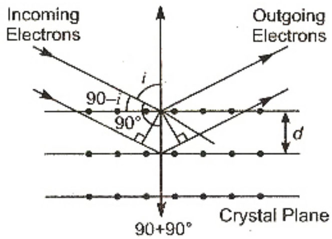
☞ **Directions :** Questions No. 1, 2 and 3 are based on the following paragraph.

Wave property of electrons implies that they will show diffraction effects. Davisson and Germer demonstrated this by diffracting electrons from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively (see figure).



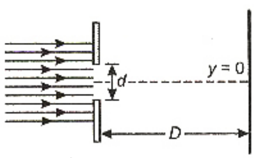
1. Electrons accelerated by potential V are diffracted from a crystal. If $d = 1$ Å and $i = 30°$, V should be about ($h = 6.6 \times 10^{-34}$ J-s, $m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C)

(a) 2000 V (b) 50 V
(c) 500 V (d) 1000 V

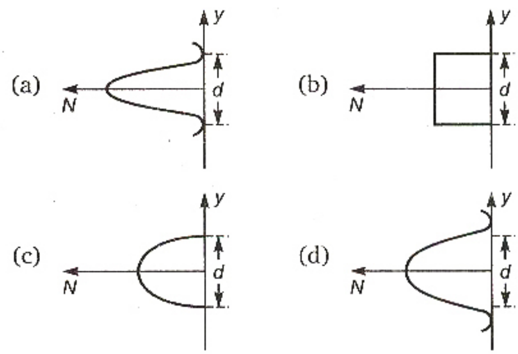
2. If a strong diffraction peak is observed when electrons are incident at an angle i from the normal to the crystal planes with distance d between them (see figure), de-Broglie wavelength λ_{dB} of electrons can be calculated by the relationship (n is an integer)

(a) $d \sin i = n\lambda_{dB}$ (b) $2d \cos i = n\lambda_{dB}$
(c) $2d \sin i = n\lambda_{dB}$ (d) $d \cos i = n\lambda_{dB}$

3. In an experiment, electrons are made to pass through a narrow slit of width d comparable to their de-Broglie wavelength. They are detected on a screen at a distance D from the slit (see figure).



Which of the following graphs can be expected to represent the number of electrons N detected as a function of the detector position y ($y = 0$ corresponds to the middle of the slit)?



4. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 kms^{-1}, the escape velocity from the surface of the planet would be

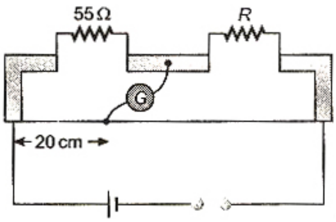
(a) 1.1 km s^{-1} (b) 11 km s^{-1}
(c) 110 km s^{-1} (d) 0.11 km s^{-1}

5. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density $\rho_2 (\rho_2 < \rho_1)$. [Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, *i.e.*, $F_{\text{viscous}} = -kv^2$ $(k > 0)$]. The terminal speed of the ball is

(a) $\sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$ (b) $\frac{Vg\rho_1}{k}$

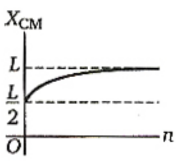
(c) $\sqrt{\frac{Vg\rho_1}{k}}$ (d) $\frac{Vg(\rho_1 - \rho_2)}{k}$

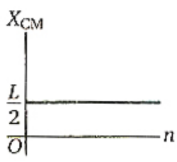
6. Shown in the figure adjacent is a meter-bridge set up with null deflection in the galvanometer. The value of the unknown resistor R is

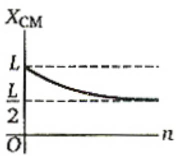


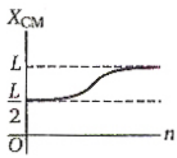
(a) 13.75 Ω (b) 220 Ω
(c) 110 Ω (d) 55 Ω

7. A thin rod of length L is lying along the x-axis with its ends at $x = 0$ and $x = L$. Its linear density (mass/length) varies with x as $k\left(\frac{x}{L}\right)^n$, where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against n, which of the following graphs best approximates the dependence of x_{CM} on n?

(a) 

(b) 

(c) 

(d) 

8. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, she measures the column length to be x cm for the second resonance. Then

(a) $18 > x$
(b) $x > 54$
(c) $54 > x > 36$
(d) $36 > x > 18$

9. The dimensions of magnetic field in M, L, T and C (Coulomb) is given as

(a) $[MLT^{-1}C^{-1}]$ (b) $[MT^2C^{-2}]$
(c) $[MT^{-1}C^{-1}]$ (d) $[MT^{-2}C^{-1}]$

10. Consider a uniform square plate of side a and mass m. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

(a) $\frac{5}{6}ma^2$ (b) $\frac{1}{12}ma^2$
(c) $\frac{7}{12}ma^2$ (d) $\frac{2}{3}ma^2$

11. A body of mass $m = 3.513$ kg is moving along the x-axis with a speed of $5.00\ ms^{-1}$. The magnitude of its momentum is recorded as

(a) $17.6\ kg\ ms^{-1}$ (b) $17.565\ kg\ ms^{-1}$
(c) $17.56\ kg\ ms^{-1}$ (d) $17.57\ kg\ ms^{-1}$

12. An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range

(a) 200 J – 500 J
(b) 2×10^5 J – 3×10^5 J
(c) 20,000 J – 50,000 J
(d) 2,000 J – 5,000 J

13. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is d. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $K_1 = 3$ and thickness $\frac{d}{3}$ while the other one has dielectric constant $K_2 = 6$ and thickness $\frac{2d}{3}$. Capacitance of the capacitor is now

(a) 1.8 pF (b) 45 pF
(c) 40.5 pF (d) 20.25 pF

14. The speed of sound in oxygen (O_2) at a certain temperature is $460\ ms^{-1}$. The speed of sound in helium (He) at the same temperature will be (*assume both gases to be ideal*)

(a) $460\ ms^{-1}$ (b) $500\ ms^{-1}$
(c) $650\ ms^{-1}$ (d) $330\ ms^{-1}$

☞ **Directions :** *Question numbers 15 to 16 are Assertion-Reason type questions. Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.*

(a) Statement I is false, Statement II is true
(b) Statement I is true, Statement II is false; Statement II is a correct explanation for Statement I
(c) Statement I is true, Statement II is true; Statement II is ***not*** a correct explanation for Statement I
(d) Statement I is true, Statement II is false

15. This question contains Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement I : Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion.

and

Statement II : For heavy nuclei, binding energy per nucleon increases with increasing Z while for light nuclei it decreases with increasing Z.

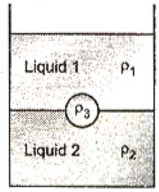
16. This question contains Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement I : For a mass M kept at the centre of a cube of side a, the flux of gravitational field passing through its sides is $4\pi GM$.

and

Statement II : If the direction of a field due to a point source is radial and its dependence on the distance r from the source is given as $\frac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.

17. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2 respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure.



Which of the following is true for ρ_1, ρ_2 and ρ_3 ?

(a) $\rho_3 < \rho_1 < \rho_2$
(b) $\rho_1 > \rho_3 > \rho_2$
(c) $\rho_1 < \rho_2 < \rho_3$
(d) $\rho_1 < \rho_3 < \rho_2$

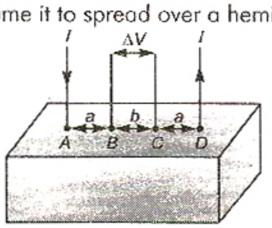
18. A working transistor with its three legs marked P, Q and R is tested using a multimeter. No conduction is found between P and Q. By connecting the common (*negative*) terminal of the multimeter to R and the other (*positive*) terminal to P or Q, some resistance is seen on the multimeter. Which of the following is true for the transistor?

(a) It is an *npn* transistor with R as base
(b) It is a *pnp* transistor with R as collector
(c) It is a *pnp* transistor with R as emitter
(d) It is an *npn* transistor with R as collector

☞ **Directions :** Questions No. 19 and 20 are based on the following paragraph.

Consider a block of conducting material of resistivity ρ shown in the figure. Current I enters at A and leaves from D. We apply superposition principle to find voltage ΔV developed between B and C. The calculation is done in the following steps :

(i) Take current I entering from A and assume it to spread over a hemispherical surface in the block



(ii) Calculate field $E(r)$ at distance r from A by using Ohm's law $E = \rho J$, where J is the current per unit area at r.
(iii) From the r dependence of $E(r)$, obtain the potential $V(r)$ at r.
(iv) Repeat (i), (ii) and (iii) for current I leaving D and superpose results for A and D.

19. ΔV measured between B and C is

(a) $\frac{\rho I}{\pi a} - \frac{\rho I}{\pi(a+b)}$ (b) $\frac{\rho I}{a} - \frac{\rho I}{(a+b)}$

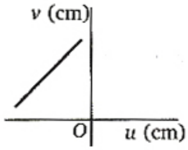
(c) $\frac{\rho I}{2\pi a} - \frac{\rho I}{2\pi(a+b)}$ (d) $\frac{\rho I}{2\pi(a-b)}$

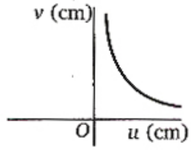
20. For current entering at A, the electric field at a distance r from A is

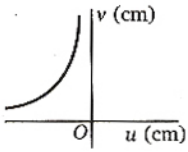
(a) $\frac{\rho I}{8\pi r^2}$ (b) $\frac{\rho I}{r^2}$

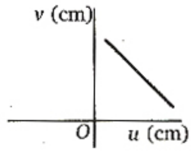
(c) $\frac{\rho I}{2\pi r^2}$ (d) $\frac{\rho I}{4\pi r^2}$

21. A student measures the focal length of a convex lens by putting an object pin at a distance u from the lens and measuring the distance v of the image pin. The graph between u and v plotted by the student should look like

(a)


(b)


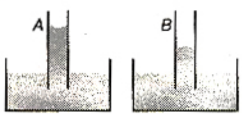
(c)


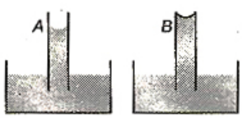
(d)


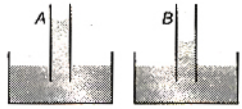
22. A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is

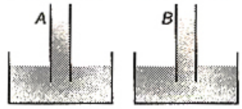
(a) 0.16 J
(b) 1.00 J
(c) 0.67 J
(d) 0.34 J

23. A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes?

(a)


(b)


(c)


(d)


24. Suppose an electron is attracted towards the origin by a force $\frac{k}{r}$, where k is a constant and r is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the n^{th} orbital of the electron is found to be r_n and the kinetic energy of the electron to be T_n. Then which of the following is true?

