UNITS & DIMENSIONS

EXERCISE – I SINGLE CORRECT

1 Fundamental quantities does not depends each other

So, In length, time and velocity here velocity is derived quantities.

2 Kilogram is not a physical quantity, its a unit.

3 Light year is a unit of distance, which is cover by light in a year.

4 PARSEC is a unit of distance.

It is used in astronmiccal science.

5 Salar day $\rightarrow$ Time far Earth to wake a complete rotation on its axis

Parallactic second [1 Parsec] $\rightarrow$ It is a distance corresponding to a parallex of one second of arc.

Leap year $\rightarrow$ A leap year is year (time) Containing one extra day.

Lunar Month $\rightarrow$ A lunar month is the time between two identical view moons of full moons.

1 Lunar month = 29.53059 days.

6 System is NOT based on unit of mass, length and time alone,

This system is based on all 7 Fundamental physical quantities and 2 supplymentary physical quantities.

7 SI unit of universal gravitational constant G is -

We know $F = \frac{GM_1M_2}{R^2}$

Here M_1 and M_2 are mass

R = Distance between them

F = Force

$$G = \frac{FR^2}{M_1M_2} = \frac{N-m^2}{kg^2}$$

So, Unit of G = $N-m^2\ kg^{-2}$

2 Unit of universal gas constant (R)

PV = nRT

P $\rightarrow$ Pressure

V $\rightarrow$ Volume

T $\rightarrow$ Temperature

R $\rightarrow$ Univ. Gas. Const.

n $\rightarrow$ No. of male

$$R = \frac{PV}{nT}$$

$$= \frac{N/m^2 \times m^3}{mol. \times K}$$

$$= \frac{N-m}{mol. \times K} = \text{Joule}\,K^{-1}mol^{-1}$$

9 Unit of impulse = V sec.

$$= kg\ \frac{m}{sec^2}\,sec$$

$$= kg\,\frac{m}{sec}$$

The unit is same as the unit of linear momentum.

10 Energy W = f × d = Nm

W = eV = electron-volt

W = p × t = Watt hour

So, kg × m/sec^2 is not the unit of energy.

11 We know $n, u, = n_2u_2$

when $n_1 > n_2$ then $u_1 < u_2$

or

when $n_1 < n_2$ then $u_1 > u_2$

Ex. 1 m = 100 cm

Here 1 < 100

m > cm

So, we can say $n \times \frac{1}{u}$

12. Unit of length is micrometer

Unit of time is mirosecond

$\because$ Velocity $= \frac{\text{Displacement}}{\text{Time tanker}}$

$$= \frac{10^{-6}m}{10^{-6}\,sec} = m/sec$$

13 $P = \frac{W}{t}$

Watt = Joule/sec.

Joule = Watt-sec.

One watt-hour = 1 watt×60×60 sec

= 3600 watt-sec

= 3600 Joule

= 3.6 × 10^3 Joule

14 $\text{Force} \times \frac{\text{Displacement}}{\text{Time}}$

$$= m^1L^1T^{-2} \times \frac{L^1}{T^1} = M^1L^2T^{-3}$$

So, Dimension of length here is 2.

15 Angular Frequency (f) $= \frac{1}{T} = M^oL^oT^{-1}$

So, here dimension in length is zero

16 Dimensionless quantity may have a unit

Ex. Angle Unit $\rightarrow$ Radian

Dimension $\rightarrow M^oL^oT^o$

17 A unitless quantity never has a nonzero dimension or unit less quantity never has a dimension

Ex. $\text{Strain} = \frac{\text{Change in length}}{\text{Original length}}$

It has no unit and dimension $\rightarrow M^oL^oT^o$

18 Only same physical quantities can be added or substracted,

It's only multiply and divided only.

So, a/b denote a new physical quantity.

19 They Can't e added or Substracted in Same expression.

20 (A, B, C)

21 (A)

21 Dimension of universal gravitational const.

$$F = \frac{GM_1M_2}{R^2}$$

$$G = \frac{FR^2}{M_1M_2} = \frac{M^1L^1T^{-2} \times L^2}{M^2} = M^{-1}L^3T^{-2}$$

22 Stefan-Constant(σ)

Unit $\rightarrow$ w/m^2-k^4 = wm^{-2}k^{-4}

23 Baltz mann's const. (k) $\rightarrow$

Unit $\rightarrow$ J/k

Dimension $= \frac{M^1L^2T^{-2}}{K^1} = M^1L^2T^{-2}K^{-1}$

24 Plank Const. (h) $\rightarrow$

Unit $\rightarrow$ J-S

Dimension $= M^1L^2T^{-2} \times T^1$

$= M^1L^2T^{-1}$

This is also a dimension of Angular momentum.

