PART–I : PHYSICS

SINGLE CORRECT TYPE

This section contains **30 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

1. A radioactive material decays by simultaneous emission of two particles with respective half lives 1620 and 810 years. The time (in years) after which one-fourth of the material remains is :-

(1*) 1080 (2) 2430

(3) 3240 (4) 4860

Ans. (1)

Sol. $\lambda = \lambda_1 + \lambda_2 \Rightarrow \frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$

$\therefore t = \frac{t_1 t_2}{t_1 + t_2} = \frac{810 \times 1620}{810 + 1620} = 540 \ year$

Thus it takes two half life to remains 1/4 th of the sample. So the time

= 2 x 540 = 1080 years

2. A non-linear triatomic gas is filled inside a vessel. If 'α' fraction of moles dissociate into individual atoms, then average degree of freedom for the mixture is : (neglect vibrational degrees of freedom)

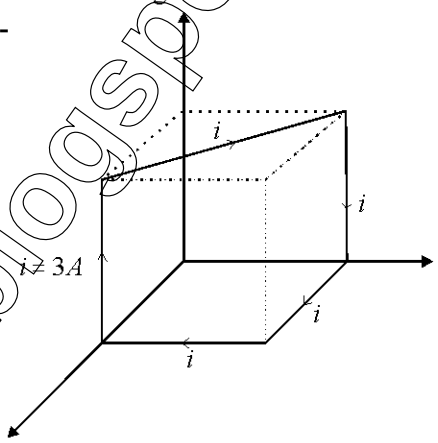
(1) $\frac{3\alpha + 6}{\alpha + 1}$ (2) $\frac{\alpha + 6}{2\alpha + 1}$

(3) $\frac{3\alpha + 6}{\alpha + 2}$ (4*) $\frac{3\alpha + 6}{2\alpha + 1}$

Ans. (4)

Sol. $f_{mix} = \frac{(n\alpha)(3)(3) + (n - n\alpha)(6)}{(n\alpha)(3) + (n - n\alpha)} = \frac{3\alpha + 6}{2\alpha + 1}$

3. A wire carrying 3A current is wrapped around a non - conducting cube of side 1m as shown below. The magntidude of net magnetic moment due to the loop will be :-



(1*) $\frac{9}{2}$ unit

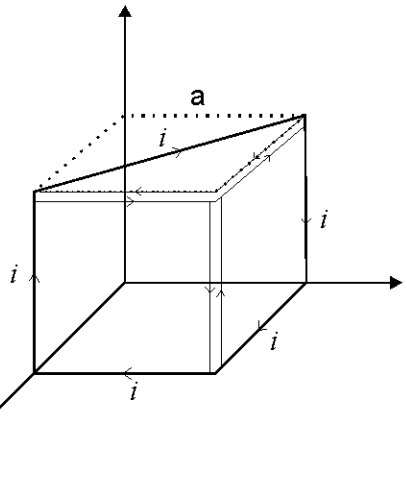
(2) $\frac{1}{\sqrt{3}}$ unit

(3) 9 unit

(4) None of these

Ans. (1)

Sol.



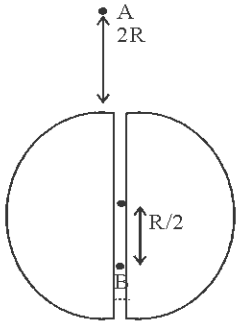
Here $\vec{M} = -ia^2\hat{i} - i\frac{a^2}{2}\hat{j} - ia^2\hat{k}$

$\vec{M} = -ia^2\left(\hat{i} + \frac{\hat{j}}{2} + \hat{k}\right)$

$\Rightarrow |\vec{M}| = \frac{3}{2}ia^2$

$= \frac{3}{2} \times 3 \times 1 \quad |\vec{M}| = \frac{9}{2}$ amphere - meter^2

4. Suppose, if a tunnel is dug along the diameter of the earth and a body of mass m is released from a point 'A' at a distance 2R above earth along the line of tunnel. If M is the mass of the earth & R is the radius of the earth. The velocity of the body during its fall when it crosses the point B at a distance $\frac{R}{2}$ below the earth centre as shown.



(1*) $\frac{5}{2}\sqrt{\frac{GM}{3R}}$ (2) $\sqrt{\frac{GM}{3R}}$

(3) $2\sqrt{\frac{GM}{3R}}$ (4) $\frac{3}{2}\sqrt{\frac{GM}{3R}}$

Ans. (1)

Sol. $V_A = \frac{-GM}{3R}$,

$V_B = \frac{-GM}{2R^3}\left(3R^2 - \frac{R^2}{4}\right) = \frac{-11GM}{8R}$

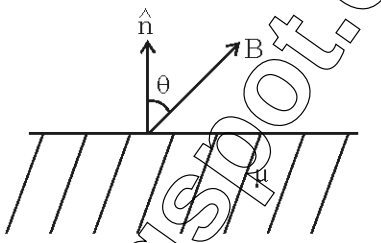
By conservation of energy

$$mV_A = mV_B + \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{-GMm}{3R} = \frac{-11GMm}{8R} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \frac{5}{2}\sqrt{\frac{GM}{3R}}$$

5. A magnetic material is placed in an external magnetic field. The magnetic field makes an angle θ with normal to the surface. If the magnetic field inside the material is B' at an angle ϕ with normal to surface then :

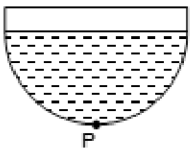


(1) $B\sin\theta = B'\sin\phi$ (2) $B\sin\theta = B'\cos\phi$

(3*) $B\cos\theta = B'\cos\phi$ (4) $B\cos\theta = B'\sin\phi$

Ans. (3)

6. A hemispherical bowl of radius 10 cm is filled with liquid of refractive index $\mu = 4/3$. A glass plate of refractive index 1.5 is placed on the top of bowl. If for the observer above the plate the shift in position of a point P on the bottom is 3 cm find the thickness of glass plate.



(1*) 1.5 cm (2) 1 cm

(3) 7 cm (4) 10 cm

Ans. (1)

Sol. Apparent depth $\frac{t_1}{\mu_1} + \frac{t_2}{\mu_2}$

$$\text{Shift} = (t_1 + t_2) - \left(\frac{t_1}{\mu_1} + \frac{t_2}{\mu_2}\right)$$

$$3.0 = (t_1 + 10) - \left(\frac{t_1}{1.5} + \frac{10 \times 3}{4}\right)$$

$$3.0 = \frac{t_1}{3} + 2.5 \qquad t_1 = 1.5 \text{ cm}$$

7. A monochromatic light is used in Young's double slit experiment when one of the slits is covered by a transparent sheet of thickness 1.8 mm, made of material of refractive index μ_1 number of fringes which shift is 18. when another sheet of thickness 3.6 mm, made of material of refractive index μ_2 is used, number of fringes which shift is 9. Relation between μ_1 and μ_2 is given by.

(1*) $4\mu_2 - \mu_1 = 3$ (2) $4\mu_1 - \mu_2 = 3$

(3) $3\mu_2 - \mu_1 = 4$ (4) $2\mu_1 - \mu_2 = 4$

Ans. **(1)**

Sol. $(\mu - 1)t = n\beta$

$$\frac{(\mu_1 - 1) \times 1.8 \times 10^{-5}}{(\mu_2 - 1) \times 3.6 \times 10^{-5}} = \frac{18\beta}{9\beta}$$

$$(\mu_1 - 1) = 4(\mu_2 - 1)$$

$$4\mu_2 - \mu_1 = 3$$

8. Electrons with de-Broglie wavelength λ fall on the target in an X-ray tube. The cutt-off wavelength of the emitted X-rays is :

(1*) $\lambda_0 = \frac{2mc\lambda^2}{h}$ (2) $\lambda_0 = \frac{2h}{mc}$

(3) $\lambda_0 = \frac{2m^2c^2\lambda^3}{h^2}$ (4) $\lambda_0 = \lambda$

Ans. **(1)**

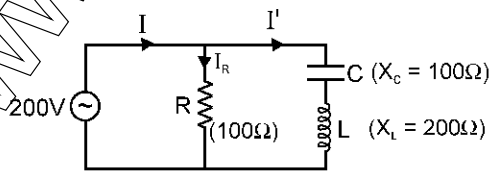
Sol. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \Rightarrow K = \frac{h^2}{2m\lambda^2}$

Wavelength,

$$\lambda_0 = \frac{hc}{K}$$

$$= \frac{hc}{(h^2/2m\lambda^2)} \Rightarrow \frac{2mc\lambda^2}{h}$$

9. In the circuit diagram shown, X_C = 100 ohms, X_L = 200 ohms & R = 100 ohms. The effective current through the source is :

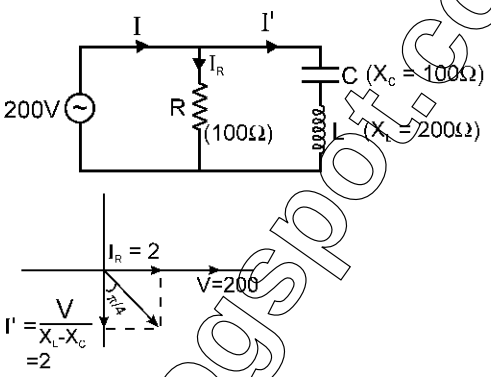


(1) 2 A (2*) $2\sqrt{2}$ A

(3) 0.5 A (4) None of these

Ans. **(2)**

Sol.

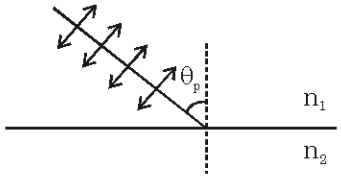


$$I_R = \frac{V}{R} = \frac{200}{100} = 2A$$

$$I' = \frac{V}{X_L - X_C} = \frac{200}{100} = 2A$$

$$I = \sqrt{I_R^2 + I'^2} = 2\sqrt{2} \text{ Amp.}$$

10. A plane polarized wave is incident on a boundary separating two media at brewster's angle. The plane of vibration of electric field is same as the plane of incidence. Then :-



(1) There is no refracted ray

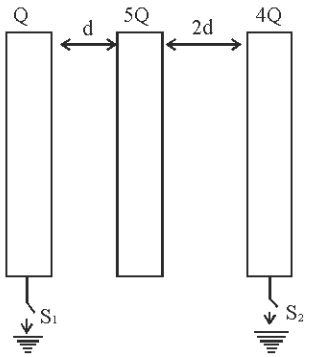
(2*) There no reflected ray.

