AD

LEVEL–I

1. Number of critical points of f (x) = $\frac{|x^2-4|}{x^2-1}$ are

(A) 1 (B) 2
(C) 3 (D) none of these

2. If the function f (x) = cos |x| – 2ax + b increases for all x $\in$ R, then

(A) a $\leq$ b (B) a = b/2
(C) a < –1/2 (D) a $\geq$ –3/2

3. Area of the triangle formed by the positive x-axis and the normal and the tangent to $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is

(A) $2\sqrt{3}$ sq. units (B) $\sqrt{3}$ sq. units
(C) $4\sqrt{3}$ sq. units (D) none of these

4. A tangent to the curve y = $\frac{x^2}{2}$ which is parallel to the line y = x cuts off an intercept from the y-axis is

(A) 1 (B) –1/3
(C) 1/2 (D) –1/2

5. A particle moves on a co-ordinate line so that its velocity at time t is v (t) = t^2 – 2t m/sec. Then distance travelled by the particle during the time interval 0 $\leq$ t $\leq$ 4 is

(A) 4/3 (B) 3/4
(C) 16/3 (D) 8/3

6. The derivative of f (x) = |x| at x = 0 is

(A) 1 (B) 0
(C) –1 (D) does not exist

7. f (x) = $-[x^2 + 3x^4 + 5x^6 + 5]$ have only ------------- value in $(-\infty,\infty)$ at x = ------------

8. If y = a log | x | + bx^2 + x has its extremum values at x = -1 and x =2 then a= -------

b = --------------

9. The value of b for which the function f (x) = sin x –bx + c is decreasing in the interval $(-\infty,\infty)$ is given by

(A) b < 1 (B) b $\geq$ 1
(C) b > 1 (D) b $\leq$ 1

10. Equation of the tangent to the curve y = $e^{-|x|}$ at the point where it cuts the line x=1

(A) is ey + x =2 (B) is x + y = e
(C) is ex + y = 1 (D) does not exist

11. The greatest and least values of the function f(x) = ax + b $\sqrt{x}$ + c, when a > 0, b > 0, c > 0 in the interval [0,1] are

(A) a+b+c and c (B) a/2 b$\sqrt{2}$+c, c
(C) $\frac{a+b+c}{\sqrt{2}}$, c (D) None of these

12. The absolute minimum value of $x^4 - x^2 - 2x + 5$
(A) is equal to 5 (B) is equal to 3
(C) is equal to 7 (D) does not exist

13. Through the point P (α, β) where $\alpha\beta>0$ the straight line $\frac{x}{a}+\frac{y}{b}=1$ is drawn so as to form with co-ordinates axes a triangle of area S. If ab >0, then the least value of S is
(A) $2\,\alpha\,\beta$ (B) $1/2\,\alpha\beta$
(C) $\alpha\,\beta$ (D) None of these

14. If $f(x) = A \ln|x| + B x^2 + x$ has its extreme values at x = 2 and x = 1 then
(A) A = 2, B = – 1/2 (B) A = – 2 , B = 1/2
(C) A = 2, B =1 (D) None of these

15. The function $2\tan^3 x - 3\tan^2 x + 12\tan x + 3$, $x \in \left(0, \frac{\pi}{2}\right)$ is
(A) increasing (B) decreasing
(C) increasing in $(0, \pi/4)$ and decreasing in $(\pi/4, \pi/2)$
(D) none of these

16. The tangent to the curve $y = 2^x$ at the point whose ordinate is 1, meets the x – axis at the point
(A) (0, ln2) (B) (ln 2, 0)
(C) (-ln2, 0) (D) (-1/ln2, 0)

17. The minimum value of ax + by, where $xy = r^2$, is (r, ab >0)
(A) $2r\sqrt{ab}$ (B) $2ab\sqrt{r}$
(C) $-2r\sqrt{ab}$ (D) None of these

18. The range of the function $f(x) = \sin^{-1}\left[x^2+\frac{1}{2}\right] + \cos^{-1}\left[x^2-\frac{1}{2}\right]$, where [.] is the greatest integer function, is
(A) $\left\{\frac{\pi}{2}, \pi\right\}$ (B) $\left\{0, \frac{\pi}{2}\right\}$ (C) $\{\pi\}$ (D) $\left(0, \frac{\pi}{2}\right)$

