Solutions
of
Optics & Modern Physics

Lesson 26^{th} to 30^{th}

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26. Reflection of Light

Introductory Exercise 26.1

1. Since $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ where c is the speed of light in vacuum hence unit of $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ is m/s.

2. Hence

$B_y = 2 \times 10^{-7}$ T sin $[500x + 1.5 \times 10^{11} t]$

Comparing this equation with the standard wave eqution $B_y = B_0 \sin [kx + \omega t]$

$$k = 500 \text{ m}^{-1} \Rightarrow k = \frac{2\pi}{\lambda}$$

$$\Rightarrow \quad \lambda = \frac{2\pi}{k} \text{ m}$$

$$\Rightarrow \quad \lambda = \frac{2\pi}{500} \text{ m} = \frac{\pi}{250} \text{ metre}$$

$$\omega = 1.5 \times 10^{11} \text{ rad/s}$$

$$\Rightarrow \quad 2\pi n = 1.5 \times 10^{11}$$

$$\Rightarrow \quad n = \frac{1.5}{2\pi} \times 10^{11} \text{ Hz}$$

Speed of the wave $v = \frac{\omega}{k} = \frac{1.5 \times 10^{11}}{500}$

$= 3 \times 10^8$ m/s

Let E_0 be the amplitude of electric field.

Then $E_0 = cB_0 = 3 \times 10^8 \times 2 \times 10^{-7}$

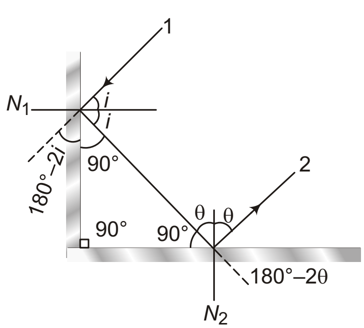
$= 60$ V/m

Since wave is propagating along x-axis and B along y-axis, hence E must be along z-axis

$\Rightarrow E = 60$ V/m $\sin [500x + 1.5 \times 10^{11} t]$

Introductory Exercise 26.2

1. Total deviation produced



$$\delta = 180° - 2i + 180° - 2\theta$$

$$\delta = 360° - 2(i + \theta)$$

From figure

$$\theta = 90° - i$$

$$\Rightarrow \quad \delta = 360° - 2[i + 90 - i]$$

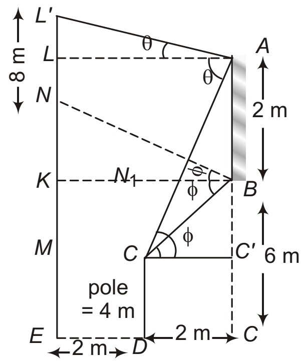
$$= 180°$$

Hence rays 1 and 2 are parallel (anti-parallel).

2. $v_0 = 2$ m/s for plane mirror $v_i = 2$ m/s.

Velocity of approach $= v_0 + v_i = 4$ m/s.

3. In figure, AB is mirror, G is ground, CD is pole and M is the man. The minimum height to see the image of top of pole is $= EN$



$= EK + KN = 6 + KN$

Now in ΔNKB,

$$\frac{NK}{KB} = \tan\phi \Rightarrow NK = KB\tan\phi$$

$$= 4\tan\phi$$

In $\Delta BC'C$ we get,

$$\tan\phi = \frac{BC'}{CC'} = \frac{2}{2} = 1$$

$$\phi = 45^\circ$$

So, $NK = 4 \times \tan 45^\circ = 4$ m

Hence in minimum height
$= 6\text{ m} + 4\text{ m} = 10\text{ m}$

In $\quad \Delta AC'C$

$$\tan\theta = \frac{4}{2} = 2$$

In $\Delta L'LA$ we get,

$$\frac{LL'}{LA} = \tan\theta$$

$$\Rightarrow \quad \frac{LL'}{4} = 2$$

$$\Rightarrow \quad LL' = 8\text{ m}$$

Maximum height $= CA + LL' = 8 + 8 = 16$ m

Introductory Exercise 26.3

1. Here $f = -10$ cm (concave mirror)

(a) $u = -25$ cm

Using mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{10} + \frac{1}{25}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{-5+2}{50}$$

$$\Rightarrow \quad v = -\frac{50}{3} = -16.7\text{ cm}$$

Hence image is real, inverted and less height of the object.

(b) Since $u = -10$ cm,

Hence object is situated on focus of the image formed at ∞.

(c) $u - 5$, $f = -10$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{10} + \frac{1}{5}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{-1+2}{10}$$

$$\Rightarrow \quad v = 10\text{ cm}$$

Hence, image is virtual, erect and two time of the object.

2. Here $u = -3$ m, $f = -\frac{1}{2}$ m,

we have,

(a) $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

$$\Rightarrow \quad \frac{1}{v} = -2 + \frac{1}{3}$$

$$\Rightarrow \quad v = -0.6\text{ m}$$

As ball moves towards focus the image moves towards $-\infty$ and image is real as the distance decreases by focal length image become virtual which moves from $+\infty$ to zero.

(b) The image of the ball coincide with ball, when $u = -R = -1$ m

Using $h = ut + \frac{1}{2} gt^2$

$$\Rightarrow \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 2}{9.8}}$$

$$= 0.639 \text{ s}$$

Similarly again images match at $t = 0.78$ s.

3. Since image is magnified, hence the mirror is concave.

Here, $m = \frac{-v}{u} \Rightarrow \frac{-v}{u} = 5$

$$\Rightarrow \quad v = -5u \quad \text{...(i)}$$

Let distance between mirror and object is x.

Since image is formed at a distance 5 m from mirror

$$v = -(5 + x) \quad \text{...(ii)}$$

From Eqs.(i) and (ii), we get

$$-(5 + x) = -5x$$

$$\Rightarrow \quad 4x = 5$$

$$\Rightarrow \quad x = 1.25$$

Hence mirror is placed at 1.25 m on right side of the object by mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

we have

$$\frac{1}{f} = -\frac{1}{6.25} - \frac{1}{1.25}$$

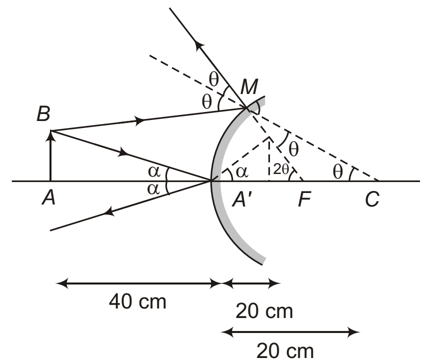
$$\Rightarrow \quad f = \frac{-6.25}{6},$$

Hence $R = 2f \Rightarrow R = -\frac{6.25}{3} = -2.08$ m

Thus mirror is concave mirror of radius of curvature 2.08 m.

4. Since the incident rays and reflected rays are parallel to each other therefore mirror is plane mirror.

5. Let us solve the first case :



By applying the geometry we can prove that,

$$PA' = v = \frac{40}{3} \text{ cm}$$

Further, in triangles ABP and $PA'B'$ we have,

$$\frac{AB}{40} = \frac{A'B'}{(40/3)}$$

$$\therefore \quad A'B' = \frac{AB}{3} = \frac{2}{3} \text{ cm}$$

Similary, we can solve other parts also.

6. Simply apply :

$$\frac{1}{v} = \frac{1}{u} = \frac{1}{f}$$

and $m = \frac{I}{o} = \frac{-v}{u}$ for lateral magnification. If magnitfication is positive, image will be virtual. If magnification is negative, image will be real.

AIEEE Corner

■ Subjective Questions (Level 1)

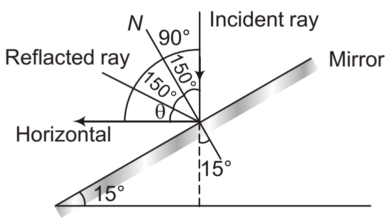
1. Here $v = 39.2$ cm, hence $v = -39.2$ cm

and magnification $m = 1$

$\Rightarrow \qquad h_i = h_o = 4.85$

Hence image is formed at 39.2 cm behind the mirror and height of image is $= 4.85$ cm.

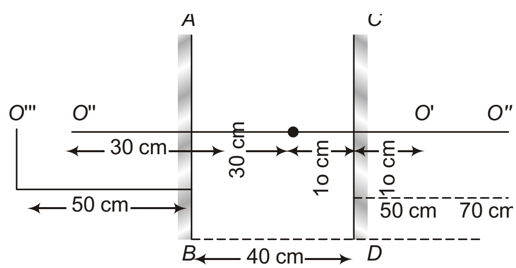
2. From figure, angle of incident $= 15°$



Let reflected ray makes an angle θ with the horizontal, then

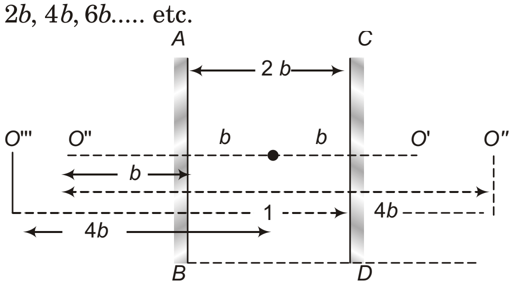
$$\theta + 15° + 15° = 90° \Rightarrow \theta = 60°$$

3.



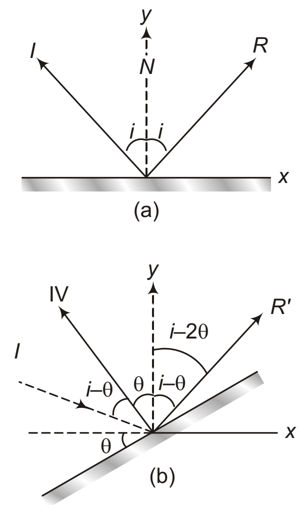
Since mirror are parallel to each other ∞ image are formed the distance of five closet to object are 20 cm, 60 cm, 80 cm, 100 cm and 140 cm.

4. The distance of the object from images are $2b, 4b, 6b$..... etc.



Hence the images distance are $2nb$, where $n = 1, 2, \ldots$. Ans.

5. Suppose mirror is rotated at angle θ about its axis perpendicular to both the incident ray and normal as shown in figure



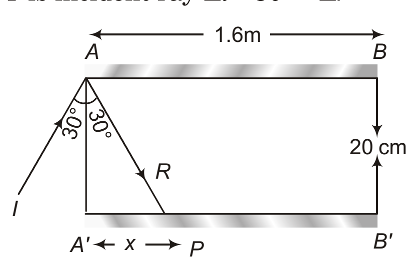
In figure (b) I remain unchanged N and R shift to N' and R'.

From figure (a) angle of rotation $= i$,

From figure (b) it is $i - 2\theta$

Thus, reflected ray has been rotated by angle 2θ.

6. I is incident ray $\angle i = 30° = \angle r$



From $\Delta\, PA'A$, we get

$$\frac{x}{20} = \tan 30° \Rightarrow x = 20 \tan 30°$$

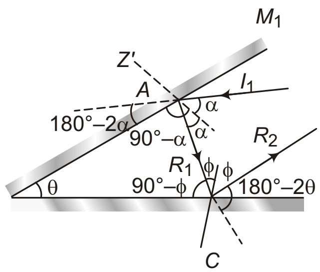
$$\text{No. of reflection} = \frac{AB}{x} = \frac{160\text{ cm}}{20\text{ cm} \times \tan 30°}$$

$$= 8\sqrt{3} \approx 14$$

Hence the reflected ray reach other end after 14 reflections.

7. The deviation produced by mirror M_1 is

$$= 180° - 2\alpha$$



and the deviation produced by mirror M_2 is

$$= 180 - 2$$

Hence total deviation

$$= 180 - 2\alpha + 180 - 2\phi$$

$$= 360 - 2(\alpha + \phi)$$

In $\Delta\, ABC$ we get,

$$90 - \alpha + \theta + 90 - \phi = 180$$

$$\Rightarrow \quad \alpha + \phi = \theta$$

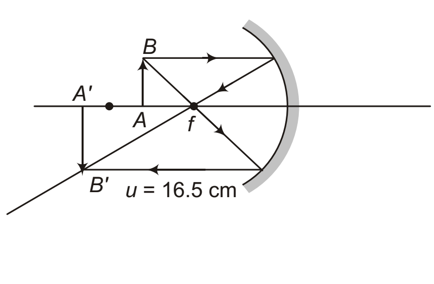
Hence deviation produces $= 180 - 2\theta$.

8. Here $f = -\dfrac{R}{2} = -\dfrac{22}{2} = -11$ cm

Object height $h_0 = 6$ mm

$$u = -16.5\text{ cm}$$

(a) The ray diagram is shown in figure



Using mirror formula, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{v} = -\frac{1}{11} + \frac{1}{16.5} = \frac{-165 + 11}{16.5 \times 11}$$

$$\Rightarrow \quad v = -\frac{16.5 \times 11}{5.5} = -33\text{ cm}$$

Hence the image is formed at 33 cm from the pole (vertex) of mirror on the object side the image is real, inverted and magnified. The absolute magnification

$$|m| = \left|\frac{v}{u}\right| = \frac{33}{16.5} = 2$$

Hence size of image is $h_i = 2 \times h_0$

$$= 2 \times 6 = 12\text{ mm}.$$

9. Here $u = -12$ cm, $f = +\dfrac{R}{2} = +10$ cm

Using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

we get

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{10} + \frac{1}{12}$$

$$= \frac{6+5}{60}$$

$$\Rightarrow \quad v = \frac{60}{11}\text{ cm} = 5.46\text{ cm}$$

The image is formed on right side of the vertex at a distance $\dfrac{60}{11}$ cm. the image is virtual and erect the absolute magnification is given by $|m| = \left|\dfrac{v}{u}\right|$

$$\Rightarrow \quad |m| = \left|\frac{60}{11 \times (-12)}\right| = \frac{5}{11}$$

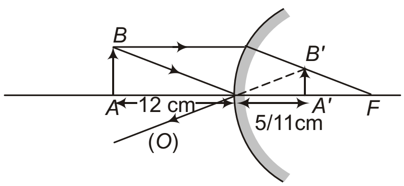
$$\because \quad m < 1$$

Hence image is de-magnified.

Height of image $h_i = |m| \times h_0$

$\Rightarrow h_i = \frac{5}{11} \times 9 = \frac{45}{11} = 4.09$ mm

The ray diagram is shown in figure



10. Here $f = -18$ cm

Let distance of object from vertex of concave mirror is u. Since image is real hence image and object lie left side of the vertex.

Magnification $m = -\frac{v}{u} = \frac{1}{9}$

$$\Rightarrow \qquad v = -\frac{u}{9}$$

By mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we have

$$-\frac{1}{u/9} - \frac{1}{u} = -\frac{1}{18} \Rightarrow -\frac{10}{u} = -\frac{1}{18}$$

$\Rightarrow u = 180$ cm (left side of the vertex).

11. Here $u = -30$ cm, since image is inverted.

Hence the mirror is concave.

$$m = \frac{1}{2} = \frac{-v}{u} \Rightarrow v = -\frac{u}{2}$$

Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$-\frac{2}{u} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{-3}{u} = \frac{1}{f}$$

$$\Rightarrow \qquad f = -\frac{u}{3} = -\frac{30}{3} = -10 \text{ cm}$$

Hence mirror is concave of focal length 10 cm.

12. Here $f = -\frac{24}{2}$ cm $= -12$ cm

(a) Since image is virtual

$$\Rightarrow \qquad m = \frac{v}{u} \Rightarrow v = mu$$

$$\Rightarrow \qquad v = 3 \times u$$

$$\Rightarrow \qquad v = 3u \text{ and } v \text{ is +ve}$$

By mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{3u} - \frac{1}{u} - \frac{1}{12}$$

$$\Rightarrow \qquad \frac{1-3}{3u} = -\frac{1}{12} \Rightarrow u = 8 \text{ cm}$$

(b) Since image is real

$$\Rightarrow \qquad m = -\frac{v}{u} = 3 \Rightarrow v = -3u$$

By using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$-\frac{1}{e} - \frac{1}{u} = -\frac{1}{12} \Rightarrow \frac{-4}{3u} = -\frac{1}{12}$$

$$\Rightarrow \qquad u = 16 \text{ cm}$$

(c) Here $m = \frac{-v}{m} = \frac{1}{3} \Rightarrow v = -\frac{u}{3}$

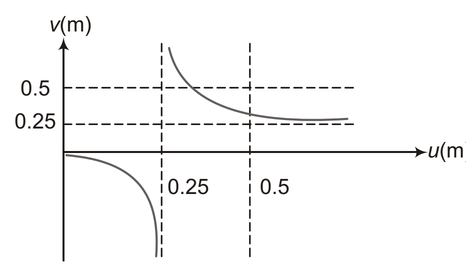
$$\Rightarrow \qquad -\frac{1}{u/3} - \frac{1}{u} = -\frac{1}{12}$$

$$\Rightarrow \qquad -\frac{4}{u} = -\frac{1}{12} \Rightarrow u = 48 \text{ cm}$$

13. We have $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow v = \frac{uf}{u-f} \quad \text{at } u = f, v = \infty$$

The variation is shown in figure



Hence focal length if assymtote of the curve.

When $u < f$, Image is virtual. It means v is negative.

When $\quad u = 2f$

$v = 2f$

$u \to 0, v \to 0$

14. Here $f = 21\text{ cm} \Rightarrow R = 2f = 42\text{ cm}$

Since the object is placed on C. Hence its image by concave mirror is formed on C. This image acts as a virtual objet for plane mirror the distance between plane mirror and virtual object $= 21$ cm.

Hence plane mirror forms its real image in front of plane mirror at 12 cm.

15. Let u is the object distance from vertex, v is the image distance for vertex and f is the focal length then distance between object and focus is $u - f$ and distance between image and focus is $v - f$ *ie*,

$$(u - f)(v - f) = uv - (u + v)f + f^2 \qquad \ldots\text{(i)}$$

Using $\frac{1}{v} + \frac{1}{u} + \frac{1}{f}$, we get

$$uv = (u + v)f \qquad \ldots\text{(ii)}$$

Putting the value of uv in RHS of Eq. (i), we get

$$(u - f)(u - f) = (v + u)f - (v + u)f + f^2$$

$$(u - f)(v - f) = f^2$$

Hence proved.

16. Let object is placed at a distance x from the convex mirror then for convex mirror

$$u - x \text{ and } f = + \frac{R}{2}$$

Let v be the distance of the image from pole (vertex) of convex mirror.

Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v} - \frac{1}{x} = \frac{2}{12} \Rightarrow v = \frac{xR}{2x + R}$$

For concave mirror

$$u' = -\left[2R + \frac{xR}{2x + R}\right] = -\left[\frac{2R^2 + 5xR}{2x + R}\right]$$

$$v' = -(2R - x) \text{ and } f' = -\frac{R}{2}$$

Using $\frac{1}{v'} + \frac{1}{u'} = \frac{1}{f'}$, we get

$$-\frac{1}{(2R - x)} - \frac{(2x + R)}{(2R^2 + 5xR)} - \frac{2}{R}$$

$\Rightarrow$ $$4R^3 - 2x^2R + 8xR^2 = 8R^3 + 16xR^2 - 10x^2R$$

$\Rightarrow$ $$4R^3 + 8xR^2 - 8x^2R = 0$$

$\Rightarrow$ $$4R[R^2 + 2xR - 2x^2] = 0$$

$\Rightarrow$ $$2x^2 - 2xR - R^2 = 0$$

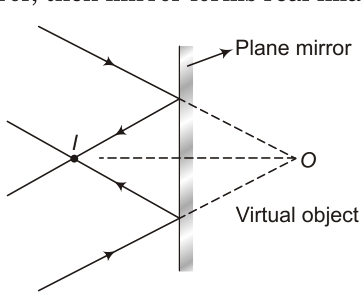
$\because$ $$R \neq 0$$

$\Rightarrow$ $$x = \frac{2R \pm 2\sqrt{3}R}{4} = \frac{[1 \pm \sqrt{3}]}{2}R$$

$\Rightarrow$ $$x = \left(\frac{1 + \sqrt{3}}{2}\right)R$$

■ Objective Questions (Level 1)

1. When convergent beam incident on a plane mirror, then mirror forms real image



2. When an object lies at the focus of a concave mirror $u = -f$ focal length of a concave mirror is negative.

Using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

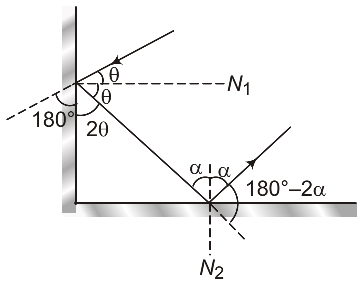
we get,

$$\frac{1}{v} - \frac{1}{f} = -\frac{1}{f} \Rightarrow v = \infty$$

also magnification $m = -\dfrac{v}{u} = \infty$.

Hence, correct option is (c) ∞, ∞.

3. Total deviation, $\delta = \delta_1 + \delta_2$



$$= 180 - 2\theta + 180 - 2\alpha$$

but $\alpha = 90 - \theta$

$\Rightarrow$ $\delta = 180 - 2\theta + 180 - 2(90 - \theta)$

$\Rightarrow$ $\delta = 180°$

Hence, option (a) is correct.

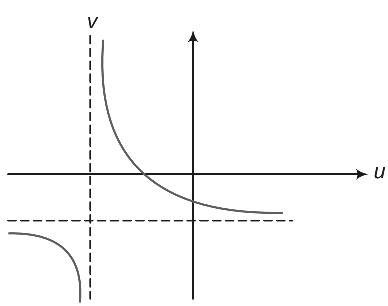
4. A concave mirror cannot from a virtual image of a virtual object.

Hence option (a) is correct.

5. For *a* concave mirror for normal sign convention if $u = -f \Rightarrow v = \infty$

and at $u = -\infty, v = -f$

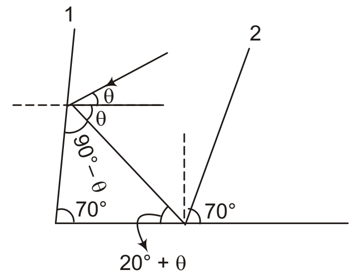
graph between u and v is



The dotted lines are the asymptotes (tangent at ∞) of the curve.

Hence correct option is (b).

6. From figure



$$20 + \theta = 70°$$

$$\theta = 70° - 20°$$

$$\theta = 50°$$

Here (1) and (2) are paralledl 11 to each other.

Hence the correct option is (a) $= 50°$.

7. The radius of curvature of convex mirror

$$R = +60 \text{ cm}.$$

Its focal length $f = \dfrac{R}{2} = +30$ cm

Magnification $m = \dfrac{v}{u} = \dfrac{1}{2}$

$\Rightarrow$ $v = \dfrac{u}{2}$

Using mirror formula, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$,

we get, $-\dfrac{1}{u/2} - \dfrac{1}{u} = \dfrac{1}{30}$

$\Rightarrow$ $\dfrac{-3}{u} = \dfrac{1}{30}$

$\Rightarrow$ $u = -90$ cm

$v = \dfrac{u}{2} = -45$ cm

Hence distance between A and B is

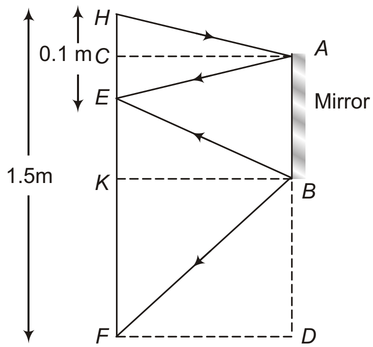
$$= 90 - 45$$

$$= 45 \text{ cm}$$

Hence the correct option is (c).

8. Here it is given that height of the boy

$$HF = 1.5\text{ m}$$



Length of mirror $= AB = 0.75$ m

The ray diagram is shown in above figure.

H is the Head of the boy and F is the feet. It also shows the paths of the rays that leaves the head of the man enter his eyes (E). After reflection from the mirror at point A, and the rays that leave his feet and enter his eyes after reflected at point B.

From figure $CE = \frac{1}{2}HE = 0.05$ m

$$CF = HF - HC = HF = CE$$
$$= 1.50 - 0.05 = 1.45\text{ m}$$

The distance of the bottom edge of mirror above the floor is

$$BP = KF = CF - KC = CD - AB$$
$$= 1.45 - 0.75 = 0.7\text{ m}$$

But according to question $BD = 0.8$ m (given) which is greater than 0.7 m, the height required to see full image. Hence the boy cannot see his feet.

Option (c) is correct

9. Since the image is magnified hence mirror is concave mirror.

Here $\quad m = -\frac{v}{u} = 3 \Rightarrow v = -3u$

$\Rightarrow \quad |v| = |-3u| = 3u$

but $\quad |v - u| = 80$

$\Rightarrow \quad |3u - u| = 80 \Rightarrow \quad u = 40$ cm'

Using mirror formula, we get

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$-\frac{1}{3u} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad f = \frac{-3u}{4}$$

$$\Rightarrow \quad f = \frac{-3 \times 40}{2} = -30 \text{ cm}$$

Mirror is concave and focal length is 30 cm.

Correct option is (a).

10. Here $m = +\frac{1}{n} = -\frac{v}{u}$

$$\Rightarrow \quad v = -\frac{u}{n}$$

From mirror formula $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$,

we get,

$$\frac{1}{f} = \frac{1}{(-u/n)} + \frac{1}{u}$$

$$\Rightarrow \quad u = -(n-1)f$$

Hence the correct option is (d).

11. Differentiating mirror formula, we get

$$\frac{dv}{dt} = \frac{v^2}{u^2}\frac{du}{dt} \quad \left[\because \text{ here } \frac{du}{dt} \text{ is } -\text{ve}\right]$$

Using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

we get $\quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

Here $\quad u = -60$ cm, $f = -24$ cm

Putting these we get, $\quad v = 40$ cm

Hence, $\quad \frac{dv}{dt} = \frac{40^2}{60^2} \times 9 = 4$ cm/s

Hence the speed of the image is 4 cm/s away from the mirror.

Hence correct option is (c).

12. The wrong statement is (d)

13. Let v_m is the speed of mirror, v_p is the speed of particle and v_p is the speed of the observer, then speed of the image measured by observer is given by

$$v_{op} = 2[v_m + v_p] - v_o$$

$$\Rightarrow \quad v_{op} = 2[10 + 4] - 2$$

$$= 28 - 2 = 26 \text{ cm/s}$$

Hence correct option is (d).

■ Assertion and Reason

1. Assertion is wrong since when a virtual object is placed at a distance less than the focal length its real image is formed.

Hence answer is (d).

2. Using mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ we get

$\frac{1}{v} - \frac{1}{20} = \frac{1}{20} \Rightarrow v = 10 \text{ cm}$

ie image is virtual exect and since $m = \frac{v}{u} = \frac{1}{2}$.

Hence image is diminished, thus assertion is true.

If $u = +20$ cm for virtual object $v = \infty$ hence reason is true but reason is not correct explanation of assertion. Hence answer is (b).

3. Using mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ we get

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

If u is front of mirror u is negative and f is negative for concave mirror.

$$\Rightarrow \quad \frac{1}{v} = -\frac{1}{f} + \frac{1}{u} \Rightarrow \quad v = \frac{uf}{f - u}$$

$$\Rightarrow \quad u \to f \Rightarrow v \to \infty$$

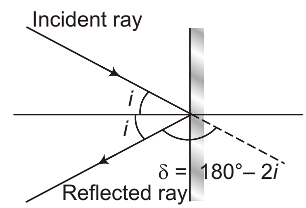
Hence assertion is true also in refractive image and object moves in opposite direction. Hence both assertion and reason are true and reason correctly explain the assertion Correct answer is (a).

4. Real view mirror of vehicles is convex mirror, hence assertion is true. It never makes real image of real object reason is also true but convex mirror is used because since its field of view is greatest. Hence both assertion and reason are true but reason is not correct explanation of assertion. Correct answer is (b).

5. Since $m = -2$ hence it is definitely a concave mirror since only concave mirror form magnified image. Since concave mirror form only real image of real object hence reason is also true. Hence it may true but when object is placed between C and F, $m < 1$.

Hence correct answer be (a) or (b).

6.

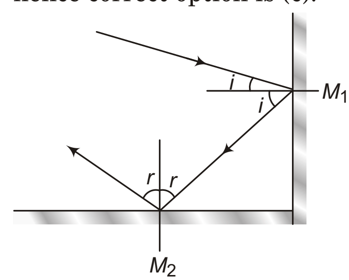


Hence assertion is true.

For normal incidence $i = 0$ hence $\delta = 180°$.

hence assertion is true but reason is false. hence correct option is (c).

7.



Deviation produced by $M_1 = 180° - 2i$
Deviation produced by $M_2 = 180° - 2r$

Total deviation produced $= 360° - 2(i + r)$

But from figure $i + r = 90°$, hence deviation $= 180°$ for any value of i.

Hence assertion is true but reason is false. Correct option is (c).

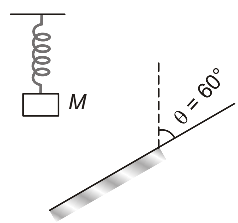
8. The correct option is (b).

9. The correct option is (a, b).

10. The correct option is (b).

■ Objective Questions (Level 2)

1. $v_{\max} = \omega A$



$$\Rightarrow \qquad v_{\max} = \sqrt{\frac{k}{m}}\, A$$

$$\because \qquad \omega^2 = \frac{k}{m} \text{ for SHM}$$

Maximum speed of insect relative to its image

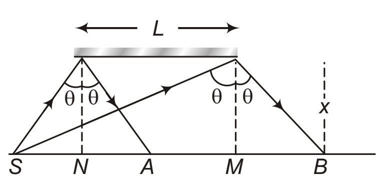
$$= 2v_{\max\perp} = 2v_{\max} \sin 60°$$

$$= A\sqrt{3}\sqrt{\frac{k}{m}}$$

Hence correct option is (c).

2. $au^n = g \downarrow$

Height $= x$



Let after time t paperndicular distance between mirror and source is x' we have from figure

$$AB = AM + MB = SM - SA + MB$$

but $SM = MB$

$$AB = 2MB - SA = 2x' \tan\phi - SA$$

$$= 2x' \tan\phi - 2x' \tan\theta$$

$$\Rightarrow \quad AB = 2x' [\tan\phi - \tan\theta]$$

$$= 2x' \left[\frac{SM}{x'} - \frac{SN}{x'}\right] = 2[SM - SN]$$

$$\Rightarrow \qquad AB = 2 \times L,$$

where $SM - SN = L =$ Length of mirror

$$\Rightarrow \quad \frac{d}{dt}[AB] = \frac{d}{dt}(2L) = 0$$

$\because$ Length of mirror is constant.

Hence the correct option is (d).

3. Here $u = -10$ cm and $v = -20$ cm

Using mirror formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \text{ we get} = \frac{dv}{v^2} - \frac{du}{u^2} = 0$$

$$\Rightarrow \qquad \frac{dv}{du} = -\frac{v^2}{u^2} = -\frac{20^2}{10^2} = -4$$

$$\Rightarrow \qquad dv = -4du$$

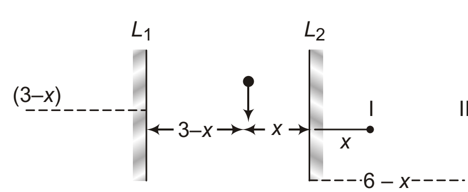
$$\Rightarrow \qquad dv = -4 - (-0.1), \text{ here } du = -0.1$$

$$\Rightarrow \qquad dv = 0.4 \text{ cm},$$

ie, 0.4 cm away from the mirror.

Hence the correct option is (a).

4. The first and second images are shown in figure but according to question

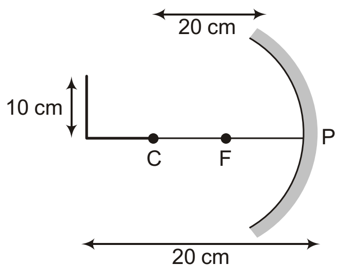


$$(6-x)-x=4$$

$\Rightarrow$ $2=2x \Rightarrow x=1\text{ m}$

Hence the correct option is (c).

5. For vertical part $-\frac{1}{20}+\frac{1}{v}=-\frac{1}{5}$



$\Rightarrow$ $v=-\frac{20}{3}$

$$|m_v|=\left|\frac{v}{u}\right|=\left|\frac{20/3}{20}\right|=\frac{1}{3}$$

$\Rightarrow$ $L_v=\frac{10}{3}\text{ cm}$

For horizontal part first end is at C hence its image is also at C *ie* at $v=-10\text{ cm}$, for other end

$$-\frac{1}{-20}+\frac{1}{v}=-\frac{1}{5}\Rightarrow v=-\frac{20}{3}$$

$\Rightarrow$ $|v|=\frac{20}{3}$

$$L_H=|v-u|=\left|\frac{20}{3}-10\right|=\frac{10}{3}$$

$\Rightarrow$ $L_H=\frac{10}{3}$

The ratio $L_V:L_H=1:1$.

Hence correct option is (c) 1 : 1.

6. Here $u=-15\text{ cm}$, $f=-10\text{ cm}$

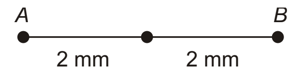
Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

We get, $v=-30\text{ cm}$

We have $m=\frac{v}{m}=\frac{(v_2-v_1)}{u_2-u_1}$

$$=\frac{dv}{du}=2$$

$\Rightarrow$ $dv=2\,du,$



$$AB=du=4\text{ mm}$$

$\Rightarrow$ $dv=2\times 4\text{ mm}\Rightarrow dv=8\text{ mm}$

Hence the correct option is (c).

7. If the mirror is rotated by an angle θ in anticlock, wise direction about an axis $\infty\perp$ to the plane mirror, the new angle of incidence becomes $i-\theta$ and angle of reflection also $i-2\theta$.

According to problem

$$i+i-2\theta=45^\circ$$

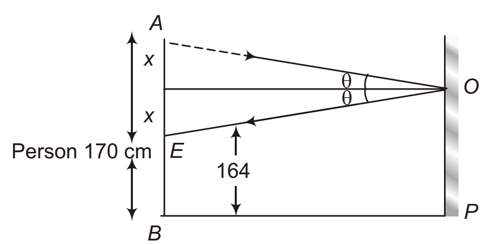
$$2i=45^\circ+2\theta=45^\circ+2\times 20^\circ=85^\circ$$

But angle of incidence = angle of reflection.

Hence the angle between origial incident and reflected ray was 85°. Similarly is the mirror is rotated clockwise the angle became 5°.

Hence correct option is (c) 85° or 5°.

8. The person see his hair if the incident ray statics from point A after reflected by mirror reach his eyes. Let O is point at minimum at a distance x below the point A.



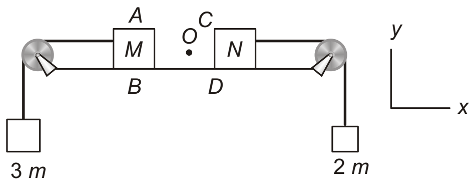
We have $2x=60\text{ cm}\Rightarrow x=3\text{ cm}$

The distance of O from P is

$$=170-3=167\text{ cm}$$

Hence correct option is (a).

9. Acceleration of block



$$a_{AB} = \frac{3mg}{3m+m} = \frac{3g}{4}$$

Acceleration of block CD :

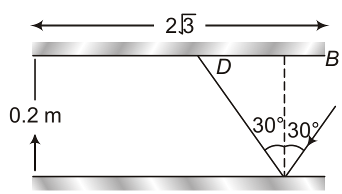
$$a_{CD} = \frac{2mg}{2m+m} = \frac{2g}{3}$$

Since the accelerations are in opposite directions relative acceleration of one image with respect to other is given by

$$a_{AB} + a_{CD} = \frac{3g}{4} + \frac{2g}{13} = \frac{17g}{12}$$

Hence the correct option is (c).

10. Here $\dfrac{BD}{0.2} = \tan 30°$

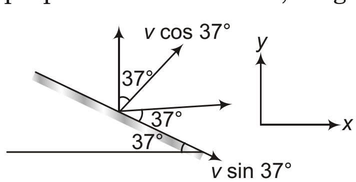


$$\Rightarrow \qquad BD = 0.2 \times \frac{1}{\sqrt{3}}$$

No. of reflections $= \dfrac{2\sqrt{3}}{0.2/\sqrt{3}} = 30$

Hence, the correct option is (b).

11. Resolving velocity along parallel to mirror and perpendicular to mirror, we get



$v_{\|} = v \sin 37°$ and $v_{\perp} = v \cos 37°$

From figure, we get

$$v_x = v \cos 37° \sin 37° + v \sin 37° \cos 37°$$

$$= 2v \cos 37° \sin 37°$$

$$v_x = 2 \times 5 \times \frac{4}{5} \times \frac{3}{5} = \frac{24}{5} = 4.8$$

$$v_y = v \cos 37° \times \cos 37°$$

$$- v \sin 37° \times \sin 37°$$

$$v_y = v\,[\cos^2 37° - \sin^2 37°]$$

$$= 5 \times \left(\frac{4}{5} + \frac{3}{5}\right)\left(\frac{4}{5} - \frac{3}{5}\right)$$

$$= 5 \times \frac{7}{5} \times \frac{1}{5} = \frac{7}{5} = 1.4$$

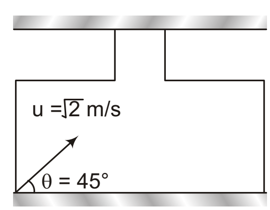
Hence velocity of image is given by

$$\vec{\mathbf{v}} = v_x\,\hat{\mathbf{i}} + v_y\,\hat{\mathbf{j}}$$

$$\Rightarrow \qquad \vec{\mathbf{v}} = 4.8\,\hat{\mathbf{i}} + 1.4\,\hat{\mathbf{j}}$$

Hence the correct option is (c).

12. Since elevator start falling freely, the relative acceleration of the particle in elevator frame $= g - g = 0$



Hence, in elevator frame path of the particle is a straight line.

The vertical component of velocity is

$$u \sin 45° = \sqrt{2} \times \frac{1}{\sqrt{2}} = 1 \text{ m/s}$$

The separation between mirror and particle in 0.5 s is

$$y = v_y t = 1 \times 0.5 = 0.5 \text{ m}$$

The separation between image of particle and particle at this moment

$$= 2y = 2 \times 0.5 \text{ m} = 1 \text{ m}$$

Hence the correct option is (b).

