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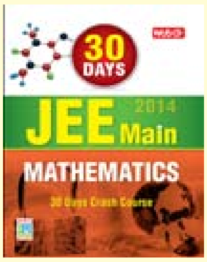

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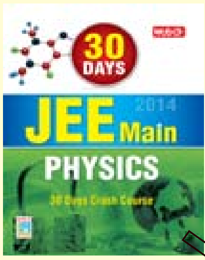

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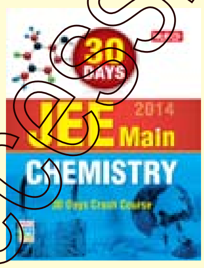

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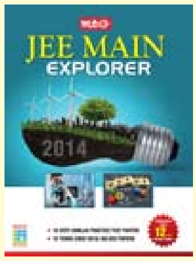

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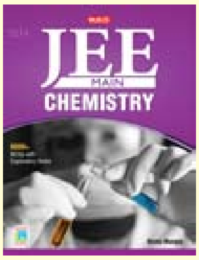

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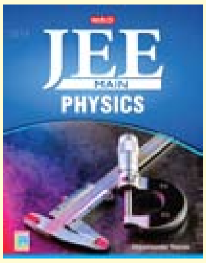

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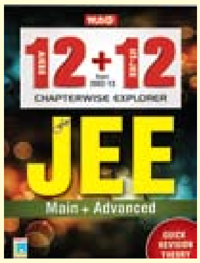

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JEE Advanced

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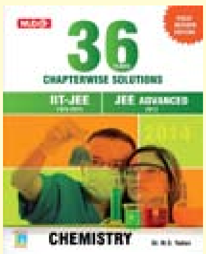

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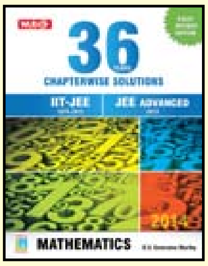

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SYLLABUS★

PHYSICS

SECTION A

Unit - 1: Physics and Measurement

Physics, technology and society, S. I. units, fundamental and derived units, least count, accuracy and precision of measuring instruments, errors in measurement.

Dimensions of physical quantities, dimensional analysis and its applications.

Unit - 2: Kinematics

Frame of reference, motion in a straight line: position-time graph, speed and velocity, uniform and non-uniform motion, average speed and instantaneous velocity.

Uniformly accelerated motion, velocity-time, position-time graphs, relations for uniformly accelerated motion.

Scalars and vectors, vector addition and subtraction, zero vector, scalar and vector products, unit vector, resolution of a vector, relative velocity, motion in a plane, projectile motion, uniform circular motion.

Unit - 3: Laws of Motion

Force and inertia, Newton's first law of motion, momentum, Newton's second law of motion, impulse, Newton's third law of motion, law of conservation of linear momentum and its applications, equilibrium of concurrent forces.

Static and kinetic friction, laws of friction, rolling friction.

Dynamics of uniform circular motion: centripetal force and its applications.

Unit - 4: Work, Energy and Power

Work done by a constant force and a variable force, kinetic and potential energies, work-energy theorem, power.

Potential energy of a spring, conservation of mechanical energy, conservative and non-conservative forces, elastic and inelastic collisions in one and two dimensions.

Unit - 5: Rotational Motion

Centre of mass of a two-particle system, centre of mass of a rigid body, basic concepts of rotational motion, moment of a force, torque, angular momentum, conservation of angular momentum and its applications, moment of inertia, radius of gyration, values of moments of inertia for simple geometrical objects, parallel and perpendicular axes theorems and their applications.

Rigid body rotation, equations of rotational motion.

Unit - 6: Gravitation

The universal law of gravitation.

Acceleration due to gravity and its variation with altitude and depth.

Kepler's laws of planetary motion.

Gravitational potential energy, gravitational potential.

Escape velocity, orbital velocity of a satellite, geostationary satellites.

Unit - 7: Properties of Solids and Liquids

Elastic behaviour, stress-strain relationship, Hooke's law, Young's modulus, bulk modulus, modulus of rigidity.

Pressure due to a fluid column, Pascal's law and its applications.

Viscosity, Stokes's law, terminal velocity, streamline and turbulent flow, Reynolds number, Bernoulli's principle and its applications.

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Surface energy and surface tension, angle of contact, application of surface tension - drops, bubbles and capillary rise.

Heat, temperature, thermal expansion, specific heat capacity, calorimetry, change of state, latent heat.

Heat transfer-conduction, convection and radiation, Newton's law of cooling.

Unit - 8: Thermodynamics

Thermal equilibrium, zeroth law of thermodynamics, concept of temperature, heat, work and internal energy, first law of thermodynamics.

Second law of thermodynamics, reversible and irreversible processes, Carnot engine and its efficiency.

Unit - 9: Kinetic Theory of Gases

Equation of state of a perfect gas, work done on compressing a gas.

Kinetic theory of gases - assumptions, concept of pressure, kinetic energy and temperature, rms speed of gas molecules, degrees of freedom, law of equipartition of energy, applications to specific heat capacities of gases, mean free path, Avogadro's number.

Unit - 10: Oscillations and Waves

Periodic motion - period, frequency, displacement as a function of time, periodic functions, simple harmonic motion (S.H.M.) and its equation, phase, oscillations of a spring - restoring force and force constant, energy in S.H.M. - kinetic and potential energies, simple pendulum - derivation of expression for its time period, free, forced and damped oscillations, resonance.

Wave motion, longitudinal and transverse waves, speed of a wave, displacement relation for a progressive wave, principle of superposition of waves, reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, beats, Doppler effect in sound.

Unit - 11: Electrostatics

Electric charges, conservation of charge, Coulomb's law-forces between two point charges, forces between multiple charges, superposition principle and continuous charge distribution.

Electric field, electric field due to a point charge, electric field lines, electric dipole, electric field due to a dipole, torque on a dipole in a uniform electric field.

Electric flux, Gauss's law and its applications to find field due to infinitely long, uniformly charged straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell.

Electric potential and its calculation for a point charge, electric dipole and system of charges, equipotential surfaces, electrical potential energy of a system of two point charges in an electrostatic field.

Conductors and insulators, dielectrics and electric polarization, capacitor, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, energy stored in a capacitor.

Unit - 12: Current Electricity

Electric current, drift velocity, Ohm's law, electrical resistance, resistances of different materials, *V-I* characteristics of ohmic and non-ohmic conductors, electrical energy and power, electrical resistivity, colour code for resistors, series and parallel combinations of resistors, temperature dependence of resistance.

Electric cell and its internal resistance, potential difference and emf of a cell, combination of cells in series and in parallel.

Kirchhoff's laws and their applications, Wheatstone bridge, metre bridge.

Potentiometer - principle and its applications.

Unit - 13: Magnetic Effects of Current and Magnetism

Biot - Savart law and its application to current carrying circular loop.

Ampere's law and its applications to infinitely long current carrying straight wire and solenoid.

Force on a moving charge in uniform magnetic and electric fields, cyclotron.

Force on a current-carrying conductor in a uniform magnetic field, force between two parallel current-carrying conductors-definition of ampere, torque experienced by a current loop in uniform magnetic field, moving coil galvanometer, its current sensitivity and conversion to ammeter and voltmeter.

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Current loop as a magnetic dipole and its magnetic dipole moment, bar magnet as an equivalent solenoid, magnetic field lines, earth's magnetic field and magnetic elements, para-, dia- and ferro- magnetic substances .
Magnetic susceptibility and permeability, hysteresis, electromagnets and permanent magnets.

Unit - 14: Electromagnetic Induction and Alternating Currents
Electromagnetic induction, Faraday's law, induced emf and current, Lenz's law, Eddy currents, self and mutual inductance. Alternating currents, peak and rms value of alternating current/ voltage, reactance and impedance, *LCR* series circuit, resonance, quality factor, power in AC circuits, wattless current.
AC generator and transformer.

Unit - 15: Electromagnetic Waves
Electromagnetic waves and their characteristics, transverse nature of electromagnetic waves, electromagnetic spectrum (radio waves, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays). Applications of electromagnetic waves.

Unit - 16: Optics
Reflection and refraction of light at plane and spherical surfaces, mirror formula, total internal reflection and its applications, deviation and dispersion of light by a prism, lens formula, magnification, power of a lens, combination of thin lenses in contact, microscope and astronomical telescope (reflecting and refracting) and their magnifying powers.

Wave optics - wavefront and Huygens principle, laws of reflection and refraction using Huygens principle, interference, Young's double slit experiment and expression for fringe width, coherent sources and sustained interference of light, diffraction due to a single slit, width of central maximum, resolving power of microscopes and astronomical telescopes, polarisation, plane polarized light, Brewster's law, uses of plane polarized light and polaroids.

Unit - 17: Dual Nature of Matter and Radiation
Dual nature of radiation, photoelectric effect, Hertz and Lenard's observations, Einstein's photoelectric equation, particle nature of light.

Matter waves-wave nature of particle, de Broglie relation, Davisson-Germer experiment.

Unit - 18: Atoms and Nuclei
Alpha-particle scattering experiment, Rutherford's model of atom, Bohr model, energy levels, hydrogen spectrum.

Composition and size of nucleus, atomic masses, isotopes, isobars, isotones, radioactivity-alpha, beta and gamma particles/rays and their properties, radioactive decay law, mass-energy relation, mass defect, binding energy per nucleon and its variation with mass number, nuclear fission and fusion.

Unit - 19: Electronic Devices
Semiconductors, semiconductor diode - *I-V* characteristics in forward and reverse bias, diode as a rectifier, *I-V* characteristics of LED, photodiode, solar cell. Zener diode, Zener diode as a voltage regulator, junction transistor, transistor action, characteristics of a transistor, transistor as an amplifier (common emitter configuration) and oscillator, logic gates (OR, AND, NOT, NAND and NOR), transistor as a switch.

Unit - 20: Communication Systems
Propagation of electromagnetic waves in the atmosphere, sky and space wave propagation, need for modulation, amplitude and frequency modulation, bandwidth of signals, bandwidth of transmission medium, basic elements of a communication system (Block Diagram only).

SECTION B

Unit - 21 : Experimental Skills
Familiarity with the basic approach and observations of the experiments and activities:

- Vernier callipers-its use to measure internal and external diameter and depth of a vessel.
- Screw gauge-its use to determine thickness/diameter of thin sheet/wire.
- Simple Pendulum-dissipation of energy by plotting a graph between square of amplitude and time.

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- Metre Scale - mass of a given object by principle of moments.
- Young's modulus of elasticity of the material of a metallic wire.
- Surface tension of water by capillary rise and effect of detergents.
- Co-efficient of Viscosity of a given viscous liquid by measuring terminal velocity of a given spherical body.
- Plotting a cooling curve for the relationship between the temperature of a hot body and time.
- Speed of sound in air at room temperature using a resonance tube.
- Specific heat capacity of a given (i) solid and (ii) liquid by method of mixtures.
- Resistivity of the material of a given wire using metre bridge.
- Resistance of a given wire using Ohm's law.
- Potentiometer :
  (i) Comparison of emf of two primary cells.
  (ii) Determination of internal resistance of a cell.
- Resistance and figure of merit of a galvanometer by half deflection method.
- Focal length of:
  (i) Convex mirror (ii) Concave mirror and (iii) Convex lens using parallax method.
- Plot of angle of deviation vs angle of incidence for a triangular prism.
- Refractive index of a glass slab using a travelling microscope.
- Characteristic curves of a *p-n* junction diode in forward and reverse bias.
- Characteristic curves of a Zener diode and finding reverse break down voltage.
- Characteristic curves of a transistor and finding current gain and voltage gain.
- Identification of Diode, LED, Transistor, IC, Resistor, Capacitor from mixed collection of such items.
- Using multimeter to:
  (i) Identify base of a transistor
  (ii) Distinguish between *npn* and *pnp* type transistor
  (iii) See the unidirectional flow of current in case of a diode and an LED.
  (iv) Check the correctness or otherwise of a given electronic component (diode, transistor or IC).

CHEMISTRY

SECTION - A (Physical Chemistry)

UNIT - 1: SOME BASIC CONCEPTS IN CHEMISTRY

Matter and its nature, Dalton's atomic theory, concept of atom, molecule, element and compound, physical quantities and their measurements in chemistry, precision and accuracy, significant figures, S.I. units, dimensional analysis, Laws of chemical combination, atomic and molecular masses, mole concept, molar mass, percentage composition, empirical and molecular formulae, chemical equations and stoichiometry.

UNIT - 2: STATES OF MATTER

Classification of matter into solid, liquid and gaseous states.

Gaseous State - Measurable properties of gases, Gas laws - Boyle's law, Charle's law, Graham's law of diffusion, Avogadro's law, Dalton's law of partial pressure, concept of absolute scale of temperature, Ideal gas equation, kinetic theory of gases (only postulates), concept of average, root mean square and most probable velocities, real gases, deviation from Ideal behaviour, compressibility factor, van der Waals equation.

Liquid State - Properties of liquids - vapour pressure, viscosity and surface tension and effect of temperature on them (qualitative treatment only).

Solid State - Classification of solids - molecular, ionic, covalent and metallic solids, amorphous and crystalline solids (elementary idea), Bragg's Law and its applications, unit cell and lattices, packing in solids (fcc, bcc and hcp lattices), voids, calculations involving unit cell parameters, imperfection in solids, electrical, magnetic and dielectric properties.

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UNIT - 3: ATOMIC STRUCTURE

Thomson and Rutherford atomic models and their limitations, nature of electromagnetic radiation, photoelectric effect, spectrum of hydrogen atom, Bohr model of hydrogen atom - its postulates, derivation of the relations for energy of the electron and radii of the different orbits, limitations of Bohr's model, dual nature of matter, de-Broglie's relationship, Heisenberg uncertainty principle, elementary ideas of quantum mechanics, quantum mechanical model of atom, its important features, concept of atomic orbitals as one electron wave functions, variation of Ψ and Ψ^2 with *r* for 1*s* and 2*s* orbitals, various quantum numbers (principal, angular momentum and magnetic quantum numbers) and their significance, shapes of *s*, *p* and *d* - orbitals, electron spin and spin quantum number, rules for filling electrons in orbitals – Aufbau principle, Pauli's exclusion principle and Hund's rule, electronic configuration of elements, extra stability of half-filled and completely filled orbitals.

UNIT - 4: CHEMICAL BONDING AND MOLECULAR STRUCTURE

Kossel - Lewis approach to chemical bond formation, concept of ionic and covalent bonds.

Ionic Bonding - Formation of ionic bonds, factors affecting the formation of ionic bonds, calculation of lattice enthalpy.

Covalent Bonding - concept of electronegativity, Fajan's rule, dipole moment, Valence Shell Electron Pair Repulsion (VSEPR) theory and shapes of simple molecules.

Quantum mechanical approach to covalent bonding - valence bond theory - its important features, concept of hybridization involving *s*, *p* and *d* orbitals, Resonance.

Molecular Orbital Theory - its important features, LCAOs, types of molecular orbitals (bonding, antibonding), sigma and pi-bonds, molecular orbital electronic configurations of homonuclear diatomic molecules, concept of bond order, bond length and bond energy.

Elementary idea of metallic bonding, hydrogen bonding and its applications.

UNIT - 5: CHEMICAL THERMODYNAMICS

Fundamentals of thermodynamics: system and surroundings, extensive and intensive properties, state functions, types of processes.

First law of thermodynamics - Concept of work, heat internal energy and enthalpy, heat capacity, molar heat capacity, Hess's law of constant heat summation, enthalpies of bond dissociation, combustion, formation, atomization, sublimation, phase transition, hydration, ionization and solution.

Second law of thermodynamics - Spontaneity of processes, ΔS of the universe and ΔG of the system as criteria for spontaneity, $\Delta G°$ (standard Gibbs energy change) and equilibrium constant.

UNIT - 6: SOLUTIONS

Different methods for expressing concentration of solution - molality, molarity, mole fraction, percentage (by volume and mass both), vapour pressure of solutions and Raoult's law - Ideal and non-ideal solutions, vapour pressure - composition plots for ideal and non-ideal solutions, colligative properties of dilute solutions - relative lowering of vapour pressure, depression of freezing point, elevation of boiling point and osmotic pressure, determination of molecular mass using colligative properties, abnormal value of molar mass, van't Hoff factor and its significance.

UNIT - 7: EQUILIBRIUM

Meaning of equilibrium, concept of dynamic equilibrium.

Equilibria involving physical processes - Solid - liquid, liquid - gas and solid - gas equilibria, Henry's law, general characteristics of equilibrium involving physical processes.

Equilibria involving chemical processes - Law of chemical equilibrium, equilibrium constants (K_p and K_c) and their significance, significance of ΔG and $\Delta G°$ in chemical equilibria, factors affecting equilibrium concentration, pressure, temperature, effect of catalyst, Le - Chatelier's principle.

Ionic equilibrium - Weak and strong electrolytes, ionization of electrolytes, various concepts of acids and bases (Arrhenius, Bronsted - Lowry and Lewis) and their ionization, acid - base equilibria (including multistage ionization) and ionization

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constants, ionization of water, pH scale, common ion effect, hydrolysis of salts and pH of their solutions, solubility of sparingly soluble salts and solubility products, buffer solutions. .

UNIT - 8 : REDOX REACTIONS AND ELECTROCHEMISTRY

Electronic concepts of oxidation and reduction, redox reactions, oxidation number, rules for assigning oxidation number, balancing of redox reactions.

Electrolytic and metallic conduction, conductance in electrolytic solutions, molar conductivities and their variation with concentration: Kohlrausch's law and its applications.

Electrochemical cells - electrolytic and galvanic cells, different types of electrodes, electrode potentials including standard electrode potential, half - cell and cell reactions, emf of a galvanic cell and its measurement, Nernst equation and its applications, relationship between cell potential and Gibbs' energy change, dry cell and lead accumulator, fuel cells.

UNIT - 9 : CHEMICAL KINETICS

Rate of a chemical reaction, factors affecting the rate of reactions -concentration, temperature, pressure and catalyst, elementary and complex reactions, order and molecularity of reactions, rate law, rate constant and its units, differential and integral forms of zero and first order reactions, their characteristics and half - lives, effect of temperature on rate of reactions - Arrhenius theory, activation energy and its calculation, collision theory of bimolecular gaseous reactions (no derivation).

UNIT - 10 : SURFACE CHEMISTRY

Adsorption - Physisorption and chemisorption and their characteristics, factors affecting adsorption of gases on solids, Freundlich and Langmuir adsorption isotherms, adsorption from solutions.

Catalysis - Homogeneous and heterogeneous, activity and selectivity of solid catalysts, enzyme catalysis and its mechanism.

Colloidal state - distinction among true solutions, colloids and suspensions, classification of colloids - lyophilic, lyophobic, multi molecular, macromolecular and associated colloids (micelles), preparation and properties of colloids - Tyndall effect, Brownian movement, electrophoresis, dialysis, coagulation and flocculation, emulsions and their characteristics.

Section - B (Inorganic Chemistry)

UNIT - 11: CLASSIFICATION OF ELEMENTS AND PERIODICITY IN PROPERTIES

Modern periodic law and present form of the periodic table, *s, p, d* and *f* block elements, periodic trends in properties of elements-atomic and ionic radii, ionization enthalpy, electron gain enthalpy, valence, oxidation states and chemical reactivity.

UNIT - 12: GENERAL PRINCIPLES AND PROCESSES OF ISOLATION OF METALS

Modes of occurrence of elements in nature, minerals, ores, steps involved in the extraction of metals - concentration, reduction (chemical and electrolytic methods) and refining with special reference to the extraction of Al, Cu, Zn and Fe, thermodynamic and electrochemical principles involved in the extraction of metals.

UNIT - 13: HYDROGEN

Position of hydrogen in periodic table, isotopes, preparation, properties and uses of hydrogen, physical and chemical properties of water and heavy water, structure, preparation, reactions and uses of hydrogen peroxide, classification of hydrides - ionic, covalent and interstitial, hydrogen as a fuel.

UNIT - 14: *s* - BLOCK ELEMENTS (ALKALI AND ALKALINE EARTH METALS)

Group - 1 and 2 Elements

General introduction, electronic configuration and general trends in physical and chemical properties of elements, anomalous properties of the first element of each group, diagonal relationships.

Preparation and properties of some important compounds - sodium carbonate, sodium hydroxide and sodium hydrogen carbonate, Industrial uses of lime, limestone, Plaster of Paris and cement, Biological significance of Na, K, Mg and Ca.

UNIT - 15: *p* - BLOCK ELEMENTS

Group - 13 to Group 18 Elements

General Introduction - Electronic configuration and general trends in physical and chemical properties of elements across the periods and down the groups, unique behaviour of the first element in each group.

Group - 13

Preparation, properties and uses of boron and aluminium, structure, properties and uses of borax, boric acid, diborane, boron trifluoride, aluminium chloride and alums.

Group - 14

Tendency for catenation, structure, properties and uses of allotropes and oxides of carbon, silicon tetrachloride, silicates, zeolites and silicones.

Group - 15

Properties and uses of nitrogen and phosphorus, allotropic forms of phosphorus, preparation, properties, structure and uses of ammonia, nitric acid, phosphine and phosphorus halides, (PCl_3, PCl_5), structures of oxides and oxoacids of nitrogen and phosphorus.

Group - 16

Preparation, properties, structures and uses of ozone, allotropic forms of sulphur, preparation, properties, structures and uses of sulphuric acid (including its industrial preparation), Structures of oxoacids of sulphur.

Group - 17

Preparation, properties and uses of hydrochloric acid, trends in the acidic nature of hydrogen halides, structures of interhalogen compounds and oxides and oxoacids of halogens.

Group - 18

Occurrence and uses of noble gases, structures of fluorides and oxides of xenon.

UNIT - 16: *d* - and *f* - BLOCK ELEMENTS

Transition Elements

General introduction, electronic configuration, occurrence and characteristics, general trends in properties of the first row transition elements - physical properties, ionization enthalpy, oxidation states, atomic radii, colour, catalytic behaviour, magnetic properties, complex formation, interstitial compounds, alloy formation, preparation, properties and uses of $K_2Cr_2O_7$ and $KMnO_4$.

Inner Transition Elements

Lanthanoids - Electronic configuration, oxidation states and lanthanoid contraction.

Actinoids - Electronic configuration and oxidation states.

UNIT - 17: COORDINATION COMPOUNDS

Introduction to coordination compounds, Werner's theory, ligands, coordination number, denticity, chelation, IUPAC nomenclature of mononuclear coordination compounds, isomerism, bonding -valence bond approach and basic ideas of crystal field theory, colour and magnetic properties, importance of coordination compounds (in qualitative analysis, extraction of metals and in biological systems).

UNIT - 18: ENVIRONMENTAL CHEMISTRY

Environmental Pollution - Atmospheric, water and soil. **Atmospheric pollution** - tropospheric and stratospheric.

Tropospheric pollutants - Gaseous pollutants: oxides of carbon, nitrogen and sulphur, hydrocarbons, their sources, harmful effects and prevention, green house effect and global warming, acid rain.

Particulate pollutants - Smoke, dust, smog, fumes, mist, their sources, harmful effects and prevention.

Stratospheric pollution - Formation and breakdown of ozone, depletion of ozone layer - its mechanism and effects.

Water Pollution - Major pollutants such as, pathogens, organic wastes and chemical pollutants, their harmful effects and prevention.

Soil Pollution - Major pollutants like pesticides (insecticides, herbicides and fungicides), their harmful effects and prevention. Strategies to control environmental pollution.

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SECTION - C (Organic Chemistry)

UNIT - 19: PURIFICATION AND CHARACTERISATION OF ORGANIC COMPOUNDS

Purification - Crystallization, sublimation, distillation, differential extraction and chromatography - principles and their applications.

Qualitative analysis - Detection of nitrogen, sulphur, phosphorus and halogens.

Quantitative analysis (Basic principles only) - Estimation of carbon, hydrogen, nitrogen, halogens, sulphur, phosphorus. Calculations of empirical formulae and molecular formulae, numerical problems in organic quantitative analysis.

UNIT - 20: SOME BASIC PRINCIPLES OF ORGANIC CHEMISTRY

Tetravalency of carbon, shapes of simple molecules - hybridization (*s* and *p*), classification of organic compounds based on functional groups: and those containing halogens, oxygen, nitrogen and sulphur, homologous series, Isomerism - structural and stereoisomerism.

Nomenclature (trivial and IUPAC)

Covalent bond fission - Homolytic and heterolytic: free radicals, carbocations and carbanions, stability of carbocations and free radicals, electrophiles and nucleophiles.

Electronic displacement in a covalent bond - Inductive effect, electromeric effect, resonance and hyperconjugation.

Common types of organic reactions - Substitution, addition, elimination and rearrangement.

UNIT - 21: HYDROCARBONS

Classification, isomerism, IUPAC nomenclature, general methods of preparation, properties and reactions.

Alkanes - Conformations: Sawhorse and Newman projections (of ethane), mechanism of halogenation of alkanes.

Alkenes - Geometrical isomerism, mechanism of electrophilic addition: addition of hydrogen, halogens, water, hydrogen halides (Markownikoff's and peroxide effect), ozonolysis, oxidation, and polymerization.

Alkynes - Acidic character, addition of hydrogen, halogens, water and hydrogen halides, polymerization.
Aromatic hydrocarbons - Nomenclature, benzene - structure and aromaticity, mechanism of electrophilic substitution: halogenation, nitration, Friedel – Craft's alkylation and acylation, directive influence of functional group in mono-substituted benzene.

UNIT - 22: ORGANIC COMPOUNDS CONTAINING HALOGENS

General methods of preparation, properties and reactions, nature of C-*X* bond, mechanisms of substitution reactions. Uses/environmental effects of chloroform, iodoform, freons and DDT.

UNIT - 23: ORGANIC COMPOUNDS CONTAINING OXYGEN

General methods of preparation, properties, reactions and uses.

Alcohols - Identification of primary, secondary and tertiary alcohols, mechanism of dehydration.

Phenols - Acidic nature, electrophilic substitution reactions: halogenation, nitration and sulphonation, Reimer - Tiemann reaction.

Ethers - Structure.

Aldehydes and Ketones - Nature of carbonyl group, nucleophilic addition to >C=O group, relative reactivities of aldehydes and ketones, important reactions such as - nucleophilic addition reactions (addition of HCN, NH_3 and its derivatives), Grignard reagent, oxidation, reduction (Wolff Kishner and Clemmensen), acidity of α-hydrogen, aldol condensation, Cannizzaro reaction, haloform reaction, chemical tests to distinguish between aldehydes and ketones.

Carboxylic acids - Acidic strength and factors affecting it.

UNIT - 24: ORGANIC COMPOUNDS CONTAINING NITROGEN

General methods of preparation, properties, reactions and uses.

Amines - Nomenclature, classification, structure basic character and identification of primary, secondary and tertiary amines and their basic character.

Diazonium Salts - Importance in synthetic organic chemistry.

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UNIT - 25: POLYMERS

General introduction and classification of polymers, general methods of polymerization - addition and condensation, copolymerization, natural and synthetic rubber and vulcanization, some important polymers with emphasis on their monomers and uses - polythene, nylon, polyester and bakelite.

UNIT - 26: BIOMOLECULES

General introduction and importance of biomolecules.

Carbohydrates - Classification: aldoses and ketoses, monosaccharides (glucose and fructose), constituent monosaccharides of oligosaccharides (sucrose, lactose, maltose).

Proteins - Elementary Idea of α - amino acids, peptide bond, polypeptides, proteins - primary, secondary, tertiary and quaternary structure (qualitative idea only), denaturation of proteins, enzymes.

Vitamins - Classification and functions.

Nucleic acids - Chemical constitution of DNA and RNA, biological functions of nucleic acids.

UNIT - 27: CHEMISTRY IN EVERYDAY LIFE

Chemicals in medicines - Analgesics, tranquilizers, antiseptics, disinfectants, antimicrobials, antifertility drugs, antibiotics, antacids, antihistamins - their meaning and common examples.

Chemicals in food - Preservatives, artificial sweetening agents - common examples.

Cleansing agents - Soaps and detergents, cleansing action.

UNIT - 28: PRINCIPLES RELATED TO PRACTICAL CHEMISTRY

Detection of extra elements (N, S, halogens) in organic compounds, detection of the following functional groups: hydroxyl (alcoholic and phenolic), carbonyl (aldehyde and ketone), carboxyl and amino groups in organic compounds.

Chemistry involved in the preparation of the following:

Inorganic compounds - Mohr's salt, potash alum.

Organic compounds - Acetanilide, p-nitroacetanilide, aniline yellow, iodoform.

Chemistry involved in the titrimetric exercises - Acids, bases and the use of indicators, oxalic acid vs $KMnO_4$, Mohr's salt *vs* $KMnO_4$.

Chemical principles involved in the qualitative salt analysis:

Cations - Pb^{2+}, Cu^{2+}, Al^{3+}, Fe^{3+}, Zn^{2+}, Ni^{2+}, Ca^{2+}, Ba^{2+}, Mg^{2+}, NH_4^+.

Anions – CO_3^{2-}, S^{2-}, SO_4^{2-}, NO_2^-, NO_3^-, Cl^-, Br^-, I^- (insoluble salts excluded).

Chemical principles involved in the following experiments:

1. Enthalpy of solution of $CuSO_4$
2. Enthalpy of neutralization of strong acid and strong base.
3. Preparation of lyophilic and lyophobic sols.
4. Kinetic study of reaction of iodide ion with hydrogen peroxide at room temperature.

MATHEMATICS

UNIT - 1 : SETS, RELATIONS AND FUNCTIONS

Sets and their representation, union, intersection and complement of sets and their algebraic properties, power set, relations, types of relations, equivalence relations, functions, one-one, into and onto functions, composition of functions.

UNIT - 2 : COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Complex numbers as ordered pairs of reals, representation of complex numbers in the form a + ib and their representation in a plane, Argand diagram, algebra of complex numbers, modulus and argument (or amplitude) of a complex number, square root of a complex number, triangle inequality, quadratic equations in real and complex number system and their solutions, relation between roots and coefficients, nature of roots, formation of quadratic equations with given roots.

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UNIT - 3 : MATRICES AND DETERMINANTS

Matrices, algebra of matrices, types of matrices, determinants and matrices of order two and three. Properties of determinants, evaluation of determinants, area of triangles using determinants. Adjoint and evaluation of inverse of a square matrix using determinants and elementary transformations, Test of consistency and solution of simultaneous linear equations in two or three variables using determinants and matrices.

UNIT - 4 : PERMUTATIONS AND COMBINATIONS

Fundamental principle of counting, permutation as an arrangement and combination as selection, Meaning of *P* (*n,r*) and *C* (*n,r*), simple applications.

UNIT - 5 : MATHEMATICAL INDUCTION

Principle of Mathematical Induction and its simple applications.

UNIT - 6 : BINOMIAL THEOREM AND ITS SIMPLE APPLICATIONS

Binomial theorem for a positive integral index, general term and middle term, properties of Binomial coefficients and simple applications.

UNIT - 7 : SEQUENCES AND SERIES

Arithmetic and Geometric progressions, insertion of arithmetic, geometric means between two given numbers. Relation between A.M. and G.M. Sum upto *n* terms of special series: $\sum n$, $\sum n^2$, $\sum n^3$. Arithmetic - Geometric progression.

UNIT - 8 : LIMITS, CONTINUITY AND DIFFERENTIABILITY

Real - valued functions, algebra of functions, polynomials, rational, trigonometric, logarithmic and exponential functions, inverse functions. Graphs of simple functions. Limits, continuity and differentiability. Differentiation of the sum, difference, product and quotient of two functions. Differentiation of trigonometric, inverse trigonometric, logarithmic, exponential, composite and implicit functions, derivatives of order upto two. Rolle's and Lagrange's Mean value theorems. Applications of derivatives: Rate of change of quantities, monotonic - increasing and decreasing functions, Maxima and minima of functions of one variable, tangents and normals.

UNIT - 9 : INTEGRAL CALCULUS

Integral as an anti - derivative. Fundamental integrals involving algebraic, trigonometric, exponential and logarithmic functions. Integration by substitution, by parts and by partial fractions. Integration using trigonometric identities.

Evaluation of simple integrals of the type

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{a^2 - x^2}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}, \int \frac{(px+q)dx}{ax^2 + bx + c}, \int \frac{(px+q)dx}{\sqrt{ax^2 + dx + c}}$$

$$\int \sqrt{a^2 \pm x^2}\, dx \text{ and } \int \sqrt{x^2 - a^2}\, dx$$

Integral as limit of a sum. Fundamental theorem of calculus. Properties of definite integrals. Evaluation of definite integrals, determining areas of the regions bounded by simple curves in standard form.

UNIT - 10: DIFFERENTIAL EQUATIONS

Ordinary differential equations, their order and degree. Formation of differential equations. Solution of differential equations by the method of separation of variables, solution of homogeneous and linear differential equations of the type:

$$\frac{dy}{dx} + p(x)y = q(x)$$

UNIT - 11: COORDINATE GEOMETRY

Cartesian system of rectangular coordinates in a plane, distance formula, section formula, locus and its equation, translation of axes, slope of a line, parallel and perpendicular lines, intercepts of a line on the coordinate axes.

Straight lines - Various forms of equations of a line, intersection of lines, angles between two lines, conditions for concurrence of three lines, distance of a point from a line, equations of internal and external bisectors of angles between two lines, coordinates of centroid, orthocentre and circumcentre of a triangle, equation of family of lines passing through the point of intersection of two lines.

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Circles, conic sections - Standard form of equation of a circle, general form of the equation of a circle, its radius and centre, equation of a circle when the end points of a diameter are given, points of intersection of a line and a circle with the centre at the origin and condition for a line to be tangent to a circle, equation of the tangent. Sections of cones, equations of conic sections (parabola, ellipse and hyperbola) in standard forms, condition for $y = mx + c$ to be a tangent and point(s) of tangency.

UNIT - 12: THREE DIMENSIONAL GEOMETRY

Coordinates of a point in space, distance between two points, section formula, direction ratios and direction cosines, angle between two intersecting lines. Skew lines, the shortest distance between them and its equation. Equations of a line and a plane in different forms, intersection of a line and a plane, coplanar lines.

UNIT - 13: VECTOR ALGEBRA

Vectors and scalars, addition of vectors, components of a vector in two dimensions and three dimensional space, scalar and vector products, scalar and vector triple product.

UNIT - 14: STATISTICS AND PROBABILITY

Measures of Dispersion - Calculation of mean, median, mode of grouped and ungrouped data. Calculation of standard deviation, variance and mean deviation for grouped and ungrouped data.

Probability - Probability of an event, addition and multiplication theorems of probability, Baye's theorem, probability distribution of a random variate, Bernoulli trials and Binomial distribution.

UNIT - 15: TRIGONOMETRY

Trigonometrical identities and equations. Trigonometrical functions. Inverse trigonometrical functions and their properties. Heights and Distances.

UNIT - 16: MATHEMATICAL REASONING

Statements, logical operations and, or, implies, implied by, if and only if. Understanding of tautology, contradiction, converse and contrapositive.

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PHYSICS

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CHAPTER 1

PHYSICS AND MEASUREMENT

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1. Let $[\in_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then

(a) $[\in_0] = [M^{-1} L^2 T^{-1} A]$ (b) $[\in_0] = [M^{-1} L^{-3} T^2 A]$

(c) $[\in_0] = [M^{-1} L^{-3} T^4 A^2]$ (d) $[\in_0] = [M^{-1} L^2 T^{-1} A^{-2}]$

(2013)

2. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is

(a) zero (b) 1% (c) 3% (d) 6%

(2012)

3. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are

(a) 4, 4, 2 (b) 5, 1, 2

(c) 5, 1, 5 (d) 5, 5, 2. *(2010)*

4. The dimension of magnetic field in M, L, T and C (coulomb) is given as

(a) $MT^{-2}C^{-1}$ (b) $MLT^{-1}C^{-1}$

(c) MT^2C^{-2} (d) $MT^{-1}C^{-1}$. *(2008)*

5. Which of the following units denotes the dimensions ML^2/Q^2, where Q denotes the electric charge?

(a) weber (Wb) (b) Wb/m^2

(c) henry (H) (d) H/m^2. *(2006)*

6. Out of the following pairs, which one does not have identical dimensions?

(a) moment of inertia and moment of a force

(b) work and torque

(c) angular momentum and Planck's constant

(d) impulse and momentum *(2005)*

7. Which one of the following represents the correct dimensions of the coefficient of viscosity?

(a) $ML^{-1}T^{-2}$ (b) MLT^{-1}

(c) $ML^{-1}T^{-1}$ (d) $ML^{-2}T^{-2}$. *(2004)*

8. The physical quantities not having same dimensions are

(a) torque and work

(b) momentum and Planck's constant

(c) stress and Young's modulus

(d) speed and $(\mu_0\varepsilon_0)^{-1/2}$. *(2003)*

9. Dimensions of $\frac{1}{\mu_0\varepsilon_0}$, where symbols have their usual meaning, are

(a) $[L^{-1}T]$ (b) $[L^{-2}T^2]$ (c) $[L^2T^{-2}]$ (d) $[LT^{-1}]$.

(2003)

10. Identify the pair whose dimensions are equal.

(a) torque and work (b) stress and energy

(c) force and stress (d) force and work.

(2002)

Answer Key

1.	(c)	**2.**	(d)	**3.**	(b)	**4.**	(d)	**5.**	(c)	**6.**	(a)
7.	(c)	**8.**	(b)	**9.**	(c)	**10.**	(a)				

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Explanations

1. **(c) :** According to Coulomb's law

$$F = \frac{1}{4\pi \epsilon_0}\frac{q_1 q_2}{r^2} \quad \therefore \quad \epsilon_0 = \frac{1}{4\pi}\frac{q_1 q_2}{Fr^2}$$

$$[\epsilon_0] = \frac{[AT][AT]}{[MLT^{-2}][L]^2} = [M^{-1}L^{-3}T^4A^2]$$

2. **(d) :** $R = \frac{V}{I}$ $\therefore$ $\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$

The percentage error in R is

$$\frac{\Delta R}{R}\times 100 = \frac{\Delta V}{V}\times 100 + \frac{\Delta I}{I}\times 100 = 3\% + 3\% = 6\%$$

3. **(b) :** (i) All the non-zero digits are significant.

(ii) All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all.

(iii)If the number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant.

(iv)The power of 10 is irrelevant to the determination of significant figures.

According to the above rules,

23.023 has 5 significant figures.

0.0003 has 1 significant figures.

2.1×10^{-3} has 2 significant figures.

4. **(d) :** Lorentz force $= |\vec{F}| = |q\vec{v}\times\vec{B}|$

$$\therefore \quad [B] = \frac{[F]}{[q][v]} = \frac{MLT^{-2}}{C\times LT^{-1}} = \frac{MLT^{-2}}{CLT^{-1}} = [MT^{-1}C^{-1}]$$

5. **(c) :** $[ML^2Q^{-2}] = [ML^2 A^{-2} T^{-2}]$

$[Wb] = [ML^2 T^{-2} A^{-1}]$

$$\left[\frac{Wb}{m^2}\right] = [MT^{-2}A^{-1}]$$

$[henry] = [ML^2 T^{-2} A^{-2}]$

$$\left[\frac{H}{m^2}\right] = [MT^{-2}A^{-2}]$$

Obviously henry (H) has dimensions $\frac{ML^2}{Q^2}$.

6. **(a) :** Moment of inertia $(I) = mr^2$

$\therefore$ $[I] = [ML^2]$

Moment of force $(C) = rF$

$\therefore$ $[C] = [r][F] = [L][MLT^{-2}]$ or $[C] = [ML^2 T^{-2}]$

Moment of inertia and moment of a force do not have identical dimensions.

7. **(c) :** Viscous force $F = 6\pi\eta rv$

$$\therefore \quad \eta = \frac{F}{6\pi rv} \quad \text{or} \quad [\eta] = \frac{[F]}{[r][v]}$$

$$\text{or} \quad [\eta] = \frac{[MLT^{-2}]}{[L][LT^{-1}]} \quad \text{or} \quad [\eta] = [ML^{-1}T^{-1}].$$

8. **(b) :** [Momentum] = $[MLT^{-1}]$

[Planck's constant] = $[ML^2T^{-1}]$

Momentum and Planck's constant do not have same dimensions.

9. **(c) :** Velocity of light in vacuum $= \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$\text{or} \quad [LT^{-1}] = \left[\frac{1}{\sqrt{\mu_0\varepsilon_0}}\right]$$

$$\text{or} \quad [L^2T^{-2}] = \left[\frac{1}{\mu_0\varepsilon_0}\right]$$

$$\therefore \quad \text{Dimensions of } \frac{1}{\mu_0\varepsilon_0} = [L^2T^{-2}]$$

10. **(a) :** Torque and work have the same dimensions.

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CHAPTER 2

KINEMATICS

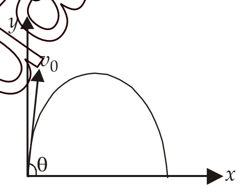
1. A projectile is given an initial velocity of $(\hat{i}+2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is
(a) $4y = 2x - 25x^2$ (b) $y = x - 5x^2$
(c) $y = 2x - 5x^2$ (d) $4y = 2x - 5x^2$ *(2013)*

2. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be
(a) 10 m (b) $10\sqrt{2}$ m (c) 20 m (d) $20\sqrt{2}$ m *(2012)*

3. An object moving with a speed of 6.25 m s^{-1}, is decelerated at a rate given by $\frac{dv}{dt} = -2.5\sqrt{v}$, where v is the instantaneous speed. The time taken by the object, to come to rest, would be
(a) 1 s (b) 2 s (c) 4 s (d) 8 s *(2011)*

4. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is
(a) $\pi\frac{v^2}{g}$ (b) $\pi\frac{v^4}{g^2}$ (c) $\frac{\pi}{2}\frac{v^4}{g^2}$ (d) $\pi\frac{v^2}{g^2}$ *(2011)*

5. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is

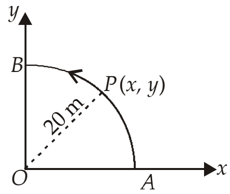


(a) $\frac{1}{2}mgv_0t^2\cos\theta\,\hat{i}$
(b) $-mgv_0t^2\cos\theta\,\hat{j}$
(c) $mgv_0t\cos\theta\,\hat{k}$ (d) $-\frac{1}{2}mg\,v_0t^2\cos\theta\,\hat{k}$
where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively. *(2010)*

6. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is
(a) $y^2 = x^2$ + constant (b) $y = x^2$ + constant
(c) $y^2 = x$ + constant (d) xy = constant *(2010)*

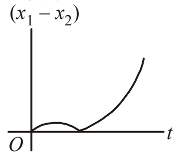
7. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point $P(R, \theta)$ on the circle of radius R is (Here θ is measured from the x-axis)
(a) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$ (b) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$
(c) $-\frac{v^2}{R}\sin\theta\,\hat{i} + \frac{v^2}{R}\cos\theta\,\hat{j}$ (d) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$ *(2010)*

8. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of P when t = 2 s is nearly
(a) 14 m s^{-2}
(b) 13 m s^{-2}
(c) 12 m s^{-2}
(d) 7.2 m s^{-2} *(2010)*



9. A particle has an initial velocity $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is
(a) 10 units (b) $7\sqrt{2}$ units
(c) 7 units (d) 8.5 units *(2009)*

10. A body is at rest at x = 0. At t = 0, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through x = 0 moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by x_2 (t) after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t?

(a)


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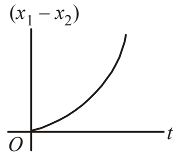

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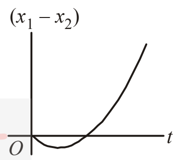
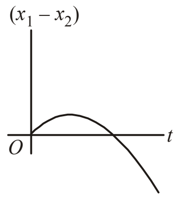


(d)


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11. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is
(a) $v_0 + g/2 + f$ (b) $v_0 + 2g + 3f$
(c) $v_0 + g/2 + f/3$ (d) $v_0 + g + f$ *(2007)*

12. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as
(a) t^3 (b) t^2 (c) t (d) $t^{1/2}$. *(2006)*

13. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 m/s^2. He reaches the ground with a speed of 3 m/s. At what height, did he bail out?
(a) 293 m (b) 111 m (c) 91 m (d) 182 m *(2005)*

14. A car, starting from rest, accelerates at the rate f through a distance s, then continues at constant speed for time t and then decelerates at the rate $f/2$ to come to rest. If the total distance traversed is 15 s, then
(a) $s = \frac{1}{2} f t^2$ (b) $s = \frac{1}{4} f t^2$
(c) $s = ft$ (d) $s = \frac{1}{6} f t^2$ *(2005)*

15. The relation between time t and distance x is $t = ax^2 + bx$ where a and b are constants. The acceleration is
(a) $-2av^3$ (b) $2av^2$ (c) $-2av^2$ (d) $2bv^3$ *(2005)*

16. A particle is moving eastwards with a velocity of 5 m/s. In 10 s the velocity changes to 5 m/s northwards. The average acceleration in this time is
(a) zero
(b) $\frac{1}{\sqrt{2}}$ ms^{-2} towards north-west
(c) $\frac{1}{\sqrt{2}}$ ms^{-2} towards north-east
(d) $\frac{1}{2}$ ms^{-2} towards north *(2005)*

17. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the time of flights in the two cases, then the product of the two time of flights is proportional to
(a) $1/R$ (b) R (c) R^2 (d) $1/R^2$. *(2005)*

18. An automobile travelling with a speed of 60 km/h, can brake to stop within a distance of 20 m. If the car is going twice as fast, *i.e.* 120 km/h, the stopping distance will be
(a) 20 m (b) 40 m (c) 60 m (d) 80 m. *(2004)*

19. A ball is released from the top of a tower of height h metre. It takes T second to reach the ground. What is the position of the ball in $T/3$ second?
(a) $h/9$ metre from the ground
(b) $7h/9$ metre from the ground
(c) $8h/9$ metre from the ground
(d) $17h/18$ metre from the ground. *(2004)*

20. If $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$, then the angle between A and B is
(a) π (b) $\pi/3$ (c) $\pi/2$ (d) $\pi/4$. *(2004)*

21. A projectile can have the same range R for two angles of projection. If T_1 and T_2 be the time of flights in the two cases, then the product of the two time of flights is directly proportional to
(a) $1/R^2$ (b) $1/R$ (c) R (d) R^2. *(2004)*

22. Which of the following statements is false for a particle moving in a circle with a constant angular speed?
(a) The velocity vector is tangent to the circle.
(b) The acceleration vector is tangent to the circle
(c) the acceleration vector points to the centre of the circle
(d) the velocity and acceleration vectors are perpendicular to each other. *(2004)*

23. A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection?
(a) yes, 60° (b) yes, 30°
(c) no (d) yes, 45°. *(2004)*

24. A car moving with a speed of 50 km/hr, can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is
(a) 12 m (b) 18 m (c) 24 m (d) 6 m. *(2003)*

25. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground ? [g = 10 m/s^2, sin30° = 1/2, cos30° = $\sqrt{3}/2$]
(a) 5.20 m (b) 4.33 m
(c) 2.60 m (d) 8.66 m. *(2003)*

26. The co-ordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by
(a) $3t\sqrt{\alpha^2 + \beta^2}$ (b) $3t^2\sqrt{\alpha^2 + \beta^2}$
(c) $t^2\sqrt{\alpha^2 + \beta^2}$ (d) $\sqrt{\alpha^2 + \beta^2}$. *(2003)*

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27. From a building two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then

(a) $v_B > v_A$

(b) $v_A = v_B$

(c) $v_A > v_B$

(d) their velocities depend on their masses. *(2002)*

28. Speeds of two identical cars are u and $4u$ at a specific instant. If the same deceleration is applied on both the cars, the ratio of the respective distances in which the two cars are stopped from that instant is

(a) 1 : 1 (b) 1 : 4 (c) 1 : 8 (d) 1 : 16. *(2002)*

29. If a body looses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest?

(a) 1 cm (b) 2 cm (c) 3 cm (d) 4 cm. *(2002)*

30. Two forces are such that the sum of their magnitudes is 18 N and their resultant is 12 N which is perpendicular to the smaller force. Then the magnitudes of the forces are

(a) 12 N, 6 N (b) 13 N, 5 N

(c) 10 N, 8 N (d) 16 N, 2 N. *(2002)*

Answer Key

1.	(c)	**2.**	(c)	**3.**	(b)	**4.**	(b)	**5.**	(d)	**6.**	(a)
7.	(d)	**8.**	(a)	**9.**	(b)	**10.**	(c)	**11.**	(c)	**12.**	(b)
13.	(a)	**14.**	(*)	**15.**	(a)	**16.**	(b)	**17.**	(b)	**18.**	(d)
19.	(c)	**20.**	(a)	**21.**	(c)	**22.**	(b)	**23.**	(a)	**24.**	(c)
25.	(d)	**26.**	(b)	**27.**	(b)	**28.**	(d)	**29.**	(a)	**30.**	(b)

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Explanations

1. **(c) :** Given : $u = \hat{i} + 2\hat{j}$

As $\vec{u} = u_x\hat{i} + u_y\hat{j}$

$\therefore$ $u_x = 1$ and $u_y = 2$

Also $x = u_x t$

and $y = u_y t - \frac{1}{2}gt^2$

$\therefore$ $x = t$

and $y = 2t - \frac{1}{2} \times 10 \times t^2 = 2t - 5t^2$

Equation of trajectory is

$$y = 2x - 5x^2$$

2. **(c) :** Let u be the velocity of projection of the stone.

The maximum height a boy can throw a stone is

$$H_{max} = \frac{u^2}{2g} = 10 \text{ m} \quad \text{...(i)}$$

The maximum horizontal distance the boy can throw the same stone is

$$R_{max} = \frac{u^2}{g} = 20 \text{ m} \quad \text{(Using (i))}$$

3. **(b) :** $\frac{dv}{dt} = -2.5\sqrt{v}$ or $\frac{1}{\sqrt{v}}dv = -2.5\, dt$

On integrating, within limit ($v_1 = 6.25$ m s^{-1} to $v_2 = 0$)

$$\therefore \int_{v_1 = 6.25 \text{ ms}^{-1}}^{v_2 = 0} v^{-1/2}dv = -2.5\int_0^t dt$$

$$2 \times [v^{1/2}]_{6.25}^{0} = -(2.5)t \Rightarrow t = \frac{-2 \times (6.25)^{1/2}}{-2.5} = 2 \text{ s}$$

4. **(b) :** $R_{max} = \frac{v^2 \sin 90°}{g} = \frac{v^2}{g}$

Area $= \pi(R_{max})^2 = \frac{\pi v^4}{g^2}$

5. **(d) :** The position vector of the particle from the origin at any time t is

$$\vec{r} = v_0 \cos\theta t\,\hat{i} + (v_0 \sin\theta t - \frac{1}{2}gt^2)\,\hat{j}$$

$\therefore$ Velocity vector, $\vec{v} = \frac{d\vec{r}}{dt}$

$$\vec{v} = \frac{d}{dt}(v_0\cos\theta t\,\hat{i} + (v_0 \sin\theta t - \frac{1}{2}gt^2)\,\hat{j})$$

$$= v_0\cos\theta\,\hat{i} + (v_0\sin\theta - gt)\,\hat{j}$$

The angular momentum of the particle about the origin is

$\vec{L} = \vec{r} \times m\vec{v}$

$\vec{L} = m(\vec{r} \times \vec{v})$

$$= m\Big[(v_0\cos\theta t\,\hat{i} + (v_0\sin\theta t - \frac{1}{2}gt^2)\,\hat{j}) \times (v_0\cos\theta\,\hat{i} + (v_0\sin\theta - gt)\,\hat{j})\Big]$$

$$= m\Big[(v_0^2\cos\theta\sin\theta t - v_0 g t^2 \cos\theta)\,\hat{k} + (v_0^2 \sin\theta\cos\theta t - \frac{1}{2}gt^2 v_0\cos\theta)(-\hat{k})\Big]$$

$$= m\Big[v_0^2\sin\theta\cos\theta t\,\hat{k} - v_0 g t^2\cos\theta\,\hat{k} - v_0^2\sin\theta\cos\theta t\,\hat{k} + \frac{1}{2}v_0 g t^2 \cos\theta\,\hat{k}\Big]$$

$$= m\Big[-\frac{1}{2}v_0 g t^2\cos\theta\,\hat{k}\Big] = -\frac{1}{2}mgv_0t^2\cos\theta\,\hat{k}$$

6. **(a) :** Here, $\vec{v} = K(y\,\hat{i} + x\,\hat{j})$

$$\vec{v} = Ky\,\hat{i} + Kx\,\hat{j} \quad \text{...(i)}$$

$$\because \vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} \quad \text{...(ii)}$$

Equating equations (i) and (ii), we get

$$\frac{dx}{dt} = Ky; \frac{dy}{dt} = Kx$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

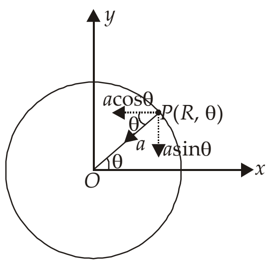
$$\frac{dy}{dx} = \frac{Kx}{Ky} = \frac{x}{y} \quad \text{...(iii)}$$

Integrating both sides of the above equation, we get

$\int y dy = \int x dx$

$y^2 = x^2$ + constant

7. **(d) :**



For a particle in uniform circular motion,

Acceleration, $a = \frac{v^2}{R}$ towards the centre

From figure,

$$\vec{a} = -a\cos\theta\,\hat{i} - a\sin\theta\,\hat{j} = -\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$$

8. **(a) :** $s = t^3 + 3$

$$\therefore v = \frac{ds}{dt} = \frac{d}{dt}(t^3 + 3) = 3t^2$$

Tangential acceleration, $a_t = \frac{dv}{dt} = \frac{d}{dt}(3t^2) = 6t$

At $t = 2$ s,

$v = 3(2)^2 = 12$ m/s, $a_t = 6(2) = 12$ m/s^2

Centripetal acceleration,

$$a_c = \frac{v^2}{R} = \frac{(12)^2}{20} = \frac{144}{20} = 7.2 \text{ m/s}^2$$

Net acceleration,

$$a = \sqrt{(a_c)^2 + (a_t)^2} = \sqrt{(7.2)^2 + (12)^2} \approx 14 \text{ m/s}^2$$

9. **(b) :** $v = u + at$

$$\vec{v} = (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j}) \times 10$$

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$\vec{v} = (3+4)\hat{i} + (4+3)\hat{j}$

$\Rightarrow \ |\vec{v}| = \sqrt{49+49} = \sqrt{98} = 7\sqrt{2}$ units

(This value is about 9.9 units close to 10 units. If (a) is given that is also not wrong).

10. **(c) :** As $u = 0$, $v_1 = at$, v_2 = constant for the other particle. Initially both are zero. Relative velocity of particle 1 w.r.t. 2 is velocity of 1 – velocity of 2. At first the velocity of first particle is less than that of 2. Then the distance travelled by particle 1 increases as

$x_1 = (1/2)\, a t_1^2$. For the second it is proportional to t. Therefore it is a parabola after crossing x-axis again. Curve (c) satisfies this.

11. **(c) :** Given : velocity $v = v_0 + gt + ft^2$

$\therefore \quad v = \dfrac{dx}{dt} \quad$ or $\quad \int_0^x dx = \int_0^t v\,dt$

or $\quad x = \int_0^t (v_0 + gt + ft^2)\,dt$

$x = v_0 t + \dfrac{gt^2}{2} + \dfrac{ft^3}{3} + C$

where C is the constant of integration

Given : $x = 0$, $t = 0$. $\therefore \ C = 0$

or $\quad x = v_0 t + \dfrac{gt^2}{2} + \dfrac{ft^3}{3}$

At $t = 1$ sec

$\therefore \quad x = v_0 + \dfrac{g}{2} + \dfrac{f}{3}.$

12. **(b) :** $v = \alpha\sqrt{x}$

or $\quad \dfrac{dx}{dt} = \alpha\sqrt{x} \quad$ or $\quad \dfrac{dx}{\sqrt{x}} = \alpha\, dt$

or $\quad \int \dfrac{dx}{\sqrt{x}} = \alpha \int dt \quad$ or $\quad 2x^{1/2} = \alpha t$

or $\quad x = \left(\dfrac{\alpha}{2}\right)^2 t^2$

or displacement is proportional to t^2.

13. **(a) :** Initially, the parachutist falls under gravity

$\therefore \ u^2 = 2ah = 2 \times 9.8 \times 50 = 980 \text{ m}^2\text{s}^{-2}$

He reaches the ground with speed

$= 3$ m/s, $a = -2 \text{ ms}^{-2}$

$\therefore \ (3)^2 = u^2 - 2 \times 2 \times h_1$

or $\ 9 = 980 - 4\, h_1$

or $\ h_1 = \dfrac{971}{4} \quad$ or $\quad h_1 = 242.75$ m

$\therefore$ Total height $= 50 + 242.75$

$= 292.75 = 293$ m.

14. **(*) :** For first part of journey, $s = s_1$,

$s_1 = \dfrac{1}{2} f t_1^2 = s$ .... (i)

$v = f\, t_1$ .... (ii)

For second part of journey,

$s_2 = vt$

or $\ s_2 = f t_1\, t$ .... (iii)

For the third part of journey,

$s_3 = \dfrac{1}{2}\left(\dfrac{f}{2}\right)(2t_1)^2 \quad$ or $\quad s_3 = \dfrac{1}{2} \times \dfrac{4 f t_1^2}{2}$

or $\ s_3 = 2s_1 = 2s$........... (iv)

$s_1 + s_2 + s_3 = 15s$

or $\ s + f\, t_1 t + 2s = 15s$

or $\ f\, t_1 t = 12s$ .... (v)

From (i) and (v),

$\dfrac{s}{12s} = \dfrac{ft_1^2}{2 \times ft_1 t}$

or $\ t_1 = \dfrac{t}{6} \quad$ or $\quad s = \dfrac{1}{2} ft_1^2 = \dfrac{1}{2} f\left(\dfrac{t}{6}\right)^2 = \dfrac{ft^2}{72}$

or $\ s = \dfrac{ft^2}{72}$

None of the given options provide this answer.

15. **(a) :** $t = ax^2 + bx$

Differentiate the equation with respect to t

$\therefore \quad 1 = 2ax\dfrac{dx}{dt} + b\dfrac{dx}{dt}$

or $\quad 1 = 2axv + bv \quad$ as $\dfrac{dx}{dt} = v$

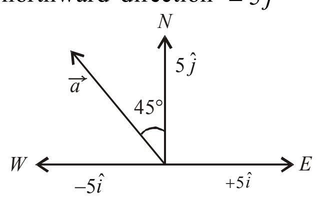
or $\quad v = \dfrac{1}{2ax + b}$

or $\quad \dfrac{dv}{dt} = \dfrac{-2a(dx/dt)}{(2ax+b)^2} = -2av \times v^2$

or $\quad$ Acceleration $= -2av^3$.

16. **(b) :** Velocity in eastward direction $= 5\hat{i}$

velocity in northward direction $= 5\hat{j}$



$\therefore \quad$ Acceleration $\vec{a} = \dfrac{5\hat{j} - 5\hat{i}}{10}$

or $\quad \vec{a} = \dfrac{1}{2}\hat{j} - \dfrac{1}{2}\hat{i} \quad$ or $\quad |\vec{a}| = \sqrt{\left(\dfrac{1}{2}\right)^2 + \left(-\dfrac{1}{2}\right)^2}$

or $\quad |\vec{a}| = \dfrac{1}{\sqrt{2}}\ \text{ms}^{-2}$ towards north-west.

17. **(b) :** Range is same for angles of projection θ and $(90 - \theta)$

$\therefore \quad t_1 = \dfrac{2u\sin\theta}{g}$ and $t_2 = \dfrac{2u\sin(90-\theta)}{g}$

$\therefore \quad t_1 t_2 = \dfrac{4u^2\sin\theta\cos\theta}{g^2} = \dfrac{2}{g} \times \left(\dfrac{u^2\sin 2\theta}{g}\right) = \dfrac{2R}{g}$

$\therefore \quad t_1\ t_2$ is proportional to R.

18. **(d) :** Let a be the retardation for both the vehicles

For automobile, $v^2 = u^2 - 2\,as$

$\therefore \quad u_1^2 - 2as_1 = 0 \Rightarrow \quad u_1^2 = 2as_1$

Similarly for car, $u_2^2 = 2as_2$

$\therefore \quad \left(\dfrac{u_2}{u_1}\right)^2 = \dfrac{s_2}{s_1} \Rightarrow \left(\dfrac{120}{60}\right)^2 = \dfrac{s_2}{20}$

or $\quad s_2 = 80$ m.

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19. **(c) :** Equation of motion : $s = ut + \frac{1}{2}gt^2$

$\therefore \quad h = 0 + \frac{1}{2}gT^2$

or $\quad 2h = gT^2$ ..... (i)

After $T/3$ sec,

$s = 0 + \frac{1}{2} \times g\left(\frac{T}{3}\right)^2 = \frac{gT^2}{18}$

or $\quad 18\ s = gT^2$ .... (ii)

From (i) and (ii),

$18\ s = 2h$

or $\quad s = \frac{h}{9}$ m from top.

$\therefore$ Height from ground $= h - \frac{h}{9} = \frac{8h}{9}$ m.

20. **(a) :** $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$

or $\quad AB\sin\theta\ \hat{n} = AB\sin(-\theta)\ \hat{n}$

or $\quad \sin\theta = -\sin\theta$

or $\quad 2\sin\theta = 0$

or $\quad \theta = 0, \pi, 2\pi.....$

$\therefore \quad \theta = \pi$.

21. **(c) :** Range is same for angles of projection θ and $(90° - \theta)$

$\therefore \quad T_1 = \frac{2u\sin\theta}{g}$ and $T_2 = \frac{2u\sin(90° - \theta)}{g}$

$\therefore \quad T_1T_2 = \frac{4u^2\sin\theta\cos\theta}{g^2} = \frac{2}{g} \times \left(\frac{u^2\sin 2\theta}{g}\right) = \frac{2R}{g}$

$\therefore \quad T_1\ T_2$ is proportional to R.

22. **(b) :** The acceleration vector acts along the radius of the circle. The given statement is false.

23. **(a) :** The person will catch the ball if his speed and horizontal speed of the ball are same

$= v_0\cos\theta = \frac{v_0}{2} \Rightarrow \cos\theta = \frac{1}{2} = \cos 60° \quad \therefore \quad \theta = 60°$.

24. **(c) :** For first case, $u_1 = 50\frac{\text{km}}{\text{hour}} = \frac{50 \times 1000}{60 \times 60} = \frac{125}{9}\frac{\text{m}}{\text{sec}}$

$\therefore$ Acceleration $a = -\frac{u_1^2}{2s_1} = -\left(\frac{125}{9}\right)^2 \times \frac{1}{2 \times 6} = -16$ m/sec^2

For second case, $u_2 = 100\frac{\text{km}}{\text{hour}} = \frac{100 \times 1000}{60 \times 60} = \frac{250}{9}\frac{\text{m}}{\text{sec}}$

$\therefore \quad s_2 = \frac{-u_2^2}{2a} = \frac{-1}{2}\left(\frac{250}{9}\right)^2 \times \left(-\frac{1}{16}\right) = 24$ m

or $\quad s_2 = 24$ m.

25. **(d) :** Height of building = 10 m

The ball projected from the roof of building will be back to roof - height of 10 m after covering the maximum horizontal range.

Maximum horizontal range $(R) = \frac{u^2\sin 2\theta}{g}$

or $\quad R = \frac{(10)^2 \times \sin 60°}{10} = 10 \times 0.866$ or $R = 8.66$ m.

26. **(b) :** $\because \quad x = \alpha t^3$

$\therefore \quad \frac{dx}{dt} = 3\alpha t^2 \Rightarrow v_x = 3\alpha t^2$

Again $y = \beta t^3$

$\therefore \quad \frac{dy}{dt} \Rightarrow v_y = 3\beta t^2 \quad \therefore \quad v^2 = v_x^2 + v_y^2$

or $\quad v^2 = (3\alpha t^2)^2 + (3\beta t^2)^2 = (3t^2)^2\ (\alpha^2 + \beta^2)$

or $\quad v = 3t^2\sqrt{\alpha^2 + \beta^2}$.

27. **(b) :** Ball A projected upwards with velocity u falls back to building top with velocity u downwards. It completes its journey to ground under gravity.

$\therefore \quad v_A^2 = u^2 + 2gh$ ....(i)

Ball B starts with downwards velocity u and reaches ground after travelling a vertical distance h

$\therefore \quad v_B^2 = u^2 + 2gh$ ....(ii)

From (i) and (ii)

$v_A = v_B$.

28. **(d) :** Both are given the same deceleration simultaneously and both finally stop.

Formula relevant to motion : $u^2 = 2\ as$

$\therefore \quad$ For first car, $s_1 = \frac{u^2}{2a}$

For second car, $s_2 = \frac{(4u)^2}{2a} = \frac{16u^2}{2a}$

$\therefore \quad \frac{s_1}{s_2} = \frac{1}{16}$.

29. **(a) :** For first part of penetration, by equation of motion,

$\left(\frac{u}{2}\right)^2 = u^2 - 2a(3)$

or $\quad 3u^2 = 24a \Rightarrow u^2 = 8a$ .... (i)

For latter part of penetration,

$0 = \left(\frac{u}{2}\right)^2 - 2ax$

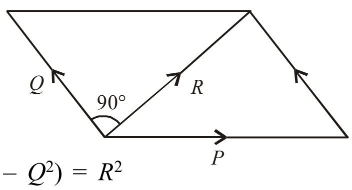
or $\quad u^2 = 8ax$ .... (ii)

From (i) and (ii)

$8ax = 8a \Rightarrow x = 1$ cm.

30. **(b) :** Resultant R is perpendicular to smaller force Q and $(P + Q) = 18$ N

$\therefore \quad P^2 = Q^2 + R^2$ by right angled triangle



or $\quad (P^2 - Q^2) = R^2$

or $\quad (P + Q)(P - Q) = R^2$

or $\quad (18)(P - Q) = (12)^2 \qquad [\because P + Q = 18]$

or $\quad (P - Q) = 8$

Hence $P = 13$ N and $Q = 5$ N.

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CHAPTER 3

LAWS OF MOTION

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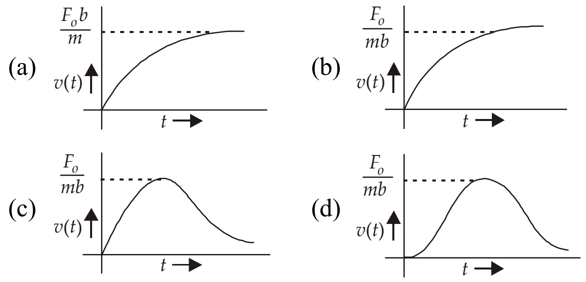
1. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is

(a) $m_1 : m_2$ (b) $r_1 : r_2$

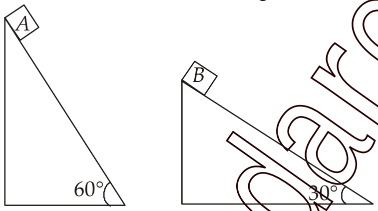
(c) 1 : 1 (d) $m_1r_1 : m_2r_2$ *(2012)*

2. A particle of mass m is at rest at the origin at time $t = 0$. It is subjected to a force $F(t) = F_oe^{-bt}$ in the x direction. Its speed $v(t)$ is depicted by which of the following curves?



(2012)

3. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?



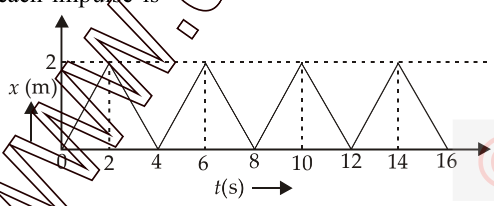
(a) 4.9 ms^{-2} in vertical direction

(b) 4.9 ms^{-2} in horizontal direction

(c) 9.8 ms^{-2} in vertical direction

(d) zero *(2010)*

4. The figure shows the position - time (x-t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is



(a) 0.2 N s (b) 0.4 N s (c) 0.8 N s (d) 1.6 N s *(2010)*

5. A body of mass m = 3.513 kg is moving along the x-axis with a speed of 5.00 ms^{-1}. The magnitude of its momentum is recorded as

(a) 17.57 kg ms^{-1} (b) 17.6 kg ms^{-1}

(c) 17.565 kg ms^{-1} (d) 17.56 kg ms^{-1}. *(2008)*

6. A block of mass m is connected to another block of mass M by a spring (massless) of spring constant k. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then a constant force F starts acting on the block of mass M to pull it. Find the force of the block of mass m.

(a) $\frac{MF}{(m+M)}$ (b) $\frac{mF}{M}$

(c) $\frac{(M+m)F}{m}$ (d) $\frac{mF}{(m+M)}$ *(2007)*

7. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m which applying the force and the ball goes upto 2 m height further, find the magnitude of the force. Consider g = 10 m/s²

(a) 22 N (b) 4 N (c) 16 N (d) 20 N. *(2006)*

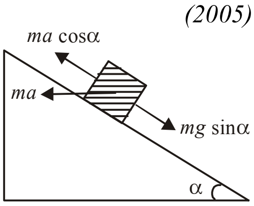
8. A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching process is completed in 0.1 s, the force of the blow exerted by the ball on the hand of the player is equal to

(a) 300 N (b) 150 N (c) 3 N (d) 30 N. *(2006)*

9. Consider a car moving on a straight road with a speed of 100 m/s. The distance at which car can be stopped is [μ_k = 0.5]

(a) 100 m (b) 400 m (c) 800 m (d) 1000 m *(2005)*

10. A block is kept on a frictionless inclined surface with angle of inclination α. The incline is given an acceleration a to keep the block stationary. Then a is equal to



(a) g (b) g tanα

(c) g/tanα (d) g cosecα *(2005)*

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11. A particle of mass 0.3 kg is subjected to a force $F = -kx$ with k = 15 N/m. What will be its initial acceleration if it is released from a point 20 cm away from the origin?
(a) 5 m/s^2 (b) 10 m/s^2 (c) 3 m/s^2 (d) 15 m/s^2
(2005)

12. A bullet fired into a fixed target loses half its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest assuming that it faces constant resistance to motion?
(a) 1.5 cm (b) 1.0 cm (c) 3.0 cm (d) 2.0 cm
(2005)

13. The upper half of an inclined plane with inclination ϕ is perfectly smooth while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the coefficient of friction for the lower half is given by
(a) $2\tan\phi$ (b) $\tan\phi$ (c) $2\sin\phi$ (d) $2\cos\phi$
(2005)

14. A smooth block is released at rest on a 45° incline and then slides a distance d. The time taken to slide is n times as much to slide on rough incline than on a smooth incline. The coefficient of friction is

(a) $\mu_s = 1 - \frac{1}{n^2}$ (b) $\mu_s = \sqrt{1 - \frac{1}{n^2}}$

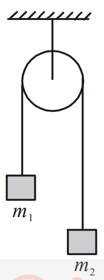
(c) $\mu_k = 1 - \frac{1}{n^2}$ (d) $\mu_k = \sqrt{1 - \frac{1}{n^2}}$ *(2005)*

15. An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two particles situated on the inner and outer parts of the ring, F_1/F_2 is

(a) 1 (b) $\frac{R_1}{R_2}$ (c) $\frac{R_2}{R_1}$ (d) $\left(\frac{R_1}{R_2}\right)^2$
(2005)

16. A block rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass of the block (in kg) is (take g = 10 m/s^2)
(a) 2.0 (b) 4.0 (c) 1.6 (d) 2.5
(2004)

17. Two masses m_1 = 5 kg and m_2 = 4.8 kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses when lift free to move? (g = 9.8 m/s^2)



(a) 0.2 m/s^2
(b) 9.8 m/s^2
(c) 5 m/s^2
(d) 4.8 m/s^2.
(2004)

18. A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most?
(a) one (b) four (c) two (d) three.
(2004)

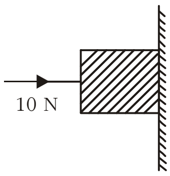
19. A rocket with a lift-off mass 3.5×10^4 kg is blasted upwards with an initial acceleration of 10 m/s^2. Then the initial thrust of the blast is
(a) 3.5×10^5 N (b) 7.0×10^5 N
(c) 14.0×10^5 N (d) 1.75×10^5 N. *(2003)*

20. A light spring balance hangs from the hook of the other light spring balance and a block of mass M kg hangs from the former one. Then the true statement about the scale reading is
(a) both the scales read M kg each
(b) the scale of the lower one reads M kg and of the upper one zero
(c) the reading of the two scales can be anything but the sum of the reading will be M kg
(d) both the scales read $M/2$ kg. *(2003)*

21. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force P is applied at the free end of the rope, the force exerted by the rope on the block is
(a) $\frac{Pm}{M+m}$ (b) $\frac{Pm}{M-m}$
(c) P (d) $\frac{PM}{M+m}$. *(2003)*

22. A marble block of mass 2 kg lying on ice when given a velocity of 6 m/s is stopped by friction in 10 s. Then the coefficient of friction is
(a) 0.02 (b) 0.03 (c) 0.06 (d) 0.01.
(2003)

23. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is



(a) 20 N (b) 50 N
(c) 100 N (d) 2 N.
(2003)

24. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of 5 m/s^2, the reading of the spring balance will be
(a) 24 N (b) 74 N
(c) 15 N (d) 49 N.
(2003)

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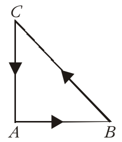
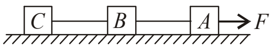
25. Three forces start acting simultaneously on a particle moving with velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle ABC (as shown). The particle will now move with velocity



(a) less than $\vec{v}$
(b) greater than $\vec{v}$
(c) $|\vec{v}|$ in the direction of the largest force BC
(d) $\vec{v}$, remaining unchanged. *(2003)*

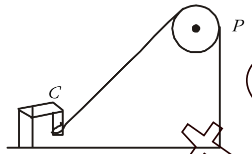
26. Three identical blocks of masses m = 2 kg are drawn by a force F = 10.2 N with an acceleration of 0.6 ms^{-2} on a frictionless surface, then what is the tension (in N) in the string between the blocks B and C?



(a) 9.2 (b) 7.8 (c) 4 (d) 9.8 *(2002)*

27. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$, then the ratio of the masses is
(a) 8 : 1 (b) 9 : 7 (c) 4 : 3 (d) 5 : 3. *(2002)*

28. One end of a massless rope, which passes over a massless and frictionless pulley P is tied to a hook C while the other end is free. Maximum tension that the rope can bear is 960 N. With what value of maximum safe acceleration (in ms^{-2}) can a man of 60 kg climb on the rope?



(a) 16 (b) 6 (c) 4 (d) 8. *(2002)*

29. When forces F_1, F_2, F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. If the force F_1 is now removed then the acceleration of the particle is
(a) F_1/m (b) F_2F_3/mF_1
(c) $(F_2 - F_3)/m$ (d) F_2/m. *(2002)*

30. A lift is moving down with acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively
(a) g, g (b) $g - a, g - a$
(c) $g - a, g$ (d) a, g. *(2002)*

31. The minimum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is
(a) 60 (b) 30 (c) 15 (d) 25. *(2002)*

Answer Key

1.	(b)	**2.**	(b)	**3.**	(a)	**4.**	(c)	**5.**	(a)	**6.**	(d)
7.	(d)	**8.**	(d)	**9.**	(d)	**10.**	(b)	**11.**	(b)	**12.**	(b)
13.	(a)	**14.**	(c)	**15.**	(b)	**16.**	(a)	**17.**	(a)	**18.**	(d)
19.	(a)	**20.**	(a)	**21.**	(d)	**22.**	(c)	**23.**	(d)	**24.**	(a)
25.	(d)	**26.**	(b)	**27.**	(b)	**28.**	(b)	**29.**	(a)	**30.**	(c)
31.	(b)										

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Explanations

1. **(b):** Centripetal acceleration, $a_c = \omega^2 r$

$\because \quad \omega = \frac{2\pi}{T}$

As $T_1 = T_2 \Rightarrow \omega_1 = \omega_2$

$\because \quad \frac{a_{c_1}}{a_{c_2}} = \frac{r_1}{r_2}$

2. **(b):** $F(t) = F_o e^{-bt}$ (Given)

$ma = F_o e^{-bt}$

$a = \frac{F_o}{m} e^{-bt}$

$\frac{dv}{dt} = \frac{F_o}{m} e^{-bt}$ or $dv = \frac{F_o}{m} e^{-bt} dt$

Integrating both sides, we get

$\int_0^v dv = \int_0^t \frac{F_o}{m} e^{-bt} dt \Rightarrow v = \frac{F_o}{m}\left[\frac{e^{-bt}}{-b}\right]_0^t = \frac{F_o}{mb}[1 - e^{-bt}]$

3. **(a) :** The acceleration of the body down the smooth inclined plane is $a = g\sin\theta$

It is along the inclined plane.

where θ is the angle of inclination

$\therefore$ The vertical component of acceleration a is

$a_{\text{(along vertical)}} = (g\sin\theta)\sin\theta = g\sin^2\theta$

For block A

$a_{A\text{(along vertical)}} = g\sin^2 60°$

For block B

$a_{B\text{(along vertical)}} = g\sin^2 30°$

The relative vertical acceleration of A with respect to B is

$a_{AB\text{(along vertical)}} = a_{A\text{(along vertical)}} - a_{B\text{(along vertical)}}$

$= g\sin^2 60° - g\sin^2 30°$

$= g\left(\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right)$

$= \frac{g}{2} = 4.9 \text{ m s}^{-2}$ in vertical direction.

4. **(c) :** Here, mass of the body, $m = 0.4$ kg

Since position-time (x-t) graph is a straight line, so motion is uniform. Because of impulse direction of velocity changes as can be seen from the slopes of the graph.

From graph,

Initial velocity, $u = \frac{(2-0)}{(2-0)} = 1 \text{ m s}^{-1}$

Final velocity, $v = \frac{(0-2)}{(4-2)} = -1 \text{ m s}^{-1}$

Initial momentum, $p_i = mu = 0.4 \times 1 = 0.4$ N s

Final momentum, $p_f = mv = 0.4 \times (-1) = -0.4$ N s

Impulse = Change in momentum $= p_f - p_i$

$= -0.4 - (0.4)$ N s $= -0.8$ N s

|Impulse| = 0.8 N s

5. **(a) :** Momentum is mv.

$m = 3.513$ kg ; $v = 5.00$ m/s

$\therefore \quad mv = 17.57 \text{ m s}^{-1}$

Because the values will be accurate up to second decimel place only, 17.565 = 17.57

6. **(d) :** Acceleration of the system $a = \frac{F}{m+M}$

Force on block of mass $m = ma = \frac{mF}{m+M}$.

7. **(d) :** Work done by hand = Potential energy of the ball

$\therefore \quad FS = mgh \Rightarrow F = \frac{mgh}{s} = \frac{0.2 \times 10 \times 2}{0.2} = 20$ N.

8. **(d) :** Force × time = Impulse = Change of momentum

$\therefore \quad \text{Force} = \frac{\text{Impulse}}{\text{time}} = \frac{3}{0.1} = 30$ N.

9. **(d) :** Retardation due to friction $= \mu g$

$\therefore \quad v^2 = u^2 + 2as$

$\therefore \quad 0 = (100)^2 - 2(\mu g)s$ or $2\mu gs = 100 \times 100$

or $\quad s = \frac{100 \times 100}{2 \times 0.5 \times 10} = 1000$ m.

10. **(b) :** The incline is given an acceleration a. Acceleration of the block is to the right. Pseudo acceleration a acts on block to the left. Equate resolved parts of a and g along incline.

$\therefore \quad ma\cos\alpha = mg\sin\alpha$ or $a = g\tan\alpha$.

11. **(b) :** $F = -kx$

or $\quad F = -15 \times \left(\frac{20}{100}\right) = -3$ N

Initial acceleration is over come by retarding force.

or $\quad m \times$ (acceleration a) = 3

or $\quad a = \frac{3}{m} = \frac{3}{0.3} = 10 \text{ ms}^{-2}$.

12. **(b) :** For first part of penetration, by equation of motion,

$\left(\frac{u}{2}\right)^2 = (u)^2 - 2f(3)$

or $\quad 3u^2 = 24f$ .... (i)

For latter part of penetration,

$0 = \left(\frac{u}{2}\right)^2 - 2fx$

or $\quad u^2 = 8fx$ .... (ii)

From (i) and (ii)

$3 \times (8fx) = 24f$

or $\quad x = 1$ cm.

13. **(a) :** For upper half smooth incline, component of g down the incline $= g\sin\phi$

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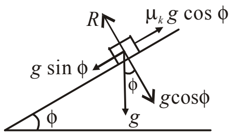
$\therefore \quad v^2 = 2(g\sin\phi)\frac{l}{2}$



For lower half rough incline, frictional retardation = $\mu_k g\cos\phi$

$\therefore$ Resultant acceleration = $g\sin\phi - \mu_k g\cos\phi$

$\therefore \quad 0 = v^2 + 2(g\sin\phi - \mu_k g\cos\phi)\frac{l}{2}$

or $\quad 0 = 2(g\sin\phi)\frac{l}{2} + 2g(\sin\phi - \mu_k\cos\phi)\frac{l}{2}$

or $\quad 0 = \sin\phi + \sin\phi - \mu_k\cos\phi$

or $\quad \mu_k\cos\phi = 2\sin\phi$

or $\quad \mu_k = 2\tan\phi.$

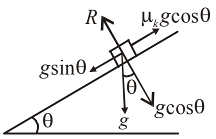
14. **(c) :** Component of g down the plane = $g\sin\theta$

$\therefore$ For smooth plane,

$$d = \frac{1}{2}(g\sin\theta)t^2 \qquad \text{........ (i)}$$

For rough plane,

Frictional retardation up the plane = $\mu_k(g\cos\theta)$



$\therefore \quad d = \frac{1}{2}(g\sin\theta - \mu_k g\cos\theta)(nt)^2$

$\therefore \quad \frac{1}{2}(g\sin\theta)t^2 = \frac{1}{2}(g\sin\theta - \mu_k g\cos\theta)n^2t^2$

or $\quad \sin\theta = n^2(\sin\theta - \mu_k\cos\theta)$

Putting $\theta = 45°$

or $\quad \sin45° = n^2(\sin45° - \mu_k\cos45°)$

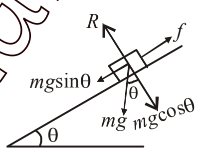
or $\quad \frac{1}{\sqrt{2}} = \frac{n^2}{\sqrt{2}}(1 - \mu_k)$

or $\quad \mu_k = 1 - \frac{1}{n^2}.$

15. **(b) :** Centripetal force on particle = $mR\omega^2$

$\therefore \quad \frac{F_1}{F_2} = \frac{mR_1\omega^2}{mR_2\omega^2} = \frac{R_1}{R_2}.$

16. **(a) :** For equilibrium of block,



$f = mg\sin\theta$

$\therefore \quad 10 = m \times 10 \times \sin30°$

or $\quad m = 2$ kg.

17. **(a) :** $\frac{a}{g} = \frac{(m_1 - m_2)}{(m_1 + m_2)} = \frac{(5 - 4.8)}{(5 + 4.8)} = \frac{0.2}{9.8}$

or $\quad a = g \times \frac{0.2}{9.8} = \frac{9.8 \times 0.2}{9.8} = 0.2 \text{ ms}^{-2}.$

18. **(d) :** Suppose he can fire n bullets per second

$\therefore$ Force = Change in momentum per second

$144 = n \times \left(\frac{40}{1000}\right) \times (1200)$

or $\quad n = \frac{144 \times 1000}{40 \times 1200}$

or $\quad n = 3.$

19. **(a) :** Initial thrust = (Lift - off mass) × acceleration = $(3.5 \times 10^4) \times (10) = 3.5 \times 10^5$ N.

20. **(a) :** Both the scales read M kg each.

21. **(d) :** Acceleration of block $(a) = \frac{\text{Force applied}}{\text{Total mass}}$

or $\quad a = \frac{P}{(M + m)}$

$\therefore$ Force on block

$= \text{Mass of block} \times a = \frac{MP}{(M + m)}.$

22. **(c) :** Frictional force provides the retarding force

$\therefore \quad \mu\, mg = ma$

or $\quad \mu = \frac{a}{g} = \frac{u/t}{g} = \frac{(6/10)}{10} = 0.06.$

23. **(d) :** Weight of the block is balanced by force of friction

$\therefore$ Weight of the block = $\mu R = 0.2 \times 10 = 2$ N.

24. **(a) :** When lift is standing, $W_1 = mg$

When the lift descends with acceleration a, $W_2 = m(g - a)$

$\therefore \quad \frac{W_2}{W_1} = \frac{m(g - a)}{mg} = \frac{9.8 - 5}{9.8} = \frac{4.8}{9.8}$

or $\quad W_2 = W_1 \times \frac{4.8}{9.8} = \frac{49 \times 4.8}{9.8} = 24$ N.

25. **(d) :** By triangle of forces, the particle will be in equilibrium under the three forces. Obviously the resultant force on the particle will be zero. Consequently the acceleration will be zero.

Hence the particle velocity remains unchanged at $\vec{v}$.

26. **(b) :** $\because$ Force = mass × acceleration

$\therefore \quad F - T_{AB} = ma$

and $T_{AB} - T_{BC} = ma$

$\therefore \quad T_{BC} = F - 2\,ma$

or $\quad T_{BC} = 10.2 - (2 \times 2 \times 0.6)$

or $\quad T_{BC} = 7.8$ N.

27. **(b) :** $\frac{a}{g} = \frac{(m_1 - m_2)}{(m_1 + m_2)}$

$\therefore \quad \frac{1}{8} = \frac{(m_1 - m_2)}{(m_1 + m_2)}$ or $\frac{m_1}{m_2} = \frac{9}{7}.$

28. **(b) :** $T - 60g = 60a$

or $\quad 960 - (60 \times 10) = 60a$

or $\quad 60a = 360$

or $\quad a = 6 \text{ ms}^{-2}.$

29. **(a) :** F_2 and F_3 have a resultant equivalent to F_1

$\therefore \quad \text{Acceleration} = \frac{F_1}{m}.$

30. **(c) :** For observer in the lift, acceleration = $(g - a)$

For observer standing outside, acceleration = g.

31. **(b) :** For no skidding along curved track,

$v = \sqrt{\mu Rg}$

$\therefore \quad v = \sqrt{0.6 \times 150 \times 10} = 30\ \frac{\text{m}}{\text{s}}.$

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CHAPTER 4

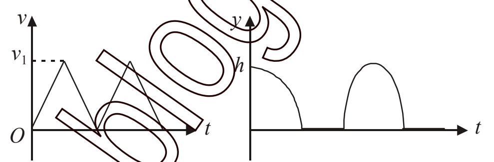
WORK, ENERGY AND POWER

1. This question has Statement-I and Statement-II. Of the four choices given after the Statements, choose the one that best describes the two Statements.
Statement-1 : A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $h\left(\frac{1}{2}mv^2\right)$ then $h = \left(\frac{m}{M+m}\right)$
Statement-2 : Maximum energy loss occurs when the particles get stuck together as a result of the collision.
(a) Statement-I is false, Statement-II is true.
(b) Statement-I is true, Statement-II is true, Statement-II is a correct explanation of Statement-I.
(c) Statement-I is true, Statement-II is true, Statement-II is not a correct explanation of statement-I.
(d) Statement-I is true, Statement-II is false. *(2013)*

2. This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.
If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.
Statement 1 : If stretched by the same amount, work done on S_1, will be more than that on S_2.
Statement 2 : $k_1 < k_2$.
(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.
(d) Statement 1 is false, Statement 2 is true. *(2012)*

3. **Statement-1 :** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.
Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.
(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true. *(2010)*

4. Consider a rubber ball freely falling from a height $h = 4.9$ m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time and the height as function of time will be

(a)


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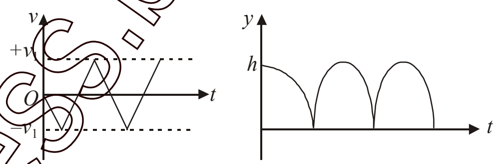
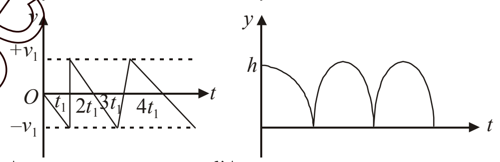
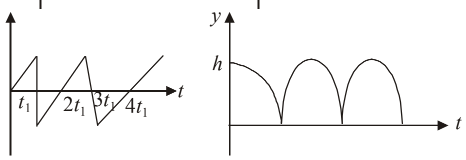


(c)


(d)


(2009)

5. A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is
(a) 0.34 J (b) 0.16 J (c) 1.00 J (d) 0.67 J. *(2008)*

6. An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range
(a) 2,000 J - 5,000 J (b) 200 J - 500 J
(c) 2×10^5 J - 3×10^5 J (d) 20,000 J - 50,000 J. *(2008)*

7. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is
(a) $K/2$ (b) K (c) zero (d) $K/4$ *(2007)*

8. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes a uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10,000 N/m. The spring compresses by
(a) 8.5 cm (b) 5.5 cm (c) 2.5 cm (d) 11.0 cm *(2007)*

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9. The potential energy of a 1 kg particle free to move along the x-axis is given by

$$V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right) \text{J}.$$

The total mechanical energy of the particle 2 J. Then, the maximum speed (in m/s) is

(a) 2 (b) $3/\sqrt{2}$ (c) $\sqrt{2}$ (d) $1/\sqrt{2}$. *(2006)*

10. A particle of mass 100 g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is

(a) 0.5 J (b) –0.5 J (c) –1.25 J (d) 1.25 J. *(2006)*

11. A bomb of mass 16 kg at rest explodes into two pieces of masses of 4 kg and 12 kg. The velocity of the 12 kg mass is 4 ms^{-1}. The kinetic energy of the other mass is

(a) 96 J (b) 144 J (c) 288 J (d) 192 J. *(2006)*

12. A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string making an angle of 45° with the initial vertical direction is

(a) $Mg(\sqrt{2}-1)$ (b) $Mg(\sqrt{2}+1)$

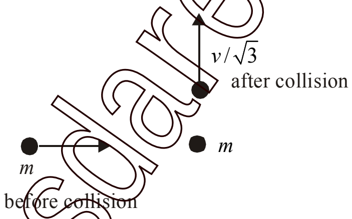
(c) $Mg\sqrt{2}$ (d) $\frac{Mg}{\sqrt{2}}$. *(2006)*

13. A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time is given by

(a) $\frac{1}{2}\frac{mv^2}{T^2}t$ (b) $\frac{1}{2}\frac{mv^2}{T^2}t^2$

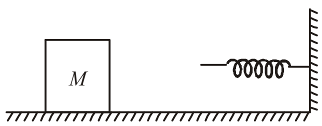
(c) $\frac{mv^2}{T^2}\cdot t$ (d) $\frac{mv^2}{T^2}\cdot t^2$ *(2005)*

14. A mass m moves with a velocity v and collides inelastically with another identical mass. After collision the first mass moves with velocity in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision



(a) $\frac{2}{\sqrt{3}}v$ (b) $\frac{v}{\sqrt{3}}$ (c) v (d) $\sqrt{3}v$ *(2005)*

15. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant K and compresses it by length L. The maximum momentum of the block after collision is



(a) zero (b) $\frac{ML^2}{K}$ (c) $\sqrt{MK}\,L$ (d) $\frac{KL^2}{2M}$ *(2005)*

16. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is

(a) 10 m/s (b) 34 m/s (c) 40 m/s (d) 20 m/s *(2005)*

17. A body of mass m, accelerates uniformly from rest to v_1 in time t_1. The instantaneous power delivered to the body as a function of time t is

(a) $\frac{mv_1t}{t_1}$ (b) $\frac{mv_1^2t}{t_1^2}$

(c) $\frac{mv_1t^2}{t_1}$ (d) $\frac{mv_1^2t}{t_1}$. *(2004)*

18. A force $\vec{F} = (5\hat{i} + 3\hat{j} + 2\hat{k})$ N is applied over a particle which displaces it from its origin to the point $\vec{r} = (2\hat{i} - \hat{j})$ m. The work done on the particle in joule is

(a) –7 (b) +7 (c) +10 (d) +13. *(2004)*

19. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table?

(a) 7.2 J (b) 3.6 J (c) 120 J (d) 1200 J. *(2004)*

20. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle, the motion of the particle takes place in a plane. It follows that

(a) its velocity is constant
(b) its acceleration is constant
(c) its kinetic energy is constant
(d) it moves in a straight line. *(2004)*

21. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to

(a) x^2 (b) e^x (c) x (d) $\log_e x$. *(2004)*

22. A spring of spring constant 5 × 10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is

(a) 12.50 N-m (b) 18.75 N-m
(c) 25.00 N-m (d) 6.25 N-m. *(2003)*

23. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to

(a) $t^{3/4}$ (b) $t^{3/2}$ (c) $t^{1/4}$ (d) $t^{1/2}$. *(2003)*

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24. Consider the following two statements.
A. Linear momentum of a system of particles is zero.
B. Kinetic energy of a system of particles is zero.
Then
(a) A does not imply B and B does not imply A
(b) A implies B but B does not imply A
(c) A does not imply B but B implies A
(d) A implies B and B implies A. *(2003)*

25. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is
(a) 16 J (b) 8 J (c) 32 J (d) 24 J. *(2002)*

26. If mass-energy equivalence is taken into account, when water is cooled to form ice, the mass of water should
(a) increase
(b) remain unchanged
(c) decrease
(d) first increase then decrease. *(2002)*

27. A ball whose kinetic energy is E, is projected at an angle of 45° to the horizontal. The kinetic energy of the ball at the highest point of its flight will be
(a) E (b) $E/\sqrt{2}$
(c) $E/2$ (d) zero. *(2002)*

Answer Key

1.	(a)	**2.**	(d)	**3.**	(b)	**4.**	(c)	**5.**	(d)	**6.**	(a)
7.	(d)	**8.**	(b)	**9.**	(b)	**10.**	(c)	**11.**	(c)	**12.**	(a)
13.	(c)	**14.**	(a)	**15.**	(c)	**16.**	(b)	**17.**	(b)	**18.**	(b)
19.	(b)	**20.**	(c)	**21.**	(a)	**22.**	(b)	**23.**	(b)	**24.**	(c)
25.	(b)	**26.**	(c)	**27.**	(c)						

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Explanations

1. **(a) :** Loss of energy is maximum when collision is inelastic.

Maximum energy loss $= \frac{1}{2}\frac{mM}{(M+m)}u^2$

$\therefore \quad f = \frac{mM}{(M+m)}$

Hence, Statement-1 is false, Statement-2 is true.

2. **(d) :** For the same force, $F = k_1x_1 = k_2x_2$ ...(i)

Work done on spring S_1 is

$W_1 = \frac{1}{2}k_1x_1^2 = \frac{(k_1x_1)^2}{2k_1} = \frac{F^2}{2k_1}$ (Using (i))

Work done on spring S_2 is

$W_2 = \frac{1}{2}k_2x_2^2 = \frac{(k_2x_2)^2}{2k_2} = \frac{F^2}{2k_2}$ (Using (i))

$\therefore \frac{W_1}{W_2} = \frac{k_2}{k_1}$

As $W_1 > W_2$ $\therefore$ $k_2 > k_1$ or $k_1 < k_2$

Statement 2 is true.

For the same extension, $x_1 = x_2 = x$ ...(ii)

Work done on spring S_1 is

$W_1 = \frac{1}{2}k_1x_1^2 = \frac{1}{2}k_1x^2$ (Using (ii))

Work done on spring S_2 is

$W_2 = \frac{1}{2}k_2x_2^2 = \frac{1}{2}k_2x^2$ (Using (ii))

$\therefore \frac{W_1}{W_2} = \frac{k_1}{k_2}$

As $k_1 < k_2$ $\therefore$ $W_1 < W_2$

Statement 1 is false.

3. **(b)**

4. **(c) :** $v = u + gt$. As the ball is dropped, $v = gt$ when coming down. v increases, makes collision, the value of v becomes +ve, decreases, comes to zero and increases. The change from $+v$ to $-v$ is almost instantaneous. Using –ve signs when coming down, (c) is correct.

Further $h = \frac{1}{2}gt^2$ is a parabola. Therefore (c).

5. **(d) :** By the law of conservation of momentum

$mu = (M + m)v$

$0.50 \times 2.00 = (1 + 0.50)\,v, \quad \frac{1.00}{1.50} = v$

Initial K.E. = (1/2) × 0.50 × (2.00)^2 = 1.00 J.

Final K.E. $= \frac{1}{2} \times 1.50 \times \frac{1.00^2}{(1.50)^2} = \frac{1.00}{3.00} = 0.33$

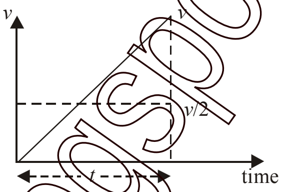
$\therefore$ Loss of energy = 1.00 – 0.33 = 0.67 J.

6. **(a) :** $v = v/2$ is average velocity

$s = 100$ m, $t = 10$ s. $\therefore$ $(v/2) = 10$ m/s.

$v_{average} = (v/2) = 10$ m/s.

Assuming an athelete has about 50 to 100 kg, his kinetic energy would have been $\frac{1}{2}mv_{av}^2$.



$(1/2)mv_{av}^2 = (1/2) \times 50 \times 100 = 2500$ J.

For 100 kg, (1/2) × 100 × 100 = 5000 J.

It could be in the range 2000 to 5000 J.

7. **(d) :** The kinetic energy of a particle is K

At highest point velocity has its horizontal component. Therefore kinetic energy of a particle at highest point is

$K_H = K\cos^2\theta = K\cos^2 60° = \frac{K}{4}$.

8. **(b) :** Let the spring be compressed by x

Initial kinetic energy of the mass = potential energy of the spring + work done due to friction

$\frac{1}{2} \times 2 \times 4^2 = \frac{1}{2} \times 10000 \times x^2 + 15x$

or $5000\,x^2 + 15x - 16 = 0$

or $x = 0.055$ m = 5.5 cm.

9. **(b) :** Total energy $E_T = 2$ J. It is fixed.

For maximum speed, kinetic energy is maximum

The potential energy should therefore be minimum.

$\because \quad V(x) = \frac{x^4}{4} - \frac{x^2}{2}$

or $\frac{dV}{dx} = \frac{4x^3}{4} - \frac{2x}{2} = x^3 - x = x(x^2 - 1)$

For V to be minimum, $\frac{dV}{dx} = 0$

$\therefore \quad x(x^2 - 1) = 0$, or $x = 0, \pm 1$

At $x = 0$, $V(x) = 0$

At $x = \pm 1$, $V(x) = -\frac{1}{4}$ J

$\therefore$ (Kinetic energy)$_{max}$ = $E_T - V_{min}$

or (Kinetic energy)$_{max}$ $= 2 - \left(-\frac{1}{4}\right) = \frac{9}{4}$ J

or $\frac{1}{2}m\,v_m^2 = \frac{9}{4}$

or $v_m^2 = \frac{9 \times 2}{m \times 4} = \frac{9 \times 2}{1 \times 4} = \frac{9}{2}$

$\therefore \quad v_m = \frac{3}{\sqrt{2}}$ m/s.

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10. (c) : Kinetic energy at projection point is converted into potential energy of the particle during rise. Potential energy measures the workdone against the force of gravity during rise.

∴ (– work done) = Kinetic energy = $\frac{1}{2}mv^2$

or (– work done)

$$= \frac{1}{2}\times\left(\frac{100}{1000}\right)(5)^2 = \frac{5\times5}{2\times10} = 1.25\text{ J}$$

∴ Work done by force of gravity = – 1.25 J

11. (c) : Linear momentum is conserved

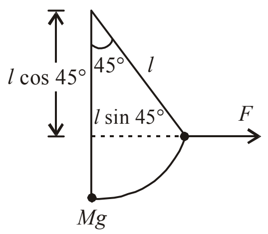
∴ $0 = m_1v_1 + m_2v_2 = (12 \times 4) + (4 \times v_2)$

or $4v_2 = -48 \Rightarrow v_2 = -12$ m/s

∴ Kinetic energy of mass $m_2 = \frac{1}{2}m_2v_2^2$

$$= \frac{1}{2}\times 4\times(-12)^2 = 288\text{ J}.$$

12. (a) : Work done in displacement is equal to gain in potential energy of mass



Work done $= F\times l\sin 45° = \frac{Fl}{\sqrt{2}}$

Gain in potential energy = $Mg(l - l\cos45°)$

$$= Mgl\left(1-\frac{1}{\sqrt{2}}\right)$$

∴ $\frac{Fl}{\sqrt{2}} = \frac{Mgl(\sqrt{2}-1)}{\sqrt{2}}$

or $F = Mg(\sqrt{2}-1)$.

13. (c) : Power = Force × velocity

$= (ma)\ (v) = (ma)\ (at) = ma^2t$

or $\text{Power} = m\left(\frac{v}{T}\right)^2(t) = \frac{mv^2}{T^2}t$

14. (a) : Let v_1 = speed of second mass after collision Momentum is conserved

Along X-axis, $mv_1\cos\theta = mv$ ....(i)

Along Y-axis, $mv_1\sin\theta = \frac{mv}{\sqrt{3}}$ .... (ii)

From (i) and (ii)

∴ $(mv_1\cos\theta)^2 + (mv_1\sin\theta)^2 = (mv)^2 + \left(\frac{mv}{\sqrt{3}}\right)^2$

or $m^2v_1^2 = \frac{4m^2v^2}{3}$

or $v_1 = \frac{2}{\sqrt{3}}v$

15. (c) : Elastic energy stored in spring $= \frac{1}{2}KL^2$

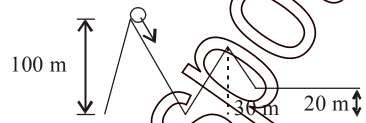
∴ kinetic energy of block $E = \frac{1}{2}KL^2$

Since $p^2 = 2ME$

∴ $p = \sqrt{2ME} = \sqrt{\frac{2M\times KL^2}{2}} = \sqrt{MK}\,L$.

16. (b) : $mgh = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$

$$= \frac{1}{2}mv^2\cdot\frac{7}{5}$$



∴ $\frac{1}{2}mv^2\left(\frac{7}{5}\right) = mg\times 80$

or $v^2 = 2 \times 10 \times 80 \times \frac{5}{7} = 1600 \times \frac{5}{7}$

or $v = 34$ m/s.

17. (b) : Acceleration $a = \frac{v_1}{t_1}$

∴ velocity $(v) = 0 + at = \frac{v_1}{t_1}t$

∴ Power P = Force × velocity = $m\,a\,v$

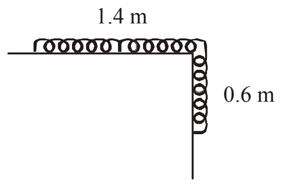
or $P = m\left(\frac{v_1}{t_1}\right)\times\left(\frac{v_1t}{t_1}\right) = \frac{mv_1^2t}{t_1^2}$.

18. (b) : Work done $= \vec{F}\cdot\vec{r}$

or work done $= (5\hat{i}+3\hat{j}+2\hat{k})\cdot(2\hat{i}-\hat{j})$

or work done = 10 – 3 = 7 J.

19. (b) : The centre of mass of the hanging part is at 0.3 m from table



mass of hanging part $= \frac{4\times0.6}{2} = 1.2$ kg

∴ $W = mgh$

$= 1.2 \times 10 \times 0.3$

$= 3.6$ J.

20. (c) : No work is done when a force of constant magnitude always acts at right angles to the velocity of a particle when the motion of the particle takes place in a plane.
Hence kinetic energy of the particle remains constant.

21. (a) : Given : Retardation ∝ displacement

or $\frac{dv}{dt} = kx$

or $\left(\frac{dv}{dx}\right)\left(\frac{dx}{dt}\right) = kx$

or $dv\ (v) = kx\ dx$

or $\int_{v_1}^{v_2} v\,dv = k\int_0^x x\,dx$

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or $\frac{v_2^2}{2} - \frac{v_1^2}{2} = \frac{kx^2}{2}$

or $\frac{mv_2^2}{2} - \frac{mv_1^2}{2} = \frac{mkx^2}{2}$

or $(K_2 - K_1) = \frac{mk}{2}x^2$

or Loss of kinetic energy is proportional to x^2.

22. **(b) :** Force constant of spring $(k) = F/x$

or $F = kx$

$\therefore$ $dW = kxdx$

or $\int dW = \int_{0.05}^{0.1} kx\,dx = \frac{k}{2}\left[(0.1)^2 - (0.05)^2\right]$

$= \frac{k}{2} \times [0.01 - 0.0025]$

or Workdone $= \frac{(5 \times 10^3)}{2} \times (0.0075) = 18.75 \text{ N-m}$.

23. **(b) :** Power $= \frac{\text{Work}}{\text{Time}} = \frac{\text{Force} \times \text{distance}}{\text{Time}} = \text{Force} \times \text{velocity}$

$\therefore$ Force × velocity = constant (K)

or $(ma)\,(at) = K$

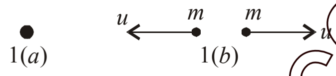
or $a = \left(\frac{K}{mt}\right)^{1/2}$

$\because$ $s = \frac{1}{2}at^2$

$\therefore$ $s = \frac{1}{2}\left(\frac{K}{mt}\right)^{1/2} t^2 = \frac{1}{2}\left(\frac{K}{m}\right)^{1/2} t^{3/2}$

or s is proportional to $t^{3/2}$.

24. **(c) :** A system of particles implies that one is discussing total momentum and total energy.



1(*a*) explodes

Total momentum = 0

But total kinetic energy $= 2\left(\frac{1}{2}\right)mu^2$

But if total kinetic energy = 0, velocities are zero.

Here *A* is true, but *B* is not true.

A does not imply *B*, but *B* implies *A*.

25. **(b) :** $W = \int_{x_1}^{x_2} F dx = \int_{0.05}^{0.15} kx\,dx$

$\therefore$ $W = \int_{0.05}^{0.15} 800x dx = \frac{800}{2}\left[x^2\right]_{0.05}^{0.15}$

$= 400\left[(0.15)^2 - (0.05)^2\right]$

or $W = 8$ J.

26. **(c) :** When water is cooled to form ice, its thermal energy decreases. By mass energy equivalent, mass should decrease.

27. **(c) :** Kinetic energy point of projection $(E) = \frac{1}{2}mu^2$

At highest point velocity = $u \cos\theta$

$\therefore$ Kinetic energy at highest point

$= \frac{1}{2}m(u\cos\theta)^2$

$= \frac{1}{2}mu^2\cos^2 45° = \frac{E}{2}$.

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CHAPTER 5

ROTATIONAL MOTION

1. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

   (a) $r\omega_0$ (b) $\frac{r\omega_0}{4}$ (c) $\frac{r\omega_0}{3}$ (d) $\frac{r\omega_0}{2}$ *(2013)*

2. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion if reversed, is

   (a) less than 3
   (b) more than 3 but less than 6
   (c) more than 6 but less than 9
   (d) more than 9 *(2011)*

3. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is

   (a) $\frac{3}{2}g$ (b) g (c) $\frac{2}{3}g$ (d) $\frac{g}{3}$ *(2011)*

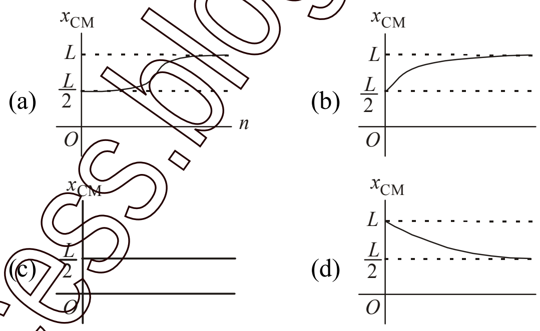
4. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc

   (a) remains unchanged
   (b) continuously decreases
   (c) continuously increases
   (d) first increases and then decreases *(2011)*

5. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of

   (a) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (b) $\frac{1}{6}\frac{l\omega}{g}$ (c) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (d) $\frac{1}{6}\frac{l^2\omega^2}{g}$ *(2009)*

6. A thin rod of length L is lying along the x-axis with its ends at $x = 0$ and $x = L$. Its linear density (mass/length) varies with x as $k(x/L)^n$ where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against n, which of the following graphs best approximates the dependence of x_{CM} on n?

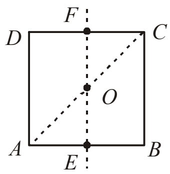


(2008)

7. Consider a uniform square plate of side a and mass m. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

   (a) $\frac{2}{3}ma^2$ (b) $\frac{5}{6}ma^2$ (c) $\frac{1}{12}ma^2$ (d) $\frac{7}{12}ma^2$ *(2008)*

8. For the given uniform square lamina $ABCD$, whose centre is O,



   (a) $I_{AC} = \sqrt{2}I_{EF}$
   (b) $\sqrt{2}I_{AC} = I_{EF}$
   (c) $I_{AD} = 3I_{EF}$
   (d) $I_{AC} = I_{EF}$ *(2007)*

9. Angular momentum of the particle rotating with a central force is constant due to

   (a) constant torque (b) constant force
   (c) constant linear momentum
   (d) zero torque *(2007)*

10. A round uniform body of radius R, mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is

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(a) $\dfrac{g\sin\theta}{1-MR^2/I}$ (b) $\dfrac{g\sin\theta}{1+I/MR^2}$

(c) $\dfrac{g\sin\theta}{1+MR^2/I}$ (d) $\dfrac{g\sin\theta}{1-I/MR^2}$ *(2007)*

11. A circular disc of radius R is removed from a bigger circular disc of radius $2R$ such that the circumferences of the discs coincide. The centre of mass of the new disc is α/R from the centre of the bigger disc. The value of a is

(a) 1/4 (b) 1/3 (c) 1/2 (d) 1/6. *(2007)*

12. Four point masses, each of value m, are placed at the corners of a square $ABCD$ of side l. The moment of inertia of this system about an axis through A and parallel to BD is

(a) ml^2 (b) $2ml^2$ (c) $\sqrt{3}\,ml^2$ (d) $3ml^2$. *(2006)*

13. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity $\omega' =$

(a) $\dfrac{\omega m}{(m+2M)}$ (b) $\dfrac{\omega(m+2M)}{m}$

(c) $\dfrac{\omega(m-2M)}{(m+2M)}$ (d) $\dfrac{\omega m}{(m+M)}$. *(2006)*

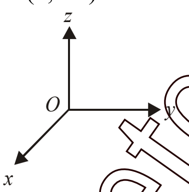
14. A force of $-F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point (1, –1) is

(a) $-F(\hat{i}-\hat{j})$

(b) $F(\hat{i}-\hat{j})$

(c) $-F(\hat{i}+\hat{j})$

(d) $F(\hat{i}+\hat{j})$.

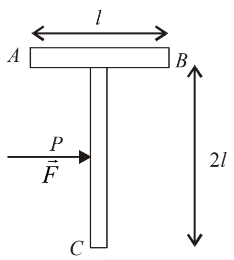


(2006)

15. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved, so as to keep the centre of mass at the same position?

(a) d (b) $\dfrac{m_2}{m_1}d$ (c) $\dfrac{m_1}{m_1+m_2}d$ (d) $\dfrac{m_1}{m_2}d$. *(2006)*

16. A T shaped object with dimensions shown in the figure, is lying on a smooth floor. A force $\vec{F}$ is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C.



(a) $\frac{4}{3}l$ (b) l (c) $\frac{3}{4}l$ (d) $\frac{3}{2}l$ *(2005)*

17. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}M$ and body C of mass $\frac{2}{3}M$. The center of mass of bodies B and C taken together shifts compared to that of body A towards

(a) body C (b) body B

(c) depends on height of breaking

(d) does not shift *(2005)*

18. The moment of inertia of a uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the center is

(a) Mr^2 (b) $\frac{1}{2}Mr^2$ (c) $\frac{1}{4}Mr^2$ (d) $\frac{2}{5}Mr^2$ *(2005)*

19. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that

(a) $I_A = I_B$ (b) $I_A > I_B$

(c) $I_A < I_B$ (d) $I_A/I_B = d_A/d_B$

where d_A and d_B are their densities. *(2004)*

20. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected?

(a) moment of inertia (b) angular momentum

(c) angular velocity (d) rotational kinetic energy. *(2004)*

21. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$ and $\vec{T}$ be the torque of this force about the origin. Then

(a) $\vec{r}\cdot\vec{T}=0$ and $\vec{F}\cdot\vec{T}\neq 0$ (b) $\vec{r}\cdot\vec{T}\neq 0$ and $\vec{F}\cdot\vec{T}=0$

(c) $\vec{r}\cdot\vec{T}\neq 0$ and $\vec{F}\cdot\vec{T}\neq 0$ (d) $\vec{r}\cdot\vec{T}=0$ and $\vec{F}\cdot\vec{T}=0$ *(2003)*

22. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is

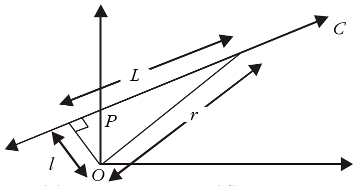
(a) $L/4$ (b) $2L$ (c) $4L$ (d) $L/2$. *(2003)*

23. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius $4R$ is made from an iron plate of thickness $t/4$. Then the relation between the moment of inertia I_X and I_Y is

(a) $I_Y = 32I_X$ (b) $I_Y = 16I_X$

(c) $I_Y = I_X$ (d) $I_Y = 64I_X$. *(2003)*

24. A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about P?



(a) mvL (b) mvl (c) mvr (d) zero. *(2002)*

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25. Moment of inertia of a circular wire of mass M and radius R about its diameter is

(a) $MR^2/2$ (b) MR^2
(c) $2MR^2$ (d) $MR^2/4$.

(2002)

26. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then maximum acceleration down the plane is for (no rolling)

(a) solid sphere (b) hollow sphere
(c) ring (d) all same. *(2002)*

27. Initial angular velocity of a circular disc of mass M is ω_1. Then two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?

(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$
(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$. *(2002)*

28. Two identical particles move towards each other with velocity $2v$ and v respectively. The velocity of centre of mass is

(a) v (b) $v/3$ (c) $v/2$ (d) zero.

(2002)

Answer Key

1.	(d)	**2.**	(b)	**3.**	(c)	**4.**	(d)	**5.**	(d)	**6.**	(b)
7.	(a)	**8.**	(d)	**9.**	(d)	**10.**	(b)	**11.**	(*)	**12.**	(d)
13.	(a)	**14.**	(d)	**15.**	(d)	**16.**	(a)	**17.**	(d)	**18.**	(b)
19.	(c)	**20.**	(b)	**21.**	(d)	**22.**	(a)	**23.**	(d)	**24.**	(d)
25.	(a)	**26.**	(d)	**27.**	(c)	**28.**	(c)				

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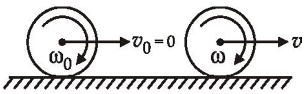
Explanations

1. **(d) :**



According to law of conservation at point of contact,

$mr^2\omega_0 = mvr + mr^2\omega$

$= mvr + mr^2\left(\frac{v}{r}\right)$

$mr^2\omega_0 = mvr + mvr$

$mr^2\omega_0 = 2mvr$

or $v = \frac{r\omega_0}{2}$

2. **(b) :** Torque exerted on pulley $\tau = FR$

or $\alpha = \frac{FR}{I}$ $\left(\because \alpha = \frac{\tau}{I}\right)$

Here, $F = (20t - 5t^2)$, $R = 2$ m, $I = 10$ kg m^2

$\therefore \alpha = \frac{(20t - 5t^2)\times 2}{10}$

$\alpha = (4t - t^2)$ or $\frac{d\omega}{dt} = (4t - t^2)dt$

$d\omega = (4t - t^2)dt$

On integrating, $\omega = 2t^2 - \frac{t^3}{3}$

At $t = 6$ s, $\omega = 0$

$\omega = \frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3}$

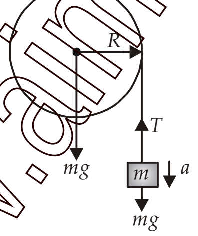
$d\theta = \left(2t^2 - \frac{t^3}{3}\right)dt$

On integration, $\theta = \frac{2t^3}{3} - \frac{t^4}{12}$

At, $t = 6$ s, $\theta = 36$ rad

$2\pi n = 36 \Rightarrow n = \frac{36}{2\pi} < 6$

3. **(c) :** The free body diagram of pulley and mass



$mg - T = ma$

$\therefore a = \frac{mg - T}{m}$ ...(i)

As per question, pulley to be consider as a circular disc.

$\therefore$ Angular acceleration of disc

$\alpha = \frac{\tau}{I}$ ...(ii)

Here, $\tau = T \times R$

and $I = \frac{1}{2}mR^2$ (For circular disc)

$\therefore T = \frac{mR\alpha}{2}$ (Using (ii))

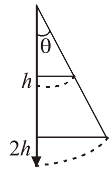
Therefore, $a = \frac{mg - \frac{mR\alpha}{2}}{m}$ (Using (i))

$ma = mg - \frac{ma}{2}$ $\left(\because \alpha = \frac{a}{R}\right)$

$\therefore a = \frac{2g}{3}$

4. **(d)**

5. **(d) :** The uniform rod of length l and mass m is swinging about an axis passing through the end. When the centre of mass is raised through h, the increase in potential energy is mgh. This is equal to the kinetic energy $= \frac{1}{2}I\omega^2$.



$\Rightarrow mgh = \frac{1}{2}\left(m\frac{l^2}{3}\right)\cdot\omega^2$. $\therefore h = \frac{l^2\cdot\omega^2}{6g}$.

6. **(b) :** $x_{C.M} = \dfrac{\int_0^L \left(\frac{k}{L^n}\cdot x^n \cdot dx\right)x}{\int_0^L \frac{k}{L^n}\cdot x^n \cdot dx}$

$\Rightarrow x_{C.M} = \dfrac{\int_0^L x^{n+1}dx}{\int_0^L x^n dx} = \frac{L^{n+2}}{n+2}\cdot\frac{(n+1)}{L^{n+1}}$

$\Rightarrow x_{C.M} = \frac{L(n+1)}{(n+2)}$

The variation of the centre of mass with x is given by

$\frac{dx}{dn} = L\left\{\frac{(n+2)1 - (n+1)}{(n+2)^2}\right\} = \frac{L}{(n+2)^2}$

If the rod has the same density as at $x = 0$ *i.e.*, $n = 0$, therefore uniform, the centre of mass would have been at $L/2$. As the density increases with length, the centre of mass shifts towards the right. Therefore it can only be (b).

7. **(a) :** For a rectangular sheet moment of inertia passing through O, perpendicular to the plate is

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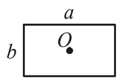
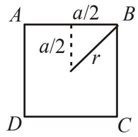
$$I_0 = M\left(\frac{a^2+b^2}{12}\right)$$



for square plate it is $\frac{Ma^2}{6}$.



$$r = \sqrt{\frac{a^2}{4}+\frac{a^2}{4}} = \frac{a}{\sqrt{2}}. \quad \therefore\ r^2 = \frac{a^2}{2}$$

$\therefore$ I about B parallel to the axis through O is

$$I_o + Md^2 = \frac{Ma^2}{6} + \frac{Ma^2}{2} = \frac{4Ma^2}{6}$$

$$I = \frac{2}{3}Ma^2$$

8. **(d) :** By perpendicular axes theorem,

$$I_{EF} = M\frac{a^2+b^2}{12} = \frac{M(a^2+a^2)}{12} = M\frac{2a^2}{12}$$

$$I_z = \frac{M(2a^2)}{12} + \frac{M(2a^2)}{12} = \frac{Ma^2}{3}.$$

By perpendicular axes theorem,

$$I_{AC} + I_{BD} = I_z \Rightarrow I_{AC} = \frac{I_z}{2} = \frac{Ma^2}{6}$$

By the same theorem $I_{EF} = \frac{I_z}{2} = \frac{Ma^2}{6}$

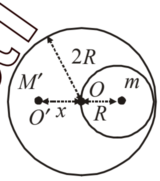
$\therefore$ $I_{AC} = I_{EF}$.

9. **(d) :** Central forces passes through axis of rotation so torque is zero.
If no external torque is acting on a particle, the angular momentum of a particle is constant.

10. **(b) :** Acceleration of a uniform body of radius R and mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal is given by

$$a = \frac{g\sin\theta}{1+\frac{I}{MR^2}}.$$

11. **(*) :** $(M' + m) = M = \pi(2R)^2 \cdot \sigma$
where σ = mass per unit area
$m = \sigma R^2 \cdot \sigma$, $M' = 3\pi R^2\sigma$



$$\frac{3\pi R^2\sigma \cdot x + \pi R^2\sigma \cdot R}{M} = 0$$

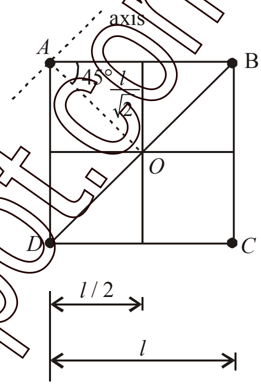
Because for the full disc, the centre of mass is at the centre O.

$$\Rightarrow \quad x = -\frac{R}{3} = \alpha R.$$

$$\therefore\ |\alpha| = \left|\frac{-1}{3}\right|.$$

The centre of mass is at $R/3$ to the left on the diameter of the original disc.
The question should be at a distance αR and not α/R.
None of the option is correct.

12. **(d) :** $AO\cos 45° = \frac{l}{2}$



$\therefore$ $AO \times \frac{1}{\sqrt{2}} = \frac{l}{2}$

or $AO = \frac{l}{\sqrt{2}}$

$I = I_D + I_B + I_C$

or $I = \frac{2ml^2}{2} + m\left(\frac{2l}{\sqrt{2}}\right)^2$

$I = \frac{2ml^2}{2} + \frac{4ml^2}{2}$

or $I = \frac{6ml^2}{2} = 3ml^2$.

13. **(a) :** Angular momentum is conserved
$\therefore$ $L_1 = L_2$
$\therefore$ $mR^2\omega = (mR^2 + 2MR^2)\omega' = R^2(m + 2M)\omega'$

or $\omega' = \frac{m\omega}{m+2M}$.

14. **(d) :** Torque $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{F} = -F\hat{k}$, $\vec{r} = \hat{i} - \hat{j}$

$$\vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 0 & 0 & -F \end{vmatrix}$$

$$= \hat{i}F - \hat{j}(-F) = F(\hat{i} + \hat{j}).$$

15. **(d) :** Let m_2 be moved by x so as to keep the centre of mass at the same position
$\therefore$ $m_1 d + m_2(-x) = 0$

or $m_1 d = m_2 x$ or $x = \frac{m_1}{m_2}d$.

16. **(a) :** It is a case of translation motion without rotation. The force should act at the centre of mass

$$Y_{cm} = \frac{(m \times 2l) + (2m \times l)}{m + 2m} = \frac{4l}{3}.$$

17. **(d) :** The centre of mass of bodies B and C taken together does not shift as no external force is applied horizontally.

18. **(b) :** $I = \frac{(\text{Mass of semicircular disc}) \times r^2}{2}$

or $I = \frac{Mr^2}{2}$.

19. **(c) :** For solid sphere, $I_A = \frac{2}{5}MR^2$

For hollow sphere, $I_B = \frac{2}{3}MR^2$

$\therefore$ $\frac{I_A}{I_B} = \frac{2MR^2}{5} \times \frac{3}{2MR^2} = \frac{3}{5}$

or $I_A < I_B$.

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20. **(b) :** Free space implies that no external torque is operating on the sphere. Internal changes are responsible for increase in radius of sphere. Here the law of conservation of angular momentum applies to the system.

21. **(d) :** $\because \quad \vec{T} = \vec{r} \times \vec{F}$

$\therefore \quad \vec{r} \cdot \vec{T} = \vec{r} \cdot (\vec{r} \times \vec{F}) = 0$

Also $\vec{F} \cdot \vec{T} = \vec{F} \cdot (\vec{r} \times \vec{F}) = 0$.

22. **(a) :** Angular momentum $L = I\omega$

Rotational kinetic energy $(K) = \frac{1}{2} I\omega^2$

$$\therefore \quad \frac{L}{K} = \frac{I\omega \times 2}{I\omega^2} = \frac{2}{\omega} \Rightarrow L = \frac{2K}{\omega}$$

$$\text{or} \quad \frac{L_1}{L_2} = \frac{K_1}{K_2} \times \frac{\omega_2}{\omega_1} = 2 \times 2 = 4$$

$$\therefore \quad L_2 = \frac{L_1}{4} = \frac{L}{4}.$$

23. **(d) :** Mass of disc $X = (\pi R^2 t)\sigma$ where σ = density

$$\therefore \quad I_X = \frac{MR^2}{2} = \frac{(\pi R^2 t\sigma)R^2}{2} = \frac{\pi R^4 \sigma t}{2}$$

Similarly, $I_Y = \frac{(\text{Mass})(4R)^2}{2} = \frac{\pi (4R)^2}{2} \frac{t}{4} \sigma \times 16 R^2$

or $\quad I_Y = 32\pi R^4 t\sigma$

$$\therefore \quad \frac{I_X}{I_Y} = \frac{\pi R^4 \sigma t}{2} \times \frac{1}{32\pi R^4 \sigma t} = \frac{1}{64}$$

$\therefore \quad I_Y = 64\, I_X$.

24. **(d) :** The particle moves with linear velocity v along line PC. The line of motion is through P. Hence angular momentum is zero.

25. **(a) :** A circular wire behaves like a ring

M.I. about its diameter $= \frac{MR^2}{2}$.

26. **(d) :** The bodies slide along inclined plane. They do not roll. Acceleration for each body down the plane = $g\sin\theta$. It is the same for each body.

27. **(c) :** Angular momentum of the system is conserved

$$\therefore \quad \frac{1}{2} MR^2 \omega_1 = 2mR^2 \omega + \frac{1}{2} MR^2 \omega$$

or $\quad M\omega_1 = (4m + M)\omega$

$$\text{or} \quad \omega = \frac{M\omega_1}{M + 4m}.$$

28. **(c) :** $v_c = \dfrac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

$$\text{or} \quad v_c = \frac{m(2v) + m(-v)}{m + m} = \frac{v}{2}.$$

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CHAPTER 6

GRAVITATION

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1. What is the minimum energy required to launch a satellite of mass *m* from the surface of a planet of mass *M* and radius *R* in a circular orbit at an altitude of 2*R*?

(a) $\frac{GmM}{3R}$ (b) $\frac{5GmM}{6R}$ (c) $\frac{2GmM}{3R}$ (d) $\frac{GmM}{2R}$ *(2013)*

2. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of *g* and *R* (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be

(a) 6.4×10^8 Joules (b) 6.4×10^9 Joules
(c) 6.4×10^{10} Joules (d) 6.4×10^{11} Joules *(2012)*

3. Two bodies of masses *m* and 4*m* are placed at a distance *r*. The gravitational potential at a point on the line joining them where the gravitational field is zero is

(a) zero (b) $-\frac{4Gm}{r}$ (c) $-\frac{6Gm}{r}$ (d) $-\frac{9Gm}{r}$ *(2011)*

4. The height at which the acceleration due to gravity becomes *g*/9 (where *g* = the acceleration due to gravity on the surface of the earth) in terms of *R*, the radius of the earth is

(a) 2*R* (b) $\frac{R}{\sqrt{2}}$ (c) *R*/2 (d) $\sqrt{2}R$ *(2009)*

5. *Directions : The following question contains statement-1 and statement-2. Of the four choices given, choose the one that best describes the two statements.*

(a) Statement-1 is true, statement-2 is false.
(b) Statement-1 is false, statement-2 is true.
(c) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.
(d) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.

Statement-1 : For a mass *M* kept at the centre of a cube of side *a*, the flux of gravitational field passing through its sides is 4π*GM*.

Statement-2 : If the direction of a field due to a point source is radial and its dependence on the distance *r* from the source is given as $1/r^2$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface. *(2008)*

6. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be

(a) 0.11 km s^{-1} (b) 1.1 km s^{-1}
(c) 11 km s^{-1} (d) 110 km s^{-1} *(2008)*

7. Average density of the earth

(a) is directly proportional to *g*
(b) is inversely proportional to *g*
(c) does not depend on *g*
(d) is a complex function of *g* *(2005)*

8. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere.
(you may take $G = 6.67 \times 10^{-11}$ Nm^2/kg^2)

(a) 6.67×10^{-9} J (b) 6.67×10^{-10} J
(c) 13.34×10^{-10} J (d) 3.33×10^{-10} J *(2005)*

9. The change in the value of *g* at a height *h* above the surface of the earth is the same as at a depth *d* below the surface of earth. When both *d* and *h* are much smaller than the radius of earth, then which of the following is correct?

(a) $d = 2h$ (b) $d = h$
(c) $d = h/2$ (d) $d = 3h/2$ *(2005)*

10. Suppose the gravitational force varies inversely as the n^{th} power of distance. Then the time period of a planet in circular orbit of radius *R* around the sun will be proportional to

(a) $R^{\left(\frac{n+1}{2}\right)}$ (b) $R^{\left(\frac{n-1}{2}\right)}$ (c) R^n (d) $R^{\left(\frac{n-2}{2}\right)}$. *(2004)*

11. If *g* is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass *m* raised from the surface of the earth to a height equal to the radius *R* of the earth is

(a) 2*mgR* (b) $\frac{1}{2}mgR$ (c) $\frac{1}{4}mgR$ (d) *mgR*. *(2004)*

12. The time period of an earth satellite in circular orbit is independent of

(a) the mass of the satellite
(b) radius of its orbit
(c) both the mass and radius of the orbit
(d) neither the mass of the satellite nor the radius of its orbit. *(2004)*

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13. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is

(a) gx (b) $\frac{gR}{R-x}$

(c) $\frac{gR^2}{R+x}$ (d) $\left(\frac{gR^2}{R+x}\right)^{1/2}$. *(2004)*

14. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, the escape velocity will be

(a) $11\sqrt{2}$ km/s (b) 22 km/s

(c) 11 km/s (d) $11/\sqrt{2}$ m/s. *(2003)*

15. Two spherical bodies of mass M and $5M$ and radii R and $2R$ respectively are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is

(a) $2.5R$ (b) $4.5R$ (c) $7.5R$ (d) $1.5R$. *(2003)*

16. The time period of a satellite of earth is 5 hour. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become

(a) 10 hour (b) 80 hour

(c) 40 hour (d) 20 hour. *(2003)*

17. The escape velocity of a body depends upon mass as

(a) m^0 (b) m^1 (c) m^2 (d) m^3. *(2002)*

18. The kinetic energy needed to project a body of mass m from the earth surface (radius R) to infinityis

(a) $mgR/2$ (b) $2mgR$

(c) mgR (d) $mgR/4$. *(2002)*

19. Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is

(a) $GMm/12R^2$ (b) $GMm/3R^2$

(c) $GMm/8R$ (d) $GMm/6R$. *(2002)*

20. If suddenly the gravitational force of attraction between Earth and a satellite revolving around it becomes zero, then the satellite will

(a) continue to move in its orbit with same velocity

(b) move tangentially to the original orbit in the same velocity

(c) become stationary in its orbit

(d) move towards the earth. *(2002)*

Answer Key

1.	(b)	**2.**	(c)	**3.**	(d)	**4.**	(a)	**5.**	(c)	**6.**	(d)
7.	(a)	**8.**	(b)	**9.**	(a)	**10.**	(a)	**11.**	(b)	**12.**	(a)
13.	(d)	**14.**	(c)	**15.**	(c)	**16.**	(c)	**17.**	(a)	**18.**	(c)
19.	(d)	**20.**	(b)								

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Explanations

1. **(b) :** Energy of the satellite on the surface of the planet is

$$E_i = \text{KE} + \text{PE} = 0 + \left(-\frac{GMm}{R}\right) = -\frac{GMm}{R}$$

If v is the velocity of the satellite at a distance $2R$ from the surface of the planet, then total energy of the satellite is

$$E_f = \frac{1}{2}mv^2 + \left(-\frac{GMm}{(R+2R)}\right)$$

$$= \frac{1}{2}m\left(\sqrt{\frac{GM}{(R+2R)}}\right)^2 - \frac{GMm}{3R}$$

$$= \frac{1}{2}\frac{GMm}{3R} - \frac{GMm}{3R} = -\frac{GMm}{6R}$$

$\therefore$ Minimum energy required to launch the satellite is

$$\Delta E = E_f - E_i = -\frac{GMm}{6R} - \left(-\frac{GMm}{R}\right)$$

$$= -\frac{GMm}{6R} + \frac{GMm}{R} = \frac{5GMm}{6R}$$

2. **(c) :** Energy required $= \dfrac{GMm}{R}$

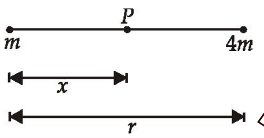
$$= gR^2 \times \frac{m}{R} \qquad \left(\because\ g = \frac{GM}{R^2}\right)$$

$= mgR$

$= 1000 \times 10 \times 6400 \times 10^3$

$= 64 \times 10^9$ J $= 6.4 \times 10^{10}$ J

3. **(d) :** Let x be the distance of the point P from the mass m where gravitational field is zero.



$$\therefore\ \frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2} \text{ or } \left(\frac{x}{(r-x)}\right)^2 = \frac{1}{4}$$

or $x = \dfrac{r}{3}$ ...(i)

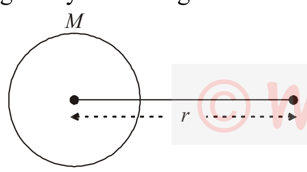
Gravitational potential at a point P is

$$= -\frac{Gm}{x} - \frac{G(4m)}{(r-x)}$$

$$= -\frac{Gm}{\left(\frac{r}{3}\right)} - \frac{G(4m)}{\left(r-\frac{r}{3}\right)} \qquad \text{(Using (i))}$$

$$= -\frac{3Gm}{r} - \frac{3G(4m)}{2r} = -9\frac{Gm}{r}$$

4. **(a) :** The acceleration due to gravity at a height h from the ground is given as $g/9$.

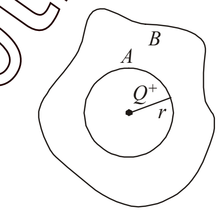


$$\frac{GM}{r^2} = \frac{GM}{R^2}\cdot\frac{1}{9}$$

$\therefore\ r = 3R$

The height above the ground is $2R$.

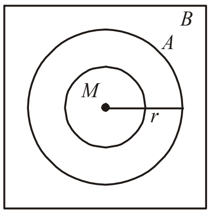
5. **(c) :** Let A be the Gaussian surface enclosing a spherical charge Q.



$$\vec{E}\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 . r^2}$$

$$\text{Flux } \phi = \vec{E}\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$



Every line passing through A, has to pass through B, whether B is a cube or any surface. It is only for Gaussian surface, the lines of field should be normal. Assuming the mass is a point mass.

$$\vec{g}, \text{gravitational field} = -\frac{GM}{r^2}$$

$$\text{Flux } \phi_g = |\vec{g}\cdot 4\pi r^2| = \frac{4\pi r^2 \cdot GM}{r^2} = 4\pi GM.$$

Here B is a cube. As explained earlier, whatever be the shape, all the lines passing through A are passing through B, although all the lines are not normal.

Statement-2 is correct because when the shape of the earth is spherical, area of the Gaussian surface is $4\pi r^2$. This ensures inverse square law.

6. **(d) :** $v_{escape} = \sqrt{\dfrac{2GM}{R}}$ for the earth

$v_e = 11$ kms^{-1}

Mass of the planet = 10 M_e, Radius of the planet = $R/10$.

$$\therefore\ v_e = \sqrt{\frac{2GM\times 10}{R/10}} = 10\times 11 = 110 \text{ km s}^{-1}$$

7. **(a) :** Density $= \dfrac{\text{Mass of earth}}{\text{Volume of earth}}$

$$\rho = \frac{M}{(4/3)\pi R^3} = \frac{3M}{4\pi R^3} \qquad \text{... (i)}$$

$$g = \frac{GM}{R^2} \qquad \text{... (ii)}$$

$$\therefore\ \frac{\rho}{g} = \frac{3M}{4\pi R^3}\times\frac{R^2}{GM} = \frac{3}{4\pi RG} \quad \text{or} \quad \rho = \frac{3}{4\pi RG}g$$

$\therefore$ Average density is directly proportional to g.

8. **(b) :** Gravitational force $F = \dfrac{Gm_1m_2}{R^2}$

$$\therefore\ dW = FdR = \frac{Gm_1m_2}{R^2}dR$$

$$\therefore\ \int_0^W dW = Gm_1m_2\int_R^\infty \frac{dR}{R^2} = Gm_1m_2\left[-\frac{1}{R}\right]_R^\infty = \frac{Gm_1m_2}{R}$$

$$\therefore\ \text{Workdone} = \frac{(6.67\times10^{-11})\times(100)\times(10\times10^{-3})}{10\times10^{-2}}$$

$= 6.67 \times 10^{-10}$ J.

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9. **(a) :** At height, $g_h = g\left(1-\frac{2h}{R}\right)$ where $h << R$

or $g - g_h = \frac{2hg}{R}$ or $\Delta g_h = \frac{2hg}{R}$ ... (i)

At depth, $g_d = g\left(1-\frac{d}{R}\right)$ where $d << R$

or $g - g_d = \frac{dg}{R}$

or $\Delta g_d = \frac{dg}{R}$ .... (ii)

From (i) and (ii), when $\Delta g_h = \Delta g_d$

$\frac{2hg}{R} = \frac{dg}{R}$ or $d = 2h$.

10. **(a) :** For motion of a planet in circular orbit,
Centripetal force = Gravitational force

$\therefore$ $mR\omega^2 = \frac{GMm}{R^n}$ or $\omega = \sqrt{\frac{GM}{R^{n+1}}}$

$\therefore$ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R^{n+1}}{GM}} = \frac{2\pi}{\sqrt{GM}} R^{\left(\frac{n+1}{2}\right)}$

$\therefore$ T is proportional to $R^{\left(\frac{n+1}{2}\right)}$.

11. **(b) :** Force on object $= \frac{GMm}{x^2}$ at x from centre of earth.

$\therefore$ Work done $= \frac{GMm}{x^2}dx$

$\therefore$ $\int$ Work done $= GMm \int_R^{2R} \frac{dx}{x^2}$

$\therefore$ Potential energy gained

$= GMm\left[-\frac{1}{x}\right]_R^{2R} = \frac{GMm \times 1}{2R}$

$\therefore$ Gain in P.E.

$= \frac{1}{2}mR\left(\frac{GM}{R^2}\right) = \frac{1}{2}mgR$ $\left[\because g = \frac{GM}{R^2}\right]$.

12. **(a) :** For a satellite
Centripetal force = Gravitational force

$\therefore$ $mR\omega^2 = \frac{GmM_e}{R^2}$ where $R = r_e + h$

or $\omega = \sqrt{\frac{GM_e}{R^3}} = \sqrt{\frac{GM_e}{(r_e+h)^3}}$

$\therefore$ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{(r_e+h)^3}{GM_e}}$

$\therefore$ T is independent of mass (m) of satellite.

13. **(d) :** For a satellite
centripetal force = Gravitational force

$\therefore$ $\frac{mv_0^2}{(R+x)} = \frac{GMm}{(R+x)^2}$

or $v_0^2 = \frac{GM}{(R+x)} = \frac{gR^2}{(R+x)}$ $\left[\because g = \frac{GM}{R^2}\right]$

or $v_0 = \sqrt{\frac{gR^2}{R+x}}$.

14. **(c) :** The escape velocity of a body does not depend on the angle of projection from earth.
It is 11 km/sec.

15. **(c) :** Let the spheres collide after time t, when the smaller sphere covered distance x_1 and bigger sphere covered distance x_2.
The gravitational force acting between two spheres depends on the distance which is a variable quantity.

The gravitational force, $F(x) = \frac{GM \times 5M}{(12R-x)^2}$

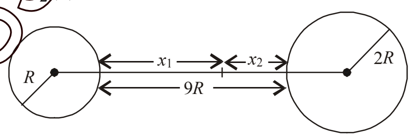
Acceleration of smaller body, $a_1(x) = \frac{G \times 5M}{(12R-x)^2}$

Acceleration of bigger body, $a_2(x) = \frac{GM}{(12R-x)^2}$

From equation of motion,

$x_1 = \frac{1}{2}a_1(x)t^2$ and $x_2 = \frac{1}{2}a_2(x)t^2$

$\Rightarrow \frac{x_1}{x_2} = \frac{a_1(x)}{a_2(x)} = 5 \Rightarrow x_1 = 5x_2$



We know that $x_1 + x_2 = 9R$

$x_1 + \frac{x_1}{5} = 9R$ $\therefore x_1 = \frac{45R}{6} = 7.5R$

Therefore the two spheres collide when the smaller sphere covered the distance of $7.5R$.

16. **(c) :** According to Kepler's law $T^2 \propto r^3$

$\therefore$ $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$ or $\frac{T_1}{T_2} = \frac{1}{8}$

or $T_2 = 8T_1 = 8 \times 5 = 40$ hour.

17. **(a) :** Escape velocity $= \sqrt{2gR} = \sqrt{\frac{2GM_e}{R}}$

Escape velocity does not depend on mass of body which escapes or it depends on m^0.

18. **(c) :** Escape velocity $v_e = \sqrt{2gR}$

$\therefore$ Kinetic energy

$= \frac{1}{2}mv_e^2 = \frac{1}{2}m \times 2gR = mgR$.

19. **(d) :** Energy = $(P.E.)_{3R} - (P.E.)_{2R}$

$= -\frac{GmM}{3R} - \left(-\frac{GmM}{2R}\right) = +\frac{GmM}{6R}$.

20. **(b) :** The centripetal and centrifugal forces disappear, the satellite has the tangential velocity and it will move in a straight line.
Compare Lorentzian force on charges in the cyclotron.

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CHAPTER 7

PROPERTIES OF SOLIDS AND LIQUIDS

1. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization.

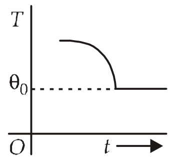
(a) $\frac{2T}{\rho L}$ (b) $\frac{\rho L}{T}$ (c) $\sqrt{\frac{T}{\rho L}}$ (d) $\frac{T}{\rho L}$ *(2013)*

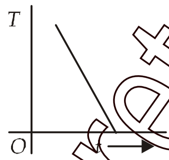
2. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density s at equilibrium position. The extension x_0 of the spring when it is in equilibrium is

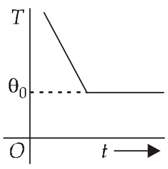
(a) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$ (b) $\frac{Mg}{k}$

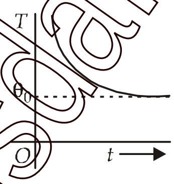
(c) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$ (d) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ *(2013)*

3. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time t will be closed to

(a) 

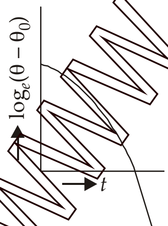
(b) 

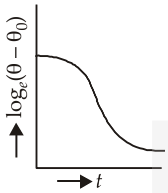
(c) 

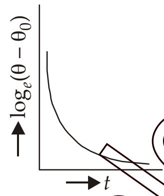
(d) 

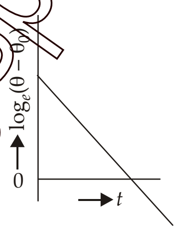
(2013)

4. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t is

(a) 

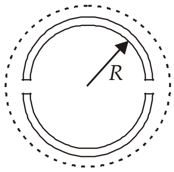
(b) 

(c) 

(d) 

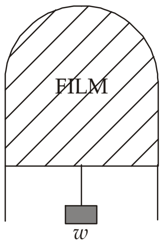
(2012)

5. A wooden wheel of radius R is made of two semicircular parts (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Young's modulus is Y, the force that one part of the wheel applies on the other part is



(a) $SY\alpha\Delta T$ (b) $\pi SY\alpha\Delta T$
(c) $2SY\alpha\Delta T$ (d) $2\pi SY\alpha\Delta T$ *(2012)*

6. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is



(a) 0.1 Nm^{-1}
(b) 0.05 Nm^{-1}
(c) 0.025 Nm^{-1}
(d) 0.0125 Nm^{-1} *(2012)*

7. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 m s^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to

(a) 5.0×10^{-3} m (b) 7.5×10^{-3} m
(c) 9.6×10^{-3} m (d) 3.6×10^{-3} m *(2011)*

8. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (Surface tension of soap solution = 0.03 N m^{-1})

(a) 4π mJ (b) 0.2π mJ (c) 2π mJ (d) 0.4π mJ *(2011)*

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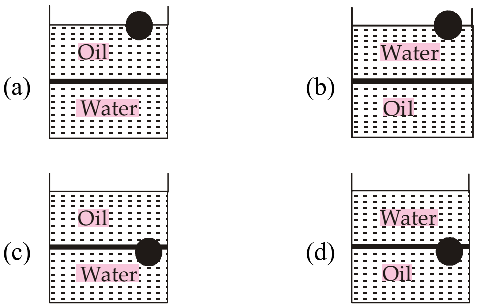
9. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is

(a) $\frac{b^2}{6a}$ (b) $\frac{b^2}{2a}$ (c) $\frac{b^2}{12a}$ (d) $\frac{b^2}{4a}$ *(2010)*

10. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position?

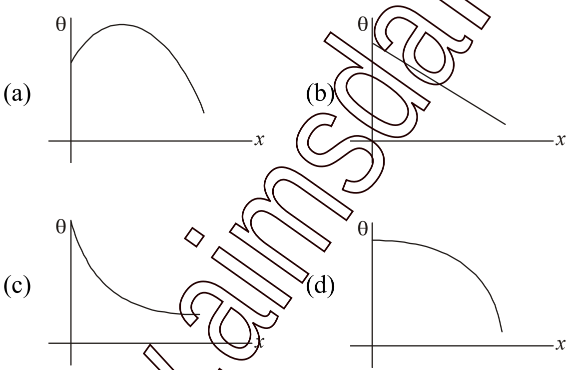


(2010)

11. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount?

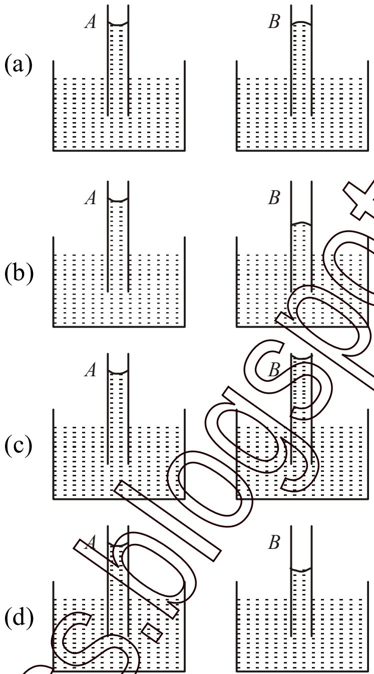
(a) F (b) $4F$ (c) $6F$ (d) $9F$ *(2009)*

12. A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figures?



(2009)

13. A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes?



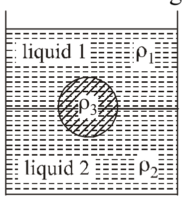
(2008)

14. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_2 ($\rho_2 < \rho_1$). Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, *i.e.*, $F_{\text{viscous}} = -kv^2$ ($k > 0$). The terminal speed of the ball is

(a) $\frac{Vg(\rho_1 - \rho_2)}{k}$ (b) $\sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$

(c) $\frac{Vg\rho_1}{k}$ (d) $\sqrt{\frac{Vg\rho_1}{k}}$ *(2008)*

15. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2 respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for ρ_1, ρ_2 and ρ_3?



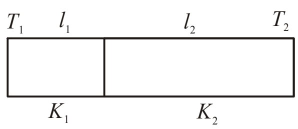
(a) $\rho_1 < \rho_3 < \rho_2$

(b) $\rho_3 < \rho_1 < \rho_2$

(c) $\rho_1 > \rho_3 > \rho_2$

(d) $\rho_1 < \rho_2 < \rho_3$ *(2008)*

16. One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of lengths l_1 and l_2 and thermal conductivities K_1 and K_2 respectively. The temperature at the interface of the two sections is



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(a) $\frac{(K_1l_1T_1 + K_2l_2T_2)}{(K_1l_1 + K_2l_2)}$ (b) $\frac{(K_2l_2T_1 + K_1l_1T_2)}{(K_1l_1 + K_2l_2)}$

(c) $\frac{(K_2l_1T_1 + K_1l_2T_2)}{(K_2l_1 + K_1l_2)}$ (d) $\frac{(K_1l_2T_1 + K_2l_1T_2)}{(K_1l_2 + K_2l_1)}$ *(2007)*

17. A wire elongates by l mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be (in mm)
(a) $l/2$ (b) l (c) $2l$ (d) zero. *(2006)*

18. If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3) find the terminal speed of a sphere of silver (density 10.5 kg/m^3) of the same size in the same liquid
(a) 0.2 m/s (b) 0.4 m/s
(c) 0.133 m/s (d) 0.1 m/s. *(2006)*

19. Assuming the sun to be a spherical body of radius R at a temperature of T K, evaluate the total radiant power, incident on earth, at a distance r from the sun.

(a) $\frac{R^2\sigma T^4}{r^2}$ (b) $\frac{4\pi r_0^2 R^2\sigma T^4}{r^2}$

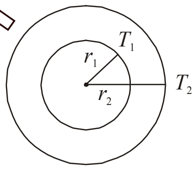
(c) $\frac{\pi r_0^2 R^2\sigma T^4}{r^2}$ (d) $\frac{r_0^2 R^2\sigma T^4}{4\pi r^2}$.

where r_0 is the radius of the earth and σ is Stefan's constant. *(2006)*

20. If S is stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is
(a) $2Y/S$ (b) $S/2Y$ (c) $2S^2Y$ (d) $\frac{S^2}{2Y}$ *(2005)*

21. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator the length of water column in the capillary tube will be
(a) 4 cm (b) 20 cm (c) 8 cm (d) 10 cm *(2005)*

22. The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperatures T_1 and T_2, respectively. The radial rate of flow of heat in a substance between the two concentric spheres is proportional to



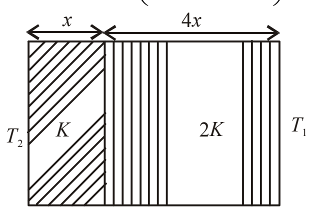
(a) $\frac{r_1r_2}{(r_2 - r_1)}$
(b) $(r_2 - r_1)$
(c) $\frac{(r_2 - r_1)}{r_1r_2}$
(d) $\ln\left(\frac{r_2}{r_1}\right)$ *(2005)*

23. If two soap bubbles of different radii are connected by a tube,
(a) air flows from the bigger bubble to the smaller bubble till the sizes become equal
(b) air flows from the bigger bubble to the smaller bubble till the sizes are interchanged
(c) air flows from the smaller bubble to the bigger
(d) there is no flow of air. *(2004)*

24. Spherical balls of radius R are falling in a viscous fluid of viscosity h with a velocity v. The retarding viscous force acting on the spherical ball is
(a) directly proportional to R but inversely proportional to v
(b) directly proportional to both radius R and velocity v
(c) inversely proportional to both radius R and velocity v
(d) inversely proportional to R but directly proportional to velocity v. *(2004)*

25. A wire fixed at the upper end stretches by length l by applying a force F. The work done in stretching is
(a) $F/2l$ (b) Fl (c) $2Fl$ (d) $Fl/2$. *(2004)*

26. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right)f$, with f equal to



(a) 1 (b) 1/2
(c) 2/3 (d) 1/3. *(2004)*

27. A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is
(a) E/c (b) $2E/c$
(c) Ec (d) E/c^2. *(2004)*

28. If the temperature of the sun were to increase from T to $2T$ and its radius from R to $2R$, then the ratio of the radiant energy received on earth to what it was previously will be
(a) 4 (b) 16
(c) 32 (d) 64. *(2004)*

29. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is
(a) 0.2 J (b) 10 J (c) 20 J (d) 0.1 J. *(2003)*

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30. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and the surroundings, and n is equal to
(a) two (b) three
(c) four (d) one. *(2003)*

31. The earth radiates in the infra-red region of the spectrum. The wavelength of the maximum intensity of the spectrum is correctly given by
(a) Rayleigh Jeans law
(b) Planck's law of radiation
(c) Stefan's law of radiation
(d) Wien's law. *(2003)*

32. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom is
(a) 10 (b) 20 (c) 25.5 (d) 5. *(2002)*

33. Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K respectively. The ratio of the energy radiated per second by the first sphere to that by the second is
(a) 1 : 1 (b) 16 : 1 (c) 4 : 1 (d) 1 : 9. *(2002)*

34. Which of the following is more close to a black body?
(a) black board paint
(b) green leaves
(c) black holes
(d) red roses. *(2002)*

Answer Key

1.	(a)	**2.**	(d)	**3.**	(d)	**4.**	(d)	**5.**	(c)	**6.**	(c)
7.	(d)	**8.**	(d)	**9.**	(d)	**10.**	(c)	**11.**	(d)	**12.**	(b)
13.	(d)	**14.**	(b)	**15.**	(a)	**16.**	(d)	**17.**	(b)	**18.**	(d)
19.	(c)	**20.**	(d)	**21.**	(b)	**22.**	(a)	**23.**	(c)	**24.**	(b)
25.	(d)	**26.**	(d)	**27.**	(b)	**28.**	(d)	**29.**	(d)	**30.**	(d)
31.	(d)	**32.**	(b)	**33.**	(a)	**34.**	(a)				

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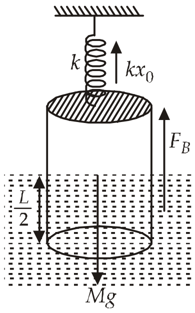
Explanations

1. **(a)**

2. **(d) :** Let k be the spring constant of spring and it gets extended by length x_0 in equilibrium position.
In equilibrium,
$kx_0 + F_B = Mg$



$$kx_0 + \sigma\frac{L}{2}Ag = Mg$$

$$x_0 = \frac{Mg - \frac{\sigma LAg}{2}}{k}$$

$$= \frac{Mg}{k}\left(1 - \frac{\sigma LA}{2M}\right)$$

3. **(d) :** According to Newton's law of cooling the option (d) represents the correct graph.

4. **(d) :** According to Newton's law of cooling

$$\frac{d\theta}{dt} = -k(\theta - \theta_0) \text{ or } \frac{d\theta}{\theta - \theta_0} = -kdt$$

Integrating both sides, we get $\int\frac{d\theta}{\theta - \theta_0} = \int -kdt$

$\log_e(\theta - \theta_0) = -kt + C$

where C is a constant of integration.

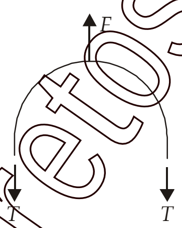
So, the graph between $\log_e(\theta - \theta_0)$ and t is a straight line with a negative slope. Option (d) represents the correct graph.

5. **(c) :** Increase in length, $\Delta L = L\alpha\Delta T$

$$\therefore \frac{\Delta L}{L} = \alpha\Delta T$$

The thermal stress developed is

$$\frac{T}{S} = Y\frac{\Delta L}{L} = Y\alpha\Delta T$$



or $T = SY\alpha\Delta T$

From FBD of one part of the wheel,

or $F = 2T$

Where, F is the force that one part of the wheel applies on the other part.

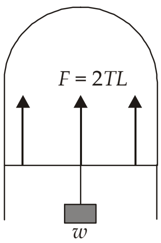
$\therefore$ $F = 2SY\alpha\Delta T$

6. **(c) :** The force due to the surface tension will balance the weight.

$F = w$

$2TL = w$

$$T = \frac{w}{2L}$$



Substituting the given values, we get

$$T = \frac{1.5\times10^{-2}\text{ N}}{2\times30\times10^{-2}\text{ m}} = 0.025\text{ Nm}^{-1}$$

7. **(d) :** Here, $d_1 = 8 \times 10^{-3}$ m

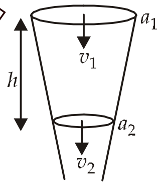
$v_1 = 0.4$ m s^{-1}

$h = 0.2$ m

According to equation of motion,

$$v_2 = \sqrt{v_1^2 + 2gh} = \sqrt{(0.4)^2 + 2\times10\times0.2}$$

≈ 2 m s^{-1}



$\therefore$ According to equation of continuity

$$a_1v_1 = a_2v_2$$

$$\pi\times\left(\frac{8\times10^{-3}}{2}\right)^2\times0.4 = \pi\times\left(\frac{d_2}{2}\right)^2\times2$$

$d_2 = 3.6 \times 10^{-3}$ m

8. **(d) :** Here, surface tension, $S = 0.03$ N m^{-1}

$r_1 = 3$ cm $= 3 \times 10^{-2}$ m, $r_2 = 5$ cm $= 5 \times 10^{-2}$ m

Since bubble has two surfaces,

Initial surface area of the bubble

$= 2 \times 4\pi r_1^2 = 2 \times 4\pi \times (3 \times 10^{-2})^2$

$= 72\pi \times 10^{-4}$ m^2

Final surface area of the bubble

$= 2 \times 4\pi r_2^2 = 2 \times 4\pi(5 \times 10^{-2})^2 = 200\pi \times 10^{-4}$ m^2

Increase in surface energy

$= 200\pi \times 10^{-4} - 72\pi \times 10^{-4} = 128\pi \times 10^{-4}$

$\therefore$ Work done $= S \times$ increase in surface energy

$= 0.03 \times 128 \times \pi \times 10^{-4} = 3.84\pi \times 10^{-4}$

$= 4\pi \times 10^{-4}$ J $= 0.4\pi$ mJ

9. **(d) :** $U = \frac{a}{x^{12}} - \frac{b}{x^6}$

$$\text{Force, } F = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{a}{x^{12}} - \frac{b}{x^6}\right)$$

$$= -\left[\frac{-12a}{x^{13}} + \frac{6b}{x^7}\right] = \left[\frac{12a}{x^{13}} - \frac{6b}{x^7}\right]$$

At equilibrium $F = 0$

$$\therefore \frac{12a}{x^{13}} - \frac{6b}{x^7} = 0 \text{ or } x^6 = \frac{2a}{b}$$

$$U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)}$$

$$= \frac{ab^2}{4a^2} - \frac{b^2}{2a} = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$$

$U(x = \infty) = 0$

$D = [U(x = \infty) - U_{\text{at equilibrium}}]$

$$= \left[0 - \left(-\frac{b^2}{4a}\right)\right] = \frac{b^2}{4a}$$

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10. (c) : As $\rho_{oil} < \rho_{water}$, so oil should be over the water. As $\rho > \rho_{oil}$, so the ball will sink in the oil but $\rho < \rho_{water}$ so it will float in the water.
Hence option (c) is correct.

11. (d) : For the same material, Young's modulus is the same and it is given that the volume is the same and the area of cross-section for the wire l_1 is A and that of l_2 is $3A$.

$V = V_1 = V_2$

$V = A \times l_1 = 3A \times l_2 \quad \Rightarrow \quad l_2 = l_1/3$

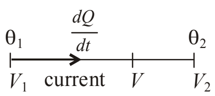
$$Y = \frac{F/A}{\Delta l/l} \Rightarrow F_1 = YA\frac{\Delta l_1}{l_1}$$

$$F_2 = Y3A\frac{\Delta l_2}{l_2}$$

Given $\Delta l_1 = \Delta l_2 = \Delta x$ (for the same extension)

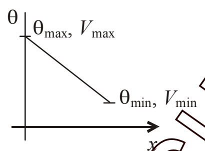
$$\therefore \quad F_2 = Y \cdot 3A \cdot \frac{\Delta x}{l_1/3} = 9 \cdot \left(\frac{YA\Delta x}{l_1}\right) = 9F_1 \text{ or } 9F.$$

12. (b) : Heat flow can be compared to charges flowing in a conductor.
Current is the same.

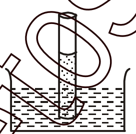


The potential difference $V_1 - V$ at any point $= I \times$ Resistance $= I \times \frac{\rho l}{A}$

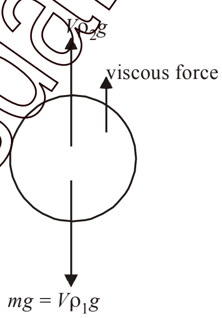
Potential difference is $\propto l$ but negative.
As l increases, potential decreases (temperature decreases) but it is a straight line function.
Potential difference is proportional to resistance (thermal as well as electric).



13. (d) : The force acting upwards $2\pi rT \simeq h\pi r^2\rho g$, the force acting down or $T \propto h$ without making finer corrections. Soap reduces the surface tension of water. The height of liquid supported decreases. But it is also a wetting agent. Therefore the meniscus will not be convex as in mercury. Therefore (d).



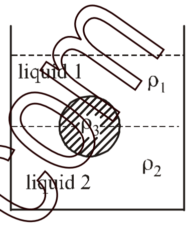
14. (b) : The forces acting on the solid ball when it is falling through a liquid are mg downwards, thrust by Archimedes principle upwards and the force due to the force of friction also acting upwards. The viscous force rapidly increases with velocity, attaining a maximum when the ball reaches the terminal velocity. Then the acceleration is zero. $mg - V\rho_2 g - kv^2 = ma$ where V is volume, v is the terminal velocity. When the ball is moving with terminal velocity $a = 0$. Therefore $V\rho_1 g - V\rho_2 g - kv^2 = 0$.



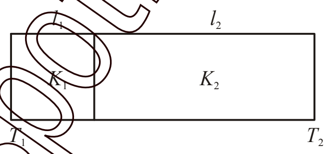
$$\Rightarrow v = \sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}.$$

15. (a) : The liquid 1 is over liquid 2. Therefore $\rho_1 < \rho_2$. If ρ_3 had been greater than ρ_2, it will not be partially inside but anywhere inside liquid 2 if $\rho_3 = \rho_2$ or it would have sunk totally if ρ_3 had been greater than ρ_2.

$\therefore \quad \rho_1 < \rho_3 < \rho_2$.



16. (d) : Let T be the temperature of the interface.
Since two section of rod are in series, rate of flow of heat in them will be equal



$$\therefore \quad \frac{K_1 A[T_1 - T]}{l_1} = \frac{K_2 A[T - T_2]}{l_2}$$

or $\quad K_1 l_2 (T_1 - T) = K_2 l_1 (T - T_2)$

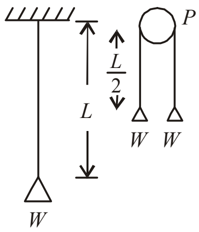
or $\quad T(K_1 l_2 + K_2 l_1) = K_1 l_2 T_1 + K_2 l_1 T_2$

$$\text{or} \quad T = \frac{K_1 l_2 T_1 + K_2 l_1 T_2}{K_1 l_2 + K_2 l_1}.$$

17. (b) : $Y = \frac{\text{Force} \times L}{A \times l} = \frac{WL}{Al}$

$\therefore \quad l = \frac{WL}{AY}$

Due to pulley arrangement, the length of wire is $L/2$ on each side and so the elongation will be $l/2$. For both sides, elongation $= l$.



18. (d) : Terminal velocity $= v$
viscous force upwards = weight of sphere downwards

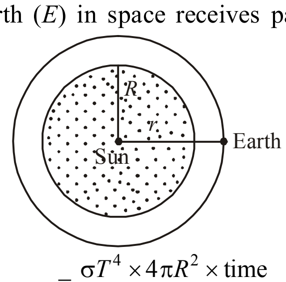
$$\text{or} \quad 6\pi\eta rv = \left(\frac{4}{3}\pi r^3\right)(\rho - \sigma)g$$

For gold and silver spheres falling in viscous liquid,

$$\therefore \quad \frac{v_g}{v_s} = \frac{\rho_g - \sigma}{\rho_s - \sigma} = \frac{19.5 - 1.5}{10.5 - 1.5} = \frac{18}{9} = \frac{2}{1}$$

$$\text{or} \quad v_s = \frac{v_g}{2} = \frac{0.2}{2} = 0.1 \text{ m/s}.$$

19. (c) : Energy radiated by sun, according to Stefan's law, $E = \sigma T^4 \times (\text{area } 4\pi R^2)$ (time)
This energy is spread around sun in space, in a sphere of radius r. Earth (E) in space receives part of this energy.



$$\frac{\text{Energy}}{\text{Area of envelope}} = \frac{\sigma T^4 \times 4\pi R^2 \times \text{time}}{4\pi r^2}$$

Energy incident per unit area on earth $= \frac{\sigma T^4 R^2 \times \text{time}}{r^2}$

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$\therefore$ Power incident per unit area on earth $= \left(\frac{R^2\sigma T^4}{r^2}\right)$

$\therefore$ Power incident on earth $= \pi r_0^{\,2} \times \frac{R^2\sigma T^4}{r^2}$

20. (d) : Energy stored per unit volume

$$= \frac{1}{2} \times \text{stress} \times \text{strain}$$

$$= \frac{\text{Stress} \times \text{stress}}{2Y} = \frac{S^2}{2Y}.$$

21. (b) : In a freely falling elevator $g = 0$
Water will rise to the full length i.e., 20 cm to tube.

22. (a) : For conduction from inner sphere to outer one,

$$dQ = -KA\frac{dT}{dr} \times (\text{time } dt)$$

or $\frac{dQ}{dt} = -K \times (4\pi r^2)\frac{dT}{dr}$

$\therefore$ Radial rate of flow $Q = -4\pi K r^2 \frac{dT}{dr}$

$\therefore$ $Q\int_{r_1}^{r_2} \frac{dr}{r^2} = -4\pi K \int_{T_1}^{T_2} dT$

or $Q\left[\frac{r_1 - r_2}{r_1 r_2}\right] = 4\pi K\left[T_2 - T_1\right]$

or $Q = \frac{4\pi K(T_1 - T_2)r_1 r_2}{(r_2 - r_1)}$

$\therefore$ Q is proportional to $\left(\frac{r_1 r_2}{r_2 - r_1}\right)$.

23. (c) : Pressure inside the bubble $= P_0 + \frac{4T}{r}$
Smaller the radius, greater will be the pressure. Air flows from higher pressure to lower pressure. Hence air flows from the smaller bubble to the bigger.

24. (b) : Retarding viscous force $= 6\pi\eta R v$
obviously option (b) holds goods.

25. (d) : Young's modulus $Y = \frac{FL}{Al}$ .... (i)

$\therefore$ $F = \frac{YAl}{L}$

or $dW = F\,dl = \frac{YAl(dl)}{L}$

or $\int dW = \frac{YA}{L}\int_0^l l\,dl = \frac{YAl^2}{2L}$

or Workdone $= \frac{YAl^2}{2L}$ .... (ii)

From (i) and (ii)

Workdone $= \frac{Fl}{2}$.

26. (d) : From first surface, $Q_1 = \frac{KA(T_2 - T)t}{x}$

From second surface, $Q_2 = \frac{(2K)A(T - T_1)t}{(4x)}$

At steady state,

$$Q_1 = Q_2 \Rightarrow \frac{KA(T_2 - T)t}{x} = \frac{2KA(T - T_1)t}{4x}$$

or $2(T_2 - T) = (T - T_1)$

or $T = \frac{2T_2 + T_1}{3}$

$\therefore$ $Q_1 = \frac{KA}{x}\left[T_2 - \frac{2T_2 + T_1}{3}\right]t$

or $\left[\frac{A(T_2 - T_1)K}{x}f\right] = \frac{KA}{x}\left[\frac{T_2 - T_1}{3}\right] \times 1$

or $f = \frac{1}{3}$

27. (b) : Initial momentum $= E/c$
Final momentum $= -E/c$

$\therefore$ Change of momentum $= \frac{E}{c} - \left(-\frac{E}{c}\right) = \frac{2E}{c}$

$\therefore$ Momentum transferred to surface $= \frac{2E}{c}$.

28. (d) : According to Stefan's law,
Radiant energy $E = (\sigma T^4) \times$ area $\times$ time

$\therefore$ $\frac{E_2}{E_1} = \frac{\sigma(2T)^4 \times 4\pi(2R)^2 \times t}{\sigma T^4 \times (4\pi R)^2 \times t} = 16 \times 4$

$\therefore$ $\frac{E_2}{E_1} = 64.$

29. (d) : Elastic energy per unit volume

$$= \frac{1}{2} \times \text{stress} \times \text{strain}$$

$\therefore$ Elastic energy

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta L}{L} \times (AL)$$

$$= \frac{1}{2}F\Delta L = \frac{1}{2} \times 200 \times 10^{-3} = 0.1\text{ J}.$$

30. (d) : According to Newton's law of cooling, rate of cooling is proportional to $\Delta\theta$.
$\therefore$ $(\Delta\theta)^n = (\Delta\theta)$ or $n = 1$.

31. (d) : Wien's law

32. (b) : $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20$ m/s.

