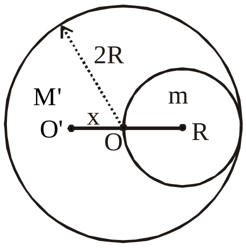
HINTS & SOLUTIONS

SECTION I - PHYSICS

1. (b) Let O be the centre of mass of the disc having radius 2R. O' is the new C.M.



Let m = mass of disc of radius R
M' = mass of disc when the disc of radius R is removed.
M = mass of disc of radius 2R

Now, $m = (\pi R^2).\sigma$,

where $\sigma = \frac{M}{\pi(2R)^2} = \frac{M}{4\pi R^2}$ = the mass per unit area

$M' = [\pi(2R)^2 - \pi R^2].\sigma$

$= 3\pi R^2 \sigma$

$M = \pi(2R)^2.\sigma = 4\pi R^2 \sigma$

We have, $\frac{M'.x + m.R}{M'+m} = 0$

($\because$ C.M. of the full disc is at the centre O)

or, $M'.x + m.R = 0$

or, $M'x = -mR$

$\Rightarrow \quad x = \left(-\frac{m}{M'}\right)R$

$= \left(-\frac{\pi R^2 \sigma}{3\pi R^2 \sigma}\right).R = \left(-\frac{1}{3}\right)R$

But $x = \frac{\alpha}{R}$

$\therefore \frac{\alpha}{R} = \left(-\frac{1}{3}\right).R$

There appears misprint in this question. There must be αR instead of $\frac{\alpha}{R}$. Then

$\alpha R = \left(-\frac{1}{3}\right)R \Rightarrow \alpha = -\frac{1}{3}$

$\therefore \quad |\alpha| = \frac{1}{3}$

2. (b) The acceleration of a solid sphere of mass M, radius R and moment of inertia I rolling down (without slipping) an inclined plane making an angle θ with the horizontal is given by

$a = \frac{g \sin\theta}{I + \frac{K^2}{R^2}}$, where, $I = MK^2$

3. (d) Central forces always act along the axis of rotation. Therefore, the torque is zero. And if there is no external torque acting on a rotating body then its angular momentum is constant.

4. (b) Let the spring be compressed by x. Clearly, Initial K.E. of block = Potential energy of spring + workdown against friction

or, $\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + fx$

or, $\frac{1}{2} \times 2 \times (4)^2 = \left(\frac{1}{2} \times 10000 \times x^2\right) + 15x$

or, $16 = 5000\,x^2 + 15x$

or, $5000\,x^2 + 15x - 16 = 0$

$\therefore \quad x = \frac{-15 \pm \sqrt{(15)^2 - 4 \times 5000 \times (-16)}}{2 \times 5000}$

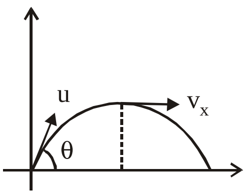
$= \frac{-15 \pm 565.88}{10000} = 0.055\,\text{m}$

(Ignoring –ve value)

$\therefore \quad x = 5.5\,\text{cm}$.

5. (d) Let K' be the K.E. at the highest point. Then

$K' = \frac{1}{2}mv_x^2$ ($\because$ $v_y = 0$ at highest point)



$= \frac{1}{2}m(u\cos\theta)^2$

$= \frac{1}{2}mu^2\cos^2\theta = K.\cos^2\theta$

$\left(\because\ K = \frac{1}{2}mu^2\right)$

or, $K' = K.\cos^2 60°$ $\quad (\because\ \theta = 60°)$

$= K.\left(\frac{1}{2}\right)^2 = \frac{K}{4}$

6. (a) In young's double slit experiment, the intensity at a point is given by

$I = I_0\cos^2\left(\frac{\phi}{2}\right)$

where, I_0 = maximum intensity

ϕ = phase difference

Also, $\phi = \frac{2\pi}{\lambda} \times$ path difference

$= \frac{2\pi}{\lambda} \times \frac{\lambda}{6} = \frac{\pi}{3}$

$\therefore\ I = I_0\cos^2\left(\frac{\pi}{6}\right)$

or, $\frac{I}{I_0} = \cos^2 30° = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$

7. (a) The two springs are in parallel.

$\therefore$ Effective spring constant,

$K = K_1 + K_2$.

Now, frequency of oscillation is given by

$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$

or, $f = \frac{1}{2\pi}\sqrt{\frac{K_1+K_2}{m}}$ ....(i)

When both K_1 and K_2 are made four times their original values, the new frequency is given by

$f' = \frac{1}{2\pi}\sqrt{\frac{4K_1+4K_2}{m}}$

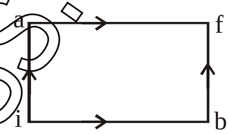
$= \frac{1}{2\pi}\sqrt{\frac{4(K_1+4K_2)}{m}} = 2\left(\frac{1}{2\pi}\sqrt{\frac{K_1+K_2}{m}}\right)$

= 2f; from (i)

8. (b) For path iaf,

Q = 50 cal

W = 20 cal



By first law of thermodynamics,

$\Delta U = Q - W = 50 - 20 = 30$ cal.

For path ibf

Q = 36 cal

W = ?

By first law of thermodynamics,

$Q = \Delta U + W$

or, $W = Q - \Delta U$

Since, the change in internal energy does not depend on the path, therefore

$\Delta U = 30$ cal

$\therefore\ W = Q - \Delta U = 36 - 30 = 6$ cal.

9. (b) The kinetic energy of a particle executing S.H.M. is given by

$K = \frac{1}{2}ma^2\omega^2\sin^2\omega t$

where, m = mass of particle

a = amplitude

ω = angular frequency

t = time

Now, average K.E. = < K >

$= <\frac{1}{2}m\omega^2a^2\sin^2\omega t>$

$= \frac{1}{2}m\omega^2a^2 <\sin^2\omega t>$

$= \frac{1}{2}m\omega^2a^2\left(\frac{1}{2}\right) \quad \left(\because\ <\sin^2\theta> = \frac{1}{2}\right)$

$$= \frac{1}{4} m\omega^2 a^2$$

$$= \frac{1}{4} ma^2 (2\pi\nu)^2 \quad (\because\ w = 2\pi\nu)$$

or, $\quad \langle K \rangle = \pi^2 m a^2 \nu^2$

10. (b) Here, $x = 2 \times 10^{-2} \cos \pi t$

Speed is given by

$$v = \frac{dx}{dt} = 2 \times 10^{-2}\ \pi \sin \pi t$$

For the first time, the speed to be maximum, $\sin \pi t = 1$

or, $\quad \sin \pi t = \sin \frac{\pi}{2}$

$$\Rightarrow \quad \pi t = \frac{\pi}{2} \qquad \text{or,} \quad t = \frac{1}{2} = 0.5 \text{ sec.}$$

11. (c) We know that power consumed in a.c. circuit is given by, $P = E_{rms}.I_{rms} \cos\phi$

Here, $E = E_0 \sin\omega t$

$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

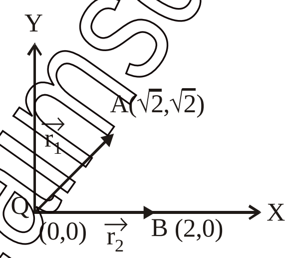
which implies that the phase difference,

$$\phi = \frac{\pi}{2}$$

$$\therefore \quad P = E_{rms}.I_{rms}.\cos\frac{\pi}{2} = 0$$

$$\left(\because \quad \cos\frac{\pi}{2} = 0\right)$$

12. (c) The distance of point $A\left(\sqrt{2}, \sqrt{2}\right)$



from the origin,

$$OA = |\vec{r}_1| = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$$

$$= \sqrt{4} = 2 \text{ units.}$$

The distance of point B(2, 0) from the origin,

$$OB = |\vec{r}_2| = \sqrt{(2)^2 + (0)^2} = 2 \text{ units.}$$

Now, potential at A, $V_A = \frac{1}{4\pi\epsilon_0}.\frac{Q}{(OA)}$

Potential at B, $V_B = \frac{1}{4\pi\epsilon_0}.\frac{Q}{(OB)}$

$\therefore$ Potential difference between the points A and B is given by

$$V_A - V_B = \frac{1}{4\pi\epsilon_0}.\frac{Q}{OA} - \frac{1}{4\pi\epsilon_0}.\frac{Q}{OB}$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{OA} - \frac{1}{OB}\right) = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{2} - \frac{1}{2}\right)$$

$$= \frac{Q}{4\pi\epsilon_0} \times 0 = 0.$$

13. (a) Required Ratio

$$= \frac{\text{Energy stored in capacitor}}{\text{Work done by the battery}}$$

$$= \frac{\frac{1}{2}CV^2}{Ce^2}$$, where C = Capacitance of capacitor

V = Potential difference,

e = emf of battery

$$= \frac{\frac{1}{2}Ce^2}{Ce^2} . (\because\ V = e)$$

$$= \frac{1}{2}$$

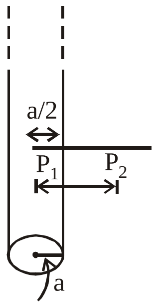
14. (a) We have, $I = I_o\left(1 - e^{-\frac{R}{L}t}\right)$

