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for IIT-JEE 2012-13

ELECTRICITY AND MAGNETISM

With Fully Solved Exercises

B.M. Sharma

Physics for IIT-JEE

ELECTRICITY & MAGNETISM

B.M. Sharma

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Physics for IIT-JEE 2012-13:
Electricity & Magnetism

B.M. Sharma

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Preface

Since the time the IIT-JEE (Indian Institute of Technology Joint Entrance Examination) started, the examination scheme and the methodology have witnessed many a change. From the lengthy subjective problems of 1950s to the matching column type questions of the present day, the paper-setting pattern and the approach have changed. A variety of questions have been framed to test an aspirant's calibre, aptitude, and attitude for engineering field and profession. Across all these years, however, there is one thing that has not changed about the IIT-JEE, i.e., its objective of testing an aspirant's grasp and understanding of the concepts of the subjects of study and their applicability at the grass-root level.

No subject can be mastered overnight; nor can a subject be mastered just by formulae-based practice. Mastering a subject is an expedition that starts with the basics, goes through the illustrations that go on the lines of a concept, leads finally to the application domain (which aims at using the learnt concept(s) in problem-solving with accuracy) in a highly structured manner.

This series of books is an attempt at coming face-to-face with the latest IIT-JEE pattern in its own format, which is going to be highly advantageous to an aspirant for securing a good rank. A thorough knowledge of the contemporary pattern of the IIT-JEE is a must. This series of books features all types of problems asked in the examination—be it MCQs (one or more than one correct), assertion-reason type, matching column type, or paragraph-based, thought-type questions. Not discounting to need for skilled and guided practice, the material in the book has been enriched with a large number of fully solved concept-application exercises so that every step in learning is ensured for the understanding and application of the subject.

This whole series of books adopts a multi-facetted approach to mastering concepts by including a variety of exercises asked in the examination. A mix of questions helps stimulate and strengthen multi-dimensional problem-solving skills in an aspirant. Each book in the series has a sizeable portion devoted to questions and problems from previous years' IIT-JEE papers, which will help students get a feel and pattern of the questions asked in the examination. The best part about this series of books is that almost all the exercises and problem have been provided with not just answers but also solutions.

Overall the whole content of the book is an amalgamation of the theme of physics with ahead-of-time problems, which an aspirant must follow to accomplish success in IIT-JEE.

B. M. Sharma

CHAPTER
1

Coulomb's Laws and Electric Field

- Electric Charge
- Charging of a Body
- Work Function of a Body
- Properties of Electric Charge
- Coulomb's Law
- Coulomb's Law in Vector Form
- Electric Field
- Different Patterns of Electric Field Lines
- Field of Ring Charge
- Field of Uniformly Charged Disk
- Field of Two Oppositely Charged Sheets
- Electric Dipole
- Electric Field Due to a Dipole
- Electric Field Intensity Due to a Short Dipole at Some General Point
- Dipole in a Uniform Electric Field

ELECTRIC CHARGE

Electric charge, like mass, is one of the fundamental attributes of the particle of which the matter is made. Charge is the physical property of certain fundamental particles (like electron, proton) by virtue of which they interact with the other similar fundamental particles.

- Charge is an intrinsic property of some fundamental particles which accompanies these particles wherever they exist.
- Charge is that property of a body/particle which is responsible for electrical force between them.

To distinguish the nature of interaction, charges are divided into two parts:

(i) positive (ii) negative.

Fig. 1.1 shows an experiment to demonstrate that there are two types of charges.

We know that matter consists of atoms. An atom consists of a central core (called nucleus) and electrons. Electrons orbit around the nucleus. Nucleus consists of neutrons and protons. Neutrons do not contain any net charge. Protons and electrons have equal charges, but of opposite nature. Protons are positively charged while electrons are negatively charged. Protons, however, are very heavy when compared with electrons, about 1836 times. Protons are imprisoned in the nucleus along with neutrons due to the strongest binding force existing in nature called 'strong or nuclear force'. Thus, protons do not travel from atom to atom. The outermost electrons may travel from atom to atom. Hence, we say that electrons are the basis of electricity.

Charge on a proton or on an electron is of indivisible nature. We designate this charge by $+e$ and $-e$, respectively. Hence, charge in or on any object is always an integral multiple of the electronic charge.

In a normal atom:

i. Number of protons = number of electrons.
ii. Protons have the basic $+e$ charge and electrons have the basic $-e$ charge.
iii. Hence, a normal atom is electrically neutral.

Electrons can travel from one atom to another and from one body to another.

If a body loses one electron, it becomes positively charged with $+e$ charge and vice versa.

A body, however, cannot lose or gain any proton, which is heavy and remains imprisoned in the nucleus, by ordinary methods.

Note: *Basic unit of charge = e, whose magnitude is equal to the magnitude of charge on an electron or proton, i.e.,* $e = 1.6 \times 10^{-19}$ *C*

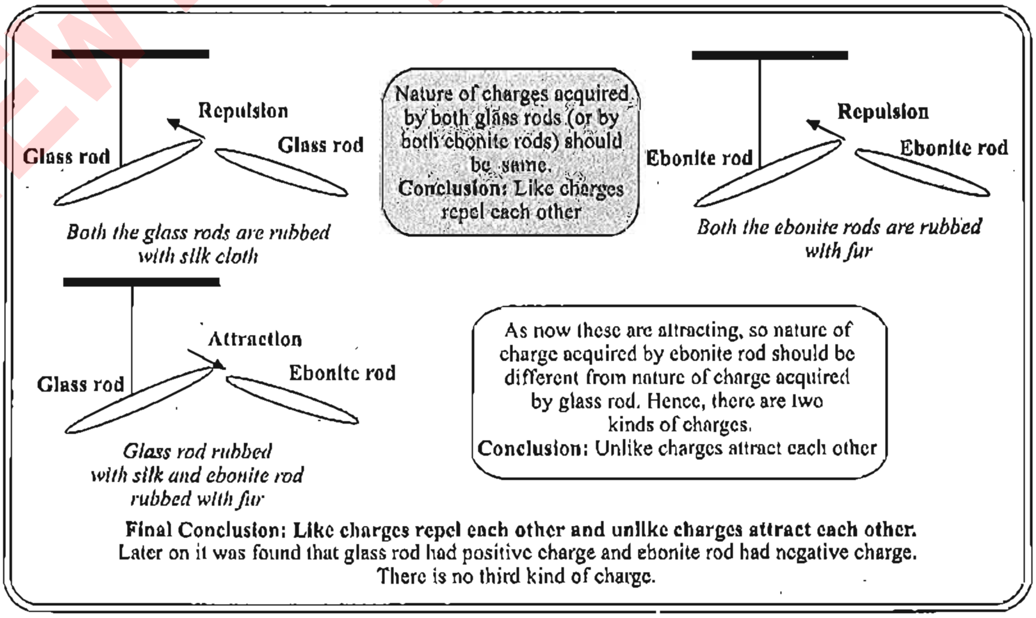
S.I. unit of charge: As mentioned above, $e = 1.6 \times 10^{-19}$ *C. In it, e stands for one electronic charge which is the basic unit of charge. C stands for "coulomb" (note the small c in "coulomb"). "coulomb" is the S.I. unit of charge.*

CHARGING OF A BODY

Ordinarily, matter contains equal number of protons and electrons. A body can be charged by the transfer of electrons or redistribution of electrons.

A body can be charged by the transfer of electrons and not due to the transfer of protons. Why?

It is because protons are inside the nucleus and it is very difficult to remove them from there. Electrons lie in the outer shells and it is easier to remove them.



Nature of charges acquired by both glass rods (or by both ebonite rods) should be same.
Conclusion: Like charges repel each other

As now these are attracting, so nature of charge acquired by ebonite rod should be different from nature of charge acquired by glass rod. Hence, there are two kinds of charges.
Conclusion: Unlike charges attract each other

Final Conclusion: Like charges repel each other and unlike charges attract each other.
Later on it was found that glass rod had positive charge and ebonite rod had negative charge.
There is no third kind of charge.

Fig. 1.1

To charge a body negatively: some electrons are given to it.
To charge a body positively: some electrons are taken from it.

WORK FUNCTION OF A BODY

'he amount of work to be done on a body in order to remove lectron from its surface. Obviously it is easier to remove an electron from a body whose work function is lower.

Let us see how bodies get charged due to friction:

As shown in Fig. 1.2, let $W_2 > W_1$.

Now, suppose A and B are rubbed together.

Net transfer of electrons will take place from A to B.

It is to be noted that mass is also affected during charging.

(Mass of negatively charged body increases and that of positively charged body decreases.)

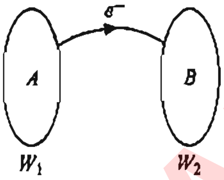


Fig. 1.2

Basically charging can be done by three methods:
1. Friction, 2. Conduction, and 3. Induction.

Charging by Friction

When two bodies are rubbed together, electrons are transferred from one body to the other making one body positively charged and the other negatively charged.

Example: When a glass rod is rubbed with silk, the rod becomes positively charged while silk gets negatively charged. However, ebonite on rubbing with wool becomes negatively charged making the wool positively charged.

Charging by Conduction

The process of charging from an already charged body can happen either by conduction or induction. Conduction from a charged body involves transfer of like charges. A positively charged body can cause more bodies to get positively charged but the sum of the total charge on all positively charged bodies will be the same as charge on initially considered charged body.

Charging by Induction

Induction is a process by which a charged body can be used to create other charged bodies without touching them or losing its own charge. If a charged body is brought near a neutral body, the charged body attracts opposite charge and repels similar charge present on the neutral body. If the neutral body is now earthed, the like charge is neutralized by the flow of charge from earth, leaving unlike charge on the body. Now, the earthing and the charging body is removed leaving the initially neutral body charged. The whole process is as shown in Fig. 1.3.

PROPERTIES OF ELECTRIC CHARGE

Quantization of Charge

Charge exists in discrete packets rather than in continuous amount, i.e., charge on any body is the integral multiple of the charge on an electron or proton.

$Q = \pm ne$, where $n = 0, 1, 2, \ldots$

Conservation of Charge

Charge is conserved, i.e., total charge on an isolated system is constant. By isolated system, we here mean a system through

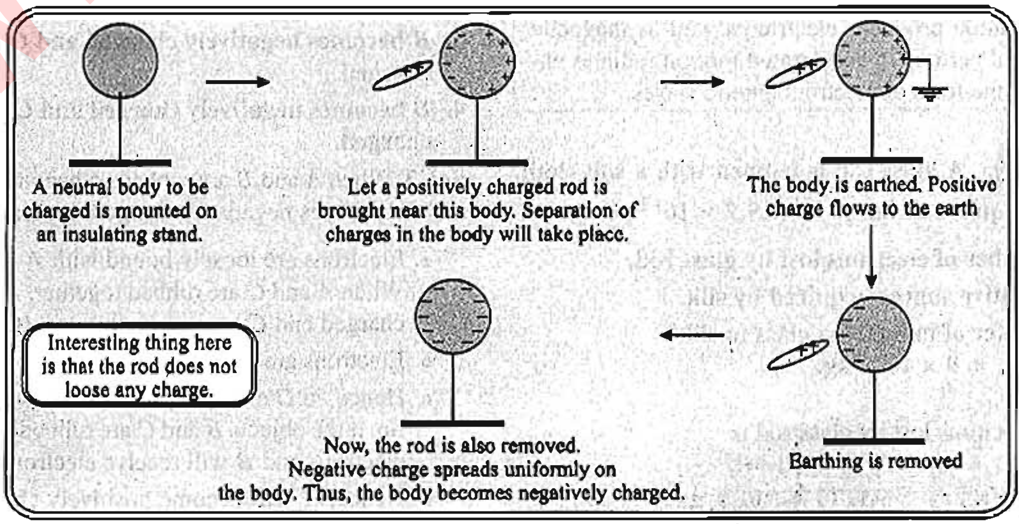


Fig. 1.3

the boundary of which no charge is allowed to escape or enter. This does not require that the amount of positive and negative charges separately be conserved.

Additivity of Charge

Total charge on a body is the algebraic sum of all the charges located anywhere on the body. While adding the charges, their sign must be taken into consideration.

For example, if a body has charges 2 C, −5 C, 4 C and 6 C (Fig. 1.4), then total charge on the body = 2 − 5 + 4 + 6 = 7 C.

Note that charges are added like real numbers. They have no direction. *So, charge is a scalar quantity.*

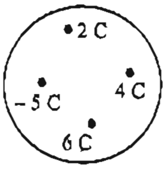


Fig. 1.4

Charge is Invariant

Charge does not depend on the speed of body.

Points to Remember

There are two types of forces which act between two charges. If the charges are stationary, there is only one type of force between them. It is called "electric" or "electrostatic" force. It is given by Coulomb's law for point charges. If the charges are moving, then two types of forces act between them. The first one is the above said electric force. The other force which emerges due to motion is called magnetic force. We shall study magnetic force in a later chapter.

Charge produces electric and magnetic fields and radiates energy: A stationary charged particle produces only electric field in the space surrounding it. A charged particle moving without acceleration produces electric as well as magnetic fields. A charged particle in accelerated motion radiates energy as well, in the form of electromagnetic waves.

Illustration 1.1 **A glass rod is rubbed with a silk cloth. The glass rod acquires a charge of $+19.2 \times 10^{-19}$ C.**

1. **Find the number of electrons lost by glass rod.**
2. **Find the negative charge acquired by silk.**
3. **Is there transfer of mass from glass to silk?**
   Given, $m_e = 9 \times 10^{-31}$kg.

Sol.

1. Number of electrons lost by glass rod is

$$n = \frac{q}{e} = \frac{19.2 \times 19^{-19}}{1.6 \times 10^{-19}} = 12$$

2. Charge on silk $= -19.2 \times 10^{-19}$ C
3. Since an electron has a finite mass $(m_e = 9 \times 10^{-31}\text{ kg})$, there will be transfer of mass from glass rod to silk cloth.
   Mass transferred $= 12 \times (9 \times 10^{-31}) = 1.08 \times 10^{-29}$ kg

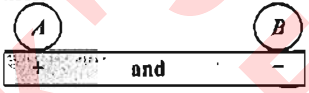
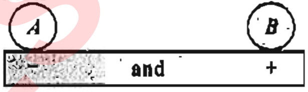
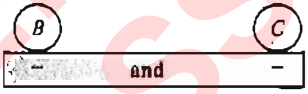
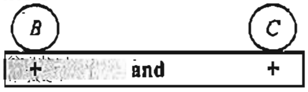
Note that mass transferred is negligibly small. This is expected because the mass of an electron is extremely small.

Illustration 1.2 **Electric charges A and B attract each other. Electric charges B and C repel each other. If A and C are held close together, they will:**

1. attract 2. repel 3. not affect each other

4. more information is needed to answer.

Sol.

Case 1	Case 2
If A and B attract each other, then 	If A and B attract each other, then 
If B and C repel each other, then 	If B and C repel each other, then 

From both cases, we see that A and C will be having unlike charges. Hence, if the charges A and C are held together, they will attract each other.

Illustration 1.3 **If an object made of substance A is rubbed with an object made of substance B, then A becomes positively charged and B becomes negatively charged. If, however, an object made of substance A is rubbed against an object made of substance C, then A becomes negatively charged. What will happen if an object made of substance B is rubbed against an object made of substance C?**

1. **B becomes positively charged and C becomes positively charged.**
2. **B becomes positively charged and C becomes negatively charged.**
3. **B becomes negatively charged and C becomes positively charged.**
4. **B becomes negatively charged and C becomes negatively charged.**

Sol. 3. When A and B are rubbed, A becomes positively charged and B becomes negatively charged. It means

- Electrons are loosely bound with A in comparison to B. When A and C are rubbed together, A becomes negatively charged and C positively charged. It means
- Electrons are loosely bound with C in comparison to A.
- Hence, in C electrons are most loosely bound. So, if the objects B and C are rubbed together, C will loose electrons and B will receive electrons.
- Hence, C will become positively charged and B will become negatively charged.

Illustration 1.4 **Objects A, B and C are three identical, insulated, spherical conductors. Originally A and B both**

+3 mC, while *C* has a charge of −6 mC. *C* are allowed to touch, then they are moved „ objects *B* and *C* are allowed to touch before ıoved apart.

jects *A* and *B* are now held near each other, they will

„attract b. repel c. have no effect on each other.

‚ıf instead objects *A* and *C* are held near each other, they will

a. attract b. repel c. have no effect on each other.

Sol.

Initially *A* *B* *C*

+3 mC +3 mC − 6 mC

- When the objects *A* and *C* are allowed to touch and then moved apart:

A *C* *A* ⟺ *C*

{(+ 3 mC + (−6 mC) = −3 mC] $-\frac{3\text{ mC}}{2}$ $-\frac{3}{2}$ mC

- When the objects *B* and *C* are allowed to touch and then moved apart:

B *C* *B* ⟺ *C*

$\left[(+3\text{ mC})+\left(-\frac{3}{2}\text{ mC}\right)=+\frac{3}{2}\text{ mC}\right]$ $+\frac{3}{4}$ mC $+\frac{3}{4}$ mC

Hence, if *A* and *B* are now held near each other, they will attract each other.

- If *A* and *C* are now held near each other, they will also attract each other.

Illustration 1.5 **Figure 1.5 shows that a positively charged rod is brought near two uncharged metal spheres *A* and *B* attached with insulated stands and placed in contact with each other.**

1. **What would happen if the rod was removed before the spheres are separated?**
2. **Would the induced charges be equal in magnitude even if the spheres had different sizes or different conductors?**
3. **What will happen if the spheres are separated first and then the rod is removed far away.**

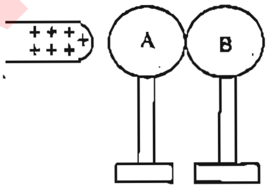


Fig. 1.5

Sol.

1. When a positively charged rod is brought near *A*, the free electrons in the sphere *A* are attracted to the rod and move in the left side of *A*. This movement leaves unbalanced positive charge on *B*. If the rod is removed before the spheres are separated, the excess electrons on sphere *A* would flow back to *B*. Both the spheres will become uncharged.

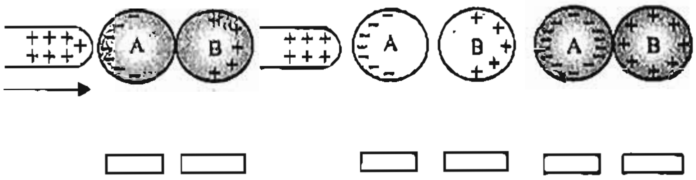


Fig. 1.6

2. Yes, net charge is conserved. Before the rod is brought near *A*, both *A* and *B* were neutral. They will remain so even if they have different sizes or materials.
3. If the rod is removed after the spheres are separated, the sphere *A* will have net negative charge and sphere *B* will have net positive charge of same magnitude.

Concept Application Exercise 1.1

1. a. How many electrons are in 1 coulomb of negative charge?
   b. Which is the true test of electrification, attraction or repulsion?
   c. Can a body have charge of 0.8 × 10^{-19} C?
2. Find the unit and dimension of permittivity of free space.
3. If only one charge is available, can it be used to obtain a charge many times greater than it in magnitude?
4. a. Can two bodies having like charges attract each other? (Yes/No)
   b. Can a charged body attract an uncharged body? (Yes/No)
   c. Two identical metallic spheres of exactly equal masses are taken; one is given a positive charge *q* and the other an equal negative charge. Their masses after charging are different. Comment on the statement.
5. A particle has charge of +10^{-12} C.
   a. Does it contain more or less number of electrons as compared to the neutral state?
   b. Calculate the number of electrons transferred to provide this charge.
6. An ebonite rod is rubbed with fur. The ebonite rod is found to have a charge of −3.2 × 10^{-8} C on it.
   a. Calculate the number of electrons transferred.
   b. What is the charge on fur after rubbing?
7. The electric charge of macroscopic bodies is actually a surplus or deficiency of electrons. Why not protons?
8. A charged rod attracts bits of dry paper which after touching the rod, often jump away from it violently. Explain.
9. A person standing on an insulating stool touches a charged insulated conductor. Will the conductor get completely discharged?
10. An electron moves along a metal tube with variable cross section. How will its velocity change when it approaches the neck of the tube (Fig. 1.7)?

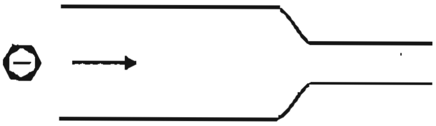


Fig. 1.7

11. Define the following statement "If there were only one electrically charged particle in the entire universe, the concept of electric charge would be meaningless".

COULOMB'S LAW

The force of interaction between two point charges is proportional to the product of magnitudes of the two charges and inversely proportional to the square of distance between them.

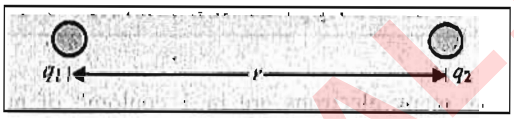


Fig. 1.8

Let two point electric charges q_1 and q_2 are at rest, separated by a distance r, then they exert a force on each other which is given by

$$F = k\frac{q_1q_2}{r^2}$$

where k is a proportionality constant known as *electrostatic force constant.*

If between the two charges there is free space (or vacuum), then $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^2 C^{-2} (in SI units)

where $\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}$ C^{2}N^{-1}m^{-2} is the absolute electric permittivity of the free space.

So, force between two charges is given as $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$ (i)

Equation (i) is applicable only for point charges placed in vacuum. Now, what happens if the two charges are placed in some medium?

In a medium, the force is given as: $F' = k'\frac{q_1q_2}{r^2}$ (ii)

where $k' = \frac{1}{4\pi\varepsilon}$ and in this ε is known as absolute electrical permittivity of medium.

Then, $F' = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}$ (iii)

The ratio $\frac{\varepsilon}{\varepsilon_0} = \varepsilon_r$ is known as relative electrical permittivity of medium.

It is also known as *dielectric constant* and denoted by K.

So, $\frac{\varepsilon}{\varepsilon_0} = \varepsilon_r = K$

The value of K for different materials: Vacuum = 1, air = 1.006 1, glass = 3 to 4, water = 81, conductor = ∞.

In general $K \geq 1$

Now, from (i) and (iii) : $\frac{F'}{F} = \frac{\varepsilon_0}{\varepsilon} = \frac{1}{K}$ =. means when the charges are placed in a medium, . creases K times.

Also, $K = \frac{F}{F'}$. So, *the dielectric constant of a medium , defined as the ratio of force between two charges when the; placed in vacuum to that when they are placed in that med, at same separation.*

Note:

- ***Coulomb's law is not valid for distances < 10^{-15} m.***
- ***Electrostatic forces are comparatively stronger than gravitational forces. Can you show this?***

(As an example—when we hold a book in our hand, electric force between hand and the book is sufficient to balance the gravitational force of earth on the book due to entire earth.)

Some Important Points

- Coulomb's law is applicable only for point charges.
- Coulomb's law is similar to Newton's gravitational law and both obey inverse square law.
- Coulomb's law obeys Newton's third law, i.e., the forces exerted by the two charges on each other are equal and opposite.
- This force acts along the line joining the two particles (called central force).
- Electrostatic force is a conservative force.

COULOMB'S LAW IN VECTOR FORM

Let q_1 and q_2 be two like charges placed at points A and B, respectively, in vacuum.

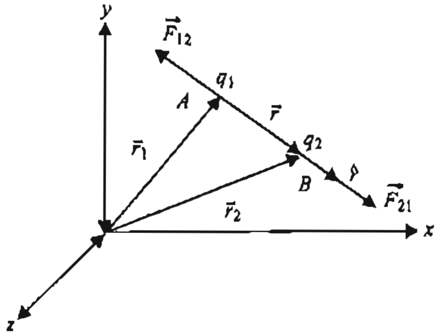


Fig. 1.9

$\vec{r}_1$ is the position vector of point A and $\vec{r}_2$ is the position vecto of point B.

Let $\vec{r}$ is vector from A to B, then $\vec{r} = \vec{r}_2 - \vec{r}_1$ and $r = |\vec{r}_2 - \vec{r}_1|$

$$\Rightarrow \qquad \hat{r} = \frac{\vec{r}}{r} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

Let $\vec{F}_{21}$ be the force on charge q_2 due to q_1; and

$\vec{F}_{12}$ be the force on charge q_1 due to q_2.

From Fig. 1.9, it is clear that $\vec{F}_{21}$ and $\hat{r}_{12}$ are in the same direction, so

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\frac{\vec{r}}{r} = \frac{q_1q_2}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}$$

$$\Rightarrow \qquad \vec{F}_{21} = \frac{q_1q_2}{4\pi\varepsilon_0}\frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3}$$

The above equations give the Coulomb's law in vector form.

As we know that charges apply equal and opposite forces on each other, so we have

$$\vec{F}_{12} = -\vec{F}_{21} \qquad \Rightarrow \qquad \vec{F}_{12} = \frac{q_1q_2}{4\pi\varepsilon_0}\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3}$$

Also, the forces due to two point charges are parallel to the line joining the point charges; such forces are called central forces and so electrostatic forces are conservative forces.

Superposition Principle

It enables us to calculate the force acting on a charge due to more than one charge.

According to superposition principle, the total force on a given charge is vector sum of all the individual forces exerted by each of the other charge.

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n$$

Another important point is that the force between two charges remains unaffected due to the presence of a third charge.

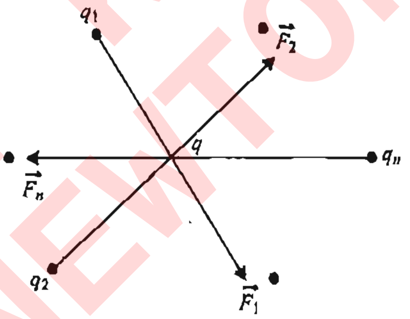


Fig. 1.10

Note:

- *Coulomb's law and principle of superposition together can explain whole of the electrostatics.*
- *Both Coulomb's law and Gravitational law describe inverse square law that involve a property of interacting particles—the charge in one case and mass in the other case.*

Illustration 1.6 Two identical conducting spheres 1 and 2 carry equal amounts of charge and are fixed a certain distance apart that is large compared with their diameters. The spheres repel each other with an electrical force of 88 mN. Suppose now that a third identical sphere 3 having an insulating handle and initially uncharged, is touched first to sphere 1 then to sphere 2 and finally removed. Find the force between spheres 1 and 2 now shown in figure d.

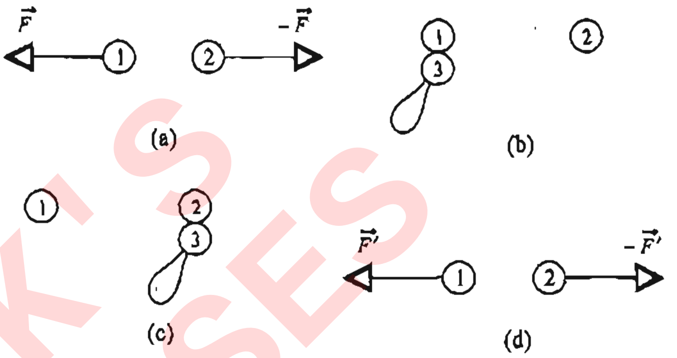


Fig. 1.11

Sol. Initial force between '1' and '2' $F = \frac{kq^2}{r^2} = 88$ mN

Charge on '1' after sphere '3' is touched with '1' = $q/2$. Same charge will be on sphere '3' also.

Charge on '2' after sphere '3' is touched with '2'= $\frac{q + q/2}{2} = \frac{3q}{4}$

Now, force between '1' and '2' in situation d:

$$F' = \frac{k(q/2)(3q/4)}{r^2} = \frac{3}{8}\frac{kq^2}{r^2} = \frac{3}{8} \times 88 = 33 \text{ mN}$$

Illustration 1.7 **Two identical He-filled spherical balloons each carrying a charge q are tied to a weight W with strings and float in equilibrium as shown in Fig. 1.12(a). Find:**

1. **the magnitude of q, assuming that the charge on each balloon acts as if it were concentrated at the centre.**
2. **the volume of each balloon.**

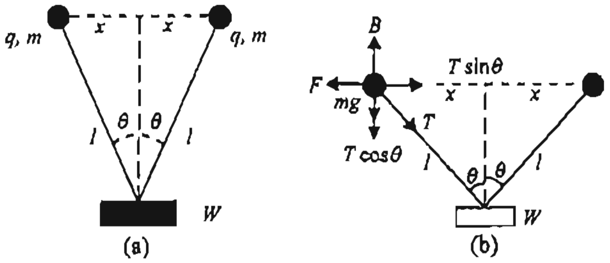


Fig. 1.12

Sol. 1. $2T\cos\theta = W$, $T\sin\theta = F$ [Fig. 1.12(b)]

$$\Rightarrow \frac{\tan\theta}{2} = \frac{F}{W} \Rightarrow F = W\frac{\tan\theta}{2}$$

$$\Rightarrow \frac{q^2}{4\pi\varepsilon_0(2x)^2} = \frac{W\tan\theta}{2} \Rightarrow q = \sqrt{8W\tan\theta\pi\varepsilon_0 x^2}$$

2. $T\cos\theta + mg = B \Rightarrow \frac{W}{2} + V\rho_{He}g = V\rho_a g$

$$\Rightarrow \qquad V = \frac{W}{2(\rho_a - \rho_{He})g}$$

Illustration 1.8 Two particles, each having a mass of 5 g and charge 10^{-7} C, stay in limiting equilibrium on a horizontal table with a separation of 10 cm between them. Find the coefficient of friction between each particle and the table, which is same between each particle and table.

Sol. Friction force f will balance the electrostatic repulsion,

i.e., $$f = F \Rightarrow \mu mg = \frac{q^2}{4\pi\varepsilon_0 r^2}$$

Fig. 1.13

$$\Rightarrow \mu \times \frac{5}{1000} \times 10 = \frac{9\times10^9\times(10^{-7})^2}{(0.10)^2} \Rightarrow \mu = 0.18$$

Illustration 1.9 A particle of mass m carrying a charge $-q_1$ starts moving around a fixed charge $+q_2$ along a circular path of radius r. Prove that period of revolution T of charge $-q_1$ is given by $T = \sqrt{\frac{16\pi^3\varepsilon_0 m r^3}{q_1 q_2}}$.

Sol. Electrostatic force on $-q_1$ due to q_2 will provide the necessary centripetal force, hence

$$\frac{kq_1q_2}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{kq_1q_2}{mr}}$$

Now, $$T = \frac{2\pi r}{v} = \sqrt{\frac{16\pi^3\varepsilon_0 m r^3}{q_1q_2}}$$

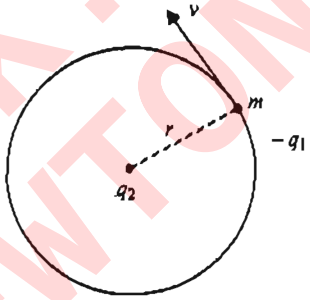


Fig. 1.14

Illustration 1.10 Consider three charges q_1, q_2 and q_3, each equal to q, at the vertices of an equilateral triangle of side l. What is the force on a charge Q placed at the centroid of the triangle?

1. $\frac{3}{4\pi\varepsilon_0}\frac{Qq}{l^2}$ 2. $\frac{\sqrt{3}}{2\pi\varepsilon_0}\frac{Qq}{l^2}$ 3. $\frac{\sqrt{3}}{4\pi\varepsilon_0}\frac{Qq}{l^2}$ 4. zero

Sol. Method 1. The resultant of three equal coplanar vectors acting at a point is zero if these vectors form a closed polygon (Fig. 1.15). Hence, the vector sum of the forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ is zero.

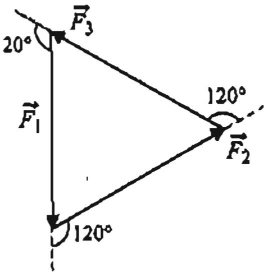


Fig. 1.15

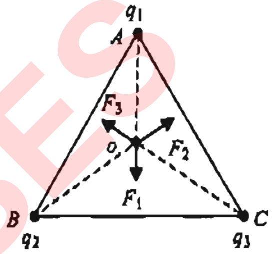


Fig. 1.16

Method 2. The forces acting on the charge Q are

$\vec{F}_1$ = force on Q due to $q_1 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_1}{AO^2}\widehat{AO}$

$\vec{F}_2$ = force on Q due to $q_2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_2}{BO^2}\widehat{BO}$

$\vec{F}_3$ = force on Q due to $q_3 = \frac{1}{4\pi\varepsilon_0}\frac{Qq_3}{CO^2}\widehat{CO}$

The resultant force is $\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Qq}{AO^2}(\widehat{AO} + \widehat{BO} + \widehat{CO}) = 0$$

(as $|q_1| = |q_2| = |q_3|$ and $|\vec{AO}| = |\vec{BO}| = |\vec{CO}|$)

Also, $\widehat{AO} + \widehat{BO} + \widehat{CO} = 0$ because these are three equal vectors in a plane making angles of 120° with each other.

Method 3. The resultant force $\sum\vec{F}$ is the vector sum of individual forces

$$\sum\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \text{ or}$$

$$\sum F_x = F_{1x} + F_{2x} + F_{3x}$$

$$= 0 + F_2\cos30^\circ - F_3\cos30^\circ \qquad \text{(i)}$$

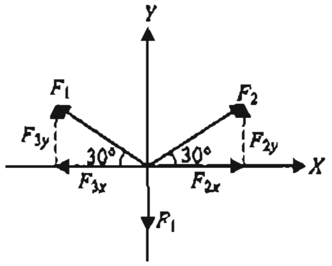


Fig. 1.17

And $$\sum F_y = F_{1y} + F_{2y} + F_{3y}$$

$$= -F_1 + F_2\sin30^\circ + F_3\sin30^\circ \qquad \text{(ii)}$$

As $|F_1| = |F_2| = |F_3| = |F|$ (say), the equations (i) and (ii) become

$\sum F_x = 0$ and $\sum F_y = 0$. Hence, resultant force $\sum\vec{F} = 0$.

Illustration 1.11 **Point charges are placed at the vertices of a square of side a as shown in Fig. 1.18. What should be sign of charge q and magnitude of the ratio $\left|\frac{q}{Q}\right|$ so that:**

1. **net force on each Q is zero?**
2. **net force on each q is zero?**

Is it possible that the entire system could be in electrostatic equilibrium?

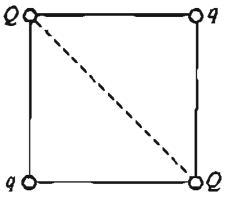
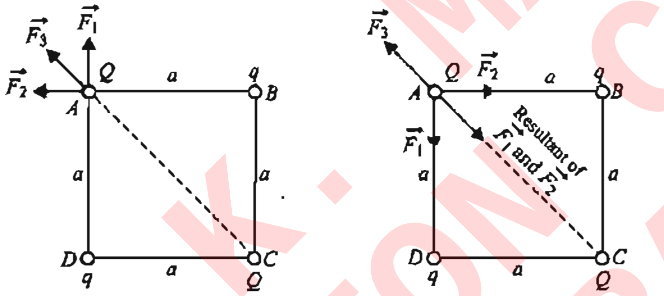


Fig. 1.18

Sol.

1. Consider the forces acting on charge Q placed at A (shown in Fig. 1.19(a) and (b))

Case I. Let the charges q and Q are of same sign.



(a)
(q and Q are of same nature)
Here, net force cannot be zero.

(b)
(q and Q are of opposite nature)
Here, net force can be zero.

Fig. 1.19

Here, $F_1=\frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}$ {force of q at D on Q at A}

$F_2=\frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}$ {force of q at B on Q at A}

$F_3=\frac{1}{4\pi\varepsilon_0}\frac{QQ}{2a^2}$ {force of Q at C on Q at A}

In Fig. 1.19(a), resultant of forces $\vec F_1$ and $\vec F_2$ will lie along $\vec F_3$ so that net force on Q cannot be zero. Hence, q and Q have to be of opposite signs.

Case II. Let the charges q and Q are of opposite sign.

In this case, as shown in Fig. 1.19(b), resultant of $\vec F_1$ and $\vec F_2$ will be opposite to $\vec F_3$ so that it becomes possible to obtain a condition of zero net force.

Let us write $\vec F_R=\vec F_1+\vec F_2$

$$\therefore\quad F_R=\sqrt{F_1^2+F_2^2}=\frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}\sqrt{2}$$

Direction of $\vec F_R$ will be along AC ($\vec F_R$, being resultant of forces of equal magnitude, bisects the angle between the two) $\vec F_R$ and $\vec F_3$ are in opposite directions. Net force on Q can be zero if their magnitudes are also equal, i.e.,

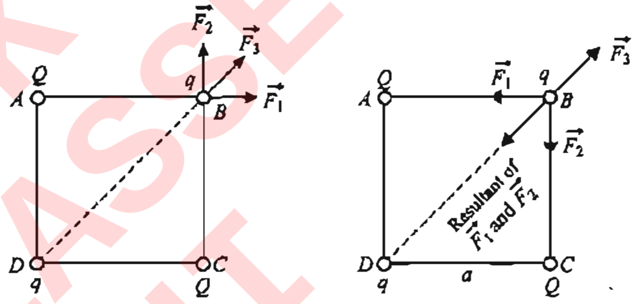
$$\frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}\sqrt{2}=\frac{1}{4\pi\varepsilon_0}\frac{QQ}{2a^2}\text{ or }\frac{Q}{4\pi\varepsilon_0 a^2}\left(\sqrt{2}q-\frac{Q}{2}\right)=0$$

$$\Rightarrow q=\frac{Q}{2\sqrt{2}}\Rightarrow\quad\left|\frac{q}{Q}\right|=\frac{1}{2\sqrt{2}}\quad\{Q\neq 0\}$$

$\therefore$ The sign of q should be negative.

2. Consider now the forces acting on charge q placed at B (see Fig. 1.20(a) and (b)).

In a similar manner, as discussed in 1, for net force on q to be zero, q and Q have to be of opposite signs. This is also shown in the given figures.



(a)
(q and Q are of same sign)
Here, net force cannot be zero.

(b)
(q and Q are of opposite sign)
Here, net force could be zero.

Fig. 1.20

Now, $F_1=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}$ {force of Q at A on q at B}

$F_2=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}$ {force of Q at C on q at B}

$F_3=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a^2}$ {force of q at D on q at B}

Referring to Fig. 1.20(b), let us write $\vec F_R=\vec F_1+\vec F_2$

$$\therefore\quad F_R=\sqrt{F_1^2+F_2^2}=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}\sqrt{2}$$

Resultant of $\vec F_1$ and $\vec F_2$, i.e., $\vec F_R$, is opposite to $\vec F_3$. Net force can become zero if their magnitudes are also equal, i.e.,

$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}\sqrt{2}=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a^2}\Rightarrow\frac{q}{4\pi\varepsilon_0 a^2}\left(\sqrt{2}Q-\frac{q}{2}\right)=0$$

$$\Rightarrow Q=\frac{q}{2\sqrt{2}}\quad\Rightarrow\quad\left|\frac{q}{Q}\right|=2\sqrt{2}\quad\{q\neq 0\}$$

$\therefore$ The sign of 'q' should be negative.

In this case, we need not to repeat the calculation as the present situation is same as previous one; we can directly write $\left|\frac{q}{Q}\right|=2\sqrt{2}$

3. The entire system cannot be in equilibrium since both conditions, i.e., $q=-\frac{Q}{2\sqrt{2}}$ and $Q=-\frac{q}{2\sqrt{2}}$ cannot be satisfied together.

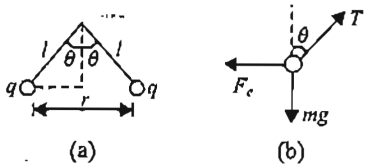
Illustration 1.12 **Two identical small charged spheres, each having a mass m, hang in equilibrium as shown in**

Fig. 1.21(a). The length of each string is l and the angle made by any string with vertical is θ. Find the magnitude of the charge on each sphere.

Sol. The forces acting on the sphere are tension in the string T; force of gravity, mg; repulsive electric force, F_e, as shown in the free body diagram of the sphere (Fig. 1.21(b)). The sphere is in equilibrium. The forces in the horizontal and vertical directions must separately add up to zero..

$$\sum F_x = T\sin\theta - F_e = 0 \qquad \text{(i)}$$

$$\sum F_y = T\cos\theta - mg = 0 \qquad \text{(ii)}$$



(a) (b)

Fig. 1.21

From equation (ii), $T = \dfrac{mg}{\cos\theta}$. Thus, we can eliminate T from equation (i) to obtain

$$F_e = mg\tan\theta \text{ or } \frac{kq^2}{r^2} = mg\tan\theta \qquad \text{(iii)}$$

where $k = \dfrac{1}{4\pi\varepsilon_0}$ and $r = 2l\ \sin\theta$.

The equation (iii) now reduces to $\dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q^2}{(2l\sin\theta)^2} = mg\tan\theta$

or $q = \sqrt{16\pi\varepsilon_0 l^2 mg\ \tan\theta\ \sin^2\theta}$

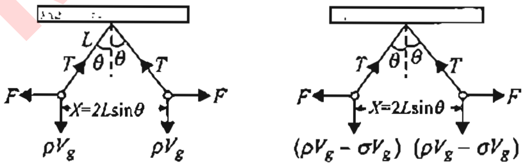
Illustration 1.13 **Two identical balls each having a density ρ are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium, each string makes an angle θ with the vertical. Now, both the balls are immersed in a liquid. As a result, the angle θ does not change. The density of liquid is σ. Find the dielectric constant of the liquid.**

Sol. Let V is the volume of each ball, then mass of each ball:

$$m = \rho V$$

When the balls are in air, from previous problem,

$$F = mg\tan\theta = \rho Vg\tan\theta \qquad \text{(i)}$$



(a) (b)

Fig. 1.22

the balls are suspended in liquid, the Coulombic force is reduced to $F' = F/K$ and apparent weight = weight − upthrust:

$$W' = (\rho Vg - \sigma Vg).$$

According to the problem, angle θ is unchanged. So,

$$F' = W'\tan\theta = (\rho Vg - \sigma Vg)\tan\theta \qquad \text{(ii)}$$

From equations (i) and (ii), we get

$$\frac{F}{F'} = K = \frac{\rho Vg}{\rho Vg - \sigma Vg} = \frac{\rho}{\rho - \sigma}$$

Illustration 1.14 **Three particles, each of mass 'm' and carrying a charge q each, are suspended from a common point by insulating massless strings, each of length 'L'. If the particles are in equilibrium and are located at the corners of an equilateral triangle of side 'a', calculate the charge q on each particle. Assume $L >> a$.**

Sol. From Fig. 1.23(b), for equilibrium of a particle along a vertical line,

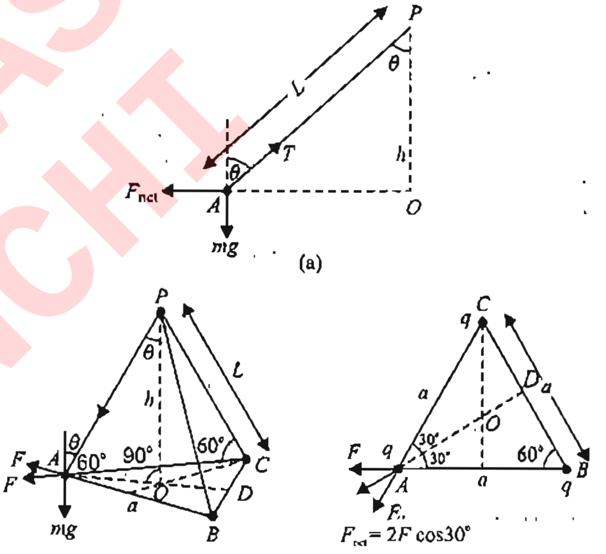
$$T\cos\theta = mg \qquad \text{(i)}$$

While for equilibrium in the plane of equilateral triangle,

$$T\sin\theta = 2F\cos 30^\circ \qquad \text{(ii)}$$

So, from equations (i) and (ii), we have

$$\tan\theta = \frac{\sqrt{3}F}{mg} \qquad \text{(iii)}$$



(b) (c)

Fig. 1.23

Here, $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{a^2}$ and $\tan\theta = \dfrac{OA}{OP} = \dfrac{OA}{\sqrt{L^2 - OA^2}}$

Also, from Fig. 1.23(c)

$$OA = \frac{2}{3}AD = \frac{2}{3}a\sin 60^\circ = \frac{a}{\sqrt{3}}$$

$$\text{So, } \tan\theta = \frac{\left(a/\sqrt{3}\right)}{\sqrt{L^2 - (a^2/3)}} = \frac{a}{\left(\sqrt{3}\right)L}\ \{\text{as } L >> a\}$$

On substituting the above values of F and $\tan\theta$ in equation (iii), we get:

$$\frac{a}{\left(\sqrt{3}\right)L} = \frac{\sqrt{3}}{mg}\frac{q^2}{4\pi\varepsilon_0 a^2}, \text{ i.e., } q = \left[\frac{4\pi\varepsilon_0 a^3 mg}{3L}\right]^{1/2}$$

Illustration 1.15 **A thin fixed ring of radius 'a' has a positive charge 'q' uniformly distributed over it. A particle of mass 'm' and having a negative charge 'Q' is placed on the**

axis at a distance of x ($x << a$) from the centre of the ring. Show that the motion of the negatively charged particle is approximately simple harmonic. Calculate the time period of oscillation.

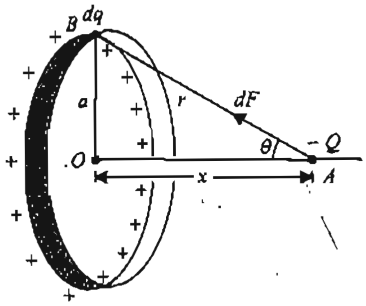


Fig. 1.24

Sol. The force on the point charge Q due to the element dq of the ring

$$dF = \frac{1}{4\pi\varepsilon_0}\frac{dq\,Q}{r^2} \text{ along } AB$$

As for every element of the ring there is symmetrically situated diametrically opposite element, the components of forces along the axis will add up while those perpendicular to it will cancel each other. Hence, net force on the charge $-Q$ is

$$F = \int dF\cos\theta = \cos\theta\int dF;$$

$$F = \frac{x}{r}\int\frac{1}{4\pi\varepsilon_0}\left[-\frac{Qdq}{r^2}\right]$$

So, $$F = -\frac{1}{4\pi\varepsilon_0}\frac{Qx}{r^3}\int dq = -\frac{1}{4\pi\varepsilon_0}\frac{Qqx}{(a^2+x^2)^{3/2}} \qquad \text{(i)}$$

$$\left\{\text{as } r = (a^2+x^2)^{1/2} \text{ and } \int dq = q\right\}$$

−ve sign shows that this force will be towards the centre of ring.

As the restoring force is not linear, the motion will be oscillatory. However, if $x \ll a$ so that $x^2 \ll a^2$,

$$F = -\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^3}x = -kx \text{ with } k = \frac{Qq}{4\pi\varepsilon_0 a^3}$$

i.e., the restoring force will become linear and so the motion is simple harmonic with time period

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4\pi\varepsilon_0 m a^3}{qQ}}$$

Illustration 1.16 **The field lines for two point charges are shown in Fig. 1.25.**

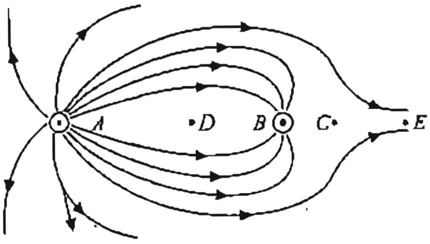


Fig. 1.25

1. **Is the field uniform?**
2. **Determine the ratio $\frac{q_A}{q_B}$.**
3. **What are the sign of q_A and q_B?**
4. **Apart from infinity, where is the neutral point?**
5. **If q_A and q_B are separated by a distance $10(\sqrt{2}-1)$ cm, find the position of neutral point.**
6. **Where will the lines which are not meeting at q_B meet?**
7. **Will a positive charge follow the line of force if free to move?**

Sol.

1. No.
2. Number of lines coming from or coming to a charge is proportional to magnitude of charge, so
$$\frac{q_A}{q_B} = \frac{12}{6} = 2$$
3. q_A is positive and q_B is negative.
4. C is the other neutral point.
5. For neutral point $E_A = E_B$
$$\frac{1}{4\pi\varepsilon_0}\frac{q_A}{(l+x)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_B}{x^2}$$

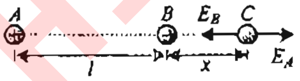


Fig. 1.26

$$\left(\frac{l+x}{x}\right)^2 = \frac{q_A}{q_B} = 2 \quad \Rightarrow \quad x = 10 \text{ cm}$$

6. At infinity.
7. No, as lines of force are curved, the direction of velocity and acceleration will be different. Hence, a charge cannot follow strictly the line of force.

Concept Application Exercise 1.2

1. **a.** A negatively charged particle is placed exactly midway between two fixed particles having equal positive charges. What will happen to the charge:
   - **i.** if it is displaced at right angle to the line joining the positive charges?
   - **ii.** if it is displaced along the line joining the positive charges?

   b. Does the Coulomb force that one charge exerts on other charges change if the other charges are brought nearby? (Yes/No)
2. **a.** Does an electric charge experience a force due to the field produced by itself? (Yes/No)

   b. Two point charges q and $-q$ are placed at a distance d apart. What are the points at which resultant electric field is parallel to line joining the two charges?
3. Two negative charges of a unit magnitude and a positive charge 'q' are placed along a straight line. At what position and value of q will the system be in equilibrium? (Negative charges are fixed.)

4. Fig. 1.27 shows three arrangements of an electron e and two protons p $(D > d)$.

a. Rank the arrangements according to the magnitude of the net electrostatic force on the electron due to the protons, largest first.

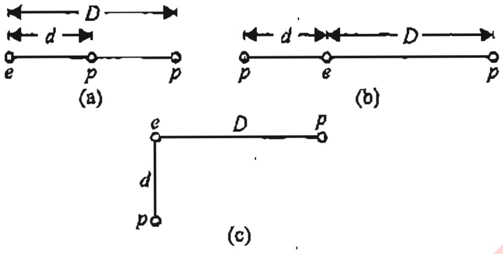


Fig. 1.27

b. In situation c, is the angle between the net force on the electron and the line labeled horizontal less than or more than 45°?

5. Fig. 1.28 shows two charge particles on an axis. The charges are free to move. At one point, however, a third charged particle can be placed such that all three particles are in equilibrium.

Fig. 1.28

a. Is that point to the left of the first two particles, to their right, or between them?

b. Should the third particle be positively or negatively charged?

c. Is the equilibrium stable or unstable?

6. In Fig. 1.29, a central particle of charge $-q$ is surrounded by two circular rings of charged particles, of radii r and R, with $R > r$. What is the magnitude and direction of the net electrostatic force on the central particle due to the other particles?

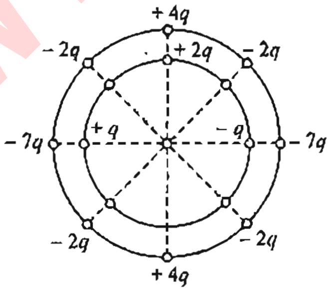


Fig. 1.29

7. Fig. 1.30 shows four situations in which particles of charge $+q$ or $-q$ are fixed in place. In each, the particles on the x-axis are equidistant from the y-axis. The particle on y-axis experiences an electrostatics force F from each of these two particles.

a. Are the magnitudes F of those forces the same or different?

b. Is the magnitude of the net force on the particle on y-axis equal to, greater than, or less than $2F$?

c. Do the x components of the two forces add or cancel?

d. Do their y components add or cancel?

e. Is the direction of the net force on the middle particle that of the canceling components or the adding components?

f. What is the direction of that net force on the middle particle?

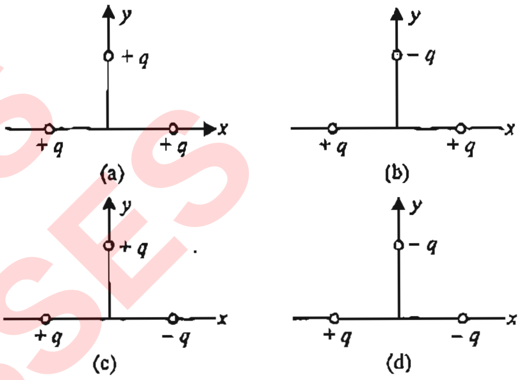


Fig. 1.30

8. Force between two point electric charges kept at a distance 'd' apart in air is F. If these charges are kept at the same distance in water, the force between the charges is F'. The ratio F'/F is equal to ________.

9. Two small balls each having charge q are suspended by two insulating threads of equal length L from a hook in an elevator. The elevator is freely falling. Calculate the angle between the two threads and tension in each thread.

10. Suppose we have a large number of identical particles, very small in size. Any two of them at 10 cm separation repel with a force of 3×10^{-10} N.

a. If one of them is at 10 cm from a group (of very small size) of n others, how strongly do you expect it to be repelled?

b. Suppose you measure the repulsion and find it 6×10^{-6} N. How many particles were there in the group?

ELECTRIC FIELD

If we place a single charge q at some point in space, it will experience no force. But if some other charge (say Q) is placed near it, q will start experiencing a force given by

$$F = \frac{kQq}{r^2}$$

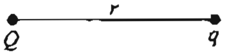


Fig. 1.31

Now, question arises, how does Q apply a force on q or how does q know the presence of Q when there is no direct contact between them.

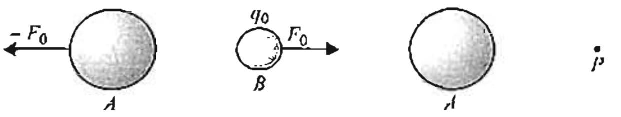
Basically, the force between two charges can be seen as a two step process:

1. Firstly, charge Q will create something around itself known as electric field.

2. Any other charge particle like q if placed at some point in that field will experience a force.

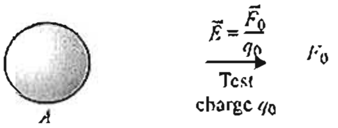
Or we can say that charges interact with each other through electric field.

So, we can define electric field as the space around a charge in which its influence can be felt by any other charged particle.



(a) How does charged body A exert a force on charged body B?

(b) Remove body B and label its former position as P.



(c) Body A sets up an electric field E at point P. E is the force per unit charge exerted.

Fig. 1.32

How to Measure Electric Field

Strength of electric field at a point in space can be measured in two measureable quantities:

1. Electric field intensity denoted by E. It is a vector quantity.
2. Electric field potential denoted by V. It is a scalar quantity.

We will first discuss them separately and then we will see what is the relation between them and how to obtain them from each other.

Electric Field Intensity E

How to find electric field intensity E at a point?

General method: Electric field intensity, E, is a vector quantity. At a point in a given space it has both magnitude and direction. Let us calculate E at some point P created due to some charges around P. Bring another small charge q_0 [test charge, generally positive] at point P. Let this charge experiences a force $\vec{F}$, then we define electric field intensity at P as force experienced per unit test charge (Fig. 1.33).

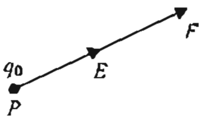


Fig. 1.33

$$\vec{E}=\lim_{q_0\to 0}\frac{\vec{F}}{q_0}.$$ The direction $\vec{E}$ will be same as that of $\vec{F}$.

Note: *Q. Why the magnitude of test charge is kept small?*
Ans. *Because otherwise it may disturb the original charge distribution and then we will get electric field due to disturbed configuration and not original.*
Q. What is the minimum possible value of q_0?
Ans. 1.6×10^{-19} C

Unit of E: N/C (newton per coulomb)

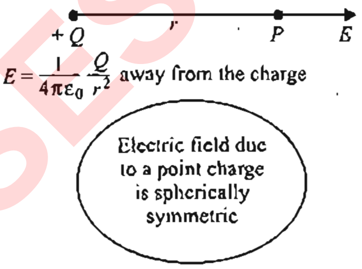
Dimensional formula of E: $\dfrac{\text{Force}}{\text{Charge}}=\dfrac{MLT^{-2}}{\text{ampere}\times\text{time}}$

$$=\frac{MLT^{-2}}{AT}=\left[MLT^{-3}A^{-1}\right]$$

Note: ***If a test charge experiences no force at a point, the electric field at that point must be zero.***

Electric field due to a point charge is illustrated in Fig. 1.34.

(i) Positive point charge



(ii) Negative point charge

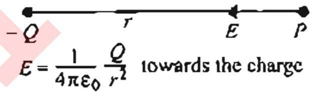


Fig. 1.34

A Point Charge in an Electric Field

What happens if a point charge q is placed at any point in an electric field which is produced by some other stationary charges. Let this electric field is $\vec{E}$. Charge q will experience a force, let this force is $\vec{F}$. Then, value of electric field at that point must be

$\vec{E}=\dfrac{\vec{F}}{q}\quad\Rightarrow\quad\vec{F}=q\vec{E}$. This is the force on q by E.

Direction of $\vec{F}$: The direction of $\vec{F}$ will be same as of $\vec{E}$ if q is +ve and opposite if q is –ve (Fig. 1.35).

(a) Positive charge q_0 placed in an electric field: force on q_0 is in same direction as $\vec{E}$.

(b) Negative charge q_0 placed in an electric field: force on q_0 is in opposite direction as $\vec{E}$.

Fig. 1.35

Note: *q has no contribution in $\vec{E}$. A charge particle is not affected due to its own field. It means a charge particle can experience force due to field produced by other charge particles, but not due to field produced by itself.*

Electric Field Intensity due to a Point Charge in Position Vector Form

Electric field at P due to charge Q: $\vec{E}=\dfrac{Q\left(\vec{r}-\vec{r}_0\right)}{4\pi\varepsilon_0\left|\vec{r}-\vec{r}_0\right|^3}$

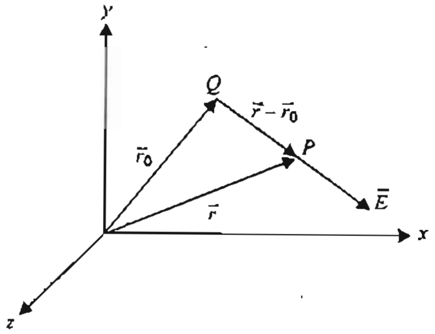


Fig. 1.36

If a charge q is placed at P, then force on this charge by Q:

$$\vec{F} = q\vec{E}$$

$$\Rightarrow \quad \vec{F} = \frac{qQ(\vec{r}-\vec{r}_0)}{4\pi\varepsilon_0 |\vec{r}-\vec{r}_0|^3}$$

Electric Field Intensity due to a Group of Charges

Using the principle of superposition, net field at point P (see Fig. 1.37)

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n$$

$$\Rightarrow \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}\hat{r}_1 + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}\hat{r}_2 + \cdots + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n^2}\hat{r}_n$$

$$\Rightarrow \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$$

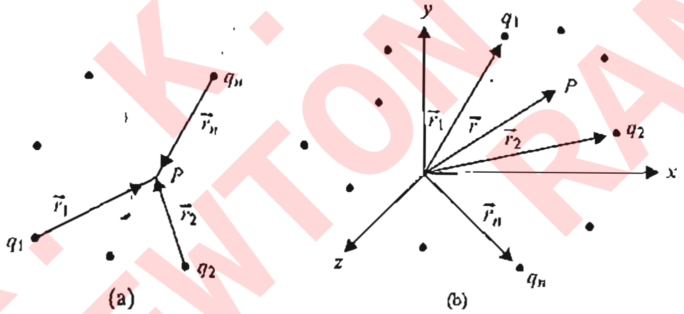


Fig. 1.37

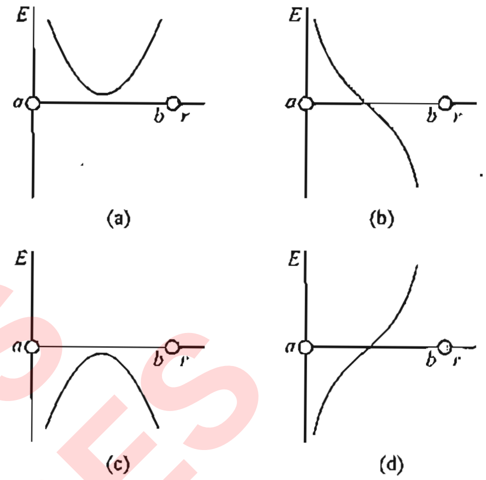
In terms of position vectors:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}$$

Illustration 1.17 **Two point-like charges a and b whose strengths are equal in absolute value are positioned at a certain distance from each other. Assuming the field strength is positive in the direction coinciding with the positive direction of the r axis, determine the signs of the charges for each distribution of the field strength between charges shown in Figs. 1.38(a), (b), (c) and (d).**

Sol.

a. As electric field tends away at a and towards at b, hence there should be + charge at a and negative charge at b, i.e., q_a is '+' and q_b is '–'.



(a) (b) (c) (d)

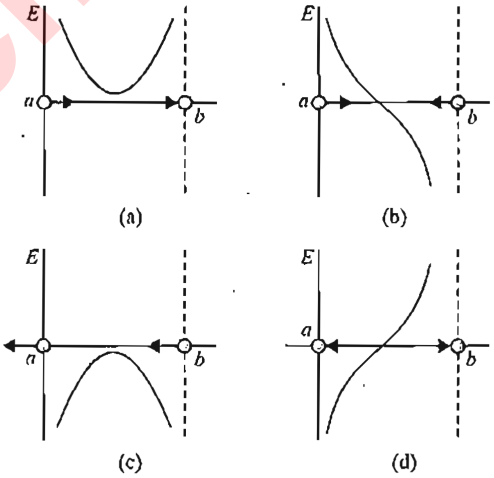
Fig. 1.38

b. The neutral point exists between a and b only when q_a and q_b both are of same sign. As direction of electric field is away from both, so both charges are positive, i.e., q_a is '+' and q_b is '+'.

Similarly, for (c) and (d) in Fig. 1.39:

c. q_a is '–' and q_b is '+'.

d. q_a is '–' and q_b is '–'.



(a) (b) (c) (d)

Fig. 1.39

Illustration 1.18 **Two identical positive point charges q are placed on the axis at $x = -a$ and $x = +a$, as shown in Fig. 1.40.**

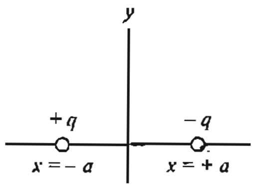


Fig. 1.40

1. **Plot the variation of E along the x-axis.**
2. **Plot the variation of E along the y-axis**

Sol.

1. Variation of E along the x-axis: 1.41(a).
2. Variation of E along the y-axis: 1.41(b)

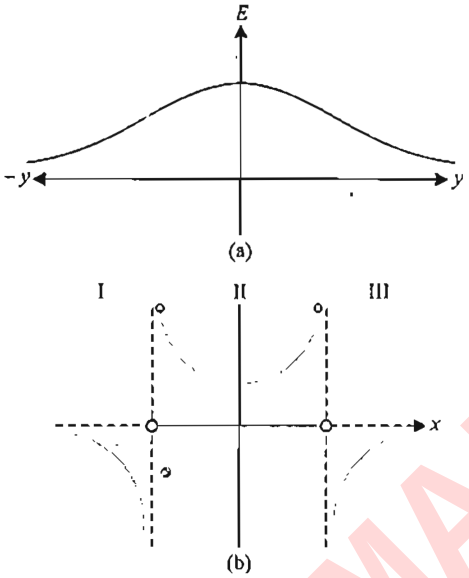


Fig. 1.41

Illustration 1.19 **In Fig. 1.42, determine the point (other than infinity) at which the electric field is zero.**

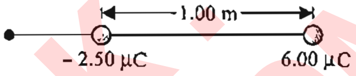


Fig. 1.42

Sol. Electric field will be zero at a point closer to the charge smaller in magnitude. Let at P electric field is zero (see Fig. 1.43). Then

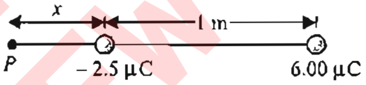


Fig. 1.43

$$\frac{k(2.5\times10^{-6})}{x^2}=\frac{k(6\times10^{-6})}{(1+x)^2}$$

$$\Rightarrow \qquad x = 1.82\text{ m}$$

Illustration 1.20 **Four charges are arranged as shown in Fig. 1.44. A point P is located at distance r from the centre of the configuration. Assuming $r >> l$, find**

1. **the magnitude of the field at point P.**
2. **the angle of its vector with x-axis.**

Sol. Electric field due to charges placed on y-axis (Fig. 1.45(a))

$$E_y = 2E_1\sin\theta = 2\,\frac{1}{4\pi\varepsilon_0}\,\frac{q}{\left(r^2+\left(\frac{l}{2}\right)^2\right)}\,\frac{l/2}{\left(r^2+\frac{l^2}{4}\right)^{1/2}}$$

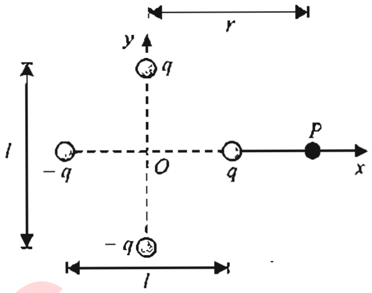


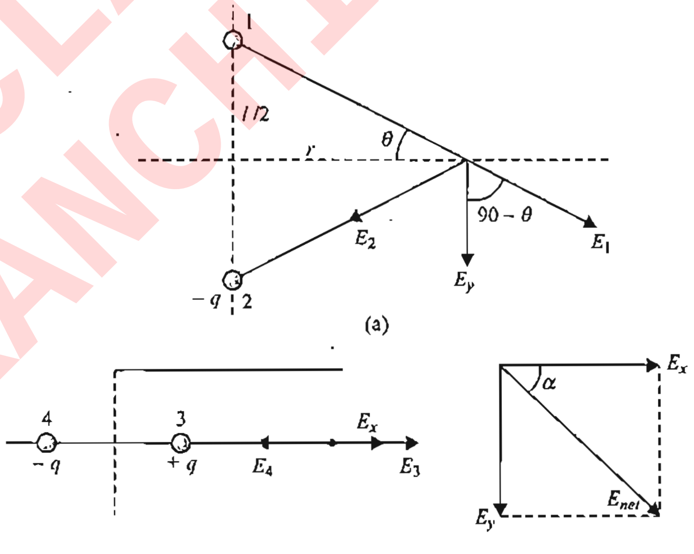
Fig. 1.44

$$E_y=\frac{1}{4\pi\varepsilon_0}\,\frac{q\,l}{\left(r^2+\frac{l^2}{4}\right)^{3/2}}=\frac{1}{4\pi\varepsilon_0}\,\frac{ql}{r^3}\ (\text{as } r>>l)$$

Electric field due to charges placed on x-axis (Fig. 1.45(b))

$$E_x=E_3-E_4=\frac{1}{4\pi\varepsilon_0}\,\frac{q}{\left(r-\frac{l}{2}\right)^2}-\frac{1}{4\pi\varepsilon_0}\,\frac{q}{\left(r+\frac{l}{2}\right)^2}$$

$$=\frac{1}{2\pi\varepsilon_0}\,\frac{ql}{r^3}$$



(b) (c)

Fig. 1.45

$$\Rightarrow \qquad E_{\text{net}}=\sqrt{E_x^2+E_y^2}=\sqrt{5}\,\frac{ql}{4\pi\varepsilon_0 r^3}$$

The angle E_{net} makes with x-axis (Fig. 1.45(c))

$$\alpha=\tan^{-1}\left(\frac{E_y}{E_x}\right)=\tan^{-1}\left(\frac{1}{2}\right)\text{ below } x\text{-axis.}$$

Illustration 1.21 **A uniform electric field E exists between two metal plates. The plate length is l and the separation of the plates is d.**

1. **An electron and a proton start from the negative plate and positive plate, respectively, and go to opposite plates. Which one of them wins this race?**

2. An electron and a proton start moving parallel to the plates towards the other end from the midpoint of the separation of plates at one end of the plates. Which of the two will have greater deviation when they come out of the plates if they start with the

a. same initial velocity,

b. same initial kinetic energy, and

c. same initial momentum.

Sol.

1. $a_e = \frac{eE}{m_e}, a_p = \frac{eE}{m_p}; d = \frac{1}{2}at^2$

$$t = \sqrt{\frac{2d}{a}} \text{ or } t = \sqrt{\frac{2md}{eE}}; \text{ So, we have } \frac{t_e}{t_p} = \sqrt{\frac{m_e}{m_p}}.$$

As $m_e < m_p$, therefore $t_e < t_p$. Hence, electron will take less time, i.e., the electron wins the race.

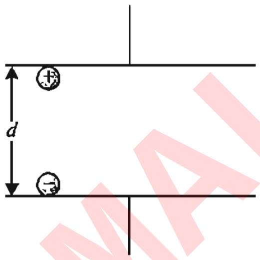


Fig. 1.46

2. Time to cross the plates $t = \frac{l}{u}$

$$\text{Deviation: } y = \frac{1}{2}at^2 = \frac{1}{2}\frac{eE}{m}\left(\frac{l}{u}\right)^2$$

$$\frac{y_e}{y_p} = \frac{m_p}{m_e} \cdot \left(\frac{u_p}{u_e}\right)^2 \quad \text{(i)}$$

a. If $u_p = u_e$, then $\frac{y_e}{y_p} = \frac{m_p}{m_e}$.

As $m_p > m_e$, therefore $y_e > y_p$.

Hence, deviation of electron will be more.

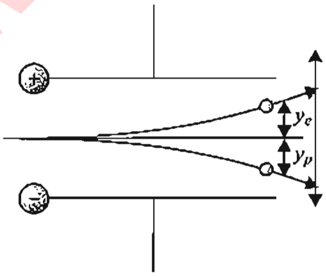


Fig. 1.47

b. From equation (i), $\frac{y_e}{y_p} = \frac{m_p u_p^2}{m_e u_e^2} = 1$ (as given)

Hence deviation of both electron and proton will be same.

c. From (i), $\frac{y_e}{y_p} = \left(\frac{m_p u_p}{m_e u_e}\right)^2 \frac{m_e}{m_p} = \frac{m_e}{m_p}$

As $m_e < m_p$, hence $y_e < y_p$.

Hence, the deviation of proton will be more.

Illustration 1.22 **A charge 10^{-9} coulomb is located at gin in free space and another charge Q at (2, 0, 0). If x-component of the electric field at (3, 1, 1) is zero, calcu the value of Q. Is the y-component zero at (3, 1, 1)?**

Sol. The electric field due to a point charge q_i at position vector from is given by

$$\vec{E}_i = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_i^3}\vec{r}_i$$

Here: $\vec{r}_1 = (3-0)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k} = 3\hat{i} + \hat{j} +$

with $r_1 = \sqrt{(3^2 + 1^2 + 1^2)} = \sqrt{11}$ m

$$\vec{r}_2 = (3-2)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k} = \hat{i} + \hat{j} + \hat{k}$$

with $r_2 = \sqrt{(1^2 + 1^2 + 1^2)} = \sqrt{3}$ m

So,

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{10^{-9}}{(11)^{3/2}}[3\hat{i} + \hat{j} + \hat{k}] \text{ and}$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(3)^{3/2}}[\hat{i} + \hat{j} + \hat{k}]$$

Hence, net field:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\left[\left(\frac{3 \times 10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}}\right)\hat{i} + \left(\frac{10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}}\right)\hat{j} + \left(\frac{10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}}\right)\hat{k}\right]$$

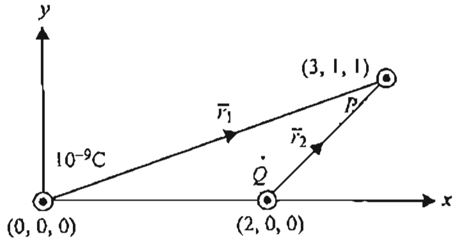


Fig. 1.48

According to given problem:

$$E_x = 0, \text{ i.e., } \frac{1}{4\pi\varepsilon_0}\left[\frac{3 \times 10^{-9}}{11\sqrt{11}} + \frac{Q}{3\sqrt{3}}\right] = 0$$

$$\text{So, } Q = -\left[\frac{3}{11}\right]^{3/2} \times 3 \times 10^{-9} \text{ coulomb}$$

And for this value of Q

$$E_y = \frac{1}{4\pi\varepsilon_0}\left[\frac{10^{-9}}{11\sqrt{11}} - \frac{(3/11)^{3/2} \times 3 \times 10^{-9}}{3\sqrt{3}}\right]$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{2 \times 10^{-9}}{11\sqrt{11}} \neq 0, \text{ i.e., } E_y \text{ is not zero.}$$

Lines of Force

This idea was given by Michael Faraday. The lines of provide a nice idea to visualise the pattern of electric fie a given space. We assume that space around a charged bo filled with some lines known as electric lines of force.

lines of force are drawn in space in such a way that tangent to the line at any point gives the direction of electric field at that point. It has been found quite convenient to visualize the electric field in terms of lines of force.

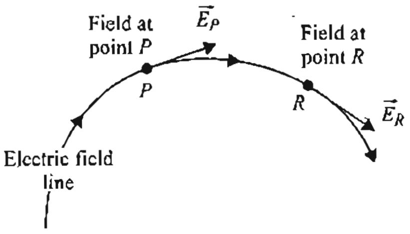


Fig. 1.49

Properties of Electric Lines of Force

- Electric lines of force start (or diverge out) from a positive charge and end (or converge) on a negative charge.
- The tangent drawn at any point on the line of force gives the direction of force acting on a positive charge at that point (see Fig. 1.50).
- In S.I. system of units, the number of electric lines of force originating or terminating on a charge of q coulomb is equal to $\frac{q}{\varepsilon_0}$, i.e., $\left(\frac{q}{\varepsilon_0}\right)$ electric lines are associated with unit charge.

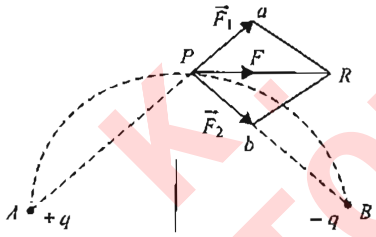


Fig. 1.50

- Two electric lines of force never cross each other, because if they do so then at the point of intersection, intensity will have two directions which is absurd.
- Electric lines of force can never be closed loops, as a line can never start and end on the same charge.
- The electric lines of force do not pass through a conductor as electric field inside a conductor is always zero.
- Lines of force have a tendency to contract longitudinally like a stretched elastic string producing attraction between opposite charges and repel each other laterally resulting in repulsion between similar charges and edge effect (curving of lines of force near the edges of a charged conductor).
- Electric lines of force end or start normally on the surface of a conductor.
- Tangent to the line of force at a point in an electric field gives the direction of intensity or force or acceleration which a positive charge will experience there but not the direction of motion always, so a positive point charge free to move in an electric field may or may not follow the line of force. It will follow the line of force if it is a straight line (as direction of velocity and acceleration will be same) and will not follow the line if it is curved as the direction of motion will be different from that of acceleration and the particle will move neither in the direction of motion nor acceleration (line of force).

The use of the electric lines of force is that we can compare the intensities at two points just by looking at the distribution of lines of force. Where the field lines are close together, E is large and where they are far apart, E is small.

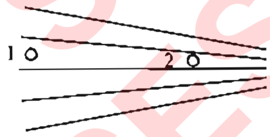
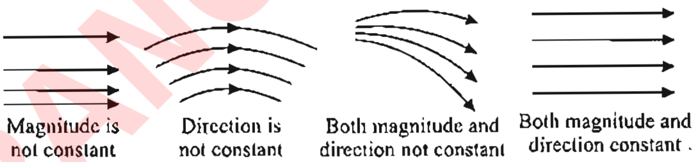
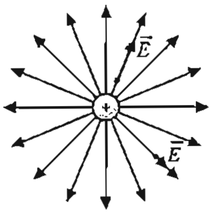


Fig. 1.51

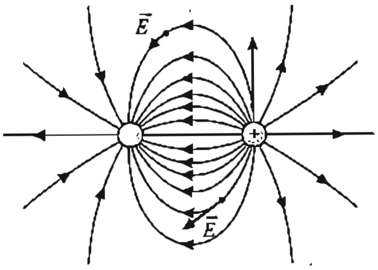
As an example in the figure electric lines of forces are shown. At point 2 the electric field intensity will be greater in comparison to that at point 1.

DIFFERENT PATTERNS OF ELECTRIC FIELD LINES

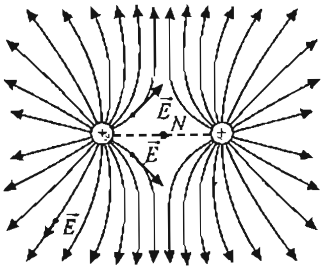

Magnitude is not constant
Direction is not constant
Both magnitude and direction not constant
Both magnitude and direction constant
(a)



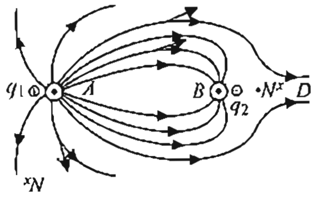
(b) A single positive charge



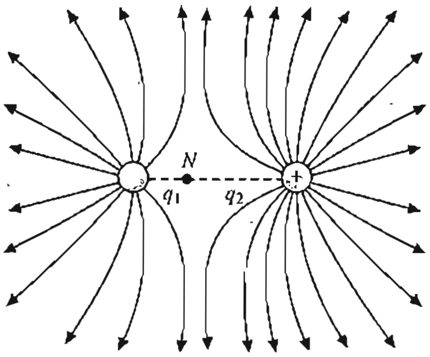
(c) A positive charge and a negative charge of equal magnitude (an electric dipole)



(d) Two equal positive charges. N is the neutral point lying at the middle of the charges.



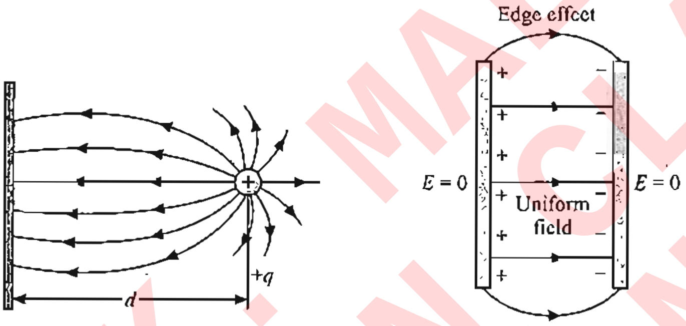
(e) A is a positive charge and B a negative charge of different magnitudes ($|q_2| < |q_1|$)



(f) Two positive charges of different magnitudes ($q_1 < q_2$)

Fig. 1.52

Note: ***Neutral point (N) is the location where the net electric field due to charges is zero. It lies near the charge of smaller magnitude.***



Parallel metal plates having dissimilar charges

Fig. 1.53

Illustration 1.23 **Fig. 1.54 shows the sketch of field lines for two point charges $2Q$ and $-Q$. The pattern of field lines can be deduced by considering the following points:**

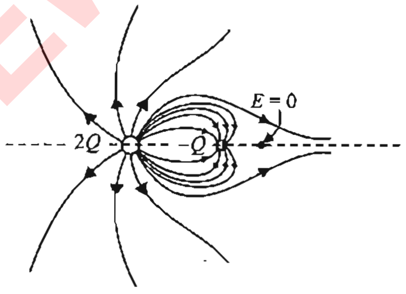


Fig. 1.54

Sol.

1. Symmetry: For every point above the line joining the two charges, there is an equivalent point below it. Therefore, the pattern must be symmetrical about the line joining the two charges.
2. Near field: Very close to a charge, its field predominates. Therefore, the lines are radial and spherically symmetric.
3. Far field: Far from the system of charges, the pattern should look like that of a single point charge of value $(2Q - -Q) = +Q$, i.e., the lines should be radially outward.
4. Null point or neutral point: There is one point at which $E = 0$. No lines should pass through this point.
   - Neutral point lies near the position of charge of smaller magnitude.
5. Number of lines: Twice as many lines leave $+2Q$ as enter $-Q$.

Note: ***Excess lines from 2Q charge will meet at infinity.***

Illustration 1.24 **Charges $+q$ and $-2q$ are fixed a distance d apart as shown in figure.**

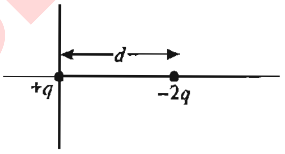


Fig. 1.55

1. **Sketch roughly the pattern of electric field lines, showing position of neutral point.**
2. **Where should a charge particle q be placed so that it experiences no force?**

Sol. Let net force on q at P is zero, then

$$\frac{kq^2}{x^2} = \frac{kq\,2q}{(d+x)^2} \Rightarrow x = \frac{d}{\sqrt{2} -}$$

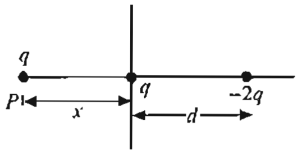


Fig. 1.56

P is the neutral potential where electric field will be zero.

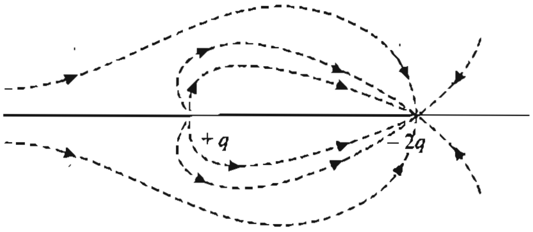


Fig. 1.57

FIELD OF RING CHARGE

A ring-shaped conductor with radius a carries a total charge Q uniformly distributed around it. Let us calculate the electric field at a point P that lies on the axis of the ring at a distance x from its center.

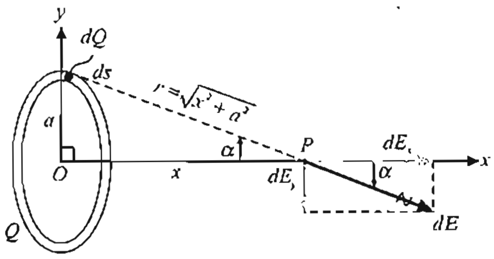


Fig. 1.58

As shown in the figure, we imagine the ring divided into infinitesimal segments each of length ds. Each segment has charge dQ and acts as a point charge source of electric field. Let $d\vec{E}$ be the electric field from one such segment; the net electric field at P is then the sum of all contributions $d\vec{E}$ from all the segments that make up the ring. (This same technique works for any situation in which charge is distributed along a line or a curve.) The calculation of $\vec{E}$ is greatly simplified because the field point P is on the symmetry axis of the ring. If we consider two ring segments at the top and bottom of the ring, we see that the contributions $d\vec{E}$ to the field at P from these segments have the same x-component but opposite y-components. Hence, the total y-component of field due to this pair of segments is zero. When we add up the contributions from all such pairs of segments, the total field $\vec{E}$ will have only a component along the ring's symmetry axis (the x-axis), with no component perpendicular to that axis (that is, no y-component or z-component). So, the field at P is described completely by its x-component E_x.

To calculate E_x note that the square of the distance r from a ring segment to the point P is $r^2 = x^2 + a^2$. Hence, the magnitude of this segment's contribution to the electric field at P is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2+a^2}$$

Using $\cos\alpha = \frac{x}{r} = \frac{x}{(x^2+a^2)^{1/2}}$, the component dE_x of this field along the x-axis is

$$dE_x = dE\cos\alpha = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}} = \frac{1}{4\pi\varepsilon_0}\frac{x\,dQ}{(x^2+a^2)^{3/2}}$$

To find the total x-component E_x of the field at P, we integrate this expression over all segments of the ring:

$$E_x = \int \frac{1}{4\pi\varepsilon_0}\frac{x\,dQ}{(x^2+a^2)^{3/2}}.$$

Since x does not vary as we move from point to point around the ring, all the factors on the right side except dQ are constant and can be taken outside the integral. The integral of dQ is just the total charge Q and we finally get

$$\vec{E} = E_x\hat{i} = \frac{1}{4\pi\varepsilon_0}\frac{xQ}{(x^2+a^2)^{3/2}}\,\hat{i} \qquad \text{(i)}$$

- Electric field is directed away from positively charged ring.
- For $x = 0$, $E = 0$. This conclusion may be arrived at by the symmetry consideration.
- At a large distance from the ring, the electric field will be zero. Hence, it should have certain maximum value between $x = 0$ and $x = \infty$ (or $x = -\infty$).
- If we maximize equation (i), we can get the value of x_m as well as E_{max}.

For maximum value of E_x:

$$\frac{d}{dx}\left\{\frac{1}{4\pi\varepsilon_0}Q\frac{x}{(x^2+a^2)^{3/2}}\right\} = 0$$

$$\frac{(x^2+a^2)^{3/2}.1 - x\,\frac{3}{2}.(x^2+a^2)^{1/2}.2x}{(x^2+a^2)^3} = 0$$

$$(x^2+a^2) - 3x^2 = 0 \quad \Rightarrow \quad x = \pm\frac{a}{\sqrt{2}}$$

and the maximum value of the electric field is

$$E_{a(\text{max})} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{3\sqrt{3}\,R^2}\right)$$

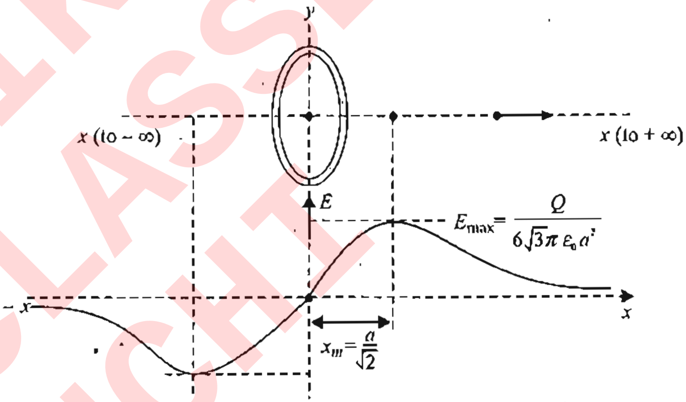


Fig. 1.59

Illustration 1.25 **If we place a negative charge (of magnitude $-q$ and mass m) at the center of a charged ring and slightly displace it along the axis of ring and release. Examine whether it will perform simple harmonic motion. If yes, then find the time period of oscillation of the particle.**

Sol. $E = \frac{kQx}{(a^2+x^2)^{3/2}}$

Force on charge $F = -qE = -\frac{kqQx}{(a^2+x^2)^{3/2}}$

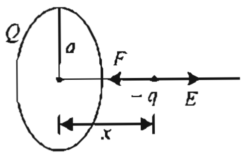


Fig. 1.60

$$a = \frac{F}{m} = \frac{-kqQx}{m(a^2+x^2)^{3/2}}$$

Hence, acceleration is opposite to displacement, so motion will be oscillatory.

But a is not directly proportional to x so motion is not SHM.

If $x << a$, then $a = -\frac{kqQx}{ma^3}$

Here $a \propto x$, so the motion will be SHM. Comparing with $a = -\omega^2 x$

We get $\omega = \sqrt{\frac{kqQ}{ma^3}}$

Illustration 1.26 **Two identical point charges having magnitude q each are placed as shown in the figure.**

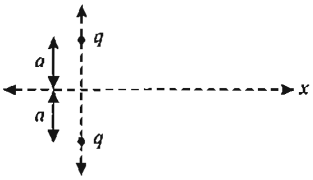


Fig. 1.61

1. **Plot the variation of electric field on x-axis.**
2. **Where will the magnitude of electric field be maximum on x-axis? Find the maximum value of electric field on x-axis.**
3. **If we place a negative charge (of magnitude $-q$ and mass m) at the mid point of charges and displaced along x-axis, examine whether it will perform simple harmonic motion. If yes, then find the time period of oscillation of the particle.**

Sol. 1.

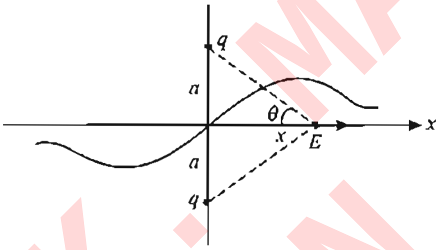


Fig. 1.62

2. Field at $x = x$: $E = 2\left[\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{(a^2+x^2)}\right]\cos\theta$

$$\Rightarrow \quad E = \frac{q}{2\pi\varepsilon_0}\frac{x}{(a^2+x^2)^{3/2}}$$

For E to be maximum, $\dfrac{dE}{dx} = 0$

Solve to get $x = \pm\dfrac{a}{\sqrt{2}} \quad \Rightarrow \quad E_{\max} = \dfrac{q}{3\sqrt{3}\pi\,\varepsilon_0 a^2}$

3. Force on particle: $F = -qE = \dfrac{-q^2}{2\pi\varepsilon_0}\dfrac{x}{(a^2+x^2)^{3/2}}$

For $x << a$, particle will execute SHM with time period

$$T = 2\pi\sqrt{\frac{2\pi\,\varepsilon_0 m a^3}{q^2}}$$

Positive electric charge Q is distributed uniformly along a line, lying along the y-axis. Let us find the electric field at point D on the x-axis at a distance r_0 from the origin.

We divide the line charge into infinitesimal segments, each of which acts as a point charge; let the length of a typical segment at height l be dl. If the charge is distributed uniformly with the linear charge density λ, then the charge dQ in a segment of length dl is $dQ = \lambda\,dl$. At point D, the differential electric field dE created by this element,

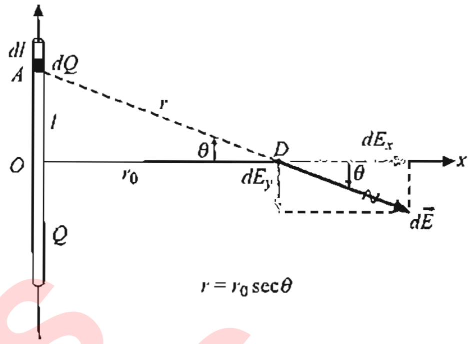


Fig. 1.63

$$dE = \frac{dQ}{4\pi\varepsilon_0 r^2} = \frac{\lambda\,dl}{4\pi\varepsilon_0 r^2} = \frac{\lambda\,dl}{4\pi\varepsilon_0 r_0^2\sec^2\theta} \qquad \text{(i)}$$

In triangle AOD; $OA = OD\tan\theta$, i.e.,

$l = r_0\tan\theta$; Differentiating this equation with respect to θ; $dl = r_0\sec^2\theta\,d\theta$

Substituting the value of dl in equation (i);

$$dE = \frac{\lambda\,d\theta}{4\pi\varepsilon_0 r_0}$$

Field dE has components dE_x, dE_y given by

$$dE_x = \frac{\lambda\cos\theta\,d\theta}{4\pi\varepsilon_0 r_0} \text{ and } dE_y = \frac{\lambda\sin\theta\,d\theta}{4\pi\varepsilon_0 r_0}$$

On integrating expression for dE_x and dE_y in limits $\theta = -\dfrac{\pi}{2}$ to $\theta = +\dfrac{\pi}{2}$, we obtain E_x and E_y. Note that as the length of wire increases, the angle θ increases; for a very long wire (infinitely long wire), it approaches $\pi/2$.

$$E_x = \int_{-\pi/2}^{+\pi/2}\frac{\lambda\cos\theta\,d\theta}{4\pi\varepsilon_0 r_0} = \frac{\lambda}{2\pi\varepsilon_0 r_0} \text{ and}$$

$$E_y = \int_{-\pi/2}^{+\pi/2}\frac{\lambda\sin\theta\,d\theta}{4\pi\varepsilon_0 r_0} = 0$$

Thus, $E = E_x = \dfrac{\lambda}{2\pi\varepsilon_0 r_0}$

Note: *Using a symmetry argument, we could have guessed that E_y would be zero; if we place a positive test charge at D, the upper half of the line of charge pushes downward on it, and the lower half pushes up with equal magnitude.*

- If the wire has finite length and the angle subtended by ends of wire at a point are θ_1 and θ_2, the limits of integration would change.

$$E_x = \int_{-\theta_1}^{+\theta_2}\frac{\lambda\cos\theta\,d\theta}{4\pi\varepsilon_0 r_0}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r_0}(\sin\theta_1 + \sin\theta_2)$$

$$E_y = \int_{-\theta_1}^{+\theta_2}\frac{\lambda\sin\theta\,d\theta}{4\pi\varepsilon_0 r_0}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r_0}(\cos\theta_1 - \cos\theta_2)$$

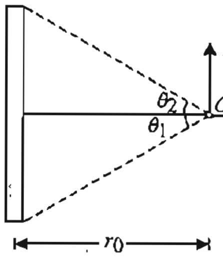


Fig. 1.64

- If we wish to determine field at the end of a long wire, we may substitute $\theta_1 = 0$ and $\theta_2 = \pi/2$ in the expressions for E_x and E_y.

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 r_0}\left[\sin(0) + \sin\left(\frac{\pi}{2}\right)\right] = \frac{\lambda}{4\pi\varepsilon_0 r_0} \text{ and}$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 r_0}\left[\cos(0) - \cos\left(\frac{\pi}{2}\right)\right] = \frac{\lambda}{4\pi\varepsilon_0 r_0}$$

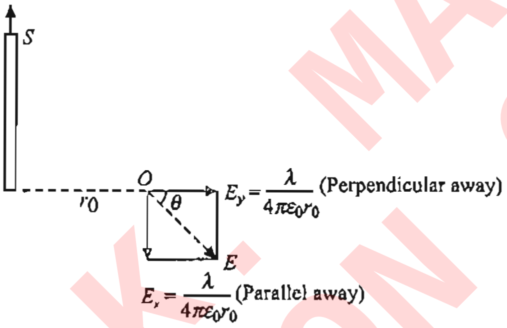


Fig. 1.65

Magnitude of resultant field $\vec{E}$:

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2} = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 r_0}$$

$\vec{E}$ makes an angle θ with the x-axis, where $\tan\theta = \frac{|E_y|}{|E_x|} = 1$; $\theta = 45°$

FIELD OF UNIFORMLY CHARGED DISK

Let us find the electric field caused by a disk of radius R with a uniform positive surface charge density (charge per unit area) σ, at a point along the axis of the disk a distance x from its center.

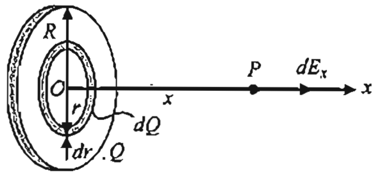


Fig. 1.66

The situation is shown in Fig. 1.66. We can represent this charge distribution as a collection of concentric rings of charge. We already know how to find the field of a single ring on its axis of symmetry, so all we have to do is to add the contribution of all the rings. As shown in the figure, a typical ring has charge dQ, inner radius r and outer radius $r + dr$. Its area dA is approximately equal to its width dr times its circumference $2\pi r$, or $dA = 2\pi r\,dr$. The charge per unit area is $\sigma = \frac{dQ}{dA}$, so the charge of ring is $dQ = \sigma(2\pi r\,dr)$, or $dQ = 2\pi\sigma r\,dr$. The field component dE_x at point P due to charge dQ of a ring of radius r

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{(2\pi\sigma r dr)x}{(x^2+r^2)^{3/2}}$$

To find the total field due to all the rings, we integrate dE_x over r. To include the whole disk, we must integrate from 0 to R (not from $-R$ to R) :

$$E_x = \int dE_x = \int_0^R dE_x = \int_0^R \frac{1}{4\pi\varepsilon_0}\frac{(2\pi\sigma r dr)x}{(x^2+r^2)^{3/2}}$$

Remember that x is a constant during the integration and that the integration variable is r. The integral can be evaluated by use of the substitution $z = x^2 + r^2$. We will let you work out the details; the result is

$$E_x = \frac{\sigma x}{2\varepsilon_0}\left[-\frac{1}{\sqrt{x^2+R^2}} + \frac{1}{x}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{(x^2+R^2)}}\right] \quad \text{(i)}$$

In this figure, the charge is assumed to be positive. At a point on the symmetry axis of a uniformly charged ring, the electric field due to the ring has no components perpendicular to the axis. Hence, at point P in the figure, $dE_y = dE_z = 0$ for each ring, and thus the total field has $E_y = E_z = 0$.

Again, we can ask what happens if the charge distribution gets very large. Suppose we keep increasing the radius R of the disk, simultaneously adding charge so that the surface charge density σ (charge per unit area) is constant. In the limit that R is much larger than the distance x of the field point from the disk ($R >> x$), i.e., the situation becomes the electric field near infinite plane sheet of charge.

From (i)

$$E_x = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{x\sqrt{1+\frac{R^2}{x^2}}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{1+\frac{R^2}{x^2}}}\right];$$

As $R >> x$, then the term $\frac{1}{\sqrt{1+\frac{R^2}{x^2}}} \to 0$

And we get $E_x = \frac{\sigma}{2\varepsilon_0}$

Our final result does not contain the distance x from the plane. This is correct but rather surprising result.

It means:

- That the electric field produced by an infinite plane sheet of charge is independent of the distance from the sheet.
- Thus, the field is uniform; its direction is everywhere perpendicular to the sheet and away from it.

- Infinite plane sheet of charge is a hypothetical case. In real practice, there is no such infinite plane sheet of charge. Again, there is no such thing as an infinite sheet of charge, but if the dimensions of the sheet are much larger than the distance x of the observation point P from the sheet, the field is very nearly the same as for an infinite sheet.

FIELD OF TWO OPPOSITELY CHARGED SHEETS

Two infinite plane sheets are placed parallel to each other, separated by a distance d (as shown in figure). The lower sheet has a uniform positive surface charge density σ, and the upper sheet has a uniform negative surface charge density $-\sigma$ with the same magnitude. Let us find the electric field between the two sheets, above the upper sheet and below the lower sheet.

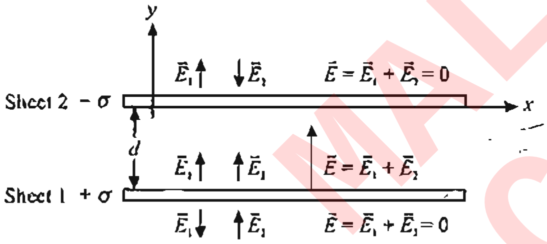


Fig. 1.67

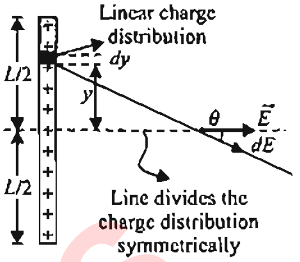
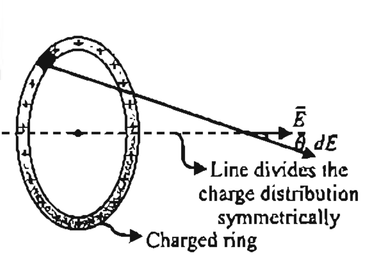
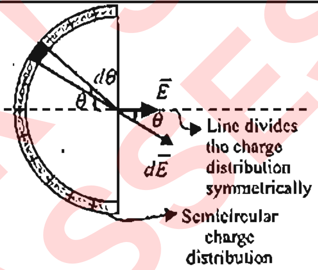
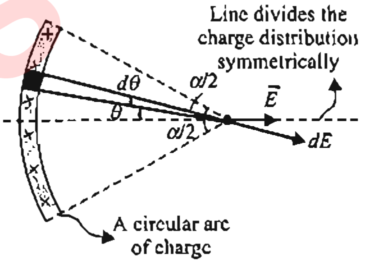
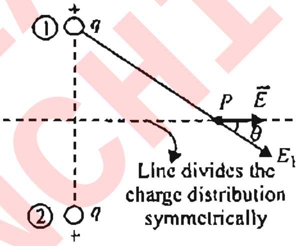
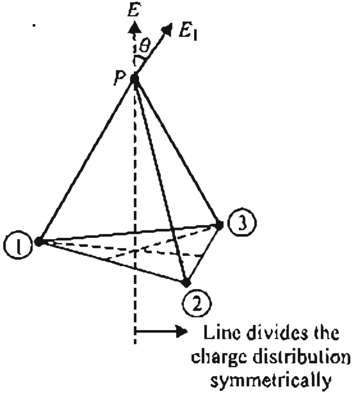
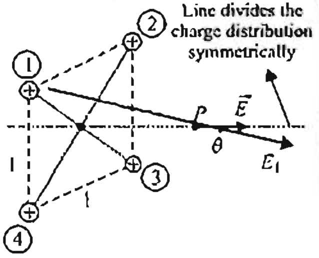
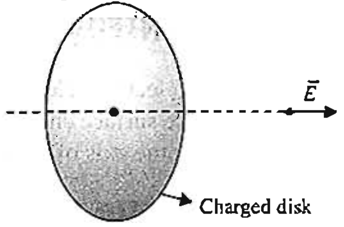
The situation described in this example in an idealization of two finite, oppositely charged sheets, like the plates shown in the figures. If the dimensions of the sheets are large in comparison to the separation d, then we can to good approximation consider the sheets to be infinite in extent. We know the field due to a single infinite plane sheet of charge. We can then find the total field by using the principle of superposition of electric fields. Let sheet 1 be the lower sheet of positive charge, and let sheet 2 be the upper sheet of negative charge; the fields due to each sheet are $\vec{E}_1$ and $\vec{E}_2$, respectively, and both have the same magnitude at all points, no matter how far from either sheet, i.e., $E_1 = E_2 = \dfrac{\sigma}{2\varepsilon_0}$.

At all points, the direction of $\vec{E}_1$ is away from the positive charge of sheet 1, and the direction of $\vec{E}_2$ is towards the negative charge of sheet 2. These fields, as well as the x- and y-axes, are shown in figure. At points between the sheets, the fields at each other and at points above the upper sheet or below the lower sheet cancel each other. Thus, the total field is

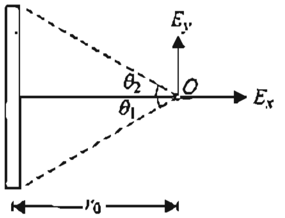
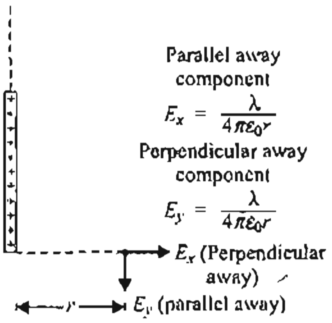
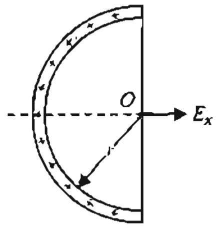
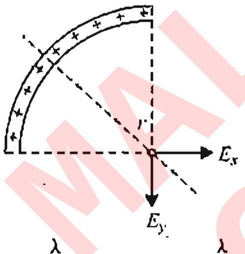
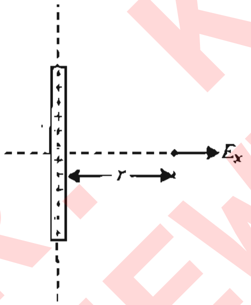
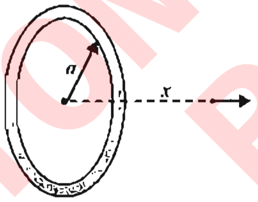
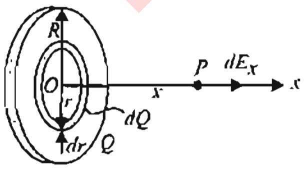
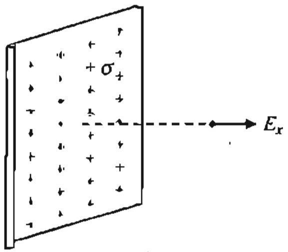
$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \begin{cases} 0 & \text{above the upper sheet} \\ \dfrac{\sigma}{\varepsilon_0}\hat{j} & \text{between the sheets} \\ 0 & \text{above the upper sheet} \end{cases}$$

Because we considered the sheets to be infinite, our result does not depend on the separation d.

Symmetry plays very important role in problem solving. Electric field is in the direction along the line which divides the charge distribution symmetrically.

 $E_{net} = \int dE \cos\theta$	 $E_{net} = \int dE \cos\theta$
 $E_{net} = \int dE \cos\theta$	 $E_{net} = \int dE \cos\theta$
Two point charges  Here, $\lvert\vec{E}_1\rvert = \lvert\vec{E}_2\rvert$ $E_{net} = 2\lvert\vec{E}_1\rvert\cos\theta$	**Three point charges at the corner of an equilateral triangle**  Here, electric field at P due to charges (1), (2) and (3) are equal, i.e., $\lvert\vec{E}_1\rvert = \lvert\vec{E}_2\rvert = \lvert\vec{E}_3\rvert$. Hence, $E_{net} = 3\lvert\vec{E}_1\rvert\cos\theta$
Four point charges at the corner of a square  The electric field at point P due to charges (1), (2), (3) and (4), $\lvert\vec{E}_1\rvert = \lvert\vec{E}_2\rvert = \lvert\vec{E}_3\rvert = \lvert\vec{E}_4\rvert$ Hence net electric field at P $\lvert\vec{E}_{net}\rvert = 4\lvert\vec{E}_1\rvert\cos\theta$	**Charged disk** 

Some Useful Results

A charged rod of fixed length having charge density λ  $E_x = \frac{\lambda}{4\pi\varepsilon_0 r_0}(\sin\theta_1 + \sin\theta_2)$ $E_y = \frac{\lambda}{4\pi\varepsilon_0 r_0}(\cos\theta_1 - \cos\theta_2)$	**Semi-infinite rod having charge density λ**  Parallel away component $E_x = \frac{\lambda}{4\pi\varepsilon_0 r}$ Perpendicular away component $E_y = \frac{\lambda}{4\pi\varepsilon_0 r}$
Semicircular ring having charge density λ $E_x = \frac{\lambda}{2\pi\varepsilon_0 r}$ $E_y = 0$  	**Quarter circular ring having charge density λ**  $E_x = \frac{\lambda}{4\pi\varepsilon_0 r} \Rightarrow E_y = \frac{\lambda}{4\pi\varepsilon_0 r}$
Infinite line charge  $E_x = \frac{\lambda}{2\pi\varepsilon_0 r}$ $E_y = 0$	**Charged ring**  $E_x = \frac{kQx}{(x^2+a^2)^{3/2}}$ $\Rightarrow \quad E_y = 0$
Charged disk  $E_x = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{(x^2+R^2)}}\right]$; $E_y = 0$	**Infinite sheet of charge**  $E_x = \frac{\sigma}{2\varepsilon_0}$ $E_y = 0$

Concept Application Exercise 1.3

1. A particle with positive charge Q is held fixed at the origin. A second particle with positive charge q is fired at the first particle, and follows the trajectory as shown in the figure. Is the angular momentum of second particle constant about some axis? Why or why not? Give reason to support your answer.

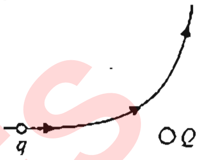


Fig. 1.68

2. Figure shows some of the electric field lines due to three point charges arranged along the vertical axis. All three charges have the same magnitude.
   a. What are the signs of each of the three charges? Explain your reasoning.
   b. At what point(s) is the magnitude of the electric field the smallest ? Explain your reasoning. Explain how the fields produced by each individual point charge combine to give a small net field at this point or points.

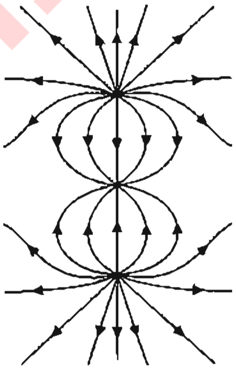


Fig. 1.69

3. Two point charges Q and $4Q$ are fixed at a distance of 12 cm from each other. Sketch lines of force and locate the neutral point, if any.
4. Is an electric field of the type shown by the electric lines in the Fig. 1.70 below physically possible?

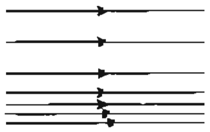


Fig. 1.70

5. Figure 1.71 shows three electric field lines. What is the direction of the electrostatic force on a positive test charge placed at
   a. points A and B?
   b. At which point, A or B, will the acceleration of the test charge be greater if the charge is released?

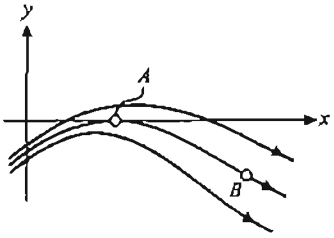


Fig. 1.71

6. A thin metallic spherical shell contains a charge Q on it. A point charge q is placed at the center of the shell and another charge q_1 is placed outside it as shown in figure. All the three charges are positive. Find the force on the charge

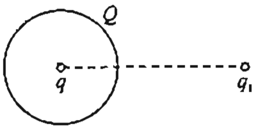


Fig. 1.72

a. at center due to all charges.
b. at center due to shell.

7. In Fig. 1.73, two particles each of charge $-q$, are arranged symmetrically about the y-axis; each producing an electric field at point P on y-axis.

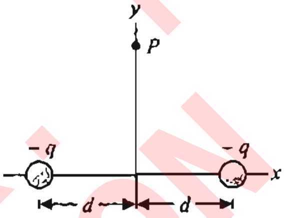
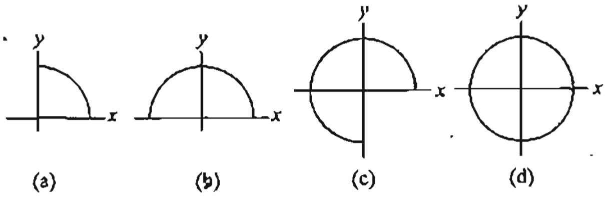


Fig. 1.73

a. Are the magnitude of the fields at P equal?
b. Is each electric field directed toward or away from the charge producing it?
c. Is the magnitude of the net electric field at P equal to the sum of the magnitudes E of the two field vectors (is it equal to $2E$)?
d. Do the x-components of those two field vectors add or cancel?
e. Do their y-components add or cancel?
f. Is the direction of the net field at P that of the canceling components or the adding components?
g. What is the direction of the net field?

8. In Fig. 1.74(a), a plastic rod in the form of circular arc with charge $+Q$ uniformly distributed on it produces an electric field of magnitude E at the center of curvature (at the origin). In figures (b), (c), and (d) more circular rods with identical uniform charges $+Q$ are added until the circle is complete. A fifth arrangement (which would be labeled e) is like that in d except that the rod in the fourth quadrant has charge $-Q$. Rank all the five arrangements according to the magnitude of the electric field at the center of curvature, greatest first.



(a) (b) (c) (d)

Fig. 1.74

9. Figure shows that E has the same value for all points in front of an infinitely charged sheet. Is this reasonable? One might think that the field should be stronger near the sheet because the charges are so much closer.

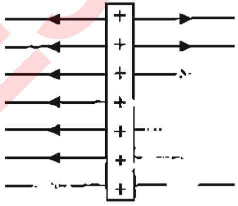


Fig. 1.75

10. Figure shows the tracks of three charged particles in a uniform electrostatic field projected parallel to plate with same velocity. Give the signs of the three charges. Which of the three particles has the highest charge to mass ratio?

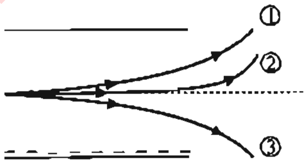


Fig. 1.76

11. Three small spheres x, y and z carry charges of equal magnitudes and with signs shown in figure. They are placed at the vertices of an isosceles triangle with the distance between x and y equal to the distance between x and z. Spheres y and z are held in place but sphere x is free to move on a frictionless surface.

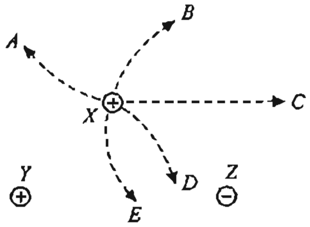


Fig. 1.77

a. What is the direction of the electric force on sphere x at the point shown in the figure?
b. Which path is sphere X likely to take when released?

12. Two identical positive charges are fixed on the y-axis, at equal distances from the origin O. A particle with a negative charge starts on the x-axis at a large distance from O, moves along the x-axis, passes through O and moves far away from O on the other side. Its acceleration a is taken as positive along its direction of motion. Plot the particle's acceleration a against its x-coordinate.

13. Electric field is defined in terms of q_0, a small positive charge. If instead the definition were in terms of a small negative charge of the same magnitude, then compared to the original field, the newly defined electric field

a. would point in the same direction and have the same magnitude.

b. would point in the opposite direction and have the same magnitude.

c. would point in the same direction and have a different magnitude.

d. would point in the opposite direction and have a different magnitude.

14. Three identical positive charges Q are arranged at the vertices of an equilateral triangle. The side of the triangle is a. Find the intensity of the field at the vertex of a regular tetrahedran of which the triangle is the base.

15. Two point charges of $+5 \times 10^{-19}$ C and $+20 \times 10^{-19}$ C are separated by a distance of 2 m. The electric field intensity will be zero at a distance $d =$ ______ from 5×10^{-19} C charge.

16. An electron (mass m_e) falls through a distance 'd' in a uniform electric field of magnitude E.

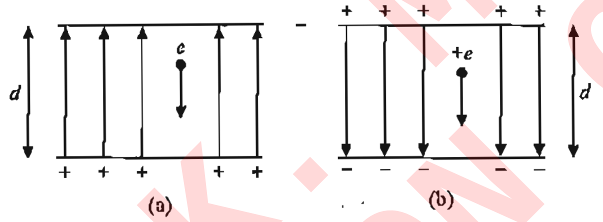


Fig. 1.78

The direction of the field is reversed keeping its magnitude unchanged and a proton (mass m_p) falls through the same distance. If the times taken by electron and proton to fall the distance d is '$t_{electron}$' and 't_{proton}', respectively, then the ratio $\frac{t_{electron}}{t_{proton}} =$ ______

17. Two charged metal plates in vacuum are 10 cm apart. A uniform electric field of intensity $(45/16) \times 10^3$ NC^{-1} is applied between the plates. An electron is released between the plates from rest at a point just outside the negative plate. Calculate

a. how long (t) will electron take to reach the other plate?

b. At what velocity (v) will it be going just before it hits the other plate?

18. A polythene piece rubbed with wool is found to have a negative charge of 3.2×10^{-7} C.

a. The number of electrons transferred is ______

b. Is there a transfer of mass from wool to polythene? (Yes/No) ______

19. Two identical point charges 'Q' are kept at a distance 'r' from each other. A third point charge is placed on the line joining the above two charges such that all the three charges are in equilibrium. The third charge

a. should be of magnitude $q = \ldots$

b. should be of sign ...

c. should be placed ...

20. If we introduce a large thin metal plate between two point charges, what will happen to the force between the charges?

21. Two point electric charges of unknown magnitude and sign are placed a certain distance apart. The electric field intensity is zero at a point not between the charges but on the line joining them. Write two essential conditions for this to happen.

22. A ball of charge q is placed in a hollow conducting uncharged sphere. After this, the sphere is connected with earth for a short time and the ball is then removed from the sphere. The ball has not been brought into contact with the sphere.

a. What charge will the sphere have after these operations? Where and how will this charge be distributed?

b. What will be the electric field inside as well on outside of sphere?

23. Two pieces of plastic, a full ring and a half ring, have the same radius and charge density. Which electric field at the center has the greater magnitude?
Define your answer.

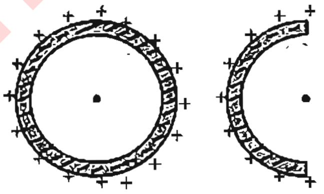


Fig. 1.79

24. A droplet of ink in an industrial ink-jet printer carries a charge of 1.6×10^{-10} C and is deflected onto paper by a force of 3.2×10^{-4} N. Find the strength of the electric field to produce this force.

25. An electric dipole of length 4 cm, when placed with its axis making an angle of 60° with a uniform electric field experiences a torque of $4\sqrt{3}$ N m. Calculate the (a) magnitude of the electric field and (b) potential energy of the dipole, if the dipole has charges of ± 8 nC.

26. An electric dipole consists of two opposite charges each of 1 μC separated by 2 cm. The dipole is placed in an external uniform field of 10^5 NC^{-1} intensity. Find

a. maximum torque exerted by the field on the dipole and

b. the work done in rotating the dipole through 180° starting from the position $\theta = 0°$.

ELECTRIC DIPOLE

- An electric dipole is a system of two equal and opposite point charges separated by a very small and finite distance.
- Fig. 1.80 shows an electric dipole consisting of two equal and opposite point charges $-q$ and $+q$ separated by a small

distance $2l$. The strength of an electric dipole is measured by a vector quantity known as *electric dipole moment*. Its magnitude is equal to the product of the magnitude of either charge and the distance between the two charges.

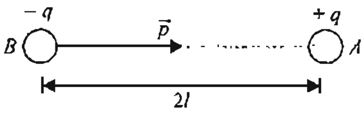


Fig. 1.80

$$\therefore \quad p = q2l$$

The direction of p is from negative charge to positive charge.

- In S.I. system of units, p is measured in coulomb-metre.

ELECTRIC FIELD DUE TO A DIPOLE

Electric Field Intensity due to an Electric Dipole at a Point on the Axial Line

- A line passing through the negative and positive charges of the electric dipole is called the axial line of the electric dipole.

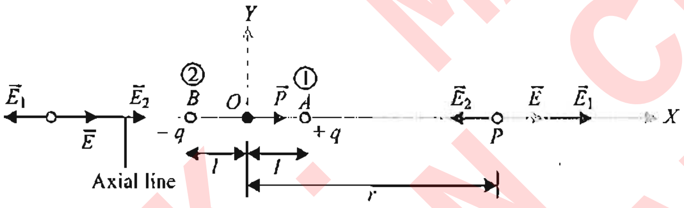


Fig. 1.81

- Suppose an electric dipole AB is located in a medium of dielectric constant K (as shown in Fig. 1.81). Let the dipole consists of two point charges of $-q$ and $+q$ coulomb separated by a short distance $2l$ meter. Let P be an observation point on the axial line such that its distance from the mid point O of the electric dipole is r. We are interested to calculate the intensity of electric field at P.

- $E_1 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r-l)^2}$ due to q at P {along the direction OX}

and $E_2 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r+l)^2}$ due to $-q$ at P {along the direction OB}

The intensities E_1 and E_2 are along the same line but in opposite directions. Since $E_1 > E_2$, hence resultant intensity E at the point P will be equal to their differences and in the direction AP. Thus,

$$E = E_1 - E_2 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r-l)^2} - \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r+l)^2}$$

$$E = \frac{q}{4\pi\varepsilon_0 K}\left[\frac{4lr}{(r^2-l^2)^2}\right] = \frac{1}{4\pi\varepsilon_0 K}\left[\frac{2(2ql)r}{(r^2-l^2)^2}\right]$$

But $2ql = p$ =electric dipole moment;

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0 K}\frac{2pr}{(r^2-l^2)^2}$$

- If l is very small compared to r $(l \ll r)$, then l^2 can be neglected in comparison to r^2. Then, the electric field intensity at the point P due to a short dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0 K}\frac{2pr}{r^4} = \frac{1}{4\pi\varepsilon_0 K}\frac{2p}{r^3}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0 K}\frac{2p}{r^3}$$

- If dipole is placed in air or vacuum, then $K = 1$ and $E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$

Note: ***The direction of electric field E is in the direction of $\vec{p}$, i.e., parallel to the axis of dipole from the negative charge towards the positive charge.***

In vector form, we can write:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}\hat{i} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$$

Electric Field Intensity due to an Electric Dipole at a Point on the Equatorial Line

An equatorial line of the electric dipole is a line perpendicular to the axial line and passing through a point mid way between charges.

- Let us now suppose that the observation point P is situated on the equatorial line of dipole such that its distance from mid-point O of the electric dipole is r (as shown in Fig. 1.82). Let us assume again that the medium between the electric dipole and the observation point has dielectric constant K.

- $E_1 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)}$ {along the direction PD}

and $E_2 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)}$ {along the direction PC}

The magnitude of E_1 and E_2 are equal but directions are different.

Net intensity: $E = E_1\cos\theta + E_2\cos\theta$

[sine components cancel out]

$$E = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)}\cos\theta + \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)}\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)} \times 2\cos\theta \text{ along } PR$$

But from the figure,

$$\cos\theta = \frac{OA}{PA} = \frac{OA}{(OP^2+OA^2)^{1/2}} = \frac{l}{(r^2+l^2)^{1/2}}$$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)} \times \frac{2l}{(r^2+l^2)^{1/2}} = \frac{1}{4\pi\varepsilon_0 K} \times \frac{2ql}{(r^2+l^2)^{3/2}}$$

But $2ql = p =$ electric dipole moment

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0 K} \times \frac{p}{(r^2+l^2)^{3/2}}$$

- If l is very small as compared to r $(l \ll r)$, then l^2 can be neglected in comparison to r^2. Then, the electric field

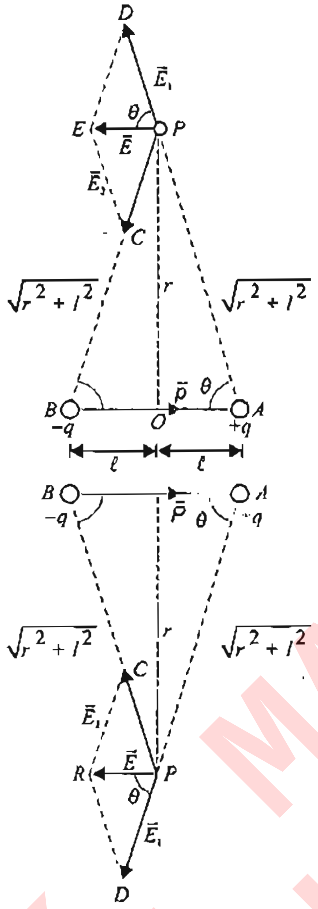


Fig. 1.82

intensity at the point P due to a short dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0 K}\frac{p}{(r^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0 K}\frac{p}{r^3}$$

- If dipole is placed in air or vacuum, then $K = 1$ and $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$

As direction of resultant electric field is along the negative x-axis, hence in vector form we can write

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\times\frac{p}{r^3}(-\hat{i}) = -\frac{1}{4\pi\varepsilon_0}\times\frac{\vec{p}}{r^3}$$

Note: *The direction of electric field E is opposite to the direction of $\vec{p}$, i.e., antiparallel to the axis of dipole from the positive charge towards the negative charge.*

ELECTRIC FIELD INTENSITY DUE TO A SHORT DIPOLE AT SOME GENERAL POINT

- Let AB be a short electric dipole of dipole moment $\vec{p}$ (directed from B to A). We are interested to find the electric field at some general point P. The distance of observation point P w.r.t. mid point O of the dipole is r and the angle made by the line OP w.r.t. axis of dipole is θ.
- We know that dipole moment of a dipole is a vector quantity. It can be resolved into two rectangular components $\vec{p}_1$ and $\vec{p}_2$ as shown in figure, so that $\vec{p} = \vec{p}_1 + \vec{p}_2$.
  The magnitude of $\vec{p}_1$ and $\vec{p}_2$ are $p_1 = p\cos\theta$ and $p_2 = p\sin\theta$.
- It is clear from figure that point P lies on the axial line of dipole with moment $\vec{p}_1$. Hence, magnitude of the electric field intensity $\vec{E}_1$ at P due to $\vec{p}_1$ is

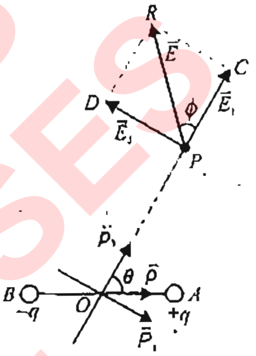


Fig. 1.83

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}\ \{\text{along } OC\} \qquad \text{(i)}$$

Similarly, P lies on the equatorial line of dipole with moment $\vec{p}_2$. Hence, magnitude of electric field intensity $\vec{E}_2$ at P due to $\vec{p}_2$ is

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}\ \{\text{opposite to } p_2\} \qquad \text{(ii)}$$

Hence, resultant intensity at P is $\vec{E} = \vec{E}_1 + \vec{E}_2$

Magnitude of $\vec{E}$ is: $E = \sqrt{(E_1^2 + E_2^2)}$ (as $\vec{E}_1$ and $\vec{E}_2$ are mutually perpendicular).

or $$E = \sqrt{\left(\frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}\right)^2 + \left(\frac{p\sin\theta}{4\pi\varepsilon_0 r^3}\right)^2}$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta} = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{1 + 3\cos^2\theta}$$

- If the resultant field intensity vector $\vec{E}$ makes an angle ϕ with the direction of $\vec{E}_1$, then

$$\tan\phi = \frac{E_2}{E_1} = \frac{(p\sin\theta/4\pi\varepsilon_0 r^3)}{(2p\cos\theta/4\pi\varepsilon_0 r^3)} = \frac{1}{2}\tan\theta$$

Illustration 1.27 **Three charges $-q$, $+2q$ and $-q$ are arranged on a line as shown in the Fig. 1.84. Calculate the field at a distance $r >> a$ on the line.**

Sol. The field at point P is superposition of fields $\vec{E}_1$, $\vec{E}_2$, $\vec{E}_3$ due to each charge.

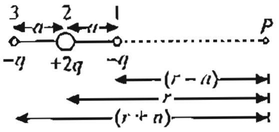


Fig. 1.84

$$\vec{E}_1 = -\frac{q}{4\pi\varepsilon_0(r-a)^2}\hat{i}; \quad \vec{E}_2 = +\frac{2q}{4\pi\varepsilon_0 r^2}\hat{i};$$

$$\vec{E}_3 = -\frac{q}{4\pi\varepsilon_0(r+a)^2}\hat{i}; \text{ Now}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{(r-a)^2} + \frac{2}{r^2} - \frac{1}{(r+a)^2}\right]\hat{i}$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}\left[-\left\{1-\left(\frac{a}{r}\right)^{-2}\right\} + 2 - \left\{1+\left(\frac{a}{r}\right)\right\}^{-2}\right]$$

If $r >> a$, we can use binomial approximation:

$$(1+\alpha)^n \simeq 1 + n\alpha + \frac{n(n-1)}{\underline{|2}}\alpha^2 + \cdots \text{ for } \alpha << 1$$

Therefore,

$$E = \frac{q}{4\pi\varepsilon_0 r^2}\left[-\left\{\left(1-2(-\frac{a}{r})\right) + \frac{-2(-2-1)}{2}\left(\frac{-a}{r}\right)^2\right\}\right.$$

$$\left. + 2 - \left\{1 - 2\frac{a}{r} + \frac{-2(-2-1)}{2}\left(\frac{a}{r}\right)^2\right\}\right] = \frac{6a^2q}{4\pi\varepsilon_0 r^4}$$

The charge in this problem may be considered as two dipoles placed close together. Such an arrangement of charge is called an electric quadrupole.

Illustration 1.28 **What is the force on a dipole of dipole moment p placed as shown in the Fig. 1.85.**

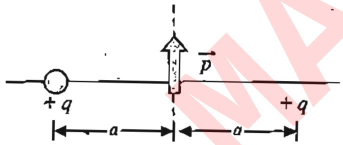


Fig. 1.85

Sol. Force on any q by dipole:

$$F = qE_{\text{dipole}} = \frac{q}{4\pi\varepsilon_0}\frac{p}{a^3} \text{ downward}$$

So from third law, force on dipole due to both charges

$$= 2F = \frac{qp}{2\pi\varepsilon_0 a^3} \text{ upward}$$

Net Force on a Dipole in a Non-Uniform Field

Suppose an electric dipole with dipole moment $\vec{P}$ is placed in a non-uniform electric field $\vec{E} = E\hat{i}$ that points along x-axis (Fig. 1.86). Let E depends only on x. The electric field at the position of negative charge is E and at the position of positive charge $(E + \Delta E)$. Net force acting on the dipole is then

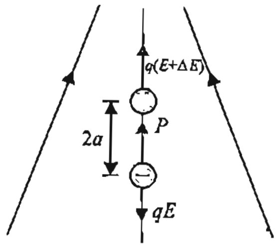


Fig. 1.86

$$F = q(E+\Delta E) - qE = q\,\Delta E = q\left[\frac{\Delta E}{\Delta x}2a\right]$$

$$\left[\text{as } \frac{\Delta E}{\Delta x} = \frac{dE}{dx}\right]$$

$$\therefore \quad F = 2aq\frac{dE}{dx} = p\frac{dE}{dx}$$

$$|\vec{F}| = \left|p\frac{d\vec{E}}{dx}\right|$$

where $\dfrac{dE}{dx}$ is the gradient of the field in the x-direction.

Illustration 1.29 **Find the force on a small electric dipole of dipole moment $\vec{p}$ due to a point charge Q placed at a distance r.**

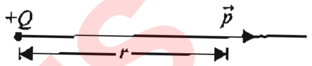


Fig. 1.87

Sol. Electric field of a point charge is a non-uniform electric field. Electric field at a distance x from the point charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2} \Rightarrow \frac{dE}{dx} = -\frac{1}{4\pi\varepsilon_0}\frac{2Q}{x^3}$$

magnitude of force on the dipole:

$$F = \left|p\frac{dE}{dx}\right|_{x=r} = \frac{1}{4\pi\varepsilon_0}\frac{2pQ}{r^3}$$

Alternatively: Same can be calculated as force on the point charge due to dipole which is same as the force on dipole due to point charge (Newton's 3rd law). The electric field of small dipole at a distance r is

$E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2p}{r^3}$. Hence, force on the point charge Q is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{2pQ}{r^3}$$

DIPOLE IN A UNIFORM ELECTRIC FIELD

Torque: When a dipole is placed in a uniform field as shown in Fig. 1.88, the net force on it: $F_R = \left[q\vec{E} + (-q)\vec{E}\right] = 0$

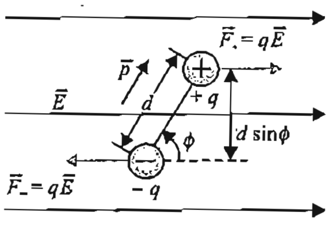


Fig. 1.88

Hence, net force on a dipole is zero in a uniform electric field. While the torque $\tau = qE \times d\sin\phi$

i.e., $\quad \tau = pE\sin\phi$ {as $p = qd$}

or $\quad \vec{\tau} = \vec{p} \times \vec{E}$ (by electric field)

and $\quad \vec{\tau} = \vec{E} \times \vec{p}$ (by us if the dipole is in equilibrium)

From the expression, it is clear that couple acting on a dipole is maximum ($= pE$) when dipole is perpendicular ($\phi = 90°$) to the field and minimum ($= 0$) when dipole is parallel ($\phi = 0°$) or antiparallel ($\phi = 180°$) to the field.

By applying a torque, electric field tends to align a dipole in its own direction.

Illustration 1.30 **An electric dipole consists of two charges of 0.1 μC separated by a distance of 2.0 cm. The dipole is placed in an external field of 10^5 NC^{-1}. What maximum torque does the field exert on the dipole?**

Sol. $\tau = pE \sin\theta = q \times 2a \times E \sin\theta$. Max. value of τ will be when $\sin\theta = 1$

$\therefore \tau_{max} = 10^{-7} \times 2 \times 10^{-2} \times 10^5 \times 1 = 2 \times 10^{-4}$ N-m

Concept Application Exercise 1.4

1. State the following statements as true / false:
   a. An electric dipole is kept in a uniform electric field at some angle with it. It experiences a force but no torque.
   b. An electric dipole may experience a net force when it is placed in a non-uniform electric field.
   c. An electric dipole is kept in a non-uniform electric field. It can experience a force and a torque.
2. Electric intensity due to an electric dipole varies with distance as $E \propto r^n$, where n is ________.
3. An electric dipole of moment $\vec{p}$ is placed at the origin along the x-axis. The electric field E at a point P, whose position vector makes an angle θ with the x-axis, will make an angle with x-axis is ________.

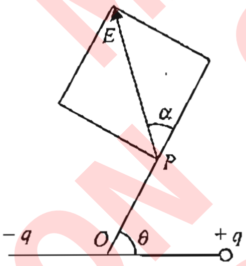


Fig. 1.89

4. Two point charges of 1 μC and −1μC are separated by a distance of 100 Å. A point P is at a distance of 10 cm from the mid point and on the perpendicular bisector of the line joining the two charges. Find the electric field at P.
5. An electric dipole consists of two opposite charges of magnitude 2×10^{-6} C each and separated by a distance of 3 cm. It is placed in an electric field of 2×10^5 NC^{-1}. Determine the maximum torque on the dipole.
6. Three charges are arranged on the vertices of an equilateral triangle as shown in Fig. 1.90. Find the dipole moment of the combination.

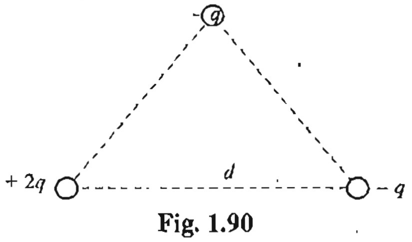


Fig. 1.90

7. The electric field at A due to dipole p is perpendicular to p. the angle θ is________

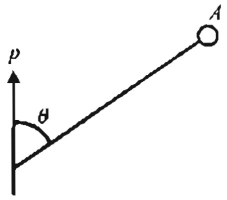


Fig. 1.91

8. A dipole lies on the x-axis, with the positive charge $+q$ at $x = +\frac{d}{2}$ and the negative charge at $-\frac{d}{2}$. Find the electric flux ϕ_E through the yz plane midway between the charges.
9. An electric dipole is formed by two particles fixed at the end of a light rod of length l. The mass of each particle is m and the charges are $-q$ and $+q$. The system is placed in such a way that the dipole axis is parallel to a uniform electric field E that exists in region. The dipole is slightly rotated about its center and released. Show that for small angular displacement motion is SHM. Evaluate its time period.

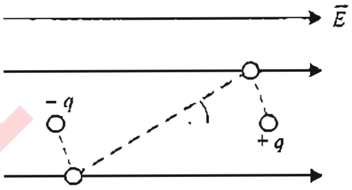


Fig. 1.92

10. A dipole consists of two particles carrying charges +2 and −2μC and masses 1 and 2 kg, respectively, separated by a distance of 6 m. It is placed in a uniform electric field of 8×10^4 Vm^{-1}. For small oscillations about its equilibrium position, find the angular frequency.
11. A small electric dipole of dipole moment P is placed near a point charge $+Q$ as shown. Then, the net force on the dipole is towards ________.

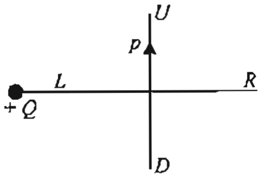


Fig. 1.93

Solved Examples

Example 1.1 **A uniformly charged wire with linear charge density λ is laid in the form of a semicircle of radius R. Find the electric field generated by the semicircle at the center.**

Sol. We consider a differential element dl on the ring, that subtends an angle $d\theta$ at the center of the ring,

$dl = R\ d\theta$. Charge on this element $= dQ = \lambda R\ d\theta$.

This element creates a field dE which makes an angle θ at the center as shown in Fig. 1.94. For each differential element

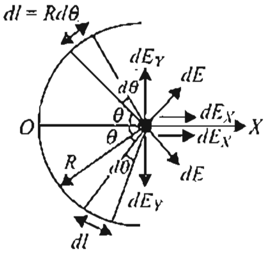


Fig. 1.94

in the upper half of the ring, there corresponds a symmetrically placed charge element in the lower half plane. The y-components of field due to these symmetric elements cancel out and x-components remain.

$$dE_x = dE\cos\theta = \frac{dQ}{4\pi\varepsilon_0 R^2}\cos\theta = \frac{\lambda(R\,d\theta)\cos\theta}{4\pi\varepsilon_0 R^2}$$

On integrating the expression for dE_x, w.r.t. angle θ, in limits $\theta = -\pi/2$ to $\theta = +\pi/2$, we obtain

$$E = \int_{-\pi/2}^{+\pi/2} \frac{\lambda R}{4\pi\varepsilon_0 R^2}\cos\theta\ d\theta = \frac{\lambda}{2\pi\varepsilon_0 R}$$

In terms of total charge, say Q, on the ring, $\lambda = \dfrac{Q}{\pi R}$ and we get $E = \dfrac{Q}{2\pi^2\varepsilon_0 R^2}$.

If we consider the wire in the form of an arc as shown in the figure, the symmetry consideration is not useful in canceling out x- and y-components of the fields, if θ_1 and θ_2 are different. We will integrate dE_x as well as dE_y in limits $\theta = -\theta_1$ to $\theta = +\theta_2$.

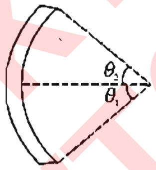


Fig. 1.95

$$E_x = \int_{-\theta_1}^{+\theta_2} \frac{\lambda R}{4\pi\varepsilon_0 R^2}\cos\theta\ d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}\ (\sin\theta_1 + \sin\theta_2)$$

$$E_y = -\int_{-\theta_1}^{+\theta_2} \frac{\lambda R}{4\pi\varepsilon_0 R^2}\sin\theta\ d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}(\cos\theta_2 - \cos\theta_1)$$

For a symmetrical arc, $\theta_1 = \theta_2$. Thus, E_y vanishes and

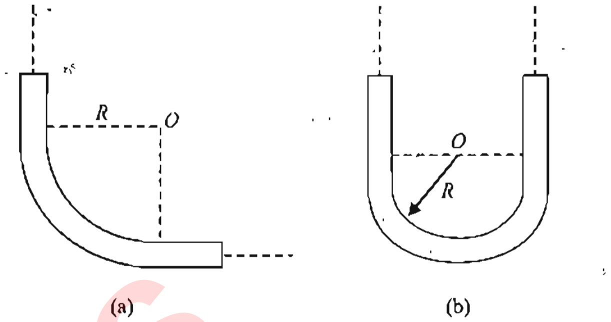
$$E_x = \frac{\lambda\ \sin\theta}{2\pi\varepsilon_0 R}$$

Example 1.2 **A long wire with a uniform charge density λ is bent in two configurations shown in figure (a) and (b). Determine the electric field intensity at point O.**

Sol. Consideration of Fig. 1.96(a)

Field due to segment (1):

$$\vec{E}_1 = \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(-\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j}$$



(a) (b)

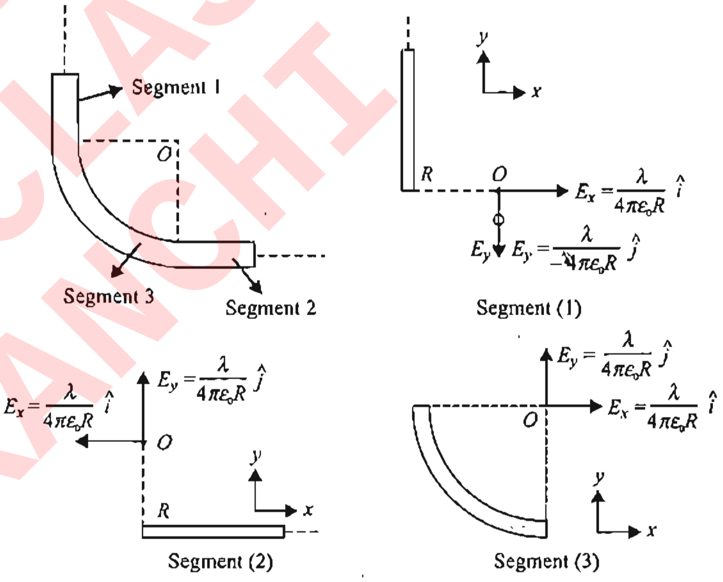
Fig. 1.96

Field due to segment (2):

$$\vec{E}_2 = \left(-\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j}$$

Field due to quarter shape wire segment (3):

$$\vec{E}_3 = \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j} \quad (\because \theta_1 = 90°\ \theta_2 = 0°)$$



Segment (1)

Segment (2) Segment (3)

Fig. 1.97

Resultant field is superposition of fields due to each part.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \qquad \text{(i)}$$

Substituting the values of $\bar{E}_1$, $\bar{E}_2$ and $\bar{E}_3$ in (i),

$$\vec{E} = \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{i} + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)\hat{j}$$

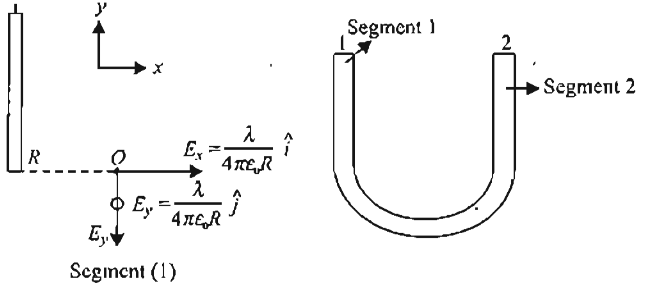


Fig. 1.98

$$|\vec{E}| = \left[\left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)^2 + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)^2\right]^{1/2} = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 R}$$

Here, $E_x = E_y = \frac{\lambda}{4\pi\varepsilon_0 R}$. Hence, the resultant field will make an angle of 45° with the axis.

b. Field due to segment 1,

$$\vec{E}_{x_1} = \frac{\lambda}{4\pi\varepsilon_0 R}\hat{i}$$

$$\vec{E}_{y_1} = -\frac{\lambda}{4\pi\varepsilon_0 R}\hat{j}$$

$$\vec{E}_1 = \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} - \hat{j}]$$

Field due to segment 2, $\vec{E}_{x_2} = -\frac{\lambda}{4\pi\varepsilon_0 R}\hat{i}$

$$\vec{E}_{y2} = -\frac{\lambda}{4\pi\varepsilon_0 R}\hat{j}$$

$$\vec{E}_2 = -\frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} + \hat{j}]$$

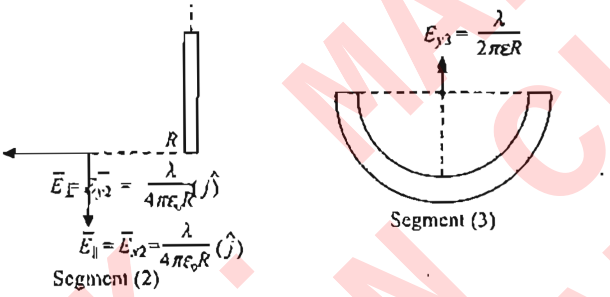


Fig. 1.99

Field due to segment 3, $\vec{E}_{x_3} = 0$, $\vec{E}_{y_3} = \frac{\lambda}{2\pi\varepsilon_0 R}\hat{j}$

$$\Rightarrow \qquad \vec{E}_3 = \frac{\lambda}{2\pi\varepsilon_0 R}\hat{j}$$

From principle of superposition of electric fields,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} - \hat{j}] - \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} + \hat{j}] + \frac{\lambda}{2\pi\varepsilon_0 R}\hat{j} = 0$$

Hence, net field is zero.

Example 1.3 **A particle having charge that of an electron and mass 1.6 × 10^{-30} kg is projected with an initial speed u at an angle 45° to the horizontal from the lower plate of a parallel plate capacitor as shown in figure. The plates are sufficiently long and have separation 2 cm. Find the maximum value of velocity of particle for it not to hit the upper plate. Take electric field between the plates = 10^3 Vm^{-1} directed upward.**

Sol. Resolving the velocity of particle parallel and perpendicular to the plate.

$$u_{\parallel} = u\cos 45° = \frac{u}{\sqrt{2}} \text{ and } u_{\perp} = u\sin 45° = \frac{u}{\sqrt{2}}$$

Force on the charged particle in downward direction normal to the plate = eE

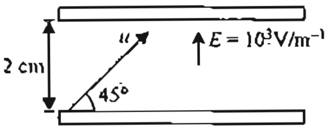


Fig. 1.100

∴ Acceleration $a = \frac{eE}{m}$, where m is the mass of charged particle.

The particle will not hit the upper plate, if the velocity component normal to plate becomes zero before reaching it, i.e.,

$0 = u_{\perp}^2 - 2ay$ with $y \le d$, where d is the distance between the plates.

∴ Maximum velocity for the particle not to hit the upper plate, (for this $y = d = 2$ cm)

$$u_{\perp} = \sqrt{2ay} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 10^3 \times 2 \times 10^{-2}}{1.6 \times 10^{-30}}}$$

$$= 2 \times 10^6 \text{ ms}^{-1}$$

$$\Rightarrow \quad u_{\max} = u_{\perp}/\cos 45° = 2\sqrt{2} \times 10^6 \text{ ms}^{-1}$$

Example 1.4 **A particle of mass m and charge q is released at rest in a uniform field of magnitude E. The uniform field is created between two parallel plates of charge densities $+\sigma$ and $-\sigma$, respectively. The particle accelerates towards the other plate a distance d away. Determine the speed at which it strikes the opposite plate.**

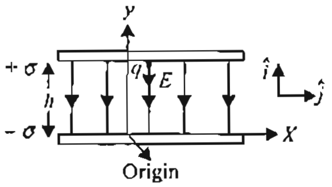


Fig. 1.101

Sol. The applied electric field is $\vec{E} = -E_0\hat{j}$

The force experienced by the charge q, $\vec{F} = q\vec{E} = -qE_0\hat{j}$

The force is constant, and so the acceleration is constant as well

$$\therefore \quad \vec{a} = \frac{\vec{F}}{m} = -\frac{qE_0}{m}\hat{j}$$

Due to constant acceleration, the particle moves in −ve y-direction; the problem is analogous to motion of a mass released from rest in a gravitational field.

From equations of motion,

$$v_y = v_{y0} + a_y t = 0 - \frac{qE_0}{m}t \qquad \text{(i)}$$

And $y = y_0 + v_0 t + \frac{1}{2}a_y t^2$; $0 = d + 0 - \frac{1}{2}\frac{qE_0}{m}t^2$ (ii)

Particle starts at $y_0 = d$ and impact occurs at $y = 0$

From equation (ii), $t = \left(\frac{2dm}{qE_0}\right)^{1/2}$

From equation (i), $v_y = -\frac{qE_0}{m}\left(\frac{2dm}{qE_0}\right)^{1/2} = -\sqrt{\frac{2qE_0 d}{m}}$

Example 1.5 **Two balls of charges q_1 and q_2 initially have a velocity of the same magnitude and direction. After**

a uniform electric field has been applied for a certain time interval, the direction of first ball changes by 60° and the velocity magnitude is reduced by half. The direction of velocity of the second ball changes thereby 90°. In what ratio will the velocity of the second ball change? Determine the magnitude of the charge-to-mass ratio of the second ball if it is equal to α_1 for the first ball. Ignore the electrostatic interaction between the balls.

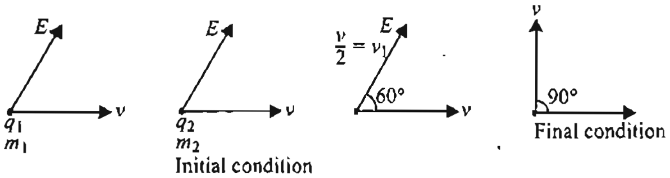


Fig. 1.102

Sol. Let the electric field on each ball be given by

$$E = E_x\hat{i} + E_y\hat{j}$$

From impulse–momentum equation, we have

Impulse = Change in momentum

Let the final velocities of the balls be v_1 and v_2. Nothing that $v_1 = v/2$, we have

$$q_1(E_x\hat{i}+E_y\hat{j})\Delta t = m_1\left(\frac{v}{2}\cos 60^\circ\hat{i}+\frac{v}{2}\sin 60^\circ\hat{j}\right) - m_1 v\hat{i} \quad \text{(i)}$$

$$q_2(E_x\hat{i}+E_y\hat{j})\Delta t = m_2\left(v_2\cos 90^\circ\hat{i} + v_2\sin 90^\circ\hat{j}\right) - m_2 v\hat{i} \quad \text{(ii)}$$

On comparing the x- and y-components on both sides of equation (i), we get

$$\frac{q_1}{m_1}E_x\,\Delta t = -\frac{3}{4}v \text{ and } \frac{q_1}{m_1}E_y\,\Delta t = \frac{\sqrt{3}}{4}v \quad \text{(iii)}$$

Similarly, for equation (ii), we get

$$\frac{q_2}{m_2}E_x\,\Delta t = -v \quad \text{and} \quad \frac{q_2}{m_2}E_y\,\Delta t = v_2 \quad \text{(iv)}$$

From equations (iii) and (iv), by dividing the equations expression for x-components, we get

$$\frac{q_1/m_1}{q_2/m_2} = \frac{3}{4} \quad \text{(v)}$$

$$\text{or} \quad \frac{q_2}{m_2} = \frac{4}{3}\frac{q_1}{m_1} = \frac{4}{3}\alpha_1$$

$$\text{Also, } \frac{q_1/m_1}{q_2/m_2} = \frac{\sqrt{3}\,v}{4v_2} \Rightarrow \frac{\sqrt{3}\,v}{4v_2} = \frac{3}{4} \Rightarrow v_2 = \frac{v}{\sqrt{3}}$$

Example 1.6 **A rigid insulated wire frame, in the form of right triangle ABC is set in a vertical plane. Two beads of equal masses m each carrying charges q_1 and q_2 are connected by a chord of length l and can slide without friction on the wires. Considering the case when the beads are stationary, determine** **(IIT-JEE, 1978)**

1. **the angle α.**
2. **the tension in the chord, and**
3. **the normal reactions on the beads if the chord is not cut. What are the values of the charges for which the beads continue to remain stationary?**

Sol. Because of equilibrium of charge q_1

$$N_1 = mg\sin 60^\circ + (T-F)\sin\alpha \ldots \quad \text{(i)}$$

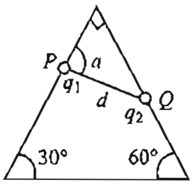


Fig. 1.103

$$\text{and } (T-F)\cos\alpha = mg\cos 60^\circ \quad \text{(ii)}$$

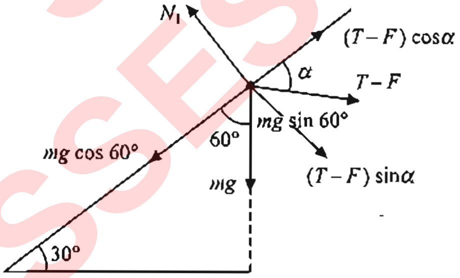


Fig. 1.104

Because of equilibrium of charge q_2

$$(T-F)\sin\alpha = mg\cos 30^\circ \quad \text{(iii)}$$

From (i) and (iii), $N_1 = mg\sin 60^\circ + mg\cos 30^\circ$ (iv)

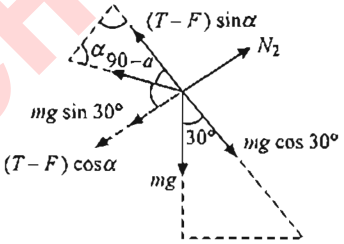


Fig. 1.105

$$\Rightarrow \quad N_1 = mg\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right) = \sqrt{3}\text{ mg}$$

From (ii) and (iv),

$$N_2 = mg\cos 60^\circ + mg\sin 30^\circ = mg\left(\frac{1}{2}+\frac{1}{2}\right) = mg$$

Also, $F = k\frac{q_1q_2}{l^2}$

Now, from equations (ii) and (iii), we get

$$(T-F)^2\cos^2\alpha + (T-F)^2\sin^2\alpha = m^2g^2\cos^2 60^\circ + m^2g^2\cos^2 30^\circ$$

$$\Rightarrow (T-F)^2 = m^2g^2\left[\frac{1}{4}+\frac{3}{4}\right] = m^2g^2$$

$$\Rightarrow \quad T - F = \pm mg \quad \text{(v)}$$

$$\Rightarrow T = mg + F = mg + k\frac{q_1q_2}{l^2} \quad \text{(vi)}$$

[Taking positive sign]

From (ii) and (v),

$$mg\cos\alpha = mg\cos 60^\circ \quad \Rightarrow \cos\alpha = \cos 60^\circ$$

When the string is cut, $T = 0$

$\therefore$ From (vi), mg $= \pm k\frac{q_1q_2}{l^2} \Rightarrow q_1q_2 = \pm\frac{mgl^2}{k}$

EXERCISES

Subjective Type

Solutions on page 1.45

1. Calculate the number of electrons in a small, electrically neutral silver pin that has a mass of 10.0 g. Silver has 47 electrons per atom, and its molar mass is 107.87 gmol^{-1}.
2. A charged particle of radius 5×10^{-7} m is located in a horizontal electric field of intensity 6.28×10^5 Vm^{-1}. The surrounding medium has coefficient of viscosity $\eta = 1.6 \times 10^5$ Nsm^{-2}. The particle starts moving under the effect of electric field and finally attains a uniform horizontal speed of 0.02 ms^{-1}. Find the number of electrons on it. Assume gravity free space.
3. Suppose that 1.00 g of hydrogen is separated into electrons and protons. Suppose also that the protons are placed at the Earth's north pole and the electrons are placed at the south pole. What is the resulting compression force on the Earth? (Given: Radius of earth is 6400 km).
4. Two identical conducting small spheres are placed with their centers 0.300 m apart. One is given a charge of 12.0 nC and the other a charge of −18.0 nC.
   a. Find the electric force exerted by one sphere on the other?
   b. If the spheres are connected by a conducting wire, find the electric force between the two after they have come to equilibrium.
5. Four equal point charges each of magnitude $+Q$ are to be placed in equilibrium at the corners of a square. What should be the magnitude and sign of the point charge that should be placed at the center of square to do this job?
6. Two point electric charges of values q and $2q$ are kept at a distance d apart from each other in air. A third charge Q is to be kept along the same line in such a way that the net force acting on q and $2q$ is zero. Find the location of the third charge from charge 'q'.
7. Two fixed point charges $+4e$ and $+e$ unit are separated by a distance 'a'. Where the third point charge should be placed from $+4e$ charge for it to be in equilibrium.
8. Two identical particles are charged and held at a distance of 1 m from each other. They are found to be attracting each other with a force of 0.027 N. Now, they are connected by a conducting wire, so that charge flows between them. When the charge flow stops, they are found to be repelling each other with a force of 0.009 N. Find the initial charge on each particle.
9. Two similarly and equally charged identical metal spheres A and B repel each other with a force of 2×10^{-5} N. A third identical uncharged sphere C is touched with A and then placed at the mid-point between A and B. Find the net electric force on C.
10. Three point charges of +2 μC, −3 μC and −3 μC are kept at the vertices A, B and C respectively, of an equilateral triangle of side 20 cm as shown in the figure. What should be the sign and magnitude of the charge (q) to be placed at the mid point (M) of side BC so that the charge at A remains in equilibrium?

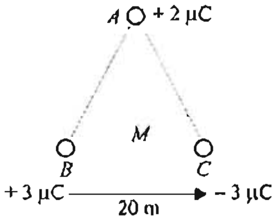


Fig. 1.106

11. Two small beads having positive charges $3q$ and q are fixed at the opposite ends of a horizontal, insulating rod, extending from the origin to the point $x = d$. As shown in figure, a third small charged bead is free to slide on the rod. At what position is the third bead in equilibrium? Can it be in stable equilibrium?

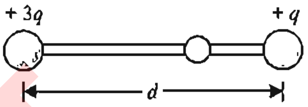


Fig. 1.107

12. A copper atom consists of copper nucleus surrounded by 29 electrons. The atomic weight of copper is 63.5 gmol^{-1}. Let us now take two pieces of copper each weighing 10 g. Let us consider one electron from one piece is transferred to another for every 1000 atoms in a piece.
    a. Find the magnitude of charge appearing on each piece.
    b. What will be the Coulomb force between the two pieces after the transfer of electrons if they are 10 cm apart?

    [Avogadro's number = 6×10^{23} mol^{-1}]
13. A flat square sheet of charge of side 50 cm carries a uniform surface charge density. An electron 0.5 cm from a point near the center of the sheet experiences a force of 1.8×10^{-12} N directed away from the sheet. Determine the total charge on the sheet.
14. Particle of mass 9×10^{-31} kg and a negative charge of 1.6×10^{-19} C is projected horizontally with a velocity of 10^6 ms^{-1} into a region between two infinite horizontal parallel plates of metal. The distance between the plates is d = 0.3 cm and the particle enters 0.1 cm below the top plate. The top and bottom plates are connected, respectively, to the positive and negative terminals of a 30 V battery. Find the components of the velocity of the particle just before it hits one of the plates.

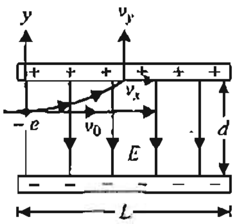


Fig. 1.108

15. A solid spherical region having a spherical cavity whose diameter 'R' is equal to the radius of the spherical region, has a total charge 'Q'. Find the electric field at a point P as shown.

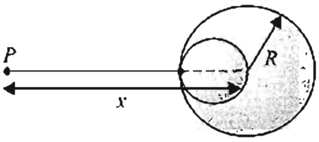


Fig. 1.109

16. A sphere of radius R has a uniform volume density ρ. A spherical cavity of radius b whose center lies at $\vec{r} = \vec{a}$ is removed from the sphere.

a. Find the electric field at any point inside the spherical cavity.

b. Find the electric field outside the cavity.

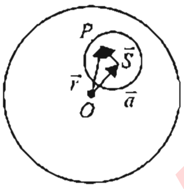


Fig. 1.110

17. A very long, solid insulating cylinder with radius R has a cylindrical hole with radius a bored along its entire length. The axis of the hole is a distance b from the axis of the cylinder, where $a < b < R$ (as shown in figure). The solid material of the cylinder has a uniform volume charge density ρ. Find the magnitude and direction of the electric field inside the hole, and show that this is uniform over the entire hole.

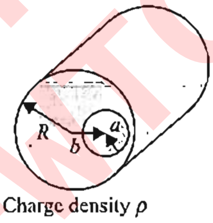


Fig. 1.111

18. Point charges q and $-q$ are located at the vertices of a square with diagonals $2l$ as shown in figure. Evaluate the magnitude of the electric field strength at a point located symmetrically with respect to the vertices of the square at a distance x from the center.

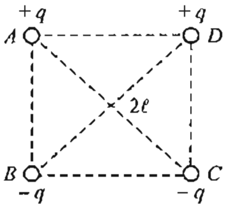


Fig. 1.112

19. Two mutually perpendicular long straight conductors carrying uniformly distributed charges of linear charge densities λ_1 and λ_2 are positioned at a distance a from each other. How does the interaction between the rods depend on a?

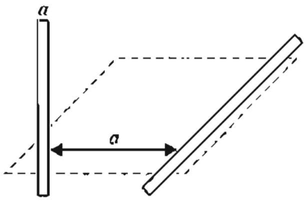


Fig. 1.113

20. A ring of radius 0.1 m is made out of a thin metallic wire of area of cross section 10^{-6} m^2. The ring has a uniform charge of π coulombs. Find the change in the radius of the ring when a charge of 10^{-8} coulomb is placed at the center of the ring. Young's modulus of the metal is 2×10^{11} Nm^{-2}.

21. A charged cork ball of mass m is suspended on a light string in the presence of a uniform electric field as shown in figure. When $E = (A\,\hat{i} + B\,\hat{j})$NC^{-1}, where A and B are positive numbers, the ball is in equilibrium at the angle θ. Find **a.** the charge on the ball and **b.** the tension in the string.

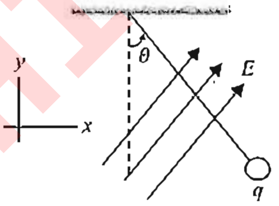


Fig. 1.114

22. A ring of radius R has charge $-Q$ distributed uniformly over it. Calculate the charge that should be placed at the center of the ring such that the electric field becomes zero at a point on the axis of the ring distant 'R' from the center of the ring.

23. Two identical small equally charged conducting balls are suspended from long threads secured at one point. The charges and masses of the balls are such that they are in equilibrium when the distance between them is a (the length of thread $L >> a$). One of the balls is then discharged. What will be the distance b ($b \ll l$) between the balls when equilibrium is restored?

24. Two point charges Q_a and Q_b are positioned at points A and B. The field strength to the right of charge Q_b on the line that passes through the two charges varies according to a law that is represented schematically in the figure accompanying the problem (without employing a definite scale). The field strength is assumed to be positive if its direction coincides with the positive direction of x-axis. The distance between the charges is l = 21 cm (Fig. 1.115). Find

a. the signs of the charges.

b. the ratio of the absolute values of charges Q_a and Q_b.

c. the coordinate x of the point where the field strength is maximum.

25. Two semicircular wires ABC and ADC each of radius 'R' are lying on x–y and x–z plane, respectively, as shown in the Fig. 1.116. If the linear charge density of the semicircular

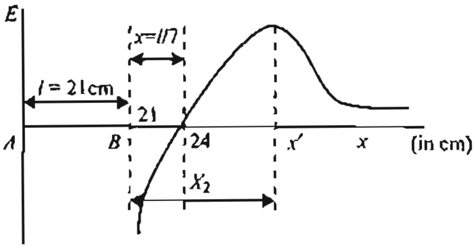


Fig. 1.115

parts and straight parts is λ, find the electric field intensity $\vec{E}$ at the origin.

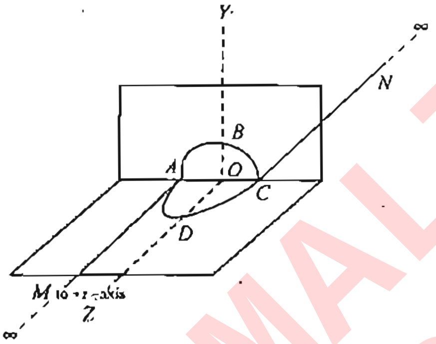


Fig. 1.116

26. An infinite wire having linear charge density λ is arranged as shown in the Fig. 1.117. A charge particle of mass m and charge q is released from point P. Find the initial acceleration of the particle (at $t = 0$) just after the particle is released.

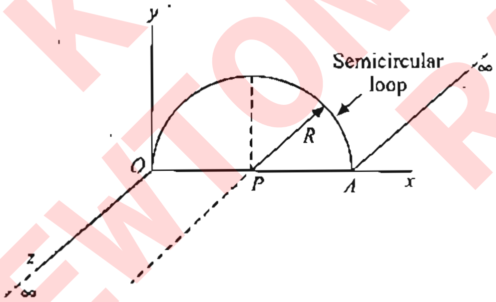


Fig. 1.117

27. Three small balls, each of mass m are suspended separately from a common point by three silk threads, each of length L. The balls are identically charged and hang at the corners of an equilateral triangle of side x. What is the charge on each ball?

28. Two similar balls, each of mass m and charge q, are hung from a common point by two silk threads, each of length l (Fig. 1.118). Prove that separation between the balls is

$$x = \left[\frac{q^2 l}{2\pi \varepsilon_0 mg}\right]^{1/3}, \text{ if } \theta \text{ is small.}$$

Find the rate $\frac{dq}{dt}$ with which the charge should leak off each sphere if their velocity of approach varies as $v = a/\sqrt{x}$, where a is a constant.

29. Three equal negative charges, $-q_1$ each, form the vertices of an equilateral triangle. A particle of mass m and a positive charge q_2 is constrained to move along a line perpendicular to the plane of triangle and through its center which is at a distance r from each of the negative charges as shown in figure. The whole system is kept in gravity free space.

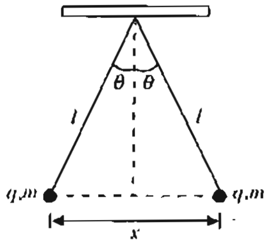


Fig. 1.118

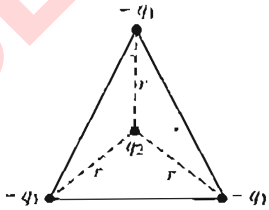


Fig. 1.119

Find the time period of vibration of the particle for small displacement from equilibrium position.

30. A ball of radius R carries a positive charge whose volume density at a point is given as $\rho = \rho_0(1 - r/R)$, where ρ_0 is a constant and r is the distance of the point from the center. Assuming the permittivities of the ball and the environment to be equal to unity, find

a. the magnitude of the electric field strength as a function of the distance r both inside and outside the ball

b. the maximum intensity $E_{\max}$ and the corresponding distance r_m.

31. The Fig. 1.120 shows two dipole moments parallel to each other and placed at a distance x apart. What is the magnitude of force of interaction? What is the nature of force, attractive or repulsive?

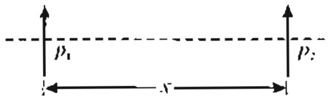


Fig. 1.120

32. Two dipoles p_1 and p_2 are placed along the same axis at a distance x apart, as shown in Fig. 1.121. What is magnitude of force of interaction? What is the nature of force, attractive or repulsive?

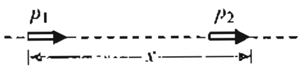


Fig. 1.121

33. A short dipole is placed along x-axis at $x = x$ (Fig. 1.122).

a. Find the force acting on the dipole due to a point charge q placed at origin.

b. Find the force on dipole if the dipole is rotated by 180° about z-axis.

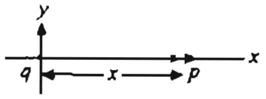


Fig. 1.122

c. Find the force on dipole if the dipole is rotated by 90° anti-clockwise about z-axis, i.e., it becomes parallel to y-axis.

Objective Type

Solutions on page 1.51

1. If a body is charged by rubbing it, its weight
   a. always decreases slightly
   b. always increases slightly
   c. may increase slightly or may decrease slightly
   d. remains precisely the same
2. In S.I. system, the value of ε_0 is
   a. $1\ C^2N^{-1}m^{-2}$
   b. $9 \times 10^9\ C^2N^{-1}m^{-2}$
   c. $\frac{1}{9 \times 10^9}\ C^2N^{-1}m^{-2}$
   d. $\frac{1}{4\pi \times 9 \times 10^9} C^2N^{-1}m^{-2}$
3. Dimensions of ε_0 are
   a. $M^{-1}L^{-3}T^4A^2$
   b. $M^0L^{-3}T^3A^3$
   c. $M^{-1}L^{-3}T^3A$
   d. $M^{-1}L^{-3}TA^2$
4. The dimensional formula of electric intensity is
   a. $MLT^{-2}A^{-1}$
   b. $MLT^{-3}A^{-1}$
   c. $ML^2T^{-3}A^{-1}$
   d. $ML^2T^{-3}A^{-2}$
5. The dielectric constant K of an insulator can be
   a. -1 b. 0 c. 0.5 d. 5
6. Choose the correct statement:
   a. The total charge of the universe is constant.
   b. The total number of the charged particles is constant.
   c. The total positive charge of the universe remains constant.
   d. The total negative charge of the universe remains constant.
7. Two neutrons are placed at some distance apart from each other. They will
   a. attract each other
   b. repel each other
   c. neither attract nor repel each other
   d. cannot say
8. When a soap bubble is charged, its size
   a. increases
   b. decreases
   c. remains the same
   d. increases if it is given positive charge and decreases if it is given negative charge
9. Two point charges certain distance apart in air repel each other with a force F. A glass plate is introduced between the charges. The force becomes F_1, where
   a. $F_1 < F$
   b. $F_1 = F$
   c. $F_1 > F$
   d. data is insufficient
10. There are two charges + 1 μC and + 5 μC. The ratio of the forces (force on one due to other) acting on them will be
    a. 1 : 1 b. 1 : 2 c. 1 : 3 d. 1 : 4
11. Two point charges Q_1 and Q_2 are 3 m apart, and their sum of charges is 10 μC. If force of attraction between them is 0.075 N, then the values of Q_1 and Q_2 respectively, are
    a. 5 μC, 5 μC
    b. 15 μC, −5 μC
    c. 5 μC, 15 μC
    d. −15 μC, 5 μC
12. A certain charge 'Q' is to be divided into two parts q and $Q - q$. What is the relationship of 'Q' to 'q' if the two parts, placed at a given distance 'r' apart are to have maximum Coulomb repulsion?
    a. $q = \frac{Q}{2}$
    b. $q = \frac{Q}{3}$
    c. $q = \frac{2Q}{2}$
    d. $q = \frac{Q}{4}$
13. Three charged particles are placed on a straight line as shown in figure. q_1 and q_2 are fixed but q_3 can be moved. Under the action of the forces from q_1 and q_2, q_3 is in equilibrium. What is the relation between q_1 and q_2?

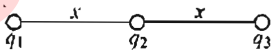


Fig. 1.123

    a. $q_1 = 4q_2$
    b. $q_1 = -q_2$
    c. $q_1 = -4q_2$
    d. $q_1 = q_2$
14. Two particles A and B (B is right of A) having charges 8×10^{-6} C and -2×10^{-6} C, respectively, are held fixed with separation of 20 cm. Where should a third charged particle be placed so that it does not experience a net electric force.
    a. 5 cm right of B
    b. 5 cm left of A
    c. 20 cm left of A
    d. 20 cm right of B
15. Five balls numbered 1, 2, 3, 4, 5 are suspended using separate threads. The balls (1, 2), (2, 4) and (4, 1) show electrostatic attraction, while balls (2, 3) and (4, 5) show repulsion. Therefore, ball 1 must be
    a. negatively charged
    b. positively charge
    c. neutral
    d. made of metal
16. Electric charges A and B repel each other. Electric charges B and C also repel other. If A and C are held close together, they will
    a. attract
    b. repel
    c. not affect each other
    d. none of these
17. Two point charges repel each other with a force of 100 N. One of the charges is increased by 10% and the other is reduced by 10%. The new force of repulsion at the same distance would be
    a. 100 N
    b. 121 N
    c. 99 N
    d. none of these

18. Three charges $+Q_1$, $+Q_2$ and q are placed on a straight line such that q is somewhere in between $+Q_1$ and $+Q_2$. If this system of charges is in equilibrium, what should be the magnitude and sign of charge q?

a. $\dfrac{Q_1Q_2}{(\sqrt{Q_1}+\sqrt{Q_2})^2}$, + ve

b. $\dfrac{Q_1+Q_2}{2}$, + ve

c. $\dfrac{Q_1Q_2}{(\sqrt{Q_1}+\sqrt{Q_2})^2}$, – ve

d. $\dfrac{Q_1+Q_2}{2}$, – ve

19. Two positive and equal charges are fixed at a certain distance. A third small charge is placed in between the two charges and it experiences zero net force due to the other two.

a. The equilibrium is stable if small charge is positive

b. The equilibrium is stable if small charge is negative

c. The equilibrium is always stable

d. The equilibrium is not stable

20. An isolated charge q_1 of mass m is suspended freely by a thread of length l. Another charge q_2 is brought near it ($r >> l$). When q_1 is in equilibrium, tension in thread will be

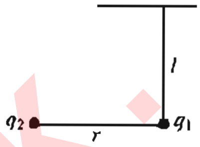


Fig. 1.124

a. mg **b.** $>mg$

c. $<mg$ **d.** none of these

21. Three equal charges, each $+q$, are placed on the corners of an equilateral triangle of side a. Then, the coulomb force experienced by one charge due to the rest of the two is

a. kq^2/a^2 **b.** $2kq^2/a^2$

c. $\sqrt{3}kq^2/a^2$ **d.** zero

22. A positively charged ball hangs from a long silk thread. Electric field at a certain point (at the same horizontal level of ball) due to this charge is E. Let us put a positive test charge q_0 at this point and measure F/q_0 on this charge. Then, E

a. $>F/q_0$ **b.** $<F/q_0$

c. $=F/q_0$ **d.** none of these

23. Electric field near a straight wire carrying a steady current is

a. proportional to the distance from the wire

b. proportional to inverse square of the distance from the wire

c. inversely proportional to the distance from the wire

d. zero

24. A force of 2.25 N acts on a charge of 15×10^{-4} C. Calculate the intensity of electric field at the point.

a. 1500 NC^{-1} **b.** 150 NC^{-1}

c. 15000 NC^{-1} **d.** none of these

25. An α particle is situated in an electric field of strength 15×10^4 NC^{-1}. Force acting on it is

a. 4.8×10^{-12} N **b.** 4.8×10^{-14} N

c. 48×10^{-14} N **d.** none of these

26. Two particles of masses in the ratio 1 : 2, with charges in the ratio 1 : 1, are placed at rest in a uniform electric field. They are released and allowed to move for the same time. The ratio of their kinetic energies will be finally

a. 2 : 1 **b.** 8 : 1 **c.** 4 : 1 **d.** 1 : 4

27. Three equal charges, each $+q$, are placed on the corners of an equilateral triangle. The electric field intensity at the centroid of the triangle is

a. kq/r^2 **b.** $3kq/r^2$

c. $\sqrt{3}kq/r^2$ **d.** zero

28. A point charge of 100 μC is placed at $3\hat{i}+4\hat{j}$ m. Find the electric field intensity due to this charge at a point located at $9\hat{i}+12\hat{j}$ m.

a. 8000 Vm^{-1} **b.** 9000 Vm^{-1}

c. 2250 Vm^{-1} **d.** 4500 Vm^{-1}

29. Electric lines of force

a. exist everywhere

b. exist only in the immediate vicinity of electric charges

c. exist only when both positive and negative charges are near one another

d. are imaginary

30. Two charges $Q_1 = 18$ μC and $Q_2 = -2$ μC are separated by a distance R and Q_1 is to the left of Q_2. The distance of the point where the net electric field is zero is

a. between Q_1 and Q_2 **b.** left of Q_1 at $R/2$

c. right of Q_2 at R **d.** right of Q_2 at $R/2$

31. Determine the electric field intensity at point P due to quadruple distribution shown in figure for $r >> a$.

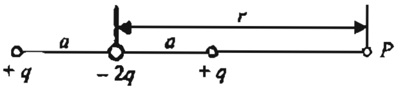


Fig. 1.125

a. 0 **b.** kqa^2/r^4

c. $6kqa^2/r^4$ **d.** $6kqa^2/r^2$

32. An oil drop, carrying six electronic charges and having a mass of 1.6×10^{-12} g, falls with some terminal velocity in a medium. What magnitude of vertical electric field is required to make the drop move upward with the same speed as it was formerly moving downward with? Ignore buoyancy.

a. 10^5 NC^{-1} **b.** 10^4 NC^{-1}

c. 3.3×10^4 NC^{-1} **d.** 3.3×10^5 NC^{-1}

33. What is the largest charge a metal ball of 1 mm radius can hold? Dielectric strength of air is 3×10^6 Vm^{-1}.

a. 3 nC **b.** 1/3 nC
c. 2 nC **d.** 1/2 nC

34. Five point charges, $+q$ each, are placed at the five vertices of a regular hexagon. The distance of center of hexagon from any of the vertices is a. The electric field at the center of the hexagon is

a. $\frac{q}{4\pi\varepsilon_0 a^2}$ **b.** $\frac{q}{8\pi\varepsilon_0 a^2}$
c. $\frac{q}{16\pi\varepsilon_0 a^2}$ **d.** zero

35. A ring of charge with radius 0.5 m has 0.002π m gap. If the ring carries a charge of +31 C, the electric field at the center is

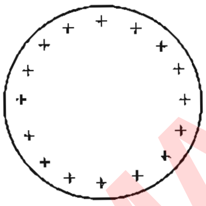


Fig. 1.126

a. 7.5×10^7 NC^{-1} **b.** 7.2×10^7 NC^{-1}
c. 6.2×10^7 NC^{-1} **d.** 6.5×10^7 NC^{-1}

36. A block of mass m containing a net negative charge $-q$ is placed on a frictionless horizontal table and is connected to a wall through an unstretched spring of spring constant k as shown. If horizontal electric field E parallel to the spring is switched on, then the maximum compression of the spring is

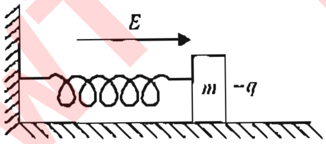


Fig. 1.127

a. $\sqrt{qE/k}$ **b.** $2qE/k$
c. qE/k **d.** zero

37. Figure shows the electric lines of force emerging from a charged body. If the electric fields at A and B are E_A and E_B, respectively, and if the distance between A and B is r, then

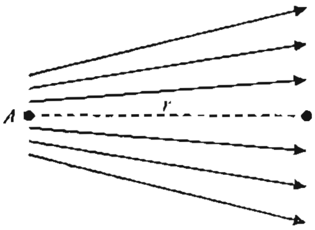


Fig. 1.128

a. $E_A > E_B$ **b.** $E_A < E_B$
c. $E_A = E_B/r$ **d.** $E_A = E_B/r^2$

38. If an electron has an initial velocity in a direction different from that of a uniform electric field, the path of the electron is

a. a straight line **b.** a circle
c. an ellipse **d.** a parabola

39. An electron is taken from a point A to point B along the path AB in a uniform electric field of intensity $E = 10$ Vm^{-1}. Side $AB = 5$ m and side $BC = 3$ m. Then, the amount of work done is

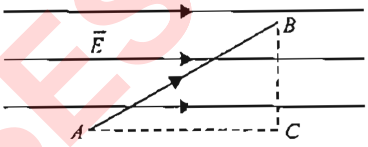


Fig. 1.129

a. 50 eV **b.** 40 eV
c. −50 eV **d.** −40 eV

40. A point charge q_1 is moved along a circular path of radius r in the electric field of another point charge q_2 at the center of the path. The work done by the electric field on the charge q_1 in half revolution is

a. zero **b.** positive
c. negative **d.** none of these

41. A spherical conducting ball is suspended by a grounded conducting thread. A positive point charge is moved near the ball. The ball will

a. be attracted to the point charge and swing toward it.
b. be repelled from the point charge and swing away from it.
c. not be affected by the point charge
d. none of these

42. Two point charges are located on the positive x-axis of a coordinate system (as shown in figure). Charge $q_1 = 1.0$ nC is 2.0 cm from the origin, and charge $q_2 = -3.0$ nC is 4.0 cm from the origin. What is the total force exerted by these two charges on a charge $q_3 = 5.0$ nC located at the origin? Gravitational forces are negligible.

a. 28μN directed to the left
b. 28 μN directed to the right
c. 196 μN directed to the left
d. 196 μN directed to the right

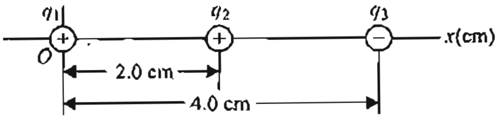


Fig. 1.130

43. Three +ve charges of equal magnitude 'q' are placed at the vertices of an equilateral triangle of side 'l'. How can the system of charges be placed in equilibrium?

a. By placing a charge $Q = \left(-\frac{q}{\sqrt{3}}\right)$ at the centroid of the triangle

b. By placing a charge $Q = \left(\frac{q}{\sqrt{3}}\right)$ at the centroid of the triangle
c. By placing a charge $Q = q$ at a distance l from all the three charges
d. By placing a charge $Q = -q$ above the plane of the triangle at a distance l from all the three charges

44. In figure, two equal positive point charges $q_1 = q_2 = 2.0\ \mu C$ interact with a third point charge $Q = 4.0\ \mu C$. Find the magnitude and direction of the net force on Q.

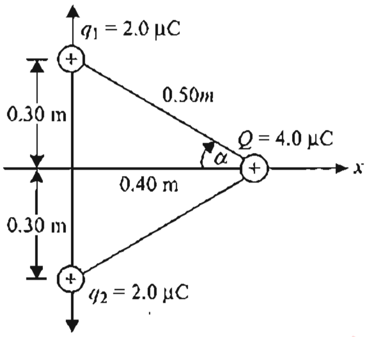


Fig. 1.131

a. 0.23 N in $+x$ direction
b. 0.46 N in $+x$ direction
c. 0.23 N in $-x$ direction
d. 0.46 N in $-x$ direction

45. Three identical spheres, each having a charge q and radius R, are kept in such a way that each touches the other two. Find the magnitude of the electric force on any sphere due to other two.

a. $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$ b. $\frac{\sqrt{3}}{4\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$
c. $\frac{\sqrt{3}}{16\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$ d. $\frac{\sqrt{5}}{16\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$

46. Five point charges, each of value $+q$, are placed on five vertices of a regular hexagon of side L. What is the magnitude of the force on a point charge of value $-q$ coulomb placed at the center of the hexagon?

a. $\frac{1}{\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$ b. $\frac{2}{\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$
c. $\frac{1}{2\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$ d. $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$

47. It is required to hold equal charges, q, in equilibrium at the corners of a square. What charge when placed at the center of the square will do this?

a. $-\frac{q}{2}\left(1+2\sqrt{2}\right)$ b. $\frac{q}{2}\left(1+2\sqrt{2}\right)$
c. $\frac{q}{4}\left(1+2\sqrt{2}\right)$ d. $-\frac{q}{4}\left(1+2\sqrt{2}\right)$

48. A point charge $q = -8.0$ nC is located at the origin. Find the electric field (in NC^{-1}) vector at the point $x = 1.2$ m, $y = -1.6$ m (as shown in Fig. 1.132).

a. $-14.4\hat{i} + 10.8\hat{j}$ b. $-14.4\hat{i} - 10.8\hat{j}$
c. $-10.8\hat{i} + 14.4\hat{j}$ d. $-10.8\hat{i} - 14.4\hat{j}$

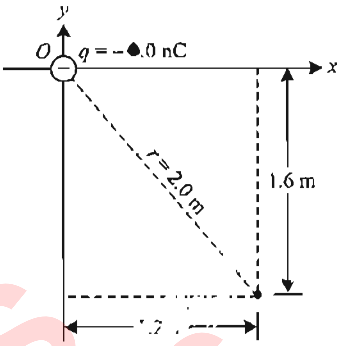


Fig. 1.132

49. A positive point charge 50 μC is located in the plane xy at a point with radius vector $\vec{r}_0 = 2\hat{i} + 3\hat{j}$. Evaluate the electric field vector $\vec{E}$ at a point with radius vector $\vec{r} = 8\hat{i} - 5\hat{j}$, where r_0 and r are expressed in meters.

a. $(1.4\hat{i} - 2.6\hat{j})\ kNC^{-1}$ b. $(1.4\hat{i} + 2.6\hat{j})\ kNC^{-1}$
c. $(2.7\hat{i} - 3.6\hat{j})\ kNC^{-1}$ d. $(2.7\hat{i} + 3.6\hat{j})\ kNC^{-1}$

50. A charge $q = 1\ \mu C$ is placed at point (3 m, 2 m, 5 m). Find the electric field vector at point P (0 m, −4 m, 3 m).

a. $-\frac{9}{343}(3\hat{i} + 6\hat{j} + 2\hat{k})\ kNC^{-1}$
b. $\frac{9}{343}(3\hat{i} - 6\hat{j} + \hat{k})\ kNC^{-1}$
c. $\frac{3}{343}(3\hat{i} + 6\hat{j} + 2\hat{k})\ kNC^{-1}$
d. $\frac{9}{343}(3\hat{i} + 6\hat{j} + 2\hat{k})\ kNC^{-1}$

51. Four identical charges Q are fixed at the four corners of a square of side a. Find the electric field at a point P located symmetrically at a distance $\frac{a}{\sqrt{2}}$ from the center of the square.

a. $\frac{Q}{2\sqrt{2}\pi\varepsilon_0 a^2}$ b. $\frac{Q}{\sqrt{2}\pi\varepsilon_0 a^2}$
c. $\frac{2\sqrt{2}\,Q}{\pi\varepsilon_0 a^2}$ d. $\frac{\sqrt{2}\,Q}{\pi\varepsilon_0 a^2}$

52. A thin glass rod is bent into a semicircle of radius r. A charge $+Q$ is uniformly distributed along the upper half and a charge $-Q$ is uniformly distributed along the lower half, as shown in Fig. 1.133. Calculate electric field E at P, the center of semicircle.

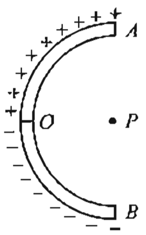


Fig. 1.133

a. $\frac{Q}{\pi^2\varepsilon_0 r^2}$ b. $\frac{2Q}{\pi^2\varepsilon_0 r^2}$

c. $\dfrac{4Q}{\pi^2 \varepsilon_0 r^2}$ d. $\dfrac{Q}{4\pi^2 \varepsilon_0 r^2}$

53. A system consists of a thin charged wire ring of radius r and a very long uniformly charged wire oriented along the axis of the ring, with one of its ends coinciding with the center of the ring. The total charge on the ring is q and the linear charge density on the straight wire is λ. Evaluate the interaction force between the ring and the wire.

a. $\dfrac{\lambda q}{4\pi \varepsilon_0 r}$ b. $\dfrac{\lambda q}{2\sqrt{2}\pi \varepsilon_0 r}$

c. $\dfrac{2\sqrt{2}\lambda q}{\pi \varepsilon_0 r}$ d. $\dfrac{4\lambda q}{\pi \varepsilon_0 r}$

54. Find the electric field vector at $P(a,\ a,\ a)$ due to three infinitely long lines of charges along x-, y- and z-axes respectively. The charge density, i.e., charge per unit length of each wire is λ.

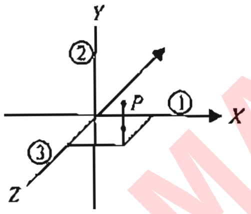


Fig. 1.134

a. $\dfrac{\lambda}{3\pi \varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$ b. $\dfrac{\lambda}{2\pi \varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$

c. $\dfrac{\lambda}{2\sqrt{2}\pi \varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$ d. $\dfrac{\sqrt{2}\lambda}{\pi \varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$

55. A particle of mass m and charge $-q$ moves diametrically through a uniformly charged sphere of radius R with total charge Q. The angular frequency of the particle's simple harmonic motion, if its amplitude $< R$, is given by

a. $\sqrt{\dfrac{1}{4\pi \varepsilon_0}\dfrac{qQ}{mR}}$ b. $\sqrt{\dfrac{1}{4\pi \varepsilon_0}\dfrac{qQ}{mR^2}}$

c. $\sqrt{\dfrac{1}{4\pi \varepsilon_0}\dfrac{qQ}{mR^3}}$ d. $\sqrt{\dfrac{1}{4\pi \varepsilon_0}\dfrac{m}{qQ}}$

56. A particle of mass m carrying a positive charge q moves simple harmonically along x-axis under the action of a varying electric field E directed along x-axis. The motion of the particle is confined between $x = 0$ and $x = 2l$. The angular frequency of the motion is ω. Then, which of the following is correct?

a. $qE = -m\omega^2(x - l)$

b. $qE = m\omega^2(x - l)$

c. Electric field to the right of origin is directed along +ve x-axis for all values of x.

d. Electric field to the right of origin is directed along −ve x-axis for all values of x.

57. A circular ring carries a uniformly distributed positive charge and lies in X–Y plane with center at origin of co-ordinate system. If at a point $(0,\ 0,\ z)$ the electric field is E, then which of the following graphs is correct?

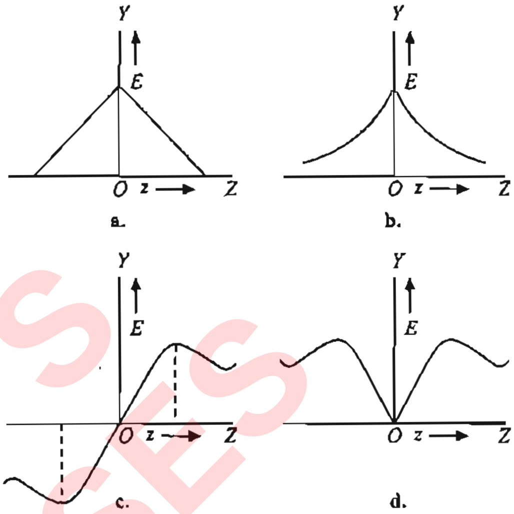


Fig. 1.135

58. Two identical positive charges are fixed on the y-axis, at equal distance from the origin O, a negatively charged particle starts on the x-axis at a large distance form O, moves along the x-axis, passes through O and moves far away from O. Its acceleration a is taken as positive along its direction of motion. The particle's acceleration a is plotted against its x-coordinate. Which of the following best represents the plot?

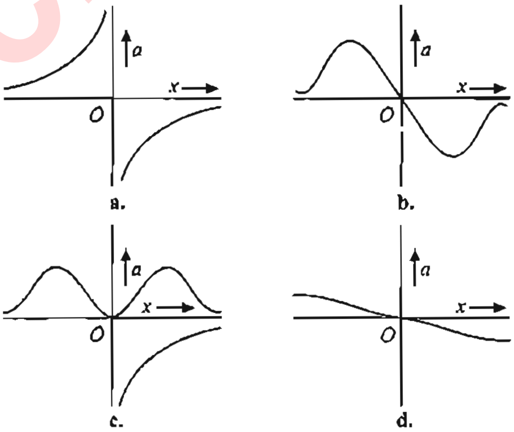


Fig. 1.136

Multiple Correct Answers Type

Solutions on page 1.57

1. A wire having a uniform linear charge density λ, is bent in the form of a ring of radius R. Point A as shown in the Fig. 1.137, is in the plane of the ring but not at the center. Two elements of the ring of lengths a_1 and a_2 subtend very small same angle at the point A. They are at distances r_1 and r_2 from the point A, respectively. $(r_2 > r_1)$

a. The ratio of charges of elements a_1 and a_2 is r_1/r_2.

b. The element a_1 produced greater magnitude of electric field at A than element a_2.

c. The elements a_1 and a_2 produce same potential at A.

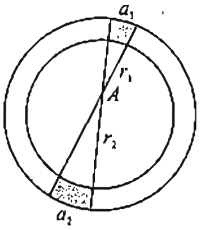


Fig. 1.137

d. The direction of net electric field produced by elements only at A is towards element a_2.

2. For the arrangement shown in the Fig. 1.138, the two positive charges, $+Q$ each, are fixed. Mark out the correct statement(s) regarding a third charged particle q placed at mid point P that can be displaced along or perpendicular to the line connecting the charges.

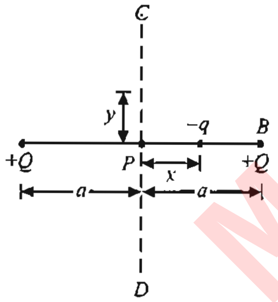


Fig. 1.138

a. The particle will perform SHM for $x << a$.
b. The particle will oscillate about P but not harmonically for any x.
c. The particle will perform SHM for $y << a$.
d. The particle will oscillate about P but not harmonically for y comparable to a.

3. A particle of mass m and charge $-q$ has been projected from ground as shown in the Fig. 1.139 below. Mark out the correct statement(s).

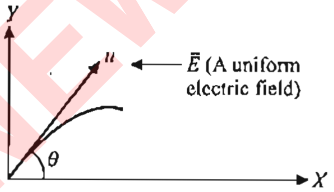


Fig. 1.139

a. The path of motion of the particle is parabolic.
b. The path of motion of the particle is a straight line.
c. Time of flight of the particle is $\frac{2u \sin\theta}{g}$.
d. Range of motion of the particle can be less than, greater than or equal to $\frac{u^2 \sin 2\theta}{g}$.

4. Two point charges of different magnitudes and of opposite signs are separated by some distance. There can be
a. only one point in space where net electric field intensity is zero
b. only two points in space where net electric potential is zero
c. infinite number of points in space where net electric field intensity is zero
d. infinite number of points in space where net electric potential is zero

5. For the arrangement shown in Fig. 1.140, the two point charges are in equilibrium. The infinite wire is fixed in the horizontal plane and the two point charges are placed one above and the other below the wire.
Considering the gravitational effect of the earth, the nature of q_1 and q_2 can be

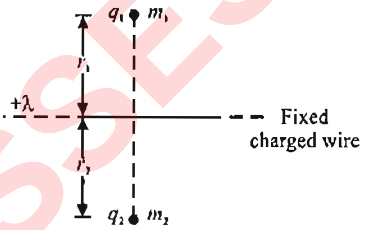


Fig. 1.140

a. $q_1 \rightarrow$ +ve, $q_2 \rightarrow$ +ve
b. $q_1 \rightarrow$ +ve, $q_2 \rightarrow$ −ve
c. $q_1 \rightarrow$ −ve, $q_2 \rightarrow$ −ve
d. $q_1 \rightarrow$ −ve, $q_2 \rightarrow$ +ve

Assertion-Reasoning Type

Solutions on page 1.57

In the following questions, each question contains Statement I (Assertion) and Statement II (Reason). Each question has four choices (a), (b), (c) and (d), out of which **ONLY ONE** is correct.