(a) $T_n \propto \frac{1}{n^2}$, $r_n \propto n^2$

(b) T_n independent of n, $r_n \propto n$

(c) $T_n \propto \frac{1}{n}$, $r_n \propto n$

(d) $T_n \propto \frac{1}{n}$, $r_n \propto n^2$

25. A wave travelling along the x-axis is described by the equation $y(x,t) = 0.005 \cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are

(a) $\alpha = 25.00\,\pi$, $\beta = \pi$

(b) $\alpha = \frac{0.08}{\pi}$, $\beta = \frac{2.0}{\pi}$

(c) $\alpha = \frac{0.04}{\pi}$, $\beta = \frac{1.0}{\pi}$

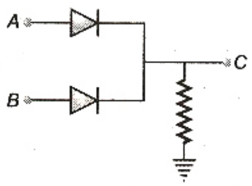
(d) $\alpha = 12.50\,\pi$, $\beta = \frac{\pi}{2.0}$

26. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area $A = 10\ cm^2$ and length $= 20$ cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is

$(\mu_0 = 4\pi \times 10^{-7}\ TmA^{-1})$

(a) $2.4\,\pi \times 10^{-5}$ H
(b) $4.8\,\pi \times 10^{-4}$ H
(c) $4.8\,\pi \times 10^{-5}$ H
(d) $2.4\,\pi \times 10^{-4}$ H

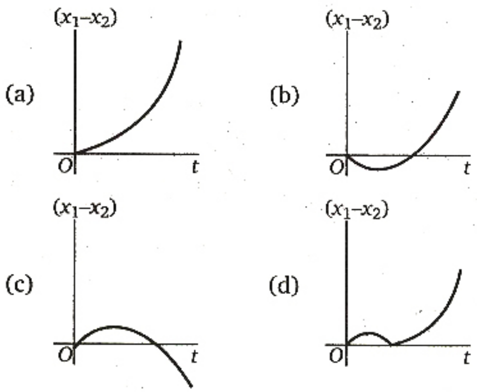
27. In the adjacent circuit, A and B represent two inputs and C represents the output,



The circuit represents

(a) NOR gate
(b) AND gate
(c) NAND gate
(d) OR gate

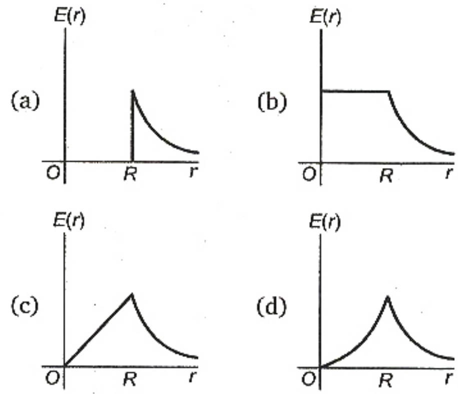
28. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t?



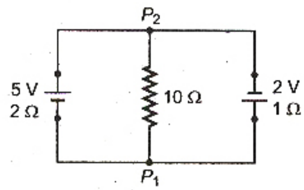
29. An experiment is performed to find the refractive index of glass using a travelling microscope. In this experiment distances are measured by

(a) a vernier scale provided on the microscope
(b) a standard laboratory scale
(c) a meter scale provided on the microscope
(d) a screw gauge provided on the microscope

30. A thin spherical shell of radius R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field $E(r)$ produced by the shell in the range $0 \le r < \infty$, where r is the distance from the centre of the shell?



31. A 5 V battery with internal resistance 2Ω and a 2V battery with internal resistance 1 Ω are connected to a 10 Ω resistor as shown in the figure.



The current in the 10 Ω resistor is

(a) 0.27 A, P_2 to P_1
(b) 0.03 A, P_1 to P_2
(c) 0.03 A, P_2 to P_1
(d) 0.27 A, P_1 to P_2

32. A horizontal overhead powerline is at a height of 4 m from the ground and carries a current of 100 A from east to west. The magnetic field directly below it on the ground is ($\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1})

(a) 2.5×10^{-7} T, southward
(b) 5×10^{-6} T, northward
(c) 5×10^{-6} T, southward
(d) 2.5×10^{-7} T, northward

33. Relative permittivity and permeability of a material are ε_r and μ_r, respectively. Which of the following values of these quantities are allowed for a diamagnetic material?

(a) $\varepsilon_r = 0.5, \mu_r = 1.5$
(b) $\varepsilon_r = 1.5, \mu_r = 0.5$
(c) $\varepsilon_r = 0.5, \mu_r = 0.5$
(d) $\varepsilon_r = 1.5, \mu_r = 1.5$

34. Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of – 0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is

(a) 3.32 mm (b) 3.73 mm
(c) 3.67 mm (d) 3.38 mm

35. An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure p_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure p_2 and temperature T_2. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be

(a) $\frac{T_1T_2(p_1V_1 + p_2V_2)}{p_1V_1T_2 + p_2V_2T_1}$ (b) $\frac{p_1V_1T_1 + p_2V_2T_2}{p_1V_1 + p_2V_2}$

(c) $\frac{p_1V_1T_2 + p_2V_2T_1}{p_1V_1 + p_2V_2}$ (d) $\frac{T_1T_2(p_1V_1 + p_2V_2)}{p_1V_1T_1 + p_2V_2T_2}$

Chemistry

1. The organic chloro compound, which shows complete stereochemical inversion during a S_N2 reaction is

(a) $(C_2H_5)_2CHCl$ (b) $(CH_3)_3CCl$
(c) $(CH_3)_2CHCl$ (d) CH_3Cl

2. The coordination number and the oxidation state of the element 'E' in the complex $[E(en)_2(C_2O_4)]NO_2$ (where (en) is ethylene diamine) are, respectively,

(a) 6 and 2 (b) 4 and 2
(c) 4 and 3 (d) 6 and 3

3. Identify the wrong statements in the following

(a) Chlorofluorocarbons are responsible for ozone layer depletion
(b) Greenhouse effect is responsible for global warming
(c) Ozone layer does not permit infrared radiation from the sun to reach the earth
(d) Acid rains is mostly because of oxides of nitrogen and sulphur

4. Phenol, when it first reacts with concentrated sulphuric acid and then with concentrated nitric acid, gives

(a) 2,4,6-trinitrobenzene
(b) *o*-nitrophenol
(c) *p*-nitrophenol
(d) nitrobenzene

5. Toluene is nitrated and the resulting product is reduced with tin and hydrochloric acid. The product so obtained is diazotised and then heated with cuprous bromide. The reaction mixture so formed contains

(a) mixture of *o*-and *p*-bromotoluenes
(b) mixture of *o*-and *p*-dibromobenzenes
(c) mixture of *o*-and *p*-bromoanilines
(d) mixture of *o*-and *m*-bromotoluenes

6. In the following sequence of reactions, the alkene affords the compound 'B'

$CH_3CH{=}CHCH_3 \xrightarrow{O_3} A \xrightarrow[Zn]{H_2O} B.$ The compound B is

(a) CH_3CH_2CHO (b) CH_3COCH_3
(c) $CH_3CH_2COCH_3$ (d) CH_3CHO

7. Larger number of oxidation states are exhibited by the actinoides than those by the lanthanoides, the main reason being

(a) 4*f* orbitals more diffused than the 5*f* orbitals
(b) lesser energy difference between 5*f* and 6*d* than between 4*f* and 5*d* orbitals
(c) more energy difference between 5*f* and 6*d* than between 4*f* and 5*d* orbitals
(d) more reactive nature of the actinoids than the lanthanoids

8. In which of the following octahedral complexes of Co (at no 27), will the magnitude of Δ_o be the highest?

(a) $[Co(CN)_6]^{3-}$ (b) $[Co(C_2O_4)_3]^{3-}$
(c) $[Co(H_2O)_6]^{3+}$ (d) $[Co(NH_3)_6]^{3+}$

9. At 80°C, the vapour pressure of pure liquid 'A' is 520 mmHg and that of pure liquid 'B' is 1000 mmHg. If a mixture solution of 'A' and 'B' boils at 80°C and 1 atm pressure, the amount of 'A' in the mixture is (1 atm = 760 mmHg)

(a) 52 mole per cent (b) 34 mole per cent
(c) 48 mole per cent (d) 50 mole per cent

10. For a reaction $\frac{1}{2}A \longrightarrow 2B$, rate of disappearance of 'A' is related to the rate of appearance of 'B' by the expression

(a) $-\frac{d[A]}{dt}=\frac{1}{2}\frac{d[B]}{dt}$ (b) $-\frac{d[A]}{dt}=\frac{1}{4}\frac{d[B]}{dt}$

(c) $-\frac{d[A]}{dt}=\frac{d[B]}{dt}$ (d) $-\frac{d[A]}{dt}=4\frac{d[B]}{dt}$

11. The equilibrium constants K_{p_1} and K_{p_2} for the reactions $X \rightleftharpoons 2Y$ and $Z \rightleftharpoons P+Q$, respectively are in the ratio of 1 : 9. If the degree of dissociation of X and Z be equal, then the ratio of total pressure at these equilibria is

(a) 1 : 36 (b) 1 : 1

(c) 1 : 3 (d) 1 : 9

12. Oxidising power of chlorine in aqueous solution can be determined by the parameters indicated below :⊖

$$\frac{1}{2}Cl_2(g) \xrightarrow{\frac{1}{2}\Delta_{diss}H^{\ominus}} Cl(g) \xrightarrow{\Delta_{EA}H^{\ominus}}$$

$$Cl^-(g) \xrightarrow{\Delta_{hyd}H^{\ominus}} Cl^-(aq)$$

The energy involved in the conversion of $\frac{1}{2}Cl_2(g)$ to $Cl^-(aq)$ (using the data,

$\Delta_{diss}H^{\ominus}_{Cl_2} = 240$ kJ mol^{-1},

$\Delta_{EA}H^{\ominus}_{Cl} = -349$ kJ mol^{-1},

$\Delta_{hyd}H^{\ominus}_{Cl} = -381$ kJ mol^{-1}) will be

(a) + 152 kJ mol^{-1} (b) − 610 kJ mol^{-1}

(c) − 850 kJ mol^{-1} (d) + 120 kJ mol^{-1}

13. Which of the following factors is of **no significance** for roasting sulphide ores to the oxides and not subjecting the sulphide ores to carbon reduction directly?

(a) Metal sulphides are thermodynamically more stable than CS_2

(b) CO_2 is thermodynamically more stable than CS_2

(c) Metal sulphides are less stable than the corresponding oxides

(d) CO_2 is more volatile than CS_2

14. Bakelite is obtained from phenol by reacting with

(a) $(CH_2OH)_2$ (b) CH_3CHO

(c) CH_3COCH_3 (d) HCHO

15. For the following three reactions I, II and III, equilibrium constants are given

I. $CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g); K_1$

II. $CH_4(g) + H_2O(g) \rightleftharpoons CO(g) + 3H_2(g); K_2$

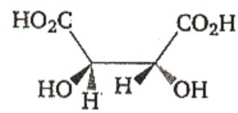
III. $CH_4(g) + 2H_2O(g) \rightleftharpoons CO_2(g) + 4H_2(g); K_3$

Which of the following relations is correct?

(a) $K_1\sqrt{K_2} = K_3$ (b) $K_2K_3 = K_1$

(c) $K_3 = K_1K_2$ (d) $K_3K_2^3 = K_1^2$

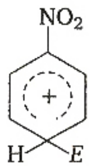
16. The absolute configuration of

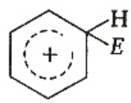


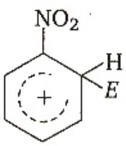
(a) S, S (b) R, R

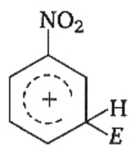
(c) R, S (d) S, R

17. The electrophile, $E^{\oplus}$ attacks the benzene ring to generate the intermediate σ -complex. Of the following, which σ-complex is of lowest energy?