(3) Reflected ray and refracted ray are both partially polarized.

(4) Reflected ray and refracted ray are polarized in a plane perpendicular to the plane of incidence.

Ans. **(2)**

11. The metal plate on the left carries charge + Q, and on the right has charge 4Q respectively as shown, The central plate has initially charge equal to 5Q. When both S_1 and S_2 switches are closed simultaneously then the charge flowed through S_1 upto steady state is :-



(1) Q

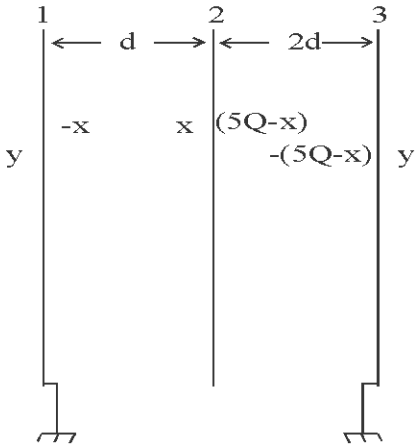
(2) $\frac{5Q}{2}$

(3) $\frac{3Q}{2}$

(4*) None of these

Ans. (4)

Sol.



$y = 0$

$\because$ the potential of both the extreme plates has to be zero

further $V_2 - V_1 = V_2 - V_3$

$$\left(\frac{x}{A\varepsilon_o}\right)d = \left(\frac{5Q-x}{A\varepsilon_o}\right)(2d)$$

$$x = 10Q - 2x$$

$$x = \frac{10Q}{3}$$

Final charge on plate (1) is $= -\frac{10Q}{3}$.

Initial charge on plate (1) is was = Q.

Charge flown through

$$S_1 = Q - \left(-\frac{10Q}{3}\right) = \frac{13Q}{3}$$

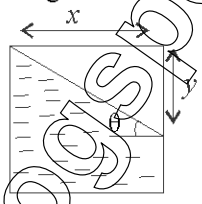
12. An open cubical tank was fully filled with water. When the tank was accelerated on a horizontal plane along one of its side was found that one third of volume of water spilled out. The acceleration was:-

(1) g/3 (2*) 2g/3

(3) 3g/2 (4) None of these

Ans. (2)

Sol. Let 'l' be the length of side $V = l^3$



$$\frac{l^3}{3} = \frac{1}{2}xyl \quad \text{[Volume spilled out]}$$

$$\frac{2}{3}l^2 = x^2\tan\theta \qquad \tan\theta = \frac{y}{x}\ x = l$$

$$\tan\theta = \frac{2}{3} = a/g$$

$$a = \frac{2}{3}g$$

13. Standing waves are set up in a string of length 240cm clamped horizontally at both ends. The separation between any two consecutive points where displacement amplitude is $3\sqrt{2}\,cm$ is 20cm. The standing waves were set by two travelling waves of equal amplitude of 3 cm. The overtone in which the string is vibrating will be :-

(1) 2nd (2) 3rd

(3) 4th (4*) 5th

Ans. (4)

Sol. $2A\sin kx = 3\sqrt{2}$

$$2\times 3\sin kx = 3\sqrt{2}$$

$$\sin kx = \frac{1}{\sqrt{2}}$$

$$\frac{2\pi}{\lambda}x = \frac{\pi}{4}; \frac{3\pi}{4}$$

$$x = \frac{\lambda}{8}; \frac{3\lambda}{8}.....$$

Distance between consecutive points

$= \frac{3\lambda}{8} - \frac{\lambda}{8} = \frac{\lambda}{4}$

$\frac{\lambda}{4} = 20\,\text{cm}$

$\Rightarrow \lambda = 80\,\text{cm}$

So, $(n+1)\frac{\lambda}{2} = 240$

$\Rightarrow (n+1)\frac{80}{2} = 240$

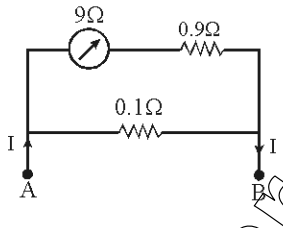
or $n + 1 = 6$

$n = 5$

So string vibrating in fifth overtone.

$\therefore (4)$

14. A milliammeter of range 10 mA and resistance 9Ω is joined in a circuit as shown. The metre gives full-scale deflection for current I when A and B are used as its terminals, i.e., current enters at A and leaves at B. The value of I is :-



(1) 100 mA (2) 900 mA

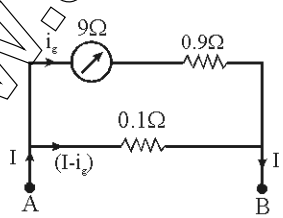
(3*) 1 A (4) 1.1 A.

Ans. **(3)**

Sol. $i_g = 10\text{mA} = 0.01\text{A}$

$V_A - V_B = (I - i_g)0.1 = i_g \times 9.9$

or $I \times 0.1 = 10 i_g$



or $I = \frac{10 \times 0.01}{0.1} = 1\text{A}$.

15. Two sound waves of slightly different frequencies have amplitude ratio $\frac{11}{9}$. What is the difference of sound levels in decibels of maximum and minimum intensities heard at a point :-

(1) 100 (2) 10

(3) 16 (4*) 20

Ans. **(4)**

Sol. $SL_1 - SL_2 = 10\log_{10}\left(\frac{I_{\max}}{I_{\min}}\right)$

$= 10\log_{10}\left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2$

$\Rightarrow SL_1 - SL_2 = 20\log_{10}\left[\frac{\frac{A_1}{A_2} + 1}{\frac{A_1}{A_2} - 1}\right]$

$= 20\log_{10} 10 = 20dB$

16. Two stationary sources A and B are sounding notes of frequency 680 Hz. An observer O moves from A to B with a constant velocity u. If the speed of sound is 340 ms^{-1}, what must be the value of u so that he identifies 10 beats per second ?

(1) 2.0 ms^{-1} (2*) 2.5 ms^{-1}

(3) 3.0 ms^{-1} (4) 3.5 ms^{-1}

Ans. **(2)**

Sol. Apparent frequency $f^1 = f_0\left(1 \pm \frac{u_{rel}}{v}\right)$

$\therefore 10 = 680\left(1 + \frac{u}{340}\right) - 680\left(1 - \frac{u}{340}\right)$

$\Rightarrow u = 2.5m/s$

17. The primary circuit of a potentiometer only contains a battery of emf E_o Volts, having zero internal resistance. The length of the potentiometer wire used is ℓ. A cell of emf E is balanced at a length $\frac{\ell}{3}$ from the positive end of the wire. If the length of the potentiometer wire is increased by $\frac{\ell}{2}$. At what distance will be the same cell give a balance point from the positive end.

(1) $\frac{2\ell}{3}$ (2*) $\frac{\ell}{2}$

(3) $\frac{\ell}{6}$ (4) $\frac{4\ell}{3}$

Ans. (2)

Sol. Let x be the desired length

Potential gradient in the first case $= \frac{E_0}{\ell}$

$\therefore \quad E = \left(\frac{\ell}{3}\right).\left(\frac{E_0}{\ell}\right) = \frac{E_0}{3}$ ...(i)

Potential gradient in second case $= \frac{E_0}{3\ell/2} = \frac{2E_0}{3\ell}$

$\therefore \quad E = (x)\frac{2E_0}{3\ell}$ ...(ii)

From equations (i) and (ii),

$\frac{E_0}{3} = \left(\frac{2E_0}{3\ell}\right)x \qquad x = \frac{\ell}{2}$

18. The angular momentum of electron in hydrogen atom is proportional to :-

(1*) $\sqrt{r}$ (2) r

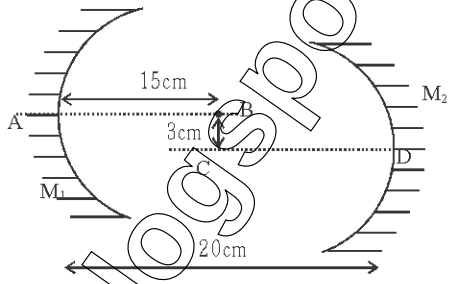
(3) r^2 (4) $\frac{1}{\sqrt{r}}$

Ans. (1)

Sol. $L = \frac{nh}{2\pi}$ and $r \propto n^2$

$n \propto \sqrt{r}$ so $L \propto \sqrt{r}$

19. M_1 and M_2 are two concave mirrors of the same focal length 10 cm. AB & CD are their principal axes respectively. An object is kept on the line AB at distance 15 cm from M_1. The distance between the mirrors is 20 cm. Considering two successive reflections, first on M_1 and then on M_2. The distance of final image from the line AB is :-



(1) 3 cm (2*) 1.5 cm

(3) 4.5 cm (4) 1 cm

Ans. (2)

Sol. Reflection through M_1

$\frac{1}{v} + \left(\frac{-1}{15}\right) = \frac{-1}{10}$

$\frac{1}{v} = \frac{+1}{15} - \frac{1}{10} = \frac{2-3}{30}$

$v = -30\,cm$

Reflection through M_2

$\frac{1}{v} + \frac{1}{10} = -\frac{1}{10}$

$\frac{1}{v} = \frac{-2}{10} \Rightarrow v = -5$

$M = \frac{-v}{u} = \frac{5}{10} = \frac{1}{2}$

$m = \frac{h_i}{h_o}, \qquad \frac{1}{2} = \frac{h_i}{h_o}$

$\Rightarrow h_i = \frac{3}{2} \qquad h_i = 1.5\,cm$

$\therefore$ Distance of image from AB = 3 - 1.5 = 1.5 cm

20. In YDSE experiment with a white light with a plane wavefront,the light reaching the screen at position y = 1mm (distance from central maxima on the screen) is passed through a prism and its spectrum is obtained. Find the missing lines in the visible region

of this spectrum. [Given D = 1m, d = 0.9mm, where symbol has usual meanings]

(1*) 600 nm (2) 700 nm

(3) 750 nm (4) 560 nm

Ans. (1)

Sol. $\text{path difference} = \frac{yd}{D} = 900\text{nm}$

Condition for missing lines

$$\text{Path Difference} = \frac{(2n-1)\lambda}{2} \Rightarrow \lambda = \frac{2\Delta x}{2n-1}$$

$$\lambda = \frac{1800}{2n-1} \text{ put } n = 1, 2, 3$$

$$\lambda = 1800\text{nm}, 600\text{nm}, 360\text{nm}$$

21. Choose the correct equation of maxwell - ampere circuital law :

(1) $\oint \vec{B}.\vec{dl} = \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$