19. The domain of $f(x) = \sqrt{\log_{1/4}\left(\frac{5x-x^2}{4}\right)} + {}^{10}C_x$ is
(A) (0, 1]U [4, 5) (B) (0, 5)
(C) {1, 4} (D) None of these

20. A function whose graph is symmetrical about the origin is given by
(A) $f(x) = e^x + e^{-x}$ (B) $f(x) = \log_e x$
(C) f (x + y) = f (x) + f (y) (D) none of these

21. Let f (x) be a function whose domain is [-5, 7]. Let g (x) = |2x + 5|, then the domain of fog (x) is
(A) [-5, 1] (B) [-4, 0] (C) [-6, 1] (D) none of these

22. $\lim_{x\to\infty} \frac{\sin^4 x - \sin^2 x + 1}{\cos^4 x - \cos^2 x + 1}$ is equal to,
(A) 0 (B) –1

(C) 1 (D) does not exist

23. Pick up the correct statement of the following where [·] is the greatest integer function,
(A) If f (x) is continuous at x = a then [f (x)] is also continuous at x = a.
(B) If f (x) is continuous at x = a then [f (x)] is differentiable at x = a.
(C) If | f (x) | is continuous at x = a then f (x) is also continuous at x = a.
(D) None of these

24. The greatest value of $f(x) = \cos(xe^{[x]} + 7x^2 - 3x)$, $x \in [-1, \infty)$ is
(A) –1 (B) 1
(C) 0 (D) none of these.

25. The equation of the tangent to the curve $f(x) = 1 + e^{-2x}$ where it cuts the line y = 2 is
(A) x + 2y = 2 (B) 2x + y = 2
(C) x – 2y = 1 (D) x – 2y + 2 = 0

26. The angle of intersection of curves $y = 4 - x^2$ and $y = x^2$ is……………………………………..

27. The greatest value of the function $f(x) = \dfrac{\sin 2x}{\sin\left(x + \frac{\pi}{4}\right)}$ on the interval $\left[0, \frac{\pi}{2}\right]$ is…………………

28. Let f(x) = x − sinx and g(x) = x − tanx, where $x \in \left(0, \frac{\pi}{2}\right)$. Then for these value of x.
(A) f(x). g(x) > 0 (B) f(x) . g(x) < 0
(C) $\dfrac{f(x)}{g(x)} > 0$ (D) none of these

29. Suppose that f(x) ≥ 0 for all x ∈ [0, 1] and f is continuous in [0, 1] and $\int_0^1 f(x)dx = 0$, then ∀ x ∈ [0, 1], f is
(A) entirely increasing (B) entirely decreasing
(C) constant (D) None of these

LEVEL−II

1. Let h (x) = f (x) + ln{f(x)} + $\{f(x)\}^2$ for every real number x, then
(A) h (x) is increasing whenever f (x) is increasing
(B) h (x) is increasing whenever f (x) is decreasing
(C) h (x) is decreasing whenever f (x) is increasing
(D) nothing can be said in general

2. Let f (x) = $a_0 + a_1x^2 + a_2x^4 + \ldots\ldots + a_nx^{2n}$, where $0 < a_0 < a_1 < a_2 < \ldots\ldots < a_n$, then f (x) has
(A) no minimum
(B) only one minimum
(C) no maximum
(D) neither a maximum nor a minimum

3. The maximum value of $\dfrac{\sin x \cos x}{\sin x + \cos x}$ in the interval $\left[0, \dfrac{\pi}{2}\right]$ is
(A) 1/2
(B) 1/4
(C) $\dfrac{1}{2\sqrt{2}}$
(D) 1/3

4. If y = $\sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots\ldots\infty}}}$, then the value of $\dfrac{dy}{dx}$ is
(A) $\sqrt{\dfrac{\sin x}{y+1}}$
(B) $\dfrac{\sin x}{y+1}$
(C) $\dfrac{\cos x}{2y+1}$
(D) $\dfrac{\cos x}{2y-1}$

5. The curve $y - e^{xy} + x = 0$ has a vertical tangent at the point
(A) (1, 1)
(B) at no point
(C) (0, 1)
(D) (1, 0)

6. A differentiable function f (x) has a relative minimum at x = 0 then the function y = f(x) + ax + b has a relative minimum at x = 0 for
(A) all a and all b
(B) all b if a = 0
(C) all b > 0
(D) all $a \geq 0$

