frac{1}{a} \right] e^{-r/a} = \frac{1}{a} \left[-\frac{2}{r} + \frac{1}{a} \right] \psi \, .$$

Now the energy of the ground state is given by $E = -me^4/8\epsilon_0^2 h^2$ and the Bohr radius is given by $a = h^2 \epsilon_0 / \pi m e^2$, so $E = -e^2/8\pi\epsilon_0 a$. The potential energy is given by $U = -e^2/4\pi\epsilon_0 r$, so

$$\begin{aligned} \frac{8\pi^2 m}{h^2} [E - U] \psi &= \frac{8\pi^2 m}{h^2} \left[-\frac{e^2}{8\pi\epsilon_0 a} + \frac{e^2}{4\pi\epsilon_0 r} \right] \psi = \frac{8\pi^2 m}{h^2} \frac{e^2}{8\pi\epsilon_0} \left[-\frac{1}{a} + \frac{2}{r} \right] \psi \\ &= \frac{\pi m e^2}{h^2 \epsilon_0} \left[-\frac{1}{a} + \frac{2}{r} \right] \psi = \frac{1}{a} \left[-\frac{1}{a} + \frac{2}{r} \right] \psi \, . \end{aligned}$$

The two terms in Schrödinger's equation obviously cancel and the proposed function ψ satisfies that equation.

Allowed values of the energy are given by $E_n = n^2h^2/8mL^2$ and you want the difference between the energies of the $n = 4$ and $n = 2$ states. Use $1\,\text{eV} = 1.60 \times 10^{-19}$ J to convert from joules to electron volts. When the electron is de-excited the energy of the photon emitted is equal to the energy difference ΔE of these states. The wavelength of the light is $\lambda = ch/\Delta E$. The electron might jump directly to the $n = 1$ state, or it might jump to the $n = 2$ state and then to the $n = 1$ state, or it might make various other jumps.

[ans: (a) 72.2 eV; (b) 13.7 nm; (c) 17.2 nm; (d) 68.7 nm; (e) 41.2 nm; (f) 68.7 nm]

The allowed values of the energy are

$$E_{nx,ny,nz} = \frac{h^2}{8m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right) .$$

Since none of the integers can be zero, the ground state has $n_x = 1$, $n_y = 1$ and $n_z = 1$. Use $1\,\text{eV} = 1.60 \times 10^{-19}\,\text{J}$ to convert the result to electron volts.

[ans: 3.08 eV]

The energy levels for hydrogen are given by $E_n = -(13.6\,\text{eV})/n^2$. The energy of the photon is the energy difference ΔE for the $n = 3$ and $n = 1$ states. Its momentum is $p = \Delta E/c$ and its wavelength is $\lambda = h/p$.

$\big[$ ans: (a) 12.1 eV; (b) $6.45 \times 10^{-27}\,\text{kg}\cdot\text{m/s}$; (c) 102 nm $\big]$

The total energy is the sum of the kinetic and potential energies. The potential energy is $U = -e^2/4\pi\epsilon_0 r$.

$\Big[$ ans: (a) 13.6 eV; (b) -27.2 eV $\Big]$

CHAPTER 40 SOLUTION FOR PROBLEM 9

(a) For $\ell = 3$, the magnitude of the orbital angular momentum is $L = \sqrt{\ell(\ell+1)}\hbar = \sqrt{3(3+1)}\hbar = \sqrt{12}\hbar = 3.46\hbar$.

(b) The magnitude of the orbital dipole moment is $\mu_{\text{orb}} = \sqrt{\ell(\ell+1)}\mu_B = \sqrt{12}\mu_B = 3.46\,\mu_B$.

(c) The largest possible value of m_ℓ is ℓ, which is +3.

(d) The corresponding value of the z component of the angular momentum is $L_z = m_\ell\hbar = +3\hbar$.

(e) The direction of the orbital magnetic dipole moment is opposite that of the orbital angular momentum, so the corresponding value of the z component of the orbital dipole moment is $\mu_{\text{orb},\,z} = -3\mu_B$.