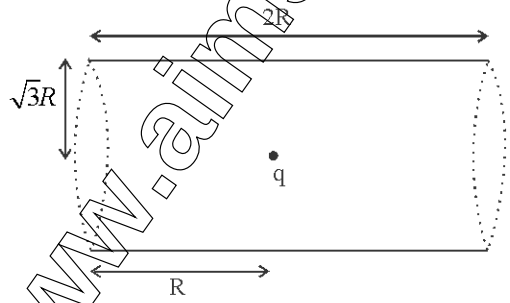
(2) $\oint \vec{B}.\vec{dl} = \mu_0 + \varepsilon_0 \frac{d\phi_E}{dt}$

(3*) $\oint \vec{B}.\vec{dl} = \mu_0\left(i_C + \varepsilon_0 \frac{d\phi_E}{dt}\right)$

(4) $\oint \vec{B}.\vec{dl} = \mu_0 i_C + \varepsilon_0 \frac{d\phi_E}{dt}$

Ans. (3)

22. In the shown figure, a point charge q is placed as shown. An imaginary cylindrical surface is also drawn as shown. The flux through curved surface of the cylinder will be :

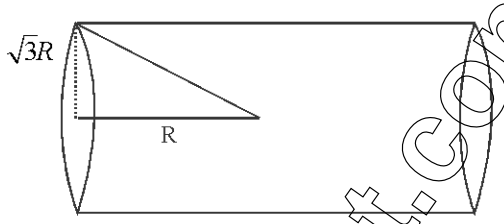


(1) $\frac{q}{\varepsilon_o}$ (2) $\frac{q}{3\varepsilon_o}$

(3*) $\frac{q}{2\varepsilon_o}$ (4) $\frac{q}{4\varepsilon_o}$

Ans. (3)

Sol.



$$\tan\theta = \frac{\sqrt{3}R}{R} \qquad \theta = 60°$$

$$\Omega = 2\pi(1 - \cos 60) = \pi \text{ str.}$$

$$\Omega^y = 4\pi - 2(\pi) = 2\pi \text{ str.}$$

$$\phi = \frac{\Omega^1}{4\pi}\left(\frac{q}{\varepsilon_o}\right) = \frac{q}{2\varepsilon_o}$$

23. A basic comunication system consists of

A. Transmitter, B. Information source,

C. User of information, D. Channel,

E. Receiver.

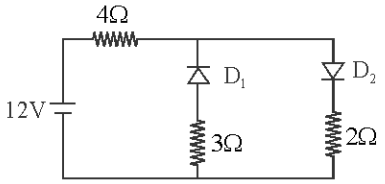
Choose the correct sequence in which there are arranged in a basic communication system :

(1) ABCDE (2*) BADEC

(3) BDACE (4) BEADC

Ans. (2)

24. The circuit has two oppositely connected ideal diodes in parallel. What is the current flowing through the Battery ?



(1) 1.71 A (2*) 2.0 A

(3) 2.31 A (4) 1.33 A

Ans. (2)

25. Begining from rest position, A solid Disk, solid Ball and a Hoop of same mass and same radius race down an inclined plane. Rank them in reaching the Bottom.

(1) Ist Rank : Ball, IInd Rank : Disk, IIIrd Rank : Hoop

(2) Ist Rank : Hoop, IInd Rank : Ball, IIIrd Rank : Disk

(3) All three will reach at same time

(4*) Information insufficient

Ans. (4)

Sol. We need to check whether they are rolling or sliding.

26. A solid iron ball and an aluminium ball of same diameter are released together in a lake. Which ball will reach the bottom first. (neglect viscosity and $\rho_{Al} < \rho_{Fe}$)

(1) Aluminium Ball

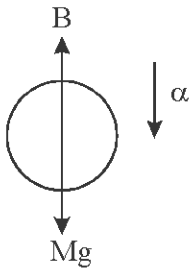
(2*) Iron Ball

(3) Both will reach at the same time

(4) Data insufficient

Ans. (2)

Sol. Upthrust will be same in both the balls.



$$mg - B = ma$$

$$a = \frac{mg - B}{m} = g - \frac{B}{m}$$

Implying greater the mass greater the acceleration

$m_{iron} > m_{aluminium}$ [for same volume]

$\therefore\ a_{iron} > a_{aluminium}$

27. Two men A and B stand facing each other on two boats having combined masses m_1 (A + boat 1) & m_2 (B + boat 2) floating on still water at a short distance 's' apart. A rope is held at its ends by both and pulled by them with a force 'F'. Find the time taken after which they meet :-

(1) $\sqrt{\frac{m_2^2 s}{m_1 F}}$ (2*) $\sqrt{\frac{2m_1 m_2 s}{(m_1+m_2)F}}$

(3) $\sqrt{\frac{m_1 m_2 s}{(m_1+m_2)F}}$ (4) $\sqrt{\frac{m_1^2 s}{m_2 F}}$

Ans. (2)

Sol. They meet at center of mass

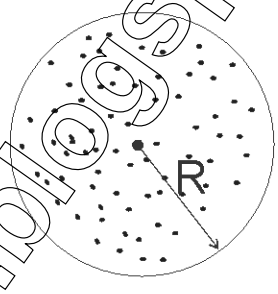
$$x_{cm} = \frac{m_2 s}{m_1 + m_2}$$

$$a_{m_1} = \frac{F}{m_1}$$

$$s = ut + \frac{1}{2}at^2$$

$$x_{cm} = \frac{1}{2}\frac{F}{m_1}t^2 \Rightarrow t = \sqrt{\frac{2m_1 m_2 s}{(m_1+m_2)F}}$$

28. In the diagram a time varying magnetic field passes through a circular region of radius "R" and the magnetic field directed outwards and it is a function of radial distance 'r' and time 't' as $B = B_0 rt$. The electric field strength at a radial distance R/2 from the center ?



(1*) $\frac{B_0 R^2}{12}$ (2) $\frac{B_0 R^2}{6}$

(3) $\frac{2B_0 R^2}{3}$ (4) $\frac{B_0 R^2}{16}$

Ans. (1)

Sol. $\int \vec{E}.\vec{dl} = -A\frac{dB}{dt} = -\frac{d\phi}{dt}$

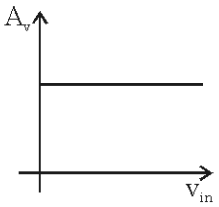
$$\phi = \int_0^{R/2} B 2\pi r\, dr = B_0 t 2\pi \left[\frac{r^3}{3}\right]_0^{R/2}$$

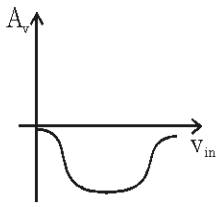
$$= 2\pi B_0 \frac{R^3}{24}(t)$$

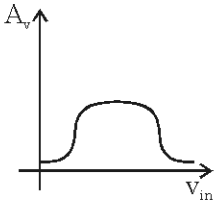
$$E\ 2\pi R/2 = -\frac{d\phi}{dt} = -B_0 2\pi \frac{R^3}{24}$$

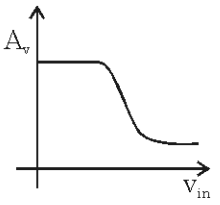
$$E = -\frac{B_0 R^2}{12}$$

29. The voltage gain of a n-p-n transistor is plotted against input voltage. The correct graph is :

(1) 

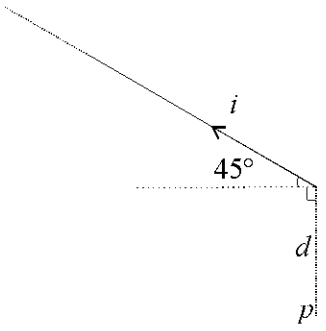
(2*) 

(3) 

(4) 

Ans. (2)

30. Find the magnitude of magnetic field at point p due to a semi - infinite wire given below :-

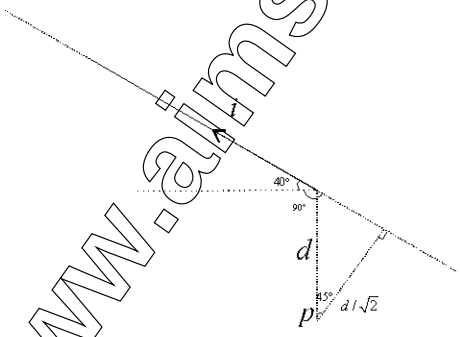


(1*) $\frac{\mu_0 i}{4\pi d}\left(\sqrt{2}-1\right)$ (2) $\frac{\mu_0 i}{4\pi d}\left(1-\sqrt{2}\right)$

(3) $\frac{\mu_0 i}{4\pi d}\left(1-\frac{1}{\sqrt{2}}\right)$ (4) $\frac{\sqrt{2}\mu_0 i}{4\pi d}$

Ans. (1)

Sol.



$$B=\frac{\mu_0 i}{4\pi d}\times\left[\sin\theta_1+\sin\theta_2\right]$$

Where d is $\perp$ distance iron wire

$$B=\frac{\mu_0 i}{4\pi d/\sqrt{2}}\times\left[\sin 90-\sin 45^\circ\right]$$

$$=\frac{\sqrt{2}\mu_o i}{4\pi d}\left[1-\frac{1}{\sqrt{2}}\right] \qquad B=\frac{\mu_0 i}{4\pi d}\left[\sqrt{2}-1\right]$$

PART–II : CHEMISTRY

SINGLE CORRECT TYPE

This section contains **30 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

31. The schrodinger wave equation for hydrogen atom is $\psi_{radial} = \frac{1}{16\sqrt{4}}\left(\frac{Z}{a_0}\right)^{3/2}\left[(\sigma-1)(\sigma^2-8\sigma+12)\right]e^{-\sigma/2}$ where a_0 & Z are coustants in which the answer can be expressed & $\sigma = \frac{2Zr}{a_0}$. Minimum & maximum distance of radial node from nucleus are

(1) $\frac{a_0}{Z}, \frac{3a_0}{Z}$ (2) $\frac{a_0}{2Z}, \frac{a_0}{Z}$

(3*) $\frac{a_0}{2Z}, \frac{3a_0}{Z}$ (4) $\frac{a_0}{2Z}, \frac{4a_0}{Z}$

Ans. (3)

