Note: For the benefit of the students, specially the aspiring ones, the question of JEE(advanced), 2013 are also given in this booklet. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '', which can be attempted as a test. For this test the time allocated in Physics, Chemistry & Mathematics are **30 minutes, 20 minutes and 30 minutes** respectively.*

FIITJEE SOLUTIONS TO JEE(ADVANCED)-2013

CODE 5

PAPER 1

Time: 3 Hours **Maximum Marks: 180**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General:

1. This booklet is your Question Paper. Do not break the seals of this booklet before being instructed to do so by the invigilators.
2. Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers and electronic gadgets are NOT allowed inside the examination hall.
3. Write your name and roll number in the space provided on the back cover of this booklet.
4. Answers to the questions and personal details are to be filled on a two-part carbon-less paper, which is provided separately. These parts should only be separated at the end of the examination when instructed by the invigilator. The upper sheet is a machine-gradable Objective Response Sheet (ORS) which will be retained by the invigilator. You will be allowed to take away the bottom sheet at the end of the examination.
5. **Using a black ball point pen darken the bubbles on the upper original sheet.** Apply sufficient pressure so that the impression is created on the bottom duplicate sheet.

B. Question Paper Format :

The question paper consists of **three parts** (Physics, Chemistry and Mathematics). Each part consists of three sections.

Section 1 constrains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

Section 2 contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**.

Section 3 contains **5 questions**. The answer to each question is a single–digit integer, ranging from 0 to 9 (both inclusive)

C. Marking Scheme:

For each question in **Section 1**, you will be awarded **2 marks** if your darken the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. No negative marks will be awarded for incorrect answers in this section.

For each question in **Section 2**, you will be awarded **4 marks** if you darken all the bubble(s) corresponding to only the correct answer(s) and zero mark if no bubbles are darkened. In all other cases, minus one (–1) mark will be awarded.

For each question in **Section 3**, you will be awarded **4 marks** if you darken the bubble corresponding to only the correct answer and zero mark if no bubbles are darkened. In all other cases, minus one (–1) mark will be awarded.

PAPER-1 [Code – 5]

JEE(ADVANCED) 2013

PART - I: PHYSICS

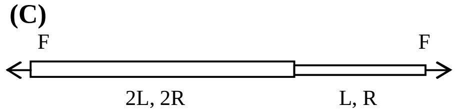
SECTION – 1: (Only one option correct Type)

This section contains **10 multiple choice questions.** Each question has four choice (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

* 1. One end of a horizontal thick copper wire of length 2L and radius 2R is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is

(A) 0.25 (B) 0.50
(C) 2.00 (D) 4.00

Sol. **(C)**



$$k_1 = \frac{\pi 4R^2 x}{2L}, k_2 = \frac{\pi R^2 y}{L}$$

$$F = k_1 x = k_2 y \Rightarrow \frac{y}{x} = \frac{k_1}{k_2} = \mathbf{2}$$

* 2. The work done on a particle of mass m by a force $K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i} + \frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$ (K being a constant of appropriate dimensions, when the particle is taken from the point (a, 0) to the point (0, a) along a circular path of radius a about the origin in the x-y plane is

(A) $\frac{2K\pi}{a}$ (B) $\frac{K\pi}{a}$
(C) $\frac{K\pi}{2a}$ (D) 0

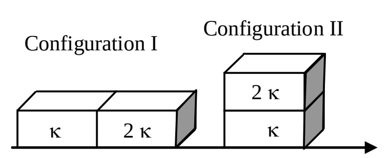
Sol. **(D)**

$$dw = \vec{F}\cdot d\vec{r} = \vec{F}\cdot(dx\hat{i} + dy\hat{j}) = K\int \frac{xdx}{(x^2+y^2)^{3/2}} + \frac{ydy}{(x^2+y^2)^{3/2}}$$

$$x^2 + y^2 = a^2$$

$$w = \frac{K}{a^3}\int_a^0 xdx + \int_0^a ydy = \frac{K}{a^3}\left(\frac{-a^2}{2} + \frac{a^2}{2}\right) = \mathbf{0.}$$

* 3. Two rectangular blocks, having identical dimensions, can be arranged either in configuration I or in configuration II as shown in the figure. One of the blocks has thermal conductivity κ and the other 2κ. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9 *s* to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is



(A) 2.0 s (B) 3.0 s
(C) 4.5 s (D) 6.0 s

Sol. **(A)**

$$R_1 = \frac{L}{\kappa A} + \frac{L}{2\kappa A} = \frac{3L}{2\kappa A}$$

$$\frac{1}{R_2} = \frac{1}{\left(\frac{L}{\kappa A}\right)} + \frac{1}{\left(\frac{L}{2\kappa A}\right)} = \frac{3\kappa A}{L}$$

$$R_2 = \frac{L}{3\kappa A}$$

$$\Delta Q_1 = \Delta Q_2$$

$$\frac{\Delta T}{R_1} t_1 = \frac{\Delta T}{R_2} t_2$$

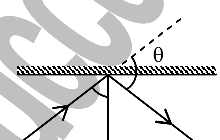
$$\Rightarrow t_2 = \frac{R_2}{R_1} t_1 = \mathbf{2\ sec.}$$

4. A ray of light travelling in the direction $\frac{1}{2}(\hat{i} + \sqrt{3}\hat{j})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{i} - \sqrt{3}\hat{j})$. The angle of incidence is

(A) 30° (B) 45°
(C) 60° (D) 75°

Sol. **(A)**

Let angle between the directions of incident ray and reflected ray be θ



$$\cos\theta = \frac{1}{2}(\hat{i} + \sqrt{3}\hat{j}) \cdot \frac{1}{2}(\hat{i} - \sqrt{3}\hat{j})$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = 120°$$

* 5. The diameter of a cylinder is measured using a Vernier callipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The 24th division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is

(A) 5.112 cm (B) 5.124 cm
(C) 5.136 cm (D) 5.148 cm

Sol. **(B)**

Main scale division (s) = .05 cm

Vernier scale division (v) = $\frac{49}{100}$ = .049

Least count = .05 – .049 = .001 cm

Diameter: 5.10 + 24 × .001

= 5.124 cm

* 6. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is

(A) 1 : 4 (B) 1 : 2 (C) 6 : 9 (D) 8 : 9

Sol. **(D)**

$PV = nRT = \frac{m}{M}RT$

$\Rightarrow PM = \rho RT$

$\frac{\rho_1}{\rho_2} = \frac{P_1M_1}{P_2M_2} = \left(\frac{P_1}{P_2}\right)\times\left(\frac{M_1}{M_2}\right) = \frac{4}{3}\times\frac{2}{3} = \frac{8}{9}$

Here ρ_1 and ρ_2 are the densities of gases in the vessel containing the mixture.

