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Educational Revolution

PHYSICS

Aakash

STUDY PACKAGE – 01

For – JEE / NEET

Units & Measurement

Chapter - 1

Units and Measurement

1

CHAPTER

AIEEE Syllabus

Physics, Technology and Society, S I units, Fundamental and derived units. Least count, accuracy and precision of measuring instruments, Errors in measurement, Significant figures, Dimensions of Physical quantities, dimensional analysis and its applications

THIS CHAPTER COVERS :

- Units
- Dimensions
- Dimensional analysis and applications
- Limitations of Dimensional method
- Accuracy and Precision
- Significant figures
- Errors in measurement
- Measuring Instruments

UNITS

Measurement of any physical quantity involves its comparison with a certain basic, reference standard called unit.

Any standard unit must have two properties

(a) Invariability

(b) Availability

The result of measurement of a physical quantity is expressed by a number (numerical measure) accompanied by a unit.

Measurement = *nu*

Fundamental quantities : The units for these quantities are called fundamental or base units. Length, mass, time, electric current etc. are some of the fundamental quantities.

Derived quantities : They can be expressed in terms of fundamental quantities. Tthe units of derived quantities are expressed in terms of fundamental units and they are called derived units. *e.g.*, velocity, force, impulse etc. are derived quantities.

International system of units (SI) :- This system of units was introduced in 1971 by the general conference on weights and measures and was internationally accepted. It has **seven** fundamental units along with **two** supplementary units.

DIMENSIONS

All the physical quantities represented by derived units can be expressed in terms of some combination of seven fundamental quantities. These seven fundamental quantities are called seven dimensions of the physical world. They are denoted with square brackets [ ].

The dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity. Note that,

1. In this type of representation the magnitudes are not considered. It is the quality of the type of the physical quantity that enters.
2. The expression which shows how and which of the base quantities represent the dimensions of a physical quantity is called dimensional formula of the given physical quantity.
3. An equation obtained by equating a physical quantity with its dimensional formula is called the dimensional equation of the physical quantity.

   e.g. $[S] = [M^0L^2T^{-2}K^{-1}]$ …(i)

DIMENSIONAL ANALYSIS AND ITS APPLICATIONS

Principle of Homogeneity of dimensions : It states that in a correct equation, the dimensions of each term added or subtracted must be same. Every correct equation must have same dimensions on both sides of the equation.

Conversion of units : The numerical value of a physical quantity in a system of units can be changed to another system of units using the equation $n[u]$ = constant *i.e.*, $n_1[u_1] = n_2[u_2]$ where n is the numerical value and u is the unit.

By knowing the conversion factors for the base quantities and dimensional formula of the derived quantity, one can convert the numerical value of a physical quantity from one system of units to other system of units.

$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$ where the dimensional formula of the physical quantity is $[M^aL^bT^c]$.

To find a relation among the physical quantities. If one knows the quantities on which a particular physical quantity depends and guesses that this dependence is of product type, method of dimensions are helpful in deducing their relation.

Suppose we want to find the relation between force, mass and acceleration. Let force depends on mass and acceleration as follows.

$F = Km^ba^c$ when K = dimensionless constant b and c are powers of mass and acceleration.

According to principle of homogeneity.

$[F] = [K]\ [m]^b\ [a]^c$

$\Rightarrow\ [MLT^{-2}] = [M^0L^0T^0]\ [M]^b\ [LT^{-2}]^c$

$\Rightarrow\ [MLT^{-2}] = M^bL^c\ T^{-2c}$

Equating the dimension on both sides we get $1 = b$, $1 = c$, $-2c = -2$.

$\Rightarrow\ b = 1$ and $c = 1$.

Now putting the values of b and c in our required equation we will get a mathematical equation $F = Kma$.

The value of K can be found experimentally.

LIMITATIONS OF DIMENSIONAL METHOD

1. Dimensional method cannot be used to derive equations involving addition and substraction.
2. Numerical constants having no dimensions cannot be obtained by method of dimensions.
3. Dimensional method fails when number of variables is more than number of equations obtained from the exponents of fundamental quantities.
4. Equations using trigonometric, exponential and logarthmic functions can not be deduced.

Some Important Points :

1. If name of unit is the name of scientist, then for *e.g.*, 5 Ampere is wrong, correct is 5 ampere. Also 5a is wrong, correct is 5A.
2. Pure numbers are dimensionless.
3. All trigonometrical ratios, powers, exponential and logarithmic functions are dimensionless.
4. All ratio of physical quantities having same dimensional formula are dimensionless. *e.g*. relative density, relative permeability, dielectric constant, angles, refractive index etc.
5. Dimensions do not depend upon magnitude.

ACCURACY AND PRECISION

Accuracy

The closeness of the measured value to the true value of the physical quantity is known as the accuracy of the measurement.

Precision

It is the measure of the extent to which successive measurements of a physical quantity differ from one another.

Suppose the true value of a measurement is 35.75 and two measured values are 35.73 and 35.725. Here 35.73 is closest to 35.75, so its accuracy is more than 35.725 but 35.725 is more precise than 35.73 because 35.725 is measured upto after 3 decimal places.

SIGNIFICANT FIGURES

The number of digits in the measured value about the correctness of which we are sure plus one more digit are called significant figures.

Rules for counting the significant figures

Rule I : All non-zero digits are significant.

Rule II : All zeros occurring between the non zero digits are significant. For example 230089 contains six significant figures.

Rule III : All zeros to the left of non zero digit are not significant. For example 0.0023 contains two significant figures.

Rule IV : If a number ends in zeros that are not to the right of a decimal, the zeros are not significant.

For example, number of significant figures in

1500 (Two)

1.5×10^3 (Two)

1.50×10^3 (Three)

1.500×10^3 (Four)

Length of an object may be represented in many ways say 5 m, 5.0 m, 500 cm, 5.00 m, 5×10^2 cm. Here 5.00 m is most precise as it contains 3 significant figures.

Rules for Arithmetic Operations with Significant Figures

Rule I : In addition or subtraction, the final result should retain as many decimal places are there are in the number with the least decimal places.

Rule II : In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures.

Rounding Off of Uncertain Digits

Rule I : The preceding digit is raised by 1 if the insignificant digit to be removed is more than 5 and is left unchanged if the later is less than 5.

Rule II : When the insignificant digit to be removed is 5 and the uncertain digit is even, 5 is simply dropped and if it is odd, then the preceding digit is raised by 1.

ERRORS IN MEASUREMENT

1. **Mean Absolute Error :-** It is given by

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + ... + |\Delta a_n|}{n}$$

$a_m = \dfrac{a_1 + a_2 + ... + a_n}{n}$ = is taken as the true value of a quantity, if the same is not known.

$\Delta a_1 = a_m - a_1$

$\Delta a_2 = a_m - a_2$

....

$\Delta a_n = a_m - a_n$

Final result of measurement may be written as :

$a = a_m \pm \Delta\overline{a}$

2. **Relative Error or Fractional Error :** It is given by

$$\frac{\overline{\Delta a}}{a_m} = \frac{\text{Mean absolute Error}}{\text{Mean value of measurement}}$$

3. **Percentage Error** $= \dfrac{\overline{\Delta a}}{a_m} \times 100\%$

4. **Combination of Errors :**

(i) **In Sum :** If $Z = A + B$, then $\Delta Z = \Delta A + \Delta B$, maximum fractional error in this case

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A+B} + \frac{\Delta B}{A+B}$$

i.e. when two physical quantities are added then the maximum absolute error in the result is the sum of the absolute errors of the individual quantities.

(ii) **In Difference :** If $Z = A - B$, then maximum absolute error is $\Delta Z = \Delta A + \Delta B$ and maximum fractional error in this case $\dfrac{\Delta Z}{Z} = \dfrac{\Delta A}{A-B} + \dfrac{\Delta B}{A-B}$

(iii) In Product : If $Z = AB$, then the maximum fractional error,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

where $\Delta Z/Z$ is known as fractional error.

(iv) In Division : If $Z = A/B$, then maximum fractional error is

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

(v) In Power : If $Z = A^n$ then $\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$

In more general form if $Z = \frac{A^x B^y}{C^q}$

then the maximum fractional error in Z is

$$\frac{\Delta Z}{Z} = x\frac{\Delta A}{A} + y\frac{\Delta B}{B} + q\frac{\Delta C}{C}$$

Applications :

1. For a simple pendulum, $T \propto l^{1/2}$ $\Rightarrow$ $\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$

2. For a sphere

$$A = 4\pi r^2, V = \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad \frac{\Delta A}{A} = 2.\frac{\Delta r}{r} \text{ and } \frac{\Delta V}{V} = 3.\frac{\Delta r}{r}$$

3. When two resistors R_1 and R_2 are connected

(a) In series

$$R_s = R_1 + R_2$$

$$\Rightarrow \quad \Delta R_s = \Delta R_1 + \Delta R_2$$

$$\frac{\Delta R_s}{R_s} = \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}$$

(b) In parallel,

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \quad \frac{\Delta R_p}{R_p^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

MEASURING INSTRUMENTS AND LEAST COUNT

The error in the measurement by an instrument is equal to least count of the instrument.

For example, a meter scale has smallest division 1 mm. This represents the least count or the absolute error in the measurement.

Let a length measured by the meter scale = 56.0 cm

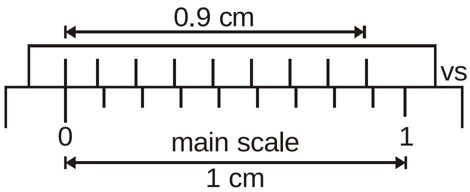
This implies that x = 56.0 cm

Absolute error Δx = 1 mm = 0.1 cm

Relative error = $\dfrac{\Delta x}{x} = \dfrac{0.1}{56.0}$

Vernier Callipers

It consists of two scales viz main scale and vernier scale. Vernier scales moves on the main scale. The least count of the instrument is the smallest distance between two consecvtive divisions and it is equal to 1 MSD – 1 VSD.



In the figure shown, 1 MSD = 0.1 cm

1 VSD = 0.09 cm

Least count = 1 MSD – 1 VSD = 0.01 cm

For the commonly used instruments, least counts are given below :

Least count of Vernier callipers = 1 MSD – 1 VSD

Screw Gauge

It contains a main scale and a circular scale. The circular scale is divided into a number of divisions. In other words, the complete rotation of circular scale is divided into a number of parts. The least count of a screw gauge is pitch/no. of circular scale divisions.

If reading on main scale is 5 cm (say) and reading on circular scale is 25 divisions, then the reading is 5 cm + 25 × least count.

Least count of spherometer and Screw Gauge = $\dfrac{\text{Pitch}}{\text{No. of CSD}}$

FUNDAMENTAL QUANTITIES, THEIR UNIT AND DIMENSIONAL FORMULA

Sl. No.	Fundamental Quantity	Fundamental Unit	Unit Symbol Used	Dimensions
1.	Mass	kilogram	kg	[M]
2.	Length	metre	m	[L]
3.	Time	second	s	[T]
4.	Temperature	Kelvin	K	[K]
5.	Electric current	Ampere	A	[A]
6.	Luminous Intensity	candela	cd	[cd]
7.	Amount of Matter	mole	mol	[mol]

SUPPLEMENTARY QUANTITY AND UNIT

Sl. No.	Supplementary Physical Quantity	Supplementary Unit	Unit Symbol Used
1.	Plane angle	radian	rad [$180^o = \pi$ radian]
2.	Solid angle	steradian	sr

Table : SI Units and Dimensions of Some Important Physical Quantities

S.No.	Quantity	SI Unit	Dimensional Formula
1.	Volume	m^3	$[M^0L^3T^0]$
2.	Density	kg m^{-3}	$[M^1L^{-3}T^0]$
3.	Velocity	ms^{-1}	$[M^0L^1T^{-1}]$
4.	Acceleration	ms^{-2}	$[M^0L^1T^{-2}]$
5.	Angular Velocity	rad s^{-1}	$[M^0L^0T^{-1}]$
6.	Frequency	s^{-1} or hertz (Hz)	$[M^0L^0T^{-1}]$
7.	Momentum	kg ms^{-1}	$[M^1L^1T^{-1}]$
8.	Force	kg ms^{-2} or newton (N)	$[M^1L^1T^{-2}]$
9.	Work, Energy	kg m^2s^{-2} or roule (J)	$[M^1L^2T^{-2}]$
10.	Power	kg m^2 s^{-3} or Js^{-1} or watt (W)	$[M^1L^2T^{-3}]$
11.	Pressure, Stress	Nm^{-2} or pascal (Pa)	$[M^1L^{-1}T^{-2}]$
12.	Modulus of Elasticity	Nm^{-2}	$[M^1L^{-1}T^{-2}]$
13.	Moment of Inertia	kg m^2	$[M^1L^2T^0]$
14.	Torque	Nm	$[M^1L^2T^{-2}]$
15.	Angular Momentum	kg m^2 s^{-1}	$[M^1L^2T^{-1}]$
16.	Impulse	Ns	$[M^1L^1T^{-1}]$
17.	Coefficient of Viscosity	kg m^{-1} s^{-1}	$[M^1L^{-1}T^{-1}]$
18.	Surface Tension	Nm^{-1}	$[M^1L^0T^{-2}]$
19.	Universal Gravitational Constant	Nm^2 kg^{-2}	$[M^{-1}L^3T^{-2}]$
20.	Latent Heat	J kg^{-1}	$[M^0L^2T^{-2}]$
21.	Specific Heat	J kg^{-1} K^{-1}	$[M^0L^2T^{-2}K^{-1}]$
22.	Thermal Conductivity	J m^{-1} s^{-1} K^{-1}	$[M^1L^1T^{-3}K^{-1}]$
23.	Electric Charge	As or coulomb (C)	$[M^1L^0T^1A^1]$
24.	Electric Potential	JC^{-1} or volt (V)	$[M^1L^2T^{-3}A^{-1}]$
25.	Electric Resistance	VA^{-1} or ohm (Ω)	$[M^1L^2T^{-3}A^{-2}]$
26.	Electric Resistivity	Ωm	$[M^1L^3T^{-3}A^{-2}]$

S.No.	Quantity	SI Unit	Dimensional Formula
27.	Electric Conductance	Ω^{-1} or siemens (S)	$[M^{-1}L^{-2}T^{3}A^{2}]$
28.	Electric Conductivity	$\Omega^{-1}\ m^{-1}$ or S m^{-1}	$[M^{-1}L^{-3}T^{3}A^{2}]$
29.	Capacitance	CV^{-1} or farad (F)	$[M^{-1}L^{-2}T^{4}A^{2}]$
30.	Inductance	Vs A^{-1} or henry (H)	$[M^{1}L^{2}T^{-2}A^{-2}]$
31.	Electric field	NC^{-1} or Vm^{-1}	$[M^{1}L^{1}T^{-3}A^{-1}]$
32.	Magnetic Induction	$NA^{-1}\ m^{-1}$ or tesla (T)	$[M^{1}L^{0}T^{-2}A^{-1}]$
33.	Magnetic Flux	Tm^{2} or weber (Wb)	$[M^{1}L^{2}T^{-2}A^{-1}]$
34.	Permittivity	$C^{2}\ N^{-1}\ m^{-2}$	$[M^{-1}L^{-3}T^{4}A^{2}]$
35.	Permeability	Tm A^{-1} or Wb $A^{-1}\ m^{-1}$	$[M^{1}L^{1}T^{-2}A^{-2}]$
36.	Planck's Constant	Js	$[M^{1}L^{2}T^{-1}]$
37.	Boltzman Constant	JK^{-1}	$[M^{1}L^{2}T^{-2}K^{-1}]$
38.	Stefan Constant	W $m^{-2}K^{-4}$	$[M^{1}L^{0}T^{-3}K^{-4}]$

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STUDY PACKAGE – 01

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Motion in One Dimension

Chapter - 2

Description of Motion in One Dimension

2

CHAPTER

AIEEE Syllabus

Frame of reference. Motion in a straight line: Position-time graph, speed and velocity. Uniform and non-uniform motion, average speed and instantaneous velocity, Uniformly accelerated motion, velocity-time, position-time graphs, relations for uniformly accelerated motion.

THIS CHAPTER COVERS :

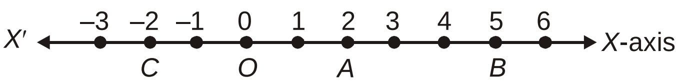
- Position
- Displacement and Velocity
- Distance and speed
- Acceleration
- Equations of Motion
- Graphs
- Motion under gravity
- Relative motion in one dimension

POSITION AND FRAME OF REFERENCE

Position of an object is specified with respect to a **reference frame.**

In a reference frame, an observer measures the position of the other object at any instant of time, with respect to a coordinate system chosen and fixed arbitrarily on the reference frame.

for example : The position of particle at *O*, *A*, *B* and *C* are Zero, +2, +5 and –2 respectively with respect to origin (*O*) of reference frame.



DISPLACEMENT AND VELOCITY

Displacement

1. The change in position of a body in a certain direction is known as displacement.
2. The distance between the initial and final position is known as **magnitude of displacement.**
3. Displacement of an object may be positive, negative or zero and it is independent of the path followed by the object.
4. Its SI unit is meter and dimensional formula is $[M^0L^1T^0]$.

Velocity

1. **Average velocity :** [<*v*>] :

   If Δx is displacement in time Δt, then average velocity in time interval Δt will be

   $$<v> = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}.$$

   Here x_f and x_i be the position of particle at time t_f and t_i ($t_f > t_i$) with respect to a given frame of reference.

2. **Instantaneous velocity (*v*) :** It is the velocity of particle at any instant of time

   Mathematically,

   $$v = \underset{\Delta t \to 0}{\text{Limit}} <v> = \underset{\Delta t \to 0}{\text{Limit}} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

DISTANCE AND SPEED

Distance

1. The total length of actual path traversed by the body between initial and final positions is called distance.
2. It has no direction and is always positive.
3. Distance covered by particle never decreases.
4. Its SI unit is meter (m) and dimensional formula is $[M^0L^1T^0]$.

Speed

1. **Average speed :** It is defined as distance travelled by particle per unit time in a given interval of time.

   If S is the distance travelled by particle in time interval *t*, then average speed in that time interval is $\frac{S}{t}$.

2. **Instantaneous speed :** The magnitude of instantaneous velocity at a given instant is called instantaneous speed at that instant.

ACCELERATION

Time rate of change of velocity is called **acceleration.**

1. Average acceleration : If Δv is change in velocity in time Δt, then average acceleration in time interval Δt is

$$< a > = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}.$$

2. **Instantaneous acceleration :** The acceleration at any instant is called **instantaneous acceleration.** Mathematically

$$a = \underset{\Delta t \to 0}{Limit} < a > = \underset{\Delta t \to 0}{Limit} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}.$$

Uniform and variable acceleration :

If the change in velocity of the particle is equal in equal intervals of time, then the acceleration of the body is said to be **uniform**. Neither direction, nor magnitude changes with respect to time.

If cnange in velocity is different in equal intervals of time, then the acceleration of the particle is known as **variable**. If either direction or magnitude or both magnitude and direction of acceleration changes with respect to time, then acceleration is **variable.**

EQUATIONS OF MOTION

General equations of motion :

$$v = \frac{dx}{dt} \Rightarrow dx = vdt \Rightarrow \int dx = \int vdt$$

$$a = \frac{dv}{dt} \Rightarrow dv = adt \Rightarrow \int dv = \int adt$$

$$a = \frac{vdv}{dx} \Rightarrow vdv = adx \Rightarrow \int vdv = \int adx$$

Equations of motion of a particle moving with uniform acceleration in straight line :

1. $v = u + at$

2. $S = ut + \frac{1}{2}at^2$

3. $v^2 = u^2 + 2aS$

4. $S_{n^{th}} = u + \frac{1}{2}a(2n - 1)$

5. $x = x_0 + ut + \frac{1}{2}at^2$

Here

u = velocity of particle at $t = 0$

S = Displacement of particle between 0 to t

$= x - x_0$ (x_0 = position of particle at $t = 0$, x = position of particle at time t)

a = uniform acceleration

v = velocity of particle at time t

$S_{n^{th}}$ = Displacement of particle in n^{th} second

Average Speed and Velocity

1. If a body covers s_1 distance with speed v_1, s_2 with speed v_2, then its average speed is

$$v_{av} = \frac{s_1 + s_2}{\frac{s_1}{v_1} + \frac{s_2}{v_2}} = \frac{\sum s}{\sum \frac{s}{v}}$$

2. If a body coves first half distance with speed v_1 and next half with speed v_2, then

Average speed = $\frac{2v_1v_2}{v_1 + v_2}$ (Harmonic mean)

3. If a body travels with uniform speed v_1 for time t_1 and with uniform speed v_2 for time t_2, then average speed

$$= \frac{v_1t_1 + v_2t_2}{t_1 + t_2} = \frac{\sum vt}{\sum t}.$$

If $t_1 = t_2 = \frac{T}{2}$ then $v_{av} = \frac{v_1 + v_2}{2}$ [T = time of journey] (Arithmatic mean)

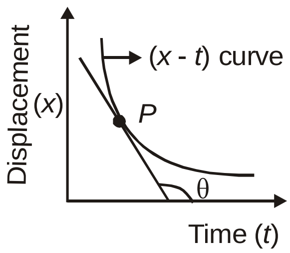
4. If body covers first one third with speed v_1, next one third with speed v_2 and remaining one third with speed v_3 then $v_{av} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$.

5. If a body moves from one point (A) to another point (B) with speed v_1 and returns back (from B to A) with speed v_2 then average velocity is 0 but average speed = $\frac{2v_1v_2}{v_1 + v_2}$.

GRAPHS

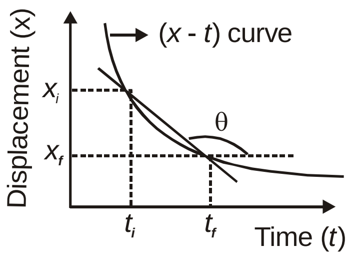
The important properties of various graphs are given below :

1. Slope of the tangent at a point on the displacement-time graph gives the instantaneous velocity at that point.



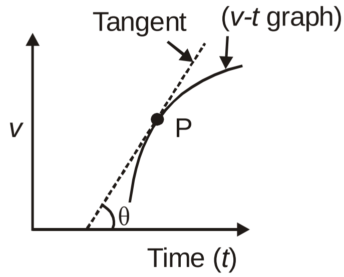
$\tan\theta = \frac{dx}{dt} = v$ (**Instantaneous Velocity** at point *P*)

2. Slope of the chord joining two points on the displacement-time graph gives the average velocity during the time interval between those points.



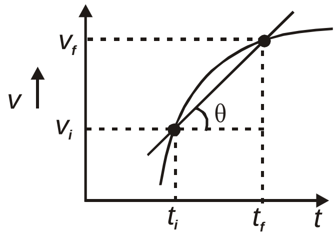
$\tan\theta = \frac{x_f - x_i}{t_f - t_i} = V_{av}$

3. Slope of the tangent at a point on the velocity-time graph gives the instantaneous acceleration at that point.



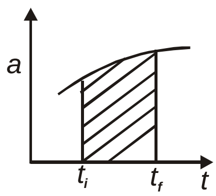
$\tan\theta = \frac{dv}{dt} = a$ (Instantaneous acceleration at *P*)

4. Slope of the chord joining two points on the velocity-time graph gives the average acceleration during the time interval between those points.



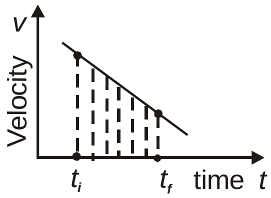
$\tan\theta = \frac{v_f - v_i}{t_f - t_i} = a_{av}$ = Average acceleration in time interval $t_f - t_i$

5. The area under the acceleration-time graph between t_i and t_f gives the change in velocity ($v_f - v_i$) between the two instants.



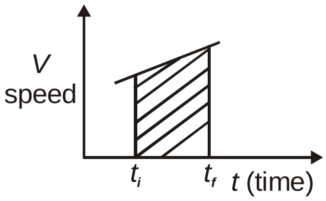
Shaded area = $v_f - v_i$ = change in velocity

6. The area under speed-time graph between t_i and t_f gives distance covered by particle in the interval $t_f - t_i$.



Shaded area = displacement in time $(t_f - t_i)$

7. The area under the velocity-time graph between t_i and t_f gives the displacement $(x_f - x_i)$ between the two instants.

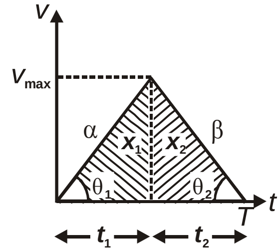


Shaded area = distance covered in time $(t_f - t_i)$

8. The displacement-time graph cannot take sharp turns because it gives two different velocities at that point.
9. The displacement-time graph cannot be symmetric about the time-axis because at an instant a particle cannot have two displacements, but the graph may be symmetric about the displacement-axis.
10. The distance-time graph is always an increasing curve for a moving body.
11. The displacement-time graph does not show the trajectory of the particle.

Applications

1. If a particle is moving with uniform acceleration and have velocity V_A at A and V_B at B, then velocity of particle midway on line AB is $V = \sqrt{\dfrac{V_A^2 + V_B^2}{2}}$.
2. If a body starts from rest with acceleration α and then retards to rest with retardation β, such that total time of journey is T, then



(a) Maximum velocity during the trip $v_{max.} = \left(\dfrac{\alpha\beta}{\alpha+\beta}\right).T$

(b) Length of the journey $L = \dfrac{1}{2}\left(\dfrac{\alpha\beta}{\alpha+\beta}\right)T^2$

(c) Average velocity of the trip = $\dfrac{v_{max.}}{2} = \dfrac{\alpha\beta T}{2(\alpha+\beta)}$ & (d) $\dfrac{x_1}{x_2} = \dfrac{\beta}{\alpha} = \dfrac{t_1}{t_2}$.

MOTION UNDER GRAVITY

Whenever a particle is thrown up or down or released from a height, it falls freely under the effect of gravitational force of earth.

The equations of motion :

1. $v = u + gt$
2. $h = h_0 + ut + \dfrac{1}{2}gt^2$ or $h - h_0 = s = ut + \dfrac{1}{2}gt^2$
3. $v^2 = u^2 + 2g(h - h_0)$ or $v^2 = u^2 + 2gs$
4. $h_{n^{th}} = u + \dfrac{g}{2}(2n - 1)$

where h = vertical displacement, $h_{n^{th}}$ = vertical displacement in n^{th} second

Following are the important cases of interest.

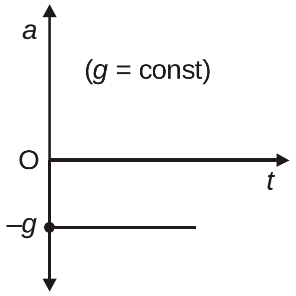
1. A particle is projected from ground with velocity u in vertically upward direction then

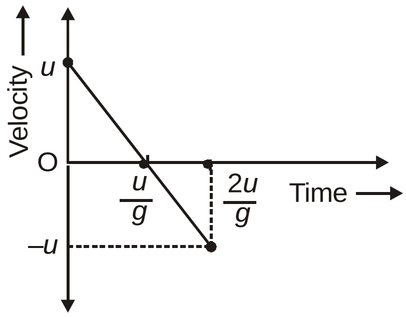
   (a) Time of ascent = Time of descent = $\dfrac{\text{Time of flight}}{2} = \dfrac{T}{2} = \dfrac{u}{8}$

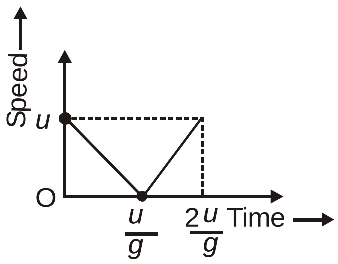
   (b) Maximum height attained = $\dfrac{u^2}{2g}$

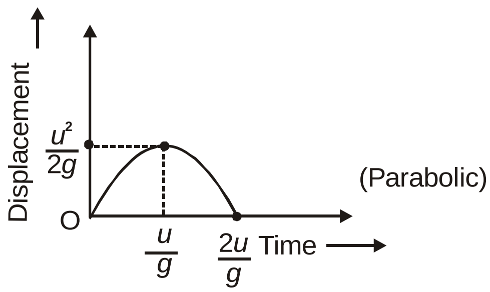
   (c) Speed of particle when it hits the ground = u

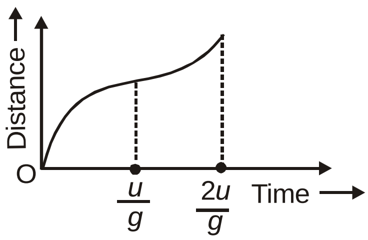
   (d) Graphs











   (e) Displacement of particle in complete journey = zero ⇒ average velocity $v_{av} = 0$

   (f) Distance covered by particle in complete journey = $\dfrac{u^2}{g}$

   ⇒ Average speed in complete journey = $\dfrac{u}{2}$

2. A body is thrown upward such that it takes t seconds to reach its highest point.

   (a) Distance travelled in $(t)^{th}$ second = distance travelled in $(t + 1)^{th}$ second.

   (b) Distance travelled in $(t - 1)^{th}$ second = distance travelled in $(t + 2)^{th}$ second.

   (c) Distance travelled in $(t - r)^{th}$ second = distance travelled in $(t + r + 1)^{th}$ second.

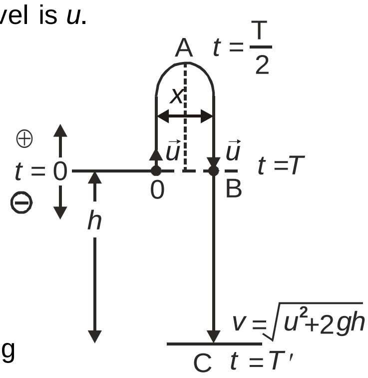
3. A body is projected upward from certain height h with initial speed u.
   (a) Its speed when it acquires the at same level is u.
   (b) Its speed at the ground level is

   $$v = \sqrt{u^2 + 2gh}$$

   (c) The time require to attain same level is

   $$T = \frac{2u}{g}$$

   (d) Total time of flight (T) is obtained by solving



Here $x = 0$ particle follows same path during ascent and descent

$$-h = +uT' + \frac{1}{2}gT'^2 \text{ or } T' = \frac{T + \sqrt{T^2 + \frac{8h}{g}}}{2}, \text{ where } T = \frac{2u}{g}$$

Some Important Points :

1. During free fall distance increases by equal amounts *i.e.,* g during 1st , 2nd, 3rd seconds of fall, *i.e.,* 4.9m, 14.7m, 24.5m are the distances travelled during 1st, 2nd and 3rd seconds respectively.
2. From the top of a tower a body is projected upward with a certain speed, 2nd body is thrown downward with same speed and 3rd is let to fall freely from same point then

   $$t_3 = \sqrt{t_1 t_2}$$

   where, t_1 = Time of flight of body projected upward
   t_2 = Time of flight of body thrown downward
   t_3 = Time of flight of body dropped.
3. If a body falls freely from a height h on a sandy surface and it buries into sand upto a depth of x, then the retardation with which body travels in the sand is

   $$a = \frac{gh}{x}$$

4. If u and v are velocity of particle at $t = t_1$ and $t = t_2$, which is moving with uniform acceleration, then average velocity of particle during time interval $(t_2 - t_1)$ is $V_{av} = \frac{u+v}{2}$.
5. For a body starting from rest and moving with uniform acceleration, the ratio of distances covered in 1s, 2s, 3s, *etc*. is $1^2 : 2^2 : 3^2$ etc., *i.e.* 1 : 4 : 9 etc.
6. A body starting from rest and moving with uniform acceleration has distances covered by it in 1st, 2nd and 3rd seconds in the ratio 1 : 3 : 5 etc. i.e., odd numbers only.
7. A body moving with a velocity v is stopped by application of brakes after covering a distance s. If the same body moves with a velocity nv, it stops after covering a distance n^2s by the application of same brake force.
8. In the absence of air resistance, the velocity of projection is equal to the velocity with which the body strikes the ground.
9. In case of air resistance, the time of ascent is less than time of descent for a body projected vertically upward.
10. For a body projected vertically upwards, the magnitude of velocity at any given point on the path is same whether the body is moving in upwards or downward direction.

RELATIVE MOTION IN ONE DIMENSION

1. If two bodies A and B are moving in straight line same direction with velocity V_A and V_B, then relative velocity of A with respect to B is $v_{AB} = v_A - v_B$. Similarly $v_{BA} = v_B - v_A$

2. If two bodies A and B are moving in straight line in opposite direction then

$v_{AB} = v_A + v_B$ (towards B)

$v_{BA} = v_B + v_A$ (towards A)

$v_{AB} = -v_{BA}$

Same concept is used for acceleration also.

3. If two cars A and B are moving in same direction with velocity v_A and v_B ($v_A > v_B$) when A is behind B at a distance d, driver in car A applies brake which causes retardation a in car A, then minimum value of d to avoid collision is $\dfrac{(v_A - v_B)^2}{2a}$ *i.e.*, $d > \dfrac{(v_A - v_B)^2}{2a}$.

4. A particle is dropped and another particle is thrown downward with initial velocity u, then
   (a) Relative acceleration is always zero
   (b) Relative velocity is always u.
   (c) Time at which their separation is x is $\dfrac{x}{u}$.

5. Two bodies are thrown upwards with same initial velocity with time gap τ. They will meet after a time t from projection of first body.

$$t = \frac{\tau}{2} + \frac{u}{g}$$

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PHYSICS

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STUDY PACKAGE – 01

For – JEE / NEET

Motion in 2 & 3 Dimension

Chapter - 3

Description of Motion in Two and Three Dimensions

3

CHAPTER

AIEEE Syllabus

Scalars and Vectors, Vector addition and Subtraction, Zero Vector, Scalar and Vector products, Unit Vector, Resolution of a Vector. Relative Velocity, Motion in a plane, Projectile Motion, Uniform Circular Motion.

THIS CHAPTER COVERS :

- Scalars and Vectors
- Types of Vector
- Vector Addition
- Vector Subtraction
- Resolution of Vectors
- Scalar and Vector product
- Relative Motion
- Projectile Motion
- Circular Motion

SCALARS AND VECTORS

Scalars

Scalars are physical quantities which are completely described by their magnitude only. For example: mass, length, time, temperature energy etc.

Vectors

Vectors are those physical quantities having both magnitude as well as direction and they obeys vector algebra (eg. parallelogram law or triangle law of vector addition). For example: displacement, velocity, acceleration, force, momentum, impulse, electric field intensity etc.

TYPES OF VECTOR

Axial and Polar Vectors

Vectors, which have some starting point or point of application are called polar vectors. *E.g.,* displacement, force but those vectors representing rotational effects and are always along the axis of rotation in accordance with right hand screw rule are axial vectors.

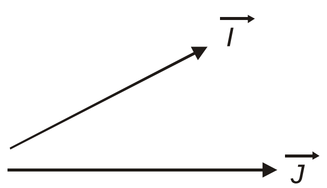
E.g., angular velocity, angular acceleration, angular displacement, torque etc.

Unit Vector : A vector having unit magnitude is called a unit vector. Thus, unit vector of a vector $\vec{V}$ is $\hat{V} = \dfrac{\vec{V}}{|\vec{V}|} = \dfrac{\vec{V}}{V}$ where $|\vec{V}| = V$ = Magnitude of $\vec{V}$.

$\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and z axis respectively. $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$.

Unit vector along a direction is unique and have no unit.

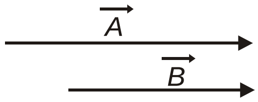
Coplanar vectors are vectors lying in same plane.



$\vec{I}$ and $\vec{J}$ are in the plane of paper so they are Coplaner

Collinear vectors are vectors having common line of action. These are of two types

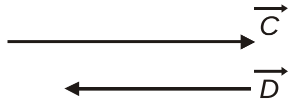
(i) Parallel or like (angle between them is 0°)



$\vec{A}$ and $\vec{B}$ are parallel vectors ($\theta = 0°$)

Parallel vectors having equal magnitude are known as equal vectors

(ii) Antiparallel or unlike (angle between them is 180°).



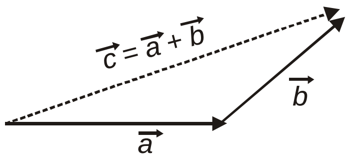
$\vec{C}$ and $\vec{D}$ are antiparallel vec

Antiparallel vectors of equal megnitude are known as negative vectors of each other.

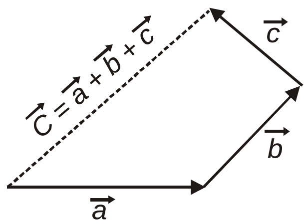
Null vector ($\vec{0}$) : Two opposite vectors added to form a null vector it has zero magnitude. If $\vec{A}$ and $\vec{B}$ are two negative vectors then $\vec{A} + \vec{B} = \vec{0}$ vector

VECTOR ADDITION

1. Triangle Law of Vector Addition : If $\vec{a}$ and $\vec{b}$ are two vectors represented as sides of a triangle in same order then other side $\vec{c}$ in opposite order is the resultant.

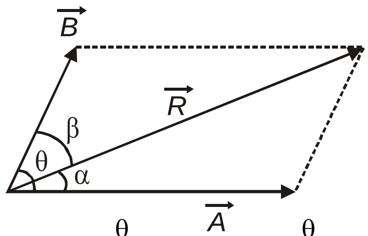


2. Polygon Law : If a number of vectors are represented as sides of a polygon in same order then the side which closes the polygon in opposite order in the resultant.



Vector addition obeys commutative law $(\vec{A} + \vec{B} = \vec{B} + \vec{A})$ and associative law $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$

3. Parallelogram Law of Vector Addition : If two vectors having common origin are represented both in magnitude and direction as the two adjacent sides of a parallelogram, then the diagonal which originates from the common origin represents the resultant of these two vectors. The result are listed below:



(a) $\vec{R} = \vec{A} + \vec{B}$ ·

(b) $|\vec{R}| = (A^2 + B^2 + 2AB\cos\theta)^{1/2}$

(c) $\tan\alpha = \dfrac{B\sin\theta}{A + B\cos\theta}, \tan\beta = \dfrac{A\sin\theta}{B + A\cos\theta}$

(d) If $|\vec{A}| = |\vec{B}| = x$ (say) , then $R = x\sqrt{2(1+\cos\theta)} = 2x\cos\dfrac{\theta}{2}$ and $\alpha = \beta = \dfrac{\theta}{2}$ *i.e.,* resultant bisect angle between $\vec{A}$ and $\vec{B}$.

(e) If $|\vec{A}| > |\vec{B}|$ then $\alpha < \beta$

(f) $R_{max} = A + B$, when $\theta = 0$ and $R_{min} = |A - B|$ when $\theta = 180°$.

(g) $R^2 = A^2 + B^2$, if $\theta = 90°$ *i.e.*, $\vec{A}$ and $\vec{B}$ are perpendicular.

(h) If $|\vec{A}| = |\vec{B}| = |\vec{R}|$ then $\theta = 120°$.

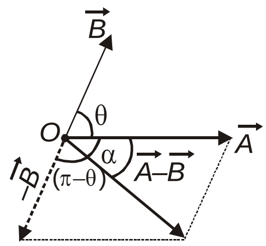
(i) If $\vec{R}$ is perpendicular to $\vec{A}$, then $\cos\theta = -\frac{A}{B}$ and $A^2 + R^2 = B^2$.

(j) For n equal vectors acting at a point such that angle between them are equal $\left(\frac{360}{n}\right)$, the resultant is zero.

VECTOR SUBTRACTION

Subtraction of vector $\vec{B}$ from vector $\vec{A}$ is simply addition of vector $-\vec{B}$ with $\vec{A}$ *i.e.,* $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

Using parallelogram law



Result : $R = |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}, \tan\alpha = \frac{B\sin(\pi-\theta)}{A + B\cos(\pi-\theta)} = \frac{B\sin\theta}{A - B\cos\theta}$

> ***Note :*** *If* $|\vec{A}| = |\vec{B}| = x$ (say), *then* $R = x\sqrt{2(1-\cos\theta)} = 2x\sin\frac{\theta}{2}$.

RESOLUTION OF VECTORS

Any vector $\vec{V}$ can be represented as a sum of two vectors $\vec{P}$ and $\vec{Q}$ which are in same plane as $\vec{V} = \lambda\vec{P} + \mu\vec{Q}$, where λ and μ are two real numbers. We say that $\vec{V}$ has been resolved in two component vector $\lambda\vec{P}$ and $\mu\vec{Q}$ along $\vec{P}$ and $\vec{Q}$ respectively.

Rectangular components in two dimensions :

$\vec{V} = \vec{V}_x + \vec{V}_y, \vec{V} = \vec{V}_x\hat{i} + \vec{V}_y\hat{j}, V = \sqrt{V_x^2 + V_y^2}$

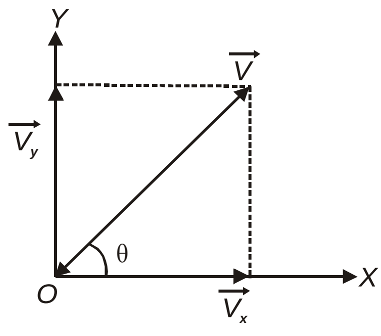
$\vec{V}_x$ and $\vec{V}_y$ are rectangular component of vector in 2-dimension.

$V_x = V\cos\theta$

$V_y = V\sin\theta = V\cos(90 - \theta)$

V_z = zero.

$\vec{V} = V\cos\theta\,\hat{i} + V\sin\theta\,\hat{j}$



> ***Note :*** *Unit vector along* $\vec{V} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

Rectangular Component in Three Dimension :

A vector $\vec{V}$ is in a space which is making α, β and γ with *x*-axis, *y*-axis and *z*-axis respectively.

$$\vec{V} = \vec{V}_x + \vec{V}_y + \vec{V}_z$$

$$|\vec{V}| = (\vec{V}_x^2 + \vec{V}_y^2 + \vec{V}_z^2)^{\frac{1}{2}}$$

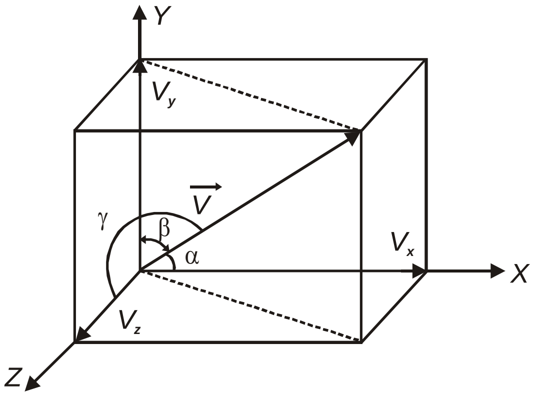
$$\vec{V} = \vec{V}_x\hat{i} + \vec{V}_y\hat{j} + \vec{V}_z\hat{k}$$

$$V_x = V\cos\alpha \Rightarrow \cos\alpha = l = \frac{V_x}{V}$$

$$V_y = V\cos\beta \Rightarrow \cos\beta = m = \frac{V_y}{V}$$

$$V_z = V\cos\gamma \Rightarrow \cos\gamma = n = \frac{V_z}{V}$$

$$\vec{V} = V\cos\alpha\,\hat{i} + V\cos\beta\hat{j} + v\cos\gamma\,\hat{k}$$



l, *m*, *n* are called direction cosines of vector $\vec{V}$.

$l^2 + m^2 + n^2 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1,\quad \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2.$

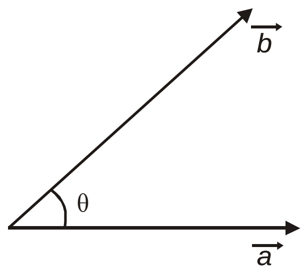
Unit vector along $\vec{V} = l\,\hat{i} + m\hat{j} + n\hat{k} = \cos\alpha\hat{i} + \cos\beta\hat{j} + \cos\gamma\hat{k}$.

SCALAR AND VECTOR PRODUCTS

Scalar (dot) Product of Two Vectors : The scalar product of two vectors $\vec{a}$ and $\vec{b}$ is defined as

$$\vec{a}\,.\,\vec{b} = ab\cos\theta$$

$$\cos\theta = \frac{\vec{a}\,.\,\vec{b}}{ab}$$



If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a}.\vec{b} = 0$

If $\theta < 90°$, then $\vec{a}.\vec{b} > 0$ and if $\vec{a}.\vec{b} < 0$, then $\theta > 90°$.

Projection of vector $\vec{a}$ on $\vec{b}$ is $(\vec{a}.\vec{b})\dfrac{\vec{b}}{b^2}$.

$$a^2 = \vec{a}.\vec{a}$$

$$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1.$$

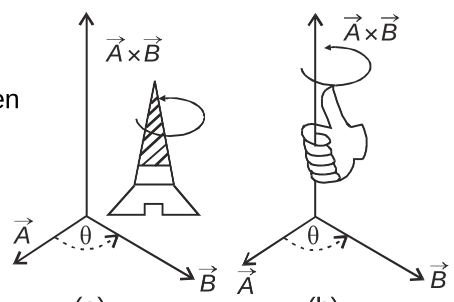
Scalar product is commutative i.e., $\vec{a}.\vec{b} = \vec{b}.\vec{a}$.

Vector Product of two Vectors :

Mathematically, if θ is the angle between vectors $\vec{A}$ and $\vec{B}$, then

$$\vec{A}\times\vec{B} = AB\,\sin\theta\,\hat{n} \qquad \ldots(i)$$

The direction of vector $\vec{A}\times\vec{B}$ is the same as that of unit vector $\hat{n}$. It is decided by any of the following two rules :



(a) (b)

(a) **Right handed screw rule :** Rotate a right handed screw from vector $\vec{A}$ to $\vec{B}$ through the smaller angle between them; then the direction of motion of screw gives the direction of vector $\vec{A}\times\vec{B}$ (Fig. a)

(b) **Right hand thumb rule :** Bend the finger of the right hand in such a way that they point in the direction of rotation from vector $\vec{A}$ to $\vec{B}$ through the smaller angle between them; then the thumb points in the direction of vector $\vec{A}\times\vec{B}$ (Fig. b)

Some Important Points :

1. The cross product of the two vectors does not obey commutative law. As discussed above

$\vec{A}\times\vec{B} = -(\vec{B}\times\vec{A})$ *i.e.*, $\vec{A}\times\vec{B} \neq (\vec{B}\times\vec{A})$

2. The cross product follows the distributive law *i.e.,*

$\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$

3. The cross product of a vector with itself is a NULL vector *i.e.,*

$\vec{A}\times\vec{A} = (A)\,(A)\sin 0^\circ\,\hat{n} = \vec{0}$

4. The cross product of two vectors represents the area of the parallelogram formed by them,

(Figure., shows a parallelogram *PQRS* whose adjacent sides $\overrightarrow{PQ}$ and $\overrightarrow{PS}$ are represented by vectors $\vec{A}$ and $\vec{B}$ respectively.

Now, area of parallelogram = *QP* × *SM* = *QP*·*AB* sin θ. Because, the magnitude of vectors $\vec{A}\times\vec{B}$ is *AB* sin θ, hence cross product of two vectors represents the area of parallelogram formed by it. It is worth noting that area vector $\vec{A}\times\vec{B}$ acts along the perpendicular to the plane of two vectors $\vec{A}$ and $\vec{B}$.

5. In case of unit vectors $\hat{i}, \hat{j}, \hat{k},$ we obtain following two important properties:

(a) $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = (1)\ (1)\ \sin 0^\circ\,(\hat{n}) = 0$

(b) $\hat{i}\times\hat{j} = (1)\,(1)\ \sin 90^\circ\,(\hat{k}) = \hat{k}$

where, $\hat{k}$ is a unit vector perpendicular to the plane of $\hat{i}$ and $\hat{j}$ in a direction in which a right hand screw will advance, when rotated from $\hat{i}$ to $\hat{j}$

Also, $-\hat{j}\times\hat{i} = -(1)\,(1)\sin 90^\circ\,(-\hat{k}) = \hat{k}$

Similarly, $\hat{j}\times\hat{k} = -\hat{k}\times\hat{j} = \hat{i}$ and $\hat{k}\times\hat{i} = -\hat{i}\times\hat{k} = \hat{j}$

6. **Cross product of two vectors in terms of their rectangular components :**

$$\vec{A}\times\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\times(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= (A_yB_z - A_zB_y)\,\hat{i} + (A_zB_x - A_xB_z)\,\hat{j} + (A_xB_y - A_yB_x)\,\hat{k}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

7. Multiplication of a vector $\vec{A}$ with a real number *m*

If m is positive real number then a parallel vector is obtained having magnitude m times the magnitude of $\vec{A}$

If m is negative real number then antiparallel vector is obtained having magnitude m times the magnitude of $\vec{A}$

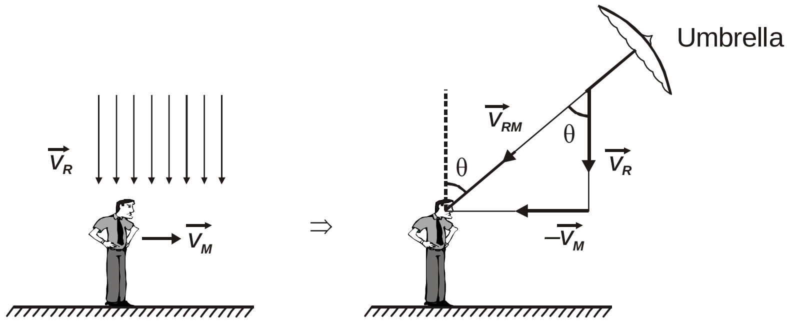
8. If angle between $\vec{A}$ and $\vec{B}$ is θ, then angle between $\vec{A}$ and $-\vec{B}$ or between $-\vec{A}$ and $\vec{B}$ is $(180° - \theta)$.

