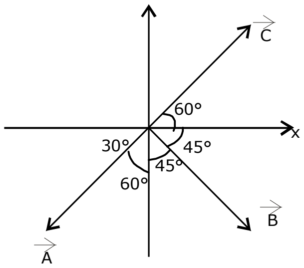
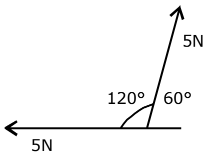
N.L.M FRICTION

EXERCISE – I SINGLE CORRECT

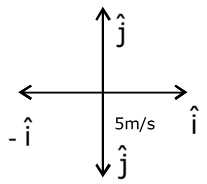
PART-III

1. 

(i) $\vec{A}$ and $\vec{B}$ = 105°
(ii) $\vec{A}$ and $\vec{C}$ = 150°
(iii) $\vec{B}$ and $\vec{C}$ =150°

2. 

Angle b/w forces= 120°

3. 

$\vec{V}_R = -5\hat{j}$

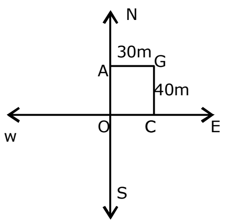
4. Given

$\vec{A} = \hat{i} + \hat{j} - \hat{k}$

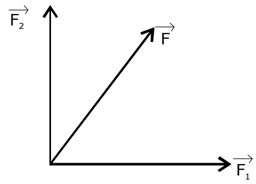
$\vec{B} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\overrightarrow{AB} = \vec{B} - \vec{A}$

$= \hat{i} - 4\hat{j} + 5\hat{k}$

5. 

Displacement
= 30m in East

6. 

$\theta = 90°$

So $|\vec{F}| = \sqrt{F_1^2 + F_2^2}$

7. $|\vec{R}| = \sqrt{30^2 + 40^2} = 50$

$\tan\alpha = = \frac{40}{30} = \frac{4}{3}$

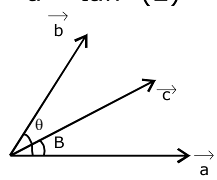
α = 53° with East

8. $|\vec{F}_2 - \vec{F}_1| = \sqrt{(250)^2 + (500)^2}$

$= 250\sqrt{5}$ N W of N

$\tan\alpha = \frac{F_1}{F_2} = \frac{500}{250}$

$a = \tan^{-1}(2)$

9. 

$|\vec{C}| = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

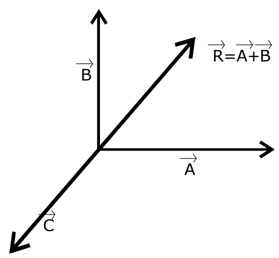
$\tan\beta = \frac{b\sin\theta}{a + b\cos\theta}$

when direction interchanged

$|\vec{C}| = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

$\tan\beta = \frac{a\sin\theta}{b + a\cos\theta}$

In both cases Magnitude is same
But direction different

10. 

$|\vec{C} + \vec{R}| \neq 0$

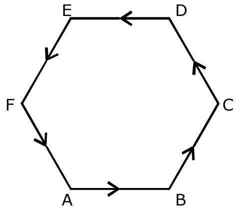
11. Resultant lies in b /w min. & maximum Values

$A - B \le R \le A + B$

So $10 - 6 \le R \le 10 + 6$

$4 \le R \le 16$

12.



$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF} + \overrightarrow{FA} = 0$

Resultant of these vectores is null vector

13. $|\vec{R}| = \sqrt{P^2 + \theta^2 2P\theta\cos\theta}$

when q = π

$\cos\theta = -1$

$|\vec{R}|_{min} = P = Q$

14. $\vec{R} = \vec{A} + \vec{B}$

$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

when $\theta = 0$

$\cos\theta = 1$

$|\vec{R}|_{max} = A + B$

15. $\vec{C} = \vec{A} + \vec{B}$

$|\vec{C}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$13^2 = 12^2 + 5^2 + 2\times12\times5\cos\theta$

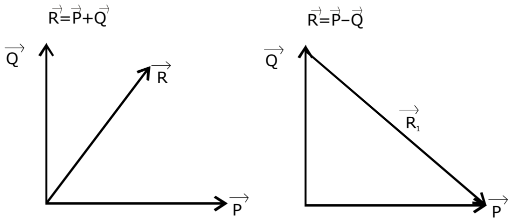
$\theta = \pi/2$

16. $\vec{P} + \vec{Q} = \vec{P} - \vec{Q}$

$\sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{P^2 + Q^2 - 2PQ\cos\theta}$

$4PQ\cos\theta = 0 \Rightarrow \theta = \pi/2$

17.



$|\vec{R}| = \sqrt{P^2 + Q^2}$ $\quad$ $|\vec{R}_1| = \sqrt{P^2 + Q^2}$

$\vec{R}$ & $\vec{R}_1$ are also perpendicular to each other are of equal lengths.

18. $\vec{A} = 3\hat{i} + 2\hat{j} + \hat{k}$

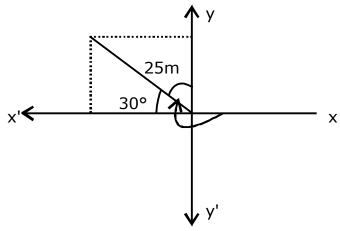
$|\vec{A}| = \sqrt{3^2 + 2^2 + 1^2}$

$|\vec{A}| = \sqrt{14}$

19. $\vec{A} = 3\hat{i} + 4\hat{j}$

$\hat{A} = \dfrac{3\hat{i} + 4\hat{j}}{5}$

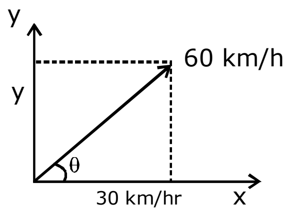
20.