(a) 

(b) 

(c) 

(d) 

18. α-D(+)-glucose and β-D-(+)-glucose are

(a) conformers (b) epimers

(c) anomers (d) enantiomers

19. Standard entropy of X_2, Y_2 and XY_3 are 60, 40 and 50 JK^{-1} mol^{-1}, respectively. For the reaction, $\frac{1}{2}X_2 + \frac{3}{2}Y_2 \longrightarrow XY_3$, $\Delta H = -30$ kJ, to be at equilibrium, the temperature will be

(a) 1250 K (b) 500 K

(c) 750 K (d) 1000 K

20. Four species are listed below

(i) HCO_3^- (ii) H_3O^+

(iii) HSO_4^- (iv) HSO_3F

Which one of the following is the correct sequence of their acid strength?

(a) (iv) < (ii) < (iii) < (i)

(b) (ii) < (iii) < (i) < (iv)

(c) (i) < (iii) < (ii) < (iv)

(d) (iii) < (i) < (iv) < (ii)

21. Which one of the following constitutes a group of the isoelectronic species?

(a) C_2^{2-}, O_2^-, CO, NO (b) $NO^+, C_2^{2-}, CN^-, N_2$

(c) $CN^-, N_2, O_2^{2-}, CO_2^{2-}$ (d) N_2, O_2^-, NO^+, CO

22. Which one of the following pairs of species have the same bond order?

(a) CN^- and NO^+ (b) CN^- and CN^+

(c) O_2^- and CN^- (d) NO^+ and CN^+

23. The ionisation enthalpy of hydrogen atom is 1.312×10^6 J mol^{-1}. The energy required to excite the electron in the atom from $n = 1$ to $n = 2$ is

(a) 8.51×10^5 J mol^{-1} (b) 6.56×10^5 J mol^{-1}

(c) 7.56×10^5 J mol^{-1} (d) 9.84×10^5 J mol^{-1}

24. Which one of the following is the correct statement?

(a) Boric acid is a protonic acid

(b) Beryllium exhibits coordination number of six

(c) Chlorides of both beryllium and aluminium have bridged chloride structures in solid phase

(d) $B_2H_6 \cdot 2NH_3$ is known as 'inorganic benzene'

25. Given $E^\circ_{Cr^{3+}/Cr} = -0.72V, E^\circ_{Fe^{2+}/Fe} = -0.42V$.

The potential for the cell $Cr|Cr^{3+}(0.1M)||Fe^{2+}(0.01M)|Fe$ is

(a) 0.26 V (b) 0.399 V

(c) – 0.339 V (d) – 0.26 V

26. Amount of oxalic acid present in a solution can be determined by its titration with $KMnO_4$ solution in the presence of H_2SO_4. The titration gives unsatisfactory result when carried out in the presence of HCl, because HCl

(a) gets oxidised by oxalic acid to chlorine

(b) furnishes H^+ ions in addition to those from oxalic acid

(c) reduces permanganate to Mn^{2+}

(d) oxidises oxalic acid to carbon dioxide and water

27. The vapour pressure of water at 20°C is 17.5 mmHg. If 18 g of glucose ($C_6H_{12}O_6$) is added to 178.2 g of water at 20°C, the vapour pressure of the resulting solution will be

(a) 17.675 mmHg (b) 15.750 mmHg

(c) 16.500 mmHg (d) 17.325 mmHg

28. Among the following substituted silanes the one which will give rise to crosslinked silicone polymer on hydrolysis is

(a) R_4Si (b) $RSiCl_3$

(c) R_2SiCl_2 (d) R_3SiCl

29. In context with the industrial preparation of hydrogen from water gas ($CO + H_2$), which of the following is the correct statement?

(a) CO and H_2 are fractionally separated using differences in their densities

(b) CO is removed by absorption in aqueous Cu_2Cl_2 solution

(c) H_2 is removed through occlusion with Pd

(d) CO is oxidised to CO_2 with steam in the presence of a catalyst followed by absorption of CO_2 in alkali

30. In a compound atoms, of element Y from ccp lattice and those of element X occupy 2/3rd of tetrahedral voids. The formula of the compound will be

(a) X_4Y_3 (b) X_2Y_3

(c) X_2Y (d) X_3Y_4

31. Gold numbers of protective colloids A, B, C and D are 0.50, 0.01, 0.10 and 0.005, respectively. The correct order of their protective powers is

(a) $D < A < C < B$ (b) $C < B < D < A$

(c) $A < C < B < D$ (d) $B < D < A < C$

32. The hydrocarbon which can react with sodium in liquid ammonia is

(a) $CH_3CH_2CH_2C \equiv CCH_2CH_2CH_3$

(b) $CH_3CH_2C \equiv CH$

(c) $CH_3CH = CHCH_3$

(d) $CH_3CH_2C \equiv CCH_2CH_3$

33. The treatment of CH_3MgX with $CH_3C \equiv C - H$ produces

(a) $CH_3 - CH = CH_2$ (b) $CH_3C \equiv C - CH_3$

(c) $CH_3 - \underset{}{\overset{H}{\overset{|}{C}}} = \overset{H}{\overset{|}{C}} - CH_3$ (d) CH_4

34. The correct decreasing order of priority for the functional groups of organic compounds in the IUPAC system of nomenclature is

(a) $-COOH, -SO_3H, -CONH_2, -CHO$

(b) $-SO_3H, -COOH, -CONH_2, -CHO$

(c) $-CHO, -COOH, -SO_3H, -CONH_2$

(d) $-CONH_2, -CHO, -SO_3H, -COOH$

35. The pK_a of a weak acid, HA, is 4.80. The pK_b of a weak base, BOH, is 4.78. The pH of an aqueous solution of the corresponding salt, BA, will be

(a) 9.58 (b) 4.79

(c) 7.01 (d) 9.22

1. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that $CD = 7$ m. From D the angle of elevation of the point A is 45°. Then, the height of the pole is

(a) $\frac{7\sqrt{3}}{2}\left(\frac{1}{\sqrt{3}-1}\right)$ m (b) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)$ m

(c) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)$ m (d) $\frac{7\sqrt{3}}{2}\left(\frac{1}{\sqrt{3}+1}\right)$ m

2. It is given that the events A and B are such that $P(A)=\frac{1}{4}$, $P\left(\frac{A}{B}\right)=\frac{1}{2}$ and $P\left(\frac{B}{A}\right)=\frac{2}{3}$. Then, $P(B)$ is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

3. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then, $P(A \cup B)$ is

(a) $\frac{3}{5}$ (b) 0

(c) 1 (d) $\frac{2}{5}$

4. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $\frac{1}{2}$. Then, the length of the semi-major axis is

(a) $\frac{8}{3}$ (b) $\frac{2}{3}$

(c) $\frac{4}{3}$ (d) $\frac{5}{3}$

5. A parabola has the origin as its focus and the line $x = 2$ as the directrix. Then, the vertex of the parabola is at

(a) (0, 2) (b) (1, 0)

(c) (0, 1) (d) (2, 0)

6. The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2+y^2+2x+4y-3=0$ is

(a) (3, – 4) (b) (– 3, 4)

(c) (– 3, – 4) (d) (3, 4)

7. Let $f : N \to Y$ be a function defined as $f(x) = 4x+3$ where $Y = \{y \in N : y = 4x+3$ for some $x \in N\}$. Then, the inverse of $f(x)$, is

(a) $g(y)=\frac{3y+4}{3}$ (b) $g(y)=4+\frac{y+3}{4}$

(c) $g(y)=\frac{y+3}{4}$ (d) $g(y)=\frac{y-3}{4}$

8. The conjugate of a complex number is $\frac{1}{i-1}$. Then, that complex number is

(a) $-\frac{1}{i-1}$ (b) $\frac{1}{i+1}$

(c) $-\frac{1}{i+1}$ (d) $\frac{1}{i-1}$

9. Let R be the real line. Consider the following subsets of the plane $R \times R$

$S = \{(x, y) : y = x+1$ and $0 < x < 2\}$

$T = \{(x, y) : x - y$ is an integer$\}$. Which one of the following is true ?

(a) Neither S nor T is an equivalence relation on R

(b) Both S and T are equivalence relations on R

(c) S is an equivalence relation on R but T is not

(d) T is an equivalence relation on R but S is not

10. The perpendicular bisector of the line segment joining $P(1, 4)$ and $Q(k, 3)$ has y-intercept – 4. Then, a possible value of k is

(a) 1 (b) 2

(c) – 2 (d) – 4

11. The solution of the differential equation $\frac{dy}{dx}=\frac{x+y}{x}$ satisfying the condition $y(1)=1$ is

(a) $y=\log x + x$ (b) $y = x\log x + x^2$

(c) $y = xe^{(x-1)}$ (d) $y = x\log x + x$

12. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80. Then, which one of the following gives possible values of a and b ?

(a) $a=0,\ b=7$ (b) $a=5,\ b=2$

(c) $a=1,\ b=6$ (d) $a=3,\ b=4$

13. The vector $\vec{\mathbf{a}}=\alpha\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\beta\hat{\mathbf{k}}$ lies in the plane of the vectors $\vec{\mathbf{b}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}$ and $\vec{\mathbf{c}}=\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and bisects the angle between $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$. Then, which one of the following gives possible values of α and β ?

(a) $\alpha=2,\ \beta=2$ (b) $\alpha=1,\ \beta=2$

(c) $\alpha=2,\ \beta=1$ (d) $\alpha=1,\ \beta=1$

14. The non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$. Then, the angle between $\vec{a}$ and $\vec{c}$ is

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

15. The line passing through the points $(5, 1, a)$ and $(3, b, 1)$ crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then,

(a) $a = 2,\ b = 8$ (b) $a = 4,\ b = 6$

(c) $a = 6,\ b = 4$ (d) $a = 8,\ b = 2$

16. If the straight lines

$$\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$$

and $$\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$$

intersect at a point, then the integer k is equal to

(a) -5 (b) 5

(c) 2 (d) -2

☞ **Directions :** *Question numbers 17 to 21 are Assertion-Reason type questions. Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.*

(a) Statement I is false, Statement II is true

(b) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I

(c) Statement I is true, Statement II is true; Statement II is *not* a correct explanation for Statement I

(d) Statement I is true, Statement II is false

17. Statement I : For every natural number $n \ge 2$,
$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}} > \sqrt{n}.$$

Statement II : For every natural number $n \ge 2$, $\sqrt{n(n+1)} < n + 1$.

18. Let A be 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by tr(A), the sum of diagonal entries of A. Assume that $A^2 = I$.

Statement I : If $A \ne I$ and $A \ne -I$, then $\det(A) = -1$.

Statement II : If $A \ne I$ and $A \ne -I$, then $\text{tr}(A) \ne 0$.

19. Statement I : $\sum_{r=0}^{n} (r+1)\,{}^nC_r = (n+2)\,2^{n-1}$.

Statement II : $\sum_{r=0}^{n} (r+1)\,{}^nC_r\, x^r = (1+x)^n + nx(1+x)^{n-1}$.

20. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number" and r be the statement "x is a rational number iff y is a transcendental number".

Statement I : r is equivalent to either q or p

Statement II : r is equivalent to $\sim (p \leftrightarrow \sim q)$

21. In a shop there are five types of ice-creams available. A child buys six ice-creams.

Statement I : The number of different ways the child can buy the six ice-creams, is ${}^{10}C_5$.

Statement II : The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row.

22. Let $f(x) = \begin{cases} (x-1)\sin \frac{1}{x-1}, & \text{if } x \ne 1 \\ 0, & \text{if } x = 1 \end{cases}$

Then, which one of the following is true ?