33. (a) : Energy radiated
$E = \sigma T^4 \times$ (area $4\pi R^2$) $\times$ time $\times$ e

$$\frac{E_1}{E_2} = \frac{(4000)^4 \times (1)^2 \times 1 \times 4\pi\sigma e}{(2000)^4 \times (4)^2 \times 1 \times 4\pi\sigma e} = \frac{1}{1}.$$

34. (a) : A good absorber is a good emitter but black holes do not emit all radiations.

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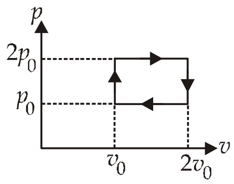
CHAPTER 8

THERMODYNAMICS

1. The above p-v diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is



(a) $4p_0v_0$ (b) p_0v_0

(c) $\left(\frac{13}{2}\right)p_0v_0$ (d) $\left(\frac{11}{2}\right)p_0v_0$ *(2013)*

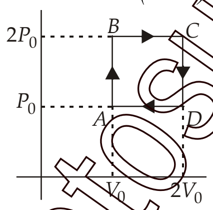
2. A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be

(a) 1200 K (b) 750 K

(c) 600 K

(d) efficiency of Carnot engine cannot be made larger than 50% *(2012)*

3. Helium gas goes through a cycle $ABCDA$ (consisting of two isochoric and two isobaric lines) as shown in figure. Efficiency of this cycle is nearly (Assume the gas to be close to ideal gas)



(a) 9.1% (b) 10.5% (c) 12.5% (d) 15.4% *(2012)*

4. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is

(a) $\frac{(T_1+T_2+T_3)}{3}$ (b) $\frac{n_1T_1+n_2T_2+n_3T_3}{n_1+n_2+n_3}$

(c) $\frac{n_1T_1^2+n_2T_2^2+n_3T_3^2}{n_1T_1+n_2T_2+n_3T_3}$ (d) $\frac{n_1^2T_1^2+n_2^2T_2^2+n_3^2T_3^2}{n_1T_1+n_2T_2+n_3T_3}$ *(2011)*

5. A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively

(a) 372 K and 310 K (b) 372 K and 330 K

(c) 330 K and 268 K (d) 310 K and 248 K *(2011)*

6. 100 g of water is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J kg^{-1} K^{-1})

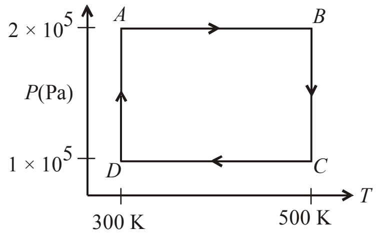
(a) 4.2 kJ (b) 8.4 kJ (c) 84 kJ (d) 2.1 kJ *(2011)*

7. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to $32V$, the efficiency of the engine is

(a) 0.25 (b) 0.5 (c) 0.75 (d) 0.99 *(2010)*

Directions: Question numbers 8, 9 and 10 are based on the following paragraph.

Two moles of helium gas are taken over the cycle $ABCDA$, as shown in the P - T diagram.



8. Assuming the gas to be ideal the work done on the gas in taking it from A to B is

(a) $200R$ (b) $300R$ (c) $400R$ (d) $500R$

9. The work done on the gas in taking it from D to A is

(a) $-414R$ (b) $+414R$ (c) $-690R$ (d) $+690R$

10. The net work done on the gas in the cycle $ABCDA$ is

(a) zero (b) $276R$ (c) $1076R$ (d) $1904R$ *(2009)*

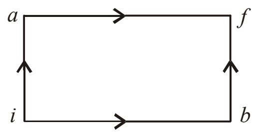
11. An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure P_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure P_2 and temperature T_2. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be

(a) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_1+P_2V_2T_2}$ (b) $\frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_2+P_2V_2T_1}$

(c) $\frac{P_1V_1T_1+P_2V_2T_2}{P_1V_1+P_2V_2}$ (d) $\frac{P_1V_1T_2+P_2V_2T_1}{P_1V_1+P_2V_2}$ *(2004, 2008)*

12. A Carnot engine, having an efficiency of $\eta = 1/10$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is
(a) 100 J (b) 99 J (c) 90 J (d) 1 J
(2007)

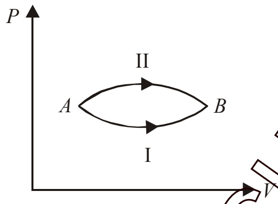
13. When a system is taken from state i to state f along the path iaf, it is found that $Q = 50$ cal and $W = 20$ cal. Along the path ibf $Q = 36$ cal. W along the path ibf is



(a) 14 cal (b) 6 cal (c) 16 cal (d) 66 cal
(2007)

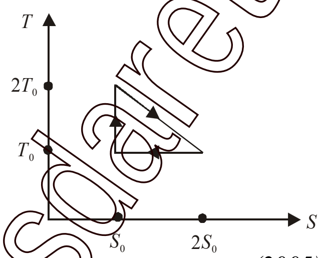
14. The work of 146 kJ is performed in order to compress one kilo mole of gas adiabatically and in this process the temperature of the gas increases by 7°C. The gas is (R = 8.3 J mol^{-1} K^{-1})
(a) monoatomic
(b) diatomic (c) triatomic
(d) a mixture of monoatomic and diatomic.
(2006)

15. A system goes from A to B *via* two processes I and II as shown in figure. If ΔU_1 and ΔU_2 are the changes in internal energies in the processes I and II respectively, then



(a) $\Delta U_2 > \Delta U_1$
(b) $\Delta U_2 < \Delta U_2$
(c) $\Delta U_1 = \Delta U_2$
(d) relation between ΔU_1 and ΔU_2 cannot be determined
(2005)

16. The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is



(a) 1/3
(b) 2/3
(c) 1/2
(d) 1/4
(2005)

17. Which of the following is incorrect regarding the first law of thermodynamics?
(a) It introduces the concept of the internal energy
(b) It introduces the concept of entropy
(c) It is not applicable to any cyclic process
(d) It is a restatement of the principle of conservation of energy
(2005)

18. Which of the following statements is correct for any thermodynamic system?
(a) The internal energy changes in all processes.
(b) Internal energy and entropy are state functions.
(c) The change in entropy can never be zero.
(d) The work done in an adiabatic process is always zero.
(2004)

19. A Carnot engine takes 3×10^6 cal of heat from a reservoir at 627°C, and gives it to a sink at 27°C. The work done by the engine is
(a) 4.2×10^6 J (b) 8.4×10^6 J
(c) 16.8×10^6 J (d) zero. *(2003)*

20. Which of the following parameters does not characterize the thermodynamic state of matter?
(a) temperature (b) pressure
(c) work (d) volume. *(2003)*

21. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_P/C_V for the gas is
(a) 4/3 (b) 2 (c) 5/3 (d) 3/2.
(2003)

22. "Heat cannot by itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of
(a) second law of thermodynamics
(b) conservation of momentum
(c) conservation of mass
(d) first law of thermodynamics. *(2003)*

23. Even Carnot engine cannot give 100% efficiency because we cannot
(a) prevent radiation (b) find ideal sources
(c) reach absolute zero temperature
(d) eliminate friction. *(2002)*

24. Which statement is incorrect?
(a) all reversible cycles have same efficiency
(b) reversible cycle has more efficiency than an irreversible one
(c) Carnot cycle is a reversible one
(d) Carnot cycle has the maximum efficiency in all cycles.
(2002)

25. Heat given to a body which raises its temperature by 1°C is
(a) water equivalent (b) thermal capacity
(c) specific heat (d) temperature gradient.
(2002)

Answer Key

1.	(c)	**2.**	(b)	**3.**	(d)	**4.**	(b)	**5.**	(a)	**6.**	(b)
7.	(c)	**8.**	(c)	**9.**	(b)	**10.**	(b)	**11.**	(b)	**12.**	(c)
13.	(b)	**14.**	(b)	**15.**	(c)	**16.**	(a)	**17.**	(b, c)	**18.**	(b)
19.	(b)	**20.**	(c)	**21.**	(d)	**22.**	(a)	**23.**	(c)	**24.**	(a)
25.	(b)										

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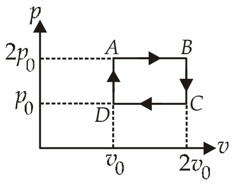
Explanations

1. **(c) :** Heat is extracted from the source in path DA and AB.



Along path DA, volume is constant.
Hence,
$\Delta Q_{DA} = nC_v\Delta T = nC_v(T_A - T_D)$
According to ideal gas equation

$$pv = nRT \quad \text{or} \quad T = \frac{pv}{nR}$$

For a monoatomic gas, $C_v = \frac{3}{2}R$

$$\therefore \quad \Delta Q_{DA} = n\left(\frac{3}{2}R\right)\left[\frac{2p_0v_0}{nR} - \frac{p_0v_0}{nR}\right] = \frac{3}{2}p_0v_0$$

Along the path AB, pressure is constant. Hence
$\Delta Q_{AB} = nC_p\Delta T = nC_p(T_B - T_A)$

For monoatomic gas, $C_p = \frac{5}{2}R$

$$\therefore \quad \Delta Q_{AB} = n\left(\frac{5}{2}R\right)\left[\frac{2p_0 2v_0}{nR} - \frac{2p_0v_0}{nR}\right] = \frac{10}{2}p_0v_0$$

$\therefore$ The amount of heat extracted from the source in a single cycle is

$$\Delta Q = \Delta Q_{DA} + \Delta Q_{AB}$$

$$= \frac{3}{2}p_0v_0 + \frac{10}{2}p_0v_0 = \frac{13}{2}p_0v_0$$

2. **(b) :** Efficiency of Carnot engine,

$$\eta = 1 - \frac{T_2}{T_1}$$

where T_1 is the temperature of the source and T_2 is the temperature of the sink.
For 1st case
$\eta = 40\%$, $T_1 = 500$ K

$$\therefore \quad \frac{40}{100} = 1 - \frac{T_2}{500} \Rightarrow \frac{T_2}{500} = 1 - \frac{40}{100} = \frac{3}{5}$$

$$T_2 = \frac{3}{5} \times 500 = 300 \text{ K}$$

For 2nd case
$\eta = 60\%$, $T_2 = 300$ K

$$\therefore \quad \frac{60}{100} = 1 - \frac{300}{T_1} \Rightarrow \frac{300}{T_1} = 1 - \frac{60}{100} = \frac{2}{5}$$

$$T_1 = \frac{5}{2} \times 300 = 750 \text{ K}$$

3. **(d) :** In case of a cyclic process, work done is equal to the area under the cycle and is taken to be positive if the cycle is clockwise.
$\therefore$ Work done by the gas
W = Area of the rectangle $ABCD = P_0V_0$
Helium gas is a monoatomic gas.

$$\therefore \quad C_V = \frac{3}{2}R \text{ and } C_P = \frac{5}{2}R$$

Along the path AB, heat supplied to the gas at constant volume,

$$\therefore \Delta Q_{AB} = nC_V\Delta T = n\frac{3}{2}R\Delta T = \frac{3}{2}V_0\Delta P = \frac{3}{2}P_0V_0$$

Along the path BC, heat supplied to the gas at constant pressure,

$$\therefore \Delta Q_{BC} = nC_P\Delta T = n\frac{5}{2}R\Delta T = \frac{5}{2}(2P_0)\Delta V = 5P_0V_0$$

Along the path CD and DA, heat is rejected by the gas

$$\text{Efficiency, } \eta = \frac{\text{Work done by the gas}}{\text{Heat supplied to the gas}} \times 100$$

$$= \frac{P_0V_0}{\frac{3}{2}P_0V_0 + 5P_0V_0} \times 100 = \frac{200}{13} = 15.4\%$$

4. **(b) :** The final temperature of the mixture is

$$T_{\text{mixture}} = \frac{T_1n_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$$

5. **(a) :** The efficiency of Carnot engine,

$$\eta = \left(1 - \frac{T_2}{T_1}\right)$$

$$\therefore \quad \frac{1}{6} = \left(1 - \frac{T_2}{T_1}\right) \qquad \left(\text{Given, } \eta = \frac{1}{6}\right)$$

$$\frac{T_2}{T_1} = \frac{5}{6} \Rightarrow T_1 = \frac{6T_2}{5} \qquad \text{...(i)}$$

As per question, when T_2 is lowered by 62 K, then its efficiency becomes $\frac{1}{3}$

$$\therefore \quad \frac{1}{3} = \left(1 - \frac{T_2 - 62}{T_1}\right)$$

$$\frac{T_2 - 62}{T_1} = 1 - \frac{1}{3}$$

$$\frac{T_2 - 62}{\frac{6}{5}T_2} = \frac{2}{3} \qquad \text{(Using (i))}$$

$$\frac{5(T_2 - 62)}{6T_2} = \frac{2}{3}$$

$5T_2 - 310 = 4T_2 \Rightarrow T_2 = 310$ K
From equation (i),

$$T_1 = \frac{6 \times 310}{5} = 372 \text{ K}$$

6. **(b) :** $\Delta Q = ms\Delta T$
Here, $m = 100$ g $= 100 \times 10^{-3}$ kg
$s = 4184$ J kg^{-1} K^{-1} and $\Delta T = (50 - 30) = 20$°C
$\therefore \quad \Delta Q = 100 \times 10^{-3} \times 4184 \times 20 = 8.4 \times 10^3$ J

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As $\Delta Q = \Delta U + \Delta W$

∴ Change in internal energy

$\Delta U = \Delta Q = 8.4 \times 10^3$ J = 8.4 kJ $(\because \Delta W = 0)$

7. **(c) :** For an adiabatic process

$TV^{\gamma-1}$ = constant

$\therefore \quad T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1} = T_2\left(\frac{32V}{V}\right)^{\gamma-1} = T_2(32)^{\gamma-1}$$

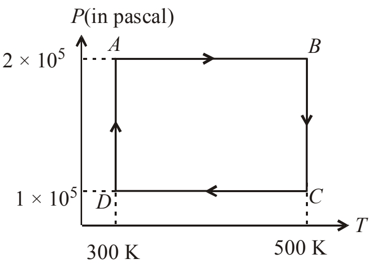
For diatomic gas, $\gamma = \frac{7}{5}$

$$\therefore \quad T_1 = T_2(32)^{\frac{7}{5}-1} = T_2(32)^{2/5} = T_2(2^5)^{2/5} = 4T_2$$

Efficiency of the engine, $\eta = 1 - \frac{T_2}{T_1} = \left(1 - \frac{1}{4}\right)$

$$\eta = \frac{3}{4} = 0.75$$

8. **(c) :**



Path AB, P is the same, ΔT is 200 K.

$PV = nRT$ for all process

$\therefore \quad P\Delta V = nR\Delta T = 2R\ 200 = 400R.$

Work done on the gas from A to $B = 400R$.

9. **(b) :** D to A, temperature remains the same.

$\therefore$ Work done by the gas $= W = nRT \ln\frac{V_2}{V_1}$

$$= nRT \ln\frac{P_1}{P_2}$$

$\Rightarrow \quad W = -600R\ (0.693) = -415.8R.$

This is the work done by the gas

$\therefore$ Work done on the gas $= +415.8R$.

Nearest to (b).

10. **(b) :** Total work done on the gas when taking from A to $B = 400R$, from C to D is equal and opposite.

They cancel each other.

For taking from D to A, work done on the gas $= +414R$.

Work done on the gas in taking it from B to C, pressure is decreased, temperature remain the same, volume increases.

$\Rightarrow \quad W_{BC} + W_{DA} = 2 \ln 2(500R - 300R).$

$\Rightarrow \quad W_{BC+DA} = (2 \ln 2) \times (200R)$

$= 400R \times 0.693 = 277R.$

$\therefore$ Work done along AB and CD cancel each other because pressure changes but temperature is the same.

Net work done on the gas of 2 moles of helium through the whole network $= 277R$ per cycle or nearest to the answer (b).

11. **(b) :** As this is a simple mixing of gas, even if adiabatic conditions are satisfied, $PV = nRT$ for adiabatic as well as isothermal changes. The total number of molecules is conserved.

$$\therefore \quad n_1 = \frac{P_1V_1}{R_1T_1},\ n_2 = \frac{P_2V_2}{RT_2}$$

Final state $= (n_1 + n_2)RT$

$$(n_1 + n_2) = \frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2} = \frac{T_2P_1V_1 + T_1P_2V_2}{RT_1T_2}$$

$$T = \frac{T_1n_1 + T_2n_2}{n_1 + n_2},\ T = \frac{T_1T_2(P_1V_1 + P_2V_2)}{T_2P_1V_1 + T_1P_2V_2}$$

12. **(c) :** For Carnot engine efficiency $\eta = \frac{Q_H - Q_L}{Q_L}$

Coefficient of performance of a refrigerator $\beta = \frac{1-\eta}{\eta}$

$$\beta = \frac{1 - \frac{1}{10}}{1/10} = 9$$

Also $\beta = \frac{Q_L}{W}$ (where W is the work done)

or $Q_L = \beta \times W = 9 \times 10 = 90$ J.

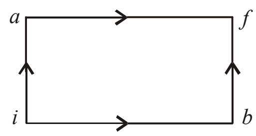
13. **(b) :** According to first law of thermodynamics for the path iaf,

$Q_{iaf} = \Delta U_{iaf} + W_{iaf}$

or $\Delta U_{iaf} = Q_{iaf} - W_{iaf}$

$= 50 - 20 = 30$ cal

For the path ibf,



$Q_{ibf} = \Delta U_{ibf} + W_{ibf}$

Since $\Delta U_{iaf} = \Delta U_{ibf}$, change in internal energy are path independent.

$Q_{ibf} = \Delta U_{iaf} + W_{ibf}$

$\therefore \quad W_{ibf} = Q_{ibf} - \Delta U_{iaf} = 36 - 30 = 6$ cal.

14. **(b) :** According to first law of thermodynamics

$\Delta Q = \Delta U + \Delta W$

For an adiabatic process, $\Delta Q = 0$

$\therefore \quad 0 = \Delta U + \Delta W$

or $\Delta U = -\Delta W$

or $nC_V\,\Delta T = -\Delta W$

$$\text{or} \quad C_V = \frac{-\Delta W}{n\Delta T} = \frac{-(-146)\times 10^3}{(1\times 10^3)\times 7}$$

$= 20.8$ J mol^{-1} K^{-1}

For diatomic gas,

$$C_V = \frac{5}{2}R = \frac{5}{2}\times 8.3 = 20.8 \text{ J mol}^{-1}\text{K}^{-1}$$

Hence the gas is diatomic.

15. **(c) :** $\Delta U_1 = \Delta U_2$, because the change in internal energy depends only upon the initial and final states A and B.

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16. **(a) :** Efficiency $\eta = 1 - \frac{Q_2}{Q_1}$

$Q_2 = T_0 (2S_0 - S_0) = T_0 S_0$

$Q_1 = T_0 S_0 + \frac{T_0 S_0}{2} = \frac{3}{2} T_0 S_0$

$\therefore \quad \eta = 1 - \frac{T_0 S_0 \times 2}{3 T_0 S_0} = 1 - \frac{2}{3} = \frac{1}{3}.$

17. **(b, c) :** Statements (b) and (c) are incorrect regarding the first law of thermodynamics.

18. **(b) :** Internal energy and entropy are state functions.

19. **(b) :** Efficiency $= 1 - \frac{T_2}{T_1} = 1 - \frac{300}{900} = 1 - \frac{1}{3} = \frac{2}{3}$

Heat energy = 3×10^6 cal = $3 \times 10^6 \times 4.2$ J

$\therefore$ Workdone by engine = (Heat energy) × (efficiency)

$= (3 \times 10^6 \times 4.2) \times \frac{2}{3}$ J

$= 8.4 \times 10^6$ J.

20. **(c) :** The work does not characterize the thermodynamic state of matter.

21. **(d) :** In an adiabatic process, $T^{\gamma} = (\text{constant}) P^{\gamma - 1}$

or $T^{\gamma/\gamma-1} = (\text{constant})\ P$

Given $T^3 = (\text{constant})\ P$

$\therefore \quad \frac{\gamma}{\gamma - 1} = 3 \Rightarrow 3\gamma - 3 = \gamma$

or $2\gamma = 3 \Rightarrow \gamma = 3/2$

N.B For monoatomic gas, $\gamma = \frac{5}{3} = 1.67$

For diatomic gas, $\gamma = \frac{7}{5} = 1.4$

when $\gamma = 1.5$, the gas must be a suitable mixture of monoatomic and diatomic gases

$\therefore \quad \gamma = 3/2.$

22. **(a) :** Second law of thermodynamics.

23. **(c) :** We cannot reach absolute zero temperature.

24. **(a) :** All reversible cycles do not have same efficiency.

25. **(b) :** Thermal capacity.

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CHAPTER 9

KINETIC THEORY OF GASES

1. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by

(a) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2$ K (b) $\frac{(\gamma-1)}{2\gamma R}Mv^2$ K

(c) $\frac{\gamma Mv^2}{2R}$ K (d) $\frac{(\gamma-1)}{2R}Mv^2$ K *(2011)*

2. One kg of a diatomic gas is at a pressure of 8×10^4 N/m^2. The density of the gas is 4 kg/m^3. What is the energy of the gas due to its thermal motion?

(a) 3×10^4 J (b) 5×10^4 J

(c) 6×10^4 J (d) 7×10^4 J *(2009)*

3. If C_P and C_V denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then

(a) $C_P - C_V = 28R$ (b) $C_P - C_V = R/28$

(c) $C_P - C_V = R/14$ (d) $C_P - C_V = R$ *(2007)*

4. Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0, while Box B contains one mole of helium at temperature (7/3) T_0. The boxes are then put into thermal contact with each other and heat flows between them until the gases reach a common final temperature. (Ignore the heat capacity of boxes). Then, the final temperature of the gases, T_f, in terms of T_0 is

(a) $T_f = \frac{5}{2}T_0$ (b) $T_f = \frac{3}{7}T_0$

(c) $T_f = \frac{7}{3}T_0$ (d) $T_f = \frac{3}{2}T_0$. *(2006)*

5. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio C_P/C_V of the mixture is

(a) 1.4 (b) 1.54 (c) 1.59 (d) 1.62 *(2005)*

6. One mole of ideal monoatomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$). What is γ for the mixture? g denotes the ratio of specific heat at constant pressure, to that at constant volume.

(a) 3/2 (b) 23/15 (c) 35/23 (d) 4/3. *(2004)*

7. 1 mole of a gas with $\gamma = 7/5$ is mixed with 1 mole of a gas with $\gamma = 5/3$, then the value of g for the resulting mixture is

(a) 7/5 (b) 2/5 (c) 24/16 (d) 12/7. *(2002)*

8. At what temperature is the r.m.s. velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C?

(a) 80 K (b) –73 K (c) 3 K (d) 20 K. *(2002)*

9. Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will

(a) increase
(b) decrease
(c) remain same
(d) decrease for some, while increase for others. *(2002)*

Answer Key

1.	(d)	**2.**	(b)	**3.**	(b)	**4.**	(d)	**5.**	(d)	**6.**	(a)
7.	(c)	**8.**	(d)	**9.**	(c)						

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Explanations

1. **(d) :** Kinetic energy of vessel $= \frac{1}{2}mv^2$
Increase in internal energy
$\Delta U = nC_V\Delta T$
where n is the number of moles of the gas in vessel.
As the vessel is stopped suddenly, its kientic energy is used to increase the temperature of the gas

$\therefore \quad \frac{1}{2}mv^2 = \Delta U$

$\frac{1}{2}mv^2 = nC_V\Delta T$

$\frac{1}{2}mv^2 = \frac{m}{M}C_V\Delta T \qquad \left(\because\ n = \frac{m}{M}\right)$

$\Delta T = \frac{Mv^2}{2C_V}$

or $\Delta T = \frac{Mv^2(\gamma-1)}{2R}$ K $\qquad \left(\because\ C_V = \frac{R}{(\gamma-1)}\right)$

2. **(b) :** The thermal energy or internal energy is $U = \frac{5}{2}\mu RT$ for diatomic gases. (5 is the degrees of freedom as the gas is diatomic)
But $PV = \mu RT$
$V = \frac{\text{mass}}{\text{density}} = \frac{1\text{ kg}}{4\text{ kg/m}^3} = \frac{1}{4}\text{ m}^3$
$P = 8 \times 10^4$ N/m².
$\therefore \quad U = \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} = 5 \times 10^4$ J

3. **(b) :** Molar heat capacity = Molar mass × specific heat capacity
So, the molar heat capacities at constant pressure and constant volume will be $28C_P$ and $28C_V$ respectively.
$\therefore \quad 28C_P - 28C_V = R \quad$ or $\quad C_P - C_V = \frac{R}{28}$

4. **(d) :** $\Delta U = 0$
$\therefore \quad 1\times\left(\frac{5}{2}R\right)(T_f - T_0) + 1\times\frac{3}{2}R\left(T_f - \frac{7}{3}T_0\right) = 0$
or $5T_f - 5T_0 + 3T_f - 7T_0 = 0$
or $8T_f = 12T_0$
or $T_f = \frac{3}{2}T_0$.

5. **(d) :** For 16 g of helium, $n_1 = \frac{16}{4} = 4$
For 16 g of oxygen, $n_2 = \frac{16}{32} = \frac{1}{2}$
For mixture of gases,

$C_V = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2}$ where $C_V = \frac{f}{2}R$

$C_P = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1 + n_2}$ where $C_P = \left(\frac{f}{2}+1\right)R$

For helium, $f = 3$, $n_1 = 4$
For oxygen, $f = 5$, $n_2 = 1/2$

$\therefore \quad \frac{C_P}{C_V} = \frac{\left(4\times\frac{5}{2}R\right) + \left(\frac{1}{2}\times\frac{7}{2}R\right)}{\left(4\times\frac{3}{2}R\right) + \left(\frac{1}{2}\times\frac{5}{2}R\right)} = \frac{47}{29} = 1.62.$

6. **(a) :** For mixture of gases,
$\frac{n_1+n_2}{\gamma_m - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$ or $\frac{1+1}{\gamma_m - 1} = \frac{1}{\frac{5}{3}-1} + \frac{1}{\frac{7}{5}-1}$

or $\frac{2}{\gamma_m - 1} = \frac{3}{2} + \frac{5}{2} = 4 \Rightarrow \gamma_m - 1 = 0.5$
$\therefore \quad \gamma_m = 1.5 = 3/2.$

7. **(c) :** For mixture of gases
$\frac{n_1+n_2}{\gamma_m - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$

$\frac{1+1}{\gamma_m - 1} = \frac{1}{\left(\frac{7}{5}-1\right)} + \frac{1}{\left(\frac{5}{3}-1\right)}$

$\frac{2}{\gamma_m - 1} = \frac{5}{2} + \frac{3}{2}$

or $\frac{2}{\gamma_m - 1} = \frac{8}{2}$
or $8\gamma_m - 8 = 4$
or $8\gamma_m = 12$
or $\gamma_m = \frac{12}{8} = \frac{24}{16}$

8. **(d) :** $v_{\text{rms}} = \sqrt{\frac{RT}{M}}$
$\therefore \quad (v_{\text{rms}})_{O_2} = (v_{\text{rms}})_{H_2}$
or $\sqrt{\frac{273+47}{32}} = \sqrt{\frac{T}{2}} \Rightarrow T = 20$ K .

9. **(c) :** It is the relative velocities between molecules that is important. Root mean square velocities are different from lateral translation.

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CHAPTER 10

OSCILLATIONS AND WAVES

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1. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5 s. In another 10 s it will decrease to α times its original magnitude where α equals
(a) 0.6 (b) 0.7 (c) 0.81 (d) 0.729
(2013)

2. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency
(a) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$ (b) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$
(c) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$ (d) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ *(2013)*

3. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively?
(a) 770 Hz (b) 188.5 Hz
(c) 178.2 Hz (d) 200.5 Hz *(2013)*

4. If a simple pendulum has significant amplitude (up to a factor of $1/e$ of original) only in the period between t = 0 s to $t = \tau$ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with b as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds
(a) b (b) $\frac{1}{b}$ (c) $\frac{2}{b}$ (d) $\frac{0.693}{b}$
(2012)

5. A cylindrical tube, open at both ends, has a fundamental frequency, f in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now
(a) $\frac{f}{2}$ (b) $\frac{3f}{4}$ (c) $2f$ (d) f
(2012)

6. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance X_0 ($X_0 > A$). If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$
(2011)

7. A mass M, attached to a horizontal spring, executes SHM with a amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is
(a) $\frac{M}{M+m}$ (b) $\frac{M+m}{M}$
(c) $\left(\frac{M}{M+m}\right)^{1/2}$ (d) $\left(\frac{M+m}{M}\right)^{1/2}$ *(2011)*

8. The transverse displacement $y(x,t)$ of a wave on a string is given by
$y(x,t) = e^{-(ax^2+bt^2+2\sqrt{ab}xt)}$
This represents a
(a) wave moving in $+x$-direction with speed $\sqrt{\frac{a}{b}}$
(b) wave moving in $-x$-direction with speed $\sqrt{\frac{b}{a}}$
(c) standing wave of frequency $\sqrt{b}$
(d) standing wave of frequency $\frac{1}{\sqrt{b}}$ *(2011)*

9. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by
$$y = 0.02\,(\text{m})\sin\left[2\pi\left(\frac{t}{0.04(\text{s})} - \frac{x}{0.50(\text{m})}\right)\right].$$
The tension in the string is
(a) 6.25 N (b) 4.0 N (c) 12.5 N (d) 0.5 N
(2010)

10. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time?
(a) $a^2T^2 + 4\pi^2v^2$ (b) aT/x
(c) $aT + 2\pi v$ (d) aT/v *(2009)*

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11. Three sound waves of equal amplitudes have frequencies $(\upsilon - 1)$, υ, $(\upsilon + 1)$. They superpose to give beats. The number of beats produced per second will be
(a) 4 (b) 3 (c) 2 (d) 1
(2009)

12. A motor cycle starts from rest and accelerates along a straight path at 2 m/s^2. At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (Speed of sound = 330 ms^{-1}).
(a) 49 m (b) 98 m (c) 147 m (d) 196 m
(2009)

13. A wave travelling along the x-axis is described by the equation $y(x, t) = 0.005 \cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are
(a) $\alpha = 12.50\pi,\ \beta = \frac{\pi}{2.0}$ (b) $\alpha = 25.00\pi,\ \beta = \pi$
(c) $\alpha = \frac{0.08}{\pi},\ \beta = \frac{2.0}{\pi}$ (d) $\alpha = \frac{0.04}{\pi},\ \beta = \frac{1.0}{\pi}$
(2008)

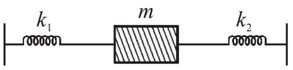
14. The speed of sound in oxygen (O_2) at a certain temperature is 460 ms^{-1}. The speed of sound in helium (He) at the same temperature will be (assume both gases to be ideal)
(a) 330 ms^{-1} (b) 460 ms^{-1}
(c) 500 ms^{-1} (d) 650 ms^{-1}. *(2008)*

15. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A\cos(\omega t + \delta)$, then
(a) $A = x_0\omega^2$, $\delta = 3\pi/4$ (b) $A = x_0$, $\delta = -\pi/4$
(c) $A = x_0\omega^2$, $\delta = \pi/4$ (d) $A = x_0\omega^2$, $\delta = -\pi/4$
(2007)

16. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metre. The time at which the maximum speed first occurs is
(a) 0.25 s (b) 0.5 s (c) 0.75 s (d) 0.125 s
(2007)

17. A particle of mass m executes simple harmonic motion with amplitude a and frequency υ. The average kinetic energy during its motion from the position of equilibrium to the end is
(a) $2\pi^2 m a^2 \upsilon^2$ (b) $\pi^2 m a^2 \upsilon^2$
(c) $\frac{1}{4} m a^2 \upsilon^2$ (d) $4\pi^2 m a^2 \upsilon^2$ *(2007)*

18. Two springs, of force constants k_1 and k_2 are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes



(a) $2f$ (b) $f/2$ (c) $f/4$ (d) $4f$
(2007)

19. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of
(a) 100 (b) 1000 (c) 10000 (d) 10
(2007)

20. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time
(a) at the highest position of the platform
(b) at the mean position of the platform
(c) for an amplitude of $\frac{g}{\omega^2}$
(d) for an amplitude of $\frac{g^2}{\omega^2}$. *(2006)*

21. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is
(a) 100 s (b) 0.01 s (c) 10 s (d) 0.1 s.
(2006)

22. Starting from the origin, a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy by 75% of the total energy?
(a) $\frac{1}{12}$ s (b) $\frac{1}{6}$ s (c) $\frac{1}{4}$ s (d) $\frac{1}{3}$ s.
(2006)

23. A string is stretched between fixed points separated by 75 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is
(a) 10.5 Hz (b) 105 Hz
(c) 1.05 Hz (d) 1050 Hz. *(2006)*

24. A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with speed v ms^{-1}. The velocity of sound in air is 300 ms^{-1}. If the person can hear frequencies upto a maximum of 10000 Hz, the maximum value of v upto which he can hear the whistle is
(a) 30 ms^{-1} (b) $15\sqrt{2}$ ms^{-1}
(c) $15/\sqrt{2}$ ms^{-1} (d) 15 ms^{-1}. *(2006)*

25. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would
(a) remain unchanged
(b) increase towards a saturation value
(c) first increase and then decrease to the original value
(d) first decrease and then increase to the original value
(2005)

26. If a simple harmonic motion is represented by $\frac{d^2x}{dt^2} + \alpha x = 0$, its time period is
(a) $2\pi\alpha$ (b) $2\pi\sqrt{\alpha}$ (c) $2\pi/\alpha$ (d) $2\pi/\sqrt{\alpha}$
(2005)

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27. Two simple harmonic motions are represented by the equations $y_1 = 0.1\sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1\cos\pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is
(a) $-\pi/3$ (b) $\pi/6$ (c) $-\pi/6$ (d) $\pi/3$. *(2005)*

28. The function $\sin^2(\omega t)$ represents
(a) a simple harmonic motion with a period $2\pi/\omega$
(b) a simple harmonic motion with a period π/ω
(c) a periodic, but not simple harmonic motion with a period $2\pi/\omega$
(d) a periodic, but not simple harmonic motion with a period π/ω *(2005)*

29. An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency?
(a) 5% (b) 20% (c) zero (d) 0.5% *(2005)*

30. When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now, some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard. if the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2?
(a) 196 Hz (b) 204 Hz (c) 200 Hz (d) 202 Hz *(2005)*

31. In forced oscillation of a particle the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force, then
(a) $\omega_1 = \omega_2$ (b) $\omega_1 > \omega_2$
(c) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large
(d) $\omega_1 < \omega_2$ *(2004)*

32. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force $F(t)$ proportional to $\cos wt$ $(\omega \neq \omega_0)$ is applied to the oscillator. The time displacement of the oscillator will be proportional to
(a) $\dfrac{m}{\omega_0^2 - \omega^2}$ (b) $\dfrac{1}{m(\omega_0^2 - \omega^2)}$
(c) $\dfrac{1}{m(\omega_0^2 + \omega^2)}$ (d) $\dfrac{m}{\omega_0^2 + \omega^2}$. *(2004)*

33. The total energy of a particle, executing simple harmonic motion is
(a) $\propto x$ (b) $\propto x^2$
(c) independent of x (d) $\propto x^{1/2}$
where x is the displacement from the mean position. *(2004)*

34. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then
(a) $T = t_1 + t_2$ (b) $T^2 = t_1^2 + t_2^2$
(c) $T^{-1} = t_1^{-1} + t_2^{-1}$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$. *(2004)*

35. The displacement y of a particle in a medium can be expressed as:
$y = 10^{-6}\sin(100t + 20x + \pi/4)$ m, where t is in second and x in meter. The speed of the wave is
(a) 2000 m/s (b) 5 m/s
(c) 20 m/s (d) 5π m/s. *(2004)*

36. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is (4/3) × 1000 kg/m³. What relationship between t and t_0 is true?
(a) $t = t_0$ (b) $t = t_0/2$ (c) $t = 2t_0$ (d) $t = 4t_0$. *(2003)*

37. A body executes simple harmonic motion. The potential energy (P.E.), the kinetic energy (K.E.) and total energy (T.E.) are measured as function of displacement x. Which of the following statement is true?
(a) K.E. is maximum when $x = 0$
(b) T.E. is zero when $x = 0$
(c) K.E. is maximum when x is maximum
(d) P.E. is maximum when $x = 0$. *(2003)*

38. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is
(a) 11% (b) 21% (c) 42% (d) 10%. *(2003)*

39. Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities, during oscillations, are equal, the ratio of amplitudes of A and B is
(a) $\sqrt{k_1/k_2}$ (b) k_2/k_1 (c) $\sqrt{k_2/k_1}$ (d) k_1/k_2. *(2003)*

40. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes $5T/3$. Then the ratio of m/M is
(a) 3/5 (b) 25/9 (c) 16/9 (d) 5/3. *(2003)*

41. A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was
(a) (256 + 2) Hz (b) (256 – 2) Hz
(c) (256 – 5) Hz (d) (256 + 5) Hz. *(2003)*

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42. A metal wire of linear mass density of 9.8 g/m is stretched with a tension of 10 kg-wt between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a permanent magnet, and it vibrates in resonance when carrying an alternating current of frequency υ. The frequency υ of the alternating source is
(a) 50 Hz (b) 100 Hz (c) 200 Hz (d) 25 Hz.
(2003)

43. The displacement of a particle varies according to the relation $x = 4(\cos\pi t + \sin\pi t)$. The amplitude of the particle is
(a) – 4 (b) 4 (c) $4\sqrt{2}$ (d) 8.
(2003)

44. The displacement y of a wave travelling in the x -direction is given by

$$y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right) \text{ metre,}$$

where x is expressed in metre and t in second. The speed of the wave-motion, in ms^{-1} is
(a) 300 (b) 600 (c) 1200 (d) 200.
(2003)

45. A child swinging on a swing in sitting position, stands up, then the time period of the swing will
(a) increase
(b) decrease
(c) reamains same
(d) increases if the child is long and decreases if the child is short. *(2002)*

46. In a simple harmonic oscillator, at the mean position
(a) kinetic energy is minimum, potential energy is maximum
(b) both kinetic and potential energies are maximum
(c) kinetic energy is maximum, potential energy is minimum
(d) both kinetic and potential energies are minimum.
(2002)

47. If a spring has time period T, and is cut into n equal parts, then the time period of each part will be
(a) $T\sqrt{n}$ (b) $T/\sqrt{n}$ (c) nT (d) T.
(2002)

48. When temperature increases, the frequency of a tuning fork
(a) increases (b) decreases
(c) remains same
(d) increases or decreases depending on the material.
(2002)

49. Length of a string tied to two rigid supports is 40 cm. Maximum length (wavelength in cm) of a stationary wave produced on it is
(a) 20 (b) 80 (c) 40 (d) 120.
(2002)

50. A wave $y = a\sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then the equation of the unknown wave is
(a) $y = a\sin(\omega t + kx)$ (b) $y = -a\sin(\omega t + kx)$
(c) $y = a\sin(\omega t - kx)$ (d) $y = -a\sin(\omega t - kx)$.
(2002)

51. A tuning fork arrangement (pair) produces 4 beats/sec with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats/sec. The frequency of the unknown fork is
(a) 286 cps (b) 292 cps
(c) 294 cps (d) 288 cps. *(2002)*

52. Tube A has both ends open while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tube A and B is
(a) 1 : 2 (b) 1 : 4 (c) 2 : 1 (d) 4 : 1.
(2002)

Answer Key

1.	(d)	**2.**	(d)	**3.**	(c)	**4.**	(c)	**5.**	(d)	**6.**	(b)
7.	(d)	**8.**	(b)	**9.**	(a)	**10.**	(b)	**11.**	(a)	**12.**	(b)
13.	(b)	**14.**	(*)	**15.**	(a)	**16.**	(b)	**17.**	(b)	**18.**	(a)
19.	(a)	**20.**	(c)	**21.**	(b)	**22.**	(b)	**23.**	(b)	**24.**	(d)
25.	(c)	**26.**	(d)	**27.**	(c)	**28.**	(d)	**29.**	(b)	**30.**	(a)
31.	(a)	**32.**	(b)	**33.**	(c)	**34.**	(b)	**35.**	(b)	**36.**	(c)
37.	(a)	**38.**	(d)	**39.**	(c)	**40.**	(c)	**41.**	(c)	**42.**	(a)
43.	(c)	**44.**	(a)	**45.**	(b)	**46.**	(c)	**47.**	(b)	**48.**	(b)
49.	(b)	**50.**	(b)	**51.**	(b)	**52.**	(c)				

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Explanations

1. **(d) :** The amplitude of a damped oscillator at a given instant of time t is given by

$A = A_0 e^{-bt/2m}$

where A_0 is its amplitude in the absence of damping, b is the damping constant.

As per question

After 5 s (*i.e.* t = 5 s) its amplitude becomes

$0.9A_0 = A_0 e^{-b(5)/2m} = A_0 e^{-5b/2m}$

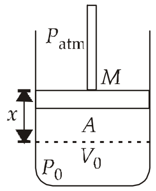
$0.9 = e^{-5b/2m}$ ...(i)

After 10 more second (*i.e.* t = 15 s), its amplitude becomes

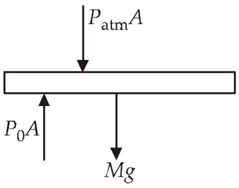
$\alpha A_0 = A_0 e^{-b(15)/2m} = A_0 e^{-15b/2m}$

$\alpha = (e^{-5b/2m})^3 = (0.9)^3$ (Using (i))

$= 0.729$

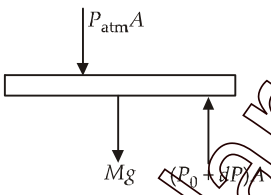
2. **(d) :** 

FBD of piston at equilibrium



$P_{atm}A + Mg = P_0A$ ...(i)

FBD of piston when piston is pushed down a distance x



$$(P_0 + dP)A - (P_{atm}A + Mg) = M\frac{d^2x}{dt^2} \quad \text{...(ii)}$$

As the system is completely isolated from its surrounding therefore the change is adiabatic.

For an adiabatic process

PV^γ = constant

$\therefore \quad V^\gamma dP + V^{\gamma-1}PdV = 0$

or $\quad dP = -\dfrac{\gamma P dV}{V}$

$$\therefore \quad dP = -\frac{\gamma P_0(Ax)}{V_0} \quad (\because dV = Ax) \quad \text{...(iii)}$$

Using (i) and (iii) in (ii), we get

$$M\frac{d^2x}{dt^2} = -\frac{\gamma P_0 A^2}{V_0}x \quad \text{or} \quad \frac{d^2x}{dt^2} = -\frac{\gamma P_0 A^2}{MV_0}$$

Comparing it with standard equation of SHM,

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

We get

$$\omega^2 = \frac{\gamma P_0 A^2}{MV_0} \quad \text{or} \quad \omega = \sqrt{\frac{\gamma P_0 A^2}{MV_0}}$$

$$\text{Frequency, } \upsilon = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0 A^2}{MV_0}}$$

3. **(c) :** Fundamental frequency of vibration of wire is

$$\upsilon = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

where L is the length of the wire, T is the tension in the wire and μ is the mass per length of the wire

As $\mu = \rho A$

where ρ is the density of the material of the wire and A is the area of cross-section of the wire.

$$\therefore \quad \upsilon = \frac{1}{2L}\sqrt{\frac{T}{\rho A}}$$

Here tension is due to elasticity of wire

$$\therefore \quad T = YA\left[\frac{\Delta L}{L}\right] \qquad \left[\text{As } Y = \frac{\text{Stress}}{\text{Strain}} = \frac{TL}{A\Delta L}\right]$$

$$\text{Hence,} \quad \upsilon = \frac{1}{2L}\sqrt{\frac{Y\Delta L}{\rho L}}$$

Here, $Y = 2.2 \times 10^{11}$ N/m^2, $\rho = 7.7 \times 10^3$ kg/m^3

$\dfrac{\Delta L}{L} = 0.01, \; L = 1.5$ m

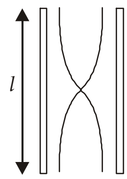
Substituting the given values, we get

$$\upsilon = \frac{1}{2 \times 1.5}\sqrt{\frac{2.2 \times 10^{11} \times 0.01}{7.7 \times 10^3}}$$

$$= \frac{10^3}{3}\sqrt{\frac{2}{7}} \text{ Hz} = 178.2 \text{ Hz}$$

4. **(c)**

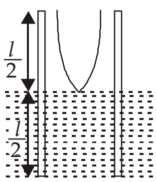
5. **(d) :** When the tube of length l is open at both ends,



$\therefore \quad f = \dfrac{v}{2l}$ ...(i)

where v is the speed of sound in air.

When the tube is dipped vertically in water and half of it is in water, it behaves closed pipe length $\dfrac{l}{2}$,



$$\therefore \quad f' = \frac{v}{4\left(\frac{l}{2}\right)} = \frac{v}{2l} = f \quad \text{(Using(i))}$$

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6. **(b)**

7. **(d) :** $T_1 = 2\pi\sqrt{\frac{M}{k}}$ ...(i)

When a mass m is placed on mass M, the new system is of mass = $(M + m)$ attached to the spring. New time period of oscillation

$$T_2 = 2\pi\sqrt{\frac{(m+M)}{k}} \quad ...(ii)$$

Consider v_1 is the velocity of mass M passing through mean position and v_2 velocity of mass $(m + M)$ passing through mean position.
Using, law of conservation of linear momentum

$$Mv_1 = (m + M)v_2$$
$$M(A_1\omega_1) = (m + M)(A_2\omega_2)$$
$$(\because\ v_1 = A_1\omega_1 \text{ and } v_2 = A_2\omega_2)$$

or $\frac{A_1}{A_2} = \frac{(m+M)}{M}\frac{\omega_2}{\omega_1}$

$$= \left(\frac{m+M}{M}\right)\times\frac{T_1}{T_2}\left(\because\ \omega_1 = \frac{2\pi}{T_1} \text{ and } \omega_2 = \frac{2\pi}{T_2}\right)$$

$$\frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}} \quad \text{(Using (i) and (ii))}$$

8. **(b) :** $y(x,t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}\,xt)}$

$$= e^{-(\sqrt{a}x + \sqrt{b}t)^2}$$

Comparing equation (i) with standard equation
$y(x, t) = f(ax + bt)$
As there is positive sign between x and t terms, hence wave travel in $-x$ direction.

$$\text{Wave speed} = \frac{\text{Coefficient of } t}{\text{Coefficient of } x} = \sqrt{\frac{b}{a}}$$

9. **(a) :** Here, linear mass density $\mu = 0.04$ kg m^{-1}
The given equation of a wave is

$$y = 0.02\sin\left[2\pi\left(\frac{t}{0.04} - \frac{x}{0.50}\right)\right]$$

Compare it with the standard wave equation
$y = A\sin(\omega t - kx)$
we get,

$$\omega = \frac{2\pi}{0.04}\text{ rad s}^{-1};\ k = \frac{2\pi}{0.5}\text{ rad m}^{-1}$$

Wave velocity, $v = \frac{\omega}{k} = \frac{(2\pi/0.04)}{(2\pi/0.5)}$ m s^{-1} ...(i)

Also $v = \sqrt{\frac{T}{\mu}}$ ...(ii)

where T is the tension in the string and μ is the linear mass density
Equating equations (i) and (ii), we get

$$\frac{\omega}{k} = \sqrt{\frac{T}{\mu}} \text{ or } T = \frac{\mu\omega^2}{k^2}$$

$$T = \frac{0.04\times\left(\frac{2\pi}{0.04}\right)^2}{\left(\frac{2\pi}{0.05}\right)^2} = 6.25\text{ N}$$

10. **(b) :** For a simple harmonic motion,

acceleration, $a = -\omega^2 x$ where ω is a constant $= \frac{2\pi}{T}$.

$$a = -\frac{4\pi^2}{T^2}\cdot x \Rightarrow \frac{aT}{x} = -\frac{4\pi^2}{T}$$

The period of oscillation T is a constant.

$\therefore\ \frac{aT}{x}$ is a constant.

11. **(a) :** The given sources of sound produce frequencies, $(\upsilon - 1)$, υ and $(\upsilon + 1)$.
For two sources of frequencies υ_1 and υ_2,
$y_1 = A\cos 2\pi\upsilon_1 t$
$y_2 = A\cos 2\pi\upsilon_2 t$
Superposing, one gets

$$y = 2A\cos 2\pi\left(\frac{\upsilon_1 - \upsilon_2}{2}\right)t\cos 2\pi\left(\frac{\upsilon_1 + \upsilon_2}{2}\right)t.$$

The resultant frequency obtained is $\frac{\upsilon_1 + \upsilon_2}{2}$ and this wave is modulated by a wave of frequency $\frac{\upsilon_1 - \upsilon_2}{2}$ (rather the difference of frequencies/2).
The intensity waxes and wanes. For a cosine curve (or sine curve), the number of beats = $\upsilon_1 \sim \upsilon_2$.

Frequencies	Mean	Beats
$\upsilon + 1$ and υ	$(\upsilon + 0.5)$ Hz	1
υ and $\upsilon - 1$	$\upsilon - 0.5$	1
$(\upsilon + 1)$ and $(\upsilon - 1)$	υ	2

Total number of beats = 4.
One should detect three frequencies, υ, $\upsilon + 0.5$ and $\upsilon - 0.5$ and each frequency will show 2 beats, 1 beat and 1 beat per second, respectively.
Total number of beats = 4

12. **(b) :** The source is at rest, the observer is moving away from the source.

$$\therefore\ f' = f\frac{(v_{\text{sound}} - v_{\text{obs}})}{v_{\text{sound}}}$$

$$\Rightarrow \frac{f'}{f}\times v_{\text{sound}} = v_{\text{sound}} - v_{\text{obs}}$$

$$\Rightarrow \frac{f'}{f}\times v_{\text{sound}} - v_{\text{sound}} = -v_{\text{obs}}$$

$$v_{\text{sound}}\left(\frac{f'}{f} - 1\right) = -v_{\text{obs}}$$

$330(0.94 - 1) = -v_{\text{obs}}$
$\Rightarrow\ v_{\text{obs}} = 330 \times 0.06 = 19.80$ ms^{-1}.

$$\therefore\ s = \frac{v^2 - u^2}{2a} = \frac{(19.80)^2}{2\times 2} = 98\text{ m}.$$

13. **(b) :** The wave travelling along the x-axis is given by $y(x, t) = 0.005\cos(\alpha x - \beta t)$.

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Therefore $\alpha = k = \frac{2\pi}{\lambda}$. As $\lambda = 0.08$ m.

$\therefore\ \alpha = \frac{2\pi}{0.08} = \frac{\pi}{0.04} \Rightarrow \alpha = \frac{\pi}{4}\times 100.00 = 25.00\pi.$

$\omega = \beta \Rightarrow \frac{2\pi}{2.0} = \beta \Rightarrow \pi$

$\therefore\ \alpha = 25.00\ \pi,\ \beta = \pi$

14. **(*) :** $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

γ for O_2 = 1 + 2/5 = 1.4;
γ for He = 1 + 2/3 = 5/3

$\frac{v_2}{v_1} = \left(\sqrt{\frac{\gamma_{He}}{4}\times\frac{32}{\gamma_{O_2}}}\right)\times 460$

$= 460\times\sqrt{\frac{5}{3}\times\frac{1}{4}\times\frac{32\times 5}{7}} = 1420\,\text{m/s}.$

* The value of the speed of sound in He should have been 965 m/s and that of O_2, about 320 m/s. The value of the velocity given for O_2 is quite high. Option not given.

15. **(a) :** Given : $x = x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$ ... (i)

Acceleration $a = A\cos(\omega t + \delta)$ ... (ii)

Velocity $v = \frac{dx}{dt}$

$v = -x_0\omega \sin\left(\omega t - \frac{\pi}{4}\right)$ ... (iii)

Acceleration $a = \frac{dv}{dt}$

$= -x_0\omega^2\cos\left(\omega t - \frac{\pi}{4}\right) = x_0\omega^2\cos\left[\pi + \left(\omega t - \frac{\pi}{4}\right)\right]$

$= x_0\omega^2\cos\left[\omega t + \frac{3\pi}{4}\right]$ ... (iv)

Compare (iv) with (ii), we get

$A = x_0\omega^2,\ \delta = \frac{3\pi}{4}.$

16. **(b) :** Given : displacement $x = 2\times 10^{-2}\cos \pi t$

Velocity $v = \frac{dx}{dt} = -2\times 10^{-2}\pi \sin \pi t$

For the first time when $v = v_{max}$, $\sin \pi t = 1$

or $\sin \pi t = \sin\frac{\pi}{2}$ or $\pi t = \frac{\pi}{2}$

or $t = \frac{1}{2}\ \text{s} = 0.5\ \text{s}.$

17. **(b) :** For a particle to execute simple harmonic motion its displacement at any time t is given by

$x(t) = a(\cos \omega t + \phi)$

where, a = amplitude, ω = angular frequency, ϕ = phase constant.

Let us choose $\phi = 0$

$\therefore\ x(t) = a\cos\omega t$

Velocity of a particle $v = \frac{dx}{dt} = -a\,\omega \sin \omega t$

Kinetic energy of a particle is $K = \frac{1}{2}mv^2$

$= \frac{1}{2}ma^2\omega^2\sin^2\omega t$

Average kinetic energy $< K >$

$= < \frac{1}{2}ma^2\omega^2\sin^2\omega t >$

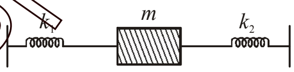
$= \frac{1}{2}m\omega^2a^2 < \sin^2\omega t >$

$= \frac{1}{2}m\omega^2a^2\left(\frac{1}{2}\right)$ $\left[\because < \sin^2\theta > = \frac{1}{2}\right]$

$= \frac{1}{4}ma^2(2\pi\upsilon)^2$ $[\because \omega = 2\pi\upsilon]$

$= \pi^2 ma^2 \upsilon^2.$

18. **(a) :** In the given figure two springs are connected in parallel. Therefore the effective spring constant is given by



$k_{eff} = k_1 + k_2$

Frequency of oscillation,

$f = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$ ... (i)

As k_1 and k_2 are increased four times

New frequency,

$f' = \frac{1}{2\pi}\sqrt{\frac{4(k_1+k_2)}{m}} = 2f$ (using (i).

19. **(a) :** $L_1 = 10\log\left(\frac{I_1}{I_0}\right)$; $L_2 = 10\log\left(\frac{I_2}{I_0}\right)$

$\therefore\ L_1 - L_2 = 10\log\left(\frac{I_1}{I_0}\right) - 10\log\left(\frac{I_2}{I_0}\right)$

or $\Delta L = 10\log\left(\frac{I_1}{I_2}\right)$ or $20\ \text{dB} = 10\log\left(\frac{I_1}{I_2}\right)$

or $10^2 = \frac{I_1}{I_2}$ or $I_2 = \frac{I_1}{100}$.

20. **(c) :** In vertical simple harmonic motion, maximum acceleration ($a\omega^2$) and so the maximum force ($ma\omega^2$) will be at extreme positions. At highest position, force will be towards mean position and so it will be downwards. At lowest position, force will be towards mean position and so it will be upwards. This is opposite to weight direction of the coin. The coin will leave contact will the platform for the first time when $m(a\omega^2) \geq mg$ at the lowest position of the platform.

21. **(b) :** Maximum velocity $v_m = a\omega = a\left(\frac{2\pi}{T}\right)$

$\therefore\ T = \frac{2\pi a}{v_m} = 2\times\frac{22}{7}\times\frac{(7\times 10^{-3})}{4.4}$

$= 10^{-2}$ sec = 0.01 sec.

22. **(b) :** During simple harmonic motion,

Kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2}m(a\,\omega\cos\omega t)^2$

Total energy $E = \frac{1}{2}ma^2\omega^2$

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$\because$ (Kinetic energy) $= \frac{75}{100}(E)$

or $\frac{1}{2}ma^2\omega^2\cos^2\omega t = \frac{75}{100}\times\frac{1}{2}ma^2\omega^2$

or $\cos^2\omega t = \frac{3}{4} \Rightarrow \cos\omega t = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

$\therefore$ $\omega t = \frac{\pi}{6}$

or $t = \frac{\pi}{6\omega} = \frac{\pi}{6(2\pi/T)} = \frac{2\pi}{6\times 2\pi} = \frac{1}{6}$ sec.

23. **(b) :** Let the successive loops formed be p and $(p + 1)$ for frequencies 315 Hz and 420 Hz

$\therefore$ $\upsilon = \frac{p}{2l}\sqrt{\frac{T}{\mu}} = \frac{pv}{2l}$

$\therefore$ $\frac{pv}{2l} = 315$ Hz and $\frac{(p+1)v}{2l} = 420$ Hz

or $\frac{(p+1)v}{2l} - \frac{pv}{2l} = 420 - 315$

or $\frac{v}{2l} = 105 \Rightarrow \frac{1\times v}{2l} = 105$ Hz

$p = 1$ for fundamental mode of vibration of string.

$\therefore$ Lowest resonant frequency = 105 Hz.

24. **(d) :** $\frac{\upsilon'}{\upsilon} = \frac{v_s}{v_s - v}$

where v_s is the velocity of sound in air.

$\frac{10000}{9500} = \frac{300}{300 - v}$

$\Rightarrow (300 - v) = 285 \Rightarrow v = 15$ m/s.

25. **(c) :** For a pendulum, $T = 2\pi\sqrt{\frac{l}{g}}$ where l is measured upto centre of gravity. The centre of gravity of system is at centre of sphere when hole is plugged. When unplugged, water drains out. Centre of gravity goes on descending. When the bob becomes empty, centre of gravity is restored to centre.

$\therefore$ Length of pendulum first increases, then decreases to original value.

$\therefore$ T would first increase and then decrease to the original value.

26. **(d) :** Standard differential equation of SHM is

$\frac{d^2x}{dt^2} + \omega^2 x = 0$

Given equation is $\frac{d^2x}{dt^2} + \alpha x = 0$

$\therefore$ $\omega^2 = \alpha$

or $\omega = \sqrt{\alpha}$

$\therefore$ $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\alpha}}$.

27. **(c) :** $v_1 = \frac{d}{dt}(y_1) = (0.1\times 100\pi)\cos\left(100\pi t + \frac{\pi}{3}\right)$

$v_2 = \frac{d}{dt}(y_2) = (-0.1\times\pi)\sin\pi t$

$= (0.1\times\pi)\cos\left(\pi t + \frac{\pi}{2}\right)$

$\therefore$ $\Delta\phi = \frac{\pi}{3} - \frac{\pi}{2} = -\frac{\pi}{6}$.

28. **(d) :** $y = \sin^2\omega t = \frac{1 - \cos 2\omega t}{2} = \frac{1}{2} - \frac{\cos 2\omega t}{2}$

It is a periodic motion but it is not SHM

$\therefore$ Angular speed = 2ω

$\therefore$ Period $T = \frac{2\pi}{\text{angular speed}} = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

Hence option (d) represents the answer.

29. **(b) :** By Doppler's effect

$\frac{\upsilon'}{\upsilon} = \frac{v_s + v_O}{v_s}$ (where v_s is the velocity of sound)

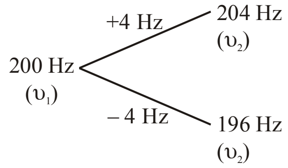
$= \frac{v + (v/5)}{v} = \frac{6}{5}$

$\therefore$ Fractional increase $= \frac{\upsilon' - \upsilon}{\upsilon} = \left(\frac{\upsilon'}{\upsilon} - 1\right) = \left(\frac{6}{5} - 1\right) = \frac{1}{5}$

$\therefore$ Percentage increase $= \frac{100}{5} = 20\%$.

30. **(a) :** Let the two frequencies be υ_1 and υ_2

υ_2 may be either 204 Hz or 196 Hz.



As mass of second fork increases, υ_2 decreases.

If υ_2 = 204 Hz, a decrease in υ_2 decreases beats/sec. But this is not given in question

If υ_2 = 196 Hz, a decrease in υ_2 increased beats/sec. This is given in the question when beats increase to 6

$\therefore$ Original frequency of second fork = 196 Hz.

31. **(a) :** In case of forced oscillations

(i) The amplitude is maximum at resonance

$\therefore$ Natural frequency = Frequency of force = ω_1

(ii) The energy is maximum at resonance

$\therefore$ Natural frequency = Frequency of force = ω_2

$\therefore$ From (i) and (ii),

$\omega_1 = \omega_2$.

32. **(b) :** In case of forced oscillations,

$x = a\sin(\omega t + \phi)$ where $a = \frac{F_0/m}{\omega_0^2 - \omega^2}$

$\therefore$ x is proportional to $\frac{1}{m(\omega_0^2 - \omega^2)}$.

33. **(c) :** Under simple harmonic motion, total energy

$= \frac{1}{2}ma^2\omega^2$

Total energy is independent of x.

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34. **(b) :** When springs are in series, $k = \frac{k_1 k_2}{k_1 + k_2}$

For first spring, $t_1 = 2\pi\sqrt{\frac{m}{k_1}}$

For second spring $t_2 = 2\pi\sqrt{\frac{m}{k_2}}$

$\therefore \quad t_1^2 + t_2^2 = \frac{4\pi^2 m}{k_1} + \frac{4\pi^2 m}{k_2} = 4\pi^2 m\left(\frac{k_1 + k_2}{k_1 k_2}\right)$

or $\quad t_1^2 + t_2^2 = \left[2\pi\sqrt{\frac{m(k_1+k_2)}{k_1 k_2}}\right]^2$

or $\quad t_1^2 + t_2^2 = T^2$.

35. **(b) :** Given wave equation :

$y = 10^{-6}\sin\left(100t + 20x + \frac{\pi}{4}\right)$m

Standard equation : $y = a\sin(\omega t + kx + \phi)$

Compare the two

$\therefore \quad \omega = 100$ and $k = 20$

$\therefore \quad \frac{\omega}{k} = \frac{100}{20} \Rightarrow \frac{2\pi n}{2\pi/\lambda} = n\lambda = v = 5$

$\therefore \quad v = 5$ m/s.

36. **(c) :** $t_0 = 2\pi\sqrt{l/g}$ .... (i)

Due to upthrust of water on the top, its apparent weight decreases

upthrust = weight of liquid displaced

$\therefore$ Effective weight $= mg - (V\sigma g) = V\rho g - V\sigma g$

$V\rho g' = Vg(\rho - \sigma)$, where σ is density of water

or $\quad g' = g\left(\frac{\rho - \sigma}{\rho}\right)$

$\therefore \quad t = 2\pi\sqrt{l/g'} = 2\pi\sqrt{\frac{l\rho}{g(\rho - \sigma)}}$ ......(ii)

$\therefore \quad \frac{t}{t_0} = \sqrt{\frac{l\rho}{g(\rho-\sigma)} \times \frac{g}{l}} = \sqrt{\frac{\rho}{\rho - \sigma}}$

$= \sqrt{\frac{4 \times 1000/3}{\left(\frac{4000}{3} - 1000\right)}} = 2$

or $\quad t = t_0 \times 2 = 2t_0$.

37. **(a) :** Kinetic energy is maximum at $x = 0$.

38. **(d) :** Let the lengths of pendulum be $(100l)$ and $(121l)$

$\therefore \quad \frac{T'}{T} = \sqrt{\frac{121}{100}} = \frac{11}{10}$

$\therefore$ Fractional change $= \frac{T' - T}{T} = \frac{11 - 10}{10} = \frac{1}{10}$

$\therefore$ Percentage change = 10%.

39. **(c) :** Maximum velocity under simple harmonic motion $v_m = a\omega$

$\therefore \quad v_m = \frac{2\pi a}{T} = (2\pi a)\left(\frac{1}{T}\right) = (2\pi a)\left(\frac{1}{2\pi}\sqrt{\frac{k}{m}}\right)$

or $\quad v_m = a\sqrt{\frac{k}{m}}$

$\because \quad (v_m)_A = (v_m)_B$

$\therefore \quad a_1\sqrt{\frac{k_1}{m}} = a_2\sqrt{\frac{k_2}{m}} \Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}$.

40. **(c) :** Initially, $T = 2\pi\sqrt{M/k}$

Finally, $\frac{5T}{3} = 2\pi\sqrt{\frac{M+m}{k}}$

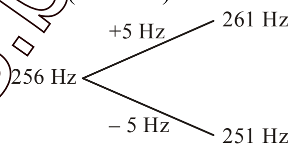
$\therefore \quad \frac{5}{3} \times 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M+m}{k}}$

or $\quad \frac{25}{9}\frac{M}{k} = \frac{M+m}{k}$

or $\quad 9m + 9M = 25M$

or $\quad \frac{m}{M} = \frac{16}{9}$.

41. **(c) :** The possible frequencies of piano are (256 + 5) Hz and (256 – 5) Hz.



For piano string, $\upsilon = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

When tension T increases, υ increases

(i) If 261 Hz increases, beats/sec increase. This is not given in the question.

(ii) If 251 Hz increases due to tension, beats per second decrease. This is given in the question.

Hence frequency of piano = (256 – 5) Hz.

42. **(a) :** At resonance, frequency of vibration of wire become equal to frequency of a.c.

For vibration of wire, $\upsilon = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

$\therefore \quad \upsilon = \frac{1}{2 \times 1}\sqrt{\frac{10 \times 9.8}{9.8 \times 10^{-3}}} = \frac{100}{2} = 50$ Hz.

43. **(c) :** $x = 4(\cos\pi t + \sin\pi t)$

$= 4 \times \sqrt{2}\left[\frac{1}{\sqrt{2}}\cos\pi t + \frac{1}{\sqrt{2}}\sin\pi t\right]$

or $\quad x = 4\sqrt{2}\left[\sin\frac{\pi}{4}\cos\pi t + \cos\frac{\pi}{4}\sin\pi t\right]$

$= 4\sqrt{2}\sin\left(\pi t + \frac{\pi}{4}\right)$

Hence amplitude $= 4\sqrt{2}$.

44. **(a) :** Given wave equation :

$y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right)$m

Standard wave equation : $y = a\sin(\omega t - kx + \phi)$

Compare them

Angular speed = ω = 600 sec^{-1}

Propagation constant = k = 2 m^{-1}

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$\frac{\omega}{k} = \frac{2\pi\upsilon}{2\pi/\lambda} = \upsilon\lambda = \text{velocity}$

$\therefore$ velocity $= \frac{\omega}{k} = \frac{600}{2} = 300\,\text{m/sec}$.

45. **(b) :** Time period will decrease.
When the child stands up, the centre of gravity is shifted upwards and so length of swing decreases. $T = 2\pi\sqrt{l/g}$.

46. **(c) :** In a simple harmonic oscillator, kinetic energy is maximum and potential energy is minimum at mean position.

47. **(b) :** For a spring, $T = 2\pi\sqrt{\frac{m}{k}}$
For each piece, spring constant = nk

$\therefore \quad T' = 2\pi\sqrt{\frac{m}{nk}}$

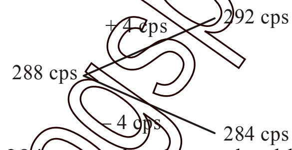
$\therefore \quad T' = 2\pi\sqrt{\frac{m}{k}} \times \frac{1}{\sqrt{n}} = \frac{T}{\sqrt{n}}$.

48. **(b) :** When temperature increases, l increases
Hence frequency decreases.

49. **(b) :** $\frac{\lambda_{\max}}{2} = 40 \Rightarrow \lambda_{\max} = 80\text{ cm}$.

50. **(b) :** Consider option (a)
Stationary wave :
$Y = a\sin(\omega t + kx) + a\sin(\omega t - kx)$
when $x = 0$, Y is not zero. The option is not acceptable.
Consider option (b)
Stationary wave :
$Y = a\sin(\omega t - kx) - a\sin(\omega t + kx)$
At $x = 0$, $Y = a\sin\omega t - a\sin\omega t$ = zero
This option holds good
Option (c) gives $Y = 2a\sin(\omega t - kx)$
At $x = 0$, Y is not zero
Option (d) gives $Y = 0$
Hence only option (b) holds good.

51. **(b) :** The wax decreases the frequency of unknown fork. The possible unknown frequencies are (288 + 4)cps and (288 – 4) cps.



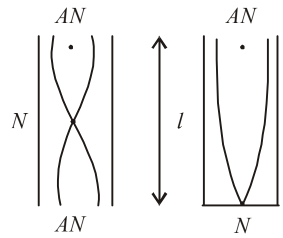
Wax reduces 284 cps and so beats should increases. It is not given in the question. This frequency is ruled out. Wax reduced 292 cps and so beats should decrease. It is given that the beats decrease to 2 from 4.
Hence unknown fork has frequency 292 cps.

52. **(c) :** In tube A, $\lambda_A = 2l$
In tube B, $\lambda_B = 4l$

$\therefore \quad \upsilon_A = \frac{v}{\lambda_A} = \frac{v}{2l}$

$\upsilon_B = \frac{v}{\lambda_B} = \frac{v}{4l}$

$\therefore \quad \frac{\upsilon_A}{\upsilon_B} = \frac{2}{1}$.



$\frac{\lambda_A}{2} = l \qquad \frac{\lambda_B}{4} = l$

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CHAPTER 11

ELECTROSTATICS

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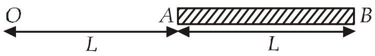
1. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then
(a) $9C_1 = 4C_2$ (b) $5C_1 = 3C_2$
(c) $3C_1 = 5C_2$ (d) $3C_1 + 5C_2 = 0$ *(2013)*

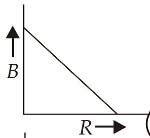
2. Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement ($y << a$) along the y-axis, the net force acting on the particle is proportional to
(a) $-\frac{1}{y}$ (b) y (c) $-y$ (d) $\frac{1}{y}$ *(2013)*

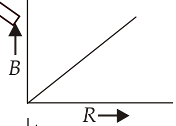
3. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at a distance L from the end A is

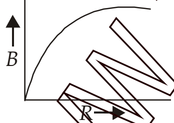


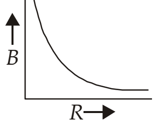
(a) $\frac{Q \ln 2}{4\pi \epsilon_0 L}$ (b) $\frac{Q \ln 2}{8\pi \epsilon_0 L}$
(c) $\frac{3Q}{4\pi \epsilon_0 L}$ (d) $\frac{Q}{4\pi \epsilon_0 L \ln 2}$ *(2013)*

4. A charge Q is uniformly distributed over the surface of non-conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure

(a) 

(b) 

(c) 

(d) 

(2012)

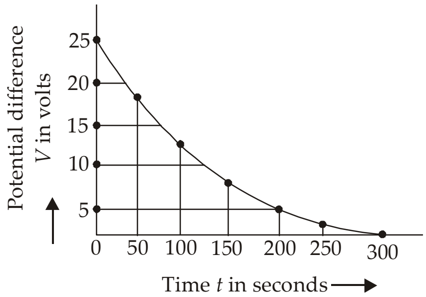
5. This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.

An insulating solid sphere of radius R has a uniformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point outside the sphere. The electric potential at infinity is zero.

Statement 1 : When a charge q is taken from the centre to the surface of the sphere, its potential energy changes by $\frac{q\rho}{3\varepsilon_0}$.

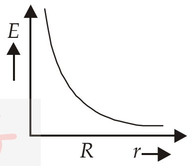
Statement 2 : The electric field at a distance $r (r < R)$ from the centre of the sphere is $\frac{\rho r}{3\varepsilon_0}$.

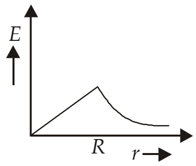
(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is false, Statement 2 is true.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.
(d) Statement 1 is true, Statement 2, is true; Statement 2 is not the correct explanation of Statement 1.
(2012)

6. 

The figure shows an experimental plot for discharging of a capacitor in an R-C circuit. The time constant t of this circuit lies between
(a) 0 and 50 sec (b) 50 sec and 100 sec
(c) 100 sec and 150 sec (d) 150 sec and 200 sec
(2012)

7. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as a function of distance from the centre. The graph which would correspond to the above will be

(a) 

(b) 

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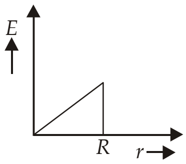
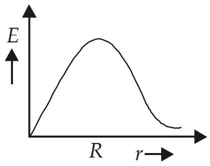
(c) 

(d) 

(2012)

8. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d(d << l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them

(a) $v \propto x^{-1/2}$ (b) $v \propto x^{-1}$ (c) $v \propto x^{1/2}$ (d) $v \propto x$ *(2011)*

9. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre; a, b are constants. Then the charge density inside the ball is

(a) $-24\pi a\varepsilon_0 r$ (b) $-6a\varepsilon_0 r$ (c) $-24\pi a\varepsilon_0$ (d) $-6a\varepsilon_0$ *(2011)*

10. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is

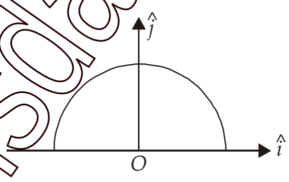
(a) 1 (b) 4 (c) 3 (d) 2 *(2010)*

11. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto $r = R$, and $\rho(r) = 0$ for $r > R$, where r is the distance from the origin. The electric field at a distance r ($r < R$) from the origin is given by

(a) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (b) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

(c) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (d) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ *(2010)*

12. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is



(a) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (b) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$

(c) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (d) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ *(2010)*

13. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then the Q/q equals

(a) $-2\sqrt{2}$ (b) -1 (c) 1 (d) $-\frac{1}{\sqrt{2}}$ *(2009)*

14. Two points P and Q are maintained at the potentials of 10 V and –4 V respectively. The work done in moving 100 electrons from P to Q is

(a) -9.60×10^{-17} J (b) 9.60×10^{-17} J

(c) -2.24×10^{-16} J (d) 2.24×10^{-16} J *(2009)*

15. Let $P(r) = \frac{Q}{\pi R^4}r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point 'p' inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is

(a) 0 (b) $\frac{Q}{4\pi\varepsilon_0 r_1^2}$ (c) $\frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$ (d) $\frac{Qr_1^2}{3\pi\varepsilon_0 R^4}$ *(2009)*

16. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1: For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement-2: The net work done by a conservative force on an object moving along a closed loop is zero.

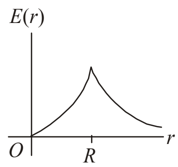
(a) Statement-1 is true, Statement-2 is false

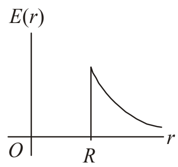
(b) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.

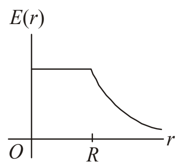
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.

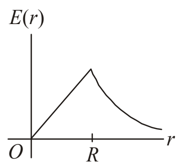
(d) Statement-1 is false, Statement-2 is true. *(2009)*

17. A thin spherical shell of radius R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field $E(r)$ produced by the shell in the range $0 \le r < \infty$, where r is the distance from the centre of the shell?

(a) 

(b) 

(c) 

(d) 

(2008)

18. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is d. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $k_1 = 3$ and thickness $d/3$ while the other one has dielectric constant $k_2 = 6$ and thickness $2d/3$. Capacitance of the capacitor is now

(a) 20.25 pF (b) 1.8 pF

(c) 45 pF (d) 40.5 pF. *(2008)*

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19. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volt. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is

(a) zero (b) $\frac{1}{2}(K-1)CV^2$

(c) $\frac{CV^2(K-1)}{K}$ (d) $(K-1)CV^2$ *(2007)*

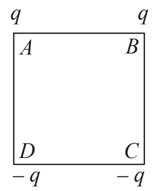
20. The potential at a point x (measured in μm) due to some charges situated on the x-axis is given by

$$V(x) = 20/(x^2 - 4) \text{ volt}$$

The electric field E at $x = 4$ μm is given by

(a) (10/9) volt/μm and in the +ve x direction
(b) (5/3) volt/μm and in the –ve x direction
(c) (5/3) volt/μm and in the +ve x direction
(d) (10/9) volt/μm in the –ve x direction *(2007)*

21. Charges are placed on the vertices of a square as shown. Let $\vec{E}$ be the electric field and V the potential at the centre. If the charges on A and B are interchanged with those on D and C respectively, then



(a) $\vec{E}$ changes, V remains unchanged
(b) $\vec{E}$ remains unchanged, V changes
(c) both $\vec{E}$ and V change
(d) $\vec{E}$ and V remain unchanged *(2007)*

22. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be

(a) 1/2 (b) 1 (c) 2 (d) 1/4 *(2007)*

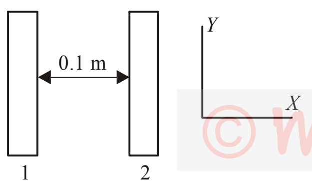
23. An electric charge 10^{-3} μC is placed at the origin (0, 0) of $X - Y$ co-ordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2,0) respectively. The potential difference between the points A and B will be

(a) 4.5 volt (b) 9 volt (c) zero (d) 2 volt *(2007)*

24. Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire then in equilibrium condition, the ratio of the magnitude of the electric fields at the surface of spheres A and B is

(a) 1 : 4 (b) 4 : 1 (c) 1 : 2 (d) 2 : 1. *(2006)*

25. Two insulating plates are both uniformly charged in such a way that the potential difference between them is $V_2 - V_1 = 20$ V. (*i.e.* plate 2 is at a higher potential). The plates are separated by $d = 0.1$ m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate 1. What is its speed when it hits plate 2? ($e = 1.6 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg)



(a) 32×10^{-19} m/s (b) 2.65×10^6 m/s
(c) 7.02×10^{12} m/s (d) 1.87×10^6 m/s. *(2006)*

26. A electric dipole is placed at an angle of 30° to a non-uniform electric field. The dipole will experience

(a) a torque only
(b) a translational force only in the direction of the field
(c) a translational force only in a direction normal to the direction of the field
(d) a torque as well as a translational force. *(2006)*

27. A fully charged capacitor has a capacitance C. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity s and mass m. If the temperature of the block is raised by ΔT, the potential difference V across the capacitance is

(a) $\frac{ms\Delta T}{C}$ (b) $\sqrt{\frac{2ms\Delta T}{C}}$

(c) $\sqrt{\frac{2mC\Delta T}{s}}$ (d) $\frac{mC\Delta T}{s}$ *(2005)*

28. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is C then the resultant capacitance is

(a) C (b) nC
(c) $(n-1)C$ (d) $(n+1)C$ *(2005)*

29. Two thin wire rings each having a radius R are placed at a distance d apart with their axes coinciding. The charges on the two rings are $+Q$ and $-Q$. The potential difference between the centers of the two rings is

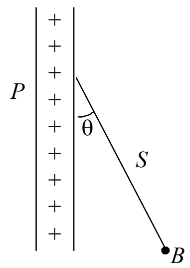
(a) zero (b) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$

(c) $\frac{QR}{4\pi\varepsilon_0 d^2}$ (d) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$ *(2005)*

30. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$ respectively. The location of a point on the x axis at which the net electric field due to these two point charges is zero is

(a) $8L$ (b) $4L$ (c) $2L$ (d) $L/4$ *(2005)*

31. A charged ball B hangs from a silk thread S, which makes an angle θ with a large charged conducting sheet P, as shown in the figure. The surface charge density σ of the sheet is proportional to



(a) sinθ
(b) tanθ
(c) cosθ
(d) cotθ *(2005)*

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32. Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium the value of q is

(a) $-\frac{Q}{4}(1+2\sqrt{2})$ (b) $\frac{Q}{4}(1+2\sqrt{2})$

(c) $-\frac{Q}{2}(1+2\sqrt{2})$ (d) $\frac{Q}{2}(1+2\sqrt{2})$. *(2004)*

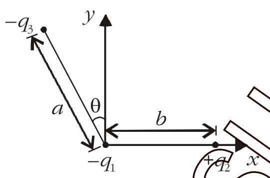
33. A charged particle q is shot towards another charged particle Q which is fixed, with a speed v. It approaches Q upto a closest distance r and then returns. If q were given a speed $2v$, the closest distances of approach would be

(a) r (b) $2r$ (c) $r/2$ (d) $r/4$. *(2004)*

34. Two spherical conductors B and C having equal radii and carrying equal charges in them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that of B but uncharged is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is

(a) $F/4$ (b) $3F/4$ (c) $F/8$ (d) $3F/8$. *(2004)*

35. Three charges $-q_1$, $+q_2$ and $-q_3$ are placed as shown in the figure. The x-component of the force on $-q_1$ is proportional to



(a) $\frac{q_2}{b^2}-\frac{q_3}{a^2}\cos\theta$

(b) $\frac{q_2}{b^2}+\frac{q_3}{a^2}\sin\theta$

(c) $\frac{q_2}{b^2}+\frac{q_3}{a^2}\cos\theta$

(d) $\frac{q_2}{b^2}-\frac{q_3}{a^2}\sin\theta$ *(2003)*

36. The work done in placing a charge of 8×10^{-18} coulomb on a condenser of capacity 100 micro-farad is

(a) 16×10^{-32} joule (b) 3.1×10^{-26} joule

(c) 4×10^{-10} joule (d) 32×10^{-32} joule. *(2003)*

37. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P at a distance $R/2$ from the centre of the shell is

(a) $\frac{2Q}{4\pi\varepsilon_0 R}$ (b) $\frac{2Q}{4\pi\varepsilon_0 R}-\frac{2q}{4\pi\varepsilon_0 R}$

(c) $\frac{2Q}{4\pi\varepsilon_0 R}+\frac{q}{4\pi\varepsilon_0 R}$ (d) $\frac{(q+Q)}{4\pi\varepsilon_0}\frac{2}{R}$ *(2003)*

38. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor

(a) decreases (b) remains unchanged

(c) becomes infinite (d) increases. *(2003)*

39. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2, the electric charge inside the surface will be

(a) $(\phi_2 - \phi_1)\varepsilon_0$ (b) $(\phi_1 + \phi_2)/\varepsilon_0$

(c) $(\phi_2 - \phi_1)/\varepsilon_0$ (d) $(\phi_1 + \phi_2)\varepsilon_0$. *(2003)*

40. Capacitance (in F) of a spherical conductor with radius 1 m is

(a) 1.1×10^{-10} (b) 10^{-6}

(c) 9×10^{-9} (d) 10^{-3}. *(2002)*

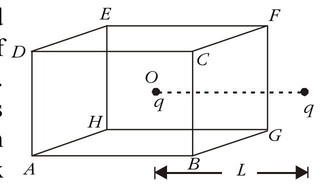
41. If a charge q is placed at the centre of the line joining two equal charges Q such that the system is in equilibrium then the value of q is

(a) $Q/2$ (b) $-Q/2$ (c) $Q/4$ (d) $-Q/4$. *(2002)*

42. If there are n capacitors in parallel connected to V volt source, then the energy stored is equal to

(a) CV (b) $\frac{1}{2}nCV^2$ (c) CV^2 (d) $\frac{1}{2n}CV^2$. *(2002)*

43. A charged particle q is placed at the centre O of cube of length L $(ABCDEFGH)$. Another same charge q is placed at a distance L from O. Then the electric flux through $ABCD$ is



(a) $q/4\pi\varepsilon_0 L$ (b) zero

(c) $q/2\pi\varepsilon_0 L$ (d) $q/3\pi\varepsilon_0 L$. *(2002)*

44. On moving a charge of 20 coulomb by 2 cm, 2 J of work is done, then the potential difference between the points is

(a) 0.1 V (b) 8 V (c) 2 V (d) 0.5 V. *(2002)*

Answer Key

1.	(c)	**2.**	(b)	**3.**	(a)	**4.**	(d)	**5.**	(b)	**6.**	(c)
7.	(b)	**8.**	(a)	**9.**	(d)	**10.**	(d)	**11.**	(c)	**12.**	(d)
13.	(a)	**14.**	(d)	**15.**	(c)	**16.**	(c)	**17.**	(b)	**18.**	(d)
19.	(a)	**20.**	(a)	**21.**	(a)	**22.**	(a)	**23.**	(c)	**24.**	(d)
25.	(b)	**26.**	(d)	**27.**	(b)	**28.**	(c)	**29.**	(d)	**30.**	(c)
31.	(b)	**32.**	(b)	**33.**	(d)	**34.**	(d)	**35.**	(b)	**36.**	(d)
37.	(c)	**38.**	(b)	**39.**	(a)	**40.**	(a)	**41.**	(d)	**42.**	(b)
43.	(*)	**44.**	(a)								

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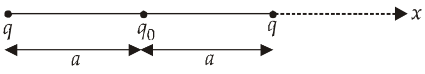
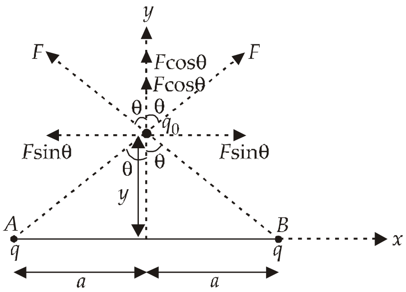
Explanations

1. **(c) :** For potential to be made zero, after connection
$120C_1 = 200C_2$
$6C_1 = 10C_2$
$3C_1 = 5C_2$

2. **(b):** The situation is as shown in the figure.



When a particle of mass m and charge $q_0\left(=\frac{q}{2}\right)$ placed is at the origin is given a small displacement along the y-axis, then the situation is shown in the figure.



By symmetry, the components of forces on the particle of charge q_0 due to charges at A and B along x-axis will cancel each other where along y-axis will add up.

$\therefore$ The net force acting on the particle is

$$F_{net} = 2F\cos\theta = 2\frac{1}{4\pi\epsilon_0}\frac{qq_0}{\left(\sqrt{y^2+a^2}\right)^2}\frac{y}{\sqrt{(y^2+a^2)}}$$

$$= \frac{2}{4\pi\epsilon_0}\frac{q\left(\frac{q}{2}\right)}{(y^2+a^2)2}\frac{y}{\sqrt{(y^2+a^2)}} \quad \left(\because q_0 = \frac{q}{2}\text{ (Given)}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q^2 y}{(y^2+a^2)^{3/2}}$$

As $y << a$

$$\therefore \quad F_{net} = \frac{1}{4\pi\epsilon_0}\frac{q^2 y}{a^3} \quad \text{or} \quad F_{net} \propto y$$

3. **(a) :**

Consider a small element of length dx at a distance x from O.

Charge on the element, $dQ = \frac{Q}{L}dx$

Potential at O due to the element is

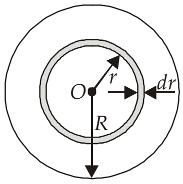
$$dV = \frac{1}{4\pi\epsilon_0}\frac{dQ}{x} = \frac{1}{4\pi\epsilon_0}\frac{Q}{Lx}dx$$

Potential at O due to the rod is

$$V = \int dV = \int_L^{2L}\frac{1}{4\pi\epsilon_0}\frac{Q}{Lx}dx$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{L}[\ln x]_L^{2L} = \frac{Q\ln 2}{4\pi\epsilon_0 L}$$

4. **(d):** Consider a elementary ring of radius r and thickness dr of a disc as shown in figure.



Charge on the ring,

$$dq = \frac{Q}{\pi R^2}(2\pi r dr) = \frac{2Qr}{R^2}dr$$

Current due to rotation of charge on ring is

$$I = \frac{dq\omega}{2\pi} = \frac{Qr\omega dr}{\pi R^2}$$

Magnetic field at the centre due to the ring element

$$dB = \frac{\mu_0 I}{2r} = \frac{\mu_0 Qr\omega dr}{2\pi R^2 r} = \frac{\mu_0 Q\omega dr}{2\pi R^2}$$

Magnetic field at the centre due to the whole disc

$$B = \int dB = \frac{\mu_0 Q\omega}{2\pi R^2}\int_0^R dr = \frac{\mu_0 Q\omega R}{2\pi R^2} = \frac{\mu_0 Q\omega}{2\pi R}$$

Since, Q and ω are constants

$$\therefore \quad B \propto \frac{1}{R}$$

Hence variation of B with R should be a rectangular hyperbola as represented in option (d).

5. **(b):** Potential at the centre of the sphere,

$$V_C = \frac{R^2\rho}{2\varepsilon_0}$$

Potential at the surface of the sphere,

$$V_S = \frac{1}{3}\frac{R^2\rho}{\varepsilon_0}$$

When a charge q is taken from the centre to the surface, the change in potential energy is

$$\Delta U = (V_C - V_S)q = \left(\frac{R^2\rho}{2\varepsilon_0} - \frac{1}{3}\frac{R^2\rho}{\varepsilon_0}\right)q = \frac{1}{6}\frac{R^2\rho q}{\varepsilon_0}$$

Statement 1 is false.
Statement 2 is true.

6. **(c) :** During discharging of a capacitor
$V = V_0 e^{-t/\tau}$
where τ is the time constant of RC circuit.
At $t = \tau$,

$$V = \frac{V_0}{e} = 0.37\,V_0$$

From the graph, $t = 0$, $V_0 = 25$ V
$\therefore \quad V = 0.37 \times 25$ V $= 9.25$ V
This voltage occurs at time lies between 100 sec and 500 sec. Hence, time constant τ of this circuit lies between 100 sec and 150 sec.

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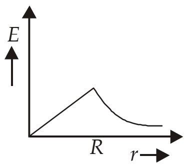
7. (b): For uniformly charged sphere

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3} \quad (\text{For } r < R)$$

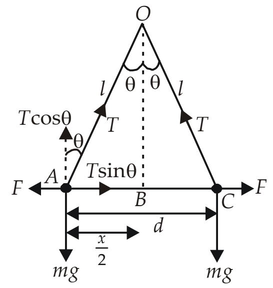
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} \quad (\text{For } r = R)$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \quad (\text{For } r > R)$$



The variation of E with distance r from the centre is as shown adjacent figure.

8. (a) : Figure shows equilibrium positions of the two sphere.



$\therefore \quad T\cos\theta = mg$

and $T\sin\theta = F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}$

$\therefore \quad \tan\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2 mg}$

When charge begins to leak from both the spheres at a constant rate, then

$$\tan\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2 mg}$$

$$\frac{x}{2l} = \frac{1q^2}{4\pi\varepsilon_0 x^2 mg} \qquad \left(\because \tan\theta = \frac{x}{2l}\right)$$

or $\frac{x}{2l} \propto \frac{q^2}{x^2}$

or $q^2 \propto x^3 \Rightarrow q \propto x^{3/2}$

$$\frac{dq}{dt} \propto \frac{3}{2}x^{1/2}\frac{dx}{dt}$$

or $v \propto x^{-1/2} \quad \left(\because \frac{dq}{dt} = \text{constant}\right)$

9. (d) : $\phi = ar^2 + b$

Electric field, $E = \frac{-d\phi}{dr} = -2ar$ ...(i)

According to Gauss's theorem,

$$\oint \vec{E}\cdot d\vec{S} = \frac{q_{\text{inside}}}{\varepsilon_0}$$

or $-2ar4\pi r^2 = \frac{q_{\text{inside}}}{\varepsilon_0}$ (Using (i))

$$q_{\text{inside}} = -8\varepsilon_0 a\pi r^3$$

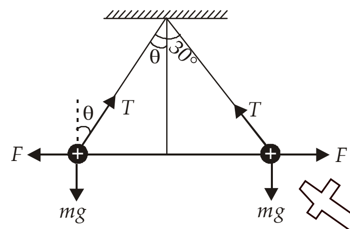
Charge density inside the ball is

$$\rho_{\text{inside}} = \frac{q_{\text{inside}}}{\frac{4}{3}\pi r^3}$$

$$\therefore \rho_{\text{inside}} = \frac{-8\varepsilon_0 a\pi r^3}{\frac{4}{3}\pi r^3}$$

$$\rho_{\text{inside}} = -6a\varepsilon_0$$

10. (d) :



Initially, the forces acting on each ball are

(i) Tension T

(ii) Weight mg

(iii) Electrostatic force of repulsion F

For its equilibrium along vertical,

$T\cos\theta = mg$ ...(i)

and along horizontal,

$T\sin\theta = F$ ...(ii)

Dividing equation (ii) by (i), we get

$$\tan\theta = \frac{F}{mg} \quad \text{...(iii)}$$

When the balls are suspended in a liquid of density σ and dielectric constant K, the electrostatic force will become $(1/K)$ times, *i.e.* $F' = (F/K)$ while weight

$mg' = mg$ – Upthrust

$= mg - V\sigma g$ [As Upthrust = $V\sigma g$]

$$mg' = mg\left[1 - \frac{\sigma}{\rho}\right] \quad \left[\text{As } V = \frac{m}{\rho}\right]$$

For equilibrium of balls,

$$\tan\theta' = \frac{F'}{mg'} = \frac{F}{Kmg[1-(\sigma/\rho)]} \quad \text{...(iv)}$$

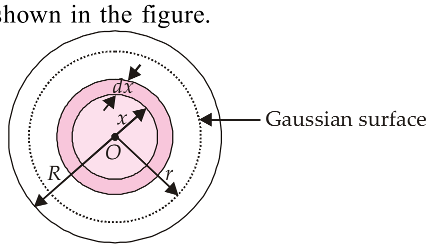
According to given problem, $\theta' = \theta$

From equations (iv) and (iii), we get

$$K = \frac{1}{\left(1 - \frac{\sigma}{\rho}\right)}$$

$$K = \frac{\rho}{(\rho - \sigma)} = \frac{1.6}{(1.6 - 0.8)} = 2$$

11. (c) : Consider a thin spherical shell of radius x and thickness dx as shown in the figure.



Volume of the shell, $dV = 4\pi x^2 dx$

Let us draw a Gaussian surface of radius $r (r < R)$ as shown in the figure above.

Total charge enclosed inside the Gaussian surface is

$$Q_{in} = \int_0^r \rho dV = \int_0^r \rho_0\left(\frac{5}{4} - \frac{x}{R}\right)4\pi x^2 dx$$

$$= 4\pi\rho_0\int_0^r\left(\frac{5}{4}x^2 - \frac{x^3}{R}\right)dx$$

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$$= 4\pi\rho_0\left[\frac{5}{12}x^3 - \frac{x^4}{4R}\right]_0^r = 4\pi\rho_0\left[\frac{5}{12}r^3 - \frac{r^4}{4R}\right]$$

$$= \frac{4\pi\rho_0}{4}\left[\frac{5}{3}r^3 - \frac{r^4}{R}\right] = \pi\rho_0\left[\frac{5}{3}r^3 - \frac{r^4}{R}\right]$$

According to Gauss's law

$$E4\pi r^2 = \frac{Q_{in}}{\varepsilon_0}$$

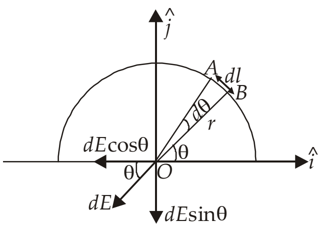
$$E4\pi r^2 = \frac{\pi\rho_0}{\varepsilon_0}\left[\frac{5}{3}r^3 - \frac{r^4}{R}\right]$$

$$E = \frac{\pi\rho_0 r^3}{4\pi r^2\varepsilon_0}\left[\frac{5}{3} - \frac{r}{R}\right]$$

$$E = \frac{\rho_0 r}{4\varepsilon_0}\left[\frac{5}{3} - \frac{r}{R}\right]$$

12. (d) : Linear charge density, $\lambda = \frac{q}{\pi r}$

Consider a small element AB of length dl subtending an angle $d\theta$ at the centre O as shown in the figure.



$\therefore$ Charge on the element,

$dq = \lambda dl$

$$= \lambda r d\theta \left(\because \ d\theta = \frac{dl}{r}\right)$$

The electric field at the centre O due to the charge element is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{\lambda r d\theta}{4\pi\varepsilon_0 r^2}$$

Resolve dE into two rectangular components

By symmetry, $\int dE\cos\theta = 0$

The net electric field at O is

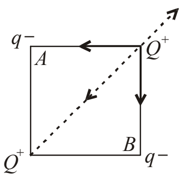
$$\vec{E} = \int_0^\pi dE\sin\theta(-\hat{j}) = \int_0^\pi \frac{\lambda r d\theta}{4\pi\varepsilon_0 r^2}\sin\theta(-\hat{j})$$

$$= -\int_0^\pi \frac{qr\sin\theta d\theta}{4\pi^2\varepsilon_0 r^3}\hat{j} \qquad \left(\because \lambda = \frac{q}{\pi r}\right)$$

$$= -\int_0^\pi \frac{q\sin\theta d\theta}{4\pi^2\varepsilon_0 r^2}\hat{j} = -\frac{q}{4\pi^2\varepsilon_0 r^2}[-\cos\theta]_0^\pi \hat{j}$$

$$= -\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$$

13. (a) : The force of repulsion by Q is cancelled by the resultant attracting force due to q^- and q^- at A and B.



Force of repulsion,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(a^2+a^2)} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2a^2}$$

Total force of attraction along the diagonal

(taking cosθ components)

$$= \frac{1}{4\pi\varepsilon_0}\left\{\frac{Qq}{a^2}\cdot\frac{1}{\sqrt{2}} + \frac{Qq}{a^2\sqrt{2}}\right\} = \frac{1}{4\pi\varepsilon_0}\left\{\frac{Qq\sqrt{2}}{a^2}\right\}$$

$$\Rightarrow \frac{Q^2}{2a^2} = -\frac{Qq\sqrt{2}}{a^2} \quad \Rightarrow \frac{Q^2}{Qq^-} = -2\sqrt{2}(a).$$

14. (d) : +10 V •··························• – 4 V
P Q

Work done in moving $100e^-$ from P to Q,

(Work done in moving 100 negative charges from the positive to the negative potential).

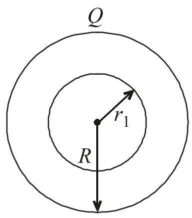
$W = (100e^-)(V_Q - V_P)$

$= (-100 \times 1.6 \times 10^{-19})(-14\text{ V}) = 2.24 \times 10^{-16}$ J.

15. (c) : If the charge density, $\rho = \frac{Q}{\pi R^4}r$,

The electric field at the point p distant r_1 from the centre, according to Gauss's theorem is

$E\cdot 4\pi r_1^2$ = charge enclosed/ε_0



$$E\cdot 4\pi r_1^2 = \frac{1}{\varepsilon_0}\int \rho dV$$

$$\Rightarrow E\cdot 4\pi r_1^2 = \frac{1}{\varepsilon_0}\int_0^{r_1}\frac{Qr}{\pi R^4}4\pi r^2 dr$$

$$\Rightarrow E = \frac{Qr_1^2}{4\pi\varepsilon_0 R^4}.$$

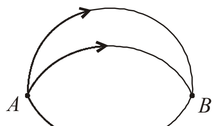
16. (c) : Work done = potential difference × charge

$= (V_B - V_A) \times q,$

V_A and V_B only depend on the initial and final positions and not on the path. Electrostatic force is a conservative force.

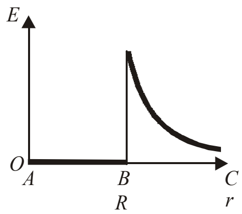
If the loop is completed, $V_A - V_A = 0$.

No net work is done as the initial and final potentials are the same.



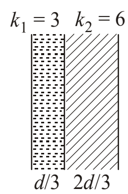
Both the statements are true but statement-2 is not the reason for statement-1.

17. (b) : The electric field for a uniformly charged spherical shell is given in the figure. Inside the shell, the field is zero and it is maximum at the surface and then decreases $\propto 1/r^2$.



$$E = \frac{Q}{4\pi\varepsilon_0\cdot r^2}$$ outside shell and zero inside.

18. (d) : $C = \frac{\varepsilon_0 A}{d} = 9\times10^{-12}$ F



With dielectric, $C = \frac{\varepsilon_0 kA}{d}$

$$C_1 = \frac{\varepsilon_0 A\cdot 3}{d/3} = 9C;$$

$$C_2 = \frac{\varepsilon_0 A\cdot 6}{2d/3} = 9C$$

$\therefore C_{\text{total}} = \frac{C_1C_2}{C_1+C_2}$ as they are in series.

$$= \frac{9C\times9C}{18C} = \frac{9}{2}\times C \text{ or } \frac{9}{2}\times9\times10^{-12}\text{ F}$$

$\Rightarrow C_{\text{total}} = 40.5$ pF.

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19. (a) : The potential energy of a charged capacitor

$$U_i = \frac{q^2}{2C}$$

where U_i is the initial potential energy.
If a dielectric slab is slowly introduced, the energy

$$= \frac{q^2}{2KC}$$

Once is taken out, again the energy increases to the old value. Therefore after it is taken out, the potential energy come back to the old value. Total work done = zero.

20. **(a) :** Given : Potential $V(x) = \frac{20}{x^2-4}$

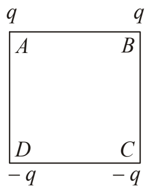
Electric field $E = \frac{-dV}{dx} = \frac{-d}{dx}\left(\frac{20}{x^2-4}\right) = \frac{40x}{(x^2-4)^2}$

At $x = 4$ μm

$$\therefore \quad E = \frac{40\times4}{[16-4]^2} = \frac{160}{144} = \frac{10}{9} \text{ V/μm}.$$

Positive sign indicate E is +ve x direction.

21. **(a) :** "Unit positive charge" will be repelled by A and B and attracted by $-q$ and $-q$ downwards in the same direction. If they are exchanged, the direction of the field will be opposite. In the case of potential, as it is a scalar, they cancel each other whatever may be their position.



∴ Field is affected but not the potential.

22. **(a) :** Let E be emf of the battery
Work done by the battery $W = CE^2$

Energy stored in the capacitor $U = \frac{1}{2}CE^2$

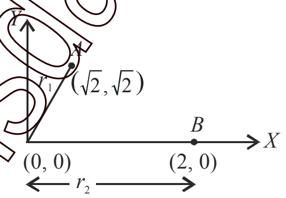
$$\therefore \quad \frac{U}{W} = \frac{\frac{1}{2}CE^2}{CE^2} = \frac{1}{2}.$$

23. **(c) :** $\vec{r}_1 = \sqrt{2}\hat{i} + \sqrt{2}\hat{j}$

$$|\vec{r}_1| = r_1 = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = 2$$

$\vec{r}_2 = 2\hat{i} + 0\hat{j}$

or $\quad |\vec{r}_2| = r_2 = 2$



Potential at point A is

$$V_A = \frac{1q}{4\pi\varepsilon_0 r_1}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{10^{-3}\times10^{-6}}{2}$$

Potential at point B is

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_2} = \frac{1}{4\pi\varepsilon_0}\frac{10^{-3}\times10^{-6}}{2}$$

∴ $V_A - V_B = 0.$

24. **(d) :** When the spherical conductors are connected by a conducting wire, charge is redistributed and the spheres attain a common potential V.

∴ Intensity $E_A = \frac{1}{4\pi\varepsilon_0}\frac{Q_A}{R_A^2}$

or $\quad E_A = \frac{1\times C_A V}{4\pi\varepsilon_0 R_A^2} = \frac{(4\pi\varepsilon_0 R_A)V}{4\pi\varepsilon_0 R_A^2} = \frac{V}{R_A}$

Similarly $E_B = \frac{V}{R_B}$

$$\therefore \quad \frac{E_A}{E_B} = \frac{R_B}{R_A} = \frac{2}{1}.$$

25. **(b) :** An electron on plate 1 has electrostatic potential energy. When it moves, potential energy is converted into kinetic energy.
∴ Kinetic energy = Electrostatic potential energy

or $\quad \frac{1}{2}mv^2 = e\Delta V$

or $\quad v = \sqrt{\frac{2e\times\Delta V}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times20}{9.11\times10^{-31}}}$

or $\quad v = 2.65 \times 10^6$ m/s.

26. **(d) :** In a non-uniform electric field, the dipole will experience a torque as well as a translational force.

27. **(b) :** Energy of capacitor = Heat energy of block

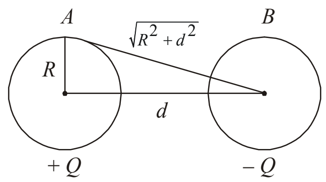
∴ $\quad \frac{1}{2}CV^2 = ms\Delta T$

or $\quad V = \sqrt{\frac{2ms\,\Delta T}{C}}.$

28. **(c) :** n plates connected alternately give rise to $(n-1)$ capacitors connected in parallel
∴ Resultant capacitance = $(n-1)C$.

29. **(d) :** $V_A = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+d^2}}$

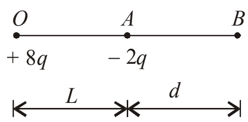
$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+d^2}}$$



$$\therefore \quad V_A - V_B = \frac{1\times Q}{4\pi\varepsilon_0}\left[\frac{2}{R} - \frac{2}{\sqrt{R^2+d^2}}\right]$$

$$= \frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right].$$

30. **(c) :** Resultant intensity = 0



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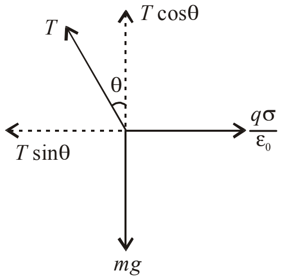
$$\frac{1}{4\pi\varepsilon_0}\frac{8q}{(L+d)^2}-\frac{1}{4\pi\varepsilon_0}\frac{2q}{d^2}=0$$

or $(L + d)^2 = 4d^2$

or $d = L$

$\therefore$ Distance from origin = $2L$.

31. **(b) :** $T\sin\theta = \sigma q/\varepsilon_0$

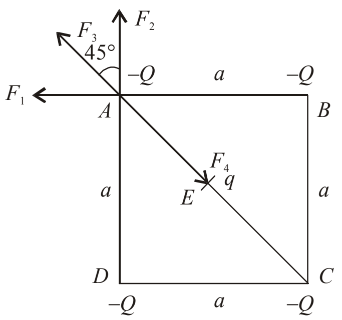


$T\cos\theta = mg$

$\therefore \quad \tan\theta = \dfrac{\sigma q}{\varepsilon_0 mg}$

$\therefore$ σ is proportional to $\tan\theta$.

32. **(b) :** Consider the four forces F_1, F_2, F_3 and F_4 acting on charge $(-Q)$ placed at A.



Distance $CA = \sqrt{2}\,a$

Distance $EA = \dfrac{\sqrt{2}\,a}{2} = \dfrac{a}{\sqrt{2}}$

For equilibrium, consider forces along DA and equate the resultant to zero

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\frac{Q\times Q}{(DA)^2}+\frac{1}{4\pi\varepsilon_0}\frac{Q\times Q}{(CA)^2}\cos 45^\circ$$

$$-\frac{1}{4\pi\varepsilon_0}\frac{Q\times q}{(EA)^2}\cos 45^\circ = 0$$

or $\dfrac{Q}{a^2}+\dfrac{Q}{2a^2}\times\dfrac{1}{\sqrt{2}}-\dfrac{q}{a^2/2}\times\dfrac{1}{\sqrt{2}}=0$

or $Q\left[1+\dfrac{1}{2\sqrt{2}}\right]=q\sqrt{2}$

or $q=\dfrac{Q}{\sqrt{2}}\left[\dfrac{2\sqrt{2}+1}{2\sqrt{2}}\right]=\dfrac{Q}{4}(1+2\sqrt{2})$.

33. **(d) :** Energy is conserved in the phenomenon

Initially, $\dfrac{1}{2}mv^2=\dfrac{kqQ}{r}$ ....(i)

Finally, $\dfrac{1}{2}m(2v)^2=\dfrac{kqQ}{r_1}$ ....(ii)

$\therefore$ From (i) and (ii)

$\dfrac{1}{4}=\dfrac{r_1}{r}\Rightarrow r_1=\dfrac{r}{4}$.

34. **(d) :** Initially, $F=\dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{d^2}$ ....(i)

when the third equal conductor touches B, the charge of B is shared equally between them

$\therefore$ Charge on $B=\dfrac{q}{2}$ = charge on third conductor.

Now this third conductor with charge $\left(\dfrac{q}{2}\right)$ touches C, their total charge $\left(q+\dfrac{q}{2}\right)$ is equally shared between them.

$\therefore$ Charge on

$C=\dfrac{3q}{4}$ = Charge of third conductor

$\therefore$ New force between B and C

$$=\frac{1}{4\pi\varepsilon_0 d^2}\left(\frac{q}{2}\times\frac{3q}{4}\right)=\frac{3}{8}F.$$

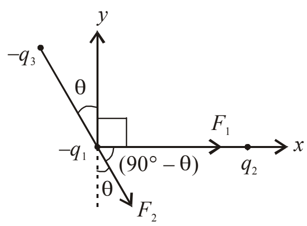
35. **(b) :** Force on $(-q_1)$ due to $q_2=\dfrac{-q_1q_2}{4\pi\varepsilon_0 b^2}$

$\therefore \quad F_1 = \dfrac{q_1q_2}{4\pi\varepsilon_0 b^2}$ along (q_1q_2)

Force on $(-q_1)$ due to $(-q_3)=\dfrac{(-q_1)(-q_3)}{4\pi\varepsilon_0 a^2}$

$F_2 = \dfrac{q_1q_3}{4\pi\varepsilon_0 a^2}$ as shown

F_2 makes an angle of $(90^\circ - \theta)$ with (q_1q_2)



Resolved part of F_2 along q_1q_2

$= F_2\cos(90^\circ - \theta)$

$=\dfrac{q_1q_3\sin\theta}{4\pi\varepsilon_0 a^2}$ along (q_1q_2)

$\therefore$ Total force on $(-q_1)$

$$=\left[\frac{q_1q_2}{4\pi\varepsilon_0 b^2}+\frac{q_1q_3\sin\theta}{4\pi\varepsilon_0 a^2}\right]\text{along } x\text{-axis}$$

$\therefore$ x-component of force $\propto\left[\dfrac{q_2}{b^2}+\dfrac{q_3}{a^2}\sin\theta\right]$.

36. **(d) :** Energy of condenser

$$=\frac{1}{2}\frac{Q^2}{C}=\frac{1}{2}\times\frac{(8\times10^{-18})^2}{(100\times10^{-6})}=32\times10^{-32}\text{ J}$$

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37. (c) : Potential at any internal point of charged shell $= \frac{q}{4\pi\varepsilon_0 R}$

Potential at P due to Q at centre $= \frac{1}{4\pi\varepsilon_0}\frac{2Q}{R}$

$\therefore$ Total potential point

$$= \frac{q}{4\pi\varepsilon_0 R} + \frac{2Q}{4\pi\varepsilon_0 R} = \frac{1}{4\pi\varepsilon_0 R}(q + 2Q).$$

38. (b) : Aluminium is a good conductor. Its sheet introduced between the plates of a capacitor is of negligible thickness. The capacity remains unchanged.

With air as dielectric, $C = \frac{\varepsilon_0 A}{d}$

With space partially filled, $C' = \frac{\varepsilon_0 A}{(d - t)} = \frac{\varepsilon_0 A}{d} = C.$

39. (a) : According to Gauss theorem,

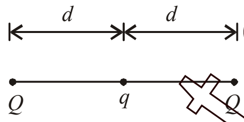
$$(\phi_2 - \phi_1) = \frac{Q}{\varepsilon_0} \Rightarrow Q = (\phi_2 - \phi_1)\varepsilon_0 .$$

The flux enters the enclosure if one has a negative charge $(-q_2)$ and flux goes out if one has a +ve charge $(+q_1)$. As one does not know whether $\phi_1 > \phi_2$, $\phi_2 > \phi_1$, $Q = q_1 \sim q_2$

40. (a) : $C = 4\pi\varepsilon_0 R = \frac{1}{9 \times 10^9} = 1.1 \times 10^{-10}$ F .

41. (d) : When the system of three charges is in equilibrium,

$$\frac{Q \times q}{4\pi\varepsilon_0 d^2} + \frac{Q \times Q}{4\pi\varepsilon_0 (2d)^2} = 0$$



or $q = -\frac{Q}{4}$.

42. (b) : Total capacity $= nC$

$\therefore$ Energy $= \frac{1}{2}nCV^2$

43. (*) : Electric flux through $ABCD$ = zero for the charge placed outside the box as the charged enclosed is zero. But for the charge inside the cube, it is $\frac{q}{\varepsilon_0}$ through all the surfaces. For one surface, it is $\frac{q}{6\varepsilon_0}$. (Option not given).

44. (a) : $W = QV$

$\therefore V = \frac{W}{Q} = \frac{2}{20} = 0.1$ volt.

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CHAPTER 12

CURRENT ELECTRICITY

1. The supply voltage to a room is 120 V. The resistance of the lead wires is 6 Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb?

(a) 10.04 Volt (b) zero Volt
(c) 2.9 Volt (d) 13.3 Volt *(2013)*

2. Two electric bulbs marked 25 W-220 V and 100 W-220 V are connected in series to a 440 V supply. Which of the bulbs will fuse?

(a) 100 W (b) 25 W
(c) neither (d) both *(2012)*

3. If a wire is stretched to make it 0.1% longer, its resistance will

(a) increase by 0.05% (b) increase by 0.2%
(c) decrease by 0.2% (d) decrease by 0.05% *(2011)*

4. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly

(a) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$ (b) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$
(c) $\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$ (d) $\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$ *(2010)*

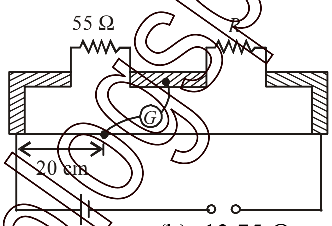
5. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1: The temperature dependence of resistance is usually given as $R = R_0(1 + \alpha\Delta t)$. The resistance of a wire changes from 100 Ω to 150 Ω when its temperature is increased from 27°C to 227°C. This implies that $\alpha = 2.5 \times 10^{-3}$/°C

Statement-2: $R = R_0(1 + \alpha\Delta T)$ is valid only when the change in the temperature ΔT is small and $\Delta R = (R - R_0) << R_0$.

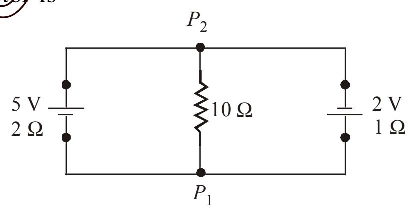
(a) Statement-1 is true, Statement-2 is false
(b) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true. *(2009)*

6. Shown in the figure below is a meter-bridge set up with null deflection in the galvanometer. The value of the unknown resistance R is



(a) 55 Ω (b) 13.75 Ω
(c) 220 Ω (d) 110 Ω. *(2008)*

7. A 5 V battery with internal resistance 2 Ω and 2 V battery with internal resistance 1 Ω are connected to a 10 Ω resistor as shown in the figure. The current in the 10 Ω resistor is

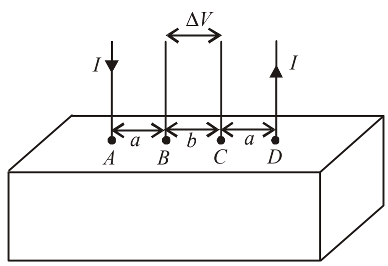


(a) 0.27 A P_1 to P_2 (b) 0.27 A P_2 to P_1
(c) 0.03 A P_1 to P_2 (d) 0.03 A P_2 to P_1. *(2008)*

Directions : Questions 8 and 9 are based on the following paragraph.

Consider a block of conducting material of resistivity ρ shown in the figure. Current I enters at A and leaves from D. We apply superposition principle to find voltage ΔV developed between B and C. The calculation is done in the following steps:

(i) Take current I entering from A and assume it to spread over a hemispherical surface in the block.
(ii) Calculate field $E(r)$ at distance r from A by using Ohm's law $E = \rho j$, where j is the current per unit area at r.
(iii) From the r dependence of $E(r)$, obtain the potential $V(r)$ at r.
(iv) Repeat (i), (ii) and (iii) for current I leaving D and superpose results for A and D.



8. ΔV measured between B and C is

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(a) $\frac{\rho I}{2\pi(a-b)}$ (b) $\frac{\rho I}{\pi a} - \frac{\rho I}{\pi(a+b)}$

(c) $\frac{\rho I}{a} - \frac{\rho I}{(a+b)}$ (d) $\frac{\rho I}{2\pi a} - \frac{\rho I}{2\pi(a+b)}$ *(2008)*

9. For current entering at A, the electric field at a distance r from A is

(a) $\frac{\rho I}{4\pi r^2}$ (b) $\frac{\rho I}{8\pi r^2}$

(c) $\frac{\rho I}{r^2}$ (d) $\frac{\rho I}{2\pi r^2}$ *(2008)*

10. The resistance of a wire is 5 ohm at 50°C and 6 ohm at 100°C. The resistance of the wire at 0°C will be

(a) 3 ohm (b) 2 ohm

(c) 1 ohm (d) 4 ohm *(2007)*

11. A material B has twice the specific resistance of A. A circular wire made of B has twice the diameter of a wire made of A. Then for the two wires to have the same resistance, the ratio l_B/l_A of their respective lengths must be

(a) 2 (b) 1

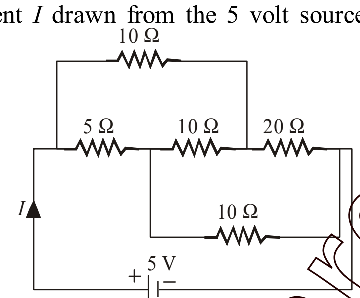
(c) 1/2 (d) 1/4. *(2006)*

12. The resistance of a bulb filament is 100 Ω at a temperature of 100ºC. If its temperature coefficient of resistance be 0.005 per ºC, its resistance will become 200 Ω at a temperature of

(a) 200ºC (b) 300ºC

(c) 400ºC (d) 500ºC. *(2006)*

13. The current I drawn from the 5 volt source will be



(a) 0.17 A (b) 0.33 A

(c) 0.5 A (d) 0.67 A. *(2006)*

14. In a Wheatstone's bridge, three resistance P, Q and R connected in the three arms and the fourth arm is formed by two resistance S_1 and S_2 connected in parallel. The condition for bridge to be balanced will be

(a) $\frac{P}{Q} = \frac{R}{S_1 + S_2}$ (b) $\frac{P}{Q} = \frac{2R}{S_1 + S_2}$

(c) $\frac{P}{Q} = \frac{R(S_1 + S_2)}{S_1 S_2}$ (d) $\frac{P}{Q} = \frac{R(S_1 + S_2)}{2S_1 S_2}$. *(2006)*

15. The Kirchhoff's first law $(\sum i = 0)$ and second law $(\sum iR = \sum E)$, where the symbols have their usual meanings, are respectively based on

(a) conservation of charge, conservation of energy

(b) conservation of charge, conservation of momentum

(c) conservation of energy, conservation of charge

(d) conservation of momentum, conservation of charge. *(2006)*

16. An electric bulb is rated 220 volt - 100 watt. The power consumed by it when operated on 110 volt will be

(a) 50 watt (b) 75 watt

(c) 40 watt (d) 25 watt. *(2006)*

17. A thermocouple is made from two metals, antimony and bismuth. If one junction of the couple is kept hot and the other is kept cold then, an electric current will

(a) flow from antimony to bismuth at the cold junction

(b) flow from antimony to bismuth at the hot junction

(c) flow from bismuth to antimony at the cold junction

(d) not flow through the thermocouple. *(2006)*

18. In a potentiometer experiment the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of 2 Ω, the balancing length becomes 120 cm. The internal resistance of the cell is

(a) 4 Ω (b) 2 Ω

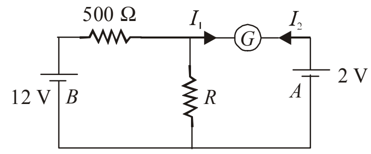
(c) 1 Ω (d) 0.5 Ω *(2005)*

19. Two sources of equal emf are connected to an external resistance R. The internal resistances of the two sources are R_1 and R_2 ($R_2 > R_1$). If the potential difference across the source having internal resistance R_2 is zero, then

(a) $R = \frac{R_1 R_2}{R_1 + R_2}$ (b) $R = \frac{R_1 R_2}{R_2 - R_1}$

(c) $R = R_2 \frac{(R_1 + R_2)}{(R_2 - R_1)}$ (d) $R = R_2 - R_1$ *(2005)*

20. In the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be



(a) 500 Ω (b) 1000 Ω

(c) 200 Ω (d) 100 Ω *(2005)*

21. An energy source will supply a constant current into the load if its internal resistance is

(a) zero

(b) non-zero but less than the resistance of the load

(c) equal to the resistance of the load

(d) very large as compared to the load resistance *(2005)*

22. The resistance of hot tungsten filament is about 10 times the cold resistance. What will be the resistance of 100 W and 200 V lamp when not in use?

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(a) 400 Ω (b) 200 Ω
(c) 40 Ω (d) 20 Ω *(2005)*

23. Two voltameters, one of copper and another of silver, are joined in parallel. When a total charge q flows through the voltmeters, equal amount of metals are deposited. If the electrochemical equivalents of copper and silver are z_1 and z_2 respectively the charge which flows through the silver voltameter is

(a) $q\frac{z_1}{z_2}$ (b) $q\frac{z_2}{z_1}$

(c) $\frac{q}{1+\frac{z_1}{z_2}}$ (d) $\frac{q}{1+\frac{z_2}{z_1}}$ *(2005)*

24. A heater coil is cut into two equal parts and only one part is now used in the heater. The heat generated will now be
(a) one fourth (b) halved
(c) doubled (d) four times *(2005)*

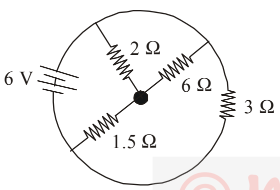
25. The thermistors are usually made of
(a) metals with low temperature coefficient of resistivity
(b) metals with high temperature coefficient of resistivity
(c) metal oxides with high temperature coefficient of resistivity
(d) semiconducting materials having low temperature coefficient of resistivity. *(2004)*

26. In a metre bridge experiment null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y. If $X < Y$, then where will be the new position of the null point from the same end, if one decides to balance a resistance of $4X$ against Y?
(a) 50 cm (b) 80 cm
(c) 40 cm (d) 70 cm. *(2004)*

27. An electric current is passed through a circuit containing two wires of the same material, connected in parallel. If the lengths and radii of the wires are in the ratio of 4/3 and 2/3, then the ratio of the currents passing through the wire will be
(a) 3 (b) 1/3
(c) 8/9 (d) 2. *(2004)*

28. The resistance of the series combination of two resistances is S. When they are joined in parallel the total resistance is P. If $S = nP$, then the minimum possible value of n is
(a) 4 (b) 3
(c) 2 (d) 1. *(2004)*

29. The total current supplied to the circuit by the battery is
(a) 1 A
(b) 2 A
(c) 4 A
(d) 6 A. *(2004)*



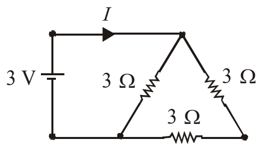
30. The electrochemical equivalent of a metal is 3.3×10^{-7} kg per coulomb. The mass of the metal liberated at the cathode when a 3 A current is passed for 2 second will be
(a) 19.8×10^{-7} kg (b) 9.9×10^{-7} kg
(c) 6.6×10^{-7} kg (d) 1.1×10^{-7} kg. *(2004)*

31. The thermo emf of a thermocouple varies with the temperature θ of the hot junction as $E = a\theta + b\theta^2$ in volt where the ratio a/b is 700°C. If the cold junction is kept at 0°C, then the neutral temperature is
(a) 700°C (b) 350°C
(c) 1400°C
(d) no neutral temperature is possible for this thermocouple. *(2004)*

32. Time taken by a 836 W heater to heat one litre of water from 10°C to 40°C is
(a) 50 s (b) 100 s (c) 150 s (d) 200 s. *(2004)*

33. The length of a given cylindrical wire is increased by 100%. Due to the consequent decrease in diameter the change in the resistance of the wire will be
(a) 200% (b) 100%
(c) 50% (d) 300%. *(2003)*

34. A 3 volt battery with negligible internal resistance is connected in a circuit as shown in the figure.
The current I, in the circuit will be



(a) 1 A (b) 1.5 A
(c) 2 A (d) (1/3) A *(2003)*

35. The length of a wire of a potentiometer is 100 cm, and the e.m.f. of its standard cell is E volt. It is employed to measure the e.m.f. of a battery whose internal resistance is 0.5 Ω. If the balance point is obtained at l = 30 cm from the positive end, the e.m.f. of the battery is

(a) $\frac{30E}{100.5}$ (b) $\frac{30E}{100-0.5}$

(c) $\frac{30E}{100}-0.5i$, where i is the current in the potentiometer wire.

(d) $\frac{30E}{100}$. *(2003)*

36. A 220 volt, 1000 watt bulb is connected across a 110 volt mains supply. The power consumed will be
(a) 750 watt (b) 500 watt
(c) 250 watt (d) 1000 watt. *(2003)*

37. The negative Zn pole of a Daniell cell, sending a constant current through a circuit, decreases in mass by 0.13 g in 30 minutes. If the electrochemical equivalent of Zn and Cu are 32.5 and 31.5 respectively, the increase in the mass of the

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positive Cu pole in this time is
(a) 0.180 g (b) 0.141 g
(c) 0.126 g (d) 0.242 g. *(2003)*

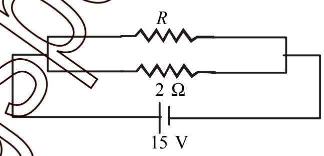
38. The thermo e.m.f. of a thermo-couple is 25 μV/°C at room temperature. A galvanometer of 40 ohm resistance, capable of detecting current as low as 10^{-5} A, is connected with the thermocouple. The smallest temperature difference that can be detected by this system is
(a) 16°C (b) 12°C
(c) 8°C (d) 20°C. *(2003)*

39. The mass of a product liberated on anode in an electrochemical cell depends on
(a) $(It)^{1/2}$ (b) It (c) I/t (d) I^2t.
(where t is the time period for which the current is passed). *(2002)*

40. If θ_i is the inversion temperature, θ_n is the neutral temperature, θ_c is the temperature of the cold junction, then
(a) $\theta_i + \theta_c = \theta_n$ (b) $\theta_i - \theta_c = 2\theta_n$
(c) $\frac{\theta_i + \theta_c}{2} = \theta_n$ (d) $\theta_c - \theta_i = 2\theta_n$. *(2002)*

41. A wire when connected to 220 V mains supply has power dissipation P_1. Now the wire is cut into two equal pieces which are connected in parallel to the same supply. Power dissipation in this case is P_2. Then $P_2 : P_1$ is
(a) 1 (b) 4 (c) 2 (d) 3. *(2002)*

42. If in the circuit, power dissipation is 150 W, then R is



(a) 2 Ω
(b) 6 Ω
(c) 5 Ω
(d) 4 Ω. *(2002)*

Answer Key

1.	(a)	**2.**	(b)	**3.**	(b)	**4.**	(a)	**5.**	(a)	**6.**	(c)
7.	(d)	**8.**	(d)	**9.**	(d)	**10.**	(d)	**11.**	(a)	**12.**	(c)
13.	(c)	**14.**	(c)	**15.**	(a)	**16.**	(d)	**17.**	(a)	**18.**	(b)
19.	(d)	**20.**	(d)	**21.**	(a)	**22.**	(c)	**23.**	(d)	**24.**	(c)
25.	(c)	**26.**	(a)	**27.**	(b)	**28.**	(a)	**29.**	(c)	**30.**	(a)
31.	(d)	**32.**	(c)	**33.**	(d)	**34.**	(b)	**35.**	(c)	**36.**	(c)
37.	(c)	**38.**	(a)	**39.**	(b)	**40.**	(c)	**41.**	(b)	**42.**	(b)

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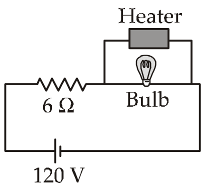
Explanations

1. **(a) :** As $P = \frac{V^2}{R}$



Here, the supply voltage is taken as rated voltage.

$\therefore$ Resistance of bulb

$$R_B = \frac{120\text{ V} \times 120\text{ V}}{60\text{ W}} = 240\ \Omega$$

Resistance of heater, $R_H = \frac{120\text{ V} \times 120\text{ V}}{240\text{ W}} = 60\ \Omega$

Voltage across bulb before heater is switched on,

$$V_1 = \frac{120\text{ V} \times 240\ \Omega}{240\ \Omega + 6\ \Omega} = 117.07\text{ V}$$

As bulb and heater are connected in parallel. Their equivalent resistance is

$$R_{eq} = \frac{(240\ \Omega)(60\ \Omega)}{240\ \Omega + 60\ \Omega} = 48\ \Omega$$

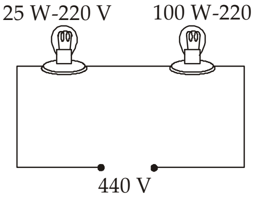
$\therefore$ Voltage across bulb after heater is switched on

$$V_2 = \frac{120\text{ V} \times 48\ \Omega}{48\ \Omega + 6\ \Omega} = 106.66\text{ V}$$

Decrease in the voltage across the bulb is

$\Delta V = V_1 - V_2 = 10.41\text{ V} \simeq 10.04\text{ V}$

2. **(b) :**



As $R = \frac{(\text{Rated voltage})^2}{\text{Rated power}}$

$\therefore$ Resistance of 25 W-220 V bulb is

$$R_1 = \frac{(220)^2}{25}\ \Omega$$

Resistance of 100 W-220 V bulb is

$$R_2 = \frac{(220)^2}{100}\ \Omega$$

When these two bulbs are connected in series, the total resistance is

$$R_s = R_1 + R_2 = (220)^2\left[\frac{1}{25} + \frac{1}{100}\right] = \frac{(220)^2}{20}\ \Omega$$

Current, $I = \frac{440}{(220)^2/20} = \frac{2}{11}$ A

Potential difference across 25 W bulb

$$= IR_1 = \frac{2}{11} \times \frac{(220)^2}{25} = 352\text{ V}$$

Potential difference across 100 W bulb

$$= IR_2 = \frac{2}{11} \times \frac{(220)^2}{100} = 88\text{ V}$$

Thus the bulb 25 W will be fused, because it can tolerate only 220 V while the voltage across it is 352 V.

3. **(b) :** Resistance of wire

$$R = \frac{\rho l}{A} \quad \text{...(i)}$$

On stretching, volume (V) remains constant.

So $V = Al$ or $A = \frac{V}{l}$

$\therefore R = \frac{\rho l^2}{V}$ (Using (i))

Taking logarithm on both sides and differentiating we get,

$$\frac{\Delta R}{R} = \frac{2\Delta l}{l} \quad (\because V \text{ and } \rho \text{ are constants})$$

or $\frac{\Delta R}{R}\% = \frac{2\Delta l}{l}\%$

Hence, when wire is stretched by 0.1% its resistance will increase by 0.2%.

4. **(a) :** Let R_0 be the resistance of both conductors at 0°C. Let R_1 and R_2 be their resistance at t°C. Then

$R_1 = R_0(1 + \alpha_1 t)$

$R_2 = R_0(1 + \alpha_2 t)$

Let R_s is the resistance of the series combination of two conductors at t°C. Then

$R_s = R_1 + R_2$

$R_{s0}(1 + \alpha_s t) = R_0(1 + \alpha_1 t) + R_0(1 + \alpha_2 t)$

where, $R_{s0} = R_0 + R_0 = 2R_0$

$\therefore 2R_0(1 + \alpha_s t) = 2R_0 + R_0 t(\alpha_1 + \alpha_2)$

$2R_0 + 2R_0\alpha_s t = 2R_0 + R_0 t(\alpha_1 + \alpha_2)$

$\therefore \alpha_s = \frac{\alpha_1 + \alpha_2}{2}$

Let R_p is the resistance of the parallel combination of two conductors at t°C. Then

$$R_p = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_{p0}(1 + \alpha_p t) = \frac{R_0(1 + \alpha_1 t)\, R_0(1 + \alpha_2 t)}{R_0(1 + \alpha_1 t) + R_0(1 + \alpha_2 t)}$$

where, $R_{p0} = \frac{R_0 R_0}{R_0 + R_0} = \frac{R_0}{2}$

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$$\therefore \frac{R_0}{2}(1+\alpha_p t) = \frac{R_0^2(1+\alpha_1 t)(1+\alpha_2 t)}{2R_0 + R_0(\alpha_1+\alpha_2)t}$$

$$\frac{R_0}{2}(1+\alpha_p t) = \frac{R_0^2(1+\alpha_1 t+\alpha_2 t+\alpha_1\alpha_2 t^2)}{R_0(2+(\alpha_1+\alpha_2)t)}$$

$$\frac{1}{2}(1+\alpha_p t) = \frac{(1+\alpha_1 t+\alpha_2 t+\alpha_1\alpha_2 t^2)}{(2+(\alpha_1+\alpha_2)t)}$$

As α_1 and α_2 are small quantities

$\therefore$ $\alpha_1\alpha_2$ is negligible

$$\therefore \frac{1}{2}(1+\alpha_p t) = \frac{1+(\alpha_1+\alpha_2)t}{2+(\alpha_1+\alpha_2)t} = \frac{1+(\alpha_1+\alpha_2)t}{2\left[1+\frac{(\alpha_1+\alpha_2)t}{2}\right]}$$

$$= \frac{1}{2}[1+(\alpha_1+\alpha_2)t]\left[1+\frac{(\alpha_1+\alpha_2)t}{2}\right]^{-1}$$

$$= \frac{1}{2}[1+(\alpha_1+\alpha_2)t]\left[1-\frac{(\alpha_1+\alpha_2)t}{2}\right]$$

[By binomial expansion]

$$= \frac{1}{2}\left[1-\frac{(\alpha_1+\alpha_2)t}{2}+(\alpha_1+\alpha_2)t-\frac{(\alpha_1+\alpha_2)^2t^2}{2}\right]$$

As $(\alpha_1+\alpha_2)^2$ is negligible

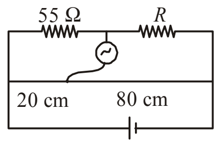
$$\therefore \frac{1}{2}(1+\alpha_p t) = \frac{1}{2}\left[1+\frac{1}{2}(\alpha_1+\alpha_2)t\right]$$

$$\alpha_p t = \frac{(\alpha_1+\alpha_2)}{2}t$$

$$\alpha_p = \frac{\alpha_1+\alpha_2}{2}$$

5. **(a) :** From the statement given, $\alpha = 2.5 \times 10^{-3}/°C$. The resistance of a wire change from 100 Ω to 150 Ω when the temperature is increased from 27°C to 227°C. It is true that α is small. But (150 – 100) Ω or 50 Ω is not very much less than 100 Ω *i.e.*, $R - R_0 << R_0$ is not true.

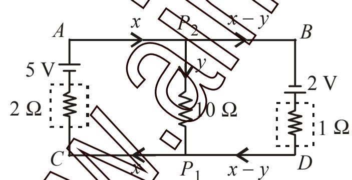
6. **(c) :**



This is a Wheatstone bridge.
If ρ_l is the resistance per unit length (in cm)

$$\frac{20\rho_l}{55} = \frac{80\rho_l}{R} \text{ or } R = \frac{80\times 55}{20} = 220\ \Omega.$$

7. **(d) :**



Applying Kirchhoff's law for the loops AP_2P_1CA and $P_2BDP_1P_2$, one gets

$-10y - 2x + 5 = 0$

$\Rightarrow 2x + 10y = 5$ ...(i)

$+ 2 - 1(x - y) + 10 \cdot y = 0$

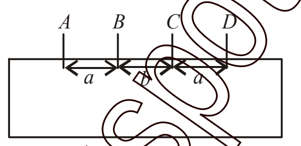
$+x - 11y = 2$ ...(ii)

$\Rightarrow 2x - 22y = 4$ ...(iii) = (ii) × 2

(i) – (iii) gives $32y = 1$

$\Rightarrow\ y = \frac{1}{32}$ A = 0.03 A from P_2 to P_1.

8. **(d) :** Current is spread over an area $2\pi r^2$. The current I is a surface current.



Current density, $j = \frac{I}{2\pi r^2}$

Resistance $= \frac{\rho l}{\text{area}} = \frac{\rho r}{2\pi r^2}$

$E = I\rho/2\pi r^2$.

$V_B - V_C = \Delta V = \int_{a+b}^{a} -E dr$

$\Rightarrow \Delta V = \frac{-I\rho}{2\pi}\int_{a+b}^{a}\frac{1}{r^2}dr = \frac{-I\rho}{2\pi}\left[-\frac{1}{r}\right]_{a+b}^{a}$

$\Delta V = \frac{I\rho}{2\pi}\left[\frac{1}{a}-\frac{1}{a+b}\right]$.

9. **(d) :** $j \times \rho = E$. $\therefore$ $E = \frac{I\rho}{2\pi r^2}$

10. **(d) :** Given : $R_{50} = 5\ \Omega$, $R_{100} = 6\ \Omega$

$R_t = R_0(1 + \alpha t)$

where R_t = resistance of a wire at t°C, R_0 = resistance of a wire at 0°C, α = temperature coefficient of resistance.

$\therefore$ $R_{50} = R_0\,[1 + \alpha\,50]$

and $R_{100} = R_0\,[1 + \alpha\,100]$

or $R_{50} - R_0 = R_0\alpha(50)$ ... (i)

$R_{100} - R_0 = R_0\alpha\,(100)$ ... (ii)

Divide (i) by (ii), we get

$$\frac{5-R_0}{6-R_0} = \frac{1}{2} \quad \text{or} \quad 10 - 2R_0 = 6 - R_0$$

or $R_0 = 4\ \Omega$.

11. **(a) :** Resistance of a wire $R = \frac{\rho l}{\pi r^2} = \frac{\rho l \times 4}{\pi D^2}$

$\because$ $R_A = R_B$

$$\therefore \frac{4\rho_A l_A}{\pi D_A^2} = \frac{4\rho_B l_B}{\pi D_B^2}$$

$$\text{or} \quad \frac{l_B}{l_A} = \left(\frac{\rho_A}{\rho_B}\right)\left(\frac{D_B}{D_A}\right)^2$$

$$= \left(\frac{\rho_A}{2\rho_A}\right)\left(\frac{2D_A}{D_A}\right)^2 = \frac{4}{2} = \frac{2}{1}$$

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12. (c) : Given : $R_{100} = 100\ \Omega$

$\alpha = 0.005°C^{-1}$

$R_t = 200\ \Omega$

$\therefore\ R_{100} = R_0[1 + 0.005\ (100)]$

or $100 = R_0[1 + 0.005 \times 100]$ ....(i)

$R_t = R_0[1 + 0.005t]$

$200 = R_0[1 + 0.005t]$ ....(ii)

Divide (i) by (ii), we get

$$\frac{100}{200} = \frac{[1+0.005\times100]}{[1+0.005t]}$$

$1 + 0.005t = 2 + 1$

or $t = 400°C$.

13. **(c) :** The equivalent circuit is a balanced Wheatstone's bridge. Hence no current flows through arm BD.

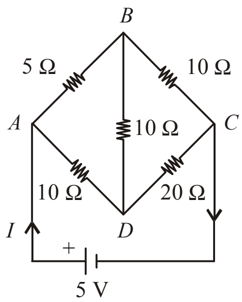
AB and BC are in series

$\therefore\ R_{ABC} = 5 + 10 = 15\ \Omega$

AD and DC are in series

$\therefore\ R_{ADC} = 10 + 20 = 30\ \Omega$

ABC and ADC are in parallel



$$\therefore\ R_{eq} = \frac{(R_{ABC})(R_{ADC})}{(R_{ABC}+R_{ADC})}$$

$$\text{or}\quad R_{eq} = \frac{15\times30}{15+30} = \frac{15\times30}{45} = 10\ \Omega$$

$$\therefore\ \text{Current}\ I = \frac{E}{R_{eq}} = \frac{5}{10} = 0.5\ \text{A}.$$

14. **(c) :** For balanced Wheatstone's bridge, $\frac{P}{Q} = \frac{R}{S}$

$$\because\ S = \frac{S_1S_2}{S_1+S_2}\quad (\because S_1 \text{ and } S_2 \text{ are in parallel})$$

$$\therefore\ \frac{P}{Q} = \frac{R(S_1+S_2)}{S_1S_2}.$$

15. **(a) :** Kirchhoff's first law $[\Sigma\, i = 0]$ is based on conservation of charge

Kirchhoff's second law $(\Sigma\, i\, R = \Sigma\, E)$ is based on conservation of energy.

16. **(d) :** Resistance of the bulb

$$(R) = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\ \Omega$$

$$\text{Power across 110 volt} = \frac{(110)^2}{484}$$

$$\therefore\ \text{Power} = \frac{110\times110}{484} = 25\ \text{W}.$$

17. **(a) :** Antimony-Bismuth couple is ABC couple. It means that current flows from A to B at cold junction.

18. **(b) :** The internal resistance of a cell is given by

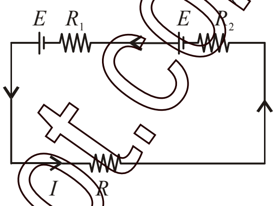
$$r = R\left(\frac{l_1}{l_2} - 1\right) = R\left(\frac{l_1-l_2}{l_2}\right)$$

$$\therefore\ r = 2\left[\frac{240-120}{120}\right] = 2\ \Omega.$$

19. **(d) :** $I = \dfrac{2E}{R_1+R_2+R}$

$\because\ E - IR_2 = 0$ (Given)

$\therefore\ E = IR_2$



or $E = \dfrac{2ER_2}{R_1+R_2+R}$

or $R_1 + R_2 + R = 2R_2$

or $R = R_2 - R_1$.

20. **(d) :** For zero deflection in galvanometer,

$I_1 = I_2$

or $\dfrac{12}{500+R} = \dfrac{2}{R}$.

$\Rightarrow 12R = 1000 + 2R \Rightarrow R = 100\ \Omega$

21. **(a) :** If internal resistance is zero, the energy source will supply a constant current.

22. **(c) :** Resistance of hot tungsten $= \dfrac{V^2}{P} = \dfrac{(200)^2}{100} = 400\ \Omega$

Resistance when not in use $= \dfrac{400}{10} = 40\ \Omega$.

23. **(d) :** The voltameters are joined in parallel.

Mass deposited $= z_1q_1 = z_2q_2$

$$\therefore\ \frac{q_1}{q_2} = \frac{z_2}{z_1} \Rightarrow \frac{q_1+q_2}{q_2} = \frac{z_1+z_2}{z_1} \quad \Rightarrow \frac{q}{q_2} = \left(1+\frac{z_2}{z_1}\right)$$

$$\text{or}\quad q_2 = \frac{q}{\left(1+\frac{z_2}{z_1}\right)}.$$

24. **(c) :** Resistance of full coil $= R$

Resistance of each half piece $= R/2$

$$\therefore\ \frac{H_2}{H_1} = \frac{V^2t}{R/2}\times\frac{R}{V^2t} = \frac{2}{1}$$

$\therefore\ H_2 = 2H_1$

Heat generated will now be doubled.

25. **(c) :** Thermistors are made of metal oxides with high temperature co-efficient of resistivity.

26. **(a) :** For meter bridge experiment,

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} = \frac{l_1}{(100-l_1)}$$

$$\text{In the first case,}\quad \frac{X}{Y} = \frac{20}{100-20} = \frac{20}{80} = \frac{1}{4}$$

In the second case,

$$\frac{4X}{Y} = \frac{l}{(100-l)} \Rightarrow \frac{4}{4} = \frac{l}{100-l} \Rightarrow l = 50\ \text{cm}.$$

27. **(b) :** Potential difference is same when the wires are put in parallel

$$V = I_1R_1 = I_1\times\frac{\rho l_1}{\pi r_1^2}$$

$$\text{Again}\quad V = I_2R_2 = I_2\times\frac{\rho l_2}{\pi r_2^2}$$

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$\therefore \quad \frac{I_1 \times \rho l_1}{\pi r_1^2} = \frac{I_2 \times \rho l_2}{\pi r_2^2} \Rightarrow \frac{I_1}{I_2} = \left(\frac{l_2}{l_1}\right)\left(\frac{r_1}{r_2}\right)^2$

or $\frac{I_1}{I_2} = \left(\frac{3}{4}\right)\left(\frac{2}{3}\right)^2 = \frac{3 \times 4}{4 \times 9} = \frac{1}{3}$.

28. (a) : In series combination, $S = (R_1 + R_2)$

In parallel combination, $P = \frac{R_1 R_2}{(R_1 + R_2)}$

$\because \quad S = nP$

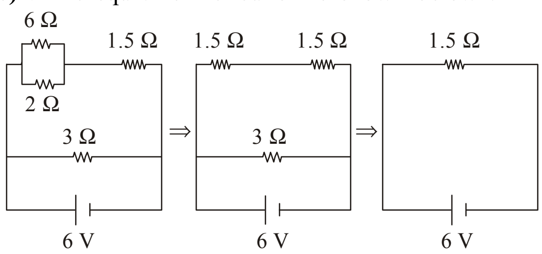
$\therefore \quad (R_1 + R_2) = n\frac{R_1 R_2}{(R_1 + R_2)} \quad \therefore \quad (R_1 + R_2)^2 = nR_1R_2$

For minimum value, $R_1 = R_2 = R$

$\therefore \quad (R + R)^2 = n(R \times R) \Rightarrow 4R^2 = nR^2$

or $n = 4$.

29. (c) : The equivalent circuits are shown below :



$I = \frac{6}{1.5} = 4$ A.

30. (a) : $m = Z\,i\,t$

or $\quad m = (3.3 \times 10^{-7}) \times (3) \times (2) = 19.8 \times 10^{-7}$ kg.

31. (d) : $E = a\theta + b\theta^2$

$\therefore \quad \frac{dE}{d\theta} = a + 2b\theta$

At neutral temperature (θ_n), $\frac{dE}{d\theta} = 0$

or $\quad 0 = a + 2b\theta_n$

or $\quad \theta_n = -\frac{a}{2b} = -\frac{1}{2} \times (700) = -350°\text{C}$

Neutral temperature is calculated to be $-350°$C

Since temperature of cold junction is 0°C, no neutral temperature is possible for this thermocouple.

32. (c) : Electrical energy is converted into heat energy

$\therefore \quad 836 \times t = 1000 \times 1 \times (40 - 10) \times (4.18)$

$[\because\ 4.18\text{ J} = 1\text{cal}]$

or $\quad t = \frac{1000 \times 30 \times 4.18}{836} = 150$ sec.

33. (d) : Let the length of the wire be l, radius of the wire be r

$\therefore$ Resistance $R = \rho\frac{l}{\pi r^2}$, ρ = resistivity of the wire

Now l is increased by 100% $\quad \therefore \ l' = l + \frac{100}{100}l = 2l$

As length is increased, its radius is going to be decreased in such a way that the volume of the cylinder remains constant.

$\pi r^2 \times l = \pi r'^2 \times l' \Rightarrow r'^2 = \frac{r^2 \times l}{l'} = \frac{r^2 \times l}{2l} = \frac{r^2}{2}$

$\therefore$ The new resistance $R'^2 = \rho\frac{l'}{\pi r'^2} = \rho\frac{2l}{\pi \times \frac{r^2}{2}} = 4R$

$\therefore$ Change in resistance $= R' - R = 3R$

$\therefore$ % change $= \frac{3R}{R} \times 100\% = 300\%$.

34. (b) : Equivalent resistance $= \frac{(3+3)\times 3}{(3+3)+3} = \frac{18}{9} = 2\ \Omega$

$\therefore \quad$ Current $I = \frac{V}{R} = \frac{3}{2} = 1.5$ A.

35. (c) : Potential gradient along wire $= \frac{E}{100}\frac{\text{volt}}{\text{cm}}$

$\therefore \quad K = \frac{E}{100}\frac{\text{volt}}{\text{cm}}$

For battery $V = E' - ir$, where E' is emf of battery.

or $\quad K \times 30 = E' - ir$, where current i is drawn from battery

or $\quad \frac{E \times 30}{100} = E' + 0.5i$ or $E' = \frac{30E}{100} - 0.5i$

36. (c) : Resistance of bulb $= \frac{V^2}{P} = \frac{(220)^2}{1000} = 48.4\ \Omega$.

Required power

$= \frac{V^2}{R} = \frac{(110)^2}{48.4} = \frac{110 \times 110}{48.4} = 250$ W.

37. (c) : According to Faraday's laws of electrolysis,

$\frac{m_{Zn}}{m_{Cu}} = \frac{Z_{Zn}}{Z_{Cu}}$ when i and t are same

$\therefore \quad \frac{0.13}{m_{Cu}} = \frac{32.5}{31.5} \Rightarrow m_{Cu} = \frac{0.13 \times 31.5}{32.5} = 0.126$ g

38. (a) : Let the smallest temperature be θ°C

$\therefore \quad$ Thermo emf $= (25 \times 10^{-6})\ \theta$ volt

Potential difference across galvanometer =

$IR = 10^{-5} \times 40 = 4 \times 10^{-4}$ volt

$\therefore \quad (25 \times 10^{-6})\theta = 4 \times 10^{-4}$

$\therefore \quad \theta = \frac{4 \times 10^{-4}}{25 \times 10^{-6}} = 16°\text{C}$.

39. (b) : According to Faraday's laws, $m \propto It$.

40. (c) : $\theta_c + \theta_i = 2\theta_n \Rightarrow \frac{\theta_i + \theta_c}{2} = \theta_n$.

41. (b) : $P_1 = \frac{V^2}{R}$

when connected in parallel,

$R_{eq} = \frac{(R/2) \times (R/2)}{\frac{R}{2} + \frac{R}{2}} = \frac{R}{4} \quad \therefore \quad P_2 = \frac{V^2}{R/4} = 4\frac{V^2}{R} = 4P_1$

$\therefore \quad \frac{P_2}{P_1} = 4$.

42. (b) : Power $= \frac{V^2}{R}$

$\therefore \quad 150 = \frac{(15)^2}{R} + \frac{(15)^2}{2} = \frac{225}{R} + \frac{225}{2} \Rightarrow R = 6\ \Omega$.

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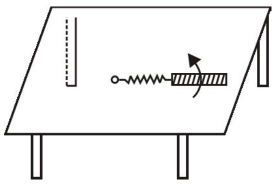
CHAPTER 13

MAGNETIC EFFECTS OF CURRENT AND MAGNETISM

1. A metallic rod of length 'l' is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is



(a) $\frac{5B\omega l^2}{2}$ (b) $\frac{2B\omega l^2}{2}$ (c) $\frac{3B\omega l^2}{2}$ (d) $\frac{4B\omega l^2}{2}$

(2013)

2. This question has Statement-I and Statement-II. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement-I : Higher the range, greater is the resistance of ammeter.

Statement-II : To increase the range of ammeter, additional shunt needs to be used across it.

(a) Statement-I is false, Statement-II is true.
(b) Statement-I is true, Statement-II is true, Statement-II is the correct explanation of Statement-I.
(c) Statement-I is true, Statement-II is true, Statement-II is not the correct explanation of Statement-I.
(d) Statement-I is true, Statement-II is false. *(2013)*

3. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to (Horizontal component of earth's magnetic induction is 3.6×10^{-5} Wb/m^2)

(a) 5.80×10^{-4} Wb/m^2 (b) 3.6×10^{-5} Wb/m^2
(c) 2.56×10^{-4} Wb/m^2 (d) 3.50×10^{-4} Wb/m^2

(2013)

4. Proton, deuteron and alpha particle of the same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are respectively r_p, r_d and r_α. Which one of the following relation is correct?

(a) $r_\alpha = r_p < r_d$ (b) $r_\alpha > r_d > r_p$
(c) $r_\alpha = r_d > r_p$ (d) $r_\alpha = r_p = r_d$ *(2012)*

5. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to

(a) induction of electrical charge on the plate.
(b) shielding of magnetic lines of force as aluminium is a paramagnetic material.
(c) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping.
(d) development of air current when the plate is placed.

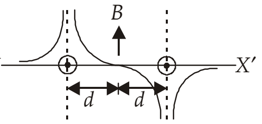
(2012)

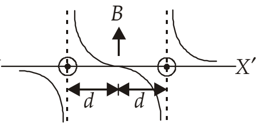
6. A current I flows in an infinitely long wire with cross-section in the form of a semicircular ring of radius R. The magnitude of the magnetic induction along its axis is

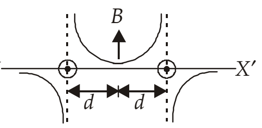
(a) $\frac{\mu_0 I}{\pi^2 R}$ (b) $\frac{\mu_0 I}{2\pi^2 R}$ (c) $\frac{\mu_0 I}{2\pi R}$ (d) $\frac{\mu_0 I}{4\pi R}$

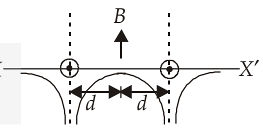
(2011)

7. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX' is given by

(a)


(b)


(c)


(d)


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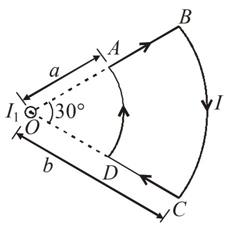
Directions : Question numbers 8 and 9 are based on the following paragraph.

A current loop $ABCD$ is held fixed on the plane of the paper as shown in the figure. The arcs BC (radius = b) and DA (radius = a) of the loop are joined by two straight wires AB and CD. A steady current I is flowing in the loop. Angle made by AB and CD at the origin O is 30°. Another straight thin wire with steady current I_1 flowing out of the plane of the paper is kept at the origin. *(2009)*



8. The magnitude of the magnetic field (B) due to loop $ABCD$ at the origin (O) is

(a) zero (b) $\frac{\mu_0 I(b-a)}{24ab}$

(c) $\frac{\mu_0 I}{4\pi}\left[\frac{b-a}{ab}\right]$ (d) $\frac{\mu_0 I}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$

9. Due to the presence of the current I_1 at the origin

(a) the forces on AB and DC are zero

(b) the forces on AD and BC are zero

(c) the magnitude of the net force on the loop is given by $\frac{I_1 I}{4\pi}\mu_0\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$

(d) the magnitude of the net force on the loop is given by $\frac{\mu_0 I I_1}{24ab}(b-a)$.

10. A horizontal overhead powerline is at a height of 4 m from the ground and carries a current of 100 A from east to west. The magnetic field directly below it on the ground is ($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})

(a) 2.5×10^{-7} T northward

(b) 2.5×10^{-7} T southward

(c) 5×10^{-6} T northward

(d) 5×10^{-6} T southward. *(2008)*

11. Relative permittivity and permeability of a material are ε_r and μ_r, respectively. Which of the following values of these quantities are allowed for a diamagnetic mateiral?

(a) $\varepsilon_r = 1.5, \mu_r = 1.5$ (b) $\varepsilon_r = 0.5, \mu_r = 1.5$

(c) $\varepsilon_r = 1.5, \mu_r = 0.5$ (d) $\varepsilon_r = 0.5, \mu_r = 0.5$

(2008)

12. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance d from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by

(a) $\frac{\mu_0}{2\pi d}(I_1^2+I_2^2)$ (b) $\frac{\mu_0}{2\pi}\left(\frac{I_1+I_2}{d}\right)^{\frac{1}{2}}$

(c) $\frac{\mu_0}{2\pi d}(I_1^2+I_2^2)^{\frac{1}{2}}$ (d) $\frac{\mu_0}{2\pi d}(I_1+I_2)$ *(2007)*

13. A charged particle moves through a magnetic field perpendicular to its direction. Then

(a) kinetic energy changes but the momentum is constant

(b) the momentum changes but the kinetic energy is constant

(c) both momentum and kinetic energy of the particle are not constant

(d) both, momentum and kinetic energy of the particle are constant *(2007)*

14. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields $\vec{E}$ and $\vec{B}$ with a velocity $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$, and comes out without any change in magnitude or direction of $\vec{v}$. Then

(a) $\vec{v} = \vec{B}\times\vec{E}/E^2$ (b) $\vec{v} = \vec{E}\times\vec{B}/B^2$

(c) $\vec{v} = \vec{B}\times\vec{E}/B^2$ (d) $\vec{v} = \vec{E}\times\vec{B}/E^2$ *(2007)*

15. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then

(a) the magnetic field at all points inside the pipe is the same, but not zero

(b) the magnetic field is zero only on the axis of the pipe

(c) the magnetic field is different at different points inside the pipe

(d) the magnetic field at any point inside the pipe is zero *(2007)*

16. A long straight wire of radius a carries a steady current i. The current is uniformly distributed across its cross section. The ratio of the magnetic field at a/2 and 2a is

(a) 1/2 (b) 1/4 (c) 4 (d) 1 *(2007)*

17. A long solenoid has 200 turns per cm and carries a current i. The magnetic field at its centre is 6.28×10^{-2} weber/m^2. Another long solenoid has 100 turns per cm and it carries a current i/3. The value of the magnetic field at its centre is

(a) 1.05×10^{-4} Wb/m^2 (b) 1.05×10^{-2} Wb/m^2

(c) 1.05×10^{-5} Wb/m^2 (d) 1.05×10^{-3} Wb/m^2. *(2006)*

18. In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be a

(a) circle (b) helix

(c) straight line (d) ellipse. *(2006)*

19. Needles N_1, N_2 and N_3 are made of a ferromagnetic, a paramagnetic and a diamagnetic substance respectively. A magnet when brought close to them will

(a) attract all three of them

(b) attract N_1 and N_2 strongly but repel N_3

(c) attract N_1 strongly, N_2 weakly and repel N_3 weakly

(d) attract N_1 strongly, but repel N_2 and N_3 weakly. *(2006)*

20. A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity then

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(a) it will turn towards right of direction of motion
(b) it will turn towards left of direction of motion
(c) its velocity will decrease
(d) its velocity will increase *(2005)*

21. A charged particle of mass m and charge q travels on a circular path of radius r that is perpendicular to a magnetic field B. The time taken by the particle to complete one revolution is

(a) $\frac{2\pi qB}{m}$ (b) $\frac{2\pi m}{qB}$
(c) $\frac{2\pi mq}{B}$ (d) $\frac{2\pi mq}{qB}$ *(2005)*

22. Two concentric coils each of radius equal to 2π cm are placed at right angles to each other. 3 ampere and 4 ampere are the currents flowing in each coil respectively. The magnetic induction in weber/m^2 at the center of the coils will be ($\mu_0 = 4\pi \times 10^{-7}$ Wb/A-m)

(a) 5×10^{-5} (b) 7×10^{-5}
(c) 12×10^{-5} (d) 10^{-5} *(2005)*

23. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milliampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 volt, the resistance in ohms needed to be connected in series with the coil will be

(a) 99995 (b) 9995
(c) 10^3 (d) 10^5 *(2005)*

24. Two thin long, parallel wires, separated by a distance d carry a current of i A in the same direction. They will

(a) attract each other with a force of $\frac{\mu_0 i^2}{(2\pi d^2)}$
(b) repel each other with a force of $\frac{\mu_0 i^2}{(2\pi d^2)}$
(c) attract each other with a force of $\frac{\mu_0 i^2}{(2\pi d)}$
(d) repel each other with a force of $\frac{\mu_0 i^2}{(2\pi d)}$ *(2005)*

25. A magnetic needle is kept in a non-uniform magnetic field. It experiences

(a) a force and a torque
(b) a force but not a torque
(c) a torque but not a force
(d) neither a force nor a torque *(2005)*

26. Two long conductors, separated by a distance d carry current I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one of them is increased to two times and its direction is reversed. The distance is also increased to $3d$. The new value of the force between them is

(a) $-2F$ (b) $F/3$ (c) $-2F/3$ (d) $-F/3$. *(2004)*

27. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is 54 μT. What will be its value at the centre of the loop?

(a) 250 μT (b) 150 μT
(c) 125 μT (d) 75 μT. *(2004)*

28. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be

(a) nB (b) n^2B
(c) $2nB$ (d) $2n^2B$. *(2004)*

29. A current i ampere flows along an infinitely long straight thin walled tube, then the magnetic induction at any point inside the tube is

(a) infinite (b) zero
(c) $\frac{\mu_0}{4\pi}\cdot\frac{2i}{r}$ tesla (d) $\frac{2i}{r}$ tesla . *(2004)*

30. The materials suitable for making electromagnets should have

(a) high retentivity and high coercivity
(b) low retentivity and low coercivity
(c) high retentivity and low coercivity
(d) low retentivity and high coercivity. *(2004)*

31. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2 s. The magnet is cut along its length into three equal parts and three parts are then placed on each other with their like poles together. The time period of this combination will be

(a) 2 s (b) $\frac{2}{3}$ s (c) $(2\sqrt{3})$ s (d) $\left(\frac{2}{\sqrt{3}}\right)$ s *(2004)*

32. An ammeter reads upto 1 ampere. Its internal resistance is 0.81 ohm. To increase the range to 10 A the value of the required shunt is

(a) 0.03 Ω (b) 0.3 Ω
(c) 0.9 Ω (d) 0.09 Ω. *(2003)*

33. A particle of charge -16×10^{-18} coulomb moving with velocity 10 ms^{-1} along the x-axis enters a region where a magnetic field of induction B is along the y -axis, and an electric field of magnitude 10^4 V/m is along the negative z -axis. If the charged particle continues moving along the x-axis, the magnitude of B is

(a) 10^3 Wb/m^2 (b) 10^5 Wb/m^2
(c) 10^{16} Wb/m^2 (d) 10^{-3} Wb/m^2. *(2003)*

34. A particle of mass M and charge Q moving with velocity $\vec{v}$ describes a circular path of radius R when subjected to a uniform transverse magnetic field of induction B. The work done by the field when the particle completes one full circle is

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(a) $\left(\frac{Mv^2}{R}\right)2\pi R$ (b) zero
(c) $BQ\ 2\pi R$ (d) $BQv\ 2\pi R$. *(2003)*

35. A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T', the ratio $\frac{T'}{T}$ is
(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 2 (d) $\frac{1}{4}$. *(2003)*

36. Curie temperature is the temperature above which
(a) a ferromagnetic material becomes paramagnetic
(b) a paramagnetic material becomes diamagnetic
(c) a ferromagnetic material becomes diamagnetic
(d) a paramagnetic material becomes ferromagnetic. *(2003)*

37. The magnetic lines of force inside a bar magnet
(a) are from north-pole to south-pole of the magnet
(b) do not exist
(c) depend upon the area of cross-section of the bar magnet
(d) are from south-pole to north-pole of the magnet. *(2003)*

38. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60°. The torque needed to maintain the needle in this position will be
(a) $\sqrt{3}W$ (b) W (c) $\left(\frac{\sqrt{3}}{2}\right)W$ (d) $2W$. *(2003)*

39. The time period of a charged particle undergoing a circular motion in a uniform magnetic field is independent of its
(a) speed (b) mass
(c) charge (d) magnetic induction. *(2002)*

40. If a current is passed through a spring then the spring will
(a) expand (b) compress
(c) remains same (d) none of these. *(2002)*

41. If an electron and a proton having same momenta enter perpendicular to a magnetic field, then
(a) curved path of electron and proton will be same (ignoring the sense of revolution)
(b) they will move undeflected
(c) curved path of electron is more curved than that of the proton
(d) path of proton is more curved. *(2002)*

42. If in a circular coil A of radius R, current I is flowing and in another coil B of radius $2R$ a current $2I$ is flowing, then the ratio of the magnetic fields, B_A and B_B, produced by them will be
(a) 1 (b) 2
(c) 1/2 (d) 4. *(2002)*

43. If an ammeter is to be used in place of a voltmeter, the we must connect with the ammeter a
(a) low resistance in parallel
(b) high resistance in parallel
(c) high resistance in series
(d) low resistance in series. *(2002)*

Answer Key

1.	(a)	**2.**	(a)	**3.**	(c)	**4.**	(a)	**5.**	(c)	**6.**	(a)
7.	(b)	**8.**	(b)	**9.**	(b)	**10.**	(d)	**11.**	(c)	**12.**	(c)
13.	(b, c)	**14.**	(b)	**15.**	(d)	**16.**	(d)	**17.**	(b)	**18.**	(c)
19.	(c)	**20.**	(c)	**21.**	(b)	**22.**	(a)	**23.**	(b)	**24.**	(c)
25.	(a)	**26.**	(c)	**27.**	(a)	**28.**	(b)	**29.**	(b)	**30.**	(b)
31.	(b)	**32.**	(d)	**33.**	(a)	**34.**	(b)	**35.**	(b)	**36.**	(a)
37.	(d)	**38.**	(a)	**39.**	(a)	**40.**	(b)	**41.**	(a)	**42.**	(a)
43.	(c)										

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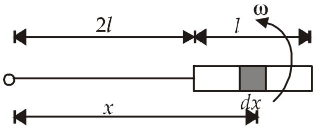
Explanations

1. **(a) :** 

Consider a element of length dx at a distance x from the fixed end of the string.
e.m.f. induced in the element is

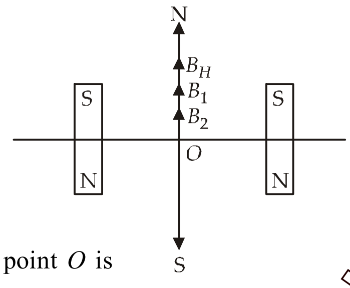
$d\varepsilon = B(\omega x)dx$

Hence, the e.m.f. induced across the ends of the rod is

$$\varepsilon = \int_{2l}^{3l} B\omega x dx = B\omega\left[\frac{x^2}{2}\right]_{2l}^{3l} = \frac{B\omega}{2}[(3l)^2 - (2l)^2] = \frac{5B\omega l^2}{2}$$

2. **(a)**

3. **(c) :** The situation is as shown in the figure.



As the point O lies on broad-side position with respect to both the magnets. Therefore,
The net magnetic field at point O is

$B_{net} = B_1 + B_2 + B_H$

$$B_{net} = \frac{\mu_0}{4\pi}\frac{M_1}{r^3} + \frac{\mu_0}{4\pi}\frac{M_2}{r^3} + B_H = \frac{\mu_0}{4\pi r^3}(M_1 + M_2) + B_H$$

Substituting the given values, we get

$$B_{net} = \frac{4\pi\times 10^{-7}}{4\pi\times(10\times 10^{-2})^3}[1.2+1] + 3.6\times 10^{-5}$$

$$= \frac{10^{-7}}{10^{-3}}\times 2.2 + 3.6\times 10^{-5}$$

$= 2.2 \times 10^{-4} + 0.36 \times 10^{-4} = 2.56 \times 10^{-4}$ Wb/m^2.

4. **(a) :** The radius of the circular path of a charged particle in the magnetic field is given by

$$r = \frac{mv}{Bq}$$

Kinetic energy of a charged particle,

$$K = \frac{1}{2}mv^2 \text{ or } v = \sqrt{\frac{2K}{m}}$$

$$\therefore \quad r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2Km}}{qB}$$

As K and B are constants

$$\therefore \quad r \propto \frac{\sqrt{m}}{q}$$

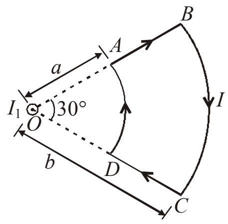
$$r_p : r_d : r_\alpha = \frac{\sqrt{m_p}}{q_p} : \frac{\sqrt{m_d}}{q_d} : \frac{\sqrt{m_\alpha}}{q_\alpha}$$

$$= \frac{\sqrt{m}}{e} : \frac{\sqrt{2m}}{e} : \frac{\sqrt{4m}}{2e} = 1 : \sqrt{2} : 1$$

$\Rightarrow \; r_\alpha = r_p < r_d$

5. **(c)** **6.** **(a)** **7.** **(b)**

8. **(b) :** O is along the line CD and AB. They do not contribute to the magnetic induction at O. The field due to DA is positive or out of the paper and that due to BC is into the paper or negative.



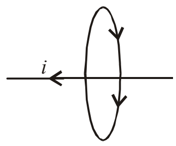
The total magnetic field due to loop $ABCD$ at O is

$B = B_{AB} + B_{BC} + B_{CD} + B_{DA}$

$$\Rightarrow B = 0 - \frac{\mu_0 I}{4\pi b}\times\frac{\pi}{6} + 0 + \frac{\mu_0 I}{4\pi a}\times\frac{\pi}{6}$$

$\Rightarrow B = \dfrac{\mu_0 I}{24ab}(b-a)$, out of the paper or positive.

9. **(b) :** The straight wire is perpendicular to the segments and the fields are parallel. There will be no force. Due to parts AB and CD, their fields are equal and opposite and their effects also cancel each other.

10.* **(d) :** 

By Ampere's theorem, $\vec{B}\cdot 2\pi d = \mu_0 i$

$$\vec{B} = \frac{\mu_0 i}{2\pi d} = \frac{4\pi\times 10^{-7}\times 100\,\text{A}}{2\pi\times 4\,\text{m}} = 50\times 10^{-7}\,\text{T}$$

$\Rightarrow \; B = 5 \times 10^{-6}$ T southwards.

* *It is assumed that this is a direct current. If it is a.c, the current at the given instant is in the given direction.*

11. **(c) :** The values of relative permeability of diamagnetic materials are slightly less than 1 and ε_r is quite high. According to the table given, one takes
$\varepsilon_r = 1.5$ and $\mu_r = 0.5$. Then the choice (c) is correct.