(When current is in growth in LR circuit)

$$= \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{5}{5}\left(1 - e^{-\frac{5}{10}\times 2}\right)$$

$$= (1 - e^{-1})$$

15. (d) Here, current is uniformly distributed across the cross-section of the wire, therefore, current enclosed in the amperean path formed at a distance $r_1\left(=\frac{a}{2}\right)$



$= \left(\frac{\pi r_1^2}{\pi a^2}\right) \times I$, where I is total current

$\therefore$ Magnetic field at

$$P_1(B_1) = \frac{\mu_0 \times \text{current enclosed}}{\text{Path}}$$

$$= \frac{\mu_0 \times \left(\frac{\pi r_1^2}{\pi a^2}\right) \times I}{2\pi r_1} = \frac{\mu_0 \times I r_1}{2\pi a^2}$$

Now, magnetic field at point P_2,

$$(B_2) = \frac{\mu_0}{2\pi} \cdot \frac{I}{(2a)} = \frac{\mu_0 I}{4\pi a}.$$

$\therefore$ Required Ratio $= \frac{B_1}{B_2} = \frac{\mu_0 I r_1}{2\pi a^2} \times \frac{4\pi a}{\mu_0 I}$

$$= \frac{2r_1}{a} = \frac{2 \times \frac{a}{2}}{a} = 1.$$

16. (d) There is no current inside the pipe and hence Ampere's law can not be applied.

17. (c) Binding energy
$= [ZM_P + (A - Z)M_N - M]c^2$
$= [8M_P + (17 - 8)M_N - M]c^2$
$= [8M_P + 9M_N - M]c^2$
$= [8M_P + 9M_N - M_0]c^2$
But the option (c) is negative of this.

18. (c) There is no change in the proton number and the neutron number as the γ-emission takes place as a result of excitation or de-excitation of nuclei.

19. (a) The current will flow through R_L when the diode is forward biased.

20. (a) Energy of a photon of frequency ν is given by $E = h\nu$.
Also, $E = pc$, where p is the momentum of photon

$$\therefore \quad h\nu = pc \Rightarrow \quad p = \frac{h\nu}{c}.$$

21. (c) We know that

$$v = \frac{dx}{dt} \Rightarrow dx = v\,dt$$

Integrating, $\int_0^x dx = \int_0^t v\,dt$

or $\quad x = \int_0^x (v_0 + gt + ft^2)\,dt$

$$= \left[v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}\right]_0^t$$

or, $\quad x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3} + c$

where c is the constant of integration.
By question,
x = 0 at t = 0.

$$\therefore \quad 0 = v_0 \times 0 + \frac{g}{2} \times 0 + \frac{f}{3} \times 0 + c$$

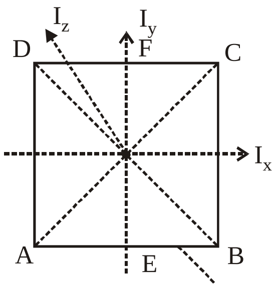
$\Rightarrow \quad c = 0.$

$$\therefore \quad x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}$$

At t = 1,

$$x = v_0 + \frac{g}{2} + \frac{f}{3}.$$

22. (d) By the theorem of perpendicular axes,
$I_z = I_x + I_y$ or, $I_z = 2\,I_y$
($\because$ $I_x = I_y$ by symmetry of the figure)



$\therefore \quad I_{EF} = \frac{I_z}{2}$ ... (i)

Again, by the same theorem
$I_z = I_{AC} + I_{BD} = 2\,I_{AC}$
($\because$ $I_{AC} = I_{BD}$ by symmetry of the figure)

$\therefore \quad I_{AC} = \frac{I_z}{2}$ ... (ii)

From (i) and (ii), we get
$I_{EF} = I_{AC}$.

23. (a) Here,
$x = x_0 \cos(\omega t - \pi/4)$
$\therefore$ Velocity,

$$v = \frac{dx}{dt} = -x_0 \omega \sin\left(\omega t - \frac{\pi}{4}\right)$$

Acceleration,

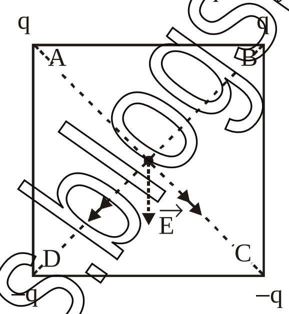
$$a = \frac{dv}{dt} = -x_0 \omega^2 \cos\left(\omega t - \frac{\pi}{4}\right)$$

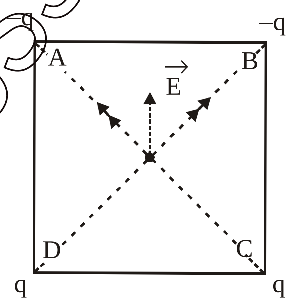
$$= x_0 \omega^2 \cos\left[\pi + \left(\omega t - \frac{\pi}{4}\right)\right] = x_0\,\omega^2 \cos\left(\omega t + \frac{3\pi}{4}\right)$$

But by question,
Acceleration, $a = A\cos(\omega t + \delta)$
Comparing the two accelerations, we get

$A = x_0 \omega^2$ and $\delta = \frac{3\pi}{4}$.

24. (a) As shown in the figure, the resultant electric fields before and after interchanging the charges will have the same magnitude but opposite directions.
Also, the potential will be same in both cases as it is a scalar quantity.





25. (b) By question,
Half life of X, $T_{1/2} = \tau_{av}$, average life of Y

$\Rightarrow \frac{\ell n 2}{\lambda_x} = \frac{1}{\lambda_Y}$ or, $\lambda_x = (\ell n 2).\lambda_Y$

$\Rightarrow \lambda_x = (0.693).\lambda_Y$

$\therefore \lambda_x < \lambda_Y$.

Now, the rate of decay is given by

$R = R_0 e^{-\lambda t}$

For X, $R_x = R_0 e^{-\lambda_x t}$

For Y, $R_y = R_0 e^{-\lambda_y t}$

Hence, $R_x > R_y$.

Thus, X will decay faster than Y.

26. (c) The efficiency (η) of a Carnot engine and the coefficient of performance (β) of a refrigerator are related as

$$\beta = \frac{1-\eta}{\eta}$$

Here, $\eta = \dfrac{1}{10}$

$$\therefore \quad \beta = \frac{1-\frac{1}{10}}{\left(\frac{1}{10}\right)} = 9.$$

Also, Coefficient of performance (β) is given by $\beta = \dfrac{Q_2}{w}$, where Q_2 is the energy absorbed from the reservoir.

or, $\quad 9 = \dfrac{Q_2}{10}$

$\therefore \quad Q_2 = 90$ J.

27. (a) Si and Ge are semiconductors but C is an insulator. Also, the conductivity of Si and Ge is more than C because the valence electrons of Si, Ge and C lie in third, fouth and second orbit repsectively.

28. (b) Here, $\vec{E}$ and $\vec{B}$ are perpendicular to each other and the velocity $\vec{v}$ does not change; therefore

$$qE = qvB \;\Rightarrow\; v = \frac{E}{B}$$

Also, $\left|\dfrac{\vec{E}\times\vec{B}}{B^2}\right| = \dfrac{E\ B\sin\theta}{B^2}$

$$= \frac{E\ B\ \sin 90^\circ}{B^2} = \frac{E}{B} = |\vec{v}| = v$$

$\therefore \quad$ Option (b) is correct.

29. (a) Here, $V(x) = \dfrac{20}{x^2-4}$ volt

We know that $E = -\dfrac{dv}{dx} = -\dfrac{d}{dx}\left(\dfrac{20}{x^2-4}\right)$

or, $\quad E = +\dfrac{40x}{(x^2-4)^2}$

At $x = 4\ \mu m$,

$$E = +\frac{40\times 4}{(4^2-4)^2} = +\frac{160}{144} = +\frac{10}{9}\ \text{volt}/\mu m.$$

Positive sign indicates that $\vec{E}$ is in +ve x-direction.

30. (d) We have to find the frequency of emitted photons. For emission of photons the transition must take place from a higher energy level to a lower energy level which are given only in options (c) and (d).
Frequency is given by

$$h\nu = -13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For transition from n = 6 to n = 2,

$$\nu_1 = \frac{-13.6}{h}\left(\frac{1}{6^2} - \frac{1}{2^2}\right) = \frac{2}{9}\times\left(\frac{13.6}{h}\right)$$

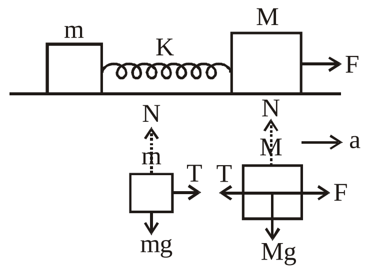
For transition from n = 2 to n = 1,

$$\nu_2 = \frac{-13.6}{h}\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = \frac{3}{4}\times\left(\frac{13.6}{h}\right).$$

$\therefore \quad \nu_1 > \nu_2$

Hence option (d) is the correct answer.

