MATHS

Q. 1. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80 . Then which one of the following gives possible values a and b?

i. a = 1, b = 6
ii. a = 3, b = 4
iii. a = 0, b = 7
iv. a = 5, b = 2

Sol.

$$Mean = \frac{\sum x}{n} = 6$$

$$Variance = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = 6.8$$

$$- \frac{a^2 + b^2 + 64 + 25 + 100}{5} - 36 - 6.8$$

$$\Rightarrow a^2 + b^2 + 189 - 180 = 34$$

$$\Rightarrow a^2 + b^2 = 25$$

Possible values of a and b is given by (2)

Q. 2. The vector $\vec{a} = a\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $+\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β?

i. $\alpha = 2, \beta = 1$
ii. $\alpha = 1, \beta = 1$
iii. $\alpha = 2, \beta = 2$
iv. $\alpha = 1, \beta = 2$

Sol.

As $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar

$\therefore [\vec{a}\ \vec{b}\ \vec{c}] = 0$

Or, $\alpha + \beta = 2$ (i)

Also $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$

$\therefore \vec{a} = \lambda(\vec{b} + \vec{c})$

$or, \vec{a} = \lambda\left(\frac{\hat{i} + 2\vec{j} + \vec{k}}{\sqrt{2}}\right)$ (ii)

But $\vec{a} = \alpha\vec{2} + 2\vec{j} + \beta\vec{k}$

Hence $\lambda = \sqrt{2}$ and $\alpha = 1, \beta = 1$

Which also satisfy (i)

$\therefore$ Correct answer is (2)

Q. 3.

The non – zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$

Then the angle between $\vec{a}$ and $\vec{c}$ is

i. $\frac{\pi}{2}$
ii. π
iii. 0
iv. $\frac{\pi}{4}$

Sol. The sign of $\vec{a}$ and $\vec{c}$ are opposite. Hence they are parallel but directions are opposite.

Therefore angle between $\vec{a}$ and $\vec{c}$ is π

$\therefore$ correct answer is (2)

Q. 4. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then

i. a = 6, b = 4
ii. a = 8, b = 2
iii. a = 2, b = 8
iv. a = 4, b = 6

Sol. Equation of line through (5, 1, a) and (3, b, 1) is

$$\frac{x-5}{-2}=\frac{y-1}{b-1}=\frac{z-a}{1-a}=\lambda$$

any point on (i) is

$$\{5-2\lambda, 1+(b-1)\lambda, a+(1-a)\lambda\} \qquad (ii)$$

$$As\left(0, \frac{17}{2}, -\frac{13}{2}\right) lies\ on\ \ (i)$$

$$5-2\lambda=0 \Rightarrow \alpha=\frac{5}{2} \qquad (iii)$$

$$1+(b-1)\times\frac{5}{2}=\frac{17}{2}$$

$$or, 2+5b-5=17$$

$$or, b=4$$

$$and\ a+(1-a)\times\frac{5}{2}=-\frac{13}{2}$$

$$or, 2a+5-5a=-13$$

$$or, a=6$$

$$\therefore Correct\ answer\ is\ (1)$$

Q. 5. If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then the integer k is equal to

i. 2
ii. 2
iii. 5
iv. 5

Sol. As the given lines intersect

$$\therefore \begin{vmatrix} 2-1 & 3-2 & 1-3 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix}=0$$

$$Or, \begin{vmatrix} 1 & 1 & -2 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix}=0$$

$$or, k=-5, 5/2$$

$$Integer\ is -5\ only$$

$$\therefore Correct\ answer\ is\ (3)$$

Q. 6. The differential of the family of circles with fixed radius 5 units and centre on the line y = 2 is

i. $(y-2)^2\,y'^2 = 25-(y-2)^2$
ii. $(x\quad 2)^2\,y'^2 = 25\quad (y\quad 2)^2$
iii. $(x-2)\,y'^2 = 25-(y-2)^2$
iv. $(y-2)\,y'^2 - 25-(y-2)^2$

Sol. The required equation of circle is

$$(x-a)^2+(y-2)^2=25 \qquad (i)$$

differentiating we get

$$2(x-a)+2(y-2)y'=0$$

$$or,\ a=x+(y-2)y' \qquad (ii)$$

putting a in (i)

$$\{x-x-(y-2)y'\}^2+(y-2)^2=25$$

$$or,\ (y-2)^2\,y'^2=25-(y-2)^2$$

$\therefore$ *The correct answer is* (1)

Q. 7. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x=cy+bz,\ y=az+cx$ *and* $z=bx+ay$. *Then* $a^2+b^2+c^2+2abc$ *is equal to*

i. 0
ii. 1
iii. 2
iv. - 1

Sol.

$$x=cy+bz \Rightarrow x-cy-bz=0 \qquad (i)$$

$$y=az+bx \Rightarrow bx-y+az=0 \qquad (ii)$$

$$z=bx+ay \Rightarrow bx+ay-z=0 \qquad (iii)$$

Eliminating x, y, z from (i), (ii) and (iii) weget

$$\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix}=0$$

$$or,\ a^2+b^2+c^2+2abc=1.$$

$\therefore$ *The correct answer is* (2)

Q. 8. Let A be a square matrix all of whose entries are integers. Then which one of the following is true?

i. If $\det A = \pm 1$, then A^{-1} exists and all its entries are integers

ii. If $\det A = \pm 1$, then A^{-1} need not exist

iii. If $\det A = \pm 1$, then A^{-1} exist but all its entries are not necessarily integers

iv. If $\det A = \pm 1$, then A^{-1} exist and all its entries are non-integers

Sol. The obvious answer is (1).

