CIRCULAR, W.P.E

EXERCISE – I SINGLE CORRECT

(A) CIRCULAR MOTION

1. **C**

$\omega_i = 0$; $\omega_f = 80$ rad/sec

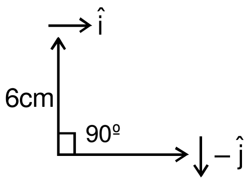
$t = 5$ sec

$\omega_f = \omega_i + \alpha t$

$\alpha = \frac{80}{5} = 16$ rad/sec^2

$\theta = \frac{1}{2}\alpha t^2 = 200$ rad

2. **D**



speed of the second hand

$v = \frac{2\pi r}{\text{time in one revolution}}$

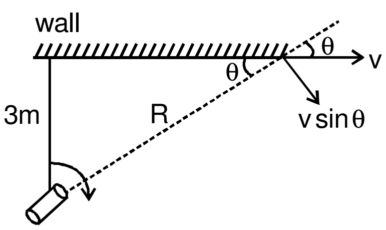
$v = \frac{2\pi 6}{60} = 2\pi\, mm/s$

Magnitude of difference of vel.= $v_i - (-v_j)$

$= v_i + v_j$; $v = \sqrt{v_i^2 + v_j^2} = \sqrt{8\pi^2}$

$= 2\sqrt{2}\pi$

3. **A**



$\omega = \frac{V_\perp}{R}$

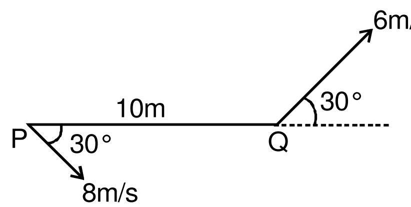
$\omega = \frac{v\sin\theta}{3/\sin\theta}$

$\omega = \frac{v\sin^2\theta}{3}$

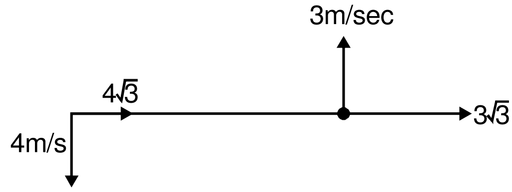
$v = 0.6$ m/s

(Given $\theta = 45°$ $\omega = 0.1$ rad/s)

4. **D**



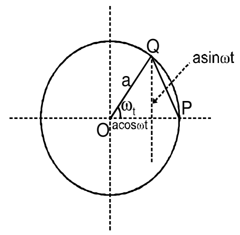
w.r.t to P



$v_{\perp rel} = 8\sin 30° + 6\sin 30° = 7$ m/s

$\omega = \frac{v_{\perp rel}}{R} = \frac{7}{10} = 0.7\, rad/sec$

5.



$PQ = \sqrt{(a - a\cos\omega t)^2 + (a\sin\omega t)^2}$

$= 2a\sin\left(\frac{\omega t}{2}\right)$

6. **C**

$\theta_A \propto t^2$ $\theta_B \propto t$

$\theta_A = k_1 t^2$ $\theta_B = k_2 t$

From given conditon calculate k_1 and k_2

$2\pi = k_1 \times \pi$ $\pi = k_2 \times 4\pi$

$k_1 = 2$ $k_2 = 1/4$

$\therefore$ $\theta_A = 2t^2$ $\theta_B = t/4$

$w_A = \frac{d\theta_A}{dt} = 4t$ $w_B = \frac{d\theta_B}{dt} = \frac{1}{4}$

$\left(\frac{d\theta_A}{dt}\right)_{t=5sec} = 20$ $\left(\frac{d\theta_B}{dt}\right)_{t=5sec} = \frac{1}{4}$

$\omega_A : \omega_B = 80 : 1$

7. **C**

$R = \frac{20}{\pi} m$; $v = 80$ m/sec

$v^2 = u^2 - 2a_t s$

$u = 0$ $s = 2(2\pi R)$

$(80)^2 = 2a_t(4\pi.\frac{20}{\pi})$

$a_t = 40$ m/s^2

8. **A**

Slope should be decreasing

$\alpha = \frac{d\omega}{dt} = \tan\theta$, if $\theta\downarrow, \alpha\downarrow$

9. **(i) A**

(i) At any moment $a_t = a_r$

$$a_t = -\frac{v^2}{R}$$

$$v\frac{dv}{ds} = -\frac{v^2}{R} \Rightarrow \frac{dv}{v} = -\frac{1}{R}ds$$

After integration $\log v = -\frac{S}{R} + C$ ...(i)

at $t = 0$, $s = 0$, $v = v_0$

$C = \log v_0$

from eq. (1) $\log\left(\frac{v}{v_0}\right) = -\frac{S}{R}$

$$v = v_0 e^{-S/R}$$

(ii) At any moment $a_t = a_v$

$$a = \sqrt{2}a_r = \sqrt{2}.\frac{v^2}{R}$$

10. **C**

$\omega = \theta^2 + 2\theta$

$$\frac{d\omega}{d\theta} = 2\theta + 2$$

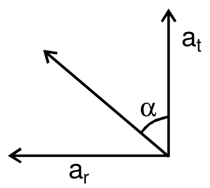
$$\alpha = \frac{\omega d\omega}{d\theta} = (\theta^2 + 2\theta).(2\theta + 2) = 12 \text{ rad/sec}^2$$

11. **B**

Given $v = a\sqrt{s}$

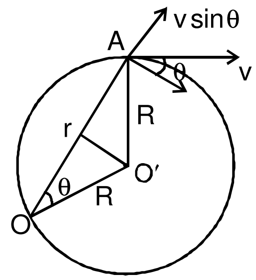
$$a_t = \frac{vdv}{ds} = a\sqrt{s}.\frac{a}{2\sqrt{s}} = \frac{a^2}{2}$$

$$a_r = \frac{v^2}{R} = \frac{a^2 s}{R}$$



$$\tan\alpha = \frac{a_r}{a_t} = \frac{2s}{R}$$

12. **D**



$$\frac{r}{2} = R\cos\theta$$

$r = 2R\cos\theta$

After differentiable

$$\frac{dr}{dt} = -2R\sin\theta\frac{d\theta}{dt}$$

$$\frac{dr}{dt} = v_{rad} = v\sin\theta$$

$$\frac{d\theta}{dt} = \omega \text{ (–ve because } \theta \text{ decreasing)}$$

$v \sin\theta = 2R\sin\theta\omega$

$v = 2R\omega = 0.4$ m/s

$a = \sqrt{a_t^2 + a_r^2}$ $\therefore \omega$ = constant

$\Rightarrow$ $a = a_r = \frac{v^2}{R} \Rightarrow a_t = 0$

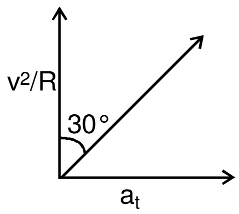
$a_r = \frac{V^2}{R} = 32$ m/s^2

13. **C**

$a_t = \sqrt{3}\,t$

$\int dV = \int \sqrt{3}tdt$

$v = \frac{\sqrt{3}t^2}{2}$



$$\tan 30° = \frac{\sqrt{3}t.R}{\left(\frac{\sqrt{3}t^2}{2}\right)^2} \Rightarrow \frac{1}{\sqrt{3}} = \frac{4t}{\sqrt{3}t^4}$$

$\Rightarrow t^4 = 4t \Rightarrow t^3 = (2)^2$

$\Rightarrow t = 2^{2/3}$ sec

14. C

The magnitude of acceleration is constant in (A) and decreasing in (B)

In (A) $\rightarrow$ r constant, $a_t = 0$;

v constant , $a_r = \frac{V^2}{R}$ constant

In (B) $\rightarrow$ r is increasing , V constant

$a_t = 0$; $a_r = \frac{V^2}{R}$ decreasing

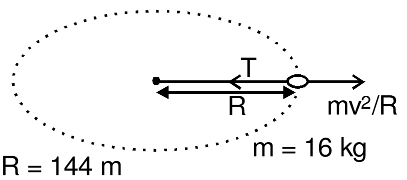
15. C

Given $\frac{m_1}{m_2} = 1$; $\frac{R_1}{R_2} = \frac{1}{2}$

If $\frac{m_1 v_1^2}{R_1} = \frac{m_2 v_2^2}{R_2}$

$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{R_1}{R_2}} = \frac{1}{\sqrt{2}}$

16. D



R = 144 m

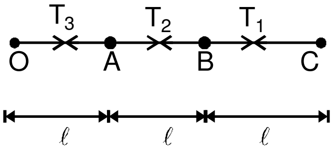
$\frac{MV^2}{R} = T$

$T_{max} = 16\,N$

$v_{max} = \sqrt{\frac{RT}{m}}$

$v_{max} = \sqrt{\frac{16 \times 144}{16}} = 12\,m/s$

17. D



$T_1 = m\omega^2 R = m\omega^2 (3l)$ ..(1)

$T_2 = T_1 + m\omega^2 (2l)$ ...(2)

$T_3 = T_2 + m\omega^2 (l)$ ...(3)