31. (d) Writing free body-diagrams for m & M,



we get
$T = ma$ and $\qquad F - T = Ma$
where T is force due to spring
$\Rightarrow \quad F - ma = Ma$
or, $\quad F = Ma + ma$

$$\therefore \quad a = \frac{F}{M+m}.$$

Now, force acting on the block of mass m is

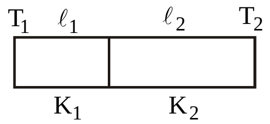
$$ma = m\left(\frac{F}{M+m}\right) = \frac{mF}{m+M}.$$

32. (c) Power of combination is given by
$P = P_1 + P_2 = (-15+5)\,D = -10\,D.$

Now, $P = \dfrac{1}{f} \Rightarrow \quad f = \dfrac{1}{P} = \dfrac{1}{-10}$ metre

$$\therefore \quad f = -\left(\frac{1}{10}\times 100\right)\text{cm} = -10\ \text{cm}.$$

33. (d) Let T be the temperature of the interface. As the two sections are in series, the rate of flow of heat in them will be equal.



$$\therefore \quad \frac{K_1A(T_1-T)}{\ell_1}=\frac{K_2A(T-T_2)}{\ell_2},$$

where A is the area of cross-section.

or, $K_1A(T_1-T)\ell_2 = K_2A(T-T_2)\ell_1$

or, $K_1T_1\ell_2 - K_1T\ell_2 = K_2T\ell_1 - K_2T_2\ell_1$

or, $(K_2\ell_1 + K_1\ell_2)T = K_1T_1\ell_2 + K_2T_2\ell_1$

$$\therefore \quad T=\frac{K_1T_1\ell_2+K_2T_2\ell_1}{K_2\ell_1+K_1\ell_2}$$

$$=\frac{K_1\ell_2T_1+K_2\ell_1T_2}{K_1\ell_2+K_2\ell_1}.$$

34. (a) We have, $L_1 = 10\log\left(\frac{I_1}{I_0}\right)$

$$L_2 = 10\log\left(\frac{I_2}{I_0}\right)$$

$$\therefore \quad L_1 - L_2 = 10\log\left(\frac{I_1}{I_0}\right) - 10\log\left(\frac{I_2}{I_0}\right)$$

or, $\Delta L = 10\log\left(\frac{I_1}{I_0}\times\frac{I_2}{I_0}\right)$

or, $\Delta L = 10\log\left(\frac{I_1}{I_2}\right)$

or, $20 = 10\log\left(\frac{I_1}{I_2}\right)$

or, $2 = \log\left(\frac{I_1}{I_2}\right)$

or, $\frac{I_1}{I_2} = 10^2$

or, $I_2 = \frac{I_1}{100}.$

$\Rightarrow$ Intensity decreases by a factor 100.

35. (b) We have,
Molar heat capacity = Molar mass × Specific heat
capacity per unit mass

$\therefore$ $C_p = 28\,C_p$ (for nitrogen)

and $C_v = 28\,C_v$

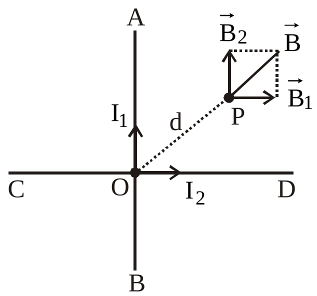
Now, $C_p - C_v = R$

or, $28\,C_p - 28\,C_v = R$

$$\Rightarrow \quad C_p - C_v = \frac{R}{28}.$$

36. (b) When a charged particle enters a magnetic field at a direction perpendicular to the direction of motion, the path of the motion is circular. In circular motion the direction of velocity changes at every point (the magnitude remains constant). Therefore, the momentum will change at every point. But kinetic energy will remain constant as it is given by $\frac{1}{2}mv^2$ and v^2 is the square of the magnitude of velocity which does not change.

37. (c) Clearly, the magnetic fields at a point P, equidistant from AOB and COD will have directions perpendicular to each other, as they are placed normal to each other.



$\therefore$ Resultant field, $B=\sqrt{B_1^2+B_2^2}$

But $B_1=\frac{\mu_0 I_1}{2\pi d}$ and $B_2=\frac{\mu_0 I_2}{2\pi d}$

$$\therefore \quad B=\sqrt{\left(\frac{\mu_0}{2\pi d}\right)^2\left(I_1^2+I_2^2\right)}$$

or, $B=\frac{\mu_0}{2\pi d}\left(I_1^2+I_2^2\right)^{1/2}$

38. (d) We know that

$R_t = R_0(1 + \alpha t)$,

$\Rightarrow \quad R_{50} = R_0(1 + 50\alpha)$ ... (i)

$R_{100} = R_0(1 + 100\alpha)$ ... (ii)

From (i), $R_{50} - R_0 = 50\alpha R_0$ ... (iii)

From (ii), $R_{100} - R_0 = 100\alpha R_0$ ... (iv)

Dividing (iii) by (iv), we get

$$\frac{R_{50} - R_0}{R_{100} - R_0} = \frac{1}{2}$$

Here, $R_{50} = 5\Omega$ and $R_{100} = 6\Omega$

$$\therefore \quad \frac{5 - R_0}{6 - R_0} = \frac{1}{2}$$

or, $6 - R_0 = 10 - 2R_0$

or, $R_0 = 4\Omega$.

39. (a) The potential energy of a charged capacitor is given by $U = \frac{Q^2}{2C}$.

If a dielectric slab is inserted between the plates, the energy is given by $\frac{Q^2}{2KC}$, where K is the dielectric constant.

Again, when the dielectric slab is removed slowly its energy increases to initial potential energy. Thus, work done is zero.

40. (b) Electronic charge does not depend on acceleration due to gravity as it is a universal constant.

So, electronic charge on earth = electronic charge on moon

$\therefore$ Required ratio = 1.

SECTION II - CHEMISTRY

41. (b) According to Kohlrausch's law, molar conductivity of weak electrolyte acetic acid (CH_3COOH) is given as follows:

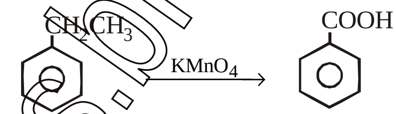
$$\Lambda^\circ_{CH_3COOH} = \Lambda^\circ_{CH_3COONa} + \Lambda^\circ_{HCl} - \Lambda^\circ_{NaCl}$$

$\therefore$ Value of Λ°_{NaCl} should also be known for calculating value of $\Lambda^\circ_{CH_3COOH}$.

42. (d) Aromatic amines are less basic than aliphatic amines. Among aliphatic amines the order of basicity is 2° > 1° > 3° ($\because$ of decreased electron density due to crowding in 3° amines)

$\therefore$ dimethylamine (2° aliphatic amine) is strongest base among given choices.

43. (d) When alkyl benzene are oxidised with alkaline $KMnO_4$, the entire alkyl group is oxidised to –COOH group regardless of length of side chain.



Ethyl benzene Benzoic aicd

44. (a)

$$\overset{7}{C}H_3 - \overset{6}{C}H_2 - \overset{5}{C}H_2 - \overset{4}{C}(CH_3)(CH_3) - \overset{3}{C}H(CH_2CH_3) - \overset{2}{C}H_2 - \overset{1}{C}H_3$$

3–ethyl –4,4–dimethyl heptane

45. (b) Diamagnetic species have no unpaired electrons

$O_2^{\,2-} \Rightarrow \sigma 1s^2, \sigma^*1s^2, \sigma{*}s^2, \sigma 2p_z^2, \pi 2p_x^{\,2}, \pi 2p_y^{\,2}, \pi^*2p_x^{\,2}, \pi^*2p_y^{\,2}$

46. (c) Reluctance of valence shell electrons to participate in bonding is called inert pair effect. The stability of lower oxidation state (+2 for group 14 element) increases on going down the group. So the correct order is

$SiX_2 < GeX_2 < PbX_2 < SnX_2$

47. (d) Chlorine reacts with excess of ammonia to produce ammonium chloride and nitrogen.

$$8NH_3 + 3Cl_2 \longrightarrow N_2 + NH_4Cl$$

48. (d) Smaller the size and higher the charge more will be polarising power of cation. So the correct order of polarising power is