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
b. Statement I is True, Statement II is True; Statement II is **NOT** a correct explanation for Statement I.
c. Statement I is True, Statement II is False.
d. Statement I is False, Statement II is True.

1. **Statement I:** If a point charge be rotated in a circle around another charge at the center of circle, the work done by electric field will be zero.
Statement II: Work done is equal to dot product of force and displacement.

2. **Statement I:** A positive point charge initially at rest in a uniform electric field starts moving along electric lines of forces. (Neglect all other forces except electric forces)
Statement II: A point charge released from rest in an electric field always moves along the lines of force.

3. **Statement I:** When a neutral body acquires +ve charge, its mass decreases.
Statement II: A body acquires +ve charge when it loses electrons.

4. **Statement I:** Two similarly charged bodies may attract each other.
   Statement II: When charge on one body (Q) is much greater than that on another (q) and they are close enough to each other, then force of attraction between Q and induced charges exceeds the force of repulsion between Q and q.
5. **Statement I:** Charge is quantized because only integral number of electrons can be transferred.
   Statement II: There is no possibility of transfer of some fraction of electron.

Comprehension Type

Solutions on page 1.57

For Problems 1–2

Two small identical conducting balls A and B of charges of + 10 μC and +30 μC, respectively, are kept at a separation of 50 cm. These balls have been connected by a wire for a short time.

1. The final charge on each of the balls A and B will be
   - **a.** 10 μC and 30 μC, respectively
   - **b.** 20 μC on each ball
   - **c.** 30 μC and 10 μC, respectively
   - **d.** –40 μC and 80 μC, respectively
2. The force of interaction between the balls is
   a. 28.8 N **b.** 32.6 N **c.** 14.4 N **d.** 72 N

For Problems 3–5

Two free point charges A and B having charges $+q$ and $+4q$, respectively, are a distance l apart. A third charge is so placed that the entire system is in equilibrium.

3. The third charge should be placed
   - **a.** left of A at a distance $\frac{l}{3}$ from A
   - **b.** right of A at a distance $\frac{l}{3}$ from B
   - **c.** between A and B at a distance $\frac{2l}{3}$ from A
   - **d.** between A and B at a distance $\frac{l}{3}$ from A
4. The third charge has magnitude and sign
   - **a.** $Q = \left(-\frac{4}{9}\right) q$
   - **b.** $Q = \left(\frac{4}{9}\right) q$
   - **c.** $Q = \left(\frac{3}{5}\right) q$
   - **d.** $Q = \left(-\frac{3}{5}\right) q$
5. Two charges of $+Q$ each are placed at two opposite corners of a square. A charge q is placed at each of the other two corners. If the resultant force on Q is zero, what should be the value of q in terms of Q?
   - **a.** $q = \frac{q}{(3\sqrt{2})}$
   - **b.** $q = \frac{Q}{(2\sqrt{2})}$
   - **c.** $q = -\frac{Q}{(2\sqrt{2})}$
   - **d.** $q = -\frac{Q}{(3\sqrt{2})}$

For Problems 6–8

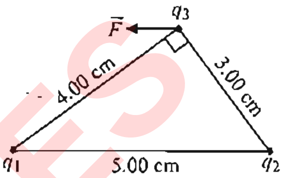


Fig. 1.141

Three charges are placed as shown in Fig. 1.141. The magnitude of q_1 is 2.00 μC, but its sign and the value of the charge q_2 are not known. Charge q_3 is +4.00 μC, and the net force on q_3 is entirely in the negative x-direction.

6. As per the condition given in the problem the sign of q_1 and q_2 will be
   a. +, + **b.** +, – **c.** –, + **d.** –, –
7. The magnitude of q_2 is
   a. $\frac{27}{64}$ μC **b.** $\frac{27}{32}$ μC **c.** $\frac{13}{32}$ μC **d.** $\frac{13}{64}$ μC
8. The magnitude of force acting on q_3 is
   a. 25.2 N **b.** 32.2 N **c.** 56.2 N **d.** 13.5 N

For Problems 9–10

A hollow conducting ball has a single positive charge $+q$ fixed at the center. The ball has no net charge.

9. The charge on the inner surface of the ball is
   a. $+2q$ **b.** $+q$ **c.** $-q$ **d.** 0
10. The charge on the outer surface of the ball is
   a. $+2q$ **b.** $+q$ **c.** $-q$ **d.** 0

For Problems 11–12

Suppose that a net charge $+q$ is placed on the ball in the previous question; the point charge remains at its center.

11. The charge on the inner surface of the ball is
   a. $+2q$ **b.** $+q$ **c.** $-q$ **d.** 0
12. The charge on the outer surface of the ball is
   a. $+2q$ **b.** $+q$ **c.** $-q$ **d.** 0

For Problems 13–14

The positive charge at the center of the ball in question 9–10 is moved off center closer to the inner surface, but it does not touch the inner surface.

13. The total charge on the inner surface of ball will
   - **a.** increase
   - **b.** decrease
   - **c.** remain the same
   - **d.** change, depending on how close the ball gets to the inner surface

14. The total charge on the outer surface of ball will

a. increase

b. decrease

c. remain the same

d. change, depending on how close the ball gets to the inner surface

For Problems 15–17

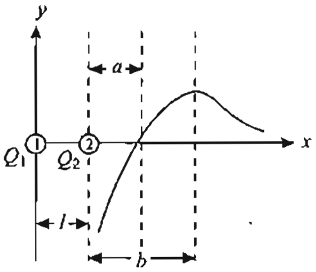


Fig. 1.142

Two point like charges Q_1 and Q_2 are positioned at points 1 and 2. The field intensity to the right of the charge Q_2 on the line that passes through the two charges varies according to a law that is represented schematically in the Fig. 1.142. The field intensity is assumed to be positive if its direction coincides with the positive direction on the x-axis. The distance between the charges is l.

15. The sign of each charge Q_1 and Q_2 is

a. +, − b. −, + c. +, + d. −, −

16. Find the ratio of the absolute values of the charges $\left|\frac{Q_1}{Q_2}\right|$.

a. $\left(\frac{a+l}{a}\right)^2$ b. $\left(\frac{l}{a}\right)^2$ c. $\left(\frac{a}{a+l}\right)^2$ d. $\left(\frac{a}{l}\right)^2$

17. Find the value of b where the field intensity is maximum.

a. $\frac{l}{(Q_1/Q_2)^{1/3}+1}$ b. $\frac{l}{(Q_1/Q_2)^{1/3}-1}$

c. $\frac{l}{(Q_2/Q_1)^{1/3}+1}$ d. $\frac{l}{(Q_2/Q_1)^{1/3}-1}$

For Problems 18–19

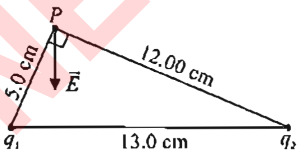


Fig. 1.143

Two charges are placed as shown in Fig. 1.143. The magnitude of q_1 is 3.00 μC, but its sign and the value of charge q_2 are not known. The direction of net electric field at point P is entirely in the negative y-direction.

18. The signs of q_1 and q_2 is

a. +, − b. −, + c. +, + d. −, −

19. Determine the magnitude of $\vec{E}$

a. 2.30×10^7 NC^{-1} b. 1.17×10^7 NC^{-1}

c. 3.55×10^7 NC^{-1} d. 4.2×10^7 NC^{-1}

For Problems 20–21

Four equal positive charges, each of value Q, are arranged at the four corners of a square of diagonal $2a$. A small body of mass m and carrying a unit positive charge is placed at a height h above the center of the square.

20. What should be the value of Q in order that this body may be in equilibrium ?

a. $\pi\varepsilon_0 \frac{mg}{2h}(h^2 + 2a^2)^{3/2}$ b. $\pi\varepsilon_0 \frac{mg}{h}(h^2 + a^2)^{3/2}$

c. $\pi\varepsilon_0 \frac{2mg}{h}(h^2 + 2a^2)^{3/2}$ d. $\pi\varepsilon_0 \frac{mg}{2h}(h^2 - a^2)^{3/2}$

21. The type of equilibrium of the point mass is

a. stable equilibrium b. unstable equilibrium

c. neutral equilibrium d. cannot be determined

For Problems 22–25

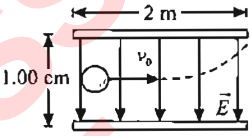


Fig. 1.144

An electron is projected with an initial speed $v_0 = 1.60 \times 10^8$ ms^{-1} into the uniform field between the parallel plates as shown in Fig. 1.144. Assume that the field between the plates is uniform and directed vertically downward, and that the field outside the plates is zero. The electron enters the field at a point midway between the plates. Mass of electron = 9.1×10^{-31} kg.

22. If the electron just misses the upper plate, the time of flight of electron upto this instant is

a. 1.25×10^{-9} sec b. 32.5×10^{-6} sec

c. 1.25×10^{-8} sec d. 32.5×10^{-8} sec

23. For condition of previous situation, the magnitude of electric field is

a. 124 NC^{-1} b. 364 NC^{-1} b. 224 NC^{-1} d. 520 NC^{-1}

24. If instead of electron, a proton were projected with the same speed, then compare the paths traveled by the electron and the proton.

a. The proton will hit the upper plate.

b. The proton will hit the lower plate.

c. The proton will not hit either plate.

d. None of these.

25. The vertical displacement traveled by the proton as it exits the region between the plates is

a. 1.6×10^{-8} m b. 3.25×10^{-8} m

c. 5.25×10^{-6} m d. 2.73×10^{-6} m

For Problems 26–27

An electron is projected as shown in Fig. 1.444, with kinetic energy K, at an angle $\theta = 45°$ between two charged plates. Ignore gravity.

26. The magnitude of the electric field, so that the electron just fails to strike the upper plate, should be greater than

a. K/qd b. $2K/qd$ c. $K/2qd$ d. infinite

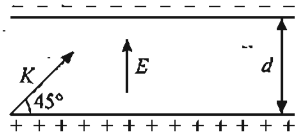


Fig. 1.145

27. At what distance from the starting point will the electron strike the lower plate?

a. d **b.** $2d$ **c.** $3d$ **d.** $4d$

For Problems 28–29

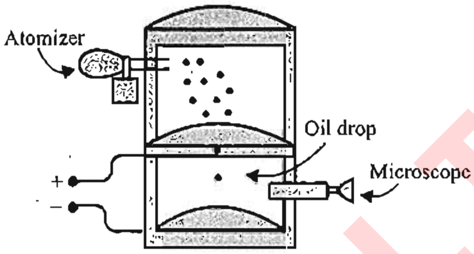


Fig. 1.146

In 1909, Robert Millikan was the first to find the charge of an electron in his now-famous oil-drop experiment. In that experiment, tiny oil drops were sprayed into a uniform electric field between a horizontal pair of oppositely charged plates. The drops were observed with a magnifying eyepiece, and the electric field was adjusted so that the upward force on some negatively charged oil drops was just sufficient to balance the downward force of gravity. That is, when suspended, upward force qE just equaled mg. Millikan accurately measured the charges on many oil drops and found the values to be whole number multiples of 1.6×10^{-19} C— the charge of the electron. For this, he won the Nobel prize.

28. If a drop of mass 1.1×10^{-14} kg remains stationary in an electric field of 1.68×10^5 NC^{-1}, what is the charge of this drop?

a. 6.40×10^{-19} **b.** 3.2×10^{-19}
c. 1.6×10^{-19} **d.** 4.8×10^{-19}

29. How many extra electrons are on this particular oil drop (given the presently known charge of the electron)?

a. 4 **b.** 3 **c.** 5 **d.** 8

Matching Column Type

Solutions on page 1.60

1. In the following Fig. 1.147, charges, each $+q$, are fixed at L and M. O is the mid point of distance LM. X- and Y- axes are as shown. Consider the situations given in column I and match them with the information in Column II:

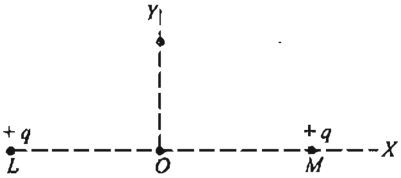


Fig. 1.147

Column I	Column II
i. Let us place a charge $+q$ at O, displace it slightly along X-axis and release. Assume that it is allowed to move only along X-axis. At position O,	**a.** force o the charge i zero
ii. Place a charge $-q$ at O. Displace it slightly along X-axis and release. Assume that it is allowed to move only along X-axis. At position O,	**b.** potentia energy o the syste is maximu
iii. Place a charge $+q$ at O. Displace it slightly along Y-axis and release. Assume that it is allowed to move only along Y-axis. At position O,	**c.** potentia energy o the syste is minimu
iv. Place a charge $-q$ at O. Displace it slightly along Y-axis and release. Assume that it is allowed to move only along Y-axis. At position O,	**d.** the charg is in equilibrium

2. Match the forces given in Column I with the properties give in Column II:

Column I	Column II
i. Electric force of nucleus on electron	**a.** Conservative
ii. Your weight, i.e., the force that earth exerts on you	**b.** Action reaction force
iii. Force between earth and sun	**c.** Depends on the nature o medium between the interacting objects.
iv. Repulsive force between two protons	**d.** Principle of superposition applies provided that other forces that are acting on the object, ar also of this nature

3. Match Column I with Column II:

Column I	Column II
i. Force on an electron in an atom	**a.** Gravitational force
ii. Force between a proton and a neutron inside nucleus	**b.** Strong force
iii. Force between a proton and proton inside nucleus	**c.** Coulomb force
iv. Conservatuve force	**d.** Electric force

4. Match the facts given in Column I with the systems given Column II:

Column I	Column II
i. Charge cannot exist	**a.** without charge
ii. Mass can exist	**b.** without mass
iii. Charge is	**c.** not conserved
iv. Mass is	**d.** conserved

5.

Column I	Column II
i. ε_0	**a.** 1
ii. K	**b.** $[M^1L^1T^{-2}]$
iii. E	**c.** $[M^{-1}L^{-3}T^4A^2]$
iv. F	**d.** $[M^1L^1T^{-3}A^{-1}]$

ANSWERS AND SOLUTIONS

Subjective Type

1. No. of atom is 10 g of silver,

$$n=\frac{6.023\times 10^{23}\times 10}{107.87}=5.58\times 10^{22}$$

No. of electron = $47n$ = 2.62×10^{24}

2. $qE = 6\pi\eta rv$

$\Rightarrow NeE = 6\pi\eta rv \Rightarrow \quad N = 6\pi\eta rv/eE$

$$\Rightarrow \quad N=\frac{6\pi\times 1.6\times 10^{-5}\times 5\times 10^{-7}\times 0.02}{1.6\times 10^{-19}\times 6.28\times 10^{5}}$$

$$=3\times 10^{11}$$

3. No. of electrons or protons in 1.00 gm of hydrogen

$$n=2\left[\frac{1}{2}\times 6.023\times 10^{23}\right]=6.023\times 10^{23}$$

Magnitude of charge on north or south pole:

$$q=ne=6.023\times 10^{23}\times 1.6\times 10^{-19}=96368\text{ C}$$

$$F=\frac{kq^2}{r^2}=\frac{9\times 10^9\times (96368)^2}{(2\times 6400\times 10^3)^2}=5.10\times 10^5\text{ N}$$

$$=510\text{ kN}$$

4. a. $F=\dfrac{9\times 10^9\times 12\times 10^{-9}\times 18\times 10^{-9}}{(0.30)^2}$

$= 2.16\times 10^{-5}$ N attractive

b. When they are connected by a conducting wire, finally charge on each will be half of total charge on both. Let q is the final charge on each, then $q=\dfrac{12-18}{2}=-3$ nC

$$F'=\frac{9\times 10^9\times (3\times 10^{-9})^2}{(0.30)^2}=9\times 10^{-7}\text{ N repulsive}$$

5. $F_3 = F_2 + 2F_1\cos 45°$

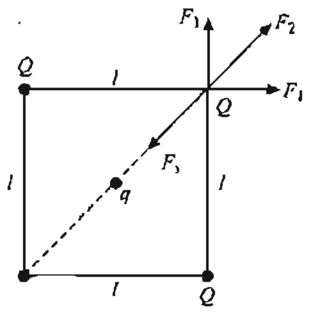


Fig. 1.148

$$\Rightarrow \quad \frac{kqQ}{(l/\sqrt{2})^2}=\frac{kQQ}{(\sqrt{2}l)^2}+2\frac{kQQ}{l^2}\frac{1}{\sqrt{2}}$$

$$\Rightarrow q=\frac{Q(2\sqrt{2}+1)}{4}$$

Q should be negative as there is attraction between Q and q.

$$\text{So, } q=\frac{-Q(2\sqrt{2}+1)}{4}$$

6. Q should be negative.

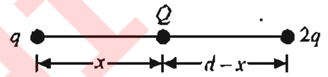


Fig. 1.149

For net force on q to be zero: $\dfrac{kqQ}{x^2}=\dfrac{kq2q}{d^2}$ (i)

For net force on $2q$ to be zero: $\dfrac{kQ2q}{(d-x)^2}=\dfrac{kq2q}{d^2}$ (ii)

From (i) and (ii),

$$\frac{1}{x^2}=\frac{2}{(d-x)^2} \quad\Rightarrow\quad x=\frac{d}{1+\sqrt{2}}$$

7. The third charge (q) is in equilibrium only when

$$F_{PA}=F_{PB} \quad\Rightarrow \frac{k(4e)q}{x^2}=\frac{k(e)q}{(a-x)^2}$$

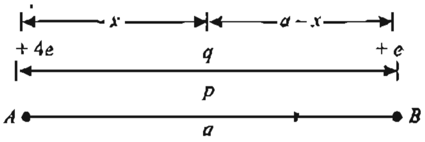


Fig. 1.150

On solving: $x=\dfrac{2}{3}a$

8. As there is attraction, so

$$\frac{9\times 10^9 Q_1Q_2\times 10^{-12}}{1^2}=-0.027 \quad\Rightarrow\quad Q_1Q_2=-3$$

–ve sign is due to attraction

$$\text{For repulsion: } \frac{9\times 10^9\times\left(\dfrac{Q_1+Q_2}{2}\right)^2}{1^2}=0.009$$

$$\Rightarrow \quad (Q_1+Q_2)^2=4$$

$$\Rightarrow \quad Q_1+Q_2=\pm 2$$

Here, we have two sets of equations.

(i) $\left\{\begin{matrix}\text{from } Q_1Q_2=-3 \\ \text{and } Q_1+Q_2=2\end{matrix}\right\} \rightarrow$ we get 3 μC and − 1 μC

(ii) $\left[\begin{matrix}\text{from } Q_1Q_2=-3 \\ \text{and } Q_1+Q_2=-2\end{matrix}\right] \rightarrow$ we get − 3 μC and 1 μC

9. Let charge on each sphere is q, then

$$F = k\,\frac{q^2}{r^2} = 2 \times 10^{-5} \quad \Rightarrow \frac{q^2}{r^2} = \frac{2 \times 10^{-5}}{9 \times 10^9}$$

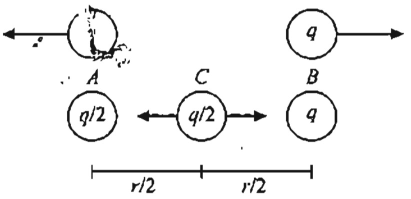


Fig. 1.151

When sphere C is touched by sphere A, charge gets equally divided between the two.

$$F_{CA} = k\,\frac{q^2/4}{r^2/4} = k\,\frac{q^2}{4} \times \frac{4}{r^2}$$

$$F_{CB} = k\,\frac{q^2/2}{r^2/4} = k\,\frac{q^2}{2} \times \frac{4}{r^2} = 2k\frac{q^2}{r^2}$$

$$F_C = F_{CB} - F_{CA} = k\,\frac{q^2}{r^2} = 2 \times 10^{-5} \text{ N towards } A.$$

10. Net electric field at A due to charges at B and C is

$$E_A = 2E_{AC}\sin 60° = 2 \times 9 \times 10^9 \times \frac{3 \times 10^{-6}}{(0.20)^2} \times \frac{\sqrt{3}}{2}$$

$$= \sqrt{3} \times 6.75 \times 10^5$$

$$\Rightarrow \qquad E_A = 1.5\sqrt{3} \times 10^{-5} \text{ NC}^{-1}$$

$$AM = \sqrt{(20)^2 - (10)^2} = \sqrt{400 - 100} = 10\sqrt{3} \text{ cm}$$

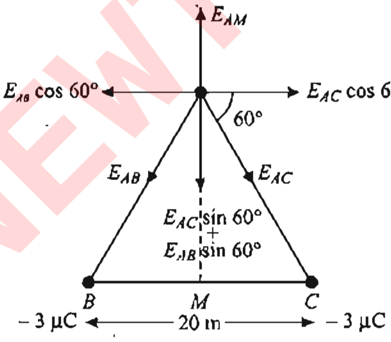


Fig. 1.152

Let the charge at M be q. Charge q should be positive so that there can be repulsion between the charges at A and M.

$$E_{AM} = 9 \times 10^9 \times \frac{q}{(10\sqrt{3}/100)^2} = 3q \times 10^{11}$$

For A to be in equilibrium:

$$E_A = E_{AM} \Rightarrow \quad \sqrt{3} \times 6.75 \times 10^5 = 3q \times 10^{11}$$

$$\Rightarrow \quad q = \frac{9\sqrt{3}}{4} \times 10^{-6} = \frac{9\sqrt{3}}{4}\mu\text{C}$$

11. For Q to be in equilibrium:

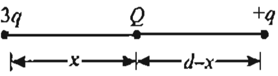


Fig. 1.153

$$\frac{k\,3q\,Q}{x^2} = \frac{k\,Qq}{(d-x)^2}$$

$$\Rightarrow \qquad x = \frac{\sqrt{3}\,d}{\sqrt{3}+1}$$

The bead can be in stable equilibrium, if it has positive charge.

12. No. of atoms in 10 gm of copper:

$$n = \frac{10}{63.5} \times 6 \times 10^{23} = 9.448 \times 10^{22}$$

Number of electrons transferred,

$$N = \frac{n \times 1}{1000} = 9.448 \times 10^{19}$$

Magnitude of charge appearing on either piece

a. $q = Ne = 9.448 \times 10^{19} \times 1.6 \times 10^{-19} = 15.12$ C

b. $F = \dfrac{kq^2}{r^2} = \dfrac{9 \times 10^9 \times (15.12)^2}{(0.10)^2} = 2.05 \times 10^{14}$ N

13. Electron is 0.5 cm from the sheet of charge and is far away from the edges of the sheet. Therefore, sheet looks effectively infinite.

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{q}{2\varepsilon_0 A} \Rightarrow \quad F = -eE = -e\left(\frac{q}{2\varepsilon_0 A}\right)$$

$$q = -\frac{2\varepsilon_0 AF}{e}$$

$$= -\frac{2\,(8.854 \times 10^{-12}) \times (0.5)^2 \times (1.8 \times 10^{-12})}{1.6 \times 10^{-19}}$$

$$= -50\,\mu\text{C}$$

14. An electron is negatively charged particle, it will be attracted by the positive plate with force $F = eE$. Hence, acceleration of electron along y-axis will be

$$a = \frac{F}{m} = \frac{eE}{m} = \frac{eV}{md} \quad \text{(i)} \left\{\text{as } E = \frac{V}{d}\right\}$$

So, from equation of motion, $v^2 = u^2 + 2as$ along x-axis,

$$v_x = v_0 = 10^6 \ (\text{ms}^{-1}) \qquad \{\text{as } a = 0\} \qquad \text{(ii)}$$

And along y-axis, $v_y^2 = 2ay_0$ {as $u = 0$ and $s = y_0$}

Now, as $y_0 = 1$ cm (given) and is given by equation (i), the electron will hit the top plate with

$$v_y = \sqrt{\frac{2y_0eV}{md}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 30 \times 1 \times 10^{-3}}{9 \times 10^{-31} \times 3 \times 10^{-3}}}$$

$$v_y = \left(4\sqrt{2}/3\right) \times 10^6 = 1.885 \times 10^6 \text{ ms}^{-1} \qquad \text{(iii)}$$

So, the electron will hit the upper plate with

$v_x = 10^6\,(\text{ms}^{-1})$ and $v_y = 1.885 \times 10^6\ \text{ms}^{-1}$

Note: *In this problem, time taken by the electron to hit the plate*

$$t=\sqrt{\frac{2y_0}{a}}=\sqrt{\frac{2y_0 m}{qE}}=\sqrt{\frac{2y_0 md}{qV}}$$

$$t=\sqrt{\frac{2\times(1\times10^{-3})\times(9\times10^{-31})\times(3\times10^{-3})}{1.6\times10^{-19}\times 30}}$$

$$=\frac{3}{2\sqrt{2}}\times10^{-9}\ s$$

And in this time electron will travel a horizontal distance:
$s = v_0t = 10^6\times1.06\times10^{-9} = 1.06\times10^{-3}\ m$

15. Volumetric charge density of given structure,

$$\rho=\frac{Q}{\frac{4}{3}\pi R^3-\frac{4}{3}\pi\left(\frac{R}{2}\right)^3} \quad\Rightarrow\quad \rho=\frac{8}{7}\frac{Q}{\frac{4}{3}\pi R^3}$$

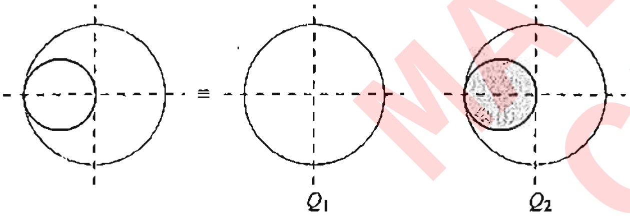


Fig. 1.154

$$Q_1=\rho\,\frac{4}{3}\pi R^3=\frac{8}{7}Q,\qquad Q_2=-\rho\,\frac{4}{3}\pi\left(\frac{R}{2}\right)^3=-\frac{1}{7}Q$$

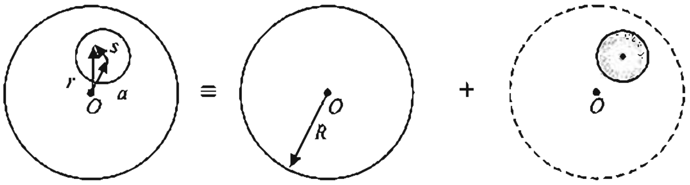
$$E_p=E_1-E_2=\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{x^2}-\frac{1}{4\pi\varepsilon_0}\frac{Q_2}{\left(x-\frac{R}{2}\right)^2}$$

$$E_p=\frac{Q}{4\pi\varepsilon_0}\left[\frac{8}{7x^2}-\frac{1}{7\left(x-\frac{R}{2}\right)^2}\right]$$

$$=\frac{Q}{28\pi\varepsilon_0}\left[\frac{8}{x^2}-\frac{1}{\left(x-\frac{R}{2}\right)^2}\right]$$

16. **a.** The field within the cavity or outside is the superposition of the field due to the original uncut sphere, plus the field due to a sphere of the size of the cavity but with a uniform negative charge density. The effective charge distribution is composed of a uniformly charged sphere of radius r, charge density ρ, superposed on it, a charge density $-\rho$ filling the cavity. Electric field $\vec{E}_1$ caused by the charge distribution $+\rho$ at a point $\vec{r}$, inside the spherical cavity.

$\vec{E}_1=\dfrac{\rho r}{3\varepsilon_0}\hat{r}=\dfrac{\rho\,\vec{r}}{3\varepsilon_0}$; where $\hat{r}$ is a unit vector in radial direction.



(1) Charge density of big sphere is σ

(2) Charge density of smaller sphere is $(-\sigma)$

Fig. 1.155

Similarly, the electric field $\vec{E}_2$ formed by the charge density $-\rho$ inside the cavity is

$$\vec{E}_2=\frac{(-\rho)\,\vec{s}}{3\varepsilon_0};\ \vec{s}=\vec{r}-\vec{a}$$

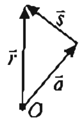


Fig. 1.156

$\vec{s}$ is the radius vector from cavity center to the point P.

$$\therefore\quad \vec{E}_2=\frac{-\rho\,(\vec{r}-\vec{a})}{3\varepsilon_0}$$

The resultant electric field inside the cavity is therefore given by the superposition of $\vec{E}_1$ and $\vec{E}_2$.

$$\vec{E}=\vec{E}_1+\vec{E}_2=\frac{\rho\,\vec{r}}{3\varepsilon_0}+\left[\frac{-\rho(\vec{r}-\vec{a})}{3\varepsilon_0}\right]$$

$$=+\frac{\rho\,\vec{a}}{3\varepsilon_0}=\text{constant}$$

$$\Rightarrow\qquad \vec{E}=\frac{\rho\,\vec{a}}{3\varepsilon_0}$$

b. (i) Electric field at points inside the large sphere but outside the cavity,

$$\vec{E}_1=\frac{\rho\,\vec{r}}{3\varepsilon_0}$$

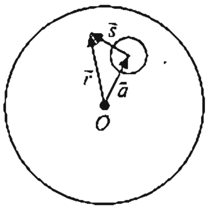


Fig. 1.157

and $\vec{E}_2=\dfrac{1}{4\pi\varepsilon_0}\dfrac{q\,\vec{s}}{s^3}=\dfrac{\left(-\frac{4}{3}\pi\rho b^3\right)(\vec{r}-\vec{a})}{4\varepsilon_0(\vec{r}-\vec{a})^3}$

The resultant electric field is therefore

$$\vec{E}=\vec{E}_1+\vec{E}_2=\frac{\rho}{3\varepsilon_0}\left[\vec{r}-\left(\frac{b}{\vec{r}-\vec{a}}\right)^3(\vec{r}-\vec{a})\right]$$

(ii) Electric field at points outside the large sphere,

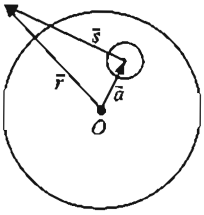


Fig. 1.158

$$\vec{E}_1 = \frac{Q_{\text{total}}}{4\pi\varepsilon_0 r^3}\vec{r} = \frac{\left(\frac{4}{3}\pi R^3\rho\right)}{4\pi\varepsilon_0 r^3}\vec{r} = \frac{R^3\rho}{3\varepsilon_0 r^3}\vec{r}$$

$$\vec{E}_2 = \frac{Q_{\text{total}}}{4\pi\varepsilon_0 r^3}\vec{r} = \frac{\left(-\frac{4}{3}\pi b^3\rho\right)}{4\pi\varepsilon_0(|\vec{r}-\vec{a}|)^3}(\vec{r}-\vec{a})$$

$$= \frac{-\rho b^3}{3\varepsilon_0(|\vec{r}-\vec{a}|)^3}(\vec{r}-\vec{a})$$

The resultant electric field,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon_0}\left[\left(\frac{R}{r}\right)^3\vec{r} - \left(\frac{b}{\vec{r}-\vec{a}}\right)^3(\vec{r}-\vec{a})\right]$$

$$E(\vec{r}) = \begin{cases} \dfrac{\rho\,\vec{a}}{3\varepsilon_0}\text{Elecric field inside the cavity} \\ \dfrac{\rho}{3\varepsilon_0}\left[\vec{r} - \left(\dfrac{b}{|\vec{r}-\vec{a}|}\right)^3(\vec{r}-\vec{a})\right] \\ \text{Electric field outside the cavity but inside the large cavity} \\ \dfrac{\rho}{3\varepsilon_0}\left[\left(\dfrac{R}{r}\right)^3\vec{r} - \left(\dfrac{b}{|\vec{r}-\vec{a}|}\right)^3(\vec{r}-\vec{a})\right] \\ \text{Electric field outside the large sphere} \end{cases}$$

17. We first find the field of a cylinder off-axis, then the electric field in a hole in a cylinder is the difference between two electric fields — that of a solid cylinder on-axis and the one off-axis.

$$\vec{r}' = \vec{r} - \vec{\mathbf{b}} \Rightarrow \Phi = 2\pi r' l\ E = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\rho}{\varepsilon_0} l\pi r'^2$$

$$E = \frac{\rho r'}{2\varepsilon_0} \quad \Rightarrow \quad \vec{\mathbf{E}} = \frac{\rho(\vec{\mathbf{r}} - \vec{\mathbf{b}})}{2\varepsilon_0}.$$

$$\vec{E}_{\text{hole}} = \vec{E}_{\text{cylinder}} - \vec{\mathbf{E}}_{\text{above}}$$

$$= \frac{\rho\,\vec{\mathbf{r}}}{2\varepsilon_0} - \frac{\rho(\vec{\mathbf{r}} - \vec{\mathbf{b}})}{2\varepsilon_0} = \frac{\rho\,\vec{\mathbf{b}}}{2\varepsilon_0} \quad \text{Note that } \vec{\mathbf{E}} \text{ is uniform.}$$

8. $E_1 = 2E\cos\theta = \dfrac{2kq(l)}{[x^2+l^2]^{3/2}}$ along AC

$E_2 = 2E\cos\theta = \dfrac{2kql}{(x^2+l^2)^{3/2}}$ along DB

Net field:

$$E_0 = \sqrt{E_1^2 + E_2^2} = \frac{2\sqrt{2}kql}{(x^2+l^2)^{3/2}} = \frac{ql}{\sqrt{2}\pi\varepsilon_0(x^2+l^2)^{3/2}}$$

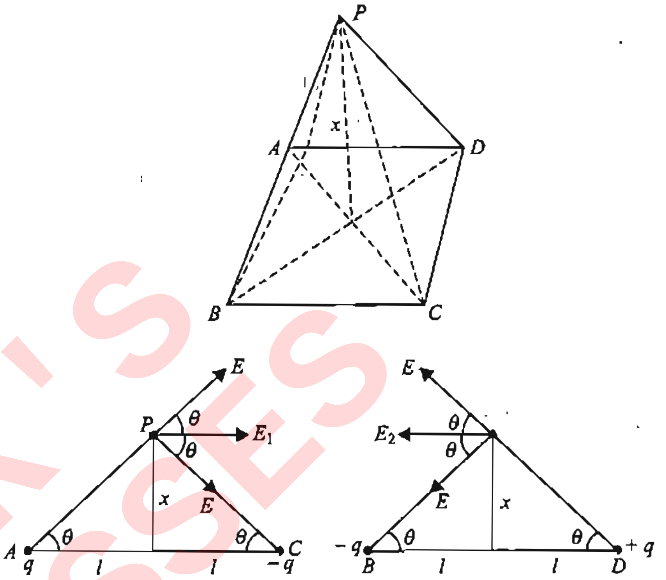


Fig. 1.159

19. Let us make two-dimensional view of the situation. Because of symmetry we can say the force on wire 2 (λ_2) will be along negative y-direction.

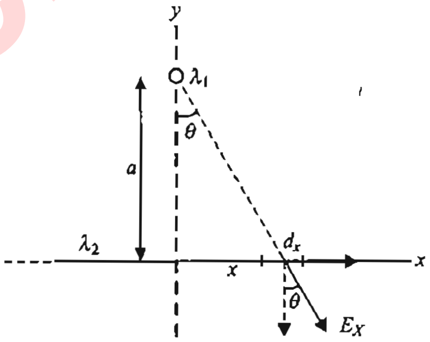


Fig. 1.160

$$dF = E_x dq\cos\theta$$

$$F = \int dF = \int \frac{\lambda_1}{2\pi\varepsilon_0\sqrt{a^2+x^2}}(\lambda_2 dx)\cos\theta$$

where $\cos\theta = \dfrac{a}{\sqrt{a^2+x^2}}$

$$F = \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0}a\int_{-\infty}^{\infty}\frac{dx}{a^2+x^2} = \frac{\lambda_1\lambda_2 a}{2\pi\varepsilon_0} \times \frac{1}{a}\left[\tan^{-1}\frac{x}{a}\right]_{-\infty}^{\infty}$$

$F = \dfrac{\lambda_1\lambda_2}{2\varepsilon_0}$ It is independent of a.

20. $q = 10^{-8}\,\text{C}, \lambda = \dfrac{Q}{2\pi r} = \dfrac{\pi}{2\pi(0.1)} = 5\ \text{Cm}^{-1}$

Take an arc of ring subtending small angle θ at the center (Fig. 1.159). Charge on this arc: $dq = \lambda r\theta$

For net force to be zero on this arc:

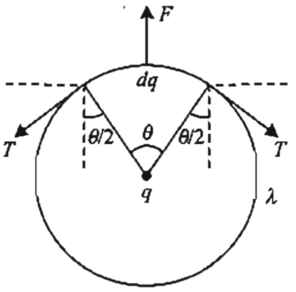


Fig. 1.161

$$2T\sin\left(\frac{\theta}{2}\right) = F,$$

$$\Rightarrow 2T\left(\frac{\theta}{2}\right) = \frac{kq\,(\lambda r d\theta)}{r^2}$$

$$\Rightarrow T = \frac{kq\lambda}{r}$$

$$= \frac{9\times10^{9}\times10^{-8}\times5}{0.1} = 4500\text{ N}$$

$$\Delta r = \frac{Tr}{AY} = \frac{4500\times0.1}{10^{-6}\times2\times10^{11}}$$

$$= 225\times10^{-5}\text{ m} = 2.25\text{ mm}$$

21. $T\cos\theta + qB = mg$

$$\Rightarrow \quad T\cos\theta = mg - qB \qquad \text{(i)}$$

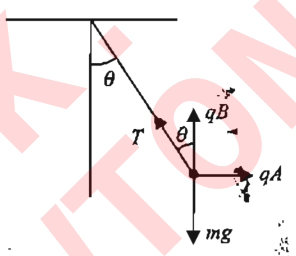


Fig. 1.162

$$T\sin\theta = qA \qquad \text{(ii)}$$

Dividing (i) and (ii),

$$\tan\theta = \frac{qA}{mg - qB}$$

a. $$\Rightarrow \quad q = \frac{mg\tan\theta}{A + B\tan\theta}$$

Put the value of q in (ii)

b. $$T = \frac{A\,mg\sec\theta}{A + B\tan\theta}$$

22. Electric field at point P due to charge of ring is

$$E = \frac{kQx}{(R^2+x^2)^{3/2}}$$

At $x = R$: $E = \dfrac{kQ}{2\sqrt{2}R^2}$ directed towards the center.

Electric field at P due to charge at center: $\dfrac{kq}{R^2}$

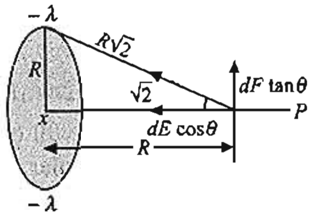


Fig. 1.163

For net field to be zero at P:

$$\frac{kq}{R^2} = \frac{kQ}{2\sqrt{2}\,R^2} \quad \Rightarrow q = \frac{Q}{2\sqrt{2}}.$$

23. $$T\sin\theta = F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2} \qquad \text{(i)}$$

$$T\cos\theta = mg \qquad \text{(ii)}$$

From (i) and (ii), $\tan\theta = \dfrac{q^2}{4\pi\varepsilon_0 a^2 mg}$

$$\frac{a/2}{L} = \frac{q^2}{4\pi\varepsilon_0 a^2 mg} \quad (\because \text{ for small } \theta, \tan\theta \approx a/2L)$$

$$\Rightarrow \quad \frac{a^3}{L} = \frac{q^2}{2\pi\varepsilon_0 mg} \qquad \text{(iii)}$$

When one of the ball is discharged, the balls come closer and touch each other and again separate due to repulsion. The charge on each ball after touching each other $= q/2$. Replacing q with $q/2$ in (iii), we get

$$\frac{b^3}{L} = \frac{(q/2)^2}{2\pi\varepsilon_0 mg} \qquad \text{(iv)}$$

From (iii) and (iv), $\dfrac{b^3}{a^3} = \dfrac{1}{4} \Rightarrow b = \dfrac{a}{2^{2/3}}$

24. **a.** Electric field near B and to the right of B is along −ve x-direction, so sign of Q_b should be −ve. There is a neutral point at $x = 24$ cm, so sign of Q_a should be opposite of Q_b. Hence, Q_a should be +ve.

b. At neutral point (at $x = 24$ cm):

$$k\frac{Q_a}{24^2} = k\frac{Q_b}{3^2} \quad \Rightarrow \left|\frac{Q_a}{Q_b}\right| = \left(\frac{24}{3}\right)^2 = 64$$

c. Electric field to the right of charges (at $x = x$)

$$E = k\frac{Q_a}{x^2} - k\frac{Q_b}{(x-21)^2}$$

For field to be maximum: $\dfrac{dE}{dx} = 0$

$$\Rightarrow \frac{kQ_a(-2)}{x^3} - \frac{kQ_b(-2)}{(x-21)^3} = 0 \quad \Rightarrow \quad x = 28\text{ cm}$$

25. Electric field due to MA: $\vec{E}_1 = \dfrac{\lambda}{4\pi\varepsilon_0 R}(\hat{i} - \hat{k})$

Electric field due to ADC: $\vec{E}_2 = \dfrac{\lambda}{2\pi\varepsilon_0 R}(-\hat{k})$

Electric field due to ABC: $\vec{E}_3 = \dfrac{\lambda}{2\pi\varepsilon_0 R}(-\hat{j})$

Electric field due to NC: $\vec{E}_4 = \dfrac{\lambda}{4\pi\varepsilon_0 R}(-\hat{i} + \hat{k})$

Net Electric field:

$$\vec{E}_0 = \vec{E}_1 + E_2 + \vec{E}_3 + \vec{E}_4 = -\frac{\lambda}{2\pi\varepsilon_0 R}(\hat{j} + \hat{k})$$

26. Electric field due to straight wires will cancel out. The net electric field will be due to semicircular wire only.

$$\text{Hence, } \vec{E}_{\text{net}} = \vec{E}_{\text{circular}} = \frac{\lambda}{2\pi\varepsilon_0 R}(-\hat{j})$$

$$\text{So, acceleration: } \vec{a} = \frac{q\vec{E}_{\text{net}}}{m} = \frac{q\lambda}{2\pi\varepsilon_0 m R}(-\hat{j})$$

27. $F = \dfrac{\sqrt{3}\,kq^2}{x^2}$

$$\tan\theta = \frac{F}{mg}$$

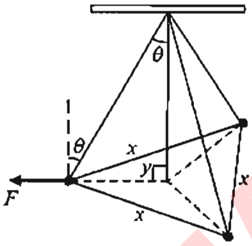


Fig. 1.164

$$\tan\theta = \frac{\sqrt{3}\,kq^2}{mg\,x^2}, \quad y = \frac{x}{\sqrt{3}}$$

$$\frac{y}{\sqrt{l^2 - y^2}} = \frac{\sqrt{3}\,q^2}{4\pi\varepsilon_0 mgx^2}$$

$$\frac{x}{\sqrt{3l^2 - x^2}} = \frac{\sqrt{3}\,q^2}{4\pi\varepsilon_0 mgx^2}$$

$$q = \left[\frac{4\pi\,\varepsilon_0 mg\,x^3}{\sqrt{3}\,[3l^2 - x^2]}\right]^{1/2}$$

$$\tan\theta = \frac{kq^2}{x^2 mg}$$

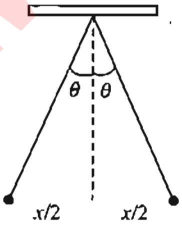


Fig. 1.165

$$\frac{x}{2l} = \frac{q^2}{4\pi\varepsilon_0 x^2 mg}$$

$$x = \left[\frac{q^2 l}{2\pi\varepsilon_0 mg}\right]^{1/3}$$

$$q^2 = \frac{2\pi\varepsilon_0 mg\,x^3}{l}$$

$$q = \sqrt{\frac{2\pi\varepsilon_0 mg}{l}}\,x^{3/2}$$

$$\frac{dq}{dt} = \sqrt{\frac{2\pi\varepsilon_0 mg}{l}}\,\frac{3}{2}\sqrt{x}\,\frac{dx}{dt}$$

$$\frac{dq}{dt} = -\frac{3}{2}a\sqrt{\frac{2\pi\varepsilon_0 mg}{l}}$$

29. $F = \dfrac{3kq_1q_2}{r^2}\cdot\dfrac{x}{r} = ma$

$$\Rightarrow \quad a = \left[\frac{3\,q_1q_2}{4\pi\varepsilon_0 mr^3}\right]x$$

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0\,m\,r^3}{3\,q_1q_2}}$$

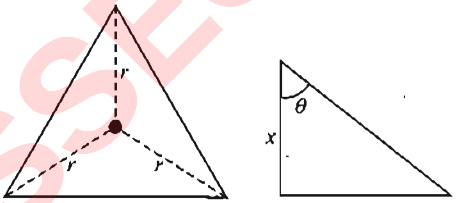


Fig. 1.166

30. Charge with in radius r,

$$q = \int_0^r \rho_0\left[1 - \frac{r}{R}\right]4\pi r^2 dr$$

$$q = 4\pi\rho_0\int_0^r\left(r^2 - \frac{r^3}{R}\right)dr$$

$$q = 4\pi\rho_0\left(\frac{r^3}{3} - \frac{r^4}{4R}\right) \qquad \text{(i)}$$

For $r < R$,

$$E = \frac{kq}{r^2} = \frac{\rho_0 r}{3\varepsilon_0}\left[1 - \frac{3r}{4R}\right] \qquad \text{(ii)}$$

Total charge: Put $r = R$ in (i)

$$Q = 4\pi\rho_0\left[\frac{R^3}{3} - \frac{R^3}{4}\right] = \frac{4\pi\rho_0 R^3}{12}$$

For $r > R$,

$$E = \frac{kQ}{r^2} = \frac{\rho_0 R^3}{12\varepsilon_0 r^2} \qquad \text{(iii)}$$

At surface, from both (ii) and (iii),

$$E_0 = \frac{\rho_0 R}{12\,\varepsilon_0}$$

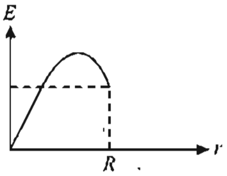


Fig. 1.167

For maximum $\dfrac{dE}{dr} = 0$ from (ii)

$$r = \frac{2}{3}R$$

$$E = \frac{\rho_0 R}{9\varepsilon_0}$$

Also, at $r = R/3$

$$E = \frac{\rho_0 R}{12\varepsilon_0}$$

31. Potential energy of dipole system

$$U = -\vec{p}_2 \cdot \vec{E}_{21} = -p_2 \frac{1}{4\pi\varepsilon_0}\frac{p_1}{x^3}\cos\pi$$

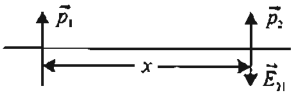


Fig. 1.168

E_{21} is the field due to p_1 at p_2 as shown.

$$U = \frac{1}{4\pi\varepsilon_0}\frac{p_1p_2}{x^3}$$

$$F = -\frac{\partial U}{\partial x} = \frac{3}{4\pi\varepsilon_0}\frac{p_1p_2}{x^4}$$

F comes out to be positive, so it is a repulsive force.

32. Potential energy of system

$$U = -\vec{p}_2 \cdot \vec{E}_{21} = -p_2\frac{1}{4\pi\varepsilon_0}\frac{2p_1}{x^3}\cos 0 = -\frac{1}{4\pi\varepsilon_0}\frac{2p_1p_2}{x^3}$$

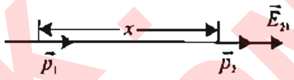


Fig. 1.169

$$F = -\frac{\partial U}{\partial x} = -\frac{6}{4\pi\varepsilon_0}\frac{p_1p_2}{x^4} = -\frac{3}{2\pi\varepsilon_0}\frac{p_1p_2}{x^4}$$

F comes out to be negative, so it is an attractive force.

33. a. $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$ (Fig. 1.170)

$$U = -pE\cos 0^\circ = -\frac{qp}{4\pi\varepsilon_0 x^2}$$

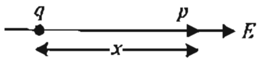


Fig. 1.170

$$F = -\frac{\partial U}{\partial x} = \frac{-pq}{2\pi\varepsilon_0 x^3}$$

–ve sign indicates that force on dipole is towards –ve x-direction or the force is attractive.

b. $U = -pE\cos 180^\circ = \frac{qp}{4\pi\varepsilon_0 x^2}$

$$F = -\frac{\partial U}{\partial x} = \frac{pq}{2\pi\varepsilon_0 x^3}$$

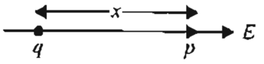


Fig. 1.171

+ve sign indicates that force on dipole is towards +ve x-direction or the force is repulsive.

c. $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{x^3}$

Let us first find force on q due to p

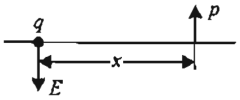


Fig. 1.172

$$F = qE = \frac{qp}{4\pi\varepsilon_0 x^3}$$

Charge q will also apply same force on dipole but in opposite direction, so force on dipole

$F = \frac{qp}{4\pi\varepsilon_0 x^3}$ along $\vec{p}$ or parallel to y-axis

Objective Type

1. c. The body may get charged either negatively or positively.

2. d. $K = \frac{1}{4\pi\varepsilon_0} = 9\times10^9$, $\varepsilon_0 = \frac{1}{4\pi\times9\times10^9}$ $\text{C}^2\text{N}^{-1}\text{m}^{-2}$

3. a. $F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$

$$[\varepsilon_0] = \frac{[q^2]}{[Fr^2]} = \frac{(\text{current}\times\text{time})^2}{MLT^{-2}L^2}$$

$$= \frac{A^2T^2}{MLT^{-2}L^2} = M^{-1}L^{-3}T^4A^2$$

4. b. $E = \frac{F}{q} = \frac{MLT^{-2}}{AT} = MLA^{-1}T^{-3}$

5. d. Dielectric constant of a material is $K \geq 1$.

6. a. Net charge of universe is constant. Positive and negative charges may separately be created or destroyed. But net sum of charge remains constant.

7. a. There will be gravitational attraction between the neutrons.

8. a. When we charge a soap bubble, there will be either net positive or net negative charge on the bubble. Due to the repulsion between like charges, size of the bubble will tend to increase.

9. a. Glass plate will act like a dielectric.

10. a. Newton's third law.

11. b. Given $Q_1 + Q_2 = 10$ (i)

And $\frac{kQ_1Q_2}{r^2} = -0.075$

$$\Rightarrow \quad \frac{9\times10^9 Q_1Q_2\times10^{-12}}{(3)^2} = -0.075$$

(Here, we have multiplied by 10^{-12} because we have considered Q_1 and Q_2 to be in μC)

$$\Rightarrow \quad Q_1Q_2 = -75 \quad \text{(ii)}$$

Solving (i) and (ii), we get $Q_1 = 15$ μC, $Q_2 = -5$ μC

12. **a.** $F = \frac{1}{4\pi\varepsilon_0}\frac{q(Q-q)}{r^2}$

For the force to be maximum:

$$\frac{dF}{dq} = \frac{1}{4\pi\varepsilon_0 r^2}\frac{d}{dq}[Qq - q^2] = 0 \Rightarrow \quad q = \frac{Q}{2}$$

And $\frac{d^2F}{dq^2}$ is negative at $q = \frac{Q}{2}$

Hence, force will be maximum if $q = Q/2$.

13. **c.** Net force on q_3: $F_1 + F_2 = 0$

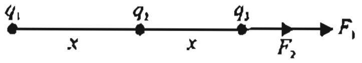


Fig. 1.173

$$\Rightarrow \quad \frac{k\,q_1q_3}{(2x)^2} + \frac{k\,q_2q_3}{x^2} = 0 \quad \Rightarrow \quad q_1 = -4q_2$$

14. **d.**

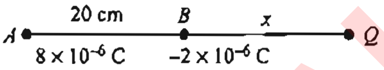


Fig. 1.174

Let third charge Q is placed at a distance x to the right of B. Then

$$\frac{k\,Q\times8\times10^{-6}}{(20+x)^2} = \frac{k\,Q(2\times10^{-6})}{x^2}$$

$$\Rightarrow \qquad x = 20 \text{ cm}$$

15. **c.** 1 2 3 4 5

\+ - +

If 1 is positively charged, then 2 should be negatively charged, then 4 should be positively charged. Now, 1 and 4 cannot attract. It means ball 1 should be neutral, because they are showing repulsion with 3 and 5, respectively.

16. **b.** A and B have like charges B and C have like charges. So, A and C also have like charges. Hence, they also repel each other.

17. **c.** $100 = kq_1q_2/r^2$ and $F = k(1.1q_1)(0.9q_2)/r^2$

$$\Rightarrow \quad \frac{F}{100} = 0.99 \Rightarrow F = 99 \text{ N}$$

18. **c.** Force on Q_2 is zero (q should be –ve)

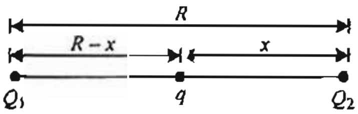


Fig. 1.175

$$\Rightarrow \quad \frac{kQ_1Q_2}{R^2} = \frac{kqQ_2}{x^2} \Rightarrow \frac{x}{R} = \sqrt{\frac{q}{Q_1}}$$

Force on q is zero: $\frac{kQ_1q}{(R-x)^2} = \frac{kqQ_2}{x^2}$

$$\Rightarrow \frac{R-x}{x} = \frac{\sqrt{Q_1}}{\sqrt{Q_2}}$$

$$\Rightarrow \frac{R}{x} = \frac{\sqrt{Q_1}+\sqrt{Q_2}}{\sqrt{Q_2}} \Rightarrow \frac{\sqrt{Q_1}}{\sqrt{q}} = \frac{\sqrt{Q_1}+\sqrt{Q_2}}{\sqrt{Q_2}}$$

$$\Rightarrow \quad q = \frac{Q_1Q_2}{(\sqrt{Q_1}+\sqrt{Q_2})^2}$$

19. **d.** The equilibrium can be stable only if on displacing the small charge slightly in any direction, the forces act on it in such a way so as to bring back the charge to its equilibrium position.

20. **b.** Initial tension: $T_1 = mg$

Final tension: $T_2\cos\theta = mg$

$$\Rightarrow \quad T_2 = \frac{mg}{\cos\theta}$$

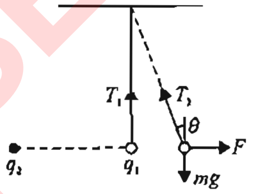


Fig. 1.176

Obviously: $T_2 > mg$

21. **c.** $F_{net} = 2F\cos30°$

$$= \frac{2kq^2}{a^2}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}\,kq^2}{a^2}$$

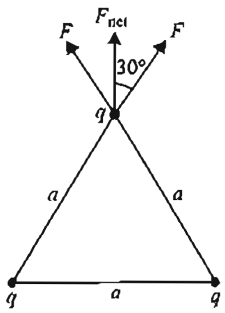


Fig. 1.177

22. **a.** $E = \frac{kQ}{r^2}$ (i)

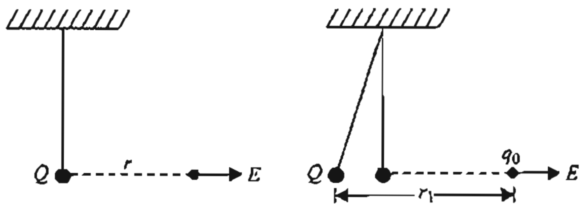


Fig. 1.178

$F = \frac{kQq_0}{r_1^2} \quad \Rightarrow \quad \frac{F}{q_0} = \frac{kQ}{r_1^2}$ (ii)

As $r_1 > r$, so from (i) and (ii) $E > \frac{F}{q_0}$

23. **d.** Net charge on a current carrying wire will be zero at any time. Because when current flows through a wire, the amount of charge entering from one end is equal to amount of charge leaving the other end.

24. **a.** $E = \dfrac{F}{q} = \dfrac{2.25}{15 \times 10^{-4}} = 1500\ \text{NC}^{-1}$

25. **b.** $F = Eq = 15 \times 10^4 \times 2 \times 1.6 \times 10^{-19} = 4.8 \times 10^{-14}\ \text{N}$

26. **a.** As charge on both particles is same, so electric force acting on them will be same. Since the particles are allowed to move for the same time, their final momentum will be same. Because

Change in momentum = impulse = force × time

So, from, $\text{KE} = \dfrac{P^2}{2m}$ we have $\text{KE} \propto \dfrac{1}{m}$

$$\frac{\text{KE}_1}{\text{KE}_2} = \frac{m_2}{m_1} = 2 : 1$$

27. **d.** Each charge will produce same magnitude of intensity, say E, at the centroid. These are directed at angles of 120° with each other. So, their vector sum will be zero.

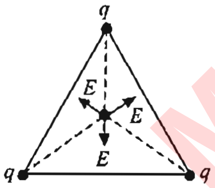


Fig. 1.179

28. **b.** $\vec{r} = (q - 3)\hat{i} + (12 - 4)\hat{j} = 6\hat{i} + 8\hat{j}$

$$\Rightarrow \quad r = \sqrt{6^2 + 8^2} = 10\ \text{m}$$

$$E = \frac{9 \times 10^9 \times 100 \times 10^{-6}}{10^2} = 9000\ \text{Vm}^{-1}$$

29. **d.** Electric lines of force is an imaginary concept, it do not exist in reality.

30. **d.** $\dfrac{k\,Q_2}{x^2} = \dfrac{k\,Q_1}{(x + R)^2}$

Fig. 1.180

$$x = \frac{R}{2}$$

31. **c.** The field at point P is superposition of fields $\vec{E}_1$, $\vec{E}_2$, $\vec{E}_3$ due to each charge.

$$\vec{E}_1 = \frac{q}{4\pi\varepsilon_0 (r - a)^2}\hat{i},$$

$$\vec{E}_2 = -\frac{2q}{4\pi\varepsilon_0 r^2}\hat{i}, \qquad \vec{E}_3 = \frac{q}{4\pi\varepsilon_0 (r + a)^2}\hat{i}$$

Now,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r - a)^2} - \frac{2}{r^2} + \frac{1}{(r + a)^2}\right]\hat{i}$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}\left[\left\{1 - \left(\frac{a}{r}\right)^{-2}\right\} - 2 + \left\{1 + \left(\frac{a}{r}\right)\right\}^{-2}\right]$$

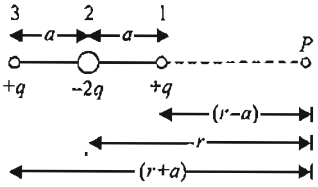


Fig. 1.181

If $r \gg a$, we can use binomial approximation:

$$(1 + \alpha)^n = 1 + n\alpha + \frac{n(n - 1)}{2!}\alpha^2 + \cdots \ (\text{for } \alpha << 1)$$

Therefore,

$$E = \frac{q}{4\pi\varepsilon_0 r^2}\left[\begin{matrix}\left\{\left(1 - 2\left(-\frac{a}{r}\right)\right) + \frac{-2(-2 - 1)}{2}\left(\frac{-a}{r}\right)^2\right\} \\ -2 + \left\{1 - 2\frac{a}{r} + \frac{-2(--2 - 1)}{2}\left(\frac{a}{r}\right)^2\right\}\end{matrix}\right]$$

$$= \frac{6a^2 q}{4\pi\varepsilon_0 r^4} = \frac{6kqa^2}{r^4}$$

32. **c.** For first case: $F = mg$
For second case: $F + mg = 6eE\ 2mg = 6eE$

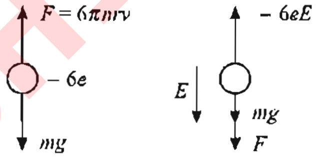


Fig. 1.182

$$\Rightarrow \quad E = \frac{mg}{3e} = \frac{1.6 \times 10^{-15} \times 10}{3 \times 1.6 \times 10^{-19}} = 3.3 \times 10^4\ \text{NC}^{-1}$$

33. **b.** Dielectric strength means the maximum electric field which a medium can bear. Here, if field becomes more than this, then charge will start leaking from the metal ball. So,

$$E = \frac{kQ}{R^2} \Rightarrow Q = \frac{ER^2}{k} = \frac{3 \times 10^6 \times (0.001)^2}{9 \times 10^9} = 1/3\ \text{nC}$$

34. **a.** $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{a^2}$

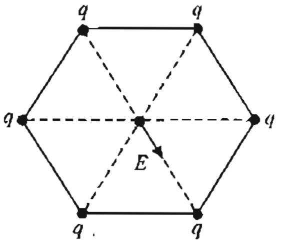


Fig. 1.183

Suppose the charge is present at the sixth vertex also, then electric field at center would be zero. Now, if charge is not present at this vertex, the electric field at center would be because of other five charges, which should be equal and opposite to the field produced due to single charge at sixth vertex.

35. b.
$$dq = \frac{Q}{2\pi r}(0.00\,2\pi)$$
$$= \frac{1}{2\pi(0.5)} \times \frac{2\pi}{1000} = 2 \times 10^{-3}\ \text{C}$$
$$E = \frac{9 \times 10^9 \times 2 \times 10^{-3}}{(0.5)^2}$$

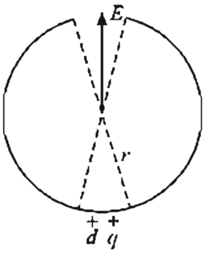


Fig. 1.184

$$\Rightarrow \quad E = 7.2 \times 10^7\ \text{NC}^{-1}$$

36. b. Force on the block: $F = qE$ towards left.
Let spring is compressed maximum by x. Then
$$Fx = \frac{1}{2}kx^2 \quad \Rightarrow \quad qEx = \frac{1}{2}kx^2 \Rightarrow x = \frac{2qE}{k}$$

37. a. Strength of electric intensity is more if field lines are closer.

38. d. We can compare this situation with that of a mass moving as a projectile.

39. b. $W_{AB} = W_{AC} + W_{CB}$

W_{CB} should be zero, because in moving from C to B, we always move perpendicular to field. Hence, force applied by field and displacement will be at 90°.
$$W_{AC} = -e(V_C - V_A)$$
$$V_C - V_A = -E \times AC$$
$$= -10 \times 4 = -40$$
$$W_{AC} = -e \times (-40) = 40e$$

So $W_{AB} = 40e$ J $= 40$ eV

40. a. Work done is zero, because force applied by q_2 on q_1 is always perpendicular to the velocity of q_1.

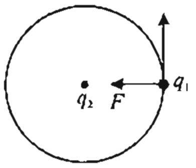


Fig. 1.185

41. a. Due to induced negative charge, there will be attraction.

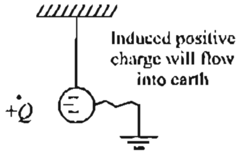


Fig. 1.186

42. a. $\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32}$
$$|\vec{F}_3| = k\left[\frac{5 \times 1}{2^2} - \frac{5 \times 3}{4^2}\right] \times \frac{10^{-18}}{10^{-4}}$$
$$= 9 \times 10^9 \times 10^{-14}\left[\frac{5}{4} - \frac{15}{16}\right] = 2.8 \times 10^{-5}\ \text{N} = 28\mu\text{N}$$

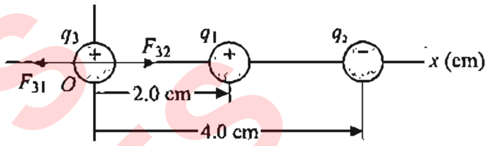


Fig. 1.187

43. a. To keep the system in equilibrium, net force experienced by charges at 'A', 'B' and 'C' should be zero. For this another charge of opposite sign should be placed at the centroid of triangle. Let this charge be '$-Q$'.

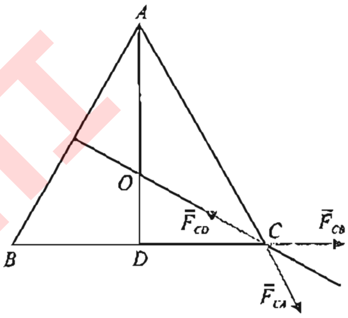


Fig. 1.188

$$AD = l\cos 30° = \frac{l\sqrt{3}}{2},\ AO = \frac{2}{3}AD = \frac{l}{\sqrt{3}}$$
$$2|\vec{F}_{CA}|\cos 30° = |\vec{F}_{CO}|$$
$$2 \times \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{l^2} \times \frac{\sqrt{3}}{2} = \frac{1}{4\pi\varepsilon_0}\frac{Q\ q}{(l/\sqrt{3})^2} \quad \Rightarrow Q = \frac{q}{\sqrt{3}}$$

44. b. $F_{\text{net}} = 2\,|F_{31}|\cos\alpha$
$$= 2 \times \frac{1}{4\pi\varepsilon_0} \times \frac{2 \times 4 \times 10^{-12}}{(0.5)^2} \times \frac{4}{5} = 0.46\ \text{N}$$

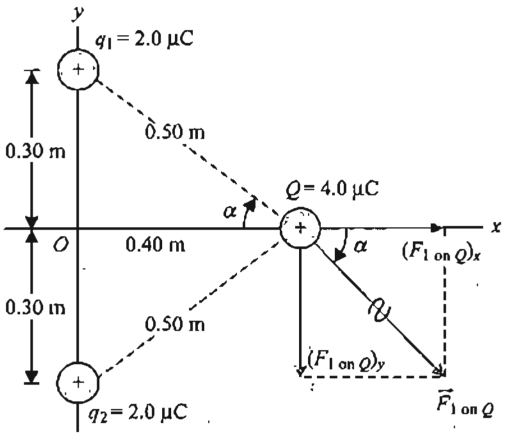


Fig. 1.189

45. **c.** For external points, a charged sphere behaves as if the whole of its charge were concentrated at its center.

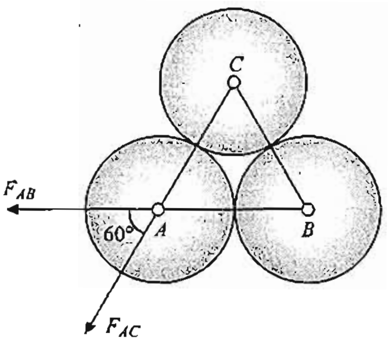


Fig. 1.190

Force on A due to B,

$$F_{AB} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4R^2} \text{ along } \overrightarrow{BA}$$

And force on A due to C

$$F_{AC} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4R^2} \text{ along } \overrightarrow{CA}$$

Now as angle between BA and CA is 60° and

$$|F_{AB}| = |F_{AC}| = F$$

$$F_A = \sqrt{F^2 + F^2 + 2F.F.\cos 60} = \sqrt{3}\,F$$

$$F_A = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{3}}{4}\left(\frac{q}{R}\right)^2$$

46. **d.** If there had been a sixth charge $+q$ at the remaininig vertex of hexagon, force due to all the six charges on $-q$ at O will be zero.

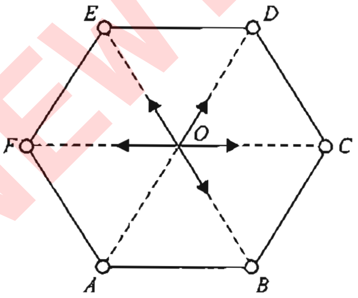


Fig. 1.191

Now if is the force due to sixth charge and due to remaining five charges, then

$$\vec{F} + \vec{f} = 0 \quad \text{i.e.,}\ \vec{F} = -\vec{f}$$

$$|\vec{F}| = |\vec{f}| = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{L^2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$$

47. **d.** $AC = \sqrt{2}l = BD$

$$\Rightarrow \qquad BD = \frac{l}{\sqrt{2}}$$

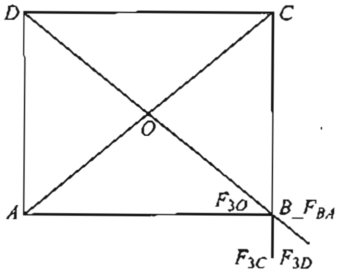


Fig. 1.192

$$12|\vec{F}_{BC}|\ \cos 45° = |\vec{F}_{BO}|$$

$$2\frac{1}{4\pi\varepsilon}\frac{q^2}{l^2} \times \frac{1}{\sqrt{2}} = \frac{1}{4\pi\varepsilon} \times \frac{Qq}{(l/\sqrt{2})^2} + \frac{1}{4\pi\varepsilon} \times \frac{q^2}{(\sqrt{2}l)^2}$$

$$\Rightarrow \qquad Q = \frac{-q}{4}(1 + 2\sqrt{2})$$

48. **c.** $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}\hat{r}$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{(1.2-0)\hat{i} + (-1.6-0)\hat{j}}{\sqrt{(1.2)^2 + (1.6)^2}}$$

$$\hat{r} = \frac{1}{2}(1.2\,\hat{i} - 1.6\,\hat{j})$$

$$\vec{E} = 9 \times 10^9 \times \frac{8 \times 10^{-9}}{2} \times \left[-\frac{1}{2}(1.2\hat{i} - 1.6\hat{j})\right]$$

$$\vec{E} = (-11\,\hat{i} + 14\hat{j})\ \text{NC}^{-1}$$

49. **c.** As $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{(r-r_0)^3}(\vec{r} - \vec{r}_0)$

$$\Rightarrow \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^2}\hat{r}$$

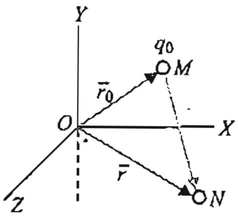


Fig. 1.193

$$\overrightarrow{OM} + \overrightarrow{MN} = \overrightarrow{ON}$$

$$\overrightarrow{MN} = \vec{r} - \vec{r}_0 = (8\hat{i} - 5\hat{j}) - (2\hat{i} + 3\hat{j})$$

$$= (6\hat{i} - 8\hat{j})$$

$$|\vec{r} - \vec{r}_0| = \sqrt{6^2 + 8^2} = 10\ \text{m}$$

$$\vec{E} = 9 \times 10^9 \times \frac{50 \times 10^{-6}}{(10)^3}(6\hat{i} - 8\hat{j})$$

$$\vec{E} = (2.7\,\hat{i} - 3.6\hat{j})\ \text{kNC}^{-1}$$

$$|\vec{E}| = 4500\ \text{NC}^{-1}$$

50. **a.** $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{(r-r_0)^3}(\vec{r} - \vec{r}_0)$

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}, \quad \overrightarrow{AB} = (\vec{r} - \vec{r}_0)$$

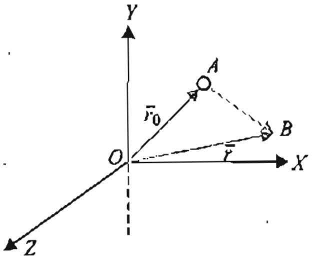


Fig. 1.194

$$\vec{AB} = (0-3)\hat{i} + (-4-2)\hat{j} + (3-5)\hat{k}$$
$$= (-3\hat{i} - 6\hat{j} - 2\hat{k}) \text{ m}$$
$$|\vec{AB}| = \sqrt{9+36+4} = 7 \text{ m}$$
$$\vec{E} = 9 \times 10^9 \times \frac{1 \times 10^{-6}}{7^3}(-3\hat{i} - 6\hat{j} - 2\hat{k})$$
$$\vec{E} = \frac{-9 \times 10^3}{343}(3\hat{i} + 6\hat{j} + 2\hat{k})$$
$$|\vec{E}| = \frac{9}{49} \times 10^3 \text{ NC}^{-1}$$

51. b. $|\vec{E}_A| = |\vec{E}_B| = |\vec{E}_C| = |\vec{E}_D| = |\vec{E}|$

$$\vec{E}_1 = 2 \times \frac{1}{4\pi\varepsilon_0}\frac{Q}{a^2} \times \sin 45°(\hat{k})$$

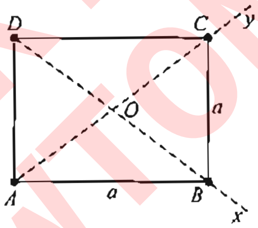


Fig. 1.195

Similarly, taking charges at A and C

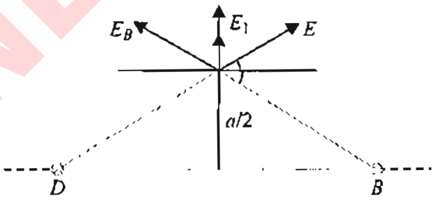


Fig. 1.196

$$\vec{E}_2 = \frac{Q}{2\sqrt{2}\pi\varepsilon_0 a^2}(\hat{k})$$
$$\vec{E}_{\text{total}} = \frac{Q}{\sqrt{2}\pi\varepsilon_0 a^2}(\hat{k})$$

52. a. Take PO as the x-axis and PA as the y-axis. Consider two elements EF and $E'F'$ of width $d\theta$ at angular distance q above and below PO, respectively. The magnitude of the field at P due to either element is

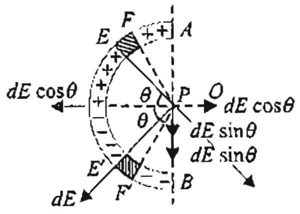


Fig. 1.197

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{r\,d\theta \times \dfrac{Q}{\pi r/2}}{r^2} = \frac{Q}{2\pi^2\varepsilon_0 r^2}d\theta$$

Resolving the fields, we find that the components alo:. PO sum up to zero and hence, the resultant field is along PB.

$\therefore$ Field at P due to pair of elements $= 2dE\sin\theta$

$$E = \int_0^{\pi/2} 2dE\sin\theta$$
$$= 2\int_0^{\pi/2}\frac{Q}{2\pi^2\varepsilon_0 r^2}\sin\theta\,d\theta = \frac{Q}{\pi^2\varepsilon_0 r^2}$$

53. a.

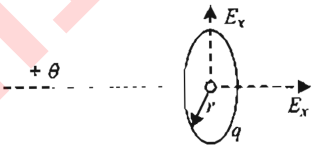


Fig. 1.198

Net force $F_{\text{net}} = qE_x$

$$F = q\frac{\lambda}{4\pi\varepsilon_0 r} = \frac{\lambda q}{4\pi\varepsilon_0 r}$$

54. b. Let us consider the electric field due to wire (3) only.

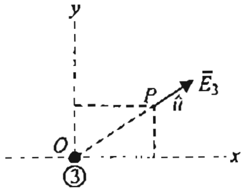


Fig. 1.199

$$\vec{E}_3 = E\hat{u}$$
$$\vec{E}_3 = \frac{\lambda}{2\pi\varepsilon_0(a^2+a^2)^{1/2}}(\hat{i}\cos 45° + \hat{j}\cos 45°)$$
$$= \frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 a}\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$$
$$\vec{E}_3 = \frac{\lambda}{4\pi\varepsilon_0 a}(\hat{i} + \hat{j})$$

Similarly, electric field due to wires (1) and (2)

$$\vec{E}_1 = \frac{\lambda}{4\pi\varepsilon_0 a}(\hat{j} + \hat{k}) \text{ and } \vec{E}_2 = \frac{\lambda}{4\pi\varepsilon_0 a}(\hat{i} + \hat{k})$$
$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$\vec{E}_{\text{net}} = \frac{\lambda}{2\pi\varepsilon_0 a}(\hat{i} + \hat{j} + \hat{k})$$

55. c.

$$F = \frac{k\,Q\,r\,q}{R^3} = -ma$$

$$a = -\left(\frac{Q\,q}{4\pi\varepsilon_0 m R^3}\right) r \Rightarrow \quad a = -\omega^2 r$$

$$\omega = \sqrt{\frac{Qq}{4\pi\varepsilon_0 m R^3}}$$

56. a. Mean position: $x = l$

$F = -k(x - 1)$, where $k = m\omega^2$

or $qE = -m\omega^2(x - 1) \Rightarrow$ At $x = 1$, $E = 0$

To the right of $x = 1$, E is –ve, so towards left and to the left of $x = 1$, E is +ve.

57. c. Following two arguments shall lead us to the right choice.

i. Electric field at the center of the ring is zero.

ii. Electric field is directed away from the ring.

58. b. Following two arguments decide the right choice.

i. Acceleration in zero at the origin and also at points which are far away from the origin.

ii. The directions of acceleration on the sides of the origin are opposite.

Multiple Correct Answers Type

1. a., b., c., d. Charge on $a_1 = (r_1\theta)\lambda$

Charge on $a_2 = (r_2\theta)\lambda$

Ratio of charges $= \frac{r_1}{r_2}$

E_1 (Field produced by a_1) $= \frac{K[(r_1\theta)\lambda]}{r_1^2} = \frac{K\theta\lambda}{r_1}$

E_2 (Field produced by a_2) $= \frac{K\theta\lambda}{r_2}$

As $r_2 > r_1$ therefore $E_1 > E_2$

i.e., Net field at A is towards a_2.

$$V_1 = \frac{K(r_1\theta)}{r_1} = K\theta\lambda$$

$$V_2 = \frac{K(r_2\theta)}{r_2} = K\theta\lambda$$

$$V_1 = V_2$$

2. c., d. The particle will oscillate or perform SHM if equilibrium is a stable one. For negative charge, equilibrium is stable if the particles are displaced along CD and unstable for displacement along AB.

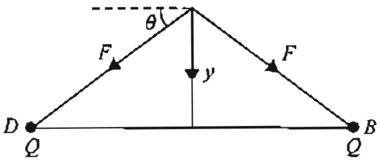


Fig. 1.200

For $y < a$, in displaced position, net resultant force towards equilibrium position is

$$F_{\text{res}} = 2F\sin\theta = \frac{2 \times Q}{4\pi\varepsilon_0(a^2 + y^2)} \times \frac{y}{(a^2 + y^2)^{1/2}}$$

$$= \frac{2Qy}{4\pi\varepsilon_0 a^3}[a \gg y]$$

$F_{\text{res}} \propto y$, so SHM for $y << a$.

For $y \simeq a$, restoring force would be there, but $F \propto y$. So, there is oscillatory motion but not SHM.

3. b., c. Along X-axis, $u_x = u\cos\theta \quad \Rightarrow \quad a_x = -\frac{qE}{m}$

Along Y-axis, $u_y = u\sin\theta \quad \Rightarrow \quad a_y = -g$

Equation of motion along X- and Y-axes would be

$$x = u\cos\theta t - \frac{qE}{2m}t^2; \quad y = u\sin\theta t - \frac{1}{2}gt^2$$

Solving above equations, we get an equation of the form $Ax + By = Ct$, which is a linear equation.

Time of flight remains unchanged as vertical motion is not affected by $\vec{E}$. Range of the particle in the present case is always less than $\frac{u^2\sin 2\theta}{g}$ whatever be the value of E.

$$R = x_{t=T} = u\cos\theta \times \frac{2u\sin\theta}{g} - \frac{1}{2}$$

$$\times \frac{qE}{m}\left[\frac{2u\sin\theta}{g}\right]^2 < \frac{u^2\sin 2\theta}{g}$$

4. a., d. E is a vector quantity; V is a scalar quantity.

5. b., c. Just draw the FBD of both charge particles and see whether force can be zero or not, on the particle.

Assertion-Reasoning Type

1. **a.** Force by electric field will be perpendicular to the displacements.
2. **c.** If the field lines are curved, then the charge particle follows the straight line path along the direction of tangent drawn to electric field lines at its starting point.
3. **a.** A body can be charged by the transfer of electrons only.
4. **a.** Reason truly explains the assertion. However, if charges are point, no induction will take place and they will never attract.
5. **a.** Both the assertion and the reason are true, and the reason is correct explanation of the assertion.

Comprehension Type

For Problems 1–2

1. b., 2. c.

Sol. Charge on each ball $= \frac{10 + 30}{2} = 20\ \mu\text{C}$

$$F = 9 \times 10^9 \times \frac{(20 \times 10^{-6})^2}{(0.50)^2} = 14.4\ \text{N}$$

For Problems 3–5

3. d., 4. a., 5. c.

Sol.

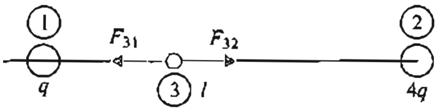


Fig. 1.201

For system to be in equilibrium net force on each charge should be zero, hence the third charge should be negative and it should be placed near q between (1) and (2).

For equilibrium of (3)

$$\frac{1}{4\pi\varepsilon_0}\frac{qQ}{x^2}=\frac{1}{4\pi\varepsilon_0}\frac{Q4q}{(l-x)^2} \Rightarrow \left(\frac{l-x}{x}\right)^2=4$$

$$\frac{l-x}{x}=2$$

$$x=\frac{l}{3}$$

For equilibrium of (1)

$$\frac{1}{4\pi\varepsilon_0}\frac{q4q}{l^2}=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{x^2}$$

$$Q=4q\left(\frac{x}{l}\right)^2=4q\left(\frac{1}{3}\right)^2=\frac{4q}{9} \Rightarrow Q=\frac{4q}{9}$$

q should be negative otherwise resultant force on Q cannot be zero.

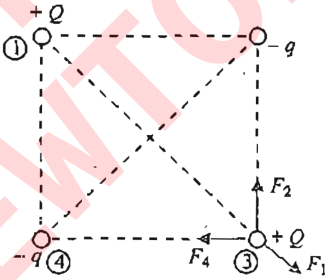


Fig. 1.202

As $|F_2|=|F_4|=F$ (say)

For force on Q to be zero

$2F\cos 45° = F_2$

$$2\frac{1}{4\pi\varepsilon_0}\frac{Qq}{l^2}\frac{1}{\sqrt{2}}=\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(\sqrt{2}l)^2}$$

$$q=\frac{Q}{2\sqrt{2}}$$

For Problems 6–8

6. c., 7. b., 8. c.

Sol. The four possible force diagrams are:

Only the last picture can result in an electric field in the $-x$-direction.

$q_1=-2.00\ \mu\text{C},\ q_3=+4.00\ \mu\text{C}$, and $q_2>0$.

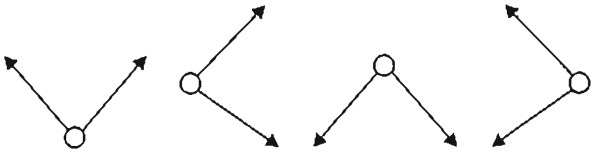


Fig. 1.203

$$E_y=0=\frac{1}{4\pi\varepsilon_0}\frac{q_1}{(0.0400\text{ m})^2}\sin\theta_1-\frac{1}{4\pi\varepsilon_0}\frac{q_2}{(0.0300\text{ m})^2}\sin\theta_2$$

$$q_2=\frac{9}{16}q_1\frac{\sin\theta_1}{\sin\theta_2}=\frac{9}{16}q_1\frac{3/5}{4/5}=\frac{27}{64}q_1=0.843\ \mu\text{C}.$$

$$F_3=q_3E_x=q_3\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{0.0016}\frac{4}{5}+\frac{q_2}{0.0009}\frac{3}{5}\right)=56.2\text{ N}$$

For Problems 9–10

9. c., 10. b.

Sol. Induced charge on the inner surface will be equal and opposite to the charge placed at the center.

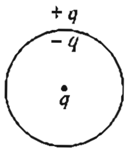


Fig. 1.204

Equal amount of opposite charge as that of inner surface will be induced on the outer surface while net charge is zero on the ball.

For Problems 11–12

11. c., 12. a.

Sol. The extra given charge will reside on the outer surface.

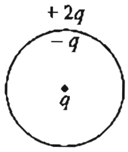


Fig. 1.205

For Problems 13–14

13. c., 14. c.

Sol. As far as net charge induced on inner and outer surfaces is concerned, it does not depend upon the location of point charge within the ball.

For Problems 15–17

15. a., 16. a., 17. b.

Sol. a. Electric field at (2) tends to $-\infty$, hence the charge at (2) should be negative. There is a neutral point to the right of charges. This is possible only when the charge at (1) should be positive.

Hence, Q_1 is positive and Q_2 is negative.

b. At neutral point, $\vec{E}_1=\vec{E}_2$

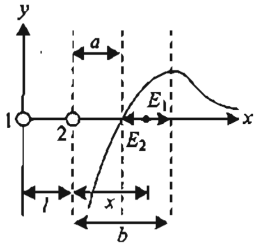


Fig. 1.206

$$\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{(a+l)^2}=\frac{1}{4\pi\varepsilon_0}\frac{Q_2}{a^2} \quad \Rightarrow \frac{Q_1}{Q_2}=\left(\frac{a+l}{a}\right)^2$$

c. Electric field at any position to the right of charges

$$E=E_1-E_2=\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{(l+x)^2}-\frac{1}{4\pi\varepsilon_0}\frac{Q_2}{x^2}$$

For maximum value of E

$$\frac{dE}{dx}=0 \Rightarrow \frac{d}{dx}\left[\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{(l+x)^2}-\frac{1}{4\pi\varepsilon_0}\frac{Q_2}{x_2}\right]=0$$

$$Q_1\frac{(-2)}{(l+x)^3}-Q_2\frac{(-2)}{x^3}=0$$

$$\frac{Q_1}{(l+x)^3}=\frac{Q_2}{x^3} \Rightarrow \left(\frac{l+x}{x}\right)^3=\frac{Q_1}{Q_2}$$

$$\frac{l+x}{x}=\left(\frac{Q_1}{Q_2}\right)^{1/3} \quad \Rightarrow \frac{l}{x}=\left(\frac{Q_1}{Q_2}\right)^{1/3}-1$$

$$x=\frac{l}{\left(\frac{Q_1}{Q_2}\right)^{1/3}-1}$$

For Problems 18–19

18. d., 19. b.

Sol. The four possible diagrams are:

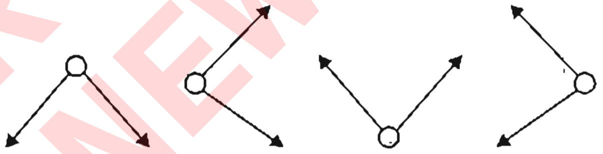


Fig. 1.207

The first diagram is the only one in which the electric field must point in the negative y-direction.

$$q_1=-3.00\ \mu\text{C}, \text{ and } q_2<0.$$

$$E_x=0=\frac{kq_1}{(0.050\text{ m})^2}\frac{5}{13}-\frac{kq_2}{(0.120\text{ m})}^2\frac{12}{13}$$

$$\Rightarrow \quad \frac{kq_2}{(0.120\text{ m})^2}=\frac{kq_1}{(0.050\text{ m})^2}\frac{5}{12}$$

$$E=E_y=\frac{kq_1}{(0.050\text{ m})^2}\frac{12}{13}+\frac{kq_2}{(0.120\text{ m})^2}\frac{5}{13}$$

$$=\frac{kq_1}{(0.05\text{ m})^2}\left(\frac{12}{13}+\left(\frac{5}{12}\right)\left(\frac{5}{13}\right)\right)$$

$$E=E_y=1.17\times10^7\text{ NC}^{-1}$$

For Problems 20–21

20. b., 21. a.

Sol. $OA=OB=OC=OD=a$

The magnitude of the electric force due to each charge is $\frac{1}{4\pi\varepsilon_0}\frac{Q}{a^2+h^2}$

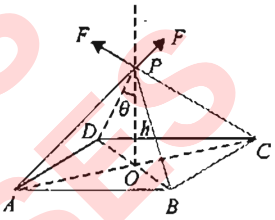


Fig. 1.208

The components of the force perpendicular to OP sum up to zero because of the symmetrical distribution of charges about OP. Hence, the resultant force at P is upward along OP. The magnitude of force is given by

$$F_{\text{up}}=4\times\frac{1}{4\pi\varepsilon_0}\frac{Q}{h^2+a^2}\cos\theta$$

$$=\frac{Q}{\pi\varepsilon_0(h^2+a^2)}\frac{h}{\sqrt{h^2+a^2}}=\frac{Qh}{\pi\varepsilon_0(h^2+a^2)^{3/2}}$$

$$F_{\text{down}}=\text{weight}=mg$$

For equilibrium, $mg=\frac{Qh}{\pi\varepsilon_0(h^2+a^2)^{3/2}}$

$$\Rightarrow \qquad Q=\pi\varepsilon_0\frac{mg}{h}(h^2+a^2)^{3/2}$$

Yes, this is a stable equilibrium as it will regain its position if displaced a little.

For Problems 22–25

22. c., 23. b., 24. c., 25. d.

Sol.

a. Passing between the charged plates, the electron feels a force upward and just misses the top plate. The distance it travels in the y-direction is 0.005 m.

$$\text{Time of flight}=t=\frac{0.0200\text{ m}}{1.60\times10^6\text{ m/s}}$$

$$=1.25\times10^{-8}\text{ s}$$

and initial y-velocity is zero.

$$\text{Now, } y=v_{0y}t+\frac{1}{2}at^2$$

$$\text{So, } 0.005\text{ m}=\frac{1}{2}a(1.25\times10^{-8}\text{s})^2$$

$$\Rightarrow \quad a=6.40\times10^{13}\text{ ms}^{-2}$$

$$\text{But also, } a=\frac{F}{m}=\frac{eE}{m_e}$$

$$\therefore \quad E=\frac{(9.11\times10^{-31}\text{ kg})(6.40\times10^{13}\text{ ms}^{-2}}{1.60\times10^{-19}\text{ C}}$$

$$=364\text{ NC}^{-1}$$

b. Since the proton is more massive, it will accelerate less, and NOT hit the plates. To find the vertical displacement when it exits the plates, we use the kinematic equations again:

$$y = \frac{1}{2}at^2 = \frac{1}{2}\frac{eE}{m_p}(1.25 \times 10^{-8}\text{s})^2$$

$$= 2.73 \times 10^{-6}\ \text{m}$$

c. As mentioned in (b), the proton will not hit one of the plates because although the electric force felt by the proton is the same as the electron felt, a smaller acceleration results for the more massive proton.

The acceleration produced by the electric force is much greater than g; it is reasonable to ignore gravity.

For Problems 26–27

26. **c.,** **27.** **d.**

Sol. $d = \dfrac{u^2 \sin^2\theta}{2a}, \quad a = \dfrac{qE}{m}$

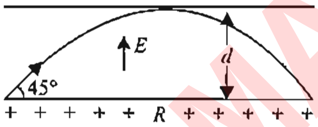


Fig. 1.209

$$d = \frac{mu^2 \sin^2 45°}{2qE}$$

$$d = \frac{K}{2qE} \Rightarrow E = \frac{K}{2dq}$$

$$R = \frac{u^2}{a} = \frac{2d}{\sin^2\theta} = 4d$$

For Problems 28–29

28. **a.,** **29.** **a.**

Sol. a. $qE = mg \Rightarrow \quad q \times 6.8 \times 10^5 = 1.08 \times 10^{-14} g$

$\Rightarrow q = 6.40 \times 10^{-19}$

b. $q = ne \Rightarrow \quad 6.4 \times 10^{-19} = n \times 1.6 \times 10^{-19} \Rightarrow \quad n = 4$

Matching Column Type

1. **i.** → **a., c., d.** **ii.** → **a., b., d.** **iii.** → **a., b., d.** **iv.** → **a., c., d.**

Sol. In case of stable equilibrium, the potential energy is minimum and in case of unstable equilibrium potential energy is maximum.

Cases (i) and (iv) are the cases of stable equilibrium, hence answers are (a), (c) and (d). Cases (ii) and (iii) are the cases of unstable equilibrium hence answers are (a), (b) and (d).

2. **i.** → **a., b., c., d.** **ii.** → **a., b., d.** **iii.** → **a., b., d.** **iv.** → **a., b., c., d.**

Electrostatic force and gravitational force are conservative, action–reaction forces. Also, they depend upon the nature of the medium between interacting objects. The principle of superposition is applicable if all the forces acting on an object have the same nature.

3. **i.** → **a., c., d.** **ii.** → **a., b.** **iii.** → **a., b., c., d.** **iv.** → **a., c., d.**

Electrostatic forces or the Coulombic force exist between charged bodies. Gravitational force exists between all bodies. Strong force exists between particles inside the nucleus.

4. **i.** → **b.** **ii.** → **a.** **iii.** → **d.** **iv.** → **c.**

Charge always needs mass to reside. Mass may or may not be charged. Charge is a coserved quantity but mass is not.

5. **i.** → **c.** **ii.** → **a.** **iii.** → **d.** **iv.** → **b.**

$$\varepsilon_0 \propto \frac{q_1q_2}{Fr^2} \Rightarrow [\varepsilon_0] = [M^1L^{-3}T^4A^2]$$

k = Dimensionless

$$E = \frac{F}{q} \Rightarrow [E] = [M^1L^1T^{-2}Q^{-1}]$$

$$= [M^1L^1T^{-2}A^{-1}T^{-1}]$$

$$= [M^1L^1T^{-3}A^{-1}]s$$

$$[F] = [M^1L^1T^{-2}]$$

CHAPTER

2

Electric Flux and Gauss's Law

ELECTRIC FLUX

The electric flux through a surface is a description of whether the electric field points into or out of the surface.

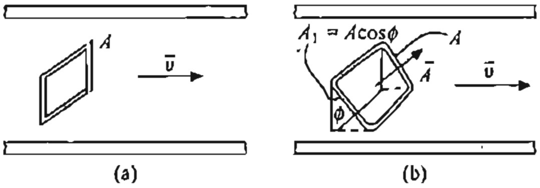
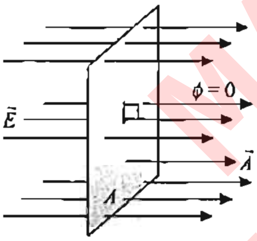


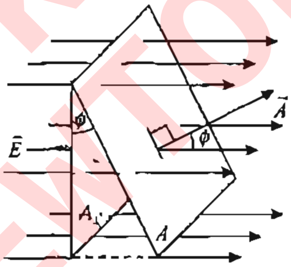
Fig. 2.1

Fig. 2.1 shows a fluid flowing steadily from left to right. Let us examine the volume flow rate dV/dt (in, say, cubic meters per second) through the wire rectangle with area A. When the area is perpendicular to the flow velocity [as shown in Fig. 2.1(a)] and the flow velocity is the same at all points in the fluid, the volume flow rate dV/dt is the area A multiplied by the flow speed v

$$\frac{dV}{dt} = vA$$



(a) Surface face on to electric field $\vec{E}$ and $\vec{A}$ parallel angle between $\vec{E}$ and $\vec{A}$ is $\phi = 0$ flux $\Phi_E = \vec{E}\cdot\vec{A} = EA$



(b) Surface tilted from face-on orientation by an angle ϕ angle between $\vec{E}$ and $\vec{A}$ is ϕ flux $\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\phi$

Fig. 2.2

When the rectangle is tilted at an angle ϕ (as shown in Fig. 2.1(b)) so that its face is not perpendicular to $\vec{v}$, the area that counts is the silhouette area that we see when we look in the direction of $\vec{v}$. This area, which is outlined in red and labeled $A_\perp$ in Fig. 2.1(b), is the projection of area A onto a surface perpendicular to $\vec{v}$. Two sides of the projected rectangle have the same length as the original one, but the other two are foreshortened by a factor of $\cos\phi$, so the projected area $A_\perp$ is equal to $A\cos\phi$. Thus, the volume flow rate through A is

$$\frac{dV}{dt} = vA\cos\phi$$

If $\phi = 90°$, $dV/dt = 0$; the wire rectangle is edge-on to the flow, and no fluid passes through the rectangle.

Also, $v\cos\phi$ is the component of the vector $\vec{v}$ perpendicular to the plane of area A. Calling this component $v_\perp$, we can rewrite the volume flow rate as $\dfrac{dV}{dt} = v_\perp A$.

We can express the volume flow rate more compactly by using the concept of vector area $\vec{A}$, a vector quantity with magnitude A and direction perpendicular to the plane of the area we are describing. The vector area $\vec{A}$ describes both the size of an area and its orientation in space. In terms of $\vec{A}$, we can write the volume flow rate of fluid through the rectangle in Fig. 2.1(b) as a scalar (dot) product:

$$\frac{dV}{dt} = \vec{v}\cdot\vec{A}$$

Using the analogy between electric field and fluid flow, we now define electric flux in the same way as we have just defined the volume flow rate of a fluid.

Roughly speaking, we can picture Φ_E in terms of the field lines passing through A (Fig. 2.2).

We generalize our definition of electric flux for a uniform electric field to

$\Phi_E = EA\cos\phi$ (electric flux for uniform, flat surface) (i)

Since $E\cos\phi$ is the component of E perpendicular to the area, we can rewrite equation (i) as

$\Phi_E = E_\perp A$ (electric flux for uniform, flat surface) (ii)

In terms of the vector area $\vec{A}$ perpendicular to the area, we can write the electric flux as the scalar product of $\vec{E}$ and $\vec{A}$:

$\Phi_E = \vec{E}\cdot\vec{A}$ (electric flux for uniform, flat surface) (iii)

A surface has two sides, so there are two possible directions for $\vec{A}$. We must always specify which direction of $\vec{A}$ we choose. For a closed surface, we will always choose the direction of $\vec{A}$ to be outward, and we will speak of the flux out of a closed surface. Thus, what we called "outward electric flux" corresponds to a positive value of Φ_E and what we called "inward electric flux" corresponds to a negative value of Φ_E.

Illustration 2.1 **Find the flux of the electric field through each of the five surfaces of the inclined plane as shown in Fig. 2.3. What is the total flux through the entire closed surface?**

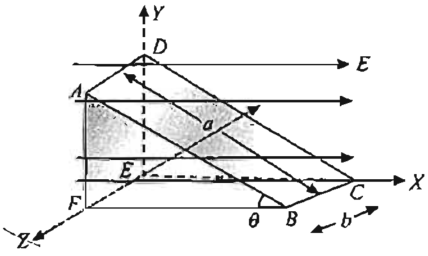


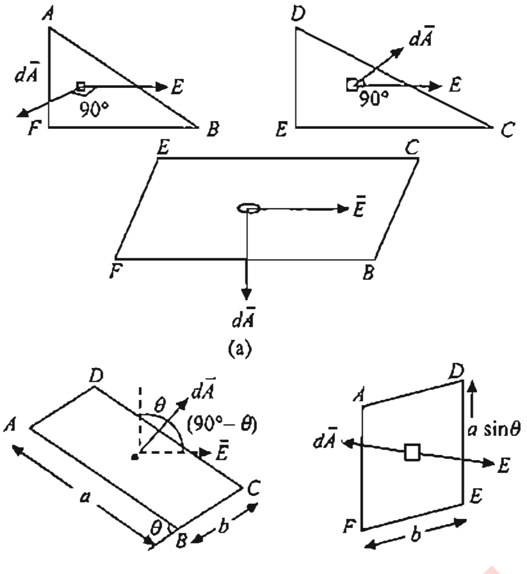
Fig. 2.3

Sol. Note that flux through the faces ABF, CDE and $BCEF$ is zero. Area vector of face ABF points in the positive z-direction, area vector of CDE points in the negative z-direction and area vector of $BCEF$ points in the negative y-direction. In all the three cases, field E is normal to area vector.

Flux through face $ABCD$ (Fig. 2.4(b)):

Magnitude of area vector of face $ABCD = ab$

$$\phi_E = \vec{E}\cdot\vec{A} = E(ab)\ \cos(90° - \theta) = Eab\ \sin\ \theta$$



(a)

(b)

(c)

Fig. 2.4

Flux through face *ADEF* (Fig. 2.4(c)):
Magnitude of area vector of face

$$ADEF = (a \ \sin\theta)b = ab\sin\theta$$

$$\phi_E = E\cos 180^\circ\,(ab\sin\theta) = -Eab \ \sin\theta$$

Thus, we obtain

$$(\phi_E)_{ABF} = 0, \ (\phi_E)_{CDE} = 0, \ (\phi_E)_{BCEF} = 0,$$

$$(\phi_E)_{ABCD} = +Eab\sin\theta \quad \text{and } (\phi_E)_{ADEF} = -Eab\sin\theta.$$

Flux is a scalar quantity, therefore total flux is algebraic sum of flux through each surface.

$$\phi_{\text{total}} = (\phi_E)_{ABF} + (\phi_E)_{CDE} + (\phi_E)_{BCEF} + (\phi_E)_{ABCD} + (\phi_E)_{ADEF}$$

$$= 0 + 0 + 0 + Eab \ \sin\theta - Eab \ \sin\theta = 0$$

- Note that the contribution to the flux for a closed surface is positive for the surface where the field is directed out (*ABCD*) and negative for the surface where the field is directed into the surface (*ADEF*).
- The net flux for this closed surface can also be seen to be zero from examination of the field lines. If the field is uniform, the number of lines that enter the closed surface equals the number of lines that come out.
- The flux of a constant vector through any closed surface is zero.

Illustration 2.2 **Consider a cylindrical surface of radius *R*, length *l*, in a uniform electric field *E*. Compute the electric flux if the axis of the cylinder is parallel to the field direction.**

Sol. We can divide the entire curved surface into three parts, right and left plane faces and curved portion of its surface. Hence, the surface integral consists of the sum of the three terms:

$$\phi_E = \oint E\cdot dA = \int_{\text{left end}} E\cdot dA + \int_{\text{right end}} E\cdot dA + \int_{\text{curved}} E\cdot dA$$

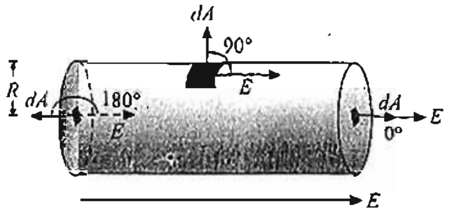


Fig. 2.5

All the area elements on the left end and electric field *E* are at an angle of 180°

$$(\phi_E)_{\text{left end}} = \oint_{\text{left end}} E\cdot dA = \oint_{\text{left end}} E\ dA\cos 180^\circ$$

$$= -E\oint_{\text{left end}} dA = -E\pi R^2$$

Note that *E* is constant over the entire plane surface of left end; therefore we take it out from the integral.

Similarly, all the area elements on the right end are parallel to electric field *E*, i.e., angle is 0°.

$$(\phi_E)_{\text{right end}} = \oint_{\text{right end}} E\cdot dA = \oint_{\text{right end}} E\ dA\cos 0^\circ$$

$$= +E\oint_{\text{right end}} dA = E\pi R^2$$

Finally, at every point on the curved surface the area vectors are perpendicular to the direction of the electric field. Thus,

$$(\phi_E)_{\text{curved}} = \oint_{\text{curved surface}} E\cdot dA = \oint_{\text{curved surface}} E\ dA\,(\cos 90^\circ) =$$

$$\text{Total flux} = (\phi_E)_{\text{right end}} + (\phi_E)_{\text{left end}} + (\phi_E)_{\text{curved surface}}$$

$$= (+E\pi R^2) + (-E\pi R^2) + 0 = 0$$

Hence, we see that in a uniform field the flux through a closed surface is zero. This is true for any shape of closed surface.

Illustration 2.3 **A point charge *q* is placed at a distance $\frac{a}{2}$ from the centre of a square of side *a* as shown in the Fig. 2.6. Calculate the electric flux passing through the square.**

Sol.

- This problem can solved by symmetry consideration and Gauss Law.

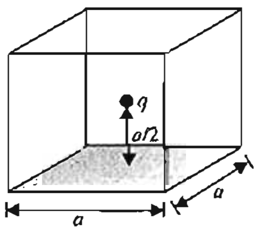


Fig. 2.6

- We can enclose the charged particle by a cube of side '*a*' and keeping the particle at the centre of the cube.
- The total flux passing through the close cube $\phi = \frac{q}{\varepsilon_0}$.

- All the six surfaces are symmetrical with respect to charge, hence they will have equal contribution of the flux. So, flux through any one face: $\phi' = \frac{\phi}{6} = \frac{q}{6\varepsilon_0}$.

Illustration 2.4 **In Fig. 2.7, shown a charge q is placed at a distance $\delta \rightarrow 0$ near one of the edges of a cube of edge l on a line of symmetry along diagonal.**

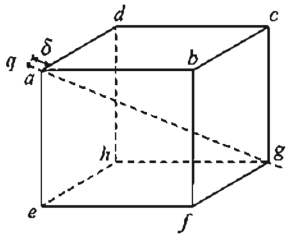


Fig. 2.7

1. **What is flux through each of the faces containing the point a?**
2. **What is the flux through the other three faces?**

Sol. Use of symmetry consideration may be useful in problems of flux calculation.

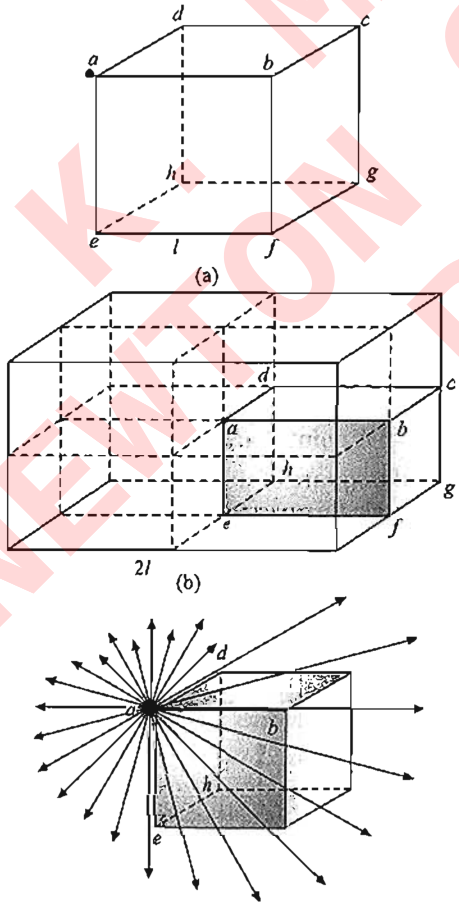


Fig. 2.8

- We can imagine a charged particle is placed at the centre of a cube of side $2l$.
- The flux enclosed with the cube $\phi = \frac{q}{\varepsilon_0}$.
- The flux passing through one of the face of the cube $\phi' = \frac{\phi}{6} = \frac{q}{6\varepsilon_0}$.
- Hence, the flux passing through the face $bcgf$ $= \frac{\phi'}{4} = \frac{q}{24\varepsilon_0}$.
- Each of the face $(efgh)$, $(bcgf)$ and $(dcgh)$ are symmetrical with respect to charge. Hence, the flux passing through each of the face is $\frac{q}{24\varepsilon_0}$.
- The electric field lines for the faces $(efgh)$, $(bcgf)$ and $(dcgh)$ are away from the faces. Hence, the flux associated with each of the faces will be positive $\left(\text{i.e., } + \frac{q}{24\varepsilon_0}\right)$.
- Hence, total flux through these sides $= \frac{3 \times q}{24\varepsilon_0} = \frac{q}{8\varepsilon_0}$.
- As $\delta \rightarrow 0$, we can say the faces $(abcd)$, $(abfe)$ and $(adhe)$ are also symmetrical about charge. Charge is slightly outside the cube.
- The number of electric field lines which are passing through the faces which do not contain the point a are same as the number of electric field lines passing through the faces containing the point a.
- Hence, same amount of flux will pass through the faces containing the point a: i.e., $\frac{q}{8\varepsilon_0}$.
- The electric field lines are towards the faces containing the point a. Hence, the flux will be negative, i.e., $\phi'' = -\frac{q}{8\varepsilon_0}$.
- Hence, the flux through each of the faces containing the point 'a' will be $\frac{\phi''}{3} = -\frac{q}{24\varepsilon_0}$.

Your Task: Repeat Illustration 2.4 if the charge is exactly at the corner of the cube given.

Concept Application Exercise 2.1

1. A charge Q is distributed uniformly on a ring of radius r. A sphere of equal radius r is constructed with its centre at the periphery of the ring. Find the flux of the electric field through the surface of the sphere.

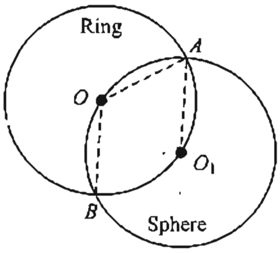


Fig. 2.9

2. Figure 2.10(a) shows an imaginary cube of edge $L/2$. A uniformly charged rod of length L moves towards left at a small

but constant speed v. At $t = 0$, the left end just touches the centre of the face of the cube opposite it. Which of the graphs shown in Fig. 2.10(b) represents the flux of the electric field through the cube as the rod goes through it?

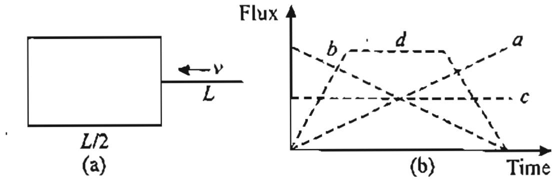


Fig. 2.10

3. A hemispherical body is placed in a uniform electric field E. What is the flux linked with the curved surface, if the field is (a) parallel to base of the body [Fig. 2.11(a)]; (b) perpendicular to base of the body [Fig. 2.11(b)]; and (c) perpendicular to the curved surface at every point as in Fig. 2.11(c).

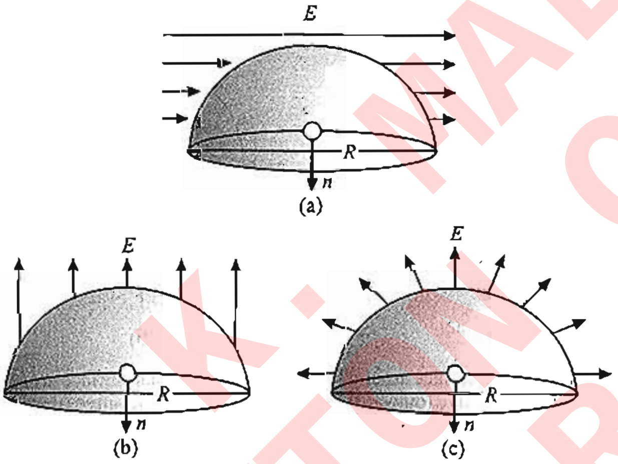


Fig. 2.11

4. What is the field in the cavity, if a conductor having a cavity is charged? Does the result depend on the shape and size of cavity or conductor?

5. Fig. 2.12 shows a closed surface which intersects a conducting sphere. If a positive charge is placed at the point P, find the sign of flux passing through the curved surface S.

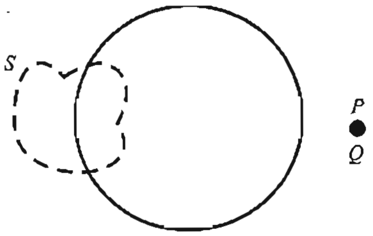


Fig. 2.12

6. In which position (A, B, C or D) of second charge the flux of the electric field through the hemisphere remains unchanged?

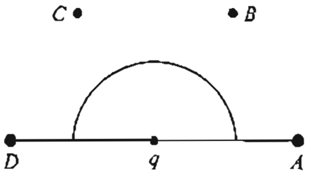


Fig. 2.13

7. A point charge Q is located just above the centre of the flat face of a hemisphere of radius R as in Fig. 2.14. What if the flux:

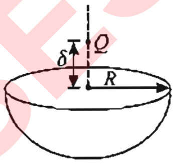


Fig. 2.14

a. through the curved surface, and

b. through the flat face?

c. Repeat parts (a) and (b) if the charge is exactly at the centre.

8. In Fig. 2.15, a cone lies in a uniform electric field E. Determine the electric flux entering the cone.

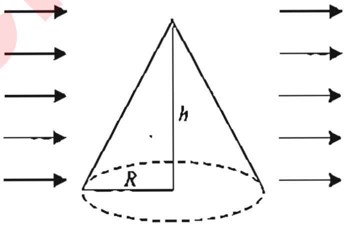


Fig. 2.15

9. A uniform electric field $a\hat{i} + b\hat{j}$ intersects a surface of area A. What is the flux through this area if the surface lies: (a) in the yz plane? (b) in the xz plane? (c) in the xy plane?

10. a. A point charge q is located a distance d from an infinite plane. Determine the electric flux through the plane due to the point charge.

b. A point charge q is located at a very small distance from the center of a very large square on the line perpendicular to the square and going through its center. Determine the approximate electric flux through the square due to the point charge.

11. Calculate the total electric flux through the paraboloidal surface due to a uniform electric field of magnitude E_0 in the direction shown in Fig. 2.16.

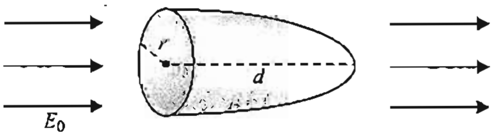


Fig. 2.16

12. Consider a closed surface of arbitrary shape as shown in Fig. 2.17. Suppose a single charge Q_1 is located at some point with in the surface and second charge Q_2 is located outside the surface

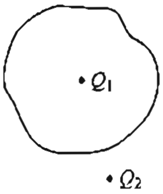


Fig. 2.17

a. What is the total flux passing through the surface due to charge Q_1?

b. What is the total flux passing through the surface due to charge Q_2?

13. If Coulomb's law involved $1/r^3$ (instead of $1/r^2$) would Gauss's law be still true?

GAUSS'S LAW

Gauss's law is an alternative to Coulomb's law. While completely equivalent to Coulomb's law, Gauss' law provides a different way to express the relationship between electric charge and electric field.

Gauss's law states that the total electric flux through a closed surface is proportional to the total electric charge enclosed within the surface. This law is useful in calculating field caused by charge distributions that have various symmetry properties.

Mathematically, Gauss's Law can be written as

$$\phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\varepsilon_0}$$

We once again emphasise that the electric field appearing in the Gauss's law is the resultant electric field due to all the charges present inside as well as outside the given closed surface. On the other hand, the charges q_{in} appearing in the law are only the charges contained within the closed surface. The contribution of the charges outside the closed surface in producing the flux is zero. A surface on which Gauss's law is applied is sometimes called the Gaussian surface (Fig. 2.18).

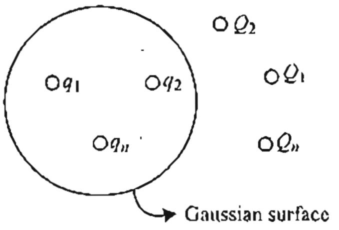
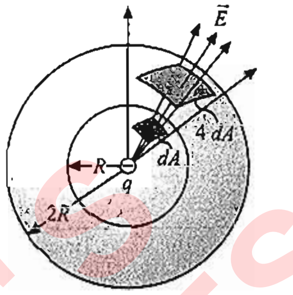
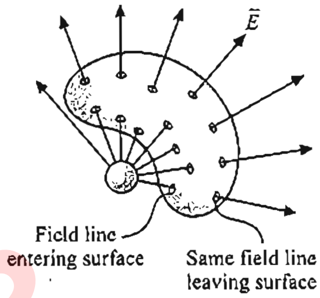


Fig. 2.18

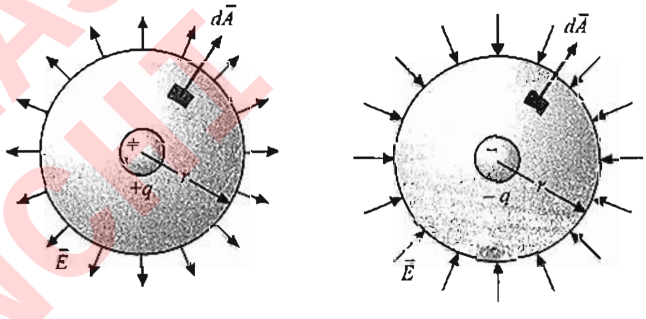
Caution: Remember that the closed surface in Gauss's law is imaginary; there need not be any material object at the position of the surface. We often refer to a closed surface used in Gauss's law as a Gaussian surface (Fig. 2.19).



Projection of an element of area dA of a sphere of radius R onto a concentric sphere of radius $2R$. The projection multiplies each linear dimension by 2, so the area element on the larger sphere is $4\,dA$. The same number of field lines and the same flux pass through these two area elements.



A point charge outside a closed surface that encloses no charge. If an electric field line from the external charge enters the surface at one point, it must leave at another. Figure illustrates this point. Electric field lines can begin or end inside a region of space only when there is charge in that region.



(a) Spherical Gaussian surface around positive charge; positive (outward) flux.

(b) Spherical Gaussian surface around negative charge; negative (inward) flux.

Fig. 2.19

Note that the electric field in the expression $\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\varepsilon_0}$ is the resultant field at any point on the Gaussian surface, whereas q_{in} is the charge enclosed by the Gaussian surface. Consider the two Gaussian surfaces A_1 and A_2 as shown in Fig. 2.20.

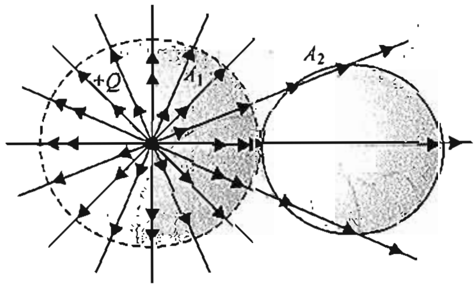


Fig. 2.20

Charge Q lies at the center of the Gaussian surface A_1. For surface A_1, the net flux through A_1 is $\frac{Q}{\varepsilon_0}$. For surface A_2, charge Q is outside A_2 so that the net flux through A_2 is zero. Note that the field lines that enter the Gaussian surface A_2 (net flux in) also leave it (net flux out).

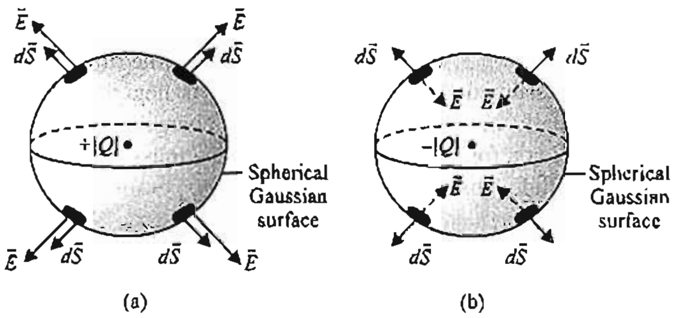


Fig. 2.21

Problem Solving Strategy

Identify the relevant concepts: Gauss's law is most useful in situations where the charge distribution has spherical or cylindrical symmetry or is distributed uniformly over a plane. In these situation, we determine the direction of $\vec{E}$ from the symmetry of the charge distribution. If we are given the charge distribution, we can use Gauss' law to find the magnitude of $\vec{E}$. Alternatively, if we are given the field, we can use Gauss's law to determine the details of the charge distribution. In either case, begin your analysis by asking the question, "What is the symmetry?"

The problem uses the following steps :

1. Select the surface that you will use with Gauss's law. We often call it a Gaussian surface. If you are trying to find the field at a particular point, then that point must lie on your Gaussian surface.

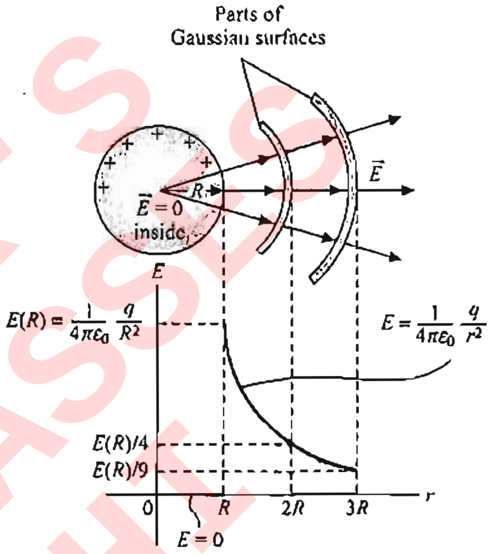
Charge Distribution	Gaussian Surface	Electric Field
Point charge	Spherical	Radial
Spherical charge distribution	Spherical	Radial
Line of charge	Cylindrical	Radial
Planer charge	Plane parallel to charge distribution	Normal to surface

2. The Gaussian surface does not have to be a real physical surface, such as a surface of a solid body. Often the appropriate surface is an imaginary geometric surface; it may be in empty space, embedded in a solid body, or both.

3. Usually, you can evaluate the integral in Gauss's law (without using a computer) only if the Gaussian surface and the charge distribution have some symmetry property. If the charge distribution has cylindrical or spherical symmetry, choose the Gaussian surface to be a coaxial cylinder or a concentric sphere, respectively.

FIELD OF A CHARGED CONDUCTING SPHERE

We place positive charge q on a solid conducting sphere with radius R (as shown in Fig. 2.22). All the charges must be on the surface of the sphere.



Under electrostatic conditions the electric field inside a solid conducting sphere is zero. Outside the sphere the electric field drops off as $1/r^2$, as though all the excess charge on sphere were concentrated at its centre.

Fig. 2.22

Selection of Gaussian Surface

The system has spherical symmetry. To take advantage of the symmetry, we take as our Gaussian surface an imaginary sphere of radius r centered on the conductor. To calculate the field outside the conductor, we take r to be greater than the conductor's radius R; to calculate field inside, we take r to be less than R. In either case, the point where we want to calculate $\vec{E}$ lies on Gaussian surface.

Electric Field Outside the Sphere

We first consider the field outside the conductor, so we choose $r > R$. The entire conductor is within the Gaussian surface, so the enclosed charge is q. The area of the Gaussian surface is $4\pi r^2$; $\vec{E}$ is uniform over the surface and perpendicular to it at each point. The flux integral $\oint E_\perp dA$ in Gauss's law is therefore just $E\,(4\pi r^2)$ which gives

$$E\,(4\pi r^2) = \frac{q}{\varepsilon_0}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad \text{(outside a charged conducting sphere)}$$

This expression for the field at any point outside the sphere $(r > R)$ is the same as for a point charge; the field due to the charged sphere is the same as though the entire charge were concentrated at its center. Just outside the surface of the sphere, where $r = R$,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

(at the surface of a charged conducting sphere)

Electric Field Inside the Sphere

We know that extra charge on a conductor lies on its outer surface. So there is no charge inside the Gaussian surface, i.e., $q_{in} = 0$ (Fig. 2.23).

$$\therefore \oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} = 0 \quad \Rightarrow \quad E4\pi r^2 = 0 \quad \Rightarrow \quad E = 0$$

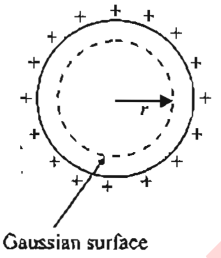


Fig. 2.23

Hence, at a point inside the sphere, electric field is zero.

FIELD OF A LINE CHARGE

Electric charge is distributed uniformly along an infinitely long, thin wire. The charge per unit length is λ (assumed positive), known as linear charge density.

Selection of Gaussian Surface

The system has cylindrical symmetry. This property suggest that we use as a Gaussian surface a cylinder with arbitrary radius r and arbitrary length l, with its ends perpendicular to the wire (Fig. 2.24). We break the surface integral for the flux Φ_E into an integral over each flat end and one over the curved side walls.

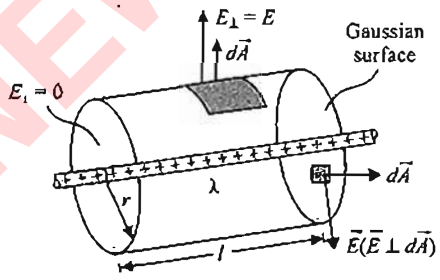


Fig. 2.24

There is no flux through the ends because $\vec{E}$ lies in the plane of the surface. To find the flux through the side walls, note that $\vec{E}$ is perpendicular to the surface at each point; by symmetry, E has the same value everywhere on the walls (curved surface). The area of the side walls is $2\pi rl$. (To make a paper cylinder with radius r and height l, you need a paper rectangle with width $2\pi r$ and height l, so area $2\pi rl$ (Fig. 2.25)). Hence, the total flux Φ_E through the entire cylinder is the sum of the flux through the side walls, which is $(E)(2\pi rl)$, and the zero flux through the two ends. Finally, we need the total enclosed charge, which is the charge per unit length multiplied by the length of wire inside the Gaussian surface, or $Q_{encl} = \lambda l$. From Gauss's law,

$$\Phi_E = (E)(2\pi rl) = \frac{\lambda \ell}{\varepsilon_0} \text{ and } E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

(field of an infinite line of charge)

We have assumed that λ is positive. If it is negative, $\vec{E}$ is directed radially inward toward the line of charge, and in the above expression for the field magnitude E we must interpret λ as the magnitude (absolute value) of the charge per unit length.

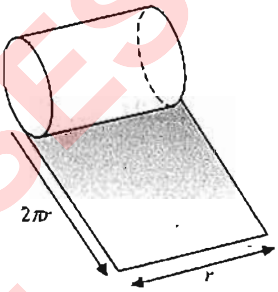


Fig. 2.25

FIELD OF AN INFINITE PLANE SHEET OF CHARGE

Let us consider a thin, flat, infinite sheet on which there is a uniform positive charge per unit area σ.

Selection of Gaussian Surface

To take advantage of these symmetry properties, we use as our Gaussian surface a cylinder with its axis perpendicular to the sheet of charge, with ends of area A (Fig. 2.26).

The charged sheet passes through the middle of the cylinder's length, so the cylinder is perpendicular to the surface; hence the flux through each end is EA. Because $\vec{E}$ is perpendicular to the charged sheet, it is parallel to the curved side walls of the cylinder, and there is no flux through these walls.

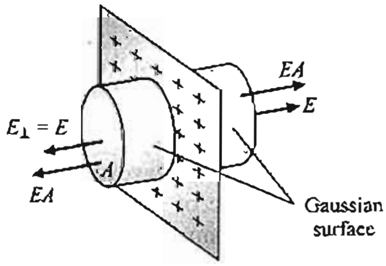


Fig. 2.26

The total flux integral in Gauss's law is then $2EA$ (EA from each end and zero from the side walls). The net charge within the Gaussian surface is the charge per unit area multiplied by the sheet area enclosed by the surface, or $Q_{encl} = \sigma A$. Hence, Gauss's law gives

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

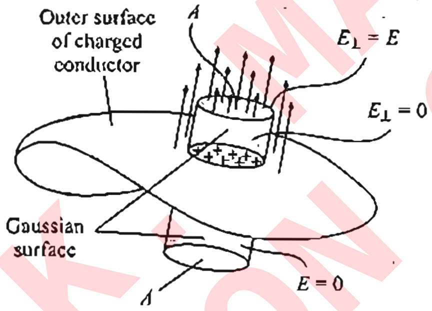
or $E = \dfrac{\sigma}{2\varepsilon_0}$ (field of an infinite sheet of charge)

If the charge density is negative, $\vec{E}$ is directed toward the sheet, the flux through the Gaussian surface in figure is negative, and in the expression $E = \dfrac{\sigma}{2\varepsilon_0}$, σ denotes the magnitude (absolute value) of the charge density.

The assumption that the sheet is infinitely large is an idealization; nothing in nature is really infinitely large. But the result $E = \dfrac{\sigma}{2\varepsilon_0}$ is a good approximation for points that are close to the sheet (compared to the sheet's dimensions) and not too near its edges. At such points, field is very nearly uniform and perpendicular to plane.

FIELD AT THE SURFACE OF A CONDUCTOR

To find a relation between σ at any point on the surface and the perpendicular component of the electric field at that point, we construct a Gaussian surface in the form of a small cylinder (as shown in Fig. 2.27).



The field just outside a charged conductor is perpendicular to the surface and its perpendicular component $E_\perp$ is equal to σ/ε_0.

Fig. 2.27

One end face, with area A, lies within the conductor and the other lies just outside. The electric field is zero at all points within the conductor. Outside the conductor the component of $\vec{E}$ perpendicular to the side walls of the cylinder is zero, and over the end face the perpendicular component is equal to $E_\perp$. (If σ is positive, the electric field points out of the conductor and $E_\perp$ is positive; if σ is negative, the field points inward and $E_\perp$ is negative.) Hence, the total flux through the surface is $E_\perp A$. The charge enclosed within the Gaussian surface is σA. So from Gauss's law,

$$E_\perp A = \frac{\sigma A}{\varepsilon_0}$$

or $E_\perp = \dfrac{\sigma}{\varepsilon_0}$ (field at the surface of a conductor)

We can check this with the results we have obtained for spherical, cylindrical, and plane surfaces.

FIELD OF A UNIFORMLY CHARGED SPHERE

Positive electric charge Q is distributed uniformly throughout the volume of an insulating sphere with radius R.

Selection of Gaussian Surface

The system is spherically symmetric. To make use of this symmetry, we choose as our Gaussian surface a sphere with radius r, concentric with the charge distribution.

Electric Field Inside the Sphere

From symmetry, the magnitude E of the electric field has the same value at every point on the Gaussian surface, and the direction of $\vec{E}$ is radial at every point on the surface. Hence, the total electric flux through the Gaussian surface is the product of E and the total area of the surface $A = 4\pi r^2$, that is, $\Phi_E = 4\pi r^2 E$.

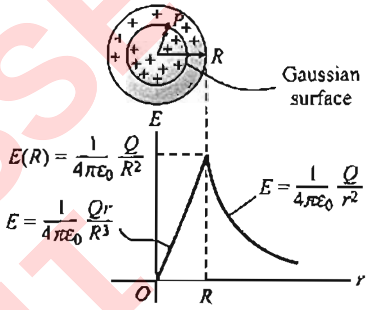


Fig. 2.28

The amount of charge enclosed within the Gaussian surface depends on the radius r. Let us first find the field magnitude inside the charged sphere of radius R; the magnitude E is evaluated at the radius of the Gaussian surface, so we choose $r < R$.

The volume charge density ρ is the charge Q divided by volume of the entire charged sphere of radius R: $\rho = \dfrac{Q}{4\pi R^3/3}$;

$$Q_{\text{encl}} = \rho V_{\text{encl}} = \left(\frac{Q}{4\pi R^3/3}\right)\left(\frac{4}{3}\pi r^3\right) = Q\frac{r^3}{R^3}$$

Then using Gauss's law, $\phi = \int E\, ds = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$

$$E\int ds = E4\pi r^2 = \frac{Q}{\varepsilon_0}\frac{r^3}{R^3}$$

or $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qr}{R^3}$

(field inside a uniformly charged sphere)

The field magnitude is proportional to the distance r of the field point from the center of the sphere (Fig. 2.28).

At the center ($r = 0$), $E = 0$.

Electric field in terms of charge density (at inside point)

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\left(\rho\frac{4}{3}\pi R^3\right)r}{R^3} = \frac{\rho r}{3\varepsilon_0}$$

$\Rightarrow$ $E = \dfrac{\rho r}{3\varepsilon_0}$ (field inside a uniformly charged sphere)

To find the field magnitude outside the charged sphere: We use a spherical Gaussian surface of radius $r > R$. This surface encloses the entire charged sphere, so $Q_{\text{encl}} = Q$ and Gauss's law gives

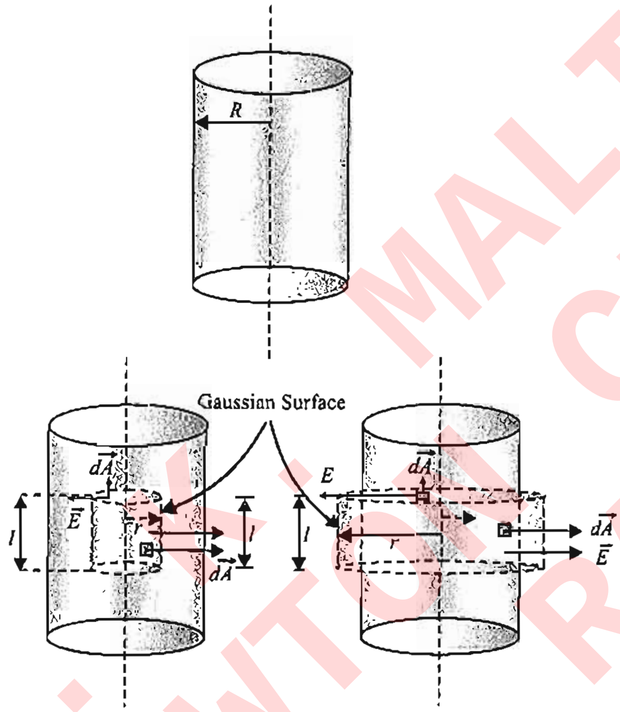
$$E4\pi r^2 = \frac{Q}{\varepsilon_0}$$

or $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ (field outside a uniformly charged sphere)

For any spherically symmetric charged body, the electric field outside the body is the same as though the entire charge were concentrated at the center.

ELECTRIC FIELD DUE TO A LONG UNIFORMLY CHARGED CYLINDER

Consider a long uniformly charged cylinder of volumetric charge density ρ and radius R.



(At point $r < R$) (At point $r > R$)

Fig. 2.29

For any point $r < R$ or $r > R$, the Gaussian surface will be cylindrical as shown in Fig. 2.29. For any point inside the cylinder ($r < R$),

$$E(2\pi r l) = \frac{q_{in}}{\varepsilon_0} = \frac{(\rho\pi r^2 l)}{\varepsilon_0}$$

$$E = \frac{\rho r}{2\varepsilon_0} \quad \Rightarrow \quad E \propto r$$

For any point outside the cylinder ($r > R$)

$$E(2\pi r l) = \frac{q_{in}}{\varepsilon_0} = \frac{(\rho\pi R^2 l)}{\varepsilon_0}$$

$$E = \frac{\rho R^2}{2\varepsilon_0 r} \quad \Rightarrow \quad E \propto \frac{1}{r}$$

Electric field inside the long uniformly charged cylinder varies lineary, i.e., $E \propto r$ and outside the cylinder the electric field varies inversely to the distance from the axis, i.e., $E \propto \frac{1}{r}$ (See Fig. 2.30).

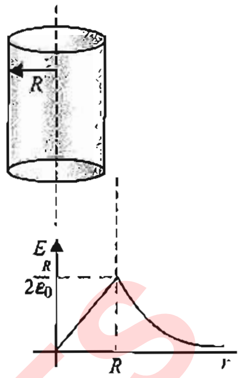


Fig. 2.30

ELECTRIC FIELD NEAR UNIFORMLY VOLUME CHARGED PLANE

Let there be charge distributed uniformly in an infinite plane of thickness d with the volume charge density ρ. Due to symmetry the electric field will be normally away and same in magnitude at same distances from the plane of symmetry.

Field Inside the Plane

Consider the Gaussian surface of the form of a cylinder of area S and thickness $2r$ ($< d$) placed symmetrically (Fig. 2.31). On the curved surface, flux of electric field will be zero as the area vector is perpendicular to the field vector. On left and right surfaces, flux is positive (outcoming flux). Hence,

$$\oint_S \vec{E}\,d\vec{S} = \frac{q_{\text{inclosed}}}{\varepsilon_0},$$

where $\oint_S \vec{E}d\vec{S} = \oint_{\text{Left}} \vec{E}d\vec{S} + \oint_{\text{Right}} \vec{E}d\vec{S} + 0$

$$\Rightarrow \oint_S \vec{E}\,d\vec{S} = E\oint_{\text{Top}} dS + E\oint_{\text{Bottom}} dS \; q_{in} = \rho V = \rho S2$$

$$\Rightarrow \oint_S \vec{E}\,d\vec{S} = 2ES = \frac{S(2r)\rho}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho r}{\varepsilon_0} \qquad \text{(field inside a uniformly charged plane)}$$

Hence, electric field inside the plane sheet $\left(r < \frac{d}{2}\right)$ is directly proportional to distance of point r from the central plane.

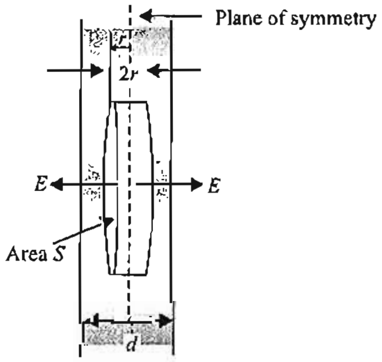


Fig. 2.31

Now, consider the Gaussian surface of the form of a cylinder of area S and thickness $2r$ ($> d$) placed symmetrically. On the curved surface, the flux of electric field will be zero as the area vector is perpendicular to the field vector. On left and right surfaces, flux is positive (outcoming flux). Hence,

$$\oint_S \vec{E}\,d\vec{S} = \frac{q_{\text{inclosed}}}{\varepsilon_0}$$

where $\oint_S \vec{E}\,d\vec{S} = \oint_{\text{Top}} \vec{E}\,d\vec{S} + \oint_{\text{Bottom}} \vec{E}\,d\vec{S} + 0$

$$\Rightarrow \oint_S \vec{E}\,d\vec{S} = E\int_{\text{Left}} dS + E\int_{\text{Right}} dS$$

$$\Rightarrow \oint_S \vec{E}\,d\vec{S} = 2ES = \frac{S\,d\,\rho}{\varepsilon_0}$$

$$\Rightarrow \quad E = \left(\frac{\rho}{\varepsilon_0}\right)\frac{d}{2} \quad \text{(field outside uniformly charged plane)}$$

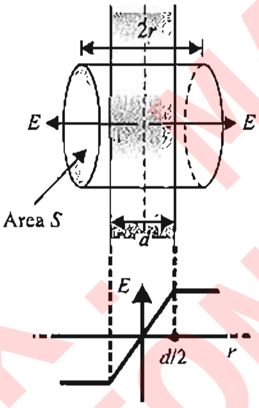


Fig. 2.32

Hence, the electric field outside the plane sheet $\left(r > \frac{d}{2}\right)$ is constant and does not depend on the distance of point r from the central plane (See Fig. 2.32).

APPENDIX

Solid angle: It is the cone subtended by an area at the point of interest (See Fig. 2.33). The magnitude of solid angle subtended by an area S at a point is defined as

$$\Omega = \int_S \frac{\cos\theta}{r^2}\,ds$$

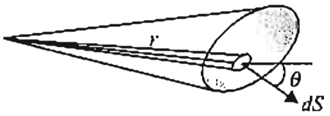


Fig. 2.33

at its center. Therefore, total solid angle around a point in space is the solid angle subtended by entire spherical surface on its center.

$$\Omega_0 = \frac{4\pi R^2}{R^2} = 4\pi \text{ steradian}$$

Solid angle subtended by a disk at a point on its axis:

Consider a coaxial area element of radius x and thickness dx (See Fig. 2.34). $\quad dS = 2\pi x dx$

Solid angle subtended by this element at point P is

$$d\Omega = \frac{dS\cos\theta}{(x^2+a^2)} \quad \Rightarrow \quad d\Omega = \frac{2\pi x dx a}{(x^2+a^2)\sqrt{a^2+x^2}}$$

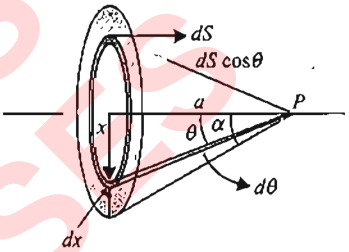


Fig. 2.34

Hence, total solid angle subtended by the disk is

$$\Omega = \pi a\int_0^R \frac{2x dx}{(x^2+a^2)^{3/2}} \quad \Rightarrow \Omega = 2\pi a\left(-\frac{1}{\sqrt{a^2+x^2}}\right)_0^R$$

$$\Rightarrow \quad \Omega = 2\pi\left(1-\frac{a}{\sqrt{a^2+R^2}}\right)$$

$\Omega = 2\pi(1-\cos\alpha)$; where α is the semi vertical angle of the cone subtended by the disk at P.

Concept Application Exercise 2.2

1. Fig. 2.36 shows the field produced by two point charges $+q$ and $-q$ of equal magnitude but opposite sign (an electric dipole). Find the electric flux through each of the closed surfaces A, B, C and D.

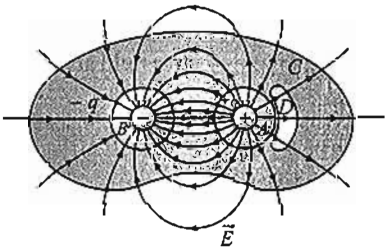
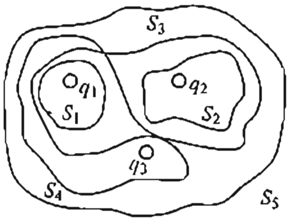


Fig. 2.35

2. The three small spheres as shown in Fig. 2.36 carry charges $q_1 = 4$ nC, $q_2 = -7.8$ nC and $q_3 = 2.4$ nC. Find the net electric flux through each of the following closed surfaces shown in cross section in the figure.

   a. S_1
   b. S_2
   c. S_3
   d. S_4
   e. S_5

   Do your answer to parts from (a) to (e) depend on how the charge is distributed over each small sphere? Why or why not?



Surface	What it encloses
S_1	q_1
S_2	q_2
S_3	q_1 and q_2
S_4	q_1 and q_3
S_5	q_1 and q_2 and q_3

Fig. 2.36

3. A conducting sphere carrying charge Q is surrounded by a spherical conducting shell.

- **a.** What is the net charge on the inner surface of the shell?
- **b.** Another charge q is placed outside the shell. Now, what is the net charge on the inner surface of the shell?
- **c.** If q is moved to a position between the shell and the sphere, what is the net charge on the inner surface of the shell?
- **d.** Are your answers valid if the sphere and shell are not concentric?

4. A solid insulating sphere of radius a carries a net positive charge $3Q$, uniformly distributed throughout its volume. Concentric with this sphere is a conducting spherical shell with inner radius b and outer radius c, and having a net charge $-Q$, as shown in Fig. 2.37.

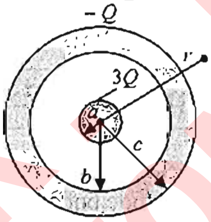


Fig. 2.37

- **a.** Consider a spherical Gaussian surface of radius $r > c$, the net charge enclosed by this surface is ______.
- **b.** The direction of the electric field at $r > c$ is ______.
- **c.** The electric field at $r > c$ is ______.
- **d.** The electric field in the region with radius r, where $c > r > b$ is ______.
- **e.** Consider a spherical Gaussian surface of radius r, where $c > r > b$, the net charge enclosed by this surface is ______.
- **f.** Consider a spherical Gaussian surface of radius r, where $b > r > a$, the net charge enclosed by this surface is ______.
- **g.** The electric field in the region $b > r > a$ is ______.
- **h.** Consider a spherical Gaussian surface of radius $r < a$. Find an expression for the net charge, Q(r) enclosed by this surface as a function of r. Note that the charge inside this surface is less than $3Q$.
- **i.** The electric field in the region $r < a$ is ______.
- **j.** The change on the inner surface of the conducting shell is ______.
- **k.** The charge on the outer surface of the conducting shell is ______.
- **l.** Make a plot of the magnitude of the electric field vs r.

5. A small conducting spherical shell with inner radius a and outer radius b is concentric with a larger conducting spherical shell with inner radius c and outer radius d. The inner shell has total charge $+2q$ and the outer shell has charge $+4q$.

- **a.** Make a plot of the magnitude of the electric field vs r.
- **b.** Calculate the electric field (magnitude and direction in terms of q and the distance r from the common centre of the two shells for (i) $r < a$; (ii) $a < r < b$ (iii) $b < r < c$; (iv) $c < r < d$; (v) $r > d$. Show you results in a graph of the radial component of $\vec{E}$ as function of r.
- **c.** What is the total charge on the
  - **i.** inner surface of the small shell;
  - **ii.** outer surface of the small shell;
  - **iii.** inner surface of the large shell;
  - **iv.** outer surface of the large shell?

6. Which of the following statements is/are correct?

- **a.** Electric field calculated by Gauss law is the field due to only those charges which are enclosed inside the Gaussian surface.
- **b.** Gauss law is applicable only when there is a symmetrical distribution of charge.
- **c.** Electric flux through a closed surface is equal to total flux due to all the charges enclosed within that surface only.

7. Which of the following statement is correct? If $E = 0$, at all points of a closed surface

- **a.** the electric flux through the surface is zero.
- **b.** the total charge enclosed by the surface is zero.
- **c.** no charge resides on the surface.

8. A hollow dielectric sphere as shown in Fig.2. 38 has inner and outer radii of R_1 and R_2, respectively.The total charge carried by the sphere is $+Q$, this charge is uniformly distributed between R_1 and R_2. Then,

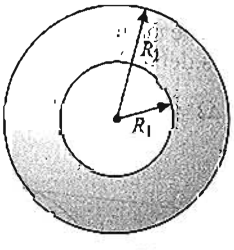


Fig. 2.38

- **a.** the electric field for $r < R_1$ is zero.
- **b.** the electric field for $R_1 < r < R_2$ is given by ______.
- **c.** the electric field for $r > R_2$ is given by ______.

9. A ring of diameter d is rotated in a uniform electric field until the position of maximum electric flux is found. The flux is found to be ϕ. What is the electric field strength?

10. Two infinite, non-conducting sheets of charge are parallel to each other, as shown in Fig. 2.39. The sheet on the left has a uniform surface charge density σ, and the one on the right has a uniform charge density $-\sigma$. Calculate the electric field at points (a) to the left of, (b) in between, and (c) to the right of the two sheets.

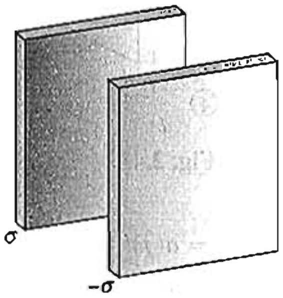


Fig. 2.39

11. S_1 and S_2 are two hollow concentric spheres enclosing charges Q and $2Q$, respectively, as shown in Fig. 2.40. What is the ratio of the electric flux through S_1 and S_2?

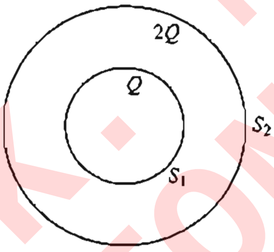


Fig. 2.40

12. A hollow half cylinder surface of radius R and length l is placed in a uniform electric field $\vec{E}$. Electric field is acting perpendicularly on the plane $ABCD$. Find the flux through the curved surface of the hollow cylindrical surface.

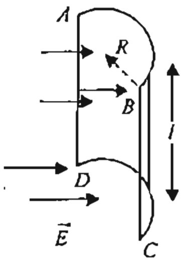


Fig. 2.41

13. Consider two concentric conducting spheres. The outer sphere is hollow and initially has a charge $-7Q$ on it. The inner sphere is solid and has a charge $+2Q$ on it.

- **a.** How much charge is on the outer surface and inner surface of the outer sphere?
- **b.** If a wire is connected between the inner and outer spheres, after electrostatic equilibrium is established how much total charge is on the outer sphere? How much charge is on the outer surface and inner surface of outer sphere? Does the electric field at the surface of the inside sphere change when the wire is connected?
- **c.** We return to original condition in (a). We now connect the outer sphere to ground with a wire and the disconnected it. How much total charge will be on the outer sphere? How much charge will be on the inner surface and outer surface of the outer sphere?

Solved Examples

Example 2.1 **A long, straight wire is surrounded by a hollow metal cylinder whose axis coincides with that of the wire. The wire has a charge per unit length of λ, and the cylinder has a net charge per unit length of 2λ. From this information, use Gauss's law to find (a) the charge per unit length on the inner and outer surfaces of the cylinder and (b) the electric field outside the cylinder, a distance r from the axis.**

Sol.

a. Inside surface: Consider a cylindrical surface within the metal. Since E inside the conducting shell is zero, the total charge inside the Gaussian surface must be zero, so the inside charge/length $= -\lambda$.

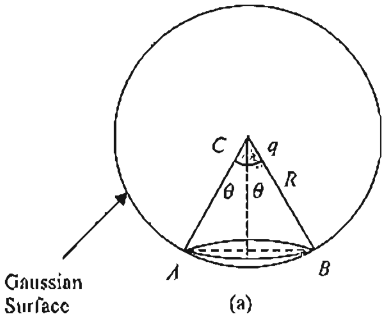
$$0 = \lambda l + q_{in} \quad \text{so} \quad \frac{q_{in}}{l} = -\lambda$$

Outside surface: The total charge on the metal/cylinder is $2\lambda l = q_{in} + q_{out}$

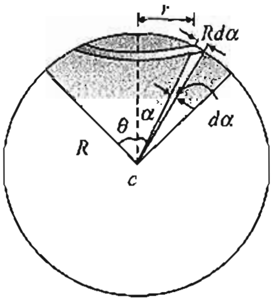
$q_{out} = 2\lambda l + \lambda l$ so the outside charge/length is 3λ

b. $E = \dfrac{2k_e(3\lambda)}{r} = \dfrac{6k_e\lambda}{r} = \dfrac{3\lambda}{2\pi\varepsilon_0 r}$ radially outward.

Example 2.2 **A point charge q is placed on the apex of a cone of semi-vertex angle θ. Show that the electric flux through the base of the cone is $\dfrac{q(1-\cos\theta)}{2\varepsilon_0}$.**



(a)



(b)

Fig. 2.42

Sol. Method 1: For point charge, Gaussian surface should be spherical. Consider a Gaussian sphere with its centre at the apex and radius the slant length of the cone. The flux through the whole sphere is q/ε_0. Therefore, the flux through the base of the cone, $\phi_E = \left(\frac{A}{A_0}\right)\cdot\frac{q}{\varepsilon_0}$

Here, A_0 = area of whole sphere = $4\pi R^2$, and

A = area of sphere below the base of the cone.

Consider a differential ring of radius r and thickness dr.

$$dA = (2\pi r)R\,d\alpha = (2\pi R\sin\alpha)R\,d\alpha \quad [\text{as } r = R\sin\alpha]$$

$$= (2\pi R^2)\sin\alpha\,d\alpha$$

$$A = \int_0^\theta (2\pi R^2)\sin\alpha\,d\alpha; \quad A = 2\pi R^2(1-\cos\theta)$$

The desired flux is $\phi_E = \left(\frac{A}{A_0}\right)\frac{q}{\varepsilon_0}$

$$= \frac{(2\pi R^2)(1-\cos\theta)}{(4\pi R^2)}\frac{q}{\varepsilon_0} = \frac{(1-\cos\theta)q}{2\varepsilon_0}$$

Method 2: Using the concept of solid angle.

Total solid angle around a point in space is 4π ste-radian. Solid angle subtended by the base of the cone at the apex of cone is $\pi = 2\pi(1-\cos\theta)$

As the flux associated with solid angle 4π is $\frac{q}{\varepsilon_0}$.

Hence, the flux associated with solid angle $2\pi(1-\cos\theta)$ is

$$\phi = \frac{q}{\varepsilon_0}\frac{2\pi(1-\cos\theta)}{4\pi} = \frac{q(1-\cos\theta)}{2\varepsilon_o}$$

Example 2.3 **A cube of side l has one corner at the origin of coordinates and extends along the positive x-, y- and z-axes. Suppose the electric field in this region is given by $E = (a + by)\hat{j}$. Determine the charge inside the cube. a and b are some constants.**

Sol. The faces $adhe$, $bcgf$, $cdhg$, $abfe$ will contribute zero flux because the area vector is normal to electric field for these faces.

Flux through face $efgh$,

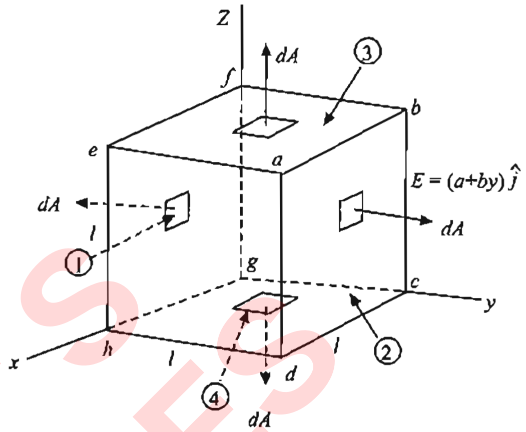


Fig. 2.43

$$\phi_1 = \int \vec{E}\cdot d\vec{A} = a(\hat{j})l^2(-\hat{j}) = -al^2$$

The field at the face $efgh$ (that lies in the yz plane, $y = 0$) is $E = a\hat{j}$ and area vector is $l^2(-\hat{j})$ (direction outward normal)

Flux through face $abcd$: $\phi_2 = (a + bl)\hat{j}\cdot l^2\hat{j} = (al^2 + bl^3)$, for this $y = l$

Net flux through the cube $= \phi_1 + \phi_2 = bl^3$

From Gauss's law, $\phi = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$, $Q_{\text{enclosed}} = \varepsilon_0\phi_E = \varepsilon_0 bl^3$

Example 2.4 **The electric field in a cubical volume is**

$$E = E_0\left(1 + \frac{z}{a}\right)\hat{i} + E_0\left(\frac{z}{a}\right)\hat{j}$$

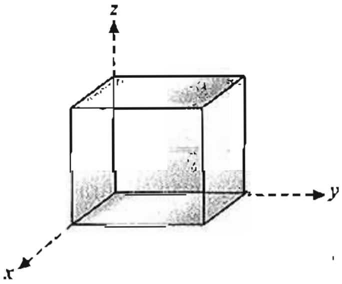
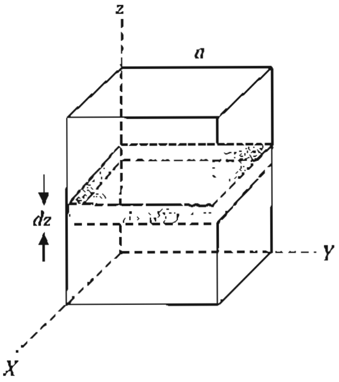


Fig. 2.44

Each edge of the cube measures d and one of the corners lies at the origin of coordinates. Determine the net charge within the cube.

Sol.



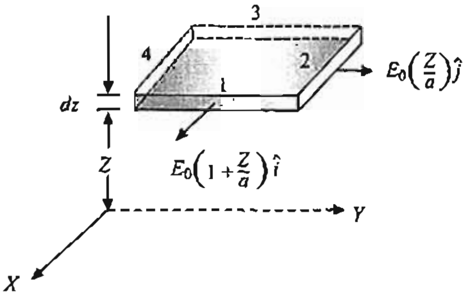


Fig. 2.45

We choose a differential slab of thickness dz, at a distance z from y-axis. The electric field varies with z-coordinate only. The field components at this position have constant magnitude. Consider faces 1 and 3. Net flux due to y-component of field is zero (area vector and field vector are perpendicular) and net flux due to x-component is also zero because net flux in through face 3 is equal to net flux out through face 1. Similarly, net flux through faces 2 and 4 are also zero. Flux through each differential slab in the cube is zero. Therefore, from Gauss's law net charge enclosed by cubical volume is zero.

Example 2.5 **Two identical metal plates each having surface area 'A', having charge 'q_1' and 'q_2' are placed facing each other at a separation 'd'. Find the charge appearing on surface (1), (2), (3), and (4). Assume the size of the plate is much larger than the separation between the plates.**

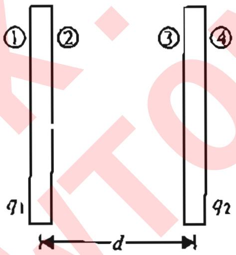


Fig. 2.46

Sol. Facing surfaces have equal and opposite charge (by Gauss theorem). Let the facing surfaces have the charge x and $-x$ (surface (2) and (3), respectively).

Then, the charge on the surfaces (1) and (2) should be $(q_1 - x)$ and x, respectively (by conservation of charge).

Facing metallic surfaces always have equal and opposite charge, hence charge appearing on surfaces (3) and (4) will be $-x$ and $(q_2 + x)$, respectively.

Let us consider a point P inside the left plate. Net electric field at P should be zero.

Net electric field at P will be due to the resultant of electric field due to charge appearing on all four surfaces.

$$\Rightarrow \quad E_1 + E_3 = E_2 + E_4$$

$$\Rightarrow \quad \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} = \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_4}{2\varepsilon_0}$$

$$\Rightarrow \quad q_1 - x + x = q_2 + x + x$$

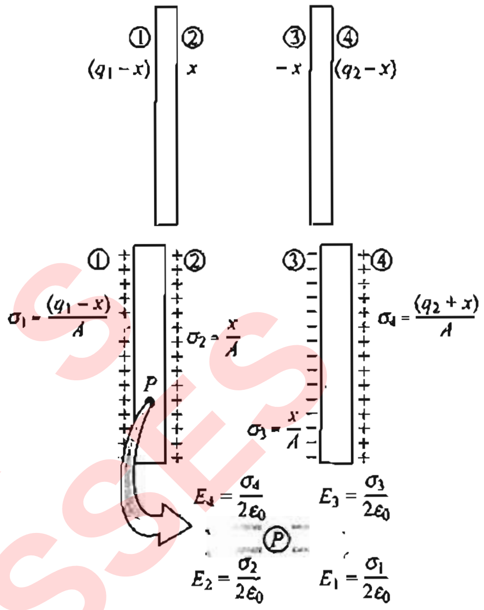


Fig. 2.47

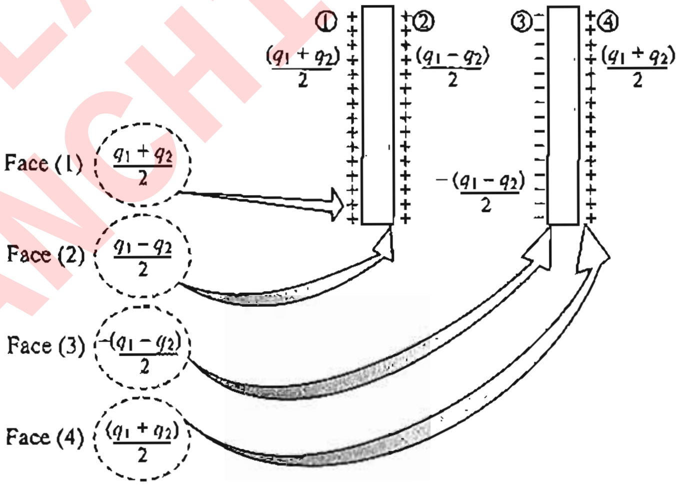


Fig. 2.48

$$\Rightarrow \quad x = \frac{q_1 - q_2}{2}$$

Hence, charge appearing on different surfaces are as shown in Fig. 2.49.

Important Note

- Facing surfaces have the equal and opposite nature of charge with magnitude 'half the difference of the charge on different plates', i.e., $\left(\frac{q_1 - q_2}{2}\right)$ in surface (2) and $\frac{(q_2 - q_1)}{2}$ or $-\left(\frac{q_1 - q_2}{2}\right)$ in surface (3).

- Outer surfaces always have equal charges of magnitude 'half the summation of charges', i.e., $\frac{(q_1+q_2)}{2}$ in each surface (1) and (4).
- If we have this type of charge distribution, then the electric field inside any metal plate will be zero.
- The charge appearing on the surfaces (2) and (3) is called bounded charge and the charge appearing on the surfaces (1) and (4) is called free charge.
- If we join second plate (right plate) with ground (Fig. 2.50) the charge appearing on the surface (4) will go to the earth and charge distribution will be

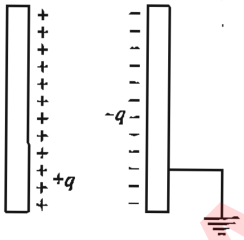


Fig. 2.50

- Any metal plate or object connected to the earth need not have zero charge. If the conducting body is isolated and connected to earth, then it will have no charge. If the conducting body is connected to earth have any charged object near to it, then the body will not have zero charge.

Example 2.6 **A point charge $+Q$ is placed at the centre of an uncharged spherical conducting shell of inner radius a and outer radius b as shown in Fig. 2.51.**

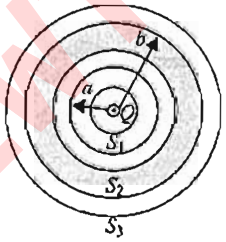


Fig. 2.51

1. **Find the electric field for $r < a$.**
2. **What is the magnitude and sign of the induced charge q' on the inner shell surface?**
3. **What is the field for $a < r < b$?**
4. **What is the electric field at points $r > b$?**
5. **What is the surface charge on the outer surface of the conductor?**

Sol.

1. Consider a Gaussian surface of radius $r < R$ inside the cavity, centred on the charge Q. From Gauss's law,

$$\phi_E = \oint EdA = E(4\pi r^2) = \frac{+Q}{\varepsilon_0}$$

From which we find the electric field to be

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

This result is the same as that of a point charge in vacuum.

2. Consider a Gaussian surface inside the conducting material, s_2. We do not know if there is a charge on the inside surface of the conductor or not. We assume that the charge is q'. If q' is zero, the result of Gauss's law will show it. Because the Gaussian surface is inside the conductor, the electric field is zero.

From Gauss's law,

$$\oint \vec{E}d\vec{A} = E(4\pi r^2)\frac{Q_{\text{enclosed}}}{\varepsilon_0} = 0$$

$$\frac{Q_{\text{enclosed}}}{\varepsilon_0} = \frac{Q+q'}{\varepsilon_0} = 0$$

which implies $q' = -Q$

There is a charge on the inside surface of the conductor. The total charge induced on the inside surface of the cavity is the negative of the charge placed at its centre.

3. The field inside a conductor in electrostatic equilibrium is always zero.
4. For E $(r > b)$, consider a Gaussian surface S_3. From Gauss's law, we have

$$\phi_E = \oint EdA = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{+Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

It was stated in the problem that the conducting sphere has no net charge. Consequently, the total charge inside our Gaussian surface S_3 is sum of charge $+Q$ and induced charges $-Q$ on the inner surface of conductor and $+Q$ on the surface. Once more we can see that field outside the sphere is same as for a point charge. The conducting sphere has no shielding effect at all. However, such a conducting shield does prevent electrostatic fields from charges outside the shell from entering it.

5. The conducting shell has no net charge, yet there is a surface charge $-Q$ on its surface. Because the net charge on the shell is zero and no charge can reside inside a conductor, there must be $+Q$ charge on the outer surface of the conductor (See Fig. 2.52).

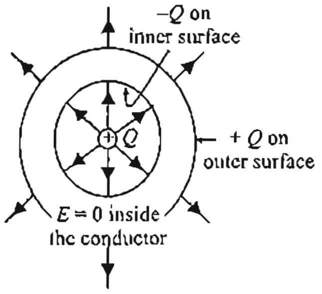


Fig. 2.52

EXERCISES

Subjective Type

Solutions on page 2.25

1. An infinite wire having charge density λ passes through one of the edges of a cube having edge length l. Find the

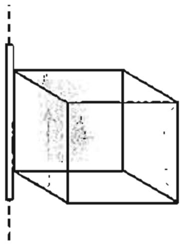


Fig. 2.53

a. total flux passing through the cube.

b. flux passing through the surfaces which are in contact with the wire.

c. flux passing through the surfaces which are not in contact with the wire.

2. It has been experimentally observed that the electric field in a large region of earth's atmosphere is directed vertically down. At an altitude of 300 m, the electric field is 60 Vm^{-1}. At an altitude of 200 m, the field is 100 Vm^{-1}. Calculate the net amount of charge contained in the cube of 100 m edge, located between 200 and 300 m altitude.

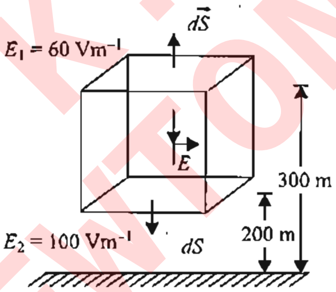


Fig. 2.54

3. A point charge Q is located on the axis of a disk of radius R at a distance b from the plane of the disk (Fig. 2.55). Show that if one-fourth of the electric flux from the charge passes through the disk, then $R = \sqrt{3}\,b$.

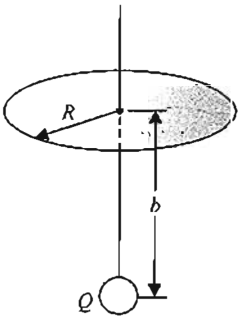


Fig. 2.55

4. A very long uniformly charged wire oriented along the axis of a circular ring of radius R rests on its center with one of the ends (as shown in Fig. 2.56). The linear charge density on the wire is λ. Evaluate the flux of the vector $\vec{E}$ across the circle area.

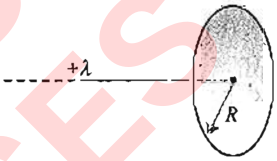


Fig. 2.56

5. Two point charges q and $-q$ are separated by a distance $2a$ (Fig. 2.57). Evaluate the flux of electric field strength vector across a circle of radius R.

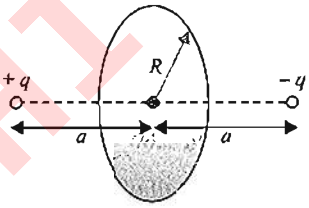


Fig. 2.57

6. An infinitely long line charge having a uniform charge per unit length λ lies a distance d from point O as shown in Fig. 2.58. Determine the total electric flux through the surface of a sphere of radius R centered at O resulting from this line charge. Consider both cases where $R < d$ and $R > d$.

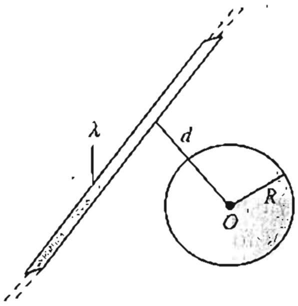


Fig. 2.58

7. Find the electric flux crossing the wire frame $ABCD$ of length l, width b and whose center is at a distance $OP = d$ from an infinite line of charge with linear charge density λ. Consider that the plane of frame is perpendicular to the line OP (Fig. 2.59).

8. A solid insulating sphere of radius R has a non-uniform charge density that varies with r according to the expression $\rho = Ar^2$, where A is a constant and $r < R$ is measured from the center of the sphere (a) show that the magnitude of the electric field outside ($r > R$) the sphere is

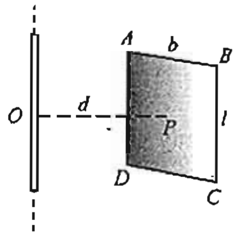


Fig. 2.59

$E = AR^5/5\varepsilon_0 r^2$. (b) Show that the magnitude of the electric field inside ($r < R$) the sphere is $E = AR^3/5\varepsilon_0$.

9. The electric field in a region is radially outward with magnitude $E = \alpha r$. Calculate the charge contained in a sphere of radius R centered at the origin. Calculate the value of charge if $\alpha = 100\ \text{V/m}^2$ and $R = 0.30$ m.

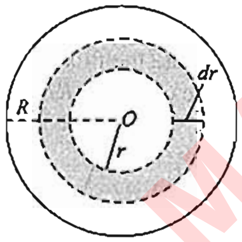


Fig. 2.60

10. A system consists of a ball of radius R carrying a spherically charge and a surrounding space filled with a charge of volume density $\rho = \frac{\alpha}{r}$ where α is a constant and r is the distance from the centre of the ball. Find the charge on the ball for which the magnitude of electric field strength is electric field strength? The dielectric constant of ball and surrounding may be taken equal to unity.

Objective Type

Solutions on page 2.28

1. Units of electric flux are
- **a.** NC^{-1}m^2
- **b.** JC^{-1}
- **c.** Vl
- **d.** Vm

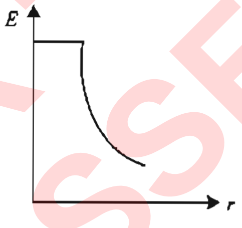
2. Positive electric flux from a closed surface indicates that electric lines of force are directed
- **a.** outwards
- **b.** inwards
- **c.** outwards or inwards
- **d.** none of these

3. A surface encloses an electric dipole. The net flux through the surface is
- **a.** zero
- **b.** positive
- **c.** negative
- **d.** infinite

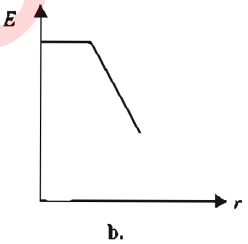
4. In a region with a uniform electric field, the number of lines of force per unit area is E. If a spherical metallic conductor is placed in the area, the field inside the conductor will be
- **a.** zero
- **b.** E
- **c.** more than E
- **d.** less than E

5. An insulated sphere of radius R has a uniform volume charge density ρ. The electric field at a point P inside the sphere at a distance r from the centre is
- **a.** $\dfrac{R\rho}{3\varepsilon_0}$
- **b.** $\dfrac{r\rho}{3\varepsilon_0}$
- **c.** zero
- **d.** $\dfrac{2}{3}\left(\dfrac{r\rho}{\varepsilon_0}\right)$

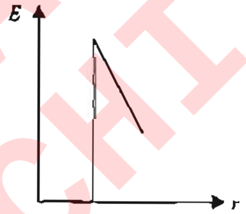
6. Which one of the following graphs shows the variation of electric field strength E with distance r from the center of a hollow conducting sphere?



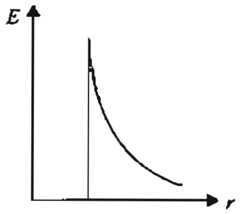
a.



b.



c.



d.

7. A cylinder of length L and radius b has its axis coincident with the x-axis. The electric field in this region is $\vec{E} = 200\,\hat{i}$. Find the flux through the left end of cylinder.
- **a.** 0
- **b.** $200\pi b^2$
- **c.** $100\pi b^2$
- **d.** $-200\pi b^2$

8. Consider two infinite parallel charged metal plates with equal and opposite charge densities $+\sigma$ and $-\sigma$. Determine the electric field in the region between the plates.
- **a.** σ/ε_0
- **b.** 0
- **c.** $\sigma/2\varepsilon_0$
- **d.** $2\sigma/\varepsilon_0$

9. Consider the Gaussian surface that surrounds part of the charge distribution shown in Fig. 2.61. Then, the contribution to the electric field at point P arises from charges
- **a.** q_1 and q_2 only
- **b.** q_3 and q_4 only
- **c.** q_1, q_2, q_3 and q_4
- **d.** none of the above

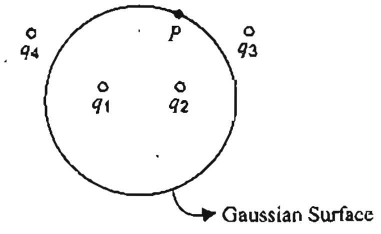


Fig. 2.61

10. Charge on an originally uncharged conductor is separated by holding a positively charged rod very closely nearby, as in Fig. 2.62. Assume that the induced negative charge on the conductor is equal to the positive charge q on the rod. Then, flux through surface S_1 is

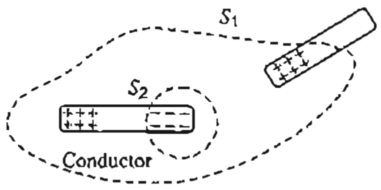


Fig. 2.62

- **a.** zero
- **b.** q/ε_0
- **c.** $-q/\varepsilon_0$
- **d.** none of these

11. A thin metallic spherical shell contains a charge Q on its surface. A point charge q_1 is placed at the centre of the shell and another charge q_2 is placed outside the shell. All the three charges are positive. Then, the force on charge q_1 is

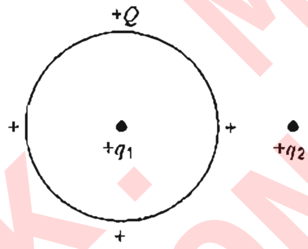


Fig. 2.63

- **a.** towards right
- **b.** towards left
- **c.** zero
- **d.** none of these

12. If one penetrates a uniformly charged spherical cloud, electric field strength
- **a.** decreases directly as the distance from the center
- **b.** increases directly as the distance from the center
- **c.** remains constant
- **d.** none of the above

13. An uncharged metal sphere is placed between two equal and oppositely charged metal plates. The nature of lines of force will be

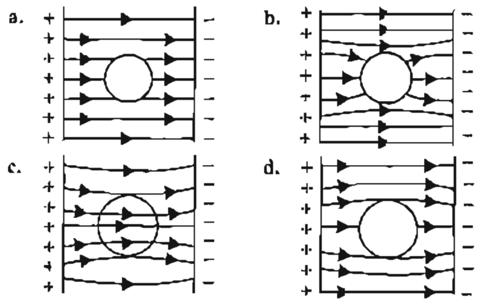


Fig. 2.64

14. A hollow metallic sphere of radius 10 cm is given a charge of 3.2×10^{-9} C. The electric intensity at a point 4 cm from the center is
- **a.** 9×10^{-9} NC^{-1}
- **b.** 288 NC^{-1}
- **c.** 2.88 NC^{-1}
- **d.** zero

15. The surface density on a copper sphere is σ. The electric field strength on the surface of the sphere is
- **a.** σ
- **b.** $\sigma/2$
- **c.** $\dfrac{\sigma}{2\varepsilon_0}$
- **d.** $\dfrac{\sigma}{\varepsilon_0}$

16. A cylinder of radius R and length l is placed in a uniform electric field E parallel to the axis of the cylinder. The total flux over the curved surface of the cylinder is
- **a.** zero
- **b.** $\pi R^2 E$
- **c.** $2\pi R^2 E$
- **d.** $E/\pi R^2$

17. A cube of side 10 cm encloses a charge of 0.1 μC at its centre. Calculate the number of lines of force through each face of the cube.
- **a.** 1.113×10^{11}
- **b.** 1.13×10^4
- **c.** 1.13×10^9
- **d.** 1883

18. Number of electric lines of force from 0.5 C of positive charge in a dielectric medium of constant 10 is
- **a.** 5.65×10^9
- **b.** 1.13×10^{11}
- **c.** 9×10^9
- **d.** 8.85×10^{-12}

19. The electric flux from a cube of edge l is ϕ. What will be its value if edge of cube is made $2l$ and charge enclosed is halved?
- **a.** 4ϕ
- **b.** 2ϕ
- **c.** $\phi/2$
- **d.** ϕ

20. In a certain region of space, there exists a uniform electric field of $2\times10^3\hat{k}$ Vm^{-1}. A rectangular coil of dimensions 10 cm × 20 cm is placed in x–y plane. The electric flux through the coil is
- **a.** zero
- **b.** 30 Vm
- **c.** 40 Vm
- **d.** 50 Vm

21. Which of the following may be discontinuous across a charged conducting surface?
- **a.** Electric potential
- **b.** Electric intensity
- **c.** Both electric potential and intensity
- **d.** None of the above

22. Consider two concentric spherical surfaces, S_1 with radius a and S_2 with radius $2a$, both centered on the origin. There is a charge $+q$ at the origin, and no other charges. Compare the flux ϕ_1 through S_1 with the flux ϕ_2 through S_2.
- **a.** $\phi_1 = 4\phi_2$
- **b.** $\phi_1 = 2\phi_2$
- **c.** $\phi_1 = \phi_2$
- **d.** $\phi_1 = \dfrac{\phi_2}{2}$

23. Under what conditions can the electric flux ϕ_E be found through a closed surface?
- **a.** If the magnitude of electric field is known everywhere on the surface.

b. If the total charge inside the surface is specified.

c. If the total charge outside the surface is specified.

d. Only if the location of each point charge inside the surface is specified.

24. Fig. 2.65 shows four charges q_1, q_2, q_3, q_4 fixed in space. Then, the total flux of electric field through a closed surface S, due to all charges q_1, q_2, q_3 and q_4, is

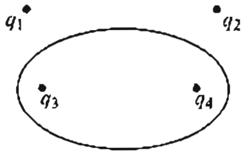


Fig. 2.65

a. not equal to the total flux through S due to charges q_3 and q_4

b. equal to the total flux through S due to charges q_3 and q_4

c. zero if $q_1 + q_2 = q_3 + q_4$

d. twice the total flux through S due to charges q_3 and q_4 if $q_1 + q_2 = q_3 + q_4$

25. If the flux of the electric field through a closed surface is zero, then

a. the electric field must be zero everywhere on the surface.

b. the total charge inside the surface must be zero

c. the electric field must be uniform throughout the closed surface

d. the charge outside the surface must be zero

26. Eight charges, 1 μC, –7 μC, –4 μC, 10 μC, 2 μC, –5 μC, –3 μC and 6 μC are situated at the eight corners of a cube of side 20 cm. A spherical surface of radius 80 cm encloses this cube. The center of the sphere coincides with the centre of the cube. Then, the total outgoing flux from the spherical surface (in units of Vm) is

a. $36\pi \times 10^3$ **b.** $684\pi \times 10^3$

c. zero **d.** none of these

27. Three charges of $q_1 = 1\times10^{-6}$ C, $q_2 = 2\times10^{-6}$ C and $q_3 = -3 \times 10^{-6}$ C have been placed as shown. Then, the net electric flux will be maximum for the surface

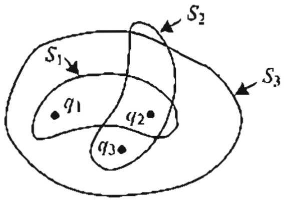


Fig. 2.66

a. S_1 **b.** S_2

c. S_3 **d.** same for all three

28. A charge q is distributed uniformly on a ring of radius 'a'. A sphere of equal radius 'a' is constructed with its center at the periphery of the ring. Calculate the flux of the electric field through the surface of the sphere.

a. $\dfrac{q}{3\varepsilon_0}$ **b.** $\dfrac{2q}{3\varepsilon_0}$

c. $\dfrac{q}{4\varepsilon_0}$ **d.** $\dfrac{3q}{4\varepsilon_0}$

29. In a region of space, the electric field is given by $\vec{E} = 8\hat{i} + 4\hat{j} + 3\hat{k}$. The electric flux through a surface of area of 100 units in x–y plane is

a. 800 units **b.** 300 units

c. 400 units **d.** 1500 units

30. A spherical shell of radius R = 1.5 cm has a charge q = 20 μC uniformly distributed over it. What is the force exerted by one half over the other half?

a. zero **b.** 10^{-2} N

c. 500 N **d.** 2000 N

31. A flat, square surface with sides of length L is described by the equations

$$x = L, 0 \le y \le L, 0 \le z \le L$$

Find the electric flux through the square due to a positive point charge q located at the origin ($x = 0$, $y = 0$, $z = 0$).

a. $\dfrac{q}{4\varepsilon_0}$ **b.** $\dfrac{q}{6\varepsilon_0}$

c. $\dfrac{q}{24\varepsilon_0}$ **d.** $\dfrac{q}{48\varepsilon_0}$

32. The electric field $\vec{E}_1$ at one face of a parallelopiped is uniform over the entire face and is directed out of the face. At the opposite face, the electric field $\vec{E}_2$ is also uniform over the entire face and is directed into that face (as shown in Fig. 2.67). The two faces in question are inclined at 30° from the horizontal, while $\vec{E}_1$ and $\vec{E}_2$ (both horizontal) have magnitudes of 2.50×10^4 N/C and 7.00×10^4 N/C . Assuming that no other electric field lines cross the surfaces of the parallelopiped, determine the net charge contained within.

a. $-67.5\,\varepsilon_0$ C **b.** $37.5\,\varepsilon_0$ C

c. $105\,\varepsilon_0$ C **d.** $-105\,\varepsilon_0$ C

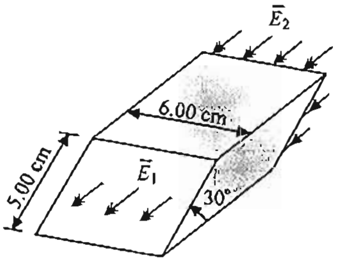


Fig. 2.67

33. A dielectric in the form of a sphere is introduced into a homogeneous electric field. A, B and C are 3 points as shown in Fig. 2.68. Then,

a. intensity at A increases while that at B and C decreases.

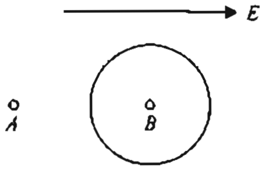


Fig. 2.68

b. intensity at A and B decreases whereas at C intensity increases

c. intensity at A and C increases and that at B decreases.

d. intensity at A, B and C decreases.

34. The electric field on two sides of a large charged plate is shown in Fig. 2.69. The charge density on the plate in S.I. Units is given by (ε_0 is the permittivity of free space in S.I. Units)

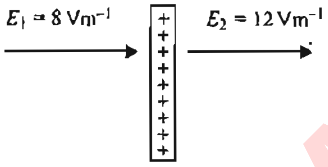


Fig. 2.69

a. $2\varepsilon_0$ **b.** $4\varepsilon_0$

c. $10\varepsilon_0$ **d.** zero

Multiple Correct Answers Type

Solutions on page 2.29

1. 10 C of charge is given to a conducting spherical shell and a −3 C point charge is placed inside the shell. For this arrangement, markout the correct statement(s).

a. The charge on the inner surface of the shell will be +3 C and it can be distributed uniformly or non-uniformly.

b. The charge on the inner surface of the shell will be +3 C and its distribution would be uniform.

c. The net charge on outer surface of the shell will be +7 C and its distribution can be uniform or non-uniform.

d. The net charge on outer surface of the shell will be +7 C and its distribution would be uniform.

2. Consider Gauss's law $\oint \vec{E} \cdot \vec{ds} = \dfrac{q}{\varepsilon_0}$.

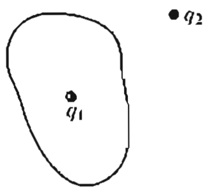


Fig. 2.70

Then, for the situation shown above at the Gaussian surface

a. $\vec{E}$ due to q_2 would be zero.

b. $\vec{E}$ due to both q_1 and q_2 would be non-zero.

c. ϕ due to both q_1 and q_2 would be non-zero.

d. ϕ due to q_2 would be zero.

3. For Gauss's law, mark out the correct statement(s).

a. If we displaced the enclosed charges (within a Gaussian surface) without crossing the boundary, then $\vec{E}$ and ϕ both remain same.

b. If we displace the enclosed charges without crossing the boundary, then $\vec{E}$ changes but ϕ remains the same.

c. If charge crosses the boundary, then both $\vec{E}$ and ϕ would change.

d. If charge crosses the boundary, then ϕ changes but $\vec{E}$ remains the same.

Assertion-Reasoning Type

Solutions on page 2.30

In the following questions, each question contains Statement I (Assertion) and Statement II (Reason). Each question has 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.

b. Statement I is True, Statement II is True; Statement II is NOT a correct explanation for Statement I.

c. Statement I is True, Statement II is False.

d. Statement I is False, Statement II is True.

1. **Statement I:** $\vec{E}$ in outside vicinity of a conductor depends only on the local charge density σ and it is independent of the other charges present anywhere on the conductor.

Statement II: $\vec{E}$ in outside vicinity of a conductor is given by $\dfrac{\sigma}{\varepsilon_0}$.

2. **Statement I:** Upon displacement of charges within a closed surface, $\vec{E}$ at any point on the surface does not change.

Statement II: The flux crossing through a closed surface is independent of the location of charge within the surface.

3. **Statement I:** If Gaussian surface does not enclose any charge, then $\vec{E}$ at any point on the Gaussian surface must be zero.

Statement II: No net charge is enclosed by Gaussian surface, so net flux passing through the surface is zero.

4. **Statement I:** For the situation shown in Fig. 2.71, if we displace the charge q within the conducting shell, then nature of distribution of charge on the outer surface of the shell does not change.

Statement II: Any conducting shell divides the entire space into two regions (inside and outside the shell), which are independent to each other in terms of electric field.

5. **Statement I:** Electric field on the surface of a conductor is more at the sharp corners.

Statement II: Surface charge density on conductor's surface is inversely proportional to the radius of curvature.

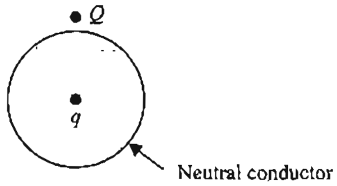


Fig. 2.71

Comprehension Type

Solutions on page 2.30

For Problems 1–4

The cube as shown in Fig. 2.72 has sides of length $L = 10.0$ cm. The electric field is uniform, has a magnitude $E = 4.00 \times 10^3$ NC^{-1}, and is parallel to the xy-plane at an angle of 37° measured from the $+x$-axis toward the $+y$-axis.

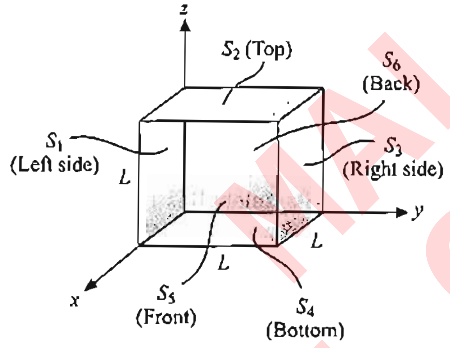


Fig. 2.72

1. Which of the surfaces have zero flux?
   - a. S_1 and S_3 b. S_5 and S_6
   - c. S_2 and S_4 d. S_1 and S_1
2. Electric flux passing through the surface S_1 is
   - a. -24 N-m^{2}C^{-1} b. 24 N-m^{2}C^{-1}
   - c. 32 N-m^{2}C^{-1} d. -32 N-m^{2}C^{-1}
3. Electric flux passing through the surface S_6 is
   - a. -24 N-m^{2}C^{-1} b. -24 N-m^{2}C^{-1}
   - c. 32 N-m^{2}C^{-1} d. -32 N-m^{2}C^{-1}
4. What is the total net electric flux through all faces of the cube?
   - a. 8 N-m^{2}C^{-1} b. -8 N-m^{2}C^{-1}
   - c. 24 N-m^{2}C^{-1} d. zero

For Problems 5–8

A cube has sides of length $L = 0.300$ m. It is placed with one corner at the origin as shown in the figure of previous problem. The electric field is not uniform, but is given by $\vec{E} = (-5.00 \text{ NC}^{-1})x\hat{i} + (3.00 \text{ NC}^{-1})z\hat{k}$.

5. Which of the surfaces have zero flux?
   - a. S_2, S_4 and S_5 b. S_1, S_3, S_4 and S_6
   - c. S_1, S_2 and S_3 d. S_2, S_3 and S_4
6. The flux passing through the surface S_5 will be
   - a. -0.135 N-m^{2}C^{-1} b. -0.054 N-m^{2}C^{-1}
   - c. 0.081 N-m^{2}C^{-1} d. 0.054 N-m^{2}C^{-1}
7. Total flux passing through the cube is
   - a. -0.135 N-m^{2}C^{-1} b. -0.054 N-m^{2}C^{-1}
   - c. 0.081 N-m^{2}C^{-1} d. zero
8. The total electric charge inside the cube is
   - a. $-0.054\,\varepsilon_0$ C b. $0.081\,\varepsilon_0$ C
   - c. $0.135\,\varepsilon_0$ C d. $0.054\,\varepsilon_0$ C

For Problems 9–10

A cube has sides of length L. It is placed with one corner at the origin as shown in the figure of previous problem. The electric field is uniform and given by $\vec{E} = -B\hat{i} + C\hat{j} - D\hat{k}$, where B, C and D are positive constants.

9. The flux passing through different surfaces (match the table) is

Surface	Flux
(i) S_1	(m) BL^2
(ii) S_2	(n) $-BL^2$
(iii) S_3	(o) CL^2
(iv) S_4	(p) $-CL^2$
(v) S_5	(q) DL^2
(vi) S_6	(r) $-DL^2$

   - a. (i, p), (ii, r), (iii, o), (iv, q), (v, n), (vi, m)
   - b. (i, r), (ii, p), (iii, q), (iv, n), (v, m), (vi, o)
   - c. (i, m), (ii, n), (iii, o), (iv, p), (v, q), (vi, r)
   - d. (i, r), (ii, q), (iii, p), (iv, o), (v, n), (vi, m)

10. Total flux passing through the cube is
    - a. $(B + C + D)L^2$ b. $2(B + C + D)L^2$
    - c. $6(B + C + D)L^2$ d. zero

For Problems 11–12

A cube of side a is placed such that the nearest face which is parallel to the y–z plane is at a distance 'a' from the origin. The electric field components are $E_x = \alpha x^{1/2}$, $E_y = E_z = 0$.

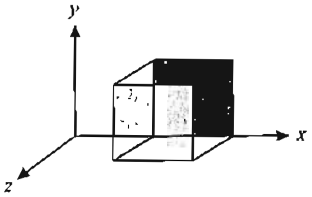


Fig. 2.73

11. Calculate the flux ϕ_E through the cube.
    - a. $\sqrt{2}\alpha a^{5/2}$ b. $-\alpha a^{5/2}$
    - c. $\left(\sqrt{2} - 1\right)\alpha a^{5/2}$ d. zero
12. Calculate the charge within the cube.
    - a. $\sqrt{2}\alpha\,\varepsilon_0 a^{5/2}$ b. $-\alpha\,\varepsilon_0 a^{5/2}$
    - c. $\left(\sqrt{2} - 1\right)\alpha\,\varepsilon_0 a^{5/2}$ d. zero

For Problems 13–18

A small conducting spherical shell with inner radius a and outer radius b is concentric with a larger conducting spherical shell

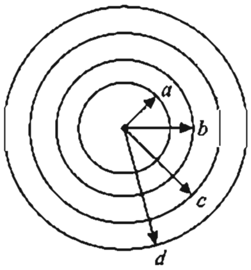


Fig. 2.74

with inner radius c and outer radius d (as shown in Fig. 2.74). The inner shell has total charge $+2q$ and the outer shell has charge $+4q$. Calculate the electric field in terms of q and the distance r from the common centre of the two shells for:

13. $r < a$

a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

14. $a < r < b$

a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

15. $b < r < c$

a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

16. $c < r < d$

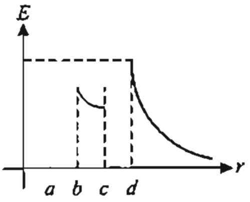
a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

17. $r > d$

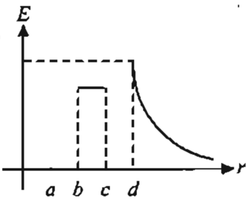
a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

18. The graph of the radial component of E as a function of r will be

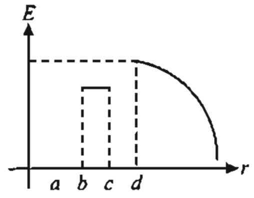
a.



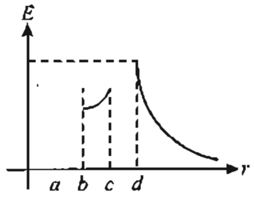
b.



c.



d.



For Problems 19–22

According to problems 13–18, what is the total charge on the

19. inner surface of the small shell?

a. zero **b.** $2q$ **c.** $-2q$ **d.** $6q$

20. outer surface of the small shell?

a. zero **b.** $2q$ **c.** $-2q$ **d.** $6q$

21. inner surface of the large shell?

a. zero **b.** $2q$ **c.** $-2q$ **d.** $6q$

22. outer surface of the large shell?

a. zero **b.** $2q$ **c.** $-2q$ **d.** $6q$

For Problems 23–28

Consider the previous problem, but now let the outer shell have charge $-4q$. As in the above problem, the inner shell has charge $+2q$. Calculate the electric field in terms of q and the distance r from the common centre of the two shells for:

23. $r < a$

a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

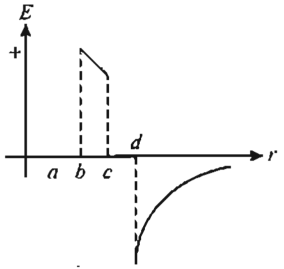
24. $a < r < b$

a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

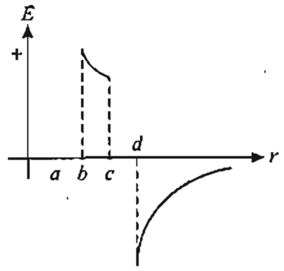
25. $b < r < c$

a. zero
b. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$
d. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

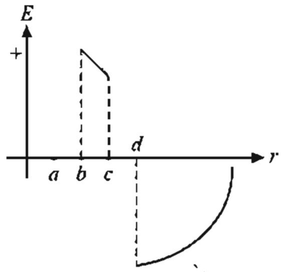
26. The graph of the radial component of E as a function of r will be



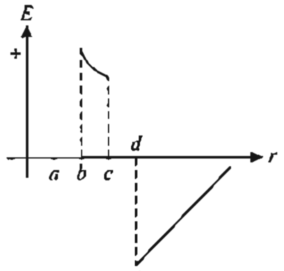
a.



b.



c.



d.