$T_1 : T_2 : T_3 = 3 : 5 : 6$

18. A

$v = r\omega$

If $r \rightarrow r/2$

$\therefore v' = \frac{v}{2} = \frac{20}{2} = 10$ cm/sec

Turn table rotating uniformly $a_t = 0$

$a_r = \frac{v^2}{R}$; $a'_r = \frac{v'^2}{R/2} = \frac{20}{2} = 10$ cm/s^2

19. D

constant speed and variable velocity

20. C

Car will not slip when moving with speed v

21. B

Given $k = as^2$

$v^2 = \frac{2a}{m}s^2$

After differentiating w.r.t s $\frac{vdv}{ds} = \frac{2as}{m} = a_t$

$a_r = \frac{v^2}{R} = \frac{2as^2}{mR}$

Total force = $\sqrt{(ma_r)^2 + (ma_t)^2}$

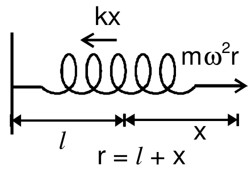
$\therefore$ Total force = $\sqrt{m^2\left(\frac{2a}{mR}s^2\right)^2 + \left(m\frac{2as}{m}\right)^2}$

$= \sqrt{\frac{4a^2s^4}{R^2} + 2a^2s^2}$

$= 2as\sqrt{\frac{s^2}{R^2} + 1}$

$= 2as\left(1 + \frac{s^2}{R^2}\right)^{1/2}$

22. B

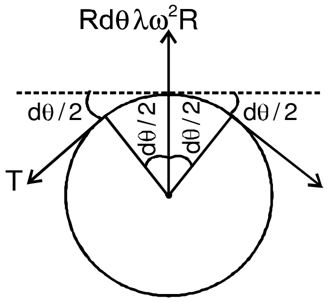


$kx = m\omega^2 r$

$kx = m\omega^2 (l + x)$

$x = \frac{m\omega^2 \ell}{k - m\omega^2}$

23. **C**



$$2T\sin\frac{d\theta}{2} = Rd\theta\lambda\omega^2R$$

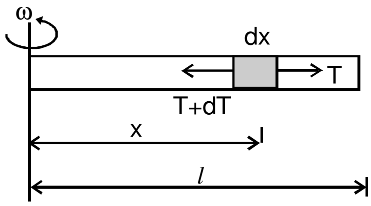
If dθ is small

$$\sin\frac{d\theta}{2} \simeq \frac{d\theta}{2}$$

$$2T\frac{d\theta}{2} = Rd\theta\lambda\omega^2R$$

$$T = \lambda\omega^2R^2$$

24 **D**



$$dT = \frac{m}{\ell}.\omega^2xdx$$

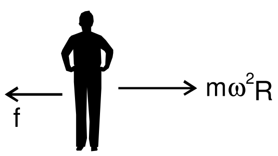
Integrate with limit x to ℓ

$$T = \int_x^{\ell}\frac{m}{\ell}\omega^2xdx$$

$$T = \frac{m\omega^2}{\ell}\left[\frac{x^2}{2}\right]_x^{\ell} = \frac{1}{2}\frac{m\omega^2}{\ell}[\ell^2 - x^2]$$

25. $mg < \dfrac{mv^2}{R}$

26.



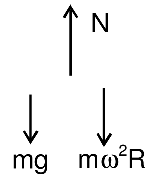
$$\mu mg \geq \frac{mv^2}{R}$$

$0.5\ mg \geq m \times (5)^2 \times R$

$$\frac{0.5\times10}{25} \geq R$$

$R \leq 0.2$ m

27. **C**



R = 10 m

m = 500 kg

$N = m\omega^2 R + mg$

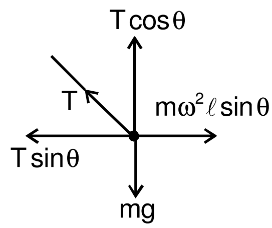
$$= \frac{mv^2}{R} + mg = \frac{500\times400}{10} + 500\times10$$

= 25 kN

28. At the highest point

velocity is zero so bob will follow the trajectory straight line

29.



$r = \ell \sin\theta$

$T \sin\theta = m\ \omega^2\ l \sin\theta$ ...(1)

$\Rightarrow$ $T = m\omega^2 l$ Ans.

$T \cos\theta = mg$ ...(2)

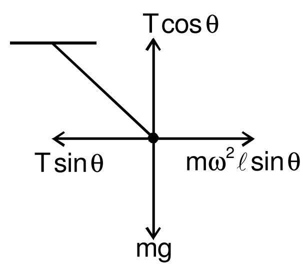
30. $v = \sqrt{Rg\tan\theta}$

$R = 10\sqrt{3}$ m, $\theta = 30°$

$$= \sqrt{10\sqrt{3}\times10\times\frac{1}{\sqrt{3}}}$$

= 10 m/sec = 36 km/hr

31.



T of simple pendulum = $2\pi\sqrt{\dfrac{\ell}{g}}$

$T \sin\theta = m\ \omega^2\ l \sin\theta$

$\Rightarrow T = m\omega^2\ l$

and $T \cos\theta = mg$

$\Rightarrow \quad T = \dfrac{mg}{\cos\theta}$

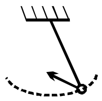
Now, $\dfrac{g}{\cos\theta} = \omega^2 l \;\Rightarrow\; \omega = \sqrt{\dfrac{g}{l\cos\theta}}$

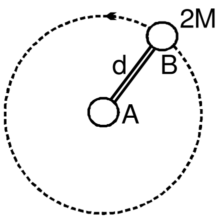
$\therefore\; T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{\ell\cos\theta}{g}}$

$\therefore\; \dfrac{T_{\text{conical Pendulum}}}{T_{\text{simple Pendulum}}} = 2\pi\sqrt{\dfrac{\ell}{g}\cos\theta} \times \sqrt{\dfrac{g}{\ell}} \times \dfrac{1}{2\pi}$

Ratio = $\sqrt{\cos\theta}$

32. In uniform circular motion
Force is towards centre

33. 

34. 

$P = \dfrac{2\pi}{\omega}$

$\Rightarrow \omega^{-1} = \dfrac{P}{2\pi}$

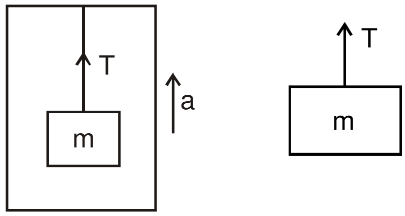
$T = 2M\,\omega^2 d = \dfrac{8\pi^2 Md}{P^2}$

35. $\because\; \omega = \dfrac{2\pi}{T}$

T is same for both cases car's
so ratio $\omega_1 : \omega_2 = 1 : 1$

(B) WORK, POWER AND ENERGY

36. $F \perp dr$
$\therefore$ W.D. = 0
Force and displacement are perpendicular to each other.

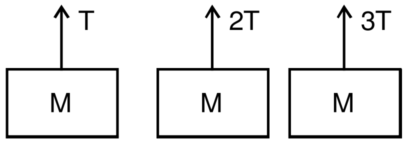
37. 

By the lift frame
$T = m(g + a)$
and

$S = \dfrac{1}{2}at^2$

$\Rightarrow W_{\text{by }T} = \vec{T}.\vec{S} = TS\cos 0°$

$= m(g + a) \times \dfrac{1}{2}at^2$

38. 

$\therefore$ (3) Mechanical Advantage

39. In case of first spring $F = k_1x_1$

$x_1 = \dfrac{F}{K_1}$ ...(1)

In case of second spring $F = K_2x_2$

$x_2 = \dfrac{F}{K_2}$ ...(2)

$\therefore\; K_1 > K_2 \Rightarrow x_2 > x_1$
$\Rightarrow$ More work is done by this force in case of second spring.

40. $f = \mu_k N$ (Tangentially)
$\Rightarrow \quad W = -2\pi r\,\mu_k N$
–ve sign indicate that f & ds is opposite

41. $V_0 = at_0$

$a = \dfrac{v_0}{t_0}$

$\therefore\; v = \dfrac{v_0}{t_0}.t$

$w = \Delta k = k_f - k_i$

$\Rightarrow \dfrac{1}{2}M\dfrac{v_0^2}{t_0^2}.t^2$

42. W.D. = $\int \vec{F}.\vec{ds}$

$= K\int [(y\hat{i} + x\hat{j}).(dx\hat{i} + dy\hat{j})]$

$= K\int (ydx + xdy)$

$= K\int_{15}^{35} d(xy) = 20K$

43. Maximum velocity will be at Mean Position
Where $F_{net} = 0$
mg = Kx
1 × 10 = 2 × 100 × x
x = 5 cm
∴ h = 20 – 5 = 15 cm

44. $w_{sp} = \frac{1}{2}kx_2^{\,2} - \frac{1}{2}kx_1^{\,2}$

$= \frac{1}{2}K(6^2 - 4^2) \times 10^{-4}$

$= \frac{1}{2} \times \frac{10}{10^{-2}} \times 20 \times 10^{-4}$ **=** 1 joule

45. Potential energy Decreases
w = –Δu

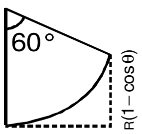
46. $\frac{1}{2}K(0.3)^2 = 10$

$\Rightarrow K = \frac{20}{0.09} = \frac{2000}{9}$

work done = $\frac{1}{2}.\frac{2000}{9}[(0.45)^2 - (0.3)^2]$

= 12.5 J

47.