Sol. Probability of finding the electron is

$\psi^2 = 0$ or $\psi = 0$ $\sigma = \frac{2Zr}{a_0}$

$(\sigma-1) = 0 \Rightarrow \sigma = 1$ $r = \frac{\sigma a_0}{2Z}$

or $r = \frac{a_0}{2Z}$ or $\sigma^2 - 8\sigma + 12 = 0$

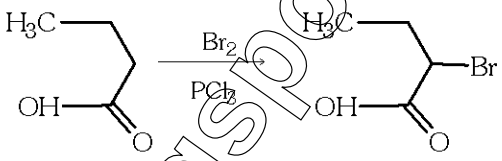
$\sigma = 6; \quad \sigma = 2$

if $\sigma = 6 \Rightarrow r = \frac{6a_0}{2Z} = \boxed{\frac{3a_0}{Z}}$

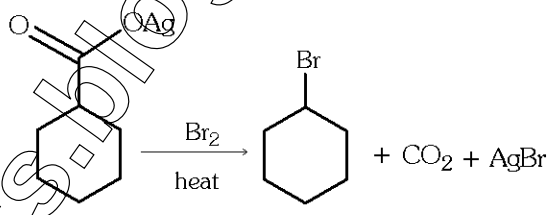
$\sigma = 2$ $r = \frac{2a_0}{2Z} = \frac{a_0}{Z}$

$\sigma = 1$ $r = \frac{a_0}{2Z} = \boxed{\frac{a_0}{2Z}}$

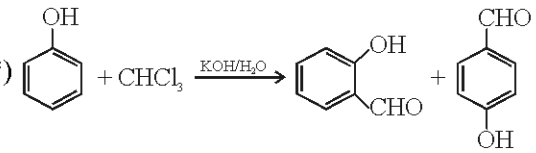
32. Identify the correct match:

(1) 

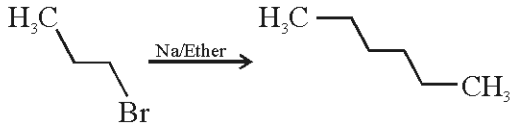
Etard reaction

(2) 

HVZ reaction

(3*) 

Reimer-Tiemann reaction

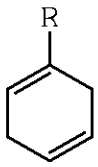
(4) 

Fitting reaction

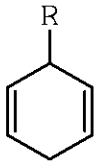
Ans. (3)

Sol. Only correct match.

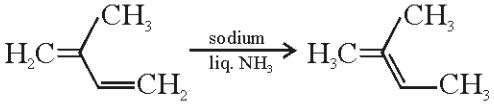
33. Identify the valid statements regarding Birch reduction

(1)  is the type of the product when

R = electron donating group.

(2)  is the type of the product when

R = electron withdrawing group.

(3) 

(4*) All are correct.

Ans. (4)

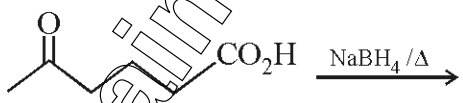
Sol. Birch reduction of aromatic ring system gives mainly unconjugated dihydroderivatives.

34. Statement-1 : LDA is a very strong nucleophile but it is not a very good base.
Statement-2 : Nucleophilicity is kinetic property but basicity is thermodynamic property.

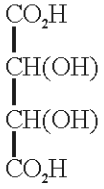
(1) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4*) Statement-1 is false, statement-2 is true.

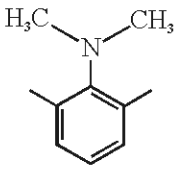
Ans. (4)

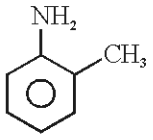
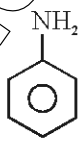
35. Which of the following options are not correct :

(1) 

Lactone is formed as product

(2) Tartaric acid  has 2 optically active stereoisomers.

(3)  is more basic than 

(4*)  is more basic than 

Ans. (4)

36. At 298K, the reduction potential of hydrogen electrode at pH = 10, will be

(1) 0.059 V (2*) –0.59V
(3) 0.59 V (4) –0.285V

Ans. (2)

Sol.

$$H^+ + e^- \rightarrow \frac{1}{2}H_2$$

$$E = E° - \frac{0.059}{1}\log\frac{1}{[H^+]} = -0.59V$$

37. A manganese (II) ion salt is oxidised by peroxodisulphate , then the oxidation number of Mn in the product formed is

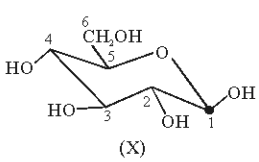
(1) +4 (2) +5
(3*) +7 (4) +3

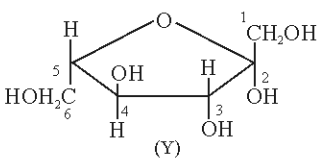
Ans. (3)

Sol. The product formed is MnO_4^-

$$2Mn^{+2} + 5S_2O_8^{-2} + 8H_2O \rightarrow 2MnO_4^- + 10SO_4^{-2} + 16H^+$$

38. A non reducing disaccharide is obtained by condensation of X & Y . Condensation takes place between X & Y respectively at -

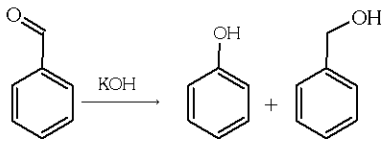

(X)

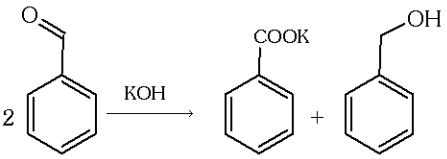

(Y)

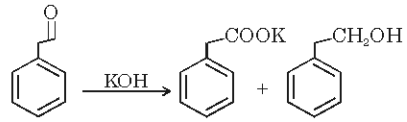
(1) C-1 & C-1 (2) C-1 & C-5
(3) C-5 & C-1 (4*) C-1 & C-2

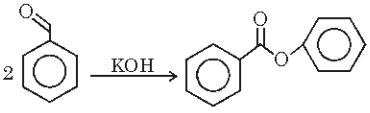
Ans. (4)

39. Cannizzaro's reaction can be exemplified by one of the following :

(1) 

(2*) 

(3) 

(4) 

Ans. **(2)**

Sol. Aromatic aldehydes that do not have α hydrogen atoms on treatment with concentrated alkali undergo self oxidation and reduction to give alcohol and salt of the corresponding carboxylic acid during Cannizzaro's reaction.

40. Atoms $_{7}X^{A}$,$_{8}Y^{B}$,$_{9}Z^{17}$ are such that $_{8}Y$ is an isobar of $_{7}X$ & atom $_{9}Z^{17}$ is isotone of $_{8}Y$. Mass no of 'X' & no of neutrons in y are respectively

(1) 8, 8 (2) 17, 7

(3) 9, 8 (4*) 16, 8

Ans. **(4)**

Sol . Isotones have same number of neutrons

$17-9=B-8$

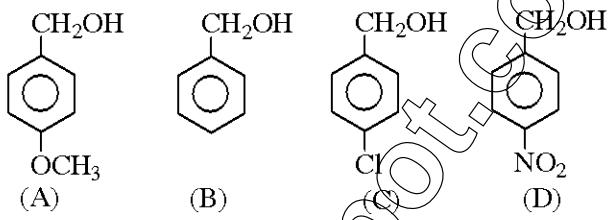
$B=16$

Isobars have same mass number

$A=B=16$

No of neutrons $=16-8=8$

41. What is the decreasing order of rate of reaction with HBr for the following benzyl alcohol and its derivative :



(A) (B) (C) (D)

(1) A > C > D > B (2) A > B > D > C

(3) D > C > B > A (4*) A > B > C > D

Ans. **(4)**

42. End centered Bravais Lattice is possible for

(1) Cubic (2) Hexagonal

(3*) Monoclinic (4) Triclinic

Ans. **(3)**

Sol. For monoclinic & orthorhombic, end centered unit cell is possible.

43. Incorrect statement among the following is

(1) PCl_3 is prepared by action of thionyl chloride on white phosphorous.

(2*) Partial hydrolysis of XeF_6 is a redox reaction.

(3) Catalyst used in Deacon's process is $CuCl_2$

(4) $FeCl_2$ is more ionic than $FeCl_3$.

Ans. **(2)**

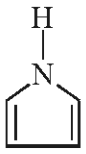
Sol. Hydrolysis of XeF_6 gives

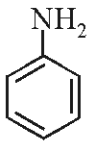
$$XeF_6 + H_2O \rightarrow XeOF_4 + 2HF$$

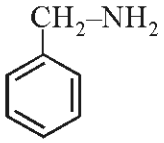
$$XeF_6 + 2H_2O \rightarrow XeO_2F_2 + 4HF$$

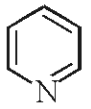
In above two reactions there is no change in oxidation number

44. Least basic compound among the following :

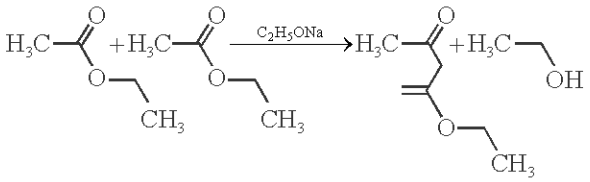
(1*) 

(2) 

(3) 

(4) 

Ans. (1)

45. 

For the above reaction the applicable facts are :

(1) It is Claisen's condensation reaction

(2) The base commonly employed is the sodium alkoxide corresponding to the OR of the ester(R'COOR) function in order to avoid ester interchange which leads to transesterification

(3) It is an example for self condensation.

(4*) All are correct.

Ans. (4)

Sol. All are valid statements for the reaction shown

46. Which of the following is an amphoteric oxide

(1*) ZnO (2) CrO_3

(3) NO (4) Mn_2O_7

Ans. (1)

Sol . It reats with both strong acids and strong bases.

47. 0.1 mole of 'XO' loses 1.806×10^{23} electrons then the oxidation state of 'X' in the product will be

(1) +4 (2*) +5

(3) +6 (4) –5

Ans. (2)

Sol. 1 mole of XO loses =

$$\frac{1.806\times10^{23}}{0.1\times6.02\times10^{23}} = 3\text{moles of electrons}$$

48. Which of the following is correct ?

(1) Silicones are good electric conductors

(2) on methylation of diborane, we get $B_2(CH_3)_6$

(3) Borax is having all planar units

(4*) Graphite is thermodynamically more stable than diamand

Ans. (4)

Sol. Silicons are good insulators , on methylation of dibrane we get $B_2H_2(CH_3)_4$

49. When the complex $K_6[(CN)_5\,Co–O–O–Co(CN)_5]$ is oxidised by bromine into $K_5[(CN)_5\,Co–O–O–Co(CN)_5]$. Then which of the following statements will be true about this change:

(1) Co(II) is oxidised to Co(III)

(2) The O–O bond length will increase

(3*) The O–O bond length will decrease

(4) '1' & '2' both are correct

Ans. (3)

Sol. In the first complex ligand is O_2^{2-} which is oxidised into O_2^{1-}.

hence O – O bond length decreases.