13. Here velocity of mirror

$$\vec{\mathbf{v}}_m = 4\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} + 8\,\hat{\mathbf{k}}$$

and velocity of object

$$\vec{\mathbf{v}}_o = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} + 5\,\hat{\mathbf{k}}$$

Since $\hat{\mathbf{k}}$ is normal to the mirror hence $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ components of image velocity remain unchanged *ie*, velocity of image can be written as

$$\vec{\mathbf{v}}_i = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} + v_{iz}\,\hat{\mathbf{k}}$$

but $\quad v_{iz} = 2u_{mz} - v_{oz} = 2 \times 8 - 5 = 11$

Hence, we get

$$\vec{\mathbf{v}}_i = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}} + 11\,\hat{\mathbf{k}} \text{ (\textit{wrt} ground)}$$

Hence, the correct option is (b).

$$3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$$

14. Only option (b) satisfy the given condition.

Here $X_0 = -2$, $X_i = +10$

Using $\dfrac{1}{X_0} + \dfrac{1}{X_i} = \dfrac{1}{f}$

we get

$$\frac{1}{10} - \frac{1}{2} = \frac{1}{f}$$

$\Rightarrow \qquad f = -2.5 \text{ cm}$

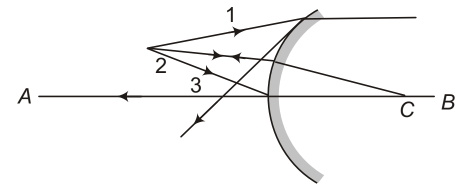
Hence, the mirror is concave.

We know that $y_i = \dfrac{fy_0}{f - x_0}$

$$= \frac{-2.5 \times 1}{-2.5 + 2} = 5 \text{ cm}$$

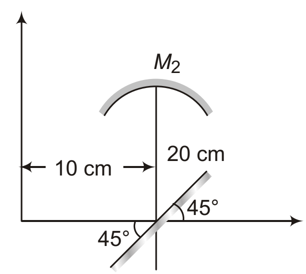
Hence, the correct option is (b).

16. There are two mistakes one in ray (1) and other in ray (3).



Hence correct option is (b).

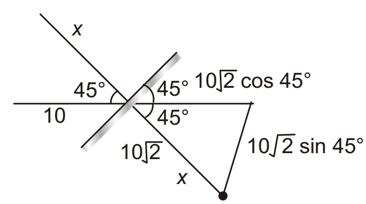
17. The image formation by plane mirror is shown as



$$\frac{10}{x} = \sin 45° \quad \Rightarrow x = 10\sqrt{2}$$

The x-coordinate is $10\sqrt{2} \cos 45° = 10$

and y-coordinate is $-10\sqrt{2} \sin 45° = -10$



Hence, the convert option if (c), (10, – 10).

18. $x_i = \dfrac{fx_0}{x_0 - f} = \dfrac{-10 \times 10}{-10 - 10} = +5 \text{ cm}$

For concave mirror $f = -10$ cm.

$$y_i = \frac{fy_0}{f - x_0} = \frac{-10 \times -20}{-10 - 10} \text{ cm}$$

$$= +10 \text{ cm}$$

Hence the coordinates of image are (5, 10).

Therefore, the correct option is (d).

19. For convex mirror $f = +10$ cm

$$x_i = \frac{fx_0}{x_0 - f} = \frac{10 \times 10}{10 - 10} = \infty$$

$$y_i = \frac{fy_0}{f - y_0} = \frac{10 \times -20}{10 - 10} = -\infty$$

Hence the correct option is (d).

20. It concave mirror is replaced by plane mirror the coordinates are (0, + 40).

Hence the correct option is (d).

More than one options are correct

1. Here $f = -20$ cm

Case 1. (if image is real) u, v and f all are –ve.

Here $m = 2 \Rightarrow v = -2u$

using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

we get, $$-\frac{1}{2u} - \frac{1}{u} = -\frac{1}{20}$$

$$\Rightarrow \quad \frac{3}{2u} = \frac{1}{20} \Rightarrow u = +30 \text{ cm}$$

Case 2. (if image is virtual)

u and f are –ve, while v is +ve

$$\Rightarrow \quad \frac{1}{2u} - \frac{1}{u} = -\frac{1}{20}$$

$$\Rightarrow \quad u = +10 \text{ cm}$$

Hence possible values of u are 10 cm, 30 cm.

The correct options are (a) and (b).

2. Magnitude of focal length spherical mirror is f and linear magnification is $\frac{1}{2}$

Since concave mirror fro inverted real image and magnification is less than unity, therefore $u > 2f$.

Hence option (a) is correct.'

If image is erect than it is a convex mirror.

Let mirror is concave hence focal length $= -f$.

Here $m = \frac{1}{2} = -\frac{v}{u}$

$$\Rightarrow \quad v = -\frac{u}{2}$$

Using mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, we get

$$-\frac{1}{u/2} - \frac{1}{u} = -\frac{1}{f}$$

$$\Rightarrow \quad \frac{-3}{u} = -\frac{1}{f}$$

$$\Rightarrow \quad u = -3f$$

Hence, if the mirror is concave the object distance will be $3f$.

Let mirror is convex, then

$$m = \frac{v}{u} = \frac{1}{2} \Rightarrow u = \frac{v}{2}$$

Using mirror formula, we get

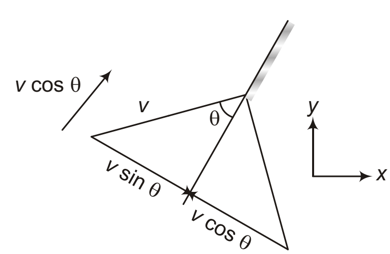
$$\frac{1}{u/2} - \frac{1}{u} = \frac{1}{f} \Rightarrow u = f$$

Hence, if mirror is convex the object distance will be f.

Hence correct options are (a), (b), (c) and (d).

3. Since by a plane mirror

speed of image = speed of object



Hence speed of image also v.

Horizontal component (along mirror)

$$= v \cos\theta$$

Vertical component ($\perp$ to mirror)

$$= v \sin\theta$$

Hence image velocity also make an angle θ with the mirror.

Resolving velocity along (y-axis *ie*, parallel to mirror) and (x-axis *ie* perpendicular to mirror).

$$\vec{\mathbf{v}}_0 = v \sin \hat{\mathbf{i}} + v \cos \hat{\mathbf{j}}$$

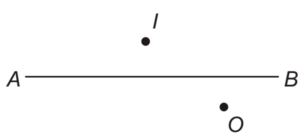
$$\vec{\mathbf{v}}_i = -v \sin \hat{\mathbf{i}} + v \cos \hat{\mathbf{j}}$$

Relative velocity of object w.r.t. image is

$$\vec{\mathbf{v}}_{0i} = \vec{\mathbf{v}}_0 - \vec{\mathbf{v}}_i = 2v \sin\theta\, \hat{\mathbf{i}}$$

Hence, correct options are (a), (b) and (d).

4.



As image is on opposite side of the principle axis (inverter image) hence the mirror is concave because convex mirror always form erect image.

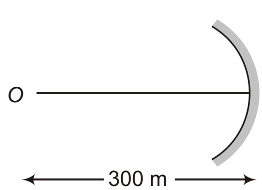
The mirror is lying to the right of O and the O lies between C and F.

If centre of curvature lies to the right hand side of O then $v < u$.

Hence, this option is incorrect.

Hence, the correct options are (a), (b) and (d).

5. Here $f = -20$ cm, $u = -30$ cm



Using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

we get,

$$v = -60$$

Different this w.r.t. time, we get

$$-\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$$

$$\Rightarrow \quad \frac{dv}{dt} = -\frac{v^2}{u^2}\left(\frac{du}{dt}\right)$$

Hence in event (1),

$$\frac{du}{dt} = -v$$

$$\Rightarrow \quad \frac{dv}{dt} = -\frac{60^2}{30^2} \times -v = 4v$$

Hence, speed of image in event (1) is $4v$.
after time y coordinate of object $y_0 = vt$

but $x_0 = -30$

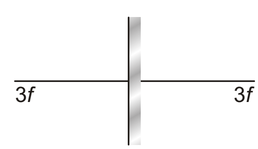
then $\quad y_i = \left|\frac{fy_0}{f - x_0}\right| = \left|\frac{-20 \times vt}{-20 + 30}\right|$

$$y_i = |-2vt| = 2vt$$

$$\Rightarrow \quad \frac{dy_i}{dt} = +2v$$

Hence, option (b) and (c) are correct.

6. For plane mirror



$$u = 3f \Rightarrow v = 3f$$

For concave mirror

$$u = -3f$$

Using mirror formula

$$\frac{1}{v} = \frac{1}{3f} - \frac{1}{f} = -\frac{2}{3f}$$

$$\Rightarrow \quad v = -1.5f$$

$$\Rightarrow \quad |v| = 1.5f$$

For convex mirror,

$$\frac{1}{v} = \frac{1}{3f} + \frac{1}{f} = \frac{4}{3f}$$

$\Rightarrow \qquad v = 0.75f$

Hence maximum distance in event (1) if image is from plane mirror and minimum distance from convex mirror

When $\quad v = 1.5f$, then $v = 1.5f$

by plane mirror

For concave mirror

$$\frac{1}{u} = \frac{1}{1.5f} - \frac{1}{f} = \frac{2}{3f} - \frac{1}{f}$$

$$= \frac{2-3}{3f} = -\frac{1}{3f}$$

$\Rightarrow \qquad v = -3f$

$\Rightarrow \qquad |v| = 3f$

For convex mirror

$$\frac{1}{v} = \frac{2}{3f} + \frac{1}{f} = \frac{5}{3f}$$

$\Rightarrow \qquad v = 0.6f$

Hence, in event (2) maximum distance of image from the concave mirror.

Hence, correct options are (a), (b) and (c).

Match the Columns

1. (a) $m = -2$, since $|m| = 2 > 1$.

Therefore mirror is concave and $\because m$ is –ve.

Hence image is real [for concave mirror m is = – ve]

Therefore,

(a) $\rightarrow$ q, r

(b) Since $m = -\frac{1}{2}$, $\because m$ is – ve

Hence mirror is concave and image is real.

(b) $\rightarrow$ q, r

(c) $m = +2$, $\because m > 1$

Hence mirror is concave and $\because m$ is + ve

Hence image is virtual.

(c) $\rightarrow$ q, s

(d) $\because 1\text{ m} = +\frac{1}{2} < 1$ and + ve

Hence the mirror is convex and image is virtual.

(d) $\rightarrow$ p, s

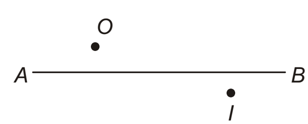
2. Plane mirror (for virtual object) $\rightarrow$ only real image

$\Rightarrow$ (a) $\rightarrow$ p

(b) $\rightarrow$ r

(c) $\rightarrow$ p

3. (a) Since object and its image are on opposite side of principle axis.



Hence mirror is concave

$\Rightarrow$ (a) $\rightarrow$ r.

(b) Similarly as for option (a).

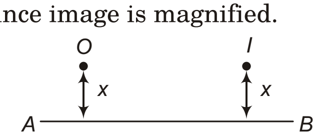
(b) $\rightarrow$ r

(c) Since image and object are of same height from AB.

Hence mirror is plane mirror.

(c) $\rightarrow$ p

(d) Since image is magnified.



Hence mirror is concave [D is. distance between O and mirror is less than the focal length].

Hence

(d) $\rightarrow$ r.

4. (a) For concave mirror M_1 focal length

$$= -20 \text{ cm}$$

When $x = 20$ cm, Mirror is M_1

$$v = \infty \text{ and magnified}$$

(a) $\rightarrow$ p, s

(b) For convex mirror M_2 of focal length $+20$ cm if X (distance of object from pole) $= 20$

Using mirror formula $\frac{1}{v} + \frac{1}{v} = \frac{1}{f}$

we get

$$\frac{1}{v} = \frac{1}{20} + \frac{1}{20} = \frac{1}{10}$$

$$\Rightarrow \quad v = 10 \text{ cm}$$

Hence image is virtual.

(b) $\rightarrow$ r

(c) $u = -30$ cm, $f = -20$ cm

$$\frac{1}{v} = \frac{1}{30} - \frac{1}{20} = \frac{2-3}{60} = -\frac{1}{60}$$

$$v = -60 \text{ cm}$$

Hence image is real.

$$m = -\frac{60}{30} = -2$$

Hence image is magnified (2 times).

$\Rightarrow$ (c) $\rightarrow$ q, s

(d) for mirror M_2 (convex) at $X = +30$ cm

image again virtual.

(d) $\rightarrow$ r

5. (a) For concave mirror $f = -20$ cm

Case I. Image is real.

$$m = 2 = -\frac{v}{u} \Rightarrow v = -2u$$

Using $\quad \frac{1}{u} - \frac{1}{u} = \frac{1}{f}$

we get, $\quad -\frac{1}{2u} - \frac{1}{u} = -\frac{1}{20}$

$$\Rightarrow \quad \frac{3}{2u} = \frac{1}{20}$$

$$\Rightarrow \quad u = 30 \text{ cm}$$

If image is virtual $v = 2v$

$$\frac{1}{2u} - \frac{1}{u} = -\frac{1}{20}$$

$$\Rightarrow \quad u = 10 \text{ cm}$$

Hence correct option are as

(a) $\rightarrow$ p, q

(b) Here $m = \frac{1}{2} < 1$

Hence image is real.

$$\Rightarrow \quad \frac{1}{2} = -\frac{v}{u} \rightarrow v = -\frac{u}{2}$$

Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$-\frac{1}{u/2} - \frac{1}{u} = -\frac{1}{20}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{1}{20} \Rightarrow u = 60 \text{ cm}$$

Hence correct option is none of these.

$\Rightarrow$ (b) $\rightarrow$ s

(c) if $m = 1$, than $u = 2f$

$$u = -40 \text{ cm}$$

Hence correct option is none of these.

$\Rightarrow$ (c) $\rightarrow$ (s)

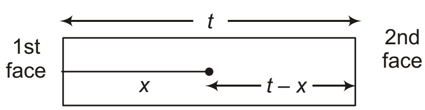
(d) Similarly as in part (b) we see that answer is none of these.

$\Rightarrow$ (d) $\rightarrow$ (s)

27 Refraction of Light

Introductory Exercise 27.1

1. Let real depth of dust particle is x and thickness of slab is t



From Ist surface

$$\mu = \frac{\text{Real depth}}{\text{App. depth}}$$

$$1.5 = \frac{x}{6\text{ cm}} \Rightarrow x = 9\text{ cm} \qquad \ldots\text{(i)}$$

From other face

$$\mu = \frac{t - x}{4} \Rightarrow t - x = 4 \times 1.5$$

$$\Rightarrow \qquad t = x + 6 = 9 + 6 = 16\text{ cm}$$

2. $${}_1\mu_2 = \frac{4}{3} \Rightarrow \frac{\mu_2}{\mu_1} = \frac{4}{3} \qquad \ldots\text{(i)}$$

$${}_2\mu_3 = \frac{3}{2} \Rightarrow \frac{\mu_3}{\mu_2} = \frac{3}{2} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\mu_3}{\mu_1} = \frac{4}{3} \times \frac{3}{2} = 2$$

3. Frequency remain same.

Let v_1 is velocity in medium (1) and v_2 in Medium (2)

We have

$$\mu_1 = \frac{c}{v_1} \text{ and } \mu_2 = \frac{c}{v_2}$$

$$\Rightarrow \qquad \frac{\mu_1}{\mu_2} = \frac{v_2}{v_1} \Rightarrow v_2 = \frac{\mu_1}{\mu_2} v_1$$

Similarly, wavelength $\lambda_2 = \frac{\mu_1}{\mu_2} \lambda_1$

4. From $v_a = n \lambda_a$

$$\Rightarrow \qquad \lambda_a = \frac{v_a}{n_a} = \frac{3 \times 10^8}{6 \times 10^{+14}} = 5 \times 10^{-7}\text{ m}$$

$$= 50\text{ nm}$$

$$\mu = \frac{\lambda_a}{\lambda_m} = \frac{500}{300} = \frac{5}{3} = 1.67$$

Introductory Exercise 27.2

1. Since light rays are coming from glass to air applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \qquad \frac{1}{v} - \frac{1.5}{10} = \frac{1 - 1.5}{-1.5}$$

$$\Rightarrow \qquad \frac{1}{v} = -\frac{1}{30} + \frac{1.5}{10} = \frac{-1 + 4.5}{30}$$

$$\Rightarrow \qquad v = \frac{30}{3.5} = 8.57\text{ cm}$$

2. $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

(a) $$\frac{1.5}{\mu} - \frac{1}{(-20)} = \frac{0.5}{6}$$

On solving $v = 45$ cm

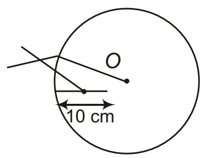
(b) $$\frac{1.5}{v} - \frac{1}{(-10)} = \frac{0.5}{6}$$

On solving we get $v = -90$ cm

(c) $\frac{1.5}{v} - \frac{1}{(-3)} = \frac{0.5}{6}$

On solving $v = -6.0$ cm

3. Light rays are coming from glass to air



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{v} - \frac{4}{3(-10)} = \frac{1 - 4/3}{(-15)}$$

$\frac{1}{v} + \frac{4}{30} = \frac{1}{45}$ on solving $v = -9$ cm

4. Applying $\frac{u_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \quad \frac{1.44}{v} - \frac{1}{\infty} = \frac{0.44}{1.25}$$

On solving $v = 0.795$ cm

5. $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\frac{1.635}{v} - \frac{1}{(-9)} = \frac{0.635}{(-2.50)}$$

on solving $v = 6.993$ cm

Lateral magnification $m = -\frac{v}{u}$

$$\Rightarrow \quad = -\frac{6.993}{9} = -0.777$$

Introductory Exercise 27.3

1. We have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\Rightarrow \quad -\frac{1}{20} + \frac{1}{60} = (1.65 - 1)\left[-\frac{1}{R} - \frac{1}{R}\right]$$

$$\Rightarrow \quad \frac{-3 + 1}{60} = 0.65 \times \frac{-2}{R}$$

$$\Rightarrow \quad R = 60 \times 0.65 = 39 \text{ cm}$$

2. Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{-50} - \frac{1}{x} = \frac{1}{30} \Rightarrow -\frac{1}{x} = \frac{1}{30} + \frac{1}{50}$$

On solving $x = -18.75$ cm

$$m = \frac{-v}{u} = \frac{50}{18.75}$$

Height of filament image $= 2 \times \frac{50}{18.75}$

$$= 5.3 \text{ cm}$$

3. $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R} + \frac{1}{R}\right]$

If lens faces becomes opposite three is no change in radius of curvature hence focal length does not change.

4. Using formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ when $u \to 0_1\ v \to 0$

when $u \to f, v \to \infty$ hence image moves from surface to ∞.

5. $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$$\Rightarrow \quad \frac{1}{f} = (1.3 - 1)\left[\frac{-1}{R} - \frac{1}{R}\right] = 0.3 \times \frac{-2}{20}$$

$$\Rightarrow \quad f = -\frac{100}{3} \text{ cm}$$

(a) When immersed in a liquid of 1.8 refractive index

$$\frac{1}{f_1} = \left(\frac{1.3}{1.8} - 1\right)\left[\frac{-2}{R}\right] = \frac{-0.5}{1.8} \times \frac{-2}{20}$$

$f' = 36$ cm

(b) The minimum distance is equal to the focal length = 36 cm

6. Using $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{v}-\frac{1}{(-20)}=\frac{12}{10}$$

On solving $v=20$ cm

Magnification $=-\frac{v}{u}=-1$

Hence the image of same size and inverted. Let the distance between second lens is x Since magnification is unity image distance also x using again

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

we get

$$\frac{1}{x}-\frac{1}{(-x)}=\frac{1}{f}=\frac{1}{10}\Rightarrow x=20\text{ cm}$$

Hence the distance between two lenses

$$=20\text{ cm}+20\text{ cm}=40\text{ cm}$$

7. $\frac{1}{v}=\frac{1}{f}+\frac{1}{u}$ ...(i)

$$\frac{1}{v_1}=\frac{1}{f}+\frac{1}{u+du} \quad \text{...(ii)}$$

$$\frac{1}{v'}-\frac{1}{v}=\frac{(u+du-u)}{(u+du)u}$$ on solving, we get

$$\frac{v-v'}{v'}=\frac{du}{u(u+du)}$$ thickness $dv=\frac{-v^2}{u^2}du$

10. Size of image $=\sqrt{6\times\frac{2}{3}}=2$ cm.

11. Let image distance is u

$$|m|=3\Rightarrow v=3u$$

$$\frac{1}{3u}+\frac{1}{u}=\frac{1}{12}\Rightarrow v=16\text{ cm}$$

12. Since image is upright and diminished hence lens is concave. Now

$$u-v=20 \quad \text{...(i)}$$

$$m=\frac{v}{u}=\frac{1}{2}$$

$$\Rightarrow \quad \frac{1}{2}=\frac{u-20}{u}$$

$\Rightarrow$ $u=40$ cm and $v=20$ cm

$\Rightarrow$ Using $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$-\frac{1}{20}+\frac{1}{40}=\frac{1}{f}\Rightarrow f=-40\text{ cm}$$

13. The image coincide itself if light falls normally on plane mirror hence object must be on focus *i.e.* + 10 cm.

8. $\frac{1}{v}+\frac{1}{u}=\frac{2(\mu_2/\mu_1)}{R_2}-\frac{2(\mu_2/\mu_1-1)}{R_1}$

$$\frac{1}{v}+\frac{1}{0.2}=\frac{2(4/3)}{0.4}-\frac{2(4/3-1)}{0.4}$$

On solving $v=12$ cm

9. Since shift in position $\Delta t=0.1$ m

Hence real depth $=(0.1+0.2)$ m

$=0.3$ m

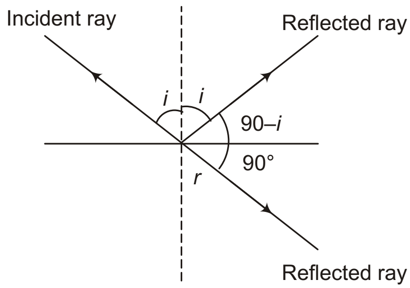
and apparent depth $=0.2$ m

$$\mu=\frac{\text{real depth}}{\text{apparent depth}}$$

$$=\frac{0.3}{0.2}=1.5$$

AIEEE Corner

1.



We have $r + 90° + i = 180° \Rightarrow r = 90 - i$

From Snell's law $1.5 = \dfrac{\sin i}{\sin r} = \dfrac{\sin i}{\sin(90 - i)}$

$\Rightarrow \quad \tan i = 1.5$

$\Rightarrow \quad i = \tan^{-1}(1.5)$

2. $n_w = \dfrac{v_{\text{air}}}{u_w} = \dfrac{343}{1498} = 0.229$

Critical angle $\theta = \sin^{-1}(0.229) = 13.2°$

3. Sped in glycrine $v_g = \dfrac{c}{n_g} = \dfrac{3 \times 10^8}{1.47}$

$$t_1 = \frac{20}{v_g} = \frac{20 \times 1.47}{3 \times 10^8} = 9.8 \times 10^{-8}\,\text{s}$$

Speed in glycrine $v_g = \dfrac{c}{n_g} = \dfrac{3 \times 10^8}{1.63}$

$$t_2 = \frac{20}{v_c} = \frac{20 \times 1.63}{3 \times 10^8} \simeq 10.8 \times 10^{-8}$$

$$t_2 - t_1 = (10.86 - 9.8) \times 10^{-8}$$
$$= 1.67 \times 10^{-8}\,\text{s}$$

4. (a) $t_1 = \dfrac{1 \times 10^{-6}\text{ m}}{v_1} = \dfrac{1 \times 10^{-6}\text{ m}}{3 \times 10^8/1.2}$

$$= \frac{1.2 \times 10^{-6}}{3 \times 10^8}$$

$\Rightarrow \quad t_1 = 0.4 \times 10^{-14}\,\text{s}$

$$t_2 = \frac{1.5 \times 10^{-6}}{3 \times 10^8} = 0.5 \times 10^{-14}\text{ s}$$

$$t_3 = \frac{1 \times 10^{-6}}{3 \times 10^8/1.8} = \frac{1.8 \times 10^{-6}}{3 \times 10^8} = 0.6 \times 10^{-14}$$

Hence t_1 is least and $t_1 = 0.4 \times 10^{-14}$ s

(b) Total number of wavelengths

$$= \frac{1\,\mu\text{m}}{\lambda/n_1} + \frac{1.5\,\mu\text{m}}{\lambda/n_2} + \frac{1\,\mu\text{m}}{\lambda/n_3}$$
$$= \frac{1000 \times 1.2\text{ nm}}{600\text{ nm}} + \frac{1.5 \times 100\text{ nm}}{600\text{ nm}}$$
$$+ \frac{1 \times 1.8 \times 1000\text{ nm}}{600\text{ nm}}$$
$$= \frac{4500}{600} = 7.5$$

5. The given wave equation is

$$E_x(y, t) = E_{ax} \sin\left[\frac{2\pi y}{5 \times 10^{-7}} - 3 \times 10^{14} \times 2\lambda t\right]$$

Comparing with standard equation

$$E_x(y, t) = E_0 \sin[ky - \omega t]$$
$$k = \frac{2\pi}{5 \times 10^{-7}}, \ \omega = 2\pi \times 3 \times 10^{14}$$
$$v = \frac{\omega}{k} = \frac{2\pi \times 3 \times 10^{14}}{2\pi/5 \times 10^{-7}} = 1.5 \times 10^8 \text{ m/s}$$

Refractive index $n = \dfrac{c}{v} = \dfrac{3 \times 10^8}{1.5 \times 10^8} = 2$

Wavelength in this way $\lambda_n = \dfrac{2\pi}{k}$

$\Rightarrow \quad \lambda_n = \dfrac{2\pi}{25/5 \times 10^7} = 5 \times 10^{-7}\,\text{m}$

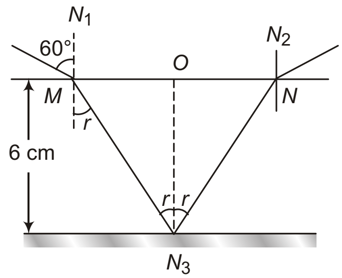
$\Rightarrow \quad \lambda_n = 500\,\text{nm}$

If vacuum, wavelength is λ then

$$\lambda_n = \frac{\lambda}{n}$$

$\Rightarrow \quad \lambda = n\lambda_n = 2 \times 500 = 1000\,\text{nm}$

6. Refraction from plane and spherical surfaces



We have $\dfrac{\sin 60^\circ}{\sin r} = 1.8$

$$\Rightarrow \quad \sin r = \frac{\sin 60^\circ}{1.8}$$

$$\Rightarrow \quad \sin r = \frac{\sqrt{3}}{2 \times 1.8} = 0.48$$

$$\Rightarrow \quad r = \sin^{-1}(0.48)$$

$$\Rightarrow \quad r \simeq 28.7^\circ$$

Now $\quad \dfrac{MO}{6} = \tan r$

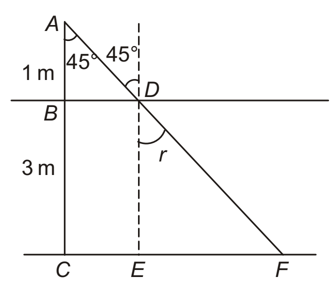
$$\Rightarrow \quad MO = 6 \tan r$$

Similarly $ON = 6 \tan r$

$\Rightarrow MN = MO + ON = 12 \tan r = 12 \tan(28.7^\circ)$

$\Rightarrow MN = 6.6\,\text{cm}$

7. From Snell's law $\dfrac{4}{3} = \dfrac{\sin 45^\circ}{\sin r}$



Solving we get $r = 32^\circ$

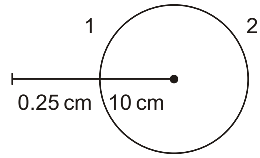
$$EF = DE \tan r = 3 \tan 32^\circ$$

$$= 1.88\,\text{m}$$

Total length of shadow $= 1 + 1.88$

$$= 2.88\,\text{m}$$

8. The situation is shown in figure



For first surface $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \quad \frac{1.5}{v} - \frac{1}{(-2.5)} = \frac{0.5}{10}$$

$$\Rightarrow \quad \frac{1.5}{v} = \frac{1}{20} - \frac{1}{2.5} = -\frac{7}{20}$$

$$\Rightarrow \quad v = -\frac{30}{7}\,\text{cm}$$

This image acts as a virtual object for 2nd surface

$$u_2 = -\left(20 + \frac{30}{7}\right) = -\frac{170}{7}\,\text{cm}$$

and $\quad R = -10\,\text{cm}$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r}$$

$$\Rightarrow \quad \frac{1}{v} + \frac{1.5}{170/7} = \frac{-0.5}{-10}$$

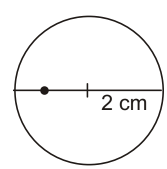
$$\Rightarrow \quad \frac{1}{v} = \frac{1}{20} - \frac{10.5}{170}$$

$$\Rightarrow \quad v = -85\,\text{cm}$$

Hence final image will produced at $-65\,\text{cm}$ from Ist surface.

9. Here $v = -1\,\text{cm}$

$R = -2\,\text{cm}$



Applying $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \frac{1}{-1} - \frac{1.5}{x} = \frac{1 - 1.5}{-2} = \frac{-0.5}{-2} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{-1.5}{x} = \frac{5}{4}$$

$$\Rightarrow \quad x = \frac{-6}{5} = -1.2\,\text{cm}$$

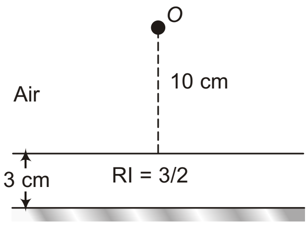
10. 

Image formed by refection acts the virtual object for the mirror.

Here shift $= t\left(1 - \frac{1}{\mu}\right)$

$$= 3\left(1 - \frac{1}{3/2}\right) = 1 \text{ cm}$$

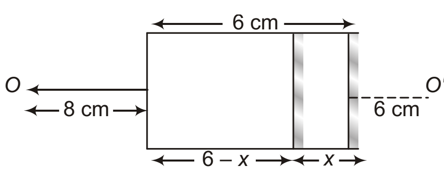
Hence object appear to the mirror

$$= (10 + 1) \text{ cm}$$

$$= 11 \text{ cm}$$

The image formed by mirror $= -11$ cm

Hence image formed by the mirror at 11 cm behind the mirror.

11. 

Step. Let shift in mirror is x then the distance of object.

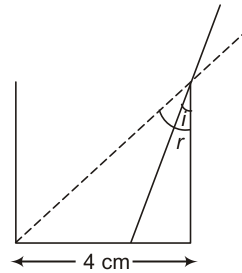
From the mirror is $= 8 + (6 - x)$.

Step II. Plane mirror form image behind the mirror at same distance as the distance object from mirror hence

$$8 + (6 - x) = x + 6 \Rightarrow x = 4 \text{ cm}$$

Step III. $\mu = \frac{\text{real depth}}{\text{app. depth}} = \frac{6}{4} = 1.5$

hence real position of the bubble inside sphere is 1.2 cm from the surface.

12. 

Here $\sin r = \frac{4}{\sqrt{4^2 + n^2}}$

$$\sin i = \frac{2}{\sqrt{2^2 + n^2}}$$

$$\Rightarrow \quad u = \frac{\sin i}{\sin r} \Rightarrow \frac{4}{3} = \frac{4\sqrt{2^2 + n^2}}{2\sqrt{4^2 + n^2}}$$

$$\Rightarrow \quad \frac{4}{9} = \frac{4 + n^2}{16 + n^2} \Rightarrow 5n^2 = 28$$

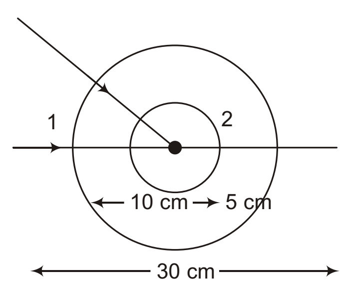
$$\Rightarrow \quad n^2 = \sqrt{\frac{28}{5}} \Rightarrow n = 2.4 \text{ cm}$$

13. Using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\frac{1}{v} - \frac{1.5}{\infty} = \frac{1 - 1.5}{-5}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{10} \Rightarrow v = 10 \text{ cm}$$

14. For first surface $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$



$$\Rightarrow \quad \frac{1.5}{v} - \frac{1}{\infty} = \frac{0.5}{10} \Rightarrow v = 30 \text{ cm}$$

For 2nd surface

$$\frac{1}{v} - \frac{1.5}{(-15)} = \frac{-0.5}{5}$$

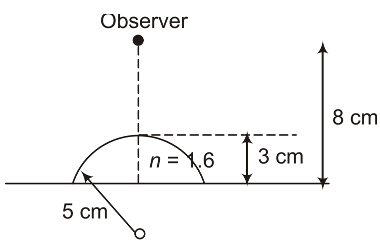
$$\Rightarrow \quad \frac{1}{v} = -\frac{1}{10} - \frac{1}{10} \Rightarrow v = -5 \text{ cm}$$

Hence the distance from first face is

$$= (10 + 5) \text{ cm}$$

$$= 15 \text{ cm}$$

15. Since rays goes from paper weight ($n = 1.6$) to air hence



$$\frac{1}{v} - \frac{1.6}{(-3)} = \frac{1-1.6}{5}$$

$$\Rightarrow \quad \frac{1}{v} = -\frac{0.6}{5} - \frac{1.6}{3}$$

On solving we get $v = -0.58 \Rightarrow |v| = 0.58$ cm

Hence the distance between observer and table top is $= (8 - 0.58)$ cm $= 7.42$ cm.

16. Let real velocity of bird $= v_B$ cm/s

Velocity of bird w.r.t. fish = 16 cm/s

Velocity of bird w.r.t. water $= \mu v_B$

But $\mu v_B + v_f = 16$ cm/s

Here $v_f = 4$ cm/s

$$\Rightarrow \quad \left(\frac{4}{3} v_B + 4\right) = 16$$

$$\Rightarrow \quad \frac{4}{3} v_B = 12 \text{ cm/s}$$

$$\Rightarrow \quad v_B = 9 \text{ cm/s}$$

17. Let the distance between the object and screen is d and let distance between object and lens is x

Using lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we get

$$\frac{1}{d-x} - \frac{1}{(-x)} = \frac{1}{16}$$

$$\Rightarrow \quad x^2 - xd + 16d = 0$$

$$\Rightarrow \quad x = \frac{d \pm \sqrt{d^2 - 64d}}{2}$$

Let $\quad x_1 = \dfrac{d + \sqrt{d^2 - 64d}}{2}$

and $\quad x_2 = \dfrac{d - \sqrt{d^2 - 64d}}{2}$

$$\Rightarrow \quad x_1 - x_2 = \sqrt{d^2 - 64d}$$

But $x_1 - x_2 = 60 \Rightarrow d^2 - 64d - 3600 = 0$

$$\Rightarrow \quad (d - 100)(d + 36) = 0$$

$$\Rightarrow \quad d = 100 \because d \neq -36$$

Hence the distance between object and screen is 100 cm.

18. $\because \frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

For identical double convex lens

$$|R_1| = |R_2| = R$$

but $\quad R_1 = R$ and $R_2 = -R$

$$\Rightarrow \quad \frac{1}{f} = (n-1)\left[\frac{1}{R} + \frac{1}{R}\right] = (n-1)\frac{2}{R}$$

$$\Rightarrow \quad f = \frac{R}{2(n-1)} \text{ hence } f_1 = \frac{R}{2(1.5-1)} = R$$

and $f_2 = \dfrac{R}{2(1.7-1)} = \dfrac{R}{1.4}$

(a) $\Rightarrow \dfrac{f_1}{f_2} = 1.4 \Rightarrow f_1 : f_2 = 1.4 : 1$

(b) For first lens

$$f_i = \frac{R}{2\left[\frac{1.5}{1.6} - 1\right]} = -\frac{1.6R}{2 \times 1} = -8R$$

hence first lens become concave (diverging)

For 2nd lens

$$f_2 = \frac{R}{2\left[\frac{1.7}{1.6} - 1\right]} = \frac{1.6R}{2 \times 0.1} = 8R$$

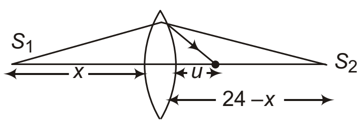
Hence 2nd lens remain **convex.**

19. Here $u = -10\,\text{cm}$ acts the virtual object for the lens $v = -15\,\text{cm}$ using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \qquad \frac{1}{-15} + \frac{1}{10} = \frac{1}{f}$$

On solving we get $f = -30\,\text{cm}$.

20. Situation is shown in figure.



For Ist source direction left to right is + ve

Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v} - \frac{1}{(-x)} = \frac{1}{9} \Rightarrow \frac{1}{v} + \frac{1}{x} = \frac{1}{9} \qquad \ldots\text{(i)}$$

For 2nd source direction right to left is + ve

$$\text{hence } \frac{-1}{v} + \frac{1}{24 - x} = \frac{1}{9} \qquad \ldots\text{(ii)}$$

On adding Eqs. (i) and (ii) we set

$$\frac{1}{x} + \frac{1}{24 - x} = \frac{2}{9} \Rightarrow x^2 - 24x + 108 = 0$$

$$\Rightarrow \qquad x = 6, 18$$

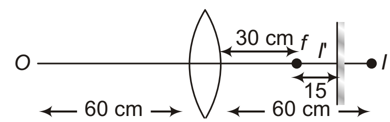
hence lens can be placed at a distance of 6 cm from any source.

21. Since the object is placed at c of first lens hence image also form at c and of same magnification *i.e.* $v = 2f$. Since two lens are separated by distance f hence the distance between 2nd lens and image is f. This image acts a virtual object for this lens using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get $\quad \frac{1}{v} - \frac{1}{f} \Rightarrow v = \frac{f}{2}$

and magnification $m_2 = -\frac{v}{u} = \frac{-f/2}{f} = \frac{-1}{2}$

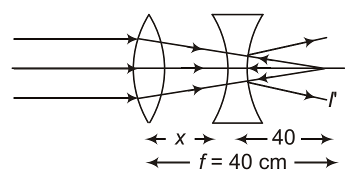
Hence image is formed at $\frac{f}{2}$ right of 2nd lens.

22. Since the object is placed at $2f$ hence image also form $2f$ by lens *i.e.*, at 60 cm. The mirror must be placed at that place that it made the final image at focus of lens. The difference is shown below.



Hence the distance between lens and mirror $= 40\,\text{cm} + 15\,\text{cm} = 45\,\text{cm}$

23. The diagram is shown in figure.

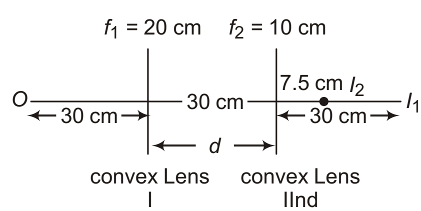


The parallel ray after refraction on convergent lens meet at focus $= 40\,\text{cm}$. Let distance between two lenses is x then using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ for diverging lens

$$\frac{1}{\infty} - \frac{1}{(40 - x)} = \frac{-1}{15}$$

$$\Rightarrow \qquad x = 25\,\text{cm}$$

24. Here $f_1 = 20\,\text{cm}$ $f_2 = 10\,\text{cm}$ and $d = 30\,\text{cm}$



For first lens using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we get

$$\frac{1}{v} + \frac{1}{30} = \frac{1}{20} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{30} \Rightarrow v = 60\,\text{cm}$$

For 2nd lens

$$\frac{1}{v} - \frac{1}{30} = \frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{30} + \frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{4}{30}$$

$$\Rightarrow \quad v = 7.5 \text{ cm}$$

Hence image formed at 7.5 cm from 2nd lens.