(a) f is neither differentiable at $x = 0$ nor at $x = 1$

(b) f is differentiable at $x = 0$ and at $x = 1$

(c) f is differentiable at $x = 0$ but not at $x = 1$

(d) f is differentiable at $x = 1$ but not at $x = 0$

23. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is

(a) -4 (b) -12

(c) 12 (d) 4

24. Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$. Then, which one of the following holds ?

(a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$

(b) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$

(c) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

(d) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

25. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have ?

(a) 7 (b) 1
(c) 3 (d) 5

26. The statement $p \to (q \to p)$ is equivalent to

(a) $p \to (p \to q)$ (b) $p \to (p \vee q)$
(c) $p \to (p \wedge q)$ (d) $p \to (p \leftrightarrow q)$

27. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is

(a) $\frac{6}{17}$ (b) $\frac{3}{17}$
(c) $\frac{4}{17}$ (d) $\frac{5}{17}$

28. The differential equation of the family of circles with fixed radius 5 unit and centre on the line $y = 2$ is

(a) $(x-2)y'^2 = 25 - (y-2)^2$
(b) $(y-2)y'^2 = 25 - (y-2)^2$
(c) $(y-2)^2 y'^2 = 25 - (y-2)^2$
(d) $(x-2)^2 y'^2 = 25 - (y-2)^2$

29. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}}\,dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}}\,dx$.

Then, which one of the following is true ?

(a) $I > \frac{2}{3}$ and $J > 2$ (b) $I < \frac{2}{3}$ and $J < 2$
(c) $I < \frac{2}{3}$ and $J > 2$ (d) $I > \frac{2}{3}$ and $J < 2$

30. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to

(a) $\frac{5}{3}$ sq unit (b) $\frac{1}{3}$ sq unit
(c) $\frac{2}{3}$ sq unit (d) $\frac{4}{3}$ sq unit

31. The value of $\sqrt{2}\int \frac{\sin x\,dx}{\sin\left(x - \frac{\pi}{4}\right)}$ is

(a) $x + \log\left|\cos\left(x - \frac{\pi}{4}\right)\right| + c$

(b) $x - \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$

(c) $x + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$

(d) $x - \log\left|\cos\left(x - \frac{\pi}{4}\right)\right| + c$

32. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent ?

(a) $8 \cdot {}^6C_4 \cdot {}^7C_4$ (b) $6 \cdot 7 \cdot {}^8C_4$
(c) $6 \cdot 8 \cdot {}^7C_4$ (d) $7 \cdot {}^6C_4 \cdot {}^8C_4$

33. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$ and $z = bx + ay$. Then, $a^2 + b^2 + c^2 + 2abc$ is equal to

(a) 2 (b) -1
(c) 0 (d) 1

34. Let A be a square matrix all of whose entries are integers. Then, which one of the following is true ?

(a) If det $(A) = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers

(b) If det $(A) \neq \pm 1$, then A^{-1} exists and all its entries are non-integers

(c) If det $(A) = \pm 1$, then A^{-1} exists and all its entries are integers

(d) If det $(A) = \pm 1$, then A^{-1} need not exist

35. The quadratic equations

$$x^2 - 6x + a = 0$$

and

$$x^2 - cx + 6 = 0$$

have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then, the common root is

(a) 1 (b) 4
(c) 3 (d) 2

24. Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$. Then, which one of the following holds ?

(a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$

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(c) $x + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$

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(a) 1 (b) 4

(c) 3 (d) 2

Answers

PHYSICS

1. (b)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (a)	**6.** (b)	**7.** (a)	**8.** (b)	**9.** (c)	**10.** (d)
11. (a)	**12.** (d)	**13.** (c)	**14.** (*)	**15.** (a)	**16.** (c)	**17.** (d)	**18.** (a)	**19.** (c)	**20.** (c)
21. (c)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (a)	**26.** (d)	**27.** (d)	**28.** (b)	**29.** (a)	**30.** (a)
31. (c)	**32.** (c)	**33.** (b)	**34.** (d)	**35.** (a)					

CHEMISTRY

1. (d)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** (d)	**7.** (b)	**8.** (a)	**9.** (d)	**10.** (b)
11. (a)	**12.** (b)	**13.** (c)	**14.** (d)	**15.** (c)	**16.** (b)	**17.** (b)	**18.** (c)	**19.** (c)	**20.** (c)
21. (b)	**22.** (a)	**23.** (d)	**24.** (c)	**25.** (a)	**26.** (c)	**27.** (d)	**28.** (b)	**29.** (d)	**30.** (a)
31. (c)	**32.** (b)	**33.** (d)	**34.** (a)	**35.** (c)					

MATHEMATICS

1. (b)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (b)	**6.** (c)	**7.** (d)	**8.** (c)	**9.** (d)	**10.** (d)
11. (d)	**12.** (d)	**13.** (d)	**14.** (d)	**15.** (c)	**16.** (a)	**17.** (b)	**18.** (d)	**19.** (b)	**20.** (*)
21. (a)	**22.** (c)	**23.** (b)	**24.** (a)	**25.** (b)	**26.** (b)	**27.** (a)	**28.** (c)	**29.** (b)	**30.** (d)
31. (c)	**32.** (d)	**33.** (d)	**34.** (c)	**35.** (d)					

Note : '*' None of the given options is correct.

1. For constructive interference, $2d\cos i = n\lambda = \frac{h}{\sqrt{2meV}}$ on substituting values we get, $V \simeq 50$ V.

2. Expression is given by $2d\cos i = n\lambda_{dB}$

3. As diffraction pattern has to be wider than slit width, so (d) is the correct option.

4. Mass of planet, $M_p = 10M_e$, where M_e is mass of earth.

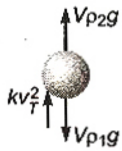
Radius of planet, $R_p = \frac{R_e}{10}$, where R_e is radius of earth.

Escape speed is given by, $v = \sqrt{\frac{2GM}{R}}$

So, for planet $v_p = \sqrt{\frac{2G \times M_p}{R_p}} = \sqrt{\frac{100 \times 2GM_e}{R_e}}$

$$= 10 \times v_e = 10 \times 11 \text{ km/s} = 110 \text{ km/s}$$

5. The forces acting on the ball are gravity force, buoyancy force and viscous force. When ball acquires terminal speed, it is in dynamic equilibrium, let terminal speed of ball is v_T. So,



$$V\rho_2 g + kv_T^2 = V\rho_1 g$$

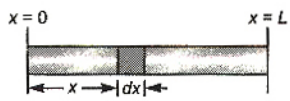
$$v_T = \sqrt{\frac{V(\rho_1 - \rho_2)g}{k}}$$

6. From balanced Wheatstone bridge concept,

$$\frac{55\,\Omega}{R} = \frac{20}{80}$$

$$\Rightarrow \quad R = 220\,\Omega$$

7. $$X_{CM} = \frac{\int x\,dm}{\int dm}$$



If $n = 0$,

then $X_{CM} = \frac{L}{2}$

As n increases, the centre of mass shift away from $x = \frac{L}{2}$ which only option (a) is satisfying.

Alternately, you can use basic concept.

$$X_{CM} = \frac{\int_0^L k\left(\frac{x}{L}\right)^n \times x\,dx}{\int_0^L k\left(\frac{x}{L}\right)^n dx} = L\left[\frac{n+1}{n+2}\right]$$

8. $l_1 = 18$ cm

$$f = \frac{v_1}{4l_1}$$

$f = \frac{3v_2}{4l_2}$, where $l_2 = x$ according to given situation

and also $v_1 < v_2$ as during summer temperature would be higher.

$$\frac{3v_2}{4l_2} = \frac{v_1}{4l_1}$$

$$l_2 = 3l_1 \times \frac{v_2}{v_1}$$

$\Rightarrow \quad x = 54 \times$ (*A* quantity greater than 1)

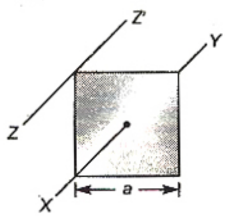
So, $\quad x > 54$

9. From $\quad F = qvB$

$\Rightarrow \quad [\text{MLT}^{-2}] = [\text{C}][\text{LT}^{-1}][\text{B}]$

$\Rightarrow \quad [\text{B}] = [\text{MC}^{-1}\text{T}^{-1}]$

10. Moment of inertia of square plate about *XY* is $\frac{ma^2}{6}$ moment of inertia about ZZ' can be computed using parallel axis theorem



$$I_{ZZ'} = I_{XY} + m\left(\frac{a}{\sqrt{2}}\right)^2$$

$$= \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{2ma^2}{3}$$

11. $m = 3.513$ kg and $v = 5.00$ m/s

So, momentum, $p = mv = 17.565$

As the number of significant digits in m is 4 and in v is 3, so, p must have 3 significant digits

$$p = 17.6 \text{ kgms}^{-1}$$

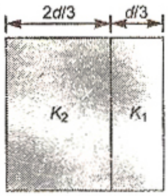
12. Question is somewhat based on approximations. Let mass of athlete is 65 kg.

Approx velocity is 10 m/s

So, $$\text{KE} = \frac{65 \times 100}{2} = 3750 \text{ J}$$

So, option (d) is most probable answer.

13. $$C = \frac{C_1 C_2}{C_1 + C_2} \quad \ldots\text{(i)}$$



where $C_1 = \dfrac{K_1 \varepsilon_0 A}{d/3}$ ...(ii)

and $C_2 = \dfrac{K_2 \varepsilon_0 A}{2\,d/3}$ ...(iii)

It is given that $\dfrac{\varepsilon_0 A}{d} = 9$ pF

On substituting Eqs. (ii) and (iii) in Eq. (i), we get the result $C_{eq} = 40.5 \mu F$

14. Speed of sound is given by, $v = \sqrt{\dfrac{\gamma RT}{M}}$

$$v_{O_2} = \sqrt{\frac{\frac{7}{5}RT}{32}} \quad \text{and} \quad v_{He} = \sqrt{\frac{\frac{5}{3}RT}{4}}$$

$$\frac{v_{O_2}}{v_{He}} = \sqrt{\frac{7 \times 3 \times 4}{5 \times 32 \times 5}}$$

$$\Rightarrow \quad v_{He} = 460 \times 10 \times \sqrt{\frac{2}{21}} \simeq 1420 \text{ m/s.}$$

No option matching.

15. Here, Statement I is correct and Statement II is wrong can be directly concluded from Binding energy/nucleon curve.

16. Correct option is (c) you can make an analogy with Gauss's law in electrostatics.

17. $\rho_1 < \rho_2$ as denser liquid acquires lowest position of vessel.

$\rho_3 > \rho_1$ as ball sinks in liquid 1 and $\rho_3 < \rho_2$ as ball doesn't sinks in liquid 2, so

$$\rho_1 < \rho_3 < \rho_2$$

18. Since no conduction is found when multimeter is connected across P and Q, it means either both P and Q are n type or p type. So, it means R is base. When R is connected to common terminal and conduction is seen when other terminal is connected to P or Q, so in means transistor is npn with R as base.