50. Assume that the decomposition of HNO_3 is

$$4HNO_3(g) \rightleftharpoons 4NO_2(g) + 2H_2O(g) + O_2(g)$$

and the reaction approaches equilibrium at 400K & 30 atm pressure. At equilibrium the partial pressure of HNO_3 is 2atm. Find K_C at 400 K $\left(R = 0.08\ \ell - atm/K - mol\right)$

(1) 4 (2) 8

(3) 16 (4*) 32

Ans . (4)

Sol . $P_{Total} = P_{HNO_3} + P_{NO_2} + P_{H_2O} + P_{O_2}$

$\because P_{NO_2} = 4P_{O_2}$ & $P_{H_2O} = 2P_{O_2}$

$\therefore \quad P_{Total} = P_{HNO_3} + 7P_{O_2}$

$\Rightarrow 30 - 2 = P_{O_2} \times 7 \Rightarrow P_{O_2} = 4$

$$K_P = \frac{\left(P_{NO_2}\right)^4 . P_{H_2O} . P_{O_2}}{P^4_{HNO_3}}$$

$$= \frac{(4\times4)^4 \times (2\times4)^2 \times 4}{2^4} = 2^{20}$$

.

$$K_P = K_C (RT)^{\Delta ng} = K_C \times (0.08 \times 400)^3$$

$$K_C = \frac{2^{20}}{(32)^3} = 32$$

51. In which species number of lone pair on iodine and number of d - orbitals used in hybridisation by iodine are same

(1) ICl_2^+ (2) ICl_2^-

(3) IF_7 (4*) ICl_4^-

Ans. (4)

Sol. ICl_2^+ - sp^3 - 2 Lp

ICl_2^- - sp^3d - 3 LP

IF_7 sp^3d^3 - 0 LP

ICl_4^- sp^3d^2 - 2 LP

52. Which of the following acid remains unaffected on heating ?

(1) Adipic acid (2) Maleic acid

(3*) Fumaric acid (4) Succinic acid

Ans. (3)

53. If one mole of a monoatomic gas $(\gamma = 5/3)$ is mixed with 1 mole of a diatomic gas $(\gamma = 7/5)$. The value of γ for the mixture

(1) 1.4 (2*) 1.5

(3) 1.53 (4) 3.07

Ans. (2)

Sol . $\left.\begin{aligned} C_V &= \tfrac{3}{2}RT \\ C_P &= \tfrac{5}{2}RT \end{aligned}\right\}$ monoatomic

$\left.\begin{aligned} C_V &= \tfrac{5}{2}RT \\ C_P &= \tfrac{7}{2}RT \end{aligned}\right\}$ diatomic

for mixture of 1 mol each

$$C_V = \frac{\frac{3}{2}RT + \frac{5}{2}RT}{2}$$

$$C_P = \frac{\frac{5}{2}RT + \frac{7}{2}RT}{2}$$

$$\frac{C_P}{C_V} = \frac{3RT}{2RT} = 1.5$$

54. The molecule $(SiH_3)_3N$ is

(1) Pyramidal and more basic than $(CH_3)_3N$

(2*) Planar and less basic than $(CH_3)_3N$

(3) Pyramidal and less basic than $(CH_3)_3N$

(4) Planar and more basic than $(CH_3)_3N$

Ans. (2)

Sol. Back bonding takes place in $(SiH_3)_3N$ due to vacent d orbitals.Hence it is planar and hence less basic.

55. Tyndall effect will be observed for

(1) Colloidal solutions in which the diameter of dispersed particles is very much smaller than the wavelength of incident light.

(2) Colloidal solutions in which the diameter of dispersed particles is not much smaller than the wavelength of incident light.

(3) The refractive indices of dispersed phase & the dispersion medium must differs

(4*) Both (2) and (3)

Ans. (4)

Sol. Conceptual

56. A student decides to demonstrate NaCl and CsCl structure in his annual practical examination. He decides to make a two - in - one structure.
He takes some wire and creates a 3 - D cubical mesh like structure and tries to locate only the lattice points and void position by small bulbs of different colour.
Na^+ = Green colour
Cl^- = Red colour
Cs^+ = Blue colour
He finally decides to show a 3 - D structure which contains only 4 NaCl or 8 CsCl molecule. Once it look like NaCl then after 1 min again CsCl and on so on. Number of extra red colour bulbs that start glowing after 1 min are
(1) 12 (2) 11
(3*) 13 (4) 14

Ans. **(3)**

Solution Those Cl^- which are kept on edgecenter and bodycenter of big cube will form common CsCl unit.

57. Which of the following is incorrect ?
(1*) Ellingham diagram can be used to find the faster reducing agent
(2) Ellingham diagram is based on thermodynamic concepts
(3) In Ellingham diagram , it is presumed that the reactions and products are in equilibrium
(4) In Ellingham diagram , ΔG increases with an increase in temperature for the formation of metal oxides

Ans. **(1)**

Sol. It can't predict the kinetics of reaction

58. The correct electronic configuration for a metal ion of d^4 configuration in weak octahedral field is
(1) $t_{2g}^4\ e_g^o$ (2*) $t_{2g}^3\ e_g^1$
(3) $e_g^4\ t_{2g}^o$ (4) $t_{2g}^2\ e_g^2$

Ans. **(2)**

Sol. In weak field , $\Delta_0 < P$ and form high spin complexes

59. At 298 K, for the following reaction
$2A + B \rightleftharpoons C$, $\Delta H°$ & $\Delta S°$ are
82.424 kJ mole^{-1} & 0.2 kJ mole^{-1} K^{-1} respectively then equilibrium constant for the above equilibrium will be (approximate value)
(1) 10^{-10} (2) 10^{-2}
(3*) 10^{-4} (4) 10^{-6}

Ans. **(3)**

Sol. $\Delta G° = 82.424 - 0.2 \times 298$
$= 22.824 kJ$
$\therefore K = 10^{-\frac{\Delta G°}{2.303RT}} = 10^{\frac{-22.824\times1000}{2.303\times8.314\times298}} = 10^{-4}$

60. Which of the following is incorrect ?
(1) Rhombic sulphur is insoluble in water but dissolves to some extent in benzene.
(2) Below 369 K , β – sulphur transforms into α – sulphur .
(3) Both Rhombic and monoclinic sulphur have S_8 molecules
(4*) α – sulphur is insoluble in CS_2

Ans. **(4)**

Sol. It is readily soluble in CS_2

PART–III : MATHEMATICS

SINGLE CORRECT TYPE

This section contains **30 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

61. Let $f(x) = \int e^{-x}(x-2)(x-4)\,dx$, then $f(x)$ decreases in the interval

(1*) (2, 4)

(2) $(-\infty, 2) \cup (4, \infty)$

(3) (0, 2)

(4) (0, 2) $\cup$ (4, ∞)

Ans. (1)

Sol. $f'(x) = e^{-x}(x-2)(x-4) < 0$

62. The value of $\int \frac{\sin 2x + 2\tan x}{\left(\cos^6 x + 6\cos^2 x + 4\right)} dx$ is equal to

(1) $2\sqrt{\frac{1+\cos^2 x}{\cos^7 x}} + c$

(2) $\tan^{-1}\frac{1}{\sqrt{2}}\left(\frac{1+\cos^2 x}{\cos^7 x}\right) + c$

(3) $\frac{1}{12}\ln\left(\frac{1+\cos^2 x}{\cos^7 x}\right) + c$

(4*) None of these

(where c is constant of integration)

Ans. (4)

Sol. $2\int \frac{(\cos x + \sec x)\sin x}{\left(\cos^6 x + 6\cos^2 x + 4\right)} dx = -2\int \frac{\left(t^2+1\right)}{t^7 + 6t^3 + 4t} dt$

Putting cosx = t

$$= -2\int \frac{\frac{1}{t^5} + \frac{1}{t^7}}{1 + \frac{6}{t^4} + \frac{4}{t^6}} dt = \frac{1}{12}\ln\left(1 + \frac{6}{t^4} + \frac{4}{t^6}\right) + c,\ t = \cos x$$

63. The value of $\int_{-6}^{6} \max\left(\left|2-|x|\right|, 4-|x|, 3\right) dx$ is

(1) 40 (2) 46

(3*) 38 (4) 30

Ans. (3)

Sol. $\int_{-6}^{6} \max\left(\left|2-|x|\right|, 4-|x|, 3\right) dx$

$$= 2\left[\int_0^1 (4-|x|)dx + \int_1^5 3dx + \int_5^6 |2-|x||\,dx\right] = 38$$

64. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$,

then $\left(BB^T A\right)^5$ is equal to

(1) $\begin{bmatrix} 2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3} \end{bmatrix}$ (2) $\frac{1}{2}\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$

(3*) $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 5 & 1 \\ 0 & 1 \end{bmatrix}$

Ans. (3)

Sol. $BB^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{4}+\frac{1}{4} & -\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4} & \frac{1}{4}+\frac{3}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

65. If $\alpha, \beta\ (\beta > \alpha)$ are the roots of $f(x) \equiv ax^2 + bx + c = 0$, $a \neq 0$ and f(x) is an even function and $I = \int_{\alpha}^{\beta} \frac{e^{f\left(\frac{f(x)}{x-\alpha}\right)}}{e^{f\left(\frac{f(x)}{x-\alpha}\right)} + e^{f\left(\frac{f(x)}{x-\beta}\right)}}$, then $|I|$ is equal to

(1) $\left|\frac{b}{a}\right|$ (2) $\left|\frac{b}{2a}\right|$

(3*) $\frac{\sqrt{b^2 - 4ac}}{|2a|}$ (4) None of these

Ans. (3)

Sol. $I=\int_{\alpha}^{\beta}\frac{e^{f\left(\frac{a(x-\alpha)(x-\beta)}{x-\alpha}\right)}}{e^{f\left(\frac{a(x-\alpha)(x-\beta)}{x-\alpha}\right)}+e^{f\left(\frac{a(x-\alpha)(x-\beta)}{x-\beta}\right)}}dx$

$=\int_{\alpha}^{\beta}\frac{e^{f(a(x-\beta))}}{e^{f(a(x-\beta))}+e^{f(a(x-\alpha))}}dx$ ....(1)

$=\int_{\alpha}^{\beta}\frac{e^{f(a(\alpha+\beta-x-\beta))}}{e^{f(a(\alpha+\beta-x-\beta))}+e^{f(a(\alpha+\beta-x-\alpha))}}dx$