12. **(c) :** The field at the same point at the same distance from the mutually perpendicular wires carrying current will be having the same magnitude but in perpendicular directions.

$$\therefore \quad B = \sqrt{B_1^2 + B_2^2} \qquad \therefore \quad B = \frac{\mu_0}{2\pi d}(I_1^2 + I_2^2)^{1/2}.$$

13. **(b, c) :** Due to Lorentzian force, $F = qv \times B$,
When a charged particle enters a field with its velocity perpendicular to the magnetic field, the motion is circular

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with $qvB = \frac{mv^2}{r}$. v constantly changes its direction (but not the magnitude). Therefore its tangential momentum changes its direction but its energy remains the same $\left(\frac{1}{2}I\omega^2 = \text{constant}\right)$. Therefore the answer is (b).

If angular momentum is taken, $I\omega$ is a constant.

As $\frac{1}{2}I\omega^2$ is also constant, (c) is the answer.

* *The questions could have been more specific, whether by "momentum" it is meant tangential momentum or angular momentum.*

14. **(b) :** When $\vec{E}$ and $\vec{B}$ are perpendicular and velocity has no changes then $qE = qvB$ *i.e.*, $v = \frac{E}{B}$. The two forces oppose each other if v is along $\vec{E}\times\vec{B}$ *i.e.*, $\vec{v} = \frac{\vec{E}\times\vec{B}}{B^2}$

As $\vec{E}$ and $\vec{B}$ are perpendicular to each other

$$\frac{\vec{E}\times\vec{B}}{B^2} = \frac{EB\sin 90°}{B^2} = \frac{E}{B}$$

For historic and standard experiments like Thomson's e/m value, if v is given only as E/B, it would have been better from the pedagogic view, although the answer is numerically correct.

15. **(d) :** Magnetic field is shielded and no current is inside the pipe to apply Ampère's law. (Compare to electric field inside a hollow sphere).

16. **(d) :** Uniform current is flowing. Current enclosed in the 1st ampèrean path is $\frac{I\cdot\pi r_1^2}{\pi R^2} = \frac{Ir_1^2}{R^2}$

$$\therefore \quad B = \frac{\mu_0 \times \text{current}}{\text{path}} = \frac{\mu_0 \cdot Ir_1^2}{2\pi r_1 R^2} = \frac{\mu_0 I r_1}{2\pi R^2}$$

Magnetic induction at a distance $r_2 = \frac{\mu_0 I}{2\pi r_2}$

$$\therefore \quad \frac{B_1}{B_2} = \frac{r_1 r_2}{R^2} = \frac{\frac{a}{2}\cdot 2a}{a^2} = 1.$$

17. **(b) :** In first case, $B_1 = \mu_0 n_1 I_1$

In second case, $B_2 = \mu_0 n_2 I_2$

$$\therefore \quad \frac{B_2}{B_1} = \frac{n_2}{n_1}\times\frac{I_2}{I_1} = \frac{100}{200}\times\frac{i/3}{i} = \frac{1}{6}$$

$$\therefore \quad B_2 = \frac{B_1}{6} = \frac{6.28\times10^{-2}}{6} = 1.05 \times 10^{-2}\ \text{Wb/m}^2.$$

18. **(c) :** Magnetic field exerts a force

$= Bev\sin\theta = Bev\sin 0 = 0$

Electric field exerts force along a straight line.

The path of charged particle will be a straight line.

19. **(c) :** Magnet will attract N_1 strongly, N_2 weakly and repel N_3 weakly.

20. **(c) :** Magnetic field applied parallel to motion of electron exerts no force on it as $\theta = 0$ and force $= Bev\sin\theta$ = zero

Electric field opposes motion of electron which carries a negative charge

$\therefore$ velocity of electron decreases.

21. **(b) :** $T = \frac{2\pi}{\omega} = \frac{2\pi r}{v}$ ....(i)

$\because$ centripetal force = magnetic force

$\therefore \quad \frac{mv^2}{r} = qvB \Rightarrow v = \frac{qBr}{m}$ ....(ii)

From (i) and (ii)

$$\therefore \quad T = \frac{2\pi r\times m}{qBr} = \frac{2\pi m}{qB}$$

22. **(a) :** Magnetic induction at centre of one coil $B_1 = \frac{\mu_0 i_1}{2r}$

Similarly $B_2 = \frac{\mu_0 i_2}{2r}$

$$\therefore \quad B^2 = B_1^2 + B_2^2 = \left(\frac{\mu_0 i_1}{2r}\right)^2 + \left(\frac{\mu_0 i_2}{2r}\right)^2 = \frac{\mu_0^2}{4r^2}(i_1^2 + i_2^2)$$

$$\therefore \quad B = \frac{\mu_0}{2r}\sqrt{i_1^2 + i_2^2} = \frac{4\pi\times10^{-7}}{2\times(2\pi\times10^{-2})}\sqrt{(3)^2+(4)^2}$$

or $B = 5 \times 10^{-5}$ Wb/m^2.

23. **(b) :** $V_{max} = \frac{150}{2} = 75$ mV

$I_{max} = \frac{150}{10} = 15\text{ mA} = I_g$

Resistance of galvanometer $G = 75/15 = 5\ \Omega$

For conversion into a voltmeter, a high resistance should be connected in series with the galvanometer

$$V = I_g(G+R) = \frac{15}{1000}(5+R) \Rightarrow 150 = 15\,\frac{(5+R)}{1000}$$

or $\quad 5 + R = \frac{150\times1000}{15} = 10000 \quad \therefore \quad R = 9995\ \Omega.$

24. **(c) :** Force of attraction between wires $= \frac{\mu_0 i^2 L}{2\pi d}$.

N.B. The options do not mention L, perhaps by slip.

25. **(a) :** A force and a torque act on a magnetic needle kept in a non-uniform magnetic field.

26. **(c) :** Initially, $F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}l$

Finally, $F' = \frac{\mu_0}{2\pi}\frac{(-2I_1)(I_2)}{3d}l$

$$\therefore \quad \frac{F'}{F} = \frac{-\mu_0}{2\pi}\frac{2I_1I_2 l}{3d}\times\frac{2\pi d}{\mu_0 I_1 I_2 l} = -\frac{2}{3}$$

$\therefore \quad F' = -2F/3.$

27. **(a) :** Field along axis of coil $B = \frac{\mu_0 i R^2}{2(R^2+x^2)^{3/2}}$

At the centre of coil, $B' = \frac{\mu_0 i}{2R}$

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$\therefore \quad \dfrac{B'}{B} = \dfrac{\mu_0 i}{2R} \times \dfrac{2(R^2+x^2)^{3/2}}{\mu_0 i R^2} = \dfrac{(R^2+x^2)^{3/2}}{R^3}$

$\therefore \quad B' = \dfrac{B \times (R^2+x^2)^{3/2}}{R^3} = \dfrac{54 \times [(3)^2+(4)^2]^{3/2}}{(3)^3} = \dfrac{54 \times 125}{27}$

or $\quad B' = 250\ \mu$T.

28. **(b) :** Initially, r_1 = radius of coil = $l/2\pi$

$\therefore \quad B = \dfrac{\mu_0 i}{2r_1} = \dfrac{2\mu_0 i \pi}{2l}$

Finally, r_2 = radius of coil $= \dfrac{l}{2\pi n}$

$\therefore \quad B' = \dfrac{\mu_0 i \times n}{2r_2} = \dfrac{n\mu_0 i \times 2\pi n}{2l} = \dfrac{2\mu_0 i n^2 \pi}{2l}$

$\therefore \quad \dfrac{B'}{B} = \dfrac{2\mu_0 i n^2 \pi}{2l} \times \dfrac{2l}{2\mu_0 i \pi} = n^2 \quad \therefore \quad B' = n^2 B.$

29. **(b) :** Magnetic field will be zero inside the straight thin walled tube according to ampere's theorem.

30. **(b) :** Materials of low retentivity and low coercivity are suitable for making electromagnets.

31. **(b) :** For a vibrating magnet, $T = 2\pi\sqrt{\dfrac{I}{MB}}$

where $I = ml^2/12$, $M = xl$, x = pole strength of magnet

$I' = \left(\dfrac{m}{3}\right)\left(\dfrac{l}{3}\right)^2 \times \dfrac{3}{12} = \dfrac{ml^2}{9 \times 12} = \dfrac{I}{9}$ (For three pieces together)

$M' = (x)\left(\dfrac{l}{3}\right) \times 3 = xl = M$ (For three pieces together)

$\therefore \quad T' = 2\pi\sqrt{\dfrac{I'}{M'B}} = 2\pi\sqrt{\dfrac{I/9}{MB}} = \dfrac{1}{3} \times 2\pi\sqrt{\dfrac{I}{MB}} = \dfrac{T}{3}$

$\therefore \quad T' = \dfrac{T}{3} = \dfrac{2}{3}$ sec.

32. **(d) :** $\dfrac{S}{S+G} = \dfrac{I_g}{I} \Rightarrow S = \dfrac{I_g G}{I - I_g}$

$\therefore \quad S = \dfrac{1 \times 0.81}{10-1} = \dfrac{0.81}{9} = 0.09\ \Omega$ in parallel.

33. **(a) :** Particle travels along x-axis. Hence $v_y = v_z = 0$
Field of induction B is along y-axis. $B_x = B_z = 0$
Electric field is along the negative z-axis.

$E_x = E_y = 0$

$\therefore$ Net force on particle $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Resolve the motion along the three coordinate axis

$\therefore \quad a_x = \dfrac{F_x}{m} = \dfrac{q}{m}(E_x + v_y B_z - v_z B_y)$

$a_y = \dfrac{F_y}{m} = \dfrac{q}{m}(E_y + v_z B_x - v_x B_z)$

$a_z = \dfrac{F_z}{m} = \dfrac{q}{m}(E_z + v_x B_y - v_y B_x)$

Since $E_x = E_y = 0$, $v_y = v_z = 0$, $B_x = B_z = 0$

$\therefore a_x = a_y = 0, \ a_z = \dfrac{q}{m}(-E_z + v_x B_y)$

Again $a_z = 0$ as the particle traverse through the region undeflected

$\therefore \quad E_z = v_x B_y$ or $B_y = \dfrac{E_z}{v_x} = \dfrac{10^4}{10} = 10^3 \dfrac{\text{Wb}}{\text{m}^2}$.

34. **(b) :** Workdone by the field = zero.

35. **(b) :** For an oscillating magnet, $T = 2\pi\sqrt{\dfrac{I}{MB}}$

where $I = ml^2/12$, $M = xl$, x = pole strength

When the magnet is divided into 2 equal parts, the magnetic dipole moment

M' = Pole strength × length $= \dfrac{x \times l}{2} = \dfrac{M}{2}$ .....(i)

$I' = \dfrac{\text{Mass} \times (\text{length})^2}{12}$

$= \dfrac{(m/2)(l/2)^2}{12} = \dfrac{ml^2}{12 \times 8} = \dfrac{I}{8}$ .....(ii)

$\therefore$ Time period $T' = 2\pi\sqrt{\dfrac{I'}{M'B}}$

$\therefore \quad \dfrac{T'}{T} = \sqrt{\dfrac{I'}{M'} \times \dfrac{M}{I}} = \sqrt{\dfrac{I'}{I} \times \dfrac{M}{M'}}$ ....(iii)

$\therefore \quad \dfrac{T'}{T} = \sqrt{\dfrac{1}{8} \times \dfrac{2}{1}} = \dfrac{1}{2}$.

36. **(a) :** A ferromagnetic material becomes paramagnetic above Curie temperature.

37. **(d) :** The magnetic lines of force inside a bar magnet are from south pole to north pole of magnet.

38. **(a) :** $W = -MB(\cos\theta_2 - \cos\theta_1)$

$= -MB(\cos 60° - \cos 0) = \dfrac{MB}{2}$

$\therefore \quad MB = 2W$ ....(i)

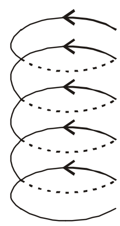
Torque $= MB \sin 60° = (2W) \sin 60°$

$= \dfrac{2W \times \sqrt{3}}{2} = \sqrt{3}\, W$.

39. **(a) :** $mR\omega^2 = BqR\omega \Rightarrow \omega = \dfrac{Bq}{m} \Rightarrow T = \dfrac{2\pi m}{Bq}$

T is independent of speed.

40. **(b) :** The spring will compress. It will be on account of force of attraction between two adjacent turns carrying currents in the same direction.



41. **(a) :** $Bqv = \dfrac{mv^2}{r} \Rightarrow r = \dfrac{mv}{Bq} = \dfrac{p}{Bq}$

r will be same for electron and proton as p, B and q are of same magnitude.

42. **(a) :** $B = \dfrac{\mu_0}{4\pi}\dfrac{2\pi I}{R} = \dfrac{\mu_0}{2}\dfrac{I}{R}$

$\therefore \quad \dfrac{B_A}{B_B} = \dfrac{I_A}{I_B} \times \dfrac{R_B}{R_A} = \left(\dfrac{1}{2}\right)\left(\dfrac{2}{1}\right) = 1$

43. **(c) :** High resistance in series with a galvanometer converts it into a voltmeter.

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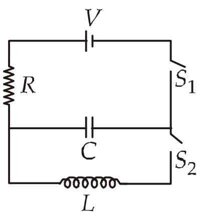
CHAPTER 14

ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENTS

1. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and $\tau = RC$ is capacitive time constant). Which of the following statement is correct?



(a) At $t = \frac{\tau}{2}$, $q = CV(1 - e^{-1})$

(b) Work done by the battery is half of the energy dissipated in the resistor

(c) At $t = \tau$, $q = CV/2$

(d) At $t = 2\tau$, $q = CV(1 - e^{-2})$ *(2013)*

2. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is

(a) 6.6×10^{-9} weber (b) 9.1×10^{-11} weber

(c) 6×10^{-11} weber (d) 3.3×10^{-11} weber *(2013)*

3. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} N A^{-1}m^{-1} due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 m s^{-1}, the magnitude of the induced emf in the wire of aerial is

(a) 1 mV (b) 0.75 mV (c) 0.50 mV (d) 0.15 mV *(2011)*

4. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is

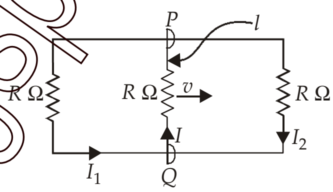
(a) $\pi\sqrt{LC}$ (b) $\frac{\pi}{4}\sqrt{LC}$ (c) $2\pi\sqrt{LC}$ (d) $\sqrt{LC}$ *(2011)*

5. A resistor R and 2 μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed. ($\log_{10}2.5 = 0.4$)

(a) 1.3×10^4 Ω (b) 1.7×10^5 Ω

(c) 2.7×10^6 Ω (d) 3.3×10^7 Ω *(2011)*

6. A rectangular loop has a sliding connector PQ of length l and resistance R Ω and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



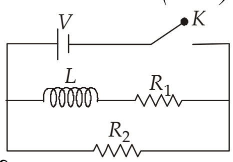
(a) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$ (b) $I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$

(c) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$ (d) $I_1 = I_2 = I = \frac{Blv}{R}$ *(2010)*

7. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$ *(2010)*

8. In the circuit shown below, the key K is closed at $t = 0$. The current through the battery is



(a) $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(b) $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(c) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = \infty$

(d) $\frac{V}{R_2}$ at $t = 0$ and $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$ *(2010)*

9. In a series LCR circuit $R = 200$ Ω and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is

(a) 242 W (b) 305 W (c) 210 W (d) zero W *(2010)*

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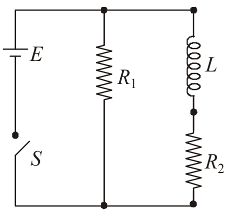
10. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 2\ \Omega$ and $R_2 = 2\ \Omega$ are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is



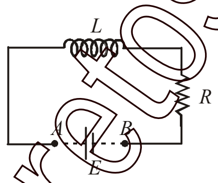
(a) $6e^{-5t}$ V (b) $\frac{12}{t}e^{-3t}$ V
(c) $6(1 - e^{-t/0.2})$ V (d) $12e^{-5t}$ V *(2009)*

11. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area $A = 10$ cm^2 and length = 20 cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is ($\mu_0 = 4\pi \times 10^{-7}$ T mA^{-1})
(a) $2.4\pi \times 10^{-4}$ H (b) $2.4\pi \times 10^{-5}$ H
(c) $4.8\pi \times 10^{-4}$ H (d) $4.8\pi \times 10^{-5}$ H. *(2008)*

12. An ideal coil of 10 H is connected in series with a resistance of 5 Ω and a battery of 5 V. 2 second after the connection is made, the current flowing in ampere in the circuit is
(a) $(1 - e^{-1})$ (b) $(1 - e)$ (c) e (d) e^{-1} *(2007)*

13. In an a.c. circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by
(a) $P = \sqrt{2}E_0I_0$ (b) $P = \frac{E_0I_0}{\sqrt{2}}$
(c) P = zero (d) $P = \frac{E_0I_0}{2}$ *(2007)*

14. An inductor ($L = 100$ mH), a resistor ($R = 100\ \Omega$) and a battery ($E = 100$ V) are initially connected in series as shown in the figure. After a long time the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is



(a) 1 A (b) (1/e) A (c) e A (d) 0.1 A. *(2006)*

15. The flux linked with a coil at any instant t is given by $\phi = 10t^2 - 50t + 250$. The induced emf at $t = 3$ s is
(a) 190 V (b) –190 V (c) –10 V (d) 10 V. *(2006)*

16. In an AC generator, a coil with N turns, all of the same area A and total resistance R, rotates with frequency ω in a magnetic field B. The maximum value of emf generated in the coil is
(a) $NAB\omega$ (b) $NABR\omega$ (c) NAB (d) $NABR$ *(2006)*

17. In a series resonant LCR circuit, the voltage across R is 100 volts and $R = 1$ kΩ with $C = 2$ μF. The resonant frequency ω is 200 rad/s. At resonance the voltage across L is
(a) 4×10^{-3} V (b) 2.5×10^{-2} V
(c) 40 V (d) 250 V. *(2006)*

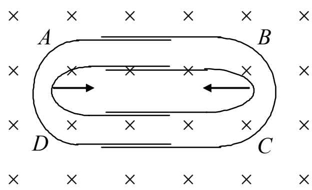
18. The phase difference between the alternating current and emf is $\pi/2$. Which of the following cannot be the constituent of the circuit?
(a) LC (b) L alone (c) C alone (d) R, L *(2005)*

19. A circuit has a resistance of 12 ohm and an impedance of 15 ohm. The power factor of the circuit will be
(a) 1.25 (b) 0.125 (c) 0.8 (d) 0.4 *(2005)*

20. The self inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of
(a) 1 μF (b) 2 μF (c) 4 μF (d) 8 μF *(2005)*

21. A coil of inductance 300 mH and resistance 2 Ω is connected to a source of voltage 2 V. The current reaches half of its steady state value in
(a) 0.15 s (b) 0.3 s (c) 0.05 s (d) 0.1 s *(2005)*

22. One conducting U tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed v, then the emf induced in the circuit in terms of B, l and v where l is the width of each tube, will be



(a) zero (b) $2Blv$ (c) Blv (d) $-Blv$ *(2005)*

23. A metal conductor of length 1 m rotates vertically about one of its ends at angular velocity 5 radian per second. If the horizontal component of earth's magnetic field is 0.2×10^{-4} T, then the e.m.f. developed between the two ends of the conductor is
(a) 5 μV (b) 50 μV (c) 5 mV (d) 50 mV. *(2004)*

24. In a LCR circuit capacitance is changed from C to $2C$. For the resonant frequency to remain unchanged, the inductance should be changed from L to
(a) $4L$ (b) $2L$ (c) $L/2$ (d) $L/4$. *(2004)*

25. In a uniform magnetic field of induction B a wire in the form of a semicircle of radius r rotates about the diameter of the circle with angular frequency ω. The axis of rotation is perpendicular to the field. If the total resistance of the circuit is R the mean power generated per period of rotation is
(a) $\frac{B\pi r^2\omega}{2R}$ (b) $\frac{(B\pi r^2\omega)^2}{8R}$
(c) $\frac{(B\pi r\omega)^2}{2R}$ (d) $\frac{(B\pi r\omega^2)^2}{8R}$. *(2004)*

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26. A coil having n turns and resistance $R\ \Omega$ is connected with a galvanometer of resistance $4R\ \Omega$. This combination is moved in time t seconds from a magnetic field W_1 weber to W_2 weber. The induced current in the circuit is

(a) $-\dfrac{W_2 - W_1}{5Rnt}$ (b) $-\dfrac{n(W_2 - W_1)}{5Rt}$

(c) $-\dfrac{(W_2 - W_1)}{Rnt}$ (d) $-\dfrac{n(W_2 - W_1)}{Rt}$. *(2004)*

27. Alternating current cannot be measured by D.C. ammeter because
(a) A.C. cannot pass through D.C. ammeter
(b) A.C. changes direction
(c) average value of current for complete cycle is zero
(d) D.C. ammeter will get damaged. *(2004)*

28. In an *LCR* series a.c. circuit, the voltage across each of the components, *L, C* and *R* is 50 V. The voltage across the *LC* combination will be
(a) 50 V (b) $50\sqrt{2}$ V
(c) 100 V (d) 0 V (zero). *(2004)*

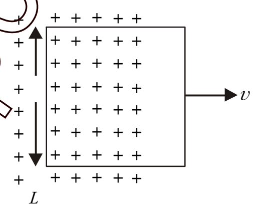
29. The core of any transformer is laminated so as to
(a) reduce the energy loss due to eddy currents
(b) make it light weight
(c) make it robust & strong
(d) increase the secondary voltage. *(2003)*

30. In an oscillating *LC* circuit the maximum charge on the capacitor is *Q*. The charge on the capacitor when the energy is stored equally between the electric and magnetic field is
(a) $Q/2$ (b) $Q/\sqrt{3}$ (c) $Q/\sqrt{2}$ (d) Q. *(2003)*

31. When the current changes from +2 A to –2 A in 0.05 second, an e.m.f. of 8 V is induced in a coil. The coefficient of self-induction of the coil is
(a) 0.2 H (b) 0.4 H (c) 0.8 H (d) 0.1 H. *(2003)*

32. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon
(a) the rates at which currents are changing in the two coils
(b) relative position and orientation of the two coils
(c) the materials of the wires of the coils
(d) the currents in the two coils. *(2003)*

33. A conducting square loop of side *L* and resistance *R* moves in its plane with a uniform velocity *v* perpendicular to one of its sides. A magnetic induction *B* constant in time and space, pointing perpendicular and into the plane at the loop exists everywhere with half the loop outside the field, as shown in figure. The induced emf is

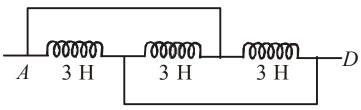


(a) zero (b) *RvB* (c) *vBL*/*R* (d) *vBL*. *(2002)*

34. In a transformer, number of turns in the primary coil are 140 and that in the secondary coil are 280. If current in primary coil is 4 A, then that in the secondary coil is
(a) 4 A (b) 2 A (c) 6 A (d) 10 A. *(2002)*

35. The power factor of an AC circuit having resistance (*R*) and inductance (*L*) connected in series and an angular velocity ω is
(a) $R/\omega L$ (b) $R/(R^2 + \omega^2L^2)^{1/2}$
(c) $\omega L/R$ (d) $R/(R^2 - \omega^2L^2)^{1/2}$. *(2002)*

36. The inductance between *A* and *D* is
(a) 3.66 H
(b) 9 H
(c) 0.66 H
(d) 1 H.



(2002)

Answer Key

1.	(d)	**2.**	(b)	**3.**	(d)	**4.**	(b)	**5.**	(c)	**6.**	(c)
7.	(d)	**8.**	(c)	**9.**	(a)	**10.**	(d)	**11.**	(a)	**12.**	(a)
13.	(c)	**14.**	(b)	**15.**	(c)	**16.**	(a)	**17.**	(d)	**18.**	(d)
19.	(c)	**20.**	(a)	**21.**	(d)	**22.**	(a)	**23.**	(b)	**24.**	(c)
25.	(b)	**26.**	(b)	**27.**	(c)	**28.**	(d)	**29.**	(a)	**30.**	(c)
31.	(d)	**32.**	(c)	**33.**	(d)	**34.**	(b)	**35.**	(b)	**36.**	(d)

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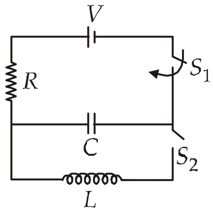
Explanations

1. (d):



As switch S_1 is closed and switch S_2 is kept open. Now, capacitor is charging through a resistor R.

Charge on a capacitor at any time t is

$$q = q_0(1 - e^{-t/\tau})$$
$$q = CV(1 - e^{-t/\tau}) \quad [\text{As } q_0 = CV]$$

At $t = \frac{\tau}{2}$

$$q = CV(1 - e^{-\tau/2\tau}) = CV(1 - e^{-1/2})$$

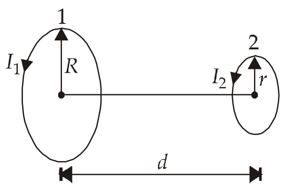
At $t = \tau$

$$q = CV(1 - e^{-\tau/\tau}) = CV(1 - e^{-1})$$

At $t = 2\tau$,

$$q = CV(1 - e^{-2\tau/\tau}) = CV(1 - e^{-2})$$

2. (b):



As field due to current loop 1 at an axial point

$$\therefore \quad B_1 = \frac{\mu_0 I_1 R^2}{2(d^2 + R^2)^{3/2}}$$

Flux linked with smaller loop 2 due to B_1 is

$$\phi_2 = B_1 A_2 = \frac{\mu_0 I_1 R^2}{2(d^2 + R^2)^{3/2}} \pi r^2$$

The coefficient of mutual inductance between the loops is

$$M = \frac{\phi_2}{I_1} = \frac{\mu_0 R^2 \pi r^2}{2(d^2 + R^2)^{3/2}}$$

Flux linked with bigger loop 1 is

$$\phi_1 = MI_2 = \frac{\mu_0 R^2 \pi r^2 I_2}{2(d^2 + R^2)^{3/2}}$$

Substituting the given values, we get

$$\phi_1 = \frac{4\pi \times 10^{-7} \times (20 \times 10^{-2})^2 \times \pi \times (0.3 \times 10^{-2})^2 \times 2}{2[(15 \times 10^{-2})^2 + (20 \times 10^{-2})^2]^{3/2}}$$

$\phi_1 = 9.1 \times 10^{-11}$ weber

3. (d) : Here, $B_H = 5.0 \times 10^{-5}$ N A^{-1} m^{-1}

$l = 2$ m and $v = 1.5$ m s^{-1}

Induced emf, $\varepsilon = B_H vl = 5 \times 10^{-5} \times 1.50 \times 2$

$= 15 \times 10^{-5}$ V $= 0.15$ mV

4. (b) : Charge on the capacitor at any instant t is

$$q = q_0 \cos\omega t \quad \text{...(i)}$$

Equal sharing of energy means

Energy of a capacitor $= \frac{1}{2}$ Total energy

$$\frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\left(\frac{1}{2}\frac{q_0^2}{C}\right) \Rightarrow q = \frac{q_0}{\sqrt{2}}$$

From equation (i)

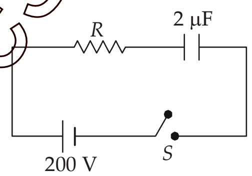
$$\frac{q_0}{\sqrt{2}} = q_0 \cos\omega t$$

$$\cos\omega t = \frac{1}{\sqrt{2}}$$

$$\omega t = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC} \quad \left(\because \ \omega = \frac{1}{\sqrt{LC}}\right)$$

5. (c) :



In case charging of capacitor through the resistance is

$$V = V_0(1 - e^{-t/RC})$$

Here, $V = 120$ V, $V_0 = 200$ V, $R = ?$

$C = 2$ μF and $t = 5$ s.

$$\therefore \quad 120 = 200(1 - e^{-5/R \times 2 \times 10^{-6}})$$

$$\text{or} \quad e^{-5/R \times 2 \times 10^{-6}} = \frac{80}{200}$$

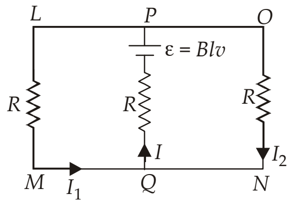
Taking the natural logarithm on both sides, we get

$$\frac{-5}{R \times 2 \times 10^{-6}} = \ln(0.4) = -0.916$$

$\Rightarrow \quad R = 2.7 \times 10^6$ Ω

6. (c) : Emf induced across PQ is $\varepsilon = Blv$.

The equivalent circuit diagram is as shown in the figure.



Applying Kirchhoff's first law at junction Q, we get

$$I = I_1 + I_2 \quad \text{...(i)}$$

Applying Kirchhoff's second law for the closed loop $PLMQP$, we get

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$-I_1R - IR + \varepsilon = 0$

$I_1R + IR = Blv$ ...(ii)

Again, applying Kirchhoff's second law for the closed loop *PONQP*, we get

$-I_2R - IR + \varepsilon = 0$

$I_2R + IR = Blv$ ...(iii)

Adding equations (ii) and (iii), we get

$2IR + I_1R + I_2R = 2Blv$

$2IR + R(I_1 + I_2) = 2Blv$

$2IR + IR = 2Blv$ (Using (i))

$3IR = 2Blv$

$I = \frac{2Blv}{3R}$ ...(iv)

Substituting this value of I in equation (ii), we get $I_1 = \frac{Blv}{3R}$

Substituting the value of I in equation (iii), we get $I_2 = \frac{Blv}{3R}$

Hence, $I_1 = I_2 = \frac{Blv}{3R}$, $I = \frac{2Blv}{3R}$

7. **(d) :** During discharging of capacitor through a resistor,

$q = q_0e^{-t/RC}$ ...(i)

The energy stored in the capacitor at any instant of time t is

$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{(q_0e^{-t/RC})^2}{C}$ (Using(i))

$= \frac{1}{2}\frac{q_0^2}{C}e^{-2t/RC} = U_0e^{-2t/RC}$ ...(ii)

where $U_0 = \frac{1}{2}\frac{q_0^2}{C}$, the maximum energy stored in the capacitor.

According to given problem

$\frac{U_0}{2} = U_0e^{-2t_1/RC}$ (Using (ii)) ...(iii)

and $\frac{q_0}{4} = q_0e^{-t_2/RC}$ (Using (i)) ...(iv)

From equation (iii), we get

$\frac{1}{2} = e^{-2t_1/RC}$

Taking natural logarithms of both sides, we get

$\ln 1 - \ln 2 = -\frac{2t_1}{RC}$ or $t_1 = \frac{RC\ln 2}{2}$ $(\because \ln 1 = 0)$

From equation (iv), we get

$\frac{1}{4} = e^{-t_2/RC}$

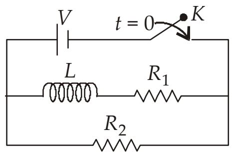
Taking natural logarithms of both sides of the above equation, we get

$\ln 1 - \ln 4 = -\frac{t_2}{RC}$

$t_2 = RC \ln 4 = 2RC \ln 2$ $(\because \ln 4 = 2\ln 2)$

$\therefore \frac{t_1}{t_2} = \frac{RC\ln 2}{2} \times \frac{1}{2RC\ln 2} = \frac{1}{4}$

8. **(c) :**

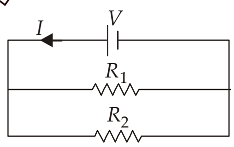


At time $t = 0$, the inductor acts as an open circuit. The corresponding equivalent circuit diagram is as shown in the figure (i).

(i)

The current through battery is $I = \frac{V}{R_2}$

At time $t = \infty$, the inductor acts as a short circuit. The corresponding equivalent circuit diagram is as shown in the figure (ii).



(ii)

$\therefore$ The current through the battery is

$I = \frac{V}{R_{eq}} = \frac{V}{\frac{R_1R_2}{R_1 + R_2}}$ $(\because R_1$ and R_2 are in parallel)

$= \frac{V(R_1 + R_2)}{R_1R_2}$

9. **(a) :** Here, $R = 200\ \Omega$, $V_{rms} = 220$ V, $\upsilon = 50$ Hz

When only the capacitance is removed, the phase difference between the current and voltage is

$\tan\phi = \frac{X_L}{R}$

$\tan 30° = \frac{X_L}{R}$ or $X_L = \frac{1}{\sqrt{3}}R$

When only the inductance is removed, the phase difference between current and voltage is

$\tan\phi' = \frac{X_C}{R}$

$\tan 30° = \frac{X_C}{R}$ or $X_C = \frac{1}{\sqrt{3}}R$

As $X_L = X_C$, therefore the given series *LCR* is in resonance.

$\therefore$ Impedance of the circuit is $Z = R = 200\ \Omega$

The power dissipated in the circuit is

$P = V_{rms}I_{rms}\cos\phi$

$= \frac{V_{rms}^2}{Z}\cos\phi$ $\left(\because I_{rms} = \frac{V_{rms}}{Z}\right)$

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At resonance, power factor $\cos\phi = 1$

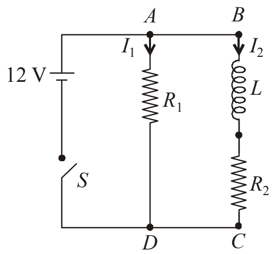
$$\therefore \quad P = \frac{V_{rms}^2}{Z} = \frac{(220\text{ V})^2}{(200\,\Omega)} = 242\text{ W}.$$

10. (d) : For the given R, L circuit the potential difference across $AD = V_{BC}$ as they are parallel.

$I_1 = E/R_1$.

$I_2 = I_0(1 - e^{-t/\tau})$ where τ = mean life or L/R.

$\tau = t_0$ (given).



$$E\text{ (across }BC) = L\frac{dI_2}{dt} + R_2 I_2$$

$I_2 = I_0(1 - e^{-t/t})$.

$$\text{But } I_0 = \frac{E}{R_2} = \frac{12}{2} = 6\text{ A}.$$

$$\tau = t_0 = \frac{L}{R} = \frac{400\times10^{-3}\text{ H}}{2\,\Omega} = 0.2\text{ s}$$

$\therefore \quad I_2 = 6(1 - e^{-t/0.2})$

Potential drop across $L = E - R_2 I_2$

$= 12 - 2 \times 6(1 - e^{-t/0.2}) = 12e^{-t/0.2} = 12e^{-5t}$ V.

11. (a) : $M = \mu_0 n_1 n_2 \pi r_1^2 l$.

From $\phi_2 = \pi r_1^2 (\mu_0 n i) n_2 l$.

$A = \pi r_1^2 = 10\text{ cm}^2$, $l = 20$ cm, $N_1 = 300$, $N_2 = 400$.

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \frac{4\pi\times10^{-7}\times300\times400\times10\times10^{-4}}{0.20}$$

$= 2.4\pi \times 10^{-4}$ H

12. (a) : During the growth of current in LR circuit is given by

$$I = I_0\left(1 - e^{-\frac{R}{L}t}\right) \quad \text{or} \quad I = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{5}{5}\left(1 - e^{-\frac{5}{10}\times2}\right)$$

$I = (1 - e^{-1})$.

13. (c) : Given : $E = E_0 \sin\omega t$

$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

Since the phase difference (ϕ) between voltage and current is $\frac{\pi}{2}$.

$\therefore$ Power factor $\cos\phi = \cos\frac{\pi}{2} = 0$

Power consumption $= E_{rms} I_{rms} \cos\phi = 0$.

14. (b) : Maximum current $I_0 = \frac{E}{R} = \frac{100}{100} = 1$ A

The current decays for 1 millisecond $= 1 \times 10^{-3}$ sec

During decay, $I = I_0 e^{-Rt/L}$

$$I = (1)e^{\frac{(-1\times10^{-3})\times100}{100\times10^{-3}}}$$

or $\quad I = e^{-1} = \frac{1}{e}$ A.

15. (c) : $\phi = 10t^2 - 50t + 250$

$$\therefore \quad \frac{d\phi}{dt} = 20t - 50$$

$$\text{Induced emf } e = \frac{-d\phi}{dt}$$

or $e = -(20t - 50) = -[(20 \times 3) - 50] = -10$ volt

or $e = -10$ volt.

16. (a) : In an a.c. generator, maximum emf $= NAB\omega$.

17. (d) : Current $I = \frac{E}{Z}$

$$\text{where } E = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonance, $X_L = X_C$

$\therefore \quad Z = R$

Again at resonance, $V_L = V_C$

$\therefore \quad E = V_R$

$$\therefore \quad I = \frac{V_R}{R} = \frac{100}{1\times10^3} = 0.1\text{ A}$$

$$\therefore \quad V_L = I L\omega = \frac{I}{C\omega} = \frac{0.1}{(2\times10^{-6})\times(200)}$$

$\therefore \quad V_L = 250$ volt.

18. (d) : R and L cause phase difference to lie between 0 and $\pi/2$ but never 0 and $\pi/2$ at extremities.

19. (c) : Power factor $\cos\phi = \frac{R}{Z} = \frac{12}{15} = 0.8$.

20. (a) : For maximum power, $L\omega = \frac{1}{C\omega}$

$$\therefore \quad C = \frac{1}{L\omega^2} = \frac{1}{10\times(2\pi\times50)^2}$$

$$= \frac{1}{10\times10^4\times(\pi)^2} = 10^{-6}\text{ F}$$

or $\quad C = 1\ \mu$F.

21. (d) : During growth of charge in an inductance,

$I = I_0(1 - e^{-Rt/L})$

$$\text{or} \quad \frac{I_0}{2} = I_0(1 - e^{-Rt/L})$$

$$\text{or} \quad e^{-Rt/L} = \frac{1}{2} = 2^{-1} \quad \text{or} \quad \frac{Rt}{L} = \ln 2 \Rightarrow t = \frac{L}{R}\ln 2$$

$$t = \frac{300\times10^{-3}}{2}\times(0.693)$$

or $\quad t = 0.1$ sec.

22. (a) : The emf induced in the circuit is zero because the two emf induced are equal and opposite when one U tube slides inside another tube.

23. (b) : Induced emf $= \frac{1}{2}B\omega l^2 = \frac{1}{2}\times(0.2\times10^{-4})(5)(1)^2$

$$\therefore \quad \text{Induced emf} = \frac{10^{-4}}{2} = \frac{100\times10^{-6}}{2} = 50\ \mu\text{V}.$$

24. (c) : At resonance, $\omega = \frac{1}{\sqrt{LC}}$

when ω is constant,

$$\therefore \quad \frac{1}{L_1 C_1} = \frac{1}{L_2 C_2} \Rightarrow \frac{1}{LC} = \frac{1}{L_2(2C)} = \frac{1}{2L_2 C}$$

$\therefore \quad L_2 = L/2$.

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25. (b) : Magnetic flux linked

$$= BA\cos\omega t = \frac{B\pi r^2 \cos\omega t}{2}$$

$\therefore$ Induced emf $e = \frac{-d\phi}{dt} = \frac{-1}{2}B\pi r^2 \omega \sin\omega t$

$\therefore$ Power $= \frac{e^2}{R} = \frac{B^2\pi^2 r^4 \omega^2 \sin^2 \omega t}{4R}$

$$= \frac{(B\pi r^2\omega)^2}{4R}\sin^2 \omega t$$

$\because$ $\langle\sin^2 \omega t\rangle = 1/2$

$\therefore$ Mean power generated

$$= \frac{(B\pi r^2\omega)^2}{4R} \times \frac{1}{2} = \frac{(B\pi r^2\omega)^2}{8R}.$$

26. (b) : Induced current $I = \frac{-n}{R'}\frac{d\phi}{dt} = \frac{-n}{R'}\frac{dW}{dt}$ where $\phi = W =$ flux $\times$ per unit turn of the coil

$\therefore$ $I = -\frac{1}{(R+4R)}\frac{n(W_2 - W_1)}{t} = -\frac{n(W_2 - W_1)}{5Rt}.$

27. (c) : Average value of A.C. for complete cycle is zero. Hence A.C. can not be measured by D.C. ammeter.

28. (d) : In an *LCR* series a.c. circuit, the voltages across components *L* and *C* are in opposite phase. The voltage across *LC* combination will be zero.

29. (a) : The energy loss due to eddy currents is reduced by using laminated core in a transformer.

30. (c) : Let *Q* denote maximum charge on capacitor. Let *q* denote charge when energy is equally shared

$\therefore$ $\frac{1}{2}\left(\frac{1}{2}\frac{Q^2}{C}\right) = \frac{1}{2}\frac{q^2}{C} \Rightarrow Q^2 = 2q^2$

$\therefore$ $q = Q/\sqrt{2}.$

31. (d) : $L = \frac{-e}{di/dt} = \frac{-8 \times 0.05}{-4} = 0.1\text{ H}.$

32. (c) : Mutual inductance between two coils depends on the materials of the wires of the coils.

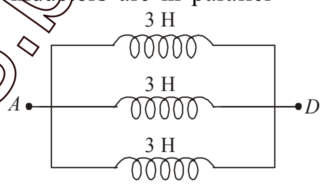
33. (d) : Induced emf = vBL.

34. (b) : $I_2 N_2 = I_1 N_1$ for a transformer

$\therefore$ $I_2 = \frac{I_1 N_1}{N_2} = \frac{4 \times 140}{280} = 2\text{ A}.$

35. (b) : Power factor $= \frac{R}{\sqrt{R^2 + L^2\omega^2}}.$

36. (d) : Three inductors are in parallel



$\therefore$ $\frac{1}{L_{(eq)}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = \frac{1}{1}$

$\therefore$ $L_{(eq)} = 1\text{ H}.$

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CHAPTER 15

ELECTROMAGNETIC WAVES

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1. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is

(a) 12 V/m (b) 3 V/m
(c) 6 V/m (d) 9 V/m *(2013)*

2. A radar has a power of 1 kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance upto which it can detect object located on the surface of the earth (Radius of earth = 6.4×10^6 m) is

(a) 16 km (b) 40 km
(c) 64 km (d) 80 km *(2012)*

3. An electromagnetic wave in vacuum has the electric and magnetic fields $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then

(a) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$ (b) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$
(c) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$ (d) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$ *(2012)*

4. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1 : Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

Statement-2 : The state of ionosphere varies from hour to hour, day to day and season to season.

(a) Statement-1 is true, statement-2 is false.
(b) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true. *(2011)*

5. The rms value of the electric field of the light coming from the sun is 720 N/C. The average total energy density of the electromagnetic wave is

(a) 3.3×10^{-3} J/m^3 (b) 4.58×10^{-6} J/m^3
(c) 6.37×10^{-9} J/m^3 (d) 81.35×10^{-12} J/m^3. *(2006)*

6. An electromagnetic wave of frequency $\upsilon = 3.0$ MHz passes from vacuum into a dielectric medium with permitivity $\varepsilon = 4.0$. Then

(a) wavelength is doubled and the frequency remains unchanged
(b) wavelength is doubled and frequency becomes half
(c) wavelength is halved and frequency remains unchanged
(d) wavelength and frequency both remain unchanged. *(2004)*

7. Consider telecommunication through optical fibres. Which of the following statements is not true?

(a) Optical fibres can be of graded refractive index.
(b) Optical fibres are subject to electromagnetic interference from outside.
(c) Optical fibres have extremely low transmission loss.
(d) Optical fibres may have homogeneous core with a suitable cladding. *(2003)*

8. Which of the following are not electromagnetic waves?

(a) cosmic rays (b) gamma rays
(c) β-rays (d) X-rays. *(2002)*

9. Electromagnetic waves are transverse in nature is evident by

(a) polarization (b) interference
(c) reflection (d) diffraction. *(2002)*

10. Infrared radiation is detected by

(a) spectrometer (b) pyrometer
(c) nanometer (d) photometer. *(2002)*

Answer Key

1.	(c)	**2.**	(d)	**3.**	(a)	**4.**	(b)	**5.**	(b)	**6.**	(c)
7.	(b)	**8.**	(c)	**9.**	(a)	**10.**	(b)				

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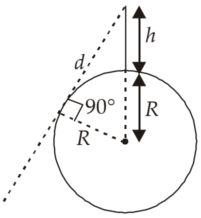
Explanations

1. **(c)** **:** In electromagnetic wave, the peak value of electric field (E_0) and peak value of magnetic field (B_0) are related by
$E_0 = B_0 c$
$E_0 = (20 \times 10^{-9}\ \text{T})\ (3 \times 10^8\ \text{m s}^{-1}) = 6\ \text{V/m}$

2. **(d)** **:** Maximum distance on earth where object can be detected is d, then



$(h + R)^2 = d^2 + R^2$
$d^2 = h^2 + 2Rh$
$\because\ h << R$
$\therefore\ d = \sqrt{2Rh}$
$d = \sqrt{2 \times 6.4 \times 10^6 \times 500} = 8 \times 10^4\ \text{m} = 80\ \text{km}$

3. **(a)** **:** The direction of polarization is parallel to electric field.
$\therefore\ \vec{X} \parallel \vec{E}$
The direction of wave propagation is parallel to $\vec{E} \times \vec{B}$.
$\therefore\ \vec{k} \parallel \vec{E} \times \vec{B}$

4. **(b)**

5. **(b)** **:** $u = \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 + \frac{1}{2\mu_0} B_{\text{rms}}^2$
$= \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 + \frac{1}{2\mu_0}\left(\frac{E_{\text{rms}}^2}{c^2}\right)$
$= \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 + \frac{1}{2\mu_0} E_{\text{rms}}^2 \varepsilon_0 \mu_0$
$= \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 + \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 = \varepsilon_0 E_{\text{rms}}^2$
$= (8.85 \times 10^{-12}) \times (720)^2$
$= 4.58 \times 10^{-6}\ \text{Jm}^{-3}$

6. **(c)** **:** During propogation of a wave from one medium to another, frequency remains constant and wavelength changes
$\mu = \sqrt{\frac{\varepsilon}{\varepsilon_0}} = \sqrt{4} = 2$
Since $\mu \propto \frac{1}{\lambda}$
$\therefore$ Wavelength is halved
Hence option (c) holds good.

7. **(b)** **:** Optical fibres are subject to electromagnetic intereference from outside.

8. **(c)** **:** β-rays are not electromagnetic waves.

9. **(a)** **:** Polarization proves the transverse nature of electromagnetic waves.

10. **(b)** **:** Infrared radiation produces thermal effect and is detected by pyrometer.

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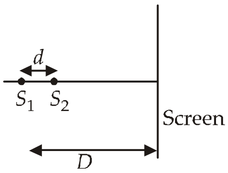
CHAPTER **16**

OPTICS

1. Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is
(a) 10 cm (b) 15 cm (c) 20 cm (d) 30 cm *(2013)*

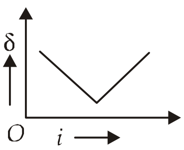
2. Two coherent point sources S_1 and S_2 are separated by a small distance 'd' as shown. The fringes obtained on the screen will be

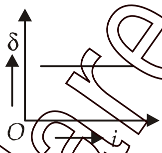


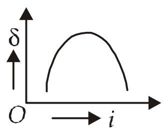
(a) concentric circles (b) points
(c) straight lines (d) semi-circles *(2013)*

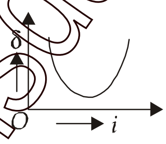
3. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is
(a) $I_0/8$ (b) I_0 (c) $I_0/2$ (d) $I_0/4$ *(2013)*

4. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by

(a) 

(b) 

(c) 

(d) 

(2013)

5. In Young's double slit experiment, one of the slit is wider than other, so that the amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference f is given by
(a) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$ (b) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$
(c) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$ (d) $\frac{I_m}{9}(4+5\cos\phi)$ *(2012)*

6. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object be shifted to be in sharp focus on film?
(a) 2.4 m (b) 3.2 m
(c) 5.6 m (d) 7.2 m *(2012)*

7. *Direction : The question has a paragraph followed by two statements, Statement-1 and Statement-2. Of the given four alternatives after the statements, choose the one that describes the statements.*

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement-1 : When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π.

Statement-2 : The centre of the interference pattern is dark.

(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true. *(2011)*

8. Let the x–z plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is
(a) 30° (b) 45°
(c) 60° (d) 75° *(2011)*

9. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m s^{-1}. The speed of the image of the second car as seen in the mirror of the first one is
(a) $\frac{1}{10}$ m s^{-1} (b) $\frac{1}{15}$ m s^{-1}
(c) 10 m s^{-1} (d) 15 m s^{-1} *(2011)*

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Directions : Questions number 10-12 are based on the following paragraph.

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

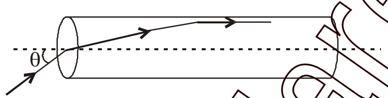
10. The initial shape of the wavefront of the beam is
(a) planar (b) convex
(c) concave
(d) convex near the axis and concave near the periphery

11. The speed of light in the medium is
(a) maximum on the axis of the beam
(b) minimum on the axis of the beam
(c) the same everywhere in the beam
(d) directly proportional to the intensity I

12. As the beam enters the medium, it will
(a) travel as a cylindrical beam
(b) diverge (c) converge
(d) diverge near the axis and converge near the periphery
(2010)

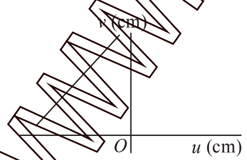
13. A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4^{th} bright fringe of the unknown light. From this data, the wavelength of the unknown light is
(a) 393.4 nm (b) 885.0 nm
(c) 442.5 nm (d) 776.8 nm *(2009)*

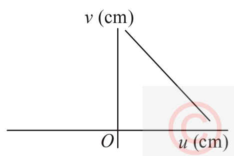
14. A transparent solid cylindrical rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the figure.

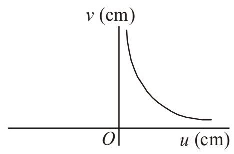


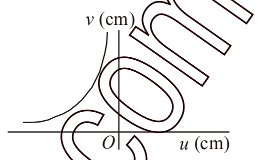
The incident angle θ for which the light ray grazes along the wall of the rod is
(a) $\sin^{-1}\left(\frac{1}{2}\right)$ (b) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
(c) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ *(2009)*

15. A student measures the focal length of a convex lens by putting an object pin at a distance u from the lens and measuring the distance v of the image pin. The graph between u and v plotted by the student should look like

(a) 

(b) 

(c) 

(d) 

(2008)

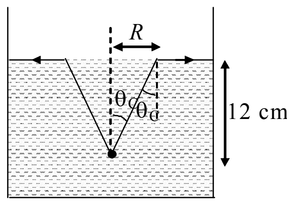
16. Two lenses of power –15 D and +5 D are in contact with each other. The focal length of the combination is
(a) + 10 cm (b) –20 cm
(c) –10 cm (d) +20 cm *(2007)*

17. In a Young's double slit experiment the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of light used) is I. If I_0 denotes the maximum intensity, $\frac{I}{I_0}$ is equal to
(a) $\frac{3}{4}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$
(2007)

18. The refractive index of glass is 1.520 for red light and 1.525 for blue light. Let D_1 and D_2 be angles of minimum deviation for red and blue light respectively in a prism of this glass. Then
(a) $D_1 > D_2$ (b) $D_1 < D_2$
(c) $D_1 = D_2$
(d) D_1 can be less than or greater than depending upon the angle of prism. *(2006)*

19. A thin glass (refractive index 1.5) lens has optical power of –5 D in air. Its optical power in a liquid medium with refractive index 1.6 will be
(a) 25 D (b) –25 D (c) 1 D (d) –1 D *(2005)*

20. A fish looking up through the water sees the outside world contained in a circular horizon. If the refractive index of water is 4/3 and the fish is 12 cm below the surface, the radius of this circle in cm is



(a) $36\sqrt{5}$ (b) $4\sqrt{5}$
(c) $36\sqrt{7}$ (d) $36/\sqrt{7}$ *(2005)*

21. Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 mm. Approximately, what is the maximum distance at which these dots can be resolved by the eye? [Take wavelength of light = 500 nm]
(a) 6 m (b) 3 m (c) 5 m (d) 1 m
(2005)

22. When an unpolarized light of intensity I_0 is incident on a polarizing sheet, the intensity of the light which does not get transmitted is

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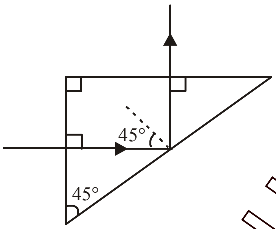
(a) zero (b) I_0
(c) $\frac{1}{2}I_0$ (d) $\frac{1}{4}I_0$ *(2005)*

23. If I_0 is the intensity of the principal maximum in the single slit diffraction pattern, then what will be its intensity when the slit width is doubled?
(a) I_0 (b) $I_0/2$ (c) $2I_0$ (d) $4I_0$ *(2005)*

24. A Young's double slit experiment uses a monochromatic source. The shape of the interference fringes formed on a screen
(a) straight line (b) parabola
(c) hyperbola (d) circle *(2005)*

25. A plano convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now this lens has been used to form the image of an object. At what distance from this lens an object be placed in order to have a real image of the size of the object?
(a) 20 cm (b) 30 cm (c) 60 cm (d) 80 cm. *(2004)*

26. A light ray is incident perpendicular to one face of a 90° prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45°, we conclude that the refractive index n



(a) $n < \frac{1}{\sqrt{2}}$ (b) $n > \sqrt{2}$
(c) $n > \frac{1}{\sqrt{2}}$ (d) $n < \sqrt{2}$. *(2004)*

27. The angle of incidence at which reflected light in totally polarized for reflection from air to glass (refractive index n), is
(a) $\sin^{-1}(n)$ (b) $\sin^{-1}(1/n)$
(c) $\tan^{-1}(1/n)$ (d) $\tan^{-1}(n)$. *(2004)*

28. The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double-slit experiment is
(a) infinite (b) five (c) three (d) zero. *(2004)*

29. To get three images of a single object, one should have two plane mirrors at an angle of
(a) 60° (b) 90°
(c) 120° (d) 30°. *(2003)*

30. The image formed by an objective of a compound microscope is
(a) virtual and diminished (b) real and diminished
(c) real and enlarged (d) virtual and enlarged. *(2003)*

31. To demonstrate the phenomenon of interference we require two sources which emit radiation of
(a) nearly the same frequency
(b) the same frequency
(c) different wavelength
(d) the same frequency and having a definite phase relationship. *(2003)*

32. An astronomical telescope has a large aperture to
(a) reduce spherical aberration
(b) have high resolution
(c) increase span of observation
(d) have low dispersion. *(2002)*

33. Which of the following is used in optical fibres?
(a) total internal reflection (b) scattering
(c) diffraction (d) refraction. *(2002)*

34. Wavelength of light used in an optical instrument are λ_1 = 4000 Å and λ_2 = 5000 Å, then ratio of their respective resolving powers (corresponding to λ_1 and λ_2) is
(a) 16 : 25 (b) 9 : 1
(c) 4 : 5 (d) 5 : 4. *(2002)*

35. If two mirrors are kept at 60° to each other, then the number of images formed by them is
(a) 5 (b) 6
(c) 7 (d) 8. *(2002)*

Answer Key

1.	(d)	**2.**	(a)	**3.**	(d)	**4.**	(d)	**5.**	(c)	**6.**	(c)
7.	(c)	**8.**	(b)	**9.**	(b)	**10.**	(a)	**11.**	(b)	**12.**	(c)
13.	(c)	**14.**	(d)	**15.**	(d)	**16.**	(c)	**17.**	(a)	**18.**	(b)
19.	(*)	**20.**	(d)	**21.**	(c)	**22.**	(c)	**23.**	(a)	**24.**	(a)
25.	(a)	**26.**	(b)	**27.**	(d)	**28.**	(b)	**29.**	(b)	**30.**	(c)
31.	(d)	**32.**	(b)	**33.**	(a)	**34.**	(d)	**35.**	(a)		

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Explanations

1. **(d):** According to lens maker's formula

$$\frac{1}{f}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

As the lens is plano-convex

$\therefore\quad R_1 = R,\ R_2 = \infty$

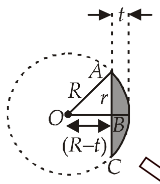
$$\therefore\quad \frac{1}{f}=\frac{(\mu-1)}{R}$$

$$\text{or}\quad f=\frac{R}{(\mu-1)} \qquad \text{...(i)}$$

As speed of light in the medium of lens is 2×10^8 m/s

$$\therefore\quad \mu=\frac{c}{v}=\frac{3\times10^8\ \text{m/s}}{2\times10^8\ \text{m/s}}=\frac{3}{2} \qquad \text{...(ii)}$$

If r is the radius and t is the thickness of lens (at the centre), the radius of curvature R of its curved surface in accordance with figure will be given by



$R^2 = r^2 + (R - t)^2$

$R^2 = r^2 + R^2 + t^2 - 2Rt$

$2Rt = r^2 + t^2$

$$R=\frac{r^2}{2t} \qquad (\because\ r >> t)$$

Here, $r = 3$ cm, $t = 3$ mm $= 0.3$ cm

$$\therefore\quad R=\frac{(3\ \text{cm})^2}{2\times0.3\ \text{cm}}=15\ \text{cm}$$

On substituting the values of m and R from Eqs. (ii) and (iii) in (i), we get

$$f=\frac{15\ \text{cm}}{(1.5-1)}=30\ \text{cm.}$$

2. **(a)** : When the screen is placed perpendicular to the line joining the sources, the fringes will be concentric circles.

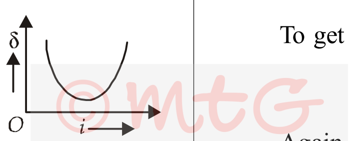
3. **(d)** : Intensity of light after passing polaroid A is

$$I_1=\frac{I_0}{2}$$

Now this light will pass through the second polaroid B whose axis is inclined at an angle of 45° to the axis of polaroid A. So in accordance with Malus law, the intensity of light emerging from polaroid B is

$$I_2=I_1\cos^2 45°=\left(\frac{I_0}{2}\right)\left(\frac{1}{\sqrt{2}}\right)^2=\frac{I_0}{4}$$

4. **(d)** : The graph between angle of deviation(δ) and angle of incidence (i) for a triangular prism is as shown in the adjacent figure.



5. **(c) :** Here, $A_2 = 2A_1$

$\because\quad$ Intensity $\propto$ (Amplitude)^2

$$\therefore\quad \frac{I_2}{I_1}=\left(\frac{A_2}{A_1}\right)^2=\left(\frac{2A_1}{A_1}\right)^2=4$$

$I_2 = 4I_1$

Maximum intensity, $I_m=\left(\sqrt{I_1}+\sqrt{I_2}\right)^2$

$$=\left(\sqrt{I_1}+\sqrt{4I_1}\right)^2=\left(3\sqrt{I_1}\right)^2=9I_1$$

$$\text{or}\quad I_1=\frac{I_m}{9} \qquad \text{...(i)}$$

Resultant intensity, $I=I_1+I_2+2\sqrt{I_1I_2}\cos\phi$

$=I_1+4I_1+2\sqrt{I_1(4I_1)}\cos\phi$

$= 5I_1 + 4I_1\cos\phi = I_1 + 4I_1 + 4I_1\cos\phi$

$= I_1 + 4I_1(1 + \cos\phi)$

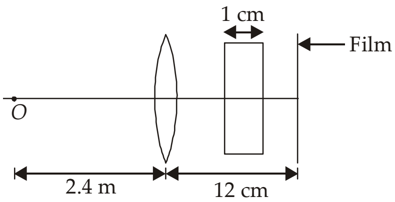
$$=I_1+8I_1\cos^2\frac{\phi}{2} \qquad \left(\because\ 1+\cos\phi=2\cos^2\frac{\phi}{2}\right)$$

$$=I_1\left(1+8\cos^2\frac{\phi}{2}\right)$$

Putting the value of I_1 from eqn. (i), we get

$$I=\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$$

6. **(c)** :



According to thin lens formula

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

Here, $u = -2.4$ m $= -240$ cm, $v = 12$ cm

$$\therefore\quad \frac{1}{f}=\frac{1}{12}-\frac{1}{(-240)}=\frac{1}{12}+\frac{1}{240}$$

$$\frac{1}{f}=\frac{21}{240}\quad \text{or}\quad f=\frac{240}{21}\ \text{cm}$$

When a glass plate is interposed between lens and film, so shift produced by it will be

$$\text{Shift}=t\left(1-\frac{1}{\mu}\right)=1\left(1-\frac{1}{1.5}\right)=1\left(1-\frac{2}{3}\right)=\frac{1}{3}\ \text{cm}$$

To get image at film, lens should form image at distance

$$v'=12-\frac{1}{3}=\frac{35}{3}\ \text{cm}$$

Again using lens formula

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$\therefore \quad \frac{21}{240} = \frac{3}{35} - \frac{1}{u'}$ or $\frac{1}{u'} = \frac{3}{35} - \frac{21}{240} = \frac{1}{5}\left[\frac{3}{7} - \frac{21}{48}\right]$

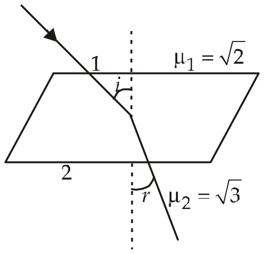
$\frac{1}{u'} = \frac{1}{5}\left[\frac{144-147}{336}\right]$ or $\frac{1}{u'} = -\frac{3}{1680}$

$u' = -560$ cm $= -5.6$ m

$|u'| = 5.6$ m

7. **(c)**

8. **(b) :**



Here, $\vec{A} = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\,\hat{k}$

$$\cos i = \frac{10}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + (-10)^2}} = \frac{10}{20}$$

$\cos i = \frac{1}{2}$ or $i = \cos^{-1}\left(\frac{1}{2}\right) = 60°$

Using Snell's law, $\mu_1 \sin i = \mu_2 \sin r$

$\sqrt{2}\sin 60° = \sqrt{3}\sin r \Rightarrow r = 45°$

9. **(b)**

10. **(a) :** As the beam is initially parallel, the shape of wavefront is planar.

11. **(b) :** Given $\mu = \mu_0 + \mu_2 I$

As $\mu = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$

$\mu = \frac{c}{v}$ or $v = \frac{c}{\mu} = \frac{c}{\mu_0 + \mu_2 I}$

As the intensity is maximum on the axis of the beam, therefore v is minimum on the axis of the beam.

12. **(c)**

13. **(c) :** For interference, by Young's double slits, the path difference $\frac{xd}{D} = n\lambda$ for bright fringes and $\frac{xd}{D} = (2n+1)\frac{\lambda}{2}$ for getting dark fringes.

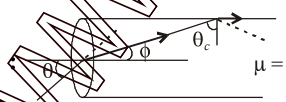
The central fringes when $x = 0$, coincide for all wavelengths.

The third fringe of $\lambda_1 = 590$ nm coincides with the fourth bright fringe of unknown wavelength λ.

$\therefore \quad \frac{xd}{D} = 3 \times 590$ nm $= 4 \times 1$ nm

$\therefore \quad \lambda = \frac{3 \times 590}{4} = 442.5$ nm.

14. **(d)**

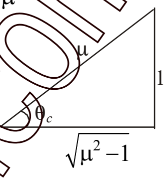


μ = ref. index of the rod

If θ_c has to be the critical angle, $\theta_c = \sin^{-1}\frac{1}{\mu}$

But $\theta_c = 90° - \phi$, $\theta_i = \theta$.

$\frac{\sin\theta_i}{\sin\phi} = \mu = \frac{2}{\sqrt{3}} \Rightarrow \frac{\sin\theta}{\cos\theta_c} = \mu.$



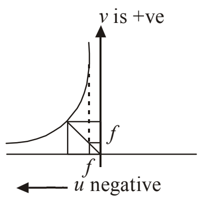
But

$\cos\theta_c = \frac{\sqrt{\mu^2-1}}{\mu} \quad \therefore \quad \sin\theta = \mu\sqrt{\frac{\mu^2-1}{\mu}} = \sqrt{\mu^2-1}.$

$\therefore \quad \theta = \sin^{-1}\sqrt{\frac{4}{3}-1} = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

So that θ_c is making total internal reflection.

15. **(d) :** According to the new cartesian system used in schools, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ for a convex lens.



u has to be negative.

If $v = \infty$, $u = f$ and if $u = \infty$, $v = f$.

A parallel beam ($u = \infty$) is focussed at f and if the object is at f, the rays are parallel. The point which meets the curve at $u = v$ gives $2f$. Therefore v is +ve, u is negative, both are symmetrical and this curve satisfies all the conditions for a convex lens.

16. **(c) :** Power of combination $= P_1 + P_2$

$= -15$ D $+ 5$ D $= -10$ D.

Focal length of combination $F = \frac{1}{P} = \frac{1}{-10\text{ D}}$

$= -0.1$ m $= -10$ cm.

17. **(a) :** In Young's double slit experiment intensity at a point is given by

$I = I_0 \cos^2\left(\frac{\phi}{2}\right)$

where ϕ = phase difference, I_0 = maximum intensity

or $\quad \frac{I}{I_0} = \cos^2\left(\frac{\phi}{2}\right) \quad$ ... (i)

Phase difference $\phi = \frac{2\pi}{\lambda} \times$ path difference

$\therefore \quad \phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{6}$ or $\phi = \frac{\pi}{3} \quad$ ... (ii)

Substitute eqn. (ii) in eqn. (i), we get

$\frac{I}{I_0} = \cos^2\left(\frac{\pi}{6}\right)$ or $\frac{I}{I_0} = \frac{3}{4}$.

18. **(b) :** Angle of minimum deviation $D = A(\mu - 1)$

$\frac{D_1 \text{ for red}}{D_2 \text{ for blue}} = \frac{\mu_R - 1}{\mu_B - 1}$

Since $\mu_B > \mu_R$,

$\therefore \quad \frac{D_1}{D_2} < 1$

$\therefore \quad D_1 < D_2$.

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19. (*) : $\frac{1}{f_a} = ({}^a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f_l} = ({}^l\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\therefore \quad \frac{f_a}{f_l} = \frac{({}^l\mu_g - 1)}{({}^a\mu_g - 1)} = \frac{(\mu_g/\mu_l) - 1}{(\mu_g - 1)}$

$= \frac{\mu_g - \mu_l}{\mu_l(\mu_g - 1)} = \frac{1.5 - 1.6}{1.6\,(1.5 - 1)}$

or $\quad \frac{P_l}{P_a} = -\frac{0.1}{1.6 \times 0.5} = \frac{-1}{8}$

$\Rightarrow \quad P_l = -\frac{P_a}{8} = -\frac{(-5)}{8} = \frac{5}{8}$

or Optical power in liquid medium $= \frac{5}{8}$ Dipotre.

N.B. : This answer is not given in the four options provided in the question.

20. (d) : For total internal reflection,

$\mu = \frac{1}{\sin\theta_C} \Rightarrow \sin\theta_C = \frac{1}{\mu} = \frac{3}{4}$

$\therefore \quad \tan\theta_c = \frac{\sin\theta_c}{\sqrt{1 - \sin^2\theta_c}}$

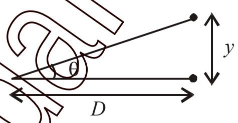
$= \frac{3/4}{\sqrt{1 - \frac{9}{16}}} = \frac{3}{4} \times \frac{4}{\sqrt{7}} = \frac{3}{\sqrt{7}}$

$\therefore \quad \frac{R}{12} = \frac{3}{\sqrt{7}} \Rightarrow R = \frac{36}{\sqrt{7}}$ cm.

21. (c) : Resolution limit $= \frac{1.22\lambda}{d}$

Again resolution limit $= \sin\theta = \theta = \frac{y}{D}$

$\therefore \quad \frac{y}{D} = \frac{1.22\lambda}{d}$



or $\quad D = \frac{yd}{1.22\lambda}$

or $\quad D = \frac{(10^{-3}) \times (3 \times 10^{-3})}{(1.22) \times (5 \times 10^{-7})} = \frac{30}{6.1} \approx 5$ m.

22. (c) : Intensity of polarized light $= I_0/2$

$\therefore$ Intensity of light not transmitted

$= I_0 - \frac{I_0}{2} = \frac{I_0}{2}$.

23. (a) : For diffraction pattern

$I = I_0\left(\frac{\sin\phi}{\phi}\right)^2$ where ϕ denotes path difference

For principal maxima, $\phi = 0$. Hence $\left(\frac{\sin\phi}{\phi}\right) = 1$

Hence intensity remains constant at I_0

$I = I_0\,(1) = I_0$.

24. (a) : Straight line fringes are formed on screen.

25. (a) : A plano-convex lens behaves like a concave mirror when its curved surface is silvered.

$\therefore \quad F$ of concave mirror so formed

$= \frac{R}{2\mu} = \frac{30}{2 \times 1.5} = 10$ cm

To form an image of object size, the object should be placed at $(2F)$ of the concave mirror

$\therefore \quad$ Distance of object from lens $= 2 \times F$

$= 2 \times 10 = 20$ cm

26. (b) : Total internal reflection occurs in a denser medium when light is incident at surface of separation at angle exceeding critical angle of the medium.

Given : $i = 45°$ in the medium and total internal reflection occurs at the glass air interface

$\therefore \quad n = \frac{1}{\sin C} > \frac{1}{\sin 45°} > \sqrt{2}$.

27. (d) : According to Brewster's law of polarization, $n = \tan i_p$ where i_p is angle of incidence

$\therefore \quad i_p = \tan^{-1}(n)$.

28. (b) : For interference maxima, $d\sin\theta = n\lambda$

$\therefore \quad 2\lambda\sin\theta = n\lambda$

or $\quad \sin\theta = \frac{n}{2}$

This equation is satisfied if $n = -2, -1, 0, 1, 2$.

$\sin\theta$ is never greater than $(+1)$, less than (-1)

$\therefore \quad$ Maximum number of maxima can be five.

29. (b) : $n = \frac{360°}{\theta^0} - 1$

$\therefore \quad 3 = \frac{360°}{\theta°} - 1 \Rightarrow 4\theta° = 360° \Rightarrow \theta° = 90°$.

30. (c) : The objective of compound microscope forms a real and enlarged image.

31. (d) : For interference phenomenon, two sources should emit radiation of the same frequency and having a definite phase relationship.

32. (b) : Large aperture leads to high resolution of telescope.

33. (a) : Total internal reflection is used in optical fibres.

34. (d) : Resolving power is proportional to λ^{-1}

$\therefore \quad \frac{R.P. \text{ for } \lambda_1}{R.P. \text{ for } \lambda_2} = \frac{\lambda_2}{\lambda_1} = \frac{5000}{4000} = \frac{5}{4}$.

35. (a) : $n = \frac{360°}{\theta°} - 1 = \frac{360°}{60°} - 1 = 5$.

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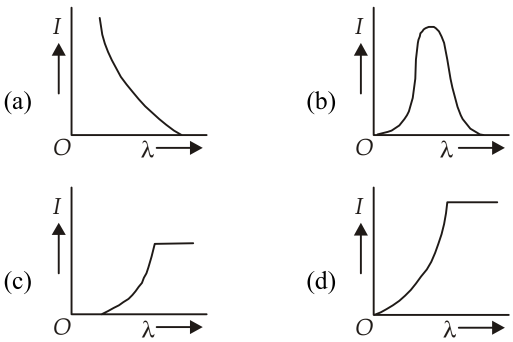
CHAPTER 17

DUAL NATURE OF MATTER AND RADIATION

1. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows



(2013)

2. This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement 1 : Davisson - Germer experiment established the wave nature of electrons.
Statement 2 : If electrons have wave nature, they can interfere and show diffraction.
(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.
(d) Statement 1 is false, Statement 2 is true. *(2012)*

3. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement-1 : A metallic surface is irradiated by a monochromatic light of frequency $\upsilon > \upsilon_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.
Statement-2 : The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.
(a) Statement-1 is true, statement-2 is false.
(b) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true. *(2011)*

4. If a source of power 4 kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called
(a) γ-rays (b) X-rays
(c) ultraviolet rays (d) microwaves *(2010)*

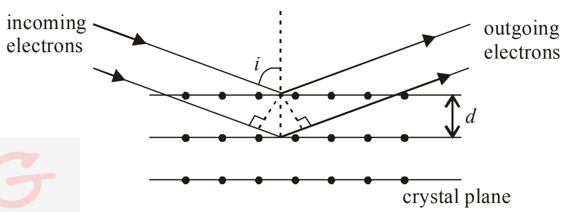
5. **Statement-1 :** When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.
Statement-2 : Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.
(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(d) Statement-1 is false, Statement-2 is true. *(2010)*

6. The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is (hc = 1240 eV nm)
(a) 3.09 eV (b) 1.41 eV
(c) 1.51 eV (d) 1.68 eV *(2009)*

Directions : Questions 7, 8 and 9 are based on the following paragraph.

Wave property of electrons implies that they will show diffraction effects. Davisson and Germer demonstrated this by diffracting electrons from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively (see figure).

(2008)



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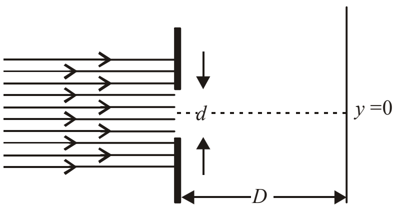
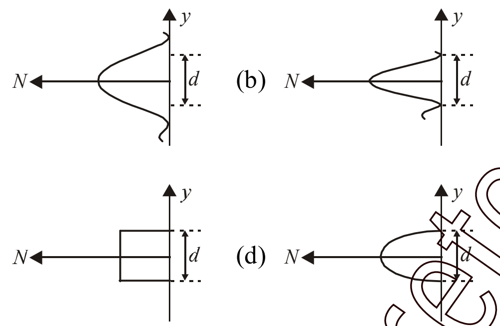
7. Electrons accelerated by potential V are diffracted from a crystal. If $d = 1$ Å and $i = 30°$, V should be about ($h = 6.6 \times 10^{-34}$ Js, $m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C)
(a) 1000 V (b) 2000 V (c) 50 V (d) 500 V.

8. If a strong diffraction peak is observed when electrons are incident at an angle i from the normal to the crystal planes with distance d between them (see figure), de Broglie wavelength λ_{dB} of electrons can be calculated by the relationship (n is an integer)
(a) $d \cos i = n\lambda_{dB}$ (b) $d \sin i = n\lambda_{dB}$
(c) $2d \cos i = n\lambda_{dB}$ (d) $2d \sin i = n\lambda_{dB}$

9. In an experiment, electrons are made to pass through a narrow slit of width d comparable to their de Broglie wavelength. They are detected on a screen at a distance D from the slit.



Which of the following graphs can be expected to represent the number of electrons N detected as a function of the detector position y ($y = 0$ corresponds to the middle of the slit)?



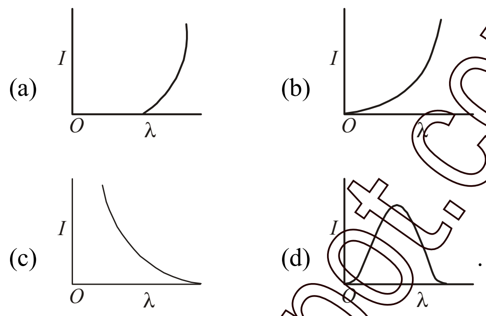
10. If g_E and g_M are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio

$$\frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}} \text{ to be}$$

(a) g_M/g_E (b) 1
(c) 0 (d) g_E/g_M *(2007)*

11. Photon of frequency υ has a momentum associated with it. If c is the velocity of light, the momentum is
(a) $h\upsilon/c$ (b) υ/c
(c) $h\upsilon c$ (d) $h\upsilon/c^2$ *(2007)*

12. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows



(2006)

13. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV, and the stopping potential for a radiation incident on this surface 5 V. The incident radiation lies in
(a) X-ray region (b) ultra-violet region
(c) infra-red region (d) visible region. *(2006)*

14. The time by a photoelectron to come out after the photon strikes is approximately
(a) 10^{-1} s (b) 10^{-4} s
(c) 10^{-10} s (d) 10^{-16} s. *(2006)*

15. If the kinetic energy of a free electron doubles, its de Broglie wavelength changes by the factor
(a) $1/\sqrt{2}$ (b) $\sqrt{2}$ (c) 1/2 (d) 2. *(2005)*

16. A photocell is illuminated by a small bright source placed 1 m away. When the same source of light is placed (1/2) m away, the number of electrons emitted by photocathode would
(a) decrease by a factor of 2
(b) increase by a factor of 2
(c) decrease by a factor of 4
(d) increase by a factor of 4 *(2005)*

17. A charged oil drop is suspended in a uniform field of 3×10^4 V/m so that it neither falls nor rises. The charge on the drop will be (take the mass of the charge $= 9.9 \times 10^{-15}$ kg and $g = 10$ m/s^2)
(a) 3.3×10^{-18} C (b) 3.2×10^{-18} C
(c) 1.6×10^{-18} C (d) 4.8×10^{-18} C. *(2004)*

18. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately
(a) 540 nm (b) 400 nm
(c) 310 nm (d) 220 nm. *(2004)*

19. According to Einstein's photoelectric equation, the plot of the kinetic energy of the emitted photo electrons from a metal *vs* the frequency, of the incident radiation gives a straight line whose slope
(a) depends on the nature of the metal used
(b) depends on the intensity of the radiation
(c) depends both on the intensity of the radiation and the metal used

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(d) is the same for all metals and independent of the intensity of the radiation. *(2004)*

20. Two identical photocathodes receive light of frequencies f_1 and f_2. If the velocities of the photoelectrons (of mass m) coming out are respectively v_1 and v_2, then

(a) $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$

(b) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{\frac{1}{2}}$

(c) $v_1^2 + v_2^2 = \frac{2h}{m}(f_1 + f_2)$

(d) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{\frac{1}{2}}$. *(2003)*

21. Sodium and copper have work functions 2.3 eV and 4.5 eV respectively. Then the ratio of the wavelengths is nearest to

(a) 1 : 2 (b) 4 : 1 (c) 2 : 1 (d) 1 : 4. *(2002)*

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Answer Key

1.	(a)	**2.**	(b)	**3.**	(d)	**4.**	(b)	**5.**	(a)	**6.**	(b)
7.	(c)	**8.**	(c)	**9.**	(a)	**10.**	(b)	**11.**	(a)	**12.**	(c)
13.	(b)	**14.**	(c)	**15.**	(a)	**16.**	(d)	**17.**	(a)	**18.**	(c)
19.	(d)	**20.**	(a)	**21.**	(c)						

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Explanations

1. (a)

2. (b) : Davisson-Germer experiment showed that electron beams can undergo diffraction when passed through atomic crystals. This shows the wave nature of electrons as waves can exhibit interference and diffraction.

3. (d) : The maximum kinetic energy of the electron
$K_{max} = h\upsilon - h\upsilon_0$
Here, υ_0 is threshold frequency.
The stopping potential is
$eV_0 = K_{max} = h\upsilon - h\upsilon_0$
Therefore, if υ is doubled K_{max} and V_0 is not doubled.

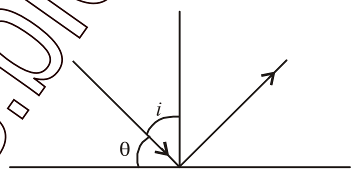
4. (b) : Here,
Power of a source, $P = 4$ kW $= 4 \times 10^3$ W
Number of photons emitted per second, $N = 10^{20}$
Energy of photon, $E = h\upsilon = \frac{hc}{\lambda}$
$\because \quad E = \frac{P}{N}$
$\frac{hc}{\lambda} = \frac{P}{N}$
or $\lambda = \frac{Nhc}{P} = \frac{10^{20} \times 6.63 \times 10^{-34} \times 3 \times 10^8}{4 \times 10^3}$
$= 4.972 \times 10^{-9}$ m $= 49.72$ Å
It lies in the X-ray region.

5. (a) : According to Einstein's photoelectric equation
$K_{max} = h\upsilon - \phi_0$
where,
υ = frequency of incident light
ϕ_0 = work function of the metal
Since $K_{max} = eV_0$
$V_0 = \frac{h\upsilon}{e} - \frac{\phi_0}{e}$
As $\upsilon_{\text{X-rays}} > \upsilon_{\text{Ultraviolet}}$
Therefore, both K_{max} and V_0 increase when ultraviolet light is replaced by X-rays.
Statement-2 is false.

6. (b) : The wavelength of light illuminating the photoelectric surface = 400 nm.
i.e., $h\upsilon = \frac{1240 \text{ eV nm}}{400 \text{ nm}} = 3.1 \text{ eV}$.
Max. kinetic energy of the electrons = 1.68 eV.
$h\upsilon = W_\phi$ + kinetic energy
$\therefore \quad W_\phi$, the work function $= h\upsilon$ – kinetic energy
$= 3.1 - 1.68$ eV $= 1.42$ eV.

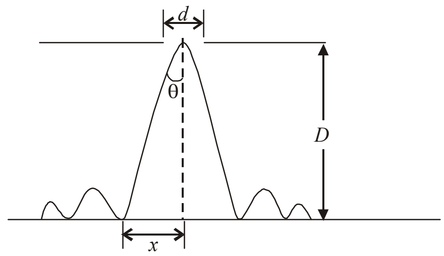
7. (c) : For electron diffraction, $d = 1$ Å, $i = 30°$
i.e., grazing angle $\theta = 60°$, $h = 6.6 \times 10^{-34}$ J s.
$m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C.
Bragg's equation for X-rays, which is also used in electron diffraction gives $n\lambda = 2d \sin\theta$.
$\therefore \quad \lambda = \frac{2 \times 1(\text{Å}) \times \sin 60°}{1}$ (assuming first order)
$\lambda = \sqrt{3}$ Å, $\quad \sqrt{V} = \frac{(12.27 \times 10^{-10})}{\sqrt{3} \times 10^{-10}}$
$V = 50.18$ Volt.

8. (c) : Bragg's relation $n\lambda = 2d \sin\theta$ for having an intensity maximum for diffraction pattern.



But as the angle of incidence is given,
$n\lambda = 2d \cos i$ is the formula for finding a peak.

9. (a) : The electron diffraction pattern from a single slit will be as shown below.



$d \sin\theta = \frac{\lambda}{2\pi}$.
The line of maximum intensity for the zeroth order will exceed d very much.

10. (b) : Since electronic charge (1.6×10^{-19} C) universal constant. It does not depend on g.
$\therefore$ Electronic charge on the moon = electronic charge on the earth
or $\frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}} = 1$.

11. (a) : Energy of a photon $E = h\upsilon$ ... (i)
Also $E = pc$ ... (ii)
where p is the momentum of a photon
From (i) and (ii), we get
$h\upsilon = pc$ or $p = \frac{h\upsilon}{c}$.

12. (c) : The graph (c) depicts the variation of λ with I.

13. (b) : For photo-electron emission,
(Incident energy E) = $(\text{K.E.})_{max}$ + (Work function ϕ)

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or $E = K_m + \phi$

or $E = 5 + 6.2 = 11.2$ eV

$= 11.2 \times (1.6 \times 10^{-19})$ J

$\therefore \quad \frac{hc}{\lambda} = 11.2 \times 1.6 \times 10^{-19}$

or $\lambda = \frac{(6.63 \times 10^{-34}) \times (3 \times 10^8)}{11.2 \times 1.6 \times 10^{-19}}$ m

or $\lambda = 1110 \times 10^{-10}$ m $= 1110$ Å.

The incident radiation lies in ultra violet region.

14. **(c) :** Emission of photo-electron starts from the surface after incidence of photons in about 10^{-10} sec.

15. **(a) :** de Broglie wavelength $\lambda = h/p = h/\sqrt{(2mK)}$

$\therefore \quad \lambda = \frac{h}{\sqrt{2mK}}$ where K = kinetic energy of particle

$\therefore \quad \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{K_1}{K_2}} = \sqrt{\frac{K_1}{2K_1}} = \frac{1}{\sqrt{2}}.$

16. **(d) :** $I = \frac{P \text{ of source}}{4\pi(\text{distance})^2} = \frac{P}{4\pi d^2}$

Here, we assume light to spread uniformly in all directions. Number of photo-electrons emitted from a surface depend on intensity of light I falling on it. Thus the number of electrons emitted n depends directly on I. P remains constant as the source is the same.

$\therefore \quad \frac{I_2}{I_1} = \frac{n_2}{n_1} \Rightarrow \frac{P_2}{P_1}\left(\frac{d_1}{d_2}\right)^2 = \frac{n_2}{n_1}$

$\therefore \quad \frac{n_2}{n_1} = \left(\frac{P}{P}\right)\left(\frac{1}{1/2}\right)^2 = \frac{4}{1}.$

17. **(a) :** For equilibrium of charged oil drop,

$qE = mg$

$\therefore \quad q = \frac{mg}{E} = \frac{(9.9 \times 10^{-15}) \times 10}{(3 \times 10^4)} = 3.3 \times 10^{-18}$ C

18. **(c) :** Let λ_m = Longest wavelength of light

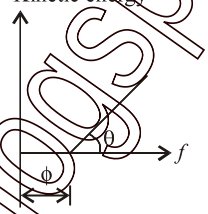
$\therefore \quad \frac{hc}{\lambda_m} = \phi(\text{work function})$

$\therefore \quad \lambda_m = \frac{hc}{\phi} = \frac{(6.63 \times 10^{-34}) \times (3 \times 10^8)}{4.0 \times 1.6 \times 10^{-19}}$

or $\lambda_m = 310$ nm.

19. **(d) :** According to Einstein's equation,

Kinetic energy $= hf - \phi$ where kinetic energy and f (frequency) are variables, compare it with equation, $y = mx + c$



$\therefore \quad$ slope of line $= h$

h is Planck's constant.

Hence the slope is same for all metals and independent of the intensity of radiation.

Option (d) represents the answer.

20. **(a) :** For photoelectric effect, according to Einstein's equation,

Kinetic energy of emitted electron $= hf -$ (work function ϕ)

$\therefore \quad \frac{1}{2}mv_1^2 = hf_1 - \phi$

$\frac{1}{2}mv_2^2 = hf_2 - \phi$

$\therefore \quad \frac{1}{2}m(v_1^2 - v_2^2) = h(f_1 - f_2)$

$\therefore \quad v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2).$

21. **(c) :** Work function $= hc/\lambda$

$\frac{W_{Na}}{W_{Cu}} = \frac{4.5}{2.3} = \frac{2}{1}.$

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CHAPTER 18

ATOMS AND NUCLEI

1. In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number $(n-1)$. If $n >> 1$, the frequency of radiation emitted is proportional to

(a) $\frac{1}{n^3}$ (b) $\frac{1}{n}$ (c) $\frac{1}{n^2}$ (d) $\frac{1}{n^{3/2}}$ *(2013)*

2. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be

(a) 3 (b) 5 (c) 6 (d) 2 *(2012)*

3. Assume that a neutron breaks into a proton and an electron. The energy released during this process is
(Mass of neutron = 1.6725×10^{-27} kg
Mass of proton = 1.6725×10^{-27} kg
Mass of electron = 9×10^{-31} kg)

(a) 7.10 MeV (b) 6.30 MeV
(c) 5.4 MeV (d) 0.73 MeV *(2012)*

4. A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by (n is an integer)

(a) $\frac{n^2h^2}{2(m_1+m_2)r^2}$ (b) $\frac{2n^2h^2}{(m_1+m_2)r^2}$

(c) $\frac{(m_1+m_2)n^2h^2}{2m_1m_2r^2}$ (d) $\frac{(m_1+m_2)^2n^2h^2}{2m_1^2m_2^2r^2}$ *(2012)*

5. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is

(a) 12.1 eV (b) 36.3 eV
(c) 108.8 eV (d) 122.4 eV *(2011)*

6. The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 when $\frac{1}{3}$ of it had decayed is

(a) 7 min (b) 14 min
(c) 20 min (d) 28 min *(2011)*

7. A radioactive nucleus (initial mass number A and atomic number Z) emits 3α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

(a) $\frac{A-Z-4}{Z-2}$ (b) $\frac{A-Z-8}{Z-4}$

(c) $\frac{A-Z-4}{Z-8}$ (d) $\frac{A-Z-12}{Z-4}$ *(2010)*

Directions : Questions number 8-9 are based on the following paragraph.

A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c.

8. The speed of daughter nuclei is

(a) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$ (b) $c\frac{\Delta m}{M+\Delta m}$

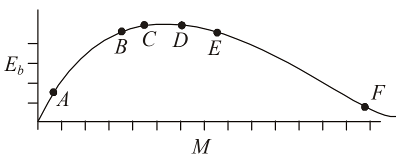
(c) $c\sqrt{\frac{2\Delta m}{M}}$ (d) $c\sqrt{\frac{\Delta m}{M}}$ *(2010)*

9. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then

(a) $E_1 = 2E_2$ (b) $E_2 = 2E_1$
(c) $E_1 > E_2$ (d) $E_2 > E_1$ *(2010)*

10. The transition from the state $n = 4$ to $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from

(a) $2 \to 1$ (b) $3 \to 2$
(c) $4 \to 2$ (d) $5 \to 4$ *(2009)*

11. 

The above is a plot of binding energy per nucleon E_b, against the nuclear mass M; A, B, C, D, E, F correspond to different nuclei. Consider four reactions:

(i) $A + B \to C + \varepsilon$ (ii) $C \to A + B + \varepsilon$

(iii) $D + E \to F + \varepsilon$ (iv) $F \to D + E + \varepsilon$

where ε is the energy released? In which reactions is ε positive?

(a) (i) and (iv) (b) (i) and (iii)
(c) (ii) and (iv) (d) (ii) and (iii) *(2009)*

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Directions : Question 12 contains statement-1 and statement-2. Of the four choices given, choose the one that best describes the two statements.

(a) Statement-1 is true, statement-2 is false.
(b) Statement-1 is false, statement-2 is true.
(c) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.
(d) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.

12. **Statement-1 :** Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion.
Statement-2 : For heavy nuclei, binding energy per nucleon increases with increasing Z while for light nuclei it decreases with increasing Z. *(2008)*

13. Suppose an electron is attracted towards the origin by a force k/r where k is a constant and r is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the n^{th} orbital of the electron is found to be r_n and the kinetic energy of the electron to be T_n. Then which of the following is true?
(a) $T_n \propto \frac{1}{n}, \; r_n \propto n^2$ (b) $T_n \propto \frac{1}{n^2}, \; r_n \propto n^2$
(c) T_n independent of n, $r_n \propto n$
(d) $T_n \propto \frac{1}{n}, \; r_n \propto n$ *(2008)*

14. Which of the following transitions in hydrogen atoms emit photons of highest frequency ?
(a) $n = 1$ to $n = 2$ (b) $n = 2$ to $n = 6$
(c) $n = 6$ to $n = 2$ (d) $n = 2$ to $n = 1$ *(2007)*

15. The half-life period of a radio-active element X is same as the mean life time of another radio-active element Y. Initially they have the same number of atoms. Then
(a) X and Y decay at same rate always
(b) X will decay faster than Y
(c) Y will decay faster than X
(d) X and Y have same decay rate initially *(2007)*

16. In gamma ray emission from a nucleus
(a) only the proton number changes
(b) both the neutron number and the proton number change
(c) there is no change in the proton number and the neutron number
(d) only the neutron number changes *(2007)*

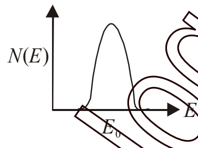
17. If M_O is the mass of an oxygen isotope $_8O^{17}$, M_P and M_N are the masses of a proton and a neutron respectively, the nuclear binding energy of the isotope is
(a) $(M_O - 17\,M_N)\,c^2$ (b) $(M_O - 8\,M_P)\,c^2$
(c) $(M_O - 8\,M_P - 9\,M_N)\,c^2$
(d) $M_O\,c^2$ *(2007)*

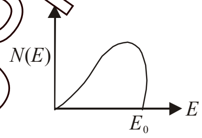
18. If the binding energy per nucleon in ^7_3Li and ^4_2He nuclei are 5.60 MeV and 7.06 MeV respectively, then in the reaction : $p + ^7_3\text{Li} \rightarrow 2\,^4_2\text{He}$, energy of proton must be
(a) 39.2 MeV (b) 28.24 MeV
(c) 17.28 MeV (d) 1.46 MeV. *(2006)*

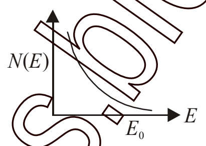
19. The 'rad' is the correct unit used to report the measurement of
(a) the rate of decay of radioactive source
(b) the ability of a beam of gamma ray photons to produce ions in a target
(c) the energy delivered by radiation to target
(d) the biological effect of radiation. *(2006)*

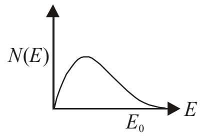
20. When $_3\text{Li}^7$ nuclei are bombarded by protons, and the resultant nuclei are $_4\text{Be}^8$, the emitted particles will be
(a) neutrons (b) alpha particles
(c) beta particles (d) gamma photons. *(2006)*

21. The energy spectrum of β-particles [number $N(E)$ as a function of β-energy E] emitted from a radioactive source is

(a) 

(b) 

(c) 

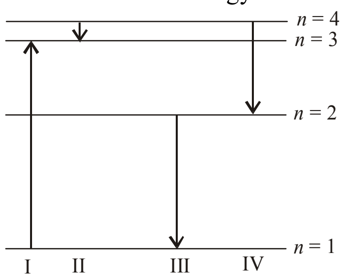
(d) 

. *(2006)*

22. An alpha nucleus of energy $\frac{1}{2}mv^2$ bombards a heavy nuclear target of charge Ze. Then the distance of closest approach for the alpha nucleus with be proportional to
(a) $1/Ze$ (b) v^2 (c) $1/m$ (d) $1/v^4$. *(2006)*

23. A nuclear transformation is denoted by $X(n, \alpha)\,^7_3\text{Li}$. Which of the following is the nucleus of element X?
(a) ^9_5B (b) $^{11}_4\text{Be}$ (c) $^{12}_6\text{C}$ (d) $^{10}_5\text{B}$ *(2005)*

24. The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy?



(a) I (b) II (c) III (d) IV *(2005)*

25. Starting with a sample of pure ^{66}Cu, 7/8 of it decays into Zn in 15 minutes. The corresponding half-life is
(a) 5 minutes (b) $7\frac{1}{2}$ minutes
(c) 10 minutes (d) 14 minutes *(2005)*

26. The intensity of gamma radiation from a given source is I. On passing through 36 mm of lead, it is reduced to $I/8$. The thickness of lead which will reduce the intensity to $I/2$ will be
(a) 18 mm (b) 12 mm
(c) 6 mm (d) 9 mm *(2005)*

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27. If radius of the $^{27}_{13}Al$ nucleus is estimated to be 3.6 fermi then the radius of $^{125}_{52}Al$ nucleus be nearly
(a) 4 fermi (b) 5 fermi
(c) 6 fermi (d) 8 fermi *(2005)*

28. An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of the closest approach is of the order of
(a) 1 Å (b) 10^{-10} cm
(c) 10^{-12} cm (d) 10^{-15} cm. *(2004)*

29. The binding energy per nucleon of deuteron $\left(^2_1H\right)$ and helium nucleus $\left(^4_2He\right)$ is 1.1 MeV and 7 MeV respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is
(a) 13.9 MeV (b) 26.9 MeV
(c) 23.6 MeV (d) 19.2 MeV. *(2004)*

30. A nucleus disintegrates into two nuclear parts which have their velocities in the ratio 2 : 1. The ratio of their nuclear sizes will be
(a) $2^{1/3}$: 1 (b) 1 : $3^{1/2}$ (c) $3^{1/2}$: 1 (d) 1 : $2^{1/3}$. *(2004)*

31. If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{++} is
(a) 30.6 eV (b) 13.6 eV
(c) 3.4 eV (d) 122.4 eV. *(2003)*

32. The wavelengths involved in the spectrum of deuterium (^2_1D) are slightly different from that of hydrogen spectrum, because
(a) size of the two nuclei are different
(b) nuclear forces are different in the two cases
(c) masses of the two nuclei are different
(d) attraction between the electron and the nucleus is different in the two cases. *(2003)*

33. Which of the following atoms has the lowest ionization potential?
(a) $^{14}_{7}N$ (b) $^{133}_{55}Cs$ (c) $^{40}_{18}Ar$ (d) $^{16}_{8}O$. *(2003)*

34. In the nuclear fusion reaction, $^2_1H + ^3_1H \rightarrow ^4_2He + n$ given that the repulsive potential energy between the two nuclei is ~ 7.7 × 10^{-14} J, the temperature at which the gases must be heated to initiate the reaction is nearly
[Boltzmann's constant k = 1.38 × 10^{-23} J/K]
(a) 10^7 K (b) 10^5 K (c) 10^3 K (d) 10^9 K. *(2003)*

35. Which of the following cannot be emitted by radioactive substances during their decay?
(a) protons (b) neutrinos
(c) helium nuclei (d) electrons. *(2003)*

36. A nucleus with Z = 92 emits the following in a sequence: α, α, $β^-$, $β^-$, α, α, α, α, $β^-$, $β^-$, α, $β^+$, $β^+$, α. The Z of the resulting nucleus is
(a) 76 (b) 78 (c) 82 (d) 74. *(2003)*

37. A radioactive sample at any instant has its disintegration rate 5000 disintegrations per minute. After 5 minutes, the rate is 1250 disintegrations per minute. Then, the decay constant (per minute) is
(a) 0.4 ln 2 (b) 0.2 ln 2
(c) 0.1 ln 2 (d) 0.8 ln 2. *(2003)*

38. When U^{238} nucleus originally at rest, decays by emitting an alpha particle having a speed u, the recoil speed of the residual nucleus is
(a) $\frac{4u}{238}$ (b) $-\frac{4u}{234}$ (c) $\frac{4u}{234}$ (d) $-\frac{4u}{238}$. *(2003)*

39. Which of the following radiations has the least wavelength?
(a) γ-rays (b) β-rays
(c) α-rays (d) X-rays. *(2003)*

40. If N_0 is the original mass of the substance of half-life period $t_{1/2}$ = 5 years, then the amount of substance left after 15 years is
(a) $N_0/8$ (b) $N_0/16$ (c) $N_0/2$ (d) $N_0/4$. *(2002)*

41. If 13.6 eV energy is required to ionize the hydrogen atom, then the energy required to remove an electron from n = 2 is
(a) 10.2 eV (b) 0 eV
(c) 3.4 eV (d) 6.8 eV. *(2002)*

42. At a specific instant emission of radioactive compound is deflected in a magnetic field. The compound can emit
(i) electrons (ii) protons
(iii) He^{2+} (iv) neutrons
The emission at the instant can be
(a) i, ii, iii (b) i, ii, iii, iv
(c) iv (d) ii, iii. *(2002)*

Answer Key

1.	(a)	**2.**	(c)	**3.**	(*)	**4.**	(c)	**5.**	(c)	**6.**	(c)
7.	(c)	**8.**	(c)	**9.**	(d)	**10.**	(d)	**11.**	(a)	**12.**	(a)
13.	(c)	**14.**	(d)	**15.**	(b)	**16.**	(c)	**17.**	(c)	**18.**	(c)
19.	(d)	**20.**	(d)	**21.**	(d)	**22.**	(c)	**23.**	(d)	**24.**	(c)
25.	(a)	**26.**	(b)	**27.**	(c)	**28.**	(c)	**29.**	(c)	**30.**	(d)
31.	(a)	**32.**	(c)	**33.**	(b)	**34.**	(d)	**35.**	(a)	**36.**	(b)
37.	(a)	**38.**	(b)	**39.**	(a)	**40.**	(a)	**41.**	(c)	**42.**	(a)

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Explanations

1. **(a)** : In a hydrogen like atom, when an electron makes an transition from an energy level with n to $n-1$, the frequency of emitted radiation is

$$\upsilon = RcZ^2\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= RcZ^2\left[\frac{n^2-(n-1)^2}{(n^2)(n-1)^2}\right] = \frac{RcZ^2(2n-1)}{n^2(n-1)}$$

As $n >> 1$

$$\therefore\ \upsilon = \frac{RcZ^2 2n}{n^4} = \frac{2RcZ^2}{n^3}$$

or $\upsilon \propto \frac{1}{n^3}$

2. **(c)** : Number of spectral lines in the emission spectra,

$$N = \frac{n(n-1)}{2}$$

Here, $n = 4$

$$\therefore\ N = \frac{4(4-1)}{2} = 6$$

3. **(*)** : Mass defect, $\Delta m = m_p + m_e - m_n$
$= (1.6725 \times 10^{-27} + 9 \times 10^{-31} - 1.6725 \times 10^{-27})$ kg
$= 9 \times 10^{-31}$ kg

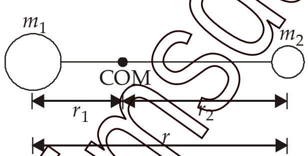
Energy released $= \Delta mc^2$
$= 9 \times 10^{-31} \times (3 \times 10^8)^2$ J

$$= \frac{9\times10^{-31}\times 9\times10^{16}}{1.6\times10^{-13}}\ \text{MeV}$$

$= 0.51$ MeV

* None of the given option is correct.

4. **(c)** : A diatomic molecule consists of two atoms of masses m_1 and m_2 at a distance r apart. Let r_1 and r_2 be the distances of the atoms from the centre of mass.



The moment of inertia of this molecule about an axis passing through its centre of mass and perpendicular to a line joining the atoms is

$$I = m_1r_1^2 + m_2r_2^2$$

As $m_1r_1 = m_2r_2$ or $r_1 = \frac{m_2}{m_1}r_2$

$\because\ r_1 + r_2 = r$

$$\therefore\ r_1 = \frac{m_2}{m_1}(r - r_1)$$

On rearranging, we get

$$r_1 = \frac{m_2 r}{m_1 + m_2}$$

Similarly, $r_2 = \frac{m_1 r}{m_1 + m_2}$

Therefore, the moment of inertia can be written as

$$I = m_1\left(\frac{m_2 r}{m_1+m_2}\right)^2 + m_2\left(\frac{m_1 r}{m_1+m_2}\right)^2$$

$$= \frac{m_1m_2}{m_1+m_2}r^2 \quad \text{...(i)}$$

According to Bohr's quantisation condition

$$L = \frac{nh}{2\pi}$$

or $L^2 = \frac{n^2h^2}{4\pi^2}$ ...(ii)

Rotational energy, $E = \frac{L^2}{2I}$

$$E = \frac{n^2h^2}{8\pi^2 I} \quad \text{(Using (ii))}$$

$$= \frac{n^2h^2(m_1+m_2)}{8\pi^2(m_1m_2)r^2} \quad \text{(Using (i))}$$

$$= \frac{n^2\hbar^2(m_1+m_2)}{2m_1m_2r^2} \quad \left(\because\ \hbar = \frac{h}{2\pi}\right)$$

In the question instead of h, $\hbar$ should be given.

5. **(c)** : Using, $E_n = -\frac{13.6Z^2}{n^2}$ eV

Here, $Z = 3$ (For Li^{++})

$$\therefore\ E_1 = -\frac{13.6(3)^2}{(1)^2}\ \text{eV}$$

$E_1 = -122.4$ eV

and $E_3 = \frac{-13.6\times(3)^2}{(3)^2} = -13.6$ eV

$\Delta E = E_3 - E_1 = -13.6 + 122.4 = 108.8$ eV

6. **(c)** : Number of undecayed atoms after time t_2,

$$\frac{N_0}{3} = N_0e^{-\lambda t_2} \quad \text{...(i)}$$

Number of undecayed atoms after time t_1,

$$\frac{2}{3}N_0 = N_0e^{-\lambda t_1} \quad \text{...(ii)}$$

Dividing (ii) by (i), we get

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$2 = e^{\lambda(t_2 - t_1)}$ or $\ln 2 = \lambda(t_2 - t_1)$

or $(t_2 - t_1) = \dfrac{\ln 2}{\lambda}$

As per question, $t_{1/2}$ = half life time = 20 min

$\therefore\ t_2 - t_1 = 20$ min $\left[\because\ t_{1/2} = \dfrac{\ln 2}{\lambda}\right]$

7. **(c) :** When a radioactive nucleus emits an alpha particle, its mass number decreases by 4 while the atomic number decreases by 2.
When a radioactive nucleus, emits a β^+ particle (or positron (e^+)) its mass number remains unchanged while the atomic number decreases by 1.

$\therefore\ {}^{A}_{Z}X \xrightarrow{3\alpha} {}^{A-12}_{Z-6}Y \xrightarrow{2e^+} {}^{A-12}_{Z-8}W$

In the final nucleus,
Number of protons, $N_p = Z - 8$
Number of neutrons, $N_n = A - 12 - (Z - 8)$
$= A - Z - 4$

$\therefore\ \dfrac{N_n}{N_p} = \dfrac{A - Z - 4}{Z - 8}.$

8. **(c) :** Mass defect, $\Delta M = \left[(M + \Delta m) - \left(\dfrac{M}{2} + \dfrac{M}{2}\right)\right]$

$= [M + \Delta m - M] = \Delta m$

Energy released, $Q = \Delta M c^2 = \Delta m c^2$ ...(i)
According to law of conservation of momentum, we get

$(M + \Delta m) \times 0 = \dfrac{M}{2} \times v_1 - \dfrac{M}{2} \times v_2$ or $v_1 = v_2$

Also, $Q = \dfrac{1}{2}\left(\dfrac{M}{2}\right)v_1^2 + \dfrac{1}{2}\left(\dfrac{M}{2}\right)v_2^2 - \dfrac{1}{2}(M + \Delta m) \times (0)^2$

$= \dfrac{M}{2}v_1^2\ (\because\ v_1 = v_2)$ ...(ii)

Equating equations (i) and (ii), we get

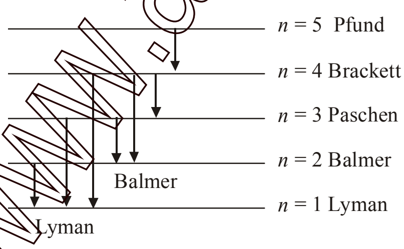
$$\left(\frac{M}{2}\right)v_1^2 = \Delta m c^2$$

$$v_1^2 = \frac{2\Delta m c^2}{M}$$

$$v_1 = c\sqrt{\frac{2\Delta m}{M}}$$

9. **(d) :** After decay, the daughter nuclei will be more stable, hence binding energy per nucleon of daughter nuclei is more than that of their parent nucleus.
Hence, $E_2 > E_1$.

10. **(d) :**



Transition 4 → 3 is in Paschen series. This is not in the ultraviolet region but this is in infrared region.
Transition 5 → 4 will also be in infrared region (Brackett).

11. **(a) :** When two nucleons combine to form a third one, and energy is released, one has fusion reaction. If a single nucleus splits into two, one has fission. The possibility of fusion is more for light elements and fission takes place for heavy elements.
Out of the choices given for fusion, only A and B are light elements and D and E are heavy elements. Therefore $A + B \to C + \varepsilon$ is correct. In the possibility of fission is only for F and not C. Therefore
$F \to D + E + \varepsilon$ is the correct choice.

12. **(a) :** Statement-1 states that energy is released when heavy nuclei undergo fission and light nuclei undergo fusion is correct. Statement-2 is wrong.
The binding energy per nucleon, B/A, starts at a small value, rises to a maximum at ^{62}Ni, then decreases to 7.5 MeV for the heavy nuclei. The answer is (a).

13. **(c) :** Supposing that the force of attraction in Bohr atom does not follow inverse square law but inversely proportional to r,

$\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{r}$ would have been $= \dfrac{mv^2}{r}$

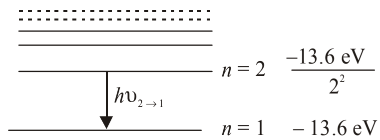
$\therefore\ mv^2 = \dfrac{e^2}{4\pi\varepsilon_0} = k \Rightarrow \dfrac{1}{2}mv^2 = \dfrac{1}{2}k.$

This is independent of n.

From $mvr_n = \dfrac{nh}{2\pi}$,

as mv is independent of r, $r_n \propto n$.

14. **(d) :**



$h\upsilon_{2\to1} = -13.6\left(\dfrac{1}{2^2} - \dfrac{1}{1^2}\right)$ eV

$= +13.6 \times \dfrac{3}{4}\text{eV} = 10.2$ eV .

Emission is $n = 2 \to n = 1$ *i.e.*, higher n to lower n.
Transition from lower to higher levels are absorption lines.

$-13.6\left(\dfrac{1}{6^2} - \dfrac{1}{2^2}\right) = +13.6 \times \dfrac{2}{9}$

This is $< E_{n=2} \to E_{n=1}$.

15. **(b) :** $T_{1/2}$, half life of $X = \tau_Y$, mean life of Y

$\dfrac{\ln 2}{\lambda_X} = \dfrac{1}{\lambda_Y} \Rightarrow \lambda_X = \lambda_Y \ln 2$

$\lambda_X > \lambda_Y$

$\therefore\ A_X = A_0 e^{-\lambda_X t}$; $A_Y = A_0 e^{-\lambda_Y t}$

X will decay faster than Y.

16. **(c) :** γ-ray emission takes place due to deexcitation of the nucleus. Therefore during γ-ray emission, there is no change in the proton and neutron number.

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17. (c) : Binding energy = $[ZM_P + (A - Z)M_N - M]c^2$
$= [8M_P + (17 - 8)M_N - M_O]c^2$
$= (8M_P + 9M_N - M_O)c^2$
[But the option given is negative of this].

18. (c) : Binding energy of
$^7_3\text{Li} = 7 \times 5.60 = 39.2$ MeV
Binding energy of $^4_2\text{He} = 4 \times 7.06 = 28.24$ MeV
$\therefore$ Energy of proton = Energy of $\left[2(^4_2\text{He}) - ^7_3\text{Li}\right]$
= 2 × 28.24 – 39.2
= 17.28 MeV.

19. (d) : The 'rad' the biological effect of radiation.

20. (d) : $_3\text{Li}^7 + {}_1\text{H}^1 \rightarrow {}_4\text{Be}^8 + {}_ZX^A$
Z for the unknown X nucleus = (3 + 1) – 4 = 0
A for the unknown X nucleus = (7 + 1) – 8 = 0
Hence particle emitted has zero Z and zero A
It is a gamma photon.

21. (d) : Graph (d) represents the variation.

22. (c) : For closest approach, kinetic energy is converted into potential energy

$$\therefore \quad \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_0} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(2e)}{r_0}$$

or $$r_0 = \frac{4Ze^2}{4\pi\varepsilon_0 mv^2} = \frac{Ze^2}{\pi\varepsilon_0 v^2}\left(\frac{1}{m}\right)$$

or r_0 is proportional to $\left(\frac{1}{m}\right)$.

23. (d) : The nuclear transformation is given by
$^A_ZX + {}^1_0n \rightarrow {}^4_2\text{He} + {}^7_3\text{Li}$
According to conservation of mass number
$A + 1 = 4 + 7$
or $A = 10$
According to conservation of charge number
$Z + 0 \rightarrow 2 + 3$
or $Z = 5$
So the nucleus of the element be $^{10}_5$B.

24. (c) : I is showing absorption photon.
From rest of three, III having maximum energy from

$$\Delta E \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

25. (a) : $$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$$

$$\therefore \quad \frac{1}{8} = \left(\frac{1}{2}\right)^{15/T} \Rightarrow \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{15/T}$$

$$\therefore \quad \frac{15}{T} = 3 \Rightarrow T = 5 \text{ min}.$$

26. (b) : $\because I = I_0e^{-kx} \Rightarrow \frac{I}{I_0} = e^{-kx}$

$\therefore \quad \ln\left(\frac{I}{I_0}\right) = -kx$

In first case

$\ln\left(\frac{1}{8}\right) = -k \times 36$

ln (2^{-3}) = – k × 36
or 3ln2 = k × 36 ....(i)

In second case, $\ln\left(\frac{1}{2}\right) = -k \times x$

or ln(2^{-1}) = – kx
or ln2 = kx .... (ii)
From (i) and (ii)
3 × (kx) = k × 36
or x = 12 mm.

27. (c) : R is proportional to $A^{1/3}$ where A is mass number
3.6 = R_0 $(27)^{1/3}$ = 3R_0, for $^{27}_{13}$Al.
Again $R = R_0(125)^{1/3}$, for $^{125}_{52}$Al

$$\therefore \quad R = \frac{(3 \cdot 6)}{3} \times 5 = 6 \text{ fermi}.$$

28. (c) : Kinetic energy is converted into potential energy at closest approach
$\therefore$ K.E. = P.E.

$$\therefore \quad 5\text{ MeV} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

or $$5 \times 10^6 \times e = \frac{(9 \times 10^9) \times (92e)(2e)}{r}$$

or $$r = \frac{9 \times 10^9 \times 92 \times 2 \times e}{5 \times 10^6}$$

$$= \frac{9 \times 10^9 \times 92 \times 2 \times (1.6 \times 10^{-19})}{5 \times 10^6}$$

$\therefore$ r = 5.3 × 10^{-14} m = 5.3 × 10^{-12} cm.

29. (c) : Total binding energy for (each deuteron)
= 2 × 1.1 = 2.2 MeV
Total binding energy for helium = 4 × 7 = 28 MeV
$\therefore$ Energy released = 28 –(2 × 2.2)
= 28 – 4.4 = 23.6 MeV.

30. (d) : Momentum is conserved during disintegration
$\therefore$ $m_1v_1 = m_2v_2$ ....(i)
For an atom, $R = R_0A^{1/3}$

$$\therefore \quad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

$$= \left(\frac{m_1}{m_2}\right)^{1/3} = \left(\frac{m_2v_2}{m_2v_1}\right)^{1/3}, \text{ from (i)}$$

$$\therefore \quad \frac{R_1}{R_2} = \left(\frac{1}{2}\right)^{1/3} = \frac{1}{2^{1/3}}.$$

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31. **(a) :** Energy $E_2 = \frac{-Z^2 E_0}{n^2} = \frac{-(3)^2 \times 13.6}{(2)^2} = -30.6$ eV

$\therefore$ energy required = 30.6 eV.

32. **(c) :** Masses of $_1H^1$ and $_1D^2$ are different. Hence the corresponding wavelengths are different.

33. **(b) :** $^{133}_{55}Cs$ has the lowest ionization potential. Of the four atoms given, Cs has the largest size. Electrons in the outer most orbit are at large distance from nucleus in a large-size atom. Hence the ionization potential is the least.

34. **(d) :** At temperature T, molecules of a gas acquire a kinetic energy $= \frac{3}{2} kT$ where k = Boltzmann's constant

$\therefore$ To initiate the fusion reaction

$\frac{3}{2} kT = 7.7 \times 10^{-14}$ J

$\therefore \quad T = \frac{7.7 \times 10^{-14} \times 2}{3 \times 1.38 \times 10^{-23}} = 3.7 \times 10^9$ K.

35. **(a) :** Protons are not emitted during radioactive decay.

36. **(b) :** The nucleus emits 8α particles *i.e.*, $8(_2He^4)$

$\therefore$ Decrease in $Z = 8 \times 2 = 16$ ....(i)

Four β^- particles are emitted *i.e.*, $4(_{-1}\beta^0)$

$\therefore$ Increase in $Z = 4 \times 1 = 4$ ....(ii)

2 positrons are emitted i.e., $2(_1\beta^0)$

$\therefore$ Decrease in $Z = 2 \times 1 = 2$ ......(iii)

$\therefore$ Z of resultant nucleus = 92 – 16 + 4 – 2 = 78.

37. **(a) :** Let decay constant per minute = λ

Disintegration rate, initially = 5000

$\therefore \quad N_0\lambda = 5000$ .... (i)

Disintegration rate, finally = 1250

$\therefore \quad N\lambda = 1250$ .... (ii)

$\therefore \quad \frac{N\lambda}{N_0\lambda} = \frac{1250}{5000} = \frac{1}{4}$

or $\quad \frac{N}{N_0} = \frac{1}{4} \Rightarrow \frac{N_0 e^{-5\lambda}}{N_0} = \frac{1}{4} \Rightarrow e^{-5\lambda} = (4)^{-1}$

$\therefore \quad 5\lambda = \ln 4 = 2\ln 2$

$\therefore \quad \lambda = \frac{2}{5}\ln 2 = 0.4 \ln 2$

38. **(b) :** Linear momentum is conserved

α-particle = ^4_2He

$U^{238} \rightarrow X^{234} + He^4$

$\therefore \quad (238 \times 0) = (238 \times v) + 4u$

or $\quad v = -\frac{4u}{234}$

39. **(a) :** Gamma rays have the least wavelength.

40. **(a) :** $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T} = \left(\frac{1}{2}\right)^{15/5} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$\therefore \quad N = N_0/8$.

41. **(c) :** $E_n = \frac{13.6}{n^2} \Rightarrow E_2 = \frac{13.6}{(2)^2} = 3.4$ eV

42. **(a) :** Neutrons are electrically neutral. They are not deflected by magnetic field.

Hence (a) represents the answer.

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CHAPTER 19

ELECTRONIC DEVICES

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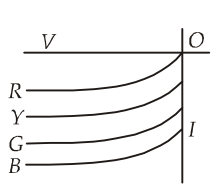
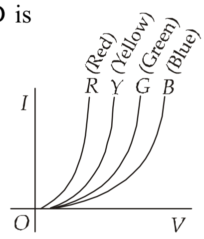
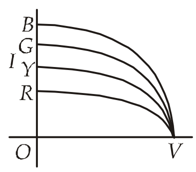
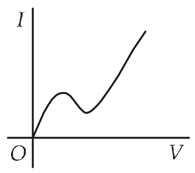
1. The I-V characteristic of an LED is

(a) 

(b) 

(c) 

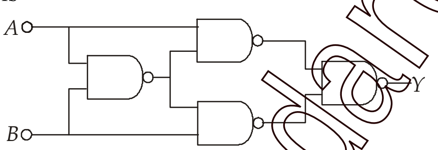
(d) 

(2013)

2. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pico farad in parallel with a load resistance 100 kilo ohm. Find the maximum modulated frequency which could be detected by it.

(a) 5.31 kHz (b) 10.62 MHz
(c) 10.62 kHz (d) 5.31 MHz *(2013)*

3. Truth table for system of four NAND gates as shown in figure is



(a)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(b)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(c)

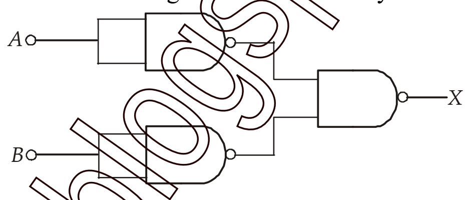
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(d)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

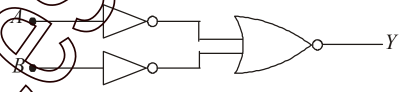
(2012)

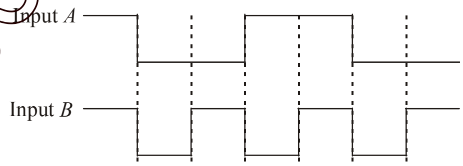
4. The combination of gates shown below yields



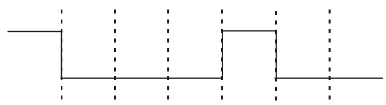
(a) NAND gate (b) OR gate
(c) NOT gate (d) XOR gate *(2010)*

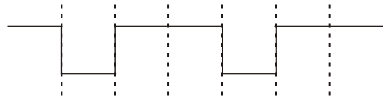
5. The logic circuit shown below has the input waveforms 'A' and 'B' as shown. Pick out the correct output waveform.

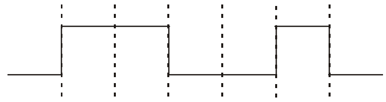


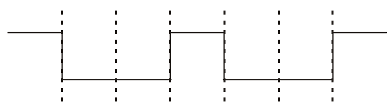


Output is

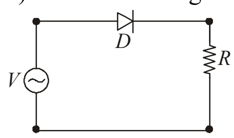
(a) 

(b) 

(c) 

(d)  *(2009)*

6. A p-n junction (D) shown in the figure can act as a rectifier.



An alternating current source (V) is connected in the circuit. The current(I) in the resistor(R) can be shown by

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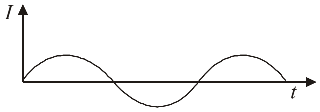
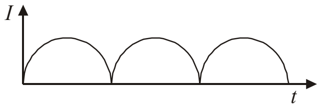
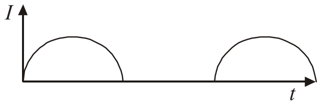
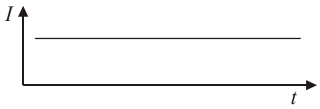
(a) 

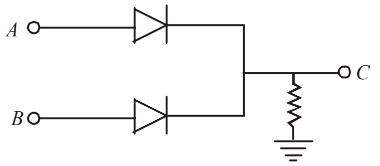
(b) 

(c) 

(d) 

(2009)

7. In the circuit below, A and B represent two inputs and C represents the output. The circuit represents

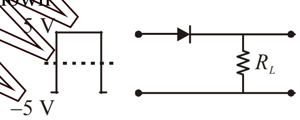


(a) OR gate (b) NOR gate
(c) AND gate (d) NAND gate. *(2008)*

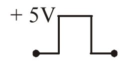
8. A working transistor with its three legs marked P, Q and R is tested using a multimeter. No conduction is found between P and Q. By connecting the common (negative) terminal of the multimeter to R and the other (positive) terminal to P or Q, some resistance is seen on the multimeter. Which of the following is true for the transistor?
(a) It is an *npn* transistor with R as collector.
(b) It is an *npn* transistor with R as base.
(c) It is a *pnp* transistor with R as collector.
(d) It is a *pnp* transistor with R as emitter. *(2008)*

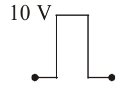
9. Carbon, silicon and germanium have four valence electrons each. At room temperature which one of the following statements is most appropriate ?
(a) The number of free electrons for conduction is significant only in Si and Ge but small in C.
(b) The number of free conduction electrons is significant in C but small in Si and Ge.
(c) The number of free conduction electrons is negligibly small in all the three.
(d) The number of free electrons for conduction is significant in all the three. *(2007)*

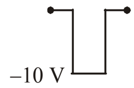
10. If in a p - n junction diode, a square input signal of 10 V is applied as shown



Then the output signal across R_L will be

(a) 

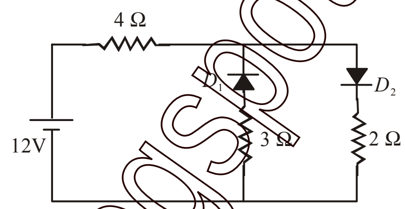
(b) 

(c) 

(d) 

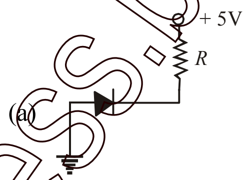
(2007)

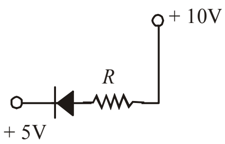
11. The circuit has two oppositely connect ideal diodes in parallel. What is the current following in the circuit?

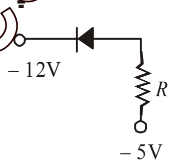


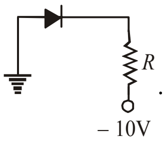
(a) 1.33 A (b) 1.71 A
(c) 2.00 A (d) 2.31 A. *(2006)*

12. In the following, which one of the diodes is reverse biased?

(a) 

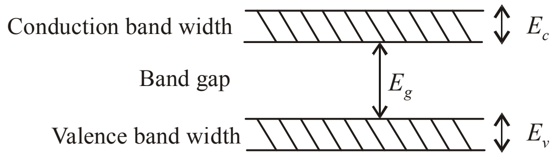
(b) 

(c) 

(d) 

(2006)

13. If the lattice constant of this semiconductor is decreased, then which of the following is correct?



(a) all E_c, E_g, E_v decrease
(b) all E_c, E_g, E_v increase
(c) E_c, and E_v increase, but E_g decreases
(d) E_c, and E_v decrease, but E_g increases. *(2006)*

14. In common base mode of a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of the base current amplification factor (β) will be
(a) 48 (b) 49 (c) 50 (d) 51. *(2006)*

15. In the ratio of the concentration of electrons that of holes in a semiconductor is 7/5 and the ratio of currents is 7/4 then what is the ratio of their drift velocities?
(a) 4/7 (b) 5/8 (c) 4/5 (d) 5/4. *(2006)*

16. A solid which is transparent to visible light and whose conductivity increases with temperature is formed by
(a) metallic binding (b) ionic binding
(c) covalent binding (d) van der Waals binding *(2006)*

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17. In a full wave rectifier circuit operating from 50 Hz mains frequency, the fundamental frequency in the ripple would be
(a) 100 Hz (b) 70.7 Hz
(c) 50 Hz (d) 25 Hz *(2005)*

18. In a common base amplifier, the phase difference between the input signal voltage and output voltage is
(a) 0 (b) $\pi/2$
(c) $\pi/4$ (d) π *(2005)*

19. The electrical conductivity of a semiconductor increases when electromagnetic radiation of wavelength shorter than 2480 nm is incident on it. The band gap in (eV) for the semiconductor is
(a) 0.5 eV (b) 0.7 eV
(c) 1.1 eV (d) 2.5 eV *(2005)*

20. When *p-n* junction diode is forward biased, then
(a) the depletion region is reduced and barrier height is increased
(b) the depletion region is widened and barrier height is reduced.
(c) both the depletion region and barrier height are reduced
(d) both the depletion region and barrier height are increased *(2004)*

21. The manifestation of band structure in solids is due to
(a) Heisenberg's uncertainty principle
(b) Pauli's exclusion principle
(c) Bohr's correspondence principle
(d) Boltzmann's law *(2004)*

22. A piece of copper and another of germanium are cooled from room temperature to 77 K, the resistance of
(a) each of them increases
(b) each of them decreases
(c) copper decreases and germanium increases
(d) copper increases and germanium decreases. *(2004)*

23. For a transistor amplifier in common emitter configuration for load impedance of 1 kΩ (h_{fe} = 50 and h_{oe} = 25) the current gain is
(a) –5.2 (b) –15.7
(c) – 24.8 (d) – 48.78. *(2004)*

24. When *npn* transistor is used as an amplifier
(a) electrons move from base to collector
(b) holes move from emitter to base
(c) electrons move from collector to base
(d) holes move from base to emitter. *(2004)*

25. In the middle of the depletion layer of a reverse-biased *p-n* junction, the
(a) electric field is zero
(b) potential is maximum
(c) electric field is maximum
(d) potential is zero. *(2003)*

26. The difference in the variation of resistance with temperature in a metal and a semiconductor arises essentially due to the difference in the
(a) crystal structure
(b) variation of the number of charge carriers with temperature
(c) type of bonding
(d) variation of scattering mechanism with temperature. *(2003)*

27. A strip of copper and another germanium are cooled from room temperature to 80 K. The resistance of
(a) each of these decreases
(b) copper strip increases and that of germanium decreases
(c) copper strip decreases and that of germanium increases
(d) each of these increases. *(2003)*

28. Formation of covalent bonds in compounds exhibits
(a) wave nature of electron
(b) particle nature of electron
(c) both wave and particle nature of electron
(d) none of these. *(2002)*

29. The part of a transistor which is most heavily doped to produce large number of majority carriers is
(a) emitter
(b) base
(c) collector
(d) can be any of the above three. *(2002)*

30. The energy band gap is maximum in
(a) metals (b) superconductors
(c) insulators (d) semiconductors. *(2002)*

31. By increasing the temperature, the specific resistance of a conductor and a semiconductor
(a) increases for both (b) decreases for both
(c) increases, decreases (d) decreases, increases. *(2002)*

32. At absolute zero, Si acts as
(a) non-metal (b) metal
(c) insulator (d) none of these. *(2002)*

Answer Key

1.	(b)	**2.**	(c)	**3.**	(d)	**4.**	(b)	**5.**	(a)	**6.**	(c)
7.	(a)	**8.**	(a)	**9.**	(a)	**10.**	(a)	**11.**	(c)	**12.**	(a)
13.	(d)	**14.**	(b)	**15.**	(d)	**16.**	(c)	**17.**	(a)	**18.**	(a)
19.	(a)	**20.**	(c)	**21.**	(b)	**22.**	(c)	**23.**	(d)	**24.**	(a)
25.	(a)	**26.**	(b)	**27.**	(c)	**28.**	(a)	**29.**	(a)	**30.**	(c)
31.	(c)	**32.**	(c)								

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Explanations

1. **(b)** : The *I-V* characteristics of a LED is similar to that of a Si junction diode. But the threshold voltages are much higher and slightly different for each colour.
Hence, the option (b) represents the correct graph.

2. **(c)** : The maximum frequency which can be detected is

$$\upsilon = \frac{1}{2\pi m_a \tau}$$

where, $\tau = CR$

Here, $C = 250$ pico farad $= 250 \times 10^{-12}$ farad

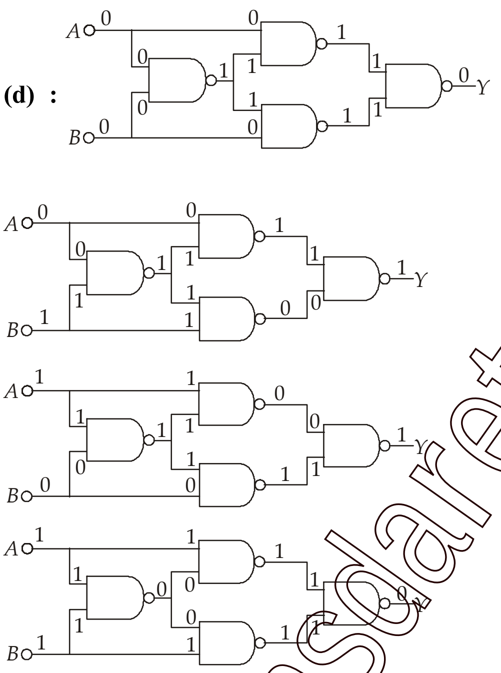
$R = 100$ kilo ohm $= 100 \times 10^3$ ohm

$m_a = 0.6$

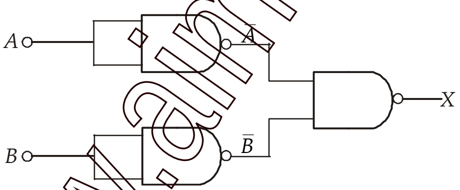
$$\therefore \quad \upsilon = \frac{1}{2\pi \times 0.6 \times 250 \times 10^{-12} \times 100 \times 10^3}$$

$= 10.61 \times 10^3$ Hz $= 10.61$ kHz.

3. **(d)** :



4. **(b)** :



The Boolean expression of the given circuit is

$X = \overline{\bar{A} \cdot \bar{B}}$

$= \bar{\bar{A}} + \bar{\bar{B}}$ (Using De Morgan theorem)

$= A + B$ (Using Boolean identity)

This is same as the Boolean expression of OR gate.

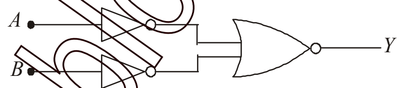
Alternative method

The truth table of the given circuit is as shown in the table

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}\bar{B}$	$X = \overline{\bar{A} \cdot \bar{B}}$
0	0	1	1	1	0
0	1	1	0	0	1
1	0	0	1	0	1
1	1	0	0	0	1

This is same as that of OR gate.

5. **(a)** :



By de Morgan's theorem, $\left(\overline{\bar{A} + \bar{B}}\right) = A \cdot B$.

A	B	$\bar{A}$	$\bar{B}$	$\bar{A} + \bar{B}$	$\overline{\bar{A} + \bar{B}}$	Verify $A \cdot B$
1	1	0	0	0	1	1
0	0	1	1	1	0	0
0	1	1	0	1	0	0
1	0	0	1	1	0	0

This is the same as AND Gate of A and B.

6. **(c)** : (a) is original wave (b) is a full-wave rectified (c) is the correct choice. The negative waves are cut off when the diode is connected in reverse bias (d) is not the diagram for alternating current.

7. **(a)** : It is OR gate. When either of them conducts, the gate conducts.

8. **(a)** : It is *npn* transistor with R as collector. If it is connected to base, it will be in forward bias.

9. **(a)** : C, Si and Ge have the same lattice structure and their valence electrons are 4. For C, these electrons are in the second orbit, for Si it is third and germanium it is the fourth orbit. In solid state, higher the orbit, greater the possibility of overlapping of energy bands. Ionization energies are also less therefore Ge has more conductivity compared to Si. Both are semiconductors.Carbon is an insulator.

10. **(a)** : The current will flow through R_L when diode is forward biased.

11. **(c)** : Since diode D_1 is reverse biased, therefore it will act like an open circuit.
Effective resistance of the circuit is $R = 4 + 2 = 6\ \Omega$.
Current in the circuit is $I = E/R = 12/6 = 2$ A.

12. **(a)** : Figure (a) represent a reverse biased diode.

13. **(d)** : E_c and E_v decrease but E_g increases if the lattice constant of the semiconductor is decreased.

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14. (b) : $\beta = \frac{I_c}{I_b} = \frac{I_c}{I_e - I_c} = \frac{5.488}{5.60 - 5.488} = \frac{5.488}{0.112} = 49.$

15. (d) : Drift velocity $v_d = \frac{I}{nAe}$

$$\frac{(v_d)_{\text{electron}}}{(v_d)_{\text{hole}}} = \left(\frac{I_e}{I_h}\right)\left(\frac{n_h}{n_e}\right) = \frac{7}{4} \times \frac{5}{7} = \frac{5}{4}.$$

16. (c) : Covalent binding.

17. (a) : Frequency of full wave rectifier = 2 × input frequency = 2 × 50 = 100 Hz.

18. (a) : In a common base amplifier, the phase difference between the input signal and output voltage is zero.

19. (a) : Band gap = Energy of photon of λ = 2480 nm

$\therefore$ Energy $= \frac{hc}{\lambda}$ J $= \frac{hc}{\lambda e}$(eV)

$\therefore$ Band gap $= \frac{(6.63 \times 10^{-34}) \times (3 \times 10^8)}{(2480 \times 10^{-9}) \times (1.6 \times 10^{-19})}$ eV

= 0.5 eV.

20. (c) : When *p-n* junction diode is forward biased, both the depletion region and barrier height are reduced.

21. (b) : Pauli's exclusion principle explains band structure of solids.

22. (c) : Copper is a conductor.
Germanium is a semiconductor.
When cooled, the resistance of copper decreases and that of germanium increases.

23. (d) : In common emitter configuration, current gain is

$$A_i = \frac{-(h_{fe})}{1 + (h_{oe})(R_L)} = \frac{-50}{1 + (25 \times 10^{-6}) \times (1 \times 10^3)}$$

$$= -\frac{50}{1 + 0.025} = \frac{-50}{1.025} = -48.78.$$

24. (a) : Electrons of *n*-type emitter move from emitter to base and then base to collector when *npn* transistor is used as an amplifier.

25. (a) : Electric field is zero in the middle of the depletion layer of a reverse baised *p-n* junction.

26. (b) : Variation of number of charge carriers with temperature is responsible for variation of resistance in a metal and a semiconductor.

27. (c) : Copper is conductor and germanium is semiconductor. When cooled, the resistance of copper strip decreases and that of germanium increases.

28. (a) : Wave nature of electron and covalent bonds are correlated.

29. (a) : The emitter is most heavily doped.

30. (c) : The energy band gap is maximum in insulators.

31. (c) : For conductor, ρ increases as temperature rises. For semiconductor, ρ decreases as temperature rises.

32. (c) : Semiconductors, like Si, Ge, act as insulators at low temperature.

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CHAPTER 20

EXPERIMENTAL SKILLS

1. A spectrometer gives the following reading when used to measure the angle of a prism.
Main scale reading : 58.5 degree
Vernier scale reading : 09 divisions
Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data
(a) 58.77 degree (b) 58.65 degree
(c) 59 degree (d) 58.59 degree *(2012)*

2. A screw gauge gives the following reading when used to measure the diameter of a wire.
Main scale reading : 0 mm
Circular scale reading : 52 divisions
Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.
The diameter of wire from the above data is :
(a) 0.52 cm (b) 0.052 cm
(c) 0.026 cm (d) 0.005 cm *(2011)*

3. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (= 0.5°), then the least count of the instrument is
(a) one minute (b) half minute
(c) one degree (d) half degree *(2009)*

4. In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance u and the image distance v, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the x-axis meets the experimental curve at P. The coordinates of P will be
(a) $(2f, 2f)$ (b) $(f/2, f/2)$
(c) (f, f) (d) $(4f, 4f)$ *(2009)*

5. An experiment is performed to find the refractive index of glass using a travelling microscope. In this experiment distances are measured by
(a) a screw gauge provided on the microscope
(b) a vernier scale provided on the microscope
(c) a standard laboratory scale
(d) a meter scale provided on the microscope. *(2008)*

6. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, she measures the column length to be x cm for the second resonance. Then
(a) $36 > x > 18$ (b) $18 > x$
(c) $x > 54$ (d) $54 > x > 36$. *(2008)*

7. Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of – 0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is
(a) 3.38 mm (b) 3.32 mm
(c) 3.73 mm (d) 3.67 mm. *(2008)*

Answer Key

1.	(b)	**2.**	(b)	**3.**	(a)	**4.**	(a)	**5.**	(b)	**6.**	(c)
7.	(a)										

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Explanations

1. **(b)** : 30 VSD = 29 MSD

$1\text{ VSD} = \frac{29}{30}\text{ MSD}$

Least count = 1 MSD – 1 VSD

$= \left(1 - \frac{29}{30}\right)\text{MSD} = \frac{1}{30} \times 0.5°$

Reading = Main scale reading + Vernier scale reading × least count

$= 58.5° + 9 \times \frac{0.5°}{30} = 58.5° + 0.15° = 58.65°.$

2. **(b)** : Least count of screw gauge

$= \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$

$= \frac{1}{100}\text{ mm} = 0.01\text{ mm}$

Diameter of wire = Main scale reading + circular scale reading × Least count

= 0 + 52 × 0.01 = 0.52 mm = 0.052 cm

3. **(a)** : Least count = $\frac{\text{value of 1 main scale division}}{\text{The number of divisions on the vernier scale}}$

as shown below.

Here n vernier scale divisions = $(n - 1)$ M.S.D.

$\therefore\ 1\text{ V.S.D.} = \frac{n-1}{n}\text{ M.S.D}$

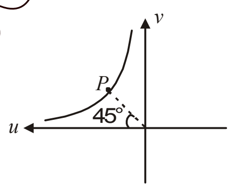
$\text{L.C.} = 1\text{ M.S.D.} - 1\text{ V.S.D}$

$= 1\text{ M.S.D} - \frac{(n-1)}{n}\text{ M.S.D.}$

$\Rightarrow\ \text{L.C.} = 0.5° - \frac{29}{30} \times 0.5°$

$\Rightarrow\ \text{L.C.} = \frac{0.5}{30} = \frac{1}{30} \times \frac{1}{2} = \frac{1}{60}° = 1\text{ min.}$

4. **(a)** : According to New Cartesian coordinate system used in our 12th classes, for a convex lens, as u is negative, the lens equation is



$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$

One has to take that u is negative again for calculation, it effectively comes to $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$

If u = radius of curvature, $2f$, $v = 2f$

i.e., $\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}.$

v and u are have the same value when the object is at the centre of curvature. The solution is (a).

According the real and virtual system, u is +ve and v is also +ve as both are real. If $u = v$, $u = 2f$ = radius of curvature.

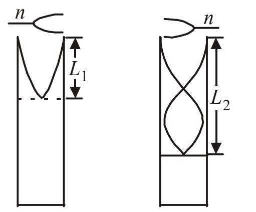
$\therefore\ \frac{1}{v} + \frac{1}{u} = \frac{1}{f}\ \Rightarrow\ \frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}.$

The answer is the same (a).

(The figure given is according to New Cartesian system).

5. **(b)** : A travelling microscope moves horizontally on a main scale provided with a vernier scale, provided with the microscope.

6. **(c)** : $v_1 = \sqrt{\frac{\gamma RT}{M}}$ assuming M is the average molar mass of the air (i.e., nitrogen) and γ is also for nitrogen.



1st resonance 2nd resonance

$v_1 = \sqrt{\frac{\gamma RT_1}{M}};\ v_2 = \sqrt{\frac{\gamma RT_2}{M}}$ where T_1 and T_2 stand for winter and summer temperatures.

$L_1 = \frac{v_1}{n} = \frac{\lambda}{4} = 18\text{ cm}.$ At temperature T_1

At T_2, summer, $v_2 > v_1$.

$L_2 = \frac{v_2}{n} = \frac{3\lambda}{4} > 3 \times 18.$

$\therefore\ L_2 > 54$ cm.

7. **(a)** : Least count of the screw gauge

$= \frac{0.5\text{ mm}}{50} = 0.01\text{ mm}$

Main scale reading = 3 mm.

Vernier scale reading = 35

∴ Observed reading = 3 + 0.35 = 3.35

zero error = –0.03

∴ actual diameter of the wire = 3.35 – (–0.03) = 3.38 mm.

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CHEMISTRY

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CHAPTER 1

SOME BASIC CONCEPTS IN CHEMISTRY

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1. The molarity of a solution obtained by mixing 750 mL of 0.5 M HCl with 250 mL of 2 M HCl will be
(a) 0.975 M (b) 0.875 M
(c) 1.00 M (d) 1.75 M *(2013)*

2. In the reaction,
$2Al_{(s)} + 6HCl_{(aq)} \rightarrow 2Al^{3+}{}_{(aq)} + 6Cl^{-}{}_{(aq)} + 3H_{2(g)}$
(a) 11.2 L $H_{2(g)}$ at STP is produced for every mole $HCl_{(aq)}$ consumed
(b) 6 L $HCl_{(aq)}$ is consumed for every 3 L $H_{2(g)}$ produced
(c) 33.6 L $H_{2(g)}$ is produced regardless of temperature and pressure for every mole Al that reacts
(d) 67.2 L $H_{2(g)}$ at STP is produced for every mole Al that reacts. *(2007)*

3. How many moles of magnesium phosphate, $Mg_3(PO_4)_2$ will contain 0.25 mole of oxygen atoms?
(a) 0.02 (b) 3.125×10^{-2}
(c) 1.25×10^{-2} (d) 2.5×10^{-2} *(2006)*

4. If we consider that 1/6, in place of 1/12, mass of carbon atom is taken to be the relative atomic mass unit, the mass of one mole of a substance will
(a) decrease twice
(b) increase two fold
(c) remain unchanged
(d) be a function of the molecular mass of the substance. *(2005)*

5. What volume of hydrogen gas, at 273 K and 1 atm. pressure will be consumed in obtaining 21.6 g of elemental boron (atomic mass = 10.8) from the reduction of boron trichloride by hydrogen?
(a) 89.6 L (b) 67.2 L
(c) 44.8 L (d) 22.4 L *(2003)*

6. With increase of temperature, which of these changes?
(a) Molality
(b) Weight fraction of solute
(c) Fraction of solute present in water
(d) Mole fraction *(2002)*

7. Number of atoms in 558.5 gram Fe (at. wt. of Fe = 55.85 g mol^{-1}) is
(a) twice that in 60 g carbon
(b) 6.023×10^{22}
(c) half that in 8 g He
(d) $558.5 \times 6.023 \times 10^{23}$ *(2002)*

Answer Key

1.	(b)	**2.**	(a)	**3.**	(b)	**4.**	(a)	**5.**	(b)	**6.**	(c)
7.	(a)										

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Explanations

1. **(b) :** $M_{mix}V_{mix} = M_1V_1 + M_2V_2$

$M_{mix} = \dfrac{M_1V_1 + M_2V_2}{V_{mix}}$

$M_{mix} = \dfrac{0.5 \times 750 + 2 \times 250}{1000}$

$M_{mix} = 0.875$ M

2. **(a) :** $2Al_{(s)} + 6HCl_{(aq)} \longrightarrow 2Al^{3+}_{(aq)} + 6Cl^{-}_{(aq)} + 3H_{2(g)}$

6 moles of HCl produced H_2 at STP = 3 × 22.4 L

∴ 1 mole of HCl will produce H_2 at STP

$= \dfrac{3 \times 22.4}{6} = 11.2$ L

3. **(b) :** 1 mole of $Mg_3(PO_4)_2$

⇒ 3 moles of Mg atom + 2 moles of P atom + 8 moles of O atom

8 moles of oxygen atoms are present in = 1 mole of $Mg_3(PO_4)_2$

0.25 mole of oxygen atoms are present in = $\dfrac{1 \times 0.25}{8}$

= 3.125×10^{-2} moles of $Mg_3(PO_4)_2$

4. **(a) :** 1 atomic mass unit on the scale of 1/6 of C-12 = 2 amu on the scale of 1/12 of C-12.

Now, atomic mass of an element

$= \dfrac{\text{Mass of one atom of the element}}{1 \text{ amu (Here on the scale of } \frac{1}{6} \text{ of C–12)}}$

$= \dfrac{\text{Mass of one atom of the element}}{2 \text{ amu (Here on the scale of } \frac{1}{12} \text{ of C–12)}}$

∴ Numerically the mass of a substance will become half of the normal scale.

5. **(b) :** $2BCl_3 + 3H_2 \rightarrow 6HCl + 2B$

or $BCl_3 + \frac{3}{2}H_2 \rightarrow 3HCl + B$

10.8 g boron requires hydrogen = $\frac{3}{2} \times 22.4$ L

21.6 g boron will require hydrogen

$= \frac{3}{2} \times \frac{22.4}{10.8} \times 21.6 = 67.2$ L

6. **(c) :** Volume increases with rise in temperature.

7. **(a) :** Fe (no. of moles) = $\dfrac{558.5}{55.85} = 10$ moles

C (no. of moles) = 60/12 = 5 moles.

(atomic weight of carbon = 12)

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CHAPTER 2

STATES OF MATTER

1. Experimentally it was found that a metal oxide has formula $M_{0.98}O$. Metal M, is present as M^{2+} and M^{3+} in its oxide. Fraction of the metal which exists as M^{3+} would be
(a) 5.08% (b) 7.01%
(c) 4.08% (d) 6.05% *(2013)*

2. For gaseous state, if most probable speed is denoted by C^*, average speed by $\bar{C}$ and mean square speed by C, then for a large number of molecules the ratios of these speed are
(a) $C^* : \bar{C} : C = 1 : 1.225 : 1.128$
(b) $C^* : \bar{C} : C = 1.225 : 1.128 : 1$
(c) $C^* : \bar{C} : C = 1.128 : 1 : 1.225$
(d) $C^* : \bar{C} : C = 1 : 1.128 : 1.225$ *(2013)*

3. Lithium forms body centred cubic structure. The length of the side of its unit cell is 351 pm. Atomic radius of the lithium will be
(a) 300 pm (b) 240 pm
(c) 152 pm (d) 75 pm *(2012)*

4. The compressibility factor for a real gas at high pressure is
(a) 1 (b) $1 + Pb/RT$
(c) $1 - Pb/RT$ (d) $1 + RT/Pb$ *(2012)*

5. In a face centred cubic lattice, atom A occupies the corner positions and atom B occupies the face centre positions. If one atom of B is missing from one of the face centred points, the formula of the compound is
(a) A_2B (b) AB_2
(c) A_2B_3 (d) A_2B_5 *(2011)*

6. 'a' and 'b' are van der Waals' constants for gases. Chlorine is more easily liquefied than ethane because
(a) a and b for Cl_2 > a and b for C_2H_6
(b) a and b for Cl_2 < a and b for C_2H_6
(c) a for Cl_2 < a for C_2H_6 but b for Cl_2 > b for C_2H_6
(d) a for Cl_2 > a for C_2H_6 but b for Cl_2 < b for C_2H_6 *(2011)*

7. The edge length of a face centred cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is
(a) 144 pm (b) 288 pm
(c) 398 pm (d) 618 pm *(2010)*

8. If 10^{-4} dm^3 of water is introduced into a 1.0 dm^3 flask at 300 K, how many moles of water are in the vapour phase when equilibrium is established?
(Given : Vapour pressure of H_2O at 300 K is 3170 Pa; $R = 8.314$ J K^{-1} mol^{-1})
(a) 1.27×10^{-3} mol (b) 5.56×10^{-3} mol
(c) 1.53×10^{-2} mol (d) 4.46×10^{-2} mol *(2010)*

9. Percentages of free space in cubic close packed structure and in body centred packed structure are respectively
(a) 48% and 26% (b) 30% and 26%
(c) 26% and 32% (d) 32% and 48% *(2010)*

10. Copper crystallizes in *fcc* with a unit cell length of 361 pm. What is the radius of copper atom?
(a) 108 pm (b) 127 pm
(c) 157 pm (d) 181 pm *(2009)*

11. In a compound, atoms of element Y form *ccp* lattice and those of element X occupy 2/3rd of tetrahedral voids. The formula of the compound will be
(a) X_3Y_4 (b) X_4Y_3
(c) X_2Y_3 (d) X_2Y *(2008)*

12. Equal masses of methane and oxygen are mixed in an empty container at 25°C. The fraction of the total pressure exerted by oxygen is
(a) 1/2 (b) 2/3
(c) $\frac{1}{3} \times \frac{273}{298}$ (d) 1/3 *(2007)*

13. Total volume of atoms present in a face-centred cubic unit cell of a metal is (r is atomic radius)
(a) $\frac{20}{3}\pi r^3$ (b) $\frac{24}{3}\pi r^3$
(c) $\frac{12}{3}\pi r^3$ (d) $\frac{16}{3}\pi r^3$ *(2006)*

14. Which one of the following statements is not true about the effect of an increase in temperature on the distribution of molecular speeds in a gas?
(a) The most probable speed increases.
(b) The fraction of the molecules with the most probable speed increases.
(c) The distribution becomes broader.
(d) The area under the distribution curve remains the same as under the lower temperature. *(2005)*

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15. An ionic compound has a unit cell consisting of A ions at the corners of a cube and B ions on the centres of the faces of the cube. The empirical formula for this compound would be
(a) AB (b) A_2B
(c) AB_3 (d) A_3B *(2005)*

16. What type of crystal defect is indicated in the diagram below?
Na^+ Cl^- Na^+ Cl^- Na^+ Cl^-
Cl^- □ Cl^- Na^+ □ Na^+
Na^+ Cl^- □ Cl^- Na^+ Cl^-
Cl^- Na^+ Cl^- Na^+ □ Na^+
(a) Frenkel defect
(b) Schottky defect
(c) Interstitial defect
(d) Frenkel and Schottky defects *(2004)*

17. In van der Waals equation of state of the gas law, the constant b is a measure of
(a) intermolecular repulsions
(b) intermolecular attraction
(c) volume occupied by the molecules
(d) intermolecular collisions per unit volume. *(2004)*

18. As the temperature is raised from 20°C to 40°C, the average kinetic energy of neon atoms changes by a factor of which of the following?
(a) 1/2 (b) $\sqrt{313/293}$
(c) 313/293 (d) 2 *(2004)*

19. A pressure cooker reduces cooking time for food because
(a) heat is more evenly distributed in the cooking space
(b) boiling point of water involved in cooking is increased
(c) the higher pressure inside the cooker crushes the food material
(d) cooking involves chemical changes helped by a rise in temperature. *(2003)*

20. According to the kinetic theory of gases, in an ideal gas, between two successive collisions a gas molecule travels
(a) in a circular path
(b) in a wavy path
(c) in a straight line path
(d) with an accelerated velocity. *(2003)*

21. How many unit cells are present in a cube-shaped ideal crystal of NaCl of mass 1.00 g?
[Atomic masses : Na = 23, Cl = 35.5]
(a) 2.57×10^{21} (b) 5.14×10^{21}
(c) 1.28×10^{21} (d) 1.71×10^{21} *(2003)*

22. Na and Mg crystallize in *bcc* and *fcc* type crystals respectively, then the number of atoms of Na and Mg present in the unit cell of their respective crystal is
(a) 4 and 2 (b) 9 and 14
(c) 14 and 9 (d) 2 and 4 *(2002)*

23. For an ideal gas, number of moles per litre in terms of its pressure P, gas constant R and temperature T is
(a) PT/R (b) PRT
(c) P/RT (d) RT/P *(2002)*

24. Kinetic theory of gases proves
(a) only Boyle's law
(b) only Charles' law
(c) only Avogadro's law
(d) all of these. *(2002)*

25. Value of gas constant R is
(a) 0.082 L atm (b) 0.987 cal mol^{-1} K^{-1}
(c) 8.3 J mol^{-1} K^{-1} (d) 83 erg mol^{-1} K^{-1} *(2002)*

Answer Key

1. (c)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (d)	**6.** (d)
7. (a)	**8.** (a)	**9.** (c)	**10.** (b)	**11.** (b)	**12.** (d)
13. (d)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (c)	**18.** (c)
19. (b)	**20.** (c)	**21.** (a)	**22.** (d)	**23.** (c)	**24.** (d)
25. (c)					

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Explanations

1. **(c) :** Let the fraction of metal which exists as M^{3+} be x

Then the fraction of metal as $M^{2+} = (0.98 - x)$

$\therefore$ $3x + 2(0.98 - x) = 2$

$x + 1.96 = 2$

$x = 0.04$

$\therefore$ % of $M^{3+} = \frac{0.04}{0.98} \times 100 = 4.08\%$

2. **(d) :** $C^* : \bar{C} : C = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{3RT}{M}}$

$= \sqrt{2} : \sqrt{\frac{8}{3.14}} = \sqrt{3}$

$\therefore$ $C^* : \bar{C} : C$ = 1 : 1.128 : 1.225

3. **(c) :** $a = 351$ pm

For *bcc* unit cell, $a\sqrt{3} = 4r$

$r = \frac{a\sqrt{3}}{4} = \frac{351 \times \sqrt{3}}{4} = 152$ pm

4. **(b) :** For real gases, $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

At high pressure, $P >> a/V^2$

Thus neglecting a/V^2 gives

$P(V - b) = RT$ or $PV = RT + Pb$

or $\frac{PV}{RT} = Z = \frac{RT + Pb}{RT}$ $\Rightarrow$ $Z = 1 + Pb/RT$

5. **(d) :** A B

$8 \times \frac{1}{8}$ $5 \times \frac{1}{2}$

Formula of the compound is A_2B_5.

6. **(d) :**

	a (dm^3 atm mol^{-2})	b(dm^3 mol^{-1})
Cl_2	6.49	0.0562
C_2H_6	5.49	0.0638

From the above values, a for Cl_2 > a for ethane (C_2H_6)

b for ethane (C_2H_6) > b for Cl_2.

7. **(a) :** In *fcc* lattice,

Given, a = 508 pm

r_c = 110 pm

$\therefore$ $110 + r_a = \frac{508}{2} \Rightarrow r_a = 144$ pm

8. **(a) :** The volume occupied by water molecules in vapour phase is $(1 - 10^{-3})$ dm^3, *i.e.*, approximately 1 dm^3.

$PV = nRT$

$3170 \times 1 \times 10^{-3} = n_{H_2O} \times 8.314 \times 300$

$n_{H_2O} = \frac{3170 \times 10^{-3}}{8.314 \times 300} = 1.27 \times 10^{-3}$ mol

9. **(c) :** The packing efficiency in a *ccp* structure = 74%

$\therefore$ Percentage free space = 100 – 74 = 26%

Packing efficiency in a body centred structure = 68%

Percentage free space = 100 – 68 = 32%

10. **(b) :** Since Cu crystallizes in *fcc* lattice,

$\therefore$ radius of Cu atom,

$r = \frac{a}{2\sqrt{2}}$ (a = edge length)

Given, a = 361 pm

$\therefore$ $r = \frac{361}{2\sqrt{2}} \approx 127$ pm

11. **(b) :** Number of Y atoms per unit cell in *ccp* lattice (N) = 4

Number of tetrahedral voids = $2N$ = 2 × 4 = 8

Number of tetrahedral voids occupied by X = 2/3 rd of the tetrahedral void = 2/3 × 8 = 16/3

Hence the formula of the compound will be

$X_{16/3}Y_4 = X_4Y_3$

12. **(d) :** Let the mass of methane and oxygen be m g.

Mole fraction of oxygen, x_{O_2}

$= \frac{\frac{m}{32}}{\frac{m}{32} + \frac{m}{16}} = \frac{m}{32} \times \frac{32}{3m} = \frac{1}{3}$

Let the total pressure be P.

$\therefore$ Partial pressure of O_2, $p_{O_2} = P \times x_{O_2}$

$= P \times \frac{1}{3} = \frac{1}{3}P$

13. **(d) :** In case of a face-centred cubic structure, since four atoms are present in a unit cell, hence volume

$V = 4\left(\frac{4}{3}\pi r^3\right) = \frac{16}{3}\pi r^3$

14. **(b) :** Most probable velocity is defined as the speed possessed by maximum number of molecules of a gas at a given temperature. According to Maxwell's distribution curves, as temperature increases, most probable velocity increases and fraction of molecule possessing most probable velocity decreases.

15. **(c) :** Number of A ions per unit cell = $\frac{1}{8} \times 8 = 1$

Number of B ions per unit cell = $\frac{1}{2} \times 6 = 3$

Empirical formula = AB_3

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16. **(b) :** When an atom or ion is missing from its normal lattice site, a lattice vacancy is created. This defect is known as Schottky defect.
Here equal number of Na^+ and Cl^- ions are missing from their regular lattice position in the crystal. So it is Schottky defect.

17. **(c) :** van der Waals constant for volume correction b is the measure of the effective volume occupied by the gas molecules.

18. **(c) :** $K_b = 3/2\ RT$

$$\frac{K_{40}}{K_{20}} = \frac{T_{40}}{T_{20}} = \frac{273+40}{273+20} = \frac{313}{293}$$

19. **(b) :** According to Gay Lussac's law, at constant pressure of a given mass of a gas is directly proportional to the absolute temperature of the gas. Hence, on increasing pressure, the temperature is also increased. Thus in pressure cooker due to increase in pressure the boiling point of water involved in cooking is also increased.

20. **(c) :** According to the kinetic theory of gases, gas molecules are always in rapid random motion colliding with each other and with the wall of the container and between two successive collisions a gas molecule travels in a straight line path.

21. **(a) :** Mass (m) = density × volume = 1.00 g
Mol. wt. (M) of NaCl = 23 + 35.5 = 58.5

Number of unit cell present in a cube shaped crystal of NaCl of

mass 1.00 g $= \frac{\rho \times a^3 \times N_A}{M \times Z} = \frac{m \times N_A}{M \times Z}$

$$= \frac{1 \times 6.023 \times 10^{23}}{58.5 \times 4}$$

(In NaCl each unit cell has 4 NaCl units. Hence $Z = 4$).
∴ Number of unit cells = 0.02573×10^{23}
= 2.57×10^{21} unit cells

22. **(d) :** *bcc* - Points are at corners and one in the centre of the unit cell.

Number of atoms per unit cell $= 8 \times \frac{1}{8} + 1 = 2$

fcc - Points are at the corners and also centre of the six faces of each cell.

Number of atoms per unit cell $= 8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$

23. **(c) :** From ideal gas equation, $PV = nRT$
∴ $n/V = P/RT$ (number of moles = n/V)

24. **(d) : Explanation of the Gas Laws on the basis of Kinetic Molecular Model**

One of the postulates of kinetic theory of gases is
Average K.E. ∝ T

or, $\frac{1}{2} mnC_{rms}^2 \propto T$ or, $\frac{1}{2} mnC_{rms}^2 = kT$

Now, $PV = \frac{1}{3} mnC_{rms}^2 = \frac{2}{3} \times \frac{1}{2} mnC_{rms}^2 = \frac{2}{3} kT$

(i) Boyle's Law :
- Constant temperature means that the average kinetic energy of the gas molecules remains constant.
- This means that the rms velocity of the molecules, C_{rms} remains unchanged.
- If the rms velocity remains unchanged, but the volume increases, this means that there will be fewer collisions with the container walls over a given time.
- Therefore, the pressure will decrease

i.e. $P \propto \frac{1}{V}$

or PV = constant.

(ii) Charles' Law :
- An increase in temperature means an increase in the average kinetic energy of the gas molecules, thus an increase in C_{rms}.
- There will be more collisions per unit time, furthermore, the momentum of each collision increases (molecules strike the wall harder).
- Therefore, there will be an increase in pressure.
- If we allow the volume to change to maintain constant pressure, the volume will increase with increasing temperature (Charles law).

(iii) Avogadro's Law
It states that under similar conditions of pressure and temperature, equal volume of all gases contain equal number of molecules. Considering two gases, we have

$P_1V_1 = \frac{2}{3} kT_1$ and $P_2V_2 = \frac{2}{3} kT_2$

Since $P_1 = P_2$ and $T_1 = T_2$, therefore

$$\frac{P_1V_1}{P_2V_2} = \frac{(2/3)kT_1}{(2/3)kT_2} \Rightarrow \frac{V_1}{V_2} = \frac{n_1}{n_2}$$

If volumes are identical, obviously $n_1 = n_2$.

25. **(c) :** Units of R
(i) in L atm ⇒ 0.082 L atm $mol^{-1}\ K^{-1}$
(ii) in C.G.S. system ⇒ 8.314×10^7 erg $mol^{-1}\ K^{-1}$
(iii) in M.K.S. system ⇒ 8.314 J $mol^{-1}\ K^{-1}$
(iv) in calories ⇒ 1.987 cal $mol^{-1}\ K^{-1}$

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CHAPTER 3

ATOMIC STRUCTURE

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1. Energy of an electron is given by $E = -2.178 \times 10^{-18}\ \text{J}\left(\frac{Z^2}{n^2}\right)$.
Wavelength of light required to excite an electron in an hydrogen atom from level $n = 1$ to $n = 2$ will be
($h = 6.62 \times 10^{-34}$ J s and $c = 3.0 \times 10^8$ m s^{-1})
(a) 8.500×10^{-7} m (b) 1.214×10^{-7} m
(c) 2.816×10^{-7} m (d) 6.500×10^{-7} m *(2013)*

2. The electrons identified by quantum numbers n and l:
(1) $n = 4, l = 1$ (2) $n = 4, l = 0$
(3) $n = 3, l = 2$ (4) $n = 3, l = 1$
can be placed in order of increasing energy as
(a) (4) < (2) < (3) < (1)
(b) (2) < (4) < (1) < (3)
(c) (1) < (3) < (2) < (4)
(d) (3) < (4) < (2) < (1) *(2012)*

3. A gas absorbs a photon of 355 nm and emits at two wavelengths. If one of the emission is at 680 nm, the other is at
(a) 1035 nm (b) 325 nm
(c) 743 nm (d) 518 nm *(2011)*

4. The energy required to break one mole of Cl—Cl bonds in Cl_2 is 242 kJ mol^{-1}. The longest wavelength of light capable of breaking a single Cl—Cl bond is ($c = 3 \times 10^8$ m s^{-1} and $N_A = 6.02 \times 10^{23}$ mol^{-1})
(a) 494 nm (b) 594 nm
(c) 640 nm (d) 700 nm *(2010)*

5. Ionisation energy of He^+ is 19.6×10^{-18} J atom^{-1}. The energy of the first stationary state ($n = 1$) of Li^{2+} is
(a) 8.82×10^{-17} J atom^{-1}
(b) 4.41×10^{-16} J atom^{-1}
(c) -4.41×10^{-17} J atom^{-1}
(d) -2.2×10^{-15} J atom^{-1} *(2010)*

6. Calculate the wavelength (in nanometre) associated with a proton moving at 1.0×10^3 m s^{-1}.
(Mass of proton = 1.67×10^{-27} kg and $h = 6.63 \times 10^{-34}$ J s)
(a) 0.032 nm (b) 0.40 nm
(c) 2.5 nm (d) 14.0 nm *(2009)*

7. In an atom, an electron is moving with a speed of 600 m/s with an accuracy of 0.005%. Certainty with which the position of the electron can be located is ($h = 6.6 \times 10^{-34}$ kg m^2 s^{-1}, mass of electron, $e_m = 9.1 \times 10^{-31}$ kg)
(a) 1.52×10^{-4} m (b) 5.10×10^{-3} m
(c) 1.92×10^{-3} m (d) 3.84×10^{-3} m *(2009)*

8. The ionization enthalpy of hydrogen atom is 1.312×10^6 J mol^{-1}. The energy required to excite the electron in the atom from $n = 1$ to $n = 2$ is
(a) 9.84×10^5 J mol^{-1}
(b) 8.51×10^5 J mol^{-1}
(c) 6.56×10^5 J mol^{-1}
(d) 7.56×10^5 J mol^{-1} *(2008)*

9. Which of the following sets of quantum numbers represents the highest energy of an atom?
(a) $n = 3, l = 0, m = 0, s = +\frac{1}{2}$
(b) $n = 3, l = 1, m = 1, s = +\frac{1}{2}$
(c) $n = 3, l = 2, m = 1, s = +\frac{1}{2}$
(d) $n = 4, l = 0, m = 0, s = +\frac{1}{2}$ *(2007)*

10. Uncertainty in the position of an electron (mass = 9.1×10^{-31} kg) moving with a velocity 300 m s^{-1}, accurate upto 0.001% will be ($h = 6.6 \times 10^{-34}$ J s)
(a) 19.2×10^{-2} m (b) 5.76×10^{-2} m
(c) 1.92×10^{-2} m (d) 3.84×10^{-2} m *(2006)*

11. According to Bohr's theory, the angular momentum of an electron in 5th orbit is
(a) $25\frac{h}{\pi}$ (b) $1.0\frac{h}{\pi}$
(c) $10\frac{h}{\pi}$ (d) $2.5\frac{h}{\pi}$ *(2006)*

12. Which of the following statements in relation to the hydrogen atom is correct?
(a) 3*s* orbital is lower in energy than 3*p* orbital.
(b) 3*p* orbital is lower in energy than 3*d* orbital.
(c) 3*s* and 3*p* orbitals are of lower energy than 3*d* orbital.
(d) 3*s*, 3*p* and 3*d* orbitals all have the same energy. *(2005)*

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13. In a multi-electron atom, which of the following orbitals described by the three quantum numbers will have the same energy in the absence of magnetic and electric fields?
(i) $n = 1, l = 0, m = 0$
(ii) $n = 2, l = 0, m = 0$
(iii) $n = 2, l = 1, m = 1$
(iv) $n = 3, 1 = 2, m = 1$
(v) $n = 3, l = 2, m = 0$
(a) (i) and (ii) (b) (ii) and (iii)
(c) (iii) and (iv) (d) (iv) and (v)
(2005)

14. The wavelength of the radiation emitted, when in a hydrogen atom electron falls from infinity to stationary state 1, would be (Rydberg constant = 1.097×10^7 m^{-1})
(a) 91 nm (b) 192 nm
(c) 406 nm (d) 9.1×10^{-8} nm
(2004)

15. Consider the ground state of Cr atom ($Z = 24$). The numbers of electrons with the azimuthal quantum numbers, $l = 1$ and 2 are, respectively
(a) 12 and 4 (b) 12 and 5
(c) 16 and 4 (d) 16 and 5
(2004)

16. Which of the following sets of quantum numbers is correct for an electron in 4*f* orbital?
(a) $n = 4, l = 3, m = +4, s = +\frac{1}{2}$
(b) $n = 4, l = 4, m = -4, s = -\frac{1}{2}$
(c) $n = 4, l = 3, m = +1, s = +\frac{1}{2}$
(d) $n = 3, l = 2, m = -2, s = +\frac{1}{2}$
(2004)

17. The orbital angular momentum for an electron revolving in an orbit is given by $\sqrt{l(l+1)} \cdot \frac{h}{2\pi}$. This momentum for an *s*-electron will be given by
(a) $+\frac{1}{2} \cdot \frac{h}{2\pi}$ (b) zero
(c) $\frac{h}{2\pi}$ (d) $\sqrt{2} \cdot \frac{h}{2\pi}$
(2003)

18. The de Broglie wavelength of a tennis ball of mass 60 g moving with a velocity of 10 metres per second is approximately
(Planck's constant, $h = 6.63 \times 10^{-34}$ J s)
(a) 10^{-33} metres (b) 10^{-31} metres
(c) 10^{-16} metres (d) 10^{-25} metres.
(2003)

19. In Bohr series of lines of hydrogen spectrum, the third line from the red end corresponds to which one of the following inter-orbit jumps of the electron for Bohr orbits in an atom of hydrogen?
(a) $3 \rightarrow 2$ (b) $5 \rightarrow 2$
(c) $4 \rightarrow 1$ (d) $2 \rightarrow 5$
(2003)

20. Uncertainty in position of a minute particle of mass 25 g in space is 10^{-5} m. What is the uncertainty in its velocity (in m s^{-1})? ($h = 6.6 \times 10^{-34}$ J s)
(a) 2.1×10^{-34} (b) 0.5×10^{-34}
(c) 2.1×10^{-28} (d) 0.5×10^{-23}
(2002)

21. In a hydrogen atom, if energy of an electron in ground state is 13.6 eV, then that in the 2nd excited state is
(a) 1.51 eV (b) 3.4 eV
(c) 6.04 eV (d) 13.6 eV
(2002)

Answer Key

1.	(b)	**2.**	(a)	**3.**	(c)	**4.**	(a)	**5.**	(c)	**6.**	(b)
7.	(c)	**8.**	(a)	**9.**	(c)	**10.**	(c)	**11.**	(d)	**12.**	(d)
13.	(d)	**14.**	(a)	**15.**	(b)	**16.**	(c)	**17.**	(b)	**18.**	(a)
19.	(b)	**20.**	(c)	**21.**	(a)						

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Explanations

1. **(b) :** $E = -2.178 \times 10^{-18} Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$E = -2.178 \times 10^{-18} \left[\frac{1}{(2)^2} - \frac{1}{(1)^2}\right]$

$E = +2.178 \times 10^{-18} \times \frac{3}{4} = 1.6335 \times 10^{-18}$ J

$E = \frac{hc}{\lambda}$

$\Rightarrow \lambda = \frac{hc}{E} = \frac{6.62 \times 10^{-34} \text{ J s} \times 3 \times 10^8 \text{ m}}{1.6335 \times 10^{-18} \text{ J}}$

$\lambda = 12.14 \times 10^{-8}$ m

or $\lambda = 1.214 \times 10^{-7}$ m

2. **(a) :** (1) $n = 4, l = 1 \Rightarrow 4p$
(2) $n = 4, l = 0 \Rightarrow 4s$
(3) $n = 3, l = 2 \Rightarrow 3d$
(4) $n = 3, l = 1 \Rightarrow 3p$

Increasing order of energy is $3p < 4s < 3d < 4p$
(4) < (2) < (3) < (1)

Alternatively,
for (1) $n + l = 5$; $n = 4$
(2) $n + l = 4$; $n = 4$
(3) $n + l = 5$; $n = 3$
(4) $n + l = 4$; $n = 3$

Lower $n + l$ means less energy and if for two subshells $n + l$ is same than lower n, lower will be the energy.
Thus correct order is (4) < (2) < (3) < (1).

