PHYSICS

STUDY MATERIAL

ELECTROSTATICS

AIEEE

NARAYANA INSTITUTE OF CORRESPONDENCE COURSES

FNS HOUSE, 63 KALU SARAI MARKET

SARVAPRIYA VIHAR, NEW DELHI-110016

PH.: (011) 32001131/32/50 • FAX : (011) 41828320

Website : www.narayanaicc.com

E-mail : info@narayanaicc.com

PREFACE

Dear Student,

Heartiest congratulations on making up your mind and deciding to be an engineer to serve the society.

As you are planning to take various Engineering Entrance Examinations, we are sure that this **STUDY PACKAGE** is going to be of immense help to you.

At NARAYANA we have taken special care to design this package according to the **Latest Pattern of AIEEE,** which will not only help but also guide you to compete for AIEEE & other State Level Engineering Entrance Examinations.

The salient features of this package include :

- Power packed division of units and chapters in a scientific way, with a correlation being there.
- Sufficient number of solved examples in Physics, Chemistry & Mathematics in all the chapters to motivate the students attempt all the questions.
- All the chapters are followed by various types of exercises (Level-I, Level-II, Level-III and Questions asked in AIEEE and other Engineering Exams).

These exercises are followed by answers in the last section of the chapter. *This package will help you to know* what to study, how to study, time management, your weaknesses and improve your performance.

We, at NARAYANA, strongly believe that quality of our package is such that the students who are not fortunate enough to attend to our Regular Classroom Programs, can still get the best of our quality through these packages.

We feel that there is always a scope for improvement. We would welcome your suggestions & feedback.

Wish you success in your future endeavours.

THE NARAYANA TEAM

ACKNOWLEDGEMENT

While preparing the study package, it has become a wonderful feeling for the NARAYANA TEAM to get the wholehearted support of our Staff Members including our Designers. They have made our job really easy through their untiring efforts and constant help at every stage.

We are thankful to all of them.

THE NARAYANA TEAM

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ELECTROSTATICS

ELECTROSTATICS

AIEEE Syllabus

Electric charge – its unit and conservation, Coulomb's law, dielectric constant, electric field, lines of force, field due to dipole and its behaviour in a uniform electric field, electric flux, Gauss's theorem and its applications. Electric potential, potential due to a point charge. Conductors and insulators, distribution of charge on conductors.

CONTENTS

- *Electric charge*
- *Coulomb's law*
- *Electrostatics field*
- *Electrostatics lines of force; Properties*
- *Electric field due to a point charge*
- *Superposition of Electric field*
- *Electric field intensity on the axis of a uniformly charged ring.*
- *Electric flux*
- *Gauss's Law.*
- *Application of Gauss's law*
- *Electric potential and potential energy*
- *Equipotential surface*

INTRODUCTION

When a glass rod is rubbed with silk, this acquires power to attract light bodies such as small pieces of paper. The bodies which acquire this power are said to be charged. If these charges do not move they are called static charges and the branch of physics which deals with static charges is called electrostatics.

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ELECTRIC CHARGE

Charge is a scalar quantity which is categorised into two types.

a) Positive charge (anciently called Vitreous)

b) Negative charge (anciently called Resinous)

A body having no charge, is said to be neutral in nature i.e. on a neutral body the sum of positive charges is equal to the sum of negative charges.

The positive charge means deficiency of electrons, whereas the negative charge on a body implies excess of electrons.

The S.I. unit of charge is coulomb (C).

I. UNITS OF CHARGE

a) S.I. unit of charge is coulomb (C). One coulomb of charge is that charge which when placed at rest in vacuum at a distance of one metre from an equal and similar stationary charge repels it and is repelled by it with a force of $9\times10^9\ \text{N}$.

b) CGS unit of charge is stat coulomb

1 coulomb = $3\times10^9\ \text{esu}$ of charge $=\frac{1}{10}$ emu of charge.

i.e. 1 coulomb $=3\times10^9$ stat coulomb $=\frac{1}{10}$ abcoulomb

c) Practical units of charge are amp × hr = 3600 coulomb and faraday $(=96500\ \text{C})$

d) Smallest unit of charge is stat coulomb and largest is faraday.

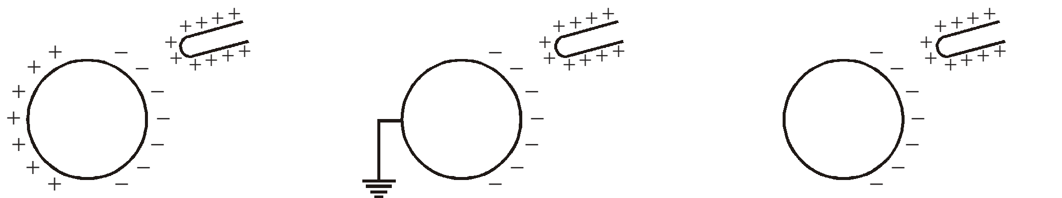
II. METHODS OF CHARGING

a) By Friction : In friction when two bodies are rubbed together electrons are transferred from one body to other. As a result one body becomes positively charged while the other negatively charged.

Examples : When glass rod is rubbed with silk, the rod becomes positively charged while the silk negatively charged.

Clouds also become charged by friction

b) By Induction : If a charged body of same sign is brought near a neutral body charged body will attract charge of opposite sign and repel charge of same sign present in neutral body. The nature as induced charge is always opposite to that of inducing charge. Charging of body by Induction is shown as :



Charged body brought near uncharged body | Uncharged body connected to earth | Uncharged body disconnected from earth

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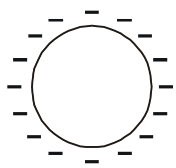
c) **By Conduction :** If a charged body is in direct contact with an uncharged body, charge flows from former to latter till both are at same potential. This flow of charges is due to mutual repulsions between same kind of charges on charged body.



Charging body
is removed

III. PROPERTIES OF ELECTRIC CHARGE

a) Like point charges repel each other and unlike charges attract each other.

b) **Charge Conservation**

The algebraic sum of all the charges in an isolated system is a constant. In crude language we can say that charge can neither be created nor be destroyed, however it can simply be transferred from one body to the other.

c) **Relativistic Invariance**

Charge on a body is relativistically invariant. i.e. charge on the body at rest equals the charge on the body at relativistic speeds. However charge density is not relativistically invariant. Mathematically

$$(q)_{\text{at rest}} = (q)_{\text{in motion}}$$

or $$(q)_{\text{at rest}} = (q)_{\text{at relativistic speeds}}$$

d) **Charge Quantisation**

Charge on a body *q* must always exist as an integral multiple of some fundamental unit of charge (called electronic charge) e , where $e = 1.6\times10^{-19}\ \text{C}$.

Mathematically, $q = \pm ne$, $n = 1, 2, 3, ...$

From here we conclude that a neutral body can have $+1\ \text{C}$ of charge when it falls deficient of 6.25×10^{18} electrons.

e) A charged body can attract light uncharged body. (Due to charging by induction)

f) Charges are always added algebraically.

For example, if a neutral body is first given a charge of $+5\ \text{C}$ and subsequently a charge of $-7\ \text{C}$, then it will finally have a charge of $+5\ \text{C} - 7\ \text{C} = -2\ \text{C}$.

EXAMPLES BASED ON ELECTRIC CHARGE

Example - 1 Calculate protonic charge in 100 cc. of water.

Solution : 1 cc = 1 gm for water as density = 1000 kg/m^3

Now no of atoms in 18 gm (atm weight) = 6.023×10^{23} and each molecule of H_2O contains 10 proton (8 of oxygen + 2 of H_2) so No. of proton in 100 gm water

$$= \left(\frac{6.023\times10^{23}}{18}\times100\right)\times10 = 3.3\times10^{25}$$

Hence protonic charge = $3.3 \times 10^{25} \times 1.6 \times 10^{-19} = 5.4 \times 10^{6}$ C

Example - 2 A copper sphere of mass 2.0 g contains about 2 x 10^{22} atoms. Atomic weight of copper is 64 and atomic number 29.

(i) How many electorns must be removed form the sphere to give it a charge of +2μC?

(ii) Determine the fraction of electrons removed.

(iii) Is there any change in mass of sphere when it is given positive charge?

Solution : (i) Number of electrons to be removed $n=\frac{Q}{e}=\frac{2\times10^{-6}}{1.6\times10^{-19}}=1.25\times10^{13}$

(ii) Total number of electrons in the sphere = 29 x 2 x 10^{22} = 5.8 x 10^{23}

Fraction of electrons removed $=\frac{1.25\times10^{13}}{5.8\times10^{23}}=2.16\times10^{-11}$

Thus 2.16 x 10^{-9}% of electrons are to be removed to give the sphere a charge of 2μC.

(iii) Yes mass decreases, when body is given a positive charge. Decrease of mass $\Delta m=9\times10^{-31}\times1.25\times10^{13}=1.125\times10^{-17}$ kg .

COULOMB'S LAW

The magnitude of the force (*F*) of attraction or repulsion between two point charges q_1 and q_2 placed in vacuum at separation *r* is

a) directly proportional to the product of the magnitude of the two charges.

$$F \propto q_1q_2 \qquad ...(i)$$

b) inversely proportional to the square of the distance of separation between them.

$$F \propto \frac{1}{r^2} \qquad ...(ii)$$

COULOMB'S LAW IN VECTOR FORM

Consider two point charges q_1 and q_2 separated by a distance *r*. If $q_1q_2 > 0$ i.e. if both q_1 and q_2 are +ve or both q_1 and q_2 are negative then the charges repel each other otherwire they attract each other.

If $\vec{F}_{12}$ is the force exerted on charge q_1 by charge q_2.

$$F_{12}=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{r}_{21} \qquad = \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{r^3}.\vec{r}_{21}$$

If $\vec{F}_{21}$ is the force exerted on q_2 due to q_1

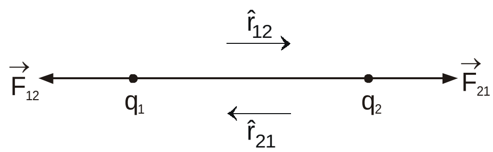
then
$$\vec{F}_{21}=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{r}_{12} \qquad = \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{r^3}.\vec{r}_{12}$$

Both $\hat{r}_{21}$ and $\hat{r}_{12}$ have same magnitude i.e. unity and are oppositely directed

$$\vec{F}_{21}=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}(-\hat{r}_{21})=-\vec{F}_{12}$$

$$\vec{F}_{21}=-\vec{F}_{12}$$

So the forces exerted by charges on each other are equal in magnitude opposite in direction



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PROBLEM SOLVING TRICKS

1. While calculating the force between the two charges from Coulomb's Law, never take into account, the sign of the two charges. The sign just indicates the nature of the force.
2. $\vec{F}_{12}$ is the force on charge 1 due to 2. $\vec{F}_{21}$ is the force on charge 2 due to 1. $\vec{F}_{ij}$ is the force on charge *i* due to *j*.
3. When we are to calculate the force on charge 1 due to charge 2, then we assume charge 2 to be fixed and vice versa, unless and otherwise stated.

   $$|\vec{F}_{12}| = m_1a_1$$

   $$|\vec{F}_{21}| = m_2a_2$$

   i.e. $m_1a_1 = m_2a_2$ (by Newton's Third Law)

   where m_1 and m_2 are the masses of charges q_1 and q_2 respectively.
4. If we are to calculate the force on charge q_0, due to assembly of charges q_1, q_2, ... q_n, then we have

   $$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + \ldots.. + \vec{F}_{0n}$$

   where $\vec{F}_{01}$ is calculated as if only q_0 and q_1 are present and all others are absent and so on for other combinations. This principle is called the **Principle of Superposition**.

Note : When two identical bodies having charges q_1 and q_2 respectively are brought in contact and separated, then the charge on each body is $\frac{q_1+q_2}{2}$.

EXAMPLES BASED ON ELECTROSTATICS FORCE

Example - 3 Atomic number of copper is 29, its atomic weight is 63.5 gm/mole. If two pieces of copper having weight 10 gm are taken and on one of the pieces 1 electron per 1000 atom is transferred to the other piece and there after this these pieces are placed 10 cm apart, then what will be the coulomb force between them.

Solution : In 1 mole copper (63.5 gm) there are 6 × 10^{23} atoms (N_A = Avogadro number = 6 × 10^{23} atoms)

Number of atoms = $\frac{6\times10^{23}\times10}{63.5}$ = 9.45 × 10^{22}

For every 1000 atoms, 1 electron is transferred therefore total number of transferred electron is

$$= \frac{9.45\times10^{22}}{1000} = 9.45\times10^{19}$$

Therefore charge on one piece is 9.45 × 10^{19} e and on the other will be (9.45 × 10^{19}e)

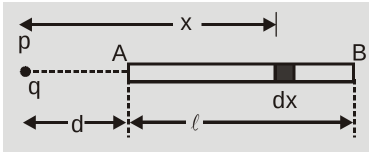
Force when they are kept 10 cm apart

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{(9.45\times10^{19})^2e^2}{(10\times10^{-2})^2} = \frac{9\times10^9\times(9.45\times10^{19})^2\times(1.6\times10^{-19})^2}{(10\times10^{-2})^2} = 2\times10^{14}\text{ N}$$

Example - 4 On an insulated rod having length L, Q charge is evenly distributed and a point charge q is placed at a distance d from one of its ends as shown. What will be the total electric force on q.

Solution : Considering a small element dx of the rod at a distance x from q, force on q due to this small element



$$dF = \frac{qdQ}{4\pi\epsilon_0 x^2}$$

where dQ is charge in small element dx and $dQ = \left(\frac{Q}{L}\right)dx$

$$\therefore\ F = dF = \frac{qQdx}{4\pi\epsilon_0 Lx^2} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{L}\int_d^{d+L}\frac{dx}{x^2}$$

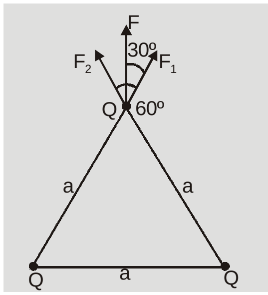
$$= \frac{1}{4\pi\epsilon_0}\frac{qQ}{L}\left[\frac{-1}{x}\right]_d^{d+L}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{qQ}{L}\left[\frac{1}{d} - \frac{1}{d+L}\right]$$

$$= \frac{qQ}{4\pi\epsilon_0 d(d+L)}$$

Example - 5 Three equal charges (Q, each) are placed on the vertices of an equilateral triangle of side a. What is the resultant force on any one charge due to the other two?

Solution : The charges are shown in fig. The resultant force



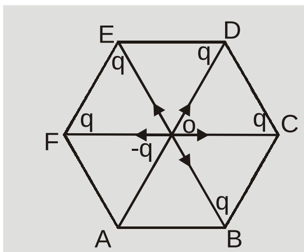
$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 60^o}$$

with $F_1 = F_2 = kQ^2/a^2$

$$F = \frac{\sqrt{3}kQ^2}{a^2}$$

From symmetry the direction is as shown along y-axis.

Example - 6 Five point charges, each of value +q are palced on five vertices of a regular hexagon of side L m. What is the magnitude of the force on a point charge of value -q coloumb placed at the centre of the hexagon?



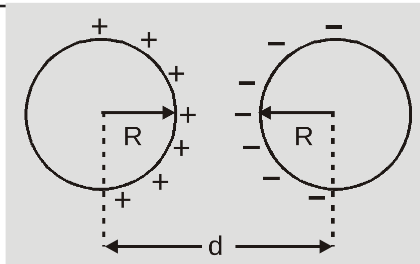
Solution : If there had been a sixth charge +q at the remaining vertex of hexagon, force due to all the six charges on -q at O will be zero (as the forces due to individual charges will balance each other), i.e., $\vec{F}_R = 0$.

Now If $\vec{f}$ is the force due to sixth charge and $\vec{F}$ due to remaining five charges, $\vec{F} + \vec{f} = 0$

i.e. $\vec{F} = -\vec{f}$ or,

$$F = f = \frac{1}{4\pi\varepsilon_0}\frac{q\ x\ q}{L^2} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{L}\right]^2$$

Example - 7 Two charged spheres of radius 'R' are kept at a distance 'd' (d > 2R). One has a charge +q and the other -q. The force between them will be-



[1] $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}$ [2] $> \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}$

[3] $< \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}$ [4] None of these

Solution : [2] Redistribution of charge will take place due to mutual attraction and hence effective distance will be less than d.

ELECTROSTATIC FIELD $(\vec{E})$

The region of space around a source charge (q) in which it can exert a force on a test charge (q_0).

Mathematically, electric field is the force experienced per unit test charge q_0 placed in the electrostatic influence of source charge q.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Electric field strength is a vector quantity directed away from a positive charge and towards the negative charge. SI unit of electric field is newton/coulomb (NC^{-1}) or volt/metre(Vm^{-1}).
The dimensional formula for E is $MLT^{-3}A^{-1}$.

Since $\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{r}$

$$\Rightarrow \quad \vec{E} = \frac{\vec{F}}{q_0} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

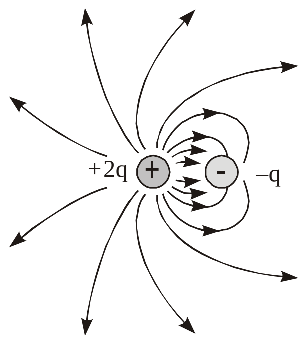
is the electric field due to a source point charge q at a distance r from it.

ELECTROSTATIC LINES OF FORCE : PROPERTIES

A line of force is an imaginary path straight or curved such that the tangent to it at any point gives the direction of electrostatic field at that point. A field line is an imaginary line along which a unit positive charge would move when set free. The lines of force are drawn such that the number of lines per unit area of cross-section, (area held normally to the field lines) is proportional to magnitude of $\vec{E}$.