$K^+ < Ca^{2+} < Mg^{2+} < Be^{2+}$

49. (d) Mass of 3.6 moles of H_2SO_4

= Moles × Molecular mass

= 3.6 × 98 g = 352.8 g

$\therefore$ 1000 ml solution has 352.8 g of H_2SO_4

Given that 29 g of H_2SO_4 is present in = 100 g of solution

$\therefore$ 352.8 g of H_2SO_4 is present in

$= \frac{100}{29} \times 352.8$ g of solution

= 1216 g of solution

Density $= \frac{\text{Mass}}{\text{Volume}} = \frac{1216}{1000} = 1.216$ g/ml

= 1.22 g/ml

50. (d) $H_2A \rightleftharpoons H^+ + HA^-$

$\therefore \quad K_1 = 1.0 \times 10^{-5} = \frac{[H^+][HA^-]}{[H_2A]}$

$HA^- \longrightarrow H^+ + A^-$

$\therefore \quad K_2 = 5.0 \times 10^{-10} = \frac{[H^+][A^-]}{[HA^-]}$

$K = \frac{[H^+]^2[A^{2-}]}{[H_2A]} = K_1 \times K_2$

$= (1.0 \times 10^{-5}) \times (5 \times 10^{-10}) = 5 \times 10^{-15}$

51. (b) Given $p_A^0 = ?$, $p_B^0 = 200\text{mm}$, $x_A = 0.6$,

$x_B = 1 - 0.6 = 0.4$, P = 290

$P = p_A + p_B = p_A^0 x_A + p_B^0 x_B$

$\Rightarrow \quad 290 = p_A^0 \times 0.6 + 200 \times 0.4$

$\therefore \quad p_A^0 = 350$ mm

52. (a) $\Delta G° = \Delta H° - T\Delta S°$

For a spontaneous reaction $\Delta G° < 0$

or $\Delta H° - T\Delta S° < 0 \quad \Rightarrow \quad T > \frac{\Delta H°}{\Delta S°}$

$\Rightarrow \quad T > \frac{179.3 \times 10^3}{160.2} > 1117.9\text{K} \approx 1118\text{K}$

53. (a) $\Delta H_R = E_f - E_b = 180 - 200 = -20$ kJ/mol

The nearest correct answer given in choices may be obtained by neglecting sign.

54. (d) $E_{cell} = 0$; when cell is completely discharged.

$E_{cell} = E°_{cell} - \frac{0.059}{2} \log\left(\frac{[Zn^{2+}]}{[Cu^{2+}]}\right)$

or $\quad 0 = 1.1 - \frac{0.059}{2} \log\left(\frac{[Zn^{2+}]}{[Cu^{2+}]}\right)$

$\log\left(\frac{[Zn^{2+}]}{[Cu^{2+}]}\right) = \frac{2 \times 1.1}{0.059} = 37.3$

$\therefore \quad \left(\frac{[Zn^{2+}]}{[Cu^{2+}]}\right) = 10^{37.3}$

55. (d) For acidic buffer $pH = pK_a + \log\left[\frac{A^-}{HA}\right]$

Given $pK_a = 4.5$ and acid is 50% ionised.

$[HA] = [A^-]$ (when acid is 50% ionised)

$\therefore \quad pH = pK_a + \log 1$

$\therefore \quad pH = pK_a = 4.5$

$pOH = 14 - pH = 14 - 4.5 = 9.5$

56. (b) From the given data we can say that order of reaction with respect to B = 1 because change in concentration of B does not change half life. Order of reaction with respect to A = 1 because rate of reaction doubles when concentration of A is doubled keeping concentration of A constant.

$\therefore$ Order of reaction = 1 + 0 = 1 and units of first order reaction are L mol^{-1} sec^{-1}.

57. (a) 4f orbital is nearer to nucleus as compared to 5f orbital therefore, shielding of 4f is more than 5f.

58. (a) Complexes with dsp^2 hybridisation are square planar. So $[PtCl_4]^{2-}$ is square planar in shape.

59. (b) The organic compounds which have chiral carbon atom and do not have plane of symmetry rotate plane polarised light.

$$\begin{array}{c} CHO \\ | \\ HO - \overset{*}{C} - H \\ | \\ CH_2OH \end{array} \quad \text{(* is asymmetric carbon)}$$

60. (b) Proteins have two types of secondary structures α-helix and β-plated sheet.

61. (b) The reaction follows Markownikoff rule which states that when unsymmetrical reagent adds across unsymmetrical double or triple bond the negative part adds to carbon atom having lesser number of hydrogen atoms.

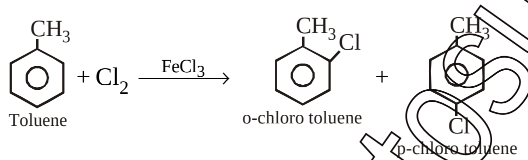
$$CH_3-C\equiv CH+HBr \rightarrow CH_3-\underset{\displaystyle Br}{\underset{|}{C}}=CH_2$$

$$\xrightarrow{HBr} CH_3-\underset{\displaystyle Br}{\underset{|}{\overset{\displaystyle Br}{\overset{|}{C}}}}-CH_3$$

2, 2-dibromo-propane

62. (a) This is carbylamine reaction.
$CH_3CH_2NH_2 + CHCl_3 + 3KOH \longrightarrow C_2H_5NC+3KCl+3H_2O$

63. (d) $FeCl_3$ is Lewis acid. In presence of $FeCl_3$ side chain hydrogen atoms of toluene are substituted.



64. (a) Nitro is electron withdrawing group, so it deactivates the ring towards electrophilic substitution.

65. (c)
(a) N_2 : bond order 3, paramagnetic
N_2^- : bond order 2.5 paramagnetic
(b) C_2 : bond order 2, diamagnetic
C_2^+ : bond order 1.5, paramagnetic
(c) NO : bond order 2.5, paramagnetic
NO^+ : bond order 3, diamagnetic
(d) O_2 : bond order 2, paramagnetic
O_2^+ : bond order 2.5, paramagnetic
$\therefore$ (c) is correct answer

66. (a) More the distance between nucleus and outer orbitals, lesser will be force of attraction on them. Distance between nucleus and 5f orbitals is more as compared to distance between 4f orbital and nucleus. So actinoids exhibit more number of oxidation states in general than the lanthanoids.

67. (d) Let the mass of methane and oxygen = m gm.
Mole fraction of O_2

$$=\frac{\text{Moles of } O_2}{\text{Moles of } O_2 + \text{Moles of } CH_4}$$

$$=\frac{m/32}{m/32+m/16}=\frac{m/32}{3m/32}=\frac{1}{3}$$

Partial pressure of O_2 = Total pressure × mole fraction if $O_2 = P\times\frac{1}{3}=\frac{1}{3}P$

68. (a) Osmotic pressure of isotonic solutions (π) are equal. For solution of unknown substance (π = CRT)

$$C_1=\frac{5.25/M}{V}$$

For solution of urea, C_2 (concentration)

$$=\frac{1.5/60}{V}$$

Given
$\pi_1=\pi_2$
$\because \pi = CRT$
$\therefore \quad C_1RT=C_2RT \quad$ or $\quad C_1=C_2$

or $\quad \dfrac{5.25/M}{V}=\dfrac{1.8/60}{V}$

$\therefore \quad M = 210$ g/mol

69. (d) Given $\Delta H = 41\ kJ\ mol^{-1} = 41000\ J\ mol^{-1}$
$T = 100°C = 273+100 = 373\ K$
$n = 1$
$\Delta U = \Delta H - \Delta nRT = 41000-(2\times 8.314\times 373)$
$= 37898.88\ J\ mol^{-1} \simeq 37.9\ kJmol^{-1}$

70. (c) Let x = solubility

$$AgIO_3 \rightleftharpoons Ag^+ + IO_3^-$$

$K_{sp}=[Ag^+][IO_3^-]=x\times x=x^2$
Given $K_{sp}=1\times 10^{-8}$

$\therefore \quad x=\sqrt{K_{sp}}=\sqrt{1\times 10^{-8}} = 1.0\times 10^4$ mol/lit
$= 1.0\times 10^{-4}\times 283$ g/lit

$$=\frac{1.0\times 10^{-4}\times 283\times 100}{1000}\text{ gm/100ml}$$

$= 2.83\times 10^{-3}$ gm/ 100 ml

71. (a) Let activity of safe working = A
Given $A_0 = 10A$

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{30}$$

$$t_{½} = \frac{2.303}{\lambda}\log\frac{A_0}{A} = \frac{2.303}{0.693/30}\log\frac{10A}{A}$$

$$= \frac{2.303\times 30}{0.693}\times \log 10 = 100 \text{ days.}$$

72. (b) Chiral conformation will not have plane of symmetry. Since twisted boat does not have plane of symmetry it is chiral.