W = Rθx F cos 0°
(by the force)

= 10 × $\frac{\pi}{3}$ × 200

Work done by g = MgR (1 – cos 60°)

$= \frac{gRM}{2}$

K.E. = RFθ – $\frac{gRM}{2}$

$\frac{1}{2}MV^2 = 10 \times \frac{\pi}{3} \times 200 - \frac{10 \times 10 \times 10}{2}$

$v^2 = 2 \times \frac{\pi}{3} \times 200 - 50$

V = 17.32 m/s

48. 2 $K.E_{man}$ = $K.E._{boy}$

$2 \times \frac{1}{2}M \times v_{man}^2 = \frac{1}{2}.\frac{M}{2}v_{boy}^2$

$V_{man} = \frac{v_{boy}}{2}$ ...(i)

$\frac{1}{2}M(v_{man} + 1)^2 = \frac{1}{2}.\frac{M}{2}v_{boy}^2$

$\Rightarrow (v_{man} + 1)^2 = \frac{V_{boy}^2}{2}$

$v_{man} = (\sqrt{2} + 1) m/sec$

49. $u_i = 0$ and $u_f = -2$
Δk = – Δu = 2 Joule

50. v = at

= $10\sqrt{3}$ m/s

In ground frame
W.D. by gravity + W.D. by normal = Δk

0 + $W.D._N$ = $\frac{1}{2}$ × 1 × $(10\sqrt{3})^2$ = 150J

51. $(W.D)_{by\ friction} + (W.D)_{by\ spring} = \Delta k = k_f - k_i = 0 - k_i$

– 0.25 × 1 × 10 × 4 – $\frac{1}{2}$ × 2.75 × 4^2 = $-\frac{1}{2} \times 1 \times v^2$

v = 8 m/s

52. Friction is present
∴ Mechanical energy is not conserved
But work energy principle conserved
Due to extrenal friction force is working on the block.

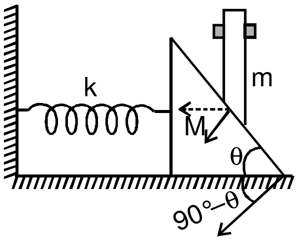
53. The block will come to rest when work done by friction becomes equal to the change in energy stored in spring.

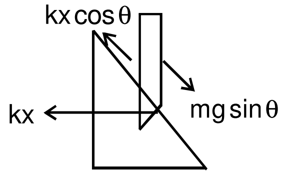
54. Work done by force F = 100 ×11 × $\frac{1}{2}$ = 550 J

Work done by the gravity = mgh
mgh = 550

$\Rightarrow h = \frac{550}{5 \times 10} = 11m$

55.



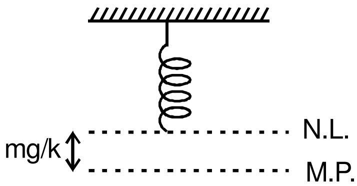


$kx \cos\theta = mg \sin\theta$

$x = mg \tan\theta / k$

$$P.E. = \frac{1}{2}kx^2 = \frac{m^2g^2\tan^2\theta}{2k}$$

56.



$K = \frac{mg}{a}$ (Given)

$$\frac{1}{2}\times m\times v^2 + \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = mg\left(\frac{mg}{k}\right)$$

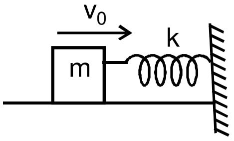
$$\frac{1}{2}\times m\times v^2 + \frac{1}{2}\times\frac{mg}{a}\times\frac{m^2g^2}{m^2g^2}\times a^2 = \frac{m^2g^2}{mg}\times a$$

$$\frac{1}{2}mv^2 + \frac{1}{2}mga = mga$$

$v^2 = ga$

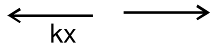
$$K.E. = \frac{1}{2}mv^2 = \frac{mga}{2}$$

57.



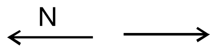
$$-\frac{1}{2}mv_0^2$$

58.



$$-\frac{1}{2}mv_0^2$$

59.



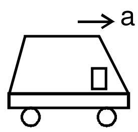
Net work done $= -\frac{1}{2}mv_0^2$

60 K.E is converted into P.E.
Mechanical energy conserved.
Block loses its K.E.

61. Velocity of block with respect to observer B is zero so
K.E of block = 0

62. P.E ↑
Due to +ve work done by N

63.



$P = F.V = (R + ma)\ V$

64. $P = F.V$
Given

$\tan\theta = \frac{1}{100}$; $v = 30$ km/hr $= 30 \times \frac{5}{18}$ m/s

$P = mg \sin\theta.\ v$ [θ is very small]

$= 30{,}000 \times 10 \times \frac{1}{100} \times 30 \times \frac{5}{18} = 25$ kw

65. $\vec{P} = \vec{F}.\vec{V}$

$P = (10\hat{i} + 10\hat{j} + 10\hat{k}).(5\hat{i} - 3\hat{j} + 6\hat{k})$

$P = (50 - 30 + 120)$

$P = 140$ J/sec

66. On comparing
$F \propto V$
$F = kV$
$P = F.V = kV^2$
Now $2P = KV'^2$
$2 \times kv^2 = kV'^2$
$\Rightarrow V'^2 = 2V^2$
$V' = \sqrt{2}V$

67. $\frac{dW}{dt} = \frac{d.K.E.}{dt} (K.E = 2t^2)$

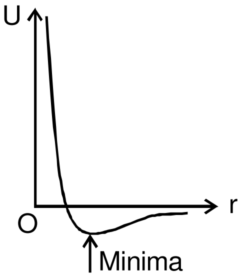
$\Rightarrow \quad P = \left(\frac{dK.E.}{dt}\right)_{at\,t=2s} = 4t = 8\text{watt}$

$\frac{1}{2}mv^2 = 2t^2$

$\Rightarrow v = 4t \qquad [\therefore m = 1 \text{ kg}]$

$\Rightarrow \frac{dv}{dt} = 2m/s^2 = a_t$

68.



For stable equilibrium

$\frac{dU}{dr} = 0 \Rightarrow r_1$ and $\left(\frac{d^2U}{dr^2}\right)_{r_1} > 0$

For stable equilibrium P.E. must be Minimum at the equilibrium position

69. $\vec{F} = -\frac{du}{dx}\hat{i} - \frac{du}{dy}\hat{j} - \frac{du}{dz}\hat{k}$

$\vec{F} = \Delta U \qquad [U = \sin(x+y)]$

$= \cos(x+y)\hat{i} + \cos(x+y)\hat{j}$

$\vec{F}_{(0,\pi/4)} = \cos\frac{\pi}{4}\hat{i} + \cos\frac{\pi}{4}\hat{j}$

$|\vec{F}| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$

70. $2x^2 - 3x - 2 = 0$

$x = \frac{3 \pm \sqrt{9+16}}{4} = \frac{3 \pm 5}{4}$

$x = -\frac{1}{2}, 2$

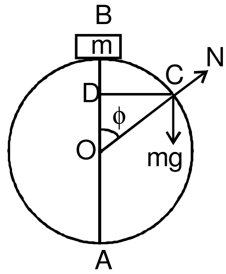
$\frac{dF}{dx} = -\frac{d^2u}{dx^2} = 4x - 3$

$\Rightarrow \quad \frac{d^2u}{dx^2} = 3 - 4x$

$\Rightarrow \quad \left(\frac{d^2u}{dx^2}\right)_{x=-\frac{1}{2}} = 3 + 4 \times \frac{1}{2}$

$= (5) > 0$ (stable)

71.



$mg\cos\phi - N = \frac{mv^2}{R}$

$N = m(g\cos\phi - \frac{v^2}{R}) \qquad ...(i)$

$\because N = 0$

$\Rightarrow \cos\phi = \frac{v^2}{Rg} \qquad ...(ii)$

By energy conservation

$\frac{1}{2}mv^2 = mg(R - R\cos\phi) \Rightarrow v^2 = 2Rg(1-\cos\varphi)$

Using (i) & (ii) $\cos\phi = \frac{2}{3}$

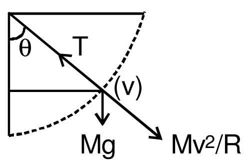
height from highest Point = BD = $R(1 - \cos\phi)$

$h = R\left(1 - \frac{2}{3}\right) = \frac{R}{3}$ Ans.

72. $\sqrt{5Rg} = \sqrt{5 \times 2.5 \times 10} = 5\sqrt{5} > 10\,m/s$

$\therefore N_2$ will be zero in part A, D, C at some point

73.



$T = \frac{Mv^2}{R} + Mg\cos\theta$

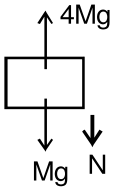
$MgR\cos\theta = \frac{1}{2}Mv^2$

$\Rightarrow Mgh = \frac{1}{2}Mv^2$

$T = \frac{2Mgh + Mgh}{R}$

Straight line

74.



$2MgR = \frac{1}{2}Mv^2$

$\Rightarrow$ $2\sqrt{gR} = V$

$\frac{mv^2}{R} = mg + N$

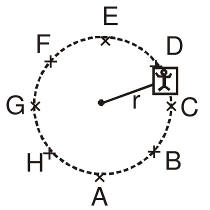
$\Rightarrow$ N = 3 mg

75. Initially be in contact with the inner wall and later with the outer wall.

76. $\sqrt{2g\ell}$

EXERCISE – II | MULTIPLE CHOICE QUESTIONS

1. B,D
(B) There are other forces on the particle
(D) The resultant of the other forces varies in magnitude as well as in direction.

