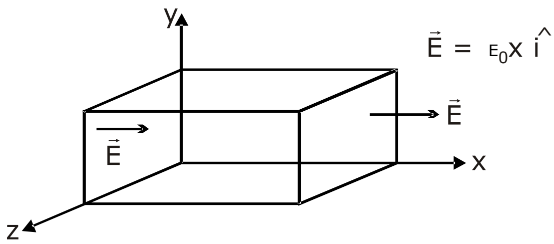
ELECTROSTATICS - 2

EXERCISE – I SINGLE CORRECT

1 [B]



Incoming flux $\phi_{in} = E_0\,(0) = 0$
Out going flux $\phi_{out} = E_0\,(a^2)$

$\Rightarrow \quad \phi_{out} - \phi_{in} = \frac{q}{\varepsilon_0}$

$q = \varepsilon_0\, E_0\, a^2$

2 [C]

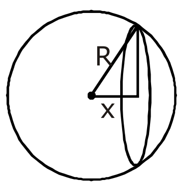
. $\vec{A} = 100\,\hat{k}$

$\vec{E} = \hat{i} + \sqrt{2}\,\hat{j} + \sqrt{3}\,\hat{k}$

$\phi = \vec{E} \bullet \vec{A}$

$\phi = 100\sqrt{3}$

3 [D]

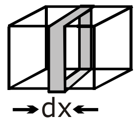


Radius of the cutting disc = $\sqrt{R^2 - x^2}$
charge on disc
$q = \sigma A$
$q = \sigma\pi\,(R^2 - x^2)$

Now $\phi = \frac{q}{\varepsilon_0} = \frac{\sigma\pi(R^2 - x^2)}{\varepsilon_0}$

4 [C]

. Flux $\phi = \frac{\Sigma q}{\epsilon_0}$



$\Sigma q = \rho a^2 dx$

$q = a^2 \int \rho\, dx$

$= a^2$ (area under curve)

$q = a^2\left(\frac{\rho_0}{8} + \frac{\rho_0}{2} + \frac{\rho_0}{8}\right)$

$q = \frac{3}{4} a^2 \rho_0$

$\phi = \frac{3/4a^2\rho_0}{\varepsilon_0} = \frac{3}{4}$

5 [A]
Charge revolve only due to electric field of inner shell.

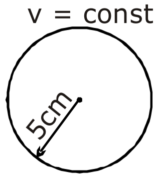
$\Rightarrow \quad \frac{mv^2}{r} = \left(\frac{2k\lambda}{r}\right)q$

$v = \sqrt{\frac{\lambda q}{2\pi\varepsilon_0 m}}$

6 [C]
Electric field lines enter and exit perpendicular to the conductor body.

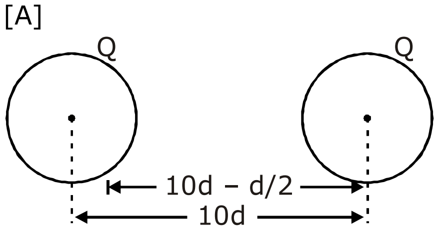
7 [B]

v = const.



$\frac{kQ}{R} = 10V \quad \Rightarrow v_{in} = 10V$

8 [A]



Force on q = $\frac{kqQ}{(10d - d/2)^2}$

9 [A]

$V_{in} = \frac{kQ}{2R^3}(3R^2 - r^2)$

$V_C = \frac{3}{2}\frac{kQ}{R}$; $V_s = \frac{kQ}{R}$

Now $V_c - V_s = \frac{kQ}{2R} = \frac{1 \times \rho}{2(4\pi\varepsilon_0)} \times \frac{4}{3}\pi R^3$

$V_c - V_s = \frac{\rho R^2}{6\varepsilon_0}$

10 [C]

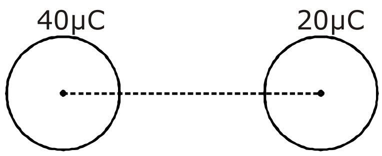
$V_C = \frac{3}{2}\frac{kQ}{R}$

Now $V = \frac{V_c}{2}$

$\frac{kQ}{r} = \frac{3}{4}\frac{kQ}{R} \Rightarrow r = \frac{4R}{3}$

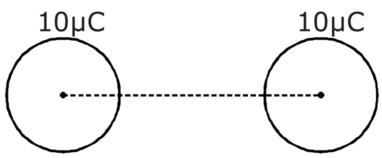
Distance from surface = $\frac{4R}{3} - R = \frac{R}{3}$

11 [A]



$F_1 = \frac{k(40)(20)}{d^2}$

After touching the charge on sphere = 10μC



$F_2 = \frac{k(10)(10)}{d^2}$

$F_1 : F_2 = 8 : 1$

12 [D]

Radius of single drop = r

then total volume = $n\frac{4}{3}\pi r^3$

Now radius of big drop = R

Volume = $\frac{4}{3}\pi R^3$

$\Rightarrow n\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$R = n^{1/3}r$ .... (i)

Charge on single drop $q = \frac{rV}{K}$

Total charge = $nq = \frac{nrV}{k}$

Final potential of big drop

$_{final} = \frac{knq}{R} = \frac{knrV}{kn^{1/3}r} = n^{2/3}V$

13 [D]

Use above formula in Q.26

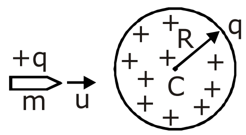
$V' = n^{2/3}V$

$= (1000)^{2/3}(1)$

$V' = 100$ V

14 [B]

Energy conservation between surface and point C



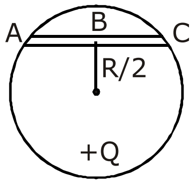
$\Rightarrow q(V_c - V_s) = \frac{1}{2}mv^2$

$\Rightarrow q\left(\frac{3kq}{2R} - \frac{kq}{R}\right) = \frac{1}{2}mv^2$

$u = \frac{q}{(4\pi\varepsilon_0 mR)^{1/2}}$

15 [A]

In the charge reach at point B then it will automatically reach at point C.