73. (c) In S_N2 mechanism transition state is pentavelent. For bulky alkyl group it will have sterical hinderance and smaller alkyl group will favour the S_N2 mechanism. So the decreasing order of reactivity of alkyl halides is

$RCH_2X > R_2CHX > R_3CX$

74. (d) $$CH_3CH_2OH \xrightarrow{P+I_2} \underset{A}{CH_3CH_2I}$$

$$\xrightarrow[\text{Ether}]{Mg} \underset{(B)}{CH_3CH_2MgI} \xrightarrow{HCHO}$$

$$H-\overset{\overset{CH_3CH_2}{|}}{\underset{\underset{H}{|}}{C}}-OMgI \xrightarrow{H_2O} H-\overset{\overset{CH_3CH_2}{|}}{\underset{\underset{H}{|}}{C}}-OH$$

(D)
n–propyl alcohol

75. (c)
(a) n = 3, ℓ = 0 means 3s-orbital
(b) n = 3, ℓ = 1 means 3p-orbital
(c) n = 3, ℓ = 2 means 3d-orbital
(d) n = 4, ℓ = 0 means 4s-orbital

Increasing order of energy among these orbitals is
3s < 3p < 4s < 3d
$\therefore$ 3d has highest energy.

76. (c) Greater the difference between electronegativity of bonded atoms, stronger will be bond.
$\therefore$ F – H ...... F is the strongest bond.

77. (c) $2Al_{(s)} + 6HCl_{(aq)} \rightarrow 2Al^{3+}_{(aq)} + 6Cl^-_{(aq)} + 3H_{2(g)}$

$\therefore$ 6 moles of HCl produces = 3 moles of H_2
$= 3 \times 22.4$ L of H_2

$\therefore$ 1 mole of HCl produces

$$= \frac{3\times 22.4}{6} = 11.2 \text{ L of } H_2$$

$\because$ 2 moles of Al produces 3 moles of H_2
$= 3 \times 22.4$ L of H_2

$\therefore$ 1 mole of Al produces

$$= \frac{3\times 22.4}{2} = 33.6 \text{ L of } H_2$$

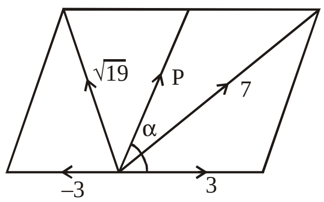
78. (a) $(NH_4)_2SO_4 + 2H_2O \longrightarrow 2H_2SO_4 + NH_4OH$
H_2SO_4 is strong acid and increases the acidity of soil.

79. (b) Spontaneity of reaction depends on tendency to acquire minimum energy state and maximum randomness. For a spontaneous process in an isolated system the change in entropy is positive.

80. (b) Isotopes are atoms of same element having same atomic number but different atomic masses. Neutron has atomic number 0 and atomic mass 1. So loss of neutron will generate isotope.

SECTION III - MATHEMATICS

81. (c) Given : Force P = Pn, Q = 3n, resultant R = 7n
& P' = Pn, Q' = (–3)n, R' = $\sqrt{19}$



We know that
$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$
$\Rightarrow (7)^2 = P^2 + (3)^2 + 2 \times P \times 3\cos\alpha$
$\Rightarrow 49 = P^2 + 9 + 6P\cos\alpha$ .....(i)
$\Rightarrow 40 = P^2 + 6P\cos\alpha$

and $\left(\sqrt{19}\right)^2 = P^2 + (-3)^2 + 2P \times -3\cos\alpha$

$\Rightarrow 19 = P^2 + 9 - 6P\cos\alpha$
$\Rightarrow 10 = P^2 - 6P\cos\alpha$ .....(ii)

Adding (i) and (ii)
$50 = 2P^2$
$\Rightarrow P^2 = 25 \Rightarrow P = 5n.$

82. (d) Given : Probabilities of aeroplane I, i.e., P(I) = 0.3 probabilities of scoring a target correctly by aeroplane II, i.e. P(II) = 0.2

$\therefore P(\overline{I}) = 1 - 0.3 = 0.7$ and $P(\overline{II}) = 1 - 0.2 = 0.8$

$\therefore$ The required probability

$= P(\overline{I} \cap II) = P(\overline{I}).P(II) = 0.7 \times 0.2 = 0.14$

83. (d) Given, $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

Apply $R_2 \to R_2 - R_1$ and $R \to R_3 - R_1$

$\therefore \quad D = \begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & y \end{vmatrix} = xy$

Hence, D is divisible by both x and y

84. (b) Given, equation of hyperbola

$\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$

We know that the equation of hyperbola is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Here, $a^2 = \cos^2\alpha$ and $b^2 = \sin^2\alpha$

We know that, $b^2 = a^2(e^2 - 1)$

$\Rightarrow \sin^2\alpha = \cos^2\alpha(e^2 - 1)$

$\Rightarrow \sin^2\alpha + \cos^2\alpha = \cos^2\alpha . e^2$

$\Rightarrow e^2 = 1 + \tan^2\alpha = \sec^2\alpha \quad \Rightarrow e = \sec\alpha$

$\therefore \quad ae = \cos\alpha . \frac{1}{\cos\alpha} = 1$

Co-ordinates of foci are $(\pm ae, 0)$ i.e. $(\pm 1, 0)$

Hence, abscissae of foci remain constant when α varies.

85. (b) Let the angle of line makes with the positive direction of z-axis is α direction cosines of line with the +ve directions of x-axis, y-axis, and z-axis is l, m, n respectively.

$\therefore \quad l = \cos\frac{\pi}{4}, m = \cos\frac{\pi}{4}, n = \cos\alpha$

as we know that, $l^2 + m^2 + n^2 = 1$

$\therefore \quad \cos^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} + \cos^2\alpha = 1$

$\Rightarrow \frac{1}{2} + \frac{1}{2} + \cos^2\alpha = 1$

$\Rightarrow \cos^2\alpha = 0 \Rightarrow \alpha = \frac{\pi}{2}$

Hence, angle with positive direction of the z-axis is $\frac{\pi}{2}$

86. (c) Using Lagrange's Mean Value Theorem

Let f(x) be a function defined on [a, b]

then, $f'(c) = \frac{f(b) - f(a)}{b - a}$ ....(i)

$c \in [a, b]$

$\therefore$ Given $f(x) = \log_e x$

$\therefore \quad f'(x) = \frac{1}{x}$

$\therefore$ equation (i) become

$\frac{1}{c} = \frac{f(3) - f(1)}{3 - 1}$

$\Rightarrow \frac{1}{c} = \frac{\log_e 3 - \log_e 1}{2} = \frac{\log_e 3}{2}$

$\Rightarrow c = \frac{2}{\log_e 3} \Rightarrow c = 2\log_3 e$

87. (d) Given $f(x) = \tan^{-1}(\sin x + \cos x)$

$f'(x) = \frac{1}{1 + (\sin x + \cos x)^2}.(\cos x - \sin x)$

$= \frac{\sqrt{2}.\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)}{1 + (\sin x + \cos x)^2}$

$= \frac{\left(\cos\frac{\pi}{4}.\cos x - \sin\frac{\pi}{4}.\sin x\right)}{1 + (\sin x + \cos x)^2}$

$\therefore \quad f'(x) = \frac{\sqrt{2}\cos\left(x + \frac{\pi}{4}\right)}{1 + (\sin x + \cos x)^2}$

if f ' (x) >O then f(x) is increasing function.

Hence f(x) is increasing, if $-\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2}$

$\Rightarrow -\frac{3\pi}{4} < x < \frac{\pi}{4}$

Hence, f(x) is increasing when $n \in \left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

88. (a) Given $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$ and $|A^2| = 25$

$\therefore \quad A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 25 & 25\alpha + 5\alpha^2 & 5\alpha + 25\alpha^2 + 5\alpha \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{bmatrix}$

$\therefore |\mathbf{A}^2| = 25\ (25\alpha^2)$

$\therefore 25 = 25\ (25\alpha^2) \Rightarrow |\alpha| = \frac{1}{5}$

89. (d) We know that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\infty$

Put $x = -1$

$\therefore \quad e^{-1} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} ....\infty$

$\therefore \quad e^{-1} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} ....\infty$

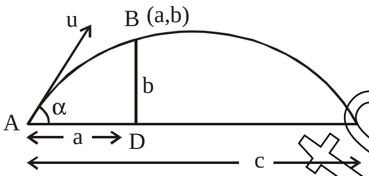
90. (b) Given $|2\hat{u} \times 3\hat{v}| = 1$ and θ is acute angle between $\hat{u}$ and $\hat{v}$, $|\hat{u}| = 1, |\hat{v}| = 1$

$\Rightarrow \quad 6|\hat{u}||\hat{v}||\sin\theta| = 1$

$\Rightarrow \quad 6|\sin\theta| = 1 \Rightarrow \sin\theta = \frac{1}{6}$

Hence, there is exactly one value of θ for which $2\hat{u} \times 3\hat{v}$ is a unit vector.