Q. 9. The quadratic equations $x^2 - 6x\,a = 0$ and $x^2 - cx + 6 = 0$ and have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is

i. 3
ii. 2
iii. 1
iv. 4

Sol.

Let the roots of $x^2 - 6x + a = 0$ be α and 4β and that of $x^2 - cx + 6 = 0$ be α and 3β

$\therefore \alpha + 4\beta = 6 \quad (i)$

$4\alpha\beta = a \quad (ii)$

$\alpha + 3\beta = c \quad (iii)$

$3\alpha\beta = 6 \quad (iv)$

Using (ii) & (iv)

$$\frac{4}{3} = \frac{a}{6} \Rightarrow a = 8$$

Then $x^2 - 6x + a = 0$ reduces to

$$x^2 - 6x + 8 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 32}}{2}$$

$$= \frac{6 \pm 2}{2} = 4, 2$$

$\therefore \alpha = 2, \beta = 1$

$\therefore$ Correct answer is (2)

Q. 10. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?

i. $6.8.{}^{7}C_{4}$
ii. $7.{}^{6}C_{4}.{}^{8}C_{4}$
iii. $8.{}^{6}C_{4}.{}^{7}C_{4}$
iv. $6.7.{}^{8}C_{4}$

Sol. M = 1, I = 4, P = 2

These letters can be arranged by

$$\frac{(1+4+2)!}{1!\,4!\,2!} = 7\ {}^{6}C_{4}\ ways$$

The remaining 8 gaps can be filled by 4 S by ${}^{8}C_{4}$ *ways*

$\therefore$ *Total no. of ways* $= 7\ {}^{6}C_{4} \quad {}^{8}C_{4}$

$\therefore$ *Correct answer is* (2)

Q. 11.

Let $I = \int_{0}^{1} \frac{\cos x}{\sqrt{x}}\,dx$. *Then which one of the following is true?*

i. $I < \frac{2}{3}$ *and* $J > 2$
ii. $I < \frac{2}{3}$ *and* $J < 2$
iii. $I > \frac{2}{3}$ *and* $J > 2$
iv. $I < \frac{2}{3}$ *and* $J > 2$

Sol.

$We\ Know \frac{\sin x}{x} < 1,\ when\ x \in (0,1)$

$\therefore \frac{\sin x}{\sqrt{x}} < \sqrt{x}$

$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}}\,dx < \int_0^1 \sqrt{x}\,dx$

$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}}\,dx < \frac{2}{3}$

$Also,\ \cos x < 1,\ when\ x \in (0,1)$

$\therefore \frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}}$

$\Rightarrow \int_0^1 \frac{\cos x}{\sqrt{x}}\,dx < \int \frac{1}{\sqrt{x}}\,dx$

$\int_0^1 \frac{\cos x}{\sqrt{x}}\,dx < 2$

$\therefore I < \frac{2}{3}\ and\ J < 2$

$\therefore Correct\ answer\ is\ (4)$

Q. 12. The area of the plane region bounded by the curve $x + 2y^2 = 0\ and\ 3y^2 = 1\ is\ equal\ to$

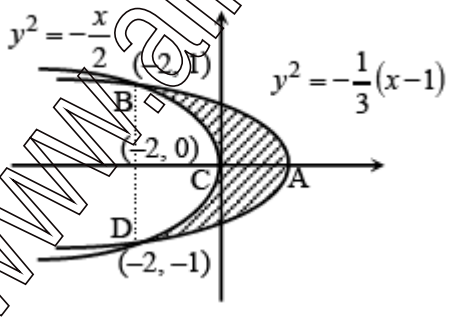
i. $\frac{2}{3}$

ii. $\frac{4}{3}$

iii. $\frac{5}{3}$

iv. $\frac{1}{3}$

Sol.



$x+2y^2=0 \Rightarrow y^2=-\frac{x}{2}$

$x+3y^2=1 \Rightarrow y^2=-\frac{1}{3}(x-1)$

$\therefore -\frac{x}{2}=-\frac{1}{3}(x-1)$

or, $-\frac{x}{2}=-\frac{x}{3}+\frac{1}{3}$

or, $\frac{x}{3}-\frac{x}{2}=\frac{1}{3}$

or, $-\frac{x}{6}=\frac{1}{3}$

or, $x=-2$

$\therefore y^2=1 \Rightarrow y=\pm 1$

Area of the region BCA

$=\left|\int_0^1 \left\{\left(-2y^2\right)-\left(1-3y^2\right)\right\}dy\right|$

$=\left|\int_0^1 \left(y^2-1\right)dy\right|$

$=\left|\left[\frac{y^3}{3}-y\right]_0^1\right|$

$=\left|\frac{1}{3}-1\right|=\frac{2}{3}$

Hence area of the region bounded by the curve is equal to $2\times\frac{2}{3}=\frac{4}{3}$

∴ Correct answer is (2)