PART C — MATHEMATICS

61. Consider the following relations :

$R = \{(x, y) \mid x, y$ are real numbers and $x = wy$ for some rational number $w\}$;

$S = \{(\frac{m}{n}, \frac{p}{q}) \mid m, n, p$ and q are integers such that $n, q \neq 0$ and $qm = pn\}$.

Then

(1) R is an equivalence relation but S is not an equivalence relation

(2) neither R nor S is an equivalence relation

(3) S is an equivalence relation but R is not an equivalence relation

(4) R and S both are equivalence relations

62. The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals

(1) 0

(2) 1

(3) 2

(4) ∞

63. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$

(1) –2

(2) –1

(3) 1

(4) 2

64. Consider the system of linear equations :

$$x_1 + 2x_2 + x_3 = 3$$
$$2x_1 + 3x_2 + x_3 = 3$$
$$3x_1 + 5x_2 + 2x_3 = 1$$

The system has

(1) infinite number of solutions

(2) exactly 3 solutions

(3) a unique solution

(4) no solution

65. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is

(1) 3

(2) 36

(3) 66

(4) 108

66. Let $f : (-1, 1) \to \mathbf{R}$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$

(1) 4

(2) – 4

(3) 0

(4) – 2

67. Let $f : \mathbb{R} \to \mathbb{R}$ be a positive increasing function with $\lim_{x \to \infty} \frac{f(3x)}{f(x)} = 1$.

Then $\lim_{x \to \infty} \frac{f(2x)}{f(x)} =$

(1) 1

(2) $\frac{2}{3}$

(3) $\frac{3}{2}$

(4) 3

68. Let p(x) be a function defined on $\mathbb{R}$ such that $p'(x) = p'(1 - x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)\,dx$ equals

(1) $\sqrt{41}$

(2) 21

(3) 41

(4) 42

69. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \ldots = a_{10} = 150$ and $a_{10}, a_{11}, \ldots$ are in an AP with common difference −2, then the time taken by him to count all notes is

(1) 24 minutes

(2) 34 minutes

(3) 125 minutes

(4) 135 minutes

70. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is

(1) y = 0

(2) y = 1

(3) y = 2

(4) y = 3

71. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is

(1) $4\sqrt{2} - 2$

(2) $4\sqrt{2} + 2$

(3) $4\sqrt{2} - 1$

(4) $4\sqrt{2} + 1$

72. Solution of the differential equation

$\cos x\,dy = y(\sin x - y)\,dx$, $0 < x < \frac{\pi}{2}$ is

(1) $\sec x = (\tan x + c)\,y$

(2) $y \sec x = \tan x + c$

(3) $y \tan x = \sec x + c$

(4) $\tan x = (\sec x + c)\,y$

73. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a} \cdot \vec{b} = 3$ is

(1) $-\hat{i} + \hat{j} - 2\hat{k}$

(2) $2\hat{i} - \hat{j} + 2\hat{k}$

(3) $\hat{i} - \hat{j} - 2\hat{k}$

(4) $\hat{i} + \hat{j} - 2\hat{k}$

74. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$

(1) $(-3, 2)$

(2) $(2, -3)$

(3) $(-2, 3)$

(4) $(3, -2)$

75. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

(1) $x = 1$

(2) $2x + 1 = 0$

(3) $x = -1$

(4) $2x - 1 = 0$

76. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is

(1) $\frac{23}{\sqrt{15}}$

(2) $\sqrt{17}$

(3) $\frac{17}{\sqrt{15}}$

(4) $\frac{23}{\sqrt{17}}$

77. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals

(1) 30°

(2) 45°

(3) 60°

(4) 75°

78. Let S be a non-empty subset of $\mathbb{R}$. Consider the following statement :

P : There is a rational number $x \in S$ such that $x > 0$.

Which of the following statements is the negation of the statement P ?

(1) There is a rational number $x \in S$ such that $x \leq 0$.

(2) There is no rational number $x \in S$ such that $x \leq 0$.

(3) Every rational number $x \in S$ satisfies $x \leq 0$.

(4) $x \in S$ and $x \leq 0 \Rightarrow x$ is not rational.

79. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \leq \alpha, \beta \leq \frac{\pi}{4}$. Then $\tan 2\alpha =$

(1) $\frac{25}{16}$

(2) $\frac{56}{33}$

(3) $\frac{19}{12}$

(4) $\frac{20}{7}$

80. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

(1) $-85 < m < -35$

(2) $-35 < m < 15$

(3) $15 < m < 65$

(4) $35 < m < 85$

81. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

(1) $\frac{5}{2}$

(2) $\frac{11}{2}$

(3) 6

(4) $\frac{13}{2}$

82. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is

(1) $\frac{1}{3}$

(2) $\frac{2}{7}$

(3) $\frac{1}{21}$

(4) $\frac{2}{23}$

83. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A ***false*** statement among the following is

(1) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

(2) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(3) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

(4) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

84. The number of 3 × 3 non-singular matrices, with four entries as 1 and all other entries as 0, is

(1) less than 4

(2) 5

(3) 6

(4) at least 7

85. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} k - 2x, & \text{if } x \le -1 \\ 2x + 3, & \text{if } x > -1 \end{cases}$$

If f has a local minimum at x = −1, then a possible value of k is

(1) 1

(2) 0

(3) $-\frac{1}{2}$

(4) −1

Directions : Questions number 86 to 90 are Assertion – Reason type questions. Each of these questions contains two statements.

Statement-1 : (Assertion) and

Statement-2 : (Reason).

Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

86. Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ... 20}.

Statement-1 : The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement-2 : If the four chosen numbers form an AP, then the set of all possible values of common difference is {±1, ±2, ±3, ±4, ±5}.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is false.

(4) Statement-1 is false, Statement-2 is true.

87. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$.

Statement-1 : $S_3 = 55 \times 2^9$.

Statement-2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is *not* a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is false.

(4) Statement-1 is false, Statement-2 is true.

88. **Statement-1 :** The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane $x - y + z = 5$.

Statement-2 : The plane $x - y + z = 5$ bisects the line segment joining A(3, 1, 6) and B(1, 3, 4).

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is *not* a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is false.

(4) Statement-1 is false, Statement-2 is true.

89. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function defined by

$$f(x) = \frac{1}{e^x + 2e^{-x}}.$$

Statement-1 : $f(c) = \frac{1}{3}$, for some $c \in \mathbb{R}$.

Statement-2 : $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all $x \in \mathbb{R}$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is *not* a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is false.

(4) Statement-1 is false, Statement-2 is true.

90. Let A be a 2 × 2 matrix with non-zero entries and let $A^2 = I$, where I is 2 × 2 identity matrix. Define

Tr(A) = sum of diagonal elements of A and

| A | = determinant of matrix A.

Statement-1 : Tr(A) = 0.

Statement-2 : | A | = 1.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is *not* a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is false.

(4) Statement-1 is false, Statement-2 is true.