7. In the Young's double slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is

(A) $(2n+1)\frac{\lambda}{2}$ (B) $(2n+1)\frac{\lambda}{4}$ (C) $(2n+1)\frac{\lambda}{8}$ (D) $(2n+1)\frac{\lambda}{16}$

Sol. **(B)**

$\frac{I_{max}}{2} = I_m \cos^2\left(\frac{\phi}{2}\right)$

$\Rightarrow \cos\left(\frac{\phi}{2}\right) = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{\phi}{2} = \frac{\pi}{4}$

$\Rightarrow \phi = \frac{\pi}{2}(2n+1)$

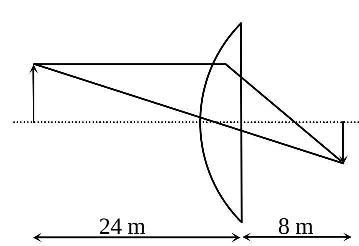
$\Rightarrow \Delta x = \frac{\lambda}{2\pi}\phi = \frac{\lambda}{2\pi}\times\frac{\pi}{2}(2n+1) = \frac{\lambda}{4}(2n+1)$

8. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is

(A) 1 m (B) 2 m (C) 3 m (D) 4 m

Sol. **(C)**

8. $\mu = \frac{\lambda_a}{\lambda_m} = \frac{3}{2}$



$\Rightarrow \frac{1}{f} = \frac{\mu-1}{R} = \frac{1}{2R}$

$\Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\Rightarrow \frac{1}{8} - \frac{1}{-24} = \frac{1}{2R}$

$\Rightarrow \frac{3+1}{24} = \frac{1}{2R}$

$\Rightarrow R = 3$ m

* 9. A particle of mass m is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{4}+\alpha$ (C) $\frac{\pi}{4}-\alpha$ (D) $\frac{\pi}{2}$

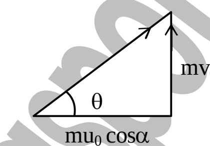
Sol. **(A)**

Velocity of particle performing projectile motion at highest point

$= v_1 = v_0\cos\alpha$

Velocity of particle thrown vertically upwards at the position of collision

$$= v_2^2 = u_0^2 - 2g\frac{u^2\sin^2\alpha}{2g} = v_0\cos\alpha$$



So, from conservation of momentum

$$\tan\theta = \frac{mv_0\cos\alpha}{mu_0\cos\alpha} = 1$$

$\Rightarrow \quad \theta = \pi/4$

10. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mW and the speed of light is 3×10^8 m/s. The final momentum of the object is

(A) 0.3×10^{-17} kg ms^{-1} (B) 1.0×10^{-17} kg ms^{-1}

(C) 3.0×10^{-17} kg ms^{-1} (B) 9.0×10^{-17} kg ms^{-1}

Sol. **(B)**

$t = 100\times10^{-9}$ sec, $P = 30\times10^{-3}$ Watt, $C = C\times10^{8}$ m/s

$$\text{Momentum} = \frac{Pt}{C} = \frac{30\times10^{-3}\times100\times10^{-9}}{3\times10^{8}} = 1.0 \times 10^{-17} \text{ kg ms}^{-1}$$

SECTION – 2 : (One or more options correct Type)

This section contains **5 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** are correct.

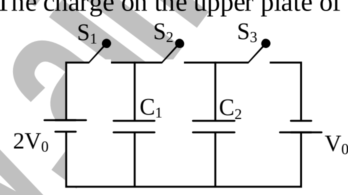
11. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time,

(A) the charge on the upper plate of C_1 is $2CV_0$.

(B) the charge on the upper plate of C_1 is CV_0.

(C) the charge on the upper plate of C_1 is 0.

(D) The charge on the upper plate of C_2 is $-CV_0$.



Sol. **(B, D)**

After switch S_1 is closed, C_1 is charged by $2CV_0$, when switch S_2 is closed, C_1 and C_2 both have upper plate charge CV_0.

When S_3 is closed, then upper plate of C_2 becomes charged by $-CV_0$ and lower plate by $+CV_0$.

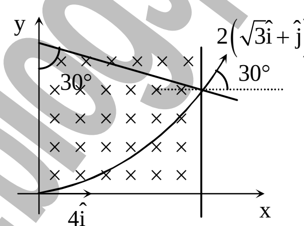
12. A particle of mass M and positive charge Q, moving with a constant velocity $u_1 = 4\hat{i}ms^{-1}$, enters a region of uniform static magnetic field normal to the x-y plane. The region of the magnetic field extends from x = 0 to x = L for all values of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $\vec{u}_2 = 2\left(\sqrt{3}\hat{i} + \hat{j}\right)$ m/s^{-1}. The correct statement(s) is (are)

(A) The direction of the magnetic field is –z direction.
(B) The direction of the magnetic field is +z direction.
(C) The magnitude of the magnetic field $\frac{50\pi M}{3Q}$ units.
(D) The magnitude of the magnetic field is $\frac{100\pi M}{3Q}$ units.

Sol. **(A, C)**

So magnetic field is along –ve, z-direction.

Time taken in the magnetic field = $10\times10^{-3} = \frac{\pi M}{6QB}$



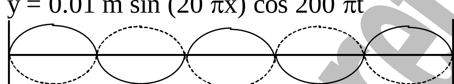
$B = \frac{\pi M}{60\times10^{-3}Q} = \frac{1000\pi M}{60Q}$

$= \frac{50\pi M}{3Q}$

* 13. A horizontal stretched string fixed at two ends, is vibrating in its fifth harmonic according to the equation y(x, t) = 0.01m sin [(62.8m^{-1})x] cos[(628s^{-1})t]. Assuming π = 3.14, the correct statement(s) is (are)
(A) The number of nodes is 5.
(B) the length of the string is 0.25 m.
(C) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01m.
(D) The fundamental frequency is 100 Hz.

Sol. **(B, C)**

y = 0.01 m sin (20 πx) cos 200 πt



no. of nodes is 6

$20\,\pi = \frac{2\pi}{\lambda}$

$\therefore \quad \lambda = \frac{1}{10}m = 0.1$ m

length of the spring = $0.5\times\frac{1}{2}$ = 0.25

Mid point is the antinode

Frequency at this mode is f = $\frac{200\pi}{2\pi} = 100$Hz

$\therefore$ Fundamental frequency = $\frac{100}{5} = 20$Hz.

* 14. A solid sphere of radius R and density ρ is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are)

(A) the net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$

(B) the net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$

(C) the light sphere is partially submerged.

(D) the light sphere is completely submerged.

Sol. **(A, D)**

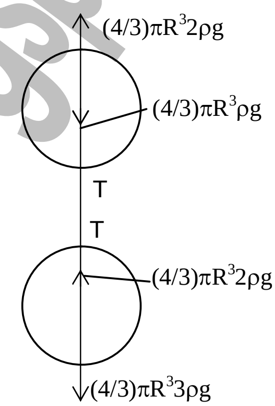
At equilibrium,

$$\frac{4}{3}\pi R^3 2\rho g = \frac{4}{3}\pi R^3 \rho g + T$$

$$T = \frac{4}{3}\pi R^3 \rho g$$

$$\therefore\ \Delta\ell = \frac{4}{3k}\pi R^3 \rho g$$

For equilibrium of the complete system, net force of buoyancy must be equal to the total weight of the sphere which holds true in the given problem. So both the spheres are completely submerged.