Dimension of c

$[t] = [c]$

$[c] = T^1$

Dimension of b

$$[v] = \left[\frac{b}{t+c}\right]$$

25 $v = at + \frac{b}{t+c}$

Same physical quantity can be added or substracted.

Dimension of a

$[v] = [at]$

$$[a] = \frac{[v]}{[t]} = \frac{L^1T^{-1}}{T^1} = L^1T^{-2}$$

Here t + c is also a Time (t)

$$[v] = \left[\frac{b}{t}\right]$$

$[b] = [v]\ [t] = L^1T^{-1} \times T^1$

$[b] = L^1$

26 $x(t) = \frac{v_o}{\alpha}[1 - e^{-\alpha t}]$

Dimension of v_o and α

Here $e^{-\alpha t}$ is dimensionless so,

$[\alpha]\ [t] = M^oL^oT^o$

$$[\alpha] = \frac{M^oL^oT^o}{T^1} = T^{-1}$$

$[\alpha] = M^oL^oT^{-1}$

Here $1 - e^{-\alpha t}$ is a number

$$[x(t)] = \frac{V_o}{\alpha}$$

$[V_o] = [L^1]\ [T^{-1}]$

$[V_o] = M^oL^1T^{-1}$

27 $P = P_o \text{ Exp } (-\alpha t^2)$

Here Exp $(-\alpha t^2)$ is a dimensionless

So, dimension of $[\alpha t^2] = M^oL^oT^o$

So, $[\alpha] = \frac{M^oL^oT^o}{T^2}$

$[\alpha] = M^oL^oT^{-2}$

$[D]\ [\alpha] = M^oL^oT^o$

$[D] = \frac{M^oL^oT^o}{L^1} = M^oL^{-1}T^o$

28 F = A sin ct + B Cos Dx

Here Sin ct and Cos Dx is dimensionless quantity

So, $[c]\ [t] = M^oL^oT^o$

$[c] = \frac{M^oL^oT^o}{T^1} = M^oL^oT^{-1}$

and dimension of A and B is same as dimension of force $[F] = M^1L^1T^{-2}$

So, $\frac{A}{B} = \frac{M^1L^1T^{-2}}{M^1L^1T^{-2}} = M^oL^oT^o$

$\frac{C}{D} = \frac{M^oL^oT^{-1}}{M^oT^{-1}T^o} = M^oL^1T^{-1}$

29 $\left(P + \frac{a}{v^2}\right)(v - b) = RT$

Dimension of a

$\because [P] = \left[\frac{a}{v^2}\right]$ Same Physical quantity can be add or substracted

$[a] = [P]\ [V^2]$

$[a] = PV^2$

30 Dimension of b

$[b] = [v]$

31 Dimension of nRT

$\text{In}\left(P + \frac{a}{V^2}\right)(v - b) = nRT$

$[PV] + [-Pb] + ... = [nRT]$

So, Dimension of [PV] = Dimension of [nRT]

$[nRT] = [M^1L^{-1}T^{-2}] \times [L^3]$

$n[RT] = [M^1L^2T^{-2}]$

It's a dimension of energy.

32 Dimensional formula of ab is–

Here

$\left[\frac{ab}{V^2}\right] = [nRT]$

$[ab] = [nRT]\ [V^2]$

$= [M^1L^2T^{-1}]\ [L^6]$

$[ab] = [M^1L^8T^{-2}]$

33 By Checking the dimension in all options

(c) Moment of Inertia $= Mr^2$

$= M^1L^2T^o$

Moment of force $= r \times F$

$= L^1 \times M^1L^1T^{-2}$

$= M^1L^2T^{-2}$

34 Find out Dimension of each physical quantity in all option.

Dimension of Torque $(\tau) = \vec{r} \times \vec{F}$

$(\tau) = L^1 \times M^1L^1T^{-2}$

It is also a dimension of Energy.

35 $F = Pt^{-1} + \alpha t$

Here F and Pt^{-1} is a same Physical quantity

$[F] = [Pt^{-1}]$

$[P] = \frac{[F]}{[t^-]} = [F \times t]$

It is a power

By find all dimension of each physical quantity in all options.

We find it is same as dimension of Impulse × distance

$M^1L^1T^{-1} \times L^1 = M^1L^2T^{-1}$

36 Action $=$ Energy × Time

$= M^1L^2T^{-2} \times T^1$

$= M^1L^2T^{-1}$

37 Dimension of Pressure $= M^1L^{-1}T^{-2}$

It is same as energy per unit volume

$= \frac{\text{Energy}}{\text{Volume}} = \frac{M^1L^2T^{-2}}{L^3}$

$= M^1L^{-1}T^{-2}$

38 Y = a sin (bt – cx)

Dimension of b

Here bt is dimensionless

$[bt] = M^oL^oT^o$

$$[b] = \frac{M^oL^oT^o}{[T^1]} = M^oL^oT^{-1}$$

It is a dimension of wave frequency.