RELATIVE MOTION IN TWO DIMENSIONS

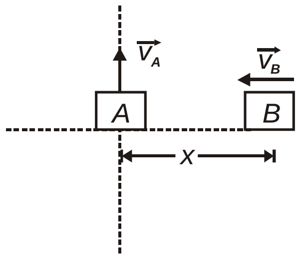
Relative velocity :

Velocity of object A w.r.t. object B is $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B, \vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

1. **Direction of Umbrella :** A person moving one straight road has to hold his umbrella opposite ot direction of relative velocity of rain. The angle θ is given by $\tan\theta = \dfrac{v_M}{v_R}$ with vertical in forward direction.



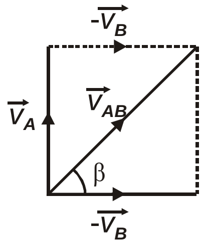
2. **Closest approach :** Two objects A and B having velocities $\vec{v}_A$ and $\vec{v}_B$ at separation x are shown in figure



The relative velocity of A with respect to B is given by

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

$$\tan\beta = \frac{v_A}{v_B}$$



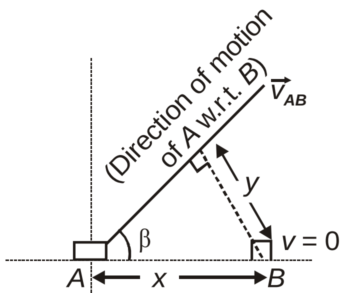
The above situation is similar to figure given below

y is the distance of closest approach.

Now, $\sin\beta = \dfrac{y}{x}$

$\Rightarrow$ $y = x\sin\beta$

$$y = \frac{x\tan\beta}{\sqrt{1+\tan^2\beta}} = \frac{xV_A}{\sqrt{V_B^2 + V_A^2}}$$

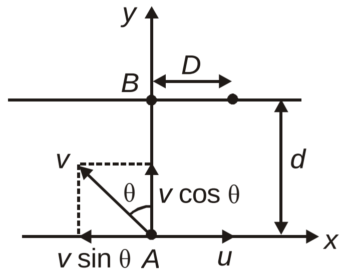


3. Crossing a river :

v = velocity of the man in still water.

θ = angle of which man swims w.r.t. normal to bank such that

$v_x = -v \sin\theta$, $v_y = v\cos\theta$



Time taken to cross the river is given by $t = \dfrac{d}{v_y} = \dfrac{d}{v\cos\theta}$

Velocity along the river

$$v'_x = u - v\sin\theta$$

Distance drifted along the river $D = t\,v'_x$

$$D = \frac{d}{v\cos\theta}(u - v\sin\theta)$$

Case I :

The Minimum time to cross the river is given by

$$\tau_{\min} = \frac{d}{v} \qquad \text{(when } \cos\theta = 1,\ \theta = 0°,\ u \perp v\text{)}$$

Distance drifted is given by

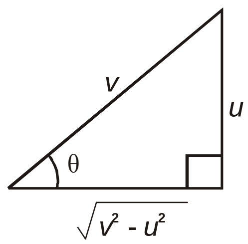
$$D = \frac{d}{v} \times u$$

Case II :

To cross the riven straight

drift $D = 0 \quad \Rightarrow \quad u - v\sin\theta = 0$

$$\sin\theta = \frac{u}{v} \quad \Rightarrow \quad (v > u)$$



Time to cross the river straight across is given by

$$t = \frac{d}{v\cos\theta} = \frac{d}{\sqrt{v^2 - u^2}}$$

PROJECTILE MOTION

An object moving in space under the influence of gravity is called projectile. Two important cases of interest are discussed below :

1. Horizontal projection :

A body of mass m is projected horizontally with a speed u from a height h at the moment $t = 0$. The path followed by it is a parabola.

It hits the ground at the moment $t = T$, with a velocity $\vec{v}$ such that

$$T = \sqrt{\frac{2H}{g}}$$

$$|\vec{v}| = \sqrt{u^2 + 2gH} = |u\hat{i} + gT\hat{j}|$$

The position at any instant t_0 is given by

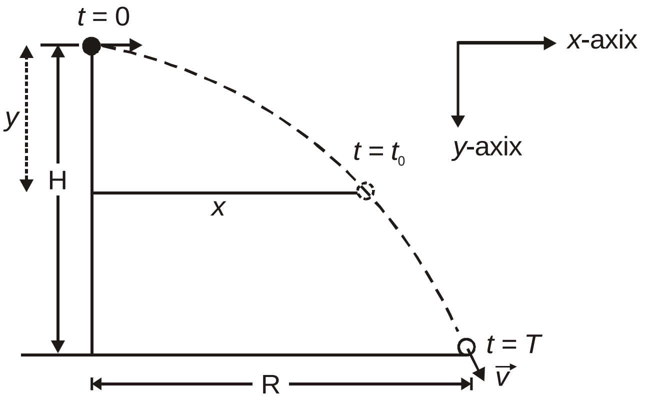
$x = ut_0$

$$y = \frac{1}{2}gt_0^2$$

$$y = \frac{gx^2}{2u^2}$$

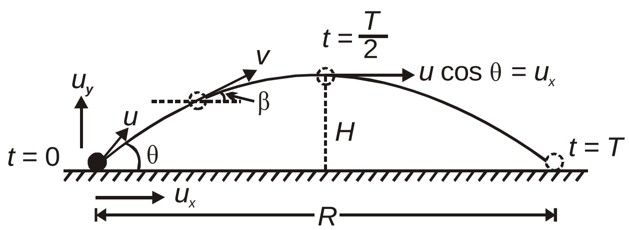
The velocity at any instant t_0 is given by

$$\overrightarrow{v_0} = u\hat{i} + gt_0\hat{j}$$



2. Oblique projection : A body of mass m is projected from ground with speed u at an angle θ above horizontal at the moment $t = 0$.

It hits the ground at a horizontal distance R at the moment $t = T$.



1. Time of flight $T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$

2. Maximum height $H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g}$

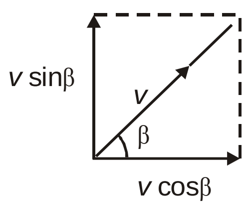
3. Horizontal range $R = u_x \times T = \frac{2u_x u_y}{g} = \frac{u^2 \sin 2\theta}{g}$

4. Equation of trajectory; $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

   or $y = x\tan\theta\left(1 - \frac{x}{R}\right)$

5. Instantaneous velocity $v = \sqrt{u^2 + (gt)^2 - 2u(gt)\sin\theta}$

   and direction of motion is such that, $\tan\beta = \tan\theta - \frac{gt}{u\cos\theta}$

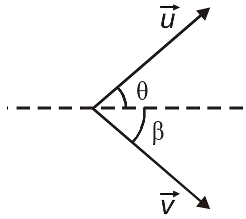


   (a) $v = \frac{u\cos\theta}{\cos\beta}$ [$\because$ Horizontal component is same every where]

   (b) $v \sin\beta = u \sin\theta - gt$

(c) When $\vec{v}$ (velocity at any instant '*t*') is perpendicular to $\vec{u}$ (initial velocity)

$\Rightarrow$ $\beta = 90° - \theta$



(i) $v = \dfrac{u\cos\theta}{\cos(90° - \theta)} = u\cot\theta$

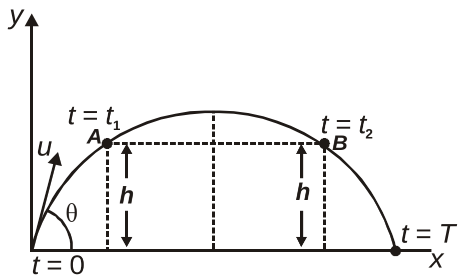
(ii) $t = \dfrac{u}{g\sin\theta}$

Applications :

1. The height attained by the particle is largest when θ = 90°. In this situation, time of flight is maximum and range is minimum (zero).
2. The horizontal range is same for complimentary angles like (θ, 90 – θ) or (45 + θ, 45 – θ). It is maximum for θ = 45°.
3. When horizontal range is maximum, $H = \dfrac{R_{\max}}{4}$
4. When *R* is range, *T* is time of flight and *H* is maximum height, then

   (a) $\tan\theta = \dfrac{gT^2}{2R}$

   (b) $\tan\theta = \dfrac{4H}{R}$
5. If *A* and *B* are two points at same level such that the object passes *A* at $t = t_1$ and *B* at $T = t_2$, then



(i) $T = \dfrac{2u\sin\theta}{g} = t_1 + t_2$

(ii) $h = \dfrac{1}{2}gt_1t_2$

(iii) Average velocity in the interval *AB* is

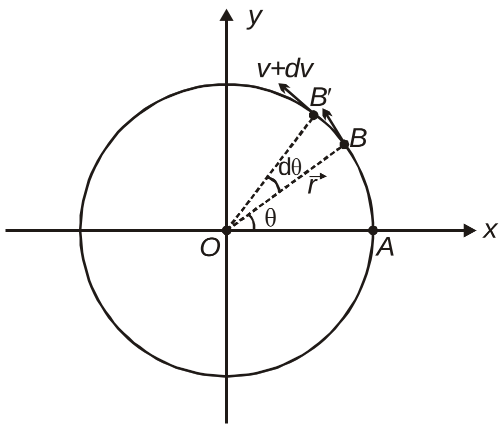
$v_{av} = u\cos\theta$ [$\because$ vertical displacement is zero]

CIRCULAR MOTION

An object of mass *m* is moving on a circular track of radius *r*. At *t* = 0, it was at *A*. At any moment of time '*t*', it has moved to *B*, such that $\angle AOB = \theta$. Let its speed at this instant be *v* and direction is along the tangent. In a small time *dt*, it moves to *B*′ such that $\angle B'OB = d\theta$.

The angular displacement vector is $\overrightarrow{d\theta} = d\theta\hat{k}$

The angular velocity vector is $\vec{\omega} = \dfrac{d\theta}{dt}\hat{k}$.



At B', the speed of the object has become $v + dv$.

The tangential acceleration is $a_t = \dfrac{dv}{dt}$

The radial (centripetal acceleration) is $a_c = \dfrac{v^2}{r} = \omega^2 r$

The angular acceleration is $\alpha = \dfrac{d\omega}{dt}$

Relations among various quantities.

1. $\vec{v} = \vec{\omega} \times \vec{r}$

2. $\vec{a} = \dfrac{d\vec{v}}{dt} = \vec{\omega} \times \dfrac{d\vec{r}}{dt} + \dfrac{d\vec{\omega}}{dt} \times \vec{r} = \overrightarrow{a_c} + \overrightarrow{a_t}$

3. $\overrightarrow{a_c} = \vec{\omega} \times \vec{v}$

4. $\overrightarrow{a_t} = \vec{\alpha} \times \vec{r}$

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PHYSICS

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STUDY PACKAGE – 01

For – JEE / NEET

Laws of Motion

Chapter - 4

Laws of Motion

4

CHAPTER

AIEEE Syllabus

Force and Inertia, Newton's First Law of motion; Momentum, Newton's Second Law of motion; Impulse; Newton's Third Law of motion. Law of conservation of linear momentum and its applications, Equilibrium of concurrent forces. Static and Kinetic friction, laws of friction, rolling friction. Dynamics of uniform circular motion: Centripetal force and its applications.

THIS CHAPTER COVERS :

- Force
- Momentum
- Newton's laws of Motion
- Frame of Reference
- Conservation of linear momentum
- Rocket propulsion
- Applications of Newton's laws of Motion
- Friction
- Motion in a horizontal circle

FORCE

Force is the action of one body upon another.

It is a vector quantity with units newton (N).

Types of Forces

Weight : Weight of a body is the force with which earth attracts it. It is also defined as force of gravity or the gravitational force.

Contact forces : Whenever two bodies come in contact they exert forces on each other, that are called contact forces.

(a) **Normal reaction (*N* or *R*) :** It is the component of contact force normal to the surface. It measures how strongly the surfaces in contact are pressed together.

(b) **Frictional force (*f*) :** It is the component of contact force parallel to the surface. It opposes the relative motion (or attempted motion) of the two surfaces in contact.

(c) **Tension :** The force exerted by the end of a taut string, rope or chain is called the tension. The direction of tension is always pulling in nature.

(d) **Spring force :** Every spring resists any attempt to change its length, the more you change its length the harder it resists. The force exerted by a spring is given by $F = -kx$, where x is the change in length and k is spring constant or stiffness constant (units N/m).

MOMENTUM

Momentum of a body is defined to be the product of its mass m and velocity $\vec{v}$. It is a vector quantity and is represented by $\vec{P}$,

$$\vec{P} = m\vec{v}$$

NEWTON'S LAWS OF MOTION

First law (Law of inertia)

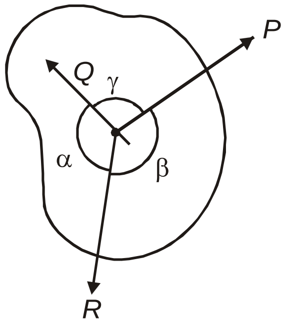
It states that a body continues to be in its state of rest or of uniform motion until and unless it is acted upon by some external force

If $F = 0, \Rightarrow v = \text{Constant}, \Rightarrow a = 0$

i.e. The body opposes any change in its state of rest or of uniform motion, it is also known as the law of inertia.

Equilibrium of Contact Forces

If three forces $\vec{P}$, $\vec{Q}$ and $\vec{R}$ action on an object such that forces are concurrent and object is in equilibrium then



$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Second Law of Motion

It states that the net force acting on a particle or a system of particles is directly proportional to the rate of change of its linear momentum.

Mathematically $\vec{F} \propto \dfrac{\overrightarrow{dp}}{dt}$

$$\vec{F} = \frac{k\overrightarrow{dp}}{dt} \text{ where } k = 1$$

$$\vec{F} = \frac{d}{dt}(m\vec{v})$$

If the mass of the system is constant, then

$$\vec{F} = m\frac{\overrightarrow{dv}}{dt}$$

$$\Rightarrow \quad \vec{F} = ma$$

$$\Rightarrow \quad \vec{F}_{inst} = m\vec{a}_{inst}$$

$$\vec{F}_{av} = m\vec{a}_{av}$$

$$= m\frac{\left(\vec{v}_f - \vec{v}_i\right)}{t}$$

Third Law of Motion

To every action there is equal and opposite reaction

Mathematically $\Rightarrow \quad \vec{F}_{12} = -\vec{F}_{21}$

Here $\overrightarrow{F_{12}}$ is force acting on body 1 due to 2 and $\overrightarrow{F_{21}}$ is force acting on body 2 due to 1.

IMPULSE

Impulse of a force is defined as the change of linear momentum imparted to the body

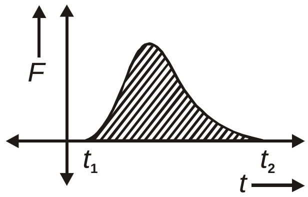
$\vec{J} = \vec{P}_2 - \vec{P}_1 = m(\vec{v}_2 - \vec{v}_1)$

If the body is acted upon by a variable force as shown in figure, then

Impulse acting on the body in the interval t_1 to t_2

$$\vec{J} = \vec{P}_2 - \vec{P}_1 = \int_{t_1}^{t_2} F\,dt$$

= Area below the $F \sim t$ graph.



FRAME OF REFERENCE

It is a set of co-ordinate axes with respect to which the position, velocity and acceleration of a particle or a system of particles can be measured.

There are two types of reference frames:

(a) Inertial frames of reference

If two frames of reference are either at rest or moving with uniform relative velocity, then they are said to be inertial frames of reference.

All Newton's laws are valid in inertial frames of reference.

(b) Non-inertial frames of reference

Accelerated frames are known as non-inertial frames of reference.

Newton's laws are not valid in non-inertial frames.

Pseudo forces (fictitious forces): It is a slight adjustment which enables us to apply Newton's laws in Non-inertial frames of reference.

If an observer is into a linearly accelerating non-inertial frame, then in order to use Newton's laws of motion to explain the behaviour of a particle or a system of particles, a pseudo force is applied opposite to the acceleration of the observer.

Mathematically, $\vec{F}_{pseudo} = -m\vec{a}$

m = mass of the particle

a = acceleration of the observer

General expression for force :

As $\vec{p} = m\vec{v}$

$$\Rightarrow \quad \vec{F} = \frac{d}{dt}(m\vec{v}) = \frac{dm}{dt}\vec{v} + \frac{m d\vec{v}}{dt}$$

Case - I :

When mass = constant, $\Rightarrow \dfrac{dm}{dt} = 0$

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

Case - II :

In variable mass system like Rocket propulsion, firing of bullets etc.

Relative speed is constant in these cases

$$\vec{F} = \vec{v}\frac{dm}{dt}$$

CONSERVATION OF LINEAR MOMENTUM

According to Newton's third law

$$\vec{f}_{ext} \propto \frac{d\vec{p}}{dt}$$

If net external force acting on the particle or system of particles is zero then the total momentum of the particle or system of particles remain conserved.

$$\frac{d\vec{p}}{dt} = 0 \quad \text{or } \vec{p} = \text{constant}$$

Applications of Conservation of Momentum

1. A block of mass *m* is at rest in a gravity free space suddenly explodes into two parts in the ratio 1 : 3, if lighter particle has a speed of 9 m/s just after explosion, then the speed of heavier particle can be calculated as shown.

$$\vec{p}_{\text{initial}} = 0$$

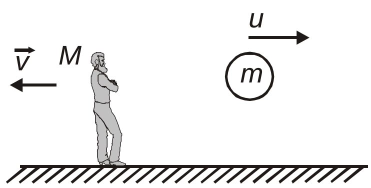
$$\vec{p}_{\text{final}} = \left(\frac{m}{4}\right)9\hat{i} + \left(\frac{3m}{4}\right)\vec{v}$$

$$\vec{p}_{\text{initial}} = \vec{p}_{\text{final}}$$

$$0 = \frac{m}{4}9\hat{i} + \frac{3m}{4}\vec{v}$$

$$\vec{v} = -3\hat{i} \text{ m/s}$$

2. A man of mass *M* standing on a frictionless floor, throws a ball of mass *m* with a speed *u* as shown in figure. The velocity of man, after he throws the ball is $\vec{v}$



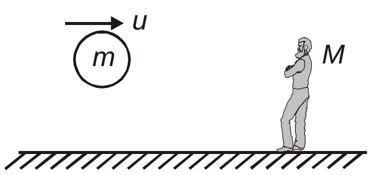
$$\vec{p}_{\text{initial}} = 0$$

$$\vec{p}_{\text{final}} = mu\hat{i} + M\vec{v}$$

As $\vec{p}_{\text{initial}} = \vec{p}_{\text{final}}$

$$\vec{v} = \frac{-m\hat{i}}{M}$$

3. A ball of mass *m* moving with constant speed of *u* m/s is caught by man of mass *M* standing on a frictionless floor. The velocity acquired by the man is $\vec{v}$



$$\vec{p}_{\text{initial}} = mu\hat{i}$$

$$\vec{p}_{\text{final}} = (m+M)\vec{v}$$

$$\vec{p}_{\text{initial}} = \vec{p}_{\text{final}}$$

$$\vec{v} = \frac{mu\hat{i}}{(m+M)}$$

4. If a gun of mass *M* fires a bullet of mass *m* with speed *v*, then recoil speed of the gun is given by $V = mv/M$.

ROCKET PROPULSION

It is an example of variable mass-system.

In a rocket, combustion chamber carries fuel and oxidising agent. When the fuel burns, a hot jet of gases emerges out forcefully from the small hole in the tail of the rocket.

Let u be the velocity of emerging gases relative to the rocket and ($\Delta M/\Delta t$) be the rate of fuel consumption, then the upthrust on the rocket is $F = u\ (\Delta M/\Delta t)$.

Let M_0 be the initial mass (rocket + fuel), M and v be the mass and velocity of the rocket at any instant t, then $\vec{v} = -\vec{u}\log_e\left(\frac{M_0}{M}\right)$

APPLICATIONS OF NEWTON'S LAWS OF MOTION

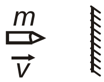
1. A machine gun fires n bullet per second with speed u and mass of each bullet is m.

   The Force required to keep the gun stationary is

   $\vec{F} = nm\vec{v}$

2. Bullets moving with a speed v hit a wall

   (i) When the bullets come to rest in wall

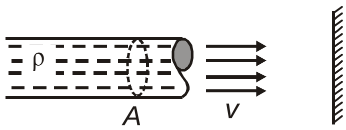


   Force on wall $F_{\text{wall}} = nmv$

   (ii) When the bullets rebound elastically

   $F_{\text{wall}} = 2nmv$

3. Liquid jet of area A moving with speed v hits a wall



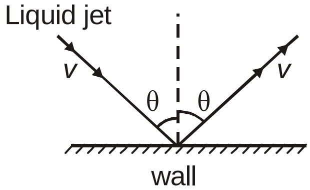
   (i) Force required be a pump to move the liquid with this speed is

   $$F = v\frac{dm}{dt} = v \times \rho Av = \rho Av^2$$

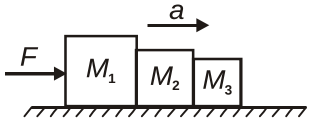
   (ii) As jet hits a vertical wall and does not rebound, the force exerted by it on the wall is, $F_{\text{wall}} = \rho Av^2$

   (iii) When water rebounds elastically, $F_{\text{wall}} = 2\rho Av^2$

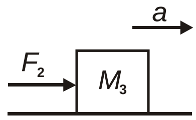
   (iv) For oblique impact as shown, $F_{\text{wall}} = 2\rho Av^2\cos\theta$



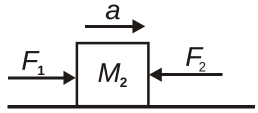
4. The blocks shown are being pushed by a force F. F_1, F_2 are contact forces between M_1 & M_2 and M_2 & M_3 respectively



$$a = \frac{F}{M_1 + M_2 + M_3}$$

FBD of M_3  $F_2 = M_3 a$

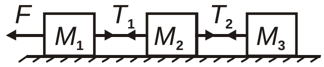
$F_1 - F_2 = M_2 a \Rightarrow F_1 = (M_3 + M_2)a$

FBD of M_2 

or $F_2 = \dfrac{M_3}{M_1 + M_2 + M_3}F, F_1 = \left(\dfrac{M_2 + M_3}{M_1 + M_2 + M_3}\right)F$

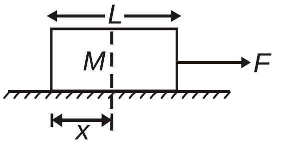
5. The strings are massless. Let T_1 and T_2 be the tensions in the two strings and 'a' be the acceleration.

$$a = \frac{F}{M_1 + M_2 + M_3}, T_1 = \frac{M_2 + M_3}{M_1 + M_2 + M_3}, T_2 = \frac{M_3 F}{M_1 + M_2 + M_3}$$

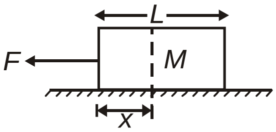


6. Tension in the block at x from left end is given as

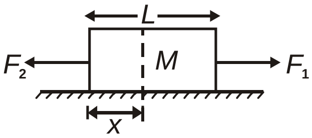
(a) $T_x = \dfrac{Mx}{L} \times \dfrac{F}{M} = \dfrac{Fx}{L}$

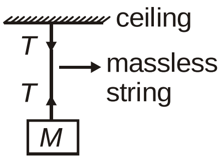


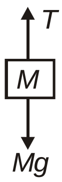
(b) $T_x = \dfrac{F(L - x)}{L}$



(c) $T_x = \dfrac{F_1 x}{L} + \dfrac{F_2(L - x)}{L}$



7. 

 FBD of M w.r.t. ground

(a) When system is stationary

$T - Mg = 0$

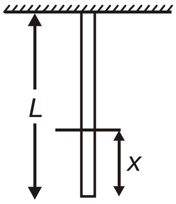
(b) System moves up with acceleration 'a'

$T = M(g + a)$

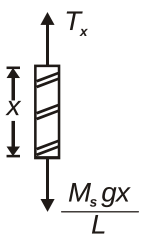
(c) System moves down with acceleration a'

$T = M(g - a')$

8. Uniform rope of mass M_s.



FBD of lower portion



(a) Stationary system

$$T_x = \frac{M_s g x}{L}$$

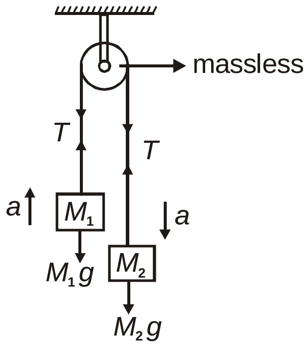
(b) If the rope is accelerating upwards, then $T_x = \frac{M_s x}{L}(g + a)$

(c) If the rope is accelerating downwards, then $T_x = \frac{M_s x}{L}(g - a)$

IMPORTANT PROBLEMS

Pulley mass systems

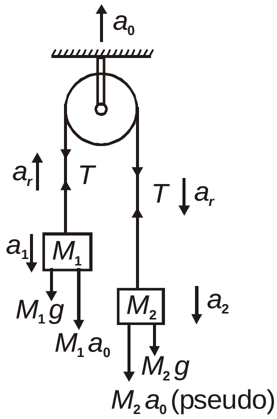
(i) Stationary pulley



$$a = \frac{M_2 - M_1}{M_2 + M_1} g$$

$$T = 2\left(\frac{M_1 M_2}{M_1 + M_2}\right) g$$

(ii) Pulley is moving upward with acceleration a_0



$$T = 2\left(\frac{M_1 M_2}{M_1 + M_2}\right)(g + a_0)$$

$$a_r = \left(\frac{M_2 - M_1}{M_2 + M_1}\right)(g + a_0)$$

$a_1 = -(a_0 + a_r)$

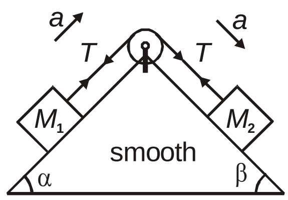
$a_2 = a_r - a_0$

a_r = relative acceleration of M_1 and M_2 w.r.t. pulley

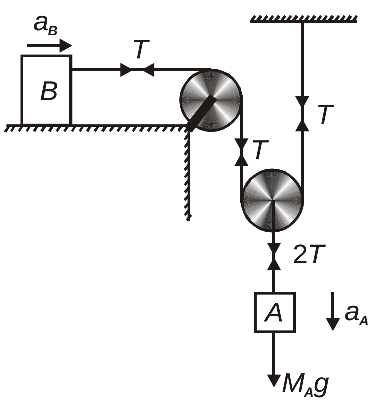
a_1, a_2 are accelerations of M_1 and M_2 w.r.t. ground.

(iii) $T = \frac{M_1 M_2}{M_1 + M_2}(\sin\alpha + \sin\beta)$

$$a = \left(\frac{M_2 \sin\beta - M_1 \sin\alpha}{M_2 + M_1}\right) g$$



(iv)



$2a_A = a_B$ ...(i)

$M_A g - 2T = M_A a_A$ ...(ii)

$T = M_B a_B$ ...(iii)

Two block system :

Case - I :

Let '*m*' does not slide down relative to wedge '*M*'

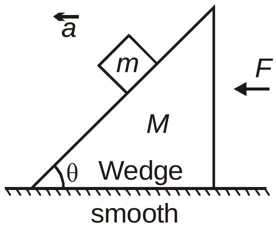
The force required is given by

$F = (M + m)g \tan\theta$

$a = g \tan\theta$ (in horizontal direction w.r.t. ground)

Contact force *R* between *m* and *M* is

$$R = \frac{Mg}{\cos\theta}$$



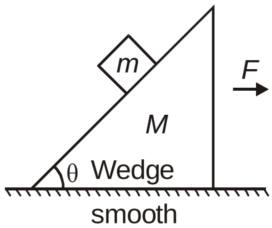
Case - II :

Minimum value of *F* so that '*m*' falls freely is given by

$F = Mg\cot\theta$

Wedge *M* moves with acceleration = $g\cot\theta$

Contact force between *M* and *m* is zero.



FRICTION

Friction force is of two types

1. Static frictions 'f_s'
2. Kinetic friction 'f_k'

Static Friction

The static friction between two contact surfaces is given by $f_s \le \mu_s N$, where *N* is the normal force between the contact surfaces and μ_s is a constant which depends on the nature of the surfaces and is called **'coefficient of limiting friction'**.

Static friction acts on stationary objects. Its values satisfy the condition

$f_s \le \mu_s N$

μ_s – Co-efficient of limiting friction

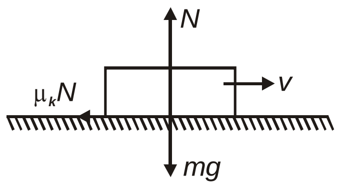
Static friction takes its peak value ($f_{s(max)} = \mu_s N$) when one surface is *'about to slide'* on the other. Static friction in this case is called limiting friction.

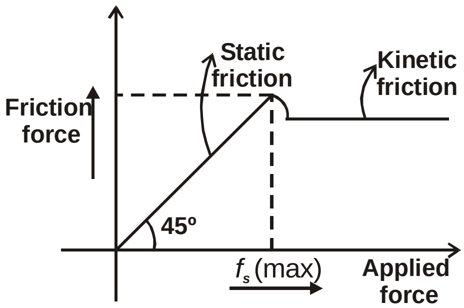
Kinetic Friction (μ_k)

It acts on the two contact surfaces only when there is relative slipiry or relative motion between two contact surfaces.

$f_k = \mu_k N$

The relative motion of a contact surface with respect to each other is opposed by a force given by $f_k = \mu_k N$, where *N* is the normal force between the contact surfaces and μ_k is a constant called **'coefficient of kinetic friction'**, which depends, largely, on the nature of the contact surfaces.

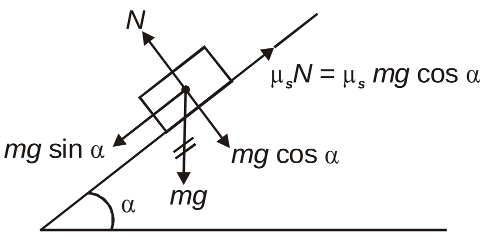




Graphical representation of variation of friction force with the force applied on a body

Angle of Repose

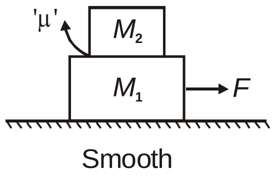
Consider a situation in which a block is placed on an inclined plane with co-efficient of friction 'μ' then the maximum value of angle of inclined plane for which the block can remain at rest is defined as angle of repose.



Two blocks placed one above the other

Case - I :

(a) $F \le \mu(M_1 + M_2)g$



Both blocks move together with same acceleration $a = \dfrac{F}{M_1 + M_2}$

$a_{max} = \mu g$

(b) $F > \mu(M_1 + M_2)g$

M_2 moves with constant acceleration $a_2 = \mu g$

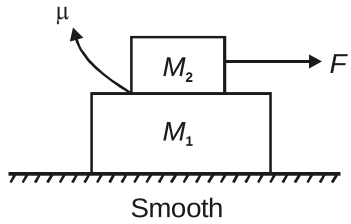
M_1 moves with acceleration $a_1 = \dfrac{F - \mu M_2 g}{M_1}$

M_2 slips backwardon M_1.

Case - II :

(a) $F \le \dfrac{\mu(M_1 + M_2)M_2}{M_1} g$, both blocks move together with acceleration $a = \dfrac{F}{M_1 + M_2}$ with $a_{max} = \dfrac{\mu M_2}{M_1} g$.

(b) $F > \dfrac{\mu(M_1 + M_2)M_2 g}{M_1}$,



M_1 moves with constant acceleration $a_1 = \dfrac{\mu M_2}{M_1} g$

M_2 moves with acceleration $a_2 = \dfrac{F - \mu M_2 g}{M_2}$

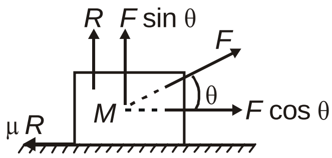
M_2 slips forward on M_1.

Minimum force required to move a body on a rough horizontal surface

$F \cos\theta > \mu R$

$F \geq \dfrac{\mu mg}{\cos\theta + \mu \sin\theta}$

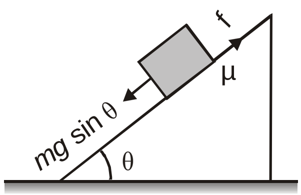
$F_{\min} = \dfrac{\mu mg}{\sqrt{1+\mu^2}}$ at $\theta = \tan^{-1}(\mu)$



Block on Inclined Plane

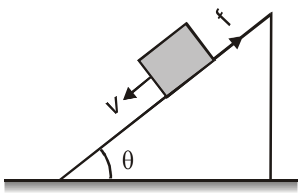
1. If $\mu \geq \tan\theta$, the block will remain stationary on the inclined plane; and the frictional force acting on the block will be equal to $mg \sin\theta$ and static in nature

   $f = mg \sin\theta$



2. If a block slides down an inclined plane with constant velocity, the frictional force acting on the block is kinetic in nature and is equal to $mg \sin\theta$

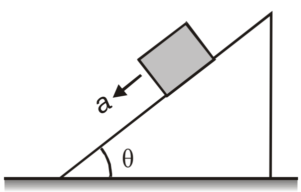
   $\Rightarrow$ $f = mg \sin\theta$



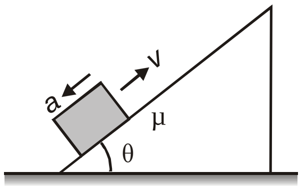
3. If $\mu < \tan\theta$, the block will slide down the plane with acceleration a equal to

   $a = g \sin\theta - \mu g \cos\theta$

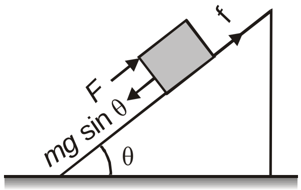
   Frictional force is kinetic in nature and is given by $f = \mu N = \mu mg \cos\theta$ (less than $mg \sin\theta$)



4. If a block is projected up the plane, retardation a is given by $a = g \sin\theta + \mu g \cos\theta$.

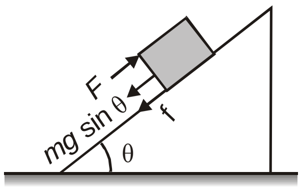


5. If $mg \sin\theta$ exceeds frictional force, the block tends to slide down. The minimum force required to prevent sliding is $F_{min} = mg \sin\theta - \mu\, mg \cos\theta$



6. If you try to push a block up the plane, frictional force and $mg \sin\theta$ both opposes the upward motion of the block, hence, the minimum force required to move up is given by

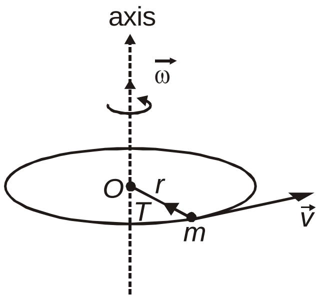
   $F_{min} = mg \sin\theta + \mu\, mg \cos\theta$



> ***Note :*** In case 5-6, if F lies between $mg \sin\theta = \mu\, mg \cos\theta$ and $mg \sin\theta + \mu\, mg \cos\theta$, the block remains stationary on the inclined plane.

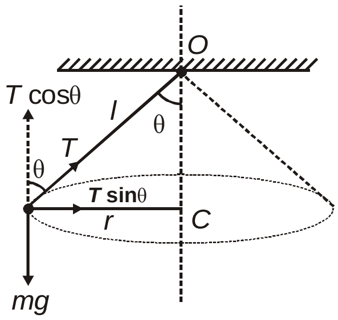
DYNAMICS OF CIRCULAR MOTION

Neglecting Gravity



T = Centripetal force = $\dfrac{mv^2}{r} = m\omega^2 r$

Considering gravity (Conical pendulum)



$T\sin\theta = m\omega^2 r$ …(1)

$T\cos\theta = mg$

$T = \dfrac{mg}{\cos\theta}$ …(2)

(a) For θ to be 90º (*i.e.*, string to be horizontal)

$T = \infty$

∴ It is not possible.

(b) $T\sin\theta = m\omega^2 r = m\omega^2 l\sin\theta$

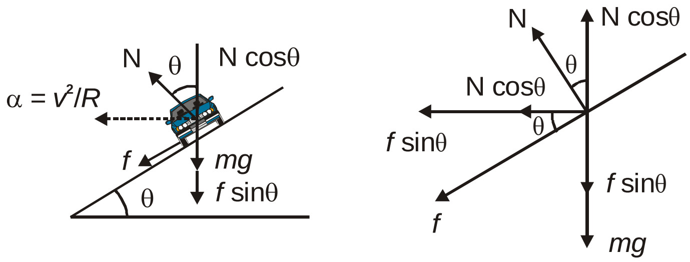
$\Rightarrow \quad T = m\omega^2 l$

(c) Time period = $2\pi\sqrt{\dfrac{l\cos\theta}{g}}$

Vehicle negotiating a curve on a banked road

The maximum velocity with which a vehicle can safely negotiate a curve of radius *r* on a rough inclined road is

$$v = \sqrt{\frac{rg(\mu + \tan\theta)}{1 - \mu\tan\theta}}$$



For a smooth surface $\mu = 0 \quad \Rightarrow v = \sqrt{rg\tan\theta}$

For a horizontal surface, $\theta = 0 \Rightarrow v = \sqrt{\mu rg}$

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PHYSICS

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STUDY PACKAGE – 01

For – JEE / NEET

Work, Energy & Power

Chapter - 5

Work, Energy and Power

5

CHAPTER

AIEEE Syllabus

Work done by a constant force and a variable force; kinetic and potential energies, work-energy theorem, power, Potential energy of a spring, conservation of mechanical energy, conservative and non-conservative forces; Elastic and inelastic collisions in one and two dimensions.

THIS CHAPTER COVERS :

- Concept of Work
- Kinetic Energy
- Work-Energy Theorem
- Potential Energy
- Conservative and non-conservative Forces
- Principle of conservation of Mechanical Energy
- Power
- Collision
- Motion in a vertical circle

CONCEPT OF WORK

Work (*W*) is energy transferred to or from an object by means of a force acting on the object. If Energy is transferred to the object, work is positive and if energy is transferred from the object, work is negative.

Mathematically

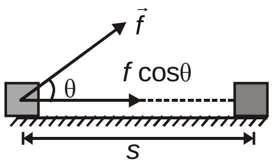
$$W = \int_{\vec{s}_1}^{\vec{s}_2} \vec{f}.\overrightarrow{ds}$$

If the force is constant, then

$$W = \vec{f}.\vec{s} \quad (\vec{s} = \vec{s}_2 - \vec{s}_1)$$

$$= fs\cos\theta$$

$$= s\,(f\cos\theta)$$



KINETIC ENERGY

Kinetic energy is associated with the state of motion of an object.

Mathematically

$$\text{K.E.} = \frac{1}{2}mv^2$$

WORK-ENERGY THEOREM

It states that, the work done on a particle or on a system of particles by the resultant force is equal to change in its kinetic energy. This is called work-energy theorem.

$$W_{\text{Total}} = \Delta\text{K.E.}$$

or

$$W_{\text{ext.}} + W_{\text{int.}} = \Delta\text{K.E}$$

where ΣW_{ext} = Work done by the external forces on the system and ΣW_{int} = Work done by the internal forces on the system.

Work Done by a Spring

When a spring is extended or compressed, the restoring force of the spring does some work.

Let l_0 be the natural length of a spring and k be the spring constant. When the length of the spring is changed from $l_0 + x_1$ to $l_0 + x_2$, the work done by spring force is given by

$$W = -\int_{x_1}^{x_2} kx dx = \frac{-1}{2}k(x_2^2 - x_1^2)$$

when $x_2 > x_1$, $W < 0$

when $x_2 < x_1$, $W > 0$

POTENTIAL ENERGY

The potential energy of a system of particles is the work the system of bodies can do by virtue of the relative position of its parts, that is, by virtue of its configuration.

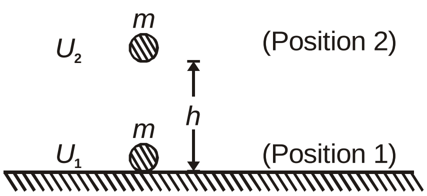
In mechanics, two types of the potential energy are of particular importance : Gravitational Potential Energy and Elastic Potential Energy.

1. **Gravitational Potential Energy (GPE)**

An object of mass *m* at a height of *h* above the earth's surface is said to have a potential energy given by the relation

$U_2 - U_2 = mgh$

where U_1 and U_2 are the respective potential energies at position 1 and 2.

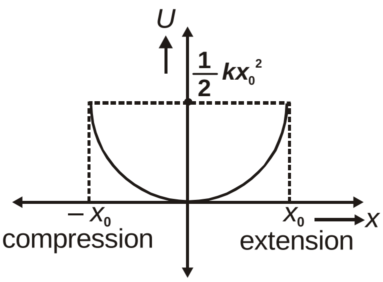


2. **Elastic Potential Energy of a Spring**

For a spring that obeys Hooke's law, $F = -kx$, the elastic potential energy is given by

$U = \frac{1}{2}kx^2$,

where *k* is the force constant of the spring and *x* is either its compression on extension, measured relative to its natural length.



CONSERVATIVE AND NON-CONSERVATIVE FORCES

Conservative Forces

The work done by a conservative force in displacing a particle from one point to another depends only on the position of the two given points and is quite independent of the actual path taken.

e.g. : Gravitational forces, electrostatic forces, etc.

The following table gives the expression for the potential energy function associated with four conservative forces

Force	Potential Energy Function
Spring force, kx	$(1/2)kx^2$
Weight (Gravitational force), mg	Mgh
Gravitational force between two particles, $\frac{-Gm_1m_2}{r^2}$ (Negative sign shows attractive force)	$\frac{-Gm_1m_2}{r}$
Electrostatic force between two points charges, $K\frac{q_1q_2}{r^2}$	$K\frac{q_1q_2}{r}$

Potential Energy Function $U(x)$ and Associated conservative force :

If we know the potential energy function $U(x)$ for a system in which a one dimensional force $F(x)$ acts, we can find the force as

$F(x) = -\frac{dU(x)}{dx}$, for one dimension $\left(\text{In three dimension } \vec{F} = \frac{-\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}\right)$

The expression says,

(i) The conservative force acts in the direction of decreasing potential energy.

(ii) The force is equal to the rate of decrease of Potential Energy.

Equilibrium Points

At equilibrium positions, the force is to be zero.

$\Rightarrow \quad -\frac{dU}{dx} = 0$ (for equilibrium)

There are three types of equilibrium

1. **Stable Equilibrium**

The equilibrium is stable at a point if the potential energy at that point is minimum. For minimum potential energy at equilibrium point x_0

$$\left(\frac{d^2U}{dx^2}\right)_{x=x_0} > 0$$

2. **Unstable Equilibrium**

At unstable equilibrium position, potential energy is maximum.

3. **Neutral Equilibrium**

If potential energy is constant with respect to distance, the equilibrium is said to be neutral.

Non-Conservative Forces

Work done by the conservative forces depends upon the actual path taken. Also the forces which are velocity dependent *i.e.* the force which depend upon the magnitude and direction of velocity like frictional force and viscous forces are in general **non-conservative forces**.

PRINCIPLE OF CONSERVATION OF MECHANICAL ENERGY

When only conservative forces are present in a system, the sum of potential energy and kinetic energy for a particle of a system of particles remains constant at all times during the motion of the particle.

Mathematically

P.E. + K.E. = constant

or $\quad$ $P.E_i + K.E_i = P.E_f + K.E_f$

POWER

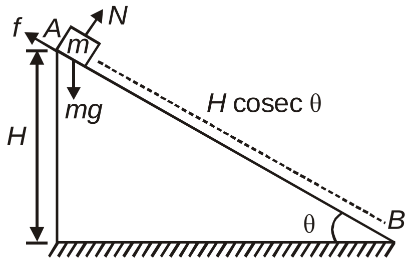
Time rate of doing work or time rate of dissipation of energy is called power.

The unit of power is watt defined as 1 watt (W) = 1 J/s

$P = \vec{F}.\vec{V} =$ It is the power generated because of force $\vec{F}$, where $\vec{V}$ is the velocity of point of application of force in the frame considered.

Applications

If the angle of an inclined plane is greater than angle of repose, the block reaches the bottom of the inclined plane starting from rest, with a speed v calculated as shown



$W_{mg} = mgH \qquad W_N = \text{zero}$

$W_{fr} = -(\mu mg\cos\theta)(H\,\text{cosec}\,\theta)$

$\Rightarrow \quad W_{Total} = mgH - \mu mgH\cot\theta$

$$\frac{1}{2}mv^2 = \mu mgH - mgH\cot\theta \Rightarrow v = \sqrt{2gH(1-\mu\cot\theta)}$$

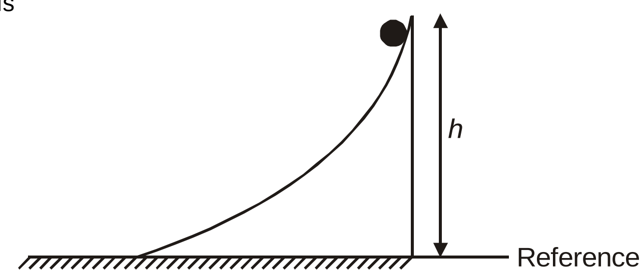
(1) A particle of mass m is dropped with zero initial velocity from the top of a frictionless curved surface.

Total mechanical energy initially at height h is

$M.E_i = mgh + 0$ ...(i)

Total mechanical energy finally is

$M.E_f = 0 + \frac{1}{2}mv_f^2$ ...(ii)



Equating equation (i) and (ii) we get

$$\frac{1}{2}mv_f^2 = mgh$$

$$v_f = \sqrt{2gh}$$

(2) A car and bus of mass M_1 and M_2 respectively are mocing on a straight road. They are stopped by application of same retarding force such that there respective stopping distances are x_1 and x_2. Consider the following situations.

(a) They are moving with same velocity,

$$\frac{1}{2}Mv^2 = F.x \quad \Rightarrow \quad x = \frac{M}{2F}V^2 \quad \Rightarrow x \propto M$$

or $\frac{x_1}{x_2} = \frac{M_1}{M_2}$

(b) They are moving with same kinetic energy

$$\frac{1}{2}Mv^2 = F.x$$

$\Rightarrow \quad x_1 = x_2$

(c) They are moving with same momentum

$$\frac{p^2}{2M} = F.x \quad (\because k = \frac{1}{2}Mv^2 = \frac{p^2}{2M})$$

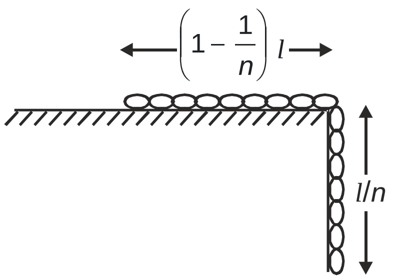
$$\Rightarrow \quad \frac{x_1}{x_2} = \frac{M_2}{M_1}$$

(3) A body moving with speed v is stopped in time 't' by delivering constant braking power.

$$P.t = \frac{1}{2}mv^2$$

$t \propto v^2$

(4) Let $\frac{1}{n}$ the part of the chain is hanging at the edge of table. Taking the horizontal surface of table as zero potential energy position,



(a) Potential energy of the chain is $\frac{-mgl}{2n^2}$

(b) Work done against gravity to pull the hanging part on the table = $\frac{mgl}{2n^2}$

(5) Two springs have spring constants k_1 and k_2

(a) They are pulled by same force,

$$F = k_1x_1 = k_2x_2$$

$$U_1 = \frac{1}{2}k_1x_1^2 = \frac{1}{2}Fx_1, \; U_2 = \frac{1}{2}Fx_2$$

$$\Rightarrow \quad \frac{U_1}{U_2} = \frac{x_1}{x_2} = \frac{k_2}{k_1}$$

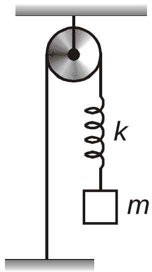
(b) They are pulled to same extension 'x',

$$F_1 = k_1 x \quad F_2 = k_2 x \qquad \Rightarrow \frac{F_1}{F_2} = \frac{k_1}{k_2}$$

$$U_1 = \frac{1}{2}k_1 x^2 \quad U_2 = \frac{1}{2}k_2 x^2$$

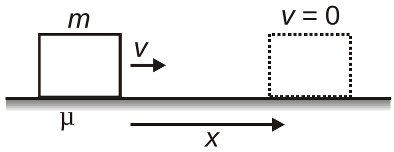
$$\Rightarrow \quad \frac{U_1}{U_2} = \frac{k_1}{k_2} = \frac{F_1}{F_2}$$

(6) A block of mass m is connected with a spring of force constant k as shown in figure. If the block is released from rest with spring relaxed, then the maximum extension in the spring is given by



$$x = \frac{2mg}{k}$$

(7) A body starts with speed v on a rough surface. The distance moved before it stops is given by

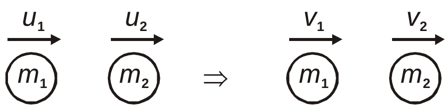


$$x = \frac{v^2}{2\mu g} \left[\text{using } \frac{1}{2}mv^2 = (\mu mg)x\right]$$

***Note :** The stopping distance does not depend on the mass of body.*

COLLISION

One dimension



($u_1 > u_2$ and $v_2 > v_1$)

Following two results should be used to deal with collision problems.