91. (a) Let B be the top of the wall whose coordinates will be (a, b). Range (R) = c



B lies on the trajectory

$y = x\tan\alpha - \frac{1}{2} g \frac{x^2}{u^2\cos^2\alpha}$

$\Rightarrow \quad b = a\tan\alpha - \frac{1}{2} g \frac{a^2}{u^2\cos^2\alpha}$

$\Rightarrow \quad b = a\tan\alpha \left[1 - \frac{ga}{2u^2\cos^2\alpha\tan\alpha}\right]$

$= a\tan\alpha \left[1 - \frac{a}{\frac{2u^2}{g}\cos^2\alpha.\frac{\sin\alpha}{\cos\alpha}}\right]$

$= a\tan\alpha \left[1 - \frac{a}{\frac{u^2.2\sin\alpha\cos\alpha}{g}}\right]$

$= a\tan\alpha \left[1 - \frac{a}{\frac{u^2\sin 2\alpha}{g}}\right]$

$= a\tan\alpha \left[1 - \frac{a}{R}\right] \quad \left(\because R = \frac{u^2\sin^2\alpha}{g}\right)$

$\Rightarrow \ b = a\tan\alpha \left[1 - \frac{a}{c}\right]$

$\Rightarrow \ b = a\tan\alpha . \left(\frac{c-a}{c}\right)$

$\Rightarrow \ \tan\alpha = \frac{bc}{a(c-a)}$

The angle of projection, $\alpha = \tan^{-1}\frac{bc}{a(c-a)}$

92. (a) Let the number of boys be x and that of girls be y.

$\Rightarrow \quad 52x + 42y = 50(x+y)$

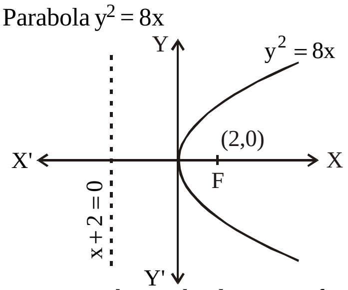
$\Rightarrow \quad 52x - 50x = 50y - 42y$

$\Rightarrow \quad 2x = 8y \Rightarrow \frac{x}{y} = \frac{4}{1}$ and $\frac{x}{x+y} = \frac{4}{5}$

Required % of boys $= \frac{x}{x+y} \times 100$

$= \frac{4}{5} \times 100 = 80\,\%$

93. (b) Parabola $y^2 = 8x$



Point must be on the directrix of parabola

$\because$ equation of directrix $x + 2 = 0 \Rightarrow x = -2$

Hence the point is (–2, 0)

94. (c) We know that equation of sphere is

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

where centre is (–u, –v, –w)

given $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$

$\therefore$ centre $\equiv (3, 6, 1)$

Coordinates of one end of diameter of the sphere are (2, 3, 5). Let the coordinates of the other end of diameter are (α, β, γ)

$\therefore\ \dfrac{\alpha+2}{2}=3,\ \dfrac{\beta+3}{2}=6,\ \dfrac{\gamma+5}{2}=1$

$\Rightarrow\ \alpha = 4,\ \beta = 9$ and $\gamma = -3$

$\therefore$ Coordinate of other end of diameter are (4, 9, –3)

95. (b) Given $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+2\hat{k}$ and

$\vec{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$

If $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then $[\vec{a}\ \vec{b}\ \vec{c}]=0$

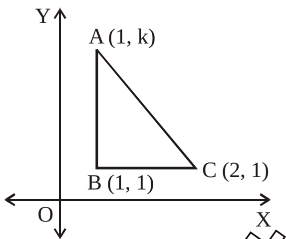
i.e. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & (x-2) & -1 \end{vmatrix}=0$

$\Rightarrow\ 1[1-2(x-2)]-1[-1-2x]+1[x-2+x]=0$

$\Rightarrow\ 1-2x+4+1+2x+2x-2=0$

$\Rightarrow\ 2x=-4\ \Rightarrow\ x=-2$

96. (a) Given : The vertices of a right angled triangle A(l, k), B(1, 1) and C(2, 1) and Area of ΔABC = 1 square unit

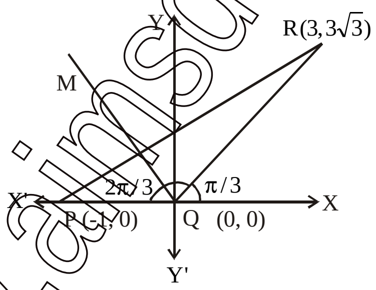


We know that, area of right angled triangle

$=\dfrac{1}{2}\times BC\times AB\ =\ 1=\dfrac{1}{2}(1)\,|(k-1)|$

$\Rightarrow\ \pm(k-1)=2\ \Rightarrow\ k=-1, 3$

97. (c) Given : The coordinates of points P, Q, R are (–1, 0), (0, 0), $(3, 3\sqrt{3})$ respectively.



Slope of QR $=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{3\sqrt{3}}{3}$

$\Rightarrow\quad \tan\theta=\sqrt{3}$

$\Rightarrow\quad \theta=\dfrac{\pi}{3}\ \Rightarrow\ \angle RQX=\dfrac{\pi}{3}$

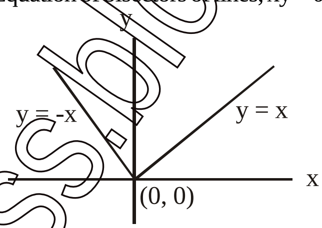
$\therefore\quad \angle RQC=\pi-\dfrac{\pi}{3}=\dfrac{2\pi}{3}$

$\therefore$ Slope of the line QM $=\tan\dfrac{2\pi}{3}=-\sqrt{3}$

$\therefore$ Equation of line QM is $(y-0)=-\sqrt{3}\,(x-0)$

$\Rightarrow\quad y=-\sqrt{3}\,x\ \Rightarrow\ \sqrt{3}\,x+y=0$

98. (a) Equation of bisectors of lines, $xy=0$ are $y=\pm x$



$\therefore$ Put $y=\pm x$ in the given equation

$my^2+(1-m^2)xy-mx^2=0$

$\therefore\ mx^2+(1-m^2)x^2-mx^2=0$

$\Rightarrow\quad 1-m^2=0\ \Rightarrow\ m=\pm 1$

99. (c) Given $f(x)=f(x)+f\left(\dfrac{1}{x}\right)$, where $f(x)=\displaystyle\int_1^x \frac{\log t}{1+t}\,dt$

$\therefore\ F(e)=f(e)+f\left(\dfrac{1}{e}\right)$

$\Rightarrow\ F(e)=\displaystyle\int_1^e \frac{\log t}{1+t}\,dt+\int_1^{1/e}\frac{\log t}{1+t}\,dt$ ....(A)

Now for solving, $I=\displaystyle\int_1^{1/e}\frac{\log t}{1+t}\,dt$

$\therefore$ Put $\dfrac{1}{t}=z\Rightarrow -\dfrac{1}{t^2}dt=dz\ \Rightarrow\ dt=-\dfrac{dz}{z^2}$

and limit for $t=1\ \Rightarrow\ z=1$ and for $t=1/e\ \Rightarrow$ $z=e$

$\therefore\ I=\displaystyle\int_1^e \frac{\log\left(\frac{1}{z}\right)}{1+\frac{1}{z}}\left(-\frac{dz}{z^2}\right)$

$=\displaystyle\int_1^e \frac{(\log 1-\log z).z}{z+1}\left(-\frac{dz}{z^2}\right)$

$=\displaystyle\int_1^e -\frac{\log z}{(z+1)}\left(-\frac{dz}{z}\right)$ $[\because\ \log 1=0]$

$=\displaystyle\int_1^e \frac{\log z}{z(z+1)}\,dz$

$\therefore\ I=\displaystyle\int_1^e \frac{\log t}{t(t+1)}\,dt$

[By property $\int_a^b f(t)dt = \int_a^b f(x)dx$]

Equation (A) be

$$F(e) = \int_1^e \frac{\log t}{1+t}dt + \int_1^e \frac{\log t}{t(1+t)}dt$$

$$= \int_1^e \frac{t.\log t + \log t}{t(1+t)}dt = \int_1^e \frac{(\log t)(t+1)}{t(1+t)}dt$$

$$\Rightarrow F(e) = \int_1^e \frac{\log t}{t}dt$$

Let $\log t = x$

$\therefore \frac{1}{t}dt = dx$

[for limit $t = 1, x = 0$ and $t = e, x = \log e = 1$]

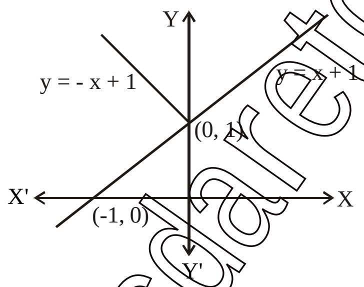
$$\therefore F(e) = \int_0^1 x\ dx$$

$$F(e) = \left[\frac{x^2}{2}\right]_0^1$$

$$\Rightarrow F(e) = \frac{1}{2}$$

100. (a) $f(x) = \min\{x+1, |x|+1\}$

$\Rightarrow f(x) = x + 1 \ \forall\ x \in R$



Hence, f(x) is differentiable everywhere for all $x \in R$.