2.


$v = \sqrt{gr}$
At A
$N = mg + \frac{mv^2}{r} = 2mg \quad [v = \sqrt{gr}]$
at E
$N + \frac{mv^2}{r} = mg$
$\Rightarrow$ N = 0
At G and C
N = mg

3. $T - Mg\cos\theta = \frac{Mv^2}{L}$
Tangential Acceleration = g sin θ

4. (A) $F \perp V$
(C) Object is at Rest But point of application of the force moves on the object.
(D) The object moves in such a way that point of application of the force remains fixed.

5. (A) The spring initially compressed and finally in its N.L.
(B) Initially stretched and then in its N.L.

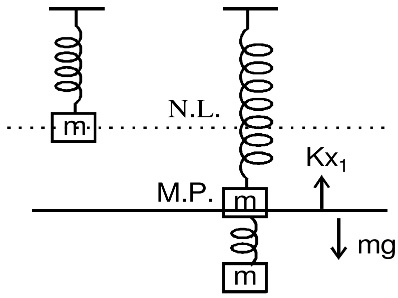
6. W.D. by force of friction
can be zero, positive & Negative

7. Total work done on a Particle positive when momentum increases & K.E increases

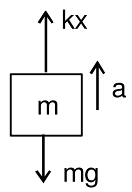
8. Total energy = E = K.E + P.E.
When speed of the particle is zero.
i.e., K.E = 0
$\Rightarrow U(x) = E$

9. Angle of Inclination

10. Only Conservative force (mg) is act.
So E.C. is done only two points
(1 and 2)

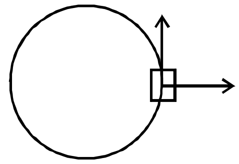
11.


M.P. $x_1 = \frac{mg}{k}$
But block further move downward due to inertia. So descending through distance
$x = \frac{2mg}{k}$

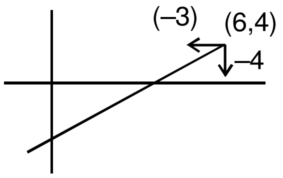


at M.P. at $\frac{x}{2} \Rightarrow F_{net} = 0$; so a = 0
at lower most point
$k\left(\frac{2mg}{k}\right) - mg = ma$
a = g

12. Particle takes speed tangentially and act as a 'Projectile' (curved path)



13. Given U = 3x + 4y
Initially particle at rest at (6,4)
So K.E = 0
$E_{total} = P.E = 3 \times 6 + 4 \times 4 = 34$ J
$F = -\frac{\partial U}{\partial y}\hat{i} - \frac{\partial U}{\partial y}\hat{j} = -3\hat{i} - 4\hat{j}$



$a = -3\hat{i} - 4\hat{j}$

$|a| = 5 m/s^2$

Let us assume particle crosses y axis after time t

$$x - 6 = -\frac{1}{2} \times 3 \times t^2$$

at y axis

$x = 0$

$\Rightarrow$ $t = 2$ sec

So $y - 4 = -\frac{1}{2} \times 4 \times (2)^2 = -8$

$y = -4m$

(P.E.) at y = – 4 and x = 0

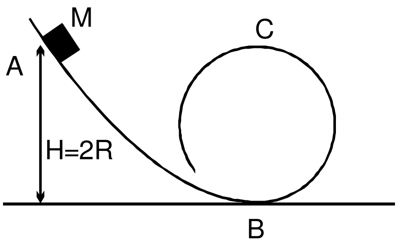
is $U_{(y = -4,\, x = 0)} = -16$ J

So. K.E. = T.E. – U

$$\frac{1}{2} MV^2 = 34 - (-16) = 50$$

$V^2 = 100 \Rightarrow V = 10$ m/s

14.



E.C between point A and B

$$Mg\,(2R) = \frac{1}{2} MV^2$$

$V = \sqrt{4gR} < \sqrt{5gR}$

$V = \sqrt{4gR} > \sqrt{2gR}$

So, doesn't complete vertical circle and break off at a height (R < H < 2R)

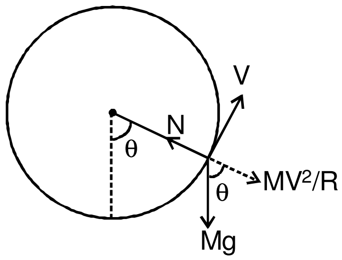
16. To complete vertical circle

speed at point B ≥ $\sqrt{5gR}$

So. E.C.

$$MgH = \frac{1}{2} M(5gR)$$

$$H = \frac{5R}{2} = 2.5\, R$$

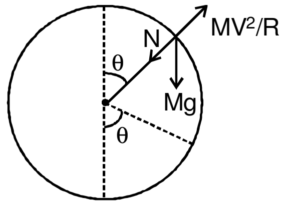


$$N = \frac{Mv^2}{R} + Mg\cos\theta$$

N_{max} at $\theta = 0°$

N is zero only

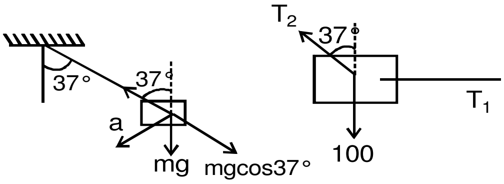
$\theta \geq \pi/2$ because in this



$$N = \frac{MV^2}{R} - Mg\cos\theta$$

EXERCISE – III | SUBJECTIVE PROBLEMS

1.



$T_2 \times \frac{4}{5} = 100$ $\quad$ $T_1 = 80$ N

$T_2 = 125$ N

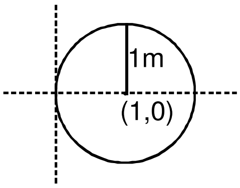
2. $\vec{v} = a\hat{i} + bt\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt} = b\hat{j}$

component of acceleration along v is

$a_t = \frac{\vec{a}.\vec{v}}{|\vec{v}|} - \frac{b^2 t}{\sqrt{a^2 + b^2 t^2}}$

3.



$a_t = \frac{\pi}{2} m/s^2$

from $S = ut + \frac{1}{2}at^2$

$\Rightarrow \pi = 0 + \frac{1}{2} \times \frac{\pi}{2} t^2$

t = 2 sec

from v = u + at

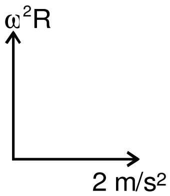
$\Rightarrow v = \frac{\pi}{2}.2 = 3.14$ m/s

4. **Do Yourself**

5. $\alpha = \frac{a_t}{R}$ $\quad$ $a_r = \frac{v^2}{R}$

$a_t = 6\ m/s^2$ $\quad$ $a_r = 8\ m/s^2$

6.



$\omega_i = 12$ rad/s

$\omega_f = \frac{2\pi \times 480}{\pi \times 60} = 16$ rad/s

$\omega_f = \omega_i + \alpha t$

$\Rightarrow \alpha = 2$ rad/sec²

12 + 2t for $t \le 2$ s

16 for $t \ge 2$ sec

at 0.5 sec

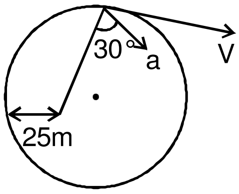
$a_t = 2m/s^2$

$\omega_f = 12 + 0.5 \times 2 = 13$ rad/sec

$\sqrt{28565} \approx 169$

at t = 3 sec only $a_r = \omega^2 r$

7.



$a_r = a \cos 30° = 25 \times \frac{\sqrt{3}}{2} m/s^2$

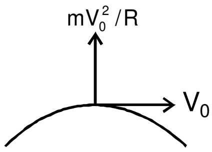
$\because\ a_r = \frac{v^2}{r}$

$\Rightarrow$ $v^2 = \frac{25 \times \sqrt{3}}{2} \times 2.5m$

$v = \left(125 \times \frac{\sqrt{3}}{4}\right)^{1/2} m/s$

$a_t = a \sin 30° = \frac{25}{2}\ m/s^2$

8.



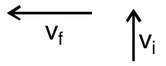
$\frac{MV^2}{R} = Mg$

$(V_0 \cos 45°)^2 = g.R$

$\Rightarrow R = \frac{V_0^2}{2g}$

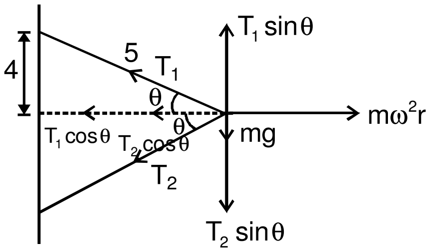
$\therefore a_c = \frac{V_0^2}{R} = 2g$

9. $\theta = \dfrac{\dfrac{V\pi R}{2V}}{R} = \dfrac{\pi}{2}$



$\therefore$ Average acceleration $= \dfrac{\vec{v}_f - \vec{v}_i}{\dfrac{\pi R}{2V}} = 2\sqrt{2}\,V^2 / \pi R$

10.