(a) Conservation of linear momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \qquad \ldots(1)$$

(b) Coefficient of restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

During the collision, there may be a loss of kinetic energy given by

$$\Delta \text{KE} = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(u_1 - u_2)^2 (1 - e^2)$$

Following are the important cases

1. Elastic collision $e = 1$

$$\Rightarrow \quad v_2 - v_1 = u_1 - u_2 \qquad \ldots(2)$$

From (1) and (2)

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2 u_2}{m_1 + m_2}$$

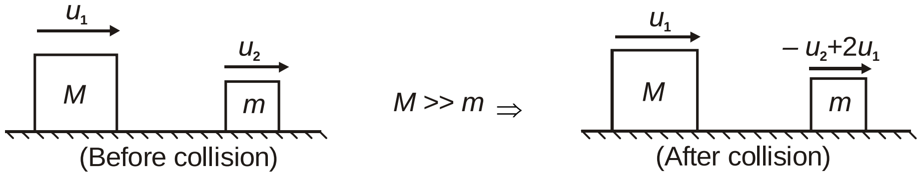
$$v_2 = \frac{m_2 - m_1}{m_2 + m_1}u_2 + \frac{2m_1 u_1}{m_2 + m_1}$$

ΔKE = 0 $\Rightarrow$ Final KE = Initial KE

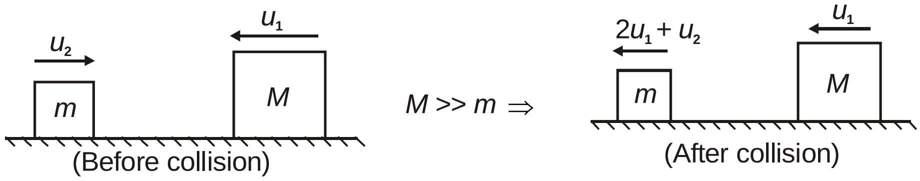
Following are the commonly asked cases for an elastic collisions.

(a) In an elastic collision, kinetic energy during the collision is not conserved. It is converted into elastic potential energy. But KE before collision = KE after collision.

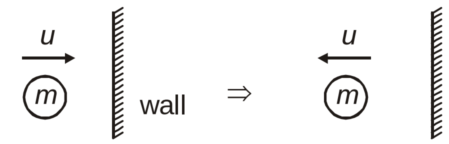
(b)



(c)

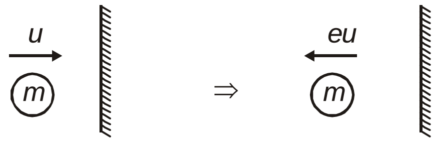


(d)



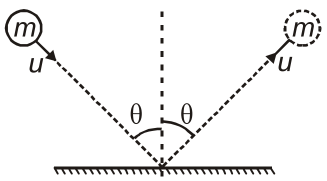
elastic collision

(e)



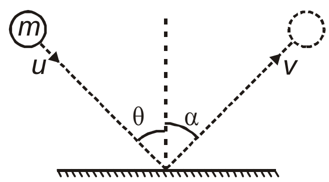
e = coefficient of restitution

(f)



Elastic collision

2. Coefficient of restitution = *e* (≠ 1)



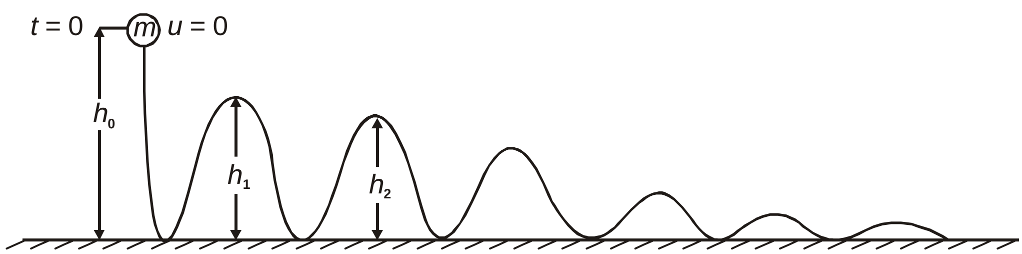
$u \sin\theta = v \sin\alpha$ ...(1)

$eu \cos\theta = v \cos\alpha$ ...(2)

$\Rightarrow$ $v = u\sqrt{\sin^2\theta + e^2\cos^2\theta}$

$\tan\alpha = \dfrac{\tan\theta}{e}$

3. A ball of mass *m* is dropped from height h_0 on an inelastic floor.



Coefficient of restitution = e

(a) Height after n^{th} bounce $h_n = e^{2n}h_0$

(b) Speed of rebound after n^{th} bounce = $e^n\sqrt{2gh_0}$

(c) Total distance travelled before the body comes to rest

$$= h_0\left[\frac{1+e^2}{1-e^2}\right]$$

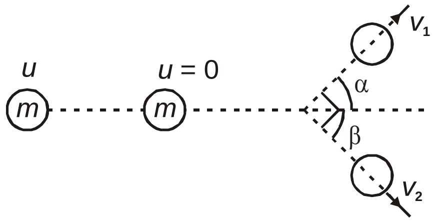
(d) The time after which the body comes to rest

$$= \sqrt{\frac{2h_0}{g}}\cdot\left[\frac{1+e}{1-e}\right]$$

(e) Average force exerted on the ground = mg

(f) Displacement of ball when it stops = h_0

4. Oblique elastic collision



(a) $\alpha + \beta = \frac{\pi}{2}$ ...(1)

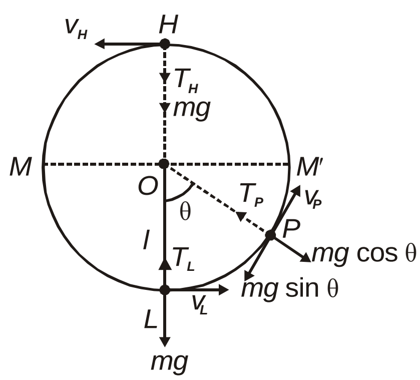
(b) $v_1\cos\alpha + v_2\cos\beta = u$ ...(2)

(c) $v_1\sin\alpha = v_2\sin\beta$ ...(3)

(d) $u^2 = v_1^2 + v_2^2$ ...(4)

MOTION IN A VERTICAL CIRCLE

A particle of mass m is tied to a string of length l whose other end is fixed. The particle can revolve about O in a verticle circle. when it is a position L (lowest point), it is given a speed V_L. Following results are useful in describing its motion.



1. $a_T = g\sin\theta$ ...(1)

2. $a_C = \frac{T_p - mg\cos\theta}{m} = \frac{v_p^2}{l}$

3. $T_p = \frac{mv_p^2}{l} + mg\cos\theta$ ...(2)

4. $T_L = mg + \frac{mv_L^2}{l}$

5. $T_H = -mg + \frac{mv_H^2}{l}$

6. $T_L - T_H = 6\,mg$ (always)

7. When $v_L \ge \sqrt{5gl}$, it completes vertical circle (Also $v_H > \sqrt{gl}$)

8. $v_L \le \sqrt{2gl}$, it oscillates between M and M'

9. $\sqrt{2gl} < v_L < \sqrt{5gl}$, it will leave the circular path somewhere between M and H.

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STUDY PACKAGE – 01

For – JEE / NEET

Rotational Motion

Chapter - 6

Rotational Motion and Moment of Inertia

6
CHAPTER

AIEEE Syllabus

Centre of mass of a two-particle system, Centre of mass of a rigid body; Basic concepts of rotational motion; moment of a force, torque, angular momentum, conservation of angular momentum and its applications; moment of inertia, radius of gyration, Values of moments of inertia for simple geometrical objects, parallel and perpendicular axes theorems and their applications. Rigid body rotation, equations of rotational motion.

THIS CHAPTER COVERS :

- Centre of mass
- Centre of mass of a rigid body
- Velocity and acceleration of centre of mass
- Rotational kinetic energy and moment of inertia
- Theorems for moment of inertia
- Moment of inertia of different objects
- Rolling of a body
- Angular momentum and its conservation

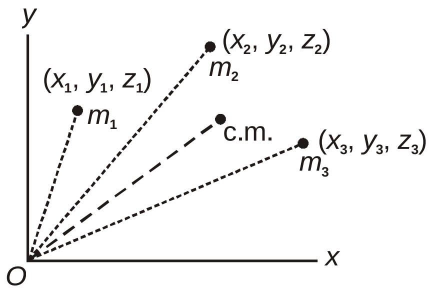
CENTRE OF MASS

It is point in a system which moves as if whole mass of the system is concentrated at the point and all external forces are acting on it. Its position is given by

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + ... + m_nx_n}{m_1 + m_2 + ... + m_n} = \frac{\sum_{i=1}^{n} m_i x_i}{M}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + ... + m_ny_n}{m_1 + m_2 + ... + m_n} = \frac{\sum_{i=1}^{n} m_i y_i}{M}$$

$$z_{cm} = \frac{m_1z_1 + m_2z_2 + ... + m_nz_n}{m_1 + m_2 + ... + m_n} = \frac{\sum_{i=1}^{n} m_i z_i}{M}$$



CENTRE OF MASS OF A RIGID BODY

Mathematically position coordinates of the centre of mass of rigid body are given by

$$x_{cm} = \frac{\int x\,dm}{\int dm}\ ;\ y_{cm} = \frac{\int y\,dm}{\int dm}\ ;\ z_{cm} = \frac{\int z\,dm}{\int dm}$$

VELOCITY AND ACCELERATION OF CENTRE OF MASS

Velocity of Centre of Mass

The instantaneous velocity of centre of mass is given by

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + ...m_n\vec{v}_n}{M}; \quad \text{or} \quad \vec{v}_{cm} = \frac{\vec{P}_{system}}{M}$$

Where $\vec{P}_{system}$ is the total linear momentum of centre of mass.

Acceleration of Centre of Mass

Differentiating $\vec{v}_{cm}$ w.r.t. time we get $\vec{a}_{cm}$ as

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ...m_n\vec{a}_n}{M}; \quad \text{or} \quad \vec{a}_{cm} = \frac{\Sigma\vec{F}_{ext}}{M}$$

Where $\Sigma\vec{F}_{ext}$ is the vector sum of forces acting on the particles of system.

Some Important Points :

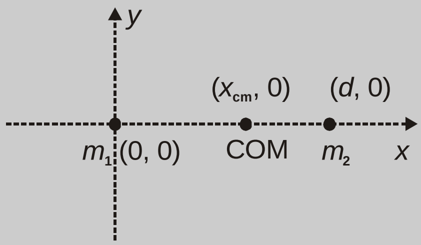
(1) The centre of mass of a system of particles lies in the region where majority of mass of the system lies.

(2) The centre of mass of a body may lie outside the body or on the body. *e.g.,* The centre of mass of a ring lies at its centre not on its body.

(3) For a *symmetrical* body of uniform density, the centre of mass lies at its geometrical centre.

(4) For a two particle system, the centre of mass lies on the line joining them and closer to the greater mass. The position co-ordinates of the centre of mass of a system of two masses as shown in the figure.

$$x_{cm} = \frac{m_1(0) + m_2 d}{m_1 + m_2} = \frac{m_2 d}{m_1 + m_2}$$



(5) The centre of mass is a point such that the mass of the system M multiplied by the acceleration (d^2R_{cm}/dt^2) of the centre of mass of the system gives the resultant of all forces acting on the system.

(6) The centre of mass of a system of two particles lies in between them on the line joining the particles.

(7) If we take the centre of mass as the origin, then the sum of the moments of the masses of the system $\Sigma\, m_i\, \vec{r}_i$ is zero.

(8) During translatory motion, the position of the centre of mass changes with time.

(9) In the absence of an external force, the velocity of the centre of mass of a body remains constant.

(10) The location of the centre of mass depends on the shape and nature of distribution of mass of the body.

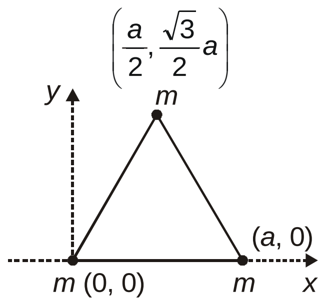
Applications

1. The position co-ordinates of the centre of mass of a system of three particles of equal masses which are placed on the vertices of an equilateral triangle of side *a* are calculated as shown.

$$x_{cm} = \frac{m(0) + m(a) + m(a/2)}{3m}$$

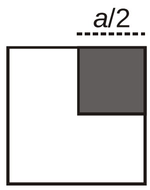
$$x_{cm} = \frac{3m\,a/2}{3m} = \frac{a}{2}$$

$$y_{cm} = \frac{m(0) + m(0) + m\left(\frac{\sqrt{3}}{2}a\right)}{3m} = \frac{a}{2\sqrt{3}}$$

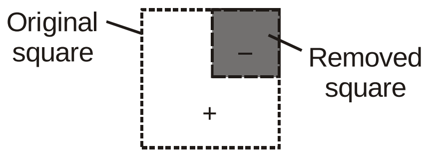


Co-ordinates of the centre of mass of the system are $\left(\frac{a}{2}, \frac{a}{2\sqrt{3}}\right)$.

2. A small square is removed from a uniform square plate of side *a*, as shown in the figure. The position centre of mass of remaining plate relative to the original centre. Can be calculated as shown



The remaining body can be assumed to be a superposition of two masses, one mass is the original mass and the other is the *negative mass* of the body which has been removed.



Here, the area of removed square will be taken *negative* and that of original square *positive.*

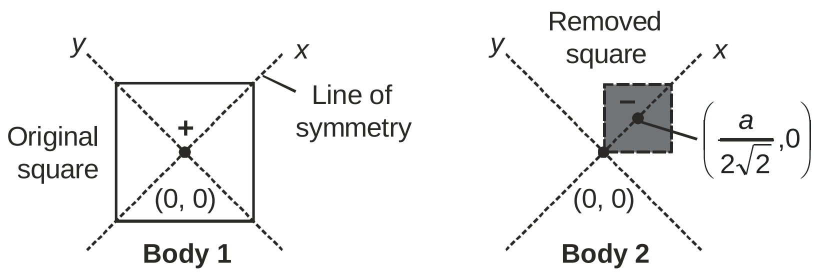
By taking the mass (or area) of removed square as negative, the coordinates of centre of mass of the remaining portion are

$$x_{cm} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} \quad \text{and} \quad y_{cm} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$

Here, $A_1 = a^2$ = area of first body (original square)

$A_2 = -a^2/4$ = area of second body (removed portion)

The remaining body has a line of symmetry, so its centre of mass must lie on this line. Choose the line of symmetry as *x*-axis with origin at the centre of original square. Now, the *x* coordinates of the two bodies are as shown in figure.



$$x_{cm} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{(a^2)\times(0) + \left(-\frac{a^2}{4}\right)\times\left(\frac{a}{2\sqrt{2}}\right)}{(a^2) + \left(-\frac{a^2}{4}\right)}$$

$\Rightarrow \quad x_{cm} = -\frac{a}{6\sqrt{2}}$; the centre of mass of remaining body lies on negative *x*-axis at a distance of $\frac{a}{6\sqrt{2}}$ from origin.

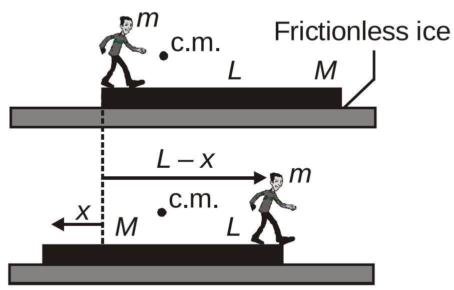
3. A man (of mass *m*) stands at the left end of a uniform sled of length *L* and mass *M*, which lies on frictionless ice. As the man then walks to the other end of the sled, the sled slides on the ice by a distance *x* calculated as shown.

   The net external force on the system (man + sled) is zero. The centre of mass of the system is to be at the same position while their movements. Obviously, the sled will shift towards left.

   Let the displacement of sled relative to ground = *x*

   The displacement of man relative to ground = $L - x$

   As the centre of mass remains stationary,



$$-\frac{Mx + m(L - x)}{M + m} = 0$$

$\Rightarrow \quad Mx = m(L - x)$

$\Rightarrow \quad x = \frac{mL}{m + M}$

SOME BASIC CONCEPTS OF ROTATIONAL MOTION

Rotation of a Rigid Body

When a rigid body rotates about a fixed axis, following parameters are needed to describe its motion :

1. **Angular velocity ($\vec{\omega}$) :** It is defined by the relation $\omega = \frac{d\theta}{dt}$. All the particles in the rigid body move in circular paths about axis of roation with angular velocity ω. Its direction is along the axis of rotation.

2. **Angular acceleration ($\vec{\alpha}$) :** It is the rate of change of angular velocity. It is given as $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$.

3. **Tangential acceleration (a_t) :** It is the rate of change of speed of a point on the rigid body. It is related to angular acceleration α as $\alpha_t = r\alpha$.
   Here, *r* is the distance of particle or point from axis of roation.

4. **Moment of inertia (*I*) :** It is the opposition offered by a rigid body to its state of rest or of uniform rotational motion.

5. **Angular momentum ($\vec{L}$) :** It is defined by the mathematical relation $\vec{L} = I\vec{\omega}$.

6. **Torque ($\vec{\tau}$) :** It is the action of a force which produces a change in its state of rest or of uniform rotational motion. By Newton's 2nd law,

$$\vec{\tau} = \frac{d\vec{L}}{dt} \text{ or } \vec{\tau} = I\vec{\alpha}$$

   If a force $\vec{F}$ acts on the rigid body at a point whose position vector with respect to axis of rotation is $\vec{r}$, the torque is given by $\vec{\tau} = \vec{r}\times\vec{F}$ or $\vec{\tau} = \vec{F}\times\perp$ distance from axis of rotation.

For uniform angular acceleration

(i) $\omega = \omega_0 + \alpha t$

(ii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

(iii) $\omega^2 = \omega_0^2 + 2\alpha\theta$

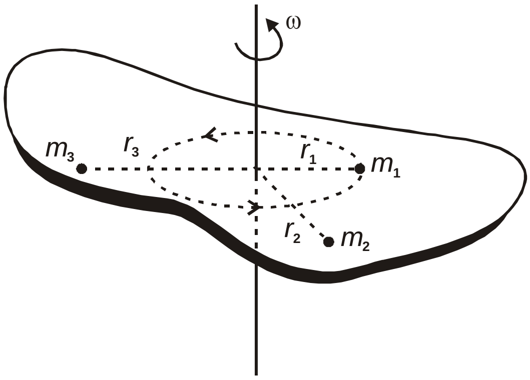
(iv) $\theta = \left(\frac{\omega + \omega_0}{2}\right)t$

ω = Angular velocity after t time

ω_0 = Initial angular velocity

ROTATIONAL KINETIC ENERGY

Consider a rigid body rotating about an axis with angular velocity 'ω'. Various particles of the body are all rotating on a circle with radius r_1, r_2 .... with angular velocity 'ω'. The total kinetic energy



K.E. $= \frac{1}{2}I\omega^2$

$I = m_1r_1^2 + m_2r_2^2 + ...$ is called moment of inertia and $\frac{1}{2}I\omega^2$ = kinetic energy of rotation.

MOMENT OF INERTIA

A body free to rotate about an axis opposes any change in its state of rest or of uniform rotation, this inherent property of a body is called moment of inertia.

Moment of inertia, in rotation motion, is the analogue of mass in linear or translational motion.

For System of Particles :

$$I = \sum_{i=1}^{N} m_i r_i^2$$

m_i = mass of i^{th} particle

r_i = radius of the i^{th} particle

For Rigid Bodies :

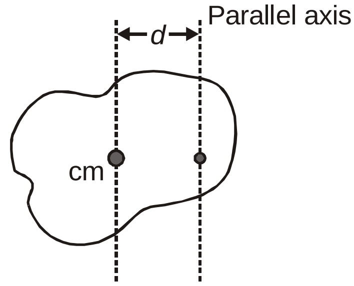
Moment of inertia of a rigid body about any axis of rotation.

Mathematically $I = \int dm\, r^2$

THEOREMS FOR MOMENT OF INERTIA

Parallel Axis Theorem

It states that the moment of inertia of a body about an axis is equal to its moment of inertia about a parallel axis through its centre of mass plus the product of the mass of the body and the square of perpendicular distance between the two axes.



$I = I_{cm} + md^2$

I_{cm} = Moment of inertia of the body about its centre of mass

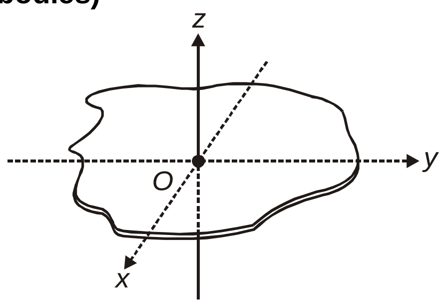
I = Moment of inertia of the body about a parallel axis

m = Total mass of the body

d = Perpendicular distance between two parallel axes.

Perpendicular Axis Theorem (Applicable only to plane-laminar bodies)

The moment of inertia of a plane lamina about an axis perpendicular to the plane of the lamina is equal to the sum of the moment of inertia of the lamina about two axes perpendicular to each other, in its own plane, and intersecting each other at the point where the perpendicular axis passes through it.



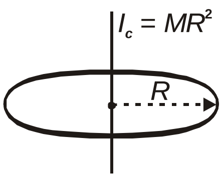
$I_z = I_x + I_y$

Where I_x, I_y and I_z are the respective moment of inertia of the body about x, y and z-axes.

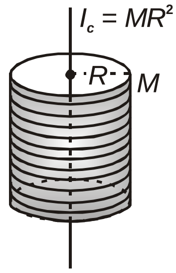
***Note :** The point O need not be the centre of mass of the body.*

MOMENT OF INERTIA OF DIFFERENT OBJECTS

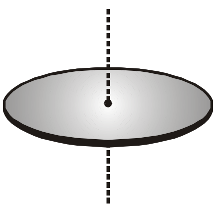
For an axis perpendicular to the plane of the ring



A hollow cylinder

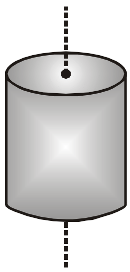


The axis perpendicular to the plane of the disc.



A disc

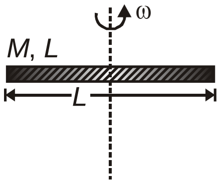
$$I_{cm} = \frac{MR^2}{2}$$



A solid cylinder

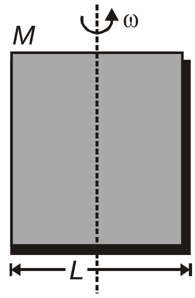
$$I_{cm} = \frac{MR^2}{2}$$

A thin rod



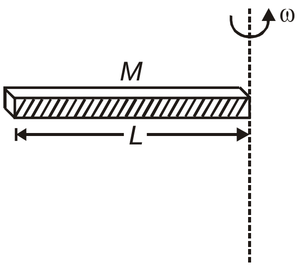
$$I_c = \frac{ML^2}{12}$$

A rectangular plate



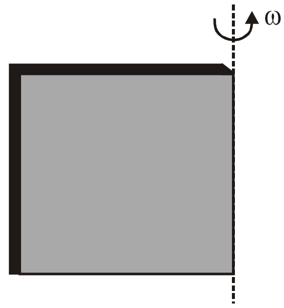
$$I_c = \frac{ML^2}{12}$$

A thin rod about a perpendicular axis through its end



$$I = \frac{ML^2}{3}$$

A rectangular plate about one edge



$$I = \frac{ML^2}{3}$$

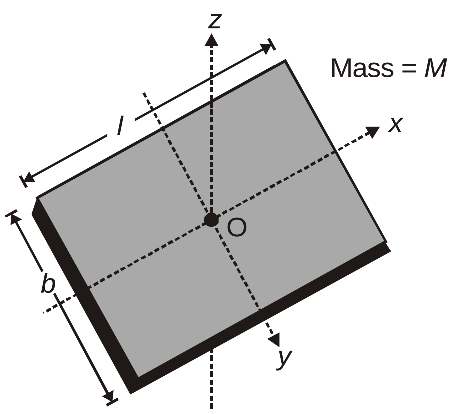
A Rectangular Plate

(a) $I_x = \dfrac{Mb^2}{12}$

(b) $I_y = \dfrac{Ml^2}{12}$

(c) $I_z = I_x + I_y$

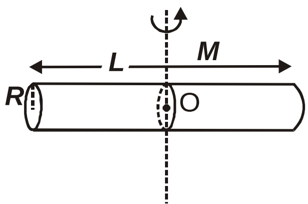
(d) $I_z = \dfrac{M(l^2+b^2)}{12}$



A Thick Rod

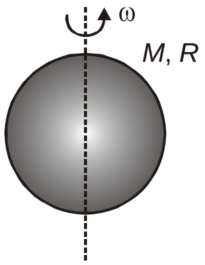
The axis is perpendicular to the rod and passing through the centre of mass

$$I_{cm} = \frac{ML^2}{12} + \frac{MR^2}{4}$$



A Solid Sphere

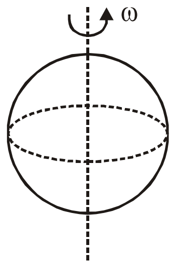
About its diameter



$I_{cm} = \frac{2}{5}MR^2$

A Hollow Sphere

About its diameter



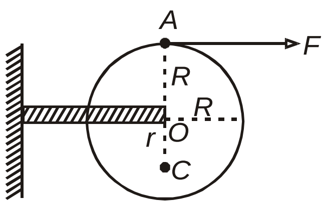
$I_{cm} = \frac{2}{3}MR^2$

RIGID BODY ROTATION

In this section, rotation of a body about a stationary fixed axis has been discussed

1. Rotating Disc

A tangential force F is applied at the periphery



$\tau = F \times R$ [about O]

$I = \frac{1}{2}mR^2$

$\alpha = \frac{\tau}{I} = \frac{2F}{MR}$

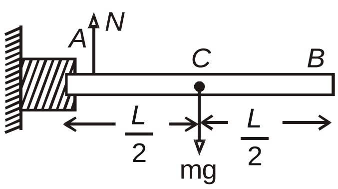
(1) Linear acceleration of A is $a_A = R\alpha = \frac{2F}{M}$ (along horizontal)

(2) Linear acceleration of B is $a_B = R\alpha = \frac{2F}{M}$ (vertically downwards)

(3) Linear acceleration of C is $a_C = r\alpha = \frac{2Fr}{MR}$ (along horizontally opposite to A)

2. Hinged Rod

The rod is released from rest from horizontal position



$\tau = mg \times \frac{L}{2}$ (about A)

$I = \frac{ML^2}{3}$

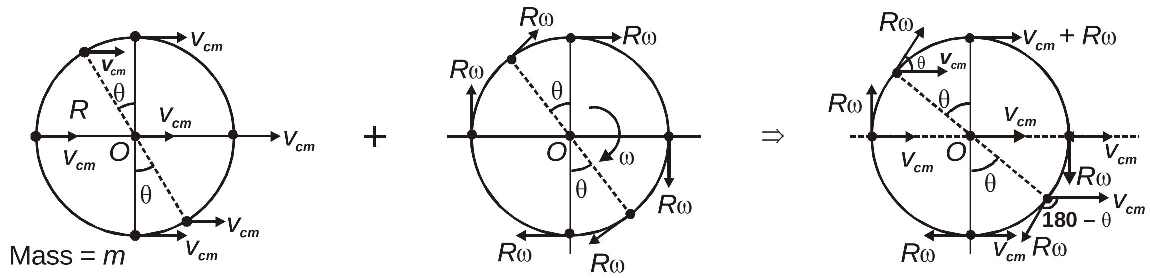
$\alpha = \frac{\tau}{I} = \frac{3g}{2L}$

(1) Linear acceleration of COM C is $a_{cm} = \frac{L}{2}\alpha = \frac{3g}{2L}$

(2) Linear acceleration of point B is $a_B = L\alpha = \frac{3g}{2}$

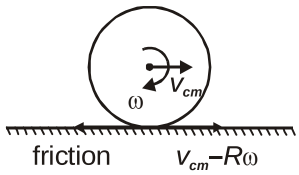
ROLLING OF A BODY

Rolling is combination of Translation and Rotation



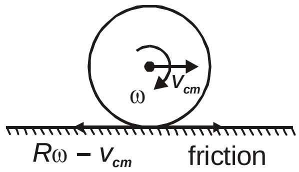
Case - I : Forward slipping

$v_{cm} > R\omega$



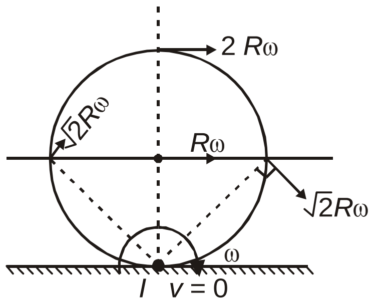
Case - II : Backward slipping

$v_{cm} < R\omega$

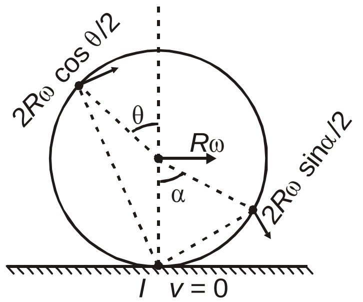


Case - III : **Pure Rolling**

$v_{cm} = R\omega$



(instantaneous centre of rotation)



(instantaneous centre of rotation)

Kinetic energy of the body during pure rolling (*E*)

E = Translational KE + Rotational KE

$= E_T + E_R$

$$= \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

$$= \frac{1}{2}\left(m + \frac{I_{cm}}{R^2}\right)v_{cm}^2 = \frac{1}{2}mv_{cm}^2\left(1 + \frac{K^2}{R^2}\right)$$ (where k is radius of gyration)

$$E = E_T\left(1+\frac{K^2}{R^2}\right)$$

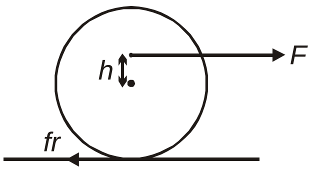
Similarly, $E = E_R\left(1+\frac{R^2}{K^2}\right)$

Type of body	K	Fraction of total energy translational $X = \left(1+\frac{K^2}{R^2}\right)^{-1}$	Fraction of total energy rotational $Y = \left(1+\frac{R^2}{K^2}\right)^{-1}$	$\frac{Y}{X}$
1. Ring or hollow cylinder	R	$\frac{1}{2} = 0.5 = 50\%$	$\frac{1}{2} = 0.5 = 50\%$	$1:1$
2. Spherical Shell	$\sqrt{\frac{2}{3}}R$	$\frac{3}{5} = 0.6 = 60\%$	$\frac{2}{5} = 0.4 = 40\%$	$2:3$
3. Disc or solid cylinder	$\frac{R}{\sqrt{2}}$	$\frac{2}{3} = 0.666 = 66.67\%$	$\frac{1}{3} = 0.333 = 33.33\%$	$1:2$
4. Solid sphere	$\sqrt{\frac{2}{5}}R$	$\frac{5}{7} = 0.714 = 71.4\%$	$\frac{2}{7} = 0.286 = 28.6\%$	$2:5$

Note : *Above values X and Y are independent of mass and radius of the body. They only depends on the type of body.*

Applications

1. A force is applied at the distance h from centre of mass as shown in figure



$$a = \frac{F\left(1+\frac{h}{R}\right)}{M\left(1+\frac{K^2}{R^2}\right)}$$

$fr = \frac{F(K^2 - hR)}{K^2 + R^2} \le \mu N$ (must be less than μmg for Rolling)

If $h < \frac{K^2}{R}$ friction is backward

$h > \frac{K^2}{R}$ friction become forward

2. If force is applied at centre of mass then (h = 0)

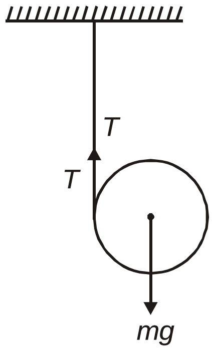
So, $a = \frac{F}{M\left(1+\frac{K^2}{R^2}\right)}$ and $fr = \frac{FK^2}{K^2+R^2} = \frac{F}{\left(1+\frac{R^2}{K^2}\right)}$

3. If force is applied at highest point ($h = R$)

$$a = \frac{2F}{M\left(1+\frac{K^2}{R^2}\right)}$$

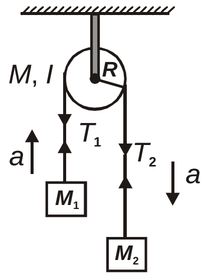
$$fr = \left(\frac{R^2 - K^2}{K^2 + R^2}\right)F \text{ forward direction}$$

4. This case is equivalent to case when object is pulled from CM an Rough surface so



$$a = \frac{mg}{m + \frac{I}{R^2}} \text{ and } T = \frac{mg}{1 + \frac{R^2}{K^2}}$$

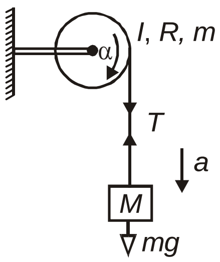
5. $a = \left(\dfrac{M_2 - M_1}{M_2 + M_1 + \frac{I}{R^2}}\right)g$



$T_2 > T_1$

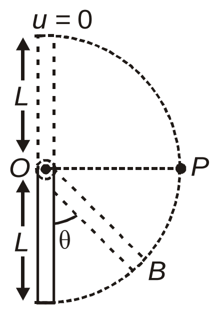
$M_2 > M_1$

6. $a = \dfrac{Mg}{M + \frac{I}{R^2}}, \alpha = \dfrac{a}{R}$



$I = mK^2$

7. The rod is released from unstable equilibrium position



(a) When at B, $Mg\dfrac{L}{2}(1+\cos\theta) = \dfrac{1}{2}\left(\dfrac{ML^2}{3}\right)\omega^2$

$$\omega = \sqrt{\frac{6g}{L}}\cos\frac{\theta}{2}$$

(b) at C, $\theta = 0°$

$$\omega = \sqrt{\frac{6g}{l}}$$

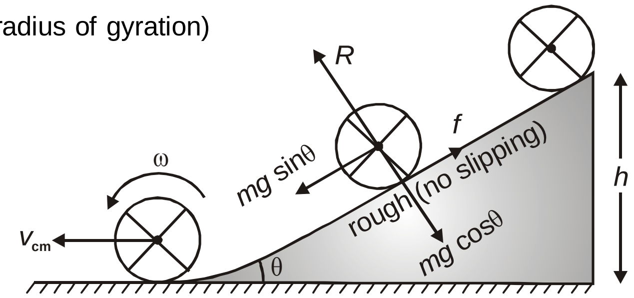
(c) at P, $\theta = 90°$, $\omega = \sqrt{\dfrac{3g}{l}}$

Rolling of a Body on an Inclined Plane

By conservation of energy

$$\underset{\text{(Total energy)}}{mgh} = \underset{\text{(Rotatory)}}{\frac{1}{2}I\omega^2} + \underset{\text{(Translatory)}}{\frac{1}{2}mv_{cm}^2}$$

Let $\beta = 1 + \frac{I}{MR^2} = 1 + \frac{K^2}{R^2}$ (where k is radius of gyration)



1. $a_{cm} = \frac{mg\sin\theta}{M + \frac{I}{R^2}} = \frac{g\sin\theta}{1 + \frac{K^2}{R^2}}$

2. $v_{cm} = \sqrt{\frac{2gh}{\beta}} = \sqrt{2gh \Big/ 1 + \frac{K^2}{R^2}}$

3. $\text{Time} = \frac{1}{\sin\theta} \cdot \sqrt{\beta \frac{2h}{g}}$ $\quad\because$ Force of friction $f = \frac{mg\sin\theta}{1 + \frac{R^2}{K^2}}$

   i.e., $t \propto \sqrt{\beta}$

4. Force of friction $f = \frac{mg\sin\theta}{1 + \frac{R^2}{K^2}}$

5. Instantaneous power $P = (mg\sin\theta)v$

6. Maximum angle of inclination for pure rolling, $\theta_{max} = \tan^{-1}\left(\frac{\mu}{1 + \frac{R^2}{K^2}}\right)$

Ring	:	$\theta_{max} = \tan^{-1}(2\mu)$,
Spherical Shell	:	$\theta_{max} = \tan^{-1}(2.5\ \mu)$
Disc	:	$\theta_{max} = \tan^{-1}(3\mu)$
Solid sphere	:	$\theta_{max} = \tan^{-1}(3.5\ \mu)$.

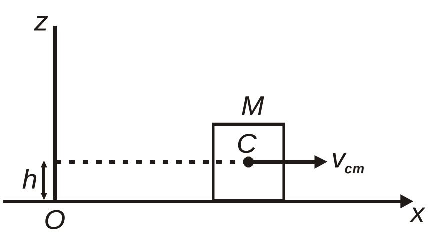
ANGULAR MOMENTUM AND ITS CONSERVATION

$L = Mv_{cm}R + I\omega$

Case - I : Pure translation

$|\vec{L}_O| = MV_{cm}h$

$L_C = 0$

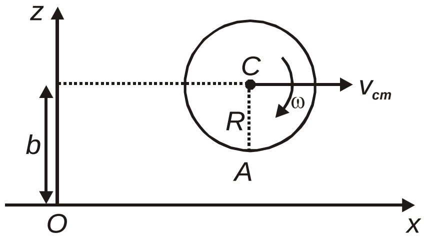


Case - II : Rolling Body

$L_c = I_c\omega$

$L_A = I_C\omega + Mv_{cm}R$

$L_O = I_c\omega + Mv_{cm}b$



Case - III : Pure rotation

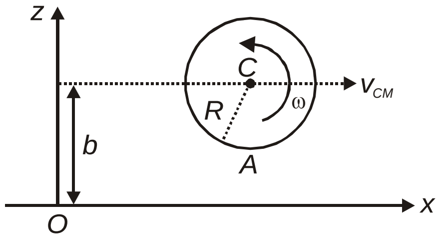
Put $v_{cm} = 0$ in the above results so $L_0 = L_A = L_C = I_C\omega$

Case - IV

$L_c = -\, I_c\omega$

$L_A = -\, I_C\omega + Mv_{cm}R$

$L_O = -\, I_c\omega + Mv_{cm}b$

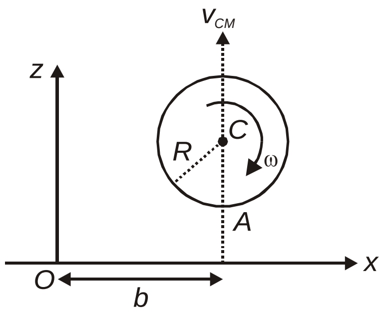


Case - V

$L_c = I_c\omega$

$L_A = I_C\omega$

$L_O = I_c\omega - Mv_{cm}b$



Conservation of Angular Momentum

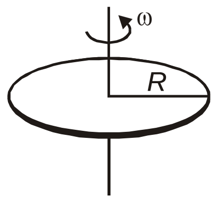
In an isolated system (no external torque) the angular momentum of the system is conserved.

$$\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$$

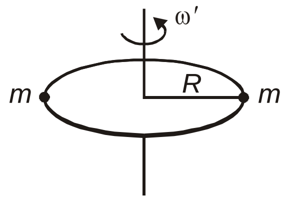
If $\vec{\tau}_{ext} = \vec{O} \Rightarrow \frac{d\vec{L}}{dt} = \vec{O} \Rightarrow \vec{L} = \text{constant}$

Applications

1. **Before** **After**



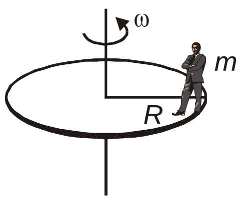
Disc
(moment of inertia I)
$L_1 = I\omega$



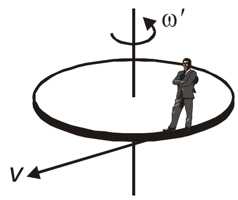
(Picks two particles at diametrically opposite points)
$L_2 = (I + 2mR^2)\omega'$

$$\Rightarrow \quad \omega' = \frac{I\omega}{I + 2mR^2}$$

2. **Before** **After**



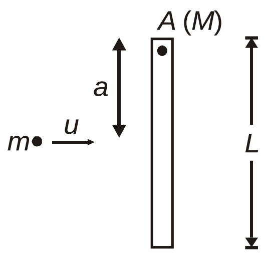
Moment of inertia of platform I
m = mass of man at rest
at edge w.r.t. platform



The man starts walking along the edge with speed v in opposite sense w.r.t. the platform then new angular velocity of the platform is

$$\omega' = \omega + \frac{mvR}{I + mR^2}$$

3. A rod is hinged at A and can rotate freely in a vertical plane. A particle of mass m moving with speed u hits and sticks to the rod.



Only conservation of angular momentum can be applied as the rod is hinged at A.

$$mua = \left[\frac{ML^2}{3} + ma^2\right]\omega$$

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Educational Revolution

PHYSICS

Aakash

STUDY PACKAGE – 01

For – JEE / NEET

Gravitation

Chapter - 7

Gravitation

7

CHAPTER

AIEEE Syllabus

The universal law of gravitation. Acceleration due to gravity and its variation with altitude and depth. Kepler's laws of planetary motion. Gravitational potential energy; gravitational potential. Escape velocity. Orbital velocity of a satellite. Geo-stationary satellites.

THIS CHAPTER COVERS :

- Universal law of gravitation
- Acceleration due to gravity
- Gravitational potential
- Gravitational Potential Energy
- Escape Velocity
- Kepler's laws
- Satellites
- Binary Star System

UNIVERSAL LAW OF GRAVITATION

Newton's Law of Gravitation

Gravitational force between two points masses or spherically symmetric m_1 and m_2 separated by distance r is $F = \frac{Gm_1m_2}{r^2}$.

G = Universal gravitational constant

= 6.67×10^{-11} Nm^{2}kg^{-2}, $[G] = [M^{-1}L^3T^{-2}]$

Some Important Points :

1. Applicable for point masses
2. Gravitational force is conservative in nature
3. Gravitational force is always attractive and central
4. Gravitational force describes the teresterial phenomena
5. Gravitational force between two bodies is due to exchange of a elementary particle known as graviton
6. It is a long range and weakest force

ACCELERATION DUE TO GRAVITY

Acceleration produced in a body due to earth's gravitational pull is called acceleration due to gravity.

As gravitational force on a body of mass m placed at the surface of earth is

$$F = \frac{GMm}{R^2}$$

$$g = \frac{F}{m} = \frac{GM}{R^2}$$

Relation between g and G :

$$g = \frac{GM}{R^2} = \frac{G \times \frac{4}{3}\pi R^3 \rho}{R^2} = \frac{4}{3}\pi G R \rho$$

$\rho \rightarrow$ density of earth = 5.5×10^3 kg/m^3

Variation in the value of *g* :

1. At height '*h*'

$$g' = \frac{g}{\left[1 + \frac{h}{R_e}\right]^2} \text{ and } g' = g\left[1 - \frac{2h}{R_e}\right] \text{ if } h << R_e$$

If $h = R_e$, $g' = \frac{g}{4}$

2. At depth '*x*'

$g' = g\left\{1 - \frac{x}{R_e}\right\}$, at the centre of earth g' = 0, weight = 0

> ***Note :*** *Mass is intrinsic property and it is not affected by variation of gravity.*

3. **Due to Rotation of Earth :**

$g' = g - R_e\omega^2\cos^2\lambda$

$\lambda \rightarrow$ angle of latitude

$g_{pole} \rightarrow$ maximum and unaffected from rotation.

$g_{equator} \rightarrow$ minimum and becomes maximum if earth stops rotation.

GRAVITATIONAL FIELD INTENSITY AND POTENTIAL (V) :

1. Gravitational field intensity at a point is the force experienced by a unit mass placed at that point

$$\vec{I} = \frac{-GM}{r^2}\hat{r}$$

2. Gravitational potential at a point is the amount of work done in bringing a unit mass from infinity to that point in the gravitational field.

$$V = -\frac{W}{m} \Rightarrow V = \frac{-GM}{r} \text{ (units J/kg)}$$

Variation of Intensity and Potential

1. For a spherical shell of mass *M* and radius *R*

Case-I : $r < R$ (internal point)

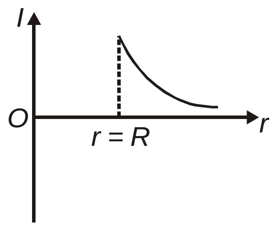
$$I_i = 0,\ V_i = -\frac{GM}{R}$$

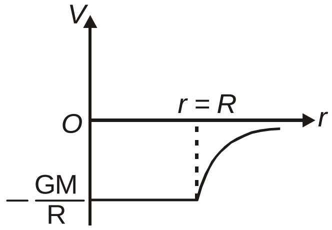
Case-II : $r < R$ (on the surface

$$I_s = -\frac{GM}{R^2}, V_s = \frac{-GM}{R}$$

Case-III : $r > R$ (outside the shell)

$$I_o = \frac{GM}{r^2}, V_o = \frac{-GM}{r}$$





2. For Uniform solid Sphere

Case-I : $r < R$ (internal point)

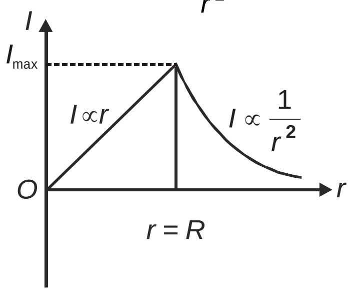
$$I_i = \frac{GMr}{R^3}, V_i = \frac{-GM}{2R^3}(3R^2 - r^2) \text{ At centre } V_c = -\frac{3GM}{2R} = \frac{3}{2}V_s$$

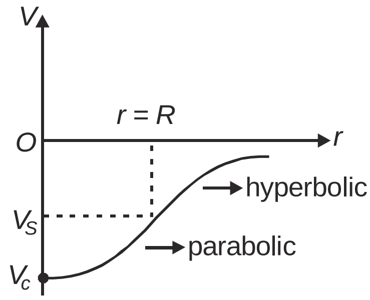
Case-II : $r = R$ (on the surface)

$$I_s = -\frac{GM}{R^2},\ V_s = \frac{-GM}{R}$$

Case-III : $r > R$ (outside the surface)

$$I_o = \frac{GM}{r^2},\ V_o = \frac{-GM}{r}$$

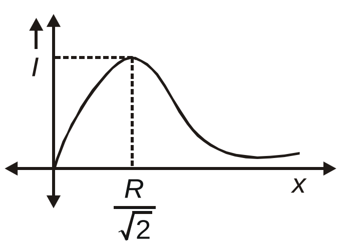


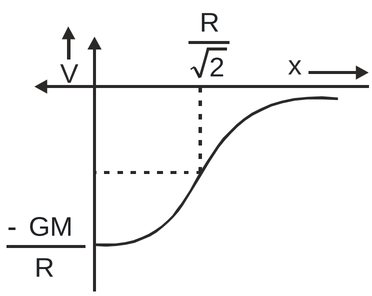


3. Gravitational intensity and potential on the axis of uniform ring of mass M radius R at distance x from centre.

$$I = \frac{GMx}{(R^2 + x^2)^{3/2}}, V = \frac{-GM}{\sqrt{R^2 + x^2}}$$

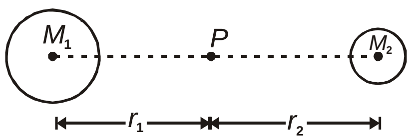
At centre $I = 0$ and I is maximum at $x = \pm\frac{R}{\sqrt{2}}$; $I_{max} = \frac{2GM}{3\sqrt{3}R^2}$





4. The point P at which gravitational field is zero between two heavenly bodies.

$$\boxed{\frac{M_1}{r_1^2} = \frac{M_2}{r_2^2}}$$



GRAVITATIONAL POTENTIAL ENERGY

Gravitational potential energy of a body at a point w.r.t earth is defined as negative of work done by gravity in bringing that body from infinity to the point of consideration.

At earth surface $U = -\frac{GM_e m}{R_e}$, At height h, $U = \frac{-GM_e m}{R_e + h}$

If a body of mass m is raised to a height h from the surface of earth, then gain of its potential energy is $\Delta U = \frac{mgRh}{R+h}$.

Energy required to escape = Escape energy = + $\frac{GM_e m}{R_e}$

ESCAPE VELOCITY

Escape velocity on the surface of planet is the minimum velocity required to escape the body from the gravitational field of planet.

To escape a body its mechanical energy should be greater than or equal to zero.

$$\frac{1}{2}mv_e^2 + \left(\frac{-GM_e m}{R_e}\right) = 0$$

$$\Rightarrow \quad v_e = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{\frac{8}{3}\pi G R_e{}^2 \rho} = \sqrt{2gR_e}$$

At earth surface v_e = 11.2 km/s

> ***Note :** Escape velocity is independent of mass of object and direction of projection.*

KEPLER'S LAWS

(1) All planets revolve around the Sun in elliptical orbit having the Sun at one focus.

If e = eccentricity of ellipse then distance of the planet from the Sun at perigee.

$r_p = (1 - e)a$

and distance of the planet from the Sun at apogee

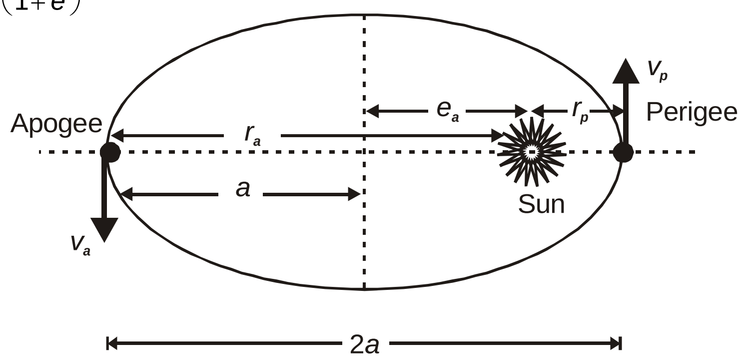
$r_a = (1 + e)a$ (a = semi major axis)

Ratio of orbital speeds at apogee and perigee are

$$\frac{v_a}{v_p} = \frac{r_p}{r_a} = \frac{1-e}{1+e}$$

Ratio of angular velocities at apogee and perigee are

$$\frac{w_a}{w_p} = \left(\frac{r_p}{r_a}\right)^2 = \left(\frac{1-e}{1+e}\right)^2$$



(2) A planet sweeps out equal area in equal time interval *i.e.* Areal speed of the planet is constant

$$\frac{dA}{dt} = \frac{1}{2}vr = \frac{L}{2m}$$ = constant ($L = mvr$ = angular momentum of planet about the Sun)

Angular momentum about the Sun 'L' of all planets and satellites is constant

(3) Square of time period is proportional to cube of semi-major axis of the elliptical orbit of the planet. *i.e.*, $T^2 \propto a^3$

SATELLITES

Important results regarding satellite motion in circular orbit.