101. (b) Given, $f(x) = \frac{1}{x} - \frac{2}{e^{2x}-1}$

$$\Rightarrow f(0) = \lim_{x\to 0} \frac{1}{x} - \frac{2}{e^{2x}-1}$$

$$= \lim_{x\to 0} \frac{(e^{2x}-1)-2x}{x(e^{2x}-1)} \quad \left[\frac{0}{0} \text{ form}\right]$$

$\therefore$ using, L'Hospital rule

$$f(0) = \lim_{x\to 0} \frac{4e^{2x}}{2(xe^{2x}2 + e^{2x}.1) + e^{2x}.2}$$

$$= \lim_{x\to 0} \frac{4e^{2x}}{4xe^{2x} + 2e^{2x} + 2e^{2x}} \left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{x\to 0} \frac{4e^{2x}}{4(xe^{2x} + e^{2x})} = \frac{4e^0}{4(0+e^0)} = 1$$

102. (c) $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$

$$\because \int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x$$

$$\therefore \left[\sec^{-1} t\right]_{\sqrt{2}}^{x} = \frac{\pi}{2}$$

$$\Rightarrow \sec^{-1} x - \sec^{-1}\sqrt{2} = \frac{\pi}{2}$$

$$\Rightarrow \sec^{-1}x - \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow \sec^{-1}x = \frac{\pi}{2} + \frac{\pi}{4}$$

$$\Rightarrow \sec^{-1}x = \frac{3\pi}{4} \Rightarrow x = \sec\frac{3\pi}{4}$$

$$\Rightarrow x = -\sqrt{2}$$

103. (c) $I = \int \frac{dx}{\cos x + \sqrt{3}\sin x}$

$$I = \int \frac{dx}{2\left[\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right]}$$

$$= \frac{1}{2}\int \frac{dx}{\left[\sin\frac{\pi}{6}\cos x + \cos\frac{\pi}{6}\sin x\right]}$$

$$= \frac{1}{2}.\int \frac{dx}{\sin\left(x + \frac{\pi}{6}\right)}$$

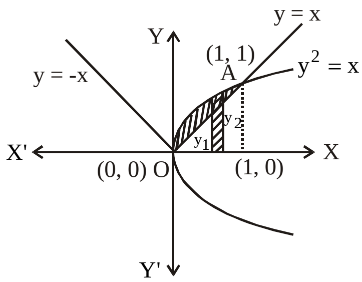
$$\Rightarrow \quad I = \frac{1}{2}.\int \operatorname{cosec}\left(x + \frac{\pi}{6}\right)dx$$

But we know that

$$\int \operatorname{cosec} x\, dx = \log|(\tan x/2)| + C$$

$$\therefore I = \frac{1}{2}.\log\tan\left(\frac{x}{2} + \frac{\pi}{2}\right) + C$$

104. (a) The area enclosed between the curves
$y^2 = x$ and $y = |x|$
From the figure, area lies between $y^2 = x$ and $y = x$



$\therefore$ Required area $= \int_0^1 (y_2 - y_1)dx$

$$= \int_0^1 (\sqrt{x} - x)dx = \left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^1$$

$\therefore$ Required area $= \frac{2}{3}\left[x^{3/2}\right]_0^1 - \frac{1}{2}\left[x^2\right]_0^1$

$$= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

105. (c) Let α and β are roots of the equation $x^2 + ax + 1 = 0$

$\alpha + \beta = -a$ and $\alpha\beta = 1$

given $|\alpha - \beta| < \sqrt{5}$

$\Rightarrow \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} < \sqrt{5}$

$\left(\because (\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta\right)$

$\Rightarrow \sqrt{a^2 - 4} < \sqrt{5} \Rightarrow a^2 - 4 < 5$
$\Rightarrow a^2 - 9 < 0 \Rightarrow a^2 < 9 \Rightarrow -3 < a < 3$
$\Rightarrow a \in (-3, 3)$

106. (b) Let the series a, ar, ar^2, ... are in geometric progression.
given, $a = ar + ar^2$
$\Rightarrow 1 = r + r^2$
$\Rightarrow r^2 + r - 1 = 0$

$\Rightarrow r = \frac{-1 \pm \sqrt{1 - 4 \times -1}}{2}$

$\Rightarrow r = \frac{-1 \pm \sqrt{5}}{2}$ (taking +ve value)

$\Rightarrow r = \frac{\sqrt{5} - 1}{2}$

107. (d) $\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$

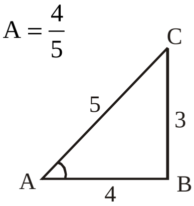
$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \frac{\pi}{2} - \text{cosec}^{-1}\left(\frac{5}{4}\right)$

$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right)$

$[\because \sin^{-1} x + \cos^{-1} x = \pi/2]$

$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \cos^{-1}\left(\frac{4}{5}\right)$ ....(1)

Let $\cos^{-1}\frac{4}{5} = A \Rightarrow \cos A = \frac{4}{5}$



$\Rightarrow A = \cos^{-1}(4/5)$

$\Rightarrow \sin A = \frac{3}{5}$

$\Rightarrow A = \sin^{-1}\frac{3}{5}$

$\therefore$ $\cos^{-1}(4/5) = \sin^{-1}(3/5)$
$\therefore$ equation (i) become,

$\sin^{-1}\frac{x}{5} = \sin^{-1}\frac{3}{5}$

$\Rightarrow \frac{x}{5} = \frac{3}{5} \Rightarrow x = 3$

108. (c) $T_{r+1} = (-1)^r.\, {}^nC_r\, (a)^{n-r}.\, (b)^r$ is an expansion of $(a - b)n$
$\therefore$ 5th term $= t_5 = t_{4+1}$
$= (-1)4.\, {}^nC_4\, (a)^{n-4}.(b)4 = {}^nC_4 \,.\, a^{n-4} \,.\, b^4$
6th term $= t_6 = t_{5+1} = (-1)^5\, {}^nC_5\, (a)^{n-5}\, (b)^5$
Given $t_5 + t_6 = 0$
$\therefore$ ${}^nC_4 \,.\, a^{n-4} \,.\, b^4 + (-\,{}^nC_5 \,.\, a^{n-5} \,.\, b^5) = 0$

$\Rightarrow \frac{n!}{4!(n-4)!}.\frac{a^n}{a^4}.b^4 - \frac{n!}{5!(n-5)!}.\frac{a^n b^5}{a^5} = 0$

$\Rightarrow \frac{n!.a^n b^4}{4!(n-5)!.a^4}\left[\frac{1}{(n-4)} - \frac{6}{5.a}\right] = 0$

or, $\frac{1}{n-4} - \frac{6}{5a} = 0 \Rightarrow \frac{a}{b} = \frac{x-4}{5}$

109. (a) Set S = {1, 2, 3, ... 12}
$A \cup B \cup C = S,\ A \cap B = B \cap C = A \cap C = \phi$
$\therefore$ The number of ways to partition

$= {}^{12}C_4 \times {}^8C_4 \times {}^4C_4 = \frac{12!}{4!8!} \times \frac{8!}{4!4!} \times \frac{4!}{4!0!}$

$= \frac{12!}{(4!)^3}$

110. (b) $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$

f(x) is defined if $-1 \le \left(\frac{x}{2} - 1\right) \le 1$ and $\cos x > 0$

or $0 \le \frac{x}{2} \le 2$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$

or $0 \le x \le 4$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$

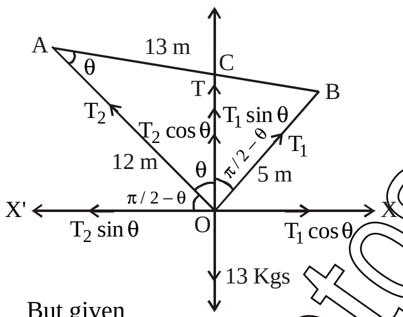
$\therefore \quad x \in \left[0, \frac{\pi}{2}\right)$

111. (a) Given : A body weighing 13 kg is suspended by two strings OB = 5m and OA = 12 m. Length of rod AB = 13 M.

Let T_1 is tension in string OB and T_2 is tension in string OA.