27. $c < r < d$

a. zero **b.** $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$

c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$ **d.** $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

28. $r > d$

a. zero **b.** $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$

c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$ **d.** $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

For Problems 29–32

According to problems 23–28, what is the total charge on the

29. inner surface of the small shell?

a. zero **b.** $2q$ **c.** $-2q$ **d.** $6q$

30. outer surface of the small shell?

a. zero **b.** $2q$ **c.** $-2q$ **d.** $6q$

31. inner surface of the large shell?

a. zero **b.** $2q$ **c.** $-2q$ **d.** $6q$

32. outer surface of the large shell?

a. zero **b.** $2q$ **c.** $-2q$ **d.** $6q$

For Problems 33–38

Consider above problem, but now let the outer shell have charge $-2q$. As in the above problem, the inner shell has charge $+2q$. Calculate the electric field in terms of q and the distance r from the common centre of the two shells for:

33. $r < a$

a. zero **b.** $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$

c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$ **d.** $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

34. $a < r < b$

a. zero **b.** $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$

c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$ **d.** $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

35. $b < r < c$

a. zero **b.** $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$

c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$ **d.** $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

36. $c < r < d$

a. zero **b.** $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$

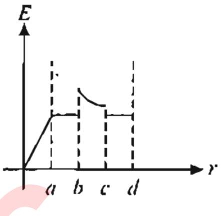
c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$ **d.** $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

37. $r > d$

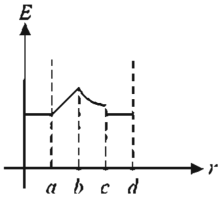
a. zero **b.** $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$

c. $-\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$ **d.** $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$

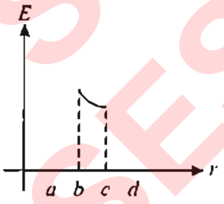
38. The graph of the radial component of E as a function of r will be



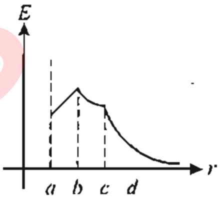
a.



b.



c.



d.

For Problems 39–42

In the previous problem, what is the total charge on the

39. inner surface of the small shell?

a. zero **b.** $2q$ **c.** $-2q$ **d.** $4q$

40. outer surface of the small shell?

a. zero **b.** $2q$ **c.** $-2q$ **d.** $4q$

41. inner surface of the large shell?

a. zero **b.** $2q$ **c.** $-2q$ **d.** $4q$

42. outer surface of the large shell?

a. zero **b.** $2q$ **c.** $-2q$ **d.** $4q$

For Problems 43–45

Two spherical cavities of radii a and b are hollowed out from the interior of a neutral conducting sphere of radius R. At the center of each cavity, a point charge is placed. Call these charges q_a and q_b.

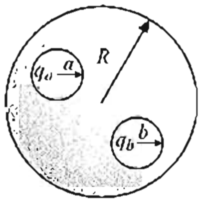


Fig. 2.75

43. Match the table

(i) σ_a	(m) $\frac{q_a+q_b}{4\pi R^2}$
(ii) σ_b	(n) $\frac{-q_a}{4\pi a^2}$
(iii) σ_R	(o) $\frac{-q_b}{4\pi b^2}$

a. (i, o), (ii, n), (iii, m) **b.** (i, n), (ii, o), (iii, m)

c. (i, m), (ii, o), (iii, n) **d.** (i, n), (ii; m), (iii, o)

44. What is the field at a distance r outside the conductor?

a. $\frac{1}{4\pi\varepsilon_0}\frac{q_b}{r^2}$ **b.** zero

c. $\frac{1}{4\pi\varepsilon_0}\frac{q_a+q_b}{r^2}$ d. $\frac{1}{4\pi\varepsilon_0}\frac{q_a}{r^2}$

45. The electric field inside the cavity of radius a at a distance r from the centre of cavity is

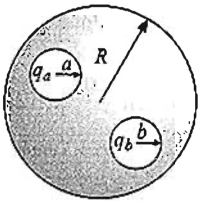


Fig. 2.76

a. $\frac{1}{4\pi\varepsilon_0}\frac{q_a}{r^2}$ b. $-\frac{1}{4\pi\varepsilon_0}\frac{q_a}{r^2}$

c. $\frac{1}{4\pi\varepsilon_0}\frac{q_a+q_b}{r^2}$ d. zero

Matching Column Type

Solutions on page 2.32

1. Column I specifies a point at distance r from the center/axis of a symmetrical distribution of charge. Column-II gives the variation of electric intensity at P as a function of r. Match the columns.

Column I	Column II
i. P lies outside a cylinder having uniform volume charge density.	**a.** $E \propto \frac{1}{r^2}$
ii. P lies inside a spherical charged conductor.	**b.** $E \propto \frac{1}{r}$
iii. P lies inside a spherical body having uniform volume charge density.	**c.** $E \propto r$
iv. P lies inside a solid cylinder having uniform volume charge density.	**d.** $E \propto r^0$
v. P lies inside a plane infinite sheet of some thickness. The sheet is charged uniformly throughout its volume.	
vi. P lies outside the plane sheet mentioned in part(v).	

2. Three identical metal plates with large surface areas are kept parallel to each other as shown in Fig. 2.82. The left most is given a charge Q, the right most a charge $-2Q$ and the middle one remains neutral. Then:

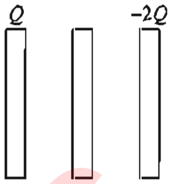


Fig. 2.77

Column I	Column II
i. The charge appearing on outer surface of right most plate	**a.** $+\frac{Q}{2}$
ii. The charge appearing on outer surface of left most plate	**b.** $-\frac{Q}{2}$
iii. The charge appearing on left surface of middle plate	**c.** $-\frac{3Q}{2}$
iv. The charge appearing on right surface of middle plate	**d.** $\frac{3Q}{2}$

3. Electric field due to

Column I	Column II
i. Infinite plane sheet of charge	**a.** 0
ii. Infinite plane sheet of uniform thickness	**b.** $\frac{\pi}{2\varepsilon_0}$
iii. Non-conducting charged solid sphere at its surface	**c.** $\frac{R\rho}{3\varepsilon_0}$
iv. Non-conducting charged solid sphere at its center	**d.** $\frac{\sigma}{\varepsilon_0}$

where symbols have their usual meaning.

ANSWERS AND SOLUTIONS

Subjective Type

1. a. If we construct these both cubes as shown in Fig. 2.78 (by dotted line),

net flux through all the four cubes $\frac{q_{\text{in}}}{\varepsilon_0}=\frac{\lambda l}{\varepsilon_0}$

So, flux through any one cube $=\frac{\lambda l}{4\varepsilon_0}$

b. Field lines will be parallel to the surfaces in contact with the wire. Hence, flux through these surfaces will be zero. There are four such surfaces.

c. There are two surfaces which are not in contact with the wire. So, the flux $\frac{\lambda l}{4\varepsilon_0}$ will be divided among these two surfaces. So,

flux through each surface $=\frac{\lambda l}{8\varepsilon_0}$.

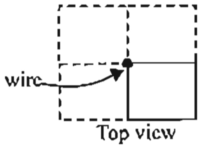


Fig. 2.78

2. According to Gauss' theorem, electric flux:

$$\phi_E = \frac{q}{\varepsilon_0} = \int \vec{E} \cdot d\vec{S}$$

The surface integral in the above equations contains six terms—the surface integral over the bottom surface, the surface integral over the top surface and surface integral over the four vertical faces.

For the bottom surface, both the vectors $\vec{E}$ and $d\vec{S}$ are in the same direction. For the top surface, they act in opposite directions while for the vertical faces, they are perpendicular to each other.

Hence,

$$\phi_E = \int_{\text{Bottom}} \vec{E}_1 d\vec{S} + \int_{\text{Top}} \vec{E}_2 d\vec{S} + 4\int_{\text{Faces}} \vec{E} d\vec{S}$$

$$= \int_{\text{Bottom}} E_1 dS \cos 0° + \int_{\text{Top}} E_2 dS \cos 180°$$

$$+ 4\int_{\text{Faces}} EdS \cos 90°$$

$$= \int_{\text{Bottom}} E_1\, dS - \int_{\text{Top}} E_2 dS = E_1 S - E_2 S = (E_1 - E_2)\, S$$

Also, $\phi_E = \dfrac{q}{\varepsilon_0}$

$$q = \varepsilon_0 (E_1 - E_2)\, S$$

$$= (8.85 \times 10^{-12} \mathrm{C^2 N^{-1} m^{-2}})$$

$$\times (100\ \mathrm{Vm^{-1}} - 60\ \mathrm{Vm^{-1}})(100\ \mathrm{m^2})$$

$$= 3.54 \times 10^{-6}\ \mathrm{C}$$

3. Solid angle subtended by disk at Q

$$W = 2\pi\,(1 - \cos\alpha)$$

$$= 2\pi\left[1 - \frac{b}{\sqrt{b^2 + R^2}}\right]$$

Flux through disk $= \dfrac{QW}{\varepsilon_0 4\pi}$

It is given to be $\dfrac{Q}{4\varepsilon_0}$

So, $\dfrac{Q}{4\varepsilon_0} = \dfrac{QW}{\varepsilon_0 4\pi}$

Solve to get $R = \sqrt{3}b$

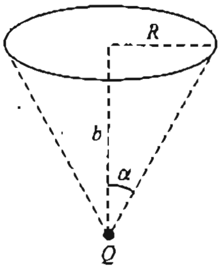


Fig. 2.79

4. The component of electric field E_y will not contribute to flux as angle between and $\vec{A}$ is 90°. Hence, flux due to $\vec{E}_y$:

$d\phi = E_x 2\pi y dy$

$$\phi = \int d\phi = \int_0^R \frac{\lambda}{4\pi\varepsilon\, y}\ 2\pi y dy$$

$$\phi = \frac{\lambda}{2\varepsilon_0} \int_0^R dy = \frac{\lambda R}{2\varepsilon_0}$$

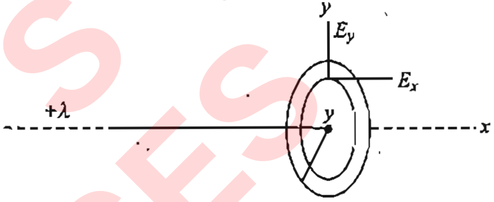


Fig. 2.80

5. $d\phi = 2E\cos\theta 2\pi y dy$

$$= 2\,\frac{1}{4\pi\varepsilon_0}\ \frac{q}{(a^2 + y^2)}\ \frac{a\ \ 2\pi y dy'}{(a^2 + y^2)^{1/2}}$$

$$\phi = \int d\phi = \frac{qa}{\varepsilon_0} \int_0^R \frac{ydy}{(a^2 + y^2)^{3/2}}$$

$$\phi = \frac{q}{\varepsilon_0}\left[1 - \frac{1}{\sqrt{1 + (R/a)^2}}\right]$$

6. For $R < d$, flux will be zero. It is because there is no charge line inside the sphere (Fig. 2.81).

For $R > d$, length inside sphere $= 2\,(\sqrt{R^2 - d^2})$ (Fig. 2.82)

So, charge inside $= 2\lambda\sqrt{R^2 - d^2}$

So, $\phi = \dfrac{2\lambda\sqrt{R^2 - d^2}}{\varepsilon_0}$

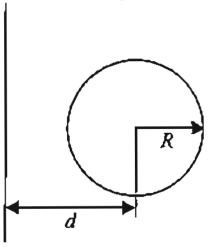


Fig. 2.81

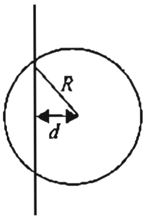


Fig. 2.82

7. The flux passing through the strip of area dA

$$d\phi = EdA\cos\theta$$

$$d\phi = \frac{\lambda}{2\pi\varepsilon_0\sqrt{d^2 + x^2}}(l\,dx)\frac{d}{\sqrt{d^2 + x^2}}$$

$$d\phi = \frac{\lambda dl}{2\pi\varepsilon_0}\ \frac{dx}{(d^2 + x^2)}$$

$$\therefore\quad \phi = \int d\phi = \frac{\lambda dl}{2\pi\varepsilon_0} \int_{-b/2}^{b/2} \frac{dx}{d^2 + x^2}$$

$$\phi = \frac{\lambda dl}{2\pi\varepsilon_0}\frac{1}{d}\left[\tan^{-1}\frac{x}{d}\right]_{-b/2}^{b/2}$$

$$\phi = \frac{\lambda l}{2\pi\varepsilon_0} \times 2\tan^{-1}\left(\frac{b}{2d}\right)$$

$$\text{or } \phi = \frac{\lambda l}{\pi\varepsilon_0}\tan^{-1}\left(\frac{b}{2d}\right)$$

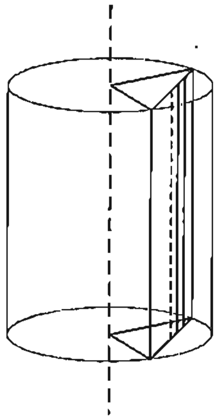


Fig. 2.83

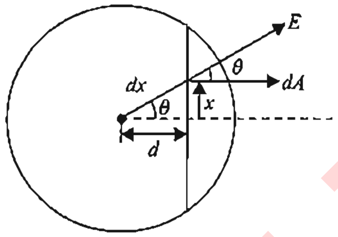


Fig. 2.84

The number of lines passing through a surface is proportional to the flux passing through the surface. The number of lines passing through the plane surface $ABCD$ is equal to the number of lines passing through the curved surface.
Amount of flux passing through the plane surface is equal to the line flux passing through the curved surface.

From Fig. 2.86, $\tan\frac{\theta}{2} = \frac{b}{2d}$

$$\theta = 2\tan^{-1}\left(\frac{b}{2d}\right)$$

2π angle corresponds to flux $\left(\frac{\lambda l}{\varepsilon_0}\right)$.
Hence, θ angle corresponds to the flux,

$$\phi = \frac{\theta}{2\pi}\left(\frac{\lambda l}{\varepsilon_0}\right) = \frac{\lambda l}{\pi\varepsilon_0}\tan^{-1}\left(\frac{b}{2d}\right)$$

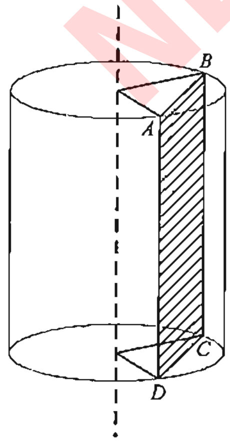


Fig. 2.85

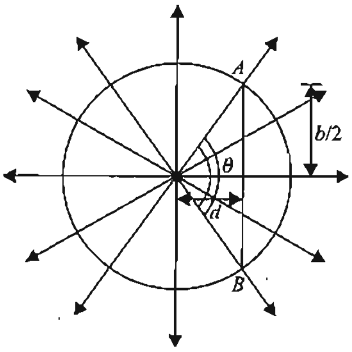


Fig. 2.86

8. $\int E\,dA = E(4\pi r^2) = \frac{q_{in}}{\varepsilon_0}$

(a) For $r > R$, $q_{in} = \int_0^R Ar^2(4\pi r^2)dr = 4\pi\frac{AR^5}{5}$

and $E = \frac{AR^5}{5\varepsilon_0 r^2}$

(b) For $r < R$, $q_{in} = \int_0^r Ar^2(4\pi r^2)dr = \frac{4\pi AR^5}{5}$ and

$E = \frac{AR^3}{5\varepsilon_0}$

9. Consider a spherical shell of radius x. The electric flux through this surface.

$$\phi = \int_s \vec{E}\cdot d\vec{S} = E_r 4\pi r^2$$

Therefore, electric flux through spherical surface of radius R will be

$$\phi = E_R 4\pi R^2$$

When $r = R$, $E_R = \alpha R$

$\therefore \quad \phi = \alpha R 4\pi R^2$

By Gauss theorem, net electric flux

$$= \frac{1}{\varepsilon_0} \times \text{charge enclosed}$$

$$\alpha R 4\pi R^2 = \frac{1}{\varepsilon_0}Q_{\text{enclosed}}$$

$\therefore \quad Q_{\text{enclosed}} = (4\pi\varepsilon_0)\cdot\alpha R^3$

Given $R = 0.30$ m, $\alpha = 100$ V/m^2

$Q_{\text{enclosed}} = \frac{1}{9\times 10^9} \times 100 \times (0.30)^3 = 3 \times 10^{-10}$ coulomb.

10. Consider a spherical surface of radius $r(< R)$ having centre at the centre of ball. If E is the magnitude of electric field strength at the surface, then electric flux through this surface $= \int_s \vec{E}\cdot d\vec{S} = \int_s EdS\cos 0°$

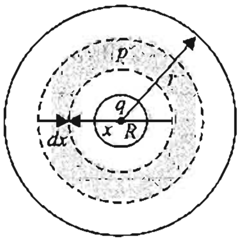


Fig. 2.87

$$= E\int_s dS = E4\pi r^2$$

By Gauss theorem, $\int_s \vec{E}.d\vec{S} = \frac{1}{\varepsilon_0}(Q_{\text{enclosed}})$ (i)

If q is charge on ball, then

$$Q_{\text{enclosed}} = q + \int_R^r \rho dv = q + \int_R^r \frac{\alpha}{x}\cdot 4\pi x^2 dx$$

$$= q + 2\pi\alpha(r^2 - r^2)$$

$\therefore$ From (i)

$$E4\pi r^2 = \frac{1}{\varepsilon_0}[q + 2\pi\alpha(r^2 - R^2)]$$

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0 r^2}[q + 2\pi\alpha(r^2 - R^2)]$$

$$\text{or } E = \frac{\alpha}{2\varepsilon_0} + \frac{1}{\varepsilon_0}\left(\frac{q}{4\pi r^2} - \frac{\alpha}{2}\frac{R^2}{r^2}\right) \quad \text{(ii)}$$

This will independent of r if second term on R.H.S. is zero

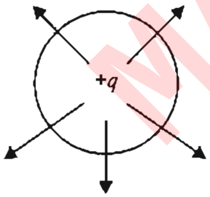
$$\text{i.e., } \frac{q}{4\pi r^2} - \frac{\alpha R^2}{2r^2} = 0$$

$$\Rightarrow q = 2\pi\alpha R^2 \quad \text{(iii)}$$

Then electric field strength E will be $E = \frac{\alpha}{2\varepsilon_0}$

Objective Type

1. **a.** $\phi = E = \text{NC}^{-1}\ \text{m}^2$
2. **a.** Outgoing flux is taken as +ve.



Positive flux

Fig. 2.88

3. **a.** Because net charge enclosed by the surface is zero.
4. **a.** Inside a conductor, electric field is zero. Because applied field is cancelled by field produced due to induced charges.
5. **b.** $E = \frac{kQr}{R^3} = \frac{1}{4\pi\varepsilon_0}\frac{(4/3)\pi R^3 \rho r}{R^3} = \frac{\rho r}{3\varepsilon_0}$
6. **d.** Inside a hollow conductor, electric field is zero. At surface, it is maximum and then decreases.
7. **d.** $\phi = EA\cos\theta$
   $= 2E\pi b^2 \cos 180° = -200\pi b^2$

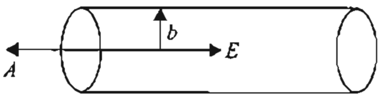


Fig. 2.89

8. **a.** $E = \frac{\sigma}{\varepsilon_0}$

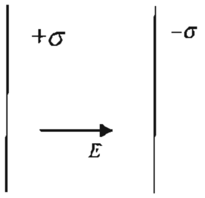


Fig. 2.90

9. **c.** Electric field at any point on Gaussian surface is due to all charges present inside or outside the surface.
10. **b.** Net charge on the conductor will be zero. So, net charge inside S_1 will be the charge on the rod. Hence, flux through S_1 is q/ε_0.
11. **c.** Electric field is zero inside the shell due to the changes outside it.
12. **a.** Inside a uniformly charged solid sphere: $E \propto r$
13. **b.** There is no electric field inside a conductor.
14. **d.** At any point inside the conductor, net electric field is zero.
15. **d.** Electric field at any point near the surface of an arbitrary conductor is $E = \sigma/\varepsilon_0$, where σ is the surface change density at that point.

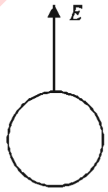


Fig. 2.91

16. **a.** At any point on the curved surface, E and area vector are perpendicular to each other.
17. **d.** $\frac{q}{6\varepsilon_0} = \frac{0.1 \times 10^{-6}}{6 \times 8.85 \times 10^{-12}}$
18. **a.** $\frac{Q}{K\varepsilon_0} = \frac{0.5}{10 \times 8.85 \times 10^{-12}}$
19. **c.** $\phi = \frac{q}{\varepsilon_0} \rightarrow$ Independent of dimensions
20. **c.** $\vec{E} = 2000\,\hat{k},\ \vec{A} = 10 \times 20 \times 10^{-4}\,\hat{k}$
   $$\phi = \vec{E}\cdot\vec{A} = 40\ \text{Vm}$$
21. **b.** Electric intensity may be zero on one side and non-zero on the other side.
22. **c.** Flux through both will be same as net charge enclosed by both is same.
23. **b.** We should know the total charge inside. There is no contribution in the flux due to outside charges.
24. **b.** Net flux is due to charges inside only.
25. **b.** $\phi = \frac{q_{\text{in}}}{\varepsilon_0} \Rightarrow 0 = \frac{q_{\text{in}}}{\varepsilon_0} \Rightarrow q_{\text{in}} = 0$
26. **c.** $1 - 7 - 4 + 10 + 2 - 5 - 3 + 6 = 0$
   Sum of all the charges is zero, so net flux is zero.
27. **a.** Charge inside $S_1 = q_1 + q_2 = 3 \times 10^{-6}$ C
   Charge inside $S_2 = q_2 + q_3 = -1 \times 10^{-6}$ C
   Charge inside S_1 is greatest. So, flux through S_1 is maximum.
28. **a.**

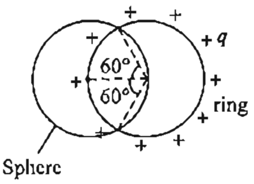


Fig. 2.92

We see that one-third part of the ring will be inside the sphere. So, flux through the sphere

$$\phi = \frac{1}{3}\left(\frac{q}{\varepsilon_0}\right)$$

29. **b.** $\vec{E} = 8\hat{i} + 4\hat{j} + 3\hat{k}$, $\vec{A} = 100\,\hat{k}$

$$\phi = \vec{E}\cdot\vec{A} = 300 \text{ units}$$

30. **d.**

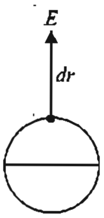


Fig. 2.93

$$E = \frac{\sigma}{2\varepsilon_0}$$

$$F = E\,dr = \frac{\sigma\,dA\,\sigma}{2\varepsilon_0}$$

$$P = \frac{F}{dA} = \frac{\sigma^2}{2\varepsilon_0}$$

$$\text{Force} = P\pi R^2 = \frac{\sigma^2}{2\varepsilon_0}\pi R^2$$

$$= \frac{Q^2}{16\pi^2R^2}\,\frac{\pi R^2}{2\varepsilon_0} = \frac{Q^2}{32\pi\varepsilon_0 R^2}$$

$$= \frac{(20\times10^{-6})^2\times 9\times 10^9}{8(1.5\times10^{-2})^2} = 2000\text{ N}$$

31. **c.**

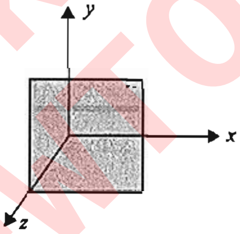


Fig. 2.94

Imagine a charge q at the center of a cube of edge length $2L$ (Fig. 2.94). Then:

$$\phi = \frac{q}{\varepsilon_0}.$$

Here, the square is one 24th of the surface area of the imaginary cube, so it intercepts 1/24 of the flux. That is,

$$\Phi = \frac{q}{24\varepsilon_0}$$

32. **a.** To find the charge enclosed, we need the flux through the parallelopiped:

$\Phi_1 = AE_1\cos 60°$

$= (0.0500\text{ m})(0.0600\text{ m})(2.50 \times 10^4\text{ NC}^{-1}\cos 60°$

$= 37.5\text{ Nm}^2\text{C}^{-1}$

$\Phi_2 = AE_2\cos 120°$

$= (0.0500\text{ m})(0.0600\text{ m})(7.00 \times 10^4\text{ NC}^{-1}\cos 60°$

$= -105\text{ Nm}^2\text{C}^{-1}$

So, the total flux is

$$\Phi = \Phi_1 + \Phi_2 = (37.5 - 105)\text{ Nm}^2\text{C}^{-1} = -67.5\text{ Nm}^2\text{C}^{-1}$$

$$q = \Phi\varepsilon_0 = (-67.5\text{ Nm}^2/\text{C}\varepsilon_0 = -5.97\times10^{-10}\text{ C}.$$

There must be a net charge (negative) in the parallelopiped since there is a net flux flowing into the surface. Also, there must be an external field or all lines would point toward the slab.

33. **c.** The dielectric gets polarized as shown.

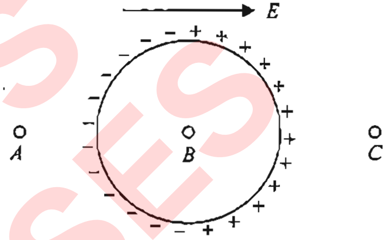


Fig. 2.95

So, intensity at points A and C will increase and at B intensity will decrease.

34. **b.** From the figure it is clear that the plate is placed in an external electric field. Let the electric field due to plate is E and E_0 be the external electric field.

$\therefore \quad E_0 + E = 12\text{ Vm}^{-1}$

$E_0 - E = 8\text{ Vm}^{-1}$

Solving equations, $E = 2\text{ Vm}^{-1}$ and $E_0 = 10\text{ Vm}^{-1}$

Now, electric field due to plate $= \dfrac{\sigma}{2\varepsilon_0} = 2$

$\therefore \sigma = 4\varepsilon_0$

Multiple Correct Answers Type

1. **a., d.** Due to induction, charge on various faces are as shown below in Fig. 2.96.

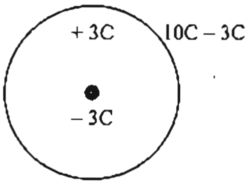


Fig. 2.96

Charge on the inner surface of shell = +3 C

Net charge on outer surface of shell = −3 C + 10 C = +7 C

Distribution of charge on inner surface would be uniform if charge is placed at the centre, otherwise non-uniform. On outer surface, charge would be always uniformly distributed as displacement of inside charges does not affect the distribution of the outer charge.

2. **b., d.** In L.H.S. of Gauss's law, $\vec{E}$ is due to all point charges present in space and ϕ depends only on the enclosed charges.

3. **b., c.** ϕ crossing through Gaussian surface does not depend on the location of charge, while $\vec{E}$ depends on it. If q crosses the boundary, then q_{enclosed} changes and hence the flux and $\vec{E}$ also change.

Assertion-Reasoning Type

1. **d.** $\vec{E}$ in outside vicinity of conductor's surface depends on all the charges present in the space, but expression $\vec{E} = \frac{\sigma}{\varepsilon_0}$.
2. **d.** Due to displacement of charges within closed surface E at any point may change. But net flux crossing the surface will not change.
3. **d.** E at any point on Gaussian surface may be due to outside charges also.
4. **a.** Statement II is true and according to that distribution of charge on outer surface is not affected by location inside of charges.
5. **a.** From $\sigma \propto \frac{1}{R}$ and electric field at the conductor's surface, $E = \frac{\sigma}{\varepsilon_0}$. We can say that Statement I is correct and Statement II is correct explanation for it.

Comprehension Type

For Problems 1–4

1. c., 2. a., 3. d., 4. d.

Sol. $\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta$, where, $\vec{A} = A\hat{n}$

$\hat{n}_{S_1} = -\hat{j}$(left), $\Phi_{S_1} = -(4 \times 10^3\ NC^{-1}(0.1\ m)^2 \cdot \cos(90° - 37°)$

$= -24\ Nm^2C^{-1}$

$\hat{n}_{S_2} = +\hat{k}$ (top),

$\Phi_{S_2} = -(4 \times 10^3\ NC^{-1})(0.1\ m)^2\ \cos 90° = 0$

$\hat{n}_{S_3} = +\hat{j}$ (right),

$\Phi_{S_3} = +(4 \times 10^3\ NC^{-1})(0.1\ m)^2\ \cos(90° - 37°)$

$= +24\ Nm^2C^{-1}$

$\hat{n}_{S_4} = -\hat{k}$ (bottom),

$\Phi_{S_4} = (4 \times 10^3\ NC^{-1})(0.1\ m)^2\ \cos 90° = 0$

$\hat{n}_{S_5} = +\hat{i}$ (front),

$\Phi_{S_5} = +(4 \times 10^3\ NC^{-1})(0.1\ m)^2\ \cos 37° = 32\ Nm^2C^{-1}$

$\hat{n}_{S_6} = -\hat{i}$ (back),

$\Phi_{S_6} = -(4 \times 10^3\ NC^{-1})(0.1\ m)^2\ \cos 37° = -32\ Nm^2C^{-1}$

The total flux through the cube must be zero; any flux entering the cube must also leave it.

For Problems 5–8

5. b., 6. a., 7. b., 8. a.

Sol. Given, $\Phi_2 = \vec{E}\cdot\hat{n}_{S_2}$

$A = (3.00\ NC^{-1}m)(0.300\ m)^2 z$, edge length $L = 0.300$ m

and $\hat{n}_{s_1} = -\hat{j} \Rightarrow \quad \Phi_1 = \vec{E}\cdot\hat{n}_{S_1}A = 0.$

$\hat{n}_{S_2} = +\hat{k}$

$\vec{E} = (-5.00\ NC^{-1}m)x\hat{i} + (3.00\ NC^{-1}m)z\hat{k}.$

$\Rightarrow \quad \phi_2(0.27\ NC^{-1}m)(0.300\ m)$

$= 0.081\ NC^{-1}\ m^2.$

$\hat{n}_{S_3} = +\hat{j} \quad \Rightarrow \quad \Phi_3 = \vec{E}\cdot\hat{n}_{S_3}A = 0.$

$\hat{n}_{S_4} = -\hat{k}$

$\Rightarrow \quad \Phi_4 = \vec{E}\cdot\hat{n}_{S_4}A = -(0.27\ NC^{-1}m)z = 0\ (z = 0).$

$\hat{n}_{S_5} = +\hat{i}$

$\Rightarrow \quad \Phi_5 = \vec{E}\cdot\hat{n}_{S_5}A = (-5.00\ NC^{-1}m)(0.300\ m)^2 x$

$= -(0.45\ NC^{-1}m)x$

$= -(0.45\ NC^{-1}m)(0.300\ m) = -(0.135\ NC^{-1}m^2.$

$\hat{n}_{S_6} = -\hat{i}$

$\Phi_6 = \vec{E}\cdot\hat{n}_{S_6}A = +(0.45\ (NC^{-1}m)x = 0\ (x = 0).$

Total flux,

$\Phi = \Phi_2 + \Phi_5 = (0.081 - 0.135)\ NC^{-1}m^2$

$= -0.054\ Nm^2C^{-1}$

$\therefore\ q = \varepsilon_0\Phi = -4.78 \times 10^{-13}$ C

For Problems 9–10

9. a., 10. d.

Sol. Given that $\vec{E} = -B\hat{i} + C\hat{j} - D\hat{k}$, $\Phi = \vec{E}\cdot\vec{A}$,

Edge length L, and

$\hat{n}_{S_1} = -\hat{j} \quad \Rightarrow \Phi_1 = \vec{E}\cdot A\hat{n}_{S_1} = -CL^2.$

$\hat{n}_{S_2} = +\hat{k} \quad \Rightarrow \Phi_2 = \vec{E}\cdot A\hat{n}_{S_2} = -DL^2.$

$\hat{n}_{S_3} = +\hat{j} \quad \Rightarrow \Phi_3 = \vec{E}\cdot A\hat{n}_{S_3} = +CL^2.$

$\hat{n}_{S_4} = -\hat{k} \quad \Rightarrow \Phi_4 = \vec{E}\cdot A\hat{n}_{S_4} = +DL^2.$

$\hat{n}_{S_5} = +\hat{i} \quad \Rightarrow \Phi_5 = \vec{E}\cdot A\hat{n}_{S_5} = -BL^2$

$\hat{n}_{S_6} = -\hat{i} \quad \Rightarrow \Phi_6 = \vec{E}\cdot A\hat{n}_{S_6} = +BL^2.$

Total flux $= \sum_{i=1}^{6}\Phi_i = 0$

For Problems 11–12

11. c., 12. c.

Sol. $f_1 = -E_1 \quad a^2 = -aa^{1/2}a^2$

$f_1 = -aa^{5/2}$

and $f_2 = E_2a^2 = a\,(2a)^{1/2}a^2$

$$\phi_2 = \sqrt{2}\alpha a^{5/2}$$

$$\phi_{net} = (\sqrt{2} - 1)\,\alpha a^{5/2}$$

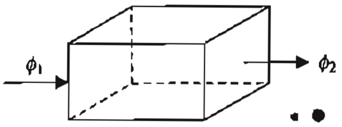


Fig. 2.97

Using Gauss theorem (Fig. 2.97),

$$\phi = \frac{q_{in}}{\varepsilon_0}$$

$\Rightarrow \quad q_{in} = \phi\,\varepsilon_0 \quad = (\sqrt{2} - 1)\,\varepsilon_0\alpha\, a^{5/2}$

For Problems 13–18

13. a., 14. a., 15. b., 16. a., 17. c., 18. a.

Sol. (i) $r < a$. $E = 0$, since $Q = 0$. (Fig. 2.98)

(ii) $a < r < b$. $E = 0$, since $Q = 0$.

(iii) $b < r < c$. Since $Q = +2q$, $E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$,

(iv) $c < r < d$. $E = 0$, since $Q = 0$.

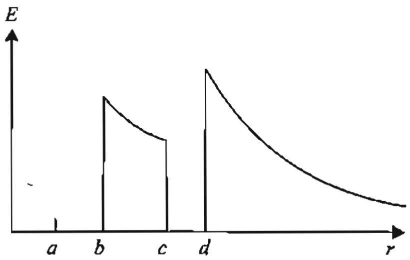


Fig. 2.98

(v) $r > d$. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$, since $Q = +6q$.

For Problems 19–22

19. **a., 20. b., 21. c., 22. d.**

Sol.(i) Small shell inner surface: $Q = 0$

(ii) Small shell outer surface: $Q = +2q$

(iii) Large shell inner surface: $Q = -2q$

(iv) Large shell outer surface: $Q = +6q$

For Problems 23–26

23. a., 24. b., 25. c., 26. c.

Sol.

i. Small shell inner surface: $Q = 0$

ii. Small shell outer surface: $Q = +2q$

iii. Large shell inner surface: $Q = -2q$

iv. Large shell outer surface: $Q = -2q$

For Problems 27–32

27. a., 28. a., 29. b., 30. a., 31. c., 32. b.

Sol. (i) $r < a$. $E = 0$, since charge enclosed is zero.

(ii) $a < r < b$. $E = 0$, since charge enclosed is zero.

(iii) $b < r < c$. $E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$, since charge enclosed is $+2q$.

(iv) $c < r < d$. $E = 0$, since charge enclosed is zero Fig. 2.99.

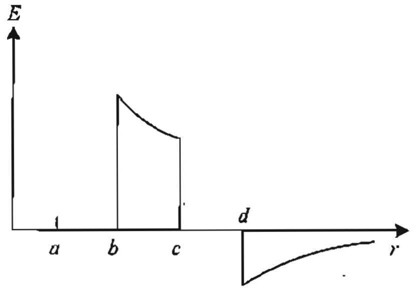


Fig. 2.99

(v) $r > d$. $E = -\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$, since charge enclosed is $-2q$.

For Problems 33–38

33. a., 34. a., 35. b., 36. a., 37. a., 38. c.

Sol.

i. $r < a$. $E = 0$, since the charge enclosed is zero (Fig. 2.100).

ii. $a < r < b$. $E = 0$, since the charge enclosed is zero.

iii. $b < r < c$. $E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$, since the charge enclosed is $2q$.

iv. $c < r < d$. $E = 0$, since the net charge enclosed is zero.

v. $r > d$. $E = 0$, since the net charge enclosed is zero.

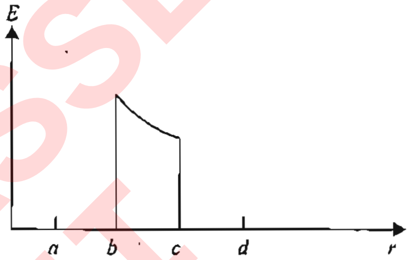


Fig. 2.100

For Problems 39–42

39. a. 40. b., 41. c., 42. a.

Sol.

i. Small shell inner surface: $Q = 0$

ii. Small shell outer surface: $Q = +2q$

iii. Large shell inner surface: $Q = -2q$

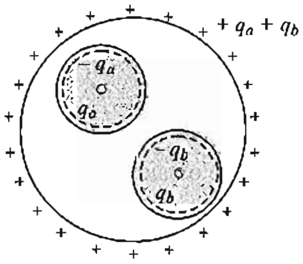
iv. Large shell outer surface: $Q = 0$

For Problems 43–45

43. b. 44. c. 45. a.

Sol. i. $\sigma_A = \frac{-q_a}{4\pi a^2}$, $\sigma_B = \frac{-q_b}{4\pi b^2}$

$$\Rightarrow \quad \sigma_R = \frac{q_a + q_b}{4\pi R^2}$$



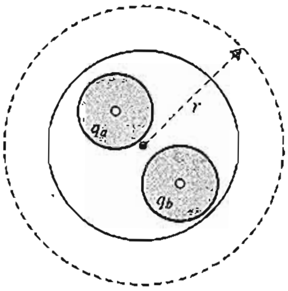
(a)

Fig. 2.101

ii. $E = \frac{1}{4\pi\varepsilon}\frac{q_a + q_b}{r^2}$ (Fig. 2.104)

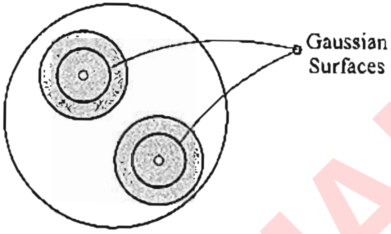
iii. $E_a = \frac{1}{4\pi\varepsilon}\frac{q_a}{r^2}$, $E_b = \frac{1}{4\pi\varepsilon}\frac{q_b}{r^2}$

iv. Electric field at the centers of cavities due to other changes is zero, hence no force will be experienced.



(b)

Fig. 2.102



(c)

Fig. 2.103

v. Electric field outside the bigger sphere will change.

Matching Column Type

1. **i. → b., ii. → d., iii. → c.**
iv. → c. **iv. → c.** **iv. → d.**

i. $E = \frac{\lambda}{2\pi\varepsilon_0 r}$.

ii. Inside electric field, $E = 0$.

iii. $E = \frac{\rho r}{3\varepsilon_0}$.

iv. $E = \frac{\lambda r}{2\pi\varepsilon_0 R^2}$.

v. Inside a plane sheet, $E \propto r$.

vi. E is independent of r outside the plane sheet.

2. **i. → b., ii. → b., iii. → c., iv. → d.**

Sol. Distribution of charge on different surfaces of the plates have been shown.

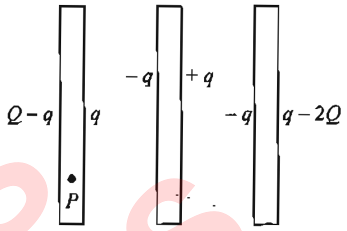


Fig. 2.104

Take a point P on left most plate. The electric field at P

$$E = \frac{Q-q}{2A\,\varepsilon_0} - \frac{q}{2A\,\varepsilon_0} + \frac{q}{2A\,\varepsilon_0}$$

$$-\frac{q}{2A\,\varepsilon_0} + \frac{q}{2A\,\varepsilon_0} - \frac{(q-2Q)}{2A\,\varepsilon_0} = 0$$

(inside a conductor, $E = 0$)

So, $Q - q - q + 2Q = 0$

or $\quad 2q = 3Q \quad \Rightarrow \quad q = \frac{3Q}{2}$

The charge appearing on outer surface of right most plate

$$= q - 2Q = \frac{3Q}{2} - 2Q = -\frac{Q}{2}$$

The charge on outer surface of left most plate

$$= Q - q = Q - \frac{3Q}{2} = -\frac{Q}{2}$$

Charge appearing on middle plate is $-q$ and $+q$

i.e., $-\frac{3Q}{2}$ and $+\frac{3Q}{2}$.

3. **i. → b., ii. → d., iii. → c., iv. → a.**

Sol. i. Infinite plane sheet of charge $= \frac{\pi}{2\,\varepsilon_0}$.

ii. Infinite plane sheet of uniform thickness $= \frac{\sigma}{\varepsilon_0}$.

iii. Non-conducting charged solid sphere at its surface

$$= \frac{R\rho}{3\,\varepsilon_0}.$$

iv. Non-conducting charged solid sphere at its centre = 0.

CHAPTER

3

Electric Potential

The concept of potential energy was introduced in mechanics in connection with such conservative forces as the gravitational force and the elastic force exerted by a spring. By using the law of conservation of energy, we were able to avoid working directly with forces when solving various problems in mechanics. The concept of potential energy is also of great value in the study of electricity. Because the electrostatic force is conservative, electrostatic phenomena can be conveniently described in terms of an electric potential energy. This idea enables us to define a scalar quantity known as *electric potential*. Because the electric potential at any point in an electric field is a scalar quantity, we can use it to describe electrostatic phenomena more simply than if we were to rely only on the electric field and electric forces.

The potential is characteristic of the field only, independent of a charged test particle that may be placed in the field. Potential energy is characteristic of the charge–field system due to an interaction between the field and a charged particle placed in the field.

ELECTRIC POTENTIAL AND ENERGY

Electrostatic force is a conservative force. Thus, when an electrostatic force acts between two or more charged particles within a system of particles, we can assign an electric potential energy U to the system.

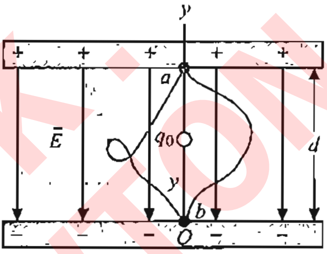
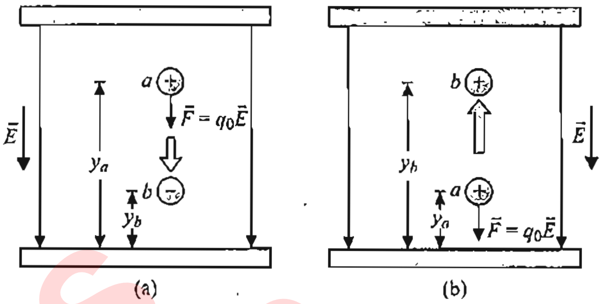


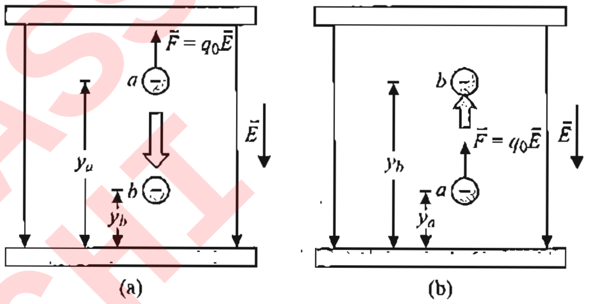
Fig. 3.1

If the system changes its configuration from an initial state 'a' to a different final state 'b' (see Fig. 3.1), let the electrostatic force does work 'W' on the particles as in Figs. 3.2 and 3.3. In a conservative field, we have a relation between change in potential energy and work done by conservative force



(a) Positive charge moves in direction of $\vec{E}$: field does positive work on charge, potential energy U decreases.
(b) Positive charge moves in direction opposite to $\vec{E}$: field does negative work on charge, potential energy U increases.

Fig. 3.2



(a) Negative charge moves in direction of $\vec{E}$: field does negative work on charge, potential energy U increases.
(b) Negative charge moves in direction opposite to $\vec{E}$: field does positive work on charge, potential energy U decreases.

Fig. 3.3

$$\Delta U = -W$$

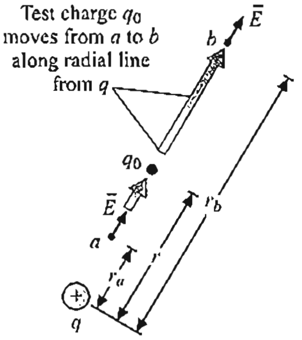
$$U_b - U_a = -W \quad \text{(i)}$$

ELECTRIC POTENTIAL ENERGY OF TWO POINT CHARGES

Let us calculate the work done on a test charge q_0 moving in the electric field caused by a single, stationary point charge q.

We will consider first a displacement along the radial line as shown in Fig. 3.4, from point a to point b. The force on q_0 is given by Coulomb's law, and its radial component is

$$F_r = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2} \quad \text{(i)}$$



Test charge q_0 moves along a straight line extending radially from charge q. As it moves from a to b, the distance varies from r_a to r_b.

Fig. 3.4

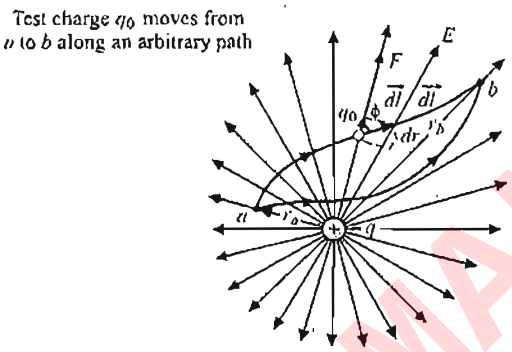
The force is not constant during the displacement, and we have to integrate to calculate the work $W_{a\to b}$ done on q_0 by this force as q_0 moves from a to b. We find

$$W_{a\to b}=\int_{r_a}^{r_b}F_r\,dr=\int_{r_a}^{r_b}\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\,dr=\frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a}-\frac{1}{r_b}\right)\qquad\text{(ii)}$$

The work done by the electric force for this particular path depends only on the end points.

In fact, the work is the same for all possible paths from a to b. To prove this, we consider a more general displacement (as shown in Fig. 3.5). The work done on q_0 during this displacement is given by

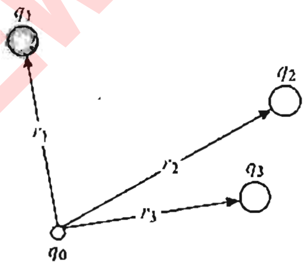
$$W_{a\to b}=\int_{r_a}^{r_b}F\cos\phi\,dl=\int_{r_a}^{r_b}\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\cos\phi\,dl\qquad\text{(iii)}$$



The work done on charge q_0 by the electric field of charge q does not depend on the path taken, but only on the distances r_a and r_b.

Fig. 3.5

But the figure shows that $\cos\phi\,dl=dr$. That is, work done during a small displacement $d\vec{l}$ depends only on the change dr in the distance r between the charges, which is the radial component of the displacement. Thus, equation (iii) is valid even for this more general displacement; the work done on q_0 by the electric field produced by q depends only on r_a and r_b not on the details of the path (Fig. 3.6). These are the needed characteristics for a conservative force, as we defined it in the section. Thus, the force on q_0 is a conservative force.



The potential energy associated with a charge q_0 at point a depends on the other charges q_1, q_2 and q_3 and on their distance r_1, r_2 and r_3 from point a.

Fig. 3.6

We see that equations (i) and (iii) are consistent if we define $\frac{qq_0}{4\pi\varepsilon_0 r_a}$ to be the potential energy U_a when q_0 is at point a, a distance r_a from q, and we define $\frac{qq_0}{4\pi\varepsilon_0 r_b}$ to be the potential energy U_b when q_0 is at point b, a distance r_b from q. Thus, the potential energy U when the test charge q_0 is at any distance r from charge q is

$$U=\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$$

(electric potential energy of two point charges q and q_0)

In case of discrete distribution charges,

$$U=\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}}+\frac{q_2q_3}{r_{23}}+\cdots\right]=\frac{1}{2}\frac{1}{(4\pi\varepsilon_0)}\sum_{i\neq j}\frac{q_iq_j}{r_{ij}}$$

[$^1/_2$ is used as each term in summation will appear twice]

ELECTRON-VOLT

It is the smallest practical unit of energy used in atomic and nuclear physics. An electron-volt is defined as the energy acquired by a particle having one quantum of charge (i.e., e) when accelerated by 1 volt, i.e.,

$$1\text{ eV}=(1.6\times10^{-19}\text{ C})\times(1\text{ JC}^{-1})=1.6\times10^{-19}\text{ J}$$

$\{$as $U=qV\}$

ELECTRIC POTENTIAL

The work done by the electric force during a displacement from a to b: $W_{a\to b}=-\Delta U=-(U_b-U_a)$. On a "work per unit charge" basis, we divide this equation by q_0, obtaining

$$\frac{W_{a\to b}}{q_0}=-\frac{\Delta U}{q_0}=-\left(\frac{U_b}{q_0}-\frac{U_a}{q_0}\right)$$

$$=-(V_b-V_a)=V_a-V_b\qquad\text{(i)}$$

where $V_a=\frac{U_a}{q_0}$ is the potential energy per unit charge at point a and similarly for V_b. We call V_a and V_b the potential at point a and potential at point b, respectively. *Thus, the work done per unit charge by the electric force when a charged body moves from a to b is equal to the potential at 'a' minus the potential at 'b'.*

The difference V_a-V_b is called the potential of a with respect to b. We sometimes abbreviate this difference as $V_{ab}=V_a-V_b$ (note the order of the subscripts). In electric circuits which we will analyze in later chapters, the potential difference between two points is often called *voltage*. Equation (i) then states that V_{ab}, *the potential of 'a' with respect to 'b', equals the work done by the electric force when a UNIT charge moves from 'a' to 'b'.*

Also, V_{ab}, the potential of a with respect to b, equals the work that must be done to move a UNIT charge slowly from b to a against the electric force.

To find the potential V due to a single point charge q, we divide U by q_0.

$$V=\frac{U}{q_0}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\qquad\text{(ii)}$$

(potential due to a point charge q at a distance r from it)

where r is the distance from the point charge q to the point at which the potential is evaluated.

Regarding potential it is worth noting that

- It is a scalar quantity having dimensions $[V]=\left[\frac{W}{q_0}\right]$

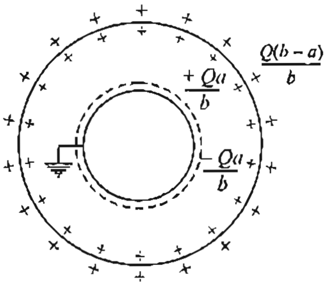


Fig. 3.44

As $b > a$, so charge on the outer surface of outer shell will be $\frac{Q(b-a)}{b} > 0$.

b. Potential of outer surface V_B = Potential due to charge on A + Potential due to charge on B.

$$V_B = V_{a,\text{out}} + V_{b,\text{bothsurfaces}} = \frac{1}{4\pi\varepsilon_0}\frac{q'}{b} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{b}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\left(-Q\frac{a}{b}\right)}{b} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{b} = \frac{Q(b-a)}{4\pi\varepsilon_0 b^2}$$

Example 3.5 **Two circular loops of radii 0.05 and 0.09 m, respectively, are put such that their axes coincide and their centers are 0.12 m apart. Charge of 10^{-6} coulomb is spread uniformly on each loop. Find the potential difference between the centers of loops.**

Sol. The potential at the center of a ring will be due to charge on both the rings and as every element of a ring is at a constant distance from the center, so

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{R_1} + \frac{q_2}{\sqrt{R_2^2 + x^2}}\right]$$

or, $$V_1 = 9\times 10^9\left[\frac{10^{-4}}{5} + \frac{10^{-4}}{\sqrt{9^2+12^2}}\right]$$

or, $$V_1 = 9\times 10^5\left[\frac{1}{5} + \frac{1}{15}\right] = 2.40\times 10^5\text{V}$$

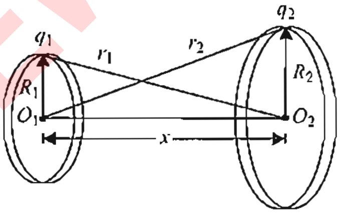


Fig. 3.45

Similarly, $$V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_2}{R_2} + \frac{q_1}{\sqrt{R_1^2 + x^2}}\right]$$

or, $$V_2 = 9\times 10^5\left[\frac{1}{9} + \frac{1}{13}\right] = \frac{198}{117}\times 10^5$$

$\Rightarrow$ $$V_2 = 1.69\times 10^5\text{ V}$$

So, $V_1 - V_2 = (2.40 - 1.69)\times 10^5 = 71$ kV

Example 3.6 **A very small sphere of mass 80 g having a charge q is held at a height of 9 m vertically above the centre of a fixed conducting sphere of radius 1 m, carrying an equal charge q. When released, it falls until it is repelled back just before it comes in contact with the sphere as in Fig. 3.46. Calculate the charge q. $[g = 10\text{ m/s}^2]$**

Sol. Keeping in mind that here both electric and gravitational potential energies are changing and for an external point a charged sphere behaves as if whole of its charge were concentrated at its centre. Applying conservation of energy between initial and final positions, we have

$$\frac{1}{4\pi\varepsilon_0}\frac{qq}{9} + mg\times 9 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{1} + mg\times 1$$

as KE is zero at both locations

or $q^2 = \frac{80\times 10^{-3}\times 10}{10^9}$ or $q = 20\sqrt{2}\,\mu\text{C}$

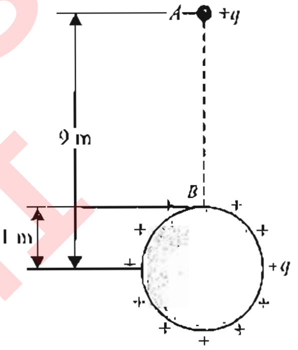


Fig. 3.46

Example 3.7 A circular ring of radius R with uniform positive charge density λ per unit length is located in the y–z plane with its center at the origin O. A particle of mass m and positive charge q is projected from the point $P\left[-\sqrt{3}R,\ 0,\ 0\right]$ on the negative x-axis directly towards O, with initial speed v. Find the smallest (non-zero) value of the speed such that the particle does not return to P?

(IIT-JEE, 1993)

Sol. As the electric field at the center of a ring is zero, the particle will not come back due to repulsion if it crosses the center (Fig. 3.47), i.e.,

$$\frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r} > \frac{1}{4\pi\varepsilon_0}\frac{qQ}{R}$$

But here, $Q = 2\pi R\lambda$ and $r = \sqrt{\left(\sqrt{3}R\right)^2 + R^2} = 2R$

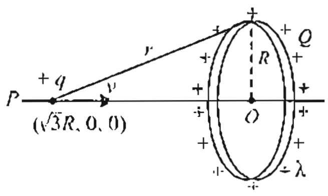


Fig. 3.47

So, $$\frac{1}{2}mv^2 > \frac{1}{4\pi\varepsilon_0}\frac{2\pi R\lambda q}{R}\left[1-\frac{1}{2}\right] \text{ or } v > \sqrt{\left(\frac{\lambda q}{2\varepsilon_0 m}\right)},$$

So, $$v_{\min} = \sqrt{\left(\frac{\lambda q}{2\varepsilon_0 m}\right)}$$

Example 3.8 **A non-conducting disk of radius a and uniform positive surface charge density σ is placed on the ground, with its axis vertical. A particle of mass m and positive charge q is dropped, along the axis of the disk, from a height H with zero initial velocity. The particle has $q/m = \varepsilon_0 g/\sigma$.**

a. Find the value of H if the particle just reaches the disk.

b. Sketch the potential energy of the particle as a function of its height and find its equilibrium position.

(IIT-JEE, 1999)

Sol.

a. Given that: a = radius of disk, σ = surface charge density, $q/m = 4\varepsilon_0 g/\sigma$

The K.E. of the particle, when it reaches the disk can be taken as zero.

Potential due to a charged disk at any axial point situated at a distance x from O.

$$V(x) = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{a^2+x^2} - x\right]$$

Hence, $V(H) = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{a^2+H^2} - H\right]$ and $V(O) = \frac{\sigma a}{2\varepsilon_0}$

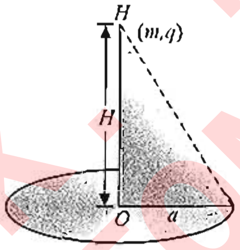


Fig. 3.48

According to law of conservation of energy,

Loss of gravitation potential energy = gain in electric potential energy

$$mgH = q\Delta V = q[V(0) - V(H)]$$

$$\Rightarrow \quad mgH = q[a - \{\sqrt{(a^2+H^2)} - H\}]\frac{\sigma}{2\varepsilon_0} \qquad \text{(i)}$$

From the given relation $\frac{\sigma}{2\varepsilon_0} = 2\,mg$ (given)

Putting this in equation (i), we get

$$mgH = 2mg[a - \{\sqrt{(a^2+H^2)} - H\}]$$

or $$H = 2[a + H - \sqrt{(a^2+H^2)}]$$

or $H = 2a + 2H - 2\sqrt{(a^2+H^2)}$ or, $2\sqrt{(a^2+H^2)} = H + 2a$

or $4a^2 + 4H^2 = H^2 + 4a^2 + 4aH$ or $3H^2 = 4aH$ or $H = \frac{4a}{3}$

[$\because H = 0$ is not valid]

b. Total potential energy of the particle at height h.

$$U(x) = mgx + qV(x) = mgx + \frac{q\sigma}{2\varepsilon_0}(\sqrt{a^2+x^2} - x)$$

$$= mgx + 2mg + [\sqrt{(a^2+x^2)} - x]$$

$$= mg\,[2\sqrt{(a^2+x^2)} - x] \qquad \text{(ii)}$$

For equilibrium: $\frac{dU}{dx} = 0$

This gives: $x = \frac{a}{\sqrt{3}}$

From equation (ii), graph between $U(h)$ and h will be as shown in Fig. 3.49.

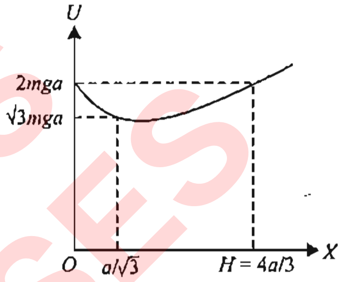
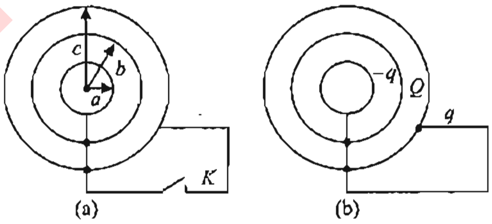


Fig. 3.49

Example 3.9 **Three concentric conducting shells of radii a, b and c are shown in Fig. 3.50. Charge on the shell of radius b is Q. If the key K is closed, find the charges on the innermost and outermost shells and ratio of charge densities of the shells. Given that $a : b : c = 1 : 2 : 3$.**

Sol. After closing the key, the innermost and outermost shells will be at the same potential. Let the charge on the outer shell be q and that on the inner shell be $-q$, the total charge on inner and outer shells is zero.



(a) (b)

Fig. 3.50

Potential on innermost shell,

$$V_a = \text{sum of potentials due to } -q, Q \text{ and } q$$

$$= -\frac{q}{a} + \frac{Q}{b} + \frac{q}{c}$$

Similarly, potential on the outermost shell,

$$\Rightarrow \quad V_c = -\frac{q}{a} + \frac{Q}{c} + \frac{q}{c}$$

As $V_a = V_c$, we have $-\frac{q}{a} + \frac{Q}{b} + \frac{q}{c} = -\frac{q}{a} + \frac{Q}{c} + \frac{q}{c}$

From the given conditions, $c = 3a$, $b = 2a$.

Equation (i) now becomes

$$-\frac{q}{a} + \frac{Q}{2a} = -\frac{q}{3a} + \frac{Q}{3a}$$

or $$q = \frac{Q}{4}$$

Thus, charge on outermost shell $= \frac{Q}{4}$

Charge on innermost shell $= -\dfrac{Q}{4}$;

$$\sigma_a = \frac{1}{4\pi a^2}\left(-\frac{Q}{4}\right); \quad \sigma_b = \frac{+Q}{4\pi b^2} = \frac{Q}{4\pi(4a^2)};$$

$$\sigma_c = \frac{1}{4\pi c^2}\left(\frac{Q}{4}\right) = \frac{+Q}{4\pi(9a^2)}$$

Example 3.10 **A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius R is mounted on an insulating stand. S_2 is initially uncharged.**

S_1 is given a charge Q, brought into contact with S_2 and removed. S_1 is recharged such that the charge on it is again Q; and it is again brought into contact with S_2 and removed. This procedure is repeated n times.

a. Find the electrostatic energy of S_2 after n such contacts with S_1.

b. What is the limiting value of this energy as $n \to \infty$?

(IIT-JEE, 1998)

Sol.

a. When the spheres S_1 and S_2 come in contact, there is transference of charges till the potentials of the two spheres become equal.

During first contact,

$V_1 = V_2$ [q_1 charge shifts from S_1 to S_2]

$$\frac{K(Q-q_1)}{r} = \frac{Kq_1}{R} \Rightarrow q_1 = Q\left(\frac{R}{R+r}\right)$$

During second contact, again $V_1 = V_2$

$$\frac{K[Q-(q_2-q_1)]}{r} = \frac{Kq_2}{R}$$

[$(q_2 - q_1)$ charge shifts from S_1 to S_2]

$$q_2 = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2\right]$$

On third contact, again $V_1 = V_2$

$$\frac{K[Q-(q_3-q_2)]}{r} = \frac{Kq_3}{R}$$

[$(q_3 - q_2)$ charge shifts from S_1 to S_2]

$$q_3 = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \left(\frac{R}{R+r}\right)^3\right]$$

On nth contact by symmetry $V_1 = V_2$

$$\frac{K[Q-(q_n-q_{n-1})]}{r} = \frac{Kq_m}{R}$$

[$(q_n - q_{n-1})$ charge shifts from S_1 to S_2]

$$q_n = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \cdots + \left(\frac{R}{R+r}\right)^n\right]$$

$$= Q\left(\frac{R}{R+r}\right)\left[1 - \frac{\{R/(R+r)^n\}}{1 - R/(R+r)}\right]$$

$$= \frac{QR}{r}\left[1 - \left(\frac{R}{R+r}\right)^n\right]$$

The electrostatic energy of S_2 after n contacts is

$$U_n = \frac{1}{2}\frac{q_n^2}{C} = \frac{1}{2} \times \frac{1}{4\pi\varepsilon_0 R} \times \left\{\frac{QR}{r}\left[1 - \left(\frac{R}{R+r}\right)^n\right]\right\}^2$$

b. The limiting value

$$\underset{n\to\infty}{\text{lt}} U_n = \underset{n\to\infty}{\text{lt}}\left[\frac{1}{2} \times \frac{1}{4\pi\varepsilon_0 R}\left\{\frac{QR}{r}\left[1 - \left(\frac{R}{R+r}\right)^n\right]\right\}^2\right]$$

$$= \frac{Q^2R}{2(4\pi\varepsilon_0)r^2}$$

Example 3.11 **Two isolated metallic solid spheres of radii R and $2R$ are charged such that both of these have same charge density σ. The spheres are located far away from each other, and connected by a thin conducting wire. Find the new charge density on the bigger sphere.**

(IIT-JEE, 1996)

Sol. For sphere of radius R (Fig. 3.51), $\sigma = \dfrac{q_1}{4\pi R^2}$

$$q_1 = \sigma \times 4\pi R^2$$

Fig. 3.51

For sphere of radius $2R$ (Fig. 3.52), $\sigma = \dfrac{q_2}{4\pi(2R)^2}$

$$q_2 = \sigma \times 16\pi R^2$$

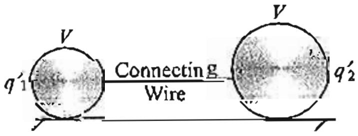


Fig. 3.52

When the two spheres are connected, then the potential on the two spheres will be same. There will be a rearrangement of charge for this to happen.

Let q'_1 and q'_2 be the new charges on the two spheres. Since the total charge remains the same,

$$q'_1 + q'_2 = q_1 + q_2 = \sigma \times 20\pi R^2 \quad \text{(i)}$$

Also, since $V_1 = V_2$

$$\frac{1}{4\pi\varepsilon_0}\frac{q'_1}{R} = \frac{1}{4\pi\varepsilon_0}\frac{q'_2}{2R}; \quad q'_1 = \frac{q'_2}{2} \quad \text{(ii)}$$

Substituting the value of q'_1 from (ii) in (i)

$$\frac{q'_2}{2} + q'_2 = \sigma \times 20\pi R^2$$

$$\Rightarrow \quad \frac{3q'_2}{2} = \sigma \times 20\pi R^2$$

$$\Rightarrow \quad \frac{q'_2}{4\pi(2R)^2} = \frac{\sigma}{3} \times \frac{5}{2}$$

New charge density on bigger sphere $= \dfrac{q'_2}{4\pi(2R)^2} = \dfrac{5\sigma}{6}$

Example 3.12

a. **A charge of Q coulomb is uniformly distributed over a spherical volume of radius R metre. Obtain an expression for the energy of the system.**

b. **What will be the corresponding expression for the energy needed to completely disassemble the planet earth against the gravitational pull amongst its constituent particles?**

Assume the earth to be a sphere of uniform mass density. Calculate this energy, given the product of the mass and the radius of the earth to be 2.5×10^{31} kg m.

c. **If the same charge of Q coulomb as in part (a) above is given to a spherical conductor of the same radius R, what will be the energy of the system? (IIT-JEE, 1992)**

Sol.

a. Let us consider a shell of the thickness dx at a distance x from the center of a sphere (Fig. 3.53).

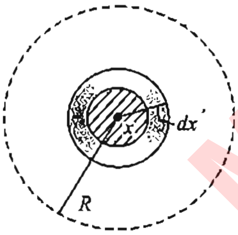


Fig. 3.53

The vol. of the shell $= \left[\frac{4}{3}\pi(x+dx)^3 - \frac{4}{3}\pi x^3\right]$

$$= \frac{4}{3}\pi\left[(x+dx)^3 - x^3\right] = \frac{4}{3}\pi x^3\left[\left(1+\frac{dx}{x}\right)^3 - 1\right]$$

$$= \frac{4}{3}\pi x^3\left[1 + \frac{3dx}{x} - 1\right] = \frac{4}{3}\pi x^3 \times \frac{3dx}{x} = 4\pi x^2 dx$$

Let ρ be the charge per unit volume of the sphere.
Charge of the shell $= dq = 4\pi x^2 \rho dx$
Potential at the surface of the sphere of radius x

$$x = \frac{1}{4\pi\varepsilon_0} \times \frac{\rho \times \frac{4}{3}\pi x^3}{x} \quad \left[\because V = k\frac{q}{r}\right]$$

$\therefore$ potential at the surface of the sphere of radius

$$x = \frac{\rho x^2}{3\varepsilon_0}.$$

Work done in bringing the charge dq on the sphere of radius

$$x = \frac{\rho x^2}{3\varepsilon_0} \times dq$$

i.e., $$dW = \frac{\rho x^2}{3\varepsilon_0} \times 4\pi x^2 \rho\, dx$$

Therefore, the work done in accumulating the charge Q over a spherical volume of radius R meter,

$$W = \int_0^R \frac{4\pi\rho^2}{3\varepsilon_0} x^4 dx = \frac{4\pi\rho^2}{3\varepsilon_0}\left[\frac{x^5}{5}\right]_0^R = \frac{4\pi\rho^2}{3\varepsilon_0}\frac{R^5}{5}$$

$$= \frac{4\pi}{3\varepsilon_0}\left(\frac{Q}{[4/3]\pi R^3}\right)^2 \frac{R^5}{5} = \frac{3Q^2}{20\pi\varepsilon_0 R}$$

This is also the energy stored in the system.

b. The above energy calculated is

$$E = \frac{3Q^2}{5 \times (4\pi\varepsilon_0)R} = \frac{3KQ^2}{5R}, \text{ where } K = \frac{1}{4\pi\varepsilon_0}.$$

In case of earth and gravitational pull, K may be replaced by G. Therefore, the energy required to disassemble the planet earth against the gravitational pull amongst its constituent particles is the work required to make earth from its constituent particles.

$$E = \frac{3GM^2}{5R} \quad [\because \; Q \text{ is replaced by } M]$$

$$\left(\text{Using } F = \frac{Kq_1q_2}{r^2} \text{ and } F = \frac{Gm_1m_2}{r^2}\right)$$

$$\text{But } g = \frac{GM}{R^2} \quad \Rightarrow \quad gMR = \frac{GM^2}{R}$$

$$\therefore \quad E = \frac{3}{5}gMR = \frac{3}{5} \times 10 \times 2.5 \times 10^{31} = 1.5 \times 10^{32}\text{ J}$$

c. During the charging process, let at any instant the spherical conductor has a charge q on its surface (Fig. 3.54). The potential at the surface $= \frac{1}{4\pi\varepsilon_0} \times \frac{q}{R}$

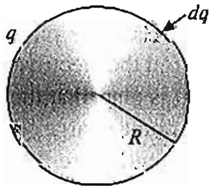


Fig. 3.54

Small amount of work done in bringing a charge dq move on the surface will be $dW = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{R} \times dq$

Therefore, total amount of work done in bringing charge Q on the surface of spherical conductor

$$W = \frac{1}{4\pi\varepsilon_0 R}\int_0^Q q\, dq = \frac{1}{4\pi\varepsilon_0 R}\left[\frac{q^2}{2}\right]_0^Q = \frac{Q^2}{(8\pi\varepsilon_0 R)}$$

Alternative solution of part (c)

If we consider the charged spherical capacitor as an isolated capacitor, then the energy stored in the capacitor is given by

$$E = \frac{1}{2}\frac{Q^2}{C} \quad (\text{where } C = \text{capacitance of the capacitor})$$

For an isolated capacitor, $C = 4\pi\varepsilon_0 R$.

$$\therefore \quad E = \frac{Q^2}{8\pi\varepsilon_0 R} = \text{energy of the system}$$

EXERCISES

Subjective Type

Solutions on page 3.33

1. Fig. 3.55 shows two equipotential lines in x–y plane for an electric field. Find the x-component (E_x) and y-component (E_y) of field in space between these lines.

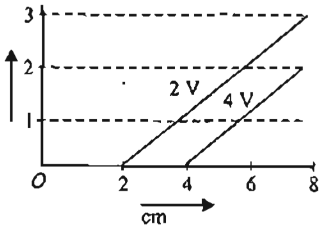


Fig. 3.55

2. The variation of electric potential for an electric field directed parallel to the x-axis is shown in the given graph (Fig. 3.56). Draw the variation of electric field strength with x-axis.

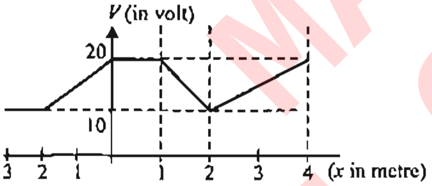


Fig. 3.56

3. Identical charges $-q$ each are placed at 8 corners of a cube of each side b, find the electrostatic potential energy of a charge $+q$ which is placed at the center of cube.
4. At a point due to a point charge, the values of electric field intensity and potential are 32 NC^{-1} and 16 JC^{-1}, respectively. Calculate
   a. magnitude of the charge, and
   b. distance of the charge from the point of observation.
5. Four charges $+q$, $-q$, $+q$ and $-q$ are placed in order on the four consecutive corners of a square of side a. Find the work done in interchanging the positions of any two neighboring charges of opposite sign.
6. Water from a tap maintained at a constant potential of V is allowed to fall by drops of radius r through a small hole into a hollow conducting sphere of radius R standing on an insulating stand until it fills the entire sphere. Find the potential of the hollow conductor, after it is completely filled with water.
7. Three point charges of 0.1 C each are placed at the corners of an equilateral triangle with side $L = 1$ m. If this system is supplied energy at the rate of 1 kW, how much time will be required to move one of the charges on to the mid-point of the line joining the other two?
8. Fig. 3.57 shows a large conducting ceiling having uniform charge density σ below which a charge particle of charge q_0 and mass m is hung from point O, through a small string of length l. Calculate the minimum horizontal velocity required for the string to become horizontal.

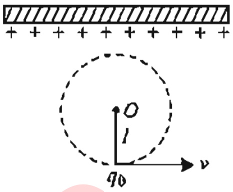


Fig. 3.57

9. A non-conducting sphere of radius $R = 5$ cm has its center at the origin O of coordinate system as shown in Fig. 3.58. Its has two spherical cavities of radius $r = 1$ cm, whose centers are at (0, 3 cm), (0, −3 cm), respectively, and solid material of the sphere has uniform positive charge density $\rho = 1/\pi$ μCm^{-3}. Calculate electric potential at point P (4 cm, 0).

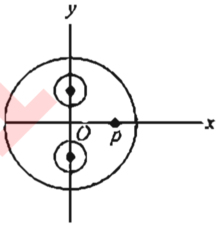


Fig. 3.58

10. Two identical thin rings, each of radius R meter, are coaxially placed at a distance of R meter from each other. If Q_1 coulomb and Q_2 coulomb are the charges uniformly spread on the two rings, find the work done in moving a charge q from the center of one ring to that of the other.
11. Fig. 3.59 shows three concentric thin spherical shells A, B and C of radii R, $2R$ and $3R$. The shell B is earthed and A and C are given charges q and $2q$, respectively. Find the charges appearing on all the surfaces of A, B and C.

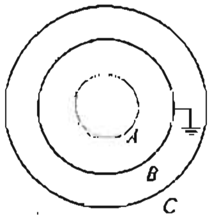


Fig. 3.59

12. Three conducting spherical shells have radii a, b and c such that $a < b < c$ (Fig. 3.60). Initially, the inner shell is uncharged, the middle shell has a positive charge Q and the outer shell has a negative charge $-Q$.
    a. Find the electric potential of the three shells.
    b. If the inner and outer shells are now connected by a wire that is insulated as it passes through the middle shell, what is the electric potential of each of the three shell? Also, what is the final charge on each shell?

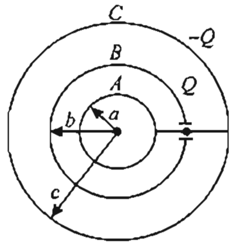


Fig. 3.60

13. Fig. 3.61 shows three concentric spherical conductors A, B and C with radii R, $2R$ and $4R$, respectively. A and C are connected by a conducting wire and B is uniformly charged (charge = $+Q$). Find

a. charges on conductors $A$and C,.
b. potentials of A and B.

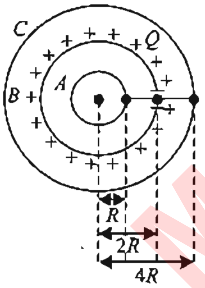


Fig. 3.61

14. Two concentric shells of radii R and $2R$ are shown in Fig. 3.62. Initially, a charge q is imparted to the inner shells. Now, key K_1 is closed and opened and then key K_2 is closed and opened. After the keys K_1 and K_2 are alternately closed n times each, find the potential difference between the shells. Note that finally the key K_2 remains closed.

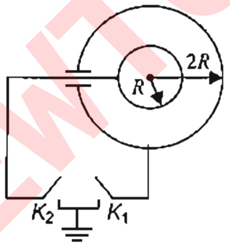


Fig. 3.62

15. Three charges each of value q are placed at the corners of an equilateral triangle. A fourth charge Q is placed at the center of the triangle.

a. Find the net force on charge 'q'.
b. If $Q = -q$, will the charges at the corners move towards the center or fly away from it?
c. For what value of Q at O will charges remain stationary?
d. In the situation (c), how much work is done in removing the charges to infinity?

(IIT-JEE, 1978)

16. An small ball of mass 2×10^{-3} kg having a charge of 1 μC is suspended by a string of length 0.8 m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball so that it can make a complete revolution.

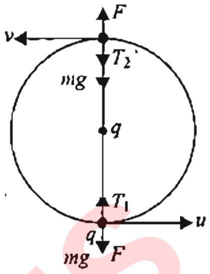


Fig. 3.63

17. Two fixed charges $-2Q$ and Q are located at the points with coordinates $(-3a,\ 0)$ and $(+3a,\ 0)$, respectively, in the x–y plane.

a. Show that all points in the x–y plane where the electric potential due to the two charges is zero lie on a circle. Find its radius and the location of its center.
b. Give the expression $V(x)$ at a general point on the x-axis and sketch the function $V(x)$ on the whole x-axis.
c. If a particle of charge $+q$ starts form rest at the center of the circle, show by a short quantitative argument that the particle eventually crosses the circle. Find its speed when it does so. **(IIT-JEE, 1991)**

18. A point charge q is located at the center O of an uncharged spherical capacitor provided with a small orifice. The inside and outside radii of the capacitor are a and b, respectively (Fig. 3.64). What amount of work has to be performed to slowly transfer the charge q from the point O through the orifice and to infinity ?

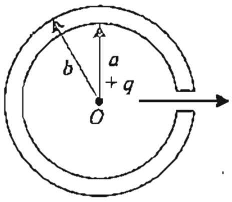


Fig. 3.64

19. Four point charges +8 mC, −1 mC, −1 mC and +8 mC are fixed at the points $-\sqrt{\frac{27}{2}}$ m, $-\sqrt{\frac{3}{2}}$ m, $+\sqrt{\frac{3}{2}}$ m and $+\sqrt{\frac{27}{2}}$ m, respectively, on the y-axis. A particle of mass 6×10^{-4} kg and charge +0.1 μC moves along the x-direction. Its speed at $x = +\infty$ is V_0. Find the least value of V_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gravity free. Given $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{Nm}^2\text{C}^{-2}$.

(IIT-JEE, 2000)

20. Charges $+q$ and $-q$ are located at the corners of a cube of side as shown in Fig. 3.65. Find the work done to separate the charges to infinite distance. **(IIT-JEE, 2003)**

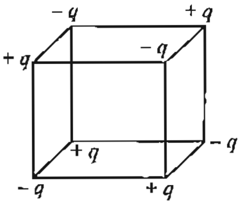


Fig. 3.65

21. Two uniformly charged large plane sheets S_1 and S_2 having charge densities σ_1 and σ_2 ($\sigma_1 > \sigma_2$) are placed at a distance d parallel to each other. A charge q_0 is moved along a line of length a ($a < d$) at an angle 45° with the normal to S_1. Calculate the work done by the electric field.

(IIT-JEE, 2004)

22. For the electrostatic charge system as shown in Fig. 3.66,

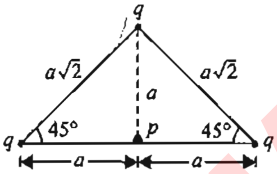


Fig. 3.66

a. find the net force on electric dipole.
b. Also, find electrostatic energy of the system.

23. Four charge particles each having charge Q are fixed at corners of base (at A, B, C and D) of a square pyramid with slant length 'a' ($AP = BP = DP = PC = a$). A charge $-Q$ is fixed at point P. A dipole with dipole moment p is placed at the center of base and perpendicular to its plane as shown in Fig. 3.67. Find

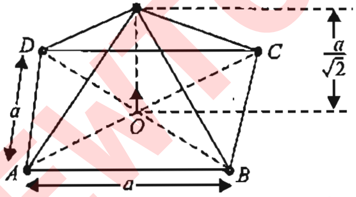


Fig. 3.67

a. force on dipole due to charge particles.
b. potential energy of the system.

24. Three identical dipoles with charges q and $-q$ and separation a are placed on the corners of an equilateral Δ of side d as shown in Fig. 3.68. Find the interaction energy of the system ($a << d$).

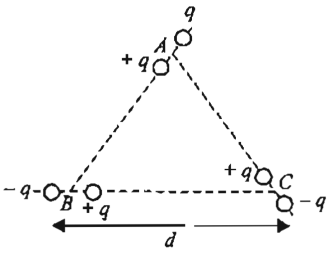


Fig. 3.68

Objective Type

Solutions on page 3.38

1. A charge q is accelerated through a potential difference of V. Find its kinetic energy.
a. qV
b. $qV/2$
c. V
d. None of these

2. The dimensional formula of electric potential is
a. $[MLT^{-2}A^{-1}]$
b. $[ML^2T^{-2}A^{-1}]$
c. $[ML^2T^{-3}A^{-1}]$
d. $[ML^2T^{-3}A^{-2}]$

3. If a conductor is electrically neutral, then
a. net charge on it should be zero
b. potential on it should be zero
c. both charge and potential should be zero
d. none of them may not be zero

4. An electron is released from rest in a region of space with a non-zero electric field. Which of the following statements is true?
a. The electron will begin moving toward a region of higher potential.
b. The electron will begin moving toward a region of lower potential.
c. The electron will begin moving along a line of constant potential.
d. Nothing can be concluded unless the direction of the electric field in known.

5. Fig. 3.66 shows two parallel surfaces A and B at the same potential, kept at a small distance r from each other. A point charge q is taken from the surface A to B. The amount of work done is

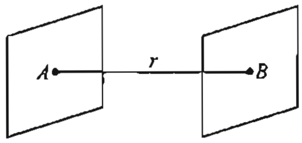


Fig. 3.69

a. $\frac{q^2}{2\pi\varepsilon_0 r}$
b. $\frac{q^2}{8\pi\varepsilon_0 r}$
c. $\frac{q^2}{4\pi\varepsilon_0 r}$
d. zero

6. Inside a hollow charged spherical conductor, the potential
a. is constant
b. varies directly as the distance from the center
c. varies inversely as the distance from the center
d. varies inversely as the square of the distance from the center

7. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. The potential at the center of the sphere is
a. 0 V
b. 10 V
c. same as at point 5 cm away from the surface
d. same as at a point 20 cm away from the surface

(IIT-JEE, 1983)

8. The electric potential at the surface of an atomic nucleus ($Z = 50$) of radius of 9×10^{-15} m is

a. 80 V **b.** 8×10^6 V

c. 9 V **d.** 9×10^5 V

9. A ball of mass 1 g carrying a charge 10^{-8} C moves from a point A at potential 600 V to a point B at zero potential. The change in its K.E. is

a. -6×10^{-6} erg **b.** -6×10^{-6} J

c. 6×10^{-6} J **d.** 6×10^{-6} erg

10. A large insulated sphere of radius r charged with Q units of electricity is placed in contact with a small insulated uncharged sphere of radius r' and is then separated. The charge on the smaller sphere will now be

a. $\dfrac{Q(r'+r)}{r'}$ **b.** $\dfrac{Q(r'+r)}{r}$

c. $\dfrac{Qr}{r'+r}$ **d.** $\dfrac{Qr'}{r'+r}$

11. Potential energy of two equal negative point charges 2 μC held 1 m apart in air is

a. 2 J **b.** 2 eV

c. 4 J **d.** 0.036 J

12. Two point charges 4 μC and −2 μC are separated by a distance of 1 m in air. At what point in between the charges and on the line joining the charges is the electric potential zero?

a. In the middle of the two charges

b. 1/3 m from 4 μC

c. 1/3 m from −2 μC

d. Nowhere the potential is zero

13. Which of the following is/are proportional to the inverse square of the distance x?

a. The potential at a distance x from an isolated point charge

b. The electric field at a distance x from an isolated point charge

c. The force per unit length between two thin, straight, infinitely long current carrying conductors, parallel to each other, separated by a distance x.

d. The electrostatic force between two large charged bodies kept at a small distance x apart.

14. Two conducting spheres of radii r_1 and r_2 have same electric field near their surfaces. The ratio of their electrical potentials is

a. $\dfrac{r_1^2}{r_2^2}$ **b.** $\dfrac{r_2^2}{r_1^2}$ **c.** $\dfrac{r_1}{r_2}$ **d.** $\dfrac{r_2}{r_1}$

15. Three charges $2q$, $-q$ and $-q$ are located at the vertices of an equilateral triangle. At the center of the triangle

a. the field is zero but potential is non-zero

b. the field is non-zero but potential is zero

c. both field and potential are zero

d. both field and potential are non-zero

16. The variation of potential with distance R from fixed point is shown in Fig. 3.70. The electric field at $R = 5$ m is

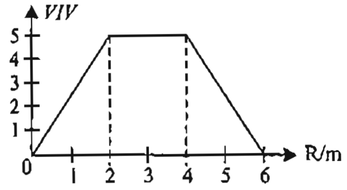


Fig. 3.70

a. 2.5 Vm^{-1} **b.** −2.5 Vm^{-1}

c. 0.4 Vm^{-1} **d.** −0.4 Vm^{-1}

17. When a 2 μC of charge is carried from a point A to point B, the amount of work done by electric field is 50 μJ. What is the potential difference and which point is at a higher potential?

a. 25 V, B

b. 25 V, A

c. 20 V, B

d. Both are at same potential

18. The work done in taking a unit positive charge from P to A is W_A and from P to B is W_B. Then

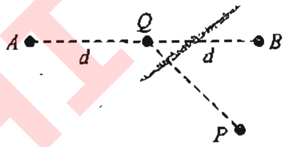


Fig. 3.71

a. $W_A > W_B$ **b.** $W_A < W_B$

c. $W_A = W_B$ **d.** $W_A + W_B = 0$

19. Four charges $+q$, $-q$, $+q$ and $-q$ are put together on four corners of a square as shown in Fig. 3.72. The work done by external agent in assembling this configuration is

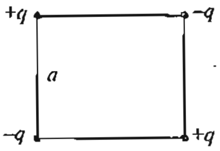


Fig. 3.72

a. zero **b.** $-2.59kq^2/a$

c. $+2.59kq^2/a$ **d.** none of these

20. For the isolated charged conductor shown in Fig. 3.73, the potentials at points A, B, C and D are V_A, V_B, V_C, and V_D, respectively. Then

a. $V_A = V_B > V_C > V_D$

b. $V_D > V_C > V_B = V_A$

c. $V_D > V_C > V_B > V_A$

d. $V_D = V_C = V_B = V_A$

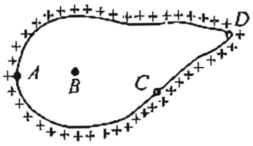


Fig. 3.73

21. The electric field in a region surrounding the origin is uniform and along the x-axis. A small circle is drawn with the center at the origin cutting the axes at points A, B, C and D having coordinates $(a, 0)$, $(0, a)$, $(-a, 0)$ and $(0, -a)$, respectively, as shown in Fig. 3.74. Then, the potential is minimum at

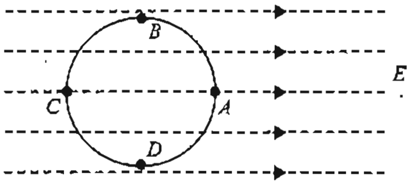


Fig. 3.74

a. A **b.** B **c.** C **d.** D

22. A small conducting sphere of radius a, carrying a charge of $+Q$, is placed inside an equal and oppositely charged conducting shell of radius b such that their centers coincide. Determine the potential at a point at a distance of c from center such that $a < c < b$.

a. $k(Q/c + Q/b)$ **b.** $k(Q/a + Q/b)$
c. $k(Q/a - Q/b)$ **d.** $k(Q/c - Q/b)$

23. Two metal spheres (radii r_1, r_2 with $r_1 < r_2$) are very far apart but are connected by a thin wire. If their combined charge is Q, then what is their common potential?

a. $kQ/(r_1 + r_2)$ **b.** $kQ/(r_1 - r_2)$
c. $-kQ/(r_1 + r_2)$ **d.** $-kQ/r_1r_2$

24. Mark correct statement

a. If E is zero at certain point, then V should be zero at that point.
b. If E is not zero at certain point, then V should not be zero at that point.
c. If V is zero at certain point, then E should be zero at that point.
d. If V is zero at certain point, then E may or may not be zero.

25. Variation in potential is maximum if one goes

a. along the line of force
b. perpendicular to the line of force
c. in any direction
d. none of these

26. An uncharged conductor A is brought near a positively charged conductor B. Then

a. the charge on B will increase but the potential of B will not change
b. the charge on B will not change but the potential of B will decrease
c. the charge on B will decrease but the potential of B will not change
d. the charge on B will not change but the potential of B will increase

27. Two spherical conductors of radii R_1 and R_2 are separated by a distance much larger than the radius of the either sphere. The spheres are connected by a conducting wire as shown in Fig. 3.75. If the charges on the spheres in equilibrium are q_1 and q_2, respectively, what is the ratio of the field strength at the surfaces of the spheres?

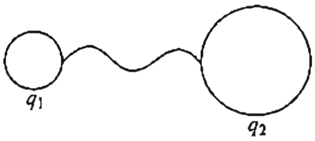


Fig. 3.75

a. R_2/R_1 **b.** R_2^2/R_1^2
c. R_1/R_2 **d.** R_1^2/R_2^2

28. There is an electric field E in x-direction. If the work done by electric field in moving a charge of 0.2 C through a distance of 2 m along a line making an angle 60° with x-axis is 4 J, then what is the value of E?

a. $\sqrt{3}$ NC^{-1} **b.** 4 NC^{-1}
c. 5 NC^{-1} **d.** 20 NC^{-1}

29. Some equipotential surfaces are shown in Fig. 3.76. The magnitude and direction of the electric field is

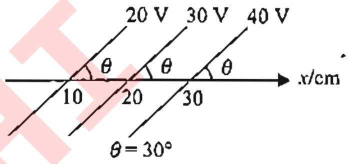


Fig. 3.76

a. 100 Vm^{-1} making angle 120° with the x-axis
b. 200 Vm^{-1} making angle 60° with the x-axis
c. 200 Vm^{-1} making angle 120° with the x-axis
d. none of the above

30. In moving from A to B along an electric field line, the electric field does 6.4×10^{-19} J of work on an electron. If ϕ_1 and ϕ_2 are equipotential surfaces, then the potential difference $V_C - V_A$ is

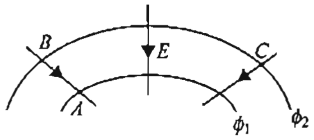


Fig. 3.77

a. -4 V **b.** 4 V
c. zero **d.** 6.4 V

31. Two identical rings P and Q of radius 0.1 m are mounted coaxially at a distance 0.5 m apart. The charges on the two rings are 2 and 4 μC, respectively. The work done in transferring a charge of 5 μC from the center of P to that of Q is

a. 1.28 J **b.** 0.72 J
c. 0.144 J **d.** 2.24 J

32. n charged drops, each of radius r and charge q, coalesce to form a big drop of radius R and charge Q. If V is the electric potential and E is the electric field at the surface of a drop, then

a. $E_{\text{big}} = n^{2/3} E_{\text{small}}$ **b.** $V_{\text{big}} = n^{1/3} V_{\text{small}}$

c. $E_{small} = n^{2/3} E_{big}$ d. $V_{big} = n^{2/3} V_{small}$

33. A small positively charged sphere is placed inside a positively charged spherical shell. What happens if the inner sphere is connected with the outer shell by a conducting wire?

- **a.** The entire charge of inner sphere will be transferred to outer shell and then both will be at same potential.
- **b.** The entire charge of inner sphere will be transferred to outer shell and then both will be at different potential.
- **c.** The entire charge of outer shell will be transferred to inner sphere and then both will be at same potential.
- **d.** Nothing can be predicted.

34. At a point in space, the electric field points towards north. In the region surrounding this point, the rate of change of potential will be zero along

a. north **b.** south
c. north-south **d.** east-west

35. A positive point charge q is carried from a point B to a point A in the electric field of a point charge $+Q$ at O. If the permittivity of free space is ε_0, the work done in the process is given by (where $a = OA$ and $b = OB$)

a. $\dfrac{qQ}{4\pi\varepsilon_0}\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$ **b.** $\dfrac{qQ}{4\pi\varepsilon_0}\left(\dfrac{1}{a}-\dfrac{1}{b}\right)$

c. $\dfrac{qQ}{4\pi\varepsilon_0}\left(\dfrac{1}{a^2}-\dfrac{1}{b^2}\right)$ **d.** $\dfrac{qQ}{4\pi\varepsilon_0}\left(\dfrac{1}{a^2}+\dfrac{1}{b^2}\right)$

36. A spherical charged conductor has surface density of charge = σ. The electric field intensity on its surface is E. If radius of surface is doubled, keeping σ unchanged, what will be electric field intensity on the new sphere?

a. $E/2$ **b.** $E/4$
c. $2E$ **d.** E

37. In the above question, if V be the electric potential of the first sphere, what would be the electric potential of the second sphere?

a. $2V$ **b.** $V/2$
c. $V/4$ **d.** V

38. Which of the following is discontinuous across a charged conducting surface?

- **a.** Electric potential
- **b.** Electric intensity
- **c.** Both electric potential and intensity
- **d.** None of the above

39. The electric field lines are closer together near object A than they are near object B. We can conclude

- **a.** the potential near A is greater than the potential near B
- **b.** the potential near A is less than the potential near B
- **c.** the potential near A is equal to the potential near B
- **d.** nothing about the relative potentials near A and B

40. As shown in Fig. 3.78, a dust particle with mass $m = 5.0 \times 10^{-9}$ kg and charge $q_0 = 2.0$ nC starts from rest at point a and moves in a straight line to point b. What is its speed v at point b?

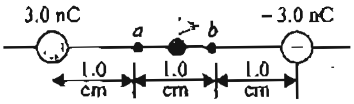


Fig. 3.78

a. 26 ms^{-1} **b.** 34 ms^{-1}
c. 46 ms^{-1} **d.** 14 ms^{-1}

41. Charges $-q$, Q and $-q$ are placed at equal distance on a straight line. If the total potential energy of the system of three charges is zero, then find the ratio Q/q.

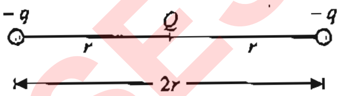


Fig. 3.79

a. 1/2 **b.** 1/4
c. 2/3 **d.** 3/4

42. ABC is a right angled triangle, where AB and BC are 25 and 60 cm, respectively. A metal sphere of 2 cm radius charged to a potential of 9×10^5 volt is placed at B as in Fig. 3.80. Find the amount of work done in carrying a positive charge of 1 coulomb from C to A.

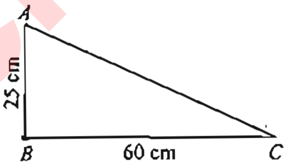


Fig. 3.80

a. 21 kJ **b.** 42 kJ
c. 14 kJ **d.** 52 kJ

43. Two charged particles having charges 1 and −1 μC and of mass 50 gm each are held at rest while their separation is 2 m. Find the speed of the particles when their separation is 1 m.

a. $\dfrac{1}{5}$ m/s **b.** $\dfrac{3}{5}$ m/s

c. $\dfrac{3}{10}$ m/s **d.** $\dfrac{2}{7}$ m/s

44. A 100 eV electron is projected directly towards a large metal plate that has surface charge density of -2.0×10^{-6} Cm^{-2}. From what distance must the electron be projected, if it is to just fail to strike that plate?

a. 0.40 mm **b.** 0.20 mm
c. 1 mm **d.** 0.30 mm

45. A solid sphere of radius R is charged uniformly. At what distance from its surface is the electrostatic potential half of the potential at the center?

a. R **b.** $R/2$ **c.** $R/3$ **d.** $2R$

46. Four identical charges are placed at the points (1, 0, 0), (0, 1, 0), (−1, 0, 0) and (0, −1, 0). Then,

a. the potential at the origin is zero

b. the electric field at the origin is not zero
c. the potential at all points on the z-axis, other than the origin, is zero
d. the field at all points on the z-axis, other than the origin, acts along the z-axis

47. When the separation between two charges is increased, the electric potential energy of the charges
a. increases
b. decreases
c. remains the same
d. may increase or decrease

48. A positive charge is moved from a low potential point A to a high potential point B. Then, the electric potential energy of the system
a. increases
b. decreases
c. will remain the same
d. nothing definite can be predicted

49. If a charge is moved against the coulomb force of an electric field, then
a. positive work is done by the electric field
b. energy is used from some outside source which does positive work
c. the strength of the field is decreased
d. the energy of the system is decreased

50. Mark the correct statement
a. An electron and a proton when released at rest in a uniform electric field experience the same force and the same acceleration.
b. Two equipotential surfaces may intersect.
c. A solid conducting sphere holds more charge than a hollow conducting sphere of the same radius.
d. No work is done in taking a positive charge from one point to another inside a negatively charged metallic sphere.

51. Two point charges Q and $-Q/4$ placed along x-axis are separated by a distance r. Take $-Q/4$ as origin and it is placed right of Q. Then, potential is zero
a. at $x = r/3$ only
b. at $x = -r/5$ only
c. both at $x = r/3$ and at $x = -r/5$
d. there exist two points on the axis where electric field is zero

52. The electric potential decreases uniformly from 120 V to 80 V as one moves on the X-axis from $x = -1$ cm to $x = +1$ cm. The electric field at the origin
a. must be equal to 20 $\mathrm{Vcm^{-1}}$
b. must be equal to 20 $\mathrm{Vm^{-1}}$
c. may be greater than 20 $\mathrm{Vcm^{-1}}$
d. may be less than 20 $\mathrm{Vcm^{-1}}$

53. Fig. 3.81 shows eight point charges arranged at the corners of a cube with sides of length d. The values of the charges are $+q$ and $-q$, as shown. This is a model of one cell of a cubic ionic crystal. In sodium chloride (NaCl), for instance, the positive ions are Na^+ and the negative ions are Cl^-. Calculate the potential energy U of this arrangement. (Take as zero the potential energy of the eight charges when they are infinitely far apart.)

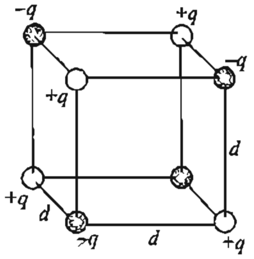


Fig. 3.81

a. $\dfrac{3q^2}{\pi\varepsilon_0 d}\left(1-\dfrac{1}{\sqrt{2}}+\dfrac{1}{3\sqrt{3}}\right)$
b. $-\dfrac{3q^2}{\pi\varepsilon_0 d}\left(1-\dfrac{1}{\sqrt{2}}+\dfrac{1}{3\sqrt{3}}\right)$
c. $\dfrac{q^2}{12\pi\varepsilon_0 d}\left(1-\dfrac{1}{\sqrt{2}}+\dfrac{1}{3\sqrt{3}}\right)$
d. $\dfrac{-q^2}{12\pi\varepsilon_0 d}\left(1-\dfrac{1}{\sqrt{2}}+\dfrac{1}{3\sqrt{3}}\right)$

54. A small sphere with mass 1.2 g hangs by a thread between two parallel vertical plates 5.00 cm apart. The plates are insulating and have uniform surface charge densities $+\sigma$ and $-\sigma$. The charge on the sphere is $q = 9 \times 10^{-6}$ C. What potential difference between the plates will cause the thread to assume an angle of 37° with the vertical as shown in Fig. 3.82.
a. 30 V
b. 12 V
c. 50 V
d. 25 V

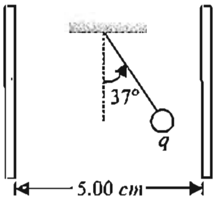


Fig. 3.82

55. A particle of mass m carrying charge 'q' is projected with velocity 'v' from point 'P' towards an infinite line of charge from a distance 'a'. Its speed reduces to zero momentarily at point Q which is at a distance $a/2$ from the line of charge. If another particle with mass m and charge '$-q$' is projected with the same velocity 'v' from P towards the line of charge, what will be its speed at Q?
a. $\sqrt{2}v$
b. $\sqrt{3}v$
c. $\dfrac{v}{\sqrt{2}}$
d. $\dfrac{v}{\sqrt{3}}$

56. Charge Q is given a displacement $\vec{r} = a\hat{i} + b\hat{j}$ in an electric field $\vec{E} = E_1\hat{i} + E_2\hat{j}$. The work done is
a. $Q\,(E_1a + E_2b)$
b. $Q\sqrt{(E_1a)^2 + (E_2b)^2}$
c. $Q\,(E_1 + E_2)\sqrt{a^2+b^2}$
d. $Q\sqrt{(E_1^2 + E_2^2)^2}\sqrt{a^2+b^2}$

57. There are two thin wire rings, each of radius R, whose axes coincide. The charges on the rings are q and $-q$. Evaluate the potential difference between the centers of the rings separated by a distance a.
a. $\dfrac{q}{\pi\varepsilon_0}\left[\dfrac{1}{R}+\dfrac{1}{\sqrt{R^2+a^2}}\right]$

b. $\frac{q}{2\pi\varepsilon_0}\left[\frac{1}{R}+\frac{1}{\sqrt{R^2+a^2}}\right]$

c. $\frac{q}{2\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+a^2}}\right]$

d. $\frac{2q}{\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+a^2}}\right]$

58. Let V_0 be the potential at the origin in an electric field $\vec{E}=E_x\hat{i}+E_y\hat{j}$. The potential at the point (x, y) is

a. $V_0-xE_x-yE_y$

b. $V_0+xE_x+yE_y$

c. $xE_x+yE_y-V_0$

d. $(\sqrt{x^2+y^2})\sqrt{E_x^2+E_y^2}-V_0$

59. A point charge q is placed inside a conducting spherical shell of inner radius $2R$ and outer radius $3R$ at a distance of R from the center of the shell. Find the electric potential at the center of the shell.

a. $\frac{1}{4\pi\varepsilon_0}\frac{q}{2R}$ **b.** $\frac{1}{4\pi\varepsilon_0}\frac{4q}{3R}$

c. $\frac{1}{4\pi\varepsilon_0}\frac{5q}{6R}$ **d.** $\frac{1}{4\pi\varepsilon_0}\frac{2q}{3R}$

60. An electric field is expressed as $E=2\hat{i}+3\hat{j}$. Find the potential difference (V_A-V_B) between two points A and B whose position vectors are given by $r_A=\hat{i}+2\hat{j}$ and $r_B=2\hat{i}+\hat{j}+3\hat{k}$.

a. –1 V **b.** 1 V **c.** 2 V **d.** 3 V

61. The potential function of an electrostatic field is given by $V=2x^2$. Determine the electric field strength at the point (2 m, 0, 3 m).

a. $\vec{E}=4\hat{i}\ (\text{NC}^{-1})$ **b.** $\vec{E}=-4\hat{i}\ (\text{NC}^{-1})$

c. $\vec{E}=8\hat{i}\ (\text{NC}^{-1})$ **d.** $\vec{E}=-8\hat{i}\ (\text{NC}^{-1})$

62. Electric field represented by equipotential surface as shown in Fig. 3.83 is

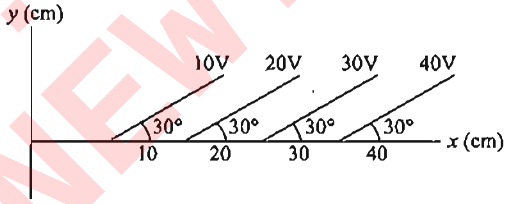


Fig. 3.83

a. $\vec{E}=200(\hat{i}+\sqrt{3}\hat{j})\text{NC}^{-1}$

b. $\vec{E}=100(\hat{i}+\sqrt{2}\hat{j})\text{NC}^{-1}$

c. $\vec{E}=100(-\hat{i}+\sqrt{3}\hat{j})\text{NC}^{-1}$

d. $\vec{E}=200(-\hat{i}+\sqrt{3}\hat{j})\text{NC}^{-1}$

63. Fig. 3.84 shows equipotential surfaces concentric at O, the magnitude of electric field at a distance r measured from O is

a. $\frac{9}{r^2}(\text{Vm}^{-1})$ **b.** $\frac{6}{r^2}(\text{Vm}^{-1})$

c. $\frac{2}{r^2}(\text{Vm}^{-2})$ **d.** $\frac{16}{r^2}(\text{Vm}^{-2})$

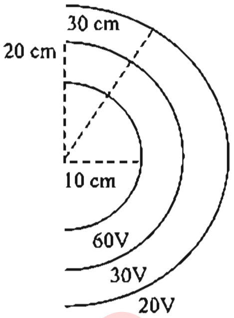


Fig. 3.84

64. A conducting sphere A of radius a, with charge Q, is placed concentrically inside a conducting shell B of radius b. B is earthed. C is the common center of A and B. Study the following statements.

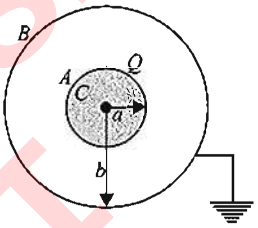


Fig. 3.85

i. The potential at a distance r from C, where $a\le r\le b$, is $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

ii. The potential difference between A and B is $\frac{1}{4\pi\varepsilon_0}Q\left(\frac{1}{a}-\frac{1}{b}\right)$

iii. The potential at a distance r from C, where $a\le r\le b$, is $\frac{1}{4\pi\varepsilon_0}Q\left(\frac{1}{r}-\frac{1}{b}\right)$

Which of the following statements are correct?

a. Only (i) and (ii) **b.** Only (ii) and (iii)

c. Only (i) and (iii) **d.** All

65. An electron having charge e and mass m starts from lower plate of two metallic plates separated by a distance d. If potential difference between the plates is V, the time taken by the electron to reach the upper plate is given by

a. $\sqrt{\frac{2md^2}{eV}}$ **b.** $\sqrt{\frac{md^2}{eV}}$

c. $\sqrt{\frac{md^2}{2eV}}$ **d.** $\frac{2md^2}{eV}$

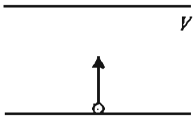


Fig. 3.86

66. There is an infinite straight chain of alternating charges q and $-q$. The distance between the two neighboring charges is equal to a. Find the interaction energy of any charge with all the other charges.

a. $-\frac{2q^2}{4\pi\varepsilon_0 a}$ **b.** $\frac{2q^2\log_e 2}{4\pi\varepsilon_0 a}$

c. $-\dfrac{2q^2 \log_e 2}{4\pi\varepsilon_0 a}$ d. None of these

67. The electric field in a certain region is A/x^3. Then, the potential at a point (x, y, z), assuming potential at infinity to be zero, is

a. zero b. $A/2x^2$
c. $3A/x^4$ d. A/x^2

68. Three identical metallic uncharged spheres A, B and C each of radius a, are kept at the corners of an equilateral triangle of side $d(d >> a)$ as shown in Fig. 3.87. The fourth sphere (of radius a), which has a charge q, touches A and is then removed to a position far away. B is earthed and then the earth connection is removed. C is then earthed. The charge on C is

a. $\dfrac{qa}{2d}\left(\dfrac{2d-a}{2d}\right)$ b. $\dfrac{qa}{2d}\left(\dfrac{2d-a}{d}\right)$
c. $-\dfrac{qa}{2d}\left(\dfrac{d-a}{d}\right)$ d. $\dfrac{2qa}{d}\left(\dfrac{d-a}{2d}\right)$

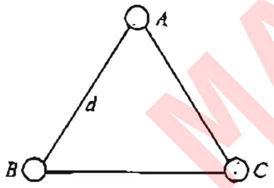


Fig. 3.87

69. A solid conducting sphere of radius 10 cm is enclosed by a thin metallic shell of radius 20 cm. A charge $q = 20\ \mu C$ is given to the inner sphere. Find the heat generated in the process when the inner sphere is connected to the shell by a conducting wire.

a. 12 J b. 9 J
c. 24 J d. zero

70. If V and u are electric potential and energy density, respectively, at a distance r from a positive point charge, then which of the following graph is correct.

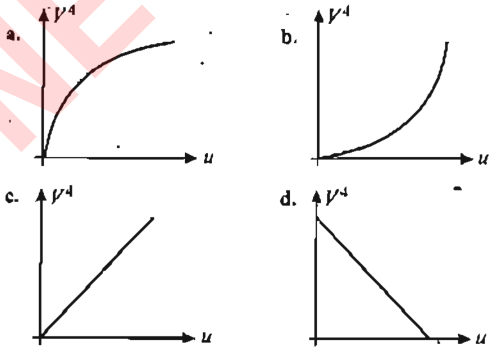


Fig. 3.88

71. Find the potential V of an electrostatic field $\vec{E} = a(y\hat{i} + x\hat{j})$, where a is a constant.

a. $axy + C$ b. $-axy + C$
c. axy d. $-axy$

72. We have three identical metallic spheres A, B, and C. A is given a charge Q and B and C are uncharged. The following processes of touching of two spheres is carried out in succession. Each process is carried out with sufficient time:

i. A and B ii. B and C
iii. C and A iv. A and B v. B and C

The final charges on the spheres are

a. $\dfrac{11Q}{32}, \dfrac{5Q}{16}, \dfrac{11Q}{32}$ b. $\dfrac{11Q}{32}, \dfrac{11Q}{32}, \dfrac{5Q}{16}$
c. $\dfrac{8Q}{8}, \dfrac{5Q}{16}, \dfrac{5Q}{16}$ d. $\dfrac{5Q}{16}, \dfrac{11Q}{32}, \dfrac{11Q}{32}$

73. The potential field depends on x- and y-coordinates as $V = x^2 - y^2$. Corresponding electric field lines in x–y plane are as

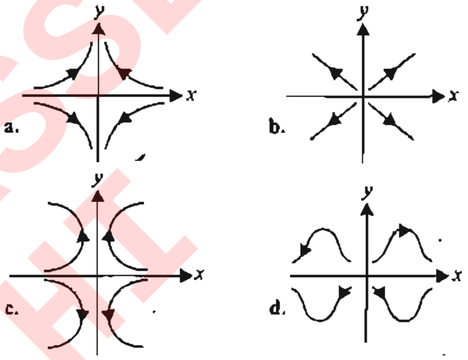


Fig. 3.89

74. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. The potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell is V. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surfaces is **(IIT-JEE, 1989)**

a. V b. $2V$ c. $4V$ d. $-2V$

Multiple Correct Answers Type

Solutions on page 3.43

1. Two infinite, parallel, non-conducting sheets carry equal positive charge density σ. One is placed in the y–z plane and the other at distance $x = a$. Take potential $V = 0$ at $x = 0$. Then:

a. For $0 \le x \le a$, potential $V_x = 0$
b. For $x \ge a$, potential $V_x = \dfrac{\sigma}{\varepsilon_0}(x - a)$
c. For $x \ge a$, potential $V_x = -\dfrac{\sigma}{\varepsilon_0}(x - a)$
d. For $x \le 0$, potential $V_x = \dfrac{\sigma}{\varepsilon_0}x$

2. A negative charge is moved by an external agent in the direction of electric field. Then:

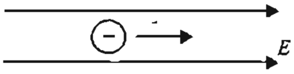


Fig. 3.90

a. potential energy of the charge increases
b. potential energy of the charge decreases
c. positive work is done by the electric field
d. negative work is done by the electric field

3. If a charged conductor is enclosed by a hollow charged conducting shell (assumed concentric and spherical in shape), and they are connected by a conducting wire, then which of the following statement(s) would be correct?
   a. Potential difference between two conductors becomes zero.
   b. If charge on inner conductor is q and on outer conductor is $2q$, then finally charge on outer conductor will be $3q$.
   c. The charge on the inner conductor is totally transferred to the outer conductor.
   d. If charge on the inner conductor is q and charge on the outer conductor is zero, then finally charge on each conductor will be $q/2$.

4. For the situation shown in Fig. 3.91, mark out the correct statement(s).

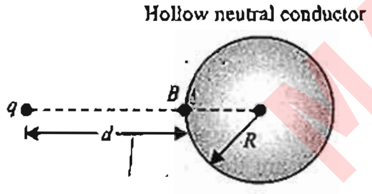


Fig. 3.91

   a. Potential of the conductor is $\dfrac{q}{4\pi\varepsilon_0(d+R)}$
   b. Potential of the conductor is $\dfrac{q}{4\pi\varepsilon_0 d}$
   c. Potential of the conductor cannot be determined as nature of distribution of induced charges is not known
   d. Potential at point B due to induced charges is $\dfrac{-qR}{4\pi\varepsilon_0(d+R)d}$

5. A spherical shell is uniformly charged by a charge q. A point charge q_0 is placed at its center. The expansion of the shell is taking place from R_1 to R_2 ($R_2 > R_1$). For this situation, mark out the correct statement(s).
   a. If an external force is acting, then work done by the external agent is negative.
   b. If no external force is acting, then energy would be released in this expansion.
   c. If no external force is acting, then energy would be dissipated in this process.
   d. The work of electric forces in this process is $\dfrac{q(q_0+q/2)}{4\pi\varepsilon_0}\left[\dfrac{1}{R_1}-\dfrac{1}{R_2}\right]$.

6. A conductor A is given a charge of amount $+Q$ and then placed inside a deep metal can B, without touching it. Then
   a. The potential of A does not change when it is placed inside B
   b. If B is earthed, $+Q$ amount of charge flows from it into the earth
   c. If B is earthed, the potential of A is reduced

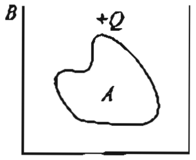


Fig. 3.92

   d. Either (b) or (c) are true, or both are true only if the outer surface of B is connected to the earth and not its inner surface

Assertion-Reasoning Type

Solutions on page 3.43

In the following questions, each question contains Statement I (Assertion) and Statement II (Reason). Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
b. Statement I is True, Statement II is True; Statement II is **NOT** a correct explanation for Statement I.
c. Statement I is True, Statement II is False.
d. Statement I is False, Statement II is True.

1. **Statement I:** Positive charge always moves from a higher potential point to a lower potential point if left free in electric field.
   Statement II: Electric potential is a vector quantity.
2. **Statement I:** The surface of a conductor is an equipotential surface.
   Statement II: Conductors allow the free flow of charge within themselves.
3. **Statement I:** Conductors having equal positive charge and volume must also have same potential.
   Statement II: Potential depends only on the charge and volume and shape of conductor.
4. **Statement I:** No work is done in taking a small positive charge from one point to other inside a positively charged metallic sphere while outside the sphere work is done in taking the charge towards the sphere. Neglect induction due to small charge.
   Statement II: Inside the sphere electric potential is same at each point, but outside it is different for different points.
5. **Statement I:** Electric potential of earth is taken to be zero as a reference.
   Statement II: The electric field produced by earth in surrounding space is zero.

Comprehension Type

Solutions on page 3.44

For Problems 1–2

A single positive point charge q is located at point P as the potential is V_0 (with $V_0 = 0$ at infinity) (see Fig. 3.93).

1. A second charge $q' = +q$ is placed equidistant from P. The potential at P is now
   a. $4V_0$ b. $2V_0$ c. $\sqrt{2}\,V_0$ d. $\dfrac{V_0}{2}$

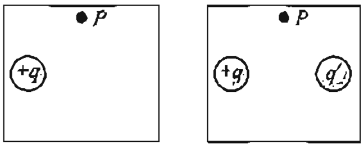


Fig. 3.93

2. Instead of positive charge, a negative charge $q' = -q$ is located. The potential at P is now

a. $2V_0$ b. $\sqrt{2}\,V_0$ c. $\dfrac{V_0}{2}$ d. 0

For Problems 3–4

Two point charges are located on the x-axis, $q_1 = -e$ at $x = 0$ and $q_2 = +e$ at $x = a$.

3. Find the work that must be done by an external force to bring a third point charge $q_3 = +e$ from infinity to $x = 2a$.

a. $\dfrac{e^2}{4\pi\varepsilon_0 a}$ b. $\dfrac{e^2}{8\pi\varepsilon_0 a}$

c. $\dfrac{-e^2}{8\pi\varepsilon_0 a}$ d. $\dfrac{-e^2}{4\pi\varepsilon_0 a}$

4. Find the total potential energy of the system of three charges.

a. $\dfrac{e^2}{4\pi\varepsilon_0 a}$ b. $\dfrac{e^2}{8\pi\varepsilon_0 a}$

c. $\dfrac{-e^2}{8\pi\varepsilon_0 a}$ d. $\dfrac{-e^2}{4\pi\varepsilon_0 a}$

For Problems 5–6

A small metal sphere, carrying a net charge of $q_1 = -2\ \mu\text{C}$, is held in a stationary position by insulating supports. A second small metal sphere, with a net charge of $q_2 = -8\ \mu\text{C}$ and mass 1.50 g, is projected toward q_1. When the two spheres are 0.800 m apart, q_2 is moving toward q_1 with speed 20 ms^{-1} as shown in Fig. 3.94. Assume that the two spheres can be treated as point charges. You can ignore the force of gravity.

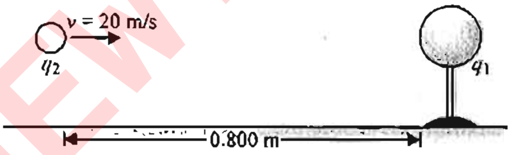


Fig. 3.94

5. What is the speed of q_2 when the spheres are 0.400 m apart?

a. $2\sqrt{10}\,\text{ms}^{-1}$ b. $2\sqrt{6}\,\text{ms}^{-1}$

c. $4\sqrt{10}\,\text{ms}^{-1}$ d. $4\sqrt{6}\,\text{ms}^{-1}$

6. How close does q_2 get to q_1?

a. 0.20 m b. 0.30 m

c. 0.10 m d. 0.15 m

For Problems 7–9

Two point charges $q_1 = +2.40$ nC and $q_2 = -6.50$ nC are 0.100 m apart. Point A is midway between them; point B is 0.080 m from q_1 and 0.060 m from q_2 as shown in Fig. 3.95. Take the electric potential to be zero at infinity. Find

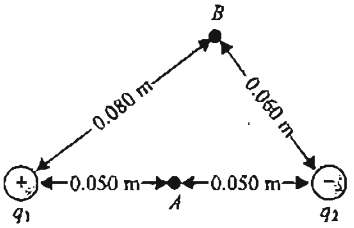


Fig. 3.95

7. the potential at point A

a. −738 V b. −323 V

c. −705 V d. −120 V

8. the potential at point B

a. −738 V b. −323 V

c. −705 V d. −120 V

9. The work done by the electric field on a charge of 2.50 nC that travels from point B to point A.

a. -8.25×10^{-8} J b. 8.25×10^{-8} J

c. 1.25×10^{8} J d. -1.25×10^{-8} J

For Problems 10–13

Four charges $+q$, $+q$, $-q$ and $-q$ are placed, respectively, at the corners A, B, C and D of a square of side a, arranged in the given order. E and F are the midpoints of sides BC and CD, respectively. O is the center of square.

10. The electric field at O is

a. $\dfrac{q}{\sqrt{2}\pi\varepsilon_0 a^2}$ b. $\dfrac{q}{\sqrt{3}\pi\varepsilon_0 a^2}$

c. $\dfrac{\sqrt{3}q}{\pi\varepsilon_0 a^2}$ d. $\dfrac{\sqrt{2}q}{\pi\varepsilon_0 a^2}$

11. The electric potential at O is

a. $\dfrac{\sqrt{2}q}{\pi\varepsilon_0 a}$ b. $\dfrac{\sqrt{3}q}{\pi\varepsilon_0 a}$

c. $\dfrac{q}{\pi\varepsilon_0 a}$ d. zero

12. The work done in carrying a charge e from O to E is

a. $\dfrac{\sqrt{2}qe}{\pi\varepsilon_0 a}$ b. $\dfrac{qe}{\pi\varepsilon_0 a}\left[\dfrac{1}{\sqrt{5}} - 1\right]$

c. $\dfrac{qe}{\pi\varepsilon_0 a}\left[\dfrac{1}{\sqrt{5}} + 1\right]$ d. zero

13. The work done in carrying a charge e from O to F is

a. $\dfrac{\sqrt{2}qe}{\pi\varepsilon_0 a}$ b. $\dfrac{qe}{\pi\varepsilon_0 a}\left[\dfrac{1}{\sqrt{5}} - 1\right]$

c. $\dfrac{qe}{\pi\varepsilon_0 a}\left[\dfrac{1}{\sqrt{5}} + 1\right]$ d. zero

For Problems 14–15

Two fixed point charges, each having charge Q, are separated by a distance $2l$. Another point charge $-q$ having the mass m is projected with a velocity v_0 from a point midway between the two charges along the perpendicular bisector of the line joining the two charges.

14. The electric field at a point situated on perpendicular bisector of the line joining the two charges at a distance x from the mid-point of the line joining the two charges.

a. $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Qx}{(x^2+l^2)^{3/2}}$ **b.** $\dfrac{1}{2\pi\varepsilon_0}\dfrac{Qx}{(x^2+l^2)^{3/2}}$

c. $\dfrac{1}{2\sqrt{2}\pi\varepsilon_0}\dfrac{Qx}{(x^2+l^2)^{3/2}}$ **d.** $\dfrac{2\sqrt{2}}{\pi\varepsilon_0}\dfrac{Qx}{(x^2+l^2)^{3/2}}$

15. Find the distance travelled by the charge $-q$ before it reverses its direction (neglect other interactions).

a. $l\left[\left(\dfrac{qQ}{qQ+\pi\varepsilon_0 lmv^2}\right)^2-1\right]^{1/2}$

b. $l\left[\left(\dfrac{qQ}{qQ+\pi\varepsilon_0 lmv^2}\right)^2+1\right]^{1/2}$

c. $l\left[\left(\dfrac{qQ}{qQ-\pi\varepsilon_0 lmv^2}\right)^2-1\right]^{1/2}$

d. $l\left[\left(\dfrac{qQ}{qQ-\pi\varepsilon_0 lmv^2}\right)^2+1\right]^{1/2}$

For Problems 16–17

A charge Q is distributed over two concentric hollow spheres of radii r and R ($R > r$) such that then surface densities are equal.

16. The charge on smaller and bigger shells are

a. $\dfrac{Qr^2}{r^2+R^2}$ and $\dfrac{QR^2}{r^2+R^2}$, respectively

b. $Q\left(1+\dfrac{r^2}{R^2}\right)$ and $Q\left(1+\dfrac{R^2}{r^2}\right)$, respectively

c. $Q\left(1-\dfrac{r^2}{R^2}\right)$ and $Q\left(1-\dfrac{R^2}{r^2}\right)$, respectively

d. $\dfrac{QR^2}{r^2+R^2}$ and $\dfrac{Qr^2}{r^2+R^2}$, respectively

17. The potential at the common center is

a. $\dfrac{\sqrt{2}}{\pi\varepsilon_0}\dfrac{Q(R+r)}{(R^2+r^2)}$ **b.** $\dfrac{1}{2\pi\varepsilon_0}\dfrac{Q(R+r)}{(R^2+r^2)}$

c. $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q(R+r)}{(R^2+r^2)}$ **d.** $\dfrac{1}{\pi\varepsilon_0}\dfrac{Q(R-r)}{(R^2+r^2)}$

For Problems 18–20

Suppose that the electric potential varies along the x-axis as shown in the graph of Fig. 3.96. Of the intervals shown (ignore the behaviour at the endpoints of the intervals)

18. The intervals in which the magnitude of electric field in x-direction is maximum are

a. $-4 \le x \le -2$ and $3 \le x \le 5$
b. $-6 \le x \le -4$ and $2 \le x \le 3$
c. $-2 \le x \le 3$ and $5 \le x \le 7$
d. $-6 \le x \le -4$ and $5 \le x \le 7$

19. The intervals in which the magnitude of electric field in x-direction is minimum are

a. $-4 \le x \le -2$ and $3 \le x \le 5$
b. $-6 \le x \le -4$ and $2 \le x \le 3$
c. $-2 \le x \le 3$ and $5 \le x \le 7$
d. $-6 \le x \le -4$ and $5 \le x \le 7$

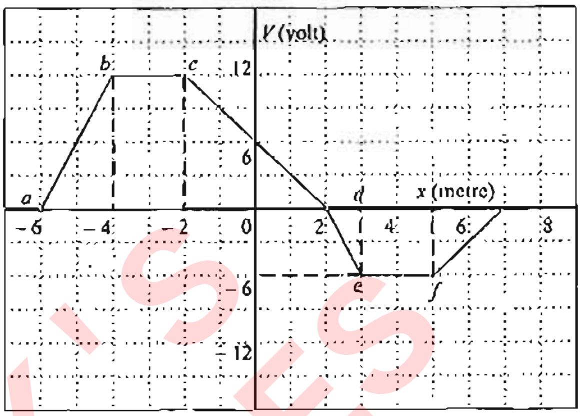
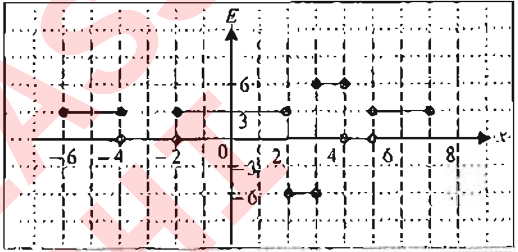


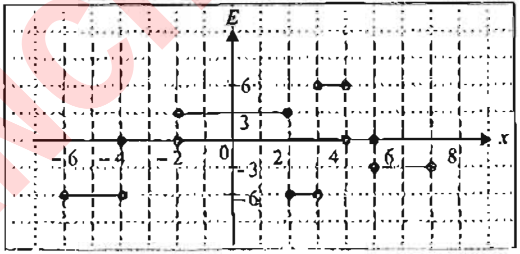
Fig. 3.96

20. The graph between E_x versus x will be

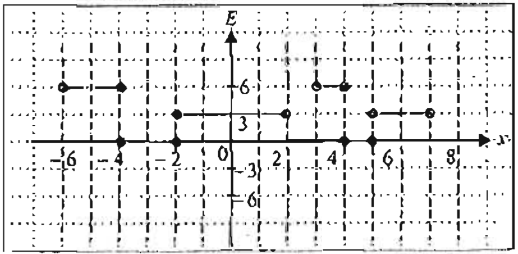
a.



b.



c.



d.

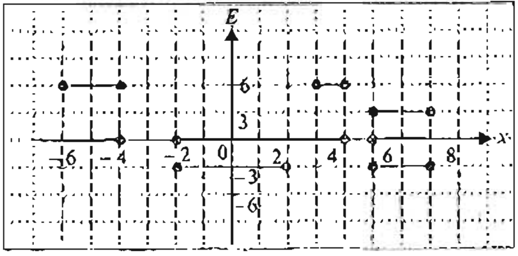


Fig. 3.97

For Problems 21–23

In moving from A to B along an electric field line, the electric field does 4.8×10^{-19} J of work on an electron in the field illustrated in Fig. 3.98. What are the differences in the electric potential?

21. The potential difference $V_B - V_A$ is

a. 3.0 V **b.** −3.0 V **c.** 2 V **d.** zero

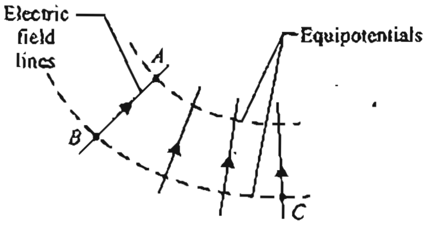


Fig. 3.98

22. The potential difference $V_C - V_A$ is
    a. 3.0 V
    b. –3.0 V
    c. 2 V
    d. zero

23. The potential difference $V_C - V_B$ is
    a. 3.0 V
    b. –3.0 V
    c. 2 V
    d. zero

For Problems 24–25

Refer the uniform electric field shown in Fig. 3.99.

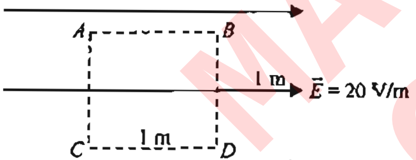


Fig. 3.99

24. The potential difference $V_A - V_C$ is
    a. zero
    b. 20 V
    c. –20 V
    d. $20\sqrt{2}$ V

25. The potential difference $V_A - V_D$ is
    a. zero
    b. 20 V
    c. –20 V
    d. $20\sqrt{2}$ V

For Problems 26–28

The electrical potential function for an electrical field directed parallel to the x-axis is shown in the given graph in Fig. 3.100.

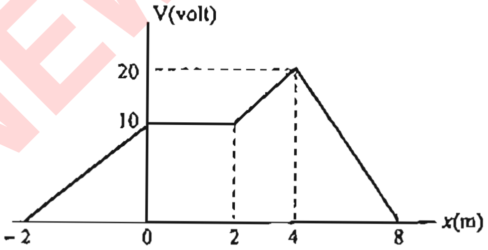
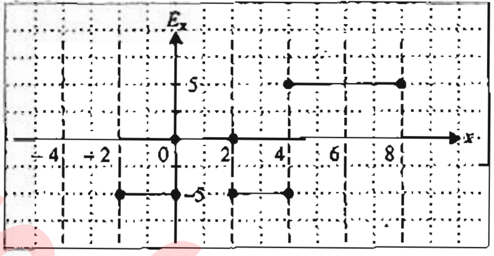


Fig. 3.100

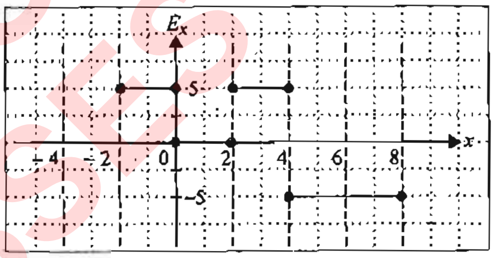
26. The intervals in which the magnitude of electric field in x-direction is maximum are
    a. $-2 \le x \le 0$, $4 \le x \le 8$ and $0 \le x \le 2$
    b. $-2 \le x \le 0$ and $0 \le x \le 2$
    c. $-2 \le x \le 0$, $2 \le x \le 4$ and $4 \le x \le 8$
    d. $0 \le x \le 2$ and $4 \le x \le 8$

27. The magnitude of electric field in x-direction in the interval $4 \le x \le 8$ is
    a. 2.5 NC^{-1}
    b. 5 NC^{-1}
    c. –2.5 NC^{-1}
    d. –5 NC^{-1}

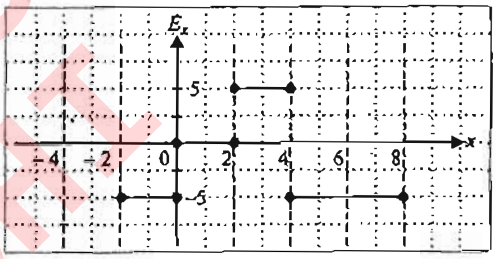
28. The graph between E_x versus x will be



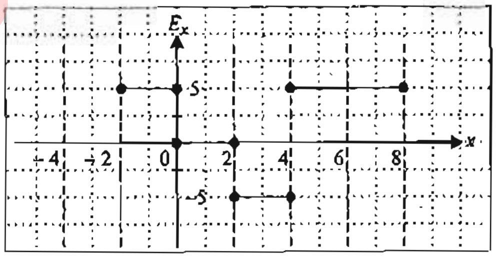
a.



b.



c.



d.

Fig. 3.101

For Problems 29–30

A uniform electric field of 100 Vm^{-1} is directed at 30° with the positive x-axis as shown in Fig. 3.102. $OA = 2$ m and $OB = 4$ m.

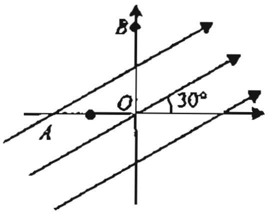


Fig. 3.102

29. The potential difference $V_O - V_A$ is
    a. $100\sqrt{3}$ V
    b. $200\sqrt{3}$ V
    c. $-100\sqrt{3}$ V
    d. $-200\sqrt{3}$ V

30. The potential difference $V_B - V_A$ is
    a. $-100\left[2+\sqrt{3}\right]$ V
    b. $100\left[2+\sqrt{3}\right]$ V
    c. $100\left[2-\sqrt{3}\right]$ V
    d. $-100\left[2-\sqrt{3}\right]$ V

For Problems 31–34

The electric potential varies in space according to the relation $V = 3x + 4y$. A particle of mass 0.1 kg starts from rest from point (2, 3.2) under the influence of this field. The charge on the particle is +1 μC. Assume V and (x, y) are in S.I. units.

31. The component of electric field in x-direction (E_x) is

a. 3 $\mathrm{Vm^{-1}}$ **b.** 4 $\mathrm{Vm^{-1}}$

c. 5 $\mathrm{Vm^{-1}}$ **d.** 8 $\mathrm{Vm^{-1}}$

32. The component of electric field in y-direction (E_y) is

a. 3 $\mathrm{Vm^{-1}}$ **b.** −4 $\mathrm{Vm^{-1}}$

c. 5 $\mathrm{Vm^{-1}}$ **d.** 8 $\mathrm{Vm^{-1}}$

33. The time taken to cross x-axis is

a. 20 s **b.** 40 s

c. 200 s **d.** 400 s

34. The velocity of the particle when it cross the x-axis is

a. $20 \times 10^{-3}\ \mathrm{ms^{-1}}$ **b.** $40 \times 10^{-3}\ \mathrm{ms^{-1}}$

c. $30 \times 10^{-3}\ \mathrm{ms^{-1}}$ **d.** $50 \times 10^{-3}\ \mathrm{ms^{-1}}$

For Problems 35–36

Three concentric spherical metallic shells A, B and C of radii a, b and c $(a < b < c)$ have surface charge densities σ, $-\sigma$ and σ, respectively.

35. If V_A, V_B and V_C are potential of shells A, B and C, respectively, match the columns

Column A	Column B
a. V_A	**i.** $\dfrac{\sigma}{\varepsilon_0}\left[\dfrac{a^2-b^2+c^2}{c}\right]$
b. V_B	**ii.** $\dfrac{\sigma}{\varepsilon_0}\left[\dfrac{a^2}{b}-b+c\right]$
c. V_C	**iii.** $\dfrac{\sigma}{\varepsilon_0}[a-b+c]$

36. If the shells A and C are at the same potential, the relation between the radii a, b and c is

a. $a = b + c$ **b.** $c = a + b$

c. $b = a + c$ **d.** $2a = b - c$

(IIT-JEE, 1990)

For Problems 37–40

We have an isolated conducting spherical shell of radius 10 cm. Some positive charge is given to it so that resulting electric field has a maximum intensity of $1.8 \times 10^6\ \mathrm{NC^{-1}}$. The same amount of negative charge is given to another isolated conducting spherical shell of radius 20 cm. Now, first shell is placed inside the second so that both are concentric as shown in Fig. 3.103. Now, answer the following questions.

37. What is electric potential at any point inside the first shell?

a. 18×10^4 V **b.** 9×10^4 V

c. 4.5×10^4 V **d.** 1.8×10^4 V

38. What is electric field intensity just inside the outer sphere?

a. 4.5×10^5 N/C **b.** 9×10^5 N/C

c. 4.5×10^4 N/C **d.** 5×10^4 N/C

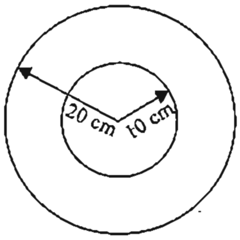


Fig. 3.103

39. What is the electrostatic energy stored in the system?

a. 1.0 J **b.** 0.045 J

c. 0.09 J **d.** 1.8 J

40. What will happen if both the spheres are connected by a conducting wire?

a. Nothing will happen

b. Some part of the energy stored in the system will convert into heat

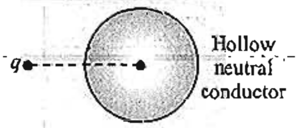
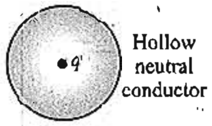
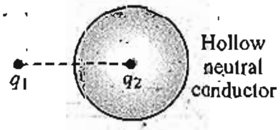
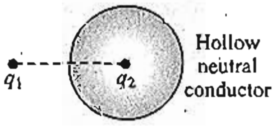
c. Charge on both spheres will be positive

d. Entire amount of the energy stored in the system will convert into heat

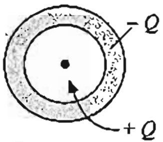
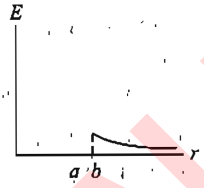
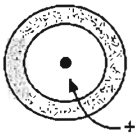
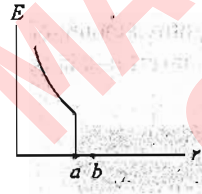
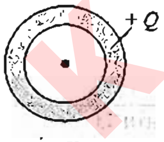
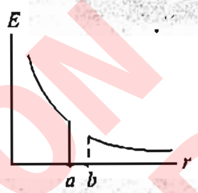
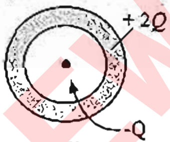
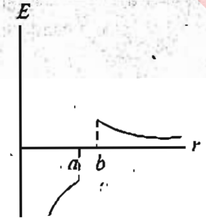
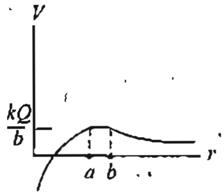
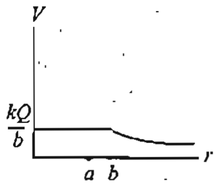
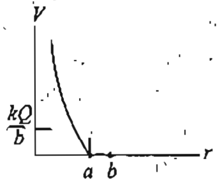
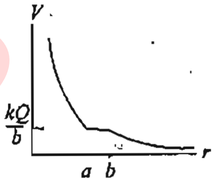
Matching Column Type

Solutions on page 3.46

1. Match the entries of Column I with entries of Column II.

Column I	Column II
i. 	**a.** $\vec{E}$ inside the conductor is zero.
ii. 	**b.** $\lvert\vec{E}\rvert$ inside the conductor is constant but not zero.
iii. 	**c.** $\lvert\vec{E}\rvert$ inside the conductor is varying.
iv. 	**e.** Potential inside the conductor is same as that of conductor.
	d. Potential inside the conductor is varying.

2. Column I shows four hollow metal spheres each with internal radius a and external radius b. You have to match these charge distributions with its corresponding E-graph and V-graph in Column II. A point charge of $+Q$ or $-Q$ is present at the center of the spheres (in figure (iii), there is no charge at the center of sphere). The charge indicated on the spherical shell itself is the net charge on the shell, that is, any induced charge distribution is not shown. Also, the label for the net charge on a conducting sphere does not necessarily indicate the actual position of the charge on or within the conductor.

Column I	Column II
i.  E ___ V ___	**a.** 
ii.  E ___ V ___	**b.** 
iii.  E ___ V ___	**c.** 
iv.  E ___ V ___	**d.** 
	e. 
	f. 
	g. 
	h. 

3. Fig. 3.104 shows three concentric thin spherical shells A, B and C of radii R, $2R$ and $3R$. The shell B is earthed and A and C are given charges q and $2q$, respectively. If the charge appearing on surfaces 1, 2, 3 and 4 are q_1, q_2, q_3, and q_4, respectivey, then match the following columns:

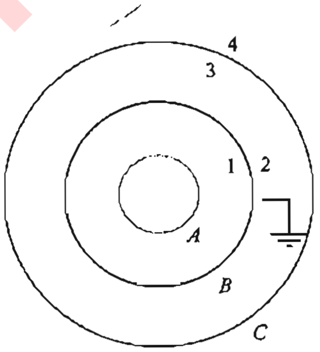


Fig. 3.104

Column I	Column II
i. q_1	**a.** $\frac{2}{3}q$
ii. q_2	**b.** $\frac{4}{3}q$
iii. q_3	**c.** $-\frac{4}{3}q$
iv. q_4	**d.** $-q$

ANSWERS AND SOLUTIONS

Subjective Type

1. $$E_x = -\frac{dV}{dx} = \frac{4-2}{(6-4)\,10^{-2}} = -100 \text{ Vm}^{-1}$$

$$E_y = -\frac{dV}{dy} = \frac{2-4}{(2-1)\,10^{-2}} = 200 \text{ Vm}^{-1}$$

2.

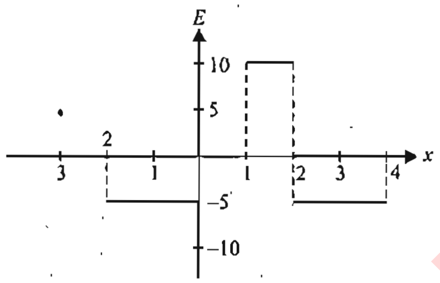


Fig. 3.105

3. Length of cube diagonal = $\sqrt{3}\,b$
Distance of center of cube from each corner,

$$r = \frac{\sqrt{3}}{2}b$$

Total P.E. of charge q at the center

$$= \frac{8q(-q)}{4\pi\varepsilon_0 r} = \frac{-8q^2}{4\pi\varepsilon_0\sqrt{3}(b/2)} = \frac{-4q^2}{\pi\varepsilon_0\sqrt{3}b}$$

4. If r is the distance of the charge Q from the point of observation, then

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = 32 \qquad \text{(i)}$$

and $$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = 16 \qquad \text{(ii)}$$

Dividing equation (ii) by equation (i): $r = 0.5$ m
Putting value of r in equation (ii), we get

$$Q = 16 \times 0.5(9 \times 10^9) = (8/9) \times 10^{-9} \text{ C}$$

5. Let U_I be the potential energy of system I and U_{II} be the potential energy of system II.
Work done = Change in P.E. = $U_{II} - U_I$.

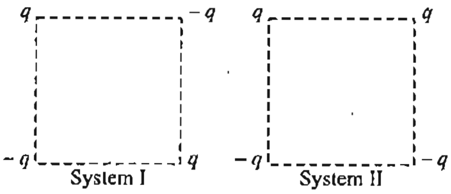


Fig. 3.106

$$U_I = -\frac{4q^2}{4\pi\varepsilon_0 a} + \frac{2q^2}{4\pi\varepsilon_0\sqrt{2}a};$$

$$U_{II} = \frac{-2q^2}{4\pi\varepsilon_0\sqrt{2}a}$$

$$\therefore \quad W = \frac{-2q^2}{4\pi\varepsilon_0\sqrt{2}a} + \frac{4q^2}{4\pi\varepsilon_0 a} - \frac{2q^2}{4\pi\varepsilon_0\sqrt{2}a}$$

$$W = \frac{q^2}{4\pi\varepsilon_0 a}\left(\frac{-4}{\sqrt{2}} + 4\right);$$

$$W = \frac{q^2}{4\pi\varepsilon_0 a}(4 - 2\sqrt{2})$$

6. Potential of a drop, $V = \dfrac{1}{4\pi\varepsilon_0\varepsilon_r}\dfrac{q}{r}$

$$q = 4\pi\varepsilon_0\varepsilon_r r\, V$$

∴ Total charge $Q = nq = 4\pi\varepsilon_0\varepsilon_r r V n$
(n = total no. of drops)

Also, $\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3 \Rightarrow n = \frac{R^3}{r^3}$

Hence, potential of hollow conductor,

$$V' = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Q}{R} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \times \frac{4\pi\varepsilon_0\varepsilon_r r V n}{R}$$

$$= \frac{r}{R} \times v \times \left(\frac{R^3}{r^3}\right)$$

$$\Rightarrow V' = \left(\frac{R}{r}\right)^2 V$$

7. As potential energy of two point charges separated by a distance r is given by $U = (q_1 q_2/4\pi\varepsilon_0 r)$, the initial and final potential energies of the system will be

$$(U_S)_I = 3 \times \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r} = 3 \times 9 \times 10^9 \times \frac{(0.1)^2}{1} = 2.7 \times 10^8 \text{ J}$$

$$(U_S)_F = 2 \times \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(r/2)} + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r} = 5 \times \frac{q^2}{4\pi\varepsilon_0 r} = 4.5 \times 10^8 \text{ J}$$

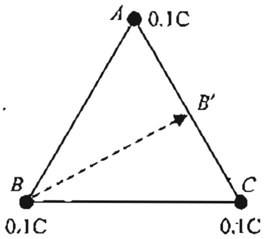


Fig. 3.107

So, work done in changing the configuration of the system

$$W = (U_S)_F - (U_S)_I$$

i.e., $W = (4.5 - 2.7) \times 10^8$ J
i.e., $W = 1.8 \times 10^8$ J
Now, as energy is supplied at the rate of 1 kW, i.e., 10^3 Js^{-1}, time required to do work W is given by

$$t = \frac{W}{P} = \frac{1.8 \times 10^8}{10^3} = 1.8 \times 10^5 \text{s} = 50 \,\text{h}$$

8. $v = \sqrt{\dfrac{2l\sigma q_0}{m\,\varepsilon_0}}$

9. $V_P = V_{\text{big, in}} + 2V_{\text{small, out}} = \dfrac{V_S}{2}\left(1 - \dfrac{r_1^2}{R^2}\right) + 2\dfrac{1}{4\pi\varepsilon_0}\dfrac{(-Q')}{r'}$

$$V_S = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{big}}}{R} = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{4}{3}\pi R^3 \rho\right)}{R} = \frac{\rho R^2}{3\varepsilon_0}$$

$$V_{\text{big, in}} = \frac{\rho R^2}{3\varepsilon_0}\left[1 - \frac{r_1^2}{R^2}\right] = \frac{1}{\pi}\frac{(5 \times 10^{-2})^2}{3\varepsilon_0}\left[1 - \left(\frac{4}{5}\right)^2\right] \times 10^{-6}$$

$$V_{\text{big, in}} = \frac{3}{\pi\varepsilon_0} \times 10^{-10}\ \text{V}$$

$$V_{\text{small, out}} = -2.\frac{1}{4\pi\varepsilon_0}\frac{\rho\frac{4}{3}\pi r^3}{r'}$$

$$= -\frac{2\rho r^3}{3\varepsilon_0 r'} = -\frac{2.1\,(1 \times 10^{-2})^3}{\pi 3\varepsilon_0 \times 5 \times 10^{-2}} \times 10^{-6}$$

$$= -\frac{2}{15\pi\varepsilon_0} \times 10^{-10}\ \text{V}$$

Hence, $V_P = \left[\dfrac{3}{\pi\varepsilon_0} - \dfrac{2}{15\pi\varepsilon_0}\right] \times 10^{-10}$

$$= \frac{43}{45\pi\varepsilon_0} \times 10^{-10}\ \text{V}$$

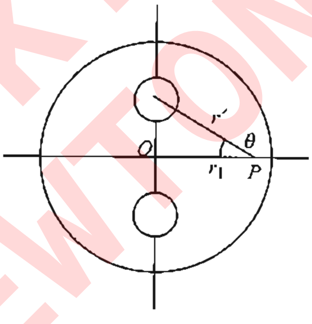


Fig. 3.108

Electric field at $P = \vec{E}_P = \vec{E}_{\text{big, in}} + 2\vec{E}_{\text{small, out}}$

$$\bar{E}_P = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{big}}}{R^3}r_1 - 2\frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{small}}}{r'^2}\cos\theta$$

$$= \frac{4 \times 10^{-8}}{3\pi\varepsilon_0} - \frac{4 \times 10^{-8}}{275\pi\varepsilon_0}$$

$$|E_P| = \frac{272 \times 4 \times 10^{-8}}{275 \times 3\pi\varepsilon_0}\ \text{Vm}^{-1}$$

0. Work done = charge × difference in potential

Let the particle moves from O to O' (Fig. 3.109)

∴ Potential at point O is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}R}\right]$$

and potential at point O' is

$$V' = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}R}\right]$$

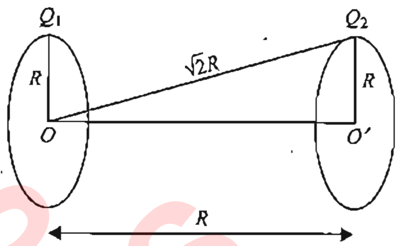


Fig. 3.109

Difference in potential

$$\Delta V = V - V'$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}R}\right] - \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}R}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{R}(Q_1 - Q_2) + \frac{1}{\sqrt{2}R}(Q_2 - Q_1)\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1 - Q_2}{R}\left(1 - \frac{1}{\sqrt{2}}\right)\right]$$

∴ W.d. $= q\Delta V = q\dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{(Q_1 - Q_2)}{\sqrt{2}R}(\sqrt{2} - 1)\right]$

$$= q\,(Q_1 - Q_2)(\sqrt{2} - 1)/\sqrt{2}R \times 4\pi\varepsilon_0$$

11. The potential of B should be zero (Fig. 3.110).

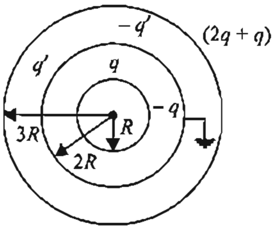


Fig. 3.110

$$v_B = 0; \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{2R} - \frac{q}{2R} + \frac{q'}{2R} - \frac{q'}{3R} + \frac{2q + q'}{3R}\right] = 0$$

12. a. Potential of shell A,

$$V_A = V_{\text{due to charge on A}} + V_{\text{due to charge on B}} + V_{\text{due to charge on C}}$$

$$= 0 + \frac{Q}{4\pi\varepsilon_0 b} - \frac{Q}{4\pi\varepsilon_0 c}$$

Potential of shell B,

$$V_B = V_{\text{due to charge on A}} + V_{\text{due to charge on B}} + V_{\text{due to charge on C}}$$

$$= 0 + \frac{Q}{4\pi\varepsilon_0 b} - \frac{Q}{4\pi\varepsilon_0 c}$$

Potential of shell C,

$$V_C = V_{\text{due to charge on A}} + V_{\text{due to charge on B}} + V_{\text{due to charge on C}}$$

$$= 0 + \frac{Q}{4\pi\varepsilon_0 c} - \frac{Q}{4\pi\varepsilon_0 c} = 0$$

b. Let the charge on inner shell be q', after inner and outer shells are connected by a conducting wire. The final charge distribution is shown in Fig. 3.111. The wire connection equalises the potential of the two shells.

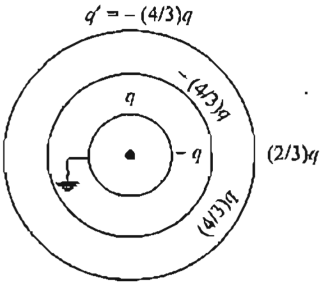


Fig. 3.111

$$V_A = V_{\text{due to charge on A}} + V_{\text{due to charge on B}} + V_{\text{due to charge on C}}$$

$$= \frac{q'}{4\pi\varepsilon_0 a} + \frac{Q}{4\pi\varepsilon_0 b} - \frac{Q}{4\pi\varepsilon_0 c} \quad \text{(i)}$$

Similarly, $V_C = V_{\text{due to charge on A}} + V_{\text{due to charge on B}}$

$$+ V_{\text{due to charge on C}}$$

$$\frac{q'}{4\pi\varepsilon_0 a} + \frac{Q}{4\pi\varepsilon_0 b} = \frac{q'}{4\pi\varepsilon_0 c} + \frac{Q}{4\pi\varepsilon_0 c} \quad \text{(ii)}$$

$$\text{or } q' = -\frac{Qa}{b}\left[\frac{c-b}{c-a}\right]$$

Charge on B, $Q_b = Q - q' + q' = Q$ [shell B is isolated and sum of induced charges is zero]

Charge on C,

$$Q_c = -Q + q' = -Q + \frac{Qa}{b}\left[\frac{c-b}{c-a}\right] = \frac{Qc}{b}\left[\frac{c-b}{c-a}\right]$$

If charge q' appears on the inner shell, an equal magnitude, opposite sign charge must appear on the outer shell in accordance with the law of conservation of charge.

13. a. Let the charges on A and C be q_1 and q_2, respectively (Fig. 3.112). From conservation of charge, we have

$$q_1 + q_2 = 0$$

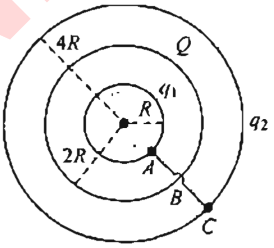


Fig. 3.112

Hence $q_1 = -q_2$

Since A and C are connected by a conducting wire, so they have same potential.

V_A = potential of A

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{4R}$$

V_C = potential of C

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{4R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{4R} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{4R}$$

Equalizing the potentials of A and C, i.e., $V_A = V_C$, we get

$$\frac{q_1}{R} + \frac{Q}{2R} + \frac{q_2}{4R} = \frac{q_1}{4R} + \frac{Q}{4R} + \frac{q_2}{4R}$$

$$\text{or } 4q_1 + 2Q = q_1 + Q$$

$$q_1 = -Q/3$$

Hence, $q_2 = Q/3$.

$$\textbf{b. } V_A = \frac{1}{4\pi\varepsilon_0 R}\left[\frac{-Q}{3} + \frac{Q}{2} + \frac{Q}{12}\right] = \frac{Q}{16\pi\varepsilon_0 R}$$

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{4R}$$

$$= \frac{1}{8\pi\varepsilon_0 R}\left[\frac{-Q}{3} + Q + \frac{Q}{6}\right] = \frac{5Q}{48\pi\varepsilon_0 R}$$

14. When K_1 is closed first time, outer sphere is earthed and the potential on it becomes zero. Let the charge on it be q_1'.

V_1' = Potential due to charge on inner sphere and that due to charge on outer sphere

$$V_1' = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{2R} + \frac{q_1'}{2R}\right] = 0 \text{ or } q_1' = -q$$

When K_2 is closed first time, the potential V_2' on inner sphere becomes zero as it is earthed. Let the new charge on inner sphere be q_2'.

$$0 = \frac{1}{4\pi\varepsilon_0}\frac{q_2'}{R} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{(2R)} \quad \Rightarrow \quad q_2' = \frac{q}{2}$$

Now, when K_1 will be closed second time, charge on outer sphere will be $-q_2'$, i.e., $-q/2$.

After one event involving closure and opening of K_1 and K_2, charge is reduced to half its initial value.

Similarly, when K_1 will be closed n^{th} time, charge on outer sphere will be $-\frac{q}{2^{n-1}}$ as each time charge will be reduced to half the previous value.

After closing K_2 n^{th} time, charge on inner shell will be negative of half the charge on outer shell, i.e., $(+q/2^n)$ and potential on it will be zero.

For potential of outer shell,

$$V_0 = \frac{1}{4\pi\varepsilon_0}\frac{(+q/2^n)}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{(-q/2^{n-1})}{2R}$$

$$V_0 = \frac{-q[-1+2]}{4\pi\varepsilon_0 2^{n+1}R} = \frac{-q}{4\pi\varepsilon_0 2^{n+1}R}$$

$$\text{Potential difference} = V_0 - V_i = \frac{-q}{4\pi\varepsilon_0 2^{n+1}R} - 0 = \frac{-q}{4\pi\varepsilon_0 2^{n+1}R}$$

15. a. Consider the situation Q is a positive charge. The resultant force on q at A will be

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\text{Here, } F_1 = F_2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}$$

$$\text{So, } (\bar{F}_1 + \bar{F}_2) = 2 \times \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\cos 30^\circ = \frac{\sqrt{3}q^2}{4\pi\varepsilon_0 a^2} \text{ along } \overrightarrow{OA}$$

$$\text{While } \vec{F}_3 = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{(a/\sqrt{3})^2} = \frac{3qQ}{4\pi\varepsilon_0 a^2} \text{ along } \overrightarrow{OA}$$

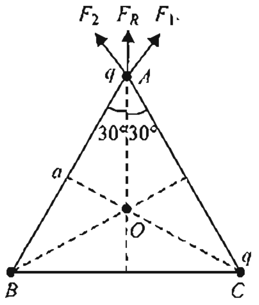


Fig. 3.113

$$\text{So, } F_R = \frac{q}{4\pi\varepsilon_0 a^2}\left[\sqrt{3}q + 3Q\right] \text{ along } \overrightarrow{OA} \qquad (1)$$

b. For $Q = -q$,

$$F_R = \frac{q}{4\pi\varepsilon_0 a^2}\left[\sqrt{3}q - 3q\right] \text{ along } \overrightarrow{OA}$$

$$F_R = \frac{\sqrt{3}q}{4\pi\varepsilon_0 a^2}\left[\sqrt{3} - 1\right] \text{ along } \overrightarrow{AO}$$

i.e., the charges q at corners A, B and C will move towards the center O.

c. Charge will remain stationary if $F_R = 0$, which in the light of equation (1) is possible only if

$$\left[\left(\sqrt{3}\right)q + 3Q\right] = 0, \text{ i.e., } Q = -\left(\frac{q}{\sqrt{3}}\right)$$

d. Potential energy of the system

$$U = \frac{1}{2} \times \frac{1}{4\pi\varepsilon_0}\sum_{i\neq j}\frac{q_i q_j}{r_{ij}} = \frac{1}{4\pi\varepsilon_0}\left[3\frac{q \times q}{a} + \frac{3q\left(-q/\sqrt{3}\right)}{\left(a/\sqrt{3}\right)}\right] = 0$$

And for finite charge distribution potential energy at infinity is always zero.

$$\therefore \quad W = U_F - U_I = 0 - 0 = 0$$

16. If the ball has to just complete the circle then the tension must vanish at the topmost point, i.e., A

From Newton's second law

$$T_2 + mg - \frac{q^2}{4\pi\varepsilon_0 l^2} = \frac{mv^2}{l} \qquad \text{(i)}$$

At the topmost point, $T_2 = 0$

$$\therefore\ mg - \frac{q^2}{4\pi\varepsilon l^2} = \frac{mv^2}{l} \qquad \text{(ii)}$$

From energy conservation

Energy at lowest point = Energy at topmost point

$$\frac{1}{2}mu^2 = \frac{1}{2}mu^2 + mg2l \qquad \text{(iii)}$$

$$v^2 = u^2 - 4gl \qquad \text{(iv)}$$

From equation (ii), $v^2 = gl - \dfrac{q^2}{4\pi\varepsilon_0 ml}$ (v)

From equation (iv) and (v),

$$u = \sqrt{5gl - \frac{q^2}{4\pi\varepsilon_0 ml}} = \left(\frac{275}{8}\right)^{1/2} = 5.86 \text{ ms}^{-1}$$

17. a. Let P be a point in the XY plane with coordinates (x, y) at which the potential due to charges $-2Q$ and $+Q$ placed at A and B, respectively, be zero (Figure 3.114).

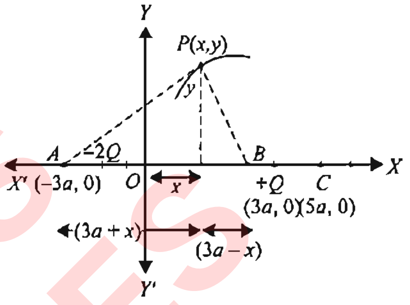


Fig. 3.114

$$\frac{K(-2Q)}{\sqrt{(3a+x)^2 + y^2}} = \frac{K(+Q)}{\sqrt{(3a-x)^2 + y^2}}$$

$$2\sqrt{(3a-x)^2 + y^2} = \sqrt{(3a+x)^2 + y^2}$$

$$\Rightarrow \quad 4\left[(3a-x)^2 + y^2\right] = \left[(3a+x)^2 + y^2\right]$$

$$4\left[9a^2 + x^2 - 6ax + y^2\right] = \left[9a^2 + x^2 + 6ax + y^2\right]$$

$$3x^2 + 3y^2 - 30ax + 27a^2 = 0$$

$$x^2 + y^2 - 10ax + 9a^2 = 0$$

$$(x - 5a)^2 + (y - 0)^2 = (4a)^2$$

This is the equation of a circle with center at $(5a, 0)$ and radius $4a$. Thus, c $(5a, 0)$ is the center of the circle.

b. For $x > 3a$:

To find $V(x)$ at any point on x-axis, let us consider a point (arbitrary) M at a distance x from the origin.

The potential at M will be

$$V(x) = \frac{K(-2Q)}{x + 3a} + \frac{K(+Q)}{(x - 3a)}, \text{ where } k = \frac{1}{4\pi\varepsilon_0}$$

$$V(x) = KQ\left[\frac{1}{x - 3a} - \frac{2}{x + 3a}\right] (\text{For } |x| > 3a)$$

Similarly, for $0 < |x| < 3a$,

$$V(x) = KQ\left[\frac{1}{3a - x} - \frac{2}{3a + x}\right]$$

Since circle of zero potential cuts the x-axis at $(a, 0)$ and $(9a, 0)$, hence $V(x) = 0$ at $x = a$ at $x = 9a$.

From the above expressions

$V(x) \to \infty$ at $x \to 3a$ and $V(x) \to -\infty$ at $x \to -3a$

$V(x) \to 0$ as $x \to \pm\infty \quad \Rightarrow \quad V(x)$ varies as $\dfrac{1}{x}$ in general.

(c) Applying energy conservation, we get

$$(\text{K.E.} + \text{P.E.})_{\text{center}} = (\text{K.E.} + \text{P.E.})_{\text{circumference}}$$

$$0 + K\left[\frac{Qq}{2a} - \frac{2Qq}{8a}\right] = \frac{1}{2}mv^2 + K\left[\frac{Qq}{6a} - \frac{2Qq}{12a}\right]$$

$$\frac{1}{2}mv^2 = \frac{KQq}{4a} \quad \Rightarrow \quad v = \sqrt{\frac{KQq}{2ma}} = \sqrt{\frac{1}{4\pi\varepsilon_0}\left(\frac{Qq}{2ma}\right)}$$

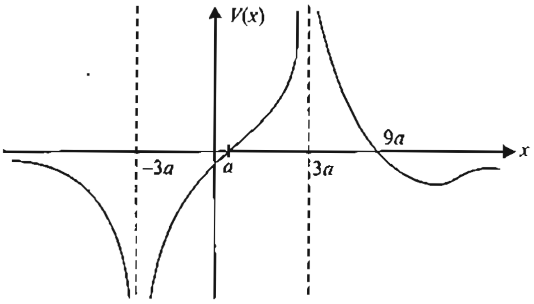


Fig. 3.115

18. Potential at the center will be $\frac{1}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$

Now, total work done in removing the entire charge from O to infinity

$$=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)dq=\frac{1}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)\int_0^q q\,dq$$

$$=\frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

19. Let the particle be, at some instant, at a point P distant x from the origin. As shown in Fig. 3.116, there are two forces of repulsion acting due to two charges of +8 mC. The net force is $2F\cos\alpha$ towards right.

Similarly, there are two forces of attraction due to two charges of −1 μC. The net force due to these force is $2F'\cos\beta$ towards left.

The net force on charge 0.1 μC is zero when.

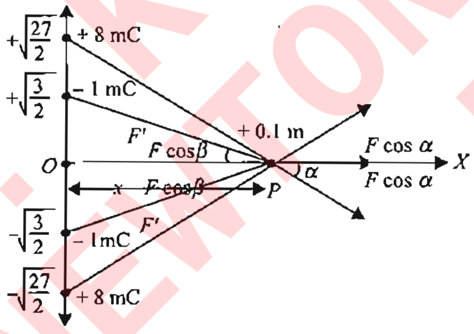


Fig. 3.116

$$2F\cos\alpha=2F'\cos\beta$$

$$\frac{K\times 8\times 10^{-6}\times 0.1\times 10^{-6}}{\sqrt{x^2+\frac{27}{2}}}\times\frac{x}{\sqrt{x^2+\frac{27}{2}}}$$

$$=\frac{K\times 1\times 10^{-6}\times 0.1\times 10^{-6}}{\sqrt{x^2+\frac{3}{2}}}\times\frac{x}{\sqrt{x^2+\frac{3}{2}}}$$

$$\frac{8}{\left[x^2+\frac{27}{2}\right]^{3/2}}=\frac{1}{\left[x^2+\frac{3}{2}\right]^{3/2}}$$

$$\left\{2^3\left[x^2+\frac{3}{2}\right]^{3/2}\right\}^{2/3}=\left\{\left[x^2+\frac{27}{2}\right]^{3/2}\right\}^{2/3}$$

$$\Rightarrow\quad 4\left(x^2+\frac{3}{2}\right)=x^2+\frac{27}{2}$$

$$3x^2=\frac{27}{2}-\frac{12}{2}\quad\Rightarrow\quad x=\pm\sqrt{\frac{5}{2}}$$

This means that we need to move the charge from $-\alpha$ to $\sqrt{\frac{5}{2}}$. Thereafter, the attractive forces will make the charge move to origin.

The electric potential of the charge at $x=\sqrt{\frac{5}{2}}$ is

$$V=\frac{2\times 9\times 10^9\times 8\times 10^{-6}}{\sqrt{\frac{5}{2}+\frac{27}{2}}}-\frac{2\times 9\times 10^9\times 10^{-6}}{\sqrt{\frac{5}{2}+\frac{3}{2}}}$$

$$=2\times 9\times 10^9\times 10^{-6}\left[\frac{8}{4}-\frac{1}{2}\right]=2.7\times 10^4\ \text{V}$$

Kinetic energy is required to overcome the force of repulsion from $\propto$ to $x=\sqrt{\frac{5}{2}}$.

The work done in this process is $W=q\,(V)$, where V = p.d. between ∞ and $x=\sqrt{\frac{5}{2}}$.

$$\text{K.E.}=0.1\times 10^{-6}\,[2.7\times 10^4-2.4\times 10]$$

$$=0.1\times 10^{-6}\times 0.3\times 10^4=3\times 10^{-4}\ \text{J}$$

20. $W_{\text{external}}=\Delta\,\text{PE}=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left[-\frac{3}{1}+\frac{3}{\sqrt{2}}-\frac{1}{\sqrt{3}}\right]\times 4$

$$=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\times\frac{4}{\sqrt{6}}\,[3\sqrt{3}-3\sqrt{6}-\sqrt{2}]$$

21. $E_1=\frac{\sigma_1}{2\varepsilon_0},\ E_2=\frac{\sigma_2}{2\varepsilon_0}$

$$E=E_1-E_2=\frac{\sigma_1-\sigma_2}{2\varepsilon_0}$$

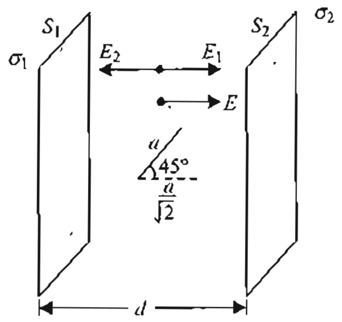


Fig. 3.117

$$W=q_0\,E\times\frac{a}{\sqrt{2}}$$

$n_1=1,\ n_2=\sqrt{2},\quad i=45°,\ r=?$

22. a. Force exerted by upper charge on dipole:

$$F_1=\frac{1}{2\pi\varepsilon_0}\frac{pq}{a^3}\ \text{(down)}$$

Force exerted by left charge on dipole:

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{pq}{a^3} \text{ (up)}$$

Force exerted by right charge on dipole:

$$F_3 = \frac{1}{4\pi\varepsilon_0}\frac{pq}{a^3} \text{ (up)}$$

Net force on the dipole due to all charges:

$$F = F_1 + F_2 + F_3 = 0$$

Hence, net force on the dipole is zero.
The total electric potential energy consists of interaction of all the three charges among themselves and interaction of these three charges with dipole. So,

$$U = 2\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{\sqrt{2}a}\right) + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a}$$
$$- \vec{p}\cdot\vec{E}_{\text{up}} - \vec{p}\cdot\vec{E}_{\text{left}} - \vec{p}\cdot\vec{E}_{\text{right}}$$

$$\vec{p}\cdot\vec{E}_{\text{left}} = \vec{p}\cdot\vec{E}_{\text{right}} = 0$$

(Because electric fields produced by left and right charges are perpendicular to p.)

$$-\vec{p}\cdot\vec{E}_{\text{up}} = -p\cdot\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\right)\cos\pi = \frac{1}{4\pi\varepsilon_0}\frac{qp}{a^2}$$

Putting the values, we get

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a}\left[2\sqrt{2}+1\right] + \frac{1}{4\pi\varepsilon_0}\frac{pq}{a^2}$$

23. **a.** Charges at A, B, C and D are placed at equilateral position of dipole. Hence, force on each of them due to dipole:

$$F_1 = \frac{Qp}{4\pi\varepsilon_0\left(a/\sqrt{2}\right)^3}$$

This force is downward on charges. Hence, force due to these charges on dipole is $4F_1$ (upwards)
Force on dipole due to charge at P:

$$F_2 = Q\frac{2pQ}{4\pi\varepsilon_0\left(a/\sqrt{2}\right)^2} \text{ (upward)}$$

Net force on dipole: $F = 4F_1 + F_2 = \dfrac{3\sqrt{2}\,Qp}{\pi\varepsilon_0 a^3}$ (upward)

b. P.E. of system

U = (10 pairs of charged particles) + (5 pairs of dipole and charged particles)
As potential energy of dipole with four charges at A, B, C and D will be zero,

$$\therefore \quad U = 4\left[\frac{1}{4\pi\varepsilon_0} \quad 2\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{\sqrt{2}a} - 4\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a}\right.$$
$$- \frac{1}{4\pi\varepsilon_0} - \Big(a$$

$$\Rightarrow \quad U = \frac{Q}{2\sqrt{2}\pi} \quad \frac{pQ}{\pi\varepsilon_0 a^2}$$

24. Potential energy of dipole system A and B:
Potential at A_1 due to dipole at B

$$V_{A_1} = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos 60^\circ}{\left[d - \frac{a}{2}\right]^2}$$

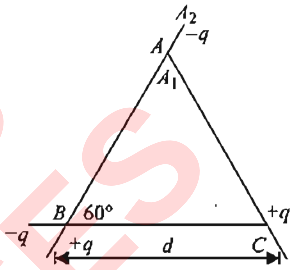


Fig. 3.118

Potential energy at $A_1 = qV_A$

Similarly, P.E. at $A_2 = -qV_A$

$$U_{1,2} = q(V_{A_1} - V_{A_2})$$

$$U_{AB} = \frac{2p\cos 60^\circ \times q}{4\pi\varepsilon_0}\left[\frac{1}{(d-a/2)^2} - \frac{1}{(d+a/2)^2}\right]$$

$$= \frac{2p\cos 60^\circ q}{4\pi\varepsilon_0}\frac{4d\frac{a}{2}}{\left(d^2 - \frac{a^2}{4}\right)^2} = \frac{2q^2a^2}{4\pi\varepsilon_0 d^3}$$

Similarly, potential energy of system A and C and B and C can be calculated which is same as U_{12}. Hence, P.E. of the system

$$U = U_{AB} + U_{BC} + U_{CA} = 3\frac{2q^2a^2}{4\pi\varepsilon_0 d^3} = \frac{6q^2a^2}{4\pi\varepsilon_0 d^3}$$

Objective Type

1. **a.** K.E. = Charge × potential difference $= qV$
2. **c.** $V = \dfrac{w}{q} = \dfrac{ML^2T^{-2}}{AT} = ML^2T^{-3}A^{-1}$
3. **a.** Electricity neutral means net charge zero. Potential of a neutral conductor may or may not be zero. It also depends upon the charge present on the surrounding bodies also.
4. **a.** Negative charge itself goes from low to high potential.
5. **d.** $W = q(V_f - V_i) = 0$ since $V_f = V_i$
6. **a.** Because electric field intensity is zero inside a spherical conductor.
7. **b.** The potential at the surface of a hollow or conducting sphere is same as the potential at the center of the sphere and any point inside the sphere.
8. **b.** Apply $V = \dfrac{kQ}{R}$ when $= Ze$
9. **c.** K.E.$_i$ + P.E.$_i$ = K.E.$_f$ + P.E.$_f$

$$0 + q \times 600 = \text{K.E.}_f + q \times 0$$

$$\text{K.E.}_f = 600 \times 10^{-8} = 6 \times 10^{-6}\text{ J}$$

10. d. Let charge on smaller sphere is q. As the potential of both will be same finally, i.e.,

$$\frac{q}{r'} = \frac{Q-q}{r} \quad \Rightarrow \quad q = \frac{Qr'}{r+r'}$$

11. d. $U = \dfrac{k\,q_1 q_2}{r}$

12. c. $\dfrac{k \times 4}{1-x} = \dfrac{k \times 2}{x} \quad \Rightarrow x = \dfrac{1}{3}$ m

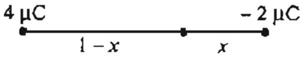


Fig. 3.119

13. b. $E = \dfrac{kQ}{x^2} \quad \Rightarrow \quad E \propto \dfrac{1}{x^2}$

14. c. $E_1 = E_2 \quad \Rightarrow \quad \dfrac{kQ_1}{r_1^2} = \dfrac{kQ_2}{r_2^2}$

$$\frac{Q_1}{Q_2} = \left(\frac{r_1}{r_2}\right)^2$$

$$\frac{V_1}{V_2} = \frac{Q_1}{r_1} \times \frac{r_2}{Q_2} = \left(\frac{r_1}{r_2}\right)^2 \frac{r_2}{r_1} = \frac{r_1}{r_2}$$

15. b. E is a vector quantity, V is a scalar quantity.

16. a. $E = -\dfrac{dV}{dr}$ = negative of the slope of V–r graph.

17. b. $W_{eq} = q\,(V_A - V_B)$

$$50 \times 10^{-6} = 2 \times 10^{-6}(V_A - V_B)$$

$$V_A - V_B = 25 \text{ V}$$

18. c. As potential at A and B is same, $V_A = V_B = \dfrac{kQ}{d}$. So, work done in both cases will be same.

19. b. $U = \varepsilon \dfrac{k\,q_1 q_2}{r}$. There will be 6 pairs, 4 on a side of square and 2 as diagonal.

20. d. A conductor is an equipotential body. Potential on it or within it is same everywhere.

21. a. Potential decreases in the direction of electric field.

22. d. $V = \dfrac{kQ}{c} - \dfrac{kQ}{b}$

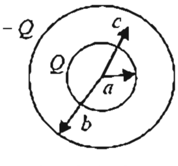


Fig. 3.120

23. a. $V_C = V_1 = V_2$

$$\frac{kq_1}{r_1} = \frac{kq_2}{r_2}$$

$$\frac{q_1}{q_2} = \frac{r_1}{r_2} \qquad \text{(i)}$$

$$q_1 + q_2 = Q \qquad \text{(ii)}$$

From (i) and (ii), we get

$$q_1 = \frac{Qr_1}{r_1 + r_2}$$

Put in V_1, we get $V_C = \dfrac{kQ}{r_1 + r_2}$

24. d. V is a scalar quantity, E is a vector quantity.

25. a. $dV = -E\,dr\cos\theta$

Along the line of force, θ is 0°, hence dV is maximum. So, the variation of potential is maximum along the line of force.

26. b. Charge will induce on A but total charge on A will remain zero. Negative charge of A will be more closer to B than positive charge on A. So potential of B will decrease.

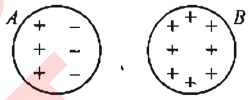


Fig. 3.121

27. a. Their potential will be same. $V_1 = V_2$

$$\frac{kq_1}{R_1} = \frac{kq_2}{R_2} \Rightarrow \frac{q_1}{q_2} = \frac{R_1}{R_2}$$

$$\frac{E_1}{E_2} = \frac{kq_1/R_1^2}{kq_2/R_2^2} = \frac{q_1}{q_2}\left(\frac{R_2}{R_1}\right)^2$$

$$= \frac{R_1}{R_2}\left(\frac{R_2}{R_1}\right)^2 = \frac{R_2}{R_1}$$

28. d. $F = qE$, $W = qE \times 2\cos 60°$

$$\Rightarrow \quad 4 = 0.2\,E \times 2 \times \frac{1}{2} \quad \Rightarrow E = 20 \text{ NC}^{-1}$$

29. c. $d = 10 \sin 30° = 5$ cm

$$E = \frac{\Delta v}{d} = \frac{30-20}{5/100} = 200 \text{ Vm}^{-1}$$

Direction of electric field will be in the direction of decreasing potential.

30. b. $W_{\text{el.}} = q(V_i - V_f)$

$$\Rightarrow \quad 6.4 \times 10^{-19} = -1.6 \times 10^{-19}(V_A - V_B)$$

$$\Rightarrow \quad V_A - V_B = -4 \text{ V}$$

$$\Rightarrow \quad V_A - V_C = -4\,V\,(\because V_B = V_C)$$

$$\Rightarrow \quad V_C - V_A = 4 \text{ V}$$

31. b.

$$V_1 = \frac{k \times 2 \times 10^{-6}}{0.1} + \frac{k \times 4 \times 10^{-6}}{\sqrt{(0.1)^2 + (0.5)^2}}$$

$$V_2 = \frac{k \times 4 \times 10^{-6}}{0.1} + \frac{k \times 2 \times 10^{-6}}{\sqrt{(0.1)^2 + (0.5)^2}}$$

Work done $= q(V_2 - V_1) \simeq 0.72$ J

32. d. $Q = nq$, $R = n^{1/3}r$

$$E_{\text{small}} = \frac{kq}{r^2},\ E_{\text{big}} = \frac{kQ}{R^2}$$

$$E_{\text{big}} = \frac{k\,n\,q}{n^{2/3}r^2} = (n^{1/3})\,\frac{kq}{r^2} = n^{1/3}\,E_{\text{small}}$$

$$V_{\text{small}} = \frac{kq}{r},\ V_{\text{big}} = \frac{kQ}{R}$$

$$V_{\text{big}} = \frac{k\,n\,q}{n^{1/3}\,r} = n^{2/3}\frac{kq}{r} = n^{2/3}\,V_{\text{small}}$$

33. **a.** We know that any extra charge resides on the outer surface. Finally both will be at same potential. It is because both are connected.

34. **d.** Equipotential surfaces are perpendicular to the electric lines of forces.

35. b. $V_A = \dfrac{Q}{4\pi\varepsilon_0 a},\ V_B = \dfrac{Q}{4\pi\varepsilon_0 b}$

$$W = q(V_A - V_B) = \frac{q\,Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

36. **d.** $E = \dfrac{\sigma}{\varepsilon_0}$ → does not depend upon radius if σ is constant.

37. **a.** $V = ER$; If R is doubled, V also gets doubled.

38. **b.** Electric intensity inside is zero, whereas outside it is not zero.

39. **d.** Potential decreases in the direction of electric field. So it depends whether the lines of forces are from A to B or from B to A.

40. c. Apply conservation of mechanical energy between point a and b: (K.E. + P.E.)$_a$ = (K.E. + P.E.)$_a$

$$\Rightarrow\ 0 + \frac{k(3\times10^{-9})q_0}{0.01} - \frac{k(3\times10^{-9})q_0}{0.02}$$

$$= \frac{1}{2}mv^2 + \frac{k(3\times10^{-9})q_0}{0.02} - \frac{k(3\times10^{-9})q_0}{0.01}$$

Put the values and get: $v = 12\sqrt{15} = 46$ m/s

41. **b.** $U = \dfrac{kqQ}{r} - \dfrac{kqQ}{r} + \dfrac{kq^2}{2r} = 0\ \Rightarrow Q/q = 1/4$

42. **b.** Find potential at A and C due to charge at B, then required work done is $W = q(V_A - V_c)$

43. **c.** Apply conservation of mechanical energy

44. **a.** $100e = \dfrac{\sigma}{2\varepsilon_0}d$. Solve to get the answer.

45. **c.** $\dfrac{kQ}{x} = \dfrac{1}{2}\left(\dfrac{3}{2}\dfrac{kQ}{R}\right) \Rightarrow x = 4R/3$

Distance from surface: $x - R = R/3$

46. **d.** All the charges are placed in x-y plane such that they form a square with origin as its center. So electric field at the origin will be zero.

47. **d.** It depends whether both charges are of same or opposite sign.

48. **a.** Because work is to be done by an external agent in moving a positive charge from low potential to high potential and this work gets stored in the form of potential energy of the system. Hence, it increases.

49. **b.** Electric field will do the negative work, because the force of electric field is opposite to the displacement. So external agent has to do positive work. So its energy will be used.

50. **d.** a. is wrong because force will be same, but acceleration will be different because masses of electrons and protons are different.

b. is wrong, because at a point there can be only one potential.

c. is wrong, because charge lies on the outer surface of a conductor always.

d. is correct because, the whole conductor will be an equipotential body.

51. **c.** $\dfrac{kQ}{4x_1} = \dfrac{kQ}{r + x_1}$

$$r + x_1 = 4x_1$$

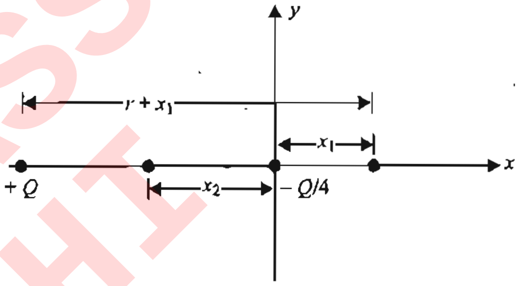


Fig. 3.122

$$x_1 = \frac{r}{3}$$

$$\frac{k\,Q}{4\,x_2} = \frac{k\,Q}{r - x_2}$$

$$x_2 = \frac{r}{5}$$

52. **c.** $E_x = \dfrac{dv}{dx} = \dfrac{120 - 80}{2} = 20\ \text{cm}^{-1}$

There may be y- and z-components of field also.

53. **a.** $U = kq^2\left(-\dfrac{3}{d} + \dfrac{3}{\sqrt{2}d} - \dfrac{1}{\sqrt{3}d}\right)$

$$+kq^2\left(-\frac{2}{d} + \frac{3}{\sqrt{2}d} - \frac{1}{\sqrt{3}d}\right) + kq^2\left(-\frac{2}{d} + \frac{2}{\sqrt{2}d} - \frac{1}{\sqrt{3}d}\right)$$

$$+ kq^2\left(-\frac{1}{d} + \frac{2}{\sqrt{2}d} - \frac{1}{\sqrt{3}d}\right) + kq^2\left(-\frac{2}{d} + \frac{1}{\sqrt{2}d}\right)$$

$$+ kq^2\left(-\frac{1}{d} + \frac{1}{\sqrt{2}d}\right) + kq^2\left(-\frac{1}{d}\right)$$

$$U = kq^2\left(-\frac{12}{d} + \frac{12}{\sqrt{2}d} - \frac{4}{\sqrt{3}d}\right)$$

$$= -\frac{12kq^2}{d}\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}}\right)$$

54. **c.** $F_e = mg\tan\theta$

$$= (1.20\ \times\ 10^{-3}\ \text{kg})\ (10\ \text{ms}^{-2})\tan(37°) = 0.0090\ \text{N}$$

(Balance forces in x- and y-directions.)
Also:

$$F_e = Eq = \frac{Vq}{d}$$

$$\therefore \quad V = \frac{Fd}{q} = \frac{(0.009\text{ N})\,(0.0500\text{ m})}{9.0 \times 10^{-6}\text{ C}} = 50.0\text{ V.}$$

55. a. $\left(0 - \frac{1}{2}mv^2\right) = q(\Delta v)$

$$\left(\frac{1}{2}mv_1^2 - \frac{1}{2}mv^2\right) = -q(\Delta v)$$

$$\frac{v_1^2 - v^2}{-v^2} = -1$$

$$v_1^2 = 2v^2$$

$$v_1 = \sqrt{2}\,v$$

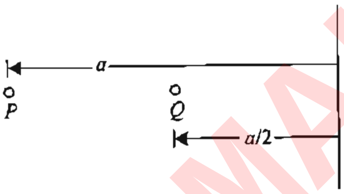


Fig. 3.123

56. a. $W = \vec{F} \cdot \vec{r} = q\vec{E} \cdot \vec{r}$

57. c. $V_A = V_1 + V_2$

$$V_A = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{1}{\sqrt{R^2 + a^2}}\right)$$

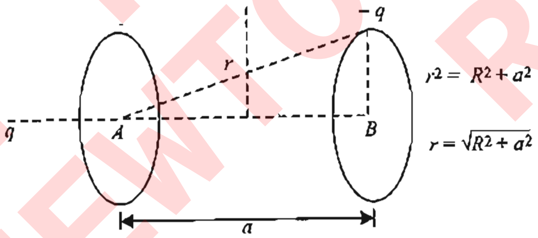


Fig. 3.124

$$V_B = -\frac{1}{4\pi}\frac{q}{R} + \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$V_A - V_B = \frac{2q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{r}\right]$$

$$V_A - V_B = \frac{2q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2 + a^2}}\right]$$

58. a. $V_{(x,y)} - V_0 = -\int_0^x E_x dx - \int_0^y E_y dy$

$$\Rightarrow V_{(x,y)} = V_0 - E_x x - E_y y$$

59. c. $V = k\,\frac{q}{R} - \frac{kq}{2R} + \frac{kq}{3R} = \frac{kq}{R}\left[1 - \frac{1}{2} + \frac{1}{3}\right]$

$$\frac{6 - 3 + 2}{6} = \frac{k\,5q}{6R}$$

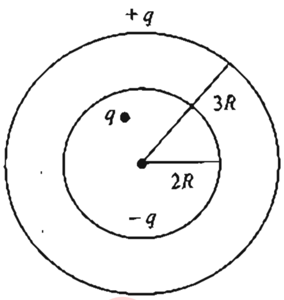


Fig. 3.125

60. a. $V_f - V_i = -\int_i^f (E_x\hat{i} + E_y\hat{j} + E_z\hat{k}) \cdot (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$

$$V_B - V_A = -\left[\int_i^f E_x dx + \int_i^f E_y dy + \int_i^f E_z dz\right]$$

$$V_B - V_A = -\left[\int_1^2 2dx + \int_2^1 3dy\right]$$

$$V_B - V_A = -[2(2-1) + 3(1-2)]$$

$$V_B - V_A = -[2-3] = 1\text{ V}$$

Hence, $V_A - V_B = -1$ V

61. d. $E_r = -\frac{dV}{dr}$

$$E_x = -4x$$

Electric field at $x = 2$ m

$$E_x = -8\text{ NC}^{-1}$$

$$\vec{E} = -8\hat{i}\text{ NC}^{-1}$$

62. c. Dotted lines will give the direction of electric field as in Fig. 3.126.

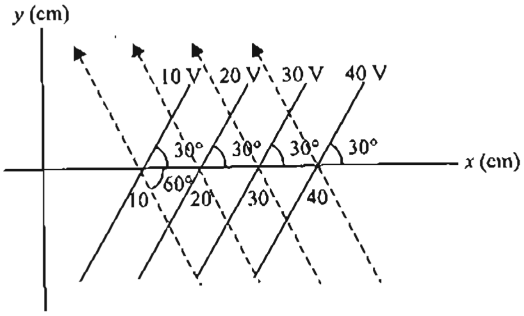


Fig. 3.126

Distance between equipotential lines

$$\Delta r = 10\cos 60° = 5\text{ cm}$$

$$|\vec{E}| = \left|\frac{\Delta V}{\Delta r}\right| = \frac{10}{5/100} = 200\text{ Vm}^{-1}$$

$$\vec{E} = |\vec{E}|\,\hat{u} = 200\,[-\cos 60°\,\hat{i} + \sin 60°\,\hat{j}]$$

$$\vec{E} = 200\left[-\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right]$$

$$\vec{E} = 100\,[-i + \sqrt{3}j]\ \text{NC}^{-1}$$

63. **b.** Equipotential lines are concentric circles,

These pattern of equipotential lines should be due to point charge at the center of circles.

Let us consider few equipotential lines

$$20 = \frac{1}{4\pi\varepsilon_0}\frac{q}{30 \times 10^{-2}} \Rightarrow \frac{q}{4\pi\varepsilon_0} = 6$$

$$60 = \frac{1}{4\pi\varepsilon_0}\frac{q}{10 \times 10^{-2}} \Rightarrow \frac{q}{4\pi\varepsilon_0} = 6$$

For point charge, electric field should be in radial direction.

The value is $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{6}{r^2}\,(\text{Vm}^{-1})$

64. **b.** $+Q$ Charge will flow into earth.

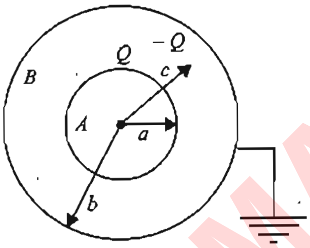


Fig. 3.127

$$v_C = \frac{kQ}{r} - \frac{kQ}{b}$$

$$v_A = \frac{kQ}{a} - \frac{kQ}{b}$$

Potential of B is zero.

65. **a.** $E = \frac{V}{d}$, $F = eE = eV/d$

$$a = \frac{F}{m} = \frac{eV}{md}$$

$$d = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2d}{a}}$$

$$t = \sqrt{\frac{2d\,md}{eV}} = \sqrt{\frac{2\,md^2}{eV}}$$

66. **c.** $U = 2kq^2\left[-\frac{1}{a} + \frac{1}{2a} - \frac{1}{3a} + \frac{1}{4a} + \cdots\right]$

$$= -\frac{2q^2}{4\pi\varepsilon_0 a}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right] = -\frac{2q^2\log_e 2}{4\pi\varepsilon_0 a}$$

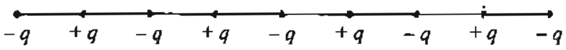


Fig. 3.128

67. **b.** $V_x - V_\infty = -\int_\infty^x E_x dx \Rightarrow \quad V_x - 0 = -\int_\infty^x \frac{A}{x^3}dx$

$$\Rightarrow \quad V_x = \frac{A}{2x^2}$$

68. **c.** $\frac{kq_1}{a} + \frac{kq}{2d} = 0 \quad \Rightarrow \quad q_1 = \frac{-qa}{2d}$

$$q_2 = -\frac{a}{d}\left[-\frac{qa}{2d} + \frac{q}{2}\right] = -\frac{aq}{2d}\left[1 - \frac{a}{d}\right]$$

$$= -\frac{aq}{2d}\,\frac{(d-a)}{d}$$

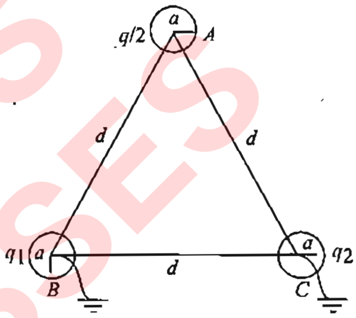


Fig. 3.129

69. **b.** On connecting, the entire amount of charge will shift to the outer sphere.

Heat generated $= U_i - U_f = \frac{q^2}{8\pi\varepsilon_0 R_1} - \frac{q^2}{8\pi\varepsilon_0 R_2}$

$$= \frac{(20 \times 10^{-6})^2 \times 9 \times 10^9}{2}\left[\frac{1}{0.10} - \frac{1}{0.20}\right]$$

$$= 9\ \text{J}$$

70. **c.** $V = \frac{kQ}{r} \quad \Rightarrow \quad u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\frac{\varepsilon_0 k^2 Q^2}{r^4}$

$$V^4 \propto u$$

71. **b.** $\vec{E} = a(y\hat{i} + x\hat{j})$

$$V_2 - V_1 = -\int (aydx + axdy)$$

$$V = -a\int (ydx + xdy) + C$$

$$V = -a\int d(xy) + C = -axy + C$$

72. **d.**
(i) $Q/2$, $Q/20$
(ii) $Q/2$, $Q/4$, $Q/4$
(ii) $3Q/8$, $Q/4$, $3Q/8$
(iv) $5Q/16$, $5Q/16$, $3Q/8$
(v) $5Q/16$, $11Q/32$, $11Q/32$

73. **a.** $V = x^2 - y^2$

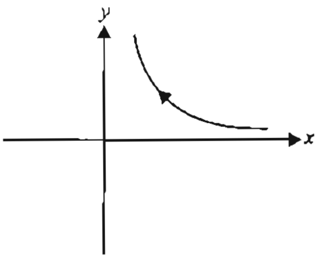


Fig. 3.130

$$E_x = -\frac{dV}{dx} = -2x$$

$$E_y = 2y$$

74. **a.** The potential inside the shell will be the same everywhere as on its surface. As we add $3Q$ charge on the surface, the potential on the surface changes by the same amount as that inside. Therefore, the potential difference remains the same.

Multiple Correct Answers Type

1. **a., b., d.** $0 \le x \le a$; $V_x = \left[-\int_0^x E_x dx\right] + V_{(0)} = 0$

(as $E_x = 0$)

$$x \ge a; \quad V_x = -\int_a^x E_x dx + V_{(a)}$$

$$V_x = \left[-\int_a^x \frac{\sigma}{\varepsilon_0} dx\right] + V_{(a)} = -\frac{\sigma}{\varepsilon_0}(x-a) = \frac{-\sigma}{\varepsilon_0}(x-a)$$

$$x \le 0; \quad V_x = -\int_0^x E_x dx + V_{(0)} \quad \left[\because \; E_x = \frac{-\sigma}{\varepsilon_0}\right]$$

$$V_x = -\left(-\frac{\sigma}{\varepsilon_0}x\right) + V_{(0)} = \frac{\sigma}{\varepsilon_0}x$$

2. **a., d.** When a negative charge moves from high potential to low potential, its potential energy increases.

$$W_{\text{el}} = q\,(v_1 - v_2)$$

As $v_1 > v_2$ and q is negative, hence $W_{\text{el}} =$ negative.