25. For lens $u = -40$ cm $f = +20$ cm using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

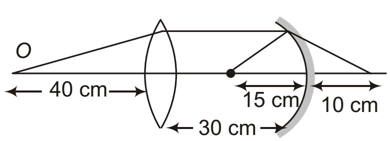
$$\Rightarrow \quad \frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{2-1}{40} = \frac{1}{40}$$

$$\Rightarrow \quad v = 40 \text{ cm}$$

Hence the image is at 40 cm right from the lens. Since distance between mirror and lens is 30 cm. Hence for mirror $v = +10$ cm, $f = -10$ cm.

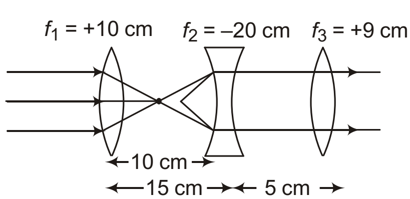
Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

$$\Rightarrow \quad \frac{1}{v} = -\frac{1}{10} - \frac{1}{10} \Rightarrow v = -5 \text{ cm}$$



Hence image is formed at 5 cm from the mirror toward lens.

26.



For first lens $v = +10$ cm

For 2nd lens $u = -5$ cm, $f = -20$ cm

Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v} = -\frac{1}{20} - \frac{1}{5}$$

$$\Rightarrow \quad v = -4 \text{ cm}$$

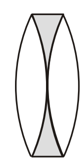
For third lens $v = -9$ cm and $f = 9$ cm

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{(-9)} = \frac{1}{9}$$

$$\Rightarrow \quad v = \infty$$

Hence the image formed at ∞ or rays become parallel.

27. Situation is shown in figure when space between two convex lenses is filled with refractive index 1.3 it become a concave lens of radii $R_1 = -30$ cm and $R_2 = +70$ cm hence it focal length is



$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\Rightarrow \quad \frac{1}{f} = (1.3-1)\left[\frac{-1}{30} - \frac{1}{70}\right]$$

$$\Rightarrow \quad f = -70 \text{ cm}$$

Hence the equivalent focal length of combination $\frac{1}{F} = \frac{1}{30} - \frac{1}{70} + \frac{1}{70}$

$\Rightarrow$ $F = 30$ cm if

$u = -90$ cm then using

$$\frac{1}{F} = \frac{1}{v} - \frac{1}{u}$$

we get $\frac{1}{30} = \frac{1}{v} + \frac{1}{90}$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{30} - \frac{1}{90} = \frac{2}{90} = \frac{1}{45}$$

$$\Rightarrow \quad v = 45 \text{ cm}$$

28. $n_{\text{ice}} = \dfrac{c}{v} = \dfrac{3 \times 10^8}{2.3 \times 10^8} = 1.30$

$$\theta_c = \sin^{-1}\left(\frac{1}{n_{\text{ice}}}\right) = \sin^{-1}\left(\frac{1}{1.30}\right)$$

$$= \sin^{-1}(0.77)$$

29. (a) Let angle of refraction in material 2 is r

then $\quad \dfrac{\sin\theta}{\sin r} = \dfrac{1.8}{1.6} = \dfrac{18}{16}$ ...(i)

For (2) to (3) interface

$$\frac{\sin r}{\sin 90^\circ} = \frac{1.3}{1.8} = \frac{13}{18}$$

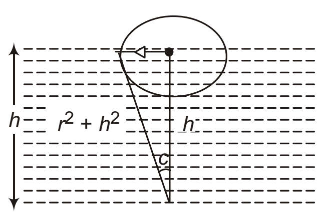
$\Rightarrow \quad \sin r = \dfrac{13}{18}$ ...(ii)

From (i) and (ii) $\sin\theta = \dfrac{18}{16} \times \dfrac{13}{18}$

$\Rightarrow \quad \theta = \sin^{-1}\left(\dfrac{13}{16}\right)$

(b) Yes, if θ decreases r also decreases and become less than the critical angle and hence light goes into material 3.

30. Let maximum height of liquid is h. From figure for critical angle C



$\mu = \dfrac{1}{\sin C} = \dfrac{\sqrt{r^2 + h^2}}{r}$ ...(i)

Here $r = 1$ cm and $\mu = \dfrac{4}{3}$ putting these values in Eq. (i). Solving we get $h = \dfrac{4}{3}$ cm

Here $\quad \sin\theta_c = \dfrac{1}{\mu_g} = \dfrac{2}{3}$

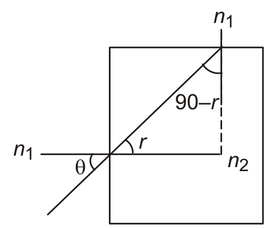
Now if water film is poured on the glass air surface. Let emergent angle at glass water surface is r, then

$$\frac{\sin\theta_c}{\sin r} = \frac{\mu_w}{\mu_g} = \frac{4 \times 2}{3 \times 3}$$

$\Rightarrow \quad \sin r = \dfrac{9}{8}\sin\theta_c = \dfrac{9}{8} \times \dfrac{2}{3} = \dfrac{3}{4}$

$\Rightarrow \quad r = \sin^{-1}\left(\dfrac{3}{4}\right)$

32. For total internal reflection at top surface



$$\frac{\sin(90^\circ - r)}{\sin 90^\circ} = \frac{n_1}{n_2}$$

$\Rightarrow \quad \cos r = \dfrac{n_1}{n_2}$

and $\quad \dfrac{\sin\theta}{\sin r} = \dfrac{n_2}{n_1}$

$\Rightarrow \quad \sin\theta = \dfrac{n_2}{n_1}\sin r$

$\Rightarrow \quad \sin\theta = \dfrac{n_2}{n_1}\sqrt{1 - \cos^2 r}$

$$= \frac{n_2}{n_1}\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2}$$

$\Rightarrow \quad \sin\theta = \sqrt{\dfrac{n_2^2 - n_1^2}{n_1^2}}$

$\Rightarrow \quad \theta = \sin^{-1}\sqrt{\left(\dfrac{n_2}{n_1}\right)^2 - 1}$

33. The deviation angle vary from 0° to θ° where $\theta = 90^\circ - c$ ...(i)

where C is the critical angle

Now, $\quad \sin c = \dfrac{\mu_w}{\mu_g} = \dfrac{4/3}{3/2} = \dfrac{8}{9}$

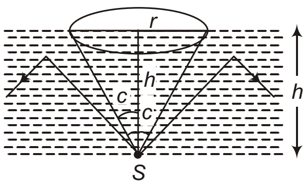
From Eq. (i) $\cos\theta = \sin C$

$\Rightarrow \quad \cos\theta = \dfrac{8}{9}$

$$\Rightarrow \qquad \theta = \cos^{-1}\left(\frac{8}{9}\right)$$

Hence deviation angle vary from 0° to $\cos^{-1}\left(\frac{8}{9}\right)$.

34.



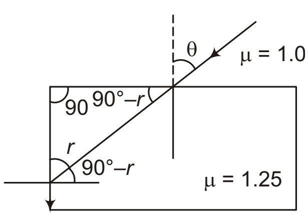
(a) Only circular patch light escapes because only those rays which are incident within a cone of semivertex angle C [Critical angle] are refracted out of the water surface. All other rays are totally internally reflected as shown in figures

(b) Now $\mu = \frac{1}{\sin C} = \frac{\sqrt{r^2 + h^2}}{r}$

or $\quad C = \sin^{-1}\left(\frac{1}{\mu}\right)$

$$= \sin^{-1}\left(\frac{r}{\sqrt{r^2 + h^2}}\right)$$

35.



For maximum angle θ the angle $90 - r$ at left surface must be equal to critical angle

$$\Rightarrow \sin(90° - r) = \frac{1}{1.25} = \frac{100}{125} = \frac{4}{5}$$

$$\Rightarrow \qquad \cos r = \frac{4}{5}$$

$$\Rightarrow \qquad \sin r = \frac{3}{5}$$

Now, $\qquad \frac{\sin\theta}{\sin r} = 1.25 = \frac{5}{4}$

$$\Rightarrow \qquad \sin\theta = \frac{5}{4}\sin r = \frac{5}{4} \times \frac{3}{5}$$

$$\Rightarrow \qquad \theta = \sin^{-1}\left(\frac{3}{4}\right)$$

36. $\mu = \dfrac{\sin\left(\dfrac{A + \delta m}{2}\right)}{\sin\dfrac{A}{2}}$

$$\sqrt{3} = \frac{\sin\left(\dfrac{A + \sin}{2}\right)}{\sin\dfrac{A}{2}} = \frac{2\sin\dfrac{A}{2}\cos\dfrac{A}{2}}{\sin\dfrac{A}{2}}$$

$$\Rightarrow \cos\frac{A}{2} = \frac{\sqrt{3}}{2} \Rightarrow \frac{A}{2} = 30°$$

$$\Rightarrow \qquad A = 60°$$

37. Here $i_1 = r_1 = 0°$.

Now, let other face angle of incidence is r_2

$\because \qquad r_1 + r_2 = A \Rightarrow 0 + r_2 = A$

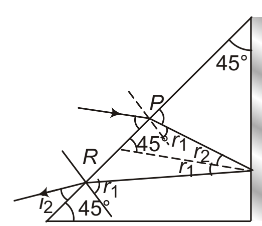
$\Rightarrow \qquad r_2 = A = 30°$

$$\frac{\sin r_2}{\sin i_2} = \frac{1}{1.5} \Rightarrow \sin i_2 = 1.4\sin r_2$$

$\Rightarrow \qquad \sin i_2 = 1.5 \times \sin 30°$

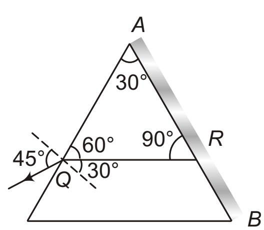
$\Rightarrow \qquad i_2 = \sin^{-1}(0.75) = 19°$

38. From figure sin $\angle OQP = \angle OQR$



Hence the ray retrace its path.

39.



The ray retrace its path from ref. by surface AB hence $\angle AR\theta = 90°$ from geometry it is clear that $r = 30°$

$$\mu = \frac{\sin i}{\sin r}$$

$$\Rightarrow \mu = \frac{\sin 45°}{\sin 30°} \Rightarrow \mu = \frac{1/\sqrt{2}}{1/2} \Rightarrow \mu = \sqrt{2}$$

40. Depends on formula.

41. The maximum angle will be $A = 2C$ where C is the critical angle

Now, $C = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.81$

Hence $A = 2C = 2 \times 41.81 = 83.62$

42. $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}}$ here $A = 60°$

$$1.5 = \frac{\sin\left(\dfrac{60° + \delta_m}{2}\right)}{\sin 30°}$$

$$0.75 = \sin\left(\frac{60° + \delta_m}{2}\right)$$

$$\Rightarrow 60° + \delta_m = 2\sin^{-1}(0.75)$$

$$\Rightarrow \delta_m = 22.8$$

and not deviation $= 180° - 22.8 = 157.2°$

(b) If the system is placed in water

$$\mu = \frac{1.5}{4/3} = \frac{4.5}{4}$$

$$\Rightarrow 60° + \delta'_m = 2\sin^{-1}(1.125 \times \sin 30°)$$

$$\Rightarrow \delta'_m = 2\sin^{-1}\left[\frac{1.125}{2}\right] - 60°$$

Net deviation $= 180 - \delta_m = 128.4°$

43. $\omega = \dfrac{\mu_V - \mu_R}{\mu_Y - 1}$

$$0.0305 = \frac{1.665 - 1.645}{\mu_y - 1}$$

On solving we get $\mu y = 1.656$

44. $\dfrac{\omega_1}{f_1} + \dfrac{\omega_2}{f_2} = 0$

$$\Rightarrow \frac{0.18}{20} + \frac{\omega_2}{-30} \Rightarrow \omega_2 = \frac{0.18 \times 30}{20}$$

$$\Rightarrow \omega_2 = 0.27$$

Now, $\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2} \Rightarrow \dfrac{1}{F} = \dfrac{1}{20} - \dfrac{1}{30}$

$$\Rightarrow F = 60\,\text{cm}$$

45. $\dfrac{\omega_1}{f_1} + \dfrac{\omega_2}{f_2} = 0$

$$\Rightarrow \frac{\omega_1}{\omega_2} = -\frac{f_1}{f_2}$$

$$\frac{3}{2} = -\frac{f_1}{f_2}$$

$$\Rightarrow f_1 = -\frac{3}{2} f_2$$

Now, $\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$

$$\Rightarrow \frac{1}{150} = \frac{1}{f_2} - \frac{2}{3f_2}$$

$f_2 = 50\,\text{cm}$ and $f_1 = -75\,\text{cm}$

46. Applying $\mu = \dfrac{\sin i_1}{\sin r_1}$

Find angle r_1 for two different refraction indices. Because $i_1 = 65°$ from both the cases.

Then again apply

$\mu = \dfrac{\sin i_2}{\sin r_2}$ and find i_2. Because $r_2 = A - r_1$

Then apply :

$\delta = (i_1 + i_2) - A$

for two refraction indices. Then difference in deviations is :

$\Delta\delta = \delta_1 - \delta_2$

Objective Questions (Level-1)

1. Endoscope is bases on total internal refraction Hence, correct option is (c)

2. Here $\mu = A + \dfrac{B}{\lambda^2}$

$\because$ μ is dimensionless.

$\Rightarrow$ $\dfrac{B}{\lambda_2} =$ dimension of μ

$\Rightarrow B = \lambda^2 \Rightarrow B$ has dimension of Area

Hence, correct option is (d).

3. Shift $= \left(1 - \dfrac{1}{\mu}\right)$

$\because \mu_R$ is minimum. than other visible colour. Red colour least raised.

correct option is (c)

4. Critical angle $\theta_C = \sin^{-1}\left(\dfrac{1}{\mu}\right)$

$\because \mu_0$ is maximum for violet colour hence θ_c for violet colour is least.

Hence correct option is (d)

5. We have $P = \dfrac{1}{f(\text{metre})}$

$= \dfrac{100}{f(\text{cm})} = 100 \times (n-1)\left[\dfrac{1}{R} + \dfrac{1}{r}\right]$

$\Rightarrow P = \dfrac{100 \times 0.6 \times 2}{10} = +12$

Hence, correct option is (a).

6. Speed of light in water $= \dfrac{c}{\mu_w}$

$\Rightarrow v_w = \dfrac{3 \times 10^8}{4/3} = 2.25 \times 10^8$ m/s

Hence correct option os (c).

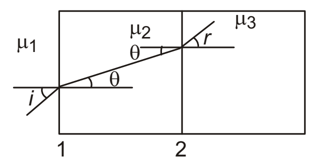
7. Due to TIR emergent beam will turn into black.

Hence correct option is (c).

8. $\because v = n\lambda$ but frequency n remain constant and v decreases hence λ decreases.

Hence correct option is (b).

9. Using Snell's law



On first and 2nd interface

$$\frac{\sin i}{\sin\theta} = \frac{\mu_2}{\mu_1} \quad \ldots\text{(i)}$$

and

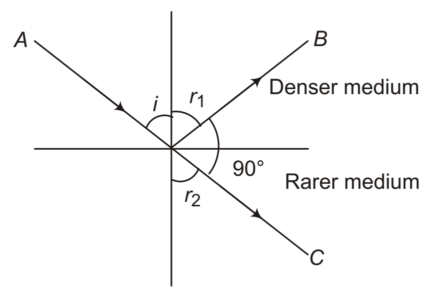
$$\frac{\sin\theta}{\sin r} = \frac{\mu_3}{\mu_2} \quad \ldots\text{(ii)}$$

Multiplying (i) and (ii), we get

$$\frac{\sin i}{\sin r} = \frac{\mu_3}{\mu_1}$$

Hence correct option is (b).

10. We have $i = r_1$ and $r_2 = 90 - i$



Now $\dfrac{\sin i}{\sin r_2} = \dfrac{1}{\mu}$

$\Rightarrow$ $\dfrac{\sin i}{\sin(90° - i)} = \dfrac{1}{\mu}$

$\Rightarrow$ $\tan i = \dfrac{1}{\mu}$ ...(i)

If C is the critical angle then $C = \sin^{-1}\left(\dfrac{1}{\mu}\right)$

$\Rightarrow$ $C = \sin^{-1}(\tan i)$

Hence correct option is (a).

11. Let angle of minimum deviation is δ_m we know that

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \quad \sqrt{2} = \frac{\sin\frac{(60° + \delta_m)}{2}}{\sin 30°}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \sin\left(\frac{60° + \delta_m}{2}\right)$$

$$\Rightarrow \quad \frac{60+\delta_m}{2} = 45° \Rightarrow \delta_m = 30°$$

Hence correct option is (a).

12. We know that

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{a^2} = (1.5-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{0.2} = (0.5-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{...(i)}$$

Let refractive index of the liquid s n_l

$$\Rightarrow \frac{1}{f_{cm}} = \left(\frac{n}{n_l} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

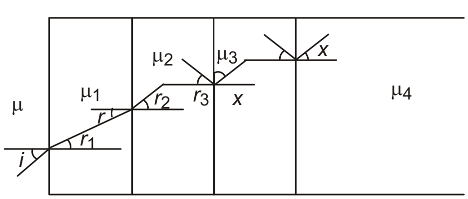
$$\Rightarrow -\frac{1}{0.5} = \left(\frac{1.5}{n_l} - 1\right) \times 10$$

$$\Rightarrow \frac{-1}{5} = \frac{1.5}{n_l} - 1$$

$$\Rightarrow \frac{1.5}{n_l} = \frac{4}{5} \Rightarrow \frac{5 \times 1.5}{4} = n_l \Rightarrow n_l = \frac{15}{8}$$

Hence correct option is (b).

13.



We have, $\frac{\sin i}{\sin r_1} = \frac{\mu_1}{\mu}$ ...(i)

$$\frac{\sin r_1}{\sin r_2} = \frac{\mu_2}{\mu_1} \qquad \text{...(ii)}$$

and $\frac{\sin r_2}{\sin r_3} = \frac{\mu_3}{\mu_2}$ ...(iii)

$$\frac{\sin r_3}{\sin x} = \frac{\mu_4}{\mu_3} \qquad \text{...(iv)}$$

Multiplying (i), (ii), (iii) and (iv), we get

$$\frac{\sin i}{\sin x} = \frac{\mu_4}{\mu} \Rightarrow \sin x = \frac{\mu}{\mu_4}\sin i$$

Hence correct option is (b)

14. Let radius of curvature of the lens is R

then $\frac{1}{f} = (n-1)\left(\frac{1}{R} + \frac{1}{R}\right) \Rightarrow f = \frac{R}{2(x-1)}$

Let focal length of one part is f'

then $\frac{1}{f'} = (n-1)\left[\frac{1}{R} - \frac{1}{\infty}\right]$

$$\Rightarrow \quad f' = \frac{R}{(n-1)} = 2f'$$

The focal length of the combination is

$$\frac{1}{F} = \frac{1}{2f} + \frac{1}{2f} + \frac{1}{2f} + \frac{1}{2f} \Rightarrow F = \frac{f}{2}$$

Hence correct option is (b).

15. Here $P = +5\text{D} \Rightarrow f + = 20\,\text{cm}$

$$\Rightarrow \quad \frac{1}{20} = (1.5-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad \text{...(i)}$$

and $-\frac{1}{100} = \left(\frac{1.5}{n_e} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ ...(ii)

Dividing Eq. (ii) by (i) $-\frac{1}{5} = \frac{\left(\frac{1.5}{n_l} - 1\right)}{0.5}$

$$\Rightarrow \quad \frac{1}{10} + 1 = \frac{1.5}{n_l}$$

$$\Rightarrow \quad n_l = \frac{5}{3}$$

Hence correct option is (b).

16. We know that

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1f_2}$$

$\because$ whole system is concave $\Rightarrow F<0$

$$\Rightarrow \quad \frac{d}{f_1f_2}>\frac{1}{f_1}+\frac{1}{f_2}\Rightarrow d>f_1+f_2$$

$$\Rightarrow \quad d>(10+20)\text{ cm}\Rightarrow d>30\text{ cm}$$

Hence only possible value of the given values is 40 cm.

Hence correct option is (d).

17. Since when $A>2\theta_c$ total light is reflect (TIR) take place hence maximum value of A is $2\theta_c$.

Hence correct option is (c).

18. $\because$ emergent ray is $\perp$ to the surface

$$\Rightarrow \quad i_2=r_2=0$$

$$\because \quad r_1+r_2=A\Rightarrow r_1=A$$

Now $\quad \mu=\dfrac{\sin i}{\sin A}$

Since i and A are small

$$\Rightarrow \quad \sin i\simeq i \text{ and } \sin A\simeq A$$

$$\Rightarrow \quad \mu=\frac{i}{A}\Rightarrow i=\mu A$$

Hence correct option is (c).

19. $$\mu=\frac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}}$$

$$\Rightarrow \quad \cot+\frac{A}{2}=\frac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}}$$

$$\Rightarrow \quad \frac{A+\delta_m}{2}=90^\circ-\frac{A}{2}$$

$$\Rightarrow \quad A+\delta_m=180^\circ-A$$

$$\Rightarrow \quad \delta_m=180^\circ-2A$$

Hence correct option is (d).

20. Minimum deviation condition

$$r_1=r_2=r\Rightarrow 90^\circ-r+90^\circ-r+A=180^\circ$$

$$\Rightarrow \quad A=2r$$

$$\Rightarrow \quad r=\frac{A}{2}=30^\circ$$

Now $\quad \mu=\dfrac{\sin i}{\sin r}$

$$\Rightarrow \quad \sqrt{2}=\frac{\sin i}{\sin 30^\circ}$$

$$\Rightarrow \quad \sin i=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad i=45^\circ$$

Hence correct option is (a).

21. $$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_1}-\frac{d}{f_1f_2} \quad \text{...(i)}$$

$$\Rightarrow \quad \frac{1}{2F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{2d}{f_1f_2} \quad \text{...(ii)}$$

$$\frac{1}{F'}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{4d}{f_1f_2} \quad \text{...(iii)}$$

On solving (i) , (ii) and (iii) we get $F'=2F$.

Correct option is (a).

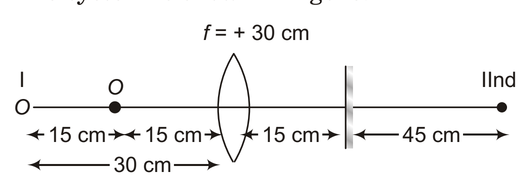
22. $$\left|\frac{1}{f}\right|=(n-1)\left|\frac{1}{R_1}-\frac{1}{R_2}\right|$$

$$\Rightarrow \quad \frac{1}{24}=(1.5-1)\left|\frac{1}{R}-\frac{1}{2R}\right|$$

$$\Rightarrow \quad \frac{1}{24}=\frac{0.5}{2R}\Rightarrow R=6 \text{ and } 2R=12$$

Hence correct option is (a).

23. The system is shown in figure.



Using $\quad \dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f}$

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{(-15)}=\frac{1}{30}\Rightarrow v=-30\text{ cm}$$

This image I_1 act as a virtual object for mirror since plane mirror form image at same distance as object. Hence the distance between object and image is $(30+45)\text{ cm} = 75\text{ cm}$.

Hence correct option is (c).

24. $\dfrac{\sin 45^\circ}{\sin r_1} = \sqrt{3}$ ...(i)

$$\frac{\sin r_1}{\sin r_2} = \sqrt{\frac{2}{3}}$$

Multiplying Eq. (i) and (ii)

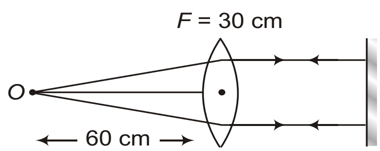
$$\frac{\sin 45^\circ}{\sin r_2} = \sqrt{2}$$

$\Rightarrow \quad \sin r_2 = \dfrac{1}{2}$

$\Rightarrow \quad r_2 = 30^\circ$

Hence correct option is (a).

25. For small angle prism



$\delta_{min} = (\mu - 1)A$ if μ increases

$\Rightarrow \quad \delta_{min} \rightarrow$ increases

Hence correct option is (a)

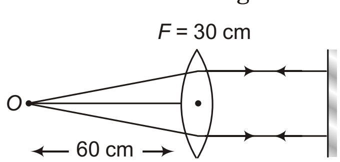
26. $\dfrac{\sin i}{\sin r} = \dfrac{n_2}{n_1} \Rightarrow \dfrac{\sin i}{\sin\left(\dfrac{i}{2}\right)} = n$

$\because r = \dfrac{i}{2}$

$\Rightarrow \quad 2\cos\dfrac{i}{2} = n \Rightarrow i = 2\cos^{-1}\left(\dfrac{n}{2}\right)$

Hence correct option is (c).

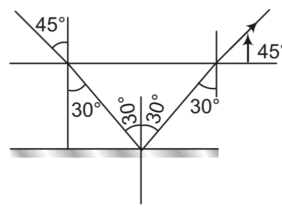
27. Situation is shown in figure.



Since image after reflection form on object itself hence the object must be placed at focus of the lens. The rays after refraction by lens becomes parallel to optic axis. Hence reflection rays follow the same path and final image form on x itself. Hence $x = 30\text{ cm}$.

Correct option is (b).

28. $\dfrac{\sin 45^\circ}{\sin r} = \sqrt{2} \Rightarrow \sin r = \dfrac{1}{2} \Rightarrow r = 30^\circ$



hence total deflection $= 45^\circ - (-45^\circ) = 90^\circ$

Hence correct option is (a).

29. $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}}$

$\Rightarrow \quad \sqrt{\dfrac{3}{2}} = \dfrac{\sin\left(\dfrac{90^\circ + \delta_m,}{2}\right)}{\sin 45^\circ}$

$\Rightarrow \quad \sin\left(\dfrac{90^\circ + \delta_m}{2}\right) = \dfrac{\sqrt{3}}{2} = \sin 60^\circ$

$\Rightarrow \quad \delta_m = 30^\circ$

Hence correct option is (c)

30. $\dfrac{1}{f} = (n-1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

here $\quad n = \dfrac{n_{\text{air}}}{n_{\text{glass}}} = \dfrac{1}{1.5}$

$\Rightarrow \dfrac{1}{f} = \left(\dfrac{1}{1.5} - 1\right)\left[\dfrac{1}{10} + \dfrac{1}{10}\right]$

$\Rightarrow f = -15\text{ cm}$. Hence lens is concave

Correct option is (a).

31. For lens $u = -12$ cm and $F = +10$ cm

We have $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \quad \frac{1}{v} + \frac{1}{12} = \frac{1}{10}$

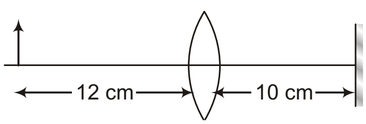
$\Rightarrow \quad v = 60$ cm

Since the distance between lens and mirror is 10 cm. Hence the image formed 50 cm from convex mirror. The rays retrace its path if image is formed at the centre of curvature of the mirror *i.e.,*

$R_{\text{convex}} = 50 \text{ cm} \Rightarrow F = \frac{R}{2} = 25 \text{ cm}$

Hence the correct option is (b)

32.



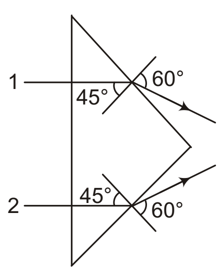
Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{(-12)} = \frac{1}{20}$

$\Rightarrow \quad v = -30$ cm

Hence the distance between mirror and this image is 40 cm. Therefore second image formed 40 cm behind the mirror.

Hence correct option is (c).

33. We have $\mu = \frac{\sin i}{\sin r}$



$\Rightarrow \quad \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sin 45°}{\sin r}$

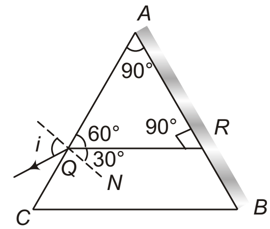
$\Rightarrow \sin r = \frac{\sqrt{3}}{2} \Rightarrow r = 60°$ (on both faces)

Hence the angle between emergent ray

$= 90° - (30 + 30°) = 30°$

Hence correct option is (b).

34.



Since the ray retrace its path hence $\angle ARQ = 90° \Rightarrow \angle RQN = \angle r = 30°$

$\mu = \frac{\sin i}{\sin r} \Rightarrow \sin i = \mu \sin r = \sqrt{2} \times \sin 30°$

$\Rightarrow \quad \sin i = \frac{1}{\sqrt{2}} \Rightarrow i = 45°$

Hence correct option is (c).

35. $f = 10$ cm

the focal length of $= \frac{1}{F} = \frac{1}{10} + \frac{1}{10}$

$\Rightarrow \quad F = 5$ cm

Here $u = -7.5$ cm, $F = 5$ cm

$\Rightarrow \quad \frac{1}{v} + \frac{1}{7.5} - \frac{1}{5} \Rightarrow \frac{1}{v} = \frac{7.5 - 5}{7.5 \times 5}$

$\Rightarrow \quad v = +15$ cm

Hence $|m| = \left|\frac{v}{u}\right| \Rightarrow \frac{\text{height of image}}{\text{height of object}} = \frac{15}{7.5}$

Height of image $= 2 \times 1$ cm $= 2$ cm

Hence correct option is (a).

36. $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

$\Rightarrow \quad \frac{1}{v} - \frac{3}{2}\frac{1}{(\infty)} = \frac{(3/2 - 1)}{-20}$

$\Rightarrow \quad v = +40$ cm

Hence correct option is (a).

37. $\frac{1}{f} = \left(\frac{n_g}{n_L} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Here $R_1 = -30$ cm and $R_2 = -50$ cm

$$\Rightarrow \quad \frac{1}{f} = \left(\frac{1.5}{1.4} - 1\right)\left(-\frac{1}{30} + \frac{1}{50}\right)$$

On solving

$$\Rightarrow \quad f = -1050 \text{ cm}$$

Hence correct option is (d).

38. From the figure, it is clear that $\mu_1 = \mu_3 < \mu_2$

Hence correct option is (b).

39. Here $A = 60°$, $\delta_m = 60°$

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin \frac{A}{2}} = \frac{\sin 60°}{\sin 30°}$$

$$\mu = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

Hence correct option is (a).

JEE Corner

Assertion and Reason

1. Due to shifting of image on refraction Shayam appear nearer to Ram and light suffer two refraction. Hence, both (a) and (b) are correct but reason does not explain the assertion.

Correct option is (b).

2. Applying lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Let u = object distance from lens

$v = d - u$ distance of image from lens.

$$\because \quad v + u = d$$

$$\Rightarrow \quad \frac{1}{d-u} - \frac{1}{-u} = \frac{1}{f}$$

$$\Rightarrow \quad u^2 - du + df = 0$$

$$\Rightarrow \quad u = \frac{d \pm \sqrt{d - 4f}}{2}$$

$\because u$ is real hence $d \geq 4f$

Thus mean distance $v = 4f$, if $u = -2f, v = 2f$. Hence both assertion and reason are true, and reason explain or may not explain assertion. Hence correct option is (a, b)

3. Correct option is (b).

4. Correct option is (c).

5. Since both assertion and reason are true built reason is not explain assertion Hence correct option is (b).

6. Using mirror formula.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Here $\quad u = -\infty$

$$f = -f \text{ (concave lens)}$$

$$\frac{1}{v} + \frac{1}{\infty} = \frac{1}{-f}$$

$$\Rightarrow \quad v = -f$$

Hence image is formed at principle focus thus assertion is false but reason is true. Hence correct option is (d).

7. $$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin A / 2}$$

Here $\quad A = 60°, \ \mu = \sqrt{2}$

$$\Rightarrow \quad \sqrt{2} = \frac{\sin\left(\frac{60 + \delta_m}{2}\right)}{\sin 30°}$$

$$\Rightarrow \quad \frac{60 + \delta_m}{2} = 450 \Rightarrow \ \delta_m = 30°$$

Hence both assertion and reason are true and reason explain assertion correctly.

Correct option is (a).

8. Focal length of combination

$$\frac{1}{F}=\frac{1}{F_{\text{convex}}}+\frac{1}{F_{\text{convex}}}=\frac{1}{f_1}-\frac{1}{f_2}$$

if $\qquad f_1 > f_2$

$\Rightarrow \qquad \frac{1}{f_1} < \frac{1}{f_2}$

$\Rightarrow F = -$ negative

Hence assertion is true. Since power is a measure of converging or divergence of a lens. Hence reason is not true. Correct option is (c).

9. Since glass slab produced a net shift. Hence v is increased. Thus magnified image is obtained but image may be real or virtual depending on the position of slab.

Correct option is (b)

10. In this case image distance of O_1 and O_2 are same from the lens.

$\because \ \frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ and reason is trure.

Hence correct option is (d).

11. Assertion is false since only ray energe if refractve index of the colour less than the prism and angle of incidence is less than critical angle but reason is true. Correct option is (a).

12. If two object is placed between pole and focus image is real hence assertion is true. Also reason is correct.

Hence correct option is (b)

13. Since both assertion and reason are true and reason explanation is correct.

Hence correct option is (a).

Objective Questions (Level 2)

■ Single option correct

1. We have $\mu = \frac{\text{Real depth}}{\text{App. depth}}$

$\Rightarrow \qquad \frac{4}{3}=\frac{1}{\text{App. depth}}$

$\Rightarrow$ App. depth $=\frac{3}{4}$

Hence the distance between bird and mirror

$$=2+\frac{3}{4}=\frac{11}{4}\text{ m}$$

Since plane mirror form image behind the mirror (for real object) at same distance as object hence the distance between bird and its image

$$=\frac{11}{4}+\frac{11}{4}=\frac{11}{2}\text{ m}$$

Correct option is (d).

2. $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

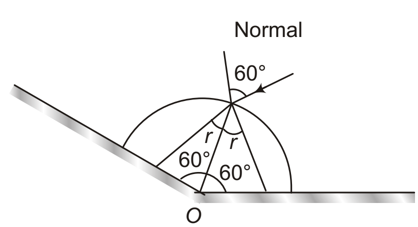
Here $\qquad \mu=-\infty \ u_1=1$

and $\qquad \mu_2=1.5$

$\Rightarrow \qquad \frac{1.5}{v}+\frac{1}{\infty}=\frac{0.5}{R} \Rightarrow v=3R$

Hence correct option is (b).

3. From figure, $r=30°$



$\because \quad r=\theta+\alpha-90=120°-90°$

Hence $\mu = \dfrac{\sin 60°}{\sin 30°} = \sqrt{3}$

The correct option is (d).

4. The lens become diverging if

$$\mu_1 - \mu_2 > \mu_2 - \mu_3$$

$\Rightarrow \quad \mu_1 + \mu_3 > 2\mu_2$

or $\quad 2\mu_2 < \mu_1 + \mu_3$

Hence correct option is (b).

5. $\dfrac{1}{v_1} + \dfrac{1}{16} = \dfrac{1}{f} \Rightarrow v_1 = \dfrac{16f}{16-f}$

$$m_1 = \frac{-v_1}{16} = \frac{-16f}{16(16-f)} = \frac{-f}{(16-f)} \quad \ldots\text{(i)}$$

and $\quad \dfrac{1}{v_2} + \dfrac{1}{6} = \dfrac{1}{f} \Rightarrow v_2 = \dfrac{6f}{6-f}$

$\mu_2 = \dfrac{v_2}{6}$ $\quad$ ($\because$ image is virtual)

$\Rightarrow \quad m_2 = \dfrac{6f}{6(6-f)} = \dfrac{f}{6-f} \quad \ldots$(ii)

But $\quad m_1 = m_2 \Rightarrow \dfrac{-f}{16-f} = \dfrac{f}{6-f}$

$\Rightarrow \quad -6 + f = 16 - f \Rightarrow 2f = 22$

$\Rightarrow \quad f = 11\,\text{cm}$

Hence correct option is (d).

6. Let real depth at any instant t of the water is h then volume of water $V = \pi R^2 h$

$\Rightarrow \quad \dfrac{dV}{dt} = \pi R^2 \dfrac{dh}{dt} \quad \ldots$(i)

Let apparent depth at this instant is h'

$\because \quad \mu = \dfrac{\text{Real depth}}{\text{Apparent depth}}$

$\Rightarrow \quad \dfrac{n_2}{n_1} = \dfrac{h}{h_1} \Rightarrow h_1 = \dfrac{n_1}{n_2} h$

Now $\quad \dfrac{dh'}{dt} = x = \dfrac{n_1}{n_2}\dfrac{dh}{dt}$

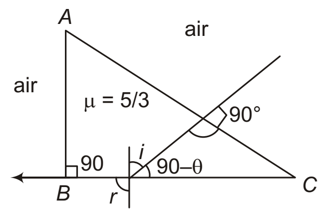
$\Rightarrow \quad \dfrac{dh}{dt} = \dfrac{xn_2}{n_1} \quad \ldots$(ii)

From Eq. (i) and (ii), we get

$$\frac{dV}{dt} = \frac{\pi R^2 x n_2}{n_1}$$

Hence correct option is (b).

7.



$\because \theta = 37° \Rightarrow i = [90 - (90 - \theta)] = \theta = 37°$

Applying Snell's law on face BC.

$$\frac{\sin 37°}{\sin r} = \frac{\mu_2}{\mu_1} = \frac{3}{5}$$

$\Rightarrow \quad \sin r = \dfrac{4}{3}\sin 37° = \dfrac{5}{3} \times \dfrac{3}{5} = 1$

$\Rightarrow \quad r = 90°$

Hence deviation $\delta = 90° + 37° = 127°$

Correct option is (b).

8. Let refractive index of liquid is μ

For position of fish w.r.t. bird is

$$\mu = \frac{\text{Real depth}}{\text{App. depth}} \Rightarrow \mu = \frac{x}{h_1} \quad \ldots\text{(i)}$$

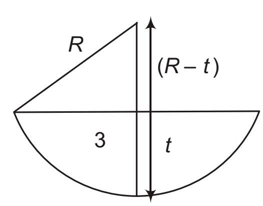
For position of bird w.r.t. fish is

$$\frac{1}{\mu} = \frac{y}{h_2} \quad \ldots(2)$$

From Eq. (i) and (ii) we get $u = \dfrac{h_2}{h_1}$

Hence correct option is (a).

9.



$\Rightarrow (R - t)^2 + 3^2 = R^2$

$\Rightarrow \quad 2Rt \simeq 9 \quad \because R >> T$

$$\Rightarrow \qquad 2R = \frac{900\,\text{mm}}{3\,\text{mm}} = 300\,\text{mm}$$

$$\Rightarrow \qquad R = 150\,\text{mm} = 15\,\text{cm}$$

Hence correct option is (a).