15. Two non-conducting solid spheres of radii R and 2R, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance 2R from the centre of the smaller sphere, along the line joining the centre of the spheres is zero. The ratio ρ_1/ρ_2 can be

(A) – 4 (B) $-\frac{32}{25}$

(C) $\frac{32}{25}$ (D) 4

Sol. **(B, D)**

At point P_1, $\frac{1}{4\pi\varepsilon_0}\frac{\rho_1(4/3)\pi R^3}{4R^2} = \frac{\rho_2 R}{3\varepsilon_0}$

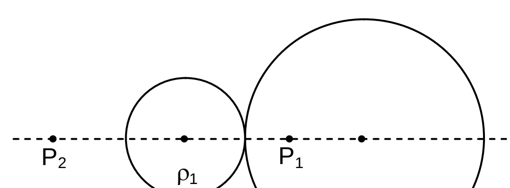
$$\frac{\rho_1 R}{12} = \frac{\rho_2 R}{3}$$

$$\frac{\rho_1}{\rho_2} = 4$$

At point P_2,

$$\frac{\rho_1(4/3)\pi R^3}{(2R)^2} + \frac{\rho_2(4/3)\pi 8R^3}{(5R)^2} = 0$$

$$\therefore\ \frac{\rho_1}{\rho_2} = -\frac{32}{25}$$



SECTION – 3 : (Integer value correct Type)

This section contains **5 questions**. The answer to each question is **a single digit integer,** ranging from 0 to 9 (*both inclusive*).

* 16. A bob of mass m , suspended by a string of length l_1 is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length l_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio l_1/l_2 is

Sol. **(5)**

The initial speed of 1st bob (suspended by a string of length l_1)is $\sqrt{5gl_1}$.

The speed of this bob at highest point will be $\sqrt{gl_1}$.

When this bob collides with the other bob there speeds will be interchanged.

$$\sqrt{gl_1} = \sqrt{5gl_2} \Rightarrow \frac{l_1}{l_2} = 5$$

* 17. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in m/s) of the particle is zero, the speed (in m/s) after 5 s is

Sol. **(5)**

Power = $\frac{dW}{dt}$ $\Rightarrow W = 0.5 \times 5 = 2.5 = KE_f - KE_i$

$2.5 = \frac{M}{2}(v_f^2 - v_i^2)$

$\Rightarrow v_f = 5$

18. The work functions of Silver and Sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for Silver to that of Sodium is

Sol. **(1)**

Slope of graph is h/e = constant

$\Rightarrow 1$

19. A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10^3 disintegrations per second. Given that ln 2 = 0.693, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is

Sol. **(4)**

$f = (1 - e^{-\lambda t}) = 1 - e^{-\lambda t} \approx 1 - (1 - \lambda t) = \lambda t$

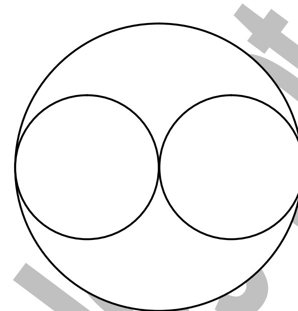
$f = 0.04$

Hence % decay ≈ 4 %

* 20. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is

Sol. **(8)**

Conservation of angular momentum about vertical axis of disc



$$\frac{50(0.4)^2}{2}\times 10 = \left[\frac{50(0.4)^2}{2} + 4(6.25)(0.2)^2\right]\omega$$

ω = 8 rad/sec

PART - II: CHEMISTRY

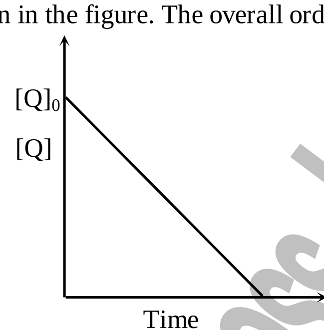
SECTION – 1
(Only One option correct Type)

This section contains **10 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

*21. In the reaction,

$P + Q \longrightarrow R + S$

the time taken for 75% reaction of P is twice the time taken for 50% reaction of P. The concentration of Q varies with reaction time as shown in the figure. The overall order of the reaction is



(A) 2
(B) 3
(C) 0
(D) 1

Sol. **(D)**

Overall order of reaction can be decided by the data given $t_{75\%} = 2t_{50\%}$

$\therefore$ It is a first order reaction with respect to P.

From graph [Q] is linearly decreasing with time, i.e. order of reaction with respect to Q is zero and the rate expression is $r = k[P]^1[Q]^0$.

Hence (D) is correct.

22. Consider the following complex ions, P, Q and R

$P = [FeF_6]^{3-}$, $Q = [V(H_2O)_6]^{2+}$ and $R = [Fe(H_2O)_6]^{2+}$

The correct order of the complex ions, according to their spin–only magnetic moment values (in B.M.) is

(A) $R < Q < P$
(B) $Q < R < P$
(C) $R < P < Q$
(D) $Q < P < R$

Sol. **(B)**

$P = Fe^{+3}$ (no. of unpaired e^- = 5)

$Q = V^{+2}$ (no. of unpaired e^- = 3)

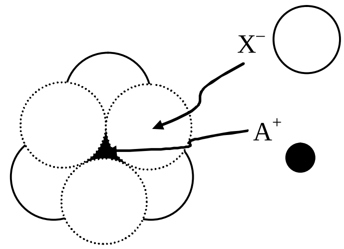
$R = Fe^{+2}$ (no. of unpaired e^- = 4)

As all ligands are weak field, hence the no. of unpaired electrons remains same in the complex ion.

$\mu = \sqrt{n(n+2)}$ B.M.

Hence (B) is correct.

23. The arrangement of X^- ions around A^+ ion in solid AX is given in the figure (not drawn to scale). If the radius of X^- is 250 pm, the radius of A^+ is



(A) 104 pm (B) 125 pm
(C) 183 pm (D) 57 pm

Sol. **(A)**

According to the given figure, A^+ is present in the octahedral void of X^-. The limiting radius in octahedral void is related to the radius of sphere as

$r_{void} = 0.414\ r_{sphere}$

$r_{A^+} = 0.414 r_{X^-}$

$= 0.414 \times 250$ pm $= 103.5$

≈ 104 pm

Hence (A) is correct.

24. Concentrated nitric acid, upon long standing, turns yellow–brown due to the formation of
(A) NO (B) NO_2
(C) N_2O (D) N_2O_4

Sol. **(B)**

$4HNO_3 \longrightarrow 2H_2O + 4NO_2 + O_2$

NO_2 remains dissolved in nitric acid colouring it yellow or even red at higher temperature.

25. The compound that does NOT liberate CO_2, on treatment with aqueous sodium bicarbonate solution, is
(A) Benzoic acid (B) Benzenesulphonic acid
(C) Salicylic acid (D) Carbolic acid (Phenol)

Sol. **(D)**

pK_a of PhOH (carbolic acid) is 9.98 and that of carbonic acid (H_2CO_3) is 6.63 thus phenol does not give effervescence with HCO_3^- ion.