1. Orbital Velocity (v_0) : Gravitational attraction of planet gives necessary centripetal force.

$$\frac{GMm}{r^2} = \frac{mv_0^2}{r}$$

$$\Rightarrow \quad v_0 = \sqrt{\frac{GM}{r}}$$

$$v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR_e^2}{R_e + h}}$$ (h = height above the surface of earth)

$$v_0 \approx \sqrt{gR_e} \approx \sqrt{\frac{GM}{R_e}}$$ (If $R_e >> h$)

Since $v_e = \sqrt{\frac{2GM}{R_e}}$

$$\Rightarrow \quad v_e = \sqrt{2}v_0$$

2. Time Period : The period of revolution of a satellite is

$$T = \frac{2\pi r}{v_0} = 2\pi r\sqrt{\frac{r}{GM}} = 2\pi\sqrt{\frac{r^3}{GM}}$$

For a satellite orbiting close to the earth's surface ($r \simeq R_e$), the time period is minimum and is given by

$$T_{\min} = 2\pi\sqrt{\frac{R_e^3}{GM}} = 2\pi\sqrt{\frac{R_e}{g}}$$

For earth R_e = 6400 km,

g = 9.8 m/s^2

$T_{\min}$ = 84.6 min.

Thus, for any satellite orbiting around the earth, its time period must be more than $2\pi\sqrt{\frac{R}{g}}$ or 84.6 minutes.

3. Potential Energy (*U*), Kinetic Energy (*K*) and Total Energy (*E*) of satellite

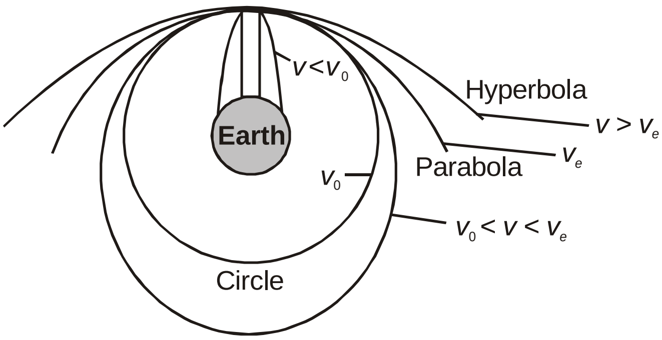
$$U = -\frac{GMm}{r}$$

$$K = \frac{GMm}{2r}$$

$$E = -\frac{GMm}{2r} \quad [K = -E \ \& \ U = 2E]$$

Bound and Unbound Trajectories

Imagine a very tall building on the surface of the earth from where a projectile is fired with a velocity v parallel to the surface of the earth. The trajectory of the projectile depends on its velocity.



(v_e = escape velocity)

(v_0 = orbital velocity, $v_e = \sqrt{2}v_0$)

Velocity	Trajectory
$v < v_0$	Projectile may not orbit the earth in an ellipical path, or it may falls back on the earth's surface.
$v = v_0$	Projectile orbits the earth in a circular path.
$v_0 < v < v_e$	Projectile orbits in an elliptical path.
$v = v_e$	Projectile does not orbit. It escapes the gravitational field of earth in a parabolic path.
$v > v_e$	Projectile does not orbit. It escapes the gravitational field of earth in a hyperbolic path.

Geostationary Satellites

(1) The plane of the orbit lies in equatorial plane of earth.

(2) Height from the earth surface is 36000 km. This orbit is called parking orbit.

(3) Orbital speed is nearly 3 km/s.

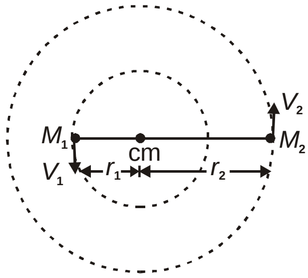
(4) Time period is equal to that of earth rotation *i.e.*, 24 hours.

Polar Satellite

Used for remote sensing, their orbit contains axis of rotation of earth. They can cover entire earth surface for viewing.

BINARY STAR SYSTEM

Two stars of mass M_1 and M_2 form a stable system when they move in circular orbit about their centre of mass under their mutual gravitational attraction.



Some Important Points :

(1) $F = \dfrac{GM_1M_2}{r^2}$, where r is distance between them (*i.e.*, $r = r_1 + r_2$)

(2) $M_1r_1 = M_2r_2$

(3) $\dfrac{GM_1M_2}{r^2} = \dfrac{M_1V_1^2}{r_1}$

(4) $\dfrac{GM_1M_2}{r^2} = \dfrac{M_2V_2^2}{r_2^2}$

(5) $r_1 = \dfrac{M_2r}{M_1+M_2}, r_2 = \dfrac{M_1r}{M_1+M_2}$

$\therefore \quad V_1 = M_2\sqrt{\dfrac{G}{(M_1+M_2)r}},$

$V_2 = M_1\sqrt{\dfrac{G}{(M_1+M_2)r}}$

when $M_1 = M_2$

$V_1 = V_2 = \sqrt{\dfrac{GM}{2r}}$

$r_1 = r_2 = \dfrac{r}{2}$

Applications

1. If a body is projected with velocity greater than escape velocity, the intersteller speed is given by

$$v_{I.S} = \sqrt{v_{\text{given}}^2 - v_e^2}$$

2. If a body is projected upwards with velocity $v = Kv_e$, where $K < 1$, maximum height attained from centre of earth 'r' = $\dfrac{R}{1-K^2}$.

From surface of the earth $h = r - R = \dfrac{RK^2}{1-K^2} = \dfrac{v^2R}{v_e^2 - v^2}$

3. If a body is dropped from rest form height h comparable to radius of earth, then maximum velocity with which it strikes the ground $v = v_e\sqrt{\frac{h}{R+h}}$

4. Time period of satellite is independent of mass of the satellite.

5. Atmosphere on a planet is possible only if $v_{\text{rms}} < v_e$. Where v_{rms} = rms speed of gas molecules.

6. If angular velocity of earth is made 17 times, object placed at equator fly off.

7. As we move from pole to equator, gravity decreases by 0.35%.

8. If orbital velocity of any satellite very close to Earth surface increases by 41.4%, then the satellite will escape to infinity.

9. If gravitation force $\propto r^n$, time period of a satellite $\propto r^{(1-n)/2}$.

❑ ❑ ❑

Learnaf

Educational Revolution

PHYSICS

Aakash

STUDY PACKAGE – 01

For – JEE / NEET

Elasticity: Properties of Matter

Chapter - 8

Solids and Fluids

AIEEE Syllabus

Elastic Behaviour, Stress-Strain relationship, Hooke's Law, Young's modulus, Bulk modulus, Modulus of rigidity. Pressure due to a fluid column; Pascal's law and its applications. Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, Reynolds number. Bernoulli's principle and its applications. Surface energy and surface tension, angle of contact, application of surface tension - drops, bubbles and capillary rise.

8

CHAPTER

THIS CHAPTER COVERS :

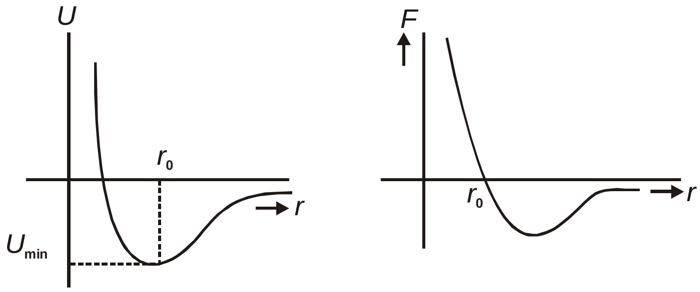
- Inter-atomic and Inter-molecular forces
- Hooke's law
- Moduli of Elasticity
- Cohesion and Adhesion
- Surface tension and surface energy
- Capillary action
- Bernoulli's theorem
- Viscosity and terminal velocity

Surface energy and surface tension angle of contact application of surfactonsia drops, bubbles and capilars rise.

INTERATOMIC AND INTERMOLECULAR FORCES

1. The force between molecules of a substance is called intermolecular force.

2.



The above graphs show the variation of potential energy and force with interatomic or intermolecular separation.

(a) For $r = \infty$, $F = 0$, $U = 0$

(b) $r > r_0$, F is attractive as r decreases from ∞ to r_0, potential energy decreases.

(c) $r = r_0$, potential energy U = Minimum, $F = 0$. This is equilibrium position.

(d) $r < r_0$, F is repulsive, therefore, potential energy increases.

Elasticity : Property of a solid by which it tries to restore its original shape by developing a restoring force in it.

Stress : Restoring force developed/Area.

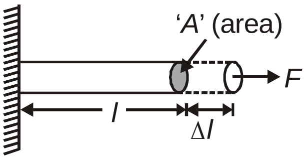
Strain : Change in dimension/original dimension.

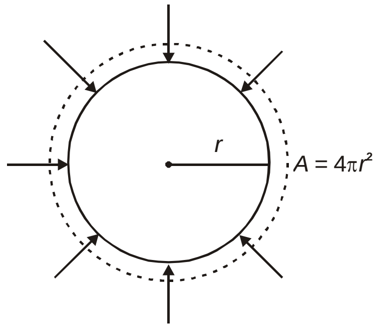
$$\text{Modulus of Elasticity} = \frac{\text{Stress}}{\text{Strain}}$$

Greater is modulus of elasticity greater is the stress developed *i.e.*, greater is the restoring force. Such a body will be more elastic.

That is why **steel is called more elastic than rubber** because its modulus of elasticity is more than that of rubber.

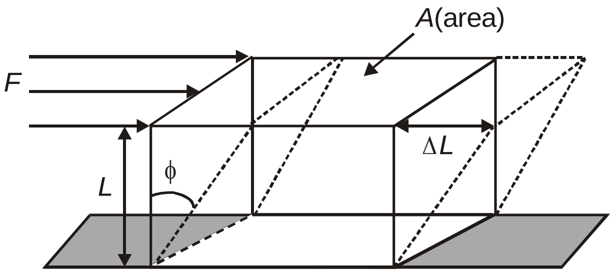
Types of Stress

(1) 

(2) 

Longitudinal Stress (Tensile) = F/A

Volumetric Stress (Compressive) = P (pressure)

(3) 

Tangential Stress or shear stress = $\frac{F}{A}$

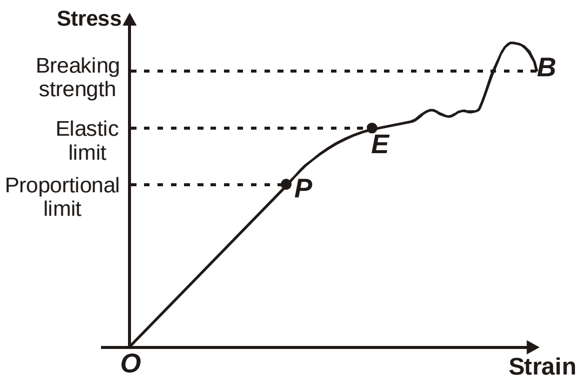
Types of Strain

(1) Longitudinal strain = $\frac{\Delta l}{l}$

(2) Volumetric strain = $-\frac{\Delta V}{V}$

(3) Shear strain = $\phi = \frac{\Delta L}{L}$

Stress - Strain Curve :



HOOKE'S LAW

Within the proportional limit stress is directly proportional to strain.

Stress $\propto$ strain

$\frac{\text{Stress}}{\text{Strain}} = \text{Elastic constant}$

(1) In region OE, material returns to original position after removal of stress.

(2) For deformation beyond E, material does not return to original size.
This phenomenon is known as elastic hysteresis.

(3) At B, fracture of the solid occurs.

Some Important points :

(1) A material having smaller value of Y is more ductile. It can be easily drawn into a wire.

(2) A material having smaller value of K (bulk modulus) is more malleable.

(3) A solid possesses all the three modulii of elasticity Y, K or η.

(4) A liquid or gas possesses a finite value of K only.

(5) For a gas, K depends on the processes by which gas expands/compresses.

For a process PV^N = constant

$$K = -\frac{dP}{dV/V} = NP$$

(i) For isothermal process $N = 1$ $\therefore K = P$

(ii) For adiabatic process $N = \gamma$ $\therefore K = \gamma P$

(iii) For isobaric process $N = 0$ $\therefore K = 0$

(iv) For isochoric process $N = \infty$ $\therefore K = \infty$

PRESSURE

Pressure is defined as the force acting on a surface per unit area. It is a scalar quantity.

$$P = \frac{F}{A} \qquad \text{SI unit Nm}^{-2}$$

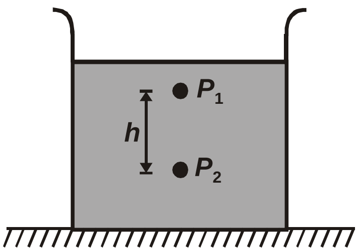
PASCAL LAW

It states that if effect of gravity is neglected, then the pressure at every point of a liquid in equilibrium is same.

The increase in pressure at any point of the enclosed liquid in equilibrium is transmitted equally to all other points of the liquid and also to the walls of the container.

Pressure difference between two points :

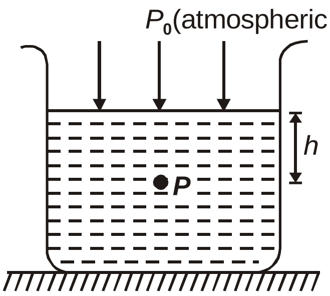
The pressure difference between two points, which are at different horizontal level is given as, $P_2 - P_1 = h\rho g$



$P_2 > P_1$ & $P_2 - P_1 = h\rho g$

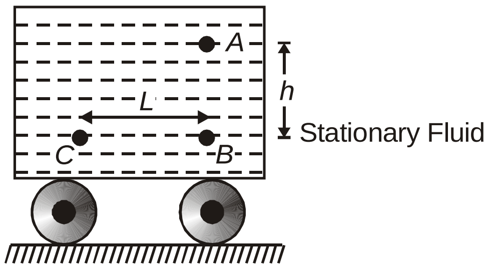
Following cases illustrate the common problems related to pressure difference :

1. $P = P_0 + h\rho g$



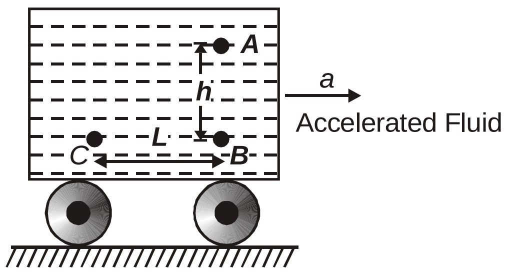
2. $P_B - P_A = h\rho g$

   $P_C - P_B = 0$

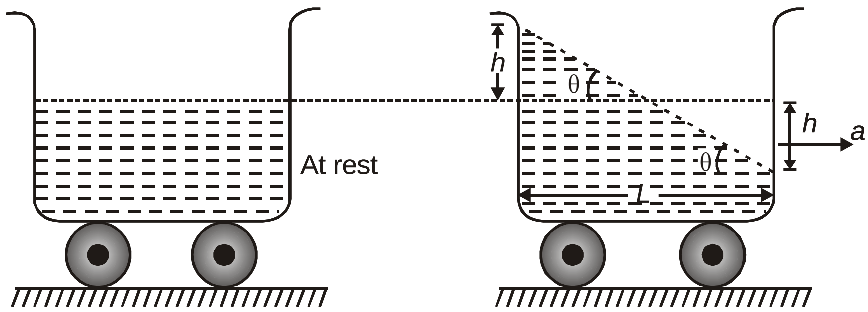


3. $P_B - P_A = h\rho g$

   $P_C - P_B = L\rho a$



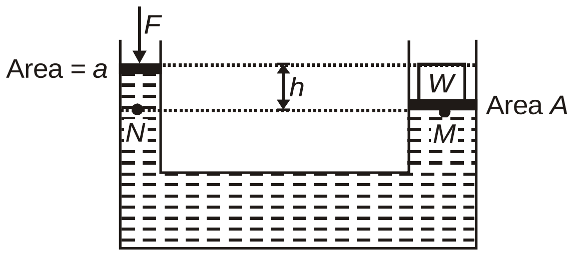
4. $\tan\theta = \dfrac{a}{g} = \dfrac{2h}{L}$



Hydraulic Lift :

It is an arrangement to lift heavy objects by applying a small force. For equilibrium of the weight W, pressure at M should be equal to pressure at N,

$$\frac{W}{A} = \frac{F}{a} + h\rho g$$

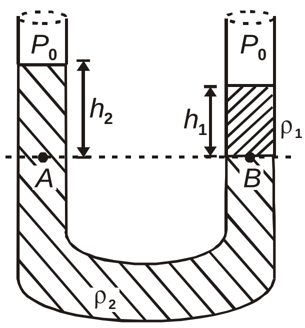


Equilibrium of Different Liquids in a U tube

1. $P_A = P_B$ (as A & B are at same level)

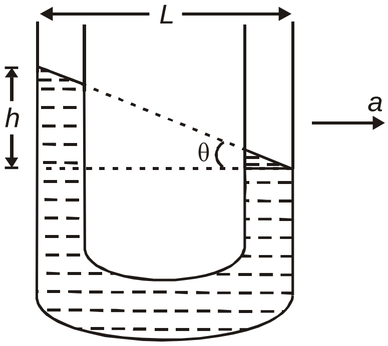
$\Rightarrow$ $P_0 + h_1\rho_1 g = P_0 + h_2\rho_2 g$ (where P_0 is atmospheric pressure)

$h_1\rho_1 g = h_2\rho_2 g$



2. When the U tube accelerates, difference of levels of liquid satisfies the relation,

$$\tan\theta = \frac{a}{g} = \frac{h}{L}$$



Buoyancy

When a body is immersed wholly or partially in a fluid, it experiences a loss of weight, due to an upward force called upthrust or buoyant force.

Archimedes Principle

It states that when a solid body is immersed wholly or partially in a liquid, then there is some apparent loss in its weight. This loss of weight is equal to weight of liquid displaced by the body.

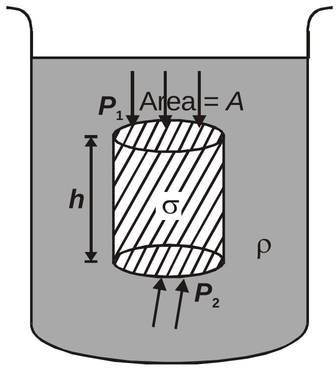
Buoyant Force

Consider a body (assumed cylinderical) of density σ and volume V immersed completely in a liquid of density ρ.

As $P_2 - P_1 = h\rho g$

$\Rightarrow$ $F_2 - F_1 = h\rho gA$

$\Rightarrow$ $F_{\mathbf{upward}} = V\rho g$ = loss of weight



Following cases are possible depending on the relation between σ and ρ.

Case - I :

$\sigma < \rho$

The body will float in the liquid with some part inside and remaining out side.

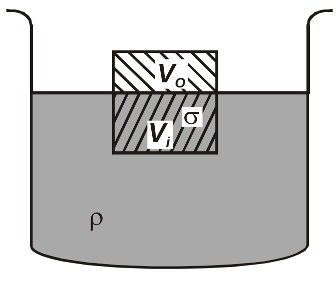
V = volume of body

V_i = volume of body inside liquid

V_0 = volume of body outside liquid

$V_i\rho g = V\sigma g$

$$\Rightarrow \frac{V_i}{V} = \frac{\sigma}{\rho} \qquad \frac{V_0}{V} = \frac{\rho - \sigma}{\rho}$$



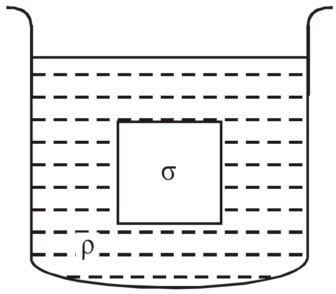
Case - II :

$\sigma = \rho$

Body floats, completely immersed in the liquid.

$V_0 = 0$

Body remains at rest wherever it is left



Case - III :

$\sigma > \rho$ (Body will sink to the bottom)

For figure-1,

R = Normal reaction between body and bottom of container

$R = V\sigma g - V\rho g$

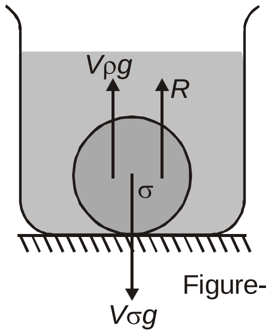


Figure-1

MODULI OF ELASTICITY

(1) Young's modulus of elasticity $Y = \dfrac{\text{Tensile stress}}{\text{Longitudinal strain}} = \dfrac{Fl}{A\Delta l}$

(2) Bulk modulus of elasticity $K = \dfrac{\text{Normal or compressive stress}}{\text{Volumetric strain}} = -V\dfrac{\Delta P}{\Delta V}$ or, $K = -V\dfrac{dP}{dV}$

Compressibility = $\dfrac{1}{K}$

(3) Modulus of rigidity η or $G = \dfrac{\text{Shear stress}}{\text{Shear strain}} = \dfrac{F}{A\phi} = \dfrac{FL}{A\Delta L}$

Some Important Points :

For a wire $Y = \dfrac{Fl}{A\Delta l}$ $\Rightarrow F = \dfrac{YA}{l}\Delta l$

i.e. a wire behaves like a spring with spring constant

$k = \dfrac{YA}{l}$ $\left(i.e., k \propto \dfrac{1}{l}\right)$

When this wire is stretched by applying an external force F, and Δl is extension produced, then

(1) Work done by external force = $F\Delta l$

(2) Work done by restoring force = $\dfrac{1}{2}F\Delta l$

(3) Heat produced = $\dfrac{1}{2}F\Delta l$

(4) Elastic potential energy stored = $\dfrac{1}{2}F\Delta l$

Energy density $U = \dfrac{1}{2}\dfrac{F\Delta l}{\text{volume}} = \dfrac{1}{2}\dfrac{F\Delta l}{Al}$

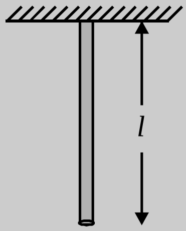
$$= \frac{1}{2}\text{ stress} \times \text{strain} = \frac{1}{2}\frac{(\text{stress})^2}{Y} = \frac{1}{2}Y(\text{strain})^2$$

(5) A rod of mass *m* and length *l* hangs from a support

Area of cross-section = *A*

Extension produced due to its own weight,

$$\Delta l = \frac{Mgl}{2AY} = \frac{\rho g l^2}{2Y} \quad (\rho = \text{density of wire})$$



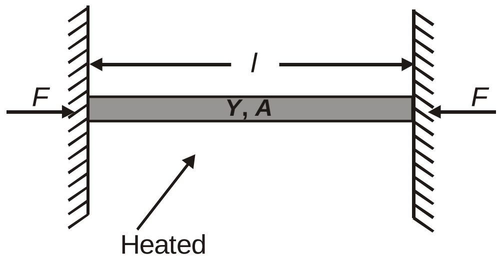
Thermal Stress : Rod Fixed between Rigid Support

If $\Delta\theta$ = Rise in temperature

Compressive strain = $\frac{\Delta l}{l} = \alpha\Delta\theta$

Compressive stress = Y × strain = $Y\alpha\Delta\theta$

$\Rightarrow$ $F = Y\alpha\Delta\theta \times A$



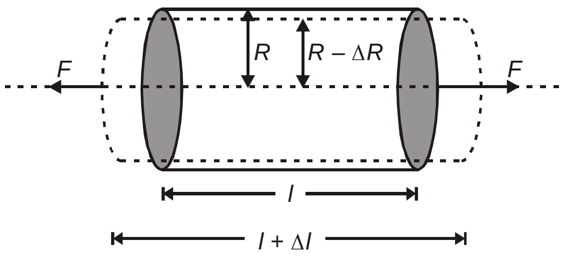
***Note :** If the rod is placed on horizontal frictionless surface, then stress developed on heating is Zero.*

Poisson's Ratio

(1) Longitudinal strain = $\frac{\Delta l}{l}$

(2) Lateral strain = $-\frac{\Delta R}{R}$

(3) Poisson's ratio $\sigma = -\frac{\Delta R / R}{\Delta l / l}$



(a) Theoretically $-1 \le \sigma \le 0.5$

(b) Practically $0 \le \sigma \le 0.5$

(c) When density of material is constant $\Rightarrow \sigma = 0.5$

(4) $\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}$

(5) $K = \frac{Y}{3(1-2\sigma)}$

(6) $\eta = \frac{Y}{2(1+\sigma)}$

(7) $\sigma = \frac{3K - 2\eta}{2\eta + 6K}$

(8) Young's modulus of a wire is numerically equal to stress required to double the length of wire.

(9) When a pressure *dP* is applied on a substance, its density changes from ρ to ρ' so that

$$\rho' = \rho\left(1 + \frac{\Delta P}{K}\right)$$

(10) The energy density of water in a lake h meter deep is

$$U = \frac{1}{2}\frac{(h\rho g)^2}{K}$$ where ρ is density of water, K is Bulk modulus.

(11) In case of a rod of length L and radius r fixed at one end. Angle of shear ϕ is related to angle of twist θ by the relation $L\phi = r\theta$.

COHESION AND ADHESION

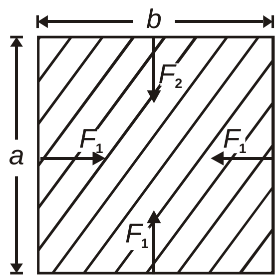
The force of attraction between similar molecules is known as cohesive force. It is very strong in solids, weak in liquids and very weak in gases.

The force between dissimilar molecules is known as adhesive force. Corresponding phenomenon is known as adhesion.

SURFACE TENSION AND SURFACE ENERGY

1. Property of a liquid due to which it behaves like a stretched membrane. A free liquid drop tries to acquire spherical shape (minimum surface area) due to surface tension.

2. Surface tension is force/length. $T = \frac{F}{l}$ (N/m)



$F = T \times l$

$F_1 = T \times a \times 2$

$F_2 = T \times b \times 2$

3. Surface energy = T × surface area

 (a) Liquid drop of radius R $\Rightarrow$ Surface Energy = $T \times 4\pi R^2$

 (b) Soap bubble of radius $R \Rightarrow$ Surface Energy = $2 \times T \times 4\pi R^2$

Angle of Contact

It is the angle between solid surface inside the liquid and the tangent drawn to the liquid surface at the point of contact.

It depends on

1. Relative cohesive and adhesive force of solid liquid pairs
2. Temperature

Application of Surface Tension

1. Work done to blow a soap bubble of radius $r = 2 \times T \times 4\pi r^2$
2. A drop of radius R breaks up into n identical drops

 work done = ΔS.E. = $[n \times 4\pi r^2 - 4\pi R^2]T$ …(1)

 $R^3 = nr^3$ …(2)

 $\Rightarrow$ Work done = $4\pi R^2 T\,[n^{1/3} - 1]$
3. n identical drops coalesce to form a single drop

 Heat produced = $4\pi R^2 T\,[n^{1/3} - 1] = mc\,\Delta\theta$

 where, c = Specific heat, m = mass = $\frac{4}{3}\pi R^3\rho$, $\Delta\theta$ = Rise in temperature.

4. A needle floats on the surface of a liquid due to surface tension.
5. Surface tension decreases with rise in temperature.
6. Surface tension decreases by adding sparingly soluble impurities like detergents.
7. Surface tension increases by adding soluble impurities like NaCl, sugar.

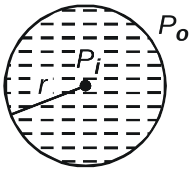
Excess pressure

If P_o = Atmospheric pressure

P_i = Inside pressure

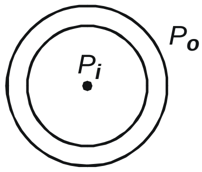
then $P_i - P_o$ = Excess pressure

Liquid drop

(1) 

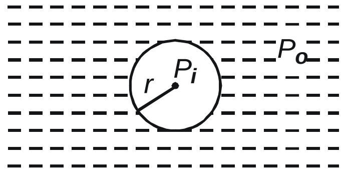
$$P_i = P_o + \frac{2T}{r}$$

Soap bubble

(2) 

$$P_i - P_o = \frac{4T}{r}$$

Air bubble

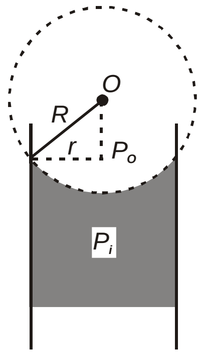
(3) 

$$P_i = P_o + \frac{2T}{r}$$

(4) Capillary tube, concave meniscus

(a) $P_i = P_o - \dfrac{2T}{R}$

(b) $F_a > \dfrac{F_c}{\sqrt{2}}$

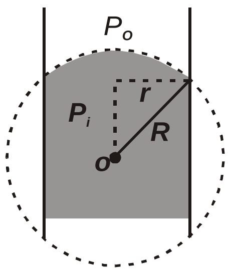


Capillary tube, Concave Meniscus

(5) Capillary tube, convex meniscus.

(a) $P_i = P_o + \dfrac{2T}{R}$

(b) $F_a < \dfrac{F_c}{\sqrt{2}}$



Convex Meniscus

Combining of Bubbles

1. If the soap bubble coalesce in vacuum, then $P_o = 0$

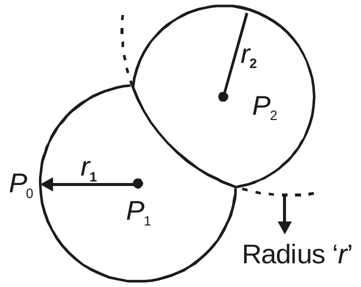
   $\Rightarrow \quad r^2 = r_1^2 + r_2^2$

2. If two soap bubbles come in contact to form a double bubble then

   r = radius of interface, $r_1 > r_2$

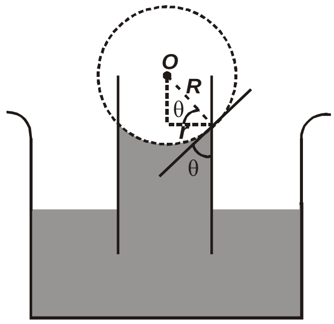
   $$\frac{1}{r} = \frac{1}{r_2} - \frac{1}{r_1}$$

The interface will be convex towards larger bubble and concave towards smaller bubble because $P_2 > P_1 > P_0$.



CAPILLARY ACTION

Rise or fall of liquid in a tube of fine diameter.



Ascent formula

$$h = \frac{2T}{R\rho g} = \frac{2T\cos\theta}{r\rho g}$$

where, θ = angle of contact (as shown in figure)

Some Important Points :

1. hR = constant and mass of liquid raised is proportional to r.
2. $h > 0$ for concave meniscus ($\theta < 90°$) *i.e.* rise
3. $h < 0$ for convex meniscus ($\theta > 90°$) *i.e.* fall
4. For water glass interface $\theta = 0°$ *i.e.*, meniscus is nearly hemispherical.
5. For mercury glass interface $\theta = 135°$.
6. Two capillary tubes of radius r_1 and r_2 ($r_2 > r_1$) are joined to form a U-tube opened at both ends. This U-tube is filled with water. The level in the two limbs will not be same due to capillary action.

   (a) Difference in level $h = \frac{2T\cos\theta}{\rho g}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$

   (b) Liquid in tube of lower radius will be at higher level.

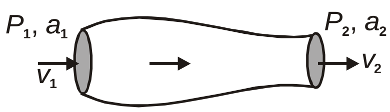
Stream Line Flow or Steady Flow

The flow of a fluid is said to be steady if all particle of the fluid passes through or cross-section with same velocity.

Turbulent Flow

Above a certain critical speed, fluid flow becomes unsteady. This irregular flow is called turbulence.

Equation of Continuity



It is based on conservation of mass. According to it,

mass entering per second = mass leaving per second

That is, $\rho_1 a_1 v_1 = \rho_2 a_2 v_2$

For incompressible liquid $\rho_1 = \rho_2 \Rightarrow a_1 v_1 = a_2 v_2 \Rightarrow v \propto \frac{1}{a}$

Energy of a Liquid

Various energies per unit mass :

1. Potential energy/mass = gh

2. Kinetic energy/mass = $\frac{1}{2}v^2$

3. Pressure energy/mass = $\frac{P}{\rho}$

Energy Heads

Various energy heads per unit mass :

1. Gravitational head = h

2. Velocity head = $\frac{v^2}{2g}$

3. Pressure head = $\frac{P}{\rho g}$

BERNOULLI'S THEOREM

It is based on conservation of energy.

For an ideal, non-viscous and incompressible liquid,

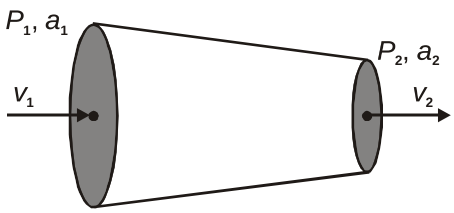
$$\frac{P_1}{\rho}+\frac{v_1^2}{2}+gh_1=\frac{P_2}{\rho}+\frac{v_2^2}{2}+gh_2 = \text{constant}$$

Applications of Bernoulli's Theorem

(1) To find rate of flow of liquid $Q = av$ [area × velocity]. Value of Q in various cases is given by

Case - (a) :

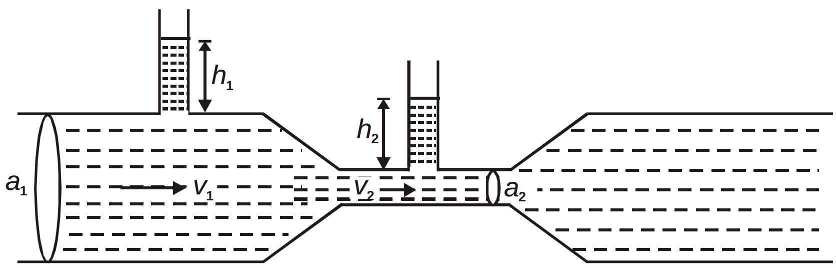
$$Q = a_1 a_2\sqrt{\frac{2(P_1-P_2)}{\rho(a_1^2-a_2^2)}}$$



Case - (b) :

Venturimeter

$$Q = a_1 a_2\sqrt{\frac{2g(h_1-h_2)}{a_1^2-a_2^2}}$$

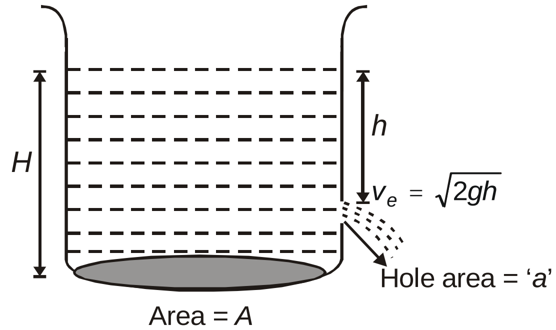


(2) Hole in a tank problem

(a) Speed of efflux $v_e = \sqrt{2gh}$ (If $a << A$)

If a is comparable to A then

$$v_e = \sqrt{2gh}\sqrt{\frac{A^2}{A^2 - a^2}}$$



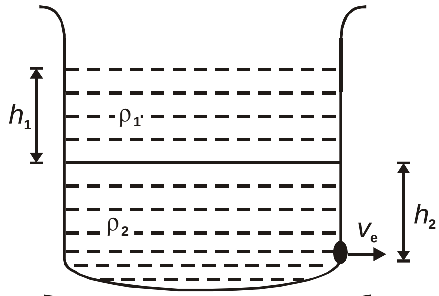
(b) Time taken by water level to fall from h_1 to h_2

$$t = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{h_1} - \sqrt{h_2}\right]$$

(c) Time taken to completely empty the container by a hole at bottom

$t \propto \sqrt{H}$ [Put $h_1 = H$, $h_2 = 0$]

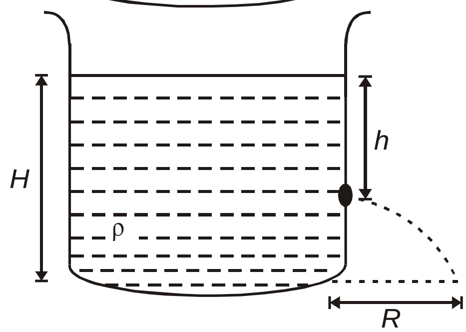
(d) $v_e = \sqrt{2g\left(h_2 + \frac{\rho_1 h_1}{\rho_2}\right)}$ in the situation shown in figure



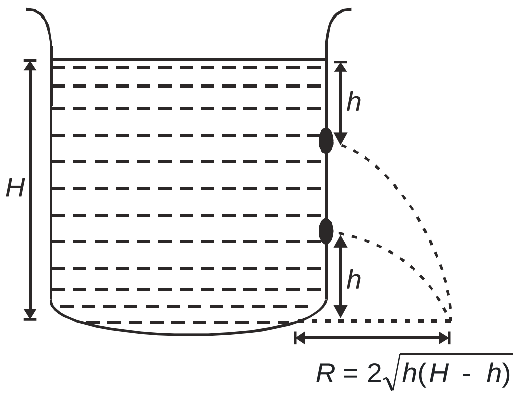
(e) Range of liquid

$$R = 2\sqrt{h(H - h)}$$

$R_{max} = H$ when $h = \frac{H}{2}$



(f)

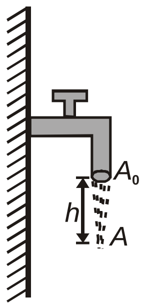


$R = 2\sqrt{h(H - h)}$ for both holes

(3) If A_0 = area of cross-section of mouth of tap

A = area of cross-section of water jet at a depth h

$A_0v_0 = Av = Q$ [rate of flow]



By Bernoulli's theorem $\frac{v_0^2}{2} + gh = \frac{v_1^2}{2}$ [$\because$ pressure is atmospheric at both points]

$\Rightarrow \quad Q = AA_0\sqrt{\frac{2gh}{A_0^2 - A^2}}$

Reynold's Number

$$N_R = \frac{\rho v D}{\eta} = \frac{\text{Inertial Force}}{\text{Viscous force}}$$

Value of N_R for various cases :

(1) $N_R < 2000$, flow is streamline

(2) $N_R > 3000$, flow is turbulent

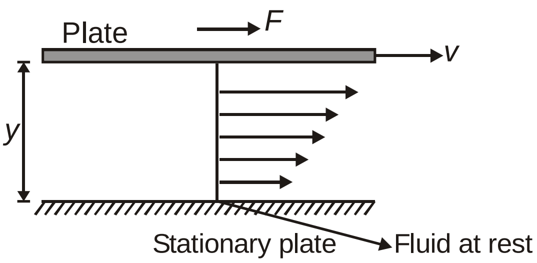
(3) $2000 < N_R < 3000$, flow is unstable

(4) When $N_R = 2000$, flow is critical

$$\frac{\rho v D}{\eta} = 2000 \Rightarrow v_c = 2000\frac{\eta}{\rho D} \text{ (Critical velocity)}$$

Viscosity & Viscous Force

1. The property of the liquid by virtue of which, it opposes the relative motion between its adjacent layers is known as viscosity. Fluid friction is due to viscosity.
2. Fluid in contact with the plate is moving with velocity v.



Viscous force is given in this case by,

$$F = -\eta A\frac{dv}{dy}$$

η = coefficient of viscosity & $\frac{dv}{dy}$ = velocity gradient

Units of η : SI $\rightarrow$ 1 Pa-s = 10 poise = 1 decapoise

C.G.S $\rightarrow$ 1 dyne/cm^2-s = 1 poise

Poiseuille's Equation

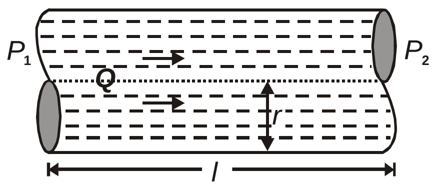
Volume flow rate across a tube with pressure difference between its ends is,

$$Q = \frac{dV}{dt} = \frac{\pi}{8}\frac{Pr^4}{\eta l}$$

Where, $P = P_1 - P_2$ = pressure difference

Comparing with $I = \dfrac{V}{R}$ $(V \to P_1 - P_2 \ \& \ I \to Q)$

$\Rightarrow$ Resistance to fluid flow $R = \dfrac{8\eta l}{\pi r^4}$



Series combination of two tubes

Two tubes of radius r_1, length l_1 and radius r_2, length l_2 are connected in series across a pressure difference of P. Length of a single tube that can replace the two tubes is found using,

$$\frac{l}{r^4} = \frac{l_1}{r_1^4} + \frac{l_2}{r_2^4}$$

STOKES LAW

When a small spherical body of radius r is moving with velocity v through a perfectly homogeneous medium having coefficient of viscosity η, it experiences a retarding force given by

$\boldsymbol{F = 6\pi\eta r v}$.

Important cases :

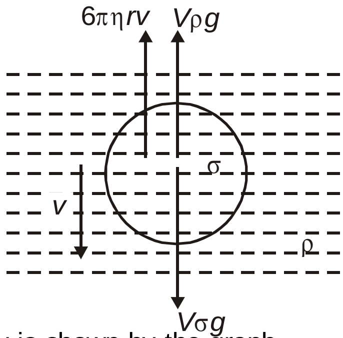
(1) A body of radius r released from rest in a fluid

If σ = density of body

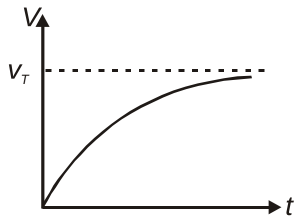
ρ = density of liquid or fluid

Terminal velocity is given by,

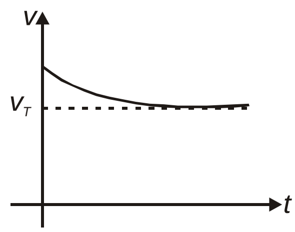
$$v_T = \frac{2}{9}\frac{r^2 g}{\eta}(\sigma - \rho)$$



Thus, velocity increases from 0 to v_T.Variation of velocity is shown by the graph.



(2) A body is thrown downwards with speed greater than v_T then its speed decreases, becomes equal to v_T.



Some Important Points :

1. With increase in temperature of water η decreases, terminal velocity increases.
2. When $\sigma < \rho$, body will move upward with terminal speed.
3. When water is replaced by glycerine, terminal speed decreases.

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Educational Revolution

PHYSICS

Aakash

STUDY PACKAGE – 01

For – JEE / NEET

Heat & Thermodynamics

Chapter - 9

Heat and Thermodynamics

9

CHAPTER

AIEEE Syllabus

Heat, temperature, thermal expansion; specific heat, calorimetry; change of state, latent heat; Thermal equilibrium, Zeroth law of thermodynamics; Heat, work and internal energy. First law of thermodynamics; Carnot engine and its efficiency

THIS CHAPTER COVERS :

- Thermal Expansion
- Coefficient of Apparent Expansion of a Liquid
- Specific Heat
- Molar Specific Heat
- Latent Heat
- Internal Energy
- First Law of Thermodynamic
- Different Thermodynamic Processes
- Work Done by System in Different Processes
- Indicator Diagram
- Slope of Isothermal and Adiabatic Curves
- Carnot Engine and Heat Pump
- Efficiency of Carnot Cycle and Carnot Theorem

HEAT

When a hot body is kept in contact with a cold body, there is a transfer of energy from hot body to cold body. The energy transferred is called heat.

ZEROTH LAW OF THERMODYNAMICS : If a body *A* is separately in thermal equilibrium with body *B* and body *C* then *B* and *C* are also in the thermal equilibrium.

"Two bodies which are in thermal equilibrium are said to have equal temperatures".

Thermal Expansion

When the temperature of a body increases, its size increases.

(1) Coefficient of linear expansion is given by

$$\alpha = \frac{\Delta L}{L\Delta T}$$

$$L_\theta = L_0 (1 + \alpha\theta)$$

(2) Coefficient of superficial expansion is given by

$$\beta = \frac{\Delta A}{A\Delta T}$$

$$A_\theta = A_0 (1 + \beta\theta)$$

(3) Coefficient of cubical expansion is given by

$$\gamma = \frac{\Delta V}{V\Delta T} \qquad \text{or } V_\theta = V_0 (1 + \gamma\,\theta)$$

$$\frac{m}{\rho_\theta} = \frac{m}{\rho_0}(1+\gamma\theta) \qquad \Rightarrow \rho_\theta \approx \rho_0(1-\gamma\theta)$$

An Isotropic body expands equally in all directions and we can obtain the following relations

$$\gamma = 3\alpha,\ \beta = 2\alpha \ \text{ or } \ \boxed{\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}}$$

Applications

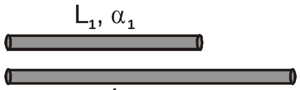
(1) If α_x, α_y, α_z are coefficient of linear expansion along *x*-axis, *y*-axis and *z*-axis, (*i.e.,* for anisotropic body) then $\gamma = \alpha_x + \alpha_y + \alpha_z$.

(2) For water γ is negative between 0°C and 4°C

(3) Density of water is maximum at 4°C. Therefore water at the bottom of lake in winter is warmer than that at the surface.

(4) Two rods of length L_1 and L_2 are kept side by side. If with increase in temperature, the difference in their lengths does not change i.e. $L_2' - L_1' = L_2 - L_1$ where $L_2' = L_1(1+\alpha_1\Delta t)$ and $L_2' = L_1(1+\alpha_2\Delta t)$

then $L_1\alpha_1 = L_2\alpha_2$ or $\dfrac{L_1}{L_1 - L_2} = \dfrac{\alpha_2}{\alpha_2 - \alpha_1}$.



(5) A vessel is completely filled with a liquid

α_v = coefficient of linear expansion of vessel

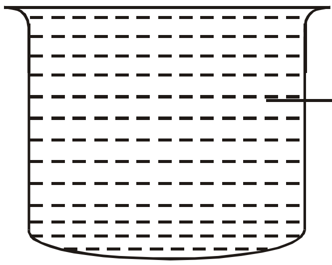
γ_l = coefficient of cubical expansion of liquid

With increase in temperature, the volume of liquid flown out is given by

$\Delta V = V_0[1 + (\gamma_l - \gamma_v)\Delta\theta] = V_0[1 + (V_l - 3\alpha_v)\Delta\theta]$

$\Rightarrow$ $\Delta V = V_0[1 + \gamma_a\Delta\theta]$, where

γ_a = coefficient of apparent expansion = $\gamma_l - \gamma_v = \gamma_l - 3\alpha_v$



(6) Variation of moment of inertia with temperature

$I' = I(1 + \beta\Delta\theta)$

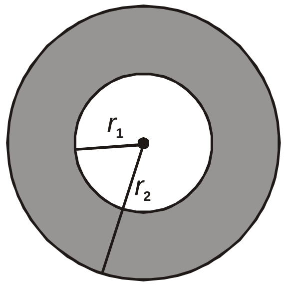
Also, $\dfrac{\Delta I}{I} = \beta\Delta\theta = -\dfrac{\Delta\omega}{\omega}$ (When Angular momentum $L = I\omega$ = Constant)

(7) As an annular disc is heated, all the dimensions (including cavity) increase.

Thus, we have

$r_1' = r_1\,(1 + \alpha\Delta\theta)$

$r_2' = r_2\,(1 + \alpha\Delta\theta)$



Annular disc

In other words, you can say that the thermal expansion is photographic i.e. as when a photograph is enlarged, all the dimensions of the photograph increase.

(8) Coefficient of volume expansion of an ideal gas at constant pressure is given by

$$\gamma = \frac{1}{T}$$

(9) A meter scale callibrated at T_1°C is used for measurement at T_2°C. Let α be the coefficient of linear expansion for scale, and gives a reading L, then error in the reading is $\Delta L = \alpha L(T_2 - T_1)$.

CALORIMETRY

Specific heat capacity : $c = \dfrac{\Delta Q}{m\Delta T}$ (cal/g/°C)

Molar heat capacity : $C = \dfrac{\Delta Q}{n\Delta T}$ (cal/mol/°C)

> ***Note :*** *Molar heat capacity = (Mol. mass) × Specific heat capacity*

Latent heat

(1) of fusion $L_f = \dfrac{Q}{m}$

(2) of vaporisation $L_v = \dfrac{Q}{m}$

Water

Specific heat C = 1 cal/gm/°C = 4.2 J/gm/°C = 4200 J/kg/°C

L_f = 80 cal/gm = 336 J/gm

L_v = 540 cal/gm = 2268 J/gm

Application

(1) To convert *m* mass of ice at 0°C into steam at 100°C, amount of heat required is

80 *m* + 100 *m* + 540 *m* = 720 *m* cal

(2) (a) Two objects having masses m_1, m_2, specific heat capacities c_1, c_2 and temperatures t_1 and t_2 are mixed. If there is no change in state during mixing, then resulting temperature of mixture,

$$t_{mix} = \frac{m_1c_1t_1 + m_2c_2t_2}{m_1c_1 + m_2c_2}$$

(b) Specific heat of the mixutre, $c_{mix} = \dfrac{m_1c_1 + m_2c_2}{m_1 + m_2}$

(3) *w* gm of water at T°C is mixed with *m* gm of ice at 0°C.

(a) $w = \dfrac{80m}{T}$ $\Rightarrow$ Whole of ice melts. Final temperature = 0°C.

(b) $w < \dfrac{80m}{T}$ $\Rightarrow$ Whole of ice will not melt. Final temperature = 0°C

Amount of ice melted, $m' = \dfrac{wT}{80}$

Amount of ice left = $m - m'$

(c) $w > \dfrac{80m}{T}$ $\Rightarrow$ Whole of ice melts. Final temperature = $\dfrac{wT - 80m}{w + m} > 0°\text{C}$

KINETIC THEORY OF GASES

Assumptions for ideal gas are

(i) Kinetic theory of gases is applicable for large number of molecules.

(ii) Intermolecular forces between two molecules is negligible.

(iii) The force due to gravity on the molecules is neglected.

(iv) The separation between the molecules is much larger as compared to their size.

(v) The molecules are perfectly elastic and all collisions between the molecules and a wall are considered to be perfectly elastic.

All gases at high temperature and low pressure behave like an ideal gas.