$\Rightarrow$ $T_1 \cos\theta + T_2 \cos\theta = m\,\omega^2 r$ ...(1)

$T_1 \sin\theta = mg + T_2 \sin\theta$ ...(2)

11. $a_t = \alpha R = 5 \times 0.5 = 2.5\ m/s^2$

Normal exert on Block, N = ma

$N = 1 \times 2.5 = 2.5$ N

$\omega = \alpha t = 5t$

Block slip when $f = m\omega^2 R$

$\mu N = m\,\omega^2 R$

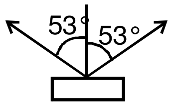
$(0.05)(2.5) = (1)(5t)^2(0.5)$

t = 0.1 sec.

12. $M\omega^2 \times 1 = 2\ mg$

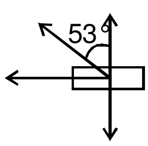
13. **Do your self**

14.

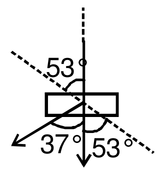


$2T \cos 53° = 20$

$T = \dfrac{50}{3}$ N



Acceleration $= \dfrac{\sqrt{\left(\dfrac{40}{3}\right)^2 + (10)^2}}{2} = \dfrac{25}{3}$

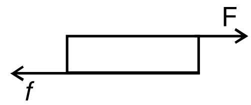


$a = g \sin 53° = 10 \times \dfrac{4}{5} = 8$

Ratio $= \dfrac{25}{24}$

15. (a) Net force on Block is zero

V= constant



$F = f = \mu mg$

so, work done by force.

$\int \vec{F}.\vec{dr} = 0$

(b) $W = \int_0^r N.dr$

$dr \perp N$

$N.dr = N\,dr \cos 90° = 0$

W = 0

(c) W = f.r

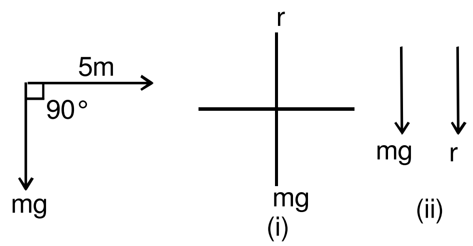
$= f\,r \cos 180° = -f_r$

$= -\mu\, mg(vt) = -\mu\, mg\, vt$

(d) work done by F = $\vec{F}.\vec{r} = F_r$

$= (\mu\, mg)(vt) = \mu\, mgvt$

16.



m = 10 kg

w = zero.

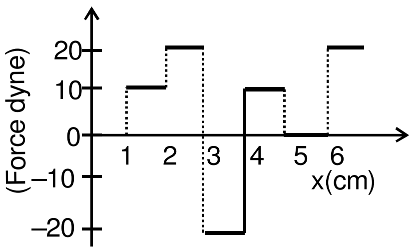
(i) $w = (-5) \times 10\,g = -500$ J

(ii) w = 500 J

17. $W = \vec{F}.\vec{r} = (-2\hat{i} + 15\hat{j} + 6\hat{k}).(10\hat{j})$

= 150 J

18.



W =10 × (2 – 1) + 20 (3 – 2) + (–20) (4 – 3) + 10 × (5 – 4)

= 20 dyne cm = 20 ergs

$= 20 \times \frac{kgm^2}{sec^2} \times \frac{1}{10^3 \times 10^4}$ = 0.2 × 10^{-5} J

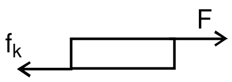
19. Work done by the resistive force

$= - mgh + \frac{1}{2}mv^2$

$= -\left(\frac{1}{1000}\right)[10 \times 10^3 - \frac{1}{2} \times 50 \times 50]$

= – 8.75 J

20.



$F = 7N\hat{i}$

and $f = 2N(-\hat{i})$

$a = \frac{7-2}{2} = \frac{5}{2}$

$S = \frac{1}{2}at^2 = \frac{1}{2}.\frac{5}{2}.(10)^2 = 125\hat{i}$

(a) $\vec{F}.\vec{r} = (7\hat{i})(125\hat{i}) = 875J$

(b) f_k r = – 2 × 125 = – 250 J

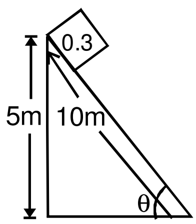
(c) $(F - f_k)r$ = 5 × 125 = 625 J

(d) ΔK = $k_f - k_i$

= work done by net force

= 625 J

21.



μ = 0.15

g = 9.8 m/s^2

$\sin\theta = \frac{1}{2}$ θ = 30°

μ g cos θ < g sin θ

Block will move on the horizontal plane.

(a) Work done by Gravitational force in round trip.

= – mgh + mgh = 0

(b) w_1 = mgh + μ mg cosθ. r

$= 0.3 \times 10 \times 5 + 0.15 \times 0.3 \times \frac{\sqrt{3}}{2} \times 10 \times 10$

= 15 + 3.82 = 18.82

(c) $w_{f_{net}} = w_{f_1} + w_{f_2}$

= – 2 × μmg cos θ.r

$= -2 \times 0.15 \times 0.3 \times 10 \times \frac{\sqrt{3}}{2} \times 10$

= – 7.64 J

(d) K.E. of a body

= mgh – μ mg cos θ

= 15 – 3.8

= 11.2 J

22. (a) $v_f = \frac{F}{m} \times t$

(b) $v'_f = v_c = \frac{F}{m}t$

(c) $\Delta k = \frac{1}{2}m(F/m \times t)^2 - \frac{1}{2}m(0)^2$

(d) $\Delta k' = \frac{1}{2}m\left(v_c + \frac{F}{m}t\right)^2 - \frac{1}{2}mv_c^2$

(e) $S = \frac{1}{2}\left(\frac{F}{m}\right)t^2$

(f) $S' = v_c t + \frac{1}{2}\left(\frac{F}{m}\right)t^2$

(g) w = F × S

w′ = F × S′

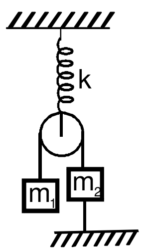
(h) K.E. is more for the ground frame.

(i) K.E. of a body is different in different different frame.

and

work-energy theorem hold for the moving observer.

23.

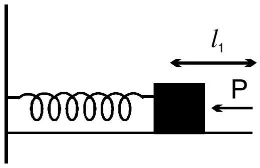


$T_1 = m_1g$

$kx = 2m_1g$

$\frac{1}{2}K.\frac{4m_1^2g^2}{K^2}$

24.



$Kx = P$

$\ell_1 = \frac{P}{K} \Rightarrow K = \frac{P}{\ell_1}$

$P(\ell_1+\ell_2) = \frac{1}{2}k\ell_2^2 - \frac{1}{2}k\ell_1^2$

$P(\ell_1+\ell_2) = \frac{1}{2}k(\ell_2+\ell_1)(\ell_2-\ell_1)$

$P = \frac{1}{2}k(\ell_2-\ell_1)$

$P = \frac{1}{2}P(\ell_2/\ell_1-1)$

$\frac{\ell_2}{\ell_1} = 3$

25. $P = 3t^2 - 2t + 1$

$dW = \int_2^4 (3t^2-2t+1)\,dt$

W.D. $= [t^3]_2^4 - [t^2]_2^4 + [t]_2^4$
$= (64-8) - (16-4) + 2$
$= 46$ J = change in K.E.

26. $P_{av} = \frac{\text{Total work done}}{\text{total time}}$

$= \frac{100\times1\times6\times9.8}{2\times60} = 49$ w

27. $P \times t = w$
$10 \times 10^3 \times t = 200 \times 10 \times 40$
$t = 8$ sec

28. $a = \frac{F}{m}$

$P = FV \Rightarrow V = \frac{P}{F} = \frac{P}{ma}$

$a = \frac{P}{mV}$

$V = \frac{F}{m}t$

$S = \frac{1}{2}\frac{F}{m}t^2$

$\frac{F}{m}t = \sqrt{\frac{2P}{m}}.\sqrt{t}$

$\frac{dv}{dt} = \frac{P}{mv}$

$vdv = \frac{P}{m}dt$

$\frac{v^2}{2} = \frac{P}{m}t$

$v = \sqrt{\frac{2P}{m}}.\sqrt{t}$

$dx = \sqrt{\frac{2P}{m}}.\sqrt{t}\,dt$

$x = \sqrt{\frac{2P}{m}}.\frac{t^{3/2}}{3/2}$

$\text{Ratio} = \frac{\frac{1}{2}\frac{F}{m}.t^2.\frac{3}{2}}{\sqrt{\frac{2P}{m}}.t^{3/2}} = \frac{3}{4}$

29. $A = 10^{-2}$ m^2
Mass coming in one second
$Av\rho = m$
$Av = 0.2$ m^3
10^{-2} V = 0.2 m^3
v = 20 m/s.
Energy required in one second

$= mgh + \frac{1}{2}mv^2$

$= 0.2 \times 1000 \times 10 \times 20 + \frac{1}{2} \times 0.2 \times 1000\,(20)^2$

= 80 kw

30. $P = F.v$
$P = mav$

$a = \frac{P}{mv} \Rightarrow a = \frac{P}{mv}$

$\frac{vdv}{dx} = \frac{P}{mv}$

$\Rightarrow \int_{u_i}^{u_f} \frac{v^3}{3} = \frac{P}{m}x = \frac{6^3-3^3}{3} = \frac{P}{m}\times252$

$$\frac{m}{p} = 4$$

$$v\frac{dv}{dt} = \frac{p}{m}$$

$$_{u_i}^{u_f}[v^2/2] = \frac{P}{m}t \Rightarrow \frac{8^2 - 3^2}{2} = \frac{t}{4}$$

t = 54 sec

31. $\vec{F} = x^2y^2\hat{i} + x^2y^2\hat{j}(N)$

act on a particle which moves in the xy plane.
If F is conservative.

$$\oint F.dr = 0$$

otherwise $\oint F.dr \neq 0$

$$w = \int x^2y^2dx + \int x^2y^2dy$$

$$\& \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y^2 & x^2y^2 & 0 \end{vmatrix}$$

$$\hat{i}\left(0 - \frac{\partial x^2y^2}{\partial z}\right) + \hat{j}\left(\frac{\partial x^2y^2}{\partial z} - 0\right) + \hat{k}\left(\frac{\partial x^2y^2}{\partial z} - \frac{\partial x^2y^2}{\partial y}\right)$$

$2(xy^2 - 2yx^2)\hat{k}$

Force is conservative if and only if x = y.