26. Sulfide ores are common for the metals
(A) Ag, Cu and Pb (B) Ag, Cu and Sn
(C) Ag, Mg and Pb (D) Al, Cu and Pb

Sol. **(A)**

Sulfide ore of Ag → Argentite (Ag_2S), Pb → Galena (PbS), Cu → Copper glance (Cu_2S)
Hence (A) is correct.

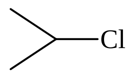
27. Methylene blue, from its aqueous solution, is adsorbed on activated charcoal at 25^0C. For this process, the correct statement is
(A) The adsorption requires activation at 25^0C.
(B) The adsorption is accompanied by a decrease in enthalpy.
(C) The adsorption increases with increase of temperature.
(D) The adsorption is irreversible.

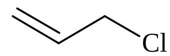
Sol. **(B)**

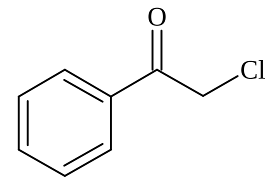
Adsorption of methylene blue on activated charcoal is physical adsorption hence it is characterised by decrease in enthalpy. Hence (B) is correct.

28. KI in acetone, undergoes S_N2 reaction with each of P, Q, R and S. The rates of the reaction vary as

$H_3C\text{-}Cl$ P


Q

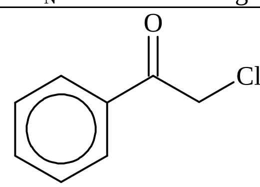
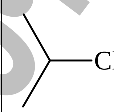

R


S

(A) P > Q > R > S
(B) S > P > R > Q
(C) P > R > Q > S
(D) R > P > S > Q

Sol. **(B)**

Relative reactivity for S_N2 reaction in the given structures is

Substrate	 (S)	CH_3Cl (P)	$H_2C{=}CH{-}CH_2Cl$ (R)	 (Q)
Relative Rates Towards S_N2	100000	200	79	0.02

*29. The standard enthalpies of formation of $CO_2(g)$, $H_2O(l)$ and glucose(*s*) at 25^0C are –400 kJ/mol, –300 kJ/mol and –1300 kJ/mol, respectively. The standard enthalpy of combustion per gram of glucose at 25^0C is

(A) +2900 kJ
(B) – 2900 kJ
(C) –16.11 kJ
(D) +16.11 kJ

Sol. **(C)**

Combustion of glucose

$$C_6H_{12}O_6 + 6O_2 \longrightarrow 6CO_2 + 6H_2O$$

$$\Delta H_{combustion} = \left(6\times\Delta H_f CO_2 + 6\times\Delta H_f H_2O\right) - \Delta H_f C_6H_{12}O_6$$

= (6 × - 400 + 6 × - 300) – (- 1300)
= - 2900 kJ/mol
= - 2900/180 kJ/g
= - 16.11 kJ/g

Hence (C) is correct.

30. Upon treatment with ammoniacal H_2S, the metal ion that precipitates as a sulfide is

(A) Fe(III)
(B) A*l*(III)
(C) Mg(II)
(D) Zn(II)

Sol. **(D)**

Among Fe^{3+}, Al^{3+}, Mg^{2+}, Zn^{2+} only Zn^{2+} is precipitated with ammonical H_2S as ZnS.

SECTION – 2
(One or More Options Correct Type)

This section contains **5 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** are correct.

*31. The initial rate of hydrolysis of methyl acetate (1 M) by a weak acid (HA, 1M) is 1/100th of that of a strong acid (HX, 1M), at 25^{o}C. The K_a of HA is
(A) 1×10^{-4} (B) 1×10^{-5}
(C) 1×10^{-6} (D) 1×10^{-3}

Sol. **(A)**

Rate in weak acid = $\frac{1}{100}(\text{rate in strong acid})$

$\therefore \quad [H^+]_{\text{weak acid}} = \frac{1}{100}[H^+]_{\text{strong acid}}$

$\therefore \quad [H^+]_{\text{weak acid}} = \frac{1}{100}M = 10^{-2}M$

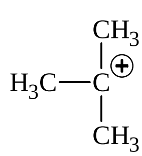
$\therefore \quad C\alpha = 10^{-2}$

$\therefore \quad K_a = 10^{-4}$

Option (A) is correct.

*32. The hyperconjugative stabilities of tert-butyl cation and 2-butene, respectively, are due to
(A) $\sigma \rightarrow p$ (empty) and $\sigma \rightarrow \pi^*$ electron delocalisations. (B) $\sigma \rightarrow \sigma^*$ and $\sigma \rightarrow \pi$ electron delocalisations.
(C) $\sigma \rightarrow p$ (filled) and $\sigma \rightarrow \pi$ electron delocalisations. (D) p(filled)$\rightarrow \sigma^*$ and $\sigma \rightarrow \pi^*$ electron delocalisations.

Sol. **(A)**



$\sigma - \pi^*(\text{antibonding})$

$\sigma - p(\text{empty})$

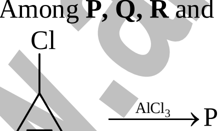
33. The pair(s) of coordination complexes/ions exhibiting the same kind of isomerism is(are)
(A) $[Cr(NH_3)_5Cl]Cl_2$ and $[Cr(NH_3)_4Cl_2]Cl$ (B) $[Co(NH_3)_4Cl_2]^+$ and $[Pt(NH_3)_2(H_2O)Cl]^+$
(C) $[CoBr_2Cl_2]^{2-}$ and $[PtBr_2Cl_2]^{2-}$ (D) $[Pt(NH_3)_3(NO_3)]Cl$ and $[Pt(NH_3)_3Cl]Br$

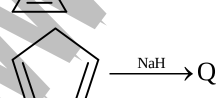
Sol. **(B, D)**

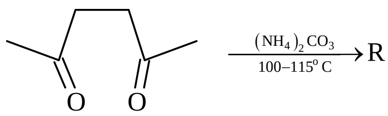
$[Co(NH_3)_4Cl_2]^+$ (an octahedral complex) and $[Pt(NH_3)_2(H_2O)Cl]^+$ (a square planar complex) will show geometrical isomerism.

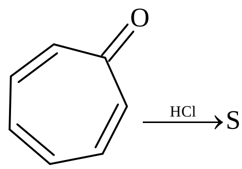
$[Pt(NH_3)_3(NO_3)]Cl$ and $[Pt(NH_3)_3Cl]Br$ will show ionization isomerism.