Pressure Exerted by the Gas

The pressure of the gas is due to continuous bombardment of the gas molecules against the walls of the container. According to kinetic theory, the pressure exerted by an ideal gas is given by

$$\boxed{P = \frac{1}{3}\frac{M}{V}\bar{v}^2}$$

M = Mass of the enclosed gas

V = Volume of the container

$\bar{v}^2$ = Mean square speed of molecules

Or $P = \frac{1}{3}\frac{Nm}{V}\bar{v}^2$ N = Number of molecule

m = Mass of the molecule

$\bar{v}^2$ = Mean square speed of molecules

Types of speed defined for a gas :

(i) Root mean square speed, $\bar{v}_{rms} = \sqrt{\frac{3RT}{M_w}} = \frac{\sqrt{3P}}{\rho} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^3 + ... {v_n}^2}{n}}$

(ii) $\bar{v}_{Avg} = \sqrt{\frac{8RT}{\pi M_w}} = \sqrt{\frac{8P}{\rho}} = \frac{v_1 + v_2 + v_3 + ... + v_n}{n}$

(iii) v_{mp} = Most probable speed is defined the speed corresponding to which there are maximum number of molecules.

$$v_{mp} = \sqrt{\frac{2RT}{M_w}} = \sqrt{\frac{2P}{\rho}}$$

ρ = Density of gas
M_w = Molecular weight
R = Gas constant
P = Pressure of gas

Order of magnitude : $v_{\text{rms}} > v_{\text{avg}} > v_{\text{mp}}$

$$V_{rms} : V_{av} : V_{mp} = \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2}$$

Kinetic Interpretation of Temperature

Translation Kinetic Energy = $\frac{3}{2}nRT$

THERMODYNAMICS

Internal Energy : Every bulk system consists of a large number of molecules. Internal energy is simply the sum of the kinetic energies and potential energies of these molecules. It is a macroscopic variable.

For gases $\Delta U = \frac{f}{2} nR\Delta T = \frac{f}{2}(P_2V_2 - P_1V_1)$

Law of equipartition of energy :

According to law of equipartition of energy, the total kinetic energy of a Thermo dynamical system consisting of a large number of particles is equally distributed among its degree of freedom and hence the average kinetic energy of a molecule associated with each degree of freedom is $\boxed{K.E. = \frac{1}{2}kT}$ *where k is Boltzmann constant and T is absolute temperature.*

Gas	Degrees of freedom (f)	$\Delta U = \frac{f}{2}nR\Delta T$	$C_V = \frac{\Delta U}{n\Delta T}$	$C_P = C_V + R$	$\gamma = \frac{C_P}{C_V}$
Monoatomic	3 (Translational)	$\frac{3}{2}nR\Delta T$	$\frac{3}{2}R$	$\frac{5}{2}R$	$\frac{5}{3}$
Diatomic or Linear Polyatomic	3(Trans) + 2(Rot)	$\frac{5}{2}nR\Delta T$	$\frac{5}{2}R$	$\frac{7}{2}R$	$\frac{7}{5}$
Non-Linear Poly atomic	3 (Trans) + 3 (Rot)	$3nR\Delta T$	$3R$	$4R$	$\frac{4}{3}$

Meanfree Path (λ)

The meanfree path is the average distance covered by a molecule between two Successive collision.

$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$ n = No. of molecules per unit volume

It is inversely proportional to density of gas.

In closed container it temperature is increased λ remain same.

First Law of Thermodynamics : It is law of conservation of energy.

Let Q heat is supplied to gas. It is used in two ways.

(1) Increasing internal energy (*i.e.,* increasing temperature).

(2) Work done by the gas during expansion (W)

$\Rightarrow Q = \Delta U + W$

Note : *(1) Q and W are path functions. They depend on the type of process.*

(2) ΔU is state function. It depends only on initial and final state of system.

$\Delta U = nC_V\Delta T$ (Always applicable, whatever may be the process. Here C_V is Molar heat capacity at constant volume)

Thermodynamic Process

(1) Melting process : (Change of state, solid to liquid)

$Q = \Delta U + W$

$mL_f = \Delta U + 0$ [W = 0 as volume remains nearly constant]

(2) Boiling process : (Change of state, liquid to gas)

$mL_v = \Delta U + P[V_2 - V_1]$

V_2 = volume of vapours

V_1 = volume of liquid

When 1 g of water vapourises isobarically at atmospheric pressure. $\Delta U = 2091$ J, $P = 1.01 \times 10^5$ Pa, $V_1 = 1$ cm^3, $V_2 = 1671$ m^3.

(3) Isochoric process :

$dV = 0 \Rightarrow W = 0$ [dV = change in volume]

$Q = nC_V\Delta T = \Delta U$

$\Rightarrow C_V = \dfrac{\Delta U}{n\Delta T}$

(4) Isobaric process :

P = constant, $dW = PdV$

$\Rightarrow W = P\Delta V = nR\Delta T$

$Q = nC_P\Delta T = \Delta U + W$

$nC_P\Delta T = nC_V\Delta T + nR\Delta T$

$\Rightarrow C_P = C_V + R$

$$\boxed{\frac{\Delta U}{\left(\frac{f}{2}\right)} = \frac{W}{1} = \frac{Q}{\left(\frac{f+2}{2}\right)}} \quad \text{or} \quad \frac{\Delta U}{1} = \frac{W}{\gamma - 1} = \frac{Q}{\gamma}$$

Fraction of total heat converted to internal energy = $\dfrac{\Delta U}{Q} = \dfrac{1}{\gamma}$

Fraction of total heat converted to work is, $\dfrac{W}{Q} = \dfrac{\gamma - 1}{\gamma}$

(5) Isothermal process :

$PV = K \Rightarrow \Delta T = 0 \Rightarrow \Delta U = 0, C = \infty$

To calculate the amount of work done by the gas in an isothermal process.

as $PV = nRT$ (Constant)

So $P = \dfrac{nRT}{V}$

Work done

$$W = Q = nRT \log_e \frac{V_2}{V_1} = 2.303\ nRT \log_{10} = \frac{V_2}{V_1}$$

(6) Adiabatic process :

$PV^{\gamma} = K$ [Equation of adiabatic process]

As $Q = 0$, $nC\Delta T = 0$ or $C = 0$

Also, $0 = nC_V\Delta T + W$ [by first law of thermodynamics]

Now, $\boxed{W = -\Delta U}$

(7) Polytropic Process

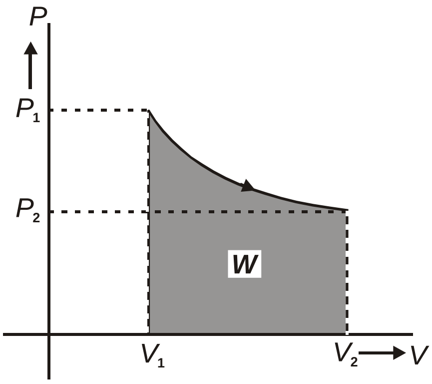
PV^n = Constant

$$W = \frac{nR\Delta T}{1-n}$$

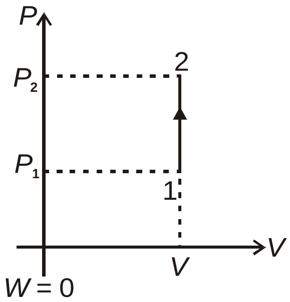
$$C = C_V + \frac{R}{1-n}$$

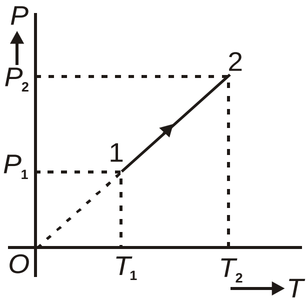
Indicator Diagram:

P-V graph of a process is called indicator diagram. Area under *P-V* graph represents the work done in a process.



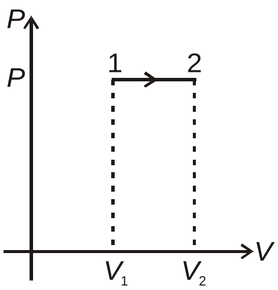
Isochoric Process :

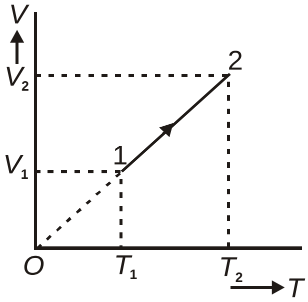




$$\text{Slope} = \frac{P}{T} = \frac{nR}{V}$$

Isobaric :

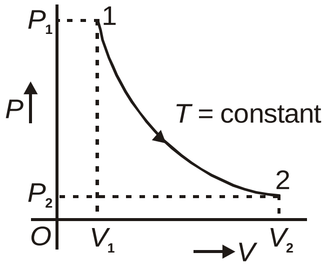




$$\text{Slope} = \frac{V}{T} = \frac{nR}{P}$$

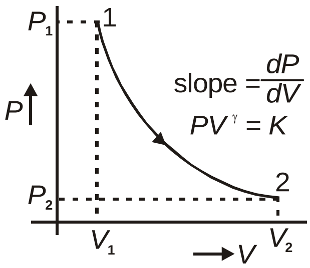
$W = P(V_2 - V_1) = P\Delta V = nR\Delta T$

Isothermal process :



$$\text{Slope} = \frac{dP}{dV} = -\frac{P}{V}$$

Adiabatic process :



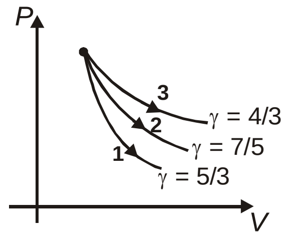
$$\text{slope} = \frac{dP}{dV} = -\frac{\gamma P}{V}$$

$$PV^{\gamma} = K$$

> ***Note :*** *Slope of adiabatic curve = g (slope of isothermal curve)*

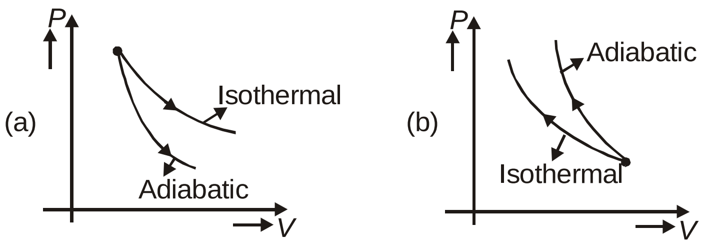
Applications

(1) P-V graph for different gases for adiabatic expansion

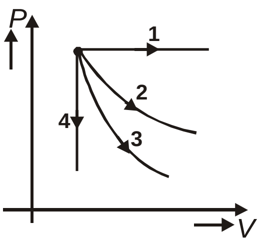


$1 \rightarrow$ monoatomic
$2 \rightarrow$ diatomic
$3 \rightarrow$ polyatomic

(2) P.V. graph for isothermal & adiabatic expansion & compression for a given gas



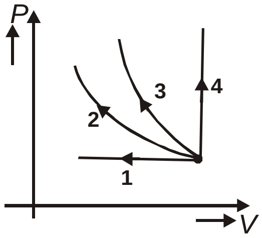
(3) **Expansion** of a gas under different processes



$1 \rightarrow$ Isobaric
$2 \rightarrow$ Isothermal
$3 \rightarrow$ Adiabatic
$4 \rightarrow$ Isochoric

$|W_1| > |W_2| > |W_3|$

(4) **Compression**



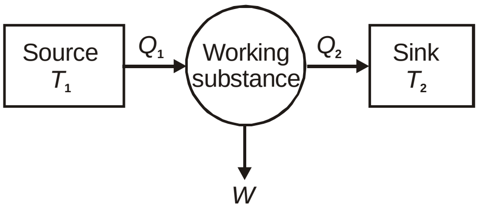
$1 \rightarrow$ Isobaric
$2 \rightarrow$ Isothermal
$3 \rightarrow$ Adiabatic
$4 \rightarrow$ Isochoric

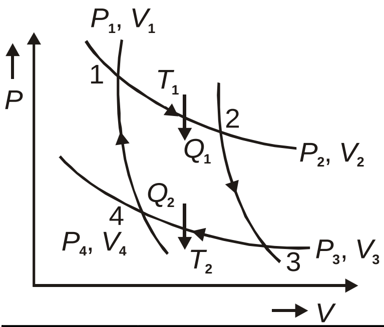
$|W_3| > |W_2| > |W_1|$

CARNOT ENGINE

Carnot Cycle :

In a carnot engine the working substance (an ideal gas) draws some heat from the source per cycle (say Q_1) performs some work W per cycle and rejects heat Q_2 to the sink per cycle.





$1 \rightarrow 2$	Isothermal Expansion	$\Delta U = 0$ $W_1 = Q_1 = nRT \ln\frac{V_2}{V_1}$ (positive)
$2 \rightarrow 3$	Adiabatic Expansion	$Q = 0$ $W_2 = -\Delta U = \frac{nR\Delta T}{1-r}$
$3 \rightarrow 4$	Isothermal Compression	$\Delta U = 0$ $W_3 = Q_2 = nRT \ln\left(\frac{V_4}{V_3}\right)$ (negative)
$4 \rightarrow 1$	Adiabatic Compression	$Q = 0$ $W_4 = -\Delta U = \frac{nR\Delta T}{1-r}$

Heat supplied = Q_1

Heat rejected = Q_2 $\Rightarrow$ $Q_1 - Q_2 = W$

$$\eta = \frac{W_{\text{total}}}{Q_{\text{supplied}}} \times 100 = \frac{Q_1 - Q_2}{Q_1} \times 100$$

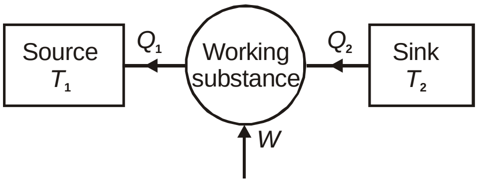
$$= \left(1 - \frac{Q_2}{Q_1}\right) \times 100$$

$$= \left(1 - \frac{T_2}{T_1}\right) \times 100 \quad \text{(for ideal engine)}$$

$$Q_1 : Q_2 : W = T_1 : T_2 : T_1 - T_2 \text{ or } \frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{W}{T_1 - T_2}$$

Carnot Theorem : The efficiency of carnot engine is maximum (<100%) for given temperatures T_1 and T_2. Any other engine working between temperature range T_1 & T_2 cannot have efficiency more than the carnot engine working between the same temperature range.

Heat Pump : In a heat pump W work is done on the working substance per cycle, Q_2 heat is absorbed by the substance from lower temperature T_2 per cycle and Q_1 heat is supplied to higher temperature $T_1 (T_1 > T_2)$ per cycle.



$$\beta = \frac{\text{Heat Supplied}}{W_{\text{total}}} = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2}$$

$$\beta = \frac{T_1}{T_1 - T_2} \text{ (for ideal pump)}$$

Relation between η and β, $\eta = \frac{1}{\beta + 1}$ and $\beta = \frac{1}{\eta} - 1$

Refrigerator : In a refrigerator, W work is done on the working substance, Q_2 heat is absorbed from lower temperature T_2 and Q_1 heat is rejected to higher temperature T_1. ($T_1 > T_2$).

Coefficient of performance $\beta = \frac{\text{heat rejected}}{W_{\text{total}}} = \frac{Q_2}{Q_1 - Q_2}$

$$\Rightarrow \beta = \frac{T_2}{T_1 - T_2}$$

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For – JEE / NEET

Transfer of Heat

Chapter - 10

Transfer of Heat

10 CHAPTER

AIEEE Syllabus

Heat transfer-conduction, convection and radiation, Newton's law of cooling

THIS CHAPTER COVERS :

- Heat Conduction
- Steady State
- Thermal Resistance
- Series and Parallel Rods
- Formation of Ice Layer
- Convection
- Radiation
- Kirchhoff's Law
- Stefan's Law
- Newton's Law of Cooling
- Wein's Displacement Law

Heat transfer can take place from one place to the other by three different processes namely conduction, convection and radiation.

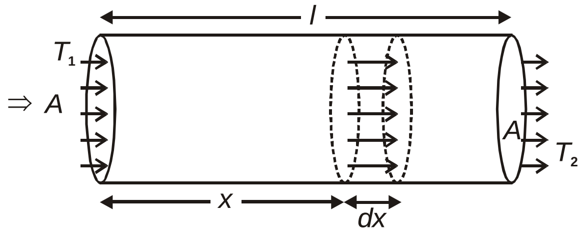
HEAT CONDUCTION

Conduction usually takes place in solids.

Steady State

When heat conduction takes place across say a rod of certain material, the state at which each cross-section of rod is at a constant temperature (which is different for different sections) is called steady state. The bar does not absorb any heat, and if the rod is completely lagged then the heat entering one end is equal to the heat leaving other end.

Law of Conduction



Rate of heat flow across any section is given by

$$\frac{dQ}{dt} = kA\frac{dT}{dx}$$

Here k = Thermal conductivity and $\frac{dT}{dx}$ is known as temperature gradient i.e. rate of change of temperature with distance.

Units of thermal conductivity are

Watt (metre-kelvin)$^{-1}$ or $Wm^{-1}K^{-1}$

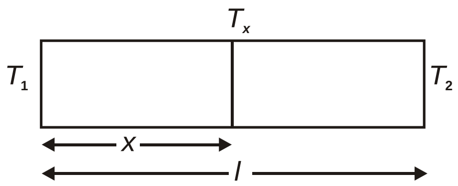
Important Results

(1) In steady state, rate of heat flow is same across any section

$$\frac{dQ}{dt} = kA\left(\frac{T_1 - T_2}{l}\right)$$

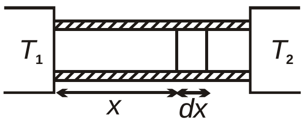
(2) If temperature at every cross section remains constant, temperature at a distance x from T_1 end is

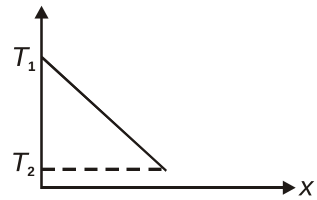
$$T_x = T_1 - \left(\frac{T_1 - T_2}{l}\right)x$$



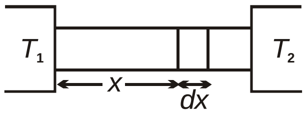
(3) Graphical Variation of T_x with x

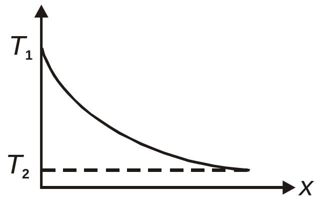
(a) Lagged rod





(b) Unlagged rod





Decreasing Order of Conductivity

For some special cases it is as follows :

(a) $K_{Ag} > K_{Cu} > K_{Al}$

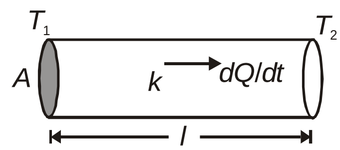
(b) $K_{solid} > K_{liquid} > K_{gas}$

(c) $K_{metals} > K_{non\text{-}metals}$

THERMAL RESISTANCE OF A ROD

In steady state $\frac{dQ}{dt} = kA\frac{(T_1 - T_2)}{l}$

$$R = \frac{l}{kA} \qquad \left[\text{as in current electricity } R = \frac{\rho l}{A} = \frac{l}{\sigma A}\right]$$



Weidman – Frenz law

$$\frac{k}{\sigma T} = \text{constant}$$

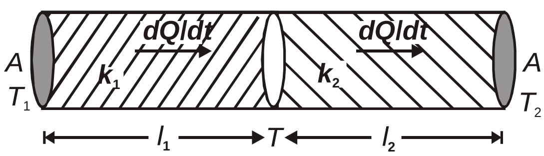
Where σ is electrical conductivity

Composite Rod :

(1) Series

In steady state

$$R = \frac{l_1}{k_1 A} + \frac{l_2}{k_2 A} = \frac{l_1 + l_2}{kA}$$



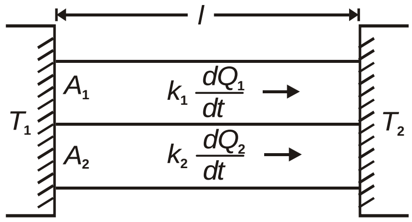
Where k = effective thermal conductivity given by

$$k=\frac{l_1+l_2}{\frac{l_1}{k_1}+\frac{l_2}{k_2}}$$

Temperature of junction $T=\dfrac{\frac{k_1}{l_1}T_1+\frac{k_2}{l_2}T_2}{\frac{k_1}{l_1}+\frac{k_2}{l_2}}$. For same geometrical dimensions,

$$T=\frac{k_1T_1+k_2T_2}{k_1+k_2}$$

(2) In parallel



$$\frac{dQ}{dt}=\frac{dQ_1}{dt}+\frac{dQ_2}{dt}$$

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2} \quad \Rightarrow \frac{k(A_1+A_2)}{l}=\frac{k_1A_1}{l}+\frac{k_2A_2}{l}$$

where k = effective coefficient of thermal conductivity given by

$$k=\frac{k_1A_1+k_2A_2}{A_1+A_2}$$

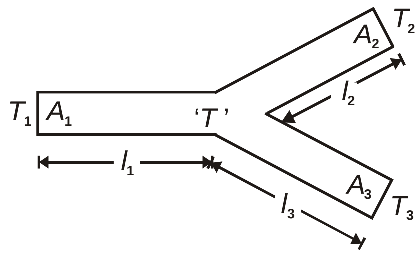
Applications

(1) If number of conductors having identical dimension are in series then equivalent thermal conductivity is harmonic mean of individuals,

$$\frac{n}{K_{eq}}=\frac{1}{K_1}+\frac{1}{K_2}+..........+\frac{1}{K_n}$$

(2) If number of conductors having identical dimension are in parallel then equivalent thermal conductivity is arithmatic mean of individuals,

$$K_{eq}=\frac{K_1+K_2+K_3+...+K_n}{n}$$

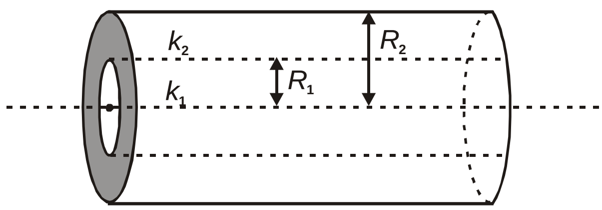


(3) $T=\dfrac{\frac{k_1A_1}{l_1}T_1+\frac{k_2A_2}{l_2}T_2+\frac{k_3A_3}{l_3}T_3}{\frac{k_1A_1}{l_1}+\frac{k_2A_2}{l_2}+\frac{k_3A_3}{l_3}}$

$$T=\frac{k_1T_1+k_2T_2+k_3T_3}{k_1+k_2+k_3} \qquad [l_1=l_2=l_3,\ A_1=A_2=A_3]$$

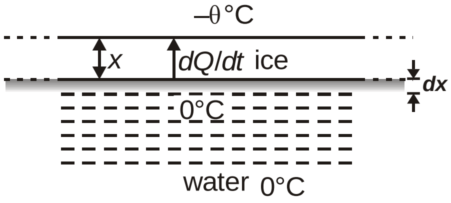
(4) $kA = k_1A_1 + k_2A_2$

If $R_1 = R$ & $R_2 = 2R$, then $k=\dfrac{k_1+3k_2}{4}$



(5) **Formation of ice**

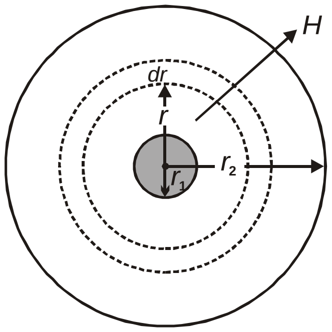
If temperature of water just below the ice layer = 0°C



$\Rightarrow$ Rate of formation $\propto \dfrac{1}{\text{instantaneous thickness}}$

Time taken to deposit x thickness, $t = \dfrac{\rho L}{k\theta}\dfrac{x^2}{2} \Rightarrow t \propto x^2$

(6) Let there be a hollow sphere of inner radius r_1 and outer radius r_2, having inner surface temperature T_1 and outer surface temperature T_2, To calculate radial rate of flow of heat '*H*', assume a spherical shell of radius r and width dr. The heat flow rate through this section is given by

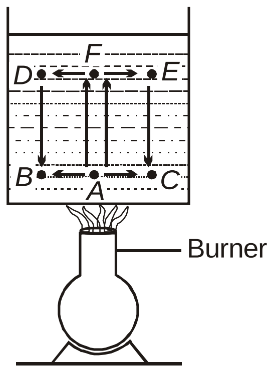


$$H = \frac{4\pi K r_1 r_2 (T_1 - T_2)}{r_2 - r_1}$$

$$H \propto \frac{r_1 r_2}{r_2 - r_1}$$

CONVECTION

In this process, heat is transferred from one place to the other by the actual movement of heated substance.



To understand the convection process consider a beaker in which some liquid is placed as shown in the figure. When liquid is heated at the bottom, the liquid expands and hence pressure at that point (A) reduces. So liquid from B & C moves toward point A. Now to take position of liquid at B & C liquid from D & E moves towards B & C respectively. At D & E liquid is supplied from region F and to fill the vacant place F liquid moves from A to F. In this way convention currents set up on the whole beaker.

Natural Convection : This type of convection results from difference in densities due to difference in temperature.

Forced Convection : Here, heated fluid is forced to move by a blower.

RADIATION

The process by which heat is transferred directly from one body to another, without requiring any medium is called radiation.

1. Radiation is the fastest mode of heat transfer as in this mode, heat energy is propagated at speed of light, *i.e.*, 3×10^8 m/s.
2. As all bodies radiate energy at all temperatures (more than θK) and at all times, radiation from a body can never be stopped.
3. Heat radiations are invisible, travel in straight line, cast shadow, affect photographic plate and can be reflected by mirrors and refracted by glasses.
4. Blue star is hotter than red star.
5. A medium which allows heat radiations to pass through it without absorbing them is called diathermanous medium. e.g. dry air.
6. A medium which partly absorbs heat rays is called athermanous medium. e.g. moist air, metals, wood, glass etc.

Glass and water vapours transmit shorter wavelengths through them but reflects longer wavelengths. This concept is utilised in **Green House Effect**. Glass transmits those waves which are emitted by a source at a temperature greater than 100ºC. So heat rays emitted from sun are able to enter through glass enclosure but heat emitted by small plants growing in the nursery gets trapped inside the enclosure.

Perfectly Black Body

A body which absorbs all the radiations incident on it is called perfectly black body.

Absorptive Power (*a*) : Absorptive power of a surface is the ratio of the radiant energy absorbed by it in a given time interval to total radiant energy incident on it in the same time interval. Absorptive power of a black body is maximum *i.e.* unity.

Emissive Power (*e*) : Emissive power of a surface is defined as the radiant energy emitted per second per unit area of the surface.

Spectral Emissive Power : The radiant energy/second/area corresponding to a definite wavelength is called spectral emissive power.

If e_λ is spectral emissive power and e is emissive power then

$$e = \int_0^\infty e_\lambda d\lambda$$

Emissive power of a surface depends on its nature and temperature. Its units are W/m^2.

KIRCHHOFF'S LAW

The ratio of emissive power to absorptive power for a given wavelength is same for all surfaces at the same temperature, and is equal to the emissive power of a perfectly black body for that wavelength at that temperature.

This implies that a good absorber is a good emitter. Following points must be remembered.

(1) Sand is rough and black. Therefore it is a good absorber as well as good emitter.

(2) A polished metal plate has a black spot. When the plate is heated strongly and taken to a dark room, spot will appear brighter than the plate.

(3) In sodium absorption spectrum, two dark lines in yellow region are found. If emission spectrum of sodium is observed, it is found to emit the corresponding lines.

(4) Fraunhoffer lines are dark lines in spectrum of sun and are formed because, the elements present in outer atmosphere absorb their characteristic wavelengths.

STEFAN'S LAW

The radiant energy emitted by a perfectly black body per second per unit area (emissive power) is directly proportional to the fourth power of the absolute temperature of the body.

$R \propto T^4 \qquad R = \sigma T^4$

$$R = \frac{\text{Power}}{\text{Area}} \Rightarrow \text{P} = A\sigma T^4$$

For other bodies $P = \varepsilon A\sigma T^4$, ε is emissivity of the body.

Rate of heat loss

For a sphere of radius r at a temperature T placed in a surrounding of temperature T_0, the rate of heat loss is $\frac{dQ}{dt} = 4\pi r^2 \varepsilon\sigma(T^4 - T_0^4)$, Where ε is emissivity.

Rate of cooling

It is rate of fall in temperature, It is given by $\frac{dT}{dt} = -\frac{3\varepsilon\sigma(T^4 - T_0^4)}{\rho sr}$

Newton's Law of Cooling

If the temperature T of a body is not much different from surrounding temperature T_0, then rate of cooling of a liquid is directly proportional to the difference in the temperature of liquid T and temperature of surroundings i.e.

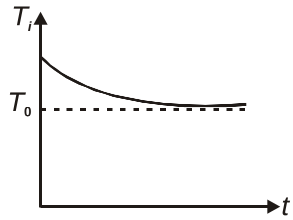
Rate of cooling $-\frac{dT}{dt} \propto (T - T_0)$

Results

(1) $T_f = T_0 + (T_i - T_0)e^{-\alpha t}$, where T_i is initial temperature, T_f is temperature after time t.

(2) Another form $\alpha t = \log\left|\frac{T_i - T_0}{T_f - T_0}\right|$

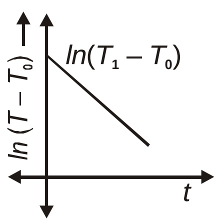
(3) $\frac{-dT}{dt} = \frac{4\varepsilon A\sigma T_0^3}{mc}(T - T_0)$, $\left[\begin{array}{l} m = \text{mass of body} \\ c = \text{specific heat} \\ A = \text{surface area} \\ \varepsilon = \text{emissivity} \end{array}\right]$



(4) Another approximate formula is

$$\frac{T_1 - T_2}{t} = \alpha\left(\frac{T_1 + T_2}{2} - T_0\right)$$

Above formula gives time 't' taken by the body to cool from T_1 to T_2. T_0 is temperature of surrounding.



Thermometric Conductivity or Diffensivity

It is defined of the ratio of thermal conductivity to the thermal capacity per unit volume of material.

$$D = \frac{K}{\rho c}$$

$K \rightarrow$ thermal conductivity

$\rho \rightarrow$ density

$c \rightarrow$ specific heat

Practical examples :

(1) Hot water loses heat in smaller duration as compared to moderate warm water.

(2) Adding milk in tea reduces the rate of cooling.

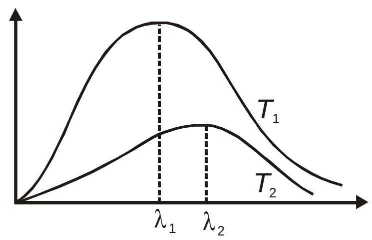
WEIN'S DISPLACEMENT LAW

This law states that the wavelength corresponding to maximum intensity for a black body is inversely proportional to the absolute temperature of the body

$$\lambda_m = \frac{b}{T}$$

where b is a constant known as Wein's constant

Results



$$\boxed{\frac{\lambda_1}{\lambda_2} = \frac{T_2}{T_1}}$$

(1) $\lambda_{max} T = b$

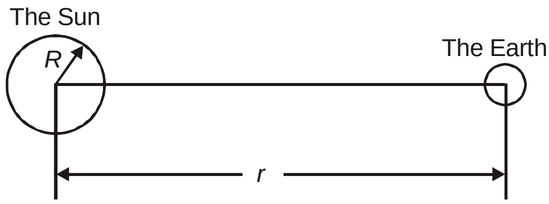
(2) $b = 2.898 \times 10^{-3}$ m-K

(3) Area under $e_\lambda - \lambda$ graph $= \sigma T^4$

(4) If the temperature of the black body is made two fold, λ_{max} becomes half, while area becomes 16 times.

(5) Temperature of the Sun,

If T = temperature of sun, then total energy radiated by sun per second = $\rho T^4 (4\pi R^2)$



Intensity at distance r from the sun (*i.e.*, on earth)

$I = S$ (Solar constant) = $\dfrac{\sigma T^4 R^2}{r^2}$ [S = 1.4 KW / m^2]

So, $T = \left[\left(\dfrac{r^2}{R^2}\right)\dfrac{S}{\sigma}\right]^{1/4}$

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Wave & Wave Motion

Chapter - 12

12

CHAPTER

Waves

AIEEE Syllabus

Wave motion. Longitudinal and transverse waves, speed of a wave. Displacement relation for a progressive wave. Principle of superposition of waves, reflection of waves, Standing waves in strings and organ pipes, fundamental mode and harmonics, Beats, Doppler effect in sound

THIS CHAPTER COVERS :

- Types of waves
- Wave function
- Velocity of wave
- Sound waves
- Superposition of waves
- Standing waves
- Free oscillation
- Forced oscillation
- Beats
- Interference
- Doppler's effect

TYPES OF WAVES

A wave is disturbance that propagates in space, transports energy and momentum from one point to another without the transport of matter.

The ripples on a water surface, the sound we hear, visible light, radio and TV signals are a few examples of waves.

There are two types of wave.

(1) **Mechanical Waves :** Require material medium (elasticity and inertia) for their propagation. These waves are also called elastic waves, water waves and sound waves are example of mechanical waves. They are of two types : Transverse and longitudital. Comparison between the two is given there :

Transverse	Longitudinal
Particles of the medium vibrate at right angles to the direction of wave motion	Particles of the medium vibrate in the direction of wave motion
Particle velocity is always perpendicular to wave velocity	Particle velocity is parallel or antiparallel to wave velocity
Waves on strings are always transverse	Can not be produced on stretched strings
These can be polarised	Can not be polarised
Do not exist in gases as they do not possess shear modulus or modulus of rigidity	Can exist in a solid, liquid or gas

(2) **Electromagnetic or non-mechanical Waves :** Do not require any material medium for their propagation, such as light and TV signals. Important points regarding these waves are :

(a) Elasticity or inertia do not affect their propagation.

(b) They are always transverse in nature.

WAVE FUNCTION

Various functions and their implications :

1. If $y = f(x + vt)$, then wave is moving in negative x-direction with velocity v.
2. If $y = f(x - vt)$, then wave is moving in positive x-direction with velocity v.
3. $y = f(x \pm vt)^2$, $y = f\left(\sqrt{x \pm vt}\right)$ or $f(x \pm vt)^3$ are valid wave equation.
4. $y = f\left(\sqrt{x} \pm \sqrt{v}\, t\right)$, $y = f(x^2 \pm v^2 t)$ or $f(x^3 \pm v^3 t)$ are not wave equation.

Here y, x, v, t stand for displacement, position, speed of wave, time respectively.

Differential Equation of Travelling Wave

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$$

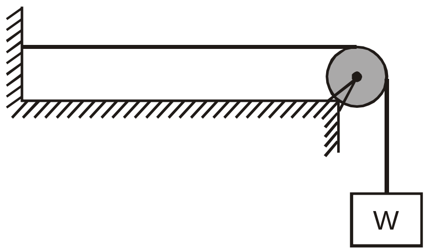
Speed of Wave Motion

(1) Speed of non-mechanical *i.e.*, electromagnetic wave in vaccum is $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

where μ_0 = absolute permeability and ε_0 = absolute permittivity

(2) Speed of mechanical waves :

(a) Transverse wave in a stretched string



T = tension in the string

μ = mass per unit length

D = diameter of pipe

ρ = density

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\text{stress}}{\text{density}}} = \frac{2}{D}\sqrt{\frac{T}{\pi\rho}}$$

$$\boxed{\mu = Ad = \frac{\pi D^2}{4}\alpha}$$

(b) Transverse wave in a solid

(i) In a long bar $v = \sqrt{\frac{Y}{\rho}}$ where Y = Young's modulus, ρ = Density of material

(ii) In an extended solid $v = \sqrt{\frac{Y + \frac{4\eta}{3}}{\rho}}$ where η = modulus of rigidity, ρ = Density of material

(c) Longitudinal Waves

(i) In liquid $v = \sqrt{\frac{K}{\rho}}$ K = bulk modulus of elasticity

ρ = density

(ii) In gases $v = \sqrt{\frac{K}{\rho}}$. For gases, K depends upon the process.

Case - I :

[Suggested by Newton] Taking isothermal process $K = P \Rightarrow v = \sqrt{\frac{P}{\rho}}$

Put P = 1 atm, ρ = 1.23 kg/m^3 $\Rightarrow v$ = 280 m/s (more than 15% error)

Case - II :

For Adiabatic $k = \gamma P$ [Corrected by Laplace]

$$\Rightarrow \quad v = \sqrt{\frac{\gamma P}{\rho}}$$. Taking $\gamma = 1.4$, we get, $\boxed{\text{For air } v = 20\sqrt{T}}$

$v = 330$ m/s

> ***Note :*** *Propagation of sound in air is adiabatic.*

Factors affecting speed of sound :

If temperature is kept constant.

(1) v is independent of pressure

(2) $v \propto \frac{1}{\sqrt{\rho}}$ or $v \propto \frac{1}{\sqrt{M}}$

(3) Velocity of a wave depends on medium.

(4) $v \propto \sqrt{T}$

(5) Velocity of sound in humid air is more because its density is very less.

(6) Velocity of sound in humid hydrogen is less than in dry hydrogen due to similar reason.

Some Important Points :

Now $v = \nu\lambda$

ν = frequency of wave, which is a constant.

λ = wavelength

$\Rightarrow \lambda \propto v$

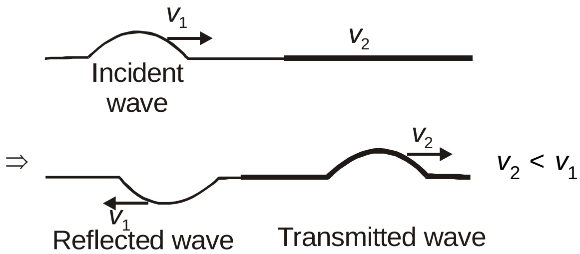
As a wave changes medium its speed and wavelength changes but frequency remains same.

Reflection of Waves

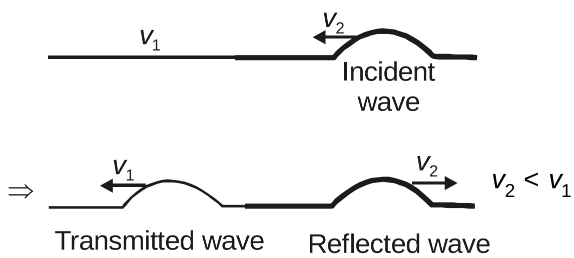
If a wave travelling in a medium of high velocity gets reflected from the surface of a medium of low velocity, it suffers a phase change π.

(1) At the interface of a rarer and denser medium.

(a) Wave is moving from rarer to denser medium.

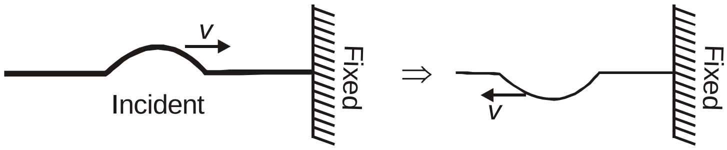


(b) Wave is moving from denser to rarer medium

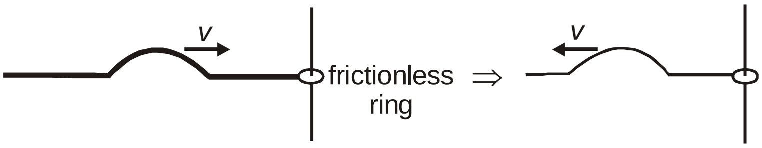


The transmitted wave is always in phase with incident wave.

(2) Reflection from fixed end :



(3) Reflection from free end :

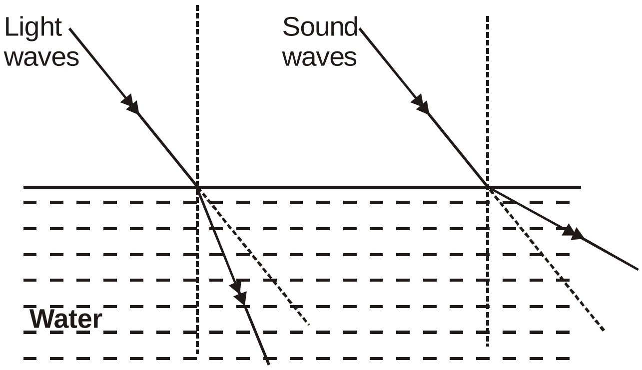


Refraction of waves

Media can be classified as

1. **Rarer Medium :** A medium in which speed of wave is greater.
2. **Denser Medium :** A medium in which speed of wave is smaller.

For example, in case of light, air is rare medium and water is denser medium as speed of light is more in air than in water.



But in case of sound air is denser medium and water is rarer medium as speed of sound in air is less than in water.

Harmonic Wave and Various Terms

If the source of the wave is a simple harmonic oscillator, then the function $f(x \pm vt)$ is sinusoidal and it represents a harmonic wave. This function, in general, can be written as,

$$y = A\sin[k(x \pm vt) + \phi]$$

or $$y = A\sin(kx \pm \omega t + \phi)$$

Various terms used to describe wave are :

1. Amplitude (A) : It is the maximum displacement of a particle in the medium from its equilibrium position.

2. Wavelength (λ) : It is the distance between the two successive points with the same phase.

3. Propagation constant or Angular wave number (k) : $k = \dfrac{2\pi}{\lambda}$

4. Wave velocity (v) : $v = \dfrac{\lambda}{T} = \lambda f = \dfrac{\omega}{k}$

5. Phase : $kx \pm \omega t + \phi$

6. Initial phase : ϕ

7. Wave number : $\dfrac{1}{\lambda}$

Relation between phase difference and path difference

$$\frac{\Delta\phi}{\Delta x} = \frac{2x}{\lambda}$$

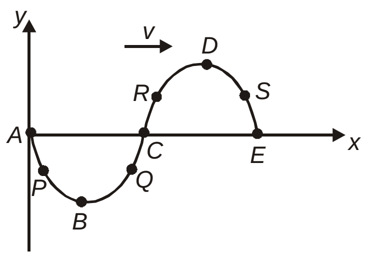
$\Delta\phi$ = Phase difference

Δx = Phase difference

Analysis of harmonic waves

1. For a transverse wave $y = A\sin(\omega t - kx)$

at $t = 0$, photograph of the wave is



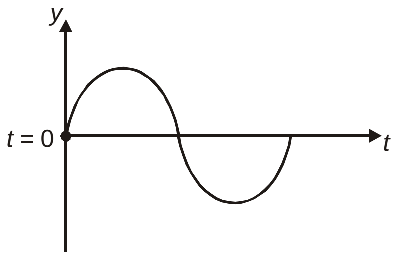
$y = A\sin(-kx)$

Particles at A, P, S, E are moving upwards

Particles at Q, C, R are moving downwards

Particles at B and D are at rest

For the particle at $x = 0$

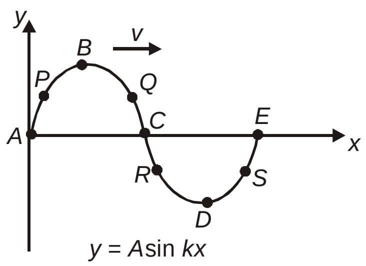


$y = A\sin\omega t$

This particle is moving upward at $t = 0$

2. For wave $y = A\sin(kx - \omega t)$

at $t = 0$, photograph of the wave is

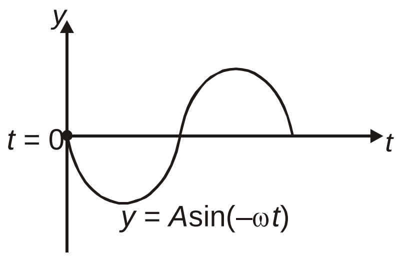


Particles at A, P, S, E are moving downwards

Particles at Q, C, R are moving upwards

Particles at B and D are at rest

For the particle at $x = 0$



This particle is moving downward at $t = 0$

Some Important Points :

1. Particle velocity, $V_{Pa} = -V\left(\frac{dy}{dx}\right)$ where V = wave velocity and $\frac{dy}{dx}$ = slope of the wave
2. In general, points with positive slope $\left(\frac{dy}{dx}\right)$ move downward.
3. The points with negative slope move upward.
4. The points with maximum slope (A, C, E) have maximum velocity.
5. The points with zero slope are at rest.

SOUND WAVES

They are mechanical and longitudinal waves. They propagate in form of compressions and rarefactions. Particle displacements can be represented by wave function

$$S = A\sin(\omega t - kx)$$

As particles oscillate, pressure variation takes place according to the wave function.

$$\Delta P = \Delta P_0 \cos(\omega t - kx),\ \Delta P_0 = \text{maximum pressure variation}$$

> ***Note :*** *Points where displacement is maximum, pressure variation is zero i.e. there is a phase difference of 90° between pressure wave and displacement wave.*

Characteristic of Sound

1. Loudness

Sensation of sound produced in human ear due to amplitude. It depends upon intensity, density of medium, presence of surrounding bodies,

(a) Intensity of Wave

$I = 2\pi^2 f^2 A^2 \rho v$

$I \propto f^2$ and $I \propto A^2$

$I = \dfrac{P}{4\pi r^2}$ P = Power of point source

$I \propto \dfrac{1}{r^2}$ (for a point source)

$I \propto \dfrac{1}{r}$ (for a line source)

(b) Intensity Level or (Sound Level) (β)

$$\beta = 10\log_{10}\left(\frac{I}{I_0}\right)\text{(dB)} \qquad \left[\begin{array}{l} I_0 = \text{minimum intensity of audible sound} = 10^{-12}\ \text{W/m}^2 \\ I = \text{measured intensity} \end{array}\right]$$

Sound level range for audible sound [0 dB to 120 dB]

$\beta_2 - \beta_1 = 10\log_{10}\left(\dfrac{I_2}{I_1}\right)$, Unit of sound level β is decibel (dB)

2. Quality : Sensation produced in human ear due to shape of wave. Quality is that characteristic of sound by which we can differentiate between the sound of same pitch and loudness coming from different sources.

3. Pitch : Sensation produced in human ear due to frequency.

(a) Pitch is the characteristics of sound that depends on frequency.

(b) Smaller the frequency smaller the pitch, higher the frequency higher the pitch.

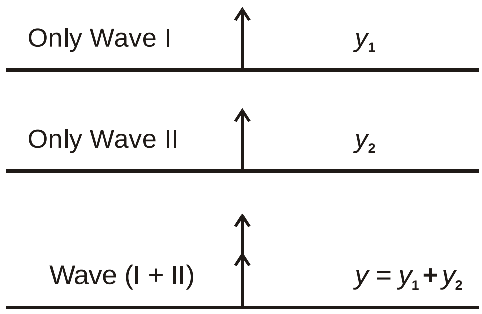
(c) Humming of mosquito has high pitch (high frequency) but low intensity (low loudness) while the roar of a lion has high intensity (loudness) but low pitch.

Classification of Waves based on Frequency :

1. **Infrasonic Wave :** Longitudinal waves having frequencies below 20 Hz are called infrasonic waves. They cannot be heard by human beings. They are produced during earth quakes. It can be heard by snakes.
2. **Audible Waves :** Longitudinal waves having frequencies lying between 20-20,000 Hz are called audible waves.
3. **Ultrasonic Waves :** Longitudinal waves having frequencies above 20,000 Hz are called ultrasonic waves. They are produced and heard by bats. They have a large energy content.
4. **Shock Waves :** A body moving with speed greater than speed of sound (supersonic speed) produces a conical disturbance called a shock waves.

SUPERPOSITION OF WAVES

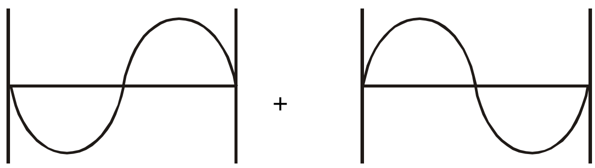
If number of waves are travelling through a medium then resultant displacement of a particle of medium is sum of individual displacements produced by individual waves in the absence of other.



Standing Waves

When two waves identical in all respects, but travelling in opposite direction along a straight line, superimpose on each other, standing waves are produced.

Let $y_1 = A\sin(\omega t - kx)$ and $y_2 = A\sin(\omega t + kx)$



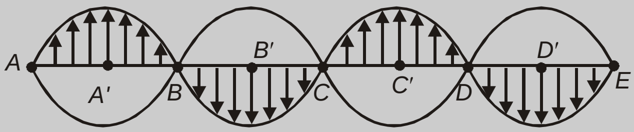
$\Rightarrow$ $y = y_1 + y_2 = 2A\cos kx \sin\omega t$

$2A\cos kx$ represents the amplitude of particle located at 'x'.

Some Important Points :

1. For $x = 0$, $x = \frac{\lambda}{2}, \lambda$ and so on, amplitude is maximum *i.e.,* $2A$. These points are called antinodes.
2. For $x = \frac{\lambda}{4}, \frac{3\lambda}{4}$, ... and so on amplitude is minimum *i.e., O*. These points are called nodes.
3. Distance between consecutive nodes = distance between consecutive antinodes = $\frac{\lambda}{2}$.
4. Distance between adjacent node and antinodes = $\frac{\lambda}{4}$.
5. All the particles in same loop *i.e.*, between two adjacent nodes vibrate in same phase.
6. Particles on the opposite side of a node vibrate in opposite phase.

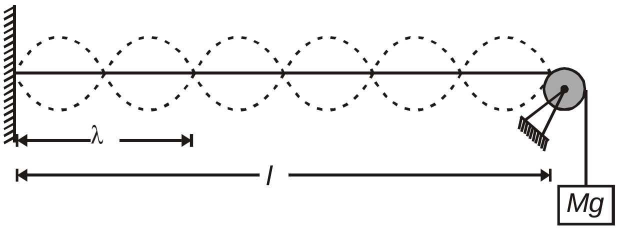
7. Node is always at rest. There is no transfer of energy across a node.
8. In longitudinal stationary waves, a point where displacement node is formed, pressure is maximum *i.e.* pressure antinode is formed.
9. All the particles of medium pass through their mean position simultaneously twice in each time period.



A, *B*, *C*, *D*, *E* are nodes.