I. PROPERTIES OF FIELD LINES

a) Field lines always come out of positive charge and enter the negative charge.
b) Field lines never cross each other.
c) Field lines never form closed loops.
d) Field lines are always directed from higher potential to lower potential.
e) Field lines never exist inside a conductor.
f) If N_1 is the number of field lines coming out of a charge q_1 and N_2 is the number of field lines entering q_2, then $\frac{|q_1|}{|q_2|} = \frac{N_1}{N_2}$



The electric field lines for a point charge +2q and a second point charge –q. Note that two lines leave the charge +2q for every one that terminates on –q.

g) If N_1 is the number of field lines coming out of a charge q_1 and N_2 is the number of field lines coming out of charge q_2,

then $$\frac{q_1}{q_2} = \frac{N_1}{N_2}$$

This relation also exists if field lines are entering both the charges

h) Tangent to field line at a point gives the direction of field at that point.

i) Field lines exhibit longitudinal (length wise) contraction, thus indicating that unlike charges attract each other. (See Figure 1)

j) Field lines exhibit lateral (sideways) expansion, thus indicating that like charges repel each other. (See Figure 2)

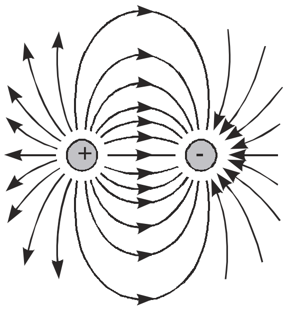


Figure 1 : The electric field lines for two equal and opposite point charges of an electric dipole. Note that the number of lines that leave the positive charge equals the number that terminate at the negative charge.

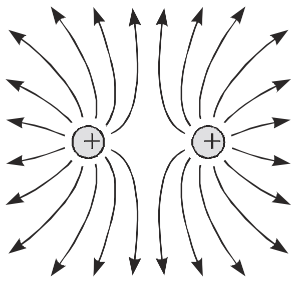


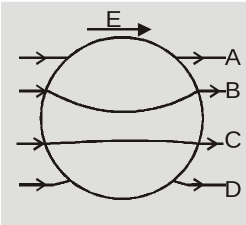
Figure 2 : The electric field lines for two equal positive point charge.

k) Field lines always enter or leave a conducting surface at right angles.

l) Since electric field inside a conductor in electrostatics is zero, electric field lines do not exist inside the conductor.

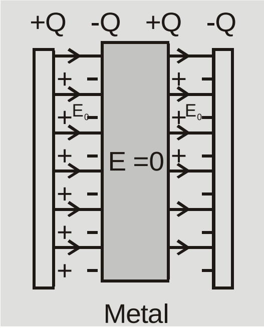
EXAMPLES BASED ON ELECTRIC LINES OF FORCE

Example - 8 A Solid metallic sphere is placed in a uniform electric field. Which of the lines A, B, C and D shows the correct path and why ?



Solution : Path (A) is wrong as lines of force do not start or end normally on the surface of a conductor. Path (B) and (C) are wrong as lines of force should not exist inside a conductor. Also lines of froce are not normal to the surface of conductor. Path (D) represents the correct situation as here line of force does not exist inside the conductor and starts and ends normally on its surface.

Example - 9 A metallic slab is introduced between the two plates of a charged parallel plate capacitor. Sketch the electric lines of force between the plates.



Solution : Keeping in mind the following properties :

1. Lines of force start from positive charge and end on negative charge.
2. Lines of force start and end normally on the surface of a conductor.
3. Lines of force do not exist inside a conductor (as field inside a conductor is zero)

The field lines between the plates are as shown in the figure.

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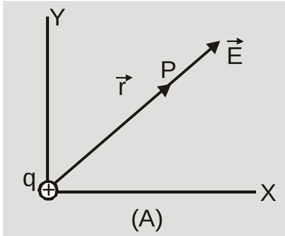
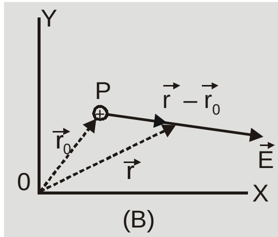
ELECTRIC FIELD DUE TO A POINT CHARGE

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \quad \Rightarrow \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}$$





If the position vector of the charge is $\vec{r}_0$, then

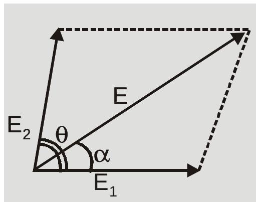
electric field at position $\vec{r}$ is $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q(\vec{r}-\vec{r}_0)}{|\vec{r}-\vec{r}_0|^3}$

SUPERPOSITION OF ELECTRIC FIELDS

The resultant electric field at any point is equal to the vector sum of electric fields at that point due to various charges.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$$

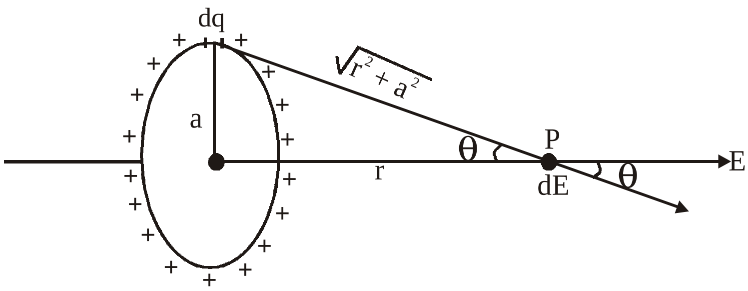
The magnitude of the resultant of two electric fields is given by



$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta}$$

The direction is given by $\tan\alpha = \dfrac{E_2\sin\theta}{E_1 + E_2\cos\theta}$

ELECTRIC FIELD INTENSITY ON THE AXIS OF A UNIFORMLY CHARGED RING



Let dE be the electric field at a point P on the axis, due to charge dq, while total charge on the ring is q.

Then $$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2+a^2}$$

As only axial components of electric field due to all elements remain, while perpendicular components are cancelled.

Therefore 'E' at Point P = $\oint dE\ \cos\theta$

$$= \int \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2+a^2}.\frac{r}{\sqrt{r^2+a^2}} = \frac{1}{4\pi\epsilon_0}\frac{r}{(r^2+a^2)^{3/2}}\int dq = \frac{1}{4\pi\epsilon_0}\frac{qr}{(r^2+a^2)^{3/2}} \quad \left[\text{as} \int dq = q\right]$$

SPECIAL CASES

(i) At r = 0 (i.e. at centre of the ring) E = 0

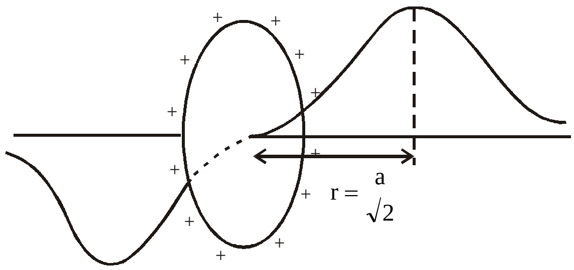
(ii) At r >>a (i.e. at for off points on the axis) $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$. It shows that the ring behaves like a point charge for far off points.

The magnitude of E increases from zero as we move on the axis starting from centre of the ring. It attains a maximum value at some point on the axis and thereafter again decreases. At the point, whose E is maximum, $\frac{dE}{dr} = 0$

$$\Rightarrow \quad \frac{d}{dr}\left[\frac{1}{4\pi\epsilon_0}\ \frac{qr}{(r^2+a^2)^{3/2}}\right] = 0$$

$$\Rightarrow \quad (r^2+a^2)^{3/2} - r.\frac{3}{2}(r^2+a^2)^{1/2}.\,2r = 0$$

$$\Rightarrow \quad r = \frac{a}{\sqrt{2}}$$



EXAMPLES BASED ON ELECTRIC FIELD

Example - 10 Charges with magnitude q are placed at 4 corners of a regular pentagon. These charges are at distance 'a' from the centre of the pentagon. Find electric field intensity at the centre of the pentagon.

Solution : Charges are placed at corners A, B, C and D of the pentagon. If charge q is placed at the fifth corner also then by symmetry the intensity $\vec{E}$ at centre O is zero.

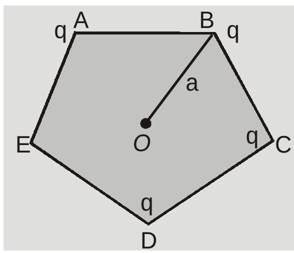
Let $\vec{E}_1$ be the resultant electric field due to charge at A, B C and D and let $\vec{E}_2$ be the elecric field due to the fifth charge.

Now, $\vec{E}_1 + \vec{E}_2 = 0$

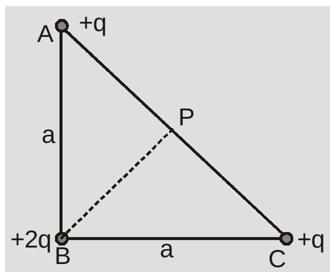
$\therefore \quad \vec{E}_1 = -\vec{E}_2$

$\therefore$ Electric field due to charges at A, B, C and D.

$\vec{E}_2 = \frac{q}{4\pi\varepsilon_0 q^2}$ along $\overline{EO}$



Example - 11 Three charges +q, +q, +2q are arranged as shown in figure. What is the field at point P (center of side AC)



Solution : The sum of fields at P due to charges at A and C add up to zero (because of equal magnitude and opposite direction) . Thus the net field at P is that due to +2q charge. Its direction is along the line BP and its magnitude is

$$E = \frac{1}{4\pi\varepsilon_0}\ \frac{2q}{(BP)^2}$$

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$$BP = \sqrt{BC^2 - PC^2} = \sqrt{a^2 - a^2/2} = a/\sqrt{2}$$

Thus $E = \dfrac{q}{\pi\varepsilon_0 a^2}$

Example - 12 An infinite plane of positive charge has a surface charge density σ. A metal ball B of mass m and charge q is attached to a thread and tied to a point A on the sheet PQ. Find the angle θ which AB makes with the plane PQ.

Solution : Due to positive charge the ball will experience electrical force F_e = qE horizontally away from the sheet while the weight of the ball will act vertically downwards and hence if T is the tension in the string, for equilibrium of ball :

Along horizontal, Tsinθ = qE

And along vertical, Tcosθ = mg

So, $\tan\theta = \dfrac{qE}{mg}$

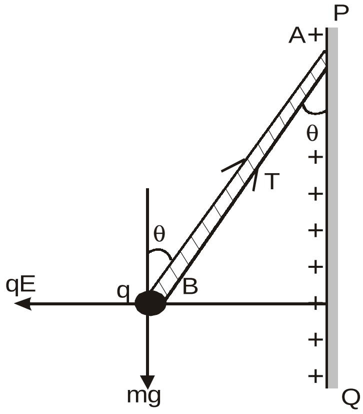
and T = $[(mg)^2 + (qE)^2]^{1/2}$

The field E produced by the sheet of charge PQ

having charge density σ is $E = \dfrac{\sigma}{2\varepsilon_0}$

So, $\tan\theta = \dfrac{q\sigma}{2\varepsilon_0 mg}$

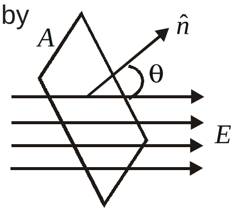
i.e., $\theta = \tan^{-1}\left[\dfrac{q\sigma}{2\varepsilon_0 mg}\right]$



ELECTRIC FLUX

The mathematical quantity related to number of lines passing through a surface is called the electric flux ϕ.
The electric flux through a surface which is perpendicular to a uniform electric field E is defined as the product of electric field E and surface area A : $\phi = EA$
Since the electric field is proportional to density of lines of force, the electric flux is proportional to number of lines of force passing through the surface area : $\phi \propto N$.
If the surface area is not perpendicular to the electric field, then the electric flux is given by

$$\phi = \vec{E}.\hat{n}A = E\cos\theta A = E_n A$$



where $\hat{n}$ is a unit vector perpendicular to the surface and E_n is the component of electric field perpendicular to the surface (normal component).

The electric flux over a curved surface over which electric field may vary in direction and magnitude can be computed by dividing the surface into large number of very small area elements. Let $\hat{n}_i$ be the unit vector perpendicular to such an area element and ΔA_i be its area. The flux of the electric field through such an area element is $\Delta\phi_i = \vec{E}_i.\vec{n}_i\,\Delta A_i$

The total flux through the surface area is found by adding flux through each area element. In the limit ,when number of area elements approaches infinity and area of each element approaches zero, the sum becomes an integral.

$$\phi = \lim_{n\to\infty} \sum_i \vec{E}_i . \hat{n}_i \, \Delta A_i = \int \vec{E} . \hat{n} \, dA .$$

Quite often we are interested in finding out the flux through a closed surface. The unit vector $\hat{n}$ in such a case is defined to be directed outward from each point. Note that when an electric line comes out of the closed surface, then $\vec{E} . \hat{n}$ is positive and if it enters the surface $\vec{E} . \hat{n}$ is negative.

The net flux through a closed surface is written as

$$\phi_{net} = \oint \vec{E} \, . \, \hat{n} \, dA = \oint E_n \, dA. \, .$$

Please note that ϕ_{net} is proportional to the net lines of force going out of the surface, ie, number of lines going out of the surface minus the number of lines going into the surface.

GAUSS LAW

I. STATEMENT

$$\phi_{net} = \oint_{surface} \vec{E} \, .\hat{n} \quad dA = \oint_S \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

where $q_{enclosed}$ is the total charge enclosed inside the surface. This important result is called Gauss's Law and can be stated as follows:

For a system of charges, the net flux through any closed surface 'S' is equal to $1/\varepsilon_0$ times the net charge inside the surface.

Example If a dipole is enclosed by a closed surface *S*, then total flux due to this charge distribution is zero.

$$\phi = \oint_S \vec{E} \cdot d\vec{A} = \frac{1}{\varepsilon_0}\left(q + (-q)\right) = 0$$

APPLICATION OF GAUSS'S LAW

ELECTRIC FIELD DUE SOME CHARGE DISTRIBUTION DERIVED BY USING GAUSS LAW.

(i) Electric field due to line charge (infinite length)

The electric field at a distance r from a line charge (density

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = k. \ \frac{2\lambda}{r}$$

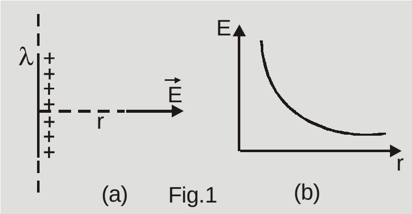
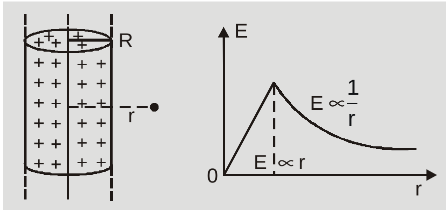


Fig.1

The direction is outward perpendicular to the line charge. The E ∝ (1/r) dependence is shown in fig. 1(b)

(ii) Electric field due to cylinder

If the line charge is a cylinder of radius R, then



(a) Fig . 1 Fig . 2 (b)

(a) Electric field outside

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad (r > R)$$

(b) Electric field on the surface

$$E = \frac{\lambda}{2\pi\varepsilon_0 R}$$

(c) Electric field inside (at a distance r from the axis)

$$E = \frac{\lambda r}{2\pi\varepsilon_0 R^2} \quad (\text{if } r \le R)$$

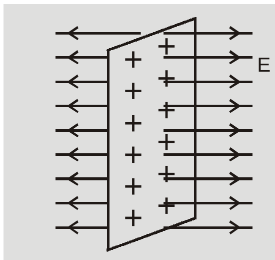
The direction of the field is outwards (normal to the axis). The dependence of the field on r is shown in figure.2 (b). Inside the charged cylinder,

$E \propto r$

outside $E \propto \frac{1}{r}$

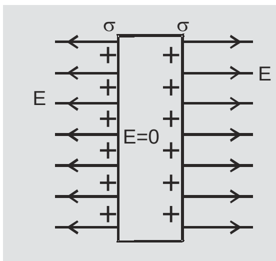
(iii) ELECTRIC FIELD DUE TO A PLANE SHEET (INFINITE DIMENSIONS)

(a) Single sheet of charge



For the surface charge density σ (coulomb/metre^2) the field at a distance r from the sheet is $E = \frac{\sigma}{2\varepsilon_0}$ directed towards outward normal (from a positively charged sheet)
The electric field does not depend on distance.

(b) Charged metal plate



Inside charged metal plate

$E = 0$

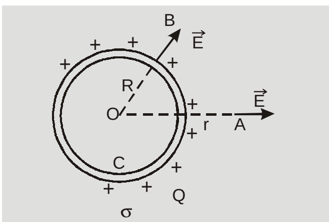
Outside charged metal plate

$$E = \frac{\sigma}{\varepsilon_0}$$

The field is normally outwards for positively charged plate and inwards for negatively charged plates.

(iv) Electric field due to a charged spherical shell

The charge Q resides on the surface of the spherical shell (radius R) .



(a) Field at outside point A

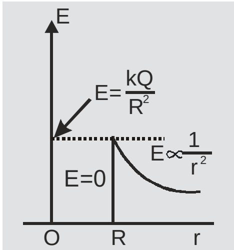
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

(b) Field at surface point B

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\hat{r}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$



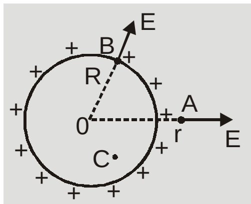
(c) Field at inside point C

$$E = 0$$

The variation of field with distance r from the centre O of the shell in shown in fig. The field at the surface is maximum. And outside the shell field varies as $E \propto 1/r^2$.

(v) ELECTRIC FIELD DUE TO CHARGED CONDUCTING SPHERE

The entire charge Q resides on the surface of a charged conductor. Any charge given to the interior, flows to the surface in less than a nanosecond.



So a charged conducting sphere behaves like spherical shell. Thus,

(a) Field outside $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

(b) Field on surface $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$

(c) Field inside $E = 0$

Special note : The surface charge density in the above case is $\sigma = Q/4\pi R^2$. In terms of σ the fields are

outside $E = \frac{\sigma}{\varepsilon_0}\left(\frac{R^2}{r^2}\right)$

on surface $E = \frac{\sigma}{\varepsilon_0}$

inside $E = 0$

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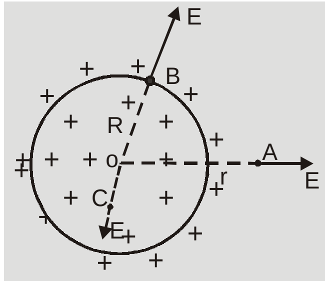
(vi) ELECTRIC FIELD DUE TO A CHARGED SPHERE (NON CONDUCTING)

In case of a charged non conducting (plastic etc.) sphere, charges do not flow. As a result, charges exist inside the sphere as well as on the surface. Assuming uniform charge distribution, the electric field, outside (point A),
on the surface (point B) and inside (point C) are as follows.