7. Let f(x) = $\begin{cases} 1+\sin x, & x < 0 \\ x^2 - x + 1, & x \geq 0 \end{cases}$ .Then
(A) f has a local maximum at x = 0
(B) f has a local minimum at x = 0
(C) f is increasing every where
(D) f is decreasing everywhere

8. Let f(x) = $x^{n+1} + a.\, x^n$, where 'a' is a positive real number, $n \in I^+$. Then x = 0 is a point of
(A) local minimum for any integer n
(B) local maximum for any integer n
(C) local minimum if n is an even integer
(D) local minimum if n is an odd integer

9. f(x) = max (sinx, cosx) $\forall\, x \in R$. Then number of critical points $\in [-2\pi, 2\pi]$ is /are ;
(A) 5
(B) 7
(C) 9
(D) none of these

10. Let $\phi(x) = (f(x))^3 - 3(f(x))^2 + 4f(x) + 5x + 3\sin x + 4\cos x$
$\forall\, x \in R$, then
(A) ϕ is increasing whenever f is increasing
(B) ϕ is increasing when ever f is decreasing
(C) ϕ is decreasing whenever f is decreasing

(D) Nothing can be said

11. A function f(x) = $\frac{x^2-3x+2}{x^2+2x-3}$ is:
(A) Maximum at x = -3
(B) Minimum at x = -3 and maximum at x = 1
(C) No point of maxima or minima
(D) Function is decreasing in it's domain.

12. Let f(x) = $\begin{cases} \sin(x^2-3x) & x \le 0 \\ 5x^2+6x & x > 0 \end{cases}$. Then f(x) has
(A) local maxima at x = 0
(B) Local minima at x = 0
(C) Global maxima at x = 0
(D) Global minima at x = 0

13. If a, b, c, d are four positive real numbers such that abcd =1, then minimum value of (1+a) (1+b) (1+c) (1+d) is
(A) 8
(B) 12
(C) 16
(D) 20

14. If f(x) + 2f(1- x) = x^2 + 2 $\forall$ $x \in R$, then f(x) is given as
(A) $\frac{(x-2)^2}{3}$
(B) $x^2 - 2$
(C) 1
(D) None of these

15. $\lim_{x \to 5\pi/4} [\sin x + \cos x]$, where [.] denotes the Integral part of x.
(A) is equal to –1
(B) is equal to –2
(C) is equal to –3
(D) Does not exist

16. If f (x) = $\frac{\ln(1+x)^{1+x}}{x^2} - \frac{1}{x}$, then the value of f (0) so that f (x) is continuous at x = 0, is;
(A) 2
(B) 1
(C)1/2
(D) None of these

17. If f (x) = $\frac{x}{1+|x|}$, then
(A) f (x) is differentiable $\forall$ $x \in R$
(B) f (x) is no where differentiable
(C) f (x) is not differentiable at finite no. of point
(D) None of these

18. If f_1 (x) = sin x + tan x, f_2 (x) = 2x then
(A) $f_1(x) > f_2(x)$ $\forall$ $x \in (0, \pi/2)$
(B) $f_1(x) < f_2(x)$ $\forall$ $x \in (0, \pi/2)$
(C) $f_1(x) - f_2(x) = 0$ has exactly one root $\forall$ $x \in (0, \pi/2)$
(D) None of these

19. .Let f (x) = $\begin{cases} |x-1|+a, & x \le 1 \\ 2x+3, & x > 1 \end{cases}$. If f (x) has a local minima at x = 1. Then exhaustive set of values of 'a' is;
(A) $a \le 4$
(B) $a \le 5$
(C) $a \le 6$
(D) $a \le 7$

20. A differentiable function f (x) has a relative minimum at x = 0 then the function y = f (x) + ax + b has a relative minimum at x = 0 for

(B) all a and all b

(D) all b > 0

(B) all b if a = 0

(D) all $a \geq 0$

21. The maximum value of $f(x) = |x \ln x|$ in $x \in (0,1)$ is;

(A) 1/e
(B) e
(C) 1
(D) none of these

22. If $f(x) = \int_0^x (t+1)\,(e^t - 1)\,(t-2)\,(t+4)\,dt$ then f (x) would assume the local minima at;

(A) x = –4
(B) x = 0
(C) x = 1
(D) x = 2.

23. $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

(A) $(0, \pi/4)$
(B) $(0, \pi/2)$
(C) $(-\pi/4, \pi/4)$
(D) none of these.