A′, *B*′, *C* ′, *D*′ are antinodes.

(1) Sonometer : In this case, transverse stationary waves are formed.



The wire vibrates in *n* loops, then

$$l = \frac{n\lambda}{2} \text{ or } \lambda = \frac{2l}{n}$$

velocity $v = \sqrt{\frac{T}{\mu}}$ where ‘μ’ is mass per unit length of wire.

$$\Rightarrow \quad \nu_n = \frac{v}{\lambda} = \frac{nv}{2l} = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$$

If the wire vibrates in simplest mode,

$$\nu_1 = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \qquad \text{[Fundamental mode, I}^{\text{st}}\text{ harmonic]}$$

For nth harmonic,

$$\nu_n = \frac{n}{2l}\sqrt{\frac{T}{\mu}} \qquad [(n-1)^{\text{th}} \text{ overtone, } n^{\text{th}} \text{ harmonic}]$$

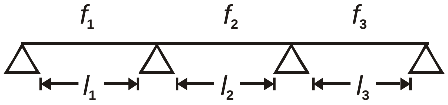
Case : A wire is to be divided in three parts whose fundamental frequencies are f_1, f_2 and f_3.

$$l_1 + l_2 + l_3 = l \qquad \ldots(1)$$

$$l_1 : l_2 : l_3 : : \frac{1}{f_1} : \frac{1}{f_2} : \frac{1}{f_3} \quad \ldots(2)$$

From (1) & (2), we get,

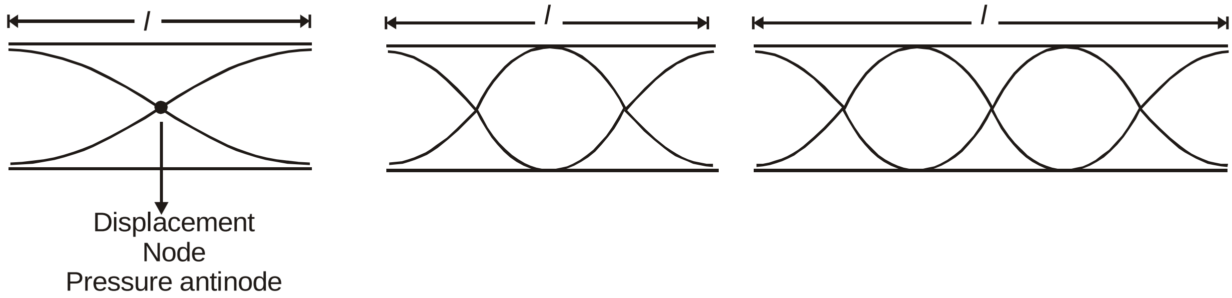
$$l_1 = \frac{f_2 f_3}{f_1 f_2 + f_2 f_3 + f_1 f_3} l$$



$$l_2 = \frac{f_1 f_3 l}{\Sigma f_1 f_2}, \; l_3 = \frac{f_1 f_2}{\Sigma f_1 f_2} l$$

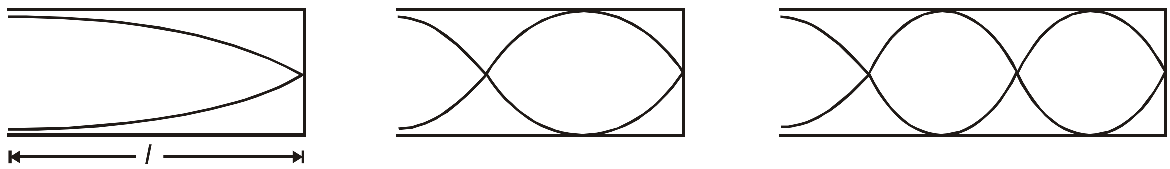
(2) Organ Pipe : In this case, longitudinal stationary waves are formed

(a) Open organ Pipe :



$l = \frac{\lambda}{2}$ or $\lambda = 2l$	$l = \lambda$	$l = \frac{3\lambda}{2}$
$\nu_1 = \frac{V}{\lambda} = \frac{V}{2l}$	$\nu = \frac{V}{l} = 2\nu_1$	$\lambda = \frac{2l}{3}$
1st harmonic or	2nd harmonic	$\nu_0 = \frac{3V}{2l} = 3\nu_1$
Fundamental mode	Ist overtone	3rd harmonic
		2nd overtone

(b) Closed organ pipe :



$l = \frac{\lambda}{4} \Rightarrow \lambda = 4l$	$l = \frac{3\lambda}{4} \Rightarrow \lambda = \frac{4l}{3}$	$l = \frac{5\lambda}{4} \quad \lambda = \frac{4l}{5}$
$\nu = \frac{V}{4l}$	$\nu = \frac{3V}{4l}$	$\nu = \frac{5V}{4l}$
Fundamental mode	Ist overtone	2nd overtone
Ist harmonic	3rd harmonic	5th harmonic

Pipe length 'l'	Fundamental Mode	Ist Overtone	(n – 1)th overtone	Ratio of Successive frequency
Open	$\nu = \frac{V}{2l}$ Ist Harmonic	$\nu = \frac{V}{l}$ 2nd Harmonic	$\nu = n\frac{V}{2l}$ nth Harmonic	1 : 2 : 3 : 4
Closed	$\nu = \frac{V}{4l}$ Ist Harmonic	$\nu = \frac{3V}{4l}$ 3rd Harmonic	$\nu = (2n-1)\frac{V}{l}$ $(2n-1)$th Harmonic	1 : 3 : 5 : 7

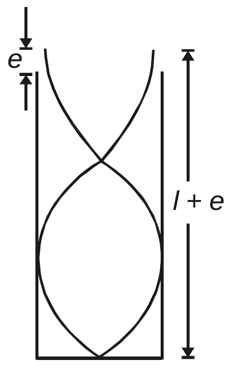
Note : *Even numbered (i.e., 2nd, 4th .....) harmonics do not exist in close organ pipe.*

End correction :

As the antinodes are formed slightly out side the open end.

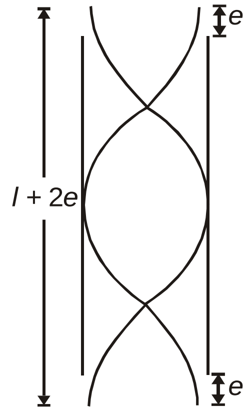
e = 0.6r = end correction. Thus, we have,

For closed organ pipe



$$\nu = \frac{(2n-1)V}{4(l+e)}$$

For open organ pipe



$$\nu = \frac{nV}{2(l+2e)}$$

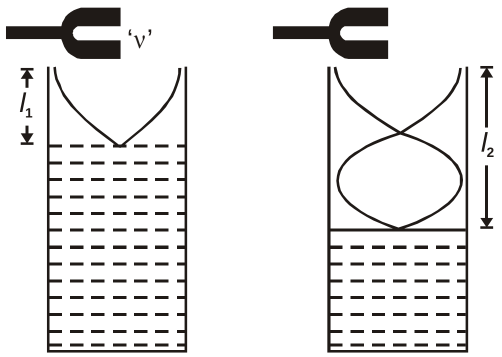
Resonance Tube:

If resonance is obtained first at length l_1.

then at length l_2, then

$\lambda = \mathbf{2(l_2 - l_1)}$

$\Rightarrow$ distance between two successive lengths is $\frac{\lambda}{2}$



BEATS

It is the phenomenon of periodic variation in intensity at a particular position on account of superposition of wave of nearly equal frequencies. When two waves of same amplitude and nearly equal frequencies ν_1 and ν_2 superimpose on each other.

(1) The amplitude at a given position varies with frequency $\left|\frac{\nu_1 - \nu_2}{2}\right|$.

(2) The intensity at a given position varies with frequency $|\nu_1 - \nu_2|$. This frequency of variation of intensity is called beat frequency.

(3) Frequency of the resulting wave is $(v_1 + v_2)/2$.

> ***Note :*** *To observe the phenomenon of beats, beat frequency should be less than 10 Hz.*

Interference

Consider two waves of same frequency and wavelength,

$y_1 = a_1 \sin(\omega t - kx)$, $I_1 = Ca_1^2$

$y_2 = a_2 \sin(\omega t - kx + \phi)$, $I_2 = Ca_2^2$

Equation of resultant wave is,

$y = y_1 + y_2 = A \sin(\omega t - kx + \theta)$, where $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$ and $\theta = \tan^{-1}\left(\frac{a_2 \sin\phi}{a_1 + a_2\cos\phi}\right)$

Resultant Intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Max. Int. : $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$ where phase difference $\phi = 2n\pi$, path difference = $2n\frac{\lambda}{2}$

Min. Int. : $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$ where phase difference $\phi = (2n + 1)\pi$, path difference = $(2n+1)\frac{\lambda}{2}$

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{\frac{I_1}{I_2}}+1}{\sqrt{\frac{I_1}{I_2}}-1}\right)^2 = \left(\frac{a_1+a_2}{a_1-a_2}\right)^2 = \left(\frac{\frac{a_1}{a_2}+1}{\frac{a_1}{a_2}-1}\right)^2$$

For equal intensity $I_{max} = 4I_0$, $I_{min} = 0$, $I = 4I_0 \cos^2 \frac{\phi}{2}$.

DOPPLER'S EFFECT

If a wave source and a receiver are moving relative to each other, the frequency observed by the receiver (f) is different from the actual source frequency (f_0) given by,

$$f = f_0\left(\frac{v \pm v_0}{v \mp v_s}\right)$$ Where v = speed of sound, v_0 = speed of observer, v_s = speed of source

Various cases :

1. Source at rest, observer moves

(a) Observer moves away from source, $f = f_0\left(\frac{v - v_0}{v}\right)$

(b) Observer moves towards source, $f = f_0\left(\frac{v + v_0}{v}\right)$

2. Observer at rest, source moves

(a) Source moves towards observer, $f = f_0\left(\frac{v}{v - v_s}\right)$

(b) Source moves away from observer, $f = f_0\left(\frac{v}{v + v_s}\right)$

3. Both move

(a) Both approaching each other

$$f = f_0\left(\frac{v+v_0}{v-v_S}\right)$$

(b) Source following the observer

$$f = f_0\left(\frac{v-v_0}{v-v_S}\right)$$

(c) Observer following the source

$$f = \left(\frac{v+v_0}{v+v_S}\right)f_0$$

(d) Both moving away from each other

$$f = \left(\frac{v-v_0}{v+v_S}\right)f_0$$

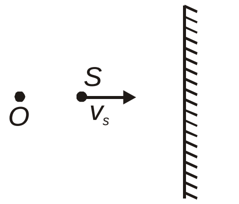
Application of beats and Doppler Effect :

Case - I :

f = frequency of source v_s = velocity of source

f' = frequency of direct sound v = velocity of sound

f'' = frequency of reflected sound



Cliff/Wall
at rest

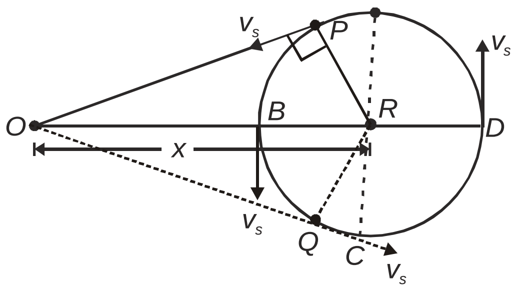
$$f' = \left(\frac{v}{v + v_s}\right)f \quad \text{(source moving away)}$$

$$f'' = \left(\frac{v}{v - v_s}\right)f \quad \text{(source moving towards)}$$

$$\text{Beat frequency} = f'' - f' = \left(\frac{v}{v - v_s} - \frac{v}{v + v_s}\right)f = \left(\frac{v \times 2v_s}{v^2 - v_s^2}\right)f$$

Case - II :

A source of frequency 'f' is revolving in a circle of radius R with speed v_s. An observer is standing at a distance x from the centre.

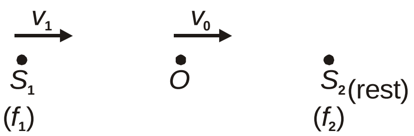


At B and D, observed frequency is 'f '. At 'P' frequency is maximum as $OP \perp PR$, *i.e.*,

$$f' = \left(\frac{v}{v - v_s}\right)f \quad OP = \sqrt{OR^2 - PR^2} = \sqrt{x^2 - R^2}$$

At Q frequency is minimum as $OQ \perp QR$, *i.e.*, $f'' = \left(\frac{v}{v + v_s}\right)f$

Case - III :



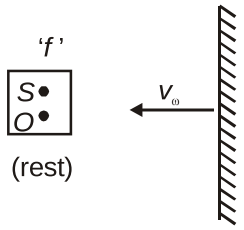
v = velocity of sound

$$f_1' = \left(\frac{v - v_0}{v - v_1}\right)f_1$$

$$f_2' = \left(\frac{v + v_0}{v}\right)f_2 \quad \text{beat frequency} = f_1' - f_2'$$

Case - IV :

Direct frequency = f



v = velocity of sound

(movable reflecting wall)

$$\text{Reflected frequency} = \left(\frac{v + v_\omega}{v - v_\omega}\right)f$$

$$\text{Beat frequency} = -f\left(\frac{v + v_\omega}{v - v_\omega} - 1\right) = f\left(\frac{2v_\omega}{v - v_\omega}\right)$$

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STUDY PACKAGE – 01

For – JEE / NEET

ASSIGNMENT

ASSIGNMENT

Objective Type Question

1. Units and Measurement

Choose the correct answer :

1. If $x = k \sin(klt)$, where x is displacement and t is time then dimensional formula for l will be (k, l = constant)

   (1) $[M^0L^0T^0]$ (2) $[M^0L^1T^0]$

   (3) $[M^0L^{-1}T^{-1}]$ (4) $[ML^{-1}T^{-1}]$

2. If y represents mass and x represents velocity, then dimensional formula for $\frac{d^3y}{dx^3}$ will be

   (1) $[M^0L^0T^0]$ (2) $[ML^{-3}T^3]$

   (3) $[M^3L^{-3}T^3]$ (4) $[M^3L^3T^{-3}]$

3. If length of a rectangle is 2.1 m and width is 1.62 m then its area will be

   (1) 3.402 m^2 (2) 3.4 m^2

   (3) 3.40 m^2 (4) 3 m^2

4. The dimensions of the ratio of angular to linear momentum is

   (1) $[M^0LT^0]$ (2) $[MLT^{-1}]$

   (3) $[ML^2T^{-1}]$ (4) $[M^{-1}L^{-1}T^{-1}]$

5. If percentage error in area of a cube is 4 then the percentage error in volume of cube will be

   (1) 6 (2) $\frac{8}{3}$

   (3) 8 (4) 4

6. Dimensional formula for $\frac{L}{R^2C}$ is (symbols have their usual meaning)

   (1) $[M^0L^0T]$ (2) $[M^0L^0T^2]$

   (3) $[M^0L^0T^0]$ (4) None of these

7. If $y = \frac{t}{\varepsilon_0 LV}$ where t is time, L is length, V is potential and ε_0 is permittivity, then dimension of y is same as that of

   (1) Resistance (2) $(\text{Current})^{-1}$

   (3) $(\text{Voltage})^{-1}$ (4) Charge

8. A new system of unit is chosen in which the unit of mass is taken as 10 kg, unit of length is taken as 1 m and unit of time is taken 10 second. 1 Joule in this new system is numerically equal to

   (1) 1 (2) 1000

   (3) 10 (4) $\frac{1}{10}$

9. Which of the following relations cannot be deduced using dimensional analysis?

   a. $y = A\sin(\omega t + kx)$

   b. $K = \frac{1}{2}I\omega^2 + mgh$

   c. $N = N_0e^{-\lambda t}$

   (1) a only (2) a and b

   (3) a, b and c (4) c only

10. If the number of particles crossing per unit volume perpendicular to x-axis per unit time is given by $N = D(N_2 - N_1)(x_2 - x_1)$ where N_2 and N_1 are number of particles per unit area at x_2 and x_1 respectively, then dimensional formula for $\frac{Dx}{t}$ is, (where t is time)

    (1) $[M^0L^2T^{-1}]$ (2) $[M^0L^{-2}T^1]$

    (3) $[M^0L^{-1}T^{-2}]$ (4) $[M^0L^{-1}T^0]$

11. The position of a particle at time t is given by the relation $x = \frac{v_0}{\alpha}(1 - C^{-\alpha t})$. The dimensional formula for $\alpha^2 v_0^3$ will be

    (1) $[L^3T^{-4}]$ (2) $[L^3T^{-5}]$

    (3) $[LT^{-2}]$ (4) $[L^2T^{-3}]$

12. If energy stored (E) in capacitor is $\frac{Q^2}{2C}$, where C is capacitance and Q is charge on it and error in measurement of C and Q are 1% and 0.5%, then maximum percentage error in energy stored (E), will be

    (1) 2 (2) 2.5

    (3) 1.5 (4) 0.5

13. In the relation $P = \frac{\alpha}{\beta} e^{\frac{-\alpha z}{k\theta}}$ if P is pressure. z is distance, k is Boltzman's constant and v is temperature. The dimensional formula of $\frac{\alpha}{\beta}$ is

(1) $[ML^{-1}T^{-2}]$ (2) $[M^0L^2T^0]$

(3) $[M^0L^2T^{-1}]$ (4) $[ML^2T^{-1}]$

14. A quantity x is defined by the equation $x = 3CB^2$ where C is capacitance and B is magnetic field, then dimension of x are

(1) $[ML^{-2}]$

(2) $[ML^{-2}T^{-2}A]$

(3) $[ML^{-2}T^{-2}A^2]$

(4) $[L^{-1}A^{-1}]$

15. Dimensional formula of a physical quantity is $[M^{-1}L^3T^{-2}]$. The errors in measuring quantities M, L and T respectively are 2%, 3% and 4%. The maximum percentage error that occurs in measuring the quantity is

(1) 9 (2) 10

(3) 14 (4) 19

16. Displacement of particle in n^{th} second moving in straight line is given by $S_n = u + a(2n - 1)$. This equation is

(1) Numerically correct only

(2) Dimensionally correct only

(3) Both numerically and dimensionally correct

(4) Neither numerically nor dimensionally correct

17. The volume V of water passing any point of a uniform tube during t seconds is related to the cross-sectional area of the tube (A) and velocity of water (v) and given by $V \propto A^a v^b t^c$, then which of the following will be true?

(1) $a = b = c$

(2) $a \neq b = c$

(3) $a = b \neq c$

(4) $a \neq b \neq c$

18. Choose the incorrect statement

(1) 0.0037218 has five significant digits

(2) 4.3500 has three significant digits

(3) 1560 has three significant digits

(4) 7.650 has four significant digits

19. If $F = ax + bt^2 + c$ where F is force, x is distance and t is time. Then what is dimension of $\frac{axc}{bt^2}$?

(1) $[M L^2 T^{-2}]$ (2) $[M L T^{-2}]$

(3) $[M^0 L^0 T^0]$ (4) $[M L T^{-1}]$

20. Which of the following is dimensionless? Here

μ_0 = permeability ε_0 = permittivity

B = magnetic field E = electric field

(1) $\mu_0\varepsilon_0 \frac{E^2}{B^2}$ (2) $\frac{\varepsilon_0 E^2}{\mu_0 B^2}$

(3) $\frac{E^2}{\mu_0\varepsilon_0 B^2}$ (4) $\frac{\mu_0 E^2}{\varepsilon_0 B^2}$

21. If force, time and velocity are treated as fundamental quantities then dimensional formula of energy will be

(1) [FTV] (2) $[FT^2V]$

(3) $[FTV^2]$ (4) $[FT^2V^2]$

22. In a new system of units, unit of mass is 10 kg, unit of length is 100 m, unit of time is 1 minutes. Then magnitude of 1 N force in new system of units will be

(1) 36 (2) 60

(3) 3.6 (4) 0.06

23. Error in measurement of volume of sphere is 1%. The error in calculation of surface area of sphere will be

(1) $\frac{2}{3}$% (2) $\frac{3}{2}$%

(3) 3% (4) $\frac{1}{3}$%

24. The period of oscillation of spring pendulum is given by $T = 2\pi\sqrt{\frac{m}{k}}$ where m is 100 gm and is know to have 0.1 gm accuracy. The period is about 2 s. The time of 100 oscillations is measured by a stop watch of least count 0.1 s. The percentage error in k is

(1) 0.1%

(2) 1%

(3) 0.2%

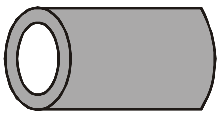
(4) 0.8%

25. A physical quantity X is related to four variables a, b, c and d as follows, $X = \frac{a^2b^3}{c\sqrt{d}}$. Errors in measurement of a, b, c, d are 1%, 3%, 2% and 2% respectively. What is percentage error in quantity X?

(1) 12% (2) 8%

(3) 14% (4) 5%

26. The inner diameter and outer diameter of a hollow cylinder are measured to be (5.3 ± 0.1) cm and (6.8 ± 0.1) respectively. Thickness of cylinder will be



(1) 0.8 ± 0.2 (2) 1.5 ± 0.1

(3) 1.5 ± 0.2 (4) 0.8 ± 0.1

27. Number of significant figure in 6.020×10^{23} is

(1) 3 (2) 24

(3) 4 (4) 1

28. Unit of self inductance is

(1) $\frac{\text{newton} \times \text{second}}{\text{coulomb} \times \text{ampere}}$ (2) $\frac{\text{joule} \times \text{second}}{\text{coulomb} \times \text{ampere}}$

(3) $\frac{\text{volt} \times \text{metre}}{\text{coulomb}}$ (4) $\frac{\text{newton} \times \text{metre}}{\text{ampere}}$

29. Dimensions of luminous flux are

(1) $M L^2 T^{-2}$ (2) $M L^2 T^{-3}$

(3) $M L^2 T^{-1}$ (4) $M L T^{-2}$

30. If x = 2.220 cm and y = 2.1 cm then $x - y$ =

(1) 0.120 cm (2) 0.12 cm

(3) 0.1 cm (4) 1.2×10^{-1} cm

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2. Description of Motion in One Dimension

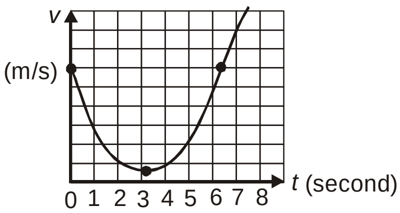
Choose the correct answer :

1. Equation of motion of a body moving along x-axis at an instant t second is given by $x = 40 + 12t - t^3$ m. Displacement of the particle before coming to rest is

   (1) 16 m (2) 56 m

   (3) 24 m (4) 40 m

2. Velocity-time graph for a particle is shown in figure. Starting from $t = 0$, at what instant t, average acceleration is zero between 0 to t?



   (1) 1 s (2) 3.5 s

   (3) 6.3 s (4) 7.3 s

3. If a particle is thrown with velocity more than 10 m/s vertically upward, then the distance travelled by the particle in last second of its ascent is

   (1) g (2) $\frac{g}{2}$

   (3) $\frac{g}{4}$ (4) $\frac{g}{8}$

4. A car starts from rest travelling with constant acceleration. If distance covered by it in 10^{th} second of its journey is 19m, what will be the acceleration of car?

   (1) 4 m/s^2 (2) 3 m/s^2

   (3) 2 m/s^2 (4) 1 m/s^2

5. A boat takes time t_1 hour to cover certain distance between two spots in a river in downstream and t_2 hours in upstream. The ratio of speed of boat to that of river is

   (1) $\frac{t_1}{t_2}$ (2) $\frac{t_1+t_2}{t_2-t_1}$

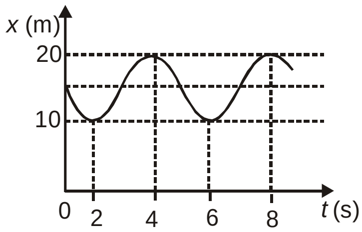
   (3) $\frac{t_2}{t_1}-\frac{t_1}{t_2}$ (4) $\frac{t_2-t_1}{t_1+t_2}$

6. A stone is dropped from the top of a tower and travels 24.5 m in the last second of its journey. The height of the tower is

   (1) 44.1 m (2) 49 m

   (3) 78.4 m (4) 72 m

7. Figure shows the position of a particle moving on the x-axis as a function of time. Choose the wrong statement



   (1) The particle has come to rest 4 times

   (2) The maximum speed is at $t = 4$ s

   (3) The average velocity is zero for $t = 2$s to $t = 6$s

   (4) Motion of particle is non-uniformly accelerated accelerated

8. The two ends of train moving with constant acceleration pass a certain point with velocities u and 2 u. The velocity with which, the middle point of the train passes the same point is

   (1) 2.5 u (2) 1.5 u

   (3) $\sqrt{5}\,u$ (4) $\sqrt{2.5}\,u$

9. A body is started from rest with acceleration 2 m/s^2 till it attains the maximum velocity then retards to rest with 3 m/s^2. If total time taken is 10 second then maximum speed attained is

   (1) 12 m/s (2) 8 m/s

   (3) 6 m/s (4) 4 m/s

10. A body moving with constant acceleration on a straight path travels 200 cm in the first two seconds and 220 cm in the next four seconds. Its velocity after 7th second from the start is

    (1) 220 cm/s (2) 100 cm/s

    (3) 50 cm/s (4) 10 cm/s

11. If a particle starting from rest has an acceleration that increases linearly with time as $a = 2t$, then the distance travelled in third sec will be

    (1) 9 m (2) $\frac{8}{3}$ m

    (3) $\frac{19}{3}$ m (4) $\frac{11}{3}$ m

12. A particle starts with velocity v_0 at at time $t = 0$ and is decelerated at a rate proportional to the square root of its speed at time t with constant of proportionality α. The total time for which it will move before coming to rest is

(1) $\sqrt{v_0}$ (2) $\dfrac{v_0^{3/2}}{\alpha}$

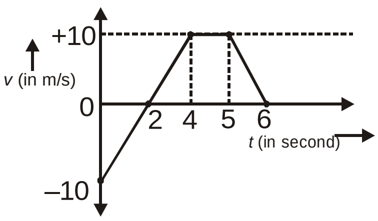
(3) $\dfrac{2v_0^{3/2}}{\alpha}$ (4) $\dfrac{2\sqrt{v_0}}{\alpha}$

13. Two balls X and Y are thrown from top of tower one vertically upward and other vertically downward with same speed. If time taken by them to reach the ground is 6 sec and 2 sec respectively, then the height of the tower and initial speed of each ball are (g = 10 m/s^2)

(1) 60 m, 15 m/s (2) 80 m, 20 m/s

(3) 60 m, 20 m/s (4) 45 m, 10 m/s

14. For the velocity ~ time (v ~ t) graph as shown in figure, the incorrect statement is



(1) The average velocity for the entire journey is 2.5 m/s

(2) The average acceleration from 1 s to 4 s is 5 m/s^2

(3) The average speed for the first 4 s is zero

(4) The acceleration at $t = 3$ s is 5 m/s^2

15. A body moves along curved path of a quarter circle. The ratio of magnitude of displacement to distance is

(1) $\dfrac{\pi}{2\sqrt{2}}$ (2) $\dfrac{\pi}{2}$

(3) $\dfrac{2\sqrt{2}}{\pi}$ (4) $\dfrac{3\pi}{2\sqrt{2}}$

16. The velocity of a particle moving along positive X-axis varies as $v = \alpha\sqrt{x}$ where α is a constant. If particle is at $x = 0$ at $t = 0$, what will be the average velocity of particle during the time it move a distance S?

(1) $\dfrac{\alpha}{2}\sqrt{S}$ (2) $\dfrac{2}{\alpha}\sqrt{S}$

(3) $\alpha\sqrt{S}$ (4) $\dfrac{\sqrt{S}}{\alpha}$

17. A ball is released from certain height h reaches ground in time T. Where will it be from the ground at time $\dfrac{3T}{4}$?

(1) $\dfrac{9h}{16}$ (2) $\dfrac{7h}{16}$

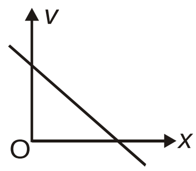
(3) $\dfrac{3h}{4}$ (4) $\dfrac{27h}{64}$

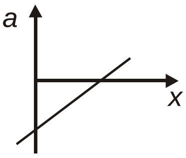
18. A particle moves along x-axis in such a way that its x-co-ordinate varies with time according to the equation $x = 8 - 4t + 6t^2$. The distance covered by particle between t = 0 to t = $\dfrac{2}{3}$ sec. is

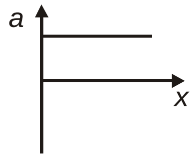
(1) Zero (2) $\dfrac{8}{3}$ m

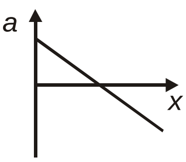
(3) 8 m (4) $\dfrac{4}{3}$ m

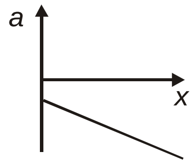
19. Draw a-x graph corresponding to given v-x graph



(1) 

(2) 

(3) 

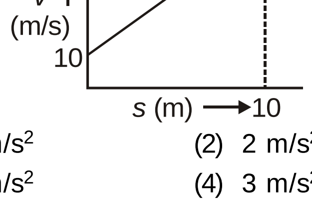
(4) 

20. A ball is dropped from rest from a bridge of 122.5 metres above a river. After two seconds, another ball is thrown straight down after it. The initial velocity of second ball, so that both hit the water at the same instant is

(1) 49 m/s (2) 55.5 m/s

(3) 26.1 m/s (4) 9.8 m/s

21. Velocity of a particle changes with position according to following curve. Acceleration of the particle at $x = 1$ m



(1) 24 m/s^2 (2) 2 m/s^2

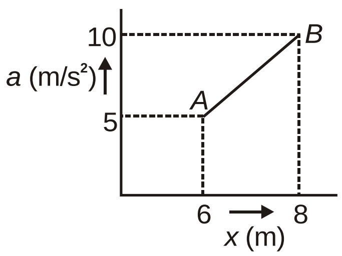
(3) 20 m/s^2 (4) 3 m/s^2

22. A particle is moving along a straight line. Select the correct statement
    (1) The sign of acceleration does not indicate that the particles speed is increasing or decreasing
    (2) The zero velocity of the particle at any instant imply zero acceleration at that instant
    (3) The sign of acceleration clearly indicates that the particle speed is increasing or decreasing
    (4) All of these

23. A ball is thrown vertically upward with a velocity u from the balloon descending with a constant velocity v. The ball will pass by the balloon after time
    (1) $\frac{u+v}{2g}$ (2) $\frac{2u+v}{g}$
    (3) $\frac{2u+2v}{g}$ (4) $\frac{(u+v)}{g}$

24. A particle passes through point A and B which are 90 m apart. It takes 6 s to cover this distance with uniform acceleration. Velocity of particle when it passes through B point is 20 m/s. What is partial velocity at point A?
    (1) 10 m/s (2) 30 m/s
    (3) 15 m/s (4) 20 m/s

25. If acceleration of a particle is varying with x according to curve. Velocity of particle at A is 2 m/s. What is the velocity of particle at point B?



    (1) 4 m/s (2) $\sqrt{34}$ m/s
    (3) $\sqrt{28}$ m/s (4) 7 m/s

26. A particle when thrown vertically upward, moves such that it passes from same height at t = 2 s and t = 10 s, the height is
    (1) g (2) $2g$
    (3) $5g$ (4) $10g$

27. A particle is thrown vertically upward. It is known that distance travelled in 6th second and 7th second is same then
    (1) The particle was thrown with 40 m/s
    (2) The particle was thrown with 70 m/s
    (3) Total vertical height is 245 m
    (4) Total vertical height is 180 m

28. Two cars are moving in the same direction with same speed 30 km/h. They are separated from each other by 5 km. A third car moving in the same direction meets the two cars after an interval of 4 minutes. The speed of the third car is
    (1) 30 km/h (2) 105 km/h
    (3) 140 km/h (4) 45 km/h

29. Velocity of particle starting from rest varies with position according to equation $v = \sqrt{\alpha x}$. What is distance travelled by particle in t second from start?
    (1) $\frac{1}{2}\alpha t^2$ (2) $\frac{1}{4}\alpha t^2$
    (3) $\frac{1}{3}\alpha t^3$ (4) $\frac{1}{6}\alpha t^3$

30. Two particles A and B are moving with equal speed V but in opposite direction such that distance between them remains constant which is D. After how much time A will reach the initial position?
    (1) $\frac{2D}{V}$ (2) $\frac{\pi D}{V}$
    (3) $\frac{2\pi D}{V}$ (4) $\frac{\pi D}{2V}$

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3. Description of Motion in Two and Three Dimensions

Choose the correct answer :

1. If speed of water in river is 4 m/s and speed of swimmer with respect to water is 3 m/s, then in which direction the swimmer must swim so that he will reach directly opposite end?
   (1) 127° with direction of river flow
   (2) 90° with direction of river flow
   (3) 143° with direction of river flow
   (4) Swimmer will never reach directly opposite end

2. Uniform circular motion is not a type of
   (1) Uniform motion
   (2) Non-uniform motion
   (3) Curvilinear motion
   (4) Variable accelerated motion

3. In case of an oblique projectile, the velocity is perpendicular to acceleration
   (1) Once only (2) Twice
   (3) Never (4) Four times

4. If the sum of two unit vectors is also a unit vector, then magnitude of their difference and angle between the two given unit vectors is
   (1) $\sqrt{2}$, 90° (2) $\sqrt{3}$, 120°
   (3) $\sqrt{2}$, 120° (4) $\sqrt{3}$, 60°

5. The angle turned by a body undergoing circular motion depends on time as $\theta = \theta_0 + \theta_1 t + \theta_2 t^2$, the ratio of angular acceleration and initial angular velocity is
   (1) $\frac{2\theta_2}{\theta_1}$ (2) $\frac{\theta_2}{\theta_1}$
   (3) $\frac{\theta_1}{2\theta_2}$ (4) Zero

6. Choose the correct statement related to a non-uniform circular motion
   (1) Tangential acceleration $= \frac{d\vec{v}}{dt}$
   (2) Tangential acceleration $= \frac{d|\vec{v}|}{dt}$
   (3) Centripetal acceleration $= \frac{d\vec{v}}{dt}$
   (4) Acceleration is independent of the radius of circle

7. For a given speed of projection, maximum range of a projectile is *R*. What will be the maximum height attained by projectile when it is projected with double speed by making 60° with vertical?
   (1) $\frac{R}{2}$ (2) $\frac{R}{4}$
   (3) R (4) $\frac{3R}{2}$

8. A particle is projected with velocity $(3\hat{i} + 4\hat{j})$ m/s from horizontal. What will be the height attained by it when velocity becomes perpendicular to acceleration?
   (1) 0.8 m (2) 5 m
   (3) 0.4 m (4) 1.6 m

9. The speed of a particle moving along a circular path is decreasing at the rate of 3 m/s^2. If the radius of circle is 4 m, then what will be the instantaneous acceleration of particle when its speed is 4 m/s?
   (1) 4 m/s^2 towards center
   (2) 3 m/s^2 along tangent
   (3) 5 m/s^2 by making 37° with tangent
   (4) 5 m/s^2 by taking 53° with tangent

10. A ball is projected so as to pass a wall at a distance *a* from the point of projection at an angle of 45° and falls at a distance *b* on the other side of the wall. If *h* is height of wall then
    (1) $h = a\sqrt{2}$ (2) $h = b\sqrt{2}$
    (3) $h = \frac{\sqrt{2}ab}{a+b}$ (4) $h = \frac{ab}{a+b}$

11. The speed of boat is 5 km/h in still water. It crosses a river of width 1 km along shortest possible path in 15 min. The velocity of river water is
    (1) 1 km/h
    (2) 3 km/h
    (3) 4 km/h
    (4) 5 km/h

12. A projectile is thrown with velocity v at an angle θ with horizontal. When the projectile is at a height equal to half of the maximum height, the vertical component of the velocity of projectile is

(1) $v\sin\theta$ (2) $3v\sin\theta$

(3) $\frac{v\sin\theta}{\sqrt{2}}$ (4) $\frac{v\sin\theta}{\sqrt{3}}$

13. When a person walks on a straight road with a speed 10 km/h, rain appears to fall verticlly downward. As he stops, the appears to fall at an angle 30º with vertical. The speed of rain with respect to ground and with respect to the person.

(1) 20 km/hr (2) $10\sqrt{3}$ km/hr

(3) 10 km/hr (4) $20\sqrt{3}$ km/hr

14. A stone is thrown at an angle θ to the horizontal reaches a maximum heigth H. What will be the time of flight of stone?

(1) $\sqrt{\frac{2H}{g}}$ (2) $2\sqrt{\frac{2H}{g}}$

(3) $\frac{2\sqrt{2H\sin\theta}}{g}$ (4) $\frac{\sqrt{2H\sin\theta}}{g}$

15. A projectile has a time of flight T and range R. If time of flight is doubled keeping angle of projection same, then what will be the new range?

(1) R (2) $2R$

(3) $\frac{R}{2}$ (4) $4R$

16. A particle moves along the positive part of the curve $y = \frac{x^2}{2}$, where $x = \frac{t^2}{2}$. What will be velocity of particle at t = 2 sec.

(1) $2\hat{i} - 4\hat{j}$ (2) $2\hat{i} + 4\hat{j}$

(3) $4\hat{i} + 2\hat{j}$ (4) $4\hat{i} - 2\hat{j}$

17. Two projectiles A and B are projected with same speed at angles 30° and 60° to horizontal, then choose the wrong statement? (symbols have their usual meaning)

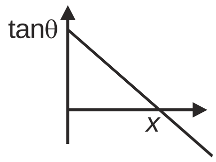
(1) $R_A = R_B$

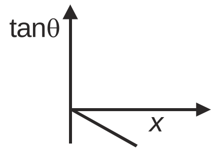
(2) $H_B = 3H_A$

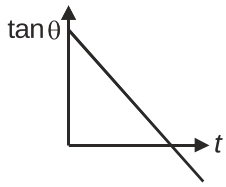
(3) $\sqrt{3}T_B = T_A$

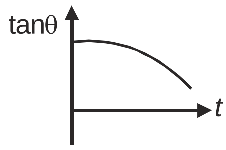
(4) All of these

18. A particle is dropped from a tower in a uniform gravitational field at t = 0. The particle is blown over by a horizontal wind with constant velocity. Slope of trajectory of particle (tan θ) with horizontal varies according to

(1)


(2)


(3)


(4)


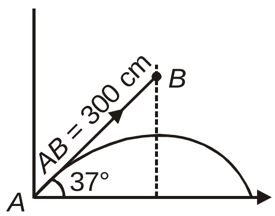
19. The X and Y co-ordinates of a particle are

$x = A\sin\omega t$ and $y = 2A\sin\left(\omega t + \frac{\pi}{2}\right)$, then the motion of the particle is

(1) Circular (2) Parabolic

(3) Rectilinear (4) Elliptical clockwise

20. A ball B is at 300 cm distance from origin on a line 37° above horizontal. Another ball A is projected directly aiming B with initial velocity 700 cm/s. At the same instant B is released from its position. How far will B has fallen when it is hit by A?



(1) 90 cm (2) 80 cm

(3) 70 cm (4) 60 cm

21. A particle is moving towards East with velocity V after some time it is moving with velocity $2V$ towards North. What is magnitude of change in velocity?

(1) V (2) $\sqrt{3}V$

(3) $\sqrt{5}V$ (4) $3V$

22. A particle is projected from ground at angle of 60º with horizontal. After one second its velocity is at angle of 45°. After one more second velocity become horizontal. With what speed the particle is projected? (g = 10 m/s^2)

(1) 10 m/s (2) $\frac{40}{\sqrt{3}}$ m/s

(3) $10\sqrt{5}$ m/s (4) 30 m/s

23. An object moves in x-y plane such that it starts from origin with initial velocity $\vec{u} = 3\hat{i} + 4\hat{j}$ (m/s) and an acceleration $\vec{a} = 2\hat{i}$ (m/s). It's position at $t = 1$ s.

(1) $23\hat{i} + 4\hat{j}$ (2) $30\hat{i} + 40\hat{j}$

(3) $31\hat{i} + 4\hat{j}$ (4) $4\hat{i} + 4\hat{j}$

24. A ball, thrown by a boy, is caught by another boy in 2s. What is the maximum height attained by the ball?

(1) 10 m (2) 5 m

(3) 20 m (4) 7.5 m

25. A stone projected with a velocity of u at angle 30° with horizontal reaches maximum height x. When it is projected with velocity u at angle 30° with vertical it reaches maximum height y. Then range of particle is

(1) $4(x + y)$ (2) $4\sqrt{xy}$

(3) $\frac{2xy}{x+y}$ (4) $4(x - y)$

26. A ball is projected with momentum p at angle θ. What is the change in momentum till it reaches highest point?

(1) $\frac{p\sin\theta}{2}$ (2) $p\cos\theta$

(3) $p\sin\theta$ (4) $\frac{p\cos\theta}{2}$

27. A particle starts with initial velocity is $(2\hat{i} + \hat{j})$ m/s. Uniform acceleration $(-\hat{i} + 3\hat{j})$ m/s^2. What is y component of velocity at the instant when x component of velocity becomes zero?

(1) 5 m/s (2) 7 m/s

(3) 6 m/s (4) 10 m/s

28. A particle is projected in such a way that, taking point of projection as origin, $y = 8t - 5t^2$ and $x = 6t$. What is the range of projectile?

(1) 48 m (2) 4.8 m

(3) 9.6 m (4) 24 m

29. A particle of mass M describes a circle of radius 1 m. The centripetal acceleration of the particle is 4. What will be the momentum of the particle?

(1) 4 M (2) 2 M

(3) 8 M (4) M

30. A projectile is thrown with an initial velocity $(3\hat{i} + a\hat{j})$ m/s. If range of projectile is doubled, the maximum height is reached by it, value of a is

(1) 1.5 m/s (2) 6 m/s

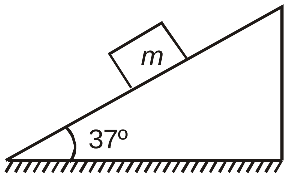
(3) 3 m/s (4) 12 m/s

❑ ❑ ❑

4. Laws of Motion

Choose the correct answer :

1. If co-efficient of friction between a fixed incline plane and the block is 1, then the acceleration of the block shown in the following figure is (g = 10 m/s^2)



(1) Zero (2) 2 m/s^2 upward
(3) 2 m/s^2 downwards (4) 4 m/s^2 downwards

2. If the net external force on a body is zero, then according to Newton's second law
(1) The body must remain at rest
(2) The body must have non-zero acceleration
(3) The body must move with uniform velocity
(4) The momentum of the particle must remain unchanged

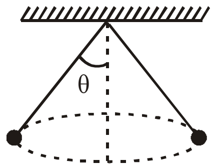
3. A cracker rocket ejecting gases at a rate of 0.02 kg/s with a velocity of 500 m/s. The force on the rocket because of the ejecting gases is
(1) Zero (2) 10 N
(3) 50 N (4) 500 N

4. The necessary condition for a particle to be in circular motion is
(1) A non-zero tangential force must be present
(2) A non-zero centripetal force must be present
(3) Zero net force can also keep the particle at rest
(4) All of these

5. Which of the following is incorrect about friction?
(1) If two bodies slip over each other than the force of friction is given by $f_k = \mu_k N$
(2) If two bodies do not slip over each other then maximum force of slatic friction that can act between the two bodies is '$f_s = \mu_s N$'.
(3) Friction force acting on a body is proportional of normal reaction between the interacting surfaces
(4) None of these

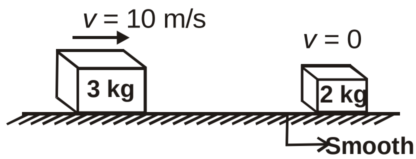
6. Internal forces can change
(1) The linear momentum but not the kinetic energy
(2) The kinetic energy but not the linear momentum
(3) Linear momentum as well as kinetic energy
(4) Neither linear momentum nor the kinetic energy

7. A block of mass m moves in a horizontal uniform circular motion (as shown in the figure) during the motion block is not in



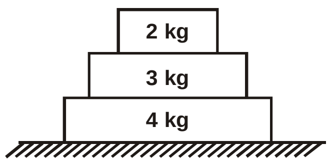
(1) Non-uniform acceleration
(2) Rotational equilibrium
(3) Accelerated motion
(4) Uniform motion

8. If the two blocks collide head on and stick together, on a frictionless horizontal surface, then the interacting impulse between the two blocks is



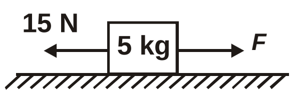
(1) 30 Ns (2) Zero
(3) 18 Ns (4) 12 Ns

9. Three blocks of masses 2 kg, 3 kg and 4 kg are placed one over the other as shown in the figure. Calculate the force exerted by the blocks of mass 2 kg and 4 kg on the 3 kg block (g = 10 m/s^2)



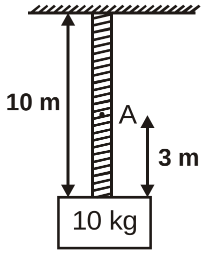
(1) 20 N, 50 N (2) 20 N, 90 N
(3) 20 N, 70 N (4) 20 N, 30 N

10. A block of mass 5 kg is placed on a horizontal surface with co-efficient of friction 'μ = 0.2', then the maximum and minimum value of force F for which the block remains at rest (g = 10 m/s^2)



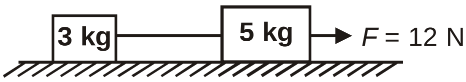
(1) 15 N, 10 N (2) 25 N, 5 N
(3) 10 N, 25 N (4) 5 N, 25 N

11. The adjoining figure shows a block of mass 10 kg connected to free end of a rope of mass 10 kg and length 10 m. The tension of the rope at point A. is (g = 10 m/s^2)



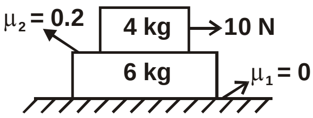
(1) 170 N (2) 30 N
(3) 130 N (4) 70 N

12. In the given arrangement two blocks are placed on a horizontal surface such that co-efficient of friction between the blocks and the floor is μ = 0.2, then tension in the thread connecting the two blocks is (g = 10 m/s^2)



(1) Zero (2) 2 N
(3) 4 N (4) 6 N

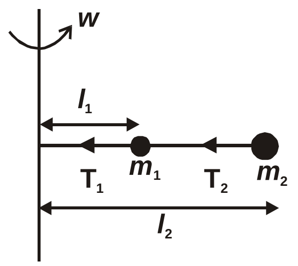
13. Two blocks of mass 4 kg and 6 kg are placed one over the other. If a force of 10 N is applied on the block of mass 4 kg, then the accelerations of the two blocks at the instant shown is (g = 10 m/s^2)



(1) a(4 kg) = a(6 kg) = 0
(2) a(4 kg) = 0.5 m/s^2 a(6 kg) = 0
(3) a(4 kg) = 2 m/s^2 a(6 kg) = 0.5 m/s^2
(4) a(4 kg) = 1 m/s^2 a(6 kg) = 1 m/s^2

14. The whole set up shown in the figure is rotating with constant angular velocity ω on a horizontal frictionless table then the ratio of tensions $\frac{T_1}{T_2}$ is

(Given $\frac{l_2}{l_1} = \frac{2}{1}$)



(1) $\frac{m_1}{m_2}$ (2) $\frac{(m_1 + 2m_2)}{2m_2}$

(3) $\frac{m_2}{m_1}$ (4) $\frac{(m_2 + m_1)}{m_2}$

15. Two friends A and B each having a mass 40 kg are playing with a ball of mass 5 kg, on a frictionless surface A throws the ball towards B with a speed of 4 m/s. Then the speeds of A and B after B catches the ball for the first time are

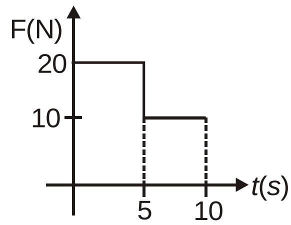
(1) $V_A = \frac{1}{2}$ m/s, $V_B = \frac{4}{15}$ m/s

(2) $V_A = \frac{4}{11}$ m/s, $V_B = 2$ m/s

(3) $V_A = \frac{4}{9}$ m/s, $V_B = \frac{4}{9}$ m/s

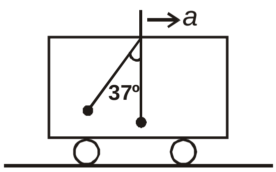
(4) $V_A = \frac{1}{2}$ m/s, $V_B = \frac{4}{9}$ m/s

16. Force is applied on an object of mass 2 kg at rest on a frictionless horizontal surface as shown in the graph. The speed of object after 1 s will be



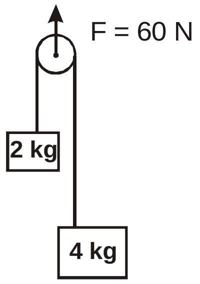
(1) 7.5 m/s (2) 12.5 m/s
(3) 10 m/s (4) 25 m/s

17. A pendulum with a bob of mass 2 kg is hanging from the ceiling of a trolley as shown in the figure. The trolley is uniformly accelerating with an acceleration a, then (g = 10 m/s^2)



(1) a = 6 m/s^2
(2) a = 7.5 m/s^2
(3) a = 4 m/s^2
(4) a = 2.5 m/s^2

18. The acceleration of blocks of mass 2 kg and 4 kg are respectively (Pulleys and threads are massless) (g = 10 m/s^2)



(1) a_1 = 5 m/s^2; a_2 = 2.5 m/s^2

(2) $a_1 = a_2 = 0$

(3) $a_1 = a_2 = \frac{20}{6}$ m/s^2

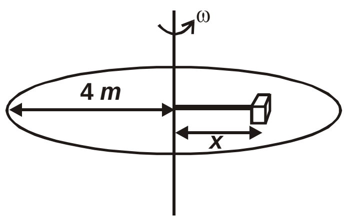
(4) a_1 = 20 m/s^2; a_2 = 5 m/s^2

19. A man is hanging from the free end of a massless rope of length 40 m. The mass of the man is 20 kg and the maximum tension that the rope can bear is 300 N. Then starting from rest, the minimum time in which man can reach the other end of the rope is (g = 10 m/s^2)