(a) Field outside

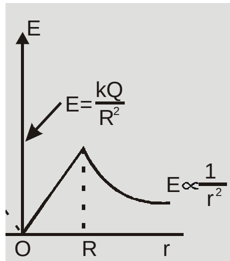
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$ directed radially outwards (for positive Q)

(b) Field on surface

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

(c) Field inside

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\ r$$



directed radially outwards for positive Q.

The variation of the electric field with distance r from the centre of the charged nonconducting sphere is as shown in fig. The field outside varies inversely as square of the distance. The field at the surface is maximum. The field inside is directly proportional to the distance.

Special note : The volume charge density in above case, is $\rho = Q/(\frac{4}{3}\pi R^3)$. In terms of ρ, the field are

outside $$E = \frac{\rho}{3\varepsilon_0}\left(\frac{R^3}{r^2}\right)$$

on surface $$E = \frac{\rho}{3\varepsilon_0}R$$

inside $$E = \frac{\rho}{3\varepsilon_0}r$$

EXAMPLE BASED ON ELECTRIC FLUX

Example - 13 If a point charge Q is located at the centre of a cube then find (i) flux through the total surface, of the cube (ii) flux through one surface.

Solution : (i) According to Gauss's law for closed surface $\phi = Q/\varepsilon_0$, (ii) Since cube is a symmetrical figure thus by symmetry the flux through each surface is $\phi' = Q/6\epsilon_0$

Example - 14 A point charge +q is located L/2 above the centre of a square having side L. Find the flux through this square.

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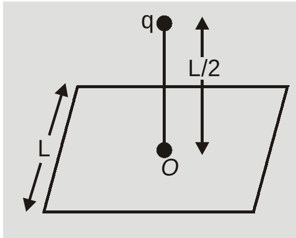
Solution : The charge q can be supposed to be situated at the centre of a cube having side L with outward flux ϕ. In this case the square is one of its face having flux $\phi/6$



$\therefore$ flux through the square $= \dfrac{q}{6\varepsilon_0}$

Example - 15 What is the value of electric flux in SI unit in Y-Z plane of area 2m^2, if intensity of electric field is $\vec{E} = (5\hat{i} + 2\hat{j})$ N/C.

Solution : $\phi = \vec{E}.\vec{dA}$

$= (5\hat{i} + 2\hat{j}).2\hat{i}$

= 10 V-m

Example - 16 2μC charge is in some Gaussion surface, given outward flux ϕ, what additional charge is needed if we want that 6ϕ flux enters into the Gaussian surface.

Solution : According to question

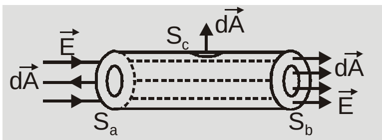
$$\frac{(2+Q)\mu C}{\epsilon_0} = -6\phi$$

or $\dfrac{(2+Q)}{\epsilon_0} = -6\left(\dfrac{2}{\epsilon_0}\right)$

Q = –14μC

Example - 17 A cylinder of length L and radius b has its axis coincident with the x axis. The electric field in this region $\vec{E} = 200\hat{i}$. Find the flux through (a) the left end of cylinder (b) the right end of cylinder (c) the cylinder curved surface, (d) the closed surface area of the cylinder.

Solution : from fig. then



(a) $\phi_a = \vec{E}.\vec{A} = EA\cos\theta$

$= 200 \times \pi b^2 \times \cos\pi$

$= -200\ \pi b^2$

(b) $\phi_b = EA\cos 0^o$

$= 200\ \pi b^2$

(c) $\phi_c = EA\cos 90^o$

$= 0$

(d) $\phi = \phi_a + \phi_b + \phi_c$

$= -200\ \pi b^2 + 200\ \pi b^2 + 0$

$= 0$

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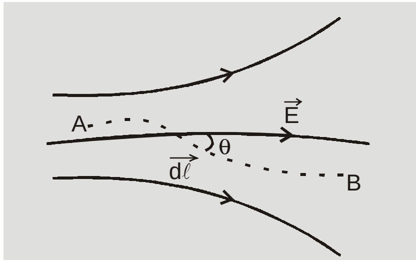
ELECTRIC POTENTIAL AND POTENTIAL ENERGY

LINE INTEGRAL OF $\vec{E}$



The line integral of electric fields is defined as the integral

$$\int_A^B \vec{E}.\vec{d\ell}$$

The value of line integral depends only on the position of points A and B, and is independent of the path between A and B

$$\int_A^B \vec{E}.\vec{d\ell} = -\int_B^A \vec{E}.\vec{d\ell}$$

Line integral for a closed path is zero $\oint \vec{E}.\vec{d\ell} = 0$

(such electric fields are called conservative fields).

Note : There exists one more type of electric field, called induced electric field. It is non-conservative for which $\oint \vec{E}.\vec{d\ell} = 0$

ELECTRIC POTENTIAL

Electric potential and potential energy are defined only for conservative fields.

Definition in terms of work done :

Potential at any point A is equal to the amount of work done (by external agent against electric field) in bringing a unit positive charge from infinity to that point.

$$V_A = \frac{W_{\infty A}}{q}$$

Unit of potential (V) = J/C or volt

Potential at a point is said to be one volt if the amount of work done in bringing one coulomb of positive charge from infinity to that point is one joule.

Since work and charge, both are scalars, the electric potential is a scalar quantity.

The dimensions of electric potential are $= \frac{ML^2T^{-2}}{TA}$

or $[V] = ML^2T^{-3}A^{-1}$

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POTENTIAL DIFFERENCE

Potential difference between two points f (final) and i (initial) is defined to be equal to the amount of work done (by external agent) in moving a unit positive charge from point i (initial) to f (final)

$$V_f - V_i = \frac{W_{if}}{q}$$

If work done in carrying a unit positive charge from point 1 to point 2 is one joule then the potential difference $V_2 - V_1$ is said to be one volt.

Potential difference may be positive or negative.

The work done against electrical forces in transporting a charge q from point i (potential V_i) to point f (potential V_f) is $W = qV$ where $V = V_f - V_i$.

RELATION BETWEEN E AND ELECTRIC POTENTIAL V

$$\Delta V = V_f - V_i = -\vec{E} \cdot \Delta\vec{\ell}$$

In one dimensions

$$E = -\frac{dV}{dr} \qquad ......(1)$$

$$V = -\int E\, dr \qquad ......(2)$$

[If electric potential is known, electric field can be determined from eq. (1) and if E is known, V can be determined from (2)]

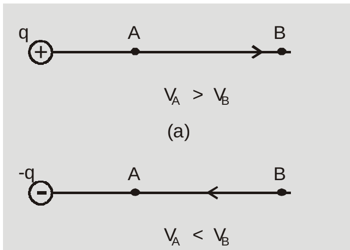
In general $\vec{E} = -\nabla V$

Electric field at any point is equal to negative of potential gradient at that point.

The electric field always points from higher potential to lower potential (see fig.)

A positive charge always moves from higher potential to lower potential.

A negative charge always moves from lower potential to higher potential.



(a)
(b)

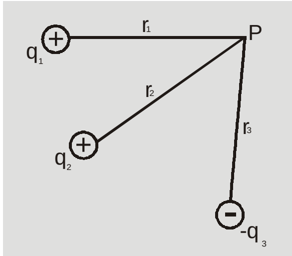
Note : In an extended region of space where electric field is zero, electrical potential is constant.

ELECTRIC POTENTIAL DUE TO POINT CHARGES

One point charge $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

Many point charges

Potential is a scalar quantity and adds like scalars. Thus potential at point P (see fig.) due to charges q_1, q_2, $-q_3$ is equal to (algebraic) sum of potentials due to individual charges.



$$V = V_1 + V_2 + V_3 + ...$$

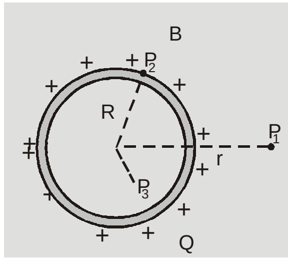
$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{-q_3}{r_3} + ...\right)$$

POTENTIAL DUE TO A CHARGED SPHERICAL SHELL

The charge resides on the shell surface. The potential at P_1, outside point, is

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

The potential at P_2, surface point is



$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$$

The potential at P_3, (inside point) is

$$V = V_{surface}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

It is constant inside the shell (same at all points inside the shell)

POTENTIAL DUE TO A CHARGED CONDUCTING SPHERE

Outside the sphere

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

At the surface of sphere $\quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

Inside the sphere $\quad V = V_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

The potential at the points inside a conducting sphere is constant. (The electric field inside is zero)

POTENTIAL DUE TO A CHARGED NON-CONDUCTING SPHERE

For a non-conducting sphere carrying uniform charge distribution, the electrical potential is given as :

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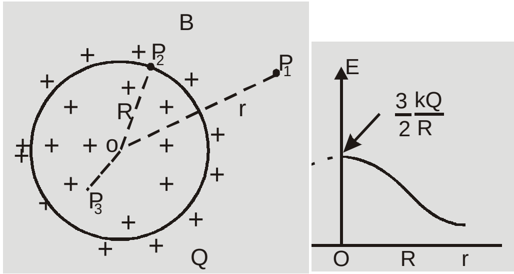
Outside (e.g. point P_1) $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

At the surface (e.g. point P_2)

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

Inside the sphere (e.g. point P_3)

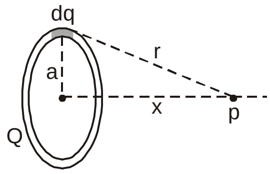
$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q(3R^2 - r^2)}{2R^3}$$



The potential at the centre of the sphere is $V_{centre} = \frac{3}{2}\frac{kQ}{R}$. This is 1.5 times the potential at the surface of the sphere $V_{surface} = kQ/R$

POTENTIAL DUE TO A RING AT A POINT LYING ON ITS AXIS

Consider a ring of charge Q, radius a, linear charge density λ. Let an element of length $d\ell$ be taken on the ring. Then,



$$dV = \frac{dq}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad dV = \frac{\lambda\, d\ell}{4\pi\varepsilon_0\sqrt{a^2 + x^2}}$$

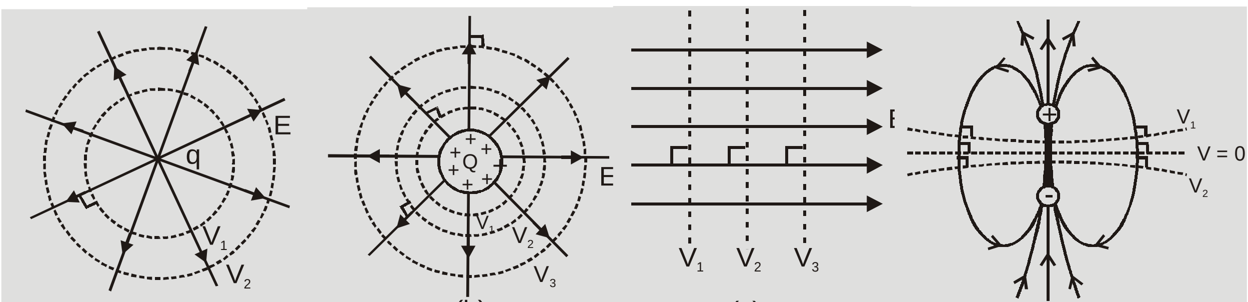
$$\Rightarrow \quad V = \frac{\lambda}{4\pi\varepsilon_0\sqrt{a^2 + x^2}}\int_0^{2\pi a} d\ell$$

$$\Rightarrow \quad V = \frac{\lambda(2\pi a)}{4\pi\varepsilon_0\sqrt{a^2 + x^2}}$$

$$\Rightarrow \quad V = \frac{Q}{4\pi\varepsilon_0\sqrt{a^2 + x^2}}$$

EQUIPOTENTIAL SURFACE

A surface on which the potential is constant is called an equipotential surface. (A curve on which the potential is constant is called equipotential curve)



(a) (b) (c) (d)

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Note : The electric field lines are perpendicular to equipotential surface (every where)

Since E = 0 inside a conductor, the entire conductor is at a constant potential.

For a point charge q and spherical charge distributions, the equipotential surfaces are spherical fig.a & b dotted lines)

For a uniform field equipotential surface is plane (see fig. c dotted lines)

For a dipole, V = 0 surface is the equatorial plane. Other equipotential surfaces are curved.

Thus, in general equipotential surface can be of any shape.

When a charge is moved on an equipotential surfaces, work done is zero

ELECTRIC STRENGTH (DIELECTRIC STRENGTH)

The electric strength of air is about 3×10^6 V/m or 3000 V/mm. This means that if the electric field exceeds this value sparking will occur in air. This sets a limit on maximum charge that can be given to a conducting sphere in air.

The electric strength sets a limit on the maximum charge that can be placed on a conductor.

ELECTRIC POTENTIAL ENERGY

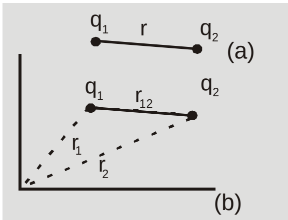
The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from an infinite distance.

U is a scalar quantity.

Dimension of [U] = ML^2T^{-2}

Unit of [U] = joule

For two charges

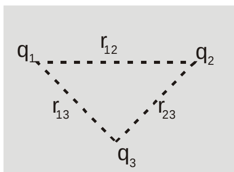


$$U = \frac{kq_1q_2}{r}$$

$$U = q_2V_1$$

$$U = \frac{kq_1q_2}{r_{12}}$$

For three charges



$$U = U_{12} + U_{23} + U_{13}$$

$$= \frac{kq_1q_2}{r_{12}} + \frac{kq_2q_3}{r_{23}} + \frac{kq_1q_3}{r_{13}}$$

For *n*-charge

$$U = \sum_{\text{all pairs}} \frac{kq_iq_j}{r_{ij}}$$

(number of pairs = ${}^nC_2$)

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ELECTRON VOLT

It is equal to the amount of energy gained by an electron when accelerated through a potential difference of one volt. It is unit of energy.

$1 \text{ eV} = 1.6 \times 10^{-19}$ joule

When a charged particle moves under the influence of an electric field, then,

Kinetic energy gained = Potential energy lost

ENERGY DENSITY

In electric field, energy stored per unit volume is called energy density. It is equal to

$$u = \frac{1}{2}\varepsilon_0 E^2$$

EXAMPLES BASED ON ELECTRIC POTENTI AL

Example - 18 A uniform electric field of magnitude E_0 and directed along positive x-axis exist in a certain region of space. If at x = 0, then potential V is zero, then what is the potential at $x = +x_0$?

Solution : For a uniform field $\Delta V = -E\Delta r$. In this case $\Delta V = V - 0$, $\Delta r = x_0 - 0$, and $E = E_0$.

Thus $V - 0 = -E(x_0 - 0)$

or $V = -Ex_0$

Note : The negative sign. As one moves along the direction of electric field, the potential falls

Example - 19 Electric field intensity is given by the relation $E = 100/x^2$ where x is in meters. Find potential difference between the points x = 10 m and x = 20 m.

Solution :
$$V = -\int_{x_1}^{x_2} E dx = -\int_{10}^{20} 100x^{-2} dx = 100\,[x^{-1}]_{10}^{20}$$

$$= 100\left[\frac{1}{10} - \frac{1}{20}\right] = 5 \text{ volt.}$$

Example - 20 In the above question if alternate charges are positive and negative then find potential x = 0.

Solution :
$$V = kq\left[\frac{1}{1} - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} ....\right] = kq\left[\frac{1}{1+1/2}\right] = \frac{2}{3}kq$$

Example - 21 A metallic charged sphere of radius r. V is the potential difference between point A on the surface and point P distant 4r from the centre of the sphere. Then the electric field at a point which is at a distance 4r from the centre of the sphere will be -

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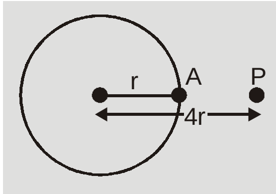
Solution : $V = V_A - V_P$



$$= \frac{kq}{r} - \frac{kq}{4r} = \frac{3kq}{4r}$$

Therefore for point P

$$E = \frac{kq}{(4r)^2} = \frac{V}{12r}$$

Example - 22 A ring of radius R has a charge +q. A charge q_0 is freed from the distance $\sqrt{3}\,R$ on its axis, when it reaches to the centre of the ring, what is it's kinetic energy ?

Solution : $$V = \frac{1}{4\pi\epsilon_0}\frac{q}{\sqrt{R^2+r^2}}$$

At $r = \sqrt{3}\,R$, $V_1 = \dfrac{kq}{2R}$

At $r = 0$, $V_2 = \dfrac{kq}{R}$

$KE = (V_2 - V_1)\, q_0$

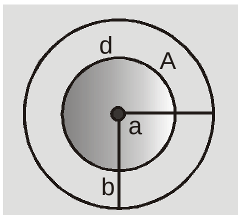
$$= \frac{kqq_0}{2R}$$

Example - 23 A solid spherical conductor carries a charge Q. It is surrounded by a concentric uncharged spherical shell. The potential difference between the surface of solid sphere and the shell is V. If a charge of –3Q is given to the shell. Then the new potential difference between the above two points will be -

Solution : Initial potential difference before charge is given to the shell

$$V_A - V_B = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right] = V$$

(ii) Final potential difference after the charge –3Q is supplied to the shell



$$V_A' = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{a} - \frac{3Q}{b}\right)$$

$$V_B' = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{b} - \frac{3Q}{b}\right)$$

$$\therefore\ V_A' - V_B' = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right] = V$$

Example - 24 In the following diagram the variation of potential with distance r is represented. What is the intensity of electric field in V/m at r = 3m ?

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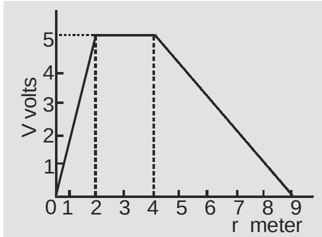


Solution : Because at r = 3m potential V = 5 volt = const.

$$\therefore\ E = -\frac{dV}{dr} = 0\ \text{V/m}$$

Example - 25 In the above example the value of E in V/m at r = 6m will be.

Solution : At r_2 = 7m, V_2 = – 2 volt

at r_1 = 5m, V_1 = 4 volt

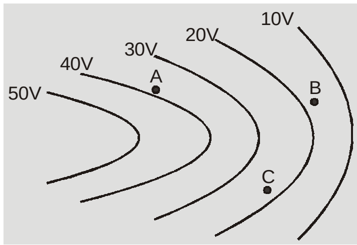
$$E = -\frac{(V_2 - V_1)}{(r_2 - r_1)} = -\frac{[2-4]}{[7-5]} = \frac{2}{2} = 1\ \text{V/m}$$

Example - 26 Electric potential for a point (x, y, z) is given by V = $4x^2$ volt. Electric field at point (1, 0, 2) is -

Solution : $E = -\frac{dV}{dx} = -8x = -8\times1 = -8$ V/m

$\therefore$ Magnitude of E = 8 V/m and direction is along.