*34. Among **P, Q, R** and **S,** the aromatic compound(s) is/are



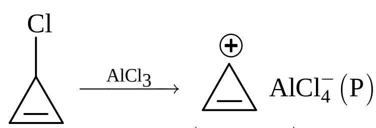




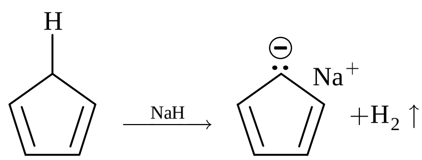


(A) P (B) Q
(C) R (D) S

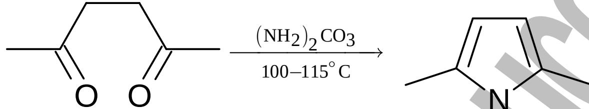
Sol. **(A, B, C, D)**



(aromatic)



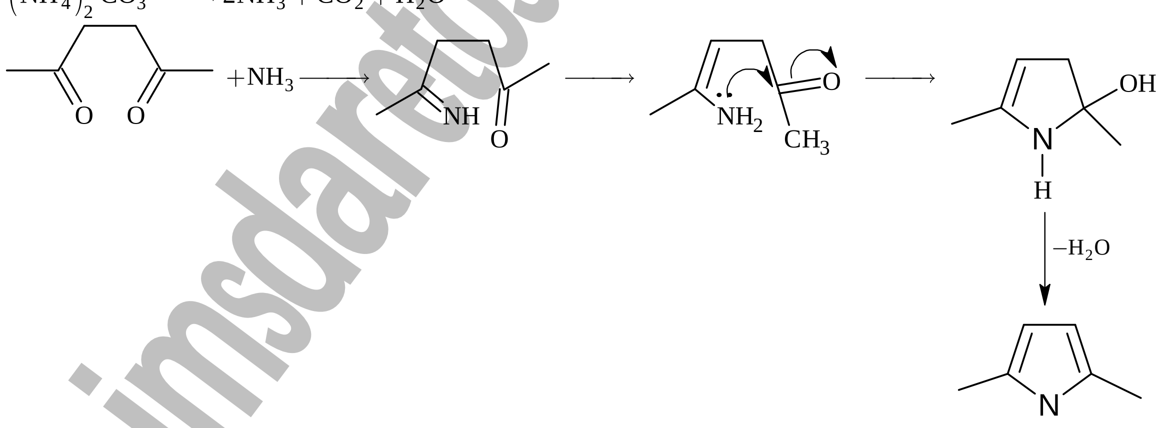
(aromatic)(Q)



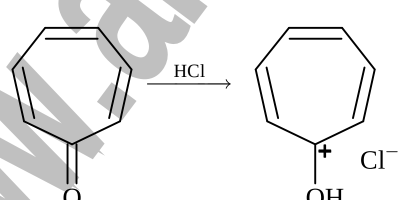
(R) aromatic

Mechanism

$(NH_4)_2CO_3 \xrightarrow{\Delta} 2NH_3 + CO_2 + H_2O$



(aromatic)



(S)aromatic

35. Benzene and naphthalene form an ideal solution at room temperature. For this process, the true statement(s) is(are)

(A) ΔG is positive
(B) ΔS_{system} is positive
(C) $\Delta S_{surroundings} = 0$
(D) $\Delta H = 0$

Sol. **(B, C, D)**

For ideal solution, $\Delta S_{system} > 0$

$\Delta S_{surrounding} = 0$

$\Delta H_{mixing} = 0$

SECTION-3 (Integer value correct Type)

This section contains **5** questions. The answer to each of the questions is a **single digit integer,** ranging from 0 to 9. (both inclusive).

*36. The atomic masses of He and Ne are 4 and 20 a.m.u., respectively. The value of the de Broglie wavelength of He gas at – 73°C is "M" times that of the de Broglie wavelength of Ne at 727° C. M is

Sol. **(5)**

Since, $\lambda = \frac{h}{mV} = \frac{h}{\sqrt{2M\ K.E}}$ (since K.E. $\propto$ T)

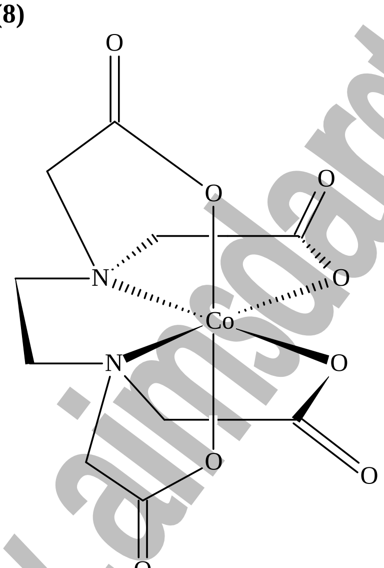
$\Rightarrow \quad \lambda \propto \frac{1}{\sqrt{MT}}$

For two gases,

$$\frac{\lambda_{He}}{\lambda_{Ne}} = \sqrt{\frac{M_{Ne}T_{Ne}}{M_{He}T_{He}}} = \sqrt{\frac{20}{4} \times \frac{1000}{200}}$$

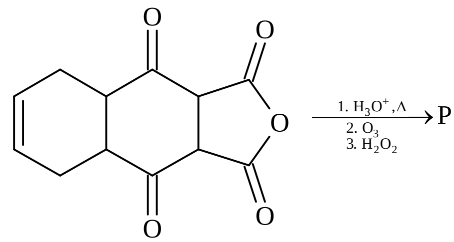
37. $EDTA^{4-}$ is ethylenediaminetetraacetate ion. The total number of N – Co – O bond angles in $[Co(EDTA)]^{1-}$ complex ion is

Sol. **(8)**

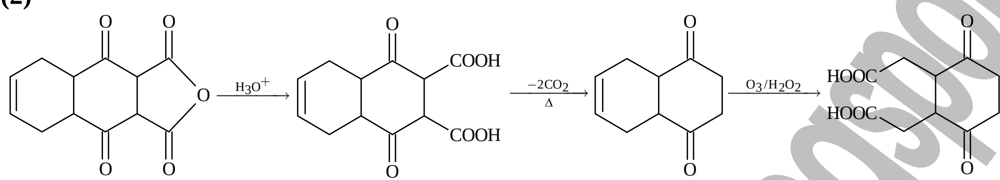


Total no. of N – Co – O bond angles is 8.

38. The total number of carboxylic acid groups in the product **P** is



Sol. **(2)**



39. A tetrapeptide has – COOH group on alanine. This produces glycine (Gly), valine (Val), phenyl alanine (Phe) and alanine (Ala), on complete hydrolysis. For this tetrapeptide, the number of possible sequences (primary structures) with – NH_2 group attached to a chiral center is

Sol. **(4)**

Because –COOH group of tetrapeptide is intact on alanine, its NH_2 must be participating in condensation.

$\therefore$ Alanine is at one terminus, $- - -$ A.

To fill the 3 blanks, possible options are:

(i) When NH_2 group attached to non chiral carbon

G	V	P
G	P	V

(ii) When NH_2 group attached to chiral carbon

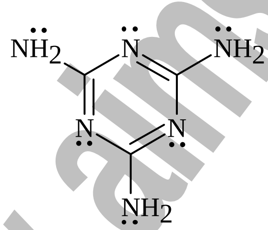
V	G	P	P	V	G
V	P	G	P	G	V

where, Glycine (G)
Valine (V)
Phenyl alanine (P)
Alanine (A)

So the number of possible sequence are 4.

40. The total number of lone-pairs of electrons in melamine is

Sol. **(6)** lone pairs



Melamine

PART - III: MATHEMATICS

SECTION – 1 : (Only one option correct Type)

This section contains 10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which** ONLY ONE is correct.