So, Dimension of $\left[\frac{b}{c}\right] = \frac{M^oL^oT^{-1}}{M^oL^{-1}T^o}$

$= M^oL^1T^{-1}$

39 Dimension of b = $M^oL^oT^{-1}$

Dimension of c

$[cx] = M^oL^oT^o$ [Dimension less]

$$[c] = \frac{M^oL^oT^o}{L^1} = M^oL^{-1}T^o$$

40 $M^1L^2T^{-2}$ is a dimension of kinetic energy.

41 Find dimension in all options.

Here stress = Load/Area

$$= \frac{M^1L^1T^{-2}}{L^2}$$

stress = $[M^1L^{-1}T^{-2}]$

By comparesion and solving we find

[a = 0] [b = 1] [c = 2]

Put these value in Equa. (1)

$[L = F^oA^1T^2]$

42 L α FAT

$L = K\ F^aA^bT^c$ .... (1)

$M^oL^1T^o = K[M^1L^1T^{-2}]\ [L^1T^{-2}]^b\ [T]^c$

$M^oL^1T^o = K[M^a]\ [L^{a+b}]\ [T^{-2a-2b+c}]$

43 By checking the dimension in all options

[Pressure] = $M^1L^{-1}T^{-2}$

44 Fundamental quantities does not depend on each other.

c = 1

$-2 = -b \Rightarrow b = 2$

and

$2a + b - 3c = 1$

$2a + 2 - 3 = 1 \Rightarrow a = 1$

45 F α Avq

$F = KA^a\ v^b\ q^c$

$= K[L^2]^a\ [L^1T^{-1}]^b\ [M^1L^{-3}]^c$

$F = K[M^cL^{2a+b-3c}\ T^{-b}]$

$M^1L^1T^{-2} = K[M^c\ L^{2a+b-3c}\ T^{-b}]$

So $F = A^1\ v^2\ q^1$

$\therefore F = Av^2q$

$$= 10^6\left[\frac{10^{-3}}{1}\right]^1\left[\frac{10^{-2}}{1}\right]^{-1}\left[\frac{1}{1}\right]^{-2}$$

$n_1 = 10^6\ [10^{-3}]\ [10^2]$

$n_1 = 10^5$

So, 10^5 N/m^2

$$= \frac{20\times100}{5\times5\times5} = 16$$

m = 16

Unit of power in new system = 16 Watt.

46 Given

$P = 10^6$ dyne/cm^2

$n_1u_1 = n_2u_2$

$$n_1\left[M_1^1L_1^{-2}T_1^{-2}\right] = 10^6\left[M_2^1L_2^{-1}T_2^{-2}\right]$$

$$n_1 = 10^6\left[\frac{M_2}{M_1}\right]^1\left[\frac{L_2}{L_1}\right]^{-1}\left[\frac{T_2}{T_1}\right]^{-2}$$

47 $n_1u_1 = n_1u_1$

$$n_1\left[M_1^1L_1^2T_1^{-3}\right] = 1\left[M_2^1L_2^2T_2^{-3}\right]$$

$$n_1 = \left[\frac{M_2}{M_1}\right]^1\left[\frac{L_2}{L_1}\right]^2\left[\frac{T_2}{T_1}\right]^{-3}$$

$$= \left[\frac{20}{1}\right]^1\left[\frac{10}{1}\right]^2\left[\frac{5}{1}\right]^{-3}$$

48 Here $\sqrt{1+\frac{2k\ell}{ma}}$ is a number.

It's a dimensionless quantity.

$$\left[\frac{2k\ell}{ma}\right] = [M^oL^oT^o]$$

$$[K] = \frac{[m][a]}{[\ell]}$$

$$= \frac{M^1L^1T^{-2}}{L^1} = M^1L^oT^{-2}$$

49 $q = 2g/am^3$

$m_1u_1 = n_2u_2$

$$n_1\left[m_1^1L_1^{-3}\right] = 2\left[M_2^1L_2^{-3}\right]$$

$$n_1 = 2\left[\frac{M_2}{M_1}\right]^1\left[\frac{L_2}{L_l}\right]^{-3}$$

$$= 2\left[\frac{10^{-3}}{1}\right]^1\left[\frac{10^{-2}}{1}\right]^{-3}$$

50 $\alpha = \frac{F}{V^2}\sin(\beta t)$

Here sin (βt) is dimensionless.