(f) The angle θ between $\vec{L}$ and the z axis is

$$\theta = \cos^{-1}\frac{L_z}{L} = \cos^{-1}\frac{3\hbar}{3.46\hbar} = 30.0^\circ .$$

(g) The second largest value of m_ℓ is $m_\ell = \ell - 1 = 2$ and the angle is

$$\theta = \cos^{-1}\frac{L_z}{L} = \cos^{-1}\frac{2\hbar}{3.46\hbar} = 54.7^\circ .$$

(h) The most negative value of m_ℓ is -3 and the angle is

$$\theta = \cos^{-1}\frac{L_z}{L} = \cos^{-1}\frac{-3\hbar}{3.46\hbar} = 150^\circ .$$

(a) All states with principal quantum number $n = 1$ are filled. The next lowest states have $n = 2$. The orbital quantum number can have the values $\ell = 0$ or 1 and of these, the $\ell = 0$ states have the lowest energy. The magnetic quantum number must be $m_\ell = 0$ since this is the only possibility if $\ell = 0$. The spin quantum number can have either of the values $m_s = -\frac{1}{2}$ or $+\frac{1}{2}$. Since there is no external magnetic field, the energies of these two states are the same. Thus, in the ground state, the quantum numbers of the third electron are either $n = 2$, $\ell = 0$, $m_\ell = 0$, $m_s = -\frac{1}{2}$ or $n = 2$, $\ell = 0$, $m_\ell = 0$, $m_s = +\frac{1}{2}$.

(b) The next lowest state in energy is an $n = 2$, $\ell = 1$ state. All $n = 3$ states are higher in energy. The magnetic quantum number can be $m_\ell = -1$, 0, or $+1$; the spin quantum number can be $m_s = -\frac{1}{2}$ or $+\frac{1}{2}$. If both external and internal magnetic fields can be neglected, all these states have the same energy. The possible states are $(2, 1, 1, +1/2)$, $(2, 1, 1, -1/2)$, $(2, 1, 0, +1/2)$, $(2, 1, 0, -1/2)$, $(2, 1, -1, +1/2)$, and $(2, 1, -1, -1/2)$.

(a) The cut-off wavelength λ_{min} is characteristic of the incident electrons, not of the target material. This wavelength is the wavelength of a photon with energy equal to the kinetic energy of an incident electron. Thus

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626 \times 10^{-34}\,\text{J}\cdot\text{s})(3.00 \times 10^{8}\,\text{m/s})}{(35 \times 10^{3}\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV})} = 3.55 \times 10^{-11}\,\text{m} = 35.5\,\text{pm}\,.$$

(b) A K_α photon results when an electron in a target atom jumps from the L-shell to the K-shell. The energy of this photon is $25.51\,\text{keV} - 3.56\,\text{keV} = 21.95\,\text{keV}$ and its wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626 \times 10^{-34}\,\text{J}\cdot\text{s})(3.00 \times 10^{8}\,\text{m/s})}{(21.95 \times 10^{3}\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV})} = 9.94 \times 10^{-11}\,\text{m} = 5.65 \times 10^{-11}\,\text{m} = 56.5\,\text{pm}\,.$$

(c) A K_β photon results when an electron in a target atom jumps from the M-shell to the K-shell. The energy of this photon is $25.51\,\text{keV} - 0.53\,\text{keV} = 24.98\,\text{keV}$ and its wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626 \times 10^{-34}\,\text{J}\cdot\text{s})(3.00 \times 10^{8}\,\text{m/s})}{(24.98 \times 10^{3}\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV})} = 4.96 \times 10^{-11}\,\text{m} = 49.6\,\text{pm}\,.$$

CHAPTER 40 HINT FOR PROBLEM 7

The magnitude of m_ℓ must be less than or equal to ℓ and n must be greater than ℓ.

[ans:]

CHAPTER 40 HINT FOR PROBLEM 19

The allowed values of the single-electron energies are given by $h^2 n^2/8mL^2$, where n is a positive integer. Two electrons can have each value of n. Find the value of n for each of the electrons so the total energy is the least possible.

[ans: 44]

CHAPTER 40 HINT FOR PROBLEM 25

Add the electrons one at time, with each electron going into the lowest-energy unfilled state. Remember that an s subshell holds 2 electrons, a p subshell holds 6, and a d subshell holds 10.

[ans: (a) 4p; (b) 4; (c) 4p; (d) 5; (e) 4p; (f) 6]

The kinetic energy of the electron must be sufficient to knock a K electron out of the atom and this energy is eV, where V is the accelerating potential. A photon associated with the minimum wavelength is emitted if the electron gives all its kinetic energy to the photon. The wavelength is $\lambda_{\text{min}} = hc/K$, where K is the electron's kinetic energy. The energy of a K_α photon is the difference in the L and K energy levels and the energy of a K_β photon is the difference in the M and K energy levels.