(b) $w_{ADC} = \int_0^a 0.dy + \int_0^a a^2x^2dx = \frac{a^5}{3}$

$$w_{ABC} = \int_0^a 0.dx + \int_0^a a^2y^2dy = \frac{a^5}{5}$$

$$w_{AC} = \int_0^a x^4dx + \int_0^a y^4dy = \frac{2a^5}{5} \quad (x = y)$$

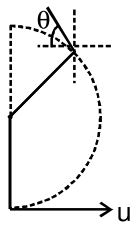
32. $F_y = -\frac{\partial x}{\partial y}$

(a) $F(y) = +\omega$

(b) $F(y) = -3ay^2 + 2by$

(c) $F(y) = -\beta U_0 \cos\beta y$

33.



$$\sqrt{3g\ell} < \sqrt{5g\ell}$$

$$\cos\theta = \frac{u^2 - 2gR}{3gR} = \frac{3gR - 2gR}{3gR} = \frac{1}{3}$$

$$mgR(1+\cos\theta) = \frac{1}{2}m(3g\ell) - \frac{1}{2}mv^2$$

$$gR.\frac{4}{3} = \frac{1}{2}(3g\ell - v^2)$$

$$\frac{8}{3}gR = 3g\ell - v'$$

$$v' = \sqrt{\frac{g\ell}{3}}$$

$$u_{min} = v\cos\theta = \frac{1}{3}\sqrt{\frac{g\ell}{3}}$$

34. $x = v\sqrt{\frac{2.2R}{g}}$

$$v = x\sqrt{\frac{g}{4R}}$$

$$\frac{1}{2}mu^2 = mg.2R + \frac{1}{2}mv^2$$

$$\frac{1}{2}mu^2 = mg.2R + \frac{1}{2}m\frac{(3R)^2g}{4R}$$

$$\frac{1}{2}u^2 = 2gR + \frac{9Rg}{8R}$$

$$u = \frac{5}{2}\sqrt{gR}$$

For x_{min} v should be min.

$\therefore u_{min} = \sqrt{5gR}$

$v = \sqrt{gR}$

$$x = \sqrt{gR}\,.\sqrt{\frac{2.2R}{2}} = 2R$$

35. $a_{av} = \frac{v_f - v_i}{\text{Total time}} = \frac{20-0}{3} = \frac{20}{3}$

EXERCISE – IV TOUGH SUBJECTIVE PROBLEMS

$F_{avg} = m\, a_{avg} = 1 \times \frac{20}{3} = \frac{20}{3}$N

0.02 kg fuel has energy = $\frac{1}{2}mv^2$

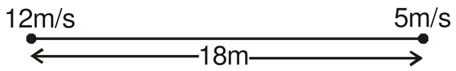
$= \frac{1}{2} \times 1 \times (20)^2$

= 200 J

1 kg fuel has energy = $\frac{200}{0.02} = 10^4$ J

Energy content per unit mass of fuel
= 10,000 J/kg

1



Given a = – Kx ⇒

$\frac{vdu}{dx} = -Kx$

$\Rightarrow \int_{12}^{5} v\,dv = -K\int_{0}^{18} x\,dx \Rightarrow 119 = K(324)$

$K = \frac{119}{324}$

Acceleration of particel at point $A = \frac{119}{324} \times 18 = \frac{119}{18}$

$a_t = \frac{119}{18}$

$a_{net} = 10\,m/s^2 = \sqrt{a_t^2 + a_N^2}$

$10 = \sqrt{\left(\frac{119}{10}\right)^2 + a_N^2}$

$\Rightarrow a_N = 7.5\,m/s^2 \Rightarrow \frac{V^2}{R} = 7.5$

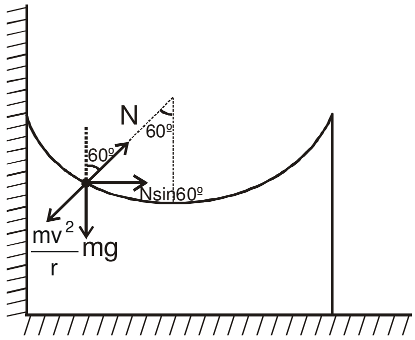
$\Rightarrow R = \frac{(5)^2}{7.5} = \frac{25}{7.5} \Rightarrow R = 3.3$ m

2 $N = \frac{mv^2}{r} + mg\cos 60º$ ...(1)

from E.C. $mg\cos 60º = \frac{1}{2}mv^2$

$v^2 = 5$

...(2)



⇒ from (1) & (2)
N = 15 N

Now force on the wedge due to wall

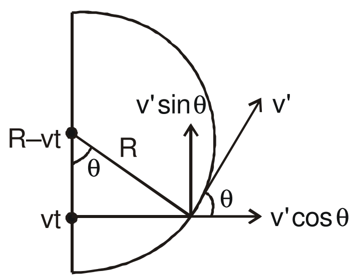
$= N \sin 60º = 15 \times \frac{\sqrt{3}}{2}$N

3 ∴ v' sin θ = v

$v' = \frac{v}{\sin\theta}$

$v' = \frac{vR}{(2Rvt - v^2t^2)^{1/2}}$

$a_N = \frac{v'^2}{R} = \frac{v^2R^2}{(2Rvt - v^2t^2)}/R$



$a_N = \frac{vR}{(2Rt - vt^2)}$

$a_t = \frac{dv'}{dt} = -\frac{(2Rv - 2v^2t)}{(2Rvt - v^2t^2)} \times \frac{1}{2(2Rvt - v^2t^2)^{1/2}}$

$a_t = \frac{-(Rv - v^2t)}{(2Rvt - v^2t^2)^{3/2}}$

4 Given

$U(x) = ax^3 - bx$

so $F(x) = -\dfrac{dU(x)}{dx} \quad = b - 3ax^2$

at maximum kinetic energy U = minimum

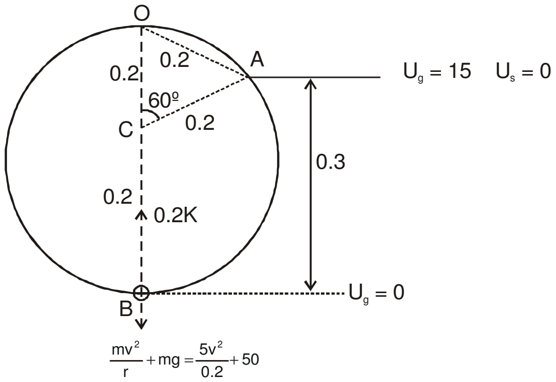
F = 0

$b - 3ax^2 \quad \Rightarrow \quad x = \sqrt{\dfrac{b}{3a}}$

5 at point B

$\Rightarrow \quad 0.2k = 50 + 25v^2 \qquad ...(1)$

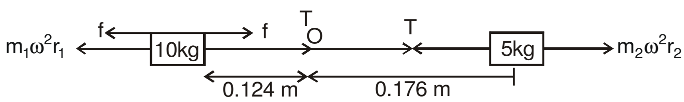
from E.C. = $15 = \dfrac{1}{2}K(0.2)^2 + \dfrac{1}{2} \times 5 \times v^2 \qquad ...(2)$



from (1) & (2) we get

k = 500 N/n

6 (i)



Now $T = m_2\,\omega^2 r_2$

...(1)

$T + f = m_1\,\omega^2 r_1$

...(2)

from (1) & (2) $\quad f = m_1\,\omega^2 r_1 - m_2\omega^2 r_2$

$f = 10.(10)^2(0.124) - 5(10)^2(0.176)$

$124 - 88$

$f = 36N$

(ii) for slipping condition friction should be maximum

$f_{max} = 50N$

$50 = m_1\omega^2 r_1 - m_2\omega^2 r_2$

$50 = 0.36\,\omega^2$

$\omega^2 = \dfrac{50}{0.36} \Rightarrow \quad \omega = 11.78\,rad/sec$

(iii)

$\Rightarrow \quad m_1\omega^2 x_1 = m_2\omega^2 x_2$

$\dfrac{x_1}{x_2} = \dfrac{m_2}{m_1} = \dfrac{5}{10} \Rightarrow \quad x_2 = 2x_1$

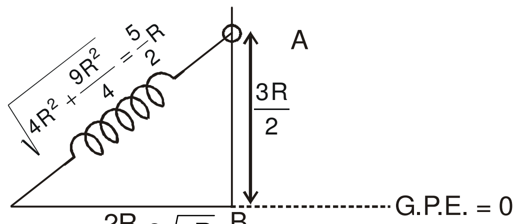
$x_1 + x_2 = 0.3$

$\Rightarrow \quad x_1 = 0.1\,m \qquad x_2 = 0.2m$

7 Extension is string $\quad x = \dfrac{5}{2}R - 2R = \dfrac{R}{2}$

Now from energy conservation between point A & B.