$\Rightarrow -25\cos 30°$

$\Rightarrow 25\sin 30°$

21.



$60\cos\theta = 30$

$\cos\theta = 1/2$

$\theta = 60°$

$y = 60\sin 60°$

$= 60\dfrac{\sqrt{3}}{2} = 30\sqrt{3}$ km / hr

22. $\vec{A} = 0.5\hat{i} + 0.8\hat{j} + c\hat{k}$

$|\hat{A}| = \sqrt{(0.5)^2 + (0.8)^2 + C^2}$

$C^2 = 1 - 0.89$

$C = \sqrt{0.11}$

23. Let $\vec{A} = 2\hat{i} + 2\hat{j}$

$\vec{B} = \hat{i} + \sqrt{3}\,\hat{j}$

θ be the angle between $\vec{A}$ & $\vec{B}$

$\cos\theta = \dfrac{\vec{A}.\vec{B}}{|\vec{A}|.|\vec{B}|} = \dfrac{2 + 2\sqrt{3}}{\sqrt{8}.2}$

$\cos\theta = \dfrac{\sqrt{3} + 1}{2\sqrt{2}}$

$\theta = 15$

24. $\vec{F}_x = 2\hat{i}$ $\quad$ $\vec{F}_y = -3\hat{j}$

$\vec{F} = \vec{F}_x + \vec{F}_y = 2\hat{i} - 3\hat{j}$

25. $\vec{A} = \hat{i} + \hat{j} + \hat{k}$

$\vec{B} = 2\hat{i} + \hat{j}$

$\vec{A}.\vec{B} = (\hat{i} + \hat{j} + \hat{k}) . (2\hat{i} + \hat{j})$

$= 2 + 1 = 3$

(b) $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{vmatrix}$

$= \hat{i}(0 - 1) - \hat{j}(0 - 2) + k(1 - 2)$

$= -\hat{i} + 2\hat{j} - \hat{k}$

26. $|\vec{A}| = 4$ $|\vec{B}| = 3$ $\theta = 60°$

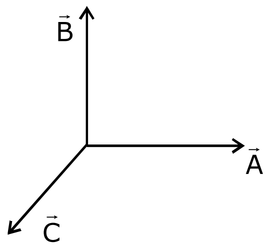
(a) $\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

$= 4 \times 3 \times \frac{1}{2} \Rightarrow 6$

(b) $\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta$

$= 4 \times 3 \times \frac{\sqrt{3}}{2} = 6\sqrt{3}$

27



$\vec{A}.\vec{B} = 0$

$\vec{A}.\vec{C} = 0$

but $\vec{B} \times \vec{C} \neq 0$

$\vec{B} \times \vec{C}$ have direction along $\vec{A}$

so $\vec{A}$ parallel with $\vec{B} \times \vec{C}$

28. AB cosθ = 8 ...(1)

AB sin θ = $8\sqrt{3}$ ...(2)

(2) ÷ (1)

tan θ = $\sqrt{3}$

θ = 60°

EXERCISE – II
MULTIPLE CHOICE QUESTIONS

1. $\Rightarrow f(x) = \frac{x-1}{x+2}$

$\Rightarrow f\{f(x)\} = \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+2}$

$\Rightarrow f\{f(x)\} = \frac{-1}{x}$

2.
$\Rightarrow f(2) = 2 \times 2 - 1 = 3$
$\Rightarrow f(1) = 1 + 2 = 3$
$\Rightarrow f(3) = 2 \times 3 - 1 = 5$

3.
$\Rightarrow y = \ell n\, x^2 + \sin x$
$\Rightarrow y = 2\ell n x + \sin x$

$\Rightarrow \frac{dy}{dx} = \frac{2}{x} + \cos x$

also $\frac{d^2y}{dx} = \frac{-2}{x^2} - \sin x$

4

$y = 7\sqrt{x} + \tan x$

$\Rightarrow \frac{dy}{dx} = \frac{1}{y} x^{-6/7} + \sec^2 x$

also $\frac{d^2y}{dx^2} = \frac{-6}{49} x^{-13/7} + 2\sec^2 x \tan x$

5

$y = e^x \tan x$

$\Rightarrow \frac{dy}{dx} = e^x \tan x + e^x \sec^2 x$

6

$y = x^2\sin^4 x + x\cos^{-2} x$

$\Rightarrow \frac{dy}{dx} = 2x\sin^4 x + 4x^2 \sin^3 x \cos x + \cos^{-2} x$

$\Rightarrow -2x\cos^{-3} x \times -\sin x$

$\Rightarrow \frac{dy}{dx} = 2x\sin^4 x + 4x^2 \sin^3 x \cos x + \cos^{-}$
$^{2}x + 2x \sin x \cos^{-3} x$

7

$y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$

$\Rightarrow \frac{dy}{dx} = \left(1 - \frac{1}{x^2}\right)\left(x - \frac{1}{x} + 1\right) + \left(x + \frac{1}{x}\right)\left(1 + \frac{1}{x^2}\right)$