⇒ Energy Conservation between A & B

$qv_A + \frac{1}{2}mv^2 = qv_B$

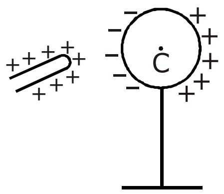
$V_B = \frac{kQ}{2R^3}\left(3R^2 - \frac{R^2}{4}\right) = \frac{11kQ}{8R}$

Now $\frac{1}{2}mv^2 = q\left[\frac{11kQ}{8R} - \frac{kQ}{R}\right]$

$v = \sqrt{\frac{6kQ}{8Rm}}$

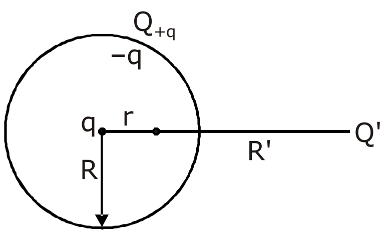
Now put $\phi = \rho \times \frac{4}{3}\pi R^3$ and $k = \frac{1}{4\pi\varepsilon_0}$

16 [C]



$V_C = +V_C$ (due to charge body)

17 [A]



$E_P = \frac{kq}{r^2}$

18 [A]

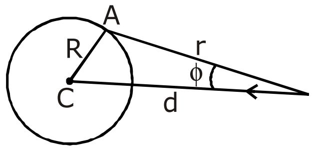
$E = \frac{kq}{r^2}$

19 [D]

20 [A]
In a conductor given charge is distributed uniformly on the surface of sphere

21 [B]
Depends on body either conductor or non-conducting.

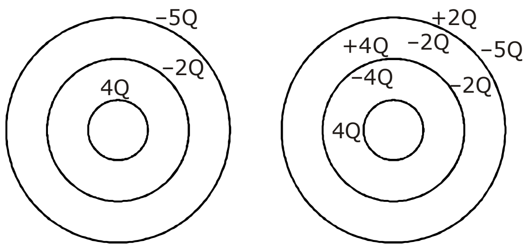
22 [B]



$d \cos \phi = r$

$V_A = V_C = \frac{kp}{d^2} = \frac{kp\cos^2\phi}{r^2}$

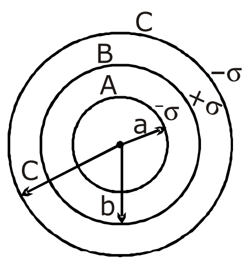
23 [D]



Charge on inner surface of outer shell
= –2Q

24 [C]
Potential of shell A is

$= \frac{kQ_A}{a} + \frac{kQ_B}{b} + \frac{kQ_C}{c}$

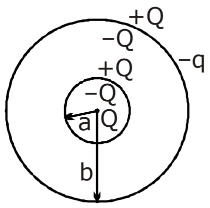


Now $Q_A = -4\pi a^2\sigma$
$Q_B = 4\pi b^2\sigma$
$Q_C = -4\pi c^2\sigma$

$k = \frac{1}{4\pi\varepsilon_0}$

25 (i) B (ii) D

(i)



Charge on inner surface of outer shell = –Q and outer surface = Q – q

(ii) Potential at every point inside = potential at surface

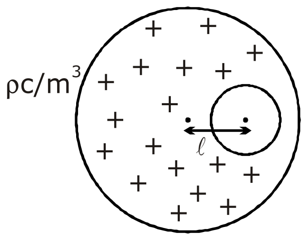
$$V_S = \frac{k(Q-q)}{b}$$

26 [A]

From notes electric field in a cavity

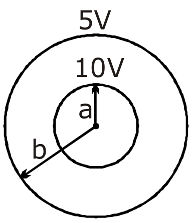
$$E = \frac{\rho}{3\varepsilon_0}\vec{\ell}$$

$$F = q\varepsilon = \frac{q\rho\vec{\ell}}{3\varepsilon_0}$$



27 [A]

Given potential at A is 10V and potential at B is 5V Now potential at centre is 10V because in hollow sphere potential is constant from centre of surface.



28 [D]

$$V_C = \frac{kQ}{1} - \frac{kQ}{2} + \frac{kQ}{4} - \frac{kQ}{8} + \ldots..$$

$$= kQ\left[1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8}\ldots..\right]$$

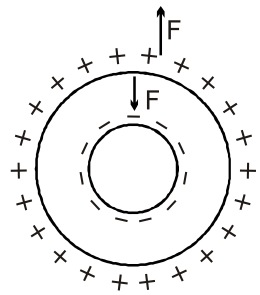
$$= kQ\left[1 + \frac{1}{4} + \frac{1}{16} + \ldots..\right] - kQ\left[\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \ldots\right]$$

$$= kQ\left\{\frac{1}{1-1/4} + \frac{1/2}{1-1/4}\right\}$$

$$V_C = \frac{Q}{6\pi\varepsilon_0}$$

29 [A]

Balancing occur only when –ve charge occur in inside conductor.



$$P_{elec.} = \frac{\sigma^2}{2\varepsilon_0}$$

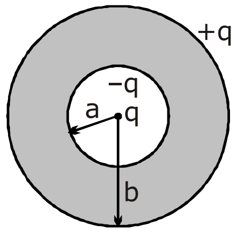
$$F = \frac{\sigma^2}{2\varepsilon_0}A$$

at equilibrium

$$\frac{\sigma^2}{2\varepsilon_0}(4\pi R^2) = \frac{\sigma^2}{2\varepsilon_0}\left(4\pi\frac{R^2}{4}\right)$$

σ' = 2σ (–ve)

30 [C]



$(W.D.)_{ext} = U_f - U_i$

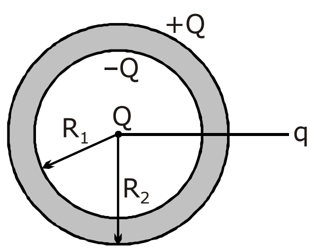
$U_i = 0$ (at ∞)