24. Let f: R→R, where $f(x) = x^3 - ax$, $a \in R$. Then set of values of 'a' so that f(x) is increasing in its entire domain is;

(A) $(-\infty, 0)$
(B) $(0, \infty)$
(C) $(-\infty, \infty)$
(D) none of these

25. The curves $y = 4x^2 + 2x - 8$ and $y = x^3 - x + 10$ touch each other at the point…………………..

26. Let f be differentiable for all x. if f (1) = -2 and $f'(x) \geq 2$ for all $x \in [1, 6]$, then

(A) f (6) < 8
(B) $f(6) \geq 8$
(C) $f(6) \geq 5$
(D) $f(6) \leq 5$

27. The function $f(x) = \dfrac{2x^2 - 1}{x^4}$ decreases in the interval……………………………………………..

28. The function $f(x) = (x + 2)\, e^{-x}$ increases in ------------------- and decreases in --------------------------------

29. The function $y = x - \cot^{-1} x - \log\left(x + \sqrt{x^2 + 1}\right)$ is increasing on

(A) $(-\infty, 0)$
(B) $(-\infty, \infty)$
(C) $(0, \infty)$
(D) R – {0}

30. Let $f : (0, \infty) \to R$ defined by $f(x) = x + \dfrac{9\pi^2}{x} + \cos x$. Then minimum value of f(x) is

(A) $10\pi - 1$
(B) $6\pi - 1$
(C) $3\pi - 1$
(D) none of these

31. Let $a, n \in N$ such that $a \geq n^3$ then $\sqrt[3]{a+1} - \sqrt[3]{a}$ is always

(A) less than $\dfrac{1}{3n^2}$
(B) less than $\dfrac{1}{2n^3}$
(C) more than $\dfrac{1}{n^3}$
(D) more than $\dfrac{1}{4n^2}$

32. The global minimum value of function $f(x) = x^3 + 3x^2 + 10x + \cos \pi x$ in [-2,3] is

(A) 0
(B) $3-2\pi$

(C) $16-2\pi$ (D) -15

33. The minimum value of the function defined by $f(x) = \text{Maximum}\{x, x+1, 2-x\}$ is
(A) 0 (B) 1/2
(C) 1 (D) 3/2

LEVEL−III

1. If the parabola $y = ax^2 + bx + c$ has vertex at (4, 2) and $a \in [1, 3]$, then difference between the extreme values of abc is equal to,
(A) 3600 (B) 144
(C) 3456 (D) None of these

2. Let α, β and γ be the roots of $f(x) = x^3 + x^2 - 5x - 1 = 0$. Then $[\alpha] + [\beta] + [\gamma]$, where [.] denotes the greatest integer function, is equal to
(A) 1 (B) – 2
(C) 4 (D) – 3

3. The number of solutions of the equation $x^3 + 2x^2 + 5x + 2\cos x = 0$ in $[0, 2\pi]$ is
(A) 0 (B) 1
(C) 2 (D) 3

4. Let S be the set of real values of parameter λ for which the equation $f(x) = 2x^3 - 3(2+\lambda)x^2 + 12\lambda x$ has exactly one local maximum and exactly one local minimum. Then S is a subset of
(A) $(-4, \infty)$ (B) $(-3, 3)$
(C) $(3, \infty)$ (D) $(-\infty, 3)$

5. Consider a function $y = f(x)$ defined parametrecally as $x = 2t + |t|$, $y = t|t|$, $\forall\, t \in R$. then function is
(A) Differentiable at x = 0 (B) non-differentiable at x = 0
(C) nothing can be said about differentiablity at x = 0
(D) None of these

6. If the line $ax + by + c = 0$ is normal to the curve $xy + 5 = 0$ then
(A) $a > 0$, $b > 0$ (B) $b > 0$, $a < 0$
(C) $a < 0$, $b < 0$ (D) $b < 0$, $a > 0$

7. The number of roots of $x^3 - 3x + 1 = 0$ in [1,2] is/are;
(A) One (B) Two
(C) Three (D)none of these

8. A cubic f(x) vanishes at x = -2 and has extrema at x = -1 and $x = \frac{1}{3}$ such that $\int_{1}^{-1} f(x)dx = \frac{14}{3}$ then f (x) = …………

9. If $g(x) = f(x) + f(1-x)$ and $f''(x) < 0$, $0 \le x \le 1$, then
(A) g(x) is decreasing in (0, 1) (B) g(x) is decreasing in $\left(0, \frac{1}{2}\right)$