Example - 27 Fig. shows lines of constant potential in a region in which an electric field is present. The value of potentials are written. At which the points A, B and C is the magnitude of the electric field greatest?



Solution : In an electric field, electric intensity E and potential V are related as

$$\vec{E} = -\frac{dV}{dr}\vec{n}, \quad \text{i.e.,} \quad E = -\frac{dV}{dr}$$

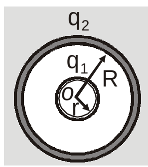
For a given line, V = constant and the potential difference between any two consecutive lines dV = $V_1 - V_2$ = 10 V = const.. So E will be maximum where the distance dr between the lines is minimum, i.e., at B (where the lines are the closest)

Example - 28 A charge Q is distributed over two concentric hollow spheres of radii r and R (> r) such that the surface densities are equal. Find the potential at the common centre.

Solution : If q_1 and q_2 are the charges on spheres of radii r and R respectively,

then $q_1 + q_2 = Q$

According to given problem $\sigma_1 = \sigma_2$ or $\frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}$ or $\frac{q_1}{q_2} = \frac{r^2}{R^2}$



So $q_1 = \dfrac{Qr^2}{(r^2+R^2)}$ and $q_2 = \dfrac{QR^2}{(r^2+R^2)}$

Now as potential inside a conducting sphere is equal to its surface, so potential at the common centre

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r}+\frac{q_2}{R}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Qr}{(R^2+r^2)}+\frac{QR}{(R^2+r^2)}\right] = \frac{1}{4\pi\varepsilon_0}\,\frac{Q(R+r)}{(R^2+r^2)}$$

EXAMPLES BASED ON ELECTRIC POTENTIAL ENERGY

Example - 29 Three charges, –q, Q, q are placed at equal distances on a straight line. If the total potential energy of the system of the three charges is zero, then Q : q = ...

Solution : Let d be the equal distance. The total potential energy of the system is,

$U = U_{12} + U_{23} + U_{31}$

$$U = \frac{1}{4\pi\in_0}\left[\frac{q_1q_2}{d}+\frac{q_2q_3}{d}+\frac{q_3q_1}{2d}\right]$$

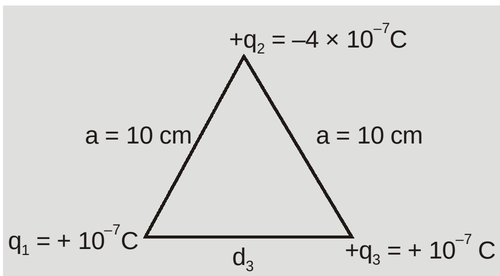
$$= \frac{1}{4\pi\in_0}\left[-\frac{qQ}{d}-\frac{Qq}{d}+\frac{q^2}{2d}\right] \text{ or, } U = \frac{1}{4\pi\in_0}\frac{q}{d}\left(-Q-Q+\frac{q}{2}\right)$$

Since U = 0

$-2Q + \dfrac{q}{2} = 0$ or, $-4Q + q = 0$ or, $4Q = q$ or, $\dfrac{Q}{q} = 1 : 4$

Example - 30 Three charges are arranged as shown in fig. Find the potential energy of the system.

Solution : The potential energy of the system is $U = U_{12} + U_{23} + U_{31}$



$$= \frac{1}{4\pi\in_0}\left[\frac{q_1(-q_2)}{a}+\frac{(-q)\times q_3}{a}+\frac{q_3q_1}{a}\right]$$

$$= \frac{1}{4\pi\in_0}\left[\frac{4\times10^{-14}}{0.1}-\frac{8\times10^{-14}}{0.1}+\frac{2\times10^{-14}}{0.1}\right]$$

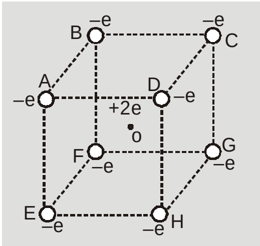
$$= \frac{1}{4\pi\in_0}\left[\frac{10^{-14}}{0.1}(-4-8+2)\right]$$

or $U = 9 \times 10^9 \times 10^{-13}\,(-10)$

$= -9 \times 10^9 \times 10^{-12} \Rightarrow -9 \times 10^{-3}$ joules

Example - 31 An electron (charge, –e) is placed at each of the eight corners of a cube of side a and α-particle (charge, +2e) at the centre of the cube. Compute the P.E of the system.

Solution : Fig., shows an electron placed at each to the eight corners of the cube and α particle at the centre. The total energy of the system is the sum of energies of each pair of charges. There are 12 pairs like A and B (separation a), 12 paris like A and C (separation, $\sqrt{2}a$), 4 pairs like A and G (separation $\sqrt{3}a$) and 8 paris like A and 0 (separation $\frac{\sqrt{3}}{2}$ a).



Hence

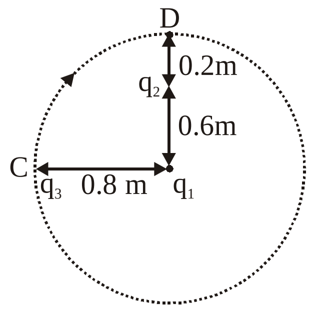
$$U = \frac{1}{4\pi\epsilon_0}\left[12\frac{(-e)(-e)}{a} + 12\frac{(-e)(-e)}{\sqrt{2}\,a} + 4\frac{(-e)(-e)}{\sqrt{3}a} + \frac{8(-e)(2e)}{\frac{\sqrt{3}}{2}a}\right]$$

$$= (9 \times 10^9)\ \frac{e^2}{a}\left[12 + \frac{12}{\sqrt{2}} + \frac{4}{\sqrt{3}} - \frac{32}{\sqrt{3}}\right]$$

$$= (9 \times 10^9)\ \frac{e^2}{a}\ (4.32) = 3.9 \times 10^{10}\left(\frac{e^2}{a}\right)\ J.$$

Example - 32 The value of q_1 and q_2 are 2×10^{-8} coulomb and 0.4×10^{-8} coulomb respectively as shown in fig. A third charge $q_3 = 0.2 \times 10^{-8}$ coulomb is moved from point C to point D along the arc of a circle. The change in the potential energy of charge will be -

Solution : Potential energy of q_3 at point C



$$U_C = k\left[\frac{q_1q_3}{0.8} + \frac{q_3q_2}{1}\right] \qquad .....(1)$$

Potential energy of q_3 at point D

$$U_D = k\left[\frac{q_1q_3}{0.8} + \frac{q_2q_3}{0.2}\right] \qquad .....(2)$$

$$\therefore\ U_D - U_C = kq_2q_3\left[\frac{1}{0.2} - \frac{1}{1}\right]$$

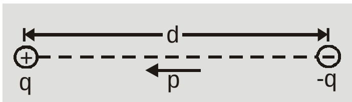
$$= 9 \times 10^9 \times 0.4 \times 10^{-8} \times 0.2 \times 10^{-8} \times 4$$

$$= 2.88 \times 10^{-7} \text{ joule}$$

Note : Note Charge potential energy charge will ne same even if we had taken any other path as electric field is conservative.)

ELECTRIC DIPOLE

An arrangement of two equal and opposite charges separated by a small distance is called an electric dipole.



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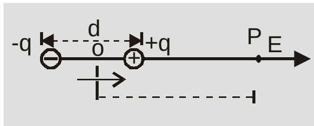
The dipole moment $\vec{p}$ is a vector quantity whose magnitude is equal to the product of magnitude of one charge and the distance between the two charges. It is directed from negative charge to positive charge.

Unit of dipole moment (p) = coulomb × metre = C.m

Dimensions of p = $M^0L^1T^1A^1$

ELECTRIC FIELD AT AN AXIAL POINT



$$\vec{E} = \frac{k2p\vec{r}}{[r^2 - (d/2)^2]^2}$$

The direction is along the axis, parallel to $\vec{p}$. The magnitude is

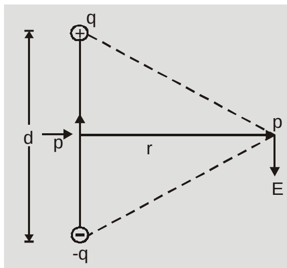
$$E = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{[r^2 - (d/2)^2]^2}$$

For r >> d,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

ELECTRIC FIELD AT AN EQUATORIAL POINT

The electric field at an equatorial point P is



$$\vec{E} = -\frac{k\vec{p}}{\left(r^2 + \left(\frac{d}{2}\right)^2\right)^{3/2}}$$

The field is directed opposite to $\vec{p}$ and the magnitude is

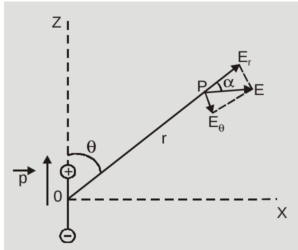
$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2 + (d/2)^2)^{3/2}}$$

For r >> d

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

ELECTRIC FIELD AT (R, θ) POINT

For $r >> d$, the radial component E_r and angular component E_θ of electric field due to a dipole, are



$$E_r = k\frac{2p\cos\theta}{r^3}$$
$$E_\theta = k\frac{p\sin\theta}{r^3}$$

The resultant field is

$$E = \sqrt{E_r^2 + E_\theta^2}$$

$$= \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$$

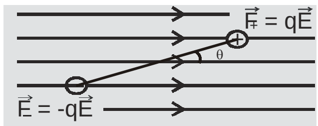
The angle α in figure is such that

$$\tan\alpha = \frac{E_\theta}{E_r} = \frac{1}{2}\tan\theta$$

FORCE AND TORQUE ON A DIPOLE PLACED IN AN ELECTRIC FIELD

(a) A positive charge +q experiences a force parallel to the electric field $\vec{F} = q\vec{E}$

(b) A negative charge –q experiences a force in a direction opposite to that of the electric field $\vec{F} = -q\vec{E}$

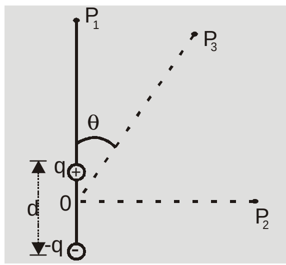


(c) The total force on a dipole placed in an uniform electric field is zero $\vec{F} = q\vec{E} + (-q\vec{E}) = 0$

(d) The torque on a dipole placed in uniform electric field is $\vec{\tau} = \vec{p}\times\vec{E} \quad \Rightarrow \quad \tau = pE\sin\theta$

ELECTRIC POTENTIAL DUE TO A DIPOLE

The potential at an axial point (P_1) is



$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2 - (d/2)^2}$$

If d << r then

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$$

where p = qd is the dipole moment

The potential at an equatorial point (P_2)

$$V = 0$$

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The potential at any arbitrary point (P_3, located at r, θ coordinates) is (for r >> d)

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{p\cos\theta}{r^2}$$

Potential at a point which is equidistant from +q and –q charge is zero. Thus potential at all points lying on the equatorial plane is zero for a dipole.

POTENTIAL ENERGY OF A DIPOLE

The work done in rotating a dipole placed in a uniform electric field E, from initial angle θ_1 to final angle θ_2 is $W = \int_{\theta_1}^{\theta_2} \tau d\theta$

$= pE\ (\cos\theta_1 - \cos\theta_2)$

where U_2, U_1 are the potential energy of the dipole in the two orientations.

Potential energy

$U_2 - U_1 = -\ pE\cos\theta_2 - (-pE\cos\theta_1)$

$= -pE\ (\cos\theta_2 - \cos\theta_1)$

The zero of the potential is taken at $\theta = 90^o$. Thus, potential energy of the dipole is

$$U = -\vec{p}.\vec{E}$$

$= -pE\cos\theta$

$U_{min} = -pE,\quad U_{max} = +\ pE$

Work done in rotating a dipole from $\theta = 0$ (aligned parallel to E), to $\theta = 180^o$ (aligned antiparallel to E) is $W = 2pE$

EXAMPLES BASED ON DIPOLE SYSTEM

Example - 33 The work required to turn an electric dipole end for end in a uniform electric field when the initial angle between $\vec{p}$ and $\vec{E}$ is θ_0 is -

Solution : $W = pE\ (\cos\theta_1 - \cos\theta_2)$, in this case $\theta_1 = \theta_0$ and $\theta_2 = \pi + \theta_0$. Thus

$W = pE\ \{\cos\theta_0 - \cos\ (\pi + \theta_0)\}$

$= 2pE\cos\theta_0$

Example - 34 Calculate the electric intensity due to a dipole of length 10 cm and having a charge of 500 μC at a point on the axis distance 20 cm from one of the charges in air .

Solution : The electric intensity on the axial line of the dipole

$$E = \frac{1}{4\pi\in_0}\frac{2pd}{(d^2 - \ell^2)^2}$$

$2l = 10$ cm $\therefore\ \ell = 5 \times 10^{-2}$ m

$d = 20 + 5 = 25$ cm $= 25 \times 10^{-2}$ m

$p = 2q\ell = 2 \times 500 \times 10^{-6} \times 5 \times 10^{-2} \Rightarrow 5 \times 10^{-3} \times 10^{-2} = 5 \times 10^{-5}$ C-m

$$\therefore \quad E = \frac{9\times10^9 \times 2\times5\times10^{-5}\times25\times10^{-2}}{10^{-8}[25^2-5^2]^2} \Rightarrow 6.25 \times 10^7 \text{ N.C.}$$

Example - 35 Calculate the electric intensity due to an electric dipole of length 10 cm having charges of 100 μC at a point 20 cm from each charge.

Solution : The electric intensity on the equatorial line of an electric dipole is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{p}{(d^2+\ell^2)^{3/2}}$

$p = 2\ell\, q$ C-m

$= 10 \times 10^{-2} \times 100 \times 10^{-6}$

$= 10^{-5}$ C-m

$d^2 + \ell^2 = (20 \times 10^{-2})^2 = 4 \times 10^{-2}$

$$\therefore \; E = \frac{9\times10^9\times10^{-5}}{(4\times10^{-2})^{3/2}}$$

$$= \frac{9\times10^9\times10^{-5}}{10^{-3}\times8} = \frac{9}{8}\times10^7 = 1.125 \times 10^7 \text{ N/C}$$

Example - 36 Find out the torque on dipole in N-m given : Electric dipole moment $\vec{P} = 10^{-7}(5\hat{i}+\hat{j}-2\hat{k})$ coulomb metre and electric field $\vec{E} = 10^7(\hat{i}+\hat{j}+\hat{k})$ Vm^{-1} is -

Solution : $\vec{\tau} = \vec{P}\times\vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 1 & -2 \\ 1 & 1 & 1 \end{vmatrix}$

$= \hat{i}(1+2)+\hat{j}(-2-5)+\hat{k}(5-1) = 3\hat{i}-7\hat{j}+4\hat{k}$

$|\vec{\tau}| = 8.6$ N-m

CHARGED LIQUID DROP

If n small drops each of radius r coalesce to form a big drop of radius R, then

(i) $\dfrac{4}{3}\pi R^3 = n\dfrac{4\pi}{3}r^3$

$\therefore \quad R = rn^{1/3}$

(ii) If each small drop has a charge q, then the charge on the big drop

$q' = nq$

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(iii) If V is the potential of the small drop, then the potential of the big drop will be

$$V' = \frac{Kq'}{r'} = \frac{Knq}{rn^{1/3}} = Vn^{2/3}$$

(iv) If E is the electric field intensity at the surface of the small drop, then the electric field intensity at the surface of the big drop will be

$$E' = \frac{Kq'}{R^2} = \frac{Knq}{R^2} = \frac{Kqn^{1/3}}{r^2} = n^{1/3}\ E$$

EXAMPLE BASED ON CHARGED LIQUID DROP

Example - 37 1000 equal drops of radius 1cm, and charge 1×10^{-6} C are fused to form one bigger drop. The ratio of potential of bigger drop to one smaller drop, and the electric field intensity on the surface of bigger drop will be respectively -

Solution : Let the potential of one smaller drop be B then potential of bigger drop, is

$V' = n^{2/3}\ V$

$$\Rightarrow \quad \frac{V'}{V} = n^{2/3} = (1000)^{2/3} = 100$$

$\therefore \quad V' : V = 100 : 1$

Also let the electric field on the surface of smaller drop be E then electric field on bigger drop is

$$E' = n^{1/3}\ E = n^{1/3}\ \frac{kq}{r^2} = (1000)^{1/3}\ \frac{9 \times 10^9 \times 1 \times 10^{-6}}{(1 \times 10^2)^2} = 9 \times 10^8\ \text{V/m}$$

FORCE ON A CHARGED SURFACE

(i) If we consider an element of the charged surface, then the charge on the element experiences a repulsive force due to the charge on the remaining part. As a result, a resultant force acts on it perpendicular to the surface in the outward direction.

(ii) The charged surface behaves as a stretched membrane.

(iii) If σ is the surface charge density, then the electric field intensity at external points close to the surface is $\frac{\sigma}{\epsilon_0}$ and at internal points close to the surface the field is zero. Thus the average intensity at the surface.

$$E = \frac{\sigma}{2\epsilon_0}$$

(iv) The repulsive force acting on a unit area of the surface will be.

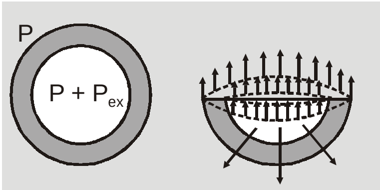
$$F = E\sigma = \frac{\sigma^2}{2\in_0} \text{ N/m}^2$$

(v) The repulsive force acts in the outwards direction. The force acting on a unit area of the surface is electrical pressure.

$$\therefore \quad P_{elec} = \frac{\sigma^2}{2\in_0}$$

$$= 2\pi K\sigma^2 \text{ N/m}^2$$

EQUILIBRIUM OF A CHARGED SOAP BUBBLE



(i) In equilibrium the air pressure inside a soap bubble is greater than the atmospheric pressure. This excess pressure is produced due to surface tension of the soap solution. If r is the radius of the bubble and T is its surface tension, then for the uncharged bubble in equilibrium

Force due to excess pressure = force produced due to surface tension

$$p_{ex} \times \pi r^2 = T(2 \times 2\pi r)$$

$$\therefore \quad p_{ex} = \frac{4T}{r}$$

(ii) If a bubble is charged, then electrical pressure due to charge acts in outwards direction on the bubble.

$$p_{elec} = \frac{\sigma^2}{2\in_0}$$

where σ is the surface charge density.