$[\beta t] = M^oL^oT^o$

$$\beta = \frac{M^oL^oT^o}{T^1} = [T^{-1}]$$

51 By checking each option.

$$\frac{V^2}{rg} = \frac{[L^1T^{-1}]^2}{[L^1][L^1T^{-2}]}$$

$$= \frac{L^2T^{-2}}{L^2T^{-2}}$$

$= [M^oL^oT^o]$

So, dimension of b.

$$[b] = \frac{[m][a]}{[k]}$$

$$= \frac{[M^1][L^1T^{-2}]}{[M^1L^oT^{-2}}$$

$[b] = L^1$

So, the unit of b — meter

$= 2 \times 10^{-3} \times 10^6$

$n_1 = 2 \times 10^3 = 2000$

So, In MKS System

$q = 2 \times 10^3$ kg/m^3

$$[\alpha] = \left[\frac{F}{V^2}\right]$$

$$= \frac{M^1L^1T^{-2}}{[L^1T^{-1}]^2} = \frac{M^1L^1T^{-2}}{L^2T^{-2}}$$

$[\alpha] = [M^1L^{-1}T^o]$

52. $\frac{EJ^2}{M^5G^2}$ $J = [ML^2T^{-1}]$

$$= \frac{[M^1L^2T^{-2}][ML^2T^{-1}]^2}{[M^1]^5[M^{-1}L^3T^2]^2}$$

$$= \frac{[M^3L^6T^{-4}]}{[M^3L^6T^{-4}]}$$

$= [M^0L^0T^0]$

Dimension of Angle $= [M^0L^0T^0]$

Energy per unit volume $= \frac{\text{Energy}}{\text{Volume}}$

$$= \frac{[M^1L^2T^{-2}]}{[L^3]}$$

$= [M^1L^{-1}T^{-2}]$

Comparing both sides

$b = 0$

$-1 = -2c \Rightarrow c = \frac{1}{2}$

$1 = a - 3b + c$

$1 = a - 0 + 1/2$

53 By checking dimension in each option

$$\text{Pressure } = \frac{F}{A} = \frac{[M^1L^1T^{-2}]}{[L^2]} = [M^1L^{-1}T^{-2}]$$

54 v α λqg

$v = k\lambda^a\ q^b\ g^c$

$[M^oL^1T^{-1}] = K[M^oL^1T^o]^a[M^1L^{-b}T^o]^b[M^oL^1T^{-2}]^c$

$[M^oL^1T^{-1}] = K[M^bL^{a-3b+c}T^{-2c}]$

$a = \frac{1}{2}$

$V = K\lambda^{1/2}q^og^{1/2}$

squaring both sides

$V^2 = kg\lambda$

55 $10^3(N) = M^1L^1T^{-2}$

$10^3 = [M]^1\ [10^3]^1\ [100]^{-2}$

$$M = \frac{10^3}{10^3 \times (100)^{-2}} = 1000\,\text{kg}$$

56 $F = KAdv^x$

$M^1L^1T^{-2} = K[L^2]\ [M^1L^{-3}]\ [L^1T^{-1}]^x$

$M^1L^1T^{-2} = K[M^1L^{-1+x}T^{-x}]$

By comparison of power

$-1 + x = 1$

$x = 2$

57 $V = g^p h^q$

$V = Kg^p h^q$

$[L^1T^{-1}] = [L^1T^{-2}]^p [L^1]^q$

$L^1T^{-1} = L^{p+q} T^{-2p}$

58 $n_2 = 6.67 \times 10^{-11}\left[\frac{M_1}{M_2}\right]^{-1}\left[\frac{L_1}{L_2}\right]^{3}\left[\frac{T_1}{T_2}\right]^{-2}$

$= 6.67 \times 10^{-11}\left[\frac{1000}{1}\right]^{-1}\left[\frac{100}{1}\right]^{3}\left[\frac{1}{1}\right]^{-2}$