$\Rightarrow \frac{dy}{dx} = 1 + 2x + \frac{2}{x^3} - \frac{1}{x^2}$

8

$y = x^2\sin x + 2x\cos x - 2\sin x$

$\Rightarrow \frac{dy}{dx} = 2x\sin x + x^2\cos x + 2\cos x - 2x\sin x - 2\cos x$

$\Rightarrow \frac{dy}{dx} = x^2 \cos x$

9

$y = x^2\cos x - 2x\sin x - 2\cos x$

$\Rightarrow \frac{dy}{dx} = 2x\cos x - x^2\sin x - 2\sin x - 2x\cos x + 2\sin x$

$\frac{dy}{dx} = -x^2 \sin x$

10

$r = (1 + \sec\theta)\sin\theta$

$\Rightarrow \frac{dx}{d\theta} = \sec\theta\tan\theta\sin\theta + (1 + \sec\theta)\cos\theta$

$\Rightarrow \frac{dr}{d\theta} = \cos\theta + \sec^2\theta$

11

$y = \frac{\sin x + \cos x}{\cos x}$

$\Rightarrow \frac{dy}{dx} = \frac{\cos x(\cos x - \sin x) - (\sin x + \cos x) \times -\sin x}{\cos^2 x}$

$\frac{dy}{dx} = \frac{1}{\cos^2 x} = \sec^2 x$

12

$y = \frac{\cot x}{1 + \cot x}$

$$\Rightarrow \frac{dy}{dx} = \frac{(1+\cot x)\times -\text{Co}\sec^2 x - \cot x(-\cos ec^2 x)}{(1+\cot x)^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-\cos ec^2 x}{(1+\cot x)^2}$$

13

$$\Rightarrow y = \frac{\cos x}{x} + \frac{x}{\cos x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x\times -\sin x - \cos x}{x^2} + \frac{\cos x + x\sin x}{\cos^2 x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x\sin x - \cos x}{x^2} + \frac{\cos x + x\sin x}{\cos^2 x}$$

14

$$p = \frac{\tan q}{1+\tan q}$$

$$\Rightarrow \frac{dp}{dq} = \frac{(1+\tan q)\sec^2 q - \tan q \sec^2 q}{(1+\tan q)^2}$$

$$\Rightarrow \frac{dp}{dq} = \frac{\sec^2 q}{(1+\tan q)^2}$$

15

$y = \sin^3 x = \sin 3x$

$\Rightarrow y = y_1 + y_2 \Rightarrow \frac{dy}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$

$\Rightarrow y_1 = \sin^3 x$

$\Rightarrow y_1 = u^3$ $\qquad u = \sin x$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$$

$\Rightarrow \frac{dy_1}{du} = 3u^2 \qquad \frac{du}{dx} = \cos x$

$\Rightarrow \frac{dy_1}{dx} = 3\sin^2 x \cos x$

$\Rightarrow y_2 = \sin 3x$

$\Rightarrow y_2 = \sin v$

$\Rightarrow \frac{dy_2}{dv} = \cos v \qquad v = 3x$

$\Rightarrow \frac{dy_2}{dx} = 3\cos 3x \qquad \frac{dv}{dx} - 3$

$$\Rightarrow \frac{dy}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 3\sin^2 x \cos x + 3\cos 3x$$

16

$y = \sin^2(x^2+1)$

$\Rightarrow y = u^2 \qquad u = \sin v \qquad v = x^2 + 1$

$\Rightarrow \frac{dy}{du} = 2u \qquad \frac{du}{dv} = \cos v$

$\Rightarrow \frac{dv}{dx} = 2x \quad \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dv}.\frac{dv}{dx}$

$\Rightarrow \frac{dy}{dx} = 2u\cos v 2x$

$\Rightarrow 4x \sin(x^2+1)\cos(x^2+1)$

17

$y = x(x^2 + 1)^{-1/2}$

$\Rightarrow y = uv$

$\Rightarrow$ Let $u = x \qquad v = (x^2+1)^{-1/2}$

$$\Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\Rightarrow \frac{du}{dx} = 1$$

$$\Rightarrow \frac{dv}{dx} = -x\left(x^2+1\right)^{-3/2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\left(x^2+1\right)^{3/2}}$$

18

$\Rightarrow q = \sqrt{2r - r^2}$

$\Rightarrow q = \sqrt{u} \qquad u = 2r - r^2$

$\Rightarrow \frac{dq}{du} = \frac{1}{2\sqrt{u}} \qquad \frac{du}{dr} = 2 - 2r$

$\Rightarrow \frac{dq}{dr} = \frac{dq}{du}.\frac{du}{dr}$

$\Rightarrow = \frac{1}{2\sqrt{u}}.(2-2r) \qquad \Rightarrow \frac{1-r}{\sqrt{2r-r^2}}$

19

$$\Rightarrow y = \left(\frac{x^2}{8} + x - \frac{1}{x}\right)^4$$

$\Rightarrow y = u^4$

$$\Rightarrow u = \frac{x^2}{8} + x - \frac{1}{x}$$

$\Rightarrow \frac{dy}{du} = 4u^3$ $\qquad \frac{du}{dx} = \frac{x}{4} + 1 + \frac{1}{x^2}$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$

$\Rightarrow = 4\left(\frac{x^2}{8} + x - \frac{1}{x}\right)^3 \left(\frac{x}{4} + 1 + \frac{1}{x^2}\right)$