41. Perpendiculars are drawn from points on the line $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z}{3}$ to the plane x + y + z = 3. The feet of perpendiculars lie on the line

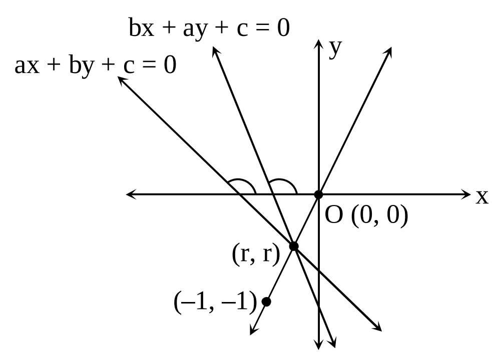
(A) $\frac{x}{5}=\frac{y-1}{8}=\frac{z-2}{-13}$
(B) $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{-5}$
(C) $\frac{x}{4}=\frac{y-1}{3}=\frac{z-2}{-7}$
(D) $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

Sol. **(D)**
Any point B on line is $(2\lambda - 2, -\lambda - 1, 3\lambda)$
Point B lies on the plane for some λ
$\Rightarrow (2\lambda - 2) + (-\lambda - 1) + 3\lambda = 3$
$\Rightarrow 4\lambda = 6 \Rightarrow \lambda = \frac{3}{2} \Rightarrow B \equiv \left(1, \frac{-5}{2}, \frac{9}{2}\right)$
The foot of the perpendicular from point (– 2, – 1, 0) on the plane is the point A (0, 1, 2)
$\Rightarrow$ D.R. of AB = $\left(1, \frac{-7}{2}, \frac{5}{2}\right) \equiv (2, -7, 5)$
Hence $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

*42. For a > b > c > 0, the distance between (1, 1) and the point of intersection of the lines ax + by + c = 0 and bx + ay + c = 0 is less than $2\sqrt{2}$, then

(A) a + b – c > 0
(B) a – b + c < 0
(C) a – b + c > 0
(D) a + b – c < 0

Sol. **(A)**
For point of intersection $(a-b)x_1 = (a-b)y_1$
$\Rightarrow$ point lie on line y = x ….. (1)
Let point is (r, r)
$\sqrt{(r-1)^2+(r-1)^2} < 2\sqrt{2}$
$\sqrt{2}\,|(r-1)| < 2\sqrt{2}$
$\Rightarrow |r - 1| < 2$
$-1 < r < 3$
$\Rightarrow$ (–1, –1) lies on the opposite side of origin for both lines
$\Rightarrow -a - b + c < 0$
$\Rightarrow a + b - c > 0$



43. The area enclosed by the curves y = sinx + cosx and y = |cosx – sinx| over the interval $\left[0, \frac{\pi}{2}\right]$ is

(A) $4\left(\sqrt{2}-1\right)$
(B) $2\sqrt{2}\left(\sqrt{2}-1\right)$
(C) $2\left(\sqrt{2}+1\right)$
(D) $2\sqrt{2}\left(\sqrt{2}+1\right)$

Sol. **(B)**

$y_1 = \sin x + \cos x = \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$

$y_2 = \sqrt{2}\left|\sin\left(\frac{\pi}{4}-x\right)\right|$

$\Rightarrow \text{Area} = \int_0^{\frac{\pi}{4}}\big((\sin x+\cos x)-(\cos x-\sin x)\big)dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\big((\sin x+\cos x)-(\sin x-\cos x)\big)dx$

$= 4-2\sqrt{2}$

44. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2},\frac{3}{4},\frac{1}{4},\frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is

(A) $\frac{235}{256}$ (B) $\frac{21}{256}$

(C) $\frac{3}{256}$ (D) $\frac{253}{256}$

Sol. **(A)**

P(at least one of them solves correctly) = 1 – P(none of them solves correctly)

$= 1-\left(\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}\times\frac{7}{8}\right) = \frac{235}{256}$

*45. Let complex numbers α and $\frac{1}{\bar{\alpha}}$ lie on circles $(x-x_0)^2+(y-y_0)^2=r^2$ and $(x-x_0)^2+(y-y_0)^2=4r^2$, respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, then $|\alpha| =$

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{7}}$ (D) $\frac{1}{3}$

Sol. **(C)**

$OB = |\alpha|$

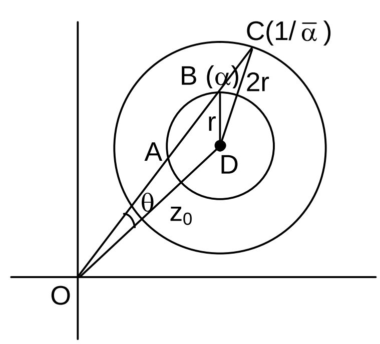
$OC = \frac{1}{|\bar{\alpha}|} = \frac{1}{|\alpha|}$

In ΔOBD

$\cos\theta = \frac{|z_0|^2+|\alpha|^2-r^2}{2|z_0||\alpha|}$

In ΔOCD

$\cos\theta = \frac{|z_0|^2+\frac{1}{|\alpha|^2}-4r^2}{2|z_0|\frac{1}{|\alpha|}}$

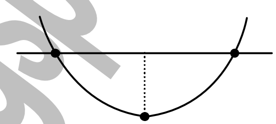


$$\frac{|z_0|^2 + |\alpha|^2 - r^2}{2|z_0||\alpha|} = \frac{|z_0|^2 + \frac{1}{|\alpha|^2} - 4r^2}{2|z_0|\frac{1}{|\alpha|}}$$

$$\Rightarrow |\alpha| = \frac{1}{\sqrt{7}}$$

46. The number of points in $(-\infty, \infty)$, for which $x^2 - x\sin x - \cos x = 0$, is
(A) 6 (B) 4
(C) 2 (D) 0

Sol. **(C)**
Let $f(x) = x^2 - x \sin x - \cos x \Rightarrow f'(x) = 2x - x \cos x$

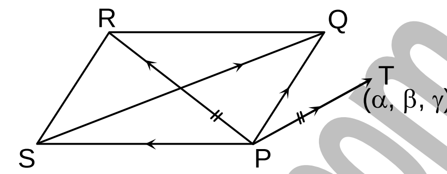


$\lim_{x\to\infty} f(x) \to \infty$
$\lim_{x\to-\infty} f(x) \to \infty$
$f(0) = -1$
Hence 2 solutions.

47. Let $f:\left[\frac{1}{2},1\right] \to \mathrm{R}$ (the set of all real numbers) be a positive, non-constant and differentiable function such that $f'(x) < 2f(x)$ and $f\left(\frac{1}{2}\right) = 1$. Then the value of $\int_{1/2}^{1} f(x)\,dx$ lies in the interval
(A) $(2e - 1, 2e)$ (B) $(e - 1, 2e - 1)$
(C) $\left(\frac{e-1}{2}, e-1\right)$ (D) $\left(0, \frac{e-1}{2}\right)$

Sol. **(D)**
Given $f'(x) - 2f(x) < 0$
$\Rightarrow f(x) < ce^{2x}$
Put $x = \frac{1}{2} \Rightarrow c > \frac{1}{e}$.
Hence $f(x) < e^{2x-1}$.
$\Rightarrow 0 < \int_{1/2}^{1} f(x)\,dx < \int_{1/2}^{1} e^{2x-1}dx$
$0 < \int_{1/2}^{1} f(x)\,dx < \frac{e-1}{2}$.