$mg\left(\dfrac{3R}{2}\right) + \dfrac{1}{2}\dfrac{4mg}{R}.\dfrac{R^2}{4} = \dfrac{1}{2}mv^2$



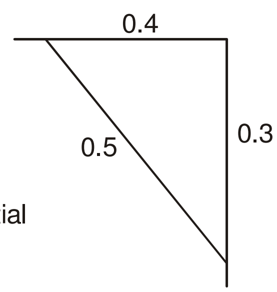
$v = 2\sqrt{gR}$

$W_f = Wmg = \Delta K$

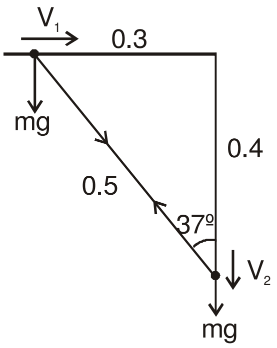
$W_f = \dfrac{1}{2}m(4gR) - \dfrac{3}{2}mgR \Rightarrow$

$W_f = \dfrac{1}{2}mgR$

8



Initial



from energy conservation

$$mgh = \frac{1}{2}m\left(v_1^2 + v_2^2\right) \quad ...(1)$$

$$1\times10\times0.1 = \frac{1}{2}\left[v_1^2 + v_2^2\right]$$

Now $x^2 + y^2 = \ell$

$$2x\,v_1 + 2y\,v_2 = 0$$

$$0.3\,v_1 = 0.4\,v_2 \Rightarrow 3v_1 = 4v_2 \quad ...(2)$$

from (1) & (2)

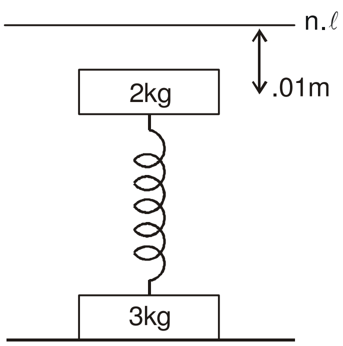
$$v_1 = \frac{4\sqrt{2}}{5}m/sec, \quad v_2 = \frac{3\sqrt{2}}{5}m/sec$$

Now $3a_1 = 4a_2$ ...(3)

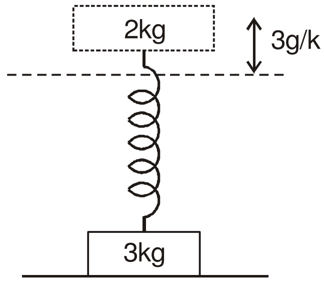
9 at equilibrium kx = mg × 2 × 10

$$k = \frac{2\times10}{.01} = 2000\,N/m$$

To just lift the 3kg block force on the 3 kg block is upward direction kx = 3g ⇒ x = 3kg/k



i.e.



from energy conservation

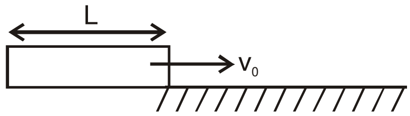
$$\frac{1}{2}k(.01+x)^2 = \frac{1}{2}k\left(\frac{3g}{k}\right)^2 + 2g(.01+x+3g/k)$$

$$\Rightarrow 1000(.01+x)^2 = \frac{1}{2}\left(\frac{90}{2000}\right) + 0.2 + 20x + \frac{6\times100}{2000}$$

after solving $x^2 = \frac{25}{40\times1000}$

x = 2.5 cm

10



(a) When x length lies on the rough surface than mass on the rough surface

$$m' = \frac{m}{L}\times x$$

$$\Rightarrow f = -\mu mg = -\mu\frac{m}{L}xg$$

(b) As the block's part enter the rough surface friction force increases so f as a function of x is.

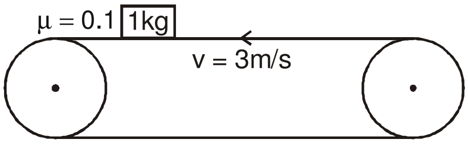
$$f = \frac{-\mu m}{L}xg, \quad a = \frac{-\mu}{L}xg$$

$v_i = v$ $\quad v_f = 0$ $\quad a = \frac{-\mu}{L}xg$

$$\Rightarrow v\frac{du}{dx} = -\frac{\mu}{L}xg \Rightarrow \int_{v_0}^{0} v\,du = -\frac{\mu}{L}g\int_0^L xdx$$

$$v^2 = \frac{\mu}{L}gL^2 \Rightarrow v = \sqrt{\mu gL}$$

11



Maximum heat liberated then all kinetic energy is loss in heat due to friction

$$\frac{1}{2}mv^2 = \text{work done by friction}$$

$$\Rightarrow \quad \frac{1}{2}v^2 = 8(0.1)(g)$$

$$v^2 = 16 \quad \Rightarrow \quad v = 4\,m/s$$

Velocity with respect to belt = 7 m/s

$v = 0 \quad u = 7$ m/s $\quad a = \mu g$

$$\Rightarrow \quad v^2 - u^2 = 2as$$

$$\Rightarrow \quad s = 49/2$$

Heat liberate = μgs = 24.5 J

When but velocity is 5 m/s then

$u = 9$ m/s $\quad v = 0 \quad a = \mu g$

$$\Rightarrow \quad s = 81/2$$

$$\Rightarrow \quad \text{Heat} = 40.5\,J$$

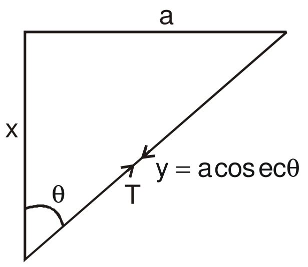
12

(a) $T = ky \Rightarrow T = \frac{2mg}{a} \times a\,\text{cosec}\,\theta$

T = 2mg cosec θ

At equilibrium

T cos θ = mg



2 mg cot θ = mg

cot θ = 1/2

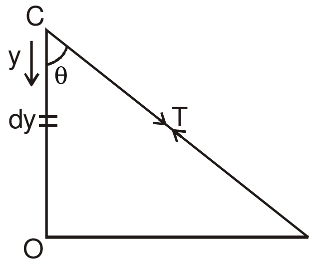
By fig $\cot\theta = \frac{x}{a}$

$$\therefore \frac{x}{a} = \frac{1}{2} \quad \Rightarrow \quad x = \frac{1}{2}$$

(b) $dF_{Tension} = Kydy$

$$F_{Tension} = \int_0^a kydy = k\left[\frac{y^2}{2}\right]_0^a = k\frac{a^2}{2} = \frac{2mg}{a} \times \frac{a^2}{2}$$

$F_{Tension} = mga$



$W_{total} = \Delta KE$

$W_{Tension} + W_{gravity} = KF$

$mga + mga = 1/2\, mv^2$

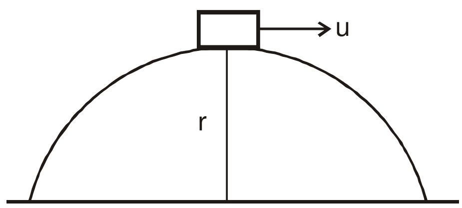
$2mga = 1/2\, mv^2$

$$v = 2\sqrt{ag} \quad \text{Ans.}$$

For maximum path $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$$x = 2a$$

13

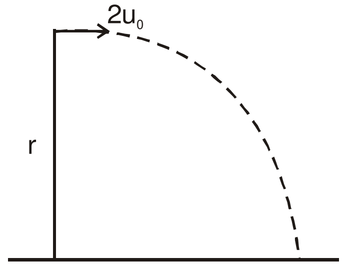


Particle leave the surface at top when

$$U = \sqrt{rg}$$

Now

$$T = \sqrt{\frac{2r}{g}}$$



$$R = 2u_0\sqrt{\frac{2r}{g}}$$

$$R = 2.\sqrt{rg}\sqrt{\frac{2r}{g}} = 2\sqrt{2}\,r$$

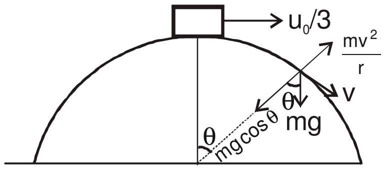
Now when $U = U_0/3$
from energy conservation

$$\frac{1}{2}m\frac{U_0^2}{9} + mgR(1-\cos\theta) = \frac{1}{2}mv^2 \qquad ...(1)$$

force balance $\frac{mv^2}{R} = mg\cos\theta$ ...(2)

from equation (1) & (2)

$$\frac{3}{2}rg\cos\theta = rg + \frac{U_0^2}{18}$$



put $U_0 = \sqrt{rg} \Rightarrow \cos\theta = \frac{19}{27}$

Height from the ground at which it leaves the hamisphere $= r\cos\theta = \frac{19}{27}r$

EXERCISE – V

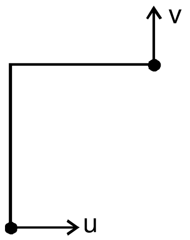
JEE QUESTIONS

1. Force is conservative
so W.D. is path independent

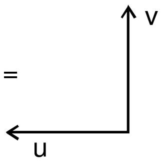
$$dw = -k\int (ydx + xdy) = -k\int_0^{a^2} d(xy)$$

$$= -ka^2$$ **Ans.**

2.