10. $\dfrac{1}{v_1} + \dfrac{1}{v_1} = \dfrac{1}{f} \Rightarrow v_1 = \dfrac{fv_1}{v_1 - f}$

$$m_1 = \frac{-v_1}{v_1} = \frac{-f}{(v_1 - f)} \qquad \ldots\text{(i)}$$

$$\frac{1}{v_2} + \frac{1}{v_2} = \frac{1}{f} \Rightarrow v_2 = \frac{fv_2}{v_2 - f}$$

$$m_2 = \frac{v_2}{v_2} = \frac{f}{\mu_2 - f} \qquad \ldots\text{(ii)}$$

$\because \quad m_1 = m_2$

$$\Rightarrow \quad \frac{-f}{v_1 - f} = \frac{f}{v_2 f} \Rightarrow f = \frac{v_1 + v_2}{2}$$

Hence the correct option is (d).

11. $\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2} - \dfrac{d}{f_1 f_2}$

$$\Rightarrow \qquad F = \frac{f_1 f_2}{f_1 + f_2 - d}$$

as d increases $f_1 + f_2 - d$ decreases hence F increases. Hence image move to right.

Correct option is (b).

12. In this case, minimum deviation of ray 1 is same as ray 2.

Hence correct option is (c).

13. For critical angle at glass air surface

$$\sin \theta_c = \frac{1}{\mu_g} = \frac{2}{3} \qquad \ldots\text{(i)}$$

Now for glass water surface.

$$\frac{\mu_w}{\mu_g} = \frac{\sin \theta_c}{\sin r} \Rightarrow \frac{4/3}{3/2} = \frac{2}{3 \times \sin r}$$

$$\Rightarrow \qquad \sin r = \frac{4}{3}$$

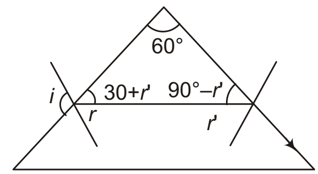
Now for water air surface

$$\frac{\mu_w}{\mu_a} = \frac{\sin r}{\sin r'} \Rightarrow \frac{4}{3} = \frac{4}{3} \times \frac{1}{\sin r'}$$

$$\Rightarrow \qquad \sin r' = 1 \Rightarrow r' = 90°$$

Hence correct option is (d).

14. For limiting angle of incident emergent ray become parallel to the 2nd face



$$\because \qquad \frac{\sin r'}{\sin 90°} = \sqrt{\frac{3}{7}} \Rightarrow r' = \sin^{-1}\sqrt{\frac{3}{7}}$$

Now $r + 30° + r' = 90°$

$$\Rightarrow \qquad r = 60° - r'$$

$$\Rightarrow \qquad r = (60° - \sin^{-1}\sqrt{3/7})$$

Now $\qquad \mu = \dfrac{\sin i}{\sin r}$

$$\Rightarrow \qquad \sqrt{\frac{7}{3}} = \frac{\sin i}{\sin[60 - \sin^{-1}\sqrt{3/7}\,]}$$

$$\Rightarrow \; i = \sin^{-1}\{\sqrt{7/3} \times \sin(60 - \sin^{-1}\sqrt{3/7}\}$$

$$= \sin^{-1}\left\{\sqrt{\frac{7}{3}} \times \sin(60° - 21°)\right\}$$

$$= \sin^{-1}\{\sqrt{7/3} \times \sin 19°\}$$

$$= \sin^{-1}\{0.49\} \simeq 30°$$

Hence correct option is (a).

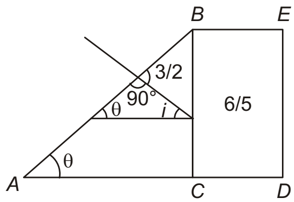
15. The image form the object itself if the rays incident parallel to optical axis on the mirror *i.e.,* image of refraction is formed at ∞. It is possible when O is placed at focus *i.e.*, $d = 10\,\text{cm}$

Hence correct option is (c).

16. The dot will appear at c for all values of μ. Since position does not in same medium.

Hence correct option is (b).

17. We have for total internal reflection



$$\frac{\sin i}{\sin 90} = \frac{6/5}{3/2}$$

$\Rightarrow$ $$\sin i = \frac{4}{5} \Rightarrow i = \sin^{-1}\left(\frac{4}{5}\right) = 53°$$

Hence the ray will not cross BC if $i > 53°$

$\Rightarrow$ $$90 + \theta + i = 180°$$

$$\theta = 90 - i \because i > 53°$$

$\Rightarrow$ $$\theta < 37°$$

Hence correct option is (a).

18. For reflection at curved surface

$$\frac{1}{v} - \frac{1}{(-x)} = \left(\frac{3}{2} - 1\right) \times \frac{1}{10}$$

$\Rightarrow$ $$v = \frac{20x}{x - 20}$$

This image act as virtual object for plane-glass-water surface

$\Rightarrow$ $$\frac{\mu_g}{\infty} - \frac{\mu_w(x-20)}{20x} = \frac{\mu_g - \mu_w}{\infty}$$

$\Rightarrow$ $$x = 20\,\text{cm}$$

Hence answer is (c).

19. The ratio of focal length in the situation II and III is 1 : 1.

Hence correct option is (c).

20. We have $\dfrac{1}{OB} - \dfrac{1}{(-OA)} = \dfrac{1}{f}$

$\Rightarrow$ $$\frac{1}{OB} + \frac{1}{OA} = \frac{1}{f}$$

$\Rightarrow$ $$f = \frac{OB.OA}{OA + OB} \quad \because OB + OA = AB$$

$\Rightarrow$ $$f = \frac{OB.OA}{AB} \qquad \ldots\text{(i)}$$

Now $AB^2 = AC^2 + BC^2$

$$(OA + OB)^2 = OC^2 + OA^2 + OB^2 + OC^2$$

$\Rightarrow$
$$OA^2 + OB^2 + 2OAOB = 2OC^2 + OA^2 + OB^2$$

$\Rightarrow$ $$OC^2 = OA\,OB \qquad \ldots\text{(ii)}$$

Putting this value in Eq. (i), we get

$\Rightarrow$ $$f = \frac{OC^2}{AB}$$

Hence correct option is (c).

21. The shift produce

$$\Delta t = t\left[1 - \frac{2}{{}_w\mu_g}\right]$$

$$= 36\left[1 - \frac{1}{9/8}\right] \because {}_w\mu_g = \frac{3/2}{4/3} = \frac{9}{8}$$

$$= \frac{36 \times 1}{9} = 4\text{ cm}$$

Hence correct option is (b).

22. $\mu = \dfrac{\text{Real depth}}{\text{App. depth}}$

$\Rightarrow$ $$\frac{4}{3} = \frac{\text{real depth}}{10.5\,\text{cm}}$$

$\Rightarrow$ Real depth $= \dfrac{4}{3} \times 10.5\,\text{cm} = 14\text{ cm}$

Hence correct option is (d).

23. $\because y_0 = +1\,\text{cm}$ and $y_i = -2\,\text{cm}$

$$m = -\frac{v}{u} \Rightarrow 2 = \frac{v}{u}$$

Now let x be the position of lens then $v = 50 - x$ and $v = (40 + x)$.

$\Rightarrow$ $$2 = \frac{50 - x}{40 + x}$$

$\Rightarrow$ $$80 + 2x = 50 - x$$

$\Rightarrow$ $$-3x = 30 \Rightarrow x = -10\text{ cm}$$

Hence correct option is (c).

24. If the plane surface of plano-convex lens is silvered it behave the concave mirror of focal length $f_m/2$

$\because \quad f_m = 10\,\text{cm}$

$\Rightarrow \quad f_e = 5\,\text{cm}$ hence $R = 10\,\text{cm}$

Correct option is (c).

25. Since lens made real and magnified image, hence it is a convex lens when lens dipped in water its focal length.

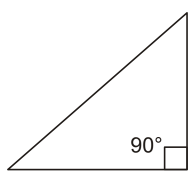
$$\frac{1}{f} = \left(\frac{\mu_w}{\mu_g} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= \left(\frac{4/3}{3/2} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{-1}{9}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$\because f$ is $-$ve lens behave as concave, hence the image is virtual and magnified.

Correct option is (c).

26. The prism transmit the light for which angle of incidence(c), $2c \le 90° \Rightarrow c \le 45°$



Hence $\mu = \dfrac{1}{\sin C} = \dfrac{1}{\sin 45°} = \sqrt{2} = 1.414$

Correct option is (b).

27. $\dfrac{1}{v} - \dfrac{1}{(-16)} = \dfrac{1}{f} \because |m| = \left|\dfrac{v}{u}\right| = 3$

For convex lens $v = 3u$

$\dfrac{1}{48} + \dfrac{1}{16} = \dfrac{1}{f} \Rightarrow f = 12\,\text{cm}$ (for real image)

Similarly when distance is 6 cm, 3 times virtual image is formed hence mirror is convex with focal length 12 cm.

Correct option is (c).

28. $v_g = n\lambda_g \Rightarrow \dfrac{c}{\mu_g} = n\lambda_g$ ...(i)

and $\quad \dfrac{c}{\mu_w} = n\lambda_w$ ...(ii)

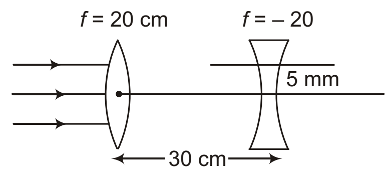
Dividing (i) and (ii)

$$\frac{\mu_w}{\mu_g} = \frac{\lambda_g}{\lambda_w} \Rightarrow \frac{4}{3\mu g} = \frac{4}{5}$$

$\Rightarrow \quad \mu_g = \dfrac{5}{3}$

Hence correct option is (a).

29. $x = \dfrac{f_1 f_2 + d(f_1 - d)}{f_1 + f_2 - d}$ and $y = \dfrac{(f_1 - d)\,\Delta}{f_1 + f_2 - d}$



Here $f_1 = f_2 = 20\,\text{cm}$, $d = 30\,\text{cm}$

and $\Delta = 55\text{m} = 0.5\,\text{cm}$

Putting these values we get

$x = 25\,\text{cm}$ and $y = 0.25\,\text{cm}$

Correct option is (b).

30. Since for each θ angle of incidence at glass-air boundry remains 0° hence there will never be total internal reflection.

Correct option is (d).

31. Diameter $= \mu \times$ Original diameter

$$= \frac{4}{3} \times 1\,\text{cm} = -\frac{4}{3}\,\text{cm}$$

Hence correct option is (a).

32. $\dfrac{1}{f_1} = (1.5 - 1)\left[\dfrac{1}{20} + \dfrac{1}{20}\right]$

$\Rightarrow \quad f_1 = 20\,\text{cm}$

Here $u_1 = -30\,\text{cm}$

$$\frac{1}{v} - \frac{1}{u_1} = \frac{1}{f_1} \Rightarrow \frac{1}{v_1} = \frac{1}{20} - \frac{1}{30} \Rightarrow v_1 = 60\,\text{cm}$$

Magnification $|m_1| = \left|-\dfrac{v}{u}\right| = \dfrac{60}{30} = +2$

(Inverted image)

For second lens. $\frac{1}{v_2}+\frac{1}{60}=\frac{1}{20} \Rightarrow v_2=+30\,\text{cm}$

Magnification $|m_2|=\left|\frac{30}{60}\right|=\frac{1}{2}$

Total magnification $= m_1 \times m_2 = 1$

Hence object size remains 3mm and it is formed at $(120+30)\,\text{cm} = 50\,\text{cm}$ from first lens.

Hence correct option is (b).

33. From left hand side refraction occur from $n_2 = 2$ to $n_1 = 1$.

$$n=\frac{n_1}{n_2}=\frac{1}{2}=0.5$$

$$\frac{n}{v}-\frac{1}{u}=\frac{n-1}{R}$$

$$\frac{0.5}{v}-\frac{1}{10}=\frac{-0.5}{10}+\frac{1}{10}=\frac{0.5}{10}$$

$$\Rightarrow \quad v = 10\,\text{cm}$$

Hence correct option is (a).

34. For lens $v=-20\,\text{cm}$, $f=+10\,\text{cm}$

Using $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we get $v=20\,\text{cm}$

This image acts virtual object for convex mirror. For mirror $v=-(x-20)$

and $v=-(20+x)$ and $f=+60\,\text{cm}$

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{-(20+x)}-\frac{1}{(x-20)}=\frac{1}{60}$$

After solving we get $x=20\,\text{cm}$.

Hence correct option is (c).

35. In this case system behave as concave mirror or focal length $=\mu \times f_e$

$$=1.5\times 20 = 30\,\text{cm}$$

Now using mirror formula

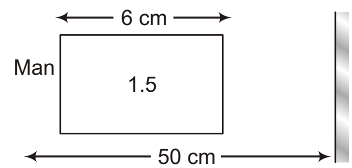
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \text{ we get}$$

$$\frac{1}{v}-\frac{1}{20}=-\frac{1}{30}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{20}-\frac{1}{30} \Rightarrow v=+60\,\text{cm}$$

Hence correct option is (d).

36. Shift in mirror $=6\left(1-\frac{1}{1.5}\right)=2\,\text{cm}$



Hence man lie at 48 cm from mirror. The distance of image from observer $=2\times 48$

$$=96\,\text{cm}$$

Hence correct option is (b).

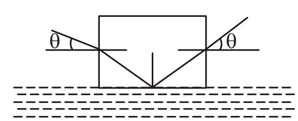
37. Using lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{(f+40)}+\frac{1}{(f+10)}=\frac{1}{f}$$

On solving we get $f=20\,\text{cm}$

Hence correct option is (c).

38.



$$\frac{\sin\theta}{\sin(90-\theta_c)}=n_1$$

$$\frac{\sin\theta}{\cos\theta_c}=n_1$$

Now $\quad \cos\theta_c<\sqrt{\frac{n_1^2-n_2^2}{n_1}}$

$$\Rightarrow \sin\theta<\sqrt{\frac{n_1^2-n_2^2}{n_1}}\times n_1$$

$$\Rightarrow \quad \sin\theta<\sqrt{n_1^2-n_2^2}$$

Hence correct option is (a).

39. For mirror $u = -1\,\text{cm}$ (taking upward direction + ve)

$$f = -2\,\text{cm}$$

$$\frac{1}{v} = -\frac{1}{f} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{v} = -\frac{1}{2} + 1 = \frac{1}{2}$$

$$\Rightarrow \quad v = 2\,\text{cm}$$

Hence mirror form virtual image behind mirror at 2 cm from pole. This image acts as virtual object for slab on see below the slab the shift is

$$\Delta t = \left(1 - \frac{1}{3/2}\right) \times 9 = 3\text{ cm}$$

Hence virtual image form on object thus correct option is (a).

40. We have $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

For real object u is + ve

$$\Rightarrow \quad \frac{\mu_2}{v} = \frac{(\mu_2 - \mu_1)}{R} + \frac{\mu_1}{u}$$

$\because \quad R$ is + ve and $\mu_1 > \mu_2$

$$\Rightarrow \quad \frac{\mu_2}{v} = -\text{ve} + \frac{\mu_1}{u}$$

$$\Rightarrow \quad v = -\text{ ve}$$

Hence, if $\mu_1 > \mu_2$ then these cannot be real image of real object.

Hence, correct option is (a).

41. At oil-concave surface

$$\frac{1}{f_1} = (1.6 - 1)\left[\frac{1}{\infty} + \frac{1}{10}\right] \Rightarrow \frac{1}{f_1} = \frac{0.6}{10}$$

At other surface light goes from oil to glass

$$\frac{1}{f_2} = \left(\frac{1.5}{1.6} - 1\right)\left[-\frac{1}{10} - \frac{1}{20}\right]$$

$$\frac{1}{f_2} = \frac{0.3}{20 \times 1.6}$$

Let focal length of combination is F

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{0.6}{10} + \frac{0.3}{20 \times 1.6}$$

$$\Rightarrow \quad F = 28.57\text{ cm}$$

Hence correct option is (d).

42. Using formula

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

For real image μ is –ve

$$\Rightarrow \quad \frac{\mu_2}{v} + \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \quad \frac{1.5}{v} = \frac{0.5}{R} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1.5}{v} = \frac{1}{2R} - \frac{1}{u} \quad v \text{ is + ve if } u > 2R$$

Hence correct option is (b).

43. $v = \frac{fu}{f - u}$ when $u < f$

u lend to lens

$\Rightarrow$ if $\quad u \to 0, v \to 0$

Hence correct option is (d).

44. Since image formed by diverging lens is always virtual.

Hence correct option is (a)

45. If the object place at first focus the image forms at ∞ *ie*, rays incident the plane surface normally and retrace its path.

Now $x = f_1 = \frac{-\mu_1 R}{\mu_2 - \mu_1} - \frac{-60}{0.5} = -120\,\text{cm}$

Hence correct option is (a).

46. $\frac{\mu_2}{v_1} - \frac{\mu_1}{u_1} = \frac{\mu_2 - \mu_1}{R}$ for $\mu_2 = 1.6$

$$\frac{1.6}{v_1} - \frac{1}{(-2)} = \frac{1.6 - 1}{1} = \frac{6}{10}$$

$$\Rightarrow \quad \frac{1.6}{v_1} = \frac{6}{10} - \frac{1}{2} = \frac{1}{10}$$

$$\Rightarrow \quad v_1 = 16\,\text{m}$$

For $\quad \mu_2 = 2$

$$\frac{2}{v_2} - \frac{1}{(-2)} = \frac{2-1}{1} = 1$$

$$\frac{2}{v_2} = 1 - \frac{1}{2} = \frac{1}{2} \Rightarrow v_2 = 4 \text{ m}$$

Hence separation between images $= v_1 - v_2$

$= (16 - 4)$ cm

$= 12$ cm

Hence correct option is (a).

47. System behave a a concave mirror

Here $u = -10, v = -40$ cm

Using mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ we get}$$

$$\frac{-1}{40} - \frac{1}{10} = \frac{1}{f}$$

$\Rightarrow \qquad f = -8$ cm

Hence correct option is (a).

48. The focal length of lens combination is $2f$

Hence $\dfrac{1}{2f} = \dfrac{(1.8-1.2)}{(1.2-1)} - \left[\dfrac{1}{\infty} + \dfrac{1}{R}\right]$

$$\frac{1}{16} = \frac{0.6}{0.2R}$$

$\Rightarrow \qquad R = 48$ cm

Hence correct option is (a).

49. If plane surface is silvered the system acts a concave mirror having focal length

$$= \frac{R}{2} = 24 \text{ cm}$$

Hence correct option is (c).

■ More than one options correct

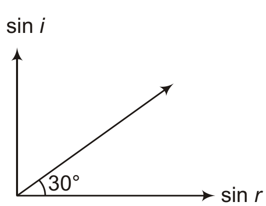
1. Since prism are identical hence if a right prism produce deviation δ inverted prism produce deviation $-\delta$.

if $n = 2m$ deviation becomes zero.

if $n = 2m + 1$ deviation produce is δ

Hence correct options are (a) and (b).

2.



$\because \; \sin i = \mu \sin r$

$\Rightarrow \qquad \mu = \tan 30° = \sqrt{3}$

if speed of light in medium x is v

then speed of light; in medium $y = \dfrac{v}{\mu} = \dfrac{v}{\sqrt{3}}$

Since y is denser w.r.t. x hence total internal reflection take place when incidence in y.

The correct options are (b) and (d).

3. The correct options are (b) and (c).

4. The lens form real image if $D \geq 4F$ (displacement method)

and $\qquad f = \dfrac{D^2 - x^2}{4D}$

and the magnification $m_1 m_2 = 1$

Hence correct options are (b), (c) and (d).

5. Deviation produced by prism

$\delta = (\mu - 1) A = (1.5 - 1) 4° = 2°$

if the mirror is rotated $\theta = 2°$ ray become horizontal after reflection from mirror.

Again if mirror is rotated by 1° reflection ray deviated by 2° from horizontal and after passing through prism again ray become horizontal.

Hence correct option are (a) and (b).

Match the Columns

1. Correct match is

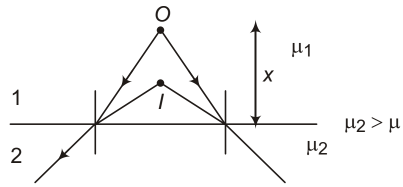
(a) $\rightarrow q, r$ (b) $\rightarrow p, s$

(c) $\rightarrow p, r$ (d) $\rightarrow p, r$

2. (a) $\rightarrow p, r$ (b) $\rightarrow q, s$

(c) $\rightarrow q, r$ (d) $\rightarrow q, r$

3.



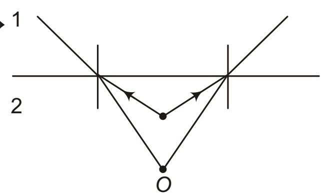
Hence image distance is less than x and virtual.

$\Rightarrow$ (a) $\rightarrow q, s$

Similarly as in (a) the correct match for

(b) $\rightarrow q, r$

(c) $\rightarrow$



(c) $\rightarrow p, s$

(d) p, r

4. (a) $\rightarrow q$,

(b) $\rightarrow r$

(c) $\rightarrow r$

(d) $\rightarrow p$

5. Since $\dfrac{1}{f} = (n-1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

here $n = 1.5$ for (a) $n = 1.4$

$$\Rightarrow \qquad \frac{1}{f} = \left(\frac{1.5}{1.4} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\Rightarrow \qquad f$ is + ve

hence

(a) > $p, s \because f$ increases, power decreases

(b) $\rightarrow q, s \because f$ is – ve and increase in magnification

Similarly

(c) $\rightarrow q, s$ and

(d) $\rightarrow p, s$

6. For real object at $2c$ convex lens form image at $2c$ similarly for virtual object concave lens does.

(a) $\rightarrow q, s$ (b) $\rightarrow q, r$

(c) $\rightarrow q, r$ (d) $\rightarrow q, r$

28 Interference and Diffraction of Light

Introductory Exercise 28.1

1. Because they are incoherent *ie*, $\Delta\phi$ does not remain constant.

2. Since laser is highly coherent and monochromatic source of light

3. $I = I_0 \cos^2 \theta/2$

$\Rightarrow \quad \frac{3I_0}{4} = I_0 \cos^2 \theta/2$

$\Rightarrow \quad \cos \theta/2 = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \frac{\theta}{2} = \frac{\pi}{6}$

$\Rightarrow \quad \theta = \frac{\pi}{3}$

$\Rightarrow \quad \Delta x = \frac{\lambda}{2\lambda} \times \phi = \frac{\lambda}{2\pi} \times \frac{\pi}{3} = \frac{\lambda}{6}$

But $\quad \Delta x = \frac{yd}{D}$

$\Rightarrow \quad y = \frac{D\,\Delta x}{d} = \frac{D}{d} \times \frac{\lambda}{6}$

$\Rightarrow y = \frac{1.2 \times 600 \times 10^{-9}}{0.25 \times 10^{-2} \times 6} = 48\ \mu\text{m}$

4. $2\mu t = \left(n - \frac{1}{2}\right)\lambda$ for minimum thickness $n = 1$

$\Rightarrow \quad t = \frac{\lambda}{4\mu} = \frac{3}{4 \times 1.5} = 0.5\,\text{cm}$

5. Here $a_1 = 3\,a$ and $a_2 = a$

$R^2 = (3a)^2 + (a)^2 + 2 \times 3a \times a \times \cos\theta$

$\Rightarrow I = 9I_0 + I_0 + 6I_0 \cos\theta$

$\Rightarrow I = [10 + 6\cos\theta]I_0$

$$I = \left[10 + 6 \times \left[2\cos^2 \frac{\theta}{2} - 1\right]\right] I_0$$

$$= \left[10 + 12\cos^2 \frac{\theta}{2} - 6\right] I_0$$

$$= \left[4 + 12\cos^2 \frac{\theta}{2}\right] I_0$$

$$= 4I_0 \left[1 + 3\cos^2 \frac{\theta}{2}\right]$$

Now, $I_{max} = \frac{I_0}{9}$

$\Rightarrow \quad I = \frac{4}{9} I_{max} \left[1 + 3\cos^2 \frac{\theta}{2}\right]$

6. $\Delta x = \frac{yd}{D} - (\mu - 1)\,t$

if $\quad t = \frac{\lambda}{2(\mu - 1)}$

$\Rightarrow \quad \Delta x = \frac{yd}{D} - \frac{\lambda}{2}$

For maxima $\Delta x = n\lambda$

$\Rightarrow \quad \frac{yd}{D} = \left(2n + \frac{1}{2}\right)\lambda$

This become minima.

For minima $\Delta x = \left(n - \frac{1}{2}\right)\lambda$

$$\frac{yd}{\Delta} - \frac{\lambda}{2} = n\lambda - \frac{\lambda}{2}$$

$\Rightarrow \frac{yd}{D} - n\lambda$ this become maxima.

Hence maxima and minima are interchanged.

7. For two slit experiment

$$d \sin\theta = n\lambda$$

$$\Rightarrow \quad \sin\theta = \frac{n\lambda}{d}$$

But $\quad \sin\theta \le 1 \Rightarrow \frac{n\lambda}{\delta} \le 1$

$$\Rightarrow \quad n \le \frac{d}{\lambda} \Rightarrow n \le \frac{4\times10^{-6}}{6\times10^{-7}} = 6.67$$

$$\Rightarrow \quad n = 6$$

8. Since amplitude of each wave is equal. The amplitude of resultant wave is zero if waves are equally displaced in phase

ie, $\quad \theta = \frac{360°}{8} = 45°$

Hence phase difference must be 45°

AIEEE Corner

Subjective Question (Level-1)

1. $R^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos\phi$

(i) $R = 2a,\ a_1 = a_2 = a$

$$4a^2 = a^2 + a^2 + 2a^2 \cos\phi$$

$$\Rightarrow \cos\phi = 1 \Rightarrow \phi = 0°$$

(ii) $2a^2 = 2a^2 + 2a^2 \cos\phi$

$$\Rightarrow \phi = 90°$$

(iii) $a^2 = 2a^2 + 2a^2 \cos\phi$

$$\Rightarrow \cos\phi = \frac{-1}{2} \Rightarrow \phi = 120°$$

(iv) $0 = 2a^2 + 2a^2 \cos\phi \Rightarrow \phi = 180°$

2. $$\frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{\left(\frac{a_1}{a_2} + 1\right)^2}{\left(\frac{a_1}{a_2} - 1\right)^2}$$

$$\Rightarrow \quad \frac{I_{\max}}{I_{\min}} = \frac{\left(\frac{5}{3} + 1\right)^2}{\left(\frac{5}{3} - 1\right)^2} = \frac{8^2}{2^2} = 16$$

$$I_{\max} : I_{\min} = 16 : 1$$

3. $I = I_{\max} \cos^2 \frac{\theta}{2}$

$$\Rightarrow \quad \frac{I_{\max}}{2} = I_{\max} \cos^2 \frac{\theta}{2}$$

$$\Rightarrow \quad \cos\frac{\theta}{2} = \cos\frac{\pi}{4}$$

$$\Rightarrow \quad \theta = \frac{\pi}{2}$$

Now, $\quad \phi = \frac{2\pi}{\lambda} \times \Delta x$

$$\frac{\pi}{2} = \frac{2\pi}{\lambda} \times \Delta x$$

$$\Rightarrow \quad \Delta x = \frac{\lambda}{4}$$

But $\quad \Delta x = \frac{yd}{D}$

$$\Rightarrow \quad y = \frac{D\,\Delta x}{d}$$

$$\Rightarrow \quad y = \frac{1\text{m} \times \lambda}{4 \times 1\text{mm}} = \frac{1 \times 500 \times 10^{-9}}{4 \times 10^{-3}}$$

$$\Rightarrow \quad y = 1.25 \times 10^{-4}\ \text{m}$$

4. $I = I_{\max} \cos^2 \frac{\theta}{2}$

(a) $\frac{I_{\max}}{2} = I_{\max} \cos^2 \frac{\theta}{2} \Rightarrow \theta = \frac{\pi}{2}$

Now $\theta = \frac{2\pi}{\lambda} \times \Delta x$

$$\Rightarrow \Delta x = \frac{\lambda}{4} \text{ and } \Delta x = \frac{yd}{D} \Rightarrow \ = \frac{D\lambda}{4d}$$

(b) $\frac{1}{4} I_{\max} = I_{\max} \cos^2 \frac{\theta}{2}$

$$\Rightarrow \cos\frac{\theta}{2} = \cos\frac{\pi}{3} \Rightarrow \quad \theta = \frac{2\pi}{3}$$

$$\phi = \frac{2\pi}{\lambda} \times \Delta x$$

$$\Rightarrow \frac{2\pi}{3} = \frac{2\pi}{\lambda} \times \Delta x \Rightarrow \Delta x = \frac{\lambda}{3}$$

Now $\Delta x = \frac{yd}{D} \Rightarrow y = \frac{D\lambda}{3d}$

5. (a) $I = I_{max} \cos^2 \frac{\theta}{2}$

$I_{max} = I_0$ and $\theta = 60°$

$$\Rightarrow \quad I = I_0 \cos^2 30° = \frac{3}{4} I_0 = 0.75 I_0$$

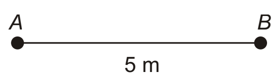
(b) $\theta = \frac{2\pi}{\lambda} \times \Delta x$

$$\Rightarrow \quad \frac{\pi}{3} = \frac{2\pi}{\lambda} \times \Delta x$$

$$\Rightarrow \quad \Delta x = \frac{\lambda}{6} = \frac{480}{6} \text{ nm}$$

$$\Rightarrow \quad \Delta x = 80 \text{ nm}$$

6. $\lambda_A = \lambda_B = 6\text{m}$



For constructive interference difference wavelength $= 0, \lambda, 2\lambda$

$\because \quad \lambda = 6 > d_{AB} = 5$

Hence only constructive interference occur at $\Delta\lambda = 0$

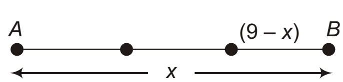
$$\Rightarrow \quad x = \frac{5}{2}\text{m} = 2.5\text{ m}$$

For destructive interference $\Delta\lambda = \lambda, \frac{3\lambda}{2}$

Only possibility at $\Delta\lambda = 6$

Which occur at $x = 1$ m and $x = 4$ m from A.

7. The wavelength $\lambda = \frac{c}{n} = \frac{3 \times 10^8}{120 \times 10^6} = 2.5$ m



For constructive interference

$x - (9 - x) = n\lambda$, where $n = 0, 1, 2\ldots$

$$x = 4.5, 5.75, 7, 8.25$$

The other points are 3.25, 2, .75

8. In Young's double slit experiment

$$w = \frac{D\lambda}{d}$$

$$\Rightarrow 2.82 \times 10^{-3}\text{m} = \frac{2.2 \times \lambda}{.460 \times 10^{-3}}$$

$$\Rightarrow \quad \lambda = 590 \text{ nm}$$

9. x_n for bright fringe is given by

$$x_n = \frac{nD\lambda}{d}$$

$$\Rightarrow \quad x_1 = \frac{D\lambda}{d} \text{ and } x_2 = \frac{2D\lambda}{d}$$

$\Delta x = \frac{D\lambda}{d}$ the angular separation

$$\sin(\Delta\theta) = \frac{\Delta x}{D} = \frac{\lambda}{d} = \frac{5 \times 10^{-7}}{2 \times 10^{-3}} = 2.5 \times 10^{-4}$$

$$(\Delta\theta) = \sin^{-1}[0.00025]$$

$$= 0.014°$$

10. When whole appratus is immersed in water

$$\lambda = n_w \lambda = \frac{4}{3} \times 700 \times 10^{-9}\text{m}$$

$$w = \frac{D\lambda}{d} = \frac{48 \times 10^{-2} \times 4 \times 7 \times 10^{-7}}{3 \times 25 \times 10^{-5}} = 0.90 \text{ mm}$$

11. $w_1 = \frac{D_1\lambda}{d}$ and $w_2 = \frac{D_2\lambda}{d}$

$$\Rightarrow w_1 - w_2 = \frac{(D_1 - D_2)\lambda}{d}$$

$$\Rightarrow 3 \times 10^{-5} = \frac{1.5 \times 10^{-2}}{10^{-3}} \times \lambda$$

$$\Rightarrow \lambda = \frac{3}{1.5} \times \frac{10^{-8}}{10^{-2}} = 2 \times 10^{-6}\text{m} = 2\mu\text{m}$$

12. For bright fringe $x_n = \frac{Dn\lambda}{d}$

For first light ($\lambda = 480$ nm) the third order Bright fringe is $x_3 = \frac{1 \times 3 \times 480 \times 10^{-9}}{5 \times 10^{-3}}$

For second light ($\lambda = 600$ nm)

$$x_3' = \frac{1 \times 3 \times 600 \times 10^{-9}}{5 \times 10^{-3}}$$

$$\Delta x = x_3' - x_3 = \frac{3\times 10^{-9}\times(600-480)}{5\times 10^{-3}}$$

$$= \frac{3\times 120}{5}\times 10^{-6}$$

$$\Delta x = 72\times 10^{-6}, \Delta x = 72\,\mu\text{m}$$

13. Fringe width :

$$\omega = \frac{\lambda D}{d}$$

$$= \frac{(500\times 10^{-9})\,(75\times 10^{-2})}{(0.45\times 10^{-3})}\text{ m}$$

$$= 0.83\times 10^{-3}\text{ m} = 0.83\text{ mm}$$

Distance between second and third dark line = one fringe width = 0.83 mm.

14. For first order bright fringe

$$x = \frac{D\lambda}{d}$$

$$\Rightarrow\quad 4.94\times 10^{-3} = \frac{3\times 600\times 10^{-9}}{d}$$

$$\Rightarrow\quad d = \frac{18\times 10^{-7}}{4.94\times 10^{-3}} = \frac{18}{5.94}\times 10^{-4}\text{m}$$

Let for wavelength λ first dark fringe is obtained at this point for first dark fringe

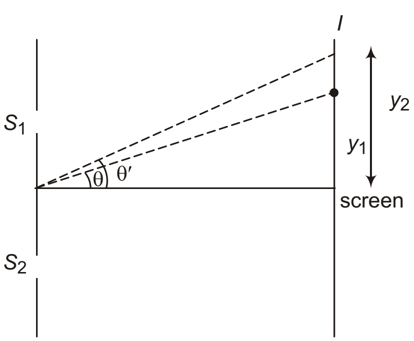
$$x = \frac{D\lambda}{2d}$$

$$\Rightarrow 4.94\times 10^{-3} = \frac{3\times\lambda\times 4.94}{2\times 18\times 10^{-4}}$$

$$\Rightarrow 2\times 18\times 10^{-7} = 3\lambda$$

$$\Rightarrow\quad \lambda = 12\times 10^{-7}\text{m}\ \lambda = 1200\text{ nm}$$

15.



For dark fringe

$$d\sin\theta = (2n-1)\frac{\lambda}{2}$$

For first dark fringe $n = 1$ and for 2nd, $n = 2$

$$d\sin\theta = \frac{\lambda}{2}$$

$$d\sin\theta' = \frac{3\lambda}{2} \Rightarrow \sin\theta = \frac{\lambda}{2d}$$

$$\Rightarrow\quad \frac{y_1}{\sqrt{D^2+y_1^2}} = \frac{\lambda}{2d}$$

$$\text{and}\quad \frac{y_2}{\sqrt{D^2+y_1^2}} = \frac{3\lambda}{2d}$$

$$\Rightarrow\quad \frac{y_1}{\sqrt{(35\times 10^{-2})^2+y_1^2}} = \frac{550\times 10^{-9}}{2\times 1.8\times 10^{-6}} \quad \ldots\text{(i)}$$

$$\text{and}\quad \frac{y_2}{\sqrt{(35\times 10^{-2})^2+y_2^2}} = \frac{3\times 550\times 10^{-9}}{2\times 1.8\times 10^{-6}}$$

On solving $y_2 - y_1 = 12.6\,\text{cm}$

16. Wavelength of source $\lambda = \dfrac{\lambda}{\sqrt{2meV}}$

$$\Rightarrow \lambda = \frac{6.62\times 10^{-34}}{\sqrt{2\times 9.1\times 10^{-31}\times 100\times 1.6\times 10^{-19}}}$$

$$\Rightarrow \lambda = 1.24\times 10^{-10}\text{m}$$

$$w = \frac{D\lambda}{d} = \frac{3\times 1.24\times 10^{-10}}{10\times 10^{-10}}\text{m} = 36.6\,\text{cm}$$

17. Given $\lambda = 546\text{ nm} = 5.46\times 10^{-7}\text{m}, D = 1\text{m}$

$d = 0.3\text{ mm} = 0.3\times 10^{-3}\text{m}$

(a) At distance $y = 10\,\text{mm} = 10\times 10^{-3}\text{m}$ from the central fringe path difference will be

$$\Delta y = \frac{y.d}{D} = \frac{10\times 10^{-3}\times 0.3\times 10^{-3}}{1}$$

$$= 3\times 10^{-6}\text{ m}$$

Corresponding phase difference

$$\phi = \frac{2\lambda}{\lambda}\times\Delta x = \frac{2\pi}{5.46\times 10^{-7}}\times 3\times 10^{-6}\text{ rad}$$

$$= 1978^\circ$$

$$I = I_0 \cos^2 \frac{\theta}{2}$$

$$= I_0 \cos^2 \left(\frac{1978^\circ}{2}\right) = I_0 \cos^2 989^\circ$$

$$I = 3 \times 10^{-4} I_0$$

(b) Fringe width $w = \frac{\lambda D}{d}$

$$w = \frac{5.46 \times 10^{-7} \times 1}{0.3 \times 10^{-3}} = 1.82\,\text{mm}$$

$$\text{Number of fringes} = \frac{10\,\text{mm}}{1.82\,\text{mm}} = 5.49$$

Hence the number of fringe is five.