Self energy of a conducting sphere = $\frac{kQ^2}{2R}$

$$\Rightarrow U_f = \frac{kq^2}{2b} - \frac{kq^2}{2a}$$

$$\Rightarrow W.D. = \frac{kq^2}{2b} - \frac{kq^2}{2a}$$

31 [C]



charge Q placed at centre so it does not feel any force ⇒ $f_{net} = 0$

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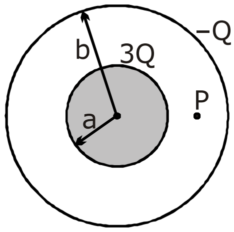
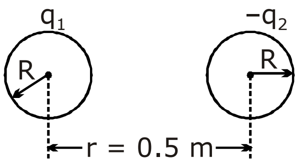
32 [C]

Electric field at point P = $\frac{k3Q}{r^2}$

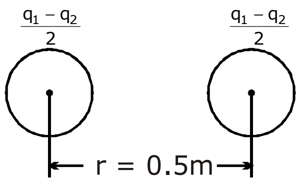


33 [B]



Given $\frac{kq_1q_2}{r^2} = 0.108$ .... (i)

Now after connecting through a wire



Given $\frac{k(q_1 - q_2)^2}{4r^2} = 0.036$ .... (2)

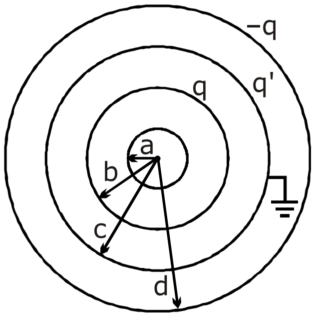
After solving equation (1) & (2) will get the answer.

34 [D]

As we connect A and B through wire with C. Then all the charge on A and B move towards C so $q_A = 0$, $q_B = 0$

$q_C = Q + q_1 + q_2$

35 [D]



b = 2a, c = 3a, d = 4a

$$\frac{kq}{3a} - \frac{kq}{4a} + \frac{kq'}{3a} = 0$$

$$q' = -\frac{q}{4}$$

Now $v_A = \frac{kq}{2a} - \frac{kq}{4a} + \frac{kq'}{3a} = 0$

$$V_A = \frac{kq}{6a}$$

$$V_A - V_C = \frac{kq}{6a} - 0 = \frac{kq}{6a}$$

36 [A]

Remain in the car, which provide electrostatic shielding.

37 [B]

Net potential of sphere = 0
(due to grounding)

$$\Rightarrow \frac{kq}{4a} + \frac{kq'}{a} = 0$$

$$q' = -\frac{q}{4}$$

38 [C]

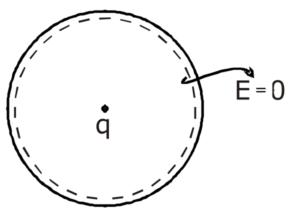
Charge distribution on outer surface of outer shell is non uniform due to presence of external charge q.

39 [A]

$$V_{outer} = \frac{kq}{4a} - \frac{kq'}{2a} = \frac{kq}{4a} - \frac{kq}{8a}$$

$$V_{outer} = \frac{kq}{8a} = \frac{q}{32\pi\varepsilon_0 a}$$

40

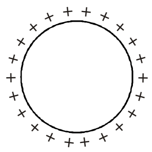


41 Gauss law is valid for all distribution either symmetric or not.

42 $\oint \vec{E} \bullet d\vec{A} = \frac{q_{in}}{\epsilon_0}$

$\vec{E}$ due to charge inside or outside charge.

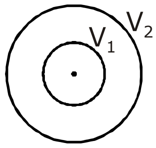
43 E depends on distribution of charge but ϕ doesn't depend on distribution q_{in}.

44 

In conducting sphere, there is no change inside it.

$\Rightarrow$ V = constant = 100 V

$E_{inside} = 0$

45 

At centre = V_1

46 Electrostatic shielding.

So $V_A - V_B$ doesn't change.

.1 [C]

$\phi = \int \vec{\varepsilon} \cdot d\vec{s}$

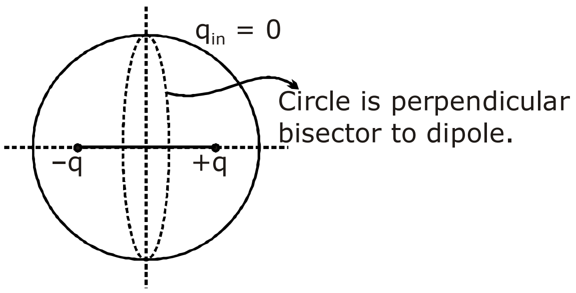
EXERCISE – II

MULTIPLE CHOICE QUESTIONS

1. $= \left(\frac{N}{C}\right)m^2$

$= \text{volt} - \text{m}$

2 [AC]



3 [C]
Electric flux due to outside charge will be zero. But elecric field will be due to all the charges.

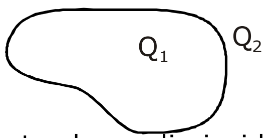
4 [AD]
Flux due to charge which is outside will be zero.

$$\oint \vec{\varepsilon}\ .\ d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

electric field due to all the charges.

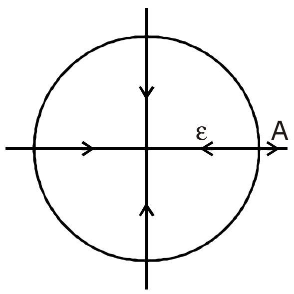
5 [ABC]

$$\oint \vec{\varepsilon}\ .\ d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$



Flux electric field due to charge lie inside or out side the surface. But ϕ is only due to charge lie inside the surface.