3. **a., b., c.** This question is based on the working principle of a generator.

$$V_{\text{I}} - V_{\text{II}} = \left(\frac{q_1}{4\pi\varepsilon_0 R_1} + \frac{q_2}{4\pi\varepsilon_0 R_2}\right) - \left(\frac{q_1 + q_2}{4\pi\varepsilon_0 R_2}\right)$$

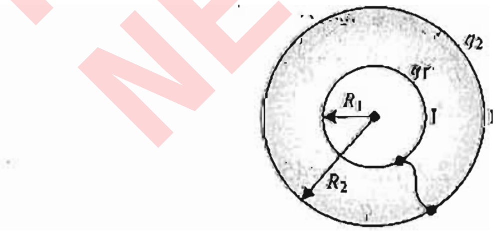


Fig. 3.131

$$V_{\text{I}} - V_{\text{II}} = \frac{q_1}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

As two conductors are connected, transfer of charge takes place from one conductor to other till both acquire same potential, i.e., here $V_{\text{I}} - V_{\text{II}} = 0$

Potential difference depends only on q_1 (charge of inner conductor), so $V_{\text{I}} - V_{\text{II}} = 0$, where $q_1 = 0$, i.e., total charge is transferred to the outer conductor.

4. **a., d.** As no charge is present inside the conductor, potential at any point inside the conductor is same as that of the potential of conductor.

So, potential of the conductor = Potential at the center

$$= V_q + V_{\text{induced charges}}$$

$$\text{i.e., } V_{\text{conductor}} = \frac{q}{4\pi\varepsilon_0(d+R)} + 0$$

As the total induced charge at conductor's surface is equal to zero and hence to the potential at center due to the induced charge.

For point B

$$V_{\text{conductor}} = V_{\text{at point B}} = V_q + V_{\text{induced charges}}$$

$$V_{\text{induced charges}} = \frac{q}{4\pi\varepsilon_0(d+R)} - \frac{q}{4\pi\varepsilon_0 d} = \frac{-qR}{4\pi\varepsilon_0 d\,(d+R)}$$

5. **a., b., d.** $v_i = \dfrac{q_0}{4\pi\varepsilon_0 R_1} \times q + \dfrac{q^2}{8\pi\varepsilon_0 R_1}$

$$v_f = \frac{qq_0}{4\pi\varepsilon_0 R_2} + \frac{q^2}{8\pi\varepsilon_0 R_2}$$

$$dU = U_f - U_i = \frac{q(q_0 + q/2)}{4\pi\varepsilon_0}\left[\frac{1}{R_2} - \frac{1}{R_1}\right] < 0$$

It means electric potential energy is decreasing and work done by electric force would be positive as given by

$$W_{\text{el}} = -dU$$

From work–energy theorem,

$$dK = W_{el} + W_{el} + \Delta H_{\text{dissipated}}$$

Now, we can answer the remaining options.

6. **a., b., c.** Because the metal can is deep, so $-Q$ charge will be induced on the inner surface and $+Q$ on the outer surface (Fig. 3.132).

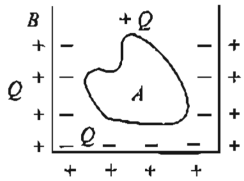


Fig. 3.132

As the can is deep, so potential of A will not change.

If metal can is earthed, then $+Q$ charge will slow into earth and remaining $-Q$ will decrease the potential of A

Assertion-Reasoning Type

1. **c.** Electric field is directed from high potential to low potential.

2. **a.** The surface of a conductor is an equipotential surface as electric field inside the conductor is zero. If there is some potential difference between two points of conductor, then charge will flow within conductor to make the potential same.

3. **d.** Potential depends not only on charge but also on the shape and size of the conductor.
4. **a.** Inside electric field is zero but not outside.
5. **c.** Earth also has some surface charge density due to which it produces electric field in the surrounding space.

Comprehension Type

For Problems 1–2

1. b., 2. d.

Sol. Potential is a scalar quantity. We see that potential at P due to both charges is same in magnitude. In problem 1, both potentials are added and in problem 2, both potentials are cancelled.

For Problems 3–4

3. b., 4. c.

Sol. The work that must be done on q_3 by an external force $\vec{F}_{ext}$ is equal to the difference between two quantities: the potential energy U associated with q_3 when it is at $x = 2a$ and the potential energy when it is infinitely far away.

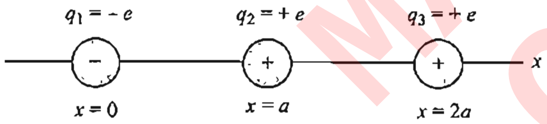


Fig. 3.133

The second of these is zero, so the work that must be done is equal to U. The distances between the charges are $r_{13} = 2a$ and $r_{23} = a$, so from

$$W = U = \frac{q_3}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$

$$= \frac{+e}{4\pi\varepsilon_0}\left(\frac{-e}{2a} + \frac{+e}{a}\right) = \frac{+e^2}{8\pi\varepsilon_0 a}$$

If q_3 is brought in from infinity along the $+x$-axis, it is attracted by q_1 but is repelled more strongly by q_2; hence positive work must be done to push q_3 to the position at $x = 2a$.

The total potential energy of the assemblage of three charges is

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{(-e)(e)}{a} + \frac{(-e)(e)}{2a} + \frac{(e)(e)}{a}\right) = \frac{-e^2}{8\pi\varepsilon_0 a}$$

For Problems 5–6

5. c., 6. b.

Sol. $E_i = K_i + U_i$

$$= \frac{1}{2}(0.0015\text{ kg})(20.0\text{ ms}^{-1})^2$$

$$+ \frac{k(2.00 \times 10^{-6}\text{C})(8.00 \times 10^{-6}\text{C})}{0.800\text{ m}}$$

$$= 0.48\text{ J}$$

$$E_i = E_f = \frac{1}{2}mv_f^2 + \frac{kq_1q_2}{r_f}$$

$$v_f = \sqrt{\frac{2(0.48\text{ J} - 0.36\text{ J})}{0.0015\text{ kg}}} = 4\sqrt{10}\text{ ms}^{-1}$$

At the closest point, the velocity is zero:

$$0.48\text{ J} = \frac{kq_1q_2}{r}$$

$$r = \frac{k(2.00 \times 10^{-6}\text{C})(8.00 \times 10^{-6}\text{C})}{0.48\text{ J}} = 0.30\text{ m}$$

For Problems 7–9

7. a., 8. c., 9. b.

Sol. At A: $V_A = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$

$$= k\left(\frac{2.40 \times 10^{-9}\text{ C}}{0.05\text{ m}} + \frac{-6.50 \times 10^{-9}\text{ C}}{0.05\text{ m}}\right) = -738\text{ V}.$$

At B: $V_B = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$

$$= k\left(\frac{2.40 \times 10^{-9}\text{ C}}{0.08\text{ m}} + \frac{-6.50 \times 10^{-9}\text{ C}}{0.06\text{ m}}\right) = -705\text{ V}$$

2. $W = q\Delta V = (2.50 \times 10^{-9}\text{ C})(-33\text{ V})$
$= -8.25 \times 10^{-8}\text{ J}$

The negative sign indicates that the work is done *on* the charge. So, the work done by the field is

$$8.25 \times 10^{-8}\text{ J}.$$

For Problems 10–13

10. d., 11. d., 12. d., 13. b.

Sol. Electric field at 'O' (Fig. 3.134)

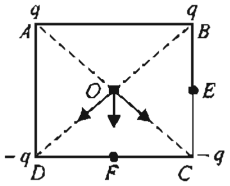


Fig. 3.134

$$E_0 = 2.2\,|\vec{E}_A|\,\cos 45°$$

$$E_0 = 4\frac{1}{4\pi\varepsilon_0}\frac{q}{(a/\sqrt{2})^2} \times \frac{1}{\sqrt{2}}$$

$$E_0 = \frac{\sqrt{2}q}{\pi\varepsilon_0 a^2}$$

Electric potential at O will be zero, i.e.,
$V_E = 0$

$$V_F = 2\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{(a^2 + a^2/4)^{1/2}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{a/2}\right]$$

$$= \frac{q}{2\pi\varepsilon_0}\left[\frac{2}{\sqrt{5}a} - \frac{2}{a}\right]$$

$$= \frac{q}{\pi\varepsilon\ a}\left[\frac{1}{\sqrt{5}} - 1\right]$$

For Problems 14–15

14. b., 15. c.

Sol. The electric field due to charges Q and Q will be directed towards perpendicular bisector of line joining charges. Hence, negative charge retards and comes to rest at some position on this line. From this position, the charge $-q$ will return back.

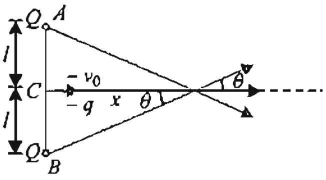


Fig. 3.135

$$E_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\ \frac{2Q}{(x^2+l^2)}\ \frac{x}{(x^2+l^2)^{1/2}}$$

$$\frac{1}{2}mv_0^2 + 2(-q)\frac{1}{4\pi\varepsilon_0}\ \frac{Q}{l} = 2(-q)\frac{1}{4\pi\varepsilon_0}\ \frac{Q}{(x^2+l^2)^{1/2}}$$

$$\frac{1}{(x^2+l^2)} = \frac{-\pi\varepsilon_0 mv^2}{qQ} + \frac{1}{l}$$

$$\frac{1}{(x^2+l^2)} = \frac{-\pi\varepsilon_0 mv^2 + qQ}{qQl}$$

$$(x^2+\lambda^2)^{1/2} = \frac{qQl}{qQ - \pi\varepsilon\,.lmv^2}$$

$$(x^2+\lambda^2) = \left(\frac{qQl}{qQ - \pi\varepsilon_0 lmv^2}\right)^2$$

$$x = \left[\left(\frac{qQl}{qQ - \pi\varepsilon_0 lmv^2}\right)^2 - l^2\right]^{1/2}$$

$$x = l\left[\left(\frac{qQ}{qQ - \pi\varepsilon_0 lmv^2}\right)^2 - 1\right]^{1/2}$$

For Problems 16–17

16. a., 17. c.

Sol. If q_1 and q_2 are the charges on the spheres of radii r and R, respectively, then, by conservation of charge

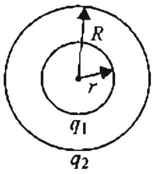


Fig. 3.136

$$q_1 + q_2 = Q \qquad \text{(i)}$$

But $\sigma_1 = \sigma_2$

$$\frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}$$

$$\frac{q_1}{q_2} = \frac{r^2}{R^2} \qquad \text{(ii)}$$

From (i) and (ii)

$$q_1 = \frac{Qr^2}{r^2+R^2} \text{ and } q_2 = \frac{QR^2}{r^2+R^2}$$

Hence, potential at common center,

$$V = V_1 + V_2$$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r} + \frac{q_2}{R}\right]$$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qr}{(R^2+r^2)} + \frac{QR}{(R^2+r^2)}\right]$$

$$V = \frac{1}{4\pi\varepsilon_0}\ \frac{Q(R+r)}{(R^2+r^2)}$$

For Problems 18–20

18. b., 19. a., 20. b.

Sol.

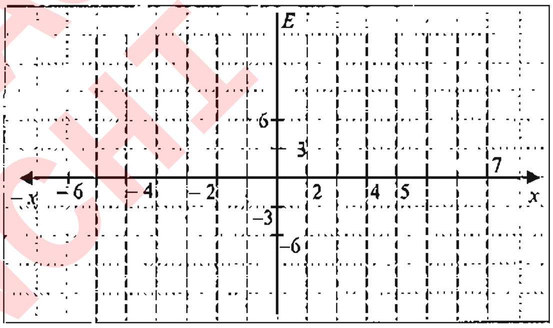


Fig. 3.137

For $-6 \le x \le -4$:

$$E = -\frac{dV}{dx} = -\frac{\Delta V}{\Delta x} = -\frac{(12-0)}{[-4-(-6)]} = -6\ \text{NC}^{-1}$$

For $-4 \le x \le -2$:

$$E = -\frac{\Delta V}{\Delta x} = 0$$

For $-2 \le x < 2$:

$$E = -\frac{\Delta V}{\Delta x} = -\frac{[0-12]}{[2-(-2)]} = 3\ \text{NC}^{-1}$$

For $2 \le x \le 3$:

$$E = -\frac{[(-6)-0]}{[3-2]} = 6\ \text{NC}^{-1}$$

For $3 \le x \le 5$: $E = 0$

For $5 \le x \le 7$:

$$E = -\frac{[0-(-6)]}{[7-5]} = -3\ \text{NC}^{-1}$$

For Problems 21–23

21. a., 22. a., 23. d.

Sol. $W_{\text{cle}} = -\Delta U = -q(V_f - V_i)$

$$4.8 \times 10^{-19} = -1.6 \times 10^{-19}(V_B - V_A)$$

$$(V_B - V_A) = -3.0\ \text{V}$$

Points B and C are on same equipotential surface.

Hence, $V_B = V_C$

$$V_C - V_A = V_B - V_A = -3.0 \text{ V}$$

$$V_C - V_B = 0$$

For Problems 24–25

24. a., 25. b.

Sol. The points A and C lies in equipotential line

$$\therefore \quad V_A = V_C$$

The points B and C lies in equipotential line

$$\therefore \quad V_B = V_D$$

$$V_A - V_D = V_A - V_B = -E\Delta x$$

$$V_A - V_B = -20\,(X_A - X_B) = -20 \times -1$$

$$V_A - V_B = 20 \text{ V}$$

$$V_A - V_D = V_A - V_B = 20 \text{ V}$$

For Problems 26–28

26. c., 27. b., 28. a.

Sol.

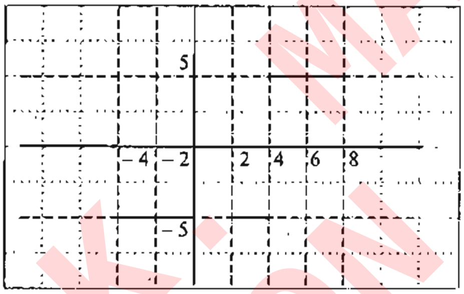


Fig. 3.138

$$E_r = -\frac{\Delta V}{\Delta r}; \quad E_x = -\frac{\Delta V}{\Delta x}$$

For $-2 \le x \le 0$; $E_x = -\dfrac{(10-0)}{[0-(-2)]} = -5 \text{ NC}^{-1}$

For $0 \le x \le 2$; $E_x = 0$

For $2 \le x \le 4$; $E_x = -\dfrac{[20-10]}{[4-2]} = -5 \text{ NC}^{-1}$

For $4 \le x \le 8$; $E_x = -\dfrac{[0-20]}{[8-4]} = 5 \text{ NC}^{-1}$

For Problems 29–30

29. c., 30. a.

Sol. $V_f - V_i = -\displaystyle\int_i^f (E_x i + E_y j + E_z k) \cdot (dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$

If electric field is constant, we can write,

$$V_f - V_i = -[E_x(x_f - x_i) + E_y(y_f - y_i) + E_z(z_f - z_i)]$$

$$V_f - V_i = -[100 \cos 30°(0+2) + 100 \sin 30°\,(4-0)]$$

$$V_B - V_A = -100\,[2 + \sqrt{3}] \text{ volt}$$

Alternatively:

In constant electric field

$$V_f - V_i = -\vec{E} \cdot \Delta\vec{r}$$

$$= -[100\,(\cos 30\,\hat{i} + \sin 30\hat{j})] \cdot [(10+2)\,i + (4-0)\,j]$$

$$= -100\,[2 + \sqrt{3}]\,V$$

Alternatively:

Drawing equipotential lines through A and B

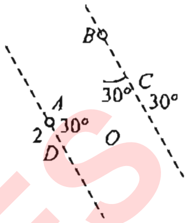


Fig. 3.139

$$DC = OB \cos 30° + OB \sin 30°$$

$$DC = 2 \times \frac{\sqrt{3}}{2} + 4 \times \frac{1}{2} = (\sqrt{3} + 2) \text{ m}$$

Potential decreases in the direction of electric field.

Hence $V_B - V_A$ should be negative.

For constant electric field

$$|E| = \left|\frac{\Delta V}{\Delta r}\right|$$

$$|\Delta V| = E\,|\Delta r| = 100\,(\sqrt{3} + 2)$$

$$V_B - V_A = -100\,(\sqrt{3} + 2)$$

For Problems 31–34

31. a., 32. b., 33. d., 34. a.

Sol. $E_x = -\dfrac{\delta V}{\delta x} = -3 \text{ Vm}^{-1}$; $E_y = -\dfrac{\delta V}{\delta y} = -4 \text{ Vm}^{-1}$

$$a_x = \frac{q\,E_x}{m} = -\frac{1 \times 10^{-6} \times 3}{10} = -3 \times 10^{-5} \text{ms}^{-2}$$

$$a_y = \frac{q\,E_y}{m} = -\frac{1 \times 10^{-6} \times 4}{10} = -4 \times 10^{-5} \text{ms}^{-2}$$

Time taken to cross x-axis:

Using $s = ut + \frac{1}{2}at^2$

$$3.2 = \frac{1}{2} \times 4 \times 10^{-5} \times t^2$$

$$t = 400 \text{ s}$$

$$v_x = a_x t = -3 \times 10^{-5} \times 400$$

$$v_x = 12 \times 10^{-3} \text{ms}^{-1}$$

$$v_y = a_y t = -4 \times 10^{-5} \times 400$$

$$v_y = 16 \times 10^{-3} \text{ms}^{-1}$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$v = 20 \times 10^{-3} \text{ms}^{-1}$$

For Problems 35–36

35. a. → iii., b. → ii., c. → i., 36. b.

Sol. $q_1 = \sigma 4\pi a^2,\ q_2 = -\sigma 4\pi b^2,\ q_3 = \sigma 4\pi c^2$

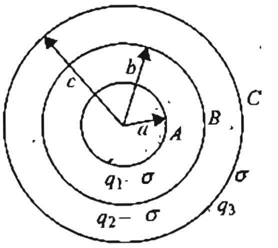


Fig. 3.140

$$V_A = \frac{kq_1}{a} + \frac{kq_2}{b} + \frac{kq_3}{c}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{\sigma 4\pi a^2}{a} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}[a - b + c]$$

Similarly $V_B = \frac{kq_1}{b} + \frac{kq_2}{b} + \frac{kq_3}{c}$,

$$V_C = \frac{kq_1}{c} + \frac{kq_2}{c} + \frac{kq_3}{c}$$

For problem 36, put $V_A = V_c$

For Problems 37–40

37. **b., 38. a., 39. c., 40. d.**

Sol. $1.8 \times 10^6 = \frac{9 \times 10^9 \times q_1}{(0.1)^2} \Rightarrow q_1 = 2 \times 10^{-6} \mu\text{ C}$

$$V = 9 \times 10^9 \times 2 \times^{-6}\left[\frac{1}{0.1} - \frac{1}{0.2}\right] = 9 \times 10^4$$

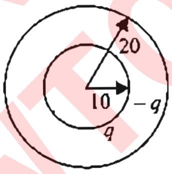


Fig. 3.141

38. $E = \frac{kq_1}{r^2} = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{(0.20)^2} = 4.5 \times 10^5$ N/C

39. Interaction energy

$$U = \frac{kq_1}{0.2}(-q_1) = \frac{9 \times 10^9 \times (2 \times 10^{-6})^2}{0.2} = -0.18 \text{ J}$$

Self-energy:

$$U_1 = \frac{9 \times 10^9 \times (2 \times 10^{-6})^2}{2 \times 0.1} = 0.18 \text{ J}$$

$$U_2 = \frac{9 \times 10^9 \times (2 \times 10^{-6})^2}{2 \times 0.2} = 0.09 \text{ J}$$

Total energy: $U + U_1 + U_2 = 0.09$ J

40. Both the charges will get neutralized and there will be no charge left on any sphere so no energy will be left in the system. It means whole amount of energy will convert into heat.

Matching Column Type

1. i. → a., d., ii. → c., e., iii. → c., e., iv. → c., e.

Sol. i. Due to q, chare will be induced on the conductor, such that net field due to q and induced chare becomes zero at any point inside the conductor.

Since $E = 0$ everywhere inside the conductor, so potential is constant inside and same as that of surface of conductor.

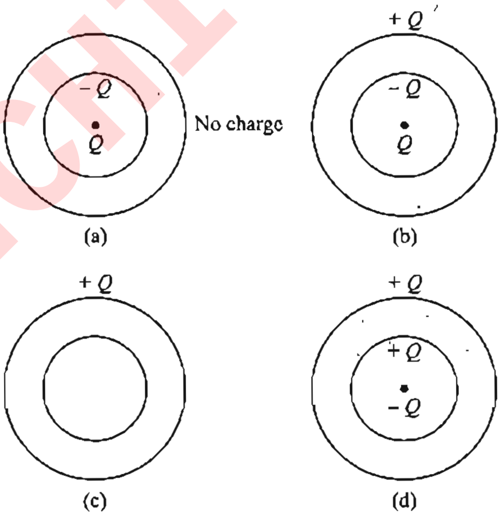
ii. Due to q, field and potential both will vary inside.

iii. Due to q_2, field and potential both will vary inside. Because inside charge system has nothing to do with outside system.

iv. Same as that of **iii.**

2. i. → b., g., ii. → c., h., iii. → a., f., iv. → d., e.

Sol. i. There is no charge on the outer surface, so not electric field at $r > a$ (Fig. 3.142(a)). E and V exist only inside $r > a$ as shown by **b.** and **g.**



(a) (b)

(c) (d)

Fig. 3.142

ii. No field from $r = 0$ to b, otherwise field at any other potential will be same as that produced by a point charge Q at center (Figure 3.142(b)).

iii. No charge inside, hence no electric field inside $r < b$. For $r \geq b$, field will be same as that produced by a potential charge at center (Fig. 3.142(c)).

iv. For $r < a$, field will be that of $-Q$ at center (Fig. 3.142(d)).

For $r > b$, field will be that of $+Q$ at center.

For $a < r < b$, no field.

3. i → d., ii. → c., iii. → b., iv. → a.

Sol. Charge induced at the surface of 'I'

$$q_1 = -q$$

Any point located on the surface of sphere 2 will have zero potential

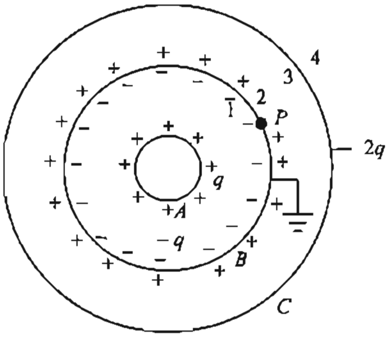


Fig. 3.143

$$V_P = 0 = V_{A,\text{out}} + V_{B,\text{surface}} + V_{C,\text{in}}$$

$$0 = k\frac{(q)}{2R} + k\frac{(-q+x)}{2R} + k\frac{(2q)}{3R}$$

Hence, $x = -\frac{4}{3}q$.

Hence, charge appearing on surface 2 is $-\frac{4}{3}q$ and charge appearing on surface 3 = $\frac{4}{3}q$ and charge appearing on surface 4

$$q_4 = 2q - \frac{4}{3}q = \frac{2q}{3}$$

CHAPTER

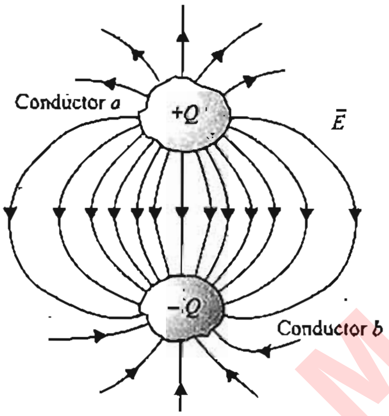
4

Capacitor and Capacitance

- Capacitor
- Units of Capacitance
- Combination of Capacitors
- Kirchhoff's Rules for Capacitors
- Sign Convention
- Dielectric
- Force on Dielectric Slab at Constant Potential Difference
- Spherical Capacitor
- Cylindrical Capacitor

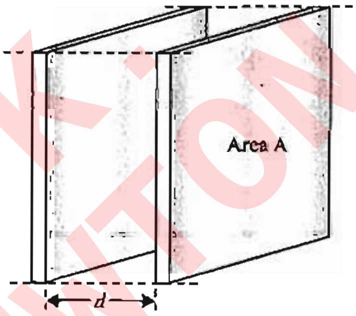
CAPACITOR

Any two conductors separated by an insulator (or a vacuum) form a capacitor (see Fig. 4.1). An electrical capacitor is not a device to store electric charge, but it stores electric energy in the form of electric field. Basic elements of a capacitor are two isolated conductors of arbitrary shape. No matter what their geometry is, these conductors are called *plates*. If these two isolated conductor plates are kept parallel to each other, the combination is called a parallel plate capacitor as shown in Fig. 4.2.



Any two conductors *a* and *b* insulated from one another form a capacitor.

Fig. 4.1



Parallel plate capacitor

Fig. 4.2

The symbol for a capacitor is –||– . Although this symbol is based upon the geometry of a parallel plate capacitor, but in fact, it is used for any type of capacitor, e.g., a parallel plate capacitor, a spherical capacitor or a cylindrical capacitor etc.

The region between the two conductor plates is generally filled by an insulating material called *dielectric*.

Practically, almost always, when a capacitor is charged, its plates have equal but opposite charges $+q$ and $-q$. However, we refer to the charge of a capacitor as being q, the absolute value of the charges on the plates. Please note it consciously that q is not the net charge on the capacitor, in fact the net charge on the capacitor is zero.

Or simply: when we say that a capacitor has charge q, or that a charge q is stored on the capacitor, it means that the conductor at higher potential has charge $+q$ and the conductor at lower potential has charge $-q$ (assuming that q is positive).

Because the plates are conductors, they are equipotential: i.e., all the points on a plate are at the same electric potential. Further, there is a potential difference between the two plates, which is denoted by V for historical reasons, rather than by ΔV.

The charge q and potential difference V for a capacitor are proportional to each other, i.e., $q = CV$.

The constant of proportionality C is called the capacitance of the capacitor. To determine the capacitance of a capacitor, electrical techniques can also be used.

The electric field at any point in the region between the conductors is proportional to the magnitude q of charge on each conductor. It follows that the potential difference V between the conductors is also proportional to q. If we double the magnitude of charge on each conductor, the charge density at each point doubles, the electric field at each point doubles and the potential difference between conductors doubles; however, the ratio of charge to potential difference does not change. This ratio is called the capacitance C of the capacitor as mentioned earlier: $C = \frac{q}{V}$ {definition of capacitance}.

Greater the capacitance C of a capacitor, the greater is the magnitude of charge q on either conductor for a given potential difference V and hence the greater is the amount of energy stored.

Value of C is independent to the charge q or potential difference V. It purely depends on the geometry and shape of the conductors forming the capacitor. It also depends upon the material medium placed between the conductors.

Note: ***A single conductor can also act as a capacitor by assuming that the other conductor is placed at infinity.***

UNITS OF CAPACITANCE

The S.I. unit of capacitance is coulomb per volt which is frequently written as **farad (F).** The symbol F is used for this. This is a very large unit. Some smaller units such as microfarad (μF), nanofarad (nF), picofarad (pF), etc. are also used frequently.

Parallel Plate Capacitor

It consists of two large plates placed parallel to each other with a separation d small in comparison to the two dimensions (length and breadth) of the plates (Fig. 4.3).

In an ideal capacitor, electric field resides in the region within the plates. No electric field is outside the plates (neglecting fringing effect for ideal case). So, the entire energy resides within the capacitor and no energy is therefore outside the capacitor. Electric field is directed from positive plate to negative plate in such a way that the lines emerge perpendicularly from positive plate and terminate perpendicularly to the negative plate.

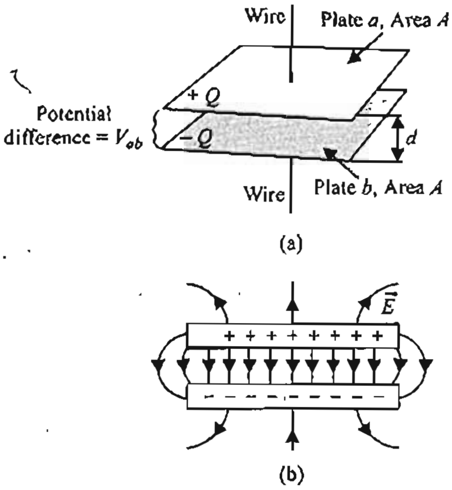
Electric field between the plates:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

Therefore, potential difference between the plates:

$$V = Ed = \frac{Qd}{A\varepsilon_0}$$

and therefore capacitance $C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$



(a) A charged parallel plate capacitor.
(b) When the separation of the plates is small compared to their size, the fringing of the electric field $\vec{E}$ at the edges is slight.

Fig. 4.3

Capacitance of a Spherical Conductor or Capacitor

As we have already said that a single conductor can also act as capacitor, here we will find the capacitance of a single isolated sphere (Fig. 4.4). For this, let a charge q is given to a spherical conductor of radius R, then potential on it

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

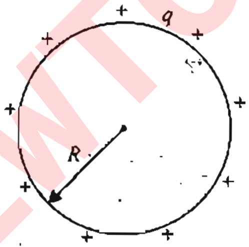


Fig. 4.4

The other conductor is supposed to be at infinity whose potential will be taken as zero.

So, the potential difference between sphere and the conductor at infinity $= V - 0 = V$

Then, capacitance $C = \frac{q}{V} = 4\pi\varepsilon_0 R$

Thus, capacitance of a spherical conductor is $C = 4\pi\varepsilon_0 R$.

Energy Stored in a Charged Conductor or Capacitor

Work has to be done in charging a conductor or capacitor against the force of repulsion by the already existing charges on it. The work is stored as a potential energy in the form of electric field of the conductor. Suppose a conductor of capacity C is to be charged up to a potential V_0 and let q_0 be the final charge on the conductor. Let the potential of the conductor when (during charging) the charge on it was q $(< q_0)$ is $V = \frac{q}{C}$.

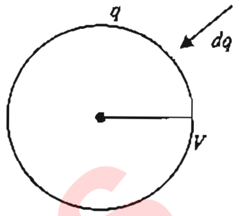


Fig. 4.5

Now, work done in bringing a small charge dq at this potential is $dW = V\,dq = \left(\frac{q}{C}\right)dq$

∴ Total work done in charging it from 0 to q_0 is

$$W = \int_0^{q_0} dW = \int_0^{q_0} \frac{q}{C}dq = \frac{1}{2}\frac{q_0^2}{C}$$

This work is stored as the potential energy.

$$\therefore \quad U = \frac{1}{2}\frac{q_0^2}{C}$$

Further, by using $q_0 = CV_0$, we can write this expression as, $U = \frac{1}{2}CV_0^2 = \frac{1}{2}q_0V_0$

In general, if a conductor (or capacitor) of capacity C is charged to a potential V by giving it a charge q, then

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV$$

Potential energy of a spherical capacitor made of a single sphere:

$$U = \frac{q^2}{2C} = \frac{q^2}{8\pi\varepsilon_0 R} \qquad (\because C = 4\pi\varepsilon_0 R)$$

Note: *It is a common misconception that electric field energy is a new kind of energy, different from the electric potential energy described before. This is not the case; it is simply a different way of interpreting electric potential energy. We can regard the energy of a given system of charges as being a shared property of all the charges, or we can think of the energy as being a property of the electric field that the charges create. Either interpretation leads to the same value of the potential energy.*

Force Between the Plates of a Parallel Plate Capacitor

Consider a parallel plate capacitor with plate area A. Suppose a positive charge $+Q$ is given to one plate and a negative charge $-Q$ to the other plate. The electric field due to only the positive plate is $E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$ at all points if the plate is large. The negative charge $-Q$ on the other plate finds itself in the field of this positive charge. Therefore, force on this plate: $F = EQ$ $= \frac{Q^2}{2A\varepsilon_0}$. This force will be attractive.

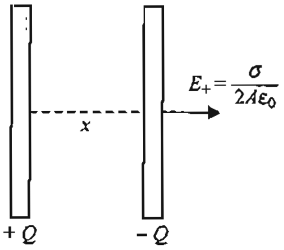


Fig. 4.6

Energy Density (Energy Per Unit Volume) in Electric Field

Consider a parallel plate capacitor of plate area A and plate separation d. Let the charge on capacitor is Q, then electric field in the region between the plates is: $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

Volume of capacitor: $V = Ad$

Energy stored: $U = \frac{Q^2}{2C} = \frac{Q^2 d}{2\varepsilon_0 A} = \left(\frac{Q}{A\varepsilon_0}\right)^2 \frac{1}{2}\varepsilon_0 Ad$

Energy per unit volume: $u = \frac{U}{V} = \left(\frac{Q}{A\varepsilon_0}\right)^2 \frac{1}{2}\frac{\varepsilon_0 Ad}{Ad}$

$$\Rightarrow \qquad u = \frac{1}{2}\varepsilon_0 E^2$$

Although we have proved the above result for a parallel plate capacitor but in general this is true for any kind of capacitor or any other kind of electric field.

Potential energy of a spherical capacitor made of a single sphere using concept of energy density:

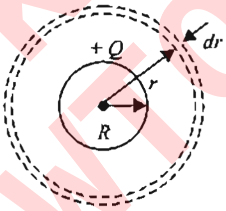


Fig. 4.7

Energy density : $u = \frac{1}{2}\varepsilon_0 E^2$

$$dU = u\,dV = \frac{1}{2}\varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\right)^2 4\pi r^2 dr$$

$$U = \frac{1}{8\pi\varepsilon_0}Q^2\int_R^\infty \frac{dr}{r^2} = \frac{Q^2}{8\pi\varepsilon_0 R}$$

Loss of Energy During Redistribution of Charge

Let two capacitors of capacitances C_1 and C_2 are charged separately up to potential differences of V_1 and V_2, respectively, as shown in Fig. 4.8(a). Now, suppose these two capacitors are connected (positive plate of one capacitor to the positive plate of other and negative plate to negative plate) with each other as shown in Fig. 4.8(b). If $V_1 \neq V_2$, then redistribution of charge will occur until the potentials of both capacitors become equal. Let V is the final common potential difference across each capacitor.

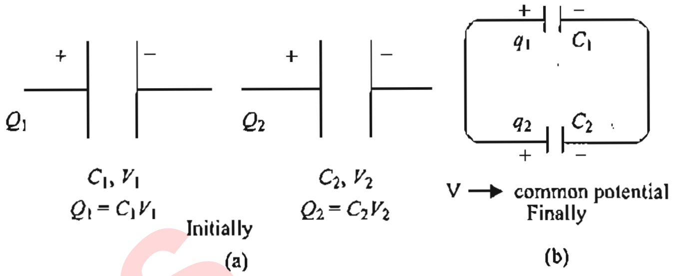


Fig. 4.8

Initial charges: $Q_1 = C_1V_1$, $Q_2 = C_2V_2$. Let final charges are q_1 and q_2 as shown in Fig. 4.8(b)

From conservation of charge: $q_1 + q_2 = Q_1 + Q_2$ (i)

And $V = \frac{q_1}{C_1} = \frac{q_2}{C_2}$ (ii)

Solving (i) and (ii), we get

$$V = \frac{Q_1 + Q_2}{C_1 + C_2}, q_1 = \frac{C_1(Q_1 + Q_2)}{C_1 + C_2}, q_2 = \frac{C_2(Q_1 + Q_2)}{C_1 + C_2}$$

We can show that in redistribution of charge energy is always lost.

Initial potential energy, $U_i = \frac{1}{2}\frac{Q_1^2}{C_1} + \frac{1}{2}\frac{Q_2^2}{C_2}$

Final potential energy,

$$U_f = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}\frac{(Q_1 + Q_2)^2}{C_1 + C_2}$$

Loss of energy:

$$\Delta U = U_i - U_f = \frac{1}{2}\left[\frac{Q_1^2}{C_1} + \frac{Q_2^2}{C_2} - \frac{(Q_1 + Q_2)^2}{C_1 + C_2}\right]$$

$$\text{or } \Delta U = \frac{1}{2C_1C_2(C_1 + C_2)}[Q_1^2C_1C_2 + Q_1^2C_2^2 + Q_2^2C_1^2$$
$$+ Q_2^2C_1C_2 - Q_1^2C_1C_2 - Q_2^2C_1C_2 - 2Q_1Q_2C_1C_2]$$

$$\Delta U = \frac{C_1^2C_2^2}{2C_1C_2(C_1 + C_2)}\left[\frac{Q_1^2}{C_1^2} + \frac{Q_2^2}{C_2^2} - \frac{2Q_1Q_2}{C_1C_2}\right]$$

$$\text{or} \quad \Delta U = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

Illustration 4.1 **Fig. 4.9 shows three conducting spherical shells A, B and C with charges $-q$, $+q/2$, $+q$, respectively. Determine the capacitance of the system between points A and C.**

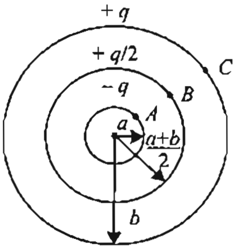


Fig. 4.9

Sol. Potential at A (V_A) = Potential due to charge on sphere A + Potential due to charge on sphere C + Potential due to charge on sphere B

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q}{a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{b} + \frac{q/2}{4\pi\varepsilon_0\,[(a+b)/2]}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a} + \frac{1}{(a+b)}\right)$$

Potential at C (V_C) = Potential due to charge on sphere A + Potential due to charge on sphere C + Potential due to charge on sphere B

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q}{b} + \frac{1}{4\pi\varepsilon_0}\frac{q}{2b} + \frac{1}{4\pi\varepsilon_0}\frac{q}{b} = \frac{q}{8\pi\varepsilon_0 b}$$

Potential difference $= V_A - V_C$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a} + \frac{1}{(a+b)}\right) - \frac{q}{8\pi\varepsilon_0 b}$$

$$= \frac{q}{8\pi\varepsilon_0}\left(\frac{a^2 - 2b^2 + ab}{ab\,(a+b)}\right)$$

Capacitance of the arrangement $= \dfrac{q}{V_A - V_C}$

$$= \frac{q}{\left[\dfrac{q}{8\pi\varepsilon_0}\left(\dfrac{a^2 - 2b^2 + ab}{ab\,(a+b)}\right)\right]} = \frac{8\pi\varepsilon_0 ab\,(a+b)}{(a^2 - 2b^2 + ab)}$$

Illustration 4.2 **Fig. 4.10 shows that a conducting sphere A of radius 'a' is surrounded by a neutral conducting spherical shell B of radius b ($> a$). Initially, switches S_1, S_2 and S_3 are open and the sphere A carries a charge Q. First, the switch S_1 is closed to connect the shell B with ground, and then S_1 is opened. Now, the switch S_2 is closed so that the sphere A is grounded and then S_2 is opened. Finally, the switch S_3 is closed to connect the spheres together. Find the heat produced on closing the switch S_3.**

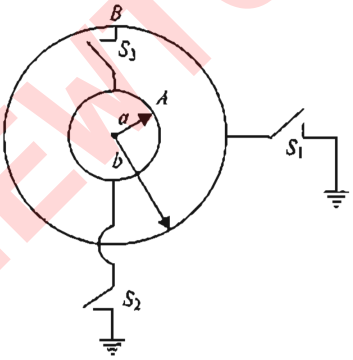


Fig. 4.10

Sol. When the outer sphere is connected to ground, charge $-Q$ resides on the inner surface of the sphere B, with no charge on its outer surface. When it is disconnected from the ground $-Q$ remains on its inner surface. Now, the sphere A is connected to the earth, so potential on its surface becomes zero.

Let the charge on the sphere A become q, then

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{a} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{b} = 0 \quad\Rightarrow\quad q = \frac{a}{b}\,Q.$$

The resulting charges on various surfaces are shown in Fig. 4.11.

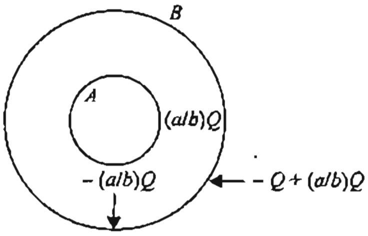


Fig. 4.11

In this position, energy stored

$$U_1 = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{1}{a}\left(\frac{a}{b}Q\right)^2 + \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{1}{b}Q^2 + \frac{1}{4\pi\varepsilon_0}\frac{1}{b}\left(\frac{a}{b}Q\right)(-Q) \qquad \text{(i)}$$

The above energy is sum of the energies of each of the individual spheres and their interaction energy (represented by the third term).

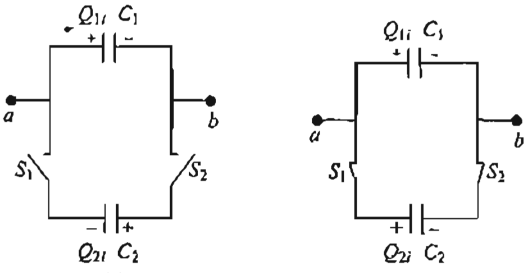
When switch S_3 is closed, total charge will appear on the outer surface of the shell B. In this position, energy stored

$$U_2 = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{1}{b}\left(\frac{a}{b} - 1\right)^2 Q^2$$

This energy is corresponding to the outer sphere only.

$$\text{Heat produced} = U_1 - U_2 = \frac{Q^2 a\,(b-a)}{8\pi\varepsilon_0 b^3}$$

Illustration 4.3 **Two capacitors C_1 and C_2 (where $C_1 > C_2$) are charged to the same initial potential difference, but with opposite polarity. The charged capacitors are removed from the battery and their plates are connected as shown in Fig. 4.12(a). The switches S_1 and S_2 are then closed, as shown in Fig. 4.12(b).**



(a) (b)

Fig. 4.12

a. Find the final potential difference between a and b after the switches are closed.

b. Find the total energy stored in the capacitors before and after the switches are closed and the ratio of the final energy to the initial energy.

Sol. a. The left-hand plates of the capacitors in Fig 4.12(b) act as an isolated system because they are not connected to the right hand plates by conductors. The charges on the left hand plates before the switches are closed are

$$Q_{1i} = C_1\,\Delta V_i \text{ and } Q_{2i} = C_2\,\Delta V_i$$

The total charge Q in the system is

$$Q = Q_{1i} + Q_{2i} = (C_1 - C_2)\,\Delta V_i \qquad \text{(i)}$$

After the switches are closed, the total charge in the system remains the same.

$$Q = Q_{1f} + Q_{2f} \qquad \text{(ii)}$$

The charges redistribute until the entire system is at the same potential ΔV_f. Thus, the final potential difference across C_1 must be the same as the final potential difference across C_2. The charges on the capacitors after the switches are closed are

$$Q_{1f} = C_1 \Delta V_f \text{ and } Q_{2f} = C_2 \Delta V_f$$

Dividing the first equation by the second, we have

$$\frac{Q_{1f}}{Q_{2f}} = \frac{C_1 \Delta V_f}{C_2 \Delta V_f} = \frac{C_1}{C_2} \quad \Rightarrow \quad Q_{1f} = \frac{C_1}{C_2} Q_{2f} \qquad \text{(iii)}$$

Combining equations (ii) and (iii), we obtain

$$Q = Q_{1f} + Q_{2f} = \frac{C_1}{C_2} Q_{2f} + Q_{2f} = Q_{2f}\left(1 + \frac{C_1}{C_2}\right)$$

$$\Rightarrow \quad Q_{2f} = Q\left(\frac{C_2}{C_1 + C_2}\right)$$

Using equation (iii) to find Q_{1f} in terms of Q, we have

$$Q_{1f} = \frac{C_1}{C_2} Q_{2f} = \frac{C_1}{C_2} Q\left(\frac{C_2}{C_1 + C_2}\right) = Q\left(\frac{C_1}{C_1 + C_2}\right)$$

Finally, voltage across each capacitor is

$$\Delta V_{1f} = \frac{Q_{1f}}{C_1} = \frac{Q\left(\dfrac{C_1}{C_1 + C_2}\right)}{C_1} = \frac{Q}{C_1 + C_2}$$

$$\text{and } \Delta V_{2f} = \frac{Q_{2f}}{C_2} = \frac{Q\left(\dfrac{C_2}{C_1 + C_2}\right)}{C_2} = \frac{Q}{C_1 + C_2}$$

As noted earlier, $\Delta V_{1f} = \Delta V_{2f} = \Delta V_f$

To express ΔV_f in terms of the given quantities C_1, C_2 and ΔV_i, we substitute the value of Q from equation (i) to obtain

$$\Delta V_f = \left(\frac{C_1 - C_2}{C_1 + C_2}\right) \Delta V_i$$

b. Before the switches are closed, the total energy stored in the capacitors is

$$U_i = \frac{1}{2} C_1 (\Delta V_i)^2 + \frac{1}{2} C_2 (\Delta V_i)^2 = \frac{1}{2}(C_1 + C_2)\,(\Delta V_i)^2$$

After the switches are closed, the total energy stored in the capacitors is

$$U_f = \frac{1}{2} C_1 (\Delta V_f)^2 + \frac{1}{2} C_2 (\Delta V_f)^2 = \frac{1}{2}(C_1 + C_2)\,(\Delta V_f)^2$$

$$= \frac{1}{2}(C_1 + C_2)\left(\frac{Q}{C_1 + C_2}\right)^2 = \frac{1}{2}\frac{Q^2}{C_1 + C_2}$$

Using equation (i), we can express this as

$$U_f = \frac{1}{2}\frac{Q^2}{(C_1 + C_2)} = \frac{1}{2}\frac{(C_1 - C_2)^2\,(\Delta V_i)^2}{(C_1 + C_2)}$$

Therefore, the ratio of the final energy stored to the initial energy stored is

$$\frac{U_f}{U_i} = \frac{\dfrac{1}{2}\dfrac{(C_1 - C_2)^2\,(\Delta V_i)^2}{(C_1 + C_2)}}{\dfrac{1}{2}(C_1 + C_2)\,(\Delta V_i)^2} = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)^2$$

Illustration 4.4 **Four identical metal plates are arranged in air at equal distance d from each other. The area of each plate is A. A battery of e.m.f. V is connected across the plates 1 and 2. Discuss the charge distribution and the find capacitance of the system between points 1 and 2 if the other two plates are connected by a conducting wire as shown in Fig. 4.13.**

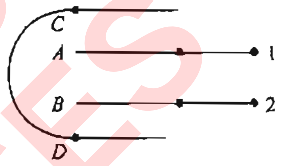


Fig. 4.13

Sol. Method 1: Imagine a battery connected to points 1 and 2, so that there is a charge $+Q$ on plate A and $-Q$ on plate B. Charge on A and B would induce a charge $-Q'$ on C while $+Q'$ on D, plates C and D remain neutral.

Consequently, charge Q on A is divided into two parts: $+Q'$ on the left side of plate A and $(Q - Q')$ on right side. Similarly, charge $-Q$ on B is also divided, $-Q'$ on right side and $-(Q - Q')$ on left side on plate B.

Due to charges on the plates, electric fields will appear in the gaps between the plates. Fields E' between AC and DB would be same, while between AB, field E would be different.

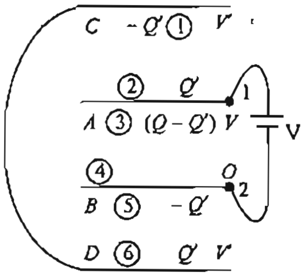


Fig. 4.14

Let the potential of plate A be V and that of plate B be zero. As the plates C and D are connected together, they will be at common potential, say V'.

The capacity of the system can be written as

$$C = \frac{Q}{V_+ - V_-} = \frac{Q}{V} \qquad \text{(i)}$$

We can write equation for facing surfaces (1) and (2)

$$Q' = C_0(V - V') \qquad \text{(ii)}$$

For facing surfaces (3) and (4)

$$(Q - Q') = C_0(V - 0) \qquad \text{(iii)}$$

For facing surfaces (5) and (6)

$$Q' = C_0(V' - 0) \qquad \text{(iv)}$$

From equations (ii), (iii) and (iv)

$$\frac{Q}{V} = \frac{3}{2} C_0 = \frac{3}{2} \frac{\varepsilon_0 A}{d} \text{ and } Q' = \frac{C_0 V}{2} = \frac{\varepsilon_0 A V}{2d}$$

Hence, equivalent capacitance is $C_{eq} = \frac{3}{2}\frac{\varepsilon_0 A}{d}$.

And charges on different surfaces are

Surfaces	**Charges**
(1), (5)	$-\frac{\varepsilon_0 A V}{2d}$
(2), (6)	$+\frac{\varepsilon_0 A V}{2d}$
(3)	$\frac{\varepsilon_0 A V}{d}$
(4)	$-\frac{\varepsilon_0 A V}{d}$

Method 2: Equivalent circuit can be draws as shown in Fig. 4.15.

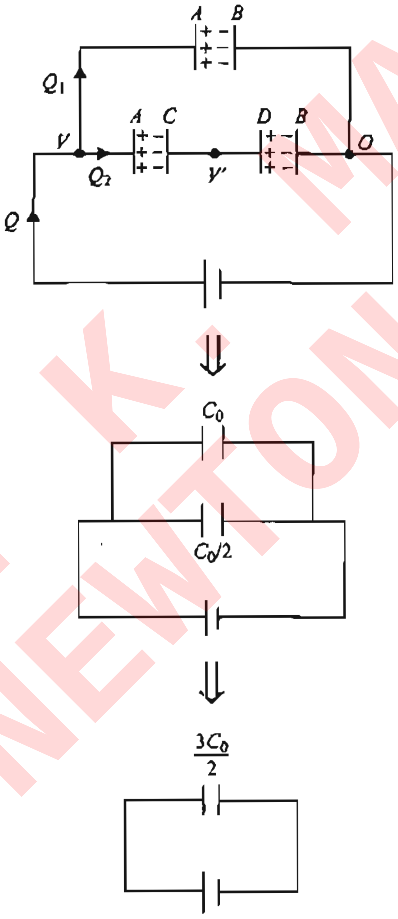


Fig. 4.15

Hence, equivalent capacitance $C_{eq} = \frac{3}{2}C_0 = \frac{3}{2}\frac{\varepsilon_0 A}{d}$

Charge supplied by battery $Q = \frac{3}{2}\frac{\varepsilon_0 A}{d} V$

Hence, $Q_1 = Q\left(\frac{C_0}{C_0 + C_0/2}\right) = \frac{2}{3}Q$ and $Q_2 = \frac{Q}{3}$

Hence, charge on different surfaces can be calculated.

Concept Application Exercise 4.1

1. Suppose the two plates of a capacitor have different areas. When the capacitor is charged by connecting it to a battery, do the charges on the two plates have equal magnitude, or can they be different? Explain your reasoning.
2. The charged plates of a capacitor attract each other, so to pull the plates farther apart requires work by some external force. What becomes of the energy added by this work ? Explain your reasoning.
3. The two plates of a capacitor are given charges $\pm Q$. The capacitor is then disconnected from the charging device so that the charges on the plates can not change, and the capacitor is immersed in a tank of oil. Does the electric field between the plates increase, decrease or stay the same? Explain your reasoning. How can this field be measured?
4. **a.** How many excess electrons must be added to one plate and removed from the other to give a 5.00 nF parallel plate capacitor 25.0 J of stored energy?
   b. How could you modify the geometry of this capacitor to get it to store 50.0 J of energy without changing the charge on its plates?
5. A capacitor of capacitance C is charged to a potential difference V from a cell and then disconnected from it. A charge $+Q$ is now given to its positive plate. Find the potential difference across the capacitor.
6. A capacitor is connected across a battery.
   a. Why does each plate receive a charge of exactly the same magnitude?
   b. Is this true even if the plates are of different size? (Yes/No)
7. Three identical large metallic plates are placed parallel to each other at very small separation as shown in the figure 4.16. The central plate is given a charge Q. What amount of charge will flow to earth when key is pressed?

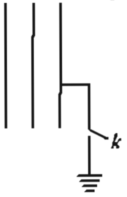


Fig. 4.16

8. The plates of a plane capacitor are drawn apart keeping them connected to a battery. Next, the same plates are drawn apart from the same initial condition keeping battery disconnected. In which case is more work done?
9. If a small charge q is moved along a closed path in the field between the plates of a parallel plate capacitor, will any work be done by the agent which moves the charge? (Yes/No)
10. At which of the two points, 1 or 2, of a charged capacitor with non-parallel plates is the surface charge density greater?

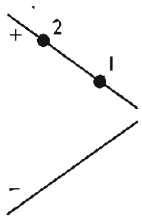


Fig. 4.17

11. A parallel plate air capacitor is connected to a battery. If plates of the capacitor are pulled farther apart, then state whether the following statements are true or false.

a. Strength of electric field inside the capacitor remains unchanged, if battery is disconnected before pulling the plates.

b. During the process, work is done by external force applied to pull the plates irrespective of whether the battery is disconnected or not.

c. Strain energy in the capacitor decreases if the battery remains connected.

COMBINATION OF CAPACITORS

Sometimes to obtain a desired value of capacitance, we group two or more than two capacitors together. In this section, we will learn how to find the equivalent capacitance when two or more than two capacitors are grouped together?

Equivalent capacitance of a group of capacitors is that value of capacitance of a single capacitor which will allow to flow (or store) the same amount of charge for a given potential difference as done by the combination.

In other words, that single capacitor will perform the same task as done by the combination in all respects.

Capacitors can be grouped in many ways, but the two common types of combinations are:

(i) Series combination and (ii) Parallel combination.

Capacitors Connected in Series

Let n capacitors are connected in series as shown in Fig. 4.18. C_{eq} is the equivalent capacitance of this system.

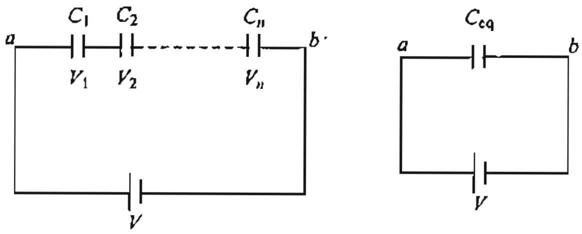


Fig. 4.18

Let a battery V is applied across the combination, then

$$V = V_1 + V_2 + \cdots + V_n$$

$$\Rightarrow \quad \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \cdots + \frac{Q}{C_n}$$

$$\Rightarrow \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

Thus, the reciprocal of the equivalent capacitance of a series combination equals the sum of the reciprocals of the individual capacitances.

Note:

- *In series combination, the equivalent capacitance is always less than any of the individual capacitance.*
- *In series combination, charge on each capacitor is same but potential is different. From $V = q/C$, larger the capacitance lesser is the potential.*

Energy in Series Combination

$$U = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \cdots + \frac{Q^2}{2C_n}$$

$$= \frac{Q^2}{2}\left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}\right) = \frac{Q^2}{2C_{eq}}$$

Thus, a single capacitor of capacitance C_{eq} will store the same amount of energy as stored by $C_1, C_2, \ldots, C_n$ when connected in series.

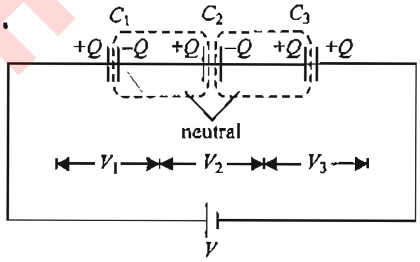


Fig. 4.19

As an example, let us take the case of a circuit consisting of three capacitors in series as shown in Fig. 4.19. Charge on each capacitor is same but potential can be different.

$$Q = C_1V_1 = C_2V_2 = C_3V_3 \qquad (1)$$

$$V = V_1 + V_2 + V_3 \qquad (2)$$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \qquad (3)$$

or
$$Q = \frac{V}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} = \frac{C_1C_2C_3V}{C_1C_2 + C_2C_3 + C_1C_3}$$

Hence,
$$V_1 = \frac{Q}{C_1} = \frac{C_2C_3V}{C_1C_2 + C_2C_3 + C_3C_1},$$

$$V_2 = \frac{Q}{C_2} = \frac{C_3C_1V}{C_1C_2 + C_2C_3 + C_3C_1}$$

and
$$V_3 = \frac{Q}{C_3} = \frac{C_1C_2V}{C_1C_2 + C_2C_3 + C_3C_1}$$

Special Case of Two Capacitors in Series

Let two capacitors C_1 and C_2 are connected in series and a potential difference V is applied across them as in Fig. 4.20.

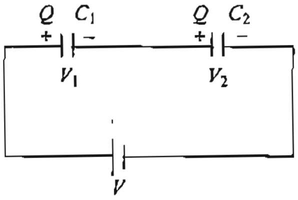


Fig. 4.20

Potential appearing on capacitors are V_1 and V_2, respectively. Q is the net charge that flows in circuit, then

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{eq} = \frac{C_1C_2}{C_1 + C_2}$$

$$V_1 + V_2 = V \qquad \text{(i)}$$

$$Q = C_1V_1 = C_2V_2 \qquad \text{(ii)}$$

From (ii), $\frac{V_1}{V_2} = \frac{C_2}{C_1}$ (iii)

It means potential will be divided on the capacitors in the inverse ratio of their capacitances.

From (i) and (iii), $V_1 = \frac{C_2V}{C_1 + C_2}$, $V_2 = \frac{C_1V}{C_1 + C_2}$

In the above discussion, take C_1 = 4 μF, C_2 = 8 μF and V = 12 V, then

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} = \frac{4 \times 8}{4 + 8} = \frac{8}{3}\ \mu\text{F},$$

$$V_1 = \frac{C_2V}{C_1 + C_2} = \frac{8 \times 12}{4 + 8} = 8\ \text{V},$$

$$V_2 = \frac{C_1V}{C_1 + C_2} = \frac{4 \times 12}{4 + 8} = 4\ \text{V}$$

Energy in C_1: $U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2} \times 4 \times (8)^2 = 128\ \mu\text{J}$

Energy in C_2: $U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2} \times 8 \times (4)^2 = 64\ \mu\text{J}$

Total energy: $U = U_1 + U_2 = 128 + 64 = 192\ \mu\text{J}$

We can find total energy by using equivalent capacitance. Equivalent capacitance will store the same amount of energy as stored by the combination on applying the same potential, i.e.,

$$U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2} \times \frac{8}{3} \times (12)^2 = 192\ \mu\text{J}$$

Capacitors Connected in Parallel

Let n capacitors are connected in parallel as shown in Fig. 4.21. C_{eq} is the equivalent capacitance of this system.

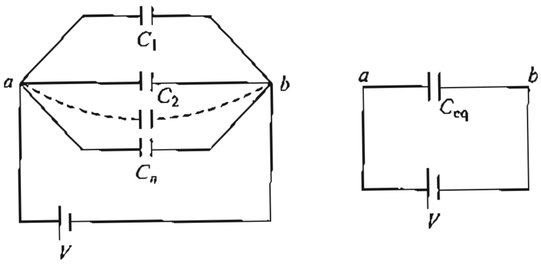


Fig. 4.21

Let total charge q flows through the battery, then

$$q = q_1 + q_2 + \cdots + q_n$$

$$\Rightarrow C_{eq}V = C_1V + C_2V + \cdots + C_nV$$

$$\Rightarrow \quad C_{eq} = C_1 + C_2 + \cdots + C_n$$

The equivalent capacitance of a parallel combination equals the sum of the individual capacitances.

Note:

- *In a parallel combination, the equivalent capacitance is always greater than any individual capacitance.*
- *In a parallel combination, the potential on each capacitor is same but charge may be different. From q = CV, greater the capacitance greater is the charge.*
- *Charge is distributed on the capacitors in the ratio of their capacitances, i.e.,*

$$q_1 : q_2 : \ldots : q_n = C_1 : C_2 : \ldots : C_n$$

Energy in Parallel Combination

$$U = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \cdots + \frac{1}{2}C_nV^2$$

$$V^2 = \frac{1}{2}(C_1 + C_2 + \cdots + C_n)V^2 = \frac{1}{2}C_{eq}V^2$$

Thus a single capacitor of capacitance C_{eq} will store the same amount of energy as stored by $C_1, C_2, \ldots, C_n$ when connected in parallel.

Special Case of Two Capacitors in Parallel

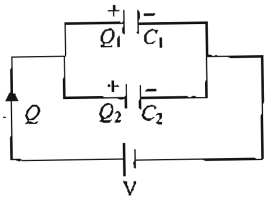


Fig. 4.22

Let two capacitors C_1 and C_2 are connected in parallel and a potential difference V is applied across them as in Fig. 4.22. Then

$$C_{eq} = C_1 + C_2$$

$$Q_1 + Q_2 = Q \qquad \text{(i)}$$

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \qquad \text{(ii)}$$

From (ii), $\frac{Q_1}{Q_2} = \frac{C_1}{C_2}$ (iii)

It means charge will be divided in the direct ratio of the capacitances.

From (i) and (iii), $Q_1 = \frac{C_1Q}{C_1 + C_2}$, $Q_2 = \frac{C_2Q}{C_1 + C_2}$

In the above discussion, take C_1 = 4 μF, C_2 = 8 μF and V = 12 V

$$C_{eq} = C_1 + C_2 = 4 + 8 = 12\ \mu\text{F},$$

$$Q_1 = C_1V = 4 \times 12 = 48\ \mu\text{C},$$

$$Q_2 = C_2V = 8 \times 12 = 96\ \mu\text{C}$$

Total charge that flows through the battery:

$$Q = Q_1 + Q_2 = 48 + 96 = 144\ \mu\text{C}$$

Also: $Q = C_{eq}V = 12 \times 12 = 144\ \mu\text{C}$,

$$Q_1 = \frac{C_1Q}{C_1 + C_2} = \frac{4 \times 144}{4+8} = 48\ \mu\text{C},$$

$$Q_2 = \frac{C_2Q}{C_1 + C_2} = \frac{8 \times 144}{4+8} = 96\ \mu\text{C}$$

Energy stored in C_1: $U_1 = \frac{1}{2}C_1V^2 = \frac{1}{2} \times 4 \times 12^2 = 288\ \mu\text{J}$

Energy stored in C_2: $U_2 = \frac{1}{2}C_2V^2 = \frac{1}{2} \times 8 \times 12^2$
$= 576\ \mu\text{J}$

Total energy: $U = U_1 + U_2 = 288 + 576 = 864\ \mu\text{J}$

Also: $U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2} \times 12 \times (12)^2 = 864\ \mu\text{J}$

Illustration 4.5 **In the circuit shown in Fig. 4.23, the P.D. between the points *a* and *b* is 4 volt. Find the e.m.f. *E* of the battery. Assume that before connecting the battery in the circuit, all the capacitors were uncharged.**

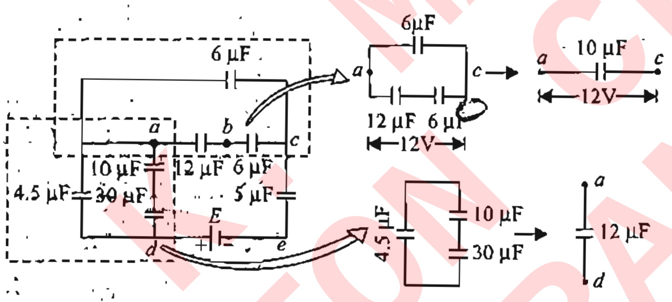


Fig. 4.23

Sol. The capacitors 12 μF and 6 μF are in series arrangement. The charge in each will be equal $q_1 = CV = 12 \times 4\ \mu\text{C}$

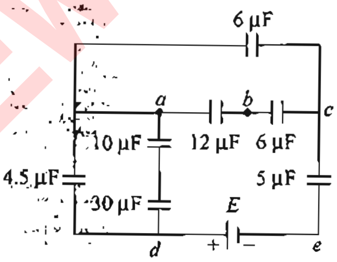


Fig. 4.24

The P.D. across 6 μF capacitors $= \frac{12 \times 4}{6}\ V = 8\ \text{V}$

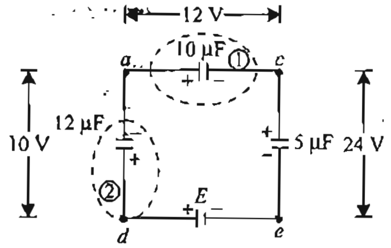


Fig. 4.25

The P.D. between points *a* and *c* $= V_a - V_c = 4 + 8 = 12$ V

The equivalent capacity in the portion (1) is 10 μF.

The charge on it $= (10 \times 12)\ \mu\text{C} = 120\ \mu\text{C}$

The portions (1) and (2) of the circuit are in series combination.

Hence, $12 \times (V_d - V_a) = 120\ \mu\text{C} \quad \Rightarrow \quad V_d - V_a = 10\ \text{V}$

The capacitor 5 μF is also in series with 10 μF, see figure. Hence, charge on it is 120 μC.

Thus, $V_c - V_e = \frac{120}{5} = 24$ V

Now, we apply KVL to circuit in Fig. 4.25. Beginning at point *d* and traversing the circuit clockwise, we get

$$-[(V_d - V_a)] + (V_c - V_a) + (V_e - V_c) + E = 0$$
$$E = (12 + 10 + 24)\ \text{V} = 46\ \text{V}$$

Illustration 4.6 **A capacitor is made of a flat plate of area *A* and a second plate having a stair-like structure as shown in Fig. 4.26. The area of each stair is *A*/3 and the height is *d*. Find capacitance of this arrangement.**

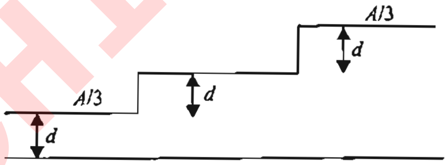


Fig. 4.26

Sol. The arrangement is a parallel combination of three capacitors. Each capacitor has a plate area $\frac{A}{3}$ and the separation between the plates as d, $2d$ and $3d$, respectively.

$$C_1 = \frac{\varepsilon_0 A/3}{d} = \frac{\varepsilon_0 A}{3d},\ C_2 = \frac{\varepsilon_0 A/3}{2d} = \frac{\varepsilon_0 A}{6d} \text{ and}$$

$$C_3 = \frac{\varepsilon_0 A/3}{3d} = \frac{\varepsilon_0 A}{9d}$$

As these three capacitances are in parallel, their equivalent capacitance is given by

$$C = C_1 + C_2 + C_3 = \frac{\varepsilon_0 A}{3d} + \frac{\varepsilon_0 A}{6d} + \frac{\varepsilon_0 A}{9d}$$

$$= \frac{\varepsilon_0 A}{3d}\left[1 + \frac{1}{2} + \frac{1}{3}\right] \Rightarrow \quad C = \frac{11\varepsilon_0 A}{18\,d}$$

Illustration 4.7 **Three capacitors of capacitances 3 μF, 6 μF and 4 μF are connected as shown across a battery of e.m.f. 6 V.**

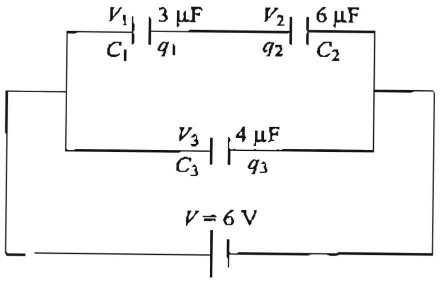


Fig. 4.27

1. Find equivalent capacitance.
2. Find potential difference and charge on each capacitor.
3. Find energy stored in each capacitor and total energy stored in the system of capacitors.

Sol.

1. C_1 and C_2 are in series, so their equivalent capacitance is:

$$C' = \frac{C_1C_2}{C_1+C_2} = \frac{3\times 6}{3+6} = 2\ \mu\text{F}$$

This C' will be in parallel with C_3, so $C_{eq} = C' + C_3$

$= 2 + 4 = 6\ \mu$F

2. 6 V will be divided across C_1 and C_2 in the inverse ratio of the capacitances. So

$$\text{p.d. across } C_1\text{: } V_1 = \frac{C_2V}{C_1+C_2} = \frac{6\times 6}{3+6} = 4\ \text{V},$$

$$\text{p.d. across } C_2\text{: } V_2 = \frac{C_1V}{C_1+C_2} = \frac{3\times 6}{3+6} = 2\ \text{V}$$

Because C_3 is connected directly across the battery without any other capacitor in between, so

p.d. across C_3: $V_3 = V = 6$ V.

Charge on C_1: $q_1 = C_1V_1 = 3\times 4 = 12\ \mu$C, Charge on C_2: $q_2 = C_2V_2 = 6\times 2 = 12\ \mu$C

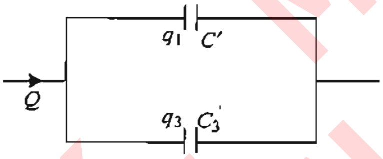


Fig. 4.28

We see that $q_1 = q_2$, and it should have been, because in series charge is same.

Charge on C_3: $q_3 = C_3V_3 = 4\times 6 = 24\ \mu$C

Alternative method to find charge: We can find charge in this way also.

Total charge that will flow through the battery:

$$Q = C_{eq}V = 6\times 6 = 36\ \mu\text{C}$$

It will be divided in direct ratio of C' and C_3, so

$$q_1 = \frac{C'Q}{C'+C_3} = \frac{2\times 36}{2+4} = 12\ \mu\text{C},$$

$$q_3 = \frac{C_3Q}{C'+C_3} = \frac{4\times 36}{2+4} = 24\ \mu\text{C}$$

3. Energy in C_1: $U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}\times 3\times 4^2 = 24\ \mu$J

Energy in C_2: $U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}\times 6\times 2^2 = 12\ \mu$J

Energy in C_3 : $U_3 = \frac{1}{2}C_3V^2 = \frac{1}{2}\times 4\times 6^2 = 72\ \mu$J

Total energy: $U = U_1 + U_2 + U_3 = 24 + 12 + 72 = 108\ \mu$J

Alternatively: $U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}\times 6\times 6^2 = 108\ \mu$J

Illustration 4.8 **Three capacitors of capacitances 4 μF, 4 μF and 8 μF are connected as shown across a battery of e.m.f. 12 V.**

1. **Find equivalent capacitance.**
2. **Find potential difference and charge on each capacitor.**
3. **Find energy stored in each capacitor and total energy stored in the system of capacitors.**

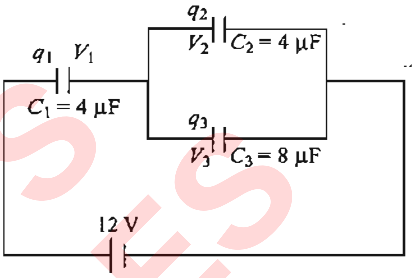


Fig. 4.29

Sol.

1. C_2 and C_3 are in parallel, so their equivalent capacitance:

$C' = C_2 + C_3 = 4 + 8 = 12\ \mu$F

This C' will be in series with C_1, so net equivalent capacitance: $C_{eq} = \frac{C_1C'}{C_1+C'} = \frac{4\times 12}{4+12} = 3\ \mu$F

2. $q_1 = C_{eq}V = 3\times 12 = 36\ \mu$C. q_1 will be divided into q_2 and q_3, so

$$q_2 = \frac{C_2q_1}{C_2+C_3} = \frac{4\times 36}{4+8} = 12\ \mu\text{C}$$

$$q_3 = \frac{C_3q_1}{C_2+C_3} = \frac{8\times 36}{4+8} = 24\ \mu\text{C}$$

$$V_1 = \frac{q_1}{C_1} = \frac{36}{4} = 9\ \text{V},\ V_2 = V_3 = \frac{q_2}{C_2} = \frac{q_3}{C_3} = 3\ \text{V}$$

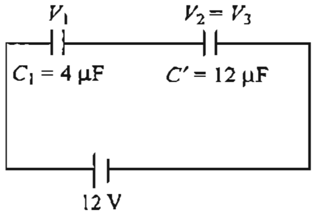


Fig. 4.30

Potentials can also be found like this: divide 12 V in inverse ratio of C_1 and C'.

$$V_1 = \frac{C'V}{C_1+C'} = \frac{12\times 12}{4+12} = 9\ \text{V},$$

$$V_2 = V_3 = \frac{C_1V}{C_1+C'} = \frac{4\times 12}{4+12} = 3\ \text{V}$$

3. $U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}\times 4\times 9^2 = 162\ \mu$J

$U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}\times 4\times 3^2 = 18\ \mu$J,

$U_3 = \frac{1}{2}C_3V_3^2 = \frac{1}{2}\times 8\times 3^2 = 36\ \mu$J

Total energy: $= U_1 + U_2 + U_3 = 216\ \mu$J

Alternatively: $U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}\times 3\times (12)^2 = 216\ \mu$J

KIRCHHOFF'S RULES FOR CAPACITORS

Kirchhoff's rules can be used to determine the p.d. and charge on the plates of a capacitor in any electric circuit. It is being explained below with the help of an example.

Illustration 4.9 **Find out charges Q_1 and Q_2 on the capacitors C_1 and C_2 in the circuit shown in Fig. 4.31.**

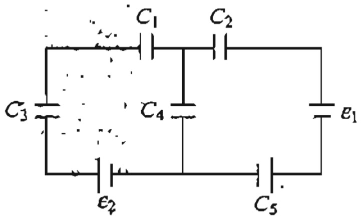


Fig. 4.31

Sol. Step I. Identify isolated regions (IRs), considering cells as continuous circuit elements and capacitances as discontinuous elements.

An IR is a continuous region or group of circuit branches in which you can travel from any one point to any other point in that region. There are four such regions in the given circuit, depicted by lines having different thickness, namely

(1) C_3 and C_1, (2) C_1, C_2 and C_4, (3) C_3, ε_2, C_4 and C_5, (4) C_2, ε_1 and C_5.

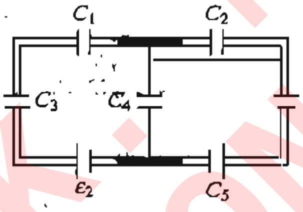


Fig. 4.32

Conservation of Charge: Electric charge will remain conserved in every isolated region.

Step II. Allocate charges on various plates of each capacitor rationally on the basis of following points:

a. Please note that cells do not supply any extra charge. They only cause charge to flow from one point to another.

b. The algebraic sum of all the plate charges in each IR must be zero (conservation of charge).

c. The two plates of each capacitor must be shown to have equal and opposite charges.

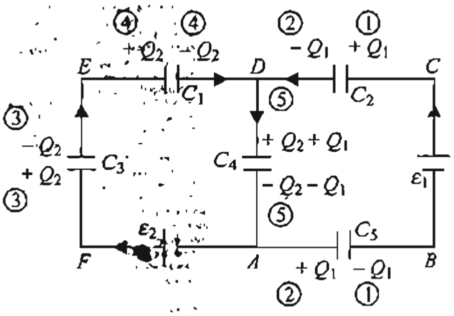


Fig. 4.33

This is done in five steps depicted by 1, 2, 3, 4 and 5 in the given circuit (Fig. 4.33).

Now, calculate and show p.d. across the plates of each capacitor, using the formula $V = \dfrac{Q}{C}$.

Note: ***Naturally, the plates having (+) charge will have positive potential and those with (−) charge will have negative potential.***

The algebraic sum of potential changes in each closed loop must be zero.

1. For determining a closed loop, consider each capacitor as well as cell as a continuous circuit element. Please note that capacitors were considered as discontinuous elements while identifying IRs in step 1.
2. A closed loop is the path travelled by you to start from any point and reach the same point again.
3. Sign convention is same as in the case of resistances.

SIGN CONVENTION

If you travel from (+)vely charged plate of a capacitor or terminal of a cell to (−)vely charged plate of a capacitor or terminal of a cell, then the sign of V or ε is (−) and vice versa.

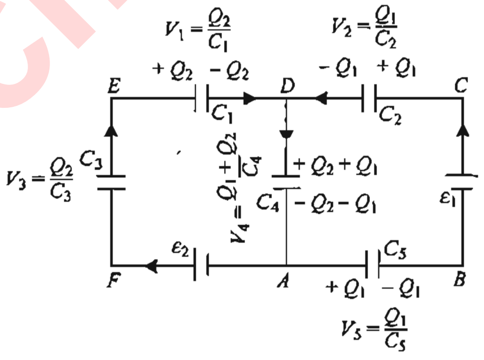


Fig. 4.34

Applying rule (2) in closed loop *ABCDA*

$$-\frac{Q_1}{C_5}+\varepsilon_1-\frac{Q_1}{C_2}-\frac{Q_1+Q_2}{C_4}=0$$

$$\Rightarrow \quad Q_1\left(\frac{1}{C_2}+\frac{1}{C_4}+\frac{1}{C_5}\right)+\frac{Q_2}{C_4}=\varepsilon_1 \qquad \text{(i)}$$

Applying rule (2) in closed loop *ADEFA*

$$\frac{Q_1+Q_2}{C_4}+\frac{Q_2}{C_1}+\frac{Q_2}{C_3}-\varepsilon_2=0$$

$$\Rightarrow \quad \frac{Q_1}{C_4}+Q_2\left[\frac{1}{C_1}+\frac{1}{C_3}+\frac{1}{C_4}\right]=\varepsilon_2 \qquad \text{(ii)}$$

Now, you have got two equations (i) and (ii) and two unknown Q_1 and Q_2 to solve. This can be done easily.

For the sake of simplicity,

If $C_1 = C_2 = C_3 = C_4 = C_5 = 1$ μF and $Q_2 = 0$ (i.e., $V_3 = 0$), what should be the values of ε_1 and ε_2? The previous two equations become

$3Q_1 = \varepsilon_1$ and $Q_1 = \varepsilon_2 \Rightarrow \varepsilon_1 = 3\varepsilon_2$

Illustration 4.10 **Consider the following circuit. Find out the charge and potential difference across capacitor C_1 of capacitance 1 μF.**

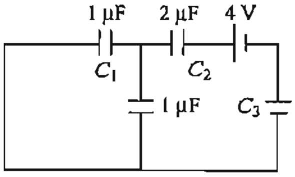


Fig. 4.35

Sol. Applying Kirchhoff's rule to the closed loop *ABFEA* (Fig. 4.36), we get

$$+\frac{Q_2-Q_1}{C_4}-\frac{Q_1}{C_1}=\frac{Q_2-Q_1}{1\times10^{-6}}-\frac{Q_1}{1\times10^{-6}}=0$$

$$Q_2=2Q_1 \qquad \text{(i)}$$

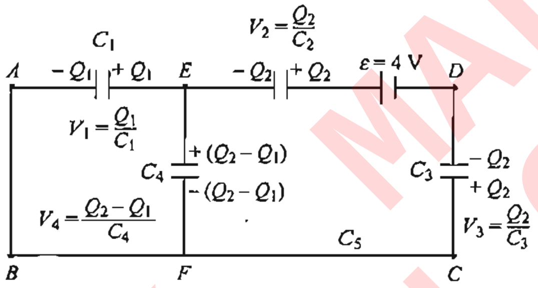


Fig. 4.36

Applying Kirchhoff's rule to closed loop *ABCDA*, we get

$$-\frac{Q_2}{C_3}+4-\frac{Q_2}{C_2}-\frac{Q_1}{C_1}=0$$

$$\Rightarrow \quad \frac{Q_1}{C_1}+Q_2\left(\frac{1}{C_2}+\frac{1}{C_3}\right)=4$$

$$\frac{Q_1}{1\times10^{-6}}+Q_2\left(\frac{1}{2\times10^{-6}}+\frac{1}{2\times10^{-6}}\right)=4$$

$$\Rightarrow \quad Q_1+Q_2=4\times10^{-6} \qquad \text{(ii)}$$

From equation (i) and (ii), we get

$$Q_1=\frac{4}{3}\mu\text{C and } V_1=\frac{4}{3}\text{ V}$$

Illustration 4.11 **In the circuit shown in fig. 4.37, find the charge on each capacitor.**

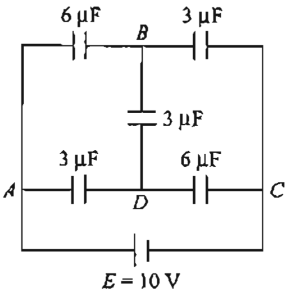


Fig. 4.37

Sol. Let total charge q flows through the battery, which gets divided at point A in q_1 and q_2. The final charge on each capacitor is as shown in Fig. 4.38. Applying KVL in the loop (1), traversing clockwise,

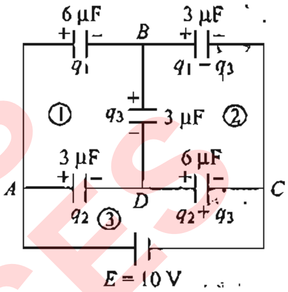


Fig. 4.38

$$-\frac{q_1}{6}-\frac{q_3}{3}+\frac{q_2}{3}=0 \quad \text{or} \quad -q_1-2q_3+2q_2=0 \qquad \text{(i)}$$

For loop (2), traversing clockwise, we have

$$-\left(\frac{q_1-q_3}{3}\right)+\frac{q_2+q_3}{6}+\frac{q_3}{3}=0$$

$$\Rightarrow \quad -2q_1+5q_3+q_2=0 \qquad \text{(ii)}$$

For loop *ABCEA*, traversing clockwise, we have

$$-\frac{q_1}{6}-\frac{(q_1-q_3)}{3}+10=0 \quad \Rightarrow \quad 3q_1-2q_3=60 \qquad \text{(iii)}$$

Solving equations (i), (ii) and (iii), we get

$$q_1=24\ \mu\text{C},\ q_2=18\ \mu\text{C},\ q_3=6\ \mu\text{C}.$$

Illustration 4.12 **Determine the equivalent capacitance between a and b. Each of the capacitor shown in Fig. 4.39 has capacity C.**

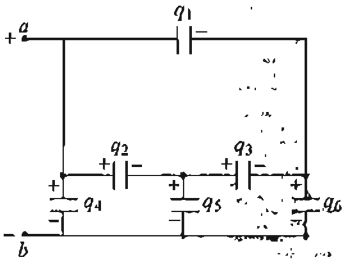


Fig. 4.39

Sol. Method 1. Let the potential difference applied across the terminals a and b be V and the charge supplied to the circuit due to this is Q. Now, equivalent capacitance of the system is the capacitance of that single capacitor which would have the same charge Q on its plates when the battery of same voltage V is applied across it. Hence,

$$C_{eq}=\frac{Q}{V} \qquad \text{(i)}$$

where, from conservation of charge, we must have

$$Q=q_1+q_2+q_4=q_4+q_5+q_6 \qquad \text{(ii)}$$

$$\Rightarrow \quad q_1+q_2=q_5+q_6 \qquad \text{(iii)}$$

and $V=\dfrac{q_4}{C}$. In a closed loop, the net potential drop must be

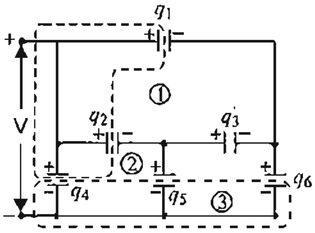


Fig. 4.40

zero, from KVL. Therefore, for loop 1, loop 2 and loop 3, we have (traversing anticlockwise)

$$\frac{q_1}{C} - \frac{q_2}{C} - \frac{q_3}{C} = 0$$

$$\frac{q_2}{C} - \frac{q_4}{C} + \frac{q_5}{C} = 0$$

and

$$\frac{q_3}{C} - \frac{q_5}{C} + \frac{q_6}{C} = 0 \quad \text{(iv)}$$

The conductor that connects the second, third and fifth capacitors is electrically neutral. Hence,

$$q_3 + q_5 - q_2 = 0 \quad \text{(v)}$$

Similarly, for first, third and sixth capacitors

$$q_1 + q_3 = q_6 \quad \text{(vi)}$$

Upon solving the equations (iii), (iv), (v) and (vi), we obtain

$q_1 = q_2 = q_5 = q_6 = \frac{q_4}{2}$, and $q_3 = 0$

Now, $C_{eq} = \frac{Q}{V} = \frac{q_1 + q_2 + q_4}{q_4/C} = 2C$

Method 2. Make the connection of point a to b (we can do this because a and b are at same potential) and note the indicated Wheatstone's bridge. Third capacitor will be useless. Now, simplify the circuit to obtain the desired result.

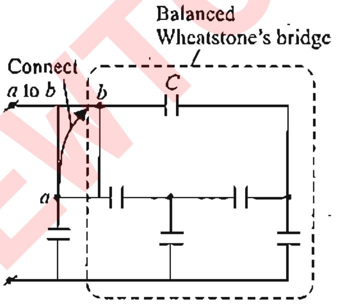


Fig. 4.41

DIELECTRIC

These are non-conductors upto a certain value of field depending upon their nature. If the field exceeds the limiting value, called dielectric strength, dielectric loses its insulating property and begins to conduct.

Dielectric Constant

When a dielectric material is placed between the plates of a capacitor, the capacitance of the capacitor increases. The ratio of the capacitance of a given capacitor with the material filling the entire space between its plates to the capacitance of the same capacitor in vacuum is called dielectric constant of that material.

Dielectric constant $K = \frac{C}{C_0}$

Dielectric in an Electric Field

Suppose a slab of dielectric material is placed in a uniform electric field $\vec{E}_0$ set up between the parallel plates of a charged capacitor. The slab becomes electrically polarized. That is, its molecules become electric dipoles oriented in the direction of the field. The net effect is the appearance of negative charge on one face of the slab and an equal positive charge on the opposite face. The net charge in the interior of the slab remains zero. The polarization charges induced on the two faces of the slab produce their own electric field $\vec{E}_0'$, which opposes the external field $\vec{E}_0$. Hence, the resultant field $\vec{E}$ within the dielectric is smaller than E_0, but points in the same direction as $\vec{E}_0$.

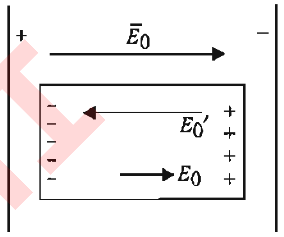


Fig. 4.42

The field in the rest of the (free) space is still $\vec{E}_0$. Hence, we conclude that when a dielectric is placed in an electric field, the field 'within' the dielectric is weakened (but not reduced to zero).

The weakening of electric field within the dielectric is illustrated in Fig. 4.43. Here, the dielectric fully fills the space between the plates. The figure (a) shows the original field. In other figures, some of the lines of force leaving the positive plate of the capacitor penetrate the dielectric; others terminate on the charges induced on the dielectric.

The reduction in the magnitude of the electric field from E_0 to E causes a reduction in the potential difference between the plates of the capacitor. If V_0 and V be the potential difference with and without the plates, then we have

$$\frac{E_0}{E} = \frac{V_0}{V} \qquad \{\because E = V/d\}$$

But $\frac{V_0}{V} = \frac{C}{C_0} = K$, where K is the dielectric constant of the slab. So, $\frac{E_0}{E} = K \Rightarrow E = \frac{E_0}{K}$.

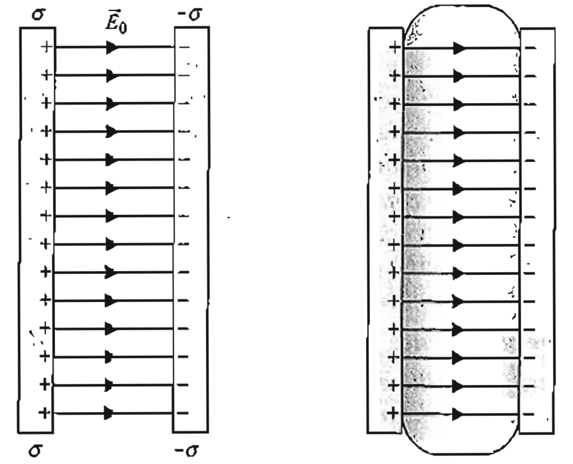
Thus, the electric field within the dielectric is reduced by a factor K.

Induced Charge on the Surface of Dielectric

Let K = dielectric constant,

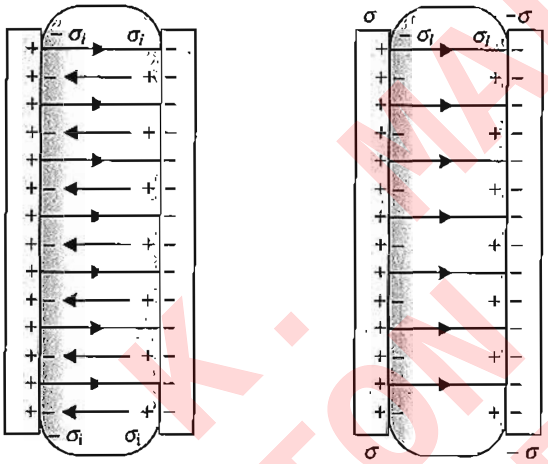
E_0 = original field in vacuum if dielectric slab was not there,

E_i = electric field induced in the dielectric slab,



(a) Electric field of magnitude E_0 between two charged plates.

(b) Introduction of a dielectric constant K.



(c) The induced surface charges and their field (thinner lines).

(d) Resultant field of magnitude E_0/K when a dielectric is between charged plates.

Fig. 4.43

E = net electric field in the dielectric slab.

$$\therefore \quad E = E_0 - E_i \qquad \text{(i)}$$

and $\frac{E_0}{E} = K$ (by definition of K of ε_r) or $E = \frac{E_0}{K}$ (ii)

From (i) and (ii), $E_0 - E_i = \frac{E_0}{K}$

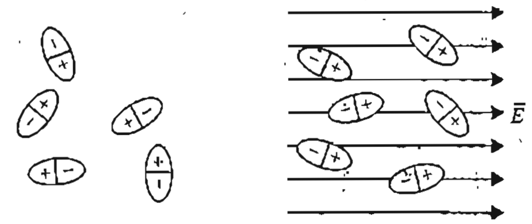
$$\Rightarrow \quad E_0K - E_iK = E_0 \quad \Rightarrow \quad E_0K - E_0 = E_iK$$

$$\Rightarrow \quad E_i = \frac{K-1}{K}E_0 \qquad \text{(iii)}$$

$$\Rightarrow \quad \frac{\sigma_i}{\varepsilon_0} = \frac{K-1}{K}\frac{\sigma}{\varepsilon_0}$$

$$\Rightarrow \quad \sigma_i = \frac{K-1}{K}\sigma$$

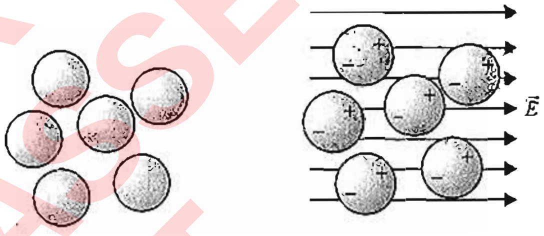
$$\Rightarrow \quad \frac{Q_i}{A} = \frac{K-1}{K}\frac{Q}{A} \Rightarrow \quad Q_i = Q\left(1 - \frac{1}{K}\right) \qquad \text{(iv)}$$



(a) Polar molecules no applied electric field

(b) Polar molecules with applied electric field

(a) Polar molecules have random orientations when there is no applied electric field.
(b) The molecules tend to line up with an applied electric field $\vec{E}$.



(a) Nonpolar molecules no applied electric field

(b) Nonpolar molecules with applied electric field

(a) Nonpolar molecules have their positive and negative charge centers at the same point.
(b) These centres becomes separated slightly by an applied electric field $\vec{E}$.

Fig. 4.44

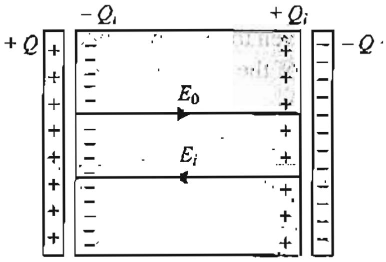


Fig. 4.45

This is irrespective of the thickness of the dielectric slab, i.e., whether it fills up the entire space between the charged plates or only part of it.

Dielectric Breakdown

If a dielectric material is subjected to a sufficiently strong electric field, dielectric breakdown takes place and the dielectric becomes a conductor (a partial ionization that permits conduction through it). This occurs when the electric field is so strong that electrons are ripped loose from their molecules and crash into other molecules, liberating even more electrons. This avalanche of moving charge, forming a spark or arc discharge, often starts quite suddenly.

Because of dielectric breakdown, capacitors always have maximum voltage ratings. When a capacitor is subjected to excessive voltage, an arc may form through a layer of dielectric, burning or melting a hole in it. This arc creates a conducting path (a short circuit) between the conductors. If a conducting path remains after the arc is extinguished, the device is rendered permanently useless and cannot be used as a capacitor.

The maximum electric field that a material can withstand without the occurrence of breakdown is called its *dielectric strength*.

Capacity of Parallel Plate Capacitor with Dielectric

Let a parallel plate capacitor has a plate area A and a separation d and a dielectric slab of thickness t and area A is inserted between the plates.

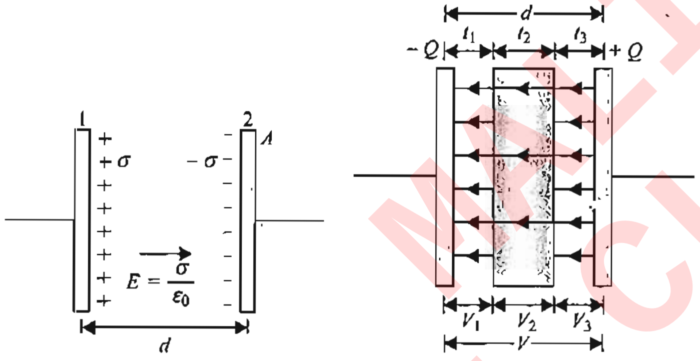


Fig. 4.46

Let Q be the charge given to the capacitor plates. The electric field between the plates of the capacitor is given by

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

The electric field in the region of dielectric slab is

$$E' = \frac{E}{K} = \frac{Q}{K\varepsilon_0 A}$$

We know that if the electric field is constant then potential difference between two points separated by distance d along the field line is Ed.

The potential difference between two plates is, therefore

$$V = V_1 + V_2 + V_3 = Et_1 + E't_2 + Et_3$$

$$= \frac{Q}{\varepsilon_0 A}t_1 + \frac{Q}{K\varepsilon_0 A}t_2 + \frac{Q}{\varepsilon_0 A}t_3$$

$$= \frac{Q}{\varepsilon_0 A}(t_1 + t_3) + \frac{Q}{K\varepsilon_0 A}t_2 = \frac{Q}{\varepsilon_0 A}(d - t) + \frac{Q}{K\varepsilon_0 A}t$$

The capacity $C = \dfrac{Q}{V_+ - V_-} = \dfrac{1}{\dfrac{(d-t)}{\varepsilon_0 A} + \dfrac{t}{K\varepsilon_0 A}}$

$$\Rightarrow \qquad C = \frac{\varepsilon_0 A}{(d-t) + \dfrac{t}{K}}$$

Note:

- *The capacitance in the above situation is independent of position of dielectric slab with respect to the plates. The capacitance depends on the thickness of the dielectric slab and dielectric constant.*
- *The dielectric constant of conducting slab (metal plate) is infinity (∞), therefore term t/K reduces to zero. If we insert a metal plate of thickness t between the plates of capacitor having area A and separation d, the capacitance will become $C = \frac{\varepsilon_0 A}{d - t}$. Also, the capacitance will be independent of position of metal plate between the plates of the capacitor.*
- *If $t << d$, then $C = \frac{\varepsilon_0 A}{d}$. Hence, if we place a thin metal plate parallel to the plates of a capacitor, the capacitance of the capacitor remains unchanged.*

If we place many dielectric slabs parallel to the plates of a capacitor as shown in Fig. 4.47, then capacitance is given by

$$C = \frac{\varepsilon_0 A}{d - (t_1 + t_2 + \cdots) + \left(\dfrac{t_1}{k_1} + \dfrac{t_2}{k_2} + \cdots\right)}$$

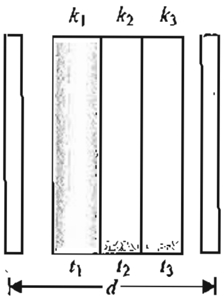


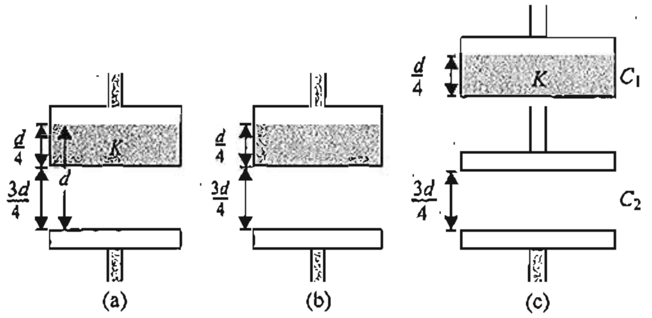
Fig. 4.47

If we introduce a number of dielectric slabs which completely fill the space between the plates, i.e., $d = t_1 + t_2 + t_3 + \cdots + t_n$, then the capacity of the capacitor will be

$$C = \frac{\varepsilon_0 A}{\left(\dfrac{t_1}{k_1} + \dfrac{t_2}{k_2} + \cdots\right)}$$

Illustration 4.13 **A parallel plate capacitor with plates of area A and plate separation d, is partially filled with a dielectric slab of constant K, as shown in Fig. 4.48. The thickness of the dielectric slab is $d/4$. What is the equivalent capacitance of this arrangement?**

Sol. (a) If we introduce a thin metal plate between the plates of the capacitor and parallel to it (figure b), the capacitance of the system will remain unchanged. The given capacitor arrangement is equivalent to the series combination of the two capacitors as shown in figure (c) because the electric potential is the same at all points on the lower surface of the dielectric. All the points on the surface of a conductor are equipotential, therefore all points on the I-shaped conductor connecting the two capacitors are equipotential. Thus, we can divide the entire region of capacitors in two parts. The capacitance of the two part capacitors are



(a) (b) (c)

Fig. 4.48

$$C_1 = \frac{AK\varepsilon_0}{d/4} \text{ and } C_2 = \frac{A\varepsilon_0}{(3d/4)}$$

Equivalent capacity, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d/4}{AK\varepsilon_0} + \frac{3d/4}{A\varepsilon_0}$

$$\Rightarrow \quad C_{eq} = \frac{4\varepsilon_0 A}{d}\left(\frac{K}{3K+1}\right).$$

Alternative method: We can use direct formula;

$$C_{eq} = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} \qquad \text{(i)}$$

Here, $t = \frac{d}{4}$. Thus, from (i)

$$C_{eq} = \frac{\varepsilon_0 A}{d - \frac{d}{4} + \frac{d/4}{K}} = \frac{\varepsilon_0 A}{\left(\frac{3d}{4} + \frac{d}{4K}\right)} = \frac{4\varepsilon_0 A}{d}\left(\frac{K}{3K+1}\right)$$

FORCE ON DIELECTRIC SLAB AT CONSTANT POTENTIAL DIFFERENCE

When a dielectric slab is placed near a charged capacitor, due to fringing effect the slab experiences a net force towards inside the capacitor.

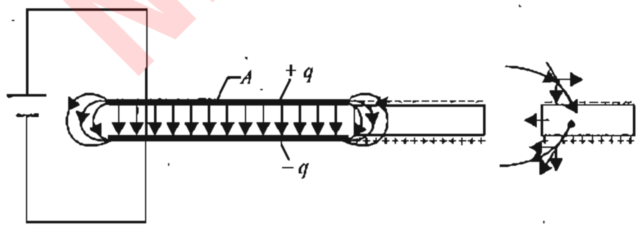
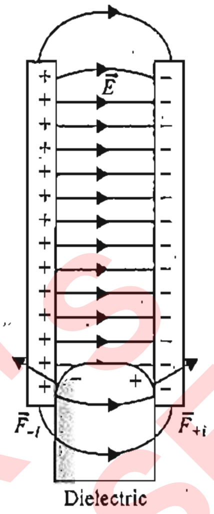


Fig. 4.49

We will find the force on a dielectric under two conditions: (i) in first case, when the battery remains connected and (ii) in second case, after the battery is disconnected.

(i) When battery remains connected: Here, V remains constant.

Let us consider a parallel plate capacitor with plates of width b and length l. The distance between the plates is d. The capacitor is connected to a battery of e.m.f. V. Let a dielectric is inserted up to distance x, then



The fringing field at the edges of the capacitor exerts forces $\vec{F}_{-l}$ and $\vec{F}_{+l}$ on the negative and positive induced surface charges of a dielectric, pulling the dielectric into the capacitor.

Fig. 4.50

$$C_1 = \frac{\varepsilon_0 b(l-x)}{d}; \quad C_2 = \frac{K\varepsilon_0 bx}{d}$$

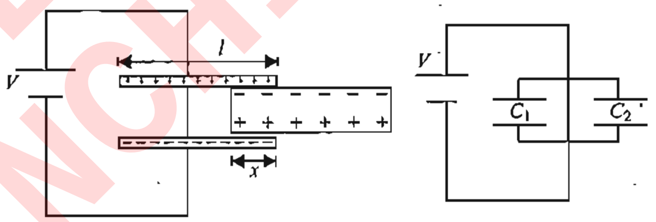


Fig. 4.51

$$C = C_1 + C_2 = \frac{\varepsilon_0 b}{d}[l + x(K-1)];$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 b}{d}[l + x(K-1)]V^2$$

Force on dielectric: $|F| = \frac{dU}{dx} = \frac{1}{2}\frac{\varepsilon_0 bV^2}{d}(K-1)$

(ii) When battery is disconnected after charging the capacitor: Here, q remains constant.

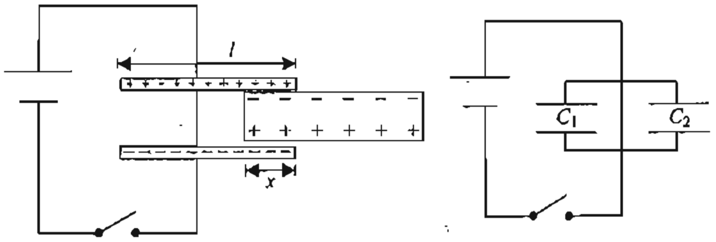


Fig. 4.52

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{q^2 d}{\varepsilon_0[l + x(K-1)]b}$$

$$F = -\frac{dU}{dx} = -\frac{q^2 d}{2\varepsilon_0 b}\frac{(K-1)(-1)}{[l + x(K-1)]^2}$$

$\therefore$ Force on dielectric: $$F = \frac{q^2 d(K-1)}{2\varepsilon_0 b[l + x(K-1)]^2}$$

Effect of Dielectric on Different Parameters

Let the entire space between the plates of a capacitor is filled with a dielectric of dielectric constant K under two conditions: (a) in first case, when the battery remains connected and (b) in second case, after the battery is disconnected.

(a) When battery remains connected (see Fig. 4.53): In this case potential difference across the plates will remain same, i.e., $V = V_0$, as battery is a source of constant potential difference.

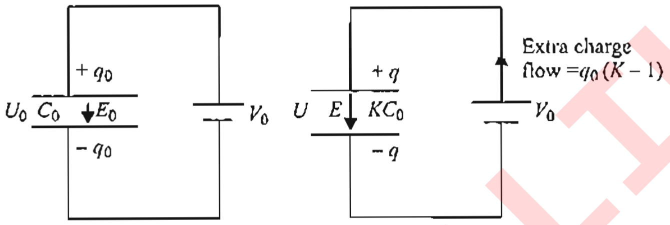


Fig. 4.53

- Capacitance increases, i.e., $C = KC_0$, as capacitance depends upon geometrical factors only.
- Charge on capacitor: $q = CV = KC_0V_0 = Kq_0$
  ($\because$ initially $q_0 = C_0V_0$)
  Thus, charge increases and becomes K times of previous charge.
- Electric field:
  $$E = \left[\frac{V}{d}\right] = \frac{V_0}{d} = E_0 \left\{\text{as } V = V_0 \text{ and } \frac{V_0}{d} = E_0\right\}$$
  Thus, electric field remains same.
- Energy stored in the capacitor:
  $$U = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)(V_0)^2 = K\frac{1}{2}C_0V_0^2 = KU_0$$
  $$\left\{\text{as } C = KC_0 \text{ and } U_0 = \frac{1}{2}C_0V_0^2\right\}$$
  Thus, energy increases and becomes K times of previous energy.

Note: *On insertion of dielectric, an extra charge of $q - q_0 = q_0(K-1)$ will flow in the circuit through the battery from the negative plate to the positive plate of the capacitor.*

(b) When battery is disconnected (see Fig. 4.54): In this case, charge on the plates will remain same, i.e., $q = q_0$, as in an isolated system charge is conserved.

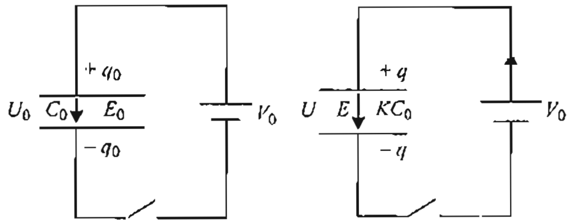


Fig. 4.54

- Capacitance increases, i.e., $C = KC_0$, as capacitance depends upon geometrical factors only.
- Potential difference between the plates:
  $$V = \frac{q}{C} = \frac{q_0}{KC_0} = \frac{V_0}{K}.$$
  So, the potential difference decreases.
- Field between the plates:
  $$E = \frac{V}{d} = \frac{V_0}{Kd} = \frac{E_0}{K} \left\{\text{as } V = \frac{V_0}{K} \text{ and } E_0 = \frac{V_0}{d}\right\}$$
  So, field decreases.
- Energy stored in the capacitor:
  $$U = \frac{q^2}{2C} = \frac{q_0^2}{2KC_0} = \frac{U_0}{K} \quad \{\text{as } q = q_0 \text{ and } C = KC_0\}$$
  So, energy decreases.

SPHERICAL CAPACITOR

It consists of two concentric spherical conducing shells of radii a and b, say $b > a$ (Fig. 4.55). The outer shell is earthed. Place a charge $+Q$ on the inner shell. It will reside on the outer surface of the shell. A charge $-Q$ will be induced on inner surface of outer shell. A charge $+Q$ will flow from outer shell to earth.

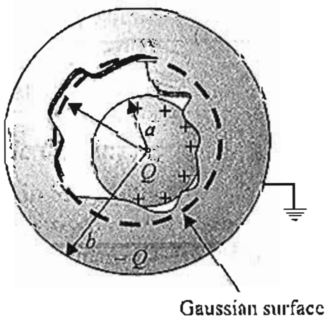


Fig. 4.55

Consider a Gaussian spherical surface of radius r such that $a < r < b$.

From Gauss's Law, electric field at distance $r > a$ is
$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

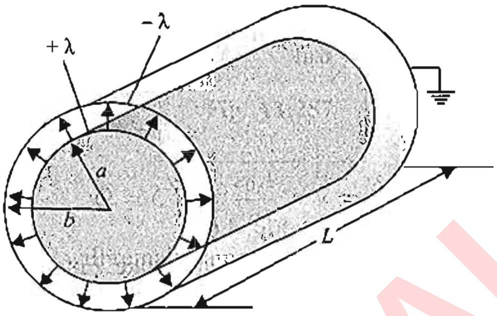
Potential difference:
$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{r} = -\int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}\,dr$$

Since $V_b = 0$
$$\Rightarrow \quad V_a = \int_a^b \frac{Q}{4\pi\,\varepsilon_0 r^2}\,dr$$
$$\Rightarrow \quad V_a = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q(b-a)}{4\pi\varepsilon_0 ab}$$

$\therefore$ Capacitance: $$C = \frac{Q}{V_a - V_b} = \frac{Q}{V_a} = \frac{4\pi\varepsilon_0 ab}{b-a}$$

CYLINDRICAL CAPACITOR

It consists of two coaxial cylinders of radii a and b, say $b > a$ (Fig. 4.56). The outer one is earthed. The cylinders are long enough so that we can neglect fringing of electric field at the ends. Electric field at a point between the cylinders will be radial and its magnitude will depend on the distance from the central axis. Consider a Gaussian surface of length y and radius r such that $a < r < b$. Flux through the plane surface is zero because electric field and area vector are perpendicular to each other.



A long cylindrical capacitor. The linear charge density λ is assumed to be positive in this figure. The magnitude of charge in a length L of either cylinder is λL.

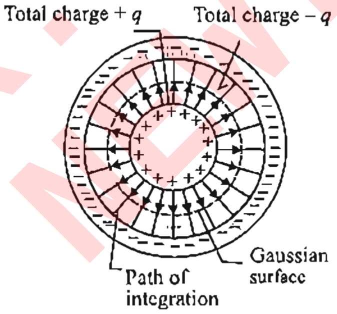
Fig. 4.56

For curved part, $\phi = \int \vec{E}\cdot d\vec{s} = \int E\,ds$

$$\Rightarrow \qquad \phi = E\int ds = E\cdot 2\pi r y$$

Charge inside the Gaussian surface, $q = \dfrac{Qy}{L}$

From Gauss's Law, $\phi = E2\pi r y = \dfrac{Qy}{L\varepsilon_0} \Rightarrow E = \dfrac{Q}{2\pi\varepsilon_0 L r}$



A cross section of a long cylindrical capacitor, showing a cylindrical Gaussian surface of radius r (that encloses the positive plate). This figure also serves to illustrate a spherical capacitor in a cross section through its center.

Fig. 4.57

Potential difference:

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{r} = -\int_a^b \frac{Q}{2\pi\varepsilon_0 L r}dr = -\frac{Q}{2\pi\varepsilon_0 L}\int_a^b \frac{1}{r}dr$$

$$\Rightarrow \quad V_a = \frac{Q}{2\pi\varepsilon_0 L}\ln\frac{b}{a} \qquad (\text{since } V_b = 0)$$

$$\text{Capacitance: } C = \frac{Q}{V_a - V_b} = \frac{Q}{V_a} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

Concept Application Exercise 4.2

1. A conductor is an extreme case of dielectric, since if an electric field is applied to a conductor, charges are free to move within the conductor to set up "induced charges". What is the dielectric constant of a perfect conductor? Is it $K = 0$, $K \to \infty$ or something in between? Explain your reasoning.
2. A capacitor of capacitance C is charged to a potential difference V_0. The terminals of the charged capacitor are then connected to those of an uncharged capacitor of capacitance $C/2$. Compute
   - **a.** the original charge of the system;
   - **b.** the final potential difference across each capacitor;
   - **c.** the final energy of the system;
   - **d.** the decrease in energy when the capacitors are connected.
   - **e.** where did the "lost" energy go?
3. A parallel plate vacuum capacitor with plate area A and separation x has charges $+Q$ and $-Q$ on its plates. The capacitor is disconnected from the source of charge, so the charge on each plate remains fixed.
   - **a.** What is the total energy stored in the capacitor?
   - **b.** The plates are pulled apart an additional distance dx. What is the change in the stored energy?
   - **c.** If F is the force with which the plates attract each other, then the change in the stored energy must equal the work $dW = F\,dx$ done in pulling the plates apart. Find an expression for F.
   - **d.** Explain why F is not equal to QE, where E is the electric field between the plates?
4. You have two identical capacitors and an external potential source.
   - **a.** Compare the total energy stored in the capacitors when they are connected to the applied potential in series and in parallel.
   - **b.** Compare the maximum amount of charge stored in each case.
   - **c.** Energy storage in a capacitor can be limited by the maximum electric field between the plates. What is the ratio of the electric field for the series and parallel combinations ?
5. A circuit has a section AB shown in fig. 4.58.

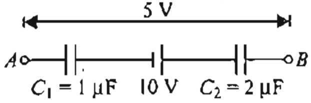


Fig. 4.58

The e.m.f. of the cell is 10 V and the capacitors have capacitances $C_1 = 1$ μF and $C_2 = 2$ μF. The potential difference $V_{AB} = 5$ V. Find the charges on the capacitors.

6. A dielectric slab is inserted at one end of a charged parallel plate capacitor (the plates being horizontal and the charging battery having been disconnected) and then released. Describe what happens. Neglect friction.

7. While a capacitor remains connected to a battery, a dielectric slab is slipped between the plates. Describe qualitatively what happens to the charge, the capacitance, the potential difference, the electric field strength and the stored energy. Is work required to insert the slab?

8. If you have several 2.0 μF capacitors, each capable of with-standing 200 V without breakdown, how would you assemble a combination which has an equivalent capacitance of
   a. 0.4 μF, and
   b. 1.2 μF, each withstanding 1000 V?

9. N identical capacitors are connected in parallel and then a potential difference of V is applied to them. Find the potential difference when these capacitors are reconnected in series, their charges being left undisturbed?

10. In the arrangement shown in Fig. 4.59, plate B is given a charge equal to 60 μC. The ratio $\frac{d_1}{d_2} = 2$. Then

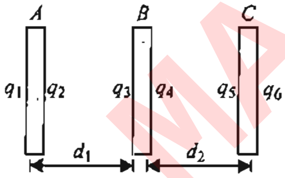


Fig. 4.59

q_1 = ________
q_2 = ________
q_3 = ________
q_4 = ________
q_5 = ________
q_6 = ________

11. In Fig. 4.60, the plate A has 100 μC charge, while the plate B has 60 μC charge.

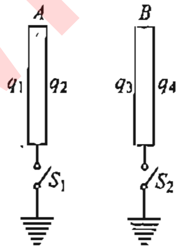


Fig. 4.60

a. When both switches are open, then
q_1 = ________ q_2 = ________
q_3 = ________ q_4 = ________

b. When only switch S_1 is closed, then
q_1 = ________
q_2 = ________
q_3 = ________
q_4 = ________

c. When switch S_2 is also closed, then
q_1 = ________
q_2 = ________
q_3 = ________
q_4 = ________

12. For the network of capacitors shown in Fig. 4.61.
   a. Find the potential of junction B,
   b. Find the potential of junction D,
   c. Find the charge on each 2 μF capacitor.

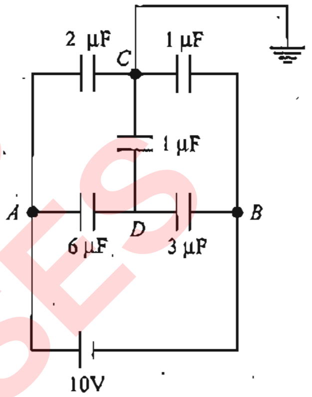


Fig. 4.61

13. In Fig. 4.62, the system is in steady state. Find

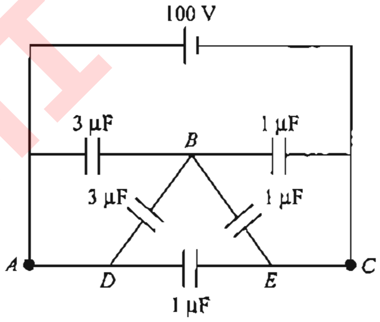


Fig. 4.62

a. $V_A - V_B$ =__________________
b. $V_B - V_C$ =__________________
c. $V_D - V_E$ =__________________
d. The energy stored in the circuit = ________

14. State the following statement as true or false.
   a. If a battery is connected across a circuit consisting of two identical capacitors and it is found that, in steady state, the two capacitors have equal charge, then the two capacitors must be in series with each other.
   b. If a battery is connected across a circuit consisting of two capacitors having different capacitances and it is found that, in steady state, the two capacitors have equal charge, then the two capacitors must be in series with each other.
   c. If a battery is connected across a circuit consisting of two identical capacitors and it is found that, in steady state, the two capacitors have equal charge, then the two capacitors may be in series with each other.

15. Find equivalent capacitance between points A and B shown in Fig. 4.63.

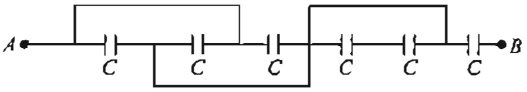


Fig. 4.63

Solved Examples

Example 4.1 **Fig. 4.64 shows two identical parallel plate capacitors connected to a battery with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity). Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. (IIT-JEE, 1983)**

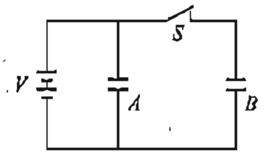


Fig. 4.64

Sol. Initially, when the switch is closed, both the capacitors A and B are in parallel and the energy stored in the capacitors is, therefore, $U_i = 2 \times \frac{1}{2}CV^2 = CV^2$ (i)

When switch S is opened, B gets disconnected from the battery. The capacitor B is now isolated, the charge on an isolated capacitor remains constant, often referred to as bound charge. On the other hand, A remains connected to battery. Hence, potential V remains constant on it.

When the capacitors are filled with dielectric, their capacitance increases to kC.

Therefore, energy stored in B changes to $\frac{1}{2}\frac{Q^2}{kC}$, where $Q = CV$ is the charge on B which remains constant, and energy stored in A changes to $\frac{1}{2}kCV^2$, where V is the potential on A which remains constant. Thus, finally, the total energy stored in the capacitors is,

$$U_f = \frac{1}{2}\frac{(CV)^2}{kC} + \frac{1}{2}kCV^2 = \frac{1}{2}CV^2\left(k + \frac{1}{k}\right) \quad \text{(ii)}$$

From (i) and (ii), we find $\frac{U_i}{U_f} = \frac{2k}{k^2+1}$

It is given that $k = 3$. Therefore, we have $\frac{U_i}{U_f} = \frac{3}{5}$

Example 4.2 **Two parallel plate capacitors of capacitance C are connected in series with a battery of e.m.f. ε. Then, one of the capacitors is filled with a dielectric of dielectric constant k.**

1. **Find the change in electric field in the two capacitors, if any.**
2. **What amount of charge flows through the battery ?**
3. **Find the change in energy stored in the circuit, if any.**

Sol.

1. Two capacitors A and B initially have same charge Q and potential $V = Q/C$. The electric field between the capacitor plates is given by $E = V/d$. Since the two capacitors are connected in series with the battery, the sum of potentials across the capacitors must be equal to ε,

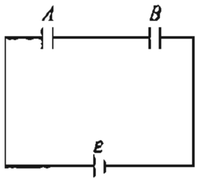


Fig. 4.65

i.e., $\varepsilon = 2V = \frac{2Q}{C}$ (i)

2. When one of the capacitors, say A, if filled with a dielectric, the capacity of A increases to $C' = kC$ while that of B remains unchanged, i.e., C.

Suppose, now charge on the capacitors becomes Q' and potentials across A and B are V'_A and V'_B, respectively, with $V'_A + V'_B = \varepsilon$.

Hence, we have $V'_A = \frac{Q'}{C'} = \frac{Q'}{kC}$ and $V'_B = \frac{Q'}{C}$

Hence, $\varepsilon = \frac{Q'}{C}\left(1 + \frac{1}{k}\right)$ (ii)

From (i) and (ii), we get $Q' = \frac{2k}{1+k}Q$

Since $k > 1$, therefore $Q' > Q$. Also, $V'_A = \frac{2}{1+k}V$ and $V'_B = \frac{2k}{1+k}V$

Thus, the electric field (or potential difference) in capacitor A decreases by a factor of $\left(\frac{2}{1+k}\right)$ while that in B increases by a factor of $\left(\frac{2k}{1+k}\right)$.

The amount of charge that flows into the circuit is given by

$$\Delta Q = Q' - Q = \left(\frac{2k}{1+k} - 1\right)Q = \frac{k-1}{k+1}Q$$

$$= \frac{1}{2}\frac{k-1}{k+1}C\varepsilon$$

Initially, the energy is given by $U_i = 2 \times \frac{1}{2}CV^2 = CV^2$

Final energy is,

$$U_f = \frac{1}{2}C'V_A'^2 + \frac{1}{2}CV_B'^2$$

$$= \frac{1}{2}kC\left(\frac{2}{1+k}\right)^2 V^2 + \frac{1}{2}C\left(\frac{2k}{1+k}\right)^2 V^2$$

$$= \frac{4k}{1+k}\left(\frac{1}{2}CV^2\right) = \frac{2k}{1+k}U_i$$

Example 4.3 **Five identical conducting plates 1, 2, 3, 4 and 5 are fixed parallel and equidistant from each other as shown in Fig. 4.66. Plates 2 and 5 are connected by a conductor while 1 and 3 are joined by another conductor. The junction of 1 and 3 and the plate 4 are connected to a source of constant e.m.f. V_0. Find**

1. **the effective capacity of the system between the terminals of the source,**

2. the charges on plates 3 and 5.

Given d = distance between any two successive plates and A = area of either face of each plate.

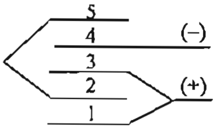


Fig. 4.66

Sol.

1. The equivalent circuit is shown in Fig. 4.67. The system consists of four capacitors, i.e., C_{12}, C_{32}, C_{34} and C_{54}. The capacity of each capacitor is $(K\varepsilon_0 A/d) = C_0$.

   The capacitors C_{12} and C_{32} are in parallel, th[illegible] capacity is $C_0 + C_0 = 2C_0$. The capacitor C_{54} is in series with the parallel combination of C_{12} and C_{32}. Hence, the resultant capacity will be $C_1 = \dfrac{C_0 \times 2C_0}{C_0 + 2C_0}$

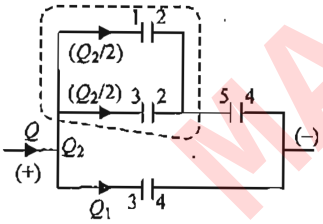


Fig. 4.67

Further, C_{34} is again in parallel with the combination of C_{12}, C_{32}, C_{54}. Hence, the effective capacity

$$C_{\text{eff.}} = C_0 + \frac{C_0 \times 2C_0}{C_0 + 2C_0} = \frac{5}{3}C_0 = \frac{5}{3}K\varepsilon_0\frac{A}{d}$$

2. Charge on the plate 5 = charge on the upper half of parallel combination, $Q_5 = V_0\left(\frac{2}{3}C_0\right) = \frac{2}{3}\frac{K\varepsilon_0 A V_0}{d}$

   Charge on plate 3 on the surface facing 4

   $$= V_0C_0 = \frac{K\varepsilon_0 A V_0}{d}$$

   Charge on plate 3 on the surface facing 2

   = [potential difference across (3 – 2)] C_0

   $$= V_0\frac{C_0}{C_0 + 2C_0}C_0 = K\varepsilon_0\frac{AV_0}{3d}$$

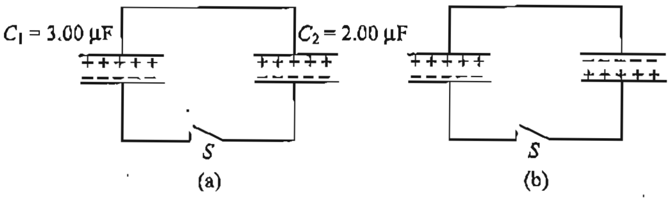
   Net charge on plate 3:

   $$Q_3 = \frac{K\varepsilon_0 A V_0}{d} + K\varepsilon_0\frac{AV_0}{3d} = \frac{K\varepsilon_0 A V_0}{d}\left[1 + \frac{1}{3}\right]$$

   $$= \frac{4}{3}K\varepsilon_0\frac{A}{d}V_0$$

Example 4.4 **Two capacitors of capacity 3.00 μF and 2.00 μF are separately charged with a 24.0 V battery. After they are fully charged, they are connected as shown in Fig. 4.68 (a) and (b).**

In each of the two arrangements, find the energy stored on each capacitor (a) before switch S is closed and (b) after switch S is closed.



(a) (b)

Fig. 4.68

Sol. a. When the switch S is open, the charges on the unconnected plates are bound, they have no place to go. Charges cannot move in open wires, a closed circuit is a must. The charges on the connected plates are held in place due to Coulomb attraction of bound charges on the unconnected plates, which prevents them from combining.

Each capacitor retains its charges and the potential difference it had when originally connected to the battery. Initially, energy stored in the capacitors,

$$U_1 = \frac{1}{2}C_1(\Delta V)^2 = \frac{1}{2}(3.00 \times 10^{-6})(24.0)^2 = 8.64 \times 10^{-4}\text{ J}$$

$$U_2 = \frac{1}{2}C_2(\Delta V)^2 = \frac{1}{2}(2.00 \times 10^{-6})(24.0)^2 = 5.76 \times 10^{-4}\text{ J}$$

The total energy of this system of capacitors is

$$U = U_1 + U_2 = 1.44 \times 10^{-3}\text{ J}$$

b. When the switch is closed, the charges can combine such that net charge is conserved. The magnitude of initial charge on each capacitor is

$$Q_1 = C_1\,\Delta V = (3.00 \times 10^{-6})(24) = 72 \times 10^{-6}\text{ C} = 72\text{ μC}$$

$$Q_2 = C_2\,\Delta V = (2 \times 10^{-6})(24) = 48 \times 10^{-6}\text{ C} = 48\text{ μC}$$

Case (i) The positive plate of C_1 is connected to negative plate of C_2, therefore net charge on both top plates is $(Q_1 - Q_2)$ and on the lower plates is $-(Q_1 - Q_2)$.

$$Q_{\text{net}} = Q_1 - Q_2 = (72 - 48) \times 10^{-6} = 24 \times 10^{-6}\text{ C}$$

$$= 24\text{ μC} \qquad \text{(i)}$$

Case (ii): The positive plate of C_1 is connected to positive plate of C_2, therefore net charge on both top plates is $(Q_1 + Q_2)$ and on the lower plates is $-(Q_1 + Q_2)$.

$$Q_{\text{net}} = Q_1 + Q_2 = (72 + 48) \times 10^{-6}\,C = 120\text{ μC} \qquad \text{(ii)}$$

After the switch is closed, the two capacitors are in parallel. Consequently, potential differences across the two capacitors are equal:

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \qquad \text{(iii)}$$

On solving equations (i) and (iii), we obtain final charges on the two capacitors in Case (i)

$$Q_1' = 14.4\text{ μC} \text{ and } Q_2' = 9.60\text{ μC}$$

The potential difference across each capacitor is now

$$\Delta V = \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} = \frac{14.4}{3.0} = \frac{9.60}{2} = 4.80\text{ V}$$

Final energy on the two capacitors,

$$U_1' = \frac{Q_1'^2}{2C_1} = \frac{(14.4 \times 10^{-6})^2}{2(3 \times 10^{-6})} = 3.46 \times 10^{-5}\text{ J}$$

and $U_2' = \dfrac{Q_2'^2}{2C_2} = \dfrac{(9.60 \times 10^{-6})^2}{2(2 \times 10^{-6})} = 2.30 \times 10^{-5}$ J

The total energy is now

$$U' = U_1' + U_2' = 5.76 \times 10^{-5}\ \text{J}$$

The energy of the system has decreased. Work was required to transfer charges between the plates when switch S was closed. This work was supplied by the electric field.

$$\Delta U = U'_{\text{tot.}} - U_{\text{tot.}} = -1.38 \times 10^{-3}\ \text{J}$$

Same charge as initial value.

Final energy on the two capacitors,

$$U_1' = \frac{Q_1'^2}{2C_1} = \frac{(72 \times 10^{-6})^2}{2(3 \times 10^{-6})} = 86.4 \times 10^{-5}\ \text{J}$$

and $$U_2' = \frac{Q_2'^2}{2C_2} = \frac{(48 \times 10^{-6})^2}{2(2 \times 10^{-6})} = 57.6 \times 10^{-5}\ \text{J}$$

Total final energy is now $U' = U_1' + U_2' = 1.44 \times 10^{-3}$ J

Example 4.5 **Two parallel plate capacitors A and B have the same separation $d = 8.85 \times 10^{-4}$ m between the plates. The plate area of A and B are 0.04 m^2 and 0.02 m^2 respectively. A slab of dielectric constant (relative permittivity) $K = 9$ has dimensions such that it can exactly fill the space between the plates of capacitor B.**

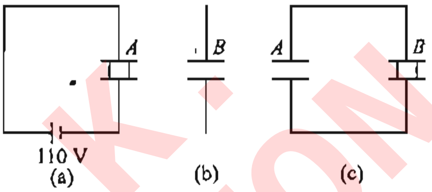


Fig. 4.69

1. **The dielectric slab is placed inside A as shown in figure (a). A is then charged to a potential difference of 110 V. Calculate the capacitance of A and the energy stored in it.**
2. **The battery is disconnected and then the dielectric slab is removed from A. Find the work done by the external agency in removing the slab from A.**
3. **The same dielectric slab is now placed inside B, filling it completely. The two capacitors A and B are then connected as shown in figure (c). Calculate the energy stored in the system.** **(IIT-JEE, 1993)**

Sol. 1. The capacitor A with dielectric can be considered as two capacitors in parallel, one having dielectric state and the other having no dielectric state. Such capacitor has an area of $\dfrac{A}{2}$. The combined capacitance is

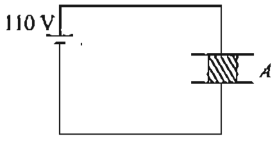


Fig. 4.70

$$C = C_1 + C_2 = \frac{(A/2)\,\varepsilon_0}{d} + \frac{(A/2)\,\varepsilon_0\,\varepsilon_r}{d} = \frac{A}{2}\frac{\varepsilon_0}{d}[1 + \varepsilon_r]$$

$$= \frac{0.04 \times 8.85 \times 10^{-12}}{2 \times 8.85 \times 10^{-4}}[1 + 9] = 2 \times 10^{-9}\ \text{F}$$

Energy stored $= \dfrac{1}{2}CV^2 = \dfrac{1}{2} \times 2 \times 10^{-9} \times (110)^2$

$$= 1.21 \times 10^{-5}\ \text{J}$$

2. Work done in removing the dielectric state = (Energy stored in capacitor without dielectric) – (Energy stored in capacitor with dielectric).

It may be noted that while taking out the dielectric the charge on the capacitor plate remains the same.

$$W = \frac{q^2}{2C'} - \frac{q^2}{2C}$$

Here, $C = 2 \times 10^{-9}$ F, $C' = \dfrac{A\varepsilon_0}{d} = \dfrac{0.04 \times 8.85 \times 10^{-12}}{8.85 \times 10^{-4}}$

$$= 0.4 \times 10^{-9}\ \text{F}$$

$$q = CV = 2 \times 10^{-9} \times 110 = 2.2 \times 10^{-7}\ \text{C}$$

$$\therefore \quad W = \frac{(2.2 \times 10^{-7})^2}{2}\left[\frac{1}{0.4 \times 10^{-9}} - \frac{1}{2 \times 10^{-9}}\right]$$

$$= 4.84 \times 10^{-5}\ \text{J}$$

3. The capacitance of $B = \dfrac{\varepsilon_0 \varepsilon_r A_B}{d}$

$$C_B = 1.8 \times 10^{-9}\ \text{F}$$

The charge on A, $q_A = 2.2 \times 10^{-7}$ C, gets distributed into two parts.

$$q_1 + q_2 = 2.2 \times 10^{-7}\ \text{C}$$

Also, the potential difference across A = p.d. across B

$$\frac{q_1}{C_A} = \frac{q_2}{C_B} \Rightarrow q_1 = \frac{C_A}{C_B}q_2 = \frac{0.4 \times 10^{-9}}{1.8 \times 10^{-9}} \Rightarrow q_2 = 0.22q_2$$

$$0.22q_2 + q_2 = 2.2 \times 10^{-7}$$

$$\Rightarrow \quad q_2 = \frac{2.2}{1.22} \times 10^{-7} = 1.8 \times 10^{-7}\ \text{C and}$$

$$q_1 = 0.4 \times 10^{-7}\ \text{C}$$

Total energy stored $= \dfrac{q_1^2}{2C_A} + \dfrac{q_2^2}{2C_B}$

$= 0.2 \times 10^{-5} + 0.9 \times 10^{-5} \quad = 1.1 \times 10^{-5}$ J

Alternatively, the combined capacitance of the two capacitors can be found. The total charge on the two capacitors is known. The energy can be found using the formula $\dfrac{Q^2}{2\,C_{\text{eq}}}$.

Example 4.6 **Two capacitors A and B with capacities 3 and 2 μF are charged to a potential difference of 100 and 180 V, respectively. The plates of the capacitors are connected as shown in Fig. 4.71 with one wire from each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged 2 μF capacitor C with lead wires falls on the free ends to complete the circuit. Calculate**

1. **the final charge on the three capacitors,**
2. **the amount of electrostatic energy stored in the system before and after the completion of the circuit.**

(IIT-JEE, 1997)

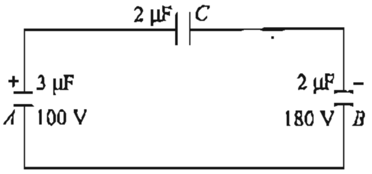


Fig. 4.71

Sol. **1.** We will attempt this question on the basis of charge conservation.

Initially:

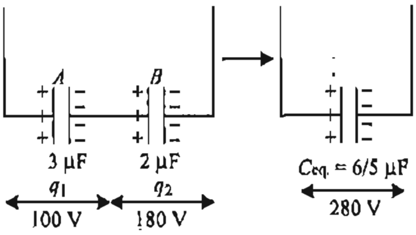


Fig. 4.72

Charge on capacitor A, $q_A = 3 \times 10^{-6} \times 100 = 3 \times 10^{-4}$ C

Charge on capacitor B, $q_B = 2 \times 10^{-6} \times 180$

$= 3.6 \times 10^{-4}$ C

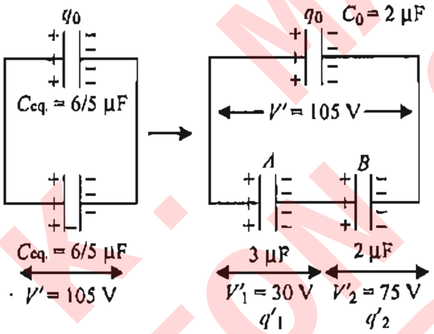


Fig. 4.73

Finally:

Let the charge on capacitor A, B and C be q_1, q_2 and q_3, respectively.

By charge conservation:

The sum of charges on plates 2 and 3 should be equal to q_A:

$$q_1 + q_2 = 3 \times 10^{-4} \qquad \text{(i)}$$

Similarly, the sum of charges on plates 4 and 5 will be equal to q_B:

$$-q_2 - q_3 = -3.6 \times 10^{-4} q_2 + q_3 = 3.6 \times 10^{-4} \qquad \text{(ii)}$$

Applying Kirchhoff's law in the loop $ACBA$, we get

$$\frac{q_1}{3 \times 10^{-6}} - \frac{q_2}{2 \times 10^{-6}} + \frac{q_3}{2 \times 10^{-6}} = 0$$

$$2q_1 - 3q_2 + 3q_3 = 0 \qquad \text{(iii)}$$

On solving (i), (ii) and (iii), we get

$q_1 = 90 \times 10^{-6}$ C, $q_2 = 210 \times 10^{-6}$ C and

$q_3 = 150 \times 10^{-6}$ C

2. Amount of electrostatic energy in the system initially

$$\frac{1}{2} \times 3 \times 10^{-6}(100)^2 + \frac{1}{2} \times 2 \times 10^{-6}(180)^2 = 4.74 \times 10^{-2}\ \text{J}$$

Amount of electrostatic energy stored finally

$$= \frac{1}{2}\frac{(90 \times 10^{-6})}{3 \times 10^{-6}} + \frac{1}{2}\frac{(210 \times 10^{-6})^2}{2 \times 10^{-6}} + \frac{1}{2}\frac{(150 \times 10^{-6})^2}{2 \times 10^{-6}}$$

$= 1.8 \times 10^{-2}$ J

Example 4.7 **A capacitor has rectangular plates of length a and width b. The top plate is inclined at a small angle as shown in Fig. 4.74. The plate separation varies from $d = y_0$ at the left to $d = 2y_0$ at the right, where y_0 is much less than a or b. Calculate the capacitance of the system.**

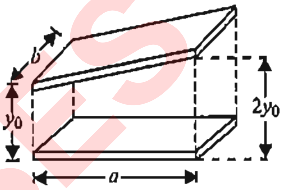


Fig. 4.74

Sol. We consider a differential strip of width dx and length b to approximate a differential capacitor of area bdx and separation $d = y_0 + \left(\frac{y_0}{a}\right)x$. All such differential capacitors are in parallel arrangement.

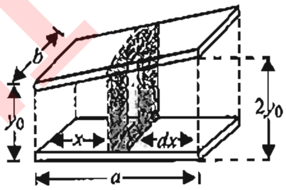


Fig. 4.75

$$dC = \frac{\varepsilon_0(bdx)}{y_0 + \left(\frac{y_0}{a}\right)x} \quad \Rightarrow \quad C = \int dC$$

$$C = \varepsilon_0 b \int_0^a \frac{dx}{\left(y_0 + \frac{y_0}{a}x\right)}$$

$$= \frac{\varepsilon_0 b}{(y_0/a)}\left[\ln\left(\frac{y_0 + \frac{y_0}{a} \times a}{y_0}\right)\right] = \frac{\varepsilon_0 ab}{y_0}\ln 2$$

We can determine expression for capacity in terms of θ as

$$d = (y_0 + x \tan\theta)$$

$$C = \int dC = \int_0^a \frac{\varepsilon_0 b\, dx}{(y_0 + x\tan\theta)}$$

$$= \frac{\varepsilon_0 b}{\tan\theta}\ln\left(\frac{y_0 + a\tan\theta}{y_0}\right)$$

For small θ, $\tan\theta = \theta \quad \Rightarrow C = \frac{\varepsilon_0 b}{\theta}\ln\left(1 + \frac{a\theta}{y_0}\right)$

Now, we can use the expansion

$$\log(1 + x) = x - \frac{1}{2}x^2 + \cdots$$

For $x < 1$, we can neglect higher powers. Thus,

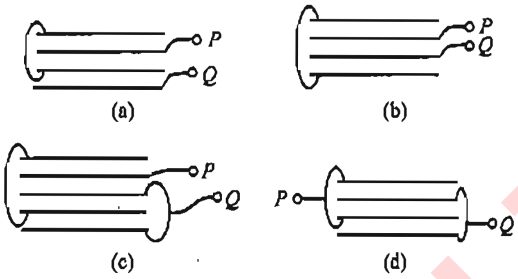
$$C = \frac{\varepsilon_0 b}{\theta}\left[\frac{a\theta}{y_0} - \frac{1}{2}\left(\frac{a\theta}{y_0}\right)^2\right] = \frac{\varepsilon_0 ab}{y_0}\left[1 - \frac{a\theta}{2y_0}\right]$$

EXERCISES

Subjective Type

Solutions on page 4.40

1. Identical metal plates are located in air at equal distance d from one another. The area of each plate is equal to A. Evaluate the capacitance of the system between P and Q if plates are interconnected as shown in Fig. 4.76.



(a) (b)

(c) (d)

Fig. 4.76

2. The capacitors in Fig. 4.77 are initially uncharged and are connected as in the diagram with switch S open. The applied potential difference is $V_{ab} = +360$ V.

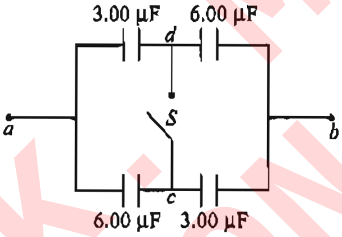


Fig. 4.77

a. What is the potential difference V_{cd}?

b. What is the potential difference across each capacitor after switch S is closed?

c. How much charge flowed through the switch when it was closed?

3. If the area of each plate is A and the successive separations are d, $2d$ and $3d$, then find the equivalent capacitance across A and B.

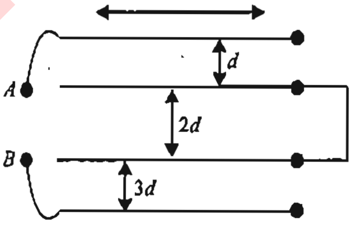


Fig. 4.78

4. Two capacitors A and B with capacities 3 and 2 μF are charged to p.d. of 100 and 180 V, respectively. The plates of the capacitors are connected as shown in Fig. 4.79. The upper plate of A is positive and that of B is negative. An uncharged capacitor C of 2 μF capacitance with lead wires falls on the free ends to complete the circuit. Calculate

a. the final charge on the three capacitors, and

b. the amount of electrostatic energy stored in the system before and after the completion of the circuit.

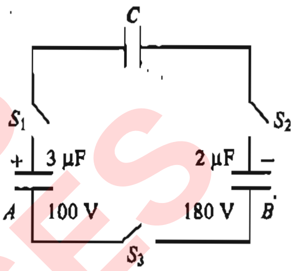


Fig. 4.79

5. Condensers with capacities $C, 2C, 3C$ and $4C$ are charged to the voltage V, $2V$, $3V$ and $4V$ correspondingly (Fig. 4.80). The circuit is closed. Find the voltage on all the condensers in equilibrium.

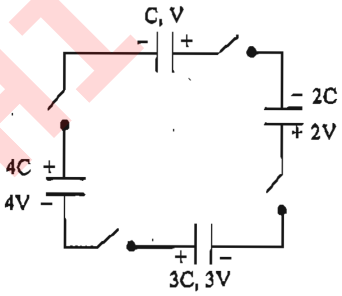


Fig. 4.80

6. In Fig. 4.81, when switch is swapped from 1 to 2, find the heat produced in the circuit.

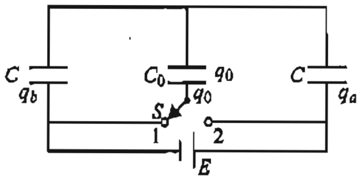


Fig. 4.81

7. Consider the network shown in Fig. 4.82. Find the effective capacity between A and B. Assume $C = 25$ μF.

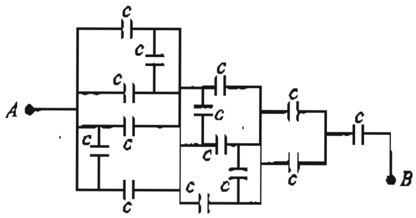


Fig. 4.82

8. A capacitor of capacitance $C_1 = 1$ μF can withstand a maximum voltage of $V_1 = 6$ kV and another capacitor of capacitance $C_2 = 2$ μF can withstand a maximum voltage of $V_2 = 4$ kV. If they are connected in series, what maximum voltage will the system withstand?

9. What charges (in μC) will flow through section B of the circuit in the direction shown when switch S is closed?

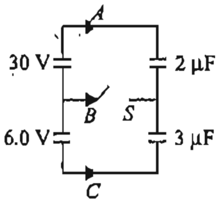


Fig. 4.83

10. A parallel plate capacitor contains a mica sheet (thickness 10^{-3} m) and a sheet of fibre (thickness 0.5×10^{-3}m). The dielectric constant of mica is 8 and that of fibre is 2.5. Assuming that the fibre breaks down when subjected to an electric field of 6.4×10^6 Vm^{-1}, find the maximum safe voltage that can be applied to the capacitor.

11. Find the potential difference between the points M and N of system shown in Fig. 4.84 if the e.m.f. is equal to $E = 110$ V and the capacitance ratio $\frac{C_2}{C_1} = 2$.

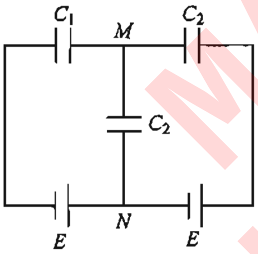


Fig. 4.84

12. Fig. 4.85 shows a network of seven capacitors. If charge on 5 μF capacitor is 10 μC, find the potential difference between points A and C.

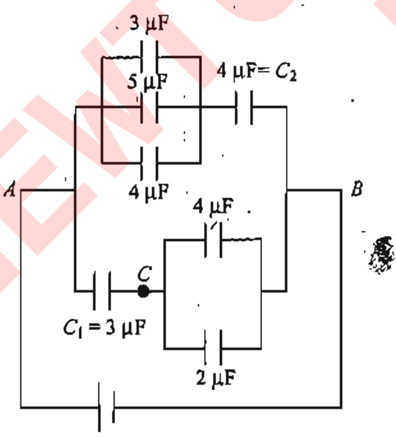


Fig. 4.85

13. In the circuit shown (Fig. 4.86) the e.m.f. of each battery is 60 V and $C_1 = 2$ μF and $C_2 = 3$ μF. Find the charges that will flow through the sections 1, 2 and 3 after the key is closed.

14. Find the potential difference between the points A and B and that between E and F of the circuit shown in Fig. 4.87.

15. Some capacitors each of capacitance 30 μF are connected as shown in Fig. 4.88. Calculate equivalent capacitance between terminals A and B.

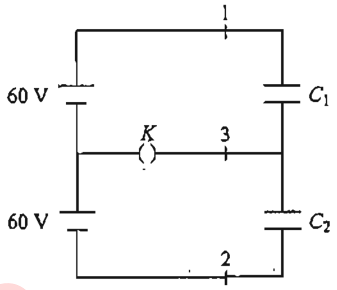


Fig. 4.86

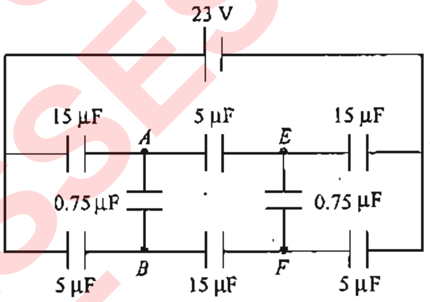


Fig. 4.87

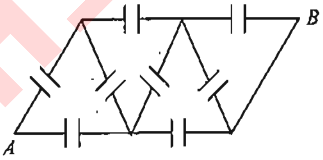


Fig. 4.88

Objective Type

Solutions on page 4.44

1. Two copper spheres of same radii, one hollow and the other solid, are charged to same potential. Then, which, if any, of the two will have more charge?

a. Hollow
b. Solid
c. Both will have the same charge
d. Nothing can be predicted

2. The distance between the plates of a parallel plate capacitor is d. A metal plate of thickness $d/2$ is placed between the plates. What will be its effect on the capacitance?

a. Capacitance will be halved
b. Capacitance will be doubled
c. Capacitance will not change
d. Capacitance will become 1.5 times original

3. Two metallic charged spheres of radii R_1 and R_2 having charges Q_1 and Q_2, respectively, are connected to each other. There is

a. no change in the energy of the system
b. an increase in the energy of the system
c. always a decrease in the energy of the system
d. a decrease in energy of the system unless $Q_1 R_2 = Q_2 R_1$

4. In the circuit of Fig. 4.89, find the charge of the condenser having capacity 5 μF.

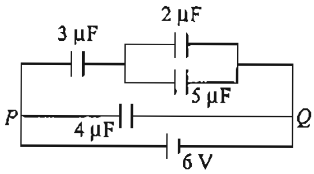


Fig. 4.89

a. 4.5 μC **b.** 9 μC
c. 7 μC **d.** 30 μC

5. In the accompanying diagram, if $C_1 = 3$ μF, $C_2 = 6$ μF, $C_3 = 9$ μF, $C_4 = 12$ μF, $C_5 = 15$ μF and $C_6 = 18$ μF, then the equivalent capacitance between the ends *A* and *B* is

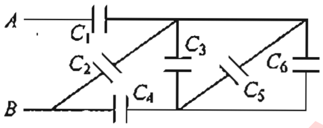


Fig. 4.90

a. 1.22 μF **b.** 5.16 μF
c. 2.25 μF **d.** 2.51 μF

6. Three capacitors of capacitances 2, 3 and 4 pF are connected in parallel. What is the charge (in pC) on each capacitor if the combination is connected to a 100 V supply?

a. 200, 300, 400 **b.** 300, 200, 400
c. 400, 300, 200 **d.** 400, 200, 300

7. In the above question, if the capacitors were connected in series, find the potential difference (in V) across each capacitor.

a. $\frac{300}{13}, \frac{600}{13}, \frac{400}{13}$ **b.** $\frac{600}{13}, \frac{300}{13}, \frac{400}{13}$
c. $\frac{300}{13}, \frac{400}{13}, \frac{600}{13}$ **d.** $\frac{600}{13}, \frac{400}{13}, \frac{300}{13}$

8. Four identical metal plates, each with a surface area *A* (on one side), are placed a distance *d* from each other as shown in Fig. 4.91. The two inner plates are connected to point *B* and the other two plates to another point *A*. Then, the capacitance of the system is

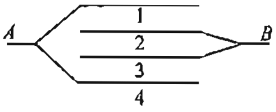


Fig. 4.91

a. $\varepsilon_0 A/d$ **b.** $2\varepsilon_0 A/d$
c. $3\varepsilon_0 A/d$ **d.** $2\varepsilon_0 A/3d$

9. We wish to obtain a capacitance of 5 μF, by using some capacitors, each of 2 μF. Then, the minimum number of capacitors required is

a. 3 **b.** 4
c. 5 **d.** not possible

10. A number of capacitors, each of equal capacitance *C*, are arranged as shown in Fig. 4.92. Equivalent capacitance between *A* and *B* is

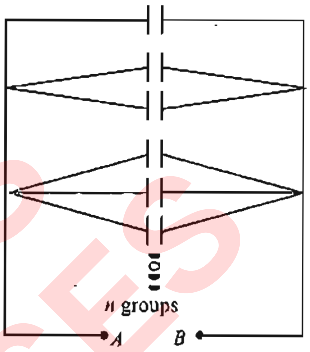


Fig. 4.92

a. n^2C **b.** $(2n + 1)C$
c. $\frac{(n-1)n}{2}C$ **d.** $\frac{(n+1)n}{2}C$

11. The plates of a parallel plate capacitor are charged up to 100 V. Now, after removing the battery, a 2 mm thick plate is inserted between the plates. Then, to maintain the same potential difference, the distance between the capacitor plates is increased by 1.6 mm. Dielectric constant of the plate is

a. 5 **b.** 1.25
c. 4 **d.** 2.5

12. Three plates *A*, *B*, *C* each of area 50 cm^2 have separation 3 mm between *A* and *B* and 3 mm between *B* and *C*. The energy stored when the plates are fully charged is

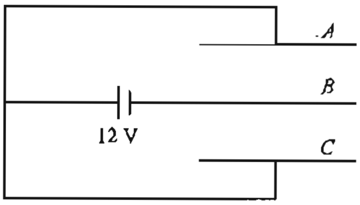


Fig. 4.93

a. 6×10^{-9} J **b.** 3.12×10^{-9} J
c. 2.12×10^{-9} J **d.** none of these

13. Four metallic plates, each with a surface area of one side *A*, are placed at a distance *d* from each other. The alternate plates are connected to points *A* and *B* as shown in Fig. 4.94. Then the capacitance of the system is:

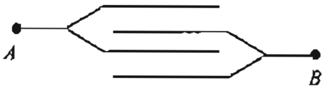


Fig. 4.94

a. $\frac{\varepsilon_0 A}{d}$ **b.** $\frac{2\varepsilon_0 A}{d}$
c. $\frac{3\varepsilon_0 A}{d}$ **d.** $\frac{4\varepsilon_0 A}{d}$

14. The capacitance of an infinite circuit formed by the repetition of the same link consisting of two identical capacitors, each with capacitance C (Fig. 4.95), is

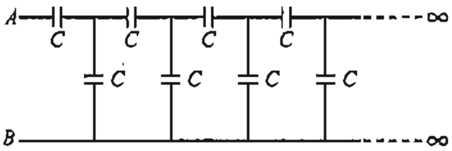


Fig. 4.95

a. zero
b. $0.618C$
c. $2.62C$
d. infinite

15. Two parallel plate capacitors, each of capacitance 40 μF, are connected in series. The space between the plates of one capacitor is filled with a dielectric of dielectric constant $K = 3$, then the equivalent capacitance of the combination is

a. 30 μF
b. 120 μF
c. 40 μF
d. 160 μF

16. For making a parallel plate capacitor you have two plates of copper, a sheet of mica (thickness = 0.10 mm, $K = 5.4$), a sheet of glass (thickness = 0.20 mm, $K = 7$) and a slab of paraffin (thickness = 1.0 cm, $K = 2$). To obtain the largest capacitance, which sheet should you place between the copper plates?

a. Mica
b. Copper
c. Glass
d. Information is insufficient

17. For configuration of media of permittivity $\varepsilon_0, \varepsilon, \varepsilon_0$ between parallel plates each of area A, as shown in Fig. 4.96, the equivalent capacitance is

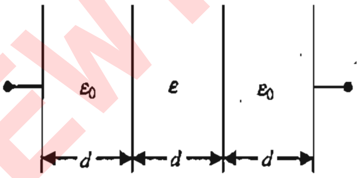


Fig. 4.96

a. $\varepsilon_0 A/d$
b. $\varepsilon\varepsilon_0 A/d$
c. $\dfrac{\varepsilon\varepsilon_0 A}{d(\varepsilon+\varepsilon_0)}$
d. $\dfrac{\varepsilon\varepsilon_0 A}{(2\varepsilon+\varepsilon_0)d}$

18. A parallel plate capacitor is connected across a battery. Now, keeping the battery connected, a dielectric slab is inserted between the plates. In this process,

a. no work is done
b. work is done by the battery and the stored energy increases
c. work is done by the external agent and the stored energy decreases
d. work is done by the battery as well as external agent but the stored energy does not change

19. When a dielectric slab is introduced between the plates of an isolated charged capacitor, it

a. increases the capacitance of the capacitor
b. decreases the electric field between the plates
c. decreases the amount of energy stored in the capacitor
d. all of the above

20. Seven capacitors, each of capacitance 2 μF, are to be combined to obtain a capacitance of 10/11 μF. Which of the following combination is possible?

a. 2 in parallel, 5 in series
b. 3 in parallel, 4 in series
c. 4 in parallel, 3 in series
d. 5 in parallel, 2 in series

21. A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32. Determine potential of the inner sphere.

a. 400 V
b. 450 V
c. 500 V
d. 300 V

22. A parallel plate capacitor has plates of area A and separation d and is charged to a potential difference V. The charging battery is then disconnected and the plates are pulled apart until their separation is $2d$. What is the work required to separate the plates ?

a. $2\varepsilon_0 AV^2/d$
b. $\varepsilon_0 AV^2/d$
c. $3\varepsilon_0 AV^2/2d$
d. $\varepsilon_0 AV^2/2d$

23. A parallel plate capacitor is charged and then disconnected from the source of potential difference. If the plates of the condenser are then moved farther apart by the use of insulated handle, which one of the following is true?

a. The charge on the capacitor increases
b. The charge on the capacitor decreases
c. The capacitance of the capacitor increases
d. The potential difference across the plates increases

24. For the section AB of a circuit shown in Fig. 4.97, $C_1 = 1$ μF, $C_2 = 2$ μF, $E = 10$ V and the potential difference $V_A - V_B = -10$ V. Charge on capacitor C_1 is

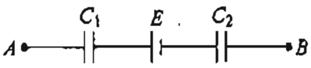


Fig. 4.97

a. 0 μC
b. 20/3 μC
c. 40/3 μC
d. none of these

25. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. What is the common potential (in V) and energy lost (in J) after reconnection?

a. 100, 6×10^{-6}
b. 200, 6×10^{-5}
c. 200, 5×10^{-6}
d. 100, 6×10^{-5}

26. Two parallel plate capacitors of capacitances C and $2C$ are connected in parallel and charged to potential difference V. The battery is then disconnected and the region between the plates of C is filled completely with a material of dielectric constant K. The common potential difference across the combination becomes

a. $\dfrac{2V}{K+2}$ **b.** $\dfrac{V}{K+2}$

c. $\dfrac{3V}{K+3}$ **d.** $\dfrac{3V}{K+2}$

27. Three capacitors A, B and C are connected in a circuit as shown in the Fig. 4.98. What is the charge in μC on the capacitor B?

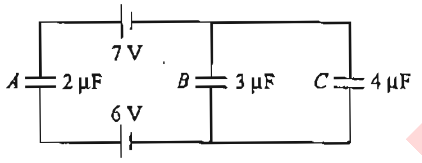


Fig. 4.98

a. 1/3 **b.** 2/3 **c.** 1 **d.** 4/3

28. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is C, then the resultant capacitance is

a. nC **b.** C

c. $(n+1)C$ **d.** $(n-1)C$

29. Three capacitors are connected as shown in Fig. 4.99. Then, the charge on capacitor C_1 is

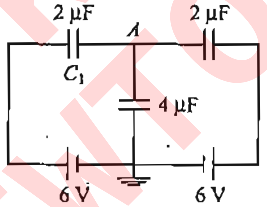


Fig. 4.99

a. 6 μC **b.** 12 μC

c. 18 μC **d.** 24 μC

30. In the above question, the potential of point A is

a. 3 V **b.** 6 V

c. 9 V **d.** zero

31. In Fig. 4.100, if the potential at point B is taken as zero, then the potential at point A will be

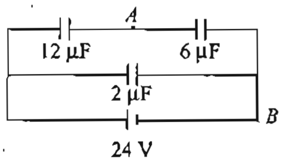


Fig. 4.100

a. 8 V **b.** 16 V

c. 24 V **d.** none of the above

32. A capacitor of capacitance $C_1 = 1$ μF charged up to a voltage $V = 110$ V is connected in parallel to the terminals of a circuit consisting of two uncharged capacitors connected in series and possessing capacitances $C_2 = 2$ μF and $C_3 = 3$ μF. Then, the amount of charge that will flow through the connecting wires is

a. 40 μC **b.** 50 μC

c. 60 μC **d.** 110 μC

33. Ten capacitors are joined in parallel and charged with a battery up to a potential V. They are then disconnected from battery and joined in series. Then, the potential of this combination will be

a. 1 V **b.** 10 V

c. 5 V **d.** 2 V

34. In Fig. 4.101, three capacitors C_1, C_2 and C_3 are joined to a battery. With symbols having their usual meaning, the correct conditions will be

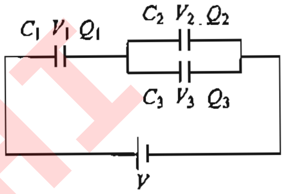


Fig. 4.101

a. $Q_1 = Q_2 = Q_3$ and $V_1 = V_2 = V_3 + V$

b. $Q_1 = Q_2 + Q_3$ and $V = V_1 + V_2 + V_3$

c. $Q_1 = Q_2 + Q_3$ and $V = V_1 + V_2$

d. $Q_2 = Q_3$ and $V_2 = V_3$

35. The cross section of a cable is shown in Fig. 4.102. The inner conductor has a radius of 10 mm and the dielectric has a thickness of 5 mm. The cable is 8 km long. Then, the capacitance of the cable is (given $\log_e 1.5 = 0.4$)

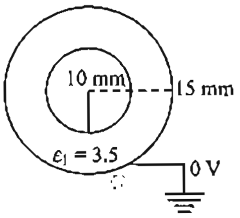


Fig. 4.102

a. 3.8 μF **b.** 1.1 μF

c. 4.8×10^{-10} F **d.** none of these

36. An uncharged parallel plate capacitor having a dielectric of dielectric constant K is connected to a similar air cored parallel plate capacitor charged to a potential V_0. The two share the charge and the common potential becomes V. The dielectric constant K is

a. $\dfrac{V_0}{V} - 1$ **b.** $\dfrac{V_0}{V} + 1$

c. $\dfrac{V}{V_0} - 1$ **d.** $\dfrac{V}{V_0} + 1$

37. Fig. 4.103 shows two identical parallel-plate capacitors connected to a battery. The switch is now opened and the free space between the plates of capacitors is filled with a dielectric of $K = 3$. The ratio of the total electrostatic energy stored in both the capacitors before and after the introduction of the dielectric is

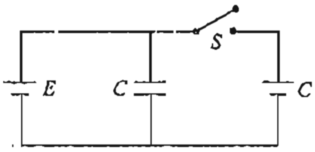


Fig. 4.103

a. 3/4 **b.** 4/5 **c.** 2/3 **d.** 3/5

38. Two identical parallel plate capacitors are connected in series and then joined in series with a battery of 100 V. A slab of dielectric constant $K = 3$ is inserted between the plates of the first capacitor. Then, the potential difference across the capacitors will be, respectively,

a. 25 V, 75 V **b.** 75 V, 25 V

c. 20 V, 80 V **d.** 50 V, 50 V

39. A parallel plate air capacitor is charged to 100 V and is then connected to an identical capacitor in parallel. The second capacitor has some dielectric medium between its plates. If the common potential is 20 V, the dielectric constant of the medium is

a. 2.5 **b.** 4 **c.** 5 **d.** 8

40. In the given network of capacitors as shown in Fig. 4.104, given $C_1 = C_2 = C_3 = 400$ pF and $C_4 = C_5 = C_6 = 200$ pF. The effective capacitance of the circuit between X and Y is

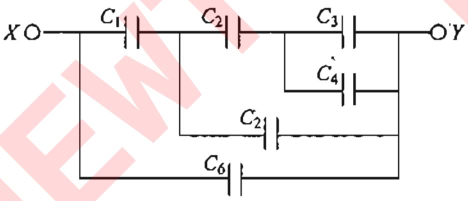


Fig. 4.104

a. 810 pF **b.** 205 pF **c.** 600 pF **d.** 410 pF

41. The work done in increasing the potential of a capacitor from V volt to $2V$ volt is W. Then, the work done in increasing the potential of the same capacitor form $2V$ volt to $4V$ volt will be

a. W **b.** $2W$ **c.** $4W$ **d.** $8W$

42. The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2 mm. The capacitor is charged by connecting it to a 400 V supply. Then the energy density of the energy stored (in Jm^{-3}) in the capacitor is (Take $\varepsilon_0 = 8.8 \times 10^{-12}\ \text{Fm}^{-1}$)

a. 0.113 **b.** 0.177

c. 0.152 **d.** none of these

43. Three identical capacitors, each of capacitance C, are connected in series with a battery of e.m.f. V and get fully charged. Now, the battery is removed and the capacitors are connected in parallel with positive terminals at one point and negative terminals at other point. Then, the common potential will be

a. V **b.** $3V$

c. $V/3$ **d.** Zero

44. In Fig. 4.105, given $C_1 = 3\ \mu$F, $C_2 = 5\ \mu$F, $C_3 = 9\ \mu$F and $C_4 = 13\ \mu$F. What is the potential difference between points A and B?

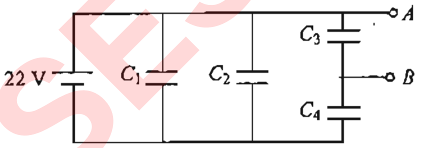


Fig. 4.105

a. 13 V **b.** 9 V

c. 0 V **d.** 11 V

45. Two capacitors of 12 and 4 μF capacitors are connected in series and charged by using a battery of 12 V e.m.f.. Now, the battery is disconnected and the charged capacitors are connected in parallel. Then, the redistributed charges on each capacitor after parallel connection will be, respectively,

a. 36 μC, 36 μC

b. 54 μC, 18 μC

c. 18 μC, 54 μC

d. none of these

46. In the combination of capacitors shown in Fig. 4.106, the potential difference across the plates of the capacitor A will be

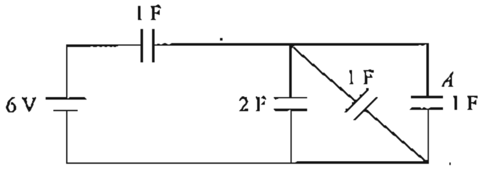


Fig. 4.106

a. 4.8 V **b.** 6 V **c.** 1.2 V **d.** 2.4 V

47. In a circuit shown in Fig. 4.107, the potential difference across the capacitor of 2 F is

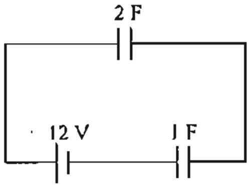


Fig. 4.107

a. 8 V **b.** 4 V **c.** 12 V **d.** 6 V

48. When a metal plate is introduced between the two plates of a charged capacitor and insulated from them, then

a. the metal plate divides the capacitor into two capacitors connected in parallel to each other

- **b.** the metal plate divides the capacitor into two capacitors connected in series with each other
- **c.** the metal plate is equivalent to a dielectric of zero dielectric constant
- **d.** capacitance of the capacitor decreases

49. The potential gradient at which dielectric of the condenser just gets punctured, is called

- **a.** dielectric constant
- **b.** dielectric strength
- **c.** dielectric resistance
- **d.** dielectric number

50. A parallel plate capacitor is charged and then isolated. What is the effect of increasing the plate separation on charge, potential and capacitance, respectively?

- **a.** Constant, decreases, decreases
- **b.** Increases, decreases, decreases
- **c.** Constant, decreases, increases
- **d.** Constant, increases, decreases

51. Six identical capacitors are joined in parallel, charged to a potential difference of 10 V, separated and then connected in series, i.e., the positive plate of one is connected to negative plate of other. Then, potential difference between free plates becomes

- **a.** 10 V
- **b.** 30 V
- **c.** 60 V
- **d.** 10/6 V

52. The effective capacitance between points X and Y in Fig. 4.108, assuming $C_2 = 10\ \mu F$ and that outer capacitors are all 4 μF each, is

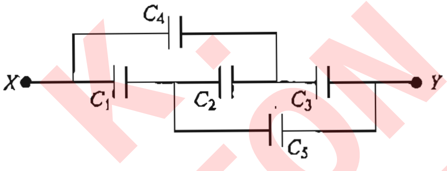


Fig. 4.108

- **a.** 1 μF
- **b.** 3 μF
- **c.** 4 μF
- **d.** 5 μF

53. The resultant capacitance between the points A and B in Fig. 4.109 is

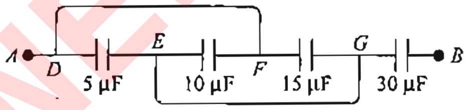


Fig. 4.109

- **a.** 15 μF
- **b.** 30 μF
- **c.** 60 μF
- **d.** 45 μF

54. Two condensers C_1 and C_2 in a circuit are joined as shown in Fig. 4.110. The potential of point A is V_1 and that of B is V_2. The potential of point D will be

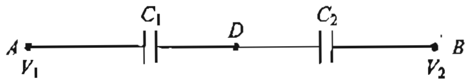


Fig. 4.110

- **a.** $\frac{1}{2}(V_1 + V_2)$
- **b.** $\frac{C_1V_2 + C_2V_1}{C_1 + C_2}$
- **c.** $\frac{C_1V_1 + C_2V_2}{C_1 + C_2}$
- **d.** $\frac{C_2V_1 - C_1V_2}{C_1 + C_2}$

55. A capacitor is charged to store an energy U. The charging battery is disconnected. An identical capacitor is now connected to the first capacitor in parallel. The energy in each of the capacitor is now

- **a.** $3U/2$
- **b.** U
- **c.** $U/4$
- **d.** $U/2$

56. Consider a parallel plate capacitor of capacity 10 μF with air filled in the gap between the plates. Now, one half of the space between the plates is filled with a dielectric of dielectric constant 4 as shown in Fig. 4.111. The capacity of the capacitor changes to

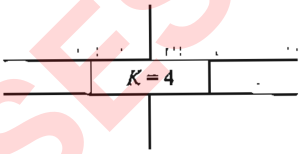


Fig. 4.111

- **a.** 25 μF
- **b.** 20 μF
- **c.** 40 μF
- **d.** 5 μF

57. A 2 μF capacitor is charged to 100 V and then its plates are connected by a conducting wire. The heat produced is

- **a.** 0.001 J
- **b.** 0.01 J
- **c.** 0.1 J
- **d.** 1 J

58. In Fig. 4.112 initial status of capacitance and their connection is shown. Which of the following is incorrect about this circuit

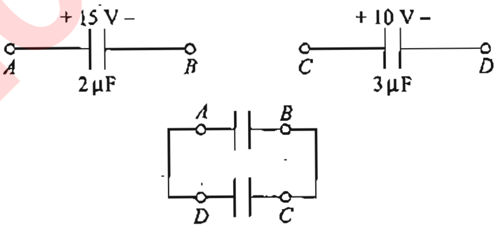


Fig. 4.112

- **a.** Final charge on each capacitor will be zero
- **b.** Final total electrical energy of the capacitance will be zero
- **c.** Total charge flown from A to D is 30 μC
- **d.** Total charge flown from A to D is −30 μC

59. A parallel plate capacitor with no dielectric has a capacitance of 0.5 μF. The space between the plates is filled with equal amounts of two dielectric materials of dielectric constants 2 and 3 as shown in Fig. 4.113. Find the capacitance of the system now.

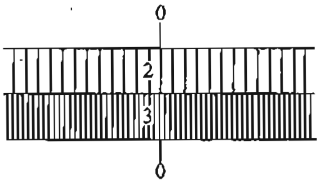


Fig. 4.113

- **a.** 1.2 μF
- **b.** 1.8 μF
- **c.** 1.25 μF
- **d.** none of these

60. Solve the above question if the dielectric materials were filled as shown in Fig. 4.114.

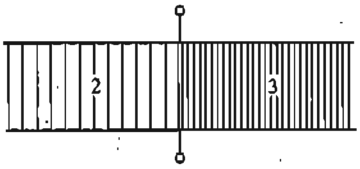


Fig. 4.114

a. 1.2 μF **b.** 1.25 μF

c. 1.80 μF **d.** None of these

61. Two metallic spheres of radii a and b are separated by a distance d as shown in Fig. 4.115. The capacity of the system is

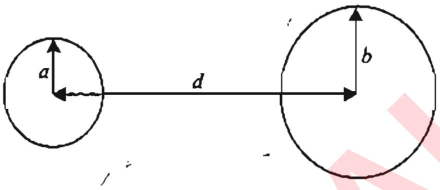


Fig. 4.115

a. $\dfrac{4\pi\varepsilon_0}{\frac{1}{a}+\frac{1}{b}-\frac{2}{d}}$ **b.** $\dfrac{4\pi\varepsilon_0}{\frac{1}{a}+\frac{1}{b}+\frac{2}{d}}$

c. $\dfrac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}+\frac{2}{d}}$ **d.** $\dfrac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}-\frac{2}{d}}$

62. A capacitor of capacitance C_0 is charged to a potential V_0 and then isolated. A small capacitor C is then charged from C_0, discharged and charged again; the process being repeated n times. Due to this, potential of the larger capacitor is decreased to V. Value of C is

a. $C_0\left(\frac{V_0}{V}\right)^{1/n}$ **b.** $C_0\left[\left(\frac{V_0}{V}\right)^{1/n}-1\right]$

c. $C_0\left[\left(\frac{V}{V_0}\right)-1\right]^n$ **d.** $C_0\left[\left(\frac{V}{V_0}\right)^n+1\right]$

63. If the current, charging a capacitor, is kept constant, then the potential difference V across the capacitor varies with time t as

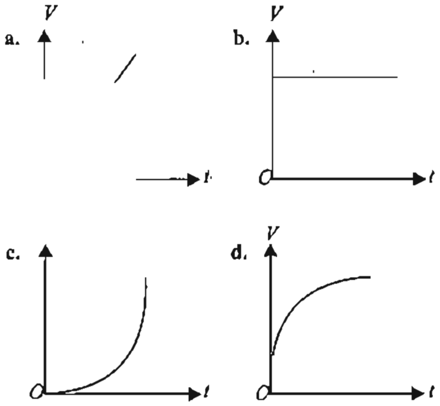


Fig. 4.116

64. A conducting sphere of radius R, carrying charge Q, lies inside an uncharged conducting shell of radius $2R$. If they are joined by a metal wire:

a. a charge $Q/3$ will flow from the sphere to the shell

b. a charge $2Q/3$ will flow from the sphere to the shell

c. a charge Q will flow from the sphere to the shell

d. $\frac{1}{8\pi\varepsilon_0}\frac{Q^2}{R}$ amount of heat will be produced

65. The plates of a parallel plate capacitor are charged with surface charge densities σ_1 and σ_2, respectively. The electric field at points:

a. inside the region between the plates will be zero

b. above the upper plate and below the lower plate will be zero

c. everywhere in the space will be zero

d. inside the region between the plates will be uniform and non-zero

66. The distance between plates of a parallel plate capacitor is $5d$. The positively charged plate is at $x = 0$ and negatively charged plate is at $x = 5d$.

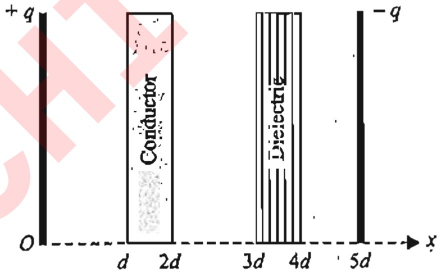
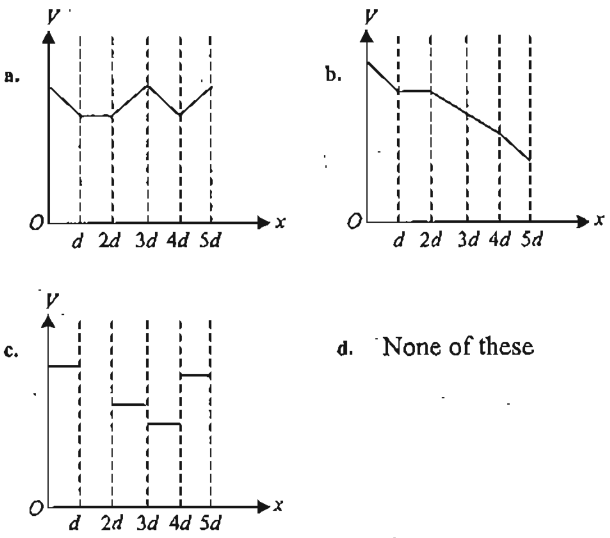


Fig. 4.117

Two slabs, one of conductor and the other of a dielectric of same thickness d, are inserted between the plates as shown in Fig. 4.118. Potential V versus distance x graph will be



d. None of these

Fig. 4.118

67. In the circuit shown in Fig. 4.119 $C = 6$ μF. The charge stored in capacitor of capacity C is

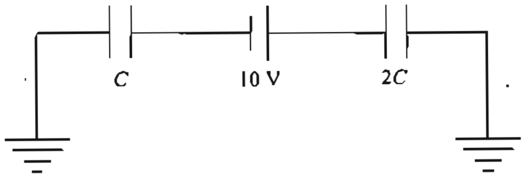


Fig. 4.119

a. zero **b.** 90 μC **c.** 40 μC **d.** 60 μC

68. One plate of a capacitor is fixed and the other is connected to a spring as shown in Fig. 4.120. Area of both the plates is A. In steady state (equilibrium), separation between the plates is $0.8d$ (spring was unstretched and the distance between the plates was d when the capacitor was uncharged). The force constant of the spring is approximately

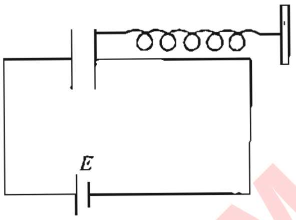


Fig. 4.120

a. $\dfrac{4\varepsilon_0 A E^2}{d^3}$ **b.** $\dfrac{2\varepsilon_0 A E}{d^2}$

c. $\dfrac{6\varepsilon_0 E^2}{A d^3}$ **d.** $\dfrac{\varepsilon_0 A E^3}{2d^3}$

69. A dielectric slab of area A and thickness d is inserted between the plates of a capacitor of area $2A$ with constant speed v as shown in Fig. 4.121. Distance between the plates is d.

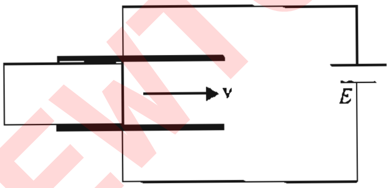


Fig. 4.121

The capacitor is connected to a battery of e.m.f. E. The current in the circuit varies with time as

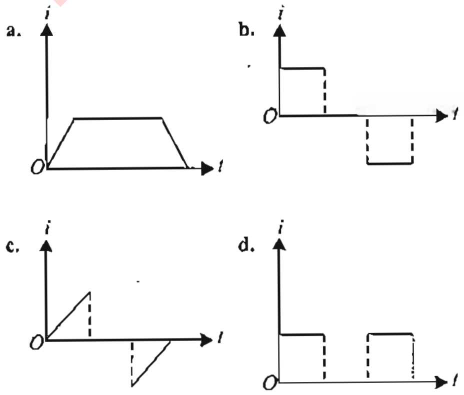


Fig. 4.122

70. A photographic flash unit consists of a xenon filled tube. It gives a flash of average power 2000 W for 0.04 s. The flash is due to discharge of a fully charged capacitor of 40 μF. The voltage to which it is charged before a flash is given by the unit is

a. 1500 V **b.** 2000 V

c. 2500 V **d.** 3000 V

71. A parallel plate capacitor is constructed using three different dielectric materials as shown in the Fig. 4.123. What is the capacitance across P and Q?

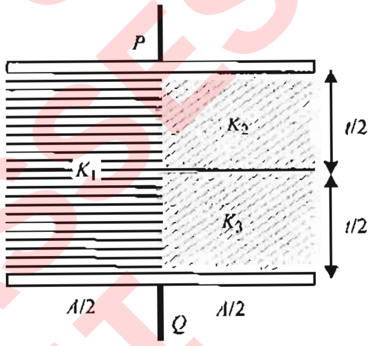


Fig. 4.123

a. $\left(\dfrac{K_1}{2}+\dfrac{K_2K_3}{K_2+K_3}\right)\dfrac{\varepsilon_0 A}{t}$

b. $\left(K_1+\dfrac{K_2K_3}{K_2+K_3}\right)\dfrac{\varepsilon_0 A}{t}$

c. $\left(K_1+\dfrac{2K_2K_3}{K_2+K_3}\right)\dfrac{\varepsilon_0 A}{t}$

d. $\left(K_1+\dfrac{K_2K_3}{2(K_2+K_3)}\dfrac{\varepsilon_0 A}{t}\right)$

72. Two square plates $(l \times l)$ and dielectric $\left(\dfrac{l}{2}\times\dfrac{t}{2}\times l\right)$ are arranged as shown in Fig. 4.124. Find the equivalent capacitance of the structure.

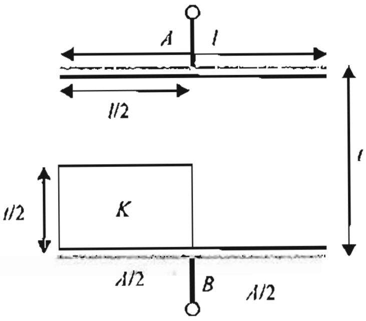


Fig. 4.124

a. $\dfrac{2\varepsilon_0 A}{t}\left(\dfrac{K+1}{K+3}\right)$ **b.** $\dfrac{2\varepsilon_0 A}{t}\left(\dfrac{K+3}{K+1}\right)$

c. $\dfrac{\varepsilon_0 A}{t}\left(\dfrac{K+1}{K+3}\right)$ **d.** $\dfrac{\varepsilon_0 A}{t}\left(\dfrac{2K+1}{2K+3}\right)$

73. The equivalent capacitance across AB (Fig. 4.125) is

a. 8 μF **b.** 12 μF

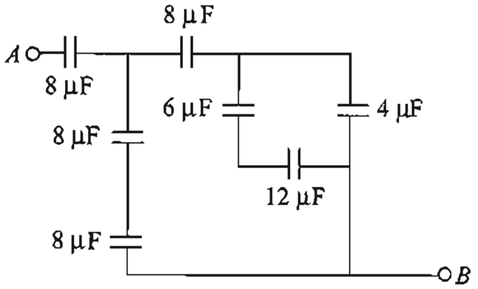


Fig. 4.125

c. 4 μF **d.** 24 μF

74. The equivalent capacitance between P and Q (Fig. 4.126) is

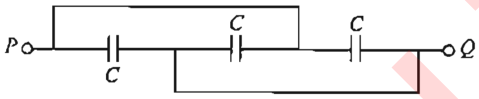


Fig. 4.126

a. $\frac{C}{3}$ **b.** $3\,C$ **c.** $2\,C$ **d.** C

75. The equivalent capacitance between P and Q (Fig. 4.127) is

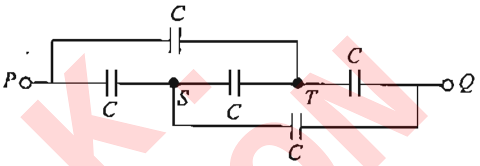


Fig. 4.127

a. $\frac{C}{3}$ **b.** $3C$ **c.** $2C$ **d.** C

76. Find capacitance between P and O (Fig. 4.128) is

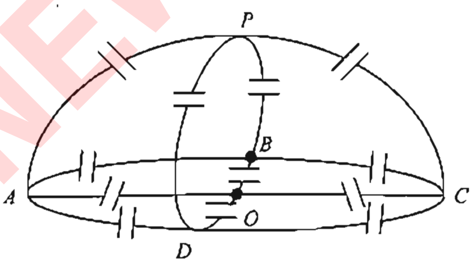


Fig. 4.128

a. $2C$ **b.** $3C$ **c.** $8C$ **d.** $6C$

77. In the circuit shown in Fig. 4.129 $C_1 = 6\ \mu F$, $C_2 = 3\ \mu F$ and battery $B = 20$ V. The switch S_1 is first closed. It is then opened and afterwards S_2 is closed. What is the final charge on C_2?

a. 120 μC **b.** 80 μC

c. 40 μC **d.** 20 μC

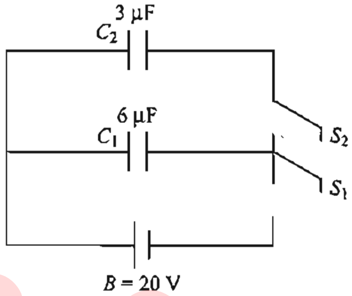


Fig. 4.129

Multiple Correct Answers Type

Solutions on page 4.49

1. To two plates of a parallel plate capacitors, charges Q_1 and Q_2 are given. The capacity of the capacitor is C. When the switch is closed, mark the correct statement(s). [Assume both Q_1, Q_2 to be +ve]

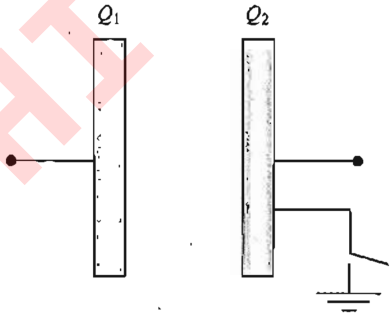


Fig. 4.130

a. The charge flown through switch is zero

b. The charge flown through switch is $Q_1 + Q_2$

c. Potential difference across the capacitor plate is Q_1/C

d. The charge of the capacitor is Q_1

2. A dielectric slab fills the lower half region of parallel plate capacitor as shown in Fig. 4.131. [Take plate area as A]

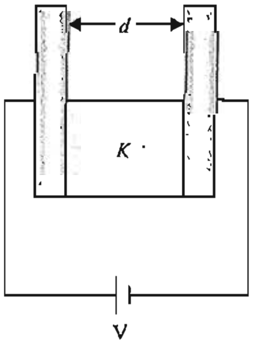


Fig. 4.131

a. Equivalent capacity of the system is $\frac{\varepsilon_0 A}{2d}(1 + K)$

b. The net charge of lower half of the left hand plate $1/K$ times the charge on upper half of the plate

c. Net charges on lower and upper halves of left hand plate are different

d. Net charges on lower half of left hand plate is $\dfrac{K\varepsilon_0 A}{2d} \times V$

3. A parallel plate air capacitor has initial capacitance C. If plate separation is slowly increased from d_1 to d_2, then mark the correct statement(s). [Take potential of the capacitor to be constant, i.e., throughout the process it remains connected to battery.]

a. Work done by electric force = − work done by external agent.

b. Work done by external force $= -\int \vec{F} \cdot \vec{dx}$, where $\vec{F}$ is the electric force of attraction between the plates at plate separation x.

c. Work done by electric force ≠ −ve of work done by external agent.

d. Work done by battery = 2 times the change in electric potential energy stored in capacitor.

4. A capacitor of 5 μF is charged to a potential of 100 V. Now, this charged capacitor is connected to a battery of 100 V with positive terminal of battery connected to negative plate of the capacitor. For the given situation, mark the correct statement(s).

a. The charge flown through 100 V battery is 500 μC.

b. The charge flown through 100 V battery is 1000 μC.

c. Heat dissipated in the circuit is 0.1 J.

d. Work done on the battery is 0.1 J.

5. Two identical capacitors with identical dielectric slabs in between them are connected in series as shown in the Fig. 4.132. Now, the slab of one capacitor is pulled out slowly with the help of an external force F at steady state as shown.

Mark out the correct statement(s).

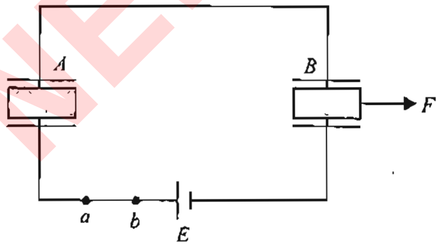


Fig. 4.132

a. During the process, charge (positive) flows from b to a.

b. During the process, charge of capacitor B is equal to charge on A at all instants.

c. Work done by F is positive, and heat may dissipate in the circuit during the process.

d. During the process, the battery has been charged.

Assertion-Reasoning Type

Solutions on page 4.50

In the following questions, each question contains Statement I (Assertion) and Statement II (Reason). Each question has four choices a., b., c. and d. out of which **ONLY ONE** is correct.

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.

b. Statement I is True, Statement II is True; Statement II is Not a correct explanation for Statement I.

c. Statement I is True, Statement II is False.

d. Statement I is False, Statement II is True.

1. Statement I: A capacitor can be given only a limited quantity of charge.

Statement II: Charge stored by a capacitor depends upon shape and size of the plates of capacitor and the surrounding medium.

2. Statement I: Capacity of a parallel plate capacitor increases when distance between the plates is decreased.

Statement II: Capacitance of a capacitor is directly proportional to distance between them.

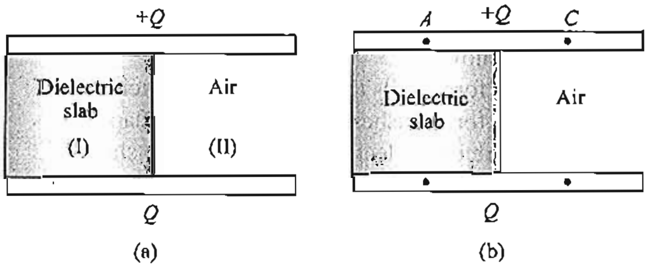
3. Statement I: The capacity of a conductor, under given circumstances, remains constant irrespective of the charge present on it.

Statement II: Capacity depends on size and shape of conductor and also on the medium.

4. Statement I: A charged plane parallel plate capacitor has half interplanar region (I) filled with dielectric slab. The other half region II has air.

Then, the magnitude of net electric field in region I is less than that in region II.

Statement II: In a dielectric medium, induced (or polarized) charges tend to reduce the electric field.



(a) (b)

Fig. 4.133

5. Statement I: A dielectric is inserted between the plates of an isolated fully charged capacitor. The dielectric completely fills the space between the plates. The magnitude of electrostatic force on either metal plate decreases, as it was before the insertion of dielectric medium.

Statement II: Due to insertion of dielectric slab in an isolated parallel plate capacitor (the dielectric completely fills the space between the plates), the electrostatic potential energy of the capacitor decreases.

Comprehension Type

Solutions on page 4.50

For Problems 1–2

An inflated balloon is covered with a conducting surface that carries a charge q. The balloon develops a leak and the radius starts to decrease, but no charge is lost from surface.

1. How does the capacitance of the balloon change as the balloon leaks?
- **a.** C increases
- **b.** C decreases
- **c.** C remains the same
- **d.** There is not enough information to answer the question

2. How does the stored electrical energy change as the balloon leaks?
- **a.** U increases
- **b.** U decreases
- **c.** U remains the same
- **d.** There is not enough information to answer the question

For Problems 3–5

Consider a parallel plate capacitor originally with a charge q_0, capacitance C_0 and potential difference ΔV_0. There is an electrostatic force of magnitude F_0 between plates, and capacitor has a stored energy U_0. The terminals of the capacitor are not connected to anything.

3. A dielectric slab with $k_e > 1$ is inserted between the plates. Which quantity/quantities increase?
- **a.** C **b.** ΔV **c.** F **d.** U

4. What is the direction of the electrostatic force on the dielectric slab while it is being inserted?
- **a.** The force pulls the slab into the capacitor.
- **b.** The force pushes the slab out of the capacitor.
- **c.** There is no electrostatic force on the slab.
- **d.** None of these

5. Later the dielectric slab is removed. What is the direction of the electrostatic force on the dielectric slab while it is being removed?
- **a.** The force pulls the slab into the capacitor
- **b.** The force pushes the slab out of the capacitor
- **c.** There is no electrostatic force on the slab
- **d.** None of these

For Problems 6–7

Fig. 4.134 shows two capacitors in series, the rigid center section of length b being movable vertically.

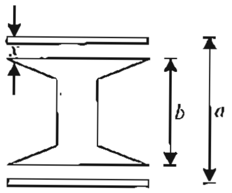


Fig. 4.134

6. The equivalent capacitance of given structure is
- **a.** $\dfrac{\varepsilon_0 A}{(a-b-x)}$ **b.** $\dfrac{\varepsilon_0 A}{(a-b)}$
- **c.** $\dfrac{\varepsilon_0 A}{(b-x)}$ **d.** $\dfrac{\varepsilon_0 A}{(a-x)}$

7. If potentials of upper and lower plates are V_1 and V_2, respectively then find the potential of rigid section.
- **a.** $V_1 - \dfrac{(V_1-V_2)x}{(a-b)}$ **b.** $V_1 - \dfrac{(V_2-V_1)x}{(a-b)}$
- **c.** $V_1 - \dfrac{(V_1-V_2)x}{(a+b)}$ **d.** $V_1 - \dfrac{(V_2-V_1)x}{(a+b)}$

For Problems 8–9

The space between plates of a parallel plate capacitor is filled with dielectric as shown in Figs. 4.135 and 4.136. The area of each plate is A and relative permittivity of dielectric is ε_r.

8. Find the capacitance across PQ in each case.

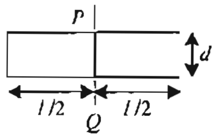


Fig. 4.135

- **a.** $\dfrac{\varepsilon_0 A}{2d}(\varepsilon_r+1)$ **b.** $\dfrac{\varepsilon_0 A}{d}(\varepsilon_r-1)$
- **c.** $\dfrac{2\varepsilon_0\varepsilon_r A}{d(\varepsilon_r-1)}$ **d.** $\dfrac{\varepsilon_0\varepsilon_r A}{d(\varepsilon_r+1)}$

9. Find the capacitance across PQ in each case.

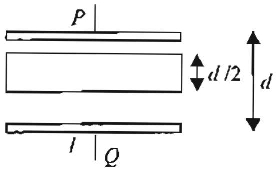


Fig. 4.136

- **a.** $\dfrac{\varepsilon_0 A}{2d}(\varepsilon_r+1)$ **b.** $\dfrac{\varepsilon_0 A}{d}(\varepsilon_r-1)$
- **c.** $\dfrac{2\varepsilon_0\varepsilon_r A}{d(\varepsilon_r-1)}$ **d.** $\dfrac{\varepsilon_0\varepsilon_r A}{d(\varepsilon_r+1)}$

For Problems 10–11

A 1 μF and a 2 μF capacitor are connected in series across a 1200 V supply.