(iii) In equilibrium, the force produced due to surface tension is equal to the sum of forces due to excess air pressure inside the bubble and the electrical pressure due to charge, i.e.

$$(p_{ex} + p_{elec})\pi r^2 = T(2 \times 2\pi r)$$

or $$p_{ex} + p_{elec} = \frac{4T}{r}$$

or $$p_{ex} + \frac{\sigma^2}{2\in_0} = \frac{4T}{r}$$

(iv) For charged bubble,

$$p_{ex} = \frac{4T}{r} - \frac{\sigma^2}{2\in_0}$$

(v) If the air pressure inside the bubble is equal to the atmospheric pressure outside, i.e., $p_{ex} = 0$ then

$$\frac{\sigma^2}{2\in_0} = \frac{4T}{r}$$

or $$\sigma = \sqrt{\frac{8\in_0 T}{r}} = \sqrt{\frac{2T}{\pi K r}}$$

(vi) If charge given to the soap bubble is q, then

$$\sigma = \frac{q}{4\pi r^2}$$

$\therefore$ The charge on the bubble

$$q = 4\pi r^2 \sigma$$

$$= 4\pi r^2 \sqrt{\frac{8\in_0 T}{r}} = 8\pi\sqrt{2\in_0 Tr^3} = \sqrt{32\pi Tr^3/K}$$

(vii) The intensity of electric field at the surface of the bubble

$$E = \sqrt{32\pi TK/r} = \sqrt{8T/\in_0 r}$$

(viii) The electric potential at the surface of bubble

$$V = \sqrt{32\pi rTK} = \sqrt{8Tr/\in_0}$$

(ix) On charging a bubble the air pressure inside it decreases because the radius of the bubble increased due to charging.

(x) A soap bubble always expands on giving any kind of charge (positive or negative)

MOTION OF A CHARGED PARTICLE IN ELECTRIC FIELD

(i) A charged particle at rest or moving experiences a force $\vec{F} = q\vec{E}$ in the presence of electric field.

The acceleration, velocity and displacement are given by

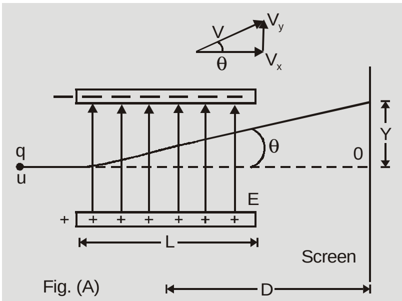


Fig. (A)

$$\vec{a} = \frac{q\vec{E}}{m}$$

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

(ii) For a charged particle with initial velocity perpendicular to the electric field

Note that $F_x = 0$, $a_x = 0$, $V_x = u$ at all times

$$F_y = qE,\ a_y = \frac{qE}{m},\ V_y = \frac{qE}{m}t$$

The displacement components are $x = ut$

$$y = \frac{1}{2}\frac{qE}{m}t^2$$

Eliminating t, $y = \frac{1}{2}\frac{qE}{mu^2}x^2$ which is the equation of a parabola.

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(iii) The path of a charged particle entering a region of electric field with initial velocity perpendicular to the field follows a parabolic trajectory.

The time spent in electric field is $t = \frac{L}{u}$ (see fig. A) The y component of velocity when it emerges out of the field region is $V_y = \frac{qEL}{mu}$

The resultant velocity $V = \sqrt{V_x^2 + V_y^2} = \sqrt{u^2 + \left(\frac{qEL}{mu}\right)^2}$

The angle at which the particle emerges

$$\tan\theta = \frac{V_y}{V_x} = \frac{qEL}{mu^2}$$

The height Y at which the particle hits the screen (see fig. A)

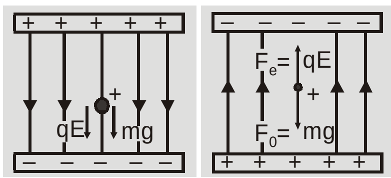
$$Y = D\tan\theta$$

$$Y = \frac{qELD}{mu^2}$$

$$Y = \frac{qELD}{2K}$$ (where K is initial kinetic energy)

EXAMPLES BASED ON MOTION OF A CHARGED PARTICLE IN ELECTRIC FIELD

Example - 38 A positively charged oil droplet remains stationary in the electric field between two horizontal plates separated by a distance of 1cm. If the charge on the drop is 9.6×10^{-10} esu and the mass of droplet is 10^{-11} g, what is the potential difference between two plates ? Now if the polarity of the plates is reversed what is the instantaneous acceleration of the droplet? [g = 9.8 m/s²]



Solution : As the droplet is at rest, its weight W = mg will be balanced by electric force F = qE

i.e., qE = mg or $V = \frac{mgd}{q}$ $V = \frac{(10^{-14})(9.8)(1\times10^{-2})}{(9.6\times10^{-10}/3\times10^{9})}$ = 3062.5 volt

Now if the polarity of the plates is reversed, both electrical and gravitational force will act downward

so, F = mg + qE = 2mg [as mg = qE]

And hence instantaneous acceleration of drop : $a = \frac{F}{m} = \frac{2mg}{m}$ = 2g = 19.6 m/s²

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ELECTROSTATIC BEHAVIOUR OF CONDUCTORS

Materials which allow flow of current through them are called **conductors**. They contain free electrons which are donated by the atoms to the material as a whole and are able to move around freely inside the material. These electrons come from the outermost orbit of atoms as they are very loosely bound to the atom.

The electrostatic behaviour of an insulated charged conductor or a conductor placed in an electric field can be summarized as follows.

a) The electric field in the interior of the conductor is zero everywhere.

b) The electric field immediately outside the surface of the conductor is perpendicular to the surface.

c) Any excess charge given to a conductor remains on the outer surface.

d) All points on and within the surface of the conductor are at the same potential.

It follows that there is no difference between the electrostatic behaviour of a solid conductor and a hollow conductor of the same shape and size.

ELECTROSTATIC SHIELDING

If there is a cavity of any shape inside a conductor, the field there will be strictly zero. This property is called **electrostatic shielding** because any thing placed inside the cavity will be completely shielded from external fields.

Examples : In a thunderstorm accompanied by lightning, it is safer to be inside a car, rather than near a tree or on the open ground.

CONDUCTOR HAVING SHARP POINTS (ACTION OF POINTS)

Another interesting result is that the charge density and the electric field tend to be relatively high on sharp points and low on plane regions of a conducting surface. This can be attributed to the fact that charge density is proportional to radius of curvature.

Glow discharges from sharp points during thunderstorms are due to this reason. The lightning rod, which has sharp points, is thus able to neutralize charged clouds and prevent lightning strokes.

INSULATORS (DIELECTRICS)

Materials which do not allow current to flow through them are called **insulators or dielectrics**. While the electrons in such materials remain bound within their individual molecules, thus preserving the overall neutrality of each molecule, they are affected by external electric fields because the positive and negative charges tend to shift in opposite directions.

Dielectric substances are of two types

1. **Non-polar dielectrics :** In such dielectrics the centre of mass of all the positive charges (protons) in a molecule coincides with the centre of mass of all the negative charges (electrons). Therefore, they are not only electrically neutral but also possess zero dipole moments.

   In the presence of an external field, the two centres of charge get slightly separated and each molecule becomes a dipole, having a small dipole moment. This is because the protons experience a force in the direction of the applied field while the electrons experience a force in the opposite direction. Thus in the presence of a field the dielectric gets polarized.

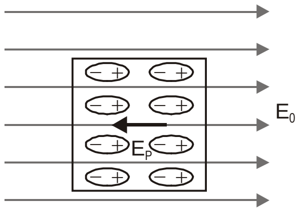
2. **Polar Dielectrics :** In polar dielectrics the centre of mass of the protons in a molecule do not coincide with the centre of mass of the electrons. This is due to the asymmetric shape of the molecule. Thus each molecule behaves as a dipole having a permanent dipole moment.

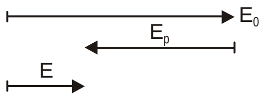
In the presence of an external applied field, these dipoles tend to align themselves along the field and their dipole moments may also increase.

a) **Dielectric Constant :** Suppose a dielectric slab is placed in a uniform electric field E_0. The electric field will polarize the slab, i.e., the positive charges of the molecules will be shifted slightly towards the right and the negative charges towards the left. The right surface of the slab gets positivelycharged and the left surface negatively charged. This sets up a field E_p in the opposite direction which is less than E_0. The net field inside the slab becomes $E = E_0 - E_p$. The ratio

$$K = \frac{E_0}{E}$$

is called the **dielectric constant** of the material.





b) **Dielectric Strength :** The dielectric strength of a dielectric is the maximum value of the electric field that can be applied to the dielectric without its electric breakdown, i.e., without liberating electrons from its atoms (or molecules).

ATMOSPHERIC ELECTRICITY

The atmosphere surrounds the earth like a blanket. The atmosphere is a system with lots of variations. Therefore only its average properties may be described. The radius of earth is about 6400 km and the atmosphere extends to about 300 km above the surface of earth which is about $\frac{1}{20}$th radius of earth.

When we go up, the temperature and density vary. At about 300 km, its density falls to 10^{-10} times its ground level value.

ELECTRICAL PROPERTIES OF ATMOSPHERE

a) At low altitudes the atmosphere is a poor conductor. The conductivity is only due to presence of ions, small nuclei of dirt, water vapour carrying static charge etc. The conductivity in low atmosphere varies a lot. It even varies from day to day.

b) At the top of stratosphere (i.e. about 50 km) the atmosphere is fairly conducting. The conductivity increases from earth's surface towards the top of stratosphere.

c) At ground level there is a vertical electric field of about 100 Vm^{-1} all over the earth. The field weakens at higher altitudes and becomes negligible a 50 km.

d) The potential drop from 50 km to earth's surface is nearly 400 kV. Most of the potential drop occurs at low altitudes.

e) The surface density of earth is $-10^{-9}\ Cm^{-2}$ the total charge being $-0.5\times10^{6}\ C$.

f) The number of protons entering the earth's surface per sec is 2×10^{7} per m^2. This is equivalent to positive charge of $+1800\ C$.

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g) To maintain the constancy of negative charge of earth and potential difference between earth and stratosphere, there are about 4×10^4 thunder storms per day world-wise. This means a storm starts off somewhere in every two second. The duration of each storm is about 1 hour.

h) Within each thunder cloud (or storm) positive charge is carried upward to a height of about 6 km while negative charges collect at about 2 km to 3 km above ground, the bottom of the cloud.

i) The total amount of negative charge may be –20 C to –30 C. At the end of storm, the negative charge bursts along narrow path from cloud to earth to maintain earth at negative potential.

j) In the last stages of a storm, there are about 200 flashes or bolts, each lasting about 2×10^{-3} sec. The peak current in each bolt is about -10^4 A in the downward direction. The calculation shows that each bolt deposits -20 C of charge on earth. After each bolt the thunder cloud gets charged again and gets ready for next bolt.

NAME	ALTITUDE RANGE IN KM	DENSITY BEHAVIOUR	TEMPERATURE BEHAVIOUR
Trophosphere	0 to 12 km	falls 1 to 10^{-1}	Falls uniformly from 290 K to 200 K
Stratosphere	12 to 50 km	falls from 10^{-3}	to rises uniformly from 200 K to 280 K
Mesosphere	50 to 80 km	falls from 10^{-3} ot 10^{-5}	falls uniformly from 280 K to 180 K
Ionosphere	80 to 300 km	falls from 10^{-5} to 10^{-10}	to rises uniformly from 180 K to 700 K

POINTS TO REMEMBER

- **Fundamental forces of nature :**

  (i) Gravitation, (ii) Electromagnetic, (iii) Nuclear, (iv) Weak
- Relative strength 1 : 10^{36} : 10^{39} : 10^{34}
- Charge is **quantised**, the quantum of charge is e = 1.6×10^{-19} C.
- The charge on the electron or proton denoted by e is the smallest charge that exists in nature. All other charges are integral multiple of this charge. (q = ne)
- When two bodies are rubbed against each other there is generally a transfer of electrons from one body to the other. A body which loses electrons is positively charged and the other negatively charged. If the body is an insulator, electric charge remains confined to the rubbed portion but in case of conductor it spreads out on the surface of the conductor.
- Electric field inside a charged conductor is zero. However there is electric field on the surface of conductor which is normal to the surface. In case of irregularly shaped conductor, the electric field E and surface charge density σ vary from point to point ($E = \dfrac{\sigma}{\varepsilon_0}$). At sharp points surface charge density and electric field have greater value. However potential at each point of the same conductor have equal value.

- Electric field lines which are tangent to electric field, originate from positive charges and end on a negative charges. They are perpendicular to the surface of conductor (or any equipotential surface) but do not pass through conductor.
- Work done in a moving a point charge q from point A to point B independent of the path taken and is equal to $q(V_B - V_A)$.
- A free stationary positive (negative) charge moves from higher (lower) to a lower (higher) potential.
- The value of dielectric constant 'k' is infinite for metals and one for vacuum.
- Work done in rotating a dipole is given by $W = U_f - U_i = pE\ (\cos\theta_i - \cos\theta_f)$
- An electric dipole always tries to align itself with the electric field because its potential energy in this configuration is minimum.
- For a charge moving in an electric field, gain (less) in kinetic energy is always equal to loss (gain) in potential energy.

  Hence, $\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) = -q(V_f - V_i)$
- Electric field intensity due to a monopole (point charge) varies inversely as a square of the distance whereas in case of dipole it varies inversely as the cube of the distance.
- Potential of an isolated conductor is inversely proportional the dielectric constant of the medium.
- Potential energy of system having two point charges :

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

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SOLVED EXAMPLES

Example - 1 A polythene piece is rubbed with wool. It developes a (-) ve charge of 3.2×10^{-7} C.

(i) Calculate the no. of electrons transfered?
(ii) What is the charge on wool after rubbing?
(iii) Calculate transfer of mass from wool to polythene

$e=-1.6\times10^{-19}C$ $\qquad m_e=9.1\times10^{-31}kg.$

Solution : Here, $q=-3.2\times10^{-7}$

(i) $n=\frac{q}{e}=\frac{-3.2\times10^{-7}}{-1.6\times10^{-19}}=2\times10^{12}$

(ii) Charge on wool after rubbing $=+3.2\times10^{-7}C$

(iii) Transfer of mass from wool to polythene $=n\times m_e$

$=2\times10^{12}\times9.1\times10^{-31}=18.2\times10^{-19}\ kg$

Example - 2 Two small charged spheres repel each other with a force $2\times10^{-3}N$. The charge on one sphere is twice that on the other. When taken 10 cm further apart, the force is $5\times10^{-4}N$. What are the charges and what was their original distance ?

Solution Let q and 2q be the charges and x the distance between them.

Then $2\times10^{-3}=\frac{1}{4\pi\varepsilon_\circ}.\frac{2q^2}{x^2}$

Again $5\times10^{-4}=\frac{1}{4\pi\varepsilon_\circ}.\frac{2q^2}{(x+0.1)^2}$

$\therefore \quad \frac{20}{5}=\frac{(x+0.1)^2}{x^2}\ or\ \ 2=\frac{x+0.1}{x}\ or\ \ 2x=x+0.1$

or $x=0.1m$

Substituting the value of x so obtained

$$2\times10^{-3}=9\times10^{9}\,\frac{2q^2}{0.1^2}\quad\left(\because\ \frac{1}{4\pi\varepsilon_\circ}=9\times10^{9}\right)$$

or $q^2=\frac{10^{-14}}{9}\quad or\quad q=\frac{10^{-7}}{3}=33.33\times10^{-9}C$

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and $2q = 66.66 \times 10^{-9} C$

So, the charges on the spheres are $33.33 \times 10^{-9} C$ and $66.66 \times 10^{-9} C$

Examples - 3 Two free charges, each +Q, are placed at a distance r form each other. A third charge q is placed on the line joining the above two charges such that all the charges are in equilibrium. What is the magnitude, sign and position of the charge q ?

Solution Let the two free charges be placed at points A and B and the third charge q be placed at a point C on AB such that $AC = x$ and $BC = r - x$.
Consider the equilibrium of the charge Q at A. The forces acting on it due to the charges at B and C must be equal and opposite. This is possible only if the charge q is negative, and

$$F_{AC} = F_{AB}$$

or $$\frac{1}{4\pi\in_\circ}\frac{Qq}{x^2} = \frac{1}{4\pi\in_\circ}\frac{Q^2}{r^2}$$

or $$q = \frac{x^2}{r^2}Q \qquad \text{............................(1)}$$

Now considering the equilibrium of charge q at C, we have

$$F_{CA} = F_{CB}$$

or $$\frac{1}{4\pi\in_\circ}\frac{Qq}{x^2} = \frac{1}{4\pi\in_\circ}\frac{Qq}{(r-x)^2}$$

or $x = r - x$

or $x = r/2$

Substituting in Eq. (1)

$$q = Q/4$$

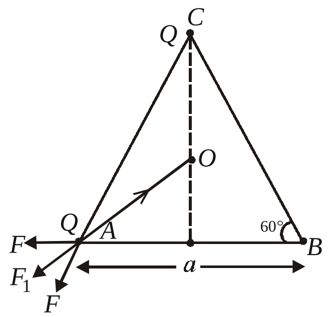
With proper sign: $q = -Q/4$

Example - 4 Three charges, each of value Q, are placed at the corners of an equilateral triangle. A fourth charge q is placed at the centroid of the triangle.

a) If $q = -Q$,will the corner charges move towards the centroid or away from it?

b) What should be the relation between Q the q so that the charges remain stationary ?

Solution: Let a be the length of each side of the triangle.



Distance $OA = \frac{2}{3}(a \sin 60^\circ)$

$$= \frac{2}{3}.a.\frac{\sqrt{3}}{2}$$

$$= a/\sqrt{3}$$

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a) The forces on the charge at A due to those at B and C have magnitude.

$$F = \frac{1}{4\pi \in_\circ} \frac{Q^2}{a^2}$$

The resultant of these forces is

$$F_1 = 2F\cos 30°$$

$$= 2.\frac{1}{4\pi\in_\circ}\frac{Q^2}{a^2}.\frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4\pi\in_\circ}\frac{Q^2}{a^2} \quad \text{along OA}$$

The force on the charge Q at A due to charge $q = -Q$ at O is

$$F_2 = \frac{1}{4\pi\in_\circ}\frac{3Q^2}{a^2} \quad \text{along AO}$$

Since $F_2 > F_1$, the charge at A will move towards O. By symmetry, the charges at B and C will also move towards O.