20

$V = \pi r^2 h$

(a) $\frac{dv}{dt} = \pi r^2 \frac{dh}{dt}$ $\qquad$ {r constnat}

$\Rightarrow \frac{dv}{dt} = 5\pi r^2$

(b) $\frac{dv}{dt} = 2\pi rh \frac{dr}{dt}$ $\qquad$ {h constant}

$\Rightarrow \frac{dv}{dt} = 10\pi rh$

(c) $\frac{dv}{dt} = \pi\left[2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right]$

$\Rightarrow \frac{dv}{dt} = 10\pi rh + 5\pi r^2$

21

$x + y = 60$
$\Rightarrow y = 60 - x$
$\Rightarrow xt = 60x - x^2$
$\Rightarrow z = 60x - x^2$

$\Rightarrow \frac{dz}{dx} = 60 - 2x$

$\Rightarrow \frac{dz}{dx} = 0 \Rightarrow x = 30$

$\Rightarrow \frac{d^2z}{dx^2} = -ve$ maximum

$\Rightarrow x = 30$ $\qquad y = 30$

22 $a^2 + 4a\lambda = 40$

$\Rightarrow h = \frac{4a - a^2}{4a}$

$\Rightarrow v = a^2h = a^2\left(\frac{48 - a^2}{4a}\right)$

$\Rightarrow v = 10a - \frac{a^3}{4}$

$\Rightarrow \frac{dv}{da} = 10 - \frac{3a^2}{4} = 0$

$a = \sqrt{\frac{40}{3}}$

23

(a) $y = \cos x$
$\Rightarrow y' = -\sin x$
$\Rightarrow y'' = -\cos x$

(b) $y = \sec x$
$\Rightarrow y' = \sec x \tan x$
$\Rightarrow y'' = \sec x(\sec^2 x) + (\sec x \tan x)$
tanx
$\Rightarrow y'' = 2\sec^3 x - \sec x$

24

$y = \cos u$ $\qquad u = \sin x$

$\Rightarrow \frac{dy}{du} = -\sin u$ $\qquad \frac{du}{dx} = \cos x$

$\Rightarrow \frac{dy}{dx} = -\sin u \cos x$

$\Rightarrow \frac{dy}{dx} = -\sin(\sin x)\cos x$

25

$y = \sin u$ $\qquad u = x - \cos x$

$\Rightarrow \frac{dy}{du} = \cos u$ $\qquad \frac{du}{dx} = 1 + \sin x$

$\Rightarrow \frac{dy}{dx} = \cos u(1 + \sin x)$

$\Rightarrow \frac{dy}{dx} = (1 + \sin x)\cos(x - \cos x)$

EXERCISE – III

SUBJECTIVE PROBLEMS

1. (A) $R = 2A\cos\theta/2$

$A = 2A\cos\theta/2$

$\theta = 120°$

(B) $A - B \le R \le A + B$

$4 \le R \le 12$

$R = 12$ N

(C) $\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta$

$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{0}{\sqrt{8}\times\sqrt{9}}$

$\cos\theta = 0 \quad \Rightarrow \theta = 90°$

(D) $\vec{R} = \vec{A} + \vec{B}$

$|\vec{R}| = \sqrt{A^2 + B^2} = \sqrt{4+1+9} = \sqrt{14}$

2. $S = t^3 - 2t^2 + 5t + 4$

(A) at t = 1

$S = 1 - 2 + 5 + 4$

$S = 8$

(B) $\frac{ds}{dt} = 3t^2 - 4t + 5$

at t = 1

$\frac{ds}{dt} = 3 - 4 + 5 = 4$

(c) $\frac{d^2s}{dt^2} = 6t - 4$

at t = 1

$\frac{d^2s}{dt^2} = 6 - 4 = 2$

3. $\vec{F}_1 = 2\hat{i} + 2\hat{j}$

$\vec{F}_2 = 3\hat{i} + 4\hat{k}$

(A) $\vec{F}_R = \vec{F}_1 + \vec{F}_2$

$= 5\hat{i} + 2\hat{j} + 4\hat{k}$

(B) $\cos\theta = \frac{\vec{F}_1.\vec{F}_2}{|\vec{F}_1||\vec{F}_2|}$

$\cos\theta = \frac{6}{2\sqrt{2}\times 5}$

$\cos\theta = \frac{3}{5\sqrt{2}}$

(c) $\vec{F}_1.\vec{F}_2 = F_1F_2\cos\theta$

$F_1\cos\theta = \frac{\vec{F}_1.\vec{F}_2}{F_2} = \frac{6}{5}$

4. S-1

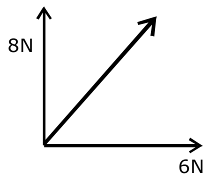
Vector quantity obeys the triangle low of Addition

S-2

$A - B \le R \le \quad A + B$

S-1 is true & S-2 is false

5.



$|\vec{R}| = 10\,N$

5. $|\vec{A}| = |\vec{B}| = 1$

$|\vec{A}\times\vec{B}| + |\vec{A}.\vec{B}| = 1$

LHS

$(AB\sin\theta)^2 + (AB\cos\theta)^2$

$\sin^2\theta + \cos^2\theta = 1$

S-2 is true

6. S-1

$\vec{A}.\vec{B} = 0$

then $\vec{A}$ & $\vec{B}$ are perpendicular to each other

$\vec{A}.\vec{C} = 0$

then $\vec{A}$ & $\vec{C}$ are perpendicular to each other

$\vec{B}\times\vec{C} = |\vec{B}||\vec{C}|\sin\theta\hat{n}$

$\hat{n}$ shows the direction which is along to $\hat{n}$.