6 [ABC]



E = 100 r

$$\oint \vec{\varepsilon}\ .\ d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

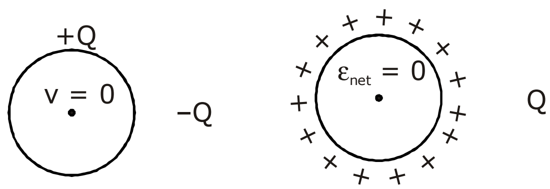
$$\varepsilon dA \cos 180° = \frac{q_{in}}{\varepsilon_0}$$

$\Rightarrow q_{in} = -ve$
$|q_{in}| = EdA\ \varepsilon_0 = 3 \times 10^{-13}$ C

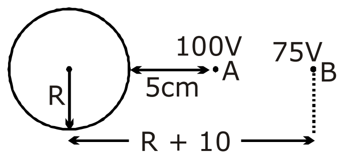
7 [D]
Electric field inside the conductor will be zero. Either external electric field is present or not. Hence potential at every point must be same. Charge distribution depends on external field

and $\sigma \propto \frac{1}{r}$ (when no electric field)

8 [D]



9 [ACD]



Given $100 = \frac{kQ}{R + 0.05}$ ..... (1)

$75 = \frac{kQ}{R + 0.10}$ ..... (2)

from (1) & (2) R = 10 cm

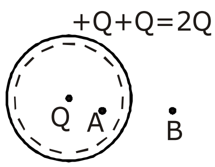
$$Q = \frac{5}{3} \times 10^{-9}\ C$$

Again $V_S = \frac{KQ}{R} = 150$ V

$$V_C = \frac{3}{2}\frac{kQ}{R} = 225\ V$$

$$E = \frac{kQ}{R^2} = 1500\ V/m$$

10 [ACD]



$\sigma = \frac{2Q}{4\pi R^2}$

$\sigma = \frac{Q}{2\pi R^2}$

ε_A only due to inside charge

$\propto \frac{1}{r^2}$

ε_B due to charge (inside + outside)

11 [AB]

In conductor given charge inside on its outer surface.

$\sigma \propto \frac{1}{r_c}$

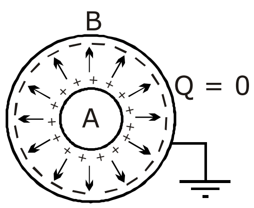
$\Rightarrow$ Potential will be same

Electric field near the surface = $\frac{\sigma}{\varepsilon_0}$

Where σ = Local charge density

$\sigma \propto \frac{1}{r}$

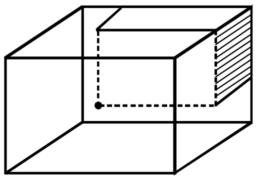
12 [ACD]



EXERCISE – III SUBJECTIVE PROBLEMS

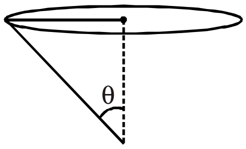
1 Because effectively electric field lives leaving the surface, so their is positive charge inside the surface.

2



$$\phi = \frac{1}{4}\left(\frac{q}{6\varepsilon_0}\right) = \frac{q}{24\varepsilon_0}$$

3



$$\cos\theta = \frac{a}{\sqrt{R^2 + a^2}}$$

So solid angle = $2\pi(1 - \cos\theta)$

$$\Omega = 2\pi\left[1 - \frac{a}{\sqrt{a^2 + R^2}}\right]$$

Let flux from 4π solid angle = ϕ
then from flux in Ω

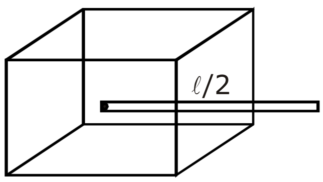
$$= \frac{\phi}{4\pi}\left[2\pi\left[1 - \frac{a}{\sqrt{a^2 + R^2}}\right]\right]$$

Now given $\frac{\phi}{4\pi} 2\pi\left[1 - \frac{a}{\sqrt{a^2 + R^2}}\right] = \frac{\phi}{4}$

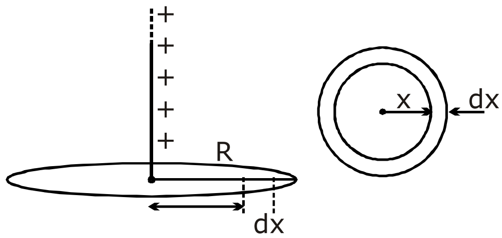
$3a^2 = R^2$

4 Flux minimum when length minimum.

$$\phi = \frac{q_{in}}{\varepsilon_0} \qquad \Rightarrow \phi = \frac{Q}{2\varepsilon_0}$$



5



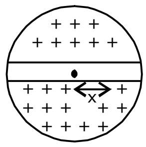
Flux passing through the area $2\pi \times dx$

$$= \frac{k\lambda}{x}(2\pi \times dx)$$

Now compute flux $\phi = 2\pi k\lambda \int_0^R dx$

$$\phi = 2\pi k\lambda R = \frac{\lambda R}{2\varepsilon_0}$$

6



Ef inside the sphere = $\frac{kQr}{R^3}$

$$F = -qE = -\frac{qKQ}{R^3}x$$

$$T = 2\pi\sqrt{\frac{mR^3}{KQq}}$$

7 Let us assume radius of smaller drop is r and bigger drop is R.

$$\Rightarrow 27\,\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$R = 3r$ ....(1)

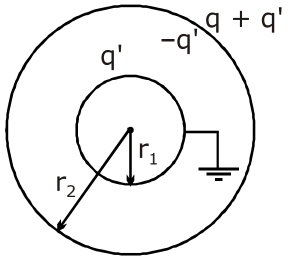
Now $\frac{kq}{r} = v_0 \Rightarrow q_0 = \frac{v_0 r}{k}$