19. Electric field at a distance r from A is, $E = \rho \times \dfrac{I}{2\pi r^2}$

$$\Rightarrow \quad \int dV = -\int E dr$$

$$\Rightarrow \quad V_C - V_B = \Delta V = -\int_a^{a+b} \frac{\rho I}{2\pi} \times \frac{dr}{r^2}$$

$$\Rightarrow \quad \Delta V = \frac{\rho I}{2\pi}\left[\frac{1}{a} - \frac{1}{a+b}\right]$$

20. From $E = \rho J = \dfrac{\rho \times I}{2\pi r^2}$

21. $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f} =$ constant, so (c) is correct graph.

22. $0.5 \times 2 + 1 \times 0 = 1.5 \times v$

[*assumed that* 2^{nd} *body is at rest*]

$$\Rightarrow \quad v = 2/3$$

$$\Delta K = K_f - K_i$$

$$= \frac{\frac{3}{2} \times \left(\frac{2}{3}\right)^2}{2} - \left(\frac{1}{2}\right) \times \frac{2^2}{2} = -\frac{2}{3} \text{ J}$$

$$= -0.67 \text{ J}$$

So, energy lost is 0.67 J.

23. Soap solution has lower surface tension, T as compared to pure water and capillary rise $h = \dfrac{2T \cos\theta}{\rho r g}$ so h is less for soap solution.

24. $\dfrac{mv^2}{r_n} = \dfrac{k}{r_n}$ given

$mvr_n = \dfrac{nh}{2\pi}$ from Bohr's theory

Solving, $r_n \propto n$ and T_n is independent of n.

25. $y(x, t) = 0.005 \cos(\alpha x - \beta t)$

$\dfrac{2\pi}{\lambda} = \alpha$ and $\dfrac{2\pi}{T} = \beta$

So, $\alpha = \dfrac{2\pi}{0.08} = 25\pi$ and $\beta = \dfrac{2\pi}{2} = \pi$

26. $M = \dfrac{\mu_0 N_1 \times N_2 \times A}{l}$

where $N_1 = 300$ turns, $N_2 = 400$ turns, $A = 10 \text{ cm}^2$ and $l = 20$ cm.

Substituting the values in the given formula, we get $M = 2.4\,\pi \times 10^{-4}$ H.

27. If we give the following inputs to A and B then corresponding output is shown in table

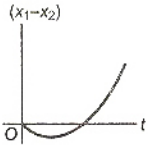
A	B	C
0	0	0
0	1	1
1	0	1
1	1	1

The above table is similar to OR gate.

28. Here, $x_2 = vt$ and $x_1 = \frac{at^2}{2}$

$$x_1 - x_2 = -\left(vt - \frac{at^2}{2}\right)$$

So, the graph would be like

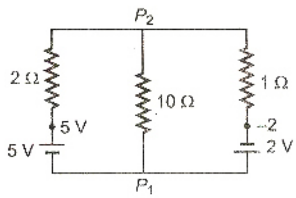


29. Factual.

30. For uniformly charged spherical shell,

$$E = 0, \quad r < R$$
$$= \frac{Q}{4\pi\varepsilon_0 r^2} \quad r \geq R$$

31. Let potential of P_1 is 0 V and potential of P_2 is V_0. Now apply KCL at P_2.



$$\frac{V_0 - 5}{2} + \frac{V_0 - 0}{10} + \frac{V_0 - (-2)}{1} = 0$$

$\Rightarrow$ $$V_0 = \frac{5}{16}$$

So, current through 10 Ω resistor is $\frac{V_0}{10}$ from P_2 to P_1.

32. $B = \frac{\mu_0 I}{2\pi R}$

Direction is given by Right hand palm value No. 1

$$B = \frac{2 \times 10^{-7} \times 100}{4} \text{ T towards south}$$

33. For diamagnatic material, $0 < \mu_r < 1$ and for any material $\varepsilon_r > 1$

34. Diameter = Main scale reading + Circular scale reading × LC + Zero error

$$= 3 + 35 \times \frac{1}{2 \times 50} + 0.03 = 3.38 \text{ mm}$$

35. As no work is done and system is thermally insulated from surrounding, it means sum of internal energy of gas in two partitions is constant *ie*, $U = U_1 + U_2$

Assuming both gases have same degree of freedom, then

$$U = \frac{f(n_1 + n_2)RT}{2} \text{ and } U_1 = \frac{fn_1RT_1}{2},$$

$$U_2 = \frac{fn_2RT_2}{2}$$

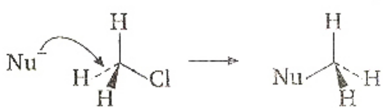
Solving we get, $T = \frac{(p_1V_1 + p_2V_2)T_1T_2}{p_1V_1T_2 + p_2V_2T_1}$

Chemistry

1. Nucleophilic substitution bimolecular (S_N2) prefers less sterically hindered site to attack. Lesser the steric hindrance better the S_N2 reaction. So ease of reaction is 1° > 2° > 3°

S_N2 involves inversion of configuration stereo chemically.

(Remember! if molecule is optically active, optical inversion, Walden inversion is known)



2. en, $CH_2 - NH_2$ / $CH_2 - NH_2$ is bidentate ligand

$C_2O_4^{2-}$ is also bidentate ligand

Hence, coordinating head are six (en : 2 × 2 = 4 + (ox) 2)

Coordination number 6

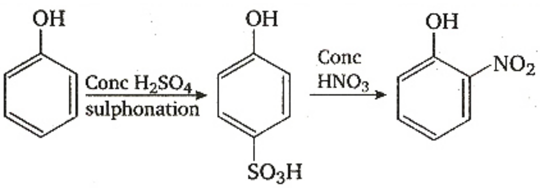
Complex can be ionised as

$[E(en)_2(C_2O_4)]NO_2 \longrightarrow [E(en)_2(C_2O_4]^+ NO_2^-$

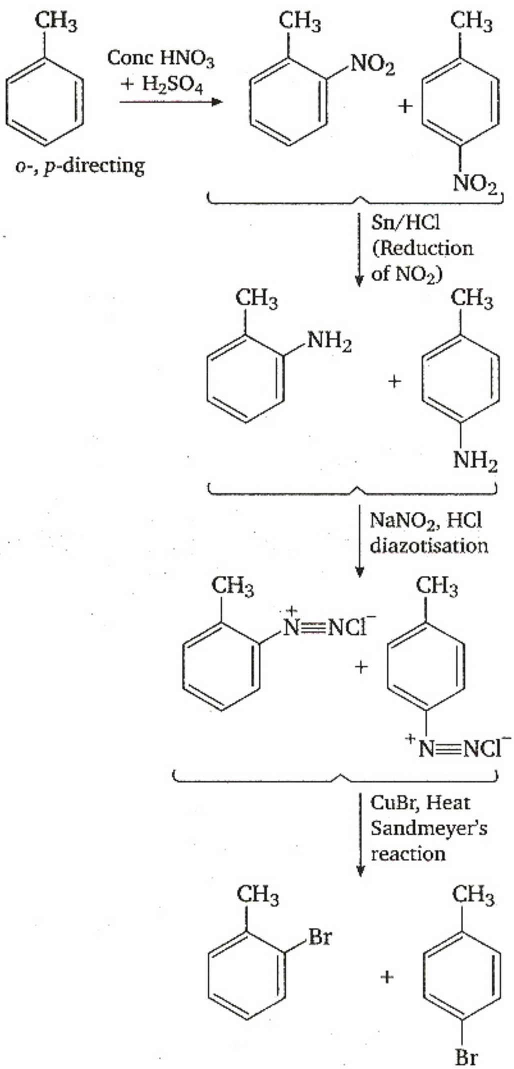
oxidation number $\quad x + 0 + (-2) = +1$

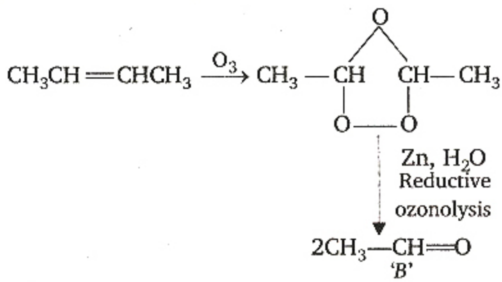
$\Rightarrow$ oxidation number = 3

3. Ozone layer permits the infrared radiation to pass through but doesn't permit the higher range of ultraviolet radiation to pass through.

4. 

First sulphonation is the means to block *para* position and to reduce the reactivity of phenolic ring against strong oxidising agent HNO_3. (The use of conc HNO_3 over phenol cause the oxidation of ring mainly). The strong acidic medium in second step cause desulphonation (ipso mechanism) also.

5. 

6. 

7. Lanthanoides : $[Xe]4f^{1-14}5d^{0-1}6s^2$

Actinoides : $[Rn]\ 5f^{1-14}6d^{0-1}7s^2$

Lathanoides and actinoides use core d and f orbitals also to show higher oxidation state. As actinoides have comparatively low energy difference between f and d orbitals show more oxidation states.

8. CFSE (crystal field splitting energy) for octahedral complex, Δ_o depends on the strength of negative ligand. Spectrochemically it has been found that the strength of splitting is as follows :

$CO > CN^- > NO_2^- > en > NH_3 > py >$

$NCS^- > H_2O > O^{2-} > OX^{2-} > OH^- > F^- >$

$Cl^- > SCN^- > S^{2-} > Br^- > I^-$

9. $p_T = p_A^\circ X_A + p_B^\circ X_B$

Mixture solution boils at 1 atm = 760 mm = total pressure

$$760 = 520\,X_A + 1000(1 - X_A)$$

$$X_A = 0.5, \quad \text{mol \% of } A = 50\%$$

10. $$\frac{1}{2}A \longrightarrow 2B$$

Remember for $\quad aA \longrightarrow bB$

$$-\frac{1}{a}\frac{d[A]}{dt} = \frac{1}{b}\frac{d[B]}{dt} = \text{Rate of reaction}$$

For the given reaction

$$-\frac{2d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt} = \text{Rate of reaction}$$

Rate of disappearance of A

$$= -\frac{d[A]}{dt} = \frac{1}{2\times 2}\frac{d[B]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$$

11. $$X \rightleftharpoons 2Y$$

$$\quad 1 \qquad 0$$

at equilibrium $\quad 1 - x \quad 2x$

Total no. of moles = $1 + x$

$$K_{p_1} = \frac{\left[\frac{2x}{1+x}\cdot p_1\right]^2}{\left(\frac{1-x}{1+x}\right)\cdot p_1} = \frac{4x^2}{(1-x)}\cdot\frac{p_1}{(1+x)}$$

$$Z \rightleftharpoons P + Q$$

$$\begin{array}{lccc} & 1 & 0 & 0 \\ \text{At equilibrium} & 1-x & x & x \end{array}$$

Similarly $\quad K_{p_2} = \dfrac{x^2}{(1-x)} \cdot \dfrac{p_2}{(1+x)}$

$$\frac{K_{p_1}}{K_{p_2}} = \frac{1}{9}$$

So, $\quad \dfrac{4 \times p_1}{p_2} = \dfrac{1}{9}$

$\Rightarrow \quad \dfrac{p_1}{p_2} = \dfrac{1}{36}$

12. $\quad \dfrac{1}{2}Cl_2(g) \longrightarrow Cl^-(aq)$

$$\Delta H = \frac{1}{2}\Delta H_{diss}(Cl_2) + \Delta H_{EA}\, Cl + \Delta H_{hyd}\,(Cl^-)$$

$$= \frac{240}{2} - 349 - 381$$

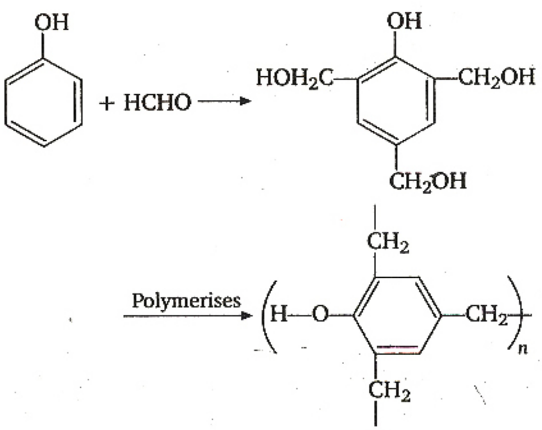
$$= -610 \text{ kJ mol}^{-1}$$

13. $2MS + C \longrightarrow 2M + CS_2 \qquad \Delta G_1 = \text{positive}$

$2MO + C \longrightarrow 2M + CO_2 \qquad \Delta G_2 = \text{negative}$

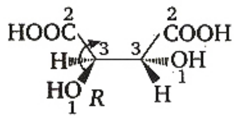
The value of ΔG for the formation of CO_2 is negative, *ie*, it is thermodynamically more stable than CS_2. Also metal sulphides are thermodynamically more stable than CS_2. Metal sulphides are more stable than the corresponding oxides, so they are roasted to convert into less stable oxides.