[ans: (a) 69.5 kV; (b) 17.8 pm; (c) 21.3 pm; (d) 18.5 pm]

CHAPTER 40 **HINT FOR PROBLEM 45**

The length of the pulse is given by $c\,\Delta t$, where Δt is the duration. The energy in the pulse is Nhf, where N is the number of photons and f is the frequency. Use $c = \lambda f$ to substitute for the frequency.

[ans: (a) 3.60 mm; (b) 5.25×10^{17}]

The Fermi-Dirac occupation probability is given by $P_{\mathrm{FD}} = 1/\left(e^{\Delta E/kT} + 1\right)$ and the Boltzmann occupation probability is given by $P_{\mathrm{B}} = e^{-\Delta E/kT}$. Let f be the fractional difference. Then

$$f = \frac{P_{\mathrm{B}} - P_{\mathrm{FD}}}{P_{\mathrm{B}}} = \frac{e^{-\Delta E/kT} - \dfrac{1}{e^{\Delta E/kT} + 1}}{e^{-\Delta E/kT}} .$$

Using a common denominator and a little algebra yields

$$f = \frac{e^{-\Delta E/kT}}{e^{-\Delta E/kT} + 1} .$$

The solution for $e^{-\Delta E/kT}$ is

$$e^{-\Delta E/kT} = \frac{f}{1-f} .$$

Take the natural logarithm of both sides and solve for T. The result is

$$T = \frac{\Delta E}{k \ln\left(\dfrac{f}{1-f}\right)} .$$

(a) Put f equal to 0.01 and evaluate the expression for T:

$$T = \frac{(1.00\,\mathrm{eV})(1.60 \times 10^{-19}\,\mathrm{J/eV})}{(1.38 \times 10^{-23}\,\mathrm{J/K}) \ln\left(\dfrac{0.010}{1-0.010}\right)} = 2.50 \times 10^{3}\,\mathrm{K} .$$

(b) Put f equal to 0.10 and evaluate the expression for T:

(a) According to Appendix F the molar mass of silver is 107.870 g/mol and the density is 10.49 $\mathrm{g/cm^3}$. The mass of a silver atom is

$$M = \frac{107.870 \times 10^{-3}\,\mathrm{kg/mol}}{6.022 \times 10^{23}\,\mathrm{mol^{-1}}} = 1.791 \times 10^{-25}\,\mathrm{kg}\,.$$

The number of atoms per unit volume is

$$n = \frac{\rho}{M} = \frac{10.49 \times 10^{3}\,\mathrm{kg/m^3}}{1.791 \times 10^{25}\,\mathrm{kg}} = 5.86 \times 10^{28}\,\mathrm{m^{-3}}\,.$$

Since silver is monovalent this is the same as the number density of conduction electrons.

(b) The Fermi energy is

$$\begin{aligned} E_F &= \frac{0.121h^2}{m}\, n^{2/3} = \frac{(0.121)(6.626 \times 10^{-34}\,\mathrm{J\cdot s})^2}{9.109 \times 10^{-31}\,\mathrm{kg}}\,(5.86 \times 10^{28}\,\mathrm{m^{-1}})^{2/3} \\ &= 8.80 \times 10^{-19}\,\mathrm{J} = 5.49\,\mathrm{eV}\,. \end{aligned}$$

(c) Since $E_F = \frac{1}{2}mv_F^2$,

$$v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2(8.80 \times 10^{-19}\,\mathrm{J})}{9.109 \times 10^{-31}\,\mathrm{kg}}} = 1.39 \times 10^{6}\,\mathrm{m/s}\,.$$

(d) The de Broglie wavelength is

$$\lambda = \frac{h}{p_F} = \frac{h}{mv_F} = \frac{6.626 \times 10^{-34}\,\mathrm{J\cdot s}}{(9.109 \times 10^{-31}\,\mathrm{kg})(1.39 \times 10^{6}\,\mathrm{m/s})} = 5.23 \times 10^{-10}\,\mathrm{m}\,.$$