10. The charged capacitors are disconnected from the line and from each other, and are now reconnected with terminals of like sign together. Find the final charge on each capacitor and voltage across each capacitor.
- **a.** Charge on capacitors: $\left(\dfrac{1400}{3}\right)$ μC and $\left(\dfrac{3200}{3}\right)$ μC; potential difference across each capacitor: $\left(\dfrac{1600}{3}\right)$ V

b. Charge on capacitors: $\left(\frac{1600}{3}\right)$ μC and $\left(\frac{3200}{3}\right)$ μC; potential difference across each capacitor: $\left(\frac{1600}{3}\right)$ V

c. Charge on each capacitor is 1600 μC and potential difference across each capacitor is 800 V

d. Charge and potential difference across each capacitor are zero

11. If the charged capacitors are reconnected with terminals of opposite sign together, find the final charge and voltage across each capacitor.

a. Charge on capacitors: $\left(\frac{1400}{3}\right)$ μC and $\left(\frac{3200}{3}\right)$ μC, potential difference across each capacitor: $\left(\frac{1600}{3}\right)$ V

b. Charge on capacitors: $\left(\frac{1600}{3}\right)$ μC and $\left(\frac{3200}{3}\right)$ μC; potential difference across each capacitor: $\left(\frac{1600}{3}\right)$ V

c. Charge on each capacitor is 1600 μC and potential difference across each capacitor is 800 V

d. Charge and potential difference across each capacitor are zero

For Problems 12–14

In the circuit shown (Fig. 4.137), when the switch S is closed, then find

Fig. 4.137

12. the common potential difference across each capacitor

a. 12 V **b.** 24 V **c.** 20 V **d.** 32 V

13. the final charge on 4 μF capacitor

a. 12 μC **b.** 24 μC **c.** 36 μC **d.** 48 μC

14. the fraction of energy lost

a. $\frac{4}{9}$ **b.** $\frac{5}{8}$ **c.** $\frac{3}{5}$ **d.** $\frac{4}{5}$

For Problems 15–17

For the arrangement shown in Fig. 4.138, when the switch S is closed, then find

15. the final charge on 6 μF capacitor

a. 12 μC **b.** 24 μC **c.** 32 μC **d.** 48 μC

16. the final potential difference across the 4 μF capacitor

a. 12 V **b.** 8 V **c.** 20 V **d.** 32 V

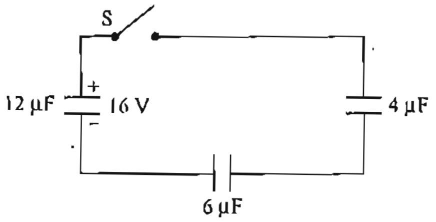


Fig. 4.138

17. the final potential difference across the 12 μF capacitor

a. $\frac{40}{3}$ V **b.** $\frac{20}{3}$ V **c.** 12 V **d.** 24 V

For Problems 18–21

In Fig. 4.139, we charge a capacitor of capacitance $C_1 = 8.0$ μF by connecting it to a source of potential difference $V_0 = 120$ V (not shown in the figure). The switch S is initially open. Once C_1 is charged the source of potential difference is disconnected.

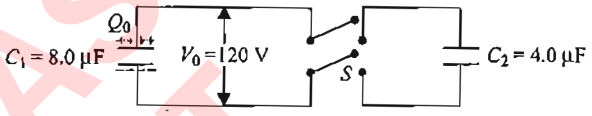


Fig. 4.139

18. The charge Q_0 on C_1, if switch S is left open, is

a. 960 μC **b.** 360 μC

c. 720 μC **d.** 520 μC

19. The energy stored in C_1, if switch S is left open, is

a. 36 mJ **b.** 96 mJ **c.** 57.6 mJ **d.** 24 mJ

20. The capacitor of capacitance $C_2 = 4.0$ μF is initially uncharged. After we close switch S, the potential difference across C_2 is

a. 80 V **b.** 48 V **c.** 36 V **d.** 72 V

21. The total energy of the system after we close switch S is

a. 72.6 mJ **b.** 48 mJ **c.** 38.4 mJ **d.** 12 mJ

For Problems 22–23

Each plate of a parallel plate air capacitor has area $S = 5 \times 10^{-3}$ m^2 and distance between the plates $d = 8.80$ mm. Plate A has positive charge $q_1 = +10^{-10}$ C and plate B has charge $q_2 = +2 \times 10^{-10}$ C. A battery of e.m.f. $E = 10$ V has its positive terminal connected to plate A and negative terminal to plate B. (Given $\varepsilon_0 = 8.8 \times 10^{-12}$)

22. Charge supplied by the battery is

a. 120 pC **b.** 100 pC **c.** 60 pC **d.** 50 pC

23. Energy supplied by the battery is

a. 10^{-9} J **b.** 5×10^{-9} J

c. 50×10^{-9} J **d.** 25×10^{-9} J

For Problems 24–26

In Fig. 4.140, for each capacitor, find

24. the charge in C_2

a. 400 μC **b.** $\left(\frac{500}{3}\right)$ μC

c. $\left(\frac{1000}{3}\right)$ μC **d.** 500 μC

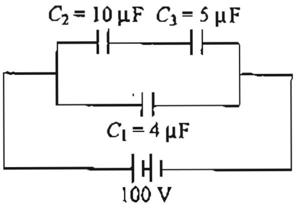


Fig. 4.140

25. the potential difference across C_2

a. 100 V **b.** $\left(\frac{100}{3}\right)$ V

c. $\left(\frac{200}{3}\right)$ V **d.** 30 V

26. the stored energy in C_2

a. $\left(\frac{1}{180}\right)$ J **b.** $\left(\frac{1}{90}\right)$ J

c. $\left(\frac{1}{15}\right)$ J **d.** $\left(\frac{1}{12}\right)$ J

For Problems 27–28

Consider Fig. 4.141.

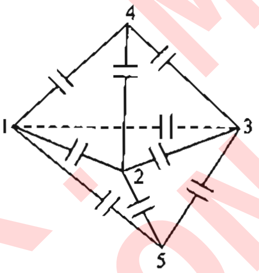


Fig. 4.141

27. The equivalent capacitance between 4 and 5 is

a. $\left(\frac{3C}{4}\right)$ **b.** $\left(\frac{3C}{2}\right)$

c. $\left(\frac{3C}{5}\right)$ **d.** $\left(\frac{5C}{4}\right)$

28. The capacitance between 1 and 3 is

a. $\left(\frac{3C}{4}\right)$ **b.** $\left(\frac{3C}{2}\right)$

c. $\left(\frac{3C}{5}\right)$ **d.** $\left(\frac{5C}{4}\right)$

For Problems 29–31

Consider Fig. 4.142.

29. The charge appearing on C_2 is

a. $E\left(\frac{C_3C_4}{C_1+C_2}\right)$ **b.** $E\left(\frac{C_1C_2}{C_1+C_2}\right)$

c. $E\left(\frac{C_1C_2}{C_3+C_4}\right)$ **d.** $E\left(\frac{C_3C_4}{C_3+C_4}\right)$

30. The potential difference $V_A - V_B$ is

a. $E\left[\frac{C_1C_4 - C_2C_3}{(C_1+C_2)(C_3+C_4)}\right]$

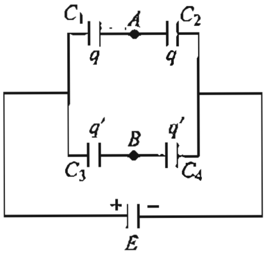


Fig. 4.142

b. $E\left[\frac{C_1C_4 + C_2C_3}{(C_1+C_3)(C_2+C_4)}\right]$

c. $E\left[\frac{C_1C_3 - C_2C_4}{(C_1+C_2)(C_3+C_4)}\right]$

d. $E\left[\frac{C_1C_3 - C_2C_4}{(C_1+C_3)(C_2+C_4)}\right]$

31. The condition for which the potential difference between A and B is zero is

a. $C_1C_2 = C_3C_4$ **b.** $C_1C_4 = C_2C_3$

c. $C_1C_3 = C_2C_4$ **d.** none of these

For Problems 32–33

Consider Fig. 4.143.

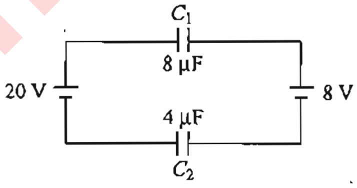


Fig. 4.143

32. The charge appearing on capacitor C_1 is

a. 16 μC **b.** 48 μC **c.** 32 μC **d.** 24 μC

33. The potential difference across C_2 is

a. 4 V **b.** 12 V **c.** 6 V **d.** 8 V

For Problems 34–35

C_1, C_2, C_3 and C_4 are four capacitors connected to a battery of constant e.m.f. equal to 12 V, as shown in Fig. 4.144. Given $C_1 = 1\ \mu F$, $C_2 = 2\ \mu F$, $C_3 = 3\ \mu F$ and $C_4 = 4\ \mu F$.

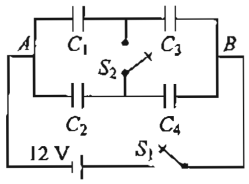


Fig. 4.144

34. Find the charge on C_1 when only S_1 is closed.

a. 16 μC **b.** 9 μC

c. 6 μC **d.** 8 μC

35. If the switch S_2 is also closed. Match the table

a. (m, q) (n, r) (o, s) (p, t)
b. (m, t) (n, s) (o, r) (p, q)

Capacitor	Charge
(m) C_1	(q) $\left(\frac{84}{5}\right)$ μC
(n) C_2	(r) $\left(\frac{72}{5}\right)$ μC
(o) C_3	(s) $\left(\frac{54}{5}\right)$ μC
(p) C_4	(t) $\left(\frac{42}{5}\right)$ μC

c. (m, t) (n, q) (o, s) (p, r)
d. (m, r) (n, s) (o, t) (p, q)

For Problems 36–38

In Fig. 4.145, each capacitance C_1 is 6.9 μF, and each capacitance C_2 is 4.0 μF.

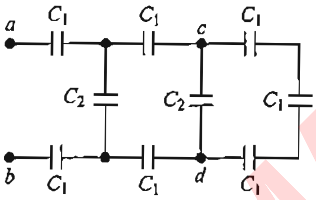


Fig. 4.145

36. The equivalent capacitance of the network between points a and b is
a. 2.3 μF **b.** 6.6 μF **c.** 4.4 μF **d.** 8.8 μF

37. The charge on C_1 nearest to a when $V_{ab} = 420$ V is
a. 840 μC **b.** 560 μC **c.** 600 μC **d.** 320 μC

38. With 420 V across a and b, the value of $(V_c - V_d)$ is
a. 24.6 V **b.** 46.7 V **c.** 18 V **d.** 72 V

For Problems 39–41

Consider Fig. 4.146.

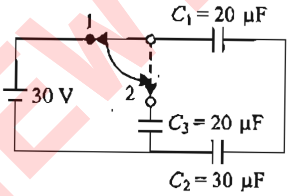


Fig. 4.146

39. In the circuit shown in the figure the switch can be shifted to positions '1' and '2'. the charge on capacitor C_1 when the switch is at position '1' is
a. 120 μC **b.** 240 μC **c.** 360 μC **d.** 80 μC

40. Now, the switch is shifted to position '2'. The charge appearing on capacitor C_3 is
a. 225 μC **b.** 135 μC **c.** 270 μC **d.** 75 μC

41. The charge on capacitor C_1 is
a. 225 μC **b.** 135 μC **c.** 270 μC **d.** 360 μC

For Problems 42–43

For the system shown in Fig. 4.147, capacitance is C. Left plate is given a charge Q_1 and right plate is uncharged. Now, switch is closed.

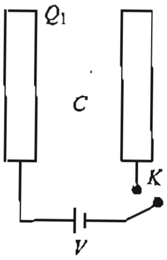


Fig. 4.147

42. Find the amount of charge that will flow through the battery before the steady state is achieved.
a. CV **b.** $CV - Q_1$
c. $CV + \frac{Q_1}{2}$ **d.** $CV - \frac{Q_1}{2}$

43. Find the charge appearing on the inner face of the left plate.
a. $CV - \frac{Q_1}{2}$ **b.** $CV + Q_1$
c. $CV + \frac{Q_1}{2}$ **d.** CV

For Problems 44–45

Consider the circuit shown in Fig. 4.148, after switch S is closed.

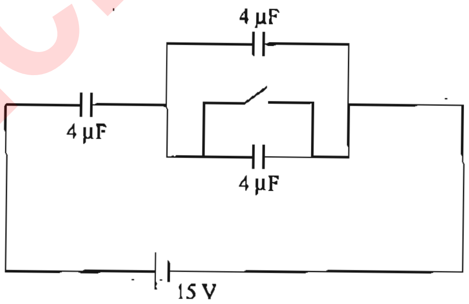


Fig. 4.148

44. What amount of charge will flow through the battery?
a. 20 μC **b.** 60 μC
c. 40 μC **d.** No charge will flow

45. What amount of charge will flow through the switch?
a. 20 μC **b.** 60 μC
c. 40 μC **d.** No charge will flow

Matching Column Type

1. Two identical capacitors are connected in series and the combination is connected with a battery, as shown. Some changes in capacitor 1 are now made independently after the steady state is achieved, listed in column I. Some effects which may occur in new steady state due to these changes on the capacitor 2 are listed in column II. Match the changes on capacitor 1 in column I with corresponding effect on capacitor 2 in column II.

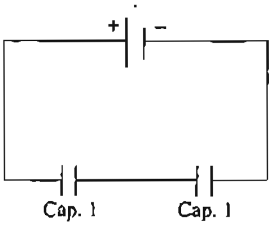


Fig. 4.149

Column I	Column II
i. A dielectric slab is inserted	**a.** Charge on the capacitor increases
ii. Separation between plates increased	**b.** Charge on the capacitor decreases
iii. A metal plate is inserted connecting both plates	**c.** Energy stored in the capacitor increases.
iv. The left plate is grounded	**d.** No change occurrs

2. Observe the circuit in Fig. 4.150 and match the following (assume q_1 q_2, and q_3 be the charges on three capacitors).

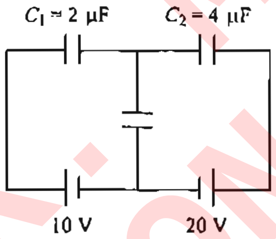


Fig. 4.150

Column I	Column II
i. q_1 (in μC)	**a.** 50
ii. q_2 (in μC)	**b.** $\frac{10}{3}$
iii. q_3 (in μC)	**c.** $\frac{140}{3}$
iv. Potential difference across 6 μF capacitor is (in volt)	**d.** $\frac{25}{3}$

3. Five identical capacitor plates, each of area A, are arranged such that adjacent plates are at d distance apart. Plates are connected to a source of e.m.f. V as shown in Fig. 4.151. Match the following:

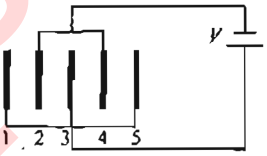


Fig. 4.151

Column I	Column II
i. Charge on plate 1	**a.** $-2\varepsilon_0 AV/d$
ii. Charge on plate 4	**b.** $+\varepsilon_0 AV/d$
iii. Potential difference between plates 2 and 3	**c.** zero
iv. Potential difference between plates 1 and 5	**d.** V

ANSWERS AND SOLUTIONS

Subjective Type

1. a.

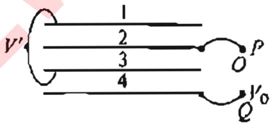


Fig. 4.152

Equivalent circuit

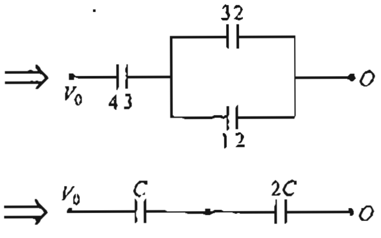


Fig. 4.153

$$C_{eq} = \frac{2C}{3} = \frac{2\varepsilon_0 A}{3d}$$

b.

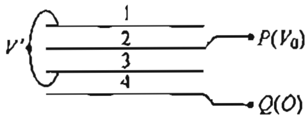


Fig. 4.154

Equivalent circuit

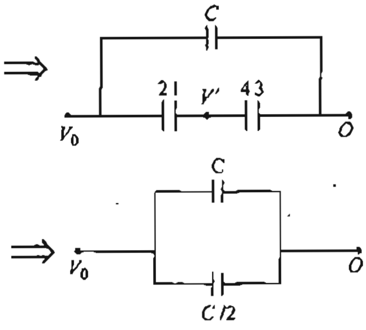


Fig. 4.155

$$C_{eq} = \frac{3}{2}C = \frac{3\varepsilon_0 A}{2d}$$

c.

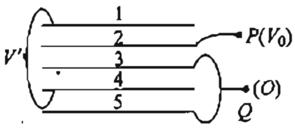


Fig. 4.156

Equivalent circuit

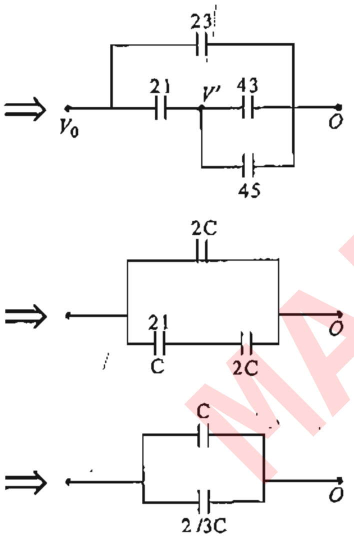


Fig. 4.157

$$C_{eq} = \frac{5C}{3} = \frac{5}{3}\frac{\varepsilon_0 A}{d}$$

2. a. Refer Fig. 4.158

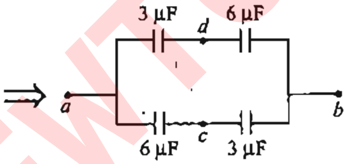


Fig. 4.158

$$V_a - V_d = \frac{6}{6+3}\times 360 = 240\text{ V}$$

and $$V_a - V_c = \frac{3}{6+3}\times 360 = 120\text{ V}$$

$$\therefore\ V_c - V_d = V_{cd} = 120\text{ V}$$

b. Refer Fig. 4.159

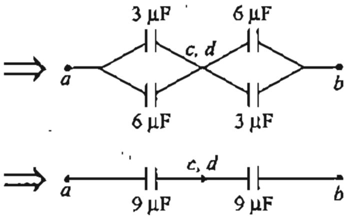


Fig. 4.159

$$V_a - V_c = V_a - V_d = V_d - V_b = V_c - V_b$$
$$= \frac{360}{2} = 180\text{ V}$$

i.e., the potential across each capacitor is 180 V

c. From the two figures it is clear that a charge +540 μC will flow through the switch from C to D.

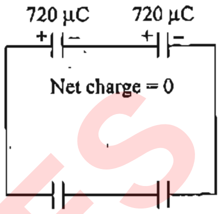


Fig. 4.160

3.

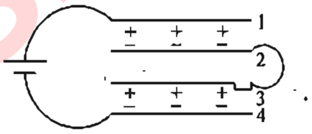


Fig. 4.161

Plates can be rearranged as shown in Fig. 4.162.

Plates 1 and 2 and plates 3 and 4 form two capacitors which are in series between A and B. Plates 2 and 3 do not form any capacitor as they are at same potential.

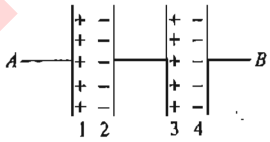


Fig. 4.162

$$\text{So, } C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{\left(\frac{\varepsilon_0 A}{d}\right)\times\left(\frac{\varepsilon_0 A}{3d}\right)}{\frac{\varepsilon_0 A}{d}+\frac{\varepsilon_0 A}{3d}}$$

$$C_{eq} = \frac{\varepsilon_0 A}{4d}$$

4. Let charge Q flows in the circuit on being completed as shown in Fig. 4.163. Applying Kirchhoff's voltage law, we can write

$$\Delta V_A + \Delta V_B = V_C$$

$$\left[100 - \frac{Q}{3\times10^{-6}}\right] + \left[180 - \frac{Q}{2\times10^{-6}}\right] = \frac{Q}{2\times10^{-6}}$$

$$\text{i.e., } Q = \frac{280\times10^{-6}\times 6}{8} = 210\,\mu\text{C}$$

a. So, the final charge on the three capacitors will be,

$$Q_A = (V_AC_A)_{\text{initial}} - Q$$
$$= 100\times3\times10^{-6} - 210\times10^{-6} = 90\,\mu\text{C}$$

$$Q_B = (V_BC_B)_{\text{initial}} - Q$$
$$= 180\times2\times10^{-6} - 210\times10^{-6} = 150\,\mu\text{C}$$

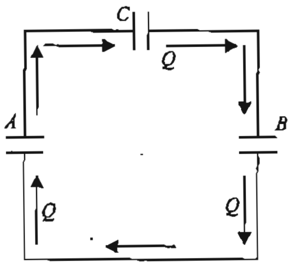


Fig. 4.163

And $Q_C = Q = 210\,\mu C$

b. Initial stored electrostatic potential energy

$$= \frac{1}{2}C_A V_A^2 + \frac{1}{2}C_B V_B^2$$

$$= \frac{1}{2} \times 3 \times 10^{-6} \times (100)^2 + \frac{1}{2} \times 2 \times 10^{-6} \times (180)^2$$

$$= 4.74 \times 10^{-2}\,\text{J}$$

Final stored electrostatic potential energy

$$= \frac{Q_A^2}{2C_A} + \frac{Q_B^2}{2C_B} + \frac{Q_C^2}{2C_C}$$

$$= \frac{(90 \times 10^{-6})^2}{2 \times 3 \times 10^{-6}} + \frac{(150 \times 10^{-6})^2}{2 \times 2 \times 10^{-6}} + \frac{(210 \times 10^{-6})^2}{2 \times 2 \times 10^{-6}}$$

$$= 1.8 \times 10^{-2}\,\text{J}$$

5.

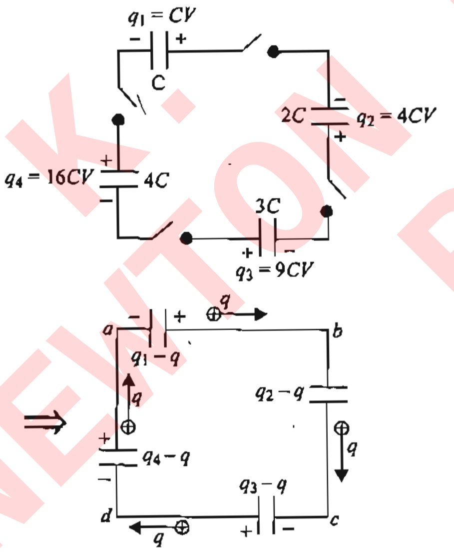


Fig. 4.164

Applying loop law in *abcda*,

$$\frac{q_1 - q}{C} + \frac{q_2 - q}{2C} + \frac{q_3 - q}{3C} + \frac{q_4 - q}{4C} = 0$$

Substituting values of q_1, q_2, q_3 and q_4, we get

$$q = \frac{24}{5}CV$$

$$|\Delta V|_C = \left|\frac{q_1 - q}{C}\right| = \frac{19}{5}\,\text{V}$$

$$|\Delta V|_{2C} = \left|\frac{q_2 - q}{2C}\right| = \frac{2}{5}\,\text{V}$$

$$|\Delta V|_{3C} = \left|\frac{q_3 - q}{3C}\right| = \frac{7}{5}\,\text{V}$$

and $$|\Delta V|_{4C} = \left|\frac{q_4 - q}{4C}\right| = \frac{14}{5}\,V$$

Charge on different capacitors are as shown in Fig. 4.165.

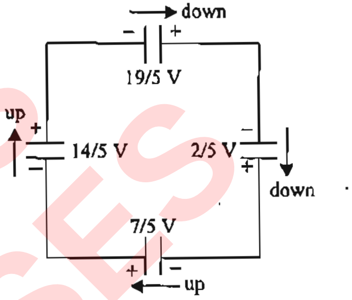
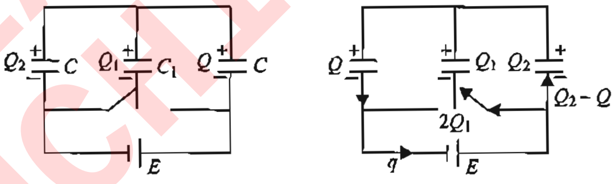


Fig. 4.165

6. For Fig. 4.166(a) $C_{eq} = \dfrac{(C + C_0)C}{2C + C_0}$ and $Q = C_{eq}E$

$$\therefore \quad Q_1 = \frac{C_0 Q}{C + C_0}, \text{ and } Q_2 = \frac{C\,Q}{C + C_0}$$



(a) (b)

Fig. 4.166

Charge flown through battery on reconnection, as in Fig. 4.166(b)

$$q = Q - Q_2 = Q - \frac{CQ}{C + C_0} = \frac{C_0 Q}{C + C_0} = \frac{C_0 E}{C + C_0}C_{eq}$$

Total energy on capacitors before and after reconnection is same. Hence, whole work done by battery will go in the form of heat. So, heat generated

$$= W_b = qE = \frac{C_0 E^2}{C + C_0} \times \frac{(C + C_0)\,C}{(2C + C_0)} = \frac{C\,C_0\,E^2}{2\,C + C_0}$$

Alternatively: Heat $= \sum \dfrac{(\Delta Q)^2}{2C} = 2\left[\dfrac{(Q - Q_2)^2}{2C}\right]$

$$+ \frac{(2\,Q_1)^2}{2C_0}$$

$$= \frac{C_0^2 Q^2}{(C + C_0)^2 C} + \frac{2\,C_0^2\,Q^2}{C_0[C + C_0]^2} = \frac{C_0^2 Q^2}{(C + C_0)^2}\left[\frac{1}{C} + \frac{2}{C_0}\right]$$

$$= \frac{E^2\,C\,C_0}{(2C + C_0)}$$

7. Vertically cross-connected capacitors in the diagram are ineffective as they all are short-circuited. So, circuits can be redrawn as

Hence, $\dfrac{1}{C_{eq}} = \dfrac{1}{4C} + \dfrac{1}{3C} + \dfrac{1}{2C} + \dfrac{1}{C}$

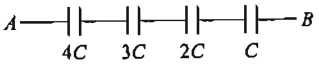


Fig. 4.167

or $C_{eq} = \frac{12}{25}C = \frac{12}{25} \times 25\ \mu F\ = 12\ \mu F$

or $C_{eq} = 12\ \mu F$

8. Maximum charge C_1 can hold

$Q_1 = C_1 V_1 = 1 \times 10^{-6} \times 6 \times 10^3$

$Q_1 = 6 \times 10^{-3}$ C

and maximum charge C_2 can hold

$Q_2 = C_2 V_2 = 2 \times 10^{-6} \times 4 \times 10^3$

$Q_2 = 8 \times 10^{-3}$ C

When connected in series, both will have equal charges and so each can have charge Q_1 which is smaller of the two.

In this case:

Voltage across $C_1 = 6$ kV

And voltage across

$C_2 = \frac{Q_1}{Q_2} = \frac{6 \times 10^{-3}}{2 \times 10^{-6}} = 3$ kV

∴ Maximum voltage across the system = 6 + 3 = 9 V

9. When switch is opened, 2 and 3 μF capaitors are in series.

So, $C_{eq} = \frac{2 \times 3}{5} = \frac{6}{5}\ \mu$ F

Hence, charge flowing

$q = CV = \frac{6}{5} \times 90 = 108\ \mu C$

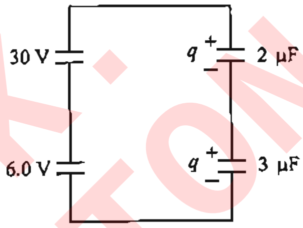


Fig. 4.168

When switch S is closed, let q_1 and q_2 be charges on the two capacitors.

So, $q_1 = 2 \times 30 = 60\ \mu C$

$q_2 = 3 \times 60 = 180\ \mu C$

Let charge q_B goes to the upper plate of 3 μF capacitor and lower plate of 2 μF capacitor. Initially, both the plates have charge $+q - q = 0$

Finally, they have charges $q_2 - q_1$.

So, $q_2 - q_1 = q_B + 0$

or $q_B = q_2 - q_1$

$= 180 - 60 = 120\ \mu C$

or $q_B = +120\ \mu C$

10. The field between the plates of a parallel plate capacitor

$E = \frac{V}{d}$ or $V = Ed$.

So, potential difference across fibre

$V_F = E_F \times d_F = 6.4 \times 10^6 \times 0.5 \times 10^{-3}$

$= 3.2$ kV

So, $q_F = C_F \times V_F$

and $q_M = C_M \times V_M$

Here, fibre and mica capacitors are in series, so charges across both are same.

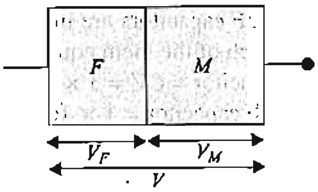


Fig. 4.169

i.e., $q_F = q_M$, $C_M V_M = C_F V_F$

or $V_M = \frac{C_F V_F}{C_M}$

$= \frac{K_F}{K_M}\left[\frac{d_M}{d_F}\right] V_F \left\{\text{as } C = \frac{\varepsilon_0 A}{d}\right\}$

or $V_M = \frac{2.5}{8}\left[\frac{1 \times 10^{-3}}{0.5 \times 10^{-3}}\right] \times 3.2\ \text{kV} = 2\ \text{kV}$

In series, $V = V_1 + V_2$

i.e., $V = V_F + V_M = 3.2\ \text{kV} + 2\ \text{kV}$

or $V = 5.2$ kV

11. Let $C_1 = C$ and $C_2 = 2C$ and charges on different capacitors have been shown. Net charge on isolated system should be zero.

Hence, $q_1 - q_2 - q_3 = 0$ (i)

$E - \frac{q_3}{2C} - \frac{q_1}{C} = 0$ (ii)

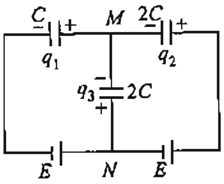


Fig. 4.170

$E - \frac{q_3}{2C} - \left(\frac{q_2 + q_3}{C}\right) = 0$ (using $q_1 = q_2 + q_3$)

Or $E - \frac{q_3}{2C} - \frac{q_2}{C} - \frac{q_3}{C} = 0$

$E - \frac{q_2}{C} - \frac{3}{2}\frac{q_3}{C}$ (iii)

$E - \frac{q_2}{2C} + \frac{q_3}{2C} = 0$

$2E - \frac{q_2}{2} + \frac{q_3}{2C} = 0$ (iv)

Solving (iii) and (iv), we get

$q_3 = \frac{2CE}{5}$

So $V_{MN} = \frac{q_3}{2C} = \frac{2CE}{5 \times 2C5} = \frac{E}{5} = \frac{110}{5} = 22$ V

So $V_{MN} = 22$ V

12. Charge across 5 μF capacitor = 10 μC.

∴ Potential difference across 5 μF capacitor

$$= \frac{q}{C} = \frac{10 \times 10^{-6}}{5 \times 10^{-6}} = 2\text{ V}$$

As 3, 4 and 5 μF capacitors are in parallel, so potential difference across each of the them equals 2 V.

Charge on 3 μF capacitor = $CV = 3 \times 10^{-6} \times 2 = 6$ μC

Charge on 4 μF capacitor = $4 \times 10^{-6} \times 2 = 8$ μC

Total charge flowing in upper branch of circuit

$$= 10\ \mu\text{C} + 6\ \mu\text{C} + 8\ \mu\text{C} = 24\ \mu\text{C}$$

Potential difference across $C_2 = \dfrac{24\ \mu\text{C}}{4\ \mu\text{F}} = 6$ V

Total potential difference across AB

$$= 6 + 2 = 8\text{ V}$$

Equivalent capacitance of lower branch of circuit

$$= \frac{6 \times 3}{6 + 3} = 2\ \mu\text{F}$$

So, charge flowing = $2 \times 10^{-6} \times 8 = 16$ μC

∴ Potential difference between A and C

$$= \frac{16\ \mu\text{C}}{3\ \mu\text{F}} = \frac{16}{3} = 5.33\text{ V}$$

13. When the key is open, 120 V is divided between C_1 and C_2 in the inverse ratio of their capacitances.

$$\therefore \quad V_1 = \frac{120}{2+3} \times 3 = 72\text{ V}$$

$$V_2 = \frac{120}{2+3} \times 2 = 48\text{ V}$$

∴ $q_1 = 72 \times 2 = 144$ μC

and $q_2 = 48 \times 3 = 144$ μC

When the key is closed, let q_1 and q_2 be the steady charge on C_1 and C_2. Then, by the loop rule

$$60 - \frac{q_1}{2 \times 10^{-6}} = 0 \quad \Rightarrow \quad q_1 = 120\ \mu\text{C}$$

$$\text{and } 60 - \frac{q_2}{3 \times 10^{-6}} = 0 \quad \Rightarrow \quad q_2 = 180\ \mu\text{C}$$

∴ Charge that flows through section 1

= 144 – 120 = 24 μC

Charge that flows through section 2

= 180 – 144 = 36 μC

Charge that flows thorugh section 3

= 24 + 36 = 60 μC

14. The distribution of charge is shown in Fig. 4.171 in compliance with the point rule. Applying loop rules

$$-\frac{q_2}{5} + \frac{q_3}{0.75} + \frac{q_1}{15} = 0$$

$$\Rightarrow \quad q_1 - 3q_2 + 20q_3 = 0. \qquad \text{(i)}$$

$$-\frac{q_2 + q_3}{15} - \frac{q_3}{0.75} + \frac{q_1 - q_3}{5} - \frac{q_3}{0.75} = 0$$

$$\Rightarrow \quad 3q_1 - q_2 - 44q_3 = 0 \qquad \text{(ii)}$$

$$23 - \frac{q_2}{5} - \frac{q_2 + q_3}{15} = \frac{q_2}{5} = 0$$

$$\Rightarrow \quad 345 = 7q_2 + q_3 \qquad \text{(iii)}$$

Solving for q_1, q_2 and q_3, we get

$$q_1 = \frac{19 \times 345}{92},\ q_2 = \frac{13 \times 345}{92},\ q_3 = \frac{345}{92}$$

∴ P.D. between A and $B = \dfrac{q_3}{0.75} = \dfrac{345}{92} \times \dfrac{4}{3} = 5$ V

P.D. between E and F is also 5 V but in the opposite direction.

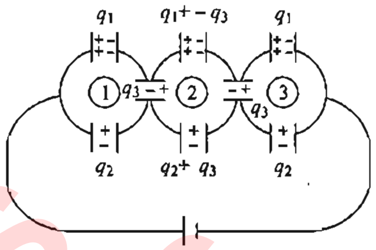


Fig. 4.171

15. $Q = Q_1 + Q_2$

Loop $ACDA$: $Q_1 = Q_2 + Q_3$

Loop $DCED$: $Q_3 + Q_1 - Q_2 + 2Q_3 = Q_2 - Q_3$

$\Rightarrow \quad Q_1 - 2Q_2 + 4Q_3 = 0$

$\Rightarrow Q_1 - 2Q_2 + 4(Q_1 - Q_2) = 0$

$\Rightarrow 5Q_1 = 6Q_2 \quad \Rightarrow 6Q = 11Q_1$

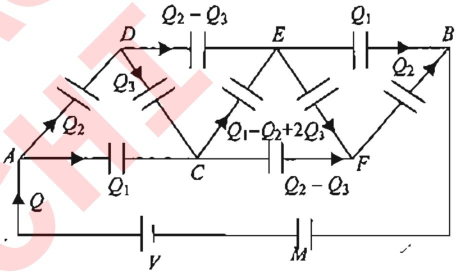


Fig. 4.172

Loop $ADEBFA$:

$CV = Q_2 + Q_2 - Q_3 + Q_3 \Rightarrow CV = Q + Q_2 - Q_3$

$CV = Q + Q_2 - (Q_1 - Q_2)$

$CV = Q - Q_1 + 2Q_2$

$CV = Q - Q_1 + 2(Q - Q_1)$

$CV = 3Q - 3Q_1$

$$CV = 3Q - 3 \times \frac{6Q}{11} = \frac{15}{11}Q$$

$$\Rightarrow \quad C_{eq} = \frac{Q}{V} = \frac{11}{15}\text{ C}$$

$$\Rightarrow C_{eq} = \frac{11}{15} \times 30 = 22\ \mu\text{F}.$$

Objective Type

1. c. New charge resides only on the outer sufaces.

2. b. $C' = \dfrac{\varepsilon_0 A}{d - t} = \dfrac{\varepsilon_0 A}{d - d/2} = \dfrac{2\,\varepsilon_0 A}{d}$

   Hence, capacitance is doubled.

3. d. Let their potentials are same.

   $$V_1 = V_2 \quad \Rightarrow \quad \frac{kQ_1}{R_1} = \frac{kQ_2}{R_2} \Rightarrow \quad Q_1R_2 = Q_2R_1$$

   If potential are same, then no flow of charge will occur, otherwise charge will flow and there will be loss of energy.

4. b. Potential on 5 μF capacitor

$$= \frac{3 \times 6}{3 + (2 + 5)} = 1.8 \text{ V}$$

So, charge on this capacitor $= CV = 5 \times 1.8 = 9\ \mu\text{F}$

5. **d.** C_3, C_5 and C_6 are in parallel and C_4 is in series with it. Then C_2 is in parallel and C_1 is in series.

6. **a.** In parallel, potential is same on each capacitor.

7. **d.** In series, charge is same on each capacitor.

8. **b.** Plate 2 and 3 will be at same potential. So, there will be two capacitors in parallel.

9. **b.**
Minimum number of capacitors is 4.

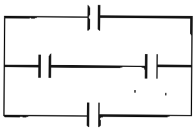


Fig. 4.173

10. **d.** $C_{eq} = C + 2C + 3C + \cdots + nC$

$$= \frac{(n+1)n}{2}C$$

11. **a.** As battery is disconnected, so charge will remain same. It is given that final potential is same. So, final capacitance should be equal to initial capacitance

$$\frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{(1.6 + d) - t(1 - 1/k)}$$

$$\Rightarrow \quad K = 5$$

12. **c.** There are two capacitors in parallel.

$$U = \frac{1}{2}CV^2,\ C = \frac{2\varepsilon_0 A}{d}$$

13. **c.** The four plates are alternately connected. They form three capacitors in parallel. Capacity of each capacitor is $\frac{\varepsilon_0 A}{d}$. So, the net capacity is $\frac{3\varepsilon_0 A}{d}$.

14. **b.** As $\infty \pm 1 = \infty$
Therefore, if one link is reduced or added to the circuit, the capacitance remains unchanged. If C_{xy} is equivalent capacitance between x and y, then the equivalent circuit is shown in figure. The capacitances C_{xy} and C connected between A and B are in parallel. Their effective capacitance

$$C_1 = C + C_{xy}$$

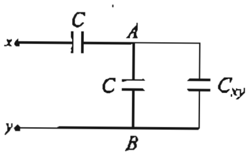


Fig. 4.174

Now, C (connected between x and A) and C_1 are in series. Therefore, effective capacitance between x and y

$$C_{xy} = \frac{CC_1}{C + C_1} \quad \Rightarrow \quad C_{xy} = \frac{C(C + C_{xy})}{C + (C + C_{xy})}$$

Solving for C_{xy}, we get

$$C_{xy} = \frac{\sqrt{5} - 1}{2}C = 0.618C$$

15. **a.** $C_1 = 40\ \mu\text{F}$, $C_2 = K \times 40 = 3 \times 40 = 120\ \mu\text{F}$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{40 \times 120}{40 + 120} = 30\ \mu\text{F}$$

16. **a.** $C = \frac{K\,\varepsilon_0 A}{d}$, find K/d for each. Capacitance will be largest for which K/d is largest.

17. **d.** $C_{eq} = \dfrac{\varepsilon_0 A}{\dfrac{d}{K_1} + \dfrac{d}{K_2} + \dfrac{d}{K_3}}$

Here, $K_1 = K_3 = 1$, $K_2 = \dfrac{\varepsilon}{\varepsilon_0}$

18. **b.** Extra charge will flow through battery, so work is done by battery. External agent will do negative work.

19. **d.** Here, battery is disconnected.

20. **d.** Check each option separately.

21. **b.** $V - 0 = \dfrac{Q}{C} = \dfrac{Q(b-a)}{k\,4\pi\varepsilon_0\,9b}$

$$V = \frac{9 \times 10^9 \times 2.5 \times 10^{-6}(0.13 - 0.12)}{32 \times 0.13 \times 0.12}$$

$$= 450 \text{ V}$$

22. **d.** $W = U_2 - U_1 = \dfrac{q^2}{2}\left[\dfrac{1}{C_2} - \dfrac{1}{C_1}\right]$

$$C_1 = \frac{\varepsilon_0 A}{d},\ C_2 = \frac{C_1}{2} = \frac{\varepsilon_0 A}{2d}$$

$$q = C_1 V = \frac{\varepsilon_0 A V}{d}$$

Solve to get, $W = \dfrac{1}{2}\dfrac{\varepsilon_0 A V^2}{d}$

23. **d.** $V = Ed$. As d increases, V also increases. Note that E remains same.

24. **c.** $V_A - \dfrac{q}{C_1} - E - \dfrac{q}{C_2} = V_B$

25. **a.** $V_C = \dfrac{C_1 V_1}{C_1 + C_2}$

$$U = \frac{1}{2}(C_1 + C_2)V_C^2$$

26. **d.** $V_C = \dfrac{Q_1 + Q_2}{C_1 + C_2} = \dfrac{CV + 2CV}{KC + 2C} = \dfrac{3V}{K + 2}$

27. **b.** Capacitors B and C are in parallel, then A is in series.

$$C_{eq} = \frac{2 \times (3 + 4)}{2 + (3 + 4)} = \frac{14}{9}\ \mu\text{F}$$

$$Q = C_{eq}V = \frac{14}{9}(7 - 6) = \frac{14}{9}\ \mu\text{C}$$

Q will be divided between B and C. So, charge on B:

$$q = \frac{3 \times 14/9}{3 + 4} = \frac{2}{3}\ \mu\text{C}$$

28. **d.** There will be $n - 1$ capacitors, all connected in parallel

29. **b.** $6 = \frac{q}{2} + \frac{2q}{4}$

$$\Rightarrow q = 6\,\mu\text{C}$$

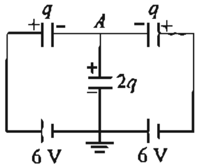


Fig. 4.175

30. **a.** $V_A - 0 = \frac{2q}{4} = 3$ V

31. **b.** Potential across 6 μF capacitor:

$$V_A - V_B = \frac{12 \times 24}{12 + 6} = 16$$

$$\Rightarrow \quad V_A - 0 = 16$$

$$\Rightarrow V_A = 16\,\text{V}$$

32. **c.** Initial charge on C_1: $Q_1 = C_1V = 110\,\mu\text{C}$

Let x charge flows through wires $\frac{Q_1 - x}{C_1} = \frac{x}{C_{eq}}$

where $C_{eq} = \frac{C_2 - C_3}{C_2 + C_3}$

33. **b.** In series, all the potentials will be added.

34. **c.** At junction A, Q_1 will divide into Q_2 and Q_3. Hence, $Q_1 = Q_2 + Q_3$.

C_2 and C_3 are in parallel, so potential on them will be same.

$V_2 = V_3$

V will divide into V_1 and V_2 (or V_3)

Hence, $V = V_1 + V_2$

or $V = V_1 + V_3$

35. **a.** $C = \frac{K\,\varepsilon_0 2\pi l}{\ln(b/a)}$

$$= \frac{3.5 \times 2\pi \times 8.85 \times 10^{-12} \times 8 \times 10^3}{\ln(15/10)}$$

$$= 3.84 \times 10^{-6}\,\text{F}$$

36. **a.** $C_1 = C$, $V_1 = V_0$, $C_2 = KC$, $V_2 = 0$, and

$V_{\text{common}} = V$

We know that $V_{\text{common}} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$$V = \frac{CV_0 + KC \times O}{C + KC}$$

$$K = \frac{V_0}{V} - 1$$

37. **d.** $U_i = \frac{1}{2}CE^2 + \frac{1}{2}CE^2 = CE^2$

$$U_f = \frac{1}{2}KCE^2 + \frac{1}{2}\frac{CE^2}{K}$$

$$= \left(K + \frac{1}{K}\right)\frac{1}{2}CE^2$$

$$\frac{U_i}{U_f} = \frac{1}{\left(K + \frac{1}{K}\right)\frac{1}{2}} = \frac{2K}{K^2 + 1}$$

$$= \frac{2 \times 3}{3^2 + 1} = \frac{6}{10} = \frac{3}{5}$$

38. **a.** $V_1 = \frac{C \times 100}{3C + C} = 25$ V

$V_2 = 100 - 25 = 75$ V

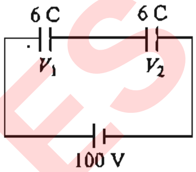


Fig. 4.176

39. **b.** $c_1 = c$, $c_2 = KC$, $v_1 = 100$ V

$V_C = 20$ V, $V_2 = 0$

$$20 = \frac{C \times 100 + KC \times 0}{C + KC}$$

$K = 4$

40. **d.** Start with C_3 and C_4 in parallel, then C_2 in series, then C_5 in parallel, then C_1, in series and finally C_6 is in parallel.

41. **c.** $W = \frac{1}{2}C[(2V)^2 - V^2] = \frac{3}{2}CV^2$

$$W' = \frac{1}{2}C[(4V)^2 - (2V)^2] = 6CV^2$$

$$W' = \frac{6 \times 2W}{3} = 4W$$

42. **b.** $A = 90\text{ cm}^2$, $d = 2$ mm

$E = \frac{V}{d}$, $V = 400$ V

$$u = \frac{1}{2}\varepsilon_0E^2 = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2$$

$$u = \frac{1}{2}(8.85 \times 10^{-12})\left(\frac{400}{2 \times 10^{-3}}\right)^2 = 0.177$$

43. **c.** In series, potential is divided.

44. **a.** 22 V get will divide into series combination of C_3 and C_4.

45. **b.** Potential across 12 μF capacitor:- $V_1 = \frac{4 \times 12}{12 + 4} = 3\,V$

Potential across 4 μF capacitor: $V_2 = \frac{12 \times 12}{12 + 4} = 9\,V$

After redistribution:

Charge on 12 μF:- $q_1 = \left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)C_1$

$$= \left(\frac{12 \times 3 + 4 \times 9}{12 + 4}\right)12 = 54\mu\text{C}$$

Similarly charge on 4 μF = 18 μC

46. **c.** 1, 1 and 2 F will be in parallel. Their equivalent is $1 + 1 + 2 = 4$ F

$$V_2 = \frac{1 \times 6}{1 + 4} = 1.2\,V$$

The same potential will be on A also.

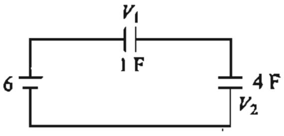


Fig. 4.177

47. **b.** 2 F and 1 F will be in series. So, potential difference across 2 F:

$$V_2 = \frac{1 \times 12}{2+1} = 4\text{ V}$$

48. **b.** The metal plate is equivalent to a dielectric of zero dielectric constant.

49. **b.** Dielectric strength, it is the maximum electric field which a material can bear.

50. **d.** As the capacitor is isolated, so charge will remain same. Now, as the separation between the plates is increased, capacitance $\left(\frac{\varepsilon_0 A}{d}\right)$ will decrease.

$V = \frac{Q}{C}$, If C decreases, V increases.

51. **c.** In series, all the potentials will be added.

52. **c.** Wheatstone's bridge will be formed.

53. **a.** 5 μF, 10 μF, 15 μF will be in parallel, then 30 μF will be in series.

54. **c.** $V_1 - V_2$ will be divided between C_1 and C_2 in series.

Potential difference across C_1:

$$V_1 - V_D = \frac{C_2(V_1 - V_2)}{C_1 + C_2}$$

$$V_D = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

55. **c.** $U = \frac{1}{2}CV^2$

Potential of each capacitor now = $V/2$

$$U' = \frac{1}{2}C\left(\frac{V}{2}\right)^2 = \frac{U}{4}$$

56. **a.** Given $\frac{\varepsilon_0 A}{d} = 10$

$$c' = \frac{\varepsilon_0 A/2}{d} + \frac{4\varepsilon_0(A/2)}{d} = 10\left[\frac{1}{2} + 2\right] = 25\ \mu\text{F}$$

57. **b.** The whole amount of energy stored in capacitor will convert into heat.

$$\text{Heat} = \frac{1}{2}CV^2 = \frac{1}{2} \times 2 \times 10^{-6} \times (100)^2 = 0.01\text{ J}$$

58. **d.** $V = \frac{Q_1 + Q_2}{C_1 + C_2} = 0$

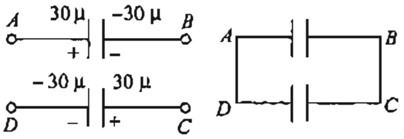


Fig. 4.178

Final potential difference = zero
Final energy = zero
Charge flow 30 μC from A to D.

59. **a.** $C = \frac{\varepsilon_0 A}{d} = 0.5$

New capacitance

$$C' = \frac{\varepsilon_0 A}{\frac{d/2}{2} + \frac{d/2}{3}} = \frac{\varepsilon_0 A}{d}\left[\frac{1}{\frac{1}{4} + \frac{1}{6}}\right]$$

$$C' = C \times 2.4 = 1.2\ \mu\text{F}$$

60. **b.** $C' = \frac{2\varepsilon_0 A/2}{d} + \frac{3\varepsilon_0 A/2}{d}$

$$C\left[1 + \frac{3}{2}\right] = \frac{5}{2} \times 0.5 = 1.25\ \mu\text{F}$$

61. **a.** $V_1 = \frac{kQ}{a} - \frac{kQ}{d},\ V_2 = \frac{-kQ}{b} + \frac{kQ}{d}$

$$C = \frac{Q}{V_1 - V_2} = \frac{4\pi\varepsilon_0}{\frac{1}{a} + \frac{1}{b} - \frac{2}{d}}$$

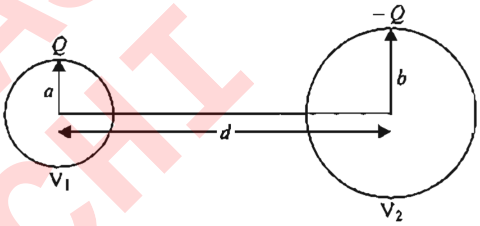


Fig. 4.179

62. **b.** Potential of larger capacitor after 1st charging

$$V_1 = \frac{C_0 V_0}{(C + C_0)}$$

After second charging

$$V_2 = \frac{C_0 V_1}{(C + C_0)}$$

$$\Rightarrow \quad V_2 = \left(\frac{C_0}{C + C_0}\right)^2 V_0$$

After n^{th} charging,

$$V_n = \left(\frac{C_0}{C + C_0}\right)^n V_0$$

$$C = C_0\left[\left(\frac{V_0}{V}\right)^{\frac{1}{n}} - 1\right]$$

63. **a.** $V' = \frac{Q/2 + CV}{C} = \frac{Q}{2C} + V$

64. **c.** The whole amount of charge will flow to the shell.

Initial energy: $U_1 = \frac{Q^2}{2C_1} = \frac{Q^2}{8\pi\varepsilon_0 R}$

Final energy: $U_2 = \frac{Q^2}{2C_2} = \frac{Q^2}{8\pi\varepsilon_0(2R)}$

Heat produced: $U_1 - U_2 = \frac{Q^2}{16\pi\varepsilon_0 R}$

65. **d.** Electric field due to large plates is uniform. We assume size of plates to be very large in comparison to the distance between them.

Also, the charge density on both plates is different.

66. **b.** Electric field is the –ve of slope of $V–x$ graph. Inside the conductor, electric field is zero, so slope of $V–x$ graph is zero. Inside dielectric, field decreases, so slope decreases.

67. **c.** $0 - \frac{Q}{C} + 10 - \frac{Q}{2C} = 0$

$$Q = \frac{20\,C}{3} = \frac{20 \times 6}{3} = 40\ \mu\text{C}$$

68. **a.** $F = Kx$

$$\frac{Q^2}{2A\varepsilon_0} = K(0.2d)$$

$$\frac{\left(\frac{\varepsilon_0 A}{0.8\,d}E\right)^2}{2A\varepsilon_0} = 0.2\,Kd$$

$$K = 3.9\,\varepsilon_0\,AE^2/d^3$$

$$K \simeq 4\,\varepsilon_0\,AE^2/d^3$$

69. **b.**

$$C = \frac{K\,\varepsilon_0 v + b}{d} + \frac{\varepsilon_0(l - vt)\,b}{d}$$

$$q = CV$$

$$I = \frac{dq}{dt} = V\frac{dc}{dt}$$

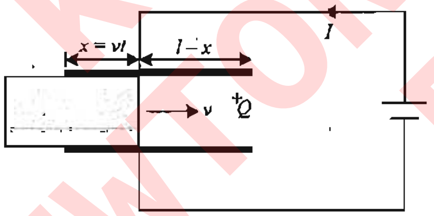


Fig. 4.180

70. **b.** $2000 \times 0.04 = \frac{1}{2}40 \times 10^{-6}V^2$

$V^2 = 4 \times 10^6 \quad \Rightarrow$ V = 2000 V

71. **a.**

$$C_1 = \frac{K_1\,\varepsilon_0\,A/2}{t}$$

$$C_2 = \frac{K_2\,\varepsilon_0\,A/2}{t/2}$$

$$C_3 = \frac{K_3\,\varepsilon_0\,A/2}{t/2}$$

Now, $C_{eq} = C_1 + \frac{C_2C_3}{C_2 + C_3}$ (see Fig. 4.181)

72. **a.** We can make equivalent circuit of given system in two ways as in Fig. 4.182(a) and (b).

73. **c.** Using the method of successive reduction (see Fig. 4.183).

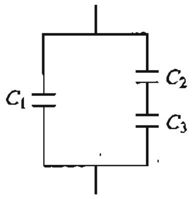


Fig. 4.181

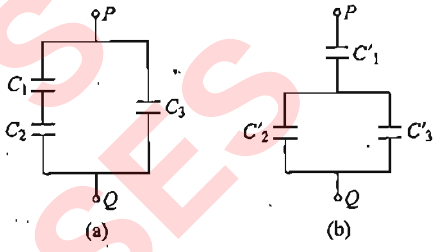


Fig. 4.182

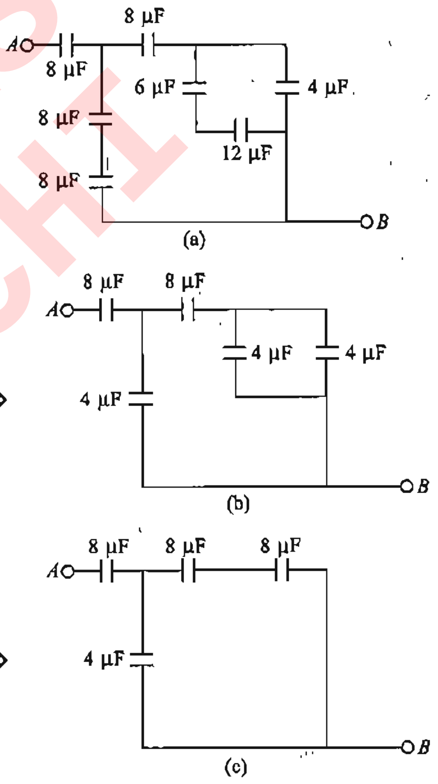
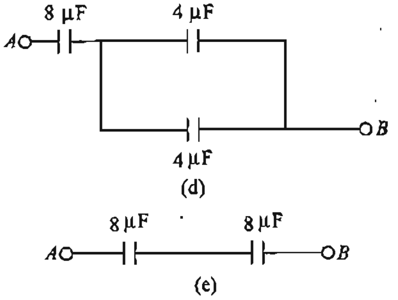
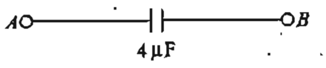


Fig. 4.183

74. **d.** Using the method of successive reduction (see Fig. 4.184).

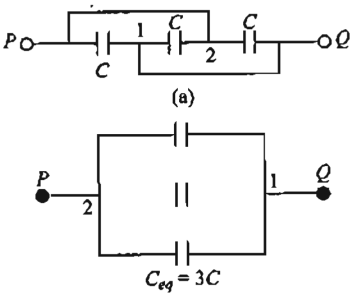




(f)

Fig. 4.183

75. b. The circuit given in Fig. 4.185 can be changed to the structure shown in Fig. 4.186.



(a)

(b)

Fig. 4.184

76. a. A, B, C, and D are equipotential points (see Fig. 4.187).

77. c. After closing S_1, charge on C_1: $q = 6 \times 20 = 120\,\mu$C

Now, S_1 is opened. On closing S_2, charge q will be distributed between C_1 and C_2 according to their capacitances.

So, charge on C_2: $q_2 = \dfrac{C_2 q}{C_1 + C_2} = \dfrac{3 \times 120}{3 + 6} = 40\,\mu$C

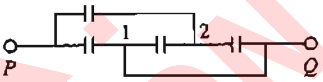
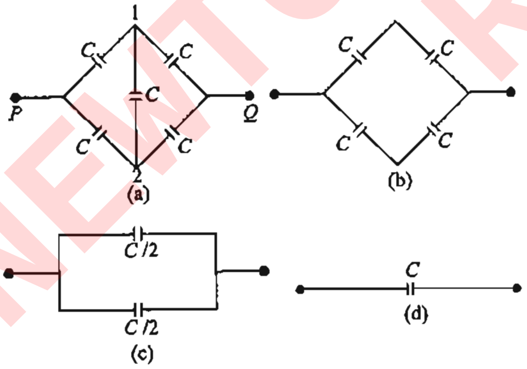


Fig. 4.185



(a)

(b)

(c)

(d)

Fig. 4.186

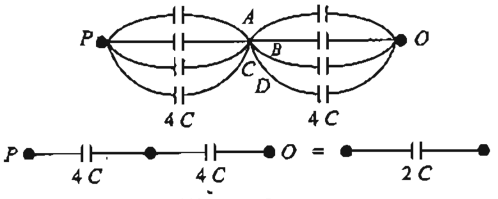
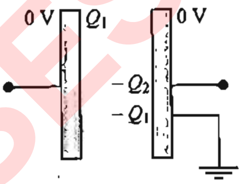


Fig. 4.187

Multiple Correct Answers Type

1. b., c., d. As the switch is closed, the charge of outer surface of 2nd plate becomes zero. From the concept in electrostatics that electric field inside the bulk of the material of conductor is zero, we can find the charges, on various faces.

So, it is clear that $Q_1 + Q_2$ charge goes from 2nd plate to earth. Charge of capacitor is Q_1 and hence its potential is $\dfrac{Q_1}{C}$.

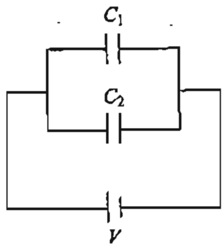


4.188

2. a., c., d. This system can be considered as two capacitors in parallel, with $C_1 = \dfrac{\varepsilon_0 A}{2d}$ and $C_2 = \dfrac{K\varepsilon_0 A}{2d}$.

C_1 is due to upper half of two plates while C_2 is due to lower half.

As potential difference across two capacitors are same, charge would be different as capacitors are different.



4.189

3. a., b., c., d. $C = \dfrac{\varepsilon_0 A}{d_1}$, $C' = \dfrac{\varepsilon_0 A}{d_2}$

Extra charge flown $= Q' - Q = (C' - C)V$

$$= \varepsilon_0 A V \left[\frac{1}{d_2} - \frac{1}{d_1}\right]$$

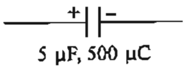
Work done by battery:

$$W_b = V \times \text{charge flown} = \varepsilon_0 A V^2 \left[\frac{1}{d_2} - \frac{1}{d_1}\right]$$

Change in P.E. of capacitor $= \Delta U = \dfrac{1}{2}(C' - C)V^2$

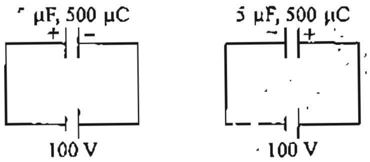
$$= \frac{1}{2}\varepsilon_0 A V^2 \left[\frac{1}{d_2} - \frac{1}{d_1}\right]$$

4. b., c. Initial condition just after connection of battery:



4.190

Condition after a long time:



4.191

It means battery supplies 1000 μC charge from its positive terminal and an equal and opposite charge enters from its negative terminal, i.e., charge flow through battery is 1000 μC.

Work done by battery, $W_{\text{battery}} = 100V \times 10^3\ \mu\text{C} = 0.1\ \text{J}$

From energy conservation law, $U_i + W_{\text{battery}} = U_f + \Delta H$

$U_i = U_f$ so $\Delta H = 0.1$ J

5. **b., c., d.** As the dielectric slab is pulled out, the equivalent capacity of the system decreases and hence charge supplied by battery decreases as potential of the system remains constant. It means charging of battery takes place and a positive charge flows from a to b. As the two capacitors are connected in series, so charge on both capacitors remains same at all instants.

From energy conservation law,

$U_i + W_{\text{ext}} = U_f +$ work done on battery $+ \Delta H$

As dielectric slab is attracted by plates of capacitors, to pull it out F has to perform some work, i.e., $W_{\text{ext}}(F) > 0$.

Assertion-Reasoning Type

1. **a.** The maximum amount of charge we can give to a capacitor depends upon the geometrical factors.

2. **c.** Capacitance is inversely proportional to distance between the plates.

3. **a.** Capacitance of a capacitor does not depend upon charge but it depends upon geometrical factors.

4. **d.** Let the electric field in regions I and II be E_1 and E_2. The potential difference across left half capacitor and half capacitor is same. Therefore, $E_1 d = E_2 d$, where d = inner planar gap.

   Hence, statement 1 is false, statement 2 is correct by definition.

   $\therefore E_1 = E_2$

5. **d.** The electrostatic force on metal of capacitor

$$= \text{pressure} \times \text{area of plate} = \frac{\sigma^2}{2\varepsilon_0} A$$

Since charge on metal plate of an "isolated" capacitor does not change, force on metal plate remain same.

Electric field decreases due to induced charge in dielectric, but this does not affect the charge distribution on isolated metal plate.

Capacitance increases, Q remains same.

$$U = \frac{Q^2}{2C}$$

Hence, U decreases.

Comprehensive Type

For Problems 1–2

1. b., 2. a.

Sol. As radius of the balloon decreases, so capacitance also decreases.

But charge remains same, so energy increases.

For Problems 3–5

3. a., 4. a., 5. a.

Sol. Capacitance will definitely increase. As the battery is disconnected, force will remain same. Potential difference and energy will decrease.

Charge on capacitor plates will apply pulling force on dielectric in both cases whether the dielectric is pushed or pulled.

For Problems 6–7

6. b., 7. a.

Sol. The circuit is equivalent to two capacitors in series.

$$C_1 = \frac{\varepsilon_0 A}{x}$$

$$C_2 = \frac{\varepsilon_0 A}{(a - b - x)}$$

Fig. 4.192

$$\frac{1}{C_{\text{eq}}} = \frac{1}{\varepsilon_0 A}[a - b - x + x] = \frac{(a-b)}{\varepsilon_0 A}$$

$$C_{\text{eq}} = \frac{\varepsilon_0 A}{(a-b)} \text{ independent of } x.$$

We can reduce this structure in following circuit (Fig. 4.193).

$$(V_1 - V) = (V_1 - V_2)\frac{C_2}{C_1 + C_2}$$

$$C_1 = \frac{\varepsilon_0 A}{x}$$

$$C_2 = \frac{\varepsilon_0 A}{(a - b - x)}$$

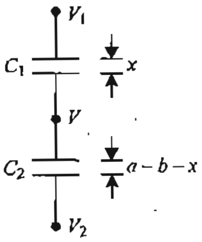


Fig. 4.193

$$V_1 - V = (V_1 - V_2)\frac{\dfrac{\varepsilon_0 A}{(a-b-x)}}{\dfrac{\varepsilon_0 A}{x} + \dfrac{\varepsilon_0 A}{a-b-x}}$$

$$V_1 - V = (V_1 - V_2)\frac{x}{(a-b)}$$

$$V = V_1 - (V_1 - V_2)\frac{x}{(a-b)}$$

$$V = (V_1 + V_2)\frac{x}{a-b}$$

For Problems 8–9

8. a., 9. c.

Sol.

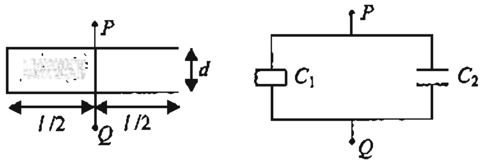


Fig. 4.194

$$C_1 = \frac{\varepsilon_0 \varepsilon_r (A/2)}{d}$$

$$C_2 = \frac{\varepsilon_0 (A/2)}{d}$$

$$C = C_1 + C_2 = \frac{\varepsilon_0 A}{2d}(\varepsilon_r + 1)$$

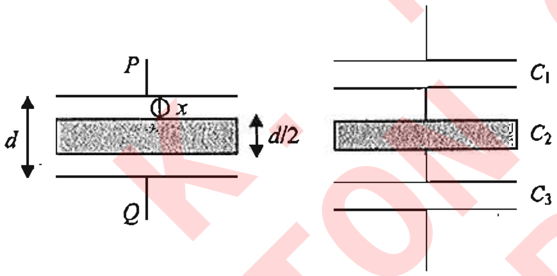


Fig. 4.195

$$C_1 = \frac{\varepsilon_0 A}{x};\ C_2 = \frac{\varepsilon_0 \varepsilon_r A}{d/2};\ C_3 = \frac{\varepsilon_0 A}{d - x - \frac{d}{2}}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow \quad C_{eq} = \frac{2\varepsilon_0 \varepsilon_r A}{d(\varepsilon_r - 1)}$$

For Problems 10–11

10. b., 11. d.

Sol.

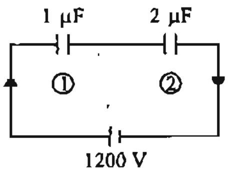


Fig. 4.196

$$1200 - \frac{q}{1} - \frac{q}{2} = 0 \quad \Rightarrow q = 800\,\mu\text{C}$$

Charge on each capacitor = 800 μC

$$V_1 = \frac{q}{C_1} = \frac{800}{1} = 800\text{ V}$$

$$V_2 = \frac{q}{C_2} = \frac{800}{2} = 400\text{ V}$$

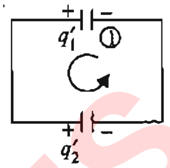


Fig. 4.197

a. $$q_1' + q_2' = 1600 \qquad \text{(i)}$$

Also, $$\frac{q_1'}{1} - \frac{q_2'}{2} = 0$$

$$\Rightarrow \quad q_2' = 2q_1'$$

From (i), $3q_1' = 1600 \Rightarrow q_1' = \frac{1600}{3}\ \mu\text{C}$

$$q_2' = \frac{3200}{3}\ \mu\text{C}$$

$$V = \frac{1600}{3}\ \text{V}$$

b. $q_1' + q_2' = 800 - 800 = 0$ (ii)

Also, $$\frac{q_1'}{1} - \frac{q_2'}{2} = 0$$

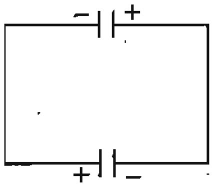


Fig. 4.198

From (ii), $q_1' = q_2' = 0$.

Therefore, potential difference across each capacitor = 0.

For Problems 12–14

12. a., 13. d., 14. a.

Sol.

$$q_0 = CV = 12 \times 16 = 192\,\mu\text{C}$$

$$\frac{(q_0 - \Delta q)}{C_1} - \frac{\Delta q}{C_2} = 0$$

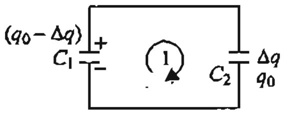


Fig. 4.199

$$\Delta q\left[\frac{C_1 + C_2}{C_1 C_2}\right] = \frac{q_0}{C_1}$$

$$\Delta q = q_0\left(\frac{C_2}{C_1 + C_2}\right) = 192\left(\frac{12 \times 4}{16}\right) = 48\,\mu\text{C}$$

Common potential difference:

$$V = \frac{576}{4} = 12\text{ V}$$

Fraction of energy lost:

$$\frac{\Delta E}{E_0} = \frac{\frac{1}{2}C_1V_0^2}{\frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2}$$

$$= \left(\frac{C_1}{C_1 + C_2}\right)\left(\frac{V_0}{V}\right)^2 = \left(\frac{4}{16}\right)\left(\frac{16}{12}\right)^2 = \frac{4 \times 16}{12 \times 12}$$

$$\Rightarrow \quad \frac{\Delta E}{E} = \frac{4}{9}$$

For Problems 15–17

15. c., **16.** b., **17.** a.

Sol.

In close loop,

$$\frac{q_0 - \Delta q}{C_1} - \frac{\Delta q}{C_2} - \frac{\Delta q}{C_2} = 0$$

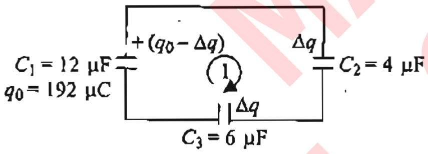


Fig. 4.200

$$\frac{q_0}{C_1} = \Delta q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

$$\Delta q = \frac{q_0}{C_1\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]}$$

$$\Delta q = \frac{192}{[1 + 3 + 2]} = 32\,\mu\text{C}$$

$$\text{P.D. across } C_2 = \frac{\Delta q}{C_2} = 8\text{ V}$$

$$\text{P.D. across } C_1 = \frac{(192 - 32)}{12} = \frac{160}{12} = \frac{40}{3}\text{ V}$$

For Problems 18–21

18. a., **19.** c., **20.** a., **21.** c.

Sol. $Q_0 = C_1 \times V_0 = 960\ \mu\text{C}$

$$U_0 = \frac{1}{2}C_1V_0^2 = \frac{1}{2} \times 8 \times 120^2 = 57.6\text{ mJ}$$

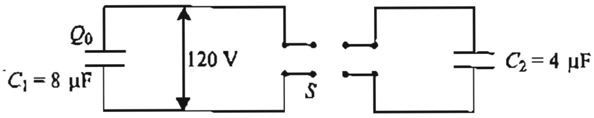


Fig. 4.201

In close loop,

$$\frac{(Q_0 - \Delta q)}{8} - \frac{\Delta q}{4} = 0$$

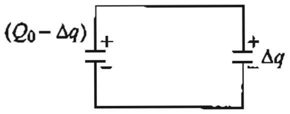


Fig. 4.202

$$Q_0 - 3\Delta q = 0 \Rightarrow \ \Delta q = \frac{960}{3} = 320\ \mu\text{C}$$

Charge on capacitors,

$$q_1 = 640\ \mu\text{C},\ q_2 = 320\ \mu\text{C}$$

$$\text{Common potential, } V = \frac{640}{8} = 80\text{ V}$$

$$C_{\text{eq}} = 8 + 4 = 12\ \mu\text{F}$$

$$U_{\text{aft}} = \frac{1}{2} \times 12 \times 80^2 = 6 \times 6400$$

$$= 38400\ \mu\text{J} = 38.4\text{ mJ}$$

For Problems 22–23

22. b., **23.** a.

Sol.

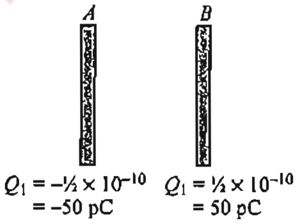


Fig. 4.203

$$C = \frac{\varepsilon_0 A}{d}$$

$$= \frac{8.8 \times 10^{-12} \times 5 \times 10^{-3}}{8.8 \times 10^{-3}} = 5 \times 10^{-12}\text{ F}$$

Charge on plate after connection with battery

$$q = CV = 5 \times 10^{-12} \times 10$$

$$= 500 \times 10^{-12}\text{ C} = 50\text{ pC}$$

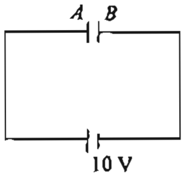


Fig. 4.204

Charge supplied by battery,

$\Delta q = (50 + 50)\text{ pC} = 100\text{ pC}$

Energy supplied by battery,

$$U_{\text{battery}} = \Delta q V = 100 \times 10 = 100\text{ pJ}$$

$$= 10^{-9}\text{ J}$$

For Problems 24–26

24. c., 25. b., 26. a.

Sol. $Q_2 = \frac{10}{3} \times 100 \Rightarrow Q_2 = \frac{1000}{3}$ μC;

$$Q_1 = 4 \times 100 = 400\,\mu\text{C}$$

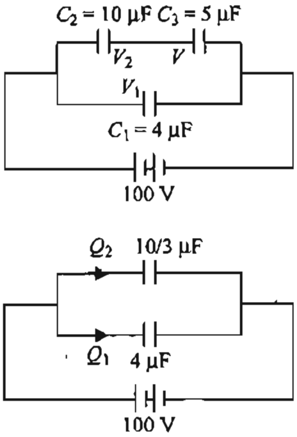


Fig. 4.205

Potential difference across 4 μF capacitor,

$$V_1 = 100\text{ V}$$

$$V_2 = \frac{1000/3}{10} = \frac{100}{3}\text{ V}$$

$$\therefore \quad V_3 = \frac{1000/3}{5} = \frac{200}{3}\text{ V}$$

Stored energy

$$U_1 = \frac{Q_1^2}{2C_1} = \frac{(400)^2}{2 \times 4} = 2 \times 10^4\,\mu\text{J}$$

$$U_2 = \frac{Q_2^2}{2C_2} = \frac{(1000/3)^2}{2 \times 10} = \frac{1}{18} \times 10^5\,\mu\text{J}$$

$$U_3 = \frac{Q_2^2}{2C_2} = \frac{(1000/3)^2}{2 \times 5} = \frac{1}{9} \times 10^5\,\mu\text{J}$$

For Problems 27–28

27. b., 28. c.

Sol. 4, 2, 5 are equipotential points.

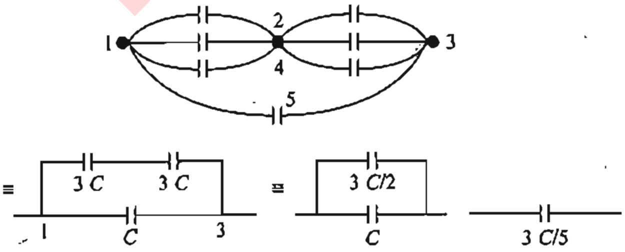


Fig. 4.206

1, 2, 3 are equipotential points.

For Problems 29–31

29. b., 30. a., 31. b.

Sol. Let charge be as shown. (Capacitors in series have the same charge.)

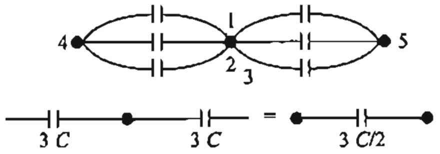


Fig. 4.207

Take loop containing C_1, C_2 and E,

$$\frac{q}{C_1} + \frac{q}{C_2} - E = 0 \Rightarrow \quad q = E\left[\frac{C_1C_2}{C_1 + C_2}\right]$$

Similarly, from loop containing C_3, C_4 and E,

$$\frac{q'}{C_3} + \frac{q'}{C_4} - E = 0 \quad \Rightarrow \quad q' = E\left[\frac{C_3C_4}{C_3 + C_4}\right]$$

Now, $V_A - V_B = \frac{q}{C_2} - \frac{q'}{C_4}$

$$= E\left[\frac{C_1}{C_1 + C_2} - \frac{C_3}{C_3 + C_4}\right]$$

$$V_A - V_B = E\left[\frac{C_1C_4 - C_3C_2}{(C_1 + C_2)(C_3 + C_4)}\right]$$

$$V_A - V_B = 0 \quad \Rightarrow \quad C_1C_4 = C_2C_3 = 0$$

or $\qquad \frac{C_1}{C_2} = \frac{C_3}{C_4}$

For Problems 32–33

32. c., 33. d.

Sol. $20 - \frac{q}{8} - 8 - \frac{q}{4} = 0$

$$12 - \frac{3q}{8} = 0 \quad \Rightarrow \quad q = 32\,\mu\text{C}$$

Potential difference across C_1, $V_1 = \frac{32}{8} = 4$ V

Across C_2, $V_2 = \frac{32}{4} = 8$ V

For Problems 34–35

34. b., 35. c.

Sol. When S_1 is closed: $-\frac{q_1}{3} - \frac{q_1}{1} + 12 = 0$

$$4q_1 = 36 \Rightarrow q_1 = 9\,\mu\text{C}$$

$$-\frac{q_4}{4} - \frac{q_4}{2} + 12 = 0$$

$$-3q_4 + 12 \times 4 = 0 \Rightarrow q_4 = 16\,\mu\text{C}$$

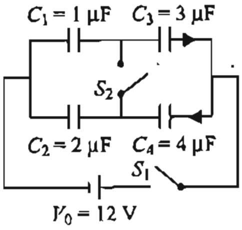


Fig. 4.208

When S_2 is also closed:

$$C_{eq} = \frac{3 \times 7}{10} = \frac{21}{10}\,\mu\text{F}$$

$$q = C_{eq}V = \frac{21}{10} \times 12 = \frac{126}{5}\ \mu C$$

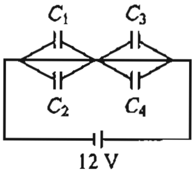


Fig. 4.209

Charge on C_3, $q_3 = q\left(\frac{C_3}{C_3 + C_4}\right)$

$$\Rightarrow \quad q_3 = \frac{126}{5} \times \frac{3}{7} = \frac{54}{5}\ \mu C$$

$$q_4 = \frac{72}{5}\ \mu C$$

$$q_1 = \left(\frac{C_1}{C_1 + C_2}\right) = \frac{126}{5}\left(\frac{1}{1+2}\right)$$

$$\Rightarrow \quad q_1 = \frac{42}{5}\ \mu C$$

$$q_2 = \frac{126}{5}\left(\frac{2}{3}\right) = \frac{84}{5}\ \mu C$$

For Problems 36–38

36. a., 37. a., 38. b.

Sol. Reducing the farthest right leg yields

$$C = \left(\frac{1}{6.0\,\mu F} + \frac{1}{6.0\,\mu F} + \frac{1}{6.0\,\mu F}\right)^{-1} = 2.0\ \mu F = \frac{C_1}{3}$$

It combines in parallel with a C_2

$$C = 4.0\ \mu F + 2.0\ \mu F = 6.0\ \mu F = C_1.$$

So, the next reduction is the same as the first: $C = 2.0\ \mu F = C_1/3$. And the next is the same as the second, leaving $3C_1$'s in series. So, $C_{eq} = 2.0\ \mu F = C_1/3$.

For the three capacitors nearest to points a and b:

$$Q_{C_1} = C_{eq}V = (2.0 \times 10^{-6}\ F)(420\ V) = 8.4 \times 10^{-4}\ C$$

$$Q_{C_2} = C_2V_2 = (4.0 \times 10^{-6}\ F)(420\ V)33 = 5.6 \times 10^{-4}\ C.$$

$$V_{cd} = \frac{1}{3}\left(\frac{420}{3}\ V\right) = 46.7\ V,$$

Since by symmetry the total voltage drop over the equivalent capacitance of the part of the circuit from the junctions between c and d is (420/3) V and the equivalent capacitance is that of three equal capacitors C_1 in series. V_{cd} is the voltage over just one of those capacitors, i.e. 1/3 of 420/3 V.

For Problems 39–41

39. c., 40. a., 41. b.

Sol. When switch is in position 1,

$$30 - \frac{q}{20} - \frac{q}{30} = 0 \quad \Rightarrow \quad q = \frac{1800}{5}$$

$$q = 360\ \mu C$$

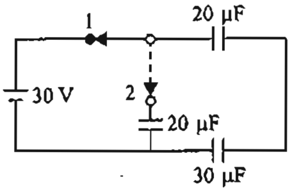


Fig. 4.210

Now, the switch is in position 2,

$$q_1' + q_3' = 360 \qquad (i)$$

Loop $ABCD$,

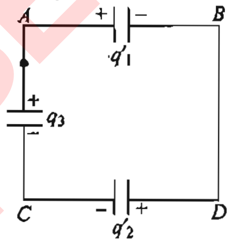


Fig. 4.211

$$-\frac{q_1'}{20} - \frac{q_1'}{30} + \frac{q_3}{20} = 0$$

$$-3q_1' - 2q_1' + 3q_3 = 0$$

$$5q_1' = 3q_3 \quad \Rightarrow \quad q_1' = \frac{3}{5}q_3$$

$$\frac{8q_3}{5} = 360$$

$$q_3 = \frac{360 \times 5}{8} = \frac{450}{2}\ \mu C \qquad (ii)$$

$$q_1' = \frac{3}{5}\frac{450}{2} = 135\ \mu C$$

$$q_3 = 225\ \mu C$$

For Problems 42–43

42. d., 43. d.

Sol. $Q = CV$

Charge flow: $\left(Q + \frac{Q_1}{2}\right) - Q_1$

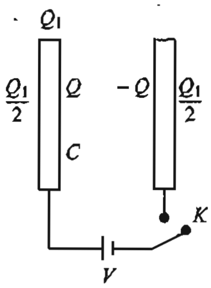


Fig. 4.212

$$Q - \frac{Q_1}{2} = CV - \frac{Q_1}{2}$$

$Q = CV$

For Problems 44–45

44. a., 45. b

Sol. $C_{eq} = \frac{8}{3}$

$$Q_1 = C_{eq} V = \frac{8}{3} \times 15 = 40\ \mu C$$

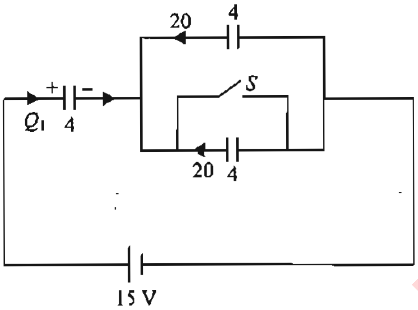


Fig. 4.213

$$Q_2 = 4 \times 15 = 60\ \mu C$$

Charge flow: $Q_2 - Q_1 = 60 - 40 = 20\ \mu C$

Matching Column Type

1. i. → a., c., ii. → b., iii., → a., c., iv. → d.

Sol. a. By inserting dielectric slab, capacitance of I increases thereby increasing charge on capacitor. Thus, more charge is flown through the battery. Energy stored in capacitor also increases.

b. By increasing separaton between the plates, capacitance of C_1 decreases. Charge on C_2 also decreases.

c. By shorting capacitor 1, only capacitor 2 remains in the circuit. Potential difference across C_2 increases thereby increasing charge on 2 as well as energy stored.

d. By earthing plate of capacitor I, potentials will change but there will be no change in potential difference, making no overall change in the circuit.

2. i. → b., ii. → c., iii. → a., iv. → d.

Sol. The charges on three plates which are in contact add to zero, because these plates taken together form an isolated system which cannot receive charges from the batteries.

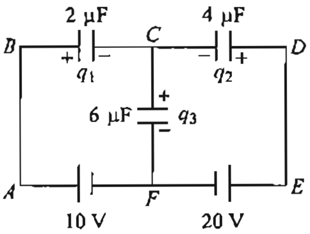


Fig. 4.214

Thus, $q_3 - q_1 - q_2 = 0$ (i)

Applying Kirchhoff's law in loop *ABCFA* and *CDEFC*

$$-\frac{q_1}{2} - \frac{q_3}{6} + 10 = 0$$

or $q_3 + 3q_1 = 60$ (ii)

and $20 - \frac{q_2}{4} - \frac{q_3}{6} = 0$

or $3q_2 + 2q_3 = 240$ (iii)

Solving the above three equations, we have

$$q_1 = \frac{10}{3}\,\mu C, \quad q_2 = \frac{140}{3}\,\mu C, \quad q_3 = 50\,\mu C$$

Potential difference across

$$6\,\mu C = \frac{q_3}{6} = \frac{50\,\mu C}{6\,\mu F}$$

$$= \frac{25}{3}\ V$$

3. i. → b., ii. → a., iii. → d., iv. → c.

Sol.

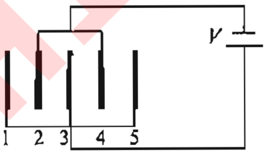


Fig. 4.215

These five plates constitute four identical capacitors in parallel, each of capacity $\frac{\varepsilon_0 A}{d}$. Now, as plate 1 is connected to positive terminal of battery and is a part of one capacitor only, so charge on it

$$q_1 = +\left(\frac{\varepsilon_0 A V}{d}\right)$$

So, **i. → b.**

However, the plate 4 is connected to negative terminal of battery and in common to two identical capacitors in parallel.

So, $q_4 = -\frac{2\varepsilon_0 A V}{d}$

Hence, **ii. → a.**

From circuit, it is obvious that between the plates 2 and 3, battery is connected so potential difference will be V. Plates 1 and 5 gets connected through connecting wire, so potential difference between 1 and 5 is zero.

Hence, **iii → d. iv. → c.**

APPENDIX

A1

Miscellaneous Assignments and Archives on Chapters 1–4

EXERCISES

Objective Type

Solutions on page A1.44

1. Three charges of equal magnitude q reside at the corners of an equilateral triangle of side length 2 m. Where must a $-4q$ charge be placed so that any charge located at it experiences no net electric force? Let P be the origin and let the distance between the $+q$ charge and P be 1.00 m.

   a. $(0, \sqrt{3})$ m **b.** $(0, \sqrt{2})$ m
   c. $(0, 2\sqrt{3})$ m **d.** $(0, 3)$ m

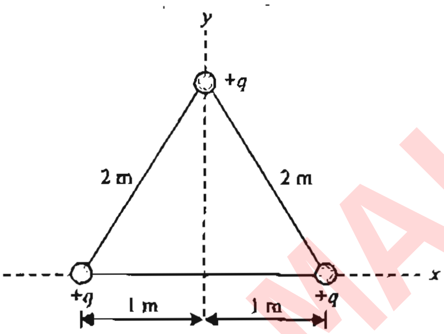


Fig. A1.1

2. A metal sphere having a radius r_1 charged to a potential V_1 is enveloped by a thin-walled conducting spherical shell of radius r_2. Determine the potential V_1 acquired by the sphere after it has been connected for a short time to the shell by a conductor.

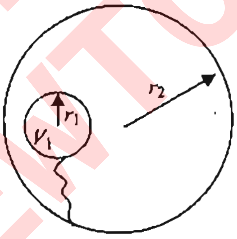


Fig. A1.2

   a. $\dfrac{V_1 r_2}{r_1}$ **b.** $\dfrac{V_1 r_1}{r_2}$
   c. V_1 **d.** $\dfrac{V_1(r_1 + r_2)}{r_2}$

3. An insulated conductor initially free from charge is charged by repeated contacts with a plate which after each contact is replenished to a charge Q from an electrophorus. If q is the charge on the conductor after the first operation, find the maximum charge which can be given to the conductor in this way.

   a. $\dfrac{Qq}{Q+q}$ **b.** $\dfrac{3Qq}{2Q+q}$
   c. $\dfrac{Qq}{Q-q}$ **d.** $\dfrac{2Qq}{Q+q}$

4. In the given circuit diagram (Fig. A1.3), the switch S_W is shifted from position 1 to position 2. Then

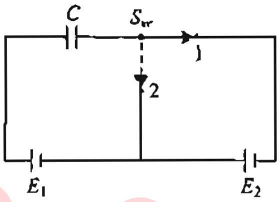


Fig. A1.3

   a. a charge of amount CE_2 will be supplied to battery E_1
   b. heat generated in the circuit is $\frac{1}{2}CE_2^2$
   c. a charge of amount CE_2 will be supplied by battery E_1
   d. heat generated in the circuit is $\frac{1}{2}CE_1E_2$

5. Find the electric dipole moment of a non-conducting ring of radius a, made of two semicircular rings having linear charge densities $-\lambda$ and $+\lambda$ as shown in Fig. A1.4.

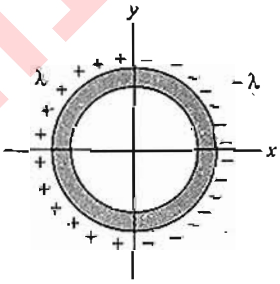


Fig. A1.4

   a. $2\lambda a^2$ **b.** $4\lambda a^2$
   c. λa^2 **d.** $2\sqrt{2}\lambda a^2$

6. Find the electric dipole moment of a non-conducting ring of radius a, having two quarter circular sections having linear charge densities λ and $-\lambda$ and arranged as shown in Fig. A1.5.

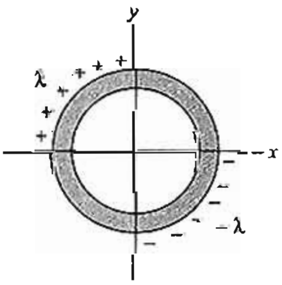


Fig. A1.5

   a. $2\lambda a^2$ **b.** $4\lambda a^2$
   c. λa^2 **d.** $2\sqrt{2}\lambda a^2$

7. For the phenomenon of "Electrostatic Induction", mark out the correct statement(s).

   a. The magnitude of net induced charge is zero.

b. The magnitude of net induced charge is equal and opposite to the magnitude of inducing charge.
c. The mass of the bodies change slightly.
d. Electrostatic induction is a permanent effect.

8. For the arrangement shown in Fig. A1.6, mark out the correct statement(s).

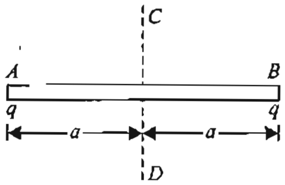


Fig. A1.6

a. For +ve test charge, P is a position of stable equilibrium for displacement along AB.
b. For +ve test charge, P is a position of unstable equilibrium for displacement along CD.
c. For –ve test charge, P is a position of unstable equilibrium for displacement along AB.
d. All of the above.

9. We can provide a non-zero net charge to a dielectric (non-conducting) sphere by a method of

a. charging by rubbing
b. charging by conduction
c. charging by induction
d. none of the above

10. Electric field intensity due to a semi-infinite wire at the point P, as shown in the Fig. A1.7, is

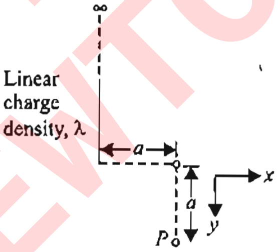


Fig. A1.7

a. $\frac{\lambda}{4\pi\varepsilon_0 d}\hat{i}\ \frac{\lambda}{4\pi\varepsilon_0 d}\hat{j}$
b. $\frac{\lambda}{4\pi\varepsilon_0 d}(1-\sin\pi/4)\,\hat{i}+\frac{\lambda}{4\pi\varepsilon_0 d}(\cos\pi/4)\,\hat{j}$
c. $\frac{\lambda}{4\pi\varepsilon_0 d}(1+\sin\pi/4)\,\hat{i}+\frac{\lambda}{4\pi\varepsilon_0 d}(\cos\pi/4)\,\hat{j}$
d. $\frac{\lambda}{4\pi\varepsilon_0 d}\sin\pi/4\,\hat{i}+\frac{\lambda}{4\pi\varepsilon_0 d}\cos\pi/4\,\hat{j}$

11. Two uncharged metal spheres A and B are in contact as shown in Fig. A1.8. A negatively charged rod is brought near to A, but not touching it. The two spheres are separated slightly and the rod is then withdrawn.
As a result of this

a. both the spheres acquire +ve charge
b. both the spheres acquire –ve charge

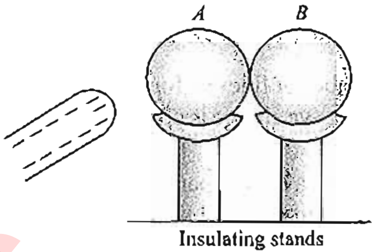


Fig. A1.8

c. A acquires –ve and B acquires +ve charge
d. A acquires +ve and B acquires –ve charge

12. Charge is distributed uniformly on the surface of a spherical balloon (an insulator) with a point charge q inside. The electric force on q is greatest when

a. it is near the inside surface of the balloon
b. it is at the center of balloon
c. it is anywhere inside (the force is same everywhere and is non-zero)
d. it is anywhere inside (the force is zero everywhere)

13. Charge is distributed on the surface of a spherical conducting shell with a point charge q inside. The electric force on q is greatest when

a. it is near the inside surface of the shell
b. it is at the center of shell
c. it is anywhere inside (the force is same everywhere and is non-zero)
d. it is anywhere inside (force is zero everywhere)

14. An infinite long tube of semicircular cross section is given as shown in Fig. A1.9. One half of the tube is given a surface charge density $+\sigma$ and the other half $-\sigma$. $\vec{E}$ at point O, on the axis, would be [The point O is somewhere near the center on the axial line.]

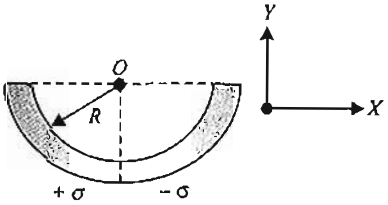


Fig. A1.9

a. $-\frac{\sigma}{\pi\varepsilon_0}\hat{i}$ **b.** $\frac{\sigma}{\pi\varepsilon_0}\hat{i}$
c. $\frac{\sigma}{\pi\varepsilon_0}\hat{j}$ **d.** $\frac{\sigma}{\pi\varepsilon_0}\hat{i}+\frac{\sigma}{\pi\varepsilon_0}\hat{j}$

15. The electric potential in a certain region of space is given by $V=-3x^2+4x$, where x is in meters and V is in volts. In this region, the equipotential surfaces are

a. planes parallel to XY plane
b. planes parallel to YZ plane
c. concentric cylinders with axis as X-axis
d. concentric spheres centered at origin

16. Mark out the correct statement about the electric lines of force.

- **a.** The number of lines of force leaving a +ve charge or entering a –ve charge is proportional to the magnitude of charge.
- **b.** The path of a charge particle in an electric field is not always along the field lines.
- **c.** The direction of electric lines of force is along the normal to the equipotential surface.
- **d.** All of the above.

17. Two hollow spherical conductors A and B are arranged as shown in Fig. A1.10. Conductor B is initially (before connection of A and B) neutral and charge on A is Q.

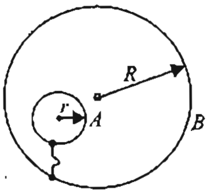


Fig. A1.10

After connecting, the potential of B is

- **a.** $\dfrac{Q}{8\pi\varepsilon_0(r+R)}$
- **b.** $\dfrac{Q}{4\pi\varepsilon_0 R}$
- **c.** $\dfrac{Q}{4\pi\varepsilon_0 R} - \dfrac{Q}{4\pi\varepsilon_0 r}$
- **d.** Information insufficient

18. A circular ring of radius R is having uniformly distributed charge Q. Find the flux crossing through a sphere of radius R having its center on the periphery of the ring.

- **a.** zero
- **b.** $\dfrac{Q}{4\varepsilon_0}$
- **c.** $\dfrac{Q}{\varepsilon_0}$
- **d.** $\dfrac{Q}{3\varepsilon_0}$

19. A point charge q is placed at the center of a closed cylindrical conductor of length l and radius R. The flux crossing through the conducting cylinder would be

- **a.** $\dfrac{ql}{2\varepsilon_0[R^2+l^2/4]^{1/2}}$
- **b.** zero
- **c.** $\dfrac{ql}{\varepsilon_0[R^2+l^2/4]^{1/2}}$
- **d.** $\dfrac{q}{\varepsilon_0}$

20. A point charge q is placed at the center of an imaginary Gaussian surface as shown in Fig. A1.11.
Find the flux crossing through the curved surface.

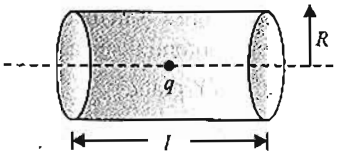
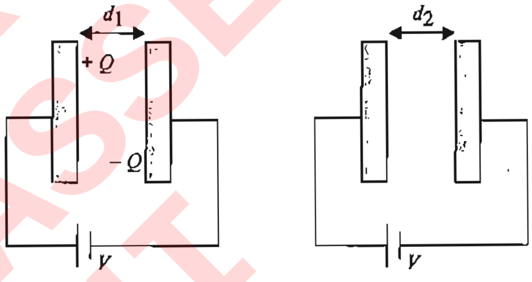


Fig. A1.11

- **a.** zero
- **b.** $\dfrac{q}{\varepsilon_0}\left[1-\dfrac{l}{\sqrt{R^2+l^2/4}}\right]$
- **c.** $\dfrac{q}{2\varepsilon_0}\left[1-\dfrac{l}{2\sqrt{l^2/4+R^2}}\right]$
- **d.** $\dfrac{ql}{2\varepsilon_0\sqrt{\dfrac{l^2}{4}+R^2}}$

21. Consider a capacitor as shown in Fig. A1.12(a). If we pull the plates of capacitor apart to a final position as shown in figure (b), then we must perform some work against electric force. For this situation, mark out the correct statement(s). [Take areas of plates as A]



(a) (b)

Fig. A1.12

- **a.** Work done is $\dfrac{Q^2}{2\varepsilon_0 A}(d_2-d_1)$ and is stored in volume $A(d_2-d_1)$.
- **b.** Work done is $+\dfrac{Q^2}{2\varepsilon_0 A}(d_2-d_1)$ and is stored in volume Ad_2.
- **c.** Work done is $+\dfrac{\varepsilon_0 AV^2}{2}\left(\dfrac{d_2-d_1}{d_1d_2}\right)$ and is stored in volume $A(d_2-d_1)$.
- **d.** Work done is $\dfrac{\varepsilon_0 AV^2}{2}\left(\dfrac{d_2-d_1}{d_1d_2}\right)$ and is stored in volume Ad_2.

22. An isolated capacitor of capacitance C is charged to a potential V. Then, a dielectric slab of dielectric constant K is inserted as shown in Fig. A1.13. The net charge on four surfaces 1, 2, 3 and 4 would be, respectively,

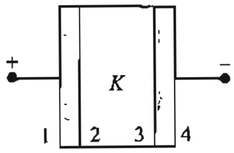


Fig. A1.13

- **a.** $0, CV, -CV, 0$
- **b.** $0, \dfrac{CV}{K}, \dfrac{-CV}{K}, 0$
- **c.** $CV, 0, 0, -CV$
- **d.** $CV, \dfrac{-CV}{K}, \dfrac{CV}{K}, -CV$

23. A parallel plate capacitor is completely immersed in a liquid dielectric having dielectric constant K as shown in Fig. A1.14. Find the force acting on a unit surface of the plate from the part of liquid dielectric in contact with that plate.

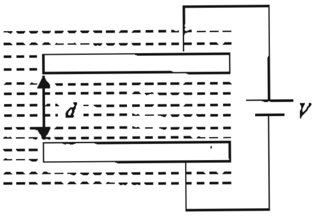


Fig. A1.14

a. $\dfrac{K\varepsilon_0 V^2}{2d^2}$ **b.** $\dfrac{\varepsilon_0(K-1)V^2}{2d^2K}$

c. $\dfrac{KV^2}{2d^2}$ **d.** $\dfrac{K(K-1)\varepsilon_0 V^2}{2d^2}$

24. A cylindrical layer of a homogeneous dielectric with the dielectric constant K in introduced into a cylindrical capacitor so that the dielectric fills the gap of width d between the plates. Mean radius of the plates is R such that $R \gg d$, the capacitor is connected to a battery of emf V. Find the force pulling the dielectric inside the capacitor.

a. $\dfrac{\varepsilon_0(K-1)\pi RV^2}{d}$ **b.** $\dfrac{\varepsilon_0(K-1)V^2R}{2d}$

c. $\dfrac{\varepsilon_0(K-1)dV^2}{2R}$ **d.** $\dfrac{\varepsilon_0(K-1)V^2R}{\pi d}$

25. Mark out the correct statement(s).

a. Capacitance of an isolated sphere depends on its charge.

b. Capacitance of a non-isolated sphere depends only on its shape and size.

c. Capacitance of a non-isolated sphere is increased because of the presence of other conducting bodies nearby it.

d. Capacitance of a conductor cannot be affected because of the presence of other conducting bodies nearby it.

26. Find the equivalent capacitance across A and B for the arrangement shown in Fig. A1.15.

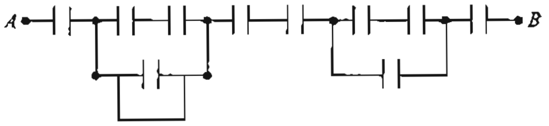


Fig. A1.15

All the capacitors are of capacitance C.

a. $\dfrac{3C}{14}$

b. $\dfrac{C}{8}$

c. $\dfrac{3C}{16}$

d. None of the above

27. Three concentric spherical conductors are as shown in Fig. A1.16. Determine the equivalent capacitance of the system between A and B.

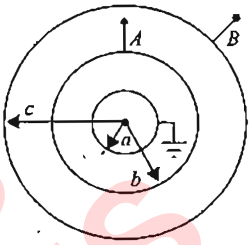


Fig. A1.16

a. $\dfrac{4\pi\varepsilon_0 ac}{c-a}$

b. $\dfrac{4\pi\varepsilon_0 bc}{c-b}+\dfrac{4\pi\varepsilon_0 ab}{b-a}$

c. $\dfrac{4\pi\varepsilon_0 bc}{c-b}+\dfrac{4\pi\varepsilon_0 abc}{ab+c(b-a)}$

d. $\dfrac{4\pi\varepsilon_0 ac}{c-a}+4\pi\varepsilon_0 c$

28. Capacitor C_1 is connected to a battery and charged till the magnitude of the charge on each plate is q_0. Then, the battery is disconnected and C_1 is connected to two other uncharged capacitors C_2 and C_3 as shown (Fig. A1.17). Final charges on the capacitors (q_1, q_2 and q_3) are related by

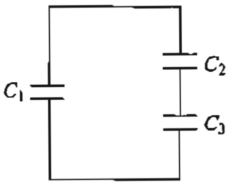


Fig. A1.17

a. $q_0 = q_1 + q_2 + q_3$

b. $q_1 + q_2 + q_3 = 0$

c. $q_0 = q_3 + q_2, q_1 = 0$

d. $q_0 = q_1 + q_2, q_2 = q_3$

29. Two identical capacitors are charged to different potentials, then they are connected to each other in such a way that the sum of charges of plates having same polarity remains constant. Mark out the correct statement.

a. Sum of charges of plates having negative polarity remains constant.

b. Mean of individual final potentials is different from mean of individual initial potentials.

c. Total energy stored in two capacitors in final state may be equal to that in the initial state.

d. Heat dissipation in the circuit can be zero.

30. Three concentric conducting spherical shells have radii r, $2r$ and $3r$ and charges q_1, q_2 and q_3, respectively. Innermost and outermost shells are earthed as shown in Fig. A1.18. The charges shown are after earthing. Select the correct alternative.

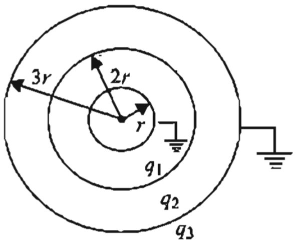
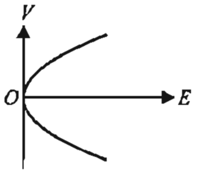


Fig. A1.18

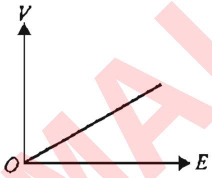
a. $q_1 + q_3 = -q_2$ **b.** $q_1 = -q_2$

c. $\dfrac{q_3}{q_2} = -\dfrac{1}{3}$ **d.** None of these

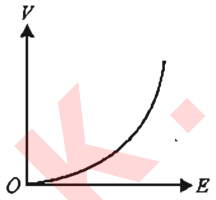
31. If at distance r from a positively charged particle, electric field strength and potential are E and V, respectively, which of the following graphs is/are correct?



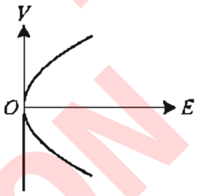
a.



b.



c.



d.

32. Two point charges are placed as shown in Fig. A1.19. Mark out the correct option(s).

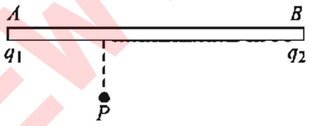


Fig. A1.19

a. If q_1 is displaced, then $\vec{E}$ at B changes instantaneously.

b. If q_1 is displaced, then $\vec{E}$ at B changes after some time (very small).

c. If q_2 is displaced, then $\vec{E}$ at P changes instantaneously.

d. If q_1 and q_2 both are displaced, then $\vec{E}$ at P changes instantaneously.

33. Two infinitely long parallel wires having linear charge densities λ_1 and λ_2, respectively, are placed at a distance R. The force per length on either wire will be

a. $k\dfrac{2\lambda_1\lambda_2}{R^2}$ **b.** $k\dfrac{2\lambda_1\lambda_2}{R}$

c. $k\dfrac{\lambda_1\lambda_2}{R^2}$ **d.** $k\dfrac{\lambda_1\lambda_2}{R}$

34. If a positively charged pendulum is oscillating in a uniform electric field as shown in Fig. A1.20. Its time period as compared to that when it was uncharged

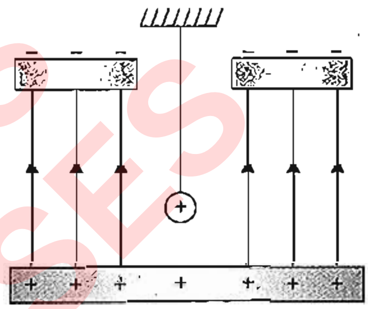


Fig. A1.20

a. will increase

b. will decrease

c. will not change

d. will first increase and then decrease

35. A uniformly charged and infinitely long line having a linear charge density λ is placed at a normal distance y from a point O. Consider a sphere of radius R with O as center and $R > y$. Electric flux through the surface of the sphere is

a. zero **b.** $\dfrac{2\lambda R}{\varepsilon_0}$

c. $\dfrac{2\lambda\sqrt{R^2 - y^2}}{\varepsilon_0}$ **d.** $\dfrac{\lambda\sqrt{R^2 + y^2}}{\varepsilon_0}$

36. A uniformly charged ring of radius 15 cm carries a total charge 60 μC. Where should a point charge 3 μC be kept on the axis of the ring so that it experiences maximum force?

a. 15 cm from the center

b. 13.8 cm from the center

c. 10.6 cm from the center

d. 7.5 cm from the center

37. A point charge $+Q$ is placed at the centroid of an equilateral triangle. When a second charge $+Q$ is placed at a vertex of the triangle, the magnitude of the electrostatic force on the central charge is 8 N. The magnitude of the net force on the central charge when a third charge $+Q$ is placed at another vertex of the triangle is

a. zero **b.** 4 N

c. $4\sqrt{2}$ N **d.** 8 N

38. Five Styrofoam balls are suspended from insulating threads. Several experiments are performed on the balls and the following observations are made:
(i) Ball A repels C and attracts B.
(ii) Ball D attracts B and has no effect on E.
(iii) A negatively charged rod attracts both A and E.
An electrically neutral styrofoam ball gets attracted if placed nearby a charged body due to induced charge. What are the charges, if any, on each ball A, B, C, D, and E?

a. $+ - + 0 +$ **b.** $+ - + + 0$
c. $+ - + 0\,0$ **d.** $- + - 0\,0$

39. A charge Q is placed at a distance of $4R$ above the center of a disk of radius R. The magnitude of flux through the disk is ϕ. Now, a hemispherical shell of radius R is placed over the disk such that it forms a closed surface. The flux through the curved surface, taking direction of area vector along outward normal as positive, is

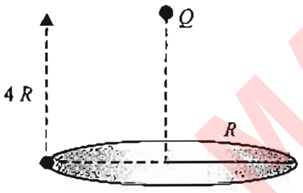


Fig. A1.21

a. zero **b.** ϕ
c. $-\phi$ **d.** 2ϕ

40. Two very large thin conducting plates having same cross-sectional area are placed as shown in Fig. A1.22. They are carrying charges Q and $3Q$, respectively.

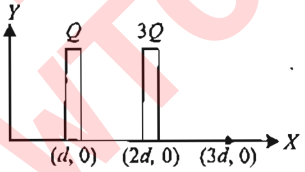
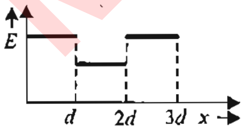
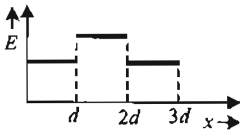


Fig. A1.22

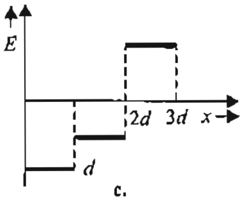
The variation of electric field as a function of x(for $x = 0$ to $x = 3d$) will be best represented by



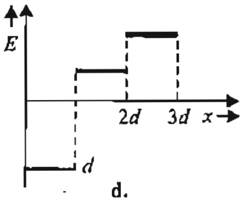
a.



b.



c.



d.

41. The negative charge $-q_2$ is fixed while positive charge q_1 as well as the conducting sphere 'S' is free to move (Fig. A1.23). If the system is released from rest,

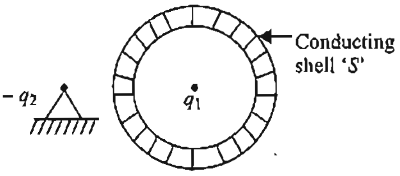


Fig. A1.23

a. both S and q_1 move towards left
b. q_1 moves towards right while S moves towards left
c. q_1 remains at rest, S moves towards left
d. both q_1 and S remain at rest

42. A charged particle of mass $m = 2$ kg and charge 1 μC is projected from a horizontal ground at an angle $\theta = 45°$ with speed 10 ms^{-1}. In space, a horizontal electric field towards the direction of projection $E = 2 \times 10^7$ NC^{-1} exists. The range of the projectile is

a. 20 m **b.** 60 m
c. 200 m **d.** 180 m

43. A wire of linear charge density λ passes through a cuboid of length l, breadth b and height h $(l > b > h)$ in such a manner that flux through the cuboid is maximum. The position of the wire is now changed, so that the flux through the cuboid is minimum. The ratio of maximum flux to minimum flux will be

a. $\dfrac{\sqrt{l^2+b^2}}{h}$ **b.** $\dfrac{\sqrt{l^2+b^2+h^2}}{h}$
c. $\dfrac{h}{\sqrt{l^2+b^2}}$ **d.** $\dfrac{h}{\sqrt{l^2+b^2+h^2}}$

44. A positively charged particle P enters the region between two parallel plates with a velocity u, in a direction parallel to the plates. There is a uniform electric field in this region. P emerges from this region with a velocity v. Taking C as a constant, v will depend on u as

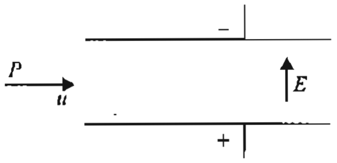


Fig. A1.24

a. $v = Cu$ **b.** $v = \sqrt{u^2 + Cu}$
c. $v = \sqrt{u^2 + \dfrac{C}{u}}$ **d.** $v = \sqrt{u^2 + \dfrac{C}{u^2}}$

45. Consider the situation shown in Fig. A1.25. We find electric field E at point P using Gauss's law and it comes out to be $E = \sigma/\varepsilon_0$. This electric field is due to

a. all the amount of charges present on both the plates
b. all the charges present on positive plate only
c. positive charge present only inside the Gaussian surface

d. positive charge present inside the Gaussian surface plus equal magnitude of negative charge present on negative plate in front of Gaussian surface

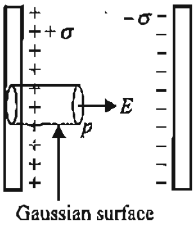


Fig. A1.25

46. Figure A1.26 shows three circular arcs, each of radius R and total charge as indicated. The net electric potential at the center of curvature is

a. $\dfrac{Q}{2\pi\varepsilon_0 R}$
b. $\dfrac{Q}{4\pi\varepsilon_0 R}$
c. $\dfrac{2Q}{\pi\varepsilon_0 R}$
d. $\dfrac{Q}{\pi\varepsilon_0 R}$

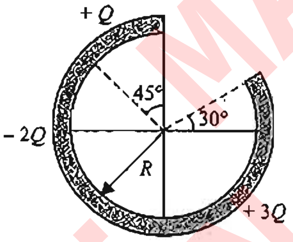


Fig. A1.26

47. Electric field intensity at an equatorial point of a dipole [dipole moment $\vec{p}$] is $\vec{E}$. The angle between $\vec{p}$ and $\vec{E}$ is

a. 90°
b. 180°
c. 0°
d. none of these

48. Four charges are placed at four corners of a square as shown (Fig. A1.27). The side of the square is a. Two charges are positive and two are negative but their magnitudes are same. Now, an external agent starts decreasing all the sides of the square slowly and at the same rate. What happens to the electrical potential energy of the system and what will be the nature of work done by the agent?

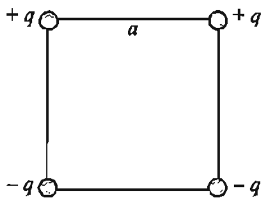


Fig. A1.27

a. Increases, positive
b. Increases, negative
c. Decreases, negative
d. Increases, positive

49. Given a metallic uniformly charged sphere. The radius of the sphere is increased keeping its potential same. What is the effect on the value of electric field intensity at its surface?

a. Increases
b. Decreases
c. Remains constant
d. Cannot say

50. One-fourth of a sphere of radius R is removed as shown (Fig. A1.28). An electric field E exists parallel to x–y plane. Find the flux through remaining curved part.

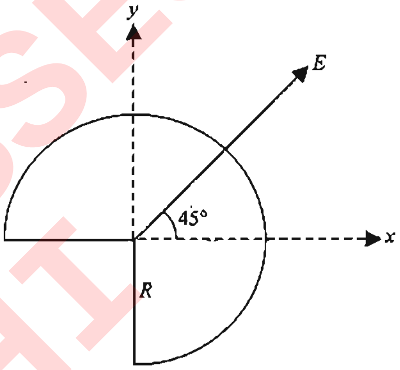


Fig. A1.28

a. $\pi R^2 E$
b. $\sqrt{2}\pi R^2 E$
c. $\pi R^2 E/\sqrt{2}$
d. none of these

51. At a distance r from a point located at origin in space, electric potential varies as $V = 10r$. Find the electric field at $\vec{r} = 3\hat{i} + 4\hat{j} - 5\hat{k}$.

a. $\sqrt{2}\,(3\hat{i} + 4\hat{j} - 5\hat{k})$
b. $-\sqrt{2}\,(3\hat{i} + 4\hat{j} - 5\hat{k})$
c. $-\sqrt{3}\,(3\hat{i} + 4\hat{j} - 5\hat{k})$
d. None of the above

52. A conducting spherical shell is earthed. A positive charge $+q_1$ is placed at the center and another small positive charge $+q_2$ is placed at a distance r from q_1 (Fig. A1.29). Ignore the effect of induced charge due to q_2 on the sphere. Then, the coulomb force on q_2 is

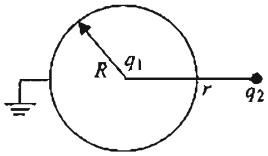


Fig. A1.29

a. zero
b. $\dfrac{q_1 q_2}{4\pi\varepsilon_0 r^2}$
c. $\dfrac{q_1 q_2}{4\pi\varepsilon_0 (r - R)^2}$
d. $\dfrac{q_1 q_2}{4\pi\varepsilon_0 (r^2 - R^2)}$

53. In the above problem, if the conducting spherical shell is not earthed but is neutral, then the force on q_2 is

a. zero **b.** $\dfrac{q_1q_2}{4\pi\varepsilon_0 r^2}$

c. $\dfrac{q_1q_2}{4\pi\varepsilon_0 (r-R)^2}$ **d.** none of these

54. Two parallel conducting plates, each of area A, are separated by a distance d. Now, the left plate is given a positive charge Q. A positive charge q of mass m is released from a point near the left plate. Find the time taken by the charge to reach the right plate.

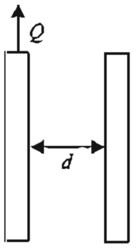


Fig. A1.30

a. $\sqrt{\dfrac{3dm\varepsilon_0 A}{qQ}}$ **b.** $\sqrt{\dfrac{4dm\varepsilon_0 A}{qQ}}$

c. $\sqrt{\dfrac{2dm\varepsilon_0 A}{qQ}}$ **d.** None of these

55. A uniform electric field of 100 Vm^{-1} is directed at 60° with the positive x-axis as shown in Fig. A1.31. If $OA = 2$ m, the potential difference $V_O - V_A$ is

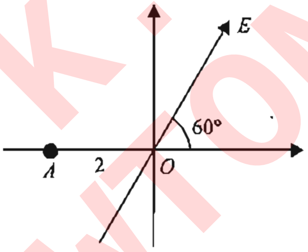


Fig. A1.31

a. 50 V **b.** 50 V

c. 100 V **d.** −100 V

56. A graph of the x-component of the electric field as a function of x in a region of space is shown in Fig. A1.32. The y and z components of the electric field are zero in this region. If the electric potential is 10 V at the origin, then potential at $x = 2.0$ m is

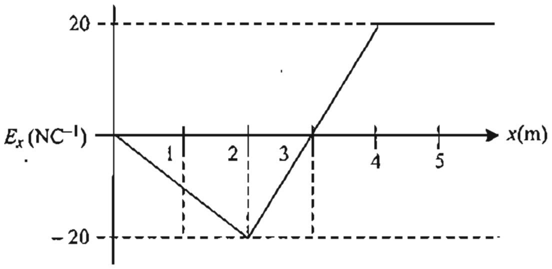


Fig. A1.32

a. 10 V **b.** 40 V

c. −10 V **d.** 30 V

57. An uncharged conductor A is brought near a positively charged conductor B. The size of the conductor A is much greater than the size of conductor B. Then,

a. the charge on B will increase but the potential of B will not change

b. the charge on B will not change but the potential of B will decrease

c. the charge on B will decrease but the potential of B will not change

d. the charge on B will not change but the potential of B will increase

58. Inside a hollow conducting sphere, which is uncharged, a charge q is placed at its center. Let electric field at a distance x from center at point p is E and potential at this point is V. Now, some positive charge Q is given to this sphere, then

a. E will remain same

b. E will increase

c. V will decrease

d. V will remain same

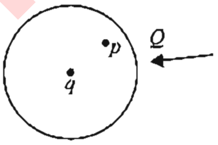


Fig. A1.33

59. A charge particle moves in a circle around an infinite line charge with the center of circle at the line and line being perpendicular to the plane of circle. Let r is the radius of circle. The velocity of the particle depends upon which power of r.

a. 1 **b.** 2

c. −1 **d.** None of these

60. Two large plates are given the charges as shown in Fig. A1.34. Now, the left plate is earthed. Find the amount of charge that will flow from earth to the plate.

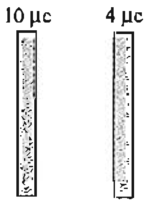


Fig. A1.34

a. 14 μC **b.** −14 μC

c. 7 μC **d.** −7 μC

61. Three identical metallic plates are kept parallel to one another at separations a and b. The outer plates are connected to the ground and the middle plate is given charge Q (Fig. A1.35). Then, charge on the right side of middle plate is

a. $Q/2$ **b.** $-\dfrac{Qb}{a+b}$

c. $\dfrac{Qb}{a+b}$ **d.** $\dfrac{Qa}{a+b}$

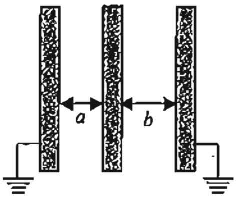


Fig. A1.35

62. An electric dipole consists of charges $\pm 2.0 \times 10^{-8}$ C separated by distance of 2.0×10^{-3} m. It is placed at a distance of 6 cm from a line charge of linear charge density 4.0×10^{-4} Cm^{-1} as shown in Fig. A1.36, such that the dipole makes an angle of $\theta = 60°$ with line AB. Find the force acting on the dipole.

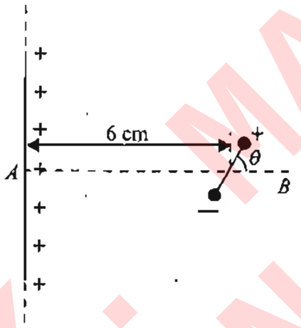


Fig. A1.36

a. 0.012 N **b.** 1.8 N

c. 0.04 N **d.** None of these

63. A 16 μF capacitor, initially charged to 5 V, is started charging at $t = 0$ by a source at the rate of $40t$ μCs^{-1}. How long will it take to raise its potential to 10 V?

a. 1 s **b.** 2 s

c. 3 s **d.** None of these

64. Find the equivalent capacitance between C and B.

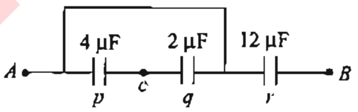


Fig. A1.37

a. 6/5 μF **b.** 5/6 μF

c. 4 μF **d.** None of these

65. A parallel plate capacitor of capacitance 10 μF is connected across a battery of emf 5 mV. Now, the space between the plates of the capacitor is filled with a dielectric material of dielectric constant $K = 5$. Then, the charge that will flow through the battery till equilibrium is reached is

a. 250 μC **b.** 250 nC

c. 200 nC **d.** 200 μC

66. Six plates of equal area A and plate separation as shown (Fig. A1.38) are arranged. Equivalent capacitance between A and B is

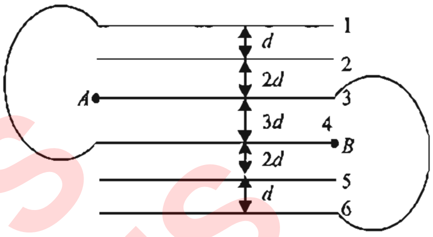


Fig. A1.38

a. $\dfrac{\varepsilon_0 A}{d}$ **b.** $\dfrac{2\varepsilon_0 A}{d}$

c. $\dfrac{3\varepsilon_0 A}{d}$ **d.** $\dfrac{\varepsilon_0 A}{4d}$

67. If area of each plate is A and the successive separations are d, $2d$, and $3d$, then equivalent capacitance across A and B is

a. $\dfrac{\varepsilon_0 A}{6d}$ **b.** $\dfrac{\varepsilon_0 A}{4d}$

c. $\dfrac{3\varepsilon_0 A}{4d}$ **d.** $\dfrac{\varepsilon_0 A}{3d}$

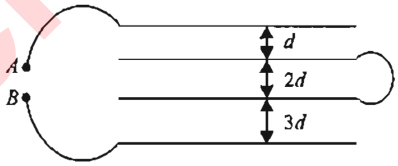


Fig. A1.39

68. Three plates A, B, and C, each of area 50 m^2, have separation 3 mm between A and B and 6 mm between B and C. The energy stored when the plates are fully charged by a 12 volt battery is

a. 2 μJ **b.** 1.6 μJ

c. 5 μJ **d.** 3.2 μJ

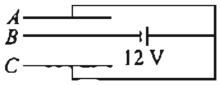


Fig. A1.40

69. A circuit has a section AB shown (Fig. A1.41). Potential difference between the points A and B (i.e., $V_A - V_B$) equals 5 V. The voltage across 2 μF capacitor is

a. 5 V **b.** 10 V **c.** 15 V **d.** zero

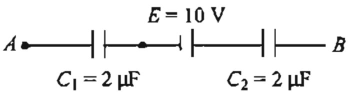


Fig. A1.41

70. In the circuit shown (Fig. A1.42), equivalent capacitance between the points X and Y is

a. 2 μF **b.** 3 μF **c.** 4 μF **d.** 5 μF

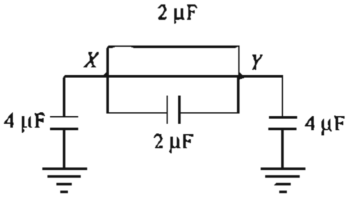


Fig. A1.42

71. An electric dipole is placed at origin in the x-y plane with its orientation along the positive x-axis. The direction of electric field

a. at $(-x_0, 0, 0)$ is along the positive x-axis.

b. at $(0, y_0, 0)$ is along the negative x-axis.

c. at $(0, 0, z_0)$ is along the negative x-axis.

d. all the above.

72. For the arrangement shown in Fig. A1.43, identify the correct statement.

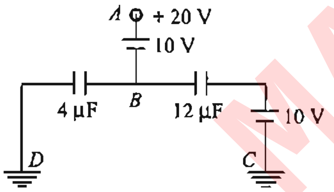


Fig. A1.43

a. the charge on the 12 μF capacitor is zero.

b. the charge on the 12 μF capacitor is 30 μC.

c. the charge on the 4 μF capacitor is 30 μC.

d. none of these

73. The electric field in a region is given by

$$\vec{E} = \frac{E_0 x}{l}\vec{i}$$

Find the charge contained inside a cubical volume bounded by the surfaces $x = 0$, $x = a$, $y = 0$, $y = a$, $z = 0$ and $z = a$. Take $E_0 = 5 \times 10^3$ N/C, $l = 0.02$ m and $a = 0.01$ m.

a. 1.1×10^{-12}C **b.** 2.2×10^{-12}C

c. 4.4×10^{-12}C **d.** 5.5×10^{-12}C

74. Three identical metal plates with large surface areas are kept parallel to each other as shown in Fig. A1.44. The leftmost plate is given a charge Q, the rightmost a charge $-2Q$ and the middle one remains neutral. Find the charge appearing on the outer surface of the rightmost plate.

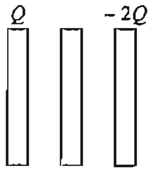


Fig. A1.44

a. $tQ/4$ **b.** $t2Q$ **c.** tQ **d.** $tQ/2$

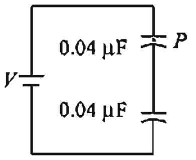


Fig. A1.45

75. The particle P shown in figure has a mass of 10 mg and a charge of -0.01 μC. Each plate has a surface area 10^{-2} m^2 on one side. What potential difference V should be applied to the combination to hold the particle P in equilibrium?

a. 43 mv **b.** 35 mv **c.** 50 mv **d.** 55 mv

76. Figure A1.46 shows two identical parallel plate capacitors connected to a battery through a switch S. Initially, the switch is closed so that the capacitors are completely charged. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant 3. Find the ratio of the initial total energy stored in the capacitors to the final total energy stored.

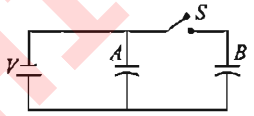


Fig. A1.46

a. 9 : 16 **b.** 5 : 9

c. 2 : 3 **d.** 3 : 5

77. Find the charge appearing on each of the three capacitors shown in Fig. A1.47 as C_A, C_B, C_C

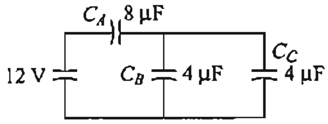


Fig. A1.47

a. 60 μC, 20 μC, 30 μC

b. 50 μC, 12 μC, 10 μC

c. 48 μC, 24 μC, 24 μC

d. 40 μC, 30 μC, 30 μC

78. Two condensers C_1 and C_2 in a circuit are joined as shown in Fig. A1.48. The potential of point A is V_1 and that of B is V_2. The potential of point D will be

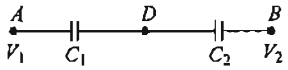


Fig. A1.48

a. $\frac{1}{2}(V_1 + V_2)$ **b.** $\frac{C_2 V_1 + C_1 V_2}{C_1 + C_2}$

c. $\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$ **d.** $\frac{C_2 V_1 - C_1 V_2}{C_1 + C_2}$

79. Two identical capacitors have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is

a. $\frac{1}{4}C(V_1^2 - V_2^2)$ **b.** $\frac{1}{4}C(V_1^2 + V_2^2)$

c. $\frac{1}{4}C(V_1 - V_2)^2$ **d.** $\frac{1}{4}C(V_1 + V_2)^2$

80. A point charge q is situated at X between two parallel plates which have a potential difference V and carry charges $+Q$ and $-Q$. What is the electric field strength at X?

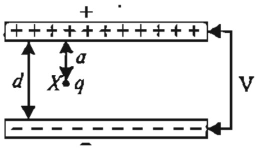


Fig. A1.49

a. $\frac{V}{d}$ **b.** $\frac{Vq}{d}$

c. $\frac{Q}{4\pi\varepsilon_0 a^2}$ **d.** $\frac{qQ}{4\pi\varepsilon_0 a^2}$

81. In the circuit shown, a capacitor of capacitance 3 μF is charged form a battery of e.m.f. 6 V with switch connected to terminal P. The switch is now connected to Q. This charges the 6 μF capacitor from the 3 μF one. What is the new potential difference across combination?

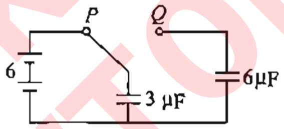


Fig. A1.50

a. 1V **b.** 2 V **c.** 4 V **d.** 6V

82. In the given arrangement of capacitors 6μC charge is added to point A. Find the charge on upper capacitor.

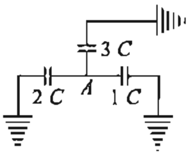


Fig. A1.51

a. 3 μC **b.** 1 μC

c. 2 μ C **d.** 6 μC

83. Two capacitors A and B with capacities C_1 and C_2 are charged to potential difference of V_1 and V_2, respectively. The plates of capacitors are connected as shown in figure with one wire from each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged capacitor of capacitance C_3 and lead wires falls on the free ends to complete circuit, then

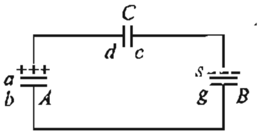


Fig. A1.52

a. final charge on each capacitor are same to each other

b. the final sum of charge on plates a & d is C_1V_1

c. the final sum of charge on plates (b) and (g) is $C_2V_2 - C_1V_1$

d. both (b) and (c) are correct

84. Two point charges q and $4q$ are held at a separation r. The electric field due to them is zero at a distance

a. $\frac{r}{\sqrt{3}}$ from charge $4q$

b. $\frac{r}{3}$ from charge $4q$s

c. $\frac{2r}{\sqrt{3}}$ from charge $4q$

d. $\frac{2r}{3}$ from charge $4q$

85. A parallel plate capacitor C is equally filled with parallel layers of materials of dielectric constants K_1 and K_2. Then the ratio of new capacitance to the previous capacitor is

a. $\frac{2K_1K_2}{K_1 + K_2}$

b. $K_1 + K_2$

c. $\frac{K_1K_2}{K_1 + K_2}$

d. none of the above

86. The time period of simple pendulum of charged bob is T as shown in the Fig. A1.53. Now a massless charge q is placed at point B, and time period of oscillation is T', then

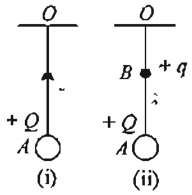


Fig. A1.53

a. $T' > T$ **b.** $T' < T$

c. $T' = T$ **d.** can't say

87. A long string with a charge of λ per unit length passes through an imaginary cube of edge a. The maximum flux of the electric field through the cube will be

a. $\frac{\lambda a}{\varepsilon_0}$ **b.** $\frac{\sqrt{2}\,\lambda a}{\varepsilon_0}$

c. $\frac{6\lambda a^2}{\varepsilon_0}$ **d.** $\frac{\sqrt{3}\,\lambda a}{\varepsilon_0}$

88. A conducting shell of radius a and charge Q is concentric with a solid sphere of charge Q and radius b $(b < a)$, then the electric potential at distance $r(b < r < a)$ from the centre is

a. $KQ\left(\frac{1}{a}+\frac{1}{b}\right)$

b. $KQ\left(\frac{1}{a}-\frac{1}{r}\right)$

c. $KQ\left(\frac{1}{a}+\frac{1}{r}\right)$

d. $\frac{KQ}{a}$ where $k=\frac{1}{4\pi\varepsilon_0}$

89. An α particle passes rapidly through the exact centre of a hydrogen molecule, moving on a line perpendicular to the internuclear axis. The distance between the nuclei is b. Where on its path does the α particle experience the greatest force? Assume that the nuclei do not move much during the passage of the α particle. Also neglect the electric field of the electrons in the molecule.

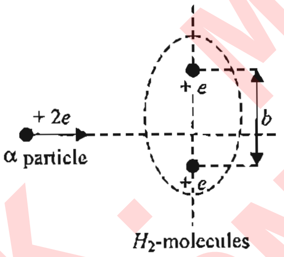


Fig. A1.54

a. $\frac{b}{2}$ **b.** $\frac{b}{2\sqrt{2}}$

c. $\frac{b}{\sqrt{2}}$ **d.** none of above

90. In the given electric field $\vec{E}=\left[\alpha(d+x)\hat{i}+E_0\hat{j}\right]$ N/C; where $\alpha = 1$ N/C hypothetical closed surface is taken as shown in Fig. A1.55. The total charged enclosed within the close surface is:

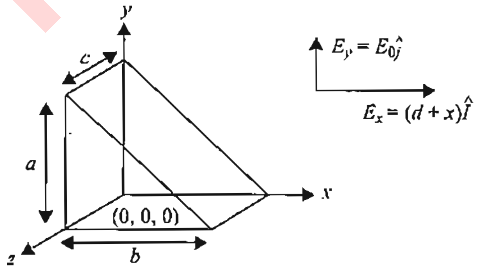


Fig. A1.55

a. $\frac{abc\,\varepsilon_0}{2}$ **b.** $\frac{acd\,\varepsilon_0}{2}$

c. $\frac{abd\,\varepsilon_0}{2}$ **d.** none of above

91. A spherical conductor A contains two spherical cavities. The total charge on the conductor itself is zero. However, there is a point charge q_b at the centre of one cavity and q_c at the centre of the other. A considerable distance r away from the centre of the spherical conductor, there is another charge q_d. Force acting on q_b, q_c and q_d are F_1, F_2, and F_3, respectively. [Assume all charges are positive]

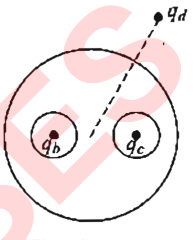


Fig. A1.56

a. $F_1 < F_2 < F_3$ **b.** $F_1 = F_2 < F_3$

c. $F_1 = F_2 > F_3$ **d.** $F_1 > F_2 > F_3$

92. Three infinite plane have a uniform surface charge distribution σ on its surface. All charges are fixed. On each of the three infinite planes, parallel to the y-z plane placed at $x = -a$, $x = 0$ and $x = a$, there is a uniform surface charge of the same density, σ. The potential difference between A and C is

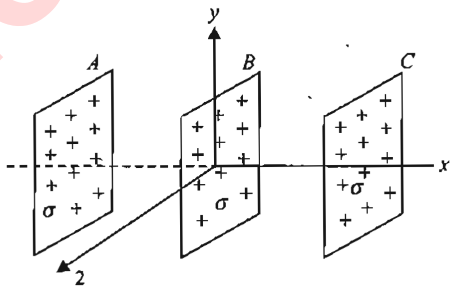


Fig. A1.57

a. $\frac{\sigma}{2\varepsilon_0}a$ **b.** $\frac{\sigma}{\varepsilon_0}a$

c. $\frac{\sigma a}{2\varepsilon_0}$ **d.** none of above

93. A capacitor of capacitance C is charged to a potential difference of V_0. The charging battery is disconnected and the capacitor is connected to a capacitor of unknown capacitance x. The potential difference across the combination is V. The value of x should be

a. $\frac{C(V_0-V)}{V}$ **b.** $\frac{C(V-V_0)}{V}$

c. $\frac{CV}{V_0}$ **d.** $\frac{CV_0}{V}$

94. In the circuit shown in Fig. A1.58, charge stored in capacitor of capacitance 3μF is

a. zero **b.** 40 μC

c. 60 μC **d.** 90 μC

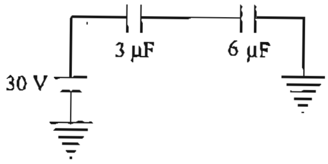


Fig. A1.58

95. A photographic flash units consists of a xenon filled tube. It gives a flash of average power 2000 W for 0.04 sec. The flash is due to discharge of a fully charged capacitor of 40 μF. The voltage to which it is charged before a flash is given by the unit is

a. 1,500 V **b.** 2,000 V
c. 2,500 V **d.** 3,000 V

96. In the circuit shown equivalent capacitance between the points X and Y is:

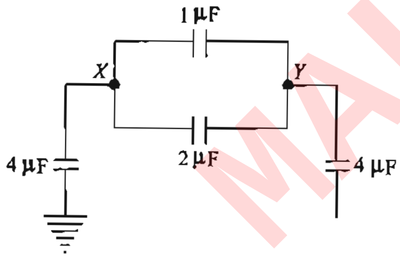


Fig. A1.59

a. 2 μF **b.** 3 μF **c.** 4 μF **d.** 5 μF

97. A parallel plate capacitor is connected to a battery. The plates are pulled apart with uniform speed. If x is the separation between the plates, then rate of change of electrostatic energy of the capacitor is proportional to

a. x^2 **b.** x **c.** $\frac{1}{x}$ **d.** $\frac{1}{x^2}$

98. Find equivalent capacitance between A and B. [Assume each conducting plate is having same dimensions and neglect the thickness of the plate, $\frac{\varepsilon_0 A}{d} = 7\,\mu F$[where A is area of plates, $A >> d$]

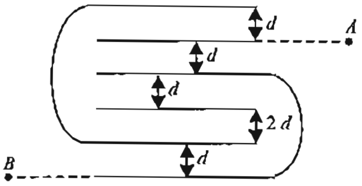


Fig. A1.60

a. $7\,\mu F$ **b.** $11\,\mu F$
c. $12\,\mu F$ **d.** $15\,\mu F$

99. If the plates of a parallel plate capacitor are not equal in area, then

a. quantity of charge on the plates will be same but nature of charge will differ

b. quantity of charge on the plates as well as nature of charge will be different

c. quantity of charge on the plates will be different but nature of charge will be same

d. quantity of charge as well as nature of charge will be same

100. Electric charges q, q, $-2q$ are placed at the corners of an equilateral triangle ABC of side l. The magnitude of electric dipole moment of the system is

a. ql **b.** $2ql$
c. $\sqrt{3}ql$ **d.** ql

101. How does the electric field strength vary when we enter a uniformly charged spherical cloud?

a. Decreases inversely as the square of the distance from the surface

b. Decreases directly as the square of the distance from the surface

c. Decreases directly as the square of the distance from the center

d. Decreases directly as the distance from the center or $E \propto r$

102. A ring of radius R carries a charge $+q$. A test charge $-q_0$ is released on its axis at a distance from its center. How much kinetic energy will be acquired by the test charge when it reaches the center of the ring?

a. $\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{R}$ **b.** $\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{2R}$
c. $\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{\sqrt{3}R}$ **d.** $\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{3R}$

103. Two points are at distances a and b ($a < b$) from a long string of charge per unit length λ. The potential difference between the points is proportional to:

a. b/a **b.** b^2/a^2
c. $\sqrt{b/a}$ **d.** log (b/a)

104. When two uncharged metal balls of radius 0.09 mm each collide, one electron is transferred between them. The potential difference between them would be:

a. 16 μV **b.** 16 pV
c. 32 μV **d.** 32 pV

105. In a certain charge distribution, all points having zero potential can be joined by a circle S. Points inside S have positive potential, and points outside S have negative potential. A positive charge, which is free to move, is placed inside S

a. it will remain in equilibrium

b. it can move inside S, but it can not cross S

c. it must cross S at some time

d. it may move, but will ultimately return to its starting point

106. A charge $+q$ is placed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$ at infinitum on the x-axis, and a charge $-q$ is placed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$ at infinitum. Here, x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it to be $Q/4\pi\varepsilon_0 r$. Then the potential at the origin due to the above system of charges is

a. 0

b. $\dfrac{q}{8\pi\varepsilon_0 x_0 \log 2}$

c. ∞

d. $\dfrac{q \log 2}{4\pi\varepsilon_0 x_0}$

107. A long string with a charge of λ per unit length passes through an imaginary cube of edge a. The maximum flux of the electric field through the cube will be

a. $\lambda a/\varepsilon_0$ **b.** $\sqrt{2}\lambda a/\varepsilon_0$

c. $6\lambda a^2/\varepsilon_0$ **d.** $\sqrt{3}\lambda a/\varepsilon_0$

108. A spring block system undergoes vertical oscillations above a large horizontal metal sheet with uniform positive charge. The time period of the oscillation is T. If the block is given a charge Q, its time period of oscillation will be

a. T

b. $> T$

c. $< T$

d. $> T$ if Q is + ve and $< T$ if Q is −ve

109. The potentials of the two plates of capacities are +10 V and −10 V. The charge on one of the plates is 40 C. The capacitance of the capacitor is

a. 2 F **b.** 4 F

c. 0.5 F **d.** 0.25 F

110. The capacity of parallel plate capacitor in air and on immersing it into oil is 50 μF and 110 μF respectively. The dielectric constant of oil is

a. 0.45 **b.** 0.55

c. 1.10 **d.** 2.20

111. An air parallel plate capacitor has capacity C. When distance between plates is doubled and capacitor is emerged in water the capacity get doubled then dielectric constant of water is

a. 1 **b.** 2 **c.** 3 **d.** 4

112. A thin aluminium sheet is placed between the plates of a parallel plates capacitor. Its capacitance will

a. increases

b. decreases

c. remain same

d. become infinite

113. A fully charged capacitor has a capacitance C. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity s and mass m. If the temperature of the block is raised by T, the potential difference V across the capacitance is

a. $\dfrac{ms\Delta T}{C}$ **b.** $\sqrt{\dfrac{2ms\Delta T}{C}}$

c. $\sqrt{\dfrac{2msC\Delta T}{s}}$ **d.** $\dfrac{mC\Delta T}{s}$

114. A capacitor of 2.5 μF is charged through a resistor of 4 MΩ. In how much time will potential drop across capacitor will become 3 times that of resistor (ln 2 = 0.693)

a. 13.86 s **b.** 6.93 s

c. 1.386 s **d.** 69.3 s

115. The work done in placing a charge of 8×10^{18} coulomb on a condenser of capacity 100 micro-farad is

a. 32×10^{-32} joule

b. 16×10^{-32} joule

c. 3.1×10^{-26} joule

d. 4×10^{-10} joule

116. A 40 μF capacitor in a defibrillator is charged to 3000 V. The energy stored in the capacitor is sent through the patient during a pulse of duration 2 ms. The power delivered to the patient is

a. 45 kW **b.** 90 kW

c. 180 kW **d.** 360 kW

117. Three charges q, q, and $-2q$ are fixed on the vertices of an equilateral triangular plate of edge length a. This plate is in equilibrium between two very large plates having surface charge density σ_1 and σ_2, respectively. Find time period of small angular oscillation about an axis passing through its centroid and perpendicular to plane. Moment of inertia of the system about this axis is I.

a. $2\pi\sqrt{\dfrac{\varepsilon_0 I}{qa|\sigma_1 - \sigma_2|}}$ **b.** $2\pi\sqrt{\dfrac{\varepsilon_0 I}{2qa|\sigma_1 - \sigma_2|}}$

c. $2\pi\sqrt{\dfrac{2\varepsilon_0 I}{\sqrt{3}qa|\sigma_1 - \sigma_2|}}$ **d.** $2\pi\sqrt{\dfrac{2\varepsilon_0 I}{qa|\sigma_1 - \sigma_2|}}$

118. The capacitor plates are fixed on an inclined plane and connected to a battery of e.m.f. E. The capacitor plates have plate area A, length l and the distance between them is d. A dielectric slab of mass m and dielectric constant K is inserted into the capacitor and tied to a mass M by a massless string as shown in the figure. Find the value of M for which the slab will stay in equilibrium. There is no friction between slab and plates.

a. $\dfrac{m}{2} + \dfrac{E^2\varepsilon_0 A(k-1)}{2Lgd}$

b. $\dfrac{m}{2} - \dfrac{E^2\varepsilon_0 A(k-1)}{2lgd}$

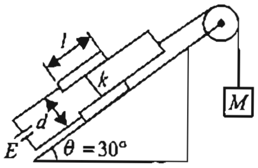


Fig. A1.61

c. $\dfrac{m}{2} + \dfrac{E^2\varepsilon_0 A(k-1)}{lgd}$

d. $\dfrac{m}{2} - \dfrac{E^2\varepsilon_0 A(k-1)}{lgd}$

119. Two plates, each of area A are placed parallel to each other at a distance d. One plate is connected to a battery of e.m.f. E and its negative is earthed. The other plate is also earthed. The charge drawn by plate is

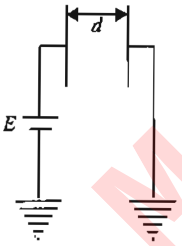


Fig. A1.62

a. $\dfrac{2\varepsilon_0 AE}{d}$ b. $\dfrac{\varepsilon_0 AE}{d}$

c. $\dfrac{\varepsilon_0 AE}{2d}$ d. $\dfrac{3\varepsilon_0 AE}{d}$

120. A particle with charge $+q$ and mass m, moving under the influence of a uniform electric field $E\hat{i}$ and a uniform magnetic field $B\hat{k}$, follows a trajectory from P to Q as shown. The velocities at P and Q are $v\hat{i}$ and $-2v\hat{j}$. Which of the following is correct?

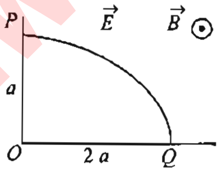


Fig. A1.63

a. $E = \dfrac{3}{4}\left(\dfrac{mv^2}{qa}\right)$

b. the rate of work done by the electric field at P is $\dfrac{3}{4}\left(\dfrac{mv^3}{a}\right)$

c. the rate of work done by the electric field at P is 0

d. the rate of work done by both the fields at Q is 0

121. A charged particle P leaves the origin with speed $v = v_0$, at some inclination with the x-axis. There is a uniform magnetic field B along the x-axis. P strikes a fixed target T on the x-axis for minimum value of $B = B_0$. P will also strike T if

a. $B = 2B_0$, $v = 2v_0$

b. $B = 2B_0$, $v = v_0$

c. $B = B_0$, $v = 2v_0$

d. $B = B_0/2$, $v = 2v_0$

122. An electron moves in a uniform magnetic field and follows a spiral path as shown in Fig. A1.64 Which of the folllowing statements is/are correct

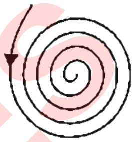


Fig. A1.64

a. Angular velocity of electron remains constant.

b. Magnitude of velocity of electron decreases continously.

c. Net force on the particle is always perpendicular to its direction of motion.

d. Magnitude of net force on the electron decreases contінously.

123. A charged particle moves in a garvity free space where an electric field of strength E and a magnetic field of induction B exist. Which of the following statement is/are correct?

a. If $E \neq 0$ and $B \neq 0$, velocity of the particle may remain constant.

b. If $E = 0$, particle cannot trace a circular path.

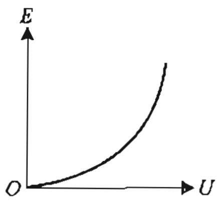
c. If $E = 0$, kinetic energy of the particle remains constant.

d. None of these.

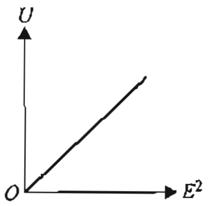
Multiple Correct Answers Type

Solutions on page A1.57

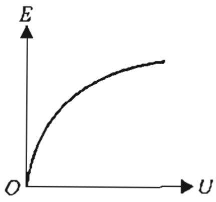
1. If at distance r from a positively charged particle, electric field strength and energy density are E and U, respectively, which of the following graphs is/are correct?



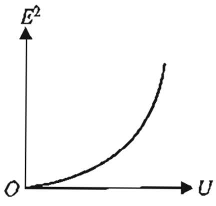
a.



b.



c.



d.

2. Mark out the incorrect statement(s).

a. A proton tends to go from a region of low electric potential to a region of high electric potential.

b. The electric potential of a negative charged conductor must be negative.

c. If $\vec{E} = 0$ at a point P, then V must be zero at P.

d. If $V = 0$ at a point P, then $\vec{E}$ must be zero at P.

3. In a uniformly charged dielectric sphere, a very thin tunnel has been made along the diameter as shown in Fig. A1.65 below. A charge particle $-q$ having mass m is released from rest at one end of the tunnel.
For the situation described, mark out the correct statement(s). [Neglect gravity.]

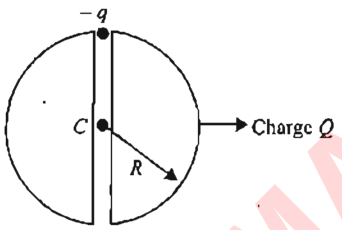


Fig. A1.65

a. Charge particle will perform SHM about center of the sphere as mean position.

b. Time period of the particle is $2\pi\sqrt{\dfrac{2\pi\varepsilon_0 m R^3}{qQ}}$.

c. Particle will perform oscillation but not SHM.

d. Speed of the particle while crossing mean position is $\sqrt{\dfrac{qQ}{4\pi\varepsilon_0 m R}}$.

4. A charge particle q is projected in an electric field produced by a fixed point charge Q as shown in Fig. A1.66. Mark out the correct statements.

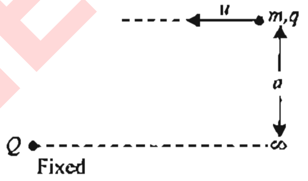


Fig. A1.66

a. The path taken by q is a straight line.

b. The path taken by q is not a straight line.

c. The minimum distance between the two particles is

$$\frac{\dfrac{qQ}{2\pi\varepsilon_0} + \sqrt{\left(\dfrac{qQ}{2\pi\varepsilon_0}\right)^2 + 4m^2u^4a^2}}{2\,mu^2}$$

d. Velocity of the particle q is changing in magnitude and direction both.

5. Four identical particles, each having mass m and charge q, are placed at the vertices of a square of side l. All the particles are free to move without any friction and released simultaneously from rest. Then,

a. at all instants, the particles remain at vertices of square whose edge length is changing

b. the configuration is changing (not remaining square) as the time passes

c. the speed of the particles when one of the particles get displaced by $\dfrac{l}{\sqrt{2}}$ is $\sqrt{\dfrac{q^2}{8\pi\varepsilon_0 ml}\left(2 + \dfrac{1}{\sqrt{2}}\right)}$

d. speed of the particles cannot be found

6. A small sphere is charged uniformly and placed at some point $A(x_0, y_0)$ so that at point $B(9\text{ m}, 4\text{ m})$ electric field strength is $\vec{E} = (54\hat{i} + 72\hat{j})\text{ NC}^{-1}$ and potential is 1800 V. Then,

a. the magnitude of charge on the sphere is 4 μC

b. the magnitude of charge on the sphere is 2 μC

c. coordinates of A are: $x_0 = -3$, $y_0 = -12$

d. coordinates of A are: $x_0 = 4$, $y_0 = -1$

7. Two conducting plates M and N, each having large surface area A (on one side), are placed parallel to each other (Fig. A1.67). The plate M is given a charge Q_1 and N a charge Q_2 ($< Q_1$). Then,

a. electric field at point A is $\dfrac{Q_1 - Q_2}{2A\varepsilon_0}$ towards right

b. electric field at point B is $\dfrac{Q_1 + Q_2}{2A\varepsilon_0}$ towards right

c. electric potential of N is greater than M

d. all of the above

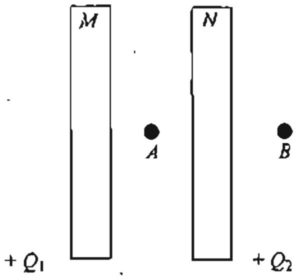


Fig. A1.67

8. A point charge q is placed within the cavity of an electrically neutral conducting shell whose outer surface has spherical shape (Fig. A1.68). Then,

a. the potential V at a point P lying outside the shell at a distance r from the center O of the outer surface depends upon the value of x

b. potential at P does not depend upon the value of x

c. a total charge q will be induced on the outer surface of the shell which will be distributed uniformly on the outer surface

d. a total charge $-q$ will be induced on the inner surface of the shell which will be distributed non-uniformly on the inner surface

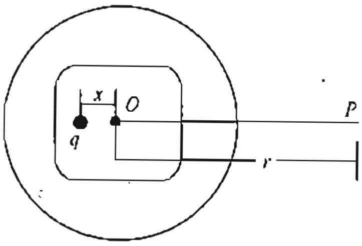


Fig. A1.68

9. In Fig. A1.69, the plates of a parallel plate capacitor have unequal charges. Its capacitance is C. P is a point outside the capacitor and close to the plate of charge $-Q$. The distance between the plates is d. Then,

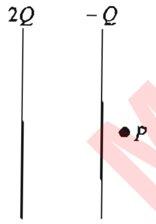


Fig. A1.69

a. a point charge at point P will experience electric force due to the capacitor

b. the potential difference between the plates will be $3Q/2C$

c. the energy stored in the electric field in the region between the plates is $\dfrac{9Q^2}{8C}$

d. the force on one plate due to the other plate is $\dfrac{Q^2}{2\pi\varepsilon_0 d^2}$

10. A particle of mass 2 kg and charge 1 mC is projected vertically with a velocity 10 ms^{-1}. There is a uniform horizontal electric field of 10^4 NC^{-1}. Then,

a. the horizontal range of the particle is 10 m

b. the time of flight of the particle is 2 s

c. the maximum height reached is 5 m

d. the horizontal range of the particle is 0

11. A thin-walled, spherical conducting shell S of radius R is given charge Q. The same amount of charge is also placed at its center C. Which of the following statements are correct?

a. On the outer surface of S, the charge density is $\dfrac{1}{2}\dfrac{Q}{\pi R^2}$.

b. The electric field is zero at all points inside S.

c. At a point just outside S, the electric field is double the field at a point just inside S, in the cavity.

d. At any point inside S (i.e., within its cavity), the electric field is inversely proportional to the square of its distance from C.

12. The electric potential at a certain distance from a point charge is 600 V and the electric field is 200 NC^{-1}. Which of the following statements will be true?

a. The work done in moving a point charge of 1 μC from the given point to a point at a distance of 9 m will be 4×10^{-4} J.

b. The distance of the given point from the charge is 3 m.

c. The potential at a distance of 9 m will be 200 V.

d. The magnitude of charge is 0.2×10^{-3} C.

13. An electric charge 2×10^{-8} C is placed at the point (1, 2, 4). At the point (3, 2, 1), the electric

a. field will increase by a factor K if the space between the points is filled with a dielectric of dielectric constant K

b. field will be along y-axis

c. potential will be 49.9 V

d. field will have no y-component

14. The electric potential in a region along the x-axis varies with x according to the relation $V(x) = 4 + 5x^2$. Then,

a. potential difference between the points $x = 1$ and $x = -2$ is 15 V

b. the force experienced by the above charge will be towards $+x$-axis

c. a uniform electric field exists in this region along the $+x$-axis

d. force experienced by a one coulomb charge at $x = -1$ m will be 10 N

15. The electric potential in the region of space is given by: $V(x) = A + Bx + Cx^2$, where V is in volts, x is in meters and A, B, C are constants. Then,

a. $\vec{E}$ varies linearly with x

b. the unit of $\vec{E}$ is newton coulomb^{-1}

c. $\vec{E}$ is in the negative x-direction

d. the electric field $\vec{E}$ in this region is constant

16. When a positively charged sphere is brought near a metallic sphere, it is observed that a force of attraction exists between the two. It means

a. the metallic sphere may be electrically neutral

b. the metallic sphere is necessarily negatively charged

c. nothing can be said about the charge of the metallic sphere

d. the metallic sphere may be negatively charged

17. A conducting sphere of radius R has a charge. Then,

a. the charge is uniformly distributed over its surface, if there is no external electric field

b. distribution of charge over its surface will be non-uniform, if an external electric field exists in the space

c. potential at every point of the sphere must be same

d. the electric field strength inside the sphere will be equal to zero only when no external electric field exists

18. A, B, and C are three large, parallel conducting plates, placed horizontally. A and C are rightly fixed and earthed (Fig. A1.70). B is given some charge. Under electrostatic and gravitational forces, B may be

a. in equilibrium if it is closer to A than to C

b. in equilibrium midway between A and C

c. B can never be in stable equilibrium

d. in equilibrium if it is closer to C than to A

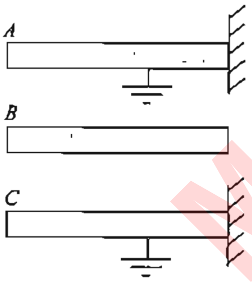


Fig. A1.70

19. A small conducting sphere of radius a mounted on an insulated handle and having a positive charge q is inserted through a hole in the wall of a hollow conducting sphere of inner radius b and outer radius c. The hollow sphere is supported on an insulating stand and is initially uncharged. The small sphere is placed at the center of the hollow sphere. Neglect any effect of the hole. Which of the following statements will be true for this system?

a. No work will be done in carrying a small charge from the inner conductor to the outer conductor.

b. The electric field at a point in the region between the spheres at a distance r from the center is equal to $q/4\pi\varepsilon_0 r^2$.

c. The electric field at a point outside the hollow sphere at a distance r from the center is $q/4\pi\varepsilon_0 r^2$.

d. The potential of the inner sphere with respect to the outer sphere is given by

$$V_{ab} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

20. Inside a uniformly charged spherical conductor, the electric

a. potential is zero everywhere

b. field is zero everywhere

c. field has the same magnitude everywhere but it is not zero

d. potential is same everywhere but not zero

21. Consider two identical charges placed distance $2d$ apart, along x-axis (Fig. A1.71). The equilibrium of a positive test charge placed at the point O midway between them is

a. stable for displacements along the x-axis

b. neutral

c. unstable for displacement along the y-axis

d. stable for displacements along the y-axis

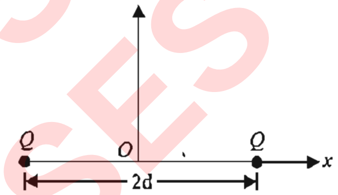


Fig. A1.71

22. Three concentric conducting spherical shells have radii r, $2r$ and $3r$ and charges q_1, q_2 and q_3, respectively. Innermost and outermost shells are earthed as shown in Fig. A1.72. Select the correct alternatives.

a. $\frac{q_3}{q_2} = -\frac{1}{3}$

b. $q_1 = \frac{-q_2}{4}$

c. $\frac{q_3}{q_1} = 3$

d. $q_1 + q_3 = -q_2$

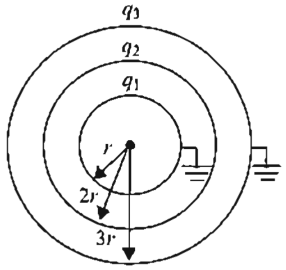


Fig. A1.72

23. Mark out the correct statements.

a. A given conducting sphere cannot be charged to a potential greater than a certain value.

b. A given conducting sphere can be charged to a potential less than a certain minimum value.

c. A given conducting sphere can be charged to any extent.

d. None of the above.

24. For the situation shown in Fig. A1.73 (assume $r >>$ length of dipole), mark out the correct statement(s).

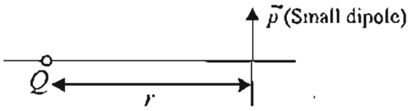


Fig. A1.73

a. Force acting on the dipole is zero.

b. Force acting on the dipole is approximately $\frac{pQ}{4\pi\varepsilon_0 r^3}$ and is acting upward.

c. Torque acting on the dipole is $\frac{pQ}{4\pi\varepsilon_0 r^2}$ in clockwise direction.

d. Torque acting on the dipole is $\frac{pQ}{4\pi\varepsilon_0 r^2}$ in anti-clockwise direction.

25. A dipole is placed in $x-y$ plane parallel to the line $y = 2x$. There exists a uniform electric field along z-axis. Net force acting on the dipole will be zero. But it can experience some torque. We can show that the direction of this torque will be parallel to the line

a. $y = 2x + 1$ **b.** $y = -2x$

c. $y = -\frac{1}{2}x$ **d.** $y = -\frac{1}{2}x + 2$

26. A hollow conducting sphere of inner radius R and outer radius $2R$ is given a charge Q as shown in the figure, then the

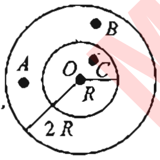


Fig. A1.74

a. potential at A and B is same

b. potential at O and B is same

c. potential at O and C is same

d. potential at A, B, C and O is same

27. An insulating spherical shell of uniform surface charge density is cut into two parts and placed at a distance d apart as shown in the Fig. A1.75.

$\vec{E}_P$ and $\vec{E}_Q$ denote the electric fields at P and Q respectively. As d (i.e., PQ) $\to \infty$

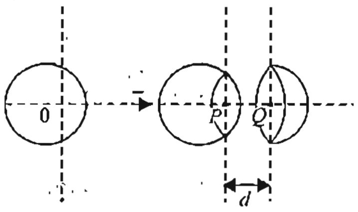


Fig. A1.75

a. $|\vec{E}_P| > |\vec{E}_Q|$

b. $|\vec{E}_P| = |\vec{E}_Q|$

c. $|\vec{E}_P| < |\vec{E}_Q|$

d. $\vec{E}_P + \vec{E}_Q = 0$

28. A point charge q is placed at origin. Let $\vec{E}_A$, $\vec{E}_B$ and $\vec{E}_C$ be the electric field at three points A (1, 2, 3), B (1, 1, −1), and C (2, 2, 2) due to charge q. Then

a. $\vec{E}_A \perp \vec{E}_B$

b. $\vec{E}_A \parallel \vec{E}_B$

c. $|\vec{E}_B| = 4|\vec{E}_C|$

d. $\vec{E}_B = 16|\vec{E}_C|$

29. Two identical parallel plate capacitors are connected in one case in parallel and in the other in series. In each case the plates of one capacitor are brought closer by a distance a and the plates of the other are moved apart by the same distance a. Then

a. total capacitance of first system increases.

b. total capacitance of first system decreases.

c. total capacitance of second system decreases

d. total capacitance of second system remains constant.

30. A charge Q is imparted to two identical capacitors in parallel. Separation of the plates in each capacitor is d_0. Suddenly, the first plate of the first capacitor and the second plate of the second capacitor starts moving to the left with speed v, then

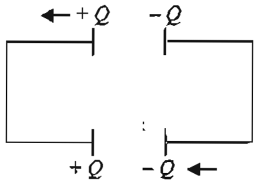


Fig. A1.76

a. charge on the two capacitors as a function of time are $\frac{Q(d_0 - vt)}{2d_0}, \frac{Q(d_0 + vt)}{2d_0}$

b. charge on the two capacitors as a function of time are $\frac{Qd_0}{2(d_0 - vt)}, \frac{Qd_0}{2(d_0 + vt)}$

c. current in the circuit will increases as time passes on.

d. current in the circuit will be constant.

31. In the circuit diagram shown, when switch is shifted from position 1 to position 2, then

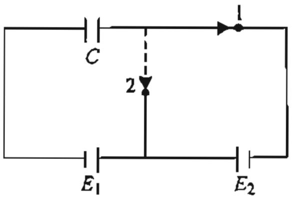


Fig. A1.77

a. extra charge drawn from battery is E_2C

b. extra charge drawn from battery is E_1E_2C

c. heat generated in the circuit is $\frac{1}{2}E_2^2C$

d. heat generated in the circuit is $\frac{1}{2}E_1E_2C$

32. A capacitor of capacitance C is charged to a potential difference V_0. The charging battery is disconnected and the capacitor is connected to a capacitor of unknown capacitance C_x. The potential difference across the combination is V, after the switch S is closed, then

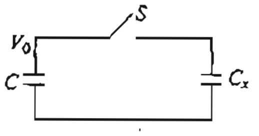


Fig. A1.78

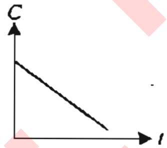
a. $C_x = \frac{C(V_0 - V)}{V}$

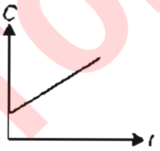
b. Final energy stored is $\frac{1}{2}CV_0V$

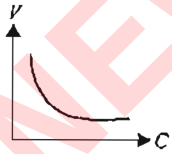
c. Heat generated in the circuit is $\frac{CV_0(V_0 - V)}{2}$

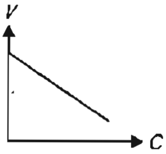
d. Heat generated in the circuit is $\frac{CV_0V}{2}$

33. A parallel plate capacitor has a dielectric slab in it. The slab just fills the space inside the capacitor. The capacitor is charged by a battery and then battery is disconnected. Now the slab is started to pull out slowly at $t = 0$. If at time t, the capacitance of the capacitor is C and potential difference between the plates capacitor is V then which of the following graphs is/are correct

a. 

b. 

c. 

d. 

34. Two plates of a parallel plate capacitors carry charges q and $-q$ and are separated by a distance x from each other. The capacitor is connected to a constant voltage source V_0. The distance between the plates is charged to $x + dx$. Then in steady state

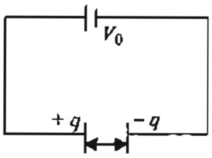


Fig. A1.79

a. Change in electrostatic energy stored in the capacitor is $\frac{-Udx}{x}$.

b. Change in electrostatic energy in the capacitor is $-\frac{Ux}{dx}$.

c. Attraction force between the plates is $1/2qE$.

d. Attraction force between the plates is qE. (where E is electric field between the plates)

35. A charged particle having a positive charge q approaches a grounded metallic spheres of radius R with a constant small speed v as shown in the Fig. A1.80. In this situation

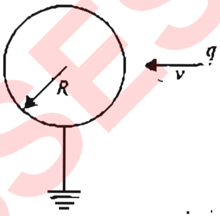


Fig. A1.80

a. As the charge draws nearer to the surface of the sphere, a current flows in to the ground.

b. As the charge draws nearer to the surface of the sphere, a current flows out of the ground in to the sphere.

c. As the charged particle draws nearer, the magnitude of current flowing in the connector joining the shell to the ground increases.

d. As the charged particle draws nearer, the magnitude of current flowing in the connector joining the sphere to the ground decreases.

36. A particle with a specific charge s starts from rest in a region where the electric field has a constant direction, but whose magnitude increase linearly with time. The particle acquires a velocity v in time t.

a. $v \propto s$

b. $v \propto \sqrt{s}$

c. $v \propto t$

d. $v \propto t^2$

Assertion-Reasoning Type

olutions on page A1.61

In the following questions, each question contains Statement I (Assertion) and Statement II (Reason). Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.

b. Statement I is True, Statement II is True; Statement II is **NOT** a correct explanation for Statement I.

c. Statement I is True, Statement II is False.

d. Statement I is False, Statement II is True.

1. Statement I: An applied electric field will polarize the polar dielectric material.

Statement II: In polar dielectrics, each molecule has a permanent dipole moment but these are randomly oriented in the absence of an externally applied electric field.

2. **Statement I:** When air between the plates of a parallel plate condenser is replaced by an insulating dielectric medium, its capacity decreases.

   Statement II: Electric field intensity between the plates with dielectric in between is reduced.

3. **Statement I:** If electric potential in certain region is constant, then the electric field must be zero in this region.

   Statement II: $\vec{E} = -\frac{dV}{dr}\hat{r}$.

4. **Statement I:** If a proton and an electron are placed in the same uniform electric field, they experience different forces.

   Statement II: Electric force on a test charge is independent of the mass of the test charge.

5. **Statement I:** Positive charge tends to move from high potential to low potential.

   Statement II: Positive charge tends to move from high electric field intensity to low electric field intensity always.

6. **Statement I:** A conducting body is connected to earth, hence it is electrically neutral.

   Statement II: The potential of a conducting body connected to earth is zero.

7. **Statement I:** For a non-uniformly charged thin circular ring with net charge zero, the electric field at any point on the axis of ring is zero.

   Statement II: For a non-uniformly charged thin circular ring with net charge zero, the electric potential at each point on the axis of ring is zero.

8. **Statement I:** A uniformly charged disk has a pin hole at its center. The electric field at the center of the disk is zero.

   Statement II: Disk can be supposed to be made up of many rings. Also, electric field at the center of a uniformly charged ring is zero.

9. **Statement I:** In a region where uniform electric field exists, the net charge within volume of any size is zero.

   Statement II: The electric flux within any closed surface in region of uniform electric field is zero.

10. **Statement I:** If the potential difference across a plane parallel plate capacitor is doubled, then the potential energy of the capacitor becomes four times under all conditions.

    Statement II: The potential energy U stored in the capacitor is $U = \frac{1}{2}CV^2$, where C and V have usual meaning.

11. **Statement I:** Total work done by non-uniform electric field on a charged particle starting from rest to any time is non-negative. (Assume no other forces act on the charged particle.)

    Statement II: The angle between electrostatic force and velocity of the charged particle released from rest in non-uniform electric field is always acute. (Assume no other forces act on the charged particle.)

12. **Statement I:** Electric lines of force cross each other.

    Statement II: Electric fields at a point superimpose to give one resultant electric field.

13. **Statement I:** A metallic shield in the form of a hollow shell may be built to block an electric field.

    Statement II: In a hollow spherical shield, the electric field inside it is zero at every point.

14. **Statement I:** If bob of a simple pendulum is kept in a horizontal electric field, its period of oscillation will remain same.

    Statement II: If bob is charged and kept in horizontal electric field, then the time period will be decreased.

15. **Statement I:** The potential of a grounded object is taken to be zero.

    Statement II: Capacitance of the earth is very large.

16. **Statement I:** Though large number of free electrons are present in a metal, yet there is no current in the absence of electric field.

    Statement II: In the absence of electric field, electrons move randomly in all directions.

17. **Statement I:** If the accelerating potential of an electron is doubled, then its velocity becomes 1:4 times.

    Statement II: It will move on a circular path with same velocity.

18. **Statement I:** The lightning conductor at the top of a high building has sharp pointed ends.

    Statement II: The surface density of charge at sharp ends is very high resulting in setting up of electric wind.

19. **Statement I:** Electric field is always directed perpendicular to an equipotential surface.

    Statement II: Equipotential surface is a surface on which at each point potential is same.

20. **Statement I:** A charged body cannot attract another uncharged body.

    Statement II: Oppositely charged bodies attract each other.

21. **Statement I:** A charged particle is free to move in an electric field. It does not move along an electric line of force.

    Statement II: Its initial position decides whether it will move along the line of force or not.

22. **Statement I:** When a body acquires positive charge, its mass decreases.

    Statement II: A body acquires positive charge when it loses electrons.

23. **Statement I:** A small metal ball is suspended in a uniform magnetic field with the help of an insulated thread. When a high-energy X-rays beam falls on the ball, then the ball will be deflected in the direction of electric field.

    Statement II: The ball will oscillate in the field.

24. **Statement I:** Two positively charged conductors are put in contact, the common potential will be less than the initial value of one of the conductors.
Statement II: Both conductors will attract each other.

25. **Statement I:** A molecule having intrinsic dipole moment is called polar molecule.
Statement II: Center of positive charge does not coincide with the negative charge in a polar molecule.

26. **Statement I:** Vehicles carrying highly inflammable materials have hanging chains, slightly touching the ground.
Statement II: The body of a vehicle gets charged when moving through air at high speed.

27. **Statement I:** A line of force has sudden breaks.
Statement II: An electrostatic line of force is a continuous curve.

28. **Statement I:** A parallel plate capacitor is connected across a battery through a key. A dielectric slab of dielectric constant K is introduced between the plates. The energy, which is stored, becomes K times.
Statement II: The plate remains uncharged or it has a constant surface charge density.

29. **Statement I:** If the distance between parallel plates of a capacitor is halved and dielectric constant becomes three times, then the capacitance becomes 6 times.
Statement II: Capacity of the capacitor does not depend upon the nature of the material.

30. **Statement I:** If three capacitors of capacitances $C_1 < C_2 < C_3$ are connected in parallel and in series, then their equivalent capacitance $C_p > C_S$.
Statement II: $\frac{1}{C_p} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ and $C_S = C_1 + C_2 + C_3$.

31. **Statement I:** A condenser of capacitance C and charge Q is connected to a battery of potential V. If the battery is removed and a dielectric slab is introduced between the plates and E is the energy of condenser, then its capacitance increases.
Statement II: On removing the battery connections, the potential energy decreases.

32. **Statement I:** A number of capacitors are connected in series with each other. If $U_1, U_2, U_3, \ldots$, respectively, be the energy stored in them, then total energy stored is $U_1 + U_2 + U_3 + \cdots$.
Statement II: Potential energy is a scalar quantity.

33. **Statement I:** When a charged capacitor is filled completely with a metallic slab, its capacity becomes very large.
Statement II The dielectric constant for metals is infinity.

34. **Statement I:** The circuits containing capacitor be handled cautiously, even when there is no current.
Statement II: A dielectric differs from an insulator.

35. **Statement I:** A metal sphere of radius 1 cm can hold a charge of 1 coulomb.
Statement II: Electric charge = (Electric potential) × (capacitance).

36. **Statement I:** If distance between the parallel plates of a capacitor is halved, then its capacitance is doubled.
Statement II: The capacitance depends upon the introduced dielectric constant.

37. **Statement I:** If the plates of parallel capacitor are not same in cross-sectional area, then quantity of charge on the plates will be same but nature of charge will be different.
Statement II: They will have the same nature of charge.

38. **Statement I:** Dielectric polarization means formation of positive and negative charges inside the dielectric.
Statement II: Free electron is formed in this process.

39. **Statement I:** If we introduce a sheet of glass between the two plates of a condenser, its potential will decrease.
Statement II: Charge will remain same.

40. **Statement I:** It is not possible to make a spherical capacitor of capacity one farad.
Statement II: It is possible for earth as its radius is 6.4×10^6 m.

41. **Statement I:** A dipole always tends to align in the direction of electric field.
Statement II: In this direction, torque acting on the dipole is zero.

42. **Statement I:** If the medium between two charges is replaced by another medium of greater dielectric constant, then the electric force between them decreases.
Statement II: Electric dipole moment varies inversely as the dielectric constant.

43. **Statement I:** The electric field due to a dipole on its axial line at a distance r is E. Then, electric field due to the same dipole on the equatorial line and at the same distance will be $\frac{E}{2}$.
Statement II: Electric field due to dipole varies inversely as the square of distance.

44. **Statement I:** A charged particle moves perpendicular to magnetic field. Its kinetic energy remains constant, but momentum changes.
Statement II: Force acts perpendicular to velocity of the particle.

45. **Statement I:** The magnetic field at the ends of a very long current carrying solenoid is half of that at the centre.
Statement II: If the solenoid is sufficiently long, the field within it is uniform.

Comprehension Type

Solutions on page A1.63

For Problems 1–3

Gauss's Law relates the net flux ϕ of an electric field through a closed surface to the net charge q_{in} that is enclosed by that surface.

$\varepsilon_0\phi = q_{\text{in}}$ or $\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{in}}$

Charge outside the surface no matter how larger or how close it may be is not included in the term q_{in} in Gauss law. The exact form or location of the charge inside the Gaussian surface is also of no concern. The electric field on the left hand side of equation, however, is the net electric field resulting from all charges, both inside and outside the Gaussian surface. This may seem to be inconsistent, but keep in mind that the electric field due to a charge outside the Gaussian surface contributes zero net flux through the surface, because as many field lines due to that charge enter the surface as leave it.

1. Fig. A1.81 shows five charged lumps of plastic and an electrically neutral coin. The cross section of Gaussian surface S is indicated. Assuming $q_1 = q_4 = +3.1$ nC, $q_2 = q_5 = -5.9$ nC and $q_3 = -3.1$ nC, the net electric flux through the surface is

a. -666 Nm^{2}C^{-1}

b. $+666$ Nm^{2}C^{-1}

c. -360 Nm^{2}C^{-1}

d. $+360$ Nm^{2}C^{-1}

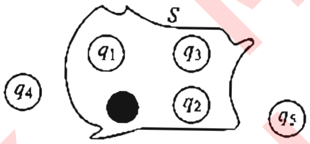


Fig. A1.81

2. A charge Q is uniformly distributed over a large plastic plate. The electric field at point P close to the center of plate is 20 Vm^{-1}. If the plastic plate is replaced by Aluminium plate of same geometrical dimensions and carrying the same charge Q, the electric field at the point P will become

a. zero **b.** 10 Vm^{-1}

c. 20 Vm^{-1} **d.** 40 Vm^{-1}

3. In an electric field due to a point charge $+Q$, a spherical closed surface is drawn as shown by dotted circle (Fig. A1.82. The electric flux through the surface drawn is zero by Gauss Law. A conducting sphere is inserted intersecting the previously drawn Gaussian surface. The electric flux through the surface

a. still remains zero

b. is non-zero but positive

c. is non-zero but negative

d. becomes infinite

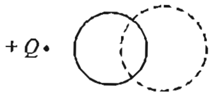


Fig. A1.82

For Problems 4–7

In many systems, equilibrium is possible due to the balancing of electric forces by other forces. We shall here consider a system which also involves force of buoyancy. A body placed in a fluid displaces fluid and it acts in the upward direction. Consequently, effective weight of the body is reduced. In Fig. A1.83, two identical helium filled balloons of small size are tied to an object of mass 1.1 g with threads of equal lengths, each 1 m. Each balloon carries a charge q that can be assumed as if it were concentrated at the center of balloon. The system floats in equilibrium as shown. Volume of helium in each balloon, in this situation, is V. The figure also shows that equilibrium separation between the balloons is 1.2 m. Helium is now replaced by another gas 'A' of a density twice that of helium. In this new situation, it is observed that volume of gas 'A' in each balloon has to be V' so as to result in equilibrium, under the condition that charge on each balloon retains its earlier value.

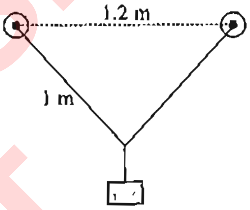


Fig. A1.83

Given density of air = 1.3 kgm^{-3}
density of He = 0.2 kgm^{-3}, g = 10 ms^{-2}, and
weight of unfilled balloon $\simeq 0$
Now answer the following questions

4. Volume of He in each balloon so that the system floats in equilibrium is

a. 500 cc **b.** 800 cc

c. 1125 cc **d.** 375 cc

5. Tension in each thread when the system is in equilibrium, with Helium filled in each balloon, will be

a. 3.44×10^{-3} N **b.** 6.88×10^{-3} N

c. 8.25×10^{-3} N **d.** 6.84×10^{-5} N

6. Charge on each balloon is nearly

a. 1.5 μC **b.** 1.2 μC

c. 0.8 μC **d.** 0.6 μC

7. Which of the following is correct?

a. In both situations, i.e., with Helium in each balloon and then with gas A in each balloon, vertical component of tension has the same value in the equilibrium condition. This value is 5.5×10^{-3} N.

b. In case of He, vertical component of tension is 5.5×10^{-3} N but with gas A, it is 11×10^{-3} N.

c. In case of He, vertical component of tension is 11×10^{-3} N but with gas A, it is 5.5×10^{-3} N.

d. In both cases, vertical component of tension has the same value and equal to 2.75×10^{-3} N.

For Problems 8–10

In a certain region, electric field E exists along x-axis which is uniform. Given $AB = 2\sqrt{3}$ m, $BC = 4$ m. Points A, B, C are in x, y plane.

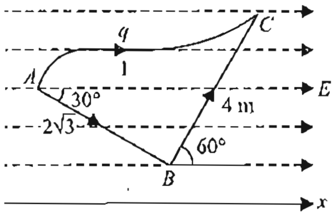


Fig. A1.84

8. Find potential difference $V_A - V_B$ between the points A and B.
   a. E b. $2E$ c. $3E$ d. $4E$

9. Find potential difference $V_C - V_B$ between the points C and B.
   a. $-E$ b. $-2E$ c. $2E$ d. $3E$

10. A charged particle q is moved from A to C as shown in path 1. What is the work done by electric field in this process?
    a. qE b. $4qE$
    c. $2qE$ d. 5qE

For Problems 11–13

We know that electric field (E) at any point in space can be calculated using the relation

$$\vec{E} = -\frac{\delta V}{\delta x}\hat{i} - \frac{\delta V}{\delta y}\hat{j} - \frac{\delta V}{\delta z}\hat{k},$$

if we know the variation of potential (V) at that point. Now, let electric potential in volt along x-axis varies as $V = 2x^2$, where x is in m. Its variation is as shown in Fig. A1.85.

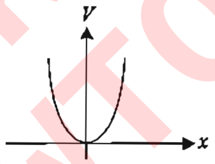
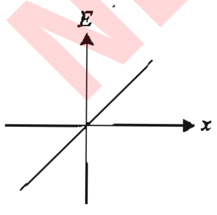
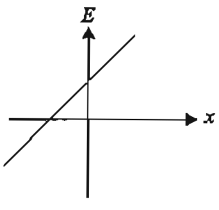


Fig. A1.85

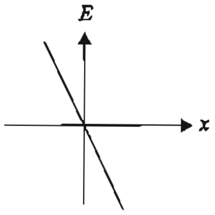
11. Draw the variation of electric field (E) along x-axis.



a.



b.



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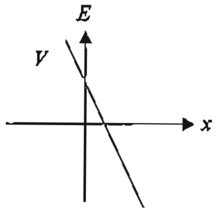


d.

12. A charge particle of mass 10 mg and charge 2.5 μC is released from rest at $x = 2$ m. Find its velocity when it crosses origin.
    a. 0.5 ms^{-1} b. 1 ms^{-1}
    c. 2 ms^{-1} d. 4 ms^{-1}

13. Will the particle perform a simple harmonic motion? Also, find the time period of its oscillations.
    a. Yes, time period = 2π s
    b. No, time period = 2π s
    c. Yes, time period = 4π s
    d. The particle will perform SHM, but time period cannot be found from the given data.

For Problems 14–17

Consider a system of two equal point charges, each $Q = 8$ μC, which are fixed at points (2 m, 0) and (−2 m, 0). Another charge q is held at a point (0, 0.1 m) on the Y-axis. Mass of the charge q is 91 mg. At $t = 0$, q is released from rest and it is observed to oscillate along Y-axis in a simple harmonic manner. It is also observed that, at $t = 0$, force experienced by it is 9×10^{-3} N. Now answer the following questions

14. Charge q is
    a. −8 μC b. −6.5 μC
    c. −5 μC d. +6.5 μC

15. Amplitude of motion is
    a. 10 cm b. 20 cm
    c. 30 cm d. 40 cm

16. Frequency of oscillation is
    a. 8 b. 10 c. 5 d. 2

17. Equation of SHM (displacement from mean position) can be expressed as
    a. $y = 0.1 \sin(10\pi t)$
    b. $y = 0.1 \sin(10\pi t + \pi/2)$
    c. $y = 0.1 \sin(5\pi t + \pi/2)$
    d. $y = 0.2 \sin(5\pi t)$

For Problems 18–21

In a certain experiment to measure the ratio of charge and mass of elementary particles, a surprising result was obtained in which two particles moved in such a way that the distance between them always remained constant. It was also noticed that this two particle system was isolated from all other particles and no force was acting on this system except the force between these two masses. After careful observation followed by intensive calculation, it was deduced that velocity of these two particles was always opposite in direction and magnitude of velocity was 10^3 ms^{-1} and 2×10^3 ms^{-1} for first and second particle, respectively, and masses of these particles were 2×10^{-30} kg and 10^{-30} kg, respectively. Distance between them came out to be 12 Å (1 Å = 10^{-10} m).

18. Acceleration of the first particle was
    a. zero

b. 4×10^{16} ms^{-2}
c. 2×10^{16} ms^{-2}
d. 2.5×10^{15} ms^{-2}

19. Acceleration of the second particle was

a. 5×10^{15} ms^{-2} b. 4×10^{16} ms^{-2}
c. 2×10^{16} ms^{-2} d. zero

20. If the first particle is stopped for a moment and then released, the velocity of center of mass of the system just after the release will be

a. $\frac{1}{3} \times 10^{-30}$ ms^{-1}
b. $\frac{1}{3} \times 10^{3}$ ms^{-1}
c. $\frac{2}{3} \times 10^{3}$ ms^{-1}
d. none of these

21. Paths of the two particles were

a. intersecting straight lines
b. parabolic
c. circular
d. straight line w.r.t. each other

For Problems 22–23

In the circuit shown in Fig A1.86, initially the switch is opened. The switch is closed now.

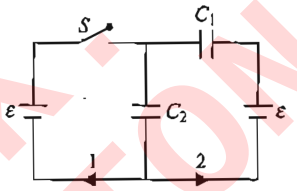


Fig. A1.86

22. The charge that will flow in direction '1' is

a. $-\frac{C_2^2\varepsilon_0}{C_1+C_2}$ b. $-\left(\frac{C_1C_2}{C_1+C_2}\right)\varepsilon_0$
c. $-\frac{C_1^2\varepsilon_0}{C_1+C_2}$ d. $C_2\varepsilon_0$

23. The charge that will flow in direction '2' is

a. $-\frac{C_2^2\varepsilon_0}{C_1+C_2}$ b. $\left(\frac{C_1C_2}{C_1+C_2}\right)\varepsilon_0$
c. $\frac{C_1^2\varepsilon_0}{C_1+C_2}$ d. $C_2\varepsilon_0$

For Problems 24–26

Capacitors $C_1 = 2\,\mu$F and $C_2 = 3\,\mu$F are connected in series to a battery of emf $\varepsilon = 120$ V, whose midpoint is earthed. The wire connecting the capacitors can be earthed through a key K. Now, key K is closed. Determine the charge flowing through the sections 1, 2 and 3 in the directions indicated in Fig. A1.87.

24. In the section 1

a. $-24\,\mu$C b. $-36\,\mu$C c. $-60\,\mu$C d. $60\,\mu$C

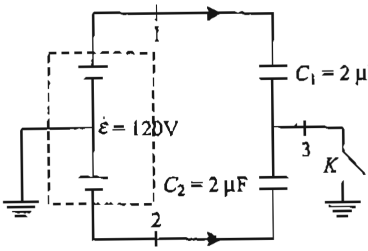


Fig. A1.87

25. In the section 2

a. $-24\,\mu$C b. $-36\,\mu$C c. $-60\,\mu$C d. $60\,\mu$C

26. In the section 3

a. $-24\,\mu$C b. $-36\,\mu$C c. $-60\,\mu$C d. $60\,\mu$C

For Problems 27–28

A researcher studying the properties of ions in the upper atmosphere wishes to construct an apparatus with the following characteristics: Using an electric field, a beam of ions, each having charge q, mass m, and initial velocity $v\hat{i}$, is turned through an angle of 90° as each ion undergoes displacement $R\hat{i} + R\hat{j}$. The ions enter a chamber as shown in Fig. A1.88 and leave through the exit port with the same speed they had when they entered the chamber. The electric field acting on the ions is to have constant magnitude.

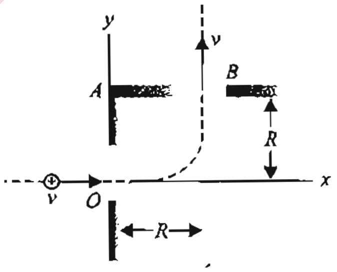


Fig. A1.88

27. Suppose the electric field is produced by two concentric cylindrical electrodes not shown in the diagram, and hence is radial. What magnitude should the field have?

a. $\frac{mv^2}{2qR}$ centered at A
b. $\frac{mv^2}{2qR}$ centered at O
c. $\frac{mv^2}{qR}$ centered at A
d. $\frac{mv^2}{qR}$ centered at O

28. If the field is produced by two flat plates and is uniform in direction, what value should the field have in this case?

a. $\frac{mv^2}{2qR}(\hat{i}+\hat{j})$
b. $\frac{mv^2}{2qR}(-\hat{i}+\hat{j})$

c. $\dfrac{2mv^2}{qR}(\hat{i}-\hat{j})$

d. $\dfrac{2mv^2}{qR}(-\hat{i}+\hat{j})$

For Problems 29–31

Two capacitors of capacities 6 and 3 μF are charged to 100 and 50 V separately and connected as shown (Fig. A1.89). Now, all the three switches S_1, S_2, and S_3 are closed.

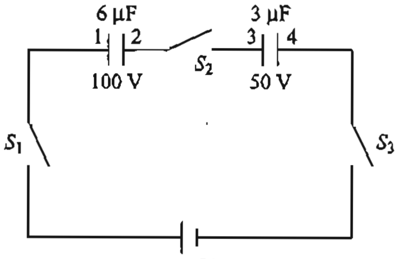


Fig. A1.89

29. Which plate(s) form an isolated system?
    - a. Plate 1 and plate 4 separately
    - b. Plate 2 and plate 3 separately
    - c. Plate 1 and plate 4 jointly
    - d. Plate 2 and plate 3 jointly

30. Charges on 6 and 3 μF capacitors in steady state will be
    - a. 400 μC, 400 μC
    - b. 700 μC, 250 μC
    - c. 800 μC, 350 μC
    - d. 300 μC, 450 μC

31. Suppose q_1, q_2, and q_3 be the magnitudes of charges flown from switches S_1, S_2, and S_3 after they are closed. Then,
    - a. $q_1 = q_3$ and $q_2 = 0$
    - b. $q_1 = q_3 = \dfrac{q_2}{2}$
    - c. $q_1 = q_3 = 3q_2$
    - d. $q_1 = q_2 = q_3$

For Problems 32–35

A and *B* are two capacitors having air as the dielectric medium between the plates. They have the same separation between the plates but the area of plates of *A* is twice that of *B*. *A* and *B* are connected in series with a 12 V dc supply as shown in Fig. A1.90.

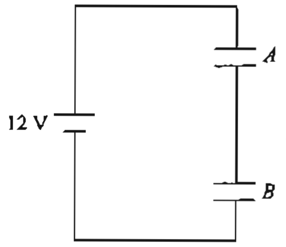


Fig. A1.90

Without removing the battery, a dielectric material ($K = 6$) is uniformly filled between the plates of capacitor *B*. The dimensions of dielectric are just enough to completely fill the space between the plates of *B*. As a result, electric energy stored in the two capacitors changes by 1.2×10^{-4} J. The capacitors are now disconnected from the battery and also from each other and the dielectric ($K = 6$) that had been filled between the plates of *B* is also removed.

Consider now the following two situations:

i. The capacitors are combined such that positive plate of *A* is connected to negative plate of *B* and vice versa.

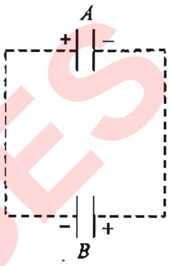


Fig. A1.91

ii. The capacitors are combined such that positive plate of *A* is connected to the positive plate of *B* and negative plate of *A* is connected to negative plate of *B*.

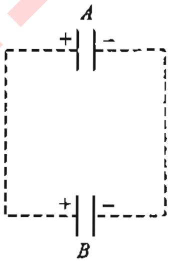


Fig. A1.92

Now answer the following questions

32. Initially, *A* and *B*, with air as the dielectric between the plates of both capacitors, are connected in series with 12 V supply. As described above, dielectric ($K = 6$) is then inserted between the plates of *B*. Consequently, electric energy stored in capacitor *B*
    - a. reduces by 10^{-5} J
    - b. increases by 2×10^{-5} J
    - c. reduces by 2×10^{-5} J
    - d. increases by 10^{-5} J

33. Work done in removing the dielectric from capacitor *B* is
    - a. 3.4×10^{-4} J
    - b. 2.7×10^{-4} J
    - c. 1.8×10^{-4} J
    - d. zero

34. In situation (i), final value of electric energy stored by the capacitor is
    - a. 4.3×10^{-4} J
    - b. 5.6×10^{-4} J
    - c. 3.2×10^{-4} J
    - d. zero

35. In situation (ii), final value of electric energy stored by the capacitors is

a. 4.3×10^{-4} J **b.** 7.3×10^{-4} J
c. 2.6×10^{-4} J **d.** zero

For Problems 36–38

An isolated parallel plate capacitor consists of two metal plates of area A and separation d. A slab of thickness t and dielectric constant $K = 2$ is inserted between the plates with its faces parallel to the plates and having the same surface area as that of plates as shown (Fig. A1.93)

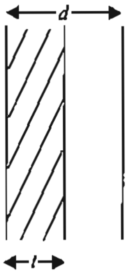


Fig. A1.93

36. The capacitance of the system is

a. $\dfrac{\varepsilon_0 A}{\left(d - \dfrac{t}{2}\right)}$ **b.** $\dfrac{\varepsilon_0 A}{\left(d + \dfrac{t}{2}\right)}$
c. $\dfrac{\varepsilon_0 A}{d - t}$ **d.** $\dfrac{\varepsilon_0 A}{d + t}$

37. For what value of t/d, will the capacitance of system be $\left(\dfrac{3}{2}\right)$ times that of the capacitance with air filing the full space?

a. $\dfrac{2}{3}$ **b.** $\dfrac{3}{2}$
c. 1 **d.** $\dfrac{1}{3}$

38. The ratio of energy in first case (with air) and second case (with dielectric) will be

a. $\dfrac{2}{3}$ **b.** $\dfrac{3}{2}$
c. 1 **d.** $\dfrac{1}{3}$

For Problems 39–43

A point charge q_1 is placed inside the Cavity 1 and another point charge q_2 is inside Cavity 2. A point charge q is placed outside the conductor.