48. Let $\overrightarrow{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overrightarrow{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT}, \overrightarrow{PQ}$ *and* $\overrightarrow{PS}$ is
(A) 5 (B) 20
(C) 10 (D) 30



Sol. **(C)**

Area of base (PQRS) = $\frac{1}{2}\left|\overrightarrow{PR}\times\overrightarrow{SQ}\right| = \frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & -2\\ 1 & -3 & -4\end{vmatrix}$

$= \frac{1}{2}\left|-10\hat{i}+10\hat{j}-10\hat{k}\right| = 5\left|\hat{i}-\hat{j}+\hat{k}\right| = 5\sqrt{3}$

Height = proj. of PT on $\hat{i}-\hat{j}+\hat{k}$ = $\left|\frac{1-2+3}{\sqrt{3}}\right| = \frac{2}{\sqrt{3}}$

Volume = $\left(5\sqrt{3}\right)\left(\frac{2}{\sqrt{3}}\right) = 10$ cu. units

*49. The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$ is

(A) $\frac{23}{25}$ (B) $\frac{25}{23}$

(C) $\frac{23}{24}$ (D) $\frac{24}{23}$

Sol. **(B)**

$\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(n^2+n+1\right)\right)$

$\cot\left(\sum_{n=1}^{23}\tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right)\right)$

$\Rightarrow \cot\left(\tan^{-1}\left(\frac{23}{25}\right)\right) = \frac{25}{23}$.

50. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x}+\sec\left(\frac{y}{x}\right)$, x > 0. Then the equation of the curve is

(A) $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$ (B) $\operatorname{cosec}\left(\frac{y}{x}\right) = \log x + 2$

(C) $\sec\left(\frac{2y}{x}\right) = \log x + 2$ (D) $\cos\left(\frac{2y}{x}\right) = \log x + \frac{1}{2}$

Sol. **(A)**

$\frac{dy}{dx} = \frac{y}{x}+\sec\frac{y}{x}$. Let y = vx

$\Rightarrow \frac{dv}{\sec v} = \frac{dx}{x}$

$\int \cos v\,dv = \int \frac{dx}{x}$

sin v = ln x + c

$\sin\left(\frac{y}{x}\right) = \ln x + c$

The curve passes through $\left(1, \frac{\pi}{6}\right)$

$\Rightarrow \sin\left(\frac{y}{x}\right) = \ln x + \frac{1}{2}$.

SECTION – 2 : (One or more options correct Type)

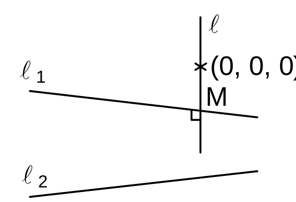
This section contains 5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which** ONE or MORE are correct.

51. A line l passing through the origin is perpendicular to the lines
$l_1 : (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$, $-\infty < t < \infty$, $l_2 : (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$, $-\infty < s < \infty$
Then, the coordinate(s) of the point(s) on l_2 at a distance of $\sqrt{17}$ from the point of intersection of l and l_1 is (are)

(A) $\left(\frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right)$ (B) (–1, –1, 0)

(C) (1, 1, 1) (D) $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

Sol. **(B, D)**



The common perpendicular is along $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -2\hat{i} + 3\hat{j} - 2\hat{k}$

Let M ≡ (2λ, –3λ, 2λ)

So, $\frac{2\lambda - 3}{1} = \frac{-3\lambda + 1}{2} = \frac{2\lambda - 4}{2} \Rightarrow \lambda = 1$

So, M ≡ (2, –3, 2)

Let the required point be P

Given that $PM = \sqrt{17}$

$\Rightarrow (3 + 2s - 2)^2 + (3 + 2s + 3)^2 + (2 + s - 2)^2 = 17$

$\Rightarrow 9s^2 + 28s + 20 = 0$

$\Rightarrow s = -2, -\frac{10}{9}$

So, P ≡ (–1, –1, 0) or $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

52. Let f(x) = xsin πx, x > 0. Then for all natural numbers n, f ′(x) vanishes at

(A) a unique point in the interval $\left(n, n+\frac{1}{2}\right)$ (B) a unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$

(C) a unique point in the interval (n, n + 1) (D) two points in the interval (n, n + 1)

Sol. **(B, C)**

We have f′(x) = sinπx + πx cosπx = 0

$\Rightarrow \tan\pi x = -\pi x$

$\Rightarrow \pi x \in \left(\frac{2n+1}{2}\pi, (n+1)\pi\right) \Rightarrow x \in \left(n+\frac{1}{2}, n+1\right) \in (n, n+1)$

*53. Let $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$. Then S_n can take value(s)

(A) 1056 (B) 1088

(C) 1120 (D) 1332

Sol. **(A, D)**

$$S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2 = \sum_{r=0}^{(n-1)} \left((4r+4)^2 + (4r+3)^2 - (4r+2)^2 - (4r+1)^2\right)$$

$$= \sum_{r=0}^{(n-1)} \left(2(8r+6) + 2(8r+4)\right)$$

$$= \sum_{r=0}^{(n-1)} (32r+20)$$

$= 16(n - 1)n + 20n$

$= 4n(4n + 1)$

$$= \begin{cases} 1056 \text{ for } n = 8 \\ 1332 \text{ for } n = 9 \end{cases}$$

54. For 3 × 3 matrices M and N, which of the following statement(s) is (are) NOT correct ?

(A) N^TMN is symmetric or skew symmetric, according as M is symmetric or skew symmetric

(B) MN – NM is skew symmetric for all symmetric matrices M and N

(C) MN is symmetric for all symmetric matrices M and N

(D) (adj M) (adj N) = adj(MN) for all invertible matrices M and N

Sol. **(C, D)**

(A) $(N^TMN)^T = N^TM^TN = N^TMN$ if M is symmetric and is $-N^TMN$ if M is skew symmetric

(B) $(MN - NM)^T = N^TM^T - M^TN^T = NM - MN = -(MN - NM)$. So, (MN – NM) is skew symmetric

(C) $(MN)^T = N^TM^T = NM \neq MN$ if M and N are symmetric. So, MN is not symmetric

(D) (adj. M) (adj. N) = adj(NM) ≠ adj (MN).

55. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio 8 : 15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. Then the lengths of the sides of the rectangular sheet are

(A) 24 (B) 32

(C) 45 (D) 60

Sol. **(A, C)**

Let the sides of rectangle be 15k and 8k and side of square be x then (15k – 2x)(8k – 2x)x is volume.

$v = 2(2x^3 - 23kx^2 + 60k^2x)$

$$\left.\frac{dv}{dx}\right|_{x=5} = 0$$

$$\left.6x^2 - 46kx + 60k^2\right|_{x=5} = 0$$

$6k^2 - 23k + 15 = 0$

$k = 3,\ k = \frac{5}{6}$. Only k = 3 is permissible.