Total charge $Q = 27q_0 = \frac{27v_0 r}{k}$

Now potential of bigger drop $v = \frac{kQ}{R}$

$$v = \frac{k \times 27 v_0 r}{3r\ k} = 9v_0$$

8

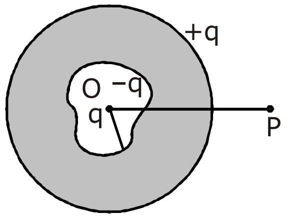


Potential of inner sphere = 0

$$\Rightarrow \quad \frac{kq}{r_2} + \frac{kq'}{r_1} = 0$$

$$q' = -q\left(\frac{r_1}{r_2}\right)$$

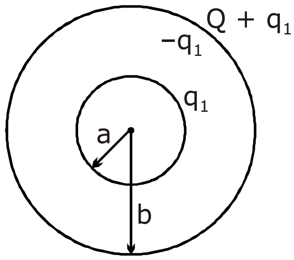
9



$$V_P = \frac{kq}{r} - \frac{kq}{r} + \frac{kq}{r}$$

$$V_P = \frac{kq}{r}$$

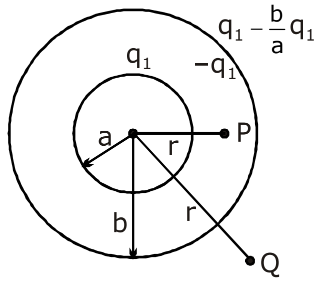
10 Charge given = Q



Potential of inner sphere = 0

$$\Rightarrow \quad \frac{kq}{b} + \frac{kq_1}{a} = 0$$

$$Q = \frac{-b}{a} q_1$$



$$V_P = \frac{kq_1}{r} - \frac{kq_1}{b} + \frac{k\left(q_1 - \frac{bq_1}{a}\right)}{b}$$

$$V_P = \frac{kq_1}{r} - \frac{kq_1}{a}$$

$$V_Q = \frac{kq_1}{r} - \frac{kq_1}{r} + \frac{k\left(q_1 - \frac{b}{a}q_1\right)}{r}$$

$$V_Q = \frac{kq_1}{r} - \frac{b}{a}\frac{kq_1}{r}$$

Q.11 From Q.6

$$q' = -q\left(\frac{r_1}{r_2}\right)$$

$$q' = Q\left[\frac{R}{3R}\right] = \frac{-Q}{3}$$

Q.12 Spheres are identical then after touching charge on each sphere is same.

I[st] A & B then $q_1 = \frac{6q - 3q}{2} = \frac{3q}{2}$

II[nd] A & C then $q_2 = \frac{3q/2 + 0}{2} = \frac{3q}{4}$

III[rd] C & B then $q_3 = \frac{3q/4 + 3q/2}{2} = \frac{9}{8}q$

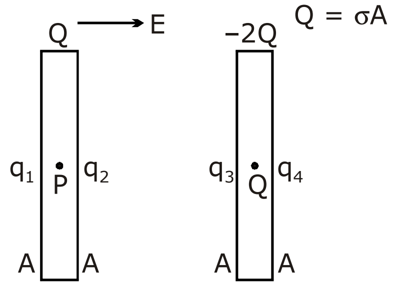
Q.13 Given $V_1 = \frac{kQ_1}{r_1}$

$$Q_1 = \frac{V_1 r_1}{k}$$

Now after connecting through wire all charge Q_1 moves on a spherical shell then

$V_2 = \frac{kQ_1}{r_2} = \frac{kV_1r_1}{kr_2} = \frac{V_1r_1}{k}$

14



P point lies inside the conductor so
Net electric field = 0

$\Rightarrow \frac{q_1}{2A\varepsilon_0} + E = \frac{q_2}{2A\varepsilon_0} + \frac{q_3}{2A\varepsilon_0} + \frac{q_4}{2A\varepsilon_0}$

from (1) & (3)

$q_1 - q_2 = -2Q - 2A\varepsilon_0 E$ .... (4)

from eq. (2) & (4)

$2q_1 = -Q - 2A\varepsilon_0 E$

$q_1 = -\frac{-Q}{2} - 2A\varepsilon_0 E$

$q_1 = \left[\frac{-\sigma}{2} - 2\varepsilon_0 E\right]A$

$\Rightarrow \quad q_1 + 2A\varepsilon_0 E = q_2 + q_3 + q_4$ ..... (1)

and $\quad q_1 + q_2 = Q$ ..... (2)

$q_3 + q_4 = Q$ ..... (3)

eq. (2) - eq. (4)

$2q_2 = 3Q + 2A\varepsilon_0 E$

$q_2 = \frac{3\sigma A}{2} + A\varepsilon_0 E$

Now $\quad q_3 = -q_2$

and $\quad q_4 = -2Q - q_3$

EXERCISE – IV — TOUGH SUBJECTIVE PROBLEMS

1. $v_A - v_B = \frac{kQ}{r} - \frac{kQ}{R} = kQ\left[\frac{R-r}{rR}\right]$

$\Rightarrow \quad v_C - v_B = \frac{kQ}{2R^3}(R^2 - r^2)$ {inderivation}

$\Rightarrow \quad v_C - v_B = \frac{kQ}{2R^3}\left(R^2 - \frac{R^2}{4}\right) = \frac{3kQ}{8R}$

Now $q(v_c - v_A) = \frac{1}{2}mv^2$

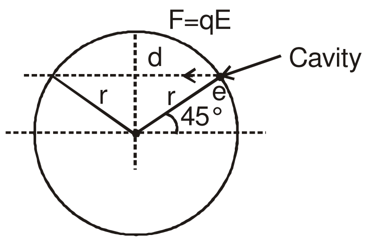
$\Rightarrow q\left[\frac{3kQ}{8R} - \frac{kQ(R-r)}{rR}\right] = \frac{1}{2}mv^2$

$\Rightarrow \quad v = \frac{2qkQ}{mR}\left[\frac{r-R}{r} + \frac{3}{8}\right]^{1/2}$

2. $F = -e\frac{\rho\vec{a}}{3\varepsilon_0} \Rightarrow a_e = \frac{e\rho a}{3\varepsilon_0 m}$

$d = \sqrt{2}\,r$

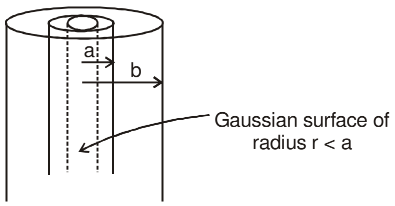
$\Rightarrow \quad$ from $d = \frac{1}{2}a_e t^2$