$I=\int_{\alpha}^{\beta}\frac{e^{f(a(x-\alpha))}}{e^{f(a(x-\alpha)}+e^{f(a(x-\beta))}}dx$ ....(2)

$2I=\int_{\alpha}^{\beta}dx\Rightarrow I=\frac{|\alpha-\beta|}{2}=\frac{\sqrt{b^2-4ac}}{2|a|}$

66. If $f(x)\geq 0\ \forall x\in(0,2)$ and $y=f(x)$ makes positive, intercepts of 2 and 1 unit on x and y-axis respectively and encloses an area of $\frac{3}{4}$ units with $x=0$, $x=2$ and $y=0$, then $\int_0^2 xf'(x)dx$ is

(1) $\frac{3}{4}$ (2) 1

(3) $\frac{5}{4}$ (4*) $-\frac{3}{4}$

Ans. (4)

Sol. $I=xf(x)\big|_0^2-\int_0^2 f(x)dx=0-\frac{3}{4}=\frac{3}{4}$

67. $P(x)$ is a polynomial such that $P(x)+P(2x)=5x^2-18$, then $\lim_{x\to 3}\frac{P(x)}{x-3}$

(1*) 6 (2) 9

(3) 18 (4) 0

Ans. (1)

Sol. Let $P(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+a_3x^{n-3}+....+a_n$

$P(2x)=a_0 2^n x^n+a_1 2^{n-1}x^{n-1}+....+a_n$

$P(x)+P(2x)=5x^2-18$

$\Rightarrow (a_0+a_0 2^n)x^n+...+2a_n=5x^2-18$

$\therefore n=2$

$(a_0+4a_0)x^2+2a_2=5x^2-18$

$5a_0=5$ and $a_2=-9$

$\therefore P(x)=x^2-9$

$\lim_{x\to 3}\frac{x^2-9}{x-3}=\lim_{x\to 3}(x+3)=6$

68. If $(1+x)^n=C_0+C_1x+C_2x^2+...+C_nx^n$, where $C_0,C_1,C_2,.....$ are bionomial coefficients and $C_r={}^nC_r$. Then $2(C_0+C_3+C_6+....+C_n)$

$+(C_1+C_4+C_7+....+C_{n-2})(1+\omega)$

$+(C_2+C_5+C_8+....+C_{n-1})(1+\omega^2)$

(where ω is the non real complex cube root of unity and n is an odd multiple of 3), is equal to

(1) 2^n-1 (2) $2^{n-1}+1$

(3) $2^{n+1}-1$ (4*) 2^n-1

Ans. (4)

Sol. $(1+\omega)^n={}^nC_0+{}^nC_1\omega+{}^nC_2\omega^2+....+{}^nC_n\omega^n$

$(1+1)^n={}^nC_0+{}^nC_1+{}^nC_2+....+{}^nC_n$

$(1+\omega)^n+(1+1)^n=2C_0+C_1(1+\omega)+C_2(1+\omega^2)+C_3(1+\omega^3)+C_4(1+\omega)$

$+C_5(1+\omega^2)+C_6(1+\omega^3)+...C_n(1+\omega^n)$

$2(C_0+C_3+C_6+...)+(C_1+C_4+C_7+...)(1+\omega)+(C_2+C_5+C_8+...)(1+\omega^2)=-\omega^{2n}+2^n$

$\Rightarrow(2^n-1)(\because n\ in\ a\ multiple\ of\ 3,\ w^n=1)$

69. For $x>-1$, let $f(x)=\int_0^{\pi/4}\log_e(1+x\tan z)dz$.

Then value of $f\left(\frac{1}{2}\right)-f\left(\frac{1}{3}\right)$ equals

(1) $\frac{\pi}{4}\log_e\left(\frac{9}{8}\right)$ (2*) $\frac{\pi}{8}\log_e\left(\frac{9}{8}\right)$

(3) $\frac{\pi}{9}\log_e\left(\frac{8}{9}\right)$ (4) $\frac{\pi}{8}\log_e\left(\frac{3}{2}\right)$

Ans. (2)

Sol. Let

$$I = f\left(\frac{1}{2}\right) - f\left(\frac{1}{3}\right) = \int_0^{\pi/4} \log_e\left(\frac{1+\frac{1}{2}\tan z}{1+\frac{1}{3}\tan z}\right) dz = \int_0^{\pi/4} \log_e\left(\frac{3}{2}\cdot\frac{2+\tan z}{3+\tan z}\right) dz$$

Replacing z by $\frac{\pi}{4} - z$, we get

$$I = \int_0^{\pi/4} \log_e\left(\frac{3}{4}\cdot\frac{3+\tan z}{2+\tan z}\right) dz$$

$$\Rightarrow 2I = \int_0^{\pi/4} \log_e\left(\frac{9}{8}\right) dz = \frac{\pi}{4}\log_e\left(\frac{9}{8}\right)$$

$$\Rightarrow I = \frac{\pi}{8}\log_e\left(\frac{9}{8}\right)$$

70. The tangents at z_1, z_2 on the circle $|z - z_0| = a$, meet at z_3. Then $\left(\frac{z_3 - z_1}{z_0 - z_1}\right)\left(\frac{z_0 - z_2}{z_3 - z_2}\right)$ is equal to

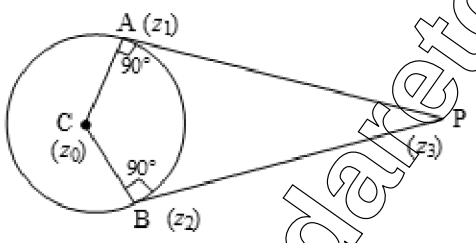
(1) 0 (2) 1

(3*) –1 (4) 2

Ans. (3)

Sol. Let z_1, z_2 are represented by A, B whereas z_0 is represented by C.

Let P represent z_3



$$\frac{z_3 - z_1}{z_0 - z_1} = \frac{PA}{AC}e^{\frac{i\pi}{2}} \; ; \; \frac{z_0 - z_2}{z_3 - z_2} = \frac{BC}{PB}e^{\frac{i\pi}{2}}$$

$$\frac{z_3 - z_1}{z_0 - z_1}\cdot\frac{z_0 - z_2}{z_3 - z_2} = \frac{PA}{\text{radius}}\cdot\frac{\text{radius}}{PB}e^{i\pi} = -1 \quad (\because PA = PB)$$

71. A letter is taken at random from the letters of the word 'STATISTICS' and another letter is taken at random from the letters of the word 'ASSISTANT'. The probability that they are the same letter is

(1) $\frac{1}{45}$ (2) $\frac{13}{90}$

(3*) $\frac{19}{90}$ (4) $\frac{5}{18}$

Ans. (3)

Sol. Letters of the word STATISTICS are AIICSSSTTT (10 letters)

Letter of the word ASSISTNAT are AAINSSSTT (9 letters)

Common letters are A, I, S and T

Probability of choosing A is $\frac{1}{10}\times\frac{2}{9} = \frac{2}{90}$

Probability of choosing I is $\frac{2}{10}\times\frac{1}{9} = \frac{2}{90}$

Probability of choosing S is $\frac{3}{10}\times\frac{3}{9} = \frac{9}{90}$

Probability of choosing T is $\frac{3}{10}\times\frac{2}{9} = \frac{6}{90}$

$\therefore$ Probability of required event =

$$\frac{2}{90} + \frac{2}{90} + \frac{9}{90} + \frac{6}{90} = \frac{19}{90}$$

72. The equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$, is

(1) $3y = 9x + 2$ (2) $y = 2x + 1$

(3) $2y = x + 8$ (4*) $y = x + 2$

Ans. (4)

Sol. General equation of tangent to the curve $y^2 = 8x$ is $y = mx + \frac{2}{m}$

Now on solving it with $xy = -1$, put discriminant = 0

$$x\left(mx + \frac{2}{m}\right) = -1 \quad \Rightarrow \quad m = 1$$

73. Number of words that can be formed using all the letters of the word 'H I P H I P H U R R A Y' in which all H's lies somewhere between R's is

(1*) $(198)7!$ (2) $(99)7!$

(3) $(99)8!$ (4) $(198)8!$

Ans. (1)

Sol. $\underbrace{(HHH),(RR),(II),(PP),AYU}_{x} = {}^{12}C_7 \cdot \frac{|7}{|2 \cdot |2} \cdot 1 = (198)7!$

74. If the equations $px^2+qx+r=0$ and $rx^2+qx+p=0$ $(p\neq r\neq 0)$ have a negative common root, then the value of $(p-q+r)$ is equal to

(1) -1 (2) 1

(3) 2 (4*) 0

Ans. (4)

Sol. $px^2+qx+r=0$

$rx^2+qx+p=0$

$\therefore x=-1$

$\therefore p-q+r=0$

75. The locus of feet of perpendicular from either foci of the ellipse $(x-y+1)^2+(2x+2y-6)^2=20$ on any tangent will be :

(1) $x^2+y^2+2x+4y+5=0$

(2) $x^2+y^2+2x+4y-5=0$

(3*) $x^2+y^2-2x-4y-5=0$

(4) $x^2+y^2-2x-4y+5=0$

Ans. (3)

Sol. $\dfrac{\left(\dfrac{x-y+1}{\sqrt{2}}\right)^2}{10}+\dfrac{\left(\dfrac{x+y-3}{\sqrt{2}}\right)^2}{5/2}=1$

Here $a^2=10$ and $b^2=5/2$ and centre is $(1, 2)$

$\because$ Locus of feet of perpendicular lie on auxiliary circle of ellipse

$\therefore$ Equation of circle is $(x-1)^2+(y-2)^2=10$

$x^2+y^2-2x-4y-5=0$

76. Let $t_r=\lfloor\underline{r}$ and $S_n=\sum_{r=1}^{n}\lfloor\underline{r}$, $n>4$, $n\in N$ then $\dfrac{S_n}{24}=a+\dfrac{\lambda}{24}$, $a,\lambda\in N$ where λ is

(1) 7 (2) 23

(3*) 9 (4) 12

Ans. (3)

Sol. $\lfloor\underline{1}+\lfloor\underline{2}+.....+\lfloor\underline{n}=9+24P \quad [n\geq 4]$

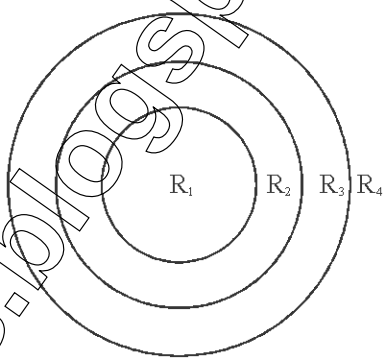
77. Concentric circles of radii 1, 2, 3, ..., 100 cm are drawn. The interior of smallest circle is coloured red and the annular regions coloured alternately green and red, such that no two adjacent regions are of the same colour. Then the total area of green regions is given by.