18. Shift due to sheet of thickness 10μ and refractive index 1.6 is

$$\Delta x_1 = \frac{(\mu - 1)\,tD}{d}$$

$$\Rightarrow \Delta x_1 = (1.6 - 1) \times \frac{10 \times 10^{-6} \times 1.5}{1.5 \times 10^{-3}}$$

$$\Rightarrow \Delta x_1 = 0.6 \times 10 \times 10^{-3} = 6 \times 10^{-3}\text{m}$$

Shift due to sheet of thickness 15μ and refractive index 1.2 is

$$\Delta x_2 = \frac{(1.2 - 1) \times 15 \times 10^{-6} \times 1.5}{1.5 \times 10^{-3}}$$

$$\Delta x_2 = 3 \times 10^{-3}\text{m}$$

Since these shifts are in opposite direction of central maxima hence net shift

$$\Delta x = \Delta x_1 - \Delta x_2 = 6 \times 10^{-3}\text{m} - 3 \times 10^{-3}\text{m}$$

$$= 3 \times 10^{-3}\text{m} = 3\text{ mm}$$

19. Let λ is the wavelength of light D is screen distance from source and d is the separation between slits (all are in metres)

$$\text{Shift} = \Delta x = \frac{(\mu - 1)\,tD}{d}$$

$$\Rightarrow \quad \Delta x = \frac{(1.6 - 1) \times 1.964 \times 10^{-6} \times D}{d}$$

$$\Rightarrow \quad \Delta x = \frac{1.1784\,D}{d} \times 10^{-6}\text{m}$$

Now when t is removed and D is doubled the distance between successive maximum (or minima) *i.e.*, fringe width

$$w = \frac{2D\lambda}{d}$$

but according to question $\Delta x = w$

$$\Rightarrow \quad \frac{1.178 \times 10^{-6} D}{d} = \frac{2D\lambda}{d}$$

$$\Rightarrow \quad \lambda = 0.589 \times 10^{-6}\text{m} = 589\text{ nm}$$

20. Let n bright fringe ($\lambda = 5500$ Å) concide with 10th

bright fringe of 6000 Å

$$\Rightarrow \quad n \times 5500\text{ Å} = 6000\text{ Å} \times 10$$

$$\Rightarrow \quad n \simeq 11$$

Similarly first bright fringe concide with 1st fringe. Now fringe width

$$w = \frac{14.74 - 12.5}{10} = 0.224\text{ mm}$$

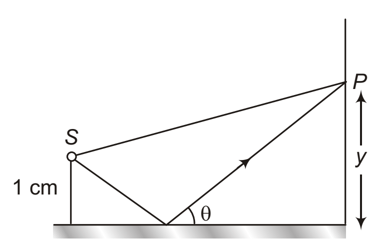
Hence position of 10th bright fringe

$$= 14.74 - 0.224 \simeq 14.55\,\text{mm}$$

Position of zero order bright fringe

$$= 12.75 - 0.224 \simeq 12.25\,\text{mm}$$

21. Here $d \simeq 1$ cm



$$D = 100\,\text{m}$$

$$\lambda = 500\text{ nm}$$

For first dark fringe

$$y = \frac{\Delta\lambda}{2d} = \frac{100 \times 500 \times 10^{-9}}{2 \times 10^{-2}}$$

$\Rightarrow \quad y = 2.5\,\text{mm}$

22. The destructive interference will be $2\mu t = n\lambda$ for thinnest $n = 1$

$\Rightarrow \quad 2\mu t = \lambda$

$\Rightarrow \quad t = \dfrac{\lambda}{2\mu} = \dfrac{650 \times 10^{-9}}{2 \times 1.42}$

$\Rightarrow \quad t = 114\text{ nm}$

23. Here $t = 0.485\,\mu\text{m}$, $t = 485\text{ nm}$, $n = 1.53$

(a) Condition for constructive interference in refractive system

$$2\mu t = \left(n - \frac{1}{2}\right)\lambda \quad \text{where, } n = 1, 2$$

For $\quad n = 1, \lambda_1 = 4\mu t = 4 \times 1.53 \times 485$

$= 2968.2\,\text{mm}$

which does not lie in visible region put

$n = 2, 3, \ldots\ldots$

we get $\lambda = 424\text{ nm}, 594\text{ nm}, \ldots$

(b) For structive interference in transmitted system

$2\mu t = n\lambda$, putting $n = 1, 2,$

only $\lambda = 495\,\text{nm}$ is lie in visible region.

24. For constructive interference

$2\mu t = \left(n - \dfrac{1}{2}\right)\lambda$ for t to be minimum $n = 1$

$\Rightarrow \quad t = \dfrac{\lambda}{4\mu} = \dfrac{6000}{4 \times 1.3} = 1154\ \text{Å}$

25. For destructive interference

$2\mu t = n\lambda$ for t to be minimum

$n = 1$

$\Rightarrow \quad t = \dfrac{\lambda}{2\mu} = \dfrac{800}{2\mu} \quad \ldots\text{(i)}$

For constructive interference

$2\mu t = (2n - 1)\dfrac{\lambda}{2}$

$2\mu \times \dfrac{800}{2\mu} = (2n - 1)\dfrac{\lambda}{2}$

$1600 = (2n - 1)\lambda$

For $n = 1$, $\lambda = 600\text{ nm}$ which does not lie in visible region

For $\quad n = 2$

$1600 = 3\lambda$

$\Rightarrow \quad \dfrac{1600}{3} = \lambda \Rightarrow \lambda = 533\text{ nm}$

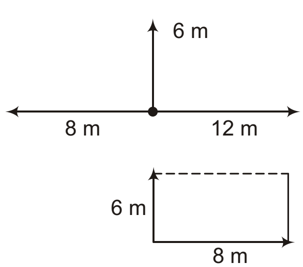
for $n = 3$, $\lambda = \dfrac{1600}{5} = 320\text{ nm}$ which does lie in visible

Hence $\lambda = 533\text{ nm}$

Objective Questions (Level-1)

■ (Single option correct)

1. Using phasor method



$$R = \sqrt{8^2 + 6^2} = 10\,\text{m}$$

Hence correct option is (b).

2. Here $\dfrac{I_1}{I_2} = B^2$

$$I_{\max} = [\sqrt{I_1} + \sqrt{I_1}]^2$$
$$I_{\min} = [\sqrt{I_1} - \sqrt{I_2}]^2$$

$$\frac{I_{min}}{I_{max}} = \left[\frac{\sqrt{\frac{I_1}{I_2}} - 1}{\sqrt{\frac{I_1}{I_2}} + 1}\right]^2 = \left[\frac{\beta - 1}{\beta + 1}\right]^2$$

$$\Rightarrow \quad \frac{(I_{max} - I_{min})}{(I_{max} + I_{min})} = \left[\frac{\beta + 1 - \beta + 1}{2\beta}\right]^2$$

$$\Rightarrow \quad \frac{I_{max} - I_{min}}{I_{max} - I_{min}} = \frac{1}{\beta^2}$$

Hence, correct option is (d).

3. For nth dark fringe

$$x_n = (2n - 1)\frac{D\lambda}{2d}$$

For 1st dark fringe $n = 1$

$$\Rightarrow \quad x_1 = \frac{D\lambda}{2d}$$

Angular position $\theta = \sin^{-1}\left(\frac{x_1}{D}\right)$

$$\Rightarrow \quad \theta = \sin^{-1}\left[\frac{\lambda}{2a}\right]$$

$$\Rightarrow \quad \theta = \sin^{-1}\left[\frac{5460 \times 10^{-10}}{2 \times 0.1 \times 10^{-3}}\right]$$

$$\Rightarrow \quad \theta = \sin^{-1}[273 \times 10^{-5}]$$

$$\Rightarrow \quad \theta = \sin^{-1}[0.00273]$$

$$\Rightarrow \quad \theta = 0.16^\circ$$

Hence correct option is (b)

4. For 10th bright fringe $x_{10} = \frac{10D\lambda}{d}$

For 6th dark fringe $x_6 = (2 \times 6 - 1)\frac{D\lambda'}{2d}$

But $\quad x_6 = x_{10}$

$$\frac{11}{2a}D\lambda' = \frac{10D\lambda}{d}$$

$$\Rightarrow \quad \frac{\lambda'}{\lambda} = \frac{20}{11}$$

But $\quad \mu = \frac{\lambda'}{\lambda} = \frac{20}{11} = 1.8$

Hence correct option is (a).

5. Shift $= \frac{(\mu - 1)tD}{d}$

$$= \frac{(1.5 - 1) \times 10 \times 10^{-6} \times 100 \times 10^{-2}}{2.5 \times 10^{-3}}$$

$$= \frac{5 \times 10^{-6}}{2.5 \times 10^{-3}} = 2 \times 10^{-3}\text{m} = 2\text{mm}$$

Hence correct option is (a).

6. $x_n = \frac{Dn\lambda}{d}$ for nth bright fringe

$x_n' = \frac{D\lambda}{2d}(2n - 1)$ for nth dark fringe

$$\Delta x = x_n - x_n' = \frac{D\lambda}{2d}[2n - 2n + 1]$$

$$\Delta x = \frac{D\lambda}{2d}$$

Hence correct option is (c).

7. Since at centre path difference for all colour is always zero hence centre will be white.

Hence correct option is (a).

8. $n_1 \times \lambda_1 = n_2 \times \lambda_2$

$$60 \times 4000 = n_2 \times 6000 \Rightarrow n_2 = 40$$

Hence correct option is (a).

9. Initial fringe width $w_1 = \frac{D\lambda}{d}$

Final fringe width $w_2 = \frac{(D - 5 \times 10^{-2})\lambda}{d}$

$$|\Delta w| = |w_2 - w_1| = \frac{\lambda}{d} \times 5 \times 10^{-2}$$

$$\Rightarrow \quad 3 \times 10^{-5} = \frac{\lambda}{10^{-3}} \times 5 \times 10^{-2}$$

$$\Rightarrow \quad \frac{3 \times 10^{-6}}{5} = \lambda \Rightarrow 6000\ \text{Å} = \lambda$$

Hence correct option is (a).

10. $\dfrac{I_{max}}{I_{min}} = \dfrac{49}{9}$

Now $\dfrac{I_{max}}{I_{min}} = \left[\dfrac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right]^2 = \left[\dfrac{\sqrt{\dfrac{I_1}{I_2}}+1}{\sqrt{\dfrac{I_1}{I_2}}-1}\right]^2$

$$\frac{I_{max}}{I_{min}} = \left[\frac{\sqrt{\dfrac{49}{9}}+1}{\sqrt{\dfrac{49}{9}}-1}\right]^2 = \left[\frac{\dfrac{7}{3}+1}{\dfrac{7}{3}-1}\right]^2 = \frac{100}{16}$$

$\Rightarrow \dfrac{I_{max}}{I_{min}} = \dfrac{25}{4}$

Hence correct option is (a)

11. $x_n = \dfrac{Dn\lambda}{d}$

$\Rightarrow \quad x_3 = \dfrac{3D\lambda}{d} = 7.5 \times 10^{-3}\text{m}$

$\Rightarrow \quad \lambda = \dfrac{7.5 \times 10^{-3} \times 0.2 \times 10^{-3}}{3 \times 1}$

$\Rightarrow \quad \lambda = 0.5 \times 10^{-6} = 500 \text{ nm}$

Hence correct option is (b).

12. Let nth fringe of 6500 Å concide with n'th fringe of 5200 Å.

$\Rightarrow x_n = \dfrac{Dn \times 6500 \text{ Å}}{d} = \dfrac{Dn'}{d} \times 5200 \text{ Å}$

$\Rightarrow \dfrac{n}{n'} = \dfrac{5200}{6500} = \dfrac{4}{5}$

It means 4th fringe of 6500 Å coincide with 5th fringe of 5200 Å hence the distance

$$x = \frac{4 \times 120 \times 10^{-2} \times 6500 \times 10^{-10}}{2 \times 10^{-3}}$$

$x = 0.156 \text{ cm}$

Hence correct option is (a).

13. Since number of minima does not depends on orientation hence $n_1 = n_2$

Hence correct option is (a).

■ Assertion and Reason

1. We have $I = 4I_0 \cos^2 \dfrac{\phi}{2}$ if $\phi = \dfrac{2\pi}{3}$

we get $I = I_0$

Hence assertion is true.

Now path difference $= \dfrac{\lambda}{2\pi} \times$phase difference

Path difference $= \dfrac{\lambda}{2\pi} \times \dfrac{2\pi}{3} = \dfrac{\lambda}{3}$

Hence reason is true.

But reason is not the explanation of assertion. Hence correct option is (b).

2. Here assertion and reason are both true but reason is not correct explanation of assertion. Correct option is (b).

3. Assertion is wrong since fringes are symmetrical *ie*, fringes obtained both above and below point O. Reason is true.

Correct option is (a).

4. Here both assertion and reason are true and reason correctly explain assertion. Hence correct option is (a).

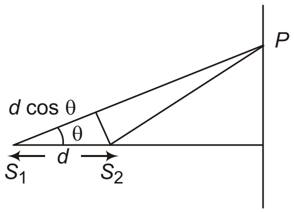
5. Both assertion and reason are true and reason correctly explain the assertion.

Hence correct option is (a).

6. Assertion is true since locus of all fringes is circle. But reason is wrong since fringes may have any shape.

Correct option is (c).

7.



Path difference at point P is $\Delta x = d\cos\theta$

The path difference decreases as θ increases.
$\because$ as $\theta \to$ increases, $\cos\theta \to$ decreases

Hence order of fringe $n = \dfrac{d\cos\theta}{\lambda}$ decreases as we go above P. Hence assertion is wrong (false).

For 11th order maxima path differenc is more hence reason is true but assertion is false correct option is (d).

8. Here assertion is true and reason is false and reason does not correctly explain assertion. Correct option is (c).

9. $\because\ d\sin\theta = n\lambda$

$$\Rightarrow \sin\theta = \frac{n\lambda}{d} \text{ but } \frac{d}{\lambda} = 4$$

$$\Rightarrow \sin\theta = \frac{n}{4}$$

Now if $\theta = 30°$

$$\Rightarrow \quad n = 4\sin 30° = 2$$

Hence assertion is true.

Also reason is true and does not correctly explain assertion correct option is (b).

10. Here assertion is false. Since shift $= \dfrac{(\mu - 1)tD}{d}$ is independent of λ.

Hence shift of red colour = shift of violet colour.

and reason is true

$$\because \quad \mu_V > \mu_R$$

Hence correct option is (d)

Objective Questions (Level 2)

1. $I = 4I_0\cos^2\dfrac{\pi x}{\alpha}$

$$\Rightarrow \quad I_0 = 4I_0\cos^2\frac{\lambda x_1}{\alpha}$$

$$\Rightarrow \quad x_1 = \frac{\alpha}{3} \quad \ldots\text{(i)}$$

and $\quad 2I_0 = 4I_0\cos^2\dfrac{\pi x_2}{\alpha}$

$$\Rightarrow \quad x_2 = \frac{\alpha}{4} \quad \ldots\text{(ii)}$$

$$\Delta x = x_1 - x_2 = \frac{\alpha}{3} - \frac{\alpha}{4} = \frac{\alpha}{12}$$

Hence correct option is (c).

2. $I = 4I_0\cos^2\left(\dfrac{\pi x}{\lambda}\right) = k\cos^2\left(\dfrac{\pi x}{\lambda}\right)$

$$I = k\cos^2\left(\frac{\pi}{\lambda}\times\lambda\right) = k \quad \ldots\text{(i)}$$

$$\frac{I}{4} = k\cos^2\left(\frac{\pi x}{\lambda}\right)$$

$$\Rightarrow \frac{k}{4} = k\cos^2\left(\frac{\pi x}{\lambda}\right)$$

$$\Rightarrow \quad \cos\frac{\pi x}{\lambda} = \frac{1}{2}$$

$$\frac{\pi x}{\lambda} = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad \frac{\pi x}{\lambda} = \frac{\pi}{3} \text{ or } \frac{\pi x}{\lambda} = \frac{2\pi}{3}$$

$$\Rightarrow \quad x = \frac{\lambda}{3} \text{ or } x = \frac{2\lambda}{3}$$

Hence correct option is (b).

3. Light of wavelength λ is strongly reflected if

$$2ut = \left(n + \frac{1}{2}\right)\lambda \quad n = 0, 1, 2 \quad \ldots\text{(i)}$$

$$\Rightarrow 2ut = 2\times 1.5\times 5\times 10^{-7}\text{m} = 1.5\times 10^{-6}\text{m}$$

Putting $\lambda = 400$ nm in eq. (1) and using eq. (ii)

$$1.5\times 10^{-6}\text{ m} = \left(n+\frac{1}{2}\right)\times 4\times 10^{-7}\text{ m}$$

$\Rightarrow \quad n = 3.25$

Putting $\quad \lambda = 700$

$$1.5\times 10^{-6}\text{ m} = \left(n+\frac{1}{2}\right)\times 7\times 10^{-7}$$

$$n = 1.66$$

Hence n can take values 2 and 3.

From (i) if $n=2$, $\lambda = \dfrac{4ut}{2\times 2+1} = 600$ nm

if $\quad n=3\ \lambda = 429$ nm

only $\lambda = 600$ is given in the options.

Hence correct option is (b).

4. $x_n = \dfrac{D\lambda n}{d} = \dfrac{100\times 10^{-2}}{0.01\times 10^{-3}}\lambda\times n$

$\Rightarrow \quad x_n = \dfrac{n\lambda}{10^{-5}} \qquad$ For $\lambda = 4000$ Å

$$x_n = \frac{n\times 4000\times 10^{-10}\text{m}}{10^{-5}} = 4\times 10^{-5}\times n$$

$$= 0.04\times n$$

$\Rightarrow x_n = 4n$ mm

Similarly for $\lambda = 7000$ Å

$x_n = 7n$ mm $n = 5, 6$

hence only $x = 5$

Passes through hole $\lambda = 5000$ Å

Hence correct option is (b).

5. $I = 4I_0\cos^2\left(\dfrac{\pi x}{\alpha}\right)$ where $\alpha = \dfrac{\Delta\lambda}{d}$

$\Rightarrow \alpha = \dfrac{1\times 6000\times 10^{-10}}{1\times 10^{-3}} = (6\times 10^{-4})$

$$75\%\ I_0 = I_0\cos^2\left(\frac{\pi x}{\alpha}\right)$$

$$\frac{3}{16} = \cos^2\frac{\pi x}{\alpha}$$

$\Rightarrow \quad \dfrac{\sqrt{3}}{4} = \cos\dfrac{\pi x}{\alpha}$

$$\pi x = \alpha\cos^{-1}\left(\sqrt{\frac{3}{4}}\right) \Rightarrow x = \frac{\alpha}{\pi}\cos^{-1}\left(\sqrt{\frac{3}{4}}\right)$$

$\Rightarrow \quad x = 0.20$ mm

Hence correct option is (d).

6. Number of fringes shifted $= \dfrac{(\mu-1)\,t}{\lambda}$

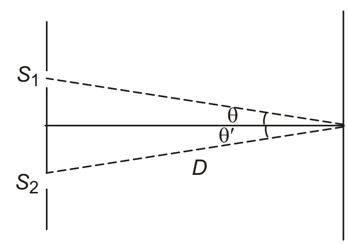
$$4 = \frac{(1.5-1)\times t}{6000\text{ Å}}$$

$\Rightarrow \quad \dfrac{4}{0.5}\times 6\times 10^{-7}\text{m} = t$

$\Rightarrow \quad t = 4.8\ \mu$m

Hence correct option is (a).

7. For nth order minima



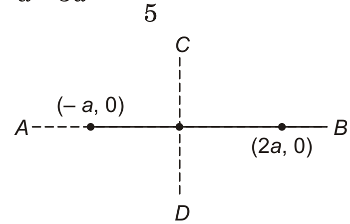
$y_n = \dfrac{(2n-1)\,D\lambda}{2d}$ for 3rd minima $n=3$

$$y_3 = \frac{5D\lambda}{2d} = \frac{5\lambda}{2\theta}$$

$\because \quad \theta \simeq \tan\theta = \dfrac{d}{D}$

Hence correct option is (b).

8. $AB = d = 3a = \dfrac{3a\times 5}{5}$



$AB = 15\lambda$

Hence total maxima $= 14\times 4 + 4$

$= 60°$

Hence correct option is (a).

9. $I = 4I_0 \cos^2\left[\frac{\pi x}{\alpha}\right]$

$\Rightarrow \; I = 4I_0 \cos^2\left[\frac{\pi}{\alpha} \times \frac{\alpha}{4}\right]$

$I = 4I_0 \cos^2 \frac{\pi}{4}$

$= \frac{4I_0}{2} = 2I_0$

$\Rightarrow \; \frac{I_0}{I} = \frac{1}{2}$

Hence correct option is (b).

10. $I = I_0 \cos^2\left[\frac{\pi}{\alpha}(\mu - 1)\, t\right]$

at $\mu = I \;\; \Rightarrow \;\; I = I_0$

Hence correct variation is (c)

11. $I = I_0 \cos^2\left(\frac{\pi x}{\lambda}\right)$

$\frac{3}{4} I_0 = I_0 \cos^2\left(\frac{\pi x}{\lambda}\right)$

$\Rightarrow \quad \frac{\pi x}{\lambda} = \frac{\pi}{6}$

$\Rightarrow \quad x = \frac{\lambda}{6}$ but $x = (\mu - 1)\, t$

$\Rightarrow \quad (1.5 - 1) \times t = \frac{\lambda}{6}$

$\Rightarrow \quad t = \frac{\lambda}{3} = \frac{6000}{3}$ Å

$\Rightarrow \quad t = 2000$ Å $= 0.2\,\mu$m

Hence correct option is (a).

12. Net shift $= (\mu_1 - 1)\, t - (\mu_2 - 1)\, t$

$= (\mu_1 - \mu_2)\, t = (1.52 - 1.40) \times t$

Net shift $= 0.12 \times 10.4\ \mu$m

$= 12 \times 104 \times 10^{-9}$ m

$= 1248 \times 10^{-9}$ m

$= 1248$ nm

Net shift $= n\lambda$

where n is + ve integer

$\Rightarrow \quad n\lambda = 1248$

Now, for $416 = \lambda,\ n = 3$

For $624 = \lambda,\ n = 2$

Hence correct option is (c).

13. $w = \frac{\lambda D}{d \times n} = \frac{6300 \text{ Å} \times 1.33 \text{ m}}{1 \text{ mm} \times 1.33}$

$= \frac{63 \times 10^{-8} \times 1.33 \text{ m}}{10^{-3} \times 1.33}$

$= 0.63$ mm

Hence correct option is (a).

14. $\Delta x = \frac{7D\lambda}{nd} - \frac{3D\lambda}{nd} = 4\left(\frac{D\lambda}{nd}\right)$

$\Delta x = 4 \times 0.63 \text{ mm} = 2.52 \text{ mm}$

Correct option is (a).

15. $\Delta x = 2$ of fridge width

$(\mu - 1)\, t = \frac{2 \times 0.63 \text{ mm}}{2}$

$t = \frac{2 \times 0.63}{0.53} = 1.57$ mm

Hence correct option is (b).

16. Since on introducing thin glass sheet fringe width does not change hence fringe width $= 0.63$ mm.

The correct option is (a).

More than one options correct

1. At centre path difference between all colours is zero hence cnetre is white since violet has least wavelength hence next to central will be violet and since intensity is different for different for colours hence there will be not a completely dark fringe

   Hence correct options, are (b), (c) and (d).

2. Correct options are (a), (c) and (d).

3. $\because I = 4I_0 \cos^2 \frac{\theta}{2}$ at centre $\theta = 0 \Rightarrow I = 4I_0$

   and at distance 4 mm above o is again maxima hence its intensity is also $4I_0$.

   Hence correct options are (a) and (c).

4. The correct options, are (a), (c) and (d)

   Since phase difference = constant

   Light should be monochromatic.

5. Since in this case fringe pattern shift upward hence the correct options are (a), (b) and (c).

6. $\because n_1\lambda_1 = n_2\lambda_2$ for maxima

   Using this option (a) is satisfied and $(2n_1 - 1)\lambda_1 = (2n_2 - 1)\lambda_2$ for minima

   Using this 3rd option is satified

   Hence correct options are (a) and (c).

Match the Columns

1. $\because I = 4I_0 \cos^2 \frac{\theta}{2}$

   if $\theta = 60°$ $I = 3I_0$

   if $\theta = 90°$ $I = 2I_0$

   if $\theta = 0°$ $I = 4I_0$

   if $\theta = 120°$ $I = I_0$

   Hence correct match is

   (a) $\rightarrow s$; (b) $\rightarrow q$; (c) $\rightarrow p$; (d) $\rightarrow r$

2. $\Delta x = \frac{\lambda}{2\pi} \times \Delta\phi$

   $\Rightarrow \Delta\phi = \frac{2\pi}{\lambda}\Delta x$ if $\Delta x = \frac{\lambda}{3}$

   $\Rightarrow \Delta\phi = \frac{2\pi}{3} = 120°$

   if $\Delta x = \frac{\lambda}{6}$

   $\Delta\phi = 60°$ if $\Delta x = \frac{\lambda}{4}, \Delta\phi = 90°$

   Using $I = 4I_0 \cos^2\left(\frac{\Delta\theta}{2}\right)$

   The correct match are

   (a) $\rightarrow q$; (b) $\rightarrow p$; (c) $\rightarrow r$; (d) $\rightarrow s$

3. Distance between third order maxima and central maxima $= \frac{3D\lambda}{d} = 3w$

   Distance between 3rd order minima and central

   Maxima $= \left(3 - \frac{1}{2}\right)\frac{D\lambda}{d} = 2.5\frac{D\lambda}{d} = 2.5\,w$

   Distance between first minima and forth order

   maxima $= \frac{4D\lambda}{d} = \frac{D\lambda}{2d} = 2.5\,w$

   Distance between first minima and forth order

   maxima $= \frac{4D\lambda}{d} - \frac{D\lambda}{2d} = 3.5\,w$

   Distance between 2nd order maxima and fifth order minima $= 4.5\,w - 2w = 2.5\,w$

   hence correct match is

   (a) $\rightarrow q$; (b) $\rightarrow p$; (c) $\rightarrow r$; (d) $\rightarrow p$

4. Since fringe shift in the division of sheet placed soure hence

(a) $\rightarrow p$

Similarly for other the correct match

(b) $\rightarrow r, s$; (c) $\rightarrow p$; (d) $\rightarrow p$

5. When $y = \dfrac{\lambda D}{2d}$ there will a dark fringe at 0.

hence (a) $\rightarrow q$

$$\text{when } y = \frac{\lambda D}{6d} = 3\left(\frac{\lambda D}{2d}\right)$$

The intensity becomes $3I$

(b) $\rightarrow p$

$$\text{when } y = \frac{\lambda D}{4d} \quad \text{Intensity} = 2I$$

(c)$\rightarrow s$

$$\text{when } y = \frac{\lambda D}{3d} \quad \text{Intensity} = I$$

(d) $\rightarrow r$

6. When a thin plate (transparent) is placed in front of S_1 zero order fringe shift above from O hence

(a) $\rightarrow r$

When S_1 is closed interference disappear and uniform illuminance is obtained on screen hence

(b) $\rightarrow p, q$

Similarly (c) $\rightarrow r, s$ and

When s is removed and two real sources s_1 and s_2 emitting light of same wavelength are placed interference disappear. Since sources become non-cohrrent hence (d) $\rightarrow p, q$

29. Modern Physics I

Introductory Exercise 29.1

1. The positron has same mass m as the electron. The reduced mass of electron positron atom is

$$\mu = \frac{m \times m}{m + m} = \frac{1}{2}\text{m}$$

$$R_H = \frac{me^4}{8\varepsilon_0^2 ch^3}$$

$$\Rightarrow \quad R_P = \frac{R_H}{2}$$

$$\frac{1}{\lambda_H} = R_H\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\frac{1}{\lambda_P} = R_P\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\Rightarrow \quad \frac{\lambda_P}{\lambda_H} = \frac{R_H}{R_P} = 2$$

$$\Rightarrow \lambda_p = 2\lambda_H = 2 \times 6563 \text{ Å} = 13126 \text{ Å}$$

$$= 1.31\,\mu\text{m}$$

$$\frac{1}{\lambda_{\text{He}}} = R_H\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \cdot z^2$$

$$\Rightarrow \quad \frac{1}{\lambda_{\text{He}}} = \frac{1}{\lambda_{\text{H}}} \cdot z^2 \Rightarrow \quad \lambda_{\text{He}} = \frac{\lambda_{\text{H}}}{2^2} = \frac{6563 \text{ Å}}{2^2}$$

$$\Rightarrow \quad \lambda_{\text{He}} = 164 \text{ nm}$$

2. $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$ for largest wavelength $n = 3$

$$\Rightarrow \quad \frac{1}{\lambda} = R\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\Rightarrow \quad \lambda = \frac{36}{5R} = \frac{36}{5 \times 1.097 \times 10^7}$$

$$\Rightarrow \quad \lambda = 656 \text{ nm}$$

3. For H-atom $r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2}$

$$u_n = \frac{e^2}{2\varepsilon_0 nh}$$

$$T_n = \frac{2\pi r_n}{u_n} = 2\pi \frac{\varepsilon_0 n^2 h^2 \times 2\varepsilon_0 nh}{\pi m e^2 \times e^2}$$

$$= \frac{4\varepsilon_0 n^3 h^3}{me^4}$$

$$r_n = \frac{1}{T_n} = \frac{me^4}{4\varepsilon_0^2 n^3 h^3}$$

$$\Rightarrow \quad r_1 = \frac{me^4}{4\varepsilon_0^2 h^3}$$

$$\Rightarrow \quad \nu_1 = \frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4}{4 \times (8.85 \times 10^{-12})^2 \times (6.6 \times 10^{-34})^3}$$

$$\Rightarrow \quad \nu_1 = 6.58 \times 10^{15} \text{ Hz}$$

$$\nu_2 = \frac{\nu_1}{2^3} = \frac{\nu_1}{8} = \frac{6.58 \times 10^{15}}{8}$$

$$= 0.823 \times 10^{15} \text{ Hz}$$

(b) $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$

$$\Rightarrow \nu = \frac{c}{\lambda} = 3 \times 10^8 \times R\left[\frac{3}{4}\right]$$

$$\Rightarrow \nu = \frac{9 \times 10^8 \times 1.097 \times 10^7}{4}$$

$$= 2.46 \times 10^{15} \text{ Hz}$$

(c) Number of revolutions

$$= v_2 \times T = 0.823 \times 10^{15} \times 1 \times 10^{-8}$$

$$= 8.23 \times 10^6 \text{ revolution}$$

4. Reduce mass $= \dfrac{m_\mu m_p}{m_\mu + m_p} = \dfrac{207\,\text{m} \times 1836\text{m}}{(207\,\text{m} + 1836\text{m})} = 186\,\text{m}$

$$r_1 = 4\pi\varepsilon_0 \left(\frac{h^2}{4\pi^2 \mu e^2}\right) = 4\pi\varepsilon_0 \left(\frac{h^2}{4\pi^2 (186m)\, e^2}\right)$$

Putting the value we get

$$r_1 = 2.55 \times 10^{-13}\text{m}$$

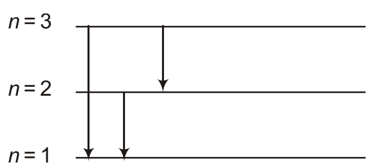
$$E_1 = \frac{-\mu e^4}{8\varepsilon_0^2 h^2} = -2810\,\text{eV}$$

Ionization energy $= -E_1 = 2.81$ keV

5. (a) $\lambda = \dfrac{h}{mv} = \dfrac{6.6 \times 10^{-34}}{46 \times 10^{-3} \times 30} = 4.8 \times 10^{-34}\text{m}$

(b) $\lambda = \dfrac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^7} = 7.3 \times 10^{-11}\text{m}$

6. (a) After absorbing 12.3 eV the atom excited to $n = 3$ state



$$\frac{1}{\lambda_L} = R\left[1 - \frac{1}{n^2}\right]$$

$$\frac{1}{\lambda_{L_1}} = R\left[1 - \frac{1}{9}\right] = \frac{8R}{9}$$

$$\Rightarrow \lambda_{L_1} = \frac{9}{8R} = \frac{9}{8 \times 1.097 \times 10^7} = 102\text{ nm}$$

$$\frac{1}{\lambda_{L_2}} = R\left[1 - \frac{1}{4}\right]$$

$$\Rightarrow \lambda_{L_2} = \frac{4}{3R} = \frac{4}{3 \times 1.097 \times 10^7} = 122\text{ nm}$$

$$\frac{1}{\lambda_B} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$$

$$\Rightarrow \lambda_B = \frac{36}{5R}$$

$$\Rightarrow \lambda_B = \frac{36}{5 \times 1.097 \times 10^7} = 653\text{ nm}$$

7. $\nu_{K_\beta} = \nu_{K_\alpha} + \nu_{L_\alpha} \Rightarrow \lambda_{L_\alpha} = \dfrac{\lambda_{K_\alpha} \times \lambda_{K_\beta}}{\lambda_{K_\alpha} - \lambda_{K_\beta}}$

$$\frac{c}{\lambda_{K_\beta}} = \frac{c}{\lambda_{K_\alpha}} + \frac{c}{\lambda_{L_\alpha}} \Rightarrow \lambda_{L_\alpha} = \frac{0.71 \times 0.63}{0.71 - 0.63}$$

$$\Rightarrow \frac{1}{\lambda_{L_\alpha}} = \frac{1}{\lambda_{K_\beta}} - \frac{1}{\lambda_{K_\alpha}}\ \lambda_{L_\alpha} = 5.59\text{ nm}$$

8. $\lambda = \dfrac{hc}{\Delta E}$

$$\Rightarrow \qquad \lambda_{K_\alpha} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{(E_1 - 2870) \times 1.6 \times 10^{-19}}$$

$$\Rightarrow 0.71 \times 10^{-9} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{(E_1 - 2870) \times 1.6 \times 10^{-19}}$$

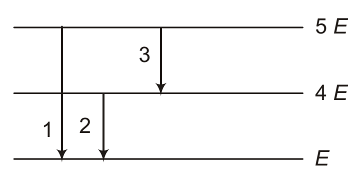
Solving this we get

$$E_1 = -\,4613\text{ eV}$$

$$\lambda_{K_B} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{(4613 - E_3) \times 1.6 \times 10^{-19}} = 0.63$$

Solving this we get $E_3 = -2650\,\text{eV}$

9. $\nu_{31} = \dfrac{4E}{h} = f$ …(i)



$$\nu_{21} = \frac{3E}{h} = \frac{3}{4}\left(\frac{4E}{h}\right) = \frac{3f}{4}$$

$$\nu_{32} = \frac{E}{h} = \frac{f}{4}$$

Introductory Exercise 29.2

1. $eV_0 = \dfrac{hc}{\lambda} - W$

$\Rightarrow\ eV_0 = \dfrac{6.6\times 10^{-34}\times 3\times 10^{8}}{2\times 10^{-7}\times 1.6\times 10^{-19}} - 4.3\text{ eV}$

$\Rightarrow\ eV_0 = 6.2\text{eV} - 4.3\text{ eV} = 1.9\text{ eV}$

$\Rightarrow\ V_0 = 1.9\text{ volt}$

2. $P = 1.5\text{mW} = 1.5\times 10^{-3}\text{ W}$

Energy of each photon

$$= \frac{6.62\times 10^{-34}\times 3\times 10^{8}}{4\times 10^{-7}}$$

$$= 4.96\times 10^{-19}\text{ J}$$

Number of photons incident per second

$$= \frac{P}{\text{Energy of each photon}} = \frac{1.5\times 10^{-3}}{4.96\times 10^{-19}}$$

$$\simeq 3\times 10^{15}$$

The number of photoelectrons produce

$$= 0.1\%\times 3\times 10^{15} = 3\times 10^{12}$$

Current $i = 3\times 10^{12}\times 1.6\times 10^{-19}\text{ A}$

$$= 4.8\times 10^{-7}\text{ A} = 0.48\ \mu\text{A}$$

3. $K_{\max} = hf - W = hf - hf_0$

$\Rightarrow K_{\max} \propto (f - f_0)$

4. $K_{\max} = \dfrac{hc}{\lambda} - W$

$$= \left[\frac{6.62\times 10^{-34}\times 3\times 10^{8}}{2\times 10^{-7}\times 1.6\times 10^{-19}} - 3\right]\text{eV}$$

$$= [6.20 - 3]\text{eV} = 3.20\text{ eV}$$

The minimum kinetic energy $= 0$.

5. $K_{\max} = h[f - f_0]$

$1.2\text{ eV} = h[f - f_0]$ ...(i)

$4.2\text{ eV} = h[1.5f - f_0]$ ...(ii)

Dividing Eq. (i) and Eq. (ii)

$$\frac{124}{42} = \frac{f - f_0}{1.5f - f_0}$$

$\Rightarrow\ 3f - 2f_0 = 7f - 7f_0$

$\Rightarrow\ 5f_0 = 4f$

$\Rightarrow\ f_0 = \dfrac{4}{5}f = 0.8f$

$\Rightarrow 1.2\times 1.6\times 10^{-19} = 6.62\times 10^{-34}\left[\dfrac{f_0}{0.8} - f_0\right]$

$\Rightarrow\ \dfrac{1.2\times 1.6\times 10^{-19}}{6.62\times 10^{-34}} = \dfrac{2f_0}{8} = \dfrac{f_0}{4}$

$\Rightarrow\ f_0 = 1.16\times 10^{15}\text{Hz}$

Subjective Questions (Level I)

1. Here $\lambda = 280\text{ nm} = 28\times 10^{-8}\text{ m}$

$E = \dfrac{hc}{\lambda} = \dfrac{6.6\times 10^{-34}\times 3\times 10^{8}}{28\times 10^{-8}}\text{ J}$

$E = \dfrac{19.8\times 10^{-34}\times 10^{16}}{28}\text{ J} = \dfrac{198\times 10^{-19}}{28}\text{ J}$

$E = \dfrac{198\times 10^{-19}}{28\times 1.6\times 10^{-19}}\text{ eV} = \dfrac{198}{28\times 1.6} \simeq 4.6\text{ eV}$

We have $E = mc^2 \Rightarrow m = \dfrac{E}{c^2} = \dfrac{198\times 10^{-19}}{28\times 9\times 10^{16}}$

$\Rightarrow\ m = 8.2\times 10^{-36}\text{ kg}$

Momentum $p = mc = 8.2\times 10^{-36}\times 3\times 10^{8}$

$$= 2.46\times 10^{-27}\text{ kg-m/s}$$

2. Intensity of light at a distance 2 m

From the source $= \dfrac{1}{4\pi\times (2)^2} = \dfrac{1}{16\pi}\text{ W/m}^2$

Let plate area is A

Energy incident on unit time is

$$E_1 = \frac{1}{16\pi} A\omega$$

Energy of each photon

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4.8 \times 10^{-7}}$$

Number of photons striking per unit area

$$n = \frac{\frac{1}{16\pi} \times A \times 4.8 \times 10^{-7}}{A \times 6.6 \times 10^{-34} \times 3 \times 10^8 \times A}$$

$= 4.82 \times 10^{16}$ per m^2 s

3. Here $p = 8.24 \times 10^{-28}$ kg-m/s

(a) Energy of photon $E = pc$

$\Rightarrow E = 8.24 \times 10^{-28} \times 3 \times 10^8$

$\Rightarrow E = 2.47 \times 10^{-19}$ J

$$\text{Energy in eV} = \frac{E \text{ in joule}}{1.6 \times 10^{-19}} = \frac{2.47 \times 10^{-19}}{1.6 \times 10^{-19}}$$

Energy (in eV) $= 1.54$ eV

(b) Wavelength

$$\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34}}{8.24 \times 10^{-28}} = 804 \text{ nm}$$

This wavelength in Infrared region.