Sample Problem 41–6 gives the fraction of silicon atoms that must be replaced by phosphorus atoms. Find the number the silicon atoms in 1.0 g, then the number that must be replaced, and finally the mass of the replacement phosphorus atoms. The molar mass of silicon is 28.086 g/mol, so the mass of one silicon atom is $(28.086\,\mathrm{g/mol})/(6.022\times10^{23}\,\mathrm{mol^{-1}}) = 4.66\times10^{-23}$ g and the number of atoms in 1.0 g is $(1.0\,\mathrm{g})/(4.66\times10^{-23}\,\mathrm{g}) = 2.14\times10^{22}$. According to Sample Problem 41–6 one of every 5×10^{6} silicon atoms is replaced with a phosphorus atom. This means there will be $(2.14\times10^{22})/(5\times10^{6}) = 4.29\times10^{15}$ phosphorus atoms in 1.0 g of silicon. The molar mass of phosphorus is 30.9758 g/mol so the mass of a phosphorus atom is $(30.9758\,\mathrm{g/mol})/(6.022\times10^{-23}\,\mathrm{mol^{-1}}) = 5.14\times10^{-23}$ g. The mass of phosphorus that must be added to 1.0 g of silicon is $(4.29\times10^{15})(5.14\times10^{-23}\,\mathrm{g}) = 2.2\times10^{-7}$ g.

Solve $P = 1/[e^{(E-E_F)/kT}+1]$ for E by taking the natural logarithm of both sides. Then evaluate $N(E) = (8\sqrt{2}\pi m^{3/2}/h^3)E^{1/2}$ for the density of states and $N_0 = N(E)P(E)$ for the density of occupied states.

[ans: (a) 6.81 eV; (b) $1.77 \times 10^{28}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-1}$; (c) $1.59 \times 10^{28}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-1}$]

The Fermi energy of a metal is given by $E_F = (3/16\sqrt{2}\pi)(h^2/m)n^{2/3}$, where m is the electron mass and n is the number of conduction electrons per unit volume. Solve for n. Now you need the number of atoms per unit volume. This the density divided by the mass of an atom. The mass of an atom is the molar mass divided by the Avogadro constant. Be sure to use consistent units.

[ans: 3]

According to Problem 24 the fraction of the conduction electrons in a metal that have energies greater than the Fermi energy is given by fract = $3kT/2E_F$, where T is the temperature on the Kelvin scale, k is the Boltzmann constant, and E_F is the Fermi energy.

$\left[\text{ans: } 4.7 \times 10^2\,\text{K}\right]$

CHAPTER 41   HINT FOR PROBLEM 33

Use $P(E) = 1/[e^{(E-E_F)/kT} + 1]$ to calculate the occupation probability. For a state at the bottom of the conduction band $E - E_F$ is half the gap for the pure semiconductor and is $0.11\,\text{eV}$ for the doped semiconductor. For the donor state $E - E_F$ is $0.11\,\text{eV} - 0.15\,\text{eV} = -0.04\,\text{eV}$.

[ans: (a) 4.79×10^{-10}; (b) 0.0140; (c) 0.824]

If a nucleus contains Z protons and N neutrons, its binding energy is $\Delta E_{\text{be}} = (Zm_H + Nm_n - m)c^2$, where m_H is the mass of a hydrogen atom, m_n is the mass of a neutron, and m is the mass of the atom containing the nucleus of interest. If the masses are given in atomic mass units, then mass excesses are defined by $\Delta_H = (m_H - 1)c^2$, $\Delta_n = (m_n - 1)c^2$, and $\Delta = (m - A)c^2$. This means $m_H c^2 = \Delta_H + c^2$, $m_n c^2 = \Delta_n + c^2$, and $mc^2 = \Delta + Ac^2$. Thus $E = (Z\Delta_H + N\Delta_n - \Delta) + (Z + N - A)c^2 = Z\Delta_H + N\Delta_n - \Delta$, where $A = Z + N$ was used. For $^{197}_{79}$Au, $Z = 79$ and $N = 197 - 79 = 118$. Hence

$$\Delta E_{\text{be}} = (79)(7.29\,\text{MeV}) + (118)(8.07\,\text{MeV}) - (-31.2\,\text{MeV}) = 1560\,\text{MeV}\,.$$

This means the binding energy per nucleon is $\Delta E_{\text{ben}} = (1560\,\text{MeV})/(197) = 7.92\,\text{MeV}$.

(a) The half-life $T_{1/2}$ and the disintegration constant are related by $T_{1/2} = (\ln 2)/\lambda$, so $T_{1/2} = (\ln 2)/(0.0108\,\mathrm{h}^{-1}) = 64.2\,\mathrm{h}$.