(1) 2 *s* (2) 4 *s*

(3) 6 *s* (4) 8 *s*

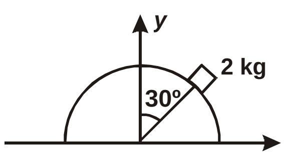
20. A disc of radius 4 m is rotating about its fixed centre with a constant angular velocity ω = 2 rad/ s (in the horizontal plane). A block is also rotating with the disc without slipping. If co-efficient of friction between the block and the disc is 0.4, then the maximum distance at which the block can rotate without slipping is (g = 10 m/s^2)



(1) 1 m (2) 2 m

(3) 3 m (4) 4 m

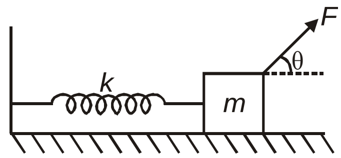
21. The magnitude of friction force acting on the block if the block is in equilibrium is (g = 10 m/s^2)



(1) $20\sqrt{3}$N (2) 20 N

(3) $10\sqrt{3}$N (4) 10 N

22. The block shown in the figure is under equilibrium on smooth surface. Spring force and contact force by ground on the block are



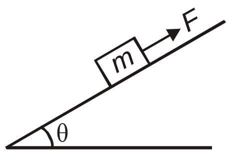
(1) $K.F\sin\theta$, $mg + F\sin\theta$ respectively

(2) $F\cos\theta$, $mg - F\sin\theta$ respectively

(3) $F\cos\theta$, $F\sin\theta$ respectively

(4) $F\sin\theta$, $F\cos\theta$ respectively

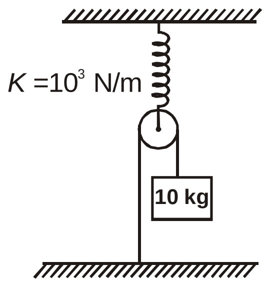
23. Force F is double the minimum force required to keep the block moving with constant speed on a smooth incline. Acceleration of the block is



(1) $\frac{F}{m}$ (2) $g\sin\theta$

(3) $g\cos\theta$ (4) $g\tan\theta$

24. For the arrangement shown in the figure. The extension in the spring, for which the block remains at rest is (g = 10 m/s^2)



(1) 20 cm (2) 25 cm

(3) 15 cm (4) 30 cm

25. Choose the incorrect statement

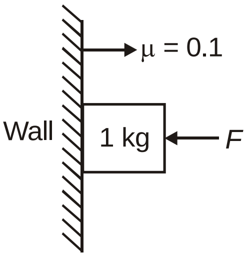
(1) Limiting friction is slightly greater than kinetic friction

(2) According to Newton's third law, forces exist in pairs

(3) Friction force is a fundamental force

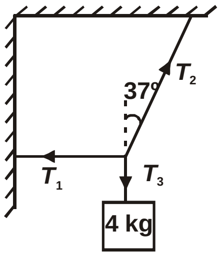
(4) Electromagnetic force between two charged particle depends on the medium between them

26. Minimum value of F to hold the block (see figure) in place at rest is



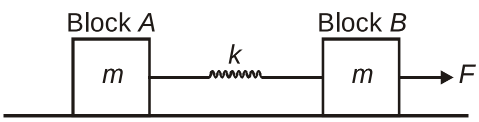
(1) 10 N (2) 100 N
(3) 50 N (4) 500 N

27. Calculate the tensions T_1, T_2 and T_3 in the three threads shown in the following figure. (All threads are mass less) (g = 10 m/s^2)



(1) 30 N, 40 N, 50 N
(2) 50 N, 30 N, 40 N
(3) 35 N, 45 N, 40 N
(4) 30 N, 50 N, 40 N

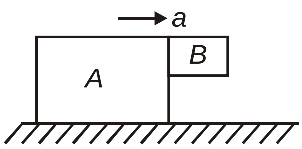
28. Magnitude of force F is double the magnitude of spring force at the instant shown. If acceleration of block B is a, then acceleration of block A is (surface is smooth)



(1) $\frac{a}{2}$ (2) $2a$
(3) $\frac{2a}{3}$ (4) Either of a or $\frac{a}{3}$

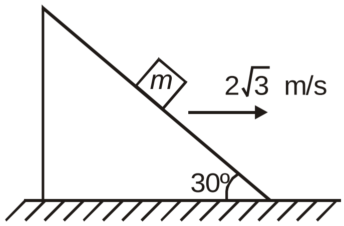
29. If a car topples over while turning
(1) Outer wheels of the car leave the road first
(2) Inner wheels of the car leave the road first
(3) Either inner or outer wheels may leave the road first depending upon the direction in which car is turning
(4) Both inner & outer wheels leave the road simultaneously

30. Block A is placed on a smooth horizontal surface. Another block B is placed in contact with A as shown in figure. The coefficient of friction between A and B is 0.5. The minimum acceleration of block A so that block B does not fall



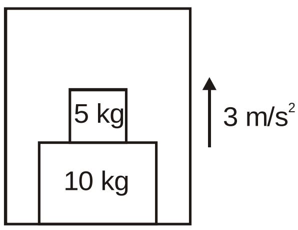
(1) 10 m/s^2
(2) 20 m/s^2
(3) 15 m/s^2
(4) 5 m/s^2

31. A block of mass m is placed on smooth triangular block as shown in the figure. The triangular block is moving horizontally with uniform speed $2\sqrt{3}$ m/s. The acceleration of block of mass m with respect to the triangular block is



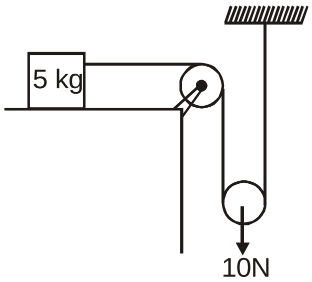
(1) 5 m/s^2
(2) 8 m/s^2
(3) 4 m/s^2
(4) 6 m/s^2

32. The force exerted on 10 kg block by floor of lift, as shown in the figure is (take g = 10 m/s^2)



(1) 180 N
(2) 150 N
(3) 195 N
(4) 135 N

33. Consider the figure, pulley and strings are massless and frictionless. The horizontal surface is smooth. The acceleration of the block



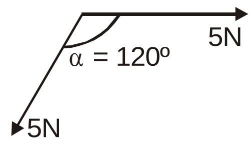
(1) 2 m/s^2

(2) 1 m/s^2

(3) 0.5 m/s^2

(4) 4 m/s^2

34. When a force of 5 N acts on a body, the acceleration produced in the body is 5 m/s^2. If two forces of magnitude 5 N each are acting on this body as shown in the figure. The acceleration of body is



(1) Zero (2) 5 m/s^2

(3) 4 m/s^2 (4) 3 m/s^2

35. A block of mass *m* is at rest on a rough inclined plane of inclination θ. The force exerted on plane by the block is

(1) $mg \sin\theta$ (2) $mg \cos\theta$

(3) mg (4) Zero

❑ ❑ ❑

5. Work, Energy and Power

Choose the Correct Answer:

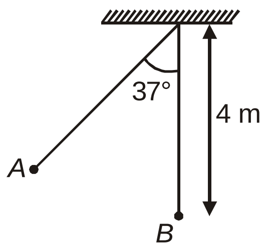
1. If a stone is thrown up vertically, returns to ground then
   (1) During the upward journey its potential energy is maximum at all points
   (2) At maximum height its potential energy is maximum
   (3) During the returns journey its potential energy is maximum at all points
   (4) At the bottom potential energy is necessarily zero

2. The total work done on a particle is equal to the change in its kinetic energy. This is applicable
   (1) Only if the conservative forces are acting on it
   (2) Only in inertial frames
   (3) Only when the observer is in non-nertial frame of reference
   (4) Always

3. A uniform chain of length L and mass M is lying on a smooth table and half of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, then the work required to pull the hanging part on to the table is
   (1) $\frac{MgL}{8}$ (2) $\frac{MgL}{3}$
   (3) $\frac{MgL}{9}$ (4) $\frac{MgL}{18}$

4. A ball is allowed to fall from rest on a horizontal floor from a height of 10 m. If there is 20% loss of energy due to impact, then after one impact ball will go upto
   (1) 10 m (2) 8 m
   (3) 4 m (4) 6 m

5. The speed at the final vertical position B of the bob of simple pendulum, when the bob is released from initial position A from rest is (g = 10 m/s^2)



   (1) 14 m/s (2) 12 m/s
   (3) 4 m/s (4) 6 m/s

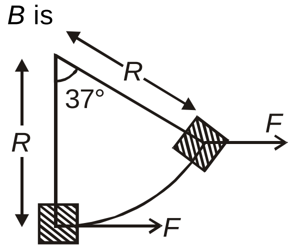
6. A motor boat is travelling with a speed of 3 m/s. If the force on it due to flow is 500 N, then the power of the boat is
   (1) 150 kW (2) $\frac{500}{3}$
   (3) 1.5 kW (4) $\frac{300}{5}$

7. A space craft of mass M is moving with velocity V and suddenly explodes into two pieces. A part of it of mass m comes to rest then the velocity of the other part will be
   (1) $\frac{MV}{M-m}$ (2) $\frac{MV}{M+m}$
   (3) $\frac{mV}{M-m}$ (4) $\frac{(m+M)V}{m}$

8. A car of mass m has engine which can deliver power P. The maximum speed that the car can attain in t seconds is
   (1) $\sqrt{\frac{3}{4}\frac{Pt}{m}}$ (2) $2\sqrt{\frac{Pt}{m}}$
   (3) $\sqrt{\frac{2Pt}{m}}$ (4) $\sqrt{\frac{Pt}{m}}$

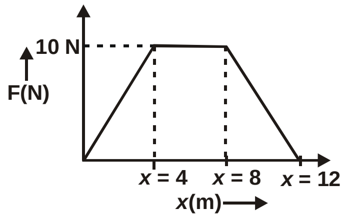
9. A particle is dropped from rest from a height h_0 on a horizontal floor. The coefficient of restitution between the floor and particle is e. Maximum height attained by the particle after first collision is
   (1) eh_0 (2) e^2h_0
   (3) e^3h_0 (4) e^4h_0

10. A block of mass m is pulled along a circular arc, by means of a constant horizontal force F as shown. Work done by this force in pulling block from A to B is



   (1) $\frac{FR}{2}$ (2) $\frac{3}{5}FR$
   (3) $\frac{\sqrt{3}}{2}FR$ (4) mgR

11. A particle of mass 10 kg is subjected to a force F which varies with distance x as shown. If it starts its journey from rest at x = 0, then its speed at x = 12 is



(1) $4\sqrt{5}$ m/s
(2) $20\sqrt{2}$ m/s
(3) $20\sqrt{3}$ m/s
(4) 40 m/s

12. A bullet of mass a and velocity b is fired into a large block of mass c. Then the final velocity of the system is

(1) $\frac{ab}{a+c}$ (2) $\frac{cb}{a+b}$
(3) $\frac{(a+b)a}{c}$ (4) $\frac{a+c}{a}b$

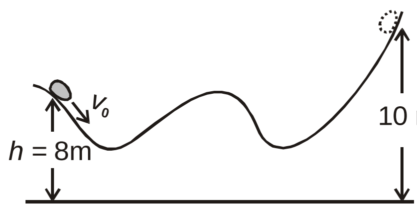
13. A body is moved along a straight line by a machine delivering constant power. The kinetic energy associated with the body in time t is proportional to

(1) $t^{1/2}$ (2) $t^{3/2}$
(3) t (4) t^2

14. A stone of mass 1 kg is tied with a string and it is whirled in vertical circle of radius 1 m. If tension at the highest point is 50 N then velocity at lower most point will be (g = 10 m/s^2)

(1) 10 m/s (2) 4 m/s
(3) 6 m/s (4) 8 m/s

15. A beagle of mass m = 4 kg runs from the left end of a curved ramp with speed V_0 = 8 m/s at height 8 m above the ground. It then slides to the right and comes to rest when it reaches a height 10 m from the ground. The maximum increase in thermal energy of the beagle -ramp system because of sliding is (g = 10 m/s^2)



(1) 0 (2) 22 J
(3) 48 J (4) 32 J

16. Two springs A and B ($K_A = 2K_B$) are stretched by applying forces of equal magnitudes if the energy stored in B is E then the energy stored in A is

(1) $\frac{E}{2}$ (2) $2E$
(3) E (4) $\frac{E}{4}$

17. The force acting on a particle of mass 1 kg which starts from rest is $f = t^3$ then the power associated with the particle is proportional to

(1) t^5 (2) t^6
(3) t^7 (4) t^8

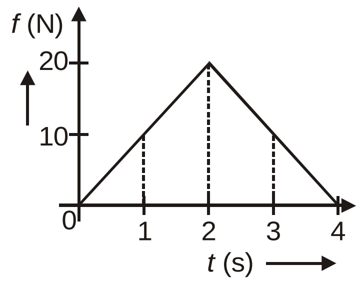
18. In a vertical spring mass system a block of mass m is initially at rest when there is no extension. Now if the mass is released suddenly, then the maximum elongation in the spring is

(1) $\frac{2mg}{k}$ (2) $\frac{mg}{k}$
(3) $\frac{3mg}{2k}$ (4) $\frac{mg}{2k}$

19. A body of mass M_1 collids elastically with another mass M_2 at test. There is maximum transfer of energy when

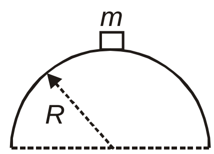
(1) $M_1 > M_2$
(2) $M_1 = M_2$
(3) $M_1 < M_2$
(4) Some for all values of M_1 and M_2

20. The given graph represents the variations of force acting on a particle (starts from rest) of mass 2 kg as a function of time, then the kinetic energy of the particle is



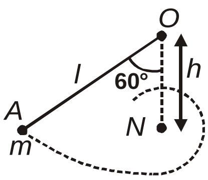
(1) 200 J (2) 400 J
(3) 800 J (4) 1600 J

21. If a particle of mass m resting at the top of a smooth hemisphere of radius R is displaced, then the angle with vertical at which it looses contact with the surface is



(1) $\tan^{-1}\left(\frac{1}{3}\right)$ (2) $\cos^{-1}\left(\frac{1}{3}\right)$
(3) $\tan^{-1}\left(\frac{2}{3}\right)$ (4) $\cos^{-1}\left(\frac{2}{3}\right)$

22. A particle of mass m attached to the end of string of length l is released from the initial position A as shown in the figure. The particle moves in a vertical circular path about O. When it is vertically below O, the string makes contact with nail N placed directly below O at distance h and rotates around it. If the particle just complete the vertical circle about N, then



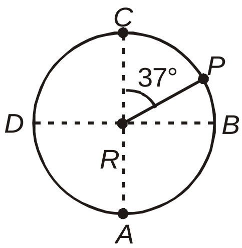
(1) $h = \frac{3l}{5}$

(2) $h = \frac{2l}{5}$

(3) $h = \frac{l}{5}$

(4) $h = \frac{4l}{5}$

23. A particle is moving along a vertical circle of radius R. The velocity of particle at P will be (assume critical condition at C)



(1) $\sqrt{\frac{7}{5}gR}$

(2) $\sqrt{2gR}$

(3) $\sqrt{\frac{3gR}{5}}$

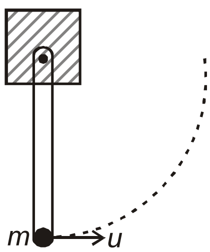
(4) $\sqrt{\frac{3}{2}gR}$

24. The position of a particle moving on a straight line under the action of a force is given as $x = 50t - 5t^2$. Here, x is in metre and t is in second. If mass of the particle is 2 kg, the work done by the force acting on the particle in first 5 s is

(1) 2500 J (2) –2500 J

(3) 5000 J (4) –5000 J

25. What minimum speed should be given to a particle of mass 'm', tied to one end of a massless rod fixed at another end, so that it completes a vertical circle? The length of the rod is 'L' and initially it is in vertical position with fixed end above the free end.



(1) $\sqrt{5gL}$ (2) $\sqrt{3gL}$

(3) $2\sqrt{gL}$ (4) $\sqrt{2gL}$

26. A bike is moving up the inclined plane with constant speed of 10 m/s. If angle of inclination is $\sin^{-1}\left(\frac{1}{50}\right)$ and engine is working at 4×10^5 watt. Then what is the resistance force to the motion of bike?

(Let mass of bike is 1×10^5 kg)

(1) 1×10^4 N

(2) 2×10^4 N

(3) 3×10^4 N

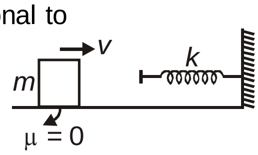
(4) 4×10^4 N

27. A car is moving on a road such that engine delivers power, proportional to velocity of the body. If initial velocity is zero then distance travelled by the car is proportional to

(1) $v^{\frac{1}{2}}$ (2) v^3

(3) $v^{\frac{3}{2}}$ (4) v^2

28. A block of mass m is moving on smooth horizontal surface. A spring of spring constant K is arranged as shown. Then maximum force applied by spring is proportional to



(1) v^2 (2) v^3

(3) v^4 (4) v

29. A force, $\vec{F} = 2x\vec{i} + 2\vec{j}$, acts on a particle placed at origin. What is the work done by this force in moving particle from origin to $P(1, 1)$

(1) 3 J (2) 4 J

(3) 5 J (4) 6 J

30. A particle is released from top of a smooth inclined plane of inclination θ. Let v be the speed of the particle after moving a distance s. Then the quantity which is constant is

(1) $v^2 - 2gs\sin\theta$

(2) $v^2 - \sqrt{2gs}\sin\theta$

(3) $v^2 + 2gs\sin\theta$

(4) $v^2 + \sqrt{2gs}\sin\theta$

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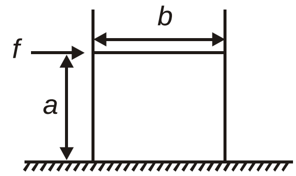
6. Rotational Motion and Moment of Inertia

Choose the correct answer :

1. A cuboidal block of height a and width b is placed on the horizontal surface with sufficient friction then for a given force



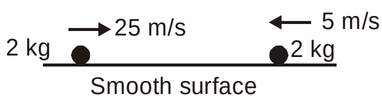
(1) Probability of toppling is more of $b > a$

(2) Probability of toppling is more of $a > b$

(3) Probability of toppling is more of $a = b$

(4) Block will not topple

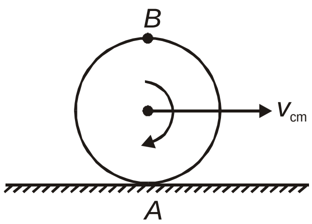
2. Calculate the velocity of the center of mass of the system of two particles each of mass 2 kg as shown in figure



(1) 5 m/s (2) 10 m/s

(3) 2.5 m/s (4) 15 m/s

3. A disc is undergoing in pure rolling on a horizontal surface, then which of the statement about angular momentum of the disc is incorrect?



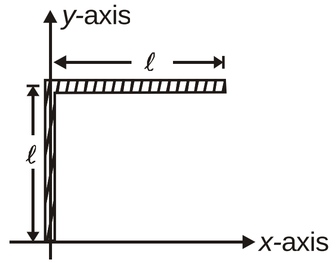
(1) Angular momentum about A is conserved under any situation

(2) Angular momentum about B is conserved only if surface is smooth

(3) Angular momentum about any point on the disc is conserved

(4) None of these

4. A rod of length 2ℓ is bent as shown in figure. Coordinates of centre of mass are



(1) $\left(\frac{2\ell}{3}, \frac{\ell}{3}\right)$ (2) $\left(\frac{\ell}{8}, \frac{\ell}{8}\right)$

(3) $\left(\frac{\ell}{4}, \frac{3\ell}{4}\right)$ (4) $\left(\frac{\ell}{3}, \frac{\ell}{6}\right)$

5. A man is standing at the centre of a rotating disc of radius R, hinged at its center. When the man starts walking from the centre towards the periphery of the disc, then

(1) Angular momentum of the system starts decreasing

(2) Kinetic energy of the system starts increasing

(3) Angular velocity of the system starts decreasing

(4) Kinetic energy first increases then decreases

6. A stick of length L mass M initially upright on a frictionless floor, starts falling, then

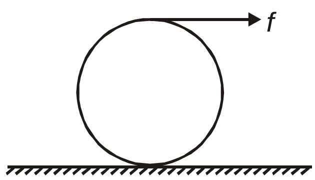
(1) Center of mass will fall vertically down

(2) Center of mass will follow a circular path

(3) Center of mass will follow any curve path

(4) All of these

7. A disc of radius R and mass M is under pure rolling when a force f is applied at the topmost point (as shown in figure) and there is sufficient friction between the disc and the horizontal surface, then



(1) $a_{cm} = \frac{f}{M}$ (2) $a_{cm} < \frac{f}{M}$

(3) $a_{cm} > \frac{f}{M}$ (4) None of these

8. A shell following a parabolic path explodes somewhere in its flight. The centre of mass of fragments will move in

(1) Tangential direction (2) Radial direction

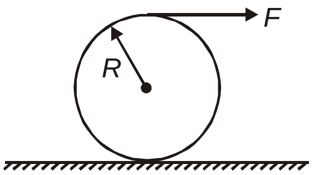
(3) Horizontal direction (4) Same parabolic path

9. If torque acting on a system about an axis of rotation is zero, then the quantity that remains constant about that axis is

(1) Force (2) Linear momentum

(3) Angular momentum (4) Angular velocity

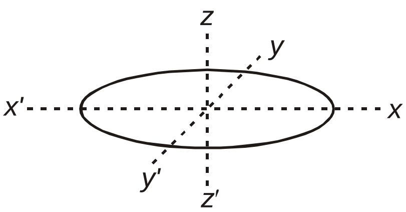
10. A ring of mass *m* and radius *R* is acted upon by a force *F* as shown in the figure, there is sufficient friction between the ring and the ground then the force of friction force necessary for pure rolling is



(1) $\frac{F}{2}$ forward (2) $\frac{F}{3}$ forward

(3) Zero (4) $\frac{F}{4}$ backward

11. An elliptical disc shown in the figure is rotated in turn about *x-x′*, *y-y′* and *z-z′* axes passing through the centre of mass of the disc. Moment of inertia of the disc is



(1) Same about all the three axes

(2) Maximum about *z-z′* axes

(3) Maximum about *y-y′* axis

(4) Same about *x-x′* and *y-y′* axis

12. A particle of mass *m* = 3 kg is projected at angle of 45° with the horizontal with a speed of $20\sqrt{2}$ m/s. The angular momentum of the particle at the highest point of tragectory about an horizontal axis passing through the origin and perpendicular to the plane of motion is

(1) 1200 Js (2) 1600 Js

(3) 1500 Js (4) 2000 Js

13. Moment of inertia of a rod of mass *m* and length *l* about an axis at a distance $\frac{l}{4}$ from one of the ends of the rod and perpendicular to its length is

(1) $\frac{19}{48}ml^2$ (2) $\frac{7}{38}ml^2$

(3) $\frac{7}{48}ml^2$ (4) $\frac{19}{38}ml^2$

14. A rod of length *L* is hinged at one end. It is brought to a horizontal position and released. The angular velocity of the rod when it is in vertical position is

(1) $\sqrt{3g/L}$ (2) $\sqrt{2g/L}$

(3) $\sqrt{g/2L}$ (4) $\sqrt{g/L}$

15. If a spherical ball is under pure rolling on a table, then the fraction of its total kinetic energy associated with rotation is

(1) $\frac{3}{5}$ (2) $\frac{2}{7}$

(3) $\frac{2}{5}$ (4) $\frac{3}{7}$

16. An automobile engine develops 10^5 W power when rotating at a speed of 1800 rad/minute. The average torque delivered by the engine is

(1) $\frac{1}{3}\times10^2$ Nm (2) $\frac{1}{3}\times10^4$ Nm

(3) $\frac{1}{3}\times10^6$ Nm (4) $\frac{1}{3}\times10^8$ Nm

17. A wheel starts from rest and attains an angular velocity of 20 rev/s after being uniformly accelerated for 20 s. The total angle in radian through which it has turned in 10 s is

(1) 200 π (2) 40 π

(3) 150 π (4) 100 π

18. A solid cylinder, a solid sphere and a hollow sphere each of mass *m* and radius *r* are released from the top of a smooth inclined plane. Then which of the bodies has minimum acceleration down the plane?

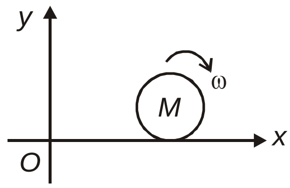
(1) Solid cylinder

(2) Solid sphere

(3) Hollow sphere

(4) All have same acceleration

19. A solid cylinder of mass *m* and radius *r* is rolling with angular speed ω on a horizontal plane. The magnitude of its angular momentum about origin *O* is



(1) $mr^2\omega$

(2) 1.5 $mr^2\omega$

(3) $2mr^2\omega$

(4) 2.5 $mr^2\omega$

20. Choose the incorrect statement

(1) The centre of mass of a two particle system lies on the line joining the two particles, being closer to the heavier particle

(2) In rolling, the point of contact of the rolling body remains at rest relative to the surface on which it is rolling

(3) Parallel axis theorem is applicable only for laminar bodies

(4) A particle moving on a straight line may have non-zero angular momentum about a point

21. Two rigid bodies *A* and *B* rotate with angular momentum L_A and L_B respectively such that $L_A/L_B = 2$. If their moment of inertia about axis of rotation are I_A and I_B respectively such that $I_B/I_A = 4$, then the ratio of their kinetic energies K_A/K_B is

(1) 25 : 4 (2) 5 : 4

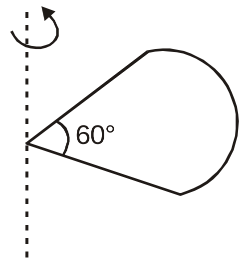
(3) 4 : 1 (4) 16 : 1

22. John is standing with folded hands at the centre of a platform rotating about its central axis. The kinetic energy of the system is K. Now, John stretches his arms so that moment of inertia of the system doubles. Kinetic energy of the system now is

(1) 2K (2) 4K

(3) K/4 (4) K/2

23. A sector is cut out of a disc of mass *m* and radius *r*. It is made to rotate about a line perpendicular to its plane and passing through the centre as shown. Its moment of inertia about the axis rotation is



(1) $\frac{mr^2}{6}$ (2) $\frac{mr^2}{2}$

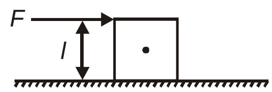
(3) $\frac{mr^2}{12}$ (4) $\frac{mr^2}{24}$

24. A solid cylinder of mass *m* and radius *r* is rotating about its logitudinal axis (vertical with angular speed 'ω'. If a disc of mass 2*m* and radius 2*r* is gently placed on it coaxially, then the new angular velocity of the system is

(1) ω (2) $\frac{\omega}{4}$

(3) $\frac{\omega}{8}$ (4) $\frac{\omega}{9}$

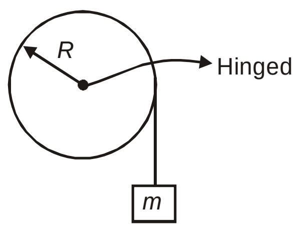
25. A cubical block of side *l* rests on a rough horizontal surface with coefficient of friction μ. A horizontal force *F* is applied on a block as shown. If there is sufficient friction between the block and the ground, then calculate the torque due to normal reaction about its centre of mass



(1) Zero (2) *Fl*

(3) $\frac{Fl}{2}$ (4) $\frac{3Fl}{2}$

26. Calculate the torque acting on the disc in the given arrangement (Radius of disc 1m and mass *m*)



(1) 2 *mg*/3 (2) *mg*/3

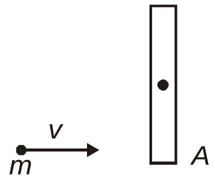
(3) *mg* (4) 4 *mg*/s

27. The speed of centre of mass of a homogenous sphere after rolling down an inclined plane of vertical height *h* from rest without sliding is

(1) $\sqrt{gh}$ (2) $\sqrt{gh/5}$

(3) $\sqrt{4gh/3}$ (4) $\sqrt{10gh/7}$

28. A rod of mass $4m$ and length l hinged at its centre is placed on a horizontal surface. A bullet of mass m moving with velocity v strikes the end A of the rod and gets embedded in it. The angular velocity with which the system rotates about its centre of mass after the bullet strikes the rod



(1) $\omega = \frac{6v}{7l}$ (2) $\omega = \frac{5v}{4l}$

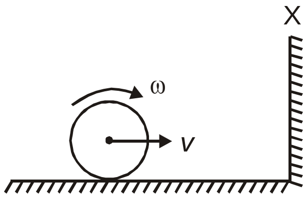
(3) $\omega = \frac{2v}{3l}$ (4) $\omega = \frac{v}{3l}$

29. Magnitude of angular momentum of a wheel changes from 2L to 3L in 5 sec. by a constant torque acting opposite to initial direction of rotation. What is the magnitude of the torque?

(1) $\frac{L}{5}$ (2) L

(3) $\frac{2L}{5}$ (4) $\frac{3L}{5}$

30. A solid cylindre is rolling without slipping with velocity of its centre of mass v and angular velocity about its centre of mass ω on a horizontal frictionless surface as shown in figure. If it collides with a frictionless vertical wall X, then after collision its velocity and angular velocity respectively become



(1) $\frac{v}{2}, \frac{\omega}{2}$

(2) $-v, -\omega$

(3) $-v, \omega$

(4) $v, -\omega$

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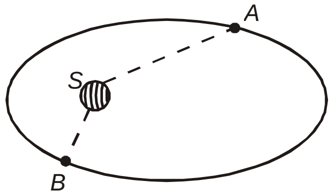
7. Gravitation

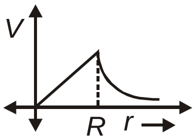
Choose the correct answer :

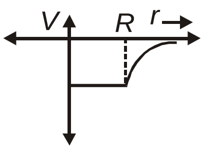
1. A planet is revolving around the Sun in an elliptical orbit. When it is at *A* and *B* at two different instants, which quantity remains constant ?

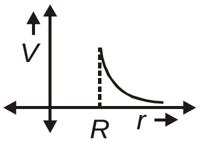


(1) Angular velocity (2) Momentum
(3) Tangential velocity (4) Areal velocity

2. Assume the Earth a homogenous uniform solid sphere of mass *M* and radius *R*. Which of the following represents the variation of gravitational potential (*V*) with distance *r* from centre ?

(1)


(2)


(3)


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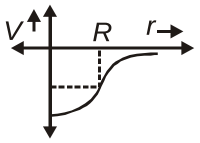


3. Force of gravitation between two masses is found to be *F*, in vacuum. If both the masses are dipped in water then, new force will be

(1) Greater than *F*
(2) Less than *F*
(3) *F*
(4) Cannot say

4. The gravitational force on a body of mass 1.5 kg situated at a point is 45 N. The gravitational field intensity at that point is

(1) 30 N/kg (2) 67.5 N/kg
(3) 46.5 N/kg (4) 43.5 N/kg

5. The magnitudes of the gravitational field at distances r_1 and r_2 from the center of a uniform solid sphere of radius *R* and mass *M* are F_1 and F_2 respectively. Then which of the following statements is/are correct?

I. $\frac{F_1}{F_2} = \frac{r_1}{r_2}$; if $r_1 < R$ and $r_2 < R$

II. $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$; if $r_1 > R$ and $r_2 > R$

III. $\frac{F_1}{F_2} = \frac{r_1}{r_2}$; if $r_1 > R$ and $r_2 > R$

(1) I and III (2) II only
(3) I only (4) I and II

6. The time period of a satellite in a circular orbit of radius *R* is *T*. The period of another satellite in a circular orbit of radius 4*R* is

(1) 4*T* (2) $\frac{T}{4}$
(3) 8*T* (4) $\frac{T}{8}$

7. A small satellite is revolving near the Earth's surface. It's orbital velocity be nearly

(1) 8 km/s (2) 11.2 km/s
(3) 4 km/s (4) 6 km/s

8. If *R* is the radius of the Earth and *g* is the acceleration due to gravity on the Earth's surface, then the mean density of the Earth is

(1) $\frac{4\pi G}{3gR}$ (2) $\frac{3\pi R}{4gG}$
(3) $\frac{3g}{4\pi RG}$ (4) $\frac{\pi Rg}{12G}$

9. If the radius of the Earth shrinks by 1.5% (mass remaining same), then the value of gravitational acceleration '*g*' changes by

(1) 2% (2) –2%
(3) 3% (4) –3%

10. A particle is projected vertically upward from the surface of Earth (radius *R*) with a kinetic energy equal to half the minimum value needed for it to escape. The height to which it rises above the surface of the Earth is

(1) R (2) $\frac{R}{2}$

(3) $\frac{R}{4}$ (4) $\frac{R}{\sqrt{2}}$

11. A body at rest starts from a point at a distance *r* (>*R*) from the center of the Earth. If *M* and *R* stand for the mass and the radius of Earth respectively, then the speed of the body when, it reaches the Earth surface is

(1) $\sqrt{\frac{2GM}{R}}$ (2) $\sqrt{\frac{2GM}{R-r}}$

(3) $\sqrt{\frac{2GM(r-R)}{Rr}}$ (4) Zero

12. The escape velocity of a body from the Earth is about 11.2 km/s. If the mass and radius of the Earth to be about 81 and 4 times the mass and radius of the Moon, then the escape velocity in km/s from the surface of the Moon will be

(1) 0.54 (2) 2.48

(3) 11 (4) 49.5

13. A satellite of the Earth is revolving in a circular orbit with a uniform speed *v*. If the gravitational force suddenly disappears, then the satellite will

(1) Continue to move with velocity *v* along the original orbit

(2) Move with a velocity *v*, tangentially to the original orbit

(3) Fall down with increasing velocity

(4) Ultimately come to rest somewhere on the original orbit

14. A satellite is revolving around the Earth in a circular orbit of radius *r*, then the correct statement is

(1) Linear momentum varies as $\frac{1}{r}$

(2) Angular momentum varies as $\frac{1}{\sqrt{r}}$

(3) Frequency of revolution varies as $\frac{1}{\sqrt{r^3}}$

(4) None of these

15. What should be the angular speed with which Earth have to rotate about its own axis so that a person on the equator would weighs $\frac{3}{5}$th as much as present?

(1) $\sqrt{\frac{2g}{5R}}$ (2) $\sqrt{\frac{2R}{5g}}$

(3) $\frac{2\sqrt{R}}{\sqrt{5g}}$ (4) $\frac{2g}{5R}$

16. If density of a planet is double that of the Earth and the radius is 1.5 times that of the Earth, then the acceleration due to gravity on the surface of the planet is

(1) $\frac{3}{4}$ times that on the surface of the Earth

(2) 3 times that on the surface of the Earth

(3) $\frac{4}{3}$ times that on the surface of the Earth

(4) 6 times that on the surface of the Earth

17. A body is thrown with a velocity equal to *n* (> 1) times the escape velocity (v_e). Velocity of the body at a very large distance away will be

(1) $v_e\sqrt{n^2-1}$ (2) $v_e\sqrt{n^2+1}$

(3) $v_e\sqrt{1-n^2}$ (4) None of these

18. The change in potential energy when a body of mass *m* is raised to height *nR* from the Earth's surface is (*R* = radius of the Earth, *g* = acceleration due to gravity)

(1) $mgR\left(\frac{n}{n-1}\right)$ (2) $nmgR$

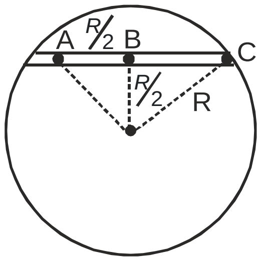
(3) $mgR\left(\frac{n}{n+1}\right)$ (4) $mgR\left(\frac{n^2}{n^2+1}\right)$

19. Three particles each of mass *m* are placed at the three corners of an equilateral triangle of side *a*. The work which should be done to increase the sides of the triangle to 2*a* is

(1) $\frac{3Gm^2}{a}$ (2) $\frac{3Gm^2}{2a}$

(3) $\frac{Gm^2}{2a}$ (4) $\frac{Gm^2}{a}$

20. Assume earth as a uniform perfect sphere of radius R as shown



A, B and C are three points on a frictionless tunnel through earth. If a body is placed successively at A, B and C, then ratio of respective normal reactions is at these points

(1) 2 : 3 : 6 (2) 3 : 2 : 6

(3) 1 : 1 : 1 (4) 4 : 2 : 1

21. Gravitational field at a distance $2R$ from the centre of a spherical shell of inner radius R, outer radius $3R$ and density d is equal to $k(G.\pi.R.d)$, where G is universal gravitational constant and k is

(1) 7/3 (2) 5/3

(3) 3 (4) 4/3

22. A system consists of a solid sphere of mass m and radius r surrounded by a concentric shell of mass m and radius $3r$. Gravitational potential at a point distant $2r$ from the common centre is (G is universal gravitational constant)

(1) $-\frac{Gm}{2r}$ (2) $-\frac{Gm}{3r}$

(3) $-\frac{5Gm}{6r}$ (4) $-\frac{Gm}{4r}$

23. Time period of a simple pendulum of length l at a height $2R$ from the earth's surface is (where R is radius of the earth)

(1) $2\pi\sqrt{\frac{l}{g}}$ (2) $\frac{2\pi}{3}\sqrt{\frac{l}{g}}$

(3) $6\pi\sqrt{\frac{l}{g}}$ (4) $\frac{\pi}{6}\sqrt{\frac{l}{g}}$

24. Choose the incorrect statement

(1) Gravitational field at the centre of the earth is zero

(2) Gravitational potential at the centre of the earth is 1.5 times the potential on its surface

(3) Radius of earth at poles is lesser than the radius at equator

(4) Distance of a geostationary satellite from earth's centre is 36,000 km

25. A system consists of two fixed particles of mass m each distant r from each other. What is the escape velocity at a point mid-way the line joining the two particles?

(1) $\sqrt{\frac{Gm}{r}}$ (2) $2\sqrt{\frac{Gm}{r}}$

(3) $2\sqrt{\frac{2Gm}{r}}$ (4) $\sqrt{\frac{Gm}{2r}}$

26. An artificial satellite moving in a circular orbit around the Earth has kinetic energy E_0. Its potential energy is

(1) $-E_0$ (2) $-2E_0$

(3) $2E_0$ (4) $-\frac{E_0}{2}$

27. Binding energy of a satellite of mass m moving with linear momentum p around earth in a circular orbit is

(1) $\frac{p^2}{m}$ (2) $\frac{p^2}{2m}$

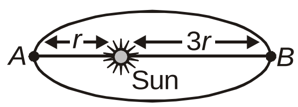
(3) $\frac{p^2}{3m}$ (4) $\frac{p^2}{4m}$

28. Gravitational potential energy of a particle of mass m at centre of the earth, if reference of the potential energy is at earth's surface, is (where R = Radius of the earth, g = Acceleration due to gravity on earth's surface)

(1) $-mgR$ (2) mgR

(3) $-\frac{mgR}{2}$ (4) $\frac{mgR}{2}$

29. A planet moves on an elliptical path around sun, with sun at one of its focii (as in figure). If kinetic energy of the planet is K when it is at a point A on the major axis of the ellipse, its kinetic energy at B the other point on the major axis is



(1) $\frac{K}{3}$ (2) $3K$

(3) $\frac{K}{9}$ (4) $9K$

30. Consider earth as a uniform solid sphere and difference between acceleration due to gravity at poles and at equator as d. If the earth starts rotating with double the angular speed, the difference becomes

(1) $2d$ (2) $3d$

(3) $4d$ (4) $9d$

❑ ❑ ❑

8. Solids and Fluids

Choose the correct answer :

1. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied?

   (1) Length 50 cm, diameter 0.5 mm

   (2) Length 100 cm, diameter 1 mm

   (3) Length 200 cm, diameter 2 mm

   (4) Length 300 cm, diameter 3 mm

2. A wire can sustain a maximum weight of 50 kg before breaking. If the wire is cut into two equal parts, then each part can sustain a maximum weight of

   (1) 25 kg (2) 50 kg

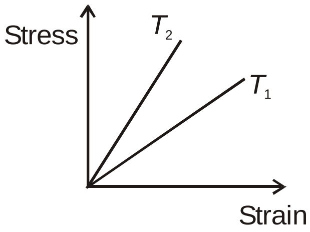
   (3) 40 kg (4) 80 kg

3. The Poisson ratio cannot have value

   (1) 0.7 (2) 0.2

   (3) 0.1 (4) 0.5

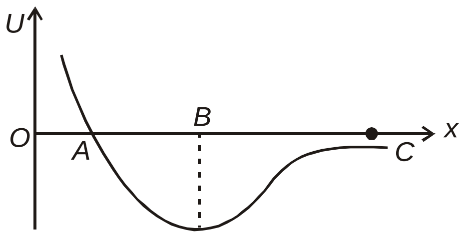
4. Figure shows graph between stress and strain for a uniform wire at two different temperatures. Then



   (1) $T_1 > T_2$ (2) $T_2 > T_1$

   (3) $T_1 = T_2$ (4) None of these

5. The potential energy U between the atoms in a diatomic molecule as a function of the distance x between the atoms is shown in figure. The atoms



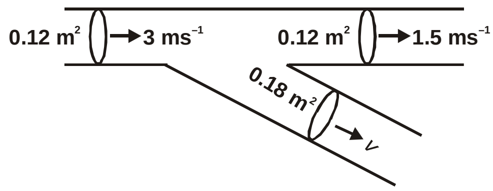
   (1) Attract each other when x is between A and B

   (2) Attract each other when x is between B and C

   (3) Attract each other when x is at B

   (4) Repel each other when x is at B

6. An incompressible liquid travels as shown in the figure. The speed v of the liquid in the lower branch is



   (1) 1 ms^{-1} (2) 1.5 ms^{-1}

   (3) 2.25 ms^{-1} (4) 3 ms^{-1}

7. The addition of soap changes the surface tension of water to σ_1 and that of salt changes to σ_2 then

   (1) $\sigma_1 > \sigma_2$ (2) $\sigma_1 < \sigma_2$

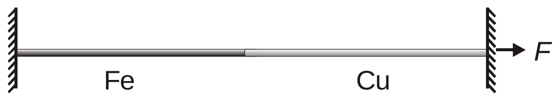
   (3) $\sigma_1 = \sigma_2$ (4) Unpredictable

8. A spherical ball contracts in volume by 0.02 %, when subjected to a normal uniform pressure of 5×10^6 N/m^2 atmosphere. The Bulk modulus of its material is

   (1) 1×10^{11} N/m^2 (2) 2×10^{10} N/m^2

   (3) 2.5×10^{10} N/m^2 (4) 1×10^{13} N/m^2

9. As shown in figure, by combining together copper and steel wires of same length and same diameter, a force F is applied at one of their end. The combined length is increased by 2 cm. The wires will have



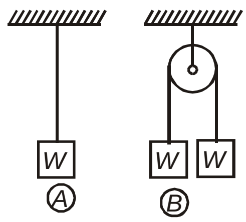
   (1) Same stress and same strain

   (2) Different stress and different strain

   (3) Different stress and same strain

   (4) Same stress and different strain

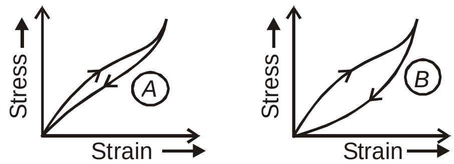
10. If in case A, elongation in wire is l. Then for same wire, elongation in case B will be



   (1) $4l$ (2) $2l$

   (3) l (4) $l/2$

11. Two different types of rubber are found to have the stress-strain curves as shown. Then



(1) *A* is suitable for shock absorber

(2) *B* is suitable for shock absorber

(3) *B* is suitable for car tyres

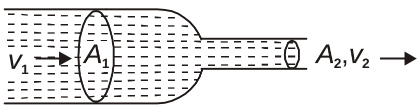
(4) None of these

12. The upper end of a wire, 1 m long and 6 mm radius, is clamped. The lower end is twisted by an angle of 30°. The angle of shear is

(1) 18° (2) 1.8°

(3) 0.18° (4) 0.018°

13. A liquid flows in the tube from left to right as shown in figure. A_1 and A_2 are the cross-sections of the portions of the tube as shown. Then the ratio of speed $\frac{v_1}{v_2}$ will be



(1) $\frac{A_1}{A_2}$ (2) $\frac{A_2}{A_1}$

(3) $\sqrt{\frac{A_2}{A_1}}$ (4) $\sqrt{\frac{A_1}{A_2}}$

14. A tank is filled with water to a height *H*. A hole is made in one of the walls at a depth *D* below the water surface. The distance *x* from the foot of the wall at which the stream of water coming out of the tank strikes the ground is given by

(1) $x = 2\,[D\,(H - D)]^{1/2}$

(2) $x = 2\,(gD)^{1/2}$

(3) $x = 2\,[D\,(H + D)]^{1/2}$

(4) None of these

15. The surface tension of a liquid is 5 N/m. If a film is held on a ring of area 0.02 m^2, its surface energy is about

(1) 5×10^{-2} J (2) 2.5×10^{-2} J

(3) 2×10^{-1} J (4) 3×10^{-1} J

16. On putting a capillary tube in a pot filled with water, the level of water rises upto a height of 4 cm in the tube. If a tube of half the diameter is used instead, the water will rise to a height of nearly

(1) 2 cm (2) 4 cm

(3) 8 cm (4) 11 cm

17. A wire suspended vertically from one end is stretched by attaching a weight 200 N to the lower end. The weight stretches the wire by 1mm. The elastic potential energy gained by the wire is

(1) 0.1 J (2) 0.2 J

(3) 0.4 J (4) 10 J

18. Two spherical soap bubbles of radii r_1 and r_2 in vacuum coalesce under isothermal condition. The resulting bubble has radius *R* such that

(1) $R = \frac{r_1 + r_2}{2}$

(2) $R = \sqrt{r_1^2 + r_2^2}$

(3) $R = \frac{r_1 - r_2}{2}$

(4) $R = \frac{r_1 r_2}{r_1 - r_2}$

19. Length of a copper wire is increased by 0.05% on applying a force. If its poisson's ratio is 0.4, then its diameter

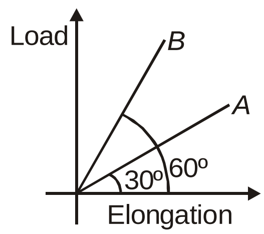
(1) Increases by 0.02%

(2) Decreases by 0.02%

(3) Increases by 0.01%

(4) Decreases by 0.01%

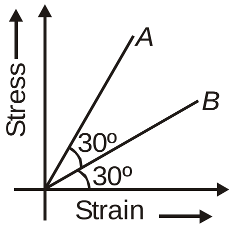
20. Load versus elongation graph for two wires *A* and *B* of same length and made of same material is shown. Ratio of radii of wire *A* to that of wire *B* is



(1) 1 : 1 (2) 1 : 3

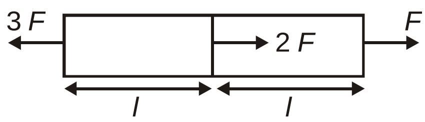
(3) $\sqrt{3} : 1$ (4) $1 : \sqrt{3}$

21. Stress-strain curves for two materials A and B are shown. If two rods of equal areas, lengths l and $3l$, made of materials A and B respectively are equally strained, potential energies stored in the rods are in the ratio



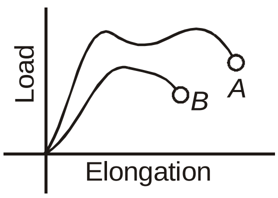
(1) 1 : 3 (2) 3 : 1
(3) 1 : 1 (4) 9 : 1

22. A bar is subjected to axial forces as shown. Find the total elongation in the bar. (E is the modulus of elasticity of the bar and A is its area of cross-section)



(1) $\frac{Fl}{AE}$ (2) $\frac{2Fl}{AE}$
(3) $\frac{3Fl}{AE}$ (4) $\frac{4Fl}{AE}$

23. Load-elongation curves for two rods A and B of same dimensions are shown. If E and D represent elasticity and ductility respectively, then



(1) $E_A = E_B$ (2) $E_A < E_B$
(3) $D_A = D_B$ (4) $D_A > D_B$

24. A pressure P is applied on a sphere. By how much should its temperature be raised to maintain its original volume? The material of sphere has Q and B as its thermal coefficient of superficial expansion and its Bulk's modulus respectively

(1) $\frac{P}{QB}$ (2) $\frac{PQ}{B}$
(3) $\frac{2P}{3QB}$ (4) $\frac{2PQ}{3B}$

25. There are two wires of same material and same length while the diameter of first wire is half the diameter of the second wire, then ratio of extensions produced in the wires by applying same load will be

(1) 1 : 8 (2) 4 : 1
(3) 1 : 1 (4) 2 : 1

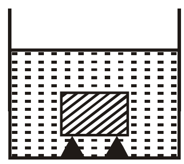
26. If breaking force and breaking stress for a given wire are x and y. Breaking force & breaking stress for another wire of same material and double the thickness are,

(1) $4x$, $4y$ respectively
(2) x, $4y$ respectively
(3) x, y respectively
(4) $4x$, y respectively

27. A metal rod of length l and cross-sectional area A is clamped between two rigid supports. Y and α represent Young's modulus and coefficient of linear expansion respectively. If the temperature of the rod is increased by t Kelvin, the force exerted by the rod on the supports is

(1) $YALt$ (2) $YAL(t + 273)$
(3) $YA\alpha t$ (4) $YA\alpha(t + 273)$

28. A cubical block of side 1m of specific gravity $K(K > 1)$ is placed at the bottom of a vessel containing water. Normal force (in newton) by the stands of the vessel on the block is
(Take g = 10 m/s^2)



(1) $10^4 (K + 1)$
(2) $10^4 (K – 1)$
(3) $10^3 (K + 1)$
(4) $10^3(K– 1)$

29. A body floats in a liquid contained in a beaker. The beaker starts accelerating upwards with 20 m/s^2. Part of the body immersed in the liquid

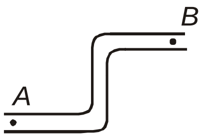
(1) Decreases
(2) Increases
(3) Remains same
(4) Depends on the density of the body

30. A spherical ball of radius r is dropped from a height. falling vertically in air with speed v. The air drag acting on the ball is proportional to

(1) vr (2) $\frac{v}{r}$

(3) $\frac{r}{v}$ (4) $\frac{1}{vr}$

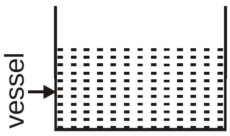
31. In the figure, an ideal fluid flows through a tube of uniform cross section. If V and P represent velocity and pressure respectively



(1) $V_A > V_B$ (2) $P_A < P_B$

(3) $P_A = P_B$ (4) $V_A = V_B$

32. A non-volatile liquid of thermal coefficient of cubical expansion 'x' is contained in a vessel of thermal coefficient of cubical expansion y. If the temperature of the system is increased, pressure on the base of the vessel



(1) Increases if $x > y$

(2) Decreases if $x > y$

(3) Remains same irrespective of values of x & y

(4) Decreases irrespective of values of x & y

33. Choose the incorrect statement

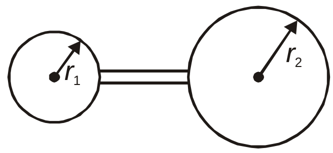
(1) A piece of ice is floating is water. The level of water does not change if the ice melts

(2) A fluid flowing out of a small hole in a vessel applies a backward thrust on the vessel

(3) To float, a body must displace liquid whose weight is greater than the weight of the body

(4) Bernoulli's theorem holds only for incompressible non-viscous fluids

34. Two soap bubbles of different radii are connected to each other as shown. Choose the most appropriate alternative



(1) Size of the bubbles remains unchanged

(2) Larger bubble bursts instantly

(3) Air flows from larger bubble into the smaller one

(4) Air flows from smaller bubble into the larger bubble

35. When two water drops coalesce to make a bigger drop

(1) Energy is absorbs

(2) The surface area of bigger drop is greater than the sum of the surface areas of both the drops

(3) The surface area of bigger drop is smaller than the surface areas of both the drops

(4) Both (1) & (3)

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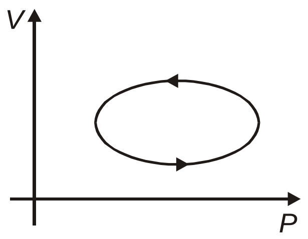
9. Heat and Thermodynamics

Choose the correct answer :

1. In the following indicator diagram, the net work done in one complete cycle is



(1) Positive (2) Negative
(3) Zero (4) Infinity

2. In a Carnot engine, when heat is rejected to the sink, temperature of sink
(1) Increases
(2) Decreases
(3) Remains constant
(4) May increase or decrease

3. In an isothermal process 30 joule of work is perfomed by gas, the amount of heat supplied to the gas is
(1) 30 J (2) –30 J
(3) 15 J (4) 60 J

4. "Heat cannot by itself flow from a body at lower temperature to a body at higher temperature" is
(1) Claussius statement of second law of thermodynamics
(2) Kelvin-Plank statement of second law of thermodynamics
(3) First law of thermodynamics
(4) Law of conservation of heat

5. If n is the degree of freedom of a gas, then the ratio of two principal specific heats $\left(\frac{C_V}{C_P}\right)$ is given by
(1) $\frac{1+n}{2}$ (2) $\frac{2+n}{2}$
(3) $\frac{n}{n+2}$ (4) $1+\frac{1}{n}$

6. The thermal capacity of 200 g of aluminium (specific heat 0.2 cal g^{-1} °C^{-1}), is
(1) 40 cal °C^{-1} (2) 4 cal °C^{-1}
(3) 8 cal °C^{-1} (4) 160 cal °C^{-1}

7. Select incorrect statement about the molar specific heat of a gaseous system
(1) Specific heat in an adiabatic process, $C_a = 0$
(2) Specific heat in an isothermal process, $C_i = \infty$
(3) Specific heat in an isobaric process, $C_p = \frac{\gamma R}{\gamma - 1}$
(4) Specific heat in an isochoric process, $C_v = \frac{R}{\gamma}$

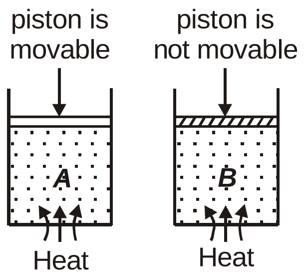
8. A certain amount of an ideal monoatomic gas needs 200 J of heat energy to raise its temperature by 10°C at constant pressure. The heat needed for the same temperature rise of the same gas but at constant volume will be
(1) 300 J (2) 120 J
(3) 2000 J (4) 400 J

9. The volume of a diatomic gas is reduced to $\frac{1}{32}$ times of its initial volume in an adiabatic process. If initial temperature of the gas is 227°C, then its final temperature is
(1) 2000°C (2) 1727°C
(3) 1000°C (4) 727°C

10. The molar heat capacity of chromium at room temperature is x J K^{-1} mol^{-1} and atomic mass is 52. Specific heat capacity of chromium is
(1) $\frac{x}{52}$ J kg^{-1} K^{-1} (2) $\frac{x}{26}$ J kg^{-1} K^{-1}
(3) $\frac{1000x}{26}$ J kg^{-1} K^{-1} (4) $\frac{1000x}{52}$ J kg^{-1} K^{-1}

11. A gas mixture contains 8 moles of oxygen and 2 moles of argon at room temperature T. The total internal energy of the mixture is
(1) 11 RT
(2) 23 RT
(3) 22 RT
(4) 26 RT

12. Two cylinders contain ideal diatomic gas. As shown in figure Same amount of heat is given to the two cylinders. If temperature rise in cylinder *B* is T_0 then temperature rise in cylinder *A* will be



(1) $\frac{7}{5}T_0$ (2) $2T_0$

(3) $\frac{3}{5}T_0$ (4) $\frac{5}{7}T_0$

13. An ideal gas absorbs *Q* heat in an isobaric process. If adiabatic exponent of the gas is γ, then what fraction of heat is used to perform work?