3. **(c) :** We know that
$E = h\upsilon = hc/\lambda$

$E = E_1 + E_2$ or $\frac{hc}{\lambda} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$

$\Rightarrow \frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \Rightarrow \frac{1}{355} = \frac{1}{680} + \frac{1}{\lambda_2}$

$\therefore \lambda_2 = \frac{355 \times 680}{680 - 355} = 742.769 \text{ nm} \approx 743$ nm

4. **(a) :** Energy required to break 1 mol of bonds = 242 kJ mol^{-1}

$\therefore$ Energy required to break 1 bond $= \frac{242 \times 10^3}{6.02 \times 10^{23}}$ J

We know that, $E = \frac{hc}{\lambda}$

Given, $c = 3 \times 10^8$ m s^{-1}

$\Rightarrow \frac{242 \times 10^3}{6.02 \times 10^{23}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda}$

$\therefore \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8 \times 6.02 \times 10^{23}}{242 \times 10^3}$

$= 0.494 \times 10^{-6}$ m = 494 nm

5. **(c) :** $I.E.(He^+) = 19.6 \times 10^{-18}$ J $atom^{-1}$
E_1 (for H) $\times Z^2 = -I.E.$
$E_1 \times 4 = -19.6 \times 10^{-18}$
E_1 (for Li^{2+}) = E_1 (for H) $\times 9$

$= \frac{-19.6 \times 10^{-18} \times 9}{4} = -44.1 \times 10^{-18}$ J $atom^{-1}$

$= -4.41 \times 10^{-17}$ J $atom^{-1}$

6. **(b) :** According to de-Broglie's equation,

$\lambda = \frac{h}{mv}$

Given, $v = 1.0 \times 10^3$ m s^{-1}

$\therefore \lambda = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1.0 \times 10^3} = 3.9 \times 10^{-10}$ m

or $\lambda \approx 0.4$ nm

7. **(c) :** Given, velocity of e^-, $v = 600$ m s^{-1}
Accuracy of velocity = 0.005%

$\therefore \Delta v = \frac{600 \times 0.005}{100} = 0.03$

According to Heisenberg's uncertainty principle,

$\Delta x \cdot m\Delta v \geq \frac{h}{4\pi}$

$\Rightarrow \Delta x = \frac{6.6 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 0.03}$

$= 1.92 \times 10^{-3}$ m

8. **(a) :** The ionisation of H-atom is the energy absorbed when the electron in an atom gets excited from first shell (E_1) to infinity (*i.e.*, E_∞)
$I.E = E_\infty - E_1$
$1.312 \times 10^6 = 0 - E_1$
$E_1 = -1.312 \times 10^6$ J mol^{-1}

$E_2 = -\frac{1.312 \times 10^6}{(2)^2} = -\frac{1.312 \times 10^6}{4}$

Energy of electron in second orbit ($n = 2$)
$\therefore$ Energy required when an electron makes transition from $n = 1$ to $n = 2$

$\Delta E = E_2 - E_1 = -\frac{1.312 \times 10^6}{4} - (-1.312 \times 10^6)$

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$= \frac{-1.312\times10^6 + 5.248\times10^6}{4} = 0.984 \times 10^6$

$\Delta E = 9.84 \times 10^5$ J mol^{-1}

9. (c) : $n = 3, l = 0$ represents 3s orbital
$n = 3, l = 1$ represents 3p orbital
$n = 3, l = 2$ represents 3d orbital
$n = 4, l = 0$ represents 4s orbital
The order of increasing energy of the orbitals is
$3s < 3p < 4s < 3d.$

10. (c) : According to Heisenberg's uncertainty principle,

$\Delta x \times \Delta p = \frac{h}{4\pi}$

$\Delta x \cdot (m\cdot\Delta v) = \frac{h}{4\pi} \Rightarrow \Delta x = \frac{h}{4\pi m\cdot\Delta v}$

Here $\Delta v = \frac{0.001}{100}\times 300 = 3\times10^{-3}$ m s^{-1}

$\therefore \Delta x = \frac{6.63\times10^{-34}}{4\times3.14\times9.1\times10^{-31}\times3\times10^{-3}}$
$= 1.92\times10^{-2}$ m

11. (d) : Angular momentum of the electron, $mvr = \frac{nh}{2\pi}$

when $n = 5$ (given)

$\therefore$ Angular momentum $= \frac{5h}{2\pi} = 2.5\frac{h}{\pi}$

12. (d) : For hydrogen the energy order of orbital is
$1s < 2s = 2p < 3s = 3p = 3d < 4s = 4p = 4d = 4f$

13. (d) : Orbitals having same $(n + l)$ value in the absence of electric and magnetic field will have same energy.

14. (a) : $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \frac{1}{\lambda} = 1.097\times10^7 \text{m}^{-1}\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$

$\therefore \lambda = 91 \times 10^{-9}$ m = 91 nm

15. (b) : $_{24}Cr \rightarrow 1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^5\,4s^1$
we know for p, $l = 1$ and for d, $l = 2$.

For $l = 1$, total number of electrons = 12 $[2p^6$ and $3p^6]$

For $l = 2$, total number of electrons = 5 $[3d^5]$

16. (c) : For 4f orbital electrons, $n = 4$

$l = 3$ (because s p d f = 0 1 2 3)
$m = +3, +2, +1, 0, -1, -2, -3$
$s = \pm 1/2$

17. (b) : The value of l (azimuthal quantum number) for s-electron is equal to zero.

Orbital angular momentum $= \sqrt{l(l+1)}\cdot\frac{h}{2\pi}$

Substituting the value of l for s-electron

$= \sqrt{0(0+1)}\cdot\frac{h}{2\pi} = 0$

18. (a) : $\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\times1000}{60\times10}$ m
$= 11.05 \times 10^{-34}$ m $= 1.105 \times 10^{-33}$ metres.

19. (b) : The electron has minimum energy in the first orbit and its energy increases as n increases. Here n represents number of orbit, i.e. 1st, 2nd, 3rd .... The third line from the red end corresponds to yellow region i.e. 5. In order to obtain less energy electron tends to come in 1st or 2nd orbit. So jump may be involved either $5 \rightarrow 1$ or $5 \rightarrow 2$. Thus option (b) is correct here.

20. (c) : According to Heisenberg uncertainty principle,

$\Delta x \cdot m\Delta v = \frac{h}{4\pi}$

$\Delta v = \frac{6.6\times10^{-34}}{4\times3.14\times25\times10^{-5}}$

$\therefore \Delta v = 2.1 \times 10^{-28}$ m s^{-1}

21. (a) : 2nd excited state will be the 3rd energy level.

$E_n = \frac{13.6}{n^2}$ eV or $E = \frac{13.6}{9} = 1.51$ eV

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CHAPTER 4

CHEMICAL BONDING AND MOLECULAR STRUCTURE

1. Stability of the species Li_2, Li_2^- and Li_2^+ increases in the order of
(a) $Li_2^- < Li_2 < Li_2^+$
(b) $Li_2 < Li_2^+ < Li_2^-$
(c) $Li_2^- < Li_2^+ < Li_2$
(d) $Li_2 < Li_2^- < Li_2^+$ (2013)

2. In which of the following pairs of molecules/ions, both the species are not likely to exist?
(a) H_2^-, He_2^{2+} (b) H_2^+, He_2^{2-}
(c) H_2^-, He_2^{2-} (d) H_2^{2+}, He_2 (2013)

3. Which one of the following molecules is expected to exhibit diamagnetic behaviour?
(a) S_2 (b) C_2
(c) N_2 (d) O_2 (2013)

4. The molecule having smallest bond angle is
(a) $AsCl_3$ (b) $SbCl_3$
(c) PCl_3 (d) NCl_3 (2012)

5. In which of the following pairs the two species are not isostructural?
(a) PCl_4^+ and $SiCl_4$ (b) PF_5 and BrF_5
(c) AlF_6^{3-} and SF_6 (d) CO_3^{2-} and NO_3^- (2012)

6. The structure of IF_7 is
(a) square pyramid (b) trigonal bipyramid
(c) octahedral (d) pentagonal bipyramid. (2011)

7. The hybridisation of orbitals of N atom in NO_3^-, NO_2^+ and NH_4^+ are respectively
(a) sp, sp^2, sp^3 (b) sp^2, sp, sp^3
(c) sp, sp^3, sp^2 (d) sp^2, sp^3, sp (2011)

8. Among the following the maximum covalent character is shown by the compound
(a) $FeCl_2$ (b) $SnCl_2$
(c) $AlCl_3$ (d) $MgCl_2$ (2011)

9. Using MO theory predict which of the following species has the shortest bond length?
(a) O_2^{2+} (b) O_2^+
(c) O_2^- (d) O_2^{2-} (2009)

10. Which one of the following constitutes a group of the isoelectronic species?
(a) N_2, O_2^-, NO^+, CO
(b) C_2^{2-}, O_2^-, CO, NO
(c) NO^+, C_2^{2-}, CN^-, N_2
(d) CN^-, N_2, O_2^{2-}, C_2^{2-} (2008)

11. Which one of the following pairs of species have the same bond order?
(a) NO^+ and CN^+ (b) CN^- and NO^+
(c) CN^- and CN^+ (d) O_2^- and CN^- (2008)

12. Which of the following hydrogen bonds is the strongest?
(a) O — H - - - F
(b) O — H - - - H
(c) F — H - - - F
(d) O — H - - - O (2007)

13. In which of the following ionization processes, the bond order has increased and the magnetic behaviour has changed?
(a) $N_2 \rightarrow N_2^+$ (b) $C_2 \rightarrow C_2^+$
(c) $NO \rightarrow NO^+$ (d) $O_2 \rightarrow O_2^+$ (2007)

14. The charge/size ratio of a cation determines its polarizing power. Which one of the following sequences represents the increasing order of the polarizing power of the cationic species, K^+, Ca^{2+}, Mg^{2+}, Be^{2+}?
(a) $Ca^{2+} < Mg^{2+} < Be^+ < K^+$
(b) $Mg^{2+} < Be^{2+} < K^+ < Ca^{2+}$
(c) $Be^{2+} < K^+ < Ca^{2+} < Mg^{2+}$
(d) $K^+ < Ca^{2+} < Mg^{2+} < Be^{2+}$ (2007)

15. Which of the following species exhibits the diamagnetic behaviour?
(a) NO (b) O_2^{2-}
(c) O_2^+ (d) O_2 (2007)

16. In which of the following molecules/ions are all the bonds not equal?
(a) SF_4 (b) SiF_4
(c) XeF_4 (d) BF_4^- (2006)

17. Among the following mixtures, dipole-dipole as the major interaction, is present in
(a) benzene and ethanol
(b) acetonitrile and acetone
(c) KCl and water
(d) benzene and carbon tetrachloride. (2006)

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18. Which of the following molecules/ions does not contain unpaired electrons?
(a) O_2^{2-} (b) B_2
(c) N_2^+ (d) O_2 *(2006)*

19. Of the following sets which one does NOT contain isoelectronic species?
(a) PO_4^{3-}, SO_4^{2-}, ClO_4^-
(b) CN^-, N_2, C_2^{2-}
(c) SO_3^{2-}, CO_3^{2-}, NO_3^-
(d) BO_3^{3-}, CO_3^{2-}, NO_3^- *(2005)*

20. Which one of the following species is diamagnetic in nature?
(a) He_2^+ (b) H_2
(c) H_2^+ (d) H_2^- *(2005)*

21. The maximum number of 90° angles between bond pair-bond pair of electrons is observed in
(a) dsp^3 hybridisation
(b) sp^3d hybridisation
(c) dsp^2 hybridisation
(d) sp^3d^2 hybridisation. *(2004)*

22. Which one of the following has the regular tetrahedral structure?
(a) XeF_4 (b) SF_4
(c) BF_4^- (d) $[Ni(CN)_4]^{2-}$
(Atomic nos.: B = 5, S = 16, Ni = 28, Xe = 54) *(2004)*

23. The bond order in NO is 2.5 while that in NO^+ is 3. Which of the following statements is true for these two species?
(a) Bond length in NO^+ is greater than in NO.
(b) Bond length in NO is greater than in NO^+.
(c) Bond length in NO^+ is equal to that in NO.
(d) Bond length is unpredictable. *(2004)*

24. The correct order of bond angles (smallest first) in H_2S, NH_3, BF_3 and SiH_4 is
(a) $H_2S < SiH_4 < NH_3 < BF_3$
(b) $NH_3 < H_2S < SiH_4 < BF_3$
(c) $H_2S < NH_3 < SiH_4 < BF_3$
(d) $H_2S < NH_3 < BF_3 < SiH_4$. *(2004)*

25. The pair of species having identical shapes for molecules of both species is
(a) CF_4, SF_4 (b) XeF_2, CO_2
(c) BF_3, PCl_3 (d) PF_5, IF_5 *(2003)*

26. Which one of the following compounds has the smallest bond angle in its molecule?
(a) SO_2 (b) OH_2
(c) SH_2 (d) NH_3 *(2003)*

27. Which of the following are arranged in an increasing order of their bond strengths?
(a) $O_2^- < O_2 < O_2^+ < O_2^{2-}$
(b) $O_2^{2-} < O_2^- < O_2 < O_2^+$
(c) $O_2^- < O_2^{2-} < O_2 < O_2^+$
(d) $O_2^+ < O_2 < O_2^- < O_2^{2-}$ *(2002)*

28. A square planar complex is formed by hybridisation of which atomic orbitals?
(a) s, p_x, p_y, d_{yz} (b) $s, p_x, p_y, d_{x^2-y^2}$
(c) s, p_x, p_y, d_{z^2} (d) s, p_y, p_z, d_{xy} *(2002)*

29. Number of sigma bonds in P_4O_{10} is
(a) 6 (b) 7
(c) 17 (d) 16 *(2002)*

30. In which of the following species is the underlined carbon having sp^3 hybridisation?
(a) $CH_3\underline{C}OOH$ (b) $CH_3\ \underline{C}H_2\ OH$
(c) $CH_3\underline{C}OCH_3$ (d) $CH_2 = \underline{C}H - CH_3$ *(2002)*

31. In which of the following species the interatomic bond angle is 109°28′?
(a) NH_3, $(BF_4)^{-1}$ (b) $(NH_4)^+$, BF_3
(c) NH_3, BF_3 (d) $(NH_2)^{-1}$, BF_3 *(2002)*

Answer Key

1.	(c)	**2.**	(d)	**3.**	(d)	**4.**	(b)	**5.**	(b)	**6.**	(d)
7.	(b)	**8.**	(c)	**9.**	(a)	**10.**	(c)	**11.**	(b)	**12.**	(c)
13.	(c)	**14.**	(d)	**15.**	(b)	**16.**	(a)	**17.**	(b)	**18.**	(a)
19.	(c)	**20.**	(b)	**21.**	(d)	**22.**	(c)	**23.**	(b)	**24.**	(c)
25.	(b)	**26.**	(c)	**27.**	(b)	**28.**	(b)	**29.**	(d)	**30.**	(b)
31.	(a)										

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Explanations

1. **(c) :**

Species	Bond order
Li_2	1
Li_2^-	0.5
Li_2^+	0.5

Higher the bond order, greater is the stability.

2. **(d) :** Species with zero bond order does not exist.

H_2^{2+} : $\sigma(1s)^0$

$\therefore$ Bond order = 0

He_2 : $\sigma(1s)^2\ \sigma^*(1s)^2$

Bond order $= \dfrac{2-2}{2} = 0$

3. **(d) :** O_2 is expected to be diamagnetic in nature but actually it is paramagnetic.

4. **(b) :** As we move down the group the size of atom increases and as size of central atom increases, lone pair-bond pair repulsion also increases. Thus bond angle decreases.
Increasing order of atomic radius :

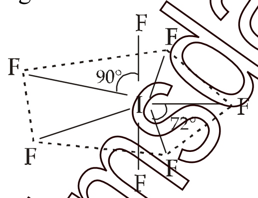
$N < P < As < Sb$

Decreasing order of bond angle :

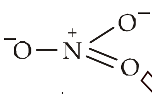
$NCl_3 > PCl_3 > AsCl_3 > SbCl_3$

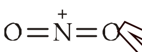
5. **(b) :** PCl_4^+ and $SiCl_4$ $\Rightarrow$ both tetrahedral
PF_5 $\Rightarrow$ trigonal bipyramidal
BrF_5 $\Rightarrow$ square pyramidal
AlF_6^{3-} and SF_6 both are octahedral, CO_3^{2-} and NO_3^- both are trigonal planar.

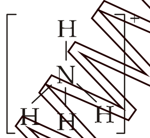
6. **(d) :** The structure is pentagonal bipyramidal having sp^3d^3 hybridisation as given below:



7. **(b) :** The structures of NO_2^-, NO_2^+ and NH_4^+ is


sp^2 hybridisation


sp hybridisation


sp^3 hybridisation

8. **(c) :** We know that, extent of polarisation $\propto$ covalent character in ionic bond.
Fajan's rule states that
(i) the polarising power of cation increases, with increase in magnitude of positive charge on the cation

$\therefore$ polarising power $\propto$ charge of cation

(ii) the polarising power of cation increases with the decrease in the size of a cation.

$\therefore$ polarising power $\propto \dfrac{1}{\text{size of cation}}$

Here the $AlCl_3$ is satisfying the above two conditions *i.e.*, Al is in +3 oxidation state and also has small size. So it has more covalent character.

9. **(a) :** According to MOT, the molecular orbital electronic configuration of

O_2^{2+} : $(\sigma 1s)^2(\sigma^* 1s)^2(\sigma 2s)^2(\sigma^* 2s)^2(\sigma 2p_z)^2(\pi 2p_x)^2 = (\pi 2p_y)^2$

$\therefore$ B.O. $= \dfrac{10-4}{2} = 3$

O_2^+ : $(\sigma 1s)^2(\sigma^* 1s)^2(\sigma 2s)^2(\sigma^* 2s)^2(\sigma 2p_z)^2(\pi 2p_x)^2 = (\pi 2p_y)^2(\pi^* 2p_x)^1$

$\therefore$ B.O. $= \dfrac{10-5}{2} = 2.5$

O_2^- : $(\sigma 1s)^2(\sigma^* 1s)^2(\sigma 2s)^2(\sigma^* 2s)^2(\sigma 2p_z)^2(\pi 2p_x)^2 = (\pi 2p_y)^2(\pi^* 2p_x)^2 = (\pi^* 2p_y)^1$

$\therefore$ B.O. $= \dfrac{10-7}{2} = 1.5$

O_2^{2-} : $(\sigma 1s)^2(\sigma^* 1s)^2(\sigma 2s)^2(\sigma^* 2s)^2(\sigma 2p_z)^2(\pi 2p_x)^2 = (\pi 2p_y)^2(\pi^* 2p_x)^2 = (\pi^* 2p_y)^2$

$\therefore$ B.O. $= \dfrac{10-8}{2} = 1.0$

$\because$ B.O. $\propto \dfrac{1}{\text{Bond length}}$,

$\therefore$ O_2^{2+} has the shortest bond length.

10. **(c) :** Number of electrons in each species are given below

N_2 = 14	CN^- = 14
O_2^- = 17	C_2^{2-} = 14
NO^+ = 14	O_2^{2-} = 18
CO = 14	NO = 15

It is quite evident from the above that NO^+, C_2^{2-}, CN^-, N_2 and CO are isoelectronic in nature. Hence option (c) is correct.

11. **(b) :** In the given pair of species, number of electron in NO^+ = number of electron in CN^- = 14 electrons.
So they are isoelectronic in nature.

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Hence bond order of these two species will be also similar which is shown below.

$NO^+ \rightarrow \sigma 1s^2\ \sigma^*1s^2\ \sigma 2s^2\ \sigma^*2s^2\ \sigma 2p_z^2\ \pi 2p_x^2\ \pi 2p_y^2$

B.O = 1/2 $[N_b - N_a]$ = 1/2 [10 – 4]

or B.O = 3

$CN^- \rightarrow \sigma 1s^2\ \sigma^*1s^2\ \sigma 2s^2\ \sigma^*2s^2\ \pi 2p_x^2\ \pi 2p_y^2\ \sigma 2p_z^2$

B.O =1/2 [10 – 4] or B.O = 3

12. **(c) :** Because of highest electronegativity of F, hydrogen bonding in F — H - - - F is strongest.

13. **(c) :** Molecular orbital configuration of

$O_2 \Rightarrow \sigma 1s^2\, \sigma^*1s^2\, \sigma 2s^2\, \sigma^*2s^2\ \sigma 2p_z^2\, \pi 2p_x^2\, \pi 2p_y^2\, \pi^*2p_x^1\, \pi^*2p_y^1$

$\Rightarrow$ paramagnetic

Bond order $= \frac{10-6}{2} = 2$

$O_2^+ \Rightarrow \sigma 1s^2\, \sigma^*1s^2\, \sigma 2s^2\, \sigma^*2s^2\ \sigma 2p_z^2\, \pi 2p_x^2\, \pi 2p_y^2\, \pi^*2p_x^1$

$\Rightarrow$ paramagnetic

Bond order $= \frac{10-5}{2} = 2.5$

$N_2 \Rightarrow \sigma 1s^2\, \sigma^*1s^2\, \sigma 2s^2\, \sigma^*2s^2\ \pi 2p_x^2\, \pi 2p_y^2\, \pi 2p_Z^2$

$\Rightarrow$ paramagnetic

Bond order $= \frac{10-4}{2} = 3$

$N_2^+ \Rightarrow \sigma 1s^2\, \sigma^*1s^2\, \sigma 2s^2\, \sigma^*2s^2\ \pi 2p_x^2\, \pi 2p_y^2\, \pi 2p_z^1$

$\Rightarrow$ paramagnetic

Bond order $= \frac{9-4}{2} = 2.5$

$C_2 \Rightarrow \sigma 1s^2\, \sigma^*1s^2\, \sigma 2s^2\, \sigma^*2s^2\ \pi 2p_x^2\, \pi 2p_y^2$

$\Rightarrow$ diamagnetic

Bond order $= \frac{8-4}{2} = 2$

$C_2^+ \Rightarrow \sigma 1s^2\, \sigma^*1s^2\, \sigma 2s^2\, \sigma^*2s^2\ \pi 2p_x^2\, \pi 2p_y^1$

$\Rightarrow$ paramagnetic

Bond order $= \frac{7-4}{2} = 1.5$

$NO \Rightarrow \sigma 1s^2\, \sigma^*1s^2\, \sigma 2s^2\, \sigma^*2s^2\ \sigma 2p_z^2\, \pi 2p_x^2\, \pi 2p_y^2\, \pi^*2p_x^1$

$\Rightarrow$ paramagnetic

Bond order $= \frac{10-5}{2} = 2.5$

$NO^+ \Rightarrow \sigma 1s^2\, \sigma^*1s^2\, \sigma 2s^2\, \sigma^*2s^2\ \sigma 2p_z^2\, \pi 2p_x^2\, \pi 2p_y^2$

$\Rightarrow$ diamagnetic

Bond order $= \frac{10-4}{2} = 3$

14. **(d) :** High charge and small size of the cations increases polarisation.

As the size of the given cations decreases as

$K^+ > Ca^{2+} > Mg^{2+} > Be^{2+}$

Hence, polarising power decreases as

$K^+ < Ca^{2+} < Mg^{2+} < Be^{2+}$

15. **(b) :** Molecular orbital configuration is

$NO \Rightarrow \sigma 1s^2\, \sigma^*1s^2\, \sigma 2s^2\, \sigma^*2s^2\ \sigma 2p_z^2\, \pi 2p_x^2\, \pi 2p_y^2\, \pi^*2p_x^1$

$\Rightarrow$ paramagnetic

$O_2 \Rightarrow \sigma 1s^2\, \sigma^*1s^2\, \sigma 2s^2\, \sigma^*2s^2\ \sigma 2p_z^2\, \pi 2p_x^2\, \pi 2p_y^2\, \pi^*2p_x^1\, \pi^*2p_y^1$

$\Rightarrow$ paramagnetic

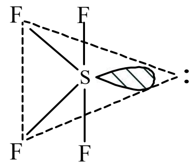
$O_2^{2-} \Rightarrow \sigma 1s^2\, \sigma^*1s^2\, \sigma 2s^2\, \sigma^*2s^2\ \sigma 2p_z^2\, \pi 2p_x^2\, \pi 2p_y^2\, \pi^*2p_x^2\, \pi^*2p_y^2$

$\Rightarrow$ diamagnetic

$O_2^+ \Rightarrow \sigma 1s^2\, \sigma^*1s^2\, \sigma 2s^2\, \sigma^*2s^2\ \sigma 2p_z^2\, \pi 2p_x^2\, \pi 2p_y^2\, \pi^*2p_x^1$

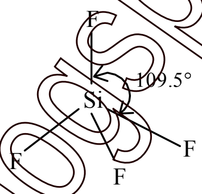
$\Rightarrow$ paramagnetic

16. **(a) :**

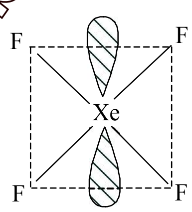


SF_4 molecule shows sp^3d hybridisation but its expected trigonal bipyramidal geometry gets distorted due to presence of a lone pair of electrons and it becomes distorted tetrahedral or see-saw with the bond angles equal to 89° and 177° instead of the expected angles of 90° and 180° respectively.

SiF_4 : sp^3 hybridisation and tetrahedral geometry.



XeF_4 : sp^3d^2 hybridisation, shape is square planar instead of octahedral due to presence of two lone pair of electrons on Xe atom.



BF_4^- : sp^3 hybridisation and tetrahedral geometry.

17. **(b) :** Dipole-dipole interactions occur among the polar molecules. Polar molecules have permanent dipoles. The positive pole of one molecule is thus attracted by the negative pole of the other molecule. The magnitude of dipole-dipole forces in different polar molecules is predicted on the basis of the polarity of the molecules, which in turn depends upon the electronegativities of the atoms present in the molecule and the geometry of the molecule (in case of polyatomic molecules, containing more than two atoms in a molecule).

18. **(a) :** The molecular orbital configuration of O_2^{2-} ion is

$KK\ \sigma(2s)^2\ \sigma^*(2s)^2\ \sigma(2p_z)^2\ \pi(2p_x)^2\ \pi(2p_y)^2\ \pi^*(2p_x)^2\ \pi^*(2p_y)^2$

Here KK represents non-bonding molecular orbital of 1s orbital.

O_2^{2-} contains no unpaired electrons.

The molecular orbital configuration of B_2 molecule is

$KK\ \sigma(2s)^2\ \sigma^*(2s)^2\ \pi(2p_x)^1\ \pi(2p_y)^1$

It contains 2 unpaired electrons.

The molecular orbital configuration of N_2^+ ion is

$KK\ \sigma(2s)^2\ \sigma^*(2s)^2\ \sigma(2p_z)^2\ \pi(2p_x)^2\ \pi(2p_y)^2\ \sigma(2p_z)^1$

It contains one unpaired electron.

The molecular orbital configuration of O_2 molecule is

$KK\ \sigma(2s)^2\ \sigma^*(2s)^2\ \pi(2p_x)^2\ \pi(2p_y)^2\ \sigma(2p_z)^2\ \pi^*(2p_x)^1\ \pi^*(2p_y)^1$

It contains 2 unpaired electrons.

19. **(c) :** Number of electrons in SO_3^{2-}

= 16 + 8 × 3 + 2 = 42

Number of electrons in CO_3^{2-} = 6 + 8 × 3 + 2= 32

Number of electrons in NO_3^- = 7 + 8 × 3 + 1= 32

These are not isoelectronic species as number of electrons are not same.

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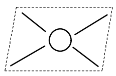
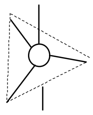
20. **(b) :** $He_2^+ \rightarrow \sigma(1s)^2\ \sigma^*(1s)^1$, one unpaired electron
$H_2 \rightarrow \sigma(1s)^2\ \sigma^*(1s)^0$, no unpaired electron
$H_2^+ \rightarrow \sigma(1s)^1\ \sigma^*(1s)^0$, one unpaired electron
$H_2^- \rightarrow \sigma(1s)^2\ \sigma^*(1s)^1$, one unpaired electron.
Due to absence of unpaired electrons, H_2 will be diamagnetic.

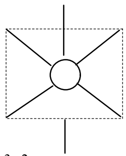
21. **(d) :**



dsp^2 hybridisation
(four 90° angles between bond pair and bond pair)

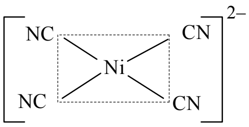


sp^3d or dsp^3 hybridisation
(six 90° angles between bond pair and bond pair)

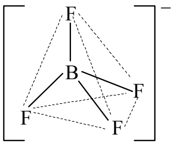


sp^3d^2 hybridisation
(twelve 90° angle between bond pair and bond pair)

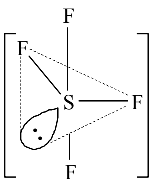
22. **(c) :**



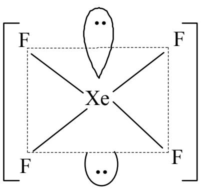
square planar



regular tetrahedral



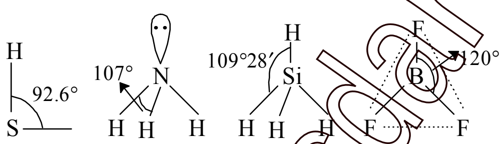
see-saw shaped



square planar

23. **(b) :** Higher the bond order, shorter will be the bond length. Thus NO^+ is having higher bond order than that of NO so NO^+ has shorter bond length.

24. **(c) :** The correct order of bond angle (smallest first) is
$H_2S < NH_3 < SiH_4 < BF_3$
$92.6° < 107° < 109°28' < 120°$



25. **(b) :** Central atom in each being *sp* hybridised shows linear shape.
F — Xe — F   O = C = O

26. **(c) :**

	SO_2	OH_2	SH_2	NH_3
Bond angle :	119.5°	104.5°	92.5°	106.5°

27. **(b) :** Molecular orbital configuration of

$O_2 \Rightarrow \sigma(1s)^2\sigma^*(1s)^2\sigma(2s)^2\sigma^*(2s)^2\sigma(2p_z)^2\pi(2p_x)^2$
$\pi(2p_y)^2\pi^*(2p_x)^1\pi^*(2p_y)^1$; B.O. $= \frac{10-6}{2} = 2$

$O_2^+ \Rightarrow \sigma(1s)^2\sigma^*(1s)^2\sigma(2s)^2\sigma^*(2s)^2\sigma(2p_z)^2\pi(2p_x)^2$
$\pi(2p_y)^2\pi^*(2p_x)^1$; B.O. $= \frac{10-5}{2} = 2.5$

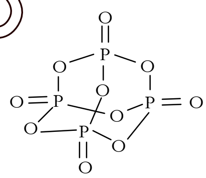
$O_2^- \Rightarrow \sigma(1s)^2\sigma^*(1s)^2\sigma(2s)^2\sigma^*(2s)^2\sigma(2p_z)^2\pi(2p_x)^2$
$\pi(2p_y)^2\pi^*(2p_x)^2\pi^*(2p_y)^1$; B.O. $= \frac{10-7}{2} = 1.5$

$O_2^{2-} \Rightarrow \sigma(1s)^2\sigma^*(1s)^2\sigma(2s)^2\sigma^*(2s)^2\sigma(2p_z)^2\pi(2p_x)^2$
$\pi(2p_y)^2\pi^*(2p_x)^2\pi^*(2p_y)^2$; B.O. $= \frac{10-8}{2} = 1$

Hence increasing order of bond order is
$O_2^{2-} < O_2^- < O_2 < O_2^+$

28. **(b) :** dsp^3 hybridisation gives square planar structure with *s*, p_x, p_y and $d_{x^2-y^2}$ orbitals with bond angles of 90°.

29. **(d) :**



No. of σ bonds = 16
No. of π bonds = 4

30. **(b) :** In molecules (a) $\left(CH_3-\overset{O}{\overset{\|}{C}}-OH\right)$, (c) $\left(CH_3-\overset{O}{\overset{\|}{C}}-CH_3\right)$ and (d) ($CH_2 = CH - CH_3$), the carbon atom has a multiple bond, only (b) has sp^3 hybridization.

31. **(a) :** Both undergoes sp^3 hybridization. The expected bond angle should be 109°28′ but actual bond angle is less than 109°28′ because of the repulsion between lone pair and bonded pairs due to which contraction occurs.

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CHAPTER 5

CHEMICAL THERMODYNAMICS

1. A piston filled with 0.04 mol of an ideal gas expands reversibly from 50.0 mL to 375 mL at a constant temperature of 37.0°C. As it does so, it absorbs 208 J of heat. The values of q and w for the process will be (R = 8.314 J/mol K) (ln 7.5 = 2.01)
(a) $q = +208$ J, $w = +208$ J
(b) $q = +208$ J, $w = -208$ J
(c) $q = -208$ J, $w = -208$ J
(d) $q = -208$ J, $w = +208$ J *(2013)*

2. The incorrect expression among the following is
(a) in isothermal process, $w_{\text{reversible}} = -nRT\ln\frac{V_f}{V_i}$
(b) $\ln K = \frac{\Delta H° - T\Delta S°}{RT}$
(c) $K = e^{-\Delta G°/RT}$
(d) $\frac{\Delta G_{\text{system}}}{\Delta S_{\text{total}}} = -T$ *(2012)*

3. The entropy change involved in the isothermal reversible expansion of 2 moles of an ideal gas from a volume of 10 dm³ to a volume of 100 dm³ at 27°C is
(a) 38.3 J mol⁻¹ K⁻¹ (b) 35.8 J mol⁻¹ K⁻¹
(c) 32.3 J mol⁻¹ K⁻¹ (d) 42.3 J mol⁻¹ K⁻¹ *(2011)*

4. The standard enthalpy of formation of NH_3 is –46 kJ mol⁻¹. If the enthalpy of formation of H_2 from its atoms is – 436 kJ mol⁻¹ and that of N_2 is –712 kJ mol⁻¹, the average bond enthalpy of N—H bond in NH_3 is
(a) – 1102 kJ mol⁻¹ (b) – 964 kJ mol⁻¹
(c) + 352 kJ mol⁻¹ (d) + 1056 kJ mol⁻¹ *(2010)*

5. For a particular reversible reaction at temperature T, ΔH and ΔS were found to be both +ve. If T_e is the temperature at equilibrium, the reaction would be spontaneous when
(a) $T = T_e$ (b) $T_e > T$
(c) $T > T_e$ (d) T_e is 5 times T *(2010)*

6. On the basis of the following thermochemical data : ($\Delta_f G°$ $H^+_{(aq)} = 0$)
$H_2O_{(l)} \rightarrow H^+_{(aq)} + OH^-_{(aq)}$; $\Delta H = 57.32$ kJ
$H_{2(g)} + \frac{1}{2}O_{2(g)} \rightarrow H_2O_{(l)}$; $\Delta H = -286.2$ kJ
The value of enthalpy of formation of OH^- ion at 25°C is
(a) – 22.88 kJ (b) – 228.88 kJ
(c) + 228.88 kJ (d) – 343.52 kJ *(2009)*

7. In a fuel cell methanol is used as fuel and oxygen gas is used as an oxidizer. The reaction is

$$CH_3OH_{(l)} + \frac{3}{2}O_{2(g)} \longrightarrow CO_{2(g)} + 2H_2O_{(l)}.$$

At 298 K standard Gibb's energies of formation for $CH_3OH_{(l)}$, $H_2O_{(l)}$ and $CO_{2(g)}$ are –166.2, –237.2 and –394.4 kJ mol⁻¹ respectively. If standard enthalpy of combustion of methanol is –726 kJ mol⁻¹, efficiency of the fuel cell will be
(a) 80 % (b) 87%
(c) 90% (d) 97% *(2009)*

8. Standard entropy of X_2, Y_2 and XY_3 are 60, 40 and 50 J K⁻¹ mol⁻¹, respectively. For the reaction, $1/2\ X_2 + 3/2\ Y_2 \rightarrow XY_3$, $\Delta H = -30$ kJ, to be at equilibrium, the temperature will be
(a) 1000 K (b) 1250 K
(c) 500 K (d) 750 K *(2008)*

9. Oxidising power of chlorine in aqueous solution can be determined by the parameters indicated below:

$$1/2\ Cl_{2(g)} \xrightarrow{1/2\ \Delta_{diss}H^\ominus} Cl_{(g)} \xrightarrow{\Delta_{eg}H^\ominus} Cl^-_{(g)} \xrightarrow{\Delta_{hyd}H^\ominus} Cl^-_{(aq)}$$

The energy involved in the conversion of $1/2\ Cl_{2(g)}$ to $Cl^-_{(aq)}$ (using data, $\Delta_{diss}H^\ominus_{Cl_2} = 240$ kJ mol⁻¹, $\Delta_{eg}H^\ominus_{Cl} = -349$ kJ mol⁻¹, $\Delta_{hyd}H^\ominus_{Cl^-} = -381$ kJ mol⁻¹) will be
(a) +120 kJ mol⁻¹ (b) +152 kJ mol⁻¹
(c) –610 kJ mol⁻¹ (d) –850 kJ mol⁻¹ *(2008)*

10. Identify the correct statement regarding a spontaneous process:
(a) Lowering of energy in the reaction process is the only criterion for spontaneity.
(b) For a spontaneous process in an isolated system, the change in entropy is positive.
(c) Endothermic processes are never spontaneous.
(d) Exothermic processes are always spontaneous. *(2007)*

11. Assuming that water vapour is an ideal gas, the internal energy change (ΔU) when 1 mol of water is vapourised at 1 bar pressure and 100°C, (given : molar enthalpy of vapourisation of water at 1 bar and 373 K = 41 kJ mol⁻¹ and R = 8.3 J mol⁻¹ K⁻¹) will be

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(a) 41.00 kJ mol^{-1} (b) 4.100 kJ mol^{-1}
(c) 3.7904 kJ mol^{-1}
(d) 37.904 kJ mol^{-1} *(2007)*

12. In conversion of limestone to lime,

$$CaCO_{3(s)} \rightarrow CaO_{(s)} + CO_{2(g)}$$

the values of $\Delta H°$ and $\Delta S°$ are +179.1 kJ mol^{-1} and 160.2 J/K respectively at 298 K and 1 bar. Assuming that $\Delta H°$ and $\Delta S°$ do not change with temperature, temperature above which conversion of limestone to lime will be spontaneous is
(a) 1118 K (b) 1008 K
(c) 1200 K (d) 845 K *(2007)*

13. $(\Delta H - \Delta U)$ for the formation of carbon monoxide (CO) from its elements at 298 K is

(R = 8.314 JK^{-1} mol^{-1})

(a) –1238.78 J mol^{-1} (b) 1238.78 J mol^{-1}
(c) –2477.57 J mol^{-1} (d) 2477.57 J mol^{-1} *(2006)*

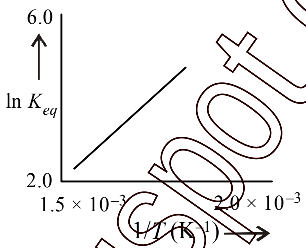
14. The enthalpy changes for the following processes are listed below:
$Cl_{2(g)} = 2Cl_{(g)}$, 242.3 kJ mol^{-1}
$I_{2(g)} = 2I_{(g)}$, 151.0 kJ mol^{-1}
$ICl_{(g)} = I_{(g)} + Cl_{(g)}$, 211.3 kJ mol^{-1}
$I_{2(s)} = I_{2(g)}$, 62.76 kJ mol^{-1}
Given that the standard states for iodine and chlorine are $I_{2(s)}$ and $Cl_{2(g)}$, the standard enthalpy of formation for $ICl_{(g)}$ is
(a) – 14.6 kJ mol^{-1} (b) –16.8 kJ mol^{-1}
(c) +16.8 kJ mol^{-1} (d) +244.8 kJ mol^{-1} *(2006)*

15. An ideal gas is allowed to expand both reversibly and irreversibly in an isolated system. If T_i is the initial temperature and T_f is the final temperature, which of the following statements is correct?
(a) $(T_f)_{irrev} > (T_f)_{rev}$
(b) $T_f > T_i$ for reversible process but $T_f = T_i$ for irreversible process
(c) $(T_f)_{rev} = (T_f)_{irrev}$
(d) $T_f = T_i$ for both reversible and irreversible processes. *(2006)*

16. The standard enthalpy of formation $(\Delta H_f°)$ at 298 K for methane, $CH_{4(g)}$ is –74.8 kJ mol^{-1}. The additional information required to determine the average energy for C – H bond formation would be
(a) the dissociation energy of H_2 and enthalpy of sublimation of carbon
(b) latent heat of vaporisation of methane
(c) the first four ionisation energies of carbon and electron gain enthalpy of hydrogen
(d) the dissociation energy of hydrogen molecule, H_2. *(2006)*

17. If the bond dissociation energies of XY, X_2 and Y_2 (all diatomic molecules) are in the ratio of 1 : 1 : 0.5 and ΔH_f for the formation of XY is –200 kJ mol^{-1}. The bond dissociation energy of X_2 will be
(a) 100 kJ mol^{-1} (b) 200 kJ mol^{-1}
(c) 800 kJ mol^{-1} (d) 400 kJ mol^{-1} *(2005)*

18. A schematic plot of ln K_{eq} versus inverse of temperature for a reaction is shown in the figure. The reaction must be



(a) exothermic
(b) endothermic
(c) one with negligible enthalpy change
(d) highly spontaneous at ordinary temperature. *(2005)*

19. Consider the reaction: $N_2 + 3H_2 \rightarrow 2NH_3$ carried out at constant temperature and pressure. If ΔH and ΔU are the enthalpy and internal energy changes for the reaction, which of the following expressions is true?
(a) $\Delta H = 0$ (b) $\Delta H = \Delta U$
(c) $\Delta H < \Delta U$ (d) $\Delta H > \Delta U$
(2005)

20. For a spontaneous reaction the ΔG, equilibrium constant (K) and $E°_{cell}$ will be respectively
(a) –ve, >1, +ve (b) +ve, >1, –ve
(c) –ve, <1, –ve (d) –ve, >1, –ve *(2005)*

21. The enthalpies of combustion of carbon and carbon monoxide are –393.5 and –283 kJ mol^{-1} respectively. The enthalpy of formation of carbon monoxide per mole is
(a) 110.5 kJ (b) 676.5 kJ
(c) –676.5 kJ (d) –110.5 kJ *(2004)*

22. An ideal gas expands in volume from 1 × 10^{-3} m^3 to 1 × 10^{-2} m^3 at 300 K against a constant pressure of 1 × 10^5 Nm^{-2}. The work done is
(a) – 900 J (b) – 900 kJ
(c) 270 kJ (d) 900 kJ *(2004)*

23. The internal energy change when a system goes from state A to B is 40 kJ/mole. If the system goes from A to B by a reversible path and returns to state A by an irreversible path what would be the net change in internal energy?
(a) 40 kJ (b) > 40 kJ
(c) < 40 kJ (d) zero *(2003)*

24. In an irreversible process taking place at constant T and P and in which only pressure-volume work is being done, the change in Gibbs free energy (dG) and change in entropy (dS), satisfy the criteria
(a) $(dS)_{V,E} < 0$, $(dG)_{T,P} < 0$
(b) $(dS)_{V,E} > 0$, $(dG)_{T,P} < 0$

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(c) $(dS)_{V,E} = 0, (dG)_{T,P} = 0$
(d) $(dS)_{V,E} = 0, (dG)_{T,P} > 0$ *(2003)*

25. The enthalpy change for a reaction does not depend upon the
(a) physical states of reactants and products
(b) use of different reactants for the same product
(c) nature of intermediate reaction steps
(d) difference in initial or final temperatures of involved substances. *(2003)*

26. If at 298 K the bond energies of C – H, C – C, C = C and H – H bonds are respectively 414, 347, 615 and 435 kJ mol^{-1}, the value of enthalpy change for the reaction
$H_2C = CH_{2(g)} + H_{2(g)} \rightarrow H_3C - CH_{3(g)}$
at 298 K will be
(a) +250 kJ (b) –250 kJ
(c) +125 kJ (d) –125 kJ. *(2003)*

27. A heat engine absorbs heat Q_1 at temperature T_1 and heat Q_2 at temperature T_2. Work done by the engine is J $(Q_1 + Q_2)$. This data
(a) violates 1st law of thermodynamics
(b) violates 1st law of thermodynamics if Q_1 is –ve
(c) violates 1st law of thermodynamics if Q_2 is –ve
(d) does not violate 1st law of thermodynamics. *(2002)*

28. If an endothermic reaction is non-spontaneous at freezing point of water and becomes feasible at its boiling point, then
(a) ΔH is –ve, ΔS is +ve
(b) ΔH and ΔS both are +ve
(c) ΔH and ΔS both are –ve
(d) ΔH is +ve, ΔS is –ve *(2002)*

29. For the reactions,
$C + O_2 \rightarrow CO_2$; $\Delta H = -393$ J
$2Zn + O_2 \rightarrow 2ZnO$; $\Delta H = -412$ J
(a) carbon can oxidise Zn
(b) oxidation of carbon is not feasible
(c) oxidation of Zn is not feasible
(d) Zn can oxidise carbon. *(2002)*

Answer Key

1.	(b)	**2.**	(b)	**3.**	(a)	**4.**	(c)	**5.**	(c)	**6.**	(b)
7.	(d)	**8.**	(d)	**9.**	(c)	**10.**	(b)	**11.**	(d)	**12.**	(a)
13.	(a)	**14.**	(c)	**15.**	(a)	**16.**	(a)	**17.**	(c)	**18.**	(a)
19.	(c)	**20.**	(a)	**21.**	(d)	**22.**	(a)	**23.**	(d)	**24.**	(b)
25.	(c)	**26.**	(d)	**27.**	(d)	**28.**	(b)	**29.**	(d)		

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Explanations

1. **(b) :** As it absorbs heat,

$\therefore \quad q = +208$ J

$$w_{rev} = -2.303 nRT \log_{10}\left(\frac{V_2}{V_1}\right)$$

$$w_{rev} = -2.303 \times (0.04) \times 8.314 \times 310 \log_{10}\left(\frac{375}{50}\right)$$

$\therefore \quad w_{rev} = -207.76 \approx -208$ J

2. **(b) :** $\Delta G° = -RT \ln K$

$\Delta G° = \Delta H° - T\Delta S°$

$\Delta H° - T\Delta S° = -RT \ln K$

$$\ln K = -\left(\frac{\Delta H° - T\Delta S°}{RT}\right)$$

3. **(a) :** Entropy change for an isothermal process is

$$\Delta S = 2.303 nR \log\left(\frac{V_2}{V_1}\right)$$

$$\Delta S = 2.303 \times 2 \times 8.314 \times \log\left(\frac{100}{10}\right)$$

$= 38.294$ J mol^{-1} K^{-1} ≈ 38.3 J mol^{-1} K^{-1}

4. **(c) :** $1/2 N_2 + 3/2 H_2 \longrightarrow NH_3$

B.E. 712 436

$$\therefore \quad (\Delta H_f°)_{NH_3} = \left[\frac{1}{2} B.E._{N_2} + \frac{3}{2} B.E._{H_2} - 3B.E._{N-H}\right]$$

$$-46 = \left[\frac{1}{2} \times 712 + \frac{3}{2} \times 436 - 3B.E._{N-H}\right]$$

$-46 = 356 + 654 - 3B.E._{N-H}$

$3B.E._{N-H} = 1056$

$$B.E._{N-H} = \frac{1056}{3} = 352 \text{ kJ mol}^{-1}$$

5. **(c) :** According to Gibb's formula,

$$\Delta G = \Delta H - T\Delta S$$

Since ΔH and ΔS both are +ve, for $\Delta G < 0$, the value of $T > T_e$.

6. **(b) :** The reaction for the formation of $OH^-_{(aq)}$ is

$$H_{2(g)} + \frac{1}{2} O_{2(g)} \longrightarrow H^+_{(aq)} + OH^-_{(aq)}$$

This is obtained by adding the two given equations.

$\therefore$ ΔH for the above reaction $= 57.32 + (-286.2)$

$= -228.88$ kJ

7. **(d) :** For the given reaction,

$$CH_3OH_{(l)} + \frac{3}{2} O_{2(g)} \longrightarrow CO_{2(g)} + 2H_2O_{(l)},$$

$\Delta H = -726$ kJ mol^{-1}

also, $\Delta G°_f[CH_3OH_{(l)}] = -166.2$ kJ mol^{-1}

$\Delta G°_f[H_2O_{(l)}] = -237.2$ kJ mol^{-1}

and $\Delta G°_f[CO_{2(g)}] = -394.4$ kJ mol^{-1}

Now, $\Delta G°_{reaction} = \sum \Delta G°_{f\,products} - \sum \Delta G°_{f\,reactants}$

$= [-394.4 + 2 \times (-237.2)] - (-166.2)$

$= -702.6$ kJ mol^{-1}

$$\% \text{ Efficiency} = \frac{\Delta G}{\Delta H} \times 100 = \frac{-702.6}{-726} \times 100$$

$= 96.77\%$

$\therefore$ Efficiency $\approx 97\%$

8. **(d) :** $\frac{1}{2} X_2 + \frac{3}{2} Y_2 \rightarrow XY_3$

$$\Delta S^0_{reaction} = \Delta S^0_{products} - \Delta S^0_{reactants}$$

$$\therefore \Delta S^0_{reaction} = \Delta S^0_{XY_3} - \frac{1}{2} \Delta S^0_{X_2} - \frac{3}{2} \Delta S^0_{Y_2}$$

$$= 50 - \frac{1}{2} \times 60 - \frac{3}{2} \times 40 = -40 \text{ J K}^{-1} \text{ mol}^{-1}$$

Using equation, $\Delta G = \Delta H - T\Delta S$

We have $\Delta H = -30$ kJ, $\Delta S = -40$ J K^{-1} mol^{-1} and at equilibrium $\Delta G = 0$. Therefore

$$T = \frac{\Delta H}{\Delta S} = \frac{-30 \times 1000}{-40} = 750 \text{ K}$$

9. **(c) :** $\frac{1}{2} Cl_{2(g)} \rightarrow Cl_{(g)}$;

$$\Delta H_1 = \frac{1}{2} \Delta_{diss} H^\ominus_{Cl_2} = \frac{240}{2} = 120 \text{ kJ mol}^{-1}$$

$Cl_{(g)} \rightarrow Cl^-_{(g)}$; $\Delta H_2 = \Delta_{eg} H^\ominus_{Cl} = -349$ kJ mol^{-1}

$Cl^-_{(g)} + aq \rightarrow Cl^-_{(aq)}$;

$\Delta H_3 = \Delta_{hyd} H^\ominus = -381$ kJ mol^{-1}

The required reaction is $\frac{1}{2} Cl_{2(g)} \rightarrow Cl^-_{(aq)}$; ΔH

Then $\Delta H = \frac{1}{2} \Delta_{diss} H^\ominus + \Delta_{eg} H^\ominus + \Delta_{hyd} H^\ominus$

$= 120 + (-349) + (-381) = -610$ kJ mol^{-1}

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10. **(b) :** In an isolated system, there is neither exchange of energy nor matter between the system and surrounding. For a spontaneous process in an isolated system, the change in entropy is positive, *i.e.* $\Delta S > 0$.
Most of the spontaneous chemical reactions are exothermic. A number of endothermic reactions are spontaneous *e.g.* melting of ice (an endothermic process) is a spontaneous reaction.
The two factors which are responsible for the spontaneity of a process are
(i) tendency to acquire minimum energy
(ii) tendency to acquire maximum randomness.

11. **(d) :** $\Delta U = \Delta H - \Delta nRT$
$= 41000 - 1 \times 8.314 \times 373$
$= 41000 - 3101.122$
$= 37898.878\ \text{J mol}^{-1} = 37.9\ \text{kJ mol}^{-1}$

12. **(a) :** For $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$
For a spontaneous process $\Delta G^\circ < 0$
i.e. $\Delta H^\circ - T\Delta S^\circ < 0$
or $\Delta H^\circ < T\Delta S^\circ$ or, $T\Delta S^\circ > \Delta H^\circ$

or $T > \dfrac{\Delta H^\circ}{\Delta S^\circ}$ *i.e.* $T > \dfrac{179.1 \times 1000}{160.2}$

or $T > 1117.9\ \text{K} \approx 1118\ \text{K}$

13. **(a) :** $\Delta H - \Delta U = \Delta n_g RT$

$C + \frac{1}{2} O_2 \rightarrow CO$

$$\Delta n_g = 1 - \left(1 + \frac{1}{2}\right) = -\frac{1}{2}$$

$$\Delta H - \Delta U = -\frac{1}{2} \times 8.314 \times 298 = -1238.78\ \text{J mol}^{-1}$$

14. **(c) :** $\frac{1}{2} I_{2\,(s)} + \frac{1}{2} Cl_{2\,(g)} \rightarrow ICl_{(g)}$

$$\Delta H_{ICl_{(g)}} = \left[\frac{1}{2}\Delta H_{I_2(s)\rightarrow I_2(g)} + \frac{1}{2}\Delta H_{I-I} + \frac{1}{2}\Delta H_{Cl-Cl}\right] - [\Delta H_{I-Cl}]$$

$$= \left[\frac{1}{2} \times 62.76 + \frac{1}{2} \times 151.0 + \frac{1}{2} \times 242.3\right] - [211.3]$$

$= [31.38 + 75.5 + 121.15] - 211.3$
$= 228.03 - 211.3 = 16.73\ \text{kJ/mol}$

15. **(a) :** If a gas was to expand by a certain volume reversibly, then it would do a certain amount of work on the surroundings. If it was to expand irreversibly it would have to do the same amount of work on the surroundings to expand in volume, but it would also have to do work against frictional forces. Therefore the amount of work have greater modulus but –ve sign.
$W_{irrev.} > W_{reve.}$; $(T_f)_{irrev.} > (T_f)_{rev.}$

16. **(a) :** $C + 2H_2 \rightarrow CH_4$; $\Delta H^\circ = -74.8\ \text{kJ mol}^{-1}$
In order to calculate average energy for C – H bond formation we should know the following data.
$C_{(graphite)} \rightarrow C_{(g)}$; ΔH_f° = enthalpy of sublimation of carbon
$H_{2(g)} \rightarrow 2H_{(g)}$; ΔH° = bond dissociation energy of H_2

17. **(c) :** Let the bond dissociation energy of XY, X_2 and Y_2 be x kJ mol^{-1}, x kJ mol^{-1} and $0.5x$ kJ mol^{-1} respectively.

$$\frac{1}{2}X_2 + \frac{1}{2}Y_2 \rightarrow XY;\ \Delta H_f = -200\ \text{kJ mol}^{-1}$$

$\Delta H_{reaction}$ = [(sum of bond dissociation energy of all reactants) – (sum of bond dissociation energy of product)]

$$= \left[\frac{1}{2}\Delta H_{X_2} + \frac{1}{2}\Delta H_{Y_2} - \Delta H_{XY}\right]$$

$$= \frac{x}{2} + \frac{0.5x}{2} - x = -200$$

$$\therefore\ x = \frac{200}{0.25} = 800\ \text{kJ mol}^{-1}$$

18. **(a) :** $\ln\dfrac{K_2}{K_1} = \dfrac{\Delta H}{R}\left[\dfrac{1}{T_1} - \dfrac{1}{T_2}\right]$

$$\ln\frac{6}{2} = \frac{\Delta H}{R}[1.5 \times 10^{-3} - 2 \times 10^{-3}]$$

$$\text{or,}\quad \ln 3 = \frac{\Delta H}{R} \times (-0.5 \times 10^{-3})$$

ΔH of reaction comes out to be negative. Hence reaction is exothermic.

19. **(c) :** $N_2 + 3H_2 \rightarrow 2NH_3$
$\Delta n = 2 - 4 = -2$
$\Delta H = \Delta U + \Delta nRT = \Delta U - 2RT$
$\therefore\ \Delta H < \Delta U$

20. **(a) :** For spontaneous process, ΔG = –ve
Now $\Delta G = -RT \ln K$
When $K > 1$, ΔG = –ve
Again $\Delta G^\circ = -nFE^\circ$
When E° = +ve, ΔG° = –ve

21. **(d) :** $C_{(s)} + O_{2(g)} \rightarrow CO_{2(g)}$; $\Delta H = -393.5\ \text{kJ mol}^{-1}$ .. (i)

$CO_{(g)} + \frac{1}{2}O_{2(g)} \rightarrow CO_{2(g)}$; $\Delta H = -283\ \text{kJ mol}^{-1}$ ... (ii)

On subtraction equation (ii) from equation (i), we get
$C_{(s)} + O_{2(g)} \rightarrow CO_{(g)}$; $\Delta H = -110.5\ \text{kJ mol}^{-1}$
The enthalpy of formation of carbon monoxide per mole
$= -110.5\ \text{kJ mol}^{-1}$

22. **(a) :** $W = -P\Delta V$
$= -1 \times 10^5(1 \times 10^{-2} - 1 \times 10^{-3})$
$= -1 \times 10^5 \times 9 \times 10^{-3} = -900\ \text{J}$

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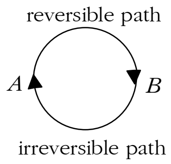
23. **(d) :**

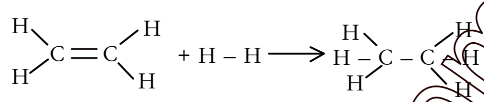


We know that for a cyclic process the net change in internal energy is equal to zero and change in the internal energy does not depend on the path by which the final state is reached.

24. **(b) :** For spontaneity, change in entropy (dS) must be positive, means it should be greater than zero.
Change in Gibbs free energy (dG) must be negative means that it should be lesser than zero. $(dS)_{V, E} > 0$, $(dG)_{T, P} < 0$.

25. **(c) :** This is according to Hess's law.

26. **(d) :**



$\Delta H_{\text{Reaction}} = \Sigma BE_{\text{reactant}} - \Sigma BE_{\text{product}}$
$= 4 \times 414 + 615 + 435 - (6 \times 414 + 347)$
$= 2706 - 2831$
$= -125$ kJ

27. **(d) :** It does not violate first law of thermodynamics but violates second law of thermodynamics.

28. **(b) :** For endothermic reaction, ΔH = +ve
Now, $\Delta G = \Delta H - T\Delta S$
For non-spontaneous reaction, ΔG should be positive
Now ΔG is positive at low temperature if ΔH is positive.
ΔG is negative at high temperature if ΔS is positive.

29. **(d) :** ΔH = negative shows that the reaction is spontaneous. Higher value for ΔH shows that the reaction is more feasible.

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CHAPTER 6

SOLUTIONS

1. K_f for water is 1.86 K kg mol^{-1}. If your automobile radiator holds 1.0 kg of water, how many grams of ethylene glycol ($C_2H_6O_2$) must you add to get the freezing point of the solution lowered to –2.8°C?
(a) 93 g (b) 39 g
(c) 27 g (d) 72 g *(2012)*

2. The density of a solution prepared by dissolving 120 g of urea (mol. mass = 60 u) in 1000 g of water is 1.15 g/mL. The molarity of this solution is
(a) 1.78 M (b) 1.02 M
(c) 2.05 M (d) 0.50 M *(2012)*

3. The degree of dissociation (α) of a weak electrolyte, A^xB^y is related to van't Hoff factor (i) by the expression
(a) $\alpha = \frac{i-1}{(x+y-1)}$ (b) $\alpha = \frac{i-1}{(x+y+1)}$
(c) $\alpha = \frac{(x+y-1)}{i-1}$ (d) $\alpha = \frac{(x+y+1)}{i-1}$ *(2011)*

4. Ethylene glycol is used as an antifreeze in a cold climate. Mass of ethylene glycol which should be added to 4 kg of water to prevent it from freezing at – 6°C will be : (K_f for water = 1.86 K kg mol^{-1}, and molar mass of ethylene glycol = 62 g mol^{-1})
(a) 804.32 g (b) 204.30 g
(c) 400.00 g (d) 304.60 g *(2011)*

5. A 5.2 molal aqueous solution of methyl alcohol, CH_3OH, is supplied. What is the mole fraction of methyl alcohol in the solution?
(a) 0.100 (b) 0.190
(c) 0.086 (d) 0.050 *(2011)*

6. On mixing, heptane and octane form an ideal solution. At 373 K, the vapour pressure of the two liquid components (heptane and octane) are 105 kPa and 45 kPa respectively. Vapour pressure of the solution obtained by mixing 25.0 g of heptane and 35 g of octane will be (molar mass of heptane = 100 g mol^{-1} and of octane = 114 g mol^{-1})
(a) 144.5 kPa (b) 72.0 kPa
(c) 36.1 kPa (d) 96.2 kPa *(2010)*

7. If sodium sulphate is considered to be completely dissociated into cations and anions in aqueous solution, the change in freezing point of water (ΔT_f), when 0.01 mol of sodium sulphate is dissolved in 1 kg of water, is (K_f = 1.86 K kg mol^{-1})
(a) 0.0186 K (b) 0.0372 K
(c) 0.0558 K (d) 0.0744 K *(2010)*

8. A binary liquid solution is prepared by mixing n-heptane and ethanol. Which one of the following statements is correct regarding the behaviour of the solution?
(a) The solution formed is an ideal solution.
(b) The solution is non-ideal, showing +ve deviation from Raoult's law.
(c) The solution is non-ideal, showing –ve deviation from Raoult's law.
(d) n-heptane shows +ve deviation while ethanol shows –ve deviation from Raoult's law.
(2009)

9. Two liquids X and Y form an ideal solution. At 300 K, vapour pressure of the solution containing 1 mol of X and 3 mol of Y is 550 mm Hg. At the same temperature, if 1 mol of Y is further added to this solution, vapour pressure of the solution increases by 10 mm Hg. Vapour pressure (in mm Hg) of X and Y in their pure states will be, respectively
(a) 200 and 300 (b) 300 and 400
(c) 400 and 600 (d) 500 and 600 *(2009)*

10. The vapour pressure of water at 20°C is 17.5 mm Hg. If 18 g of glucose ($C_6H_{12}O_6$) is added to 178.2 g of water at 20°C, the vapour pressure of the resulting solution will be
(a) 17.325 mm Hg (b) 17.675 mm Hg
(c) 15.750 mm Hg (d) 16.500 mm Hg
(2008)

11. At 80°C, the vapour pressure of pure liquid A is 520 mm of Hg and that of pure liquid B is 1000 mm of Hg. If a mixture solution of A and B boils at 80°C and 1 atm pressure, the amount of A in the mixture is (1 atm = 760 mm of Hg)
(a) 50 mol percent (b) 52 mol percent
(c) 34 mol percent (d) 48 mol percent
(2008)

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12. A 5.25% solution of a substance is isotonic with a 1.5% solution of urea (molar mass = 60 g mol^{-1}) in the same solvent. If the densities of both the solutions are assumed to be equal to 1.0 g cm^{-3}, molar mass of the substance will be
(a) 210.0 g mol^{-1} (b) 90.0 g mol^{-1}
(c) 115.0 g mol^{-1} (d) 105.0 g mol^{-1}
(2007)

13. A mixture of ethyl alcohol and propyl alcohol has a vapour pressure of 290 mm at 300 K. The vapour pressure of propyl alcohol is 200 nm. If the mole fraction of ethyl alcohol is 0.6, its vapour pressure (in mm) at the same temperature will be
(a) 360 (b) 350
(c) 300 (d) 700 *(2007)*

14. The density (in g mL^{-1}) of a 3.60 M sulphuric acid solution that is 29% H_2SO_4 (molar mass = 98 g mol^{-1}) by mass will be
(a) 1.45 (b) 1.64
(c) 1.88 (d) 1.22 *(2007)*

15. 18 g of glucose ($C_6H_{12}O_6$) is added to 178.2 g of water. The vapour pressure of water for this aqueous solution at 100°C is
(a) 759.00 torr (b) 7.60 torr
(c) 76.00 torr (d) 752.40 torr *(2006)*

16. Density of a 2.05 M solution of acetic acid in water is 1.02 g/mL. The molality of the solution is
(a) 1.14 mol kg^{-1} (b) 3.28 mol kg^{-1}
(c) 2.28 mol kg^{-1} (d) 0.44 mol kg^{-1} *(2006)*

17. Equimolal solutions in the same solvent have
(a) same boiling point but different freezing point
(b) same freezing point but different boiling point
(c) same boiling and same freezing points
(d) different boiling and different freezing points. *(2005)*

18. Two solutions of a substance (non electrolyte) are mixed in the following manner. 480 mL of 1.5 M first solution + 520 mL of 1.2 M second solution. What is the molarity of the final mixture?
(a) 1.20 M (b) 1.50 M
(c) 1.344 M (d) 2.70 M *(2005)*

19. Benzene and toluene form nearly ideal solutions. At 20°C, the vapour pressure of benzene is 75 torr and that of toluene is 22 torr. The partial vapour pressure of benzene at 20°C for a solution containing 78 g of benzene and 46 g of toluene in torr is
(a) 50 (b) 25
(c) 37.5 (d) 53.5 *(2005)*

20. If α is the degree of dissociation of Na_2SO_4, the vant Hoff's factor (i) used for calculating the molecular mass is
(a) 1 + α (b) 1 – α
(c) 1 + 2α (d) 1 – 2α *(2005)*

21. Which one of the following statements is false?
(a) Raoult's law states that the vapour pressure of a component over a solution is proportional to its mole fraction.
(b) The osmotic pressure (π) of a solution is given by the equation ($\pi = MRT$, where M is the molarity of the solution.
(c) The correct order of osmotic pressure for 0.01 M aqueous solution of each compound is $BaCl_2 > KCl > CH_3COOH >$ sucrose.
(d) Two sucrose solutions of same molality prepared in different solvents will have the same freezing point depression. *(2004)*

22. Which of the following liquid pairs shows a positive deviation from Raoult's law?
(a) Water - hydrochloric acid
(b) Benzene - methanol
(c) Water - nitric acid
(d) Acetone - chloroform *(2004)*

23. To neutralise completely 20 mL of 0.1 M aqueous solution of phosphorous acid (H_3PO_3), the volume of 0.1 M aqueous KOH solution required is
(a) 10 mL (b) 20 mL
(c) 40 mL (d) 60 mL *(2004)*

24. 6.02×10^{20} molecules of urea are present in 100 ml of its solution. The concentration of urea solution is
(a) 0.001 M (b) 0.01 M
(c) 0.02 M (d) 0.1 M *(2004)*

25. Which one of the following aqueous solutions will exhibit highest boiling point?
(a) 0.01 M Na_2SO_4 (b) 0.01 M KNO_3
(c) 0.015 M urea (d) 0.015 M glucose *(2004)*

26. If liquids A and B form an ideal solution, the
(a) enthalpy of mixing is zero
(b) entropy of mixing is zero
(c) free energy of mixing is zero
(d) free energy as well as the entropy of mixing are each zero.
(2003)

27. 25 mL of a solution of barium hydroxide on titration with a 0.1 molar solution of hydrochloric acid gave a litre value of 35 mL. The molarity of barium hydroxide solution was
(a) 0.07 (b) 0.14
(c) 0.28 (d) 0.35 *(2003)*

28. In a 0.2 molal aqueous solution of a weak acid HX, the degree of ionization is 0.3. Taking K_f for water as 1.85, the freezing point of the solution will be nearest to
(a) – 0.480°C (b) – 0.360 °C
(c) – 0.260 °C (d) + 0.480 °C *(2003)*

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29. In mixture A and B components show –ve deviation as

(a) $\Delta V_{mix} > 0$

(b) $\Delta H_{mix} < 0$

(c) $A - B$ interaction is weaker than $A - A$ and $B - B$ interaction

(d) $A - B$ interaction is stronger than $A - A$ and $B - B$ interaction.

(2002)

30. Freezing point of an aqueous solution is (–0.186)°C. Elevation of boiling point of the same solution is K_b = 0.512°C, K_f = 1.86°C, find the increase in boiling point.

(a) 0.186°C

(b) 0.0512°C

(c) 0.092°C

(d) 0.2372°C

(2002)

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Answer Key

1.	(a)	**2.**	(c)	**3.**	(a)	**4.**	(a)	**5.**	(c)	**6.**	(b)
7.	(c)	**8.**	(b)	**9.**	(c)	**10.**	(a)	**11.**	(a)	**12.**	(a)
13.	(b)	**14.**	(d)	**15.**	(d)	**16.**	(c)	**17.**	(c)	**18.**	(c)
19.	(a)	**20.**	(c)	**21.**	(d)	**22.**	(b)	**23.**	(c)	**24.**	(b)
25.	(a)	**26.**	(a)	**27.**	(b)	**28.**	(a)	**29.**	(b,d)	**30.**	(b)

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ALL SUBJECTS CLASS 1 TO 12 STUDY MATERIAL WITH COMPLETE SOLUTIONS ARE AVAILABLE [154 of 353] [9440345996]

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Explanations

1. **(a) :** $K_f = 1.86$ K kg mol^{-1}
$\Delta T_f = 0 - (-2.8) = 2.8°C$
Mass of solvent = 1.0 kg
Mass of solute = ?
Molecular mass of solute = 62
$\Delta T_f = K_f \times m$

$$m = \frac{\frac{\text{Weight of solute}}{\text{Molecular mass of solute}}}{\text{Mass of solvent (g)}} \times 1000$$

$$m = \frac{w/62}{1000} \times 1000 = \frac{w}{62}$$

$\Delta T_f = K_f \times m$

$$2.8 = 1.86 \times \frac{w}{62} \Rightarrow w = \frac{62 \times 2.8}{1.86} = 93 \text{ g}$$

2. **(c) :** Mass of solute taken = 120 g
Molecular mass of solute = 60 u
Mass of solvent = 1000 g
Density of solution = 1.15 g/mL
Total mass of solution = 1000 + 120 = 1120 g

Volume of solution = $\frac{\text{Mass}}{\text{Density}} = \frac{1120}{1.15}$ mL

$$\text{Molarity} = \frac{\frac{\text{Mass of solute}}{\text{Molecular mass of solute}}}{\text{Volume of solution}} \times 1000$$

$$= \frac{120/60}{1120/1.15} \times 1000 = \frac{2 \times 1000 \times 1.15}{1120} = 2.05 \text{ M}$$

3. **(a) :** $A^xB^y \longrightarrow xA^{y+} + yB^{x-}$
$1 - \alpha \qquad x\alpha \qquad y\alpha$
$i = 1 - \alpha + x\alpha + y\alpha = 1 + \alpha(x + y - 1)$

$$\therefore \quad \alpha = \frac{i-1}{(x+y-1)}$$

4. **(a) :** $\Delta T_f = K_f \times m = K_f \times \frac{w_2 \times 1000}{w_1 \times m_2}$
w_1 and w_2 = wt. of solvent and solute
m_2 = molecular wt. of solute
$\Delta T_f = 0 - (-6) = 6$

$$\therefore \quad 6 = \frac{1.86 \times w_2 \times 1000}{4000 \times 62}$$

$$w_2 = \frac{6 \times 62 \times 4000}{1000 \times 1.86} = 800 \text{ g}$$

5. **(c) :** Mole fraction of solute = $\frac{n}{N+n}$
n = number of moles of solute
N = number of moles of solvent
Here solute is methyl alcohol, solvent is water.
Given $n = 5.2$, $N = \frac{1000}{18}$

$$\therefore \quad \text{Mole fraction} = \frac{5.2}{5.2 + \frac{1000}{18}}$$

$$= \frac{5.2 \times 18}{93.6 + 1000} = \frac{93.6}{1093.6} = 0.0855 \approx 0.086$$

6. **(b) :** Given, $p°_{\text{heptane}} = 105$ kPa
$p°_{\text{octane}} = 45$ kPa
$w_{\text{heptane}} = 25$ g
$w_{\text{octane}} = 35$ g

$$n_{\text{heptane}} = \frac{25}{100} = 0.25$$

$$n_{\text{octane}} = \frac{35}{114} = 0.30$$

$$x_{\text{heptane}} = \frac{0.25}{0.25 + 0.30} = 0.45$$

$$x_{\text{octane}} = \frac{0.30}{0.25 + 0.30} = 0.54$$

$p_{\text{Total}} = x_{\text{heptane}}\, p°_{\text{heptane}} + x_{\text{octane}}\, p°_{\text{octane}}$
$= 0.45 \times 105 + 0.54 \times 45$
$= 47.25 + 24.3 = 71.55 \approx 72$ kPa

7. **(c) :** Depression in freezing point, $\Delta T_f = i \times K_f \times m$
For sodium sulphate, $i = 3$

$$m = \frac{0.01}{1 \text{ kg}} = 0.01 \text{ m.}$$

Given, $K_f = 1.86$ K kg mol^{-1}
$\therefore \quad \Delta T_f = 3 \times 1.86 \times 0.01 = 0.0558$ K

8. **(b) :** The solution containing n-heptane and ethanol shows non-ideal behaviour with positive deviation from Raoult's law. This is because the ethanol molecules are held together by strong H-bonds, however the forces between n-heptane and ethanol are not very strong, as a result they easily vapourise showing higher vapour presure than expected.

9. **(c) :** $P_T = p°_X x_X + p°_Y x_Y$
where, P_T = Total presure
$p°_X$ = Vapour pressure of X in pure state
$p°_Y$ = Vapour pressure of Y in pure state
x_X = Mole fraction of X = 1/4
x_Y = Mole fraction of Y = 3/4
(i) When $T = 300$ K, $P_T = 550$ mm Hg

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$\therefore\ 550 = p_X^\circ\left(\frac{1}{4}\right) + p_Y^\circ\left(\frac{3}{4}\right)$

$\Rightarrow\ p_X^\circ + 3p_Y^\circ = 2200$ .....(1)

(ii) When at $T = 300$ K, 1 mole of Y is added,
$P_T = (550 + 10)$ mm Hg
$\therefore\ x_X = 1/5$ and $x_Y = 4/5$

$\Rightarrow\ 560 = p_X^\circ\left(\frac{1}{5}\right) + p_Y^\circ\left(\frac{4}{5}\right)$

or $p_X^\circ + 4p_Y^\circ = 2800$ ....(2)

On solving equations (1) and (2), we get
$p^\circ_Y = 600$ mm Hg and $p^\circ_X = 400$ mm Hg

10. **(a) :** In solution containing non-volatile solute, pressure is directly proportional to its mole fraction.
P_{solution} = vapour pressure of its pure component × mole fraction in solution

$\therefore\ P_{\text{sol}} = P^\circ X_{\text{solvent}}$
Let A be the solute and B the solvent

$$\therefore\ X_B = \frac{n_B}{n_A + n_B} = \frac{\frac{178.2}{18}}{\frac{18}{180} + \frac{178.2}{18}}$$

$$X_B = \frac{9.9}{9.94} = 0.99$$

Now $P_{\text{solution}} = P^\circ X_{\text{solvent}} = 17.5 \times 0.99$
$P_{\text{solution}} = 17.325$

11. **(a) :** We have, $P_A^\circ = 520$ mm Hg
and $P_B^\circ = 1000$ mm Hg
Let mole fraction of A in solution $= X_A$
and mole fraction of B in solution $= X_B$
Then, at 1 atm pressure *i.e.* at 760 mm Hg

$P_A^\circ X_A + P_B^\circ X_B = 760$ mm Hg

$P_A^\circ X_A + P_B^\circ (1 - X_A) = 760$ mm Hg

$\Rightarrow\ 520\ X_A + 1000 - 1000\ X_A = 760$ mm Hg

$\Rightarrow\ X_A = \frac{1}{2}$ or 50 mol percent

12. **(a) :** Isotonic solutions have same osmotic pressure.
$\pi_1 = C_1RT,\ \ \pi_2 = C_2RT$

For isotonic solution, $\pi_1 = \pi_2$

$\therefore\ C_1 = C_2$

or, $\frac{1.5/60}{V} = \frac{5.25/M}{V}$

[Where M = molecular weight of the substance]

or, $\frac{1.5}{60} = \frac{5.25}{M}$ or $M = 210$

13. **(b) :** According to Raoult's law,

$$P = P_A + P_B = P_A^\circ x_A + P_B^\circ x_B$$

or $290 = P_A^\circ \times (0.6) + 200 \times (1 - 0.6)$

or $290 = 0.6 \times P_A^\circ + 0.4 \times 200$

or $P_A^\circ = 350$ mm

14. **(d) :** 3.6 M solution means 3.6 mole of H_2SO_4 is present in 1000 mL of solution.
$\therefore$ Mass of 3.6 moles of H_2SO_4 = 3.6 × 98 g = 352.8 g
$\therefore$ Mass of H_2SO_4 in 1000 mL of solution = 352.8 g
Given, 29 g of H_2SO_4 is present in 100 g of solution
$\therefore$ 352.8 g of H_2SO_4 is present in

$\frac{100}{29} \times 352.8 = 1216$ g of solution

Now, density $= \frac{\text{Mass}}{\text{Volume}} = \frac{1216}{1000} = 1.216$ g/mL

15. **(d) :** $\frac{p^\circ - p_s}{p_s} = \frac{n}{N}$

$$\frac{760 - p_s}{p_s} = \frac{18/180}{178.2/18} = \frac{1/10}{9.9}$$

$\Rightarrow\ 760 - p_s = \frac{1}{99}p_s \ \Rightarrow\ 760 \times 99 - 99p_s = p_s$

$\Rightarrow\ 100p_s = 760 \times 99 \ \Rightarrow\ p_s = \frac{760 \times 99}{100} = 752.4$ torr

16. **(c) :** Molality, $m = \frac{M}{1000d - MM_2} \times 1000$

where M = molarity, d = density, M_2 = molecular mass

$m = \frac{2.05}{1000 \times 1.02 - 2.05 \times 60} = \frac{2.05}{897}$

$= 2.28 \times 10^{-3}$ mol g^{-1} = 2.28 mol kg^{-1}

17. **(c) :** According to Raoult's law equimolal solutions of all the substances in the same solvent will show equal elevation in boiling points as well as equal depression in freezing point.

18. **(c) :** Total millimoles of solute
= 480 × 1.5 + 520 × 1.2 = 720 + 624 = 1344
Total volume = 480 + 520 = 1000

Molarity of the final mixture $= \frac{1344}{1000} = 1.344$ M

19. **(a) :** According to Raoult's law, $P_B = P^\circ{}_B X_B$
$P_B^\circ = 75$ torr

$$X_B = \frac{78/78}{(78/78) + (46/92)} = \frac{1}{1 + 0.5} = \frac{1}{1.5}$$

$$P_B = 75 \times \frac{1}{1.5} = 50 \text{ torr}$$

20. **(c) :** $Na_2SO_4 \rightleftharpoons 2Na^+ + SO_4^{2-}$

1	0	0
$1 - \alpha$	2α	α

Vant Hoff factor (i) $= \frac{1 - \alpha + 2\alpha + \alpha}{1} = 1 + 2\alpha$

21. **(d) :** The extent of depression in freezing point varies with the number of solute particles for a fixed solvent only and it's a characteristic feature of the nature of solvent also.
$\Delta T_f = k_f \times m$
For different solvents, value of k_f is also different. So, for two different solvents the extent of depression may vary even if number of solute particles be dissolved in them.

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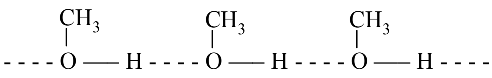
22. (b) : In solutions showing positive deviation, the observed vapour pressure of each component and total vapour pressure are greater than predicted by Raoult's law, *i.e.*

$$p_A > p_A^\circ x_A;\ p_B > p_B^\circ x_B;\ p > p_A + p_B$$

In solution of methanol and benzene, methanol molecules are held together due to hydrogen bonding as shown below:



On adding benzene, the benzene molecules get in between the molecules of methanol, thus breaking the hydrogen bonds. As the resulting solution has weaker intermolecular attractions, the escaping tendency of alcohol and benzene molecules from the solution increases. Consequently the vapour pressure of the solution is greater than the vapour pressure as expected from Raoult's law.

23. (c) : H_3PO_3 is a dibasic acid.

N_1V_1 (acid) $= N_2V_2$ (base)

$0.1 \times 2 \times 20 = 0.1 \times 1 \times V_2$

$\therefore\ V_2 = \frac{0.1 \times 2 \times 20}{0.1 \times 1} = 40$ mL

24. (b) : Moles of urea $= \frac{6.02 \times 10^{20}}{6.02 \times 10^{23}} = 10^{-3}$ moles

Concentration (molarity) of solution

$= \frac{\text{no. of moles of solute}}{\text{no. of litres of solution}} = \frac{10^{-3}}{100} \times 1000 = 0.01$ M

25. (a) : Elevation in boiling point is a colligative property which depends upon the number of solute particles.

Greater the number of solute particles in a solution, higher the extent of elevation in boiling point.

$$Na_2SO_4 \rightarrow 2Na^+ + SO_4^{2-}$$

26. (a) : For ideal solutions, $\Delta H_{mix} = 0$, neither heat is evolved nor absorbed during dissolution.

27. (b) : $Ba(OH)_2$ $\quad$ HCl

$M_1V_1 = M_2V_2$

$M_1 \times 25 = 0.1 \times 35$

or, $M_1 = \frac{0.1 \times 35}{25} = 0.14$

28. (a) : $HX \rightleftharpoons H^+ + X^-$

1	0	0
1 – 0.3	0.3	0.3

Total number of moles after dissociation

$= 1 - 0.3 + 0.3 + 0.3 = 1.3$

$$\frac{K_f\,(\text{observed})}{K_f\,(\text{experimental})} = \frac{\text{no. of moles after dissociation}}{\text{no. of moles before dissociation}}$$

or, $\frac{K_f\,(\text{observed})}{1.85} = \frac{1.3}{1}$

or, K_f (observed) $= 1.85 \times 1.3 = 2.405$

$\Delta T_f = K_f \times$ molality $= 2.405 \times 0.2 = 0.4810$

Freezing point of solution $= 0 - 0.481 = -0.481°C$

29. (b, d) : For negative deviation, from Raoult's law, $\Delta V_{mix} < 0$ and $\Delta H_{mix} < 0$. Here $A - B$ attractive force is greater than $A - A$ and $B - B$ attractive forces.

30. (b) : $\Delta T_b = K_b \frac{W_B}{M_B \times W_A} \times 1000$

$$\Delta T_f = K_f \frac{W_B}{M_B \times W_A} \times 1000$$

$\frac{\Delta T_b}{\Delta T_f} = \frac{K_b}{K_f}$ or $\frac{\Delta T_b}{0.186} = \frac{0.512}{1.86}$ or, $\Delta T_b = 0.0512°C$

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CHAPTER 7

EQUILIBRIUM

1. How many litres of water must be added to 1 litre of an aqueous solution of HCl with a pH of 1 to create an aqueous solution with pH of 2?
(a) 9.0 L (b) 0.1 L
(c) 0.9 L (d) 2.0 L *(2013)*

2. The equilibrium constant (K_c) for the reaction $N_{2(g)} + O_{2(g)} \rightarrow 2NO_{(g)}$ at temperature T is 4×10^{-4}. The value of K_c for the reaction, $NO_{(g)} \rightarrow \frac{1}{2} N_{2(g)} + \frac{1}{2} O_{2(g)}$ at the same temperature is
(a) 2.5×10^2 (b) 4×10^{-4}
(c) 50.0 (d) 0.02 *(2012)*

3. The pH of a 0.1 molar solution of the acid HQ is 3. The value of the ionization constant, K_a of this acid is
(a) 1×10^{-3} (b) 1×10^{-5}
(c) 1×10^{-7} (d) 3×10^{-1} *(2012)*

4. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm. Some of the CO_2 is converted into CO on the addition of graphite. If the total pressure at equilibrium is 0.8 atm, the value of K is
(a) 1.8 atm (b) 3 atm
(c) 0.3 atm (d) 0.18 atm *(2011)*

5. At 25°C, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions start precipitating in the form of $Mg(OH)_2$ from a solution of 0.001 M Mg^{2+} ions?
(a) 8 (b) 9
(c) 10 (d) 11 *(2010)*

6. Three reactions involving $H_2PO_4^-$ are given below:
(i) $H_3PO_4 + H_2O \longrightarrow H_3O^+ + H_2PO_4^-$
(ii) $H_2PO_4^- + H_2O \longrightarrow HPO_4^{2-} + H_3O^+$
(iii) $H_2PO_4^- + OH^- \longrightarrow H_3PO_4 + O^{2-}$
In which of the above does $H_2PO_4^-$ act as an acid?
(a) (i) only (b) (ii) only
(c) (i) and (ii) (d) (iii) only *(2010)*

7. Solubility product of silver bromide is 5.0×10^{-13}. The quantity of potassium bromide (molar mass taken as 120 g mol^{-1}) to be added to 1 litre of 0.05M solution of silver nitrate to start the precipitation of AgBr is
(a) 5.0×10^{-8} g (b) 1.2×10^{-10} g
(c) 1.2×10^{-9} g (d) 6.2×10^{-5} g *(2010)*

8. In aqueous solution the ionisation constants for carbonic acid are
$$K_1 = 4.2 \times 10^{-7} \text{ and } K_2 = 4.8 \times 10^{-11}$$
Select the correct statement for a saturated 0.034M solution of the carbonic acid.
(a) The concentration of H^+ is double that of CO_3^{2-}.
(b) The concentration of CO_3^{2-} is 0.034 M.
(c) The concentration of CO_3^{2-} is greater than that of HCO_3^-.
(d) The concentration of H^+ and HCO_3^- are approximately equal.
(2010)

9. The correct order of increasing basicity of the given conjugate bases ($R = CH_3$) is
(a) $RCOO^- < HC \equiv C^- < NH_2^- < R^-$
(b) $RCOO^- < HC \equiv C^- < R^- < NH_2^-$
(c) $R^- < HC \equiv C^- < RCOO^- < NH_2^-$
(d) $RCOO^- < NH_2^- < HC \equiv C^- < R^-$ *(2010)*

10. Solid $Ba(NO_3)_2$ is gradually dissolved in a 1.0×10^{-4} M Na_2CO_3 solution. At what concentration of Ba^{2+} will a precipitate begin to form?(K_{sp} for $BaCO_3 = 5.1 \times 10^{-9}$)
(a) 4.1×10^{-5} M (b) 5.1×10^{-5} M
(c) 8.1×10^{-8} M (d) 8.1×10^{-7} M *(2009)*

11. Four species are listed below :
(i) HCO_3^- (ii) H_3O^+
(iii) HSO_4^- (iv) HSO_3F
Which one of the following is the correct sequence of their acid strength?
(a) iii < i < iv < ii
(b) iv < ii < iii < i
(c) ii < iii < i < iv
(d) i < iii < ii < iv *(2008)*

12. The pK_a of a weak acid, (HA), is 4.80. The pK_b of a weak base, BOH is 4.78. The pH of an aqueous solution of the corresponding salt, BA, will be
(a) 9.22 (b) 9.58
(c) 4.79 (d) 7.01 *(2008)*

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13. For the following three reactions (i), (ii) and (iii), equilibrium constants are given

(i) $CO_{(g)} + H_2O_{(g)} \rightleftharpoons CO_{2(g)} + H_{2(g)}$; K_1

(ii) $CH_{4(g)} + H_2O_{(g)} \rightleftharpoons CO_{(g)} + 3H_{2(g)}$; K_2

(iii) $CH_{4(g)} + 2H_2O_{(g)} \rightleftharpoons CO_{2(g)} + 4H_{2(g)}$; K_3

Which of the following relation is correct?

(a) $K_3 \cdot K_2^3 = K_1^2$ (b) $K_1\sqrt{K_2} = K_3$

(c) $K_2 K_3 = K_1$ (d) $K_3 = K_1 K_2$ *(2008)*

14. The equilibrium constants K_{P_1} and K_{P_2} for the reactions $X \rightleftharpoons 2Y$ and $Z \rightleftharpoons P + Q$, respectively are in the ratio of 1 : 9. If degree of dissociation of X and Z be equal then the ratio of total pressures at these equilibria is

(a) 1 : 9 (b) 1 : 36

(c) 1 : 1 (d) 1 : 3 *(2008)*

15. In a saturated solution of the sparingly soluble strong electrolyte $AgIO_3$ (molecular mass = 283) the equilibrium which sets in is $AgIO_{3(s)} \rightleftharpoons Ag^+_{(aq)} + IO^-_{3(aq)}$.

If the solubility product constant K_{sp} of $AgIO_3$ at a given temperature is 1.0×10^{-8}, what is the mass of $AgIO_3$ contained in 100 mL of its saturated solution?

(a) 1.0×10^{-4} g (b) 28.3×10^{-2} g

(c) 2.83×10^{-3} g (d) 1.0×10^{-7} g

(2007)

16. The pK_a of a weak acid (H*A*) is 4.5. The pOH of an aqueous bufferd solution of H*A* in which 50% of the acid is ionized is

(a) 7.0 (b) 4.5

(c) 2.5 (d) 9.5

(2007)

17. The first and second dissociation constants of an acid H_2A are 1.0×10^{-5} and 5.0×10^{-10} respectively. The overall dissociation constant of the acid will be

(a) 0.2×10^5 (b) 5.0×10^{-5}

(c) 5.0×10^{15} (d) 5.0×10^{-15} *(2007)*

18. Given the data at 25°C,

$Ag + I^- \rightarrow AgI + e^-$; $E° = 0.152$ V

$Ag \rightarrow Ag^+ + e^-$; $E° = -0.800$ V

What is the value of log K_{sp} for AgI?

$$\left(2.303\frac{RT}{F} = 0.059 \text{ V}\right)$$

(a) – 8.12 (b) + 8.612

(c) – 37.83 (d) –16.13 *(2006)*

19. The equilibrium constant for the reaction,

$SO_{3(g)} \rightleftharpoons SO_{2(g)} + \frac{1}{2}O_{2(g)}$

is $K_c = 4.9 \times 10^{-2}$. The value of K_c for the reaction

$2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}$ will be

(a) 416 (b) 2.40×10^{-3}

(c) 9.8×10^{-2} (d) 4.9×10^{-2} *(2006)*

20. Phosphorus pentachloride dissociates as follows in a closed reaction vessel,

$PCl_{5(g)} \rightleftharpoons PCl_{3(g)} + Cl_{2(g)}$

If total pressure at equilibrium of the reaction mixture is P and degree of dissociation of PCl_5 is x, the partial pressure of PCl_3 will be

(a) $\left(\frac{x}{x+1}\right)P$ (b) $\left(\frac{2x}{1-x}\right)P$

(c) $\left(\frac{x}{x-1}\right)P$ (d) $\left(\frac{x}{1-x}\right)P$ *(2006)*

21. An amount of solid NH_4HS is placed in a flask already containing ammonia gas at a certain temperature and 0.50 atm. pressure. Ammonium hydrogen sulphide decomposes to yield NH_3 and H_2S gases in the flask. When the decomposition reaction reaches equilibrium, the total pressure in the flask rises to 0.84 atm. The equilibrium constant for NH_4HS decomposition at this temperature is

(a) 0.30 (b) 0.18

(c) 0.17 (d) 0.11 *(2005)*

22. Among the following acids which has the lowest pK_a value?

(a) CH_3COOH (b) $(CH_3)_2CH - COOH$

(c) HCOOH (d) CH_3CH_2COOH *(2005)*

23. What is the conjugate base of OH^-?

(a) O_2 (b) H_2O

(c) O^- (d) O^{2-} *(2005)*

24. Hydrogen ion concentration in mol/L in a solution of pH = 5.4 will be

(a) 3.98×10^8 (b) 3.88×10^6

(c) 3.68×10^{-6} (d) 3.98×10^{-6} *(2005)*

25. For the reaction,

$2NO_{2(g)} \rightleftharpoons 2NO_{(g)} + O_{2(g)}$

($K_c = 1.8 \times 10^{-6}$ at 184°C)

(R = 0.0831 kJ/(mol.K)). When K_p and K_c are compared at 184°C it is found that

(a) K_p is greater than K_c

(b) K_p is less than K_c

(c) $K_p = K_c$

(d) whether K_p is greater than, less than or equal to K_c depends upon the total gas pressure.

(2005)

26. The exothermic formation of ClF_3 is represented by the equation:

$Cl_{2(g)} + 3F_{2(g)} \rightleftharpoons 2ClF_{3(g)}$; $\Delta H = -329$ kJ

Which of the following will increase the quantity of ClF_3 in an equilibrium mixture of Cl_2, F_2 and ClF_3?

(a) Increasing the temperature

(b) Removing Cl_2

(c) Increasing the volume of the container

(d) Adding F_2 *(2005)*

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27. The solubility product of a salt having general formula MX_2, in water is 4×10^{-12}. The concentration of M^{2+} ions in the aqueous solution of the salt is
(a) 2.0×10^{-6} M (b) 1.0×10^{-4} M
(c) 1.6×10^{-4} M (d) 4.0×10^{-10} M *(2005)*

28. Consider an endothermic reaction $X \rightarrow Y$ with the activation energies E_b and E_f for the backward and forward reactions, respectively. In general
(a) $E_b < E_f$ (b) $E_b > E_f$
(c) $E_b = E_f$
(d) there is no definite relation between E_b and E_f. *(2005)*

29. The molar solubility (in mol L^{-1}) of a sparingly soluble salt MX_4 is s. The corresponding solubility product is K_{sp}. s is given in terms of K_{sp} by the relation
(a) $s = (K_{sp}/128)^{1/4}$ (b) $s = (128K_{sp})^{1/4}$
(c) $s = (256K_{sp})^{1/5}$ (d) $s = (K_{sp}/256)^{1/5}$ *(2004)*

30. The equilibrium constant for the reaction,
$N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$
at temperature T is 4×10^{-4}. The value of K_c for the reaction :
$NO_{(g)} \rightleftharpoons \frac{1}{2}N_{2(g)} + \frac{1}{2}O_{2(g)}$ at the same temperature is
(a) 2.5×10^2 (b) 50
(c) 4×10^{-4} (d) 0.02 *(2004)*

31. For the reaction, $CO_{(g)} + Cl_{2(g)} \rightleftharpoons COCl_{2(g)}$, the K_p/K_c is equal to
(a) $1/RT$ (b) RT
(c) $\sqrt{RT}$ (d) 1.0 *(2004)*

32. What is the equilibrium expression for the reaction
$P_{4\,(s)} + 5O_{2\,(g)} \rightleftharpoons P_4O_{10\,(s)}$?
(a) $K_c = \dfrac{[P_4O_{10}]}{[P_4][O_2]^5}$ (b) $K_c = \dfrac{[P_4O_{10}]}{5[P_4][O_2]}$
(c) $K_c = [O_2]^5$ (d) $K_c = \dfrac{1}{[O_2]^5}$ *(2004)*

33. The conjugate base of $H_2PO_4^-$ is
(a) PO_4^{3-} (b) P_2O_5
(c) H_3PO_4 (d) HPO_4^{2-} *(2004)*

34. When rain is accompanied by a thunderstorm, the collected rain water will have a pH value
(a) slightly lower than that of rain water without thunderstorm
(b) slightly higher than that when the thunderstorm is not there
(c) uninfluenced by occurrence of thunderstorm
(d) which depends on the amount of dust in air. *(2003)*

35. Which one of the following statements is not true?
(a) The conjugate base of $H_2PO_4^-$ is HPO_4^{2-}.
(b) pH + pOH = 14 for all aqueous solutions.
(c) The pH of 1×10^{-8} M HCl is 8.
(d) 96,500 coulombs of electricity when passed through a $CuSO_4$ solution deposits 1 gram equivalent of copper at the cathode. *(2003)*

36. The correct relationship between free energy change in a reaction and the corresponding equilibrium constant K_c is
(a) $\Delta G = RT \ln K_c$ (b) $-\Delta G = RT \ln K_c$
(c) $\Delta G^\circ = RT \ln K_c$ (d) $-\Delta G^\circ = RT \ln K_c$ *(2003)*

37. The solubility in water of a sparingly soluble salt AB_2 is 1.0×10^{-5} mol L^{-1}. Its solubility product will be
(a) 4×10^{-15} (b) 4×10^{-10}
(c) 1×10^{-15} (d) 1×10^{-10} *(2003)*

38. For the reaction equilibrium,
$N_2O_{4(g)} \rightleftharpoons 2NO_{2(g)}$
the concentrations of N_2O_4 and NO_2 at equilibrium are 4.8×10^{-2} and 1.2×10^{-2} mol L^{-1} respectively. The value of K_c for the reaction is
(a) 3.3×10^2 mol L^{-1}
(b) 3×10^{-1} mol L^{-1}
(c) 3×10^{-3} mol L^{-1}
(d) 3×10^3 mol L^{-1} *(2003)*

39. Consider the reaction equilibrium:
$2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}$; $\Delta H^\circ = -198$ kJ.
On the basis of Le Chatelier's principle, the condition favourable for the forward reaction is
(a) lowering of temperature as well as pressure
(b) increasing temperature as well as pressure
(c) lowering the temperature and increasing the pressure
(d) any value of temperature and pressure. *(2003)*

40. In which of the following reactions, increase in the volume at constant temperature does not affect the number of moles at equilibrium?
(a) $2NH_3 \rightarrow N_2 + 3H_2$
(b) $C_{(g)} + (1/2)\,O_{2(g)} \rightarrow CO_{(g)}$
(c) $H_{2(g)} + O_{2(g)} \rightarrow H_2O_{2(g)}$
(d) None of these. *(2002)*

41. Change in volume of the system does not alter the number of moles in which of the following equilibria?
(a) $N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$
(b) $PCl_{5(g)} \rightleftharpoons PCl_{3(g)} + Cl_{2(g)}$
(c) $N_{2(g)} + 3H_{2(g)} \rightleftharpoons 2NH_{3(g)}$
(d) $SO_2Cl_{2(g)} \rightleftharpoons SO_{2(g)} + Cl_{2(g)}$ *(2002)*

42. For the reaction
$CO_{(g)} + (1/2)\,O_{2(g)} = CO_{2(g)}$, K_p/K_c is
(a) RT (b) $(RT)^{-1}$
(c) $(RT)^{-1/2}$ (d) $(RT)^{1/2}$ *(2002)*

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43. Let the solubility of an aqueous solution of $Mg(OH)_2$ be x then its K_{sp} is
(a) $4x^3$ (b) $108x^5$
(c) $27x^4$ (d) $9x$ *(2002)*

44. Species acting as both Bronsted acid and base is
(a) $(HSO_4)^{-1}$ (b) Na_2CO_3
(c) NH_3 (d) OH^{-1} *(2002)*

45. 1 M NaCl and 1 M HCl are present in an aqueous solution. The solution is
(a) not a buffer solution with pH < 7
(b) not a buffer solution with pH > 7
(c) a buffer solution with pH < 7
(d) a buffer solution with pH > 7. *(2002)*

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Answer Key

1.	(a)	**2.**	(c)	**3.**	(b)	**4.**	(a)	**5.**	(c)	**6.**	(b)
7.	(c)	**8.**	(d)	**9.**	(a)	**10.**	(b)	**11.**	(d)	**12.**	(d)
13.	(d)	**14.**	(b)	**15.**	(c)	**16.**	(d)	**17.**	(d)	**18.**	(d)
19.	(a)	**20.**	(a)	**21.**	(d)	**22.**	(b)	**23.**	(d)	**24.**	(d)
25.	(a)	**26.**	(d)	**27.**	(b)	**28.**	(a)	**29.**	(d)	**30.**	(b)
31.	(a)	**32.**	(d)	**33.**	(d)	**34.**	(a)	**35.**	(c)	**36.**	(d)
37.	(c)	**38.**	(c)	**39.**	(c)	**40.**	(d)	**41.**	(a)	**42.**	(c)
43.	(a)	**44.**	(a)	**45.**	(a)						

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Explanations

1. (a) : Initial concentration of aq. HCl solution with pH 1 = 10^{-1} M
Final concentration of this solution after dilution = 10^{-2} M

$MV = M_1(V_1 + V_2)$
$10^{-1} \times 1 = 10^{-2}(1 + V_2)$
$\frac{0.1}{0.01} = 1 + V_2$
$10 = 1 + V_2$
$\Rightarrow V_2 = 9$ L

2. (c) : $N_{2(g)} + O_{2(g)} \longrightarrow 2NO_{(g)}, K_c = 4 \times 10^{-4}$ ... (i)

By multiplying the equation (i) by $\frac{1}{2}$

$\frac{1}{2}N_{2(g)} + \frac{1}{2}O_{2(g)} \longrightarrow NO_{(g)}$ ... (ii)

$K'_c = \sqrt{K_c} = \sqrt{4 \times 10^{-4}} = 2 \times 10^{-2}$

By reversing the equation (ii), we get

$NO_{(g)} \longrightarrow \frac{1}{2}N_{2(g)} + \frac{1}{2}O_{2(g)}$

$K''_c = \frac{1}{K'_c} = \frac{1}{2 \times 10^{-2}} = 50.0$

3. (b) : pH = 3
Molarity = 0.1 M
$[H^+] = \sqrt{K_a C}$
$H^+ = 10^{-pH} = 10^{-3}$
$10^{-3} = \sqrt{K_a \times 0.1}$ or $10^{-6} = K_a \times 0.1$
$\therefore K_a = 10^{-5}$

4. (a) : $CO_{2\,(g)} + C_{(s)} \rightleftharpoons 2CO_{(g)}$
0.5 atm
$0.5 - P$ $\quad 2P$
Total pressure = $0.5 - P + 2P = 0.8$
$P = 0.3$

$K_P = \frac{P^2_{CO}}{P_{CO_2}} = \frac{(2P)^2}{(0.5-P)} = \frac{(0.6)^2}{(0.5-0.3)} = 1.8$ atm

5. (c) : $(K_{sp})_{Mg(OH)_2} = [Mg^{2+}][OH^-]^2$
$1 \times 10^{-11} = [0.001][OH^-]^2$
$\Rightarrow [OH^-]^2 = \frac{10^{-11}}{10^{-3}} = (10^{-8})$
$\Rightarrow [OH^-] = 10^{-4}$
pOH = 4
Thus, pH = 14 – 4 = 10

6. (b) : In equation (ii), $H_2PO_4^-$ acts as a proton donor and thus, acts as an acid.

7. (c) : Given, $(K_{sp})_{AgBr} = 5.0 \times 10^{-13}$
The required equation is,
$KBr + AgNO_3 \longrightarrow AgBr + KNO_3$
Given, $[AgNO_3] = 0.05$ M
$\Rightarrow [Ag^+] = [NO_3^-] = 0.05$ M
$[Ag^+][Br^-] = (K_{sp})_{AgBr}$
$\Rightarrow 0.05 \times [Br^-] = 5 \times 10^{-13}$

$\Rightarrow [Br^-] = \frac{5 \times 10^{-13}}{5 \times 10^{-2}} = 1 \times 10^{-11}$ M

$\because [K^+] = [Br^-] = [KBr]$
$\therefore [KBr] = 1 \times 10^{-11}$ M

Molarity = $\frac{n_{KBr}}{V_{solution}(L)}$

$1 \times 10^{-11} = \frac{w_{KBr}/120}{1}$ (Mol. wt. of KBr = 120)

$\Rightarrow w_{KBr} = 1 \times 10^{-11} \times 120 = 120 \times 10^{-11}$
$w_{KBr} = 1.2 \times 10^{-9}$ g

8. (d) : $H_2CO_3 \rightleftharpoons H^+ + HCO_3^-; K_1 = 4.2 \times 10^{-7}$
$HCO_3^- \rightleftharpoons H^+ + CO_3^{2-}; K_2 = 4.8 \times 10^{-11}$
$\because K_1 >> K_2$, so H_2CO_3 ionises more than HCO_3^- and hence, contribution of H^+ is mostly due to ionisation of carbonic acid, thus the concentrations of H^+ and HCO_3^- are approximately equal.

9. (a) : The order of acidity can be explained on the basis of the acidity of the acids of the given conjugate base. Stronger is the acid, weaker is the conjugate base. Since *R*COOH is the strongest acid amongst all, $RCOO^-$ is the weakest base. Due to *sp* hybridised carbon, acetylene is also acidic and hence a weak base but stronger than $RCOO^-$. As sp^3 carbon is less electronegative than sp^3 nitrogen, R^- is more basic than NH_2^-.

10. (b) : K_{sp} for $BaCO_3 = [Ba^{2+}][CO_3^{2-}]$
given, $[CO_3^{2-}] = 1 \times 10^{-4}$ M (from Na_2CO_3)
$K_{sp} = 5.1 \times 10^{-9}$
$\therefore 5.1 \times 10^{-9} = [Ba^{2+}] \times [10^{-4}]$
$\Rightarrow [Ba^{2+}] = 5.1 \times 10^{-5}$ M
Thus, when $[Ba^{2+}] = 5.1 \times 10^{-5}$ M, $BaCO_3$ precipitate will begin to form.

11. (d) : HSO_3F is the super acid. Its acidic strength is greater than any given species. The pK_a value of other species are given below :

$HCO_3^- \rightarrow 10.25$
$H_3O^+ \rightarrow -1.74$
$HSO_4^- \rightarrow 1.92$

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Lesser the pK_a value, higher will be its acidic strength. Hence sequence of acidic strength will be

$HSO_3F > H_3O^+ > HSO_4^- > HCO_3^-$

12. **(d) :** Given that pK_a = 4.8 and pK_b = 4.78

$\therefore$ pH = 7 + 1/2 (pK_a – pK_b)

= 7 + 1/2 (4.80 – 4.78) = 7.01

13. **(d) :** $CO_{(g)} + H_2O_{(g)} \rightleftharpoons CO_{2(g)} + H_{2(g)}$

$$K_1 = \frac{[CO_2][H_2]}{[CO][H_2O]} \quad .... (i)$$

$CH_{4(g)} + H_2O_{(g)} \rightleftharpoons CO_{(g)} + 3H_{2(g)}$

$$K_2 = \frac{[CO][H_2]^3}{[CH_4][H_2O]} \quad .... (ii)$$

$CH_{4(g)} + 2H_2O_{(g)} \rightleftharpoons CO_{2(g)} + 4H_{2(g)}$

$$K_3 = \frac{[CO_2][H_2]^4}{[CH_4][H_2O]^2} \quad .... (iv)$$

From equations (i), (ii) and (iii) ; $K_3 = K_1 \times K_2$

14. **(b) :** $X \rightleftharpoons 2Y$; $Z \rightleftharpoons P + Q$

	X	$2Y$	Z	P	Q
Initial mol.	1	0	1	0	0
At equilibrium	1 – a	2a	1–a	a	a

$$K_{P_1} = \frac{P_Y^2}{P_X} = \frac{\left(\frac{2\alpha}{1+\alpha}P_1\right)^2}{\left(\frac{1-\alpha}{1+\alpha}P_1\right)}$$

$$K_{P_2} = \frac{P_P P_Q}{P_Z} = \frac{\left(\frac{\alpha}{1+\alpha}P_2\right)\left(\frac{\alpha}{1+\alpha}P_2\right)}{\left(\frac{1-\alpha}{1+\alpha}P_2\right)}$$

$$\Rightarrow K_{P_1} = \frac{4\alpha^2 P_1}{1-\alpha^2} \quad ... (i)$$

$$\Rightarrow K_{P_1} = \frac{4\alpha^2 P_1}{1-\alpha^2} \quad ... (ii)$$

Given is $\frac{K_{P_1}}{K_{P_2}} = \frac{1}{9}$ ... (iii)

Substituting values of from equation (i) and (ii) into (iii), we get

$$\frac{\frac{4\alpha^2 P_1}{1-\alpha^2}}{\frac{\alpha^2 P_2}{1-\alpha^2}} = \frac{1}{9} \Rightarrow \frac{4P_1}{P_2} = \frac{1}{9} \Rightarrow \frac{P_1}{P_2} = \frac{1}{36}$$

15. **(c) :** $AgIO_3 \rightleftharpoons \underset{S}{Ag^+} + \underset{S}{IO_3^-}$ [S = Solubility]

$K_{sp} = S^2$

or, $S^2 = 1.0 \times 10^{-8}$ or, $S = 1.0 \times 10^{-4}$ mol/L

$= 1.0 \times 10^{-4} \times 283$ g/L

$$= \frac{1.0 \times 10^{-4} \times 283}{1000} \text{ g/L}$$

$$= \frac{1.0 \times 10^{-4} \times 283 \times 100}{1000} \text{ g/100mL}$$

= 28.3 × 10^{-4} g/100 mL

= 2.83 × 10^{-3} g/100 mL

16. **(d) :** For acidic buffer, $pH = pK_a + \log\frac{[A^-]}{[HA]}$

When the acid is 50% ionised, $[A^-] = [HA]$

or pH = pK_a + log1 or pH = pK_a

Given pK_a = 4.5 $\therefore$ pH = 4.5

$\therefore$ pOH = 14 – 4.5 = 9.5

17. **(d) :** $H_2A \rightleftharpoons H^+ + HA^-$;

$$K_1 = \frac{[H^+][HA^-]}{[H_2A]} = 1 \times 10^{-5}$$

$HA^- \rightleftharpoons H^+ + A^{2-}$; $K_2 = 5 \times 10^{-10} = \frac{[H^+][A^{2-}]}{[HA^-]}$

$$K = \frac{[H^+]^2[A^{2-}]}{[H_2A]} = K_1 \times K_2 = 1 \times 10^{-5} \times 5 \times 10^{-10}$$

$= 5 \times 10^{-15}$

18. **(d) :** $AgI_{(s)} + e^- \longrightarrow Ag_{(s)} + I^-$, $E°$ = – 0.152 V

$Ag_{(s)} \longrightarrow Ag^+ + e^-$, $E°$ = – 0.800 V

$AgI_{(s)} \longrightarrow Ag^+ + I^-$, $E°$ = – 0.952 V

$$E°_{cell} = \frac{0.059}{n}\log K \quad i.e. \quad -0.952 = \frac{0.059}{1}\log K_{sp}$$

or, $\log K_{sp} = -\frac{0.952}{0.059} = -16.135$

19. **(a) :** $SO_{3(g)} \rightleftharpoons SO_{2(g)} + 1/2\, O_{2(g)}$

$$\frac{[SO_2][O_2]^{1/2}}{[SO_3]} = K_c = 4.9 \times 10^{-2} \quad ... (i)$$

$SO_{3(g)} + 1/2\, O_{2(g)} \rightleftharpoons SO_{3(g)}$

$$\frac{[SO_3]}{[SO_2][O_2]^{1/2}} = K'_c = \frac{1}{4.9 \times 10^{-2}} \quad ... (ii)$$

For $2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}$

$$\frac{[SO_3]^2}{[SO_2]^2[O_2]} = K_c'^2 = \frac{1}{4.9 \times 4.9 \times 10^{-4}}$$

$$= \frac{10000}{24.01} = 416.49$$

20. **(a) :** Given $PCl_{5\,(g)} \rightleftharpoons PCl_{3\,(g)} + Cl_{2\,(g)}$

	PCl_5	PCl_3	Cl_2
t = 0	1	0	0
t_{eq}	1 – x	x	x

Total number of moles = 1 – x + x + x = 1 + x

Thus partial pressure of PCl_3 = $\left(\frac{x}{1+x}\right)P$

21. **(d) :** $NH_4HS_{(s)} \rightleftharpoons NH_{3\,(g)} + H_2S_{(g)}$

	NH_4HS	NH_3	H_2S
Initial pressure	0	0.5	0
At equi.	0	0.5 + x	x

Total pressure = 0.5 + 2x = 0.84 $\therefore$ x = 0.17 atm

$K_p = p_{NH_3} \times p_{H_2S}$ = (0.5 + 0.17)(0.17) = 0.11 atm^2

22. **(b) :** Higher the pK_a value, weaker is the acid. Hence, strongest acid has lowest pK_a value.

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23. **(d) :** Conjugate base of OH^- is O^{2-}.
$OH^- \rightleftharpoons O^{2-} + H^+$

24. **(d) :** pH = – log[H^+]
[H^+] = antilog (–pH) = antilog(–5.4) =3.98×10^{-6}

25. **(a) :** $K_p = K_c (RT)^{\Delta n}$
$\Delta n = 3 - 2 = 1$
$K_p = K_c (0.0831 \times 457)^1$
$\therefore \quad K_p > K_c$

26. **(d) :** $Cl_{2(g)} + 3F_{2(g)} \rightleftharpoons 2ClF_{3(g)}$; $\Delta H = -329$ kJ
Favourable conditions:
(i) As the reaction is exothermic, hence decrease in temperature will favour the forward reaction.
(ii) Addition of reactants or removal of product will farour the forward reaction.
(iii) Here $\Delta n = 2 - 4 = -2$ (*i.e.*, –ve) hence decrease in volume or increase in pressure will favour the forward reaction.

27. **(b) :** $MX_{2(s)} \rightleftharpoons \underset{s}{M^{2+}_{(aq)}} + \underset{2s}{2X^-_{(aq)}}$
$K_{sp} = s \cdot (2s)^2 = 4s^3$
$4 \times 10^{-12} = 4s^3$ or, $s^3 = 1 \times 10^{-12}$
or, $s = 1 \times 10^{-4}$ M $\Rightarrow$ $[M^{2+}] = 1 \times 10^{-4}$ M

28. **(a) :** For endothermic reaction, ΔH = +ve
$\Delta H = E_f - E_b$, it means $E_b < E_f$

29. **(d) :** MX_4 (solid) $\rightleftharpoons \underset{s}{M^{4+}_{(aq)}} + \underset{4s}{4X^-_{(aq)}}$
Solubility product, $K_{sp} = s \times (4s)^4 = 256\ s^5$.
$$\therefore \quad s = \sqrt[5]{\frac{K_{sp}}{256}} = \left(\frac{K_{sp}}{256}\right)^{1/5}$$

30. **(b) :** $N_{2(g)} + O_{2(g)} \rightleftharpoons 2NO_{(g)}$
$$K_c = \frac{[NO]^2}{[N_2][O_2]} = 4 \times 10^{-4}$$
$$NO_{(g)} \rightleftharpoons \frac{1}{2}N_{2(g)} + \frac{1}{2}O_{2(g)}$$
$$K'_c = \frac{[N_2]^{1/2}[O_2]^{1/2}}{[NO]} = \frac{1}{\sqrt{K_c}} = \frac{1}{\sqrt{4 \times 10^{-4}}}$$
$$= \frac{1}{2 \times 10^{-2}} = \frac{100}{2} = 50$$

31. **(a) :** $CO_{(g)} + Cl_{2(g)} \rightleftharpoons COCl_{2(g)}$
$\Delta n = 1 - 2 = -1$
$$K_p = K_c (RT)^{\Delta n}, \quad \therefore \frac{K_p}{K_c} = (RT)^{-1} = \frac{1}{RT}$$

32. **(d) :** $P_{4\,(s)} + 5O_{2\,(g)} \rightleftharpoons P_4O_{10\,(s)}$
$$K_c = \frac{[P_4O_{10\,(s)}]}{[P_{4(s)}][O_{2(g)}]^5}$$
We know that concentration of a solid component is always taken as unity.
$$K_c = \frac{1}{[O_2]^5}$$

33. **(d) :** Conjugate base is formed by the removal of H^+ from acid.
$H_2PO_4^- \rightarrow HPO_4^{2-} + H^+$

34. **(a) :** Due to thunderstorm, temperature increases. As temperature increases, [H^+] also increases, hence pH decreases.

35. **(c) :** pH of an acid cannot exceed 7. Here we should also consider [H^+] that comes from H_2O.
Now $[H^+] = [H^+]_{\text{from HCl}} + [H^+]_{\text{from } H_2O}$
$= 10^{-8} + 10^{-7} = 10^{-8} + 10 \times 10^{-8} = 11 \times 10^{-8}$
$\therefore \quad pH = -\log(11 \times 10^{-8}) = 6.9587$

36. **(d) :** $\Delta G = \Delta G° + 2.303\ RT \log K_c$
At equilibrium, $\Delta G = 0$
$\Delta G° = -2.303\ RT \log K_c$

37. **(c) :** $AB_2 \rightleftharpoons A^{2+} + 2B^-$
$S = 1.0 \times 10^{-5}$ mol L^{-1}
$K_{sp} = [A^{2+}][B^-]^2 = 1.0 \times 10^{-5} \times (1.0 \times 10^{-5})^2$
$= 1.0 \times 10^{-15}$

38. **(c) :** $[N_2O_4] = 4.8 \times 10^{-2}$ mol L^{-1}
$[NO_2] = 1.2 \times 10^{-2}$ mol L^{-1}
$$K_c = \frac{[NO_2]^2}{[N_2O_4]} = \frac{1.2 \times 10^{-2} \times 1.2 \times 10^{-2}}{4.8 \times 10^{-2}}$$
$= 0.3 \times 10^{-2} = 3 \times 10^{-3}$ mol L^{-1}

39. **(c) :** The conversion of SO_2 to SO_3 is an exothermic reaction, hence decrease the temperature will favour the forward reaction. There is also a decrease in volume or moles in product side. Thus the reaction is favoured by low temperature and high pressure. (Le-Chatelier's principle).

40. **(d) :** For those reactions, where $\Delta n = 0$, increase in volume at constant temperature does not affect the number of moles at equilibrium.

41. **(a) :** In this reaction the ratio of number of moles of reactants to products is same *i.e.* 2 : 2, hence change in volume will not alter the number of moles.

42. **(c) :** $K_p = K_c (RT)^{\Delta n}$; $\Delta n = 1 - \left(1 + \frac{1}{2}\right) = 1 - \frac{3}{2} = -\frac{1}{2}$
$$\therefore \quad \frac{K_p}{K_c} = (RT)^{-1/2}$$

43. **(a) :** $Mg(OH)_2 \rightarrow \underset{x}{[Mg^{2+}]} + \underset{2x}{2[OH^-]}$
$K_{sp} = [Mg^{2+}][OH^-]^2$
or, $K_{sp} = (x) \times (2x)^2 = x \times 4x^2 = 4x^3$

44. **(a) :** According to Bronsted-Lowry concept, a Bronsted acid is a substance which can donate a proton to any other substance and a Bronsted base is a substance which can accept a proton from any other substance.
$(HSO_4)^-$ can accept and donate a proton.
$(HSO_4)^- + H^+ \rightarrow H_2SO_4$
$(HSO_4)^- - H^+ \rightarrow SO_4^{2-}$

45. **(a) :** HCl is a strong acid and its salt do not form buffer solution. As the resultant solution is acidic, hence pH is less than 7.

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CHAPTER 8

REDOX REACTIONS AND ELECTROCHEMISTRY

1. Consider the following reaction.

$$xMnO_4^- + yC_2O_4^{2-} + zH^+ \rightarrow xMn^{2+} + 2yCO_2 + \frac{z}{2}H_2O$$

The values of x, y and z in the reaction are, respectively

(a) 5, 2 and 8 (b) 5, 2 and 16
(c) 2, 5 and 8 (d) 2, 5 and 16 *(2013)*

2. Given

$E^\circ_{Cr^{3+}/Cr} = -0.74$ V; $E^\circ_{MnO_4^-/Mn^{2+}} = 1.51$ V

$E^\circ_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33$ V; $E^\circ_{Cl/Cl^-} = 1.36$ V

Based on the data given above, strongest oxidising agent will be

(a) MnO_4^- (b) Cl^-
(c) Cr^{3+} (d) Mn^{2+} *(2013)*

3. The standard reduction potentials for Zn^{2+}/Zn, Ni^{2+}/Ni, and Fe^{2+}/Fe are – 0.76, – 0.23 and – 0.44 V respectively. The reaction $X + Y^{2+} \rightarrow X^{2+} + Y$ will be spontaneous when

(a) X = Ni, Y = Zn (b) X = Fe, Y = Zn
(c) X = Zn, Y = Ni (d) X = Ni, Y = Fe *(2012)*

4. The reduction potential of hydrogen half-cell will be negative if

(a) $p(H_2)$ = 1 atm and $[H^+]$ = 2.0 M
(b) $p(H_2)$ = 1 atm and $[H^+]$ = 1.0 M
(c) $p(H_2)$ = 2 atm and $[H^+]$ = 1.0 M
(d) $p(H_2)$ = 2 atm and $[H^+]$ = 2.0 M *(2011)*

5. The Gibb's energy for the decomposition of Al_2O_3 at 500°C is as follows :

$$2/3Al_2O_3 \longrightarrow 4/3Al + O_2, \Delta_rG = +966 \text{ kJ mol}^{-1}.$$

The potential difference needed for electrolytic reduction of Al_2O_3 at 500°C is at least

(a) 5.0 V (b) 4.5 V
(c) 3.0 V (d) 2.5 V *(2010)*

6. Given : $E^\circ_{Fe^{3+}/Fe} = -0.036$ V, $E^\circ_{Fe^{2+}/Fe} = -0.439$ V. The value of standard electrode potential for the change, $Fe^{3+}_{(aq)} + e^- \rightarrow Fe^{2+}_{(aq)}$ will be

(a) – 0.072 V (b) 0.385 V
(c) 0.770 V (d) – 0.270 V *(2009)*

7. Amount of oxalic acid present in a solution can be determined by its titration with $KMnO_4$ solution in the presence of H_2SO_4. The titration gives unsatisfactory result when carried out in the presence of HCl, because HCl

(a) oxidises oxalic acid to carbon dioxide and water
(b) gets oxidised by oxalic acid to chlorine
(c) furnishes H^+ ions in addition to those from oxalic acid
(d) reduces permanganate to Mn^{2+}. *(2008)*

8. Given $E^\circ_{Cr^{3+}/Cr} = -0.72$ V, $E^\circ_{Fe^{2+}/Fe} = -0.42$ V

The potential for the cell

$Cr \mid Cr^{3+}(0.1\text{ M}) \parallel Fe^{2+}(0.01\text{ M}) \mid Fe$ is

(a) –0.26 V (b) 0.26 V
(c) 0.339 V (d) –0.339 V *(2008)*

9. The cell,

$Zn \mid Zn^{2+}(1\text{ M}) \parallel Cu^{2+}(1\text{ M}) \mid Cu$ (E°_{cell} = 1.10 V)

was allowed to be completely discharged at 298 K. The relative concentration of Zn^{2+} to Cu^{2+} $\left(\frac{[Zn^{2+}]}{[Cu^{2+}]}\right)$ is

(a) 9.65×10^4 (b) antilog(24.08)
(c) 37.3 (d) $10^{37.3}$ *(2007)*

10. The equivalent conductances of two strong electrolytes at infinite dilution in H_2O (where ions move freely through a solution) at 25°C are given below:

$\Lambda^\circ_{CH_3COONa}$ = 91.0 S cm²/equiv.

Λ°_{HCl} = 426.2 Scm²/equiv.

What additional information/quantity one needs to calculate Λ° of an aqueous solution of acetic acid?

(a) Λ° of chloroacetic acid ($ClCH_2COOH$)
(b) Λ° of NaCl
(c) Λ° of CH_3COOK
(d) The limiting equivalent conductance of H^+ ($\lambda^\circ_{H^+}$). *(2007)*

11. Resistance of a conductivity cell filled with a solution of an electrolyte of concentration 0.1 M is 100 Ω. The conductivity of this solution is 1.29 S m⁻¹. Resistance of the same cell when filled with 0.2 M of the same solution is 520 Ω. The molar conductivity of 0.02 M solution of the electrolyte will be

(a) 124×10^{-4} S m² mol⁻¹
(b) 1240×10^{-4} S m² mol⁻¹
(c) 1.24×10^{-4} S m² mol⁻¹
(d) 12.4×10^{-4} S m² mol⁻¹ *(2006)*

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12. The molar conductivities Λ°_{NaOAc} and Λ°_{HCl} at infinite dilution in water at 25°C are 91.0 and 426.2 S cm²/mol respectively. To calculate Λ°_{HOAc}, the additional value required is
(a) $\Lambda^\circ_{H_2O}$ (b) Λ°_{KCl}
(c) Λ°_{NaOH} (d) Λ°_{NaCl} *(2006)*

13. Which of the following chemical reactions depicts the oxidising behaviour of H_2SO_4?
(a) $2HI + H_2SO_4 \rightarrow I_2 + SO_2 + 2H_2O$
(b) $Ca(OH)_2 + H_2SO_4 \rightarrow CaSO_4 + 2H_2O$
(c) $NaCl + H_2SO_4 \rightarrow NaHSO_4 + HCl$
(d) $2PCl_5 + H_2SO_4 \rightarrow 2POCl_3 + 2HCl + SO_2Cl_2$. *(2006)*

14.

Electrolyte	KCl	KNO_3	HCl	NaOAc	NaCl
(S cm² mol⁻¹)	149.9	145.0	426.2	91.0	126.5

Calculate molar conductance of acetic acid using appropriate molar conductances of the electrolytes listed above at infinite dilution in H_2O at 25°C.
(a) 517.2 (b) 552.7
(c) 390.7 (d) 217.5 *(2005)*

15. Aluminium oxide may be electrolysed at 1000°C to furnish aluminium metal (At. Mass = 27 amu; 1 Faraday = 96,500 Coulombs). The cathode reaction is

$$Al^{3+} + 3e^- \rightarrow Al^0$$

To prepare 5.12 kg of aluminium metal by this method would require
(a) 5.49×10^7 C of electricity
(b) 1.83×10^7 C of electricity
(c) 5.49×10^4 C of electricity
(d) 5.49×10^{10} C of electricity *(2005)*

16. The highest electrical conductivity of the following aqueous solutions is of
(a) 0.1 M acetic acid
(b) 0.1 M chloroacetic acid
(c) 0.1 M fluoroacetic acid
(d) 0.1 M difluoroacetic acid. *(2005)*

17. The $E^\circ_{M^{3+}/M^{2+}}$ values for Cr, Mn, Fe and Co are – 0.41, +1.57, 0.77 and +1.97 V respectively. For which one of these metals the change in oxidation state from +2 to +3 is easiest?
(a) Cr (b) Mn
(c) Fe (d) Co *(2004)*

18. In a cell that utilizes the reaction,
$Zn_{(s)} + 2H^+_{(aq)} \rightarrow Zn^{2+}_{(aq)} + H_{2(g)}$
addition of H_2SO_4 to cathode compartment, will
(a) lower the E and shift equilibrium to the left
(b) lower the E and shift the equilibrium to the right
(c) increase the E and shift the equilibrium to the right
(d) increase the E and shift the equilibrium to the left. *(2004)*

19. The limiting molar conductivities Λ° for NaCl, KBr and KCl are 126, 152 and 150 S cm² mol⁻¹ respectively. The Λ° for NaBr is
(a) 128 S cm² mol⁻¹ (b) 176 S cm² mol⁻¹
(c) 278 S cm² mol⁻¹ (d) 302 S cm² mol⁻¹ *(2004)*

20. The standard e.m.f. of a cell, involving one electron change is found to be 0.591 V at 25°C. The equilibrium constant of the reaction is (F = 96,500 C mol⁻¹, R = 8.314 JK⁻¹mol⁻¹)
(a) 1.0×10^1 (b) 1.0×10^5
(c) 1.0×10^{10} (d) 1.0×10^{30} *(2004)*

21. Consider the following E° values.
$E^\circ_{Fe^{3+}/Fe^{2+}} = +0.77$ V ; $E^\circ_{Sn^{2+}/Sn} = -0.14$ V
Under standard conditions the potential for the reaction
$Sn_{(s)} + 2Fe^{3+}_{(aq)} \rightarrow 2Fe^{2+}_{(aq)} + Sn^{2+}_{(aq)}$ is
(a) 1.68 V (b) 1.40 V
(c) 0.91 V (d) 0.63 V *(2004)*

22. In a hydrogen-oxygen fuel cell, combustion of hydrogen occurs to
(a) generate heat
(b) create potential difference between the two electrodes
(c) produce high purity water
(d) remove adsorbed oxygen from electrode surface. *(2004)*

23. Among the properties (*A*) reducing (*B*) oxidising (*C*) complexing, the set of properties shown by CN⁻ ion towards metal species is
(a) *A, B* (b) *B, C*
(c) *C, A* (d) *A, B, C.* *(2004)*

24. Standard reduction electrode potentials of three metals *A*, *B* and *C* are +0.5 V, –3.0 V and –1.2 V respectively. The reducing power of these metals are
(a) $B > C > A$ (b) $A > B > C$
(c) $C > B > A$ (d) $A > C > B$ *(2003)*

25. For a cell reaction involving a two-electron change, the standard e.m.f. of the cell is found to be 0.295 V at 25°C. The equilibrium constant of the reaction at 25°C will be
(a) 1×10^{-10} (b) 29.5×10^{-2}
(c) 10 (d) 1×10^{10} *(2003)*

26. For the redox reaction:

$$Zn_{(s)} + Cu^{2+}\,(0.1\text{ M}) \rightarrow Zn^{+}\,(1\text{M}) + Cu_{(s)}$$

taking place in a cell, E°_{cell} is 1.10 volt. E_{cell} for the cell will be $\left(2.303\frac{RT}{F} = 0.0591\right)$
(a) 2.14 V (b) 1.80 V
(c) 1.07 V (d) 0.82 V *(2003)*

27. When during electrolysis of a solution of $AgNO_3$, 9650 coulombs of charge pass through the electroplating bath, the mass of silver deposited on the cathode will be
(a) 1.08 g (b) 10.8 g
(c) 21.6 g (d) 108 g *(2003)*

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28. The heat required to raise the temperature of body by 1ºC is called
(a) specific heat
(b) thermal capacity
(c) water equivalent
(d) none of these. *(2002)*

29. Which of the following reaction is possible at anode?
(a) $2\,Cr^{3+} + 7H_2O \rightarrow Cr_2O_7^{2-} + 14H^+$
(b) $F_2 \rightarrow 2F^-$
(c) $(1/2)\,O_2 + 2H^+ \rightarrow H_2O$
(d) None of these. *(2002)*

30. What will be the emf for the given cell,
$Pt\,|\,H_2\,(P_1)\,|\,H^+_{(aq)}\,||\,H_2\,(P_2)\,|\,Pt$
(a) $\frac{RT}{F}\log\frac{P_1}{P_2}$ (b) $\frac{RT}{2F}\log\frac{P_1}{P_2}$
(c) $\frac{RT}{F}\log\frac{P_2}{P_1}$ (d) none of these. *(2002)*

31. If ϕ denotes reduction potential, then which is true?
(a) $E^\circ_{cell} = \phi_{right} - \phi_{left}$
(b) $E^\circ_{cell} = \phi_{left} + \phi_{right}$
(c) $E^\circ_{cell} = \phi_{left} - \phi_{right}$
(d) $E^\circ_{cell} = -(\phi_{left} + \phi_{right})$ *(2002)*

32. Conductivity (unit Siemen's S) is directly proportional to area of the vessel and the concentration of the solution in it and is inversely proportional to the length of the vessel then the unit of the constant of proportionality is
(a) $S\ m\ mol^{-1}$ (b) $S\ m^2\ mol^{-1}$
(c) $S^{-2} m^2\ mol$ (d) $S^2\ m^2\ mol^{-2}$ *(2002)*

33. Which of the following is a redox reaction?
(a) $NaCl + KNO_3 \rightarrow NaNO_3 + KCl$
(b) $CaC_2O_4 + 2HCl \rightarrow CaCl_2 + H_2C_2O_4$
(c) $Mg(OH)_2 + 2NH_4Cl \rightarrow MgCl_2 + 2NH_4OH$
(d) $Zn + 2AgCN \rightarrow 2Ag + Zn(CN)_2$ *(2002)*

34. When $KMnO_4$ acts as an oxidising agent and ultimately forms $[MnO_4]^{-1}$, MnO_2, Mn_2O_3, Mn^{2+} then the number of electrons transferred in each case respectively is
(a) 4, 3, 1, 5 (b) 1, 5, 3, 7
(c) 1, 3, 4, 5 (d) 3, 5, 7, 1 *(2002)*

35. EMF of a cell in terms of reduction potential of its left and right electrodes is
(a) $E = E_{left} - E_{right}$ (b) $E = E_{left} + E_{right}$
(c) $E = E_{right} - E_{left}$ (d) $E = -(E_{right} + E_{left})$ *(2002)*

Answer Key

1.	(d)	**2.**	(a)	**3.**	(c)	**4.**	(c)	**5.**	(d)	**6.**	(c)
7.	(d)	**8.**	(b)	**9.**	(d)	**10.**	(b)	**11.**	(a)	**12.**	(d)
13.	(a)	**14.**	(c)	**15.**	(a)	**16.**	(d)	**17.**	(a)	**18.**	(c)
19.	(a)	**20.**	(c)	**21.**	(c)	**22.**	(b)	**23.**	(c)	**24.**	(a)
25.	(d)	**26.**	(c)	**27.**	(b)	**28.**	(b)	**29.**	(a)	**30.**	(b)
31.	(a)	**32.**	(b)	**33.**	(d)	**34.**	(c)	**35.**	(c)		

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Explanations

1. **(d)** : $2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$
$\therefore \quad x = 2, y = 5, z = 16$

2. **(a)** : Greater the reduction potential of a substance, stronger is the oxidising agent.
$\therefore$ MnO_4^- is the strongest oxidising agent.

3. **(c)** : The elements with high negative value of standard reduction potential are good reducing agents and can be easily oxidised.
Thus X should have high negative value of standard potential than Y so that it will be oxidised to X^{2+} by reducing Y^{2+} to Y.
$X = Zn, \quad Y = Ni$
$Zn + Ni^{2+} \rightarrow Zn^{2+} + Ni$
Alternatively, for a spontaneous reaction $E°$ must be positive.
$E° = E°_{\text{reduced species}} - E°_{\text{oxidised species}}$
$= -0.23 - (-0.76)$
$\Rightarrow \quad E° = +0.53$ V

4. **(c)** : $2H^+_{(aq)} + 2e^- \longrightarrow H_{2\,(g)}$

$$E_{red} = E°_{red} - \frac{0.0591}{n}\log\frac{p_{H_2}}{[H^+]^2}$$

$$E_{red} = 0 - \frac{0.0591}{2}\log\frac{2}{(1)^2}$$

E_{red} will only be negative when $p_{H_2} > [H^+]$. So option (c) is correct.

5. **(d)** : The ionic reactions are :
$2/3Al_2^{3+} + 4e^- \longrightarrow 4/3Al$
$2/3O_3^{2-} \longrightarrow O_2 + 4e^-$
Thus, no. of electrons transferred = 4 = n
$\Delta G = -nFE = -4 \times 96500 \times E$
or $966 \times 10^3 = -4 \times 96500 \times E$

$$\Rightarrow \quad E = -\frac{966 \times 10^3}{4 \times 96500} = -2.5 \text{ V}$$

6. **(c)** : Given,
$Fe^{3+} + 3e^- \longrightarrow Fe \;;\quad E_1^\circ = -0.036$ V
$Fe^{2+} + 2e^- \longrightarrow Fe \;;\quad E_2^\circ = -0.439$ V
Required equation is
$Fe^{3+} + e^- \longrightarrow Fe^{2+} \;;\quad E_3^\circ = ?$
Applying $\Delta G° = -nFE°$
$\therefore \quad \Delta G_3^\circ = \Delta G_1^\circ - \Delta G_2^\circ$
$(-n_3FE_3^\circ) = (-n_1FE_1^\circ) - (-n_2FE_2^\circ)$
$E_3^\circ = 3E_1^\circ - 2E_2^\circ = 3 \times (-0.036) - 2 \times (-0.439)$
$E°_3 = -0.108 + 0.878 = 0.77$ V

7. **(d)** : Oxalic acid present in a solution can be determined by its titration with $KMnO_4$ solution in the presence of H_2SO_4.

$$2KMnO_4 + 3H_2SO_4 + 5\begin{array}{c}COOH\\|\\COOH\end{array} \longrightarrow K_2SO_4 + 2MnSO_4 + 10CO_2 + 5H_2O$$

Titration cannot be done in the presence of HCl because $KMnO_4$ being a strong oxidizing agent oxidises HCl to Cl_2 and get itself reduced to Mn^{2+}. So actual amount of oxalic acid in solution cannot be determined.

8. **(b)** : $Cr \rightarrow Cr^{3+} + 3e^- \qquad E°_{red} = -0.72$ V
$Fe^{2+} + 2e^- \rightarrow Fe \qquad E°_{red} = -0.42$ V
$2Cr + 3Fe^{2+} \rightarrow 2Cr^{3+} + 3Fe$
$E°_{cell} = E°_{cathode} - E°_{anode} = -0.42 - (-0.72)$
$E°_{cell} = 0.3$
According to Nernst equation,

$$E_{cell} = E°_{cell} - \frac{0.059}{n_{cell}}\log_{10}\frac{[Cr^{3+}]^2}{[Fe^{2+}]^3}$$

$$E_{cell} = 0.3 - \frac{0.059}{6}\log_{10}\frac{(0.1)^2}{(0.01)^3}$$

$$E_{cell} = 0.3 - \frac{0.059}{6}\log_{10}10^4$$

$E_{cell} = 0.3 - 0.039$
$\therefore \quad E_{cell} = 0.261$ V

9. **(d)** : $Zn + Cu^{2+} \longrightarrow Zn^{2+} + Cu$

$$E_{cell} = E°_{cell} - \frac{0.059}{2}\log\frac{[Zn^{2+}]}{[Cu^{2+}]}$$

When the cell is completely discharged, $E_{cell} = 0$

$$0 = 1.1 - \frac{0.059}{2}\log\frac{[Zn^{2+}]}{[Cu^{2+}]}$$

or $\log\frac{[Zn^{2+}]}{[Cu^{2+}]} = \frac{2 \times 1.1}{0.059}$ or, $\log\frac{Zn^{2+}}{Cu^{2+}} = 37.3$

or $\frac{Zn^{2+}}{Cu^{2+}} = 10^{37.3}$

10. **(b)** : According to Kohlrausch's law, the molar conductivity at infinite dilution ($\Lambda°$) for weak electrolyte, CH_3COOH is

$$\Lambda^\circ_{CH_3COOH} = \Lambda^\circ_{CH_3COONa} + \Lambda^\circ_{HCl} - \Lambda^\circ_{NaCl}$$

So, for calculating the value of $\Lambda^\circ_{CH_3COOH}$, value of Λ°_{NaCl} should also be known.

11. **(a)** : $\kappa = \frac{1}{R}\left(\frac{l}{a}\right)$ *i.e.*, $1.29 = \frac{1}{100}\left(\frac{l}{a}\right)$

$l/a = 129$ m^{-1}
$R = 520\ \Omega$ for 0.2 M, $C = 0.02$ M

$$\lambda_m = \kappa \times \frac{1000}{\text{molarity}} = \frac{1 \times 129}{520} \times \frac{1000}{0.02} \times 10^{-6} \text{ m}^3$$

$= 124 \times 10^{-4}$ S m^2 mol^{-1}

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12. (d) : $CH_3COONa + HCl \rightarrow CH_3COOH + NaCl$

From the reaction,

$$\Lambda^\circ_{CH_3COONa} + \Lambda^\circ_{HCl} = \Lambda^\circ_{CH_3COOH} + \Lambda^\circ_{NaCl}$$

or, $\Lambda^\circ_{CH_3COOH} = \Lambda^\circ_{CH_3COONa} + \Lambda^\circ_{HCl} - \Lambda^\circ_{NaCl}$

Thus to calculate the value of $\Lambda^\circ_{CH_3COOH}$ one should know the value of Λ°_{NaCl} along with $\Lambda^\circ_{CH_3COONa}$ and Λ°_{HCl}.

13. (a) : In the reaction,

$$\underset{+1-1}{2HI} + \underset{+1\ +6\ -2}{H_2SO_4} \rightarrow \underset{0}{I_2} + \underset{+4}{SO_2} + \underset{+1\ \ -2}{2H_2O}$$

I[st] half reaction : $\underset{-1}{2HI} \rightarrow \underset{0}{I_2}$

In this reaction oxidation number of I increases by one, thus this is an oxidation reaction and HI behaves as a reducing agent.

II[nd] half reaction : $\underset{+6}{H_2SO_4} \rightarrow \underset{+4}{SO_2}$

In this reaction oxidation number of S decreases by two, thus this is a reduction reaction and H_2SO_4 behaves as oxidising agent.

14. (c) : $\Lambda^\infty_{AcOH} = \Lambda^\infty_{AcONa} + \Lambda^\infty_{HCl} - \Lambda^\infty_{NaCl}$

$= 91.0 + 426.2 - 126.5$

$= 390.7$ S cm^2 mol^{-1}

15. (a) : From Faraday's 1st law,

$W = Z \times Q$ [W = weight, Z = electrochemical equivalent, Q = quantity of electricity]

Now $E = Z \times F$ [E = equivalent weight , F = Faraday]

or $W = \dfrac{E}{F} \times Q$

or $Q = \dfrac{W \times F}{E}$ or $Q = \dfrac{W \times F}{\frac{A}{n}}$

[A = Atomic weight , n = valency of ion]

or $Q = \dfrac{n \times w \times F}{A}$

$= \dfrac{3 \times 5.12 \times 10^3 \times 96500}{27} = 5.49 \times 10^7$ C

16. (d) : Higher the acidity, higher will be the tendency to release protons and hence lighter will be the electrical conductivity. Difluoroacetic acid will be strongest acid due to electron withdrawing effect of two fluorine atoms so as it will show maximum electrical conductivity.

17. (a) : $Cr^{2+} \mid Cr^{3+} = +0.41$ V

$Mn^{2+} \mid Mn^{3+} = -1.57$ V

$Fe^{2+} \mid Fe^{3+} = -0.77$ V

$Co^{2+} \mid Co^{3+} = -1.97$ V

More is the value of oxidation potential more is the tendency to get oxidised.

As Cr will have maximum oxidation potential value, therefore its oxidation will be easiest.

18. (c) : $Zn_{(s)} + 2H^+_{(aq)} \rightleftharpoons Zn^{2+}_{(aq)} + H_{2\,(g)}$

$$E_{cell} = E^\circ_{cell} - \frac{0.059}{2}\log\frac{[Zn^{2+}] \times p_{H_2}}{[H^+]^2}$$

On adding H_2SO_4 the $[H^+]$ will increase therefore E_{cell} will also increase and the equilibrium will shift towards the right.

19. (a) : $\Lambda^\circ_{NaCl} = \Lambda^\circ_{Na^+} + \Lambda^\circ_{Cl^-}$ ... (i)

$\Lambda^\circ_{KBr} = \Lambda^\circ_{K^+} + \Lambda^\circ_{Br^-}$ ... (ii)

$\Lambda^\circ_{KCl} = \Lambda^\circ_{K^+} + \Lambda^\circ_{Cl^-}$ ... (iii)

Equation (i) + (ii) – (iii)

$\Lambda^\circ_{NaBr} = \Lambda^\circ_{Na^+} + \Lambda^\circ_{Br^-}$

$= 126 + 152 - 150 = 128$ S cm^2 mol^{-1}

20. (c) : $E_{cell} = E^\circ_{cell} - \dfrac{0.0591}{n}\log K_c$

$0 = 0.591 - \dfrac{0.0591}{1}\log K_c$

$\Rightarrow\ -0.591 = -0.0591 \log K_c$

$\Rightarrow\ \log K_c = \dfrac{0.591}{0.0591} = 10$

$\therefore\ K_c = \text{antilog } 10 = 1 \times 10^{10}$

21. (c) : $E^\circ_{cell} = E^\circ_{Sn/Sn^{2+}} + E^\circ_{Fe^{3+}/Fe^{2+}}$

$= 0.14 + 0.77 = 0.91$ V

22. (b) : Direct conversion of chemical energy to electric energy can be made considerably more efficient (*i.e.* upto 75%) than the 40% maximum now obtainable through burning of fuel and using the heat to form steam for driving turbines. Furthermore, the water obtained as a byproduct may be used for drinking by the astronauts.

At anode : $2H_{2(g)} + 4OH^-_{(aq)} \rightarrow 4H_2O_{(l)} + 4e^-$

At cathode : $O_{2(g)} + 2H_2O_{(l)} + 4e^- \rightarrow 4OH^-_{(aq)}$

$2H_{2(g)} + O_{2(g)} \rightarrow 2H_2O_{(l)}$

23. (c) : CN^- ions act both as reducing agent as well as good complexing agent.

24. (a) :

	A	*B*	*C*
E°_{red}	+0.5 V	–3.0 V	–1.2 V

More is the value of reduction potential, more is the tendency to get reduced, *i.e.* less is the reducing power.

The reducing power follows the following order:

$B > C > A$.

25. (d) : $E^\circ_{cell} = \dfrac{0.0591}{n}\log K_c$

$0.295 = \dfrac{0.0591}{2}\log K_c$

or, $0.295 = 0.0295 \log K_c$

or, $K_c = \text{antilog } 10$ or $K_c = 1 \times 10^{10}$

26. (c) : $E_{cell} = E^\circ_{cell} - \dfrac{0.0591}{n}\log\dfrac{1}{0.1}$

Here $n = 2$, $E^\circ_{cell} = 1.10$ V

$E_{cell} = 1.10 - \dfrac{0.0591}{2}\log 10$

$E_{cell} = 1.10 - 0.0295 = 1.0705$ V

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27. (b) : The mass of silver deposited on the cathode

$$= \frac{108 \times 9650}{96500} = 10.8 \text{ g}$$

28. (b) : It is also known as heat capacity.

29. (a) : Here Cr^{3+} is oxidised to $Cr_2O_7^{2-}$.

30. (b) : $2H^+ + 2e^- \longrightarrow H_2(P_2)$

$H_2(P_1) \longrightarrow 2H^+ + 2e^-$

Overall reaction : $H_2(P_1) \longrightarrow H_2(P_2)$

$$E = E° - \frac{RT}{nF} \log \frac{P_2}{P_1} = 0 - \frac{RT}{nF} og \frac{P_2}{P_1} = \frac{RT}{nF} \log \frac{P_1}{P_2}$$

31. (a) : $E_{cell} = E_{right\ (cathode)} - E_{left\ (anode)}$.

32. (b) : $S \propto A$ (A = area)

$S \propto C$ (C = concentration)

$S \propto \frac{1}{L}$ (L = length)

Combining we get, $S \propto \frac{AC}{L}$

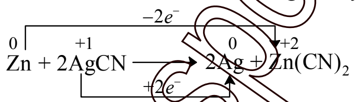
or $S = K\frac{AC}{L}$ [K = constant of proportionality]

or $K = \frac{SL}{AC}$

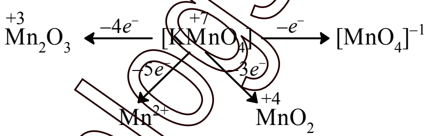
$\therefore$ Unit of $K = \dfrac{S \times m}{m^2 \times \dfrac{mol}{m^3}} = \dfrac{S \times m \times m^3}{m^2 \times mol}$

$= S\ m^2\ mol^{-1}$

33. (d) : The oxidation states show a change only in reaction (d).



34. (c) :



35. (c) : E_{cell} = Reduction potential of cathode (right) – reduction potential of anode (left)

$= E_{right} - E_{left}$.

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CHAPTER 9

CHEMICAL KINETICS

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1. The rate of a reaction doubles when its temperature changes from 300 K to 310 K. Activation energy of such a reaction will be (R = 8.314 J K^{-1} mol^{-1} and log 2 = 0.301)
(a) 60.5 kJ mol^{-1} (b) 53.6 kJ mol^{-1}
(c) 48.6 kJ mol^{-1} (d) 58.5 kJ mol^{-1} *(2013)*

2. For a first order reaction, $(A) \rightarrow$ products, the concentration of A changes from 0.1 M to 0.025 M in 40 minutes. The rate of reaction when the concentration of A is 0.01 M is
(a) 3.47×10^{-4} M/min (b) 3.47×10^{-5} M/min
(c) 1.73×10^{-4} M/min (d) 1.73×10^{-5} M/min *(2012)*

3. The rate of a chemical reaction doubles for every 10°C rise of temperature. If the temperature is raised by 50°C, the rate of the reaction increases by about
(a) 10 times (b) 24 times
(c) 32 times (d) 64 times *(2011)*

4. Consider the reaction :
$Cl_{2(aq)} + H_2S_{(aq)} \longrightarrow S_{(s)} + 2H^+_{(aq)} + 2Cl^-_{(aq)}$
The rate of reaction for this reaction is
rate = $k[Cl_2][H_2S]$
Which of these mechanism is/are consistent with this rate equation?
A. $Cl_2 + H_2S \longrightarrow H^+ + Cl^- + Cl^+ + HS^-$ (slow)
$Cl^+ + HS^- \longrightarrow H^+ + Cl^- + S$ (fast)
B. $H_2S \Leftrightarrow H^+ + HS^-$ (fast equilibrium)
$Cl_2 + HS^- \longrightarrow 2Cl^- + H^+ + S$ (slow)
(a) A only (b) B only
(c) Both A and B (d) Neither A nor B *(2010)*

5. The time for half life period of a certain reaction $A \longrightarrow$ Products is 1 hour. When the initial concentration of the reactant A is 2.0 mol L^{-1}, how much time does it take for its concentration to come from 0.50 to 0.25 mol L^{-1} if it is a zero order reaction?
(a) 1 h (b) 4 h
(c) 0.5 h (d) 0.25 h *(2010)*

6. The half-life period of a first order chemical reaction is 6.93 minutes. The time required for the completion of 99% of the chemical reaction will be
(log 2 = 0.301)
(a) 230.3 minutes (b) 23.03 minutes
(c) 46.06 minutes (d) 460.6 minutes *(2009)*

7. For a reaction $\frac{1}{2}A \rightarrow 2B$ rate of disappearance of A is related to the rate of appearance of B by the expression
(a) $-\frac{d[A]}{dt} = 4\frac{d[B]}{dt}$
(b) $-\frac{d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt}$
(c) $-\frac{d[A]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$
(d) $-\frac{d[A]}{dt} = \frac{d[B]}{dt}$ *(2008)*

8. Consider the reaction, $2A + B \rightarrow$ products. When concentration of B alone was doubled, the half-life did not change. When the concentration of A alone was doubled, the rate increased by two times. The unit of rate constant for this reaction is
(a) s^{-1} (b) L mol^{-1} s^{-1}
(c) no unit (d) mol L^{-1} s^{-1}. *(2007)*

9. The energies of activation for forward and reverse reactions for $A_2 + B_2 \rightleftharpoons 2AB$ are 180 kJ mol^{-1} and 200 kJ mol^{-1} respectively. The presence of a catalyst lowers the activation energy of both (forward and reverse) reactions by 100 kJ mol^{-1}. The enthalpy change of the reaction ($A_2 + B_2 \rightarrow 2AB$) in the presence of a catalyst will be (in kJ mol^{-1})
(a) 20 (b) 300
(c) 120 (d) 280 *(2007)*

10. The following mechanism has been proposed for the reaction of NO with Br_2 to form NOBr.
$NO_{(g)} + Br_{2(g)} \rightleftharpoons NOBr_{2(g)}$
$NOBr_{2(g)} + NO_{(g)} \rightarrow 2NOBr_{(g)}$
If the second step is the rate determining step, the order of the reaction with respect to $NO_{(g)}$ is
(a) 1 (b) 0
(c) 3 (d) 2 *(2006)*

11. Rate of a reaction can be expressed by Arrhenius equation as : $k = Ae^{-E/RT}$. In this equation, E represents

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(a) the energy above which all the colliding molecules will react
(b) the energy below which colliding molecules will not react
(c) the total energy of the reacting molecules at a temperature, T
(d) the fraction of molecules with energy greater than the activation energy of the reaction. *(2006)*

12. A reaction was found to be second order with respect to the concentration of carbon monoxide. If the concentration of carbon monoxide is doubled, with everything else kept the same, the rate of reaction will be
(a) remain unchanged
(b) tripled
(c) increased by a factor of 4
(d) doubled. *(2006)*

13. $t_{1/4}$ can be taken as the time taken for the concentration of a reactant to drop to 3/4 of its initial value. If the rate constant for a first order reaction is k, the $t_{1/4}$ can be written as
(a) $0.10/k$ (b) $0.29/k$
(c) $0.69/k$ (d) $0.75/k$ *(2005)*

14. A reaction involving two different reactants can never be
(a) unimolecular reaction
(b) first order reaction
(c) second order reaction
(d) bimolecular reaction. *(2005)*

15. The rate equation for the reaction $2A + B \rightarrow C$ is found to be : rate = $k[A]\,[B]$. The correct statement in relation to this reaction is that the
(a) unit of k must be s^{-1}
(b) $t_{1/2}$ is a constant
(c) rate of formation of C is twice the rate of disappearance of A
(d) value of k is independent of the initial concentrations of A and B. *(2004)*

16. In a first order reaction, the concentration of the reactant, decreases from 0.8 M to 0.4 M in 15 minutes. The time taken for the concentration to change from 0.1 M to 0.025 M is
(a) 30 minutes (b) 15 minutes
(c) 7.5 minutes (d) 60 minutes. *(2004)*

17. In the respect of the equation $k = Ae^{-E_a/RT}$ in chemical kinetics, which one of the following statements is correct?
(a) k is equilibrium constant.
(b) A is adsorption factor.
(c) E_a is energy of activation.
(d) R is Rydberg constant. *(2003)*

18. For the reaction system:
$$2NO_{(g)} + O_{2(g)} \rightarrow 2NO_{2(g)},$$
volume is suddenly reduced to half its value by increasing the pressure on it. If the reaction is of first order with respect to O_2 and second order with respect to NO_2, the rate of reaction will
(a) diminish to one-fourth of its initial value
(b) diminish to one-eighth of its initial value
(c) increase to eight times of its initial value
(d) increase to four times of its initial value. *(2003)*

19. The rate law for a reaction between the substances A and B is given by rate = $k\,[A]^n\,[B]^m$. On doubling the concentration of A and halving the concentration of B, the ratio of the new rate to the earlier rate of the reaction will be as
(a) $\dfrac{1}{2^{m+n}}$ (b) $(m+n)$
(c) $(n-m)$ (d) $2^{(n-m)}$ *(2003)*

20. The formation of gas at the surface of tungsten due to adsorption is the reaction of order
(a) 0 (b) 1
(c) 2 (d) insufficient data. *(2002)*

21. The differential rate law for the reaction,
$$H_2 + I_2 \rightarrow 2HI \text{ is}$$
(a) $-\dfrac{d[H_2]}{dt} = -\dfrac{d[I_2]}{dt} = -\dfrac{d[HI]}{dt}$
(b) $\dfrac{d[H_2]}{dt} = \dfrac{d[I_2]}{dt} = \dfrac{1}{2}\dfrac{d[HI]}{dt}$
(c) $\dfrac{1}{2}\dfrac{d[H_2]}{dt} = \dfrac{1}{2}\dfrac{d[I_2]}{dt} = -\dfrac{d[HI]}{dt}$
(d) $-2\dfrac{d[H_2]}{dt} = -2\dfrac{d[I_2]}{dt} = \dfrac{d[HI]}{dt}$ *(2002)*

22. For the reaction $A + 2B \rightarrow C$, rate is given by $R = [A]\,[B]^2$ then the order of the reaction is
(a) 3 (b) 6
(c) 5 (d) 7 *(2002)*

23. Units of rate constant of first and zero order reactions in terms of molarity M unit are respectively
(a) s^{-1}, Ms^{-1} (b) s^{-1}, M
(c) Ms^{-1}, s^{-1} (d) M, s^{-1} *(2002)*

Answer Key

1.	(b)	**2.**	(a)	**3.**	(c)	**4.**	(a)	**5.**	(d)	**6.**	(c)
7.	(c)	**8.**	(b)	**9.**	(a)	**10.**	(d)	**11.**	(b)	**12.**	(c)
13.	(b)	**14.**	(a)	**15.**	(d)	**16.**	(a)	**17.**	(c)	**18.**	(c)
19.	(d)	**20.**	(a)	**21.**	(d)	**22.**	(a)	**23.**	(a)		

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Explanations

1. **(b) :** As $r = k[A]^n$

$$\frac{r_2}{r_1} = \frac{k_2}{k_1}$$

Since $\frac{r_2}{r_1} = 2$ (Given)

$$\therefore \frac{k_2}{k_1} = 2$$

$$\log_{10}\frac{k_2}{k_1} = \frac{E_a}{2.303\,R}\left[\frac{T_2 - T_1}{T_1 T_2}\right]$$

$$\log 2 = \frac{E_a}{2.303 \times 8.314 \times 10^{-3}}\left[\frac{310-300}{310 \times 300}\right]$$

$$E_a = \frac{0.3010 \times 2.303 \times 8.314 \times 10^{-3} \times 93 \times 10^3}{10}$$

E_a = 53.6 kJ mol^{-1}

2. **(a) :** For the first order reaction

$$k = \frac{2.303}{t}\log\frac{a}{a-x}$$

a = 0.1 M, $a - x$ = 0.025 M, t = 40 min

$$k = \frac{2.303}{40}\log\frac{0.1}{0.025} = \frac{2.303}{40}\log 4 = 0.0347\ \text{min}^{-1}$$

$[A] \longrightarrow$ product

Thus, rate = $k[A]$

rate = 0.0347 × 0.01 M min^{-1}

= 3.47 × 10^{-4} M min^{-1}

3. **(c) :** $\frac{\text{Rate at }50°\text{C}}{\text{Rate at }T_1°\text{C}} = 2^{\frac{50}{10}} = 2^5 = 32$ times.

4. **(a) :** The rate equation depends upon the rate determining step. The given rate equation is only consistent with the mechanism A.

5. **(d) :** For a zero order reaction, $t_{1/2}$ is given as

$$t_{1/2} = \frac{[A_0]}{2k} \quad \text{or} \quad k = \frac{[A_0]}{2t_{1/2}}$$

Given, $t_{1/2}$ = 1 hr, $[A_0]$ = 2M

$$\therefore \quad k = \frac{2}{2 \times 1} = 1\ \text{mol L}^{-1}\ \text{hr}^{-1}$$

Integrated rate law for zero order reaction is

$[A] = -kt + [A_0]$

Here, $[A_0]$ = 0.5 M and $[A]$ = 0.25 M

$\Rightarrow$ 0.25 = $-t$ + 0.5 $\Rightarrow$ t = 0.25 hours

6. **(c) :** Given, $t_{1/2}$ = 6.93 min

$\lambda = \frac{0.693}{t_{1/2}}$ (for 1st order reaction)

$$= \frac{0.693}{6.93}$$

Since reaction follows 1st order kinetics,

$$t = \frac{2.303}{\lambda}\log\frac{[A_0]}{[A]}$$

where $[A_0]$ = initial concentration

and $[A]$ = concentration of A at time t.

$\because$ Reaction is 99% complete, $\therefore \frac{[A_0]}{[A]} = \frac{100}{1}$

or $t = \frac{2.303 \times 6.93}{0.693} \times \log(100)$

= 23.03 × 2 log(10) = 46.06 minutes.

7. **(c) :** For this reaction,

$$\text{Rate} = -\frac{1}{1/2}\frac{d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt}$$

$$-\frac{d[A]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$$

8. **(b) :** Rate = $k\,[A]^x\,[B]^y$

When $[B]$ is doubled, keeping $[A]$ constant half-life of the reaction does not change.

Now, for a first order reaction $t_{1/2} = \frac{0.693}{k}$

i.e. $t_{1/2}$ is independent of the concentration of the reactant. Hence the reaction is first order with respect to B. Now when $[A]$ is doubled, keeping $[B]$ constant, the rate also doubles. Hence the reaction is first order with respect to A.

$\therefore$ Rate = $[A]^1\,[B]^1$ $\therefore$ order = 2

Now for a nth order reaction, unit of rate constant is (L)$^{n-1}$ (mol)$^{1-n}$ s^{-1} when n = 2, unit of rate constant is L mol^{-1} s^{-1}.

9. **(a) :** $\Delta H_R = E_f - E_b = 180 - 200 = -20$ kJ mol^{-1}

The correct answer for this question should be –20 kJ mol^{-1}. But no option given is correct. Hence we can ignore sign and select option (a).

10. **(d) :** $NO_{(g)} + Br_{2(g)} \rightleftharpoons NOBr_{2(g)}$

$NOBr_{2(g)} + NO_{(g)} \rightarrow 2NOBr_{(g)}$ [rate determining step]

Rate of the reaction (r) = $K\,[NOBr_2]\,[NO]$

where $[NOBr_2] = K_C\,[NO][Br_2]$

$r = K \cdot K_C \cdot [NO][Br_2][NO]$

$r = K'\,[NO]^2\,[Br_2]$

The order of the reaction with respect to $NO_{(g)}$ = 2

11. **(b) :** $k = Ae^{-E/RT}$

where E = activation energy, *i.e.* the minimum amount of energy required by reactant molecules to participate in a reaction.

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12. **(c) :** Given $r_1 = \frac{dx}{dt} = k[CO]^2$

$r_2 = k[2CO]^2 = 4k[CO]^2$

Thus, according to the rate law expression doubling the concentration of CO increases the rate by a factor of 4.

13. **(b) :** $t_{1/4} = \frac{2.303}{k}\log\frac{4}{3} = \frac{0.29}{k}$

14. **(a) :** Generally, molecularity of simple reactions is equal to the sum of the number of molecules of reactants involved in the balanced stoichiometric equation. Thus, a reaction involving two different reactants can never be unimolecular. But a reaction involving two different reactants can a first order reaction. For example, for the following reaction

$RCl + H_2O \rightarrow ROH + HCl$

Expected rate law :

Rate = $k[RCl][H_2O]$ expected order = 1 + 1 = 2

But actual rate law :

Rate = $k'[RCl]$ actual order = 1

Here water is taken in excess, hence its concentration may be taken constant.

Here the molecularity of the reaction = 2 and the order of the reaction = 1.

15. **(d) :** $2A + B \rightarrow C$

rate = $k[A][B]$

The value of k (velocity constant) is always independent of the concentration of reactant and it is a function of temperature only.

For a second order reaction, unit of rate constant, k is L mol^{-1} sec^{-1} for a second order reaction,

$$t_{1/2} = \frac{1}{ka}$$

i.e. $t_{1/2}$ is inversely proportional to initial concentration.

$$2A + B \rightarrow C$$

$$\text{Rate} = -\frac{1}{2}\frac{d[A]}{dt} = -\frac{d[B]}{dt} = \frac{d[C]}{dt}$$

i.e. rate of formation of C is half the rate of disappearance of B.

16. **(a) :** The concentration of the reactant decreases from 0.8 M to 0.4 M in 15 minutes,

i.e. $t_{1/2}$ = 15 minute.

Therefore, the concentration of reactant will fall from 0.1 M to 0.025 M in two half lives.

i.e. $2t_{1/2} = 2 \times 15 = 30$ minutes.

17. **(c) :** In Arrhenius equation, $k = Ae^{-E_a/RT}$

k = rate constant, A = frequency factor

T = temperature, R = gas constant, E_a = energy of activation.

This equation can be used for calculation of energy of activation.

18. **(c) :** Rate$_1 = k[NO]^2[O_2]$

When volume is reduced to 1/2, concentration becomes two times.

Rate$_2 = k[2NO]^2[2O_2]$

$$\frac{\text{Rate}_1}{\text{Rate}_2} = \frac{k[NO]^2[O_2]}{k[2NO]^2[2O_2]} \text{ or } \frac{\text{Rate}_1}{\text{Rate}_2} = \frac{1}{8}$$

$\therefore$ Rate$_2$ = 8 Rate$_1$

19. **(d) :** Rate$_1 = k[A]^n[B]^m$

On doubling the concentration of A and halving the concentration of B

Rate$_2 = k[2A]^n[B/2]^m$

Ratio between new and earlier rate

$$= \frac{k[2A]^n[B/2]^m}{k[A]^n[B]^m} = 2^n \times \left(\frac{1}{2}\right)^m = 2^{n-m}$$

20. **(a)**

21. **(d) :** $H_2 + I_2 \rightarrow 2HI$

When 1 mole of H_2 and 1 mole of I_2 reacts, 2 moles of HI are formed in the same time interval.

Thus the rate may be expressed as

$$\frac{-d[H_2]}{dt} = \frac{-d[I_2]}{dt} = \frac{1}{2}\frac{d[HI]}{dt}$$

The negative sign signifies a decrease in concentration of the reactant with increase of time.

22. **(a) :** Order is the sum of the power of the concentrations terms in rate law expression.

$R = [A]\cdot[B]^2$

Thus, order of reaction = 1 + 2 = 3

23. **(a) :** Unit of $K = (\text{mol L}^{-1})^{1-n}\text{ s}^{-1}$,

where n = order of reaction

$n = 0 \Rightarrow$ zero order reaction

$n = 1 \Rightarrow$ first order reaction

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CHAPTER 10

SURFACE CHEMISTRY

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1. The coagulating power of electrolytes having ions Na^+, Al^{3+} and Ba^{2+} for arsenic sulphide sol increases in the order :
(a) $Al^{3+} < Na^+ < Ba^{2+}$ (b) $Al^{3+} < Ba^{2+} < Na^+$
(c) $Na^+ < Ba^{2+} < Al^{3+}$ (d) $Ba^{2+} < Na^+ < Al^{3+}$
(2013)

2. According to Freundlich adsorption isotherm, which of the following is correct?
(a) $\frac{x}{m} \propto p^1$ (b) $\frac{x}{m} \propto p^{1/n}$
(c) $\frac{x}{m} \propto p^0$
(d) All the above are correct for different ranges of pressure.
(2012)

3. Which of the following statements is incorrect regarding physisorption?
(a) It occurs because of van der Waals forces.
(b) More easily liquefiable gases are adsorbed readily.
(c) Under high pressure it results into multi molecular layer on adsorbent surface.
(d) Enthalpy of adsorption ($\Delta H_{\text{adsorption}}$) is low and positive.
(2009)

4. Gold numbers of protective colloids *A*, *B*, *C* and *D* are 0.50, 0.01, 0.10 and 0.005, respectively. The correct order of their protective powers is
(a) $B < D < A < C$
(b) $D < A < C < B$
(c) $C < B < D < A$
(d) $A < C < B < D$ *(2008)*

5. In Langmuir's model of adsorption of a gas on a solid surface
(a) the rate of dissociation of adsorbed molecules from the surface does not depend on the surface covered
(b) the adsorption at a single site on the surface may involve multiple molecules at the same time
(c) the mass of gas striking a given area of surface is proportional to the pressure of the gas
(d) the mass of gas striking a given area of surface is independent of the pressure of the gas.
(2006)

6. The disperse phase in colloidal iron (III) hydroxide and colloidal gold is positively and negatively charged, respectively. Which of the following statements is NOT correct?
(a) Magnesium chloride solution coagulates, the gold sol more readily than the iron (III) hydroxide sol
(b) Sodium sulphate solution causes coagulation in both sols
(c) Mixing of the sols has no effect
(d) Coagulation in both sols can be brought about by electrophoresis
(2005)

7. The volume of a colloidal particle, V_c as compared to the volume of a solute particle in a true solution V_s could be
(a) ~ 1 (b) $\sim 10^{23}$
(c) $\sim 10^{-3}$ (d) $\sim 10^{3}$ *(2005)*

8. Which one of the following characteristics is not correct for physical adsorption?
(a) Adsorption on solids is reversible
(b) Adsorption increases with increase in temperature
(c) Adsorption is spontaneous
(d) Both enthalpy and entropy of adsorption are negative.
(2003)

Answer Key

1.	(c)	**2.**	(d)	**3.**	(d)	**4.**	(d)	**5.**	(c)	**6.**	(c)
7.	(d)	**8.**	(b)								

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Explanations

1. **(c) :** For a negatively charged sol, like As_2S_3, greater the positive charge on cations, greater is the coagulating power.

2. **(d) :** According to Freundlich adsorption isotherm

$$\frac{x}{m} = kp^{1/n}$$

$1/n$ can have values between 0 to 1 over different ranges of pressure.

3. **(d) :** Physical adsorption is an exothermic process (*i.e.*, $\Delta H = -\text{ve}$) but its value is quite low because the attraction of gas molecules and solid surface is weak van der Waals forces.

4. **(d) :** The different protecting colloids differ in their protecting powers. Zsigmondy introduced a term called Gold number to describe the protective power of different colloids. Smaller the value of gold number greater will be protecting power of the protective colloid. Thus

$$\text{protective power of colloid} \propto \frac{1}{\text{Gold number}}$$

5. **(c) :** Assuming the formation of a monolayer of the adsorbate on the surface of the adsorbent, it was derived by Langmuir that the mass of the gas adsorbed per gram of the adsorbent is related to the equilibrium pressure according to the equation:

$$\frac{x}{m} = \frac{aP}{1+bP}$$

where x is the mass of the gas adsorbed on m gram of the adsorbent, P is the pressure and a, b are constants.