(b) For equilibrium of the charges

$$F_1 = F_2$$

or $\sqrt{3}\frac{Q^2}{a^2} = \frac{3Qq}{a^2}$

or $q = Q/\sqrt{3}$

with proper sign $q = -Q/\sqrt{3}$

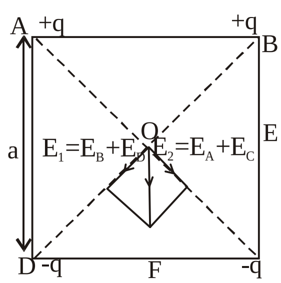
Example - 5 Four charges $+q, +q, -q$ and $-q$ are placed respectively at the corners A,B,C and D of a square of side a , arranged in the given order. Calculate the electric potential and intensity at O, the centre of the square. If E and F are the midpoints of sides BC and CD respectively, what will be the work done in carrying a charge e from O to E and from O to F.

Solution Potential at O

$$V_\circ = \frac{q}{4\pi\in_\circ}\left(\frac{1}{OA} + \frac{1}{OB} - \frac{1}{OC} - \frac{1}{OD}\right)$$

$$= 0\,(as\ \ OA = OB = OC = OD)$$

Intensity at O :



Let E_A, E_B, E_C and E_D be the intensities at O due to the charges at A,B,C, and D respectively, Their directions are as shown. We have

$$E_A = E_B = E_C = E_D = \frac{1}{4\pi\in_o}\frac{q}{(a/\sqrt{2})^2} = \frac{1}{4\pi\in_o}\frac{2q}{a^2}$$

$$\therefore \quad E_1 = E_B + E_D = \frac{1}{4\pi\in_o}\frac{4q}{a^2}$$

and $E_2 = E_A + E_C = \frac{1}{4\pi \in_o}\frac{4q}{a^2}$

$$\therefore E = \sqrt{E_1^2 + E_2^2} = \frac{1}{4\pi\in_o}\frac{4\sqrt{2}q}{a^2}$$

Work done in carrying charge e from O to E

Potential at E,

$$V_E = \frac{q}{4\pi\in_o}\left[\frac{1}{AE}+\frac{1}{BE}-\frac{1}{DE}-\frac{1}{CE}\right]$$

But AE = DE and BE = CE

$$\therefore V_E = 0$$

$$W(0 \to E) = e[V_E - V_0] = 0$$

Work done in carrying charge e from 0 to F

Potential at F, $V_F = \frac{q}{4\pi\in_o}\left(\frac{1}{AF}+\frac{1}{BF}-\frac{1}{DF}-\frac{1}{CF}\right)$

$$= \frac{2q}{4\pi\in_o}\left(\frac{1}{AF}-\frac{1}{DF}\right)$$

$$= \frac{2q}{4\pi\in_o}\left(\frac{1}{a\sqrt{5/2}}-\frac{q}{a/2}\right)$$

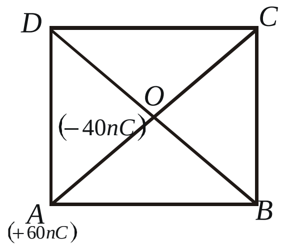
$$= \frac{1}{4\pi\in_o}\frac{4q}{a}\left(\frac{1}{\sqrt{5}}-1\right)$$

$$W(O \to F) = e(V_F - V_o) = \frac{1}{4\pi\in_o}\frac{4eq}{a}\left(\frac{1}{\sqrt{5}}-1\right)$$

Example - 6 A charge of 60nC(nanocoulomb) is placed at the corner A of a square ABCD of side 10cm. Another charge of -40nC is located at the centre of the square. Find the work done in carrying a charge of +5nC from the corner C to the cornerr B of square.

Solution Work done per unit charge is always equal to potential difference. Hence we will calculate the potential at C, and the potential at B and take the potential difference between them. This will give the work done per unit charge

From geometry of the figure



$OA = OB = OC = OD = 5\sqrt{2}\,cm - 5\sqrt{2}\times10^{-2}\,m$

Potential at C

$$= \frac{+60\times10^{-9}}{4\pi\varepsilon_o 10\sqrt{2}\times10^{-2}} + \frac{-40\times10^{-9}}{4\pi\varepsilon_o 5\sqrt{2}\times10^{-2}}$$

$$= \frac{9\times10^9\times60\times10^9}{10\sqrt{2}\times10^{-5}} - \frac{9\times10^9\times40\times10^9}{5\sqrt{2}\times10^{-2}} \qquad \left(\because \frac{1}{4\pi\varepsilon_o} = 9\times10^9\right)$$

$$= 2700\sqrt{2} - 3600\sqrt{2} = -900\sqrt{2} = -1272.6V$$

Potential at $B = \frac{60\times10^{-9}}{4\pi\varepsilon_o\times10\times10^{-2}} + \frac{-40\times10^{-9}}{4\pi\varepsilon_o\times5\sqrt{2}\times10^{-2}}$

$$= 5400 - 3600\sqrt{2} = 5400 - 5090.4 = 309.6V$$

Potential difference between B and C $= V_B - V_C$

$\therefore$ Work done on unit charge from C to B = 1582.2J

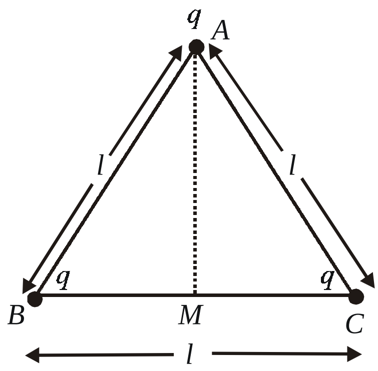
$\therefore$ Work done in taking 5nC from C to B $= 1582.2 \times 5 \times 10^{-9} = 7.91 \times 10^{-6}\, J$

Example - 7 Three point charges, each of 0.1C, are placed at the corners of an equilateral triangle of side 1m. If this system is supplied energy at the rate of 1kW, how much time will be required to move one of the charges on to the midpoint of the line joining the other two ?

Solution Initially the three charges are placed at the three corners A,B,C of the triangle. Initial potential energy

$$U_i = \frac{1}{4\pi \in_o}\left[\frac{q^2}{l} + \frac{q^2}{l} + \frac{q^2}{l}\right]$$

$$= \frac{3q^2}{4\pi \in_o l}$$



Suppose the charge at A is moved to the midpoint M of BC. The final potential energy of the system

$$U_f = \frac{1}{4\pi \in_o}\left[\frac{q^2}{l/2} + \frac{q^2}{l/2} + \frac{q^2}{l}\right]$$

$$= \frac{5q^2}{4\pi \in_o l}$$

Increase in energy

$$\Delta U = U_f - U_i = \frac{q^2}{4\pi \in_o l}(5-3)$$

$$= \frac{1}{4\pi \in_o} \cdot \frac{2q^2}{l}$$

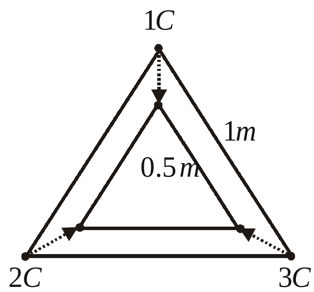
$$= \frac{9 \times 10^9 \times 2 \times (0.1)^2}{1}$$

$$= 18 \times 10^7\, J$$

Energy supplied per second = 1000 J

Time taken $= \dfrac{18 \times 10^7}{1000} = 50\ hrs$.

Example - 8 Three point charges 1C, 2C and 3C are placed at the corners of an equilateral triangle of side 1m. Calculate the work required to move these charges to the corners of a smaller equilateral triangle of side 0.5m, as shown in the figure.



Solution Work required = (P.E of the charges on the inner triangle) – (P.E of the charges on the outer triangle)

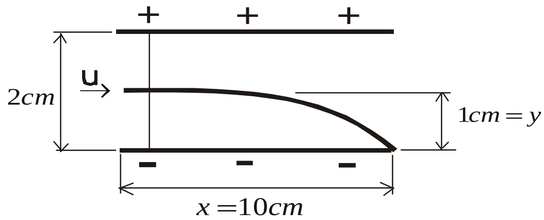
$$= \frac{1}{4\pi \in_o}\left[\frac{1\times2}{0.5} + \frac{1\times3}{0.5} + \frac{2\times3}{0.5} - \frac{1\times2}{1} - \frac{1\times3}{1} - \frac{2\times3}{1}\right]$$

$$= 9 \times 10^9 [4 + 6 + 12 - 2 - 3 - 6]$$

$$= 9.9 \times 10^{10}\, J$$

Example - 9 A particle having a charge of $1.6\times10^{-19}C$ enters between the plates of a parallel plate capacitor. The initial velocity of the particle is parallel to the plates. A potential difference of 300V is applied to the capacitor plates. If the length of the capacitor plates is 10cm and they are separated by 2cm, calculate the greatest initial velocity for which the particle will not be able to come out of the plates. The mass of the particle is $12\times10^{-24}kg$.

Solution Here, V=300V, d=2cm



$$\Rightarrow E=\frac{V}{d}=\frac{300}{2\times10^{-2}}=15000\ V/M$$

Now, $y=\frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x}{u}\right)^2$

$$\Rightarrow\ u^2=\frac{1}{2}\frac{qEx^2}{my}$$

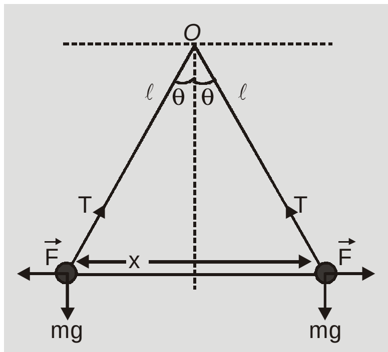
$$x=10cm=0.1m$$
$$y=1cm=0.01m$$

$$\Rightarrow\ u^2=\frac{1.6\times10^{-19}\times15000\times(0.1)^2}{2\times12\times10^{-24}\times(0.01)}$$

$$\therefore\ u=10^4 m/s$$

Example - 10 Two similar small balls having mass m and charge q are suspended by silk strings having length ℓ, according to the figure. If in the figure θ is an acute angle then for equilibrium what will the distance between the centre of the two balls.

Solution : The force & acting on the system are as follows.
T is tension in string, F is coulomb force and mg is weight.
For equilibrium,
T cos θ = mg and T sin θ = F



$$\therefore\quad \tan\theta=\frac{F}{mg}=\frac{1}{4\pi\epsilon_0}\frac{q^2}{x^2\,mg}$$

If θ is small

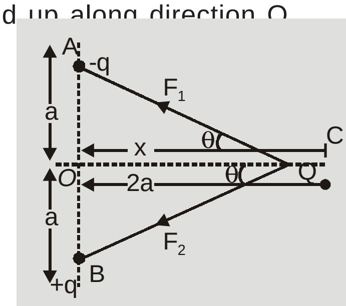
$$\therefore\ \tan\theta\simeq\sin\theta\approx\frac{x}{2\ell}$$

$$\therefore\ \frac{x}{2\ell}=\frac{1}{4\pi\epsilon_0}\frac{q^2}{x^2\,mg}$$

$$x=\left\{\frac{q^2\ell}{2\pi\epsilon_0\,mg}\right\}^{1/3}$$

Example - 11 Two similar negative charges –q are situated at point (0, a) & (0, –a) along Y-axis. A positive charge Q is left from point (2a, 0). Analyse the motion of Q.

Solution : Due to symmetry the y components of forces acting on Q due to charges –q at A and B will balance each other and the x components will add up along direction O
If at any instant Q is at a distance x from O
F = F_1 cos θ + F_2 cos θ = $2F_1$ cosθ [$\because$ $F_1 = F_2$]



$$=2\frac{1}{4\pi\epsilon_0}\left[\frac{-qQ}{(a^2+x^2)}\right]\left[\frac{x}{(a^2+x^2)^{1/2}}\right]$$

$$F=-\frac{1}{4\pi\epsilon_0}\frac{2qQx}{(a^2+x^2)^{3/2}}\quad\text{i.e. } F\propto -x$$

$\because$ F $\propto$ –x, the motion is oscillatory having amplitude 2a but it will not be S.H.M.
If x << a

$$F = -\frac{1}{4\pi\epsilon_0}\frac{2qQ}{a^3}x = -kx$$

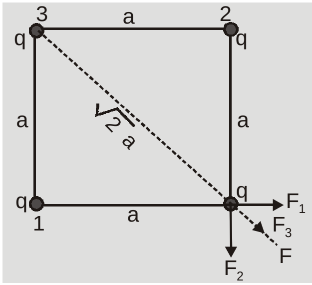
and the motion is S.H.M. with time period T, where

$$T = 2\pi\sqrt{\frac{m}{k}} \quad = 2\pi\sqrt{\frac{4\pi\epsilon_0\, ma^3}{2Qq}}$$

Example - 12 Consider four equal charges (q, each) placed on the corners of a square with side a. Determine the magnitude and direction of the resultant force on the charge on lower right corner.

Solution : The forces on the charge on lower right corner due to charges 1, 2, 3 are $F_1 = kq^2/a^2$, $F_2 = kq^2/a^2$, $F_3 = kq^2/2a^2$

The resultant of F_1 and F_2 is



$$F_{12} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 90^o}$$

$$= \sqrt{2}kq^2/a^2$$

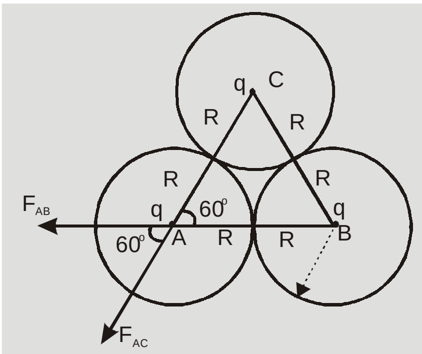
This is in the direction parallel to F_3. Therefore the total force on the said charge is $F = F_{12} + F_3$

$$F = \frac{1}{2}\frac{kq^2}{a^2}(1+2\sqrt{2})$$ The direction of F is 45º below the horizontal line

Example-13 Three identical spheres each having a charge q and radius R, are kept in such a way that each touches the other two. Find the magnitude of the electric force on any sphere due to other two.

Solution : For external points a charged sphere behaves as if the whole of its charge was concentrated at its centre.

Force on A due to B is



$$F_{AB} = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{(2R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4R^2}\text{along}\,\overrightarrow{BA}$$

Force on A due to C.

$$F_{AC} = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{(2R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4R^2}\text{along}\,\overrightarrow{CA}$$

Now, angle between BA and CA is 60^0 and $F_{AB} = F_{AC} = F$.

$$F_A = \sqrt{F^2 + F^2 + 2FF\cos 60} = \sqrt{3}F = \frac{1}{4\pi\varepsilon_0}\frac{(\sqrt{3})}{4}\left[\frac{q}{R}\right]^2$$

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Example - 14 Along x-axis at positions x = 1, x = 2, x = 4 and x = 8 charges q is placed. What will be electric field at x = 0 due to these charges. What will be the value of electric field if the charges are alternately positive and negative.

...............|.....................|...............|.....................|.............∞

x = 1 x = 2 x = 4 x = 8

Solution : By superposition theory

$$E = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{8^2}+.....\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+.....\right]$$

Terms in the braket are G.P. with first term a = 1 and common ratio r = $\frac{1}{4}$ .Its sum

$$S = \frac{a}{1-r}$$

$$\therefore\ E = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{1-1/4}\right] = \left[\frac{1}{4\pi\epsilon_0}\right]\frac{4}{3}q$$

If the charges are alternately positive and negative

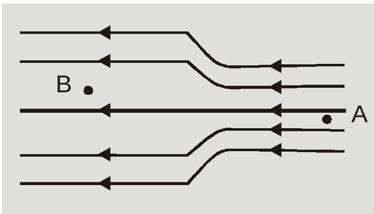
$$E = \frac{q}{4\pi\epsilon_0}\left[1-\frac{1}{4}+\frac{1}{16}-\frac{1}{64}+....\right]$$

where a = 1, r = –1/4

$$E = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{1-(-1/4)}\right]$$

$$= \left[\frac{q}{4\pi\epsilon_0}\right]\frac{4q}{5}$$

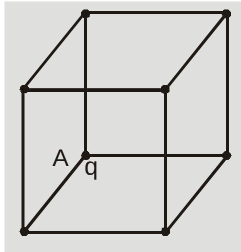
Example - 15 Fig. shows field lines of an electric field, the line spacing perpendicular to the page is same every where. If the magnitude of the field at A is 40 N/C, then what is the magnitude of the field at B ?



Solution : From the diagram we notice that density of lines at B is approximately half of that at A. Since the density of field lines is proportional to the strength of field, we expect B = 20 N/C.

Example - 16 A point charge q is placed at a corner of a cube with side L. Find flux through entire surface and flux through each face.

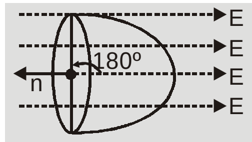
Solution : A corner of a cube can be supposed to be the centre of a big cube made up of 8 such cubes, therefore flux through it is $q/8\in_0$. The direction of E is parallel to the three faces that pass through this face, thus flux through these is zero.



Flux through each of the other three faces

$$=\frac{1}{3}\left(\frac{q}{8\in_0}\right)=\frac{q}{24\in_0}$$

Example - 17 According to the figure, a hemi-spherical object is located in an electric field. Find the outward flux through its curved surface.



Solution : Total outward flux $\phi = \phi_{CS} + \phi_n$

where ϕ_{CS} = flux through curved surface and

ϕ_n = flux through circular base

$\because$ No charge is associated with this surface

$\therefore\ \phi_{CS} = -\phi_n$

$\because$ curved surface perpendicular electric field

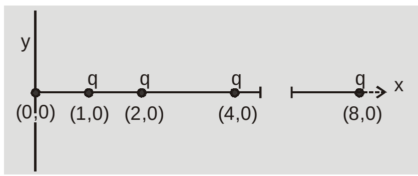
$$\phi_n = E\times\pi R^2\cos 180^o$$

$$= -E\ \pi R^2$$

$\therefore\ \phi_{CS} = -\phi_n = E\pi R^2$

Example - 18 Infinite charges of magnitude q are placed at coordinates x = 1m, 2m, 8m respectively along the x-axis. Find the value of potential at x = 0 due to these charges.