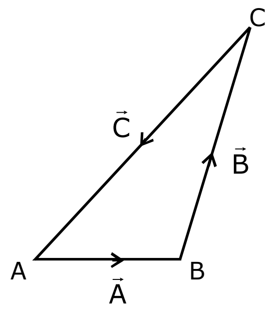
so $\vec{A}$ is parellel to $\vec{B}\times\vec{C}$

S-2

$\vec{A}\perp\vec{B}$ & $\vec{A}\perp\vec{C}$

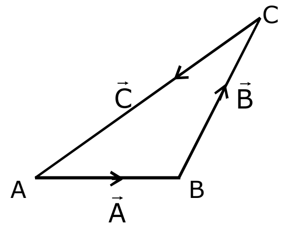
so $\vec{A}$ is perpendicular to plane formed by $\vec{B}$ & $\vec{C}$ true

7.



$\vec{A}+\vec{B}+\vec{C}=0$
statement - 1 is true.

7.



$\vec{A}+\vec{B}+\vec{C}=0$
Statement - 2 is true.

8. $\vec{A}=\hat{i}+\hat{j}$

$\vec{B}=\hat{k}$

$\vec{A}.\vec{B}=|\vec{A}||\vec{B}|\cos\theta$

$0 = \cos\theta$

$\theta = \pi/2$

S-2

$\vec{A}.\vec{B}=|\vec{A}||\vec{B}|\cos\theta$

$\cos\theta=\dfrac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$

9. Statement - 1
Distance is a scalar quantity because of it has only Magnitude
Statement - 2
Distace is the length of path traversed

10. (i) $\vec{A}.\vec{B}=|\vec{A}||\vec{B}|\cos\theta$

$\vec{B}.\vec{A}=|\vec{A}||\vec{B}|\cos\theta$

(ii) $\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$
Because in this magnitude is same but direction is different.

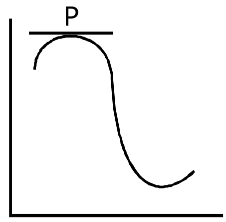
iii. let $\vec{A}$ & $\vec{B}$ be two non- zero vector

$\vec{A}\times\vec{B} = AB\sin\theta\hat{n}$

if $\theta = 0$ then $\vec{A}\times\vec{B}=\vec{0}$

then $\vec{A}$ & $\vec{B}$ are collinear

10.



$\tan\theta = m = \text{slope} = 0$

11. (i) $\vec{A}.\vec{B}=25$ unit

(ii) $\vec{c}=\lambda\vec{A}$

$|\vec{c}|=\lambda|\vec{A}|$

$|\vec{B}|=\lambda|\vec{A}|$

$25 = \lambda\, 5$

$\lambda = 5$

$\vec{C}=5(3\hat{i}+4\hat{j})=(15\hat{i}+20\hat{j})$

$\vec{A}\,||\,\vec{B}$

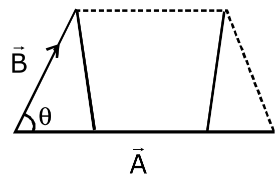
$\vec{A}\times\vec{B}=AB\sin\theta\,\hat{n}$

$\theta = 0°$

$\vec{A}\times\vec{B}=\vec{0}$

(iv) $\vec{A}=3\hat{i}+2\hat{j}$ $\quad\vec{B}=2\hat{i}-2\hat{k}$

Area of parallelogram = $|\vec{A}||\vec{B}|\sin\theta$



$|\vec{A}|=\sqrt{9+4}=\sqrt{13}$

$|\vec{B}|=\sqrt{4+4}=\sqrt{8}$

$\sin\theta=\dfrac{\vec{A}\times\vec{B}}{|\vec{A}||\vec{B}|}$

$|\vec{A}||\vec{B}|\sin\theta=\vec{A}\times\vec{B}=\sqrt{68}$

(V) $\vec{F} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$|\vec{F}| = \sqrt{4+9+36} = \sqrt{49} = 7$

(vi) $\vec{R} = \hat{i} + \hat{j} + \hat{k}$

$\hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

$\hat{R} = \frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$

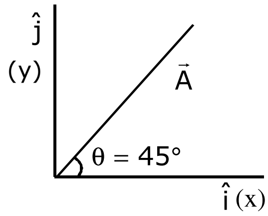
$\vec{A} \perp \vec{B}$

then $\vec{A}.\vec{B} = 0$

$\theta = 90°$

(viii)