6. **(c) :** Opposite charges attract each other. Hence on mixing coagulation of two sols may be take place.

7. **(d) :** For true solution the diameter range is 1 to 10 Å and for colloidal solution diameter range is 10 to 1000 Å.

$$\frac{V_c}{V_s} = \frac{(4/3)\pi r_c^3}{(4/3)\pi r_s^3} = \left(\frac{r_c}{r_s}\right)^3$$

Ratio of diameters = $(10/1)^3 = 10^3$

$V_c/V_s \simeq 10^3$

8. **(b) :** During adsorption, there is always decrease in surface energy which appears as heat. Therefore adsorption always takes place with evolution of heat, *i.e.* it is an exothermic process and since the adsorption process is exothermic, the physical adsorption occurs readily at low temperature and decreases with increasing temperature. (Le Chatelier's principle).

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CHAPTER 11

NUCLEAR CHEMISTRY*

1. Which of the following nuclear reactions will generate an isotope?
(a) β-particle emission
(b) Neutron particle emission
(c) Positron emission
(d) α-particle emission *(2007)*

2. A radioactive element gets spilled over the floor of a room. Its half-life period is 30 days. If the initial velocity is ten times the permissible value, after how many days will it be safe to enter the room?
(a) 100 days (b) 1000 days
(c) 300 days (d) 10 days *(2007)*

3. In the transformation of $^{238}_{92}U$ to $^{234}_{92}U$, if one emission is an α-particle, what should be the other emission(s)?
(a) Two β^-
(b) Two β^- and one β^+
(c) One β^- and one γ
(d) One β^+ and one β^- *(2006)*

4. A photon of hard gamma radiation knocks a proton out of $^{24}_{12}Mg$ nucleus to form
(a) the isotope of parent nucleus
(b) the isobar of parent nucleus
(c) the nuclide $^{23}_{11}Na$
(d) the isobar of $^{23}_{11}Na$ *(2005)*

5. Hydrogen bomb is based on the principle of
(a) nuclear fission (b) natural radioactivity
(c) nuclear fusion (d) artificial radioactivity. *(2005)*

6. The half-life of a radioisotope is four hours. If the initial mass of the isotope was 200 g, the mass remaining after 24 hours undecayed is
(a) 1.042 g (b) 2.084 g
(c) 3.125 g (d) 4.167 g *(2004)*

7. Consider the following nuclear reactions:
$$^{238}_{92}M \rightarrow {}^{X}_{Y}N + 2\,{}^{4}_{2}He \ ; \ {}^{X}_{Y}N \rightarrow {}^{A}_{B}L + 2\beta^+$$
The number of neutrons in the element L is
(a) 142 (b) 144
(c) 140 (d) 146 *(2004)*

8. The half-life of a radioactive isotope is three hours. If the initial mass of the iosotope were 256 g, the mass of it remaining undecayed after 18 hours would be
(a) 4.0 g (b) 8.0 g
(c) 12.0 g (d) 16.0 g *(2003)*

9. The radionucleide $^{234}_{90}Th$ undergoes two successive β-decays followed by one α-decay. The atomic number and the mass number respectively of the resulting radionucleide are
(a) 92 and 234 (b) 94 and 230
(c) 90 and 230 (d) 92 and 230 *(2003)*

10. β-particle is emitted in radioactivity by
(a) conversion of proton to neutron
(b) form outermost orbit
(c) conversion of neutron to proton
(d) β-particle is not emitted. *(2002)*

11. If half-life of a substance is 5 yrs, then the total amount of substance left after 15 years, when initial amount is 64 grams is
(a) 16 g (b) 2 g
(c) 32 g (d) 8 g *(2002)*

Answer Key

1.	(b)	2.	(a)	3.	(a)	4.	(c)	5.	(c)	6.	(c)
7.	(b)	8.	(a)	9.	(c)	10.	(c)	11.	(d)		

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Explanations

1. **(b) :** The atoms of the some elements having same atomic number but different mass numbers are called isotopes.

$${}^{A}_{Z}X \xrightarrow{-\alpha} {}^{A-4}_{Z-2}Y; \quad {}^{A}_{Z}X \xrightarrow{-\beta} {}^{A}_{Z+1}Y$$

$${}^{A}_{Z}X \xrightarrow{-{}^{1}_{0}n} {}^{A-1}_{Z}X \quad {}^{A}_{Z}X \xrightarrow{+\beta^+} {}^{A}_{Z-1}Y$$

2. **(a) :** Let A be the activity for safe working.

Given $A_0 = 10\ A$

$A_0 \times N_0$ and $A \times N$

$$t = \frac{2.303}{\lambda}\log\frac{N_0}{N} = \frac{2.303}{\lambda}\log\frac{A_0}{A}$$

$$= \frac{2.303}{0.693/30}\log\frac{10A}{A} = \frac{2.303\times 30}{0.693}\log 10$$

$$= \frac{2.303\times 30}{0.693} = 99.69 \text{ days} \approx 100 \text{ days}$$

3. **(a) :** ${}_{92}U^{238} \xrightarrow{-\alpha} {}_{90}A^{234} \xrightarrow{-\beta} {}_{91}B^{234} \xrightarrow{-\beta} {}_{92}U^{234}$

Thus in order to get ${}_{92}U^{234}$ as end product 1α and 2β particles should be emitted.

4. **(c) :** ${}^{24}_{12}Mg + \gamma \rightarrow {}^{23}_{11}Na + {}^{1}_{1}p$

5. **(c) :** Hydrogen bomb is based on the principal of nuclear fusion. In hydrogen bomb, a mixture of deuterium oxide and tritium oxide is enclosed in a space surrounding an ordinary atomic bomb. The temperature produced by the explosion of the atomic bomb initiates the fusion reaction between ${}^{3}_{1}H$ and ${}^{2}_{1}H$ releasing huge amount of energy.

6. **(c) :** $t_{1/2} = 4$ hours

$$n = \frac{T}{t_{1/2}} = \frac{24}{4} = 6\ ;\quad N = N_0\left(\frac{1}{2}\right)^n$$

or, $N = 200 \times \left(\frac{1}{2}\right)^6 = 3.125$ g

7. **(b) :** ${}^{238}_{92}M \rightarrow {}^{230}_{88}N + 2\,{}^{4}_{2}He$

${}^{230}_{88}N \rightarrow {}^{230}_{86}L + 2\beta^+$

Therefore, number of neutrons in element L
$= 230 - 86 = 144$

8. **(a) :** $t_{1/2} = 3$ hours, $n = T/t_{1/2} = 18/3 = 6$

$$N = N_0\left(\frac{1}{2}\right)^n \Rightarrow N = 256\left(\frac{1}{2}\right)^6$$

$\Rightarrow N = 4.0$ g

9. **(c) :** ${}_{90}Th^{234} \xrightarrow{-2\beta} {}_{92}X^{234} \xrightarrow{-\alpha} {}_{90}Th^{230}$

Elimination of 1α and 2β particles give isotope.

10. **(c) :** Since the nucleus does not contain β-particles, it is produced by the conversion of a neutron to a proton at the moment of emission.

$${}_{0}n^1 \rightarrow {}_{+1}p^1 + {}_{-1}e^0$$

11. **(d) :** $t_{1/2} = 5$ years, $\quad n = \frac{T}{t_{1/2}} = \frac{15}{5} = 3$

$$n = N_0\left(\frac{1}{2}\right)^n = 64\left(\frac{1}{2}\right)^3 = 8 \text{ g}$$

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CHAPTER 12

CLASSIFICATION OF ELEMENTS AND PERIODICITY IN PROPERTIES

1. The first ionisation potential of Na is 5.1 eV. The value of electron gain enthalpy of Na^+ will be
(a) + 2.55 eV (b) – 2.55 eV
(c) – 5.1 eV (d) – 10.2 eV *(2013)*

2. Which of the following represents the correct order of increasing first ionization enthalpy for Ca, Ba, S, Se and Ar?
(a) Ca < Ba < S < Se < Ar
(b) Ca < S < Ba < Se < Ar
(c) S < Se < Ca < Ba < Ar
(d) Ba < Ca < Se < S < Ar *(2013)*

3. The increasing order of the ionic radii of the given isoelectronic species is
(a) S^{2-}, Cl^-, Ca^{2+}, K^+ (b) Ca^{2+}, K^+, Cl^-, S^{2-}
(c) K^+, S^{2-}, Ca^{2+}, Cl^- (d) Cl^-, Ca^{2+}, K^+, S^{2-}
(2012)

4. Which one of the following orders presents the correct sequence of the increasing basic nature of the given oxides?
(a) $Al_2O_3 < MgO < Na_2O < K_2O$
(b) $MgO < K_2O < Al_2O_3 < Na_2O$
(c) $Na_2O < K_2O < MgO < Al_2O_3$
(d) $K_2O < Na_2O < Al_2O_3 < MgO$ *(2011)*

5. The correct sequence which shows decreasing order of the ionic radii of the element is
(a) $O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$
(b) $Al^{3+} > Mg^{2+} > Na^+ > F^- > O^{2-}$
(c) $Na^+ > Mg^{2+} > Al^{3+} > O^{2-} > F^-$
(d) $Na^+ > F^- > Mg^{2+} > O^{2-} > Al^{3+}$ *(2010)*

6. Following statements regarding the periodic trends of chemical reactivity of the alkali metals and the halogens are given. Which of these statements gives the correct picture?
(a) The reactivity decreases in the alkali metals but increases in the halogens with increase in atomic number down the group.
(b) In both the alkali metals and the halogens the chemical reactivity decreases with increase in atomic number down the group.
(c) Chemical reactivity increases with increase in atomic number down the group in both the alkali metals and halogens.
(d) In alkali metals the reactivity increases but in the halogens it decreases with increase in atomic number down the group.
(2006)

7. The decreasing values of bond angles from NH_3 (106°) to SbH_3 (101°) down group-15 of the periodic table is due to
(a) increasing bond-bond pair repulsion
(b) increasing *p*-orbital character in sp^3
(c) decreasing lone pair-bond pair repulsion
(d) decreasing electronegativity. *(2006)*

8. The increasing order of the first ionisation enthalpies of the elements B, P, S and F (lowest first) is
(a) F < S < P < B (b) P < S < B < F
(c) B < P < S < F (d) B < S < P < F *(2006)*

9. Which one of the following sets of ions represents a collection of isoelectronic species?
(a) K^+, Cl^-, Ca^{2+}, Sc^{3+}
(b) Ba^{2+}, Sr^{2+}, K^+, S^{2-}
(c) N^{3-}, O^{2-}, F^-, S^{2-}
(d) Li^+, Na^+, Mg^{2+}, Ca^{2+} *(2006)*

10. In which of the following arrangements the order is NOT according to the property indicated against it?
(a) $Al^{3+} < Mg^{2+} < Na^+ < F^-$ - increasing ionic size
(b) B < C < N < O - increasing first ionisation enthalpy
(c) I < Br < F < Cl - increasing electron gain enthalpy (with negative sign)
(d) Li < Na < K < Rb - increasing metallic radius
(2005)

11. Based on lattice energy and other considerations which one of the following alkali metal chlorides is expected to have the highest melting point?
(a) LiCl (b) NaCl
(c) KCl (d) RbCl *(2005)*

12. Lattice energy of an ionic compound depends upon
(a) charge on the ion only
(b) size of the ion only
(c) packing of the ion only
(d) charge and size of the ion. *(2005)*

13. Which among the following factors is the most important in making fluorine the strongest oxidising agent?
(a) Electron affinity
(b) Ionization energy
(c) Hydration enthalpy
(d) Bond dissociation energy *(2004)*

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14. The formation of the oxide ion $O^{2-}_{(g)}$ requires first an exothermic and then an endothermic step as shown below.

$O_{(g)} + e^- = O^-_{(g)}$; $\Delta H° = -142$ kJmol^{-1}

$O^-_{(g)} + e^- = O^{2-}_{(g)}$; $\Delta H° = 844$ kJmol^{-1}

This is because

(a) oxygen is more electronegative
(b) oxygen has high electron affinity
(c) O^- ion will tend to resist the addition of another electron
(d) O^- ion has comparatively larger size than oxygen atom.

(2004)

15. Which one of the following sets of ions represents the collection of isoelectronic species?

(a) K^+, Ca^{2+}, Sc^{3+}, Cl^-
(b) Na^+, Ca^{2+}, Sc^{3+}, F^-
(c) K^+, Cl^-, Mg^{2+}, Sc^{3+}
(d) Na^+, Mg^{2+}, Al^{3+}, Cl^-.

(Atomic nos.: F = 9, Cl = 17, Na = 11, Mg = 12, Al = 13, K = 19, Ca = 20, Sc = 21)

(2004)

16. Which one of the following ions has the highest value of ionic radius?

(a) Li^+ (b) B^{3+}
(c) O^{2-} (d) F^- *(2004)*

17. Which one of the following groupings represents a collection of isoelectronic species?

(At. nos.: Cs-55, Br-35)

(a) Na^+, Ca^{2+}, Mg^{2+} (b) N^{3-}, F^-, Na^+
(c) Be, Al^{3+}, Cl^- (d) Ca^{2+}, Cs^+, Br *(2003)*

18. According to the periodic law of elements, the variation in properties of elements is related to their

(a) atomic masses
(b) nuclear masses
(c) atomic numbers
(d) nuclear neutron-proton number ratios. *(2003)*

19. Which is the correct order of atomic sizes?

(a) Ce > Sn > Yb > Lu
(b) Sn > Ce > Lu > Yb
(c) Lu > Yb > Sn > Ce
(d) Sn > Yb > Ce > Lu.

(At. Nos. : Ce = 58, Sn = 50, Yb = 70 and Lu = 71)

(2002)

Answer Key

1.	(c)	**2.**	(d)	**3.**	(b)	**4.**	(a)	**5.**	(a)	**6.**	(d)
7.	(c)	**8.**	(d)	**9.**	(a)	**10.**	(b)	**11.**	(b)	**12.**	(d)
13.	(d)	**14.**	(c)	**15.**	(a)	**16.**	(c)	**17.**	(b)	**18.**	(c)
19.	(a)										

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Explanations

1. **(c) :** Electron gain enthalpy = – Ionisation potential
$= -5.1$ eV

2. **(d) :** Ionization enthalpy decreases from top to bottom in a group while it increases from left to right in a period.

3. **(b) :** For isoelectronic species as effective nuclear charge increases, ionic radii decreases. Nuclear charge is maximum of the specie with maximum protons. Order of nuclear charge:

	Ca^{2+} >	K^+ >	Cl^- >	S^{2-}
Protons :	20	19	17	16
Electrons :	18	18	18	18

Thus increasing order of ionic radii
$Ca^{2+} < K^+ < Cl^- < S^{2-}$

4. **(a) :** While moving from left to right in periodic table basic character of oxide of elements will decrease.
$\therefore \xrightarrow[\text{Increasing basic strength}]{Al_2O_3 < MgO < Na_2O}$
And while descending in the group basic character of corresponding oxides increases.
$\therefore \xrightarrow[\text{Increasing basic strength}]{Na_2O < K_2O}$
$\therefore$ Correct order is
$Al_2O_3 < MgO < Na_2O < K_2O$

5. **(a) :** All the given species are isoelectronic. Among isoelectronic species, anions generally have greater size than cations.
Also greater, the nuclear charge (Z) of the ion, smaller the size. Thus the order is :
$O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$

6. **(d) :** All the alkali metals are highly reactive elements since they have a strong tendency to lose the single valence s-electron to form unipositive ions having inert gas configuration. This reactivity arises due to their low ionisation enthalpies and high negative values of their standard electrode potentials. However, the reactivity of halogens decreases with increase in atomic number due to following reasons:
(a) As the size increases, the attraction for an additional electron by the nucleus becomes less.
(b) Due to decrease in electronegativity from F to I, the bond between halogen and other elements becomes weaker and weaker.

7. **(c) :**

	NH_3	PH_3	AsH_3	SbH_3
Bond angle	106.5°	93.5°	91.5°	91.3°

The bond angle in ammonia is less than 109° 28′ due to repulsion between lone pairs present on nitrogen atom and bonded pairs of electrons. As we move down the group, the bond angles gradually decrease due to decrease in bond pair lone pair repulsion.

8. **(d) :**

Element:	B	S	P	F
I.E. (eV):	8.3	10.4	11.0	17.4

In general as we move from left to right in a period, the ionisation enthalpy increases with increasing atomic number. The ionisation enthalpy decreases as we move down a group. P($1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^3$) has a stable half filled electronic configuration than S ($1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^4$). For this reason, ionisation enthalpy of P is higher than S.

9. **(a) :** $K^+ = 19 - 1 = 18\ e^-$
$Cl^- = 17 + 1 = 18\ e^-$
$Ca^{2+} = 20 - 2 = 18\ e^-$
$Sc^{3+} = 21 - 3 = 18\ e^-$
Thus all the species are isoelectronic.

10. **(b) :** As we move from left to right across a period, ionisation enthalpy increases with increasing atomic number. So the order of increasing ionisation enthalpy should be $B < C < N < O$.
But N($1s^2\ 2s^2\ 2p^3$) has a stable half filled electronic configuration. So, ionization enthalpy of nitrogen is greater than oxygen.
So, the correct order of increasing the first ionization enthalpy is $B < C < O < N$.

11. **(b) :** In case of halides of alkali metals, melting point decreases going down the group because lattice enthalpies decreases as size of alkali metal increases. But LiCl has lower melting point in comparison to NaCl due to covalent nature. Thus, NaCl is expected to have the highest melting point among given halides.

12. **(d) :** The value of lattice energy depends on the charges present on the two ions and the distance between them.

13. **(d) :** The bond dissociation energy of F – F bond is very low. The weak F – F bond makes fluorine the strongest oxidising halogen.

14. **(c) :** The addition of second electron in an atom or ion is always endothermic.

15. **(a) :** Isoelectronic species are those which have same number of electrons.
$K^+ = 19 - 1 = 18$; $Ca^{2+} = 20 - 2 = 18$
$Sc^{3+} = 21 - 3 = 18$; $Cl^- = 17 + 1 = 18$
Thus all these ions have 18 electrons in them.

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16. **(c)** : This can be explained on the basis of z/e $\left\{\frac{\text{nuclear charge}}{\text{no. of electrons}}\right\}$, whereas z/e ratio increases, the size decreases and when z/e ratio decreases the size increases.

For Li^+, $\frac{z}{e} = \frac{3}{2} = 1.5$

For B^{3+}, $\frac{z}{e} = \frac{5}{2} = 2.5$

For O^{2-}, $\frac{z}{e} = \frac{8}{10} = 0.8$

For F^-, $\frac{z}{e} = \frac{9}{10} = 0.9$

Hence, O^{2-} has highest value of ionic radius.

17. **(b)** : Isoelectronic species are the neutral atoms, cations or anions of different elements which have the same number of electrons but different nuclear charge. Number of electrons in $N^{3-} = 7 + 3 = 10$.

Number of electrons in $F^- = 9 + 1 = 10$

Number of electrons in $Na^+ = 11 - 1 = 10$

18. **(c)** : According to modified modern periodic law, the properties of elements are periodic functions of their atomic numbers.

19. **(a)** : Generally as we move from left to right in a period, there is regular decrease in atomic radii and in a group as the atomic number increases the atomic radii also increases. Thus the atomic radius of Sn should be less than lanthanides. La > Sn. But due to lanthanide contraction, in case of lanthanides there is a continuous decrease in size with increase in atomic number. Hence the atomic radius follow the given trend : Ce > Sn > Yb > Lu.

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CHAPTER 13

GENERAL PRINCIPLES AND PROCESSES OF ISOLATION OF METALS

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1. Which method of purification is represented by the following equation?

$Ti_{(s)} + 2I_{2(g)} \xrightarrow{523\text{ K}} TiI_{4(g)} \xrightarrow{1700\text{ K}} Ti_{(s)} + 2I_{2(g)}$

(a) Cupellation (b) Poling
(c) Van Arkel (d) Zone refining *(2012)*

2. Which of the following factors is of no significance for roasting sulphide ores to the oxides and not subjecting the sulphide ores to carbon reduction directly?
(a) CO_2 is more volatile than CS_2.
(b) Metal sulphides are thermodynamically more stable than CS_2.
(c) CO_2 is thermodynamically more stable than CS_2.
(d) Metal sulphides are less stable than the corresponding oxides.
(2008)

3. During the process of electrolytic refining of copper, some metals present as impurity settle as 'anode mud'. These are
(a) Sn and Ag (b) Pb and Zn
(c) Ag and Au (d) Fe and Ni.
(2005)

4. Which one of the following ores is best concentrated by froth-flotation method?
(a) Magnetite (b) Cassiterite
(c) Galena (d) Malachite.
(2004)

5. When the sample of copper wth zinc impurity is to be purified by electrolysis, the appropriate electrodes are

	cathode	**anode**
(a)	pure zinc	pure copper
(b)	impure sample	pure copper
(c)	impure zinc	impure sample
(d)	pure copper	impure sample.

(2002)

6. Cyanide process is used for the extraction of
(a) barium (b) aluminium
(c) boron (d) silver.
(2002)

7. The metal extracted by leaching with a cyanide is
(a) Mg (b) Ag
(c) Cu (d) Na. *(2002)*

8. Aluminium is extracted by the electrolysis of
(a) bauxite
(b) alumina
(c) alumina mixed with molten cryolite
(d) molten cryolite.
(2002)

Answer Key

1.	(c)	**2.**	(a)	**3.**	(c)	**4.**	(c)	**5.**	(d)	**6.**	(d)
7.	(b)	**8.**	(c)								

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Explanations

1. **(c) :** Van Arkel method which is also called as vapour-phase refining is used for preparing ultrapure metals like titanium, zirconium, thorium and uranium.

2. **(a) :** Oxidising roasting is a very common type of roasting in metallurgy and is carried out to remove sulphur and arsenic in the form of their volatile oxides. CS_2 is more volatile than CO_2. So option (a) is of no significance for roasting sulphide ores to their oxides. The reduction process is on the thermodynamic stability of the products and not on their volatility.

3. **(c) :** In the electrolytic refining of copper the more electropositive impurities like Fe, Zn, Ni, Co, etc. dissolve in the solution and less electropositive impurities such as Ag, Au and Pt collect below the anode in the form of anodic mud.

4. **(c) :** Froth-flotation method is used for the concentration of sulphide ores. The method is based on the preferential wetting properties with the frothing agent and water. Here galena (PbS) is the only sulphide ore.

5. **(d) :** The impure metal is made anode while a thin sheet of pure metal acts as cathode. On passing the current, the pure metal is deposited on the cathode and equivalent amount of the metal gets dissolved from the anode.

6. **(d) :** Gold and silver are extracted from their native ores by Mac-Arthur forrest cyanide process.

7. **(b) :** Silver ore forms a soluble complex with NaCN from which silver is precipitated using scrap zinc.

$$Ag_2S + 2NaCN \rightarrow Na[Ag(CN)_2] \xrightarrow{Zn} Na_2[Zn(CN)_4] + Ag\downarrow$$

sod. argentocyanide (soluble)

8. **(c) :** Aluminium is obtained by the electrolysis of the pure alumina (20 parts) dissolved in a bath of fused cryolite (60 parts) and fluorspar (20 parts).

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CHAPTER 14

HYDROGEN

1. Very pure hydrogen (99.9%) can be made by which of the following processes?
(a) Mixing natural hydrocarbons of high molecular weight.
(b) Electrolysis of water.
(c) Reaction of salt like hydrides with water.
(d) Reaction of methane with steam. *(2012)*

2. In context with the industrial preparation of hydrogen from water gas ($CO + H_2$), which of the following is the correct statement?
(a) CO is oxidised to CO_2 with steam in the presence of a catalyst followed by absorption of CO_2 in alkali.
(b) CO and H_2 are fractionally separated using differences in their densities.
(c) CO is removed by absorption in aqueous Cu_2Cl_2 solution.
(d) H_2 is removed through occlusion with Pd. *(2008)*

3. Which one of the following processes will produce hard water?
(a) Saturation of water with $CaCO_3$.
(b) Saturation of water with $MgCO_3$.
(c) Saturation of water with $CaSO_4$.
(d) Addition of Na_2SO_4 to water. *(2003)*

Answer Key

1.	(b)	**2.**	(a)	**3.**	(c)

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Explanations

1. **(b) :** Dihydrogen of high purity is usually prepared by the electrolysis of water using platinum electrodes in presence of small amount of acid or alkali.

$$2H_2O_{(l)} \xrightarrow{\text{electrolysis}} 2H_{2(g)} + O_{2(g)}$$

Dihydrogen is collected at cathode.

2. **(a) :** Carbon monoxide is oxidised to carbon dioxide by passing the gases and steam over an iron oxide or cobalt oxide or chromium oxide catalyst at 673 K resulting in the production of more H_2.

$$CO + H_2O \xrightarrow[673\ K]{Fe_2O_3,\ CoO} CO_2 + H_2$$

$$CO_2 \xrightarrow[\text{alkali}]{NaOH} Na_2CO_3$$

CO_2 is absorbed in alkali (NaOH).
The entire reaction is called water gas shift reaction.

3. **(c) :** Permanent hardness is introduced when water passes over rocks containing the sulphates or chlorides of both of calcium and magnesium.

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CHAPTER 15

s-BLOCK ELEMENTS

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1. Which of the following on thermal decomposition yields a basic as well as an acidic oxide?
(a) $KClO_3$ (b) $CaCO_3$
(c) NH_4NO_3 (d) $NaNO_3$ *(2012)*

2. The set representing the correct order of ionic radius is
(a) $Li^+ > Be^{2+} > Na^+ > Mg^{2+}$
(b) $Na^+ > Li^+ > Mg^{2+} > Be^{2+}$
(c) $Li^+ > Na^+ > Mg^{2+} > Be^{2+}$
(d) $Mg^{2+} > Be^{2+} > Li^+ > Na^+$ *(2009)*

3. The ionic mobility of alkali metal ions in aqueous solution is maximum for
(a) K^+ (b) Rb^+
(c) Li^+ (d) Na^+ *(2006)*

4. Beryllium and aluminium exhibit many properties which are similar. But, the two elements differ in
(a) exhibiting maximum covalency in compounds
(b) forming polymeric hydrides
(c) forming covalent halides
(d) exhibiting amphoteric nature in their oxides. *(2004)*

5. One mole of magnesium nitride on the reaction with an excess of water gives
(a) one mole of ammonia
(b) one mole of nitric acid
(c) two moles of ammonia
(d) two moles of nitric acid. *(2004)*

6. Several blocks of magnesium are fixed to the bottom of a ship to
(a) keep away the sharks
(b) make the ship lighter
(c) prevent action of water and salt
(d) prevent puncturing by under-sea rocks. *(2003)*

7. In curing cement plasters water is sprinkled from time to time. This helps in
(a) keeping it cool
(b) developing interlocking needle-like crystals of hydrated silicates
(c) hydrating sand and gravel mixed with cement
(d) converting sand into silicic acid. *(2003)*

8. The solubilities of carbonates decrease down the magnesium group due to a decrease in
(a) lattice energies of solids
(b) hydration energies of cations
(c) inter-ionic attraction
(d) entropy of solution formation. *(2003)*

9. The substance not likely to contain $CaCO_3$ is
(a) a marble statue (b) calcined gypsum
(c) sea shells (d) dolomite. *(2003)*

10. A metal M readily forms its sulphate MSO_4 which is water-soluble. It forms its oxide MO which becomes inert on heating. It forms an insoluble hydroxide $M(OH)_2$ which is soluble in NaOH solution. Then M is
(a) Mg (b) Ba
(c) Ca (d) Be. *(2002)*

11. KO_2 (potassium super oxide) is used in oxygen cylinders in space and submarines because it
(a) absorbs CO_2 and increases O_2 content
(b) eliminates moisture
(c) absorbs CO_2
(d) produces ozone. *(2002)*

Answer Key

1. (b)	**2.** (b)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (b)
7. (b)	**8.** (b)	**9.** (b)	**10.** (d)	**11.** (a)	

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Explanations

1. **(b) :** $CaCO_3 \xrightarrow{\Delta} \underset{\text{(basic)}}{\underset{\text{metal oxide}}{CaO}} + \underset{\text{(acidic)}}{\underset{\text{non-metal oxide}}{CO_2}}$

2. **(b) :** Moving from left to right in a period, the ionic radii decrease due to increase in effective nuclear charge as the additional electrons are added to the same shell, however from top to bottom the ionic radii increase with increasing atomic number and presence of additional shells. Also Li and Mg are diagonally related and hence the order is
$$Na^+ > Li^+ > Mg^{2+} > Be^{2+}.$$

3. **(b) :** The alkali metal ion exist as hydrated ions $M^+(H_2O)_n$ in the aqueous solution. The degree of hydration, decreases with ionic size as we go down the group. Hence Li^+ ion is mostly hydrated *e.g.* $[Li(H_2O)_6]^+$. Since the mobility of ions is inversely proportional to the size of the their hydrated ions, hence the increasing order of ionic mobility is
$Li^+ < Na^+ < K^+ < Rb^+$

4. **(a) :** Beryllium has the valency +2 while aluminium exhibits its valency as +3.

5. **(c) :** $Mg_3N_2 + 6H_2O \rightarrow 3Mg(OH)_2 + 2NH_3$

6. **(b) :** Magnesium, on account of its lightness, great affinity for oxygen and toughness is used in ship. Being a lighter element, magnesium makes the ship lighter when it is fixed to the bottom of the ship.

7. **(b) :** Water develops interlocking needle-like crystals of hydrated silicates. The reactions involved are the hydration of calcium aluminates and calcium silicates which change into their colloidal gels. At the same time, some calcium hydroxide and aluminium hydroxides are formed as precipitates due to hydrolysis. Calcium hydroxide binds the particles of calcium silicate together while aluminium hydroxide fills the interstices rendering the mass impervious.

8. **(b) :** The stability of the carbonates of the alkaline earth metals increases on moving down the group. The solubility of carbonate of metals in water is generally low. However they dissolve in water containing CO_2 yielding bicarbonates, and this solubility decreases on going down in a group with the increase in stability of carbonates of metals, and decrease in hydration energy of the cations.

9. **(b) :** The composition of gypsum is $CaSO_4{\cdot}2H_2O$. It does not have $CaCO_3$.

10. **(d) :** Be forms water soluble $BeSO_4$, water insoluble $Be(OH)_2$ and BeO. $Be(OH)_2$ is insoluble in NaOH giving sodium beryllate Na_2BeO_2.

11. **(a) :** $4KO_2 + 2CO_2 \rightarrow 2K_2CO_3 + 3O_2$

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CHAPTER 16

p-BLOCK ELEMENTS

1. Which of the following exists as covalent crystals in the solid state?
(a) Phosphorus (b) Iodine
(c) Silicon (d) Sulphur *(2013)*

2. Which of the following is the wrong statement?
(a) Ozone is diamagnetic gas.
(b) ONCl and ONO^- are not isoelectronic.
(c) O_3 molecule is bent.
(d) Ozone is violet-black in solid state. *(2013)*

3. Boron cannot form which one of the following anions?
(a) BF_6^{3-} (b) BH_4^-
(c) $B(OH)_4^-$ (d) BO_2^-
(2011)

4. Which of the following statement is wrong?
(a) The stability of hydrides increases from NH_3 to BiH_3 in group 15 of the periodic table.
(b) Nitrogen cannot form $d\pi$-$p\pi$ bond.
(c) Single N — N bond is weaker than the single P – P bond.
(d) N_2O_4 has two resonance structure.
(2011)

5. Which of the following statements regarding sulphur is incorrect?
(a) S_2 molecule is paramagnetic.
(b) The vapour at 200°C consists mostly of S_8 rings.
(c) At 600°C the gas mainly consists of S_2 molecules.
(d) The oxidation state of sulphur is never less than +4 in its compounds.
(2011)

6. The bond dissociation energy of B – F in BF_3 is 646 kJ mol^{-1} whereas that of C – F in CF_4 is 515 kJ mol^{-1}. The correct reason for higher B – F bond dissociation energy as compared to that of C – F is
(a) smaller size of B-atom as compared to that of C-atom
(b) stronger σ bond between B and F in BF_3 as compared to that between C and F in CF_4
(c) significant $p\pi$-$p\pi$ interaction between B and F in BF_3 whereas there is no possibility of such interaction between C and F in CF_4.
(d) lower degree of $p\pi$-$p\pi$ interaction between B and F in BF_3 than that between C and F in CF_4.
(2009)

7. Which one of the following reactions of xenon compounds is not feasible?
(a) $XeO_3 + 6HF \rightarrow XeF_6 + 3H_2O$
(b) $3XeF_4 + 6H_2O \rightarrow 2Xe + XeO_3 + 12HF + 1.5O_2$
(c) $2XeF_2 + 2H_2O \rightarrow 2Xe + 4HF + O_2$
(d) $XeF_6 + RbF \rightarrow Rb[XeF_7]$ *(2009)*

8. In which of the following arrangements, the sequence is not strictly according to the property written against it?
(a) $CO_2 < SiO_2 < SnO_2 < PbO_2$: increasing oxidising power
(b) HF < HCl < HBr < HI : increasing acid strength
(c) $NH_3 < PH_3 < AsH_3 < SbH_3$:increasing basic strength
(d) B < C < O < N : increasing first ionization enthalpy
(2009)

9. Which one of the following is the correct statement?
(a) $B_2H_6 \cdot 2NH_3$ is known as 'inorganic benzene'.
(b) Boric acid is a protonic acid.
(c) Beryllium exhibits coordination number of six.
(d) Chlorides of both beryllium and aluminium have bridged chloride structures in solid phase.
(2008)

10. Among the following substituted silanes the one which will give rise to cross linked silicone polymer on hydrolysis is
(a) R_3SiCl (b) R_4Si
(c) $RSiCl_3$ (d) R_2SiCl_2 *(2008)*

11. The stability of dihalides of Si, Ge, Sn and Pb increases steadily in the sequence
(a) $PbX_2 \ll SnX_2 \ll GeX_2 \ll SiX_2$
(b) $GeX_2 \ll SiX_2 \ll SnX_2 \ll PbX_2$
(c) $SiX_2 \ll GeX_2 \ll PbX_2 \ll SnX_2$
(d) $SiX_2 \ll GeX_2 \ll SnX_2 \ll PbX_2$. *(2007)*

12. Identify the incorrect statement among the following.
(a) Br_2 reacts with hot and strong NaOH solution to give NaBr and H_2O.
(b) Ozone reacts with SO_2 to give SO_3.
(c) Silicon reacts with $NaOH_{(aq)}$ in the presence of air to give Na_2SiO_3 and H_2O.
(d) Cl_2 reacts with excess of NH_3 to give N_2 and HCl.
(2007)

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13. Regular use of the following fertilizers increases the acidity of soil?
(a) Ammonium sulphate
(b) Potassium nitrate
(c) Urea
(d) Superphosphate of lime *(2007)*

14. A metal, *M* forms chlorides in +2 and +4 oxidation states. Which of the following statements about these chlorides is correct?
(a) MCl_2 is more volatile than. MCl_4.
(b) MCl_2 is more soluble in anhydrous ethanol than MCl_4.
(c) MCl_2 is more ionic than MCl_4.
(d) MCl_2 is more easily hydrolysed than MCl_4. *(2006)*

15. What products are expected from the disproportionation reaction of hypochlorous acid?
(a) $HClO_3$ and Cl_2O (b) $HClO_2$ and $HClO_4$
(c) HCl and Cl_2O (d) HCl and $HClO_3$ *(2006)*

16. Which of the following statements is true?
(a) H_3PO_3 is a stronger acid than H_2SO_3.
(b) In aqueous medium HF is a stronger acid than HCl.
(c) $HClO_4$ is a weaker acid than $HClO_3$.
(d) HNO_3 is a stronger acid than HNO_2. *(2006)*

17. Heating an aqueous solution of aluminium chloride to dryness will give
(a) $AlCl_3$ (b) Al_2Cl_6
(c) Al_2O_3 (d) $Al(OH)Cl_2$ *(2005)*

18. The number and type of bonds between two carbon atoms in calcium carbide are
(a) one sigma, one pi
(b) one sigma, two pi
(c) two sigma, one pi
(d) two sigma, two pi. *(2005)*

19. The structure of diborane (B_2H_6) contains
(a) four 2*c*-2*e* bonds and two 3*c*-2*e* bonds
(b) two 2*c*-2*e* bonds and four 3*c*-2*e* bonds
(c) two 2*c*-2*e* bonds and two 3*c*-3*e* bonds
(d) four 2*c*-2*e* bonds and four 3*c*-2*e* bonds *(2005)*

20. The molecular shapes of SF_4, CF_4 and XeF_4 are
(a) the same with 2, 0 and 1 lone pairs of electrons on the central atom respectively
(b) the same with 1, 1 and 1 lone pair of electrons on the central atoms respectively
(c) different with 0, 1 and 2 lone pairs of electrons on the central atom respectively
(d) different with 1, 0 and 2 lone pairs of electrons on the central atom respectively *(2005)*

21. The number of hydrogen atom(s) attached to phosphorus atom in hypophosphorous acid is
(a) zero (b) two
(c) one (d) three. *(2005)*

22. The correct order of the thermal stability of hydrogen halides (H – *X*) is
(a) HI > HBr > HCl > HF
(b) HF > HCl > HBr > HI
(c) HCl < HF > HBr < HI
(d) HI > HCl < HF > HBr *(2005)*

23. In silicon dioxide
(a) each silicon atom is surrounded by four oxygen atoms and each oxygen atom is bonded to two silicon atoms
(b) each silicon atom is surrounded by two oxygen atoms and each oxygen atom is bonded to two silicon atoms
(c) silicon atom is bonded to two oxygen atoms
(d) there are double bonds between silicon and oxygen atoms. *(2005)*

24. Which of the following oxides is amphoteric in character?
(a) CaO (b) CO_2
(c) SiO_2 (d) SnO_2 *(2005)*

25. The soldiers of Napolean army while at Alps during freezing winter suffered a serious problem as regards to the tin buttons of their uniforms. White metallic tin buttons got converted to grey powder. This transformation is related to
(a) an interaction with nitrogen of the air at very low temperatures
(b) a change in the crystalline structure of tin
(c) a change in the partial pressure of oxygen in the air
(d) an interaction with water vapour contained in the humid air. *(2004)*

26. Aluminium chloride exists as dimer, Al_2Cl_6 in solid state as well as in solution of non-polar solvents such as benzene. When dissolved in water, it gives
(a) $Al^{3+} + 3Cl^-$ (b) $[Al(H_2O)_6]^{3+} + 3Cl^-$
(c) $[Al(OH)_6]^{3-} + 3HCl$ (d) $Al_2O_3 + 6HCl$. *(2004)*

27. Among Al_2O_3, SiO_2, P_2O_3 and SO_2 the correct order of acid strength is
(a) $SO_2 < P_2O_3 < SiO_2 < Al_2O_3$
(b) $SiO_2 < SO_2 < Al_2O_3 < P_2O_3$
(c) $Al_2O_3 < SiO_2 < SO_2 < P_2O_3$
(d) $Al_2O_3 < SiO_2 < P_2O_3 < SO_2$. *(2004)*

28. The states of hybridisation of boron and oxygen atoms in boric acid (H_3BO_3) are respectively
(a) sp^2 and sp^2 (b) sp^2 and sp^3
(c) sp^3 and sp^2 (d) sp^3 and sp^3. *(2004)*

29. Which one of the following statements regarding helium is incorrect?
(a) It is used to fill gas in balloons instead of hydrogen because it is lighter and non-inflammable.
(b) It is used as a cryogenic agent for carrying out experiments at low temperatures.
(c) It is used to produce and sustain powerful superconducting magnets.
(d) It is used in gas-cooled nuclear reactors. *(2004)*

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30. Glass is a
(a) micro-crystalline solid
(b) super-cooled liquid
(c) gel
(d) polymeric mixture. *(2003)*

31. Graphite is a soft solid lubricant extremely difficult to melt. The reason for this anomalous behaviour is that graphite
(a) is a non-crystalline substance
(b) is an allotropic form of diamond
(c) has molecules of variable molecular masses like polymers
(d) has carbon atoms arranged in large plates of rings of strongly bound carbon atoms with weak interplate bonds. *(2003)*

32. Which one of the following pairs of molecules will have permanent dipole moments for both members?
(a) SiF_4 and NO_2 (b) NO_2 and CO_2
(c) NO_2 and O_3 (d) SiF_4 and CO_2 *(2003)*

33. Which one of the following substances has the highest proton affinity?
(a) H_2O (b) H_2S
(c) NH_3 (d) PH_3 *(2003)*

34. Which one of the following is an amphoteric oxide?
(a) ZnO (b) Na_2O
(c) SO_2 (d) B_2O_3 *(2003)*

35. Concentrated hydrochloric acid when kept in open air sometimes produces a cloud of white fumes. The explanation for it is that
(a) concentrated hydrochloric acid emits strongly smelling HCl gas all the time
(b) oxygen in air reacts with the emitted HCl gas to form a cloud of chlorine gas
(c) strong affinity of HCl gas for moisture in air results in forming of droplets of liquid solution which appears like a cloudy smoke
(d) due to strong affinity for water, concentrated hydrochloric acid pulls moisture of air towards itself. This moisture forms droplets of water and hence the cloud. *(2003)*

36. What may be expected to happen when phosphine gas is mixed with chlorine gas?
(a) The mixture only cools down
(b) PCl_3 and HCl are formed and the mixture warms up
(c) PCl_5 and HCl are formed and the mixture cools down
(d) $PH_3 \cdot Cl_2$ is formed with warming up. *(2003)*

37. Which one of the following statements is correct?
(a) Manganese salts give a violet borax test in the reducing flame.
(b) From a mixed precipitate of AgCl and AgI, ammonia solution dissolves only AgCl.
(c) Ferric ions give a deep green precipitate on adding potassium ferrocyanide solution.
(d) On boiling a solution having K^+, Ca^{2+} and HCO_3^- ions we get a precipitate of $K_2Ca(CO_3)_2$. *(2003)*

38. Alum helps in purifying water by
(a) forming Si complex with clay particles
(b) sulphate part which combines with the dirt and removes it
(c) coagulating the mud particles
(d) making mud water soluble. *(2002)*

39. In case of nitrogen, NCl_3 is possible but not NCl_5 while in case of phosphorus, PCl_3 as well as PCl_5 are possible. It is due to
(a) availability of vacant *d* orbitals in P but not in N
(b) lower electronegativity of P than N
(c) lower tendency of H-bond formation in P than N
(d) occurrence of P in solid while N in gaseous state at room temperature. *(2002)*

40. In XeF_2, XeF_4, XeF_6 the number of lone pairs on Xe are respectively
(a) 2, 3, 1 (b) 1, 2, 3
(c) 4, 1, 2 (d) 3, 2, 1. *(2002)*

41. Which of the following statements is true?
(a) HF is less polar than HBr.
(b) Absolutely pure water does not contain any ions.
(c) Chemical bond formation takes place when forces of attraction overcome the forces of repulsion.
(d) In covalency transference of electron takes place. *(2002)*

42. When H_2S is passed through Hg_2S we get
(a) HgS (b) HgS + Hg_2S
(c) Hg_2S + Hg (d) Hg_2S. *(2002)*

Answer Key

1.	(c)	**2.**	(None)	**3.**	(a)	**4.**	(a)	**5.**	(d)	**6.**	(c)
7.	(a)	**8.**	(c)	**9.**	(d)	**10.**	(c)	**11.**	(c)	**12.**	(d)
13.	(a)	**14.**	(c)	**15.**	(d)	**16.**	(d)	**17.**	(b)	**18.**	(b)
19.	(a)	**20.**	(d)	**21.**	(b)	**22.**	(b)	**23.**	(a)	**24.**	(d)
25.	(b)	**26.**	(b)	**27.**	(d)	**28.**	(b)	**29.**	(a)	**30.**	(b)
31.	(d)	**32.**	(c)	**33.**	(c)	**34.**	(a)	**35.**	(c)	**36.**	(c)
37.	(b)	**38.**	(c)	**39.**	(a)	**40.**	(d)	**41.**	(c)	**42.**	(c)

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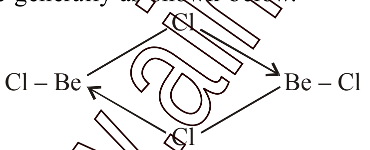
Explanations

1. **(c)**
2. **(None) :** All the statements are correct.
3. **(a) :** Due to non-availability of *d*-orbitals, boron is unable to expand its octet. Therefore, the maximum covalency of boron cannot exceed 4.
4. **(a) :** Thermal stability decreases gradually from NH_3 to BiH_3. So the stability also decreases.

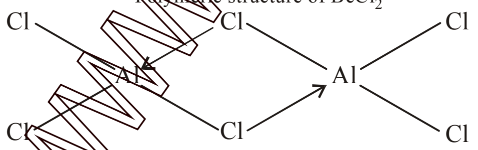
	NH_3	PH_3	AsH_3	SbH_3	BiH_3
Decomposition temperature	1300°C	440°C	280°C	150°C	room temp.

The size of the central atom increases from N to Bi therefore, the tendency to form a stable covalent bond with small atom like hydrogen decreases and therefore, stability decreases.

5. **(d) :** Sulphur exhibits –2, +2, +4, +6 oxidation states but +4 and +6 are more common.
6. **(c) :** In BF_3, B is sp^2 hybridised and has a vacant 2*p*-orbital which overlaps laterally with a filled 2*p*-orbital of F forming strong $p\pi$-$p\pi$ bond. However in CF_4, C does not have any vacant *p*-orbitals to undergo π-bonding. Thus $B.E._{B-F} > B.E._{C-F}$.
7. **(a) :** The reaction is not feasible because XeF_6 formed will further produce XeO_3 by getting hydrolysed.
$XeF_6 + 3H_2O \longrightarrow XeO_3 + 3H_2F_2$.
8. **(c) :** In group15 hydrides, the basic character decreases on going down the group due to decrease in the availability of the lone pair of electrons because of the increase in size of elements from N to Bi. Thus, correct order of basicity is
$NH_3 > PH_3 > AsH_3 > SbH_3$.
9. **(d) :** Boric acid is a weak monobasic acid ($K_a = 1.0 \times 10^{-9}$). It is a notable part that boric acid does not act as a protonic acid (*i.e.*, proton donor) but behaves as a Lewis acid by accepting a pair of electrons from OH^- ions.
$B(OH)_3 + 2H_2O \rightarrow [B(OH)_4]^- + H_3O^+$
$BeCl_2$ like Al_2Cl_6 has a bridged polymeric structure in solid phase generally as shown below.



Polymeric structure of $BeCl_2$



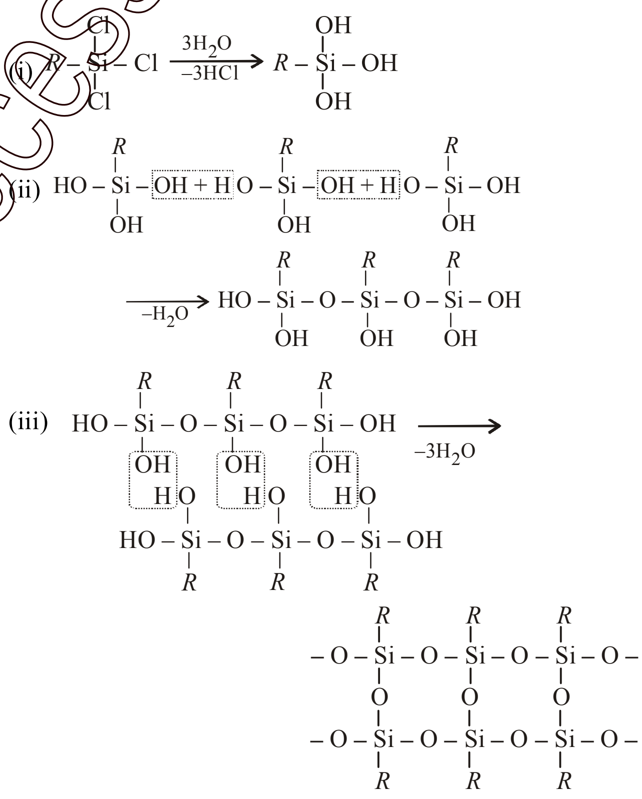
Polymeric structure of Al_2Cl_6

Beryllium exhibits coordination number of four as it has only four available orbitals in its valency shell.
Also,

$$3B_2H_6 + 6NH_3 \longrightarrow 3[BH_2(NH_3)_2]^+BH_4^-$$
or
$$B_2H_6 \cdot 2NH_3$$
or
$$B_2H_6 \cdot 2NH_3 \xrightarrow{\text{Heat, 450 K}} 2B_3N_3H_6 + 12H_2$$
Borazine

Borazine has structure similar to benzene and therefore, it is called inorganic benzene. Hence option (d) is correct.

10. **(c) :** $RSiCl_3$ on hydrolysis gives a cross linked silicone. The formation can be explained in three steps



Cross linked silicone

11. **(c) :** Due to the inert pair effect (the reluctance of ns^2 electrons of outermost shell to participate in bonding) the stability of M^{2+} ions (of group IV elements) increases as we go down the group.
12. **(d) :** $3Br_2 + 6NaOH \rightarrow 5NaBr + NaBrO_3 + 3H_2O$
$O_3 + SO_2 \rightarrow O_2 + SO_3$
$Si + 2NaOH + O_2 \rightarrow Na_2SiO_3 + H_2O$

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Cl_2 reacts with excess of ammonia to produce ammonium chloride and nitrogen.

$$2NH_3 + 3Cl_2 \longrightarrow N_2 + 6HCl$$
$$6NH_3 + 6HCl \longrightarrow 6NH_4Cl$$
$$\overline{8NH_3 + 3Cl_2 \longrightarrow N_2 + 6NH_4Cl}$$

13. (a) : $(NH_4)_2SO_4 + 2H_2O \longrightarrow \underset{\text{Strong acid}}{(2H^+ + SO_4^{2-})} + \underset{\text{Weak base}}{2NH_4OH}$

$(NH_4)_2SO_4$ on hydrolysis produces strong acid H_2SO_4, which increases the acidity of the soil.

14. (c) : The elements of group 14 show an oxidation state of +4 and +2. The compounds showing an oxidation state of +4 are covalent compound and have tetrahedral structures. *e.g.* $SnCl_4$, $PbCl_4$, $SiCl_4$, etc. whereas those which show +2 oxidation state are ionic in nature and behave as reducing agent. *e.g.* $SnCl_2$, $PbCl_2$, etc.

Further as we move down the group, the tendency of the element to form covalent compound decreases but the tendency to form ionic compound increases.

15. (d) : $3HClO_{4(aq)} \rightarrow HClO_{3(aq)} + 2HCl_{(aq)}$

It is a disproportionation reaction of hypochlorous acid where the oxidation number of Cl changes from +1 (in ClO^-) to +5 (in ClO_3^-) and –1 (in Cl^-).

16. (d) : Higher is the oxidation state of the central atom, greater is the acidity.

Hence, $HClO_4$ is a stronger acid than $HClO_3$.

HNO_3 is a stronger acid than HNO_2.

Now, greater is the electronegativity and higher is the oxidation state of the central atom, greater is the acidity. Hence H_2SO_3 is a stronger acid than H_3PO_3.

Due to higher dissociation energy of H – F bond and molecular association due to hydrogen bonding in HF, HF is a weaker acid than HCl.

17. (b) : Aluminium chloride in aqueous solution exists as ion pair.

$2AlCl_3 + aq. \rightarrow [AlCl_2(H_2O)_4]^+_{(aq)} + [AlCl_4(H_2O)_2]^-_{(aq)}$

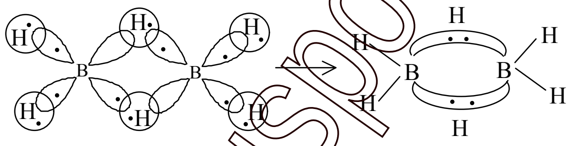
The crystallisation of $AlCl_3$ from aqueous solution, therefore, yields an ionic solid of composition $[AlCl_2(H_2O)_4]^+[AlCl_4(H_2O)_2]^- \cdot xH_2O$. This compound decomposes at about 190°C to give the non-ionic dimer Al_2Cl_6.

$$[AlCl_2(H_2O)_4]^+ [AlCl_4(H_2O)_2]^- \cdot xH_2O \xrightarrow[190°C]{heat} Al_2Cl_6 + H_2O$$

18. (b) : Calcium carbide is ionic carbide having $[:C \equiv C:]^{2-}$.

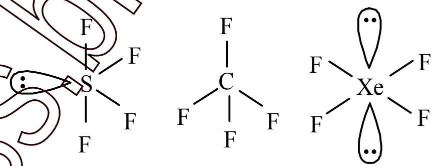
$$Ca^{2+} [:C \overset{1\sigma}{\underset{2\pi}{\equiv}} C:]^{2-}$$

19. (a) : According to molecular orbital theory, each of the two boron atoms is in sp^3 hybrid state. Of the four hybrid orbitals, three have one electron each while the fourth is empty. Two of the four orbitals of each of the boron atom overlap with two terminal hydrogen atoms forming two normal B – H σ-bonds. One of the remaining hybrid orbital (either filled or empty) of one of the boron atoms, 1*s* orbital of hydrogen atoms (bridge atom) and one of hybrid orbitals of the other boron atom overlap to form a delocalised orbital covering the three nuclei with a pair of electrons. Such a bond is known as three centre two electron (3*c* – 2*e*) bonds.



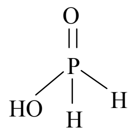
Structure of diborane

20. (d) : SF_4 (sp^3d, trigonal bipyramidal with one equatorial position occupied by 1 lone pair), CF_4 (sp^3, tetrahedral, no lone pair), XeF_4 (sp^3d^2, square planar, two lone pairs).



(SF_4) (CF_4) (XeF_4)

21. (b) : Hypophosphorous acid

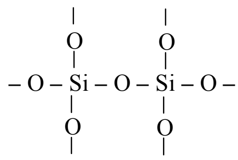


Number of hydrogen atom(s) attached to phosphorus atom = 2.

22. (b) : As the size of the halogen atom increases from F to I, H – *X* bond length in H*X* molecules also increases from H – F to H – I (H – F < H – Cl < H – Br < H – I).

The increase in H – *X* bond length decreases the strength of H – *X* bond from H – F to H – I (H – F > H – Cl > H – Br > H – I). The decrease in the strength of H – *X* bond is evident from the fact that H – *X* bond dissociation energies decrease from H – F to H – I. Due to successive decrease in the strength of H – *X* bond from H – F to H – I, thermal stability of H*X* molecules also decreases from HF to HI (HF > HCl > HBr > HI).

23. (a) : Silicon dioxide exhibits polymorphism. It is a network solid in which each Si atom is surrounded tetrahedrally by four oxygen atoms.



24. (d): CaO-basic, CO_2 and SiO_2-acidic, SnO_2-amphoteric, as it reacts both with acids and bases.

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$SnO_2 + 4HCl \rightarrow SnCl_4 + 2H_2O$
$SnO_2 + 2NaOH \rightarrow Na_2SnO_3 + H_2O$

25. **(b) :** Grey tin is very brittle and easily crumbles down to a powder in very cold climates.

$$\underset{\text{(cubic)}}{\text{Grey tin}} \rightleftharpoons \underset{\text{(tetragonal)}}{\text{White tin}}$$

The change of white tin to grey tin is accompanied by increase in volume. This is called tin disease or tin plague.

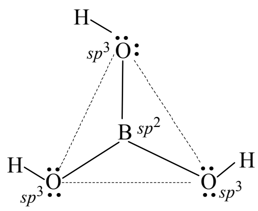
26. **(b) :** $Al_2Cl_6 + 12H_2O \rightleftharpoons 2[Al(H_2O)_6]^{3+} + 6Cl^-$

27. **(d) :** Acidity of the oxides of non metals increases with the electronegativity and oxidation number of the element.

$$Al_2O_3 < SiO_2 < P_2O_3 < SO_2$$

Al_2O_3 is amphoteric. SiO_2 is slightly acidic whereas P_2O_3 and SO_2 are the anhydrides of the acids H_3PO_3 and H_2SO_3.

28. **(b) :**



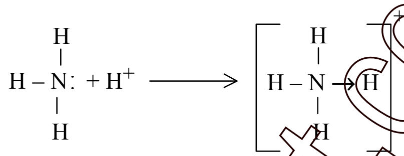
29. **(a) :** Helium is twice as heavy as hydrogen, its lifting power is 92 percent of that of hydrogen.
Helium has the lowest melting and boiling points of any element which makes liquid helium an ideal coolant for many extremely low-temperature applications such as superconducting magnets, and cryogenic research where temperatures close to absolute zero are needed.

30. **(b) :** Glass is a transparent or translucent amorphous supercooled solid solution (supercooled liquid) of silicates and borates, having a general formula $R_2O{\cdot}MO{\cdot}6SiO_2$ where R = Na or K and M = Ca, Ba, Zn or Pb.

31. **(d) :** Graphite has a two-dimensional sheet like structure and each carbon atom makes a use of sp^2 hybridisation.
The above layer structure of graphite is less compact than that of diamond. Further, since the bonding between the layers involving only weak van der Waal's forces, these layers can slide over each other. This gives softness, greasiness and lubricating character of graphite.

32. **(c) :** NO_2 and O_3 both have unsymmetrical structures, so they have permanent dipole moment.

33. **(c) :** Ammonia is a Lewis base, accepting proton to form ammonium ion as it has tendency to donate an electron pair.



34. **(a) :** ZnO is an amphoteric oxide and dissolves readily in acids forming corresponding zinc salts and alkalies forming zincates.

$ZnO + H_2SO_4 \rightarrow \underset{\text{zinc sulphate}}{ZnSO_4} + H_2O$

$ZnO + 2NaOH \rightarrow \underset{\text{sodium zincate}}{Na_2ZnO_2} + H_2O$

35. **(c) :** HCl gas in presence of moisture in air forms droplets of liquid solution in the form of cloudy smoke.

36. **(c) :** Phosphine burns in the atmosphere of chlorine and forms phosphorus pentachloride.

$$PH_3 + 4Cl_2 \rightarrow PCl_5 + 3HCl$$

37. **(b) :** The solubility product of AgCl, AgBr and AgI at the room temperature are 2.8×10^{-10}, 5.0×10^{-13} and 8.5×10^{-17} respectively. Thus, AgI is the least soluble silver halide.
The lattice energies of AgBr and AgI are even higher because of greater number of electrons in their anions. Consequently, they are even less soluble than AgCl. Due to greater solubility of AgCl than AgI, ammonia solution dissolves only AgCl and forms a complex.

$$AgCl + 2NH_4OH \rightarrow \underset{\text{Diammine silver chloride}}{[Ag(NH_3)_2]Cl} + 2H_2O$$

38. **(c) :** The negatively charged colloidal particles of impurities get neutralised by the Al^{3+} ions and settle down and pure water can be decanted off.

39. **(a) :** ${}_7N = 1s^2\ 2s^2\ 3p^3$
${}_{15}P = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^3$
In phosphorus the 3d-orbitals are available.

40. **(d) :**

XeF_2	sp^3d	3 lone pairs
XeF_4	sp^3d^2	2 lone pairs
XeF_6	sp^3d^3	1 lone pair

41. **(c) :** Due to the higher electronegativity of F, HF is more polar than HBr pure water contains H^+ and OH^- ions. In covalency, sharing of electrons between two non-metal atoms takes place.

42. **(c)**

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CHAPTER 17

d- and *f*-BLOCK ELEMENTS

1. Which of the following arrangements does not represent the correct order of the property stated against it?
(a) Sc < Ti < Cr < Mn : number of oxidation states
(b) $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$: paramagnetic behaviour
(c) $Ni^{2+} < Co^{2+} < Fe^{2+} < Mn^{2+}$: ionic size
(d) $Co^{3+} < Fe^{3+} < Cr^{3+} < Sc^{3+}$: stability in aqueous solution.
(2013)

2. Four successive members of the first row transition elements are listed below with atomic numbers. Which one of them is expected to have the highest $E°_{M^{3+}/M^{2+}}$ value?
(a) Co (*Z* = 27) (b) Cr (*Z* = 24)
(c) Mn (*Z* = 25) (d) Fe (*Z* = 26) *(2013)*

3. Iron exhibits +2 and +3 oxidation states. Which of the following statements about iron is incorrect?
(a) Ferrous compounds are relatively more ionic than the corresponding ferric compounds.
(b) Ferrous compounds are less volatile than the corresponding ferric compounds.
(c) Ferrous compounds are more easily hydrolysed than the corresponding ferric compounds.
(d) Ferrous oxide is more basic in nature than the ferric oxide.
(2012)

4. The outer electronic configuration of Gd (Atomic No : 64) is
(a) $4f^3 5d^5 6s^2$ (b) $4f^8 5d^0 6s^2$
(c) $4f^4 5d^4 6s^2$ (d) $4f^7 5d^1 6s^2$ *(2011)*

5. In context of the lanthanoids, which of the following statement is not correct?
(a) There is a gradual decrease in the radii of the members with increasing atomic number in the series.
(b) All the members exhibit +3 oxidation state.
(c) Because of similar properties the separation of lanthanoids is not easy.
(d) Availability of 4*f* electrons results in the formation of compounds in +4 state for all the members of the series.
(2011)

6. The correct order of $E°_{M^{2+}/M}$ values with negative sign for the four successive elements Cr, Mn, Fe and Co is
(a) Cr > Mn > Fe > Co (b) Mn > Cr > Fe > Co
(c) Cr > Fe > Mn > Co (d) Fe > Mn > Cr > Co
(2010)

7. In context with the transition elements, which of the following statements is incorrect?
(a) In addition to the normal oxidation states, the zero oxidation state is also shown by these elements in complexes.
(b) In the highest oxidation states, the transition metals show basic character and form cationic complexes.
(c) In the highest oxidation states of the first five transition elements (Sc to Mn), all the 4*s* and 3*d* electrons are used for bonding.
(d) Once the d^5 configuration is exceeded, the tendency to involve all the 3*d* electrons in bonding decreases.
(2009)

8. Knowing that the chemistry of lanthanoids (Ln) is dominated by its +3 oxidation state, which of the following statements is incorrect?
(a) Because of the large size of the Ln(III) ions the bonding in its compounds is predominantly ionic in character.
(b) The ionic sizes of Ln(III) decrease in general with increasing atomic number.
(c) Ln(III) compounds are generally colourless.
(d) Ln(III) hydroxides are mainly basic in character.
(2009)

9. In which of the following octahedral complexes of Co (At. no. 27), will the magnitude of Δ_{oct} be the highest?
(a) $[Co(NH_3)_6]^{3+}$ (b) $[Co(CN)_6]^{3-}$
(c) $[Co(C_2O_4)_3]^{3-}$ (d) $[Co(H_2O)_6]^{3+}$ *(2008)*

10. Larger number of oxidation states are exhibited by the actinoids than those by the lanthanoids, the main reason being
(a) more reactive nature of the actinoids than the lanthanoids
(b) 4*f* orbitals more diffused than the 5*f* orbitals
(c) lesser energy difference between 5*f* and 6*d* than between 4*f* and 5*d* orbitals
(d) more energy difference between 5*f* and 6*d* than between 4*f* and 5*d*-orbitals.
(2008)

11. The actinoids exhibit more number of oxidation states in general than the lanthanoids. This is because

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(a) the 5*f* orbitals extend further from the nucleus than the 4*f* orbitals
(b) the 5*f* orbitals are more buried than the 4*f* orbitals
(c) there is a similarity between 4*f* and 5*f* orbitals in their angular part of the wave function
(d) the actinoids are more reactive than the lanthanoids. *(2007)*

12. Identify the incorrect statement among the following:
(a) 4*f*-and 5*f*-orbitals are equally shielded.
(b) *d*-Block elements show irregular and erratic chemical properties among themselves.
(c) La and Lu have partially filled *d*-orbitals and no other partially filled orbitals.
(d) The chemistry of various lanthanoids is very similar. *(2007)*

13. The "spin-only" magnetic moment [in units of Bohr magneton, (μ_B)] of Ni^{2+} in aqueous solution would be (atomic number of Ni = 28)
(a) 2.84 (b) 4.90
(c) 0 (d) 1.73 *(2006)*

14. Nickel (Z = 28) combines with a uninegative monodentate ligand X^- to form a paramagnetic complex $[NiX_4]^{2-}$. The number of unpaired electron(s) in the nickel and geometry of this complex ion are, respectively
(a) one, tetrahedral (b) two, tetrahedral
(c) one, square planar (d) two, square planar. *(2006)*

15. Which of the following factors may be regarded as the main cause of lanthanide contraction?
(a) Poor shielding of one of 4*f*-electron by another in the subshell.
(b) Effective shielding of one of 4*f*-electrons by another in the subshell.
(c) Poorer shielding of 5*d* electrons by 4*f*-electrons.
(d) Greater shielding of 5*d* electrons by 4*f*-electrons. *(2006)*

16. The lanthanide contraction is responsible for the fact that
(a) Zr and Y have about the same radius
(b) Zr and Nb have similar oxidation state
(c) Zr and Hf have about the same radius
(d) Zr and Zn have the same oxidation state. *(2005)*

17. Calomel (Hg_2Cl_2) on reaction with ammonium hydroxide gives
(a) $HgNH_2Cl$ (b) $NH_2 - Hg - Hg - Cl$
(c) Hg_2O (d) HgO *(2005)*

18. The oxidation state of chromium in the final product formed by the reaction between KI and acidified potassium dichromate solution is
(a) +4 (b) +6
(c) +2 (d) +3 *(2005)*

19. Heating mixture of Cu_2O and Cu_2S will give
(a) $Cu + SO_2$ (b) $Cu + SO_3$
(c) $CuO + CuS$ (d) Cu_2SO_3 *(2005)*

20. The correct order of magnetic moments (spin only values in B.M.) among is
(a) $[MnCl_4]^{2-} > [CoCl_4]^{2-} > [Fe(CN)_6]^{4-}$
(b) $[MnCl_4]^{2-} > [Fe(CN)_6]^{4-} > [CoCl_4]^{2-}$
(c) $[Fe(CN)_6]^{4-} > [MnCl_4]^{2-} > [CoCl_4]^{2-}$
(d) $[Fe(CN)_6]^{4-} > [CoCl_4]^{2-} > [MnCl_4]^{2-}$.
(Atomic nos.: Mn = 25, Fe = 26, Co = 27) *(2004)*

21. Cerium (Z = 58) is an important member of the lanthanoids. Which of the following statements about cerium is incorrect?
(a) The common oxidation states of cerium are +3 and +4.
(b) The +3 oxidation state of cerium is more stable than +4 oxidation state.
(c) The +4 oxidation state of cerium is not known in solutions.
(d) Cerium (IV) acts as an oxidising agent. *(2004)*

22. Excess of KI reacts with $CuSO_4$ solution and then $Na_2S_2O_3$ solution is added to it. Which of the statements is incorrect for this reaction?
(a) Cu_2I_2 is formed. (b) CuI_2 is formed.
(c) $Na_2S_2O_3$ is oxidised. (d) Evolved I_2 is reduced. *(2004)*

23. Of the following outer electronic configurations of atoms, the highest oxidation state is achieved by which one of them?
(a) $(n-1)d^8ns^2$ (b) $(n-1)d^5ns^1$
(c) $(n-1)d^3ns^2$ (d) $(n-1)d^5ns^2$. *(2004)*

24. For making good quality mirrors, plates of float glass are used. These are obtained by floating molten glass over a liquid metal which does not solidify before glass. The metal used can be
(a) mercury (b) tin
(c) sodium (d) magnesium. *(2003)*

25. Which one of the following nitrates will leave behind a metal on strong heating?
(a) Ferric nitrate
(b) Copper nitrate
(c) Manganese nitrate
(d) Silver nitrate. *(2003)*

26. The radius of La^{3+} (Atomic number of La = 57) is 1.06 Å. Which one of the following given values will be closest to the radius of Lu^{3+} (Atomic number of Lu = 71)?
(a) 1.60 Å (b) 1.40 Å
(c) 1.06 Å (d) 0.85 Å *(2003)*

27. What would happen when a solution of potassium chromate is treated with an excess of dilute nitric acid?
(a) Cr^{3+} and $Cr_2O_7^{2-}$ are formed.
(b) $Cr_2O_7^{2-}$ and H_2O are formed.
(c) CrO_4^{2-} is reduced to +3 state of Cr.
(d) CrO_4^{2-} is oxidised to +7 state of Cr. *(2003)*

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28. The number of *d*-electrons retained in Fe^{2+} (At. no. Fe = 26) ions is
(a) 3 (b) 4
(c) 5 (d) 6 *(2003)*

29. The atomic numbers of vanadium (V), chromium (Cr), manganese (Mn) and iron (Fe) are respectively 23, 24, 25 and 26. Which one of these may be expected to have the highest second ionisation enthalpy?
(a) V (b) Cr
(c) Mn (d) Fe *(2003)*

30. A reduction in atomic size with increase in atomic number is a characteristic of elements of
(a) high atomic masses
(b) *d* -block
(c) *f* -block
(d) radioactive series. *(2003)*

31. A red solid is insoluble in water. However it becomes soluble if some KI is added to water. Heating the red solid in a test tube results in liberation of some violet coloured fumes and droplets of a metal appear on the cooler parts of the test tube. The red solid is
(a) $(NH_4)_2Cr_2O_7$ (b) HgI_2
(c) HgO (d) Pb_3O_4. *(2003)*

32. How do we differentiate between Fe^{3+} and Cr^{3+} in group III?
(a) By taking excess of NH_4OH solution.
(b) By increasing NH_4^+ ion concentration.
(c) By decreasing OH^- ion concentration.
(d) Both (b) and (c). *(2002)*

33. The most stable ion is
(a) $[Fe(OH)_3]^{3-}$ (b) $[Fe(Cl)_6]^{3-}$
(c) $[Fe(CN)_6]^{3-}$ (d) $[Fe(H_2O)_6]^{3+}$ *(2002)*

34. Arrange Ce^{3+}, La^{3+}, Pm^{3+} and Yb^{3+} in increasing order of their ionic radii.
(a) $Yb^{3+} < Pm^{3+} < Ce^{3+} < La^{3+}$
(b) $Ce^{3+} < Yb^{3+} < Pm^{3+} < La^{3+}$
(c) $Yb^{3+} < Pm^{3+} < La^{3+} < Ce^{3+}$
(d) $Pm^{3+} < La^{3+} < Ce^{3+} < Yb^{3+}$ *(2002)*

35. Most common oxidation states of Ce (cerium) are
(a) +2, +3 (b) +2, +4
(c) +3, +4 (d) +3, +5 *(2002)*

36. Which of the following ions has the maximum magnetic moment?
(a) Mn^{2+} (b) Fe^{2+}
(c) Ti^{2+} (d) Cr^{2+} *(2002)*

Answer Key

1.	(b)	**2.**	(a)	**3.**	(c)	**4.**	(d)	**5.**	(d)	**6.**	(b)
7.	(b)	**8.**	(c)	**9.**	(b)	**10.**	(c)	**11.**	(a)	**12.**	(a)
13.	(a)	**14.**	(b)	**15.**	(a)	**16.**	(c)	**17.**	(a)	**18.**	(d)
19.	(a)	**20.**	(a)	**21.**	(c)	**22.**	(b)	**23.**	(b)	**24.**	(a)
25.	(d)	**26.**	(d)	**27.**	(b)	**28.**	(d)	**29.**	(b)	**30.**	(c)
31.	(b)	**32.**	(d)	**33.**	(b)	**34.**	(a)	**35.**	(c)	**36.**	(a)

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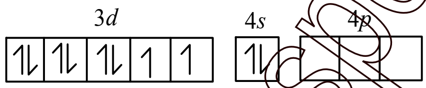
Explanations

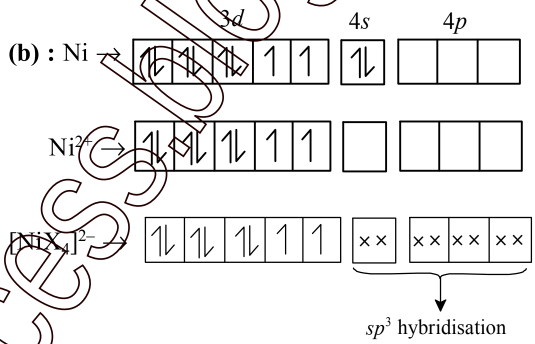
1. **(b) :** Number of unpaired electrons in Fe^{2+} is less than Mn^{2+}, so Fe^{2+} is less paramagnetic than Mn^{2+}.
2. **(a)**
3. **(c) :** Ferrous oxide is more basic, more ionic, less volatile and less easily hydrolysed than ferric oxide.
4. **(d) :** The electronic configuration of $_{64}Gd = [Xe]^{54}\ 4f^{7}\ 5d^{1}\ 6s^{2}$
5. **(d) :** Availability of $4f$ electrons does not result in the formation of compounds in +4 state for all the members of the series.
6. **(b) :** $E^\circ_{Mn^{2+}/Mn} = -1.18$ V
$E^\circ_{Cr^{2+}/Cr} = -0.91$ V
$E^\circ_{Fe^{2+}/Fe} = -0.44$ V
$E^\circ_{Co^{2+}/Co} = -0.28$ V
7. **(b) :** When the transition metals are in their highest oxidation state, they no longer have tendency to give away electrons, thus they are not basic but show acidic character and form anionic complexes.
8. **(c) :** Ln^{3+} compounds are generally coloured in the solid state as well as in aqueous solution. Colour appears due to presence of unpaired f-electrons which undergo f-f transition.
9. **(b) :** Strong field ligand such as CN^-, usually produce low spin complexes and large crystal field splittings. H_2O is a weaker field ligand than NH_3 and $C_2O_4^{2-}$ therefore
$\Delta_{oct}[Co(H_2O)_6]^{3+} < \Delta_{oct}[Co(C_2O_4)]^{3-} < [Co(NH_3)_6]^{3+}$
Common ligands in order of increasing crystal field strength are given below :
$I^- < Br^- < Cl^- < F^- < OH^- < C_2O_4^{2-} < H_2O < NH_3 < en < NO_2^- < CN^-$
10. **(c) :** Actinoids show different oxidation states such as +2, +3, +4, +5, +6 and +7. However +3 oxidation state is most common among all the actinoids.
The wide range of oxidation states of actinoids is attributed to the fact that the $5f$, $6d$ and $7s$ energy levels are of comparable energies. Therefore all these three subshells can participate.
11. **(a) :** As the distance between the nucleus and $5f$ orbitals (actinides) is more than the distance between the nucleus and $4f$ orbitals (lanthanides) hence the hold of nucleus on valence electron decreases in actinides. For this reason the actinoides exhibit more number of oxidation states in general.
12. **(a) :** The decrease in the force of attraction exerted by the nucleus on the valency electrons due to presence of electrons in the inner shells is called shielding effect. An $4f$ orbital is nearer to the nucleus than $5f$ orbitals. Hence shielding of $4f$ is more than $5f$.
13. **(a) :** $_{28}Ni \rightarrow [Ar]\ 3d^8\ 4s^2$



Number of unpaired electrons $(n) = 2$

$\mu = \sqrt{n(n+2)} = \sqrt{2(2+2)} = \sqrt{8} \approx 2.84$

14. **(b) :**



Number of unpaired electrons = 2
Geometry = tetrahedral.

15. **(a) :** As we proceed from one element to the next element in the lanthanide series, the nuclear charge, *i.e.* atomic number increases by one unit and the addition of one electron occurs at the same time in $4f$-energy shell. On account of the very diffused shapes of f-orbitals, the $4f$-electrons shield each other quite poorly from the nuclear charge. Thus, the effect of nuclear charge increase is somewhat more than the changed shielding effect. This brings the valence shell nearer to the nucleus and hence the size of atom or ion goes on decreasing as we move in the series. The sum of the successive reactions is equal to the total lanthanide contraction.
16. **(c) :** In each vertical column of transition elements, the elements of second and third transition series resemble each other more closely than the elements of first and second transition series on account of lanthanide contraction. The pairs of elements such as Zr-Hf, Mo-W, Nb-Ta, etc; possess almost the same properties.
17. **(a) :** Calomel on reaction with ammonium hydroxide turns black. The black substance is a mixture of mercury and mercuric amino chloride.

$$Hg_2Cl_2 + 2NH_4OH \rightarrow \underbrace{Hg\begin{matrix} NH_2 \\ Cl \end{matrix} + Hg}_{\text{Black}} + NH_4Cl + 2H_2O$$

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18. (d) :

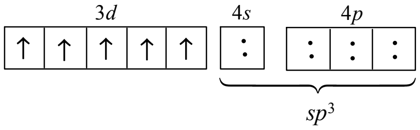
$$\overset{+6}{Cr_2}O_7^{2-} + 14H^+ + 6I^- \longrightarrow \overset{+3}{2Cr^{3+}} + 7H_2O + 3I_2$$

19. (a) : $Cu_2S + 2Cu_2O \rightarrow 6Cu + SO_2$

This is an example of auto-reduction.

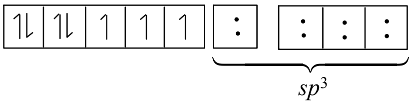
20. (a) :

$[MnCl_4]^{2-} \rightarrow$



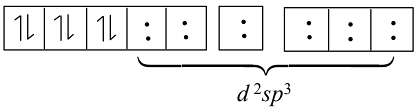
Number of unpaired electrons = 5

$[CoCl_4]^{2-} \rightarrow$



Number of unpaired electrons = 3

$[Fe(CN)_6]^{4-} \rightarrow$



Number of unpaired electrons = 0

Magnetic moment = $n\sqrt{n+2}$

where n = number of unpaired electrons.

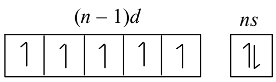
i.e. greater the number of unpaired electrons, greater will be the paramagnetic character.

21. (c) : +4 oxidation state of cerium is also known in solutions.

22. (b) : $4\overset{-1}{KI} + 2CuSO_4 \longrightarrow \overset{0}{I_2} + Cu_2I_2 + 2K_2SO_4$

$$\overset{0}{I_2} + 2Na_2\overset{+2}{S_2}O_3 \longrightarrow Na_2\overset{+2.5}{S_4}O_6 + 2Na\overset{-1}{I}$$

23. (b) :



$(n-1)d^5\ ns^2$ can achieve the maximum oxidation state of +7.

24. (a) : Mercury is such a metal which exists as liquid at room temperature.

25. (d) : When heated at red heat, $AgNO_3$ decomposes to metallic silver.

$$2AgNO_3 \rightarrow 2Ag + 2NO_2 + O_2$$

26. (d) : Due to lanthanide contraction, the ionic radii of Ln^{3+} (lanthanide ions) decreases from La^{3+} to Lu^{3+}. Thus the lowest value (here 0.85 Å) is the ionic radius of Lu^{3+}.

27. (b) : Dilute nitric acid converts chromate into dichromate and H_2O.

$$2K_2CrO_4 + 2HNO_3 \rightarrow K_2Cr_2O_7 + 2KNO_3 + H_2O$$

or, $$\underset{\text{yellow}}{2CrO_4^{2-}} \overset{H^+}{\rightleftharpoons} \underset{\text{orange}}{Cr_2O_7^{2-}} + H_2O$$

28. (d) : ${}_{26}Fe = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^6\ 4s^2$

$Fe^{2+} = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^6$

The number of d-electrons retained in Fe^{2+} = 6.

29. (b) : The second ionisation potential values of Cu and Cr are sufficiently higher than those of neighbouring elements. This is because of the electronic configuration of Cu^+ which is $3d^{10}$ (completely filled) and of Cr^+ which is $3d^5$ (half-filled), *i.e.*, for the second ionisation potentials, the electron is to be removed from very stable configurations.

30. (c) : With increase in atomic number *i.e.* in moving down a group, the number of the principal shell increases and therefore, the size of the atom increases. But in case of f-block elements there is a steady decrease in atomic size with increase in atomic number due to lanthanide contraction.

As we move through the lanthanide series, $4f$ electrons are being added one at each step. The mutual shielding effect of f electrons is very little. This is due to the shape of the f-orbitals. The nuclear charge, however increases by one at each step. Hence, the inward pull experienced by the $4f$ electrons increases. This causes a reduction in the size of the entire $4f^n$ shell.

31. (b) : The precipitate of mercuric iodide dissolves in excess of potassium iodide forming a complex, K_2HgI_4.

$$HgI_2 + 2KI \rightarrow K_2HgI_4$$

HgI_2 on heating liberates I_2 gas.

$$HgI_2 \rightleftharpoons Hg + \underset{\text{violet vapours}}{I_2}$$

32. (d) : NH_4^+ ions are increased to suppress release of OH^- ions, hence solubility product of $Fe(OH)_3$ is attained. Colour of precipitate is different.

33. (b) : A more basic ligand forms stable bond with metal ion, Cl^- is most basic amongst all.

34. (a) : According to their positions in the periods, these values are in the order:

	Yb^{3+} <	Pm^{3+} <	Ce^{3+} <	La^{3+}
At. Nos.	70	61	58	57
Ionic radii (pm)	86	98	103	106

Ionic size decreases from La^{3+} to Lu^{3+} due to lanthanide contraction.

35. (c) : The common stable oxidation state of all the lanthanides is +3. The oxidation states of +2 and +4 are also exhibited and these oxidation states are only stable in those cases where stable $4f^0$, $4f^7$ or $4f^{14}$ configurations are achieved. Ce^{4+} is stable due to $4f^0$ configuration.

36. (a) : Mn^{2+} $(3s^2 3p^6 3d^5)$ has the maximum number of unpaired electrons (5) and therefore has maximum moment.

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CHAPTER 18

COORDINATION COMPOUNDS

1. Which of the following complex species is not expected to exhibit optical isomerism?
(a) $[Co(en)(NH_3)_2Cl_2]^+$ (b) $[Co(en)_3]^{3+}$
(c) $[Co(en)_2Cl_2]^+$ (d) $[Co(NH_3)_3Cl_3]$ *(2013)*

2. Which among the following will be named as dibromidobis (ethylene diamine) chromium(III bromide?
(a) $[Cr(en)_2Br_2]Br$ (b) $[Cr(en)Br_4]^-$
(c) $[Cr(en)Br_2]Br$ (d) $[Cr(en)_3]Br_3$ *(2012)*

3. The magnetic moment (spin only) of $[NiCl_4]^{2-}$ is
(a) 1.82 BM (b) 5.46 BM
(c) 2.82 BM (d) 1.41 BM *(2011)*

4. Which of the following facts about the complex $[Cr(NH_3)_6]Cl_3$ is wrong?
(a) The complex involves d^2sp^3 hybridisation and is octahedral in shape.
(b) The complex is paramagnetic.
(c) The complex is an outer orbital complex.
(d) The complex gives white precipitate with silver nitrate solution. *(2011)*

5. Which one of the following has an optical isomer?
(a) $[Zn(en)_2]^{2+}$ (b) $[Zn(en)(NH_3)_2]^{2+}$
(c) $[Co(en)_3]^{3+}$ (d) $[Co(H_2O)_4(en)]^{3+}$ *(2010)*

6. A solution contains 2.675 g of $CoCl_3 \cdot 6NH_3$ (molar mass = 267.5 g mol^{-1}) is passed through a cation exchanger. The chloride ions obtained in solution were treated with excess of $AgNO_3$ to give 4.78 g of AgCl (molar mass = 143.5 g mol^{-1}). The formula of the complex is (At. mass of Ag = 108 u)
(a) $[CoCl(NH_3)_5]Cl_2$ (b) $[Co(NH_3)_6]Cl_3$
(c) $[CoCl_2(NH_3)_4]Cl$ (d) $[CoCl_3(NH_3)_3]$ *(2010)*

7. Which of the following pairs represents linkage isomers?
(a) $[Cu(NH_3)_4][PtCl_4]$ and $[Pt(NH_3)_4][CuCl_4]$
(b) $[Pd(PPh_3)_2(NCS)_2]$ and $[Pd(PPh_3)_2(SCN)_2]$
(c) $[Co(NH_3)_5(NO_3)]SO_4$ and $[Co(NH_3)_5(SO_4)]NO_3$
(d) $[PtCl_2(NH_3)_4]Br_2$ and $[PtBr_2(NH_3)_4]Cl_2$ *(2009)*

8. Which of the following has an optical isomer?
(a) $[Co(NH_3)_3Cl]^+$ (b) $[Co(en)(NH_3)_2]^{2+}$
(c) $[Co(H_2O)_4(en)]^{3+}$ (d) $[Co(en)_2(NH_3)_2]^{3+}$ *(2009)*

9. The coordination number and the oxidation state of the element E in the complex $[E(en)_2(C_2O_4)]NO_2$ (where (en) is ethylene diamine) are, respectively
(a) 6 and 3 (b) 6 and 2
(c) 4 and 2 (d) 4 and 3 *(2008)*

10. Which of the following has a square planar geometry?
(a) $[PtCl_4]^{2-}$ (b) $[CoCl_4]^{2-}$
(c) $[FeCl_4]^{2-}$ (d) $[NiCl_4]^{2-}$.
(At. nos.: Fe = 26, Co = 27, Ni = 28, Pt = 78) *(2007)*

11. How many EDTA (ethylenediaminetetraacetic acid) molecules are required to make an octahedral complex with a Ca^{2+} ion?
(a) Six (b) Three
(c) One (d) Two *(2006)*

12. In $Fe(CO)_5$, the Fe – C bond possesses
(a) π-character only (b) both σ and π characters
(c) ionic character (d) σ-character only. *(2006)*

13. The IUPAC name for the complex $[Co(NO_2)(NH_3)_5]Cl_2$ is
(a) nitrito-N-pentaamminecobalt(III) chloride
(b) nitrito-N-pentaamminecobalt(II) chloride
(c) pentaammine nitrito-N-cobalt(II) chloride
(d) pentaammine nitrito-N-cobalt(III) chloride. *(2006)*

14. The value of the 'spin only' magnetic moment for one of the following configurations is 2.84 BM. The correct one is
(a) d^4 (in strong ligand field)
(b) d^4 (in weak ligand field)
(c) d^3 (in weak as well as in strong fields)
(d) d^5 (in strong ligand field) *(2005)*

15. Which one of the following cyano complexes would exhibit the lowest value of paramagnetic behaviour?
(a) $[Cr(CN)_6]^{3-}$ (b) $[Mn(CN)_6]^{3-}$
(c) $[Fe(CN)_6]^{3-}$ (d) $[Co(CN)_6]^{3-}$ *(2005)*

16. Which of the following compounds shows optical isomerism?
(a) $[Cu(NH_3)_4]^{2+}$ (b) $[ZnCl_4]^{2-}$
(c) $[Cr(C_2O_4)_3]^{3-}$ (d) $[Co(CN)_6]^{3-}$ *(2005)*

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17. The IUPAC name of the coordination compound $K_3[Fe(CN)_6]$ is
(a) potassium hexacyanoferrate (II)
(b) potassium hexacyanoferrate (III)
(c) potassium hexacyanoiron (II)
(d) tripotassium hexacyanoiron (II) *(2005)*

18. The oxidation state of Cr in $[Cr(NH_3)_4Cl_2]^+$ is
(a) +3 (b) +2
(c) +1 (d) 0 *(2005)*

19. Which one of the following has largest number of isomers?
(a) $[Ru(NH_3)_4Cl_2]^+$ (b) $[Co(NH_3)_5Cl]^{2+}$
(c) $[Ir(PR_3)_2H(CO)]^{2+}$ (d) $[Co(en)_2Cl_2]^+$
(R = alkyl group, en = ethylenediamine)
(2004)

20. Coordination compounds have great importance in biological systems. In this context which of the following statements is incorrect?
(a) Chlorophylls are green pigments in plants and contain calcium.
(b) Haemoglobin is the red pigment of blood and contains iron.
(c) Cyanocobalamine is B_{12} and contains cobalt.
(d) Carboxypeptidase-A is an enzyme and contains zinc.
(2004)

21. Which one of the following complexes is an outer orbital complex?
(a) $[Fe(CN)_6]^{4-}$ (b) $[Mn(CN)_6]^{4-}$
(c) $[Co(NH_3)_6]^{3+}$ (d) $[Ni(NH_3)_6]^{2+}$
[Atomic nos.: Mn = 25, Fe = 26, Co = 27, Ni = 28]
(2004)

22. The coordination number of a central metal atom in a complex is determined by
(a) the number of ligands around a metal ion bonded by sigma bonds
(b) the number of ligands around a metal ion bonded by pi-bonds
(c) the number of ligands around a metal ion bonded by sigma and pi-bonds both
(d) the number of only anionic ligands bonded to the metal ion.
(2004)

23. One mole of the complex compound $Co(NH_3)_5Cl_3$, gives 3 moles of ions on dissolution in water. One mole of the same complex reacts with two moles of $AgNO_3$ solution to yield two moles of AgCl (s). The structure of the complex is
(a) $[Co(NH_3)_5Cl]Cl_2$
(b) $[Co(NH_3)_3Cl_2]\cdot 2NH_3$
(c) $[Co(NH_3)_4Cl_2]Cl\cdot NH_3$
(d) $[Co(NH_3)_4Cl]Cl_2\cdot NH_3$ *(2003)*

24. Ammonia forms the complex ion $[Cu(NH_3)_4]^{2+}$ with copper ions in alkaline solutions but not in acidic solutions. What is the reason for it?
(a) In acidic solutions hydration protects copper ions.
(b) In acidic solutions protons coordinate with ammonia molecules forming NH_4^+ ions and NH_3 molecules are not available.
(c) In alkaline solutions insoluble $Cu(OH)_2$ is precipitated which is soluble in excess of any alkali.
(d) Copper hydroxide is an amphoteric substance.
(2003)

25. In the coordination compound, $K_4[Ni(CN)_4]$, the oxidation state of nickel is
(a) –1 (b) 0
(c) +1 (d) +2 *(2003)*

26. The type of isomerism present in nitropentamine chromium (III) chloride is
(a) optical (b) linkage
(c) ionization (d) polymerisation. *(2002)*

27. $CH_3 - Mg - Br$ is an organometallic compound due to
(a) Mg – Br bond (b) C – Mg bond
(c) C – Br bond (d) C – H bond. *(2002)*

Answer Key

1.	(d)	**2.**	(a)	**3.**	(c)	**4.**	(c)	**5.**	(c)	**6.**	(b)
7.	(b)	**8.**	(d)	**9.**	(a)	**10.**	(a)	**11.**	(c)	**12.**	(b)
13.	(d)	**14.**	(a)	**15.**	(d)	**16.**	(c)	**17.**	(b)	**18.**	(a)
19.	(d)	**20.**	(a)	**21.**	(d)	**22.**	(a)	**23.**	(a)	**24.**	(b)
25.	(b)	**26.**	(b)	**27.**	(b)						

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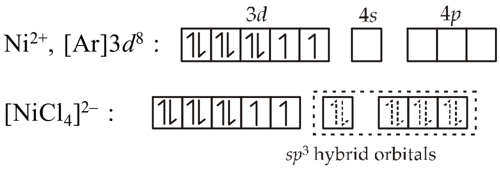
Explanations

1. **(d) :** $[Co(NH_3)_3Cl_3]$ will not exhibit optical isomerism due to presence of plane of symmetry.

2. **(a)**

3. **(c) :** In the paramagnetic and tetrahedral complex $[NiCl_4]^{2-}$, the nickel is in +2 oxidation state and the ion has the electronic configuration $3d^8$. The hybridisation scheme is as shown in figure.



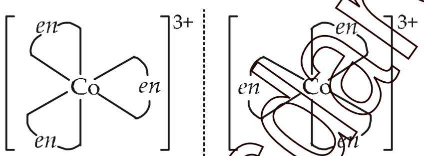
$\mu = \sqrt{n(n+2)}$ BM

$= \sqrt{(2(2+2)} = \sqrt{8}$

$= 2.82$ BM

4. **(c) :** The complex $[Cr(NH_3)_6]Cl_3$ involves d^2sp^3 hybridization as it involves $(n-1)d$ orbitals for hybridization. It is an inner orbital complex.

5. **(c) :** Optical isomers rarely occur in square planar complexes due to the presence of axis of symmetry.
Optical isomerism is common in octahedral complexes of the general formula, $[Ma_2b_2c_2]^{n\pm}$, $[Mabcdef]^{n\pm}$, $[M(AA)_3]^{n\pm}$, $[M(AA)_2a_2]^{n\pm}$, $[M(AA)_2ab]^{n\pm}$ and $[M(AB)_3]^{n\pm}$. Thus, among the given options, only $[Co(en)_3]^{3+}$ shows optical isomerism.



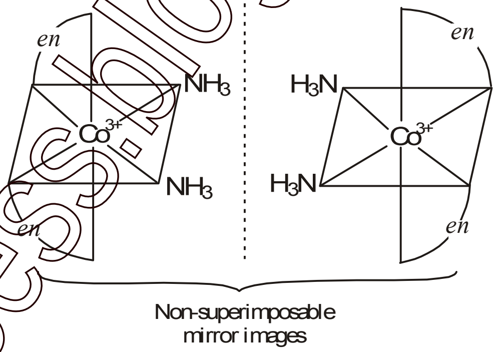
6. **(b) :** No. of moles of $CoCl_3 \cdot 6NH_3 = \frac{2.675}{267.5} = 0.01$

No. of moles of $AgCl = \frac{4.78}{143.5} = 0.03$

Since 0.01 moles of the complex $CoCl_3 \cdot 6NH_3$ gives 0.03 moles of AgCl on treatment with $AgNO_3$, it implies that 3 chloride ions are ionisable, in the complex. Thus, the formula of the complex is $[Co(NH_3)_6]Cl_3$.

7. **(b) :** Linkage isomerism is exhibited by compounds containing ambidentate ligand.
In$[Pd(PPh_3)_2(NCS)_2]$, the linkage of NCS and Pd is through N.
In $[Pd(PPh_3)_2(SCN)_2]$, the linkage of SCN and Pd is through S.

8. **(d) :** Optical isomerism is usually exhibited by octahedral compounds of the type $[M(AA)_2B_2]$, where (AA) is a symmetrical bidentate ligand. Square planar complexes rarely show optical isomerism on accout of presence of axis of symmetry.
Thus among the given options, $[Co(en)_2(NH_3)_2]^{3+}$ exhibits optical isomerism.



9. **(a) :** In the given complex $[E(en)_2(C_2O_4)]^+NO_2^-$ ethylene diamine is a bidentate ligand and $(C_2O_4^{2-})$ oxalate ion is also bidentate ligand. Therefore co-ordination number of the complex is 6 *i.e.*, it is an octahedral complex.
Oxidation number of E in the given complex is

$x + 2 \times 0 + 1 \times (-2) = +1$

$\therefore\ x = 3$

10. **(a) :** In 4-coordinate complexes Pt, the four ligands are arranged about the central 2-valent platinum ion in a square planar configuration.

11. **(c) :** EDTA, which has four donor oxygen atoms and two donor nitrogen atoms in each molecule forms complex with Ca^{2+} ion. The free acid H_4EDTA is insoluble and the disodium salt Na_2H_2EDTA is the most used reagent.
$Ca^{2+} + [H_2EDTA]^{2-} \rightarrow [Ca(EDTA)]^{2-} + 2H^+$

12. **(b) :** In a metal carbonyl, the metal carbon bond possesses both the σ- and π-character. A σ-bond between metal and carbon atom is formed when a vacant hybrid bond of the metal atom overlaps with an orbital of C atom of carbon monoxide containing a lone pair of electrons.
Formation of π-bond is caused when a filled orbital of the metal atom overlaps with a vacant antibonding π* orbital of C atom of CO. This overlap is also called back donation of electrons by metal atom to carbon.

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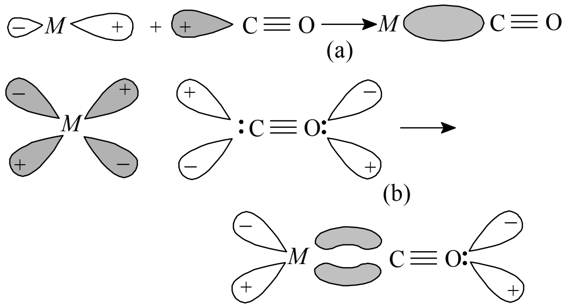


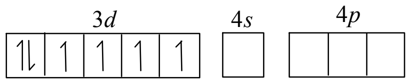
(a) The formation of the metal ← carbon σ-bond using an unshared pair of the C atom. (b) The formation of the metal → carbon π-bond.

The π-overlap is perpendicular to the nodal plane of σ-bond.

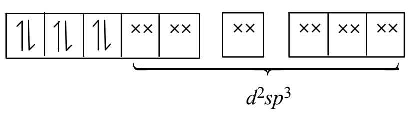
13. **(d) :** $[Co(NO_2)(NH_3)_5]Cl_2$
pentaaminenitrito-N-cobalt(III) chloride

14. **(a) :** Spin only magnetic moment $= \sqrt{n(n+2)}$ B.M.
Where n = no. of unpaired electron.
Given, $\sqrt{n(n+2)} = 2.84$
or, $n(n+2) = 8.0656$
or, $n = 2$
In an octahedral complex, for a d^4 configuration in a strong field ligand, number of unpaired electrons = 2

15. **(d) :** $[Co(CN)_6]^{3-}$
Co → [Ar] $3d^7\ 4s^2$
Co^{3+} → [Ar] $3d^6\ 4s^0$

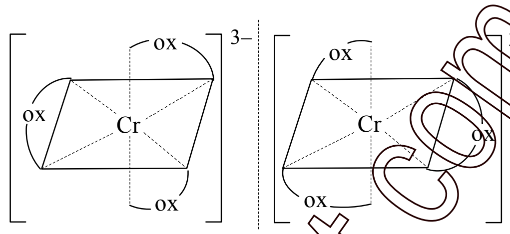


In presence of strong field ligand CN^- pairing of electrons takes place.



There is no unpaired electron, so the lowest value of paramagnetic behaviour is observed.

16. **(c) :** Optical isomers rarely occur in square planar complexes on account of the presence of axis of symmetry. Optical isomerism is very common in octahedral complexes having general formulae:
$[Ma_2b_2c_2]^{n\pm}$, $[Mabcdef]^{n\pm}$, $[M(AA)_3]^{n\pm}$, $[M(AA)_2a_2]^{n\pm}$, $[M(AA)_2ab]^{n\pm}$ and $[M(AB)_3]^{n\pm}$
(where AA = symmetrical bidentate ligand and AB = unsymmetrical bidentate ligand).



17. **(b) :** $K_3[Fe(CN)_6]$
Potassium hexacyanoferrate(III)

18. **(a) :** Let the oxidation state of Cr in $[Cr(NH_3)_4Cl_2]^+ = x$
$x + 4(0) + 2(-1) = +1$
$x - 2 = +1$ or, $x = +1 + 2 = +3$

19. **(d) :** $[Co(en)_2Cl_2]^+$ shows geometrical as well as optical isomerism.

20. **(a) :** Chlorophyll are green pigments in plants and contains magnesium instead of calcium.

21. **(d) :**

Complex ion	Hybridization of central ion
$[Fe(CN)_6]^{4-}$	d^2sp^3 (inner)
$[Mn(CN)_6]^{4-}$	d^2sp^3 (inner)
$[Co(NH_3)_6]^{3+}$	d^2sp^3 (inner)
$[Ni(NH_3)_6]^{2+}$	sp^3d^2 (outer)

22. **(a) :** The number of atoms of the ligands that are directly bound to the central metal atom or ion by coordinate bonds is known as the coordination number of the metal atom or ion.
Coordination number of metal = number of σ bonds formed by metal with ligands.

23. **(a) :** Given reactions can be explained as follows:
$[Co(NH_3)_5Cl]Cl_2 \rightleftharpoons [Co(NH_3)_5Cl]^{2+} + 2Cl^- \Rightarrow 3$ ions.
$[Co(NH_3)_5Cl]Cl_2 + 2AgNO_3 \rightarrow [Co(NH_3)_5Cl](NO_3)_2 + 2AgCl$

24. **(b) :** In acidic solution, NH_3 forms a bond with H^+ to give NH_4^+ ion which does not have a lone pair on N atom. Hence it cannot act as a ligand.

25. **(b) :** Let the oxidation number of Ni in $K_4[Ni(CN)_4] = x$
$1 \times 4 + x \times (-1) \times 4 = 0 \Rightarrow 4 + x - 4 = 0 \Rightarrow x = 0$

26. **(b) :** The nitro group can attach to metal through nitrogen as ($-NO_2$) or through oxygen as nitrito (–ONO).

27. **(b) :** Compounds that contain at least one carbon-metal bond are called organometallic compounds.

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CHAPTER 19

ENVIRONMENTAL CHEMISTRY

1. The gas leaked from a storage tank of the Union Carbide plant in Bhopal gas tragedy was
   (a) phosgene (b) methylisocyanate
   (c) methylamine (d) ammonia *(2013)*

2. Identify the wrong statement in the following.
   (a) Acid rain is mostly because of oxides of nitrogen and sulphur.
   (b) Chlorofluorocarbons are responsible for ozone layer depletion.
   (c) Greenhouse effect is responsible for global warming.
   (d) Ozone layer does not permit infrared radiation from the sun to reach the earth.
   (2008)

3. The smog is essentially caused by the presence of
   (a) O_2 and O_3
   (b) O_2 and N_2
   (c) oxides of sulphur and nitrogen
   (d) O_3 and N_2. *(2004)*

Answer Key

1.	(b)	**2.**	(d)	**3.**	(c)

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Explanations

1. **(b)**

2. **(d) :** The thick layer of ozone called ozoneplanket which is effective in absorbing harmful ultraviolet rays given out by the sun acts as a protective shield. It does not permit the ultra violet rays from sun to reach the earth.

3. **(c) :** Photochemical smog is caused by oxides of sulphur and nitrogen.

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CHAPTER 20

PURIFICATION AND CHARACTERISATION OF ORGANIC COMPOUNDS

1. 29.5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method and the evolved ammonia was absorbed in 20 mL of 0.1 M HCl solution. The excess of the acid required 15 mL of 0.1 M NaOH solution for complete neutralization. The percentage of nitrogen in the compound is

(a) 29.5 (b) 59.0
(c) 47.4 (d) 23.7 *(2010)*

2. The ammonia evolved from the treatment of 0.30 g of an organic compound for the estimation of nitrogen was passed in 100 mL of 0.1 M sulphuric acid. The excess of acid required 20 mL of 0.5 M sodium hydroxide solution for complete neutralization. The organic compound is

(a) acetamide (b) benzamide
(c) urea (d) thiourea. *(2004)*

3. In a compound C, H and N atoms are present in 9 : 1 : 3.5 by weight. Molecular weight of compound is 108. Molecular formula of compound is

(a) $C_2H_6N_2$ (b) C_3H_4N
(c) $C_6H_8N_2$ (d) $C_9H_{12}N_3$. *(2002)*

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Answer Key

1.	(d)	**2.**	(c)	**3.**	(c)

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Explanations

1. **(d) :** The % of N according to Kjeldahl's method $= \dfrac{1.4 \times N_1 \times V}{w}$

N_1 = Normality of the standard acid = 0.1 N
w = Mass of the organic compound taken
= 29.5 mg = 29.5 × 10^{-3} g
V = Volume of N_1 acid neutralised by ammonia
= (20 – 15) = 5 mL.

$$\Rightarrow \quad \%N = \frac{1.4 \times 0.1 \times 5}{29.5 \times 10^{-3}} = 23.7$$

2. **(c) :** Equivalents of NH_3 evolved

$$= \frac{100 \times 0.1 \times 2}{1000} - \frac{20 \times 0.5}{1000} = \frac{1}{100}$$

Percent of nitrogen in the unknown organic compound = $\dfrac{1}{100} \times \dfrac{14}{0.3} \times 100 = 46.6$

Percentage of nitrogen in urea $(NH_2)_2CO$

$$= \frac{14 \times 2}{60} \times 100 = 46.6$$

∴ The compound must be urea.

3. **(c) :**

C		H		N
9	:	1	:	3.5
$\frac{9}{12}$	:	$\frac{1}{1}$	:	$\frac{3.5}{14}$
$\frac{3}{4}$	:	$\frac{1}{1}$	:	$\frac{1}{4}$
3	:	4	:	1

Empirical formula = C_3H_4N
$(C_3H_4N)_n = 108$
$(12 \times 3 + 1 \times 4 + 14)_n = 108$
$54n = 108 \Rightarrow n = 108/54 = 2$
Molecular formula = $C_6H_8N_2$

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CHAPTER 21

SOME BASIC PRINCIPLES OF ORGANIC CHEMISTRY

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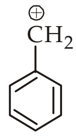
1. The order of stability of the following carbocations is

$CH_2{=}CH-\overset{\oplus}{C}H_2$ (I) ; $CH_3-CH_2-\overset{\oplus}{C}H_2$ (II) ;



III

(a) III > I > II (b) III > II > I
(c) II > III > I (d) I > II > III *(2013)*

2. How many chiral compounds are possible on monochlorination of 2-methyl butane?
(a) 2 (b) 4
(c) 6 (d) 8 *(2012)*

3. Identify the compound that exhibits tautomerism.
(a) 2-Butene (b) Lactic acid
(c) 2-Pentanone (d) Phenol *(2011)*

4. Out of the following, the alkene that exhibits optical isomerism is
(a) 2-methyl-2-pentene (b) 3-methyl-2-pentene
(c) 4-methyl-1-pentene (d) 3-methyl-1-pentene *(2010)*

5. The IUPAC name of *neo*-pentane is
(a) 2-methylbutane (b) 2,2-dimethylpropane
(c) 2-methylpropane (d) 2,2-dimethylbutane *(2009)*

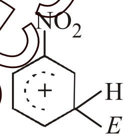
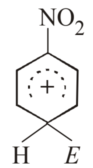
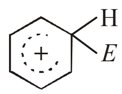
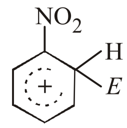
6. The number of stereoisomers possible for a compound of the molecular formula $CH_3 - CH = CH - CH(OH) - Me$ is
(a) 3 (b) 2
(c) 4 (d) 6 *(2009)*

7. The alkene that exhibits geometrical isomerism is
(a) propene (b) 2-methylpropene
(c) 2-butene (d) 2-methyl-2-butene *(2009)*

8. Arrange the carbanions,
$(CH_3)_3\bar{C}$, $\bar{C}Cl_3$, $(CH_3)_2\bar{C}H$, $C_6H_5\bar{C}H_2$
in order of their decreasing stability
(a) $C_6H_5\bar{C}H_2 > \bar{C}Cl_3 > (CH_3)_3\bar{C} > (CH_3)_2\bar{C}H$
(b) $(CH_3)_2\bar{C}H > \bar{C}Cl_3 > C_6H_5\bar{C}H_2 > (CH_3)_3\bar{C}$
(c) $\bar{C}Cl_3 > C_6H_5\bar{C}H_2 > (CH_3)_2\bar{C}H > (CH_3)_3\bar{C}$
(d) $(CH_3)_3\bar{C} > (CH_3)_2\bar{C}H > C_6H_5\bar{C}H_2 > \bar{C}Cl_3$ *(2009)*

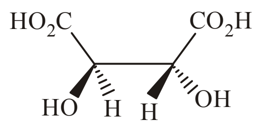
9. The correct decreasing order of priority for the functional groups of organic compounds in the IUPAC system of nomenclature is
(a) $-CONH_2$, $-CHO$, $-SO_3H$, $-COOH$
(b) $-COOH$, $-SO_3H$, $-CONH_2$, $-CHO$
(c) $-SO_3H$, $-COOH$, $-CONH_2$, $-CHO$
(d) $-CHO$, $-COOH$, $-SO_3H$, $-CONH_2$ *(2008)*

10. The electrophile, $E^{\oplus}$ attacks the benzene ring to generate the intermediate σ-complex. Of the following, which σ-complex is of lowest energy?

(a) 
(b) 
(c) 
(d) 

(2008)

11. The absolute configuration of



is

(a) *S*, *R* (b) *S*, *S*
(c) *R*, *R* (d) *R*, *S* *(2008)*

12. Which one of the following conformations of cyclohexane is chiral?
(a) Boat (b) Twist boat
(c) Rigid (d) Chair *(2007)*

13. Increasing order of stability among the three main conformations (*i.e.* eclipse, anti, gauche) of 2-fluoroethanol is
(a) eclipse, gauche, anti
(b) gauche, eclipse, anti
(c) eclipse, anti, gauche
(d) anti, gauche, eclipse. *(2006)*

14. The increasing order of stability of the following free radicals is

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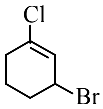
(a) $(CH_3)_2\dot{C}H < (CH_3)_3\dot{C} < (C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C}$
(b) $(C_6H_5)_3\dot{C} < (C_6H_5)_2\dot{C}H < (CH_3)_3\dot{C} < (CH_3)_2\dot{C}H$
(c) $(C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C} < (CH_3)_3\dot{C} < (CH_3)_2\dot{C}H$
(d) $(CH_3)_2\dot{C}H < (CH_3)_3\dot{C} < (C_6H_5)_3\dot{C} < (C_6H_5)_2\dot{C}H$ *(2006)*

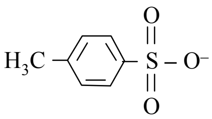
15. $CH_3Br + Nu^- \rightarrow CH_3 - Nu + Br^-$
The decreasing order of the rate of the above reaction with nucleophiles (Nu^-) *A* to *D* is
[Nu^- = (*A*) PhO^-, (*B*) AcO^-, (*C*) HO^-, (*D*) CH_3O^-]
(a) $D > C > A > B$ (b) $D > C > B > A$
(c) $A > B > C > D$ (d) $B > D > C > A$. *(2006)*

16. The IUPAC name of the compound shown below is



(a) 2-bromo-6-chlorocyclohex-1-ene
(b) 6-bromo-2-chlorocyclohexene
(c) 3-bromo-1-chlorocyclohexene
(d) 1-bromo-3-chlorocyclohexene. *(2006)*

17. The decreasing order of nucleophilicity among the nucleophiles is
(1) $CH_3C(=O)-O^-$ (2) CH_3O^-
(3) CN^- (4)



(a) 1, 2, 3, 4 (b) 4, 3, 2, 1
(c) 2, 3, 1, 4 (d) 3, 2, 1, 4 *(2005)*

18. Due to the presence of an unpaired electron, free radicals are
(a) chemically reactive
(b) chemically inactive
(c) anions
(d) cations *(2005)*

19. Among the following four structures I to IV,

$C_2H_5-CH(CH_3)-C_3H_7$ (I) $CH_3-C(=O)-CH(CH_3)-C_2H_5$ (II) $H-C^{\oplus}H_2$ (with H above and below: $CH_3^{\oplus}$) (III) $C_2H_5-CH(CH_3)-C_2H_5$ (IV)

it is true that
(a) all four are chiral compounds
(b) only I and II are chiral compounds
(c) only III is a chiral compound
(d) only II and IV are chiral compounds. *(2003)*

20. The reaction :
$$(CH_3)_3C-Br \xrightarrow{H_2O} (CH_3)_3C-OH$$
(a) elimination reaction
(b) substitution reaction
(c) free radical reaction
(d) displacement reaction. *(2002)*

21. Which of the following does not show geometrical isomerism?
(a) 1,2-dichloro-1-pentene
(b) 1,3-dichloro-2-pentene
(c) 1,1-dichloro-1-pentene
(d) 1,4-dichloro-2-pentene. *(2002)*

22. A similarity between optical and geometrical isomerism is that
(a) each forms equal number of isomers for a given compound
(b) if in a compound one is present then so is the other
(c) both are included in stereoisomerism
(d) they have no similarity. *(2002)*

23. Racemic mixture is formed by mixing two
(a) isomeric compounds
(b) chiral compounds
(c) meso compounds
(d) optical isomers. *(2002)*

24. Arrangement of $(CH_3)_3C-$, $(CH_3)_2CH-$, CH_3CH_2- when attached to benzyl or an unsaturated group in increasing order of inductive effect is
(a) $(CH_3)_3C- < (CH_3)_2CH- < CH_3CH_2-$
(b) $CH_3CH_2- < (CH_3)_2CH- < (CH_3)_3C-$
(c) $(CH_3)_2CH- < (CH_3)_3C- < CH_3CH_2-$
(d) $(CH_3)_3C- < CH_3CH_2- < (CH_3)_2CH-$. *(2002)*

Answer Key

1.	(a)	2.	(a)	3.	(c)	4.	(d)	5.	(b)	6.	(c)
7.	(c)	8.	(c)	9.	(c)	10.	(c)	11.	(c)	12.	(b)
13.	(a)	14.	(a)	15.	(a)	16.	(c)	17.	(d)	18.	(a)
19.	(b)	20.	(b)	21.	(c)	22.	(c)	23.	(d)	24.	(b)

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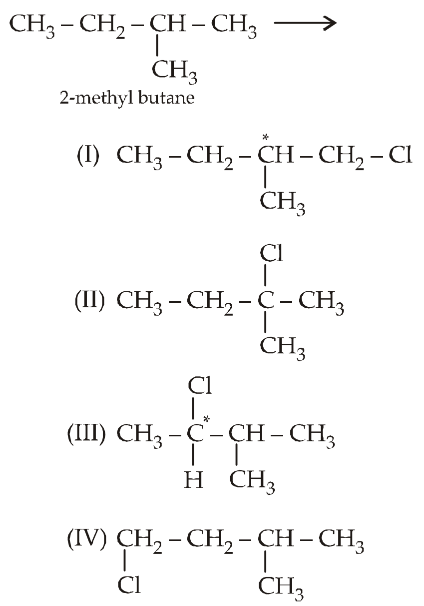
Explanations

1. **(a) :** Greater the number of resonating structures a carbocation possess, greater is its stability.

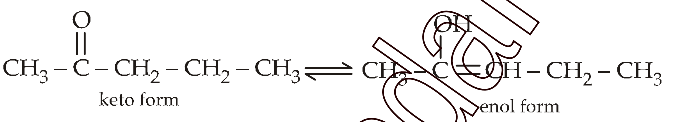
2. **(a) :**



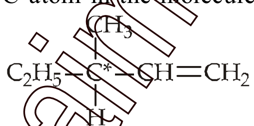
Possible products on chlorination

Out of four possible isomers only I and III are chiral.

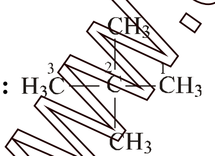
3. **(c) :** The type of isomerism in which a substance exists in two readily interconvertible different structures leading to a dynamic equilibrium is known as tautomerism.
2-pentanone exhibits tautomerism.



4. **(d) :** 3-Methyl-1-pentene exhibits optical isomerism as it has an asymmetric C-atom in the molecule.



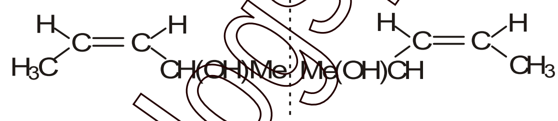
3-methyl-1-pentene

5. **(b) :** 

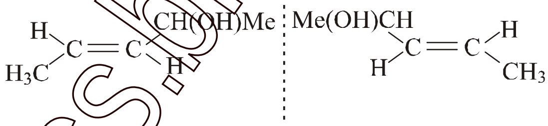
neo-pentane or 2,2-dimethylpropane

6. **(c) :** The given compound has a C = C group and one chiral (*) carbon,

$H_3C-HC=CH-\overset{*}{C}H(OH)-Me$



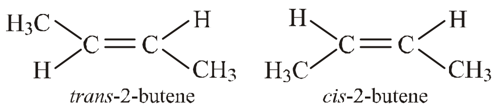
d, *l* isomers of *cis*-form



d, *l* isomers of *trans*-form

∴ Total stereoisomers = 4.

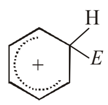
7. **(c) :** When two groups attached to a double bonded carbon atom are same, the compound does not exhibit geometrical isomerism.
Compounds in which the two groups attached to a double bonded carbon are different, exhibit geometrical isomerism, thus, only 2-butene exhibits *cis-trans* isomerism.



8. **(c) :** The groups having +*I* effect decrease the stability while groups having –*I* effect increase the stability of carbanions. Benzyl carbanion is stabilized due to resonance. Also, out of 2° and 3° carbanions, 2° carbanions are more stable, thus the decreasing order of stability is :

$$\bar{C}Cl_3 > C_6H_5\bar{C}H_2 > (CH_3)_2\bar{C}H > (CH_3)_3\bar{C}.$$

9. **(c) :** The order of preference of functional groups is as follows:
$-SO_3H > -COOH > -COOR > -COX > -COCl > -CONH_2$
$> -CHO > -CN > \ \rangle C{=}O > -OH > -SH > -NH_2$
$> \ \rangle C{=}C\langle \ > -C\equiv C- > -NO_2 > -NO > -X$

10. **(c) :** 

This structure will be of lowest energy due to resonance stabilisation of +ve charge. In all other three structures, the presence of electron-withdrawing NO_2 group will destabilize the +ve charge and hence they will have greater energy.

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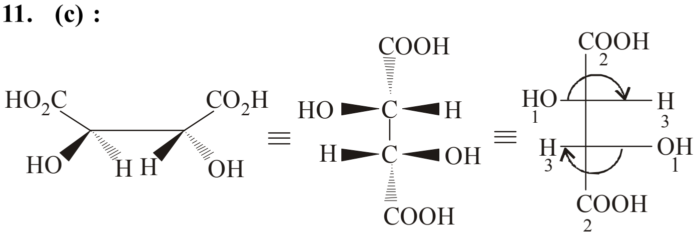
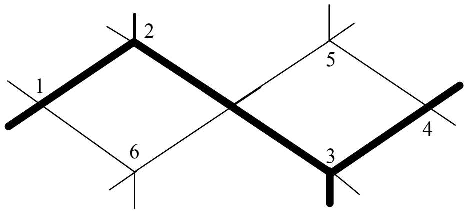
11. (c) :



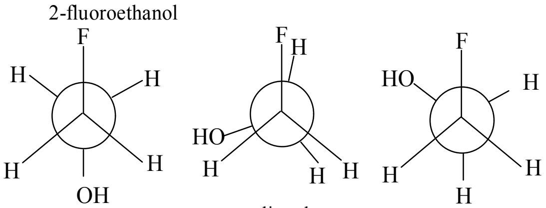
(*R*, *R*)

12. (b) : The twist boat conformation of cyclohexane is optically active as it does not have any plane of symmetry.

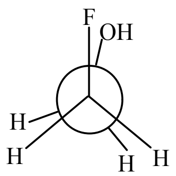


13. (a) : $CH_2(F)-CH_2(OH)$

2-fluoroethanol



anti or staggered   eclipsed   skew or gauche



fully eclipsed

The anti conformation is most stable in which F and OH groups are far apart as possible and minimum repulsion between two groups occurs.

In fully eclipsed conformation F and OH groups are so close that the steric strain is maximum, hence this conformation is most unstable. The order of stability of these conformations is anti > gauche > partially eclipsed > fully eclipsed

14. (a) : On the basis of hyperconjugation effect of the alkyl groups, the order of stability of free radical is as follows: tertiary > secondary > primary.

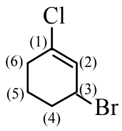
Benzyl free radicals are stabilised by resonance and hence are more stable than alkyl free radicals. Further as the number of phenyl group attached to the carbon atom holding the odd electron increases, the stability of a free radical increases accordingly.

i.e. $(CH_3)_2\dot{C}H < (CH_3)_3\dot{C} < (C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C}$

15. (a) : If the nucleophilic atom or the centre is same, nucleophilicity parallels basicity, *i.e.* more basic the species stronger is the nucleophile.

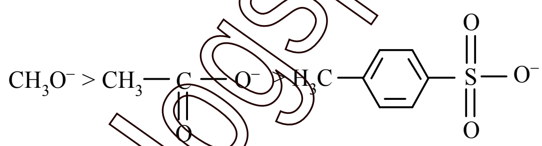
$$CH_3O^- > HO^- > PhO^- > AcO^-$$

Here, the nucleophilic atom *i.e.* O is the same in all these species. This order can be easily explained on the general concept that a weaker acid has a stronger conjugate base.

16. (c) :  3-bromo-1-chlorocyclohexene

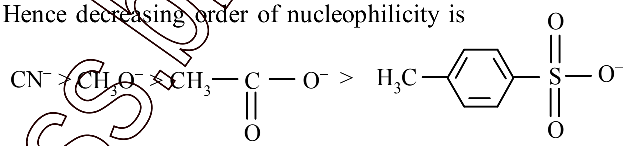
17. (d) : Strong bases are generally good nucleophile.

If the nucleophilic atom or the centre is the same, nucleophilicity parallels basicity, *i.e.*, more basic the species, stronger is the nucleophile. Hence basicity as well as nucleophilicity order is



Now CN^- is a better nucleophile than CH_3O^-.

Hence decreasing order of nucleophilicity is



18. (a) : Free radicals are highly reactive due to presence of an unpaired electron. They readily try to pair-up the odd electrons.

19. (b) : A chiral object or compound can be defined as the one that is not superimposable on its mirror image, or we can say that all the four groups attached to a carbon atom must be different. Only I and II are chiral compounds.

(I) $C_2H_5-\overset{*}{C}H(CH_3)-C_3H_7$ (II) $CH_3CO-\overset{*}{C}H(CH_3)-C_2H_5$

20. (b) : This is an example of nucleophilic substitution reaction.

$$\underset{\text{Substrate}}{(CH_3)_3C-Br} + \underset{\text{Nucleophile}}{OH^-} \longrightarrow (CH_3)_3C-OH + \underset{\text{Leaving group}}{Br^-}$$

21. (c) : $(Cl)_2C=C(H)-CH_2-CH_2-CH_3$

Condition for geometrical isomerism is presence of two different atoms of groups attached to each carbon atom containing double bond.

Identical groups (Cl) on C - 1 will give only one compound. Hence it does not show geometrical isomerism.

22. (c) : Both involves compounds having the same molecular and structural formulae, but different spatial arrangement of atoms or groups.

23. (d) : An equimolar mixture of two *i.e.* dextro and laevorotatory optical isomers is termed as racemic mixture or *dl* form or (±) mixture.

24. (b) : $-CH_3$ group has $+I$ effect, as number of $-CH_3$ group increases the inductive effect increases.

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CHAPTER 22

HYDROCARBONS

1. A gaseous hydrocarbon gives upon combustion 0.72 g of water and 3.08 g of CO_2. The empirical formula of the hydrocarbon is
(a) C_7H_8 (b) C_2H_4
(c) C_3H_4 (d) C_6H_5 *(2013)*

2. Which branched chain isomer of the hydrocarbon with molecular mass 72 u gives only one isomer of mono substituted alkyl halide?
(a) Neopentane (b) Isohexane
(c) Neohexane (d) Tertiary butyl chloride *(2012)*

3. 2-Hexyne gives *trans*-2-hexene on treatment with
(a) Li/NH_3 (b) $Pd/BaSO_4$
(c) $LiAlH_4$ (d) Pt/H_2 *(2012)*

4. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of 44 u. The alkene is
(a) ethene (b) propene
(c) 1-butene (d) 2-butene. *(2010)*

5. The treatment of CH_3MgX with $CH_3C \equiv C - H$ produces
(a) CH_4
(b) $CH_3 - CH = CH_2$
(c) $CH_3C \equiv C - CH_3$
(d) $CH_3 - \overset{H}{\overset{|}{C}} = \overset{H}{\overset{|}{C}} - CH_3$ *(2008)*

6. The hydrocarbon which can react with sodium in liquid ammonia is
(a) $CH_3CH_2C \equiv CCH_2CH_3$
(b) $CH_3CH_2CH_2C \equiv CCH_2CH_2CH_3$
(c) $CH_3CH_2C \equiv CH$
(d) $CH_3CH = CHCH_3$
(2008)

7. In the following sequence of reactions, the alkene affords the compound *B*
$CH_3CH = CHCH_3 \xrightarrow{O_3} A \xrightarrow[Zn]{H_2O} B.$
The compound *B* is
(a) CH_3CHO (b) CH_3CH_2CHO
(c) CH_3COCH_3 (d) $CH_3CH_2COCH_3$ *(2008)*

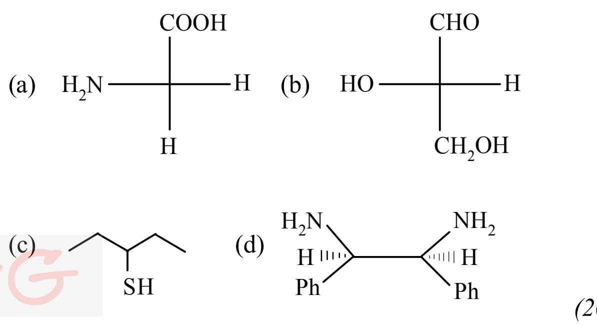
8. Toluene is nitrated and the resulting product is reduced with tin and hydrochloric acid. The product so obtained is diazotised and then heated with cuprous bromide. The reaction mixture so formed contains
(a) mixture of *o*- and *m*-bromotoluenes
(b) mixture of *o*- and *p*-bromotoluenes
(c) mixture of *o*- and *p*-dibromobenzenes
(d) mixture of *o*- and *p*-bromoanilines. *(2008)*

9. Presence of a nitro group in a benzene ring
(a) deactivates the ring towards electrophilic substitution
(b) activates the ring towards electrophilic substitution
(c) renders the ring basic
(d) deactivates the ring towards nucleophilic substitution. *(2007)*

10. The reaction of toluene with Cl_2 in presence of $FeCl_3$ gives predominantly
(a) *m*-chlorobenzene
(b) benzoyl chloride
(c) benzyl chloride
(d) *o*- and *p*-chlorotoluene. *(2007)*

11. Which of the following reactions will yield 2,2-dibromopropane?
(a) $CH_3 - CH = CH_2 + HBr \rightarrow$
(b) $CH_3 - C \equiv CH + 2HBr \rightarrow$
(c) $CH_3CH = CHBr + HBr \rightarrow$
(d) $CH \equiv CH + 2HBr \rightarrow$ *(2007)*

12. Which of the following molecules is expected to rotate the plane-polarised light?



(2007)

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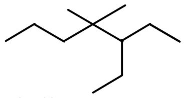
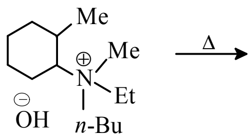
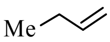
13. The IUPAC name of  is

(a) 3-ethyl-4-4-dimethylheptane
(b) 1,1-diethyl-2,2-dimethylpentane
(c) 4,4-dimethyl-5,5-diethylpentane
(d) 5,5-diethyl-4,4-dimethylpentane. *(2007)*

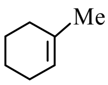
14. The compound formed as a result of oxidation of ethyl benzene by $KMnO_4$ is
(a) benzyl alcohol (b) benzophenone
(c) acetophenone (d) benzoic acid. *(2007)*

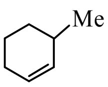
15. 

The alkene formed as a major product in the above elimination reaction is

(a) 

(b) $CH_2 = CH_2$

(c) 

(d) 

(2006)

16. Acid catalyzed hydration of alkenes except ethene leads to the formation of
(a) primary alcohol
(b) secondary or tertiary alcohol
(c) mixture of primary and secondary alcohols
(d) mixture of secondary and tertiary alcohols. *(2005)*

17. Of the five isomeric hexanes, the isomer which can give two monochlorinated compounds is
(a) *n*-hexane (b) 2,3-dimethylbutane
(c) 2,2-dimethylbutane (d) 2-methylpentane. *(2005)*

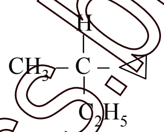
18. Reaction of one molecule of HBr with one molecule of 1, 3-butadiene at 40°C gives predominantly
(a) 3-bromobutene under kinetically controlled conditions
(b) 1-bromo-2-butene under thermodynamically controlled conditions
(c) 3-bromobutene under thermodynamically controlled conditions
(d) 1-bromo-2-butene under kinetically controlled conditions. *(2005)*

19. 2-Methylbutane on reacting with bromine in the presence of sunlight gives mainly
(a) 1-bromo-2-methylbutane
(b) 2-bromo-2-methylbutane
(c) 2-bromo-3-methylbutane
(d) 1-bromo-3-methylbutane. *(2005)*

20. Which one of the following is reduced with zinc and hydrochloric acid to give the corresponding hydrocarbon?
(a) Ethyl acetate (b) Acetic acid
(c) Acetamide (d) Butan-2-one *(2004)*

21. Amongst the following compounds, the optically active alkane having lowest molecular mass is
(a) $CH_3 - CH_2 - CH_2 - CH_3$

(b) $CH_3 - CH_2 - \overset{\overset{\large CH_3}{|}}{CH} - CH_3$

(c) 

(d) $CH_3 - CH_2 - C \equiv CH$ *(2004)*

22. Which one of the following has the minimum boiling point?
(a) *n*-Butane (b) 1-Butyne
(c) 1-Butene (d) Isobutene *(2004)*

23. On mixing a certain alkane with chlorine and irradiating it with ultraviolet light, it forms only one monochloroalkane. This alkane could be
(a) propane (b) pentane
(c) isopentane (d) neopentane. *(2003)*

24. Butene-1 may be converted to butane by reaction with
(a) Zn - HCl (b) Sn - HCl
(c) Zn - Hg (d) Pd/H_2. *(2003)*

25. What is the product when acetylene reacts with hypochlorous acid?
(a) CH_3COCl (b) $ClCH_2CHO$
(c) Cl_2CHCHO (d) ClCHCOOH. *(2002)*

26. Which of these will not react with acetylene?
(a) NaOH (b) ammonical $AgNO_3$
(c) Na (d) HCl *(2002)*

Answer Key

1.	(a)	**2.**	(a)	**3.**	(a, c)	**4.**	(d)	**5.**	(a)	**6.**	(c)
7.	(a)	**8.**	(b)	**9.**	(a)	**10.**	(d)	**11.**	(b)	**12.**	(b)
13.	(a)	**14.**	(d)	**15.**	(d)	**16.**	(b)	**17.**	(b)	**18.**	(b)
19.	(b)	**20.**	(d)	**21.**	(c)	**22.**	(d)	**23.**	(d)	**24.**	(d)
25.	(c)	**26.**	(a)								

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Explanations

1. **(a) :** Moles of water produced = $\frac{0.72}{18} = 0.04$

Moles of CO_2 produced = $\frac{3.08}{44} = 0.07$

Equation for combustion of an unknown hydrocarbon, C_xH_y is

$$C_xH_y + \left(x + \frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$$

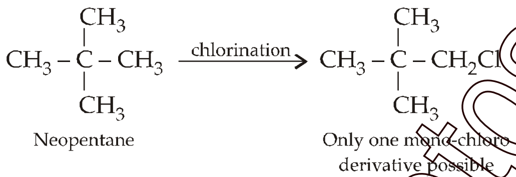
$\Rightarrow$ $x = 0.07$ and $\frac{y}{2} = 0.04$

$\therefore$ $y = 0.08$

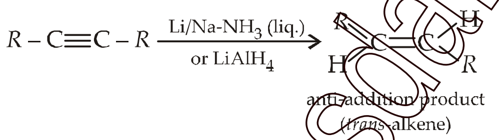
$$\frac{x}{y} = \frac{0.07}{0.08} = \frac{7}{8}$$

$\therefore$ The empirical formula of the hydrocarbon is C_7H_8.

2. **(a) :** As the molecular mass indicates it should be pentane and neopentane can only form one mono substituted alkyl halide as all the hydrogens are equivalent in neopentane.



3. **(a, c) :** For *trans* products we should take Na or Li metal in NH_3 or $EtNH_2$ at low temperature or $LiAlH_4$ as reducing agent (anti-addition).



4. **(d) :** $RCH{=}CHR$ (Symm. alkene) $\xrightarrow[\text{(ii) Zn, } H_2O]{\text{(i) } O_3}$ $2RCHO$

Molecular mass of $RCHO = 44$

$\Rightarrow$ $R + 12 + 1 + 16 = 44$

Mol. mass of $R = 44 - 29 = 15$

This is possible, only when R is —CH_3 group.

$\therefore$ The aldehyde is CH_3CHO and the symmetrical alkene is $CH_3HCCHCH_3$.

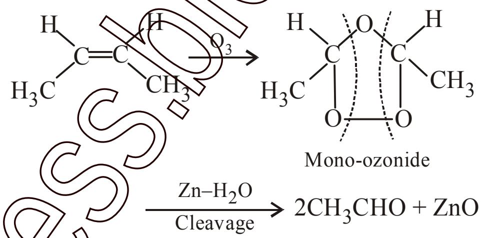
$CH_3CH{=}CHCH_3$ (2-Butene) $\xrightarrow[\text{(ii) Zn, } H_2O]{\text{(i) } O_3}$ $2CH_3CHO$ (Acetaldehyde)

5. **(a) :** Grignard reagent reacts with compounds having active or acidic hydrogen atom to give alkane.

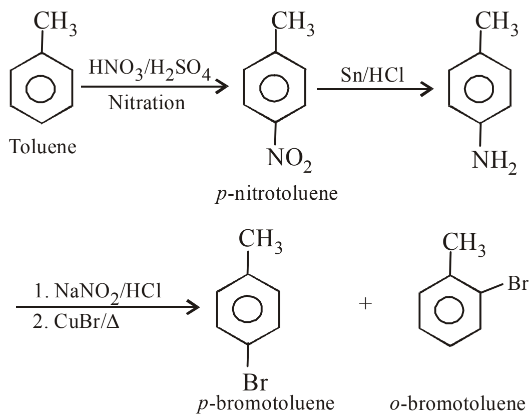
$CH_3MgX + CH_3C \equiv C{-}H \rightarrow CH_4 + CH_3C \equiv CMgX$

6. **(c) :** Terminal alkynes react with sodium in liquid ammonia to yield ionic compounds *i.e.* sodium alkylides.

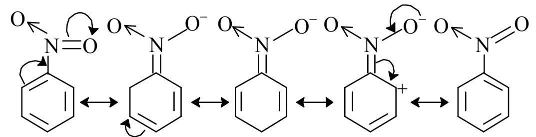
7. **(a) :** The complete reaction sequence is as follows



8. **(b) :** The reaction sequence is as follows :

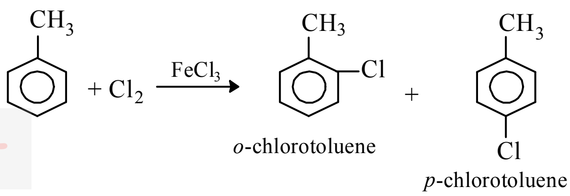


9. **(a) :**



From the resonating structures of it can be seen that the nitrogroup withdrawn electrons from the rings and hence it deactivates the benzene ring for further electophilic substitution.

10. **(d) :**



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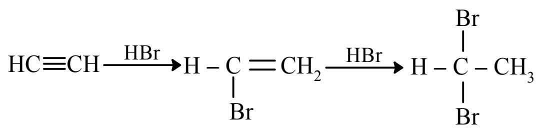
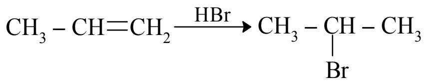
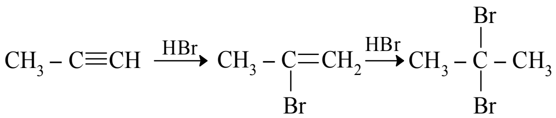
11. (b) :



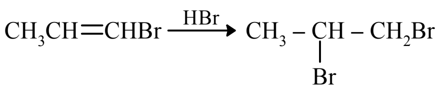
1, 1-dibromoethene



2-bromopropane

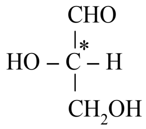


2, 2-dibromopropane



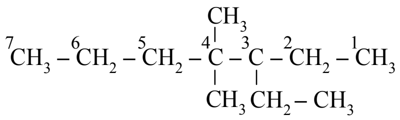
1, 2-dibromopropane

12. (b) :



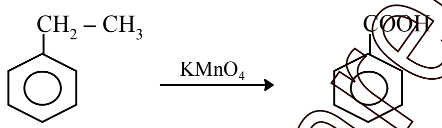
Due to the presence of chiral carbon atom, it is optically active, hence it is expected to rotate plane of polarized light.

13. (a) :



3-ethyl-4, 4-dimethylheptane

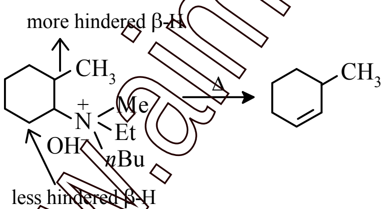
14. (d) :



Ethyl benzene Benzoic acid

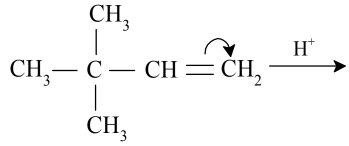
When oxidises with alkaline $KMnO_4$ or acidic $Na_2Cr_2O_7$, the entire side chain (in benzene homologues) with atleast one H at α-carbon, regardless of length is oxidised to –COOH.

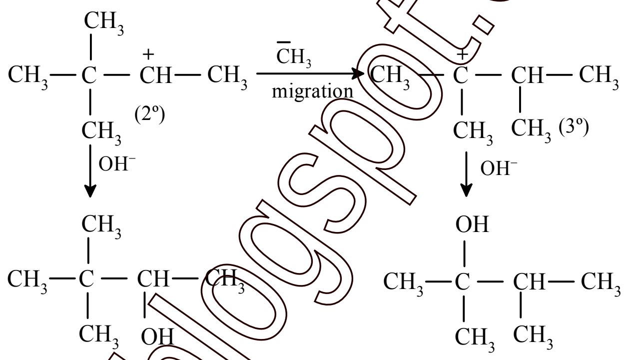
15. (d) :



In Hofmann elimination reaction, it is the less sterically hindered β-hydrogen that is removed and hence less substituted alkene is the major product.

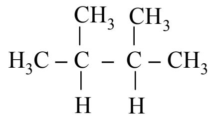
16. (b) :





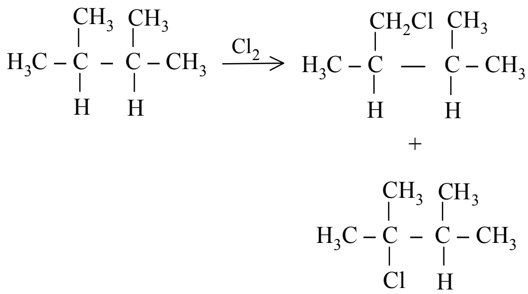
(secondary alcohol) (tertiary alcohol)

17. (b) : The number of monohalogenation products obtained from any alkane depends upon the number of different types of hydrogen it contains.



2,3-dimethylbutane

2,3-dimethylbutane has two types of hydrogen atoms so on monochlorination gives only two monochlorinated compounds.



18. (b) : 1,2-addition product is kinetically controlled product while 1,4-addition product is thermodynamically controlled product and formed at comparatively higher temperature.

$$CH_2{=}CH-CH{=}CH_2 \xrightarrow[40°]{HBr} \underset{\text{1,4-addition}}{CH_2(Br)CH{=}CHCH_3} + \underset{\text{1,2-addition}}{CH_3CH(Br)-CH{=}CH_2}$$

Therefore, 1-bromo-2-butene will be the main product under thermodynamically controlled conditions.

19. (b) : The reactivity order of abstraction of H atoms towards bromination of alkane is 3°H > 2°H > 1°H.

So, $\underset{\text{2-methylbutane}}{(CH_3)_2CHCH_2CH_3} \xrightarrow[127°C]{Br_2/h\upsilon} \underset{\text{2-bromo-2-methylbutane}}{(CH_3)_2C(Br)CH_2CH_3}$

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20. **(d) :** Butan-2-one will get reduced into butane when treated with zinc and hydrochloric acid following Clemmensen reaction whereas Zn/HCl do not reduce ester, acid and amide.

$$\underset{\text{Butan-2-one}}{CH_3-CH_2-\overset{O}{\overset{\|}{C}}-CH_3} \xrightarrow[HCl]{Zn-Hg} \underset{\text{Butane}}{CH_3-CH_2CH_2-CH_3}$$

21. **(c) :** $CH_3-\overset{H}{\underset{C_2H_5}{C^*}}-\triangleleft$

Optically active due to presence of chiral carbon atom.

22. **(d) :** Among the isomeric alkanes, the normal isomer has a higher boiling point than the branched chain isomer. The greater the branching of the chain, the lower is the boiling point. The *n*-alkanes have larger surface area in comparison to branched chain isomers (as the shape approaches that of a sphere in the branched chain isomers). Thus, intermolecular forces are weaker in branched chain isomers, therefore, they have lower boiling points in comparison to straight chain isomers.

23. **(d) :** The number of monohalogenation products obtained from any alkene depends upon the number of different types of hydrogen it contains.
Compound containing only one type of hydrogen gives only one monohalogenation product.

$\underset{\text{propane}}{CH_3CH_2CH_3}$ — two types of hydrogen (two monohalogenation product)

$\underset{\text{pentane}}{CH_3CH_2CH_2CH_2CH_3}$ — two types of hydrogen (two monohalogenation product)

$\underset{\text{isopentane}}{CH_3-\overset{CH_3}{\overset{|}{CH}}-CH_2-CH_3}$ — three types of hydrogen (three monohalogenation product)

$\underset{\text{neopentane}}{H_3C-\overset{CH_3}{\underset{CH_3}{C}}-CH_3}$ — one type of hydrogen (one monohalogenation product)

Thus the given alkane should be neopentane.

24. **(d) :** $\underset{\text{Butene - 1}}{H_3C-CH_2-CH=CH_2} \xrightarrow{Pd/H_2} \underset{\text{Butane}}{H_3C-CH_2-CH_2-CH_3}$

25. **(c) :** $CH\equiv CH \xrightarrow{HOCl} \begin{matrix}CHOH\\ \|\\ CHCl\end{matrix} \xrightarrow{HOCl} \begin{matrix}CH(OH)_2\\ |\\ CHCl_2\end{matrix} \xrightarrow{-H_2O} \begin{matrix}CHO\\ |\\ CHCl_2\end{matrix}$

26. **(a) :** Acetylene does not react with NaOH because product would be the stronger acid H_2O and the stronger base $(CH_3-C\equiv\bar{C})$.
Acetylene reacts with the other three as:

$$\begin{matrix}CH\\ |||\\ CNa\end{matrix} \xleftarrow[NH_{3(l)}]{Na} \begin{matrix}CH\\ |||\\ CH\end{matrix} \xrightarrow{HCl} \begin{matrix}CH_2\\ \|\\ CHCl\end{matrix} \xrightarrow{HCl} \begin{matrix}CH_3\\ |\\ CHCl_2\end{matrix}$$

$$\begin{matrix}CH\\ |||\\ CH\end{matrix} \xrightarrow{[AgNO_3 + NH_4OH]} \underset{\text{white ppt.}}{\begin{matrix}CAg\\ |||\\ CAg\end{matrix}} + NH_4NO_3$$

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CHAPTER 23

ORGANIC COMPOUNDS CONTAINING HALOGENS

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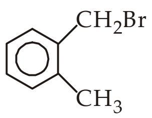
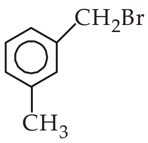
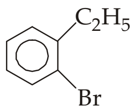
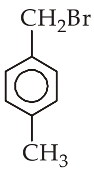
1. Compound (A), C_8H_9Br, gives a white precipitate when warmed with alcoholic $AgNO_3$. Oxidation of (A) gives an acid (B), $C_8H_6O_4$. (B) easily forms anhydride on heating. Identify the compound (A).

(a) 

(b) 

(c) 

(d) 

(2013)

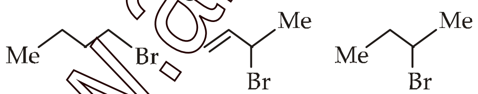
2. A solution of (–) – 1 – chloro – 1 – phenylethane in toluene racemises slowly in the presence of a small amount of $SbCl_5$, due to the formation of
   (a) free radical (b) carbanion
   (c) carbene (d) carbocation. *(2013)*

3. An unknown alcohol is treated with the "Lucas reagent" to determine whether the alcohol is primary, secondary or tertiary. Which alcohol reacts fastest and by what mechanism?
   (a) Tertiary alcohol by S_N2.
   (b) Secondary alcohol by S_N1.
   (c) Tertiary alcohol by S_N1.
   (d) Secondary alcohol by S_N2. *(2013)*

4. Iodoform can be prepared from all except
   (a) isopropyl alcohol (b) 3-methyl-2-butanone
   (c) isobutyl alcohol (d) ethyl methyl ketone. *(2012)*

5. What is DDT among the following?
   (a) A fertilizer (b) Biodegradable pollutant
   (c) Non-biodegradable pollutant
   (d) Greenhouse gas *(2012)*

6. Consider the following bromides :



(A) (B) (C)

The correct order of S_N1 reactivity is
   (a) $A > B > C$ (b) $B > C > A$
   (c) $B > A > C$ (d) $C > B > A$ *(2010)*

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7. Which of the following on heating with aqueous KOH produces acetaldehyde?
   (a) CH_3COCl (b) CH_3CH_2Cl
   (c) CH_2ClCH_2Cl (d) CH_3CHCl_2 *(2009)*

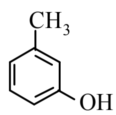
8. The organic chloro compound, which shows complete stereochemical inversion during a S_N2 reaction, is
   (a) CH_3Cl (b) $(C_2H_5)_2CHCl$
   (c) $(CH_3)_3CCl$ (d) $(CH_3)_2CHCl$ *(2008)*

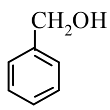
9. Which of the following is the correct order of decreasing S_N2 reactivity?
   (a) $R_2CH\,X > R_3C\,X > RCH_2\,X$
   (b) $RCH\,X > R_3C\,X > R_2CH\,X$
   (c) $RCH_2\,X > R_2CH\,X > R_3C\,X$
   (d) $R_3C\,X > R_2CH\,X > RCH_2\,X$
   (X is a halogen) *(2007)*

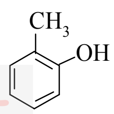
10. Reaction of *trans*-2-phenyl-1-bromocyclopentane on reaction with alcoholic KOH produces
    (a) 4-phenylcyclopentene
    (b) 2-phenylcyclopentene
    (c) 1-phenylcyclopentene
    (d) 3-phenylcyclopentene. *(2006)*

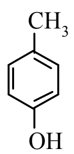
11. Fluorobenzene (C_6H_5F) can be synthesised in the laboratory
    (a) by heating phenol with HF and KF
    (b) from aniline by diazotization followed by heating the diazonium salt with HBF_4
    (c) by direct fluorination of benzene with F_2 gas
    (d) by reacting bromobenzene with NaF solution. *(2006)*

12. The structure of the compound that gives a tribromoderivative on treatment with bromine water is

(a) 

(b) 

(c) 

(d) 

(2006)

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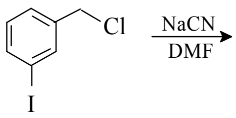
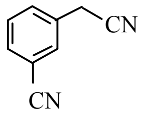
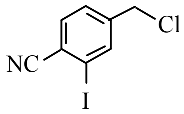
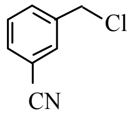
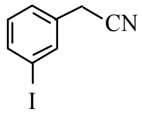
13. The structure of the major product formed in the following reaction is



(a)  (b) 

(c)  (d)  *(2006)*

14. Elimination of bromine from 2-bromobutane results in the formation of
(a) equimolar mixture of 1 and 2-butene
(b) predominantly 2-butene
(c) predominantly 1-butene
(d) predominantly 2-butyne. *(2005)*

15. Alkyl halides react with dialkyl copper reagents to give
(a) alkenes (b) alkyl copper halides
(c) alkanes (d) alkenyl halides. *(2005)*

16. Tertiary alkyl halides are practically inert to substitution by S_N2 mechanism because of
(a) insolubility (b) instability
(c) inductive effect (d) steric hindrance. *(2005)*

17. Which of the following compounds is not chiral?
(a) 1-chloropentane
(b) 2-chloropentane
(c) 1-chloro-2-methylpentane
(d) 3-chloro-2-methylpentane *(2004)*

18. Acetyl bromide reacts with excess of CH_3MgI followed by treatment with a saturated solution of NH_4Cl gives
(a) acetone
(b) acetamide
(c) 2-methyl-2-propanol
(d) acetyl iodide. *(2004)*

19. Which of the following will have a meso-isomer also?
(a) 2-chlorobutane
(b) 2,3-dichlorobutane
(c) 2,3-dichloropentane
(d) 2-hydroxypropanoic acid *(2004)*

20. The compound formed on heating chlorobenzene with chloral in the presence of concentrated sulphuric acid is
(a) gammexene (b) DDT
(c) freon (d) hexachloroethane. *(2004)*

21. Bottles containing C_6H_5I and $C_6H_5CH_2I$ lost their original labels. They were labelled *A* and *B* for testing. *A* and *B* were separately taken in a test tube and boiled with NaOH solution. The end solution in each tube was made acidic with dilute HNO_3 and then some $AgNO_3$ solution was added. Substance *B* gave a yellow precipitate. Which one of the following statements is true for this experiment?
(a) *A* was C_6H_5I
(b) *A* was $C_6H_5CH_2I$
(c) *B* was C_6H_5I
(d) Addition of HNO_3 was unnecessary. *(2003)*

Answer Key

1.	(a)	**2.**	(d)	**3.**	(c)	**4.**	(c)	**5.**	(c)	**6.**	(b)
7.	(d)	**8.**	(a)	**9.**	(c)	**10.**	(d)	**11.**	(b)	**12.**	(a)
13.	(d)	**14.**	(b)	**15.**	(c)	**16.**	(d)	**17.**	(a)	**18.**	(c)
19.	(b)	**20.**	(b)	**21.**	(a)						

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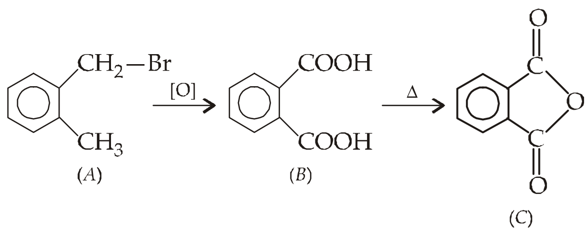
Explanations

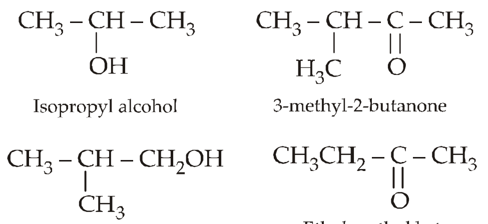
1. **(a) :**



2. **(d) :** A carbocation intermediate is formed during racemisation.

3. **(c) :** In Lucas test, turbidity appears immediately with tertiary alcohol by S_N1 mechanism.

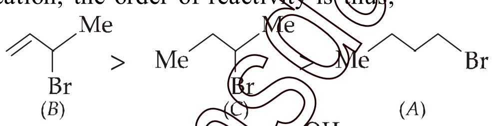
4. **(c) :** The compounds with $CH_3-\underset{\underset{O}{||}}{C}-$ or $CH_3-\underset{\underset{OH}{|}}{CH}-$ group form iodoform.



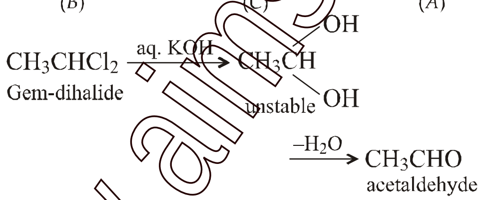
Thus all the compounds except isobutyl alcohol will form iodoform.

5. **(c)**

6. **(b) :** S_N1 reaction rate depends upon the stability of the carbocation, as carbocation formation is the rate determining step. Compound (*B*), forms a 2° allylic carbocation which is the most stable, the next stable carbocation is formed from (*C*), it is a 2° carbocation, (*A*) forms the least stable 1° carbocation, the order of reactivity is thus,

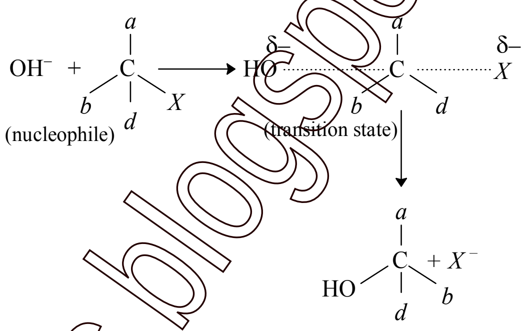


7. **(d) :**



8. **(a) :** In S_N2 reactions, the nucleophile attacks from back side resulting in the inversion of molecule. Also, as we move from 1° alkyl halide to 3° alkyl halide, the crowding increases and +*I* effect increases which makes the carbon bearing halogen less positively polarised and hence less readily attacked by the nucleophile.

9. **(c) :** S_N2 mechanism occurs as

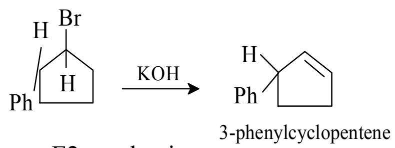


In S_N2 reaction, in the transition state there will be five groups attached to the carbon atom at which reaction occurs. Thus there will be crowding in the transition state, and the bulkier the group, the more the reaction will be hindered sterically. Hence S_N2 reaction is favoured by small groups on the carbon atom attached to halogens. So the decreasing order of reactivity of halide is

$$RCH_2X > R_2CHX > R_3CX$$

(primary) (secondary) (tertiary)

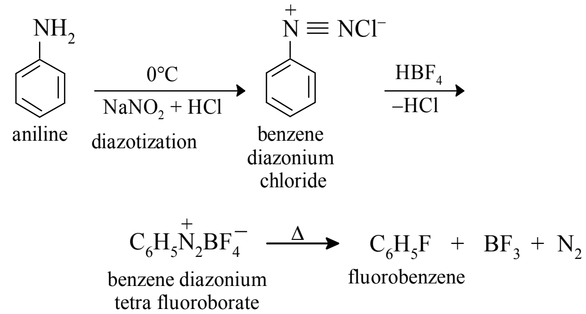
10. **(d) :**



It follows E2 mechanism.

Hughes and Ingold proposed that bimolecular elimination reactions take place when the two groups to be eliminated are *trans* and lie in one plane with the two carbon atoms to which they are attached *i.e.* E2 reactions are stereoselectively *trans*.

11. **(b) :**



12. **(a) :** Since the compound on treatment with Br_2-water gives a tribromoderivative, therefore it must be *m*-cresol, because it has two *ortho* and one *para* position free with respect to OH group and hence can give tribromoderivative.

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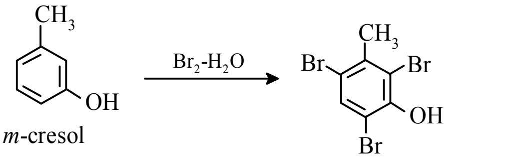
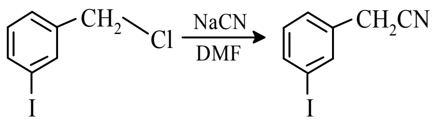


m-cresol → 2,4,6-tribromo-3-methylphenol

13. (d) :



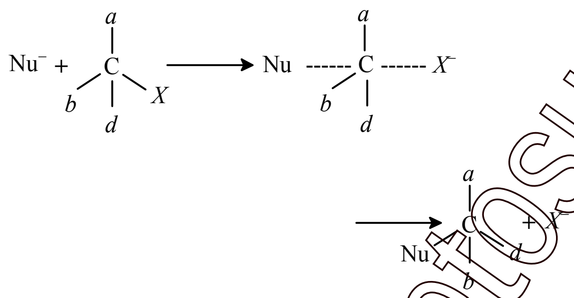
14. (b) : $CH_3CH_2CH(Br)CH_3$ (2-bromobutane) $\xrightarrow{\text{KOH (alc.)}}$ $CH_3CH{=}CHCH_3$ (2-butene (80%)) + $CH_3CH_2CH{=}CH_2$ (1-butene (20%))

In elimination reaction of alkyl halide major product is obtained according to Saytzeff's rule, which states that when two alkenes may be formed, the alkene which is most substituted one predominates.

15. (c) : In Corey House synthesis of alkane, alkyl halide reacts with lithium dialkyl cuprate.

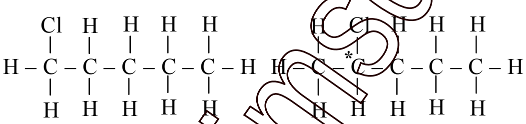
$$R_2CuLi + R'X \rightarrow RR' + RCu + LiX$$

16. (d) :

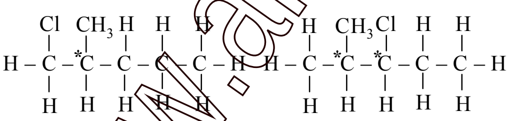


In an S_N2 reaction, in the transition state, there will be five groups attached to the carbon atom at which reaction occurs. Thus there will be crowding in the transition state, and presence of bulky groups make the reaction sterically hindered.

17. (a) : To be optically active the compound or structure should possess chiral or asymmetric centre.

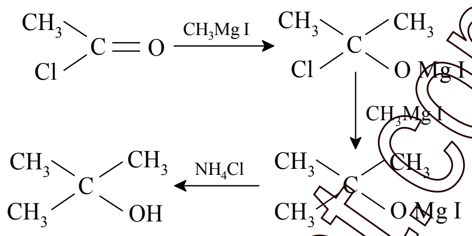


1-chloropentane 2-chloropentane



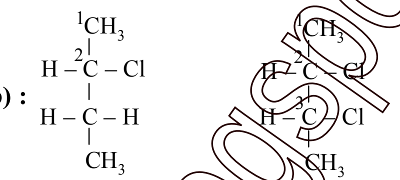
1-chloro-2-methylpentane 3-chloro-2-methylpentane

18. (c) :

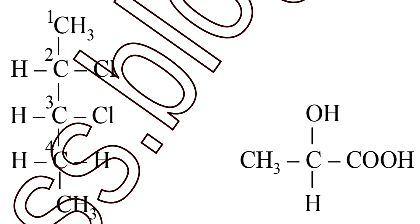


2-methyl-2-propanol

19. (b) :



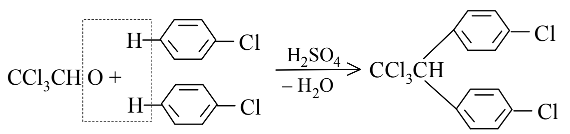
2-chlorobutane 2,3-dichlorobutane



2,3-dichloropentane 2-hydroxypropanoic acid

2,3-dichlorobutane have meso isomer due to the presence of plane of symmetry.

20. (b) : DDT is prepared by heating chlorobenzene and chloral with concentrated sulphuric acid.



1,1,1-trichloro-2,2 bis (*p*-chlorophenyl) ethane or DDT

21. (a) :

A: $C_6H_5I \xrightarrow{NaOH} C_6H_5ONa \xrightarrow{HNO_3/H^+} C_6H_5OH \xrightarrow{AgNO_3}$ No yellow precipitate

B: $C_6H_5CH_2I \xrightarrow{NaOH} C_6H_5CH_2ONa \xrightarrow{HNO_3/H^+} C_6H_5CH_2OH \xrightarrow{AgNO_3}$ Yellow precipitate

Thus *A* must be C_6H_5I.

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CHAPTER 24

ALCOHOLS, PHENOLS AND ETHERS

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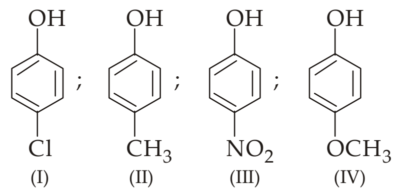
1. Arrange the following compounds in order of decreasing acidity.



(a) IV > III > I > II (b) II > IV > I > III
(c) I > II > III > IV (d) III > I > II > IV *(2013)*

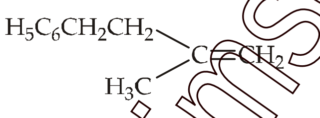
2. *Ortho*-nitrophenol is less soluble in water than *p*- and *m*-nitrophenols because
(a) *o*-nitrophenol shows intramolecular H-bonding
(b) *o*-nitrophenol shows intermolecular H-bonding
(c) melting point of *o*-nitrophenol is lower than those of *m*- and *p*-isomers
(d) *o*-nitrophenol is more volatile in steam than those of *m*- and *p*-isomers. *(2012)*

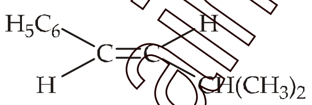
3. Sodium ethoxide has reacted with ethanoyl chloride. The compound that is produced in this reaction is
(a) diethyl ether (b) 2-butanone
(c) ethyl chloride (d) ethyl ethanoate. *(2011)*

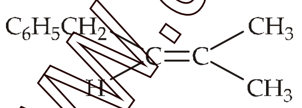
4. Phenol is heated with a solution of mixture of KBr and $KBrO_3$. The major product obtained in the above reaction is
(a) 2-bromophenol (b) 3-bromophenol
(c) 4-bromophenol (d) 2, 4, 6-tribromophenol. *(2011)*

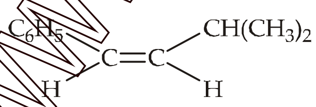
5. The main product of the following reaction is

$$C_6H_5CH_2CH(OH)CH(CH_3)_2 \xrightarrow{Conc.H_2SO_4}$$

(a) 

(b) 

(c) 

(d) 

(2010)

6. From amongst the following alcohols the one that would react fastest with conc.HCl and anhydrous $ZnCl_2$, is
(a) 1-Butanol (b) 2-Butanol
(c) 2-Methylpropan-2-ol (d) 2-Methylpropanol. *(2010)*

7. The major product obtained on interaction of phenol with sodium hydroxide and carbon dioxide is
(a) benzoic acid (b) salicylaldehyde
(c) salicylic acid (d) phthalic acid. *(2009)*

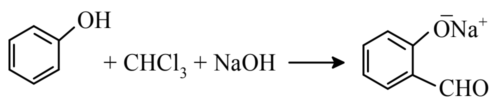
8. Phenol, when it first reacts with concentrated sulphuric acid and then with concentrated nitric acid, gives
(a) nitrobenzene (b) 2, 4, 6-trinitrobenzene
(c) *o*-nitrophenol (d) *p*-nitrophenol. *(2008)*

9. In the following sequence of reactions,

$$CH_3CH_2OH \xrightarrow{P + I_2} A \xrightarrow[ether]{Mg} B \xrightarrow{HCHO} C \xrightarrow{H_2O} D$$

the compound *D* is
(a) propanal (b) butanal
(c) *n*-butyl alcohol (d) *n*-propyl alcohol. *(2007)*

10. HBr reacts with $CH_2 = CH - OCH_3$ under anhydrous conditions at room temperature to give
(a) CH_3CHO and CH_3Br
(b) $BrCH_2CHO$ and CH_3OH
(c) $BrCH_2 - CH_2 - OCH_3$
(d) $H_3C - CHBr - OCH_3$. *(2006)*

11. 

The electrophile involved in the above reaction is
(a) dichloromethyl cation $(\overset{\oplus}{C}HCl_2)$
(b) dichlorocarbene $(: CCl_2)$
(c) trichloromethyl anion $(\overset{\ominus}{C}Cl_3)$
(d) formyl cation $(\overset{\oplus}{C}HO)$ *(2006)*

12. Phenyl magnesium bromide reacts with methanol to give
(a) a mixture of anisole and Mg(OH)Br
(b) a mixture of benzene and Mg(OMe)Br
(c) a mixture of toluene and Mg(OH)Br
(d) a mixture of phenol and Mg(Me)Br. *(2006)*

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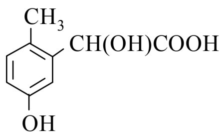
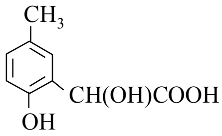
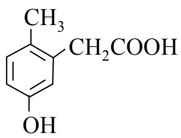
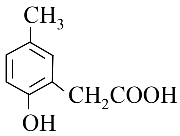
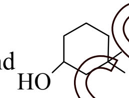
13. *p*-cresol reacts with chloroform in alkaline medium to give the compound *A* which adds hydrogen cyanide to form the compound *B*. The latter on acidic hydrolysis gives chiral carboxylic acid. The structure of the carboxylic acid is

(a) 
(b) 
(c) 
(d)  *(2005)*

14. The best reagent to convert pent-3-en-2-ol into pent-3-en-2-one is
(a) acidic permanganate
(b) acidic dichromate
(c) chromic anhydride in glacial acetic acid
(d) pyridinium chlorochromate. *(2005)*

15. Among the following compounds which can be dehydrated very easily?
(a) $CH_3CH_2CH_2CH_2CH_2OH$
(b) $CH_3CH_2CH_2CH(OH)CH_3$
(c) $CH_3CH_2C(CH_3)(OH)CH_2CH_3$
(d) $CH_3CH_2CH(CH_3)CH_2CH_2OH$ *(2004)*

16. The IUPAC name of the compound  is
(a) 3,3-dimethyl-1-hydroxy cyclohexane
(b) 1,1-dimethyl-3-hydroxy cyclohexane
(c) 3,3-dimethyl-1-cyclohexanol
(d) 1,1-dimethyl-3-cyclohexanol *(2004)*

17. For which of the following parameters the structural isomers C_2H_5OH and CH_3OCH_3 would be expected to have the same values? (Assume ideal behaviour)
(a) Heat of vaporisation
(b) Vapour pressure at the same temperature
(c) Boiling points
(d) Gaseous densities at the same temperature and pressure *(2004)*

18. During dehydration of alcohols to alkenes by heating with concentrated H_2SO_4 the initiation step is
(a) protonation of alcohol molecule
(b) formation of carbocation
(c) elimination of water
(d) formation of an ester. *(2003)*

19. An ether is more volatile than an alcohol having the same molecular formula. This is due to
(a) dipolar character of ethers
(b) alcohols having resonance structures
(c) inter-molecular hydrogen bonding in ethers
(d) inter-molecular hydrogen bonding in alcohols. *(2003)*

Answer Key

1.	(d)	**2.**	(a)	**3.**	(d)	**4.**	(d)	**5.**	(b)	**6.**	(c)
7.	(c)	**8.**	(None of the option is correct)			**9.**	(d)	**10.**	(d)	**11.**	(b)
12.	(b)	**13.**	(b)	**14.**	(d)	**15.**	(c)	**16.**	(c)	**17.**	(d)
18.	(a)	**19.**	(d)								

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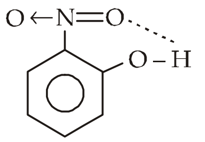
Explanations

1. **(d) :** Electron donating groups (—CH_3 and —OCH_3) decrease while electron withdrawing groups (—NO_2 and —Cl) increase the acidity.
Since —OCH_3 is a stronger electron donating group than —CH_3 and —NO_2 is stronger electron withdrawing group than —Cl, therefore order of decreasing acidity is III > I > II > IV.

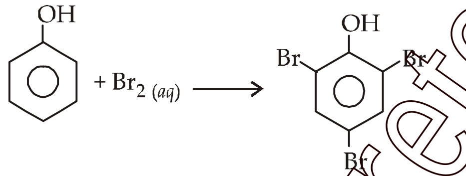
2. **(a) :** *o*-Nitrophenol is stable due to intramolecular hydrogen bonding.



It is difficult to break the H-bonding when dissolved in water thus less soluble.

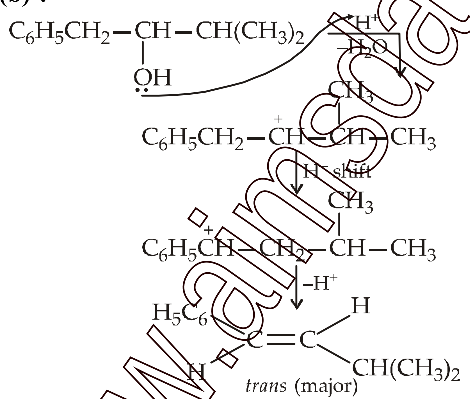
3. **(d) :** $CH_3-\underset{\underset{O}{\|}}{C}-Cl + C_2H_5ONa \longrightarrow CH_3-\underset{\underset{O}{\|}}{C}-OC_2H_5 + NaCl$
Ethyl ethanoate

4. **(d) :** $KBr_{(aq)} + KBrO_{3(aq)} \longrightarrow Br_{2(aq)}$
This bromine reacts with phenol gives 2,4,6-tribromophenol.



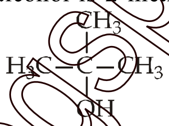
2,4,6-tribromophenol

5. **(b) :**



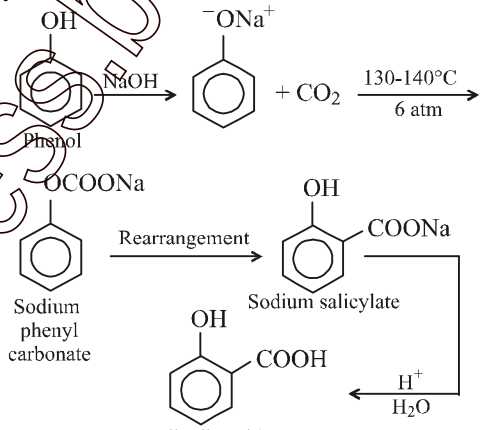
The preferential formation of this compound is due to conjugation in the compound.