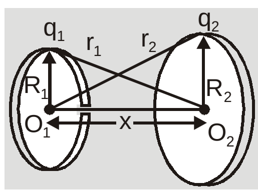
Solution : Resultant potential at x = 0



$$V = kq\left[\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+....\right] = kq\left[\frac{1}{1-1/2}\right]$$

$= 2kq$ [$\therefore$ Sum of above G.P., S. = $\frac{a}{1-r}$]

Example - 19 Two circular loops of radius 0.05 m and 0.09 m respectively are put such that their axes coincide and their centres are 0.12 m apart. Charge of 10^{-6} coulomb is spread uniformly on each loop. Find the potential difference between the centre of loops.



Solution : The potential at the centre of a ring will be due to charge on both the rings and as every element of a ring is at a constant distance from the centre,

$$\text{So, } V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{R_1}+\frac{q_2}{\sqrt{R_2^2+x}}\right] = 9\times 10^9\left[\frac{10^{-4}}{5}+\frac{10^{-4}}{\sqrt{9^2+12^2}}\right] = 2.40\times 10^5\ V$$

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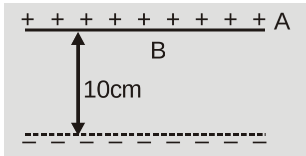
Similarly, $V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_2}{R_2} + \frac{q_1}{\sqrt{R_1^2 + x^2}}\right]$ or $V_2 = 9 \times 10^5 \left[\frac{1}{9} + \frac{1}{13}\right] = \frac{198}{117} \times 10^5$

$= 1.69 \times 10^5$ V

So, $V_1 - V_2 = (2.40 - 1.69) \times 10^5 = 71$ kV

Example - 20 An oil drop 'B' has charge 1.6×10^{-19} C and mass 1.6×10^{-14} kg. If the drop is in equilibrium position, then what will be potential diff. between the plates. [The distance between the plates is 100mm]

Solution : For equilibrium, electric force = weight of drop



$\Rightarrow \quad qE = mg \quad$ or $q = \frac{V}{d} = mg$

$\Rightarrow \quad V = \frac{mgd}{q} = \frac{1.6 \times 10^{-14} \times 9.8 \times 10 \times 10^{-3}}{1.6 \times 10^{-19}} = 10^4$ volt

EXERCISES

LEVEL - I

1 An electric field can deflect :
(1) X-rays (2) Neutrons
(3) α-particles (4) γ-rays

2 Relative permittivity of mica is :
(1) one (2) less than one
(3) more then one (4) infinite

3 Two identical metallic sphere are charged with 10 and -20 units of charge. If both the spheres are first brought into contact with each other and then are placed to their previous positions, then the ratio of the force in the two situations will be :-
(1) -8 : 1 (2) 1 : 8
(3) -2 : 1 (4) 1 : 2

4 Two equal and like charges when placed 5 cm apart experience a repulsive force of 0.144 newton. The magnitude of the charge in microcoloumb will be :
(1) 0.2 (2) 2
(3) 20 (4) 12

5 For an electrostatic system which of the statement is always true :
[i] electric lines are parallel to metallic surface.
[ii] electric field inside a metallic surface is zero.
[iii] electric lines of force are perpendicular to equi-potential surface.
(1) (i) and (ii) only (2) (ii) and (iii) only
(3) (i) and (iii) only (4) (i), (ii) and (iii)

6 A negative charge is placed at some point on the line joining the two +Q charges at rest. The direction of motion of negative charge will depend upon the :
(1) position of negative charge alone
(2) magnitude of negative charge alone
(3) both on the magnitude and position of negative charge
(4) magnitude of positive charge.

7 If Q = 2 coloumb and force on it is F = 100 newton, then the value of field intensity will be :
(1) 100 N/C (2) 50 N/C
(3) 200 N/C (4) 10 N/C

8 A force of 3000 N is acting on a charge of 3 coloumb moving in a uniform electric field. The potential difference between two point at a distance of 1 cm in this field is :
(1) 10V (2) 90V
(3) 1000V (4) 9000V

9 If we move in a direction opposite to the electric lines of force :
(1) electrical potential decreases. (2) electrical potential increases.
(3) electrical potential remains uncharged (4) nothing can be said.

10 Two infinite linear charges are placed parallel at 0.1 m apart. If each has charge density of 5m C/m, then the force per unit length of one of linear charges in N/m is :
(1) 2.5 (2) 3.25
(3) 4.5 (4) 7.5

11 The electric field intensity due to a uniformly charged sphere is zero :
(1) at the centre (2) at infinity

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(3) at the centre and at infinite distance (4) on the surface

12 Two spheres of radii 2 cm and 4 cm are charged equally, then the ratio of charge density on the surfaces of the spheres will be -
(1) 1 : 2 (2) 4 : 1
(3) 8 : 1 (4) 1 : 4

13 Total charge on a sphere of radii 10 cm is 1 μC. The maximum electric field due to the sphere in N/C will be -
(1) 9 x 10^{-5} (2) 9 x 10^{3}
(3) 9 x 10^{5} (4) 9 x 10^{15}

14 A charged water drop of radius 0.1 μm is under equilibrium in some electric field. The charge on the drop is equivalent to electronic charge. The intensity of electric field is (g = 10 m/s^2)-
(1) 1.61 NC^{-1} (2) 26.2 NC^{-1}
(3) 262 NC^{-1} (4) 1610 NC^{-1}

15 Two large sized charged plates have a charge density of +σ and -σ. The resultant force on the proton located midway between them will be -
(1) $\sigma e/\in_0$ (2) $\sigma e/2\in_0$
(3) $2\sigma e/\in_0$ (4) zero

16 The distance between two plates is 2 cm, when an electric potential of 10 volt is applied to both the plates, then the value of electric field will be -
(1) 20 N/C (2) 500 N/C
(3) 5 N/C (4) 250 N/C

17 The charge density of an insulating infinite surface is (e/π) C/m^2 then the field intensity at a nearby point in volt/meter will be -
(1) 2.88 x 10^{-12} (2) 2.88 x 10^{-10}
(3) 2.88 x 10^{-9} (4) 2.88 x 10^{-19}

18 Two objects A and B are charged with equal charge The potential of A relative to B will be -
(1) more (2) equal
(3) less (4) indefinite

19 The potential due to a point charge at distance r is -
(1) proportional to r. (2) inversely proportional to r.
(3) proportional to r^2. (4) inversely proportional to r^2

20 The dimensions of potential difference are -
(1) $ML^2T^{-2}Q^{-1}$ (2) $MLT^{-2}Q^{-1}$
(3) $MT^{-2}Q^{-2}$ (4) $ML^2T^{-1}Q^{-1}$

21 An object is charged with positive charge. The potential at that object will be -
(1) positive only (2) negative only
(3) zero always (4) may be positive, negative or zero.

22 In H atom, an electron is rotating around the proton in an orbit of radius r. Work done by an electron in moving once around the proton along the orbit will be -
(1) ke/r (2) ke^2/r^2
(3) $2\pi re$ (4) zero

23 The potential at 0.5 Å from a proton is -
(1) 0.5 volt (2) 8μ volt
(3) 28.8 volt (4) 2 volt

24 Two metallic spheres which have equal charges, but their radii are different, are made to touch each other and then separated apart. The potential the spheres will be -
(1) same as before (2) more for bigger
(3) more for smaller (4) equal

25 A conducting shell of radius 10 cm is charged with 3.2 x 10^{-1} C. The electric potential at a distance 4cm from its centre in volt be -
(1) 9 x 10^{-9} (2) 288
(3) 2.88 x 10^{-8} (4) zero

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Level - II

1 Two charges of +1 μC & + 5 μC are placed 4 cm apart, the ratio of the force exerted by both charges on each other will be -

(1) 1 : 1 (2) 1 : 5
(3) 5 : 1 (4) 25 : 1

2 If an electron is placed in a uniform electric field, then the electron will :

(1) experience no force.
(2) moving with constant velocity in the direction of the field.
(3) move with constant velocity in the direction opposite to the field.
(4) accelerate in direction opposite to field.

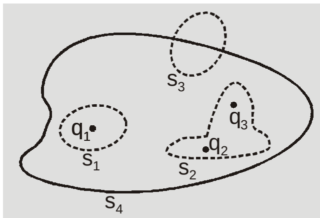
3 A body has 80 microcoulomb of charge. Number of additional electrons on it will be :

(1) 8×10^{-5} (2) 80×10^{15}
(3) 5×10^{14} (4) 1.28×10^{-17}

4 Two identical metallic balls carry charges of $+20\mu C$ and $-10\mu C$. They are put in contact and again separated to the same distance as before. What will be the ratio of initial to final force between them ? Ignore the nature of force.

(1) 2 : 1 (2) 4 : 1
(3) 8 : 1 (4) 16 : 1

5 Three charges $q_1 = 1 \times 10^{-6}$, $q_2 = 2 \times 10^{-6}$, $q_3 = -3 \times 10^{-6}$ C have been placed, as shown in figure, in four surfaces S_1, S_2, S_3 and S_4 electrical flux emitted from the surface S_2 in N–m²/C will be -

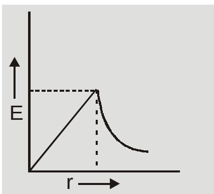


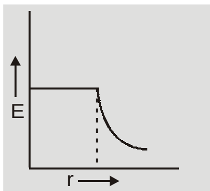
(1) $36\pi \times 10^3$ (2) $-36\pi \times 10^3$
(3) $36p \times 10^9$ (4) $-36p \times 10^9$

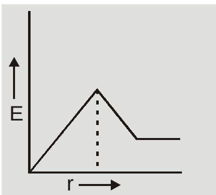
6 The intensity of an electric field at some point distant r from the axis of infinite long pipe having charges per unit length as q wil be :

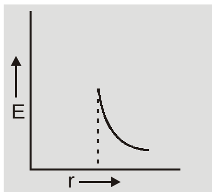
(1) proportional to r^2 (2) proportional to r^3
(3) inversely proportional to r. (4) inversely proportional to r^2.

7 Which of the following represents the correct graph for electric field intensity and the distance r from the centre of a hollow charged metal sphere or solid metallic conductor of radius R :

(1) 

(2) 

(3) 

(4) 

8 The electric field near the conducting surface of a uniform charge density σ will be -

(1) σ / ϵ_0 and parallel to surface. (2) $2\sigma / \epsilon_0$ and parallel to surface.
(3) σ / ϵ_0 and perpendicular to surface. (4) $2\sigma / \epsilon_0$ and perpendicular to surface.

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9 Two parallel charged plates have a charge density +σ and -σ. The resultant force on the proton located outside the plates at some distance will be -

(1) $2\sigma e/\epsilon_0$ (2) $\sigma e/\epsilon_0$

(3) $\sigma e/2\epsilon_0$ (4) zero

10 In electrostatics the potential is equivalent to -
(1) temperature in heat (2) height of levels in liquids
(3) pressure in gases (4) all of the above

11 Two parallel plates have charges + Q and - Q, with potential difference V between them. If the distance between the plates is increased then the potential difference will -
(1) decrease (2) increase
(3) be same as before. (4) depend upon the metal of plates

12 An uncharged conductor A is brought close to another charged conductor B, then the charge on B
(1) will increase but potential will be constant. (2) will be constant but potential will increase
(3) will be constant but potential decreases. (4) and the potential both are constant.

13 Two points (0, a) and (0, -a) have charges q and -q respectively then the electrical potential at origin will be-

(1) zero (2) kq/a

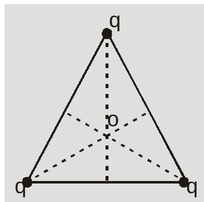
(3) kq/2a (4) $kq/4a^2$

14 The charges of same magnitude q are placed at four corners of a square of side a. The value of potential at the centre of square will be -

(1) 4kq/a (2) $4\sqrt{2}kq/a$

(3) $4kq\sqrt{2a}$ (4) $kq/a\sqrt{2}$

15 Three equal charges are placid at the three corners of an isosceles triangle as shown in the figure. The statement which is true fro electric potential V and the field intensity E at the centre of the triangle -



(1) V = 0, E = 0 (2) V = 0, E ≠ 0

(3) V ≠ 0, E = 0 (4) V ≠ 0, E ≠ 0

16 A wire of 5 m length carries a steady current. If it has an electric field of 0.2 V/m, the potential difference across the wire in volt will be -

(1) 25 (2) 0.04

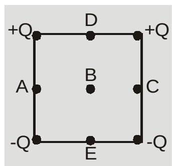
(3) 1.0 (4) none of the above

17 A nucleus has a charge of + 50e. A proton is located at a distance of 10^{-12} m. The potential at this point in volt will be -

(1) 14.4×10^4 (2) 7.2×10^4

(3) 7.2×10^{-12} (4) 14.4×10^8

18 For the arrangement of charges shown in the figure, potential is zero at -



(1) A, B and C (2) D, B and E

(3) B only (4) A, B, C, D, and E

19 A spherical charged conductor has σ as the surface density of charge. The electric field on its surface is E. If the radius of the sphere is doubled keeping the surface density of charge unchanged, what will be the electric field on the surface of the new sphere ?
(a) E/4 (b) E/2
(c) E (d) 2E

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20 The electric potential inside a uniformly charged sphere has the value which -

(1) increase with distance from the centre.
(2) decreases with distance from the centre.
(3) is equal at all the points.
(4) is zero at all the points.

21 Two spheres of radii R and 2R are charged and then connected by a conducting wire, then the charge will

(1) flow from smaller sphere to the bigger sphere.
(2) flow from bigger sphere to the smaller sphere
(3) not flow.
(4) oscillate between the spheres.

22 The potential difference between two spheres of radii r_1 and r_2 is zero. The ratio of their charges Q_1/Q_2 will be-

(1) r_1/r_2
(2) r_2/r_1
(3) r_1^2/r_2^2
(4) r_1^3/r_2^3

23 The potential on the conducting spheres of radii r_1 and r_2 is same, the ratio of their charge densities will be-

(1) r_1/r_2
(2) r_2/r_1
(3) r_1^2/r_2^2
(4) r_2^2/r_1^2

24 64 charged drops coalesce to from a bigger charged drop. The potential of bigger drop will be times that of smaller drop -

(1) 4
(2) 16
(3) 64
(4) 8

25 The electric potential outside a uniformly charged sphere at a distance 'r' is ('a' being the radius of the sphere)-

(1) directly proportional to a^3
(2) directily proportional to r.
(3) inversely proportional to r.
(4) inversely proportional to a^3.

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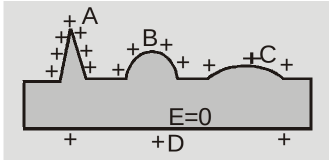
Level - III

1. In the following figure an isolated charged conductor is shown. The correct statement will be -



(1) $E_A > E_B > E_C > E_D$ (2) $E_A < E_B < E_C < E_D$
(3) $E_A = E_B = E_C = E_D$ (4) $E_B = E_C$ and $E_A > E_D$

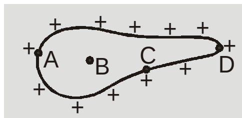
2. If the above question, the potential has correct relations as given -
(1) $V_A > V_B > V_C > V_D$ (2) $V_A > V_B \geq V_C > V_D$
(3) $V_D = V_C = V_B = V_A$ (4) $V_C < V_B > V_A > V_D$

3. In the above question, the surface charge densities have the correct relation is -
(1) $\sigma_A > \sigma_B > \sigma_C > \sigma_D$ (2) $\sigma_A = \sigma_B = \sigma_C = \sigma_D$
(3) $\sigma_D > \sigma_C > \sigma_B > \sigma_A$ (4) $\sigma_C < \sigma_B > \sigma_A > \sigma_D$

4. Van-dee Graph generator is used in -
(1) nuclear power establishments
(2) in accelerating charged particles to very high potential
(3) lighting and heating
(4) lighting only

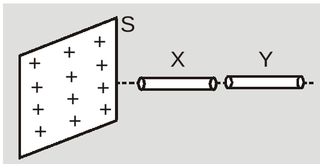
5. Two identical pith-balls of mass m and having charge q are suspended from a point by weight-less strings of length 'ℓ'. If both the strings make an angle of 'θ' with the vertical, then the distance between the balls will be (tanking θ to be small) -
(1) $(q^2\ell/2\pi\in_0 mg)^{1/3}$ (2) $(q^2\ell/4\pi\in_0 mg)^{1/3}$
(3) $(q\ell^2/4\pi\in_0 mg)^{1/3}$ (4) $(q\ell^2/2\pi\in_0 mg)^{1/3}$

6. For the isolated charged conductor shown in fig. the potential at points A, B, C and D are V_A, V_B, V_C and V_D respectively. Then -



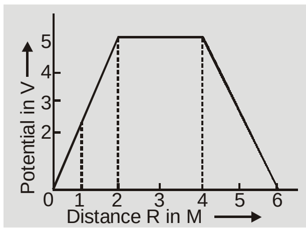
(1) $V_A = V_B > V_C > V_D$ (2) $V_D > V_C > V_B = V_A$
(3) $V_D > V_C > V_B > V_A$ (4) $V_D = V_C = V_B = V_A$

7. A non conducting sheet S is given a uniform charge density s. Two uncharged thin and small metal rods X and Y are placed near the sheet as shown. Then, the correct statement is -



(1) S attracts both X and Y
(2) X attracts both S and Y
(3) Y attracts both S and X
(4) all of the above

8. The variation of potential with distance R from fixed point is shown in fig. The electric field at R = 5m is -



(1) 2.5 V/m (2) –2.5 V/m
(3) 2/5 V/m (4) –2/5 V/m

9. A charged spherical conductor of radius R carries a charge A point test charge q_0 is placed at a distance x from the surface of the conductor . The force experienced by the test charge will be proportional to
(a) $(R+x)^2$ (b) $\frac{1}{(R+x)^2}$
(c) $(R-x)^2$ (d) $\frac{1}{(R-x)^2}$

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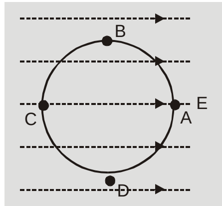
10. Two charges 9e and 3e are placed at a distance r. The distance of the point where the electric field intensity will be zero is -

(1) $r/(1+\sqrt{3})$ from 3e charge
(2) $r/(1+\sqrt{3})$ from 9e charge
(3) $r/(1-\sqrt{3})$ from 3e charge
(4) $r/(1+1/\sqrt{3})$ from 3e charge