(1) 1000 π sq. cm (2*) 5050 π sq. cm

(3) 4950 π sq. cm (4) None of these

Ans. (2)

Sol.



$\text{Area}=\pi\left[\left(2^2-1^2\right)+\left(4^2-3^2\right)+....\left(100^2-99^2\right)\right]$

$=\pi[3+7+11+.....+199]$

$=5050\pi$

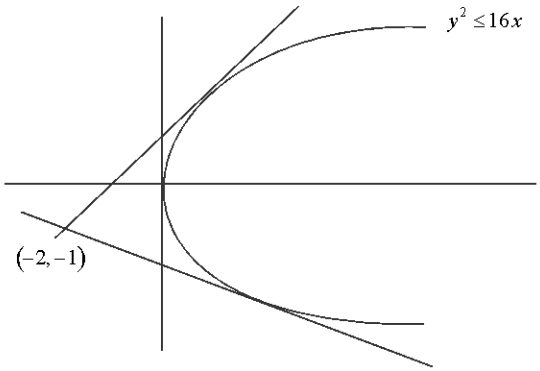
78. Let S be the set all points (x, y) satisfying $y^2 \le 16x$. For points in S let maximum and minimum value of $\frac{y+1}{x+2}$ be M and m respectively, then (m + M) is.

(1) 1 (2) – 2

(3) 0 (4*) $\frac{1}{2}$

Ans. (4)

Sol.



S be the set of points inside the parabola.

and $\frac{y+1}{x+2}$ is $\frac{y-(-1)}{x-(-2)}$

which is slope of line joining (x, y) and (–2, –1)

$\therefore$ points (x, y) should the taken on parabola and then make tangents .

now eqⁿ of tangent in slope form

$y = mx + \frac{a}{m}$

$y = mx + \frac{4}{m}$

$-1 = -2m + \frac{4}{m}$

$-m = -2m^2 + 4$

$2m^2 - m - 4 = 0$

$m = \frac{1 \pm \sqrt{1+32}}{4}$

$m = \frac{1-\sqrt{33}}{4}$ $\qquad M = \frac{1+\sqrt{33}}{4}$

$\Rightarrow m + M = \frac{1}{2}$

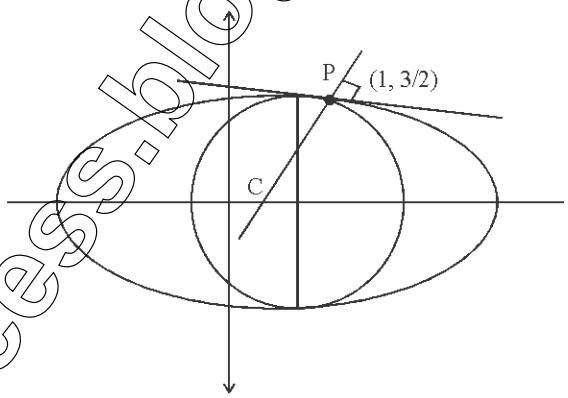
79. The ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ has a double contact with a circle at the extremity of latus rectum and point of contacts lie in first and fourth quadrant then the product of length of intercepts made by the circle on coordinate axes is

(1*) $\frac{3\sqrt{55}}{2}$ (2) $\frac{3\sqrt{51}}{2}$

(3) $3\sqrt{55}$ (4) $3\sqrt{51}$

Ans. (1)

Sol.



By symmetry centre of circle lies on X-axis

$\therefore$ Normal at P is $\frac{4x}{1} - \frac{3y}{3/2} = 1$

$\Rightarrow c \equiv \left(\frac{1}{4}, 0\right)$

$\therefore$ radius $= \sqrt{\left(1-\frac{1}{4}\right)^2 + \left(\frac{3}{2}\right)^2} = \frac{3\sqrt{5}}{4}$

$\therefore$ equation of circle is

$\left(x - \frac{1}{4}\right)^2 + y^2 = \left(\frac{3\sqrt{5}}{4}\right)^2$

$\Rightarrow x^2 - \frac{x}{2} + y^2 + \frac{1}{16} - \frac{45}{16} = 0$

$x^2 - \frac{x}{2} + y^2 - \frac{44}{16} = 0$

$\therefore x - \text{int ercept} = 2\sqrt{g^2 - c}$

$= 2\sqrt{\frac{1}{16} + \frac{44}{16}} = \frac{2}{4}\sqrt{45}$

$= \frac{1}{2} 3\sqrt{5} = \frac{3\sqrt{5}}{2}$

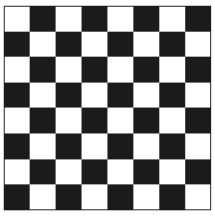
$$y\text{ - intercept} = 2\sqrt{f^2 - c}$$

$$= 2\sqrt{\frac{44}{16}} \qquad = \frac{2\times 2}{4}\sqrt{11}$$

$$= \sqrt{11}$$

$$\therefore \text{ product} = \frac{3\sqrt{5}}{2}\times\sqrt{11} = \frac{3\sqrt{55}}{2}$$

80. The ratio of number of rectangles (not square) and number of squares in a chess board is.



(1) 2 : 1 (2) 1 : 2

(3*) $\frac{91}{17}$ (4) None of these

Ans. (3)

Sol. No of (rectangles + squares)

$= {}^9C_2 \times {}^9C_2 = 1296$

and no. of squares

$= {}^8C_1 \times {}^8C_1 + {}^7C_1 \times {}^7C_1 + \ldots\ldots\ldots {}^1C_1 \times {}^1C_1$

$= 1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + 8^2$

$= 204$

$\therefore$ No of pure rectangles = 1296 − 204

= 1092

$\therefore$ ratio $= \frac{1092}{204} = \frac{91}{17}$

81. If x > 0, then greatest value of the expression $\frac{x^{50}}{1+x+x^2+\ldots\ldots+x^{100}}$ is

(1) $\frac{1}{102}$ (2*) $\frac{1}{101}$

(3) $\frac{1}{100}$ (4) None of these

Ans. (2)

Sol. A.M ≥ G.M

$$\Rightarrow \frac{1+x+x^2+\ldots\ldots+x^{100}}{101} \geq \left(1.x.x^2\ldots\ldots\ldots x^{100}\right)^{\frac{1}{101}}$$

$$\Rightarrow \frac{1+x+x^2+\ldots\ldots\ldots+x^{100}}{101} \geq x^{\frac{100\times 101}{2}\times\frac{1}{101}}$$

$$\Rightarrow \frac{1+x+x^2+\ldots\ldots x^{100}}{101} \geq x^{50}$$

$$\Rightarrow \frac{1}{101} \geq \frac{x^{50}}{1+x+x^2+\ldots\ldots+x^{100}}$$

$\therefore$ Greatest value $= \frac{1}{101}$

82. If the eccentricity of the hyperbola $x^2 - y^2\sec^2\theta = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\theta + y^2 = 25$, then smallest positive value of θ is $\frac{\pi}{P}$, where twice the value of 'p' is

(1*) 8 (2) 12

(3) 16 (4) 24

Ans. (1)

Sol. Eccentricity of the hyperbola $x^2 - y^2\sec^2\theta = 5$ is

$$e_1 = \sqrt{\frac{1+\sec^2\theta}{\sec^2\theta}} = \sqrt{1+\cos^2\theta}$$

Eccentricity of the ellipse $x^2\sec^2\theta + y^2 = 25$ is

$$e_2 = \sqrt{\frac{\sec^2\theta - 1}{\sec^2\theta}} = |\sin\theta| \quad \text{Given } e_1 = \sqrt{3}e_2$$

$$\Rightarrow 1+\cos^2\theta = 3\sin^2\theta \Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}}$$

$\therefore$ Least positive value of θ is $\frac{\pi}{4}$ $\therefore P = 4 \Rightarrow 2P = 8$

83. For any x, y $\in$ R, xy > 0 then the minimum value of

$$\frac{2x}{y^3}+\frac{x^3y}{3}+\frac{4y^2}{9x^4} \text{ equals}$$

(1) $2^{\frac{1}{3}}$ (2*) 2

(3) $3^{\frac{1}{3}}$ (4) 3

Ans. (2)

Sol. As x, y $\in$ R and xy > 0, so x and y will be of same sign.

All the quantities $\frac{2x}{y^3}, \frac{x^3y}{3}, \frac{4y^2}{9x^4}$ are positive.

A.M. $\geq$ G.M.

$$\Rightarrow \frac{2x}{y^3}+\frac{x^3y}{3}+\frac{4y^2}{9x^4} \geq 3\left(\left(\frac{2x}{y^3}\right)\left(\frac{x^3y}{3}\right)\left(\frac{4y^2}{9x^4}\right)\right)^{\frac{1}{3}}$$

$$= 3 \times \frac{2}{3} = 2$$

84. If the ordered pair (x, y) satisfies $x^2 + y^2 = 9$, then the largest value of the expression $x^2 + 2y^2 + 4x$ equals

(1*) 22 (2) 27

(3) 24 (4) 36

Ans. (1)

Sol. Expression = $x^2 + 2y^2 + 4x = 9 + y^2 + 4x$

$= 9 + 9\sin^2\theta + 4(3\cos\theta) = 9 + 9(1 - \cos^2\theta) + 12\cos\theta$

$= 18 - 3[3\cos^2\theta - 4\cos\theta]$

$$= 18 - 3 \times 3\left[\cos^2\theta - \frac{4}{3}\cos\theta\right]$$

$$= 18 - 9\left[\left(\cos\theta - \frac{2}{3}\right)^2 - \frac{4}{9}\right]$$

$$= 18 + 4 - 9\left(\cos\theta - \frac{2}{3}\right)^2 = 22 - 9\left(\cos\theta - \frac{2}{3}\right)^2$$

Clearly maximum value = 22

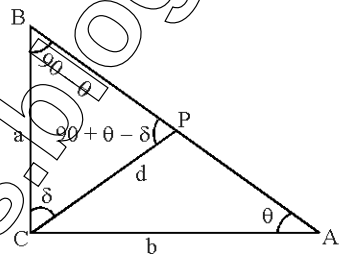
85. The legs of a right angle triangle are 'a' and 'b'. The line segment of length 'd' connecting the vertex of the right angle to a point 'P' of the hypotenuse enclose an angle δ with the leg a. The quantities a, b, d and δ are correctly related as

(1) $\frac{1}{2d} = \frac{\cos\delta}{a} + \frac{\sin\delta}{b}$ (2) $\frac{2}{d} = \frac{\cos\delta}{a} + \frac{\sin\delta}{b}$

(3*) $\frac{1}{d} = \frac{\cos\delta}{a} + \frac{\sin\delta}{b}$ (4) $\frac{2}{d} = \frac{\cos\delta}{b} + \frac{\sin\delta}{a}$

Ans. (3)

Sol.