6. **(c) :** The reagent, conc.HCl and anhydrous $ZnCl_2$ is Lucas reagent, which is used to distinguish between 1°, 2° and 3° alcohols.
3° alcohol + Lucas reagent ⟶ Immediate turbidity.
2° alcohol + Lucas reagent ⟶ Turbidity after 5 mins.
1° alcohol + Lucas reagent ⟶ No reaction.
Thus, the required alcohol is 2-methylpropan-2-ol, *i.e.*,

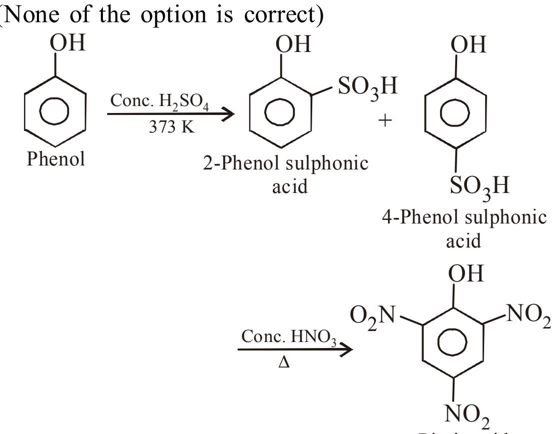


(3° alcohol)

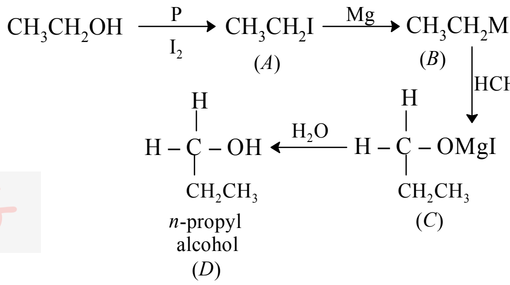
7. **(c) :** The reaction of phenol with NaOH and CO_2 is known as Kolbe-Schmidt or Kolbe's reaction. The product formed is salicylic acid.



8. **:** (None of the option is correct)



9. **(d) :**



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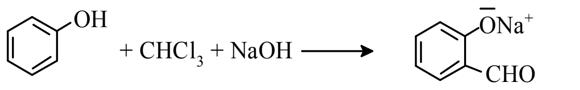
10. (d) : Methyl vinyl ether is a very reactive gas. It is hydrolysed rapidly by dilute acids at room temperature to give methanol and aldehyde.

However, under anhydrous conditions at room temperature, it undergoes many addition reactions at the double bond.

$$H_2C{=}CH-OCH_3 \xrightarrow{H^+} H_2C(H)-\overset{\oplus}{C}H-OCH_3 \xrightarrow{Br^-} H_3C-CH(Br)-OCH_3$$

11. (b) :



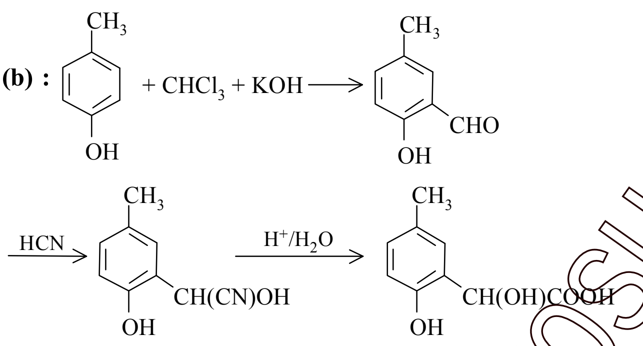
(Reimer-Tiemann reaction)

The electrophile is dichlorocarbene, $:CCl_2$ generated from chloroform by the action of a base.

$OH^- + CHCl_3 \rightleftharpoons HOH + :CCl_3^- \rightarrow Cl^- + :CCl_2$

12. (b) : $CH_3OH + C_6H_5MgBr \rightarrow C_6H_6 + Mg(OCH_3)Br$

13. (b) :

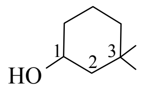


14. (d) : Pyridinium chlorochromate oxidises an alcoholic group selectively in the presence of carbon-carbon double bond.

15. (c) : The ease of dehydration of alcohols is tertiary > secondary > primary according to the order of stability of the carbocations.

$$CH_3-CH_2-\underset{OH}{\overset{CH_3}{C}}-CH_2-CH_3 \xrightarrow{H^+} CH_3-CH_2-\underset{\oplus}{\overset{CH_3}{C}}-CH_2-CH_3$$

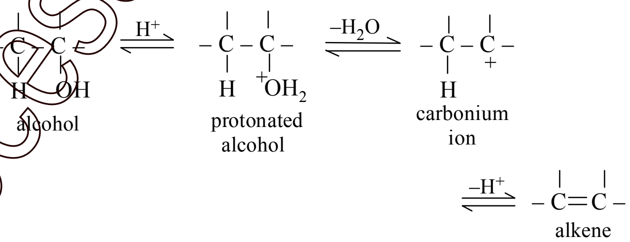
The more stable carbocation is generated thus more easily it will be dehydrated.

16. (c) :  → 3,3-dimethyl-1-cyclohexanol

17. (d) : Vapour density $= \frac{\text{Molecular weight}}{2}$

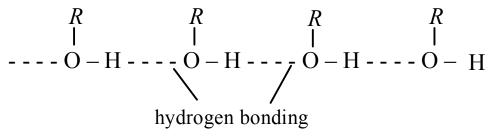
As both the compounds have same molecular weights, both will have the same vapour density. Hence, gaseous density of both ethanol and dimethyl ether would be same under identical conditions of temperature and pressure. The rest of these three properties; vapour pressure, boiling point and heat of vaporization will differ as ethanol has hydrogen bonding whereas ether does not.

18. (a) : Dehydration of alcohol to alkene in presence of concentrated H_2SO_4 involves following steps :



Thus, the initiation step is protonation of alcohol.

19. (d) : The reason for the lesser volatility of alcohols than ethers is the intermolecular association of a large number of molecules due to hydrogen bonding as – OH group is highly polarised.



No such hydrogen bonding is present in ethers.

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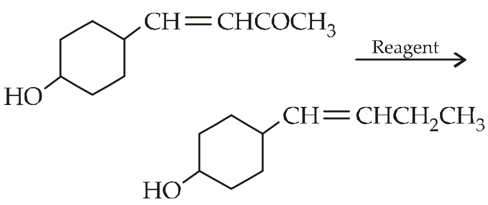
CHAPTER 25

ALDEHYDES, KETONES AND CARBOXYLIC ACIDS

1. An organic compound *A* upon reacting with NH_3 gives *B*. On heating, *B* gives *C*. *C* in presence of KOH reacts with Br_2 to give $CH_3CH_2NH_2$. *A* is
(a) CH_3CH_2COOH
(b) CH_3COOH
(c) $CH_3CH_2CH_2COOH$
(d) $CH_3-CH(CH_3)-COOH$ *(2013)*

2. In the given transformation, which of the following is the most appropriate reagent?



(a) Zn-Hg/HCl (b) Na, liq. NH_3
(c) $NaBH_4$ (d) NH_2-NH_2, OH^- *(2012)*

3. Silver mirror test is given by which one of the following compounds?
(a) Acetaldehyde (b) Acetone
(c) Formaldehyde (d) Benzophenone *(2011)*

4. Ozonolysis of an organic compound gives formaldehyde as one of the products. This confirms the presence of
(a) two ethylenic double bonds
(b) a vinyl group
(c) an isopropyl group
(d) an acetylenic triple bond. *(2011)*

5. The strongest acid amongst the following compounds is
(a) CH_3COOH
(b) $HCOOH$
(c) $CH_3CH_2CH(Cl)CO_2H$
(d) $ClCH_2CH_2CH_2COOH$ *(2011)*

6. Trichloroacetaldehyde was subjected to Cannizzaro's reaction by using NaOH. The mixture of the products contains sodium trichloroacetate ion and another compound. The other compound is
(a) 2,2,2-trichloroethanol
(b) trichloromethanol
(c) 2,2,2-trichloropropanol
(d) chloroform *(2011)*

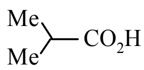
7. In Cannizzaro reaction given below

$$2PhCHO \xrightarrow{:OH^-} PhCH_2OH + PhCOO^-$$

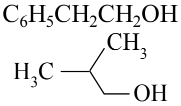
the slowest step is
(a) the attack of $:OH^-$ at the carboxyl group
(b) the transfer of hydride to the carbonyl group
(c) the abstraction of proton from the carboxylic group
(d) the deprotonation of $PhCH_2OH$. *(2009)*

8. A liquid was mixed with ethanol and a drop of concentrated H_2SO_4 was added. A compound with a fruity smell was formed. The liquid was
(a) CH_3OH (b) HCHO
(c) CH_3COCH_3 (d) CH_3COOH *(2009)*

9. The compound formed as a result of oxidation of ethyl benzene by $KMnO_4$ is
(a) benzyl alcohol (b) benzophenone
(c) acetophenone (d) benzoic acid. *(2007)*

10. The correct order of increasing acid strength of the compounds
(*A*) CH_3CO_2H (*B*) $MeOCH_2CO_2H$
(*C*) CF_3CO_2H (*D*)  is

(a) $B < D < A < C$ (b) $D < A < C < B$
(c) $D < A < B < C$ (d) $A < D < C < B$. *(2006)*

11. Among the following the one that gives positive iodoform test upon reaction with I_2 and NaOH is
(a) $CH_3CH_2CH(OH)CH_2CH_3$
(b) $C_6H_5CH_2CH_2OH$
(c)  (d) $PhCHOHCH_3$ *(2006)*

12. The increasing order of the rate of HCN addition to compounds *A* - *D* is
A. HCHO *B*. CH_3COCH_3
C. $PhCOCH_3$ *D*. PhCOPh.
(a) $A < B < C < D$ (b) $D < B < C < A$
(c) $D < C < B < A$ (d) $C < D < B < A$ *(2006)*

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13. Which one of the following undergoes reaction with 50% sodium hydroxide solution to give the corresponding alcohol and acid?
(a) Phenol (b) Benzaldehyde
(c) Butanol (d) Benzoic acid *(2004)*

14. On mixing ethyl acetate with aqueous sodium chloride, the composition of the resultant solution is
(a) $CH_3COOC_2H_5 + NaCl$
(b) $CH_3COONa + C_2H_5OH$
(c) $CH_3COCl + C_2H_5OH + NaOH$
(d) $CH_3Cl + C_2H_5COONa$. *(2004)*

15. Consider the acidity of the carboxylic acids:
(i) PhCOOH
(ii) *o*-$NO_2C_6H_4COOH$
(iii) *p*-$NO_2C_6H_4COOH$
(iv) *m*-$NO_2C_6H_4COOH$
Which of the following order is correct?
(a) i > ii > iii > iv (b) ii > iv > iii > i
(c) ii > iv > i > iii (d) ii > iii > iv > i *(2004)*

16. Rate of the reaction,

$$R-C(=O)-Z + Nu^{\ominus} \longrightarrow R-C(=O)-Nu + Z^{\ominus}$$

is fastest when Z is
(a) Cl (b) NH_2
(c) OC_2H_5 (d) $OCOCH_3$. *(2004)*

17. In the anion $HCOO^-$ the two carbon-oxygen bonds are found to be of equal length. What is the reason for it?
(a) Electronic orbitals of carbon atom are hybridised.
(b) The C $=$ O bond is weaker than the C – O bond.
(c) The anion $HCOO^-$ has two resonating structures.
(d) The anion is obtained by removal of a proton from the acid molecule. *(2003)*

18. The general formula $C_nH_{2n}O_2$ could be for open chain
(a) diketones (b) carboxylic acids
(c) diols (d) dialdehydes *(2003)*

19. When $CH_2 = CH - COOH$ is reduced with $LiAlH_4$, the compound obtained will be
(a) $CH_3 - CH_2 - COOH$
(b) $CH_2 = CH - CH_2OH$
(c) $CH_3 - CH_2 - CH_2OH$
(d) $CH_3 - CH_2 - CHO$. *(2003)*

20. The IUPAC name of $CH_3COCH(CH_3)_2$ is
(a) isopropylmethyl ketone
(b) 2-methyl-3-butanone
(c) 4-methylisopropyl ketone
(d) 3-methyl-2-butanone. *(2003)*

21. On vigorous oxidation by permanganate solution, $(CH_3)_2C = CH - CH_2 - CHO$ gives
(a) $CH_3-C(OH)(CH_3)-CH(OH)-CH_2CH_3$
(b) $(CH_3)_2COOH + CH_3CH_2COOH$
(c) $(CH_3)_2CH-OH + CH_3CH_2CH_2OH$
(d) $(CH_3)_2C=O + CH_3CH_2CHO$. *(2002)*

22. $CH_3CH_2COOH \xrightarrow[\text{red P}]{Cl_2} A \xrightarrow{\text{alc. KOH}} B$
What is B ?
(a) CH_3CH_2COCl (b) CH_3CH_2CHO
(c) $CH_2 = CHCOOH$ (d) $ClCH_2CH_2COOH$ *(2002)*

23. Which of the following compounds has wrong IUPAC name?
(a) $CH_3 - CH_2 - CH_2 - COO - CH_2CH_3$ → ethyl butanoate
(b) $CH_3 - CH(CH_3) - CH_2 - CHO$ → 3-methylbutanal
(c) $CH_3 - CH(OH) - CH(CH_3) - CH_3$ → 2-methyl-3-butanol
(d) $CH_3 - CH(CH_3) - C(=O) - CH_2 - CH_3$ → 2-methyl-3-pentanone *(2002)*

Answer Key

1.	(a)	**2.**	(d)	**3.**	(a, c)	**4.**	(b)	**5.**	(c)	**6.**	(a)
7.	(b)	**8.**	(d)	**9.**	(d)	**10.**	(c)	**11.**	(d)	**12.**	(c)
13.	(b)	**14.**	(a)	**15.**	(d)	**16.**	(a)	**17.**	(c)	**18.**	(b)
19.	(b)	**20.**	(d)	**21.**	(b)	**22.**	(c)	**23.**	(c)		

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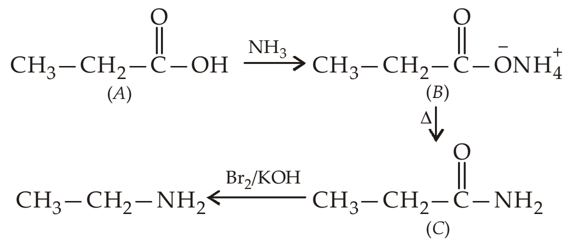
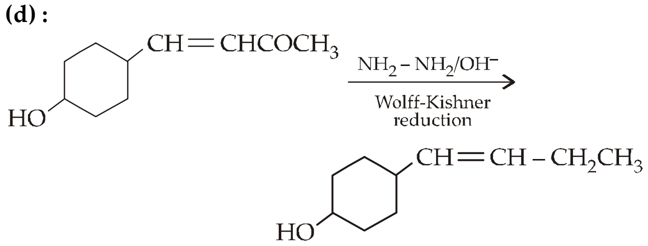
Explanations

1. **(a) :**

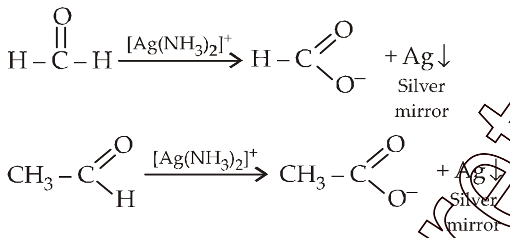


2. **(d) :**

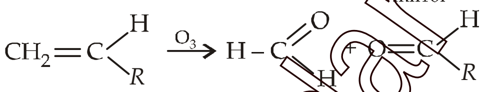


– OH group and alkene are acid-sensitive groups so Clemmensen reduction cannot be used and $NaBH_4$ reduces to – CHOH only.

3. **(a, c) :** Formaldehyde and acetaldehyde can be oxidised by Tollen's reagent to give silver mirror.



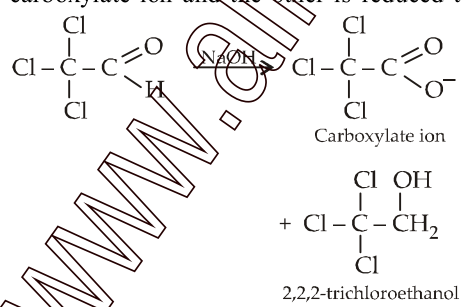
4. **(b) :**



Vinyl group ($CH_2=CH-$) on ozonolysis gives formaldehyde.

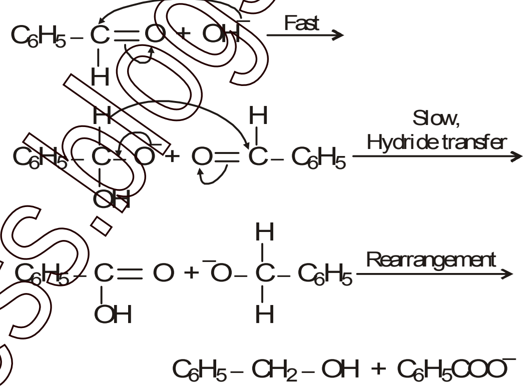
5. **(c) :** $CH_3CH_2CH(Cl)COOH$ is the strongest acid due to $-I$ effect of Cl.

6. **(a) :** In Cannizzaro's reaction one molecule is oxidised to carboxylate ion and the other is reduced to alcohol.



7. **(b) :** Rate determining step is always the slowest step. In case of Cannizzaro reaction, H-transfer to the carbonyl group is the rate determining step and hence the slowest.

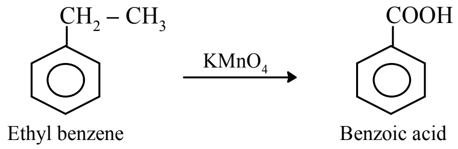
Mechanism :



8. **(d) :** Since the compound formed has a fruity smell, it is an ester, thus the liquid to which ethanol and conc. H_2SO_4 are added must be an acid.

$$\therefore\ CH_3COOH + C_2H_5OH \xrightarrow{\text{conc.}H_2SO_4} CH_3COOC_2H_5 + H_2O.$$

9. **(d) :**



When oxidises with alkaline $KMnO_4$ or acidic $Na_2Cr_2O_7$, the entire side chain (in benzene homologues) with atleast one H at α-carbon, regardless of length is oxidised to – COOH.

10. **(c) :** Effect of substituent on the acid strength of aliphatic acids:

(i) Acidity decreases as the $+I$-effect of the alkyl group increases.

$HCOOH > CH_3COOH > (CH_3)_2CHCOOH > (CH_3)_3CCOOH$

(ii) Acidity decreases as the $-I$-effect as well as number of halogen atoms decreases.

$FCH_2COOH > ClCH_2COOH > BrCH_2COOH > ICH_2COOH > CH_3COOH$

$F_3CCOOH > F_2CHCOOH > FCH_2COOH > CH_3COOH$

(iii) Electron donating substituents like – R, – OH, – NH_2 etc. tend to decrease while electron withdrawing substituents like $-NO_2$, –CHO, etc. tend to increase the acid strength of substituted acid.

On the basis of given information the relative order of increasing acid strength of the given compounds is

$(CH_3)_2COOH < CH_3COOH < CH_3OCH_2COOH < CF_3COOH$

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11. **(d) :** Iodoform test is given by only the compounds containing CH_3CO – or CH_3CHOH – group.

$$PhCHOHCH_3 + 4I_2 + 6NaOH \xrightarrow{\Delta} CHI_3 + PhCOONa + 5NaI + 5H_2O$$

12. **(c) :** Addition of HCN to carbonyl compounds is a characteristic nucleophilic addition reaction of carbonyl compounds.

Order of reactivity:

$$\begin{matrix}H\\H\end{matrix}\!\!>C=O \;>\; \begin{matrix}R\\H\end{matrix}\!\!>C=O \;>\; \begin{matrix}R\\R\end{matrix}\!\!>C=O$$

The lower reactivity of ketones over aldehydes is due to +*I*-effect of the alkyl (*R*) group and steric hindrance. As the size of the alkyl group increases, the reactivity of the ketones further decreases.

$$\begin{matrix}CH_3\\CH_3\end{matrix}\!\!>C=O \;>\; \begin{matrix}(CH_3)_2CH\\(CH_3)_2CH\end{matrix}\!\!>C=O \;>\; \begin{matrix}(CH_3)_3\\(CH_3)_3\end{matrix}\!\!>C=O$$

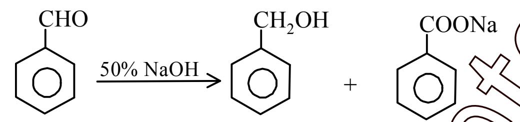
The aromatic aldehydes and ketones are less reactive than their aliphatic analogous. This is due to the +*R* effect of the benzene ring.

$$PhCHO > PhCOCH_3 > PhCOPh$$

From the above information, it is clear that increasing order of the rate of HCN addition to compounds HCHO, CH_3COCH_3, $PhCOCH_3$ and PhCOPh is

$PhCOPh < PhCOCH_3 < CH_3COCH_3 < HCHO$.

13. **(b) :** Benzaldehyde will undergo Cannizzaro reaction on treatment with 50% NaOH to produce benzyl alcohol and benzoic acid as it does not contain α-hydrogen.

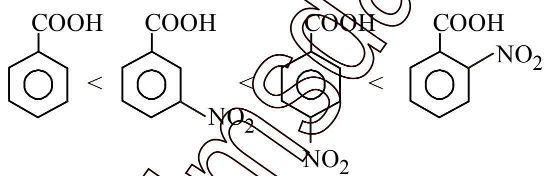


Benzaldehyde Benzyl alcohol Sodium benzoate

14. **(a) :** $CH_3COOC_2H_5 + NaCl_{(aq)} \rightarrow$ no reaction

i.e., the resultant solution contains ethyl acetate and sodium chloride.

15. **(d) :**



Electron withdrawing group increases the acidity of benzoic acid, *o*-isomer will have higher acidity then corresponding *m* and *p* isomer due to *ortho*-effect.

As *M* group (*i.e.* NO_2) at *p*-position have more pronounced electron withdrawing effect than as – NO_2 group at *m*-position (–*I* effect)

∴ Correct order of acidity is ii > iii > iv > i.

16. **(a) :** $R-C(=O)-Z + Nu^{\ominus} \longrightarrow R-C(=O)-Nu + Z^{\ominus}$

Reactivity of the acid derivatives decreases as the basicity of the leaving group increases. The basicity of the leaving group increases as

$$Cl^- < RCOO^- < RO^- < NH_2^-$$

Secondly least stabilization by resonance due to ineffective overlapping between the 3*p* orbital of Cl and 2*p* orbital of carbon.

17. **(c) :** $HCOO^-$ exists as

$$H-\overset{\overset{O}{\|}}{C}-O^- \longleftrightarrow H-\overset{\overset{O^-}{|}}{C}=O$$

So, the two carbon-oxygen bonds are found to be of equal length.

18. **(b) :** Diketones - $C_nH_{2n-2}O_2$, Carboxylic acid - $C_nH_{2n}O_2$

Diols - $C_nH_{3n}O_2$, Dialdehydes - $C_nH_nO_2$.

19. **(b) :** $LiAlH_4$ is a strong reducing agent, it reduces carboxylic group into primary alcoholic group without affecting the basic skeleton of compound.

$$CH_2=CH-COOH \xrightarrow[{[H^+]}]{LiAlH_4} CH_2=CH-CH_2OH$$

20. **(d) :** $H-\overset{H}{\underset{H}{\overset{|}{\underset{|}{C^1}}}}-\overset{O}{\overset{\|}{C^2}}-\overset{H}{\underset{CH_3}{\overset{|}{\underset{|}{C^3}}}}-\overset{H}{\underset{H}{\overset{|}{\underset{|}{C^4}}}}-H$

3-methyl-2-butanone

21. **(b) :** Aldehydic group gets oxidised to carboxylic group.

Double bond breaks and carbon gets oxidised to carboxylic group.

22. **(c) :** $CH_3CH_2COOH \xrightarrow[\text{red P, –HCl}]{Cl_2} \underset{(A)}{CH_3CHClCOOH} \xrightarrow[\text{–HCl}]{\text{alc. KOH}} \underset{(B)}{CH_2=CHCOOH}$

23. **(c) :** $\overset{1}{C}H_3-\underset{OH}{\underset{|}{\overset{2}{C}H}}-\underset{CH_3}{\underset{|}{\overset{3}{C}H}}-\overset{4}{C}H_3$

3-methyl-2-butanol

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CHAPTER 26

ORGANIC COMPOUNDS CONTAINING NITROGEN

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1. A compound with molecular mass 180 is acylated with CH_3COCl to get a compound with molecular mass 390. The number of amino groups present per molecule of the former compound is
(a) 6 (b) 2
(c) 5 (d) 4 *(2013)*

2. In the chemical reaction,
$CH_3CH_2NH_2 + CHCl_3 + 3KOH \rightarrow (A) + (B) + 3H_2O$,
the compounds (*A*) and (*B*) are respectively
(a) C_2H_5NC and 3KCl (b) C_2H_5CN and 3KCl
(c) $CH_3CH_2CONH_2$ and 3KCl
(d) C_2H_5NC and K_2CO_3. *(2007)*

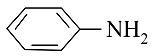
3. Which one of the following is the strongest base in aqueous solution?
(a) Methylamine (b) Trimethylamine
(c) Aniline (d) Dimethylamine *(2007)*

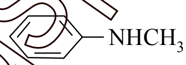
4. An organic compound having molecular mass 60 is found to contain C = 20%, H = 6.67% and N = 46.67% while rest is oxygen. On heating it gives NH_3 alongwith a solid residue. The solid residue gives violet colour with alkaline copper sulphate solution. The compound is
(a) CH_3NCO (b) CH_3CONH_2
(c) $(NH_2)_2CO$ (d) $CH_3CH_2CONH_2$ *(2005)*

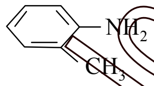
5. Reaction of cyclohexanone with dimethylamine in the presence of catalytic amount of an acid forms a compound if water during the reaction is continuously removed. The compound formed is generally known as
(a) a Schiff's base (b) an enamine
(c) an imine (d) an amine *(2005)*

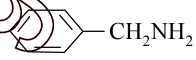
6. Amongst the following the most basic compound is
(a) benzylamine (b) aniline
(c) acetanilide (d) *p*-nitroaniline *(2005)*

7. Which one of the following methods is neither meant for the synthesis nor for separation of amines?
(a) Hinsberg method (b) Hofmann method
(c) Wurtz reaction (d) Curtius reaction *(2005)*

8. Which of the following is the strongest base?
(a) 

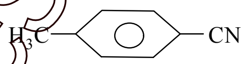
(b) 

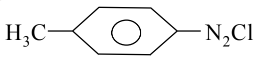
(c) 

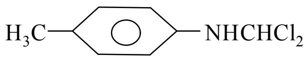
(d) 

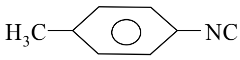
(2004)

9. Which one of the following does not have sp^2 hybridized carbon?
(a) Acetone (b) Acetic acid
(c) Acetonitrile (d) Acetamide *(2004)*

10. The reaction of chloroform with alcoholic KOH and *p*-toluidine forms
(a) 

(b) 

(c) 

(d) 

(2003)

11. Ethyl isocyanide on hydrolysis in acidic medium generates
(a) ethylamine salt and methanoic acid
(b) propanoic acid and ammonium salt
(c) ethanoic acid and ammonium salt
(d) methylamine salt and ethanoic acid. *(2003)*

12. The correct order of increasing basic nature for the bases NH_3, CH_3NH_2 and $(CH_3)_2NH$ is
(a) $CH_3NH_2 < NH_3 < (CH_3)_2NH$
(b) $(CH_3)_2NH < NH_3 < CH_3NH_2$
(c) $NH_3 < CH_3NH_2 < (CH_3)_2NH$
(d) $CH_3NH_2 < (CH_3)_2NH < NH_3$ *(2003)*

13. When primary amine reacts with chloroform in ethanolic KOH then the product is
(a) an isocyanide (b) an aldehyde
(c) a cyanide (d) an alcohol. *(2002)*

Answer Key

1.	(c)	**2.**	(a)	**3.**	(d)	**4.**	(c)	**5.**	(b)	**6.**	(a)
7.	(c)	**8.**	(d)	**9.**	(c)	**10.**	(d)	**11.**	(a)	**12.**	(c)
13.	(a)										

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Explanations

1. **(c) :** No. of amino groups $= \frac{390-180}{42} = 5$

2. **(a) :** $CH_3CH_2NH_2 + CHCl_3 + 3KOH \longrightarrow C_2H_5NC + 3KCl + 3H_2O$
This is called carbylamine reaction.

3. **(d) :** The increasing order of basicity of the given compounds is
$(CH_3)_2NH > CH_3NH_2 > (CH_3)_3N > C_6H_5NH_2$
Due to the $+I$ effect of alkyl groups, the electron density on nitrogen increases and thus the availability of the lone pair of electrons to proton increases and hence the basicity of amines also increases. So aliphatic amines are more basic than aniline. In case of tertiary anine $(CH_3)_3N$, the covering of alkyl groups over nitrogen atom from all sides makes the approach and bonding by a proton relatively difficult, hence the basicity decreases. Electron withdrawing $(C_6H_5 -)$ groups decreases electron density on nitrogen atom and thereby decreasing basicity.

4. **(c) :**

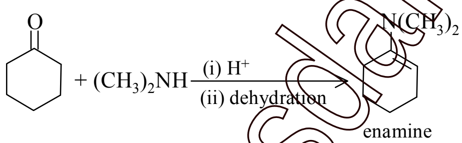
Element ratio	Simplest	Percentage of atom	Relative no.
C	20.00	1.67	1
H	6.67	6.67	4
N	46.67	3.33	2
O	26.66	1.67	1

The molecular formula is CH_4N_2O.
So, the compound is H_2NCONH_2.

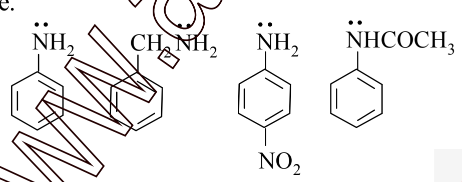
$$2NH_2CONH_2 \xrightarrow{\Delta} \underset{\text{biuret}}{NH_2CONHCONH_2} + NH_3\uparrow$$

Biuret gives violet colour with alkaline copper sulphate solution.

5. **(b) :**



6. **(a) :** Due to resonance of electron pair in aniline, basic strength decreases. In benzylamine electron pair is not involved in resonance. Further the presence of electron donating groups in the benzene ring increase the basic strength while electron withdrawing group decrease the basic strength of substituted aniline.



I II III IV

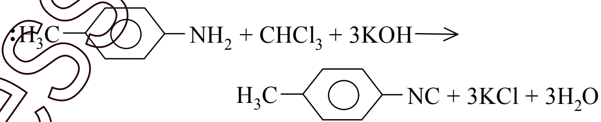
Decreasing order of basic strength is II > I > IV > III.

7. **(c) :** In Wurtz reaction alkyl halide reacts with sodium metal in the presence of dry ether to give alkane.

8. **(d) :** In this compound, the non-bonding electron pair of nitrogen does not take part in resonance. In other three compounds, the non-bonding electron pair of nitrogen is delocalized into benzene ring by resonance, as a result the electron density on the N atom decreases, due to which basicity decreases.

9. **(c) :** $\overset{sp^3}{CH_3} - \overset{sp^2}{CO} - \overset{sp^3}{CH_3}$ (Acetone) ; $\overset{sp^3}{CH_3} - \overset{sp^2}{COOH}$ (Acetic acid)
$\overset{sp^3}{CH_3} - \overset{sp}{C} \equiv N$ (Acetonitrile) ; $\overset{sp^3}{CH_3} - \overset{sp^2}{CONH_2}$ (Acetamide)

10. **(d) :**



The above reaction is known as carbylamine reaction and is generally used to convert primary amine into isocyanide.

11. **(a) :** Alkyl isocyanides are hydrolysed by dilute mineral acids to form primary amines.

$$\underset{\text{Ethylisocyanide}}{C_2H_5N \overrightarrow{=} C} + 2H_2O \xrightarrow{H^+} \underset{\text{Ethylamine}}{C_2H_5NH_2} + \underset{\text{Methanoic acid}}{HCOOH}$$

12. **(c) :** Except the amines containing tertiary butyl group, all lower aliphatic amines are stronger bases than ammonia because of +I (inductive) effect. The alkyl groups, which are electron releasing groups, increase the electron density around the nitrogen thereby increasing the availability of the lone pair of electrons to proton or Lewis acids and making the amine more basic. The observed order in the case of lower members is found to be as secondary > primary > tertiary. This anomalous behaviour of tertiary amines is due to steric factors *i.e.* crowding of alkyl groups cover nitrogen atom from all sides and thus makes it unable for protonation.
Thus the relative strength is in order
$(CH_3)_2NH > CH_3NH_2 > NH_3$.

13. **(a) :** When a primary amine reacts with chloroform with ethanolic KOH, then a bad smell compound isocyanide is formed. This is called carbylamine reaction and this reaction is used as a test of primary amines.

$$\underset{\text{Primary amine}}{RNH_2} + CHCl_3 + 3KOH \xrightarrow{\Delta} \underset{\text{Isocyanide}}{RNC} + 3KCl + 3H_2O$$

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CHAPTER 27

POLYMERS

1. The species which can best serve as an initiator for the cationic polymerization is
(a) HNO_3 (b) $AlCl_3$
(c) BuLi (d) $LiAlH_4$ *(2012)*

2. The polymer containing strong intermolecular forces *e.g.*, hydrogen bonding is
(a) natural rubber (b) teflon
(c) nylon-6,6 (d) polystyrene. *(2010)*

3. Bakelite is obtained from phenol by reaction with
(a) HCHO (b) $(CH_2OH)_2$
(c) CH_3CHO (d) CH_3COCH_3 *(2008)*

4. Which of the following is fully fluorinated polymer?
(a) Neoprene (b) Teflon
(c) Thiokol (d) PVC *(2005)*

5. Which of the following is a polyamide?
(a) Teflon (b) Nylon-6,6
(c) Terylene (d) Bakelite *(2005)*

6. Nylon threads are made of
(a) polyvinyl polymer
(b) polyester polymer
(c) polyamide polymer
(d) polyethylene polymer. *(2003)*

7. Polymer formation from monomers starts by
(a) condensation reaction between monomers
(b) coordinate reaction between monomers
(c) conversion of monomer to monomer ions by protons
(d) hydrolysis of monomers. *(2002)*

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Answer Key

1.	(b)	**2.**	(c)	**3.**	(a)	**4.**	(b)	**5.**	(b)	**6.**	(c)
7.	(a)										

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Explanations

1. **(b) :** Cationic polymerisation is initiated by use of strong Lewis acids such as H_2SO_4, HF, $AlCl_3$, $SnCl_4$ or BF_3 in H_2O.

2. **(c) :** Nylon-6,6 involves amide (CONH) linkage therefore, it will also have very strong inter molecular hydrogen bonding between $>NH \cdots OC<$ group of two polyamide chains.

3. **(a) :** Bakelite is a thermosetting polymer which is made by reaction between phenol and HCHO.

4. **(b) :** Neoprene : $\left[CH_2 - CH = \underset{|}{\overset{Cl}{C}} - CH_2 \right]_n$

Teflon : $\left[CF_2 - CF_2 \right]_n$

Thiokol :

$$-CH_2 \left[CH_2 - S - S - CH_2 \right]_n CH_2 - S - S - CH_2CH_2 -$$

PVC : $\left[CH_2 - \underset{Cl}{\underset{|}{CH}} \right]_n$

5. **(b) :** Polymers having amide linkages (–CONH) are known as polyamides.

$$\underset{\text{Hexamethylene diamine}}{n(H_2N(CH_2)_6NH_2)} + \underset{\text{Adipic acid}}{n(HOOC(CH_2)_4COOH)} \longrightarrow$$

$$\underset{\text{Nylon-6, 6}}{-\!\!\left(HN - (CH_2)_6 - \overset{\text{amide linkage}}{NHCO} - (CH_2)_4 - CO \right)_n}$$

6. **(c) :** Nylon threads are polyamides. They are the condensation polymers of diamines and dibasic acids.

$$\underset{\text{Adipic acid}}{n\,HOOC(CH_2)_4COOH} + \underset{\text{Hexamethylene diamine}}{n\,H_2N(CH_2)_6NH_2} \xrightarrow[\text{high pressure}]{280°C}$$

$$\underset{\text{Nylon (polyamide)}}{HO\left[OC - (CH_2)_4 - CONH(CH_2)_6NH \right]_n}$$

7. **(a) :** Polymerisation takes place either by condensation or addition reactions.

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CHAPTER 28

BIOMOLECULES

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1. Synthesis of each molecule of glucose in photosynthesis involves
   (a) 6 molecules of ATP
   (b) 18 molecules of ATP
   (c) 10 molecules of ATP
   (d) 8 molecules of ATP *(2013)*

2. Which of the following compounds can be detected by Molisch's test?
   (a) Sugars (b) Amines
   (c) Primary alcohols (d) Nitro compounds *(2012)*

3. Which one of the following statements is correct?
   (a) All amino acids are optically active.
   (b) All amino acids except glycine are optically active.
   (c) All amino acids except glutamic acid are optically active.
   (d) All amino acids except lysine are optically active. *(2012)*

4. The presence or absence of hydroxy group on which carbon atom of sugar differentiates RNA and DNA.
   (a) 1^{st} (b) 2^{nd}
   (c) 3^{rd} (d) 4^{th} *(2011)*

5. The two functional groups present in a typical carbohydrate are
   (a) –OH and –COOH
   (b) –CHO and –COOH
   (c) $>C{=}O$ and –OH
   (d) –OH and –CHO *(2009)*

6. α-D-(+)-glucose and β-D-(+)-glucose are
   (a) enantiomers (b) conformers
   (c) epimers (d) anomers. *(2008)*

7. The secondary structure of a protein refers to
   (a) fixed configuration of the polypeptide backbone
   (b) α-helical backbone
   (c) hydrophobic interactions
   (d) sequence of α-amino acids. *(2007)*

8. The pyrimidine bases present in DNA are
   (a) cytosine and adenine
   (b) cytosine and guanine
   (c) cytosine and thymine
   (d) cytosine and uracil. *(2006)*

9. The term anomers of glucose refers to
   (a) isomers of glucose that differ in configurations at carbons one and four (C-1 and C-4)
   (b) a mixture of (D)-glucose and (L)-glucose
   (c) enantiomers of glucose
   (d) isomers of glucose that differ in configuration at carbon one (C-1). *(2006)*

10. In both DNA and RNA, heterocyclic base and phosphate ester linkages are at
    (a) C_5' and C_2' respectively of the sugar molecule
    (b) C_2' and C_5' respectively of the sugar molecule
    (c) C_1' and C_5' respectively of the sugar molecule
    (d) C_5' and C_1' respectively of the sugar molecule *(2005)*

11. Insulin production and its action in human body are responsible for the level of diabetes. This compound belongs to which of the following categories?
    (a) A co-enzyme (b) A hormone
    (c) An enzyme (d) An antibiotic *(2004)*

12. Which base is present in RNA but not in DNA?
    (a) Uracil (b) Cytosine
    (c) Guanine (d) Thymine *(2004)*

13. Identify the correct statement regarding enzymes.
    (a) Enzymes are specific biological catalysts that can normally function at very high temperatures ($T \sim 1000$ K).
    (b) Enzymes are normally heterogeneous catalysts that are very specific in action.
    (c) Enzymes are specific biological catalysts that cannot be poisoned.
    (d) Enzymes are specific biological catalysts that possess well-defined active sites. *(2004)*

14. The reason for double helical structure of DNA is operation of
    (a) van der Waal's forces
    (b) dipole-dipole interaction
    (c) hydrogen bonding
    (d) electrostatic attractions. *(2003)*

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15. Complete hydrolysis of cellulose gives
(a) D-fructose (b) D-ribose
(c) D-glucose (d) L-glucose. *(2003)*

16. The functional group, which is found in amino acid is
(a) – COOH group (b) – NH_2 group
(c) – CH_3 group (d) both (a) and (b). *(2002)*

17. RNA is different from DNA because RNA contains
(a) ribose sugar and thymine
(b) ribose sugar and uracil
(c) deoxyribose sugar and thymine
(d) deoxyribose sugar and uracil. *(2002)*

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Answer Key

1.	(b)	**2.**	(a)	**3.**	(b)	**4.**	(b)	**5.**	(c)	**6.**	(d)
7.	(b)	**8.**	(c)	**9.**	(d)	**10.**	(c)	**11.**	(b)	**12.**	(a)
13.	(d)	**14.**	(c)	**15.**	(c)	**16.**	(d)	**17.**	(b)		

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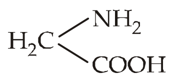
Explanations

1. **(b) :** $6CO_2 + 18ATP + 12NADPH + 6RuBP \longrightarrow 6RuBP + Glucose + 18ADP + 18P + 12NADP^+$
One molecule of glucose is formed from $6CO_2$ by utilising 18ATP and 12NADPH.

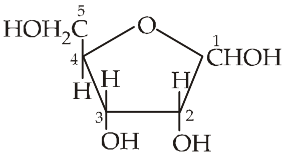
2. **(a) :** Molisch's test is a sensitive chemical test for the presence of carbohydrates, based on the dehydration of carbohydrate by sulphuric acid to produce an aldehyde, which condenses with two molecules of phenol resulting in red or purple coloured compound.

3. **(b) :** Glycine is optically inactive while all other amino acids are optically active.

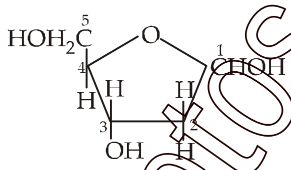


Glycine (optically inactive)

4. **(b) :** The sugar molecule found in RNA is D-ribose while the sugar in DNA is D-2-deoxyribose. The sugar D-2-deoxyribose differs from ribose only in the substitution of hydrogen for an – OH group at 2-position as shown in figure.



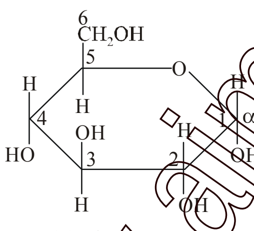
D-ribose (RNA)



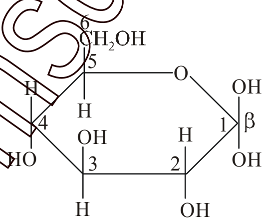
D-2-deoxyribose (DNA)

5. **(c) :** Carbohydrates are essentially polyhydroxy aldehydes and polyhydroxy ketones. Thus the two functional groups present are $>C{=}O$ (aldehyde or ketone) and –OH.

6. **(d) :** Structures of α-D-(+)-glucose and β-D-(+)-glucose are :



α-D-(+)-glucopyranose



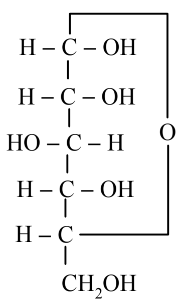
β-D-(+)-glucopyranose

A pair of stereoisomers which differ in configuration at C-1 are known as anomers.

7. **(b) :** Secondary structure of proteins is mainly of two types.
(i) α-helix : This structure is formed when the chain of α-amino acid coils as a right handed screw (called α-helix) because of the formation of hydrogen bonds between amide groups of the same peptide chain.
(ii) β-plated sheet : In this structure the chains are held together by a very large number of hydrogen bonds between $C{=}O$ and NH of different chains.

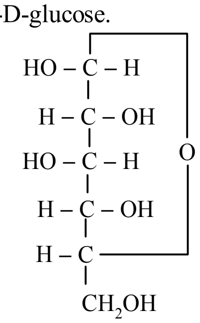
8. **(c) :** DNA contains cytosine and thymine as pyrimidine bases and guanine and adenine as purine bases.

9. **(d) :** Due to cyclic hemiacetal or cyclic hemiketal structures, all the pentoses and hexoses exist in two stereoisomeric forms *i.e.* α form in which the OH at C_1 in aldoses and C_2 in ketoses lies towards the right and β form in which it lies towards left. Thus glucose, fructose, ribose, etc., all exist in α and β form. Glucose exists in two forms α-D-glucose and β-D glucose.
α-D-(+) glucose ⇌ equilibrium mixture ⇌ β-(D)- (+) glucose
As a result of cyclization the anomeric (C-1) becomes asymmetric and the newly formed – OH group may be either on left or on right in Fischer projection thus resulting in the formation of two isomers (anomers). The isomers having – OH group to the left of the C-1 is designated β-D-glucose and other having – OH group on the right as α-D-glucose.



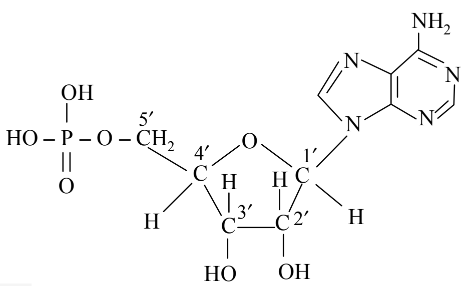
α-D-glucose

and



β-D-glucose

10. **(c) :**



11. **(b) :** Insulin is a proteinaceous hormone secreted by β-cells by islet of Langerhans of pancreas in our body.

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12. **(a) :** RNA contains cytosine and uracil as pyrimidine bases while DNA has cytosine and thymine. Both have the same purine bases *i.e.* guanine and adenine.

13. **(d) :** Enzymes are shape selective specific biological catalysts which normally functions effectively at body temperature.

14. **(c) :** The two polynucleotide chains or strands of DNA are linked up by hydrogen bonding between the nitrogenous base molecules of their nucleotide monomers.

Adenine $=$ Thymine (two hydrogen bonds)   Cytosine $\equiv$ Guanine (three hydrogen bonds)

15. **(c) :** $\underset{\text{Cellulose}}{(C_6H_{10}O_5)_n} + nH_2O \xrightarrow{H^+} \underset{\text{D-glucose}}{nC_6H_{12}O_6}$

Cellulose is a straight chain polysaccharide composed of D-glucose units which are joined by β-glycosidic linkages. Hence cellulose on hydrolysis produces only D-glucose units.

16. **(d) :** An amino acid is a bifunctional organic molecule that contains both a carboxyl group, –COOH, as well as an amino group, $-NH_2$.

17. **(b) :**

		DNA	RNA
(a)	Pyrimidine derivatives	Cytosine Thymine	Cytosine Uracil
(b)	Purine derivatives	Adenine Guanine	Adenine Guanine
(c)	Sugar	Deoxyribose	Ribose

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CHAPTER 29

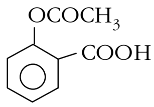
CHEMISTRY IN EVERYDAY LIFE

1. Aspirin in known as
(a) phenyl salicylate (b) acetyl salicylate
(c) methyl salicylic acid (d) acetyl salicylic acid *(2012)*

2. Buna-N synthetic rubber is a co-polymer of
(a) $H_2C=CH-\underset{\displaystyle |}{\overset{Cl}{C}}=CH_2$ and $H_2C=CH-CH=CH_2$
(b) $H_2C=CH-CH=CH_2$ and $H_5C_6-CH=CH_2$
(c) $H_2C=CH-CN$ and $H_2C=CH-CH=CH_2$
(d) $H_2C=CH-CN$ and $H_2C=CH-\underset{Cl}{\underset{|}{C}}=CH_2$ *(2009)*

3. Which one of the following types of drugs reduces fever?
(a) Analgesic
(b) Antipyretic
(c) Antibiotic
(d) Tranquiliser *(2005)*

4. Which of the following could act as a propellant for rockets?
(a) Liquid hydrogen + liquid nitrogen
(b) Liquid oxygen + liquid argon
(c) Liquid hydrogen + liquid oxygen
(d) Liquid nitrogen + liquid oxygen. *(2003)*

5. The compound 

is used as
(a) antiseptic (b) antibiotic
(c) analgesic (d) pesticide. *(2002)*

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Answer Key

1.	(d)	**2.**	(c)	**3.**	(b)	**4.**	(c)	**5.**	(c)

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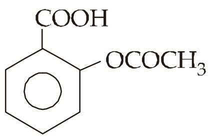
Explanations

1. **(d) :** Aspirin -



Acetyl salicylic acid

2. **(c) :** Buna-N is a co-polymer of butadiene and acrylonitrile.

$$nCH_2{=}CH-CH{=}CH_2 + nCH_2{=}\underset{\text{Acrylonitrile}}{\overset{CN}{\overset{|}{CH}}} \longrightarrow$$
(1,3-Butadiene)

$$-\!\!\left[CH_2-CH{=}CH-CH_2-CH_2-\overset{CN}{\overset{|}{CH}}\right]\!\!-_n$$
Buna-N

3. **(b) :** An antipyretic is a drug which is responsible for lowering temperature of the feverish organism to normal but has no effect on normal temperature states.

4. **(c) :** Liquid hydrogen (because of its low mass and high enthalpy of combustion) and liquid oxygen (as it is a strong supporter of combustion) are used as an excellent fuel for rockets.

5. **(c) :** The compound is acetyl salicylic acid (Aspirin). Drugs which relieve or decrease pain are termed analgesics.

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CHAPTER 30

PRINCIPLES RELATED TO PRACTICAL CHEMISTRY

1. Which of the following reagents may be used to distinguish between phenol and benzoic acid?
   (a) Aqueous NaOH (b) Tollen's reagent
   (c) Molisch reagent (d) Neutral $FeCl_3$ *(2011)*

2. Biuret test is not given by
   (a) proteins (b) carbohydrates
   (c) polypeptide (d) urea *(2010)*

3. The compound formed in the positive test for nitrogen with the Lassaigne solution of an organic compound is
   (a) $Fe_4[Fe(CN)_6]_3$
   (b) $Na_3[Fe(CN)_6]$
   (c) $Fe(CN)_3$
   (d) $Na_4[Fe(CN)_5NOS]$. *(2004)*

Answer Key

1. (d) 2. (b) 3. (a)

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Explanations

1. **(d) :** Phenol gives violet colouration with neutral ferric chloride solution.
Benzoic acid gives buff coloured (pale dull yellow) precipitate with neutral ferric chloride solution.

2. **(b) :** Biuret test is used to characterise the presence of —CONH group in a compound.

3. **(a) :** $3Na_4[Fe(CN)_6] + 4Fe^{3+} \rightarrow \underset{\text{Prussian blue}}{Fe_4[Fe(CN)_6]_3} + 12Na^+$

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MATHEMATICS

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CHAPTER 1

SETS, RELATIONS AND FUNCTIONS

1. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is
(a) 211 (b) 256 (c) 220 (d) 219 *(2013)*

2. Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty is
(a) 2^5 (b) 5^3 (c) 5^2 (d) 3^5 *(2012)*

3. Let R be the set of real numbers.
Statement-1 : $A = \{(x, y) \in R \times R : y - x$ is an integer$\}$ is an equivalence relation on R.
Statement-2 : $B = \{(x, y) \in R \times R : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation on R.
(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2011)*

4. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is
(a) $(-\infty, 0)$ (b) $(-\infty, \infty) - \{0\}$
(c) $(-\infty, \infty)$ (d) $(0, \infty)$ *(2011)*

5. Consider the following relations:
$R = \{(x, y) | x, y$ are real numbers and $x = wy$ for some rational number $w\}$;
$S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) \middle| m, n, p \text{ and } q \text{ are integers such that } n, q \neq 0 \text{ and } qm = pn\right\}$.
Then
(a) R is an equivalence relation but S is not an equivalence relation
(b) neither R nor S is an equivalence relation
(c) S is an equivalence relation but R is not an equivalence relation
(d) R and S both are equivalence relations *(2010)*

6. Let $f(x) = (x + 1)^2 - 1$, $x \geq -1$.
Statement-1 : The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$.
Statement-2 : f is bijection.
(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2009)*

7. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then
(a) $A = C$ (b) $B = C$
(c) $A \cap B = \phi$ (d) $A = B$ *(2009)*

8. For real x, let $f(x) = x^3 + 5x + 1$, then
(a) f is onto R but not one-one
(b) f is one-one and onto R
(c) f is neither one-one nor onto R
(d) f is one-one but not onto R *(2009)*

9. Let R be the real line. Consider the following subsets of the plane $R \times R$:
$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$
$T = \{(x, y) : x - y \text{ is an integer}\}$.
Which one of the following is true?
(a) T is an equivalence relation on R but S is not
(b) Neither S nor T is an equivalence relation on R
(c) Both S and T are equivalence relations on R
(d) S is an equivalence relation on R but T is not *(2008)*

10. Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$ where
$Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$.
Show that f is invertible and its inverse is
(a) $g(y) = \frac{y-3}{4}$ (b) $g(y) = \frac{3y+4}{3}$
(c) $g(y) = 4 + \frac{y+3}{4}$ (d) $g(y) = \frac{y+3}{4}$ *(2008)*

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11. The set $S = \{1, 2, 3, ..., 12\}$ is to be partitioned into three sets A, B, C of equal size. Thus $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is

(a) $\frac{12!}{(4!)^3}$ (b) $\frac{12!}{(4!)^4}$

(c) $\frac{12!}{3!(4!)^3}$ (d) $\frac{12!}{3!(4!)^4}$. *(2007)*

12. Let W denote the words in the English dictionary. Define the relation R by :
$R = \{(x, y) \in W \times W|$ the words x and y have at least one letter in common$\}$.
Then R is

(a) not reflexive, symmetric and transitive
(b) reflexive, symmetric and not transitive
(c) reflexive, symmetric and transitive
(d) reflexive, not symmetric and transitive. *(2006)*

13. Let $R = \{(3, 3) (6, 6) (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$ be a relation on the set $A = \{3, 6, 9, 12\}$. The relation is

(a) reflexive and symmetric only
(b) an equivalence relation
(c) reflexive only
(d) reflexive and transitive only. *(2005)*

14. Let $f : (-1, 1) \to B$, be a function defined by
$f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$,
then f is both one-one and onto when B is the interval

(a) $\left[0, \frac{\pi}{2}\right)$ (b) $\left(0, \frac{\pi}{2}\right)$

(c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. *(2005)*

15. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

Interval	**Function**
(a) $[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(b) $(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$
(c) $(-\infty, -4]$	$x^3 + 6x^2 + 6$
(d) $\left(-\infty, \frac{1}{3}\right]$	$3x^2 - 2x + 1$

(2005)

16. A real valued function $f(x)$ satisfies the functional equation $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$ where a is a given constant and $f(0) = 1$. $f(2a - x)$ is equal to

(a) $f(x)$ (b) $-f(x)$
(c) $f(-x)$ (d) $f(a) + f(a - x)$. *(2005)*

17. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is

(a) not symmetric (b) transitive
(c) a function (d) reflexive. *(2004)*

18. The range of the function $F(x) = {}^{7-x}P_{x-3}$ is

(a) $\{1, 2, 3, 4\}$ (b) $\{1, 2, 3, 4, 5, 6\}$
(c) $\{1, 2, 3\}$ (d) $\{1, 2, 3, 4, 5\}$. *(2004)*

19. If $f : R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is

(a) $[0, 1]$ (b) $[-1, 1]$
(c) $[0, 3]$ (d) $[-1, 3]$. *(2004)*

20. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then

(a) $f(x) = f(-x)$ (b) $f(2 + x) = f(2 - x)$
(c) $f(x + 2) = f(x - 2)$ (d) $f(x) = -f(-x)$. *(2004)*

21. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

(a) $[1, 2]$ (b) $[2, 3)$ (c) $[2, 3]$ (d) $[1, 2)$. *(2004)*

22. The function $f(x) = \log(x + \sqrt{x^2+1})$ is

(a) an odd function
(b) a periodic function
(c) neither an even nor an odd function
(d) an even function. *(2003)*

23. A function f from the set of natural numbers to integers defined by $f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\frac{n}{2}, & \text{when } n \text{ is even} \end{cases}$ is

(a) onto but not one-one
(b) one-one and onto both
(c) neither one-one nor onto
(d) one-one but not onto. *(2003)*

24. Domain of definition of the function
$f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$, is

(a) $(-1, 0) \cup (1, 2)$
(b) $(1, 2) \cup (2, \infty)$
(c) $(-1, 0) \cup (1, 2) \cup (2, \infty)$
(d) $(1, 2)$. *(2003)*

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25. If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is

(a) $\frac{7(n+1)}{2}$ (b) $7n(n + 1)$

(c) $\frac{7n(n+1)}{2}$ (d) $\frac{7n}{2}$. *(2003)*

26. Which one is not periodic?

(a) $|\sin 3x| + \sin^2 x$ (b) $\cos\sqrt{x} + \cos^2 x$

(c) $\cos 4x + \tan^2 x$ (d) $\cos 2x + \sin x$. *(2002)*

27. The period of $\sin^2\theta$ is

(a) π^2 (b) π (c) π^3 (d) $\pi/2$. *(2002)*

28. The domain of $\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$ is

(a) [1, 9] (b) [–1, 9]

(c) [–9, 1] (d) [–9, –1]. *(2002)*

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Answer Key

1. (d)	**2.** (d)	**3.** (a)	**4.** (a)	**5.** (c)	**6.** (b)
7. (b)	**8.** (b)	**9.** (a)	**10.** (a)	**11.** (a)	**12.** (b)
13. (d)	**14.** (c)	**15.** (d)	**16.** (b)	**17.** (a)	**18.** (c)
19. (d)	**20.** (c)	**21.** (b)	**22.** (a)	**23.** (b)	**24.** (c)
25. (c)	**26.** (b)	**27.** (b)	**28.** (a)		

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Explanations

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1. **(d) :** $A \times B$ will have $2 \times 4 = 8$ elements.
The number of subsets having atleast 3 elements
$= {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8$
$= 2^8 - ({}^8C_0 + {}^8C_1 + {}^8C_2) = 256 - 1 - 8 - 28 = 219$

2. **(d) :** $X = \{1, 2, 3, 4, 5\}$; $Y \subseteq X$, $Z \subseteq X$, $Y \cap Z = \phi$
Number of ways $= 3^5$.

3. **(a) :** $y - x$ = integer and $z - y$ = integer
$\Rightarrow \quad z - x$ = integer
$\therefore \quad (x, y) \in A$ and $(y, z) \in A \Rightarrow (x, z)$
$\Rightarrow$ Transitive
Also $(x, x) \in A$ is true $\Rightarrow$ Reflexive
As $(x, y) \in A \Rightarrow (y, x) \Rightarrow$ Symmetric
Hence A is a equivalence relation but B is not.
$(0, y)$ is in B but $(y, 0)$ is not in B.

4. **(a) :** $f(x) = \dfrac{1}{\sqrt{|x| - x}}$
$|x| - x > 0 \Rightarrow |x| > x$
Thus x must be –ve. $\therefore \quad x \in (-\infty, 0)$.

5. **(c) :** We have $(x, x) \in R$ for $w = 1$ implying that R is reflexive.
For $a \neq 0$, $(a, 0) \notin R$ for any w but $(0, a) \in R$. Thus R is not symmetric.
Hence R is not an equivalence relation.
As $\left(\frac{m}{n}, \frac{m}{n}\right) \in S$ since $mn = mn$, S is reflexive.
$\left(\frac{m}{n}, \frac{p}{q}\right) \in S \Rightarrow qm = pn$
But this can be written as $np = mq$,
giving $\left(\frac{p}{q}, \frac{m}{n}\right) \in S$. Thus S is symmetric.
Again, $\left(\frac{m}{n}, \frac{p}{q}\right) \in S$ and $\left(\frac{p}{q}, \frac{a}{b}\right) \in S$
means $qm = pn$ and $bp = aq$.
i.e. $\frac{m}{n} = \frac{p}{q}$ and $\frac{p}{q} = \frac{a}{b}$ i.e. $\frac{m}{n} = \frac{a}{b}$
Thus $\left(\frac{m}{n}, \frac{a}{b}\right) \in S$
This means S is transitive.

6. **(b) :** The solution of $f(x) = f^{-1}(x)$ are given by
$f(x) = x$, which gives $(x + 1)^2 - 1 = x$
$\Rightarrow (x + 1)^2 - (x + 1) = 0 \Rightarrow (x + 1)x = 0$
$x = -1, 0$
But as no co-domain of f is specified, nothing can be said about f being ONTO or not.

7. **(b) :** Let $x \in C$
Suppose $x \in A \Rightarrow x \in A \cap C$
$\Rightarrow \quad x \in A \cap B \quad (\because A \cap C = A \cap B)$
Thus $x \in B$
Again suppose $x \notin A \Rightarrow x \in C \cup A$
$\Rightarrow x \in B \cup A \Rightarrow x \in B$
Thus in both cases $x \in C \Rightarrow x \in B$
Hence $C \subseteq B$ ....(1)
Similarly we can show that $B \subseteq C$ ....(2)
Combining (1) and (2) we get $B = C$.

8. **(b) :** The function is $f : R \to R$
$f(x) = x^3 + 5x + 1$
Let $y \in R$ then $y = x^3 + 5x + 1$
$\Rightarrow \quad x^3 + 5x + 1 - y = 0$
As a polynomial of odd degree has always at least one real root, corresponding to any $y \in$ co-domain there $\exists$ some $x \in$ domain such that $f(x) = y$. Hence f is ONTO.
Also f is continuous on R, because it's a polynomial function
$f'(x) = 3x^2 + 5 > 0$
$\therefore \quad f$ is strictly increasing
Hence f is one-one also.

9. **(a) :** To be an equivalence relation the relation must be all – reflexive, symmetric and transitive.
$T = \{(x, y) : x - y \in Z\}$ is
reflexive – for $(x, x) \in Z$ i.e. $x - x = 0 \in Z$
symmetric – for $(x, y) \in Z \Rightarrow x - y \in Z$
$\Rightarrow \quad y - x \in Z$ i.e. $(y, x) \in Z$
transitive – for $(x, y) \in Z$ and $(y, w) \in Z$
$\Rightarrow \quad x - y \in Z$ and $y - w \in Z$, giving
$x - w \in Z$ i.e. $(x, w) \in Z$.
$\therefore \quad T$ is an equivalence relation on R.
$S = \{(x, y) : y = x + 1, 0 < x < 2\}$ is not reflexive for $(x, x) \in S$ would imply $x = x + 1$
$\Rightarrow \quad 0 = 1$ (impossible)
Thus S is not an equivalence relation

10. **(a) :** Let $f(x_1) = f(x_2)$, $x_1, x_2 \in N$
$\Rightarrow \quad 4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$
Thus $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. Hence the function is one-one. Let $y \in Y$ be a number of the form $y = 4k + 3$, for some $k \in N$, then $y = f(x)$
$\Rightarrow \quad 4k + 3 = 4x + 3 \Rightarrow x = k \in N$
Thus corresponding to any $y \in Y$ we have $x \in N$. The function then is onto.
The function, being both one-one and onto is invertible.
$y = 4x + 3 \Rightarrow x = \dfrac{y-3}{4} \quad \therefore \quad f^{-1}(x) = \dfrac{x-3}{4}$
or $g(y) = \dfrac{y-3}{4}$ is the inverse of the function.

11. **(a) :** Number of ways $= {}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4 = \dfrac{12!}{(4!)^3}$.

12. **(b) :** Given relation R such that
$R = \{(x, y) \in W \times W \mid$ the word x and y have atleast one letter in common$\}$
where W denotes set of words in English dictionary
Clearly $(x, x) \in R\ \forall\ x \in W$
$\therefore$ (x, x) has every letter common $\therefore$ R is reflexive
Let $(x, y) \in R$ then $(y, x) \in R$ as x and y have atleast one letter in common. $\Rightarrow$ R is symmetric.
But R is not transitive
$\therefore$ Let x = DON, y = NEST, z = SHE
then $(x, y) \in R$ and $(y, z) \in R$. But $(x, z) \notin R$.
$\therefore$ R is reflexive, symmetric but not transitive.

13. **(d) :** For $(3, 9) \in R$, $(9, 3) \notin R$
$\therefore$ relation is not symmetric which means our choice (a) and (b) are out of court. We need to prove reflexivity and transitivity.
For reflexivity $a \in R$, $(a, a) \in R$ which is hold *i.e.* R is reflexive.
Again,
for transitivity of $(a, b) \in R$, $(b, c) \in R$
$\Rightarrow (a, c) \in R$
which is also true in $R = \{(3, 3)(6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$.

14. **(c) :** For $x \in (-1, 1)$ we have

$$f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\therefore\ f(\tan\theta) = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) \quad (\text{By } x = \tan\theta)$$

$$= \tan^{-1}\tan 2\theta = 2\tan^{-1}x$$

$$\Rightarrow -\frac{\pi}{2} < \tan^{-1}\left(\frac{2x}{1-x^2}\right) < \frac{\pi}{2}.$$

15. **(d) :** $f(x) = x^3 + 6x^2 + 6$

$f'(x) = 3x^2 + 12x = 3x(x+4)$

$f'(x) > 0 \Rightarrow x < -4 \cup x > 0$

the interval $x < -4$ *i.e* $(-\infty, -4]$ matched correctly
and after checking others we find that $f(x) = 3x^2 - 2x + 1$
$\Rightarrow f'(x) > 0$ for $x > 1/3$ which is not given in the choice.

16. **(b) :** Given $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$ ...(*)
let $x = 0 = y$
$f(0) = (f(0))^2 - (f(a))^2$
$1 = 1 - (f(a))^2 \Rightarrow f(a) = 0$
$\therefore$ $f(2a - x) = f(a - (x - a))$
$= f(a)f(x - a) - f(a + x - a)f(0)$
By using (*)
$= 0 - f(x)(1) = -f(x)$
$(\because f(a) = 0,\ f(0) = 1)$

17. **(a) :** R is a function as $A = \{1, 2, 3, 4\}$ and $(2, 4) \in R$ and $(2, 3) \in R$
R is not reflexive as $(1, 1) \notin R$
R is not symmetric as $(2, 3) \in R$ but $(3, 2) \notin R$
R is not transitive as $(1, 3) \in R$ and $(3, 1) \in R$ but $(1, 1) \notin R$.

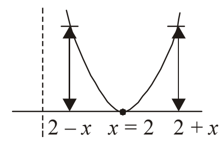
18. **(c) :** $F(x)$ to be defined for $x \in N$.
(i) $\therefore\ 7 - x > 0 \Rightarrow x < 7$
(ii) $x - 3 \geq 0 \Rightarrow x \geq 3$
(iii) $x - 3 \leq 7 - x \Rightarrow x \leq 5$
$\therefore$ from (i), (ii), (iii)
$x = 3, 4, 5$
$\therefore$ $F(3) = {}^4P_0$, $F(4) = {}^3P_1$, $F(5) = {}^2P_2$
$\therefore$ $\{1, 2, 3\}$ is required range

19. **(d):** Let $f(x) = g(x) + 1$

$$\text{where } g(x) = 2\left[\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x\right]$$

$= 2\sin(x - 60°)$
$\therefore$ $-2 \leq 2\sin(x - 60°) \leq 2$
$-1 \leq 2\sin(x - 60°) + 1 \leq 3$

20. **(c):** If $y = f(x)$ is symmetrical about the line $x = \alpha$ then $f(x + \alpha) = f(x - \alpha)$



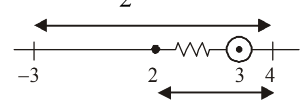
$\therefore$ $f(x + 2) = f(x - 2)$

21. **(b) :** $f(x) = \dfrac{p(x)}{q(x)}$ (say)

then Domain of $f(x)$ is $D_f\, p(x) \cap D_f\, q(x)$, $q(x) \neq 0$

now D_f of $p(x)$ is $-\dfrac{\pi}{2} \leq \sin^{-1}(x - 3) \leq \dfrac{\pi}{2}$

$\Rightarrow -\sin\dfrac{\pi}{2} \leq x - 3 \leq \sin\dfrac{\pi}{2}$



$\Rightarrow\ 2 \leq x \leq 4$ ...(i)
Again $9 - x^2 > 0 \Rightarrow x^2 < 9$
$|x| < 3$
i.e. $-3 < x < 3$ ...(ii)
From (i) and (ii) we have
$\therefore$ $2 \leq x < 3$ is correct Domain

22. **(a) :** $f(x) = \log\left[\sqrt{x^2+1} + x\right]$

$$\therefore\ f(-x) = \log\left[\sqrt{1+x^2} - x\right]$$

$$= -\log\left[\frac{1}{\sqrt{1+x^2} - x}\right] = -\log\left[\frac{\sqrt{1+x^2}+x}{1}\right]$$

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$= -f(x) \Rightarrow f(x) + f(-x) = 0$

$\Rightarrow f(x)$ is an odd function.

23. **(b) :** If n is odd, let $n = 2k - 1$

Let $f(2k_1 - 1) = f(2k_2 - 1)$

$$\Rightarrow \frac{2k_1 - 1 - 1}{2} = \frac{2k_2 - 1 - 1}{2}$$

$\Rightarrow k_1 = k_2$

$\Rightarrow f(n)$ is one-one functions if n is odd

Again, If $n = 2k$ (i.e. n is even)

Let $f(2k_1) = f(2k_2)$

$$\Rightarrow -\frac{2k_1}{2} = -\frac{2k_2}{2}$$

$\Rightarrow k_1 = k_2$

$\Rightarrow f(n)$ is one-one if n is even

Again $f(n) = \dfrac{n-1}{2}$

$f'(n) = \dfrac{1}{2} > 0 \ \forall n \in N$ if n is odd

and $f'(n) = \dfrac{-1}{2} < 0 \ \forall n \in N$ if n is even

Now all such function which are either increasing or decreasing in the stated domain are said to be onto function. Finally $f(n)$ is one-one onto function.

24. **(c) :** Let $g(x) = \dfrac{3}{4 - x^2}$ $\quad \therefore x \neq \pm 2$

$\therefore D_f\, g(x) = R - \{-2, 2\}$

$f(x) = \log_{10}(x^3 - x)$ $\quad \therefore x^3 - x > 0$

$x(x + 1)(x - 1) > 0$

```
 –   +   –   +
 ----+---+---+----
    –1   0   1
```

$\therefore x \in (-1, 0) \cup (1, \infty)$

$\therefore$ Domain of $f(x)$ is $(-1, 0) \cup (1, 2) \cup (2, \infty)$

25. **(c) :** Let $x = 0 = y$ $\quad \therefore f(0) = 0$

and $x = 1, y = 0$ $\quad \therefore f(1 + 0) = f(1) + f(0) = 7$ (given)

$x = 1, y = 1$ $\quad \therefore f(1 + 1) = 2f(1) = 2(7)$

$\therefore f(2) = 2(7)$

$x = 1, y = 2$ $\quad \therefore f(3) = f(1) + f(2)$

$= 7 + 2(7) = 3(7)$

and so on.

$$\therefore \sum_{r=1}^{n} f(r) = f(1) + f(2) + f(3) + ... + f(n)$$

$$= 7(1 + 2 + 3 + ... + n) = \frac{7n(n+1)}{2}$$

26. **(b) :** Period of $|\sin 3x|$ is $\dfrac{\pi}{3}$ and period of $\sin^2 x$ is π

(a) Same as the period of $|\sin x|$ or $\dfrac{1 - \cos 2x}{2}$ whose period is π

Now period of $|\sin 3x| + \sin^2 x$ is the L.C.M of their periods

$\therefore$ L.C.M of $\left(\dfrac{\pi}{3}, \pi\right) = \dfrac{\text{LCM } (\pi, \pi)}{\text{HCF } (3, 1)} = \pi$

(c, d) Similarly we can say that $\cos 4x + \tan^2 x$ and $\cos 2x + \sin x$ are periodic function.

(b) Now $\cos^2 x$ is periodic with period π and for period of $\cos\sqrt{x}$ let us take.

$f(x) = \cos\sqrt{x}$

Let $f(x + T) = f(x)$

$\Rightarrow \cos\sqrt{T + x} = \cos\sqrt{x}$

$\Rightarrow \sqrt{T + x} = 2n\pi \pm \sqrt{x}$

which gives no value of T independent of x

$\therefore f(x)$ cannot be periodic

Now say $g(x) = \cos^2 x + \cos\sqrt{x}$ which is sum of a periodic and non periodic function and such function have no period.

So, $\cos\sqrt{x} + \cos^2 x$ is non periodic function.

27. **(b) :** Let $f(\theta) = \sin^2\theta = |\sin\theta|$

Period of $|\sin\theta|$ is π

28. **(a) :** If $y = \sin^{-1} a$, then $-1 \leq a \leq 1$

$\therefore -1 \leq \log_3\left(\dfrac{x}{3}\right) \leq 1$ $\quad \left[\text{as } y = \sin^{-1}\left[\log_3\left(\dfrac{x}{3}\right)\right]\right]$

$\Rightarrow \dfrac{1}{3} \leq \dfrac{x}{3} \leq 3^1$

$\Rightarrow 1 \leq x \leq 9$

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CHAPTER 2

COMPLEX NUMBERS

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1. If z is a complex number of unit modulus and argument θ, then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals

(a) $\frac{\pi}{2}-\theta$ (b) θ (c) $\pi - \theta$ (d) $-\theta$

(2013)

2. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies

(a) either on the real axis or on a circle not passing through the origin.
(b) on the imaginary axis.
(c) either on the real axis or on a circle passing through the origin.
(d) on a circle with centre at the origin. *(2012)*

3. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals

(a) (1, 0) (b) (–1, 1)
(c) (0, 1) (d) (1, 1) *(2011)*

4. The number of complex numbers z such that $|z-1| = |z+1| = |z-i|$ equals

(a) 0 (b) 1 (c) 2 (d) ∞

(2010)

5. If $\left|Z-\frac{4}{Z}\right| = 2$, then the maximum value of $|Z|$ is equal to

(a) $\sqrt{5}+1$ (b) 2 (c) $2+\sqrt{2}$ (d) $\sqrt{3}+1$

(2009)

6. The conjugate of a complex number is $\frac{1}{i-1}$. Then that complex number is

(a) $\frac{1}{i-1}$ (b) $\frac{-1}{i-1}$ (c) $\frac{1}{i+1}$ (d) $\frac{-1}{i+1}$

(2008)

7. If $|z+4| \leq 3$, then the maximum value of $|z+1|$ is

(a) 6 (b) 0 (c) 4 (d) 10.

(2007)

8. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11}+i\cos\frac{2k\pi}{11}\right)$ is

(a) i (b) 1 (c) –1 (d) $-i$.

(2006)

9. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of $\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+\ldots+\left(z^6+\frac{1}{z^6}\right)^2$ is

(a) 18 (b) 54 (c) 6 (d) 12.

(2006)

10. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to

(a) $-\pi$ (b) $\pi/2$ (c) $-\pi/2$ (d) 0.

(2005)

11. If $\omega = \frac{z}{z-(1/3)i}$ and $|\omega| = 1$, then z lies on

(a) a circle (b) an ellipse
(c) a parabola (d) a straight line. *(2005)*

12. If the cube roots of unity are 1, ω, ω^2 then the roots of the equation $(x-1)^3 + 8 = 0$, are

(a) –1, –1, –1
(b) $-1, -1+2\omega, -1-2\omega^2$
(c) $-1, 1+2\omega, 1+2\omega^2$
(d) $-1, 1-2\omega, 1-2\omega^2$. *(2005)*

13. Let z, ω be complex numbers such that $\bar{z}+i\bar{\omega} = 0$ and $z\omega = \pi$. Then arg z equals

(a) $3\pi/4$ (b) $\pi/2$ (c) $\pi/4$ (d) $5\pi/4$.

(2004)

14. If $z = x - iy$ and $z^{1/3} = p + iq$, then $\frac{\left(\frac{x}{p}+\frac{y}{q}\right)}{(p^2+q^2)}$ is equal to

(a) 2 (b) –1 (c) 1 (d) –2.

(2004)

15. If $|z^2 - 1| = |z|^2 + 1$, then z lies on

(a) a circle (b) the imaginary axis
(c) the real axis (d) an ellipse. *(2004)*

16. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then

(a) $x = 2n$, where n is any positive integer
(b) $x = 4n+1$, where n is any positive integer
(c) $x = 2n+1$, where n is any positive integer
(d) $x = 4n$, where n is any positive integer. *(2003)*

17. If z and ω are two non-zero complex numbers such that $|z\omega| = 1$, and $\text{Arg}(z) - \text{Arg}(\omega) = \pi/2$, then $\bar{z}\omega$ is equal to

(a) –1 (b) i (c) $-i$ (d) 1.

(2003)

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18. Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex further, assume that the origin, z_1 and z_2 form an equilateral triangle, then

(a) $a^2 = 2b$ (b) $a^2 = 3b$
(c) $a^2 = 4b$ (d) $a^2 = b$. *(2003)*

19. z and ω are two nonzero complex number such that $|z| = |\omega|$ and Arg z + Arg $\omega = \pi$ then z equals

(a) $\bar{\omega}$ (b) $-\bar{\omega}$ (c) ω (d) $-\omega$ *(2002)*

20. If $|z-4| < |z-2|$, its solution is given by

(a) $\text{Re}(z) > 0$ (b) $\text{Re}(z) < 0$
(c) $\text{Re}(z) > 3$ (d) $\text{Re}(z) > 2$. *(2002)*

21. The locus of the centre of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally (z, z_1 & z_2 are complex numbers) will be

(a) an ellipse (b) a hyperbola
(c) a circle (d) none of these *(2002)*

Answer Key

1.	(b)	**2.**	(c)	**3.**	(d)	**4.**	(b)	**5.**	(a)	**6.**	(d)
7.	(a)	**8.**	(d)	**9.**	(d)	**10.**	(d)	**11.**	(d)	**12.**	(d)
13	(a)	**14.**	(d)	**15.**	(b)	**16.**	(d)	**17.**	(c)	**18.**	(b)
19.	(b)	**20.**	(c)	**21.**	(b)						

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Explanations

1. **(b) :** Note that $\frac{1+z}{1+\bar{z}} = \frac{1+z}{1+\frac{1}{z}} = z$

Observe that $|z^2| = 1 = z\bar{z}$

Then the arg of the number $\frac{1+z}{1+\bar{z}}$ is just the argument of z and that's θ.

2. **(c) :** $z \ne 1$, $\frac{z^2}{z-1}$ is real.

If z is a real number, then $\frac{z^2}{z-1}$ is real.

Let $z = x + iy$

$\therefore \frac{(x^2-y^2+2xiy)((x-1)-iy)}{(x-1)^2+y^2}$ is real

$\Rightarrow -y(x^2-y^2) + 2xy(x-1) = 0$

$\Rightarrow y(x^2+y^2-2x) = 0 \Rightarrow y = 0$ or $x^2+y^2-2x = 0$

$\therefore$ z lies on real axis or on a circle passing through origin.

3. **(d) :** $(1+\omega)^7 = (-\omega^2)^7 = -\omega^{14} = -\omega^{12}\omega^2$

$= -\omega^2 = 1 + \omega = A + B\omega$ given

Hence on comparison, we have $(A, B) = (1, 1)$.

4. **(b) : 1st solution :**

$|z-1| = |z+1| = |z-i|$ reads that the distance of desired complex number z is same from three points in the complex plane -1, 1 and i. These points are non-collinear, hence the desired number is the centre of the (unique) circle passing through these three non-collinear points.

2nd solution :

We resort to definition of modulus.

$|z-1| = |z+1| \Rightarrow |z-1|^2 = |z+1|^2$

$\Rightarrow (z-1)(\bar{z}-1) = (z+1)(\bar{z}+1)$

$\Rightarrow z\bar{z} - z - \bar{z} + 1 = z\bar{z} + z + \bar{z} + 1$

$\Rightarrow z + \bar{z} = 0$ (z being purely imaginary)

Thus $x = 0$

Again, $|z-1|^2 = |z-i|^2$

$\Rightarrow (x-1)^2 + y^2 = x^2 + (y-1)^2$

$\Rightarrow 1 + y^2 = (y-1)^2$ (because $x = 0$)

$\Rightarrow 1 + y^2 = y^2 - 2y + 1$

$\therefore y = 0$

Thus, (0, 0) is the desired point.

5. **(a) :** We have for any two complex numbers α and β

$||\alpha| - |\beta|| \le |\alpha - \beta|$

Now $\left||Z| - \frac{4}{|z|}\right| \le \left|Z - \frac{4}{Z}\right| \Rightarrow \left||Z| - \frac{4}{|Z|}\right| \le 2$

Set $|Z| = r > 0$, then $\left|r - \frac{4}{r}\right| \le 2$

$\Rightarrow -2 \le r - \frac{4}{r} \le 2$

The left inequality gives

$r^2 + 2r - 4 \ge 0$

The corresponding roots are

$r = \frac{-2 \pm \sqrt{20}}{2} = \pm\sqrt{5}$

Thus $r \ge \sqrt{5} - 1$ or $r \le -1 - \sqrt{5}$

implies that $r \ge \sqrt{5} - 1$ (As $r > 0$) ...(i)

Again consider the right inequality

$r - \frac{4}{r} \le 2 \Rightarrow r^2 - 2r - 4 \le 0$

The corresponding roots are

$r = \frac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}$

Thus $1 - \sqrt{5} \le r \le 1 + \sqrt{5}$

But $r > 0$, hence $r \le 1 + \sqrt{5}$ ...(ii)

(i) and (ii) gives $\sqrt{5} - 1 \le r \le \sqrt{5} + 1$

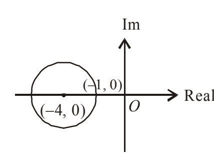
So, the greatest value is $\sqrt{5} + 1$.

6. **(d) :** $\bar{z} = \frac{1}{i-1}$

We have $z = \overline{(\bar{z})}$ giving $z = \frac{1}{\bar{i}-1} = \frac{1}{-i-1} = \frac{-1}{i+1}$

7. **(a) :** z lies on or inside the circle with centre (–4, 0) and radius 3 units.

Hence maximum distance of z from (–1, 0) is 6 units.



8. **(d) :** $\sum_{k=1}^{n}\left(\sin\frac{2k\pi}{n+1} + i\cos\frac{2k\pi}{n+1}\right)$

$\therefore \quad = \sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right) = -i$

9. **(d) :** $z^2 + z + 1 = 0$

$\Rightarrow z = \omega, \omega^2$

$\therefore \left(z+\frac{1}{z}\right)^2 + \left(z^2+\frac{1}{z^2}\right)^2 + \ldots\ldots + \left(z^6+\frac{1}{z^6}\right)^2$

$= 4(\omega + \omega^2)^2 + 2(\omega^3 + \omega^3)^2$

$= 4(-1)^2 + 2(2^2) = 4 + 8 = 12$

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10. **(d) :** *Method I :* Let $z_1 = \cos\theta_1 + i\sin\theta_1$,
$z_2 = \cos\theta_2 + i\sin\theta_2$
$\therefore\ z_1 + z_2 = (\cos\theta_1 + \cos\theta_2) + i(\sin\theta_1 + \sin\theta_2)$
Now $|z_1 + z_2| = |z_1| + |z_2|$
$\Rightarrow \sqrt{(\cos\theta_1+\cos\theta_2)^2+(\sin\theta_1+\sin\theta_2)^2} = 1+1$
$\Rightarrow 2(1 + \cos(\theta_1 - \theta_2)) = 4$ (by squaring)
$\Rightarrow \cos(\theta_1 - \theta_2) = 1 \Rightarrow \theta_1 - \theta_2 = 0 \quad (\because \cos 0° = 1)$
$\Rightarrow \text{Arg } z_1 - \text{Arg } z_2 = 0.$

11. **(d) :** Given $\omega = \dfrac{3z}{3z-i} \quad \therefore\ |\omega| = \dfrac{3|z|}{|3z-i|}$
$\Rightarrow |3z-i| = 3|z|$
$\Rightarrow |3(x)+i(3y-1)| = |3(x+iy)| \quad (z = x+iy)$
$\Rightarrow (3x)^2 + (3y-1)^2 = 9(x^2+y^2) \Rightarrow 6y - 1 = 0$ which is straight line.

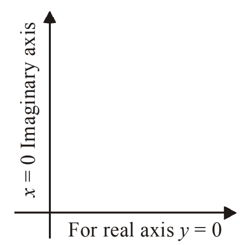
12. **(d) :** ***Method (1) :*** (By making the equation from the given roots)
Let us consider $x = -1, -1, -1$
$\therefore$ Required equation from given roots is
$(x + 1)(x + 1)(x + 1) = 0$
$(x + 1)^3 = 0$ which does not match with the given equation $(x - 1)^3 + 8 = 0$ so $x = -1, -1, -1$ cannot be the proper choice.
Again consider $x = -1, -1 + 2\omega, -1 - 2\omega^2$
$\therefore$ Required equation from given roots is
$\Rightarrow (x + 1)(x + 1 - 2\omega)(x + 1 + 2\omega^2) = 0$
$\Rightarrow (x + 1)[(x + 1)^2 + (x + 1)(2\omega^2 - 2\omega) - 4\omega^3] = 0$
$\Rightarrow (x + 1)[(x + 1)^2 + 2(x + 1)(\omega^2 - \omega) - 4] = 0$
$\Rightarrow (x + 1)^3 + 2(x + 1)^2(\omega^2 - \omega) - 4(x + 1) = 0$
which cannot be expressed in the form of given equation $(x - 1)^3 + 8 = 0$. Now consider the roots
$x_i = -1, 1 - 2\omega, 1 - 2\omega^2 \quad (i = 1, 2, 3)$
and the equation with these roots is given by
$x^3 -$ (sum of the roots)$x^2 + x$(Product of roots taken two at a time) $-$ Product of roots taken all at a time $= 0$
Now sum of roots $x_1 + x_2 + x_3$
$= -1 + 1 - 2\omega + 1 - 2\omega^2 = 3$
Product of roots taken two at a time
$= -1 + 2\omega - 1 + 2\omega^2 + 1 + 2(\omega^2 + \omega) + 4\omega^3 = 3$
Product of roots taken all at a time
$= (-1)[(1 - 2\omega)(1 - 2\omega^2)] = -7$
$\therefore$ Required equation is $x^3 - 3x^2 + 3x + 7 = 0$
$\Rightarrow x^3 - 3x^2 + 3x - 1 + 8 = 0 \Rightarrow (x - 1)^3 + 8 = 0$ which matched with given equation.

Method 2 (by taking cross checking)
As $(x - 1)^3 + 8 = 0$ ...(*)
and $x = -1$ satisfies $(x - 1)^3 + 8 = 0$
i.e. $(-2)^3 + 8 = 0 \Rightarrow 0 = 0$
Similarly for $1 - 2\omega$ we have $(x - 1)^3 + 8 = 0$
$\Rightarrow (1 - 2\omega - 1)^3 + 8 = 0$
$\Rightarrow (-2\omega)^3 + 8 = 0 \Rightarrow -8 + 8 = 0$ and for $1 - 2\omega^2$ we have $(1 - 2\omega^2 - 1)^3 + 8 = 0$
$\Rightarrow \omega^6(-8) + (8) = 0 \Rightarrow 0 = 0$
$\therefore\ -1, 1 - 2\omega, 1 - 2\omega^2$ are roots of $(x - 1)^3 + 8 = 0$ but on the other hand the other roots does not satisfies the equation $(x - 1)^3 + 8 = 0$.

13 **(a) :** $\bar{z} + i\bar{\omega} = 0$
$\Rightarrow \bar{z} = -i\bar{\omega} \Rightarrow z = i\omega$
$\Rightarrow \omega = -iz \quad \therefore\ \arg(-iz^2) = \pi$
$\Rightarrow \arg(-i) + 2\arg(z) = \pi$
$\Rightarrow 2\arg(z) = \pi + \pi/2 = 3\pi/2$
$\arg(z) = 3\pi/4$

14. **(d) :** $z^{1/3} = p + iq$
$\Rightarrow x - iy = (p + iq)^3$
$\Rightarrow x - iy = p^3 - 3pq^2 + i(3p^2q - q^3)$
$\Rightarrow x = p^3 - 3pq^2$ and $y = -(3p^2q - q^3)$
$\dfrac{x}{p} = p^2 - 3q^2$ and $\dfrac{y}{q} = -(3p^2 - q^2) \quad (*)$
Adding the equations of (*) we get
$\dfrac{x}{p} + \dfrac{y}{q} = -2(p^2 + q^2)$

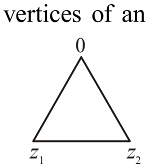
15. **(b) :** $|z^2 - 1| = |z|^2 + 1$
$\Rightarrow$ Let $z = x + iy$
$\Rightarrow (x - 1)^2 + y^2 = (x^2 + y^2) + 1$
$\Rightarrow 2x = 0$
$x = 0$
$\Rightarrow z$ lies on imaginary axis.



16. **(d) :** Given $\left(\dfrac{1+i}{1-i}\right)^x = 1$
$\Rightarrow \left(\dfrac{2i}{2}\right)^x = 1$
$\Rightarrow i^x = 1 \quad \Rightarrow \quad i^x = (i)^{4n}$
$\Rightarrow x = 4n,\ n \in I^+$

17. **(c) :** $|z\omega| = 1 \Rightarrow |z||\omega| = 1$ So $|z| = \dfrac{1}{|\omega|}$ ... (1)
Again $Arg(z) - Arg(\omega) = \dfrac{\pi}{2}$
$\therefore\ \dfrac{z}{\omega} = \left|\dfrac{z}{\omega}\right| i = |z|^2 i$ from (1)
$\therefore\ \dfrac{z}{\omega} = z\bar{z}\, i \Rightarrow \bar{z}\,\omega = \dfrac{1}{i} = -i.$

18. **(b) :** As z_1, z_2 are roots of $z^2 + az + b = 0$
$\therefore\ z_1 + z_2 = -a,\ z_1z_2 = b$
Again $0, z_1, z_2$ are vertices of an equilateral triangle



$\therefore\ 0^2 + z_1^2 + z_2^2 = 0z_1 + z_1z_2 + z_20 = 0$
$z_1^2 + z_2^2 = z_1z_2$
$\Rightarrow (z_1 + z_2)^2 = 3z_1z_2$
$a^2 = 3b$

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19. **(b) :** Let $|z| = |\omega| = r$

$\therefore\ z = re^{i\alpha}$ and $\omega = re^{i\beta}$

where $\alpha + \beta = \pi$ (given)

Now $\bar{z} = re^{i\alpha} = re^{i(\pi - \beta)}$

$= re^{i\pi} \cdot e^{-i\beta}$

$= -re^{-i\beta}$

$= -\bar{\omega}$

20. **(c) :** $|z - 4| < |z - 2|$

or $|a - 4 + ib| < |(a - 2) + ib|$ by taking $z = a + ib$

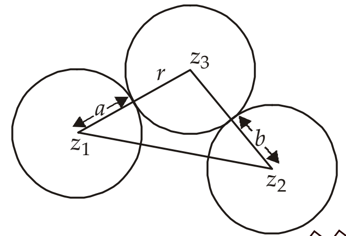
$\Rightarrow\quad (a - 4)^2 + b^2 < (a - 2)^2 + b^2$

$\Rightarrow\quad -8a + 4a < -16 + 4$

$\Rightarrow\quad 4a > 12 \quad \Rightarrow \quad a > 3$

$\Rightarrow\quad \text{Re}(z) > 3$

21. **(b) :**



$z_1z_3 - z_3z_2 = (a + r) - (b + r)$

$= a - b =$ a constant, which represent a hyperbola

Since, A hyperbola is the locus of a point which moves in such a way that the difference of its distances from two fixed points (foci) is always constant.

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CHAPTER 3

MATRICES AND DETERMINANTS

1. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3 × 3 matrix A and $|A| = 4$, then α is equal to

(a) 11 (b) 5 (c) 0 (d) 4 *(2013)*

2. The number of values of k, for which the system of equations $(k + 1)x + 8y = 4k$, $kx + (k + 3)y = 3k - 1$, has no solution, is

(a) 1 (b) 2 (c) 3 (d) infinite *(2013)*

3. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to

(a) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ (c) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ *(2012)*

4. Let P and Q be 3 × 3 matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to

(a) 0 (b) – 1 (c) – 2 (d) 1 *(2012)*

5. Let A and B be two symmetric matrices of order 3.

Statement-1 : $A(BA)$ and $(AB)A$ are symmetric matrices.

Statement 2 : AB is symmetric matrix if matrix multiplication of A with B is commutative.

(a) Statement-1 is true, Statement-2 is false.

(b) Statement-1 is false, Statement-2 is true.

(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2011)*

6. The number of values of k for which the linear equations

$4x + ky + 2z = 0$

$kx + 4y + z = 0$

$2x + 2y + z = 0$

possess a non-zero solution is

(a) 1 (b) zero (c) 3 (d) 2 *(2011)*

7. Consider the system of linear equations

$x_1 + 2x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 3$

$3x_1 + 5x_2 + 2x_3 = 1$

The system has

(a) infinite number of solutions

(b) exactly 3 solutions

(c) a unique solution

(d) no solution *(2010)*

8. The number of 3 × 3 non-singular matrices, with four entries as 1 and all other entries as 0, is

(a) less than 4 (b) 5

(c) 6 (d) at least 7 *(2010)*

9. Let A be a 2 × 2 matrix with non-zero entries and let $A^2 = I$, where I is 2 × 2 identity matrix. Define $Tr(A)$ = sum of diagonal elements of A and $|A|$ = determinant of matrix A.

Statement-1 : $Tr(A) = 0$.

Statement-2 : $|A| = 1$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true. *(2010)*

10. Let A be a 2 × 2 matrix

Statement-1: adj (adj A) = A

Statement-2: $|\text{adj } A| = |A|$

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1. *(2009)*

11. Let a, b, c be such that $b(a + c) \neq 0$. If

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$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0,$$

then the value of n is
(a) any even integer (b) any odd integer
(c) any integer (d) zero *(2009)*

12. Let A be a 2 × 2 matrix with real entries. Let I be the 2 × 2 identity matrix. Denote by $tr(A)$, the sum of diagonal entries of A. Assume that $A^2 = I$.
Statement-1 : If $A \neq I$ and $A \neq -I$, then det $A = -1$.
Statement-2 : If $A \neq I$ and $A \neq -I$, then $tr(A) \neq 0$.
(a) Statement-1 is true, Statement-2 is false
(b) Statemen-1 is false, Statement-2 is true
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1 *(2008)*

13. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$ and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to
(a) 1 (b) 2 (c) – 1 (d) 0 *(2008)*

14. Let A be a square matrix all of whose entries are integers. Then which one of the following is true?
(a) If det $A = \pm 1$, then A^{-1} need not exist
(b) If det $A = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers
(c) If det $A \neq \pm 1$, then A^{-1} exists and all its entries are non-integers
(d) If det $A = \pm 1$, then A^{-1} exists and all its entries are integers *(2008)*

15. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$ then D is
(a) divisible by x but not y
(b) divisible by y but not x
(c) divisible by neither x nor y
(d) divisible by both x and y. *(2007)*

16. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals
(a) 1/5 (b) 5 (c) 5^2 (d) 1. *(2007)*

17. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true?
(a) $A = B$ (b) $AB = BA$
(c) either A or B is a zero matrix
(d) either A or B is an identity matrix. *(2006)*

18. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, a, $b \in N$. Then
(a) there cannot exist any B such that $AB = BA$
(b) there exist more than one but finite number B's such that $AB = BA$
(c) there exists exactly one B such that $AB = BA$
(d) there exist infinitely many B's such that $AB = BA$. *(2006)*

19. If $A^2 - A + I = 0$, then the inverse of A is
(a) A (b) $A + I$ (c) $I - A$ (d) $A - I$. *(2005)*

20. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathematical induction
(a) $A^n = 2^{n-1}A + (n - 1)I$
(b) $A^n = nA - (n - 1)I$
(c) $A^n = 2^{n-1}A - (n - 1)I$
(d) $A^n = nA + (n - 1)I$. *(2005)*

21. If $a^2 + b^2 + c^2 = -2$ and
$$f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$
then $f(x)$ is a polynomial of degree
(a) 0 (b) 1 (c) 2 (d) 3. *(2005)*

22. The system of equations $\alpha x + y + z = \alpha - 1$, $x + \alpha y + z = \alpha - 1$, $x + y + \alpha z = \alpha - 1$ has no solutions, if α is
(a) either –2 or 1 (b) –2
(c) 1 (d) not –2. *(2005)*

23. Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. The only correct statement about the matrix A is
(a) A^{-1} does not exist
(b) $A = (-1)I$, where I is a unit matrix
(c) A is a zero matrix (d) $A^2 = I$. *(2004)*

24. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$ and $10(B) = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$. If B is the inverse of matrix A, then α is
(a) 2 (b) –1 (c) –2 (d) 5. *(2004)*

25. If $a_1, a_2, a_3, \ldots, a_n, \ldots$ are G.P., then the value of the determinant
$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix},$$ is
(a) 2 (b) 1 (c) 0 (d) –2. *(2004)*

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26. If 1, ω, ω^2 are the cube roots of unity,

then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to

(a) 1 (b) ω (c) ω^2 (d) 0. *(2003)*

27. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

(a) $\alpha = a^2 + b^2, \beta = 2ab$
(b) $\alpha = a^2 + b^2, \beta = a^2 - b^2$
(c) $\alpha = 2ab, \beta = a^2 + b^2$
(d) $\alpha = a^2 + b^2, \beta = ab$. *(2003)*

28. If $a > 0$ and discriminant of $ax^2 + 2bx + c$ is –ve, then

$\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is

(a) +ve (b) $(ac - b^2)(ax^2 + 2bx + c)$
(c) –ve (d) 0. *(2002)*

29. If l, m, n are the p^{th}, q^{th} and r^{th} term of a GP, all positive, then

$\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals

(a) –1 (b) 2 (c) 4 -(d) 0. *(2002)*

Answer Key

1.	(a)	**2.**	(a)	**3.**	(b)	**4.**	(a)	**5.**	(d)	**6.**	(d)
7.	(d)	**8.**	(d)	**9.**	(c)	**10.**	(a)	**11.**	(b)	**12.**	(a)
13.	(a)	**14.**	(d)	**15.**	(d)	**16.**	(a)	**17.**	(b)	**18.**	(d)
19.	(c)	**20.**	(b)	**21.**	(c)	**22.**	(b)	**23.**	(d)	**24.**	(d)
25.	(c)	**26.**	(d)	**27.**	(a)	**28.**	(c)	**29.**	(d)		

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Explanations

1. **(a) :** $P=\begin{bmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$

Let $P = 1(12-12)-\alpha(4-6)+3(4-6) = 2\alpha - 6$

Also, $\det(\text{adj } A) = (\det A)^2$

$\Rightarrow \quad 2\alpha - 6 = 16 \Rightarrow 2\alpha = 22. \quad \therefore \quad \alpha = 11$

Remark : $\det(\text{adj } A) = (\det A)^{n-1}$, where A is a matrix of order n.

2. **(a) :** The equation is $\begin{bmatrix}k+1 & 8\\ k & k+3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4k\\ 3k-1\end{bmatrix}$

For no solution of $AX = B$ a necessary condition is $\det A = 0$.

$\Rightarrow \begin{vmatrix}k+1 & 8\\ k & k+3\end{vmatrix}=0$

$\Rightarrow \quad (k+1)(k+3)-8k = 0 \Rightarrow k^2+4k+3-8k = 0$

$\Rightarrow \quad k^2-4k+3 = 0 \Rightarrow (k-1)(k-3) = 0 \quad \therefore \quad k = 1, 3$

For $k = 1$, the equation becomes

$2x + 8y = 4$, $x + 4y = 2$

which is just a single equation in two variables.

$x + 4y = 2$ It has infinite solutions.

For $k = 3$, the equation becomes

$4x + 8y = 12$, $3x + 6y = 8$

which are parallel lines. So no solution in this case.

3. **(b) :** $A=\begin{bmatrix}1 & 0 & 0\\ 2 & 1 & 0\\ 3 & 2 & 1\end{bmatrix}$, $Au_1=\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}$, $Au_2=\begin{bmatrix}0\\ 1\\ 0\end{bmatrix}$

Let $u_1=\begin{bmatrix}a\\ b\\ c\end{bmatrix}$

$Au_1=\begin{bmatrix}1\\ 0\\ 0\end{bmatrix} \Rightarrow a = 1,\ 2a + b = 0$
$\Rightarrow b = -2,\ 3a + 2b + c = 0 \Rightarrow c = 1$

Let $u_2=\begin{bmatrix}p\\ q\\ r\end{bmatrix}$

$Au_2=\begin{bmatrix}0\\ 1\\ 0\end{bmatrix} \Rightarrow p = 0,\ 2p + q = 1 \Rightarrow q = 1,$
$3p + 2q + r = 0 \Rightarrow r = -2$

$u_1+u_2=\begin{bmatrix}1\\ -2\\ 1\end{bmatrix}+\begin{bmatrix}0\\ 1\\ -2\end{bmatrix}=\begin{bmatrix}1\\ -1\\ -1\end{bmatrix}$

4. **(a) :** $P^3 = Q^3$, $P^2Q = Q^2P$, $PQ^2 = P^2Q$

$\Rightarrow \quad P(P^2+Q^2) = (Q^2+P^2)Q$

$\Rightarrow \quad P(P^2+Q^2) = (P^2+Q^2)Q$

$P \neq Q \Rightarrow P^2 + Q^2$ is singular.

Hence, $|P^2 + Q^2| = 0$

5. **(d) :** Let $A(BA) = P$

Then $P^T = (ABA)^T = A^TB^TA^T$ (Transversal rule)

$= ABA = P$

Thus P is symmetric.

Again, $A(BA) = (AB)A$ by associativity.

Also $(AB)^T = B^TA^T = BA = AB$

($\because$ A and B are commutative)

$\Rightarrow \quad AB$ is also symmetric.

6. **(d) :** For the system to possess non-zero solution,

we have $\begin{vmatrix}4 & k & 2\\ k & 4 & 2\\ 2 & 2 & 1\end{vmatrix}=0$

which on expansion gives $k^2 - 6k + 8 = 0$

$\Rightarrow \quad (k-2)(k-4) = 0. \quad \therefore \quad k = 2, 4$

7. **(d) :** $x_1 + 2x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 3$

$3x_1 + 5x_2 + 2x_3 = 1$

A quick observation tells us that the sum of first two equations yields

$(x_1 + 2x_2 + x_3) + (2x_1 + 3x_2 + x_3) = 3 + 3$

$\Rightarrow \quad 3x_1 + 5x_2 + 2x_3 = 6$

But this contradicts the third equation, *i.e.*,

$3x_1 + 5x_2 + 2x_3 = 1$

As such the system is inconsistent and hence it has no solution.

8. **(d) :** $A=\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}$

Let $A = (a_1b_2c_3 + a_2c_1b_3 + a_3b_1c_2) - (a_1c_2b_3 + a_2b_1c_3 + a_3c_1b_2)$

If any of the terms be non-zero, then det A will be non-zero and all the elements of that term will be 1 each.

Number of non-singular matrices $= {}^6C_1 \times {}^6C_1 = 36$

We can also exhibit more than 6 matrices to pick the right choice.

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9. **(c) :** Let $A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$

$$A^2 = \begin{bmatrix} \alpha^2+\beta\gamma & \beta(\alpha+\delta) \\ \gamma(\alpha+\delta) & \delta+\beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Which gives $\alpha + \delta = 0$ and $\alpha^2 + \beta\gamma = 1$

So we have $Tr(A) = 0$

$\det A = \alpha\delta - \beta\gamma = -\alpha^2 - \beta\gamma = -(\alpha^2 + \beta\gamma) = -1$

Thus statement-1 is true but statement-2 is false.

10. **(a) :** We have $\text{adj}(\text{adj } A) = |A|^{n-2}A$

Here $n = 2$, which gives $\text{adj}(\text{adj } A) = A$

The statement-1 is true.

Again $|\text{adj } A| = |A|^{n-1}$

Here $n = 2$, which gives $|\text{adj } A| = |A|$

Thus statement-2 is also true. But statement-2 doesn't explain statement-1.

11. **(b) :** $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c-1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^n \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c-1 \\ a & -b & c \end{vmatrix} = 0$$

$$\Rightarrow D + (-1)^n \begin{vmatrix} a+1 & a-1 & a \\ b+1 & b-1 & -b \\ c+1 & c-1 & c \end{vmatrix} = 0$$

(Changing rows to columns)

$$\Rightarrow D + (-1)^n \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c+1 & c-1 \end{vmatrix} = 0$$

(Changing columns in cyclic order doesn't change the determinant)

$\Rightarrow \quad D + (-1)^n D = 0 \quad \Rightarrow \quad \{1 + (-1)^n\}D = 0$

Now $D = \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} \neq 0$

$$= \begin{vmatrix} a & 2 & a-1 \\ -b & 2 & b-1 \\ c & -2 & c+1 \end{vmatrix} \quad C_2 \to C_2 - C_3$$

$$= \begin{vmatrix} a+c & 0 & a+c \\ -b+c & 0 & b+c \\ c & -2 & c+1 \end{vmatrix} \quad R_1 \to R_1 + R_3, \; R_2 \to R_2 + R_3$$

Expanding along 2nd column

$D = 2\{(a + c)(b + c) - (a + c)(c - b)\}$

$= 2(a + c)2b$

$= 4b(a + c) \neq 0$ (By hypothesis)

Now $\{1 + (-1)^n\}D = 0 \quad \Rightarrow \quad 1 + (-1)^n = 0$

Which mean n = odd integer.

12. **(a) :** Let $A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$. We have

$$A^2 = \begin{bmatrix} \alpha^2+\beta\gamma & \beta(\alpha+\delta) \\ \gamma(\alpha+\delta) & \delta^2+\beta\gamma \end{bmatrix}$$

$$A^2 = I = \begin{bmatrix} \alpha^2+\beta\gamma & \beta(\alpha+\delta) \\ \gamma(\alpha+\delta) & \delta^2+\beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

giving $\alpha^2 + \beta\gamma = 1 = \delta^2 + \beta\gamma$

and $\gamma(\alpha + \delta) = \beta(\alpha + \delta) = 0$

As $A \neq I$, $A \neq -I$, we have $\alpha = -\delta$

$$\det A = \begin{vmatrix} \sqrt{1-\beta\gamma} & \beta \\ \gamma & -\sqrt{1-\beta\gamma} \end{vmatrix} = -1 + \beta\gamma - \beta\gamma = -1$$

Statement-1 is therefore true.

$\text{tr}(A) = \alpha + \delta = 0 \quad \{\alpha = -\delta\}$

Statement-2 is false because $\text{tr}(A) = 0$

13. **(a) :** System of equations

$x - cy - bz = 0$

$cx - y + az = 0$

$bx + ay - z = 0$

has non trivial solution if the determinant of coefficient matrix is zero

$$\Rightarrow \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$\Rightarrow 1(1 - a^2) + c(-c - ab) - b(ca + b) = 0$

$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

14. **(d) :** Each entry of A is an integer, so the cofactor of every entry is an integer. And then each entry of adjoint is integer. Also $\det A = \pm 1$ and we know that

$$A^{-1} = \frac{1}{\det A}(adj\ A)$$

This means all entries in A^{-1} are integers.

15. **(d) :** $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

(Apply $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$)

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix} = 1(xy - 0) = xy$$

Hence D is divisible by both x and y.

16. **(a) :** $A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$

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$$A^2 = \begin{bmatrix} 25 & 25\alpha+5\alpha^2 & 5\alpha+5\alpha+25\alpha^2 \\ 0 & \alpha^2 & 25\alpha+5\alpha^2 \\ 0 & 0 & 25 \end{bmatrix}$$

Given $|A^2| = 25$, $625\alpha^2 = 25 \Rightarrow |\alpha| = \frac{1}{5}$.

17. **(b) :** Give $A^2 - B^2 = (A + B)(A - B)$

$\Rightarrow \quad 0 = BA - AB$

$\Rightarrow \quad BA = AB$

18. **(d) :** $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$

Now $AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} a & 2b \\ 3a & 4b \end{pmatrix}$ ... (i)

and $BA = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} a & 2a \\ 3b & 4b \end{pmatrix}$ ... (ii)

As $AB = BA \Rightarrow 2a = 2b \Rightarrow a = b$

$\therefore \quad B = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} = aI_2 \Rightarrow \exists$ infinite value of $a = b \in N$

19. **(c) :** $A^2 - A + I = 0 \Rightarrow I = A - A \cdot A$

$IA^{-1} = AA^{-1} - A(AA^{-1})$, $A^{-1} = I - A$.

20. **(b) :** $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

$\therefore \quad A^2 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$, $A^3 = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ so $A^n = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$

and $nA - (n - 1)I = \begin{pmatrix} n & 0 \\ n & n \end{pmatrix} - \begin{pmatrix} n-1 & 0 \\ 0 & n-1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix} = A^n$.

21. **(c) :** Applying $C_2 \to C_2 + C_3 + C_1$

$f(x) = 1 + 2x + x(a^2 + b^2 + c^2)$

$$\begin{vmatrix} 1+a^2x & 1 & (1+c^2)x \\ (1+a^2)x & 1 & (1+c^2)x \\ (1+a^2)x & 1 & 1+c^2x \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$ and using $a^2 + b^2 + c^2 = -2$ we have

$$(1+2x-2x)\begin{vmatrix} 1-x & 0 & 0 \\ 0 & 0 & x-1 \\ (1+a^2)x & 1 & 1+c^2x \end{vmatrix} = (1-x)^2$$

$= x^2 - 2x + 1 \quad \therefore$ degree of $f(x)$ is 2.

22. **(b) :** For no solution $|A| = 0$ and (adj $A)(B) \neq 0$

Now $|A| = 0 \Rightarrow \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$

$\Rightarrow \alpha^3 - 3\alpha + 2 = 0 \Rightarrow (\alpha - 1)^2(\alpha + 2) = 0$

$\Rightarrow \alpha = 1, -2$.

But for $\alpha = 1$, $|A| = 0$ and (adj $A)(B) = 0$

$\Rightarrow$ for $\alpha = 1$ there exist infintiely many solution.

Also the each equation becomes

$x + y + z = 0$ again for $\alpha = -2$

$|A| = 0$ but (adj $A)(B) \neq 0 \Rightarrow \exists$ no solution.

23. **(d) :** (i) $|A| = 1 \therefore A^{-1}$ does not exist is wrong statement

(ii) $(-1)\, I = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \neq A \Rightarrow$ (b) is false

(iii) A is clearly a non zero matrix $\therefore$ (c) is false

We left with (d) only.

24. **(d) :** Given $A^{-1} = B = 10\, A^{-1} = 10\, B$

$\Rightarrow \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix} = 10\, A^{-1}$.

$\Rightarrow \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix} (A) = 10I$

$\Rightarrow \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$

$= \begin{pmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{pmatrix}$ ...(*)

$\Rightarrow -5 + \alpha = 0$

(equating A_{21} entry both sides of (*))

$\Rightarrow \alpha = 5$

25. **(c) :** $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \ldots \frac{a_n}{a_{n-1}} = r$

which means $a_n, a_{n+1}, a_{n+2} \in$ G.P.

$\Rightarrow a_{n+1}^2 = a_n\, a_{n+2}$

$\Rightarrow 2 \log a_{n+1} - \log a_n - \log a_{n+2} = 0$...(i)

Similarly $2 \log a_{n+4} - \log a_{n+3} - \log a_{n+5} = 0$ ...(ii)

and $2 \log a_{n+7} - \log a_{n+6} - \log a_{n+8} = 0$ ...(iii)

Using $C_1 \to C_1 + C_3 - 2C_2$

we get $\Delta = 0$

26. **(d) :** As ω is cube root of unity $\therefore \omega^3 = \omega^{3n} = 1$

$\therefore \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$

$= (\omega^{3n} - 1) - \omega^n(\omega^{2n} - \omega^{2n}) + \omega^{2n}(\omega^n - \omega^n) = 0$

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27. **(a) :** $A^2 = AA = \begin{pmatrix} a & b \\ b & a \end{pmatrix}\begin{pmatrix} a & b \\ b & a \end{pmatrix}$

$= \begin{pmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix}$

28. **(c) :** $C_1 \to xC_1 + C_2 - C_3$

$= \frac{1}{x}\begin{vmatrix} 0 & b & ax+b \\ 0 & c & bx+c \\ ax^2+2bx+c & bx+c & 0 \end{vmatrix}$

$= \frac{(ax^2+2bx+c)}{x}[b^2x + bc - acx - bc]$

$= (b^2 - ac)(ax^2 + 2bx + c)$

$= (+\text{ve})(-\text{ve}) < 0$

29. **(d) :** Let A be the first term and R be the common ratio of G. P.

$\therefore \quad l = t_p = AR^{p-1}$

$\Rightarrow \quad \log l = \log A + (p-1)\log R$

Similarly, $\log m = \log A + (q-1)\log R$

and $\log n = \log A + (r-1)\log R$

$\therefore \begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$

$= \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix}$

$= \begin{vmatrix} \log A - \log R & p & 1 \\ \log A - \log R & q & 1 \\ \log A - \log R & r & 1 \end{vmatrix} + \begin{vmatrix} p\log R & p & 1 \\ q\log R & q & 1 \\ r\log R & r & 1 \end{vmatrix}$

$c_1 \propto c_3 \qquad c_1 \propto c_2$

$= 0 + 0$

$= 0$

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CHAPTER 4

QUADRATIC EQUATIONS

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1. The real number k for which the equation $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1]
(a) lies between 2 and 3
(b) lies between –1 and 0
(c) does not exist
(d) lies between 1 and 2 *(2013)*

2. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in R$ have a common root, then $a : b : c$ is
(a) 3 : 2 : 1 (b) 1 : 3 : 2
(c) 3 : 1 : 2 (d) 1 : 2 : 3 *(2013)*

3. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has
(a) exactly one real root.
(b) exactly four real roots.
(c) infinite number of real roots.
(d) no real roots. *(2012)*

4. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re $z = 1$, then it is necessary that
(a) $|\beta| = 1$ (b) $\beta \in (1, \infty)$
(c) $\beta \in (0, 1)$ (d) $\beta \in (-1, 0)$ *(2011)*

5. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$
(a) –2 (b) –1 (c) 1 (d) 2 *(2010)*

6. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x. The expression $3b^2x^2 + 6bcx + 2c^2$ is
(a) less than $4ab$ (b) greater than $-4ab$
(c) less than $-4ab$ (d) greater than $4ab$ *(2009)*

7. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is
(a) 2 (b) 1 (c) 4 (d) 3 *(2008)*

8. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is
(a) $(3, \infty)$ (b) $(-\infty, -3)$
(c) $(-3, 3)$ (d) $(-3, \infty)$. *(2007)*

9. If the roots of the quadratic equation $x^2 + px + q = 0$ are tan30° and tan15°, respectively then the value of $2 + q - p$ is
(a) 2 (b) 3 (c) 0 (d) 1. *(2006)*

10. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than –2 but less than 4, lie in the interval
(a) $-2 < m < 0$ (b) $m > 3$
(c) $-1 < m < 3$ (d) $1 < m < 4$. *(2006)*

11. The value of a for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value is
(a) 0 (b) 1 (c) 2 (d) 3. *(2005)*

12. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals
(a) 3 (b) –2 (c) 1 (d) 2. *(2005)*

13. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval
(a) $(6, \infty)$ (b) $(5, 6]$
(c) $[4, 5]$ (d) $(-\infty, 4)$. *(2005)*

14. If the equation $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x = 0$, $a_1 \neq 0$, $n \geq 2$, has a positive root $x = \alpha$, then the equation $na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots + a_1 = 0$ has a positive root, which is
(a) smaller than α (b) greater than α
(c) equal to α (d) greater than or equal to α. *(2005)*

15. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation
(a) $x^2 + 18x - 16 = 0$ (b) $x^2 - 18x + 16 = 0$
(c) $x^2 + 18x + 16 = 0$ (d) $x^2 - 18x - 16 = 0$. *(2004)*

16. If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$ then its roots are
(a) 0, –1 (b) –1, 1 (c) 0, 1 (d) –1, 2. *(2004)*

17. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of q is
(a) 3 (b) 12 (c) 49/4 (d) 4. *(2004)*

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18. The value of a for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is
(a) $-2/3$ (b) $1/3$ (c) $-1/3$ (d) $2/3$. *(2003)*

19. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in
(a) geometric progression
(b) harmonic progression
(c) arithmetic-geometric progression
(d) arithmetic progression. *(2003)*

20. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is
(a) 4 (b) 1 (c) 3 (d) 2. *(2003)*

21. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation whose roots are α/β and β/α is
(a) $3x^2 - 25x + 3 = 0$ (b) $x^2 + 5x - 3 = 0$
(c) $x^2 - 5x + 3 = 0$ (d) $3x^2 - 19x + 3 = 0$. *(2002)*

22. Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then
(a) $a + b + 4 = 0$ (b) $a + b - 4 = 0$
(c) $a - b - 4 = 0$ (d) $a - b + 4 = 0$. *(2002)*

23. Product of real roots of the equation $x^2 + |x| + 9 = 0$
(a) is always positive (b) is always negative
(c) does not exist (d) none of these. *(2002)*

24. If p and q are the roots of the equation $x^2 + px + q = 0$, then
(a) $p = 1, q = -2$ (b) $p = 0, q = 1$
(c) $p = -2, q = 0$ (d) $p = -2, q = 1$. *(2002)*

25. If a, b, c are distinct +ve real numbers and $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ is
(a) less than 1 (b) equal to 1
(c) greater than 1 (d) any real no. *(2002)*

Answer Key

1.	(c)	**2.**	(d)	**3.**	(d)	**4.**	(b)	**5.**	(b)	**6.**	(b)
7.	(a)	**8.**	(c)	**9.**	(b)	**10.**	(c)	**11.**	(b)	**12.**	(c)
13.	(d)	**14.**	(a)	**15.**	(b)	**16.**	(a)	**17.**	(c)	**18.**	(d)
19.	(b)	**20.**	(a)	**21.**	(d)	**22.**	(a)	**23.**	(c)	**24**	(a)
25.	(a)										

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Explanations

1. **(c) :** Let $f(x) = 2x^3 + 3x + k$, $f'(x) = 6x^2 + 3 > 0$
Thus f is strictly increasing. Hence it has atmost one real root. But a polynomial equation of odd degree has atleast one root. Thus the equation has exactly one root. Then the two distinct roots, in any interval whatsoever is an impossibility. No such (c) exists.

2. **(d) :** In the equation $x^2 + 2x + 3 = 0$, both the roots are imaginary.
Since $a, b, c \in R$, we have $\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$
Hence $a : b : c :: 1 : 2 : 3$

3. **(d) :** $e^{\sin x} - e^{-\sin x} - 4 = 0$
$\Rightarrow (e^{\sin x})^2 - 4e^{\sin x} - 1 = 0 \Rightarrow t^2 - 4t - 1 = 0$
$\Rightarrow t = \frac{4 \pm \sqrt{16+4}}{2} = 2 \pm \sqrt{5}$
i.e., $e^{\sin x} = 2 + \sqrt{5}$ or $\underbrace{2 - \sqrt{5}}_{-\text{ve}}$ (neglected)
$\sin x = \ln(2 + \sqrt{5}) > 1$ $\therefore$ No real roots.

4. **(b) :** Let roots be $1 + ai$, $1 + bi$, then we have, $(a \in R)$
$(1 + ai) + (1 + bi) = -\alpha \Rightarrow 2 + (a + b)i = -\alpha$
$(1 + ai)(1 + bi) = \beta$
Comparing we have, $\alpha = -2$ and $a = -b$
Now $(1 + ai)(1 - ai) = \beta$
$\Rightarrow 1 + a^2 = \beta \Rightarrow \beta = 1 + a^2$
As $a^2 \geq 0$ we have $\beta \in (1, \infty)$

5. **(b) :** We have $x^2 - x + 1 = 0$ giving $x = \frac{1 \pm i\sqrt{3}}{2}$
Identifying these roots as ω and ω^2,
we have $\alpha = \omega$, $\beta = \omega^2$. We can also take the other way round that would not affect the result.
Now $\alpha^{2009} + \beta^{2009} = \omega^{2009} + \omega^{4018}$
$= \omega^{3k+2} + \omega^{3m+1}$ $(k, m \in N)$
$= \omega^2 + \omega = -1.$ $(\because \omega^{3k} = 1)$

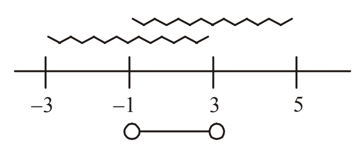
6. **(b) :** The roots of $bx^2 + cx + a = 0$ are imaginary means $c^2 - 4ab < 0 \Rightarrow c^2 < 4ab$
Again the coeff. of x^2 in
$3b^2x^2 + 6bcx + 2c^2$ is +ve, so the minimum value of the expression
$= -\frac{36b^2c^2 - 4(3b^2)(2c^2)}{4(3b^2)} = \frac{12b^2c^2}{12b^2} = -c^2$
As $c^2 < 4ab$ we have $-c^2 > -4ab$
Thus the minimum value is $-4ab$.

7. **(a) :** Let α and 4β be the root of
$x^2 - 6x + a = 0$
and α and 3β be those of the equation $x^2 - cx + 6 = 0$
From the relation between roots and coefficients
$\alpha + 4\beta = 6$ and $4\alpha\beta = a$
$\alpha + 3\beta = c$ and $3\alpha\beta = 6$
we obtain $\alpha\beta = 2$ giving $a = 8$
The first equation is $x^2 - 6x + 8 = 0 \Rightarrow x = 2, 4$
For $\alpha = 2$, $4\beta = 4 \Rightarrow 3\beta = 3$
For $\alpha = 4$, $4\beta = 2 \Rightarrow 3\beta = 3/2$ (not an integer)
So the common root is $\alpha = 2$.

8. **(c) :** $x^2 + ax + 1 = 0$
Let roots be α and β, then $\alpha + \beta = -a$ and $\alpha\beta = 1$
$|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$, $|\alpha - \beta| = \sqrt{a^2 - 4}$
Since, $|\alpha - \beta| < \sqrt{5} \Rightarrow \sqrt{a^2 - 4} < \sqrt{5}$
$\Rightarrow a^2 - 4 < 5 \Rightarrow a^2 < 9 \Rightarrow -3 < a < 3.$

9. **(b) :** $\alpha = \tan 30°$, $\beta = \tan 15°$ are roots of the equation $x^2 + px + q = 0$
$\therefore \tan\alpha + \tan\beta = -p$ and $\tan\alpha \cdot \tan\beta = q$
using $\tan\alpha + \tan\beta = \tan(\alpha + \beta)$
$(1 - \tan\alpha\tan\beta)$
$\Rightarrow -p = 1 - q \Rightarrow q - p = 1 \Rightarrow 2 + q - p = 3$

10. **(c) :** Let α, β are roots of the equation
$(x^2 - 2mx + m^2) = 1$



$\Rightarrow x = m \pm 1 = m + 1, m - 1$
Now $-2 < m + 1 < 4$ ...... (i)
and $-2 < m - 1 < 4$ .... (ii)
$\begin{cases} \Rightarrow -3 < m < 3 & \text{......(A)} \\ \text{and } -1 < m < 5 & \text{.......(B)} \end{cases}$
By (A) & (B) we get $-1 < m < 3$ as shown by the number line.

11. **(b) :** Let $f(a) = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= (a - 2)^2 + 2(a + 1)$
$\therefore f'(a) = 2(a - 2) + 2$
For Maxima | Minima $f'(a) = 0$
$\Rightarrow 2[a - 2 + 1] = 0 \Rightarrow a = 1$
Again $f''(a) = 2$,
$f''(1) = 2 > 0 \Rightarrow$ at $a = 1$, $f(a)$ will be least.

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12. **(c) :** Let α, α + 1 are consecutive integer

∴ $(x + \alpha)(x + \alpha + 1) = x^2 - bx + c$

Comparing both sides we get ⇒ $-b = 2\alpha + 1$

$c = \alpha^2 + \alpha$

∴ $b^2 - 4c = (2\alpha + 1)^2 - 4(\alpha^2 + \alpha) = 1$.

13. **(d) :** Given $x^2 - 2kx + k^2 + k - 5 = 0$

Roots are less than 5 ⇒ $D \geq 0$

⇒ $(-2k)^2 \geq 4(k^2 + k - 5) \Rightarrow k \leq 5$ ...(A)

Again $f(5) > 0$

⇒ $25 - 10k + k^2 + k - 5 > 0$

⇒ $k^2 - 9k + 20 > 0 \Rightarrow (k-4)(k-5) > 0$

⇒ $k < 4 \cup k > 5$ ...(B)

Also $\frac{\text{sum of roots}}{2} < 5 \Rightarrow k < 5$ ...(C)

from (A), (B), (C) we have

$k \in (-\infty, 4)$ as the choice gives number $k < 5$ is (d).

14. **(a) :** If possible say

$f(x) = a_0x^n + a_1x^{n-1} + ... + a_nx \quad \therefore f(0) = 0$

Now $f(\alpha) = 0$ (∵ $x = \alpha$ is root of given equation)

∴ $f'(x) = na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + ... + a_1 = 0$ has at least one root in]0, α [

⇒ $na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + ... + a_1 = 0$

has a +ve root smaller than α.

15. **(b) :** Let the two number be α, β

∴ $\frac{\alpha+\beta}{2} = 9$ and $\sqrt{\alpha\beta} = 4$

∴ Required equation

$x^2 - 2(\text{Average value of } \alpha, \beta)x + \sqrt{G.M}^2 = 0$

$x^2 - 2(9)x + 16 = 0$

16. **(a) :** As $1 - p$ is root of $x^2 + px + 1 - p = 0$

⇒ $(1-p)^2 + p(1-p) + (1-p) = 0$

$(1-p)[1 - p + p + 1] = 0$

⇒ $p = 1$

∴ Given equation becomes $x^2 + x = 0$

⇒ $x = 0, -1$

17. **(c) :** As $x^2 + px + q = 0$ has equal roots ∴ $p^2 = 4q$

and one root of $x^2 + px + 12 = 0$ is 4.

∴ $16 + 4p + 12 = 0$ ∴ $p = -7$

∴ $p^2 = 4q \Rightarrow q = \frac{49}{4}$

18. **(d) :** Let α, 2α are roots of the given equation

∴ sum of the roots

$\alpha + 2\alpha = 3\alpha = \frac{1-3a}{a^2-5a+3}$ ...(i)

and product of roots

$\alpha(2\alpha) = 2\alpha^2 = \frac{2}{a^2-5a+3}$ ...(ii)

By (i) and (ii) we have

$$\frac{9\alpha^2}{2\alpha^2} = \frac{(1-3a)^2}{(a^2-5a+3)^2} \times \frac{a^2-5a+3}{2}$$

⇒ $9(a^2 - 5a + 3) = (1 - 3a)^2$

⇒ $a = \frac{2}{3}$

19. **(b) :** Given α + β

$= \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$

⇒ $2a^2c = bc^2 + ab^2$

⇒ $\frac{2a}{b} = \frac{c}{a} + \frac{b}{c} \Rightarrow \frac{c}{a}, \frac{a}{b}, \frac{b}{c} \in$ A.P

⇒ reciprocals are in H.P

20. **(a) :** Given $x^2 - 3|x| + 2 = 0$

If $x \geq 0$ *i.e.* $|x| = x$

∴ The given equation can be written as

$x^2 - 3x + 2 = 0$

⇒ $(x-1)(x-2) = 0$

⇒ $x = 1, 2$

Similarly for $x < 0$, $x^2 - 3|x| + 2 = 0$

⇒ $x^2 + 3x + 2 = 0$

⇒ $x = -1, -2$

Hence 1, –1, 2, –2 are four solutions of the given equation.

21. **(d) :** We need the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ which are reciprocal of each other, which means product of roots is $\frac{\alpha}{\beta}\frac{\beta}{\alpha} = 1$. In our choice (a) and (d) have product of roots 1, so choices (b) and (d) are out of court. In the problem choice, None of these is not given. If out of four choices only one choice satisfies that product of root is 1 then you select that choice for correct answer. Now for proper choice we proceed as,

$\alpha \neq \beta$, but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$,

Changing α, β by x

∴ α, β are roots of $x^2 - 5x + 3 = 0$

⇒ $\alpha + \beta = 5, \alpha\beta = 3$

now, $S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{19}{3}$ and product $\frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$

∴ Required equation,

$x^2 - (\text{sum of roots})\, x + \text{product of roots} = 0$

⇒ $x^2 - \frac{19}{3}x + 1 = 0$

⇒ $3x^2 - 19x + 3 = 0$ is correct answer.

22. **(a) :** Let α, β are roots of $x^2 + bx + a = 0$

∴ $\alpha + \beta = -b$ and $\alpha\beta = a$

again let γ, δ are roots of $x^2 + ax + b = 0$

∴ $\gamma + \delta = -a$ and $\gamma\delta = b$

Now given

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$\alpha - \beta = \gamma - \delta$

$\Rightarrow (\alpha - \beta)^2 = (\gamma - \delta)^2$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$

$\Rightarrow b^2 - 4a = a^2 - 4b$

$\Rightarrow b^2 - a^2 = -4(b - a)$

$\Rightarrow (b - a)(b + a + 4) = 0$

$\Rightarrow b + a + 4 = 0$ as $(a \neq b)$

23. **(c) :** $x^2 + |x| + 9 = 0$

$\Rightarrow |x|^2 + |x| + 9 = 0$

$\Rightarrow \exists$ no real roots $(\because D < 0)$

24 **(a) :** Given $S = p + q = -p$ and product $pq = q$

$\Rightarrow q(p - 1) = 0$

$\Rightarrow q = 0, p = 1$

Now if $q = 0$ then $p = 0 \Rightarrow p = q$

If $p = 1$, then $p + q = -p$

$q = -2p$

$q = -2(1)$

$q = -2$

$\Rightarrow p = 1$ and $q = -2$

25. **(a) :** In such type of problem if sum of the squares of number is known and we needed product of numbers taken two at a time or needed range of the product of numbers taken two at a time. We start square of the sum of the numbers like

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow 2(ab + bc + ca) = (a + b + c)^2 - (a^2 + b^2 + c^2)$

$\Rightarrow ab + bc + ca = \dfrac{(a + b + c)^2 - 1}{2} < 1$

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CHAPTER 5

PERMUTATIONS AND COMBINATIONS

1. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is
(a) 5 (b) 10 (c) 8 (d) 7
(2013)

2. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is
(a) 630 (b) 879 (c) 880 (d) 629
(2012)

3. **Statement-1 :** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.
Statement-2 : The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.
(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(2011)

4. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$

and $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$.

Statement-1 : $S_3 = 55 \times 2^9$.
Statement-2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.
(a) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation of statement-1.
(b) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.
(c) Statement-1 is true, statement-2 is false.
(d) Statement-1 is false, statement-2 is true. *(2010)*

5. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is
(a) 3 (b) 36 (c) 66 (d) 108
(2010)

6. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangements is
(a) at least 500 but less than 750
(b) at least 750 but less than 1000
(c) at least 1000
(d) less than 500 *(2009)*

7. In a shop there are five types of ice-creams available. A child buys six ice-creams.
Statement-1 : The number of different ways the child can buy the six ice-creams is ${}^{10}C_5$.
Statement-2 : The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row.
(a) Statement-1 is true, Statement-2 is false
(b) Statemen-1 is false, Statement-2 is true
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(2008)

8. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?
(a) $7 \cdot {}^6C_4 \cdot {}^8C_4$ (b) $8 \cdot {}^6C_4 \cdot {}^7C_4$
(c) $6 \cdot 7 \cdot {}^8C_4$ (d) $6 \cdot 8 \cdot {}^7C_4$ *(2008)*

9. The sum of the series
${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots - \ldots\ldots + {}^{20}C_{10}$ is
(a) 0 (b) ${}^{20}C_{10}$ (c) $-{}^{20}C_{10}$ (d) $\frac{1}{2}\,{}^{20}C_{10}$.
(2007)

10. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote is
(a) 5040 (b) 6210 (c) 385 (d) 1110.
(2006)

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11. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number
(a) 602 (b) 603 (c) 600 (d) 601.
(2005)

12. The value of $^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is
(a) $^{56}C_4$ (b) $^{56}C_3$ (c) $^{55}C_3$ (d) $^{55}C_4$.
(2005)

13. How many ways are there to arrange the letters in the word GARDEN with the vowels in alphabetical order?
(a) 360 (b) 240 (c) 120 (d) 480.
(2004)

14. Then number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is
(a) 3^8 (b) 21 (c) 5 (d) $^8C_3$.
(2004)

15. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by
(a) 30 (b) $5! \times 4!$ (c) $7! \times 5!$ (d) $6! \times 5!$.
(2003)

16. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is
(a) 196 (b) 280 (c) 346 (d) 140.
(2003)

17. If ^nC_r denotes the number of combinations of n things taken r at a time, then the expression $^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals
(a) $^{n+2}C_{r+1}$ (b) $^{n+1}C_r$ (c) $^{n+1}C_{r+1}$ (d) $^{n+2}C_r$.
(2003)

18. Number greater than 1000 but less than 4000 is formed using the digits 0, 2, 3, 4 repetition allowed is
(a) 125 (b) 105 (c) 128 (d) 625.
(2002)

19. Five digit number divisible by 3 is formed using 0, 1, 2, 3, 4, 6 and 7 without repetition. Total number of such numbers are
(a) 312 (b) 3125 (c) 120 (d) 216.
(2002)

20. The sum of integers from 1 to 100 that are divisible by 2 or 5 is
(a) 3000 (b) 3050 (c) 3600 (d) 3250.
(2002)

21. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 are
(a) 216 (b) 375 (c) 400 (d) 720.
(2002)

Answer Key

1.	(a)	**2.**	(b)	**3.**	(c)	**4.**	(c)	**5.**	(d)	**6.**	(c)
7.	(b)	**8.**	(a)	**9.**	(d)	**10.**	(a)	**11.**	(d)	**12.**	(a)
13.	(a)	**14.**	(b)	**15.**	(d)	**16.**	(a)	**17.**	(a)	**18.**	(c)
19.	(d)	**20.**	(b)	**21.**	(d)						

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Explanations

1. (a) : **1st solution :** ${}^{n+1}C_3 - {}^{n}C_3 = 10$

$\Rightarrow \frac{(n+1)n(n-1)}{6} - \frac{n(n-1)(n-2)}{6} = 10$

$\Rightarrow 3n(n-1) = 60 \Rightarrow n(n-1) = 20 \Rightarrow n^2 - n - 20 = 0$

$\Rightarrow (n-5)(n+4) = 0 \quad \therefore n = 5$

2nd solution : ${}^{n+1}C_3 - {}^{n}C_3 = 10$

$\Rightarrow {}^{n}C_2 = 10 \Rightarrow \frac{n(n-1)}{2} = 10$

$\Rightarrow n^2 - n - 20 = 0. \quad \therefore n = 5$

Here we have used ${}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$

2. (b) : Number of ways in which one or more balls can be selected from 10 white, 9 green, 7 black balls is
$= (10 + 1)(9 + 1)(7 + 1) - 1$
$= 880 - 1 = 879$ ways

3. (c) : $x_1 + x_2 + x_3 + x_4 = 10$
The number of positive integral solution is ${}^{6+4-1}C_{4-1} = {}^{9}C_3$

It is the same as the number of ways of choosing any 3 balls from 9 different places.

4. (c) : $S_1 = \sum j(j-1)\,{}^{10}C_j$

$= \sum j(j-1)\cdot\frac{10(10-1)}{j(j-1)}\cdot{}^{8}C_{j-2}$

$= 9\times 10\sum_{j=2}^{10}{}^{8}C_{j-2} = 90\times 2^8$

$S_2 = \sum_{j=1}^{10} j\cdot{}^{10}C_j = 10\sum_{j=1}^{10}{}^{9}C_{j-1} = 10\times 2^9$

$S_3 = \sum_{j=1}^{10} j^2\cdot{}^{10}C_j = \sum_{j=1}^{10}(j(j-1)+j)\cdot{}^{10}C_j$

$= \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j + \sum_{j=1}^{10} j\cdot{}^{10}C_j$

$= 90\cdot 2^8 + 10\cdot 2^9 = (45 + 10)2^9 = 55\cdot 2^9.$

Then statement-1 is true and statement-2 is false.

5. (d) : The number of ways $= ({}^{3}C_2)\times({}^{9}C_2) = 3\times\frac{9\times 8}{2} = 108$

6. (c) : Out of 6 novels, 4 novels can be selected in ${}^{6}C_4$ ways.
Also out of 3 dictionaries, 1 dictionary can be selected in ${}^{3}C_1$ ways.
Since the dictionary is fixed in the middle, we only have to arrange 4 novels which can be done in 4! ways.
Then the number of ways $= {}^{6}C_4\cdot{}^{3}C_1\cdot 4!$

$= \frac{6\cdot 5}{2}0\cdot 3\cdot 24 = 1080 = 1080$

7. (b) : We have to find the number of integral solutions if $x_1 + x_2 + x_3 + x_4 + x_5 = 6$
and that equals ${}^{5+6-1}C_{5-1} = {}^{10}C_4$
Thus Statement-1 is false.
Number of different ways of arranging 6A's and 4B's in a row

$= \frac{\lfloor 10}{\lfloor 6\times\lfloor 4} = {}^{10}C_4 =$ Number of different ways the child can buy the six ice-creams.

$\therefore$ Statement-2 is true
So, Statement-1 is false, Statement-2 is true.

8. (a) : Leaving S, we have 7 letters M, I, I, I, P, P, I.
way of arranging them $= \frac{\lfloor 7}{\lfloor 2\lfloor 4} = 7\cdot 5\cdot 3$

And four S can be put in 8 places in ${}^{8}C_4$ ways.
The required number of ways $= 7\cdot 5\cdot 3\cdot{}^{8}C_4 = 7\cdot{}^{6}C_4\cdot{}^{8}C_4$.

9. (d) : $\because\ {}^{20}C_0 + {}^{20}C_1x + \ldots\ldots + {}^{20}C_{10}x^{10} + \ldots\ldots + {}^{20}C_{20}x^{20} = (1+x)^{20}$

After putting $x = -1$, we get

${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots + {}^{20}C_{10} - {}^{20}C_{11} - {}^{20}C_{12} + \ldots\ldots + {}^{20}C_{20} = 0$

$2({}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots - {}^{20}C_9) + {}^{20}C_{10} = 0$

${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots - {}^{20}C_9 + {}^{20}C_{10} = \frac{1}{2}{}^{20}C_{10}$

10. (a) : A voter can vote one candidate or two or three or four candidates
$\therefore$ Required number of ways
$= {}^{10}c_1 + {}^{10}c_2 + {}^{10}c_3 + {}^{10}c_4 = 385$

11. (d) : S A C H I N (S Fixed)
No. of word start with A = 5!
No. of word start with C = 5!
No. of word start with H = 5!
No. of word start with I = 5!
No. of word start with N = 5!
Total words = 5! + 5! + 5! + 5! + 5! = 5(5 !) = 600

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Now add the rank of SACHIN so required rank of SACHIN = 600 + 1 = 601.

12. (a) $^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$

Putting r = 6, 5, 4, 3, 2, 1 we get

$^{50}C_4 + {}^{50}C_3 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

$\left(\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}\right)$

$= {}^{51}C_4 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$
$= {}^{52}C_4 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$
$= {}^{53}C_4 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3 = {}^{54}C_4 + {}^{54}C_3 + {}^{55}C_3$
$= {}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4$

13. (a) : Number of letters = 6

Number of vowels = 2 namely A.E these alphabets can be arrange themselves by 2! ways

$\therefore$ Number of words = $\frac{6!}{2!} = 360$

14. (b) : (i) Each box must contain at least one ball since no box remains empty so we have the following cases

Box	Number of balls				
I	1,	1,	1,	2,	2,
I	1,	2,	3,	3,	2,
III	6,	5,	4,	3,	4,

$\therefore$ Number of ways

$3 \times \frac{1 \times 3!}{2!} + 3! \times 2$

$= 9 + 6 \times 2 = 21$

As [1, 1, 6] [2, 3, 4] [2, 2, 4] have case ways and [1, 2, 5] [1, 3, 4] have equal number of ways of arranging the balls in the different boxes.

(ii): Let the number of balls in the boxes are x, y, z respectively then $x + y + z = 8$ and no box is empty so each $x, y, z \geq 1$

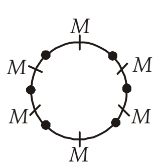
$\Rightarrow l + m + n + 3 = 8$ where $l = x - 1$, $m = y - 1$, $n = z - 1$

i.e. $(l + 1) + (m + 1) + (n + 1) = 8$ are non negative integers

$\therefore$ Required number of ways $= {}^{n+r-1}C_r$

$= {}^{3+5-1}C_5 = {}^7C_5 = {}^7C_2$

15. (d) : Number of women = 5



Number of men = 6

Number of ways of 6 men at a round Table is $(n - 1)! = (6 - 1)! = 5!$

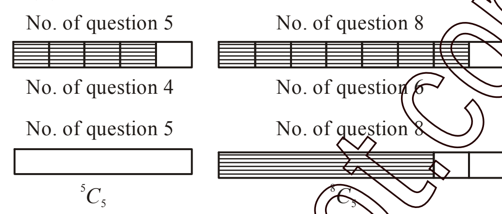
Now we left with six places between the men and there are 5 women, these 5 women can be arranged themselves by $^6P_5$ way.

$\therefore$ Required number of ways $= 5! \times {}^6P_5 = 5! \times 6!$

16. (a) : Case (i) :

Required ways for first case $= {}^5C_4 \times {}^8C_6 = 140$

Case (ii):



$\therefore$ Required ways for case (ii) $= {}^5C_5 \times {}^8C_5$

$= \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$

Total number of ways = 140 + 56
= 196

17. (a) : Consider $^nC_{r-1} + 2\,{}^nC_r + {}^nC_{r+1}$

$= ({}^nC_{r-1} + {}^nC_r) + ({}^nC_r + {}^nC_{r+1})$

$= {}^{n+1}C_r + {}^{n+1}C_{r+1}$

$= {}^{n+2}C_{r+1}$

18. (c) : Let number of digits formed x.

$\therefore$ $1000 < x < 4000$, which means left extreme digit will be either 2 or 3.

$\therefore$ Required numbers $= {}^2C_1 \times H\ T\ U$

where H = Hundred place

$= {}^2C_1 \times 4 \times 4 \times 4$ T = Ten's place

= 128 U = Unit place

19. (d)

20. (b) : Set of numbers divisible by 2 are 2, 4, 6, ...100

Set of numbers divisible by 5 are 5, 10, 15, ...100

Set of numbers divisible by 10 are 10, 20, 30, ...100

Now sum of numbers divisible by 2 is given by

$S_{50} = \frac{50}{2}[2 + 100]$ using $S_n = \frac{n}{2}[a + l]$

$S_{50} = 25[102]$

Similarly, $S_{20} = \frac{20}{2}[5 + 100] = 10 \times 105 = 1050$

and $S_{10} = \frac{10}{2}[10 + 100] = 5 \times 110$

$\therefore$ Required sum $= 25 \times 102 + 1050 - 550$
$= 25[102 + 42 - 22]$
$= 25 \times 122$
$= 3050$

21. (d) : Odd numbers are 1, 3, 5, 7

We have to fill up four places like *TH H T U*

(Case: If repetition is allow)

$^5C_1\ 6^2\ {}^4C_1 = 5 \times 6^2 \times 4$

$= 5 \times 36 \times 4$

$= 720$

CHAPTER 6

MATHEMATICAL INDUCTION AND ITS APPLICATION

1. **Statement-1 :** For every natural number $n \geq 2$,

$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\ldots+\frac{1}{\sqrt{n}} > \sqrt{n}$.

Statement-2 : For every natural number $n \geq 2$,

$\sqrt{n(n+1)} < n+1$.

(a) Statement-1 is true, Statement-2 is false

(b) Statemen-1 is false, Statement-2 is true

(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(2008)

2. Let $S(k) = 1 + 3 + 5 + \ldots + (2k - 1) = 3 + k^2$. Then which of the following is true?

(a) $S(k) \Rightarrow S(k - 1)$ (b) $S(k) \Rightarrow S(k + 1)$

(c) $S(1)$ is correct

(d) principle of mathematical induction can be used to prove the formula. *(2004)*

3. If $a_n = \sqrt{7+\sqrt{7+\sqrt{7+\ldots}}}$ having n radical signs then by methods of mathematical induction which is true

(a) $a_n > 7, \forall n \geq 1$ (b) $a_n > 3, \forall n \geq 1$

(c) $a_n < 4, \forall n \geq 1$ (d) $a_n < 3, \forall n \geq 1$.

(2002)

Answer Key

1. (d) **2.** (b) **3.** (b)

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Explanations

1. **(d) :** Statement-1

Let $P(n): \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+...+\frac{1}{\sqrt{n}}>\sqrt{n}$

Step 1 : For $n = 2$, $P(2): \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}>\sqrt{2}$ is true

Step 2 : Assume $P(n)$ is true for $n = k$, i.e.

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+...+\frac{1}{\sqrt{k}}>\sqrt{k} \quad ...(i)$$

Step 3 : For $n = k + 1$, we have to show that

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}>\sqrt{k+1} \quad ...(ii)$$

By Assumption step, we get

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+...+\frac{1}{\sqrt{k}}>\sqrt{k}$$

Adding $\frac{1}{\sqrt{k+1}}$ on both sides, we get

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+...+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}>\sqrt{k}+\frac{1}{\sqrt{k+1}} \quad ...(iii)$$

Statement-2

For $n = k$

$\sqrt{k(k+1)}<k+1$

$\Rightarrow \sqrt{k}\sqrt{k+1}<\sqrt{k+1}\sqrt{k+1} \Rightarrow \sqrt{k}<\sqrt{k+1}$

$\because \sqrt{k+1}>\sqrt{k}$ For $k \geq 2$

$\Rightarrow 1>\frac{\sqrt{k}}{\sqrt{k+1}} \Rightarrow \sqrt{k}>\frac{k}{\sqrt{k+1}}$

Multiplying by $\sqrt{k}$

$\Rightarrow \sqrt{k}>\frac{(k+1)-1}{\sqrt{k+1}} \Rightarrow \sqrt{k}>\sqrt{k+1}-\frac{1}{\sqrt{k+1}}$

$$\Rightarrow \sqrt{k}+\frac{1}{\sqrt{k+1}}>\sqrt{k+1} \quad ...(iv)$$

From (iii) & (iv)

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}>\sqrt{k+1}$$

hence (ii) is true for $n = k + 1$

hence $P(n)$ is true for $n \geq 2$

So, Statement-1 and Statement-2 are correct but Statement-2 is not explanation of Statement-1

2. **(b) :** $S(k) = 1 + 3 + ... + (2k - 1) = 3 + k^2$ ...(i)

When $k = 1$, L.H.S of $S(k) \neq$ R.H.S of $S(k)$

So $S(1)$ is not true.

Now $S(k + 1)$; $1 + 3 + 5 + ... + (2k - 1) + (2k + 1)$

$= 3 + (k + 1)^2$ ...(ii)

Let $S(k)$ is true $\therefore$ $1 + 3 + 5 + ... + (2k - 1) = k^2 + 3$

$\Rightarrow 1 + 3 + 5 + ... + (2k - 1) + (2k + 1)$

$= 3 + k^2 + 2k + 1 = (k + 1)^2 + 3$

$\Rightarrow S(k + 1)$ true $\therefore$ $S(k) \Rightarrow S(k + 1)$

3. **(b) :** $a_n = \sqrt{7 + a_n}$

$\Rightarrow a_n^2 - a_n - 7 = 0$

$\therefore a_n = \frac{1 \pm \sqrt{1 + 28}}{2}$

$= \frac{1 \pm \sqrt{29}}{2} > 3$

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CHAPTER 7

BINOMIAL THEOREM

1. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is
(a) 120 (b) 210 (c) 310 (d) 4 *(2013)*

2. If n is a positive integer, then $(\sqrt{3}+1)^{2n}-(\sqrt{3}-1)^{2n}$ is
(a) an even positive integer.
(b) a rational number other than positive integers.
(c) an irrational number.
(d) an odd positive integer. *(2012)*

3. The coefficient of x^7 in the expansion of $(1-x-x^2+x^3)^6$ is
(a) –144 (b) 132 (c) 144 (d) –132 *(2011)*

4. The remainder left out when $8^{2n}-(62)^{2n+1}$ is divided by 9 is
(a) 2 (b) 7 (c) 8 (d) 0 *(2009)*

5. **Statement-1 :** $\sum_{r=0}^{n}(r+1)\,{}^nC_r=(n+2)2^{n-1}$
Statement-2 : $\sum_{r=0}^{n}(r+1)\,{}^nC_r\,x^r=(1+x)^n+nx(1+x)^{n-1}$
(a) Statement-1 is true, Statement-2 is false
(b) Statemen-1 is false, Statement-2 is true
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1 *(2008)*

6. In the binomial expansion of $(a-b)^n$, $n \ge 5$, the sum of 5th and 6th terms is zero, then a/b equals
(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$ (c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$. *(2007)*

7. If the expansion in powers of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0+a_1x+a_2x^2+a_3x^3+\ldots$, then a_n is
(a) $\frac{b^n-a^n}{b-a}$ (b) $\frac{a^n-b^n}{b-a}$
(c) $\frac{a^{n+1}-b^{n+1}}{b-a}$ (d) $\frac{b^{n+1}-a^{n+1}}{b-a}$. *(2006)*

8. For natural numbers m, n if $(1-y)^m(1+y)^n=1+a_1y+a_2y^2+\ldots$, and $a_1=a_2=10$, then (m, n) is
(a) (20, 45) (b) (35, 20)
(c) (45, 35) (d) (35, 45). *(2006)*

9. If the coefficient of x^7 in $\left[ax^2+\left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax-\left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation
(a) $a+b=1$ (b) $a-b=1$
(c) $ab=1$ (d) $\frac{a}{b}=1$. *(2005)*

10. If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+x)^{3/2}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$ may be approximated as
(a) $3x+\frac{3}{8}x^2$ (b) $1-\frac{3}{8}x^2$
(c) $\frac{x}{2}-\frac{3}{8}x^2$ (d) $-\frac{3}{8}x^2$. *(2005)*

11. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same if α equals
(a) –3/10 (b) 10/3 (c) –5/3 (d) 3/5. *(2004)*

12. The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is
(a) $(-1)^{n-1}(n-1)^2$ (b) $(-1)^n(1-n)$
(c) $(n-1)$ (d) $(-1)^{n-1}n$. *(2004)*

13. If $s_n=\sum_{r=0}^{n}\frac{1}{{}^nC_r}$ and $t_n=\sum_{r=0}^{n}\frac{r}{{}^nC_r}$, then $\frac{t_n}{s_n}$ is equal to
(a) $n-1$ (b) $\frac{1}{2}n-1$ (c) $\frac{1}{2}n$ (d) $\frac{2n-1}{2}$. *(2004)*

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14. If x is positive, the first negative term in the expansion of $(1 + x)^{27/5}$ is
(a) 5^{th} term (b) 8^{th} term (c) 6^{th} term (d) 7^{th} term. *(2003)*

15. The number of integral terms in the expansion of $(\sqrt{3}+\sqrt[8]{5})^{256}$ is
(a) 33 (b) 34 (c) 35 (d) 32. *(2003)*

16. The positive integer just greater than $(1 + .0001)^{1000}$ is
(a) 4 (b) 5 (c) 2 (d) 3. *(2002)*

17. r and n are positive integers $r > 1$, $n > 2$ and coefficient of $(r + 2)^{th}$ term and $3r^{th}$ term in the expansion of $(1 + x)^{2n}$ are equal, then n equals
(a) $3r$ (b) $3r + 1$ (c) $2r$ (d) $2r + 1$. *(2002)*

18. The coefficients of x^p and x^q in the expansion of $(1 + x)^{p+q}$ are
(a) equal
(b) equal with opposite signs
(c) reciprocals of each other
(d) none of these. *(2002)*

19. If the sum of the coefficients in the expansion of $(a + b)^n$ is 4096, then the greatest coefficient in the expansion is
(a) 1594 (b) 792 (c) 924 (d) 2924. *(2002)*

Answer Key

1.	(b)	**2.**	(c)	**3.**	(a)	**4.**	(a)	**5.**	(c)	**6.**	(b)
7.	(d)	**8.**	(d)	**9.**	(c)	**10.**	(d)	**11.**	(a)	**12.**	(b)
13.	(c)	**14.**	(d)	**15.**	(a)	**16.**	(c)	**17.**	(c)	**18.**	(a)
19.	(c)										

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Explanations

1. **(b) :** $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$

$=\left\{\frac{(x^{1/3}+1)(x^{2/3}-x^{1/3}+1)}{x^{2/3}-x^{1/3}+1}-\frac{(\sqrt{x}+1)(\sqrt{x}-1)}{\sqrt{x}(\sqrt{x}-1)}\right\}^{10}$

$=(x^{1/3}-x^{-1/2})^{10}\ \therefore\ T_{r+1}=(-1)^r\,{}^{10}C_r x^{\frac{20-5r}{6}}$

Thus $\frac{20-5r}{6}=0 \Rightarrow r=4 \quad \therefore$ Term $={}^{10}C_4=210$.

2. **(c) :** $(\sqrt{3}+1)^{2n}-(\sqrt{3}-1)^{2n}$

$=2\left[{}^{2n}C_1\cdot(\sqrt{3})^{2n-1}+{}^{2n}C_3\cdot(\sqrt{3})^{2n-3}+....\right]$,

an irrational number.

3. **(a) :** $(1-x-x^2+x^3)^6=((1-x)(1-x^2))^6$

$=(1-x)^6(1-x^2)^6$

$=(1-{}^6C_1x+{}^6C_2x^2-{}^6C_3x^3+{}^6C_4x^4-{}^6C_5x^5+{}^6C_6x^6)$

$(1-{}^6C_1x^2+{}^6C_2x^4-{}^6C_3x^6+{}^6C_4x^8-{}^6C_5x^{10}+{}^6C_6x^{12})$

Coeff. of $x^7=(-{}^6C_1)(-{}^6C_3)+(-{}^6C_3)({}^6C_2)+(-{}^6C_5)(-{}^6C_1)$

$=6\cdot 20-20\cdot 15+6\cdot 6=120-300+36=-144$

4. **(a) :** Using Modulo Arithmetic

$8=-1$ (modulo 9) Also $62=-1$ (modulo 9)

$\Rightarrow 8^{2n}-(62)^{2n+1}=[(-1)^{2n}-(-1)^{2n+1}]$mod 9

$=(1+1)$mod 9 $=2$ mod 9 $\Rightarrow$ Remainder $=2$

5. **(c) :** $\sum_{r=0}^{n}(r+1)\,{}^nC_r=\sum_{r=0}^{n}r\cdot{}^nC_r+\sum_{r=0}^{n}{}^nC_r$

$=\sum_{r=0}^{n}r\cdot\frac{n}{r}\cdot{}^{n-1}C_{r-1}+\sum_{r=0}^{n}{}^nC_r$

$=n\cdot 2^{n-1}+2^n=2^{n-1}(n+2)$

Thus Statement-1 is true.

Again $\sum_{r=0}^{n}(r+1)\,{}^nC_rx^r=\sum_{r=0}^{n}r\cdot{}^nC_rx^r+\sum{}^nC_rx^r$

$=n\sum_{r=0}^{n}{}^{n-1}C_{r-1}x^r+\sum_{r=0}^{n}{}^nC_rx^r$

$=nx(1+x)^{n-1}+(1+x)^n$

Substitute $x=1$ in the above identity to get

$\sum(r+1)\,{}^nC_r=n\cdot 2^{n-1}+2^n$

Statement-2 is also true & explains Statement-1 also.

6. **(b) :** ${}^nC_4a^{n-4}(-b)^4=-\left({}^nC_5a^{n-5}(-b)^5\right)\Rightarrow\frac{a}{b}=\frac{n-4}{5}$.

7. **(d) :** From given

$\frac{1}{(1-ax)(1-bx)}=(1-ax)^{-1}(1-bx)^{-1}$

$=(a_0+a_1x+.....+a_nx^n+.....)(1-bx)^{-1}$

$=(1+ax+a^2x^2+......+a^{n-1}x^{n-1}+a^nx^n+......)$

$(1+bx+b^2x^2 .... +b^nx^n+.......)$

$\Rightarrow (a_0+a_1x+......+a_nx^n+.......)$

$=1+x(a+b)+x^2(a^2+ab+b^2)+x^3(a^3+a^2b+ab^2+b^3)+....+....+....+x^n(a^n+a^{n-1}b+a^{n-2}b^2+....+ab^{n-1}+b^n)+....$

On comparing the coefficient of x^n both sides we have

$a_n=a^n+a^{n-1}b+a^{n-2}b^2+......+a\,b^{n-1}+b^n$

$=\frac{(a^n+a^{n-1}b+a^{n-2}b^2+.....+ab^{n-1}+b^n)(b-a)}{b-a}$

(Multiplying and dividing by $b-a$)

$=\frac{b^{n+1}-a^{n+1}}{b-a}$.

8. **(d) :** $(1-y)^m(1+y)^n$

$=1+a_1y+a_2y_2+a_3y^3+......+$ ...... (*)

Differentiating w.r.t. y both sides of (*) we have

$-m(1-y)^{m-1}(1+y)^n+(1-y)^m n(1+y)^{n-1}$

$=a_1+2a_2y+3a_3y^2+4a_4y^3+.......$

$\Rightarrow n(1+y)^{n-1}(1-y)^m-m(1-y)^{m-1}(1+y)^n$

$=a_1+2a_2y+3a_2y^2+4a_4y^3+.......$ .....(**)

Again differentiating (**) with respect to y we have

$[n(n-1)(1+y)^{n-2}(1-y)^m+n(1+y)^{n-1}(-m)(1-y)^{m-1}]$

$-[m(1+y)^n(m-1)(1-y)^{m-2}(1-y)^{m-1}n(1+y)+n-1]$

$=2a_2+6a_3y+.......$ .... (***)

Now putting $y=0$ in (**) and (***) we get

$n-m=a_1=10$ (A)

and $m^2+n^2-(m+n)-2mn=2a_2=20$ ....(B)

Solving (A) and (B)

$n=45,\ m=35$

$\therefore\ (m,n)=(35,45)$

9. **(c) :** T_{r+1} of $\left(ax^2+\frac{1}{bx}\right)^{11}={}^{11}C_r(ax^2)^r\left(\frac{1}{bx}\right)^{11-r}$

T_{r+1} of $\left(ax-\frac{1}{bx^2}\right)^{11}={}^{11}C_r(ax)^r\left(-\frac{1}{bx^2}\right)^{11-r}$

$\therefore$ Coeff. of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}={}^{11}C_5\frac{a^6}{b^5}$

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and coefficient of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11} = {}^{11}C_6 \frac{a^5}{b^6}$

Now ${}^{11}C_5 \frac{a^6}{b^5} = {}^{11}C_6 \frac{a^5}{b^6} \quad \therefore \quad ab = 1.$

10. (d) : $\frac{(1+x)^{3/2} - \left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$

$= \frac{\left(1+\frac{3}{2}x+\frac{3}{2}\frac{1}{2}\frac{1}{2!}x^2+...\right)-\left(1+3\cdot\frac{1}{2}x+\frac{3\cdot 2}{2!}\cdot\frac{1}{4}x^2+....\right)}{(1-x)^{1/2}}$

$= -\frac{3}{8}x^2(1-x)^{-1/2} = -\frac{3}{8}x^2\left[1+\frac{1}{2}x+\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{1}{2!}x^2+...\right]$

$= -\frac{3}{8}x^2 + \text{higher powers of } x^2.$

11. (a) : Coefficient of middle term in $(1 + \alpha x)^4$ = coefficient of middle term in $(1 - \alpha x)^6$

$\therefore \ {}^4C_2\alpha^2 = {}^6C_3(-\alpha)^3 \Rightarrow \alpha = -\frac{3}{10}$

12. (b): $(1 + x)(1 - x)^n = (1 - x)^n + x(1 - x)^n$

$\therefore$ Coefficient of x^n is $= (-1)^n + (-1)^{n-1}\,{}^nC_1$

$= (-1)^n [1 - n]$

13. (c): $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$

$t_n = \sum_{r=0}^{n} \frac{n-(n-r)}{{}^nC_{n-r}} \Rightarrow t_n = n\sum_{r=0}^{n}\frac{1}{{}^nC_r} - \sum_{r=0}^{n}\frac{n-r}{{}^nC_{n-r}}$

$t_n = n\sum\frac{1}{{}^nC_r} - \sum_{r=0}^{n}\frac{r}{{}^nC_r}$ replacing $n - r$ by r

$t_n = ns_n - t_n$

$\therefore \frac{t_n}{s_n} = \frac{n}{2}$

14. (d) : General term in the expansion of $(1+x)^{\frac{27}{5}}$

$T_{r+1} = \frac{n(n-1).....(n-r+1)}{r!}x^r$

$\therefore n - r + 1 < 0 \Rightarrow \frac{27}{5}+1 < r \Rightarrow r > \frac{32}{5}$

$\Rightarrow r > 6$

15. (a) : $(3^{1/2} + 5^{1/8})^{256}$

$T_{r+1} = {}^{256}C_r\,(\sqrt{3})^{256-r}\,5^{\frac{r}{8}}$

For integral terms $\frac{256-r}{2}, \frac{r}{8}$ are both positive integer

$\therefore r = 0, 8, 16, ...256$

$\therefore 256 = 0 + (n - 1)8$ using $t_n = a + (n - 1)d$

$\therefore \frac{256}{8} = n - 1 \quad \therefore n = \frac{256}{8} + 1$

$n = 32 + 1 \Rightarrow n = 33$

16. (c) : Let $R = \left(1 + \frac{1}{10^4}\right)^{1000}$

$= 1 + 1000\left(\frac{1}{10^4}\right)^1 + 1000\,\frac{999}{2}\left(\frac{1}{10^4}\right)^2 + ... + \left(\frac{1}{10^4}\right)^{10^3}$

$< 1 + \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + ...\infty = \frac{10}{9}$

$\therefore \quad R < \frac{10}{9}$

$\therefore$ The positive integer just greater than $\frac{10}{9}$ is 2.

17. (c) : Given $r > 1$, $n > 2$ and Coefficient of T_{r+2} = Coefficient of T_{3r} in $(1 + x)^{2n}$

$\Rightarrow {}^{2n}C_{r+1} = {}^{2n}C_{3r-1}$

$\Rightarrow 3r - 1 = r + 1$ and $\left\{\begin{array}{l} 2n - 3r + 1 = r + 1 \\ \Rightarrow 2n = 4r \\ n = 2r \end{array}\right.$

$2r = 2$

$\because {}^nC_x = {}^nC_y$

$r = 1 \quad \Rightarrow x + y = n$

or $x = y$

18. (a) : In the expansion of $(1 + x)^{p+q}$

$T_{r+1} = {}^{p+q}C_r x^r$

$\therefore$ Coefficient of $x^p = {}^{p+q}C_p$

$= \frac{(p+q)!}{p!(p+q-p)!} = \frac{(p+q)!}{p!q!}$ ...(i)

Also coefficient of x^q in $(1 + x)^{p+q}$ is

$= {}^{p+q}C_q$

$= \frac{(p+q)!}{q!(p+q-q)!}$

$= \frac{(p+q)!}{q!p!}$ ....(ii)

$\therefore$ By (i) and (ii)

Coefficient of x^p in $(1 + x)^{p+q}$ = Coefficient of x^q in $(1 + x)^{p+q}$

19. (c) : Consider $(a + b)^n = C_0 a^n + C_1 a^{n-1}b + C_2 a^{n-2}b^2 + ... + C_n b^n$

Putting $a = b = 1$

$\therefore \quad 2^n = C_0 + C_1 + C_2 + ... + C_n$

$2^n = 4096 = 2^{12}$

$\Rightarrow \quad n = 12$ (even)

Now $(a + b)^n = (a + b)^{12}$

as $n = 12$ is even so coefficient of greatest term is

${}^nC_{\frac{n}{2}} = {}^{12}C_{\frac{12}{2}} = {}^{12}C_6$

$= \frac{12}{6}\times\frac{11}{5}\cdot\frac{10}{4}\frac{9}{3}\cdot\frac{8}{2}\cdot\frac{7}{1} = \frac{11\times 9\cdot 8\cdot 7}{3\cdot 2\cdot 1} = 11 \times 3\cdot 4 \cdot 7 = 924$

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CHAPTER 8

SEQUENCES AND SERIES

1. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ..., is

(a) $\frac{7}{9}(99-10^{-20})$ (b) $\frac{7}{81}(179+10^{-20})$

(c) $\frac{7}{9}(99+10^{-20})$ (d) $\frac{7}{81}(179-10^{-20})$ *(2013)*

2. **Statement 1 :** The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400) is 8000.

Statement 2 : $\sum_{k=1}^{n}(k^3-(k-1)^3)=n^3$ for any natural number n.

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement- 2 is a correct explanation for Statement 1. *(2012)*

3. If 100 times the 100th term of an A.P. with non-zero common difference equals the 50 times its 50th term, then the 150th term of this A.P. is

(a) 150 (b) zero
(c) – 150 (d) 150 times its 50th term *(2012)*

4. A man saves ₹ 200 in each of the first three months of his service. In each of the subsequent months his saving increases by ₹ 40 more than the saving of immediately previous month. His total saving from the start of service will be ₹ 11040 after

(a) 20 months (b) 21 months
(c) 18 months (d) 19 months *(2011)*

5. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an A.P. with common difference –2, then the time taken by him to count all notes is

(a) 24 minutes (b) 34 minutes
(c) 125 minutes (d) 135 minutes *(2010)*

6. The sum to infinity of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+......$ is

(a) 3 (b) 4 (c) 6 (d) 2 *(2009)*

7. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is

(a) 4 (b) – 4 (c) – 12 (d) 12 *(2008)*

8. The sum of the series $\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-....$ upto infinity is

(a) $e^{-\frac{1}{2}}$ (b) $e^{+\frac{1}{2}}$ (c) e^{-2} (d) e^{-1}. *(2007)*

9. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression is equals

(a) $\sqrt{5}$ (b) $\frac{1}{2}(\sqrt{5}-1)$

(c) $\frac{1}{2}(1-\sqrt{5})$ (d) $\frac{1}{2}\sqrt{5}$. *(2007)*

10. Let $a_1, a_2, a_3, ...$ be terms of an A.P. If $\frac{a_1+a_2+...+a_p}{a_1+a_2+...+a_q}=\frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals

(a) 41/11 (b) 7/2 (c) 2/7 (d) 11/41. *(2006)*

11. If $a_1, a_2, ..., a_n$ are in H.P., then the expression $a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$ is equal to

(a) $n(a_1 - a_n)$ (b) $(n - 1)(a_1 - a_n)$
(c) na_1a_n (d) $(n - 1)a_1a_n$. *(2006)*

12. If the coefficients of r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms in the binomial expansion of $(1 + y)^m$ are in A.P., then m and r satisfy the equation

(a) $m^2 - m(4r - 1) + 4r^2 + 2 = 0$
(b) $m^2 - m(4r + 1) + 4r^2 - 2 = 0$
(c) $m^2 - m(4r + 1) + 4r^2 + 2 = 0$
(d) $m^2 - m(4r - 1) + 4r^2 - 2 = 0$. *(2005)*

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13. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$ then x, y, z are in
(a) H.P.
(b) Arithmetic-Geometric progression
(c) A.P.
(d) G.P. *(2005)*

14. If $a_1, a_2, a_3, \ldots, a_n, \ldots$ are in G.P., then the determinant

$$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix},$$ is equal to

(a) 0 (b) 1 (c) 2 (d) 4. *(2005)*

15. The sum of the series

$1 + \frac{1}{4\cdot 2!} + \frac{1}{16\cdot 4!} + \frac{1}{64\cdot 6!} + \ldots\, \infty$ is

(a) $\frac{e+1}{\sqrt{e}}$ (b) $\frac{e-1}{\sqrt{e}}$ (c) $\frac{e+1}{2\sqrt{e}}$ (d) $\frac{e-1}{2\sqrt{e}}$. *(2005)*

16. Let T_r be the r^{th} term of an A.P. whose first term is a and common difference is d. If for some positive integers m, n, $m \neq n$, $T_m = \frac{1}{n}$, and $T_n = \frac{1}{m}$, then $a - d$ equals

(a) $1/mn$ (b) 1 (c) 0 (d) $\frac{1}{m} + \frac{1}{n}$. *(2004)*

17. The sum of first n terms of the series

$1^2 + 2\cdot 2^2 + 3^2 + 2\cdot 4^2 + 5^2 + 2\cdot 6^2 + \ldots$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd, the sum is

(a) $\frac{n(n+1)^2}{4}$ (b) $\frac{n^2(n+1)}{2}$

(c) $\frac{3n(n+1)}{2}$ (d) $\left[\frac{n(n+1)}{2}\right]^2$. *(2004)*

18. The sum of series $\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots$ is

(a) $\frac{(e-1)^2}{2e}$ (b) $\frac{(e^2-1)}{2e}$

(c) $\frac{(e^2-1)}{2}$ (d) $\frac{(e^2-2)}{e}$. *(2004)*

19. If the system of linear equations $x + 2ay + az = 0$, $x + 3by + bz = 0$, $x + 4cy + cz = 0$ has a non-zero solution, then a, b, c
(a) are in G.P. (b) are in H.P.
(c) satisfy $a + 2b + 3c = 0$ (d) are in A.P. *(2003)*

20. Let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a, b, c are in A.P., then $f'(a)$, $f'(b)$ and $f'(c)$ are in
(a) G.P. (b) H.P.
(c) Arithmetic-Geometric Progression
(d) A.P. *(2003)*

21. The sum of the series $\frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - \ldots$ upto ∞ is equal to
(a) $\log_e 2 - 1$ (b) $\log_e 2$
(c) $\log_e (4/e)$ (d) $2\log_e 2$. *(2003)*

22. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3)
(a) lie on an ellipse
(b) lie on a circle
(c) are vertices of a triangle
(d) lie on a straight line. *(2003)*

23. Let R_1 and R_2 respectively be the maximum ranges up and down on an inclined plane and R be the maximum range on the horizontal plane. Then, R_1, R, R_2 are in
(a) A.P.
(b) G.P.
(c) H.P.
(d) Arithmetic-Geometric Progression (A.G.P.). *(2003)*

24. If 1, $\log_9(3^{1-x} + 2)$, $\log_3[4\cdot 3^x - 1]$ are in A.P. then x equals
(a) $\log_3 4$ (b) $1 - \log_3 4$
(c) $1 - \log_4 3$ (d) $\log_4 3$. *(2002)*

25. $1^3 - 2^3 + 3^3 - 4^3 + \ldots + 9^3 =$
(a) 425 (b) -425 (c) 475 (d) -475. *(2002)*

26. Sum of infinite number of terms in GP is 20 and sum of their square is 100. The common ratio of GP is
(a) 5 (b) 3/5 (c) 8/5 (d) 1/5. *(2002)*

27. The value of $2^{1/4}\cdot 4^{1/8}\cdot 8^{1/16}\ldots\infty$ is
(a) 1 (b) 2 (c) 3/2 (d) 4. *(2002)*

28. Fifth term of a GP is 2, then the product of its 9 terms is
(a) 256 (b) 512 (c) 1024 (d) none. *(2002)*

Answer Key

1.	(b)	2.	(d)	3.	(b)	4.	(b)	5.	(b)	6.	(a)
7.	(c)	8.	(d)	9.	(b)	10.	(d)	11.	(d)	12.	(b)
13.	(a)	14.	(a)	15.	(c)	16.	(c)	17.	(b)	18.	(a)
19.	(b)	20.	(d)	21.	(c)	22.	(d)	23.	(c)	24.	(c)
25.	(a)	26.	(b)	27.	(b)	28.	(b)				

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Explanations

1. **(b) :** $t_r = \underbrace{0.77777....7}_{r \text{ terms}}$

$= \frac{7}{10} + \frac{7}{10^2} + ... + \frac{7}{10^r} = \frac{7}{9}(1-10^{-r})$

$S_{20} = \sum_{r=1}^{20} t_r = \frac{7}{9}\left(20 - \sum_{r=1}^{20} 10^{-r}\right) = \frac{7}{9}\left\{20 - \frac{1}{9}(1-10^{-20})\right\}$

$= \frac{7}{81}(179 + 10^{-20})$

2. **(d) :Statement 1 :**

$1 + (1 + 2 + 4) + (4 + 6 + 9) + ... + (361 + 380 + 400)$ is 8000

Statement 2 : $\sum_{k=1}^{n} (k^3 - (k-1)^3) = n^3$

Statement 1 : $T_1 = 1$, $T_2 = 7 = 8 - 1$,

$T_3 = 19 = 27 - 8 \Rightarrow T_n = n^3 - (n-1)^3$

$\therefore$ Statement 2 is a correct explanation of statement 1.