11. The electric field in a region surrounding the origin is uniform and along the x-axis. A small circle is drawn with the centre at the origin cutting the axes at points A, B, C, D having coordinates (a, 0); (0, a); (–a, 0); (0, –a) respectively as shown in fig. Then the potential is minimum at -

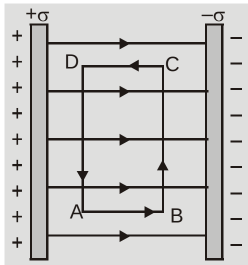


(1) A
(2) B
(3) C
(4) D

12. If an electron has an initial velocity in a direction different from that of an electric field the path of the electron is-

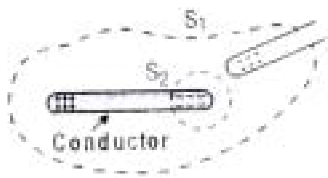
(1) a straight line
(2) a circle
(3) an ellipse
(4) a parabola

13. Consider equal and oppositely charged oppositely charge large parallel plates, with charge density $\pm\sigma$. A small charge q_0 is moved along the rectangular path ABCDA where side AB = x and side BC = y. Then correct statement(s) is (are) -



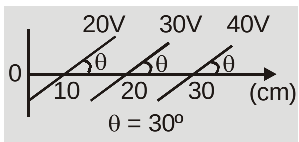
(1) work done by electric field along path AB is positive and equal to $q_0\sigma x/\varepsilon_0$.
(2) work done by electric field along path BC is zero
(3) work done by electric field along the path ABCDA is zero
(4) all of the above

14. Charge on an originally uncharged conductor is separated by holding a positively charged rod very closely nearby, as in Fig. Assume that the induced negative charge on the conductor is equal to the positive charge q on the rod then, flux through surface S_1 is -



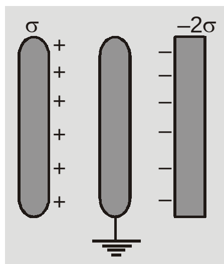
(1) zero
(2) q_0/ε_0
(3) $-q_0/\varepsilon_0$
(4) none of the above

15. Some equipotential surfaces are shown in the figure. The magnitude and direction of the electric field is-



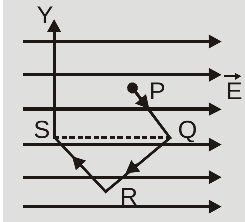
(1) 100 V/m making angle 120^0 with the x-axis
(2) 100 V/m making angle 60^0 with the x-axis
(3) 200 V/m making angle 120^0 with the x-axis
(4) none of the above

16. The metal plate on the left in fig. carries a surface charge of +s per unit area. The metal plate on the right has a surface charge of -2s per unit area. It is assumed that the plates are large and the central plate is connected to zero. Then the charge densities on the left and right surface of the central plate are, respectively -



(1) $-\sigma, +\sigma$
(2) $-2\sigma, +2\sigma$
(3) $-\sigma, +2\sigma$
(4) none of the above

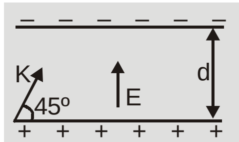
17 Point charge q moves from point P to point S along the path PQRS (as shown in Fig.) in a uniform electric field E pointing co-parallel to the positive direction of X-axis. The coordinates of the points P, Q, R, and S are (a, b, 0), (2a, 0, 0), (a, -b. 0) and (0, 0, 0) respectively. The work done by the field in the above process is given by the expression-



(1) qEa (2) –qEa

(3) $qEa\sqrt{2}$ (4) $qE\sqrt{(2a)^2+b^2}$

18 An electron is projected as in fig. with kinetic energy K, at an angle q = 45^0 between two charged plates. The magnitude of the electric field so that the electron just fails to strike the upper plate, should be greater than -



(1) K/qd (2) 2K/qd

(3) K/2qd (4) infinite

19 Two plastic rods of equal lengths ($L = \pi R$) one of charge q and other of charge -q, form a circle of radius R in an xy plane. The charge is distributed uniformly on both rods. Then the electric field at the centre of circle is-

(1) zero (2) $q/4\pi\varepsilon_0 R^2$

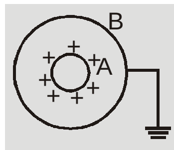
(3) $q/2\pi^2\varepsilon_0 R^2$ (4) $q/\pi^2\varepsilon_0 R^2$

20 Two identical thin rings, each of radius R metre are coaxially placed at distance R metre apart. If Q_1 and Q_2 coulomb are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to that of the other is-

(1) zero (2) $q(Q_1-Q_2)\left(\sqrt{2}-1\right)/\sqrt{2}(4\pi\varepsilon_0 R)$

(3) $q\sqrt{2}(Q_1-Q_2)/4\pi\varepsilon_0 R$ (4) $q(Q_1+Q_2)\left(\sqrt{2}+1\right)/\sqrt{2}\,4\pi\varepsilon_0 R$

21 A and B are concentric conducting spherical shells. A is given a positive charge while B is earthed. Then-



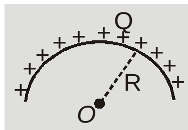
(1) A and B both will have the same charge densities

(2) the potential inside A and outside B will zero

(3) the electric field between A and B is none zero

(4) the electric field inside A and outside B is non zero.

22 A semicircular ring of radius R is given a uniform charge Then the electric field and electric potential at its centre will be -



(1) $\frac{Q}{4\pi\epsilon_0 R^2}, \frac{Q}{4\pi\epsilon_0 R}$ (2) $\frac{Q}{2\epsilon_0 \pi^2 R^2}, \frac{Q}{4\pi\epsilon_0 R}$

(3) $\frac{Q}{4\pi\epsilon_0 R}, \frac{Q}{2\pi\epsilon_0 R}$ (4) zero, zero

23 Mark the wrong statement -

(1) Equipotential surface never cross and other

(2) For a uniformly charged nonconducting sphere, the electric potential at the centre of the sphere is 1.5 times that at the surface

(3) If potential in a certain region in non zero constant, then the electric field in that region will also be non zero constant

(4) Inside a spherical charged shell, the electric field is zero but the electric potential is the same as that at the surface.

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24 A charge +q is fixed at each of the points $x = x_0,\ x = 3x_0,\ x = 5x_0 ...$ ad inf. of the x-axis and a charge -q is fixed at each of the points x= $2x_0$, x=$4x_0$, x=$6x_0$..... an inf. Here x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it be $Q/4\pi\in_0$ r. Then, the potential at te origin due to the above system of charge is

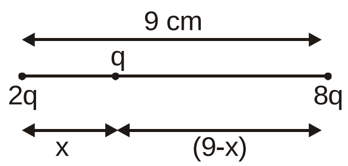
(1) 0

(2) $\frac{q}{8\pi\varepsilon_0 x_0 \log 2}$

(3) ∞

(4) $\frac{g \log 2}{4\pi\varepsilon_0 x_0}$

25 In the follwoing fig. where the change q must be kept so that the potential energy of the system will be minimum?



(1) 3 cm

(2) 2 cm

(3) 4 cm

(4) none of these

PROBLEMS ASKED IN VARIOUS EXAMS

1 Two identical conductors of copper and aluminum are placed in an identical electric field. The magnitude of induced charge in the aluminum will be - **[AIIMS 1999]**

(1) zero (2) greater than in copper

(3) less than in copper (4) equal to that in copper

2 The study of the effects associated with electric charge at rest is known as - **[AIIMS 1999]**

(1) electrostatics (2) magnetostatic

(3) electromagnetism (4) none of these

3 Two electric charges 12μC and –6μC are placed 20 cm apart in air. There will be a point P at which electric potential is zero on the line joining these two charges and outside, excluding the region between them. The distance of P from –6μC charge is - **[EAMCET ENGG. 2000]**

(1) 0.10 m (2) 0.15m

(3) 0.20m (4) 0.25m

4 The displacement $\vec{r}$ of a charge Q in an electric field $\vec{E} = e_1\hat{i} + e_2\hat{j} + e_3\hat{k}$ is $\vec{r} = a\hat{i} + b\hat{j}$. The work done is - **[EAMCET ENGG. 2000]**

(1) $Q(ae_1 + be_2)$ (2) $Q\sqrt{(ae_1)^2 + (be_2)^2}$

(3) $Q(e_1 + e_2)\sqrt{a^2 + b^2}$ (4) $(\sqrt{e_1^2 + e_2^2})\ (a + b)$

5 An electron of mass m and charge e is accelerated from rest through a potential difference V in vacuum . The final speed of the electron will be - **[MP PMT 2000]**

(1) $V\sqrt{e/m}$ (2) $\sqrt{eV/m}$

(3) $\sqrt{2eV/m}$ (4) 2eV/m

6 An insulated charge conducting sphere of radius 5 cms has potential of 10 V at the surface. What is the potential at centre - **[AIIMS 2000]**

(1) 10 V (2) zero

(3) same as that at 5 cms from the surface (4) same as that at 25 cms from the surface

7 Let E_a be the electric field due to a dipole in its axial plane distant 1 and let E_q be the field in the equatorial plane distant ℓ. The relation between E_a and E_q is - **[AIIMS 2000]**

(1) $E_a = E_q$ (2) $E_a = 2E_q$

(3) $E_q = 2E_a$ (4) $E_a = 3E_q$

8 A point positive charge of 'Q' unit is moved round another positive charge of Q unit on a circular path. If the radius of the circle is r, the work done on the charge Q' in making one complete revolution is - **[HARAYANA PMT 2000, BHU MED. 2000]**

(1) $\frac{Q}{4\pi\varepsilon_0 r}$ (2) $\frac{QQ'}{4\pi\varepsilon_0 r}$

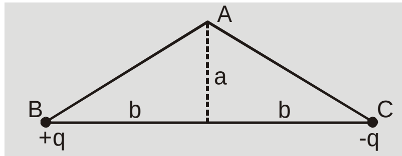
(3) $\frac{Q'}{4\pi\varepsilon_0 r}$ (4) zero

9 Three point charges Q_1 , Q_2 and Q_3 in that order are placed equally spaced along k straight line. Q_2 and Q_3 are equal in magnitude but opposite in sign. If the net force on Q_3 is zero, the value of Q_1 is - **[UPSEAT 2000]**

(1) $Q_1 = |Q_3|$ (2) $Q_1 = \sqrt{2}\,|Q_3|$

(3) $Q_1 = 2\,|Q_3|$ (4) $Q_1 = 4\,|Q_3|$

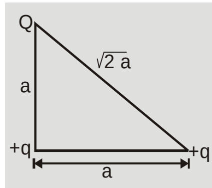
10 As shown in the figure, charge +q and –q are placed at the vertices B and C of an isosceles triangle. The potential at the vertex A is - **[MP PET 2000]**



(1) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{\sqrt{a^2+b^2}}$ (2) zero

(3) $\frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{a^2+b^2}}$ (4) $\frac{1}{4\pi\varepsilon_0}\frac{(-q)}{\sqrt{a^2+b^2}}$

11 Three charges Q, +q and +q are placed at the vertices of a right-angled isosceles triangle as shown. The net electrostatic energy of the configuration is zero if Q is equal to - **[IIT SCREENING 2000]**



(1) $\frac{-q}{1+\sqrt{2}}$ (2) $\frac{-2q}{2+\sqrt{2}}$

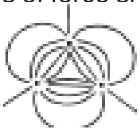
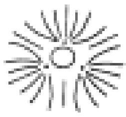
(3) –2q (4) +q

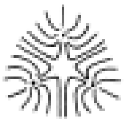
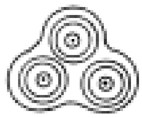
12 Two point charges (+Q) and (–2Q) are fixed on the X-axis at positions a and 2a from origin respectively. At what positions on the axis, the resultant electric field is zero - **[MP PET 2001]**

(1) only x = $\sqrt{2}$ a (2) only x = $-\sqrt{2}$ a

(3) both x = ± $\sqrt{2}$ a (4) x = $\frac{3a}{2}$ only

13 Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in - **[IIT SCREENING 2001]**

(1)  (2) 

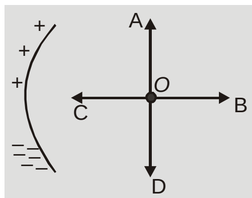
(3)  (4) 

14 A uniform electric field pointing in positive x-direction exists in a region. Let A be the origin, B be the point on the x-axis at x = +1 cm and C be the point on the y-axis at y = +1 cm. Then the potentials at the points A, B and C satisfy - **[IIT SCREENING 2001]**

(1) $V_A < V_B$ (2) $V_A > V_B$

(3) $V_A < V_C$ (4) $V_A > V_C$

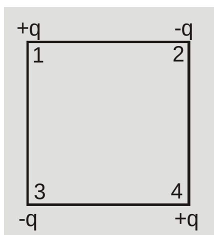
15 Charge density on upper half is λ and in lower half charge density is –λ. Direction of electric field at O is **[UPSEAT 2001]**



(1) along OA (2) along OB

(3) along OC (4) along OD

16 The work done in placing four charges at the corners of a square as shown in the figure, will be - **[RPET 2002]**



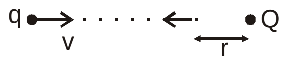
(1) $(4-\sqrt{2})\frac{Kq^2}{a}$ (2) $(4+\sqrt{2})\frac{Kq^2}{a}$

(3) $(4-\sqrt{2})\frac{Kq^2}{a^2}$ (4) $(4+\sqrt{2})\frac{Kq^2}{a^2}$

17 Two spherical conductors B and C having equal radii and carrying equal charges on them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that of B but uncharged is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is - **[AIEEE 2004]**

(1) 3F/8 (2) 3F/4
(3) F/8 (4) F/4

18 A charged particle 'q' is shot towards another charged particle 'Q', which is fixed, with a speed 'v'. It approaches 'Q' upto a closest distance r and then returns. If q were given a speed of '2v', the closest distances of approach would be - **[AIEEE 2004]**



(1) r/4 (2) 2 r
(3) r/2 (4) r

19 Four charged equal to –Q are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium the value of q is - **[AIEEE 2004]**

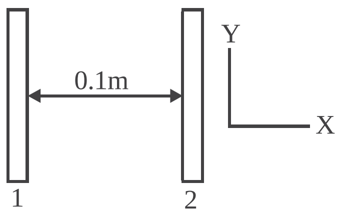
(1) $\frac{Q}{2}(1+2\sqrt{2})$ (2) $\frac{Q}{4}(1+2\sqrt{2})$

(3) $-\frac{Q}{2}(1+2\sqrt{2})$ (4) $-\frac{Q}{4}(1+2\sqrt{2})$

20 An electric-dipole is placed at an angle of 30º to a non-uniform electric field. The dipole will experience

(1) a torque only **[AIEEE 2006]**
(2) a translational force only in the direction of the field
(3) a translational force only in a direction normal to the direction of the field
(4) a torque as well as a translational force

21 Two insulating plates are both uniformly charged in such a way that the potential difference between them is $V_2 - V_1 = 20$ V. (i.e. plate 2 is at a higher potential). The plates are separated by d = 0.1 m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate 1. What is its speed when it hits plate 2 ? ($e = 1.6 \times 10^{-19}$C, $m_e = 9.11 \times 10^{-31}$ kg)
[AIEEE 2006]



(1) 32×10^{-19} m/s (2) 2.65×10^{6} m/s
(3) 7.02×10^{12} m/s (4) 1.87×10^{6} m/s

22 Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire then in equilibrium condition, the ratio of the magnitude of the electric fields at the surfaces of spheres A and B is
[AIEEE 2006]

(1) 1 : 4 (2) 4 : 1
(3) 1 : 2 (4) 2 : 1

23 An electric charge 10^{-3} μC is placed at the origin (0,0) of X-Y co-ordinate system. Two points A and B are situated at $(\sqrt{2},\sqrt{2})$ and $(2,0)$ respectively. The potential difference between the points and A and B will be
[AIEEE 2007]

(1) 9 V (2) zero
(3) 2 V (4) 4.5 V

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24 Charge are placed on the vertices of a square as shown in Fig. Let $\vec{E}$ is along (2). Potential at centre remains same

[AIEEE 2007]

(1) $\vec{E}$ remains unchanged, V changes
(2) Both $\vec{E}$ and V change
(3) $\vec{E}$ and V remain unchanged
(4) $\vec{E}$ changes, V remain changed

25 The potential at a poit x (measured in μm) due to some chargfes situated on the x-axis is given by $V(x) = 20/(x^2 - 4)$ volts. The electric field E at x = 4 μm is given by

[AIEEE 2007]

(1) $\frac{5}{3}$ volt/μm and in the -ve x direction
(2) $\frac{5}{3}$ volt/μm and in the +ve x direction
(3) $\frac{10}{9}$ volt/μm and in the -ve x direction
(4) $\frac{10}{9}$ volt/μm and in the +ve x direction

ANSWERS

EXERCISE

LEVEL - I

1. (3)	2. (3)	3. (1)	4. (1)	5. (2)
6. (1)	7. (3)	8. (1)	9. (2)	10. (3)
11. (3)	12. (2)	13. (3)	14. (3)	15. (1)
16. (2)	17. (3)	18. (4)	19. (2)	20. (1)
21. (4)	22. (4)	23. (3)	24. (4)	25. (3)

LEVEL - II

1. (1)	2. (4)	3. (3)	4. (3)	5. (2)
6. (3)	7. (4)	8. (3)	9. (4)	10. (4)
11. (2)	12. (3)	13. (1)	14. (2)	15. (3)
16. (3)	17. (2)	18. (1)	19. (3)	20. (2)
21. (1)	22. (1)	23. (2)	24. (2)	25. (3)

LEVEL - III

1. (1)	2. (3)	3. (1)	4. (2)	5. (1)
6. (4)	7. (4)	8. (1)	9. (2)	10. (1)
11. (1)	12. (4)	13. (4)	14. (2)	15. (3)
16. (3)	17. (2)	18. (3)	19. (4)	20. (2)
21. (3)	22. (2)	23. (3)	24. (4)	25. (1)

PROBLEMS ASKED IN VARIOUS EXAMS

1. (4)	2. (1)	3. (3)	4. (1)	5. (3)
6. (1)	7. (2)	8. (4)	9. (4)	10. (2)
11. (2)	12. (2)	13. (3)	14. (2)	15. (4)
16. (1)	17. (1)	18. (3)	19. (4)	20. (4)
21. (2)	22. (2)	23. (2)	24. (4)	25. (4)

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