(b) At time t, the number of undecayed nuclei remaining is given by

$$N = N_0\, e^{-\lambda t} = N_0\, e^{-(\ln 2)t/T_{1/2}}\,.$$

Substitute $t = 3T_{1/2}$ to obtain

$$\frac{N}{N_0} = e^{-3\ln 2} = 0.125\,.$$

In each half-life, the number of undecayed nuclei is reduced by half. At the end of one half-life, $N = N_0/2$, at the end of two half-lives, $N = N_0/4$, and at the end of three half-lives, $N = N_0/8 = 0.125N_0$.

(c) Use

$$N = N_0\, e^{-\lambda t}\,.$$

10.0 d is 240 h, so $\lambda t = (0.0108\,\mathrm{h}^{-1})(240\,\mathrm{h}) = 2.592$ and

$$\frac{N}{N_0} = e^{-2.592} = 0.0749\,.$$

Since the electron has the maximum possible kinetic energy, no neutrino is emitted. Since momentum is conserved, the momentum of the electron and the momentum of the residual sulfur nucleus are equal in magnitude and opposite in direction. If p_e is the momentum of the electron and p_S is the momentum of the sulfur nucleus, then $p_S = -p_e$. The kinetic energy K_S of the sulfur nucleus is $K_S = p_S^2/2M_S = p_e^2/2M_S$, where M_S is the mass of the sulfur nucleus. Now, the electron's kinetic energy K_e is related to its momentum by the relativistic equation $(p_e c)^2 = K_e^2 + 2K_e mc^2$, where m is the mass of an electron. See Eq. 37–54. Thus

$$\begin{aligned} K_S &= \frac{(p_e c)^2}{2M_S c^2} = \frac{K_e^2 + 2K_e mc^2}{2M_S c^2} = \frac{(1.71\,\text{MeV})^2 + 2(1.71\,\text{MeV})(0.511\,\text{MeV})}{2(32\,\text{u})(931.5\,\text{MeV/u})} \\ &= 7.83 \times 10^{-5}\,\text{MeV} = 78.3\,\text{eV}\,, \end{aligned}$$

where $mc^2 = 0.511\,\text{MeV}$ was used.

Let f_{24} be the abundance of ^{24}Mg, f_{25} be the abundance of ^{25}Mg, and f_{26} be the abundance of ^{26}Mg. Let M_{24}, M_{25}, and M_{26} be the masses of the nuclei. Then the average atomic mass is $f_{24}M_{24} + f_{25}M_{25} + f_{26}M_{26}$. Furthermore, the abundances must sum to 1: $F_{24} + f_{25} + f_{26} = 1$. Solve these two equation simultaneously for f_{25} and f_{26}.

[ans:]

Calculate the mass of the undecayed nuclei at $t = 14.0\,\mathrm{h}$ and at $t = 16.0\,\mathrm{h}$. The difference is the mass of the nuclei that decayed between those two times. The number of undecayed nuclei at time t is given by $N = N_0 e^{-\lambda t}$, where N_0 is the number at $t = 0$ and λ is the disintegration constant, which is related to the half-life by $\lambda = (\ln 2)/T_{1/2}$. Multiply by the mass of a nucleus to obtain $m = Me^{-\lambda t}$ for the mass of undecayed nuclei at time t. Here M is the mass of the sample.

[ans: 265 mg]

Assume the ^{238}U nucleus is initially at rest and the ^{234}Th nucleus is in its ground state. Then the disintegration energy is given by $Q = K_{\text{Th}} + K_\alpha$, where K_{Th} is the kinetic energy of the recoiling ^{234}Th nucleus and K_α is the kinetic energy of the alpha particle (4.196 MeV). Linear momentum is conserved, so $0 = p_{\text{Th}} + p_\alpha$, where p_{Th} is the momentum of the ^{234}Th nucleus and p_α is the momentum of the alpha particle. Use $K_{\text{Th}} = p_{\text{Th}}^2/2m_{\text{Th}}$ and $K_\alpha = p_\alpha^2/2m_\alpha$ along with $p_{\text{Th}} = -p_\alpha$, where m_{Th} and m_α are the masses, to show that $K_{\text{Th}} = (m_\alpha/m_{\text{Th}})K_\alpha$.