3. **(b) :** $100(a + 99d) = 50(a + 49d)$

$\Rightarrow a + 149d = 0$ *i.e.*, $T_{150} = 0$

4. **(b) :** Let it happens after n months.

$3 \times 200 + \frac{n-3}{2}\{2 \times 240 + (n-4)40\} = 11040$

$\Rightarrow \left(\frac{n-3}{2}\right)(480 + 40n - 160) = 11040 - 600 = 10440$

$\Rightarrow n^2 + 5n - 546 = 0 \Rightarrow (n + 26)(n - 21) = 0$

$\therefore n = 21$.

5. **(b) :** We have $a_1 + a_2 + ... + a_n = 4500$

$\Rightarrow a_{11} + a_{12} + ... + a_n = 4500 - 10 \times 150 = 3000$

$\Rightarrow 148 + 146 + ... = 3000$

$\Rightarrow \frac{n-10}{2} \cdot (2 \times 148 + (n - 10 - 1)(-21)) = 3000$

Let $n - 10 = m$

$\Rightarrow m \times 148 - m(m-1) = 3000$

$\Rightarrow m^2 - 149m + 3000 = 0$

$\Rightarrow (m - 24)(m - 125) = 0$

$\therefore m = 24, 125$,

giving $n = 34, 135$

But for $n = 135$, we have

$a_{135} = 148 + (135 - 1)(-2) = 148 - 268 < 0$

But a_{34} is positive.

Hence, $n = 34$ is the only answer.

6. **(a) :** Let $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...$ ...(1)

$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^1} + \frac{6}{3^1} + \frac{10}{3} + ...$ ...(2)

Subtracting (2) from (1), we get

$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...$

$= \frac{4}{3}\left[1 + \frac{1}{3} + ... \text{to } \infty\right] = \frac{4}{3} \cdot \frac{1}{1 - \frac{1}{3}} = \frac{4}{3} \cdot \frac{1}{2/3} = 2$

$\Rightarrow \frac{2}{3}S = 2 \therefore S = 3$

7. **(c) :** Let the G.P. be $a, ar, ar^2, ar^3, ...$

we have $a + ar = 12$ ...(1)

$ar^2 + ar^3 = 48$ ...(2)

on division we have

$\frac{ar^2(1+r)}{a(1+r)} = \frac{48}{12} \Rightarrow r^2 = 4$

$\therefore r = \pm 2$

But the terms are alternately positive and negative,

$\therefore r = -2$

Now $a = \frac{12}{1+r} = \frac{12}{1-2} = \frac{12}{-1} = -12$ From (1)

8. **(d) :** $\because e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - ....$ upto infinity

Then put $x = 1$, we get

$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - ....$ upto infinity.

9. **(b) :** Given, $a = ar + ar^2 \Rightarrow r^2 + r - 1 = 0$

$\Rightarrow r = \frac{-1+\sqrt{5}}{2}$.

10. **(d) :** Given $a_1, a_2, a_3, ...$ be terms of A.P.

$\frac{a_1 + a_2 + ...a_p}{a_1 + a_2 + ...a_q} = \frac{p^2}{q^2}$

$\Rightarrow \frac{\frac{p}{2}[2a_1 + (p-1)d]}{\frac{q}{2}[2a_1 + (q-1)d]} = \frac{p^2}{q^2}$

$\Rightarrow \frac{2a_1 + (p-1)d}{2a_1 + (q-1)d} = \frac{p}{q}$

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$\Rightarrow$ $[2a_1 + (p-1)d]q = p[2a_1 + (q-1)d]$

$\Rightarrow$ $2a_1(q-p) = d[(q-1)p - (p-1)q]$

$\Rightarrow$ $2a_1(q-p) = d(q-p) \Rightarrow 2a_1 = d$

$\Rightarrow$ $\dfrac{a_6}{a_{21}} = \dfrac{a_1+5d}{a_1+20d} = \dfrac{a_1+10a_1}{a_1+40a_1}$

$\Rightarrow$ $\dfrac{a_6}{a_{21}} = \dfrac{11}{41}$

11. **(d) :** Given $a_1, a_2, \ldots a_n$ are in H.P.

$\Rightarrow$ $\dfrac{1}{a_1}, \dfrac{1}{a_2} \ldots\ldots \dfrac{1}{a_n} \in$ A.P.

$\Rightarrow$ $\dfrac{1}{a_2} - \dfrac{1}{a_1} = d$

$\Rightarrow$ $a_1a_2 = \dfrac{a_1-a_2}{d} = \dfrac{a_1}{d} - \dfrac{a_2}{d}$ ... (i)

$a_2a_3 = \dfrac{a_2}{d} - \dfrac{a_3}{d}$ ... (ii)

$\vdots$

$a_{n-1}a_n = \dfrac{a_{n-1}}{d} - \dfrac{a_n}{d}$ ... (n)

Adding (i), (ii) .... (n) equations we get

$a_1a_2 + a_2a_3 + a_3a_4 + \ldots\ a_{n-1}\,a_n = \dfrac{a_1}{d} - \dfrac{a_n}{d}$

Also $\dfrac{1}{a_n} = \dfrac{1}{a_1} + (n-1)d$

$\Rightarrow$ $\dfrac{a_1-a_n}{d} = (n-1)a_1a_n$

$\therefore$ $a_1a_2 + a_2a_3 + \ldots\ a_{n-1}\,a_n = (n-1)a_1a_n.$

12. **(b) :** $T_{r+1} = {}^mC_r\,y^r$. $\therefore\ {}^mC_{r-1} + {}^mC_{r+1} = 2 \times {}^mC_r$

$\Rightarrow \dfrac{m!}{(r-1)!(m-r+1)!} + \dfrac{m!}{(r+1)!(m-r-1)!} = 2\dfrac{m!}{r!(m-r)!}$

$\Rightarrow \dfrac{r(r+1)}{(r+1)!(m-r+1)!} + \dfrac{(m-r+1)(m-r)}{(r+1)!(m-r+1)!}$

$= \dfrac{2(r+1)(m-r+1)}{(r+1)!(m-r+1)!}$

$\Rightarrow$ $r(r+1) + (m-r+1)(m-r) = 2(r+1)(m-r+1)$

$\Rightarrow r(r+1) + (m-r)^2 + m - r - 2(r+1)(m-(r-1))=0$

$\Rightarrow r(r+1) + m^2 + r^2 - 2mr + m - r + 2(r^2-1)$
$- 2m(r+1) = 0$

$\Rightarrow m^2 - m(4r+1) + 4r^2 - 2 = 0.$

13. **(a) :** Given $|a| < 1, |b| < 1, |c| < 1,\ a, b, c \in$ A.P.

and $\sum\limits_{n=0}^{\infty} a^n = \dfrac{1}{1-a},\ \sum\limits_{n=0}^{\infty} b^n = \dfrac{1}{1-b},\ \sum\limits_{n=0}^{\infty} c^n = \dfrac{1}{1-c}$

$\therefore x = \dfrac{1}{1-a},\ y = \dfrac{1}{1-b},\ z = \dfrac{1}{1-c}$

$\Rightarrow a = \dfrac{x-1}{x},\ b = \dfrac{y-1}{y},\ c = \dfrac{z-1}{z}$

as $a, b, c \in$ AP $\therefore 2b = a + c$

$2\left(\dfrac{y-1}{y}\right) = \dfrac{x-1}{x} + \dfrac{z-1}{z} \Rightarrow \dfrac{2}{y} = \dfrac{1}{x} + \dfrac{1}{z}$

$\Rightarrow x, y, z \in$ H.P.

14. **(a) :** Let t_r denote the r^{th} term of G.P. with first term b and common ratio R

$\therefore\ t_r = bR^{r-1}$. $\therefore\ \log r = \log b + (r-1)\log R$

Now from given determinant we have

$$\begin{vmatrix} \log b+(r-1)\log R & \log b+r\log R & \log b+(r+1)\log R \\ \log b+(r+2)\log R & \log b+(r+3)\log R & \log b+(r+4)\log R \\ \log b+(r+5)\log R & \log b+(r+6)\log R & \log b+(r+7)\log R \end{vmatrix}$$

$\Rightarrow$ using (applying $C_2 \to 2C_2 - (C_1 + C_3)$)

$$= \frac{1}{2}\begin{vmatrix} \log b+(r-1)\log R & 0 & \log b+\log R(r+1) \\ \log b+(r+2)\log R & 0 & \log b+(r+4)\log R \\ \log b+(r+5)\log R & 0 & \log b+(r+7)\log R \end{vmatrix}$$

$= \dfrac{1}{2} \times 0 = 0$

15. **(c) :** $1 + \dfrac{1}{4(2!)} + \dfrac{1}{16(4!)} + \dfrac{1}{64(6!)} + \ldots\infty$

$= 1 + \dfrac{1}{2^2\,2!} + \dfrac{1}{2^4(4!)} + \dfrac{1}{2^6(6!)} + \ldots\infty$

$= \dfrac{1}{2}\left[2\left(1 + \dfrac{1}{2^2\,2!} + \dfrac{1}{2^4(4!)} + \dfrac{1}{2^6(6!)} + \ldots\infty\right)\right]$

$= \dfrac{1}{2}\left[2\left(1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \dfrac{x^6}{6!} + \ldots\infty\right)\right]$

$= \dfrac{1}{2}\left[e^x + e^{-x}\right]$ where $x = 1/2$

$= \dfrac{1}{2}\left[e^{1/2} + e^{-1/2}\right] = \dfrac{e+1}{2\sqrt{e}}$

16. **(c) :** $T_m = a + (m-1)\,d = \dfrac{1}{n}$ ...(i)

$T_n = a + (n-1)\,d = \dfrac{1}{m}$ ...(ii)

Now $T_m - T_n = \dfrac{1}{n} - \dfrac{1}{m} = (m-n)\,d$

$\Rightarrow d = \dfrac{1}{mn}$ and $a = \dfrac{1}{mn}$

$\therefore\ a - d = 0$

17. **(b):** As S_n is needed for n is odd let $n = 2k + 1$

$\therefore\ S_n = S_{2k+1}$

= Sum up to $2k$ terms + $(2k+1)^{th}$ term

$= \dfrac{2k(2k+1)^2}{2}$ + last term

$= \dfrac{(n-1)n^2}{2} + n^2$ as $n = 2k + 1$ $= \dfrac{n^2(n+1)}{2}$

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18. (a) : $e^x + e^{-x} = 2\left[1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\infty\right]$

$\frac{e + e^{-1}}{2} - 1 = \frac{1}{2!} + \frac{1}{4!} + \ldots\infty$

$\frac{(e-1)^2}{2e} = \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots\infty$

19. (b) : For non trivial solution the determinant of the coefficient of various term vanish

i.e. $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$

$\Rightarrow (3bc - 4bc) - 2a(c - b) + a(4c - 3b) = 0$

$\Rightarrow \frac{2ac}{a + c} = b$

$\Rightarrow a, b, c \in$ H.P.

20. (d) : Let the polynomial be $f(x) = ax^2 + bx + c$

given $f(1) = f(-1) \Rightarrow b = 0$

$\therefore f(x) = ax^2 + c$

now $f'(x) = 2ax$

$\therefore f'(a) = 2a^2, f'(b) = 2ab, f'(c) = 2ac$

as $a, b, c \in$ A.P.

$\Rightarrow a^2, ab, ac \in$ A.P. $\Rightarrow 2a^2, 2ab, 2ac \in$ A.P.

$\Rightarrow f'(a), f'(b), f'(c) \in$ A.P.

21. (c): $s = \frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - \ldots.\infty$

Let $s_1 = \frac{1}{1\cdot 2} + \frac{1}{3\cdot 4} + \frac{1}{5\cdot 6} + \ldots.\infty$

$\therefore t_n = \frac{1}{(2n-1)(2n)} = \frac{1}{2n-1} - \frac{1}{2n}$

$\therefore s_n = \Sigma t_n = \sum\left(\frac{1}{2n-1} - \frac{1}{2n}\right)$

$= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \ldots..\infty$

$= \log_e 2$ ...(A)

Again $s_2 = \frac{1}{2\cdot 3} + \frac{1}{4\cdot 5} + \frac{1}{6\cdot 7} + \ldots.\infty$

$t'_n = \frac{1}{(2n)(2n+1)}$

$s_2 = \Sigma t_n' =$

$\sum \frac{1}{(2n)(2n+1)} = \sum\left(\frac{1}{2n} - \frac{1}{2n+1}\right)$

$= \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots.\infty$

$= -\left[-\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \ldots\infty\right]$

$= -[\log_e 2 - 1]$

$= 1 - \log_e 2$ ...(B)

Now $s = s_1 - s_2 = (A) - (B)$

$= \log_e 2 - 1 + \log_e 2 = \log(4/e)$

22. (d) : Let $x_1 = a \therefore x_2 = ar, x_3 = ar^2$

and $y_1 = b \therefore y_2 = br, y_3 = br^2$

Now $A(a, b), B(ar, br), C(ar^2, br^2)$

Now slope of $AB = \frac{b(1-r)}{a(1-r)} = \frac{b}{a}$ and

slope of $BC = \frac{br(1-r)}{ar(1-r)} = \frac{b}{a}$

as slope of AB = slope of BC

$\therefore AB \parallel BC$, but point B is common so

A, B, C are collinear.

23. (c) : Let θ be the angle of inclination of plane to horizontal and u be the velocity of projection of the projectile

$\therefore R_1 = \frac{u^2}{g(1+\sin\theta)}, R_2 = \frac{u^2}{g(1-\sin\theta)}$

$\therefore \frac{1}{R_1} + \frac{1}{R_2} = \frac{2g}{u^2} = \frac{2}{R}$

$\Rightarrow R_1, R, R_2 \in$ H.P.

24. (c) : As $1, \frac{1}{2}\log_3(3^{1-x} + 2), \log_3(4\cdot 3^x - 1) \in$ A.P.

$\Rightarrow \log_3(3^{1-x} + 2) = \log_3(4\cdot 3^x - 1) + 1$

$\Rightarrow 3^{1-x} + 2 = (4\cdot 3^x - 1) \times 3 \because \log_3 3 = 1.$

$\Rightarrow 3^{1-x} + 2 = 12\cdot 3^x - 3$

$\Rightarrow 3^x[(3^{1-x}) + 2] = 12\cdot 3^{2x} - 3\cdot 3^x$

(multiplying 3^x both side)

$\Rightarrow 12t^2 - 5t - 3 = 0$ where $t = 3^x$

$\Rightarrow (3t + 1)(4t - 3) = 0$

$\Rightarrow t = -1/3, t = 3/4$

$\Rightarrow 3^x = -1/3$ which is not possible

and $t = \frac{3}{4} \Rightarrow 3^x = \frac{3}{4}$

$\Rightarrow x\log_3 3 = \log_3 3 - \log_3 4$

(By taking logarithm at the base 3 both sides)

$\Rightarrow x = 1 - \log_3 4$

25. (a) : $(1^3 + 3^3 + 5^3 + \ldots.. + 9^3) - (2^3 + 4^3 + 6^3 + 8^3)$

$= (1^3 + 3^3 + 5^3 + \ldots. + 9^3) - 2^3(1^3 + 2^3 + 3^3 + 4^3)$

$= [1^3 + 3^3 + \ldots.. + (2n-1)^3]_{n = \text{odd} = 5} - 2^3[1^3 + 2^3 + \ldots. + n^3]_{n = \text{even} = 4}$

$= [2n(n+1)(n+2)(n+3) - 12n(n+1)(n+2) + 13n(n+1) - n]_{n = 5\,(\text{odd})} - 2^3\left[\frac{n^2(n+1)^2}{4}\right]_{n=4\,(\text{even})}$

(Remember this result)

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$= [2 \times 5 \times 6 \times 7 \times 8 - 12 \times 5 \times 6 \times 7 + 13 \times 5 \times 6 - 5] - 2^3\left(\frac{16 \times 25}{4}\right)$

$= [3750 - 5(505)] - 2 \times 16 \times 25$

$= 1225 - 800 = 425$

26. **(b) :** Let terms of G.P. are $a, ar, ar^2, \ldots.$

$\therefore S_\infty = \frac{a}{1-r}$ where a = first term, r = common ratio

$S_\infty = 20$

According to question $\frac{a}{1-r} = 20$

$\Rightarrow a = 20(1 - r)$ ...(i)

Also $\frac{a^2}{1-r^2} = 100$

$\Rightarrow \frac{a}{1-r} \cdot \frac{a}{1+r} = 100$

$\Rightarrow a = 5(1 + r)$ ...(ii)

Solving (i) and (ii) we have $r = 3/5$

27. **(b) :** $S_\infty = 2^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+\ldots\infty} = 2^\lambda$(say) ...(*)

Where $\lambda = \frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+\ldots.\infty$ ... (A)

$\frac{\lambda}{2} = 0 + \frac{1}{8}+\frac{2}{16}+\frac{3}{32}+\frac{4}{64}+\ldots\infty$ ... (B)

Now $(B) - (A) \Rightarrow \frac{\lambda}{2} = \frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\ldots$

$\frac{\lambda}{2} = \frac{a}{1-r} = \frac{1}{4} \times \frac{2}{1} \quad \therefore \lambda = 1$

so $S_\infty = 2^1$

28. **(b) :** Let first term of a G.P. is a and common ratio r

$\therefore t_5 = ar^4 = 2$

$\therefore \prod_{i=1}^{9} a_i = a \cdot ar \cdot ar^2 \ldots.ar^8$

$= a^9 r^{\frac{8\times 9}{2}}$

$= a^9 r^{36}$

$= (ar^4)^9$

$= 2^9 = 512$

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CHAPTER 9

DIFFERENTIAL CALCULUS

1. $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to

(a) 1/2 (b) 1 (c) 2 (d) –1/4

(2013)

2. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is

(a) 3000 (b) 3500 (c) 4500 (d) 2500

(2013)

3. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to

(a) $\frac{1}{2}$ (b) 1 (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

(2013)

4. Consider the function, $f(x) = |x - 2| + |x - 5|$, $x \in R$

Statement 1 : $f'(4) = 0$

Statement 2 : f is continuous in [2, 5], differentiable in (2, 5) and $f(2) = f(5)$.

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

(b) Statement 1 is true, Statement 2 is false.

(c) Statement 1 is false, Statement 2 is true.

(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1. *(2012)*

5. If $f : R \to R$ is a function defined by

$f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$, where $[x]$ denotes the greatest integer function, then f is

(a) discontinuous only at non-zero integral values of x.

(b) continuous only at $x = 0$.

(c) continuous for every real x.

(d) discontinuous only at $x = 0$. *(2012)*

6. Let $a, b \in R$ be such that the function f given by $f(x) = \ln|x| + bx^2 + ax$, $x \neq 0$ has extreme values at $x = -1$ and $x = 2$.

Statement 1 : f has local maximum at $x = -1$ and at $x = 2$.

Statement 2 : $a = \frac{1}{2}$ and $b = \frac{-1}{4}$

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

(b) Statement 1 is true, Statement 2 is false.

(c) Statement 1 is false, Statement 2 is true.

(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1. *(2012)*

7. A spherical balloon is filled with 4500 π cubic metres of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic metres per minute, then the rate (in metres per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is

(a) 2/9 (b) 9/2 (c) 9/7 (d) 7/9

(2012)

8. $\frac{d^2x}{dy^2}$ equals to

(a) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (b) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

(c) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (d) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ *(2011)*

9. $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$

(a) equals $-\sqrt{2}$ (b) equals $\frac{1}{\sqrt{2}}$

(c) does not exist (d) equals $\sqrt{2}$ *(2011)*

10. The values of p and q for which the function

$$f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$$

is continuous for all x in R, are

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(a) $p = -\frac{3}{2}, q = \frac{1}{2}$ (b) $p = \frac{1}{2}, q = \frac{3}{2}$
(c) $p = \frac{1}{2}, q = -\frac{3}{2}$ (d) $p = \frac{5}{2}, q = \frac{1}{2}$ *(2011)*

11. Let $f : (-1, 1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$, $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0)$ =
(a) 4 (b) -4 (c) 0 (d) -2 *(2010)*

12. Let $f : R \to R$ be a positive increasing function with $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x\to\infty} \frac{f(2x)}{f(x)} =$
(a) 1 (b) 2/3 (c) 3/2 (d) 3 *(2010)*

13. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} k - 2x, & \text{if } x \le -1 \\ 2x + 3, & \text{if } x > -1 \end{cases}$$

If f has a local minimum at $x = -1$, then a possible value of k is
(a) 1 (b) 0 (c) $-1/2$ (d) -1 *(2010)*

14. Let $f : R \to R$ be a continuous function defined by $f(x) = \frac{1}{e^x + 2e^{-x}}$.

Statement-1 : $f(c) = 1/3$, for some $c \in R$.

Statement-2 : $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all $x \in R$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation of Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true. *(2010)*

15. Let $f(x) = x|x|$ and $g(x) = \sin x$.

Statement-1 : gof is differentiable at $x = 0$ and its derivative is continuous at that point.

Statement-2 : gof is twice differentiable at $x = 0$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1. *(2009)*

16. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that $x = 0$ is the only real root of $P'(x) = 0$. If $P(-1) < P(1)$, then in the interval $[-1, 1]$:
(a) $P(-1)$ is not minimum but $P(1)$ is the maximum of P
(b) $P(-1)$ is the minimum but $P(1)$ is not the maximum of P
(c) neither $P(-1)$ is the minimum nor $P(1)$ is the maximum of P
(d) $P(-1)$ is the minimum and $P(1)$ is the maximum of P *(2009)*

17. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals
(a) 1 (b) log 2 (c) $-\log 2$ (d) -1 *(2009)*

18. Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$. Then which one of the following holds?
(a) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$
(b) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$
(c) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$
(d) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ *(2008)*

19. Let $f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1} & \text{if } x \ne 1 \\ 0 & \text{if } x = 1 \end{cases}$

Then which one of the following is true?
(a) f is differentiable at $x = 1$ but not at $x = 0$
(b) f is neither differentiable at $x = 0$ nor at $x = 1$
(c) f is differentiable at $x = 0$ and at $x = 1$
(d) f is differentiable at $x = 0$ but not at $x = 1$ *(2008)*

20. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have?
(a) 5 (b) 7
(c) 1 (d) 3 *(2008)*

21. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p + q)$ is
(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$
(c) $\sqrt{2}$ (d) 2. *(2007)*

22. The function $f : R - \{0\} \to R$ given by

$$f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$$

can be made continuous at $x = 0$ by defining $f(0)$ as
(a) 0 (b) 1
(c) 2 (d) -1. *(2007)*

23. Let $f : R \to R$ be a function defined by $f(x) = \min\{x + 1, |x| + 1\}$. Then which of the following is true?
(a) $f(x)$ is differentiable everywhere
(b) $f(x)$ is not differentiable at $x = 0$
(c) $f(x) \ge 1$ for all $x \in R$
(d) $f(x)$ is not differentiable at $x = 1$. *(2007)*

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24. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(c) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$. *(2007)*

25. A value of c for which conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3] is

(a) $\log_3 e$ (b) $\log_e 3$

(c) $2\log_3 e$ (d) $\frac{1}{2}\log_e 3$. *(2007)*

26 If $x^m \cdot y^n = (x + y)^{m+n}$, then dy/dx is

(a) $\frac{y}{x}$ (b) $\frac{x+y}{xy}$

(c) xy (d) $\frac{x}{y}$. *(2006)*

27. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is

(a) $\frac{3}{2}x^2$ (b) $\sqrt{\frac{x^3}{8}}$

(c) $\frac{1}{2}x^2$ (d) πx^2. *(2006)*

28. The set of points where $f(x) = \frac{x}{1+|x|}$ is differentiable, is

(a) $(-\infty, 0) \cup (0, \infty)$ (b) $(-\infty, -1) \cup (-1, \infty)$

(c) $(-\infty, \infty)$ (d) $(0, \infty)$. *(2006)*

29. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2, 0) and (3, 0) is

(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/6$ (c) $\pi/4$. *(2006)*

30. The function $g(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at

(a) $x = 2$ (b) $x = -2$

(c) $x = 0$ (d) $x = 1$. *(2006)*

31. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is

(a) 1/4 (b) 41 (c) 1 (d) 17/7. *(2006)*

32. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is

(a) $\frac{1}{18\pi}$ cm/min (b) $\frac{1}{36\pi}$ cm/min

(c) $\frac{5}{6\pi}$ cm/min (d) $\frac{1}{54\pi}$ cm/min. *(2005)*

33. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then $\lim_{x\to\alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to

(a) 0 (b) $\frac{a^2}{2}(\alpha-\beta)^2$

(c) $\frac{1}{2}(\alpha-\beta)^2$ (d) $\frac{-a^2}{2}(\alpha-\beta)^2$. *(2005)*

34. The normal to the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ at any point θ is such that

(a) it makes angle $\frac{\pi}{2}+\theta$ with x-axis

(b) it passes through the origin

(c) it is at a constant distance from the origin

(d) it passes through $\left(a\frac{\pi}{2}, -a\right)$. *(2005)*

35. If f is a real-valued differentiable function satisfying $|f(x) - f(y)| \le (x-y)^2$, $x, y \in R$ and $f(0) = 0$, then $f(1)$ equals

(a) 1 (b) 2 (c) 0 (d) – 1. *(2005)*

36. Let f be the differentiable for $\forall\ x$. If $f(1) = -2$ and $f'(x) \ge 2$ for [1, 6], then

(a) $f(6) < 8$ (b) $f(6) \ge 8$

(c) $f(6) = 5$ (d) $f(6) < 5$. *(2005)*

37. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h\to 0}\frac{1}{h}f(1+h) = 5$, then $f'(1)$ equals

(a) 4 (b) 3 (c) 6 (d) 5. *(2005)*

38. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ is

(a) ab (b) $2ab$ (c) a/b (d) $\sqrt{ab}$ *(2005)*

39. If $2a + 3b + 6c = 0$, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval

(a) (2, 3) (b) (1, 2) (c) (0, 1) (d) (1, 3). *(2004)*

40. A function $y = f(x)$ has a second order derivative $f''(x) = 6(x - 1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is

(a) $(x + 1)^3$ (b) $(x - 1)^3$

(c) $(x - 1)^2$ (d) $(x + 1)^2$. *(2004)*

41. Let $f(x) = \frac{1-\tan x}{4x-\pi}$, $x \ne \frac{\pi}{4}$, $x \in \left[0, \frac{\pi}{2}\right]$. $f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) –1. *(2004)*

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42. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b, are

(a) $a \in \mathbf{R}$, $b = 2$ (b) $a = 1$, $b \in \mathbf{R}$
(c) $a \in \mathbf{R}$, $b \in \mathbf{R}$(d) $a = 1$ and $b = 2$. *(2004)*

43. Let $f(a) = g(a) = k$ and their n^{th} derivatives $f^n(a)$, $g^n(a)$ exist and are not equal for some n. Further if

$$\lim_{x\to a}\frac{f(a)g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)} = 4,$$

then the value of k is

(a) 2 (b) 1
(c) 0 (d) 4. *(2003)*

44. $\lim_{x\to\pi/2}\frac{[1-\tan(x/2)][1-\sin x]}{[1+\tan(x/2)][\pi-2x]^3}$ is

(a) 0 (b) 1/32 (c) ∞ (d) 1/8. *(2003)*

45. The value of

$$\lim_{n\to\infty}\frac{1+2^4+3^4+....+n^4}{n^5}-\lim_{n\to\infty}\frac{1+2^3+3^3+....+n^3}{n^5}$$ is

(a) zero (b) 1/4 (c) 1/5 (d) 1/30. *(2003)*

46. The real number x when added to its inverse gives the minimum value of the sum at x equal to

(a) 1 (b) –1 (c) –2 (d) 2. *(2003)*

47. If $f(x)=\begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x\neq 0 \\ 0, & x=0 \end{cases}$, then $f(x)$ is

(a) continuous for all x, but not differentiable at $x = 0$
(b) neither differentiable not continuous at $x = 0$
(c) discontinuous everywhere
(d) continuous as well as differentiable for all x. *(2003)*

48. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals

(a) 1 (b) 2 (c) 1/2 (d) 3. *(2003)*

49. If $f(x) = x^n$, then the value of

$$f(1)-\frac{f'(1)}{1!}+\frac{f''(1)}{2!}-\frac{f'''(1)}{3!}+....+\frac{(-1)^n f^n(1)}{n!}$$ is

(a) 2^{n-1} (b) 0 (c) 1 (d) 2^n. *(2003)*

50. If $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=k$, the value of k is

(a) –1/3 (b) 2/3 (c) –2/3 (d) 0. *(2003)*

51. If $2a + 3b + 6c = 0$ ($a, b, c \in R$) then the quadratic equation $ax^2 + bx + c = 0$ has

(a) At least one in (0, 1)
(b) At least one root in [2, 3]
(c) At least one root in [4, 5]
(d) none of these *(2002)*

52. Let $f(2) = 4$ and $f'(2) = 4$ then $\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}$ equals

(a) 2 (b) –2
(c) –4 (d) 3. *(2002)*

53. $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+3}\right)^{\frac{1}{x}}$

(a) e^4 (b) e^2
(c) e^3 (d) 1. *(2002)*

54. If $f(x + y) = f(x) \cdot f(y) \ \forall \ x, y$ and $f(5) = 2$, $f'(0) = 3$, then $f'(5)$ is

(a) 0 (b) 1
(c) 6 (d) 2. *(2002)*

55. $\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is

(a) 1 (b) –1
(c) 0 (d) does not exist. *(2002)*

56. The maximum distance from origin of a point on the curve

$x = a\sin t - b\sin\left(\frac{at}{b}\right)$

$y = a\cos t - b\cos\left(\frac{at}{b}\right)$, both $a, b > 0$ is

(a) $a - b$ (b) $a + b$
(c) $\sqrt{a^2+b^2}$ (d) $\sqrt{a^2-b^2}$. *(2002)*

57. If $f(1) = 1$, $f'(1) = 2$, then $\underset{x\to 1}{\mathrm{Lt}}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ is

(a) 2 (b) 4
(c) 1 (d) 1/2. *(2002)*

58. $f(x)$ and $g(x)$ are two differentiable function on [0, 2] such that $f''(x)-g''(x)=0$, $f'(1)=2g'(1)=4$, $f(2)=3g(2)=9$ then $f(x) - g(x)$ at $x = 3/2$ is

(a) 0 (b) 2
(c) 10 (d) 5. *(2002)*

59. f is defined in [–5, 5] as

$f(x)=\begin{cases} x, \text{ if } x \text{ is rational and} \\ -x, \text{ if } x \text{ is rational. Then} \end{cases}$

(a) $f(x)$ is continuous at every x, except $x = 0$
(b) $f(x)$ is discontinuous at every x, except $x = 0$
(c) $f(x)$ is continuous everywhere
(d) $f(x)$ is discontinuous everywhere. *(2002)*

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60. $\lim_{x \to 0} \frac{\log x^n - [x]}{[x]}$, $n \in N$, ([x] denotes greatest integer less than or equal to x)
(a) has value – 1
(b) has value 0
(c) has value 1
(d) does not exist *(2002)*

61. If $y = (x + \sqrt{1 + x^2})^n$, then $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is
(a) n^2y
(b) $-n^2y$
(c) $-y$
(d) $2x^2y$. *(2002)*

Answer Key

1.	(c)	**2.**	(b)	**3.**	(d)	**4.**	(a)	**5.**	(c)	**6.**	(a)
7.	(a)	**8.**	(b)	**9.**	(c)	**10.**	(a)	**11.**	(b)	**12.**	(a)
13.	(d)	**14.**	(a)	**15.**	(b)	**16.**	(a)	**17.**	(d)	**18.**	(b)
19.	(b)	**20.**	(c)	**21.**	(c)	**22.**	(b)	**23.**	(a)	**24.**	(d)
25.	(c)	**26.**	(a)	**27.**	(c)	**28.**	(c)	**29.**	(a)	**30.**	(a)
31.	(b)	**32.**	(a)	**33.**	(b)	**34.**	(a), (c)	**35.**	(c)	**36.**	(b)
37.	(d)	**38.**	(b)	**39.**	(c)	**40.**	(b)	**41.**	(a)	**42.**	(b)
43.	(d)	**44.**	(b)	**45.**	(c)	**46.**	(a)	**47.**	(a)	**48.**	(b)
49.	(b)	**50.**	(b)	**51.**	(a)	**52.**	(c)	**53.**	(d)	**54.**	(c)
55.	**(a)**	**56.**	**(a)**	**57.**	(a)	**58.**	(d)	**59.**	(b)	**60.**	(d)
61.	(a)										

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Explanations

1. **(c) :** $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x(\tan 4x)}$

$= \lim_{x\to 0}\frac{1-\cos 2x}{x^2\cdot\left(\frac{\tan 4x}{x}\right)}(3+\cos x)$

$= \lim_{x\to 0}\left(\frac{2\sin^2 x}{x^2}\right)\cdot\left(\frac{x}{\tan 4x}\right)(3+\cos x)$

$= 2\times\frac{1}{4}\times 4 = 2$

2. **(b) :** $\frac{dP}{dx} = 100 - 12\sqrt{x}$

Integrating, we have, $dP = (100-12\sqrt{x})dx$

$P = 100x - 12\cdot\frac{2}{3}\cdot x^{3/2} + \lambda$

$P = 100x - 8x^{3/2} + \lambda$

$P(0) = 2000 = \lambda. \quad \therefore \quad \lambda = 2000$

$P(25) = 100\times 25 - 8\times 25^{3/2} + 2000 = 3500.$

3. **(d) :** $y = \sec(\tan^{-1}x)$

$\frac{dy}{dx} = \sec(\tan^{-1}x)\cdot\tan(\tan^{-1}x)\cdot\frac{1}{1+x^2}$

$\left.\frac{dy}{dx}\right|_{x=1} = \sqrt{2}\cdot 1\cdot\frac{1}{2} = \frac{1}{\sqrt{2}}$

4. **(a) :** $f(x) = |x-2| + |x-5|$

$\Rightarrow \quad f(x) = \begin{cases} 7-2x, & x<2 \\ 3, & 2\le x\le 5 \\ 2x-7, & x>5 \end{cases}$

Statement-1 : $f'(4) = 0$. True

Statement-2 : f is continuous in [2, 5], differentiable in (2, 5) and $f(2) = f(5)$. True

But Statement 2 is not a correct explanation for statement 1.

5. **(c) :** $f: R\to R,\ f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$

$= [x]\cos\left(\pi x - \frac{\pi}{2}\right) = [x]\sin\pi x$

Let n be an integer.

$\lim_{x\to n^+} f(x) = 0, \ \lim_{x\to n^-} f(x) = 0$

$\therefore \quad f(n) = 0$

$\Rightarrow f(x)$ is continuous for every real x.

6. **(a) :** $f(x) = \ln|x| + bx^2 + ax,\ x\ne 0$ has extreme values at $x = -1,\ x = 2$.

$\Rightarrow \ f'(x) = \frac{1}{x} + 2bx + a$

$f'(-1) = 0$ and $f'(2) = 0$ [Given]

$\Rightarrow \ -1-2b+a = 0 \ \Rightarrow \ b = -\frac{1}{4}$

and $\frac{1}{2} + 4b + a = 0 \ \Rightarrow \ a = \frac{1}{2}$

$f''(x) = -\frac{1}{x^2} + 2b = -\frac{1}{x^2} - \frac{1}{2} = -\left(\frac{1}{x^2}+\frac{1}{2}\right) < 0$

for all $x \in R - \{0\}$

$\Rightarrow f$ has a local maximum at $x = -1,\ x = 2$

$\therefore$ **Statement 1 :** f has local maxima at $x = -1,\ x = 2$

$\therefore$ **Statement 2 :** $a = \frac{1}{2},\ b = -\frac{1}{4}$

7. **(a) :** $\frac{dv}{dt} = -72\pi\ \text{m}^3/\text{min},\ v_0 = 4500\pi$

$v = \frac{4}{3}\pi r^3 \quad \therefore \quad \frac{dv}{dt} = \frac{4}{3}\pi\times 3r^2\times\frac{dr}{dt}$

After 49 min, $v = v_0 + 49\cdot\frac{dv}{dt} = 4500\pi - 49\times 72\pi$

$= 4500\pi - 3528\pi = 972\pi$

$\Rightarrow 972\pi = \frac{4}{3}\pi r^3 \Rightarrow r^3 = 243\times 3 = 3^6 \ \Rightarrow \ r = 9$

$\therefore \ -72\pi = 4\pi\times 81\times\frac{dr}{dt} \ \Rightarrow \ \frac{dr}{dt} = -\frac{18}{81} = -\frac{2}{9}$

Thus, radius decreases at a rate of $\frac{2}{9}$ m/min

8. **(b) :** $\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\}$

$= \frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\}\cdot\frac{dx}{dy} = -\left(\frac{dy}{dx}\right)^{-2}\frac{d^2y}{dx^2}\cdot\left(\frac{dy}{dx}\right)^{-1}$

$= -\left(\frac{dy}{dx}\right)^{-3}\frac{d^2y}{dx^2}$

9. **(c) :** Let $x = 2 + h$

$\lim_{h\to 0}\frac{\sqrt{1-\cos 2h}}{h} = \lim_{h\to 0}\frac{|\sin h|}{h}$

RHL = 1, LHL = –1. Thus limit doesn't exist.

10. **(a) :** $f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x<0 \\ q, & x=0 \\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}, & x>0 \end{cases}$

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$$\lim_{x\to 0^+} f(x) = \frac{1}{2}$$

Again, $\lim_{x\to 0^-} f(x) = \frac{\sin(p+1)+\sin x}{x} = p+2$

Now, $p + 2 = q = 1/2$

$\therefore$ $p = -3/2$, $q = 1/2$.

11. **(b) :** $g(x) = \{f(2f(x) + 2\}^2$

We have on differentiation with respect to x,

$g'(x) = 2f(2f(x) + 2) \cdot f'(2f(x) + 2) \cdot 2f'(x)$

Let $x = 0$

$g'(0) = 2f(2f(0) + 2) \cdot f'(2f(0) + 2) \cdot 2f'(0)$

$= 2f(0) \cdot f'(0) \cdot 2f'(0) = (-2)(1)(2) = -4.$

12. **(a) :** As f is a positive increasing function, we have $f(x) < f(2x) < f(3x)$

Dividing by $f(x)$ leads to $1 < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}$

As $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$, we have by Squeez theorem

or Sandwich theorem, $\lim_{x\to\infty} \frac{f(2x)}{f(x)} = 1.$

13. **(d) :** $\lim_{x\to 1+} f(x) = 1$

As $f(-1) = k + 2$

As f has a local miminum at $x = -1$

$f(-1^+) \ge f(-1) \ge f(-1^-) \Rightarrow 1 \ge k + 2$

$\Rightarrow k + 2 \le 1. \quad \therefore \quad k \le -1$

Thus $k = -1$ is a possible value.

14. **(a) :** Using A.M.-G.M. inequality,

$\frac{e^x + 2e^{-x}}{2} \ge \sqrt{e^x \cdot 2e^{-x}}$. Thus, $e^x + 2e^{-x} \ge 2\sqrt{2}$

Then $\frac{1}{e^x + 2e^{-x}} \le \frac{1}{2\sqrt{2}}$

As $\frac{1}{e^x + 2e^{-x}}$ is always positive, we have

$0 < \frac{1}{e^x + 2e^{-x}} \le \frac{1}{2\sqrt{2}}$

Observe that $f(0) = 1/3$. Thus such that $f(c) = 1/3$.

Using extreme-value theorem, we can say that as f is continuous, f will attain a value 1/3 at some point. Here we are able to identify the point as well.

15. **(b) :** $gof(x) = g(f(x)) = \sin(x|x|)$

$$= \begin{cases} -\sin x^2, & x < 0 \\ \sin x^2, & x \ge 0 \end{cases}$$

Let the composite function $gof(x)$ be denoted by $H(x)$.

Then $H(x) = \begin{cases} -\sin x^2, & x < 0 \\ \sin x^2, & x \ge 0 \end{cases}$

$$LH'(0) = \lim_{h\to 0^-} \frac{H(0-h) - H(0)}{-h}$$

$$= \lim_{h\to 0^-} \frac{-\sin h^2}{-h} = \lim_{h\to 0^-} \frac{\sin h^2}{h^2} \cdot h = 1 \cdot 0 = 0$$

$$RH'(0) = \lim_{h\to 0^+} \frac{H(0+h) - H(0)}{h}$$

$$= \lim_{h\to 0^+} \frac{\sin h^2 - 0}{h} = \lim_{h\to 0^+} \left(\frac{\sin h^2}{h^2}\right) h$$

$= 1 \cdot 0 = 0$

Thus $H(x)$ is differentiable at $x = 0$

Also $H'(x) = \begin{cases} -2x\cos x^2 & , x < 0 \\ 0 & , x = 0 \\ 2x\cos x^2 & , x > 0 \end{cases}$

$H'(x)$ is continuous at $x = 0$ for

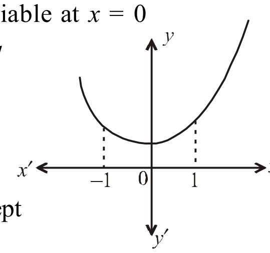
$H'(0) = LH'(0) = RH'(0)$

Again $H''(x) = \begin{cases} -2\cos x^2 + 4x^2 \sin x^2 & , x < 0 \\ 2\cos x^2 - 4x^2 \sin x^2 & , x \ge 0 \end{cases}$

$LH''(0) = -2$ and $RH''(0) = 2$

Thus $H(x)$ is NOT twice differentiable at $x = 0$

16. **(a) :** $P(x) = x^4 + ax^3 + bx^2 + cx + d$



$P'(x) = 4x^3 + 3ax^2 + 2bx + c$

$P'(0) = 0 \quad \Rightarrow \quad c = 0$

Also $P'(x) = x(4x^2 + 3ax + 2b)$

As $P'(x) = 0$ has no real roots except $x = 0$, we have

D of $4x^2 + 3ax + 2b$ is less than zero

i.e., $(3a)^2 - 4 \cdot 4 \cdot 2b < 0$

then $4x^2 + 3ax + 2b > 0 \quad \forall\ x \in R$

(If $a > 0$, $b^2 - 4ac < 0$ then $ax^2 + bx + c > 0\ \forall\ x \in R$)

So $P'(x) < 0$ if $x \in [-1, 0)$ *i.e.*, decreasing

and $P'(x) > 0$ if $x \in (0, 1]$ *i.e.*, increasing

Max. of $P(x) = P(1)$

But minimum of $P(x)$ doesn't occur at $x = -1$, *i.e.*, $P(-1)$ is not the minimum.

17. **(d) :** $x^{2x} - 2x^x \cot y - 1 = 0$ .....(i)

At $x = 1$ we have

$1 - 2\cot y - 1 = 0$

$\Rightarrow \cot y = 0 \quad \therefore \quad y = \pi/2$

Differentiating (i) w.r.t. x, we have

$$2x^{2x}(1+\ln x) - 2[x^x(-\mathrm{cosec}^2\, y)\frac{dy}{dx} + \cot y \cdot x^x(1+\ln x)] = 0$$

At $P(1, \pi/2)$ we have

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$$2(1+\ln 1)-2[1(-1)\left(\frac{dy}{dx}\right)_P+0]=0$$

$$\Rightarrow\ 2+2\left(\frac{dy}{dx}\right)_P=0$$

$$\therefore\ \left(\frac{dy}{dx}\right)_P=-1$$

18. (b) : Denote $x^3 - px + q$ by $f(x)$

i.e. $f(x) = x^3 - px + q$

Now for expression, $f'(x) = 0$, i.e. $3x^2 - p = 0$

$$x=-\sqrt{\frac{p}{3}},\sqrt{\frac{p}{3}}$$

$f''(x) = 6x$

$$f''\left(-\sqrt{\frac{p}{3}}\right)<0\ \Rightarrow\ f''\left(\sqrt{\frac{p}{3}}\right)>0$$

Thus maxima at $-\sqrt{\frac{p}{3}}$ and minima at $\sqrt{\frac{p}{3}}$.

19. (b) : By definition

$$f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h},\ \text{if the limit exists.}$$

$$\therefore\ \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$=\lim_{h\to 0}\frac{(1+h-1)\sin\frac{1}{(1+h-1)}-0}{h}=\lim_{h\to 0}\sin\frac{1}{h}$$

As the limit dosen't exist,

$\therefore$ it is not diffentiable at $x = 1$

Again $f'(0)=\lim_{h\to 0}\frac{f(h)-f(0)}{h}$, if the limit exists

$$\therefore\ \lim_{h\to 0}\frac{f(h)-f(0)}{h}=\lim_{h\to 0}\frac{(h-1)\sin\frac{1}{h-1}+\sin 1}{h}$$

But this limit dosen't exist. Hence it is not differentiable at $x = 0$.

20. (c) : Let $f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560$

$\therefore f'(x) = 7x^6 + 70x^4 + 48x^2 + 30$

$\Rightarrow f'(x) > 0\ \forall\ x \in R$

i.e. $f(x)$ is an strictly increasing function.

so it can have at the most one solution. It can be shown that it has exactly one solution.

21. (c) : Let $p = \cos\theta$, $q = \sin\theta$

$0 \le \theta \le \pi/2$

$p + q = \cos\theta + \sin\theta$

$\Rightarrow$ maximum value of $(p + q) = \sqrt{2}$

Second method

By using A.M $\ge$ G.M., $\frac{p^2+q^2}{2}\ge pq\ \Rightarrow\ pq\le\frac{1}{2}$

$(p + q)^2 = p^2 + q^2 + 2pq \Rightarrow (p+q)\le\sqrt{2}$.

22. (b) : $f(0)=\lim_{x\to 0}\left[\frac{1}{x}-\frac{2}{e^{2x}-1}\right]$

$$=\lim_{x\to 0}\frac{e^{2x}-1-2x}{x(e^{2x}-1)}\ \left(\frac{0}{0}\text{form}\right)$$

By using L' Hospital rule

$$f(0)=\lim_{x\to 0}\frac{2e^{2x}-2}{(e^{2x}-1)+2xe^{2x}}\ \left(\frac{0}{0}\text{form}\right)$$

Again use L' Hospital rule

$$f(0)=\lim_{x\to 0}\frac{4e^{2x}}{4e^{2x}+4xe^{2x}}=1$$

23. (a) : $f(x) = \min\{x + 1, |x| + 1\}$

$\Rightarrow f(x) = x + 1, x \in R$

Hence $f(x)$ is differentiable for all $x \in R$.

24. (d) : $f'(x)=\frac{1}{1+(\sin x+\cos x)^2}.(\cos x-\sin x)$

$$f'(x)=\frac{\cos x-\sin x}{2+\sin 2x}$$

If $f'(x) > 0$ then $f(x)$ is increasing function

For $-\frac{\pi}{2}<x<\frac{\pi}{4}, \cos x>\sin x$

Hence $y = f(x)$ is increasing in $\left(-\frac{\pi}{2},\frac{\pi}{4}\right)$.

25. (c) : By LMVT,

$$f'(c)=\frac{f(b)-f(a)}{b-a}=\frac{f(3)-f(1)}{3-1}$$

$$f'(c)=\frac{\log_e 3-\log_e 1}{2}=\frac{1}{2}\log_e 3$$

$$\Rightarrow\ \frac{1}{c}=\frac{1}{2}\log_e 3=\frac{1}{2\log_3 e}\quad\therefore\ c=2\log_3 e.$$

26. (a) : $x^m \times y^n = (x + y)^{m+n}$

Taking log both sides we get

$m \log x + n \log y = (m + n) \log(x + y)$

Differentiating w.r.t. x we get

$$\frac{m}{x}+\frac{n}{y}\frac{dy}{dx}=\frac{m+n}{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow\ \frac{dy}{dx}\left(\frac{n}{y}-\frac{m+n}{x+y}\right)=\frac{m+n}{x+y}-\frac{m}{x}$$

$$\Rightarrow\ \frac{dy}{dx}\left(\frac{nx+ny-my-ny}{y(x+y)}\right)=\frac{mx+nx-mx-my}{x(x+y)}$$

$$\Rightarrow\ \frac{dy}{dx}=\left(\frac{nx-my}{nx-my}\right)\frac{y}{x}=\frac{y}{x}\Rightarrow\frac{dy}{dx}=\frac{y}{x}.$$

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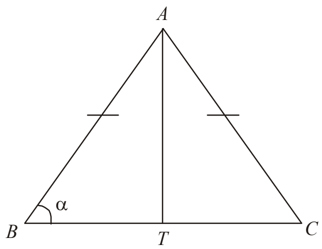
27. **(c) :** $AT = x \sin\alpha$

$BT = x\cos\alpha$

Area of triangle

ABC $= \frac{1}{2}$ base × height



$= \frac{1}{2}(2BT)(AT)$

$= \frac{1}{2}(2x^2\cos\alpha\sin\alpha)$

$= \frac{1}{2}x^2\sin 2\alpha \le \frac{1}{2}x^2$ as $-1 \le \sin 2\alpha \le 1$

$\therefore$ Maximum are of $\Delta ABC = \frac{1}{2}x^2$

28. **(c) :** Given $f(x) = \frac{x}{1+|x|}$

$$\Rightarrow f(x) = \begin{cases} \frac{x}{1-x}, & x < 0 \\ \frac{x}{1+x}, & x \ge 0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} \frac{1}{(1-x)^2}, & x < 0 \\ \frac{1}{(1+x)^2}, & x \ge 0 \end{cases}$$

$f'(x)$ is finite quantity $\forall\ x \in R$

$\therefore f'(x)$ is differentiable $\forall\ x \in (-\infty, \infty)$

29. **(a) :** Given equation $y = x^2 - 5x + 6$, given points (2, 0), (3, 0)

$\therefore \frac{dy}{dx} = 2x - 5$

say $m_1 = \left(\frac{dy}{dx}\right)_{\substack{x=2\\y=0}} = 4 - 5 = -1$

and $m_2 = \left(\frac{dy}{dx}\right) at_{\substack{x=3\\y=0}} = 6 - 5 = 1$

since $m_1 m_2 = -1$

$\Rightarrow$ tangents are at right angle i.e. $\frac{\pi}{2}$

30. **(a) :** Let $g(x) = \frac{x}{2} + \frac{2}{x}$

$\therefore \quad g'(x) = \frac{1}{2} - \frac{2}{x^2}$

for maxima and minima $g'(x) = 0 \Rightarrow x = \pm 2$

Again $g''(x) = \frac{4}{x^3} > 0$ for $x = 2$

< 0 for $x = -2 \quad \therefore x = 2$ is point of minima

31. **(b) :** For the range of the expression

$\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7} = y = \frac{ax^2 + bx + c}{px^2 + qx + r}$,

[find the solution of the inequality

$Ay^2 + By + K \ge 0$]

where $A = q^2 - 4pr = -3$, $B = 4ar + 4PC - 2bq = 126$

$K = b^2 - 4ac = -123$

i.e., solve $-3y^2 + 126 + y - 123 \ge 0$

$\Rightarrow \quad 3y^2 - 126y + 123 \le 0$

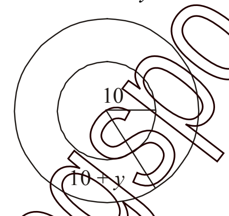
$\Rightarrow \quad y^2 - 42y + 41 \le 0$

$\Rightarrow \quad (y-1)(y-42) \le 0$

$\Rightarrow \quad 1 \le y \le 42$

$\Rightarrow$ maximum value of y is 42

32. **(a) :** $v = \frac{4}{3}\pi(y+10)^3$ where y is thickness of ice



$\Rightarrow \frac{dv}{dt} = 4\pi(10+y)^2\frac{dy}{dt}$

$\left(\frac{dy}{dt}\right)_{\text{at } y=5} = \frac{50}{4\pi(15)^2} \qquad \left(\text{as } \frac{dv}{dt} = 50 \text{ cm}^3/\text{min.}\right)$

$= \frac{1}{18\pi}$ cm/min.

33. **(b) :** As α is root of $ax^2 + bx + c = 0$

$\therefore a\alpha^2 + b\alpha + c = 0$. Now

$$\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$$

$$= \lim_{x\to\alpha}\frac{2\sin^2\left(\frac{ax^2+bx+c}{2}\right)}{(x-\alpha)^2}$$

$$= \lim_{x\to\alpha}\frac{2\sin^2\left[\frac{a(x-\alpha)(x-\beta)}{2}\right]}{a^2\left[\frac{(x-\alpha)^2(x-\beta)^2}{4}\right]} \times \frac{a^2(x-\beta)^2}{4}$$

$$= \lim_{x\to\alpha}\left[\frac{\sin\left(\frac{a(x-\alpha)(x-\beta)}{2}\right)}{\frac{a(x-\alpha)(x-\beta)}{2}}\right]^2 \times \frac{a^2(x-\beta)^2}{2}$$

$$= 1 \times \frac{a^2}{2}(\alpha-\beta)^2.$$

34. **(a), (c) :** $\frac{dy}{dx} = \frac{dy}{d\theta}\cdot\frac{d\theta}{dx} = \tan\theta$ = slope of tangent

$\therefore$ Slope of normal to the curve $= -\cot\theta$

$= \tan(90 + \theta)$.

Now equation of normal to the curve

$$[y - a(\sin\theta - \theta\cos\theta)] = -\frac{\cos\theta}{\sin\theta}(x - a(\cos\theta + a\sin\theta))$$

$\Rightarrow x\cos\theta + y\sin\theta = a(1)$

Now distance from (0, 0) to $x\cos\theta + y\sin\theta = a$ is

distance $(d) = \frac{(0+0-a)}{1}$

$\therefore$ distance is constant $= a$.

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35. **(c) :** Given $|f(x)-f(y)| \le (x-y)^2$

$$\lim_{x\to y}\left|\frac{f(x)-f(y)}{x-y}\right| \le \lim_{x\to y}|x-y|$$

$\Rightarrow |f'(x)| \le 0,\ f(x) = 0 \quad (|f'(x)| < 0,\ \text{not possible})$

$\Rightarrow f(x) = k$ (by integration)

$\Rightarrow f(x) = 0 \qquad \because f(0) = 0$

$\Rightarrow f(x)\ (\forall\ x \in R) = 0 \quad \therefore f(1) = 0.$

36. **(b) :** Let if possible $f'(x) = 2$ for
$\Rightarrow f(x) = 2x + c$ (Integrating both side w.r.t. x)
$\therefore f(1) = 2 + c,\ -2 = 2 + c$
$\Rightarrow c = -4 \quad \therefore f(x) = 2x - 4$
$\therefore f(6) = 2 \times 6 - 4 = 8 \quad \therefore f(6) \ge 8.$

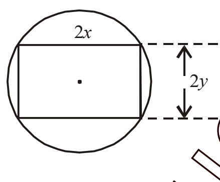
37. **(d) :** As $f(x)$ is differenatiable at $x = 1$

$5 = \lim_{h\to 0}\frac{f(1+h)}{h}$ assumes 0/0 form

$5 = \lim_{h\to 0}\frac{f'(1)}{1} \quad \therefore f'(1) = 5.$

38. **(b) :** Any point on the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$



is $(a\cos\theta,\ b\sin\theta)$ so the area of rectangle inscribed in the ellipse is given by
$A = (2a\cos\theta)(2b\sin\theta)$

$\therefore\ A = 2ab\sin 2\theta \Rightarrow \frac{dA}{d\theta} = 4ab\cos 2\theta$

Now for maximum area

$\frac{dA}{d\theta} = 0 \Rightarrow \theta = \frac{\pi}{4}$ and $\left(\frac{d^2A}{d\theta^2}\right)_{\theta=\pi/4} = -8ab\sin 2\theta$

as $\frac{d^2A}{d\theta^2} < 0.\quad \therefore$ Area is maximum for $\theta = \pi/4.$

$\therefore$ sides of rectangle are $\frac{2a}{\sqrt{2}},\ \frac{2b}{\sqrt{2}}$

Required area $= 2ab.$

39. **(c) :** Let $f(x) = \frac{ax^3}{3}+\frac{bx^2}{2}+cx$

Note : In such type of problems we always consider $f(x)$ as the integration of L.H.S of the given equation without constant.

Here integration of $ax^2 + bx + c$ is $\frac{ax^3}{3}+\frac{bx^2}{2}+cx$ called it by $f(x)$. Now use the intervals in $f(x)$ if $f(x)$ satisfies the given condition then at least one root of the equation $ax^2 + bx + c = 0$ must lies in that interval.

Now $f(x) = \frac{ax^3}{3}+\frac{bx^2}{2}+cx$

$f(0) = 0$ and $f(1) = \frac{a}{3}+\frac{b}{2}+c$

$= \frac{2a+3b+6c}{6}$
$= 0$ given $2a + 3b + 6c = 0$
$\therefore\ x = 0$ and $x = 1$ are roots of

$f(x) = \frac{ax^3}{3}+\frac{bx^2}{2}+cx = 0$

$\therefore$ at least one root of the equation $ax^2 + bx + c = 0$ lies in (0, 1)

40. **(b) :** Given $f''(x) = 6(x-1)$

$\Rightarrow \quad f'(x) = \frac{6(x-1)^2}{2}+c$

$\Rightarrow \quad 3 = 3 + c \qquad \left[\begin{array}{l}\because f(x) = y = 3x+5\\ f'(x) = 3\ \forall\ x \in R\end{array}\right]$

$\Rightarrow \quad c = 0$

so $\quad f'(x) = 3(x-1)^2$

$\Rightarrow \quad f(x) = (x-1)^3 + c_1$ as curve passes through (2, 1)

$\Rightarrow \quad 1 = (2-1)^3 + c_1 \Rightarrow c_1 = 0$

$\therefore \quad f(x) = (x-1)^3$

41. **(a) :** $\text{Lt}_{x\to\pi/4}\frac{1-\tan x}{4x-\pi}$ putting $4x - \pi = t$

$$\therefore\ \text{Lt}_{x\to\pi/4}\frac{(1-\tan x)\times(1+\tan x)}{(1+\tan x)\left[-4\left(\frac{\pi}{4}-x\right)\right]}$$

$$\text{Lt}_{x\to\pi/4} -\frac{\tan\left(\frac{\pi}{4}-x\right)\times(1+\tan x)}{4\left(\frac{\pi}{4}-x\right)} = -1/2$$

42. **(b) :** $e^2 = \left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}$ (1^∞ form)

$e^2 = e^{\text{Lt}_{x\to\infty}\left[1+\frac{a}{x}+\frac{b}{x^2}-1\right](2x)}$

$e^2 = e^{2a}$

$\Rightarrow 2a = 2 \ \therefore\ a = 1$ and $b \in R$

43. **(d) :** $\text{Lt}_{x\to a}\frac{f(a)\,g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)} = 4$

$\Rightarrow \text{Lt}_{x\to a}\frac{f(a)[g(x)-f(x)]}{g(x)-f(x)} = 4 \Rightarrow \text{Lt}_{x\to a} f(a) = 4$

$k = 4$

44. **(b) :** $\text{Lt}_{x\to\pi/2}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)(1-\sin x)}{4.\left(\frac{\pi-2x}{4}\right)(\pi-2x)^2}$

$$\text{Lt}_{x\to\pi/2}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{4.\left(\frac{\pi}{4}-\frac{x}{2}\right)}\ \frac{1-\cos\left(\frac{\pi}{2}-x\right)}{(\pi-2x)^2}$$

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$$\underset{x\to\pi/2}{\text{Lt}} \frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{4.\left(\frac{\pi}{4}-\frac{x}{2}\right)} \cdot \frac{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{4^2\left(\frac{\pi-2x}{4}\right)^2} = \frac{1}{4}\times\frac{2}{16} = \frac{1}{32}$$

45. (c) : $\underset{n\to\infty}{\text{Lt}} \dfrac{1^4+2^4+3^4+\ldots+n^4}{n^5} - \underset{n\to\infty}{\text{Lt}} \dfrac{1^3+2^3+\ldots+n^3}{n^5}$

$$= \underset{n\to\infty}{\text{Lt}} \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30\cdot n^5} - \underset{n\to\infty}{\text{Lt}} \frac{n^2(n+1)^2}{4\times n^5}$$

$$\left[\text{Using } 1^4+2^4+\ldots+n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{n(n+1)(6n^3+9n^2+n-1)}{30}\right]$$

$$= \frac{6}{30} - 0$$

$$= \frac{1}{5}$$

46. (a) : $f(x) = x + 1/x$

$f'(x) = 1 - 1/x^2$ and $f''(x) = \dfrac{2}{x^3}$

now $f'(x) = 0$

$\Rightarrow x = \pm 1 \quad \therefore \; f''(1) > 0$

$\Rightarrow x = 1$ is point of minima.

47. (a) : Given $f(x) = \begin{cases} x\, e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$\underset{x\to 0^+}{\text{Lt}} f(x) = \underset{x\to 0^+}{\text{Lt}} x e^{-2/x} = 0$ ...(A)

and $\underset{x\to 0^-}{\text{Lt}} f(x) = \underset{x\to 0^-}{\text{Lt}} e^{\left[-\frac{1}{x}+\frac{1}{x}\right]} = 0$ ...(B)

As $\underset{x\to 0}{\text{LHL}} = \underset{x\to 0}{\text{RHL}}$ $\therefore f(x)$ is continuous at $x = 0$

Again RHD at $x = 0$ is

$$\underset{x\to 0^+}{\text{Lt}} \frac{(0+h)e^{-\left(\frac{1}{h}+\frac{1}{h}\right)} - 0}{h} = 0$$

also we have L.H.D at $x = 0$

is $\dfrac{(0-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)} - 0}{-h} = 1$

so L.H.D $\neq$ R.H.D at $x = 0$

$\therefore f(x)$ is non differentiable at $x = 0$

48. (b) : For maximum and minima $f'(x) = 0$

$\Rightarrow 6x^2 - 18ax + 12a^2 = 0$ and $f''(x) = 12x - 18a$

$f'(x) = 0$

$\Rightarrow x = a, 2a$ and $f''(a) < 0$ and $f''(2a) > 0$

Now $p = a$ and $q = 2a$ and $p^2 = q$

$\Rightarrow a^2 = 2a \Rightarrow a^2 - 2a = 0$

$\Rightarrow a(a-2) = 0 \Rightarrow a = 0,\ a = 2$

49. (b) : $f(x) = x^n \;\therefore f(1) = 1 = {}^nC_0$

$\therefore f'(x) = nx^{n-1}$ so $-f'(1) = -n = -{}^nC_1$

$f''(x) = n(n-1)x^{n-2}$ so $\dfrac{f''(1)}{2!} = \dfrac{n(n-1)}{2!} = {}^nC_2$

$f^n(x) = n(n-1)\ldots 1 \;\therefore \dfrac{f^n(1)(-1)^n}{n!} = (-1)^n\, {}^nC_n$

$\therefore f(1) - \dfrac{f'(1)}{1!} + \dfrac{f''(1)}{2!} - \dfrac{f'''(1)}{3!} + \ldots + \dfrac{(-1)^n f^n(1)}{n!} =$

${}^nC_0 - {}^nC_1 + {}^nC_2 \ldots + (-1)^n C_n$

Now $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$ ...(i)

Putting $x = -1$ in both side of (i) we get

$0 = C_0 - C_1 + C_2 - C_3 + \ldots$

50. (b) : $\underset{x\to 0}{\text{Lt}} \dfrac{\log(3+x) - \log(3-x)}{x} = k$

$$\therefore k = \underset{x\to 0}{\text{Lt}} \frac{\log\left(1+\frac{x}{3}\right) - \log\left(1-\frac{x}{3}\right)}{x}$$

$$k = \underset{x\to 0}{\text{Lt}} \frac{\log\left(1+\frac{x}{3}\right)}{\frac{x}{3}\times 3} + \underset{x\to 0}{\text{Lt}} \frac{\log\left(1-\frac{x}{3}\right)}{-\frac{x}{3}\times 3}$$

$$k = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

51. (a) : Let us consider $f(x) = \dfrac{ax^3}{3} + \dfrac{bx^2}{2} + cx$

$\therefore f(0) = 0$ and $f(1) = \dfrac{a}{3} + \dfrac{b}{2} + c$

$= \dfrac{2a+3b+6c}{6} = 0$ given.

As $f(0) = f(1) = 0$ and $f(x)$ is continuous and differentiable also in [0, 1].

$\therefore$ By Rolle's theorem $f'(x) = 0$

$\Rightarrow ax^2 + bx + c = 0$ has at least one root in the interval (0, 1).

52. (c) : $\underset{x\to 2}{\text{Lt}} \dfrac{xf(2) - 2f(x) + 2f(2) - 2f(2)}{x-2}$

$\underset{x\to 2}{\text{Lt}} \dfrac{(x-2)f(2) - 2[f(x) - f(2)]}{x-2}$

$= \lim\limits_{x\to 2} [f(2) - 2f'(x)]$

$= 4 - 2\times 4 = -4$

53. (d) : We have $\lim\limits_{x\to\infty} \left(\dfrac{x^2+5x+3}{x^2+x+3}\right)^{1/x}$

$$= \lim_{x\to\infty} \left(\frac{1+\frac{5}{x}+\frac{3}{x^2}}{1+\frac{1}{x}+\frac{3}{x^2}}\right)^{1/x} = 1^0 = 1$$

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54. (c) : Given $f(x+y) = f(x)f(y)$

$\therefore\ f(0+0) = (f(0))^2$

$\Rightarrow\ f(0) = 0$ or $f(0) = 1$ but $f(0) \neq 0$

Now $f'(x) = \underset{h\to 0}{\text{Lt}} \dfrac{f(x+h)-f(x)}{h} = \underset{h\to 0}{\text{Lt}} \dfrac{f(x)f(h)-f(x)}{h}$

$$f'(x) = f(x)\underset{h\to 0}{\text{Lt}} \frac{f(h)-1}{h}$$

$$\therefore\ f'(0) = f(0)\underset{h\to 0}{\text{Lt}} \frac{f(h)-1}{h}$$

$$3 = \underset{h\to 0}{\text{Lt}} \frac{f(h)-1}{h}\quad (\because f(0) = 1)$$

Now $f'(x) = f(x)\underset{h\to 0}{\text{Lt}} \dfrac{f(h)-1}{h}$

$\therefore\ f'(5) = f(5)\times 3 = 2\times 3 = 6$

55. (a) : $\underset{x\to 0}{Lt} \dfrac{\sqrt{2}\sqrt{\sin^2 x}}{x\sqrt{2}} = \underset{x\to 0}{Lt} \dfrac{\sin x}{x} = 1.$

56. (a) :

Let $A(0,0)$, $B(x, y) = \begin{cases} a\sin t - b\sin\dfrac{at}{b} = x \\ a\cos t - b\cos\dfrac{at}{b} = y \end{cases}$

$$\therefore\ \sqrt{x^2+y^2} = AB = \sqrt{a^2(\sin^2 t + \cos^2 t) + b^2\left(\sin^2\left(\frac{at}{b}\right) + \cos^2\left(\frac{at}{b}\right)\right) - 2ab\cos\left(t - \frac{at}{b}\right)}$$

$= \sqrt{a^2+b^2-2ab\cos\alpha}$ (since $|\cos\alpha| \le 1$)

$\le \sqrt{a^2+b^2-2ab} = a - b.$

57. (a) : $\underset{x\to 1}{\text{Lt}} \dfrac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ (0/0 form)

$$\underset{x\to 1}{\text{Lt}} \frac{1}{2\sqrt{f(x)}}\times\frac{2\sqrt{x}}{1}\times f'(x)$$

$$= \frac{2\times 1\times 2}{2} = 2$$

58. (d) : As $f''(x) - g''(x) = 0$

$\Rightarrow\ f'(x) - g'(x) = k$

$f'(1) - g'(1) = k\ \therefore\ k = 2$

So $f'(x) - g'(x) = 2$

$\Rightarrow\ f(x) - g(x) = 2x + k_1$

$f(2) - g(2) = 4 + k_1$

$k_1 = 2$

So $f(x) - g(x) = 2x + 2$

$\therefore\ [f(x)-g(x)]_{x=\frac{3}{2}} = \dfrac{2\times 3}{2} + 2 = 5$

59. (b)

60. (d) : $\underset{x\to 0}{\text{Lt}} \dfrac{\log x^n - [x]}{[x]} = \underset{x\to 0}{\text{Lt}} \dfrac{n\log x}{[x]} - 1$

which does not exist as $\underset{x\to 0}{\text{Lt}} \dfrac{\log x}{[x]}$ does not exist

61. (a) : $y_1 = n\left[x+\sqrt{1+x^2}\right]^{n-1}\left[1+\dfrac{x}{\sqrt{1+x^2}}\right]$

$y_1 = n\left[x+\sqrt{1+x^2}\right]^n\cdot\dfrac{1}{\sqrt{1+x^2}}$

$y_1 = \dfrac{ny}{\sqrt{1+x^2}}\ \left(y_1 = \dfrac{dy}{dx}\right)$

$\Rightarrow y_1^2(1+x^2) = n^2y^2$

$\Rightarrow y_1^2(2x) + (1+x^2)(2y_1y_2) = 2yy_1n^2$

$\Rightarrow y_2(1+x^2) + xy_1 = n^2y$

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CHAPTER 10

INTEGRAL CALCULUS

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1. If $\int f(x)dx = \psi(x)$ then $\int x^5 f(x^3)dx$ is equal to

(a) $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)dx + C$

(b) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + C$

(c) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^3\psi(x^3)dx\right] + C$

(d) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^3)dx\right] + C$ *(2013)*

2. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t|\,dt, x \in R$, which are parallel to the line $y = 2x$, are equal to

(a) ± 2 (b) ± 3

(c) ± 4 (d) ± 1 *(2013)*

3. The area (in sq. units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis and lying in the first quadrant is

(a) 36 (b) 18 (c) $\frac{27}{4}$ (d) 9 *(2013)*

4. **Statement-I :** The value of the integral $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$ is equal to $\pi/6$.

Statement-II : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$.

(a) Statement-I is true, Statement-II is true, Statement-II is **not** a correct explanation for Statement-I.

(b) Statement-1 is true, Statement-II is false.

(c) Statement-I is false, Statement-II is true.

(d) Statement-I is true Statement-II is true, Statement-II is a **correct** explanation for Statement-I. *(2013)*

5. If the integral $\int\frac{5\tan x}{\tan x - 2}dx = x + a\ln|\sin x - 2\cos x| + k$, then a is equal to

(a) 1 (b) 2 (c) -1 (d) -2 *(2012)*

6. If $g(x) = \int_0^x \cos 4t\,dt$, then $g(x+\pi)$ equals

(a) $g(x) - g(\pi)$ (b) $g(x)\cdot g(\pi)$

(c) $\frac{g(x)}{g(\pi)}$ (d) $g(x) + g(\pi)$ *(2012)*

7. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is

(a) $\frac{20\sqrt{2}}{3}$ (b) $10\sqrt{2}$

(c) $20\sqrt{2}$ (d) $\frac{10\sqrt{2}}{3}$ *(2012)*

8. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is

(a) $\frac{1}{2}\ln 18$ (b) ln 18

(c) 2 ln 18 (d) ln 9 *(2012)*

9. Let I be the purchase value of an equipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T-t)$; where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is

(a) $I - \frac{k(T-t)^2}{2}$ (b) e^{-kT}

(c) $T^2 - \frac{I}{k}$ (d) $I - \frac{kT^2}{2}$ *(2011)*

10. If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then $y(\ln 2)$ is equal to

(a) 13 (b) -2

(c) 7 (d) 5 *(2011)*

11. The value of $\int_0^1\frac{8\log(1+x)}{1+x^2}dx$ is

(a) $\frac{\pi}{2}\log 2$ (b) log2

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(c) $\pi \log 2$ (d) $\frac{\pi}{8}\log 2$ *(2011)*

12. For $x \in \left(0, \frac{5\pi}{2}\right)$, define $f(x) = \int_0^x \sqrt{t}\sin t\,dt$ Then f *has*

(a) local minimum at π and local maximum at 2π.
(b) local maximum at π and local minimum at 2π.
(c) local maximum at π and 2π.
(d) local minimum at π and 2π. *(2011)*

13. The area of the region enclosed by the curves $y = x$, $x = e$, $y = 1/x$ and the positive x-axis is

(a) 3/2 square units (b) 5/2 square units
(c) 1/2 square units (d) 1 square units *(2011)*

14. Let $p(x)$ be a function defined on R such that $p'(x) = p'(1 - x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)\,dx$ equals

(a) $\sqrt{41}$ (b) 21 (c) 41 (d) 42 *(2010)*

15. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is

(a) $4\sqrt{2} - 2$ (b) $4\sqrt{2} + 2$
(c) $4\sqrt{2} - 1$ (d) $4\sqrt{2} + 1$ *(2010)*

16. The area of the region bounded by the parabola $(y - 2)^2 = x - 1$, the tangent to the parabola at the point (2, 3) and the x-axis is

(a) 6 (b) 9
(c) 12 (d) 3 *(2009)*

17. $\int_0^{\pi}[\cot x]dx$, where [.] denotes the greatest integer function, is equal to

(a) 1 (b) -1
(c) $-\pi/2$ (d) $\pi/2$ *(2009)*

18. The value of is $\sqrt{2}\int \frac{\sin x\,dx}{\sin\left(x - \frac{\pi}{4}\right)}$

(a) $x - \log\left|\cos\left(x - \frac{\pi}{4}\right)\right| + c$ (b) $x + \log\left|\cos\left(x - \frac{\pi}{4}\right)\right| + c$
(c) $x - \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$ (d) $x + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$ *(2008)*

19. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to

(a) $\frac{4}{3}$ (b) $\frac{5}{3}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$ *(2008)*

20. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}}dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}}dx$.

Then which one of the following is true?

(a) $I > \frac{2}{3}$ and $J < 2$ (b) $I > \frac{2}{3}$ and $J > 2$
(c) $I < \frac{2}{3}$ and $J < 2$ (d) $I < \frac{2}{3}$ and $J > 2$ *(2008)*

21. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is

(a) 1/6 (b) 1/3 (c) 2/3 (d) 1. *(2007)*

22. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$ equals

(a) $\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$ (b) $\log\tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + C$
(c) $\frac{1}{2}\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$ (d) $\frac{1}{2}\log\tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + C$. *(2007)*

23. The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2 - 1}} = \frac{\pi}{2}$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $2\sqrt{2}$ (c) 2 (d) π. *(2007)*

24. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{\log t}{1 + t}dt$,
Then $F(e)$ equals

(a) 1 (b) 2 (c) 1/2 (d) 0. *(2007)*

25. The value of $\int_1^a [x]\,f'(x)\,dx$, $a > 1$, where $[x]$ denotes the greatest integer not exceeding x is

(a) $a\,f(a) - \{f(1) + f(2) + ... + f([a])\}$
(b) $[a]\,f(a) - \{f(1) + f(2) + ... + f([a])\}$
(c) $[a]\,f([a]) - \{f(1) + f(2) + ... + f(a)\}$
(d) $a\,f([a]) - \{f(1) + f(2) + ... + f(a)\}$. *(2006)*

26. $\int_{-3\pi/2}^{-\pi/2} [(x + \pi)^3 + \cos^2(x + 3\pi)]dx$ is equal to

(a) $\frac{\pi^4}{32}$ (b) $\frac{\pi^4}{32} + \frac{\pi}{2}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{2} - 1$. *(2006)*

27. $\int_0^{\pi} x f(\sin x)dx$ is equal to

(a) $\pi\int_0^{\pi} f(\cos x)dx$ (b) $\pi\int_0^{\pi} f(\sin x)dx$
(c) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)dx$ (d) $\pi\int_0^{\pi/2} f(\cos x)dx$. *(2006)*

28. The value of the integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9 - x} + \sqrt{x}}dx$ is

(a) 1/2 (b) 3/2 (c) 2 (d) 1. *(2006)*

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29. $\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}+....+\frac{1}{n}\sec^2 1\right]$ equals

(a) $\frac{1}{2}\text{cosec}1$ (b) $\frac{1}{2}\sec 1$
(c) $\frac{1}{2}\tan 1$ (d) $\tan 1$. *(2005)*

30. The value of $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx$, $a > 0$, is

(a) $\pi/2$ (b) $a\pi$ (c) 2π (d) π/a *(2005)*

31. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is

(a) 1 : 2 : 3 (b) 1 : 2 : 1
(c) 1 : 1 : 1 (d) 2 : 1 : 2. *(2005)*

32. The area enclosed between the curve $y = \log_e(x + e)$ and the coordinate axes is

(a) 2 (b) 1 (c) 4 (d) 3. *(2005)*

33. If $I_1=\int_0^1 2^{x^2}dx$, $I_2=\int_0^1 2^{x^3}dx$, $I_3=\int_1^2 2^{x^2}dx$ and $I_4=\int_1^2 2^{x^3}dx$ then

(a) $I_1 > I_2$ (b) $I_2 > I_1$ (c) $I_3 > I_4$ (d) $I_3 = I_4$. *(2005)*

34. Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x-axis and the ordinates $x=\frac{\pi}{4}$ and $x=\beta>\frac{\pi}{4}$ is $\left(\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is

(a) $\left(\frac{\pi}{4}-\sqrt{2}+1\right)$ (b) $\left(\frac{\pi}{4}+\sqrt{2}-1\right)$
(c) $\left(1-\frac{\pi}{4}+\sqrt{2}\right)$ (d) $\left(1-\frac{\pi}{4}-\sqrt{2}\right)$. *(2005)*

35. Let $F : R \to R$ be a differentiable function having $f(2) = 6$, $f'(2)=\left(\frac{1}{48}\right)$. Then $\lim_{x\to 2}\int_6^{f(x)}\frac{4t^3}{x-2}dt$ equals

(a) 36 (b) 24 (c) 18 (d) 12. *(2005)*

36. $\int\left\{\frac{(\log x-1)}{1+(\log x)^2}\right\}^2 dx$ is equal to

(a) $\frac{x}{x^2+1}+C$ (b) $\frac{\log x}{(\log x)^2+1}+C$
(c) $\frac{x}{(\log x)^2+1}+C$ (d) $\frac{xe^x}{1+x^2}+C$. *(2005)*

37. The area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and the x-axis is

(a) 3 (b) 2 (c) 1 (d) 4. *(2004)*

38. If $f(x)=\frac{e^x}{1+e^x}$, $I_1=\int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$ and $I_2=\int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$, then the value of $\frac{I_2}{I_1}$ is

(a) –1 (b) –3 (c) 2 (d) 1. *(2004)*

39. If $\int_0^{\pi} x f(\sin x)dx = A\int_0^{\pi/2} f(\sin x)dx$, then A is

(a) $\pi/4$ (b) π (c) 0 (d) 2π. *(2004)*

40. The value of $I=\int_0^{\pi/2}\frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}dx$ is

(a) 2 (b) 1 (c) 0 (d) 3. *(2004)*

41. The value of $\int_{-2}^{3}|1-x^2|\,dx$ is

(a) 7/3 (b) 14/3 (c) 28/3 (d) 1/3. *(2004)*

42. $\int\frac{dx}{\cos x-\sin x}$ is equal to

(a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|+C$

(b) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right|+C$

(c) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+C$

(d) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+C$. *(2004)*

43. If $\int\frac{\sin x}{\sin(x-\alpha)}dx = Ax+B\log\sin(x-\alpha)+C$, then value of (A, B) is

(a) $(-\sin\alpha, \cos\alpha)$ (b) $(\cos\alpha, \sin\alpha)$
(c) $(\sin\alpha, \cos\alpha)$ (d) $(-\cos\alpha, \sin\alpha)$ *(2004)*

44. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}e^{r/n}$ is

(a) $1 - e$ (b) $e - 1$ (c) e (d) $e + 1$. *(2004)*

45. Let $\frac{d}{dx}F(x)=\left(\frac{e^{\sin x}}{x}\right)$, $x>0$.

If $\int_1^4\frac{3}{x}e^{\sin x^3}dx = F(k)-F(1)$, then one of the possible values of k is

(a) 16 (b) 63
(c) 64 (d) 15. *(2003)*

46. The value of $\lim_{x\to 0}\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}$ is

(a) 2 (b) 1 (c) 0 (d) 3 *(2003)*

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47. The value of the integral $I=\int_0^1 x(1-x)^n dx$ is

(a) $\frac{1}{n+2}$ (b) $\frac{1}{n+1}-\frac{1}{n+2}$

(c) $\frac{1}{n+1}+\frac{1}{n+2}$ (d) $\frac{1}{n+1}$. *(2003)*

48. If $f(a+b-x)=f(x)$, then $\int_a^b x f(x)dx$ is equal to

(a) $\frac{a+b}{2}\int_a^b f(x)dx$ (b) $\frac{b-a}{2}\int_a^b f(x)dx$

(c) $\frac{a+b}{2}\int_a^b f(a+b-x)dx$

(d) $\frac{a+b}{2}\int_a^b f(b-x)dx$. *(2003)*

49. If $f(y)=e^y$, $g(y)=y$; $y>0$ and

$$F(t)=\int_0^t f(t-y)g(y)dy\text{, then}$$

(a) $F(t)=e^t-(1+t)$ (b) $F(t)=te^t$

(c) $F(t)=te^{-t}$ (d) $F(t)=1-e^t(1+t)$. *(2003)*

50. Let $f(x)$ be a function satisfying $f'(x)=f(x)$ with $f(0)=1$ and $g(x)$ be a function that satisfies $f(x)+g(x)=x^2$. Then the value of the integral $\int_0^1 f(x)g(x)dx$ is

(a) $e+\frac{e^2}{2}-\frac{3}{2}$ (b) $e-\frac{e^2}{2}-\frac{3}{2}$

(c) $e+\frac{e^2}{2}+\frac{5}{2}$ (d) $e-\frac{e^2}{2}-\frac{5}{2}$. *(2003)*

51. The area of the region bounded by the curves $y=|x-1|$ and $y=3-|x|$ is

(a) 3 sq. units (b) 4 sq. units

(c) 6 sq. units (d) 2 sq. units. *(2003)*

52. $\lim_{n\to\infty}\frac{1^p+2^p+3^p+...+n^p}{n^{p+1}}$ is

(a) $\frac{1}{p+1}$ (b) $\frac{1}{1-p}$

(c) $\frac{1}{p}-\frac{1}{p-1}$ (d) $\frac{1}{p+2}$. *(2002)*

53. The area bounded by the curves $y=\ln x$, $y=\ln|x|$, $y=|\ln x|$ and $y=|\ln|x||$ is

(a) 4 sq. units (b) 6 sq. units

(c) 10 sq. units (d) none of these *(2002)*

54. $\int_{-\pi}^{\pi}\frac{2x(1+\sin x)}{1+\cos^2 x}dx$ is

(a) $\pi^2/4$ (b) π^2 (c) 0 (d) $\pi/2$. *(2002)*

55. If $y=f(x)$ makes +ve intercept of 2 and 0 unit x and y and encloses an area of 3/4 square unit with the axes then $\int_0^2 x f'(x)dx$ is

(a) 3/2 (b) 1 (c) 5/4 (d) –3/4. *(2002)*

56. $\int_{\pi}^{10\pi}|\sin x|dx$ is

(a) 20 (b) 8 (c) 10 (d) 18. *(2002)*

57. $I_n=\int_0^{\pi/4}\tan^n x\,dx$, then $\lim_{n\to\infty} n\left[I_n+I_{n-2}\right]$ equals

(a) 1/2 (b) 1 (c) ∞ (d) 0. *(2002)*

58. $\int_0^{\sqrt{2}}[x^2]dx$ is

(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$

(c) $\sqrt{2}-1$ (d) $\sqrt{2}-2$. *(2002)*

Answer Key

1.	(b)	**2.**	(d)	**3.**	(d)	**4.**	(c)	**5.**	(b)	**6.**	(a, d)
7.	(a)	**8.**	(c)	**9.**	(d)	**10.**	(c)	**11.**	(b)	**12.**	(b)
13.	(a)	**14.**	(b)	**15.**	(a)	**16.**	(b)	**17.**	(c)	**18.**	(d)
19.	(a)	**20.**	(c)	**21.**	(a)	**22.**	(c)	**23.**	(*)	**24.**	(c)
25.	(b)	**26.**	(c)	**27.**	(d)	**28.**	(b)	**29.**	(c)	**30.**	(a)
31.	(c)	**32.**	(b)	**33.**	(a)	**34.**	(c)	**35.**	(c)	**36.**	(c)
37.	(c)	**38.**	(c)	**39.**	(b)	**40.**	(a)	**41.**	(c)	**42.**	(d)
43.	(b)	**44.**	(b)	**45.**	(c)	**46.**	(b)	**47.**	(b)	**48.**	(a, c)
49.	(a)	**50.**	(b)	**51.**	(b)	**52.**	(a)	**53.**	(a)	**54.**	(b)
55.	(d)	**56.**	(d)	**57.**	(b)	**58.**	(c)				

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Explanations

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1. **(b) :** Let $x^3 = u$, then $3x^2dx = du$

Also suppose $\int f(x)dx = \psi(x)$

Now $\int x^5 f(x^3)dx = \frac{1}{3}\int u\, f(u)du$

$= \frac{1}{3}\left[u\int f(u)du - \int\left(\int f(u)\right)du\right]$

$= \frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + C$

2. **(d) :** $\frac{dy}{dx} = |x| = 2. \quad \therefore\ x = \pm 2$

We can solve for y to get

$y_1 = \int_0^2 |t|\,dt = \int_0^2 t\,dt = \left.\frac{t^2}{2}\right|_0^2 = 2$

and $y_2 = \int_0^{-2} |t|\,dt = -\int_0^{-2} t\,dt = -2$

Tangents are $y - 2 = 2(x - 2)$ and $y + 2 = 2(x + 2)$

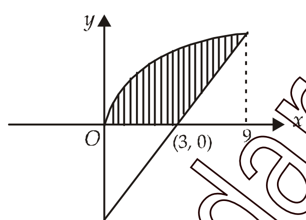
Then the x intercepts are obtained by putting $y = 0$.

We then get $x = \pm 1$

3. **(d) :** Solving $y = \sqrt{x}$ with $2y - x + 3 = 0$, we have

$2\sqrt{x} - x + 3 = 0 \Rightarrow (\sqrt{x} - 3)(\sqrt{x} + 1) = 0$

$\therefore\ x = 1, 9$



Area $= \int_0^3 [(2y + 3) - y^2]\,dy = \left. y^2 + 3y - \frac{y^3}{3}\right|_0^3$

$= 9 + 9 - 9 = 9$

4. **(c) :** $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{1}{1 + \sqrt{\cot x}}$

Adding, $2I = \int_{\pi/6}^{\pi/3}\left(\frac{1}{1 + \sqrt{\tan x}} + \frac{\sqrt{\tan x}}{1 + \sqrt{\tan x}}\right)dx$

$= \int_{\pi/6}^{\pi/3} 1\cdot dx = \left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{\pi}{6} \qquad \therefore\ I = \frac{\pi}{12}$

Again Statement-II is true.

5. **(b) :** $\int \frac{5\tan x}{\tan x - 2}dx = x + a\ln|\sin x - 2\cos x| + k$

Differentiating both sides, we get

$\frac{5\tan x}{\tan x - 2} = 1 + \frac{a(\cos x + 2\sin x)}{\sin x - 2\cos x}$

$\Rightarrow \frac{5\sin x}{\sin x - 2\cos x} = \frac{\sin x(1 + 2a) + \cos x(a - 2)}{\sin x - 2\cos x}$

$\Rightarrow a = 2$

6. **(a, d) :** $g(x) = \int_0^x \cos 4t\, dt$

$\Rightarrow g(x) = \left[\frac{\sin 4t}{4}\right]_0^x = \frac{\sin 4x}{4}$

$\Rightarrow g(x + \pi) = \frac{\sin 4(x + \pi)}{4} = \frac{\sin 4x}{4}$

$\Rightarrow g(\pi) = 0 \Rightarrow g(x + \pi) = g(x) + g(\pi)$ or $g(x) - g(\pi)$.

7. **(a) :** $x^2 = \frac{y}{4}$, $x^2 = 9y$

Area bounded by the parabolas and $y = 2$

$= 2\times\int_0^2\left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right)dy = 5\int_0^2\sqrt{y}\,dy$

$= 5\times\frac{(y)^{3/2}}{3/2} = \frac{10}{3}\times 2\sqrt{2} = \frac{20\sqrt{2}}{3}$

8. **(c) :** $\frac{d(p(t))}{dt} = 0.5p(t) - 450$

$\int_{850}^{p}\frac{2dp}{p - 900} = \int_0^t dt \Rightarrow 2\ln\frac{p - 900}{-50} = t$

$\Rightarrow p = 900 - 50\cdot e^{t/2}$

If $p = 0$, then $\frac{900}{50} = e^{t/2} \Rightarrow t = 2\ln 18$

9. **(d) :** $\frac{dV}{dt} = -k(T - t)$

On integration, $V = \frac{k(T - t)^2}{2} + \alpha$

At $t = 0$, $V(t) = I \Rightarrow I = \frac{kT^2}{2} + \alpha$

$\therefore\ \alpha = I - \frac{kT^2}{2}$

As $t = T$, we have $V(T) = \alpha = I - \frac{kT^2}{2}$

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10. (c) : $\frac{dy}{dx} = y + 3 \Rightarrow \frac{dy}{y+3} = dx$

As $y(0) = 2$, we have $\ln 5 = C$

Now $\ln(y + 3) = x + \ln 5$

As $x = \ln 2$ we have

$\ln(y + 3) = \ln 2 + \ln 5 = \ln 10$

$\Rightarrow y + 3 = 10 \Rightarrow y = 7.$

11. (b) : $I = \int_0^1 \frac{8\ln(1+x)}{1+x^2}dx$

Let $J = \int_0^1 \frac{\ln(1+x)}{1+x^2}dx$

Let $x = \tan\theta \Rightarrow J = \int_0^{\pi/4} \ln(1+\tan\theta)d\theta$

Now $J = \int_0^{\pi/4} \ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$

Adding $2J = \int_0^{\pi/4} \ln(1+\tan\theta) + \ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$

$= \int_0^{\pi/4} \ln\left\{(1+\tan\theta)\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)\right\}d\theta$

$2J = \int_0^{\pi/4} (\ln 2)d\theta = \frac{\pi}{4}\ln 2 \Rightarrow 8J = 4\frac{\pi}{4}\ln 2$

$\Rightarrow I = 8J = \pi \ln 2.$

12. (b) : $f(x) = \int_0^x \sqrt{t}\sin t\,dt$

$f'(x) = \sqrt{x}\sin x$

$f''(x) = \sqrt{x}\cos x + \frac{1}{2}x^{-1/2}\sin x$

$f''(\pi) = -\sqrt{\pi} < 0\,;\; f''(2\pi) = \sqrt{2\pi} > 0$

Thus at π maximum and at 2π minimum.

13. (a) : $\text{Area} = \frac{1}{2} + \int_1^e \frac{dx}{x} = \frac{1}{2} + \ln x\,\Big|_1^e = \frac{3}{2}$

14. (b) : $p'(x) = p'(1 - x)$

On integration,

$p(x) = -p(1 - x) + k,$

k being the constant of integration.

Set $x = 0$ to obtain $p(0) = -p(1) + k$

$\Rightarrow 1 = -41 + k. \therefore k = 42$

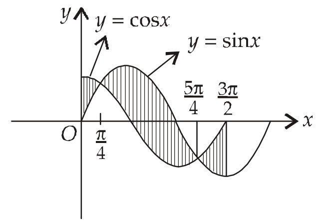
Now, $I = \int_0^1 p(x)dx = \int_0^1 p(1-x)dx$

On adding we get

$2I = \int_0^1 p(x) + p(1-x)\,dx = \int_0^1 k dx = \int_0^1 42\,dx = 42.$

Thus $I = 21.$

15. (a) :



The desired area =

$\int_0^{\pi/4}(\cos x - \sin x)dx + \int_{\pi/4}^{5\pi/4}(\sin x - \cos x)dx + \int_{5\pi/4}^{3\pi/2}(\cos x - \sin x)dx$

$= 2[\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{5\pi/4}$

(As the first and third integrals are equal in magnitude)

$= 2\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right) + \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$

$= \frac{8}{\sqrt{2}} - 2 = 4\sqrt{2} - 2$

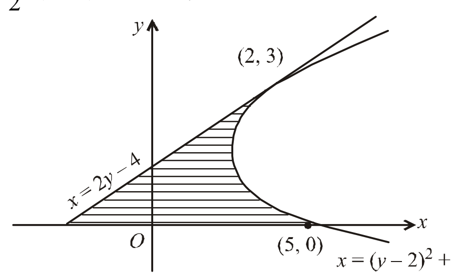
16. (b) : $(y - 2)^2 = x - 1$

Differentiating w.r.t. x, we have $2(y - 2)y' = 1$

$\Rightarrow y' = \frac{1}{2(y-2)}$ at $(2, 3)$, $y' = 1/2$

The equation of the tangent to the parabola at (2, 3) is

$y - 3 = \frac{1}{2}(x - 2) \Rightarrow x - 2y + 4 = 0$



The area of the bounded region

$= \int_0^3 [(y-2)^2 + 1 - (2y-4)]dy$

$= \int_0^3 (y^2 - 6y + 9)\,dy = \int_0^3 (y-3)^2\,dy$

(Let $3 - y = t$)

$= \int_0^3 (3-y)^2\,dy = \int_0^3 t^2 dt = \left[\frac{t^3}{3}\right]_0^3 = \frac{3^3}{3} = 9$

17. (c) : $I = \int_0^\pi [\cot x]dx$

$I = \int_0^\pi [\cot(\pi - x)]dx = \int_0^\pi [-\cot x]dx$

Adding we have

$2I = \int_0^\pi \{[\cot x] + [-\cot x]\}dx$

$2I = \int_0^\pi (-1)dx = -\pi \quad \therefore I = -\pi/2$

Note that $[x] + [-x] = 0,\ x \in Z\ = -1,\ x \notin Z.$

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18. **(d) :** $\sqrt{2}\int \frac{\sin x}{\sin\left(x-\frac{\pi}{4}\right)}dx$

$$= \sqrt{2}\int \frac{\sin\left(x-\frac{\pi}{4}+\frac{\pi}{4}\right)}{\sin\left(x-\frac{\pi}{4}\right)}dx$$

$$= \sqrt{2}\int\left[\cos\frac{\pi}{4}+\cot\left(x-\frac{\pi}{4}\right)\sin\frac{\pi}{4}\right]dx$$

$$= \sqrt{2}\cdot\frac{1}{\sqrt{2}}x+\sqrt{2}\cdot\frac{1}{\sqrt{2}}\ln\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$$

$$= x+\ln\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$$

c being a constant of integration.

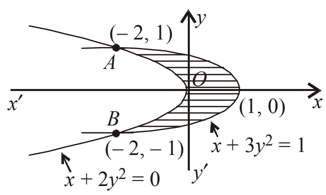
19. **(a) :** Solution $x + 2y^2 = 0$ and $x + 3y^2 = 1$ we have

$1 - 3y^2 = -2y^2 \Rightarrow y^2 = 1 \quad \therefore \quad y = \pm 1$

$y = -1 \Rightarrow x = -2$

$y = 1 \Rightarrow x = -2$

The bounded region is as under



The desired area $= 2\int_0^1[(1-3y^2)-(-2y^2)]dy$

$$= 2\int_0^1(1-y^2)dy = 2\left[y-\frac{y^3}{3}\right]_0^1$$

$$= 2\times\frac{2}{3}=\frac{4}{3} \text{ sq. units}$$

20. **(c) :** In the interval of integration $\sin x < x$

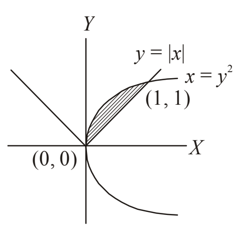
$$I=\int_0^1\frac{\sin x}{\sqrt{x}}dx<\int_0^1\frac{x}{\sqrt{x}}dx=\int_0^1\sqrt{x}dx=\left[\frac{2}{3}x^{3/2}\right]_0^1=\frac{2}{3}$$

$\therefore \; I < \frac{2}{3}$

Also $J=\int_0^1\frac{\cos x}{\sqrt{x}}dx<\int_0^1\frac{1}{\sqrt{x}}dx=[2\sqrt{x}]_0^1=2$

$\therefore \quad J < 2$

21. **(a) :** Required area



$$=\int_0^1(\sqrt{x}-x)dx$$

$$=\left[\frac{2}{3}x^{3/2}-\frac{x^2}{2}\right]_0^1=\frac{2}{3}-\frac{1}{2}=\frac{1}{6}.$$

22. **(c) :** $\frac{1}{2}\int\frac{dx}{\sin\left(x+\frac{\pi}{6}\right)}=\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+c.$

23. **(*) :** $\left[\sec^{-1}t\right]_{\sqrt{2}}^{x}=\frac{\pi}{2}$

$$\sec^{-1}x-\sec^{-1}\sqrt{2}=\frac{\pi}{2} \Rightarrow \sec^{-1}x=\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$$

$x=-\sqrt{2}$. There is no correct option.

24. **(c) :** $F(x)=\int_1^x\frac{\ln t}{1+t}dt+\int_1^{1/x}\frac{\ln t}{1+t}dt$

$$F(x)=\int_1^x\left(\frac{\ln t}{1+t}+\frac{\ln t}{(1+t)t}\right)dt=\int_1^x\frac{\ln t}{t}dt=\frac{1}{2}(\ln x)^2$$

$F(e) = 1/2$.

25. **(b) :** $\int_2^a[x]f'(x)dx$, say $[a] = K$ such that $a > 1$

$$=\int_1^2 1f'(x)dx+\int_2^3 2f'(x)dx+\ldots\ldots+\int_{K-1}^{K}(K-1)f'(x)dx+\int_K^a Kf'(x)dx$$

$= f(2) - f(1) + 2[f(3) - f(2)] + 3[f(4) - f(3)] + \ldots$
$(K - 1)\,[f(K) - f(K - 1)] + K[f(a) - f(K)]$

$= - [f(1) + f(2) + \ldots\ldots + f(K)] + K f(a)$

$= [a]\, f(a) - [f(1) + f(2) + \ldots\ldots + f([a])]$

26. **(c) :** Let $I=\int_{-3\pi/2}^{-\pi/2}\left[(x+\pi)^3+\cos^2(x+3\pi)\right]dx$

Putting $x + \pi = z$

also $x=\frac{-\pi}{2}\Rightarrow z=\frac{\pi}{2}$ and $x=\frac{-3\pi}{2}\Rightarrow z=\frac{-\pi}{2}$

$\therefore \quad dx = dz$

and $x + 3\pi = z + 2\pi$

$$\therefore \quad I=\int_{-\pi/2}^{\pi/2}[z^3+\cos^2(2\pi+z)]dz=\int_{-\pi/2}^{\pi/2}z^3dz+\int_{-\pi/2}^{\pi/2}\cos^2 z\,dz$$

$= 0$ (an odd function) $+2\int_0^{\pi/2}\cos^2 z\,dz$

$$=0+2\frac{1}{2}\times\frac{\pi}{2}$$

$\left\{\text{using fact } \int_0^{\pi/2}\sin^n x\,dx\right.$

$$=\begin{cases}\frac{n-1}{n}\cdot\frac{n-3}{n-2}\ldots\ldots\frac{1}{2}\times\frac{\pi}{2} & \text{if } n=2m\\ \frac{n-1}{n}\frac{n-3}{n-2}\ldots\ldots\frac{2}{2} & \text{if } n=2m+1\end{cases}$$

$$=\frac{\pi}{2}$$

27. **(d) :** Let $I=\int_0^{\pi}xf(\sin x)dx$ ..... (i)

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$$I = \int_0^{\pi} (\pi - x) f(\sin x)dx \quad \text{..... (ii)}$$

using $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

By (i) & (ii) on adding

$$\therefore\ I = \frac{\pi}{2}\int_0^{\pi} f(\sin x)dx = 2\frac{\pi}{2}\int_0^{\frac{\pi}{2}} f(\sin x)dx$$

[using $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$ if $f(2a-x) = f(x)$]

$$= \pi\int_0^{\frac{\pi}{2}} f\left(\sin(\frac{\pi}{2}-x)\right)dx = \pi\int_0^{\frac{\pi}{2}} f(\cos x)dx$$

28. (b) : Using fact

$$\int_a^b \frac{f(x)}{f(a+b+x)+f(x)} = \int_a^b f(x)dx = \frac{b-a}{2}$$

$$\therefore \quad \int_3^6 \frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}dx = \frac{6-3}{2} = \frac{3}{2}$$

29. (c) : $\lim_{n\to\infty}\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\left(\frac{4}{n^2}\right) + \ldots + \frac{1}{n}\sec^2 1$

$$= \lim_{n\to\infty}\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\left(\frac{4}{n^2}\right) + \ldots + \frac{n}{n^2}\sec^2\left(\frac{n^2}{n^2}\right)$$

$$= \lim_{n\to\infty}\sum_{r=1}^{r=n}\left(\frac{r}{n^2}\right)\sec^2\left(\frac{r}{n}\right)^2 = \lim_{n\to\infty}\sum_{r=0}^{r=n}\frac{1}{n}\left(\frac{r}{n}\right)\sec^2\left(\frac{r}{n}\right)^2$$

$$= \int_0^1 x\sec^2(x^2)dx = \frac{1}{2}\tan 1.$$

30. (a) : Let $f(x) = \int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx \quad (a>0)$ ...(1)

$$\therefore\ f(x) = \int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^{-x}}dx \quad \therefore\ \int_a^b f(x)dx = \int_a^b f(a+b-x)$$

$$\therefore\ f(x) = \int_{-\pi}^{\pi}\frac{a^x\cos^2 x}{1+a^x}dx \quad \text{... (2)}$$

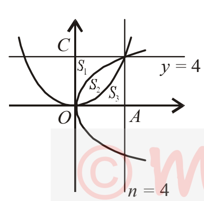
$$2f(x) = \int_{-\pi}^{\pi}\cos^2 x\,dx = 2\int_0^{\pi}\cos^2 x\,dx$$

$$= 2\times 2\int_0^{\pi/2}\cos^2 x\,dx,\ 2f(x) = 4\times\frac{1}{2}\times\frac{\pi}{2}$$

[By using $\int_0^{\pi/2}\sin^n x\,dx$
$= \frac{n-1}{n}\cdot\frac{n-3}{n-2}\ldots\frac{1}{2}\times\frac{\pi}{2}$ if n is even]

$$f(x) = \frac{\pi}{2}$$

31. (c) : Total area $= 4\times 4 = 16$ sq. units

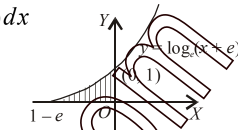


Area of $S_3 = \int_0^4 \frac{x^2}{4}dx = \frac{16}{3} = S_1$

$\therefore\ S_2 = 16 - \frac{16}{3}\times 2 = \frac{16}{3}$.

$\therefore\ S_1 : S_2 : S_3$ is $1 : 1 : 1$.

32. (b) : Required area $= \int_{1-e}^{0}\log_e(x+e)dx$



$$= \int_1^e \log z\,dz$$

$$= \left[z(\log_e z - 1)\right]_1^e = 1.$$

33. (a) : For $0 < x < 1$, $x^2 > x^3$ $\therefore$ $2^{x^2} > 2^{x^3}$

and for $1 < x < 2$, $x^3 > x^2$ $\therefore$ $2^{x^3} > 2^{x^2}$

i.e. $2^{x^2} < 2^{x^3} \Rightarrow I_3 < I_4$

as $2^{x^2} > 2^{x^3}$

$$\therefore\ \int_0^1 2^{x^2}dx > \int_0^1 2^{x^3}dx$$

$\therefore\ I_1 > I_2$.

34. (c) : According to question

$$\int_{\pi/4}^{B} f(x)dx = \int_{\pi/4}^{B(>\pi/4)}\left(B\sin B + \frac{\pi}{4}\cos B + B\sqrt{2}\right)$$

$$f(B) = \sin B + B\cos B - \frac{\pi}{4}\sin B + \sqrt{2}$$

$$\therefore\ f\left(\frac{\pi}{2}\right) = 1 - \frac{\pi}{4} + \sqrt{2}.$$

35. (c) : $\lim_{x\to 2}\int_6^{f(x)}\frac{4t^3}{x-2}dt$ (0/0) form,

$$= \lim_{x\to 2}\frac{f'(x)\times 4(f(x))^3}{1}$$

$$= 4f'(2)\times(f(2))^3 = \frac{1}{48}\times 4\times 6\times 6\times 6 = 18.$$

36. (c) : Method by cross check

Consider $f(x) = \frac{x}{(\log x)^2 + 1}$

$$\therefore\ f'(x) = \frac{1+(\log x)^2 - \frac{2x\log x}{x}}{(1+(\log x)^2)^2}$$

$$\therefore\ f'(x) = \frac{1+(\log x)^2 - 2\log x}{(1+\log^2 x)^2} = \left(\frac{(1-\log x)}{(1+\log x)^2}\right)^2$$

$$\therefore\ \int\left(\frac{(1-\log x)^2}{1+(\log x)^2}\right)dx = \int f'(x)\,dx = f(x)$$

$$\therefore\ \int\left(\frac{1-\log x}{1+(\log x)^2}\right)^2 dx = \frac{x}{1+(\log x)^2}$$

Hence (c) is correct answer and we can check the other choices by the similar argument.

37. (c) : $y = \begin{cases} x-2 & \text{if } x>2 \\ 0 & \text{if } x=0 \\ 2-x & \text{if } x<2 \end{cases}$

Required area = Area of ΔLAB + Area of ΔMBC

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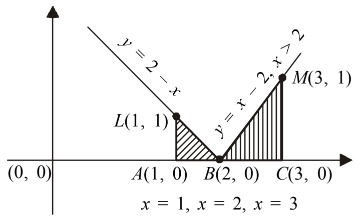

$= \frac{1}{2}[AL \times AB + BC \times CM] = \frac{1}{2}\ [1 \times 1 + 1 \times 1] = 1$

38. **(c) :** As $f(x) = \dfrac{e^x}{1+e^x}$

$\therefore \quad f(a) = \dfrac{e^a}{1+e^a}$ and $f(-a) = \dfrac{e^{-a}}{1+e^{-a}}$

$\therefore \quad f(-a) + f(a) = 1$

Now $\displaystyle\int_{f(-a)}^{f(a)} x\, g\{x(1-x)\}\, dx =$

$\displaystyle\int_{f(-a)}^{f(a)} (1-x) g\{(1-x)(x)\}\, dx$

using $\displaystyle\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$\Rightarrow \quad 2\displaystyle\int_{f(-a)}^{f(a)} x\, g\{x(1-x)\}\, dx = \int_{f(-a)}^{f(a)} g\{(1-x)x\}\, dx$

$\Rightarrow \quad 2I_1 = I_2$

$\therefore \quad \dfrac{I_2}{I_1} = \dfrac{2}{1}$

39. **(b) :** $\displaystyle\int_0^{\pi} x f(\sin x)dx = A\int_0^{\frac{\pi}{2}} f(\sin x)dx$

or $A\displaystyle\int_0^{\frac{\pi}{2}} f(\sin x)dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)dx = \int_0^{\pi} xf(\sin x)dx$

$\Rightarrow A\displaystyle\int_0^{\frac{\pi}{2}} f(\sin x)dx = \frac{\pi}{2}\times 2\int_0^{\frac{\pi}{2}} f(\sin x)dx$

$\Rightarrow A\displaystyle\int_0^{\frac{\pi}{2}} f(\sin x) = \pi\int_0^{\frac{\pi}{2}} f(\sin x)dx$

$\Rightarrow A = \pi$

40. **(a) :** $\displaystyle\int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{(\sin x + \cos x)^2}}dx = \int_0^{\pi/2} (\sin x + \cos x)dx$

$= \left(-\cos x + \sin x\right)_0^{\frac{\pi}{2}}$

$= 1 - (-1) = 2$

41. **(c) :** $\displaystyle\int_{-2}^{3} \left|1 - x^2\right| dx = \int_{-2}^{3} \left|(1-x)(1+x)\right| dx$

Putting $1 - x^2 = 0 \ \therefore\ x = \pm 1$

Points $-2, -1, 1, 3$

$\therefore\ |1 - x^2| = \begin{cases} 1-x^2 & \text{if } \quad |x|<1 \\ (1-(1-x^2)) & \text{if } x<-1 \text{ and } x \ge 1 \end{cases}$

$\therefore \quad \displaystyle\int_{-2}^{3} \left|(1-x^2)\right| dx$

$= \displaystyle\int_{-2}^{-1} (x^2-1)dx + \int_{-1}^{1} (1-x^2)dx + \int_{1}^{3} (x^2-1)dx$

$= \dfrac{4}{3} + 2\left(\dfrac{2}{3}\right) + \dfrac{20}{3} = \dfrac{28}{3}$

42. **(d) :** $\displaystyle\int \frac{1}{a\cos x - b\sin x}dx$ where $a = b = 1$

let $\quad a = r\cos\theta = 1$

$\quad b = r\sin\theta = 1$

$\therefore \quad r = \sqrt{2}$

$\theta = \tan^{-1}(b/a)$

$= \dfrac{1}{\sqrt{2}}\displaystyle\int \frac{1}{\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x}dx$

$= \dfrac{1}{\sqrt{2}}\displaystyle\int \frac{1}{\cos(x+\pi/4)}dx$

$= \dfrac{1}{\sqrt{2}}\displaystyle\int \frac{1}{\sin\left(\frac{\pi}{2}+x+\frac{\pi}{4}\right)}dx$

$= \dfrac{1}{\sqrt{2}}\displaystyle\int \frac{1}{2\sin\left(\frac{x}{2}+\frac{3\pi}{8}\right)\cos\left(\frac{x}{2}+\frac{3\pi}{8}\right)}dx$

$= \dfrac{1}{2\sqrt{2}}\displaystyle\int \frac{\sec^2\left(3\frac{\pi}{8}+\frac{x}{2}\right)}{\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)}dx$

$= \dfrac{1}{2\sqrt{2}} \times 2\log\left|\tan\left(\dfrac{x}{2}+\dfrac{3\pi}{8}\right)\right| + c$

$= \dfrac{1}{\sqrt{2}}\log\left|\tan\left(\dfrac{x}{2}+\dfrac{3\pi}{8}\right)\right| + c$

43. **(b) :** $\displaystyle\int \frac{\sin x}{\sin(x-\alpha)}dx = Ax + B\log\sin(x-\alpha) + C$

$\Rightarrow$ Differentiating *w.r.t.* x both sides

$\Rightarrow \dfrac{\sin x}{\sin(x-\alpha)} = A + \dfrac{B\cos(x-\alpha)}{\sin(x-\alpha)}$

$\Rightarrow \sin x = A\sin(x-\alpha) + B\cos(x-\alpha)$

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$\sin x = A(\sin x \cos\alpha - \cos x \sin\alpha) + B(\cos x \cos\alpha + \sin x \sin\alpha)$

$\sin x = \sin x(A\cos\alpha + B\sin\alpha) + \cos x(B\cos\alpha - A\sin\alpha)$

Now solving $A\cos\alpha + B\sin\alpha = 1$

and $B\cos\alpha - A\sin\alpha = 0$

$(A, B) = (\cos\alpha, \sin\alpha)$

44. (b) : $\underset{n\to\infty}{\text{Lt}} \sum_{r=1}^{r=n} \frac{1}{n} e^{\frac{r}{n}}$

$= \int_0^1 e^x dx = e - 1$

45. (c): Given $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = F(k) - F(1)$

$\Rightarrow \int_1^4 \frac{3x^2}{x^3} e^{\sin x^3} dx = F(k) - F(1)$

$\Rightarrow \int_1^{64} \frac{e^{\sin z}}{z} dz = F(k) - F(1)$ where $(x^3 = z)$

$\Rightarrow [F(z)]_1^{64} = F(k) - F(1)$

$\Rightarrow F(64) - F(1) = F(k) - F(1)$

$\Rightarrow k = 64$

46. (b) : $\underset{x\to 0}{\text{Lt}} \frac{(\tan t)_0^{x^2}}{x \sin x}$

$= \underset{x\to 0}{\text{Lt}} \frac{\tan x^2}{x \sin x}$

$= \underset{x\to 0}{\text{Lt}} \frac{\tan x^2}{x^2 \frac{\sin x}{x}}$

$= \underset{x\to 0}{\text{Lt}} \frac{\tan x^2}{x^2} \frac{1}{\underset{x\to 0}{\text{Lt}} \frac{\sin x}{x}} = 1 \times 1 = 1$

47. (b) : $\int_0^1 x(1-x)^n \, dx$

Putting $x = \sin^2\theta$

$dx = 2\sin\theta\cos\theta \, d\theta$

and $x = 0, \theta = 0$

$x = 1, \theta = \pi/2$

$\therefore \int_0^1 x(1-x)^n dx = \int_0^{\pi/2} \sin^2\theta \cos^{2n}\theta \,(2\sin\theta\cos\theta) d\theta$

$= 2\int_0^{\frac{\pi}{2}} \sin^3\theta \cos^{2n+1}\theta \, d\theta$

$\left[\text{Using } \int_0^{\pi/2} \sin^{2n+1}\theta \cos^{2n+1}\theta \, d\theta = \frac{[(2n)(2n-2)...2][(2n)(2n-2)...2]}{(4n+2)(4n)(4n-2)....2}\right]$

$\therefore 2\int_0^{\pi/2} \sin^3\theta \cos^{2n+1}\theta \, d\theta$

$= \frac{2[2 \times (2n)(2n-2)(2n-4)...4.2]}{(2n+4)(2n+2)(2n)(2n-2)...4.2}$

$= \frac{2 \times 2 \times 1}{(2n+4)(2n+2)}$

$= \frac{1}{(n+2)(n+1)}$

$= \frac{1}{n+1} - \frac{1}{n+2}$ (by partial fraction)

48. (a), (c) : Let $I = \int_a^b x f(x) \, dx$

$I = \int_a^b (a+b-x) f(a+b-x) \, dx$

$I = \int_a^b (a+b) f(a+b-x) \, dx - \int_a^b x f(a+b-x) \, dx$

$I = \int_a^b (a+b) f(x) dx - \int_a^b x f(x) dx$

$\therefore I = \frac{a+b}{2} \int_a^b f(x) \, dx = \frac{a+b}{2} \int_a^b f(a+b-x) dx$

49. (a) : From given $F(t) = \int_0^t f(t-y) g(y) \, dy$

$= \int_0^t e^{t-y} y \, dy$ (By replacing $y \to t - y$ in $f(y)$)

$F(t) = -\int_t^0 (t-\theta)e^{\theta} d\theta = \int_0^t (t-\theta)e^{\theta} d\theta$

$= (t\, e^{\theta})_0^t - [(\theta - 1)\, e^{\theta}]_0^t$

$= t(e^t - 1) - (t-1)e^t - 1$

$= e^t(t - t + 1) - t - 1$

$= e^t - (t + 1)$

50. (b) : As $f(x) = f'(x)$ and $f(0) = 1$

$\Rightarrow \frac{f'(x)}{f(x)} = 1$

$\Rightarrow \log(f(x)) = x$

$\Rightarrow f(x) = e^x + k$

$\Rightarrow f(x) = e^x$ as $f(0) = 1$

Now $g(x) = x^2 - e^x$

$\therefore \int_0^1 f(x) g(x) \, dx = \int_0^1 e^x(x^2 - e^x) \, dx$

$= \int_0^1 x^2 e^x dx - \int_0^1 e^{2x} dx$

$= [(x^2 - 2x + 2)e^x]_0^1 - \left(\frac{e^{2x}}{2}\right)_0^1$

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$= (e-2) - \left(\frac{e^2-1}{2}\right) = e - \frac{e^2}{2} - \frac{3}{2}$

Using $\int f^n(x)e^x\,dx = e^x[f^n(x) - f_1^n(x) + f_2^n(x) + \ldots + (-1)^n f_n(x)]$

where $f_1, f_2, \ldots f_n$ are derivatives of first, second ...n^{th} order.

51. (b) : Required area

$= \int_{-1}^{0}(3+x)-(-x+1) + \int_{0}^{1}(3-x)-(-x+1)dx + \int_{1}^{2}(3-x)-(x-1)\,dx$

$= \int_{-1}^{0} 2(1+x)\,dx + \int_{0}^{1} 2\,dx + \int_{1}^{2}(4-2x)\,dx$

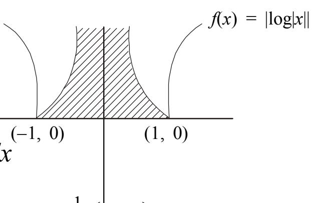
$= 4$ sq. units

52. (a) : $\lim_{n\to\infty} \frac{1^p + 2^p + \ldots + n^p}{n^p} \times \frac{1}{n}$

$= \lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^p + \left(\frac{2}{n}\right)^p + \ldots + \left(\frac{n}{n}\right)^p\right]$

$= \lim_{n\to\infty} \sum_{r=1}^{r=n} \frac{1}{n}\left(\frac{r}{n}\right)^p = \int_0^1 x^p\,dx = \frac{1}{p+1}$

53. (a) : Required Area



$= 2\int_0^1 \left|\log|x|\right| dx$

$= 2\left[\left(x\left|\log|x|\right|\right)\right]_0^1 - \int_0^1\left(-\frac{1}{x}\right)\cdot x\,dx$

$= 2[(1-0) + (x)_0^1] = 4$ sq. units.

54. (b) : $2\int_{-\pi}^{\pi}\frac{x}{1+\cos^2 x}dx + 2\int_{-\pi}^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$

$= 0 + 2\int_{-\pi}^{\pi}\frac{x\sin x}{1+\cos^2 x}dx = 2\cdot 2\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}$

$= 4\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx = 4\times\frac{\pi}{2}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$

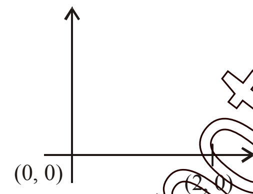
$\left(\text{by using } \int_0^{\pi} xf(\sin x)\,dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx\right)$

$= 4\frac{\pi}{2}\times 2\times\int_0^{\pi/2}\frac{\sin x}{1+\cos^2 x}dx$

$= 4\pi\,(\tan^{-1}\cos x)_{\frac{\pi}{2}}^{0}$ (By putting $\cos x = t$)

$= 4\pi\times\left(\frac{\pi}{4} - 0\right) = \pi^2$

55. (d) : Given $\int_0^2 f(x)\,dx = 3/4$



$\therefore \int_0^2 x\,f'(x)\,dx = x\int_0^2 f'(x)\,dx - \int_0^2 f(x)\,dx$

$= (x\cdot f(x))_0^2 - 3/4 = 2f(2) - \frac{3}{4}$

$= 0 - \frac{3}{4}$ [$\because f(2) = 0$, curve having intercept 2 units on x-axis.]

$= -3/4$

56. (d) : $\int_{\pi}^{10\pi}|\sin x|\,dx$

$= \int_0^{10\pi}|\sin x|\,dx - \int_0^{\pi}|\sin x|\,dx$

$= 10\times 2 - 1\times 2 = 18$ (Using period of $|\sin x| = \pi$)

57. (b) : $I_n = \int_0^{\pi/4}\tan^n x\,dx$

$I_{n-2} = \int_0^{\pi/4}\tan^{n-2} x\,dx$

$\therefore I_n + I_{n-2} = \int_0^{\pi/4}\tan^n x\,dx + \int_0^{\pi/4}\tan^{n-2} x\,dx$

$= \int_0^{\pi/4}\tan^{n-2} x\times(\sec^2 x - 1)\,dx + \int_0^{\pi/4}\tan^{n-2} x\,dx$

$= \int_0^{\pi/4}\tan^{n-2} x\sec^2 x\,dx$

$I_n + I_{n-2} = \frac{1}{n+1}$

$\therefore n(I_n + I_{n-2}) = \frac{1}{1 + 1/n}$

$\therefore \lim_{n\to\infty} n(I_n + I_{n-2}) = 1$

58. (c) : $\int_0^{\sqrt{2}}[x^2]dx = \int_0^1[x^2]dx + \int_1^{\sqrt{2}}[x^2]dx$

$= 0 + \int_1^{\sqrt{2}} 1\,dx = \sqrt{2} - 1$

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CHAPTER 11

DIFFERENTIAL EQUATIONS

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1. Solution of the differential equation
$\cos x dy = y(\sin x - y)dx$, $0 < x < \pi/2$ is
(a) $\sec x = (\tan x + c)y$ (b) $y \sec x = \tan x + c$
(c) $y \tan x = \sec x + c$ (d) $\tan x = (\sec x + c)y$ *(2010)*

2. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where c_1 and c_2 are arbitrary constants, is
(a) $y'' = y'y$ (b) $yy'' = y'$
(c) $yy'' = (y')^2$ (d) $y' = y^2$ *(2009)*

3. The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ satisfying the condition $y(1) = 1$ is
(a) $y = x \ln x + x$ (b) $y = \ln x + x$
(c) $y = x \ln x + x^2$ (d) $y = x e^{(x-1)}$ *(2008)*

4. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is
(a) $(x - 2)^2 y'^2 = 25 - (y - 2)^2$
(b) $(x - 2) y'^2 = 25 - (y - 2)^2$
(c) $(y - 2) y'^2 = 25 - (y - 2)^2$
(d) $(y - 2)^2 y'^2 = 25 - (y - 2)^2$ *(2008)*

5. The differential equation of all circles passing through the origin and having their centres on the x-axis is
(a) $y^2 = x^2 + 2xy\frac{dy}{dx}$
(b) $y^2 = x^2 - 2xy\frac{dy}{dx}$
(c) $x^2 = y^2 + xy\frac{dy}{dx}$
(d) $x^2 = y^2 + 3xy\frac{dy}{dx}$ *(2007)*

6. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants is of
(a) second order and second degree
(b) first order and second degree
(c) first order and first degree
(d) second order and first degree. *(2006)*

7. If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is
(a) $x\log\left(\frac{y}{x}\right) = cy$ (b) $y\log\left(\frac{x}{y}\right) = cx$
(c) $\log\left(\frac{x}{y}\right) = cy$ (d) $\log\left(\frac{y}{x}\right) = cx.$ *(2005)*

8. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where $c > 0$, is a parameter, is of order and degree as follows
(a) order 1, degree 1 (b) order 1, degree 2
(c) order 2, degree 2 (d) order 1, degree 3. *(2005)*

9. The solution of the differential equation
$ydx + (x + x^2y)dy = 0$ is
(a) $\frac{1}{xy} + \log y = C$ (b) $-\frac{1}{xy} + \log y = C$
(c) $-\frac{1}{xy} = C$ (d) $\log y = Cx.$ *(2004)*

10. The differential equation for the family of curves $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant is
(a) $(x^2 - y^2)y' = 2xy$ (b) $2(x^2 + y^2)y' = xy$
(c) $2(x^2 - y^2)y' = xy$ (d) $(x^2 + y^2)y' = 2xy.$ *(2004)*

11. If $x = e^{y + e^{y + \ldots \text{ to } \infty}}$, $x > 0$ then $\frac{dy}{dx}$ is
(a) $\frac{1-x}{x}$ (b) $\frac{1}{x}$
(c) $\frac{x}{1+x}$ (d) $\frac{1+x}{x}.$ *(2004)*

12. The solution of the differential equation
$(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$ is
(a) $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$
(b) $xe^{\tan^{-1} y} = \tan^{-1} y + k$
(c) $xe^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$
(d) $(x - 2) = ke^{-\tan^{-1} y}.$ *(2003)*

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13. The degree and order of the differential equation of the family of all parabolas whose axis is x-axis, are respectively

(a) 1, 2 (b) 3, 2
(c) 2, 3 (d) 2, 1. (2003)

14. The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{\frac{2}{3}} = 4\frac{d^3y}{dx^3}$ are

(a) $1, \frac{2}{3}$ (b) 3, 1 (c) 3, 3 (d) 1, 2. *(2002)*

15. The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$

(a) $\frac{1}{4}e^{-2x}$ (b) $\frac{1}{4}e^{-2x} + cx + d$
(c) $\frac{1}{4}e^{-2x} + cx^2 + d$ (d) $\frac{1}{4}e^{-2x} + c + d$. *(2002)*

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Answer Key

1.	(a)	**2.**	(c)	**3.**	(a)	**4.**	(d)	**5.**	(a)	**6.**	(d)
7.	(d)	**8.**	(d)	**9.**	(b)	**10.**	(a)	**11.**	(a)	**12.**	(a)
13.	(a)	**14.**	(c)	**15.**	(b)						

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Explanations

1. (a) : 1st solution:

$\cos x\, dy = y(\sin x - y)dx$

$\Rightarrow \cos x\, dy = y \sin x\, dx - y^2\, dx$

$\Rightarrow \cos x\, dy - y \sin x\, dx = -y^2\, dx$

$\Rightarrow d(y\cos x) = -y^2\, dx \Rightarrow \dfrac{d(y\cos x)}{(y\cos x)^2} = -\dfrac{dx}{\cos^2 x}$

On integration, we have

$\Rightarrow -\sec x = -y \tan x + yk$

$\Rightarrow \sec x = y(\tan x + C)$ where C is a constant

2nd solution:

$\dfrac{dy}{dx} = \dfrac{y(\sin x - y)}{\cos x} \Rightarrow \dfrac{dy}{dx} = y\tan x - y^2 \sec x$

$\Rightarrow \dfrac{dy}{dx} - y\tan x = -y^2\sec x$

$\Rightarrow \dfrac{1}{y^2}\dfrac{dy}{dx} - \dfrac{1}{y}\tan x = -\sec x$

Setting, $-\dfrac{1}{y} = v$, we have

$\dfrac{dv}{dx} + (\tan x)v = -\sec x$, which is linear in v.

$\text{I.F.} = e^{\int \tan x\, dx} = e^{\ln \sec x} = \sec x$

The solution is

$v \times \sec x = \int -\sec^2 x\, dx + k$

$\Rightarrow v\sec x = -\tan x + k$

$\Rightarrow -\dfrac{\sec x}{y} = -\tan x - C \Rightarrow \sec x = y(\tan x + C)$

2. (c) : $y = c_1 e^{c_2 x}$

Differentiating w.r.t. x, we get

$y' = c_1 c_2 e^{c_2 x} = c_2 y$ ...(i)

Again differentiating w.r.t. x

$y'' = c_2 y'$ ...(ii)

From (i) and (ii) upon division

$\dfrac{y'}{y''} = \dfrac{y}{y'} \Rightarrow y''y = (y')^2$

Which is the desired differential equation of the family of curves.

3. (a) : 1st Method (Homogeneous equation):

Let $y = vx$, so that $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

We have $v + x\dfrac{dv}{dx} = \dfrac{x + vx}{x} = 1 + v$

$\Rightarrow x\dfrac{dv}{dx} = 1 \Rightarrow dv = \dfrac{dx}{x}$

$\Rightarrow v = \ln x + \ln k$

As $v = y/x$ we have $y = x \ln x + (\ln k)x$

At $x = 1$, $y = 1$ giving

$1 = 0 + (\ln k)$ $\therefore$ $\ln k = 1$, Then $y = x \ln x + x$

2nd Method (Inspection) :

Rewriting the equation

$\dfrac{dy}{dx} = \dfrac{x + y}{x}$ as

$xdy - ydx = xdx$

We have $\dfrac{xdy - ydx}{x^2} = \dfrac{dx}{x}$

$d\left(\dfrac{y}{x}\right) = \dfrac{dx}{x}$

On integration $\dfrac{y}{x} = \ln x + k$

$\Rightarrow y = x \ln x + kx$

As before, evaluating constant, $y = x \ln x + x$

4. (d) : The equation to circle is

$(x - \alpha)^2 + (y - 2)^2 = 25$ ...(1)

Differentiation w.r.t. x

$(x - \alpha) + (y - 2)\dfrac{dy}{dx} = 0$

$\Rightarrow x - \alpha = -(y - 2)\dfrac{dy}{dx}$ ...(2)

From (1) and (2) on eliminate 'α'

$(y - 2)^2\left(\dfrac{dy}{dx}\right)^2 + (y - 2)^2 = 25$

$\Rightarrow (y - 2)^2(y')^2 = 25 - (y - 2)^2$

5. (a) : General equation of all such circles is

$(x - h)^2 + (y - 0)^2 = h^2$ .... (i) where h is parameter

$\Rightarrow (x - h)^2 + y^2 = h^2$

Differentiating, we get $2(x - h) + 2y\dfrac{dy}{dx} = 0$

$h = x + y\dfrac{dy}{dx}$ to eliminate h, putting value of h in equation .... (i) ,

$\therefore$ we get $y^2 = x^2 + 2xy\dfrac{dy}{dx}$.

6. (d) : Given $A x^2 + B y^2 = 1$

As solution having two constants, $\therefore$ order of differential equation is 2 so our choices (b) & (c) are discarded from the list, only choices (a) and (b) are possible

Again $A x^2 + B y^2 = 1$ .... (*)

$\Rightarrow -\dfrac{A}{B} = \dfrac{y}{x}\dfrac{dy}{dx}$ .... (i)

Differentiating (*) w.r.t. x

Again on differentiating

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$$-\frac{A}{B} = y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 \qquad \text{......(ii)}$$

By (i) and (ii) we get

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = y\left(\frac{dy}{dx}\right)$$

$\Rightarrow$ order 2 degree 1.

7. **(d) :** $x\frac{dy}{dx} = y(\log y - \log x + 1)$

$\therefore \frac{dy}{dx} = \frac{y}{x}\left(\log\left(\frac{y}{x}\right) + 1\right)$ Now put $\frac{y}{x} = v$

$\therefore v \log v \; dx = x \; dv$

$\Rightarrow \frac{dv}{v\log v} = \frac{dx}{x} \Rightarrow \log\left(\frac{y}{x}\right) = cx.$

8. **(d) :** $y^2 = 2c(x + \sqrt{c})$ ...(i)

$\therefore 2yy_1 = 2c$ $\qquad \therefore yy_1 = c$

Now putting $c = yy_1$ in (i) we get

$$y^2 = 2\cdot yy_1\left(x + \sqrt{yy_1}\right) \Rightarrow (y^2 - 2xyy_1)^2 = 4(yy_1)^3$$

$\Rightarrow (y^2 - 2xyy_1)^2 = 4y^3y_1^3 \Rightarrow$ order 1, degree 3.

9. **(b) :** $y\,dx = -(x^2y + x)\,dy$

$\Rightarrow ydx + xdy = -x^2y\,dy$

$$\Rightarrow \frac{ydx + xdy}{(xy)^2} = \frac{-dy}{y} \Rightarrow \frac{d(xy)}{(xy)^2} = -\frac{dy}{y}$$

$$\Rightarrow d\left(-\frac{1}{xy}\right) = -\frac{dy}{y}$$

$$\Rightarrow -\frac{1}{xy} = -\log y + C$$

$$\Rightarrow -\frac{1}{xy} + \log y = C$$

10. **(a) :** Given family of curve is

$x^2 + y^2 - 2ay = 0$ ...(1)

$$\Rightarrow 2a = \frac{x^2 + y^2}{y}$$

Also from (1), $2x + 2yy' - 2a\,y' = 0$

$$\Rightarrow 2x + 2yy' - \left(\frac{x^2 + y^2}{y}\right)y' = 0$$

$\Rightarrow 2xy + y'(2y^2 - x^2 - y^2) = 0 \Rightarrow y'(x^2 - y^2) = 2xy$

11. **(a) :** $x = e^{y+e^{y+e^{y\ldots}}} \Rightarrow x = e^{y+x}$

Differentiate *w.r.t.* x after taking logarithm both sides

$\therefore \frac{1}{x} = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1-x}{x}$

12. **(a) :** From the given equation

$$(1 + y^2)\frac{dx}{dy} + 1x = e^{\tan^{-1}y}$$

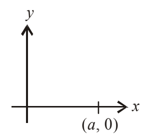
$$\Rightarrow \frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{e^{\tan^{-1}y}}{1+y^2} \Rightarrow x\cdot\text{I.F.} = \int y\cdot\text{I.F.}\,dy$$

where $\text{I.F} = e^{\int\frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

$$\Rightarrow x\,e^{\tan^{-1}y} = \frac{e^{2\tan^{-1}y}}{2} + c$$

$$\Rightarrow x\,e^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$$

13. **(a):** As axis of parabola is x-axis which means focus lies on x-axis. Equation of such parabola's is given by



$y^2 = 4a(x - k)$ ... (i)

$\Rightarrow 2yy_1 = 4a$ (by differentiating (i) w.r.t. x)

$\Rightarrow y\frac{dy}{dx} = 2a$ ... (ii)

$$\Rightarrow y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

(by differentiating (ii) w.r. to x)

$\Rightarrow$ order 2 and degree 1 (Concept: Exponent of highest order derivative is called degree and order of that derivative is called order of the differential equation.)

$\Rightarrow 2x\,e^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$

14. **(c) :** $\left(1 + 3\frac{dy}{dx}\right)^{\frac{2}{3}} = 4\left(\frac{d^3y}{dx^3}\right)$

$$\Rightarrow \left(1 + 3\frac{dy}{dx}\right)^2 = \left[4\frac{d^3y}{dx^3}\right]^3$$

$\therefore$ highest order is 3 whose exponent is also 3.

15. **(b) :** Given $\frac{d^2y}{dx^2} = e^{-2x}$

$$\therefore \frac{dy}{dx} = \frac{e^{-2x}}{-2} + c$$

$$\therefore y = \frac{e^{-2x}}{4} + cx + d$$

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CHAPTER 12

TWO DIMENSIONAL GEOMETRY

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1. The circle passi ough (1, –2) and touching the axis of x at (3, 0) also passes through the point

(a) (2, –5) (b) (5, –2)
(c) (–2, 5) (d) (–5, 2) *(2013)*

2. Given : A circle, $2x^2 + 2y^2 = 5$ and a parabola $y^2 = 4\sqrt{5}x$.

Statement-1 : An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

Statement-2 : If the line, $y = mx + \frac{\sqrt{5}}{m} (m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$.

(a) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1. *(2013)*

3. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is

(a) $\sqrt{3}y = x - \sqrt{3}$ (b) $y = \sqrt{3}x - \sqrt{3}$
(c) $\sqrt{3}y = x - 1$ (d) $y = x + \sqrt{3}$ *(2013)*

4. The equation of the circle passing through the focii of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and having centre at (0, 3) is

(a) $x^2 + y^2 - 6y + 7 = 0$ (b) $x^2 + y^2 - 6y - 5 = 0$
(c) $x^2 + y^2 - 6y + 5 = 0$ (d) $x^2 + y^2 - 6y - 7 = 0$ *(2013)*

5. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1), (1, 1) and (1, 0) is

(a) $2 - \sqrt{2}$ (b) $1 + \sqrt{2}$
(c) $1 - \sqrt{2}$ (d) $2 + \sqrt{2}$ *(2013)*

6. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals

(a) 6 (b) 11/5 (c) 29/5 (d) 5 *(2012)*

7. **Statement 1 :** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$

Statement 2 : If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1. *(2012)*

8. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is

(a) 6/5 (b) 5/3
(c) 10/3 (d) 3/5 *(2012)*

9. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is

(a) $4x^2 + y^2 = 8$ (b) $x^2 + 4y^2 = 16$
(c) $4x^2 + y^2 = 4$ (d) $x^2 + 4y^2 = 8$ *(2012)*

10. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is

(a) – 2 (b) – 1/2
(c) – 1/4 (d) – 4 *(2012)*

11. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if

(a) $a = 2c$ (b) $|a| = 2c$
(c) $2|a| = c$ (d) $|a| = c$ *(2011)*

12. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement-1 : The ratio $PR : RQ$ equals

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Statement-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.
(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2011)*

13. The shortest distance between line $y - x = 1$ and curve $x = y^2$ is
(a) $\frac{8}{3\sqrt{2}}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\frac{3\sqrt{2}}{8}$ *(2011)*

14. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point $(-3, 1)$ and has eccentricity $\sqrt{\frac{2}{5}}$ is
(a) $3x^2 + 5y^2 - 15 = 0$ (b) $5x^2 + 3y^2 - 32 = 0$
(c) $3x^2 + 5y^2 - 32 = 0$ (d) $5x^2 + 3y^2 - 48 = 0$ *(2011)*

15. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is
(a) $y = 0$ (b) $y = 1$
(c) $y = 2$ (d) $y = 3$ *(2010)*

16. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is
(a) $x = 1$ (b) $2x + 1 = 0$
(c) $x = -1$ (d) $2x - 1 = 0$ *(2010)*

17. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point $(13, 32)$. The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is
(a) $\frac{23}{\sqrt{15}}$ (b) $\sqrt{17}$ (c) $\frac{17}{\sqrt{15}}$ (d) $\frac{23}{\sqrt{17}}$ *(2010)*

18. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if
(a) $-85 < m < -35$ (b) $-35 < m < 15$
(c) $15 < m < 65$ (d) $35 < m < 85$ *(2010)*

19. The shortest distance between the line $y - x = 1$ and the curve $x = y^2$ is
(a) $\frac{2\sqrt{3}}{8}$ (b) $\frac{3\sqrt{2}}{5}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\frac{3\sqrt{2}}{8}$ *(2009)*

20. The differential equation which represents the family of curves $y = c_1e^{c_2x}$, where c_1 and c_2 are arbitrary constants, is
(a) $y'' = y'y$ (b) $yy'' = y'$
(c) $yy'' = (y')^2$ (d) $y' = y^2$ *(2009)*

21. Three distinct points A, B and C are given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point $(1, 0)$ to the distance from the point $(-1, 0)$ is equal to 1/3. Then the circumcentre of the triangle ABC is at the point
(a) $\left(\frac{5}{4}, 0\right)$ (b) $\left(\frac{5}{2}, 0\right)$
(c) $\left(\frac{5}{3}, 0\right)$ (d) $(0, 0)$ *(2009)*

22. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point $(4, 0)$. Then the equation of the ellipse is
(a) $x^2 + 12y^2 = 16$ (b) $4x^2 + 48y^2 = 48$
(c) $4x^2 + 64y^2 = 48$ (d) $x^2 + 16y^2 = 16$ *(2009)*

23. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through P, Q and $(1, 1)$ for
(a) all except one value of p
(b) all except two values of p
(c) exactly one value of p
(d) all values of p *(2009)*

24. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is
(a) $\frac{5}{3}$ (b) $\frac{8}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$ *(2008)*

25. The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is
(a) $(3, 4)$ (b) $(3, -4)$
(c) $(-3, 4)$ (d) $(-3, -4)$ *(2008)*

26. A parabola has the origin as its focus and the line $x = 2$ as the directrix. Then the vertex of the parabola is at
(a) $(2, 0)$ (b) $(0, 2)$ (c) $(1, 0)$ (d) $(0, 1)$ *(2008)*

27. The perpendicular bisector of the line segment joining $P(1, 4)$ and $Q(k, 3)$ has y-intercept -4. Then a possible value of k is
(a) -4 (b) 1 (c) 2 (d) -2 *(2008)*

28. The normal to a curve at $P(x, y)$ meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a
(a) circle (b) hyperbola
(c) ellipse (d) parabola. *(2007)*

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29. Consider a family of circles which are passing through the point (–1, 1) and are tangent to x-axis. If (h, k) are the coordinate of the centre of the circles, then the set of values of k is given by the interval
(a) $-\frac{1}{2} \le k \le \frac{1}{2}$ (b) $k \le \frac{1}{2}$
(c) $0 \le k \le \frac{1}{2}$ (d) $k \ge \frac{1}{2}$. *(2007)*

30. If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines $xy = 0$, then m is
(a) 1 (b) 2 (c) –1/2 (d) –2. *(2007)*

31. Let $P = (-1, 0)$, $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR is
(a) $\frac{\sqrt{3}}{2}x + y = 0$ (b) $x + \sqrt{3}y = 0$
(c) $\sqrt{3}x + y = 0$ (d) $x + \frac{\sqrt{3}}{2}y = 0$. *(2007)*

32. Let $A(h, k)$, $B(1, 1)$ and $C(2, 1)$ be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1 square unit, then the set of values which 'k' can take is given by
(a) {–1, 3} (b) {–3, –2}
(c) {1, 3} (d) {0, 2}. *(2007)*

33. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is
(a) (2, 4) (b) (–2, 0)
(c) (–1, 1) (d) (0, 2). *(2007)*

34. For the hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant when α varies ?
(a) abscissae of vertices (b) abscissae of foci
(c) eccentricity (d) directrix. *(2007)*

35. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}$, $x > 0$ and $y = 3x$, $x > 0$, then a belongs to
(a) $\left(0, \frac{1}{2}\right)$ (b) $(3, \infty)$
(c) $\left(\frac{1}{2}, 3\right)$ (d) $\left(-3, -\frac{1}{2}\right)$. *(2006)*

36. Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of chord of the circle C that subtend an angle of $2\pi/3$ at its centre is
(a) $x^2 + y^2 = \frac{3}{2}$ (b) $x^2 + y^2 = 1$
(c) $x^2 + y^2 = \frac{27}{4}$ (d) $x^2 + y^2 = \frac{9}{4}$. *(2006)*

37. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, then the equation of the circle is
(a) $x^2 + y^2 + 2x - 2y - 47 = 0$
(b) $x^2 + y^2 + 2x - 2y - 62 = 0$
(c) $x^2 + y^2 - 2x + 2y - 62 = 0$
(d) $x^2 + y^2 - 2x + 2y - 47 = 0$. *(2006)*

38. In an ellipse, the distance between its focii is 6 and minor axis is 8. Then its eccentricity is,
(a) 3/5 (b) 1/2
(c) 4/5 (d) $1/\sqrt{5}$. *(2006)*

39. The locus of the vertices of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is
(a) $xy = \frac{105}{64}$ (b) $xy = \frac{3}{4}$
(c) $xy = \frac{35}{16}$ (d) $xy = \frac{64}{105}$. *(2006)*

40. A straight line through the point $A(3, 4)$ is such that its intercept between the axes is bisected at A. Its equation is
(a) $x + y = 7$ (b) $3x - 4y + 7 = 0$
(c) $4x + 3y = 24$ (d) $3x + 4y = 25$. *(2006)*

41. If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then
(a) $3a^2 - 2ab + 3b^2 = 0$ (b) $3a^2 - 10ab + 3b^2 = 0$
(c) $3a^2 + 2ab + 3b^2 = 0$ (d) $3a^2 + 10ab + 3b^2 = 0$. *(2005)*

42. The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is
(a) a circle (b) an ellipse
(c) a hyperbola (d) a parabola. *(2005)*

43. An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is
(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{4}$. *(2005)*

44. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is
(a) $2ax + 2by - (a^2 - b^2 + p^2) = 0$
(b) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$
(c) $2ax + 2by - (a^2 + b^2 + p^2) = 0$
(d) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$. *(2005)*

45. A circle touches the x-axis and also touches the circle with centre at (0, 3) and radius 2. The locus of the centre of the circle is
(a) a circle (b) an ellipse
(c) a parabola (d) a hyperbola. *(2005)*

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46. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q then the line $5x + by - a = 0$ passes through P and Q for
(a) no value of a
(b) exactly one value of a
(c) exactly two values of a
(d) infinitely many values of a. *(2005)*

47. If a vertex of a triangle is (1, 1) and the mid points of two sides through this vertex are (–1, 2) and (3, 2), then the centroid of the triangle is
(a) $\left(\frac{-1}{3}, \frac{7}{3}\right)$ (b) $\left(-1, \frac{7}{3}\right)$
(c) $\left(\frac{1}{3}, \frac{7}{3}\right)$ (d) $\left(1, \frac{7}{3}\right)$. *(2005)*

48. If non-zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ always passes through a fixed point. That point is
(a) (–1, –2) (b) (–1, 2)
(c) $\left(1, -\frac{1}{2}\right)$ (d) (1, –2). *(2005)*

49. The line parallel to the x-axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is
(a) below the x-axis at a distance of 2/3 from it
(b) below the x-axis at a distance of 3/2 from it
(c) above the x-axis at a distance of 2/3 from it
(d) above the x-axis at a distance of 3/2 from it. *(2005)*

50. Let P be the point (1, 0) and Q a point on the locus $y^2 = 8x$. The locus of mid point of PQ is
(a) $x^2 - 4y + 2 = 0$ (b) $x^2 + 4y + 2 = 0$
(c) $y^2 + 4x + 2 = 0$ (d) $y^2 - 4x + 2 = 0$. *(2005)*

51. The eccentricity of an ellipse, with its centre at the origin, is 1/2. If one of the directrices is $x = 4$, then the equation of the ellipse is
(a) $4x^2 + 3y^2 = 12$ (b) $3x^2 + 4y^2 = 12$
(c) $3x^2 + 4y^2 = 1$ (d) $4x^2 + 3y^2 = 1$. *(2004)*

52. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then
(a) $d^2 + (2b - 3c)^2 = 0$ (b) $d^2 + (3b + 2c)^2 = 0$
(c) $d^2 + (2b + 3c)^2 = 0$ (d) $d^2 + (3b - 2c)^2 = 0$. *(2004)*

53. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is
(a) $x^2 + y^2 + x + y = 0$ (b) $x^2 + y^2 - x + y = 0$
(c) $x^2 + y^2 - x - y = 0$ (d) $x^2 + y^2 + x - y = 0$. *(2004)*

54. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameters of a circle of circumference 10π, then the equation of the circle is
(a) $x^2 + y^2 + 2x + 2y - 23 = 0$
(b) $x^2 + y^2 - 2x - 2y - 23 = 0$
(c) $x^2 + y^2 - 2x + 2y - 23 = 0$
(d) $x^2 + y^2 + 2x - 2y - 23 = 0$. *(2004)*

55. A variable circle passes through the fixed point $A(p, q)$ and touches x-axis. The locus of the other end of the diameter through A is
(a) $(y - p)^2 = 4qx$ (b) $(x - q)^2 = 4py$
(c) $(x - p)^2 = 4qy$ (d) $(y - q)^2 = 4px$. *(2004)*

56. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is
(a) $2ax - 2by + (a^2 + b^2 + 4) = 0$
(b) $2ax + 2by - (a^2 + b^2 + 4) = 0$
(c) $2ax - 2by + (a^2 + b^2 + 4) = 0$
(d) $2ax - 2by - (a^2 + b^2 + 4) = 0$. *(2004)*

57. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then c equals
(a) 3 (b) – 1 (c) 1 (d) – 3. *(2004)*

58. If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product, then c has the value
(a) 2 (b) –1 (c) 1 (d) –2. *(2004)*

59. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is –1 is
(a) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$
(b) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = 1$
(c) $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$
(d) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$. *(2004)*

60. Let $A(2, -3)$ and $B(-2, 1)$ be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line
(a) $3x + 2y = 5$ (b) $2x - 3y = 7$
(c) $2x + 3y = 9$ (d) $3x - 2y = 3$. *(2004)*

61. The normal to the curve $x = a(1 + \cos\theta)$, $y = a \sin\theta$ at θ always passes through the fixed point
(a) (0, 0) (b) $(0, a)$
(c) $(a, 0)$ (d) (a, a). *(2004)*

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62. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

(a) $\left(\frac{-9}{8}, \frac{9}{2}\right)$ (b) (2, –4)

(c) (2, 4) (d) $\left(\frac{9}{8}, \frac{9}{2}\right)$ *(2004)*

63. If the equation of the locus of point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then $c =$

(a) $a_1^2 - a_2^2 + b_1^2 - b_2^2$

(b) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$

(c) $\sqrt{(a_1^2 + b_1^2 - a_2^2 - b_2^2)}$

(d) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$. *(2003)*

64. Locus of centroid of the triangle whose vertices are $(a \cos t, a \sin t)$, $(b \sin t, -b \cos t)$ and $(1, 0)$, where t is a parameter, is

(a) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

(b) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$

(c) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$

(d) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$. *(2003)*

65. If the pairs of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then

(a) $p = -q$ (b) $pq = 1$

(c) $pq = -1$ (d) $p = q$. *(2003)*

66. A square of side a lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle α $(0 < \alpha < \pi/4)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is

(a) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$

(b) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$

(c) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

(d) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$. *(2003)*

67. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq. units. Then the equation of the circle is

(a) $x^2 + y^2 + 2x - 2y = 47$

(b) $x^2 + y^2 - 2x + 2y = 47$

(c) $x^2 + y^2 - 2x + 2y = 62$

(d) $x^2 + y^2 + 2x - 2y = 62$. *(2003)*

68. If the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then

(a) $r < 2$ (b) $r = 2$

(c) $r > 2$ (d) $2 < r < 8$. *(2003)*

69. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then

(a) $t_2 = -t_1 + \frac{2}{t_1}$ (b) $t_2 = t_1 - \frac{2}{t_1}$

(c) $t_2 = t_1 + \frac{2}{t_1}$ (d) $t_2 = -t_1 - \frac{2}{t_1}$. *(2003)*

70. The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide. Then the value of b^2 is

(a) 5 (b) 7 (c) 9 (d) 1. *(2003)*

71. A triangle with vertices (4, 0), (–1, –1), (3, 5) is

(a) isosceles and right angled

(b) isosceles but not right angled

(c) right angled but not isosceles

(d) neither right angled nor isosceles *(2002)*

72. The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length $3a$ is

(a) $x^2 + y^2 = 9a^2$ (b) $x^2 + y^2 = 16a^2$

(c) $x^2 + y^2 = 4a^2$ (d) $x^2 + y^2 = a^2$. *(2002)*

73. The centre of the circle passing through (0, 0) and (1, 0) and touching the circle $x^2 + y^2 = 9$ is

(a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, -\sqrt{2}\right)$

(c) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \frac{3}{2}\right)$ *(2002)*

74. Locus of mid point of the portion between the axes of $x \cos\alpha + y \sin\alpha = p$ where p is constant is

(a) $x^2 + y^2 = \frac{4}{p^2}$ (b) $x^2 + y^2 = 4p^2$

(c) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{2}{p^2}$ (d) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$. *(2002)*

75. The point of lines represented by $3ax^2 + 5xy + (a^2 - 2)y^2 = 0$ and $\perp$ to each other for

(a) two values of a (b) $\forall$ a

(c) for one value of a (d) for no values of a. *(2002)*

76. The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is

(a) $4 \le x^2 + y^2 \le 64$ (b) $x^2 + y^2 \le 25$

(c) $x^2 + y^2 \ge 25$ (d) $3 \le x^2 + y^2 \le 9$ *(2002)*

77. If the pair of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ intersect on the y-axis then

(a) $2fgh = bg^2 + ch^2$ (b) $bg^2 \ne ch^2$

(c) $abc = 2fgh$ (d) none of these *(2002)*

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78. If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subtends an angle of measure 45° at the major segment of the circle then value of m is

(a) $2 \pm \sqrt{2}$ (b) $-2 \pm \sqrt{2}$
(c) $-1 \pm \sqrt{2}$ (d) none of these *(2002)*

79. Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are

(a) $x = \pm(y + 2a)$ (b) $y = \pm(x + 2a)$
(c) $x = \pm(y + a)$ (d) $y = \pm(x + a)$ *(2002)*

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Answer Key

1.	(b)	**2.**	(a)	**3.**	(a)	**4.**	(d)	**5.**	(a)	**6.**	(a)
7.	(d)	**8.**	(c)	**9.**	(b)	**10.**	(a)	**11.**	(d)	**12.**	(a)
13.	(c)	**14.**	(c)	**15.**	(d)	**16.**	(c)	**17.**	(d)	**18.**	(b)
19.	(d)	**20.**	(c)	**21.**	(a)	**22.**	(a)	**23.**	(a)	**24.**	(b)
25.	(d)	**26.**	(c)	**27.**	(a)	**28.**	(b, c)	**29.**	(d)	**30.**	(a)
31.	(c)	**32.**	(a)	**33.**	(b)	**34.**	(b)	**35.**	(c)	**36.**	(d)
37.	(d)	**38.**	(a)	**39.**	(a)	**40.**	(c)	**41.**	(c)	**42.**	(c)
43.	(b)	**44.**	(c)	**45.**	(c)	**46.**	(a)	**47.**	(d)	**48.**	(d)
49.	(b)	**50.**	(d)	**51.**	(b)	**52.**	(c)	**53.**	(c)	**54.**	(c)
55.	(c)	**56.**	(b)	**57.**	(d)	**58.**	(a)	**59.**	(d)	**60.**	(c)
61.	(c)	**62.**	(d)	**63.**	(d)	**64.**	(a)	**65.**	(c)	**66.**	(c)
67.	(b)	**68.**	(d)	**69.**	(d)	**70.**	(b)	**71.**	(a)	**72.**	(c)
73.	(b)	**74.**	(d)	**75.**	(a)	**76.**	(a)	**77.**	(a)	**78.**	(c)
79.	(b)										

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Explanations

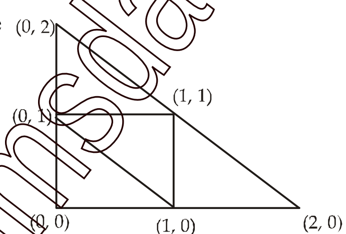
1. **(b) :** The system of circles touches the line $y = 0$ at the point $(3, 0)$ is given by $\{(x-3)^2 + y^2\} + \lambda y = 0$
As the circle passes through $(1, -2)$, we can determine λ which gives $4 + 4 - 2\lambda = 0$ $\therefore$ $\lambda = 4$
The circle is $(x-3)^2 + y^2 + 4y = 0$. A simple calculation shows that $(5, -2)$ lies on the circle.

2. **(a) :** Let a tangent to the parabola be
$y = mx + \frac{\sqrt{5}}{m}$ $(m \neq 0)$
As it is a tangent to the circle $x^2 + y^2 = 5/2$, we have
$\left(\frac{\sqrt{5}}{m}\right) = \frac{\sqrt{5}}{\sqrt{2}}\sqrt{1+m^2} \Rightarrow (1+m^2)m^2 = 2$
which gives $m^4 + m^2 - 2 = 0$ $\Rightarrow$ $(m^2-1)(m^2+2) = 0$
As $m \in R$, $m^2 = 1$ $\therefore$ $m = \pm 1$
Also $m = \pm 1$ does satisfy $m^4 - 3m^2 + 2 = 0$
Hence common tangents are
$y = x + \sqrt{5}$ and $y = -x - \sqrt{5}$

3. **(a) :** As the slope of incident ray is $-\frac{1}{\sqrt{3}}$ so the slope of reflected ray has to be $\frac{1}{\sqrt{3}}$.
The point of incidence is $(\sqrt{3}, 0)$. Hence the equation of reflected ray is $y = \frac{1}{\sqrt{3}}(x - \sqrt{3})$.
$\therefore \sqrt{3}y - x = -\sqrt{3}$. $\therefore x - \sqrt{3}y - \sqrt{3} = 0$

4. **(d) :** Foci are given by $(\pm ae, 0)$
As $a^2e^2 = a^2 - b^2 = 7$ we have equation of circle as
$(x-0)^2 + (y-3)^2 = (\sqrt{7}-0)^2 + (0-3)^2$
$\therefore$ $x^2 + y^2 - 6y - 7 = 0$

5. **(a) :** The triangle whose sides midpoints are given to be $(0, 1)$, $(1, 0)$ and $(1, 1)$ happen to be a right angled triangle with vertices as shown.



1st solution : x-coordinate of incentre
$= \frac{ax_1 + bx_2 + cx_3}{a+b+c} = \frac{2\times 2 + 2\sqrt{2}\times 0 + 2\times 0}{2+2+2\sqrt{2}}$
$= \frac{4}{4+2\sqrt{2}} = \frac{2}{2+\sqrt{2}} = 2 - \sqrt{2}$
2nd solution : $r = (s-a)\tan\frac{A}{2}$
$= \left(\frac{4+2\sqrt{2}}{2} - 2\sqrt{2}\right)\tan\frac{\pi}{4} = 2 - \sqrt{2}$

6. **(a) :** $A(1, 1)$; $B(2, 4)$
$P(x_1, y_1)$ divides line segment AB in the ratio $3 : 2$
$x_1 = \frac{3(2)+2(1)}{5} = \frac{8}{5}$ $y_1 = \frac{3(4)+2(1)}{5} = \frac{14}{5}$
$2x + y = k$ passes through $P(x_1, y_1)$
$\therefore 2\times\frac{8}{5} + \frac{14}{5} = k \Rightarrow k = 6$

7. **(d) : Statement 1 :**
$y^2 = 16\sqrt{3}x$, $y = mx + \frac{4\sqrt{3}}{m}$
$\frac{x^2}{2} + \frac{y^2}{4} = 1$, $x = m_1 y + \sqrt{4m_1^2 + 2}$
$\Rightarrow y = \frac{x}{m_1} - \sqrt{4 + \frac{2}{m_1^2}}$ $m = \frac{1}{m_1}$
Now, $\left(\frac{4\sqrt{3}}{m}\right)^2 = \left(-\sqrt{4+\frac{2}{m_1^2}}\right)^2$
$\Rightarrow \frac{48}{m^2} = 4 + \frac{2}{m_1^2} = 4 + 2m^2 \Rightarrow \frac{24}{m^2} = 2 + m^2$
$\Rightarrow$ $m^4 + 2m^2 - 24 = 0$ ...(1)
$\Rightarrow$ $(m^2+6)(m^2-4) = 0 \Rightarrow m = \pm 2$
Statement 2 : If $y = mx + \frac{4\sqrt{3}}{m}$ is a common tangent to $y^2 = 16\sqrt{3}x$ and ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 - 24 = 0$
From (1), statement 2 is a correct explanation for statement 1.

8. **(c) :** Let the equation of the circle is $(x-1)^2 + (y-k)^2 = k^2$
It passes through $(2, 3)$
$\therefore$ $1 + 9 + k^2 - 6k = k^2$
$\Rightarrow k = \frac{5}{3} \Rightarrow$ diameter $= \frac{10}{3}$

9. **(b) :** $(x-1)^2 + y^2 = 1$, $r = 1$ $\Rightarrow$ $a = 2$
and $x^2 + (y-2)^2 = 4$, $r = 2$ $\Rightarrow b = 4$
$\Rightarrow \frac{x^2}{16} + \frac{y^2}{4} = 1 \Rightarrow x^2 + 4y^2 = 16$

10. **(a) :** $y = mx + c$ $\Rightarrow$ $2 = m + c$
Co-ordinates of P & Q : $P(0, c)$, $Q(-c/m, 0)$
$\frac{1}{2}\times|c|\times\left|\frac{c}{m}\right| = A \Rightarrow \frac{c^2}{2m} = A$

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$\Rightarrow \dfrac{(2-m)^2}{2m} = A \Rightarrow \dfrac{m^2-4m+4}{2m} = A \Rightarrow \dfrac{m}{2} - 2 + \dfrac{2}{m} = A$

$\because \dfrac{dA}{dm} = 0$

$\Rightarrow \dfrac{1}{2} - \dfrac{2}{m^2} = 0 \Rightarrow \dfrac{1}{2} = \dfrac{2}{m^2} \Rightarrow m^2 = 4 \Rightarrow m = \pm 2$

11. (d) : The centres and radii are

$\left(x - \dfrac{a}{2}\right)^2 + y^2 = \dfrac{a^2}{4}, \quad x^2 + y^2 = c^2$

Centre $\left(\dfrac{a}{2}, 0\right)$ and (0, 0) & radius $= \dfrac{a}{2}$ and c

$\sqrt{\left(\dfrac{a}{2}\right)^2 + (0-0)} = \left|\left|\dfrac{a}{2}\right| \pm c\right| \Rightarrow \left|\dfrac{a}{2}\right| = \left|\left|\dfrac{a}{2}\right| \pm c\right|$

$\Rightarrow \left|\dfrac{a}{2}\right| = c - \left|\dfrac{a}{2}\right|. \quad \therefore \ |a| = c.$

12. (a) : In triangle OPQ, O divides PQ in the ratio of $OP : OQ$ which is $2\sqrt{2} : \sqrt{5}$ but it fails to divide triangle into two similar triangles.

13. (c) : Let P be (y^2, y)

Perpendicular distance from P to $x - y + 1 = 0$ is

$\dfrac{|y^2 - y + 1|}{\sqrt{2}}$

As $|y^2 - y + 1| = y^2 - y + 1$ $(\because y^2 - y + 1 > 0)$

Minimum value $= \dfrac{1}{\sqrt{2}} \cdot \dfrac{(4ac - b^2)}{4a}$

$= \dfrac{1}{\sqrt{2}} \cdot \dfrac{4-1}{4 \cdot 1} = \dfrac{3}{4\sqrt{2}}$

14. (c) : Let the ellipse be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

(–3, 1) lies on it $\Rightarrow \dfrac{9}{a^2} + \dfrac{1}{b^2} = 1$

Also $b^2 = a^2\left(1 - \dfrac{2}{5}\right) \Rightarrow 5b^2 = 3a^2$

Upon solving we get $a^2 = \dfrac{32}{7}, \ b^2 = \dfrac{32}{5}$

The equation to ellipse becomes

$3x^2 + 5y^2 = 32$

15. (d) : $y = x + \dfrac{4}{x^2}$

On differentiation, $\dfrac{dy}{dx} = 1 - \dfrac{8}{x^3}$

As the tangent is parallel to x-axis, we have

$1 - \dfrac{8}{x^3} = 0 \Rightarrow x^3 = 8. \quad \therefore \ x = 2$

So, $y = 2 + \dfrac{4}{2^2} = 2 + 1 = 3$

Thus (2, 3) is the point of contact and equation of the tangent is $y = 3$.

16. (c) : From a property of the parabola, the perpendicular tangents intersect at the directrix.

The equation of directrix is $x = -1$, hence this is the locus of point P.

17. (d) : As the line passes through (13, 32), we have

$\dfrac{13}{5} + \dfrac{32}{b} = 1 \Rightarrow \dfrac{32}{b} = 1 - \dfrac{13}{5} = -\dfrac{8}{5} \Rightarrow b = -20$

Thus the line is $\dfrac{x}{5} - \dfrac{y}{20} = 1$, *i.e.*, $4x - y = 20$

The equation of line parallel to $4x - y = 20$ has slope 4.

Thus $-\dfrac{3}{c} = 4. \quad \therefore \ c = -\dfrac{3}{4}.$

Then the equation to line k is $4x - y = -3$

The distance between lines k and c is

$\dfrac{20+3}{\sqrt{4^2+1^2}} = \dfrac{23}{\sqrt{17}}$

18. (b) : The circle is $x^2 + y^2 - 4x - 8y - 5 = 0$

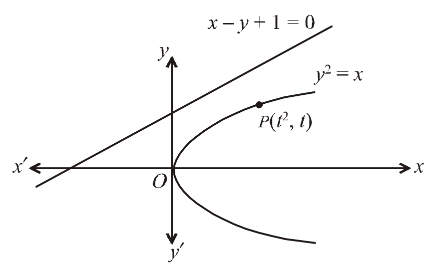
$\Rightarrow (x-2)^2 + (y-4)^2 = 5^2$

Length of perpendicular from centre (2, 4) on the line $3x - 4y - m = 0$ should be less than radius.

$\Rightarrow \dfrac{|6 - 16 - m|}{5} < 5 \Rightarrow (10 + m) < 25$

$\Rightarrow -25 < 10 + m < 25 \Rightarrow -35 < m < 15$

19. (d) :



Let $P(t^2, t)$ be a point on $y^2 = x$

The slope of normal at P

$= -\dfrac{1}{\text{the slope of tangent at } P} = -\dfrac{1}{1/2t} = -2t$

The shortest distance between the curves occur along the common normal. Thus $-2t = -1$

$\Rightarrow t = 1/2$

Thus the point P is (1/4, 1/2)

The required shortest distance

$= \dfrac{\left(\dfrac{1}{4} - \dfrac{1}{2} + 1\right)}{\sqrt{2}} = \dfrac{3}{4\sqrt{2}} = \dfrac{3\sqrt{2}}{8}$

20. (c) : $y = c_1 e^{c_2 x}$

Differentiating w.r.t. x, we get

$y' = c_1 c_2 e^{c_2 x} = c_2 y$ ...(i)

Again differentiating w.r.t. x

$y'' = c_2 y'$ ....(ii)

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From (i) and (ii) upon division

$$\frac{y'}{y''} = \frac{y}{y'} \Rightarrow \quad y''y = (y')^2$$

which is the desired differential equation of the family of curves.

21. **(a) :** Let P be a general point (x, y) such that

$\frac{PM}{PN} = \frac{1}{3}$ where $M \equiv (1, 0)$ and $N \equiv (-1, 0)$

we have $\frac{\sqrt{(x-1)^2 + y^2}}{\sqrt{(x+1)^2 + y^2}} = \frac{1}{3}$

$\Rightarrow \quad 9[(x - 1)^2 + y^2] = (x + 1)^2 + y^2$

which reduces to

$8x^2 + 8y^2 - 20x + 8 = 0$

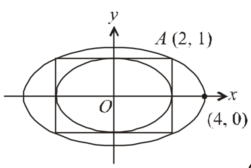
$$\Rightarrow \quad x^2 + y^2 - \frac{10}{4}x + 1 = 0$$

$$\Rightarrow \quad x^2 + y^2 - \frac{5}{2}x + 1 = 0$$

The locus is a circle with centre (5/4, 0)

As points A, B, C lie on this circle, the circumcentre of triangle ABC is (5/4, 0).