$= 6.67 \times 10^{-11} \times 10^3$

$n_2 = 6.67 \times 10^{-8}$

$n_2 = 13600\left[\frac{M_1}{M_2}\right]^{1}\left[\frac{L_1}{L_2}\right]^{-3}$

$= 13600\left[\frac{1000}{1}\right]^{1}\left[\frac{100}{1}\right]^{-3}$

$n_2 = 13.6\ gcm^{-3}$

By Comparison of powers

$p + q = 1$ $-1 = -2P$

$q = 1/2$ $p = 1/2$

59 $G = 6.67 \times 10^{-11}$

$n_1u_1 = n_2u_2$

$6.67 \times 10^{-11}\ [M_1^{-1}L_1^3T_1^{-2}] = n_2[M_2^{-1}L_2^3T_2^{-2}]$

60 $13600\ kgm^{-3}$

$n_1u_1 = n_2u_2$

$13600[M_1^1L_1^{-3}] = n_2[M_2^1L_2^{-3}]$

61 $g = 10\ ms^{-2}$

$n_1u_1 = n_2u_2$

$10[L_1]^1[T_1]^{-2} = n_2[L_2]^1[T_2]^{-2}$

$n_2 = 10\left[\frac{L_1}{L_2}\right]^{1}\left[\frac{T_1}{T_2}\right]^{-2}$

$n_2 = 10\left[\frac{1}{1000}\right]^{1}\left[\frac{1}{3600}\right]^{-2}$

$n_2 = 129600$

$M^0L^1T^0 = K[M^cL^{a+b-c}T^{-a-2b-2c}]$

By comparison of powers

$a = 2$

$b = -1$

$c = 0$

So, $\ell = c^2/g$

$\because$ Momentum (P) = mv = kg m/sec.

$P = kg\frac{m}{sec}\times\frac{m}{m}\times\frac{sec}{sec}$

62 $\ell \alpha c^a g^b p^c$

$\ell = kc^ag^bp^c$

$[M^0L^1T^0] = K[L^1T^{-1}]^a\ [L^1T^{-1}]^b\ [M^1L^{-1}T^{-2}]^6$

63 In new system

Length	$\to$ m	2m
Velocity	$\to$ m/sec.	2m/sec
Force	$\to$ kgm/sec^2	2kgm/sec^2

$P = kg\frac{m}{sec^2}\times\frac{m}{(m/sec)}$

In new system

$P^1 = \left(2kg\frac{m}{sec^2}\right)\times\frac{(2m)}{(2m/sec)}$

$P^1 = (2kg\,m/sec) = 2P$

So, Here unit of momentum is doubled.

$= \left(2kg\frac{m}{sec^2}\right)(2m)$

$= 4kg\frac{m^2}{sec^2}$

So, Unit of Energy is 4 times.

64 Unit of Energy = $kg\frac{m^2}{sec^2}$

$= \left(kg\frac{m}{sec^2}\right)\times(m)$

Now unit of force and length are doubled.

65 $K.E. = \frac{1}{2}mv^2$

Dimension = $M^1L^2T^{-2}$

Now M.L are doubled

$= (2M)^1\ (2L)^2\ (T^{-2})$

$= 8\ M^1L^2T^{-2}$

So, K.E. will become 8 times.

$(Perimeter)_2 = 2\pi r_2 = 2\pi(4r)$

$= 8\pi r$

$$\frac{(\text{Area})_1}{(\text{Area})_2} = \frac{\pi r^2}{16\pi r^2} = \frac{1}{16}$$

$$\frac{(\text{Perimeter})_1}{(\text{Perimeter})_2} = \frac{2\pi r}{8\pi r} = \frac{1}{4}$$