[ans: 4.269 MeV]

The number of ^{238}U nuclei in the rock is given by M/m, where M is the total mass of that isotope and m is the mass of a single nucleus (238 u). A similar expression holds for ^{206}Pb. A ^{206}Pb nucleus is created for every ^{238}U nucleus lost so the number of ^{238}U nuclei in the rock is the sum of the number of nuclei of both types now present. Use $N = N_0 e^{-\lambda t}$ to find the age t of the rock. The disintegration constant λ is related to the half-life by $\lambda = (\ln 2)/T_{1/2}$.

[ans: (a) 1.06×10^{19}; (b) 0.624×10^{19}; (c) 1.68×10^{19}; (d) 2.97×10^{9} y]

(a) If X represents the unknown fragment, then the reaction can be written

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \to ^{83}_{32}\mathrm{Ge} + ^{A}_{Z}X\,,$$

where A is the mass number and Z is the atomic number of the fragment. Conservation of charge yields $92 + 0 = 32 + Z$, so $Z = 60$. Conservation of mass number yields $235 + 1 = 83 + A$, so $A = 153$. Look in Appendix F or G for nuclides with $Z = 60$. You should find that the unknown fragment is $^{153}_{60}\mathrm{Nd}$.

(b) and (c) Ignore the small kinetic energy and momentum carried by the neutron that triggers the fission event. Then $Q = K_{\mathrm{Ge}} + K_{\mathrm{Nd}}$, where K_{Ge} is the kinetic energy of the germanium nucleus and K_{Nd} is the kinetic energy of the neodymium nucleus. Conservation of momentum yields $p_{\mathrm{Ge}} + p_{\mathrm{Nd}} = 0$, where p_{Ge} is the momentum of the germanium nucleus and p_{Nd} is the momentum of the neodymium nucleus. Since $p_{\mathrm{Nd}} = -p_{\mathrm{Ge}}$, the kinetic energy of the neodymium nucleus is

$$K_{\mathrm{Nd}} = \frac{p_{\mathrm{Nd}}^2}{2M_{\mathrm{Nd}}} = \frac{p_{\mathrm{Ge}}^2}{2M_{\mathrm{Nd}}} = \frac{M_{\mathrm{Ge}}}{M_{\mathrm{Nd}}}K_{\mathrm{Ge}}\,.$$

Thus the energy equation becomes

$$Q = K_{\mathrm{Ge}} + \frac{M_{\mathrm{Ge}}}{M_{\mathrm{Nd}}}K_{\mathrm{Ge}} = \frac{M_{\mathrm{Nd}} + M_{\mathrm{Ge}}}{M_{\mathrm{Nd}}}K_{\mathrm{Ge}}$$

and

$$K_{\mathrm{Ge}} = \frac{M_{\mathrm{Nd}}}{M_{\mathrm{Nd}} + M_{\mathrm{Ge}}}Q = \frac{153\,\mathrm{u}}{153\,\mathrm{u} + 83\,\mathrm{u}}(170\,\mathrm{MeV}) = 110\,\mathrm{MeV}\,.$$

Similarly,

$$K_{\mathrm{Nd}} = \frac{M_{\mathrm{Ge}}}{M_{\mathrm{Nd}} + M_{\mathrm{Ge}}}Q = \frac{83\,\mathrm{u}}{153\,\mathrm{u} + 83\,\mathrm{u}}(170\,\mathrm{MeV}) = 60\,\mathrm{MeV}\,.$$

The mass conversion factor can be found in Appendix C.

(d) The initial speed of the germanium nucleus is

$$v_{\mathrm{Ge}} = \sqrt{\frac{2K_{\mathrm{Ge}}}{M_{\mathrm{Ge}}}} = \sqrt{\frac{2(110\times10^{6}\,\mathrm{eV})(1.60\times10^{-19}\,\mathrm{J/eV})}{(83\,\mathrm{u})(1.661\times10^{-27}\,\mathrm{kg/u})}} = 1.60\times10^{7}\,\mathrm{m/s}\,.$$

(e) The initial speed of the neodymium nucleus is

$$v_{\mathrm{Nd}} = \sqrt{\frac{2K_{\mathrm{Nd}}}{M_{\mathrm{Nd}}}} = \sqrt{\frac{2(60\times10^{6}\,\mathrm{eV})(1.60\times10^{-19}\,\mathrm{J/eV})}{(153\,\mathrm{u})(1.661\times10^{-27}\,\mathrm{kg/u})}} = 8.69\times10^{6}\,\mathrm{m/s}\,.$$