For the situation described above, answer the following questions

39. The charge on outer surface of the conductor would be

a. $Q + q_1 + q_2$ and non-uniformly distributed.
b. $Q + q_1 + q_2$ and its uniform or non-uniform distribution depends upon locations of q_1 and q_2.
c. $Q + q_1 + q_2$ and would be distributed uniformly.
d. $Q + q_1 + q_2$ and the distribution depends upon the locations of q_1, q_2 and q.

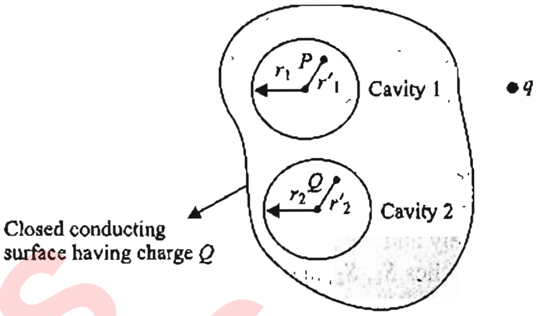


Fig. A1.94

40. If q_1 is at the center of Cavity 1, then $\vec{E}$ at a point S, at a distant r from center of Cavity 1 ($r > r_1$), due to induced charge on the surface of Cavity 1 is

a. $\dfrac{q_1}{4\pi\varepsilon_0 r^2}$ away from center of Cavity 1
b. $\dfrac{q_1}{4\pi\varepsilon_0 r_1^2}$ away from center of Cavity 1
c. zero
d. $\dfrac{q_1}{4\pi\varepsilon_0 r^2}$ towards center of Cavity 1

41. $\vec{E}$ inside the conductor at point S, distant r from point charge q, due to charge on outer surface of the conductor would be

a. $\dfrac{Q + q_1 + q_2}{4\pi\varepsilon_0 r^2}$ away from charge q
b. $\dfrac{q}{4\pi\varepsilon_0 r^2}$ towards charge q
c. zero
d. cannot be determined

42. If charge q_2 is at point Q (inside Cavity 2), then $\vec{E}$ at the center of Cavity 2 due to induced charge on the surface of Cavity 2 would be

a. $\dfrac{q_2}{4\pi\varepsilon_0 r_2'^2}$ towards q_2
b. $\dfrac{q_2}{4\pi\varepsilon_0 r_2'^2}$ away from q_2
c. zero
d. cannot be determined

43. If the potential of the conductor is V_0 and charge q_2 is placed at center of Cavity 2, then potential at point Q is

a. $\dfrac{q_2}{4\pi\varepsilon_0 r'_2} + V_0$
b. $\dfrac{q_2}{4\pi\varepsilon_0}\left(\dfrac{1}{r'_2} + \dfrac{1}{r_2}\right) + V_0$
c. $\dfrac{q_2}{4\pi\varepsilon_0}\left(\dfrac{1}{r'_2} - \dfrac{1}{r_2}\right) + V_0$
d. V_0

For Problems 44–47

Four concentric hollow spheres of radii R, $2R$, $3R$, and $4R$ are given the charges as shown in Fig. A1.95. Then, the conductors

1 and 3, 2 and 4 are connected by conducting wires (both the connections are made at the same time).

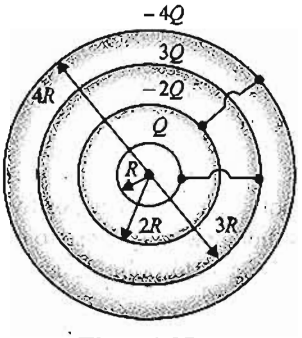


Fig. A1.95

44. The charge on inner surface of 3rd conductor is

a. $-6Q/5$ **b.** $6Q/5$
c. $-2Q$ **d.** $+2Q$

45. The charge on 4th conductor is

a. $+\frac{22\,Q}{5}$ **b.** $+\frac{11\,Q}{3}$
c. $-\frac{11\,Q}{3}$ **d.** $-\frac{22\,Q}{5}$

46. The potential of conductor 1 is

a. $\frac{3Q}{40\,\pi\varepsilon_0 R}$ **b.** $-\frac{19Q}{40\,\pi\varepsilon_0 R}$
c. $-\frac{3Q}{40\,\pi\varepsilon_0 R}$ **d.** $-\frac{19Q}{40\,\pi\varepsilon_0 R}$

47. The potential of conductor 2 is

a. $-\frac{Q}{8\pi\,\varepsilon_0 R}$ **b.** $\frac{Q}{8\pi\,\varepsilon_0 R}$
c. $\frac{Q}{32\pi\,\varepsilon_0 R}$ **d.** $-\frac{Q}{32\pi\,\varepsilon_0 R}$

For Problems 48–51

Some cell walls in the human body have a layer of negative charge on the inside surface and a layer of positive charge of equal magnitude on the outside surface. Consider one such cell having the thickness of cell wall 10^{-10} m. The charge densities on the walls are $-\sigma$ and $+\sigma$ Cm^{-2}, respectively, and the relative permittivity of the cell wall material is 5. Take volume based on above information, answer the following questions.

48. Which wall is at a higher potential?

a. Inner
b. Outer
c. Both are at the same potential
d. Cannot be determined from the given information

49. Determine the electric field intensity in between the cell walls.

a. $\frac{\sigma}{5\varepsilon_0}$ NC^{-1}
b. $\frac{\sigma}{2\,\varepsilon_0}C$ NC^{-1}
c. $\frac{\sigma}{5\,\varepsilon_0}\times 10^{-5}$ NC^{-1}
d. $\frac{\sigma}{2\,\varepsilon_0}\times 10^{-5}$ NC^{-1}

50. The potential difference between inside and outside walls of the cell is

a. $\frac{\sigma}{5\,\varepsilon_0}\times 10^{-5}$ V
b. $\frac{\sigma}{2\,\varepsilon_0}\times 10^{-5}$ V
c. $\frac{\sigma}{5\,\varepsilon_0}\times 10^{-10}$ V
d. $\frac{\sigma}{2\,\varepsilon_0}\times 10^{-10}$ V

51. The energy stored in the cell wall is

a. $\frac{\sigma^2}{5\,\varepsilon_0}\times 10^{-5}$ J **b.** $\frac{\sigma^2}{2\,\varepsilon_0}\times 10^{-15}$ J
c. $\frac{\sigma^2}{2\,\varepsilon_0}\times 10^{-16}$ J **d.** $\frac{\sigma^2}{\varepsilon_0}\times 10^{-16}$ J

For Problems 52–53

Fig. A1.96 shows a parallel plate capacitor with plate area A and plate separation d. A potential difference is being applied between the plates. The battery is then disconnected, and a dielectric slab of dielectric constant K is placed in between the plates of the capacitor as shown.

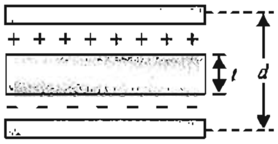


Fig. A1.96

Now, answer the following questions based on above information.

52. The electric field in the gaps between the plates and the dielectric slab will be

a. $\frac{\varepsilon_0 AV}{d}$ **b.** $\frac{V}{d}$ **c.** $\frac{KV}{d}$ **d.** $\frac{V}{d-t}$

53. The electric field in the dielectric slab is

a. $\frac{V}{Kd}$ **b.** $\frac{KV}{d}$ **c.** $\frac{V}{d}$ **d.** $\frac{KV}{t}$

For Problems 54–55

A parallel plate capacitor is connected to a battery as shown in Fig. A1.97. A thin conducting plate is inserted mid way between the two plates. Takes plate area as A.

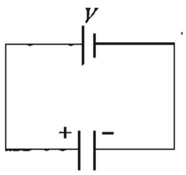


Fig. A1.97

54. What is the capacitance of the system after the plate is inserted?

a. $\frac{\varepsilon_0 A}{d}$

b. $\frac{\varepsilon_0 A}{2d}$

c. $\frac{2\varepsilon_0 A}{d}$

d. cannot say anything

55. If the thin plate and upper plate are shortened, the extra charge flown through the battery is

a. $\frac{2\varepsilon_0 AV}{d}$ b. zero

c. $\frac{\varepsilon_0 AV}{d}$ d. $\frac{\varepsilon_0 AV}{2d}$

For Problems 56–58

Two plates of a parallel plate capacitors are connected to a battery as shown in Fig. A1.98. The separation between the plates is increased from x to $x + dx$ (very slowly). Take plate area as A.

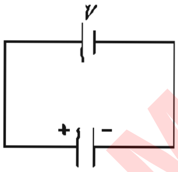


Fig. A1.98

56. What is the change in electrostatic energy stored in the capacitor?

a. $-\frac{\varepsilon_0 AV^2}{2} \times \frac{dx}{x^2}$

b. $\frac{\varepsilon_0 AV^2 dx}{2x^2}$

c. $\frac{\varepsilon_0 AV^2}{2x}$

d. remains constant

57. The charge flown out from positive terminal of battery and work done by battery is [during the process]

a. $-\frac{\varepsilon_0 AVdx}{x^2}, \frac{\varepsilon_0 AV^2 dx}{x^2}$

b. $\frac{\varepsilon_0 AVdx}{x^2}, \frac{\varepsilon_0 AV^2 dx}{x^2}$

c. $\frac{\varepsilon_0 AVdx}{x^2}, -\frac{\varepsilon_0 AV^2 dx}{x^2}$

d. $-\frac{\varepsilon_0 AVdx}{x^2}, -\frac{\varepsilon_0 AV^2 dx}{x^2}$

58. Work done by external agent is

a. $\frac{\varepsilon_0 AV^2 dx}{x^2}$

b. $-\frac{\varepsilon_0 AV^2}{2} \times \frac{dx}{x^2}$

c. $\frac{\varepsilon_0 AV^2}{2} \times \frac{dx}{x^2}$

d. cannot be calculated as force with which the plates have been pulled is not given

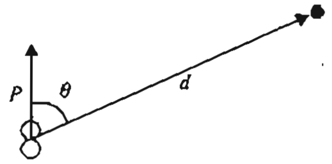


Fig. A1.99

For Problems 59–61

A simple electric dipole consists of a positive and negative charge of equal magnitude held very close to one another. The components of the electric field pointing away from a dipole has magnitude $E = \frac{2kp\cos\theta}{d^3}$, where d is the distance from the center of the dipole to the point in question, $K = 9.0 \times 10^9$ Nm^{2}C^{-2} is a universal constant and q is the magnitude of the dipole moment vector, which specifies the strength and direction of the dipole. Here, θ denotes the angle between the dipole moment vector and d, the displacement vector (from the dipole to the point in question).

A student performs an experiment to determine if a mystery object is an electric dipole. (The mystery object is only a few millimeters long.) Using a sophisticated instrument, the student measures the component of the electric field pointing away from the object, at various distances from the center of the object. By taking each measurement along an imaginary line emanating outward from the center of mystery object, he ensures that "θ" stays the same throughout the experiment. Table A1.1 shows the electric field he found at various distances.

Table A1.1

Trial	Distance (m)	Field (NC^{-1})
1	0.010	3.6×10^{-10}
2	0.020	9.0×10^{-11}
3	0.030	4.0×10^{-11}

59. From the given information, can we calculate the electrostatic force that would act on a point charge $q = 25 \times 10^{-7}$, held at the location where the student measured the electric field in trial 2?

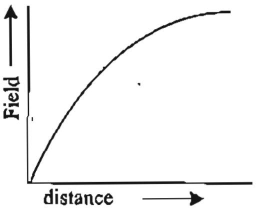
a. Yes, because we can use, $E = \frac{2kp\cos\theta}{r^3}$ with $r = 0.02$ m.

b. Yes, because we know the electric field at the relevant point.

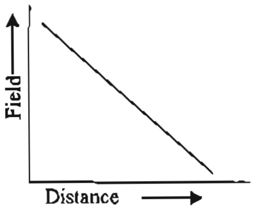
c. No, because the formula $F = \frac{kq_1q_2}{r^2}$ does not apply, and we do not know the dipole moment.

d. No, because even though $F = \frac{kq_1q_2}{r^2}$ applies, we do not know the dipole moment.

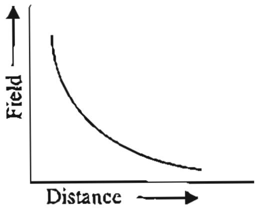
60. Which of the following graph best expresses how the electric field measured by the student varies with distance from the mystery object?



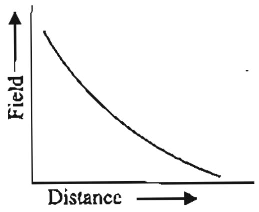
a.



b.



c.



d.

61. Consider the electric field produced by a dipole. If the dipole moment and the distance from the dipole are both doubled, while θ is kept the same, the electric field component pointing away from the dipole decreases by a factor of

a. 2 **b.** 4 **c.** 8 **d.** 1

For Problems 62–63

The electric field in a certain region of space obeys $E_y \neq 0$, $E_x = E_z = 0$ and $\frac{\partial \vec{E}}{\partial x} \neq 0$, $\frac{\partial \vec{E}}{\partial y} = \frac{\partial \vec{E}}{\partial z} = 0$

62. The net force on an electric dipole oriented parallel to the x-axis in this field is

a. directed along the x-axis
b. directed along the y-axis
c. directed along the z-axis
d. none of the above

63. The net torque on an electric dipole parallel to the x-axis in this field is

a. directed along the x-axis
b. directed along the y-axis
c. directed along the z-axis
d. none of the above

For Problems 64–65

An electric dipole of length 2 cm is placed with its axis making an angle of 60° to a uniform electric field of 10^5 NC^{-1}. If it experiences a torque of $8\sqrt{3}$ Nm, calculate the

64. magnitude of charge on the dipole.

a. 4 mC **b.** 6 mC **c.** 8 mC **d.** 12 mC

65. potential energy of the dipole.

a. –8 J **b.** 6 J **c.** –12 J **d.** 8 J

For Problems 66–67

Refer the quadrupole distribution shown in Fig. A1.100 for $r > a$ and answer the following questions.

66. Electric field at point P is

a. $\frac{3}{2}\frac{qa^2}{\pi\varepsilon_0 r^4}$ **b.** $\frac{3}{4}\frac{qa^2}{\pi\varepsilon_0 r^4}$

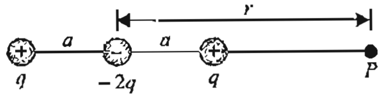


Fig. A1.100

c. $\frac{5}{2}\frac{qa^2}{\pi\varepsilon_0 r^4}$ **d.** $\frac{5}{4}\frac{qa^2}{\pi\varepsilon_0 r^4}$

67. Electric potential at point P is

a. $\frac{3}{2}\frac{q\,a^2}{\pi\varepsilon_0 r^3}$ **b.** $\frac{5}{2}\frac{qa^2}{\pi\varepsilon_0 r^3}$

c. $\frac{q\,a^2}{2\pi\varepsilon_0 r^3}$ **d.** $\frac{2q\,a^2}{\pi\varepsilon_0 r^3}$

For Problems 68–71

Fig. A1.101 shows an electric dipole in a uniform electric field with magnitude 5.0×10^5 NC^{-1} directed parallel to the plane of the figure. The charges are $\pm 1.6 \times 10^{-19}$ C; both lie in the plane and are separated by 0.125 nm $= 0.125 \times 10^{-9}$ m. (Both the charge magnitude and distance are typical of molecular quantities.) $\left(\text{see that: } \sin 37° = \frac{3}{5}, \cos 37° = \frac{4}{5}\right)$

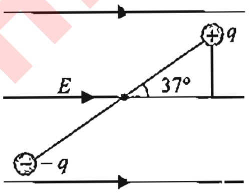


Fig. A1.101

68. Find the net force exerted by the field on the dipole.

a. zero **b.** $2\,qE$
c. qE **d.** $\sqrt{2}\,qE$

69. Find the magnitude and direction of the electric dipole moment.

a. 2×10^{-29} C m from positive to negative charge.
b. 2×10^{-29} C m from negative to positive charge.
c. 4×10^{-29} C m from positive to negative charge.
d. 4×10^{-29} C m from negative to positive charge.

70. Find the magnitude and direction of torque by us.

a. 6.0×10^{-24} N-m out of the page
b. 6.0×10^{-24} N-m into the page
c. 3.0×10^{-24} N-m out of the page
d. 3.0×10^{-24} N-m into the page

71. Find the potential energy of the system in the position shown.

a. 3×10^{-24} J **b.** 5×10^{-24} J
c. 8×10^{-24} J **d.** 4×10^{-24} J

For Problems 72–73

A positive point charge q is fixed at origin. A dipole with a dipole moment $\vec{p}$ is placed along the x-axis far away from the origin with $\vec{p}$ pointing along positive x-axis.

72. Find the kinetic energy of the dipole when it reaches a distance d from the origin.

a. $\frac{3}{4\pi\varepsilon_0}\frac{pq}{d^2}$ **b.** $\frac{3}{2\pi\varepsilon_0}\frac{pq}{d^2}$

c. $\frac{1}{2\pi\varepsilon_0}\frac{pq}{d^2}$ **d.** $\frac{1}{4\pi\varepsilon_0}\frac{pq}{d^2}$

73. Find the force experienced by the charge q at this moment.

a. $\frac{1}{4\pi\varepsilon_0}\frac{pq}{d^2}$ **b.** $\frac{1}{2\pi\varepsilon_0}\frac{pq}{d^2}$

c. $\frac{1}{2\sqrt{2}\pi\varepsilon_0}\frac{pq}{d^2}$ **d.** $\frac{\sqrt{2}}{\pi\varepsilon_0}\frac{pq}{d^2}$

(IIT-JEE, 2003)

For Problems 74–75

A large sheet carries uniform surface charge density σ. A rod of length $2l$ has a linear charge density l on one half and – l on the other half. The rod is hinged at mid point O and makes an angle θ with the normal to the sheet.

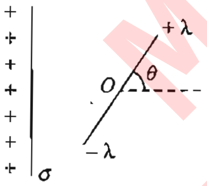


Fig. A1.102

74. What is the net force experienced by the rod?

a. $\frac{\sigma\lambda l}{2\varepsilon_0}$ **b.** $\frac{\sigma\lambda l}{\varepsilon_0}$

c. zero **d.** none of these

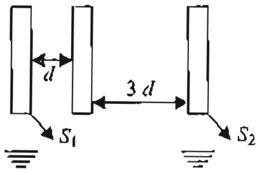
75. The torque experienced by the rod is

a. $\frac{\sigma\lambda l^2}{2\varepsilon_0}\cos\theta$ **b.** $\frac{\sigma\lambda l^2\sin\theta}{2\varepsilon_0}$

c. $\frac{\sigma\lambda l}{2\varepsilon_0}\cos^2\theta$ **d.** $\frac{\sigma\lambda l\sin^2\theta}{2\varepsilon_0}$

For Problems 76–78

Three metallic plates out of which middle is given charge Q as shown in Figure given below. The outer plates can be earthed with the help of switches S_1 and S_2. The area of each plates is same.



Answer the following question based on the following passage.

76. The charge appearing on the outer surface of extreme left plate is

a. $-(Q/2)$ **b.** $(Q/2)$ **c.** Q **d.** $-Q$

77. The charge that will flow to earth when only switch S_1 is connected to earth is

a. $-(Q/2)$ **b.** $(Q/2)$ **c.** Q **d.** $-Q$

78. The charge that will flow to earth through S_2 when both the switches S_1 and S_2 are grounded simultaneously is

a. Q **b.** $-Q$ **c.** $\frac{3}{4}Q$ **d.** $\frac{Q}{4}$

For Problems 79–81

Three concentric spherical conductors A, B and C of radii R, $2R$ and $4R$ respectively. A and C is shorted and B is uniformly charged (charge $+Q$).

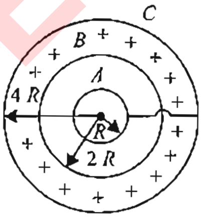


Fig. A1.103

79. Charge on conductor A is

a. $Q/3$ **b.** $-Q/3$ **c.** $2Q/3$ **d.** $-\frac{2Q}{3}$

80. Potential at A is

a. $\frac{Q}{4\pi\varepsilon_0 R}$ **b.** $\frac{Q}{16\pi\varepsilon_0 R}$

c. $\frac{Q}{20\pi\varepsilon_0 R}$ **d.** $\frac{5Q}{48\pi\varepsilon_0 R}$

81. Potential at B is

a. $\frac{Q}{4\pi\varepsilon_0 R}$ **b.** $\frac{Q}{16\pi\varepsilon_0 R}$

c. $\frac{Q}{48\pi\varepsilon_0 R}$ **d.** $\frac{5Q}{48\pi\varepsilon_0 R}$

For Problems 82–84

Two capacitors of capacity 6 μF and 3 μF are charged to 100 V and 50 V separately and connected as shown in Fig. A1.104. Now all the three switches S_1, S_2 and S_3 are closed.

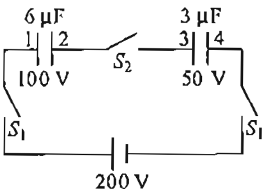


Fig. A1.104

82. Which plate(s) form an isolated system

a. plate 1 and plate 4 separately.

b. plate 2 and plate 3 separately.

c. plate 1 and plate 4 jointly.

d. plate 2 and plate 3 jointly.

83. Charges on 6 μF and 3 μF capacitors in steady state will be

a. 400 μC, 400 μC **b.** 700 μC, 250 μC
c. 800 μC, 350 μC **d.** 300 μC, 450 μC

84. Suppose q_1, q_2 and q_3 be the magnitudes of charges flown from switches S_1, S_2 and S_3 after they are closed. Then

a. $q_1 = q_3$ and $q_2 = 0$
b. $q_1 = q_3 = \frac{q_2}{2}$
c. $q_1 = q_3 = 3q_2$
d. $q_1 = q_2 = q_3$

For Problems 85–87

Most capacitors have a non-conducting material between their conducting plates. Placing a solid dielectric between the plates of capacitor serves following three functions:

i. It solves the problems of maintaining two large metal sheets at a very small separation without actual contact.
ii. It increases the maximum possible potential difference which can be applied between the plates of capacitor without the dielectric breakdown.
iii. It increases the capacity of the capacitor. When a dielectric material is inserted between the plates (keeping the charge to be constant) the electric field and hence potential difference decreases by a factor K. Electric field decreases because an induced charge of the opposite sign appears on each surface of dielectric.

85. Two metal plates having charges Q and $-Q$ face each other at some separation and are dipped into an oil-tank. If all oil is pumped out, the electric field between the plates will be

a. increased **b.** decreased
c. remain same **d.** become zero

86. A dielectric slab is inserted between the plates of an isolated charged capacitor. Which of the following quantities will remain same.

a. the electric field in the capacitor
b. the charge on capacitor
c. the potential difference between the plates
d. stored energy in the capacitor

87. In a parallel plate capacitor, the region between the plates is filled by a dielectric slab. The capacitor is connected to a cell and the slab is taken out then,

a. some charge is drawn from cell.
b. some charge is returned to the cell.
c. the potential difference across the capacitor is reduced.
d. no work is done by an external agent in taking the slab out.

For Problems 88–90

In Fig. A1.105 $m_A = m_B = 1$ kg

Block A is neutral while block B carries charge -1 C. Sizes of A and B are negligible. Block B is released from rest at a distance 1.8 m from A. Initially spring is neither compressed or stretched.

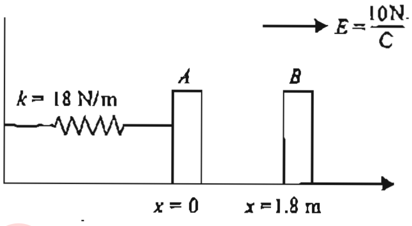


Fig. A1.105

88. In collision, between A and B is perfectly inelastic, what is the velocity of combined mass just after collision.

a. 6 m/s **b.** 3 m/s **c.** 9 m/s **d.** 12 m/s

89. Equilibrium position of the combined mass is at $x =$ ______ m.

a. $-\frac{2}{9}$ **b.** $-\frac{1}{3}$ **c.** $-\frac{5}{9}$ **d.** $-\frac{7}{9}$

90. The amplitude of oscillation of the combined mass will be

a. $\frac{2}{3}m$ **b.** $\frac{\sqrt{124}}{3}m$
c. $\frac{\sqrt{72}}{9}m$ **d.** $\frac{\sqrt{106}}{9}m$

Matching Column Type

Solutions on page A1.73

1. A conducting shell of inner radius R_1 and outer radius R_2 is given a charge $+Q$. A point charge q_1 is placed inside the shell and q_2 is placed outside the shell. Then, for various locations of q_1 and q_2 match the entries of Column I with the entries of Column II.

Column I	Column II
i. If q_1 is at center and $q_2 = 0$, then $\vec{E}$ at center of shell due to charge on outer surface of shell is	**a.** $\frac{q_1}{4\pi\varepsilon_0 r^2}$
ii. If q_1 is not at center and q_2 is at distance r from the center, then $\vec{E}$ at the inner surface of shell (at a point closest to q_2) due to charge on outer surface of shell is	**b.** $\frac{q_2}{4\pi\varepsilon_0(r-R_1)^2}$
iii. If q_1 is at center and q_2 is at distance r from the center, then $\vec{E}$ at a point distant r_2 (> r) from the center of shell due to outer surface charge is	**c.** zero
iv. If q_1 is not at center and $q_2 = 0$, then $\vec{E}$ at point P (P is at a distance r from q_1) due to charge of inner surface of shell is	**d.** cannot be determined

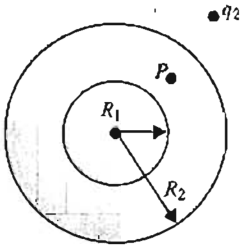


Fig. A1.106

2. For the situation shown in Fig. A1.107, match the entries of Column I with entries of Column II.

Column I	Column II
i. If we displace the inside charge,	**a.** distribution of charge on inner surface of conductor is uniform.
ii. If we displace the outside charge keeping the inside charge at center,	**b.** distribution of charge on inner surface of conductor is non-uniform.
iii. If both the charges are displaced,	**c.** distribution of charge on outer surface of conductor is uniform.
iv. If outside charge is not present and inside charge is at center,	**d.** distribution of charge on outer surface of conductor is non-uniform.

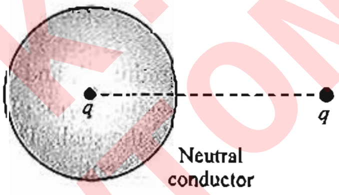


Fig. A1.107

3.

Column I	Column II
i. When a dielectric slab is gradually inserted between the plates of an isolated parallel plate capacitor,	**a.** the electric potential energy of the system decreases.
ii. When a dielectric slab is gradually inserted between the plates of a parallel plate capacitor and its potential is kept constant,	**b.** work done by external agent is positive.
iii. When the plates of a parallel plate capacitor are pulled apart keeping its potential constant,	**c.** work done by battery is positive.
iv. When the plates of a parallel plate capacitor are pulled apart, keeping its charge constant,	**d.** work done by external agent is negative.

4. A capacitor of capacitance C is charged to a potential V. Now, it is connected to a battery of emf E as shown in Fig. A1.108. Based on this information, match the entries of Column I with entries of Column II in the following Table.

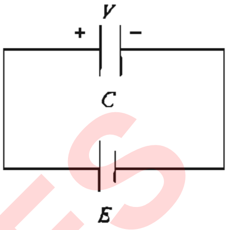


Fig. A1.108

Column I	Column II
i. If $V = E$, then	**a.** charge flows in the circuit.
ii. If $V > E$, then	**b.** no charge flows in the circuit.
iii. If $V < E$, then	**c.** non-zero thermal energy will be dissipated in the circuit.
iv. If the plates of capacitor are shorted, then	**d.** outer surfaces of the plates of capacitor have zero charge.

5. In Fig. A1.109, the separation between the plates of C_1 is slowly increased to double of its initial value. Now, match the entries in columns I and II.

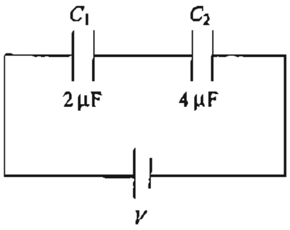


Fig. A1.109

Column I	Column II
i. The potential difference across C_1	**a.** increases
ii. The potential difference across C_2	**b.** decreases
iii. The energy stored in C_1	**c.** increases by a factor of 6/5
iv. The energy stored in C_2	**d.** decreases by a factor of 18/25

6. Two point charges of 10^{-8} C and -10^{-8} C are placed 0.1 m apart (Fig. A1.110). Match the electric field intensity for points in Column I with corresponding values in Column II.

Column I	Column II
i. Total electric field intensity at point P	**a.** 3.2×10^4 NC^{-1} along AQ
ii. Total electric field intensity at point Q	**b.** 7.2×10^4 NC^{-1} along PB
iii. Total electric field intensity at point R	**c.** 4000 NC^{-1}
iv. Electric field intensity at point Q due to charge at B.	**d.** 9×10^3 NC^{-1}

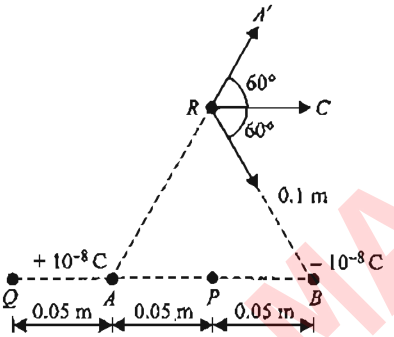


Fig. A1.110

7.

Column I	Column II
i. Spherical charged conductor	**a.** At the surface, electric field is continuous and maximum.
ii. Spherical non-conductor having uniform volume distribution of charge	**b.** At the surface, electric field is discontinuous.
iii. Charged ring	**c.** Electric field is uniform.
iv. Infinite sheet of charge	**d.** At the center, electric field is zero.

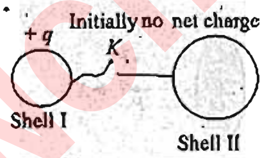
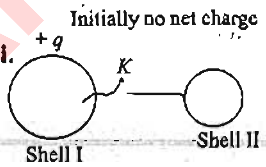
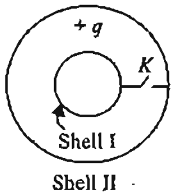
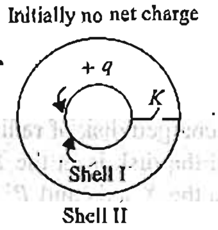
8.

Column I	Column II
i. Infinite sheet of charge	**a.** uniform non-zero electric field intensity
ii. At the center of uniformly charged solid sphere	**b.** uniform decreasing potential
iii. Due to infinite line charge assuming potential to be zero at infinity	**c.** infinite potential everywhere
iv. Inside a charged conducting sphere	**d.** maximum magnitude of potential

9. A solid conducting sphere of radius a is placed inside a conducting shell of radius b so that both are concentric. Now, the shell is given a charge Q. Match the following:

Column I	Column II
i. Charge appearing on the inner sphere	**a.** zero
ii. Charge appearing on the inner sphere after it is earthed	**b.** Q
iii. Electric field intensity (E) inside the inner sphere (before earthing)	**c.** Qa/b
iv. Electric field intensity (E) inside the inner sphere after it is earthed	**d.** $\frac{1}{4\pi\varepsilon_0}\frac{Q}{ab}$

10. In the following table, Column I gives certain situations involving two thin conducting shells connected by a conducting wire via a key K. In all situations, one sphere has net charge $+q$ and other sphere has no net charge. After the key K is pressed, Column II gives some resulting effect. Match the figures in Column I with the statements in Column II.

Column I	Column II
i. 	**a.** Charge flows through the connecting wire.
ii. 	**b.** Potential energy of system of spheres decreases.
iii. 	**c.** No heat is produced.
iv. 	**d.** Sphere I has no charge after equilibrium is reached.

11. Capacitors with capacitances C, $2C$, $3C$ and $4C$ are charged to the voltages V, $2V$, $3V$, and $4V$ respectively. Circuit is closed. Now match the following. (Assume voltages across capacitors in equilibrium are V_1, V_2, V_3 and V_4, respectively.)

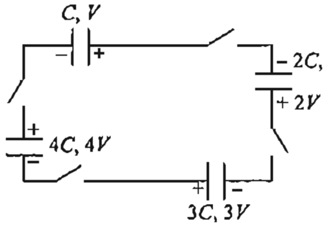


Fig. A1.111

Column I	**Column II**
i. V_1	**a.** $\frac{2}{5}$ volt
ii. V_2	**b.** $\frac{7}{5}$ volt
iii. V_3	**c.** $\frac{19}{5}$ volt
iv. V_4	**d.** $\frac{14}{5}$ volt

12. Fig. A1.112(a) A shows a uniformly charged ring of radius R. Its axis is along the X-axis and the ring is in the YZ plane. Point P can be anywhere on the X-axis and P' in the XY plane.

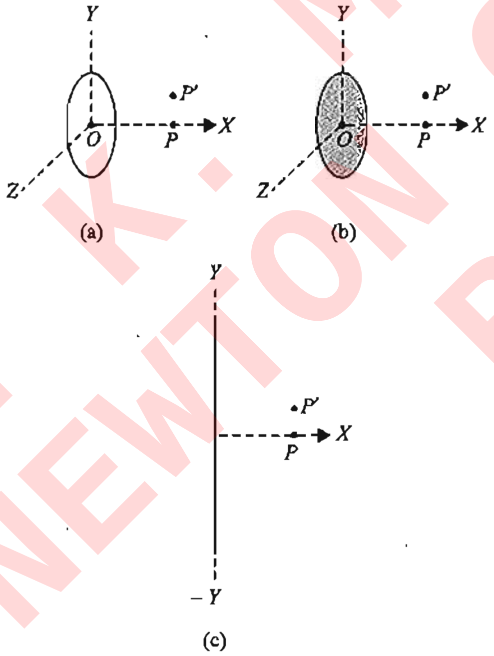


Fig. A1.112

Fig. A1.112(b) shows a uniformly charged disk of radius R. Its axis is along the X-axis and the disk is in the YZ plane. Point P can be anywhere on the X-axis and P' in the XY plane.

Fig. A1.112(c) shows an infinite line charged uniformly, placed along y-axis. P is a point on X-axis and P' in the XY plane.

E_x, E_y, and E_z are the components of electric field along x-, y-, and z-axis and E_p and $E_{p'}$ are net electric fields at P, and P', respectively.

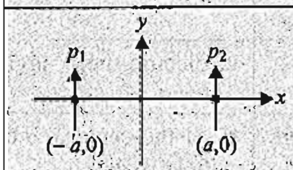
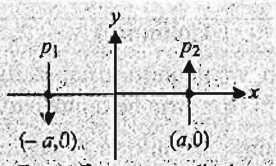
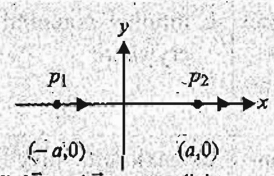
Now, match the following columns.

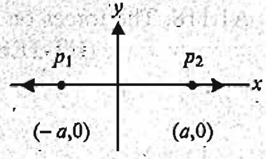
Column I	**Column II**
i. $E_x = 0$, $E_y = 0$, $E_z = 0$	**a.** Point P' in figure (c)
ii. $E_x \neq 0$, $E_y = 0$, $E_z = 0$	**b.** Point P' in figure (b)
iii. $E_x \neq 0$, $E_y \neq 0$, $E_z = 0$	**c.** Point P in figure (c)
iv. $E_P = E_{P'}$	**d.** Point P in figures (a) and (b).

13. An electric dipole is placed in a uniform external electric field. θ is the angle between the dipole moment and the field direction. In general, the dipole rotates under a torque. With reference to the behaviour of the dipole in an electric field, match Column I with Column II.

Column I	**Column II**
i. Potential energy of the dipole is maximum	**a.** conserved
ii. Angular acceleration of the dipole is maximum	**b.** not conserved
iii. Angular momentum of the dipole	**c.** $\theta = 180°$
iv. Kinetic energy of the dipole	**d.** $\theta = 90°$

14. In each situation of Column I, two electric dipoles having dipole moments $\vec{p}_1$ and $\vec{p}_2$ of same magnitude (that is $p_1 = p_2$) are placed on x-axis symmetrically about origin in different orientations as shown. In Column II, certain inferences are drawn for these two dipoles. Then, match the different orientations of dipoles in Column I with the corresponding results in Column-II.

Column I	**Column II**
 **i.** ($\vec{p}_1$ and $\vec{p}_2$ are perpendicular to x-axis as shown)	**a.** The torque on one dipole due to other is zero.
 **ii.** ($\vec{p}_1$ and $\vec{p}_2$ are perpendicular to x-axis as shown)	**b.** The potential energy of one dipole in electric field of other dipole is negative.
 **iii.** ($\vec{p}_1$ and $\vec{p}_2$ are parallel to x-axis as shown)	**c.** There is one straight line in x–y plane (not at infinity) which is equipotential.

Column I	**Column II**
 **iv.** ($\vec{p}_1$ and $\vec{p}_2$ are parallel to x-axis as shown)	**d.** Electric field at origin is zero.

15.

Column I	**Column II**
i. $\lvert \vec{E}_{\text{axial}} \rvert$ for a dipole	**a.** $\dfrac{\lvert \vec{P} \rvert}{4\pi\varepsilon_0 r^3}$
ii. $\lvert \vec{E}_{\text{equipotential}} \rvert$ for a dipole	**b.** $\dfrac{2\lvert \vec{P} \rvert}{4\pi\varepsilon_0 r^3}$
iii. When dipole is in uniform electric field when $\theta = 0°$	**c.** $F_{\text{net}} = 0$, $\tau_{\text{net}} = 0$
iv. When dipole is in non-uniform electric field when $\theta = 0°$	**d.** $F_{\text{net}} \neq 0$, $\tau_{\text{net}} = 0$

16. A certain electric field is given as

$$E = -[(2xy + z^2)\hat{i} + (2yz + x^2)\hat{j} + (2zx + y^2)\hat{k}]$$

Column I	**Column II**
i. Work done by electric field in taking a unit charge from (0, 0, 0) to (3, 4, 0) along a straight line	**a.** – 36 units
ii. Work done by electric field in taking a charge from (3, 4, 0) to (0, 0, 0) along a straight line	**b.** + 36 units
iii. Work done by external agent in taking a charge from (3, 4, 0) to (6, 8, 0) without change in K.E.	**c.** zero units
iv. Net charge enclosed in a sphere of radius 5 units centred at the origin.	**d.** +252 units

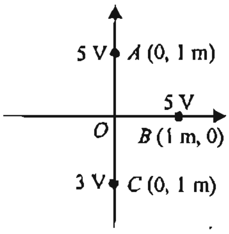


Fig. A1.113

17. An uniform electric field exists in straight line in the X-Y plane. The potential at different point in region are shown in the figure. Now match the following list if charge particles P (mass = 10^{-6} kg). Charge = $-2\sqrt{2}$ μC are released from a origin.

Column I	**Column II**
i. Co-ordinate of position of particle P at time $t = 2\sqrt{2}$ sec.	**a.** $(4\sqrt{2}m, 4\sqrt{2}m)$
ii. Co-ordinate of position of particle θ at time $t = 2$ sec.	**b.** $(-4\sqrt{2}m, -4\sqrt{2}m)$
iii. Distance travelled by particle Q in 2 sec is	**c.** 8 m
iv. Distance travelled by particle P in 2 sec is	**d.** 4 m

Archives

Solutions on page A1.76

Fill in the Blanks Type

1. Five identical capacitor plates, each of area A, are arranged such that adjacent plates are a distance d apart. The plates are connected to a source of e.m.f. V as shown in Fig. A1.114. **(IIT-JEE, 1984)**

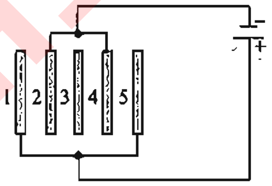


Fig. A1.114

The charge on plate 1 is ________ and that on plate 4 is ________.

2. Fig. A1.115 shows line of constant potential in a region in which an electric field is present. The values of the potential are written in brackets. Of the points A, B, and C, the magnitude of the electric field is greater at the point ________. **(IIT-JEE, 1984)**

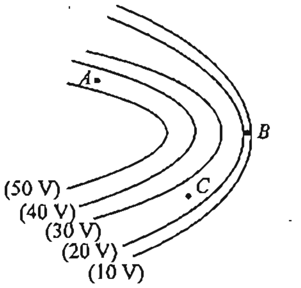


Fig. A1.115

3. Two small balls having equal positive charges Q (coulomb) on each are suspended by two insulating strings of equal length L (metre) from a hook fixed to a stand. The whole setup is taken in a satellite into space where there is no gravity (state of weightlessness). The angle between the two strings is ________ and the tension in each string is ________ newton. **(IIT-JEE, 1986)**

4. Two parallel plate capacitors of capacitances C and $2C$ are connected in parallel and charged to potential difference V. The battery is then disconnected and the region between the plates of the capacitor C is completely filled with a material of dielectric constant K. The potential difference across the capacitors now becomes ________. **(IIT-JEE, 1988)**

5. A point charge q moves from point P to point S along the path $PQRS$ in a uniform electric field E pointing parallel to the positive direction of the X-axis (Fig. A1.116). The coordinates of the points P, Q, R, and S are $(a, b, 0)$, $(2a, 0, 0)$ $(a, -b, 0)$, and $(0, 0, 0)$, respectively. The work done by the field in the above process is given by the expression ________. **(IIT-JEE, 1989)**

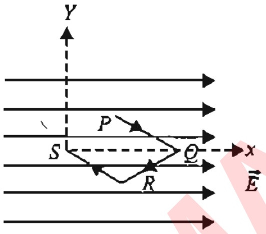


Fig. A1.116

6. The electric potential V at any point x, y, z (all in metres) in space is given by $V = 4x^2$ V. The electric field at the point (1 m, 0, 2 m) is ________Vm^{-1}. **(IIT-JEE, 1992)**

7. Five point charges, each of value $+q$ coulomb, are placed on five vertices of a regular hexagon of side L metres. The magnitude of the force on the point charge of value $-q$ coulomb. placed at the center of the hexagon is ________ newton.

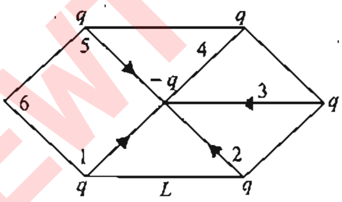


Fig. A1.117

(IIT-JEE, 1992)

True or False

1. The work done in carrying a point charge from one point to another in an electrostatic field depends on the path along which the point charge is carried. **(IIT-JEE, 1981)**
2. Two identical metallic spheres of exactly equal masses are taken. One is given a positive charge Q coulombs and the other an equal negative charge. Their masses after charging are different. **(IIT-JEE, 1983)**
3. A small metal ball is suspended in a uniform electric field with the help of an insulated thread. If a high-energy X-ray beam falls on the ball, the ball will be deflected in the direction of the field. **(IIT-JEE, 1983)**
4. Two protons A and B are placed in between the two plates of a parallel plate capacitor charged to a potential difference V as shown in Fig. A1.118. The forces on the two protons are identical. **(IIT-JEE, 1986)**

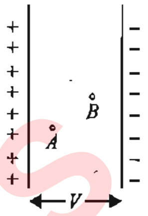


Fig. A1.118

5. A ring of radius R carries a uniformly distributed charge $+Q$. A point charge $-q$ is placed on the axis of the ring at a distance 2R from the center of the ring and released from rest. The particle executes a simple harmonic motion along the axis of the ring. **(IIT-JEE, 1988)**

Single Correct Answers Type

1. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. The potential at the center of the sphere is

- **a.** zero
- **b.** 10 V
- **c.** same as at a point 5 cm away from the surface
- **d.** same as at a point 25 cm away from the surface

(IIT-JEE, 1983)

2. Two point charges $+q$ and $-q$ are held fixed at $(-d, 0)$ and $(d, 0)$, respectively, of a x–y coordinate system. Then,

- **a.** electric field E at all points on the x-axis has the same direction.
- **b.** electric field at all points on y-axis is along x-axis.
- **c.** work has to be done in bringing a test charge from ∞ to the origin.
- **d.** the dipole moment is $2qd$ along the x-axis.

(IIT-JEE, 1995)

3. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to a potential difference $2V$. The charging battery is now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is

a. zero **b.** $\frac{3}{2}CV^2$

c. $\frac{25}{6}CV^2$ **d.** $\frac{9}{2}CV^2$

(IIT-JEE, 1995)

4. An electron of mass m_e, initially at rest, moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p, also initially at rest, takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio t_2/t_1 is nearly equal to

a. 1 **b.** $(m_p/m_e)^{1/2}$

c. $(m_e/m_p)^{1/2}$ **d.** 1836

(IIT-JEE, 1997)

5. A non-conducting ring of radius 0.5 m carries a total charge of 1.11×10^{-10} C distributed non-uniformly on its circumference producing an electric field E everywhere in space. The value of the integral $\int_{l=\infty}^{l=0} -\vec{E}\cdot d\vec{l}$ ($l = 0$ being center of the ring) in volts is

a. +2 **b.** −1

c. −2 **d.** zero

(IIT-JEE, 1997)

6. Two identical metal plates are given positive charges Q_1 and $Q_2 (< Q_1)$, respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is

a. $(Q_1 + Q_2)/(2C)$

b. $(Q_1 + Q_2)/C$

c. $(Q_1 - Q_2)/C$

d. $(Q_1 - Q_2)/(2C)$

(IIT-JEE, 1999)

7. For the circuit shown in Fig. A1.119, which of the following statements is true?

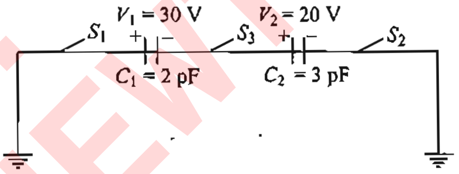


Fig. A1.119

a. With S_1 closed, $V_1 = 15$ V, $V_2 = 20$ V.

b. With S_3 closed, $V_1 = V_2 = 25$ V.

c. With S_1 and S_2 closed, $V_1 = V_2 = 0$.

d. With S_1 and S_2 closed, $V_1 = 30$ V, $V_2 = 20$ V.

(IIT-JEE, 1999)

8. Three charges Q, $+q$, and $+q$ are placed at the vertices of a right angled isosceles triangle as shown (Fig. A1.120). The net electrostatic energy of the configuration is zero if Q is equal to

a. $\dfrac{-q}{1+\sqrt{2}}$ **b.** $\dfrac{-2q}{2+\sqrt{2}}$

c. $-2q$ **d.** $+q$

(IIT-JEE, 2000)

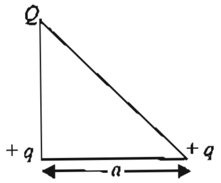


Fig. A1.120

9. A parallel plate capacitor of area A, plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants k_1, k_2, and k_3 as shown (Fig. A1.121). If a single dielectric material is to be used to have the same capacitance C in this capacitor, then its dielectric constant k is given by

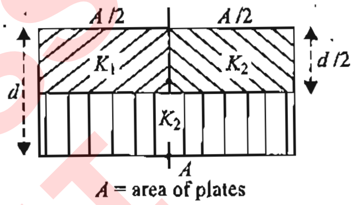


Fig. A1.121

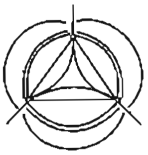
a. $\dfrac{1}{k} = \dfrac{1}{k_1} + \dfrac{1}{k_2} + \dfrac{1}{2k_3}$

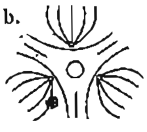
b. $\dfrac{1}{k} = \dfrac{1}{k_1 + k_2} + \dfrac{1}{2k_3}$

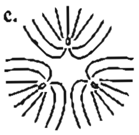
c. $k = \dfrac{k_1 k_2}{k_1 + k_2} + 2k_3$

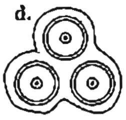
d. $k = k_1 + k_2 + 2k_3$ **(IIT-JEE, 2000)**

10. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in

a.


b.


c.


d.


(IIT-JEE, 2001)

11. Consider the situation shown in Fig. A1.122. The capacitor A has a charge q on it whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is **(IIT-JEE, 2001)**

a. zero **b.** $q/2$

c. q **d.** $2q$

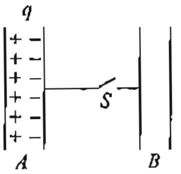


Fig. A1.122

12. A uniform electric field pointing in positive x-direction exists in a region. Let A be the origin, B be the point on the x-axis at $x = +1$ cm and C be the point on the y-axis at $y = +1$ cm. Then, the potentials at the points A, B and C satisfy **(IIT-JEE, 2001)**

a. $V_A < V_B$ **b.** $V_A > V_B$
c. $V_A < V_C$ **d.** $V_A > V_C$

13. Two equal point charges are fixed at $x = -a$ and $x = +a$ on the x-axis. Another point charge Q is placed at the origin. The change in the electrical potential energy of Q, when it is displaced by a small distance x along the x-axis, is approximately proportional to **(IIT-JEE, 2002)**

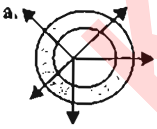
a. x **b.** x^2 **c.** x^3 **d.** $1/x$

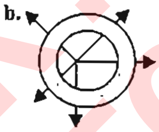
14. Two identical capacitors have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is

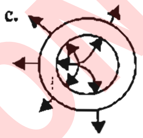
a. $\frac{1}{4}C(V_1^2 - V_2^2)$ **b.** $\frac{1}{4}C(V_1^2 + V_2^2)$
c. $\frac{1}{4}C(V_1 - V_2)^2$ **d.** $\frac{1}{4}C(V_1 + V_2)^2$

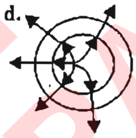
(IIT-JEE, 2002)

15. A metallic shell has a point charge 'q' kept inside its cavity. Which one of the following diagrams correctly represents the electric lines of force? **(IIT-JEE, 2003)**









16. Six charges of equal magnitude, 3 positive and 3 negative, are to be placed on $PQRSTU$ corners of a regular hexagon, such that field at the center is double that of what it would have been if only one +ve charge is placed at R.

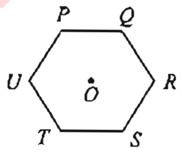


Fig. A1.123

a. +, +, +, −, −, − **b.** −, +, +, +, −, −
c. −, +, +, −, +, − **d.** +, −, +, −, +, −

(IIT-JEE, 2004)

17. A Gaussian surface in Fig. A1.124 is shown by dotted line. The electric field on the surface will be

a. due to q_1 and q_2 only
b. due to q_2 only
c. zero
d. due to all **(IIT-JEE, 2004)**

18. Three infinitely long charge sheets are placed as shown in Fig. A1.125. The electric field at point P is

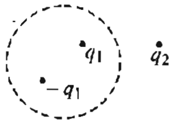


Fig. A1.124

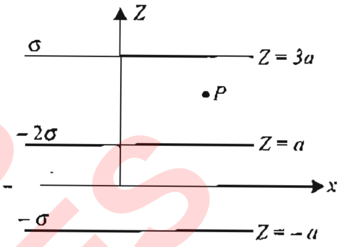


Fig. A1.125

a. $\frac{2\sigma}{\varepsilon_0}\hat{k}$ **b.** $\frac{4\sigma}{\varepsilon_0}\hat{k}$
c. $-\frac{2\sigma}{\varepsilon_0}\hat{k}$ **d.** $-\frac{4\sigma}{\varepsilon_0}\hat{k}$

(IIT-JEE, 2005)

19. A long hollow conducting cylinder is kept coaxially inside another long hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral. Then

a. a potential difference appears between the two cylinders when a charge density is given to the inner cylinder.
b. a potential difference appears between the two cylinders when a charge density is given to the outer cylinder.
c. no potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders.
d. no potential difference appears between the two cylinders when same charge density is given to both the cylinders. **(IIT-JEE, 2007)**

20. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then **(IIT-JEE, 2007)**

a. negative and distributed uniformly over the surface of the sphere
b. negative and appears only at the point on the sphere closest to the point charge
c. negative and distributed non-uniformly over the entire surface of the sphere
d. zero

21. A spherical portion has been removed from a solid sphere having a charge distributed uniformly in its volume as shown in Fig. A1.126. The electric field inside the emptied space is **(IIT-JEE, 2007)**

a. zero everywhere
b. non-zero and uniform
c. non-uniform
d. zero only at its center

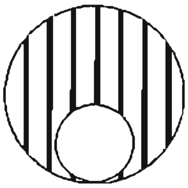


Fig. A1.126

22. Positive and negative point charges of equal magnitude are kept at $(0, 0, a/2)$ and $(0, 0, -a/2)$, respectively. The work done by the electric field when another positive point charge is moved from $(-a, 0, 0)$ to $(0, a, 0)$ is

- **a.** positive
- **b.** negative
- **c.** zero
- **d.** depends on the path connecting the initial and final positions **(IIT-JEE, 2007)**

23. Consider a system of three charges $\frac{q}{3}$, $\frac{q}{3}$ and $-\frac{2q}{3}$ placed at points A, B, and C, respectively, as shown in Fig. A1.127. Take O to be the center of the circle of radius R and angle $CAB = 60°$. Then **(IIT-JEE, 2008)**

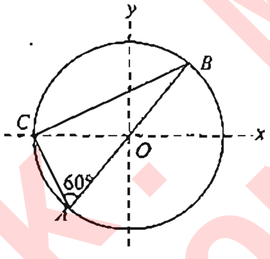


Fig. A1.127

- **a.** the electric field at point O is $\frac{q}{8\pi\varepsilon_0 R^2}$ directed along the negative x-axis.
- **b.** the potential energy of the system is zero.
- **c.** the magnitude of the force between the charges at C and B is $\frac{q^2}{54\pi\varepsilon_0 R^2}$.
- **d.** the potential at point O is $\frac{q}{12\pi\varepsilon_0 R}$.

24. A disk of radius $a/4$ having a uniformly distributed charge $6C$ is placed in the x-y plane with its centre at $(-a/2, 0, 0)$. A rod of length a carrying a uniformly distributed charge $8C$ is placed on the x-axis from $x = a/4$ to $x = 5a/4$. Two point charges $-7C$ and $3C$ are placed at $(a/4, -a/4, 0)$ and $(-3a/4, 3a/4, 0)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm a/2$, $y = \pm a/2$, $z = \pm a/2$. The electric flux through this cubical surface is **(IIT-JEE, 2009)**

- **a.** $\frac{-2C}{\varepsilon_0}$
- **b.** $\frac{2C}{\varepsilon_0}$
- **c.** $\frac{10C}{\varepsilon_0}$
- **d.** $\frac{12C}{\varepsilon_0}$

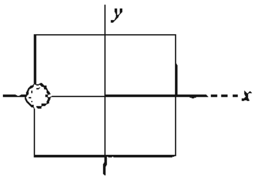


Fig. A1.128

25. Three concentric metallic spherical shells of radii R, $2R$, $3R$, are given charges Q_1, Q_2, Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$ is **(IIT-JEE, 2009)**

- **a.** 1:2:3
- **b.** 1:3:5
- **c.** 1:4:9
- **d.** 1:8:18

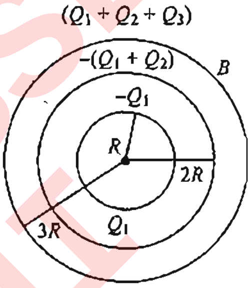


Fig. A1.129

Assertion-Reasoning Type

- **a.** Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
- **b.** Statement I is True, Statement II is True; Statement II is NOT a correct explanation for Statement I.
- **c.** Statement I is True, Statement II is False.
- **d.** Statement I is False, Statement II is True.

Statement I: For practical purposes, the earth is used as a reference at zero potential in electrical circuits.
Statement II: The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by $\frac{Q}{4\pi\varepsilon_0 R}$.

Comprehension Type

For Problems 1–3

The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density $\rho(r)$ [charge per unit volume] is dependent only on the radial distance r from the center of the nucleus as shown in Fig. A1.130. The electric field is only along the radial direction.

1. The electric field at $r = R$ is

- **a.** independent of a
- **b.** directly proportional to a
- **c.** directly proportional to a^2
- **d.** inversely proportional to a

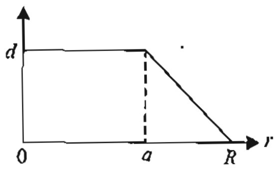


Fig. A1.130

2. For $a = 0$, the value of d (maximum value of ρ as shown in the Fig. A1.130) is

a. $\dfrac{3Ze}{4\pi R^3}$ **b.** $\dfrac{3Ze}{\pi R^3}$

c. $\dfrac{4Ze}{3\pi R^3}$ **d.** $\dfrac{Ze}{3\pi R^3}$

3. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies

a. $a = 0$ **b.** $a = \dfrac{R}{2}$

c. $a = R$ **d.** $a = \dfrac{2R}{3}$

Multiple Correct Answers Type

1. Two equal negative charges $-q$ are fixed at points $(0, \pm a)$ on y-axis. A positive charge Q is released from rest at the point $(2a, 0)$ on the x-axis. The charge Q will

a. execute simple harmonic motion about the origin

b. move to the origin and remain at rest

c. move to infinity

d. execute oscillatory but not simple harmonic motion

(IIT-JEE, 1984)

2. A parallel plate air capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with the capacitor are given by Q_0, V_0, E_0 and U_0, respectively. A dielectric slab is now introduced to fill the space between the plates with battery still in connection. The corresponding quantities now given by Q, V, E and U are related to the previous one as

a. $Q > Q_0$ **b.** $V > V_0$

c. $E > E_0$ **d.** $U > U_0$

3. A charge q is placed at the center of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to **(IIT-JEE, 1987)**

a. $-\dfrac{Q}{2}$ **b.** $-\dfrac{Q}{4}$ **c.** $+\dfrac{Q}{4}$ **d.** $+\dfrac{Q}{2}$

4. A parallel plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved farther apart by means of insulating handles:

a. the charge on the capacitor increases.

b. the voltage across the plates increases.

c. the capacitance increases.

d. the electrostatic energy stored in the capacitor increases. **(IIT-JEE, 1987)**

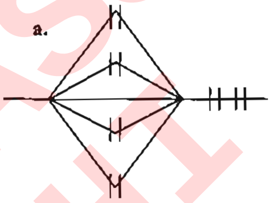
5. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and the outer surface of the hollow shell be V. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surfaces is

a. V **b.** $2V$ **c.** $4V$ **d.** $-2V$

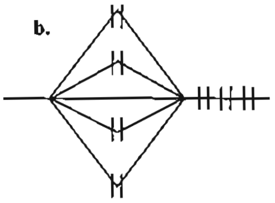
(IIT-JEE, 1989)

6. Seven capacitors, each of capacitance 2 μF, are to be connected in a configuration to obtain an effective capacitance of 10/11 μF. Which of the combination (s) shown in the given graphs will achieve the desired result?

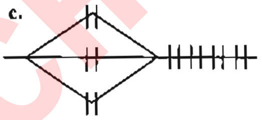
a.



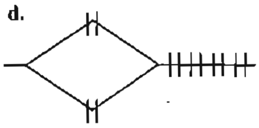
b.



c.



d.



(IIT-JEE, 1990)

7. A parallel plate capacitor of plate area A and plate separation d is charged to potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. If Q, E and W denote, respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted) and work done on the system, in question, in the process of inserting the slab, then **(IIT-JEE, 1991)**

a. $Q = \dfrac{\varepsilon_0 AV}{d}$

b. $Q = \dfrac{\varepsilon_0 KAV}{d}$

c. $E = \dfrac{V}{Kd}$

d. $W = \dfrac{\varepsilon_0 AV^2}{2d}\left[1 - \dfrac{1}{K}\right]$

8. Two identical thin rings, each of radius R metres, are coaxially placed a distance R metres apart. If Q_1 coulomb, and Q_2 coulomb, are, respectively, the charges uniformly spread on the two rings, the work done in moving a charge q from the center of one ring to that of the other is

a. zero

b. $\dfrac{q(Q_1 - Q_2)(\sqrt{2} - 1)}{(4\sqrt{2}\pi\varepsilon_0 R)}$

c. $\dfrac{q\sqrt{2}(Q_1 + Q_2)}{(4\pi\varepsilon_0 R)}$

d. $\dfrac{q(Q_1 + Q_2)(\sqrt{2}+1)}{(4\sqrt{2}\pi\varepsilon_0 R)}$ **(IIT-JEE, 1992)**

9. The magnitude of electric field $\vec{E}$ in the annular region of a charged cylindrical capacitor **(IIT-JEE, 1996)**

a. is same throughout.

b. is higher near the outer cylinder than near the inner cylinder.

c. varies as $1/r$, where r is the distance from axis.

d. varies as $1/r^2$ where r is the distance from the axis.

10. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path(s) shown in Fig. A1.131 as **(IIT-JEE, 1996)**

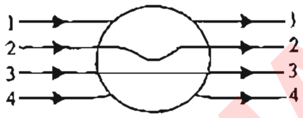


Fig. A1.131

a. 1 **b.** 2 **c.** 3 **d.** 4

11. A dielectric slab of thickness d is inserted in a parallel plate capacitor whose negative plate is at $x = 0$ and positive plate is $x = 3d$. The slab is equidistant from the plates. The capacitor is given some charge. As x goes from 0 to $3d$,

a. the magnitude of the electric field remains the same.

b. the direction of the electric field remains the same.

c. the electric potential increases continuously.

d. the electric potential increases at first, then decreases and again increases. **(IIT-JEE, 1998)**

12. A charge $+q$ is fixed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$ and so on, on the x-axis, and a charge $-q$ is fixed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$ and so on. Here, x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it to be $Q/(4\pi\varepsilon_0 r)$. Then, the potential at the origin due to the above system of charges is **(IIT-JEE, 1998)**

a. 0 **b.** $\dfrac{q}{8\pi\varepsilon_0 x_0 \ln 2}$

c. ∞ **d.** $\dfrac{q \ln 2}{4\pi\varepsilon_0 x_0}$

13. A positively charged thin metal ring of radius R is fixed in the xy plane with its center at the origin O. A negatively charged particle P is released from rest at the point $(0, 0, z_0)$, where $z_0 > 0$. Then, the motion of P is

a. periodic, for all values of z_0 satisfying $0 < z_0 < \infty$

b. simple harmonic, for all values of z_0 satisfying $0 < z_0 \le R$

c. approximately simple harmonic, provided $z_0 << R$

d. such that P crosses O and continues to move along the negative z-axis towards $z = -\infty$.

(IIT-JEE, 1998)

14. A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its center **(IIT-JEE, 1998)**

a. increases as r increases, for $r < R$

b. decreases as r increases, for $0 < r < \infty$

c. decreases as r increases, for $R < r < \infty$

d. is discontinuous at $r = R$

15. An elliptical cavity is carved within a perfect conductor (Fig. A1.132). A positive charge q is placed at the center of the cavity. The points A and B are on the cavity surface as shown in the figure. Then

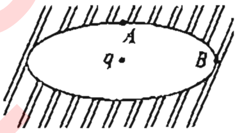


Fig. A1.132

a. electric field near A in the cavity = electric field near B in the cavity

b. charge density at A = charge density at B

c. potential at A = potential at B

d. total electric field flux through the surface of the cavity is q/ε_0. **(IIT-JEE, 1999)**

16. The electrostatic potential (ϕ_r) of a spherical symmetric system, kept at origin, is shown in Fig. A1.133, and given as **(IIT-JEE, 2006)**

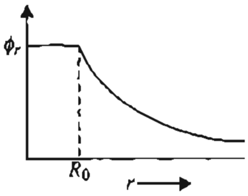


Fig. A1.133

$$\phi_r = \frac{q}{4\pi\varepsilon_0 r}\ (r \ge R_0), \phi_r = \frac{q}{4\pi\varepsilon_0 R_0}\ (r \le R_0)$$

Which of the following option(s) is/are correct?

a. For spherical region $r \le R_0$, total electrostatic energy stored is zero.

b. Within $r = 2R_0$, total charge is q.

c. There will be no charge anywhere except at $r = R_0$.

d. Electric field is discontinuous at $r = R_0$.

Integer Answer Type

1. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = kr^a$ wher k and a are constants and r is the distance from its centre. If the electric field at $r = R/2$ is 1/8 times that at $r = R$, find the value of a. **(IIT-JEE, 2009)**

ANSWERS AND SOLUTIONS

Objective Type

1. **c.** (Force on Q due to both charges $+q$ on x-axis will be cancelled. Now, net force on Q due to q and $-4q$ on y-axis should be zero.

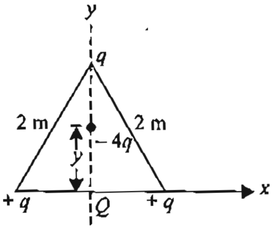


Fig. A1.134

$$\Rightarrow \quad \frac{kQq}{(2\cos 30°)^2} = \frac{kQ4q}{y^2} \quad \Rightarrow \quad y = 2\sqrt{3}\text{ m}$$

2. **b.** Charge on metal sphere: $V_1 = \frac{kq}{r_1} \Rightarrow q = \frac{V_1 r_1}{k}$

On connecting the sphere with shell, entire charge will be transferred to the shell. So, potential of sphere now:

$$V_2 = \frac{kq}{r_2} = \frac{V_1 r_1}{r_2}$$

3. **c.** Let capacitance of conductor is C_1 and that of plate is C_2 (Fig. A1.135). After first operation:

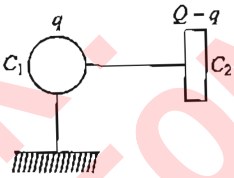


Fig. A1.135

$$\frac{q}{C_1} = \frac{Q-q}{C_2} \qquad \text{(i)}$$

Let q_0 is the maximum charge that can be transferred to the conductor (Fig. A1.136). Then,

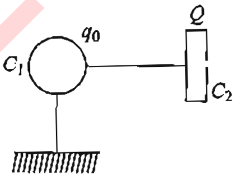


Fig. A1.136

$$\frac{q_0}{C_1} = \frac{Q}{C_2} \qquad \text{(ii)}$$

From (i) and (ii), $q_0 = \frac{Qq}{Q-q}$

4. **b.** Initially, when the switch is closed on position 1, the capacitor C is connected in series with batteries E_1 and E_2.

From KVL, we have

$$\frac{Q_i}{C} - E_2 + E_1 = 0$$

or $$Q_i = (E_2 - E_1)C \qquad \text{(i)}$$

Depending upon the sign of $(E_2 - E_1)$, charge Q_i on the left plate may be positive (if $E_2 > E_1$), or negative (if $E_2 < E_1$); charge on right plate would be equal and opposite.

When the switch is moved to position 2, the left plate (earlier having charge $+Q_i$), will now have charge

$$Q_f = -E_1 C \qquad \text{(ii)}$$

The net charge flow through the circuit is

$$\Delta Q = Q_f - Q_i = [-E_1 - (E_2 - E_1)]C = -E_2 C$$

We can say that a net positive charge equal to $E_2 C$ is pulled by the battery of e.m.f. E_1 from the left plate of the capacitor, which flows through battery E_1 and is transferred to the right plate of the capacitor. Work done by battery E_1 in the process of charge transfer is

$$\Delta W = E_1 E_2 C \qquad \text{(iii)}$$

A part of this work changes the energy of the capacitor:

$$\Delta W_C = \frac{Q_f^2}{2C} - \frac{Q_i^2}{2C} = \frac{1}{2}E_1^2 C - \frac{1}{2}(E_2 - E_1)^2 C$$

$$= \frac{1}{2}(2E_1 E_2 - E_2^2)C$$

and the remaining part is lost as Joule heating:

$$H = \Delta W - \Delta W_C = \frac{1}{2}E_2^2 C$$

5. **b.** From Fig. A1.137

1. $dp = (\lambda a\, d\theta)\, 2a\cos\theta$

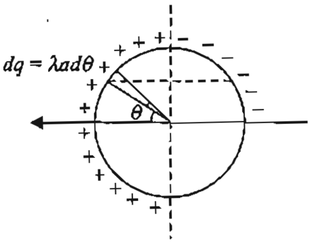


Fig. A1.137

$$p = \int dp = 2\lambda a^2 \int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta$$

$$= 2\lambda a^2\, [\sin\theta]_{-\pi/2}^{\pi/2}$$

$$|p| = 4\lambda a^2$$

2. $dp = (\lambda a\, d\theta)\, 2a\cos\theta$ (Fig. A1.138)

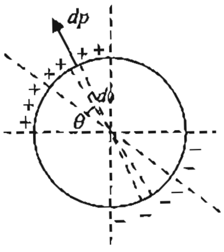


Fig. A1.138

$$p = \int dp = 2\lambda a^2 \int_{-\pi/4}^{\pi/4} \cos\theta \, d\theta$$

$$= 2\lambda a^2 \, [\sin\theta]_{-\pi/4}^{\pi/4}$$

$$= 2\lambda a^2 \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$$

$$p = 2\sqrt{2}\lambda a^2$$

6. d. $dp = \sigma\,(2\pi a \sin\theta)\, a\, d\theta \;\; 2a\cos\theta$

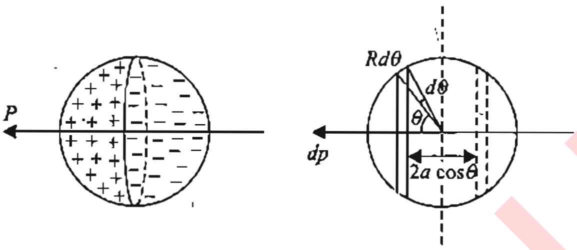


Fig. A1.139

$$dp = 4\sigma\pi a^3 \sin\theta \cos\theta \, d\theta$$

$$p = \int dp = 4\sigma\pi a^3 \int_0^{\pi/2} \sin\theta \cos\theta \, d\theta$$

Let $\sin\theta = t$

$\Rightarrow \cos\theta d\theta = dt$

$$\therefore \quad p = 4\pi\sigma a^3 \int_0^1 t \, dt$$

$$p = 2\pi\sigma a^3$$

7. a. The most general case is as shown in Fig. A1.140. No net charge is induced as no flow of charge into or out from the body is taking place. So, mass of the bodies also remain unaffected. It is a temporary effect as when we take away the charged body, the redistribution of charge in the neutral body disappears. We can also consider other cases like charge enclosed by a conducting shell, etc.

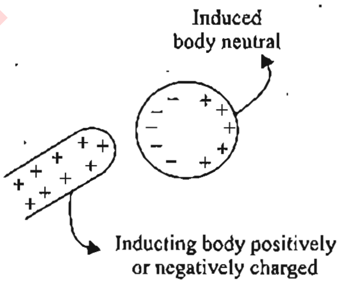


Fig. A1.140

8. d. Equilibrium would be stable, when the particle displaced from equilibrium position comes back to its equilibrium position under the action of restoring forces and equilibrium would be unstable, when the particle has no tendency to come back to equilibrium position when displaced from the equilibrium position.

9. a. For providing a non-zero net charge, transfer of electron has to take place from given charged conducting body to the dielectric sphere (which has to be charged), which is not possible in induction and conduction.

10. b. From the expression for $\vec{E}$ due to straight wire (Fig. A1.141),

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 d}(\sin\theta_2 + \sin\theta_1)$$

$$\text{And} \quad E_y = \frac{\lambda}{4\pi\varepsilon_0 d}(\cos\theta_1 - \cos\theta_2)$$

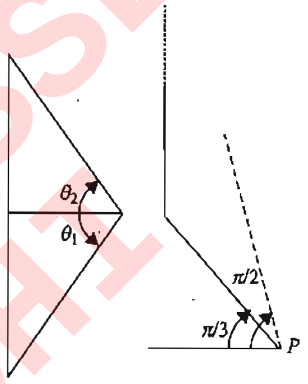


Fig. A1.141

For the given configuration,

$$\theta_1 = -\frac{\pi}{4} \text{ and } \theta_2 = \frac{\pi}{2}$$

$$\text{So,} \quad \vec{E} = \frac{\lambda}{4\pi\varepsilon_0 d}\left[\left(1 - \sin\frac{\pi}{4}\right)\hat{i} + \cos\frac{\pi}{4}\,\hat{j}\right]$$

11. c. Due to induction, redistribution of charge will take place as shown in Fig. A1.142.

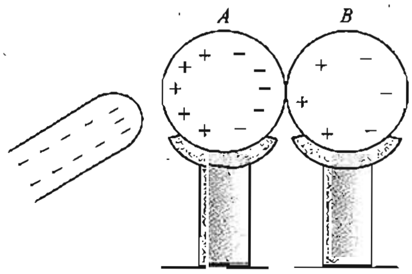


Fig. A1.142

Due to lesser distance from rod, A would have more negative potential as compared to potential of B. Since spheres are in contact, transfer of electrons takes place from A to B [negative charge moves from low potential to a high potential]. As spheres would be separated, A acquires some net positive charge while B has some net negative charge. Due to withdrawal of rod, the charges appearing on A and B get distributed over their surfaces.

12. c. Due to q, charge gets distributed on the inner and outer surfaces of sphere as shown in Fig. A1.143. On the inner surface, q' (non-uniform or uniform distribution) on outer surface $-q'$ (non-uniform or uniform distribution) and Q is uniformly distributed.

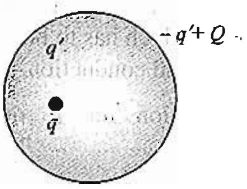


Fig. A1.143

$$\vec{E}_q = \vec{E}_{q'} + \vec{E}_{-q'} + \vec{E}_Q = 0$$

$$\vec{E}_{q'} = -\vec{E}_{-q'} \text{ and } \vec{E}_Q = 0$$

So, $\quad \vec{F}_q = q\vec{E}_q = 0,$

whatever be the location of charge q.

13. b. The charge distribution is as shown in Fig. A1.144. Out of these charges, the charge on the outer surface, $(q+Q)$, would be always uniformly distributed, so $\vec{E}$ due to charge on $q+Q$ at any inside point is zero. [In another reasoning, conducting shell divides the space into two regions, inside and outside, which are independent of each other in terms of electric field].

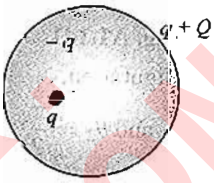


Fig. A1.144

Now, electric field at the location of point charge q is only due to induced charge on the inner surface of the conductor. As charge q moves from center to the inner surface, $\vec{E}_{-q}$ increases as charge density becomes more near to q.

14. b. $dE_x = 2dE\sin\theta$ and $dE_y = 0$ (Fig. A1.145)

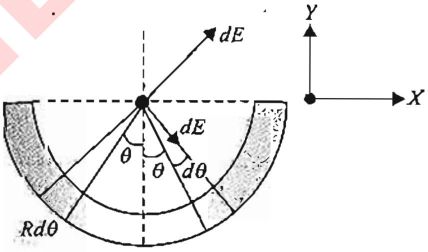


Fig. A1.145

$$dE = \frac{\sigma \times Rd\theta}{2\pi\varepsilon_0 R} = \frac{\sigma\, d\theta}{2\pi\varepsilon_0}$$

$$E_x = \int dE_x = \int_0^{\pi/2} \frac{\sigma}{\pi\varepsilon_0}\sin\theta\, d\theta = \frac{\sigma}{\pi\varepsilon_0}$$

15. b. $\vec{E} = -\dfrac{\partial V}{\partial x}\hat{i} - \dfrac{\partial V}{\partial y}\hat{j} - \dfrac{\partial V}{\partial z}\hat{k}$

$$\vec{E} = (-6x+4)\,\hat{i}$$

So, the equipotential surfaces would be planes parallel to YZ plane, as $\vec{E}$ is perpendicular to equivalent surface.

16. a. All the statements are general properties of electric lines of force.

17. d. The total charge of the inner conductor transfers to the outer one,

$$V_B = \frac{q}{4\pi\varepsilon_0 R}$$

18. d. The part of the ring enclosed by sphere subtends an angle of 120° $\left(\dfrac{2\pi}{3}\right)$ at its center.

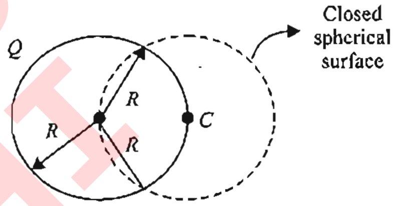


Fig. A1.146

So, charge enclosed by the sphere is $-\dfrac{Q}{2\pi} \times \dfrac{2\pi}{3} = \dfrac{Q}{3}$

So, flux crossing through the sphere is $\dfrac{Q}{3\varepsilon_0}$.

19. b. Equal and opposite charges get induced on the inner surface of conductor, so the net charge enclosed by the surface is zero and hence flux crossing through the surface is zero.

20. d. $\quad \phi = \phi_{ps_1} + \phi_{cs} + \phi_{ps_2}$

$$\phi = 2\phi_{ps} + \phi_{cs} \quad [\because \phi_{ps_1} = \phi_{ps_2}]$$

$$\phi_{cs} = \phi - 2\phi_{ps} = \frac{q}{\varepsilon_0} - 2\phi_{ps}$$

To find ϕ_{ps}, use integration technique.

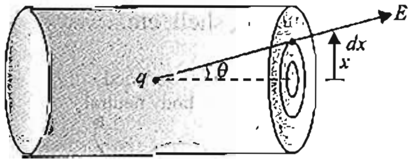


Fig. A1.147

$$\vec{E} = \frac{q}{4\pi\varepsilon_0} \times \frac{1}{(x^2+l^2/4)}$$

$$d\phi_{ps} = \vec{E}\cdot\vec{ds} = E \times 2\pi x\, dx \times \cos\theta$$

$$= \frac{q}{2\varepsilon_0} \times \frac{x\,dx}{(x^2+l^2/4)} \times \frac{l/2}{\sqrt{x^2+l^2/4}}$$

$$= \frac{ql}{4\varepsilon_0} \times \frac{x\,dx}{(x^2+l^2/4)^{3/2}}$$

$$\phi_{ps} = \frac{ql}{4\varepsilon_0}\int_0^R \frac{x\,dx}{(x^2+l^2/4)^{3/2}} = \frac{q}{2\varepsilon_0} - \frac{ql}{4\varepsilon_0\sqrt{R^2+l^2/4}}$$

21. **c.** This work done by us is stored in the capacitor in the volume $A(d_2 - d_1)$ where new electric field is created.

If you calculate the work done by using the expression $W_{\text{ext}} - W_{\text{el}} = dU = \frac{\varepsilon_0 A V^2}{2}\left[\frac{1}{d_2} - \frac{1}{d_1}\right] < 0$, you may get confused. Here, battery is also doing work, so from energy conservation principle.

$$W_{\text{ext}} + W_{\text{el}} + W_{\text{battery}} = 0$$

22. **b.** Due to polarization, charge on dielectric slab would be $CV\left(1 - \frac{1}{K}\right)$.

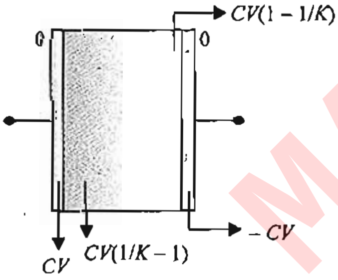


Fig. A1.148

So, charge on 1 = 0

$$\text{Charge on 2} = CV - CV\left(1 - \frac{1}{K}\right) = \frac{CV}{K}$$

$$\text{Charge on 3} = CV\left(1 - \frac{1}{K}\right) - CV = -\frac{CV}{K}$$

Charge on 4 = 0

23. **d.** The resultant force acting per unit area of each plate can be written as $F = F_0 - F'$, where F_0 is the force acting on unit area of plate due to other plate and F' is the force acing on unit area of plate from the dielectric.

$$\text{Now, } F = \frac{\dfrac{q^2}{2\varepsilon_0\varepsilon A}}{A} = \frac{\left(\dfrac{\varepsilon_0\varepsilon A V}{d}\right)^2}{2\varepsilon_0\varepsilon A} \times \frac{1}{A}$$

$$F = \frac{\varepsilon_0\varepsilon V^2}{2d^2}$$

Also, $F_0 = F \times \varepsilon$

$$\text{So, } F' = F_0 - \frac{F_0}{\varepsilon} = F_0\left(1 - \frac{1}{\varepsilon}\right)$$

$$= \varepsilon F\left(1 - \frac{1}{\varepsilon}\right) = \frac{\varepsilon(\varepsilon - 1)\varepsilon_0 V^2}{2d^2}$$

24. **a.** Use the formula, $F = -\frac{dU}{dx}$, where $U = \frac{q^2}{2C}$

Treat the cylindcal capacitor as a parallel plate capacitor as $d << R$.

25. **c.** Refer concepts and formulate Isolated sphere and non-isolated sphere.

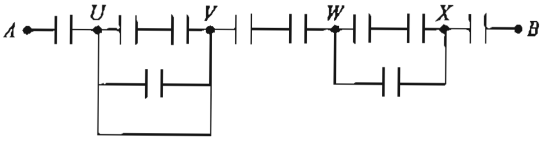


Fig. A1.149

26. **a.** Charge on capacitor, $Q = C(E - V)$ (Fig. A1.149)

Equivalent circuit can be drawn as (Fig. A1.150)

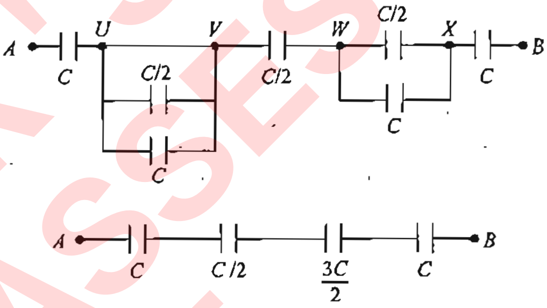


Fig. A1.150

[As U and V are at same potential, remove all capacitors between these points]

$$\Rightarrow \quad C_{\text{eq}} = \frac{3C}{14}$$

27. **c.** The innermost conductor is at 0 potential (the same potential as we assumed for infinity).

The conductors with radius a and b make one capacitor, between b and c other capacitor and c makes a capacitor with its other plate at infinity. So, equivalent diagram can be drawn as

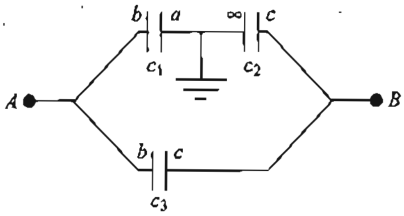


Fig. A1.151

$$\text{Here, } C_1 = \frac{4\pi\varepsilon_0 ab}{b - a}$$

$$C_2 = 4\pi\varepsilon_0 c$$

$$C_3 = \frac{4\pi\varepsilon_0 bc}{c - b}$$

$$\therefore \quad C_{\text{eq}} = C_3 + \frac{C_1 C_2}{C_1 + C_2} = 4\pi\varepsilon_0\left[\frac{bc}{c - b} + \frac{abc}{ab + c(b - a)}\right]$$

28. **d.** Initial situation after the reconnection is shown in Fig. A1.152(a) and the final situation in Fig. A1.152(b).

The charge transferred by C_1 is $q_0 - q_1$ and the capacitors C_2 and C_3 are in series, so $q_2 = q_3$.

Other way to solve the questions is to equalize the potentials across C_1 and series combination of C_2 and C_3.

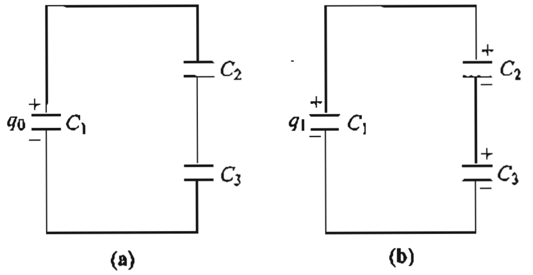


Fig. A1.152

29. a. Common potential: $V_C = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$

Since $C_1 = C_2$, so $V_C = \dfrac{V_1 + V_2}{2}$

Here, +ve plates of both are connected and −ve plates of both are connected. So, sum of charge on −ve plates will remain constant.

30. a. Potential of outer sphere $V = \dfrac{kq_1}{3r} + \dfrac{kq_2}{3r} + \dfrac{kq_3}{3r} = 0$

$\Rightarrow q_1 + q_3 = -q_2$

31. b. If magnitude of the charge on the positively charged particle is q, then

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

And, $\quad U = \dfrac{1}{2}\varepsilon_0 E^2$

It means, $E \propto V^2$. Hence, the curve between E and V^2 will be a straight line passing through the origin. Hence, the option (b) is correct.

Since $V^2 \propto E$, therefore the curve between V and E will be a parabola which is symmetric about E-axis. Since the particle is positively charged, therefore the potential cannot be negative. Hence, the graph between V and E will be as shown in figure (a). It means, the option (a) is wrong.

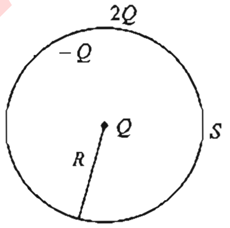


Fig. A1.153

Since $U = \dfrac{1}{2}\varepsilon_0 E^2$, therefore the graph between E and U will again be a parabola which is symmetric about U-axis.

Since $V \propto \dfrac{1}{r}$ and $U \propto \dfrac{1}{r^4}$, therefore $V^4 \propto U$ or $(V^2)^2 \propto U$. Hence, the graph between V^2 and U will be a parabola which is symmetric about U-axis. But neither V nor U can be negative. Hence, the part of the curve lying in positive quadrant only, is possible. Hence, the graph between V^2 and U will be as shown in figure (b). Therefore, the option (d) is wrong.

32. b. Electric field travels with the speed of light and takes finite time to propagate. So, if charge is displaced, then $\vec{E}$ due to this charge at any point changes after some time.

33. b. Electric field due to one wire $= \dfrac{1}{4\pi\varepsilon_0}\dfrac{2\lambda_1}{R}$

Charge per unit length on other wire: $q_2 = \lambda_2 l$

$$\Rightarrow \quad q_2 = \lambda_2 \times 1 = \lambda_2$$

Force per metre length on other wire

$$F = q_2E = \lambda_2 E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda_1\lambda_2}{R} = K \times \frac{2\lambda_1\lambda_2}{R}$$

34. a. $T = 2\pi\sqrt{\dfrac{l}{g_{\text{eff}}}}$

Here, $g_{\text{eff}} = g - \dfrac{qE}{m}$

$\Rightarrow$ g_{eff} will decrease.

Hence, T will increase.

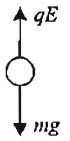


Fig. A1.154

35. c. Electric flux $\displaystyle\oint_S \vec{E}\cdot d\vec{S} = \frac{q_{\text{in}}}{\varepsilon_0}$

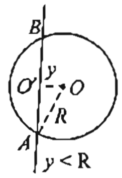


Fig. A1.155

q_{in} is the charge enclosed by the Gaussian surface which, in the present case, is the surface of given sphere. As shown, length AB of the line lies inside the sphere.

In $\Delta OO'A$, $R^2 = y^2 + (O'A)^2$

$O'A = \sqrt{R^2 - y^2}$ and $AB = 2\sqrt{R^2 - y^2}$

Charge on length $AB = 2\sqrt{R^2 - y^2} \times \lambda$

$\therefore$ electric flux $\displaystyle\oint_S \vec{E}\cdot d\vec{S} = \frac{2\lambda\sqrt{R^2 - y^2}}{\varepsilon_0}$

36. c. Field strength at a point on the axis a distance x from the center can be expressed as

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(R^2 + x^2)^{3/2}}$$

Point charge 3 μC has to be kept at such a point on the axis at which field strength due to the ring has a maximum value.

For E to be maximum, $\dfrac{dE}{dx} = 0$ and we obtain $x = \dfrac{R}{\sqrt{2}}$

So, the point charge has to be kept a distance

$x = \dfrac{R}{\sqrt{2}} = \dfrac{15}{\sqrt{2}} = 1.6$ cm from the center.

37. **d.** $R = \sqrt{8^2 + 8^2 + 2 \times 8 \times 8 \cos 120^\circ} = 8$ N

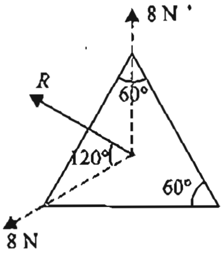


Fig. A1.156

38. **c.** From (i), A and C both are charged, either positively or negatively.

From (ii), B is charged and D and E have no charge. From (iii), A is positively charged.

Therefore, from (i), B is negatively charged.

39. **c.** After covering with a hemispherical shell;

$$\phi_{\text{shell}} + \phi_{\text{disc}} = 0 \text{ (from Gauss law)}$$

$$\phi_{\text{shell}} = -\phi_{\text{disc}} = -\phi$$

40. **c.** Using the formula for electric field produced by large sheet, $E = \dfrac{Q}{2A\varepsilon_0}$

We get: $E_A = \dfrac{4Q}{2A\varepsilon_0}(-\hat{i})$; $E_B = \dfrac{2Q}{2A\varepsilon_0}(-\hat{i})$

$$E_C = \frac{4Q}{2A\varepsilon_0}(+\hat{i})$$

41. **c.** Net force on q_1 is zero, while that on the conducting sphere is towards the left due to attraction of $-q_2$ (Fig. A1.109).

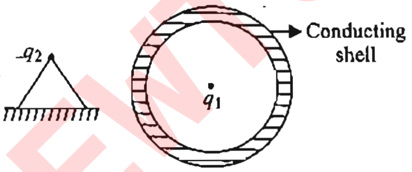


Fig. A1.157

42. **a.** $a_x = \dfrac{qE}{m} = \dfrac{10^{-6} \times 2 \times 10^7}{2} = 10 \text{ ms}^{-2}$

Time of flight $T = \dfrac{2u \sin\theta}{g} = \dfrac{2 \times 10 \times \dfrac{1}{\sqrt{2}}}{10}$

$$T = \sqrt{2} \text{ sec}$$

Hence, Range $R = u_x T + \dfrac{1}{2} a_x T^2$

$$R = 10 \cos 45^\circ \times T + \frac{1}{2} a_x T^2$$

$$= \frac{10}{\sqrt{2}} \times \sqrt{2} + \frac{1}{2} \times 10 \times 2 \Rightarrow \quad R = 20 \text{ m}$$

43. **b.** From Gauss law, $\phi = \dfrac{q}{\varepsilon_0}$

So, $\dfrac{\phi_{\max}}{\phi_{\min}} = \dfrac{Q_{\max}}{Q_{\min}} = \dfrac{\lambda(l^2 + b^2 + h^2)^{1/2}}{\lambda h}$

$$= \frac{\sqrt{l^2 + b^2 + h^2}}{h}$$

44. **d.** l = length of the each tube

$u_x = u$ = constant

Time of travel between the plates $t = \dfrac{l}{u}$

Let a = constant acceleration in y-direction

So, $v_y = at$ when the particle emerges from the plates

So, $v^2 = u_x^2 + v_y^2 \quad = u^2 + a^2 t^2$

$$= u^2 + a^2 \frac{l^2}{u^2} = u^2 + \frac{C}{u^2} \text{ (where } a^2 l^2 = C\text{)}$$

So, $v = \sqrt{u^2 + \dfrac{C}{u^2}}$

45. **a.** According to Gauss's law, electric field at any point on Gaussian surface is due to all the charges present inside or outside.

46. **a.** $V = V_1 + V_2 + V_3$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} + \frac{1}{4\pi\varepsilon_0}\left(\frac{-2Q}{R}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{3Q}{R}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{R}\right) = \frac{Q}{2\pi\varepsilon_0 R}$$

47. **b.** At equatorial point, $\vec{E}$ is opposite to $\vec{P}$. So, the required angle is 180°.

48. **c.** Electrical potential energy of system:

$$U = -2\left[\frac{kq^2}{\sqrt{2}a}\right] = \frac{-\sqrt{2}kq^2}{a}$$

(four pairs will cancel each other)

If a decreases, U also decreases and if U decreases, the agent will do negative work.

49. **b.** $V = \dfrac{kQ}{R}$, $E = \dfrac{kQ}{R^2}$

$\dfrac{V}{E} = R \quad \Rightarrow \quad E = \dfrac{V}{R}$

If R increases, E decreases.

50. **c.** From Fig. A1.158

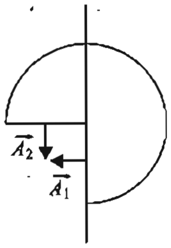


Fig. A1.158

$$\vec{A}_1 = -\frac{\pi R^2}{2}\hat{i}$$

$$\vec{A}_2 = -\frac{\pi R^2}{2}\hat{j}$$

$$\vec{E} = E\cos 45^\circ\,\hat{i} + E\sin\,45^\circ\,\hat{j} = \frac{E}{\sqrt{2}}\hat{i} + \frac{E}{\sqrt{2}}\hat{j}$$

$$\phi = \vec{E}\cdot(\vec{A}_1 + \vec{A}_2) = \frac{-E}{\sqrt{2}}\frac{\pi R^2}{2} - \frac{E}{\sqrt{2}}\frac{\pi R^2}{2}$$

$$= \frac{-\pi R^2 E}{\sqrt{2}}$$

This is the flux entering. So, flux leaving $= \dfrac{\pi R^2 E}{\sqrt{2}}$

51. **b.** $V = 10r = 10\sqrt{x^2+y^2+z^2}$

$$E_x = -\frac{dv}{dx} = -\frac{10(2x)}{2\sqrt{x^2+y^2+z^2}}$$

$$= \frac{-10x}{\sqrt{x^2+y^2+z^2}} = \frac{-10\times 3}{\sqrt{3^2+4^2+5^2}} = -3\sqrt{2}$$

Similarly, $E_y = -4\sqrt{2}$

$$E_z = 5\sqrt{2};\ \vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

52. **a.** There is no charge on the outer surface. Hence, no force on q_2 (Fig. A1.159).

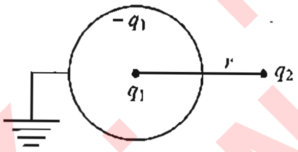


Fig. A1.159

53. **b.** The charge on outer surface of shell will apply a force on q_2

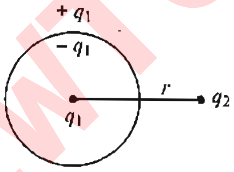


Fig. A1.160

$$F = \frac{kq_1q_2}{r^2}$$

54. **b.**

$$E = \frac{Q}{2A\varepsilon_0}$$

$$a = \frac{qE}{m} = \frac{qQ}{2Am\varepsilon_0}$$

$$d = \frac{1}{2}at^2;\ t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2d\,2Am\varepsilon_0}{qQ}}$$

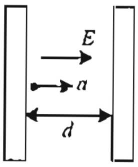


Fig. A1.161

55. **d.** $OB = OA\cos 60^\circ = 2\times\dfrac{1}{2} = 1$ m

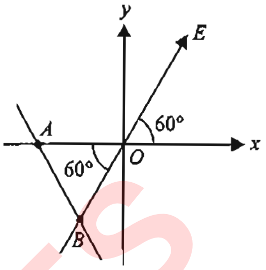


Fig. A1.162

$$V_B - V_O = E(OB) = 100\times 1 = 100\text{ V}$$

$$V_A - V_O = 100\text{ V}\ [\because\ V_B = V_A]$$

$$V_O - V_A = -100\text{ V}$$

56. **d.** $V_B - V_A = -\displaystyle\int E_x dx = -$[Area under E_x-x curve]

$$V_B - 10 = -\frac{1}{2}\times 2\times(-20) = 20$$

$$V_B = 30\text{ V}$$

57. **b.** Charge on B will remain same as it is not touched with any other body. Charge induced on A will decrease the potential of B (Fig. A1.163).

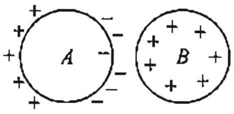


Fig. A1.163

58. **a.** Electric field produced due to charge Q will be zero at p. But potential produced by Q at p will not be zero.

59. **d.** $\dfrac{\lambda}{2\pi\varepsilon_0 r} = \dfrac{mv^2}{r}$ or $v = \sqrt{\dfrac{\lambda}{2\pi\varepsilon_0 m}}$ independent of r (see Fig. A1.164).

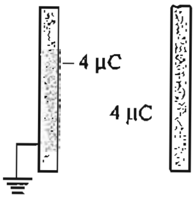


Fig. A1.164

60. **b.** Final charges will be as shown in Fig. A1.165.
So, charge flowing from earth to plate
= final charge – initial charge
= –4 – 10 = –14 μC

61. **d.** $V = \dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2}$ (1)

$Q_1 + Q_2 = Q$ (2)

where $C_1 = \dfrac{\varepsilon_0 A}{a}$ and $C_2 = \dfrac{\varepsilon_0 A}{b}$

Solve to find $Q_2 = \dfrac{Qa}{a+b}$

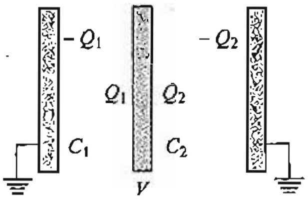


Fig. A1.165

62. c. $P = 2 \times 10^{-8} \times 2 \times 10^{-3} = 4 \times 10^{-11}$ Cm

$$E = \frac{\lambda}{2\pi\varepsilon_0 x}, F = p\cos\theta\frac{dE}{dx}$$

$$F = -\frac{p\cos\theta\lambda}{2\pi\varepsilon_0 x^2} = \frac{-4 \times 10^{-11} \times \cos 60° \times 4 \times 10^{-4}}{\frac{1}{2 \times 9 \times 10^9} \times \left(\frac{6}{100}\right)^2}$$

$$= 0.04 \text{ N}$$

63. b. $q_i = CV_i = 16 \times 5 = 80\ \mu\text{C}, q_f = CV_f = 16 \times 10$

$= 160\ \mu\text{C}$

Given $\frac{dq}{dt} = 40t \Rightarrow \quad \int_{q_i}^{q_f} dq = \int_0^t 40t$

$$\Rightarrow \quad q_f - q_i = \frac{40t^2}{2}$$

$$\Rightarrow 160 - 80 = 20t^2 \quad \Rightarrow \quad t = 2 \text{ s}$$

64. c. p and q are in parallel and then r in series.

65. c. The charge on capacitor before dielectric,

$$q_1 = CV = 50 \times 10^{-9}\ C$$

Final charge on capacitor after dielectric,

$$q_2 = (KC)V = 5 \times 50 \times 10^{-9}$$
$$= 250 \times 10^{-9}\ \text{C}$$

Charge flow from battery, $\Delta q = q_2 - q_1 = 200$ nC

66. a. Given plates can be rearranged as shown:

$$C_1 = \frac{\varepsilon_0 A}{d}; C_2 = \frac{\varepsilon_0 A}{2d}$$

$$C_3 = \frac{\varepsilon_0 A}{3d}; C_4 = \frac{\varepsilon_0 A}{2d}; C_5 = \frac{\varepsilon_0 A}{d}$$

C_1 and C_2 are in series and its effective capacity

$$= \frac{\frac{\varepsilon_0 A}{d} \times \frac{\varepsilon_0 A}{2d}}{\frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{2d}} = \frac{\varepsilon_0 A}{3d}$$

Effective capacitance of C_4 and $C_5 = \frac{\varepsilon_0 A}{3d}$

$$\therefore \quad C_{AB} = \frac{\varepsilon_0 A}{3d} + \frac{\varepsilon_0 A}{3d} + \frac{\varepsilon_0 A}{3d} = \frac{\varepsilon_0 A}{d}$$

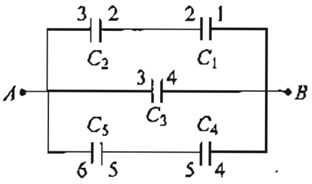


Fig. A1.166

67. b. Plates can be rearranged as shown in Fig. A1.168.

Plates 1 and 2 and plates 3 and 4 form two capacitors which are in series between A and B. Plates 2 and 3 do not form any capacitor as they are at same potential.

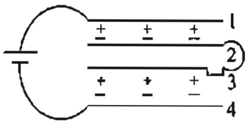


Fig. A1.167

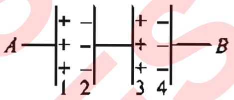


Fig. A1.168

$$\text{So, } C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\left(\frac{\varepsilon_0 A}{d}\right) \times \left(\frac{\varepsilon_0 A}{3d}\right)}{\frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{3d}}$$

$$C_{eq} = \frac{\varepsilon_0 A}{4d}$$

68. b. Given circuit is equivalent to (see Fig. A1.169)

$$C_1 = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \times 50 \times 10^{-4}}{3 \times 10^{-3}} = \frac{5}{3}\varepsilon_0$$

$$\text{and } C_2 = \frac{\varepsilon_0 \times 50 \times 10^{-4}}{6 \times 10^{-3}} = \frac{5}{6}\varepsilon_0$$

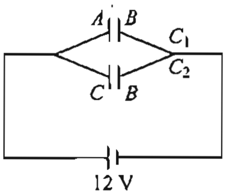


Fig. A1.169

Effective capacity,

$$C_{eq} = C_1 + C_2 = \left(\frac{5}{3} + \frac{5}{6}\right)\varepsilon_0 = \frac{5}{2}\varepsilon_0$$

Hence, energy stored $U = \frac{1}{2}CV^2 = \frac{1}{2} \times \frac{5}{2}\varepsilon_0 \times 12^2$

$$= \frac{5}{4} \times 8.85 \times 10^{-12} \times 12^2 = 1.6 \times 10^{-9}\ \text{J} = 1.6\ \text{nJ}$$

69. a. Let the charge following through section AB is Q (Fig. A1.170).

Applying Kirchhoff's law,

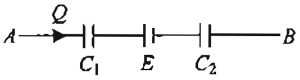


Fig. A1.170

$$V_A - \frac{Q}{C_1} + E - \frac{Q}{C_2} = V_B$$

$$V_A - V_B + E = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$5 + 10 = Q\left(\frac{C_1 + C_2}{C_1 C_2}\right) = Q\left(\frac{1+2}{2}\right)$$

$$Q=\frac{15\times 2}{3}=10\,\mu\text{C}$$

So, potential difference across 2 μF capacitor

$$V=\frac{Q}{C_2}=\frac{10\ \mu\text{C}}{2\ \mu\text{F}}=5\ \text{V}$$

70. d. Circuit is redrawn as shown in Fig. A1.171.

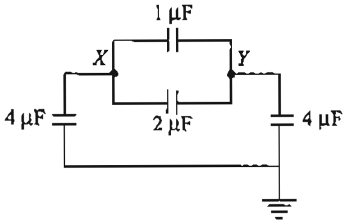


Fig. A1.171

In the circuit, 4 μF and 4 μF capacitors are connected in series. Combination of this is in parallel with 1 μF and 2 μF capacitors.

Hence, choice (d) is correct.

71. d. Field on the axis of the dipole lies in the direction of dipole.

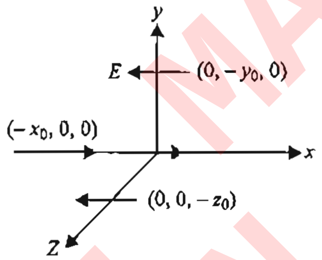


Fig. A1.172

Field on the bisector of the dipole lies opposite to the direction of dipole.

72. a. The potential of point B is +10 V, therefore, no potential difference exists across 12 μF capacitor, hence $q_{12}=0$.

Charge on the 4 μF capacitor is

$q_4=(4)(10)=40\ \mu\text{C}$.

73. b. Given:

$E=\frac{E_0}{l}\vec{i}$, $l=2$ cm, a = 1 cm,

$E_0=5\times 10^3$ N/c

We see that flux passes mainly through surface area *ABDC* and *EFGH*.

As the *AEFB* and *CHGD* are parallel to the flux again in $ABDC=0$;

Hence the flux only passes through the surface area $EFGHE=0$

$$flux=E_0\frac{a}{l}\times\text{area}=5\times10^3\times\frac{a}{l}\times a^2$$

$$=5\times10^3\times\frac{a^3}{l}$$

$$=5\times10^3\times\frac{(0.01)^3}{2\times10^{-2}}=2.5\times10^{-1}$$

So, $q=\varepsilon_0$ flux

$=8.85\times10^{-12}\times2.5\times10^{-1}$

$=22.125\times10^{-13}=2.2125\times10^{-12}$ C

74. d. Consider the gaussian surface the induced charge be as shown in the fig.

The net field at p due to all the charges is zero

$$-2Q+\frac{q}{2A\varepsilon_0}(\text{left})+\frac{q}{2A\varepsilon_0}(\text{right})+q-\frac{q}{2A\varepsilon_0}(\text{right})=0$$

$$\Rightarrow -2q+q-Q+q=0$$

$$\Rightarrow\quad q=\frac{3}{2}Q$$

charge on the right side of right most plate =

$$-2Q+Q=-2Q+\frac{3}{2}Q=-\frac{Q}{2}$$

75. a. Electrical force is balanced by the weight of the mass

$mg=qE$ $\quad E=$ electric field

or $mg=q.\frac{V}{d}$ (where d = separation at the plates)

or $mg=q.\frac{V}{2}\left(\because\ \frac{\varepsilon_0 A}{d}=C\quad\frac{1}{d}=\frac{C}{\varepsilon_0 A}\right)$

or $V=\frac{2mg\varepsilon_0 A}{qC}$

$$=\frac{10^{-6}\times10\times2.2\times10^{-12}\times10^{-2}}{10^{-5}\times4\times10^{-8}}$$

$$=\frac{10^{-6}\times9.8\times8.88\times10^{-12}\times100\times10^{-4}}{10^{-8}\times0.04\times10^{-6}}$$

$V=43$ mV

76. d. Initial total energy $=\frac{1}{2}CV^2+\frac{1}{2}CV^2=CV^2$

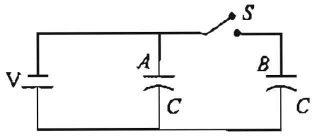


Fig. A1.173

when the switch is open dielectric induced.

Then capacitance $C=KC=3C$

Then energy stored in $C=\frac{1}{2}3CV^2=\frac{3}{2}CV^2$

Since switch is open so change be same in B so energy in $B=\frac{1}{2}\frac{C}{3}V^2$.

So, Total final energy $=\frac{3}{2}CV^2+\frac{1}{6}CV^2$

$$=\frac{9CV^2+1CV^2}{6}=\frac{10}{6}CV^2$$

So, Required Ratio $=\frac{CV^2}{\frac{10}{6}CV^2}=\frac{3}{5}=3:5$

77. c. $C_A=8\mu\text{F}$, $C_B=4\ \mu\text{F}$, $C_C=4\ \mu\text{F}$, $C_{eq}=4\ \mu\text{F}$

Since b and c are parallel and are in series with A

So, $q_1=8\times6=48\ \mu\text{C}$, $q_2=4\times6=24\ \mu\text{C}$,

$q_3=4\times6=24\ \mu\text{C}$

78. c. If V be the potential at D, then

$$\frac{V_1 - V}{V - V_2} = \frac{C_2}{C_1}$$

or, $C_1V_1 - C_1V = C_2V - C_2V_2$

or $C_1V_1 + C_2V_2 = (C_1 + C_2)V$

or, $V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$

79. c. Loss of energy

$$= \frac{1}{2}C\left[V_1^2 + V_2^2\right] - \frac{1}{2}(2C)\left[\frac{V_1 + V_2}{2}\right]^2$$

$$= \frac{C}{4}\left[2V_1^2 + 2V_2^2 - V_1^2 - V_2^2 - 2V_1V_2\right]$$

$$= \frac{C}{4}\left[V_1^2 + V_2^2 - 2V_1V_2\right] = \frac{C}{4}(V_1 - V_2)^2$$

80. a. The electric field strength in a parallel-plate capacitor with applied voltage of V and separation d is given by, $E = V/d$.

81. b. Initially at switch position Q, by conservation of charge, the voltage V across the two capacitors is thus given by,

$q = q_1 + q_2 = C_1V + C_2V = (C_1 + C_2)V$

$\Rightarrow \quad 18 = (3 + 6)V$

$\Rightarrow \quad V = (18/9) = 2$ V

82. c. $Q_3 = 6\left(\dfrac{3}{1 + 2 + 3}\right) = 3\ \mu C$

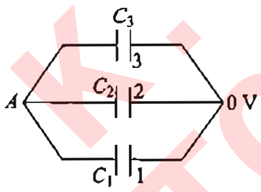


Fig. A1.174

83. d.

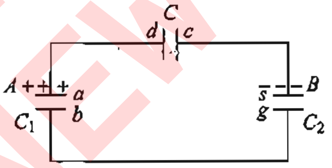


Fig. A1.175

Charge on positive plate of $A = C_1V_1$

Charge on negative plate of $B = -C_2V_2$

When d plate is of capacitor C is connected with the plate of A, then the total charge of (d, a) plate system will be C_1V_1 (conservation of charge).

Similarly on (C, S) plates total charge will be $-C_2V_2$

The total charge on b, g plate system will be $+C_2V_2 - C_1V_1$.

84. d. Let A be the point, where electric field is zero, which is at distance x from $4q$.

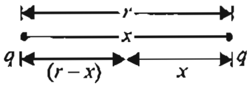


Fig. A1.176

$$\frac{1}{4\pi\varepsilon_0}\frac{4q}{x^2} = \frac{q}{4p\varepsilon_0(r - x)^2}$$

$$= \frac{2}{x} = \frac{1}{(r - x)}$$

$$2(r - x) = x$$

$$2r - 2x = x$$

$$x = \frac{2r}{x}$$

85. a.

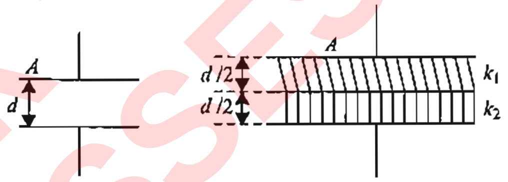


Fig. A1.177

$$C_0 = \frac{\varepsilon_0 A}{d}$$

$$C_1 = \frac{k_1\varepsilon_0 A}{d/2} \Rightarrow C_2 = \frac{k_2\varepsilon_0 A}{d/2}$$

C_1 and C_2 are in series

Equivalent dielectric constant $K = \dfrac{2k_1k_2}{k_1 + k_2}$

Which is the ratio of capacitor.

86. c. $+q$ will not charge restoring torque.

87. d. Max. length of string = $\sqrt{3}a$

$$\phi = \frac{\sqrt{3}\lambda a}{\varepsilon_0}$$

88. c. Radius of solid sphere of b, having charge enclosed in shell of radius q, charge Q

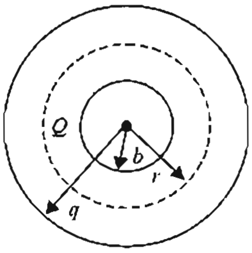


Fig. A1.178

(v_1) Potential due to solid sphere at distance $r = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r}$.

Potential inside the spherical shell.

$$(v_2) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{b}$$

Then potential due to both

$$v = \frac{1}{4\pi\varepsilon_0}\theta\left(\frac{1}{q} + \frac{1}{r}\right) = k\theta\left(\frac{1}{q} + \frac{1}{b}\right)$$

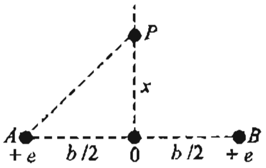


Fig. A1.179

89. b.

$$E_P = \frac{2Kex}{\left(x^2+\frac{b^2}{4}\right)^{3/2}}$$

For maximum E_P

$$\frac{dE_P}{dx}=0 \quad \Rightarrow x=\frac{b}{2\sqrt{2}}$$

90. a. $\phi_{ABCD} = -acd$ unit
$\phi_{CDEF} = -bcE_0$ unit

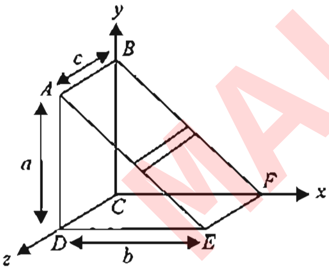


Fig. A1.180

$$\phi_{ABEF} = bcE_0 + c\int_o^a (d+x)\,dy$$

$$= +bcE_0 + acd + c\int_o^a x dy$$

$$= +bcE_0 + acd + \frac{ca}{b}\int_o^b x dx$$

$$\left[\text{ Since } \frac{x}{b}+\frac{y}{a}=1 \quad \Rightarrow \frac{dx}{b}=\frac{-dy}{a}\right]$$

$$= \left[+bcE_0 + acd + \frac{acb}{2}\right] \text{unit}$$

Using Gauss law

$$\phi_{net} = \frac{q_{in}}{\varepsilon_0} \quad \Rightarrow q_{in} = \frac{abc\,\varepsilon_0}{2}$$

91. b. $F_3 = \dfrac{k(a_b+q_d)}{r^2}$

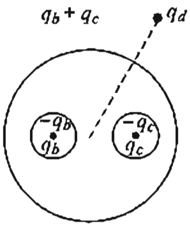


Fig. A1.181

$F_1 = F_2 =$ zero

92. d. $\Delta V = -\int_A^B \vec{E}\cdot dl =$ zero

93. a. Initial charge on capacitor C
$q = CV_0$

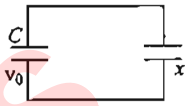


Fig. A1.182

When it is connected to uncharged capacitor of capacitance x then charge q gets distributed on both capacitors

So $q = q_1 + q_2$

or $x = \dfrac{C(V_0-V)}{V}$

Hence, choice (a) is correct.

94. c. Circuit is redrawn as shown in Fig. A1.183. Equivalent capacity of combination

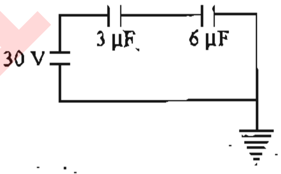


Fig. A1.183

$$C_{eq} = \frac{3\times 6}{9} = 2\mu F$$

Hence, charge $q = C_{eq}.\,V$
$q = 2\ \mu F \times 30\ V = 60\ \mu C$
$q = 60\ \mu C$
Hence, choice c is correct.

95. b. Energy given out by flash unit
$\Delta H = P \times t = 2000 \times 0.04$, $\Delta H = 80$ J
Energy of capacitor when fully charged

$$= \frac{1}{2}\times 40\times 10^{-6}\times V^2$$

or $\dfrac{1}{2}\times 40\times 10^{-6}\times V^2 = 80$

$$V^2 = \frac{2\times 80}{40\times 10^{-6}} = 4\times 10^6$$

$V = 2\times 10^3 = 2000$ V

Hence, choice (b) is correct

96. d. Circuit is redrawn as shown in Fig. A1.184

In the circuit 4 μF and 4μF capacitors are connected in series. Combination of this is in parallel with 1μF and 2μF capacitors.

Hence, $C_{eq} = 5\mu F$

97. d. The capacitance of capacitor $C = \dfrac{\varepsilon_0 A}{x}$

Charge on capacitor

$$q = CV = \frac{\varepsilon_0 AV}{x};\ U = \frac{1}{2}qV = \frac{1}{2}\frac{\varepsilon_0 AV^2}{x}$$

$$\text{So, } \frac{dU}{dt} = \frac{1}{2}\varepsilon_0 AV^2\left(-\frac{1}{x^2}\right)\frac{dx}{dt}$$

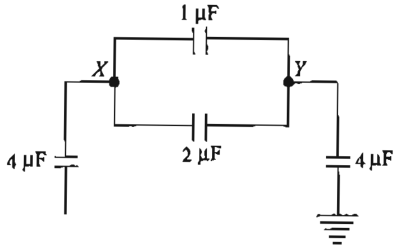


Fig. A1.184

$$= \frac{1}{2}\varepsilon_0 A V^2 v\left(-\frac{1}{x^2}\right)$$

From above expression $\frac{dU}{dt} \propto \frac{1}{x^2}$

98. b.

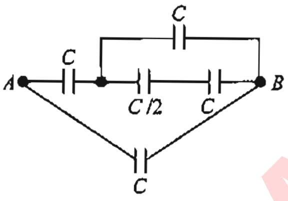


Fig. A1.185

$$C_{eq} = \frac{\frac{4C}{3} \times C}{\frac{7C}{3}} + C = \frac{11\,C}{7}$$

99. a. The total charge on each plate will have to be same. They should also carry opposite charges.

100. c. There will be two dipoles inclined to each other at an angle of 60°. The dipole moment of each dipole will be $q\lambda$. The resultant dipole moment

$$= \sqrt{(ql)^2 + (ql)^2 + 2\,(ql)\,(ql)\cos 60^\circ} = \sqrt{3}\,ql$$

101. d. $E = \frac{1}{4\pi\varepsilon_0}\left(\frac{\frac{4}{3}\pi r^3 \rho}{r^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{4}{3}\pi r\rho\right)$ or $E \propto r$

102. b. Potential V_1, due to a ring is given by:

$$V_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{\sqrt{R^2 + r^2}}$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{q}{\sqrt{R^2 + 3R^2}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{2R}$$

At the centre, $V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

$$\text{KE} = q_0(V_2 - V_1) = \frac{1}{4\pi\varepsilon_0}\left(\frac{qq_0}{2R}\right)$$

103. d. $E = \frac{\lambda}{2\pi\varepsilon_0 r} = -\frac{\partial V}{\partial r}$ or, $\int_{V_a}^{V_b} dV = -\int_a^b \frac{\lambda}{2\pi\varepsilon_0}\frac{dr}{r}$

$$V_a - V_b = \frac{\lambda}{2\pi\varepsilon_0}\log\left(\frac{b}{a}\right)$$

Hence $V_a - V_b \propto \log\frac{b}{a}$.

104. c. Potentials of the two spheres = $\pm e/c$

Potential difference = $(2e/c)$

or $\Delta V = \frac{2e}{4\pi\varepsilon_0 r}$

$$= 9 \times 10^9 \times \frac{2 \times 1.6 \times 10^{-19}}{9 \times 10^{-5}} = 32\,\mu\text{ V}$$

105. c. A positive charge which is free to move will always move from higher to lower potential.

106. d. $V = K\left[\frac{q}{x_0} - \frac{q}{2x_0} + \frac{q}{3x_0} - \frac{q}{4x_0} + \cdots\right]$

$$= \left(\frac{q}{x_0}\right) K\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{x_0}\log 2$$

107. d. The maximum length of the string which can fit into the cube $\sqrt{3}a$, is equal to its body diagonal. The total charge inside the cube is $\sqrt{3}a\lambda$, and hence the total flux through the cube is $\sqrt{3}a\lambda/\varepsilon_0$.

108. a. Though effective value of g will change but as the time period of spring block system does not depend on g, so time period will remain same, i.e., T.

109. a. $C = \frac{q}{V} = \frac{40}{20} = 2$ F

110. d. $C = C_0 K \Rightarrow K = \frac{C}{C_0} = \frac{110}{50} = 2.20$

111. d. $C = \frac{\varepsilon_0 A}{d}$

It is doubled, it becomes $(C/2)$

$$2C = \frac{C}{2}K$$

$$K = 4$$

112. c. On placing this aluminium sheet, it will form two capacitors in series, say initial separation is d.

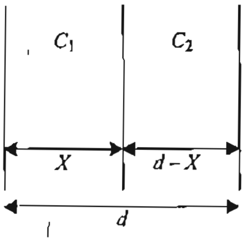


Fig. A1.186

$$C_1 = \frac{\varepsilon_0 A}{x}, \quad C_2 = \frac{\varepsilon_0 A}{d - x}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\frac{\varepsilon_0 A}{x} \cdot \frac{\varepsilon_0 A}{d - x}}{\frac{\varepsilon_0 A}{x} + \frac{\varepsilon_0 A}{d - x}}$$

$$= \frac{\varepsilon_0 A}{x\,(d - x)\left[\frac{1}{x} + \frac{1}{d - x}\right]} = \frac{\varepsilon_0 A}{x + d - x}$$

$$C_{eq} = \frac{\varepsilon_0 A}{d}$$

Hence, capacitance remains unchanged.

113. **b.** $\frac{1}{2}CV^2$ = mass × specific heat × change in temperature

$$V^2 = \frac{2 \times m \times s \times \Delta t}{C}; V = \sqrt{\frac{2ms\,\Delta t}{C}}$$

114. **a.** $\frac{q}{C} + IR - E = 0$ (Applying loop rule)

$$\frac{q}{C} = 3IR; 4 \qquad I = \frac{I_{max}}{4}$$

$$I = I_{max}e^{-t/RC}$$

$$\Rightarrow \frac{I_{max}}{4} = I_{max}\,e^{-t/RC}$$

$$\log 4 = \frac{t}{RC}$$

$$\Rightarrow \quad t = 2RC \log 2$$

$$t = 2 \times 4 \times 10^6 \times 2.5 \times 10^{-6} \times 0.693$$

$$t = 13.86 \text{ sec}$$

Work done is stored in the form of potential energy.

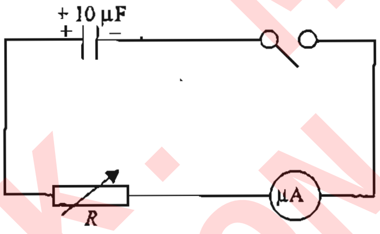


Fig. A1.187

115. **a.** $u = \frac{q^2}{2C} = \frac{(8 \times 10^{-18})^2}{2 \times 100 \times 10^{-6}} = \frac{64 \times 10^{-36}}{2 \times 10^{-4}} = 32 \times 10^{-32}$ Joule

116. **b.** Power = $\frac{\text{Energy}}{\text{Time}} = \frac{(1/2)\,CV^2}{\text{Time}}$

$$= \frac{40 \times 10^{-6} \times (3000)^2}{2 \times 2 \times 10^{-3}} = \frac{40 \times 10^{-6} \times 9 \times 10^6}{4 \times 10^{-3}}$$

$$= 9 \times 10 = 90 \text{ kW}$$

117. **c.** Net dipole moment = $\sqrt{3}\,qa$

$$\tau = -\sqrt{3}\;qa\frac{|\sigma_1 - \sigma_2|\theta}{2\varepsilon_0} = I\alpha$$

$$\frac{\sqrt{3}qa|\sigma_1 - \sigma_2|}{2\varepsilon_0} = I\left(\frac{2\pi}{T}\right)^2$$

$$T = 2\pi\sqrt{\frac{2\varepsilon_0 I}{\sqrt{3}qa\,|\sigma_1 - \sigma_2|}}$$

118. **a.** For equilibrium

$T = Mg$

And $T = F + mg \sin 30°$

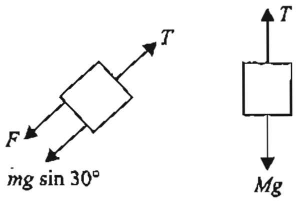


Fig. A1.188

$$\Rightarrow Mg = F + \frac{mg}{2}$$

$$\Rightarrow \quad m = \frac{m}{2} + \frac{F}{g} = \frac{m}{2} + \frac{E^2\varepsilon_0 A\,(k-1)}{2lgd}$$

119. **b.** The circuit can be redrawn as shown in Fig. A1.244. Charge on capacitor

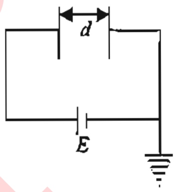


Fig. A1.189

$$Q = CE = \frac{\varepsilon_0 AE}{d}$$

Choice (a), (c), and (d) are wrong.

120. **a.** Applying W.E. theorem,

$W_{\text{Efield}} + W_{\text{ext}} = \Delta KE$

$$\Rightarrow \quad \frac{\sigma}{2\varepsilon_0}q\,(b-a) + W_{\text{ext}} = 0$$

$$\Rightarrow \quad W_{\text{ext}} = \frac{\sigma}{2\varepsilon_0}q(a-b)$$

121. **b.** Equivalent circuit is shown in Fig. A1.186.

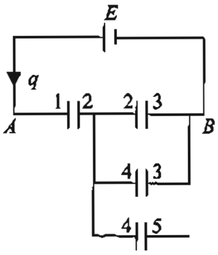


Fig. A1.190

Equivalent at capacitance = $\frac{C.2C}{C+2C} = \frac{2C}{3} = \frac{2}{3}\frac{\varepsilon_0 A}{d}$

$$q = CV = \frac{2}{3}\frac{\varepsilon_0 AE}{d}$$

122. **b.** Initially, charge $q' = CV$ and energy stored in capacitor = $\frac{1}{2}CV^2$

Finally, charge $q_2 = C \times 2V = 2CV$

Charge flown through the batter) is

$q' = q_2 - (-q) = 3CV$

Energy drawn from battery

$E_b = q' \times V' = 3CV \times 2V = 6CV^2$

Energy stored in capacitor = $\frac{1}{2}C\,(2V)^2 \Rightarrow E_2 = 2CV^2$
Change in potential energy of capacitor = $2CV^2$t $\frac{1}{2}CV^2$
$\Delta E_C = (3\,/\,2)\;CV^2$

Energy lost in heat is

$E_H = 6CV^2 - (3/2)CV^2 = (9/2)CV^2$

The ratio of $E_H/E_2 = 9/4 = 2.25$

123. **c.** Due to a solid hemispherical charge of $2Q$ the field will be $\frac{1}{4\pi\varepsilon_0} \times \frac{2Q}{d^2}$ along x-axis. So due to one hemisphere the component of field along x-direction will be $\frac{1}{4\pi\varepsilon_0} \times \frac{Q}{d^2}$.

And hence the net field will be more than $\frac{1}{4\pi\varepsilon_0} \times \frac{Q}{d^2}$.

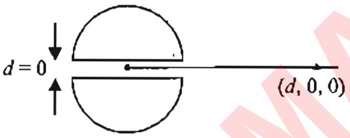


Fig. A1.191

Multiple Correct Answers Type

1. **b., c.** Energy density, $u = \frac{1}{2}\varepsilon_0 E^2$

2. **a., b., c., d.** Positive charge moves from a region of high electric potential to a region of low electric potential, while for negative charge it reverses. If some charge is present near the conductor, then its potential gets affected.

For Fig. A1.192 $\vec{E} = 0$ at point P but V is not

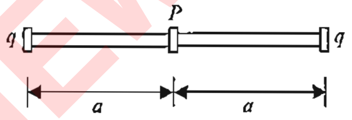


Fig. A1.192

For Fig. A1.193, $V = 0$ at point P but E is not.

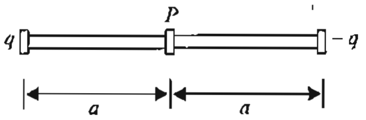


Fig. A1.193

3. **a., b., d.** At a center, the force experienced by the charge particle is zero, so it is a position of equilibrium. As we displace the charge from the equilibrium position, electric force starts acting on it towards equilibrium position and hence equilibrium is a stable one.

At a distance x from the center of sphere (equilibrium position), force experienced by the charge particle is

$F = \frac{qQx}{4\pi\varepsilon_0 R^3}$ [for $x < R$].

As $F \propto x$, so it performs SHM about the center.

Time period can be calculated by using $F = m\omega^2 x$

4. **b., c., d.** The path traced by q is shown in Fig. A1.194, the path is curvilinear and acceleration is due to the force exerted by Q on q.

The separation between them is minimum if relative velocity of the particle along the line joining them is zero. Let d be the minimum separation between them. As torque about Q is zero, so angular momentum remains conserved.

$mua = mvd \quad \Rightarrow \quad v = \frac{ua}{d}$

From energy conservation law, $\frac{mu^2}{2} = \frac{mv^2}{2} + \frac{1}{4\pi\varepsilon_0}\frac{qQ}{d}$

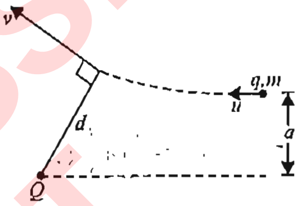


Fig. A1.194

5. **a., c.** Since the situation is having symmetry, the particles move symmetrically along the diagonal shown in Fig. A1.195.

The energy conservation law is to be used.

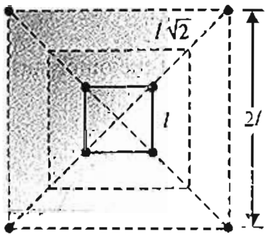


Fig. A1.195

$$U_i = \frac{1}{4\pi\varepsilon_0}\left[\frac{4q^2}{l} + \frac{2q^2}{\sqrt{2}l}\right]$$

$$U_f = \frac{1}{4\pi\varepsilon_0}\left[\frac{4q^2}{2l} + \frac{2q^2}{\sqrt{2} \times \sqrt{2}l}\right]$$

$$U_i = U_f + \frac{4mv^2}{2}$$

$$2mv^2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q^2}{l} + \frac{q^2}{l\sqrt{2}}\right]$$

$$v = \left[\frac{q^2}{8\pi\varepsilon_0 ml}\left(2 + \frac{1}{\sqrt{2}}\right]\right)^{1/2}$$

6. **a.,c.** $\vec{E} = 54\hat{i} + 74\hat{j},\ E = 90\ \text{NC}^{-1}$

$$90 = \frac{9 \times 10^9 Q}{r^2} \qquad \text{(i)}$$

$$V = 1800 = \frac{9 \times 10^9 Q}{r} \qquad \text{(ii)}$$

From equations (i) and (ii),

$$r = 20\,M, \quad q = 4\,\mu\text{C}$$

Now, $$\frac{(9 - x_0)\hat{i} + (4 - y_0)\hat{j}}{20} = \frac{54\hat{i} + 72\hat{j}}{90}$$

$$\Rightarrow \quad x_0 = -3, \quad y_0 = -12$$

7. a., b. Electric field lines will be from M to N, so potential of N will be less than that of M.

8. b., c., d. $V_P = \dfrac{kq}{r}$ (independent of x)

9. a., b., c. i. $E = \dfrac{2Q}{2A\varepsilon_0} + \dfrac{Q}{2A\varepsilon_0} \Rightarrow E = \dfrac{3Q}{2A\varepsilon_0}$

$$E = \frac{3}{2}\frac{Q}{Cd} \Rightarrow Ed = \frac{3Q}{2C} = V$$

ii. $F = E\,(-Q);\ F = \left(\dfrac{2Q}{2A\varepsilon_0}\right) \times \dfrac{(-Q)}{1} = -\dfrac{Q^2}{A\varepsilon_0}$

$$\Rightarrow F = \frac{Q^2}{A\,\varepsilon_0}$$

iii. Energy $= \dfrac{1}{2}\varepsilon_0\, E^2\, Ad = \dfrac{1}{2}\varepsilon_0 \left(\dfrac{3Q}{2Cd}\right)^2 Ad = \dfrac{9}{8}\dfrac{Q^2}{C}$

10. a., b., c. Time of flight $(t) = \dfrac{2u}{g} = \dfrac{2 \times 10}{10} = 2$ sec

$$H = \frac{u^2}{2g} = \frac{10^2}{2 \times 10} = 5\text{ m}$$

$$R = 0 + \frac{1}{2}\left(\frac{qE}{m}\right)t^2 = \frac{1}{2} \times \frac{10^{-3} \times 10^4 \times 2 \times 2}{2} = 10\text{ m}$$

11. a., c., d. Charge density at the outer surface (Fig. A1.196)

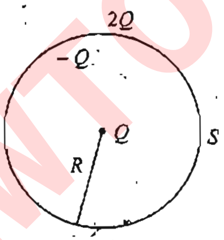


Fig. A1.196

$$= \frac{2Q}{4\pi R^2} = \frac{Q}{2\pi R^2}$$

$$E_{\text{inside}} = \frac{Q}{4\pi\varepsilon_0 R^2},\ E_{\text{outside}} = \frac{2Q}{4\pi\varepsilon_0 R^2}$$

$$\therefore \quad E_{\text{outside}} = 2E_{\text{inside}}$$

12. a., b., c. $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r} = 600$ volt

And $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2} = 200\text{ NC}^{-1}$

Hence, $\dfrac{V}{E} = r = \dfrac{600}{200} = 3$ metre

Now, $\dfrac{q}{4\pi\varepsilon_0 r} = 600;\ \dfrac{q}{4\pi\varepsilon_0 r'} = V'$

$$\therefore \quad \frac{v'}{600} = \frac{r}{r'} = \frac{3}{9}$$

or $\quad V' = 200$ volt

$$W = q\,(V - V') = 10^{-6}(600 - 200)$$
$$= 4 \times 10^{-4}\text{ J}$$

13. c., d. $\vec{r}_P = \hat{i} + 2\hat{j} + 4\hat{k};\ \vec{r}_q = 3\hat{i} + 2\hat{j} + \hat{k}$

$$|\vec{r}_q - \vec{r}_P| = \sqrt{[(2)^2 + (-3)^2]} = \sqrt{13}$$

As $\quad q = 2 \times 10^{-8}$ coulomb, hence

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}_q - \vec{r}_p|}$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-8}}{\sqrt{13}} = 49.9 \text{ volt}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}_q - \vec{r}_P|^2}\frac{(\vec{r}_q - \vec{r}_P)}{|\vec{r}_q - \vec{r}_P|}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}_q - \vec{r}_P|^3}(2\hat{i} - 3\hat{k})$$

[By filling a dielectric of dielectric constant K, electric field gets decreased as $E = \dfrac{E_0}{K}$ and $K > 1$.]

14. a., c., d. $V(x) = 4 + 5x^2$

$V(x = 1) = 9$ V

And $V\,(x = -2) = 24$ V

Hence, $\Delta V = 15$ V

$$E = -(\Delta V/\Delta x) = -10\,x$$

$\therefore \quad E$ (at $x = -1$ m) $= 10\text{ NC}^{-1}$

So, $F = qE = +10$ N (along +ve x-axis).

15. a., b., c. $E = -(\Delta V / \Delta x) = -B - 2Cx$,

i.e., E varies linearly with x and is along negative x-direction. It is also clear that B has got dimensions or units of E, i.e., NC^{-1}.

16. a., d. Suppose a positively charged sphere is brought near an uncharged metallic sphere, then on nearer surface of the uncharged sphere, negative charge is induced and on farther surface, positive charge is induced. Hence, a force of attraction will be observed between these two spheres. Therefore, if a force of attraction is observed between a positively charged sphere and a metallic sphere, it cannot be concluded that the metallic sphere is necessarily negatively charged. Therefore, option (b) is wrong and options (a) and (d) are correct.

17. a., b., c. If there is no external electric field, then the charge given to a conducting sphere gets uniformly distributed over its surface. Therefore, option (a) is correct.

If an external electric field exists, then the charge gets distributed over the surface of the sphere in such a way that the electric field inside the sphere can become equal to zero. Hence, distribution of the charge on the surface of sphere will be non-uniform.

Therefore, option (b) is correct. Obviously, option (d) is wrong.

Since electric field inside the conducting sphere is equal to zero, therefore, potential difference between two points in the sphere is equal to zero. It means, the potential is same at every point of the sphere. Therefore, option (c) is correct.

18. a., c. As A and C are earthed, they are connected to each other. Hence, '$A + B$' and '$B + C$' are two capacitors with the same potential difference. If B is closer to A than to C, then the capacitance $C_{AB} > C_{BC}$. The upper surface of B will have greater charge than the lower surface. As the force of attraction between the plates of a capacitor is proportional to Q^2, there will be a net upward force on B. This can balance its weight.

19. b.,c.,d. Charge on inner sphere can be supposed to be concentrated as a point charge at the center, hence electric field at a point in the region between the spheres at a distance r from the center $= (q/4\pi\varepsilon_0 r^2)$. Due to induction, equal and opposite charges will appear on the inner and outer surfaces of the outer sphere. Hence, net charge which can be supposed to be placed at the center $= q$ and electric field due to it at a point outside the hollow sphere, at a distance r from center $= (q/4\pi\varepsilon_0 r^2)$.

Potential of inner sphere $= \dfrac{q}{4\pi\varepsilon_0}\left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right)$

And potential of outer sphere $= \dfrac{q}{4\pi\varepsilon_0 c}$

Potential of inner sphere with respect to the outer sphere

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

20. b., d. Potential is same everywhere ($\neq 0$) inside a uniformly charged spherical conductor.

i.e., $\Delta V = 0$

As $E = -(\Delta V/\Delta x)$

Hence, electric field is zero everywhere.

21. a., c. If positive test charge is displaced along x-axis, then net force will always act in a direction opposite to that of displacement and the test charge will always come back to its original position. But if test charge is displaced along y-axis, it will never come back to its original position and will fly away along y-axis.

22. b., c., d. Potential of innermost shell is zero.

$$\frac{q_1}{r}+\frac{q_2}{2r}+\frac{q_3}{3r}=0$$

or $$6q_1+3q_2+2q_3=0 \qquad \text{(i)}$$

Similarly, potential on outermost shell is also zero.

$$\frac{q_1}{3r}+\frac{q_2}{3r}+\frac{q_3}{3r}=0$$

or $$q_1+q_3=-q_2 \qquad \text{(ii)}$$

Solving equations (i) and (ii), we get

$$q_1=-\frac{q_2}{4},\quad \frac{q_3}{q_1}=3 \text{ and } \frac{q_3}{q_2}=-\frac{3}{4}$$

∴ Options (b), (c) and (d) are correct.

23. a., b. If the charge is given to a conducting sphere, then an electric field is established in the surrounding space. Magnitude of electric field is maximum just outside the sphere. This maximum electric field may be increased to the dielectric strength of the surrounding medium. Therefore, there is a limiting value of maximum charge which can be given to the conducting sphere. Hence, option (c) is wrong. Obviously, the conducting sphere cannot be charged to a potential greater than a certain value. Hence, option (a) is correct. It can be easily said that option (b) is also correct.

24. b., c. The situation is shown in the Fig. A1.197.

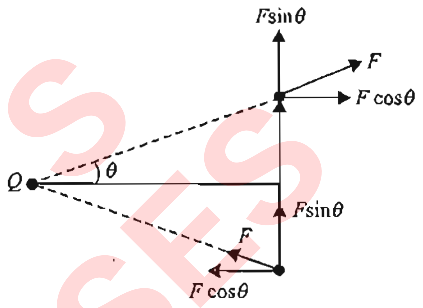


Fig. A1.197

$$\vec{F}_{\text{net}} = 2F\sin\theta \uparrow = \frac{PQ}{4\pi\varepsilon_0 r^3}$$

$$\vec{\tau} = F\cos\theta \times 2a \text{ in clockwise direction}$$

$$= \frac{PQ}{4\pi\varepsilon_0 r^2}$$

25. c.,d. Torque will be perpendicular to the line $y = 2x$ and it should be in x–y plane, because electric field is in z-direction. The lines in options (c) and (d) both are perpendicular to $y = 2x$.

26. a., b., c., d. Points A and B lies within same metal hence $V_A = V_B$; The potential inside a hollow sphere is same as potential at the surface hence $V_A = V_B = V_C = V_0$.

27. b., d. The electric field inside any point of the sphere is zero.

28. a., c. $\vec{E}_A$ is along $\overrightarrow{OA}$ and $\overrightarrow{OA} = \hat{i}+2\hat{j}+3\hat{k}$

$\vec{E}_B$ is along $\longrightarrow OB = \hat{i}+\hat{j}-\hat{k}$

Since $\overrightarrow{OA}\,\overrightarrow{OB}(\hat{i}+2\hat{j}+3\hat{k}).(\hat{i}+\hat{j}-\hat{k})=0$

So $\overrightarrow{OA}\perp\overrightarrow{OB} \Rightarrow \vec{E}_A\perp\vec{E}_B$

So, choice (a) is correct.

Since $E_B = \dfrac{kq}{|OB|^2} = \dfrac{kq}{3}$

$E_C = \dfrac{kq}{|OC|^2} = \dfrac{kq}{12}$

So $\dfrac{E_B}{E_C} = 4$ or $|\vec{E}_B| = 4|\vec{E}_C|$

So, choice (c) is correct.

Choice (b) and (d) are wrong from above explanation.

29. a., d. When capacitors are connected in parallel, initial capacitance is $C = \dfrac{2\varepsilon_0 A}{d}$. After the distance between the plates is changed, the capacitance becomes

$$C = \frac{\varepsilon_0 A}{d+a}+\frac{\varepsilon_0 A}{d-a} \text{ or}$$

$$C = \frac{2\varepsilon_0 A}{d-(a^2/d)}$$

which is greater than initial one. Hence **a.** is correct and **b.** is wrong.

When capacitors are connected in series.

$$\frac{1}{C} = \frac{2d}{\varepsilon_0 A}$$

After the distance between the plates is changed

$$\frac{1}{C} = \frac{d+a}{\varepsilon_0 A} + \frac{d-a}{\varepsilon_0 A} = \frac{2d}{\varepsilon_0 A}$$

That is, the capacitance remains unchanged. Hence **d.** is correct and choice **c.** is wrong.

30. a., d. Let q_1 and q_2 be the instantaneous charges on capacitors. Since they are in parallel,

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \text{and } q_1 + q_2 = Q$$

$$C_1 = \frac{\varepsilon_0 A}{d_0 + vt}, \ C_2 = \frac{\varepsilon_0 A}{d_0 - vt}$$

$$\text{So } \frac{q_1}{q_2} = \frac{C_1}{C_2} = \frac{d_0 - vt}{d_0 + vt} \Rightarrow q_2 \left(\frac{d_0 - vt}{d_0 + vt}\right) + q_2 = 0$$

$$\text{So } q_2 = \frac{Q\,(d_0 + vt)}{2d_0} \text{ and } q_1 = \frac{Q\,(d_0 - vt)}{2d_0}$$

Hence, choice (a) is correct and choice (b) is wrong.

$$i = \frac{-dq_1}{dt} \text{ or } \frac{dq_2}{dt} \text{ or } i = \frac{Q\,v}{2d_0}$$

which does not depends on time. So choice (d) is correct and choice (c) is wrong.

31. a., b., c. In position 1 Stored energy $= \frac{1}{2}(E_1 - E_2)^2 C$

In position 2 Stored energy $= \frac{1}{2} C E_1^2$

Additional energy drawn from battery

$\Delta q = E_1 C - (E_1 - E_2)C = E_2 C$

So choice (a) is correct.

Extra energy drawn from battery

$= E_1 \Delta q = E_1 E_2 C$

So choice (b) is correct.

Heat produced = Loss in stored energy + extra energy drawn from battery

$$= \frac{1}{2}(E_1 - E_2)^2 C - \frac{1}{2} E_1^2 C + E_1 E_2 C = \frac{1}{2} E_2^2 C.$$

Choice (c) is correct and choice (d) is wrong.

32. a., b., c.

Sol. When capacitor C is charged to a potential difference of V_0, it has a charge q_0 but when C and C_x are connected by closing the switch S, q_0 is shared by two capacitors. Let q_1 and q_2 be the charged of C and C_x then

$$q_0 = q_1 + q_2$$

$$C V_0 = C V + C_x V$$

$$\text{or } C_x = \frac{C\,(V_0 - V)}{V}$$

Hence, (a) is correct.

Final energy stored

$$U_f = \frac{1}{2} C V^2 + \frac{1}{2} C_x V^2$$

$$= \frac{1}{2} C V^2 + \frac{1}{2} \frac{C(V_0 0 - v)}{V} V^2$$

$U_f = \frac{1}{2} C V_0 V$. So choice (b) is correct.

Initial energy stored

$$U_i = \frac{1}{2} C V_0^2$$

Heat generated $= U_i - U_f = \frac{1}{2} C V_0^2 - \frac{1}{2} C V_0 V$

$$\Rightarrow \quad H = \frac{C V_0 (V_0 - V)}{2}$$

So, choice (c) is correct.

Choice (d) is wrong which is obvious from above explanation.

33. a., c. Let the length, width and thickness of slab be l, b and d respectively. K be the dielectric constant of dielectric.

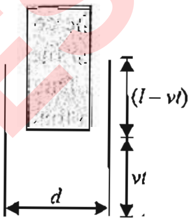


Fig. A1.198

At time, t length vt of the slab has been pulled out, so at this instant the capacitor may be considered as a parallel combination of two capacitors as shown. The capacitance of combination is

$$C = \frac{\varepsilon_0 vtb}{d} + \frac{\varepsilon_0 K b\,(l - vt)}{d};$$

$$C = \frac{\varepsilon_0 b}{d}[vt + K(l - vt)]$$

$$\Rightarrow C = \frac{\varepsilon_0 b}{d}[Kl - (K - 1)vt]$$

This equation is of straight line, having positive intercept and negative slope. Hence choice (a) is correct and choice (b) is wrong.

Since battery is disconnected, so charge on capacitor remains constant.

$$V = \frac{q}{C} \text{ or } V \propto \frac{1}{C}$$

Hence, current between V and C will be rectangular hyperbola.

So, choice (c) is correct and choice (d) is wrong.

34. a., c. Initial stored energy $U_i = \frac{1}{2} \frac{\varepsilon_0 A V_1^2}{x}$

Final stored energy $U_f = \frac{1}{2} \frac{\varepsilon_0 A V_0^2}{2(x + dx)}$

$$\text{so } \Delta U = U_f - U_i = \frac{1}{2} \varepsilon_0 A V_0^2 \left[\frac{1}{x + dx} - \frac{1}{x}\right]$$

$$= \frac{1}{2} \varepsilon_0 A V_0^2 \left[\frac{x - x - dx}{x(x + dx)}\right]$$

$$= -\frac{1}{2} \frac{\varepsilon_0 A V_0^2}{x^2} dx = -\frac{1}{2} \frac{\varepsilon_0 A V_0^2}{x} \cdot \frac{dx}{x} = -\frac{U\,dx}{x}$$

So, choice (a) is correct and choice 9b0 is wrong.

$$F = -\frac{dU}{dx} = -\frac{U}{x}$$

$$F = -\frac{1}{2} \frac{\varepsilon_0 A}{x} \cdot \frac{V_0^2}{x} = -\frac{1}{2}\left(\frac{\varepsilon_0 A}{x} \cdot V_0\right) \cdot \frac{V_0}{x} = -\frac{1}{2} q E$$

So magnitude of attractive force is $\frac{1}{2} q E$.

So, choice (c) is correct and choice (d) is wrong.

35. a., c. $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}+\frac{1}{4\pi\varepsilon_0}\frac{q}{(R_0-vt)}=0$

where is the initial distance of the charged particle.

$$Q=\frac{Rq}{R_0-vt}\Rightarrow\frac{dQ}{dt}=i=\frac{Rqv}{(R_0-vt)^2}$$

36. a., d.

Sol. $E=at(a=\text{constant})\ F=QE$

$a=F/m=QE/m=Es=ast$

$a=\frac{dv}{dt}=ast$

or $v=\frac{1}{2}ast^2v\propto s$ and t^2

Assertion-Reasoning Type

1. **a.** Electric field aligns the randomly oriented dipoles in its own direction, thus producing some net dipole moment in the material.
2. **d.** Capacitance increases and electric field decreases.
3. **a.** If $E=0$ in some region, then potential is same everywhere in that region.
4. **d.** When an electron and a proton are placed in the same uniform electric field, they experience equal forces but different accelerations. Hence, $m_ea_e=m_pa_p$
   Since $m_pa_p=1837m_e$, so $a_p<a_e$.
5. **c.** Where E is high, V may be low.
6. **d.** An electrically neutral body means charge on the body is zero. A body connected to earth may possess some charge.
7. **d.** For a non-uniformly charged thin circular ring with net zero charge, electric potential at each point on its axis is zero. Hence, electric field at each point on its axis must be perpendicular to the axis. Therefore, assertion is false and reason is true.
8. **a.** The electric field due to disk is superposition of electric fields due to its constituent rings as given in reason. Assertion is true, reason is true; reason is a correct explanation for assertion.
9. **a.** (Moderate) Electric flux within any closed surface in region of uniform field is zero because the total number of electric lines of force entering the closed surface equals that leaving the surface. Hence, from Gauss theorem the net charge enclosed within such a closed surface is zero.
10. **d.** If potential difference across an isolated charged capacitor is doubled by doubling separation between plates, the energy stored in capacitor from $U=\frac{Q^2}{2C}$ becomes double of previous value. Hence, statement I is false.
11. **c.** From work–energy theorem,
    Final K.E. – Initial K.E. = work done by non-uniform electric field
    As initial K.E. = 0 and final K.E. cannot be negative.
    ∴ work done by non-uniform electric field on a charged particle starting from rest is non-negative.
    Hence, statement I is true.
    Consider a situation in which two point charges $+Q$ are fixed some distance apart. At some distance left of equilibrium point O, a charge $+q$ is released from rest. After the charge $+q$ moving towards right crosses O, it experiences a force towards left.
    Hence, statement II is false.

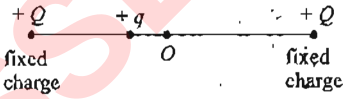


Fig. A1.199

12. **b.** Electric lines of force never cross each other because if electric lines of force cross each other, then the electric field at the point of intersection will have two directions simultaneously which is not possible.
13. **a.** In a hollow spherical shield (hollow), the charge is present on its surface but charge is zero at every point inside the hollow sphere. Hence, the metallic shield in the form of a hollow shell may be built to block an electric field.
14. **b.** When the bob is placed in an electric field, the bob time period of simple pendulum having charged bob is decreased because there will be an increase in the restoring force.
15. **a.** Earth is a good conductor of very large size. The capacitance of earth is very large. If some charge is given to earth or some charge drawn from earth, it does not affect the original potential. Thus, the potential of grounded object is supposed to be zero.
16. **a.** Free electrons present in the metal are moving randomly in all directions, in absence of electric field. Hence, the average velocity of electrons is zero. Because of it the current does not flow in the metal in the absence of electric field.
17. **c.** If V is the accelerating potential and v is the velocity of electron, then

$eV=\frac{1}{2}mv^2$ Or $v\propto\sqrt{V}$

$$\therefore\quad\frac{v_1}{v_2}=\sqrt{\frac{V_1}{V_2}}$$

Here, $V_1=V,\ V_2=2V.$

$$\therefore\quad\frac{v_1}{v_2}=\sqrt{\frac{V}{2V}}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow\quad v_2=\sqrt{2}v_1=1.4v_1$$

Reason is a false statement, as conditions must be discussed for electron to move on circular path.

18. **a.** We know that surface density of a charge is very large on the sharp ends of a conductor. Hence the lightning conductor conducts charges of lighting to earth and there is no harm to the building.

19. **b.** On an equipotential surface, the potential at each point is same throughout.

Let $\vec{E}$ be electric field at a point on an equipotential surface. Then, small work done in moving a test charge q_0 through a small displacement along the surface.

$$dW = \vec{E} \cdot d\vec{r} = (q_0 \vec{E}) \cdot d\vec{r}$$

Since work in moving a test charge along an equipotential surface is always zero,

$$\therefore \quad (-q_0 \vec{E}) \cdot d\vec{r} = 0 \text{ or } \vec{E} \cdot d\vec{r} = 0$$

Hence, electric field is directed perpendicular to the surface.

20. **d.** A charged body and an uncharged body can attract each other. When such bodies are placed near each other, the induced charges of opposite kind are produced on the uncharged body.

21. **d.** The charged particle may or may not move along an electric line of force. If the charged particle was initially at rest, it will move along an electric line of force. In case the charged particle has some initial velocity making certain angle with a line of force, then its resultant path will not be along the line of force.

22. **a.** If a body acquires positive charge, then it means it has lost a few electrons. In this way, its mass decreases.

23. **c.** We know that when a high-energy X-rays beam falls on metallic ball, the ball will emit photoelectrons. So, the ball will acquire positive charge because of which it will be deflected in the direction of electric field till equilibrium is reached.

24. **c.** We know that common potential will be between two initial values, so it will be less than the potential of one of the conductors. Since both have positive charge, so they will repulse each other.

25. **a.** A polar molecule has intrinsic dipole moment or permanent dipole moment, so it is called an electric dipole. In a polar molecule, the center of positive and negative charges does not coincide with each other because of asymmetric shape of molecule.

26. **a.** It is true that body of a vehicle is charged when the vehicle is moving through air at high speed. Because of it, the vehicles which are carrying highly inflammable material have hanging chains, which touch slightly the ground. This chain transfers the charge to the ground (earth). Hence, there is no harm to the vehicle.

27. **a.** The tangent at a point on the electric line of force tells the direction of electric field changes from point to point. So, the lines of force are curved lines. Further, they are continuous curves and cannot have sudden breaks otherwise it will indicate the absence of electric field at the break point.

28. **c.** If a dielectric slab of dielectric constant K is filled in between the plates of a condenser while charging it, the potential difference between the plates does not change, but the capacity becomes K time. Therefore,

$$V' = V; C' = KC$$

Energy stored in the capacitor

$$\therefore \quad U' = \frac{1}{2} C'V'^2$$

$$= \frac{1}{2}(KC)(V^2) = \left(\frac{1}{2}CV^2\right) K = KU$$

Thus, energy stored becomes K times.
Surface charge density,

$$\sigma' = \frac{q'}{A} = \frac{C'V'}{A} = \frac{KCV}{A} = K\frac{q}{A} = K\sigma$$

29. **b.** The capacitance of a capacitor

$$C = \frac{K\varepsilon_0 A}{d} \propto \frac{K}{d}$$

$$\text{Hence, } \frac{C_1}{C_2} = \frac{K_1}{d_1} \times \frac{d_2}{K_2} = \frac{K}{d} \times \frac{\frac{d}{2}}{3K} = \frac{1}{6}$$

$$\text{or} \quad C_2 = 6C_1$$

Again, capacity of a capacitor $C = \frac{Q}{V}$

Therefore, capacity of a capacitor does not depend upon the nature of the material.

30. **c.** In series combination,

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

In parallel combination, $C_P = C_1 + C_2 + C_3$
Thus, it is obvious that $C_P > C_S$.

31. **b.** If the medium of dielectric constant K is filled in between the plates of a condenser after removing the connection of battery from the plates of the condenser, then capacitance increases K times. Also, the potential energy reduces to $\frac{1}{K}$ times.

32. **b.** The total energy stored in series combination of capacitors is the sum of energies stored in the individual capacitors, i.e., $U = U_1 + U_2 + U_3 + \cdots$. It is also true that energy is a scalar quantity.

33. **d.** From the relation,

$$C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

where, t is the thickness of dielectric and $K = \infty$. If the capacitor is filled completely with a metallic slab, then thickness becomes equal to the distance between plates i.e., $t = d$. Hence C will be equal to infinity. It means that when a capacitor is filled completely with metal, the capacitor will be short circuited. Hence, it cannot work as a capacitor.

34. **b.** A capacitor does not discharge itself. In case the capacitor is connected in a circuit containing a source of high voltage, the capacitor charges itself to a very high potential. So, if a person handles it without discharging, he may get a severe shock.
Dielectrics and insulators are the same.

35. **d.** Potential at the surface of metallic sphere is given by

$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ (Here, $r = 1$ cm $=10^{-2}$ m, $q = 1$ C)

$\therefore \quad V = 9 \times 10^9 \times \frac{1}{10^{-2}}$ or $V = 9 \times 10^{11}$ V

Now, this large potential will ionise the air surrounding the metallic sphere, therefore the charge will leak away.

Statement II follows from relation $Q = CV$,
i.e., electric charge = capacitance × electric potential.

36. **b.** The capacitance of parallel plate capacitor is given by,

$$C = \frac{\varepsilon_0 A}{d}$$

where A is area of the plates and d is the distance between plates.

$$\text{Hence, } \frac{C'}{C} = \frac{\frac{\varepsilon_0 A}{d}}{\left(\frac{\frac{\varepsilon_0 A}{d}}{2}\right)} = 2 \quad \Rightarrow \quad C' = 2C$$

When a dielectric of dielectric constant K is introduced in between the plates, then the capacitance

$$C = \frac{K\varepsilon_0}{d} \quad \Rightarrow \quad C \propto K$$

i.e., C depends on induced dielectric constant.

37. **d.** The total charge on each plate will have the charge in same quantity, as charge is independent of area. But the charge will be of opposite nature.

38. **c.** When an electric field is applied to the dielectric, each molecule of dielectric gets polarised, i.e., centers of gravity of positive and negative charges get displaced from each other. On the left face, a net negative charges $-q_i$ appears. Thus, electric dipoles are produced inside. This is the dielectric polarization.

39. **b.** When a glass sheet is introduced between the plates of a condenser, then capacitance will increase. Since no battery connection is made, so charge will remain same.

Hence, from the relation $V = \frac{q}{C}$ or $V \propto \frac{1}{C}$, potential decreases.

40. **c.** The capacitance of a spherical conductor of radius r is given by $C = 4\pi\varepsilon_0 r$.
The radius of spherical conductor having capacitance 1 F is given by

$$r = \frac{C}{4\pi\varepsilon_0} = 1 \times 9 \times 10^9 = 9 \times 10^9 \text{ m} = 9 \times 10^6 \text{ km}$$

Since one cannot have a spherical conductor of such a big radius, even greater than earth, so it is not possible to make a spherical capacitance of 1 F and earth itself cannot have capacitance of 1 F.

41. **b.** Torque acting on the dipole is also zero when it is oppositely directed w.r.t. electric field.

42. **c.** In a medium, $F_m = \frac{F}{K}$
From above expression, it is quite clear that greater the value of K smaller is the force between the two charges. Electric dipole moment is directly proportional to dielectric constant.

43. **c.** We know that for an electric dipole,

$$E_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2p}{r^3}\right) \text{ and } E_{\text{equatorial}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{p}{r^3}\right)$$

$$\text{Hence, } \frac{E_{\text{axial}}}{2} = E_{\text{equatorial}}$$

$$\text{or} \qquad E_{\text{equatorial}} = \frac{E}{2}$$

Reason is false as electric field due to dipole varies inversely as cube of distance, i.e., $E \propto \frac{1}{r^3}$.

44. **a.** Magnetic field acting on particle is given by

$$F = q(v \times B)$$

Power associated with force $P = Ev = 0$
(Since F is perpendicular to v)
Magnetic force does no work and hence K.E. remains constant. But force acts so momentum changes. So both assertion and reason are true and Reason gives explanation for assertion.

45. **b.** Statement I and Statement II are true statements but Statement II do not explain Statement I.

Comprehension Type

For Problems 1–3

1. a., 2. c., 3. b.

Sol. $\phi = \frac{q_{en}}{\varepsilon_0} = \frac{q_1 + q_2 + q_3}{\varepsilon_0}$

$$= \frac{(+3.1 \times 10^{-9} - 5.9 \times 10^{-9} - 3.1 \times 10^{-9})}{8.85 \times 10^{-12}}$$

$$= -666 \text{ Nm}^2\text{C}^{-1}$$

2. c. By replacing the plastic plate with Aluminium plate of same dimensions, the charge density (i.e., σ) does not get affected. Hence, electric field will remain constant.

3. b. Due to induction, some positive charge will lie within Gaussian surface drawn and hence flux becomes positive.

For Problems 4–7

4. a., 5. b., 6. c., 7. a.

Sol. $2[m_1 g - B] + m_2 g = 0$

$$2[V\rho_{He} g - V\rho_{air} g] = -m_2 g$$

$$V = \frac{-m_2}{2[\rho_{He} - \rho_{air}]} = \frac{1.1 \times 10^{-3}}{2[1.3 - 0.2]} = \frac{10^{-3}}{2} \text{ m}^3$$

$$= \frac{10^3}{2}\text{cm}^3 = 500\ \text{cm}^3$$

$$T\cos\theta = \frac{kq^2}{r^2};\ T\ (0.6) = \frac{9\times10^9\times q^2}{(1.2)^2} \qquad \text{(i)}$$

$$T\ \sin\theta = \frac{m_2 g}{2};\ T\ (0.8) = \frac{1.1\times10^{-3}\times10}{2} \qquad \text{(ii)}$$

From (i) and (ii),

$$\frac{0.6}{0.8} = \frac{9\times10^9\times q^2}{(1.2)^2}\times\frac{2}{1.1\times10^{-3}\times10}$$

$$\Rightarrow \quad q = 8.12\times10^{-7}\ \text{C} = 0.8\ \mu\text{C}$$

$$\text{and } T = \frac{9\times10^9\times q^2}{(1.2)^2\times(0.6)} = 6.875\times10^{-3}\ \text{N}$$

Also, $T\sin\theta = 5.5\times10^{-3}\,\text{N}$

For Problems 8–10

8. c. From Fig. A1.200

Sol.

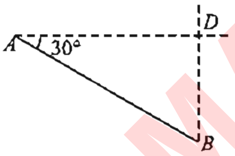


Fig. A1.200

$$V_A - V_B = V_A - V_D = E\ AD$$

$$= E\ AB\ \cos\ 30^\circ = E2\sqrt{3}\frac{\sqrt{3}}{2} = 3E$$

9. b. $V_C - V_B = V_F - V_B = E\ (BF)$ (Fig. A1.201)

$$= -E(BC)\ \cos 60^\circ = -E\times4\times\frac{1}{2} = -2E$$

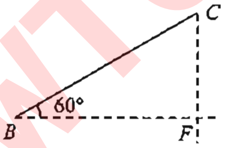


Fig. A1.201

10. d. $W_{el} = q\ (V_A - V_C)$

$$= q\ (V_A - V_B) + (V_B - V_C) = q\ (3E + 2E) = 5qE$$

For Problems 11–13

11. c., 12. c., 13. a.

Sol. $E = -\dfrac{dV}{dx} = -4x$

12. c. $F = qE = 2.5\times10^{-6}(-4x) = -10^{-5}x$

$$W = \int_2^0 F\,dx = -10^{-5}\int_2^0 x\,dx$$

$$\frac{1}{2}mv^2 = 10^{-5}\left[\frac{x^2}{2}\right]_0^2 \quad \Rightarrow V = 2\ \text{ms}^{-1}$$

13. a. $F = -10^{-5}x$

$ma = -10^{-5}x \quad \Rightarrow \quad 10\times10^{-6}a = -10^{-5}x$

$\Rightarrow a = -x$

$$\omega^2 = 1,\ \omega = 1\ \text{rad.} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{1} = 2\pi\ \text{s}$$

For Problems 14–17

14. c., 15. a., 16. c., 17. b.

Sol. Since $Q = +8\ \mu$C, if q is a positive charge, resultant force on it due to Q at A and Q at B will be along positive Y-axis and it would move away along Y-axis. But the charge q here is observed to oscillate. This is possible only if q is a negative charge so that resultant force on it due to Q at A and Q at B is towards O. Under the action of this force, q moves towards O, crosses O and as it is moving along negative Y direction, resultant force on it will again be towards O. This force retards the motion of q along negative Y-axis. It comes to rest at some point and then moves back towards O and so on (Fig. A1.202).

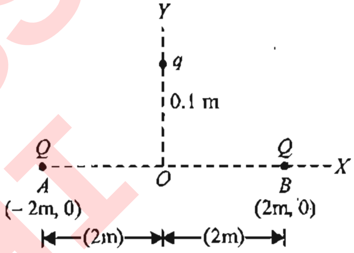


Fig. A1.202

Force applied by Q on q has a magnitude

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{y^2 + a^2}$$

Force applied by Q at A on q can be resolved into rectangular components: F cos θ and F sin θ. Similarly, force applied by Q at B on q can be resolved into components: F cos θ and F sin θ. F sin θ components of the two forces balance each other so that the net force on q is $2F$ cos θ towards O.

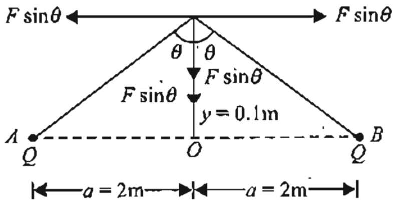


Fig. A1.203

$$\therefore \text{ Net force on } q,\ F_n = 2\frac{1}{4\pi\varepsilon_0}\frac{Qq}{y^2+a^2}\cos\theta$$

$$F_n = \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{y^2+a^2}\frac{y}{(y^2+a^2)^{1/2}} = \frac{1}{4\pi\varepsilon_0}\frac{2Qqy}{(y^2+a^2)^{3/2}} \qquad (1)$$

For $y << a$,

$$\text{net force on } q,\ F_n = \frac{1}{4\pi\varepsilon_0}\frac{2Qqy}{a^3} \qquad (2)$$

Here, $Q = 8\mu\text{C} = 8\times10^{-6}\,\text{C}$

At $t = 0$, q is at $y = 0.1$ m. Obviously $y < a$ (= 2 m). Since the motion is simple harmonic, we can use the approximation $y << a$ so that net force, from equation (2), will be proportional to displacement (y). Initially, i.e., at $y = 0$ m, force on q is 9×10^{-3} N.

Using equation (i).

$$9 \times 10^{-3} = (9 \times 10^{9})\frac{2\,(8 \times 10^{-6})\,q\,(0.1)}{(2)^3}$$

$$\Rightarrow \qquad q = 5 \times 10^{-6} C = 5\,\mu\text{C}$$

This, in fact, is the magnitude of q. We know that q, as explained earlier, is a negative charge. Hence, $q = -5\,\mu\text{C}$.

So, correct option is (c).

At $t = 0$, q is released at a point 0.1 m from O on Y-axis. As it oscillates, its other extreme position will be 0.1 m from O on the negative Y-axis, assuming undamped simple harmonic motion. Hence, amplitude of oscillations is 0.1 m or 10 cm.

So, correct option is (a).

From Fig. A1.203,

$$F_n = \frac{1}{4\pi\varepsilon_0}\frac{2Qqy}{a^3} = ky$$

where $\qquad k = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2Qq}{a^3}$

Thus, $F_n \propto y$

We also know that F_n always acts towards O (mean position). Time period of resulting SHM will be $T = 2\pi\sqrt{\dfrac{m}{k}}$

or frequency $= \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$

$$\therefore \quad f = \frac{1}{2\pi}\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{2Qq}{ma^3}}$$

$$f = \frac{1}{(2 \times 3.14)}\sqrt{(9 \times 10^{9})\frac{(2)(8 \times 10^{-6})(5 \times 10^{-6})}{(91 \times 10^{-6})(2)^3}} = 5$$

$[m = 91 \text{ mg} = 91 \times 10^{-6} \text{ kg}]$

Thus, the correct option is (c).

In SHM, equation of displacement from mean position can be expressed as $y = a\sin(\omega t + \phi)$

Here, $a = 0.1$ m; $\omega = 2\pi f = 2\pi \times 5 = 10\pi$

$$y = 0.1\sin(10\pi t + \phi)$$

But at $t = 0$, $y = 0.1$ (given)

Hence, $0.1 = 0.1\sin\phi$ or $\sin\phi = 1$

$$\Rightarrow \phi = \frac{\pi}{2};\; y = 0.1\sin(10\pi t + \pi/2)$$

Thus, the correct option is (b).

For Problems 18–21

18. d., 19. a., 20. c., 21. c.

Sol. The two particles move in different circles (Fig. A1.204)

The mutual interaction force provides the required centripetal force to the particle. As magnitude of the interaction force is same,

$$F_{12} = \frac{m_1 v_1^2}{r_1} \text{ and } F_{21} = \frac{m_2 v_2^2}{r_2}$$

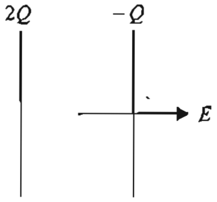


Fig. A1.204

$$\left|\vec{F}_1\right| = \left|\vec{F}_2\right| \Rightarrow \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2}$$

Putting values, we get $r_2 = 2r_1$

Also, $r_1 + r_2 = 12 \times 10^{-12} m$ (given)

$$r_1 = 4 \times 10^{-12} \text{ m};\; r_2 = 8 \times 10^{-12} \text{ m}$$

Acceleration of first particle $= \dfrac{v_1^2}{r_1} = \dfrac{(10^3)^2}{(4 \times 10^{-12})}$

$= 2.5 \times 10^{15}\text{ ms}^{-2}$

19. a. Acceleration of second particle is

$$\frac{v_2^2}{r_2} = \frac{(2 \times 10^3)^2}{(8 \times 10^{-12})} = 5 \times 10^{15}\text{ ms}^{-2}$$

20. c. Just after release, $V_{CM} = \dfrac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

$$= \frac{(2 \times 10^{-30})(0) + (10^{-30})(2 \times 10^{3})}{3 \times 10^{-30}} = \frac{2}{3} \times 10^{3}\text{ ms}^{-1}$$

21. c. From Fig. A1.205

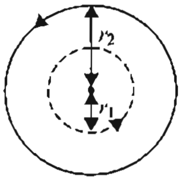


Fig. A1.205

Since the distance between them always remains constant but they move with different velocities, therefore they must move in different circles with common center as shown in the Fig. A1.205.

For Problems 22–23

22. b., 23. d.

Sol. Charge on capacitors C_1 and C_2 before closing the switch S (Fig. A1.206),

$$q_0 = \left(\frac{C_1C_2}{C_1 + C_2}\right)\varepsilon$$

After closing S, charge in C_2 (final charge)

$$q_2 = C_2\varepsilon$$

In loop $ABDEFGA$, $\varepsilon - \dfrac{q_1}{\varepsilon} - \varepsilon = 0 \quad \Rightarrow q_1 = 0$

Final charge on C_1, $q_1 = 0$

To make final charge in C_1, the charge q_0 will flow towards battery.

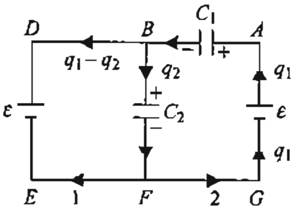


Fig. A1.206

Hence, charge flown towards direction 2,

$$\Delta q_2 = -\left(\frac{C_1C_2}{C_1+C_2}\right)\varepsilon$$

To make charge on capacitor C_2 to final value q_2, the charge flow into capacitor

$$\Delta q = C_2\varepsilon - \frac{C_1C_2}{C_1+C_2}\varepsilon = C_2\varepsilon\left[1-\frac{C_1}{C_1+C_2}\right]$$

$$\Delta q_B = \frac{C_2^2\varepsilon}{C_1+C_2}$$

At junction F,

$$\Delta q_1 = \Delta q - \Delta q_2 = \frac{C_2\varepsilon}{C_1+C_2}[C_2+C_1] = C_2\varepsilon$$

Hence, charge flown in the direction of 1 = $\Delta q = C_2\varepsilon$

Alternate Method: (Fig. A1.207)

In loop $ABDEG$, $-\frac{(q_0+\Delta q_1)}{C_1} - \varepsilon + \varepsilon = 0$

$$\Delta q_1 = -q_0 = -\frac{C_1C_2\varepsilon}{C_1+C_2}$$

In loop $BFEDB$, $-\frac{(q_0+\Delta q_1)}{C_2} + \varepsilon = 0$

$$\Delta q = C_2\varepsilon - q_0$$

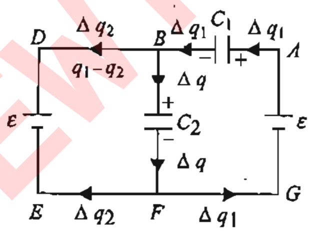


Fig. A1.207

At junction F, $\Delta q_2 = \Delta q_1 - \Delta q_1 = -q_0 - (C_2\varepsilon - q_0)$

$$= -C_2\varepsilon$$

Hence charge flow, in the direction of:

$$(1) = -\frac{C_1C_2\varepsilon}{C_1+C_2}$$

$$(2) = C_2\varepsilon$$

For Problems 24–26

24. a., 25. b., 26. d.

Sol. Before earthing charge on each capacitor (Fig. A1.208),

$$q_0 = 120 \times \frac{6}{5}$$

$$q_0 = 24 \times 6 = 144\ \mu C$$

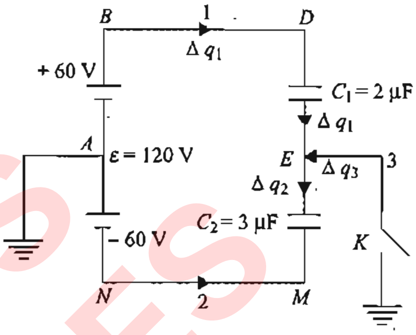


Fig. A1.208

After earthing,

Path $ABDE$, $0 + 60 - \frac{(q_0+\Delta q_1)}{2} = 0$

$$\Delta q_1 = 120 - q_0 = 24\ \mu C$$

Path $ANME$, $0 - 60 + \frac{(q_0+\Delta q_2)}{3} = 0$

$$\Delta q_2 = 180 - q_0 = 36\ \mu C$$

At junction E, $\Delta q_1 + \Delta q_3 = \Delta q_2$

$$\Delta q_3 = \Delta q_2 - \Delta q_1 = 36 + 24 = 60\ \mu C$$

Hence, the charge flown in the direction

$$(1) = -24\ \mu C$$
$$(2) = -36\ \mu C$$
$$(3) = +60\ \mu C$$

For Problems 27–28

27. c., 28. b.

Sol. Concentric cylindrical electrodes will produce radial electric field. As ion is entering at O and leaving at B, hence the path followed by ion should be circular and centered at A. Required centripetal force should be provided by force on the ion due to electric field.

Hence, $qE = \frac{mv^2}{R} \quad \Rightarrow \quad E = \frac{mv^2}{qR}$

As final velocity along x-axis becomes zero and finally the ion starts moving towards y direction, hence the electric field should have component towards x- and y-directions respectively.

For x-component of electric field (using $v_x^2 = u_x^2 + 2a_x\,\Delta x$):

$$0 = v^2 - 2\left(\frac{qE_x}{m}\right)R \quad \Rightarrow E_x = \frac{mv^2}{2qR}$$

For y-component of electric field (again using $v_y^2 = u_y^2 + 2a\,\Delta y$)

$$v^2 = 0 + 2\left(\frac{qE_y}{m}\right)R \quad \Rightarrow \quad E_y = \frac{mv^2}{2qR}$$

Hence, net electric field is

$$\vec{E} = -\frac{mv^2}{2qR}\hat{i} + \frac{mv^2}{2qR}\hat{j} = \frac{mv^2}{2qR}(-\hat{i}+\hat{j})$$

For Problems 29–31

29. d., 30. b., 31. d.

Sol. Plates 2 and 3 are joined together and they are neither connected to any of the terminals of the battery nor to any other source of charge. So, they jointly form an isolated system.

30. b. Before closing the switches, charges are shown in figure below.

After closing the switch, let q charge goes from battery. Then, charge on each will increase by q.

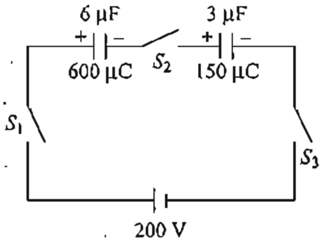


Fig. A1.209

Applying Kirchhoff's Voltage Law,

$$200 - \left(\frac{600+q}{6}\right) - \left(\frac{150+q}{3}\right) = 0$$

$$\Rightarrow q = 100\,\mu\text{C}$$

So, charge on 6 μF capacitor will be 700 μC and 3 μF capacitor will be 250 μC.

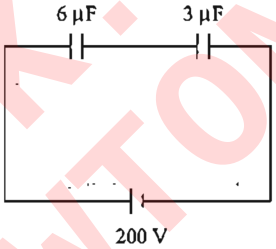


Fig. A1.210

Hence, option (b) is correct.

31. d. From all the switches, 100 μC of charge will flow.

For Problems 32–35

32. a., 33. b., 34. d., 35. d.

Sol. Let capacitance of A is C_1 and that of B is C_2. Both are with air.

Given $C_1 = 2C_2$

Before insertion of dielectric:

$$C_{eq_1} = \frac{2}{3}C_2$$

$$\text{After insertion: } C_{eq1} = \frac{6C_2}{C_1 + 6C_2} = \frac{3}{2}C_2$$

$$\Delta U = \frac{1}{2}(C_{eq2} - C_{eq1})\ V_1^2$$

$$1.2 \times 10^{-4} = \frac{1}{2}\left[\frac{3}{2}C_2 - \frac{2}{3}C_2\right](12)^2$$

$$C_2 = 2 \times 20^{-6}.\ \text{F} = 2\ \mu\text{F}$$

Now before insertion of dielectric (Fig. A1.211):

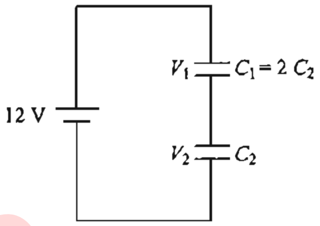


Fig. A1.211

$$V_2 = \frac{2 \times 12}{2+1} = 8\ V$$

$$\text{Energy in } B = U_1 = \frac{1}{2}C_2V_2^2 = 64 \times 10^{-6}\ \text{J}$$

After insertion of dielectric (Fig. A1.212):

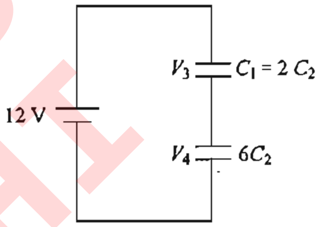


Fig. A1.212

$$V_4 = \frac{2 \times 12}{2+6} = 3\ V$$

$$\text{Energy in } B = U_2 = \frac{1}{2}6\,C_2V^2 = 54 \times 10^{-6}\ \text{J}$$

$$\text{Decrease in energy} = U_1 - U_2 = 10^{-5}\ \text{J}$$

33. b. Since dielectric is removed after disconnecting the battery, so charge on it will remain same.

$$Q = C_{eq2}\ V = \frac{3}{2}C_2V = 36 \times 10^{-6}\ C$$

Work done in removing the dielectric = change in energy of B

$$= \frac{Q^2}{2\,C_2} - \frac{Q^2}{2 \times 6\,C_2} = 2.7 \times 10^{-4}\ \text{J}$$

34. d. Before connecting as in situation (ii), both capacitors will carry equal charge. They will get neutralised on connecting as in (i). So, net charge on any capacitor will become zero. Hence, final electrical energy stored on them will be zero.

35. a. Charges acquired by capacitors after the whole process described is completed (Fig. A1.213).

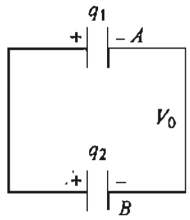


Fig. A1.213

$$Q = \frac{3}{2}C_2 \times 12 = 36 \times 10^{-6}\,C$$

For situation (ii), let q_1 and q_2 are the final charges. V_0 is common potential.

$$q_1 + q_2 = 36 + 36 = 72\,\mu\text{C}$$

$$V_0 = \frac{q_1}{C_1} = \frac{q_2}{C_2},\ q_1 = 48\,\mu\text{C}, \quad q_2 = 24\,\mu\text{C}$$

Final electric energy

$$= \frac{1}{2}\left[\frac{q_1^2}{C_1} + \frac{q_2^2}{C_2}\right] = 432 \times 10^{-6}\text{ J} = 4.3 \times 10^{-4}\text{ J}$$

For Problems 36–38

36. a., 37. a., 38. b.

36. a. System is equivalent to two capacitors

$$C_1 = \frac{K\varepsilon_0 A}{t},\ C_2 = \frac{\varepsilon_0 A}{d - t}$$

Since both are connected in series, so

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{t}{K\varepsilon_0 A} + \frac{(d-t)}{\varepsilon_0 A}$$

$$= \frac{t + Kd - Kt}{K\varepsilon_0 A} = \frac{K(d-t) + t}{K\varepsilon_0 A}$$

or $$C = \frac{K\varepsilon_0 A}{K(d-t) + t}$$

or $$C = \frac{\varepsilon_0 A}{(d-t) + \frac{t}{K}} = \frac{\varepsilon_0 A}{(d-t) + \frac{t}{2}}$$

or $$C = \frac{\varepsilon_0 A}{d - \frac{t}{2}}$$

37. a. According to question, $\frac{\varepsilon_0 A}{d - \frac{t}{2}} = \frac{3}{2}\frac{\varepsilon_0 A}{d}$

Solving, we get $\frac{t}{d} = \frac{2}{3}$

38. b. If charge q on the capacitor remains unchanged, then

$$U_i = \frac{1}{2}\frac{q^2}{C_a},\ U_f = \frac{q^2}{2C_{eq}}$$

or $$\frac{U_i}{U_f} = \frac{C_{eq}}{C_a} = \frac{d}{t} = \frac{3}{2}$$

For Problems 39–43

39. a., 40. d., 41. b., 42. d., 43. c.

Sol. The charge distribution on various surfaces is as shown in the following figure. $-q_1$ on the surface of Cavity 1 will spread uniformly if q_1 is at the center, otherwise the distribution would be non-uniform. Same is the case with $-q_2$.

The charge appearing on outer surface of the conductor is $q_1 + q_2 + Q$ which would be non-uniformly distributed as radius of curvature at various points of conductor's surface is different. The presence of q_1 and q_2 (and their location) have no effect on the distribution of charge on outer surface. The presence of q will change the distribution of charge on outer surface, but still it remains non-uniform.

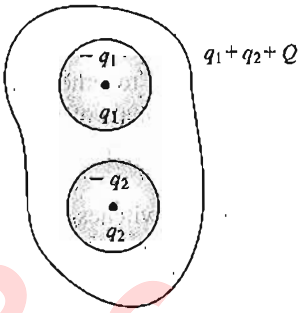


Fig. A1.214

40. d. Electric field intensity outside the cavity due to q_1 and $-q_1$ would be zero.

$$\vec{E}_{-q_1} + \vec{E}_{-q_1} = 0$$

$$\vec{E}_{-q_1} = -\vec{E}_{q_1} = \frac{q_1}{4\pi\varepsilon_0 r^2} \text{ towards center of Cavity 1.}$$

41. b. $\vec{E}$ inside the conductor due to outside charges = 0

$$\vec{E}_q + \vec{E}_{q_1+q_2+Q} = 0$$

$$\Rightarrow \vec{E}_{q_1+q_2+Q} = -\vec{E}_q$$

$$= \frac{q}{4\pi\varepsilon_0 r^2} \text{ towards } q.$$

42. d. If q_2 is at point Q, then induced charge $-q_2$ would be non-uniformly distributed. So, we cannot determine $\vec{E}$ due to $-q_2$ at any inside point.

43. c.

Sol. Potential at point Q = Potential of conductor + Potential due to q_2 + Potential due to $-q_2$

$$= V_0 + \frac{q_2}{4\pi\varepsilon_0 r_2'} + \frac{-q_2}{4\pi\varepsilon_0 r_2} = \frac{q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_2'} - \frac{1}{r_2}\right) + V_0$$

For Problems 44–47

44. b., 45. d., 46. c., 47. b.

Sol. Let the charge distribution be as shown in Fig. A1.215.

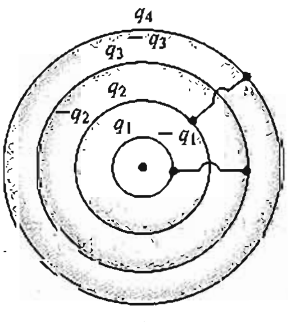


Fig. A1.215

From Gauss's theorem, we know that facing surfaces of the conductor acquire equal and opposite charges.

i.e., $V_1 = V_3$ and $V_2 = V_4$

$$q_1+q_3-q_2=+4Q \qquad \text{(i)}$$

Now, $$q_2-q_1+q_4-q_3=-6Q \qquad \text{(ii)}$$

$$V_1=\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{R}+\frac{q_2-q_1}{2R}+\frac{q_3-q_2}{3R}+\frac{q_4-q_3}{4R}\right]$$

$$V_2=\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{2R}+\frac{q_2-q_1}{2R}+\frac{q_3-q_2}{3R}+\frac{q_4-q_3}{4R}\right]$$

$$V_3=\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{3R}+\frac{q_2-q_1}{3R}+\frac{q_3-q_2}{3R}+\frac{q_4-q_3}{4R}\right]$$

$$V_4=\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{4R}+\frac{q_2-q_1}{4R}+\frac{q_3-q_2}{3R}+\frac{q_4-q_3}{4R}\right]$$

From $V_1=V_3$, $q_1=-\frac{q_2}{3}$

From $V_2=V_4$, $q_2=-\frac{q_3}{2}$

On solving equation (2), $q_1=\frac{2Q}{5}$, $q_2=-\frac{6Q}{5}$, and $q_3=\frac{12Q}{5}$

Substituting these values in equation (ii), $q_4=-2Q$

44. b. Charge on the inner surface of 3rd conductor

$$=-q_2=\frac{6Q}{5}$$

45. d. Charge on 4th conductor

$$=q_4-q_3=-2Q-\frac{12Q}{5}=\frac{-22Q}{5}$$

46. c. Potential of conductor 1, $V_1=\frac{-3Q}{40\pi\varepsilon_0 R}=V_3$

47. b. Potential of conductor 2, $V_2=\frac{Q}{8\pi\varepsilon_0 R}=V_4$

For Problems 48–51

48. b., 49. a., 50. c., 51. d.

Sol. Inner wall is negatively charged, so it is at a lower potential (Fig. A1.216).

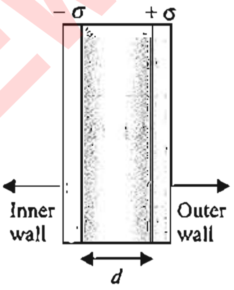


Fig. A1.216

49. a. Area of cell wall is, $A=\frac{V}{d} \quad \Rightarrow \quad A=10^{-5}\text{ m}^2$

$$E=\frac{\sigma}{5\varepsilon_0}\text{NC}^{-1}$$

50. c. $dV=-\vec{E}\cdot\vec{dr}$

$$V_{\text{outer}}-V_{\text{inner}}=\frac{\sigma}{5\varepsilon_0}\times10^{-10}\text{ volt}$$

51. d. $U=\frac{1}{2}\varepsilon_0E^2\times$ volume

$$U=\frac{1}{2}(5\varepsilon_0)\left[\frac{\sigma}{5\varepsilon_0}\right]^2\times10^{-15}\text{ J}=\frac{\sigma^2}{\varepsilon_0}\times10^{-16}\text{ J}$$

For Problems 52–53

52. b., 53. a.

Sol. Charge on the plate of capacitor is (Fig. A1.217),

$$Q=CV=\frac{\varepsilon_0A}{d}\times V$$

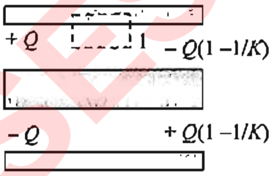


Fig. A1.217

From Gauss's theory, for surface 1 E_1 (Electric field between plates and dielectric slab) $=\frac{Q}{\varepsilon_0A}=\frac{V}{d}$

53. a. Inside the dielectric, $E_2=\frac{E_1}{K}=\frac{V}{Kd}$

For Problems 54–55

54. c., 55. c.

Sol. This can be treated as a system of two capacitors in series (Fig. A1.218),

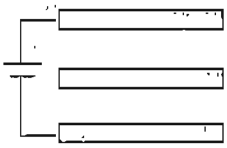


Fig. A1.218

$$C_{\text{eq}}=\frac{C_1C_1}{C_1+C_2}$$

where $C_1=C_2=\frac{2\varepsilon_0A}{d}$

55. c. Initially, charge supplied by battery, $q_i=\frac{\varepsilon_0A}{d}\times V$

After shorting, $C_{\text{eq}}=\frac{2\varepsilon_0A}{d}$, so $q_f=\frac{2\varepsilon_0AV}{d}$

So, extra charge flow $=q_f-q_i=\frac{\varepsilon_0AV}{d}$

For Problems 56–58

56. a., 57. d., 58. c.

Sol. $C_i=\frac{\varepsilon_0d}{x}$, $C_f=\frac{\varepsilon_0A}{x+dx}$

$$U_i=\frac{C_iV^2}{2},\ U_f=\frac{C_fV^2}{2}$$

$$dU=U_f-U_i=\frac{\varepsilon_0AV^2}{2}\left[\frac{1}{x+dx}-\frac{1}{x}\right]$$

$$=-\frac{\varepsilon_0AV^2}{2}\times\frac{dx}{x^2}\quad[x>>dx]$$

57. d. $q_i = C_i V,\ q_f = C_f V$

So, required charge, $q = q_f - q_i$

$$= \varepsilon_0 A V\left[\frac{1}{x+dx} - \frac{1}{x}\right]$$

$$= -\frac{\varepsilon_0 A V\, dx}{x^2}$$

Negative sign implies that charge is entering into the battery from its positive terminal.

Work done by battery $= qv = -\dfrac{\varepsilon_0 A V^2 dx}{x^2}$

Negative sign means that work is done on the battery.

58. c. From work–energy theorem

$$dK = 0 = W_{\text{el}} + W_{\text{ext}} + W_{\text{battery}}$$

$$W_{\text{el}} = \frac{\varepsilon_0 A V^2}{2} \times \frac{dx}{x^2}$$

For Problems 59–61

59. b., 60. b., 61. b.

Sol. The electrostatic force generated by an electric field E on a point charge q is always

$$F = qE$$

No matter what object generates that electric field. Think of the mystery object as creating an electric field, which then pushes on q. If we know the field, we need not know what kind of object created the field.

60. b. As discussed in previous problem, the data indicate an inverse square relationship, doubling the distance decreases the field by a factor of 4. Only graph (d) captures this insight. Graph (c) represents a regular inverse proportionality, in which doubling the distance cuts the field in half.

61. b. According to the formula $E = \dfrac{2kp\cos\theta}{d^3}$, doubling the distance from the dipole decreases the field by a factor $2^3 = 8$. (Mathematically, that is because doubling d increases the denominator by 8). By similar reasoning, doubling the dipole moment p, increases the field by a factor of 2. So, when we "turn on" both of these effects at once, the field decreases by a factor of 4.

If this quick and dirty reasoning does not make sense, you can reason in steps as follows. Suppose the electric field has strength E_0. First, we double p. This increases the field strength to $2E_0$. Next, we double d. This decreases the field by a factor of 8, to $\dfrac{2E_0}{8} = \dfrac{E_0}{4}$. The order of the steps makes no difference, you get the same answer either way. Therefore, this step by step reasoning works even, in real life, the dipole moment and the distance get doubled simultaneously.

For Problems 62–63

62. a., 63. d.

Sol. $\dfrac{\partial E}{\partial x}$ is not zero, it means E_y may change, as we move along x-axis. So, $-q$ and $+q$ may find different forces along y-axis, hence net force on dipole may be along y-axis (Fig. A1.219).

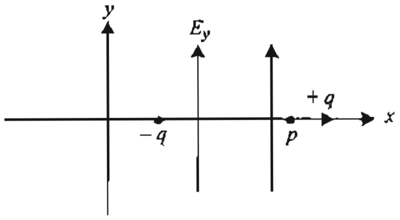


Fig. A1.219

Obviously, forces along y-axis will rotate the dipole in x–y plane, producing torque along z-axis.

For Problems 64–65

64. c., 65. a.

Sol. Given, $2l = 2$ cm $= 2 \times 10^{-2}$ m (Fig. A1.220)

$\theta = 60°$, $E = 10^5$ NC^{-1}, $\tau = 8\sqrt{3}$ Nm

$\Rightarrow \quad \tau = pE\sin\theta$

$\Rightarrow \quad 8\sqrt{3} = (q\ 2l)\ E\sin\theta$

$\Rightarrow \quad q = 8 \times 10^{-3}$ C $= 8$ mC

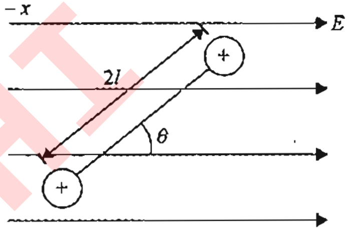


Fig. A1.220

Potential energy of dipole $U = -pE$

$$U = -pE\cos\theta = -q \times 2l \times E\cos\theta$$

$\Rightarrow \quad U = -8$ J

For Problems 66–67

66. a., 67. c.