$\therefore$ Volume of Material $= \dfrac{256}{3}\pi - \dfrac{108}{3}$

$= \dfrac{148\pi}{3}$

(b) $(120)^{1/2} = (100+20)^{1/2}$

$= 10\left(1+\dfrac{20}{100}\right)^{1/2}$

$= 10\left(1+\dfrac{1}{5}\right)^{1/2}$

$= 10\left(1+\dfrac{1}{10}-\dfrac{1}{200}\right)$

$= 10.95$

66 $(\text{Area})_1 = \pi r_1^2 = \pi r^2$

$(\text{Perimeter})_1 = 2\pi r_1 = 2\pi r$

$(\text{Area})_2 = \pi r_2^2 = \pi(4r)^2$

$= 16\pi r^2$

67 External Volume $= \dfrac{4}{3}\pi r^3$

$= \dfrac{4}{3}\pi(4)^3 = \dfrac{256\pi}{3}$

Internal volume $= \dfrac{4}{3}\pi r^3$

$= \dfrac{4}{3}\pi(3)^3 = \dfrac{108\pi}{3}$

68 (a) $(99)^{1/2}$

$= (100-1)^{1/2}$

$= 10\left(1-\dfrac{1}{100}\right)^{1/2}$

$= 10\left(1-\dfrac{1}{200}-\dfrac{1}{80000}\right)$

$= 9.9598$

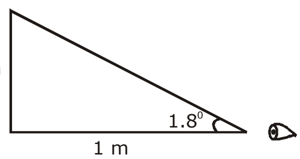
(c) $(126)^{1/3} = (125+1)^{1/3}$

$= 5\left(1+\dfrac{1}{125}\right)^{1/3}$

$= 5\left(1+\dfrac{1}{125\times 3}-\dfrac{1}{9\times 125^2}\right)$

$= 5.01329$

.69

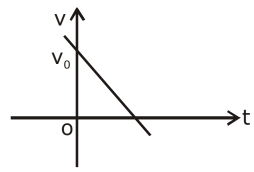


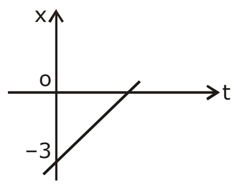
$\tan 1.8° = \dfrac{h}{1}$

Small Angle Approximation

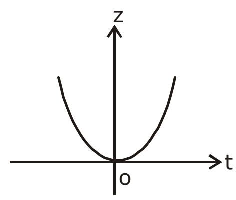
$\tan 1.8° \approx 1.8°$

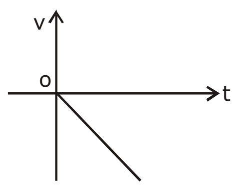
70 (i) $v = v_0 - at$ (ii) $x = 4t-3$



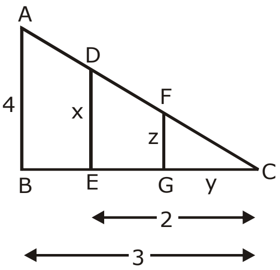


(iii) $x = 4at^2$ (iv) $v = -gt$





.71



$1.8 \times \dfrac{\pi}{180} = h$

$h = 0.01\ \pi m$

$= \pi$ cm

By Similar triangle concept

In ΔABC and ΔDEC

$$\frac{AB}{DE} = \frac{BC}{EC}$$

$$\frac{4}{x} = \frac{3}{2} \Rightarrow x = \frac{8}{3}$$

In ΔABC and ΔFGC

$$\frac{AB}{FG} = \frac{BC}{GC}$$

$$\frac{4}{2} = \frac{3}{y}$$

$$y = \frac{3}{2}$$

EXERCISE – II

MULTIPLE CHOICE QUESTIONS

1 Radian is a unit of Angle.

It is dimensionless.

Cos ax is dimensionless.

$[ax] = M^0L^0T^0$

$a = \frac{M^0L^0T^0}{L^1} = M^0L^{-1}T^0$

$[a] = [M^0L^{-1}T^0]$

From eq. (1)