$\sqrt{2}\,r = \frac{1}{2}\frac{e\rho a}{3\varepsilon_0 m}t^2 \Rightarrow t = \sqrt{\frac{6\sqrt{2}\,r\varepsilon_0 m}{e\rho a}}$

3. for $r < a$

$\Rightarrow \oint \vec{E}.\vec{ds} = \frac{q_{in}}{\varepsilon_0}$



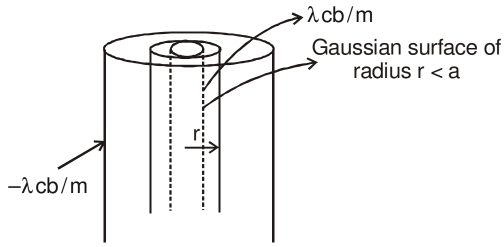
$q_{in} = 0$

$\Rightarrow E = 0$

for $b > r > a$

$\oint \vec{E}.\vec{ds} = q_{in}\varepsilon_0$

$2\pi r\ell.E = \frac{\lambda l}{\varepsilon_0}$

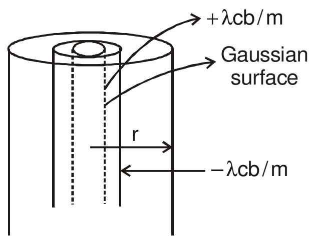


$E = \frac{2k\lambda}{r}$

(small part of length l of long cyclinder)

for $r > b$

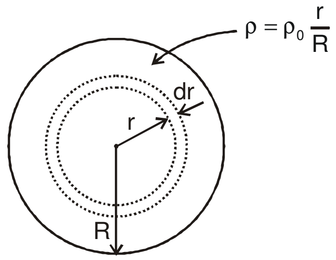
$q_{in} = 0$ (because equal & opposite charge on cyclinder)



$\Rightarrow E = 0$

4. (a) $dq = (4\pi r^2\,dr)\ \rho_0\frac{r}{R}$

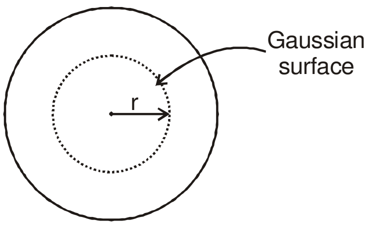
$\Rightarrow \quad \int_0^Q dq = \int_0^R 4\pi r^2 dr\,\rho_0\frac{r}{R}$



$Q = \frac{4\pi\rho_0}{R}\int_0^R r^3 dr \Rightarrow Q = \frac{4\pi\rho_0}{R}.\frac{R^4}{4} = \pi\rho_0 R^3$...(1)

(b) $\oint \vec{E}.\vec{ds} = \frac{q_{in}}{\varepsilon_0}$

$$\int_0^{q_{in}} dq_{in} = \int_0^r (4\pi r^2 dr)\frac{\rho_0 r}{R} \Rightarrow q_{in} = \frac{\pi\rho_0 r^4}{R}$$



$$\Rightarrow \quad E\,(4\pi r^2) = \frac{\pi\rho_0 r^4}{R\varepsilon_0} \Rightarrow E = \frac{\rho_0 r^2}{4R\varepsilon_0} \quad ...(2)$$

from eq. (1) and (2)

$$E = \frac{kQr^2}{R^4}$$

5. $E = \frac{v}{d} = \frac{500}{d}$

$a = \frac{qE}{m} = \frac{500e}{dm}$

from $v^2 - u^2 = 2ad$

$v^2 = \frac{1000e}{m} \Rightarrow v = \sqrt{\frac{1000e}{m}}$ (velocity of one electron)

Given $i = \frac{q}{t} = \frac{ne}{t} \Rightarrow t = \frac{ne}{i}$

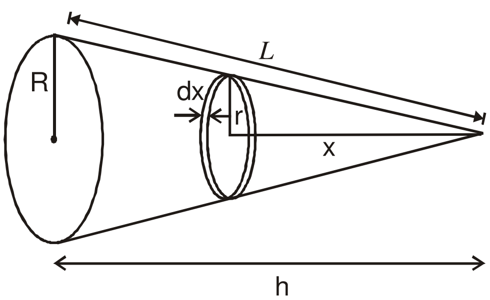
Now Impulse = change in momentum

$F.t = n\,mv \Rightarrow F = \frac{nmv}{t} = \frac{nm}{ne}i\sqrt{\frac{1000e}{m}} \Rightarrow F = 7.5 \times 10^{-9}$ N.

6. $dq = \sigma\, 2\,\pi\, r$
dx

$$dv = \frac{K\sigma 2\pi r dx}{(r^2 + x^2)^{1/2}}$$

$$\Rightarrow \quad \frac{r}{x} = \frac{R}{n} \Rightarrow r = \frac{xR}{h}$$



$$dv = \frac{K\sigma 2\pi xRdx}{n\left[\frac{x^2R^2}{n^2} + x^2\right]^{1/2}} \Rightarrow dv = \frac{\sigma 2\pi KRdx}{(R^2 + h^2)^{1/2}}$$

after integration

$$v = \frac{\sigma 2\pi KRL}{(R^2 + h^2)^{1/2}} \Rightarrow \sigma = \frac{Q}{\pi R(R^2 + h^2)^{1/2}} \quad \& \; R^2 + h^2 = L^2$$

$$v = \frac{Q}{2\pi\varepsilon_0 L}$$

So now energy required = $qv = \frac{qQ}{2\pi\varepsilon_0 L}$

7. When net electric field is zero.

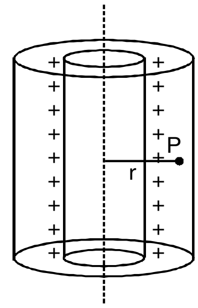
$$\Rightarrow \frac{Kqa}{(a^2 + a^2)^{3/2}} = \frac{K(2/5)^{-3/2}qa}{(b^2 + a^2)^{b/2}}$$

$$\Rightarrow \frac{2}{5}(b^2 + a^2) = 2a^2 = \frac{b}{a} = 2$$

EXERCISE – V

JEE QUESTIONS

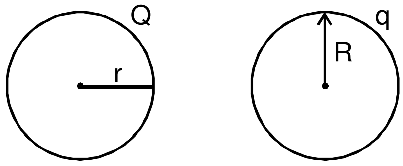
1.