Sol. Electric field at P is (Fig. A1.221)

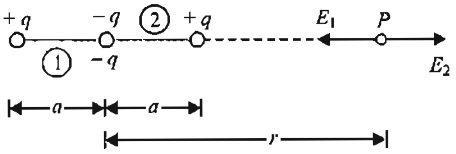


Fig. A1.221

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p_2}{(r-a/2)^3} - \frac{1}{4\pi\varepsilon_0}\frac{2p_1}{(r+a/2)^3}$$

$$= \frac{1}{4\pi\varepsilon_0} 2qa\left[\frac{1}{(r-a/2)^3} - \frac{1}{(r+a/2)^3}\right]$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\left(\frac{6qa^2}{r^4}\right)$$

Calculation of electric potential (Fig. A1.222):

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

$$V_p = V_1 + V_2$$

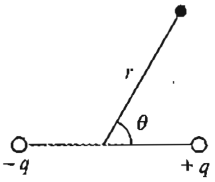


Fig. A1.222

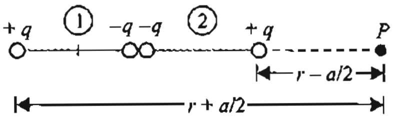


Fig. A1.223

$$= \frac{1}{4\pi\varepsilon_0}\frac{p\cos\pi}{(r+a/2)^2} + \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{(r-a/2)^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}\left[-\frac{1}{\left(1+\frac{a}{2r}\right)^2} + \frac{1}{\left(1-\frac{a}{2r}\right)^2}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}\left[-\left(1+\frac{a}{2r}\right)^{-2} + \left(1-\frac{a}{2r}\right)^{-2}\right]$$

$$\Rightarrow \qquad V_p = \frac{1}{4\pi\varepsilon_0}\frac{2pa}{r^3}$$

For Problems 68–71

68. a., 69. b., 70. a., 71. c.

Sol. Since the field is uniform, the forces on the two charges are equal and opposite, and the total force is zero.

The magnitude p of the electric dipole moment $\vec{p}$ is

$$p = qd = 1.6 \times 10^{-19} \times 0.125 \times 10^{-9}$$
$$= 2.0 \times 10^{-29}\ \text{C m}$$

The direction of $\vec{p}$ is from the negative to the positive charge, 145° clockwise from the electric field direction (as shown in Fig. A1.224).

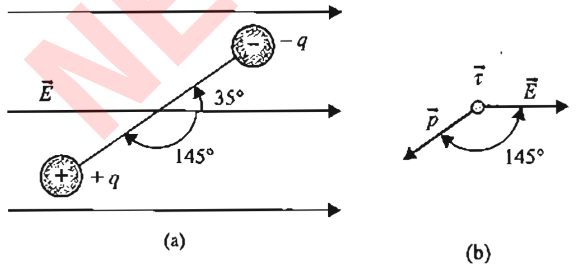


Fig. A1.224

The magnitude of the torque is

$$\tau = qE\sin\phi = (2.0 \times 10^{-29} C)(5.0 \times 10^5 \text{NC}^{-1})$$
$$\sin(180° - 37°) = 6.0 \times 10^{-24}\text{Nm}$$

From the right-hand rule for vector products, the direction of the torque $\vec{\tau} = \vec{p} \times \vec{E}$ is out of the page. This corresponds to a counterclockwise torque that tends to align $\vec{p}$ with $\vec{E}$.

The potential energy is

$$U = -pE\cos\phi$$
$$= -(2.0 \times 10^{-29}\text{Cm})(5.0 \times 10^5\ \text{NC}^{-1})\cos(180° - 37°)$$
$$= 8.0 \times 10^{-24}\ \text{J}$$

For Problems 72–73

72. d., 73. b.

Sol. Conserving energy (Fig. A1.225),

$$(-\vec{p}\vec{E})_{\text{initial}} + (KE)_{\text{initial}} = \text{constant}$$

$$0 + 0 = -p \times \frac{1}{4\pi\varepsilon_0}\frac{q}{d^2} + KE$$

$$KE = \frac{pq}{4\pi\varepsilon_0 d^2}$$

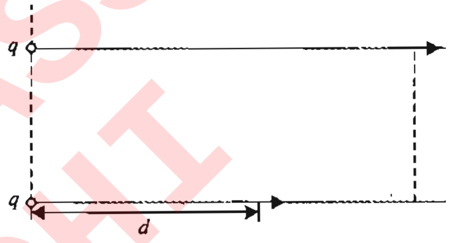


Fig. A1.225

$$|F| = \left|\frac{dU}{dx}\right| = \frac{pq}{4\pi\varepsilon_0}\frac{d}{dx}\left(\frac{1}{x^2}\right)$$

$$= \frac{pq}{4\pi\varepsilon_0 0}(-2x^{-3}) = \frac{pq}{2\pi\varepsilon_0}\frac{1}{x^3}$$

Force on dipole $|F| = \frac{1}{2\pi\varepsilon_0}\frac{pq}{d^3}$

For Problems 74–75

74. c., 75. b.

Sol. Net charge on the rod is zero, so the net force will also be zero. Location of charge will not matter as the electric field produced by a large sheet is uniform.

75. b. $\tau = 2\int_0^l (\lambda\, dx\, E)\, x\sin\theta$

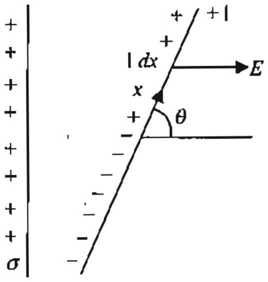


Fig. A1.226

$$\Rightarrow \tau = \frac{2\lambda\sigma\sin\theta}{2\varepsilon_0}\int_0^l x\,dx = \frac{\sigma\lambda l^2\sin\theta}{2\varepsilon_0}$$

For Problems 76–78

76. b., 77. c., 78. d.

Sol. Field inside the conductor is zero.

77. c. Charge distribution is shown in Fig. A1.227.

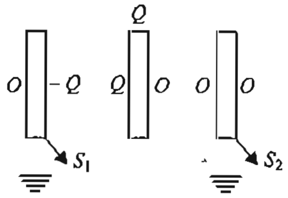


Fig. A1.227

78. d. $\frac{(Q-q)}{\varepsilon_0 A}3d = \frac{qd}{\varepsilon_0 A}$

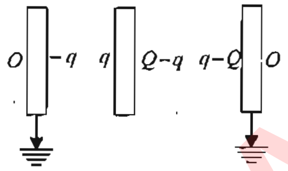


Fig. A1.228

$$\Rightarrow \quad q = \frac{3Q}{4}$$

For Problems 79–81

79. b., 80. b., 81. d.

Sol.

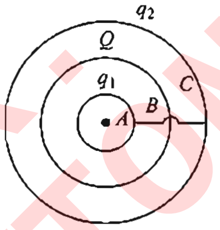


Fig. A1.229

$$q_1 + q_2 = 0$$

$$v_A = \frac{kq_1}{R} + \frac{kQ}{2R} + \frac{kq_2}{4R}$$

$$v_C = \frac{kq_1}{4R} + \frac{kQ}{4R} + \frac{kq_2}{4R}$$

$$v_A = v_C$$

$$q_1 = -\frac{Q}{3} \text{ and } q_2 = \frac{Q}{3}$$

80. b. $v_A = k\left[\frac{-Q}{3R} + \frac{Q}{2R} + \frac{Q}{12R}\right] = \frac{Q}{16\pi\varepsilon_0 R}$

81. d. $v_B = k\left[\frac{-Q}{6R} + \frac{Q}{2R} + \frac{Q}{12R}\right] = \frac{5Q}{48\pi\varepsilon_0 R}$

For Problems 82–84

82. d., 83. b., 84. 84. d.

Sol. Plate 2 and 3 are joined together and they are neither connected to any of the terminals of the battery nor to any other source of charge. So, they jointly form an isolated system.

83. b. Before closing the switches, charges are shown in Fig. A1.230. After closing the switch, let q charge goes from battery. Then charge on each will increase by q.

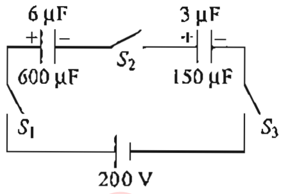


Fig. A1.230

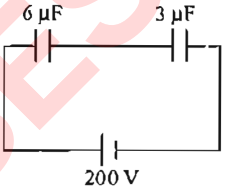


Fig. A1.231

Applying Kirchoff's Voltage Law

$$200 - \left(\frac{600+q}{6}\right) - \left(\frac{150+q}{3}\right) = 0$$

Or $q = 100$ μC.

So, charge on 6 μF capacitor will be 70 μC and 3 μF capacitor will be 250 μC.

Hence, choice (b) is correct.

84. d. From all the switches 100 μC of charge will flow.

For Problems 85–87

85. a., 86. b., 87. b.

Sol. When oil is pumped out then plates act as air capacitor between which electric field is more than electric field in dielectric capacitor. So choice (a) is correct and other choices are wrong.

86. b. Since charged capacitor is isolated, hence, charge is conserved. So, choice (b) is correct and other choices are wrong.

87. b. Since capacitor is connected to cell so potential difference across capacitor remains constant. When slab is pulled out capacitance decreases.

Charge on capacitor $Q = CV$

So charge on capacitor decreases and some charge will be returned to cell.

Choices (a), (c) and (d) are wrong.

For Problems 88–90

88. b., 89. c., 90. d.

Sol. Electrostatic force on block A is zero, while on block B

$F = qE = 1 \times 10 = 10$ N

This force acts along negative direction of X-axis.

Acceleration of block B

$$a = \frac{F}{q} = \frac{10}{1} = 10 \text{ m/s}^2$$

But before collision, velocity of block B,

$$v = \sqrt{2aS}$$

or $v = \sqrt{2 \times 10 \times 1.8} = 6$ m/s
Now, from conservation of linear momentum

$$(m_A + m_B)v = m_A v_A + m_B v_B$$

$$(1+1)v = 0 + 1 \times 6 \text{ or } v = \frac{6}{2} = 3 \text{ m/s}$$

89. c. At equilibrium, net force on system is zero. Let x_0 be the compression in the spring in equilibrium, then

Kx_0 = electrostatic force
$18x_0 = 10$
or $x_0 = \frac{10}{18} = \frac{5}{9}$

So, equilibrium position will be $x = -\frac{5}{9}$ m

90. d. Angular frequency of S.H.M. will be

$$\omega = \sqrt{\frac{k}{m_A + m_B}} = \sqrt{\frac{18}{2}} = 3 \text{ rad/ sec}$$

At $x = \frac{5}{9}$ m. Speed is 3 m/s

Therefore from $v = \omega\sqrt{A^2 - x^2}$

or $3 = 3\sqrt{A^2 - x^2}$or $1 = 1\sqrt{A^2 - x^2}$

or $A^2 - x^2 = 1$ or $A^2 = 1 + x^2$

$= 1 + \left(\frac{5}{9}\right)^2 = 1 + \frac{25}{81} = \frac{106}{81}$ or $A = \frac{\sqrt{106}}{9}$ m

Hence, choice (d) is correct.

Matching Column Type

1. i. → **c.**; **ii.** → **b.**; **iii.** → **d.**; **iv.** → **a.**

Sol. i. Since $q_2 = 0$, so charge $Q + q_1$ on outer surface will be distributed uniformly. Hence, electric field of $Q + q_1$ at center will be zero.

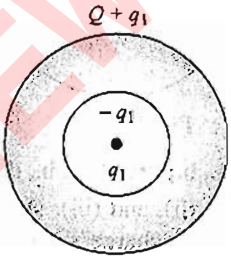


Fig. A1.232

ii. Since q_2 is not zero, so $Q + q_1$ will be induced non-uniformly on outer surface. But due to q_2 and $Q + q_1$, net field at any point inside the outer surface should be zero.

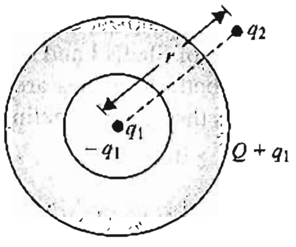


Fig. A1.233

$$E_{q_2} + E_{Q+q_1} = 0$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_2}{(r-R_1)^2} + E_{Q+q_1} = 0$$

$$E_{Q+q_1} = -\frac{1}{4\pi\varepsilon_0}\frac{q_2}{(r-R_1)^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(r-R_1)^2} \text{ radially outward}$$

iii. Since $Q + q_1$ will be induced non-uniformly, so it is difficult to determine its electric field at an outside point.

iv.

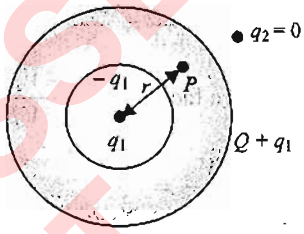


Fig. A1.234

Net field at P due to q_1 and $-q_1$ should be zero.

$$E_{q_1} + E_{(-q_1)} = 0$$

$$E_{(-q_1)} = -E_{-q_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}$$

2. i. → **b., d.;** **ii.** → **a., d.;** **iii.** → **b., d.;** **iv.** → **a., c.**

If any charge is present outside, then charge on outer surface will be distributed non-uniformly. But if there is no charge present outside, then charge on outer surface will be distributed uniformly. This is irrespective of location of the charge inside.

If charge is at center, then charge on inner surface will be distributed uniformly, but if charge is displaced from center, then charge on inner surface will be distributed non-uniformly. This is irrespective of whether the charge is present outside or not.

3. i. → **a., d.;** **ii.** → **c., d.;** **iii.** → **a., b.;** **iv.** → **b.**

i. When we insert the dielectric slowly, we have to apply the force on dielectric in opposite direction, so we have to do negative work. Because of this negative work done, energy of system decreases.

ii. Here, again work done by external agent is negative, but here battery will supply some energy (or the battery will do positive work) due to which there is overall increase in energy of system.

iii. $U = \frac{1}{2}CV^2$, V remains same and C decreases so U also decreases. Work done by external agent will be positive in pulling apart the plates against the attractive force between plates.

iv. $U = \frac{q^2}{2C}$, q remains same and C decreases so U increases. Work done by external agent will be positive in pulling apart the plates against the attractive force between plates.

4. i. → b., d.; ii. → a., c., d.; iii. → a., c., d.; iv. → a., c., d.

Charge on the outer surfaces of the plates of capacitor will be zero always. If $V = E$, then no charge will flow in the circuit and hence no thermal energy will be dissipated. But if $V \neq E$, then charge will flow in the circuit and thermal energy will be dissipated.

5. i. → a., c.; ii. → b.; iii. → b., d.; iv. → b.

i. Initial potential difference across C_1:

$$V_1 = \frac{4V}{2+4} = \frac{2V}{3}$$

On doubling the distance between plates, C_1 becomes half. So, final potential difference across C_1:

$$V_1' = \frac{4V}{1+4} = \frac{2V}{5}$$

This increases by a factor $\frac{V_1'}{V_1} = \frac{4\,V/5}{2\,V/3} = \frac{6}{5}$

ii. Across C_2:

Initially: $V_2 = \frac{2V}{2+4} = \frac{V}{3}$

Finally: $V_2' = \frac{1\,V}{1+4} = \frac{V}{5}$

This decreases by a factor of

$$\frac{V_2'}{V_2} = \frac{V/5}{V/3} = \frac{3}{5}$$

iii. Energy in C_1

Initially: $U_1 = \frac{1}{2} \times 2\left(\frac{2V}{3}\right)^2 = \frac{4V^2}{9}$

Finally: $U_1' = \frac{1}{2} \times 1 \times \left(\frac{4V}{5}\right)^2 = \frac{8V^2}{25}$

This decreases by a factor

$$\frac{U_1'}{U_1} = \frac{8\,V^2/25}{4\,V^2/9} = \frac{18}{25}$$

iv. Energy in C_2

Initially: $U_2 = \frac{1}{2}\,4\left(\frac{V}{3}\right)^2 = \frac{2V^2}{9}$

Finally: $U_2' = \frac{1}{2}\,4\left(\frac{V}{3}\right)^2 = \frac{2V^2}{25}$

This decreases by a factor $\frac{U_2'}{U_2} = \frac{9}{25}$

6. i. → b.; ii. → a.; iii. → d.; iv. → c.

i. Total electric field intensity at point $P = 3.2 \times 10^4$ NC^{-1} along AQ

ii. Total electric field intensity at point $Q = 7.2 \times 10^4$ NC^{-1} along PB

iii. Total electric field intensity at point

$$R = 9 \times 10^3 \text{ NC}^{-1}$$

iv. Electric field at point Q due to charge at B is 4000 NC^{-1}.

7. i. → b., d.; ii. → a., d.; iii. → d.; iv. → c.

Sol. In case of hollow or solid conducting sphere of radius R, for an internal point (i.e., $r < R$) electric field is zero. At the surface, E can be either minimum $(= 0)$ or maximum $\left(= \frac{Q}{4\pi\varepsilon_0 R^2}\right)$. It is worth mentioning here that one should take $E = 0$ at the surface for dealing internal behavior and $\left(= \frac{Q}{4\pi\varepsilon_0 R^2}\right)$ for dealing external behavior. So, electric field is discontinuous.

Hence, i. → b., d.

In case of spherical volume distribution of charge, inside the sphere electric field $E_{\text{in}} = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}$. From this expression, it is clear that electric field is zero at the center. At the surface, E is continuous and maximum $\left(= \frac{Q}{4\pi\varepsilon_0 R^2}\right)$.

So, ii. → a., d.

In case of charged ring, electric field is zero at the center. In case of infinite sheet of charge, electric field $E = \frac{\sigma}{2\varepsilon_0}$, which is constant.

So, iii → d. iv. → c.

8. i. → a., b.; ii. → d.; iii. → c.; iv. → d.

Sol. Potential difference to a line charge

$$V_2 - V_1 = -\frac{\lambda}{2\pi\varepsilon_0}(\ln r_2 - \ln r_1)$$

$$V_2 = V_1 - \frac{\lambda}{2\pi\varepsilon_0}\ln r_2 + \frac{\lambda}{2\pi\varepsilon_0}\ln r_1$$

At $r_1 = \infty$, $V_1 = 0$

So, at any value of r_2, V_2 is infinite.

9. i. → a.; ii. → c.; iii. → a.; iv. → a.

Sol. $\frac{kq}{a} + \frac{k\,Q}{b} = 0 \;\Rightarrow\; q = -\frac{Qa}{b}$

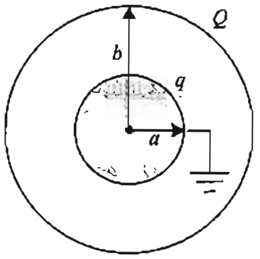


Fig. A1.235

10. i. → a., b.; ii. → a., b.; iii. → a., b., d.; iv. → c., d.

Sol. In situation (i), (ii), and (iii), shells I and II are not at same potential. Hence, charge shall flow from Sphere I to Sphere II till both acquire same potential.

If charge flows, the potential energy of system decreases and heat is produced.

In situations (i) and (ii) charges shall divide in some fixed ratio, but in situation (iii) complete charge shall be transferred to Shell II for potential of Shells I and II to be same.

In situation (iv) both the shells are at same potential, hence no charge flows through connecting wire.

11. i. → c.; ii. → a.; iii. → b.; iv. → d.

Sol. From Fig. A1.236

When circuit is closed, let charge q flows in the circuit; then applying loop law in $ABCDA$

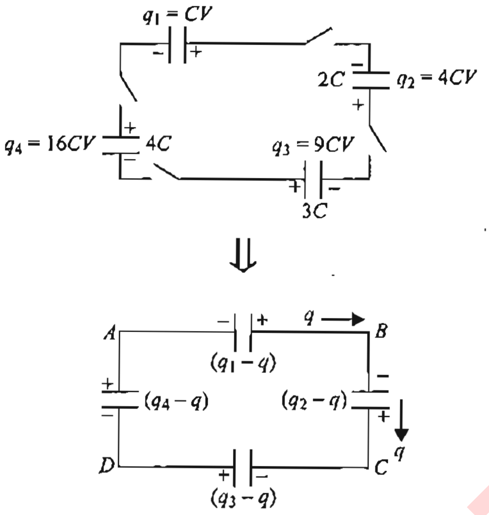


Fig. A1.236

$$\frac{q_1 - q}{C} + \frac{q_2 - q}{2C} + \frac{q_3 - q}{3C} + \frac{q_4 - q}{4C} = 0$$

Substituting the value of q_1, q_2, q_3 and q_4 we get

$$q = \frac{24}{5} CV$$

Hence, $V_1 = \frac{q_1 - q}{C} = \frac{19}{5} V$; $V_2 = \frac{q_2 - q}{2C} = \frac{2}{5} V$

$V_3 = \frac{q_3 - q}{3C} = \frac{7}{5} V$; $V_4 = \frac{q_4 - q}{4C} = \frac{14}{5} V$

12. i. → d.; ii. → a., c., d.; iii. → b.; iv. → a., c.

In this case, electric field is totally zero because in (a) and (b) particle P can be at origin.

For points P, P' in Fig. A1.112 (c) electric field will be along x-axis only. Also, for point P in Fig. A1.112 (a) and (b) E_x will remain non-zero while other two components of electric field will be zero.

iii. → b. Only in the case of disk x–y components of electric field will be present as per given statements.

iv. → a., c. For the two points P and P' in Fig. A1.112(c), the field will be same as these are points are at equal distances from infinite line of charge.

13. i. → c.; ii. → d.; iii. → b.; iv. → b.

Sol. (i) $U = -\vec{p} \cdot \vec{E}$. So, when $\theta = 180°$, $U = pE$.

ii. Angular acceleration is maximum when torque is maximum. $\vec{\tau} = \vec{p} \times \vec{E}$ is maximum for $\theta = 90°$.

iii. Angular momentum will not be conserved as there is no external force on the system but torque will act on the system.

iv. During movement of dipole, total energy remains conserved. Only K.E. cannot be conserved.

14. i. → a., c.; ii. → a., b., c., d.; iii. → a., b., c.; iv. → a., d.

Sol. The electric field due to one dipole at center of other dipole is parallel to that dipole in all cases. Hence, torque on dipole is zero in all cases.

In cases (ii) and (iii), the electic field at second dipole due to first is along the second dipole, hence electrostatic potential energy of second dipole is negative.

In cases (i) and (ii), x-axis is the line of zero potential. In case (iii), y-axis is the line of zero potential. In cases (ii) and (iii), electric field at origin is zero.

15. i. → b.; ii. → a.; iii. → c.; iv. → d.

Sol. i. $\vec{E}_{\text{axial}}$ for a short dipole $= \frac{2|\vec{P}|}{4\pi\varepsilon_0 r^3}$

ii. $\vec{E}_{\text{potential}}$ for a dipole $= \frac{|\vec{P}|}{4\pi\varepsilon_0 r^3}$

iii. Net force $= qE - qE = 0$.

Torque $= PE \sin 0° = 0$

iv. Net force $\neq 0$. as the value of E is different at both ends. But as the angle $\theta = 0°$, hence the torque is zero.

16. i. → a.; ii. → b.; iii. → d.; iv. → c.

Sol. From question: $\frac{\partial V}{\partial x} = -(2xy + z^2)$

$$\frac{\partial V}{\partial y} = -(2yz + x^2)$$

$$\frac{\partial V}{\partial z} = -(2zx + y^2)$$

$$V = x^2y + y^2z + z^2x + C$$

Using this we can find out the works

Further $E(x, y, z) = E(-x, -y, -z)$

Direction of field at x, y, z is same as direction of field at $(-x, -y, -z)$ and direction of area vector is just opposite. That net flux will turn out to be zero.

17. i. → b.; ii. → a.; iii. → c.; iv. → d.

Sol. $\vec{E} = -\hat{i} - \hat{j} \Rightarrow \vec{E} = \sqrt{2}$ N/C

$$a_P = 2 \text{ m/s}^2 \quad a_Q = 4 \text{ m/s}^2$$

$$S_P = \frac{1}{2} \times 2 \times 4 \times 2 = 8 \text{ m}$$

$$S_Q = \frac{1}{2} \times 4 \times 4 = 8 \text{ m}$$

Archives

Fill in the Blanks Type

1. For plate 1 (Fig. A1.237)

$$q = CV = \frac{\varepsilon_0 A}{d} \times V$$

For plate 2:

$$2q = \frac{2\varepsilon_0 A}{d} \times V$$

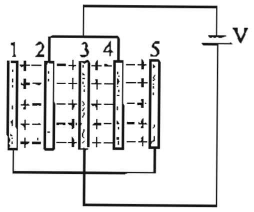


Fig. A1.237

2. It is greatest at point B since at B the equipotential surfaces are closest.

3. Where there is no gravitational force, then in this case only electrostatic force of repulsion is acting which will take the two balls as far as possible.

The angle between the two strings will be 180°.

The tension in the string will be equal to the electrostatic force of repulsion

$$T = \frac{1}{4\pi\varepsilon_0} \times \frac{Q \times Q}{(2L)^2} = \frac{1}{4\pi\varepsilon_0} \times \frac{Q^2}{4L^2}$$

4. Initially, Charge on capacitance $C = q_1 = CV$

Charge on capacitance $2C = q_2 = 2CV$

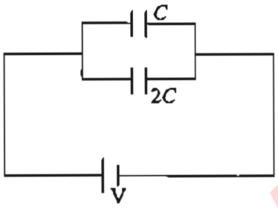


Fig. A1.238

Finally, Charge on capacitance $C = q_1' = KCV'$

Charge on capacitance $2C = q_2' = 2CV'$

Since charge will not change

$$\Rightarrow \quad CV + 2CV = KCV' + 2CV' \Rightarrow \quad V' = \frac{3V}{k+2}$$

5. Since electric field is conservative in nature, the work done by the field along $PQRS$ will be same as along P to M to S (Fig. A1.239).

$$\text{Work done from P to M} = \vec{F} \cdot \overrightarrow{PM}$$
$$= F\,(PM)\ \cos 90° = 0$$

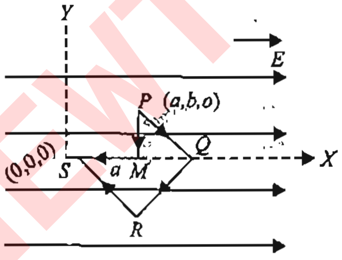


Fig. A1.239

$$\text{Work done from } M \text{ to } S = \vec{F} \cdot \overrightarrow{MS} = F(MS) \cos 180°$$
$$= -qEa$$

6. $V = 4x^2$ V

The electric potential changes only along x-axis

We know that

$$E_x = \frac{-dV}{dx} \Rightarrow \quad E_x = -\frac{d}{dx}(4x^2) = -8x$$

The electric field at point (1, 0, 2) will be (here $x = 1$)

$$E_x = -8\text{ V}$$

7. If we place a charge q at the sixth vertex of the regular hexagon, then the net force on the charge $-q$ placed at the center of hexagon will be zero due to symmetry.

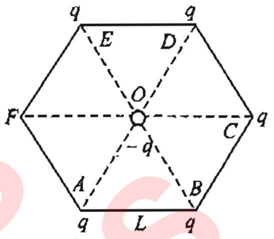


Fig. A1.240

The force on charge $-q$ due to the charge q placed on the sixth vertex, balances the net force on charge $-q$ due to the other five charges placed at the five vertices. The force on charge $-q$ due to charge q placed at the sixth vertex will be

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q \times q}{L^2}$$

where L is the distance of the center of hexagon from any vertex (directed from to C).

The magnitude of force on the point charge of value $-q$ coulomb placed at the center of the hexagon is $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{L^2}$ directed from O to F.

True or False

1. Let us consider two points A and B in an electric field. Let the potentials at A and B be V_A and V_B, respectively.

Now, by the definition of potential difference, the potential difference between two points B and A is the amount of work done in carrying a unit positive charge from A to B between the two points.

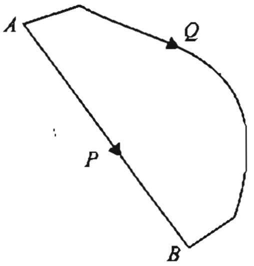


Fig. A1.241

Mathematically, for path $A \to P \to B$,

$$\frac{W_{APB}}{q} = V_B - V_A$$

$$W_{APB} = q\,(V_B - V_A) \qquad \text{(i)}$$

For path $A \to Q \to B$,

$$\frac{W_{AQB}}{q} = V_B - V_A$$

$$W_{AQB} = q\,(V_B - V_A) \qquad \text{(ii)}$$

Since the R.H.S. of equations (i) and (ii) is the same,

$$\Rightarrow \qquad W_{APB} = W_{AQB} \qquad \text{(iii)}$$

2. The statement is true. The metallic sphere which gets negatively charged gains electrons and hence its mass increases. The metallic sphere which gets positively charged loses electrons and hence its mass decreases.
3. When a high-energy X-ray beam falls, it will knock out electrons from the small metal ball making it positively charged. Therefore, the ball will be deflected in the direction of electric field.
4. The electric field produced between the parallel plate capacitor is uniform. The force acing on a charged particle placed in an electric field is given by $F = qE$.

   In the case of two protons, q and E are equal and hence force will be equal.

   The statement is true.
5. Force on charge $-q$ due to small charge dq situated at length dl

$$dF = k\frac{qdq}{5R^2}$$

Resolving this force into two parts $dF\cos\theta$ and $dF\sin\theta$ as shown in Fig. A1.242.

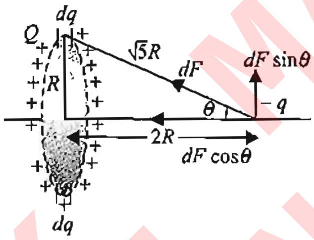


Fig. A1.242

If we take another diametrically opposite length dl, the charge on it being dq, then the force on charge $-q$ by this small charge dq will be

$$dF = k\frac{qdq}{5R^2}$$

Again, resolving this force, we find $dF\sin\theta$ cancels out with $dF\sin\theta$ of the previous force and $dF\cos\theta$ components add up.

$$F = \int_0^{2\pi R} dF\cos\theta = \int_0^{2\pi R}\frac{kqdq}{5R^2}\times\frac{2R}{\sqrt{5}R}$$

Charge on length $2\pi R = Q$

Charge on length $dl = \dfrac{Q\,dl}{2\pi R} = dq$

$$F = \int_0^{2\pi R}\frac{2kq}{5\sqrt{5}R^2}\times\frac{Q\,dl}{2\pi R}$$

$$= \frac{2kQq}{5\sqrt{5}\times 2\pi R^3}\times 2\pi R = \frac{2kQq}{5\sqrt{5}R^2}$$

This is not an equation of simple harmonic motion.

∴ the statement is false.

Single Correct Answers Type

1. **b.** Correct option is (b). Concept: The potential at the surface of a sphere is same as the potential at the center of the sphere.
2. **b.** If we take a point M on the X-axis as shown in the Fig. A1.243, then the net electric field is in X-direction.

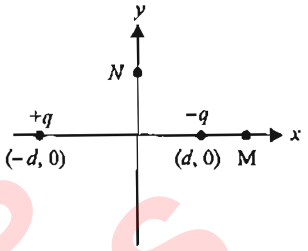


Fig. A1.243

∴ option (a) is incorrect.

If we take a point N on Y-axis, we find net electric field along $+X$-direction. The same will be true for any point on Y-axis. Therefore, (b) is a correct option.

∴ C is incorrect. The direction of dipole moment is from −ve to +ve. Therefore, (d) is incorrect.

3. **b.** Energy stored, $U = \frac{1}{2}C_{eq}V_{net}^2 = \frac{1}{2}(3C)V^2$ (Fig. A1.244)

$$= \frac{3}{2}CV^2$$

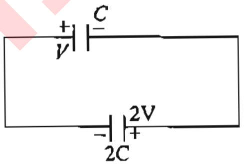


Fig. A1.244

∴ (b) is the correct option.

4. **b.** Let the distance to be travelled be x. Let the strength of uniform electric field be E.

For the electron:

$$u = 0, s = x, a = \frac{eE}{m_e}, t = t_1$$

$$S = ut + \frac{1}{2}at^2 \quad\Rightarrow\quad x = \frac{1}{2}\frac{eE}{m_e}\times t_1^2 \qquad \text{(i)}$$

For the proton:

$$u = 0, s = x, a = \frac{eE}{m_p}, t = t_2$$

$$S = ut + \frac{1}{2}at^2 \quad\Rightarrow\quad x = \frac{1}{2}\frac{eE}{m_p}\times t_2^2 \qquad \text{(ii)}$$

From equations (i) and (ii),

$$\frac{t_2^2}{t_1^2} = \frac{m_p}{m_e} \quad\Rightarrow\quad \frac{t_2}{t_1} = \left[\frac{m_p}{m_e}\right]^{1/2}$$

∴ (b) is the correct option.

5. **a.**

$$V_0 = K\frac{q}{R}, V_\alpha = 0$$

$$\int_{l=\alpha}^{l=o} -\vec{E}\cdot\vec{dl} = V_0 - V_\infty$$

$$= \frac{Kq}{r} - 0$$

$$= \frac{9 \times 10^9 \times 1.11 \times 10^{-10}}{0.5}$$

$$= 2\ \text{V}$$

(a) is the correct option.

6. **d.** For the capacitor to get charged upto 0.75 V, the charge on the plates should be

$$q = CV$$

$$= 10^{-5} \times 0.75 = 0.75 \times 10^{-5}\ \text{C}$$

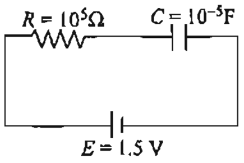


Fig. A1.245

Using the equation of charging of capacitor

$$q = CE\,[1 - e^{-t/RC}]$$

$$\Rightarrow \quad 0.75 \times 10^{-5} = 10^{-5} \times 1.5 \left[1 - e^{\frac{t}{10^5 \times 10^{-5}}}\right]$$

$$\Rightarrow \quad \frac{1}{2} = [1 - e^{-t}] \quad \Rightarrow \quad e^{-t} = \frac{1}{2}$$

Taking log on both sides,

$$-t = -\ln 2$$

$$\Rightarrow \quad t = 0.693\ \text{s}$$

(b) is the correct option.

7. **b.** With the closing of switch S_3, the potential across C_1 and C_2 would become identical to the average of V_1 and V_2, i.e., (30 V + 20 V)/2 = 25 V.

$\therefore$ (b) is the correct option.

8. **b.** (b) is the correct option.

Here, we have $\frac{Qq}{a} + \frac{q^2}{a} + \frac{Qq}{a\sqrt{2}} = 0$

or $Q\sqrt{2} + q\sqrt{2} + Q = 0$ or $Q(\sqrt{2} + 1) = -q\sqrt{2}$

$$\Rightarrow Q = -\frac{q\sqrt{2}}{\sqrt{2} + 1} = -\frac{2q}{2 + \sqrt{2}}$$

9. **b.** (b) is the correct option.

The effective capacitance is given by

$$\frac{\varepsilon_0 A}{d}\left[\frac{1}{(k_1 + k_2)} + \frac{1}{2k_3}\right]^{-1}$$

The capacitance of a single capacitor will be $\frac{\varepsilon_0 A}{d} k$

$$k = \left[\frac{1}{(k_1 + k_2)} + \frac{1}{2k_3}\right]^{-1} \text{ or } \frac{1}{k} = \frac{1}{(k_1 + k_2)} + \frac{1}{2k_3}.$$

10. **c.** Option (a) is not possible because all the three charges are positive and the electric lines of force will expand laterally and not contract longitudinally.

Option (b) is not possible as electric lines of force are continuous lines but there are three lines which end up abruptly somewhere in between the electric field which is not possible. Option (d) makes no sense.

$\therefore$ option (c) is correct.

11. **a.** When S is closed, there will be no shifting of negative charge from plate A to B as the charge $-q$ is held by the charge $+q$. Neither there will be any shifting of charge from B to A. Correct option is (a).

12. **b.** As we move along the direction of electric field, potential decreases.

$$V_A < V_B$$

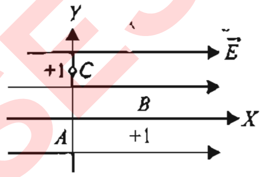


Fig. A1.246

Correct option is (b).

13. **b.** $U_i = \frac{2Qq}{4\pi\varepsilon_0(a)}$; $U_f = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{a + x} + \frac{1}{a - x}\right]$;

$U_i - U_f = \frac{(Qqx^2)}{2\pi\varepsilon_0 a^3}$ [for $x << a$, x^2 can be neglected in comparison to a^2 in denominator]

14. **c.** Initially,

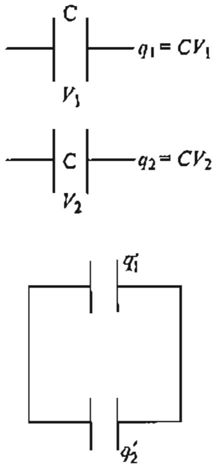


Fig. A1.247

Initial energy $= \frac{1}{2}C(V_1^2 + V_2^2) q_1' + q_2' = CV_1 + CV_2$

$$\frac{q_1'}{C} = \frac{q_2'}{C} \Rightarrow \quad q_1' = q_2' \text{ (Charge conservation)}$$

$$q_1' = \frac{C(V_1 + V_2)}{2}$$

Final energy $= \frac{C(V_1 + V_2)^2}{4}$

$\therefore$ change in energy = Initial energy – Final energy

$$= \frac{1}{2}C(V_1^2 + V_2^2) - \frac{C}{4}(V_1^2 + V_2^2 + 2V_1 V_2)$$

$$= \frac{C}{4}[2V_1^2 + 2V_2^2 - V_1^2 - V_2^2 - 2V_1V_2] = \frac{C}{4}(V_1 - V_2)^2$$

15. **c.** Electric field is perpendicular to the equipotential surfaces and is zero everywhere inside the metal.

16. **c.** $|\vec{E}| = \frac{kq}{r^2}$

Electric field due to P on O is cancelled by electric field due to S on O (Fig. A1.248).

Similarly, electric field due to Q to O is cancelled by electric field due to T and O. The electric field due to R on O in the same direction as that of U and O. Therefore, the net electric field is $2\vec{E}$.

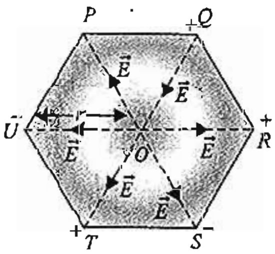


Fig. A1.248

17. **d.** The flux through the Gaussian surface is due to the charges inside the Gaussian surface. But the electric field on the Gaussian surface will be due to the charges present in the Gaussian surface and outside it. It will be due to all the charges.

18. **c.** Fig. A1.249 shows the electric fields due to the sheets 1, 2 and 3 at point P. The direction of electric fields is according to the charge on the sheets (away from positively charged sheet and towards the negatively charged sheet and perpendicular).

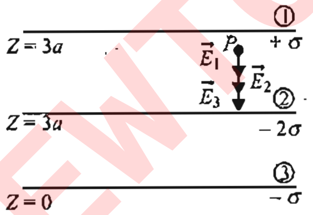


Fig. A1.249

The total electric field

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$
$$= E_1(-\hat{k}) + E_2(-\hat{k}) + E_3(-\hat{k})$$
$$= \left[\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right](-\hat{k})$$
$$= -\frac{2\sigma}{\varepsilon_0}\hat{k}$$

∴ (c) is the correct option.

19. **a.** When a charge density is given to the inner cylinder, the potential developed at its surface is different from that on the outer cylinder. This is because the potential decreases with distance from a charged conducting cylinder when the point of consideration is outside the cylinder.

But when a charge density is given to the outer cylinder, it will change its potential by the same amount as that of the inner cylinder. Therefore, no potential difference will be produced between the cylinders in this case.

20. **d.** When a positive point charge is placed outside a conducting sphere, a rearrangement of charge takes place on the surface. But the total charge on the sphere is zero as no charge has left or entered the sphere.

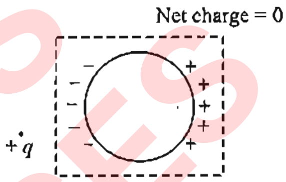


Fig. A1.250

21. **b.** Let us consider a uniformly charged solid sphere without any cavity. Let the charge per unit volume be σ and O be the center of the sphere. Let us consider a uniformly charged sphere of negative charged density σ having its center at O'. Also, let OO' be equal to a (Fig. A1.251).

Let us consider on arbitrary point P in the small sphere. The electric field due to charge on big sphere $\vec{E}_1 = \frac{\sigma}{3\varepsilon_0}\overrightarrow{OP}$

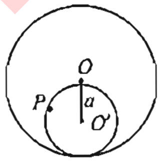


Fig. A1.251

Also, the electric field due to small sphere

$$\vec{E}_2 = \frac{\sigma}{3\varepsilon_0}\overrightarrow{PO}$$

∴ the total electric field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{3\varepsilon_0}[\overrightarrow{OP} + \overrightarrow{PO}] = \frac{\sigma}{3\varepsilon_0}\overrightarrow{OO'}$$

This will have a finite value which will be uniform.

22. **c.** The charges make an electric dipole. Points A and B lie on the equatorial plane of the dipole. Therefore, potential at A = potential at $B = 0$

$$W = q(V_A - V_B) = q \times 0 = 0$$

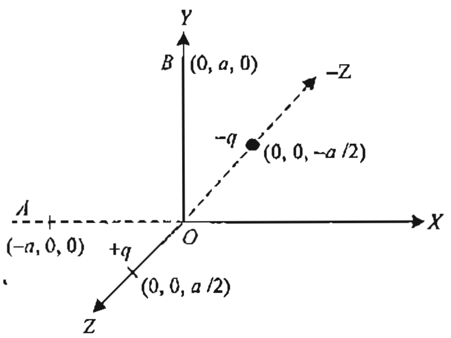


Fig. A1.252

23. c. $F_{BC} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\left(\frac{q}{3}\right)\left(\frac{2q}{3}\right)}{(R/\sqrt{3})^2} = \dfrac{q^2}{54\pi\varepsilon_0 R^2}$

24. a. Total charge enclosed by cube is $-2C$. Hence electric flux through the cube is $\dfrac{-2C}{\varepsilon_0}$.

25. b. $\frac{Q_1}{4\pi R^2} = \frac{Q_1+Q_2}{4\pi(2R)^2} = \frac{Q_1+Q_2+Q_3}{4\pi(3R)^2}$

$\Rightarrow Q_1 : Q_2 : Q_3 :: 1 : 3 : 5$

Assertion-Reasoning Type

1. b. Both are basic facts and have no relation with each other

Comprehension Type

For Problems 1–3

1. a., 2. b., 3. c.

Sol. Net charge with in $r < R$ is constant hence electric field is independent of a.

$$q = \int_0^R \frac{d}{R}(R-x)4\pi x^2 dx = Ze.$$

$$d = \frac{3Ze}{\pi R^3}$$

If within a sphere ρ is constant, then $E \propto r$.

Multiple Correct Answers Type

1. d. Let us consider the positive charge Q at any instant of time t at a distance x from the origin. It is under the influence of two forces, $F_1(=F)$ and $f_2\,(=F)$. On resolving these two forces we find that $F\sin\theta$ cancels out. The resultant force is

$$F_R = 2F\cos\theta = 2\times\frac{kQq}{\sqrt{x^2+a^2}}\times\frac{x}{\sqrt{x^2+a^2}} = \frac{2kQqx}{(x^2+a^2)^{3/2}}$$

Since F_R is not proportional to x, the motion is not simple harmonic. The charge Q will accelerate till the origin and gain velocity. At the origin, the net force is zero but due to momentum it will cross the origin and move towards left. As it comes on negative x-axis, the force is again towards the origin.

2. a., d. (Fig. A1.253)

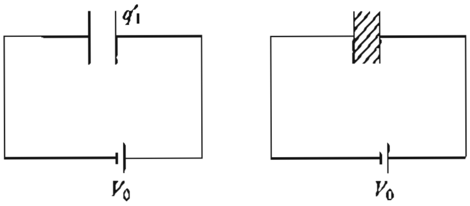


Fig. A1.253

Before introducing dielectric slab:

P.d. $= V_0$

Capacitance $= C$

Charge, $q_0 = CV_0$

P.E. $U_0 = \frac{1}{2}CV_0^2$

After introduction of dielectric slab

P.d. = V_0

Capacitance = KC [K is the dielectric constant of slab; $K > 1$]

New charge, $Q = KCV_0$

New P.E., $U = \frac{1}{2}KCV_0^2$

$\therefore$ Correct options are (a) and (d).

3. b. q has to be negative for equilibrium.

Considering equilibrium of 1

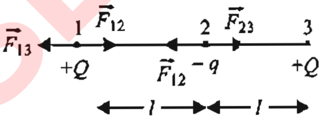


Fig. A1.254

$$F_{13} = F_{12}$$

$$\frac{KQ\times Q}{(2l)^2} = \frac{kQq}{l^2} \text{ or } q = \frac{Q}{4}$$

4. b., d. From Fig. A1.255

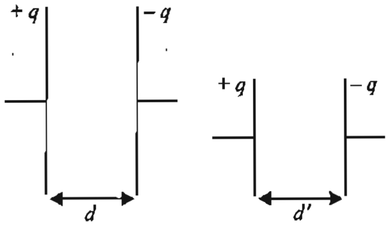


Fig. A1.255

Charge on plate is q	Charge on plate is q
$C = \dfrac{\varepsilon_0 A}{d}$	$C' = \dfrac{\varepsilon_0 A}{d'} \Rightarrow C' < C$
$q = CV \Rightarrow V = \dfrac{q}{C}$	$V' = \dfrac{q}{C'} \Rightarrow V' > V$
$U = \dfrac{1}{2}q\times V$	$U' = \dfrac{1}{2}qV' \Rightarrow U' > U$

$\therefore$ Options (b) and (d) are correct.

5. a. The potential inside the shell will be the same everywhere as on its surface. As we add $3Q$ charge on the surface, the potential on the surface changes by the same amount as that inside. Therefore, the potential difference remains the same.

(a) is the correct answer.

6. a. The equivalent capacitance (Fig. A1.256)

$$\frac{1}{C_{eq}} = \frac{1}{2}+\frac{1}{2}+\frac{1}{2\times5} = \frac{11}{10}$$

$$C_{eq} = \frac{11}{10}\ \mu\text{F}$$

$\therefore$ (a) is the correct option.

7. a., c., d. $C = \dfrac{\varepsilon_0}{d}A,\quad C' = \dfrac{K\varepsilon_0 A}{d}$

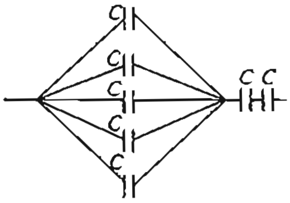


Fig. A1.256

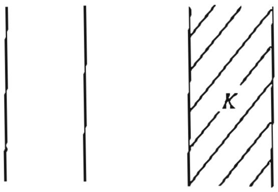


Fig. A1.257

$$Q = CV = \frac{\varepsilon_0 A}{d} \times V = \frac{\varepsilon_0 A V}{d}$$

[Q will remain same as no charge is leaving or entering the plates during the process of slab insertion]

$$Q = C'V' = C'E'd$$

$$E' = \frac{Q}{C'd} = \frac{\frac{\varepsilon_0 A V}{d}}{\frac{K\varepsilon_0 A}{d}\times} d = \frac{V}{Kd}$$

Work done is the change in energy stored

$$W = \frac{1}{2}C'V'^2 - \frac{1}{2}CV^2$$

$$= \frac{1}{2}\frac{K\varepsilon_0 A}{d} \times \frac{V^2}{K^2} - \frac{1}{2}\frac{\varepsilon_0 A}{d} \times V^2 \quad \left[\because V' = E'd = \frac{V}{K}\right]$$

$$= \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2\left[\frac{1}{K} - 1\right]$$

∴ (a), (c) and (d) are correct options.

8. **b.** The work done in moving a charge from A to B

$$W = (\text{T.P.E.})_A - (\text{T.P.E.})_B$$

T.P.E = Total potential energy

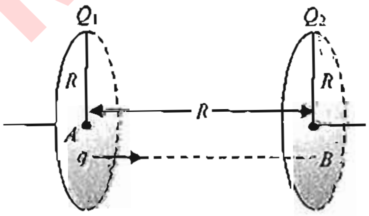


Fig. A1.258

$(\text{T.P.E.})_A$ = [P.E. due to Q_1 + P.E. due to Q_2]

$$= \left[\left(\frac{Q_1}{4\pi\varepsilon_0 R}\right) \times q + \left(\frac{Q_2}{4\pi\varepsilon_0\sqrt{R^2 + R^2}}\right)q\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_1 + \frac{Q_2}{\sqrt{2}}\right]$$

$(\text{T.P.E.})_B$ = [P.E. due to Q_2 + P.E. due to Q_1]

$$= \left[\left(\frac{Q_2}{4\pi\varepsilon_0 R}\right)q + \left(\frac{Q_1}{4\pi\varepsilon_0\sqrt{R^2 + R^2}}\right)q\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_2 + \frac{Q_1}{\sqrt{2}}\right]$$

$$\therefore\ W = \frac{q}{4\pi\varepsilon_0 R}\left[Q_1 + \frac{Q_2}{\sqrt{2}} - Q_2 - \frac{Q_1}{\sqrt{2}}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 R}\left[Q_1\left(1 - \frac{1}{\sqrt{2}}\right) - Q_2\left(1 - \frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{q(Q_1 - Q_2)}{4\pi\varepsilon_0 R}\left(\frac{\sqrt{2} - 1}{\sqrt{2}}\right)$$

∴ (b) is the correct option.

9. **c.** Let λ be the charge per unit length. Let us consider a Gaussian surface (dotted cylinder) (Fig. A1.259).

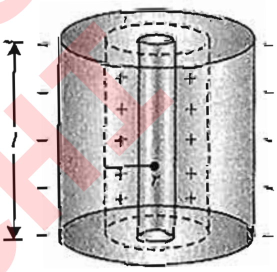


Fig. A1.259

Applying Gauss's law

$$\phi = \oint_s \vec{E}\cdot\vec{ds} = \frac{\lambda l}{\varepsilon_0}$$

For the flat portions of Gaussian surface, the angle between electric field and surface is 90°. Hence, flux through flat portions is zero. By symmetry, the electric field on the curved surface is same throughout.

The angle between $\vec{E}$ and $\vec{ds}$ is 0° (for curved surface)

$$E\int ds = \frac{\lambda l}{\varepsilon_0} \Rightarrow E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$E \propto \frac{1}{r}$$

∴ (c) is the correct option.

10. **d.** The electric lines of force cannot enter the metallic sphere as electric field inside the solid metallic sphere is zero. Also, the origination ad termination of the electric lines of force from the metallic surface is normal (directed towards the center)

(d) is the correct option.

11. **b., c.** In regions I and III, there will be electric field $\vec{E}_0$ directed from + to −. In region II, due to orientation of dipoles, there is an electric field $\vec{E}_k$ present in opposite direction of $\vec{E}_0$. But since $\vec{E}_0$ is also present, the net electric field is $E_0 - E_k$ in the direction of $\vec{E}_0$ as shown in Fig. A1.260. ($\because\ E_0 > E_k$)

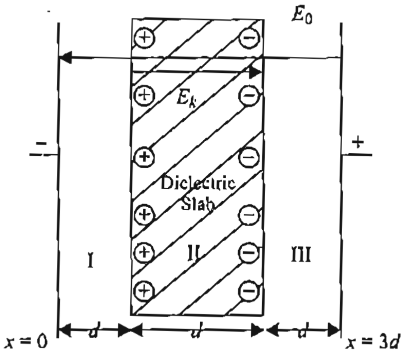


Fig. A1.260

Also, please note that when one moves opposite to the direction of electric field, the potential always increases. The stronger the electric field, the more is the increase in potential. Since in region II, the electric field is less as compared to I and III, therefore the increase in potential will be less but there has to be increase in potential in all regions from $x = 0$ to $x = 3d$.

12. **d.** Potential at origin will be given by

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{x_0} - \frac{1}{2x_0} + \frac{1}{3x_0} - \frac{1}{4x_0} + \cdots\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{1}{x_0}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right] = \frac{q}{4\pi\varepsilon_0 x_0}\ln(2)$$

13. **(a, c)** Let Q be the charge on the ring, the negative charge $-q$ is released from point P (0, 0, Z_0). The electric field at P due to the charged ring will be along positive z-axis and its magnitude will be

$$E = \frac{1}{4\pi\varepsilon_0}\frac{QZ_0}{(R^2 + Z_0^2)^{3/2}}$$

Therefore, force on charge P will be towards center as shown, and its magnitude is

$$F_e = qE = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{(R^2 + Z_0^2)^{3/2}}Z_0 \quad \text{(i)}$$

Similarly, when it crosses the origin, the force is again towards center O.

Thus, the motion of the particle is periodic for all values of Z_0 lying between 0 and ∞.

Secondly, if $Z_0 << R$, $(R^2 + Z_0^2)^{3/2} \to R^3$

$$F_e = \frac{1}{4p\varepsilon_0} \times \frac{Qq}{R^3} \times Z_0 \text{ [from equation (i)]}$$

i.e., the restoring force $F_e \propto -Z_0$. Hence, the motion of the particle will be simple harmonic. (Here, negative sign implies that the force is towards its mean position.)

14. **a., c.** The expressions of the electrical field are:

Inside the sphere ($r < R$), $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{R^3}r$;

Outside the sphere ($R < r < \infty$), $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$

Hence, E increases for $r < R$ and decreases for $R < r < \infty$.

15. **c., d.** When two points are connected with a conducting path in electrostatic condition, then the potential of the two points is equal. Therefore (c) is the correct option. Option (d) follows from Gauss's law.

(a) and (b) are dependent on the curvature which are different at points A and B.

16. **a., b., c., d.**

For $r > R_0$, $E = -\dfrac{d\phi}{dr} = \dfrac{Q}{4\pi\varepsilon_0 r^2}$

∴ Charge enclosed by concentric spherical surface of r

$$= 2R_0 = \varepsilon_0\,\phi_E\,4\pi r^2$$

$$= \varepsilon_0\frac{Q}{4\pi\varepsilon_0 r^2}4\pi r^2 = Q$$

For $r < R_0$, $E = -\dfrac{dV}{dr} = 0$ and for $r > R_0$

$$E = -\frac{dV}{dr} = 4\pi\varepsilon_0 r^2 \text{ (Here, } V = \phi)$$

Integer Answer Type

$\rho = kr^a$

$$E\left(r - \frac{R}{2}\right) = \frac{1}{8}E(r - R)$$

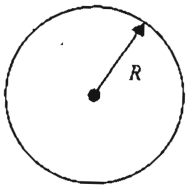


Fig. A1.261

$$\frac{q_{\text{enclosed}}}{4\pi\varepsilon_0(R/2)^2} = \frac{1}{8}\frac{Q}{4\pi\varepsilon_0 R^2}$$

$32q_{\text{enclosed}} = Q$

$\text{enclosed} = Q$

$$q_{\text{enclosed}} = \int_0^{R/2} kr^a 4\pi r^2 dr = \frac{4\pi k}{(a+3)}\left(\frac{R}{2}\right)^{(a+3)}$$

$$Q = \frac{4\pi k}{(a+3)}R^{(a+3)}$$

$$\frac{Q}{q_{\text{enclosed}}} = 2^{a+3}$$

$2^{a+3} = 32$

$a = 2$

CHAPTER

5

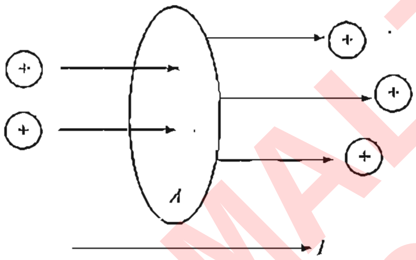
Electric Current and Circuits

- Electric Current
- Current Density
- Drift Velocity
- Structural Model for Electrical Conductor
- Mobility
- Temperature Coefficient of Resistivity
- Validity and Failure of Ohm's Law
- Electromotive Force and Potential Difference
- Internal Resistance of a Cell
- Combination of Resistance
- Kirchhoff's Law: Kirchhoff's Laws For Electrical Networks
- Wheatstone Bridge: Balanced Wheatstone Bridge
- Combination of Cells
- Superposition Principle
- Charging

In electrostatics, our discussion of electric phenomena has been focused on charges at rest. In the previous chapter we treated the concept of electric potential, which is measured in volt. Now we will see that this voltage acts like an "electrical pressure" that can produce a flow of charge or current, which is measured in ampere (or simply, amp and abbreviated as A) and that the resistance that restrains this flow is measured in ohm (Ω).

ELECTRIC CURRENT

An isolated metallic conductor, say a wire, contains a few electrons which are moving at random with high speeds. These are called conduction electrons. The rate at which these electrons pass from left to right through a point in a wire is the same as the rate at which they pass from right to left through the same point, i.e., net rate is zero.



Charges in motion through an area A. The time rate at which charge flows through the area is defined as the current I. The direction of the current is the direction in which positive charges flow when free to do so.

Fig. 5.1

To define the current mathematically, suppose charged particles are moving perpendicular to a surface of area A as in Fig. 5.1 (this area could be the cross-sectional area of a wire, for example). The current is defined as the rate at which electric charge flows through this surface. If ΔQ is the amount of charge that passes through this area in time interval Δt, then average current, I_{avg}, over this time interval through this area is the ratio of the charge to the time interval,

i.e., $$I_{avg} = \frac{\Delta Q}{\Delta t} \qquad \text{(i)}$$

It is possible for the rate at which the charge flows to vary with time. We define the instantaneous current I as the limit of the preceding expression as Δt goes to zero;

$$I \equiv \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \qquad \text{(ii)}$$

Points to Remember

- The particles flowing through a surface can be charged positively or negatively, or we can have two or more types of particles moving, with charges of both the signs in the flow. Conventionally, we define the direction of the current as the direction of flow of positive charge.
- In a common conductor such as copper, the current is in physical state due to the motion of the negatively charged electrons. Therefore, when we speak of current in such a conductor, the direction of the current is opposite to the direction of flow of electrons.
- On the other hand, if one considers a beam of positively charged protons in a particle accelerator, then the current is in the direction of the motion of the protons.
- In some cases—gases and electrolytes, for example, the current is the result of the flow of both positive and negative charged particles.
- It is common to refer to a moving charged particle (whether it is positive or negative) as a mobile charge carrier. For example, the charge carriers in a metal are electrons.

When a wire is connected to a battery, an electric field is set up at every point within the wire. This field exerts a force on each conduction electron. Although the electrons are continuously accelerated by the field, but due to their frequent collisions with the atoms of the wire, they on an average simply drift at a small constant speed in the direction opposite to the field. Thus, there is a net flow of change in the wire at a small rate. The total charge passing through any cross-section per second is the electric current in the wire.

In the steady state of current through each section of the conducting loop would be same – no matter what is the location or orientation of area of that section see Fig. 5.2 This is because of the fact that charge is conserved.

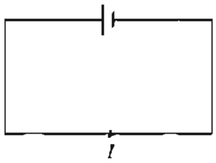


Fig. 5.2

Unit of Electric Current

It is ampere (A) in SI system.

"The current is said to be one ampere when one coulomb of charge flows past any cross-section of a conductor every second."

Statement of Ohm's Law

"The electrical current in any conductor is proportional to the potential difference between its ends, other factors remaining constant."

The ratio of the potential difference to current is termed as the resistance of the conductor.

Accordingly, $R = \frac{V}{I} \quad \Rightarrow \quad V = IR$

where, I = current, V = p.d., and R = resistance

The resistance of the conductor is the opposition offered b the conductor to the flow of electric current passing through it.

The resistance R not only depends on the material of th conductor but also on the dimensions of the conductor.

The resistance of an ohmic conducting wire is found to b proportional to its length ℓ and inversely proportional to its cross section area A.

i.e., $$R = \rho\frac{\ell}{A} \qquad \text{(iii)}$$

where the constant of proportionality ρ is called the *resistivity* of the material, which has the unit ohm metre (Ωm). To understand the relationship between resistance and resistivity, we should know that ρ depends on the properties of the material and on temperature. On the other hand, the resistance R of a particular conductor depends on its size and shape as well as on the resistivity of the material.

The inverse of resistivity is defined as *conductivity* σ. Hence, the resistance of an ohmic conductor can be expressed in terms of its conductivity as

$$R = \frac{\ell}{\sigma A}, \quad \text{where } \sigma = \frac{1}{\rho}$$

Resistance and resistivity: Resistivity is a property of a substance, whereas resistance is a property of an object. We have seen similar pairs of variables before. For example, density is a property of a substance, whereas mass is a property of an object. Equation (iii) which is given above relates resistance to resistivity.

SI Unit of Resistance

The unit of resistance is ohm (Ω).

"The resistance of a conductor is said to be 1 ohm if a current of 1 ampere flows through it when the p.d. across its ends is 1 volt."

CURRENT DENSITY

It is the current flowing per unit area of the cross-section of the medium. The medium may be a conductor or a beam of charged particles. Unit of current density is A/m^2 in SI units.

Note:

While electric current is a scalar quantity, electric current density is a vector quantity.

If we take a localized view and study the flow of charge through a cross section of the conductor at a particular point, to describe this flow, we can used the current density J, which has the same direction as that of velocity of positive charge. We can write the amount of current through the elements of area dS as $di = \vec{J}d\vec{S}$, where $d\vec{S}$ is the area vector of the element, perpendicular to the area of the element. The total current through the surface will be $i = \int \vec{J}d\vec{S}$.

If the current is uniform across the surface and perpendicular to it, then J is uniform over the area and parallel to $d\vec{S}$.

$$\therefore \qquad i = \int JdS = J\int dS = JS \Rightarrow J = \frac{I}{S}$$

In case of conductors as $V = IR$ and by definitions,

$$E = \frac{V}{L} \text{ and } R = \rho\frac{L}{S}. \text{ So, } (EL) = I\rho\frac{L}{S}$$

i.e., $$J = \frac{I}{S} = \frac{1}{\rho}E \text{ or } \vec{j} = \sigma\vec{E} \quad \left\{\text{with } \sigma = \frac{1}{\rho}\right\} \qquad \text{(i)}$$

i.e., in the case of conductors, current density is proportional to electric field $\vec{E}$.

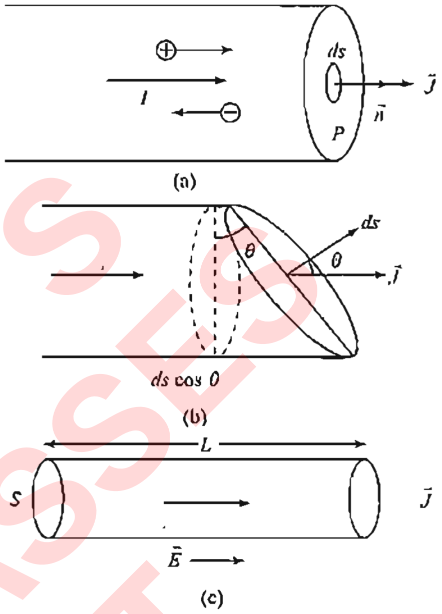
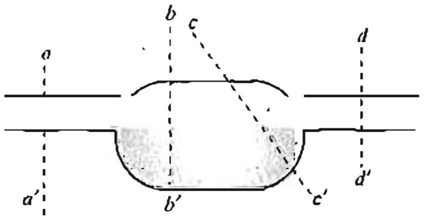


Fig. 5.3

Points to Remember

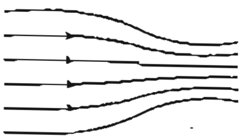
- If the current has not reached a steady state, i.e., the flow of charge is not constant, then the current through different cross sections at a particular instant may have different values.
- Electric current may be distributed non-uniformly over the surface through which it passes. Hence, to characterize current in greater detail, current density vector $\vec{J}$ is introduced.
- Current density, $\vec{J}$, tells us how charge flows at a certain point and its direction tells us about the direction of the flow of charge at that point, while the current describes how charge flows through an extended object.



Current through all sections would be same in steady state

Fig. 5.4

- The direction of current density is same as that of the velocity of +ve charge or opposite to the direction of the velocity of –ve charge.
- Current density can be represented by a similar set of lines known as stream lines. The spacing of the stream lines suggest the value of current density. Narrower stream lines mean more current density, spaced stream lines mean less current density (Fig. 5.5).



A conductor having
non uniform cross-section

Fig. 5.5

This in turn implies that in the case of conductors:

a. Direction of current density is the same as that of electric field.

b. If electric field is uniform (i.e., is constant) current density will be constant

c. If the electric field is zero (as in electrostatics inside a conductor), current density and hence current will be zero.

Illustration 5.1 **In a hydrogen discharge tube, the number of protons drifting across a cross-section per second is 1.0×10^{18}, while the number of electrons drifting in the opposite direction across another cross-section is 2.7×10^{18} per second. Find the current flowing in the tube.**

Sol. As electrons and protons are moving in the opposite directions, they will effectively produce current in the same direction and the total current in the tube is $I = (n_p + n_e)e/t$

$$= (1.0\times10^{18} + 2.7\times10^{18})\times 1.6\times10^{-19}/1$$
$$= 3.7\times1.6\times10^{-1}\text{ A} = 0.592\text{ A}$$

Illustration 5.2 **You need to produce a set of cylindrical copper wires 2.5 long that will have a resistance of 0.125 Ω each. What will be the mass of each of these wires? (Density of copper is 8.9×10^3 Kg/m^3, resistivity of copper is 1.72×10^{-8} Ωm).**

Sol. Given $L = 2.5$ m, $R = 0.125\ \Omega$.

To find out the mass of each wire we need to find out the volume of one of the wire as

Mass = Density × Volume

$$\text{Volume} = \text{Area}\times\text{Length. } [\because\ R = \frac{\rho L}{A}\ \Rightarrow\ A = \frac{\rho L}{R}]$$

$$\text{Volume} = AL = \frac{\rho L^2}{R} = \frac{1.72\times10^{-8}\times(2.5)^2}{0.125}$$

$$= \frac{1.72\times10^{-8}\times6.25}{0.125} = 1.72\times10^{-7}\times5$$

$$= 8.6\times10^{-7}\text{ m}^3$$

$$m = d\times v = 8.9\times10^{-3}\times8.6\times10^{-7}$$

$$= 9\times8.6\times10^{-4} = 1.72\times10^{-7}\text{ kg} = 7.74\text{ g}$$

Hence, the mass of each wire is desired to be 7.74 g.

Illustration 5.3 **Consider a wire of length l, area of cross-section A and resistivity ρ with resistance 10 Ω. Its length is increased by applying a force and it becomes four times of its original value. Find the changed resistance of the wire.**

Sol. Here $l_1 = l$, $A_1 = A$, and $R = 10\ \Omega$. Similarly, $l_2 = 4l$ and $R_2 = ?$ Resistivity is same in each case as the material is same. The volume of the wire will remain the same even after the increase in the length.

$$A_1l_1 = A_2l_2 \quad\Rightarrow\quad A_2 = \frac{A_1l_1}{l_2} = \frac{Al}{4l} = \frac{A}{4}$$

The formula used for measuring resistance of wire $R = \rho\frac{l}{A}$.

$$\text{Using this formula in both cases, } R_1 = \rho\frac{l_1}{A_1} = \frac{\rho l}{A} \qquad \text{(i)}$$

$$\text{and} \qquad R_2 = \rho\frac{l_2}{A_2} = \rho\frac{4l}{A/4} = 16\,\rho\,\frac{l}{A} \qquad \text{(ii)}$$

$$\text{Dividing equation (ii) by (i) } \frac{R_2}{R_1} = 16 \quad\Rightarrow\quad R_2 = 160\ \Omega$$

Illustration 5.4 **Consider a wire of length l, area of cross-section A, and resistivity ρ where resistance is 10 Ω. Its length is increased by applying a force on it and its length increases four times of its horizontal length. Find the changed resis-tance of the wire.**

Sol. Here $l_1 = l$, $A_1 = A$, and $R = 10\ \Omega$. Similarly, $l = 4l + l = 5l$, $A_2 = ?$ and $R_2 = ?$ The resistivity of the material will remain the same as wire in both the cases is same. The volume of the wire will be same as only the shape of the wire has changed

$$A_1l_1 = A_2l_2 \quad\Rightarrow\quad A = \frac{A_1l_1}{A_2l_2} = \frac{Al}{5l} = \frac{A}{5} \qquad \text{(i)}$$

The formula used for measuring the resistance of wire.

$$R = \frac{\rho l}{A}$$

Applying this formula to both situations one by one we have.

$$R_1 = \rho\frac{l_1}{A_1} = \frac{\rho l}{A}, \text{ and}$$

$$R_2 = \rho\frac{l_2}{A_2} = \rho\frac{5l}{A/5} = 25\,\frac{\rho l}{A} \qquad \text{(ii)}$$

$$\text{Dividing equation (ii) by (i) we have } \frac{R_2}{R_1} = 25$$

$$\Rightarrow \qquad R_2 = 25\,R_1 = 250\ \Omega$$

Illustration 5.5 **A wire of mass m, length l, density d, and area of cross-section A is stretched in such a way that its length increases by 10% of its original value. Express the changed resistance in percentage.**

Sol. Given mass m, length $l_1 = l$, density d, and area of cross-section $A_1 = A$. Let ρ be the resistivity and R_1 be the resistance of the wire. Mass of wire m = volume × density = $Al\times d = Al\,d$.

∴ area of cross-section $A_1 = \frac{m}{ld}$, the resistance of the wire

$$R_1 = \rho\frac{l}{A_1} = \rho\frac{l^2 d}{m} = \left(\frac{\rho d}{m}\right) l^2 = k\,l^2 \qquad \text{(i)}$$

Let l_2 be the new length

$$\therefore \quad l_2 = l + \frac{10}{100} l = l + 0.1 l = 1.1 l$$

Let R_2 be the resistance of the wire after stretching, then

$$R_2 = k l_2^2 \qquad \text{(ii)}$$

Dividing equation (ii) by (i),

$$\frac{R_2}{R_1} = \frac{l_2^2}{l_1^2} = \frac{(1.1)^2 l_1^2}{l_1^2} = 1.21$$

$$\Rightarrow \quad R_2 = 1.21\ R_1 = R_1 + 0.21\ R_1$$

$$\Rightarrow \quad R_2 - R_1 = 0.21\,R_1$$

Hence, the percentage charge in the resistance

$$\frac{R_2 - R_1}{R_1} \times 100 = 21\ \%$$

Illustration 5.6 **A uniform copper wire of mass 2.23×10^{-3} kg carries a current of 1 A when 1.7 V is applied across it. Calculate the length and the area of cross-section. If the wire is uniformly stretched to double its length; calculate the new resistance. Density of copper is 8.92×10^{3} kg/m^3 and resistivity is 1.7×10^{-8} Ω-m.**

Sol. As, m = volume × density = $(L \times S) \times d$

[as volume = $L \times S$]

$$\text{So,} \quad L \times S = \frac{m}{d} = \frac{2.23 \times 10^{-3}}{8.92 \times 10^{3}} = \frac{1}{4} \times 10^{-6} \qquad \text{(i)}$$

$$\text{And as,} \quad V = IR, \text{ i.e.,} \quad R = \frac{V}{I} = \frac{1.7}{1} = 1.7\ \Omega \qquad \text{(ii)}$$

But as by definition, $R = \rho\,(L/S)$

$$\Rightarrow \quad \frac{L}{S} = \frac{R}{\rho} = \frac{1.7}{1.7 \times 10^{-8}} = 10^{8} \qquad \text{(iii)}$$

Solving equations (i) and (iii) for L and S we get, L = 5 m and $S = 5 \times 10^{-8}\ \text{m}^2$

When the wire is stretched uniformly to double its length, the volume will remain unchanged, i..e,

$$SL = S'(2L) \quad \text{so} \quad S' = S/2$$

and hence the new resistance will be

$$R' = \rho\frac{(2L)}{(S/2)} = 4\rho\frac{L}{S} = 4R = 4 \times 1.7 = 6.8\,\Omega$$

DRIFT VELOCITY

Under the normal conditions of temperature and pressure and without the influence of any external electrostatic field, the motion of free electrons in a conductor is due to the thermal energy and random.

Now, when this conductor is placed in an external field, these free electrons start experiencing electric force and start moving under the influence of this force (see Fig. 5.6). However, although they are free to move, they are not able to move in a straight line because they encounter other electrons, ions, atoms, or molecules in their way (see Fig. 5.7). Hence they experience collisions after collisions, but are able to drift in a particular direction because of this external field. The drifting of these free electrons over some period of time is called drift velocity.

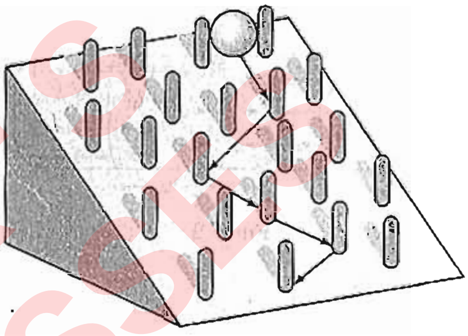


Fig. 5.6

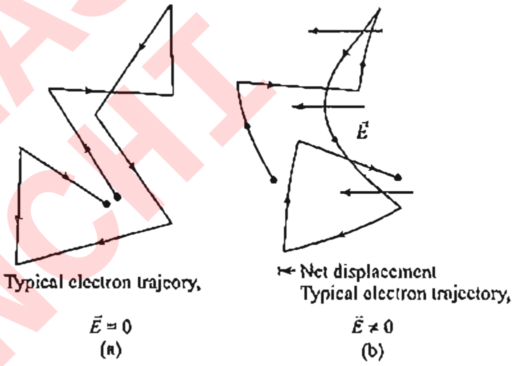


Fig. 5.7

Relation Between Drift Velocity and Current

Let A = area of cross-section of the conductor, e = charge on each electron, v_d = drift velocity, n = number of free electrons per unit volume, and I = current, then

Total number of electrons between cross-sections P and Q which are v_d distance apart = (Volume between P and Q) × n = $Av_d n = nAv_d$

∴ total charge in this volume,

$$nAv_d e = neAv_d$$

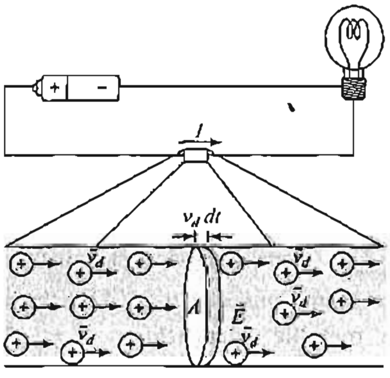


Fig. 5.8

Now, the electron which is present at the cross-section Q, will reach the cross-section P after one second because P and Q are so selected that the distance between them is v_d which is the drift speed of the electron. Therefore, $neAv_d$ is the charge which will pass through the cross-section at P (where P can be any point on the conductor). Hence this is the electric current which flows through the conductor. (see Fig. 5.9)

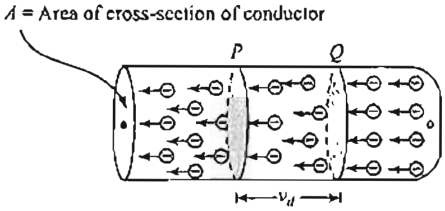


Fig. 5.9

$$\therefore \qquad I = neAv_d$$

Also, accordingly current density, $J = \frac{I}{A} = nev_d$

The equation, $J = nev_d$, can be written in vector form as follows: $\bar{J} = nq\bar{v}_d$ where q is the charge of the charge carrier and v_d is the average drift velocity. This equation is correct for both the signs of q. If $q > 0$, $\bar{v}_d$ is in the direction of electric field $\bar{E}$ and $\bar{J}$ is in the direction of $\bar{E}$. If $q < 0$ ($q = -e$ for electrons), as it is in metallic conductor, $\bar{v}_d$ is opposite to $\bar{E}$, and $\bar{J} = -ne\bar{v}_d$ continues to be in the direction of $\bar{E}$.

Illustration 5.7 **A copper wire has a square cross-section of 6 mm on a side. The wire is 10 m long and carries a current of 3.6 A. The density of free electrons is 8.5 × 10^{28}/m^3. Find the magnitude of (a) the current density in the wire; (b) the electric field in the wire. (c) how much time is required for an electron to travel the length of the wire? (ρ, electrical resistivity, is 1.72 × 10^{-8} Ω m)**

Sol. Given $r = 6$ mm, $l = 10$ m, $I = 3.6$ A, $n = 8.5 \times 10^{28}$/m^3.

(a) To find the current density, formula used should be $J = \frac{I}{A}$

$$\Rightarrow \qquad J = \frac{3.6}{(6\times10^{-3})^2} = \frac{3.6}{36\times10^{-6}} = 10^5 \text{ A/m}^2$$

(b) To find electric field,

$$E = \rho J = 1.72 \times 10^{-8} \times 10^5 = 1.72 \times 10^{-3} \text{ V/m}$$

(c) Time taken,

$$t = \frac{l}{v_d} = \frac{lneA}{I}$$

$$= \frac{10 \times 8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times (6 \times 10^{-3})^2}{3.6}$$

$$= 9.1 \times 10^6 \, S \text{ (approx)}$$

Illustration 5.8 **Consider a wire of length 0.1 m with an area of cross-section 1 mm^2 connected to 5 V. Find the current flowing through the metallic wire where μ = 45 × 10^{-6} m^2 V^{-1} s^{-1}, $e = 1.6 \times 10^{-19}$ C, $n = 8 \times 10^{28}$ m^{-3}.**

Sol. Given $A = 1$ mm$^2 = 10^{-6}$ m^2, $l = 0.1$ m, $V = 5$ V

Due to the applied potential difference across the wire an electric field is set up in the conductor.

Formula used, $E = \frac{V}{l}$, $= \frac{5}{0.1} = 50 \text{ V m}^{-1}$

The current flowing through the wire is given by

$$I = nAev_d = nAe\mu E \qquad \left[\text{where, } \mu = \frac{v_d}{E}\right]$$

$$= 8 \times 10^{28} \times 10^{-6} \times 1.6 \times 10^{-19} \times 5 \times 10^{-6} \times 50$$

$$= 400 \times 8 \times 10^{-3} = 3.2 \text{ A}$$

Illustration 5.9 **A current, 16 A, is made to pass through a conductor where the number density of free electrons is 4 × 1028 m^{-3} and its area of cross-section is 10^{-5} m^2. Find out the value of the drift velocity of free electrons.**

Sol. Given $I = 16$ A, $A = 10^{-5}$ m^2, $n = 4 \times 10^{28}$ m^{-3}.

Also $\qquad e = 1.6 \times 10^{-19}$ C

Formula for drift velocity

$$v_d = \frac{I}{neA} = \frac{16}{4\times10^{28}\times1.6\times10^{-19}\times10^{-5}}$$

$$= \frac{4}{1.6\times10^4} = \frac{1}{4}\times10^{-3}$$

$$= 0.25 \times 10^{-3} = 2.5 \times 10^{-4} \text{ ms}^{-1}$$

Illustration 5.10 **(a) Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area 1.0 × 10^{-7} m^2 carrying a current of 1.5 A. Assume that each copper atom contributes roughly one conduction electron. The density of copper is 9.0 × 10^3 kg/m^3, and its atomic mass is 63.5 u. (b) Compare the drift speed obtained above with speed of propagation of electric field along the conductor which causes the drift motion.**

Sol.

(a) The direction of drift velocity of conduction electrons is opposite to the electric field direction, i.e., the electrons drif in the direction of increasing potential. The drift speed v_d is given by eq. $v_d = (I/neA)$.

Now, $e = 1.6 \times 10^{-10}$ C, $A = 1.0 \times 10^{-7}$ m^2, $I = 1.5$ A, th density of conduction electrons, n is equal to the number o atoms per cubic metre (assuming one conduction electron pe Cu atom is reasonable from its valence electron count o one). A cubic metre of copper has a mass of 9.0 × 10^3 kg Since 6.0 × 10^{23} copper atoms have a mass of 63.5 g.

$$\text{so} \qquad n = \frac{6.0\times10^{23}}{63.5}\times9.0\times10^6 = 8.5 \times 10^{28}$$

which gives,

$$v_d = \frac{1.5}{8.5\times10^{28}\times1.6\times10^{-19}\times1.0\times10^{-7}}$$

$$= 1.1 \times 10^{-3} \text{ ms}^{-1} = 1.1 \text{ mms}^{-1}$$

(b) An electric field travelling along the conductor has a speed of an electromagnetic wave, namely equal to 3.0×10^8 ms^{-1}. The drift speed is, in comparison, extremely small; smaller by a factor of 10^{-11}.

A STRUCTURAL MODEL FOR ELECTRICAL CONDUCTOR

Consider a conductor as a regular array of atoms containing free electrons (sometimes called conduction electrons). Such electrons are free to move through the conductor. In the absence of an electric field, the free electrons move in random directions with average speeds of the order of 10^6 m/s. The situation is similar to the motion of the gas molecules confined in a vessel that we studied in kinetic theory of gases. In fact, the conduction electrons in a metal are often called electros gas.

Conduction electrons are not totally free because they are confined to the interior of the conductor and undergo frequent collisions with the array of atoms. The collisions are the predominant mechanism contributing to the resistivity of a metal at normal temperatures. Note that there is no current in a conductor in the absence of an electric field because the average velocity of the free electrons is zero. On an average, just as many electrons move in one direction as in the opposite direction, so there is no net flow of charge.

However, the situation is modified, when an electric field is applied to the metal. In addition to the random thermal motion, the free electrons drift slowly in a direction opposite that of the electric field, with an average drift speed of v_d, which is much less (typically 10^{-4} m/s) than the average speed between collisions (typically 10^6 m/s).

In our structural model, we shall assume that the excess kinetic energy acquired by the electrons in the electric field is lost to the conductor in the collision process. The energy given up to the atoms in the collisions increases the total vibrational energy of the atoms, causing the conductor to warm up. The model also assumes that an electron's motion after a collision is independent of its motion before the collision.

On the basis of our model, we now take the first step towards obtaining an expression of the drift speed. Let a mobile charged particle of mass m and charge q is subjected to an electric field $\vec{E}$. For electrons in a metal $\vec{F}_e = e\vec{E}$. The motion of the electron can be determined from Newton's second law, $\sum \vec{F} = m_e\vec{a}$. The acceleration of the electron is

$$\vec{a} = \frac{\sum \vec{F}}{m_e} = \frac{\vec{F}_e}{m_e} = \frac{-e\vec{E}}{m_e} \qquad \text{(i)}$$

The acceleration, which occurs for only a short time interval between collisions, changes the velocity of the electron. Because the force is constant, the acceleration is constant, and we can model the electron as a particle under constant acceleration. If $\vec{v}_0$ is the velocity of the electron just after a collision, at which we define the time as $t = 0$, the velocity of the electron at time t is

$$\vec{v} = \vec{v}_0 + \vec{a}t = \vec{v}_0 - \frac{e\vec{E}}{m_e}t \qquad \text{(ii)}$$

The motion of the electron through the metal is characterized by a very number of collisions per second. Consequently, we consider the average value of $\vec{v}$ over a time interval compared with the time interval between collisions, which gives us the drift velocity $\vec{v}_d$. Because the velocity of the electron after a collision is assumed to be independent of its velocity before the collision, the initial velocities are randomly distributed in direction, so that the average value of $\vec{v}_0$ is zero. In the second term on the right of equation (ii), the charge, electric field, and the mass are all constant. Therefore, the only factor affected by the averaging process is the time t. The average value of this term is $\left(-e\vec{E}/m_e\right)\tau$, where τ is the average time interval between collisions. Therefore, eq. (ii) becomes, after the averaging process,

$$\vec{v}_d = \frac{-e\vec{E}}{m_e}\tau \qquad \text{(iii)}$$

Substituting the magnitude of this drift velocity (the drift speed) into equation $I_{avg} = \frac{\Delta Q}{\Delta t} = nev_d A$, we have

$$I = nev_d A = ne\left(\frac{eE}{m_e}\tau\right)A = \frac{ne^2E}{m_e}\tau A \qquad \text{(iv)}$$

According to Ohm's law, the current is related to the macroscopic variables of potential difference and resistance:

$$I = \frac{\Delta V}{R}$$

Incorporating equation $R = \rho\frac{\ell}{A}$, we can write this expression as

$$I = \frac{\Delta V}{\left(\rho\frac{\ell}{A}\right)} = \frac{\Delta V}{\rho\ell}A$$

In the conductor, the electric field is uniform, so we use eq. $\Delta V = E\ell$, to substitute for the magnitude of the potential difference across the conductor:

$$I = \frac{E\ell}{\rho\ell}A = \frac{E}{\rho}A \qquad \text{(v)}$$

Setting the two expressions for the current, equations (iv) and (v), equal we solve for the resistivity:

$$I = \frac{ne^2E}{m_e}\tau A = \frac{E}{\rho}A \rightarrow \rho = \frac{m_e}{ne^2\tau} \qquad \text{(vi)}$$

According to this structural model, resistivity does not depend on the electric field or, equivalently, on the potential difference, but depends only on fixed parameters associated with the material and the electron. This feature is characteristic of a conductor obeying Ohm's law. The model shows that the resistivity can be calculated from a knowledge of the density of the electrons, their charge and mass, and the average time interval τ between collisions.

We can also write current density as

$$J = nev_d = (ne)\left(\frac{eE}{m}\tau\right) = \frac{ne^2}{m}\tau E$$

Illustration 5.11 **A copper wire of cross-sectional area 3.00 $\times 10^{-6}$ m^2 carries a current 10.0 A. Find**

a. **The drift speed of the electrons in the wire. Assume that each copper atom contributes one free electron to the body of material.**

b. **The average time between collisions for electrons in the copper at 20 °C. The density of copper is 8.95 g/cm^3, motor mass of copper 63.5 g/mol, Avagadro number 6.02×10^{-23} electron / mol and resistivity of copper.**

Sol.

a. The volume occupied by 63.5 g of copper

$$V = \frac{M}{\rho} = \frac{63.5}{8.95} = 7.09 \text{ cm}^3/\text{mol}$$

As each copper atom contributes one free electron to the body of the material, the density of free electrons is

$$n = \frac{6.02\times10^{23}}{7.09\times10^{-6}} = 8.48\times10^{28} \text{ electron}/\text{m}^3$$

The drift speed

$$v_d = \frac{I}{neA};$$

$$v_d = \frac{10.0}{8.48\times10^{28}\times1.60\times10^{-19}\times300\times10^{-6}}$$

$$= 2.46\times10^{-4} \text{ m/s}$$

b. Average time between collision for electrons

$$\tau = \frac{m_e}{ne^2\rho}$$

$$= \frac{9.10\times10^{-31}}{8.48\times10^{28}\times(1.6\times10^{-19})^2\times1.7\times10^{-8}}$$

$$= 2.5\times10^{-14} \text{ s}$$

MOBILITY

As we have seen, conductivity arises from mobile charge carriers. In metals, these mobile charge carriers are electrons; in an ionized gas, they are electron and positive charged ions; in an electrolyte, there can be both positive an negative ions. In a semiconductor material such as germanium or silicon, conduction is partly due to electrons and partly due to electron vacancies called holes. Holes are sites of missing electrons which act like positive charges.

An important quantity is the mobility μ defined as the magnitude of the drift velocity per unit electric field

$$\mu = \frac{v_d}{E}$$

Illustration 5.12 **Consider a conductor of length 40 cm where a potential difference of 10 V is maintained between the ends of conductor. Find out the mobility of the electrons provided the drift velocity of the electrons is 5×10^{-6} ms^{-1}.**

Sol. Given $L = 40$ cm, $V = 10$ V, $v_d = 5\times10^{-6}$ ms^{-1}

To find the electron mobility we need the value of the electric field which can be obtained using following formula

$$E = \frac{V}{l} \Rightarrow E = \frac{10}{0.4} = 25 \text{ V/m}$$

Also the formula used for electron mobility:

$$\mu = \frac{v_d}{E} = \frac{5\times10^{-6}}{25} = \frac{1}{5}\times10^{-6}$$

$$= 0.2\times10^{-6} \simeq 2\times10^{-7} \text{ m}^2\text{V}^{-1}\text{s}^{-1}$$

Effect of temperature on resistivity and resistance: As long as the temperature of the material is constant, the resistivity of material also remains constant. As the temperature of material increases, the relaxation time (t) decreases the resistivity increases and hence resistance also increases.

As the number of electrons per unit volume goes up, the resistance decreases. This can be easily visualized, since if the density of electrons increases, more electrons can flow in response to the potential difference and hence the current will increase. Therefore, the resistance will decrease.

TEMPERATURE COEFFICIENT OF RESISTIVITY

Let us study the equation

$$R_T = R_{T_0}[1 + \alpha(T - T_0) + \beta(T - T_0)^2]$$

where α and β both are called temperature coefficients of resistance, though different in magnitude.

Temperature Coefficients of Resistance

In case of pure metals, β is negligibly small, so the resistance varies linearly with the rise of temperature (see Fig. 5.10).

$$R_T \simeq R_{T0}[1 + \alpha(T - T_0)]$$

where R_T = resistance at temperature T

R_{T0} = resistance at temperature T_0 (called reference temperature).

α = a constant for a given metal and for a given reference temperature and is called temperature coefficient of resistance. Its unit is per degree temperature (°C^{-1}).

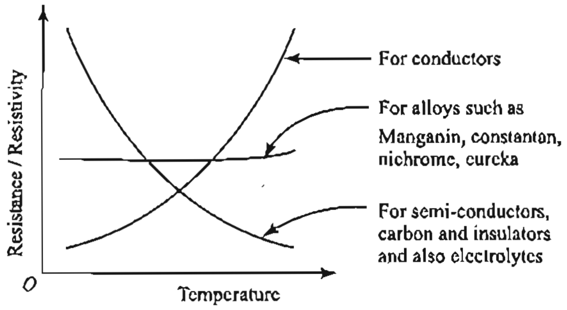


Fig. 5.10

T_0 is some reference temperature generally either 0°C or 20°C. α is (+) for metals, .004 per degree for *Cu*, (+) but very small (almost zero) for alloys such as mangain (α= .00001 per degree), nichrome, constantan, eureka, and (–) for semiconductors and insulators.

Let $\rho_T = \rho_0[1+\alpha(T-T_0)]$ (see Fig. 5.11)

$$\Rightarrow \quad \rho_T - \rho_0 = \alpha\rho_0(T-T_0)$$

$$\Rightarrow \quad \alpha = \frac{1}{\rho_0}\times\frac{(\rho_T-\rho_0)}{(T-T_0)} \quad \Rightarrow \quad \rho = \frac{1}{\rho}\times\frac{d\rho}{dT}$$

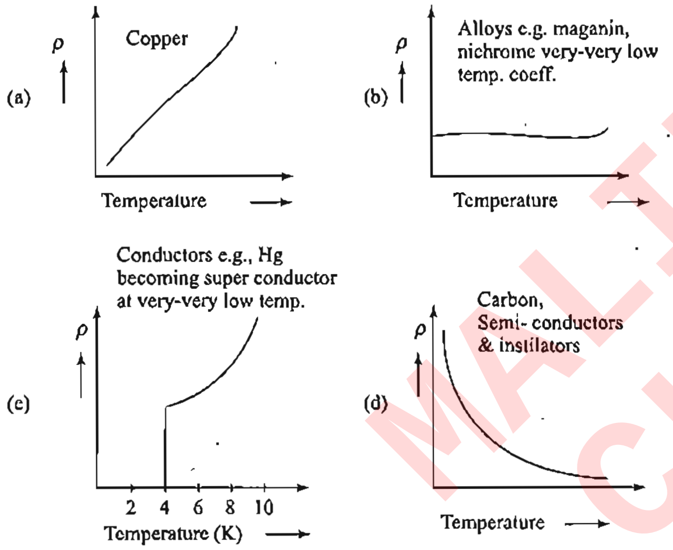


Fig. 5.11

These observations can be understood qualitatively using the relation for resistivity: $\rho = \frac{m}{ne^2\tau}$

For metals, the number of free electrons is fixed. As temperature increases, the amplitude of vibration of atoms/ions increases and collisions of electrons with them become more effective and frequent, resulting in the decrease in τ and hence, increase in ρ. Thus for metals, ρ increases with temperature.

Finding Coefficient of Resistance

i. $\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$ where, R_2 = Resistance at t_2 °C and

R_1 = Resistance at t_1 °C

Derivation

$$R_1 = R_0(1+\alpha t_1) \qquad \text{(i)}$$

$$R_2 = R_0(1+\alpha t_2) \qquad \text{(ii)}$$

Dividing equation (ii) by equation (i) we have

$$\frac{R_2}{R_1} = \frac{1+\alpha t_2}{1+\alpha t_1} \quad \Rightarrow \quad \frac{R_2}{R_1} = (1+\alpha t_2)(1+\alpha t_1)^{-1}$$

$$\frac{R_2}{R_1} = (1+\alpha t_2)\,(1-\alpha t_1)$$

(Using binomial theorem and thus neglecting higher terms)

$$\frac{R_2}{R_1} = (1+\alpha t_2)(1-\alpha t_1)$$

(since α^2 is very small and hence neglected)

$$R_2 = R_1 + R_1\alpha(t_2 - t_1) \quad \Rightarrow \quad \alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$$

ii. $\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$; where R_2 = Resistance at t_2 °C

Derivation

$$R_1 = R_0(1+\alpha t_1) \qquad \text{(i)}$$

$$R_2 = R_0(1+\alpha t_2) \qquad \text{(ii)}$$

Dividing equation (ii) by equation (i), we have

$$\frac{R_2}{R_1} = \frac{1+\alpha t_2}{1+\alpha t_1} \quad \Rightarrow \quad R_2 + R_2\alpha t_1 = R_1 + R_1\alpha t_2$$

$$R_2 - R_1 = \alpha(R_1 t_2 - R_2 t_1) \quad \Rightarrow \quad \alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

iii. $\alpha = \frac{R_{t_1} - R_0}{R_0 t}$; where R_t = Resistance at t °C and

R_0 = Resistance at 0 °C

Derivation

$$R_t = R_0(1+\alpha t) \quad \Rightarrow \quad R_t - R_0 = R_0\alpha t$$

$$\Rightarrow \quad \alpha = \frac{R_t - R_0}{R_0 t}$$

Illustration 5.13 **A copper coil has a resistance of 20.0 Ω at 0°C and a resistance of 26.4 Ω at 80°C. Find out the temperature coefficient of resistance of copper.**

Sol. $R_{80°C} = R_{0°C}[1+\alpha\Delta T]$

$$\Rightarrow \quad 26.4\,\Omega = 20.0\,\Omega[1+\alpha\times(80-0)]$$

$$\Rightarrow \quad \frac{26.4}{20} = 1 + 80\,\alpha$$

On solving, $\alpha = 4\times10^{-3}\,°C^{-1}$.

Illustration 5.14 **A metallic wire has a resistance of 120 Ω at 20° C. Find the temperature at which the resistance of same metallic wire rises to 240 Ω where the temperature coefficient of wire is 2×10^{-5} °C^{-1}.**

Sol. Given $R_1 = R_{20}$, $R_2 = R_T = 120\ \Omega$, t = ?, $\alpha = 2\times10^{-4}$ °C^{-1}, t_1 = 20°C, $t_2 = T$ = ?.

Formula used to find the temperature coefficient of resistance

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)} \quad \Rightarrow \quad \alpha = \frac{R_T - R_{20}}{R_{20}(T-20)}$$

$$2\times10^{-4}=\frac{240-120}{120(T-20)} \Rightarrow T-20=\frac{120}{120\times2\times10^{-4}}$$

$$\Rightarrow \quad T=\frac{1}{2\times10^{-4}}+20=0.5\times10^{4}+20=5020°\text{C}$$

VALIDITY AND FAILURE OF OHM'S LAW

Ohm's law is not a law of nature, i.e., it is not a universal law which would apply everywhere, under all conditions. Ohm's law is obeyed by metallic conductors, which accordingly are called ohmic conductors, that too at about normal working temperatures. At very high currents/voltages, even ohmic conductors do not follows this law as shown in the following sketch. Semiconductors also do not follow ohm's law as shown Figs. 5.12 and 5.13.

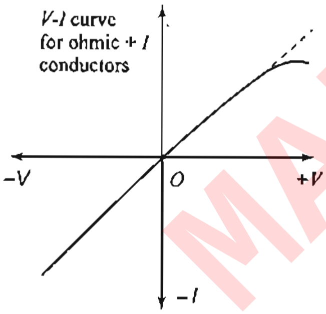
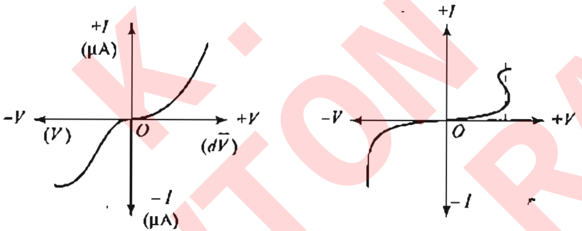


Fig 5.12



V–I curve of a Diode junction

V–I curve of a Thyrister

Fig. 5.13

Thus, Ohm's law is not followed in the following cases:

Materials: (i) Vacuum tubes (ii) Crystal rectifiers (iii) Transistors (iv) Thermistors, thyristors (v) Superconductors

Conditions: (i) At very high temperatures (ii) At very low temperatures (Superconductivity) (iii) At very high potential differences

Concept Application Exercise 5.1

1. How many electrons per second pass through a section of wire carrying a current of 0.7 A?

2. A current of 7.5 A is maintained in a wire for 45 s. In this time (a) how much charge and (b) how many electrons flow through the wire?

3. If 0.6 mol of electrons flow through a wire in 45 min, what are (a) the total charge that passes through the wire and (b) the magnitude of the current?

4. A typical copper wire might have 2×10^{21} free electrons in 1cm of its length. Suppose that the drift speed of the electrons along the wire is 0.05 cm/s. How many electrons would pass through a given cross-section of the wire each second. How large a current would be flowing in the wire?

5. A coil of wire has a resistance of 25.00 Ω at 20°C and a resistance of 25.17 Ω at 35°C. What is its temperature coefficient of resistance?

6. A metal wire of diameter 2 mm and of length 300 m has a resistance of 1.6424 Ω at 20 °C and 2.415 Ω at 15°C. Find the values of α, R_0, and ρ_0, where the zero subscript refers to 0°C, and $\rho_{20°C}$. Identify the metal.

7. It is desired to make a 20 Ω coil of wire which has a zero thermal coefficient of resistance. To do this, a carbon resistor of resistance R_1 is placed in series with an iron resistor of resistance R_2. The proportions of iron and carbon are so chosen that $R_1 + R_2 = 20\,\Omega$ for all temperatures near 20°C. How long are R_1 and R_2?

8. A resistance thermometer measures temperature by the increase in resistance of a wire of high temperature. If the wire is platinum and has a resistance of 10 Ω at 20°C and a resistance of 35 Ω in a hot furnace, what is the temperature of the furnace? (α for platinum is 0.0036 °C^{-1})

9. Though the drift velocity of electrons responsible for current in a conductor under ordinary circumstances is very small even then lights in a room turn on immediately after the switch is closed. Explain why and how?

10. a. A steady current flows in a metallic conductor of non-uniform cross-section. State which of the quantities current, current density, electric field, and drift velocity remains constant?

b. A steady current passes through a cylindrical conductor. Is there an electric field inside the conductor?

11. A potential difference V is applied to copper wire of diameter d and length L. What is the effect on the electron drift speed of doubling (a) voltage V; (b) length L; and (c) diameter d?

12. The current-voltage graphs for a given specimen at two different temperatures T_1 and T_2 are shown in Fig. 5.14. (a) is the specimen ohmic; (b) at which temperature resistance is greater; and (c) which temperature is higher ?

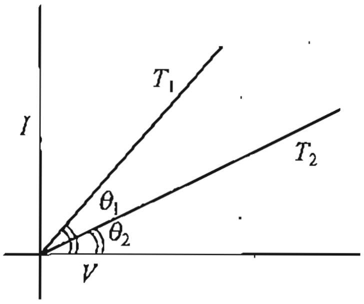


Fig. 5.14

13. A wire has a resistance R. What will be its resistance if (a) it is double on itself and (b) it is stretched so that (i) its length is doubled (ii) its radius is halved.

14. The current in a wire varies with time according to the equation $i = 4 + 2t$, where i is in ampere and t is in sec. Calculate the quantity of charge which has passed through a cross-section of the wire during the time $t = 2$ s to $t = 6$ s.

15. The following table gives the length of three copper rods, their diameters, and the potential differences between their ends. Rank the rods according to (a) the magnitude of the electric field within them, (b) the current density within them, and (c) the drift speed of electrons through them, greatest first.

Rod	Length	Diameter	P.D.
1	L	$3d$	V
2	$2L$	d	$2V$
3	$3L$	$2d$	$2V$

16. V-I graph for a metallic wire at two different temperatures T_1 and T_2 is as shown in the Fig. 5.15. Which of the two temperatures T_1 and T_2 is higher and why?

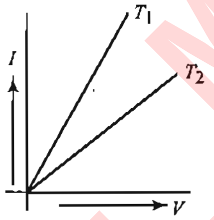


Fig. 5.15

17. The voltage-current variation of two metallic wires X and Y at constant temperature are as shown in the Fig. 5.16. Assuming that the wires have the same length and the same diameter, explain which of the two wires will have larger resistivity.

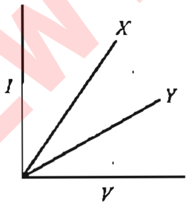


Fig. 5.16

18. What is the drift speed of the conduction electrons in a copper wire with radius $r = 900$ μm when it has a uniform current $i = 17$ mA? Assume that each copper atom contributes one conduction electron to the current and the current density is uniform across the wire's cross section.

19. A beam contains 2.0×10^8 doubly charged positive ions per cubic centimeter, all of which are moving north with a speed of 1.0×10^5 m/s. (a) What are the magnitude and direction of the current density $\vec{J}$? (b) Can you calculate the total current i in this ion beam? If not, what additional information is needed.

20. The current density across a cylindrical conductor of radius R varies in magnitude according to the equation $J = J_0\left(1 - \frac{r}{R}\right)$, where r is the distance from the central axis. Thus, the current density is a maximum J_0 at that axis ($r = 0$) and decreases linearly to zero at the surface ($r = R$). Calculate the current in terms of J_0 and the conductor's cross-sectional area $A = \pi R^2$.

21. Figure 5.17 shows a conductor of length l having a circular cross-section.

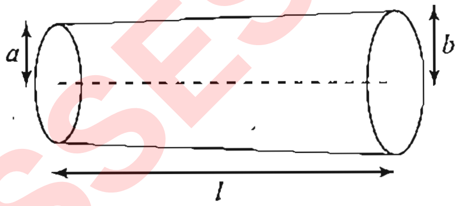


Fig. 5.17

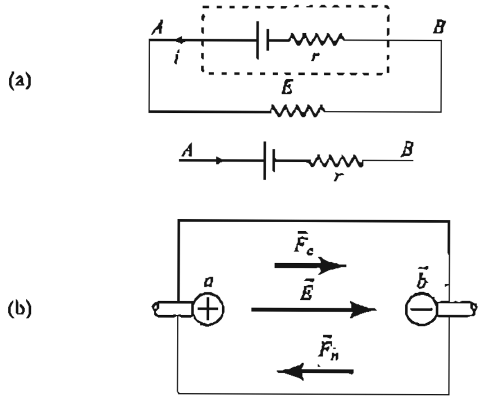
The radius of cross-section varies linearly from a to b. The resistivity of the material is ρ. Assuming that b-$a \leftrightarrow l$, find the resistance of the conductor.

22. If a copper wire is stretched to make it 0.1% longer, what is the percentage change in its resistance? **(IIT-JEE, 1978)**

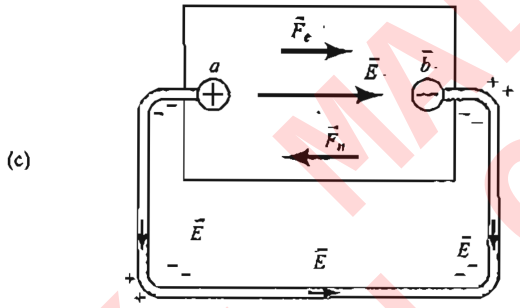
23. A copper wire having cross-sectional area of 0.5 mm^2 and a length of 0.1 m is initially at 25°C and is thermally insulated from the surrounding. If a current of 10 A is set up in this wire, (a) find the time in which the wire will start melting. The change of resistance with the temperature of the wire may be neglected. (b) What will this time be, if the length of the wire is doubled? **(IIT-JEE, 1979)**

ELECTROMOTIVE FORCE AND POTENTIAL DIFFERENCE

A device which can supply energy to charge carriers and thereby maintain their flow in a circuit is called a set of electromotive force (abbreviated as an e.m.f). A voltaic cell, a thermocouple, and a coil rotating in a magnetic field are some common examples of the seats of e.m.f. Thus, a seat of emf is a device in which energy is converted from non-electrical to electrical form and in which charge carriers are passed on from lower potential to higher potential. The e.m.f of such a device is measured by the rate at which the energy is converted from non-electrical to electrical form during the passage of unit charge. If ΔW = energy converted into the electrical form when Δq amount of electricity passes through it, then e.m.f (ε) of the device is given by $\varepsilon = (\Delta W/\Delta q)$ joule per coulomb or volt. Thus, charge carriers carrying Δq amount of charge receive $\Delta W = \varepsilon\Delta q$ joules of energy while passing through a seat of e.m.f from lower (negative) potential plate to the higher (positive) potential plate (see Fig. 5.18).



Source of emf
not connected to a circuit:
electric-field force $\vec{F}_e$
has the same magnitude as
non-electrostatic force $\vec{F}_n$



Source of emf connected to a circuit:
electric-field force $\vec{F}_e$ has the same
magnitude as non-electrostatic force $\vec{F}_n$

Fig. 5.18

The potential difference between any two points is defined as the energy given up by charge carriers carrying unit charge and is, therefore, equal to the work done by the charge carriers carrying unit charge from one point to the other.

If ΔW is the energy given up by charge carriers carrying Δq charge then the potential difference between the ends of a current carrying conductor is $V = (\Delta W)/(\Delta q)$ joule per coulomb or volt.

$$\Rightarrow \quad V_{ab} = \varepsilon \qquad \text{(i)}$$

Now let's make a complete circuit by connecting a wire path resistance R to the terminals of a source [as shown in Fig. 5.18(c)]. The potential difference between terminals a and b sets up an electric field within the wire; this causes current to flow around the loop from a to b, from higher to lower potential. Notice that where the wire bends, equal amounts of positive and negative charge persist on the "inside" and "outside" of the bend. These charges exert the forces that cause the current to follow the bends in the wire.

From $V = IR$ (equation of relationship between voltage, current, and resistance), the potential difference between the ends of the wire in Fig. 5.18(a) is given by $V_{ab} = IR$. Combining with equation (i), we have,

$$\varepsilon = V_{ab} = IR \qquad \text{(ii)}$$

That is, when a positive charge q flows around the circuit, the potential rise ε as it passes through the ideal source is numerically equal to the potential drop $V_{ab} = IR$ as it passes through the remainder of the circuit. Once ε and R are known, this relation determines the current in the circuit.

Difference Between e.m.f (E) and Potential Difference (V)

As the name suggests potential difference (p.d.) between two points is the difference in their potentials. Terminal p.d. of a cell, however, has a special meaning. It means p.d. between the terminals of a cell in closed circuit, i.e., when the current is flowing through the cell. E.m.f is the p.d. between the terminals of a cell in open circuit, i.e., when no current flows through the cell.

INTERNAL RESISTANCE OF A CELL

Real sources in a circuit do not behave in exactly the way we have described. The potential difference across a real source in a circuit is not equal to the e.m.f. The reason is that charge moving through the material of any real source encounters resistance. We call this the internal resistance of the source, denoted by r. If this resistance behaves according to Ohm's law, r is constant, and is independent of the current I. As the current moves through r, it experiences an associated drop in the potential which is equal to Ir. Thus, when a current is flowing through a source from the negative terminal b to the positive terminal a, the potential difference V_{ab} between the terminals is $V_{ab} = \varepsilon - Ir$.

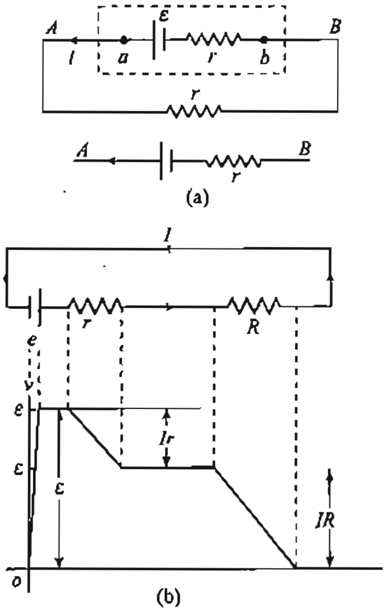


Fig. 5.19

A cell of emf ε always has some internal resistance r.

If a current I is drawn from the cell, a potential drop Ir occurs inside the cell and therefore, if the potential difference between the terminals of the battery is V, then

$$\varepsilon - V = Ir \;\Rightarrow\; \varepsilon - Ir \;\Rightarrow\; V_A - V_B = \varepsilon - Ir$$

If the external resistance is R, then

$$V = IR \;\Rightarrow\; \varepsilon - Ir \;\Rightarrow\; I = \left(\frac{\varepsilon}{R+r}\right)$$

Variation of the potential difference in different parts of the circuit is shown in Fig. 5.19.

COMBINATION OF RESISTANCES

Resistances in Series

If resistances are connected as shown in Fig. 5.20 such that the current flowing through them is the same, the resistances are said to be in series.

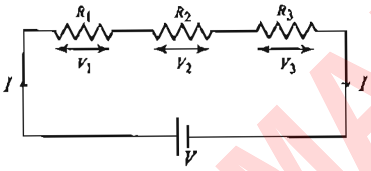


Fig. 5.20

If I = current flowing through the resistances then potential drop across each is

$$V_1 = IR_1, \quad V_2 = IR_2, \quad V_3 = IR_3$$

Adding, $V_1 + V_2 + V_3 = I(R_1 + R_2 + R_3)$ (i)

but $V_1 + V_2 + V_3 = V$, so we get from equation (i) as

$$V = I(R_1 + R_2 + R_3)$$

where V = Potential difference across the combinations. Also from Ohm's law $V = IR$ (ii)

From equations (i) and (ii), we get $R = R_1 + R_2 + R_3$ where R is known as *equivalent resistance*.

Resistances in Parallel

If the resistances are connected between the same two points such that the potential drop across each resistance is same, then the resistances are said to be in parallel (Fig. 5.21).

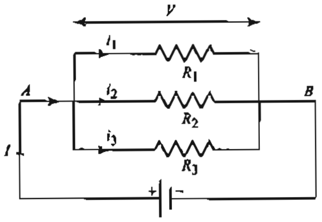


Fig. 5.21

In this case, if V is the potential difference between the points A and B, then

$$V = i_1R_1, \; V = i_2R_2, \quad \text{and} \quad V = i_3R_3$$

$$\Rightarrow \quad i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad \text{and} \quad i_3 = \frac{V}{R_3}$$

$\therefore$ total current flowing through the battery

$$i = i_1 + i_2 + i_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

i.e., $$i = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Also by Ohm's law, $i = V/R$, where R is the equivalent resistance between A and B. So we get $\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$

Voltage Divider

In a series circuit, current through each resistor is the same (see Fig. 5.22).

$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$$

$$I = \frac{V}{R_1 + R_2 + R_3}$$

$$V_1 = \frac{VR_1}{R_1 + R_2 + R_3}, \; V_2 = \frac{VR_2}{R_1 + R_2 + R_3},$$

$$V_3 = \frac{VR_3}{R_1 + R_2 + R_3}$$

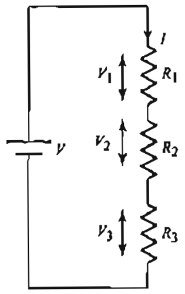


Fig. 5.22

Current Divider for Two Resistances

From Fig. 5.23, we have,

$$I = I_1 + I_2 \qquad \text{(i)}$$

$$I_1R_1 = I_2R_2 \qquad \text{(ii)}$$

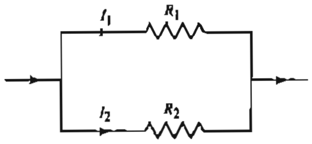


Fig. 5.23

On solving equations (i) and (ii), we get,

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I; \quad I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I$$

The division of current in the branches of a parallel circuit is inversely proportional to their resistances.

Current Divider for Three Resistances

The division of current in the branches of a parallel circuit is inversely proportional to their resistances (see Fig. 5.24).

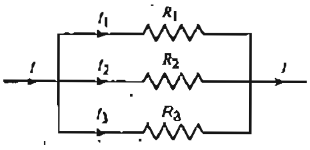


Fig. 5.24

$$I = I_1 + I_2 + I_3$$

$$I_1R_1 = I_2R_2 = I_3R_3$$

and
$$R_{eq} = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

$$I_1 = I\left[\frac{R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

$$I_2 = I\left[\frac{R_1R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

$$I_3 = I\left[\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

It is easy to remember the expressions for I_1, I_2, and I_3. Notice which resistance is missing in the numerator.

In all parts of Fig. 5.25, the resistances R_1, R_2, and R_3 are connected in parallel between two the points M and N.

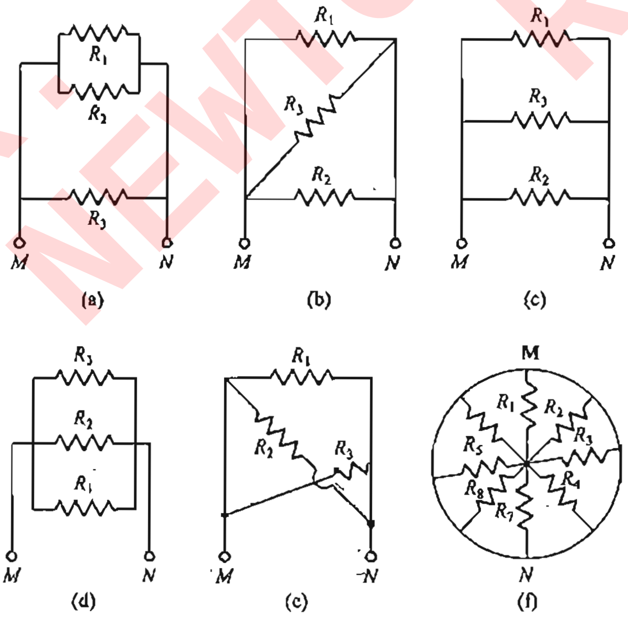


Fig. 5.25

Calculation of Effective Resistance

Method of Successive Reduction

It is the most common method to determine the equivalent resistance. This method is applicable only when we are able to identify resistances in series or in parallel. The method is based on the simplification of the circuit by successive reduction of the series and parallel combinations.

To calculate the equivalent resistance between the points M and N, the network shown in Fig. 5.26, may be successively reduced as described step by step.

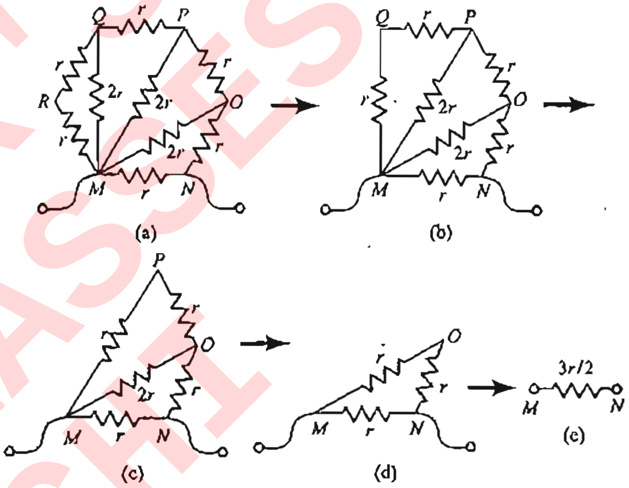


Fig. 5.26

Method of Equipotential Reduction

This method is based on identifying the points of equal potential and connecting them. By doing so the electric resistance network reduces to an arrangement of series and parallel combinations that can be easily solved by the successive reduction method.

Now the question arises how to identify the points of same potential?

In this section, we will discuss the method to calculate the equivalent resistance and capacitance using symmetry techniques.

There are various kind of symmetry considerations. The most common are:

- parallel axis of symmetry,
- perpendicular axis of symmetry,
- shifted symmetry or shifted asymmetry, and
- path symmetry.

We will discuss each one of these in the following sections:

Parallel Axis of Symmetry: It is along the direction of current flow.

Let us discuss this concept by an example.

In the circuit shown in Fig. 5.27 (a), even though the resistors 1 and 2 do not appear to be connected in parallel, but they can be treated as parallel, why? For explaining this, we have to use the concept of symmetry. Note that the circuit is symmetric about the line MN. Therefore, all characteristics such as potential and current should also be symmetrical.

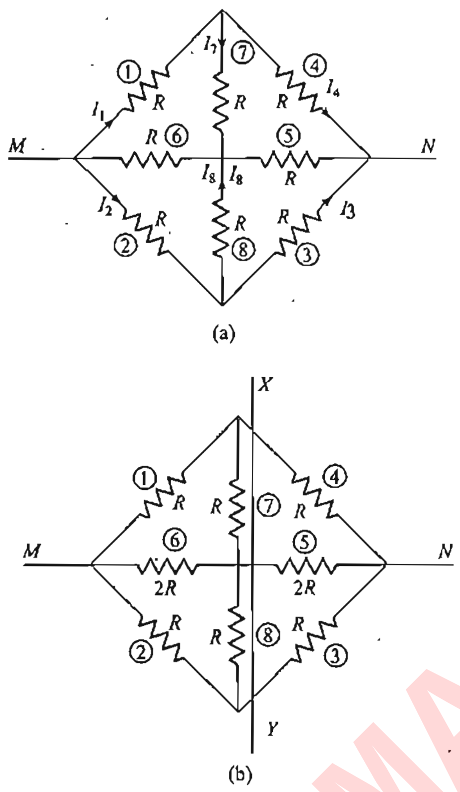


Fig. 5.27

From this, it means that current in 1 (I_1) = current in 2 (I_2), Current in 3(I_3) = current in 4(I_4), Current in 7 (I_7) = current in 8 (I_8), hence potential difference across 1 is equal to the potential difference across 2.

Note that the conditions for 1 and 2 to be in parallel are satisfied. Hence we can consider them to be in parallel, however if one of the resistors had a different value, we cannot use this method.

Perpendicular Axis of Symmetry: It is perpendicular to the direction of flow of current.

Consider the circuit diagram given in Fig. 5.27 (b). The circuit has perpendicular axis of symmetry about *XY*.

The perpendicular axis of symmetry means, that the circuit diagram is symmetric except for the fact that the input and output are reversed. That is only the flow of current will not be a mirror image about this particular axis. For example, in the above case, element 1 and 4 are symmetric about *XY*, but the current flow condition is not a mirror image. The current flow condition is in the same direction.

This implies that current into 6 = current out of 5, current into 1 = current out of 4, etc.

Perpendicular axis of symmetry is a very powerful principle. In fact just by looking at the circuit, we can easily say that since the circuit has perpendicular axis of symmetry about *XY*, no current will flow in elements 7 and 8.

Therefore, we can ignore these two elements completely, in some cases, we may not be able to use symmetry to simplify the circuit, but we can find out some of the characteristics of current /potential based on symmetry. We should always look out for these characteristics and use them as much as possible.

Illustration 5.15 **In the network shown in Fig. 5.28 find the equivalent resistance across the points *M* and *M'*.**

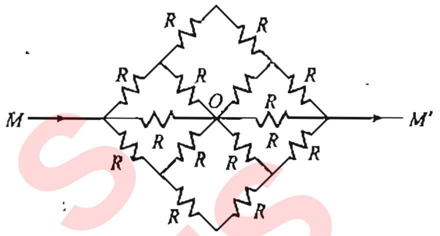


Fig. 5.28

Sol.

i. The axis *MM'* is the parallel axis of symmetry, and the axis *NN'* is the perpendicular axis of symmetry.

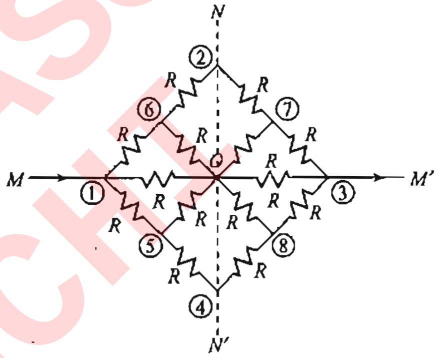


Fig. 5.29

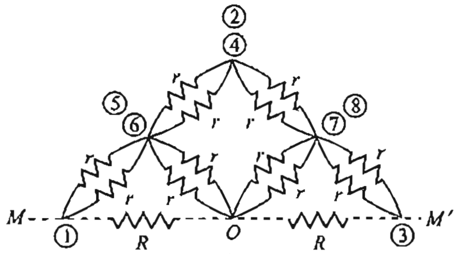
ii. Points lying on the perpendicular axis of symmetry may have same potential. In the given network, points 2, 0, and 4 are at the same potential.

iii. Points lying on the parallel axis of symmetry can never have the same potential.

iv. The network can be folded about the parallel axis of symmetry, and the overlapping nodes have same potential.

Thus, as shown in Fig. 5.29, the following points have same potential, (5 and 6), (2, 0, 4), (7 and 8).

After folding the network about the axis *AA'* the circuit may be simplified by using the method of successive reduction (Fig. 5.30).



(a)

(Contd.)

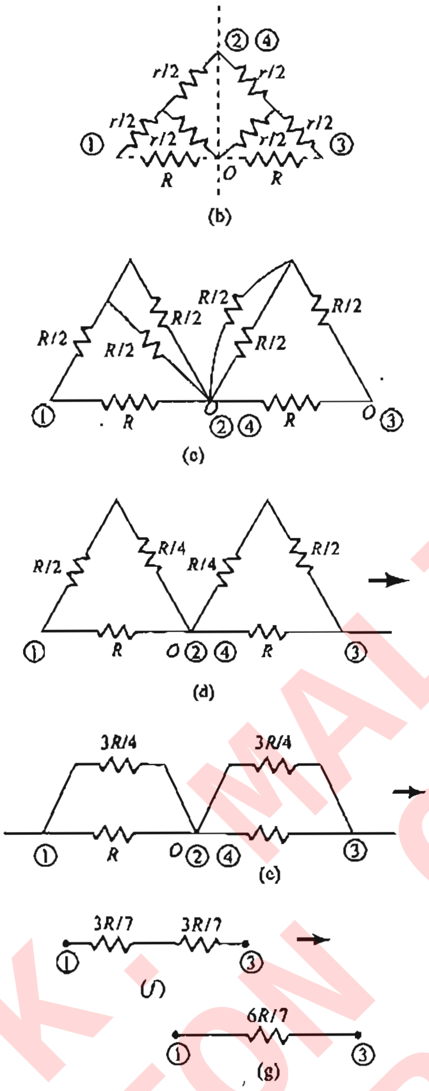
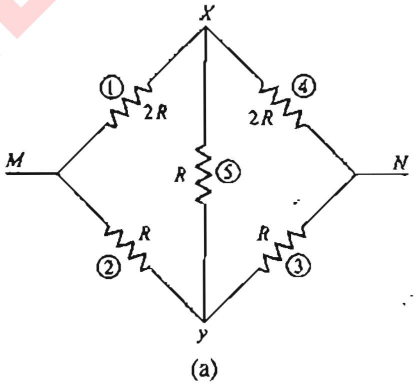


Fig. 5.30

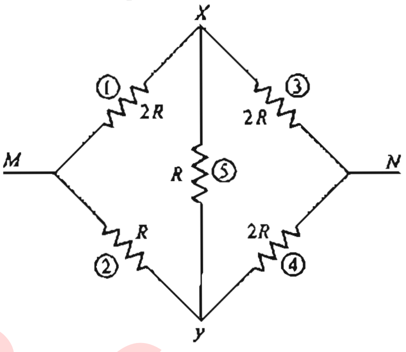
Shifted Symmetry

Shifted symmetry is the same as the parallel axis of symmetry and the perpendicular axis of symmetry principles, except that the symmetry is shifted. [Figs. 5.31(a) and (b)]



(a)

Fig. 5.31



(b)

Fig. 5.31

Consider the following situation:

In this case, the system has a perpendicular axis of symmetry about *MN*, therefore we can say that current in 1 must be equal to the current out of 4.

In Fig. 5.31 (b) if we interchange the positions of 3 and 4 the diagram still has perpendicular axis of symmetry of *xy*, but the positions have been interchanged. Now we can say that the current in 1 = current in 4.

Path Symmetry

Path symmetry is also very powerful method that one can use. In case of path symmetry: "*If all paths from one point to another point have the same configuration of resistance or capacitance, then the charge or current into the beginning of the path must be the same*".

Illustration 5.16 **Consider a more complex example where you have resistors on all edges of a cube. The resistors are all the same. Then find the equivalent resistance between the edges *A* and *B* as shown in Fig. 5.32.**

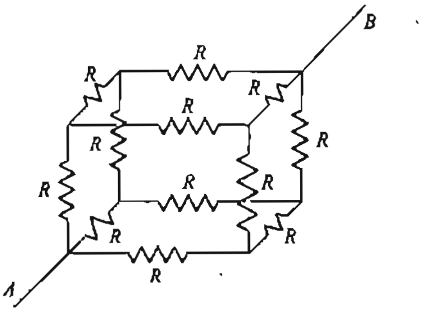


Fig. 5.32

Sol. Let us number the corners as 1, 2, 3, ··· and so on as shown in Fig. 5.33.

a (1 – 8 – 7 – 6), *b*(1 – 2 – 7 – 6), *c*(1 – 8 – 5 – 6), *d*(1 – 2 – 3 – 6), *e*(1 – 4 – 5 – 6), *f* (1 – 4 – 3 – 6)

Now for each of these paths we have identical resistances. Let the current entering at point *A* be *I*. The current will be distributed equally among all the three parts. Therefore, we can say that current is

$$I_{12} = I_{14} = I_{18} = I/3$$

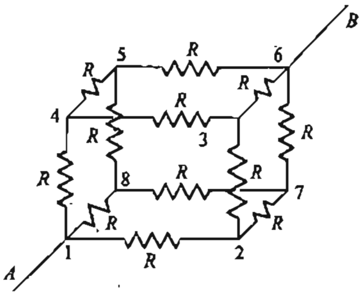


Fig. 5.33

Now look at all paths from 2 to 6. We have the following paths $g(2-7-6)$ and $h(2-3-6)$.

Since they also have the same resistances, we can assume that current in 2—3 and current in 2—7 must be same. Therefore

$$I_{27} = I_{23} = \frac{1}{2}\left(\frac{I}{3}\right) = \frac{I}{6}$$

Similarly we can say that $I_{43} = I_{45} = I_{85} = I_{87} = I/6$

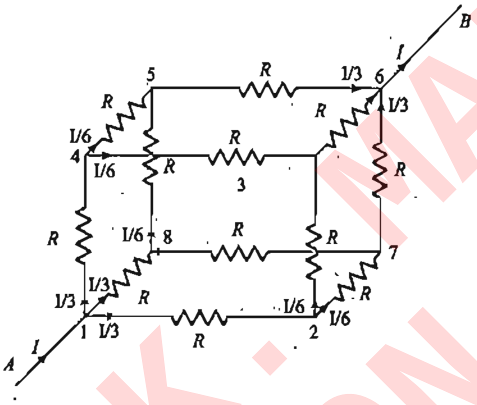


Fig. 5.34

Therefore to calculate the potential difference across 1—6 (see Fig. 5.34) the path A_{1276} we have

$$V_1 - \frac{I}{3}R - \frac{I}{6}R - \frac{I}{3}R = V_6 \Rightarrow V_1 - V_6 = I\left[\frac{R}{3} + \frac{R}{6} + \frac{R}{3}\right] = \frac{5}{6}IR$$

Hence $\dfrac{(V_1 - V_6)}{I} = \dfrac{5}{6}R$

Therefore equivalent resistance is $\frac{5}{6}R$.

Illustration 5.17 **In the given circuits [Figs. 5.35 (a) and (b)] 1 and 2 calculate the resistance between the points MM'.**

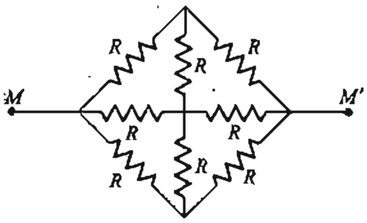


Fig. 5.35 (a)

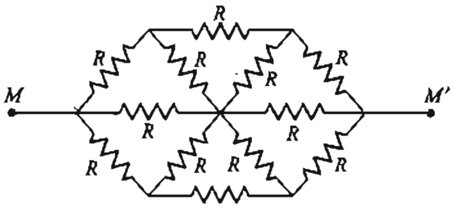
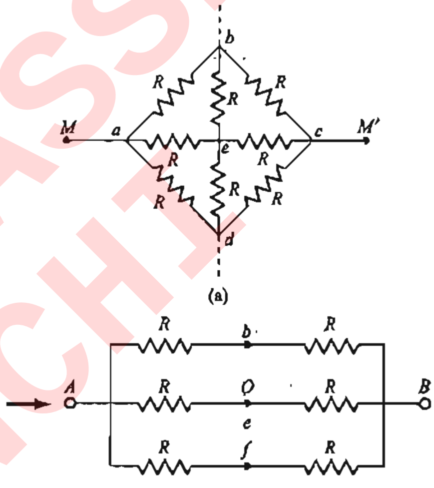


Fig. 5.35 (b)

Sol.

1. When a circuit is symmetrical about a line ac (by symmetry we mean that the two parts are mirror images of each others), then the potential and current must also be symmetrical [see Fig. 5.36 (a)]. Therefore, current in ab and ad is same. Current in dc and bc is also same. Potentials of the points b, e, and d are same. The equivalent circuit is redrawn, the equivalent resistance is 2/3 R. Note that there is no current in branches be and ed.



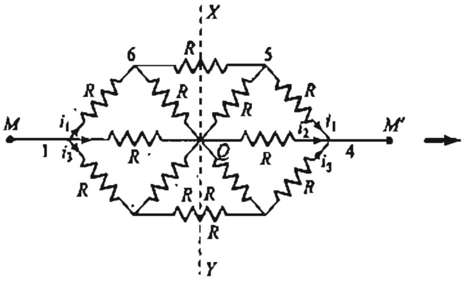
(a)

(b)

Fig. 5.36

Another symmetry is visible along the line bd. The current flow is not a mirror image in branches ab and bc because the flow is in the same direction. This is called asymmetric condition. The special thing about this asymmetry is that current incoming at b is equal to outgoing current, similar situation exists at b and d also. Thus resistors in branches be and de are ineffective.

2. In Fig. 5.37 there is asymmetry along line xy. The current reaching O (i_2) is equal to the outgoing current that means there is no mingling of current from upper branch and lower branch into the middle branch.



(a)

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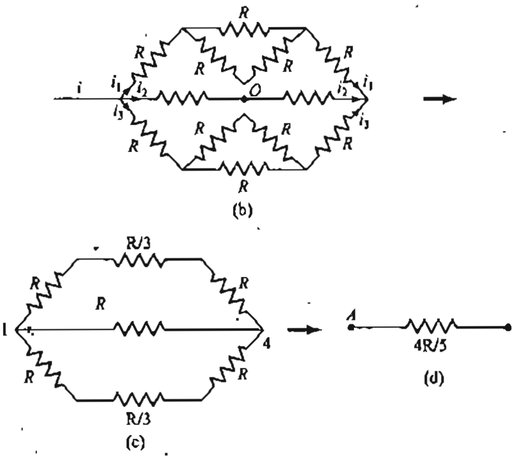


Fig. 5.37

The resulting circuit is simple enough, the equivalent resistance is $4R/5$.

Illustration 5.18 **In the Fig. 5.38 (a), the resistances are connected as shown. Determine the equivalent resistance between points *A* and *D*.**

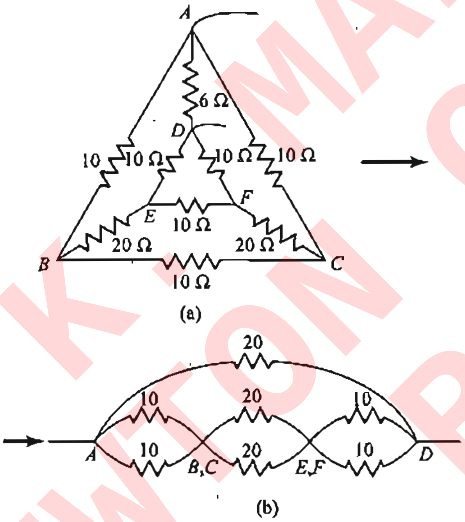


Fig. 5.38

Sol. Points *B* and *C*, and *E* and *F* are at the same potential, so the circuit can be redrawn as shown in figure. Thus, the equivalent resistance is 1Ω. There exists parallel axis of symmetry. The points across the parallel axis of symmetry can be treated as equipotenital points.

KIRCHHOFF'S LAW: KIRCHHOFF'S LAWS FOR ELECTRICAL NETWORKS

As indicated in the preceding section, some simple circuits can be analyzed using $\Delta V = IR$ and the rules for series and parallel combinations of resistors. Resistors, however, can be connected so that the circuits formed cannot be reduced to a single equivalent resistor. Consider the circuit in Fig. 5.39, for example. If either battery were removed from this circuit, the resistors could be combined with the simple, either series or parallel, combination. With both the batteries present, however, that cannot be done.

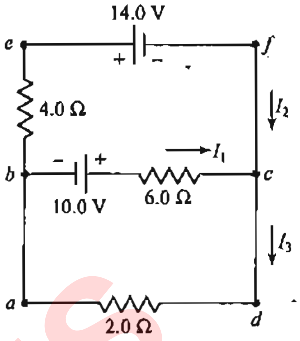


Fig. 5.39

The procedure for analyzing such circuits is greatly simplified by the use of two simple rules called Kirchhoff's rules:

- At any junction, the sum of the currents must be equal to zero:

$$\sum_{\text{junction}} = 0$$

  This rule is often referred to as the junction rule. In Fig. 5.39 there are junctions at *b* and *c*.

- The sum of the potential differences across each element around any closed circuit loop must be zero:

$$\sum_{\text{loop}} = 0$$

This rule is usually called the loop rule. In Fig. 5.39 we can identify three loops: abcda, aefda, and befcb.

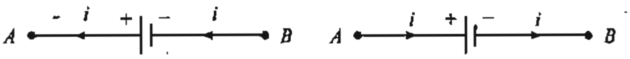
Kirchhoff's rules are generally used to determine the current in each element in the circuit. By using these rules, we first draw the circuit diagram and assume a direction for the current in each device of the circuit. We draw an arrow representing that direction next to the device and assign a symbol to each independent current, such as I_1, I_2, and so on. Figure 5.39 shows the three different currents that exist in this circuit. Keep in mind that currents in devices connected in series are the same, so the currents in these devices will have the same assigned symbol.

The junction rule is a statement of conservation of charge. The amount of charge that enters a given point in a circuit in a time interval must also leave that point in the same time interval because the charge cannot build up or disappear at a point. Current with a direction into the junction is entered into the junction rule as $+ I$, whereas current with a direction out of a junction is entered as $- I$. If we apply the rule to the junction. We have $I_1 - I_2 - I_3 = 0$.

The loop rule is equivalent to the law of conservation of energy, suppose a charge moves around any closed loop in a circuit (the charge starts and ends at the same point). In this case, the circuit must gain as much energy as it loses. In this isolated system model for the system of the circuit, no energy is transferred across the boundary of the system (ignoring energy transfer by radiation and heat into the air from warm circuit elements), but energy transformations do occur within the system. The energy of the circuit may decrease due to a potential drop – *IR* as a charge moves through a resistor or as a result of having the charge move in the reverse direction through an emf. In the latter case, electric potential energy is converted to chemical energy as the battery is charged. the potential energy increases when the charge moves through a battery in the same direction as the emf.

Some useful conventions to be followed in circuit analysis

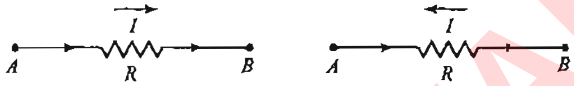
While discharging, current is drawn from the battery, the current comes out from positive terminal and enters negative terminal.



Discharging Battery Charging a Battery

Fig. 5.40

While charging of battery current is forced from positive terminal of the battery to negative terminal.



If we traverse a resistor in the direction of current, the change in potential is-IR.
$V_B - V_A = - I_R$

If we traverse a resistor in the direction opposite to the direction of current, the change in potential is + IR.
$V_A - V_B = + IR$

Fig. 5.41

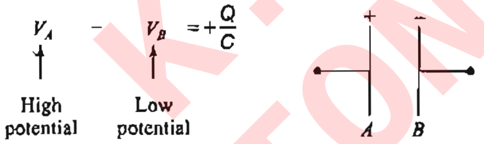


Fig. 5.42

If a capacitor is traversed from negative plate to positive plate, the charge in potential is + *Q*/*C*.

Illustration 5.19 **Calculate the current through each resistance in the given circuit (see Fig. 5.43). Also calculate the potential difference between the points *a* and *b*.**

$$E_1 = 6\text{ V}, \quad E_2 = 8\text{ V}, \quad E_3 = 10\text{ V},$$

$$R_1 = 5\,\Omega, \quad R_2 = 10\,\Omega, \quad R_3 = 4\,\Omega$$

Assume that all the cells have no internal resistance.

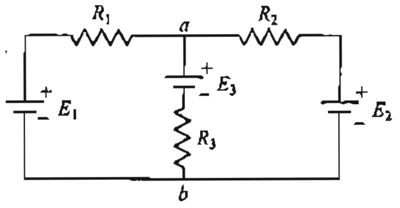


Fig. 5.43

Sol. The process of solving a circuit involves three steps:

i. Assume unknowns ($x, y, \cdots$) for currents in different branches of the circuit. Use the Kirchhoff's current law at the junctions so that the number of unknowns introduced is minimum.

Let x be the current through R_1 and y be the current through R_3 as shown in Fig. 5.44. Kirchhoff's current law at the junction a gives a current $(x - y)$ through R_2.

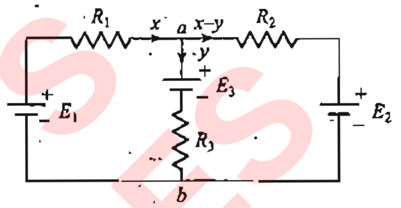


Fig. 5.44

ii. Select as many loops as the number of unknowns introduced for currents. Apply Kirchhoff's voltage law through every loop.

Going anticlockwise through the loop containing R_1 and R_3 (starting from junction *a*)

$$+ x R_1 - E_1 + y R_3 + E_3 = 0$$

$$\Rightarrow 5x + 4y = -4 \qquad (1)$$

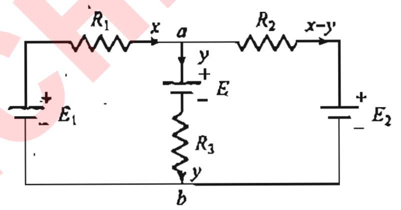


Fig. 5.45

Going clockwise through the loop containing R_2 and R_3 (starting from junction *a*)

$$-E_3 - 5x - 7y = 1\, yR_3 + E_2 + (x + y)R_2 = 0 \qquad (2)$$

(i) Solve equations (1) and (2) some currents may come out to be negative. This simply means that their directions was incorrectly assumed. So the signs of the currents will give us the correct direction of each current.

Solving (1) and (2), we get

$$x = \frac{-24}{55}\text{ A} \quad \text{and} \quad y = \frac{-5}{11}\text{ A}$$

$$x - y = \frac{+1}{55}\text{ A}$$

The signs indicate that the direction of x and y was assumed incorrectly while the direction of $(x - y)$ was correct.

The current i (through R_1) $= \frac{25}{55}$ A towards left

The current i (through R_2) $= \frac{1}{55}$ A towards right

The current i (through R_3) $= \frac{5}{11}$ A towards right

The current directions are shown in the circuit diagram

Fig. 5.46.

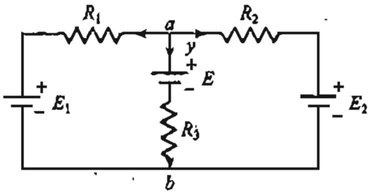


Fig. 5.46

iii. Potential difference between *a* and *b*

The p.d. between any two points in a circuit so calculated by adding changes in potential while going through any path from one point to the other point.

Hence let us go from *b* to *a* through R_3.

$$V_a - V_b = +yR_3 + E_3 = \left(\frac{-5}{11}\right)\times 4 + 10 = \frac{90}{11}\text{ V}$$

Illustration 5.20 **Find the current in the branch CD in the circuit given in Fig. 5.47.**

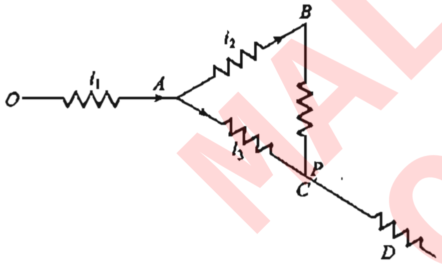


Fig. 5.47

Sol. Current in AB = current in $BP = i_2$, current in $AP = i_3$

∴ the current in $CD = i_2 + i_3$ (using Kirchoff's junction law).

Illustration 5.21 **In the circuit shown in Fig. 5.48, find the current through the branch *BD*.**

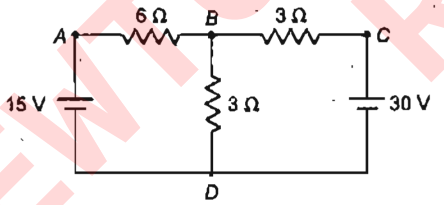


Fig. 5.48

Sol. To find out the current in a particular branch in a circuit where resistances and voltages are given, lower loop rule is applicable. In order to find out the solution, the given circuit is directed into parts or close loops which are analysed using Kirchhoff's second law and corresponding sign conventions.

Assume the currents in the circuit as shown in the Fig. 5.48.

Applying KVL along the loop *ABDA* and moving in clockwise direction

$$-6I_1 - 3I_2 + 15 = 0 \quad \Rightarrow \quad 2I_1 + I_2 = 5 \qquad \text{(i)}$$

While moving in the direction of current corresponding *IR*, products are taken as negative and if the negative terminal of the battery comes first then emf is taken as +ve (Fig. 5.49).

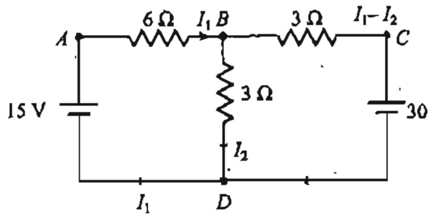


Fig. 5.49

Applying KVL along the loop *BCDB*, we get

$$-3(I_1 + I_2) - 30 + 3I_2 = 0$$

$$\Rightarrow \qquad -I_1 + 2I_2 = 10 \qquad \text{(ii)}$$

Solving equations (i) and (ii) for I_2, we get $I_2 = 5\text{ A}$

Illustration 5.22 **In the circuit shown in Fig. 5.50, determine the voltage drop between *A* and *D*.**

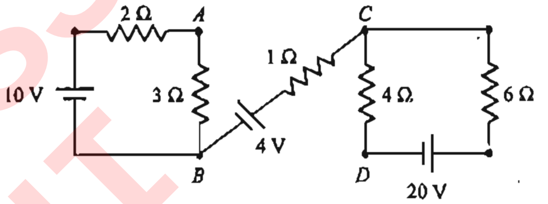


Fig. 5.50

Sol. We need to divide this circuit into three parts. We have left loop, right loop, and the central part. To find out voltage drop between points *A* and *D* we have to apply Kirchhoff's second law to these circuits one by one.

Also let us assume that current I_1 flows in the left circuit in the clockwise direction and current I_2 flows in the right circuit in the anticlockwise direction.

Direction of both the currents is decided by the battery present in the circuit as current flows from positive to negative.

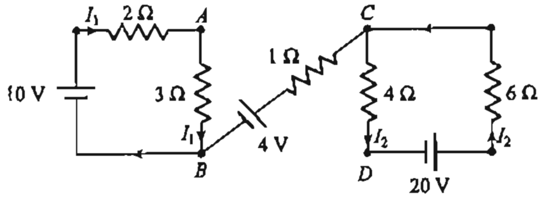


Fig. 5.51

Applying Kirchoff's second law to left loop,

$$-2I_1 - 3I_1 + 10 = 0 \quad \Rightarrow \quad -5I_1 = -10 \quad \Rightarrow \quad I_1 = 2\text{ A}$$

Now applying Kirchoff's second law to right loop,

$$4I_2 + 6I_2 - 20 = 0 \quad \Rightarrow \quad 10I_2 = 20 \quad \Rightarrow \quad I_2 = 2\text{ A}$$

Applying between the points *A* and *D*,

$$V_A - 3\,I_1 + 4 - 4\,I_2. \quad \Rightarrow \quad V_A = V_D = 3I_1 + 4I_2 - 4$$

Putting values of I_1 and I_2 in the above expression

$$V_A - V_D = 3.2 + 4.2 - 4 = 6 + 8 - 4 = 10\text{ V}$$

Illustration 5.23 **Find the current through 12 Ω resistor in Fig. 5.52(a).**

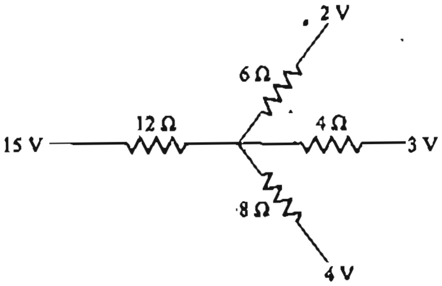


Fig. 5.52 (a)

Sol. Let V be the potential at P then applying KCL at junction P.

$$I = I_1 + I_2 + I_3 \Rightarrow \frac{15-V}{12} = \frac{V-2}{6} + \frac{V-3}{4} + \frac{V-4}{8}$$

$$15 - V = 2(V-2) + 3(V-3) + 1.5(V-4) 7.5\,V = 39$$

or $$V = \frac{39}{7.5} = 5.2 \text{ V} \quad \text{and} \quad I = \frac{15-5.2}{12} = \frac{4.9}{6} \text{ A}$$

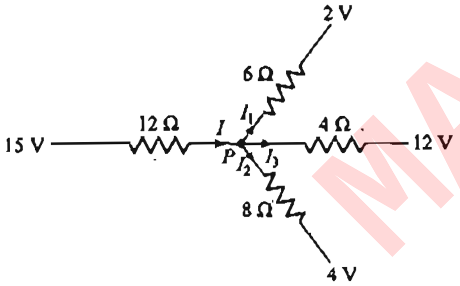
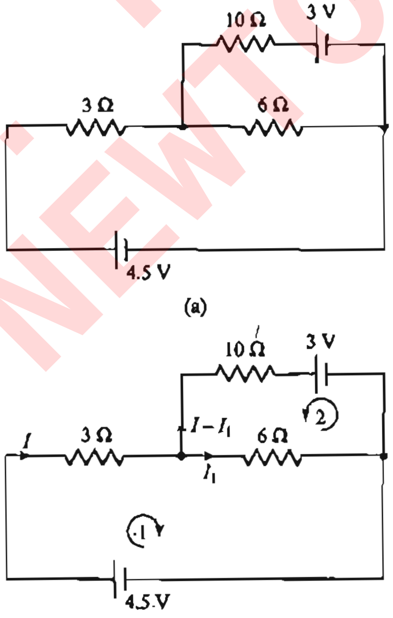


Fig. 5.52 (b)

Illustration 5.24 **Find the current in each part of the circuit.**

Sol. Apply loop law in Fig. 5.53 (b).



(a)

(b)

Fig. 5.53

$$(1) -3I - 6I_1 + 4.5 = 0 \text{ or } I + 2I_1 = 1.5 \qquad \text{(i)}$$

$$(2)\ 10(I - I_1) + 3 - 6I_1 = 0$$

or $$10I - 10I_1 = -3 \qquad \text{(ii)}$$

Solving equations (i) and (ii) we get

$$I = \frac{1}{2} \text{ A}$$

and $$I - I_1 = \frac{1}{2} - \frac{1}{2} = 0$$

WHEATSTONE BRIDGE: BALANCED WHEATSTONE BRIDGE

Wheatstone bridge is also known as a metre bridge or slide wire bridge.

If 4 resistances P, Q, R, and S are joined as shown in Fig. 5.54, both the keys (K_1 and K_2) are on and no current flows through the galvanometer (i.e., $I_g = 0$), then the combination of resistances is called a balanced wheatstone bridge. Then

$$\frac{P}{Q} = \frac{R}{S}$$

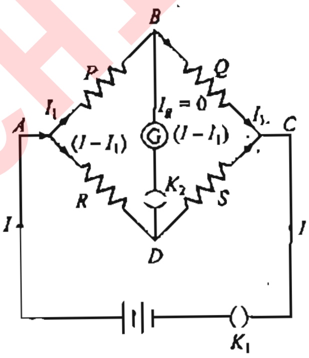


Fig. 5.54

If, say, S is an unknown resistance, then $S = R \times \frac{Q}{P}$

Now if we are given R, P, and Q

or R and ratio $\frac{Q}{P}$ then we can calculate the value of S easily.

Proof: Applying Loop Rule to loop $ABDA$ (moving in clockwise direction).

$$-I_1P + (I - I_1)R = 0 \qquad \{\because I_g = 0\}$$

or $$I_1P = (I - I_1)R \qquad \text{(i)}$$

Applying loop rule to loop $BCDB$

$$-I_1Q + (I - I_1)S = 0$$

or $$I_1Q = (I - I_1)S \qquad \text{(ii)}$$

Dividing equation (i) by (ii), we get $\frac{P}{Q} = \frac{R}{S}$

The balanced wheatstone bridge method is an accurate method because:

i. We do not have to read out deflection but we only have to see that needle remains at dead zero.

ii. It is not affected by internal resistance of cells, resistances of galvanometers, etc.

This is the principle used in the metre bridge or in the slide-wire bridge.

Other circuits which can form Wheatstone bridge are:

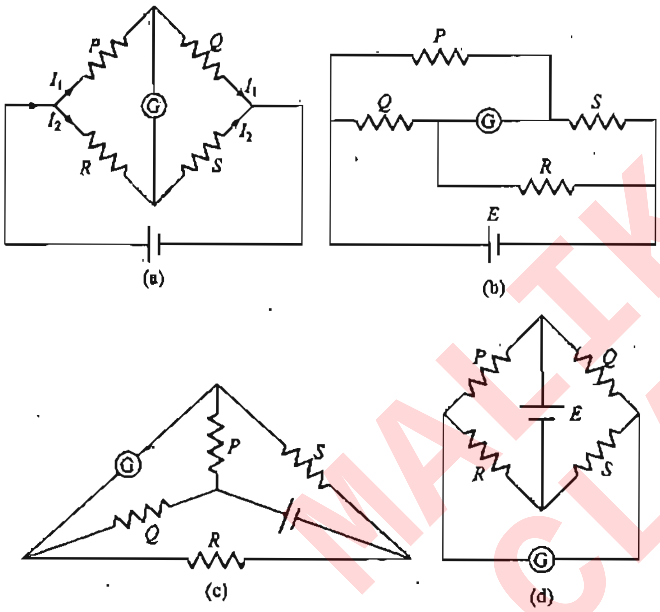


Fig. 5.55

All the four circuits, (a), (b), (c), and (d) represent a Wheatstone bridge network.

COMBINATION OF CELLS

Series Grouping

Suppose n cells each of emf E and internal resistance r are connected in series as shown in Fig. 5.56. Then

Net emf = nE; Total resistance = $nr + R$

$$\therefore \text{ current in the circuit } I = \frac{\text{net e.m.f}}{\text{total resistance}} \text{ or } i = \frac{nE}{nr+R}$$

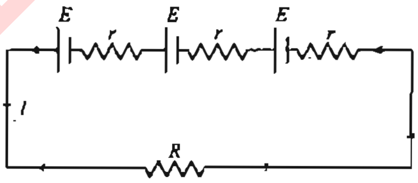


Fig. 5.56

Note:

If polarity of m cells is reversed, then equivalent e.m.f = (n – 2m)E. While total resistance is still nr + R

$$i = \frac{(n-2m)E}{nr+R}$$

If the same current passes through every resistor in a given branch, irrespective of the presence of sources in that branch, the resistors are in series even though they are not directly connected to each other. Same is true about capacitors.

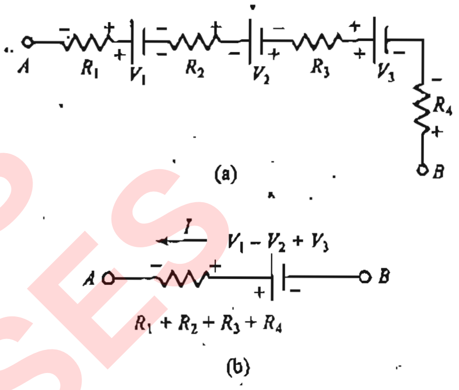


Fig. 5.57

Parallel Grouping

Case 1: If E and r of each cell are different but still the positive terminals of all cells are connected at one junction while negative terminals at the other.

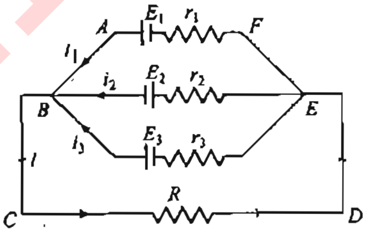


Fig. 5.58

Applying Kirchhoff's second law in loop $ABCDEFA$,

$$E_1 - iR - i_1r_1 = 0 \text{ or } i_1 = -\frac{iR}{r_1} + \frac{E_1}{r_1} \quad \text{(i}$$

Similarly, we can write,

$$i_2 = -i\frac{R}{r_2} + \frac{E_2}{r_2} \quad \text{(ii)}$$

Adding all above equations, we have,

$$(i_1 + i_2 + \cdots + i_n) = -iR\sum\left(\frac{1}{r}\right) + \sum\left(\frac{E}{r}\right)$$

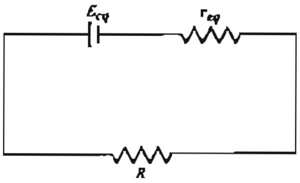


Fig. 5.59

But $\quad i_2 + i_2 + \cdots + i_n = i$

$$\therefore \quad i = -iR\sum\left(\frac{1}{r}\right) + \sum\left(\frac{E}{r}\right)$$

$$\therefore \quad i=\frac{\sum\left(\frac{E}{r}\right)}{1+R\sum\left(\frac{1}{r}\right)}=\frac{E_{eq.}}{R_{eq.}}$$

where, $$E_{eq}=\frac{\sum\left(\frac{E}{r}\right)}{\sum\left(\frac{1}{r}\right)} \quad \text{and} \quad R_{eq}=R+\frac{1}{\sum\left(\frac{1}{r}\right)}$$

From the above the expression, we can see that, $i=\frac{E}{R+r/n}$ if n cell of same emf E and internal resistance r are connected in parallel. This is because,

$$\sum\left(\frac{E}{r}\right)=\frac{nE}{r} \text{ and } \sum\left(\frac{1}{r}\right)=\frac{n}{r}$$

$$\therefore \quad i=\frac{nE/r}{1+nR/r}=\frac{E}{R+r/n}$$

We can also write

$$i=\frac{(E_1/r_1)-(E_2/r_2)+(E_3/r_3)}{1+R\left(\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}\right)}$$

Mixed Grouping

The situation is shown in Fig 5.60.

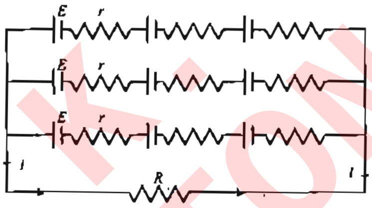


Fig. 5.60

There are n identical cells in a row and number of rows are m. E.m.f of each cell is E and internal resistance is r. Treating each row as a single cell of e.m.f nE and internal resistance nr, we have

Net e.m.f = nE, Total internal resistance = $\frac{nr}{m}$

Total external resistance = R

∴ current through the external resistance R is,

$$i=\frac{nE}{R+\frac{nr}{m}}$$

This expression after some rearrangement can also be written as,

$$i=\frac{mnE}{(\sqrt{mR}-\sqrt{nr})^2+2\sqrt{mnrR}}$$

From this expression we see that i is maximum when,

or $$\sqrt{mR}=\sqrt{nR} \text{ or } R=\frac{nr}{m}$$

or total external resistance = total internal resistance.

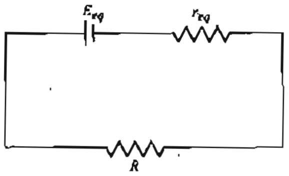


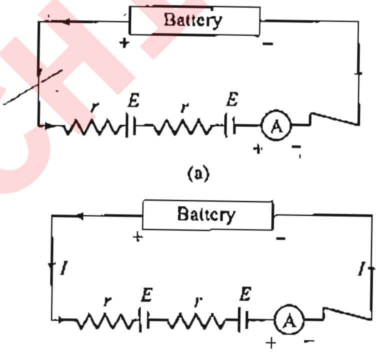
Fig. 5.61

Thus, we can say that the current and hence the power transferred to the load is maximum when the load resistance is equal to internal resistance. This is known as *maximum power transfer theorem.*

Illustration 5.25 **12 cells each having the same emf are connected in series and are kept in a closed box. Some of the cells are wrongly connected. This battery is connected in series with an ammeter and two cells identical with the others. The current is 3 A when the cells and the battery aid each other and 2 A when the cells and battery oppose each other. How many cells in the battery are wrongly connected?**

Sol. Let n be the cells in the battery that are wrongly connected, then

$$E_B=(12-n)E-nE=(12-2n)\,E \quad \text{and}$$
$$r_B=12\,r$$



(a)

(b)

Fig. 5.62

So according to the given problem as shown in Figs. 5.62 (a) and (b),

$$\frac{(12-2n)\,E+2E}{12r+2r}=3 \qquad \text{(i)}$$

and $$\frac{(12-2n)\,E-2E}{12r+2r}=2 \qquad \text{(ii)}$$

Dividing equation (i) by equation (ii), $\frac{14-2n}{10-2n}=\frac{3}{2}$, i.e., $n=1$

This means that in the battery only one cell is wrongly connected.

SUPERPOSITION PRINCIPLE

Concepts

Whenever a circuit has more than one cell or battery. The superposition principle may be used to find current and voltages. This principle is based on the fact that every cell or battery acts independently of the presence of others.

According to this principle, the total current I in the circuit equals the algebraic sum of currents (I_1, I_2, ⋯, I_n) produced by each source (cell or battery), taken one at a time.

Mathematically,

$$I = I_1 + I_2 + \cdots + I_n$$

The superposition splits the original two source problem into two one source problems. Instead of solving a difficult two source problem, we can solve two simple problems.

Determine the current I in the 2 Ω resistor.

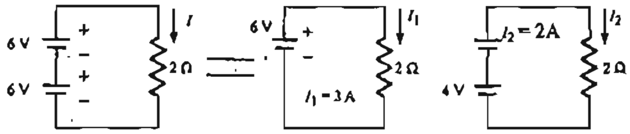


Fig. 5.63

$$I = I_1 + I_2 = \frac{6}{2} + \frac{4}{2} = 3 + 2 = 5 \text{ A}$$

Find the value of I in the circuit shown in Fig. 5.64.

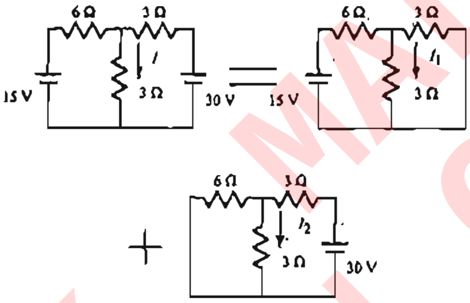


Fig. 5.64

$$I_1 = 1\,A, \quad I_2 = 4A \quad \Rightarrow \quad I = I_1 + I_2 = 1 + 4 = 5\,A$$

The situation where superposition principle is not valid:

In the circuit shown in Fig. 5.65, find the current I flowing through the 2 Ω resistor.

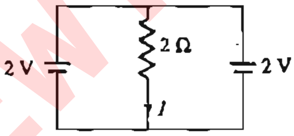


Fig. 5.65

Many students come out with an answer $I = 2\,A$ with a wrong reason.

Each battery contributes a current of 1 A, as there are two batteries so the total current in the 2 Ω resistor is 2 A.

The circuit may be split up into two parts as shown in Fig. 5.66.

With each battery the current in the 2 Ω resistor is zero. It happens so because when we remove one battery from a branch, then that particular branch gets short circuited, as there is no other resistance present in that branch.

Note that in such a situation the principle of superposition is not applicable.

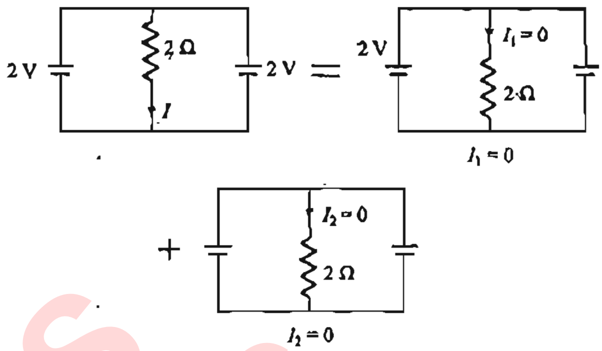


Fig. 5.66

Conditions for the Applicability of Principal of Superposition:

Whenever a cell or a battery is present in a branch there must be some resistance (internal or external or both) present in that branch in order to apply the superposition principle.

In practical situations it always happen because we can never have an ideal cell or a battery with zero resistance.

In this case the current flowing through the 2 Ω resistor is 1 A, since voltage drop across it is 2V (correct).

Concept Application Exercise 5.2

1. Calculate the value of the electric currents I_1, I_2, and I_3 in the given electrical network.

   a. I_1 = —

   b. I_2 = —

   c. I_3 = —

   d.

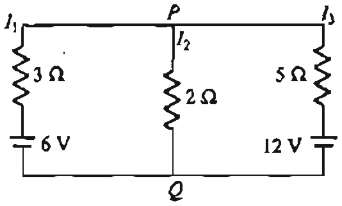


Fig. 5.67

2. Determine the voltage drop across the resistor R_1 in the circuit given below with $E = 65$ V, $R_1 = 50$ Ω, $R_2 = 100$ Ω, $R_3 = 100$ Ω, and $R_4 = 300$ Ω.

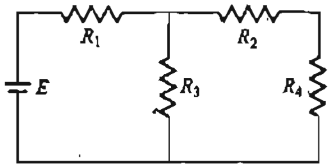


Fig. 5.68

3. A 20 V battery of internal resistance 1 Ω is connected to three coils of 12 Ω, 6 Ω, and 4 Ω in parallel, a resistor of 5 Ω and a reversed battery (e.m.f. 8 V and internal resistance 2 Ω) as shown in Fig. 5.69.

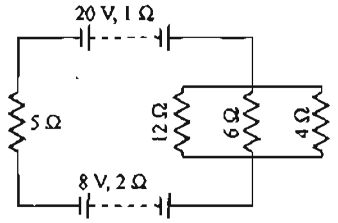


Fig. 5.69

Calculate

a. the current in the circuit,

b. current in resistor of 12 Ω coil and

c. p.d. across each battery.

4. Potential difference across terminals of a cell were measured (in volts) against different currents (in ampere) flowing through the cell. A graph was drawn which was a straight line *ABC*. Using the data given in the graph (see Fig. 5.70), determine

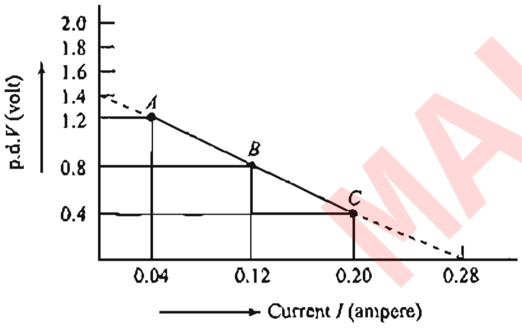


Fig. 5.70

a. the e.m.f, and

b. the internal resistance of the cell.

5. Find the current in each resistor in the circuit as shown in Fig. 5.71.

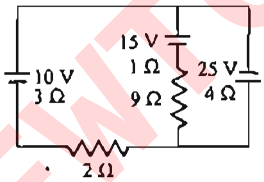


Fig. 5.71

6. The given Wheatstone bridge is showing no deflection in the galvanometer joined between the points *B* and *D* shown in Fig. 5.72. Calculate the value of *R*.

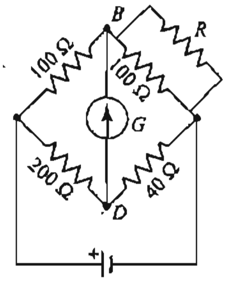


Fig. 5.72

7. When 10 identical cells in series are connected to the ends of a resistance of 59 Ω, the current is found to be 0.25 A. But when the same cells being connected in parallel, are joined to the ends of a resistance of 0.05 Ω, the current is 25 A. Calculate the internal resistance and the e.m.f of each cell.

8. Find the minimum number of cells required to produce a current of 1.5 A through a resistance of 30 Ω. Given that the e.m.f of each cell is 1.5 V and the internal resistance is 1 Ω.

9. The current in a simple series circuit is 5 A. When an additional resistance of 2 Ω is introduced, the current is reduced to 4 A. Calculate the resistance of the original circuit. Assume that the applied potential difference is the same in both the cases.

10. Calculate the resistance between the terminals *A* and *B* of the network shown in Fig. 5.73.

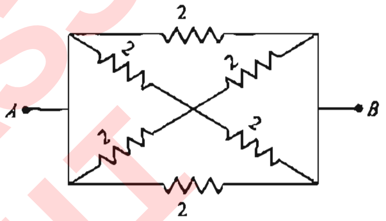


Fig. 5.73

11. In the circuit diagram given below (Fig. 5.74), the cells E_1 and E_2 have e.m.f.'s of 4 V and 8 V and internal resistances 0.5 Ω and 1.0 Ω, respectively. Calculate the current through 6 Ω resistance.

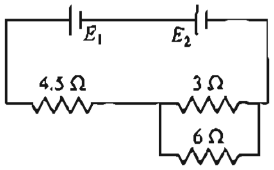


Fig. 5.74

12. Determine the currents I_1, I_2, and I_3 for the network shown below (Fig. 5.75).

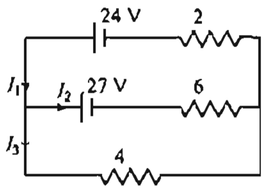


Fig. 5.75

a. $I_1 =$—,

b. $I_2 =$—,

c. $I_3 =$—.

13. Find the current supplied by the source in Fig. 5.76. The resistors are mounted around a cylindrical form.

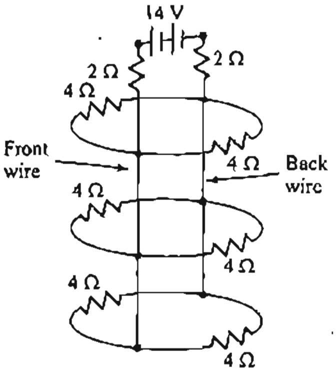


Fig. 5.76

14. A parallel combination of an 8 Ω resistor and an unknown resistor R is connected in series with a 16 Ω resistor and a battery. This circuit is then disassembled and the three resistors are then connected in series with each other and the same battery. In both arrangements, the current through the 8 Ω resistor is the same. What is the unknown resistance R ?

15. For the resistor network shown in Fig. 5.77, the potential drop between a and b is 12 V. Find the current through each resistor.

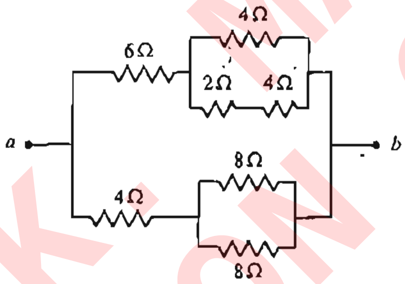


Fig. 5.77

a. Current through resistance of 6 Ω is ______.

b. Current through resistance of 2 Ω is ______.

c. Current through resistance of 8 Ω is ______.

16. For the circuit in Fig. 5.78, find the potential difference between points a and b.

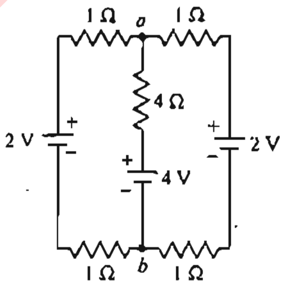


Fig. 5.78

17. Find the current in each resistor of the circuit shown in Fig. 5.79.

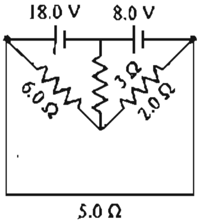


Fig. 5.79

a. Current through resistance of 6 Ω is ______.

b. Current through resistance of 3 Ω is ______.

c. Current through resistance of 2 Ω is ______.

d. Current through resistance of 5 Ω is ______.

18. Given that 5.0 A passes along the branch from C to B in Fig. 5.80. What is the voltage of points A, D, E, F, and G ?

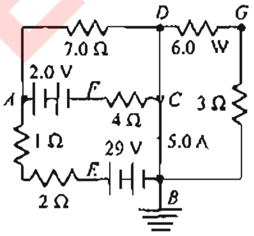


Fig. 5.80

a. V_A = — b. V_D = — c. V_G = —

19. In the circuit shown in Fig. 5.81 switch S is closed at time $t = 0$.

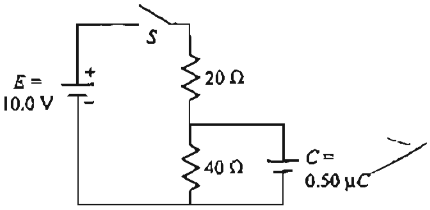


Fig. 5.81

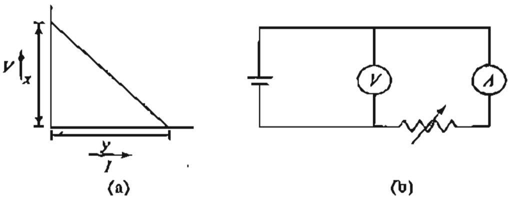
a. What is the current I_0 leaving the battery at t = 0, immediately after the switch is closed ?

b. What is the current I a "long time" later?

c. What charge has accumulated on the capacitor after this long time ?

d. If, finally, switch S is opened again, how long will it take after the switch is opened for the capacitor to lose 80% of its charge ?

20. In an experiment, a graph (Fig. 5.82) was plotted of the potential difference V between the terminals of a cell again circuit current I by varying load rheostat. Find the internal conductance of the cell.



(a) (b)

Fig. 5.82

21. Each resistance in the circuit (Fig. 5.83) is of 2,000 Ω.

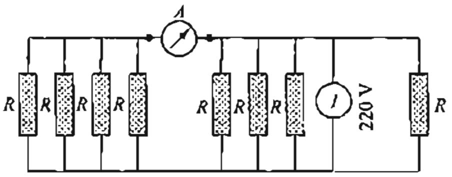
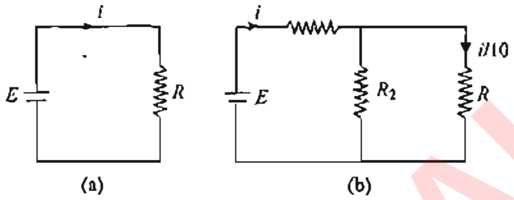


Fig. 5.83

The combination is put across a supply of 200 V. Find the reading of ammeter.

22. Consider the circuits shown in the Fig. 5.84. Both the circuits are taking same current from battery but current through R in the second circuit is $\frac{1}{10}$ th of current through R in the first circuit. If R is 11 Ω, then find the value of R_1 and R_2.



(a) (b)

Fig. 5.84

23. Calculate the current through the resistance connected across MN and the current supplied by each of the battery in the circuit diagram shown in Fig. 5.85.

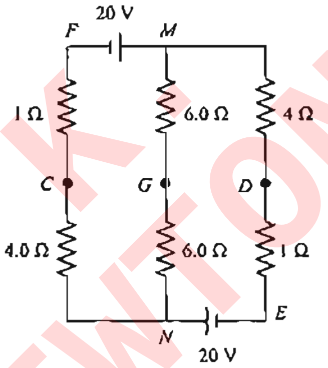


Fig. 5.85

24. A hemispherical network of radius a is made by using a conducting wire of resistance per unit length r. Find the equivalent resistance across OP.

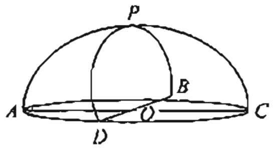


Fig. 5.86

25. A battery of e.m.f 2 V and internal resistance 0.1 Ω is being charged with a current of 5 A. In what direction will the current flow inside the battery? What is the potential difference between the two terminals of the battery?

(IIT-JEE, 1980)

26. How a battery is to be connected so that the shown rheostat will behave like a potential divide?

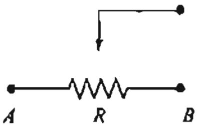


Fig. 5.87

Also indicate the points about which output can be taken.

(IIT-JEE, 2003)

CHARGING

Let us assume that the capacitor in the network shown in the Fig. 5.88 is uncharged for $t < 0$. The switch is connected to position 1 at $t = 0$. Now, C is getting charged.

If the charge on capacitor at time t is q.

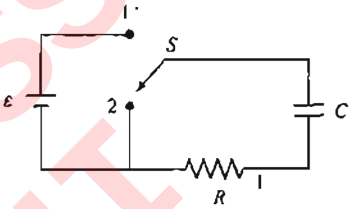


Fig. 5.88

Writing the loop rule,

$$\frac{q}{C} + IR - E = 0 \quad \Rightarrow \quad R\frac{dq}{dt} = E - \frac{q}{C}$$

$$\Rightarrow \quad RC\frac{dq}{dt} = EC - q$$

$$\Rightarrow \quad \frac{dq}{EC - q} = \frac{dt}{RC}$$

Integrating,

$$\int_0^q \frac{dq}{EC - q} = \frac{1}{RC}\int_0^q dt \quad \Rightarrow \quad \ln - |EC - q|_0^q = \frac{1}{RC}\int_0^t dt$$

$$\Rightarrow \quad \ln\left|\frac{EC - q}{EC}\right| = \frac{-t}{RC} \qquad q = EC[1 - e^{-t/RC}]$$

$\Rightarrow$ At $t = 0$, $q = 0$

and at $t = \infty$, $q = E\,C$ (maximum charge)

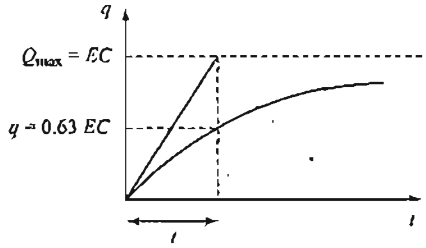


Fig. 5.89

Thus, $$q = q_{max}\left[1-e^{\frac{-t}{RC}}\right]$$

$$i = \frac{dq}{dt} = \frac{q_{max}}{RC}e^{-t/Rc} = \frac{E}{R}e^{-t/RC}$$

$$i = i_{max}e^{-t/RC} \text{ where } i_{max} = \frac{E}{R}$$

Time Constant (τ)

It is the time during which the charge would have been completed, had the growth rate been as it began initially. Numerically it is equal to RC.

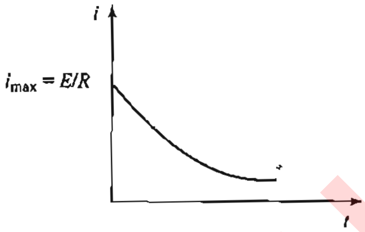


Fig. 5.90

Discharging

Consider the same arrangement as we had in previous case with one difference that the capacitor has charge q_0 for $t < 0$ and the switch is connected to position 2 at $t = 0$. If the charge on capacitor is q after the switch is flipped to 2

$$\frac{q}{C} - IR = 0\left(I = -\frac{dq}{dt}\right)$$

$$\Rightarrow \quad R\frac{dq}{dt} = \frac{-q}{C}$$

$$\Rightarrow \quad \frac{dq}{q} = \frac{-1}{RC}dt$$

Integrating, at $t = 0$, $q = q_0$ $\quad t = t$, $q = q$

$$\int_{q_0}^{q}\frac{dq}{q} = -\frac{1}{RC}\int_{q_0}^{q} dt \Rightarrow \ln\left|\frac{q}{q_0}\right| = -\frac{t}{RC}$$

or $\quad q = q_0 \times e^{-t/RC}$

$$i = \frac{q_0}{RC}e^{-t/RC} \Rightarrow i = \frac{EC}{RC}e^{-t/RC} \Rightarrow i = i_0 e^{-t/RC}$$

Illustration 5.25 **What is the dimensional formula of RC.**

Sol. [T]

The quantity RC has the same unit as time and is called time constant and represented by the symbol τ (Fig. 5.91).

Analysis of RC-Circuits at the initial and the infinite time.

The current in the circuit just after closing the switch, i.e., $t = 0$

$$I = \frac{\varepsilon}{R}e^{-t/RC} = \frac{\varepsilon}{R}e^{-0} \Rightarrow I_0 = \frac{\varepsilon}{R}$$

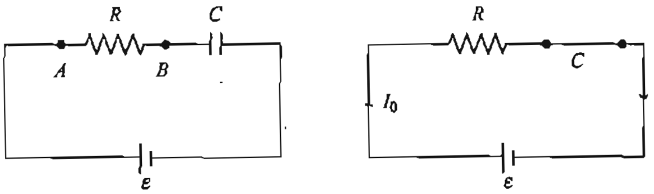


Fig. 5.91

The capacitor in the circuit is acting as if a conducting wire is connected in place of the capacitor.

Hence, a capacitor acts as a conducting wire (or short circuit) just after the closing of the switch.

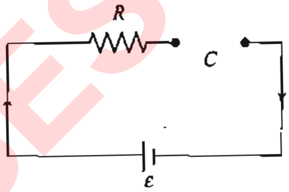


Fig. 5.92

At $\quad t = 8$

$$I = \frac{\varepsilon}{R}e^{-t/RC} = \frac{\varepsilon}{R}e^{-\infty} \Rightarrow I_8 \approx 0$$

After long time ($t = 8$), no current is flowing through the circuit. The capacitor is acting as the circuit is broken from this position.

Hence, after long time the capacitor acts as open circuit or a resistance of infinite value.

Potential difference across resistance is function of time.

$$V_{AB} = IR \Rightarrow V_{AB} = \left(\frac{\varepsilon}{R}e^{-\frac{t}{RC}}\right)R$$

$$V_{AB} = \varepsilon e^{-\frac{t}{RC}}$$

At $\quad t = 0, \quad V_{AB} = \varepsilon e^{-0}, V_{AB} = \varepsilon$

At $\quad t = 8, \quad V_{AB} = \varepsilon e^{-\infty}, V_{AB} = 0$

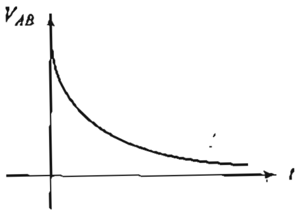


Fig. 5.93

Discharging of a charged capacitor (**Fig. 5.94**)

$$q = q_0 e^{-t/RC}$$

$$I = -\frac{dq}{dt} \Rightarrow I = \frac{q_0}{RC}e^{-t/RC}$$

$$I = \frac{(q_0/C)}{R}e^{-t/RC}$$

At $t = 0$, $I_0 = \frac{(q_0/C)}{R}$

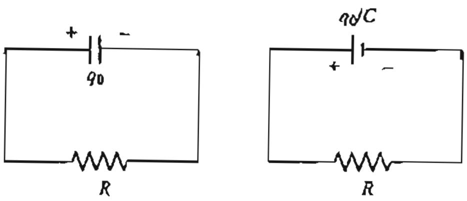


Fig. 5.94

At $t = 0$ the capacitor is acting as a battery of e.m.f $= q_0/C$. As the time passes the charge on the capacitor keeps on decreasing. The potential difference across the capacitor keeps on decreasing, hence the capacitor acts as a battery of decreasing e.m.f.

Hence, when a charged capacitor is connected with resistance, the charge capacitor acts as a battery of decreasing e.m.f.

At $t = 8$

$$\Rightarrow \quad I = \frac{q_0}{RC}e^{-t/RC} = \frac{q_0}{RC}e^{-\infty} \Rightarrow I_8 = 0$$

Charging of the Capacitor—Other Approach

Applying Kirchhoff's loop equation in the given circuit (Fig. 5.95)

$$-\frac{q}{C} - IR + \varepsilon = 0 \qquad \text{(i)}$$

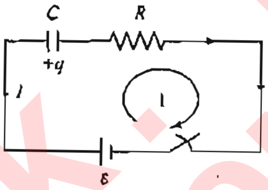


Fig. 5.95

Differentiating equation (i) with respect to time on both the sides

$$-\frac{1}{C}\frac{dq}{dt} - R\frac{dI}{dt} + 0 = 0 \Rightarrow -\frac{1}{C}I - R\frac{dI}{dt} = 0$$

$$\Rightarrow \quad \frac{dI}{I} = -\frac{1}{RC}dt$$

Integrating both sides

$$\int_{I_0}^{I}\frac{dI}{I} = -\frac{1}{RC}\int_0^t dt \Rightarrow [\ln I]_{I_0}^{I} = -\frac{1}{RC}[t]_0^t$$

$$\ln\frac{I}{I_0} = -\frac{t}{RC} \Rightarrow \frac{I}{I_0} = e^{-\frac{t}{RC}} \Rightarrow I = I_0 e^{-\frac{t}{RC}}$$

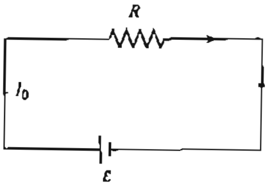


Fig. 5.96

At $t = 0$, the uncharged capacitor in the circuit with a battery acts as a conducting wire

$$\Rightarrow \quad I_0 = \frac{\varepsilon}{R}$$

Hence $\quad I = \frac{\varepsilon}{R}e^{-t/RC}$

Illustration 5.27 **Consider the circuit shown in Fig. 5.97. Find out the steady state current in the 4 Ω resistor. Assume the internal resistance of the 8 V battery to be negligible.**

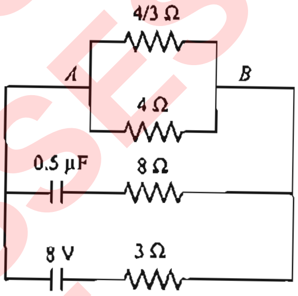


Fig. 5.97

Sol. The equivalent resistance between points A and B

$$\frac{1}{R_P} = \frac{3}{4} + \frac{1}{4} = 1$$

$$\therefore \quad R_P = 1\ \Omega$$

In the steady state, the current will not pass through 0.5 μF capacitor as it offers infinite resistance to steady state current or direct current.

So the total resistance offered by the circuit = 1 + 3 = 4 Ω

∴ the current from the battery $I = \frac{8}{4} = 2$ A. Hence potential difference between points A and $B = I \times RP = 2$ V

Therefore, the current through 4 Ω resistor $\frac{2}{4} = 0.5$ A.

Illustration 5.28 **Find out the potential difference between the points x and y in Fig. 5.98.**

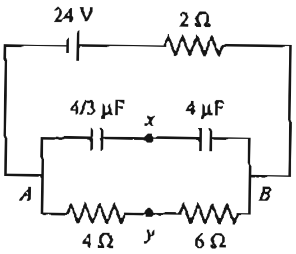


Fig. 5.98

Sol. Given $E = 24$ V, $R = 2\ \Omega$, $R_1 = 4\ \Omega$, $R_2 = 6\ \Omega$, $C_1 = \frac{4}{3}$ μF, $C_2 = 4$ μF.

As the capacitor offers a very high resistance to the current in the steady state, so the current is prevented to pass through the capacitors.

Now, the total resistance in the circuit

$$R_{eq} = R + R_1 + R_2 = 2 + 4 + 6 = 12\ \Omega$$

Hence the net current in the circuit $= \dfrac{E}{R_{eq}} = \dfrac{24}{12} = 2\,\text{A}$

Therefore, the terminal potential difference across

$$AB = 24 - 2 \times 2 = 20\ \text{V}$$

Note:

As the capacitors and resistances are in parallel, so the potential difference of 20 V is available to both the capacitors and the resistors.

Let's first of all discuss about resistors. Resistances are in the ratio 2 : 3. So the potential across them will also bear the same ratio. Potential across $y_A = 20 \times \dfrac{2}{5} = 8\,\text{V}$, Similarly the potential difference across $y_B = 20 \times \dfrac{3}{5} = 12\,\text{V}$

In the same manner, capacitors bear the ratio 1 : 3 of their capacitances. Hence accordingly potential difference across

$Ay = \dfrac{4}{3}\ \mu\text{F}$ capacitor $= 20 \times \dfrac{1}{4} = 5\,\text{V}$

Similarly the potential difference across By, i.e., 4 μF capacitor $= 20 \times \dfrac{3}{4} = 15\,\text{V}$

∴ the potential difference between the points x and $y = 15 - 12 = 3\text{V}$ or $8 - 5 = 3$ V.

(Subtract the potential difference with a common reference point.)

Illustration 5.29 **Consider the circuit shown in Fig. 5.99 where a battery of emf 4 V and a capacitor of capacitance 1 μF is connected to a combination of resistances. Find out the steady state current in the circuit.**

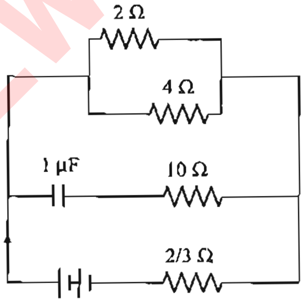


Fig. 5.99

Sol. Let us consider I to be the steady state current through the circuit.

In the steady state current I is constant in the circuit and the capacitor offers infinite resistance. So the resistance 10 Ω becomes ineffective in the circuit. So in this case, the equivalent resistance of resistors 2 Ω and 4 Ω are connected in parallel.

$$= \frac{1}{R_p} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} \Rightarrow R_p = \frac{4}{3}\ \Omega$$

∴ the total resistance of the circuit $= \dfrac{4}{3} + \dfrac{2}{3} = \dfrac{6}{3} = 2\Omega$

Hence the steady state current in the circuit $= \dfrac{4}{2} = 2\,\text{A}$.

Illustration 5.30 **A resistor with resistance 10 MΩ is connected in series with a capacitor with capacitance 1.0 μ F and a battery with emf 12.0 V, as in Fig. 5.100. Before the switch is closed at time $t = 0$, the capacitor is uncharged.**

a. **What is the time constant?**

b. **What fraction of the final charge is on the plates at time = 46 s?**

c. **What fraction of the initial remains at $t = 46$ s?**

Sol.

a. The time constant is
$t = RC = (10 - 106\ \text{W})\ (1.0\ ´\ 10^{-9}\ \text{F}) = 10\ \text{s}$

b. The fraction f the final capacitor charge is q/Q_f.

$$\frac{q}{Q_f} = 1 - e^{-t/RC} = 1 - e^{-(46s)/(10s)} = 0.99$$

The capacitor is 99 percent charged after a time equal to 4.6 RC, or 4.6 time constants.

c. $$\frac{i}{I_0} = e^{-4.6} = 0.010$$

After 4.6 time constants, the current has decreased to 1.0 percent of its initial value.

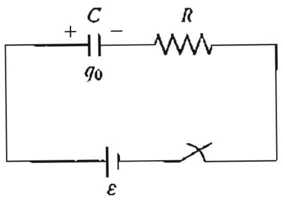


Fig. 5.100

Illustration 5.31 **The resistor and the capacitor described in previous example are reconnected as shown in Fig. 5.101. The capacitor is originally given a charge of 5.0 μC, then discharged by closing the switch at $t = 0$.**

a. **At what time will the charge be equal to 0.50 μC?**

b. **What is the current at this time?**

Sol.

a. For the time t gives

$$t = -RC \ln \frac{q}{Q_0}$$

$$= -(10 \times 10^6\ \Omega)(1.0 \times 10^{-6}\ \text{F}) \ln \frac{0.50\mu\text{C}}{5.0\mu\text{C}} = 23\text{s}$$

This is 2.3 times the constant $\tau = RC = 10$ s

b. $Q_0 = 5.0\ \mu\text{C} = 5.0 \times 10^{-6}\ \text{C}$

$$\Rightarrow \qquad i = -\frac{Q_0}{RC}e^{-t/RC} = -\frac{5.0\times10^{-6}\,\text{C}}{10\,s}\,e^{-23}$$

$$= -5.0\times10^{-8}\,\text{C}$$

The current has the opposite sign when the capacitor is discharging than when it is charging.

Consider a situation when a charged capacitor is connected with a resistance and a battery.

If a charged capacitor is connected across a battery with a resistance

Final charge on capacitor is $q_f = CE$

if $\qquad q_f > q_0$

Charge supplied by battery is $\Delta q = (CE - q_0)$

if $\qquad q_f > q_0$

Charge supplied to the battery is $\Delta q = (CE - q_0)$

let $\qquad q_0 < CE$

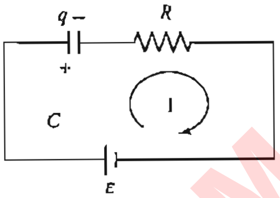


Fig. 5.101

Applying Kirchhoff's loop law in the circuit

$$-\frac{q}{C} - IR + \varepsilon = 0 \qquad \text{(i)}$$

Given $\qquad I = \dfrac{dq}{dt} \qquad$ (ii)

$$-\frac{1}{C}q - R\left(\frac{dq}{dt}\right) + \varepsilon = 0 \quad \Rightarrow \quad -RC\frac{dq}{dt} = q - C\varepsilon$$

$$\Rightarrow \quad \frac{dq}{(q - C\varepsilon)} = \left(-\frac{1}{RC}\right)dt$$

Integrating both sides

$$\int_{q_0}^{q}\frac{dq}{(q - C\varepsilon)} = \left(-\frac{1}{RC}\right)\int_0^t dt \qquad [\ln(q - C\varepsilon)]_{q_0}^{q} = -\frac{t}{RC}$$

$$\Rightarrow \quad \ln\left(\frac{q - C\varepsilon}{q_0 - C\varepsilon}\right) = -\frac{t}{RC}$$

After solving we get $q = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) + q_0\,e^{-t/RC}$

This formula looks like the summation of the formulae of standard case of charging and discharging.

Note:

We will come across the following integration very frequently. So, remember the result as such.

If $\qquad \displaystyle\int_0^x \frac{dx}{a - bx} = \int_0^t c\,dt$ ***then*** $\quad x = \dfrac{a}{b}(1 - e^{-bct})$

And if $\displaystyle\int_{x_0}^{x}\frac{dx}{a - bx} = \int_0^t c\,dt$ ***then*** $\quad x = \dfrac{a}{b} - \left(\dfrac{a}{b} - x_0\right)e^{-bct}$

Here a, b, and c are constants.

Similarly, if x increases from x_2 to x_1 exponentially, then $x - t$ equation is,

$$x = x_2 + (x_1 - x_2)(1 - e^{-kt})\,;$$

$$q = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) + q_0\,e^{-t/RC}$$

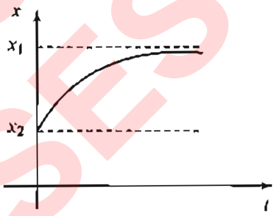


Fig. 5.102

Another approach to analyse the above condition. The charge further supplied by battery $\Delta q = (C\varepsilon - q_0)$

Hence, transition charge $= \Delta q = (C\varepsilon - q_0)$

Hence, charge at any time

$$q = q_0 + (C\varepsilon - q_0)\,(1 - e^{-t/RC})$$

$$q = C\varepsilon\,(1 - e^{-t/RC})\,q_0 + e^{-t/RC}$$

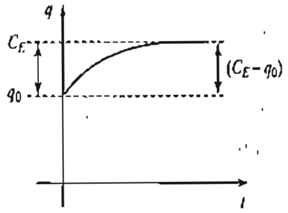


Fig. 5.103

The expression is the same as the one we have calculated in the previous section.

Sometimes a physical quantity x decreases from x_1 to x_2, exponentially, then the x-t equation is like,

$$x = x_2 + (x_1 - x_2)e^{-kt}$$

Here, K is a constant.