Let P_0 be the initial power output, P be the final power output, k be the multiplication factor, t be the time for the power reduction, and $t_{\rm gen}$ be the neutron generation time. Then according to the result of Problem 23,

$$P = P_0\, k^{t/t_{\rm gen}}\,.$$

Divide by P_0, then take the natural logarithm of both sides of the equation and solve for $\ln k$. You should obtain

$$\ln k = \frac{t_{\rm gen}}{t}\,\ln\frac{P}{P_0}\,.$$

Hence

$$k = e^{\alpha}\,,$$

where

$$\alpha = \frac{t_{\rm gen}}{t}\,\ln\frac{P}{P_0} = \frac{1.3\times10^{-3}\,{\rm s}}{2.6000\,{\rm s}}\,\ln\frac{350.00\,{\rm MW}}{1200.0\,{\rm MW}} = -6.161\times10^{-4}\,.$$

This yields $k = .99938$.

(a) The mass of a carbon atom is $(12.0\,\mathrm{u})(1.661 \times 10^{-27}\,\mathrm{kg/u}) = 1.99 \times 10^{-26}\,\mathrm{kg}$, so the number of carbon atoms in 1.00 kg of carbon is $(1.00\,\mathrm{kg})/(1.99 \times 10^{-26}\,\mathrm{kg}) = 5.02 \times 10^{25}$. (The mass conversion factor can be found in Appendix C.) The heat of combustion per atom is $(3.3 \times 10^{7}\,\mathrm{J/kg})/(5.02 \times 10^{25}\,\mathrm{atom/kg}) = 6.58 \times 10^{-19}\,\mathrm{J/atom}$. This is $4.11\,\mathrm{eV/atom}$.

(b) In each combustion event, two oxygen atoms combine with one carbon atom, so the total mass involved is $2(16.0\,\mathrm{u}) + (12.0\,\mathrm{u}) = 44\,\mathrm{u}$. This is $(44\,\mathrm{u})(1.661 \times 10^{-27}\,\mathrm{kg/u}) = 7.31 \times 10^{-26}\,\mathrm{kg}$. Each combustion event produces $6.58 \times 10^{-19}\,\mathrm{J}$ so the energy produced per unit mass of reactants is $(6.58 \times 10^{-19}\,\mathrm{J})/(7.31 \times 10^{-26}\,\mathrm{kg}) = 9.00 \times 10^{6}\,\mathrm{J/kg}$.

(c) If the Sun were composed of the appropriate mixture of carbon and oxygen, the number of combustion events that could occur before the Sun burns out would be $(2.0 \times 10^{30}\,\mathrm{kg})/(7.31 \times 10^{-26}\,\mathrm{kg}) = 2.74 \times 10^{55}$. The total energy released would be $E = (2.74 \times 10^{55})(6.58 \times 10^{-19}\,\mathrm{J}) = 1.80 \times 10^{37}\,\mathrm{J}$. If P is the power output of the Sun, the burn time would be $t = E/P = (1.80 \times 10^{37}\,\mathrm{J})/(3.9 \times 10^{26}\,\mathrm{W}) = 4.62 \times 10^{10}\,\mathrm{s}$. This is 1460 y.

The rate of spontaneous fission decays is given by λN, where λ is the disintegration constant for that type decay and N is the number of ^{235}U nuclei in the sample. The disintegration constant is related to the half-life by $\lambda = (\ln 2)/T_{1/2}$ and the number of nuclei in the sample is M/m, where M is the mass of the sample and m is the mass of a nucleus (235 u). The ratio of the decay rates is equal to the ratio of the disintegration constants and also to the reciprocal of the ratio of the half-lives.

[ans: (a) 16 fissions/day; (b) 4.3×10^8]

Consider 1.00 g of ^{90}Sr, which produces thermal energy at a rate of 0.93 W. The rate of thermal energy generation is given by $P = Q_{\text{eff}}R$, where R is the fission rate. The fission rate is $R = \lambda N$, where N is the number of λ is the disintegration constant and is related to the half-life by $\lambda = (\ln 2)/T_{1/2}$. The number of nuclei is given by M/m, where M is 1.00g and m is the mass of a ^{90}Sr nucleus.