14. Bakelite is obtained from phenol by reacting with HCHO in the acidic or alkaline medium

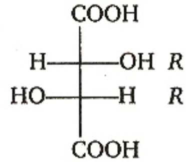


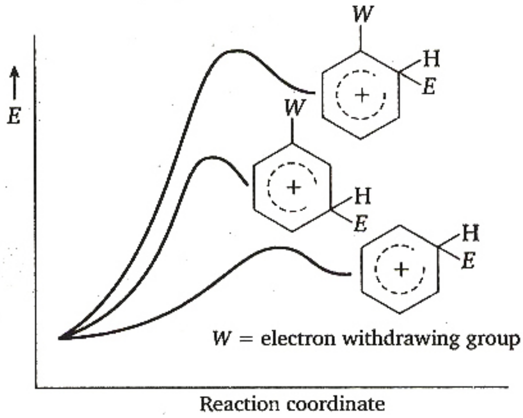
bakelite

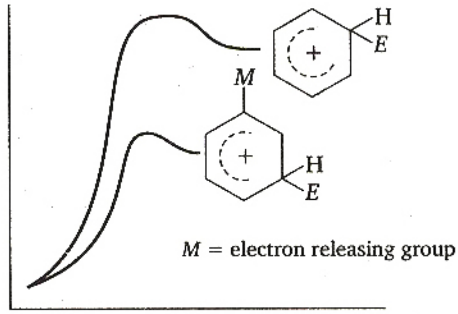
15. As equation 'III' is obtained on adding equation 'I' and equation 'II', so $K_3 = K_1 \cdot K_2$

16. 

or

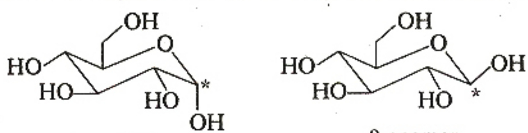


17. 



electron withdrawing group destabilises the arenium ion σ–complex.

18. α-D(+)glucose and β-D-(+)glucose are anomers



α-anomer $\qquad$ β-anomer

19. $\quad \dfrac{1}{2}X_2 + \dfrac{3}{2}Y_2 \longrightarrow XY_3$

$$\Delta S_{reaction} = S_{products} - S_{reactants}$$

$$\Delta S_{reaction} = 50 - \left(\frac{3}{2} \times 40 + \frac{1}{2} \times 60\right)$$

$$= -40 \text{ J mol}^{-1}$$

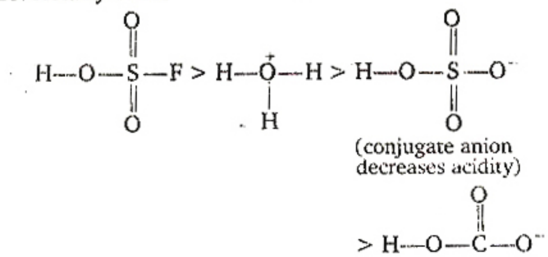
$$\Delta G = \Delta H - T\Delta S$$

At equilibrium as $\Delta G = 0$

$$\Delta H = T\Delta S$$

$$T = \frac{\Delta H}{\Delta S} = \frac{30 \times 10^3}{40} = 750 \text{ K}$$

20. Acidity order



21. Isoelectronic species have same number of electrons, NO^+, C_2^{2-}, CN^- and N_2 all have 14 electrons.

22. CN^- and NO^+ both have same number of electrons and same bond order (3).

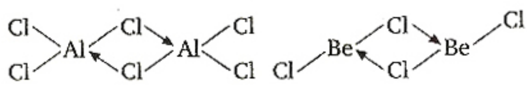
23. Ionisation enthalpy of hydrogen atom is 1.312×10^6 J mol^{-1}. It suggest that the energy of electron in the ground state (first orbit) is -1.312×10^6 J mol^{-1}

$$\Delta E = E_2 - E_1$$

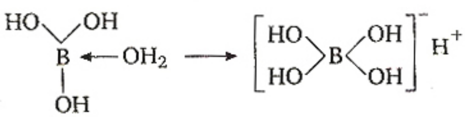
$$= -\frac{1.312 \times 10^6}{2^2} - \left(\frac{-1.312 \times 10^6}{1}\right)$$

$$= 9.84 \times 10^5 \text{ J mol}^{-1}$$

24. Chlorides of both beryllium and aluminium have bridged structures in solid phase



Boric acid is not a protonic acid



Borazole, inorganic benzene contains $B_3N_3H_6$

25. $Cr | Cr^{3+}(0.1M) || Fe^{2+}(0.01M) | Fe$

Oxidation half cell

$$Cr \longrightarrow Cr^{3+} + 3e^- \times 2$$

Reduction half cell

$$Fe^{2+} + 2e^- \longrightarrow Fe \times 3$$

Net cell reaction :

$$2Cr + 3Fe^{2+} \longrightarrow 2Cr^{3+} + 3Fe \qquad n = 6$$

$$E^\circ_{cell} = E^\circ_{oxid} + E^\circ_{red}$$

$$= 0.72 - 0.42 = 0.30 \text{ V}$$

$$E_{cell} = E^\circ_{cell} - \frac{0.0591}{n} \log \frac{[Cr^{3+}]^2}{[Fe^{2+}]^3}$$

$$= 0.30 - \frac{0.0591}{6} \log \frac{(0.1)^2}{(0.01)^3}$$

$$= 0.30 - \frac{0.0591}{6} \log \frac{10^{-2}}{10^{-6}}$$

$$= 0.30 - \frac{0.0591}{6} \log 10^4$$

$$E_{cell} = 0.2606 \text{ V}$$

26. Titration of oxalic acid by $KMnO_4$ in the presence of HCl gives unsatisfactory result because HCl is a better reducing agent than oxalic acid and HCl reduces preferably MnO_4^- to Mn^{2+}

27. Moles of glucose $= \frac{18}{180} = 0.1$

Moles of $H_2O = \frac{178.2}{18} = 9.9$

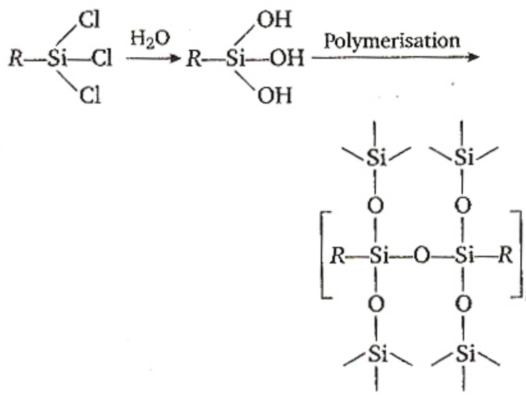
According to Raoult's law

$$\frac{p^\circ - p_s}{p^\circ} = X_{solute}$$

$$\frac{17.5 - p_s}{17.5} = \frac{0.1}{10}$$

So, $\qquad p_s = 17.325$ mmHg

28. $RSiCl_3$ give cross linked silicone polymer on hydrolysis



29. CO is oxidised to CO_2 with steam in the presence of a catalyst followed by absorption of CO_2 in alkali

$$CO + H_2 \xrightarrow[\text{catalyst}]{\text{Steam}} CO_2 + 2H_2$$
$$\downarrow KOH$$
$$K_2CO_3$$

30. Suppose atoms of element Y in ccp = 100

Number of tetrahedral voids = 2×100

Number of element $X = \frac{2}{3} \times 200 = \frac{400}{3}$

$$\frac{X}{Y} = \frac{400}{300}$$

Formula = X_4Y_3

31. Higher the gold number, lesser will be the protective power of colloid.

32. $CH_3CH_2C \equiv CH \xrightarrow[\Delta]{Na/Liq\ NH_3}$

$$CH_3CH_2C \equiv \bar{C}Na^+$$

Considering the options given it appears correct. Na/liq NH_3 is known for metal dissolved reduction. Actually it is truth that Na liq NH_3 reduces internal triple bond and terminal double bond and do not reduce the terminal alkyne due to such alkylide formation.

33. Terminal alkyne has acidic hydrogen which is enough to protonate the Grignard reagent.

$$CH_3MgX + CH_3C \equiv CH \longrightarrow CH_4 + CH_3C \equiv CMgX$$

34. The correct decreasing order of priority for the functional groups of organic compounds in the IUPAC system is

$$—COOH > —SO_3H > —COOR > COCl > —CONH_2 > —CN > —CH{=}O$$

35. For salt of weak acid and weak base

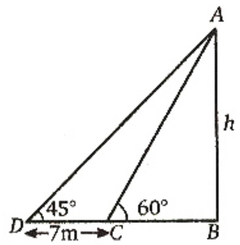
$$pH = -\frac{1}{2}[\log K_a + \log K_w - \log K_b]$$
$$= \frac{1}{2}pK_a + \frac{1}{2}pK_w - \frac{1}{2}pK_b$$
$$= \frac{1}{2} \times 4.80 + 7 - \frac{1}{2} \times 4.78$$
$$= 7.01$$

Mathematics

1. In ΔABC, $BC = h \cot 60°$

and in ΔABD, $BD = h \cot 45°$

Since, $BD - BC = DC$



$$\Rightarrow h \cot 45° - h \cot 60° = 7$$

$$\therefore \quad h = \frac{7}{\cot 45° - \cot 60°} = \frac{7}{\left(1 - \frac{1}{\sqrt{3}}\right)}$$

$$= \frac{7\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{7\sqrt{3}}{2}(\sqrt{3}+1) \text{ m}$$

2. Given that, $P(A) = \frac{1}{4}$, $P\left(\frac{A}{B}\right) = \frac{1}{2}$ and $P\left(\frac{B}{A}\right) = \frac{2}{3}$

We know, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ ... (i)

and $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$ ... (ii)

$$\therefore \quad P(B) = \frac{P\left(\frac{B}{A}\right) \cdot P(A)}{P\left(\frac{A}{B}\right)} = \frac{\left(\frac{2}{3}\right)\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)} = \frac{1}{3}$$

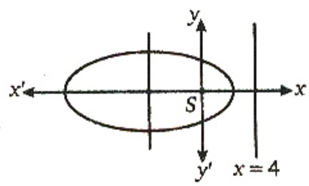
3. $\because \quad A = \{4, 5, 6\}$ and $B = \{1, 2, 3, 4\}$

$\therefore \quad A \cap B = \{4\}$

$\because \ P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A \cup B) = \frac{3}{6} + \frac{4}{6} - \frac{1}{6} = 1$$

4. Since, $\frac{a}{e} - ae = 4$ and $e = \frac{1}{2}$

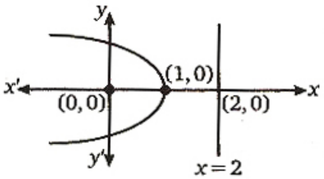


$$\therefore \quad 2a - \frac{a}{2} = 4$$

$\Rightarrow \quad \frac{3a}{2} = 4$

$\Rightarrow \quad a = \frac{8}{3}$

5. From the figure, it is clear that vertex of the parabola at (1, 0).



6. Given equation can be rewritten as

$$(x+1)^2 + (y+2)^2 = (2\sqrt{2})^2$$

Let required point be $Q(\alpha, \beta)$.