We have $\frac{a}{\sin(90°+\theta-\delta)} = \frac{d}{\cos\theta}$ (By sine rule in ΔBCP)

$$\Rightarrow \frac{a}{d} = \frac{\cos(\theta-\delta)}{\cos\theta} = \frac{\cos\theta\cos\delta+\sin\theta\sin\delta}{\cos\theta}$$

$$\Rightarrow \frac{a}{d} = \cos\delta + \tan\theta\sin\delta \quad \text{....(1)}$$

or $\frac{1}{d} = \frac{\cos\delta}{a} + \frac{\sin\delta}{a}\tan\theta$

But $\tan\theta = \frac{a}{b}$

$$\Rightarrow \frac{1}{d} = \frac{\cos\delta}{a} + \frac{\sin\delta}{a}\left(\frac{a}{b}\right) \text{.....} \quad \text{Hence } \frac{1}{d} = \frac{\cos\delta}{a} + \frac{\sin\delta}{b}$$

86. The expression
$(\alpha\tan\gamma + \beta\cot\gamma)(\alpha\cot\gamma + \beta\tan\gamma) - 4\alpha\beta\cot^2 2\gamma$ is :
(1) independent of α, β (2) dependent of γ, α
(3) dependent on γ (4*) dependent on α, β

Ans. (4)

Sol. Expression
$= (a\tan g + b\cot g)(a\cot g + b\tan g) - 4a\,b\cot^2 2g$

$= a^2 + (\tan^2 g + \cot^2 g)\,ab + b^2 - 4ab\,\dfrac{\cos^2 2\gamma}{\sin^2 2\gamma}$

$= a^2 + b^2 + ab$

$\left[\left(\dfrac{\sin^2\gamma}{\cos^2\gamma} + \dfrac{\cos^2\gamma}{\sin^2\gamma}\right) - \dfrac{4(\cos^2\gamma - \sin^2\gamma)^2}{4\sin^2\gamma\cos^2\gamma}\right]$

$= a^2 + b^2 + ab$

$\left[\dfrac{(\sin^4\gamma + \cos^4\gamma) - (\cos^4\gamma + \sin^4\gamma - 2\sin^2\gamma\cos^2\gamma)}{\sin^2\gamma\cos^2\gamma}\right]$

$= a^2 + b^2 + 2ab = (a + b)^2$,

Which is independent of γ and dependent on α, β.

$\therefore$ options (4) is correct.

87. If m is selected at random from set {1, 2,……10} and the probability that the quadratic equation $2x^2 + 2mx + m + 1 = 0$ has real roots , is

(1) $\dfrac{1}{8}$ (2) $\dfrac{1}{9}$

(3*) $\dfrac{4}{5}$ (4) $\dfrac{1}{2}$

Ans. (3)

Sol. $2x^2 + 2mx + m + 1 = 0$

for real roots, $D \geq 0$

$\Rightarrow 4m^2 - 8(m+1) \geq 0$

$m^2 - 2m - 2 \geq 0$

$\therefore (m-1)^2 \geq 3$

$\Rightarrow m \geq 1 + \sqrt{3} \simeq 2.7$

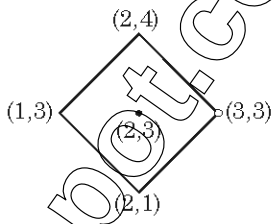
$\Rightarrow m = 3, 4, 5, 6, 7, 8, 9, 10$ are favourable cases

required $\Rightarrow$ probability is $\dfrac{8}{10} = \dfrac{4}{5}$

88. The area enclosed within $|x - 2| + |y - 3| = 1$ is
(1) 1 sq. unit (2*) 2 sq. units
(3) 3 sq. units (4) none of these

Ans. (2)

Sol. $A = \dfrac{1}{2}d_1 d_2 = 2$



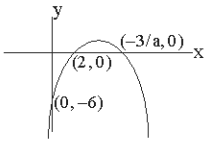
89. The value of 'a' for which the quadratic expression $ax^2 + |2a - 3|x - 6$ is positive for exactly two integral values of x is

(1) $\left(-\dfrac{3}{4}, -\dfrac{3}{5}\right)$ (2*) $\left(-\dfrac{3}{4}, -\dfrac{3}{5}\right]$

(3) $\left[-\dfrac{3}{4}, -\dfrac{3}{5}\right]$ (4) None of these

Ans. (2)

Sol. Since expression is positive only for two integer values of x, therefore parabola $y = ax^2 + |2a - 3|x$ will open downward.



$a < 0$ and $y = ax^2 + (3 - 2a)x - 6$

or $y = (ax + 3)(x - 2)$

$4 < -\dfrac{3}{a} \leq 5 \Rightarrow -\dfrac{3}{4} < a \leq -\dfrac{3}{5}$.

90. For what values of r, $x_1 + x_2 + x_3 + x_4 = r$ has no solutions such that $-4 \leq x_1, 7 \leq x_2, 5 \leq x_3, x_4 \geq 10$ and $x_i \in I, \forall\ i = 1, 2, 3, 4$

(1) $r = 18$ (2) $r > 18$

(3*) $r < 18$ (4) cannot be determined

Ans. (3)

Sol. $x_1 + x_2 + x_3 + x_4 = r$

$x_1 \geq -4, x_2 \geq 7, x_3 \geq 5, x_4 \geq 10$

$(y_1 - 4) + (y_2 + 7) + (y_3 + 5) + (y_4 + 10) = r$

$y_1 + y_2 + y_3 + y_4 = r - 18$

$r - 18 \geq 0 \Rightarrow$ has integral solution but $r < 18$ will not have any solutions

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

FULL SYLLABUS TEST-1

SCORE-1

JEE (MAIN) 2017

ENTHUSIAST COURSE
(Class - XII)

Time : 3 Hours | **Date : 09-01-17** | **Max. Marks : 360**

Student's Form No. : ..... **Batch :** .....

Student's Name : .....

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

GENERAL INSTRUCTIONS

A. General:

1. This Question Paper contains **90 questions**.
2. No additional sheets will be provided for rough work.
3. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets in any form are not allowed.
4. The answer sheet, a machine-gradable Objective Response Sheet (ORS), is provided separately.
5. Do not Tamper / mutilate the ORS or this booklet.
6. Do not break the seals of the question – paper booklet before instructed to do so by the invigilators.
7. How to fill response on ORS (Assuming answer is A)

	(1)	(2)	(3)	(4)
Correct Method	●	○	○	○
Wrong Method	⊗	⊘	⊙	◐

B. Filling the top-half of the ORS:

8. Write your ALLEN Form Number, Name, Mobile Number, School, Class and section with pen in appropriate boxes. Do not write these any where else.
9. Darken the appropriate bubbles below your ALLEN Form number with Pen.

C. Question paper format and Marking scheme:

10. The question paper consists of **3 PARTS**.

(i) **Part-I :** consists of **30 questions of Physics**. For each question in Section I, you will be awarded **4 marks** if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. In all other cases minus one **(–1) mark** will be awarded.

(ii) **Part-II :** consists of **30 questions of Chemistry**. For each question in Section II, you will be awarded **4 marks** if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. In all other cases minus one **(–1) mark** will be awarded.

(iii) **Part-III** : consists of **30 questions of Maths**. For each question in Section III, you will be awarded **4 marks** if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. In all other cases minus one **(–1) mark** will be awarded.

Sarvatra : 11, Samachar Jagat, Opp Vidyasharam School, JLN Marg, Jaipur 302017
Sarvatra-1 : S-2, Sri Gopal Nagar, Khandal Tower, Near Gurjar Ki Thadi, Jaipur 302015
Sarvatra-2 : B 41-42, Nityanand Nagar, Gandhi Path, Vaishali Nagar, Jaipur 302021
Sarvatra-3 : KK Tower, 1st Floor, Opp Doodh Mandi, Jhotwara Road, Bani Park, Jaipur 302012
Sarvatra-4 : 506, Surya Nagar, adjoining Gangotri Garden, Gopalpura Bypass, Jaipur 302015
Sarvatra-5 : C-1, Near C-2 Plaza Apex Circle, Malviya Nagar, Jaipur 302017
Sarvatra-6 : B-3 and B-4, sirsi Road, Hamuman Nagar, Jaipur 302021
Sarvatra-7 : E 49-50, 10-B Scheme, Gopalpura Bypass, Jaipur 302015
Call: +91-0141-5195100; Email: jaipur@allen.ac.in|Website: http://jaipur.allen.ac.in

Useful Data:

1 amu	=	1.66×10^{-27} kg	R	=	0.082 L-atm K^{-1} mol^{-1}
h	=	6.626×10^{-34} J s	N_A	=	6.022×10^{23}
m_e	=	9.1×10^{-31} kg	e	=	1.6×10^{-19} C
c	=	3.0×10^{8} m s^{-1}	F	=	96500 C mol^{-1}
R_H	=	2.18×10^{-18} J	$4\pi\epsilon_0$	=	1.00×10^{-10} C^2 N^{-1} m^{-1}

Atomic No. : H=1, He=2, Li=3, Be=4, B=5, C=6, N=7, O=8, F=9, Na=11, Mg=12, Al = 13, Si = 14, P = 15, S = 16, Cl = 17, Ar =18, K=19, Ca=20,Cr=24, Mn=25, Fe=26, Co=27, Ni=28, Cu=29, Zn=30, As=33, Br = 35, Ag = 47, Si = 21, Sn = 50, Ti = 22,I = 53, Xe = 54, Ba = 56, Pb = 82, U = 92, V = 23.

Atomic masses: H =1, He=4, Li=7, Be=9, B=11, C=12, N=14, O=16, F=19, Na=23, Mg=24, Al=27, Si=28, P=31, S=32, Cl=35.5, K=39, Ca=40, Cr=52, Mn=55, Fe=56, Co=59, Ni=58.7, Cu=63.5, Zn = 65.4, As = 75, Br = 80, Ag = 108, Sn = 118.7, I = 127, Xe = 131, Ba = 137, Pb = 207, U = 238.