$[M^0L^0T^1] = a[M^1L^0T^0]^a\ [M^1L^0T^{-2}]^b\ [M^0L^1T^0]^2$

$[M^0L^0T^1] = a[M^{a+b}L^cT^{-2b}$

2 $U_x = K(1-\cos ax)$

$U_x = K-K\cos ax$

$[K] = [U_x]$

$[K] = [M^1L^2T^{-2}]$

3 $T\ \alpha\ m^aK^b\ell^c$ $\qquad K = \frac{\text{Force}}{\text{Length}}$

$T = a\ m^aK^b\ell^c$

$[k] = \left[\frac{M^1L^1T^{-2}}{L^1}\right] = [M^1L^0T^{-2}]$

By comparison of power

$a + b = 0$

$c = 0$

$-2b = 1$

$b = -1/2$

$a = \frac{1}{2}$ and $c = 0$

So $T = a\ m^{1/2}\ k^{-1/2}\ \ell^0$

$T = a\sqrt{\frac{m}{k}}$

4 $P = \frac{nRT}{v-b} - \frac{a}{T^{1/2}(v+b)}$

Here,

Dimension of P = Dimension of

$\left[\frac{a}{T^{1/2}V(v+b)}\right]$

& $[v] = [b]$

So, $[b] = [M^0L^3T^0]$

$[p] = \frac{[a]}{K^{1/2}L^3 \times L^3}$

$[a] = [p]\ [K^{1/2}L^6]$

$= [M^1L^{-1}T^{-2}]\ [K^{1/2}L^6]$

$[a] = [M^1L^5T^{-2}K^{1/2}]$

By Comparison

$b - c = 0$

$a + 3c = 0$

$-1 = -2c$

$c = 1/2,\ b = 1/2$

$a + \frac{3}{2} = 0$

5 $x = a\sin\omega t$

$\omega = r^am^bG^c$

$\omega = kr^am^bG^c$

$[M^0L^0T^{-1}] = k[M^1L^1T^0]^a\ [M^1L^0T^0]^b\ [M^{-1}L^3T^2]^c$

$[M^0L^0T^{-1}] = k[M^{b-c}\ L^{a+3c}\ T^{-2c}]$

$a = -3/2$

$\omega = kr^{-3/2}\ m^{1/2}\ G^{1/2}$

$\omega = k\sqrt{\frac{Gm}{r^3}}$

$= k[M^1L^2T^{-1}]^a\ [L^1T^{-1}]^b\ [M^{-1}L^3T^{-2}]^c$

$M^1L^0T^0 = k^p[M^{a-c}L^{2a+b+3c}T^{-a-b-2c}]$

By Comparison -

$a - c = 1$

$2a+b+3c = 0$

$-a-b-2c = 0$

Similarly

$L\ \alpha\ h^a\ c^b\ G^c$

$M^0L^1T^0 = k[h]^a\ [c]^b\ [G]^c$

$M^0L^1T^0 = k[M^1L^2T^{-1}]^a\ [L^1T^{-1}]^b\ [m^{-1}L^3T^{-2}]^c$

$M^0L^1T^0 = k[M^{a-c}L^{2a+b+3c}T^{-a-b-2c}]$

By Comparison -

$a - c = 0$ $\qquad a = 1/2$

$2a + b + 3c = 1$ $\qquad b = -3/2$

$-a - b - 2c = 0$ $\qquad c = 1/2$

$[L] = [h^{1/2}C^{-3/2}G^{1/2}]$

By Comparison -

$a - c = 0$ $\quad a = 1/2$

$2a + b + 3c = 0$ $\quad b = -5/2$

$-a - b - 2c = 1$ $\quad c = 1/2$

Comparing both sides..

$b - c = 0 \Rightarrow b = c$

$a + 3c = 1$

$-2c = -1 \Rightarrow c = 1/2$

$\therefore b = 1/2$ and $a = -1/2$

6 Dimension of $[c] = [L^1 T^{-1}]$

Dimension of $[G] = [M^{-1}L^3T^{-2}]$

Dimension of $[h] = [M^1L^2T^{-1}]$

$[M] \;\alpha\; h^a c^b G^c$ ......(1)

$M^1L^0T^0 = k[h]^a [c]^b [G]^c$

On solving -

$a = 1/2$ $\quad b = 1/2$ $\quad c = -1/2$

From ...(i)

$M = h^{1/2} c^{1/2} G^{-1/2}$

Then $[T] \;\alpha\; h^a c^b G^c$

$M^0L^0T^1 = k[H^1L^2T^{-2}] [L^1T^{-1}]^b [M^{-1}L^3T^{-2}]^c$

$M^0L^0T^1 = k[M^{a-c}L^{2a+b+3c}T^{-a-b-2c}]$

7 $V_0 \;\alpha\; R^a M^b G^c$

$V_0 = KR^a M^b G^c$

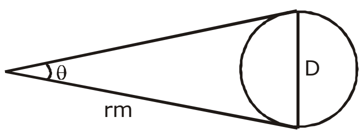
$[LT^{-1}] = K[L]^a [M]^b [M^{-1}L^3T^{-2}]^c$

$[LT^{-1}] = K[M^{b-c}L^{a+3c}T^{-2c}]$

$\therefore V_0 = K R^{-1/2} M^{1/2} G^{1/2}$

$$V_0 = k\sqrt{\frac{Gm}{R}}$$

8 Take small angle approximation



$$\text{Sin } \theta = \frac{D}{rm}$$

$$\text{Sin } 0.50° = \frac{D}{rm}$$

$$0.50 \times \frac{\pi}{180} = \frac{D}{384000}$$

$$D = 0.50 \times \frac{\pi}{180} \times 384000$$

$D = 3349.33$

$D \approx 3350$ km.

9 $(1 + x)^u \approx 1+nx, \; |x| << 1$

(a) $\sqrt{99} = (99)^{1/2}$

$= (100-1)^{1/2}$

$$= 10\left(1 - \frac{1}{100}\right)^{1/2}$$

$$= 10\left(1 - \frac{1}{200}\right)$$

$= 9.95$

(b) $\frac{1}{1.01} = (1.01)^{-1}$

$= (1+0.01)^{-1}$

$$= \left(1 + \frac{1}{100}\right)^{-1}$$

$$= \left(1 - \frac{1}{100}\right)$$

$= 0.99$

10 (a) sin 8°

Using small angle approximation.

sin 8° $\approx$ 8°

$$= \frac{8\pi}{180}$$

$= 0.139 \approx 0.14$

(b) tan 5°

Using small Angle Approximation

tan 5° $\approx$ 5°

$$= \frac{5\pi}{180}$$

$= 0.087 \approx 0.09$

EXERCISE – III SUBJECTIVE PROBLEMS

1 By checking the dimension in all options.

(A) [Raynables No.] = [Co-efficient of Friction] $= M^0L^0T^0$

(B) [Latent heat] = [Gravitational potential] $= M^0L^2T^{-2}$

(C) [Curie] = [Frequency of light wave] $= M^0L^0T^{-1}$

(D) [Plank's constant] = $[M^1L^2T^{-1}]$

[Torque] = $[M^1L^2T^{-2}]$

2 $x = 3yz^2$

$$[y] = \frac{[x]}{[z^2]}$$

[x] = dimension of capacitance.

$$C = \frac{Q}{V} = \frac{Q}{W}Q \qquad [W = Qv]$$

$$B = \frac{F}{IL} = \frac{Ft}{QL}$$

$$[B] = [z] = \frac{[MLT^{-2}][T]}{[Q][L]}$$

$[z] = [MT^{-1}Q^{-1}]$

$$[y] = \frac{[M^{-1}L^{-2}T^2Q^2]}{[MT^{-1}Q^{-1}]^2} = [M^{-3}L^{-2}T^4Q^4]$$

$$C = \frac{Q^2}{W}$$

$$[c] = [x] = \frac{Q^2}{[ML^2T^{-2}]} = [M^{-1}L^{-2}T^2Q^2]$$

[z] = dimension of magnetic induction.