Then, mid point of $P(1, 0)$ and $Q(\alpha, \beta)$ is the centre of the circle.

$\therefore \frac{\alpha+1}{2} = -1$ and $\frac{\beta+0}{2} = -2$

$\Rightarrow \quad \alpha = -3$ and $\beta = -4$

$\therefore$ Required point is $(-3, -4)$.

7. $\because Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$

$\therefore \quad Y = \{7, 8, \ldots, \infty\}$

Let $\quad y = 4x + 3 \Rightarrow x = \frac{y-3}{4}$

Inverse of $f(x)$ is

$g(y) = \frac{y-3}{4}$

8. Let $z = \frac{1}{i-1}$

$\therefore \quad \bar{z} = \overline{\left(\frac{1}{i-1}\right)} = \frac{1}{-i-1} = -\frac{1}{i+1}$

9. Since, $(1, 2) \in S$ but $(2, 1) \notin S$

$\therefore S$ is not symmetric.

Hence, S is not equivalence relation.

Given, $T = \{(x, y) : x - y \in I\}$

Now, $x - x = 0 \in I$, it is reflexive relation

$x - y \in I$

$\Rightarrow y - x \in I$, it is symmetric relation.

Let $x - y = I_1$ and $y - z = I_2$

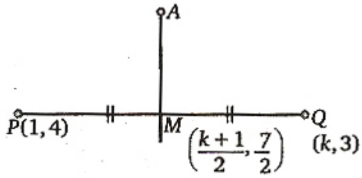
Now, $x - z = (x - y) + (y - z) = I_1 + I_2 \in I$

$\therefore T$ is also transitive.

Hence, T is an equivalence relation.

10. $\because$ Slope of $PQ = \frac{4-3}{1-k} = \frac{1}{1-k}$

$\therefore$ Slope of $AM = (k-1)$



$\therefore$ Equation of AM is

$$y - \frac{7}{2} = (k-1)\left[x - \left(\frac{k+1}{2}\right)\right]$$

For y-intercept, $\quad x = 0, \; y = -4$

$\therefore \quad -4 - \frac{7}{2} = -(k-1)\left(\frac{k+1}{2}\right)$

$\Rightarrow \quad \frac{15}{2} = \frac{k^2-1}{2}$

$\Rightarrow \quad k^2 - 1 = 15$

$k^2 = 16$

$\Rightarrow \quad k = -4$

11. Given equation can be rewritten as $\frac{dy}{dx} - \frac{1}{x} \cdot y = 1$

Now, $\quad \text{IF} = e^{-\int \frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$

$\therefore$ Required solution is $y\left(\frac{1}{x}\right) = \int \frac{1}{x} dx = \log x + c$

Since, $y(1) = 1 \Rightarrow 1 = \log 1 + c \Rightarrow c = 1$

$\therefore \quad y = x \log x + x$

12. According to the given condition

$$6.80 = \frac{(6-a)^2 + (6-b)^2 + (6-8)^2 + (6-5)^2 + (6-10)^2}{5}$$

$\Rightarrow \quad 34 = (6-a)^2 + (6-b)^2 + 4 + 1 + 16$

$\Rightarrow \quad (6-a)^2 + (6-b)^2 = 13 = 9 + 4$

$\Rightarrow \quad (6-a)^2 + (6-b)^2 = 3^2 + 2^2$

$\Rightarrow \quad a = 3, \; b = 4$

13. Given that, $\vec{\mathbf{b}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\vec{\mathbf{c}} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$

The equation of bisector of $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ is

$$\vec{\mathbf{r}} = \lambda(\hat{\mathbf{b}} + \hat{\mathbf{c}})$$

$$= \lambda\left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}} + \frac{\hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{2}}\right)$$

$$= \frac{\lambda}{\sqrt{2}}(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) \quad \text{...(i)}$$

Since, point C lies on $\overrightarrow{\mathbf{AB}}$.

$$\therefore \quad \frac{\lambda}{\sqrt{2}}(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) = (\hat{\mathbf{i}} + \hat{\mathbf{j}}) + \mu(\hat{\mathbf{j}} + \hat{\mathbf{k}})$$

On equating the coefficient of $\hat{\mathbf{i}}$ both sides, we get

$$\frac{\lambda}{\sqrt{2}} = 1 \Rightarrow \lambda = \sqrt{2}$$

On putting $\lambda = \sqrt{2}$ in Eq. (i), we get

$$\vec{\mathbf{r}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

Since, the given vector $\vec{\mathbf{a}}$ represents the same bisector equation $\vec{\mathbf{r}}$.

$\therefore \quad \alpha = 1$ and $\beta = 1$

Alternate solution

Since, $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are coplanar.

$$\therefore \quad \begin{vmatrix} \alpha & 2 & \beta \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow \alpha(1-0) - 2(1-0) + \beta(1-0) = 0$

$\Rightarrow \alpha + \beta = 2$ which is possible for $\alpha = 1, \beta = 1$.

14. Since, $\vec{\mathbf{a}} = 8\vec{\mathbf{b}}$ and $\vec{\mathbf{c}} = -7\vec{\mathbf{b}}$

$\therefore \vec{\mathbf{a}}$ is parallel to $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ is anti-parallel to $\vec{\mathbf{b}}$.

$\Rightarrow \vec{\mathbf{a}}$ and $\vec{\mathbf{c}}$ are anti-parallel.

$\Rightarrow$ Angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{c}}$ is π.

15. Equation of line passing through $(5, 1, a)$ and $(3, b, 1)$ is

$$\frac{x-3}{5-3} = \frac{y-b}{1-b} = \frac{z-1}{a-1} \quad \text{... (i)}$$

Point $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$ satisfies Eq. (i), we get

$$\therefore \quad -\frac{3}{2} = \frac{\frac{17}{2} - b}{1-b} = \frac{-\frac{13}{2} - 1}{a-1}$$

$$\Rightarrow \quad a - 1 = \frac{\left(-\frac{15}{2}\right)}{\left(-\frac{3}{2}\right)} = 5 \quad \Rightarrow \quad a = 6$$

Also, $-3(1-b) = 2\left(\frac{17}{2} - b\right)$

$\Rightarrow \quad 3b - 3 = 17 - 2b$

$\Rightarrow \quad 5b = 20 \Rightarrow b = 4$

16. Given, $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ ... (i)

and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ ... (ii)

Since, lines intersect at a point

$$\therefore \quad \begin{vmatrix} k & 2 & 3 \\ 3 & k & 2 \\ 1 & 1 & -2 \end{vmatrix} = 0$$

$\Rightarrow k(-2k - 2) - 2(-6 - 2) + 3(3 - k) = 0$

$\Rightarrow -2k^2 - 5k + 25 = 0$

$\Rightarrow 2k^2 + 5k - 25 = 0$

$\Rightarrow 2k^2 + 10k - 5k - 25 = 0$

$\Rightarrow 2k(k + 5) - 5(k + 5) = 0$

$\Rightarrow k = \frac{5}{2}, -5$

Hence, integer value of k is -5.

17. We have,

$n(n+1) = n^2 + n < n^2 + n + n + 1 = (n+1)^2$

$\Rightarrow \quad \sqrt{n(n+1)} < n + 1, \quad \forall \quad n \geq 2$

$\Rightarrow \quad \sqrt{n} < \sqrt{n+1}$

$\Rightarrow \quad \frac{1}{\sqrt{n}} < \frac{1}{\sqrt{n+1}}; \quad n \geq 2$

Statement II is true.

Also, $\frac{1}{\sqrt{1}} > \frac{1}{\sqrt{n}}, \frac{1}{\sqrt{2}} > \frac{1}{\sqrt{n}}, \frac{1}{\sqrt{3}} > \frac{1}{\sqrt{n}}, \ldots,$

$$\frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n}}, \forall n \geq 2$$

On adding all of them, we get

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{n}} > \frac{n}{\sqrt{n}} = \sqrt{n}, \ \forall n \geq 2$$

Clearly, statements I and II are true and statement II is a correct explanation of statement I.

18. Let $\quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\therefore \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (\because A^2 = I)$$

$$\Rightarrow \quad \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad b(a + d) = 0, c(a + d) = 0$

and $a^2 + bc = 1, bc + d^2 = 0$

$\Rightarrow \quad a = 1, d = -1, b = c = 0$

If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, then

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$A \neq I,\ A \neq -I$

$\det A = -1$ (Statement I is true)

Statement II $\operatorname{tr}(A) = 1 - 1 = 0$, Statement II is false.

19. Since, $\sum_{r=0}^{n} {}^nC_r \cdot x^r = (1+x)^n$

On multiplying by x, we get

$$\sum_{r=0}^{n} {}^nC_r \cdot x^{r+1} = x(1+x)^n$$

On differentiating w.r.t. x, we get

$$\sum_{r=0}^{n} (r+1)\, {}^nC_r \cdot x^r = (1+x)^n + nx(1+x)^{n-1}$$

$\therefore$ Statement II is true.

If $x = 1$, then

$$\sum_{r=0}^{n} (r+1)\, {}^nC_r = 2^n + n(2)^{n-1} = (n+2)\, 2^{n-1}$$

$\therefore$ Statement I is true and Statement II is a correct explanation of Statement I.

20. p : x is an irrational number

q : y is a transcendental number

r : x is a rational number, iff y is a transcendental number

$\Rightarrow r : \sim p \leftrightarrow q$

$\therefore S_1 : r \equiv q \vee p$

and $S_2 : r \equiv \sim(p \leftrightarrow \sim q)$

p	q	$\sim p$	$\sim q$	$r\ \sim p \leftrightarrow q$	Statement I $q \vee p$	$(p \leftrightarrow \sim q)$	Statement II $\sim(p \leftrightarrow \sim q)$
T	T	F	F	F	T	F	T
T	F	F	T	T	T	T	F
F	T	T	F	T	T	T	F
F	F	T	T	F	F	F	T

It is clear from the table that r is not equivalent to either of the statements.

Hence, none of the given option is correct.

21. Since, the number of ways that child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row.

$\therefore$ Number of ways to arrange 6A's and 4B's in a row

$$= \frac{10!}{6!\,4!} = {}^{10}C_4$$

and number of integral solution of the equation

$x_1 + x_2 + x_3 + x_4 + x_5 = 6$

$= {}^{6+5-1}C_{5-1} = {}^{10}C_4 \neq {}^{10}C_5$

Statement I is false and statement II is true.