22. **(a) :** The given ellipse is $\frac{x^2}{4} + \frac{y^2}{1} = 1$



i.e., the point A, the corner of the rectangle in 1st quadrant, is (2, 1). Again the ellipse circumscribing the rectangle passess through the point (4, 0), so its equation is

$$\frac{x^2}{16} + \frac{y^2}{b^2} = 1$$

$A(2, 1)$ lies on the above ellipse

$$\Rightarrow \quad \frac{4}{16} + \frac{1}{b^2} = 1 \Rightarrow \quad \frac{1}{b^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

$\Rightarrow \quad b^2 = 4/3$

Thus the equation to the desired ellipse is

$$\frac{x^2}{16} + \frac{3}{4}y^2 = 1 \quad \Rightarrow x^2 + 12y^2 = 16$$

23. **(a) :** The radical axis, which in the case of intersection of the circles is the common chord, of the circles

$S_1 : x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and

$S_2 : x^2 + y^2 + 2x + 2y - p^2 = 0$ is $S_1 - S_2 = 0$

$\Rightarrow \quad x + 5y + 2p - 5 + p^2 = 0$ ...(i)

If there is a circle passing through P, Q and (1, 1) it's necessary and sufficient that (1, 1) doesn't lie on PQ,

i.e., $1 + 5 + 2p - 5 + p^2 \neq 0$

$\Rightarrow \quad p^2 + 2p + 1 \neq 0 \Rightarrow \quad (p + 1)^2 \neq 0 \quad \therefore \quad p \neq -1$

Thus for all values of p except '–1' there is a circle passing through P, Q and (1, 1).

24. **(b) :** Obviously the major axis is along the x-axis

The distance between the focus and the corresponding directrix

$$= \left|\frac{a}{e} - ae\right| = 4$$

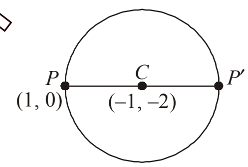
$$\Rightarrow \quad \frac{a}{e} - ae = 4 \qquad \text{(note that } \frac{a}{e} > ae\text{)}$$

$$\Rightarrow \quad a\left(\frac{1}{e} - e\right) = 4 \Rightarrow \quad a\left(2 - \frac{1}{2}\right) = 4$$

$$\Rightarrow \quad a \cdot \frac{3}{2} = 4 \;\therefore\; a = \frac{8}{3}$$

Remark : The question should have read "*The corresponding directrix*" in place of "*the directrix*".

25. **(d) :** The centre C of the circle is seen to be $(-1, -2)$. As C is the mid point P and P', the coordinate of P' is given by



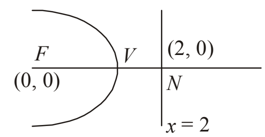
$P' \equiv (2 \times -1 - 1, 2 \times -2 - 0)$

$\equiv (-3, -4)$

Remark : If P be (α, β) and $C(h, k)$ then

$P' \equiv (2h - \alpha, 2k - \beta)$

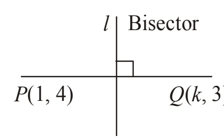
26. **(c) :** The vertex is the mid point of FN, that is, vertex = (1, 0)



27. **(a) :** The slope of

$$l = -\frac{1}{\text{the slope of the original line } PQ}$$

$$= -\frac{1}{\frac{3-4}{k-1}} = (k-1)$$



The midpoint $= \left(\frac{k+1}{2}, \frac{7}{2}\right)$

The equation to the bisector l is

$$\left(y - \frac{7}{2}\right) = (k-1)\left(x - \frac{k+1}{2}\right)$$

As $x = 0$, $y = -4$ satisfies it, we have

$$\left(-4 - \frac{7}{2}\right) = (k-1)\left(0 - \frac{k+1}{2}\right) \Rightarrow \quad -\frac{15}{2} = -\frac{k^2-1}{2}$$

$\Rightarrow k^2 - 1 = 15 \quad \Rightarrow \quad k^2 = 16 \quad \therefore \quad k = \pm 4.$

28. **(b, c) :** Equation of normal at $P(x, y)$ is

$$Y - y = -\frac{dx}{dy}(X - x) \quad \Rightarrow G \equiv \left(x + y \cdot \frac{dy}{dx}, 0\right)$$

$$\left|x + y\frac{dy}{dx}\right| = |2x| \quad \Rightarrow \quad y\frac{dy}{dx} = x \quad \text{or} \quad y\frac{dy}{dx} = -3x$$

$ydy = xdx$ or $ydy = -3xdx$

After integrating, we get

$$\frac{y^2}{2} = \frac{x^2}{2} + c \text{ or } \frac{y^2}{2} = -\frac{3x^2}{2} + c$$

$\Rightarrow x^2 - y^2 = -2c$

or $3x^2 + y^2 = 2c \Rightarrow x^2 - y^2 = c_1$ or $3x^2 + y^2 = c_2$.

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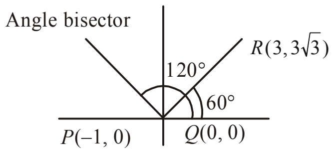
29. **(d) :** Equation of circle $(x - h)^2 + (y - k)^2 = k^2$
It is passing through (–1, 1) then
$(-1 - h)^2 + (1 - k)^2 = k^2 \Rightarrow h^2 + 2h - 2k + 2 = 0,$
$D \geq 0$
$2k - 1 \geq 0 \Rightarrow k \geq \frac{1}{2}.$

30. **(a) :** Sum of the slopes $= -\frac{\text{co-efficient of } xy}{\text{co-efficient of } y^2}$

$\therefore$ Sum of slopes $= -\frac{(1-m^2)}{m} = 0$

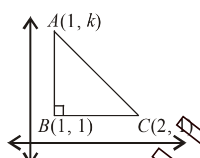
$\Rightarrow m = \pm 1.$

Second method
Equation of bisectors of lines $xy = 0$ are $y = \pm x$
Put $y = \pm x$ in
$my^2 + (1 - m^2)xy - mx^2 = 0$, we get
$(1 - m^2)x^2 = 0 \Rightarrow m = \pm 1.$



31. **(c) :** Slope of the required angle bisector is $\tan 120° = -\sqrt{3}$
Hence equation of the angle bisector is
$y = -\sqrt{3}(x - 0)$
$\Rightarrow \sqrt{3}x + y = 0$

32. **(a) :** $\frac{1}{2} \times |k - 1| \times 1 = 1$
$k = -1, 3.$



33. **(b) :** Let P is the required point, then P lies on directrix $x = -2$ of $y^2 = 8x$
Hence $P \equiv (-2, 0)$.

34. **(b) :** $\because$ $b^2 = a^2(e^2 - 1) \Rightarrow \sin^2\alpha = \cos^2\alpha\,(e^2 - 1) \Rightarrow \tan^2\alpha + 1 = e^2 \Rightarrow e^2 = \sec^2\alpha$
Vertices $\equiv (\pm\cos\alpha, 0)$
Coordinate of focii are $(\pm ae, 0) \equiv (\pm 1, 0)$
$\Rightarrow$ if α varies then the abscissa of foci remain constant.

35. **(c) :** Given lines are $y = \frac{x}{2}(x > 0)$
and $y = 3x$ $(x > 0)$ using (a, a^2) in these lines

$a^2 - \frac{a}{2} > 0$ ... (i)

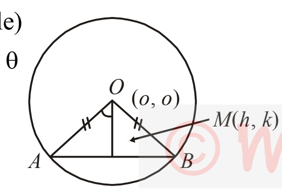
and $a^2 - 3a < 0$ ... (ii)

Solving (i) and (ii) we get $\frac{1}{2} < a < 3$

36. **(d) :** Let AB is chord of circle and $M(h, k)$ be mid point of AB $\angle AOM = 60°$
Now $OA = OB = 3$ and
$OM \perp AB$ (By properties of circle)
Now $OM = \sqrt{h^2 + k^2}$, $OM = r\cos\theta$

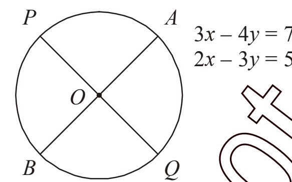
$\sqrt{h^2 + k^2} = 3\cos 60°$

$\sqrt{h^2 + k^2} = \frac{3}{2}$



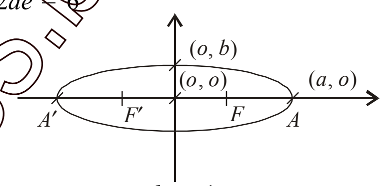
$\Rightarrow h^2 + k^2 = \frac{9}{4} \Rightarrow x^2 + y^2 = \frac{9}{4}$

37. **(d) :** Let $OA = r$
Given area = 49 π



$\Rightarrow$ $\pi r^2 = 49\pi$
$r = 7$
Point of intersection of AB and PQ is (1, –1)
$\therefore$ equation of circle is $(x - 1)^2 + (y + 1)^2 = 7^2$
$\Rightarrow$ $x^2 + y^2 - 2x + 2y - 47 = 0$

38. **(a) :** Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (a > b)$
Given $2b = 8$ ... (i)
and $2ae = 6$ ... (ii)



By (i) and (ii) we have $\frac{b}{ae} = \frac{4}{3}$

$\Rightarrow \frac{b^2}{a^2} = \frac{16}{9}e^2$

$\Rightarrow 1 - e^2 = \frac{16}{9}e^2$ $(\because\ b^2 = a^2(1 - e^2)$ as $a > b)$

$\Rightarrow e = \frac{3}{5}$

39. **(a) :** Let h, k be the locus of the vertex of family of parabola

$y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$

$\therefore$ $k = \frac{a^3h^2}{3} + \frac{a^2h}{2} = 2a$

$\Rightarrow \frac{3k}{a^3} = h^2 + \frac{3h}{2a} - \frac{6}{a^2}$

$\Rightarrow \frac{3}{a^3}\left(k + \frac{35a}{16}\right) = \left(h + \frac{3}{4a}\right)^2$

i.e., $\left\{x^2 = \frac{3}{a^3}y, \text{ where } x = h + \frac{3}{4a}, y = k + \frac{35a}{16}\right\}$

$\Rightarrow$ vertex is $\left(\frac{-3}{4a}, \frac{-35a}{16}\right)^2$

$\therefore$ $hk = \left(\frac{-3}{4a}\right)\left(\frac{-35a}{16}\right)$

$\Rightarrow hk = \frac{105}{64}$

$\Rightarrow xy = \frac{105}{64}$ (taking $h = x$, $k = y$)

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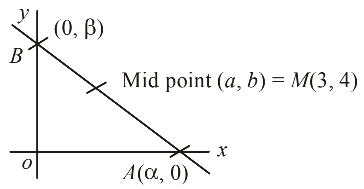
40. **(c) :** Now the equation of line which meet the x-axis and y-axis



at $A(\alpha, 0)$ B $(0, \beta)$ is given by

$\frac{x}{\alpha}+\frac{y}{\beta}=1$

where $\alpha = 2a = 6$

and $\beta = 2b = 8$

$\therefore$ required equation be $\frac{x}{6}+\frac{y}{8}=y$

$\Rightarrow$ $4x + 3y = 24$

41. **(c) :**

$ax^2+2(a+b)xy+by^2=0$ (*)

Let θ be the angle between the lines represent by *

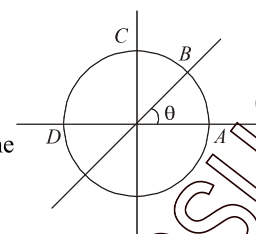
$\therefore \ \tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$

$\tan\theta = \left|\frac{2\sqrt{(a+b)^2-ab}}{a+b}\right|$

$\{3 \text{ area } \widehat{OAB} = \text{ Area of } \widehat{DBC}\}$

Now $\theta = 45$

{$\because$ area of one sector = 3 time the area of another sector}



$\therefore \ \tan 45° = \left|\frac{2\sqrt{(a+b)^2-ab}}{a+b}\right|$

$\Rightarrow 3a^2 + 2ab + 3b^2 = 0.$

42. **(c) :** Given $y = \alpha x + \beta$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\therefore \ b^2x^2 - a^2y^2 = a^2b^2$

$\Rightarrow b^2x^2 - a^2(\alpha x+\beta)^2 = a^2b^2$

by using $y = a\alpha + \beta$

$\Rightarrow x^2(b^2 - a^2\alpha^2) - 2a^2\alpha\beta x + (-\beta^2a^2 - a^2b^2) = 0$ ....(*)

Now the line $y = \alpha x + \beta$ will be tangent to circle if both roots if (*) are equal

$\therefore$ keeping $D = 0$ in (*) we have

$4\alpha^2a^4\beta^2 = 4(b^2 - a^2\alpha^2)(-\beta^2a^2 - a^2b^2)$

$\Rightarrow \alpha^2a^2\beta^2 = (b^2 - a^2\alpha^2)(-\beta^2 - b^2)$

$\Rightarrow \alpha^2a^2\beta^2 = -b^2\beta^2 + \beta^2a^2\alpha^2 - b^4 + a^2\alpha^2b^2$

$\Rightarrow a^2\alpha^2b^2 = b^2(b^2+\beta^2) \Rightarrow a^2\alpha^2 = b^2+\beta^2$

$\Rightarrow a^2x^2 - y^2 = b^2$

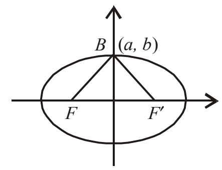
43. **(b) :** $F'(-ae, 0)$, $F(ae, 0)$

Slope of $BF = \frac{b}{ae} = m_1$ (say)

Slope of $BF' = \frac{b}{-ae} = m_2$ (say)

Now $m_1 \times m_2 = -1 \ \Rightarrow \frac{b}{ae}\times\frac{b}{-ae} = -1$

$\Rightarrow b^2 = a^2e^2 \ \Rightarrow a^2 - a^2e^2 = a^2e^2$ $\left(\because e^2 = 1-\frac{b^2}{a^2}\right)$



$\Rightarrow 1 - e^2 = e^2, \ 2e^2 = 1, e = \pm\frac{1}{\sqrt{2}}.$

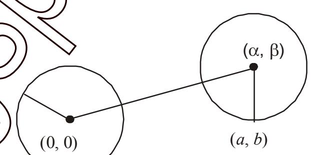
44. **(c) :** Let locus of the centre of circle be (α, β).

If C_1, C_2 are centres of the circles with radii r_1, r_2 respectively then $(C_1C_2)^2 = r_1^2 + r_2^2$

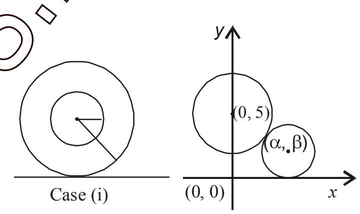
$\Rightarrow \ \alpha^2 + \beta^2 = p^2 + (\alpha - a)^2 + (\beta - b)^2$

$\Rightarrow \ p^2 + a^2 + b^2 - 2a\alpha - 2b\beta = 0$

$\Rightarrow \ 2ax + 2by - (a^2 + b^2 + p^2) = 0.$



45. **(c) :** Let locus of centre be α, β then according to given, if r_1, r_2 are radii of circles then



Internal touch. This case does not exist as centre of circle is (0, 3) and radius is 2.

$C_1C_2 = r_2 \pm r_1$

$\Rightarrow \ (\alpha - 0)^2 + (\beta - 3)^2 = |\beta \pm 2|$

$\Rightarrow \ \alpha^2 + \beta^2 - 6\beta + 9 = \beta^2 + 4 + 4\beta$

and $\ \alpha^2 + \beta^2 - 6\beta + 9 = \beta^2 - 4\beta + 4$

$\Rightarrow \ \alpha^2 - 10\beta + 5 = 0$ and $x^2 = 2\beta + 5$

$\Rightarrow \ x^2 = 10y - 5$ and $x^2 = 2y - 5$

Both are parabolas but $x^2 = 2y - 5$ does not exist.

46. **(a) :** As the line passes through P and Q which are the point of intersection of two circles. It means given line is the equation of common chord and the equation of common chord of two intersecting circle is

$S_1 - S_2 = 0$

$= 5ax + (c - d)y + a + 1 = 0.$

Now $5ax + (c - d)y + a + 1 = 0$ and $5x + by - a = 0$ represent same equation.

$\therefore \ \frac{5a}{5} = \frac{c-d}{b} = \frac{a+1}{-a}$

$\Rightarrow a^2 + a + 1 = 0$ and $\frac{c-d}{b}+1 = -\frac{1}{a}$

$\Rightarrow \left(a+\frac{1}{2}\right)^2+\frac{3}{4} = 0$ and $-(c - d + b) = b/a$

$d - b - c = +b/a$ has no solution. $\therefore$ No value of a exist.

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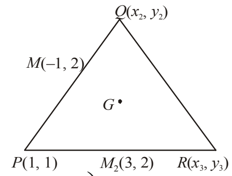
47. **(d) :** $\therefore\ x_2 = 2(-1) - 1 = -3$

$y_2 = 2 \times 2 - 1 = 3$

$x_3 = 3 \times 2 - 1 = 5$

$y_3 = 2 \times 2 - 1 = 3$



$\therefore$ Centroid $G = \left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$

$= \left(\dfrac{1-3+5}{3}, \dfrac{1+3+3}{3}\right) = \left(1, \dfrac{7}{3}\right).$

48. **(d) :** Let us take the two set of values of $a = 1,\ b = 1/2,\ c = 1/3$

and $a = 1/2,\ b = 1/3,\ c = 1/4$

Putting these value in the given equation we get

$x + 2y + 3 = 0$ and $2x + 3y + 4 = 0$ ...(*)

Solving the equations of (*) we have $x = 1,\ y = -2$

$(1, -2)$ is required point on the line.

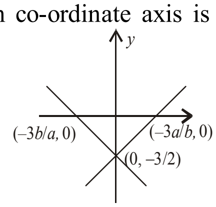
49. **(b) :** Intercepts made by the lines with co-ordinate axis is

$(-3b/a, 0),\ (0, -3/2)$

and $(0, -3/2)$

and $(3a/b, 0)$ and

Common intercept is $(0, -3/2)$.



50. **(d) :** Let $M(x', y')$ be point of locus mid point of PQ.

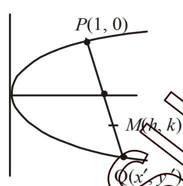
$\Rightarrow \dfrac{x'+1}{2} = h,\ \dfrac{y'+0}{2} = k,$

$\therefore\ x' = 2h - 1,\ y' = 2k$

Now (x', y') lies on $y^2 = 8x$

$\Rightarrow\ (2k)^2 = 8(2h - 1)$

$\Rightarrow\ y^2 = 2(2x - 1) \Rightarrow y^2 - 4x + 2 = 0.$



51. **(b) :** Equation of directrix $x = 4$ which is parallel to y-axis so axis of the ellipse is x-axis. Let equation of ellipse be

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1\ (a > b)$

Again $e = 1/2$ and $e^2 = 1 - \dfrac{b^2}{a^2}$

$\Rightarrow\ \left(\dfrac{b}{a}\right)^2 = 1 - 1/4 = 3/4$ ...(*)

Also the equation of one directrix is $x = 4$

$\therefore$ equation of directrix $x = \dfrac{a}{e}$

$\therefore\ 4 = \dfrac{a}{e}$

$\Rightarrow\ a = 2$ $(\because\ e = 1/2)$

Further $b^2 = \dfrac{a^2 \times 3}{4}$ by (*)

$b^2 = \dfrac{4 \times 3}{4} = 3$

Hence equation of ellipse is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$\Rightarrow \dfrac{x^2}{4} + \dfrac{y^2}{3} = 1$ or $3x^2 + 4y^2 = 12$

52. **(c) :** The point of intersection of parabola's $y^2 = 4ax$ and $x^2 = 4ay$ are $A(0, 0),\ B(4a, 4a)$

as the line $2bx + 3cy + 4d = 0$ passes through these points

$\therefore\ d = 0$ and $2b(4a) + 3c(4a) = 0$

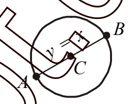
$\Rightarrow 2b + 3c = 0 \ \Rightarrow (2b + 3c)^2 + d^2 = 0$

53. **(c) :** Given circle $x^2 + y^2 - 2x = 0$ ... (1)

and line be $y = x$ ... (2)

Solving (1) and (2) we get

$x = 0,\ 1\ \therefore\ y = 0,\ 1$



$\therefore\ A(0, 0),\ B(1, 1)$ and equation of circle in the diameter form is $(x - 0)(x - 1) + (y - 0)(y - 1) = 0$

$\Rightarrow x^2 + y^2 - (x + y) = 0$

54. **(c) :** As per given condition centre of the circle is the point of intersection of the $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$

$\therefore$ centre is $(1, -1)$

Also circumference of the circle be given $2\pi r = 10\pi$

$\therefore\ r = 5$

$\therefore$ Required equation of circle is

$(x - 1)^2 + (y + 1)^2 = 5^2$

or $x^2 + y^2 - 2x + 2y - 23 = 0$

55. **(c) :** Equation of circle AB as diameter is given by

$(x - p)(x - \alpha) + (y - q)(y - \beta) = 0$

$\Rightarrow x^2 + y^2 - x(p + \alpha) - y(q + \beta) + p\alpha + q\beta = 0$ ...(1)

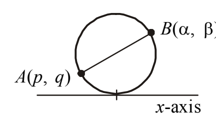
Now (1) touches axis of x so put $y = 0$ in (1) we have

$x^2 - x(p + \alpha) + p\alpha + q\beta = 0$ ...(2)

and $D = 0$ in equation (2)

$\therefore\ (p + \alpha)^2 = 4[p\alpha + q\beta]$

$\Rightarrow (p - \alpha)^2 = 4q\beta$



Now $\alpha \to x,\ \beta \to y$

$\therefore\ (p - x)^2 = 4q(y)$ which required locus of one end point of the diameter.

56. **(b) :** Let the equation of circle cuts orthogonally the circle $x^2 + y^2 = 4$ is

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

$\therefore\ 2g_1g_2 + 2f_1f_2 = c_1c_2$ (where $(-g, -f)$ are point of locus)

$\Rightarrow c = -4$

Again circle (i) passes through (a, b), so

$a^2 + b^2 + 2ga + 2fb + 4 = 0$

Now replacing g, f by x, y respectively

$\therefore\ 2ax + 2by - (a^2 + b^2 + 4) = 0$

57. **(d) :** The equation $ax^2 + 2hxy + by^2 = 0$

$= (y - m_1x)(y - m_2x)$

$\Rightarrow\ m_1 + m_2 = -\dfrac{2h}{b} = \dfrac{1}{4c}$ ...(*)

$m_1m_2 = \dfrac{3}{2}c$

and $3x + 4y = 0 \Rightarrow m_1 = -3/4$

$\therefore\ m_2 = -\dfrac{2}{c}$

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Now by (*) we have

$$-\left(\frac{3}{4}+\frac{2}{c}\right) = \frac{1}{4c}$$

$$\Rightarrow \quad -\frac{3}{4} = \frac{1}{4c}+\frac{2}{c} \quad \Rightarrow \quad -\frac{3}{4} = \frac{1}{4c}+\frac{8}{4c}$$

$$\Rightarrow \quad -\frac{3}{4} = \frac{9}{4c} \quad \therefore \; c = -3$$

58. **(a) :** If m_1 and m_2 are slope of the lines then by given condition
$m_1 + m_2 = 4m_1m_2$

$$\Rightarrow \quad -\frac{2c}{7} = -\frac{4}{7}$$

$$\Rightarrow \quad c = 2$$

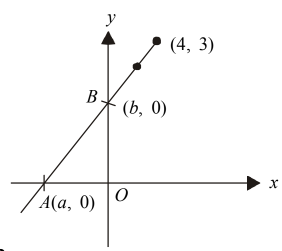
By using $ax^2 + 2hxy + by^2 = 0$

$$\Rightarrow \quad m_1 + m_2 = \frac{-2h}{b} \text{ and } m_1m_2 = \frac{a}{b}$$

59. **(d) :** Given $OA + OB = -1$

i.e. $a + b = -1$

$\therefore$ equation of the line be



$$\frac{x}{a}-\frac{y}{1+a} = 1$$

$$\Rightarrow \quad \frac{4}{a}-\frac{3}{1+a} = 1$$

$\Rightarrow$ $a = \pm 2$ (as $a = 2$ $b = -3$ and $a = -2$ $b = 1$)

so equation are $\frac{x}{2}-\frac{y}{3} = 1$ and $\frac{x}{-2}+\frac{y}{1} = 1$

60. **(c) :** Let locus of point $C(h, k)$ and centroid (α, β)

As (α, β) lies on $2x + 3y = 1$ $\therefore$ $2\alpha + 3\beta = 1$

Now centroid of ABC is

$$\left(\frac{2+(-2)+h}{3}, \frac{-3+1+k}{3}\right) \text{ or } \left(\frac{h}{3}, \frac{k-2}{3}\right)$$

$$\therefore \quad 2\left(\frac{h}{3}\right)+\frac{3(k-2)}{3} = 1$$

$$\Rightarrow \quad 2h + 3k = 9 \quad \Rightarrow \quad 2x + 3y = 9$$

61. **(c) :** The equation of normal at θ is

$$y - y_1 = -\frac{1}{\frac{dy}{dx}}(x - x_1)$$

$$\Rightarrow \quad y - a\sin\theta = \frac{\sin\theta}{\cos\theta}(x - a(1-\cos\theta))$$

which passes through $(a, 0)$

62. **(d) :** Given $y^2 = 18x$ and $\frac{dy}{dt} = 2\frac{dx}{dt}$

$$\therefore \quad 2y\frac{dy}{dt} = 18\frac{dx}{dt}$$

$$\Rightarrow 2y\frac{dy}{dt} = \frac{18}{2}\frac{dy}{dt}$$

$$\Rightarrow y = 9/2 \quad \therefore \; x = \frac{y^2}{18} = \frac{81}{72} = \frac{9}{8}$$

So the required point is $x = \frac{9}{8}$, $y = 9/2$

63. **(d) :** Let α, β is the point of locus, equidistant from (a_1, b_1) and (a_2, b_2) is given by

$(\alpha - a_1)^2 + (\beta - b_1)^2 = (\alpha - a_2)^2 + (\beta - b_2)^2$

$\Rightarrow a_1^2 + b_1^2 - 2a_1\alpha - 2b_1\beta - a_2^2 - b_2^2 + 2a_2\alpha + 2b_2\beta = 0$

$\Rightarrow 2(a_2 - a_1)\alpha + 2(b_2 - b_1)\beta + a_1^2 + b_1^2 - b_2^2 - a_2^2 = 0$

$$\Rightarrow (a_2 - a_1)x + (b_2 - b_1)y + \frac{1}{2}(a_1^2 + b_1^2 - a_2^2 - b_2^2) = 0$$

$$\Rightarrow c = -\frac{1}{2}[a_1^2 + b_1^2 - a_2^2 - b_2^2]$$

64. **(a) :** Let (h, k) be the co-ordinate of centroid

$$\therefore \quad h = \frac{a\cos t + b\sin t + 1}{3},$$

$$k = \frac{a\sin t - b\cos t + 0}{3}$$

$\Rightarrow$ $3h - 1 = a\cos t + b\sin t$ ...(i)

$3k = a\sin t - b\cos t$ ...(ii)

By squaring (i) and (ii) then adding we get

$(3h - 1)^2 + (3k)^2 = a^2(\cos^2 t + \sin^2 t) + b^2(\cos^2 t + \sin^2 t)$

Replacing (h, k) by (x, y) we get choice (a) is correct.

65. **(c) :** Given equations are

$x^2 - 2qxy - y^2 = 0$ ...(1)

$x^2 - 2pxy - y^2 = 0$ ...(2)

Joint equation of angle bisector of the line (i) and (ii) are same

$$\therefore \quad \frac{x^2 - y^2}{1+1} = \frac{xy}{-q}$$

$\Rightarrow$ $qx^2 + 2xy - qy^2 = 0$ ...(3)

Now (2) and (3) are same, taking ratio of their coefficients

$$\therefore \quad \frac{1}{q} = \frac{-p}{1}$$

$\Rightarrow$ $pq = -1$

66. **(c) :** According to the problem square lies above x-axis

Now equation of AB using two point form. We get

$y - y_1 = m(x - x_1)$

$$(y - a\sin\alpha) = -\frac{\cancel{a}(\cos\alpha - \sin\alpha)}{\cancel{a}(\cos\alpha + \sin\alpha)}[x - a\cos\alpha]$$

$\Rightarrow y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha)$

$= a\sin\alpha(\cos\alpha + \sin\alpha) + a\cos\alpha(\cos\alpha - \sin\alpha)$

$= a(\sin^2\alpha + \cos^2\alpha)$

$= a(1)$

67. **(b) :** Co-ordinate of centre may be $(1, -1)$ or $(-1, 1)$ but $1, -1$ satisfies the given equations of diameter, so choices (a) and (d) are out of court.

Again $\pi R^2 = 154$, $R^2 = 49$ $\therefore$ $R = 7$

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$\therefore$ Required equation of circle be

$(x - 1)^2 + (y + 1)^2 = 49$

$\Rightarrow x^2 + y^2 - 2x + 2y = 47$

68. **(d) :** $(x - 1)^2 + (y - 3)^2 = r^2 \therefore C_1(1, 3)$ and $r_1 = t_1 = r$

$(x - 4)^2 + 1(y + 1)^2 = 9 \therefore C_2(4, -1)$ and $r_2 = t_2 = 3$

so $C_1C_2 = \sqrt{(4-1)^2 + (3+1)^2} = 5$

Now for intersecting circles

$r_1 + r_2 > C_1C_2$ and $|r_1 - r_2| < C_1C_2$

$\therefore \Rightarrow r + 3 > 5$ and $|r - 3| < 5$

$\Rightarrow \quad r > 2$ and $-5 < r - 3 < 5$

$\Rightarrow \quad r > 2$ and $-2 < r < 8$

$\Rightarrow \quad r \in (2, 8)$

69. **(d) :** Since the normal at $(bt_1^2, 2bt_1)$, on parabola $y^2 = 4bx$ meet the parabola again at $(bt_2^2, 2bt_2)$

$\therefore \quad t_1x + y = 2bt_1 + bt_1^3$ passes through $(bt_2^2, 2bt_2)$

$\Rightarrow \quad t_1bt_2^2 + 2bt_2 = 2bt_1 + bt_1^3$

$\Rightarrow \quad t_1(t_2^2 - t_1^2) = 2(t_1 - t_2)$

$\Rightarrow \quad t_1(t_2 + t_1) = -2$

$\Rightarrow \quad t_2 + t_1 = -\dfrac{2}{t_1}$

$\Rightarrow \quad t_2 = -\dfrac{2}{t_1} - t_1$

70. **(b) :** Eccentricity for $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

is $b^2 = a^2(1 - e^2)$

and eccentricity for $\dfrac{x^2}{\frac{144}{25}} - \dfrac{y^2}{\frac{81}{25}} = 1$ is

$$\left\{\begin{aligned} e_1 &= \frac{a_1^2 + b_1^2}{a_1^2} \\ \therefore e_1 &= \sqrt{1 + \frac{81}{144}} = \frac{15}{12} \end{aligned}\right.$$

Again foci $= a_1e_1 = \dfrac{12}{5} \times \dfrac{15}{12} = 3$

$\therefore$ focus of hyperbola is $(3, 0) = (ae, 0)$

so focus of ellipse $(ae, 0) = (4e, 0)$

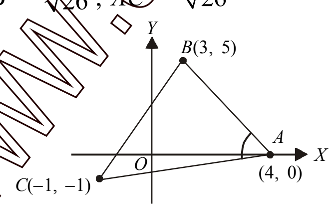
As their foci are same $\therefore 4e = 3 \therefore e = 3/4$

$\therefore e^2 = 1 - \left(\dfrac{b}{a}\right)^2 = 1 - \dfrac{b^2}{16}$

or $\dfrac{b^2}{16} = 1 - e^2 = 1 - \dfrac{9}{16}$

$\Rightarrow \quad b^2 = 7$

71. **(a) :** $AB = \sqrt{26}$, $AC = \sqrt{26}$



$\therefore \quad \triangle ABC$ is isosceles

Again product of the slope of $\overline{AC}$ and $\overline{AB}$

$= \dfrac{1}{5} \times (-5) = -1$

$\Rightarrow \quad AC \perp AB$

$\Rightarrow \quad$ right at A

72. **(c) :** Given median of the equilateral triangle is $3a$.

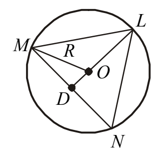
In $\triangle LMD$, $(LM)^2 = (LD)^2 + (MD)^2$

$(LM)^2 = 9a^2 + \left(\dfrac{LM}{2}\right)^2$

$\Rightarrow \dfrac{3}{4}(LM)^2 = 9a^2 \qquad \therefore (LM)^2 = 12a^2$

Again in triangle OMD, $(OM)^2 = (OD)^2 + (MD)^2$

$R^2 = (3a - R)^2 + \left(\dfrac{LM}{2}\right)^2$



$\Rightarrow R^2 = 9a^2 + R^2 - 6aR + 3a^2$

$\Rightarrow 6aR = 12a^2$

$R = 2a$

So equation of circle be

$(x - 0)^2 + (y - 0)^2 = R^2 = (2a)^2$

$\Rightarrow x^2 + y^2 = 4a^2$

73. **(b) :** Let equation of circle be $x^2 + y^2 + 2gx + 2fy + c = 0$.

As it passes through $(0, 0)$ so $c = 0$

and as it passes $(1, 0)$ so $-g = \dfrac{1}{2}$.

Now $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 = 9$ touches each other.

$\therefore$ equation of common tangent is $2gx + 2fy - 9 = 0$ and distance from the centre of circle $x^2 + y^2 = 9$ to the common tangent is equal to the radius of the circle $x^2 + y^2 = 9$

$\therefore \quad \dfrac{|0 + 0 - 9|}{\sqrt{4g^2 + 4f^2}} = 3$

$\Rightarrow 3^2 = 4(g^2 + f^2)\ 9$

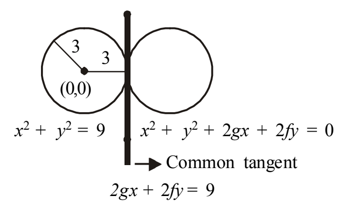
$= 4\left(\dfrac{1}{4} + f^2\right)$

$9 = 1 + 4f^2$

$\therefore \quad f^2 = 2$

$f = \pm\sqrt{2}$

$\therefore \quad -f = \pm\sqrt{2}$



$\therefore \quad$ Centre of the required circle be $\left(\dfrac{1}{2}, \sqrt{2}\right), \left(\dfrac{1}{2}, -\sqrt{2}\right)$

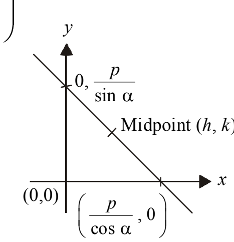
74. **(d) :** $\therefore (h, k)$ is $\left(\dfrac{p}{2\cos\alpha}, \dfrac{p}{2\sin\alpha}\right)$

$\therefore \quad \cos\alpha = \dfrac{p}{2h}$,

$\sin\alpha = \dfrac{p}{2k}$

$\Rightarrow \quad \sin^2\alpha + \cos^2\alpha$

$= \dfrac{p^2}{4h^2} + \dfrac{p^2}{4k^2} = \dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{4}{p^2}$



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75. **(a) :** Using fact : Pair of lines $Ax^2 + 2hxy + By^2 = 0$ are $\perp$ to each other if $A + B = 0$

$\Rightarrow \quad 3a + a^2 - 2 = 0$

$\Rightarrow \quad a^2 + 3a - 2 = 0$

$\Rightarrow$ There exist two value of a as $D > 0$

$\therefore \; a = \dfrac{-3 \pm \sqrt{17}}{2}$

76. **(a) :** Let (α, β) is the centre of the circle whose radius is 3.

$\therefore$ Equation of such circle be

$(x - \alpha)^2 + (y - \beta)^2 = 3^2$

$\Rightarrow \quad \alpha^2 + \beta^2 - 2\alpha x - 2\beta y + 25 = 9$

$\Rightarrow \quad x^2 + y^2 - 2x^2 - 2y^2 + 25 = 9$

$\Rightarrow \quad x^2 + y^2 = 25 - 9$

$\Rightarrow \quad x^2 + y^2 = 16$ and $x^2 + y^2 = 25$

$\Rightarrow \quad 4 \le x^2 + y^2 \le 64$

77. **(a) :** As $s = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a pair of line

$$\therefore \quad \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

or $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ ...(1)

Now say point of intersection on y axis be $(0, y_1)$ and point of intersection of pair of line be obtained by solving the equations

$$\frac{\partial s}{\partial x} = 0 = \frac{\partial s}{\partial y}$$

$\therefore \quad \dfrac{\partial s}{\partial x} = 0 \Rightarrow ax + by + g = 0 \Rightarrow$

and $\quad \dfrac{\partial s}{\partial y} = 0 \Rightarrow hx + by + f = 0 \Rightarrow$

$$\begin{cases} hy_1 + g = 0 \\ by_1 + f = 0 \end{cases} \quad (*)$$

On compairing the equation given in (*) we get

$bg = fh$ and $bg^2 = fgh$ ...(2)

Again $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

meet at y-axis $\quad \therefore \; x = 0$

$\Rightarrow \quad by^2 + 2fy + c = 0$ whose roots must be equal

$\therefore \quad f^2 = bc$

$af^2 = abc$ ...(3)

Now using (2) and (3) in equation (1) we have

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow \quad (abc - af^2) + (fgh - bg^2) + fgh - ch^2 = 0$

$\Rightarrow \quad 0 + 0 + fgh - ch^2 = 0$

$\therefore \quad ch^2 = fgh$ ...(4)

Now adding (2) and (4)

$2fgh = ch^2 + bg^2$

78. **(c) :** Equation of chord $y = mx + 1$

Equation of circle $x^2 + y^2 = 1$

Joint equation of the curve through the intersection of line and circle be given by $x^2 + y^2 = (y - mx)^2$,

$\Rightarrow \quad x^2(1 - m^2) + 2mxy = 0$

Now $\tan\theta = \pm \dfrac{2\sqrt{h^2 - ab}}{a + b}$ where $\begin{cases} a = 1 - m^2 \\ h = m, b = 0 \end{cases}$

$\tan 45° = \pm \dfrac{2\sqrt{m^2 - 0}}{1 - m^2}$

$\Rightarrow \quad 1(1 - m^2) = \pm 2m$

$\Rightarrow \quad m^2 \pm 2m - 1 = 0$

$\Rightarrow \quad m = \pm 1 \pm \sqrt{2}$

$\Rightarrow \quad m = 1 \pm \sqrt{2}$ and $-1 \pm \sqrt{2}$

79. **(b) :** Let common tangent to the curves be

$y = mx + c$ ...(1)

$= mx + \dfrac{a}{m}$

and $\quad \therefore \; y^2 = 8ax = 4(2a)x$

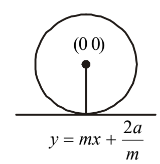
$\therefore$ Equation of tangent to parabola

$y = mx + \dfrac{2a}{m}$ ...(2)

which is also tangent to the circle

$x^2 + y^2 = 2a^2 = (\sqrt{2}a)^2$

Now Distance from (0, 0) to the tangent line = Radius of circle



$\therefore \quad \sqrt{2}\,a = \pm \dfrac{2a}{m} \times \dfrac{1}{\sqrt{1 + m^2}}$

$\Rightarrow \quad m^2(1 + m^2) - 2 = 0$

$\Rightarrow \quad (m^2 - 1)(m^2 + 2) = 0$

$\Rightarrow \quad m = \pm 1$

Required equation of tangent $y = mx + \dfrac{2a}{m}$

$= \pm (x + 2a)$

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CHAPTER 13

THREE DIMENSIONAL GEOMETRY

1. If the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar, then k can have

(a) exactly three values (b) any value
(c) exactly one value (d) exactly two values *(2013)*

2. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is

(a) $\frac{5}{2}$ (b) $\frac{7}{2}$ (c) $\frac{9}{2}$ (d) $\frac{3}{2}$ *(2013)*

3. An equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is

(a) $x - 2y + 2z - 1 = 0$ (b) $x - 2y + 2z + 5 = 0$
(c) $x - 2y + 2z - 3 = 0$ (d) $x - 2y + 2z + 1 = 0$ *(2012)*

4. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then k is equal to

(a) 9/2 (b) 0 (c) – 1 (d) 2/9 *(2012)*

5. **Statement-1 :** The point $A(1, 0, 7)$ is the mirror image of the point $B(1, 6, 3)$ in the line

$$\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$$

Statement-2 : The line : $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ bisects the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$.

(a) Statement-1 is true, Statement-2 is false.
(b) Statement-1 is false, Statement-2 is true.
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. *(2011)*

6. If the angle between the line $x=\frac{y-1}{2}=\frac{z-3}{\lambda}$ and the plane $x+2y+3z=4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$ then λ equals

(a) 2/5 (b) 5/3
(c) 2/3 (d) 3/2 *(2011)*

7. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals

(a) 30º (b) 45º
(c) 60º (d) 75º. *(2010)*

8. **Statement-1 :** The point $A(3, 1, 6)$ is the mirror image of the point $B(1, 3, 4)$ in the plane $x - y + z = 5$.

Statement-2 : The plane $x - y + z = 5$ bisects the line segment joining $A(3, 1, 6)$ and $B(1, 3, 4)$.

(a) Statement -1 is true, Statement-2 is true; Statement-2 is a correct explanation of Statement 1.
(b) Statement-1 is true, Statement-2 is true; Statement 2 is not a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true. *(2010)*

9. Let the $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ line lie in the plane $x + 3y - \alpha z + \beta = 0$. Then (α, β) equals

(a) (–6, 7) (b) (5, –15)
(c) (–5, 5) (d) (6, –17) *(2009)*

10. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are

(a) $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$ (b) $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$
(c) $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$ (d) 6, –3, 2 *(2009)*

11. If the straight lines

$$\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3} \text{ and } \frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$$

intersect at a point, then the integer k is equal to

(a) – 2 (b) – 5
(c) 5 (d) 2 *(2008)*

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12. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then
(a) $a = 8, b = 2$ (b) $a = 2, b = 8$
(c) $a = 4, b = 6$ (d) $a = 6, b = 4$ *(2008)*

13. Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x-axis, then cos α equals
(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$. *(2007)*

14. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vectors $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals
(a) – 4 (b) –2 (c) 0 (d) 1. *(2007)*

15. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the coordinates of the other end of the diameter are
(a) (4, 3, 5) (b) (4, 3, –3)
(c) (4, 9, –3) (d) (4, –3, 3). *(2007)*

16. If a line makes an angle of $\pi/4$ with the positive directions of each of x-axis and y-axis, then the angle that the line makes with the positive direction of the z-axis is
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$. *(2007)*

17. The image of the point (–1, 3, 4) in the plane $x - 2y = 0$ is
(a) $\left(-\frac{17}{3}, -\frac{19}{3}, 4\right)$ (b) (15, 11, 4)
(c) $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$ (d) $\left(\frac{9}{5}, -\frac{13}{5}, 4\right)$. *(2006)*

18. The two lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are perpendicular to each other if
(a) $aa' + cc' = -1$ (b) $aa' + cc' = 1$
(c) $\frac{a}{a'} + \frac{c}{c'} = -1$ (d) $\frac{a}{a'} + \frac{c}{c'} = 1$. *(2006)*

19. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is
(a) 90° (b) 0° (c) 30° (d) 45°. *(2005)*

20. The plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius
(a) 1 (b) 3 (c) $\sqrt{2}$ (d) 2. *(2005)*

21. If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$, the value of λ is
(a) $-\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{-4}{3}$ (d) $\frac{3}{4}$. *(2005)*

22. The distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and the plane $\vec{r}\cdot(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is
(a) $\frac{10}{3\sqrt{3}}$ (b) $\frac{10}{9}$
(c) $\frac{10}{3}$ (d) $\frac{3}{10}$. *(2005)*

23. If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the midpoint of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ then a equals
(a) 1 (b) –1 (c) 2 (d) –2. *(2005)*

24. The intersection of the spheres $x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and $x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the sphere and the plane
(a) $x - y - 2z = 1$ (b) $x - 2y - z = 1$
(c) $x - y - z = 1$ (d) $2x - y - z = 1$. *(2004)*

25. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The co-ordinates of each of the points of intersection are given by
(a) (3a, 2a, 3a), (a, a, 2a) (b) (3a, 2a, 3a), (a, a, a)
(c) (3a, 3a, 3a), (a, a, a) (d) (2a, 3a, 3a), (2a, a, a). *(2004)*

26. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is
(a) 7/2 (b) 5/2
(c) 3/2 (d) 9/2 *(2004)*

27. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals
(a) 3/5 (b) 1/5
(c) 2/3 (d) 2/5. *(2004)*

28. If the straight lines $x = 1 + s$, $y = -3 - \lambda s$, $z = 1 + \lambda s$ and $x = \frac{t}{2}$, $y = 1 + t$, $z = 2 - t$, with parameters s and t respectively, are coplanar, then λ equals
(a) –1/2 (b) –1 (c) –2 (d) 0. *(2004)*

29. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if
(a) k = 1 or –1 (b) k = 0 or –3
(c) k = 3 or –3 (d) k = 0 or –1. *(2003)*

30. The two lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d$ will be perpendicular, if and only if

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(a) $aa' + bb' + cc' = 0$
(b) $(a + a')(b + b') + (c + c') = 0$
(c) $aa' + cc' + 1 = 0$
(d) $aa' + bb' + cc' + 1 = 0$. *(2003)*

31. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $(1, \vec{a}, \vec{a}^2)$, $(1, \vec{b}, \vec{b}^2)$ and $(1, \vec{c}, \vec{c}^2)$ are non-coplanar, then the product abc equals
(a) –1 (b) 1 (c) 0 (d) 2. *(2003)*

32. A tetrahedron has vertices at O (0, 0, 0), A (1, 2, 1), B (2, 1, 3) and C (–1, 1, 2). Then the angle between the faces OAB and ABC will be
(a) $\cos^{-1}(17/31)$ (b) 30°
(c) 90° (d) $\cos^{-1}(19/35)$. *(2003)*

33. The radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$ is
(a) 2 (b) 3
(c) 4 (d) 1. *(2003)*

34. The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is
(a) $11\frac{3}{4}$ (b) 13
(c) 39 (d) 26. *(2003)*

35. Two systems of rectangular axes have the same origin. If a plane cuts them at distance a, b, c and a', b', c' from the origin, then
(a) $\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}-\frac{1}{c'^2}=0$
(b) $\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$
(c) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$
(d) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$. *(2003)*

36. The d.r. of normal to the plane through (1, 0, 0), (0, 1, 0) which makes an angle $\pi/4$ with plane $x + y = 3$ are
(a) $1, \sqrt{2}, 1$ (b) $1, 1, \sqrt{2}$
(c) 1, 1, 2 (d) $\sqrt{2}, 1, 1$. *(2002)*

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Answer Key

1.	(d)	**2.**	(b)	**3.**	(c)	**4.**	(a)	**5.**	(d)	**6.**	(c)
7.	(c)	**8.**	(b)	**9.**	(a)	**10.**	(b)	**11.**	(b)	**12.**	(d)
13.	(c)	**14.**	(b)	**15.**	(c)	**16.**	(b)	**17.**	(d)	**18.**	(a)
19.	(a)	**20.**	(a)	**21.**	(b)	**22.**	(a)	**23.**	(d)	**24.**	(d)
25.	(b)	**26.**	(a)	**27.**	(a)	**28.**	(c)	**29.**	(b)	**30.**	(c)
31.	(a)	**32.**	(d)	**33.**	(b)	**34.**	(b)	**35.**	(c)	**36.**	(b)

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Explanations

1. **(d) :** For the lines to be coplanar

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

Expanding, we get $1(1 + 2k) + 1(1 + k^2) - 1(2 - k) = 0$
$\Rightarrow k^2 + 1 + 2k + 1 - 2 + k = 0$
$\Rightarrow k^2 + 3k = 0 \quad \Rightarrow \quad k(k + 3) = 0 \quad \therefore \quad k = 0, -3$
So there are two values of k.

2. **(b) :** The planes are
$4x + 2y + 4z = 16, \;\; 4x + 2y + 4z = -5$

Distance between planes $= \dfrac{16-(-5)}{\sqrt{4^2+2^2+4^2}} = \dfrac{21}{6} = \dfrac{7}{2}$

3. **(c) :** Equation of a plane parallel to $x - 2y + 2z - 5 = 0$ and at a unit distance from origin is $x - 2y + 2z + k = 0$

$\Rightarrow \quad \dfrac{|k|}{3} = 1 \Rightarrow |k| = 3$
$\therefore\ x - 2y + 2z - 3 = 0$
or $\ x - 2y + 2z + 3 = 0$

4. **(a) :** $\dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-1}{4} = r_1$

and $\dfrac{x-3}{1} = \dfrac{y-k}{2} = \dfrac{z}{1} = r_2$

or $2r_1 + 1 = r_2 + 3,\ 3r_1 - 1 = 2r_2 + k,\ 4r_1 + 1 = r_2$
$\Rightarrow 2r_1 - r_2 = 2$, and $4r_1 - r_2 = -1$

$-2r_1 = 3 \Rightarrow r_1 = \dfrac{-3}{2}$ and $r_2 = -5$

$\therefore\ -\dfrac{9}{2} - 1 = -10 + k \Rightarrow k = 10 - \dfrac{11}{2} = \dfrac{9}{2}$

5. **(d) :** The direction ratios of the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$ is $(0, 6, -4)$.
The direction ratios of the given line is (1, 2, 3).
As $1.0 + 6.2 - 4.3 = 0$ we have the lines as perpendicular
Also the midpoint of AB lies on the given line, so statement 1 and statement 2 are true but statement 2 is not a correct explanation of statement 1.
Statement '2' holds even if the line is not perpendicular. This situation is possible.

6. **(c) :** $\dfrac{x-0}{1} = \dfrac{y-1}{2} = \dfrac{z-3}{\lambda}$
$x + 2y + 3z = 4$

Angle between line and plane (by definition)

$$= \sin^{-1}\left(\frac{1\cdot1+2\cdot2+\lambda\cdot3}{\sqrt{1+4+9}\sqrt{1+4+\lambda^2}}\right) = \sin^{-1}\left(\frac{5+3\lambda}{\sqrt{14}\sqrt{5+\lambda^2}}\right)$$

So, $\dfrac{(5+3\lambda)^2}{14(5+\lambda^2)} + \dfrac{5}{14} = 1 \quad (\sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow \quad \dfrac{(5+3\lambda)^2}{5+\lambda^2} + 5 = 14$

$\Rightarrow (5 + 3\lambda)^2 + 5(5 + \lambda^2) = 14(5 + \lambda^2)$
$\Rightarrow 25 + 30\lambda + 9\lambda^2 + 25 + 5\lambda^2 = 70 + 14\lambda^2$
$\Rightarrow 30\lambda + 50 = 70$
$\Rightarrow \quad 30\lambda = 20 \quad \therefore \quad \lambda = 2/3$

7. **(c) :** We have $l = \dfrac{1}{\sqrt{2}}, m = -\dfrac{1}{2}$

As $l^2 + m^2 + n^2 = 1$, we have $n^2 = \dfrac{1}{4} \Rightarrow n = \dfrac{1}{2}$

We take positive values, so $n = \dfrac{1}{2}$

$\Rightarrow \quad \cos\theta = \dfrac{1}{2}. \quad \therefore \quad \theta = 60°.$

8. **(b) :** Let the image be (a, b, c)
Thus by image formula, we have

$$\frac{a-1}{1} = \frac{b-3}{-1} = \frac{c-4}{1} = -2\left(\frac{1-3+4-5}{3}\right)$$

$\Rightarrow \quad \dfrac{a-1}{1} = \dfrac{b-3}{-1} = \dfrac{c-4}{1} = 2$

$\therefore\ (a, b, c) = (3, 1, 6)$
Again the midpoint of $A(3, 1, 6)$ and $B(1, 3, 4)$ is (2, 2, 5) & the equation of the plane is $x - y + z = 5$.
As the point lies on the plane, so the plane bisects the segment AB. But it does not explain statement-1.

9. **(a) :** The line is $\dfrac{x-2}{3} = \dfrac{y-1}{-5} = \dfrac{z+2}{2}$
The direction ratios of the line are (3, –5, 2).
As the line lies in the plane $x + 3y - \alpha z + \beta = 0$,
we have $(3)(1) + (-5)(3) + 2(-\alpha) = 0$
$\Rightarrow -12 - 2\alpha = 0. \quad \therefore \quad \alpha = -6$
Again (2, 1, –2) lies on the plane
$\Rightarrow 2 + 3 + 2\alpha + \beta = 0$
$\Rightarrow \beta = -2\alpha - 5 = 12 - 5 = 7$
Hence (α, β) is (–6, 7).

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10. (b) : Let the vector $\overrightarrow{PQ}$ be $(x_1 - x_2, y_1 - y_2, z_1 - z_2)$

we have $x_1 - x_2 = 6$

$y_1 - y_2 = -3$

$z_1 - z_2 = 2$

Length of PQ

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$= \sqrt{6^2 + 3^2 + 2^2} = \sqrt{36 + 9 + 4} = 7$$

The direction cosines of $\overrightarrow{PQ}$ are

$$\left\langle \frac{x_1 - x_2}{PQ}, \frac{y_1 - y_2}{PQ}, \frac{z_1 - z_2}{PQ} \right\rangle$$

i.e., $\left\langle \frac{6}{7}, -\frac{3}{7}, \frac{2}{7} \right\rangle$

11. (b) : As the lines intersect,

we have

which on solving given $2k^2 + 5k - 25 = 0$

$\Rightarrow \quad 2k^2 + 10k - 5k - 25 = 0$

$\Rightarrow \quad 2k(k + 5) - 5(k + 5) = 0$

$\Rightarrow \quad (2k - 5)(k + 5) = 0$

$\therefore \quad k = -5, \frac{5}{2}$

12. (d) : The equation of the line passing through $(3, b, 1)$ and $(5, 1, a)$ is

$$\frac{x-5}{2} = \frac{y-1}{1-b} = \frac{z-a}{a-1} = \mu \text{ (say)}$$

The line crosses the yz plane where $x = 0$, i.e.

$-5 = 2\mu \quad \therefore \quad \mu = -\frac{5}{2}$

Again $y = \mu(1-b) + 1 = \frac{17}{2}$

$\Rightarrow \quad -\frac{5}{2}(1-b) + 1 = \frac{17}{2} \quad \Rightarrow \quad -\frac{5}{2}(1-b) = \frac{15}{2}$

$\Rightarrow \quad (1-b) = -3 \quad \therefore \quad b = 4$

Again $z = \mu(a-1) + a = -\frac{13}{2}$

$\Rightarrow \quad -\frac{5}{2}(a-1) + a = -\frac{13}{2} \quad \Rightarrow \quad -\frac{3}{2}a + \frac{5}{2} = -\frac{13}{2}$

$\Rightarrow \quad -\frac{3}{2}a = -9 \Rightarrow a = 6$

13. (c) : Direction of the line, $L = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 3 & 2 \end{vmatrix}$

$= 3\hat{i} - 3\hat{j} + 3\hat{k}$.

Then $\cos\alpha = \frac{3}{\sqrt{9+9+9}} = \frac{1}{\sqrt{3}}$.

Second method

If direction cosines of L be l, m, n, then

$2l + 3m + n = 0, \quad l + 3m + 2n = 0$

After solving, we get, $\frac{l}{3} = \frac{m}{-3} = \frac{n}{3}$

$\therefore \ l : m : n = \frac{1}{\sqrt{3}} : -\frac{1}{\sqrt{3}} : \frac{1}{\sqrt{3}} \Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

14. (b) : $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \ \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and

$\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$

$[\vec{a} \ \vec{b} \ \vec{c}] = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix} = 0$$

$\Rightarrow 1(1 - 2x + 4) - 1(-1 - 2x) + 1(x - 2 + x) = 0$

$\Rightarrow 5 - 2x + 1 + 2x + 2x - 2 = 0$

$\Rightarrow x = -2.$

15. (c) : Centre of sphere $\equiv (3, 6, 1)$

Let the other end of diameter is (α, β, γ)

$3 = \frac{\alpha + 2}{2} \Rightarrow \alpha = 4, \quad 6 = \frac{\beta + 3}{2} \Rightarrow \beta = 9$

$1 = \frac{\gamma + 5}{2} \Rightarrow \gamma = -3.$

16. (b) : Let required angle is θ

$\because \quad l = \cos\frac{\pi}{4}, m = \cos\frac{\pi}{4}$ then $n = \cos\theta$

We know that $l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad \cos^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} + \cos^2\theta = 1$

$\Rightarrow \quad \frac{1}{2} + \frac{1}{2} + \cos^2\theta = 1$

$\Rightarrow \quad \cos^2\theta = 0 \Rightarrow \theta = \pi/2$

Thus required angle is $\pi/2$.

17. (d) : Image of point (x', y', z') in $ax + by + cz + d = 0$ is given by

$$\frac{x - x'}{a} = \frac{y - y'}{b} = \frac{z - z'}{c} = \frac{-2(ax' + by' + cz' + d)}{a^2 + b^2 + c^2}$$

$\Rightarrow \quad \frac{x+1}{1} = \frac{y-3}{-2} = \frac{z-4}{0} = \frac{-2(-1-6)}{5}$

$\therefore \quad x = \frac{9}{5}, y = \frac{-13}{5}, z = 4$

18. (a) : Fact : Two lines $\frac{x - x_1}{a_1} = \frac{x - y_1}{b_1} = \frac{z - z_1}{c_1}$

and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$

are $\perp$ if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

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Given lines can be written as $\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$ ...(i)

and $\frac{x-b'}{a'} = \frac{y}{1} = \frac{z-d'}{c'}$ ...(ii)

As lines are perpendicular

$\therefore \quad a\,a' + 1 + c\,c' = 0$

$\Rightarrow \quad a\,a' + c\,c' = -1$

19. (a) : From given line

$\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ and $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta = \frac{6-24+18}{\sqrt{3^2+2^2+(-6)^2}\sqrt{2^2+(-12)^2+(-3)^2}} = 0$$

$\therefore\ \theta = 90°$.

20. (a) : Centre of sphere is 1/2, 0, –1/2

R = Radius of sphere is $\sqrt{g^2+f^2+w^2-c}$

$$= \sqrt{\frac{1}{4}+\frac{1}{4}+2} \quad \therefore \quad R = \sqrt{\frac{5}{2}}$$

d = ⊥ distance from centre to the plane is equal to

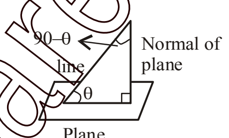
$$d = \left|\frac{\frac{1}{2}+0+\frac{1}{2}-4}{\sqrt{1^2+2^2+1^2}}\right|, \quad d = \frac{3}{\sqrt{6}}.$$

$\therefore$ Radius of the circle

= Radius of sphere – perpendicular distance from centre of sphere to plane

$$= \sqrt{\left(\sqrt{\frac{5}{2}}\right)^2 - \left(\frac{9}{6}\right)} = \sqrt{\frac{15}{6}-\frac{9}{6}} = 1.$$

21. (b) : Angle between the line and plane is same as the angle between the line and normal to the plane



$$\therefore \quad \cos(90-\theta) = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\Rightarrow \frac{1}{3} = \frac{(1\times 2 + 2\times(-1)) + 2\sqrt{\lambda}}{\sqrt{1^2+2^2+2^2}\sqrt{2^2+1^2+\lambda}}$$

$$\Rightarrow \lambda = \frac{5}{3}.$$

22. (a) : $d = \left|\frac{\vec{a}\cdot\vec{n} - d}{|\vec{n}|}\right|$

$$\therefore\ d = \left|\frac{(2i-2j+3k)\cdot(i+5j+k)-(-5)}{\sqrt{1^2+5^2+1^2}}\right|,$$

$$d = \frac{10}{3\sqrt{3}}.$$

23. (d) : Centre of spheres are (–3, 4, 1) and (5, –2, 1)

M (1, 1, 1)

C_1 (–3, 4, 1) C_2 (5, –2, 1)

using mid point in the equation

$2ax - 3ay + 4az + 6 = 0$

$\Rightarrow 2a - 3a + 4a + 6 = 0 \Rightarrow a = -2$.

24. (d) : Equation of the plane of intersection of two spheres $S_1 = 0 = S_2$ is given by $S_1 - S_2 = 0$

$\Rightarrow \quad 10x - 5y - 5z = 5$

$\Rightarrow \quad 2x - y - z = 1$

25. (b) : Given $AB = \frac{x}{1} = \frac{y+a}{1} = \frac{z}{1}$

$CD : \frac{x+a}{2} = \frac{y}{1} = \frac{z}{1}$

Let $P \equiv (r, r-a, r)$ and $Q = (2\lambda - a, \lambda, \lambda)$

Direction ratios of PQ are $r - 2\lambda + a,\ r - \lambda - a,\ r - \lambda$

According to question direction ratios of PQ are (2, 1, 2)

$$\therefore \underset{(i)}{\frac{r-2\lambda+a}{2}} = \underset{(ii)}{\frac{r-\lambda-a}{1}} = \underset{(iii)}{\frac{r-\lambda}{2}}$$

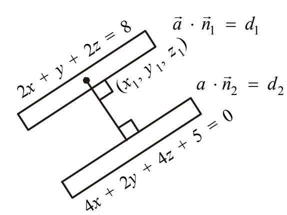
(ii) and (iii) $\Rightarrow r - \lambda = 2a$ ...(1)

(i) and (iii) $\Rightarrow \lambda = a \qquad r = 3a,\ \lambda = a$

$\therefore\ p \equiv (3a, 2a, 3a)$ and $Q \equiv (a, a, a)$.

26. (a) : Let x_1, y_1, z_1 be any point on the plane $2x + y + 2z - 8 = 0$

$\therefore\ 2x_1 + y_1 + 2z_1 - 8 = 0$



$$\therefore\ d = \frac{|2(2x+y+2z-8)+21|}{\sqrt{4^2+2^2+4^2}} = \frac{21}{6} = \frac{7}{2}$$

27. (a) : If a line makes the angle α, β, γ with x, y, z axis respectively then

$l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad 2l^2 + m^2 = 1$ or $2n^2 + m^2 = 1$

$\Rightarrow \quad 2\cos^2\theta = 1 - \cos^2\beta \ (\alpha = \gamma = \theta)$

$2\cos^2\theta = \sin^2\beta$

$\Rightarrow \quad 2\cos^2\theta = 3\sin^2\theta$ (given $\sin^2\beta = 3\sin^2\theta$)

$\Rightarrow \quad 5\cos^2\theta = 3$

28. (c) : From the given lines we have

$$\frac{x-1}{1} = \frac{y+3}{\lambda} = \frac{z-1}{\lambda} = s \qquad \ldots (A)$$

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and $\frac{x-0}{1}=\frac{y-1}{2}=\frac{z-2}{-2}$ ...(*B*)

As lines (*A*) and (*B*) are coplanar

$$\therefore \begin{vmatrix} 1 & -4 & -1 \\ 1 & -\lambda & \lambda \\ 1 & 2 & -2 \end{vmatrix} = 0$$

$\Rightarrow (2\lambda - 2\lambda) + 4(-2-\lambda) - 1(2+\lambda) = 0$

$\Rightarrow 5\lambda = -10 \quad \therefore \lambda = -2$

29. **(b) :** Using fact, two lines

$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ are coplanar if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow k^2 + 3k = 0$

$k = 0$ or $k = -3$

30. **(c) :** Given lines can be written as

$\frac{x-b}{a}=\frac{y-0}{1}=\frac{z-d}{c}$ and

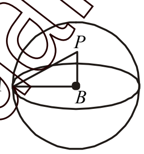
$\frac{x-b'}{a'}=\frac{y-0}{1}=\frac{z-d}{c'}$

$\therefore$ Required condition of perpendicularity is

$aa' + cc' + 1 = 0$

31. **(a) :** As vectors $(1,\vec{a},\vec{a}^2)$, $(1,\vec{b},\vec{b}^2)$, $(1,\vec{c},\vec{c}^2)$ are non coplanar.

$$\therefore \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0 \quad \text{... (A)}$$



now $\begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix} = 0$

On solving, we get

$$\Rightarrow (1+abc)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$\Rightarrow (1+abc) = 0$ by using (A)

32. **(d) :** Concept using angle between the faces is equal to the angle between their normals.

$\therefore$ Vector $\perp$ to the face *OAB* is $\overrightarrow{OA}\times\overrightarrow{OB}$

$= 5i - j - 3k$ and vector $\perp$ to the face *ABC* is $\overrightarrow{AB}\times\overrightarrow{AC} = i - 5j - 3k$

$\therefore$ Let θ be the angle between the faces *OAB* and *ABC*

$$\therefore \cos\theta = \frac{(5i-j-3k)\cdot(i-5j-3k)}{|5i-j-3k||i-5j-3k|}$$

$\cos\theta = \frac{19}{35} \quad \therefore \theta = \cos^{-1}\left(\frac{19}{35}\right)$

33. **(b) :** The radius and centre of sphere $x^2+y^2+z^2+2x-2y-4z-19=0$ is

$\sqrt{1^2+1^2+4+19} = 5$ and centre $(-1, 1, 2)$

PB $\perp$ from centre to the plane

$$\frac{|-1+2+4+7|}{\sqrt{1+2^2+2^2}} = 4$$

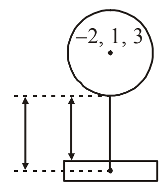
Now $(AB)^2 = AP^2 - PB^2$

$= 25 - 16$

$= 9$

$\therefore AB = 3$

34. **(b) :** In order to determine the shortest distance between the plane and sphere, we find the distance from the centre of sphere to the plane-Radius of sphere



$12x + 4y + 3z - 327 = 0$

$\therefore$ Centre of sphere is $(-2, 1, 3)$

Required distance is

$$\frac{|-24+4+9-327|}{\sqrt{12^2+4^2+3^2}} - \sqrt{(2)^2+1^2+3^2+155}$$

$= 26 - 13 = 13$ units.

35. **(c) :** Now equation of the plane through $(a, 0, 0)$ $(0, b, 0)$ $(0, 0, c)$ is

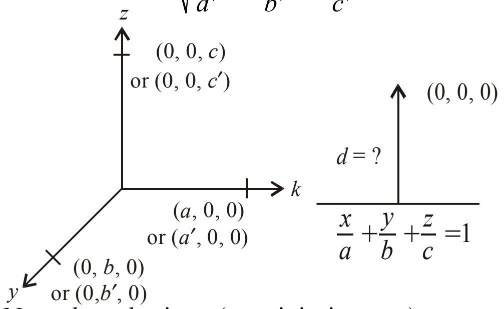
$$\Rightarrow \frac{x}{x\text{-Intercept}}+\frac{y}{y\text{-Intercept}}+\frac{z}{z\text{-Intercept}}=1 \quad \text{...(*)}$$

$\Rightarrow \frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$

So the distance from (0, 0, 0) to this plane to the plane (*) is given by

$$d_1 = \frac{|0+0+0-1|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} = \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$$

Similarly, $d_2 = \dfrac{1}{\sqrt{\dfrac{1}{a'^2} + \dfrac{1}{b'^2} + \dfrac{1}{c'^2}}}$



Now $d_1 = d_2$ given (as origin is same)

$$\Rightarrow \frac{1}{\sqrt{\dfrac{1}{a^2} + \dfrac{1}{b^2} + \dfrac{1}{c^2}}} = \frac{1}{\sqrt{\dfrac{1}{a'^2} + \dfrac{1}{b'^2} + \dfrac{1}{c'^2}}}$$

$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} - \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$$

36. (b) : Let *DR*'s of normal to plane are a, b, c

$\therefore \quad a(x-1) + b(y) + c(z) = 0 \qquad \text{...(*)}$

$\Rightarrow \quad a(0-1) + b(1) + c(0) = 0$

(by using (0, 1, 0) in (*))

$\Rightarrow \quad -a + b = 0 \Rightarrow a = b$

Also angle between (*) and $x + y + 0z = 3$ is $\pi/4$

$\therefore \quad \cos\dfrac{\pi}{4} =$

$$\frac{a+a}{\sqrt{1^2+1^2}\sqrt{a^2+b^2+c^2}} = \frac{2a}{\sqrt{2}\sqrt{2a^2+c^2}}$$

$\Rightarrow \quad 2a^2 + c^2 = 4a^2$

$\Rightarrow \quad c = \pm\sqrt{2}\,a$

$\therefore \quad$ *DR's* a, b, c *i.e.* $a, a, \pm\sqrt{2}a$

$\therefore \quad$ Required *DR*'s are $1, 1, \sqrt{2}$ or $1, 1, -\sqrt{2}$

Hence $1, 1, \sqrt{2}$ match with choice (b)

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CHAPTER 14

VECTOR ALGEBRA

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1. If the vectors $\overrightarrow{AB}=3\hat{i}+4\hat{k}$ and $\overrightarrow{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle *ABC*, then the length of the median through *A* is

(a) $\sqrt{45}$ (b) $\sqrt{18}$ (c) $\sqrt{72}$ (d) $\sqrt{33}$ *(2013)*

2. Let $\overrightarrow{AB}=3\hat{i}+4\hat{k}$ be two unit vectors. If the vectors $\vec{c}=\hat{a}+2\hat{b}$ and $\vec{d}=5\hat{a}+4\hat{b}$ are perpendicular to each other, then the angle between is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$ *(2012)*

3. Let *ABCD* be a parallelogram such that $\overrightarrow{AB}=\vec{q}, \overrightarrow{AD}=\vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex *B* to the side *AD*, then $\vec{r}$ is given by

(a) $\vec{r}=\vec{q}-\left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$ (b) $\vec{r}=-3\vec{q}+\frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

(c) $\vec{r}=3\vec{q}-\frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$ (d) $\vec{r}=-\vec{q}+\left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$ *(2012)*

4. If $\vec{a}=\frac{1}{\sqrt{10}}\left(3\hat{i}+\hat{k}\right)$ and $\vec{b}=\frac{1}{7}\left(2\hat{i}+3\hat{j}-6\hat{k}\right)$, then the value of $\left(2\vec{a}-\vec{b}\right)\cdot\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}+2\vec{b}\right)\right]$ is

(a) 5 (b) 3 (c) –5 (d) –3 *(2011)*

5. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$ and $\vec{a}\cdot\vec{d}=0$. Then the vector $\vec{d}$ is equal to

(a) $\vec{b}+\left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$ (b) $\vec{c}-\left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$

(c) $\vec{b}-\left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$ (d) $\vec{c}+\left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$ *(2011)*

6. Let $\vec{a}=\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a}\times\vec{b}+\vec{c}=0$ and $\vec{a}\cdot\vec{b}=3$ is

(a) $-\hat{i}+\hat{j}-2\hat{k}$ (b) $2\hat{i}-\hat{j}+2\hat{k}$

(c) $\hat{i}-\hat{j}-2\hat{k}$ (d) $\hat{i}+\hat{j}-2\hat{k}$ *(2010)*

7. If the vectors $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}+4\hat{k}$ and $\vec{c}=\lambda\hat{i}+\hat{j}+\mu\hat{k}$ are mutually orthogonal, then (λ, μ) =

(a) (–3, 2) (b) (2, –3)

(c) (–2, 3) (d) (3, –2) *(2010)*

8. Let $\vec{a}=\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$. Then the vector satisfying $\vec{a}\times\vec{b}+\vec{c}=0$ and $\vec{a}\cdot\vec{b}=3$ is

(a) $-\hat{i}+\hat{j}-2\hat{k}$ (b) $2\hat{i}-\hat{j}+2\hat{k}$

(c) $\hat{i}-\hat{j}-2\hat{k}$ (d) $\hat{i}+\hat{j}-2\hat{k}$ *(2010)*

9. If the vectors $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}+4\hat{k}$ and $\vec{c}=\lambda\hat{i}+\hat{j}+\mu\hat{k}$ are mutually orthogonal, then (λ, μ) =

(a) (–3, 2) (b) (2, –3)

(c) (–2, 3) (d) (3, –2) *(2010)*

10. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar **vectors** and *p*, *q* are real numbers, then the equality $\left[3\vec{u}\ p\vec{v}\ p\vec{w}\right]-\left[p\vec{v}\ \vec{w}\ q\vec{u}\right]-\left[2\vec{w}\ q\vec{v}\ q\vec{u}\right]=0$ holds for

(a) exactly two values of (*p*, *q*)

(b) more than two but not all values of (*p* , *q*)

(c) all values of (*p*, *q*)

(d) exactly one value of (*p*, *q*) *(2009)*

11. The non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are related by $\vec{a}=8\vec{b}$ and $\vec{c}=-7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is

(a) π (b) 0 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$ *(2008)*

12. The vector $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$ lies in the plane of the vectors $\vec{b}=\hat{i}+\hat{j}$ and $\vec{c}=\hat{j}+\hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β?

(a) α = 1, β = 1 (b) α = 2, β = 2

(c) α = 1, β = 2 (d) α = 2, β = 1 *(2008)*

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13. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u}\times 3\hat{v}$ is a unit vector for
(a) no value of θ
(b) exactly one value of θ
(c) exactly two values of θ
(d) more than two values of θ *(2007)*

14. The values of a, for which the points A, B, C with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $a\hat{i}-3\hat{j}+\hat{k}$ respectively are the vertices of a right-angled triangle at c are
(a) 2 and 1 (b) –2 and –1
(c) –2 and 1 (d) 2 and –1. *(2006)*

15. If $(\vec{a}\times\vec{b})\times\vec{c}=\vec{a}\times(\vec{b}\times\vec{c})$, where $\vec{a},\vec{b}$ and $\vec{c}$ are any three vectors such that $\vec{a}\cdot\vec{b}\neq 0$, $\vec{b}\cdot\vec{c}\neq 0$, then $\vec{a}$ and $\vec{c}$ are
(a) inclined at an angle of $\pi/3$ between them
(b) inclined at an angle of $\pi/6$ between them
(c) perpendicular
(d) parallel. *(2006)*

16. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar vector and λ is a real number then $\left[\lambda(\vec{a}+\vec{b})\ \lambda^2\vec{b}\ \lambda\vec{c}\right]=\left[\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{b}\right]$ for
(a) no value of λ
(b) exactly one value of λ
(c) exactly two values of λ
(d) exactly three values of λ. *(2005)*

17. Let a, b and c be distinct non-negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then c is
(a) the arithmetic mean of a and b
(b) the geometric mean of a and b
(c) the harmonic mean of a and b
(d) equal to zero. *(2005)*

18. Let $\vec{a}=\hat{i}-\hat{k}$, $\vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}$ and $\vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$. Then $\left[\vec{a},\vec{b},\vec{c}\right]$ depends on
(a) only x (b) only y
(c) neither x nor y (d) both x and y. *(2005)*

19. For any vector $\vec{a}$, the value of $(\vec{a}\times\hat{i})^2+(\vec{a}\times\hat{j})^2+(\vec{a}\times\hat{k})^2$ is equal to
(a) $\vec{a}^2$ (b) $3\vec{a}^2$
(c) $4\vec{a}^2$ (d) $2\vec{a}^2$. *(2005)*

20. If C is the mid point of AB and P is any point outside AB, then
(a) $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=\vec{0}$ (b) $\overrightarrow{PA}+\overrightarrow{PB}+2\overrightarrow{PC}=\vec{0}$
(c) $\overrightarrow{PA}+\overrightarrow{PB}=\overrightarrow{PC}$ (d) $\overrightarrow{PA}+\overrightarrow{PB}=2\overrightarrow{PC}$ *(2005)*

21. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be non-zero vectors such that $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the acute angle between the vectors $\vec{b}$ and $\vec{c}$, then $\sin\theta$ equals
(a) $\frac{2}{3}$ (b) $\frac{\sqrt{2}}{3}$
(c) $\frac{1}{3}$ (d) $\frac{2\sqrt{2}}{3}$. *(2004)*

22. Let $\vec{u}$, $\vec{v}$, $\vec{w}$ be such that $|\vec{u}|=1$, $|\vec{v}|=2$, $|\vec{w}|=3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}$, $\vec{w}$ are perpendicular to each other then $|\vec{u}-\vec{v}+\vec{w}|$ equals
(a) $\sqrt{14}$ (b) $\sqrt{7}$ (c) 2 (d) 14. *(2004)*

23. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a}+2\vec{b}+3\vec{c}$, $\lambda\vec{b}+4\vec{c}$ and $(2\lambda-1)\vec{c}$ are non-coplanar for
(a) all except two values of λ
(b) all except one value of λ
(c) all values of λ
(d) no value of λ. *(2004)*

24. A particle is acted upon by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ which displace it from a point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The work done in standard units by the forces is given by
(a) 25 (b) 30 (c) 40 (d) 15. *(2004)*

25. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ (λ being some non-zero scalar) then $\vec{a}+2\vec{b}+6\vec{c}$ equals
(a) $\lambda\vec{c}$ (b) $\lambda\vec{b}$ (c) $\lambda\vec{a}$ (d) 0. *(2004)*

26. $\vec{a},\vec{b},\vec{c}$ are three vectors, such that $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{b}|=2$, $|\vec{c}|=3$, then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is equal to
(a) –7 (b) 7 (c) 1 (d) 0. *(2003)*

27. The vectors $\overrightarrow{AB}=3\hat{i}+4\hat{k}$ and $\overrightarrow{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is
(a) $\sqrt{72}$ (b) $\sqrt{33}$ (c) $\sqrt{288}$ (d) $\sqrt{18}$. *(2003)*

28. Let $\vec{u}=\hat{i}+\hat{j}$, $\vec{v}=\hat{i}-\hat{j}$ and $\vec{w}=\hat{i}+2\hat{j}+3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}\cdot\hat{n}=0$ and $\vec{v}\cdot\hat{n}=0$, then $|\vec{w}\cdot\hat{n}|$ is equal to
(a) 1 (b) 2 (c) 3 (d) 0. *(2003)*

29. If $\vec{u},\vec{v}$ and $\vec{w}$ are three non-coplanar vectors, then $(\vec{u}+\vec{v}-\vec{w})\cdot(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})$ equals
(a) $\vec{u}\cdot\vec{v}\times\vec{w}$ (b) $\vec{u}\cdot\vec{w}\times\vec{v}$
(c) $3u\cdot\vec{u}\times\vec{w}$ (d) 0. *(2003)*

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30. Consider A, B, C and D with position vectors $7\hat{i}-4\hat{j}+7\hat{k}$, $\hat{i}-6\hat{j}+10\hat{k}$, $-\hat{i}-3\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+5\hat{k}$ respectively. Then $ABCD$ is a
(a) rhombus
(b) rectangle
(c) parallelogram but not a rhombus
(d) square. *(2003)*

31. If $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ then $\vec{a}+\vec{b}+\vec{c}=$
(a) abc (b) -1 (c) 0 (d) 2. *(2002)*

32. $\vec{a}=3\hat{i}-5\hat{j}$ and $\vec{b}=6\hat{i}+3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c}=\vec{a}\times\vec{b}$ then $|\vec{a}|:|\vec{b}|:|\vec{c}|=$
(a) $\sqrt{34}:\sqrt{45}:\sqrt{39}$ (b) $\sqrt{34}:\sqrt{45}:39$
(c) $34:39:45$ (d) $39:35:34$. *(2002)*

33. $3\lambda\vec{c}+2\mu(\vec{a}\times\vec{b})=0$ then
(a) $3\lambda+2\mu=0$ (b) $3\lambda=2\mu$
(c) $\lambda=\mu$ (d) $\lambda+\mu=0$. *(2002)*

34. If $|\vec{a}|=5, |\vec{b}|=4, |\vec{c}|=3$ thus what will be the value of $|\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}|$, given that $\vec{a}+\vec{b}+\vec{c}=0$
(a) 25 (b) 50 (c) -25 (d) -50. *(2002)*

35. If $\vec{a}, \vec{b}, \vec{c}$ are vectors show that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=7$, $|\vec{b}|=5$, $|\vec{c}|=3$ then angle between vector $\vec{b}$ and $\vec{c}$ is
(a) 60º (b) 30º (c) 45º (d) 90º. *(2002)*

36. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $[\vec{a}\ \vec{b}\ \vec{c}]=4$ then $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]=$
(a) 16 (b) 64 (c) 4 (d) 8. *(2002)*

37. If $|\vec{a}|=4$, $|\vec{b}|=2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$ then $(\vec{a}\times\vec{b})^2$ is equal to
(a) 48 (b) 16
(c) $\vec{a}$ (d) none of these *(2002)*

Answer Key

1.	(d)	**2.**	(a)	**3.**	(d)	**4.**	(c)	**5.**	(b)	**6.**	(a)
7.	(a)	**8.**	(d)	**9.**	(a)	**10.**	(a)	**11.**	(a)	**12.**	(a)
13.	(b)	**14.**	(a)	**15.**	(d)	**16.**	(a)	**17.**	(b)	**18.**	(c)
19.	(d)	**20.**	(d)	**21.**	(d)	**22.**	(a)	**23.**	(a)	**24.**	(c)
25.	(d)	**26.**	(a)	**27.**	(b)	**28.**	(c)	**29.**	(a)	**30.**	(*)
31.	(c)	**32.**	(b)	**33.**	(b)	**34.**	(a)	**35.**	(a)	**36.**	(a)
37.	(b)										

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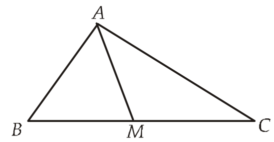
Explanations

1. **(d) :** $\overrightarrow{AM} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$



$= \frac{1}{2}\{(3, 0, 4) + (5, -2, 4)\}$

$= \frac{1}{2}(8, -2, 8) = (4, -1, 4)$

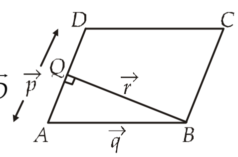
$\therefore |\overrightarrow{AM}| = \sqrt{4^2 + 1^2 + 4^2} = \sqrt{33}$

2. **(a) :** $\vec{c} = \hat{a} + 2\hat{b},\ \vec{d} = 5\hat{a} - 4\hat{b}$

$\therefore\ \vec{c}\cdot\vec{d} = 0 \Rightarrow (\hat{a} + 2\hat{b})\cdot(5\hat{a} - 4\hat{b}) = 5 - 4\hat{b}\cdot\hat{a} + 10\hat{b}\cdot\hat{a} - 8$

$\Rightarrow 6\hat{b}\cdot\hat{a} - 3 = 0 \Rightarrow \hat{b}\cdot\hat{a} = \frac{1}{2}\quad \therefore \theta = \frac{\pi}{3}$

3. **(d) :** $\vec{r} = \overrightarrow{BA} + \overrightarrow{AQ}$



$= -\vec{q} +$ projection of $\overrightarrow{BA}$ across $\overrightarrow{AD}$

$= -\vec{q} + \frac{(\vec{p}\cdot\vec{q})\vec{p}}{(\vec{p}\cdot\vec{p})}$

4. **(c) :** $(2\vec{a} - \vec{b})\cdot\{(\vec{a}\times\vec{b})\times(\vec{a} + 2\vec{b})\}$

$= (2\vec{a} - \vec{b})\cdot\{(\vec{a}\times\vec{b})\times\vec{a} + 2(\vec{a}\times\vec{b})\times\vec{b}\}$

$= (2\vec{a} - \vec{b})\cdot\{(\vec{a}\cdot\vec{a})\,\vec{b} - (\vec{a}\cdot\vec{b})\vec{a} + 2(\vec{a}\cdot\vec{b})\vec{b} - 2(\vec{b}\cdot\vec{b})\vec{a}\}$

$= (2\vec{a} - \vec{b})\cdot(\vec{b} - 2\vec{a}) = -4\vec{a}\cdot\vec{a} - \vec{b}\cdot\vec{b} = -5$

5. **(b) :** $\vec{a}\cdot\vec{b} \neq 0$ (given)

$\vec{a}\cdot\vec{d} = 0$

Now, $\vec{b}\times\vec{c} = \vec{b}\times\vec{d} \Rightarrow \vec{a}\times(\vec{b}\times\vec{c}) = \vec{a}\times(\vec{b}\times\vec{d})$

$\Rightarrow (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})c = (\vec{a}.\vec{d})\vec{b} - (\vec{a}.\vec{b})\vec{d}$

$\Rightarrow (\vec{a}.\vec{b})\vec{d} = -(\vec{a}.\vec{c})\vec{b} + (\vec{a}.\vec{b})\vec{c}$

$\Rightarrow \vec{d} = -\frac{(\vec{a}\cdot\vec{c})\vec{b}}{(\vec{a}.\vec{b})} + \vec{c}$

6. **(a) :** We have $\vec{a}\times\vec{b} + \vec{c} = 0$

Multiplying vectorially with $\vec{a}$, we have

$\vec{a}\times(\vec{a}\times\vec{b}) + \vec{a}\times\vec{c} = 0$

$\Rightarrow (\vec{a}\cdot\vec{b})\vec{a} - (\vec{a}\cdot\vec{a})\vec{b} + \vec{a}\times\vec{c} = 0$

$\vec{a}\times\vec{c} = (\hat{j} - \hat{k})\times(\hat{i} - \hat{j} - \hat{k}) = -2\hat{i} - \hat{j} - \hat{k}$

Thus, $3(\hat{j} - \hat{k}) - 2\vec{b} - 2\hat{i} - \hat{j} - \hat{k} = 0$

$\therefore\ \vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$

7. **(a) :** $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + 4\hat{k}$, $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$

$\vec{a}$ and $\vec{c}$ are orthogonal $\Rightarrow \vec{a}\cdot\vec{c} = 0$ giving $\lambda - 1 + 2\mu = 0$

Also $\vec{b}$ and $\vec{c}$ are orthogonal $\Rightarrow 2\lambda + 4 + 4\mu = 0$

Solving the equation we get $\lambda = -3, \mu = 2$.

8. **(d) :** We have $[l\vec{a}\ m\vec{b}\ n\vec{c}] = lmn[\vec{a}\,\vec{b}\,\vec{c}]$ for scalars l, m, n.

Also $[\vec{a}\,\vec{b}\,\vec{c}] = [\vec{b}\,\vec{c}\,\vec{a}] = [\vec{c}\,\vec{a}\,\vec{b}]$ (cyclic)

And $[\vec{a}\,\vec{b}\,\vec{c}] = -[\vec{a}\,\vec{c}\,\vec{b}]$ (Interchange of any two vectors)

$[3\vec{u}\ p\vec{v}\ p\vec{w}] - [p\vec{v}\ \vec{w}\ q\vec{u}] - [2\vec{w}\ q\vec{v}\ q\vec{u}] = 0$

$\Rightarrow 3p^2[\vec{u}\ \vec{v}\ \vec{w}] - pq[\vec{u}\ \vec{v}\ \vec{w}] + 2q^2[\vec{u}\ \vec{v}\ \vec{w}] = 0$

$\Rightarrow (3p^2 - pq + 2q^2)[\vec{u}\ \vec{v}\ \vec{w}] = 0$

As $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar, $[\vec{u}\ \vec{v}\ \vec{w}] \neq 0$

Hence $3p^2 - pq + 2q^2 = 0$, $p, q \in R$

As a quadratic in p, roots are real

$\Rightarrow q^2 - 24q^2 \geq 0 \Rightarrow -23q^2 \geq 0$

$\Rightarrow q^2 \leq 0 \qquad \Rightarrow q = 0$

And thus $p = 0$

Thus $(p, q) \equiv (0, 0)$ is the only possibility.

9. **(a) :** We have $\vec{a}\times\vec{b} + \vec{c} = 0$

Multiplying vectorially with , we have

$\vec{a}\times(\vec{a}\times\vec{b}) + \vec{a}\times\vec{c} = 0$

$\Rightarrow (\vec{a}\cdot\vec{b})\vec{a} - (\vec{a}\cdot\vec{a})\vec{b} + \vec{a}\times\vec{c} = 0$

$\vec{a}\times\vec{c} = (\hat{j} - \hat{k})\times(\hat{i} - \hat{j} - \hat{k}) = -2\hat{i} - \hat{j} - \hat{k}$

Thus, $3(\hat{j} - \hat{k}) - 2\vec{b} - 2\hat{i} - \hat{j} - \hat{k} = 0$

$\therefore\ \vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$

10. **(a) :** $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + 4\hat{k}$,

$\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$

$\vec{a}$ and $\vec{c}$ are orthogonal $\Rightarrow \vec{a}\cdot\vec{c} = 0$ giving $\lambda - 1 + 2\mu = 0$

Also $\vec{b}$ and $\vec{c}$ are orthogonal $\Rightarrow 2\lambda + 4 + 4\mu = 0$

Solving the equation we get $\lambda = -3, \mu = 2$.

11. **(a) :** $\vec{a} = 8\vec{b}$

$\vec{c} = -7\vec{b}$

$\vec{a}$ and $\vec{b}$ are parallel and $\vec{b}$ and $\vec{c}$ are antiparallel.

Thus $\vec{a}$ and $\vec{c}$ are antiparallel.

Hence the angle between $\vec{a}$ and $\vec{c}$ is π.

12. **(a) :** $\vec{a}$ lies in the plane of $\vec{b}$ and $\vec{c}$. Also $\vec{a}$ bisects the angle $\vec{b}$ and $\vec{c}$. Thus $\vec{a} = \lambda(\vec{b} + \vec{c})$

$\alpha\hat{i} + 2\hat{j} + \beta\hat{k} = \lambda\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} + \frac{\hat{j} + \hat{k}}{\sqrt{2}}\right) = \lambda\left(\frac{\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{2}}\right)$

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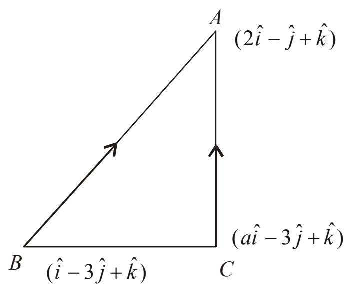
on comparison, $\lambda = \sqrt{2}\alpha, \lambda = \sqrt{2}$ and $\lambda = \sqrt{2}\beta$

Thus $\alpha = 1$ and $\beta = 1$

13. **(b) :** $|2\hat{u} \times 3\hat{v}| = 1 \Rightarrow 6|\hat{u}||\hat{v}||\sin\theta| = 1 \Rightarrow \sin\theta = \frac{1}{6}$

$2\hat{u} \times 3\hat{v}$ is a unit vector for exactly one value of θ.

14. **(a) :** Now $\overrightarrow{CA} \cdot \overrightarrow{CB} = 0$



where $\overrightarrow{CA} = (2 - a)\hat{i} - 2\hat{j}$

and $\overrightarrow{CB} = (1 - a)\hat{i} - 6\hat{k}$

$\Rightarrow \quad a^2 - 3a + 2 = 0 \Rightarrow (a - 2)(a - 1) = 0$

$\Rightarrow \quad a = 1, 2$

15. **(d) :** Given $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$

$\Rightarrow \quad (\vec{b} \cdot \vec{c})\vec{a} = (\vec{a} \cdot \vec{b})\vec{c} \quad \Rightarrow \quad \lambda_1 \vec{a} = \lambda_2 \vec{c}$

$(\lambda_1 = \vec{b} \cdot \vec{c}, \lambda_2 = \vec{a} \cdot \vec{b}$ are scalar quantities)

$\Rightarrow \quad \vec{a} \parallel \vec{c}$

16. **(a) :** From given

$\lambda^2(\vec{a} + \vec{b}) \cdot (\lambda\vec{b} \times \lambda\vec{c}) = \vec{a} \cdot (\vec{b} + \vec{c}) \times \vec{b}$

$\Rightarrow \lambda^2 \vec{a} \cdot (\lambda\vec{b} + \lambda\vec{c}) + \lambda^2 \vec{b} \cdot (\lambda\vec{b} \times \vec{c}) = \vec{a} \cdot (\vec{c} \times \vec{b})$

$\Rightarrow \lambda^4 [a \ b \ c] = -[a \ \ b \ \ c] \Rightarrow \lambda^4 + 1 = 0$

$\Rightarrow (\lambda^2)^2 + 1 = 0 \qquad D < 0$

$\Rightarrow$ No value of λ exist on real axis.

17. **(b) :** We are given that points lies in the same plane. We know that the vector L, M, N are coplanar if

$L \cdot (\vec{M} \times \vec{N}) = 0 \Rightarrow \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0 \Rightarrow c = \sqrt{ab}$

$\therefore$ C is G.M. of a and b.

18. **(c) :** $[a, \ b, \ c] = \vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$= \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix} \quad C_3 \to C_3 + C_1$

$= \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ y & x & 1+x \end{vmatrix} = 1(1) = 1$

which is independent of x and y.

19. **(d) :** Let $\vec{a} = a_1 i + b_1 j + c_1 k$

$\therefore \ \vec{a}^2 = a_1^2 + b_1^2 + c_1^2 \qquad \therefore \ \vec{a} \times \vec{i} = -b_1 k + c_1 j$

$\therefore \ (\vec{a} \times \vec{i})^2 = b_1^2 + c_1^2$

Similarly $(\vec{a} \times \hat{j})^2 = a_1^2 + c_1^2$

$(\vec{a} \times \hat{k})^2 = a_1^2 + b_1^2$

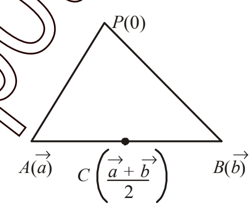
$\therefore (\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2 = 2(a_1^2 + b_1^2 + c_1^2)$

$= 2\vec{a}^2$.

20. **(d) :** Let P is origin

Let $\overrightarrow{PA} = \vec{a}, \ \overrightarrow{PB} = \vec{b}$

$\therefore \ \overrightarrow{PC} = \frac{\vec{a} + \vec{b}}{2}$



Now $\overrightarrow{PA} + \overrightarrow{PB} = \vec{a} + \vec{b}$

$= 2\left(\frac{\vec{a} + \vec{b}}{2}\right) = 2\overrightarrow{PC}$.

21. **(d) :** $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \frac{1}{3}|b||c|\vec{a}$ (As given)

$\Rightarrow \left(\vec{a} \cdot \vec{c}\right)\vec{b} - \left(\vec{b} \cdot \vec{c}\right)\vec{a} \quad = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow -\vec{b} \cdot \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}| \Rightarrow \cos\theta = -1/3$

$\sin\theta = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

22. **(a):** Given $\frac{\vec{v} \cdot \vec{u}}{|u|} = \frac{\vec{u} \cdot \vec{w}}{|u|}$ and $\vec{v} \cdot \vec{w} = 0$

Also $|\vec{u}| = 1, \ |\vec{v}| = 2, \ |\vec{w}| = 3$

Now $|\vec{u} - \vec{v} + \vec{w}|^2$

$= \vec{u}^2 + \vec{v}^2 + \vec{w}^2 - 2\vec{u} \cdot \vec{v} - 2\vec{v} \cdot \vec{w} + 2\vec{u} \cdot \vec{w}$

$= 1 + 4 + 9 + 0 = 14$

23. **(a) :** Using the condition of coplanarity of three vectors

$\therefore \begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & 2\lambda - 1 \end{vmatrix} = 0$

$\Rightarrow \lambda = 0, \frac{1}{2}$.

24. **(c) :** Total force $\vec{F} = \vec{F}_1 + \vec{F}_2 = 7i + 2j - 4k$ and displacement

$\vec{d} = \vec{d}_2 - \vec{d}_1$

$= (5 - 1)i + (4 - 2)j + (k - 3k)$

$\therefore \ W.D = \vec{F} \cdot \vec{d} = 28 + 4 + 8 = 40$

25. **(d):** As $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$

$\therefore \ \vec{a} + 2\vec{b} = P\vec{c}$

and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a} \ \therefore \ \vec{b} + 3\vec{c} = Q\vec{a}$ ...(i)

Now by (i) and (ii) we have

$\vec{a} - 6\vec{c} = P\vec{c} - 2Q\vec{a}$

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$\Rightarrow \vec{a}(1 + 2Q) + \vec{c}(-6 - P) = 0$
$\Rightarrow 1 + 2Q = 0$ and $-P - 6 = 0$
$Q = -1/2,\ P = -6$
Putting these value either in (1) or in (ii) we get
$\vec{a} + 2\vec{b} + 6\vec{c} = 0$

26. (a) : $\vec{a} + \vec{b} + \vec{c} = 0$
Consider $(a + b + c)^2$
$= a^2 + b^2 + c^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$$\Rightarrow (\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = -\frac{(a^2 + b^2 + c^2)}{2}$$

$$= -\frac{(1^2 + 2^2 + 3^2)}{2}$$

$= -7$

27. (b) : Median through any vertex divide the opposite side into two equal parts
$\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$

$$\Rightarrow \overrightarrow{AD} = \frac{1}{2}[\overrightarrow{AB} + \overrightarrow{AC}]$$

$$= \frac{1}{2}[8i - 2j + 8k]$$

$\therefore \left|\overrightarrow{AD}\right| = \sqrt{33}$

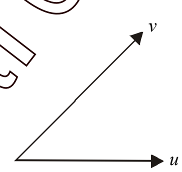
28. (c) : $\hat{n} \parallel \vec{u} \times \vec{v} \quad \because \vec{u}\cdot\hat{n} = 0 = \vec{v}\cdot\vec{n}$

now $\hat{n} = \dfrac{\vec{u} \times \vec{v}}{|\vec{u}||\vec{v}|}$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}(-2\hat{k}) = -\hat{k}$$

Now $\vec{w}\cdot\hat{n} = \left|(i + 2\hat{j} + 3\hat{k})\cdot(-\hat{k})\right|$
$= |-3| = 3$

29. (a) : $(\vec{u} + \vec{v} - \vec{w})\cdot[\vec{u} \times \vec{v} - \vec{u} \times \vec{w} + \vec{v} \times \vec{w}]$
$\because \vec{v} \times \vec{v} = 0$
$\vec{u}\cdot(\vec{u} \times \vec{v}) - \vec{u}\cdot(\vec{u} \times \vec{w}) + \vec{u}\cdot(\vec{v} \times \vec{w})$
$+\vec{v}\cdot\vec{u} \times \vec{v} - \vec{v}\cdot(\vec{u} \times \vec{w}) + v\cdot(\vec{v} \times \vec{w})$
$-\vec{w}\cdot(\vec{u} \times \vec{v}) + \vec{w}\cdot(\vec{u} \times \vec{w}) - \vec{w}\cdot(\vec{v} \times \vec{w})$
$= \vec{u}\cdot(\vec{v} \times \vec{w}) + \vec{v}\cdot(\vec{w} \times \vec{u}) - \vec{w}\cdot(\vec{u} \times \vec{v})$
$= \vec{u}\cdot(\vec{v} \times \vec{w}) + \vec{u}\cdot(\vec{v} \times \vec{w}) - \vec{u}\cdot(\vec{v} \times \vec{w})$
$= \vec{u}\cdot(\vec{v} \times \vec{w}) \qquad (\because [a\ b\ c] = [b\ c\ a] = [c\ a\ b])$



30. (*) : $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 6\hat{i} + 2\hat{j} - 3\hat{k}$
$\therefore |\overrightarrow{AB}| = \sqrt{49} = 7$
Similarly $\overrightarrow{BC} = 2\hat{i} - 3\hat{j} + 6\hat{k}$
$\therefore |\overrightarrow{BC}| = \sqrt{49} = 7$
$\overrightarrow{CD} = -6\hat{i} - 2\hat{j} - \hat{k} \quad |\overrightarrow{CD}| = \sqrt{41}$
$\overrightarrow{DA} = -2\hat{i} + 3\hat{j} - 2\hat{k} \quad |\overrightarrow{DA}| = \sqrt{17}$

31. (c) : If possible say $\vec{a} + \vec{b} + \vec{c} = 0$
$\Rightarrow \vec{b} + \vec{c} = -\vec{a}$
$\vec{a} \times (\vec{b} + \vec{c}) = -\vec{a} \times \vec{a}$
$\Rightarrow \vec{a} \times \vec{b} = \vec{c} \times \vec{a}$
Similarly $\vec{b} \times \vec{c} = \vec{c} \times \vec{a}$
$\therefore \vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

32. (b) : Given $\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -5 & 0 \\ 6 & 3 & 0 \end{vmatrix}$

$\therefore \vec{c} = 39\hat{k}$
Now $|\vec{a}| = \sqrt{34}, |\vec{b}| = \sqrt{45}$ and
$|\vec{c}| = |39\vec{k}| = 39$
$\therefore |\vec{a}| : |\vec{b}| : |\vec{c}| = \sqrt{34} : \sqrt{45} : 39$

33. (b) : $3\lambda\,\vec{c} = 2\mu(\vec{b} \times \vec{a})$
$\Rightarrow$ either $3\lambda = 2\mu$ or $\vec{c} \parallel \vec{b} \times \vec{a}$
but $3\lambda = 2\mu$

34. (a) : We have $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow (\vec{a} + \vec{b} + \vec{c})^2 = 0$
$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$
$\Rightarrow 25 + 16 + 9 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$
$\Rightarrow 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = -50$
$\Rightarrow (\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = -25$
$\therefore (\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 25$

35. (a) : Given $\vec{a} + \vec{b} + \vec{c} = 0$, we need angle between $\vec{b}$ and $\vec{c}$ so consider $\vec{b} + \vec{c} = -\vec{a}$
$\Rightarrow b^2 + c^2 + 2|b|\,|c| \cos\theta = a^2$

$$\Rightarrow \cos\theta = \frac{a^2 - b^2 - c^2}{2|b||c|} = \frac{49 - 25 - 9}{2 \times 5 \times 3} = \frac{1}{2}$$

$\therefore \theta = 60°$

36. (a) : Consider $[\vec{a} \times \vec{b}\ \ \vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}]$
$= (\vec{a} \times \vec{b})\cdot[\vec{k} \times (\vec{c} \times \vec{a})]$ where $k = \vec{b} \times \vec{c}$
$= \vec{a} \times \vec{b}\cdot[(\vec{k}\cdot\vec{a})\vec{c} - (\vec{k}\cdot\vec{c})\vec{a}]$
$= (\vec{a} \times \vec{b})\cdot\left[[(b \times c)\cdot a]\vec{c} - [(\vec{b} \times \vec{c})\cdot c]\vec{a}\right]$
$= (\vec{a} \times \vec{b})\cdot[(\vec{b} \times \vec{c})\cdot\vec{a}]\vec{c} - 0 = \left((\vec{b} \times \vec{c})\cdot\vec{a}\right)\left[(\vec{a} \times \vec{b})\cdot\vec{c}\right]$
$= \left[\vec{a}\cdot(\vec{b} \times \vec{c})\right]\left[\vec{c}\cdot(\vec{a} \times \vec{b})\right] = \left[\vec{a}\cdot(\vec{b} \times \vec{c})\right]^2 = 16$

37. (b) : Using fact: $(\vec{a} \times \vec{b})^2 = a^2b^2 - (\vec{a}\cdot\vec{b})^2$
$= a^2b^2 - a^2b^2\cos^2\theta$

$$= (4 \times 2)^2 - (4 \times 2)^2 \cos^2\frac{\pi}{6}$$

$$= 64 \times \sin^2\frac{\pi}{6} = 64 \times \frac{1}{4} = 16$$

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CHAPTER 15

STATISTICS

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1. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given?
(a) median (b) mode
(c) variance (d) mean *(2013)*

2. Let $x_1, x_2, ..., x_n$ be n observations, and let be their arithmetic mean and σ^2 be their variance.
Statement 1 : Variance of $2x_1, 2x_2, ..., 2x_n$ is $4\sigma^2$.
Statement 2 : Arithmetic mean of $2x_1, 2x_2, ..., 2x_n$ is 4.
(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1. *(2012)*

3. If the mean deviation about the median of the numbers $a, 2a, ..., 50a$ is 50, then $|a|$ equals
(a) 4 (b) 5 (c) 2 (d) 3 *(2011)*

4. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is
(a) $\frac{5}{2}$ (b) $\frac{11}{2}$ (c) 6 (d) $\frac{13}{2}$ *(2010)*

5. **Statement-1 :** The variance of first n even natural numbers is $\frac{n^2-1}{4}$
Statement-2 : The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$
(a) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is false, Statement 2 is true.
(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1. *(2009)*

6. If the mean deviation of number $1, 1+d, 1+2d, ..., 1+100d$ from their mean is 255, then the d is equal to
(a) 20.0 (b) 10.1
(c) 20.2 (d) 10.0 *(2009)*

7. The mean of the numbers $a, b, 8, 5, 10$ is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b?
(a) $a = 3, b = 4$ (b) $a = 0, b = 7$
(c) $a = 5, b = 2$ (d) $a = 1, b = 6$ *(2008)*

8. The average marks of boys in class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is
(a) 80 (b) 60
(c) 40 (d) 20. *(2007)*

9. Suppose a population A has 100 observations 101, 102, ..., 200, and another population B has 100 observations 151, 152, ..., 250. If V_A and V_B represent the variances of the two populations, respectively, then V_A/V_B is
(a) 1 (b) 9/4
(c) 4/9 (d) 2/3. *(2006)*

10. Let $x_1, x_2, ..., x_n$ be n observations such that $\sum x_1^2 = 400$ and $\sum x_i = 80$. Then a possible value of n among the following is
(a) 18 (b) 15
(c) 12 (d) 9. *(2005)*

11. In a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is approximately
(a) 20.5 (b) 22.0
(c) 24.0 (d) 25.5. *(2005)*

12. In a series of $2n$ observations, half of them equal a and remaining half equal $-a$. If the standard deviation of the observations is 2, then $|a|$ equals
(a) 2 (b) $\sqrt{2}$
(c) $\frac{1}{n}$ (d) $\frac{\sqrt{2}}{n}$. *(2004)*

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13. Consider the following statements :
(1) Mode can be computed from histogram
(2) Median is not independent of change of scale
(3) Variance is independent of change of origin and scale.
Which of these is/are correct?
(a) only (1) and (2) (b) only (2)
(c) only (1) (d) (1), (2) and (3). *(2004)*

14. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set
(a) is decreased by 2
(b) is two times the original median
(c) remains the same as that of the original set
(d) is increased by 2. *(2003)*

15. In an experiment with 15 observations on x, the following results were available.

$$\sum x^2 = 2830, \ \sum x = 170$$

One observation that was 20 was found to be wrong and was replaced by the correct value 30. Then the corrected variance is
(a) 188.66 (b) 177.33
(c) 8.33 (d) 78.00. *(2003)*

16. In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average of the girls?
(a) 73 (b) 65
(c) 68 (d) 74. *(2002)*

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Answer Key

1.	(c)	**2.**	(b)	**3.**	(a)	**4.**	(b)	**5.**	(c)	**6.**	(b)
7.	(a)	**8.**	(a)	**9.**	(a)	**10.**	(a)	**11.**	(c)	**12.**	(a)
13.	(a)	**14.**	(b)	**15.**	(d)	**16.**	(b)				

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Explanations

1. **(c) : 1st solution :** Variance doesn't change with the change of origin.

2nd solution : $\sigma_1^2 = \frac{1}{n}\sum (x_i - \bar{x})^2$

$\sigma_2^2 = \frac{1}{n}\sum \{(x_i + 10) - (\bar{x} + 10)\}^2$ Hence $\sigma_1^2 = \sigma_2^2$

2. **(b) :** $x_1, x_2, x_3, \ldots x_n$, A.M. = $\bar{x}$, Variance = σ^2

Statement 2 : A.M. of $2x_1, 2x_2, \ldots, 2x_n$

$$= \frac{2(x_1 + x_2 + \ldots + x_n)}{n} = 2\bar{x}$$

Given A.M. $= 4\bar{x}$ $\therefore$ Statement 2 is false.

3. **(a) :** Median is the mean of 25th and 26th observation.

$$M = \frac{25a + 26a}{2} = 25.5a$$

$$MD(M) = \frac{\sum |r_i - M|}{N}$$

$$\Rightarrow 50 = \frac{1}{50}\{2|a| \times (0.5 + 1.5 + \ldots + 24.5)\}$$

$$\Rightarrow 2500 = 2|a| \cdot \frac{25}{2} \cdot 25. \quad \therefore |a| = 4$$

4. **(b) : 1st solution:**

$\left.\begin{array}{l}\sigma_1^2 = 4 \\ \sigma_2^2 = 5\end{array}\right|\begin{array}{l}\bar{x} = 2 \\ \bar{y} = 4\end{array}$

We have $\frac{\sum x_i}{5} = 2 \Rightarrow \sum x_i = 10$

Similarly, $\sum y_i = 20$

$$\sigma_1^2 = \left(\frac{1}{5}\sum x_i^2\right) - \bar{x}^2 \Rightarrow 4 = \frac{1}{5}\sum x_i^2 - 4$$

$$\Rightarrow \frac{1}{5}\sum x_i^2 = 8. \quad \therefore \sum x_i^2 = 40.$$

$$\sigma_2^2 = \left(\frac{1}{5}\sum y_i^2\right) - \bar{y}^2 \Rightarrow 5 = \frac{1}{5}\sum y_i^2 - 16$$

$$\Rightarrow \frac{1}{5}\sum y_i^2 = 21. \quad \therefore \sum y_i^2 = 105$$

$$\sigma^2 = \frac{1}{10}\left(\sum x_i^2 + \sum y_i^2\right) - \left(\frac{\bar{x} + \bar{y}}{2}\right)^2$$

$$= \frac{1}{10}(40 + 105) - 9 = \frac{145 - 90}{10} = \frac{55}{10} = \frac{11}{2}.$$

2nd solution :

$\sigma_1^2 = 4,\ n_1 = 5,\ \bar{x}_1 = 2$

$\sigma_2^2 = 5,\ n_2 = 5,\ \bar{x}_2 = 4$

$$\bar{x}_{12} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{5 \times 2 + 5 \times 4}{10} = 3$$

$d_1 = (\bar{x}_1 - \bar{x}_{12}) = -1,\ d_2 = (\bar{x}_2 - \bar{x}_{12}) = 1$

$$\sigma = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$$

$$= \sqrt{\frac{5.4 + 5.5 + 5.1 + 5.1}{10}} = \sqrt{\frac{55}{10}} = \sqrt{\frac{11}{2}}$$

$$\therefore \sigma^2 = \frac{11}{2}$$

5. **(c) :** Sum of first n even natural numbers

$= 2 + 4 + 6 + \ldots + 2n = 2(1 + 2 + \ldots + n)$

$= 2 \cdot \frac{n(n+1)}{2} = n(n+1)$

Mean $(\bar{x}) = \frac{n(n+1)}{n} = n + 1$

Variance $= \frac{1}{n}(\sum x_i)^2 - (\bar{x})^2$

$$= \frac{1}{n}(2^2 + 4^2 + \ldots + (2n)^2) - (n+1)^2$$

$$= \frac{1}{n} \cdot 2^2(1^2 + 2^2 + \ldots + n^2) - (n+1)^2$$

$$= \frac{4}{n} \cdot \frac{n(n+1)(2n+1)}{6} - (n+1)^2$$

$$= \frac{2}{3} \cdot (n+1)(2n+1) - (n+1)^2$$

$$= \frac{(n+1)}{3}[2(2n+1) - 3(n+1)]$$

$$= \frac{(n+1)}{3} \cdot (n-1) = \frac{n^2 - 1}{3}$$

6. **(b) :** The numbers are $1, 1 + d, 1 + 2d, \ldots, 1 + 100d$.

The numbers are in A.P.

Then mean = 51st term = $1 + 50d = \bar{x}$ (say)

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Mean deviation (M.D.) $= \frac{1}{n}\sum_{i=1}^{101} |x_i - \bar{x}|$

$= \frac{1}{101}[50d + 49d + 48d + ... + d + 0 + d + 2d + ... + 50d]$

$+ d + 0 + d + 2d + ... + 50d]$

$= \frac{1}{101} \cdot 2d(1 + 2 + ... + 50)$

$= \frac{1}{101} \cdot 2d \cdot \frac{50 \cdot 51}{2} = \frac{50 \cdot 51}{101} d$

But M.D. = 255 (given)

$\Rightarrow \frac{50 \cdot 51}{101} d = 255$

$\Rightarrow d = \frac{101 \times 255}{50 \times 51} = \frac{101 \times 255}{2550} = 10.1$

7. **(a) :** The mean of a, b, 8, 5, 10 is 6

$\Rightarrow \frac{a + b + 8 + 5 + 10}{5} = 6$

$\Rightarrow a + b + 23 = 30 \quad \Rightarrow \quad a + b = 7$ ...(1)

Again variance $= \frac{\Sigma(x_i - A)^2}{n} = 6.8$

$\Rightarrow \frac{(a-6)^2 + (b-6)^2 + 4 + 1 + 16}{5} = 6.8$

$\Rightarrow a^2 + b^2 - 12(a + b) + 36 + 21 + 72 = 5 \times 6.8 = 34$

$\Rightarrow a^2 + b^2 - 12 \times 7 + 72 + 21 = 34$

$\therefore a^2 + b^2 = 25$ ...(2)

using (1) we have

$a^2 + (7 - a)^2 = 25 \quad \Rightarrow \quad a^2 + 49 - 14a + a^2 = 25$

$\Rightarrow a^2 - 7a + 12 = 0 \quad \therefore \quad a = 3, 4$ also $b = 3, 4$

8. **(a) :** Let x and y are number of boys and girls in a class respectively.

$\frac{52x + 42y}{x + y} = 50$

$\Rightarrow x = 4y \Rightarrow \frac{x}{y} = \frac{4}{1}$ and $\frac{x}{x + y} = \frac{4}{5}$

Required percentage $= \frac{x}{x + y} \times 100 = \frac{4}{5} \times 100 = 80\%$.

9. **(a) :** Series A = 101, 102, 103, ..., 200

Series B = 151, 152, 153, ..., 250

Series B is obtained by adding a fixed quantity to each item of series A, we know that variance is independent of change of origin both series have the same variance so ratio of their variances is 1.

10. **(a) :** Using well known fact that root mean square of number $\geq$ A.M. of the numbers

$\Rightarrow \sqrt{\frac{400}{n}} \geq \frac{80}{n} \Rightarrow \frac{20}{\sqrt{n}} \geq \frac{80}{n} \Rightarrow \sqrt{n} \geq 4 \Rightarrow n \geq 16$

$\Rightarrow n = 17$ but not given in choice.

$\therefore n = 18$ is correct number.

11. **(c) :** Using fact, mode = 3 median – 2 mean

$= 3 \times 22 - 2 \times 21 = 3(22 - 14) = 3 \times 8 = 24.$

12. **(a) :** According to problem

X	Value of X	d = value of $X - \bar{X}$	$(X - \bar{X})^2$
x_1	a	a	a^2
x_2	a	a	a^2
–	–	–	–
–	–	–	–
–	–	–	–
–	–	–	–
x_n	a	a	–
x_{n+1}	$-a$	$-a$	a^2
x_{n+2}	$-a$	$-a$	a^2
–	–	–	–
–	–	–	–
–	–	–	–
x_{n+n}	$-a$	$-a$	a_2
$\Sigma X = 0$			$\sum(X - \bar{X})^2 = 2na^2$

$\therefore \bar{X} = \frac{\Sigma X}{N} = \frac{0}{2n} = 0$

Now $SD = \sqrt{\frac{\sum(X - \bar{X})^2}{N}} = \sqrt{\frac{\sum X^2}{N} - (\bar{X})^2}$

$2 = \sqrt{\frac{2na^2}{2n} - 0}\,; 2 = \sqrt{a^2} \quad ; \quad 2 = |a|$

13. **(a) :** Mode can be computed by histogram

Median will be changed if data's are changed so (2) is correct.

Variance depends on change of scale so (3) is not correct.

14. **(b) :** Total number of observations are 9 which is odd which means median is 5th item now we are increasing 2 in the last four items which does not effect its value. The new median remain unchanged.

15. **(d) :** $\sum x = 170$ and $\sum x^2 = 2830$

Increase in $\sum x = 10$ and $\sum x' = 170 + 10 = 180$

Increase in $\sum x^2 = 900 - 400 = 500$ then

$\sum x'^2 = 2830 + 500 = 3330$

$= \frac{1}{15} \times 3330 - \left(\frac{1}{15} \times 180\right)^2 = 222 - (12)^2 = 78$

16. **(b) :** Using $\bar{x} = \frac{(x_1 + x_2 + ... + x_{100})}{100} = 72$

$\therefore x_1 + ... + x_{100} = 7200$ ...(i)

Again $\frac{x_1 + x_2 + ... + x_{70}}{70} = 75$

$x_1 + ... + x_{70} = 75 \times 70$ ...(ii)

$\therefore$ Average of 30 girls $= \frac{7200 - 5250}{30} = 65$

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CHAPTER 16

PROBABILITY

1. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is

(a) $\frac{10}{3^5}$ (b) $\frac{17}{3^5}$ (c) $\frac{13}{3^5}$ (d) $\frac{11}{3^5}$ *(2013)*

2. Three numbers are chosen at random without replacement from {1, 2, 3, ..., 8}. The probability that their minimum is 3, given that their maximum is 6, is

(a) $\frac{1}{4}$ (b) $\frac{2}{5}$ (c) $\frac{3}{8}$ (d) $\frac{1}{5}$ *(2012)*

3. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is

(a) $P(C|D) < P(C)$ (b) $P(C|D) = \frac{P(D)}{P(C)}$

(c) $P(C|D) = P(C)$ (d) $P(C|D) \geq P(C)$ *(2011)*

4. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval

(a) $\left[0, \frac{1}{2}\right]$ (b) $\left(\frac{11}{12}, 1\right]$

(c) $\left(\frac{1}{2}, \frac{3}{4}\right]$ (d) $\left(\frac{3}{4}, \frac{11}{12}\right]$ *(2011)*

5. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is

(a) 1/3 (b) 2/7 (c) 1/21 (d) 2/23 *(2010)*

6. Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ..., 20}.

Statement-1 : The probability that the chosen numbers when arranged in some order will form an A.P. is $\frac{1}{85}$.

Statement-2 : If the four chosen numbers form an A.P., then the set of all possible values of common difference is {±1, ±2, ±3, ±4, ±5}.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation of Statement-1.

(b) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true. *(2010)*

7. In a binomial distribution $B\left(n, p = \frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than

(a) $\frac{1}{\log_{10} 4 + \log_{10} 3}$ (b) $\frac{9}{\log_{10} 4 - \log_{10} 3}$

(c) $\frac{4}{\log_{10} 4 - \log_{10} 3}$ (d) $\frac{1}{\log_{10} 4 - \log_{10} 3}$ *(2009)*

8. One ticket is selected at random from 50 tickets numbered 00, 01, 02, …., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals

(a) $\frac{1}{7}$ (b) $\frac{5}{14}$ (c) $\frac{1}{50}$ (d) $\frac{1}{14}$ *(2009)*

9. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) 0 (d) 1 *(2008)*

10. It is given that the events A and B are such that $P(A) = \frac{1}{4}, P(A|B) = \frac{1}{2}$ and $P(B|A) = \frac{2}{3}$. Then $P(B)$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$ *(2008)*

11. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is

(a) 8/729 (b) 8/243

(c) 1/729 (d) 8/9. *(2007)*

12. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the

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first misses the target. The probability that the target is hit by the second plane is

(a) 0.2 (b) 0.7
(c) 0.06 (d) 0.14. *(2007)*

13. At a telephone enquiry system the number of phone calls regarding relevant enquiry follow Poisson distribution with a average of 5 phone calls during 10-minute time intervals. The probability that there is at the most one phone call during a 10-minute time period is

(a) $\frac{6}{5^e}$ (b) $\frac{5}{6}$ (c) $\frac{6}{55}$ (d) $\frac{6}{e^5}$. *(2006)*

14. A random variable X has Poisson distribution with mean 2. The $P(X > 1.5)$ equals

(a) 0 (b) $2/e^2$
(c) $3/e^2$ (d) $1-\frac{3}{e^2}$. *(2005)*

15. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is

(a) 1/9 (b) 2/9
(c) 7/9 (d) 8/9. *(2005)*

16. Let A and B be two events such that $P(\overline{A\cup B})=\frac{1}{6}$, $P(A\cap B)=\frac{1}{4}$ and $P(\bar{A})=\frac{1}{4}$, where $\bar{A}$ stands for complement of event A. Then events A and B are

(a) equally likely but not independent
(b) equally likely and mutually exclusive
(c) mutually exclusive and independent
(d) independent but not equally likely. *(2005)*

17. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is:

(a) 128/256 (b) 219/256
(c) 37/256 (d) 28/256. *(2004)*

18. A random variable X has the probability distribution:

X :	1	2	3	4	5	6	7	8
$P(X)$:	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events E = {X is a prime number} and F = {$X < 4$}, the probability $P(E \cup F)$ is:

(a) 0.35 (b) 0.77
(c) 0.87 (d) 0.50. *(2004)*

19. The probability that A speaks truth is 4/5, while this probability for B is 3/4. The probability that they contradict each other when asked to speak on a fact is

(a) 7/20 (b) 1/5
(c) 3/20 (d) 4/5. *(2004)*

20. The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively, then $P(X = 1)$ is

(a) 1/16 (b) 1/8 (c) 1/4 (d) 1/32. *(2003)*

21. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is

(a) 3/5 (b) 1/5
(c) 2/5 (d) 4/5. *(2003)*

22. Events A, B, C are mutually exclusive events such that $P(A)=\frac{3x+1}{3}$, $P(B)=\frac{1-x}{4}$ and $P(C)=\frac{1-2x}{2}$. Then set of possible values of x are in the interval

(a) $\left[\frac{1}{3},\frac{2}{3}\right]$ (b) $\left[\frac{1}{3},\frac{13}{3}\right]$
(c) $[0, 1]$ (d) $\left[\frac{1}{3},\frac{1}{2}\right]$. *(2003)*

23. A die is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is

(a) 8/3 (b) 3/8
(c) 4/5 (d) 5/4. *(2002)*

24. A and B are events such that $P(A \cup B) = 3/4$, $P(A \cap B) = 1/4$, $P(\bar{A}) = 2/3$ then $P(\bar{A}\cap B)$ is

(a) 5/12 (b) 3/8
(c) 5/8 (d) 1/4. *(2002)*

25. A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is 1/2, 1/3 and 1/4. Probability that the problem is solved is

(a) 3/4 (b) 1/2
(c) 2/3 (d) 1/3. *(2002)*

Answer Key

1. (d)	**2.** (d)	**3.** (d)	**4.** (a)	**5.** (b)	**6.** (c)
7. (d)	**8.** (d)	**9.** (d)	**10.** (c)	**11.** (b)	**12.** (d)
13. (d)	**14.** (d)	**15.** (a)	**16.** (d)	**17.** (d)	**18.** (b)
19. (a)	**20.** (d)	**21.** (c)	**22.** (d)	**23.** (d)	**24.** (a)
25. (a)					

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Explanations

1. (d) : P(correct answer) = 1/3

The required probability $= {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right) + {}^5C_5\left(\frac{1}{3}\right)^5$

$$= \frac{5\times 2}{3^5} + \frac{1}{3^5} = \frac{11}{3^5}$$

2. (d) : 3 numbers are chosen from {1, 2, 3, ..., 8} without replacement. Let A be the event that the maximum of chosen numbers is 6.

Let B be the event that the minimum of chosen numbers is 3.

$$P(B/A) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1\cdot 1\cdot 2}{{}^8C_3}}{\frac{{}^5C_2}{{}^8C_3}} = \frac{2}{10} = \frac{1}{5}$$

3. (d) : $P(C\mid D) = \dfrac{P(C\cap D)}{P(D)}$ as $C \subset D$, $P(C) \subset P(D)$.

$\therefore \quad P(C \cap D) = P(C)$

We have, $P(C\mid D) = \dfrac{P(C)}{P(D)}$

As $0 < P(D) \le 1$ we have $P(C|D) \ge P(C)$

4. (a) : Probability of at least one failure

$= 1 - P(\text{no failure}) = 1 - p^5$

Now $1 - p^5 \ge \dfrac{31}{32}$

$\Rightarrow \quad p^5 \le \dfrac{1}{32}$ thus $p \le \dfrac{1}{2} \therefore\ p \in [0, 1/2]$

5. (b) : $n(S) = {}^9C_3 = \dfrac{9\times 8\times 7}{6} = 84$

$n(E) = {}^3C_1 \cdot {}^4C_1 \cdot {}^2C_1 = 3 \times 4 \times 2 = 24.$

The desired probability $= \dfrac{24}{84} = \dfrac{2}{7}.$

6. (c) : Number of A.P.'s with common difference 1 = 17

Number of A.P.'s with common difference 2 = 14

Number of A.P.'s with common difference 3 = 11

Number of A.P.'s with common difference 4 = 8

Number of A.P.'s with common difference 5 = 5

Number of A.P.'s with common difference 6 = $\dfrac{2}{57}$

The total number of ways $n(S) = {}^{20}C_4$

The desired probability $= \dfrac{57}{{}^{20}C_4} = \dfrac{57\times 24}{20\times 19\times 18\times 17} = \dfrac{1}{85}$

Now statement-2 is false and statement-1 is true.

7. (d) : Probability of at least one success

$= 1 -$ No success $= 1 - {}^nC_n q^n$

where $q = 1 - p = 3/4$

we want $1 - \left(\frac{3}{4}\right)^4 \ge \dfrac{9}{10}$

$\Rightarrow \quad \dfrac{1}{10} \ge \left(\dfrac{3}{4}\right)^4 \Rightarrow \left(\dfrac{3}{4}\right)^4 \le \dfrac{1}{10}$

Taking logarithm on base 10 we have

$n \log_{10}(3/4) \le \log_{10} 10^{-1}$

$\Rightarrow n(\log_{10}3 - \log_{10}4) \le -1$

$\Rightarrow n(\log_{10}4 - \log_{10}3) \ge 1$

$\Rightarrow \quad n \ge \dfrac{1}{\log_{10}4 - \log_{10}3}$

8. (d) : Any number in the set

S = {00, 01, 02, ..., 49} is of the form ab where $a \in$ {0, 1, 2, 3, 4} and $b \in$ {0, 1, 2, ..., 9} for the product of digits to be zero, the number must be of the form either $x0$ which are 5 in numbers, because $x \in$ {0, 1, 2, 3, 4}

or of the form $0x$ which are 10 in numbers because $x \in$ {0, 1, 2, ..., 9}

The only number common to both = 00

Thus the number of numbers in S, the product of whose digits is zero = 10 + 5 – 1 = 14

Of these the number whose sum of digits is 8 is just one, i.e. 08

The required probability = 1/14.

9. (d) : A = {4, 5, 6}

Also B = {1, 2, 3, 4}

We have $A \cup B$ = {1, 2, 3, 4, 5, 6} = S

Where S is the sample space of the experiment of throwing a die. $P(S) = 1$, for it is a sure event.

Hence $P(A \cup B) = 1$

10. (c) : From the definition of independence of events

$$P(A/B) = \frac{P(A\cap B)}{P(B)}$$

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Then $P(B) \cdot P(A/B) = P(A \cap B)$ ...(1)
Interchanging the role of A and B in (1)
$P(A)P(B/A) = P(B \cap A)$ ...(2)
As $A \cap B = B \cap A$, we have from (1) and (2)
$P(A)P(B/A) = P(B)P(A/B)$

$$\Rightarrow \frac{1}{4}\cdot\frac{2}{3} = P(B)\cdot\frac{1}{2} \Rightarrow P(B) = \frac{1}{4}\cdot\frac{2}{3}\cdot 2 = \frac{1}{3}$$

11. **(b) :** Possibility of getting 9 are (5, 4), (4, 5), (6, 3), (3, 6)

Probability of getting score 9 in a single throw $= p = \frac{4}{36} = \frac{1}{9}$

Required probability = probability of getting score 9 exactly twice $= {}^3C_2\left(\frac{1}{9}\right)^2 \times \left(\frac{8}{9}\right) = \frac{8}{243}$.

12. **(d) :** $P(\text{I}) = 0.3, P(\overline{\text{I}}) = 1 - 0.3 = 0.7,$

$P(\text{II}) = 0.2, P(\overline{\text{II}}) = 1 - 0.2 = 0.8$

Required probability $= P(\overline{\text{I}} \cap \text{II}) = P(\overline{\text{I}})P(\text{II})$
$= (0.7)(0.2) = 0.14.$

13. **(d) :** We know that poission distribution is given by

$P(x = r) = \frac{e^{-\lambda}\lambda^r}{r!}$ where $\lambda = 5$

Now $P(x = r \le 1) = P(x = 0) + P(x = 1)$

$$= \frac{e^{-\lambda}}{0!} + \frac{\lambda_e^{-\lambda}}{1!} = e^{-5}(1+5) = \frac{6}{e^5}.$$

14. **(d) :** $P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!}$ $(\lambda = \text{mean})$
$\therefore P(X = r > 1.5) = P(2) + P(3) + \ldots \infty$
$= 1 - [P(0) + P(1)]$

$$= 1 - \left[e^{-2} + \frac{e^{-2} \times 2^2}{2}\right] = 1 - \frac{3}{e^2}.$$

15. **(a) :** No. of houses = 3 = No. of favourable cases
No. of applicants = 3,
$\therefore$ Total number of events $= 3^3$
(because each candidate can apply by 3 ways)

Required probability $= \frac{3}{3^3} = \frac{1}{9}$.

16. **(d) :** $P(\overline{A \cup B}) = 1 - P(A \cup B)$

$= 1 - P(A) - P(B) + P(A \cap B)$

$\frac{1}{6} = 1 - \frac{3}{4} - P(B) + \frac{1}{4}$

$P(B) = \frac{1}{2} - \frac{1}{6} \Rightarrow P(B) = \frac{4}{12} = \frac{1}{3}$

now $P(A \cap B) = \frac{1}{4} = \frac{1}{3} \times \frac{3}{4} = P(A)P(B)$

so even are independent but not equally likely as $P(A) \ne P(B)$.

17. **(d) :** Given $np = 4$ and $npq = 2$

$q = \frac{npq}{np} = \frac{2}{4} = \frac{1}{2}$ so $p = 1 - 1/2 = \frac{1}{2}$

Now $npq = 2 \therefore n = 8$

$\therefore$ BD is given by $P(X = r) = {}^8C_r\, p^r\, q^{n-r}$

$\therefore$ $P(X = r = 2) = {}^8C_2\left(\frac{1}{2}\right)^8 = \frac{28}{256}$

18. **(b) :** From the given table prime numbers are 2, 3, 5, 7
'E' denote prime
'F' denote the number < 4
$\therefore$ $P(E) = P(2$ or 3 or 5 or 7)
(Events 2, 3, 5, 7 are $M.E$)
$= P(2) + P(3) + P(5) + P(7) = .62$
$P(F) = P(1$ or 2 or 3) (events 1, 2, 3 are $m.E.$)
$= P(1) + P(2) + P(3) = .50$
$P(E \cap F) = P(2$ or 3) $= P(2) + P(3) = .35$
$P(E \cup F) = P(E) + P(F) - P(E \cap F)$
$= .62 + .50 - .35 = .77$

19. **(a) :** $P(A) = \frac{4}{5}$ $\therefore P(\bar{A}) = \frac{1}{5}$

$P(B) = \frac{3}{4}$ $\therefore P(\bar{B}) = \frac{1}{4}$

Now we needed $P(A)\,P(\bar{B}) + P(B)\,P(\bar{A})$

$= \frac{4}{5}\times\frac{1}{4} + \frac{3}{4}\times\frac{1}{5} = \frac{7}{20}$

20. **(d) :** Given mean $np = 4$, $npq = 2$

$\Rightarrow \frac{npq}{np} = \frac{2}{4} \therefore q = p = \frac{1}{2}$ and $n = 8$

Now $P(X = r) = {}^8C_r\left(\frac{1}{2}\right)^8$

(Use $p(X = r) = {}^nC_r\, p^r q^{n-r}$

$\therefore p(X = 1) = {}^8C_1\left(\frac{1}{2}\right)^8 = \frac{8}{16 \times 16} = \frac{1}{32}$

21. **(c) :** No. of horses = 5

$\therefore$ Probability that A can't win the race $= \frac{4}{5}\times\frac{3}{4}$

Probability that 'A' must win the race $= 1 - P(\bar{A})P(\bar{B})$

$= 1 - \frac{12}{20} = \frac{2}{5}$

22. **(d) :** A, B, C are mutually exclusive
$\therefore$ $0 \le P(A) + P(B) + P(C) \le 1$ ...(i)
$0 \le P(A), P(B), P(C) \le 1$ ...(ii)
Now on solving (i) and (ii) we get

$\frac{1}{3} \le x \le \frac{1}{2}$

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ALL SUBJECTS CLASS 1 TO 12 STUDY MATERIAL WITH COMPLETE SOLUTIONS ARE AVAILABLE [344 of 353] [9440345996]

23. **(d) :** $n = 5\ p = q = 1/2 \quad P(X = r) = {}^5C_r\left(\frac{1}{2}\right)^n$

	x_i	f_i	$f_i x_i$	$f_i x_i^2$
(0)	0	$\left(\frac{1}{2}\right)^5$	0	0
(1)	${}^5C_1\left(\frac{1}{2}\right)^5$	$\frac{1\times 5}{32}$	$\frac{5}{32}$	
(2)	${}^5C_1\left(\frac{1}{2}\right)^5$	$\frac{2\times 10}{32}$	$2^2\frac{10}{32}$	
(3)	${}^5C_3\left(\frac{1}{2}\right)^5$	$\frac{3\times 10}{32}$	$3^2\frac{10}{32}$	
(4)	${}^5C_4\left(\frac{1}{2}\right)^5$	$\frac{4\times 5}{32}$	$4^2\frac{5}{32}$	
(5)	${}^5C_5\left(\frac{1}{2}\right)^5$	$5\times\frac{1}{32}$	$5^2\frac{1}{32}$	
	$\Sigma f_i = 1$	$\Sigma f_i x_i = \frac{80}{32}$	$\Sigma f_i x_i^2 = \frac{240}{32}$	

$\bar{x} = \text{mean} = \frac{5}{2}$

Now variance $= \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$

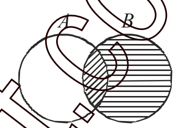
$= \frac{240}{32} - \frac{25}{4}$

$= \frac{40}{32} = \frac{5}{4}$

24. **(a) :** Given $P(A \cup B) = 3/4$

$P(A \cap B) = 1/4$

$P(\bar{A}) = 2/3$



$\therefore \quad P(B) = \frac{2}{3}$

$P(\bar{A} \cap \bar{B}) = P(B) - P(A \cap B)$

By using $P(B) = P(A \cup B) + P(A \cap B) - P(A)$

$\therefore \quad P(\bar{A} \cap B) = \frac{2}{3} - \frac{1}{4} = 5/12$

25. **(a) :** Given $P(A) = 1/2 \quad \therefore P(\bar{A}) = 1/2$

$P(B) = 1/3 \quad \therefore P(\bar{B}) = \frac{2}{3}$

$P(C) = \frac{1}{4} \quad \therefore P(\bar{C}) = \frac{3}{4}$

Now problem will be solved if any one of them will solve the problem.

$\therefore$ P(at least one of them solve the problem)

= 1 – probability none of them can solve the problem.

or $P(A \cup B \cup C) = 1 - P(\bar{A})\,P(\bar{B})\,P(\bar{C})$

$= 1 - \frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4} = 3/4$

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CHAPTER 17

TRIGONOMETRY

1. If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then
(a) $2x = 3y = 6z$ (b) $6x = 3y = 2z$
(c) $6x = 4y = 3z$ (d) $x = y = z$ *(2013)*

2. $ABCD$ is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then AB is equal to
(a) $\dfrac{p^2 + q^2\cos\theta}{p\cos\theta + q\sin\theta}$ (b) $\dfrac{p^2 + q^2}{p^2\cos\theta + q^2\sin\theta}$
(c) $\dfrac{(p^2 + q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$ (d) $\dfrac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ *(2013)*

3. The expression $\dfrac{\tan A}{1 - \cot A} + \dfrac{\cot A}{1 - \tan A}$ can be written as
(a) $\sec A \operatorname{cosec} A + 1$ (b) $\tan A + \cot A$
(c) $\sec A + \operatorname{cosec} A$(d) $\sin A \cos A + 1$ *(2013)*

4. In a ΔPQR, if $3 \sin P + 4 \cos Q = 6$ and $4 \sin Q + 3\cos P = 1$, then the angle R is equal to
(a) $\pi/4$ (b) $3\pi/4$ (c) $5\pi/6$ (d) $\pi/6$ *(2012)*

5. If $A = \sin^2 x + \cos^4 x$, then for all real x
(a) $1 \le A \le 2$ (b) $\dfrac{3}{4} \le A \le \dfrac{13}{16}$
(c) $\dfrac{3}{4} \le A \le 1$ (d) $\dfrac{13}{16} \le A \le 1$ *(2011)*

6. Let $\cos(\alpha + \beta) = \dfrac{4}{5}$ and let $\sin(\alpha - \beta) = \dfrac{5}{13}$, where $0 \le \alpha,\ \beta \le \dfrac{\pi}{4}$. Then $\tan 2\alpha =$
(a) $\dfrac{25}{16}$ (b) $\dfrac{56}{33}$ (c) $\dfrac{19}{12}$ (d) $\dfrac{20}{17}$ *(2010)*

7. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is
(a) there is a regular polygon with $r/R = 1/2$
(b) there is a regular polygon with $r/R = 1/\sqrt{2}$
(c) there is a regular polygon with $r/R = 2/\sqrt{3}$
(d) there is a regular polygon with $r/R = \sqrt{3}/2$ *(2010)*

8. Let A and B denote the statements
$A : \cos\alpha + \cos\beta + \cos\gamma = 0$
$B : \sin\alpha + \sin\beta + \sin\gamma = 0$
If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\dfrac{3}{2}$, then
(a) A is false and B is true
(b) both A and B are true
(c) both A and B are false
(d) A is true and B is false *(2009)*

9. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that $CD = 7$ m. From D the angle of elevation of the point A is 45°. Then the height of the pole is
(a) $\dfrac{7\sqrt{3}}{2}\dfrac{1}{\sqrt{3}+1}$ m (b) $\dfrac{7\sqrt{3}}{2}\dfrac{1}{\sqrt{3}-1}$ m
(c) $\dfrac{7\sqrt{3}}{2}(\sqrt{3}+1)$ m (d) $\dfrac{7\sqrt{3}}{2}(\sqrt{3}-1)$ m *(2008)*

10. The value of is $\cot\left(\operatorname{cosec}^{-1}\dfrac{5}{3} + \tan^{-1}\dfrac{2}{3}\right)$
(a) $\dfrac{5}{17}$ (b) $\dfrac{6}{17}$ (c) $\dfrac{3}{17}$ (d) $\dfrac{4}{17}$ *(2008)*

11. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB (= a) subtends an angle of 60° at the foot of the tower, and the angle of elevation of the top of the tower from A or B is 30°. The height of the tower is
(a) $a/\sqrt{3}$ (b) $a\sqrt{3}$
(c) $2a/\sqrt{3}$ (d) $2a\sqrt{3}$. *(2007)*

12. The largest interval lying in $\left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$ for which the function, $f(x) = 4^{-x^2} + \cos^{-1}\left(\dfrac{x}{2} - 1\right) + \log(\cos x)$ is defined, is
(a) $\left[-\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$ (b) $\left[0, \dfrac{\pi}{2}\right)$
(c) $[0, \pi]$ (d) $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$. *(2007)*

13. If $\sin^{-1}\left(\dfrac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\dfrac{5}{4}\right) = \dfrac{\pi}{2}$, then the values of x is
(a) 4 (b) 5 (c) 1 (d) 3. *(2007)*

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14. If $0 < x < \pi$ and $\cos x + \sin x = 1/2$, then $\tan x$ is

(a) $\frac{(1-\sqrt{7})}{4}$ (b) $\frac{(4-\sqrt{7})}{3}$

(c) $-\frac{(4+\sqrt{7})}{3}$ (d) $\frac{(1+\sqrt{7})}{4}$. *(2006)*

15. The number of value of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is

(a) 4 (b) 6 (c) 1 (d) 2. *(2006)*

16. If in a ΔABC, the altitudes from the vertices A, B, C on opposite sides are in H.P., then $\sin A$, $\sin B$, $\sin C$ are in

(a) H.P.
(b) Arithmetic-Geometric progression
(c) A.P.
(d) G.P. *(2005)*

17. If $\cos^{-1} x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to

(a) 4 (b) $2\sin 2\alpha$

(c) $-4\sin^2\alpha$ (d) $4\sin^2\alpha$. *(2005)*

18. In a triangle ABC, let $\angle C = \frac{\pi}{2}$. If r is the inradius and R is the circumradius of the triangle ABC, then $2(r + R)$ equals

(a) $a + b$ (b) $b + c$

(c) $c + a$ (d) $a + b + c$ *(2005)*

19. In a triangle PQR, if $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then

(a) $b = a + c$ (b) $b = c$

(c) $c = a + b$ (d) $a = b + c$ *(2005)*

20. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60º and when he retires 40 meters away from the tree the angle of elevation becomes 30º. The breadth of the river is

(a) 40 m (b) 30 m

(c) 20 m (d) 60 m. *(2004)*

21. The sides of a triangle are $\sin\alpha$, $\cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is

(a) 120º (b) 90º

(c) 60º (d) 150º. *(2004)*

22. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$ then the difference between the maximum and minimum values of u^2 is given by

(a) $(a + b)^2$ (b) $2\sqrt{a^2+b^2}$

(c) $2(a^2 + b^2)$ (d) $(a - b)^2$. *(2004)*

23. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -21/65$, and $\cos\alpha + \cos\beta = -27/65$, then the value of $\cos\frac{\alpha-\beta}{2}$ is

(a) $\frac{6}{65}$ (b) $\frac{3}{\sqrt{130}}$ (c) $-\frac{3}{\sqrt{130}}$ (d) $-\frac{6}{65}$. *(2004)*

24. If in a triangle ABC, $a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides a, b and c

(a) are in G.P. (b) are in H.P.

(c) satisfy $a + b = c$ (d) are in A.P. *(2003)*

25. In a triangle ABC, medians AD and BE are drawn. If $AD = 4$, $\angle DAB = \pi/6$ and $\angle ABE = \pi/3$, then the area of the ΔABC is

(a) 16/3 (b) 32/3

(c) 64/3 (d) 8/3. *(2003)*

26. The upper 3/4th portion of a vertical pole subtends an angle $\tan^{-1}(3/5)$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is

(a) 40 m (b) 60 m

(c) 80 m (d) 20 m. *(2003)*

27. The sum of the radii of inscribed and circumscribed circles for an n sides regular polygon of side a, is

(a) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ (b) $a\cot\left(\frac{\pi}{2n}\right)$

(c) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$ (d) $a\cot\left(\frac{\pi}{n}\right)$. *(2003)*

28. The trigonometric equation $\sin^{-1}x = 2\sin^{-1}a$, has a solution for

(a) all real values (b) $|a| < \frac{1}{2}$

(c) $|a| \geq \frac{1}{\sqrt{2}}$ (d) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$. *(2003)*

29. In a triangle with sides a, b, c, $r_1 > r_2 > r_3$ (which are the ex-radii) then

(a) $a > b > c$ (b) $a < b < c$

(c) $a > b$ and $b < c$ (d) $a < b$ and $b > c$. *(2002)*

30. $\cot^{-1}\left[(\cos\alpha)^{\frac{1}{2}}\right] + \tan^{-1}\left[(\cos\alpha)^{\frac{1}{2}}\right] = x$

then $\sin x =$

(a) 1 (b) $\cot^2(\alpha/2)$

(c) $\tan\alpha$ (d) $\cot(\alpha/2)$ *(2002)*

31. The number of solutions of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is

(a) 2 (b) 3 (c) 0 (d) 1. *(2002)*

Answer Key

1.	(d)	**2.**	(d)	**3.**	(a)	**4.**	(d)	**5.**	(c)	**6.**	(b)
7.	(c)	**8.**	(b)	**9.**	(c)	**10.**	(b)	**11.**	(a)	**12.**	(b)
13.	(d)	**14.**	(c)	**15.**	(a)	**16.**	(c)	**17.**	(d)	**18.**	(a)
19.	(c)	**20.**	(c)	**21.**	(a)	**22.**	(d)	**23.**	(c)	**24.**	(d)
25.	(*)	**26.**	(a)	**27.**	(a)	**28.**	(*)	**29.**	(a)	**30.**	(a)
31.	(b)										

Explanations

1. **(d) :** As x, y, z are in A.P. $\Rightarrow 2y = x + z$ … (i)

$\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are in A.P., then

$2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$

$$2\tan^{-1}y = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$

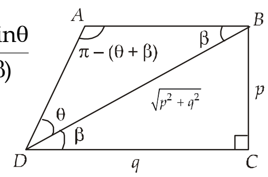
Thus $y^2 = xz$ … (ii)

From (i) and (ii), we get $x = y = z$.

Remark : $y \neq 0$ is implicit to make any of the choice correct.

2. **(d) :** Using sine rule in triangle ABD, we get

$$\frac{AB}{\sin\theta} = \frac{BD}{\sin(\theta+\beta)} \Rightarrow AB = \frac{\sqrt{p^2+q^2}\sin\theta}{\sin(\theta+\beta)}$$



As $\tan\beta = \frac{p}{q}$, we have

$\sin(\theta + \beta) = \sin\theta \cos\beta + \cos\theta \sin\beta$

$$= \sin\theta \cdot \frac{q}{\sqrt{p^2+q^2}} + \cos\theta \cdot \frac{p}{\sqrt{p^2+q^2}} = \frac{p\cos\theta + q\sin\theta}{\sqrt{p^2+q^2}}$$

We then get $AB = \dfrac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$

3. **(a) :** $\dfrac{\tan A}{1-\cot A} + \dfrac{\cot A}{1-\tan A}$

$$= \frac{\sin^2 A}{\cos A(\sin A - \cos A)} + \frac{\cos^2 A}{\sin A(\cos A - \sin A)}$$

$$= \frac{\sin^3 A - \cos^3 A}{(\sin A - \cos A)\cos A \sin A}$$

$$= \frac{(\sin A - \cos A)(\sin^2 A + \sin A\cos A + \cos^2 A)}{(\sin A - \cos A)\sin A\cos A}$$

$$= \frac{1+\sin A\cos A}{\sin A\cos A} = 1 + \sec A\,\mathrm{cosec}A$$

4. **(d) :** $3\sin P + 4\cos Q = 6$

$4\sin Q + 3\cos P = 1$

$\Rightarrow 16 + 9 + 24(\sin(P+Q)) = 37$

$\Rightarrow 24(\sin(P+Q)) = 12$

$$\Rightarrow \sin(P+Q) = \frac{1}{2} \Rightarrow \sin R = \frac{1}{2} \Rightarrow R = \frac{5\pi}{6} \text{ or } \frac{\pi}{6}$$

But if $R = \frac{5\pi}{6}$ then $P < \frac{\pi}{6}$ and then $3\sin P < \frac{1}{2}$

and so $3\sin P + 4\cos Q < \frac{1}{2} + 4\ (\neq 6)$ Thus, $R = \frac{\pi}{6}$.

5. **(c) :** $A = \sin^2 x + \cos^2 x$

We have $\cos^4 x \le \cos^2 x$

$\sin^2 x = \sin^2 x$

Adding $\sin^2 x + \cos^4 x \le \sin^2 x + \cos^2 x$

$\therefore A \le 1$.

Again $A = t + (1-t)^2 = t^2 - t + 1,\ t \ge 0$,

where minimum is 3/4

Thus $\frac{3}{4} \le A \le 1$.

6. **(b) :** $\cos(\alpha+\beta) = 4/5$ giving $\tan(\alpha+\beta) = 3/4$

Also $\sin(\alpha-\beta) = 5/13$ given $\tan(\alpha-\beta) = 5/12$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)} = \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}} = \frac{36+20}{48-15} = \frac{56}{33}$$

7. **(c) :** We have $\frac{r}{R} = \cos\frac{\pi}{n}$,

Let $\cos\frac{\pi}{n} = \frac{1}{\sqrt{2}}$.

Thus we get $\frac{\pi}{n} = \frac{\pi}{4}$

i.e., $n = 4$, acceptable.

$\cos\frac{\pi}{n} = \frac{1}{2} \Rightarrow \frac{\pi}{n} = \frac{\pi}{3}$. $\therefore n = 3$, acceptable.

$\cos\frac{\pi}{n} = \frac{\sqrt{3}}{2} \Rightarrow \frac{\pi}{n} = \frac{\pi}{6}$. $\therefore n = 6$, acceptable.

But $\cos\frac{\pi}{n} = \frac{2}{3}$ will produce no value of n.

As $\frac{1}{2} < \frac{2}{3} < \frac{1}{\sqrt{2}} \Rightarrow \cos\frac{\pi}{3} < \cos\frac{\pi}{n} < \cos\frac{\pi}{4}$

$\Rightarrow \frac{\pi}{3} > \frac{\pi}{n} > \frac{\pi}{4} \Rightarrow 3 < n < 4$ (impossible)

8. **(b) :** $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -3/2$

$\Rightarrow (\cos\beta\cos\gamma + \sin\beta\sin\gamma) + (\cos\gamma\cos\alpha + \sin\gamma\sin\alpha) + (\cos\alpha\cos\beta + \sin\alpha\sin\beta) = -3/2$

$\Rightarrow 2(\cos\beta\cos\gamma + \cos\gamma\cos\alpha + \cos\alpha\cos\beta) + 2(\sin\beta\sin\gamma + \sin\gamma\sin\alpha + \sin\alpha\sin\beta) + 3 = 0$

$\Rightarrow \{\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2(\cos\alpha\cos\beta + \cos\beta\cos\gamma + \cos\gamma\cos\alpha)\} + \{\sin^2\alpha + \sin^2\beta + \sin^2\gamma + 2(\sin\alpha\sin\beta + \sin\beta\sin\gamma + \sin\gamma\sin\alpha)\} = 0$

$\Rightarrow (\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2 = 0$

Which yields simultaneously

$\cos\alpha + \cos\beta + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$

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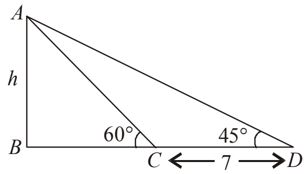
9. **(c) : 1st Solution :**

Let height of the pole AB be h. Then

$BC = h \cot 60° = h/\sqrt{3}$

$BD = h \cot 45° = h$

As $BD - BC = CD$



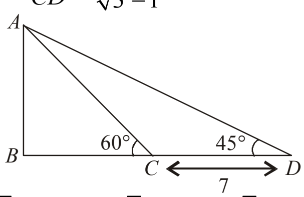
$\Rightarrow\ h - \dfrac{h}{\sqrt{3}} = 7 \ \Rightarrow\ h(\sqrt{3}-1) = 7\sqrt{3}$

$\Rightarrow\ h = \dfrac{7\sqrt{3}}{\sqrt{3}-1} = \dfrac{7\sqrt{3}(\sqrt{3}+1)}{2} = \dfrac{7\sqrt{3}}{2}(\sqrt{3}+1)\text{m}$

2nd Solution :

We use the fact that the ratio of distance of B from D and that of B from C i.e. BD to BC is $\sqrt{3}:1$

$\dfrac{BD}{BC} = \sqrt{3}$, so that $\dfrac{BD}{CD} = \dfrac{\sqrt{3}}{\sqrt{3}-1}$



Then $BD = \dfrac{\sqrt{3}}{\sqrt{3}-1}CD = \dfrac{\sqrt{3}}{\sqrt{3}-1}\cdot 7 = \dfrac{7\sqrt{3}}{2}(\sqrt{3}+1)$

As $AB = BD$, the height of the pole $= \dfrac{7\sqrt{3}}{2}(\sqrt{3}+1)\text{m}$

10. **(b) :** $\cot\left(\operatorname{cosec}^{-1}\dfrac{5}{3} + \tan^{-1}\dfrac{2}{3}\right)$

$= \cot\left(\tan^{-1}\dfrac{3}{4} + \tan^{-1}\dfrac{2}{3}\right)$

$= \cot\left(\tan^{-1}\dfrac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right) = \cot\left(\tan^{-1}\dfrac{17}{6}\right) = \dfrac{6}{17}$

11. **(a) :** OP = Tower

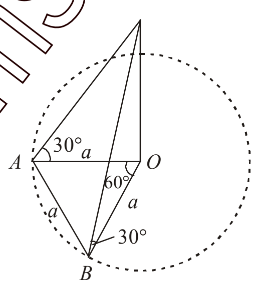
OAB is equilateral triangle

$\therefore\ OA = OB = AB = a$

In $\Delta\, AOP$,

$\tan 30° = \dfrac{OP}{OA}$

$\Rightarrow\ OP = \dfrac{a}{\sqrt{3}}$



12. **(b) :** $f(x)$ is defined if $-1 \le \dfrac{x}{2} - 1 \le 1$ and $\cos x > 0$

or $0 \le x \le 4$ and $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$ $\quad\therefore\ 0 \le x < \dfrac{\pi}{2}$.

13. **(d) :** $\sin^{-1}\left(\dfrac{x}{5}\right) + \sin^{-1}\left(\dfrac{4}{5}\right) = \dfrac{\pi}{2}$

$\Rightarrow\ \sin^{-1}\left(\dfrac{x}{5}\right) = \dfrac{\pi}{2} - \sin^{-1}\left(\dfrac{4}{5}\right)$

$\dfrac{x}{5} = \sin\left(\dfrac{\pi}{2} - \sin^{-1}\left(\dfrac{4}{5}\right)\right)$

$\dfrac{x}{5} = \cos\left(\sin^{-1}\dfrac{4}{5}\right) = \cos\left(\cos^{-1}\dfrac{3}{5}\right) = \dfrac{3}{5} \Rightarrow x = 3.$

14. **(c) :** $0 < x < \pi$

Given $\cos x + \sin x = \dfrac{1}{2}$

$\Rightarrow\ 1 + \sin 2x = \dfrac{1}{4}$ (By squaring both sides)

$\dfrac{2\tan x}{1+\tan^2 x} = \dfrac{-3}{4}$

$\Rightarrow\ 3\tan^2 x + 8\tan x + 3 = 0$

$\tan x = \dfrac{-8 \pm \sqrt{64-36}}{6} = \dfrac{-4 \pm \sqrt{7}}{3}$

$\because\ \tan x < 0 \Rightarrow \tan x = \dfrac{-4-\sqrt{7}}{8}$

15. **(a) :** $2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow\ \sin x = \dfrac{1}{2}, \sin x \ne -3$

there $\sin x = \dfrac{1}{2}$, we know that each trigonometrical function assumes same value twice in $0 \le x \le 360°$.

In our problem $0° \le x \le 540°$. So number of values are 4 like 30°, 150°, 390°, 510°.

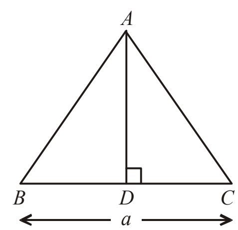
16. **(c) :** Altitude from A to BC is AD

Area of $\Delta = \dfrac{1}{2}AD \times BC$

$\therefore\ \dfrac{2\cdot \text{Area of } \Delta}{a} = AD$

$\therefore$ Altitudes are in H.P.

$\therefore\ \dfrac{2\Delta}{a}, \dfrac{2\Delta}{b}, \dfrac{2\Delta}{c} \in \text{HP}$



$\Rightarrow\ \dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c} \in \text{H.P.} \quad \Rightarrow\ a, b, c \in \text{A.P.}$

17. **(d) :** Using $\cos^{-1}A - \cos^{-1}B$

$= \cos^{-1}\left(AB + \sqrt{(1-A^2)}\sqrt{(1-B^2)}\right)$

$\therefore\ \cos^{-1}x - \cos^{-1}\dfrac{y}{2} = \alpha$

$\Rightarrow\ \dfrac{xy}{2} + \sqrt{1-x^2}\sqrt{1-\dfrac{y^2}{4}} = \cos\alpha$

$\Rightarrow\ \left(\cos\alpha - \dfrac{xy}{2}\right)^2 = (1-x^2)\left(1-\dfrac{y^2}{4}\right)$

$\Rightarrow\ 4x^2 - 4xy\cos\alpha + y^2 = 4(1-\cos^2\alpha) = 4\sin^2\alpha.$

18. **(a) :** $\dfrac{c}{\sin C} = 2R$

$\therefore\ c = 2R$ ...(A)($\because C = 90°$) and

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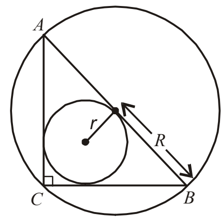
$\tan\frac{C}{2} = \frac{r}{s-c}$



$\therefore\ r = (s - c)$

$\left(\tan\frac{C}{2} = \tan 45° = 1\right)$

$= \frac{a+b+c}{2} - c\ ;\ 2r = a + b - c$ ...(B)

adding (A) and (B) we get $2(r + R) = a + b$.

19. (c) : $\angle R = 90°\ \therefore\ \angle P + \angle Q = 90°$

$\therefore\ \frac{P}{2} = \frac{90}{2} - \frac{Q}{2},\ \frac{P}{2} = 45 - \frac{Q}{2}$

$\Rightarrow\ \tan\frac{P/2}{1} = \frac{1-\tan Q/2}{1+\tan Q/2}$

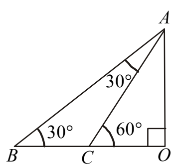
$\Rightarrow\ \tan\frac{P}{2} + \tan\frac{Q}{2} = 1 - \tan\frac{P}{2}\cdot\tan\frac{Q}{2}$

$\Rightarrow\ -\frac{b}{a} = 1 - \frac{c}{a}$

$\left(\because\ \tan\frac{P}{2},\ \tan\frac{Q}{2}\text{ are roots of } ax^2 + bx + c = 0\right)$

$\Rightarrow\ \frac{c-b}{a} = 1 \Rightarrow c = a + b.$

20. (c) : Breadth of river $OC = AC\cos 60°$



$= 40\cos 60°$

21. (a) : If $a^2 = \sin^2\alpha,\ b^2 = \cos^2\alpha,\ c^2 = 1 + \sin\alpha\cos\alpha$

then $\cos c = \frac{-\sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha}\ \therefore\ \cos c = -1/2$

22. (d) : $u^2 = a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) +$

$2\sqrt{\left(a^4+b^4\right)\sin^2\theta\cos^2\theta + a^2b^2\left(\sin^4\theta+\cos^4\theta\right)}$

$= a^2 + b^2 + 2\sqrt{a^2b^2 + \left(a^2-b^2\right)^2\sin^2\theta\cos^2\theta}$

$= a^2 + b^2 + 2 + \sqrt{a^2b^2 + \left(\frac{a^2-b^2}{2}\right)\sin 2\theta}$

$\therefore\ u^2$ will be maximum or minimum according as $\theta = \pi/4$ or $\theta = 0°$

$\therefore$ Max. $u^2 = 2(a^2 + b^2)$ and

Min. $u^2 = a^2 + b^2 + 2ab = (a + b)^2$

Now Maximum u^2 – Minimum u^2

$= 2(a^2 + b^2) - (a^2 + b^2 + 2ab)$

$= a^2 + b^2 - 2ab = (a - b)^2$

23. (c) : $\sin\alpha + \sin\beta = -\frac{21}{65}$

and $\cos\alpha + \cos\beta = -\frac{27}{65}$

by squaring and adding we get

$2(1 + \cos\alpha\cos\beta + \sin\alpha\sin\beta) = \frac{(21)^2+(27)^2}{(65)^2}$

$2[1 + \cos(\alpha - \beta)] = \frac{1170}{(65)^2}$

$\cos^2\frac{(\alpha-\beta)}{2} = \frac{1170}{4\times65\times65} = \frac{130\times9}{(130)\times(130)} = \frac{9}{130}$

$\therefore\ \cos\frac{\alpha-\beta}{2} = \frac{3}{\sqrt{130}}$

As $\pi < \alpha - \beta < 3\pi$ then $\cos\left(\frac{\alpha-\beta}{2}\right)$ = negative

24. (d) : $2a\cos^2\frac{C}{2} + 2c\cos^2 A/2 = 3b$ (from given)

$\Rightarrow\ a(1 + \cos C) + c(1 + \cos A) = 3b$

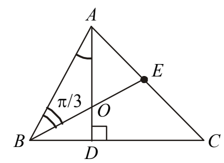
$\Rightarrow\ a + c + a\cos C + c\cos A = 3b$

$(a\cos C + c\cos A = b$ projection formula)

$\Rightarrow\ a + c + b = 3b$

$\Rightarrow\ a + c = 2b$

25. (*) : $\frac{OB}{AO} = \tan 30°$

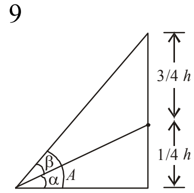


$\Rightarrow\ OB = \frac{OA}{\sqrt3} = \frac{8\sqrt3}{9}$

Area of triangle $ADB = \frac{1}{2}\times\frac{8\sqrt3}{9}\times 4 = \frac{16\sqrt3}{9}$

Area of triangle $ABC = 2\times\frac{16\sqrt3}{9} = \frac{32\sqrt3}{9}$

26. (a) : $\alpha = A + \beta$



$\therefore\ \beta = A - \alpha$

$\tan\beta = \frac{\tan A - \tan\alpha}{1 - \tan A\tan\alpha}$

$\Rightarrow\ \frac{3}{5} = \frac{\frac{h}{40} + \left(-\frac{h}{160}\right)}{1 - \left(\frac{h}{40}\right)\left(-\frac{h}{160}\right)}$

$\Rightarrow\ h^2 - 200h + 6400 = 0$

$\Rightarrow\ (h - 40)(h - 160) = 0$

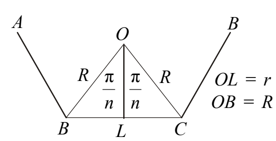
$\Rightarrow\ h = 40$ or $h = 160$

27. (a) : If R be the radius of circumcircle of regular polygon of n sides, and r be the radius of inscribed circle then

$R = \frac{a}{2}\ \text{cosec}\ \frac{\pi}{2n}$ and $r = \frac{a}{2}\cot\frac{\pi}{n}$

$\therefore\ R + r = \frac{a}{2}\left(\text{cosec}\frac{\pi}{n} + \cot\frac{\pi}{n}\right)$

$= \frac{a}{2}\left(\frac{1+\cos\frac{\pi}{n}}{\sin\frac{\pi}{n}}\right) = \frac{a}{2}\cot\frac{\pi}{2n}$



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28. **(*) :** $\sin^{-1}x = 2\sin^{-1}a$

$\Rightarrow -\frac{\pi}{2} \le 2\sin^{-1}a \le \frac{\pi}{2}$ $\quad \left[\because \sin^{-1}x = 2\sin^{-1}a \text{ and } -\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}\right]$

$\Rightarrow -\frac{\pi}{4} \le \sin^{-1}a \le \frac{\pi}{4}$

$\Rightarrow \sin\left(-\frac{\pi}{4}\right) \le a \le \sin\left(\frac{\pi}{4}\right)$

$\Rightarrow -\frac{1}{\sqrt{2}} \le a \le \frac{1}{\sqrt{2}}$

$\Rightarrow |a| \le \frac{1}{\sqrt{2}}$ No choice is matched.

29. **(a) :** As $r_1 > r_2 > r_3$

$\Rightarrow \frac{\Delta}{s-a} > \frac{\Delta}{s-b} > \frac{\Delta}{s-c}$

$\Rightarrow \frac{s-a}{\Delta} < \frac{s-b}{\Delta} < \frac{s-c}{\Delta}$

$\Rightarrow a > b > c$

30. **(a) :** Using $\tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2} = x$

$\therefore \quad \sin x = \sin\frac{\pi}{2} = 1$

31. **(b) :** $\tan x + \sec x = 2\cos x$

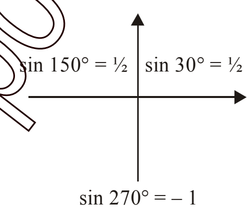
$1 + \sin x = 2\cos^2 x$

$1 + \sin x = 2(1 - \sin^2 x)$

$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$

$\Rightarrow (2\sin x - 1)(1 + \sin x) = 0$

$\Rightarrow \sin x = \frac{1}{2}, \sin x = -1$



so there are three solution like $x = 30°, 150°, 270°$

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CHAPTER 18

MATHEMATICAL LOGIC

1. Consider :
Statement-1 : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.
Statement-2 : $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a tautology.
(a) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1. *(2013)*

2. The negation of the statement "If I become a teacher, then I will open a school", is
(a) Neither I will become a teacher nor I will open a school.
(b) I will not become a teacher or I will open a school.
(c) I will become a teacher and I will not open a school.
(d) Either I will not become a teacher or I will not open a school. *(2012)*

3. Consider the following statements
P : Suman is brilliant
Q : Suman is rich
R : Suman is honest
The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as
(a) $\sim Q \leftrightarrow \sim P \wedge R$
(b) $\sim (P \wedge \sim R) \leftrightarrow Q$
(c) $\sim P \wedge (Q \leftrightarrow \sim R)$
(d) $\sim (Q \leftrightarrow (P \wedge \sim R))$
(2011)

4. Let S be a non-empty subset of R. Consider the following statement:
P : There is a rational number $x \in S$ such that $x > 0$.
Which of the following statements is the negation of the statement P ?
(a) There is a rational number $x \in S$ such that $x \leq 0$.
(b) There is no rational number $x \in S$ such that $x \leq 0$.
(c) Every rational number $x \in S$ satisfies $x \leq 0$.
(d) $x \in S$ and $x \leq 0 \Rightarrow x$ is not rational. *(2010)*

5. **Statement-1 :** $\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.
Statement-2 : $\sim (p \leftrightarrow \sim q)$ is a tautology.
(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(b) Statement-1 is true, Statement-2 is false
(c) Statement-1 is false, Statement-2 is true
(d) Statement-1 is true, Statement-2 is true; Statement-2 is correct explanation for Statement-1 *(2009)*

6. The statement $p \rightarrow (q \rightarrow p)$ is equivalent to
(a) $p \rightarrow (p \leftrightarrow q)$
(b) $p \rightarrow (p \rightarrow q)$
(c) $p \rightarrow (p \vee q)$
(d) $p \rightarrow (p \wedge q)$ *(2008)*

7. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the statement "x is a rational number iff y is a transcendental number".
Statement-1 : r is equivalent to either q or p.
Statement-2 : r is equivalent to $\sim (p \leftrightarrow \sim q)$.
(a) Statement-1 is true, Statement-2 is false
(b) Statemen-1 is false, Statement-2 is true
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1 *(2008)*

Answer Key

1. (a)	2. (c)	3. (d)	4. (c)	5. (b)	6. (c)
7. (a)					

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Explanations

1. **(a) : 1st solution :** Let's prepare the truth table for the statements.

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \wedge q$	$(p \wedge \sim q) \wedge (\sim p \wedge q)$
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	F	F

Then Statement-1 is fallacy.

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to p$	$(p \to q) \to (\sim q \to p)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Then Statement-2 is tautology.

2nd solution : $\sim (\sim p \vee q) \wedge \sim (\sim q \vee p)$

$\equiv \sim ((\sim p \vee q) \vee (\sim q \vee p)) \equiv \sim ((p \to q) \vee (q \to p)) \equiv \sim T$

Thus Statement-1 is true because its negation is false.

$((p \to q) \to (\sim q \to \sim p) \wedge ((\sim q \to \sim p) \to (p \to q))$

$= ((\sim p \vee q) \to (q \vee \sim p) \wedge ((q \vee \sim p) \to (\sim p \vee q))$

$\equiv T \wedge T \equiv T$. Then Statement-2 is true.

2. **(c) :** The given statement is

"If I become a teacher, then I will open a school"

Negation of the given statement is

" I will become a teacher and I will not open a school"

$(\because \sim (p \to q) = p \wedge \sim q)$

3. **(d) :** The statement can be written as $P \wedge \sim R \Leftrightarrow Q$

Thus the negation is $\sim (Q \leftrightarrow P \wedge \sim R)$

4. **(c) :** The given statement is

P : at least one rational $x \in S$ such that $x > 0$.

The negation would be : There is no rational number $x \in S$ such that $x > 0$

which is equivalent to all rational numbers $x \in S$ satisfy $x \leq 0$.

5. **(b) :** Let's prepare the truth table

p	q	$\sim q$	$p \leftrightarrow q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$
T	T	F	T	F	T
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	T	F	T

As the column for $\sim(p \leftrightarrow \sim q)$ and $(p \leftrightarrow q)$ is the same, we conclude that $\sim(p \leftrightarrow \sim q)$ is equivalent to $(p \leftrightarrow q)$.

$\sim(p \leftrightarrow \sim q)$ is NOT a tautology because it's statement value is not always true.

6. **(c) :** Let's simplify the statement

$p \to (q \to p) = \sim p \vee (q \to p) = \sim p \vee (\sim q \vee p)$

$= - p \vee p \vee \sim q = p \to (p \vee q)$

7. **(a) :** The given statement $r \equiv \sim p \leftrightarrow q$

The Statement-1 is $r_1 \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$

The Statement-2 is

$$r_2 \equiv \sim (p \leftrightarrow \sim q) = (p \wedge q) \vee (\sim q \wedge \sim p)$$

we can establish that $r = r_1$

Thus Statement-1 is true but Statement-2 is false.

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