4. We have $c = f\lambda \Rightarrow f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{6 \times 10^{-7} \text{m}}$

$\Rightarrow \quad f = 5 \times 10^{14}$ Hz

We have $p = \frac{E}{t} \Rightarrow E = pt$ power per sec = energy

$\Rightarrow \quad P = E$

$\Rightarrow \quad 75 = (h \times v) \times n$

$\Rightarrow \quad = \frac{75}{6.6 \times 10^{-34} \times 5 \times 10^{14}}$

$\Rightarrow \quad n = 2.3 \times 10^{20}$ photons/sec

5. (a) $E = 2.45 \text{ MeV} = 2.45 \times 1.6 \times 10^{-19} \times 10^6$ J

$$E = h\nu \Rightarrow \nu = \frac{E}{h} = \frac{3.92 \times 10^{-13}}{6.6 \times 10^{-34}}$$

$\Rightarrow \nu = 5.92 \times 10^{20}$ Hz

(b) We have $c = \nu\lambda \Rightarrow \lambda = \frac{c}{\nu}$

$$\Rightarrow \lambda = \frac{3 \times 10^8}{5.92 \times 10^{20}} = 5.06 \times 10^{-13} \text{ m}$$

6. We have $\quad p = \sqrt{2mK}$

$\Rightarrow \quad p_1 = \sqrt{2mK_1}$ and $p_2 = \sqrt{2mK_2}$

$$\Rightarrow \quad \frac{p_1}{p_2} = \sqrt{\frac{K_1}{K_2}}$$

$$\Rightarrow \quad \frac{1}{2} = \sqrt{\frac{K_1}{K_2}} \qquad [\because p_2 = 2p_1]$$

$\Rightarrow \quad K_2 = 4K_1$

(b) $E_1 = p_1 c$ and $E_2 = p_2 c \Rightarrow E_2 = 2p_1 c$

$\Rightarrow E_2 = 2E_1$

7. (a) Since power = energy per unit line

let n be the number of photons

$$\Rightarrow P = nE \Rightarrow 10 = n \times \frac{nc}{\lambda}$$

$$\Rightarrow n = \frac{10 \times \lambda}{hc} = \frac{10 \times 500 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^8}$$

$\Rightarrow n = 2.52 \times 10^{19}$

(b) Force exerted on that surface

$$F = \frac{P}{c} = \frac{10}{3 \times 10^8} = 3.33 \times 10^{-8} \text{ N}$$

8. Absorbing (power) light $= 70\%$ of incident light

$\Rightarrow P_a = 70\%$ of 10 W $= 7$ W

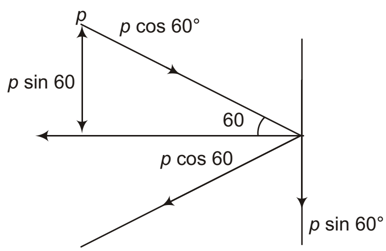
Refractive power $= 30\%$ of 10 W

$\Rightarrow P_R = 3$ W

$$\text{The force exerted} = \frac{P_a}{c} + \frac{2P_R}{c} = \frac{7 + 2 \times 3}{c} = \frac{13}{3 \times 10^8}$$

$= 4.3 \times 10^{-8}$ N

9. Force = rate of change of momentum



$$F = \frac{\Delta p}{\Delta t} \text{ here } \Delta t = 1\text{ s}$$

$$F = 2p\cos 60°$$

$$F = p$$

$$\Rightarrow \quad F = \frac{nh}{\lambda}$$

where n is number of photons striking per second

$$\Rightarrow F = \frac{1 \times 10^{19} \times 6.63 \times 10^{-34}}{663 \times 10^{-9}} = 10^{-8}\text{ N}$$

10. Here output energy = 60 W/s

$$\text{Pressure } p = \frac{2 \times 60}{3 \times 10^{8}} = 4 \times 10^{-7}\text{ N}$$

de-Broglie wavelength

11. Here $m = 5$ g $\Rightarrow$ m $= 5 \times 10^{-3}$kg, $v = 340$ m/s

by de-Broglie hypothesis wavelength

$$\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{5 \times 10^{-3} \times 340}$$

$$\Rightarrow \quad \lambda = 3.9 \times 10^{-34}\text{ m}$$

Since λ is too small. No wave like property is exhibit.

12.(a) $\lambda_e = \dfrac{h}{m_e v} = \dfrac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 4.7 \times 10^{6}}$

$$= 1.55 \times 10^{-10}\text{ m}$$

(b) $\lambda_p = \dfrac{6.6 \times 10^{-34}}{1836 \times 9.1 \times 10^{-31} \times 4.7 \times 10^{-6}}$

$$= 8.44 \times 10^{-14}\text{m}$$

13. (a) $\lambda = \dfrac{h}{p} \Rightarrow p = \dfrac{h}{\lambda} = \dfrac{6.6 \times 10^{-34}}{2.8 \times 10^{-10}}$

$$\Rightarrow p = 2.37 \times 10^{-24}\text{ kg-m/s}$$

(b) $\because p^2 = 2m_e K \Rightarrow K = \dfrac{p^2}{2m_e}$

$$\Rightarrow K = \frac{(2.37 \times 10^{-24})^2}{2 \times 9.1 \times 10^{-31}}$$

$$\Rightarrow K = 3.07 \times 10^{-18}\text{ J}$$

$$K\text{ (in eV)} = \frac{K\text{ in J}}{1.6 \times 10^{-19}} = \frac{3.07 \times 10^{-18}}{1.6 \times 10^{-19}}$$

$$\Rightarrow \quad K = 19.2\text{ eV}$$

14. Here $T = 273° + 20° = 293$ K

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \Rightarrow \lambda = \frac{h}{Mv_{\text{rms}}}$$

$$\Rightarrow \lambda = \frac{h}{\sqrt{3\,MRT}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{3 \times 1836 \times 9.1 \times 10^{-31} \times 8.31 \times 293}}$$

$$\Rightarrow \lambda = 1.04\text{ Å}$$

15. For hydrogen like atom

$$E = -K \text{ Here } E = -3.4\text{ eV}$$

$$\Rightarrow \quad K = 3.4\text{ eV} = 3.4 \times 1.6 \times 10^{-19}\text{ J}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e K}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}}}$$

$$\Rightarrow \lambda = 6.663\text{ Å}$$

16. In Bohr model the velocity of electron in nth orbit is given by

$$U_n = \frac{e^2}{2\varepsilon_0 nh}$$

Putting the values of e, ε_0, h and $n = 1$, we get

$$U_1 = 2.19 \times 10^{7}\text{ m/s and } U_4 = \frac{2.19 \times 10^{6}}{4}\text{ m/s}$$

$$\Rightarrow \lambda_1 = \frac{h}{m_e v_1} \text{ and } \lambda_4 = \frac{h}{m_e u_4} = \frac{h}{m_e \frac{u_1}{4}} = 4\,\lambda_1$$

$$\Rightarrow \qquad \lambda_1 = \frac{6.6\times 10^{-34}}{9.1\times 10^{-31}\times 2.19\times 10^{6}}$$

$$= 3.32\times 10^{-10}\text{m}$$

$$\Rightarrow \qquad \lambda_4 = 4\lambda_1 = 4\times 3.332\times 10^{-10}\text{m}$$

$$= 13.28\times 10^{-10}\text{ m}$$

$$= 1.33\times 10^{-9}\text{m}$$

The radius of first Bohr orbit

$$r_1 = 0.529\times 10^{-10}$$

The radius of fourth Bohr orbit

$$r_4 = 16\times 0.529\times 10^{-10}$$

$$\Rightarrow \qquad 2\pi r_1 = 2\times 3.14\times 0.529\times 10^{-10}$$

$$\approx 3.32\times 10^{-10}\text{m} = \lambda_1$$

Bohr Atomic Model and Emission Spectrum

17. For hydrogen like atom we can write

$$e_n = \frac{-z^2}{n^2}(13.6\,\text{eV})$$

For lithium atom $z = 3$ we get

$$E_n = \frac{-9}{n^2}(13.6\,\text{eV}) = \frac{-122.4}{n^2}\text{eV}$$

The ground state energy is for $n = 1$

$$\Rightarrow \qquad E_1 = \frac{-122.4}{1^2}\text{eV} = -122.4\text{ eV}$$

Ionization potential $= -E_1 = 122.4$ eV

18. For hydrogen atom we can write

(a) $E = -K \Rightarrow K = 3.4$ eV

(b) PE $= -2K = -2\times 3.4 = -6.8$ eV

Since potential energy depends upon refrence hence it will changed.

19. Binding energy of an electron in He-atom is $E_0 = 24.6\,\text{eV}$. *ie*, the energy required to remove one electron from He-atom $= 24.6$ eV

Now, He-atom becomes He^+ and energy of He^+ ion is given by

$$E_n = \frac{-z^2(13.6)}{n^2} \text{ for } He^+ \; z = 2, \text{ we get}$$

$$E_1 = -4\times 13.6 = -54.4\text{ eV}.$$

Hence energy required to remove this electron $= 54.4$ eV, thus total energy $= 24.6 + 54.4 = 79$ eV

20. For hydrogen atom $E_n = \dfrac{-13.6\,\text{eV}}{n^2}$

Putting $n = 3$, we get

$$E_3 = \frac{-13.6\,\text{eV}}{9} = -1.51\,\text{eV}$$

Hence hydrogen atom is in third excited state the angular momentum

$$L = \frac{nh}{2\pi} = \frac{3h}{2\pi} = \frac{3\times 6.62\times 10^{-34}}{2\times 3.14}$$

$$\Rightarrow L = 3.16\times 10^{-34}\,\text{kg-m}^2/\text{s}$$

21. We have $\lambda = \dfrac{hc}{\Delta E} = \dfrac{6.6\times 10^{-34}\times 3\times 10^{8}}{\Delta E}$

$$\Rightarrow \Delta E = \frac{6.6\times 10^{-34}\times 3\times 10^{8}}{1023\times 10^{-10}}\text{ (in Joule)}$$

$$\Rightarrow \Delta E = \frac{6.6\times 10^{-34}\times 3\times 10^{8}}{1023\times 10^{-10}\times 1.6\times 10^{-19}}\text{(in eV)}$$

$$\simeq 12.1\,\text{eV}$$

$\Rightarrow E_n - E_1 = 12.1\,\text{eV}$ But $E_1 = -13.6\,\text{eV}$

$\Rightarrow E_n \simeq -1.51\,\text{eV}$

For H-atom $E_n = \dfrac{-13.6}{n^2} \Rightarrow -1.51 = -\dfrac{13.6}{n^2}$

$$\Rightarrow \quad n = 3$$

Hence atom goes to 3rd excited state. The possible transition are ($3\to 2, 3\to 1$, and $2\to 1$) *ie*, 3 transitions are possible and the largest wavelength $= 1023$ Å

(From $3\to 1$)

22. For hydrogen like atom

$$E_n = \frac{-z^2}{n^2}(13.6)\text{ eV}$$

For Li^{++} $z = 3$

$$\Rightarrow E_n = \frac{-122.4}{n^2}\text{eV} \Rightarrow E_1 = -122.4\text{ eV}$$

$$E_3 = \frac{-122.4}{9}\text{eV} = -13.6\text{ eV}$$

$$\Delta E = E_3 - E_1 = (122.4 - 13.6)\text{ eV} = 108.8\text{ eV}$$

$$\lambda = \frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{108.8 \times 1.6 \times 10^{-19}} = 113.74\ \text{Å}$$

23. The excited state energy He^+ atom will be equal to the sum of energies of the photons having wavelength 108.5 nm and 30.4 nm.

$$\Rightarrow E_n - E_1 = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{10^{-10}}\left[\frac{1}{1085} + \frac{1}{304}\right]$$

$$\Rightarrow \quad E_n - E_1 = 83.7 \times 10^{-19}\text{ J}$$

$$= \frac{83.7 \times 10^{-19}\text{ J}}{1.6 \times 10^{-19}\text{ J/eV}} = 52.3\text{ eV} \quad \ldots\text{(i)}$$

For He^+ atom $E_n = \dfrac{-z^2(13.6)}{n^2}$eV

For He^+ $z = 2$

$$E_n = \frac{54.4}{n^2}\text{ eV}$$

$$\Rightarrow \quad E_1 = -54.4\text{ eV}$$

$$\Rightarrow \quad E_n - E_1 = 54.4\left(1 - \frac{1}{n^2}\right)\text{eV} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$52.3 = 54.4\left(1 - \frac{1}{n^2}\right)$$

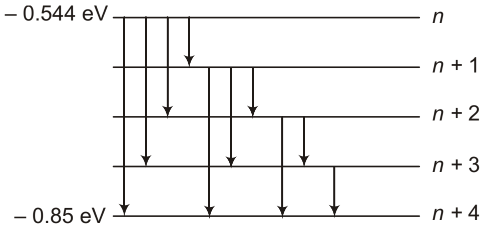
$$\Rightarrow \quad 1 - \frac{1}{n^2} = 0.96$$

$$\Rightarrow \quad n = 5$$

24. For hydrogen like atom

$$E_n = \frac{-z^2}{n^2}(13.6)\text{ eV}$$

Let for ten transitions quantum numbers of energy levels are n, $n + 1$, $n + 2$, $n + 3$ and $n + 4$



$$\frac{-z^2(13.6)\text{ eV}}{n^2} = -0.85\text{ eV} \quad \ldots\text{(i)}$$

$$\frac{-z^2(13.6)\text{ eV}}{(n+4)^2} = -0.544\text{ eV} \quad \ldots\text{(ii)}$$

Dividing Eq. (i) and Eq. (ii)

$$\frac{(n+4)^2}{n^2} = \frac{0.85}{0.544} = 1.5625$$

$$\Rightarrow \quad \frac{n+4}{n} = 1.25$$

$$\Rightarrow \quad 1 + \frac{4}{n} = 1.25$$

$$\Rightarrow \quad \frac{4}{n} = 0.25$$

$$\Rightarrow \quad n = \frac{4}{0.25} = 16$$

Putting this value of n in Eq. (i)

$$\frac{-z^2(13.6)}{(16)^2} = -0.85$$

$$\Rightarrow \quad z^2 = \frac{256 \times 0.85}{13.6}$$

$$\Rightarrow \quad z^2 = 16 \Rightarrow z = 4$$

Hence atom no. of atom is = 4

We know that $\Delta E = \dfrac{hc}{\lambda}$

$\Rightarrow \lambda = \dfrac{hc}{\Delta E}$ for smallest wavelength ΔE is maximum

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{[-0.544 - (-0.85)] \times 1.6 \times 10^{-19}}$$

$$\lambda = 40441\ \text{Å}$$

25. Here $E_1 = -15.6\,\text{eV}$

(a) Hence ionization potential

$= -E_1 = 15.6\,\text{eV}$

(b) We have $\lambda = \dfrac{hc}{\Delta E}$ for short wavelength

ΔE is maximum

$$\Rightarrow \lambda = \frac{6.6\times 10^{-34}\times 3\times 10^{8}}{0-(-5.3)\times 1.6\times 10^{-19}} \simeq 2335\ \text{Å}$$

(c) Excitation potential for $n = 3$ state is

$= E_3 - E_1 = -3.08 + 15.6 = 12.52\ \text{V}$

(d) From $n = 3$ to $n = 1$

$\Delta E = E_3 - E_1 = -3.08 + 15.6 = 12.52\,\text{eV}$

We know $\lambda = \dfrac{hc}{\Delta E}$

$$\Rightarrow \frac{1}{\lambda} = \frac{\Delta E}{hc} = \frac{12.52\times 1.6\times 10^{-19}}{6.6\times 10^{-34}\times 3\times 10^{8}}$$

$$= 1.01\times 10^{7}\ (\text{m}^{-1})$$

26. (a) $E_1 = -6.52\,\text{eV}$

$\lambda = 860\ \text{nm} = 8600\ \text{Å}$

Energy of this photon

$$= \frac{12375\,(\text{eV})}{8600} = 1.44\ \text{eV}$$

hence internal energy of atom after absorbing this photon is given by

$E_i = E_1 + 1.44\ \text{eV} = -6.52 + 1.44$

$= -5.08\ \text{eV}$

(b) $\lambda_2 = \dfrac{12375\,(\text{eV})}{4200} = 2.95\ \text{eV}$

hence internal energy of the atom after emission of this photon is given by

$E_i = E_1 - 2.95\,\text{eV} = (-2.68 - 2.95)\ \text{eV}$

$\Rightarrow \qquad E_i = -5.63\ \text{eV}$

27. Heve $U = \dfrac{1}{2} m\omega^2 r^2 \Rightarrow F = \left|\dfrac{dU}{dr}\right| = m^2\omega^2 r$

But $\qquad \dfrac{mv^2}{r} = m^2\omega^2 r$

$\Rightarrow \qquad v^2 = m\omega^2 r^2$

But by Bohr's postulate $mvr = \dfrac{nh}{2\pi}$

$$\Rightarrow \quad m^2v^2r^2 = \frac{n^2h^2}{4\pi^2}$$

$$\Rightarrow \quad m^3\omega^2 r^4 = \frac{n^2h^2}{4\pi^2}$$

$$\Rightarrow \quad r^4 = \frac{n^2h^2}{4\pi^2 m^3\omega^2}$$

$$\Rightarrow \quad r \propto \sqrt{n}$$

28. $\dfrac{1}{\lambda_{K_\alpha}} = R(z-1)^2\left[1 - \dfrac{1}{2^2}\right] = \dfrac{1}{\lambda_0}$

$$\frac{1}{\lambda_{K_\beta}} = R(z-1)^2\left[1 - \frac{1}{3^2}\right]$$

$$\Rightarrow \quad \frac{\lambda_{K_\beta}}{\lambda_{K_\alpha}} = \frac{3\times 9}{4\times 8} = \frac{27}{32}$$

$$\Rightarrow \quad \lambda_{K_\beta} = \frac{27}{32}\lambda_{K_\alpha} = \frac{27}{32}\lambda_0$$

29. $\lambda_0 = \dfrac{hc}{eV}$

$$\Rightarrow \lambda_{0_1} = \frac{6.6\times 10^{-34}\times 3\times 10^{8}}{1.6\times 10^{-19}\times 25\times 10^{3}}$$

$\Rightarrow \lambda_{0_1} = 49.5\,\text{pm}$

$\Rightarrow \lambda_{0_2} = 2\lambda_{0_1} = 2\times 49.5\,\text{pm} = 99\ \text{pm}$

$[1\,\text{pm} = 10^{-12}\text{m}]$

30. $f_{K\alpha} = (2.48\times 10^{15})\ \text{Hz}\,(z-1)^2$

$$\Rightarrow \frac{3\times 10^{8}}{\lambda_{K\alpha}} = 2.48\times 10^{15}(2-1)^2$$

$$\Rightarrow \frac{3\times 10^{8}}{0.76\times 10^{-10}\times 2.58\times 10^{15}} = (z-1)^2$$

$(z-1) \simeq 40 \Rightarrow z = 41$

31. $\lambda_i = \dfrac{hc}{eV}$ $\quad \Delta\lambda = 26\,\text{pm}$ when $V_f = 1.5\ \text{V}$

$\Rightarrow \qquad \lambda_f = (\lambda_i - 26)\ \text{pm}$

$$\lambda_f = \frac{hc}{e\times 1.5\,V} = \frac{1}{1.5}\frac{hc}{eV} = \frac{12}{1.5} = \frac{1}{1.5}\lambda_i$$

$$\Rightarrow \quad (\lambda_i - 26) = \frac{2}{3}\lambda_i$$

$$\Rightarrow \quad 3\lambda_i - 26 \times 3 = 2\lambda_i$$

$$\Rightarrow \quad \lambda_i = 78 \text{ pm}$$

$$\Rightarrow 78 \times 10^{-12} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times V}$$

$$\Rightarrow V = \frac{6.6 \times 3 \times 10^{-26}}{1.6 \times 78 \times 10^{-12} \times 10^{-19}}$$

$$\Rightarrow V = 15865 \text{ volt}$$

32. $\sqrt{V} = a(z - b)$

$$\Rightarrow \quad \frac{c}{\lambda} = a^2(z-b)^2$$

$$\Rightarrow \quad \frac{1}{\lambda} = \frac{a^2}{c}(z-b)^2$$

$$\Rightarrow \quad \frac{1}{887 \text{ pm}} = \frac{a^2}{c}(13-b)^2 \qquad \ldots\text{(i)}$$

and $$\frac{1}{146 \text{ pm}} = \frac{a^2}{z}(30-b)^2 \qquad \ldots\text{(ii)}$$

Dividing Eq. (i) and Eq. (ii)

$$\frac{146}{887} = \left[\frac{(13-b)}{(30-b)}\right]^2$$

$$\Rightarrow \quad 2.5 = \frac{30-b}{13-b}$$

$$\Rightarrow \quad 32.5 - 2.5b = 30 - b$$

$$2.5 = 1.5b$$

$$b = \frac{5}{3}$$

$$\frac{1}{\lambda_{26}} = \frac{a^2}{c}\left(26 - \frac{5}{3}\right)^2$$

$$\frac{1}{\lambda_{26}} = \frac{\phi\left(26 - \frac{5}{3}\right)^2}{887 \text{ pm} \times \left(13 - \frac{5}{3}\right)^2}$$

$$\Rightarrow \quad \lambda_{26} = \frac{887 \text{ pm} \times \left(13 - \frac{5}{3}\right)^2}{\left(26 - \frac{5}{3}\right)^2}$$

$$= 887 \text{ pm} \times \frac{34^2}{(73)^2}$$

$$\simeq 198 \text{ pm}$$

33. $\sqrt{f} = \sqrt{\frac{3RC}{4}}(z-1)$

$$\sqrt{4.2 \times 10^{18}} = \sqrt{\frac{3 \times 1.1 \times 10^{7} \times 3 \times 10^{8}}{4}}(z-1)^2$$

$$\Rightarrow \quad \frac{4.2 \times 10^{18} \times 4}{9 \times 1.1 \times 10^{15}} = (z-1)^2$$

$$\Rightarrow (z-1) = 41 \Rightarrow z = 42$$

34. $P = Vi = 40 \text{ kW} \times 10 \text{ mA} = 400 \text{ W}$

$$\% \text{ of } P = \frac{400 \times 1}{100} = 4 \text{ W}$$

(a) Total power of X-rays = 4 W

(b) Heat produced per second

$$= 400 - 4 = 396 \text{ J/s}$$

Photoelectric effect

35. Einstein photo electric equation is

$$K_{\max} = h\nu - W$$

$$\Rightarrow eV_0 = \frac{hc}{\lambda} - W \quad \because K_{\max} = eV_0$$

$$\Rightarrow 10.4 \text{ eV} = \frac{12375}{\lambda(\text{Å})} - 1.7 \text{ eV}$$

$$\Rightarrow \lambda(\text{Å}) = \frac{12375}{12.1} = 1022 \text{ Å}$$

For H-atom $\lambda = \frac{hc}{\Delta E} \Rightarrow \Delta E = \frac{12375}{1022} = 12.1 \text{ eV}$

This difference equal to $n = 3 \rightarrow n = 1$ transition.

36. $K_{\max} = h\nu - W$

$$\Rightarrow K_{\max} = \frac{6.6 \times 10^{-34} \times 1.5 \times 10^{15}}{1.6 \times 10^{-19}} - 3.7$$

$$\Rightarrow K_{\max} = 6.18 - 3.7 = 2.48 \text{ eV}$$

37. Here work function

$$W(\text{in eV}) = \frac{12375}{5000 \text{ Å}} = 2.475 \text{ eV}$$

$$K_{\max} = eV_0 = 3 \text{ eV}$$

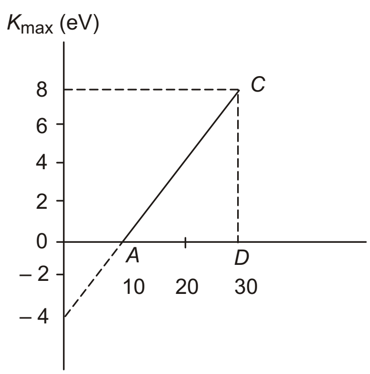
$$K_{\max} = \frac{hc}{\lambda} - W$$

$$\Rightarrow \quad 3 = \frac{12375}{\lambda(\text{in Å})} - 2.475$$

$$\Rightarrow \quad \lambda = \frac{12375}{5.475} = 2260 \text{ Å}$$

38. Comparing the given graph with

$$K_{\max} = h\nu - W$$



$f = 1 \times 10^{14}$ Hz

(a) ν_o = threshold frequency $\theta = 10 \times 10^{14}$ Hz

$= 10^{15}$ Hz

(b) $W = 4$ eV

(c) h = slope of the graph

$$= \frac{CD}{AD} = \frac{8 \text{ eV}}{20 \times 10^{14} \text{Hz}}$$

$$\Rightarrow h = \frac{8 \times 1.6 \times 10^{-19}}{2 \times 10^{15}} = 6.4 \times 10^{-34} \text{ J-s}$$

39. Here $\frac{v_{1(\max)}}{u_{2(\max)}} = \frac{3}{1}$

Using Einstein equation, $K_{\max} = \frac{hc}{\lambda} - W$, we get

$$\frac{1}{2} m v_{\max}^2 = \frac{hc}{\lambda} - W$$

where m is the mass of photoelectron

$$\Rightarrow \quad \frac{1}{2} m [v_{1(\max)}]^2 = \frac{hc}{\lambda_1} - W \quad \text{...(i)}$$

$$\text{and} \quad \frac{1}{2} m [v_{2(\max)}]^2 = \frac{hc}{\lambda_2} - W \quad \text{...(ii)}$$

Dividing Eq. (i) and Eq. (ii), we get

$$\left(\frac{v_{1\max}}{v_{2\max}}\right)^2 = \frac{\frac{hc}{\lambda_1} - W}{\frac{hc}{\lambda_2} - W} \Rightarrow (3)^2 = \frac{\frac{hc}{\lambda_1} - W}{\frac{hc}{\lambda_2} - W}$$

$$\Rightarrow \quad \frac{9hc}{\lambda_2} - \frac{hc}{\lambda_1} = 8W$$

$$\Rightarrow \quad hc\left[\frac{9}{6000} - \frac{1}{3000}\right] = 8W$$

$$\Rightarrow \quad \frac{6.6 \times 10^{-34} \times 3 \times 10^8 \times 7}{6000 \times 10^{-10} \times 1.6 \times 10^{-19}} = 8W$$

$$\Rightarrow \quad W = 1.81 \text{ eV}$$

Putting the value of W in Eq. (i)

$$\frac{1}{2} m(u_{1\max})^2 = \frac{hc}{3000 \times 10^{-10}} - 1.81 \times 1.6 \times 10^{-19}$$

$$\frac{1}{2} m(u_{1\max})^2 = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{3 \times 10^7} - 2.896 \times 10^{-19}$$

$$\frac{1}{2} m(u_{1\max})^2 = 6.62 \times 10^{-19} - 2.896 \times 10^{-19}$$

$$\frac{1}{2} m(u_{1\max})^2 = 3.724 \times 10^{-19}$$

$$\Rightarrow (u_{1\,max})^2 = \frac{3.724 \times 10^{-19} \times 2}{9.1 \times 10^{-31}}$$

$$-1.81 \times 1.6 \times 10^{-19} \quad \because \quad m_e = 9.1 \times 10^{-31}$$

$$\Rightarrow u_{1\,max} = 9 \times 10^5 \text{ m/s}$$

and $v_{2\,max} = \frac{1}{3} v_{1\,max} = 3 \times 10^5$ m/s

40. Here intensity $I = 2$ W/m^2 and

Area $A = 1 \times 10^{-4}$ m^2

Energy incident per unit time on the metal surface

$$E = IA = 2 \times 10^{-4} \text{ W}$$

$$= 2 \times 10^{-4} \text{ J/s} = \frac{2 \times 10^{-4}}{1.6 \times 10^{-19}} \frac{\text{eV}}{\text{s}}$$

$$= \frac{2 \times 10^{15}}{1.6} \text{ eV/s}$$

Energy of each photon $= 10.6$ eV

Number of photons incident on surface

$$= \frac{2 \times 10^{15}}{1.6 \times 10.6}$$

Number of photoelectrons emitted

$$= \frac{0.53}{100} \times \frac{2 \times 10^{15}}{1.6 \times 10.6}$$

$$= 6.25 \times 10^{11} \text{ per second}$$

Minimum KE $= 0$

Maximum KE $= (10.6 - 5.6)$ eV $= 5$ eV

41. $K_{max} = \frac{hc}{\lambda} - W$

$\Rightarrow$

$$K_{max} = \frac{6.6 \times 10^{34} \times 3 \times 10^8}{180 \times 10^{-9}} - 2 \times 1.6 \times 10^{-19}$$

$$\frac{1}{2} m_e v_{max}^2 = \frac{6.6 \times 3 \times 10^{-18}}{18} - 2 \times 1.6 \times 10^{-19}$$

$$\frac{1}{2} m_e v_{max}^2 = 11 \times 10^{-19} - 3.2 \times 10^{-19}$$

$$= 7.2 \times 10^{-19} \text{ J}$$

$$v_{max} = \sqrt{\frac{2 \times 7.2 \times 10^{-19}}{9.1 \times 10^{-31}}} = 1.25 \times 10^6 \text{ m/s}$$

$$r = \frac{mv_{max}}{eB} = \frac{9.1 \times 10^{-31} \times 1.25 \times 10^6}{1.6 \times 10^{-19} \times 5 \times 10^{-5}}$$

$r = 0.148$ m

42. The given equation is

$$E = (100 \text{ v / m}) [\sin ((5 \times 10^{15}) t + \sin (8 \times 10^{15}) t]$$

Light consist of two different frequencies.

Maximum frequency

$$= \frac{8 \times 10^{15}}{2\pi} = 1.27 \times 10^{15} \text{ Hz}$$

For maximum KE we will use Einstein's equation

$$(\text{KE})_{max} = h\nu - W$$

$$= \frac{1.27 \times 10^{15} \times 6.62 \times 10^{-34}}{1.6 \times 10^{-19}} - 2$$

$$\Rightarrow (\text{KE})_{max} = 3.27 \text{ eV}$$

43. Here $E = E_0 \sin 1.57 \times 10^7 (x - ct)$ frequency of the wave

$$\nu = \frac{1.57 \times 3 \times 10^{15}}{2 \times 3.14} = 0.75 \times 10^{15}$$

We have

$$eV_0 = \left(\frac{6.6 \times 10^{-34} \times 0.75 \times 10^{15}}{1.8 \times 10^{-19}} - 1.9 \right) \text{eV}$$

$$\Rightarrow \quad V_0 = 1.2 \text{ V}$$

Objective Questions (Level-1)

1. Einstein photo electric equation is

$$K_{max} = h\nu - W$$

its slope $= h =$ planck constant which is same for all metals and independent of intensity of radiation.

Hence correct option is (d).

2. Since current is directly proportional to intensity therefore as current is increased intensity is increased since $\lambda_{min} \propto \frac{1}{V}$, if V is decreased λ_{min} is increased.

Hence correct option is (c).

3. For hydrogen atom (Bohr's model) nth orbital speed $v_n = \frac{e^2}{2\varepsilon_0 nh}$

For first orbit $n = 1$

$$\Rightarrow \quad v_1 = \frac{e^2}{2\varepsilon_0 \times h}$$

$$= \frac{(1.6 \times 10^{-19})^2}{2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}}$$

$$\Rightarrow \quad v \simeq \left(\frac{1}{137}\right) \times 3 \times 10^8 = \frac{c}{137}$$

Hence correct option is (c).

4. $_{86}A^{22} \xrightarrow[3\alpha]{} {}_{80}X^{210} \xrightarrow{4\beta} {}_{84}B^{210}$

Hence correct option is (b)

5. $\lambda_{min}(\text{in Å}) = \frac{12375}{V(\text{in volt})} = \frac{12375}{20 \times 1000} \simeq 0.62$ Å

Hence correct option is (c).

6. We have $\frac{1}{2} m_e v_{max}^2 = \text{eV}$

$$\Rightarrow \quad v_{max} = \sqrt{\frac{2eV}{m}}$$

$$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 18 \times 1000}{9.1 \times 10^{-31}}}$$

$$\Rightarrow \quad v_{max} \simeq 8 \times 10^7 \text{ m/s}$$

Hence correct option is (a).

7. For hydrogen atom $v_n = \frac{e^2}{2\varepsilon_0 nh}$

$$\Rightarrow \quad v_2 = \frac{e^2}{2\varepsilon_0 \times 2h}, v_3 = \frac{e^2}{2\varepsilon_0 \times 3h}$$

$$\Rightarrow \quad \frac{v_2}{v_3} = \frac{3}{2}$$

Let λ_2 and λ_3 are the de-Broglie wavelengths

$$\Rightarrow \quad \frac{\lambda_2}{\lambda_3} = \frac{\frac{h}{mv_2}}{\frac{n}{mv_3}} = \frac{v_3}{v_2}$$

$$\Rightarrow \quad \frac{\lambda_2}{\lambda_3} = \frac{2}{3}$$

Hence correct option is (a).

8. For hydrogen like atom

$$E_n = \frac{-z^2}{n^2}(13.6 \text{ eV})$$

For ground state $n = 1$

$$\Rightarrow \quad E_1 = -z^2 \times 13.6 \text{ eV}$$

But $\quad E_1 = -122.4$ eV

$$\Rightarrow \quad -122.4 \text{ eV} = -z^2 \times 13.6 \text{ eV}$$

$$\Rightarrow \quad z^2 = 9$$

$$\Rightarrow \quad z = 3$$

Hence it is Li^{2+}

The correct option is (c).

9. $\lambda_{min} = \frac{hc}{eV} \Rightarrow \frac{\Delta\lambda_{min}}{\lambda_{min}} \times 100 = -\frac{\Delta V}{V} \times 100$

Percentage change in $\lambda_{min} = -2\%$

Hence λ_{min} is decreased by 2%

correct option is (c)

10. $E_n = \frac{-z^2}{n^2}(13.6 \text{ eV})$ for first excited state $n = 2$

$$\Rightarrow \quad E_2 = \frac{-z^2}{4}(13.6) \text{ eV}$$

$$\Rightarrow \quad -13.6 \text{ eV} = \frac{-z^2}{4} \times 13.6 \text{ eV}$$

$$\Rightarrow \quad z = 2$$

Hence it is He^+

Correct option is (a).

11. $\lambda_{min}(\text{in Å}) = \frac{12375}{V(\text{in volt})}$

$$\Rightarrow \quad V = \frac{12375}{1} = 12.375 \times 10^3 \text{ V}$$

$$\Rightarrow \quad V = 12.4 \text{ eV}$$

Hence correct option is (c).

12. We have $\lambda = \frac{h}{p}$

$$\Rightarrow \quad p = \frac{h}{\lambda} = \frac{6.62 \times 10^{-34}}{0.5 \times 10^{-10}}$$

$\Rightarrow \lambda = 13.26 \times 10^{-24}$ kg-m/s

Hence correct option is (c).

13. $W = \frac{hc}{\lambda_0} \Rightarrow \lambda_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 1.6}$

$$\Rightarrow \quad \lambda_0 = 7750 \text{ Å}$$

Hence correct option is (a).

14. For H-like atom Balmer series is $\frac{1}{\lambda} = z^2 R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$

For third Balmer line $n = 5$

$$\Rightarrow \frac{1}{1085 \times 10^{-10}} = z^2 \times 1.09 \times 10^7 \left[\frac{1}{4} - \frac{1}{5^2}\right]$$

$$\Rightarrow \quad z^2 = \frac{100 \times 1000}{1085 \times 1.097 \times 21} = 4$$

$$\Rightarrow \quad z = 2$$

Binding energy $= (13.6) \times z^2 \text{eV} = 13.6 \times 4$ eV

$= 54.4$ eV

Hence correct option is (a).

15. If $V_1 = 0$ then total energy = KE

$$\Rightarrow \quad \text{KE} = 13.6 \, \text{eV}$$

and the energy difference between two states = 10.2 eV

Hence total energy in this state

$= 13.6 + 10.2$

$= 23.8$ eV

The correct option is (c).

16. $eV_0 = \frac{hc}{3300} - W$ ...(i)

$2eV_0 = \frac{hc}{2200}$ ...(ii)

Substracted Eq. (i) from Eq. (ii), we get

$$eV_0 = hc \times \frac{3300 - 2200}{3300 \times 2200}$$

$$= \frac{hc}{3 \times 2200 \times 10^{-10}}$$

$$V_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{3 \times 2200 \times 1.6 \times 10^{-19} \times 10^{-10}}$$

$$= \frac{30}{16} = \frac{15}{8}$$

$$\Rightarrow \quad V_0 = \frac{15}{8} \text{ V}$$

Hence the correct option is (c).

17. $E = \frac{hc}{\lambda} - W$ ...(i)

$$4E = \frac{hc}{\lambda/3} - W$$

$\Rightarrow \quad 4E = \frac{3hc}{\lambda} - W$ ...(ii)

From Eqs. (i) and (ii)

$$4\left(\frac{hc}{\lambda} - W\right) - \frac{3hc}{\lambda} - W$$

$$\Rightarrow \quad \frac{hc}{\lambda} = 3W$$

$$\Rightarrow \quad W = \frac{hc}{3\lambda}$$

Hence correct option is (b).

18. By Moseley's law

$\sqrt{f} = a(z - b)$ for K_α line $b = 1$

$\sqrt{f} = a(z - 1)$

$\Rightarrow \sqrt{f} = a(31 - 1) = a \times 30$ ...(i)

and $\quad \sqrt{f'} = a(51 - 1) = 9 \times 50$

$$\Rightarrow \quad \sqrt{\frac{f}{f'}} = \frac{3}{5}$$

$$\Rightarrow \quad f' = \frac{25f}{9}$$

Hence correct option is (a).

19. $\sqrt{f} = \sqrt{(RC)}\,(z - 1)\sqrt{\left[\frac{1}{12} - \frac{1}{n^2}\right]}$

for K_α, $n = 2$

K_β, $n = 3$

$$\sqrt{f_\alpha} = \sqrt{RC}\,(z-1)\sqrt{\left[1-\frac{1}{2^2}\right]} = \sqrt{RC}\,(z-1)\times\sqrt{\frac{3}{4}}$$

$$\sqrt{f_\beta} = \sqrt{RC}\,(z-1)\sqrt{1-\frac{1}{9}} = \sqrt{RC}\,(z-1)\times\sqrt{\frac{8}{9}}$$

$$\frac{\sqrt{f_\beta}}{\sqrt{f_\alpha}} = \sqrt{\frac{8\times 4}{9\times 3}} = \sqrt{\frac{32}{27}},$$

Correct option is (a).