22. Now, $f'(1^-) = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$

$$= \lim_{h\to 0} \frac{(1-h-1)\cdot \sin\left(\frac{1}{1-h-1}\right) - 0}{-h}$$

$$= \lim_{h\to 0} \sin\left(-\frac{1}{h}\right) = -\lim_{h\to 0} \sin\frac{1}{h}$$

and $\quad f'(1^+) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$

$$= \lim_{h\to 0} \frac{(1+h-1)\sin\left(\frac{1}{1+h-1}\right) - 0}{h} = \lim_{h\to 0} \sin\frac{1}{h}$$

$\therefore f'(1^-) \neq f'(1^+)$

f is not differentiable at $x = 1$.

Again, Now

$$f'(0^-) = \lim_{h\to 0} \frac{(0+h+1)\sin\left(\frac{1}{0+h+1}\right) - \sin 1}{-h},\ \left(\frac{0}{0}\right)$$

$$= \lim_{h\to 0} \frac{-\left\{(h+1)\cos\left(\frac{1}{h+1}\right) \times \left(\frac{1}{(h+1)^2}\right)\right\} + \sin\left(\frac{1}{h+1}\right)}{-1}$$

(using L' Hospital's rule)

$= \cos 1 - \sin 1$

and

$$f'(0^+) = \lim_{h\to 0} \frac{(0+h-1)\sin\left(\frac{1}{0+h-1}\right) - \sin 1}{h},\ \left(\frac{0}{0}\right)$$

$$= \lim_{h\to 0} \frac{(h-1)\cos\left(\frac{1}{h-1}\right)\left(\frac{-1}{(h-1)^2}\right) + \sin\left(\frac{1}{h-1}\right)}{1}$$

(using L' Hospital's rule)

$= \cos 1 - \sin 1$

$\Rightarrow f'(0^-) = f'(0^+)$

$\therefore$ f is differentiable at $x = 0$.

23. Since, $a + ar = a(1+r) = 12$ ...(i)

and $ar^2 + ar^3 = ar^2(1+r) = 48$ ...(ii)

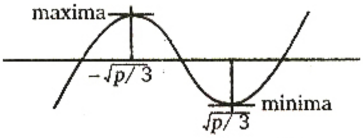
From Eqs. (i) and (ii),

$$r^2 = 4 \Rightarrow r = -2$$

On putting the value of r in Eq. (i), we get

$$a = -12$$

24. Let $f(x) = x^3 - px + q$, then $f'(x) = 3x^2 - p$



Put $f'(x) = 0 \Rightarrow x = \sqrt{\frac{p}{3}}, -\sqrt{\frac{p}{3}}$

Now, $f''(x) = 6x$

$\therefore$ At $x = \sqrt{\frac{p}{3}}, f''(x) = 6\sqrt{\frac{p}{3}} > 0$, minima

and at $x = -\sqrt{\frac{p}{3}}, f''(x) < 0$, maxima

25. Let $f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560$

$f'(x) = 7x^6 + 70x^4 + 48x^2 + 30 > 0, \quad \forall \quad x \in R$

$\therefore$ $f(x)$ is increasing.

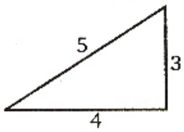
$\therefore$ $f(x) = 0$ has only one solution.

26.

p	q	$q \to p$	$p \to (q \to p)$	$(p \vee q)$	$p \to (p \vee q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	T	F	T

$\therefore$ Statement $p \to (q \to p)$ is equivalent to $p \to (p \vee q)$.

27. Since, $\text{cosec}^{-1}\left(\frac{5}{3}\right) = \tan^{-1}\left(\frac{3}{4}\right)$



$$\therefore \cot\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right)$$

$$= \cot\tan^{-1}\left[\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{1}{2}}\right] = \cot\tan^{-1}\left[\frac{\left(\frac{17}{12}\right)}{\left(\frac{1}{2}\right)}\right]$$

$$= \cot\left\{\tan^{-1}\left(\frac{17}{6}\right)\right\} = \frac{6}{17}$$

28. The equation of family of circles with centre on $y = 2$ and of radius 5 is

$$\Rightarrow (x-\alpha)^2 + (y-2)^2 = 5^2 \quad ...(i)$$

$$x^2 + \alpha^2 - 2\alpha x + y^2 + 4 - 4y = 25$$

On differentiating w.r.t. x, we get

$$2x - 2\alpha + 2y\frac{dy}{dx} - 4\frac{dy}{dx} = 0$$

$$\Rightarrow \alpha = x + \frac{dy}{dx}(y-2)$$

On putting the value of α in Eq. (i), we get

$$\left(x - x - \frac{dy}{dx}(y-2)\right)^2 + (y-2)^2 = 5^2$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 (y-2)^2 = 25 - (y-2)^2$$

$$\Rightarrow y'^2(y-2)^2 = 25 - (y-2)^2$$

29. Since, $I = \int_0^1 \frac{\sin x}{\sqrt{x}}dx < \int_0^1 \frac{x}{\sqrt{x}}dx$,

because in $x \in (0,1)$, $x > \sin x$.

$$I < \int_0^1 \sqrt{x}\,dx = \frac{2}{3}[x^{3/2}]_0^1$$

$$\Rightarrow I < \frac{2}{3}$$

and $J = \int_0^1 \frac{\cos x}{\sqrt{x}}dx < \int_0^1 x^{-\frac{1}{2}}dx = 2$

$$J < 2.$$

30. Given equations of curves are

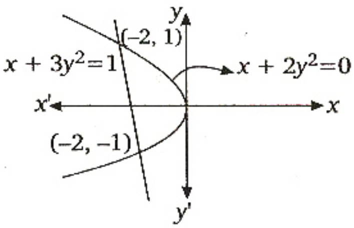
$$x + 3y^2 = 1 \quad ...(i)$$

and $$x + 2y^2 = 0 \quad ...(ii)$$

On solving Eqs. (i) and (ii), we get

$$y = \pm 1 \text{ and } x = -2$$

$\therefore$ Required area $= \left|\int_{-1}^{1}(x_1 - x_2)dy\right|$



$$= \left|\int_{-1}^{1}(1 - 3y^2 + 2y^2)dy\right| = \left|\int_{-1}^{1}(1 - y^2)dy\right|$$

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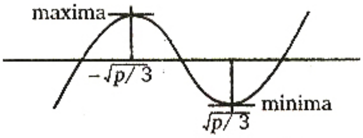

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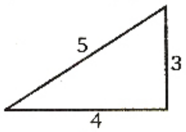

$$\therefore \cot\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right)$$

$$= \cot\tan^{-1}\left[\frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{1}{2}}\right] = \cot\tan^{-1}\left[\frac{\left(\frac{17}{12}\right)}{\left(\frac{1}{2}\right)}\right]$$

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$$\Rightarrow \left(\frac{dy}{dx}\right)^2 (y - 2)^2 = 25 - (y - 2)^2$$

$$\Rightarrow y'^2(y - 2)^2 = 25 - (y - 2)^2$$

29. Since, $I = \int_0^1 \frac{\sin x}{\sqrt{x}}\,dx < \int_0^1 \frac{x}{\sqrt{x}}\,dx$,

because in $x \in (0, 1)$, $x > \sin x$.

$$I < \int_0^1 \sqrt{x}\,dx = \frac{2}{3}[x^{3/2}]_0^1$$

$$\Rightarrow I < \frac{2}{3}$$

and $J = \int_0^1 \frac{\cos x}{\sqrt{x}}\,dx < \int_0^1 x^{-\frac{1}{2}}dx = 2$

$$J < 2.$$

30. Given equations of curves are

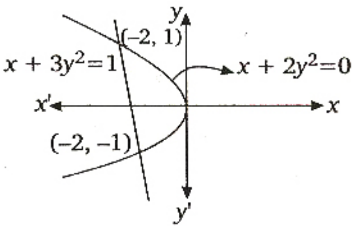
$$x + 3y^2 = 1 \quad ...(i)$$

and $$x + 2y^2 = 0 \quad ...(ii)$$

On solving Eqs. (i) and (ii), we get

$$y = \pm 1 \text{ and } x = -2$$

$\therefore$ Required area $= \left|\int_{-1}^{1}(x_1 - x_2)\,dy\right|$



$$= \left|\int_{-1}^{1}(1 - 3y^2 + 2y^2)\,dy\right| = \left|\int_{-1}^{1}(1 - y^2)\,dy\right|$$

$$= \left| 2\int_0^1 (1-y^2)\,dy \right| = \left| 2\left[y - \frac{y^3}{3} \right]_0^1 \right|$$

$$= \left| 2\left(1-\frac{1}{3}\right) \right| = \frac{4}{3} \text{ sq unit}$$

31. Let $I = \sqrt{2}\int \frac{\sin x}{\sin\left(x - \frac{\pi}{4}\right)}\,dx$

Put $x - \frac{\pi}{4} = t \Rightarrow dx = dt$

$$\therefore \quad I = \sqrt{2}\int \frac{\sin\left(\frac{\pi}{4}+t\right)dt}{\sin t}$$

$$= \sqrt{2}\int \left[\frac{1}{\sqrt{2}}\cot t + \frac{1}{\sqrt{2}} \right] dt$$

$$= t + \log|\sin t| + c$$

$$= x + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$$

32. Given word is MISSISSIPPI.

Here, I = 4 times, S = 4 times, P = 2 times

M = 1 time.

_M_I_I_I_I_P_P_

$\therefore$ Required number of words $= {}^8C_4 \times \frac{7!}{4!\,2!}$

$$= {}^8C_4 \times \frac{7 \times 6!}{4!2!} = 7 \cdot {}^8C_4 \cdot {}^6C_4$$

33. Given equations are $x - cy - bz = 0$

$cx - y + az = 0$

and $bx + ay - z = 0$

For non-zero solution

$$\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$$\Rightarrow 1(1-a^2) + c(-c-ab) - b(ac+b) = 0$$

$$\Rightarrow 1 - a^2 - c^2 - abc - abc - b^2 = 0$$

$$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$$

34. As $\det(A) = \pm 1$, A^{-1} exists and

$$A^{-1} = \frac{1}{\det(A)}(\text{adj } A) = \pm(\text{adj } A)$$

All entries in adj (A) are integers.

$\therefore$ A^{-1} has integer entries.

35. Let the roots of $x^2 - 6x + a = 0$ be α, 4β and that of $x^2 - cx + 6 = 0$ be α, 3β.

$\therefore$ $\alpha + 4\beta = 6$ and $4\alpha\beta = a$

and $\alpha + 3\beta = c$ and $3\alpha\beta = 6 \Rightarrow a = 8$

$\Rightarrow$ $\frac{a}{6} = \frac{4}{3} \Rightarrow a = 8$

$\Rightarrow$ $x^2 - 6x + 8 = 0$

$\Rightarrow (x-4)(x-2) = 0$

$\Rightarrow$ $x = 2, 4$

and $x^2 - cx + 6 = 0$

$\Rightarrow$ $2^2 - 2c + 6 = 0$

$\Rightarrow$ $c = 5$

$\therefore$ $x^2 - 5x + 6 = 0$

$\Rightarrow$ $x = 2, 3$

Common root is 2.