(1) $\frac{1}{\gamma}$ (2) $\frac{\gamma-1}{\gamma}$

(3) $\frac{\gamma}{\gamma+1}$ (4) $\frac{1}{\gamma+1}$

14. The change in volume (in cm^3) of a metal sphere of radius 10 cm, when its temperature is increased from 10°C to 60°C is (Given that coefficient of volume expansion of the sphere is 6.9×10^{-5} °C^{-1})

(1) 2.3π

(2) 4.6π

(3) 6.9π

(4) 9.2π

15. A pendulum clock has a steel rod to suspend a bob in its pendulum. The clock is calibrated at 25°C. When the temperature is increased to 125°C, the percentage change in its time period is

($\alpha_{steel} = 1.2 \times 10^{-5}$ / °C)

(1) 0.06%

(2) – 0.01%

(3) 0.03%

(4) Zero

16. 100 g of ice (latent heat 80 cal g^{-1}) at 0°C is mixed with 300 g of water (specific heat 1 cal g^{-1}°C^{-1}) at 80ºC. The final temperature of the mixture will be

(1) 0° C (2) 40° C

(3) 80° C (4) < 0° C

17. A copper block of mass 80 g is heated to 100°C and placed on a block of ice at 0°C. The specific heat of copper is 0.1 cal g^{-1} °C^{-1} and latent heat of ice is 80 cal g^{-1}. The amount of ice melted is

(1) 4 g (2) 10 g

(3) 12 g (4) 16 g

18. A diatomic gas undergoes a process given by the equation $P = KV$ where *K* is a constant. The molar specific heat of the gas for the process is

(1) R (2) $\frac{5R}{2}$

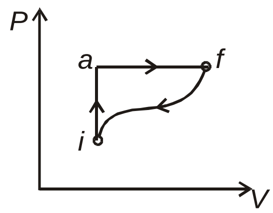
(3) $4R$ (4) $3R$

19. 8 g of He at T_0 is mixed with 32 g of O_2 at 3.2 T_0 at constant volume. The temperature of the mixture is

(1) T_0 (2) $2T_0$

(3) $2.5T_0$ (4) $4T_0$

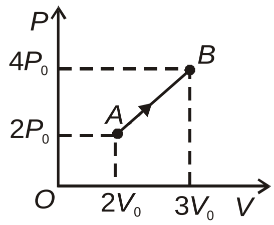
20. If Q_{iaf} = 80 cal, W_{iaf} = 60 cal and W = –30 cal for the curved path $f \to i$, then value of Q for path $f \to i$, will be



(1) 50 cal (2) 30 cal

(3) –30 cal (4) –50 cal

21. The *P*-*V* diagram of 2 g of Helium gas for a certain process $A \to B$ is shown in figure. Heat given to the body during process $A \to B$ is



(1) $4P_0V_0$ (2) $12P_0V_0$

(3) $6P_0V_0$ (4) $15P_0V_0$

22. An ideal heat engine operates in Carnot cycle between 227°C and 27°C. It absorb 5×10^4 cals per cycle at the higher temperature. The amount of heat converted into work per cycle is

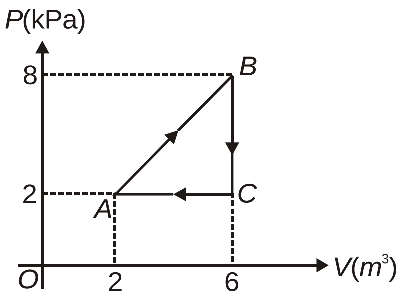
(1) 2×10^4 cals

(2) 4×10^4 cals

(3) 1.6×10^4 cals

(4) 1.2×10^4 cals

23. For the process as shown below the heat supplied to the system is 30 kJ per cycle. The efficiency of the cycle is



(1) 40% (2) 60%

(3) 50% (4) 30%

24. When an ideal diatomic gas is heated at constant pressure, the fraction of the heat energy supplied which increases the internal energy of the gas is

(1) $\frac{5}{7}$ (2) $\frac{2}{5}$

(3) $\frac{3}{5}$ (4) $\frac{1}{7}$

25. A body is placed in a liquid in just immersed condition. If the temperature of the system is increased, then Bouyant force will

(given that $\gamma_L > 3\alpha_s$)

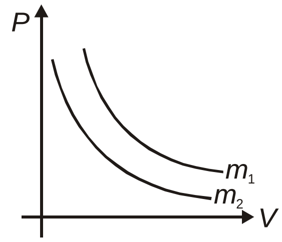
(1) Decrease

(2) Increase

(3) First increases and then decreases

(4) Remains same

26. The *P*-*V* diagrams of two different masses m_1 and m_2 for an ideal gas at constant temperature *T* is given in figure. Then



(1) $m_1 = m_2$ (2) $m_1 > m_2$

(3) $m_1 < m_2$ (4) Data is insufficient

27. Two mole of an ideal monoatomic gas is expanded in such a manner that it absorbs heat of 5*R* and increase in temperature is 1ºC where *R* is universal gas constant. The work done by the gas is equal to

(1) 5*R* (2) 3*R*

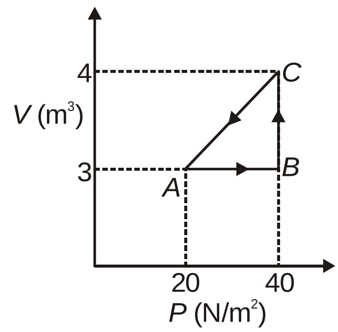
(3) 2*R* (4) –8*R*

28. The root mean square speed of hydrogen molecule is v_1 at temperature *T* K. The most prabole speed of Oxygen molecule is v_2 at same temperature. Then $\frac{v_1}{v_2}$ is equal to

(1) $2\sqrt{6}$ (2) 4

(3) $3\sqrt{6}$ (4) 2

29. P-V diagram of an ideal gas is given in the figure



Work done on the gas in process *CA* is

(1) 60 J (2) 70 J

(3) 30J (4) 20 J

30. The number of degrees of freedom for a diatomic gas, taking into account. Vibrational mode, is

(1) 2 (2) 5

(3) 6 (4) 7

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10. Transfer of Heat

Choose the correct answer :

1. Woolen clothes are used in winter season because woolen clothes
   (1) Absorb heat from surrounding
   (2) Are good sources for producing heat
   (3) Transfer heat from surroundings to body
   (4) Trap air which is bad conductors of heat

2. For a body the ratio of absorptive power to the reflective power is
   (1) Always greater than 1
   (2) Always less than 1
   (3) Always 1
   (4) May be more than 1 or less than 1

3. If a polished plate with rough black paintings is heated to a high temperature and taken to a dark room, then
   (1) Paintings will appear brighter than the plate
   (2) Paintings will appear darker than the plate
   (3) Both will appear equally bright
   (4) Both will not be visible

4. One can determine the temperature of a star
   (1) Using Kirchhoff's law
   (2) Using Ohm's law
   (3) Using Wein's displacement law
   (4) Using Kepler's law

5. The S.I. unit of Stefan's constant is
   (1) W m^{-2} K^{-1}
   (2) W m^{-2} K^{-4}
   (3) J m^{-2} s^{-1} K^{4}
   (4) J m^{-2} K^{-4}

6. For cooking the food, which of the following type of utensils are suitable?
   (1) Having high conductivity and low specific heat
   (2) Having low conductivity and low specific heat
   (3) Having high conductivity and high specific heat
   (4) Having low conductivity and high specific heat

7. Select correct statement related to heat
   (1) Heat is possessed by a body
   (2) Hot water contains more heat as compared to cold water
   (3) Heat is an energy which flows due to temperature difference
   (4) Heat is a fluid

8. Two identical rods of a metal are welded as shown in figure (1). A certain amount of heat flows through them in 16 min. If the rods are welded as shown in the figure (2) then the same amount of heat will flow in

(1) (2)

   (1) 1 minute (2) 4 minutes
   (3) 16 minutes (4) 2 minutes

9. In which of the following process, convection does not take place primarily?
   (1) Boiling of water
   (2) Heating of air above a furnace
   (3) Heating of a car placed in sunlight
   (4) Sea and land breeze

10. A black body, which is at a high temperature T, radiates energy at the rate of E. If the temperature falls to $T/2$, then the rate of radiated energy will be
    (1) E (2) $E/4$
    (3) $E/64$ (4) $E/16$

11. Star A emits radiation of maximum intensity at a wavelength of 5000 Å and it has temperature 1227°C. If star B has temperature 2727°C, then the maximum intensity would be observed at
    (1) 4000 Å (2) 3500 Å
    (3) 3000 Å (4) 2500 Å

12. A black piece of iron is heated continuously, which of the following is the correct sequence of its observed colours?
    (1) Red, yellow, orange (2) Red, yellow, black
    (3) White, yellow, red (4) Red, yellow, white

13. According to Kirchhoff's law
    (1) Good reflectors are good emitters
    (2) Good absorbers are good reflectors
    (3) Good reflectors are good absorbers
    (4) Good emitters are good absorbers

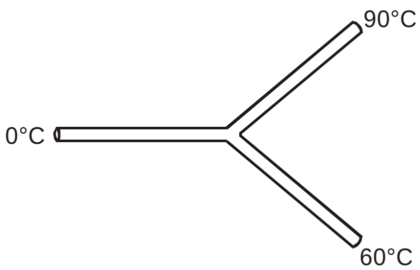
14. A body cools from 70°C to 50°C in 20 minutes. The time it takes to cool from 50°C to 30°C will be (Given that temperature of surroundings is 30°C)
    (1) 20 minutes (2) 60 minutes
    (3) 180 minutes (4) Infinite

15. A sphere, a cube and a thin circular plate, all made of the same mass and finish are heated to a temperature of 800°C. Which of these objects will lose heat at minimum rate, when left in air at room temperature?
    (1) The sphere
    (2) The cube
    (3) The circular plate
    (4) All will radiate heat at same rate

16. Four rods with different radii r and length l are used to connect two reservoirs of heat at different temperatures. Which one will conduct heat at maximum rate?
    (1) r = 1 cm, l = 1 m (2) r = 2 cm, l = 2 m
    (3) r = 3 cm, l = 3 m (4) r = 4 cm, l = 4 m

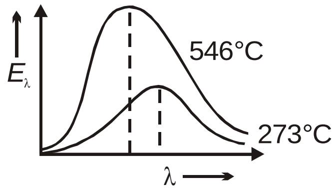
17. A slab consists of two layers of A and B of same thickness and having thermal conductivities in the ratio 1 : 4 in series. If the free face of B is at 150°C and that of A at 25°C, the temperature of the interface is
    (1) 125°C (2) 50°C
    (3) 100°C (4) 75°C

18. Three rods made of the same material and having the same cross-section have been joined as shown in the figure. Each rod is of same length. The temperatures of free ends are shown in the figure. The temperature of the junction will be



    (1) 50°C (2) 30°C
    (3) 20°C (4) 75°C

19. The spectra of a black body at temperatures 273°C and 546°C are shown in the figure. If A_1 and A_2 be the areas under the two curves respectively, the value of $\frac{A_2}{A_1}$ is



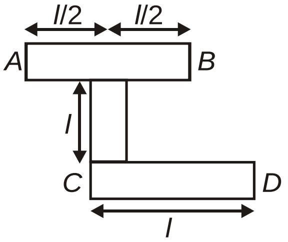
    (1) $\frac{81}{16}$ (2) $\frac{16}{1}$
    (3) $\frac{27}{8}$ (4) $\frac{16}{81}$

20. Two metal spheres A and B have radii r and $4r$ respectively. They are heated to 4000 K each and allowed to cool down. The respective ratio of their rates of cooling is
    (1) 1 : 4 (2) 4 : 1
    (3) 1 : 256 (4) 1 : 16

21. Space between two thin spherical concentric shells of radii r_1 and r_2 ($r_1 < r_2$) is filled with material of thermal conductivity K. Inner and outer shells are maintained at temperatures T_1 and T_2 respectively ($T_1 < T_2$). Rate of heat flow radially through the material is
    (1) $\frac{4\pi K r_2^2 (T_1 - T_2)}{r_2 - r_1}$
    (2) $\frac{4\pi K (r_1 - r_2)(T_1 - T_2)}{r_1 r_2}$
    (3) $\frac{4\pi K r_1 r_2 (T_1 - T_2)}{r_2 - r_1}$
    (4) $\frac{4\pi K r_1^2 (T_1 - T_2)}{r_2 (r_1 - r_2)}$

22. Choose the incorrect statement
    (1) Newton's law is a special case of Stefan's law
    (2) A hot metal ball in vacuum can loose heat only by radiation
    (3) Heating by convection is not possible in solids
    (4) Thermal conductivity of a rod depends on its length

23. Three rods of identical dimensions are arranged as shown.



Conductivity of the material of rods is k, area of cross-section of rods is A, length of each rod is l. Thermal resistance of the system between points A and D is, (Assume there are no losses due to radiation)

(1) $\frac{3l}{kA}$ (2) $\frac{l}{3kA}$

(3) $\frac{5l}{2kA}$ (4) $\frac{3l}{2kA}$

24. A solid and a hollow cube of same side, made of same material are heated to equal temperature

(1) Both cubes emit different amount of radiation per unit time in the beginning

(2) Both cubes absorb equal amount of radiation from the surroundings in the beginning

(3) Only initial rate of cooling is same for both the cubes

(4) Rate of cooling at any time is same for both the cubes

25. A body radiates energy at a rate of 20 W at temperature 127°C. If the temperature of the body is increased by 800°C, then it will radiate at the rate of

(1) 1620 W (2) 1500 W

(3) 410 W (4) 81 W

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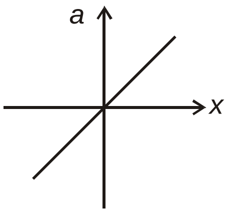
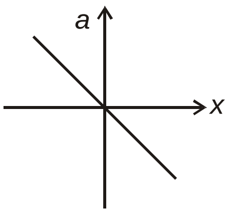
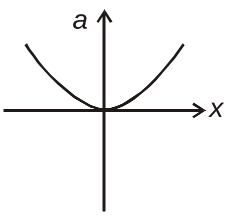
11. Oscillations

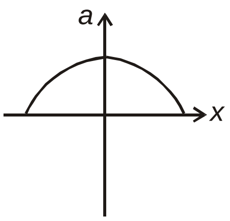
Choose the correct answer :

1. The correct variation of acceleration '*a*' with displacement *x* from mean position of a particle executing SHM is

(1)


(2)


(3)


(4)


2. A simple pendulum suspended from the ceiling of a stationary lift has period T_0. When the lift descends at steady speed, the period is T_1. When it descends with constant downward acceleration, the period is T_2. Which one of the following is true?

(1) $T_0 = T_1 = T_2$ (2) $T_0 = T_1 < T_2$
(3) $T_0 = T_1 > T_2$ (4) $T_0 < T_1 < T_2$

3. A particle is executing S.H.M. between $x = \pm a$. The time taken to go from 0 to $\frac{a}{2}$ is T_1 and to go from $\frac{A}{2}$ to A is T_2, then

(1) $T_1 < T_2$ (2) $T_1 > T_2$
(3) $T_1 = T_2$ (4) $T_1 = 2T_2$

4. A simple harmonic motion has amplitude *A* and time period *T*. The maximum velocity will be

(1) $4AT$ (2) $\frac{2A}{T}$
(3) $2\pi\sqrt{\frac{A}{T}}$ (4) $\frac{2\pi A}{T}$

5. The kinetic energy and potential energy of a particle executing S.H.M. will be equal when displacement in terms of amplitude '*a*' will be

(1) $\frac{a}{2}$ (2) $\frac{a}{\sqrt{2}}$
(3) $\frac{a\sqrt{2}}{3}$ (4) $a\sqrt{2}$

6. Two identical pendulums oscillate with a constant phase difference $\frac{\pi}{4}$ and same amplitude. The maximum velocity of one is *v*. The maximum velocity of the other will be

(1) v (2) $\sqrt{2}v$
(3) $2v$ (4) $\frac{v}{\sqrt{2}}$

7. A particle is executing SHM with time period *T*. Starting from mean position, time taken by it to complete $\frac{5}{8}$ oscillations, is

(1) $\frac{T}{12}$ (2) $\frac{T}{6}$
(3) $\frac{5T}{12}$ (4) $\frac{7T}{12}$

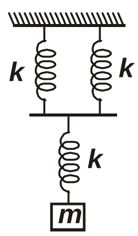
8. The motion of a particle is given by
$y = A \sin \omega t + B \cos \omega t$.
The motion of the particle is

(1) Not simple harmonic
(2) Simple harmonic with amplitude $A + B$
(3) Simple harmonic with amplitude $\frac{A+B}{2}$
(4) Simple harmonic with amplitude $\sqrt{A^2+B^2}$

9. Acceleration of a particle (of mass *m*) executing SHM is given by $a = -bx$ where *x* is the displacement from mean position and *b* is a positive constant. The time period of vibration is

(1) $2\pi\sqrt{\frac{b}{m}}$ (2) $\frac{2\pi}{b}$
(3) $\frac{2\pi}{\sqrt{b}}$ (4) $\frac{\pi}{b}$

10. A block of mass *m* hangs from three light springs having same spring constant *k*. If the mass is slightly displaced vertically, time period of oscillation will be



(1) $2\pi\sqrt{\frac{m}{3k}}$
(2) $2\pi\sqrt{\frac{3m}{2k}}$
(3) $2\pi\sqrt{\frac{2m}{3k}}$
(4) $2\pi\sqrt{\frac{3k}{m}}$

11. A 100 gm mass stretches a particular spring 9.8 cm. How large a mass must be attached to the spring if its period of vibration is to be 6.28 sec?

(1) 1000 gm (2) 10^5 gm

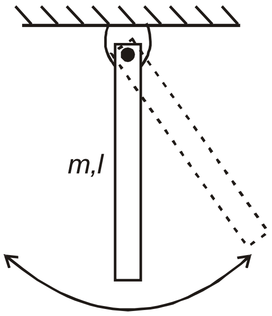
(3) 10^7 gm (4) 10^4 gm

12. If a second's pendulum is moved to a planet where acceleration due to gravity is 4 times, then the length of the second's pendulum on the planet should be

(1) 2 times (2) 4 times

(3) 8 times (4) 15 times

13. A rod of mass *m* and length *l* is suspended from its end. Time period of oscillations is



(1) $2\pi\sqrt{\frac{l}{g}}$ (2) $2\pi\sqrt{\frac{2l}{g}}$

(3) $2\pi\sqrt{\frac{2l}{3g}}$ (4) $2\pi\sqrt{\frac{l}{3g}}$

14. A body executes S.H.M. with an amplitude *A*. The displacement from the mean position at which potential energy of the body becomes one-fourth of its total energy is

(1) $\frac{A}{4}$

(2) $\frac{A}{2}$

(3) $\frac{3A}{4}$

(4) Some other fraction of *A*

15. A simple pendulum with a metallic bob has a time period *T*. The bob is now immersed in a non-viscous liquid and oscillated. If the density of the liquid is 1/4 that of metal, then the time period of the pendulum will be

(1) $\frac{T}{\sqrt{3}}$ (2) $\frac{2T}{\sqrt{3}}$

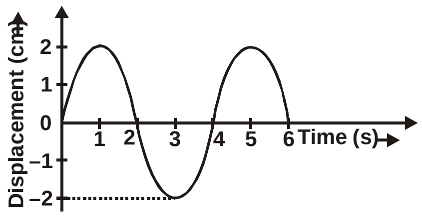
(3) $\frac{4}{3}T$ (4) $\frac{2}{3}T$

16. The amplitude and the periodic time of a SHM with initial phase zero are 5 cm and 6 sec respectively. At a distance of 2.5 cm away from the mean position the phase will be

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$

(3) $\frac{5\pi}{12}$ (4) $\frac{\pi}{3}$

17. The given figure shows the displacement-time graph of a simple harmonic oscillator. The amplitude, time period and initial phase of the oscillator are respectively



(1) 1 cm, 2 s, $\frac{\pi}{2}$ (2) 2 cm, 4 s, $\frac{\pi}{4}$

(3) 2 cm, 4 s, zero (4) 2 cm, 2 s, zero

18. Select wrong statement about simple harmonic motion

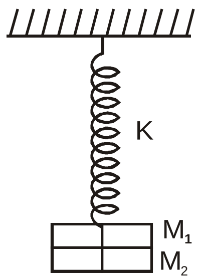
(1) The body is uniformly accelerated

(2) The velocity of the body changes harmonically at all instants

(3) The amplitude of oscillation is symmetric about the equilibrium position

(4) The frequency of oscillation is independent of amplitude

19. Two masses M_1 and M_2 are suspended from the ceiling by a massless spring of force constant K. Initially the system is at equilibrium of M_1 is gently removed, then amplitude of vibration of the system will be



(1) $\frac{(M_1+M_2)g}{K}$

(2) $\frac{M_1g}{K}$

(3) $\frac{M_2g}{K}$

(4) $\frac{(M_2-M_1)g}{K}$

20. Two simple harmonic motions are given by

$y_1 = 5\sin(\omega t + \pi/3)$, $y_2 = 5(\sin\omega t + \sqrt{3}\cos\omega t)$

Ratio of their amplitudes is

(1) 1 : 2 (2) 1 : 4

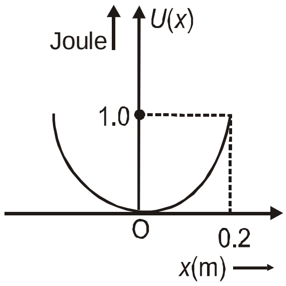
(3) 1 : 5 (4) 1 : 8

21. Two pendulum of length 1.21 m and 1.0 m start vibrating from their mean position in same phase. After how many vibrations of the longer pendulum, the two will be in phase

(1) 10 (2) 11

(3) 20 (4) 21

22. A particle of mass 4 kg moves simple harmonically and its PE (U) varies with position x is as shown. The period of oscillation is



(1) $\frac{2\pi}{25}$ s

(2) $\frac{\pi\sqrt{2}}{5}$ s

(3) $\frac{4\pi}{5}$ s

(4) $\frac{2\pi\sqrt{2}}{5}$ s

23. If a graph is plotted between velocity (v) and displacement (y) of a particle executing SHM from mean position, then the nature of the graph is

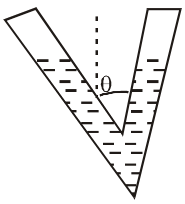
(1) Straight line

(2) Parabola

(3) Ellipse

(4) Hyperbola

24. In the figure shown, if length of liquid column is l, then time period will be



(1) $2\pi\sqrt{\frac{l}{2g\cos\theta}}$

(2) $2\pi\sqrt{\frac{l}{2g\sin\theta}}$

(3) $2\pi\sqrt{\frac{l}{(1+\cos\theta)g}}$

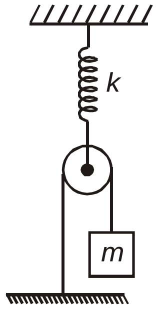
(4) $2\pi\sqrt{\frac{l}{2(1+\sin\theta)g}}$

25. Starting from the origin a body oscillates simple harmonically with a period of 2 second. After what time will its kinetic energy be 75% of the total energy?

(1) $\frac{1}{6}$ s (2) $\frac{1}{4}$ s

(3) $\frac{1}{3}$ s (4) $\frac{1}{12}$ s

26. In given figure. If spring is light and pulley is massless, then time period of oscillations of block of mass m suspended will be



(1) $2\pi\sqrt{\frac{m}{k}}$ (2) $4\pi\sqrt{\frac{m}{k}}$

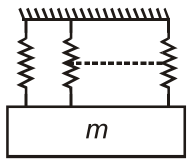
(3) $\pi\sqrt{\frac{m}{k}}$ (4) $2\pi\sqrt{\frac{2m}{k}}$

27. Two particles executing SHM of same frequency, meet at $x = +A/2$, while moving in opposite direction. Phase difference between the particles is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$

(3) $\frac{5\pi}{6}$ (4) $\frac{2\pi}{3}$

28. A spring of spring constant k is cut into n equal parts. A block of mass m is attached to these parts of the spring as shown



Time period of the oscillation of the block will be

(1) $2\pi\sqrt{\frac{m}{k}}$ (2) $2\pi\sqrt{\frac{m}{nk}}$

(3) $\frac{2\pi}{n}\sqrt{\frac{m}{k}}$ (4) $2\pi\sqrt{\frac{nm}{k}}$

29. Instantaneous displacement of a harmonic oscillator is given by $y = A\cos\left(\omega t + \frac{\pi}{6}\right)$. Its speed is maximum at time

(1) $\frac{\pi}{3\omega}$ (2) $\frac{4\pi}{3\omega}$

(3) $\frac{\pi}{6\omega}$ (4) Both (1) & (2)

30. A cylindrical cork piece of density σ, base area A and height h floats in a liquid of density ρ. The cork is slightly depressed and then released. The cork oscillates

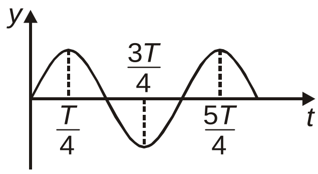
(1) Simple harmonically with time period $2\pi\sqrt{\frac{\sigma h}{\rho g}}$

(2) Simple harmonically with time period $2\pi\sqrt{\frac{\rho h}{\sigma g}}$

(3) With time period $2\pi\sqrt{\frac{\sigma h}{\rho g}}$, non harmonically

(4) With time period $2\pi\sqrt{\frac{\rho h}{\sigma g}}$, non harmonically

31. Displacement time graph for a particle executing S.H.M. is as shown



(1) Velocity of the particle is positive for $\frac{T}{4} < t < \frac{3T}{4}$

(2) Velocity of the particle is negative for $\frac{T}{2} < t < T$

(3) Acceleration of the particle is positive for $\frac{T}{4} < t < \frac{3T}{4}$

(4) Acceleration of the particle is positive for $\frac{T}{2} < t < T$

32. Which of these represents oscillatory motion? (F, k, x represent force on the particle, a positive constant quantity and displacement of the particle respectively)

(1) $F = kx^2$ (2) $F = -kx^2$

(3) $F = -kx^3$ (4) $F = kx^3$

33. A spring-mass system and a simple pendulum are taken in a tunnel at a depth $d = \frac{3R}{4}$ down the earth from the surface of the earth, where R = radius of the earth. Time periods of the spring-mass system and the simple pendulum become n and m times respectively

(1) $n = 1,\ m = \sqrt{\frac{4}{3}}$

(2) $n = \sqrt{\frac{4}{3}},\ m = 2$

(3) $n = 1$, $m = 2$

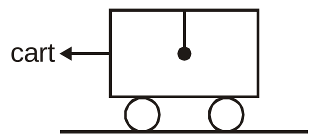
(4) $n = 2,\ m = \sqrt{\frac{4}{3}}$

34. A simple pendulum has bob made of material of specific gravity $\frac{4}{3}$. Its time period of oscillation in air is T. When it is made to oscillate in a liquid, its time period of oscillation is double. Specific gravity of the liquid is

(1) 1 (2) 2

(3) $\frac{1}{2}$ (4) $\frac{3}{4}$

35. The simple pendulum shown here has time period T if the cart moves with constant speed. With what acceleration should the cart move so that the time period is $\frac{T}{\sqrt{2}}$? (g is acceleration due to gravity)



(1) g (2) $\sqrt{2}\,g$

(3) $\sqrt{3}\,g$ (4) $2\,g$

❑ ❑ ❑

12. Waves

Choose the correct answer :

1. A wave transports

(1) Medium (2) Velocity

(3) Energy (4) Temperature

2. The phenomenon of sound propagation in air is

(1) An isothermal process (2) An adiabatic process

(3) An isobaric process (4) None of these

3. Beat frequency is defined as

(1) The sum of frequencies

(2) The number of times intensities become maximum or minimum in one second

(3) The average of frequencies

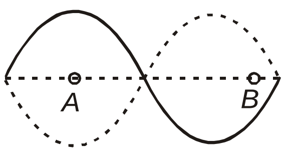
(4) Both (1) & (2)

4. Which of the following characteristics of sound help us in identifying two persons taking in a room without seeing them ?

(1) Loudness (2) Pitch

(3) Quality (4) Intensity

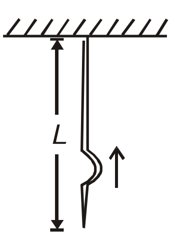
5. In a standing wave particles at the positions *A* and *B*, have a phase difference of



(1) 0 (2) $\frac{\pi}{2}$

(3) $\frac{5\pi}{6}$ (4) π

6. A transverse wave pulse is generated at lower end of a hanging rope of uniform linear mass density and length *L*. Time taken by pulse to travel from lower end to the ceiling is



(1) $\sqrt{\frac{L}{g}}$

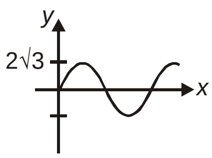
(2) $2\sqrt{\frac{L}{g}}$

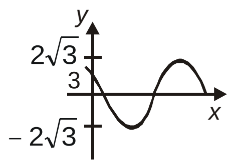
(3) $\sqrt{\frac{L}{2g}}$

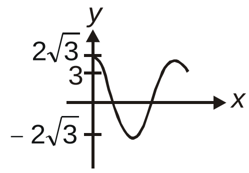
(4) $\sqrt{\frac{3l}{g}}$

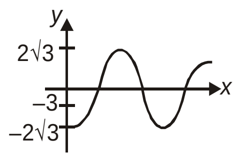
7. Identify the figure, which correctly represent the given wave function at $t = 0$?

$$y = 2\sqrt{3}\sin\pi(2x - 3t)$$

(1) 

(2) 

(3) 

(4) 

8. A wave is represented by $x = 4\cos\left(8t - \frac{y}{2}\right)$, where *x* and *y* are in metre and *t* in sec. The frequency of the wave in Hz is

(1) $\frac{4}{\pi}$ (2) $\frac{8}{\pi}$

(3) $\frac{2}{\pi}$ (4) $\frac{\pi}{4}$

9. The temperature at which the velocity of sound in air becomes double its velocity at 0°C is

(1) 435°C (2) 694°C

(3) 781°C (4) 819°C

10. Ten tunning forks are arranged in increasing order of frequency in such a way that any two consecutive tunning forks produces 4 beats/s. The highest frequency is twice that of the lowest. Possible highest and lowest frequencies (in Hz) are

(1) 80 & 40 (2) 100 & 50

(3) 44 & 22 (4) 72 & 36

11. A tunning fork of unknown frequency produces 4 beats per second when sounded with another tunning fork of frequency 254 Hz. It gives the same number of beats/s when loaded with wax. The unknown frequency is

(1) 258 (2) 254

(3) 250 (4) Can't be determined

12. A train moves towards a stationary observer with speed 40 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the trains's speed is reduced to 20 m/s, the frequency registered is f_2. If the speed of sound is 340 m/s, then the ratio of $\frac{f_1}{f_2}$ is

(1) $\frac{15}{16}$ (2) $\frac{1}{2}$

(3) 2 (4) $\frac{16}{15}$

13. A tunning fork vibrating with a sonometer having 20 cm wire produces 5 beats/s. When the length of the wire is changed to 21 cm the beat frequency does not change. The frequency of the tunning fork must be

(1) 200 Hz (2) 210 Hz

(3) 205 Hz (4) 215 Hz

14. A closed pipe of length 10 cm has its fundamental frequency half that of the second overtone of an open pipe. The length of the open pipe must be

(1) 10 cm (2) 20 cm

(3) 30 cm (4) 40 cm

15. The string of a violin has a fundamental frequency of 440 Hz. If the violin string is shortend by one fifth, its fundamental frequency will be changed to

(1) 440 Hz (2) 880 Hz

(3) 550 Hz (4) 2200 Hz

16. The speed of sound in hydrogen at NTP is 1270 m/s. Then the speed in m/s in a mixture of hydrogen and oxygen in the ratio 4 : 1 by volume will be

(1) 635 (2) 318

(3) 158 (4) 1270

17. Two identical sources of sound S_1 and S_2 produce intensity I_0 at a point P equidistant from each source. If intensity of S_1 is reduced to 64 % of initial value, then resultant intensity at P would be

(1) $0.25I_0$

(2) $0.01I_0$

(3) $0.81I_0$

(4) $0.64I_0$

18. A dog is barking at the rate 4π mW in a spherical distribution. Sound level at distance 100 m from the dog is

(1) 5 dB (2) 20 dB

(3) 50 dB (4) 70 dB

19. Two strings of same material are stretched to the same tension. If their radii are in the ratio 1 : 2, then respective wave velocities will be in ratio

(1) 4 : 1 (2) 2 : 1

(3) 1 : 2 (4) 1 : 4

20. In a resonance tube two successive positions of resonance are obtained at 15 cm and 48 cm. If the frequency of the fork is 500 Hz, then the velocity of sound is

(1) 330 m/s (2) 300 m/s

(3) 1000 m/s (4) 360 m/s

21. The sound intensity level at a point 4 m from the point source is 10 dB, then the sound level at a distance 2 m from the same source will be

(1) 26 dB (2) 16 dB

(3) 23 dB (4) 32 dB

22. If velocity of sound in a gas (γ = 1.5) at STP is 600 m/s then rms velocity of the gas molecules at STP will be

(1) 400 m/s (2) 600 m/s

(3) $600\sqrt{2}$ m/s (4) $300\sqrt{2}$ m/s

23. A transverse wave has amplitude A, maximum particle speed B, angular wave number C. Speed of the wave is

(1) $\frac{B}{AC}$ (2) $\frac{A}{BC}$

(3) $\frac{AB}{C}$ (4) $\frac{CB}{A}$

24. Two waves given by : $y_1 = 5 \sin \omega t$ m and $y_2 = 5 \sin (\omega t + \phi)$ m superimpose to give resultant wave of amplitude 5 m. Then ϕ is equal to

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$ (4) $\tan^{-1}\frac{4}{3}-\frac{2\pi}{3}$

25. Strings A and B of a guitar produce 4 beats in 2 seconds. If the tension in string B is slightly decreased beat frequency is found to be 4 Hz. If the frequency of A is 300 Hz, then the original frequency of B is

(1) 296 Hz

(2) 298 Hz

(3) 302 Hz

(4) 304 Hz

26. A string vibrates according to the equation $y = 10\sin\left(\frac{2\pi x}{3}\right)\cos(20\pi t),$ where x and y are in cm and t is in sec. The distance between two adjacent nodes is

(1) 1 cm (2) 1.5 cm

(3) 2 cm (4) 2.5 cm

27. If consecutive frequencies emitted from an organ pipe are 50 Hz, 75 Hz, 100 Hz, 125 Hz, then the frequency of the 10th overtone is

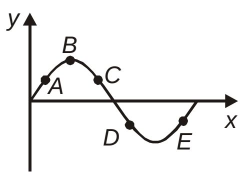
(1) 200 Hz

(2) 250 Hz

(3) 275 Hz

(4) 300 Hz

28. The diagram shows portion of a transverse wave travelling in +x direction. Choose the correct option



(1) Point D has acceleration in negative y direction

(2) Point C has acceleration in negative x direction

(3) Point A and point C have velocities in opposite directions

(4) Point A and point E have velocities in opposite directions

29. An organ pipe closed at one end is in resonance in its fifth harmonic with fork of frequency f_1. Now it is opened at one end and the frequency of tuning fork is decreased slowly from f_1 and again resonance is obtained at frequency f_2. In this case, the organ pipe vibrates in n^{th} harmonic then

(1) $n = 2,\ f_2 = \frac{5}{4}f_1$

(2) $n = 3,\ f_1 = \frac{5}{4}f_2$

(3) $n = 3,\ f_2 = \frac{4}{5}f_1$

(4) $n = 2,\ f_1 = \frac{5}{4}f_2$

30. A source of sound is moving in horizontal x-y plane on a circle of radius R with centre at origin. A wind is blowing along negative x-direction. An observer at x = 2R will hear the actual frequency of the source when the source is at

(1) (± R, 0)

(2) y = + R, –R (0, ± R)

(3) Depends on the wind velocity

(4) Will never hear the actual frequency

31. A source of sound is moving towards an observer then

(1) Velocity of sound in the medium is increased

(2) Wavelength of sound in the medium is decreased

(3) Frequency of the sound source is decreased

(4) Amplitude of the vibration of the particles is increased

32. A train moving towards a hill with speed of 110 m/s sounds a horn of frequency 700 Hz. If the velocity of sound in air is 330 m/s, the frequency of reflected sound as heard by the driver of the train is

(1) 1200 Hz

(2) 1300 Hz

(3) 1400 Hz

(4) 1500 Hz

33. Standing waves can't be produced

(1) When incident wave gets reflected from a wall

(2) On a string clamped at both the ends

(3) On a string clamped at one end and free at the other

(4) When two identical waves with a phase difference of π are moving in same direction

34. Two uniform strings P and Q made of copper are vibrated under same tension. If the second overtone of P is equal to fourth harmonic of Q and if the radius of P is twice that of Q, the ratio of lengths of the strings is

(1) $\frac{3}{4}$ (2) $\frac{3}{8}$

(3) $\frac{5}{4}$ (4) $\frac{5}{8}$

35. Power of a sound from the speaker of a stereo is 25×10^{-3} W. By turning the knob of the volume control, the power of the sound is increased by 25×10^{-2} W. The power increase in decibels compared to the original power is approximately

(1) 90 dB

(2) 10 dB

(3) 100 dB

(4) 20 dB

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ANSWERS

1. Units and Measurement

1.	(3)	2.	(2)	3.	(2)	4.	(1)	5.	(1)	6.	(3)	7.	(2)
8.	(3)	9.	(3)	10.	(4)	11.	(2)	12.	(1)	13.	(1)	14.	(1)
15.	(4)	16.	(2)	17.	(1)	18.	(2)	19.	(2)	20.	(1)	21.	(1)
22.	(3)	23.	(1)	24.	(3)	25.	(1)	26.	(3)	27.	(3)	28.	(2)
29.	(2)	30.	(3)										

2. Description of Motion in One Dimension

1.	(1)	2.	(3)	3.	(2)	4.	(3)	5.	(2)	6.	(1)	7.	(2)
8.	(4)	9.	(1)	10.	(4)	11.	(3)	12.	(4)	13.	(3)	14.	(3)
15.	(3)	16.	(1)	17.	(2)	18.	(4)	19.	(1)	20.	(3)	21.	(1)
22.	(1)	23.	(3)	24.	(1)	25.	(2)	26.	(4)	27.	(4)	28.	(2)
29.	(2)	30.	(2)										

3. Description of Motion in Two and Three Dimensions

1.	(4)	2.	(1)	3.	(1)	4.	(2)	5.	(1)	6.	(2)	7.	(1)
8.	(1)	9.	(4)	10.	(4)	11.	(2)	12.	(3)	13.	(2)	14.	(2)
15.	(4)	16.	(2)	17.	(3)	18.	(2)	19.	(2)	20.	(1)	21.	(3)
22.	(2)	23.	(4)	24.	(2)	25.	(2)	26.	(3)	27.	(2)	28.	(3)
29.	(2)	30.	(2)										

4. Laws of Motion

1.	(1)	2.	(4)	3.	(2)	4.	(2)	5.	(3)	6.	(3)	7.	(4)
8.	(4)	9.	(1)	10.	(2)	11.	(3)	12.	(2)	13.	(4)	14.	(2)
15.	(4)	16.	(3)	17.	(2)	18.	(1)	19.	(2)	20.	(1)	21.	(4)
22.	(2)	23.	(2)	24.	(1)	25.	(3)	26.	(2)	27.	(4)	28.	(4)
29.	(2)	30.	(2)	31.	(1)	32.	(3)	33.	(2)	34.	(2)	35.	(3)

5. Work, Energy and Power

1.	(2)	2.	(4)	3.	(1)	4.	(2)	5.	(3)	6.	(3)	7.	(1)
8.	(3)	9.	(2)	10.	(2)	11.	(4)	12.	(1)	13.	(3)	14.	(1)
15.	(3)	16.	(1)	17.	(3)	18.	(1)	19.	(2)	20.	(2)	21.	(4)
22.	(4)	23.	(1)	24.	(2)	25.	(3)	26.	(2)	27.	(4)	28.	(4)
29.	(1)	30.	(1)										

6. Rotational Motion and Moment of Inertia

1.	(2)	2.	(2)	3.	(3)	4.	(3)	5.	(3)	6.	(1)	7.	(3)
8.	(4)	9.	(3)	10.	(3)	11.	(2)	12.	(1)	13.	(3)	14.	(1)
15.	(2)	16.	(2)	17.	(4)	18.	(4)	19.	(2)	20.	(3)	21.	(4)
22.	(4)	23.	(3)	24.	(4)	25.	(2)	26.	(2)	27.	(4)	28.	(1)
29.	(2)	30.	(3)										

7. Gravitation

1.	(4)	2.	(4)	3.	(3)	4.	(1)	5.	(4)	6.	(3)	7.	(1)
8.	(3)	9.	(3)	10.	(1)	11.	(3)	12.	(4)	13.	(2)	14.	(3)
15.	(1)	16.	(2)	17.	(1)	18.	(3)	19.	(2)	20.	(3)	21.	(1)
22.	(3)	23.	(3)	24.	(4)	25.	(3)	26.	(2)	27.	(2)	28.	(3)
29.	(3)	30.	(3)										

8. Solids and Fluids

1.	(1)	2.	(2)	3.	(1)	4.	(3)	5.	(2)	6.	(1)	7.	(1)
8.	(3)	9.	(4)	10.	(3)	11.	(2)	12.	(3)	13.	(2)	14.	(1)
15.	(3)	16.	(3)	17.	(2)	18.	(2)	19.	(2)	20.	(4)	21.	(3)
22.	(4)	23.	(4)	24.	(3)	25.	(2)	26.	(4)	27.	(3)	28.	(2)
29.	(3)	30.	(1)	31.	(4)	32.	(4)	33.	(3)	34.	(4)	35.	(3)

9. Heat and Thermodynamics

1.	(1)	2.	(3)	3.	(1)	4.	(1)	5.	(3)	6.	(1)	7.	(4)
8.	(2)	9.	(2)	10.	(4)	11.	(2)	12.	(4)	13.	(2)	14.	(2)
15.	(1)	16.	(2)	17.	(2)	18.	(4)	19.	(2)	20.	(4)	21.	(4)
22.	(1)	23.	(1)	24.	(4)	25.	(4)	26.	(2)	27.	(3)	28.	(1)
29.	(3)	30.	(4)										

10. Transfer of Heat

1.	(4)	2.	(4)	3.	(1)	4.	(3)	5.	(2)	6.	(1)	7.	(3)
8.	(2)	9.	(3)	10.	(4)	11.	(3)	12.	(4)	13.	(4)	14.	(4)
15.	(1)	16.	(4)	17.	(1)	18.	(1)	19.	(1)	20.	(2)	21.	(3)
22.	(4)	23.	(3)	24.	(2)	25.	(1)						

11. Oscillations

1.	(2)	2.	(2)	3.	(1)	4.	(4)	5.	(2)	6.	(1)	7.	(4)
8.	(4)	9.	(3)	10.	(2)	11.	(4)	12.	(2)	13.	(3)	14.	(2)
15.	(2)	16.	(1)	17.	(3)	18.	(1)	19.	(2)	20.	(1)	21.	(1)
22.	(4)	23.	(3)	24.	(2)	25.	(1)	26.	(2)	27.	(4)	28.	(3)
29.	(4)	30.	(1)	31.	(4)	32.	(3)	33.	(3)	34.	(1)	35.	(3)

12. Waves

1.	(3)	2.	(2)	3.	(2)	4	(3)	5.	(4)	6.	(2)	7.	(1)
8.	(1)	9.	(4)	10.	(4)	11.	(1)	12.	(4)	13.	(3)	14.	(3)
15.	(3)	16.	(1)	17.	(3)	18.	(3)	19.	(2)	20.	(1)	21.	(2)
22.	(3)	23.	(1)	24.	(1)	25.	(2)	26.	(2)	27.	(3)	28.	(3)
29.	(4)	30.	(1)	31.	(2)	32.	(3)	33.	(4)	34.	(2)	35.	(2)

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