(C) g(x) is decreasing in $\left(\frac{1}{2}, 1\right)$

(D) g(x) is increasing in (0, 1)

10. Let $g'(x) > 0$ and $f'(x) < 0$ $\forall$ x $\in$R then

(A) g(f(x + 1)) > g(f(x –1))

(B) f(g(x – 1)) < f(g(x + 1))

(C) g(f(x + 1) < g(f(x – 1))

(D) g(g′(x + 1)) < g(g(x + 1))

11. The function $f(x) = \dfrac{ax+b}{(x-1)(x-4)}$ has a local maxima at (2, –1) then

(A) b = 1, a = 0

(B) a = 1, b = 0

(C) b = –1, a = 0

(D) None of these

12. $f_1(x) = 2x, \quad f_2(x) = 3\sin x - x - \cos x$, then for x $\in$ (0, π/2):

(A) $f_1(x) < f_2(x)$

(B) $f_1|x| < f_2|x|$

(C) $f_1(x) > f_2(x)$

(D) $f_1|x| > f_2|x|$

13. y = f(x) is a parabola, having its axis parallel to y – axis. If the line y = x touches this parabola at x = 1 then

(A) $f''(1) - f'(0) = 1$

(B) $f''(0) - f'(1) = 1$

(C) $f''(1) + f'(0) = 1$

(D) $f''(0) + f'(1) = 1$

14. If f(x) = $2e^x - ae^{-x} + (2a+1)x - 3$ is increasing for all values of 'x' then

(A) a $\in$ (–∞, ∞) – {0}

(B) a $\in$ (–∞, 0]

(C) a $\in$ (0, ∞)

(D) a $\in$ [0, ∞)

15. If 2a + 3b + 6c = 0, then equation $ax^2 + bx + c = 0$ has roots in the interval

(A) (0, 1)

(B) (2, 3)

(C) (1, 2)

(D) (0, 2)

16. The equation $3x^2 + 4ax + b = 0$ has at least one root in (0, 1) if

(A) 4a + b + 3 = 0

(B) 2a + b + 1 = 0

(C) b = 0, a = -4/3

(D) None of these

17. If f(x) satisfies the conditions of Rolle's theorem in [1, 2] then $\int_1^2 f'(x)\,dx$ is equal to

(A) 3

(B) 0

(C) 1

(D) –1

18. If f(x) = $x^2 e^{-x^2/a^2}$ is a non-decreasing function then for a > 0;

(A) x $\in$ [a, 2a)

(B) x $\in$ (–∞, –a] $\cup$ [0, a]

(C) x $\in$ (–a, 0)

(D) None of these

19. The function $f(x) = \dfrac{x}{1 + x\tan x}$ has

(A) One point of minimum in the interval (0, π/2)

(B) One point of maximum in the interval (0, π/2)

(C) No point of maximum, no point of minimum in (0, π/2)

(D) Two points of maximum in (0, π/2)

20. The number of solutions of the equation $a^{f(x)} + g(x) = 0$, where a > 0, g(x) $\neq$ 0 and has minimum value of ½ is

(A) 1

(B) 2

(C) 4

(D) 0

ANSWERS

LEVEL −I

1.	A	2.	C	3.	A	4.	D
5.	C	6.	D	7.	0	8.	2, −1/2
9.	C	10.	A	11.	A	12.	B
13.	C	14.	D	15.	A	16.	D
17.	A	18.	C	19.	C	20.	D
21.	C	22.	C	23.	C	24.	B
25.	B	26.	$2\sqrt{2}$	27.	$\sqrt{2}$	28.	B
29.	C						

LEVEL −II

1.	A	2.	B	3.	C	4.	D
5.	D	6.	B	7.	A	8.	C
9.	B	10.	A	11.	C	12.	B
13.	C	14.	A	15.	B	16.	C
17.	C	18.	A	19.	B	20.	B
21.	A	22.	D	23.	C	24.	A
25.	$3, 34; -\frac{1}{3}, -\frac{74}{9}$			26.	B	27.	$\left(-\frac{1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$
28.	(0, 1); R − (0, 1)			29.	B	30.	B
31.	A	32.	D	33.	C		

LEVEL −III

1.	C	2.		3.	A	4.	D
5.	A	6. A, C		7.	A	8.	$-x^3 - x^2 + x - 2$
9.	C	10.	C	11.	B	12.	C
13.	C	14.	D	15.	A	16.	B
17.	B	18.	B	19.	B	20.	D