$\varepsilon_p = \frac{2k\lambda}{r}$

2. $\frac{k(Q-q)}{r} = \frac{kq}{R} \Rightarrow q = \frac{QR}{R+r}$

Now



$\Rightarrow \quad \frac{k(Q-q')}{r} = \frac{K(q+q')}{R}, \quad q' = \frac{QR-qr}{R+r}$

Total charge = q + q′ = $\frac{QR}{R+r} + \frac{QR-qr}{(R+r)}$

$= \frac{2QR}{R+r} - \frac{rQR}{(R+r)(R+r)} = \frac{QR}{(R+r)}\left[2-\frac{r}{R+r}\right] = \frac{QR}{(R+r)}\left[1+\frac{R}{R+r}\right]$

After 3rd touching

$q'' = \frac{RQ}{R+r}\left[1+\frac{R}{R+r}+\left(\frac{R}{R+r}\right)^2\right]$

so after nth touch

$q_n = \frac{RQ}{R+r}\left[1+\frac{R}{R+r}+\left(\frac{R}{R+r}\right)^2 \ldots\ldots\ldots\right] \Rightarrow$

$q_n = Qa\left[\frac{1-a^n}{1-a}\right]$; $a = \frac{R}{R+r}$

Potential energy = $\frac{KQ^2}{2R}$

(b) as m → ∞ then

$q_\infty = \frac{Qa}{1-\frac{R}{R+r}} = \frac{\frac{QR}{R+r}}{\frac{r}{R+r}} = \frac{QR}{r}$ energy = $\frac{Kq_\infty^2}{2R}$

3. **(i)** Point A & B is in conductor

So ε = 0 but v = const. then

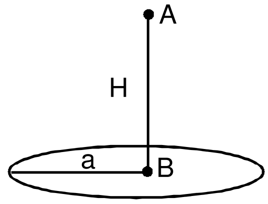
$v_A = v_B$

flux through the surface of the cavity

$\phi = \frac{q}{\varepsilon_0}$

(ii) $v_A = \frac{\sigma}{2\varepsilon_0}[\sqrt{a^2+h^2} - h] \Rightarrow v_B = \frac{\sigma}{2\varepsilon_0}a$

Now from E.C.



$mgh + qv_A = qv_B$

$mgh = \frac{\sigma q}{2\varepsilon_0}[-\sqrt{a^2+h^2} + h + a] \Rightarrow$

$\frac{h}{2} = [a + h - \sqrt{a^2+h^2}]$

after solving H = $\frac{4a}{3}$

(ii) Potential Energy at any point

$U = qV + mgh = q\frac{\sigma}{2\varepsilon_0}[\sqrt{a^2+h^2} - h] + mgh$

$U = mg[2\sqrt{a^2+h^2} - 2h + h] \Rightarrow$

$U = mg[2\sqrt{a^2+h^2} - h]$

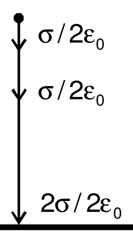
at equilibrium position $\frac{dU}{dh} = 0$

$h = \frac{a}{\sqrt{3}}$ ∴ at $h = 0 \Rightarrow 2\,mga$

U at $h = \frac{a}{\sqrt{3}} = \sqrt{3}\,mga$

4. E.f. lines originate & terminate perpendicular to surface.

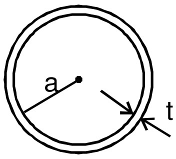
5.



$E_{net} = \frac{2\sigma}{\varepsilon_0} \downarrow$

6. $V' = \frac{KQ}{a} = V$

volume = $4\pi a^2 t$

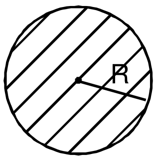


Now bubble of radius R then

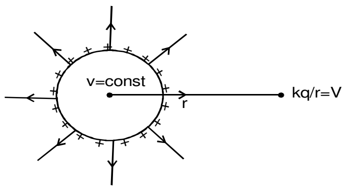
$V'' = \frac{kQ}{R}$

Volume is const. $\Rightarrow \frac{4}{3}\pi R^3 = 4\pi a^3 t$

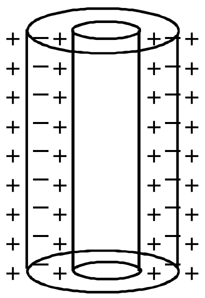
$R = (3a^2t)^{1/3} \Rightarrow V'' = \frac{Va}{(3a^2t)^{1/3}} = \left(\frac{a}{3t}\right)^{1/3} V$



7. **A.B.C.D**

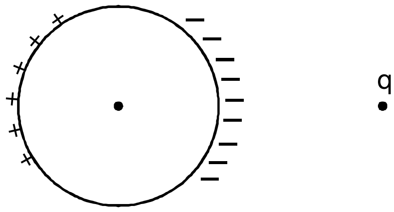


8. **A**



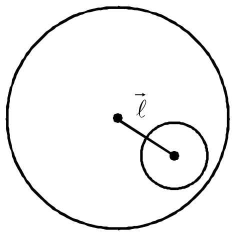
Potential difference occur when charge is given to inner cylinder.