So, the sides are 45 and 24.

SECTION – 3 : (Integer value correct Type)

This section contains 5 questions**. The answer to each question is** single digit integer**, ranging from 0 to 9 (*both inclusive*).**

56. Consider the set of eight vectors $V = \left\{ a\hat{i} + b\hat{j} + c\hat{k} \,;\, a, b, c \in \{-1, 1\} \right\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is ________

Sol. **(5)**

Let (1, 1, 1), (–1, 1, 1), (1, –1, 1), (–1, –1, 1) be vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ rest of the vectors are $-\vec{a}, -\vec{b}, -\vec{c}, -\vec{d}$ and let us find the number of ways of selecting co–planar vectors.
Observe that out of any 3 coplanar vectors two will be collinear (anti parallel)
Number of ways of selecting the anti parallel pair = 4
Number of ways of selecting the third vector = 6
Total = 24
Number of non co–planar selections = ${}^8C_3 - 24 = 32 = 2^5$, p = 5
Alternate Solution:
Required value = $\frac{8 \times 6 \times 4}{3!}$
$\therefore$ p = 5

57. Of the three independent events E_1, E_2, and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations (α – 2β) p = αβ and (β – 3γ) p = 2βγ. All the given probabilities are assumed to lie in the interval (0, 1). Then $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ = __________

Sol. **(6)**

Let P (E_1) = x, P (E_2) = y and P (E_3) = z
then (1 – x) (1 – y) (1 – z) = p
x (1 – y) (1 – z) = α
(1 – x) y(1 – z) = β
(1 – x) (1 – y) (1 – z) = γ

so $\frac{1-x}{x} = \frac{p}{\alpha}$ $\quad x = \frac{\alpha}{\alpha + p}$

similarly z = $\frac{\gamma}{\gamma + p}$

so $\frac{P(E_1)}{P(E_3)} = \frac{\frac{\alpha}{\alpha+p}}{\frac{\gamma}{\gamma+p}} = \frac{\frac{\gamma+p}{\gamma}}{\frac{\alpha+p}{\alpha}} = \frac{1+\frac{p}{\gamma}}{1+\frac{p}{\alpha}}$

also given $\frac{\alpha\beta}{\alpha - 2\beta} = p = \frac{2\beta\gamma}{\beta - 3\gamma} \Rightarrow \beta = \frac{5\alpha\gamma}{\alpha + 4\gamma}$

Substituting back $\left(\alpha - 2\left(\frac{5\alpha\gamma}{\alpha + 4\gamma}\right)\right)p = \frac{\alpha \cdot 5\alpha\gamma}{\alpha + 4\gamma}$

$\Rightarrow$ αp – 6pγ = 5αγ

$$\Rightarrow \left(\frac{p}{\gamma}+1\right)=6\left(\frac{p}{\alpha}+1\right) \Rightarrow \frac{\frac{p}{\gamma}+1}{\frac{p}{\alpha}+1}=6.$$

*58. The coefficients of three consecutive terms of $(1 + x)^{n+5}$ are in the ratio 5 : 10 : 14. Then n = _______

Sol. **(6)**

Let T_{r-1}, T_r, T_{r+1} are three consecutive terms of $(1 + x)^{n+5}$

$T_{r-1} = {}^{n+5}C_{r-2}(x)^{r-2}$, $T_r = {}^{n+5}C_{r-1}x^{r-1}$, $T_{r+1} = {}^{n+5}C_r x^r$

Where, ${}^{n+5}C_{r-2} : {}^{n+5}C_{r-1} : {}^{n+5}C_r = 5 : 10 : 14$.

So $\frac{{}^{n+5}C_{r-2}}{5}=\frac{{}^{n+5}C_{r-1}}{10}=\frac{{}^{n+5}C_r}{14}$

So from $\frac{{}^{n+5}C_{r-2}}{5}=\frac{{}^{n+5}C_{r-1}}{10} \Rightarrow n - 3r = -3$ ….. (1)

$\frac{{}^{n+5}C_{r-1}}{10}=\frac{{}^{n+5}C_r}{14} \Rightarrow 5n - 12r = -30$ ….. (2)

From equation (1) and (2) n = 6

*59. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then k – 20 = ________

Sol. **(5)**

Clearly, $1 + 2 + 3 + \ldots + n - 2 \le 1224 \le 3 + 4 + \ldots n$

$\Rightarrow \frac{(n-2)(n-1)}{2} \le 1224 \le \frac{(n-2)}{2}(3+n)$

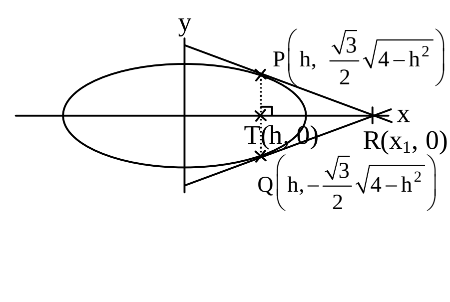
$\Rightarrow n^2 - 3n - 2446 \le 0$ and $n^2 + n - 2454 \ge 0$

$\Rightarrow 49 < n < 51 \Rightarrow n = 50$

$\therefore \frac{n(n+1)}{2}-(2k+1)=1224 \Rightarrow k = 25 \Rightarrow k - 20 = 5$

*60. A vertical line passing through the point (h, 0) intersects the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h)$ = area of the triangle PQR, $\Delta_1 = \max\limits_{1/2 \le h \le 1} \Delta(h)$ and $\Delta_2 = \min\limits_{1/2 \le h \le 1} \Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$ ________

Sol. **(9)**



$\frac{x^2}{4}+\frac{y^2}{3}=1$

$y = \frac{\sqrt{3}}{2}\sqrt{4-h^2}$ at x = h

Let $R(x_1, 0)$

PQ is chord of contact, so $\frac{xx_1}{4}=1 \Rightarrow x = \frac{4}{x_1}$

which is equation of PQ, x = h

so $\frac{4}{x_1} = h \quad \Rightarrow x_1 = \frac{4}{h}$

$\Delta(h)$ = area of $\Delta PQR = \frac{1}{2} \times PQ \times RT$

$= \frac{1}{2} \times \frac{2\sqrt{3}}{2}\sqrt{4-h^2} \times (x_1 - h) = \frac{\sqrt{3}}{2h}(4 - h^2)^{3/2}$

$\Delta'(h) = \frac{-\sqrt{3}\left(4+2h^2\right)}{2h^2}\sqrt{4-h^2}$ which is always decreasing

so Δ_1 = maximum of $\Delta(h) = \frac{45\sqrt{5}}{8}$ at $h = \frac{1}{2}$

Δ_2 = minimum of $\Delta(h) = \frac{9}{2}$ at h = 1

so $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 = \frac{8}{\sqrt{5}} \times \frac{45\sqrt{5}}{8} - 8 \cdot \frac{9}{2} = 45 - 36 = 9$.

******