$\vec{A} = \hat{i} + \hat{j}$



$\tan\theta = \frac{y}{x} = \frac{1}{1} = 1$

$\theta = 45°$

(ix) $\vec{A} + \vec{B} + \vec{C} = 0$

given $\vec{A}.(\vec{B}\times\vec{C})$

$-(\vec{B}\times\vec{C}).(\vec{B}\times\vec{C})$

$-(\vec{B}).(\vec{B}\times\vec{C}) - (\vec{C}).(\vec{B}\times\vec{C})$

$-0-0=0$

EXERCISE – IV TOUGH SUBJECTIVE PROBLEMS

Sol.1 $R^2 = P^2 + Q^2 + 2PQ\cos60°$

$7Q^2 = P^2 + Q^2 + PQ$

$P^2 - 6Q^2 + PQ = 0$

$$\left(\frac{P}{Q}\right)^2 + \left(\frac{Q}{Q}\right) - 6 = 0$$

$$\left(\frac{P}{Q}+3\right)\left(\frac{P}{Q}-2\right) = 0$$

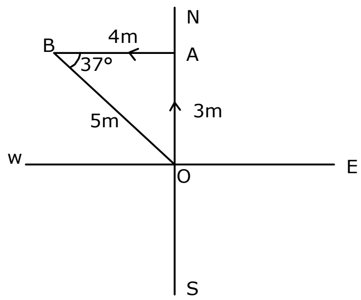
$$\frac{P}{Q} = 2$$

Sol.2 $P2 = F_1^2 + F_2^2$

$Q^2 = F_1^2 + F_2^2$

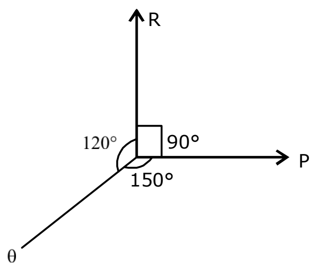
$P^2 + Q^2 = 2(F_1^2 + F_2^2)$

Sol.3



Displacement = 0

Sol.4

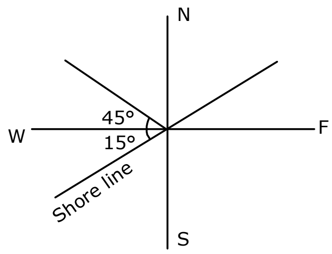


$$\frac{P}{\sin120°} = \frac{\theta}{\sin90°} = \frac{R}{\sin150°}$$

$$\frac{P}{\sqrt{3}/2} = \frac{\theta}{1} = \frac{R}{\frac{1}{2}}$$

P : Q R = $\sqrt{3}$: 2 : 1

Sol. 5



projection on Shore line = 18 cos60°
= 9 km/h

Sol.6 $\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$

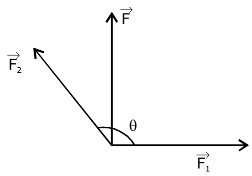
$\vec{B} = 4\hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{AB} = 3\hat{i} + 4\hat{j}k$

$$\hat{AB} = \frac{3\hat{i}+4\hat{j}}{5}$$

$$\vec{V} = 10\left(\frac{3\hat{i}+4\hat{j}}{5}\right) = 6\hat{i} + 8\hat{j}$$

Sol.7 $|\vec{F}_2| = |\vec{F}_2|$



$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

$\alpha = 90°$

then $F_1 + F_2\cos\theta - 0$

$$\cos\theta = -\frac{1}{2}$$

$\theta = 120°$

Sol.8 $|\hat{a} - \hat{b}| = \sqrt{a^2 + b^2 - 2ab\cos\theta}$

$|\hat{a} - \hat{b}| = 1$ $\theta = 60°$

$|\hat{a} - \hat{b}| = 1$

Sol.9 $x = t^3 - 6t^2 + 3t + 7$

$\vec{v} = \frac{dx}{dt} = 3t^2 - 12t + 3$

Sol.10 $\left|\vec{F}_1\right| = \left|\vec{F}_2\right| = \text{dyne}$

$\theta = 120°$

$\left|\vec{F}\right| = 2\left|\vec{F}_1\right| \cos\theta/2$

10 dyne

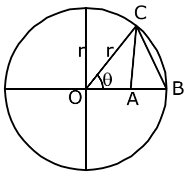
Sol.11 $\left|\vec{A} + \vec{B}\right| = \left|\vec{A} - \vec{B}\right|$