$\Delta V =$

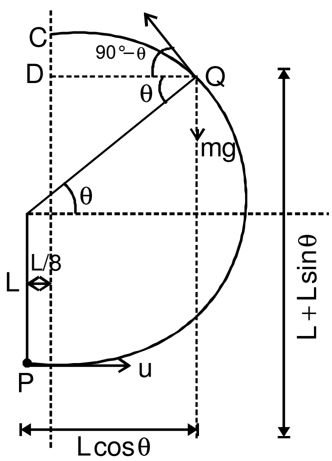


$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\ell$$

$v^2 = u^2 - 2g\ell$

$$|\vec{v}| = \sqrt{v^2 + u^2} = \sqrt{2(u^2 - g\ell)}$$

3.



T at Q = 0

$$\therefore\ mg\sin\theta = \frac{mv^2}{L} \quad ...(i)$$

$$v^2 = u^2 - 2gh = u^2 - 2gL(1 + \sin\theta) \quad ...(ii)$$

$$QD = \frac{1}{2}\text{ (Range)}$$

$$\left(L\cos\theta - \frac{L}{8}\right) = \frac{v^2\sin 2(90-\theta)}{2g} = \frac{v^2\sin 2\theta}{2g} \quad ...(iii)$$

$$\Rightarrow \left(\cos\theta - \frac{1}{8}\right) = \frac{v^2}{2gL}\sin 2\theta$$

$$\Rightarrow \left(\cos\theta - \frac{1}{8}\right) = \frac{v^2}{gL}\sin\theta\cos\theta$$

Now, Put $\frac{v^2}{gL} = \sin\theta$

$$\therefore \left(\cos\theta - \frac{1}{8}\right) = \sin^2\theta.\cos\theta$$

$$= (1 - \cos^2\theta)\cos\theta$$

$$\cos\theta - \frac{1}{8} = \cos\theta - \cos^3\theta$$

$$\cos^3\theta = \frac{1}{8} \text{ or } \cos\theta = \frac{1}{2} \text{ or } \theta = 60°$$

$\therefore\quad \theta = 60°$

$$\therefore\quad v^2 = gL\sin 60° = \frac{\sqrt{3}}{2}gL$$

Now from eq.(2)

$$u^2 = v^2 + 2gL(1 + \sin\theta)$$

$$= \frac{\sqrt{3}}{2}gL + 2gL\left(1 + \frac{\sqrt{3}}{2}\right)$$

$$u = \sqrt{gL\left(2 + \frac{3\sqrt{3}}{2}\right)}$$

4. $N = m\,\alpha\,L$

$\Rightarrow\ f = \mu N = \mu\, m\,\alpha\, L$

$\Rightarrow\ \mu\, m\,\alpha\, L = m\omega^2 L$

$\Rightarrow\ \omega^2 = \mu\alpha$

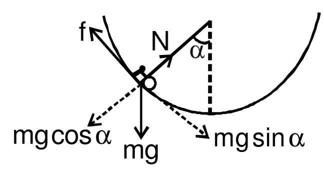
$\Rightarrow\ \omega = \alpha t$

$\Rightarrow\ t = \sqrt{\frac{u}{\alpha}}$

5. $N = \frac{mv^2}{R} - mg$

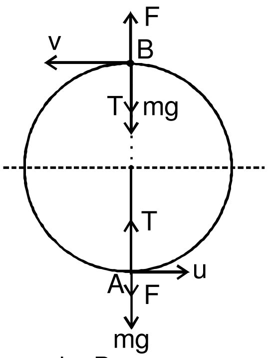
For $N_{max} = R_{min}$

6.



$f = \mu N = \mu mg\cos\alpha$

⇒ μ mg cos α = mg sin α
⇒ cot α = 3

7. 

$F = \frac{kq^2}{\ell^2}$

at point B

$F - mg - T = \frac{mv^2}{\ell}$

$\because T = 0$

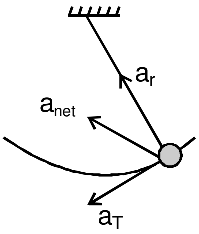
$\Rightarrow F\ell - mg\ell = mv^2$

E.C. between A & B

$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + 2mg\ell$

$\Rightarrow u = \sqrt{\frac{F\ell + 3mg\ell}{m}}$

$\Rightarrow u = 5.8$ m/s

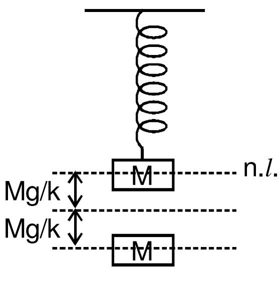
8. 

9. $F(x) = -kx + ax^2$

$F = -\frac{du}{dx} = -du$

$= -\int kxdx + \int ax^2dx$

$= -\frac{kx^2}{2} + \frac{ax^3}{3}$

10. 

Maximum Extension = $\frac{2Mg}{K}$

11. Ball will lose contact with inner sphere A

$\cos\theta = \frac{2}{3}$

⇒ $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

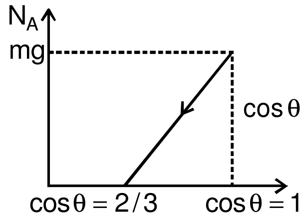
Now, N starts acting towards the centre and make contact with outer sphere.

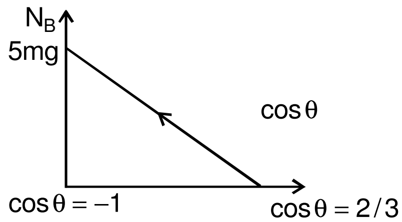
$\therefore \theta \le \cos^{-1}\left(\frac{2}{3}\right)$

$N_A = mg\,(3\cos\theta - 2)$; $N_B = 0$

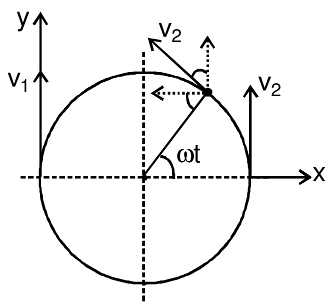
and for $\theta \ge \cos^{-1}\left(\frac{2}{3}\right)$

$N_A = 0$ and $N_B = mg(2 - 3\cos\theta)$





12. **Nahi**

13. 

$P = m\,|\vec{v}_2 - \vec{v}_1|$

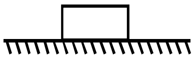
$= m[-v_2 \sin\omega t\,\hat{i} + v_2\cos\omega t - v_2)\hat{j}]$

14. $F = kx$

$$\frac{du}{dx} = -kx$$

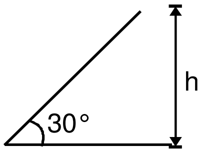
$$u = -\frac{kx^2}{2}$$

15.



$$\frac{1}{2}mv^2 = \mu mg\ell$$

(Decreases)



$$\frac{1}{2}mv^2 - mgh = \mu mg\ell\cos\theta$$

(Decreases)

16. **(D)**

$$\frac{1}{2}5mg\ell = \frac{1}{2}m\frac{5g\ell}{4} + mg(1-\cos\theta)$$

$\cos\theta = -7/8$

Hence, $3\pi/4 < \theta < \pi$

17. **8**

$a = g/3$, $T = 4.8$ N, $S = 1/2\ at^2 = 5/3$ m $\Rightarrow W = TS = 8$ (in joule)

18. **D**

$T = m\omega^2\ell$

$324 = 0.5\ \omega^2\ (0.5)$

$\omega = 36$ Radian/S

19. Applying energy conservation

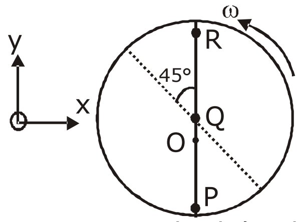
$$\frac{1}{2}kx^2 + \mu Nx = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}\times 2\times(0.06)^2 + 0.1\times 1.8\times 0.06$$

$$= \frac{1}{2}\times 0.18\times\left(\frac{N}{10}\right)^2$$

or $N = 4$

20. **C**

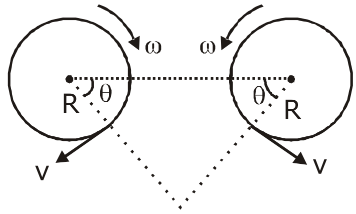


At 45° P & Q both land in unshaded region.

21. A

$V_2 - V_1 = V_{Rel}$

$$\therefore \left|\vec{V}_2 - \vec{V}_1\right| = 2R\sin\frac{(2\theta)}{2}$$



$= 2\ R\sin\theta = \left|2R\sin\omega t\right|$