9. **D**



Net charge on sphere = 0

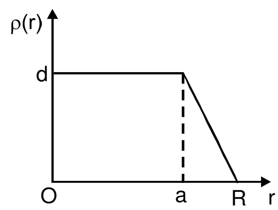
10. **B**



$\vec{E} = \frac{\rho\vec{\ell}}{3\varepsilon_0}$

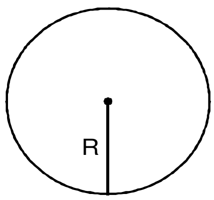
11. **B**

Theory



12. **A**

Total charge on the nucleus = Ze



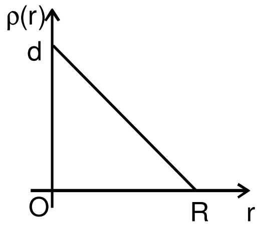
E.f at surface = $\frac{K(Ze)}{R^2}$ (independent of a)

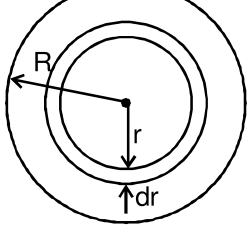
13. **B**

for a = 0

$\rho(r) = -\frac{dr}{R} + d \Rightarrow dq = 4\pi r^2 \rho(r)\, dr$

$Q = \int_0^R 4\pi r^2 \rho(r)\, dr = 4\pi \int_0^R r^2 \left(-\frac{dr}{R} + d\right) dr$





$$4\pi\int_0^R -\frac{dr^3}{R}dr + 4\pi\int_0^R dr^2 dr = Ze \Rightarrow$$

$$4\pi\left[\frac{-dR^3}{4}+\frac{dR^3}{3}\right] = Ze \quad = \quad \frac{3Ze}{\pi R^3} = d$$

14. **C**

In a non-conductor if ρ is uniform then $\varepsilon = \frac{\rho r}{3\varepsilon_0}$

We make the nucleus uniformly charge distributed then $a = R$ in which $\rho = d$ (uniform)

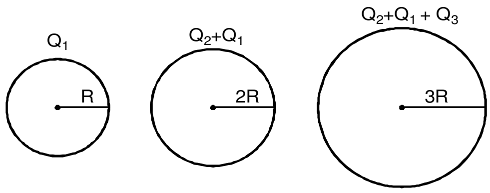
We make the nucleus uniformly charge distributed then $a = R$ in which $\rho = d$ (uniform)

15. **B**

$$\sigma = \frac{Q_1}{4\pi R^2} \quad \sigma = \frac{Q_2+Q_1}{16\pi R^2} \quad \sigma = \frac{Q_2+Q_1+Q_3}{36\pi R^2}$$

from I, II $\quad 1 = \frac{4Q_1}{Q_2+Q_1} \Rightarrow \frac{Q_2+Q_1}{4Q_1} = 1$

$$\frac{Q_2}{Q_1} = 3 \qquad \text{...(1)}$$



from II, III

$$1 = \frac{9(Q_2+Q_1)}{4(Q_2+Q_1+Q_3)}$$

from eq. (1) $\quad Q_2 = 3Q_1$

then $\frac{Q_3}{Q_1} = 5 \qquad \text{...(2)}$

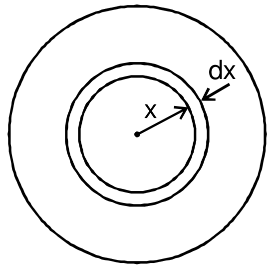
from (1) & (2)

$Q_1 : Q_2 : Q_3 = 1 : 3 : 5$

16. Given $\rho = Kr^a$

$$q_{in} = k4\pi\int_0^r x^{2+a}dx = \frac{4\pi k r^{3+a}}{3+a}$$

Now $\oint \vec{\varepsilon}.d\vec{s} = \frac{q_{in}}{\varepsilon_0}$



$$\varepsilon(4\pi r^2) = \frac{4\pi k r^{3+a}}{\varepsilon_0(3+a)}, \quad \varepsilon = \frac{kr^{a+1}}{\varepsilon_0(3+a)}$$

According to given condition

$$\frac{1}{8}\frac{K}{\varepsilon_0}\frac{R^{a+1}}{(3+a)} = \frac{KR^{a+1}}{\varepsilon_0(3+a)2^{a+1}}$$

$2^{a+1} = 8 \Rightarrow a = 2$

17. **A** $\quad \phi = \frac{q_{in}}{\varepsilon_0}$

q_{in} = charge on half disc + charge of point

$$\left(\frac{a}{4}, -\frac{a}{4}, 0\right)$$

+ charge of rod in the cubical surface

$$= \frac{6}{2} + (-7) + \frac{8}{\left(\frac{5a}{4}-\frac{a}{4}\right)}\left(\frac{a}{2}-\frac{a}{4}\right) = -2C \Rightarrow \phi = \frac{-2C}{\varepsilon_0}$$

18. **A** $\quad F = P_{elec.}A \Rightarrow F = \frac{\sigma^2}{2\varepsilon_0}.\pi R^2 \Rightarrow F \propto \frac{\sigma^2 R^2}{\varepsilon_0}$

19. **C** From the given figure

$$|\vec{A}| = a.\frac{a}{\sqrt{2}} = \frac{a^2}{\sqrt{2}}$$

$\vec{A} = a^2\hat{i} - a^2\hat{k}$ flux = $\vec{\varepsilon}.\vec{A} = \varepsilon_0 a^2$

20. **A,B,C,D** Initially

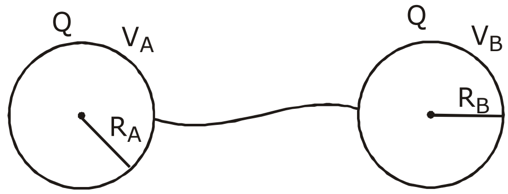


Fig - 1

$R_A > R_B$
$V_A < V_B$

Fig - 2

(A) In conductor $E_{net} = 0$
(B) from fig : 2 $Q_A > Q_B$ (To make the potential same)

(C) $\frac{KQ_A}{R_A} = \frac{KQ_B}{R_B}$ (after connecting)

$$\frac{K\sigma_A 4\pi R_A^2}{R_A} = \frac{K\sigma_B 4\pi R_B^2}{R_B} \Rightarrow \frac{\sigma_A}{R_A} = \frac{\sigma_B}{R_B}$$

(D) Use $\varepsilon = \frac{\sigma}{\varepsilon_0}$

21. C,D