$\therefore \quad T_2 \sin\theta = T_1 \cos\theta$ ....(i)

and $\quad T_1 \sin\theta + T_2 \cos\theta = 13$ ....(ii)



But given

OC = CA = CB

$\therefore \quad \angle AOC = \angle OAC = \theta$ (let)

and $\angle COB = \angle OBC$

Now in Δ AOB

$\sin\theta = \sin A = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$

Now putting the value of $\sin\theta$ and $\cos\theta$ in equation (i) and (ii) we get

$T_2 \cdot \frac{5}{13} = T_1 \frac{12}{13}$ and $T_1 \cdot \frac{5}{13} + T_2 \cdot \frac{12}{13} = 13$

$\Rightarrow \quad 12T_1 - 5T_2 = 0$ ...(iii)

$\Rightarrow \quad 5T_1 + 12T_2 = 169$ ....(iv)

Solving equation (iii) and (iv)

$60T_1 - 25T_2 = 0$

$\underset{-}{60}T_1 \pm 144T_2 = \underset{-}{}169 \times 12$

$-169\, T_2 = -169 \times 12$

$\Rightarrow \quad T_2 = 12$ and $T_1 = 5$

$\therefore$ Tensions in strings are 5kg and 12 kg

112. (b) A pair of fair dice is thrown, the sample space S = (1, 1), (1, 2) (1, 3) .... = 36

Possibility of getting 9 are (5, 4), (4, 5), (6, 3), (3, 6)

$\therefore$ Possibility of getting score 9 in a single throw

$= \frac{4}{36} = \frac{1}{9}$

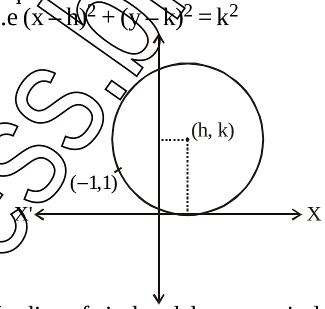
$\therefore$ Probability of getting score 9 exactly twice

$= {}^3C_2 \times \left(\frac{1}{9}\right)^2 . \left(1 - \frac{1}{9}\right) = \frac{3!}{2!} \times \frac{1}{9} \times \frac{1}{9} \times \frac{8}{9}$

$= \frac{3.2!}{2!} \times \frac{1}{9} \times \frac{1}{9} \times \frac{8}{9} = \frac{8}{243}$

113. (d) Equation of circle whose centre is (h, k)

i.e $(x - h)^2 + (y - k)^2 = k^2$



(radius of circle = k because circle is tangent to x-axis)

Equation of circle passing through (–1, +1)

$\therefore (-1 - h)^2 + (1 - k)^2 = k^2$

$\Rightarrow 1 + h^2 + 2h + 1 + k^2 - 2k = k^2$

$\Rightarrow h^2 + 2h - 2k + 2 = 0$

$D \ge 0$

$\therefore (2)^2 - 4 \times 1.(-2k + 2) \ge 0$

$\Rightarrow 4 - 4(-2k + 2) \ge 0 \Rightarrow 1 + 2k - 2 \ge 0$

$\Rightarrow k \ge \frac{1}{2}$

114. (c) Let the direction cosines of line L be l, m, n, then

$2l + 3m + n = 0$ ....(i)

and $l + 3m + 2n = 0$ ....(ii)

on solving equation (i) and (ii), we get

$\frac{l}{6-3} = \frac{m}{1-4} = \frac{n}{6-3} \quad \Rightarrow \quad \frac{l}{3} = \frac{m}{-3} = \frac{n}{3}$

Now $\frac{l}{3} = \frac{m}{-3} = \frac{n}{3} = \frac{\sqrt{l^2 + m^2 + n^2}}{\sqrt{3^2 + (-3)^2 + 3^2}}$

$\because l^2 + m^2 + n^2 = 1$

$\therefore \frac{l}{3} = \frac{m}{-3} = \frac{n}{3} = \frac{1}{\sqrt{27}}$

$\Rightarrow l = \frac{3}{\sqrt{27}} = \frac{1}{\sqrt{3}}, m = -\frac{1}{\sqrt{3}}, n = \frac{1}{\sqrt{3}}$

Line L, makes an angle α with +ve x-axis

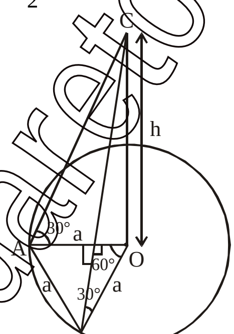
$\therefore l = \cos\alpha$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

115. (a) General equation of circles passing through origin and having their centres on the x-axis is
$x^2 + y^2 + 2gx = 0$ ...(i)
On differentiating w.r.t x, we get
$$2x + 2y\,.\,\frac{dy}{dx} + 2g = 0 \Rightarrow g = -\left(x + y\frac{dy}{dx}\right)$$
$\therefore$ equation (i) be
$$x^2 + y^2 + 2\left\{-\left(x + y\frac{dy}{dx}\right)\right\}.x = 0$$
$$\Rightarrow x^2 + y^2 - 2x^2 - 2x\frac{dy}{dx}.y = 0$$
$$\Rightarrow y^2 = x^2 + 2xy\frac{dy}{dx}$$

116. (c) Since, p and q are positive real numbers
$p^2 + q^2 = 1$ (Given)
Using AM $\geq$ GM
$$\therefore \left(\frac{p+q}{2}\right)^2 \geq \sqrt{(pq)^2} \quad = \frac{p^2+q^2+2pq}{4} \geq pq$$
$$\frac{1+2pq}{4} \geq pq \qquad 1 + 2pq \geq 4pq$$
$$1 \geq 2pq \qquad \text{or,} \quad 2pq \leq 1$$
$$pq \leq \frac{1}{2} \qquad \text{or,} \quad pq \leq \frac{1}{2}$$
Now, $(p+q)^2 = p^2 + q^2 + 2pq$
$$\Rightarrow (p+q)^2 \leq 1 + 2 \times \frac{1}{2} \Rightarrow p + q \leq \sqrt{2}$$

117. (a) In the ΔAOB, $\angle$AOB = 60°, and $\angle$OBA = $\angle$OAB (since OA = OB = AB radius of same circle). $\therefore$ ΔAOB is a equilateral triangle. Let the height of tower is h m. Given distance between two points A & B lie on boundary of circular park, subtends an angle of 60° at the foot of the tower AB i.e. AB = a. A tower OC stands at the centre of a circular park. Angle of elevation of the top of the tower from A and B is 30°. In Δ OAX

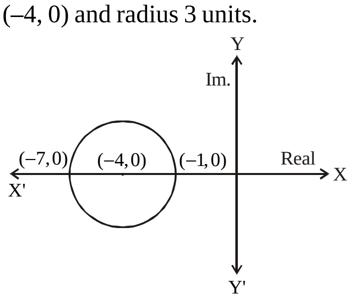


$$\tan 30° = \frac{h}{a}$$
$$\therefore \quad \angle OBA = \angle AOB = \angle OAB = 60°$$
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{a} \Rightarrow h = \frac{a}{\sqrt{3}}$$

118. (d) We know that, $(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1 x + {}^{20}C_2 x^2 + ... {}^{20}C_{10} x^{10} + ... {}^{20}C_{20} x^{20}$
Put x = –1, $(0) = {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... + {}^{20}C_{10} - {}^{20}C_{11} .... + {}^{20}C_{20}$
$$\Rightarrow 0 = 2[{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9] + {}^{20}C_{10}$$
$$\Rightarrow {}^{20}C_{10} = 2[{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9 + {}^{20}C_{10}]$$
$$\Rightarrow {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... + {}^{20}C_{10} = \frac{1}{2}\,{}^{20}C_{10}$$

119. (b,c) Equation of normal at p(x, y) is
$$Y - y = -\frac{dx}{dy}(X - x)$$
Coordinate of G at X axis is (X, 0) (let)
$$\therefore 0 - y = -\frac{dx}{dy}(X - x)$$
$$\Rightarrow y\frac{dy}{dx} = X - x \Rightarrow X = x + y\frac{dy}{dx}$$
$\therefore$ Co-ordinate of G $\left(x + y\frac{dy}{dx}, 0\right)$
Given distance of G from origin = twice of the abscissa of p.
$$\therefore \left|x + y\frac{dy}{dx}\right| = |2x|$$
$$\Rightarrow x + y\frac{dy}{dx} = 2x \quad \text{or} \quad x + y\frac{dy}{dx} = -2x$$
$$\Rightarrow y\frac{dy}{dx} = x \quad \text{or} \quad y\frac{dy}{dx} = -3x$$
$$\Rightarrow ydy = xdx \quad \text{or} \quad ydy = -3xdx$$
On Integrating
$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + c_1 \quad \text{or} \quad \frac{y^2}{2} = -\frac{3x^2}{2} + c_2$$
$$\Rightarrow x^2 - y^2 = -2c_1 \quad \text{or} \quad 3x^2 + y^2 = 2c_2$$
$\therefore$ the curve is a hyperbola and ellipse both

120. (a) z lies on or inside the circle with centre (–4, 0) and radius 3 units.



From the Argand diagram maximum value of $|z + 1| = 6$
Second method : $|z + 1| = |z + 4 - 3|$
$\leq |z + 4| + |-3| \leq |3| + |-3|$
$\Rightarrow |z + 1| = 6$