3 By checking pimension in all option -

[Torque] = [work] = $[M^1L^2T^{-2}]$

and

[Light year] = [wavelength] = $[M^0L^1T^0]$

4 $$V = L\frac{dI}{dt}$$

$$L = V\frac{dt}{dI} = \frac{\text{volt} - \text{sec}}{\text{Amp.}} \qquad \text{(B)}$$

$$\frac{\text{Volt}}{\text{Amp}} = \text{ohms}$$

L = ohms–sec. (D)

$\phi = Li$

$$L = \frac{\phi}{i} = \frac{\text{webt}}{\text{Amp}} \qquad \text{(A)}$$

$$L = \frac{\text{Joule}}{(\text{Amp})^2} \qquad \text{(C)}$$

5 Unit of $E_0 = \dfrac{c^2}{N - m^2}$

$$[E_0] = \frac{I^2T^4}{ML^3} = [M^{-1}L^{-3}T^4I^2]$$

Unit of No. = V/A^2

$[u] = [M^1L^1T^{-2}I^{-2}]$

$$\therefore \quad \left[\frac{1}{2}t_0E^2\right] = [t_0][E]^2$$

$= [M^{-1}L^{-3}T^4I^2]\ [MLT^{-3}I^{-1}]^2$

$= [M^1L^{-1}T^{-2}]$

6 Dimension of $1/2\ t_0\ E^2 = [t_0]\ [E^2]$

$[t_0] = [M^{-1}L^{-3}T^4I^2]$

$[E] = [M^1L^1T^{-3}I^{-1}]$

7 $$x = t_0L\frac{\Delta v}{\Delta t}$$

$$[x] = \frac{[M^{-1}L^{-3}T^4I^2][L][M^1L^2T^3I^{-1}]}{[T^1]} = [I^1]$$

Dimension of current

$$P = \left[\frac{\alpha}{\beta}\right] = \left[\frac{k\theta}{z\beta}\right]$$

$$[\beta] = \left[\frac{k\theta}{zp}\right] \quad [k\theta] = [\text{Energy}]$$

$$[\beta] = \frac{[ML^2T^{-2}]}{[L][ML^{-1}T^{-2}]} = [M^0L^2T^0]$$

8 $$P = \frac{\alpha}{\beta}\exp\left(\frac{-\alpha z}{k\theta}\right)$$

exponent is dimensionless quantity

$$\left[\frac{\alpha z}{k\theta}\right] = [M^0L^0T^0]$$

$$[\alpha] = \left[\frac{k\theta}{z}\right]$$

9 By checking dimension all option

[Dipole moment] = $[M^0L^1T^1A^1]$

[Electric Flux] = $[M^1L^3T^{-3}I^{-1}]$

[Electric field] = $[M^1L^1T^{-3}I^{-1}]$

10 Dimension of N = [L–3]

[M] = [M]

$[t_0]$ = $[M^{-1}L^{-3}T^2Q^2]$

$[e^2]$ = $[Q^2]$

$$\left[\sqrt{\frac{Ne^2}{mt_0}}\right] = \sqrt{\frac{[N][e^2]}{[m][t_0]}}$$

$$= \sqrt{\frac{L^{-3}Q^2}{M.\ M^{-1}L^{-3}T^2Q^2}}$$

$$= \sqrt{\frac{1}{T^2}}$$

$$= \frac{1}{T} = [\omega_p]$$

11 $\omega_p = \sqrt{\frac{Ne^2}{mt_0}}$

$$= \sqrt{\frac{4\times10^{27}\times(1.6\times10^{-19})^2}{10^{-30}\times10^{-11}}}$$

$$= \sqrt{10.24\times10^{30}}$$

ω_p = 3.2 × 10^{15}

$$f_p = \frac{3.2\times10^{15}}{2\pi} = 0.51\times10^{15}$$

c = fd

$$\lambda = \frac{c}{f}$$

$$= \frac{3\times10^8}{0.51\times10^{15}} = 5.88\times10^{-7}$$

= 588 × 10^{-9}

$\approx$600 nm