20. $\frac{1}{\lambda} = R\left[\frac{1}{2^2}-\frac{1}{n^2}\right]$ here $n = 3$

$$\Rightarrow \frac{1}{\lambda} = R\left[\frac{1}{2^2}-\frac{1}{3^2}\right] = \left(\frac{9-4}{9\times 4}\right)R = \frac{5R}{36}$$

$$\Rightarrow \lambda = \frac{36}{5R}$$

Hence correct option is (c)

21. $\lambda_D = \frac{h}{\sqrt{2\,meV}}$ and $\lambda_{\min} = \frac{hc}{eV}$

$$\Rightarrow \frac{\lambda_D}{\lambda_{\min}} = \frac{1}{c}\sqrt{\frac{eV}{2m}}$$

$$= \frac{1}{3\times 10^8}\sqrt{\frac{1.8\times 10^{11}\times 10000}{2}}$$

$$\Rightarrow \frac{\lambda_D}{\lambda_{\min}} = \frac{3\times 10^7}{3\times 10^8} = \frac{1}{10}$$

Hence correct option is (c).

22. $5\,eV_0 = \frac{hc}{\lambda} - W$ ...(i)

$eV_0 = \frac{hc}{3\lambda} - W$ ...(ii)

From Eqs. (i) and (ii)

$$\frac{5hc}{3\lambda} - 5W = \frac{hc}{\lambda} - W$$

$$\Rightarrow \frac{5hc}{3\lambda} - \frac{hc}{\lambda} = 4\,W$$

$$\Rightarrow \frac{hc}{\lambda}\frac{(5-3)}{3} = 4\,W$$

$$\Rightarrow \frac{2hc}{3\lambda} = 4\,W$$

$$\Rightarrow W = \frac{hc}{6\lambda}$$

Hence correct option is (a).

23. $eV_0 = h[2V_0 - V_0] = hV_0$ ...(i)

$eV = h[3V_0 - V_0] = h\times 2V_0 = 2hV_0$ ...(ii)

From Eqs. (i) and (ii)

$eV = 2\times eV_0 \Rightarrow V = 2V_0$

Hence correct option is (b).

24. For H-atom Lyman series is

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2}-\frac{1}{n^2}\right]$$

$$\Rightarrow v = \frac{c}{\lambda} = RC\left[\frac{1}{1^2}-\frac{1}{n^2}\right]$$

For H-like atom

$$\nu' = RC\left[\frac{1}{12}-\frac{1}{n^2}\right]\times z^2$$

For Li^{++}, $n = 3 \Rightarrow \nu' = \nu\times 3^2 = 9\,v$

Hence correct option is (c).

25. Ground state energy of H-atom $= -13.6\,\text{eV}$

For Li^{++} atom $E_n = \frac{(-13.6)\,eV}{n^2}\times z^2$

$$\Rightarrow -13.6\,\text{eV} = \frac{-13.6\,\text{eV}\times 9}{n^2}$$

$$\Rightarrow n = 3$$

Hence correct option is (c).

26. $\frac{1}{2}mv_1^2 = h\nu_1 - W$ ...(i)

$\frac{1}{2}mv_2^2 = h\nu_2 - W$ ...(ii)

$$\Rightarrow \frac{1}{2}m[v_1^2 - v_2^2] = h[\nu_1 - \nu_2]$$

$$\Rightarrow v_1^2 - v_2^2 = \frac{2h}{m}[\nu_1 - \nu_2]$$

Hence correct option is (b).

27. For Lyman series

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2}-\frac{1}{n^2}\right]$$

For largest wavelength $n = 2$

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$\Rightarrow \quad \frac{1}{\lambda} = R\left[1 - \frac{1}{4}\right]$$

$$\Rightarrow \quad \lambda = \frac{4}{3R}$$

For He^+ atom

$$\frac{1}{\lambda_{He}} = R(z)^2\left[\frac{1}{2^2} - \frac{1}{n^2}\right] = 4R\left[\frac{1}{4} - \frac{1}{n^2}\right]$$

$$\Rightarrow \quad \frac{3R}{4} = 4R\left[\frac{1}{4} - \frac{1}{n^2}\right]$$

$$\Rightarrow \quad \frac{3}{16} = \frac{1}{4} - \frac{1}{n^2}$$

$$\Rightarrow \quad \frac{1}{n^2} = \frac{1}{4} - \frac{3}{16} = \frac{4-3}{16}$$

$$\Rightarrow \quad \frac{1}{n^2} = \frac{1}{16}$$

$$\Rightarrow \quad n = 4$$

Hence correct option is (b).

28. We have $\sqrt{\frac{1}{\lambda}} = \sqrt{\frac{3R}{4}} \times (z-1)$

$$\Rightarrow \sqrt{\frac{1}{\lambda}} = \sqrt{\frac{3 \times 1.0973 \times 10^7}{4}} \times (92-1)$$

$$\Rightarrow \lambda = \frac{4}{3 \times 91 \times 91 \times 1.0973 \times 10^7} = 0.15 \text{ Å}$$

Hence correct option is (c).

29. For K_α, K_β and L_α of X-rays

$$\nu_{K_\beta} = \nu_{K_\alpha} + \nu_{L_\alpha}$$

$$\Rightarrow \quad Y_2 = Y_1 + Y_3$$

Hence correct option is (b).

30. We have $\lambda = \frac{h}{\sqrt{2mqV}}$

$$\lambda_p = \frac{h}{\sqrt{2m_p eV}} \text{ and } \lambda_\alpha = \frac{h}{\sqrt{2m_{\alpha\,(2e)} V}}$$

$$\Rightarrow \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{2m_\alpha}{m_p}} = \sqrt{\frac{2 \times 4m_p}{m_p}} = \sqrt{8} = 2\sqrt{2}$$

Hence correct option is (c).

31. We have, $\lambda_e = \frac{h}{\sqrt{2m_e E_1}}$, $\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha E_2}}$

$$\lambda_p = \frac{h}{\sqrt{2m_p E_3}} \quad \because m_\alpha > m_p > m_e$$

and $\quad \lambda_e = \lambda_\alpha = \lambda_p \quad \because E_1 > E_3 > E_2$

Hence correct option is (a).

32. KE in ground state $= 13.6$ eV

Total energy in $n = \infty$ = KE + Energy difference between $n = \infty$ to $n = 1$

$\Rightarrow$ Total energy

$$= 13.6 \text{ eV} + 13.6 \text{ eV}$$

$$= 27.2 \text{ eV}$$

Hence correct option is (b).

33. Here $P = 1000$ W, $\nu = 880$ kHz $= 880 \times 10^3$Hz

Let n is the number of photon p emitted per second

$$\Rightarrow \quad n = \frac{P}{h\nu} = \frac{1000}{6.62 \times 10^{-34} \times 880 \times 10^3}$$

$$= 1.7 \times 10^{30}$$

Correct option is (b).

34. Here $\lambda = 3000$ Å $= 3 \times 10^{-7}$m

Energy of incident radiation $E = \frac{hc}{\lambda}$ joule

$$\Rightarrow \quad E = \frac{hc}{\lambda \times 1.6 \times 10^{-19}} \text{(in eV)}$$

$$E = \frac{6.62 \times 3 \times 10^{-26}}{3 \times 1.6 \times 10^{-26}} = 4.125 \text{ eV}$$

$\because E <$ work function hence no emission of electrons it means sphere remain natured.

Hence correct option is (c).

35. $\because E_n = \frac{-13.6 \text{ eV}}{n^2}$ for $n = 5$

$$E_5 = \frac{-13.6 \text{ eV}}{5^2} = -0.54 \text{ eV}$$

Hence correct option is (a).

36. $K_{\max} = E - W = (6.2 - 4.2)\,\text{eV} = 2\,\text{eV}$

$\Rightarrow K_{\max} = 2 \times 1.6 \times 10^{-19}$ J

$= 3.2 \times 10^{-19}$ J

Hence correct option is (b).

37. $\lambda = \dfrac{6.62 \times 10^{-34}}{5200 \times 10^{-10}} = 9.1 \times 10^{-31} \times v$

$$\Rightarrow \qquad v = \frac{6.625 \times 10^{-27}}{5.2 \times 9.1 \times 10^{-31}} \simeq 1400\,\text{m/s}$$

Hence correct option is (c).

38. $K_{\max} = \left[\dfrac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{3000 \times 10^{-10} \times 1.6 \times 10^{-19}} - 1\right] \text{eV}$

$K_{\max} = 3.14\ \text{eV} = 3.14 \times 1.6 \times 10^{-19}$ J

$\Rightarrow \dfrac{1}{2} m_e v_{\max}^2 = 3.14 \times 1.6 \times 10^{-19}$

$\Rightarrow v_{\max} = \sqrt{\dfrac{3.14 \times 3.2 \times 10^{-19}}{9.1 \times 10^{-31}}} \simeq 10^{6}\,\text{m/s}$

Hence correct option is (d).

JEE Corner

1. Here both assertion and reason are true and reason explain correctly assertion. Correct option is (a).

2. For photon $E = \dfrac{hc}{\lambda}$ and $p = \dfrac{E}{c}$

$\Rightarrow$ If λ is doubled, E and p are reduced to half. Hence assertion is true. Since speed of photon is always c. Hence reason is false. Hence correct option is (c).

3. If frequency is increased keeping intensity constant photoelectron emitted the plate reach other plate in less time hence saturation current can be increased. Reason can be true or not hence correct option is (a, b).

4. Here both assertion and reason is true and reason correctly explain assertion. Hence correct option is (a).

5. Here assertion is true since possible transition are $6 \to 3, 6 \to 4, 6 \to 5, 5 \to 3, 5 \to 4$, and $4 \to 3$. According to reason total $\dfrac{n(n-1)}{2}$ transition has $n = 3 \Rightarrow \dfrac{6 \times (3-1)}{2} = 6$ it may explain or may not explain assertion

Hence correct option is (a, b)

6. We have

$$eV_0 = h[\nu - \nu_0]$$

$$\Rightarrow \qquad V_0 = \frac{h}{e}\nu_0 - \frac{h}{e}\nu_0 \qquad \ldots\text{(i)}$$

if $\nu \to 2\nu_0$ ν_0 does not become double hence assertion is false but reason is true. Hence correct option is (d).

7. Here both assertion and reason are true and reason may or may not explain assertion correct option is (a, b).

8. Here assertion and reason are both true.

$\because \quad \lambda_{\min} = \dfrac{hc}{eV}$ if $V \to$ increases

$\lambda_{\min} \to$ decreases

but reason is not correct explanation of assertion hence correct option is (b).

9. $\because \quad E_n = \dfrac{-13.6}{n_2}$

$\Rightarrow \qquad E_2 > E_1$

Hence assertion is true and $E = -K = +\dfrac{v}{2}$

$\Rightarrow v$ is more in $n = 2$

Here reason is also true but it is not correct explanation of assertion hence correct option is (b).

10. Here assertion is false but reason is true. Hence correct option is (a).

■ Objective Questions (Level 2)

1. $F_a = F_c \Rightarrow \dfrac{GmM}{r^2} = \dfrac{mv^2}{r}$ ...(i)

and $mvr = \dfrac{nh}{2\pi}$ ...(ii)

From Eq. (ii)

$\Rightarrow v = \dfrac{nh}{2\pi mr}$

Putting this value in Eq. (i)

$\dfrac{GM}{r} = \dfrac{h^2h^2}{4\pi^2m^2r^2}$

$\Rightarrow \quad r = \dfrac{n^2h^2}{4\pi^2m^2GM}$

$\text{KE} = \dfrac{1}{2}mv^2 = \dfrac{1}{2}\dfrac{m \times GM}{r} = \dfrac{GMm}{2r}$

$\text{PE} = \dfrac{-GMm}{r}$

$\Rightarrow E = \text{KE} + \text{PE}$

$\Rightarrow E = \dfrac{-GMm}{2r} = \dfrac{-GMm \times 4\pi^2m^2GM}{2n^2h^2}$

$\Rightarrow E = \dfrac{-2\pi^2G^2M^2m^3}{n^2h^2}$ for ground state $n = 1$

$\Rightarrow E = \dfrac{-2\pi^2G^2M^2m^3}{n^2}$

Hence correct option is (b).

2. We have $\mu_n = i_nA_n = \dfrac{e}{T_n} \times \pi r_n^2$

$\Rightarrow \mu_n = \dfrac{e \times \pi \times r_n^2 \times u_n}{2\pi r_n} = \dfrac{e\,u_nr_n}{2}$

$\Rightarrow \mu_n = e \times \dfrac{v_1}{n} \times \dfrac{r_1 \times n^2}{2} = \dfrac{e\,v_1r_1}{2} \times n$

$\Rightarrow \quad \mu_2 = \dfrac{ev_1r_1 \times 2}{2}$

and $\quad \mu_1 = \dfrac{e\,v_1r_1}{2} \times 1$

$\Rightarrow \quad \mu_1 = \dfrac{\mu_2}{2}$

Hence magnetic moment decreases two times correct option is (b).

3. For H-like atom $E_n = \dfrac{-(13.6)Z^2\text{eV}}{n^2}$

Here $E_2 = \dfrac{-(13.6)z^2}{4}$ and $E_1 = -13.6\, z^2$

$E_2 - E_1 = 40.8$ eV

$\Rightarrow \quad 13.6Z^2 \times \left[1 - \dfrac{1}{4}\right] = 40.8$

$\Rightarrow \quad Z^2 = \dfrac{40.8 \times 4}{13.6 \times 3} \Rightarrow Z^2 = 4$

$\Rightarrow \quad Z = 2$

Energy needed to remove the electron from ground state is

$-E_1 = +(13.6) \times Z^2 = +13.6 \times 4 = 54.4$ eV

Hence correct option is (a).

$$i_n = \frac{e}{T_n} = \frac{e}{\frac{2\pi r_n}{u_n}} = \frac{ev_n}{2\pi r_n}$$

$\because \quad u_n \propto \dfrac{1}{n}$ and $r_n \propto \dfrac{1}{n^2}$

$\Rightarrow \quad i_n \propto \dfrac{1}{n^3} \Rightarrow \dfrac{i_2}{i_1} = \dfrac{1}{2^3}$

$\Rightarrow \quad i_1 = 8i_2$

Hence current increases 8 times correct option is (c).

5. Since five dark lines are possible hence atom is excited to $n = 6$ state.

The number of transition in emission line $= \dfrac{n(n-1)}{2}$

Number of emission transition $= \dfrac{6 \times 5}{2} = 15$

Hence correct option is (c).

6. $A_n = \pi r_n^2$ for hydrozen atom $r_n = kn^2$

where k is constant.

$\Rightarrow \quad A_n = \pi k^2 n^4$

$\Rightarrow \quad A_1 = \pi k^2 \propto \dfrac{A_n}{A_1} = n^4$

Taking log both sides $\log\left(\frac{A_n}{A_1}\right) = 4\log_n$

Hence it is a straight line with slope = 4

Correct option is (b)

7. For hydrogen atom $i_n \propto \frac{1}{n^3}$ and

$$B_n \propto \frac{i_n}{r_n} \propto B_n = k\frac{1}{n^5} \qquad [\because r_n \times n^2]$$

$$\Rightarrow \qquad B_2 = \frac{k}{2^5} \text{ and } B_1 = k$$

$$\Rightarrow \qquad \frac{B_2}{B_1} = \frac{1}{2^5} = \frac{1}{32}$$

$$\Rightarrow \qquad B_1 = 32\,B_2$$

Hence magnatic field increases 32 times.

The correct option is (d).

8. For H-atom Lyman series is given by

$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$ for first line $n = 2$

$$\Rightarrow \qquad \frac{1}{\lambda} = R\left[1 - \frac{1}{2^2}\right]$$

$$\Rightarrow \qquad \frac{1}{\lambda} = \frac{3R}{4}$$

Momentum of photon $P_p = \frac{h}{\lambda}$

Let momentum of atom p_A

$\because$ Initial momentum was zero. Hence using momentum conservation law, we get

$$p_A = p_B \Rightarrow Mv = \frac{h}{\lambda} = \frac{3hR}{4}$$

$$\Rightarrow \qquad v = \frac{3hR}{4M}$$

Hence the correct option is (a).

9. Light wave equation is

$$200\text{ V/m} \sin(1.5 \times 10^{15}\text{sec}^{-1})\,t$$
$$\times \cos(0.5 \times 10^{15}\text{sec}^{-1})\,t$$

Here maximum frequency $= \frac{1.5 \times 10^{15}}{2\pi}$

Maxmum incident energy

$$= \frac{1.5 \times 10^{15}}{2\pi} \times \frac{6.6 \times 10^{-34}}{1.6 \times 10^{-19}}$$

$$= 0.98\text{ eV}$$

Since work function $= 2\text{ eV} >$ maximum energy hence no emission of electrons.

Thus correct option is (d).

10. Since in Balmer series of H-like atom wavelengths (in visible region) are found same or smaller hence the gas was initially in second excited state.

Correct option is (c).

11. For H-atom $T_H = 2\pi n^3$

and for H-like atom

$$T_x = \frac{2\pi n^3}{z^2}$$

For H-atom in ground state $T_H = 2\pi$

For H-like atom in first excited state

$$T_x = \frac{(2\pi) \times 2^3}{z^2} = \frac{2\pi \times 8}{z^2}$$

But $T_H = 2T_x \Rightarrow 2\pi = \frac{2 \times 2\pi \times 8}{z^2}$

$$\Rightarrow \qquad z^2 = 16$$

$$\Rightarrow \qquad z = 4$$

Hence correct option is (c).

12. For K_α line of X-ray

$$\frac{1}{\lambda} = \frac{a^2}{c}(z - 1)^2$$

$\because z$ (atomic No.) for Pb^{204}, Pb^{206}, Pb^{208} are same hence $\lambda_1 = \lambda_2 = \lambda_3$.

Hence correct option is (c).

13. The correct option is (d).

14. Since $E_n = \frac{-13.6\text{ eV}}{n^2}$

$$\Rightarrow E_1 = -13.6\text{ eV}$$

and first excited state $E_2 = \frac{-13.6}{4}\text{eV}$

$\Rightarrow \quad E_2 = -3.4 \text{ eV}$

$$\Delta E = E_2 - E_1 = 10.2 \text{ if } K < 10.2 \text{ eV}$$

The electron collide elastically with H-atom in ground state.

The correct option is (c).

15. For Lyman series $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$ here $n = 3$

$$\Rightarrow \quad \frac{1}{\lambda} = R\left[1 - \frac{1}{9}\right]$$

$$\Rightarrow \quad \frac{1}{\lambda} = \frac{8R}{9}$$

$$P_{\text{photon}} = \frac{h}{\lambda} = \frac{8Rh}{9}$$

But $\quad P_{\text{Photon}} = P_{\text{H-atom}}$

$$\Rightarrow \quad \frac{8Rh}{9} = M_p \times v$$

$$\Rightarrow v = \frac{8 \times 1.097 \times 10^7 \times 6.6 \times 10^{-34}}{9 \times 1837 \times 9.1 \times 10^{-31}}$$

$\Rightarrow v = 4$ m/s

Hence correct option is (a).

16. Power $= VI = 150 \times 10^3 \times 10 \times 10^{-3} = 1500$ W

The 99% power heated the target hence

Heating power $= \frac{99}{100} \times 1500 = 15 \times 99$ W

The rate at which target is heated per sec. (in cal)

$$= \frac{15 \times 99}{4.2} \simeq 355 \qquad \left[\because 1\text{J} = \frac{1}{4.2}\text{cal}\right]$$

Hence correct option is (c).

17. $E_n = -\frac{13.6 \text{ eV} \times z^2}{n^2}$

$$E_3 = -\frac{13.6 \text{ eV}}{9} z^2 \text{ and } E_4 = \frac{-13.6 \text{ eV}}{16} z^2$$

$$\Delta E = E_4 - E_3 = (13.6) \text{ eV} \times z^2 \left[\frac{1}{9} - \frac{1}{16}\right]$$

$$\Rightarrow \Delta E = \frac{13.6(\text{eV}) \times 2^2 \times 7}{16 \times 9} = 32.4 \text{ eV}$$

$$\Rightarrow z^2 = \frac{16 \times 32.4 \times 9}{13.6 \times 7} = 49$$

$\Rightarrow z = 7$

Hence correct option is (d).

18. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_p K}} = \frac{h}{\sqrt{2m_p eV}}$

$\Rightarrow$

$$10^{-13} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1836 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}}$$

$$\Rightarrow \quad V = 8.15 \times 10^4 \text{ volt}$$

Hence correct option is (b).

19. Since $E_n \propto \frac{1}{n^2}$ and $L_n \propto n$

Hence $\quad E_n \propto \frac{1}{L_n^{\,2}}$

The correct option is (d).

20. Since $\mu_n = \frac{e u_n r_n}{2\pi} \because u_n \propto \frac{1}{n}$ and $r_n \propto n^2$

$\Rightarrow \mu_n = kn$

Where k is constant for H-atom

For ground state $\quad \mu_1 = k \times 1 = k \qquad$ ...(i)

For third excited state $n = 4$

$$\mu_2 = k \times 4 = 4k \qquad \text{...(ii)}$$

From Eqs. (i) and (ii) we get $\mu_2 = 4\mu_1$

Hence correct option is (d).

21. By conservation of momentum

$$M_H v = (M_H + M_H) v' \Rightarrow v' = \frac{v}{2}$$

Let initial KE of H-atom $= K$

Final KE of each-H-atom $= \frac{K}{2}$

For excitation $\frac{K}{2} = E_2 - E_1 = \frac{-13.6}{4} + 13.6$

$$\Rightarrow \quad \frac{K}{2} = 10.2 \text{ eV}$$

$$\Rightarrow \quad K = 20.4 \text{ eV}$$

Hence correct option is (a).

22. We know that for H-like atom

$$E_n = -K_n \Rightarrow K_n = 3.4 \text{ eV}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e K}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}}}$$

$$\Rightarrow \lambda = 6.6 \text{ Å}$$

Hence both options (a) and (b) are correct.

Hence answer is (c) both are correct.

23-25. $0.6e = \dfrac{hc}{4950 \times 10^{-10}} - W$ ...(i)

$$1.1\, e = \frac{hc}{\lambda_2} - W \quad \text{...(ii)}$$

Subtracting Eq. (i) from Eq. (ii) we get

$$0.5\, e = hc\left[\frac{1}{\lambda_2} - \frac{1}{4950 \times 10^{-10}}\right]$$

$$\Rightarrow \frac{0.5 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34} \times 3 \times 10^{8}} + \frac{1}{4950 \times 10^{-10}} = \frac{1}{\lambda_2}$$

$$\Rightarrow \lambda_2 \simeq 4111 \text{ Å}$$

From Eq. (i)

$$W = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{4.95 \times 10^{-7} \times 1.6 \times 10^{-19}} \simeq 1.9 \text{ eV}$$

23. $W = 1.9$ eV

Hence correct option is (c).

24. $\lambda = 4111$ Å

Hence correct option is (c)

25. Since magnatic field does not change the KE of electrons hence retarding potential remain same.

Hence correct option is (c).

26. $(\text{KE})_{\max} = 5 \text{ eV} - 3 \text{ eV} = 2 \text{ eV}$

$$= 2 \times 1.6 \times -19 \text{ J}$$

$$\lambda_{\min} = \frac{h}{p} = \frac{h}{\sqrt{2m_e K}}$$

$$\Rightarrow \lambda_{\min} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 2 \times 1.6 \times 10^{-19}}}$$

$$= 8.69 \text{ Å}$$

Hence correct option is (b).

27. Photo emission will stop when potential of sphere becomes stopping potential

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = 2 \text{ V}$$

Since $\quad (\text{KE})_{\max} = eV_0$

hence $\quad V_0 = 2$

$$\Rightarrow \quad q = 8\pi\varepsilon_0 r \text{ coulomb}$$

Hence correct option is (b)

28. Let t be the time for photo emission

$$\frac{1}{4\pi\varepsilon_0} \times \frac{q}{r} \times t = 2$$

$$t = \frac{8\pi\varepsilon_0 r}{q}$$

Intensity of light at 0.8 from source

$$I = \frac{3.2 \times 10^{-3}}{4\pi \times (0.8)^2} \simeq 4 \times 10^{-4} \text{ W/m}^2$$

Energy incident on the sphere in unit time

$$E_1 = \pi \times (8 \times 10^{-3})^2 \times 4 \times 10^{-4} = 8.04 \times 10^{-8} \text{ W}$$

Energy of each photon

$$E_2 = 5 \times 1.6 \times 10^{-19} = 8 \times 10^{-19} \text{ J}$$

Total number of photons incident on the sphere per second

$$\eta = \frac{E_1}{E_2} = \frac{8.04 \times 10^{-8}}{8 \times 10^{-19}} = 10^{11}$$

Since 10^6 photons emit one electron.

Hence the total number of photoelectron per sec is $\quad n_2 = \dfrac{n_1}{10^1} = \dfrac{10^{11}}{10^6} = 10^5$

Therefore,

$$q = n_2 \times e \times t = 10^5 \times 1.6 \times 10^{-19} \times t$$

$$\Rightarrow \quad \frac{9 \times 10^9 \times 1.6 \times 10^{-14} \times t}{r} = 2$$

$$t = \frac{2 \times 8 \times 10^{-3}}{9 \times 1.6 \times 10^{-5}} \Rightarrow t = 111\text{ s}$$

Hence correct option is (c).

More than one options are correct

1. Since $\lambda_0 = \frac{hc}{eV}$ if v increases λ_0 decreases hence the interval between $\lambda_{K\alpha}$ and λ_0 as well as $\lambda_{K\beta}$ and λ_0 increases.

The correct options are (b) and (c).

2. $R \propto n^2, V \propto \frac{1}{n}$ and $E \propto \frac{1}{n^2}$ for Bohr model of H-atom

$$\Rightarrow \quad VR \propto n \text{ and } \frac{V}{E} \propto n$$

Hence, the correct options are (a) and (c).

3. For Bohr model of H-atom

$$L \propto n, \quad r \propto n^2 \quad \text{and} \quad T \propto n^3$$

Hence $\frac{rL}{T}$ is independent of n

$$\frac{L}{T} \propto \frac{1}{n^2} \text{ and } \frac{T}{r} \propto n, L \propto n^3$$

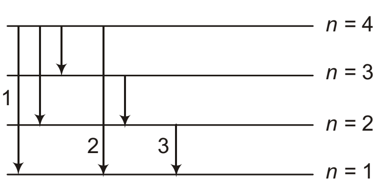
Hence correct options are (a), (b) and (c).

4. $\because \lambda = \frac{h}{mv}$ and $\lambda = \frac{h}{\sqrt{2mK}}$

Hence heavy particle has smallest wavelength when speed and KE both particle are same.

The correct options are (a) and (c).

5. Since there are six different wavelength



Hence, final state will be $n = 4$.

Since two wavelengths are longer than λ_0 [(From $n = 4 \to 3$ and $n = 3 \to 2$)]

Hence initial state was $n = 2$

and there are three transitions shown as (1), (2) and (3) belonging to Lymen series.

Hence correct options are (a), (b) and (d).

6. $\because \sqrt{f} = a(z - b)$

$\sqrt{f}$ *versus* z is a straight line

$$\because \quad f = \frac{c}{\lambda} \Rightarrow \sqrt{\frac{1}{\lambda}} = \frac{a}{\sqrt{c}}(z - b)$$

hence $\sqrt{\frac{1}{\lambda}}$ *versus* z is a straight line

$$f = a^2(z - b)$$

$\Rightarrow \log f = \log a^2 + \log(z - b)$ which is a straight line

Hence correct options are (a), (b) and (c).

Match the Columns :

1. Lymen series lies in UV region, Balmer series lies in visible region and Paschen and Brackelt series lie in infrared region. Hence

(a)	r
(b)	q
(c)	p
(d)	p

2. For H-atom $E_n = \frac{-13.6}{n^2}$ eV

$$\Rightarrow \quad E_2 = \frac{-13.6}{2^2}\text{ eV} = \frac{-13.6}{4}\text{ eV}$$

Ionization energy from first excited state of H-atom

$$E = -E_2 = \frac{13.6}{4}\text{ eV} \qquad \text{...(i)}$$

For He^+ ion

$$E_{H(\text{He})} = \frac{-(13.6)\text{ eV}}{n^2} \times Z^2 \text{ for } He^+ \; z = 2$$

$$\Rightarrow E_n(\text{He}) = \frac{-13.6\text{ eV}}{n^2} \times 4$$

Ionization energy of He^+ atom from ground state $= (13.6)\text{ eV} \times 4 = 4E \times 4$ from Eq. (i)

$$= 16E$$

$$E_2(\text{He}) = \frac{-(13.6)\,\text{eV} \times 4}{4} = -(13.6)\,\text{eV}$$

But $E_2 = -K_2 \Rightarrow K_2 = (13.6)$

and $\overline{U} = -2K = -2 \times (13.6) = -2 \times 4E = -8E$

From Eq. (i)

KE in ground state of He^+ ion

$$= (13.6)\,\text{eV} \times 4$$
$$= 4E \times 4 = 16E$$

Ionisation energy from Ist executive state

$$= -\left(\frac{-13.6\,\text{eV}}{4}\right) \times 4 = 13.6\,\text{eV} = 4E$$

Hence correct match are

(a) $\longrightarrow$	s
(b) $\longrightarrow$	r
(c) $\longrightarrow$	s
(d) $\longrightarrow$	p

3. $K_{\max} = h\nu - W$ and $V_0 = \frac{h}{e}\nu - \frac{W}{e}$

Slope of line -1 is h, $Y_1 = W$

Slope of line -2 is $\frac{h}{e}$, $Y_2 = \frac{W}{e}$

Hence correct match are

(a) $\longrightarrow$	q
(b) $\longrightarrow$	p
(c) $\longrightarrow$	r
(d) $\longrightarrow$	s

4. $T = \frac{2\pi r}{v} \because r \propto \frac{n^2}{z} \Rightarrow U \propto \frac{z}{n}$

$$\Rightarrow \quad T \propto \frac{n^3}{z^2}$$

$\Rightarrow (a) \to r$

$\because \quad L \propto n \Rightarrow \text{(b)} \to s$

$\because \quad V \propto \frac{z}{n} \Rightarrow \text{(c)} \to s$

$\because \quad R \propto \frac{n^2}{z} \Rightarrow \text{(d)} \to q$

5. Balmer series is

$$\frac{1}{\lambda_B} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right] n = 3, 4, 5 \ldots$$

For 2nd line $n = 4$ and $\lambda_B = \lambda$

$$\Rightarrow \quad \frac{1}{\lambda} = \left(\frac{1}{4} - \frac{1}{16}\right)R$$

$$\Rightarrow \quad \lambda = \frac{16}{3R}$$

$$\frac{1}{\lambda_{B_1}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$\Rightarrow \quad \lambda_{B_1} = \frac{36}{5R} = \frac{36}{5\left(\frac{16}{3\lambda}\right)} = \frac{36 \times 3\lambda}{16 \times 5}$$

$$\Rightarrow \quad \lambda_{B_1} = \left(\frac{27}{20}\right)\lambda$$

$\Rightarrow$ (a)$\to p$

$$\frac{1}{\lambda_{B_3}} = R\left[\frac{1}{2^2} - \frac{1}{5^2}\right] = \frac{R \times (25-4)}{25 \times 4} = \frac{21R}{100}$$

$$\Rightarrow \lambda_{B_3} = \frac{100}{21R} = \frac{100}{21 \times \left(\frac{16}{3\lambda}\right)}$$

$$= \frac{100 \times 3\lambda}{21 \times 16} = \frac{25\lambda}{28}$$

$\Rightarrow$ (b)$\to$(s)

For Lyman series

$$\frac{1}{\lambda_L} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right] \quad n = 2, 3, 4$$

$$\Rightarrow \frac{1}{\lambda_{L_1}} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$\Rightarrow \lambda_{L_1} = \frac{4}{3R}$$

$$\Rightarrow \lambda_{L_1} = \frac{5}{\left(\frac{16}{\lambda}\right)} = \frac{\lambda}{4} \Rightarrow \text{(c)} \to \text{q}$$

$$\frac{1}{\lambda_{L_2}} = R\left[1 - \frac{1}{9}\right] = \frac{8R}{9}$$

$$\Rightarrow \lambda_{L_2} = \frac{9}{8R} = \frac{9}{8 \times \frac{16}{3\lambda}} = \frac{9 \times 3\lambda}{16 \times 8} = \frac{27\lambda}{128}$$

$\Rightarrow$ (d) $\to s$

6. The proper match are

(a)$\rightarrow s$

$\because$ X-ray is inverse process of photoelectric effect [high energy electrons convert in electromagnetic radiation]

$\Rightarrow$ (b) $\rightarrow p$

$\because \qquad \lambda_c \propto \dfrac{1}{V} \Rightarrow$ (c)$\rightarrow q$

$\because$ Wavelength of continuos X-ray depends on voltage

$\Rightarrow$ (d) $\rightarrow q$

7. $\because (\text{KE})_{\max} \propto f$ and stopping potential $\propto f$

$\Rightarrow$ (a) $\rightarrow p, r$

$\because$ stopping potental $\propto f$ hence it remains same

$\Rightarrow$ (b)$\rightarrow s$

$\because$ Current is directly proportional to Intensity. Hence sat current increases but stopping potential does not chase

$\Rightarrow$ (c) $\rightarrow q, s$

$\because (\text{KE})_{\max} = hf - W$ and stopping potential

$$= \left(\frac{hf}{e} - \frac{W}{e}\right)$$

If W is decreased $(\text{KE})_{\max}$ and stopping potential increased

$\Rightarrow$ (d) $\rightarrow p, r$

Modern Physics II

Introductory Exercise 30.1

1. $R_0 = \lambda N_0 = 8000\,\text{Bq}$

$$R_1 = \lambda N = 1000\,\text{Bq}$$

$$\Rightarrow \quad \frac{R_1}{R_0} = \frac{1}{8} = \frac{N}{N_0} = e^{-\lambda t} \Rightarrow \log 8 = \lambda t$$

$$\Rightarrow \quad 3 \log 2 = \frac{\log 2}{T_{1/2}} \times 9$$

$$\Rightarrow \quad T_{1/2} = \frac{9}{3} = 3 \text{ days}$$

Average, life $= 1.44 \times T_{1/2} = 1.44 \times 3 = 4.33$ days

2. $R_0 = \lambda N_0$

$$\Rightarrow \quad 40 \times 3.7 \times 10^{10} \times 10^{-6} = \frac{0.693}{64.8} \times N_0$$

$$\Rightarrow \quad N_0 = \frac{40 \times 64.8 \times 3.7 \times 10^4}{0.693}$$

$$= 13.83 \times 10^7$$

Now, $\quad N = N_0 e^{-\lambda t}$

$$N_{10} = N_0 e^{\frac{-0.693 \times 10}{64.8}}$$

$$= 13.83 \times 10^7 \times e^{\frac{-0.693 \times 10}{64.8}}$$

$$N_{12} = 13.83 \times 10^7 \times e^{\frac{-0.693 \times 12}{64.8}}$$

$$\Rightarrow \quad N_{10} - N_{12} = 13.83 \times 10^7 \left[e^{\frac{-0.693 \times 10}{64.8}} - e^{\frac{-0.693 \times 12}{64.8}} \right]$$

$$\approx 9.47 \times 10^9 \text{ nuclei}$$

3. (a) $\quad R_0 = 10\,\text{mCi}, R = 8\text{ mCi} \Rightarrow \frac{R_0}{R} = e^{\lambda t}$

$$\Rightarrow \quad \log \frac{10}{8} = \lambda \times 4 \times 3600$$

$$\Rightarrow \quad \lambda = \frac{0.223}{4 \times 3600}$$

$$\Rightarrow \quad \lambda = 1.55 \times 10^{-5}/\text{s}$$

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{1.55 \times 10^{-5}} = 12.4\text{ h}$$

(b) $R_0 = 10\,\text{mci} = 10 \times 3.7 \times 10^{10} \times 10^{-3}$ Bq

$$= 3.7 \times 10^8 \text{ Bq}$$

$$R_0 = \lambda N_0$$

$$\Rightarrow N_0 = \frac{R_0}{\lambda} = \frac{3.7 \times 10^8}{1.55 \times 10^{-5}}$$

$$= 2.39 \times 10^{13} \text{ (atoms)}$$

(c) $R = R_0 e^{-\lambda t}$

$$= 10\,\text{mCi} \times e^{-\frac{0.693}{12.4} \times 30} = 1.87 \text{ mCi}$$

4. $R_0 = \lambda N_0$

$$\Rightarrow \lambda = \frac{R_0}{N_0} = \frac{6 \times 10^{11}\text{Bq}}{10^{15}}$$

$$= 6 \times 10^{-4}/\text{s}$$

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{6 \times 10^{-4}} = 1.16 \times 10^3\text{s}$$

5. $N_x = N_y = N_0$

$T_{1/2x} = 50\,\text{min}$ and $T_{1/2\,y} = 100\,\text{min}$

$$N_x = (N_0)\left(\frac{1}{2}\right)^n = N_0 \left(\frac{1}{2}\right)^4 = \frac{N_0}{16}$$

$$N_y = N_0 \left(\frac{1}{2}\right)^n = N_0 \left(\frac{1}{2}\right)^2 = \frac{N_0}{4}$$

$$\frac{N_x}{N_y} = \frac{\frac{N_0}{16}}{\frac{N_0}{4}} = \frac{1}{4}$$

Introductory Exercise 30.2

1. $\Delta E = \Delta M c^2$

Here, $P = 10^9$ J/s $= 10^9 \times 24 \times 60$ J/day

$\Rightarrow \Delta M = \dfrac{10^9 \times 24 \times 60}{(3 \times 10^8)^2} = 9.6 \times 10^{-4}$ kg

2. Number of fission $= \dfrac{10^9 \text{ J/s}}{200 \times 10^6 \times 1.6 \times 10^{-19}}$

$= 3.125 \times 10^{19}$

3. Given reaction is

$_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_2He^4$

$\Delta M = (238.050784 - 234.043593$

$- 4.002602)$ u

$\Delta M = 0.0004589$ u

$\Delta E = \Delta M \times 931.5$ MeV

$= 0.0004589 \times 931.5$

$= 4.27$ MeV

4. Complete reactions are

(a) $_3Li^6 + {}_1H^2 \rightarrow {}_4Be^7 + {}_0n^1$

(b) $_{17}Cl^{35} + {}_1H^1 \rightarrow {}_{16}S^{32} + {}_2He^4$

(c) $_4Be^9 + {}_2He^4 \rightarrow 3({}_2He^4) + {}_0n^1$

(d) $_{35}Br^{79} + {}_1H^2 \rightarrow {}_{36}Kr^{79} + 2({}_0n^1)$

AIEEE Corner

■ Subjective Questions (Level 1)

Radioactivity

1. (a) Initially the rate of distingration is

$-\left(\dfrac{dN}{dt}\right)_0 = \lambda N_0$

After 5 min $\dfrac{-dN}{dt} = \lambda N$

$\Rightarrow \dfrac{N_0}{N} = \dfrac{\left(\dfrac{dN}{dt}\right)_0}{\dfrac{dN}{dt}} = \dfrac{4750}{2700} = 1.76$

Now $N = N_0 e^{-\lambda t}$ or $\lambda = -\dfrac{\log\left(\dfrac{N}{N_0}\right)}{t}$

$\Rightarrow \lambda = \dfrac{2.3026}{t}\log_{10}\dfrac{N_0}{N}$

$= \dfrac{2.3026}{5}\log_{10}(1.76)$

$\Rightarrow \lambda = 0.113$ /min

(b) Half-life $= \dfrac{0.693}{\lambda} = \dfrac{0.693}{0.113} = 6.132$ min

2. We have $A = \lambda N$

$6 \times 10^{11} = \lambda \times 1 \times 10^{15}$

$\Rightarrow \lambda = 6 \times 10^{-4}\ \text{s}^{-1}$

$T_{1/2} = \dfrac{0.693}{\lambda} = \dfrac{0.693}{6 \times 10^{-4}} = 1155$ s

$= 19.25$ min

3. $A = \lambda N$

$A = 8$ Ci $= 8 \times 3.7 \times 10^{10}$ decay/s

$\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{5.3 \text{ yr}}$

$= \dfrac{0.693}{5.3 \times 365 \times 24 \times 60 \times 60 \text{s}}$

$\Rightarrow N = \dfrac{A}{\lambda}$

$= \dfrac{8 \times 3.7 \times 10^{10} \times 5.3 \times 365 \times 24 \times 3600}{0.693}$

$\Rightarrow N = 7.2 \times 10^{19}$

6.023×10^{23} nuclei $= 60$ g

$$1 = \frac{60 \text{ g}}{6.023 \times 10^{23}}$$

Hence 7.2×10^{19} nuclei

$$= \frac{60 \times 7.2 \times 10^{19}}{6.023 \times 10^{23}} = 7.11 \times 10^{-3} \text{ g}$$

4. Number of decay per second

$$\frac{m}{M} \times N_A \times \lambda = \frac{1}{238} \times 6 \times 10^{23} \times \frac{0.693}{4.5 \times 10^9 \text{ yr}}$$

$$= \frac{6 \times 10^{23}}{238} \times \frac{0.693}{4.5 \times 10^9 \times 365 \times 24 \times 60 \times 60 \text{s}}$$

$$= 1.23 \times 10^4 \text{ decay/s}$$

5. Probability of decay

$$P = (1 - e^{-\lambda t}) = (1 - e^{-t/T_{\text{mean}}})$$

$$\Rightarrow P = (1 - e^{-5/10}) = 1 - e^{-2} = 0.39$$

6. Since initially no $\overset{206}{\text{Pb}}$ nuclei is present and after time t the ratio of $\dfrac{N_v}{N_{\text{Pb}}} = 3$

It means $\dfrac{1}{4}$ of original $\overset{238}{\text{U}}$ nuclei decays.