$A^2 + B^2 3AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$

$4AB\cos\theta = 0$

$\theta = \pi/2$

Sol.12

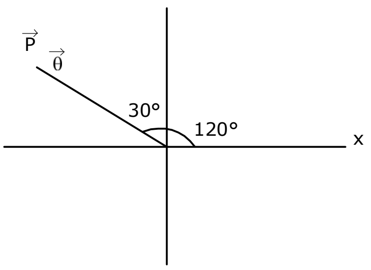


Displacement BC = $\sqrt{AB^2 + AC^2}$

$\sqrt{(r - r\cos\theta)^2 + r^2\sin^2\theta}$

BC = 2rsinθ/2

Sol.13

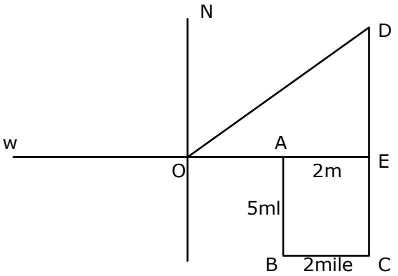


Angle b/w $\vec{P} \,\&\, \vec{\theta}$ is 0

so Resultant = $\sqrt{P^2 + \theta^2 + 2PQ\cos\theta}$

$\left|\vec{R}\right| = P + Q$

Sol.14



$OD^2 = OE^2 + ED^2$

$OD = \sqrt{3^2 + 4^2} = 5\text{miles}$

Sol.15 $\vec{A} = 2\hat{i} + 3\hat{j}$

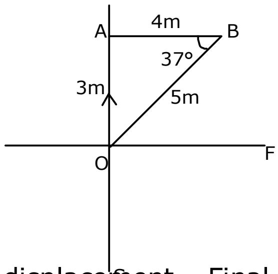
$\vec{B} = \hat{j}$

$\vec{A}.\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta$

$\cos\theta = \frac{3}{\sqrt{13}}$

tanθ=2/3 $\Rightarrow$ Q = $\tan^{-1}(2/3)$

Sol.16



displacement = Final position –initial position

Displacement = 0 – 0 = 0

Sol17. $\vec{A} = 3\hat{i} + 2\hat{j} + 8\hat{k}$

$\vec{B} = 2\hat{i} + x\hat{j} + \hat{k}$

$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\cos\theta$ θ = 90°

$\vec{A}.\vec{B} = 0$

6 + 2x + 8 = 0

x = –7

Sol.18 $\vec{A} = a_1\hat{i} + a_2\hat{j}$

$\vec{B} = 4\hat{i} - 3\hat{j}$

$|\hat{A}| = 1 \Rightarrow a_1^2 + a_2^2 = 1$

$\vec{A}.\vec{B} = 0$

$4a_1 - 3a_2 = 0$

check the option

Sol.19 $\vec{B} = x\vec{a}$

if x is –ve direction of $\vec{B}$ change

if x is +ve direction of $\vec{B}$ same as $\vec{a}$

$\vec{B} \,\&\, \vec{a}$ are colinear vector

EXERCISE – V JEE QUESTIONS

1 $|\vec{A}| = 3$ $|\vec{B}| = 3$

(a) For $|\vec{R}| = 1$

$\therefore$ 1 = 9 + 16 + 24 cosθ

cosθ = –1 θ = π

(b) For $|\vec{R}| = 5$

25 = 9 + 16 + 24 cosθ

cosθ = 0

θ = 90

(c) For $|\vec{R}| = 7$

49 = 9 + 16 + 24 cosθ

24 = 24cosθ

cosθ = 1

θ = 0

2. $R = \sqrt{P^2 + Q^2}$ q = 90°

when θ = 180°

$R^1 = \sqrt{P^2 + Q^2 - 2pQ = \theta}$

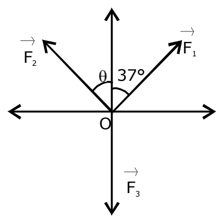
Given $R^1 = \frac{R}{12}$

$\frac{R}{12} = \sqrt{P^2 + Q^2 - 2PQ}$

$P^2 + Q^2 - 4PQ = 0$

$\left(\frac{P}{Q}\right)^2 - 4\left(\frac{P}{Q}\right) + 1 = 0$

$\frac{P}{Q} = 21\sqrt{3}$

3. 

(a) x - direction

F_1 = sin37° = F_2 sin θ

$10 \times \frac{3}{5} = 6\sin\theta$

θ = 90

y = direction

$|\vec{F}_3| = |\vec{F}_1|\cos 37°$

$|\vec{F}_3| = 10 \times 4/5 = 8\text{Nt}$

(b) $|\vec{F}_2| = 6\sin\theta(-\hat{i}) + 6\cos\theta(\hat{j})$

= $-6\hat{i}$ θ = 90°

4. In x-direction

10cos30° + F_2sin30° = 15cos37°

$F_2 = 2(12 - 5\sqrt{3})\text{N}$

$\vec{F}_2 = F_2\sin 30\hat{i} + F_2\cos 30\hat{j}$

$= (12 - 5\sqrt{3})\hat{i} + (12 - 5\sqrt{3})\sqrt{3}\hat{j}$

In y -direction

$F_2\cos 30° + 10\sin 30° + 15\sin 37° = F_1$

$F_1 = (12\sqrt{3} - 1)$

$\vec{F}_1 = -(12\sqrt{3} - 1)\hat{j}$

5. $\vec{F}_1 = 2\hat{i} + a\hat{j} - 3\hat{k}$

$\vec{F}_2 = 5\hat{i} + c\hat{j} - b\hat{k}$

$\vec{F}_3 = b\hat{i} + 5\hat{j} - 7\hat{k}$

$\vec{F}_4 = c\hat{i} + 6\hat{j} - a\hat{k}$

$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$

In x-direction

2+5+b+c= 0 ⇒ b + c = –7 ....(1)

in y-direction

a + c + 5 + 6 = 0

a + c = –11......92)

in z–direction

–3 –b –7 –a = 0

a + b = –10 ...(3)

ON solving

a = –7, b = –3, c = –4

6. $\vec{F}_1 = 5P$ in the direction OY = $5P\hat{j}$

$\vec{F}_2 = 4P$ in the direction OX = $4p\hat{i}$

$\vec{A} = 3a\hat{i} + 4a\hat{j}$

$\hat{A} = \frac{3a\hat{i} + 4a\hat{j}}{5a}$

10P in the direction OA

$= 10 \times \hat{A} = 10\left(\frac{3\hat{i} + 4\hat{j}}{5}\right)$

$\vec{F}_3 = 6\hat{i} + 8\hat{j}$

$\vec{B} = -a\hat{i} + a\hat{j}$

direction $\hat{AB} = \frac{-4a\hat{i} - 3a\hat{j}}{5a} = \frac{-4\hat{i} - 3\hat{j}}{5}$

15 P in the direction $\overline{AB}$

$\vec{F}_4 = -12\hat{i} - 9\hat{j}$

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

$= -2p\hat{i} + 4p3$

$|\vec{F}_{net}| = \sqrt{4P^2 + 16P^2} = \sqrt{20}P$

$\tan\theta = \frac{y}{x} = \frac{4P}{-2P} = -2$

q = tan^{-1} (–2) with x-axis

7. $|\vec{R}| = \sqrt{(10)^2 + (6)^2 - 2\times10\times6\times\frac{1}{2}}$ θ = 60°

$= 2\sqrt{10}$

$\tan\alpha = \frac{B\sin\theta}{A - B\cos\theta} = \frac{3\sqrt{3}}{7}$

$\cos\theta = \frac{7}{2\sqrt{19}} = \theta = \cos^{-1}\left[\frac{7}{2\sqrt{19}}\right]$