[ans: (a) 1.2 MeV; (b) 3.2 kg]

The barrier height is $q^2/4\pi\epsilon_0 d$, where q is the charge on a nucleus and d is the center-to-center separation of the nuclei. That is, $d = 2r$, where r is the radius of a nucleus. Use $r = r_0 A^{1/3}$, where A is the mass number and $r_0 = 1.2\,\text{fm}$, to compute the radius.

[ans: 1.41 MeV]

The number of fusion events per unit time is P/Q, where P is the rate of energy radiation and Q is the energy produced per event. P is given in Problem 35 and Q is given in Section 43–7. Each event produces 2 neutrinos. The fraction of the neutrinos that reach Earth is equal to the ratio of the cross-sectional area of Earth to the surface area of a sphere with radius equal to the Earth-Sun distance.

[ans: (a) $1.8 \times 10^{38}\,\mathrm{s}^{-1}$; (b) $8.2 \times 10^{28}\,\mathrm{s}^{-1}$]

CHAPTER 44 SOLUTION FOR PROBLEM 11

(a) The conservation laws considered so far are associated with energy, momentum, angular momentum, charge, baryon number, and the three lepton numbers. The rest energy of the muon is 105.7 MeV, the rest energy of the electron is 0.511 MeV, and the rest energy of the neutrino is zero. Thus the total rest energy before the decay is greater than the total rest energy after. The excess energy can be carried away as the kinetic energies of the decay products and energy can be conserved. Momentum is conserved if the electron and neutrino move away from the decay in opposite directions with equal magnitudes of momenta. Since the orbital angular momentum is zero, we consider only spin angular momentum. All the particles have spin $\hbar/2$. The total angular momentum after the decay must be either $\hbar$ (if the spins are aligned) or zero (if the spins are antialigned). Since the spin before the decay is $\hbar/2$, angular momentum cannot be conserved. The muon has charge $-e$, the electron has charge $-e$, and the neutrino has charge zero, so the total charge before the decay is $-e$ and the total charge after is $-e$. Charge is conserved. All particles have baryon number zero, so baryon number is conserved. The muon lepton number of the muon is +1, the muon lepton number of the muon neutrino is +1, and the muon lepton number of the electron is 0. Muon lepton number is conserved. The electron lepton numbers of the muon and muon neutrino are 0 and the electron lepton number of the electron is +1. Electron lepton number is not conserved. The laws of conservation of angular momentum and electron lepton number are not obeyed and this decay does not occur..

(b) Analyze the decay in the same way. You should find that only charge is not conserved.

(c) Here you should find that energy and muon lepton number cannot be conserved.

(a) Look at the first three lines of Table 44–5. Since the particle is a baryon, it must consist of three quarks. To obtain a strangeness of -2, two of them must be s quarks. Each of these has a charge of $-e/3$, so the sum of their charges is $-2e/3$. To obtain a total charge of e, the charge on the third quark must be $5e/3$. There is no quark with this charge, so the particle cannot be constructed. In fact, such a particle has never been observed.

(b) Again the particle consists of three quarks (and no antiquarks). To obtain a strangeness of zero, none of them may be s quarks. We must find a combination of three u and d quarks with a total charge of $2e$. The only such combination consists of three u quarks.

The kinetic energy K of a relativistic particle is related to it speed v by $K = (\gamma - 1)mc^2$, where m is its mass and $\gamma = 1/\sqrt{1-(v/c)^2}$. Solve for v. Since the energy is so much greater than the rest energy you should find the algebraic solution and then make a binomial expansion in powers of the ratio of the rest energy to the total energy.

$\left[\text{ans: } 0.0266\,\mathrm{m/s.}\right]$

CHAPTER 44 HINT FOR PROBLEM 17

Use conservation of charge, baryon number, and angular momentum to find the values of these quantities for the unknown particle. Conservation of angular momentum requires that if an odd number of fermions enter the reaction then an odd number must leave and if an even number (including 0) enter then an even number must leave.

[ans: (a) K^+; (b) $\overline{n}$; (c) K^0]

CHAPTER 44 HINT FOR PROBLEM 31

The number of molecules in an excited state with energy E above the ground state is given by $N = N_0 e^{-E/kT}$, where N_0 is the number in the ground state, k is the Boltzmann constant, and T is the temperature on the Kelvin scale. Put N/N_0 equal to 0.25 and solve for E.

[ans: (a) 256 μeV; (b) 4.84 mm]