Hence $\quad N = N_0 e^{-\lambda t}$

$$\left(N_0 - \frac{N_0}{4}\right) = N_0\, e^{-\lambda}$$

$$\frac{3}{4} = e^{-\lambda t}$$

$$\Rightarrow \quad t = \frac{\log 4 - \log 3}{\lambda}$$

$$\Rightarrow \quad t = \frac{(\log 4 - \log 3)}{0.693} \times 4.5 \times 10^9 \text{ yr}$$

$$\Rightarrow \quad t = 1.88 \times 10^9 \text{ yr}$$

7. $(R_1)_{0\text{P}_{32}} = \lambda_1 4 N_0$

$(R_2)_{0\text{P}_{33}} = \lambda_2\, N_0$

$$R_0 = (R_1)_{0\text{P}_{32}} + (R_2)_{0\text{P}_{33}}$$

$$= N_0\,(4\lambda_1 + \lambda_2) = 3 \text{ mCi}$$

$(R_1)_{t\text{P}_{32}} = \lambda_1 N = \lambda_1 4 N_0\, e^{-\lambda_1 t}$

$(R_1)_{t\text{P}_{32}} = \lambda_1 \times 4 N_0\, e^{-\frac{\log 2}{14} \times 60 \times 365}$

$(R_2)_{t\text{P}_{33}} = \lambda_2 \times N_0\, e^{-\frac{\log 2}{25} \times 60 \times 365}$

$R = (R_1)_{t\text{P}_{32}} + (R_2)\, t_{\text{P}_{33}}$

$$= N_0 \left[4\lambda_1 e^{-\frac{\log 2}{14} \times 60 \times 365} + \lambda_2\, e^{-\frac{\log 2}{25} \times 60 \times 365}\right]$$

$$= \frac{3 \text{ mCi}}{(4\lambda_1 + \lambda_2)} \left[4\lambda_1\, e^{-\frac{\log 2 \times 60 \times 365}{14}} + \lambda_2 e^{-\frac{\log 2}{25} \times 60 \times 365}\right]$$

$$= \frac{3 \text{ mCi}}{\left[\dfrac{4 \times \log 2}{14} + \dfrac{\log 2}{25}\right]} \left[\frac{4 \log 2}{14} e^{-\frac{\log 2 \times 60 \times 365}{14}} + \frac{\log 2}{25} \times e^{-\frac{\log 2}{25} \times 60 \times 365}\right]$$

$$\approx 0.205 \text{ mCi}$$

8. Complete reactions are

(a) ${}_{88}\text{Ra}^{226} \longrightarrow \alpha + {}_{86}\text{RN}^{222}$

(b) ${}_{8}\text{O}^{19} \longrightarrow {}_{9}\text{F}^{19} + e + \bar{\nu}$

(c) ${}_{13}\text{Al}^{25} \longrightarrow {}_{12}\text{Mg}^{25} + e^{+} + \nu$

9. Only reaction (b) is possible.

10. $\Delta E = (7 \times 1.000783 + 7 \times 1.00867 - 14.00307)$

$$\times\, 931.5 \text{ MeV}$$

$$\Rightarrow \quad \Delta E = 104.72 \text{ MeV}$$

11. $\Delta E = [8m_p + 8m_n - m\,({}_8O^{16})] \times 931.5$

$$= (8 \times 1.007825 + 8 \times 1.008665 - 15.994915)$$

$$\times\, 931.5 = 127.6 \text{ MeV}$$

12. (a) Number of nuclei in kg

$$= \frac{6.023 \times 10^{23}}{235} \times 1$$

Energy

$$= \frac{6.023 \times 10^{23}}{235} \times 200 \times 10^6 \times 1.6 \times 10^{-19}$$

$$= 8.09 \times 10^{13} \text{J}$$

(b) Mass $= \dfrac{8.09 \times 10^{13}\,\text{J}}{30 \times 10^3\,\text{J/g}}$

$= \dfrac{8.09 \times 10^{13}}{30 \times 10^3}\,\text{g}$

$= \dfrac{8.09}{3} \times 10^9 \times \dfrac{1}{10^3}\,\text{kg}$

$= 2.7 \times 10^6\,\text{kg}$

13. Applying conservation of momentum

$$M_\alpha v_\alpha = M_{\text{Ti}} v_{\text{Ti}}$$

$$\Rightarrow \quad v_{\text{Ti}} = \frac{M_\alpha v_\alpha}{M_{\text{Ti}}}$$

$$K_\alpha = \frac{1}{2} M_\alpha v_\alpha^2 = 6.802\,\text{MeV}$$

$$K_{\text{Ti}} = \frac{1}{2} M_{\text{Ti}}\, v_{\text{Ti}}^2 = \frac{1}{2} M_{\text{Ti}} \frac{M_\alpha^2 v_\alpha^L}{M_{\text{Ti}}^2}$$

$$\Rightarrow \quad K_{\text{Ti}} = \frac{M_\alpha}{M_{\text{Ti}}} \left(\frac{1}{2} M_\alpha v_\alpha^2 \right)$$

$$= \frac{4}{208} \times 6.802\,\text{MeV}$$

$$= \frac{1}{52} \times 6.802 = 0.1308\,\text{MeV}$$

14. Power $= 100\,\text{MW} = 10^8\,\text{W} = 10^8\,\text{J/s}$

$$= \frac{10^8}{1.6 \times 10^{-13}\,\text{J/MeV}} = \frac{10^{21}}{1.6} \frac{\text{MeV}}{\text{s}}$$

Energy per fission $= 185\,\text{MeV}$

Hence number of fissions $= \dfrac{10^{21}}{1.6 \times 185}$ / s.

Number of nuclei in 1 kg
$U^{235} = \dfrac{6.023 \times 10^{26}}{235}$

Hence $t = \dfrac{6.023 \times 10^{26} \times 1.6 \times 185}{235 \times 10^{21}}$

$= 8.78$ days

15. (a) The given reaction is

$${}_1H^2 + {}_1H^2 \longrightarrow {}_1H^3 + {}_1H^1$$

$$\Delta M = [2m({}_1H^2) - m({}_1H^3) - m({}_1H^1)]$$

$$= [2(2.014102) - 3.016049 - 1.007825]u$$

$$\Delta M = 0.000433\,\text{u}$$

$$Q = \Delta M \times 931.5 = 0.000433 \times 931.5$$

$$\approx 4.05\,\text{MeV}$$

(b) $\Delta M = (2 \times 2.014102 - 3.016049 - 1.008665) \times 931.5$

$\Delta M = 3.25\,\text{MeV}$

(c) $\Delta M = (2.014102 + 3.016049 - 4.002603 - 1.008665) \times 931.5$

$\Delta M = 17.57\,\text{MeV}$

16. The given reaction is

$$\text{He}^4 + \text{He}^4 \longrightarrow \text{Be}^8$$

$$\Delta M = (2 \times 4.0026 - 8.0053)\,\text{u}$$

$$= (8.0052 - 8.0053)\,\text{u}$$

$$\Delta M = -(0.0001)\,\text{u}$$

$\because \Delta M$ is negative this reaction is not energetically favourable

$$\Delta E = \Delta M \times 931.5$$

$$= -1 \times 10^{-4} \times 931.5\,\text{MeV}$$

$$= -93.15\,\text{keV}$$

17. Number of nuclei in 1 kg water

$$= \frac{6.023 \times 10^{26}}{18}$$

Heavy water $= \dfrac{6.023 \times 10^{26}}{18} \times \dfrac{1.5 \times 10^{-2}}{100}$

$$= \frac{6.023 \times 1.5}{18} \times 10^{22}$$

Energy realesed per fission

$= (2 \times 2.014102 - 3.016049 - 1.007825) \times 931.5$

$= 4.33 \times 10^{-3} \times 931.5 \times 10^6 \times 1.6 \times 10^{-19}\,\text{J}$

Hence total energy $= \dfrac{6.023 \times 1.5 \times 10^{22}}{18} \times 4.33 \times 931.5 \times 1.6 \times 10^{-16}$

$= 3200\,\text{MJ}$

Objective Questions (Level-1)

1. Since during β^- decay a neutron in the nucleus is transformed into a proton, an electron and an antineutrino as $n \to P + e^- + \bar{\gamma}$.

   Hence Correct option is (c).

2. Since nuclear force is same for all nucleons. Hence $F_1 = F_2 = F_3$

   Correct option is (a)..

3. Given reaction is ${}_{90}X^{200} \longrightarrow {}_{80}Y^{168}$

   Difference in mass number $= 200 - 168 = 32$

   Hence Number of α-particles $= \dfrac{32}{4} = 8$

   Difference in atomic number $= 10$

   hence number of β-particles $= 6$

   Hence correct option is (d).

4. The reaction is

   ${}_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{54}Xe^{138} + {}_{38}Sr^{94} + 3({}_{0}n^{1})$

   The correct option is (b) three neutrons.

5. The reactions are $A \to \beta + \alpha$ and $\beta \to C + 2\beta$

   After one α atomic number reduced by 2 and after 2β atomic number increased by.

   Hence A and C are isotopes Correct option is (d).

6. Here $m_p = 1.00785\,u$, $m_n = 1.00866\,u$

   and $m_\alpha = 4.00274$ u

   $\Delta m = 2(m_p + m_n) - m_\alpha$

   $\Rightarrow \Delta m = [2(1.00785 + 1.00866) - 4.00274]\,u$

   $\Rightarrow \Delta m = 0.03028$ u

   $\Delta E = \Delta m \times 931.5\,\text{MeV}$

   $= 0.3028 \times 931.5 = 28.21\,\text{MeV}$

   Hence correct option is (c).

7. $N = N_0 - \dfrac{7}{8}N_0 = \dfrac{1}{8}N_0$

   But $N = N_0\left(\dfrac{1}{2}\right)^n \Rightarrow \dfrac{1}{8}N_0 = N_0\left(\dfrac{1}{2}\right)^n$

   $\Rightarrow \quad n = 3$

   $\Rightarrow \quad 3 \times T_{1/2} = 8$ s

   $\Rightarrow \quad T_{1/2} = \dfrac{8}{3}$ s

   Hence correct option is (d).

8. $N = N_0\, e^{-\lambda t}$ for mean life $t = \dfrac{1}{\lambda}$

   $\Rightarrow N = N_0\, e^{-\lambda \times \frac{1}{\lambda}} = \dfrac{N_0}{e}$

   Hence the fraction disintegrated

   $= \dfrac{N_0 - N}{N_0} = \left(1 - \dfrac{1}{e}\right)$

   Correct option is (b).

9. $N = N_0 - \dfrac{7}{8}N_0 = \dfrac{1}{8}N_0$

   But $N = N_0\left(\dfrac{1}{2}\right)^n$

   $\Rightarrow \dfrac{1}{8} = \left(\dfrac{1}{2}\right)^n \Rightarrow n = 3$

   Hence half life $T_{1/2} = \dfrac{15\,\text{min}}{3} = 5\,\text{min}$

   Correct option is (a).

10. Since radioactive substance loses half of its activity in 4 days it means its half life

    $T_{1/2} = 4$ days

    Now $A = 5\%$ of A_0

    $\Rightarrow \quad A = \dfrac{1}{20}A_0$

    $\Rightarrow \quad \dfrac{A}{A_0} = \dfrac{1}{20}$

    But $\quad A = A_0\, e^{-\lambda t} \Rightarrow \dfrac{1}{20} = e^{-\lambda t}$

    $\Rightarrow \quad \log 20 = \lambda t \Rightarrow t = \dfrac{\log 20}{\lambda}$

But $\lambda = \frac{\log_e 2}{T_{1/2}}$

$$\Rightarrow t = T_{1/2}\frac{\log_e 20}{\log_e 2} = T_{1/2}\frac{\log_{10} 20}{\log_{10} 2} = 4.32 \times 4$$

$\Rightarrow t = 17.3$ days

Hence correct option is (c).

11. Total energy released per sec

$$= 1.6\,\text{MW} = 1.6 \times 10^6\ \text{J/s}$$

Energy released per fission $= 200\,\text{MeV}$

$$= 200 \times 10^6 \times 1.6 \times 10^{-19}\ \text{J}$$

$$= 2 \times 1.6 \times 10^{-11}\ \text{J}$$

Number of fission per second

$$= \frac{1.6 \times 10^6}{2 \times 1.6 \times 10^{-11}} = 5 \times 10^{16}/\text{s}$$

Hence correct option is (a).

12. $\because R = R_0 A^{1/3}$

Volume $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$

Mass of nucleus $= A \times 1.67 \times 10^{-27}$ kg

$$\text{Density } \rho = \frac{\text{mass}}{\text{volume}} = \frac{A \times 1.67 \times 10^{-27}\ \text{kg}}{\frac{4}{3}\pi R_0^3 A}$$

$$\Rightarrow \quad \rho = \frac{1.67 \times 10^{-27}\ \text{kg}}{\frac{4}{3}\pi R_0^3}$$

$\because \rho$ is independent of A, hence ratio of densities $\frac{\rho_1}{\rho_2} = 1$.

Correct option is (d).

13. $N = N_0 e^{-\lambda t} \Rightarrow \frac{N}{N_0} = e^{-\lambda t}$

$$\Rightarrow \quad \frac{N}{N_0} = e^{-\frac{0.693}{6.93}\times 10} = e^{-1} = \frac{1}{e}$$

Fractional change

$$= \frac{N_0 - N}{N_0} = \left(1 - \frac{1}{e}\right) \simeq 0.63$$

Hence correct option is (b).

14. Since radioactive substance reduce to about 6% it means $N = \frac{N_0}{16}$

We have $\quad N = (N_0)\left(\frac{1}{2}\right)^n$

$$\Rightarrow \quad \frac{1}{16} = \left(\frac{1}{2}\right)^n \Rightarrow n = 4$$

$4 \times T_{1/2} = 2\,h \Rightarrow T_{1/2} = 30$ min

Hence correct option is (a).

15. Probability of a nucleus for survival of time t.

$$p_{(\text{survival})} = \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$$

For one mean life $t = \frac{1}{\lambda}$

$$\Rightarrow P_{\text{survival}} = e^{-\lambda \times \frac{1}{\lambda}} = e^{-1} = \frac{1}{e}$$

Hence Correct option is (a).

■ Assertion and Reason

1. Here both assertion and reason are true but reason does not explain assertion. Hence correct option is (b).

2. Here assertion is false but reason is true since for heavier nucleus binding energy per nucleon is least.

Correct option is (d).

3. Here both assertion and reason are true but reason is not correct explanation of assertion. Hence correct option is (b).

4. Here a ssertion is true but reason is false since electromagnetic waves are produced by accelerating charge particles.

Correct option is (c).

5. Here assertion is wrong since β-decay process is $n \to p + e^- + \bar{v}$

   but reason is true hence correct option is (d).

6. Here assertion is true but reason is false. Correct option is (c).

7. Here both assertion and reason are true and reason may or may not be true. Correct option is (a, b)

8. Both assertion and reason are true but reason is not correct explanation of assertion. Hence correct option is (b).

9. Here reason is true but assertion is false

   $\because$ 1 amu = 931.5 MeV

   Correct option is (d)

10. Both assertion and reason are true but reason does not correctly explain assertion. Hence correct option is (b).

11. Here both assertion and reason are true and reason may or may not be correct explanation of assertion .

    Hence correct options are (a, b).

Objective Questions (Level 2)

■ Single option correct

1. Let initially substance have N_i nuclei then

$$N = N_i e^{-\lambda t}$$

$$\frac{dN}{dt} = -\lambda N_i e^{-\lambda t}$$

At $\quad t = t$

we get

$$\left(\frac{dN}{dt}\right)_{t=t} = -\lambda N_i e^{-\lambda t} = N_0 \qquad \text{...(i)}$$

At $\quad t = 4t$

$$\left(\frac{dN}{dt}\right)_{t=4t} = -\lambda N_i e^{-4\lambda t} = \frac{N_0}{16} \qquad \text{...(ii)}$$

Dividing Eq. (i) and Eq. (ii) we get

$$e^{3\lambda t} = 16 \qquad \text{...(iii)}$$

Now at $t = \left(\frac{11}{2}\right)t$

$$\left(\frac{dN}{dt}\right)_{t=\frac{11t}{2}} = -\lambda N_i e^{\frac{-11\lambda}{2}t}$$

$$= -\lambda N_i e^{\frac{-8\lambda t}{2}} \times e^{\frac{-3\lambda t}{2}}$$

$$= \frac{-\lambda N_i e^{-4\lambda t}}{\sqrt{e^{3\lambda t}}} = \frac{N_0}{16 \times \sqrt{16}}$$

From Eqs. (ii) and (iii)

$$= \frac{N_0}{64}$$

Hence, correct option is (b).

2. We have $\lambda = \lambda_1 + \lambda_2 = \frac{\log 2}{30} + \frac{\log 2}{60}$

$$\Rightarrow \quad \lambda = \frac{\log 2}{20}$$

Now $\quad N = N_0 e^{-\lambda t}$

$$\Rightarrow \quad \frac{N_0}{4} = N_0 e^{\frac{-t \log 2}{20}}$$

$$\Rightarrow \quad \log 4 = \frac{t}{20} \log 2$$

$$\Rightarrow \quad 2 \log 2 = \frac{t}{20} \log 2$$

$$\Rightarrow \quad t = 40 \text{ yr}$$

Hence correct option is (c).

3. From graph it is clear that number of nucleons in X is N_3 and binding energy per nucleon is E_3 for Y nucleon is N_2 and BE per nucleon is E_2.

   Hence $X + Y = E_3 N_3 + E_2 N_2$

   Similerly $W = E_1 N_1$

The reaction is $W \rightarrow X + Y$

The energy released is
$(E_3 N_3 + E_2 N_2 - E_1 N_1)$

Hence Correct option is (b).

4. Energy $= (110 \times 8.2 + 90 \times 8.2 - 200 \times 7.4)$

$$= 200 \times (8.2 - 7.4)$$

$$= 200 \times 0.8 = 160\,\text{MeV}$$

Hence correct option is (d).

5. The reaction is

${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + \text{energy}$

Energy $= (4 \times 7 - 2 \times 1.1)$ MeV

$= (28 - 4.4) = 23.6$ MeV

Hence Correct option is (b).

6. Total energy released per second

$= 16 \times 10^6$ W

$= 16 \times 10^6$ J/s

Energy per fission $= 200$ MeV

$= 200 \times 10^6 \times 1.6 \times 10^{-19}$

$= 2 \times 1.6 \times 10^{-11}$ J

$\because$ Efficiency $= 50\%$

Hence power (energy converted per second)

$$= 2 \times 1.6 \times 10^{-11} \times \frac{50}{100} = 1.6 \times 10^{-11}\,\text{J}$$

$$\text{Number of fission} = \frac{16 \times 10^6}{1.6 \times 10^{-11}} = 10^{18}/\text{s}$$

Hence correct option is (d).

7. $\dfrac{dN}{dt} = A - \lambda N$

$\because$ After time N become

conservation $\Rightarrow \dfrac{dN}{dt} = 0$

$$\Rightarrow \quad N = \frac{A}{\lambda} = \frac{A}{\frac{\log 2}{T}} = \frac{AT}{\log 2}$$

Hence correct option is (d).

8. By conservation of momentum

$$M_H v = (M_H + M_H) v'$$

$$\Rightarrow \quad v' = \frac{v}{2}$$

Let initial KE of H-atom $= K$

Final KE of each H-atom $= \dfrac{K}{2}$

For excitation

$$\frac{K}{2} = E_2 - E_1 = \left(\frac{-13.6}{4} + 13.6\right)\text{eV}$$

$$\Rightarrow \quad \frac{K}{2} = 10.2\,\text{eV}$$

$$\Rightarrow \quad K = 2 \times 10.2 \times 1.6 \times 10^{-19}$$

$$\Rightarrow \quad \frac{1}{2} M_H u^2 = 2 \times 10.2 \times 1.2 \times 1.6 \times 10^{-19}$$

$$\Rightarrow u_H = \sqrt{\frac{2 \times 1.2 \times 10.2 \times 2 \times 1.6 \times 10^{-19}}{1.673 \times 10^{27}}}$$

$$= 6.25 \times 10^4\,\text{m/s}$$

Hence correct option is (c).

9. Let us suppose just before the death no radioactive atoms were present hence original activity A_0 is given as

$$A_0 = \lambda N_0 \qquad \ldots\text{(i)}$$

After death the radioactivity decreases exponentially *ie*,

$$A = \frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} \qquad \ldots\text{(ii)}$$

Dividing eq. (ii) by eq. (i) we get

$$\frac{A}{A_0} = e^{-\lambda t}$$

or $\lambda t = \log \dfrac{A_0}{A}$ or $t = \dfrac{1}{\lambda} \log \dfrac{A_0}{A}$

Now $A_0 = 15$ decay/min/gram

$$A = \frac{375}{200}\,\text{decay/min/g}$$

but $\quad \lambda = \dfrac{0.693}{5730\,\text{yr}}$

$$t = \frac{5730}{0.693} \times \log \frac{15 \times 200}{375} = \frac{5730}{0.693} \times \log \frac{200}{25}$$

$$\Rightarrow \qquad t = \frac{5730}{0.693} \times \log 8$$

$$= \frac{5730}{0.693} \times 3 \log 2 = 5730 \times 3$$

$$\Rightarrow \qquad t = 17190 \text{ yr}$$

Hence correct option is (c).

10. $N_P = N_0\, e^{-\lambda(t_1 + t)}$ ...(i)

and $\quad N_Q = N_0\, e^{-\lambda t}$

Now $\quad A_P = \lambda N_P$ and $A_Q = \lambda N_Q$

$$\Rightarrow \qquad \frac{A_P}{A_Q} = \frac{N_P}{N_Q} = e^{-\lambda t_1}$$

$$\Rightarrow \qquad \lambda t_1 = \log\left(\frac{A_Q}{A_P}\right)$$

$$\Rightarrow \qquad t_1 = \frac{1}{\lambda} \log\left(\frac{A_Q}{A_P}\right) = T \log\left(\frac{A_Q}{A_P}\right)$$

Hence correct option is (b).

11. The given reactions are

$${}_1\text{H}^2 + {}_1\text{H}^2 \rightarrow {}_1\text{H}^3 + p$$

$${}_1\text{H}^2 + {}_1\text{H}^3 \rightarrow {}_2\text{He}^4 + n$$

$$\Rightarrow 3\,{}_1\text{H}^2 \rightarrow {}_2\text{He}^4 + n + p$$

Mass defect

$$\Delta m = (3 \times 2.014 - 4.001 - 1.007 - 1.008)$$

$$\Delta m = 0.026 \text{ amu}$$

Energy Released

$$= 0.026 \times 931 \text{ MeV} = 3.87 \times 10^{-12} \text{ J}$$

This energy produced by the three deutronatoms. Total energy released by 10^{40} deutrons

$$= \frac{10^{40}}{3} \times 3.87 \times 10^{-12} \text{ J} = 1.29 \times 10^{28} \text{ J}$$

The average power $P = 10^{16}$ W $= 10^{16}$ J/s

Therefore total time to exhaust all deutrons of the star will be

$$t = \frac{1.29 \times 10^{28}}{10^{16}} \text{ s} \approx 10^{12} \text{ s.}$$

Hence correct option is (c).

12. $N_1 = N_0\, e^{-\lambda_1 t} = N_0\, e^{\frac{-\log 2}{t_1} t}$ ...(i)

$$N_2 = N_0\, e^{-\lambda_2 t} = N_0\, e^{\frac{-\log 2}{t_2} t} \qquad \text{...(ii)}$$

$$R_1 = \lambda_1 N_1 \qquad \text{...(iii)}$$

and $\quad R_2 = \lambda_2 = \lambda_2 N_2$ ...(iv)

Let after time t, $R_1 = R_2$ then

$$\Rightarrow \qquad \frac{R_1}{R_1} = 1$$

$$\Rightarrow \qquad \frac{\lambda_1}{\lambda_2} \times \frac{N_1}{N_2} = 1$$

$$\Rightarrow \qquad \frac{\lambda_1}{\lambda_2} = \frac{N_2}{N_1} = e^{\log 2 \left[\frac{t_2 - t_1}{t_1 t_2}\right] \times t}$$

$$\Rightarrow \qquad \log \frac{t_2}{t_1} = t \times \log 2 \times \left(\frac{t_2 - t_1}{t_1 t_2}\right)$$

$$\Rightarrow \qquad t_1 = \frac{t_1 t_2}{0.693\,(t_2 - t_1)} \times \log \frac{t_2}{t_1}$$

Hence Correct option is (a).

13. The given reaction is ${}_Z X^{232} \longrightarrow {}_{90}Y^A + \alpha$

$$\Rightarrow \qquad {}_Z X^{232} \longrightarrow {}_{90}Y^A + {}_2\text{He}^4$$

$$\Rightarrow \qquad Z = 92 \text{ and } A = 228$$

$\because$ Initially X is in rest hence momentum of α-particle after decay will be equal and opposite of Y.

$$\Rightarrow \qquad M_Y v_Y = M_\alpha v_\alpha$$

$$\Rightarrow \qquad v_Y = \frac{M_\alpha}{M_Y} v_\alpha$$

Total kinetic energy

$$K_T = \frac{1}{2}(M_\alpha v_\alpha^2 + M_Y v_Y^2)$$

$$\Rightarrow \qquad K_T = \frac{1}{2}\left[M_\alpha v_\alpha^2 + M_y \frac{M_\alpha^2 v_\alpha^2}{M_Y^2}\right]$$

$$\Rightarrow \quad K_T = \frac{1}{2} M_\alpha v_\alpha^2 \left[1 + \frac{M_\alpha}{M_Y}\right]$$

$$\Rightarrow \quad K_T = K_\alpha \left[1 + \frac{4}{228}\right]$$

$$\Rightarrow \quad K_\alpha = \frac{232}{228} K_T$$

Hence Correct option is (b).

14. Energy of emitted photon $= 7$ MeV

$$= 7 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$= 11.2 \times 10^{-13} \text{ J}$$

$$\text{Momentum of photon} = \frac{11.2 \times 10^{-13} \text{ J}}{3 \times 10^8 \text{ m/s}}$$

$$= \frac{11.2}{3} \times 10^{-21} \text{ kg-m/s}$$

$\because$ Initial nucleus is stationary

Applying conservation of momentum principle

$$O = \vec{\mathbf{P}}_{\text{nuc}} + \vec{\mathbf{P}}_{\text{photon}} \Rightarrow \vec{\mathbf{P}}_{\text{nuc}} = -\vec{\mathbf{P}}_{\text{photon}}$$

$$\Rightarrow |\vec{\mathbf{P}}_{\text{nuc}}| = |-\vec{\mathbf{P}}_{\text{photon}}|$$

$$\Rightarrow P_{\text{nuc}} = \frac{11.2}{3} \times 10^{-21} \text{ kg-m/s}$$

Mass of nucleus $= 24$ amu

$$= 24 \times 1.66 \times 10^{-27} \text{ kg}$$

But $\quad P_{\text{nuc}}^2 = 2mK_{\text{nuc}}$

$$\Rightarrow K_{\text{nuc}} = \frac{P_{\text{nuc}}^2}{2m} = \frac{11.2 \times 11.2 \times 10^{-42}}{9 \times 2 \times 24 \times 1.66 \times 10^{-27}} \text{ Joule}$$

$$\Rightarrow K_{\text{nucc}} = \frac{11.2 \times 11.2 \times 10^{-42}}{18 \times 24 \times 1.66 \times 10^{-27} \times 1.6 \times 10^{-19}} \text{ eV}$$

$$\approx 1.1 \text{ keV}$$

Hence correct option is (b).

15. Let time interval between two instants is t_1 then

$$N_1 = N_0 \, e^{-\lambda(t + t_1)}$$

and $\quad N_2 = 2N_0 \, e^{-\lambda t}$

$$A_1 = N_1 \lambda = \lambda N_0 \, e^{-\lambda(t + t_1)}$$

$$A_2 = N_2 \lambda = \lambda(2N_0) \, e^{-\lambda t}$$

$$\Rightarrow \quad \frac{A_1}{A_2} = \frac{1}{2} \times e^{-\lambda t_1}$$

$$\Rightarrow \quad \frac{2A_1}{A_2} = e^{-\lambda t_1}$$

$$\Rightarrow \quad \log\left(\frac{A_2}{2A_1}\right) = \lambda t_1$$

$$\Rightarrow \quad t_1 = \frac{1}{\lambda} \log\left(\frac{A_2}{2A_1}\right)$$

$$\Rightarrow \quad t_1 = \frac{T}{\log 2} \log\left(\frac{A_2}{2A_1}\right)$$

Hence Correct option is (c).

16. The given reaction is

$${}_1\text{H}^2 + {}_1\text{H}^2 \longrightarrow {}_1\text{H}^3 + {}_1\text{H}^1$$

$$\Delta M = [2m\,({}_1\text{H}^2) - m\,({}_1\text{H}^3) - m\,({}_1\text{H}^1)]$$

$$= (2 \times 2.014102 - 3.016049 - 1.007825) \text{ amu}$$

$$= 4.33 \times 10^{-3} \text{ amu}$$

$$\Delta E = 4.33 \times 10^{-3} \times 931.5 \text{ MeV}$$

$$= 4 \text{ MeV}$$

Hence correct option is (c).

17. Number of fusion required to generate 1 kWh

$$= \frac{1 \times 10^3 \times 3600}{4 \times 10^6 \times 1.6 \times 10^{-19}}$$

$$= \frac{36 \times 10^{18}}{6.4} = 5.6 \times 10^{18} \approx 10^{18}$$

Hence correct option is (b).

18. The energy released $= 4$ MeV

This energy produced by two atoms.

Hence energy produced per atom

$$= 2 \text{ MeV} = 2 \times 1.6 \times 10^{-13} \text{ J}$$

Hence number of atom fused to produced 1 kWJ

$$= \frac{36 \times 10^5}{2 \times 1.6 \times 10^{-13}} = \frac{18}{1.6} \times 10^{18}$$

Mass of deutrium which contain

$$\frac{18}{1.6} \times 10^{18} \text{ atom}$$

$$= \frac{18 \times 10^{18}}{1.6 \times 6.02 \times 10^{23}} = 3.7 \times 10^{-5} \text{ kg.}$$

Hence correct option is (c).

■ More Than one Option is Correct

1. $x = N_0,\ y = \lambda N_0$

$$\Rightarrow \quad \frac{x}{y} = \frac{N_0}{\lambda N_0} = \frac{1}{\lambda}$$

where λ is decay constant

Hence $\frac{x}{y}$ is constant throught.

$$\because \quad \frac{x}{y} = \frac{1}{\lambda} = \frac{1}{\frac{0.693}{T}} = \frac{T}{0.693}$$

$$\Rightarrow \quad \frac{x}{y} > T,\ xy = \lambda (N_0)^2$$

For one half life $N = \frac{N_0}{2}$

$$\Rightarrow (xy)_T = \lambda \left(\frac{N_0}{2}\right)^2 = \frac{\lambda N_0^2}{4} = \frac{xy}{4}$$

Hence correct options are (a), (b) and (d).

2. The correct options are (a), (b), (c) and (d).

3. A nucleus in excited state emits a high energy photon called as γ-ray. The reaction is

$$X^* \longrightarrow X + \gamma$$

Hence by gamma radiation atomic number and mass number are not changed. Since after emission of one α atomic number reduced by $2({}_2\alpha^4)$ and after 2β atomic number is increased by (2). Hence correct options are (a), (b) and (c).

4. Here half lives are T and $2T$ and $N_x = N_0$, $N_y = N_0$ after $4T$ for first substance = 4 half lives and after $4T$ the second substance = 2 Half lines.

$$\Rightarrow \quad N_x = N_0 \left(\frac{1}{2}\right)^4 = \frac{N_0}{16}$$

$$N_y = N_0 \left(\frac{1}{2}\right)^2 = \frac{N_0}{4}$$

$$\Rightarrow \quad x = \frac{N_x}{N_y} = \frac{N_0/16}{N_0/4} = \frac{1}{4}$$

Let their activity are R_x and R_y.

$$\Rightarrow \quad R_x = \lambda_x N_x \text{ and } R_y = \lambda_y N_y$$

$$\Rightarrow \quad y = \frac{R_x}{R_y} = \frac{\lambda_x}{\lambda_y} \times \frac{N_x}{N_y} = \frac{\frac{0.693}{T}}{\frac{0.693}{2T}} \times \frac{1}{4}$$

$$\Rightarrow \quad \frac{R_x}{R_y} = y = \frac{1}{2}$$

Hence correct options are (b) and (c).

5. Since nuclear forces are vary short range charge independent, no electromagnetic and they exchange ($n \to p$ or $p \to n$). Hence the correct options are (a), (b), (c) and (d).

6. $\because R = R_0 A^{1/3}$

$$\rho = \frac{M}{4/3\ \pi R^3} = \frac{A \times 1.67 \times 10^{-27} \text{ kg}}{\frac{4}{3} \pi R_0^3 A}$$

$\Rightarrow$ ρ is independent of A.

$$\text{But } \rho = \frac{1.67 \times 10^{-27} \text{ kg}}{\frac{4}{3} \times 3.14 \times (1.3 \times 10^{-15})^3}$$

$$= 1.8 \times 10^{17} \text{ kg/m}^3$$

Hence correct options are (b) and (c).

Match the Columns

1. Here $N_0 = x$, let λ is the decay constant.

$$\Rightarrow \quad N = N_0 e^{-\lambda t} = x e^{-\lambda t}$$

$$\left|\frac{dN}{dt}\right| = \lambda x e^{-\lambda t} \quad \text{but} \quad \left|\frac{dN}{dt}\right|_{t=0} = y$$

$$\Rightarrow \quad y = \lambda x e^0 = \lambda x$$

$$\Rightarrow \quad \lambda = \frac{y}{x}$$

Hence

(a) $\longrightarrow$ s

Half life $T_{1/2} = \dfrac{\log 2}{\lambda} = \dfrac{\log 2}{\frac{y}{x}} = \dfrac{x}{y}\log 2$

Hence

(b) $\longrightarrow$ p

We have activity $R = \lambda N = \lambda x e^{-\lambda t}$

at $t = \dfrac{1}{\lambda}$

$$R = \lambda x e^{-\lambda \times \frac{1}{\lambda}} = \lambda x e^{-1} = \frac{\lambda x}{e}$$

but $\lambda x = y$

$$\Rightarrow \quad R = \frac{y}{e}$$

Hence

(c) $\longrightarrow$ r

Number of nuclei after time $t = \dfrac{1}{\lambda}$

$$N = x e^{-\lambda \times \frac{1}{\lambda}} = \frac{x}{e}$$

Hence

(d) $\longrightarrow$ s

Thus correct match is

(a) $\longrightarrow$ s

(b) $\longrightarrow$ r

(c) $\longrightarrow$ r

(d) $\longrightarrow$ s

2. In reaction $P + P \rightarrow Q$ energy is released

$\because$ Binding energy increases when two or more lighter nucleus combine to form haviour nucleus.

Hence correct match is

(a) $\longrightarrow$ p

Similerly for reaction

$P + P + R \rightarrow R$

Correct match is

(b) $\longrightarrow$ p

for reaction

$P + R = 2Q$ from graph BE per nucleon increases.

Hence energy is released.

Correct match is

(c) $\longrightarrow$ p

For the reaction

$P + Q = R$

We will check energy process if BE per nucleon is given. Hence data is not sufficient correct match is

(d) $\longrightarrow$ s

3. Since A and B are radioactive nuclei of $(A + B)$ decreases with time. Hence correct match is

(a) $\longrightarrow$ q

$\because A$ is converted into B and B is converted into C and decay rate of $A \rightarrow B$ and $B \rightarrow C$ are not known. Hence correct match is

(b) $\longrightarrow$ s

Since at time passes A is converted to B and B is converted to C. Hence nuclei of $(B + C)$ increases. Correct match is

(c) $\longrightarrow$ p

Similerly the correct match for (d) is

(d) $\longrightarrow$ s

4. After emission of 1α particle mass no decreased by 4 but after emission of 1β particle atomic number will increase or decrease by 1. Hence for (a)

(a) $\longrightarrow$ p, s

(b) $\longrightarrow$ p, r

(c) $\longrightarrow$ s

(d) $\longrightarrow$ q, r

5. For

(a) $\longrightarrow$ p

Since BE per nucleon of heavy nuclei is about 7.2 MeV. Hence

(b) $\longrightarrow$ s

X-ray photon have wavelength about 1 Å the energy of this wavelength is of order of 10 keV.

(c) $\longrightarrow$ r

$\because$ Visible light energy of order of ≈ 2 eV. Hence

(d) $\longrightarrow$ q