8. $\vec{r} = \vec{a} - \vec{b} + \vec{c}$

$\vec{r} = 11\hat{i} + 5\hat{j} - 7\hat{k}$

(b) $\vec{r} = 11\hat{i} + 5\hat{j} = 7\hat{k}$

let $\vec{B} = \hat{k}$

$\cos\theta = \frac{\vec{r}.\vec{B}}{|\vec{r}||\vec{B}|} = \frac{-7}{\sqrt{195}}$

$\theta = \cos^{-1}\left(\frac{-7}{\sqrt{195}}\right)$

(c) $\cos\theta = \frac{-10 + 8 - 18}{\sqrt{77}\sqrt{77}}$

$\cos\theta = \frac{-20}{\sqrt{1309}}$

$\theta = \cos^{-1}\left(\frac{-20}{\sqrt{1309}}\right)$

10. $\vec{r} = (t^2 - 4)\hat{i} + (t - 4)\hat{j}$

(a) $\vec{r} = x\hat{i} + y\hat{j}$

x = $t^2 - 4$ ....(1) y = t – 4 ...(2)

Eleminating t

x = $(y + 4)^2 - 4$

x = y^2 + 16 + 8y – 4

⇒ y^2 + 8y +12 = x

(b) Cross the x-axis y = 0

t – 4 = 0

t = 4 sec

cross the y-axis x = 0

$t^2 - 4 = 0$

t = ± 2 sec

11. $\vec{r} = (t^2 + t)\hat{i} + (3t - 2)\hat{j} + (2t^3 - 4t^2)\hat{k}$

$\frac{d\vec{r}}{dt} = (2t + 1)\hat{i} + 3\hat{j} + (6t^2 - 8t)\hat{k} = \vec{v}$

(a) when t = 2 sec

$\vec{v} = 5\hat{i} + 3\hat{j} + 8\hat{k}$

(b) $\frac{d\vec{v}}{dt} = 2\hat{i} + o + (12 + -3)\hat{k}$

t = 2 sec

$\vec{a} = 2\hat{i} + 16\hat{k}$

(c) speed = $|\vec{v}| = \sqrt{25 + 9 + 64}$

= $7\sqrt{2}$

(d) Magnitude of acc. $= |\vec{a}|$

$= \sqrt{4 + 256}$

= $2\sqrt{65}$

12. m=3kg $\vec{r} = 6t\hat{i} = t^3\hat{j} + \cos t\hat{k}$

$\frac{d\vec{r}}{dt} = 6\hat{i} - 3t^2\hat{j} - \sin t\hat{k} = \vec{v}$

speed = $|\vec{v}|$

At = t = π

speed = $\sqrt{36 + 9\pi 4}$ = $3\sqrt{4 + \pi^4}$

$\vec{a} = \frac{d\vec{v}}{dt} = 0 - 6t\hat{j} - \cos t\hat{k}$

at t = π/2 $\vec{a} = -3\pi\hat{j}$

$|\vec{a}| = 3\pi$

$\vec{F} = -18t\hat{j} - 3\cos t\hat{k}$

13. t = 1 $v = \frac{1}{\sqrt{3}}$ by slope

t = 1.5 v = u + at

$v = \frac{1}{\sqrt{3}} + 0 \times 1.5$

$v = \frac{1}{\sqrt{3}}$ n/s

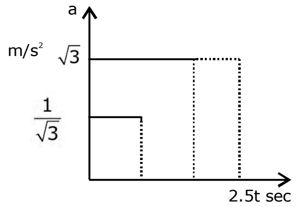
(b) $a_1 = \tan 30° = \frac{1}{\sqrt{3}}$ m/s^2

$a_2 = \tan 60° = \sqrt{3}$ m/s^2

$A_{avg} = \frac{a_1 + a_2}{2}$

$= \sqrt{3}/2 \ m/s^2$

(c)



14. $V_x = 50 - 16t$ $y = 100 - 4t^2$
$v_y = -8t$
when y = 0 t = 5 sec
At t = 5 sec $v_x = -30$ $v_y = -40$

$\vec{v} = v_x\hat{i} + v_y\hat{j} = -30\hat{i} - 40\hat{j}$

$a_x = \frac{dv_x}{dt} = -16$ $a_y = \frac{dy_y}{dt} = -8$

$\vec{a} = a_x\hat{i} + a_y\hat{j}$

$\vec{a} = -16\hat{i} - 8\hat{j}$

15. $v = at - bt^2$

(a) $= \frac{dv}{dt} = a - 2bt$

when velocity maximum acc. (a) = 0

$t = \frac{a}{2b}$

at $t = \frac{a}{2b}$

velocity $= \frac{a \times a}{2b} - b\frac{a^2}{4b^2}$

$v = \frac{a^2}{4b}$

16. $\vec{F} = 3t^2 - 4t + 1$
m = 1kg

$a = \frac{F}{m} = 3t^2 - 4t + 1 = \frac{dv}{dt}$

$\int_{\theta}^{v} dv = \int_0^t (3t^2 - 4t + 1)dt$

$\Rightarrow v = t^3 = 2t^2 + t = \frac{dx}{dt}$

$\int_0^x dx = \int_0^x (t^3 - 2t^2 + t)dt$

$x = \left[\frac{t^4}{4} - \frac{2}{3}t^3 + \frac{t^2}{2}\right]_0^2$

x = 2/3 m