A capacitor offers zero resistance in a circuit when it is uncharged, i.e., it can be assumed as short circuited and it offers infinite resistance when it is fully charged.

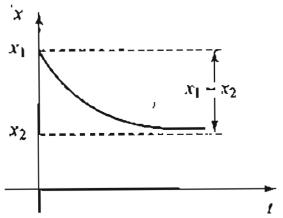


Fig. 5.104

Equivalent Time Constant

To find the equivalent time constant of a circuit, following steps are followed:

i. Short – circuit the battery.

ii. Find net resistance across the capacitor (say it R_{net}).

iii. $\tau_C = (R_{net})C$

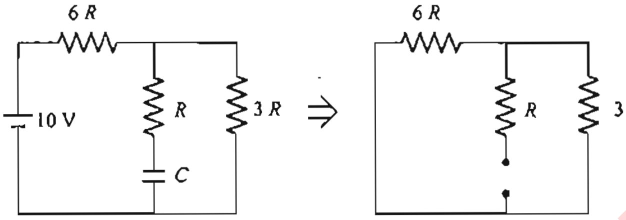


Fig. 5.105

For example, in the circuit shown in Fig. 5.105, after short circuiting the battery $3R$ and $6R$ are in parallel, so their combined resistance is $\frac{(6R)(3R)}{6R+3R} = 2R$

Now this $2R$ is in series with the remaining R.

Hence, $R_{net} = 2\,R + R = 3\,R \quad \Rightarrow \quad \tau_C = (R_{net})\,C = 3\,RC$

Alternate method of finding current in the circuit and the charge on the capacitor at any time t

In a complicated C-R circuit it is easy to find current in the circuit and charge stored in the capacitor at time $t = 0$ and $t = 8$. But to find the current and the charge at time t the following steps may be followed.

i. Find equivalent time constant (t_c) of the circuit.

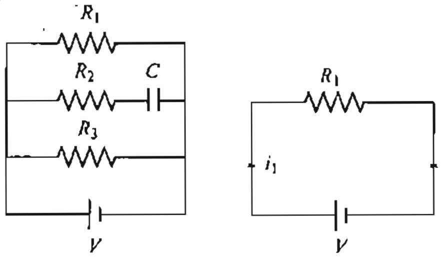
ii. Find steady state charge q_0 (at time $t = 8$) on the capacitor.

iii. Charge on the capacitor at any time t is $q = q_0(1 - e^{-t/RC})$.

By differentiating it w.r.t. time we can find the current through the capacitor at time t. Then by using Kirchhoff's laws we can calculate currents in other parts of the circuit also.

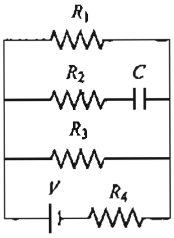
Illustration 5.32 **Calculate the current in branch R_1, R_2, and R_3 in the circuit shown in Fig. 5.106 (a).**

Sol. The circuit shown in Figs. 5.106 (a) can be considered as combination of the circuits shown in Figs. 5.106 (b), (c), and (d).



(a) (b)

Fig. 5.106



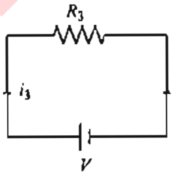
(c)

Fig. 5.106

In Fig. 5.106 (b) $i_1 = \frac{V}{R_1}$ = constant. In Fig. 5.106 (c) $i_2 = i_0 e^{-t/RC}$

where $i_0 = \frac{V}{R_2}$ and $\tau_C = CR_2$. In Fig. 5.106 (d) $i_3 = \frac{V}{R_3}$

Current through the battery, $i = i_1 + i_2 + i_3$



(d)

Fig. 5.106

Note:

Due to the presence of R_4, it cannot be broken into three simple parallel circuits.

Illustration 5.33 **Calculate the current flowing through the capacitor branch in the circuit shown in Fig. 5.107, initially and finally.**

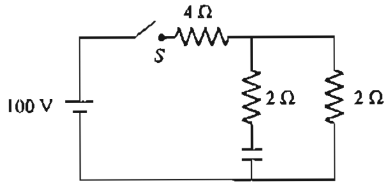
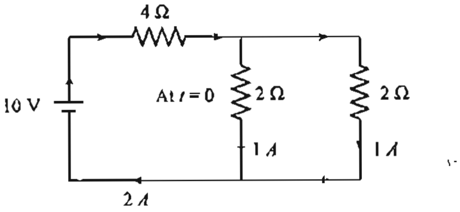


Fig. 5.107

Sol. At time $t = 0$, when the switch is closed it becomes as shown in Fig. 5.107.

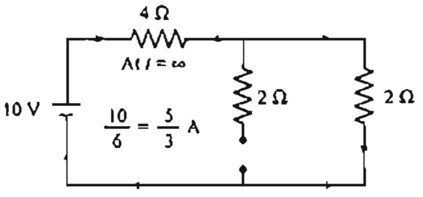
When the capacitor is fully charged, i.e., at $t = 8$, the circuit becomes as in Fig. 5.108 (a).

This means that no current flows through the capacitor.



(a)

Fig. 5.108 (*Contd.*)



(b)

Fig. 5.108

Concept Application Exercise 5.3

1. Consider the circuit shown in Fig. 5.109. If the switch is closed at $t = 0$, then calculate the values of I, I_2, and I_2 at
(a) $t = 0$ (b) $t = \infty$

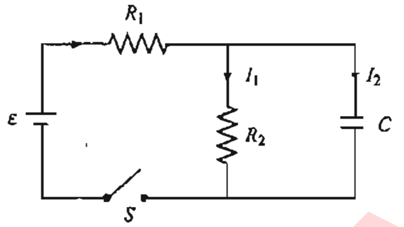


Fig. 5.109

2. Figure 5.110 shows three sections of the circuit that are to be connected in turn to the same battery via a switch. The resistors are all identical, as are capacitors. Rank the sections according to

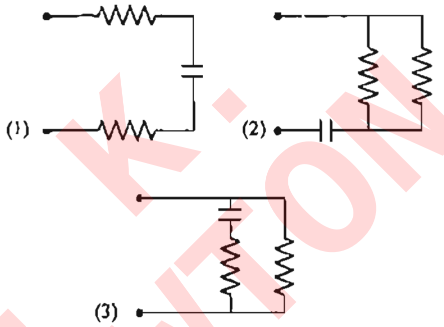


Fig. 5.110

a. the final (equilibrium) charge on the capacitor ______.

b. the time required for the capacitor to reach 50 percent of its final charge, greatest first ______.

3. Determine the current through the battery in the circuit shown in Fig. 5.111.

a. Immediately after the switch S is closed ______.

b. After a long time ______.

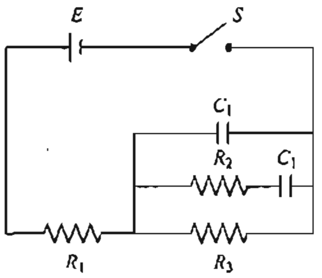
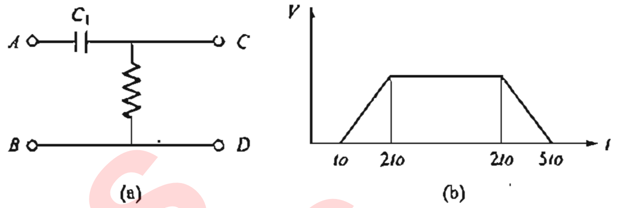


Fig. 5.111

4. A varying voltage is applied to the clamps AB as shown in Fig. 5.112. Such that the voltage across the capacitor plates varies as shown in the figure. Plot the time depending of voltage across the clamps CD.



(a) (b)

Fig. 5.112

5. Consider the network shown in Fig. 5.113, initially, the switch S_1 is closed at S_2 is open

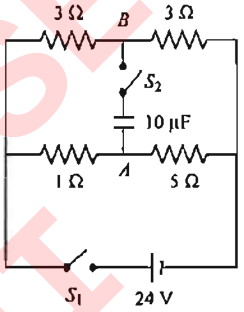


Fig. 5.113

a. Calculate $V_A - V_B$.

b. When S_2 is also closed, what is $V_A - V_B$
(i) just after closing (ii) after long time

6. In the circuit shown in Fig. 5.114, $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $C_1 = 1\ \mu F$, $C_2 = 2\ \mu F$ and $E = 6$ V. Calculate the charge on each capacitor in the steady state.

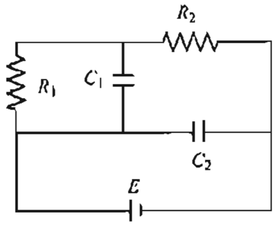


Fig. 5.114

7. The plates of a 50 μF capacitor charged to 400 μC are connected through a resistance of 1.0 kΩ. Find the charge remaining on the capacitor 1 s after the connection is made.

8. The electric field between the plates of a parallel-plate capacitor of capacitance 2.0 μF drops to one third of its initial value in 4.4 ms when the plates are connected by a thin wire. Find the resistance of the wire.

9. The time constant of an R-C circuit during discharge is that time in which the charge on the condenser plates, as compared to maximum charge (q_0), becomes $\frac{q}{q_0} \times 100$ which is equal to ______.

10. After how many time constants will the energy stored the capacitor reach one-half of its equilibrium value?

Solved Examples

Example 6.1 **The gap between the two plane plates of a capacitor equal to d is filled with gas. One of the plate emits n_0 electrons per second. It is moving in an electric field, which ionizes gas molecules. This way each electron produces α new electrons (and ions) along a unit length of its path. Find the electronic current at the opposite plate, neglecting the ionization of gas molecules formed by ions, and the electronic current density at the plate possessing a higher potential. Assume that n_i electrons per unit volume per second are formed.**

Sol. Let the number of electrons at a distance x be n, then the increase in number of electron per unit length is

$$\frac{dn}{dx} = \alpha n \qquad \text{(i)}$$

or, $$\frac{dn}{n} = \alpha \times dx \Rightarrow \int_{n_0}^{n} \frac{dn}{n} = \alpha \int_0^d dx$$

$$\Rightarrow \ln\left(\frac{n}{n_0}\right) = \alpha d$$

or $$n = n_e e^{\alpha d} \qquad \text{(ii)}$$

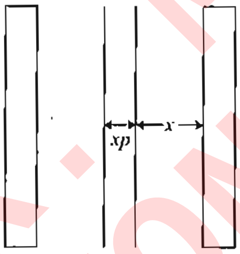


Fig. 5.115

Hence the number of electrons reaching other plate per unit time is $n_0 e^{\alpha d}$.

$\because$ electronic current at opposite plate = charge reaching per unit time

or $\quad i = en$

or $\quad i = en_0 e^{\alpha d} \qquad \text{(iii)}$

Here $\quad n_0 = (n_i A dx)$

and $\quad n = (n_i A dx)e^{\alpha x}$

So $$I = \int en = \int_0^d n_i A e^{\alpha x}\, dx$$

$$\Rightarrow \quad I = en_i A\left(\frac{e^{\alpha d - 1}}{\alpha}\right)$$

Hence $$J = \frac{I}{A} = \frac{en_i}{\alpha}(e^{\alpha d} - 1)$$

Example 5.2 **Switch S of circuit shown in Fig. 5.116 is in position 1 for a long time. At instant $t = 0$, it is thrown from position 1 to 2. Calculate thermal power $P_1(t)$ and $P_2(t)$ generated across resistances R_1 and R_2, respectively.**

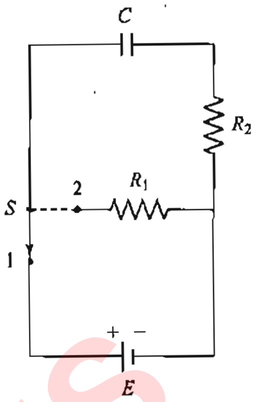


Fig. 5.116

Sol. Initially the switch was in position 1 for a long time, therefore, initially the capacitor was fully charged and potential difference across capacitor at $t = 0$ was equal to e.m.f E of the battery.

Initial charge on capacitor, $q_0 = CE$

When the switch is thrown to position 2, the capacitor starts to discharge through resistances R_1 and R_2.

To calculate thermal power $P_1(t)$ and $P_2(t)$ generated across R_1 and R_2, respectively, current I at time t through the circuit must be known.

Let at instant t, the charge remaining on the capacitor be q and let the current through the circuit be I.

Applying Kirchhoff's voltage law on the mesh in the circuit of Fig. 5.177.

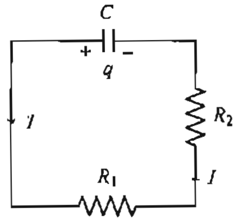


Fig. 5.117

$$\frac{q}{C} - IR_2 - IR_1 = 0 \Rightarrow I = \frac{q}{(R_1 + R_2)C}$$

Since, the capacitor is discharging, therefore,

$$I = -\frac{dq}{dt} \qquad \text{(i)}$$

From equation (i),

$$\frac{dq}{q} = -\frac{dt}{(R_1 + R_2)C} \qquad \text{(ii)}$$

Knowing that at $t = 0$, $q = q_0 = CE$, integrating equation (ii),

$$\int_{q=CE}^{q=?} \frac{dq}{q} = -\int_{t=0}^{t} \frac{dt}{(R_1 + R_2)C}$$

$$\Rightarrow \quad \log\frac{q}{CE} = -\frac{t}{(R_1 + R_2)C}$$

$$\Rightarrow \quad q = CEe^{-t/(R_1 + R_2)C}$$

But $$I = -\frac{dq}{dt}$$

Therefore $$I = \frac{E}{(R_1 + R_2)} e^{-t/(R_1+R_2)C}$$

Hence, thermal power across R_1 is $P_1 = I^2R_1$

$$\Rightarrow \qquad P_1 = \frac{E^2R_1}{(R_1+R_2)^2} e^{-2t/(R_1+R_2)C}$$

Similarly, thermal power across R_2, $P_2 = I^2R_2$

$$\Rightarrow \qquad P_2 = \frac{E^2R_2}{(R_1+R_2)^2} e^{-2t/(R_1+R_2)C}$$

Example 5.3 **A charged capacitor C_1 is discharged through a resistance R by putting switch S in position 1 of the circuit shown in Fig. 5.118. When the discharge current reduces to i_0, the switch is suddenly shifted to position 2. Calculate the amount of heat liberated in resistor R starting from this instant. Also calculate, current i through the circuit as a function of time.**

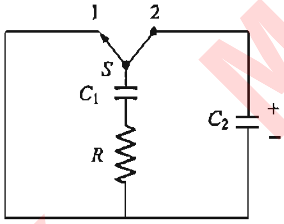
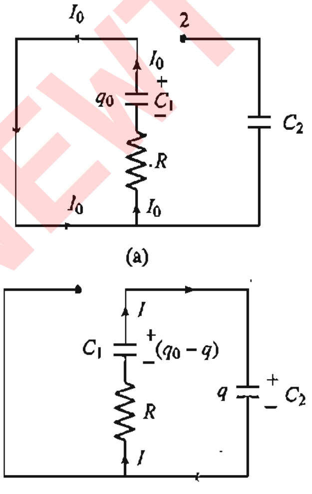


Fig. 5.118

Sol. Let the charge on capacitor C_1 be q_0 when the switch was shifted from position 1 to position 2. Just before shifting of switch the circuit was as shown in Fig. 5.119 (a).



(a)

(b)

Fig. 5.119

$$\frac{q_0}{C} - I_0R = 0 \quad \Rightarrow \quad q_0 = I_0RC_1$$

When the switch is shifted from position 1 to position 2 the capacitor C_1 continues to be discharged while C_2 starts charging.

Let at time t, after shifting of switch to position 2, charge on capacitor C_2 be q and let current through the circuit be I.

∴ therefore, charge remaining on C_1 is equal to $(q_0 - q)$ as shown in Fig. 5.119 (b)

Applying Kirchhoff's voltage law on the circuit shown in Fig. 5.119 (b).

$$\frac{q}{C_2} + IR - \frac{(q_0 - q)}{C_1} = 0$$

$$IR = \frac{q_0 - q}{C_1} - \frac{q}{C_2} = \frac{(q_0C_2 - qC_2) - qC_2}{C_1C_2}$$

But current, $I = dq/dt$ (Rate of increase of charge on C_2)

$$R\frac{dq}{dt} = \frac{q_0C_2 - q(C_1 + C_2)}{C_1C_2}$$

$$\Rightarrow \qquad \frac{dq}{q_0C_2 - q(C_1+C_2)} = \frac{dt}{RC_1 \times C_2}$$

But at $t = 0$, $q = 0$, $\int_0^q \frac{dq}{q_0C_2 - q(C_1+C_2)} = \int_0^t \frac{dt}{RC_1.C_2}$

From the above equation,

$$q = \left(\frac{q_0C_2}{C_1+C_2}\right)\left[1 - e^{-\left(\frac{C_1+C_2}{RC_1C_2}\right)t}\right]$$

Subtracting,

$$q_0 = I_0RC_1$$

$$\Rightarrow \qquad q = I_0\frac{RC_1C_2}{C_1+C_2}\left[1 - e^{-\left(\frac{C_1+C_2}{RC_1C_2}\right)t}\right]$$

But current, $I = \frac{dq}{dt} \quad \Rightarrow \quad I = I_0e^{-\left(\frac{C_1+C_2}{RC_1C_2}\right)t}$

In a steady state the common potential difference across capacitors is given by,

$$V = \frac{q_0 + 0}{C_1 + C_2} \qquad \left(\text{using } V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)$$

$$V = \frac{I_0RC_1}{C_1 + C_2}$$

Initially energy stored in C_1 was $U_1 = \frac{q_0^2}{2C_1} = \frac{1}{2}I_0^2R^2C_1$

In steady state, energy stored in two capacitors is,

$$U_2 = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1 + C_2)\frac{I_0^2R^2C_1^2}{(C_1+C_2)^2}$$

$$= \frac{I_0^2R^2C_1^2}{2(C_1+C_2)}$$

Heat generated across resistor R = loss of energy stored in capacitors during redistribution of charge

$$= U_1 - U_2 = \frac{I_0^2 R^2 C_1 C_2}{2(C_1 + C_2)}$$

Example 5.4 **The capacitor shown in the Fig. 5.120 has been charged to a potential difference of V volt so that it carries a charge CV with both the switches S_1 and S_2 remaining open. Switch S_1 is closed at $t = 0$. At $t = R_1 C$ switch S_1 is opened and S_2 is closed. Find the charge on the capacitor at $t = 2R_1 C + R_2 C$.**

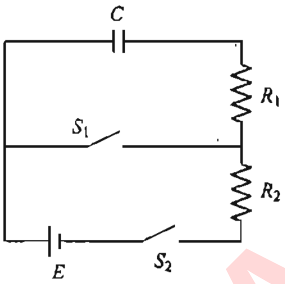


Fig. 5.120

(IIT-JEE, 1978)

Sol. At $t = R_1 C$: $q_1 = CV\left(e^{-\frac{R_1 C}{R_1 C}}\right) = \frac{CV}{e}$

Now, $$\int_{q_1}^{q} \frac{dq}{CE - q} = \int_{R_1 C}^{2R_1 C + R_2 C} \frac{dt}{(R_1 + R_2)C}$$

$$\Rightarrow \quad q = CE\left(1 - \frac{1}{e}\right) + \frac{CV}{e^2}$$

Example 5.5 **In the circuit shown in Fig. 5.121 E, F, G, and H are cells of e.m.f 2, 1, 3, and 1 V, respectively. The resistances 2, 1, 3, and 1 W are their respective internal resistances. Calculate**

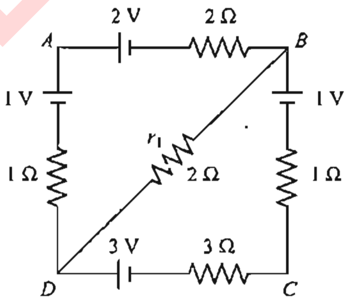


Fig. 5.121

i. the potential difference between B and D, and
ii. the potential differences across the terminals of each of the cells G and H. **(IIT-JEE, 1984)**

Sol. Suppose a current i_1 goes in the branch BAD. therefore a current i_2 in the branch DCB will be $i_1 - i_2$ from the junction law. The circuit with the currents shown is redrawn in Fig. 5.122. Applying the loop law to $BADB$ we get,

$$(2\rho)i_1 - 2\text{ V} + 1\text{ V} + (1\ \Omega)i_1 + (2\ \Omega)(i_1 - i_2) = 0$$

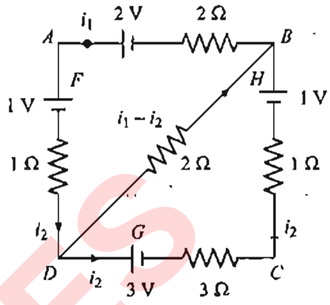


Fig. 5.122

Applying the same law to the loop $DCBD$, we get

$$-3\text{ V} + (3\ \Omega)i_2 + (1\ \Omega)i_2 + 1\text{ V} - (2\ \Omega)(i_1 - i_2) = 0$$

or $$-(2\ \Omega)i_1 + (6\ \Omega)i_2 = 2\text{ V}$$

From equations (i) and (ii),

$$i_1 = \frac{5}{13}\text{ A},\ i_2 = \frac{6}{13}\text{ A}$$

$$\therefore \quad i - i_2 = -\frac{1}{13}\text{ A}$$

The current in BD is from B to D (opposite to assumption).

i. $V_B - V_D = (2\ \Omega)\left(\frac{1}{13}\text{ A}\right) = \frac{2}{13}\text{ V}$

ii. Potential differences across the cell G

$$V_c - V_D = -(3\ \Omega)i_2 + 3\text{ V} = \left(3\text{ V} - \frac{18}{13}\text{ V}\right) = \frac{21}{13}\text{ V}$$

Potential difference across the cell H

$$V_c - V_B = (1\Omega)i_2 + 1\text{V} = (1\Omega)\left(\frac{6}{13}\text{ A}\right) + 1\text{ V} = \frac{19}{13}\text{ V}$$

$(\because V = E + ir)$

Example 5.6 **In the circuit shown in Fig. 5.123, E_1 = 3 V, E_2 = 2 V, E_3 = 1 V, and $R = r_1 = r_2 = r_3 = 1\ \Omega$.**

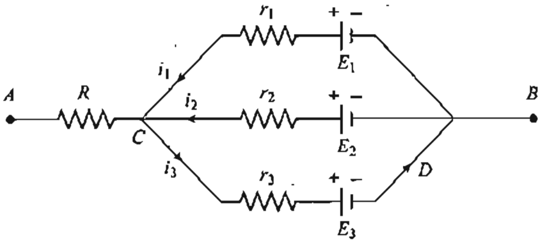


Fig. 5.123

i. Find the potential difference between the points A and B and the currents through each branch.
ii. If r_2 is short circuited and the point A is connected to point B, find the currents through E_1, E_2, E_3, and the resistor R. **(IIT-JEE, 1981)**

Sol.

i. Applying Kirchoff's law in *PQRUP* starting from *P* moving clockwise

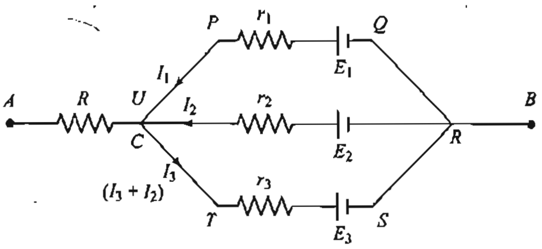


Fig. 5.124

$$I_1r_1 - E_1 + E_2 - I_2r_2 = 0 \Rightarrow I_1 \times 1 - 3 + 2 - I_2 \times 1 = 0$$

$$\Rightarrow \quad I_1 - I_2 = 1 \qquad \text{(i)}$$

Applying Kirchoff's law in *URSTU* starting from *U* moving clockwise.

$$I_2r_2 - E_2 + E_3 - I_3r_3 = 0 \Rightarrow I_2 \times 1 - 2 + 1 + I_3 = 0$$

$$\Rightarrow \quad I_2 + I_3 = 0$$

$$\Rightarrow \quad I_2 + I_1 + I_2 = 1$$

$$\Rightarrow \quad I_1 + 2I_2 = 1 \qquad \text{(ii)}$$

Subtracting equation (i) from (ii)

$$I_1 + 2I_2 - I_1 + I_2 = 0$$

$$\Rightarrow \quad I_2 = 0 \quad \Rightarrow \quad I_1 = 1\text{A}$$

∴ therefore, current through branch *PQ* is 1 A. Current through branch *UR* is 0 A. Current through branch *TS* is 1 A p.d. from *A* to *B*

$$V_A - O \times R + I_1r_1 - E_1 = V_B$$

$$\therefore \quad V_A - V_B = E1 - I_1r_1 = 3 - 1 = 2\text{ V}$$

ii. Applying Kirchoff's law in *PQRUP* starting from *P* moving clockwise

$$I_1r_1 - E_1 + E_2 = 0 \quad \Rightarrow \quad I_1 - 3 + 2 = 0$$

$$\Rightarrow \quad I_1 = 1\text{ A}$$

Applying Kirchoff's law in *URSTU* starting from *U* moving clockwise

$$-E_2 + E_3 - I_3r_3 = 0$$

$$\Rightarrow \quad -2 + 1 - I_3 = 0 \quad \Rightarrow \quad I_3 = -1\text{ A}$$

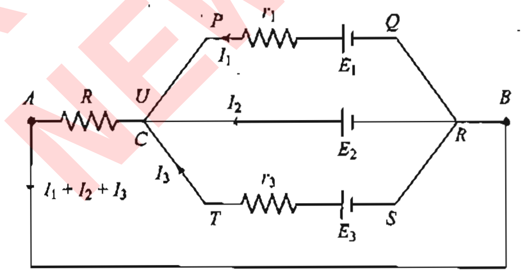


Fig. 5.125

The –ve sign of I_3 indicates that the direction of current in the branch *UTSR* is opposite to that assumed. Applying Kirchoff's law in *AURBA* starting from *A* moving clockwise is $(I_1 + I_2 + I_3)R - E_2 = 0 \Rightarrow (1 + I_2 - 1)R = 2$

$\Rightarrow I_2 = 2$ A

Example 5.7 **Calculate the steady state current in the 2 Ω resistor shown in the circuit in Fig. 5.126. The internal resistance of the battery is negligible and the capacitance of the condenser *C* is 0.2 microfarad.**

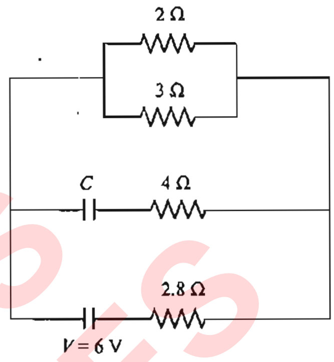


Fig. 5.126

Or

Two resistors, 400 Ω and 800 Ω are connected in series with a 6 V battery. It is desired to measure the current in the circuit. An ammeter of a 10 Ω resistance is used for this purpose. What will be the reading in the ammeter? Similarly, if a voltmeter of 10,000 Ω resistance is used to measure the potential difference across the 400 Ω resistor, what will be the reading in the voltmeter. **(IIT-JEE, 1982)**

Sol. When the current becomes steady then the branch containing the capacitor will be ineffective as no current will be flowing through it. The circuit can be redrawn as it is clear from the Fig. 5.127 that resistance 2 Ω and 3 Ω are in parallel.

$$\therefore \quad 2I_1 = 3I_2 \text{ (as p.d. across the two resistors will be same)}$$

$$I_2 = \frac{2}{3} I_1 \qquad \text{(i)}$$

Applying Kirchhoff's law in loop *ABCDEFGA* starting from *A* in the clockwise direction

$$-I_1 \times 2 - 1 \times 2.8 + 6 = 0 \quad \Rightarrow \quad -I_1 \times 2 - (I_1 + I_2) \times 2.8 + 6 = 0$$

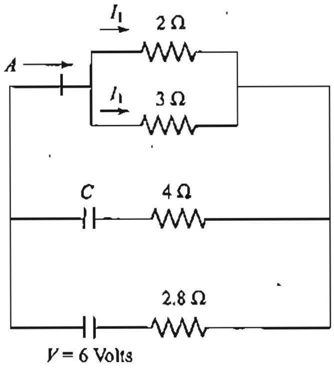


Fig. 5.127

$$\Rightarrow \quad -2I_1 - 2.8\left[I_1 + \frac{2}{3}I_1\right] + 6 = 0$$

$$\Rightarrow \quad 6 = \left[2 + 2.8 \times \frac{5}{3}\right] I_1$$

$$\Rightarrow \quad 6 = 5.67 \times I_1$$

$$\therefore \quad I_1 = \frac{6}{6.67} \quad \text{or} \quad I_1 = 0.9\text{ A}$$

Applying Kirchhoff's law moving in clockwise direction starting from battery we get

$$+6 - 10I - 400\,I - 800\,I = 0$$

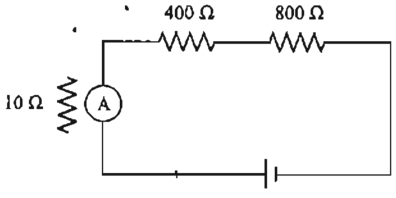


Fig. 5.128

$\therefore \quad 6 = 1210I$

$\therefore \quad I = \dfrac{6}{1210} = 4.96\times10^{-3}$ A

The voltmeter and 400 Ω resistor are in parallel and hence p.d. will be same.

$\therefore\ 10{,}000\ I_1 = 400\ I_2$ (ii)

Applying Kirchoff's law in loop *ABCDEA* starting from *A* in clockwise direction.

$$-400\ I_2 - 800\ I + 6 = 0$$

$\therefore \quad 6 = 400\ I_2 - 800(I_1 + I_2)$

$\therefore \quad 6 = 400\ I_2 + 800(.04\ I_1 + I_2)$

From equation (ii) putting the value of I_1

$6 = 1232\ I_2$

$\therefore \quad I_2 = 4.87\times10^{-3}$ A

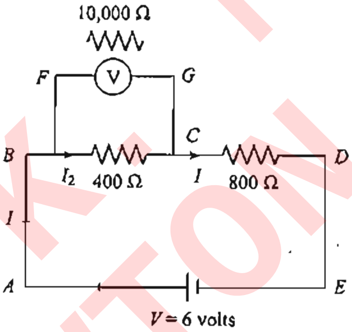


Fig. 5.129

∴ potential drop across 400 Ω resistor

$= I_2\times400 = 4.87\times10^{-3}\times400 = 1.948\text{ V} = 1.95\text{ V}$

∴ the reading measured by voltmeter = 1.95 V.

Example 5.8 **A part of a circuit is in steady state along with the current flowing in the branches. Value of each resistance is shown in Fig. 5.130. Calculate the energy stored in the capacitor *C* (4μF).**

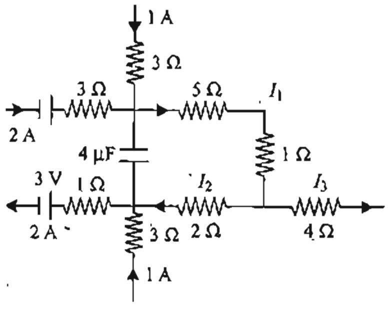


Fig. 5.130 (IIT-JEE, 1986)

Sol. Applying Kircholl's first law at junction *M*, we get the current $I_1 = 3$ A.

Applying Krichoff's first law at junction *P*, we get current $I_2 = 1$ A

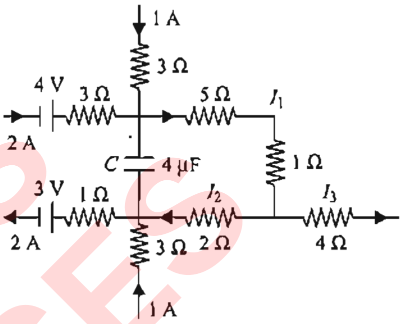


Fig. 5.131

Moving the loop from *MNO* to *P* $\therefore\ v_M - 5\times i_1 - 2\times i_2 = v_P$

$\therefore \quad v_M - v_P = 6i_1 + 2i_2 = 6\times3 + 2\times1 = 20$ V

Energy stored in the capacitor

$$= \frac{1}{2}CV^2 = \frac{1}{2}\times4\times10^{-6}\times20\times20 = 8\times10^4\text{ J}$$

Example 5.9 **An infinite ladder network of resistance is constructed with a 1 Ω and 2 Ω resistance, as shown in Fig. 5.132.**

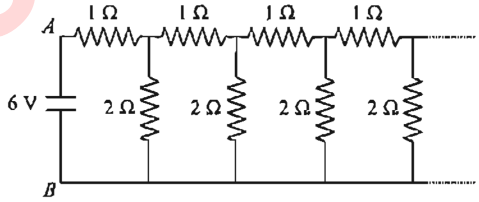


Fig. 5.132

The 6 V battery between *A* and *B* has negligible internal resistance:

- **i. Show that the effective resistance between *A* and *B* is 2 Ω.**
- **ii. What is the current that passes through the 2 Ω resistance nearest to the battery?** **(IIT-JEE 1987)**

Sol. Let the effective resistance between point *C* and *D* be *R* then the circuit can be redrawn as shown in Fig. 5.133

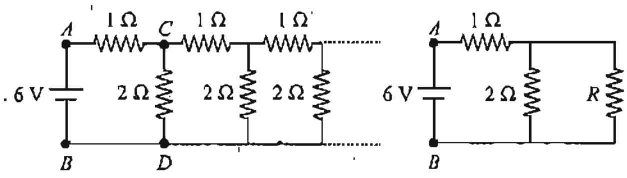


Fig. 5.133

The effective resistance between *A* and *B* is

$$R_{eq} = 1 + \frac{2\times R}{R+2}$$

This resistance R_{eq} can be taken as R because if we add one identical item in infinite item then the result will almost be the same.

$$\therefore \quad 1+\frac{2\times R}{R+2}=R$$

$$\Rightarrow \quad R+2+2R=R^2+2R$$

$$\Rightarrow \quad R^2-R-2=0$$

$$\Rightarrow \quad R-2R+R-2=0$$

$$\Rightarrow \quad R^2-2R+R-2=0$$

$$\Rightarrow \quad R(R-2)+1(R-2)=0$$

$$\Rightarrow \quad [R+1][R-2]=0$$

$$\Rightarrow \quad R=2\,\Omega$$

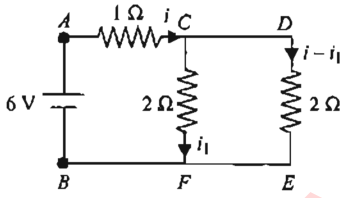


Fig. 5.134

Applying Kirchoff's law in the two loops we get $6-i-2i_l=0$

$\Rightarrow \quad i=6-2i_l$ (i)

$\quad -2(i-i_l)\,i-2i_l+=0$ (ii)

From equations (i) and (ii), we get $-2(6-2i_l)+2i_l+2i_l=0$

$$\Rightarrow \quad -12+4i_l+4i_l=0$$

$$\Rightarrow \quad i_1=\frac{12}{8}=\frac{3}{2}=1.5\text{ A}$$

Example 5.10 **All resistances in the diagram below (see Fig. 5.135) are in ohms.**

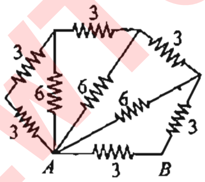


Fig. 5.135

Find the effective resistance between the point A and B.

(IIT-JEE, 1979)

Sol. The given system can be reduced as shown in Fig. 5.136.

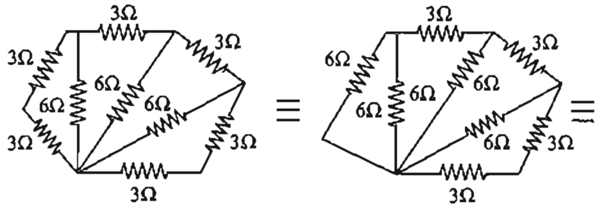


Fig. 5.136 (*Contd.*)

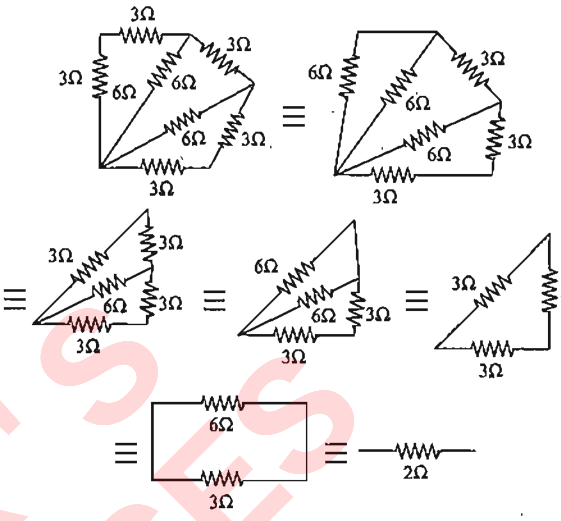


Fig. 5.136

Example 5.11 **In the diagram shown in Fig. 5.137 find the potential difference between the points A and B and between the points B and C in the steady state.**

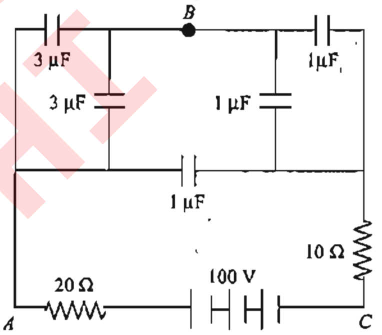
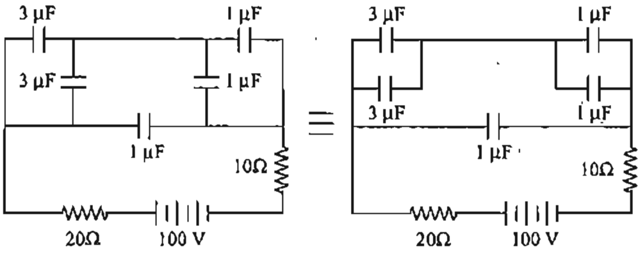


Fig. 5.137

(IIT-JEE, 1979)

Sol. Applying Kirchoff's law in loop $AQBRC$:

$$-\frac{q}{6}-\frac{q}{2}+100=0 \Rightarrow q=150\,\mu\text{C}$$



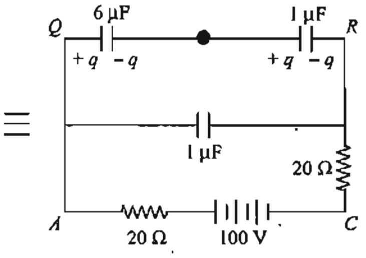


Fig. 5.138

$\therefore$ the potential difference between $AB = \frac{150}{6} = 25\text{V}$ and potential difference between $BC = 100 - 27 = 75\text{V}$.

Example 5.12 **In the given circuit (Fig. 5.139): $E_1 = 3E_2 = 2E_3 = 6$ V, $R_1 = 2R_4 = 6\ \Omega$, $R_3 = 2R_2 = 4\ \Omega$, $C = 5\ \mu$F.**

Find the current in R$_3$ and the energy stored in the capacitor.

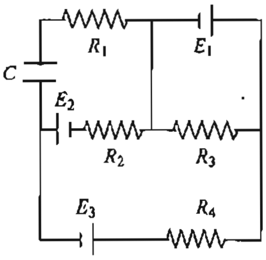


Fig. 5.139

(IIT-JEE, 1988)

Sol. Applying Kirchhoff's law in $ABFGA$ $6 - (i_1 + i_2)\,4 = 0$ (i)
Applying Kirchhoff's law in BCDEFB

$$i_2 \times 3 - 3 - 2 + 2i_2 + (i_2 + i_1)\,4 = 0 \quad \text{(ii)}$$

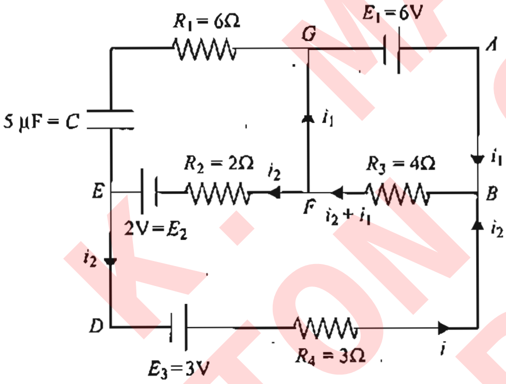


Fig. 5.140

Putting the value of $4\,(i_1 + i_2) = 6$ in equation (ii) we get

$$3i_2 - 5 + 2i_2 + 6 = 0$$

$$\Rightarrow \quad i_2 = -\frac{1}{5}\ \text{A}$$

Substituting this value in equation (i) we get

$$i_1 = 1.5 - \left(-\frac{1}{5}\right) = 1.7\ \text{A}$$

Therefore, current in $= R_3 = i_1 - i_2 1.7 - 0.2 = 1.5$ A
To find the p.d. across the capacitor $V_E - 2 - 0.2 \times 2 = V_G$
$\therefore \quad V_E - V_G = 2.4$ V
$\therefore$ energy stored in capacitor is

$$= \frac{1}{2}CV^2 = \frac{1}{2} \times 5 \times 10^{-6} \times (2.4)^2 = 1.44 \times 10^{-5}\ \text{J}$$

Example 5.13 **Find the e.m.f (V) and internal resistance (r) of a single battery which is equivalent to a parallel combination of two batteries of e.m.f.s V_1 and V_2 and internal resistance r_1 and r_2, respectively, with polarities as shown in Fig. 5.141.**

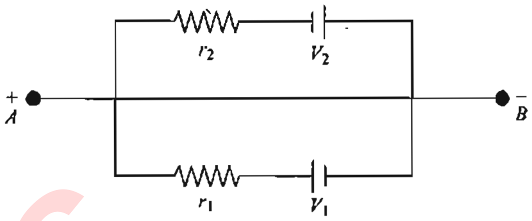


Fig. 5.141

(IIT-JEE, 1997)

Sol. The equivalent resistance $r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$

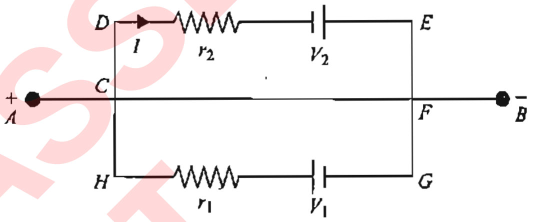


Fig. 5.142

This is the internal resistance of the single battery which is equivalent to a parallel combination of the two batteries. Now applying Kirchoff's law in loop $HCDEFGH$ moving clockwise starting from D

$$- Ir_2 + V_2 + V_1 - Ir_1 = 0$$

$$\Rightarrow \quad I = \frac{V_1 + V_2}{r_1 + r_2}$$

Now applying Kirchoff's law in branch $ACHGB$

$$V_A + Ir_1 - V_1 = V_B$$

$$\Rightarrow \quad V_A - V_B = V_1 - Ir_1$$

$$\Rightarrow \quad V_A - V_B = V_1 - \frac{(V_1 + V_2)r_1}{r_1 + r_2} = \frac{V_1 r_1 + V_1 r_2 - V_1 r_1 - V_2 r_1}{r_1 + r_2}$$

$$= \frac{V_1 r_2 - V_2 r_1}{r_1 + r_2}$$

This is the potential difference of the new battery.

Example 5.14 **A leaky parallel plate capacitor is filled completely with a material having a dielectric constant k as 5 and electrical conductivity $\sigma = 7.4 \times 10^{-12}\ \Omega^{-1}\text{m}^{-1}$. If the charge on the plane at instant $t = 0$ is $q = 8.85\ \mu$C, then calculate the leakage current at the instant $t = 12$ s.**

(IIT-JEE, 1997)

Sol. $q_0 = 8.85 \times 10^{-6}\ C$ at $t = 0$; $q = q$ at $t = 12$ s

$$\text{Now,} \quad I = \frac{V}{R} = \frac{AV}{\rho l} \quad \text{(i)}$$

R = Resistance V= Potential difference at t seconds.

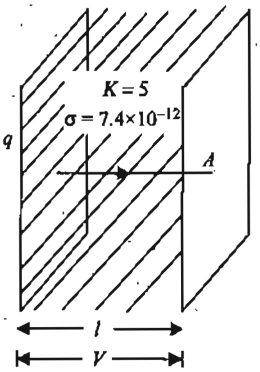


Fig. 5.143

$$\Rightarrow \quad -\frac{dq}{dt}=\frac{AV}{\rho l} \quad \Rightarrow \quad -\frac{dq}{dt}=\frac{AV}{\rho l}\frac{q}{c} \qquad (\because q=CV)$$

$$\Rightarrow \quad -\frac{dq}{q}=\frac{A\,ldt}{\rho lK\varepsilon_0 A} \qquad \left(\because C=\frac{K\varepsilon_0 A}{l}\right)$$

$$\Rightarrow \quad -\frac{dq}{dt}=\frac{\sigma}{K\varepsilon_0}dt \qquad \left(\because \sigma=\frac{1}{\rho}\right)$$

$$\frac{\sigma}{K\varepsilon_0}=\frac{7.4\times10^{-12}}{5\times8.85\times10^{-12}}=0.1672$$

$$\frac{dq}{q}=0.1672\,dt$$

On integrating $\int_{q_0}^{q}\frac{dq}{q}=-0.1672\int_0^t dt$

$$\Rightarrow \quad \log_e\frac{q}{q_0}=-0.1672\,t$$

$$q=q_0e^{-0.1672\,t}$$

When $t = 12$ s

$$q=\frac{q_0}{e^{-0.1672t}}=\frac{8.85\times10^{-6}}{e^{-0.1672t\times1L}}$$

From equation (i) $=\frac{8.85}{7.439}\times10^{-6}=1.1896\times10^{-6}\,\text{C}$

$$\therefore \quad I=\frac{\sigma A}{l}\times\frac{ql}{K\varepsilon_0 A}=\frac{\sigma}{K\varepsilon_0}\times q$$

$$=0.1672\times1.1896\times10^{-6}=0.199\,\mu\text{A}$$

Alternatively : The problem can be treated as discharging of CR circuit. For which $q=q_0e^{-t/\tau}$, where q_0 = initial charge; q = charge at time; and t τ_c = time constant.

$$\Rightarrow \quad \tau_c=CR=\frac{K\varepsilon_0 A}{l}\times\frac{\rho l}{A}\,\frac{K\varepsilon_0}{\sigma}$$

$$=\frac{5\times8.85\times10^{-12}}{7.4\times10^{-12}}=5.98\text{ s}$$

$$\therefore \quad q=q_0e^{-t/5.98}$$

$$\therefore \text{ current } \quad I=\left(-\frac{dq}{dt}\right)=\frac{q_0}{5.98}e^{-t/5.98}=\frac{8.85\times10^{-6}}{5.98}e^{-12/5.98}$$

$$=0.199\,\mu\text{A}$$

Example 5.15 **In the circuit shown in Fig. 5.144, the battery is an ideal one, with emf V. The capacitor is initially uncharged. The switch S is closed at time $t = 0$.**

a. Find the charge Q on the capacitor at time t.

b. Find the current in AB at time t, what is its limiting value as $t \to \infty$.

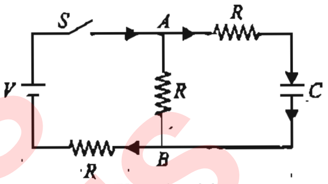


Fig. 5.144

(IIT-JEE, 1998)

Sol. Let at any time t charge on capacitor C be Q and currents are as shown in Fig. 5.145. Since charge Q will increase with time t, therefore,

$$i_1=\frac{dQ}{dt}$$

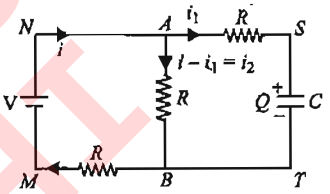


Fig. 5.145

a. Applying Kirchoff's second law in the loop $MNABM$

$$V=(i-i_1)R+iR \text{ or } V=2ir-i_lR \qquad (1)$$

Similarly, applying Kirchoff's second law in loop $MNSTM$, we have

$$V=i_1R+\frac{Q}{C}+iR\,.$$

Eliminating i from equation (1) and (2), we get

$$V=3i_1R+\frac{2Q}{C} \quad \text{or} \quad i_1=3i_1R=V-\frac{2Q}{C}$$

$$\text{or} \qquad i_1=\frac{1}{3R}\left(V-\frac{2Q}{C}\right)$$

$$\text{or } \frac{dQ}{dt}=\frac{1}{3R}\left(V-\frac{2Q}{C}\right) \quad \text{or} \quad \frac{dQ}{V-\frac{2Q}{C}}=\frac{dt}{3R}$$

$$\text{or } \int_0^Q\frac{dQ}{V-\frac{2Q}{C}}=\int_0^t\frac{dt}{3R}$$

This equation gives $Q=\frac{CV}{2}(1-e^{-2t/3RC})$

b. $i_1=\frac{dQ}{dt}=\frac{V}{3R}e^{-2t/3RC}$

from equation (1) we get:

$$i=\frac{V+i_1R}{2R}=\frac{V+\frac{V}{3}e^{-2t/3RC}}{2R}$$

Current through AB:

$$i_2 = i - i_1 = \frac{V + \frac{V}{3}e^{-2t/3RC}}{2R} - \frac{V}{R}e^{-2t/3RC}$$

$$i_2 = \frac{V}{2R} - \frac{V}{6R}e^{-2t/3RC}$$

$$\Rightarrow \quad i_2 = \frac{V}{2R} \text{ as } t \to \infty$$

Example 5.16 **In the given circuit, the switch S is closed at time $t = 0$. The charge Q on the capacitor at any instant t is given by $Q(t) = Q(1 - e^{-\alpha t})$. Find the value of Q_0 and α in terms of given parameters as shown in the circuit in Fig. 5.145.**

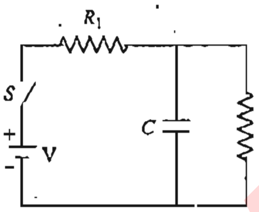


Fig. 5.146

(IIT-JEE, 2005)

Sol. At any instant of time t, the current flowing the loops are given in the Fig. 5.147.

Applying Kirchoff's law in loop $ADEFA$.

$$V - IR_1 - \frac{\theta}{c} = 0 \qquad (1)$$

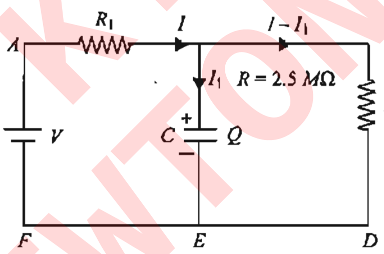


Fig. 5.147

Applying Kirchoff's law in eq. $BCDEB$

$$-\frac{\theta}{C} + (I - I_1)R_2 = 0 \qquad (2)$$

From equation (1)

$$IR_1 = V - \frac{\theta}{C}$$

Substituting in the above values in equation (2)

$$-\frac{\theta}{C} + \frac{R_2}{R_1} = \left(V - \frac{\theta}{C}\right) - R_2\frac{d\theta}{dt} = 0$$

$$\Rightarrow \quad -\frac{\theta}{C} + \frac{VR_2}{R_1} - \theta\frac{R_2}{R_1C} - R_2\frac{d\theta}{dt} = 0$$

$$\Rightarrow \quad \frac{d\theta}{dt} = \frac{V}{R_1} - \frac{\theta}{CR_2}\left[1 + \frac{R_2}{R_1}\right] = \frac{V}{R_1} - \theta\left[\frac{R_1 + R_2}{CR_1R_2}\right]$$

$$\Rightarrow \quad \frac{d\theta}{dt} = \left(\frac{R_1 + R_2}{CR_1R_2}\right)\theta = \frac{V}{R_1}$$

This is a first order linear differential equation whose integrating factor is $e^{\left[\frac{R_1+R_2}{CR_1R_2}t\right]}$. On solving

$$\Rightarrow \quad \theta e\left(\frac{R_1 + R_2}{CR_1R_2}\right)t = \int e\left(\frac{R_1 + R_2}{CR_1R_2}\right)t \times \frac{V}{R_1}dt + C_{10}$$

Given that at $t = 0$, $\theta = 0$

$$\therefore \quad C_1 = \frac{-CVR_2}{R_1 + R_2} \quad \Rightarrow \quad \theta = \frac{CVR_2}{R_1 + R_2}\left[1 - e^{-\left(\frac{R_1+R_2}{R_1+R_2}\right)t}\right]$$

On comparing with $\theta = \theta_0[1 - e^{-\alpha t}]$ we get

$$\theta_0 = \frac{CVR_2}{R_1 + R_2} \text{ and } \alpha = \frac{R_1 + R_2}{CR_1 + R_2}$$

EXERCISES

Subjective Type

Solutions on page 5.61

1. Find the e.m.f.'s e_1 and e_2 in the circuit of Fig. 5.148. Also find the potential difference of point b relative to point a.

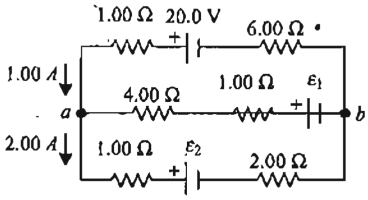
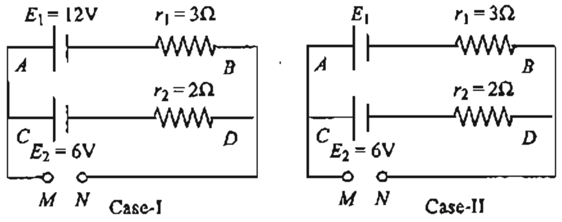


Fig. 5.148

2. What is the potential difference between the points M and N for the circuits shown in Figs. 5.149 (a) and (b) for Case I and Case II.



(a) (b)

Fig. 5.149

3. At room temperature (27 °C) the resistance of a heating element is 100 Ω. What is the temperature of the element if the resistance is found to be 117 Ω? Given that the temperature coefficient of the material of the resistor is $1.70 \times 10^{-4}\,°C^{-1}$.

4. A platinum wire has a resistance of 10 Ω at 0°C and 20 Ω at 273°C. Find the value of temperature coefficient of platinum.

5. A metal wire of diameter 4 mm and length 100 m has a resistance of 0.408 Ω at 10°C and 0.508 Ω at 120°C. Find the value of

- **i.** temperature coefficient of resistance,
- **ii.** its resistance at 0°C, and
- **iii.** its resistivities at 0°C and 120°C.

6. **i.** A car has a fresh storage battery of e.m.f 12 V and internal resistance 5.0×10^{-2} Ω. If the starter motor draws a current of 90 A, what is the terminal voltage of the battery when the starter is on?

ii. After a long use, the internal resistance of the storage increases to 500 Ω. What maximum current can be drawn from the battery? Assume the emf of the battery to remain unchanged.

iii. If the discharged battery is charged by an external e.m.f source, is the terminal voltage of the battery during charging greater or less than its e.m.f 12 V.

7. A storage battery of e.m.f 8.0 V and internal resistance 0.5 Ω is being charged by a 120 V dc supply using a series resistor of 15.5 Ω. What is the terminal voltage of the battery during charging? What is the purpose of having a series resistor in the charging circuit?

8. Find the resistance R_{AB} of the frame made of a thin wire. Assume that the number of successively embedded equilateral triangles (with sides decreasing to half) tends to infinity (see Fig. 5.150).

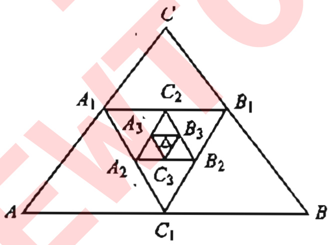


Fig. 5.150

Side *AB* is equal to *a* and the resistance per unit length of wire is *r*.

9. A network consisting of three resistors, three batteries, and a capacitor is shown in Fig. 5.151. Find the charge on the capacitor *C* in a steady state.

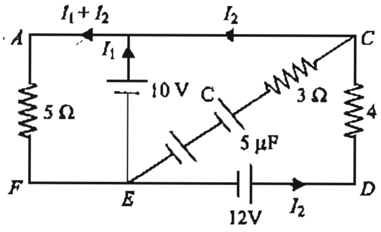


Fig. 5.151

10. Calculate equivalent resistance between *A* and *B* of the circuit shown in Fig. 5.152.

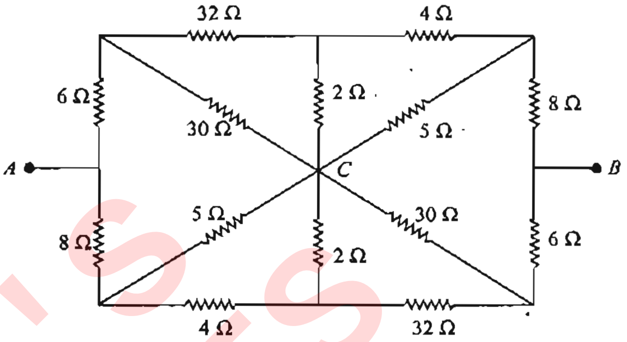


Fig. 5.152

11. The circuit shown in Fig. 5.153 contains three resistors R_1 = 100 Ω, R_2 = 50 Ω, and R_3 = 20 Ω and cells of e.m.f's E_1 = 2 V and E_2. The ammeter indicates a current of 50 mA. Determine the currents in the resistors and the e.m.f of the second cell. The internal resistance of the ammeter and of the cells should be neglected.

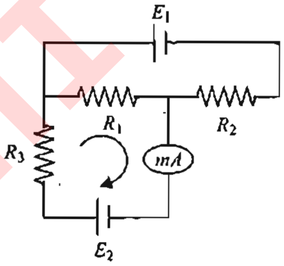


Fig. 5.153

12. In the given circuit of Fig. 5.154 all batteries have e.m.f 10 V and internal resistance negligible. All resistors are in ohms. Calculate the current in the right most 2 Ω resistor.

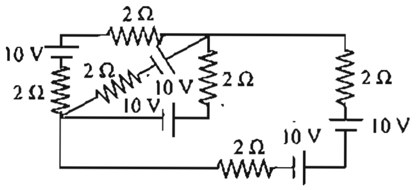


Fig. 5.154

13. In the circuit diagram shown in Fig. 5.155 find the current through the 1 Ω resistor.

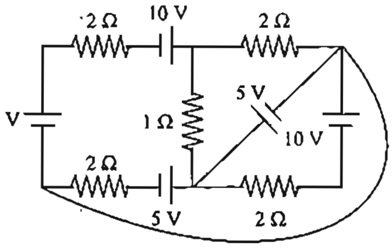


Fig. 5.155

14. The circuit shown in Fig. 5.156 is in steady state.

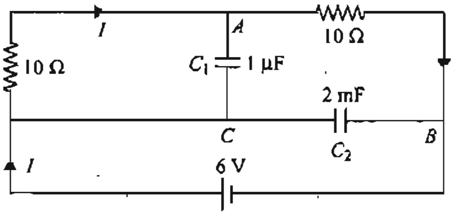


Fig. 5.156

Find the charge on the capacitors C_1 and C_2, respectively.

15. Consider an infinite ladder of network shown in Fig. 5.157. A voltage is applied between points A and B. If the voltage is halved after each section, find the ratio of $\frac{R_1}{R_2}$.

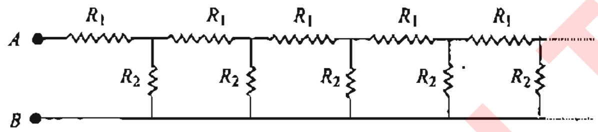


Fig. 5.157

Suggest a method to terminate it after a few sections without introducing much error in its attenuation.

16. For a circuit shown in Fig. 5.158 switch S_1 is closed at $t = 0$, then at $t = (2R_2 + R_1)C$, S_1 is opened and S_2 is closed.

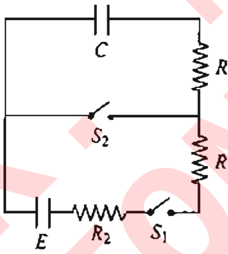


Fig. 5.158

a. find the charge on capacitor at $t = (2R_2 + 2R_1)C$.

b. Find current through R_2 (adjacent to battery) at $t = (3R_1 + 2R_2)C$.

17. Find the potential difference between the plates of the capacitor C in the circuit shown in Fig. 5.159. The internal resistances of sources can be neglected.

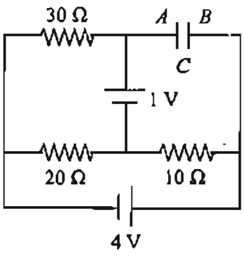


Fig. 5.159

18. Analyze the circuit given in Fig. 5.160 in the steady state condition. Charge on the capacitor in this state is $q_0 = 16\,\mu\text{C}$.

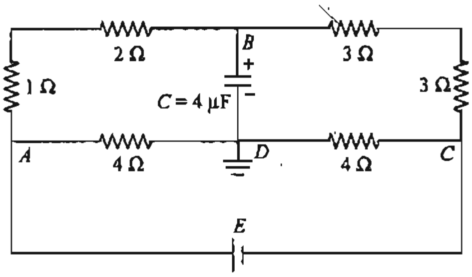


Fig. 5.160

a. Find the current in each branch.

b. Find the e.m.f of the battery.

c. If in the beginning the battery is removed and the nodes A and C are shortened, then find the duration in which charge on the capacitor becomes 5.92 μC.

19. Eleven equal wires each of resistance 2 Ω form the edges of an incomplete skeleton cube. Find the total resistance between points A and B of the vacant edge.

20. i. What is the potential difference between points a and b in Fig. 5.161 when switch S is open ?

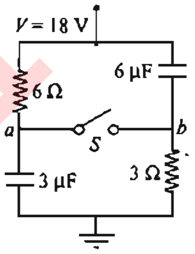


Fig. 5.161

ii. Which point, a or b, is at the higher potential?

iii. What is the final potential of point b when switch S is closed?

iv. How much does the charge on each capacitor change when S is closed?

21. In the circuit shown in Fig. 5.162, C is a parallel plate air capacitor having plate of area $A = 50\text{ cm}^2$ each and distance $d = 1$ mm apart. R_1, R_2, and R_3 are resistors having resistances 3 Ω, 2 Ω, and 1 Ω, respectively. Two identical sources each of e.m.f V and of negligible internal resistance are connected as shown in Fig. 5.162. If a dielectric strength of air is $E_0 = 3 \times 10^6\text{ Vm}^{-1}$, calculate the maximum safe value of V.

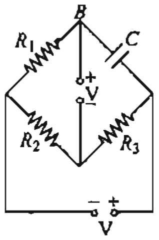


Fig. 5.162

22. The circuit shown in the Fig. 5.163 is in steady state. Calculate

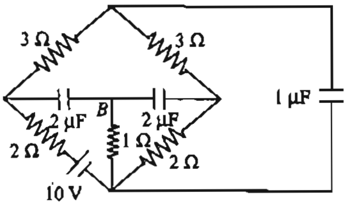


Fig. 5.163

i. energy stored in the capacitors shown in the Fig. 5.163, and

ii. the rate at which battery supplies energy.

23. The given *RC* circuit has two switchs S_1 and S_2. Switch S_2 is closed and S_1 is open till the capacitor is fully charged to q_0. Then S_2 is opened and S_1 is closed simultaneously till the charge on capacitor remains $q_0/2$ for which it takes time t_1. Now S_1 is again opened and S_2 is closed till charge on capacitor becomes $3q_0/4$. It takes time t_2 (see Fig. 5.164 for reference).

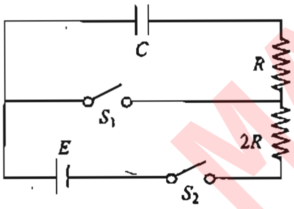


Fig. 5.164

Find the ratio t_1/t_2.

24. For the circuit arrangement shown in Fig. 5.165

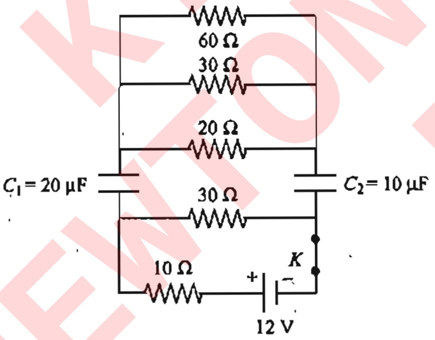


Fig. 5.165

a. Find the potential difference across each capacitor in the steady state condition.

b. Also, find the current through the 60 Ω resistor just after the instant when the key *K* is opened.

25. Find the equivalent resistance between points *A* and *B* of the circuit shown in Fig. 5.166.

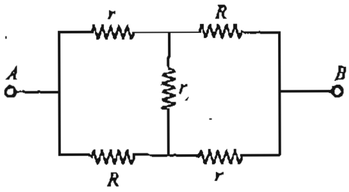


Fig. 5.166

Objective Type

Solutions on page 5.67

1. A battery of internal resistance *r* having no load resistance has an e.m.f *E* volt. What is the observed e.m.f across the terminals of the battery when a load resistance *R* (= *r*) is connected to its terminals ?

a. 2 *E* volt
b. *E* volt
c. $\frac{E}{2}$ volt
d. $\frac{E}{4}$ volt

2. Figure 5.167 represents a load consisting of three identical resistances connected to an electric energy source of e.m.f 12 V and internal resistance 0.6 Ω. The ammeter reads 2 A. The magnitude of each resistance is

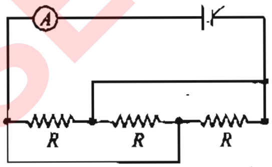


Fig. 5.167

a. 3.6 Ω
b. 7.2 Ω
c. 16.2 Ω
d. 10.8 Ω

3. In the circuit shown in Fig. 5.168, the current *I* has a value equal to

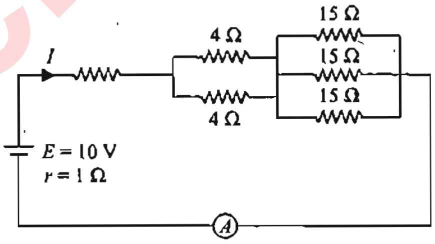


Fig. 5.168

a. 1 A
b. 2 A
c. 4 A
d. 3.5 A

4. Figure 5.169 represents a part of closed circuit. The potential difference ($V_A - V_B$) is

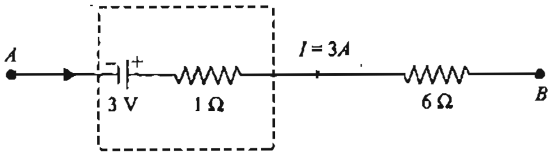


Fig. 5.169

a. 24 V
b. 0 V
c. 6 V
d. 18 V

5. Figures 5.170 and 5.171 show two squares, *X* and *Y*, cut from a sheet of metal, of uniform thickness *t*. *X* and *Y* have sides of length *L* and 2*L*, respectively.

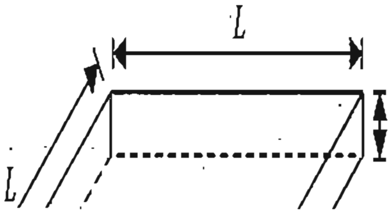


Fig. 5.170

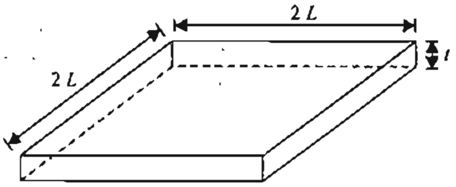


Fig.5.171

The resistance R_x and R_y of the square are measured between the opposite faces shaded in the Figs. 5.170 and 5.171.

What is value of $\frac{R_X}{R_Y}$?

a. 1/4 b. 1/2
c. 1 d. 2

6. In the circuit shown in Fig. 5.172, the magnitudes and the direction of the flow of current, respectively, would be

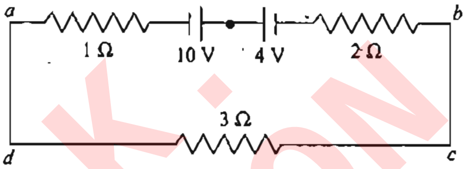


Fig. 5.172

a. $\frac{7}{3}$ A from *a* to *b* via *e* b. $\frac{7}{3}$ A from *b* to *a* via *e*
c. 1 A from *b* to *a* via *e* d. 1 A from *a* to *b* via *e*

7. A cell of e.m.f *E* volt with no internal resistance is connected to a wire whose cross-section changes. The wire has three sections of equal length. The middle section has a radius *a* whereas the radius of the outer two sections is 2*a*. The ratio of the potential difference across the section *AB* to the potential difference across the section *CA* is

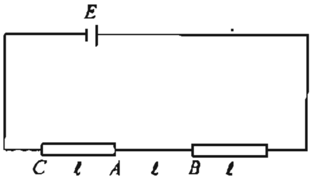


Fig. 5.173

a. 5 b. 4
c. 1/2 c. 1/4

8. The plot represents the flow of current through a wire at three different times. The ratio of charges flowing through the wire at different times is (see Fig. 5.174)

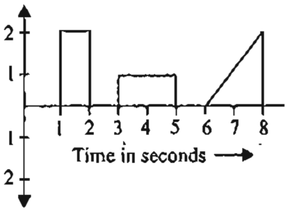


Fig. 5.174

a. 2 : 1 : 2 b. 1 : 3 : 3
c. 1 : 1 : 1 d. 2 : 3 : 4

9. Two cell *A* and *B*, each of e.m.f 2 V, are connected in series to an external resistance *R* = 1 Ω. If the internal resistance of cell is 1.9 Ω and that of *B* is 0.9 Ω, what is the potential difference between the terminals of cell *A*?

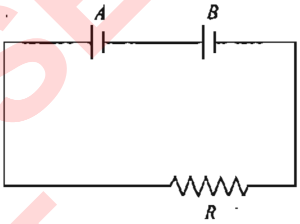


Fig. 5.175

a. 2V b. 3.8 V
c. 0 d. None of the above

10. Two resistors of resistances 200 kΩ and 1 MΩ, respectively, form a potential divider with outer junctions maintained at potentials of + 3 V and – 15 V.

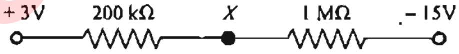
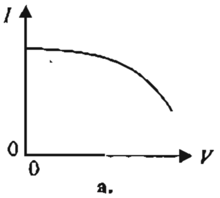


Fig. 5.176

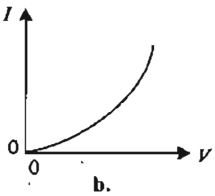
What is the potential at the junction *X* between the resistors?

a. + 1 V b. 0V
c. –0.6 V d. – 12 V

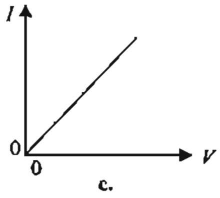
11. Some early electric light bulbs used carbon filaments, the resistances of which decreased as their temperature increased. Which of the following graphs best represents the way in which *I*, the current through such a bulb, would depend upon *V*, the potential differed across it?



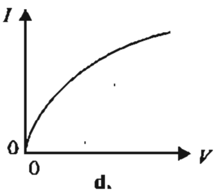
a.



b.



c.



d.

12. A cell is connected to a uniform resistance wire XY and Y is earthed as shown in Fig. 5.177

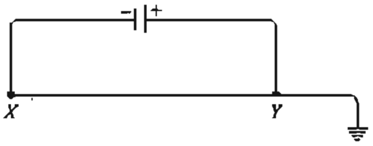
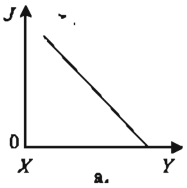
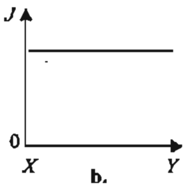


Fig. 5.177

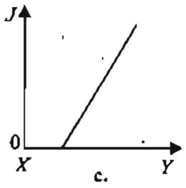
Which one of the options of the graphs show that current density J varies along XY?



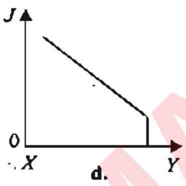
a.



b.



c.



d.

13. The equivalent resistance between A and B in the network in Fig. 5.178 is

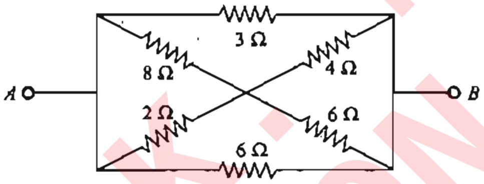


Fig. 5.178

a. $\frac{4}{3}\,\Omega$
b. $\frac{3}{2}\,\Omega$
c. $3\,\Omega$
d. $2\,\Omega$

14. In the circuit shown here in Fig. 5.179, $E_1 = E_2 = E_3 = 2$ V and $R_1 = R_2 = 4\ \Omega$. The current flowing between points A and B through battery E_2 is

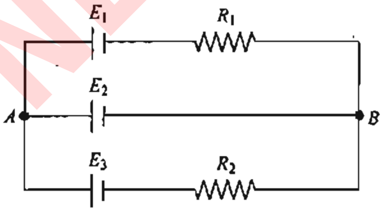


Fig. 5.179

a. zero
b. 2 A from A to B
c. 2 A from B to A
d. None of the above

15. An electric current flows along an insulated strip PQ of a metallic conductor. The current density in the strip varies as shown in the graph of Fig. 5.180.

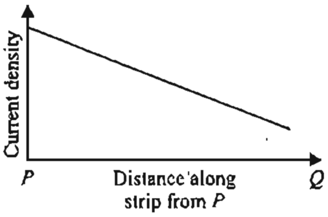


Fig. 5.180

Which one of the following statements could explain this variation ?

a. The strip is narrower at P than at Q.
b. The strip is narrower at q than at P.
c. The potential gradient along the strip is uniform.
d. The resistance per unit length of the strip is constant.

16. Figure 5.181 shows a thick copper rod X and a thin copper wire Y joined in series. They carry a current which is sufficient to make Y much hotter than X.

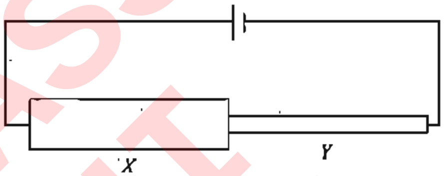


Fig. 5.181

Which one of the following is correct?

Number of density conduction electrons	Mean time between collisions of the electrons
a. Same in X and Y	Less X in than in Y
b. Same in X and Y	Same in X and Y
c. More in X than Y	More in X than in Y
d. More in X than Y	Less in X than in Y
	Same in X and Y

17. If a copper wire is stretched to make it 0.1 % longer. The percentage change in its resistance is

a. 0.2% increase
b. 0.2% decrease
c. 0.1% increase
d. 0.1% decrease

18. The mass of the three wires of copper are in the ratio 1 : 3 : 5. And their length are in ratio 5 : 3 : 1. The ratio of their electrical resistance is

a. 1 : 3 : 5
b. 5 : 3 : 1
c. 1 : 15 : 125
d. 125 : 15 : 1

19. A steady current flows in a metallic conductor of non-uniform cross-section. The quantity/quantities constant along the length of the conductor is/are

a. current, electric field and drift speed
b. drift speed only
c. current and drift speed
d. current only

20. In the part of a circuit shown in Fig. 5.182, the potential difference $(V_G - V_H)$ between points G and H will be

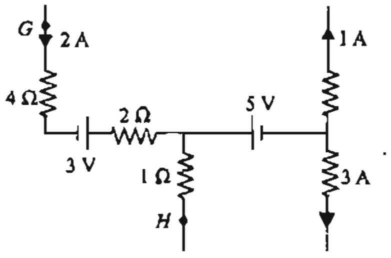


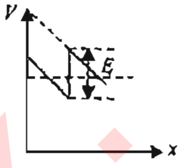
Fig. 5.182

a. 0V b. 15V
c. 7 V d. 3 V

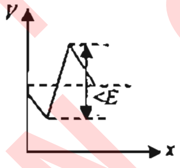
21. Is it possible that a battery has some constant value of e.m.f but the potential difference between the plates is zero?

a. Not possible
b. Yes, if another identical battery is joined in series
c. Yes, possible if another battery is joined in opposition
d. Yes, possible if another similar battery is joined in parallel

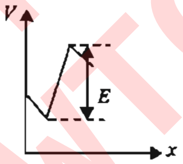
22. The two ends of a uniform conductor with some resistance are joined to a cell of e.m.f E and some internal resistance r. Starting from the midpoint P of the conductor, we move in the direction of current and return to P while moving through the complete circuit. The potential V at every point on the path is plotted against the distance covered (x). Which of the following graphs best represents the resulting curve?



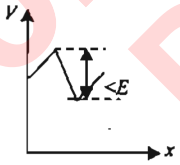
a.



b.



c.



d.

23. In Fig. 5.183 shown, if a battery is connected between points A and B, e.m.f E = 18 V, the current flowing through the battery is

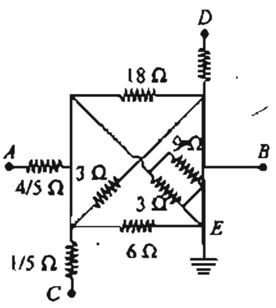


Fig. 5.183

a. 10 A b. 20 A c. 5 A d. 15 A

24. Two square metal plates A and B are of the same thickness and material. The side of B is twice that of A. These are connected as shown in Fig. 5.184 (series connection). If R_A and R_B are the resistances of A and B, respectively, then $\frac{R_A}{R_B}$ is

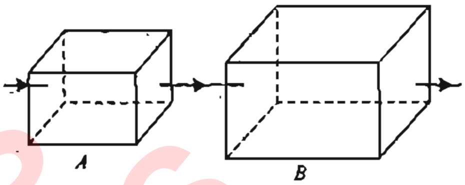


Fig. 5.184

a. 1 : 2 b. 2 : 1
c. 1 : 1 d. 4 : 1

25. In a gas discharge tube, 3×10^{18} electrons are flowing per sec from left to right and 2×10^{18} protons are flowing per second from right to left through a given cross-section. Find the magnitude and the direction of current through the cross section.

a. 0.80 A (Right to Left) b. 0.40 A (Right to Left)
c. 0.80 A (Left to Right) d. 0.40 A (Left to Right)

26. Find out the value of current through 2 Ω resistance for the given circuit in Fig. 5.185.

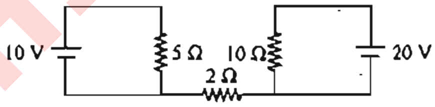


Fig. 5.185

a. zero b. 2 A
c. 5 A d. 4 A

27. The length of a given cylindrical wire is increased by 100%. Due to the consequent decrease in the diameter, the change in the resistance of the wire will be **(AIEEE, 2003)**

a. 300% b. 200%
c. 100% d. 50%

28. The resistance of the series combination of two resistance is S. When they are joined in parallel, the total resistance is P. If $S = nP$, then the minimum possible value of n is **(AIEEE, 2004)**

a. 4 b. 3
c. 2 d. 1

29. An electric current is passed through a circuit containing two wires of the same material, connected in parallel. If the lengths and the radii of the wires are in the ratio of 4/3 and 2/3, then the ratio of the currents passing through the wires will be **(AIEEE, 2004)**

a. 3 b. 1/3
c. 8/9 d. 2

30. The Kirchhoff's first law (Σi = 0) and second law ($\Sigma iR = \Sigma E$), where the symbols have their usual meanings, are respectively based on **(AIEEE, 2006)**

a. conservation of momentum, conservation of energy
b. conservation of charge, conservation of energy

c. conservation of charge, conservation momentum

d. conservation of energy, conservation of charge.

31. The resistance of a bulb filament is 100 Ω at a temperature of 100°C. If its temperature coefficient of resistance be 0.005 Ω per °C, then its resistance will become 200 Ω at a temperature of **(AIEEE-2006)**

a. 500°C b. 200°C

c. 300°C d. 400°C

32. When a current I is set up in a wire of radius r, then the drift velocity is V_d. If the same current is set up through a wire of radius $2r$, then drift velocity will be

a. $4V_d$ b. $2V_d$

c. $V_d/2$ d. $V_d/4$

33. The resistance of a metallic conductor increases with temperature due to

a. change in carrier density

b. change in the dimensions of the conductor

c. increase in the number of collisions among the carriers

d. increase in the rate of collisions between the carriers and the vibrating atoms of the conduct

34. A straight conductor of uniform cross-section carries a current I. Let s be the specific charge of an electron. The momentum of all the free electrons per unit length of the conductor, due to their drift velocities only, is

a. Is b. I/s

c. $\sqrt{I/s}$ d. I/s^2

35. Current flows through a metallic conductor whose area of cross-section increases in the direction of the current. If we move in this direction then,

a. the current will change

b. the carrier density will change

c. the drift velocity will increase

d. the drift velocity will decrease

36. A no conducting ring of radius R has charge Q distributed unevenly over it. If it rotates with an angular velocity ω, the equivalent current will be

a. 0 b. $Q\omega$

c. $Q\dfrac{\omega}{2\pi}$ d. $Q\dfrac{\omega}{2\pi R}$

37. All the edges of a block with parallel faces are unequal. Its longest edge is twice its shortest edge. The ratio of the maximum to minimum resistance between parallel faces is

a. 2 b. 4 c. 8

d. Indeterminate unless the length of the third edge is specified.

38. The e.m.f of a cell is ε and its internal resistance is r. Its terminals are connected to a resistance R. The potential difference between the terminals is 1.6 V for $R = 4\,\Omega$, and 1.8 V for $R = 9\,\Omega$. Then,

a. $\varepsilon = 1$ V, $r = 1\,\Omega$ b. $\varepsilon = 2$ V, $r = 1\,\Omega$

c. $\varepsilon = 2$ V, $r = 2\,\Omega$ d. $\varepsilon = 2.5$ V, $r = 0.5\,\Omega$

39. N identical cells are connected to form a battery. When the terminals of the battery are joined directly (short-circulated), current I flows in the circuit. To obtain the maximum value of I,

a. All the cells should be joined in series.

b. All the cells should be joined in parallel.

c. Two rows of $N/2$ cells each should be joined in parallel.

d. $\sqrt{N}$ rows of $\sqrt{N}$ cells each should be joined in parallel, given that $\sqrt{N}$ is an integer.

40. n identical cells, each of e.m.f ε and internal resistance r, are joined in series to form a closed circuit. The potential difference across any one cell is

a. zero b. ε

c. $\dfrac{\varepsilon}{n}$ d. $\dfrac{n-1}{n}\varepsilon$

41. n identical cells, each of e.m.f ε and internal resistance r, are joined in series to form a closed circuit. One cell A is joined with reversed polarity. The potential difference across each cell, except A, is

a. $\dfrac{2\varepsilon}{n}$ b. $\dfrac{n-1}{n}\varepsilon$

c. $\dfrac{n-2}{n}\varepsilon$ d. $\dfrac{2n}{n-2}\varepsilon$

42. In question 41 , the potential difference across A is

a. $\dfrac{2\varepsilon}{n}$ b. $\varepsilon\left(1-\dfrac{1}{n}\right)$

c. $2\varepsilon\left(1-\dfrac{1}{n}\right)$ d. $\varepsilon\left(\dfrac{n-2}{n}\right)$

43. A potential divider is used to give outputs of 2 V and 3 V from a 5 V source, as shown in Fig. 5.186.

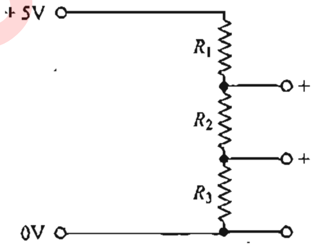


Fig. 5.186

Which combination of resistances, R_1, R_2, and R_3 gives the correct voltages?

	R_1 kΩ	R_2 kΩ	R_3 kΩ
a	1	1	2
b	2	1	2
c	3	2	2
d	3	2	3

44. Five resistors are connected between points A and B as shown in Fig. 5.187. A current of 10 A flows from A to B. Which of the following is correct ?

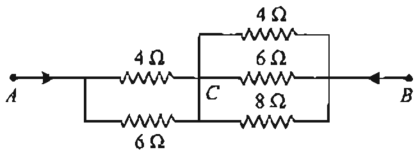


Fig. 5.187

a. $V_{AC} = V_{CB}$ b. $V_{AC} > V_{CB}$
c. $V_{AC} < V_{CB}$ d. $V_{CB} = 24$ V

45. Figure 5.188 shows a potential divider circuit which, by adjustment of the position of the contact *X*, can be used to provide a variable potential difference between the terminals *p* and *Q*. What are the limits of this potential difference ?

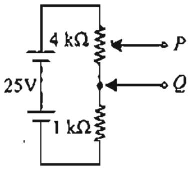


Fig. 5.188

a. 0 and 20 mV b. 5 mV and 25 mV
c. 0 and 20 V d. 0 and 25 V

46. The current through the 8 Ω resistor (shown in Fig. 5.189) is

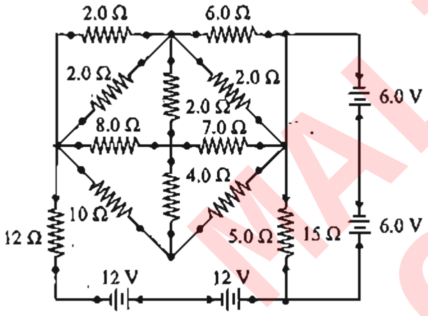


Fig. 5.189

a. 4 A b. 2 A c. zero d. 2.5 A

47. In the network shown in Fig. 5.190, the potential difference across *A* and *B* is

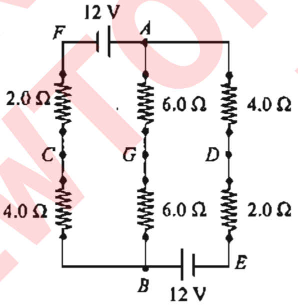


Fig. 5.190

a. 6V b. zero c. 2V d. 4V

48. What resistor should be connected in parallel with the 20 Ω resistor in branch *ADC* in the circuit shown in Fig. 5.191 so that potential difference between *B* and *D* may be zero

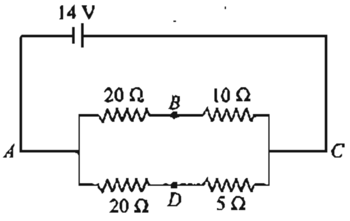


Fig. 5.191

a. 20 Ω b. 10 Ω
c. 5 Ω c. 15 Ω

49. Three resistors are connected as shows in Fig. 5.192. the points *X* and *Y* are connected to a source of direct current.

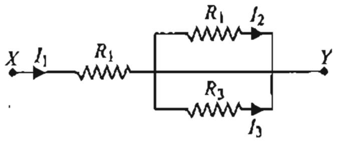


Fig. 5.192

The ratio I_1/I_3 is

a. $\dfrac{R_3 + R_1}{R_1}$ b. $\dfrac{R_2 + R_1}{R_1}$ c. $\dfrac{R_2 R_3}{R_1(R_2 + R_3)}$

d. Dependent on the internal resistance of the source and independent of R_1.

50. Find out the value of resistance *R* in Fig. 5.193.

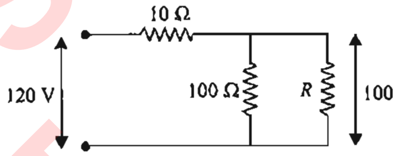


Fig. 5.193

a. 100 Ω b. 200 Ω
c. 50 Ω d. 150 Ω

51. Three resistances are joined together to form a letter *Y*, as shown in Fig. 5.194. If the potentials of the terminals *A*, *B*, and *C* are V_1, V_2, and V_3, respectively, then determine the potential of the node *O*.

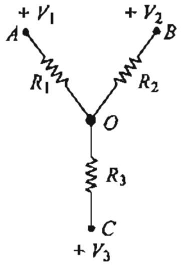


Fig. 5.194

a. $\left[\dfrac{V_1}{R_1^2} + \dfrac{V_2}{R_2^2} + \dfrac{V_3}{R_3^2}\right]\left[\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}\right]^{-2}$

b. $\left[\dfrac{V_1}{R_1} + \dfrac{V_2}{R_2} + \dfrac{V_3}{R_3}\right]\left[\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}\right]^{-1}$

c. $\left[\dfrac{V_1}{R_1} + \dfrac{V_2}{R_2} + \dfrac{V_3}{R_3}\right][R_1 + R_2 + R_3]$

d. $\left[\dfrac{V_1}{R_1^2} + \dfrac{V_2}{R_2^2} + \dfrac{V_3}{R_3^2}\right][R_1^2 + R_2^2 + R_3^2]$

52. In Fig. 5.195, the value of resistors to the connected between *C* and *D*, so that the resistance of the entire circuit between *A* and *B* does not change with the number of elementary sets and is

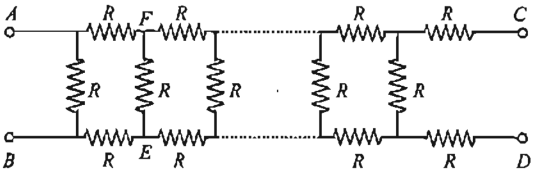


Fig. 5.195

a. R b. $R(\sqrt{3}-1)$ c. $3R$ d. $R(\sqrt{3}+1)$

53. A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raised by ΔT in a time t. A number N of similar cells is now connected in series with a wire of the same material and cross-section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t, the value of N is

a. 4 b. 6 c. 8 d. 9

54. The effective resistance between point P and Q of the electrical circuit shown in Fig. 5.196 is

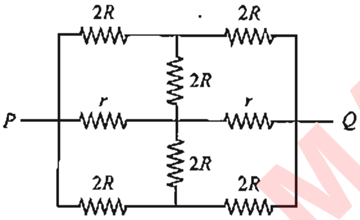


Fig. 5.196

a. $\dfrac{2Rr}{R+r}$ b. $\dfrac{8R(R+r)}{3R+r}$ c. $2r+4R$ d. $\dfrac{5R}{2}+2r$

55. Figure 5.197 shows a wheatstone bridge circuit. Which of the following correctly shows the currents I_1, I_2, and I_3 in the correct decreasing order of magnitude?

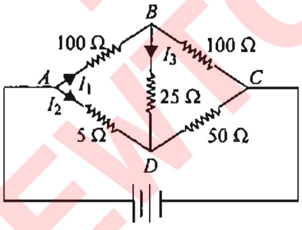


Fig. 5.197

a. I_1, I_2, I_3 b. I_2, I_3, I_1 c. I_2, I_1, I_3 d. I_3, I_2, I_1

56. Figure 5.198 below shows an unbalanced wheatstone bridge. What is the direction of conventional current between B and D?

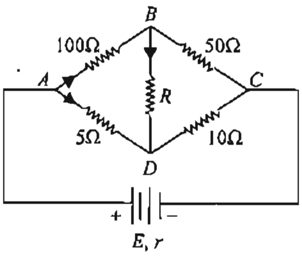


Fig. 5.198

a. B to D b. D to B

c. Depends on the value of emf E of the cell

d. Depends on the internal resistance of the cell

57. For a cell, a graph is plotted between the potential difference V across the terminals of the cell and the current I drawn from the cell (see Fig. 5.199). The e.m.f and internal resistance of the cell is E and r, respectively.

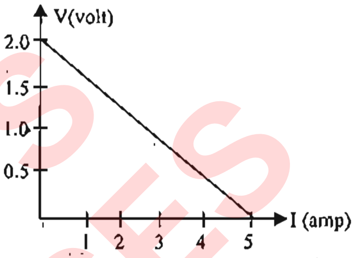


Fig. 5.199

a. $E=2\text{V}, r=0.5\,\Omega$ b. $E=2\text{V}, r=0.4\,\Omega$

c. $E>2\text{V}, r=0.5\,\Omega$ d. $E>2\text{V}, r=0.4\,\Omega$

58. A $1\,\mu\text{F}$ capacitor holding a charge of 1×10^{-5} C is connected to a $10\,\Omega$ resistor via a switch.

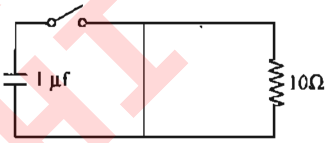


Fig. 5.200

What current will flow after the switch is closed?

a. 0 b. 10^{-5} A

c. 1 A d. 10 A

59. In the given circuit in Fig. 5.201, with steady current, the potential drop across the capacitor must be

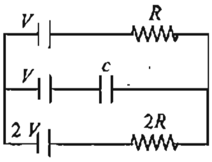


Fig. 5.201

a. V b. $V/2$

c. $V/3$ d. $2V/3$

60. In the given circuit of Fig. 5.202, with steady current, the potential drop across the capacitor must be

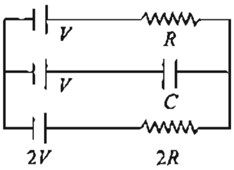


Fig. 5.202

a. V b. $V/2$

c. $V/3$ d. $2V/3$

61. The capacitive time constant of the RC circuit shown in Fig. 5.203.

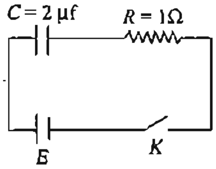


Fig. 5.203

a. Zero b. Infinity
c. 2 s d. 2 μs

62. A capacitor is charged to certain potential difference and then discharged through a resistor R. It takes 2μs for current through take 4μs for current to become half its initial value, if

a. C is doubled b. R is doubled
c. either [a] or [b] d. both R and C are doubled

63. For the arrangement shown in Fig. 5.204, the switch is closed at $t = 0$. The time after which the current becomes 2.5μA is given by ($\mu_2 = 0.69$)

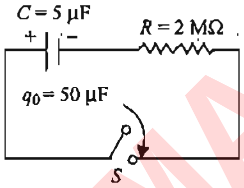


Fig. 5.204

a. 10 s b. 5 s
c. 7 s d. 0.693

64. A capacitor discharges through a resistance. The stored energy μ_0 in one capacitive time constant falls to

a. u_0/e^2 b. eu_0
c. u_0/e d. None of these

65. When the switch is closed, then initial current through 1Ω resistor is (see Fig. 5.205)

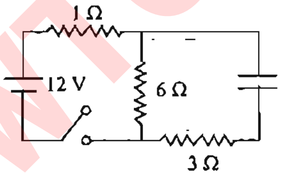


Fig. 5.205

a. 12 A b. 4 A
c. $\frac{10}{7}$ A d. 3 A

66. A capacitor C is connected to the two equal resistances as shown in Fig. 5.206. What is the ratio of time constant during charging and discharging of the capacitance?

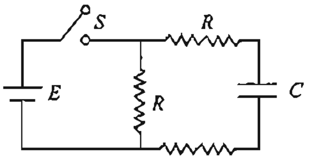


Fig. 5.206

a. 1:1 b. 2 : 1
c. 1 : 2 d. 4 : 1

67. Current through the battery, at instant when the switch S is closed is (see Fig. 5.207)

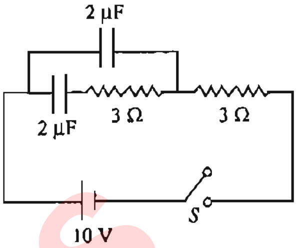


Fig. 5.207

a. zero b. 2A
c. 4A d. 5A

68. When the switch is closed, then final charge on the 3μF capacitor in the stead state is (see Fig. 5.208)

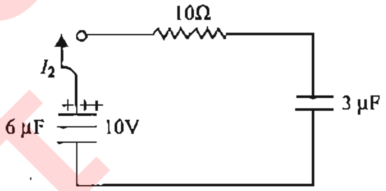


Fig. 5.208

a. 10μC b. 20μC
c. 30 μC d. 40 μC

69. In Fig. 5.209, $r = 10$ Ω and $C = 2$ μF. The value of the steady state current I is

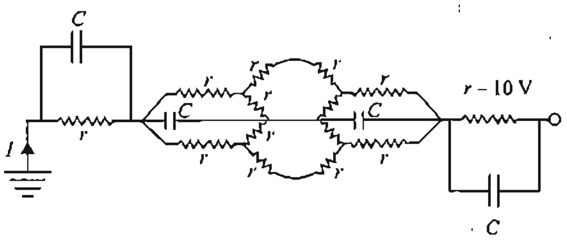


Fig. 5.209

a. 2 A b. 1 A
c. 0 d. None of these

70. Find equivalent resistance between points A and B in the Fig. 5.210 when they are in the steady state.

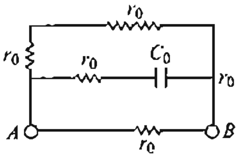


Fig. 5.210

a. $\frac{3}{4}r_0$ b. $\frac{4}{3}r_0$
c. $\frac{5}{3}r_0$ d. None of these

71. The equivalent resistance between the point A and B in Fig. 5.211 at steady state will be

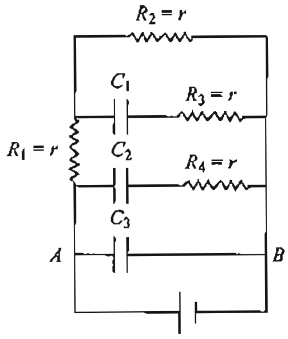


Fig. 5.211

a. $2r$ b. $\frac{3}{5}r$

c. $\frac{5}{3}r$ d. None of these

72. n resistors each of resistance R are joined with a capacitor s of capacity C (each) and a battery of e.m.f E as shown in the Fig. 5.212. In steady state condition, ratio of charges stored in the first and last capacitor is

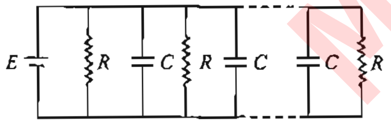


Fig. 5.212

a. $n : 1$ b. $(n-1) : R$

c. $(n^2 + 1) : (n^2 - 1)$ d. 1 : 1

73. At a steady state, the energy stored in capacitor is

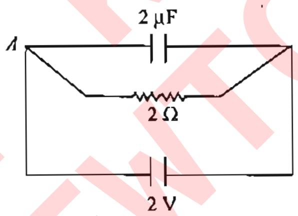


Fig. 5.213

a. 4×10^{-6} J b. 2J

c. 4J d. Zero

74. In the circuit shown in the Fig. 5.214, when the switch is closed, the capacitor charges with a time constant

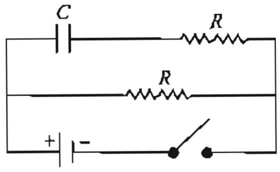


Fig. 5.214

a. RC b. $2RC$

c. $\frac{1}{2}RC$ d. $RC \log 2$

75. In the question 74, if the switch is opened after the capacitor has been charged, it will discharge with a time constant

a. RC b. $2RC$

c. $\frac{1}{2}RC$ d. $RC \ln 2$

76. A capacitor is charged and then made to discharge through a resistance. The time constant is τ. In what time will the potential difference across the capacitor decrease by 10%

a. $\tau \ln (0.1)$ b. $\tau \ln (0.9)$

c. $\tau \ln (10/9)$ d. $\tau \ln (11/10)$

77. In the question 76, after how many time constants will the potential difference across the capacitor fall to 10% of its initial value?

a. 2 b. 2.303

c. $\frac{1}{0.693}$ d. $\frac{1}{0.37}$

78. A capacitor charges from a cell through a resistance. The time constant is τ. In what time will the capacitor collect 10% of its final charge?

a. $\tau \ln (0.1)$ b. $\tau \ln (0.9)$

c. $\tau \ln (10/9)$ d. $\tau \ln (11/10)$

79. In the question 78, after how many time constants will the charge on the capacitor be 10% less then its final charge?

a. 2 b. 2.303

c. $\frac{1}{0.693}$ d. $\frac{1}{0.37}$

80. The charge on a capacitor decrease η times in time t, when it discharges through a circuit with a time constant τ,

a. $t = \eta \tau$ b. $\tau = \tau \ln \eta$

c. $t = \tau(\ln \eta - 1)$ d. $t = \tau \ln \left(1 - \frac{1}{\eta}\right)$

81. What is the charge stored on each capacitor C_1 and C_2 in the circuit show below?

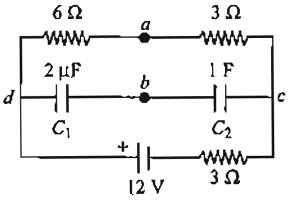


Fig. 5.215

a. 6 μC, 6 μC b. 6 μC, 3 μC

c. 3 μC, 6 μC d. 3 μC, 3 μC

82. In the circuit shown in Fig. 5.216, find the maximum energy stored on the capacitor. Initially the capacitor was uncharged.

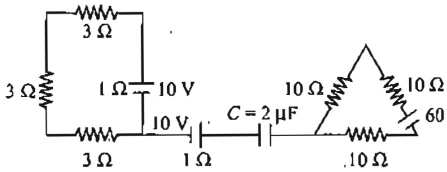


Fig. 5.216

a. 150 μC b. 100 μC c. 50 μC d. zero

83. A charge capacitor is allowed to discharge through a resistor by closing the key at the instant $t = 0$ (see Fig. 5.217). At the instant $t = (\ln 4)\mu$s, the reading of the ammeter falls half the initial value. The resistance of the ammeter is equal to

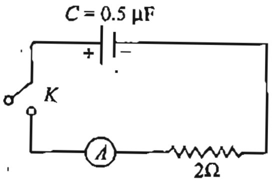


Fig. 5.217

a. 1 MΩ   b. 1 Ω   c. 2 Ω   d. 2 MΩ

84. In the circuit shown in Fig. 5.218, switch S is closed at time $t = 0$, Let I_1 and I_2 be the currents at any finite time t, then ratio I_1/I_2

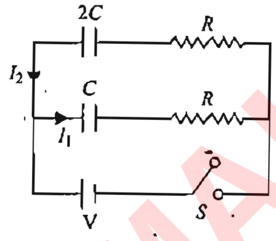


Fig. 5.218

a. is constant
b. increase with time
c. decreases with time
d. first increases, then decreases

85. In the circuit shown in Fig. 5.219, $C_1 = 2C_2$. Capacitor C_1 is charged to a potential of V. The current in the circuit just after the switch S is closed is

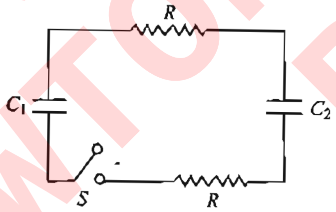


Fig. 5.219

a. 0   b. $2V/R$
c. ∞   d. $V/2R$

86. A capacitor of capacitance 2μF is connected as shown in Fig. 5.220 the internal resistance of the cell is 0.05. The amount of charge on the capacitor plates is

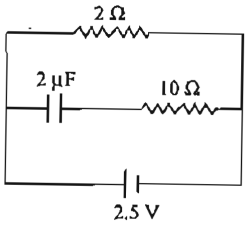


Fig. 5.220

a. Zero   b. 2μC   c. 4μC   d. 6μC

87. In the circuit given in Fig. 5.221 switch S is at position 1 fo long time. Find the total heat generated in resistor o resistance $(2r_0)$, when the switch S is shifted from position to position 2.

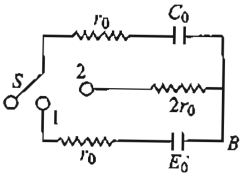


Fig. 5.221

a. $\frac{C_0E_0^2}{2}$ b. $C_0E^2{}_0$ c. $\frac{C_0E_0^2}{3}$ d. None

88. A capacitor of capacitance 3 μF is first charged by connectin it across a 10 V battery by closing key K_1, then it is allowe to get discharged through 2 Ω and 4 Ω resistors by closin the key K_2 (see Fig. 5.222). The total energy dissipated in th 2 Ω resistor is equal to

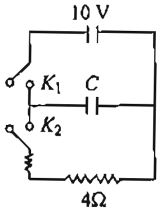


Fig. 5.222

a. 0.5 mJ   b. 0.05 mJ
c. 0.15 mJ   d. None of these

89. In the circuit in Fig. 5.223, if no current flows through th galvanometer when the key k is closed, the bridge balanced. The balancing condition for bridge is

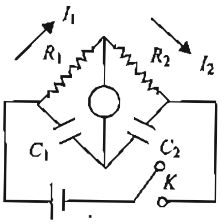


Fig. 5.223

a. $\frac{C_1}{C_2} = \frac{R_1}{R_2}$ b. $\frac{C_1}{C_2} = \frac{R_2}{R_1}$

c. $\frac{C_1^2}{C_2^2} = \frac{R_1^2}{R_2^2}$ d. $\frac{C_2^1}{C_2^2} = \frac{R_2}{R_1}$

90. A capacitor of capacitance C has charge Q. It is connec to an identical capacitor through a resistance. The h produced in the resistance is

a. $\frac{Q^2}{2C}$ b. $\frac{Q^2}{4C}$

c. $\frac{Q^2}{8C}$

d. Dependent on the value of the resistance

91. In the circuit shown in Fig. 5.224, the cell is ideal, with e.m.f = 15 V. Each resistance is of 3 Ω. The potential difference across the capacitor is

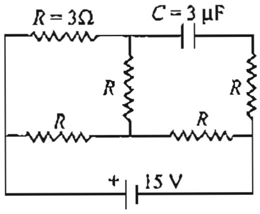


Fig. 5.224

a. 0 **b.** 9V
c. 12 V **d.** 15 V

92. A conductor of area of cross-section A having charge carriers, each having a charge q is subjected to a potential V. The number density of charge carriers in the conductor I n and the charge carriers along with their random motion are moving with a velocity v. A current I flows in the conductor. If $\bar{j}$ is the current density, then

a. $|\vec{j}| = nqV$, in the direction of current flow.

b. $|\vec{j}| = nq\text{v}$, in the direction opposite to current flow.

c. $|\vec{j}| = nqV$, in the direction perpendicular to current flow.

d. $|\vec{j}| = nq\text{v}$, in the direction of current flow.

93. A conductor of area of cross-section A having charge carriers, each having a charge q is subjected to a potential V. The number density of charge carriers in the conductor is n and the charge carriers (along with their random motion) are moving with a velocity v. If s is the conductivity of the conductor and τ is the average relaxation time, then

a. $\tau = \frac{m}{nq^2\sigma}$ **b.** $\tau = \frac{m\sigma}{nq^2}$

c. $\tau = \frac{2m\sigma}{nq^2}$ **d.** $\tau = \frac{1}{2}\frac{m\sigma}{nq^2}$

94. The temperature coefficient of resistance of conductor varies as a $T = 3T^2 + 2T$. If R_0 is resistance at $T = 0$ and R be resistance at T then

a. $R = R_0(6T + 2)$ **b.** $R = 2R_0(3 + 2T)$
c. $R = R_0(1 + T^2 + T^3)$ **d.** $R = R_0(1 - T + T^2 + T^3)$

95. A straight conductor of uniform cross-section carries a time varying current which varies at the rate $\frac{dI}{dt} = I$. If s is the specific charge that is carried by each charge carrier of the conductor and ℓ is the length of the conductor then the total force experienced by all the charge carriers per unit length of the conductor due to their drift velocities only is

a. $F = \dot{I}s$ **b.** $F = \frac{\dot{I}}{2\sqrt{Is}}$

c. $F = \frac{\dot{I}}{s}$ **d.** $F = \frac{2I\dot{I}}{s}$

96. A block of metal is made in the cuboid form with all edges of unequal length. The shortest length is one-third the longest one. If R_{max} and R_{min} are the maximum and minimum resistance between parallel faces then,

a. $\frac{R_{max}}{R_{min}} = 4$ **b.** $\frac{R_{max}}{R_{min}} = 9$

c. $\frac{R_{max}}{R_{min}} = 3$ **d.** Data insufficient

97. Sixteen resistors each of resistance 16Ω are connected in the circuit as shown in Fig. 5.225. The net resistance between AB is

a. 1 Ω **b.** 2 Ω
c. 3 Ω **d.** 4 Ω

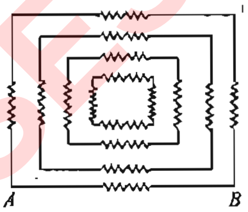


Fig. 5.225

98. The circuit diagram shown in the Fig. 5.226 consists of a large number of elements (each element has two resistors R_1 an R_2). The resistance of the resistors in each subsequent element differs by a factor of $k = \frac{1}{2}$ from the resistances of the resistors in the previous elements. The equivalent resistance between A and B shown in Fig. 5.226 is

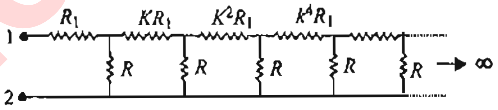


Fig. 5.226

a. $\frac{R_1 - R_2}{2}$ **b.** $\frac{(R_1 - R_2) + \sqrt{6R_1R_2}}{2}$

c. $\frac{(R_1 - R_2) + \sqrt{R_1^2 + R_2^2 + 6R_1R_2}}{2}$

d. None of these

99. The resistance of all the wires between any two adjacent dots is R. The equivalent resistance between A and B as shown in Fig. 5.227 is

a. $\frac{7}{3}R$ **b.** $\frac{7}{6}R$

c. $\frac{14}{8}R$ **d.** None of these

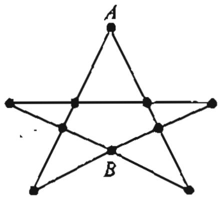


Fig. 5.227

100. There is an infinite wire grid with cells in the form of equilateral triangles. The resistance of each wire between neighboring joint connections is R_0. The net resistance of the whole grid between the points A and B as shown in Fig. 5.228 is

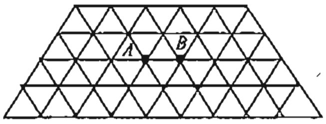


Fig. 5.228

a. R_0 b. $\frac{R_0}{2}$ c. $\frac{R_0}{3}$ d. $\frac{R_0}{4}$

101. The equivalent resistance between A and B (of the circuit as shown in Fig. 5.229) is

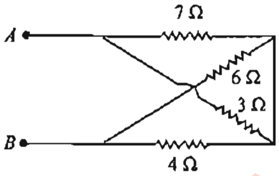


Fig. 5.229

a. 4.5 Ω b. 12 Ω c. 5.4 Ω d. 20 Ω

102. For the circuit shown in Fig. 5.230 the equivalent resistance between A and C is

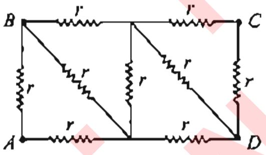


Fig. 5.230

a. $\frac{12}{11}r$ b. $\frac{13}{11}r$ c. $\frac{14}{11}r$ d. $\frac{15}{11}r$

103. In a series RC circuit a steady state charge of 10μC is established in time of 10 ms. If 1 ms is the time constant of the circuit and 3μC is the charge at any instant for the growth part then the decay charge in the circuit is

a. $q_d = 3\mu C$ b. $q_d = 7\mu C$
c. $q_d = -3\mu C$ d. $q_d = -7\mu C$

104. $ABCD$ is square (see Fig. 5.231) where each side is a uniform wire of resistance 1Ω. A point E lies on CD such that if a uniform wire of resistance 1Ω is connected across AE and constant potential difference is applied across A and C then B and E are equipotential.

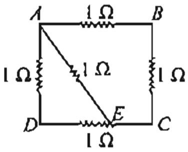


Fig. 5.231

a. $\frac{CE}{ED} = 1$ b. $\frac{CE}{ED} = 2$
c. $\frac{CE}{ED} = \frac{1}{\sqrt{2}}$ d. $\frac{CE}{ED} = \sqrt{2}$

105. In the circuit shown in Fig. 5.232 each battery is 5 V and has an internal resistance of 0.2 Ω. The reading of the voltmeter is V. Then V equals.

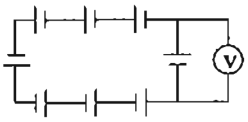


Fig. 5.232

a. 5V b. 10V c. 15V d. Zero

Multiple Correct Answer Type

Solutions on page 5.76

1. In the network shown in Fig. 5.233, points A, B, and C are at potentials of 70 V, 0, and 10 V, respectively.

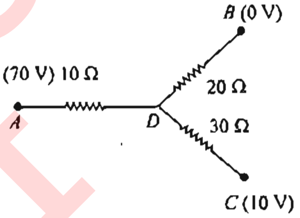


Fig. 5.233

a. Point D is at a potential of 40V.
b. The currents in the sections AD, DB, DC are in the ratio 3 : 2 : 1
c. The currents in the sections AD, DB, DC are in the ratio 1: 2: 3.
d. The network draws a total power of 200 W.

2. When some potential difference is maintained between A and B, current I enters the network at A and leaves at B see Fig. 5.234.

a. The equivalent resistance between A and B is 8 Ω

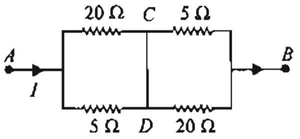


Fig. 5.234

b. C and D are at the same potential.
c. No current flows between C and D
d. Current $3I/5$ flows from D to C.

3. In the circuit shown in Fig. 5.235, the cell has e.m.f = 10V and internal resistance = 1Ω

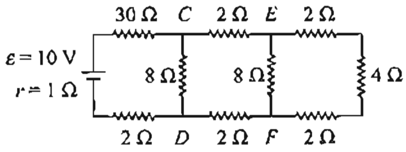


Fig. 5.235

a. The current through the 3 – Ω resistor is 1 A
b. The current through the 3 – Ω resistor is 0.5 A
c. The current through the 4 – Ω resistor is 0.5 A
d. The current through the 4 – Ω resistor is 0.25 A

4. In the circuit shown in the Fig. 5.236, some potential difference is applied between A and B. The equivalent resistance between A and B is R.

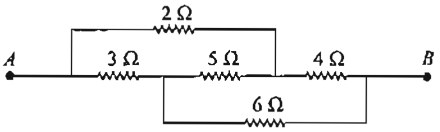


Fig. 5.236

a. No current flows through the 5 – Ω resistor.

b. $R = 15\ \Omega$

c. $R = 12.5\ \Omega$

d. $R = \frac{18}{5}\ \Omega$

5. A battery of e.m.f E and internal resistance r is connected R. Resistance R can be adjusted to any value greater than or equal to zero. A graph is plotted between the current passing through the resistance (I) and potential difference (V) across it. Select the correct alternatives

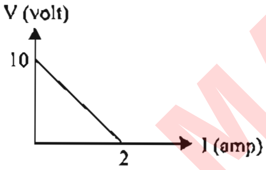


Fig. 5.237

a. internal resistance of the battery is 5Ω

b. emf of the battery is 10 V

c. maximum current which can be taken from the battery is 2 A

d. V-I graph can never be a straight lines as shown in the Fig. 5.237

6. In the given circuit (Fig. 5.238)

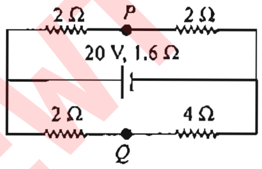


Fig. 5.238

a. The current through the battery is 5 A

b. P and Q are at the same potential

c. P is 2 V higher than Q

d. Q is 2 V higher than P

7. For the batteries shown in Fig. 5.239, R_1, R_2, and R_3 are the internal resistances of E_1, E_2, and E_3, respectively. Then, which of the following is/are correct?

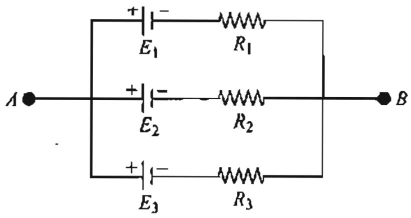


Fig. 5.239

a. Equivalent internal resistance R of the system is given by: $\frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$.

b. If $E_3 = \frac{(E_1R_2 + E_2R_1)}{(R_1 + R_2)}$, equivalent e.m.f of the batteries will be equal to E_3.

c. Equivalent e.m.f of the battery is equal to $E = (E_1 + E_2 + E_3)/3$.

d. Equivalent emf of the battery not only depends upon values of E_1, E_2 and E_3 but depends upon values of R_1, R_2, and R_3 also.

8. A single battery is connected to three resistances as shown in Fig. 5.240.

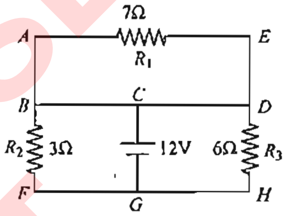


Fig. 5.240

a. The current through 7 Ω resistance is 4 A.

b. The current through 3 Ω resistance is 4 A.

c. The current through 6 Ω resistance is 2 A.

d. The current through 7 Ω resistance is 0.

9. The charge flowing in a conductor varies with time as $Q = at - bt^2$. Then, the current

a. decreases linearly with time

b. reaches a maximum and then decreases

c. falls to zero after time $t = \frac{a}{2b}$

d. changes at a rate $-2b$

10. The potential difference between the points A and B in the circuit shown in Fig. 5.241 is 16 V. Then;

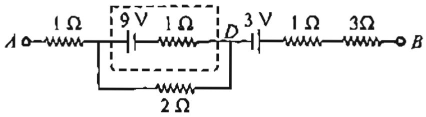


Fig. 5.241

a. the current through the 2 Ω resistance is 3.5 A

b. the current through the 4 Ω resistance is 2.5 A

c. the current through the 3 Ω resistance is 1.5 A

d. the potential difference between the terminals of the 9 V battery is 7 V

11. In the circuit shown in Fig. 5.242, mark the correct options.

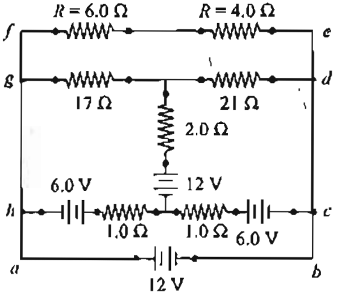


Fig. 5.242

a. Potential drop across R_1 is 3.2 V.
b. Potential drop across R_2 is 5.4 V.
c. Potential drop across R_1 is 7.2 V.
d. Potential drop across R_2 is 4.8 V.

12. In the given circuit (as shown in Fig. 5.243)

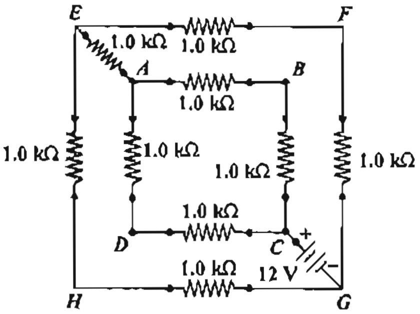


Fig. 5.243

a. The equivalent resistance between C and G is 3 kΩ.
b. The current provided by the source is 4 mA.
c. The current provided by the source is 8 mA.
d. Voltage across points G and E is 4 V.

13. Study the following circuit diagram in Fig. 5.244 and mark the correct options.

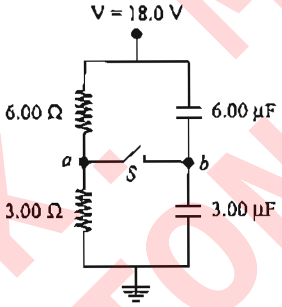


Fig. 5.244

a. The potential of point a with respect to point b in figure when switch S is open is – 6 V.
b. The points a and b, are at the same potential, when S is opened.
c. The charge flows through switch S when it is closed is 54 μC.
d. The final potential of b with respect to ground when switch S is closed is 8 V.

14. The capacitor C is initially without charge. X is now joined to Y for a long time, during which H_1 heat is produced in the resistance R. X is now joined to Z for a long time, during which H_2 heat is produced in (see Fig. 5.245) R.

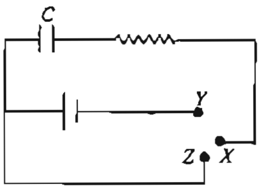


Fig. 5.245

a. $H_1 = H_2$
b. $H_1 = \frac{1}{2} H_2$
c. $H_1 = 2H_2$
d. The maximum energy stored in C at any time is H_1.

15. In the question 14, the energy supplied by the cell during charging is equal to

a. H_1
b. H_2
c. $2H_2$
d. $H_1 + H_2$

16. In the circuit shown in Fig. 5.246, the cell is ideal, with e.m.f = 2V. The resistance of the coil of the galvanometer G is 1 Ω. Then

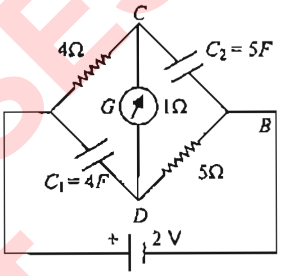


Fig. 5.246

a. no current flows in G
b. 0.2 A current flows in G
c. potential difference across C_1 is 1 V
d. potential difference across C_2 is 1.2 V

17. In the circuit given in Fig. 5.247 the resistances $R_1 = R_2 = R_3 = R_4 = 4$ Ω and $R_5 = R_6 = R_7 = R_8 = 12$ Ω and the capacitors $C_1 = C_2 = C_3 = C_4 = 1$μF and $C_5 = C_6 = C_7 = C_8 = 3$ μF, $C_9 = 5$ μF, are arranged with a battery of e.m.f ε. Point O is earthed

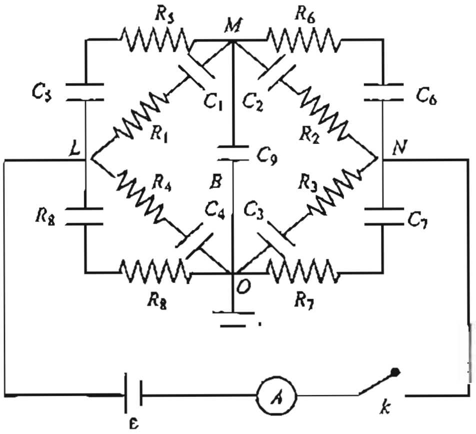


Fig. 5.247

a. The reading of the ammeter long time after closing the switch is 2 A.
b. If just after closing the key the ammeter reads 2A then the value of ε is 6 V.
c. The charge on C_1 capacitor at steady state is 3 μC.
d. The heat developed in the circuit long time after closing the key is 72μJ.

18. A number of resistors R_1, R_2, R_3, ⋯ are connected in series such that R_s is the equivalent resistance of series combination. A current I is flowing in the circuit due to a potential V applied across the circuit. V_1, V_2, V_3, ⋯ are potential across R_1, R_2, R_3, ⋯ respectively.

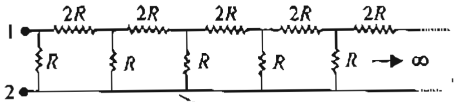
a. Same current I will flow through each resistor.

b. $V_1 + V_2 + V_3 + \cdots = V$

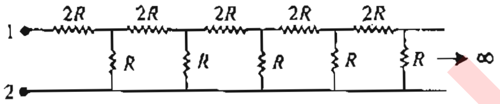
c. $V_1 = \left(\frac{R_1}{R_s}\right)V;\ V_2 = \left(\frac{R_2}{R_s}\right)V;\ V_3 = \left(\frac{R_3}{R_s}\right)V; \cdots$

d. Data insufficient

19. Two circuits (as shown in Fig. 5.248) are called Circuit A and Circuit B. The equivalent resistance of Circuit A is x and that of Circuit B is y between 1 and 2.



Circuit A



Circuit B

Fig. 5.248

a. $y > x$

b. $y = (\sqrt{3} + 1)R$

c. $xy = 2R^2$

d. $x - y = 2R$

20. A cube is made of twelve identical wires each of resistance R_0. The equivalent resistance between the two points lying on the diagonal corners of cube is x and the equivalent resistance between the two opposite corners of a face of cube is y.

a. $\frac{x}{y} = \frac{10}{9}$

b. $x - y = \frac{R_0}{12}$

c. $\frac{x}{y} = \frac{10}{7}$

d. $x - y = \frac{R_0}{4}$

Assertion–Reasoning Type

Solutions on page 5.79

In the following questions, each question contains STATEMENT I (Assertion) and STATEMENT II (Reason). Each question has four choices a, b, c, and d out of which **ONLY ONE** is correct.

a. Statement I is True, Statement II is True ; Statement II is a correct explanation for Statement I.

b. Statement I is True, Statement II is True ; Statement II is NOT a correct explanation for Statement I.

c. Statement I is True, Statement II is False.

d. Statement I is False, Statement II is True.

1. Statement I: When a wire is stretched so that its diameter is halved then its resistance become 16 times.

Statement II: Resistance of wire decrease with increase in length.

2. Statement I: The value of temperature coefficient of resistance is positive for metals.

Statement II: The temperature coefficient of resistance for insulator is also positive.

3. Statement I: When an insulated wire is bent, its resistivity increases

Statement II: On bending, the velocity of electron decreases.

4. Statement I: If the radius of copper wire carrying a current is doubled, then the drift velocity of the electrons will become one fourth.

Statement II: Drift velocity will change according to the relation, $I = neAv_d$.

5. Statement I: A wire of resistance R is bent in the form of a circle. The resistance between two points on circumference of the wire or at the end of diameter is $R/4$.

Statement II: The resistance between the two points on circumference of the circle will be the parallel combination of two resistances of upper and lower parts of the circle.

6. Statement I: The equivalent resistance in series combination is larger than even the largest individual resistance.

Statement II: The equivalent resistance of the parallel combination is smaller than even the smallest resistance.

7. Statement I: If a wire is stretched to increase its length n times then its resistance also become n times.

Statement II: Resistance of the wire is directly proportional to its length.

8. Statement I: Two unequal resistances are connected in parallel across a cell, then current through the smaller resistor is more.

Statement II: More current will flow through a larger resister.

9. Statement I: Two unequal resistances are connected in series across a cell, then potential drop across the larger resistance is more.

Statement II: The current will be same in both unequal resistances.

10. Statement I: A piece of copper and other of germanium are cooled from room temperature to 100 K conductivity of copper increases and that of germanium decreases.

Statement II: Copper has positive temperature coefficient where as germanium has negative temperature coefficient.

11. Statement I: Current flows in a conductor only when there is an electric field within the conductor.

Statement II: The drift velocity of electrons in the presence of electric field decreases.

12. Statement I: If the length of a conductor is doubled, the drift velocity will become half of the original value (keeping potential difference unchanged).

Statement II: At constant potential difference drift velocity is inversely proportional to the length of the conductor.

Comprehension Type

Solutions on page 5.80

For Problems 1 – 3

Find the current supplied by the battery in the circuit in each case (i), (ii), and (iii) as shown in Fig. 5.249.

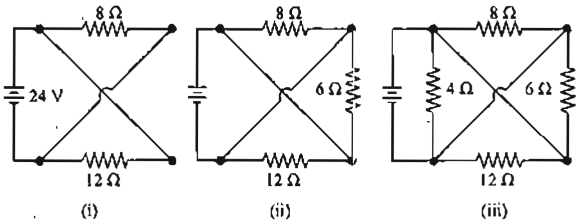


Fig. 5.249

1. a. 4 A b. 3 A c. 12 A d. 5 A
2. a. 9 A b. 4 A c. 12 A d. 3 A
3. a. 5 A b. 15 A c. 10 A d. 25 A

For Problems 4 – 6

Fig. 5.250 shows two ideal voltmeters and an ammeter which are connected across the various circuit elements. If the voltmeter connected across 9 Ω resistance reads 4.5 V, then answer the following problems.

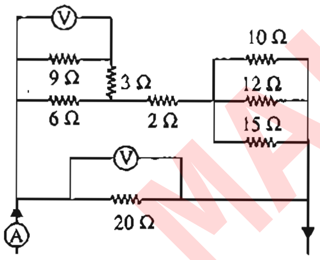


Fig. 5.250

4. The current through 12 Ω resistance is
   a. 0.1 A b. 0.75 A c. 0.5 A d. 1.25 A
5. The reading of the voltmeter connected across 20 Ω resistance
   a. 15 V b. 10 V c. 5 V d. 22.5 V
6. The reading of the ammeter is
   a. 0.5 A b. 2.25 A c. 1.5 A d. 0.1 A

For Problems 7 – 9

A network of resistance is constructed with R_1 and R_2 as shown in Fig. 5.251. The potential at the points 1, 2, 3 ⋯ N are V_1, V_2, V_3, ⋯ V_n, respectively, each having a potential K times smaller than the previous one.

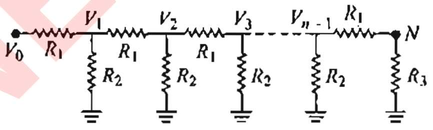


Fig. 5.251

7. The ratio $\frac{R_1}{R_2}$ is
   a. $k^2 - \frac{1}{k}$ b. $\frac{k}{k-1}$
   c. $k - \frac{1}{k^2}$ d. $\frac{(k-1)^2}{k}$
8. The ratio $\frac{R_2}{R_3}$ is
   a. $\frac{(k-1)^2}{k}$ b. $k^2 - \frac{1}{k}$ c. $\frac{k}{k-1}$ d. $k - \frac{1}{k^2}$
9. The current that passes through the resistance R_2 nearest to the V_0 is
   a. $\frac{(k-1)^2}{k}\frac{V_0}{R_3}$ b. $\frac{(k+1)^2}{k}\frac{V_0}{R_3}$
   c. $\left(k + \frac{1}{k^2}\right)\frac{V_0}{R_3}$ d. $\left(k - \frac{1}{k^2}\right)\frac{V_0}{R_3}$

For Problems 10 – 12

Relation between current in conductor and time is shown in Fig. 5.252, then determine:

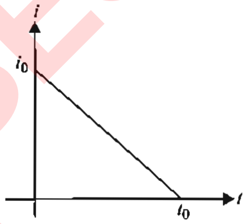


Fig. 5.252

10. Total charge flown through the conductor is
    a. $i_0t_0/2$ b. i_0t_0 c. $i_0t_0/4$ d. $2i_0t_0$
11. Write the expression of current in terms of time.
    a. $i = i_0\frac{t}{t_0}$ b. $i = i_0\left(1 + \frac{t}{t_0}\right)$
    c. $i = i_0\left(\frac{t}{t_0} - 1\right)$ d. $i = i_0\left(1 - \frac{t}{t_0}\right)$
12. If the resistance of conductor is R, then total heat dissipated across resistance R is
    a. $\frac{i_0^2Rt_0}{2}$ b. $\frac{i_0^2Rt_0}{4}$ c. $\frac{i_0^2Rt_0}{3}$ d. $i_0^2Rt_0$

For Problems 13 – 16

Consider the circuit shown in Fig. 5.253.

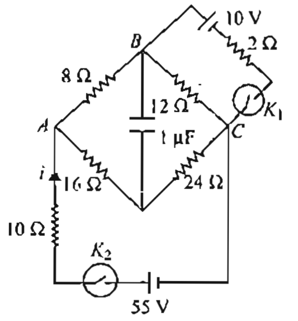


Fig. 5.253

13. Find the current i flowing through the circuit when the key K_1 is open and K_2 is closed.
    a. 27 / 14 A b. 23 / 12 A
    c. 33 / 14 A d. 35 / 11 A
14. Find the net change on the capacitor when K_1 is open
    a. 0 b. 4.8 μC c. 2.4 μC d. 1.8 μC

15. What is the change in the current i, when K_1 is closed ?
a. 1.8 A b. 0.14 A
c. 0.34 A d. 2.3 A

16. The charge of the capacitor when K_1 is closed
a. 7.2 μC b. 9.5 μC
c. 4.8 μC d. 1.2 μC

For Problems 17 – 19

Consider the circuit shown in Fig. 5.254.

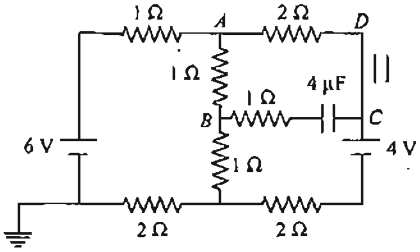


Fig. 5.254

17. The value of i_1 is
a. 7/9 A b. 14/13 A c. 14/3 A d. 17/23 A

18. The potential of point B is
a. 27/34 V b. 46/13 V c. 1/2 V d. 61/49 V

19. The charge in capacitor is
a. 2 μC b. 4 μC c. 6 μC d. 8 μC

Matching Column Type

Solutions on page 5.82

1. A capacitor of capacitance 0.1 μF is connected to a battery of e.m.f 8 V (as shown in Fig. 5.255) under the steady state condition.

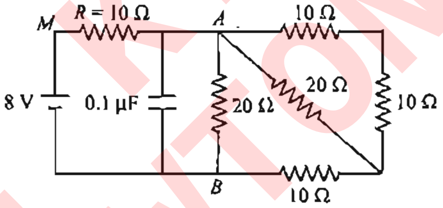


Fig. 5.255

Column I	Column II
i. Charge on the capacitor	a. 0.4 μC
ii. Charge in AC branch	b. 0.2 A
iii. Current in AB branch	c. 0.1 A
iv. Current in R connected between M and N.	d. 0.4 A

2. A network consisting of three resistors, three batteries, and a capacitor is shown in Fig. 5.256.

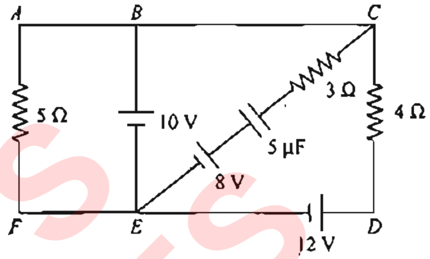


Fig. 5.256

Column I	Column II
i. Current in branch EB is	a. 10 μC
ii. Current in branch CB is	b. 0.5 A
iii. Current in branch ED is	c. 1.5 A
iv. Charge on capacitor is	d. 5 μC

3. A circuit is shown in Fig. 5.257. R is a non zero variable with finite resistance. e is some unknown emf with polarities as shown. Match the columns

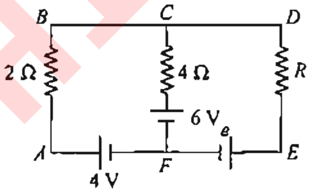


Fig. 5.257

Column I	Column II
i. Current passing through 4 Ω resistance can be zero	a. possible if $e = 6$ V
ii. Current passing through 4 Ω resistance can be from F to C direction	b. possible if $e > 6$ V
iii. Current passing through 4 Ω resistance can be from C to F direction	c. possible if $e < 6$ V
iv. Current passing through 2 Ω resistance will be from B to A direction	d. possible for any value of e from zero to infinity

ANSWERS AND SOLUTIONS

Subjective Type

1. From the given currents in the diagram, the current through the middle branch of the circuit must be 1.00 A (the difference between 2.00 A and 1.00 A). We now use Kirchoff's rules, passing counterclockwise around the top loop:

200 V − (1.00 A) (6.00 Ω + 1.00 Ω) + (1.00 A) (4.00 Ω + 1.00 Ω) − $\varepsilon_1 = 0$

$\Rightarrow \quad \varepsilon_1 = 18.0$ V

Now traveling around the external loop of the circuit:

20.0 V − (1.00 A) (6.00 Ω + 1.00 Ω) − (2.00 A) (1.00 Ω + 2.00 Ω) − $\varepsilon_2 = 0 \quad \Rightarrow \quad \varepsilon_2 = 7.0$ V

$$V_{ab} = -(1.00\text{ A})(4.00\,\Omega + 1.00\,\Omega) + 18.0\text{ V}$$

$$= +13.0\text{ V}$$

So, $\quad V_{ba} = -13.0$ V

2. Case I: Current in the circuit $= \dfrac{12-6}{3+2} = \dfrac{6}{5} = 1.2$ A

$$\therefore \quad V_A - V_B = 12 - 3 \times 1.2 = 12 - 3.6 = 8.4 \text{ V}$$

$$V_c - V_o = 6 + 2 \times 1.2 = 6 + 2.4 = 8.4 \text{ V}$$

Hence $V_m - V_n = 8.4$ V.

Case II: $\dfrac{12+6}{3+2} = \dfrac{18}{5} = 3.6$ A

$$V_A - V_B = 12 - 3.6 \times 3 = 12 - 10.8 = 1.2 \text{ V}$$

$$V_P - V_C = 6 - 3.6 \times 2 = 6 - 7.2 = -1.2 \text{ V}$$

$$V_C - V_D = +1.2 \text{ V}$$

$$\Rightarrow \quad V_m - V_n = 1.2 \text{ V}$$

3. Temperature coefficients of the material is given by

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$$

Here $\quad \alpha = 1.70 \times 10^{-4}\,°\text{C}^{-1}$

$t_1 = 27°\text{C}, R_1 = 100\ \Omega$, and $R_2 = 117\ \Omega$

$$\therefore \quad 1.70 \times 10^{-4} = \frac{177 - 100}{100(t_2 - 27)}$$

$$\text{Or} \quad (t_2 - 27) = \frac{177 - 100}{100 \times 1.70 \times 10^{-4}} = \frac{17}{1.70 \times 10^{-2}} = 1000$$

$$\text{Or} \quad t_2 = 1000 + 27 = 1027\ °\text{C}$$

4.

$$\alpha = \frac{R_t - R_0}{R_0 t} = \frac{R_{20} - R_0}{R_0 \times 273}$$

$$= \frac{20 - 10}{10 \times 273} = \frac{1}{273} = \frac{1}{273}\,°\text{C}^{-1} \text{ or K}^{-1}$$

5. Given that:

$$r = \frac{4}{2} \text{ mm} = 2 \text{ mm} = 2 \times 10^{-3} \text{m}$$

$$\ell = 100 \text{m}, t_1 = 10\ °\text{C}, t_2 = 120\ °\text{C}$$

$$R_{t_1} = 0.408\ \Omega, R_{t_2} = 0.508\ \Omega$$

i. Temperature coefficient of resistance is given by

$$\alpha = \frac{R_{t_1} - R_{t_2}}{1 + \alpha t} = \frac{0.508 - 0.408}{0.408(120 - 10)} = 2.2 \times 10^{-3}\,\text{C}^{-1}$$

ii. We know that, $R_{t_1} = R_0(1 + \alpha t_1)$

$$\text{or} \quad R_0 = \frac{Rt_1}{1 + \alpha t_1} = \frac{0.408}{1 + 2.2 \times 10^{-3} \times 10} = \frac{0.408}{1.022} = 0.4\ \Omega$$

iii. Resistivity at 0 °C is given by

$$\rho_0 = \frac{R_0 A}{\ell} = \frac{R_0 \pi r^2}{\ell}$$

$$= \frac{0.4 \times 3.14 \times \left(2 \times 10^{-3}\right)^2}{100} \Omega\text{m} = 5.02 \times 10^{-8}\ \Omega\text{m}$$

Resistivity at 120°C is given by:

$$\rho_{20} = \rho_0(1 + \alpha t)$$

$$= 5.02 \times 10^{-8}(1 + 2.2 \times 10^{-3} \times 120)\ \Omega\text{m}$$

$$= 5.02 \times 10^{-8} \times 1.264\ \Omega\text{m} = 5.34 \times 10^{-8}\ \Omega\text{m}$$

6. i. Terminal voltage of the battery is given by,

$$V = E - IR$$

Here $\quad E = 12\text{V}, I = 90\text{A}, r = 5.0 \times 10^{-2}\ \Omega$

$$\therefore \quad V = 121 - 90 \times 5.0 \times 10^{-2} = 12 - 4.5 = 7.5 \text{ V}$$

ii. The maximum current can be drawn from the battery by short circuiting it. At that time, $V = 0$, hence

$$E - I_m r = 0 \quad \text{or} \quad I_m = \frac{E}{r} = \frac{12 \text{ V}}{500} = \text{A} = 24 \text{ mA}$$

Obviously, on short-circuiting, the battery will be discharged and will need recharging.

iii. During charge, the current flows in the opposite direction, i.e., from positive to negative terminal inside the cell. Hence during the charging,

$$V = E + Ir \quad \text{or} \quad V > E > 12 \text{ V}$$

This means that the terminal voltage of the battery during charging is greater than its e.m.f 12 V.

7. Given that $E = 8.0$ V, $V = 120$ V, $r = 0.5\ \Omega$, and

$$R = 15.5\ \Omega$$

Current in the circuit during charging is given by

$$I = \frac{\text{Total voltage}}{\text{Total resistance}} = \frac{V - E}{R + r} = \frac{120 - 8}{15.5 + 0.5} = \frac{112}{16}$$

$$= 7\ \Omega$$

During charging, the current flows inside the battery in a direction opposite to that during discharge. Hence the terminal voltage of the battery during charging is

$$= E + Ir = 8 + 7 \times 0.5 = 11.5 \text{ V}$$

A series resistor in the charging circuit limits the current drawn from the external source. In its absence, the current will be dangerously high.

8. Let $R_{AB} = x$ is equivalent resistance of system between A and B. As the resistance of a conductor is directly proportional to length, the equivalent resistance between A_1 and B_1 will be $\dfrac{x}{2}$. Therefore, the equivalent circuit becomes as given below in Fig. 5.258.

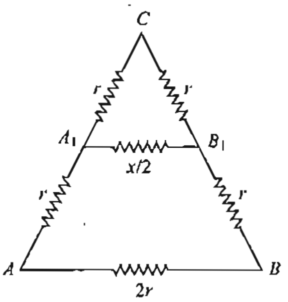


Fig. 5.258

Let $AB = 2r$, then $A_1C = CB_1 = AA_1 = BB_1 = r$.

In the circuit $2r$ and $\dfrac{x}{2}$ are in parallel between A_1 and B_1, then their effective resistance is

$$R_1 = \frac{2r\frac{x}{2}}{2r+\frac{x}{2}} = \frac{2rx}{4r+x}$$

Now R_1 is in series with AA_1 and BB_1, therefore, their effective resistance is

$$R_2 = R_1 + 2r = \frac{2rx}{4r+x} + 2r$$

R_2 is in parallel with $2r$ (of AB), so the net effective resistance across AB is

$$x = \frac{R_2 \times 2r}{R_2 + 2r} = \frac{\left(\frac{2rx}{4r+x}+2r\right)}{\left(\frac{2rx}{4r+x}+2r\right)+2r}$$

$$\Rightarrow \quad 3x^2 + 4rx - 8r^2 = 0 \text{ or } x = \frac{-4r+\sqrt{6r^2+4\times 3\times 8r^2}}{2\times 3}$$

As x cannot be negative

$$\therefore \quad x = \frac{-4r \pm \sqrt{16r^2+96r^2}}{6} = \frac{(2\sqrt{7}-2)}{3}r$$

But $\quad r = \frac{a}{2}\rho$

$$\therefore \quad x = \frac{2\left(\sqrt{7}-1\right)}{3} \times \frac{a}{2}\rho = 0.55\,a\rho$$

9. When a steady state is reached, no current passes through the capacitor and therefore, there is no current in the CE branch of the network.

Considering the loop $ABEFA$, $5(I_1 + I_2) = 10$

or $\quad I_1 + I_2 = 2 \quad$ (i)

Considering the loop $BCDEB$, $4I_2 = 12 - 10 = 2$

$\Rightarrow \quad I_2 = 0.5$ A

$\therefore \quad I_1 = 2 - 0.5 = 1.5$ A

To find the charge on the capacitor, we must know the potential difference across the plates. Consider the closed loop $CEDCE$

$$-12 + 4I_2 + 3\times(0) - V_c + 8 = 0$$

or $\quad -12 + 2 - V_c + 8 = 0 \quad$ or $\quad V_c = -2\text{V}$

The negative sign indicates that the plate of the capacitor nearer to E is negative and the one nearer to C is positive.

$\therefore$ charge on the capacitor,

$$Q = CV$$

$$= 5\mu\text{F} \times 2.0\text{ V} = 10\,\mu\text{C}$$

10. In the given circuit of Fig. 5.259 there is no series or parallel combination of resistances. Therefore, to calculate its equivalent resistance, a battery of voltage V is to be connected across terminals A and B.

If the circuit draws a current I from the battery, its equivalent resistance will be equal to $R = V/I$.

Given that the combination is symmetric about centre C. Therefore, current through various components will be as shown in figure.

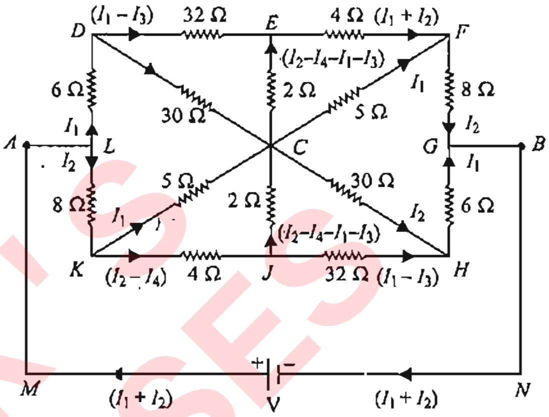


Fig. 5.259

Applying Kirchhoff's voltage law on mesh $DCKLD$

$$30I_3 - 4I_4 - 8I_2 + 6I_1 = 0 \quad \text{(i)}$$

For mesh $DECD$,

$$32(I_2 - I_3) - 2(I_2 - I_4 - I_1 + I_3) - 30I_3 = 0 \quad \text{(ii)}$$

For mesh $KCJK$,

$$5I_4 - 2(I_2 - I_4 - I_1 + I_3) - 4(I_2 - I_4) = 0 \quad \text{(iii)}$$

For mesh $ALKJHGBNMA$,

$$8I_2 + 4(I_2 - I_4) + 32(I_1 - I_3) + 6I_1 - V = 0 \quad \text{(iv)}$$

Solving equations (i), (ii), (iii), and (iv)

$$I_1 = \frac{V}{42}, I_2 = \frac{V}{21}, I_3 = \frac{V}{84}, I_4 = \frac{V}{42}$$

Equivalent resistance,

$$R = \frac{V}{I} = 14\,\Omega$$

But the total current drawn by the circuit from battery is equal to

$$I = I_1 + I_2$$

$$I = \frac{V}{14}$$

$\therefore$ equivalent resistance,

$$R = \frac{V}{I} = 14\,\Omega$$

11. Applying KVL in the loop of $ABCD$

$$E_1 = (I + 0.05)R_1 + IR_2 \quad \Rightarrow \quad I = -20\text{ A}$$

Current through $R_1 = 30$ mA towards right

Current through $R_2 = 20$ mA towards left.

Applying KVL in loop $BGEF$

$$E_2 = (I + 0.05)100 + (0.05)20 =$$

12. The simplified circuit is shown in Fig. 5.260. We have to find i.

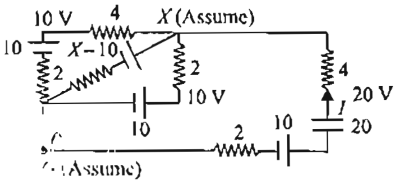


Fig. 5.260

Let the potential of point P be O. Potential at other points are shown in Fig. 5.260. Apply Kirchoff's current law at X_X.

$$\frac{x-10}{4}+\text{[illegible]}+\frac{x-20}{4}+\frac{(x-10)-0}{2}=0$$

$x-10+$ [illegible] $-20+x-20+2x-20=0$

$$\Rightarrow \qquad 6x=70 \quad \Rightarrow \quad x=\frac{35}{3}\text{ V}, I=\frac{20-\frac{35}{3}}{4}=\frac{25}{12}\text{ A}$$

13. $$\frac{(v-10)-10}{2}+\frac{v-0}{2}+\frac{v-5}{1}=0$$

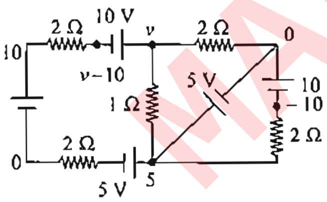


Fig. 5.261

$$\frac{v-20}{2}+\frac{v}{2}+v-5=0$$

$$v-20+v+2(v-5)=0=4v-20-10=0$$

$$v=\frac{30}{4}=\frac{15}{2}$$

$$v-5=\frac{15}{2}-5=\frac{15-10}{2}=\frac{5}{2}$$

$$i=\frac{5/2}{1}=\frac{5}{2}\text{ A}$$

14. In steady state, current will flow as shown in the Fig. 5.262.

$I=6/(10+10)=0.30$ A

P.d. across C_1 is same as that across 10 Ω on left side.

So $\quad V_1=10I=3$ V

Charge on C_1: $q_1=C_1V_1=1\times3=3\ \mu$C

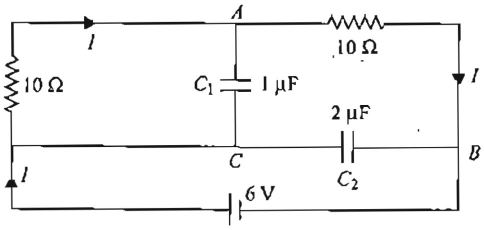


Fig. 5.262

We can see that p.d. across C_2 is 6 V.

So charge on C_2: $q_2=C_2V_2=2\times6=12\ \mu$C

15. Voltage across $AB=V$, Voltage across $A'B'=\dfrac{V}{2}$

i.e., Voltage across $R_2=\dfrac{V}{2}$

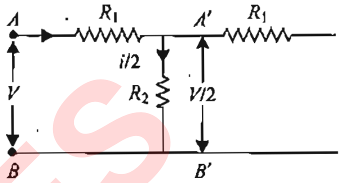


Fig. 5.263

Now from Kirchhoff's law it is obvious that voltage across

$$R_1=V-\frac{V}{2}=\frac{V}{2}$$

When the voltage is halved, current is also halved, i.e., curren in R_2 is half of that in R_1. So $R_1i=R_2\dfrac{i}{2} \Rightarrow \dfrac{R_1}{R_2}=\dfrac{1}{2}$

Now the attenuation produced by the circuit on termination b a resistance will not be affected if equivalent resistance R becomes independent of number of sections in the circuit This is only possible if the terminating resistance R_0 is itsel equal to equivalent resistance (Fig. 5.263). The equivalent resistance of R_0 and R_2 is $R'=\dfrac{R_0R_2}{R_0+R_2}$.

R_1 is in series with it, so equivalent resistance between A and B is $R_1+R'=R_1+\dfrac{R_0R_2}{R_0+R_2}$

According to proposition $R_0=R_1+\dfrac{R_0R_2}{R_0+R_2}$

Solving for R_0, we get $R_0=\dfrac{R_1}{2}\left[1+\sqrt{\left(1+\dfrac{4R_2}{R_1}\right)}\right]$

Thus the circuit may be terminated after a few sections if resistance R_0 is connected in parallel as shown in Fig. 5.263.

16. a. For $t=0$ to $t=(2R_2+R_1)$C, capacitor gets charged from all the three resistors. So

$$q=CE\left[1-e^{-\frac{t}{(2R_2+R_1)C}}\right], \text{ put } t=(2R_2+R_1)\text{ C}$$

$$q_1=CE[1-e^{-1}]=\frac{CE(e-1)}{e}$$

Now battery is disconnected and the capacitor gets discharged through R_1 after S_1 is opened and S_2 is closed. So

$$\int_{q_1}^{q_2}\frac{dq}{q}=-\int_{(2R_2+R_1)C}^{(2R_2+2R_1)C}\frac{dt}{R_1C}$$

$$\Rightarrow \quad q_2=\frac{q_1}{e}=\frac{CE(e-1)}{e^2}$$

b. Since battery is disconnected at this time, so there is no current in R_2.

17. The current distribution is indicated in Fig. 5.264 below. When the condenser has been fully charged there will be no current in this branch.

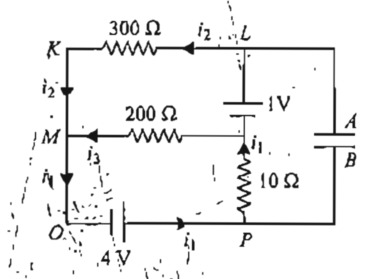


Fig. 5.264

Applying Kirchhoff's law to meshes *NLKMN* and *PNMOP*:

$$30i_2 - 20i_3 = 1 \quad \text{(i)}$$

$$10i_1 + 20i_3 = 4 \quad \text{(ii)}$$

Applying Kirchoff's first law of junction *M*

$$i_2 + i_3 = i_1 \quad \text{(iii)}$$

Solving equations (i), (ii), and (iii), we get

$$i_2 = i_3 = 0.1\,\text{A};\ i_1 = 0.2\,\text{A}$$

Considering mesh *PBALP*, we have

$$V_{AB} - 1 = -10i_1 = -10 \times 0.2 = -2$$

$$V_{AB} = -2 + 1 = -1\,\text{V}$$

18. For part (a) and (b)

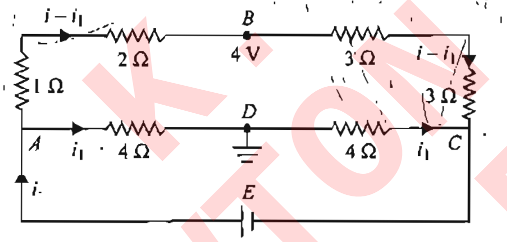


Fig. 5.265 (a)

For *BAD*: $V_B + 2(i - i_1) + 1(i - i_1) - 4i_1 = V_D$

$\Rightarrow \quad 3i - 7i_1 = -4$

For *BCD*: $6i - 10i_1 = 4$

$\Rightarrow \quad i_1 = 3\,\text{A},\ i = 17/3\,\text{A}$

Current in $AC = i_1 = 3\text{A}$, In $ABC = i - i_1 = 8/3$ A

To find *E*: $\quad E = 8i_1 = 24\,\text{V}$

$$i = 2i_3 = i_1 + i_2 \quad \text{(i)}$$

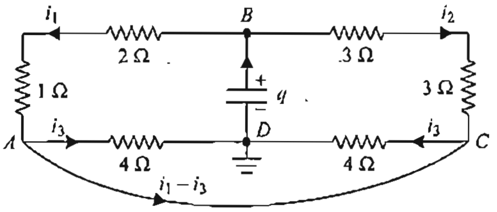


Fig. 5.265 (b)

$$\frac{q}{C} = 2i_1 + i_1 + 4i_3 \Rightarrow \frac{q}{C} = 3i_1 + 4i_3 \quad \text{(ii)}$$

Also $\quad \dfrac{q}{C} = 3i_2 + 3i_2 + 4i_3 \Rightarrow \dfrac{q}{C} = 6i_2 + 4i_3 \quad$ (iii)

From equations (ii) and (iii)

$$\frac{q}{C} = 3i_1 + 2i$$

From equations (i) and (iii)

$$\frac{q}{2C} = 3i_2 + i$$

$$\Rightarrow \frac{q}{C} + \frac{q}{2C} = 3(i_1 + i_2) + 3i \Rightarrow \frac{3q}{2C} = 6i$$

$$\Rightarrow \quad i = \frac{q}{4C} \Rightarrow \frac{-dq}{dt} = \frac{q}{4C} \Rightarrow q = q_0 e^{-t/4C}$$

$$\Rightarrow \quad 5.92 = 16e^{-\frac{t}{4\times4\times15^{-6}}} \Rightarrow t = 16\,\mu\text{s}$$

19. Let a battery of emf *E* is applied between the points *A* and *B*. Let a current *I* enters through point *A*. If R_{AB} is equivalent resistance between the points *A* and *B*, then from Ohm's law

$$R_{AB} I = E'. \quad \text{(i)}$$

The distribution of currents, keeping in mind symmetry condition, is shown in Fig. 5.266.

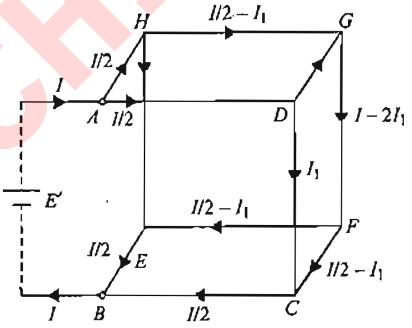


Fig. 5.266

Let R (= 2 Ω) be the resistance of each wire.

Applying Kirchhoff's second law to mesh *DGFC*, we get

$$\left(\frac{I}{2} - I_1\right)R + (I - 2I_1)R + \left(\frac{I}{2} - I_1\right)R - I_1 R = 0$$

$$\text{or } 2\left(\frac{I}{2} - I_1\right) + (I - 2I_1) - I_1 = 0$$

$$\text{or} \quad 2I - 5I_1 = 0 \text{ or } I_1 = \frac{2}{5}I \quad \text{(ii)}$$

Applying Kirchhoff's second law to external circuit *AHEBE'*, we get $\dfrac{I}{2}R + I_1 R + \dfrac{I}{2}R = E'$

$$IR + \frac{2}{5}IR = E' \quad \text{or} \quad \frac{7}{5}IR = E' \quad \text{(iii)}$$

Comparing (i) and (iii), we get

$$R_{AB} = \frac{7}{5}R = \frac{7}{5} \times 2 = 2.4\,\Omega$$

20. **i.** In steady state, no current will flow through the circuit if S is opened. So the potential of a will be 18 V, and that of b will be zero.

Hence $V_a - V_b = 18 - 0 = 18\text{ V}$

ii. Obviously a is at the higher potential.

iii. When S is closed, finally in steady state, current I will flow as shown in Fig. 5.267.

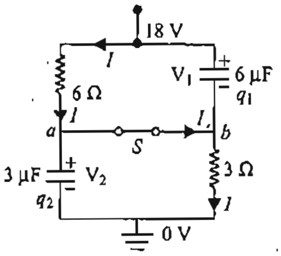


Fig. 5.267

$$I = \frac{18}{6+3} 2\text{ A}, V_1 = 6I = 6 \times 2 = 12\text{ V}$$

$$V_b = 18 - V_1 = 18 - 12 = 6\text{ V}$$

iv. $q_1 = C_1V_1 = 6 \times 12 = 72\ \mu\text{C}$,

$V_1 + V_2 = 18 \Rightarrow V_2 = 6\text{V}, q_2 = C_2V_2 = 3 \times 6 = 18\ \mu\text{C}$

Before closing S, the potential on each capacitor is 18 V and charge: $q_{10} = 6 \times 18 = 108\ \mu\text{C}$,

$q_{20} = 3 \times 18 = 54\ \mu\text{C}$

Change in charge; $q_1 - q_{10} = -36\ \mu\text{C}$ and

$q_2 - q_{20} = -3.6\ \mu\text{C}$

21. Due to sources, currents flow through resistance R_1, R_2, and R_3 and capacitor gets charged. Due to charge, an electric field is established in the capacitor whose magnitude cannot exceed di- electric strength E_0 of air. Maximum safe value of V corresponds to the maximum possible charge on capacitor. Let the maximum possible charge on capacitor be q_0. Then the electric field inside the capacitor,

$$E_0 = \frac{q_0}{A\varepsilon_0}$$

$$q_0 = A\varepsilon_0 E_0 = 15{,}000\ \varepsilon_0\text{C and } C = \frac{\varepsilon_0 A}{d} = 5\varepsilon_0\text{ Farad}$$

Since, in steady state no current flows through the capacitor, therefore, current through various parts of the circuit will be as shown in Fig. 5.268.

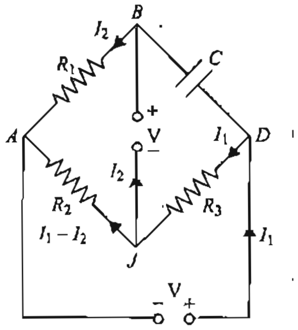


Fig. 5.268

Now analysing the circuit in a steady state.

First applying Kirchhoff's voltage law on mesh $ABJA$,

$-I_2R_1 + V + R_2(I_1 - I_2) = 0 \Rightarrow 2I_1 - 5I_2 = -V$

For mesh $AJDFGA$:

$$-R_2(I_1 - I_2) = -R3I_1 + V = 0$$

$$\Rightarrow \quad 3I_1 - 2I_2 = V$$

From equations (i) and (ii): $I_1 = \frac{7V}{11}$ and $I_2 = \frac{5V}{11}$

Now applying Kirchhoff's voltage law on mesh $BDJB$:

$$\frac{q}{C} + I_1R_3 - V = 0 \Rightarrow q = \frac{4CV}{11} = \frac{20}{11}\varepsilon_0\text{V}$$

But maximum possible value of q is $q_0 = 15{,}000\ \varepsilon_0$

∴ maximum safe value of

$$V = \frac{11q_0}{20\varepsilon_0} = 8250\text{ V} = 8.25\text{ kV}$$

22. Since, in steady state no current flows through the capacitors, therefore, the current through 1 Ω resistor becomes zero. Current through resistors and charge on capacitors will be as shown in Fig. 5.269.

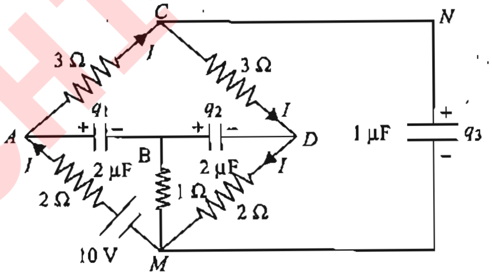


Fig. 5.269

Applying KVL on mesh $MACDA$

$2I + 3I + 3I + 2I - 10 = 0 \Rightarrow I = 1\text{ A}$

Mesh $MABM$: $10 - 2I - \frac{q_1}{2 \times 10^{-6}} = 0 \Rightarrow q_1 = 16\ \mu\text{C}$

Mesh $MBDM$: $-\frac{q_2}{2 \times 10^{-6}} - 2I = 0 \Rightarrow q_2 = -4\ \mu\text{C}$

Mesh $MDCNM$: $2I + 3I - \frac{q_3}{(1 \times 10^{-6})} \Rightarrow q_3 = 5\ \mu\text{C}$

Energy stored in capacitors, $U = \sum \frac{q^2}{2C}$

$$= \frac{q_1^2}{2 \times (2 \times 10^{-6})} + \frac{q_2^2}{2 \times (2 \times 10^{-6})} + \frac{q_3^2}{2 \times (1 \times 10^{-6})}$$

$$= 80.5 \times 10^{-6}\text{ J}$$

Rate of supply of energy be battery is $P = EI$

$$= 10 \times 1 = 10\text{ W}$$

23. $q_0 = CE, q_0/2 = q_0e^{-t_1/RC} \Rightarrow t_1 = RC\ln 2$

Now $\int_{q_0/2}^{3q_0/4} \frac{dq}{CE - q} = \int_0^{t_2} \frac{dt}{3RC} \Rightarrow t_2 = 3RC\ln 2$

From here we get: $t_1/t_2 = 1/3$

24. In the steady state, no current passes through upper three resistors. So $I = \frac{12}{10+30} = 0.3$ A

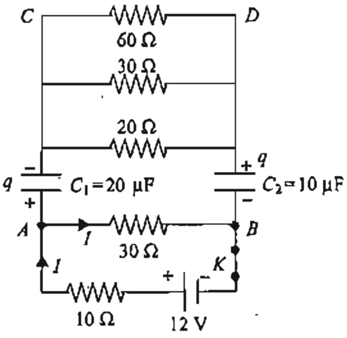


Fig. 5.270

Potential difference between A and B:

$$V = 30 \times 0.3 = 9 \text{ V}$$

Now in loop $ACDBA$: $\frac{q}{C_1} + \frac{q}{C_2} = 9$

$$\frac{q}{20} + \frac{q}{10} = 9 \quad \Rightarrow \quad q = 60\,\mu\text{C}$$

a. Potential difference across $C_1 : V_1 = \frac{q}{C_1} = \frac{60}{20} = 3\text{ V}$

Potential difference across $C_2 : V_2 = \frac{q}{C_2} = \frac{60}{10} = 6\text{ V}$

b. After K is opened, 12 V will be out of circuit, capacitors will act as batteries as shown in Fig. 5.271.

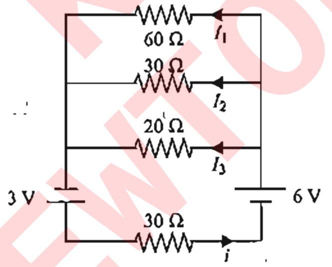


Fig. 5.271

Now one can find, $I_1 = 0.375$ A

25. $R_{eq} = \frac{E}{I}$

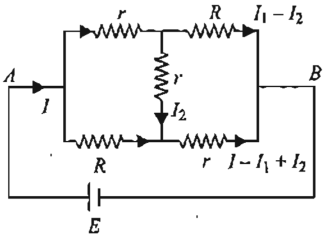


Fig. 5.272

$$E = rI_1 + R(I_1 - I_2) \Rightarrow E = (r+R)\,I_1 - RI_2 \quad (1)$$

$$rI_1 + rI_2 = R\,(I - I_1) \Rightarrow (r + \text{P})I_1 + rI_2 = RI \quad (2)$$

$$rI_2 + r\,(I - I_1 + I_2) = R\,(I_1 - I_2)$$

$$\Rightarrow \quad (r+R)\,I_1 - (R+2r)\,I_2 = rI \quad (3)$$

$$2 - 3 \Rightarrow (3r+R)\,I_2 = (R-r)\,I \Rightarrow I_2 = \frac{(R-r)I}{3r+R}$$

Put the above value in equation (2) hence, we get:

$$(r+R)\,I_1 + \frac{r(R-r)}{3r+R}I = R\,I$$

$$\Rightarrow \quad I_1 = \frac{(R+r)I}{3r+R} \quad \text{(after simplification)}$$

Put I_1 and I_2 in equation (1)

$$R_{eq} = \frac{E}{I} = \frac{(R+r)^2}{3r+R} - \frac{R(R-r)}{3r+R} = \frac{r(r+3R)}{3r+R}$$

Objective Type

1. c. $V = E - \frac{E}{R+r}$ or $V = E - \frac{E}{R+R}R$

or $V = E - \frac{E}{2}$ or $V = \frac{E}{2}$

2. c. $2 = \frac{12}{\frac{R}{3} + 0.6}$ or $\frac{R}{3} + 0.6 = 6$

Or $\frac{R}{3} = 5.4$ or $R = 16.2\,\Omega$

3. a. The equivalent resistance of resistors:

$$R = 2 + \frac{4}{2} + \frac{15}{3} = 9\,\Omega$$

$$I = \frac{E}{r+R} = \frac{10}{1+9} = 1\text{ A}$$

4. d. $V_A + 3 - 3 - 18 = V_B$ or $V_A - B_B = 18$ V

5. c. The resistance of the square X is given byz

$$R_X = \rho\frac{L}{Lt} = \frac{\rho}{t}$$

where ρ is the resistivity on the metal.

Similarly, for R_Y,

$$R_Y = \rho\frac{2L}{(2L)t} = \frac{\rho}{t} \text{ same as before.}$$

Hence, $\frac{R_X}{R_Y} = 1$

6. d. e.m.f = 6 V; Total resistance = 6 Ω

$$I = \frac{6}{6}\text{ A} = 1\text{ A}$$

For the direction of current, look at the direction of e.m.f of cell of 10 V.

7. b. In series combination,

I is constant.

$\therefore \quad V \propto R$

$$\frac{V_{AB}}{V_{CA}} = \frac{R_{AB}}{R_{CA}} = \frac{\rho \frac{l}{\pi a^2}}{\rho \frac{l}{\pi (2a)^2}}$$

or $$\frac{V_{AB}}{V_{CA}} = \frac{4}{1}$$

8. c. Charge = are under the current -time graph

$$q_1 = 2 \times 1 = 2,\ q_2 = 1 \times 2 = 2$$

and $$q_3 = \frac{1}{2} \times 2 \times 2 = 2$$

$$q_1 : q_2 : q_3 = 2 : 2 : 2 = 1 : 1 : 1$$

9. c. $E = 4$ V

Total resistance = $(1 + 0.9 + 1.9)\ \Omega = 3.8\ \Omega$

Now, $I = \frac{4}{3.8}$ A

Again, terminal potential difference across A

$$= 2 - \frac{4}{3.8} \times 1.9 = 2 - 2 = 0$$

10. b. Current I through the resistors is

$$I = \frac{3-(-15)}{200+1000} \text{ mA} = 0.015 \text{ mA}$$

Potential at X is thus

$$V_X = 3 - (200 \times 10^3)(0.0.15 \times 10^{-3}) = 0$$

11. d. Resistance is the gradient of V–I graph. If the resistance decreases with the temperature rise (which occurs when voltage is increased), the graph becomes less steep in the I-axis.

12. b. The current I through the resistance wire XY is the same. Since the wire is uniform, its cross-sectional area A is constant throughout its length. Hence, the current density is

$$J = \frac{1}{A}$$

which is uniform through the wire XY.

13. a. The equivalent of the network is given in Fig. 5.273.

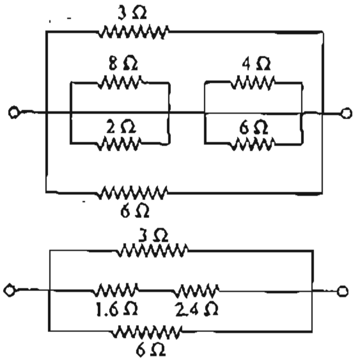


Fig. 5.273

The equivalent of the above network is as under:

The equivalent of the above network is a parallel combination of 3 Ω, 4 Ω, and 6 Ω

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{4} + \frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{R} = \frac{8+6+4}{24} \quad \text{or} \quad R = \frac{24}{12}\Omega = \frac{4}{3}\Omega.$$

14. b. For loop (1)

$$2 + 2 - 4I = 0$$

$$4I_1 = 4 \text{ or } I_1 = 1 \text{ A}$$

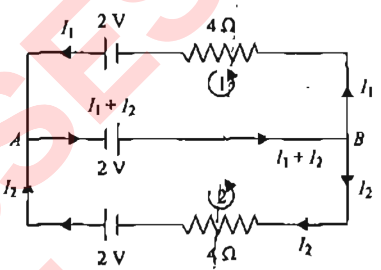


Fig. 5.274

For loop (2)

$$-2 - 2 + 4I_2 = 0$$

or $$4I_2 = 4 \text{ or } I_2 = 1 \text{ A}$$

So, the current from A to B is 2 A.

15. a. The current density at P is higher than at Q. For the same current flowing through the metallic conductor PQ, the cross sectional area at P is narrower than at Q. The resistance per unit length r is given by

$$r = \rho \frac{1}{A}$$

where ρ is the resistivity and A is the cross-sectional area of the conductor PQ.

Thus, r is inversely proportional to the cross-sectional area A of the conductor.

16. c. The number density n of conduction electrons in the copper is a characteristic of the copper and is about 10^{29} at room temperature for both the copper rod X and the thin copper wire Y.

Both X and Y carry the same current I since they are joined in series.

From $\quad I = nA_{vq}$

where q is the electron charge of 1.6×10^{-19}C, v is the drift velocity in the conductor and A is the cross-sectional area of the conductor.

We may conclude that rod X has a lower drift velocity of electrons compared to wire Y since rod X has larger cross-sectional area. This is so because the electrons in X collide more often with one another and with the copper ions when drifting towards the positive end. Thus, the mean time between collisions of the electrons is more in X than in Y.

17. a. For a given wire, $R = \dfrac{\rho L}{s}$

with $L \times s$ = volume = V = constant

So that, $R = \dfrac{\rho L}{s}$

$$\frac{\Delta R}{R} = 2\frac{\Delta L}{L} = 2(0.1\%)$$

$$= 0.2\% \text{ increase}$$

18. d

$$\frac{\rho\ell}{A} = \frac{\rho\ell^2}{A\ell} = \frac{\rho\ell^2}{V} = \frac{\rho\ell^2}{m/d}$$

$$R = \frac{\rho\ell}{A} = \frac{\rho\ell^2}{A\ell} = \frac{\rho\ell^2}{V} = \frac{\rho\ell^2}{m/d}$$

$$\frac{\rho d\ell^2}{m} \quad \text{or,} \quad R\alpha\frac{\ell^2}{m}$$

$$R_1 : R_2 : R_3 = \frac{\ell_1^2}{m_1} : \frac{\ell_2^2}{m_2} : \frac{\ell_3^2}{m_3} = \frac{25}{1} : \frac{9}{3} : \frac{1}{5}$$

$$= 125 : 15 : 1$$

19. d. When a steady current flows in a metallic conductor of non uniform cross-section then the drift speed is $V_d = \dfrac{I}{n_e A}$ and electric field

$$E = \frac{I}{\sigma A} \quad \Rightarrow \quad V_d \propto \frac{1}{A} \text{ and } E \propto \frac{1}{A}$$

$\Rightarrow$ Only current remain constant.

20. c. The potential difference between the point P and the earth (E_1) is 15 V. As the current through 5 Ω resistance is 2 A, therefore, potential difference between Q and $E_2 = 5 \times 2 = 10$ V. Hence the total potential difference between P and $Q = 5$ V.

21. c. If an identical battery is connected in opposition, net emf = $E - E$ = and the current through circuit will be zero, although each one of them has constant emf.

22. b. If we move from P to B, potential will decrease. From B to N, there is no change in the potential difference. From N to M potential will increase, but increase in potential will be $E - Ir (< E)$.

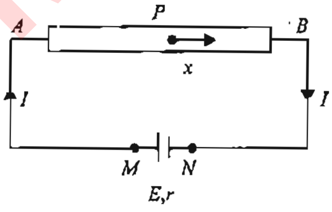


Fig. 5.275

Finally we return to the same potential at P.

23. a. 9 Ω and 3 Ω are in parallel. Their equivalent is

$$\frac{9 \times 3}{9+3} = \frac{9}{4}\,\Omega$$

Now 18 Ω, 3 Ω, 6 Ω, and $\dfrac{9}{4}$ Ω are in parallel. Their equivalent will be 1 Ω. This will be in series with 4/5 Ω. So, net $R_{eq} = 1 + \dfrac{4}{5} = \dfrac{9}{5}\,\Omega$, $I = \dfrac{E}{R_{eq}} = \dfrac{18}{9/5} = 10\,\Omega$.

Resistances near C and D will have no current in them.

24. c. Both the length and the cross-sectional area are doubled. So, resistance remains unaffected.

25. d. Since current is rate of flow of charge in the direction in which positive charge will move, then current due to the electron will be

$$i_e = \frac{n_e q_e}{t} = 3 \times 10^{18} \times 1.6 \times 10^{-19} = 0.48 \text{ A}$$

(Opposite to the motion of electrons, i e., right to left)

$$= 0.32 \text{ A (Right to left)}$$

total $\quad I = i_e + i_p$

$$= 0.48 + 0.32 = 0.80 \text{ A (Right to left)}$$

26. a. The current in 2 Ω resistor will be zero because it is not a part of any closed loop.

27. a. New length is $2l$, if the original length is l. Clearly, the new cross-sectional area is $\dfrac{\alpha}{2}$, if a is the initial cross-sectional area. This is because the volume of the wire has to remain constant.

Now, $\quad R' = \rho\dfrac{2l}{9/2} = 4\,R$

Increase in resistance = $4R - R = 3R$

Percentage increase in resistance = $\dfrac{3R}{R} \times 100 = 300$

28. a. $S = R_1 + R_2, \quad P = \dfrac{R_1 R_2}{R_1 + R_2} \quad \therefore \quad S = nP,$

$$\therefore \quad R_1 + R_2 + \frac{n R_1 R_2}{R_1 + R_2}$$

or $\quad (R_1 + R_2)^2 = nR_1R_2$

For the minimum value of n, $R_1 = R_2$.

$\therefore \quad (2R_1)^2 = nR_1{}^2 \quad$ or $\quad n = 4$

29. b. $\dfrac{I_1}{I_2} = \dfrac{R_2}{R_1}, \quad \dfrac{I_1}{I_2} = \dfrac{\rho L_2}{A_2} \times \dfrac{A_1}{\rho L_1}$

or $\quad \dfrac{I_1}{I_2} = \left(\dfrac{L_2}{L_1}\right)\left(\dfrac{A_1}{A_2}\right)$ or $\dfrac{I_1}{I_2} = \left(\dfrac{L_2}{L_1}\right)\left(\dfrac{\pi r_1{}^2}{\pi r_2{}^2}\right)$

$$= \frac{L_2 r_1{}^2}{L_1 r_2{}^2} = \frac{3}{4}\left(\frac{2}{3}\right)^2 = \frac{3}{4} \times \frac{4}{9} = \frac{1}{3}.$$

30. b. According to Kirchhoff's first law, a junction can act neither as source of charge nor as sin k or charge. This supports law conservation of charges. According to Kirchhoff's second law, the energy per unit charge transferred to the moving charges is equal to the energy per unit charge transferred from them. This supports law of conservation of energy.

31. d. Using $R_t = R_o\,(1 + at)$ twice, we get

$$100 = R_o(1 + a \times 100)$$

and $$200 = R_o\,(1 + a \times t)$$

Dividing $$2 = \frac{1 + \alpha \times t}{1 + \alpha \times 100}$$

or $$2 + 200a = 1 + at$$

$$1 = a(t - 200)$$

or $$t - 200 = \frac{1}{\alpha} = \frac{1}{0.005} = 200$$

Note:

Strictly speaking, we should use the following formula.

$$R_t = R_o\, e^{at}$$

However, if we read the mind of the examiner from the given options, we find that we need not use this formula.

32. d. $I = nAev_d$ or $n_d \propto \frac{1}{A}$ or $n_d \propto \frac{1}{\pi r^2}$

33. d. We know that resistivity $\rho \propto \frac{1}{\tau}$, where τ is the relaxation time. On increasing temperature, τ deceases so resistivity or resistance increases.

34. b. I = Avne, No. of free electrons per unit length = $1 \times A \times n$. Momentum of each free electron = mv.

$\therefore$ momentum per unit length = Anmv = $\frac{1}{e}$ m

$$= \frac{I}{(e/m)} = \frac{I}{s}$$

35. d. $I = neAv_d \Rightarrow v_d = \frac{I}{ne\,A}$

$\Rightarrow v_d \propto \frac{I}{A}$ so as A increases v_d decreases

36. c. With each rotation, charge Q crosses any fixed Point P near the ring. Number of rotations per second = $\omega/2\pi$

$\therefore$ charge crossing P per second = current = $\frac{Q\omega}{2\pi}$

37. b. Let the edges be $2l$, a, and l, in decreasing order.

$$R_{\max} = \rho\frac{2l}{al} = \frac{2\rho}{a}$$

$$R_{\min} = \rho\frac{l}{2la} = \frac{\rho}{a} \quad \frac{R_{\max}}{R_{\min}} = 4$$

38. b. Current in the circuit = $i = \frac{\varepsilon}{R + r}$

p.d. across cell = p.d. across $R = iR = \frac{\varepsilon R}{R + r}$

Set up two equations with the given data and solve for ε, r.

39. b. For series connection,

$$I_{\max} = \frac{N\varepsilon}{Nr} = \frac{\varepsilon}{r}$$

For parallel connection,

$$I_{\max} = \frac{\varepsilon}{(r/n)} = \frac{N\varepsilon}{r}$$

40. a. Current in circuit

$$i = \frac{n\varepsilon}{nr} = \frac{\varepsilon}{r}$$

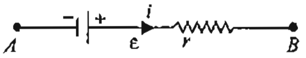


Fig. 5.276

The equivalent circuit of one cell is shown in the Fig. 5.276

The p.d across the cell is $V_A - V_B = -\varepsilon + ir = -\varepsilon + \frac{\varepsilon}{r}.r = 0$

41. a. See the Fig. 5.277

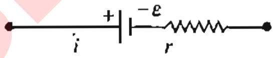


Fig. 5.277

$$i = \frac{(n-2)\varepsilon}{nr}$$

$$V_B - V_A = -ir + \varepsilon$$

$$= e - \frac{(n-2)\varepsilon}{nr}r = \varepsilon\left[1 - \frac{n-2}{n}\right] = \frac{2\varepsilon}{n}$$

42. c. For the cell A, the current i flows opposite to the direction of its emf.

$$\text{p.d} = \varepsilon + ir = \varepsilon + \frac{(n-2)\varepsilon}{nr}r = \varepsilon\left[1 + \frac{n-2}{n}\right]$$

$$= \varepsilon\left(\frac{2n-2}{n}\right)$$

43. b. For resistors in series connection, current (I) is the same through the resistors. In other words, ratio of the voltage drop across each resistor with its resistance is the same. That is

$$I = \frac{5-3}{R_1} = \frac{3-2}{R_2} = \frac{2}{R_3}$$

i.e., $R_1 : R_2 : R_3 = 2 : 1 : 2$.

44. b $$R_{46} = \frac{4 \times 6}{4 + 6}\,\Omega = \frac{24}{10}\,\Omega = 2.4\,\Omega$$

$$\frac{1}{R_{468}} = \frac{1}{4} + \frac{1}{6} + \frac{1}{8}$$

$$\frac{1}{R_{468}} = \frac{12 + 8 + 6}{48}$$

or $$R_{468} = \frac{48}{26}\,\Omega = \frac{24}{13}\,\Omega = 1.85\,\Omega$$

In series combination, I is constant. So, V is more for higher resistance.

45. c. The lower limit is zero volt (0 V) when X is at the lower end of the 4 kΩ resistor.

The upper limit is the potential difference across the 4 kΩ resistor when X is at the upper end of the 4 kΩ resistor. That is

$$v=\left(\frac{25}{1K+4K}\right)4K=\left(\frac{25}{5}\right)4=20\text{ V}$$

Thus, the limits are 0 and 20 V.

46. c. Notice the polarities of the batteries. The batteries will cancel each other and finally there will be no current anywhere in the circuit.

47. b. By symmetry, we see that the current in left and right arms should be same. It means no current should flow from A to B.

48. a. $$\frac{\frac{20R}{20+r}}{5}=\frac{20}{10}\quad\text{or}\quad\frac{20R}{20+R}=10$$

or $\quad 20R=200+10R$

or $\quad 10R=200$ or $R=20\ \Omega$

49. d. The potential difference across R_2 and R_3 is the same which is given by

Potential difference $= R_2I_2 = R_3I_3$

$\Rightarrow \quad I_2/I_3 = R_3/R_3$

Sum of the current I_2 and I_3 is I_1.

So, $\quad I_1/I_3 = 1+\dfrac{I_2}{I_3}+1+\dfrac{R_3}{R_2}$ (independent of R_1).

50. a.

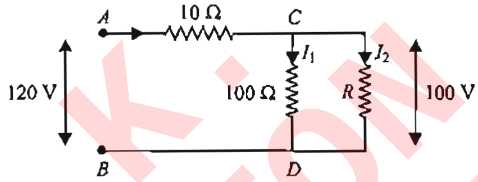


Fig. 5.278

Potential difference across C and D = 100 V

Hence $\quad I_1=\dfrac{100}{100}=1\text{ A},$

$\quad V_{AC}=120-100=20\text{ V}$

And $\quad I=\dfrac{20}{10}=2\text{A},$

Hence $\quad I_2=2-1=1$ A

$\quad R=100\times1=100\ \Omega$

51. b. Applying junction rule to O

$$-I_1-I_2-I_3=0\quad\Rightarrow\quad I_1+I_2+I_3=0 \qquad \text{(i)}$$

Now if V_0 is the potential at point O then by Ohm's law for resistances R_1, R_2, and R_3, respectively,

we have,

$(V_0-V_1)=I_1R_1,\ (V_0-V_2)=I_2R_2$

$(V_0-V_3)=I_3R_3$

$$I=\frac{(V_0-V_1)}{R_1};\ I_2=\frac{(V_0-V_2)}{R_2};$$

and $\quad I_3=\dfrac{(V_0-V_3)}{R_3};$

So substituting these values of I_1, I_2, and I_3 in equation (i), we get

$$V_0\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\left[\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right]=0$$

$$V_0=\left[\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right]\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]^{-1}$$

52. b. Let resistor to be connected across CD is x. Then the equivalent resistance across EF should be x and also across AB should be x. So we get $\dfrac{(2R+x)R}{3R+x}=x$

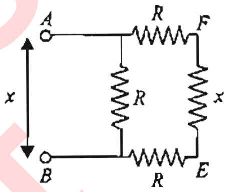


Fig. 5.279

Solve to get $x=\sqrt{3}-1\ \Omega$

53. b. Let R be the resistance of wire Let R' be the resistance of wire Energy released in t second $=\dfrac{(3V^2)}{R}\times t$

$\Rightarrow \quad R'=2R \qquad$ (∵ Length is twice)

∴ Energy released in t-seconds $=\dfrac{(NV^2)}{2R}\times t$

But $\quad Q=mc\Delta T$

$$\therefore\quad Q'=\frac{(N^2V^2)}{2R}\times t\quad \therefore mc\Delta T=\frac{(9V^2)}{R}\times t \qquad \text{(i)}$$

Applying $Q'=m'c\Delta T$

$$2mc\Delta T=\frac{(N^2V^2)}{2R}\times t \qquad \text{(ii)}$$

Dividing equation (ii) by equation (i)

$$\frac{mc\Delta T}{2mc\Delta T}=\frac{9V^2\times t/R}{N^2V^2\,t/2R}\quad\therefore\ \frac{1}{2}=\frac{9\times2}{N^2}$$

$\Rightarrow \quad N^2=18\times2 \quad \therefore N=6$

54. a. The circuit is symmetrical about the axis POQ. Therefore, the equivalent circuit is drawn

$$\therefore\quad \frac{1}{R_{PQ}}=\frac{1}{4R}+\frac{1}{4R}+\frac{1}{2r}$$

$$\Rightarrow\quad R_{PQ}=\frac{2Rr}{R+r}$$

55. **c.** The bridge is balanced and the current in the ADC is larger than in the part ABC. Also $I_3 = 0$.

56. **b.** The p.d. across AD is less than that across AB. So, the potential of D is higher than that of B.

57. **b.** $r = \dfrac{V}{I} = \dfrac{2}{5} = 0.4\ \Omega$

when $i = 0$, the potential reading is 2 V. Hence emf = 2 V

58. **d.** The voltage across the capacitor is

$$V = \frac{Q}{C} = \frac{1\times10^{-5}}{1\times10^{-6}} = 10\text{ V}$$

Thus, the current flowing through the resistor after the switch is closed will be

$$I = \frac{V}{R} = \frac{10}{10}\text{ A} = 1\text{ A}$$

59. **c. Method I:** In this method there will be no current flowing in branch BE in a steady condition. Let I be the current flowing in the loop $ABCDEFA$. Applying Kirchoff's law in the loop moving in anticlockwise direction starting from C. $+2V - I(2R) - I(R) - V = 0$ $V = 3IR \Rightarrow I = V/3R$ (1)

Applying Kirchoff's law in the circuit $ABEFA$ we get on moving in anticlockwise direction starting from B $+ V + V_{cap} - IR - V = 0$ where V_{cap} is the p.d. across capacitor

$$\therefore \quad V_{cap} = IR = \left(\frac{V}{3R}\right)\times R = \frac{V}{3}$$

Method II: Let us consider A to be at OV. Then point B, C, and D will be at V, V, and 2V volt, respectively. Let the current be flowing in clockwise direction. Applying Kirchoff's law in the outer loop we get $V - IR - I(2R) - 2V = 0$ $\therefore I = -V/3R$. The minus sign here indicates that the current is in the opposite direction to what we have assumed. Applying Kirchoff's law from A to E via B we get $V_A + V + IR = V_E$

$$\therefore \quad 0 + V + \frac{V}{3R}\times R = V_E = \frac{4V}{3}$$

Again applying Kirchoff's law from A to E via C, we get $V_A + V + V_{cap} = V_c$

$$\therefore \quad V_{cap} = \frac{V}{3}$$

60. **c.** In the steady state conduction, no current will flow through the capacitor C.

Current in the outer circuit,

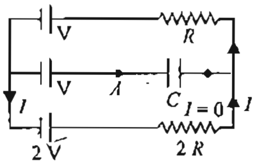


Fig. 5.280

$$I = \frac{2V - V}{2R + R} = \frac{V}{3R}$$

Potential difference between A and B.

$$V_A - V + V + IR = V_B$$

$$\therefore \quad V_B - V_A = IR = \left(\frac{V}{3R}\right)R = \frac{V}{3}$$

Note:

In this problem, charge stored in the capacitor can also be asked, which is equal to $q = C.\ \dfrac{V}{3}$ with positive charge on B side and negative on A side because $V_B > V_A$.

61. **b.** Time constant $= RC = R\times\infty = \infty$ $(\because C = \infty)$

62. **c.** Time constant $= RC$. So, if either R or C is doubled, time will be doubled.

63. **c.** In the case of discharging $I = I_0 e^{-t/Rc}$

or $2.5\times10^{-6} = \dfrac{q_0}{RC}e^{-t/RC}$

or $2.5\times10^{-6} = 5\times10^{-6}\,e^{-t/10}$ or $e^{t/10} = 2$

Taking log on both sides, we get

$\dfrac{t}{10} = \log 2$ or $t = 10\log 2 = 6.9 \cong 7\text{ s}$

64. **a.** $\dfrac{q^2}{2C} = \dfrac{q_0^2 e^{-2t/RC}}{2C} = u_0 e^{-2} = \dfrac{u_0}{e^2}$

65. **b.** At $t = 0$, the capacitor behaves as a short circuit. The corresponding circuit is shown in Fig. 5.281.

According to the loop rule,

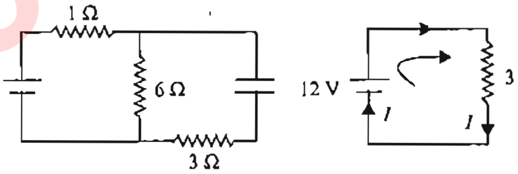


Fig. 5.281

$$12 - 3I = 0 \quad \therefore I = 4\text{ A}$$

66. **c.** During charging, $T_1 = RC$

During discharging, $T_2 = 2RC$

$$\therefore \quad \text{ratio} = \frac{T_1}{T_2} = \frac{1}{2} = 1:2$$

67. **d.** At $t = 0$, the potential difference across the top 2μF capacitor = 0

$$\therefore \quad I = \frac{10}{2} = 5\text{ A}$$

68. **d.** In the steady state, no current is flowing through capacitors. According to the loop rule,

$$\therefore \quad \frac{q_0 - q}{C_1} - \frac{q}{C_2} = 0$$

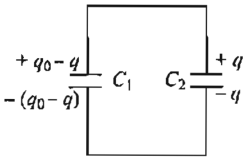


Fig. 5.282

or $\dfrac{C_1V_0 - q}{C_1} - \dfrac{q}{C_2} = 0$

Here $C_1 = 6\,\mu F$, $C_2 = 3\,\mu F$ and $V_0 = 10$ V

$$\therefore \quad 10 - \frac{q}{6} - \frac{q}{3} = 0 \quad \Rightarrow \quad q = 10 \times \frac{18}{9} = 20\,\mu C$$

69. d. $R = \dfrac{5r}{2} + 2r = 4.5r = 45\,\Omega,\ I = \dfrac{10}{45}$ A

70. a. In steady state, the current through capacitor branch is zero. So, capacitor branch may be removed.

$$R_{eq} = \frac{r_0 \times 3r_0}{r_0 + 3r_0} = \frac{3}{4} r_0$$

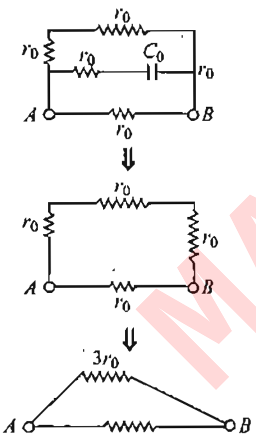


Fig. 5.283

71. a. In the steady state, no current is flowing through the capacitor branch, so the capacitor branches may be removed from the circuit, the equivalent circuit is shown in Fig. 5.284.

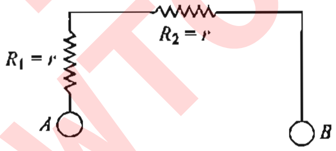


Fig. 5.284

$$\therefore \quad R_{AB} = R_1 + R_2 = r + r = 2r$$

72. d $U_{\text{initial}} = \dfrac{1}{q} CE^2;\ U_{last} = \dfrac{1}{q} CE^2$

$$\therefore \quad \frac{U_{\text{initial}}}{U_{\text{last}}} = 1:1$$

73. a. $V_A - V_B =$ e.m.f of the cell $= 2$V

$$\therefore \quad U = \frac{1}{2} CV^2 = \frac{1}{2} \times 2 \times 10^{-6} \times 2^2$$

$$= 4 \times 10^{-6}\ \text{J}$$

74. a. The resistance in the middle plays no part in the charging process of C, as it does not alter either the potential difference across the RC combination or the current through it.

75. b. C discharges through $R + R$ in series.

76. c. $Q = Q_0 e^{-t/\tau}$ and potential difference across C is proportional to Q. For the p.d. to fall by 10% Q must fall by 10%.

$$Q = 0.9\,Q_0 = Q_0 e^{-t/\tau}$$

$$\text{or} \quad e^{t/\tau} = \frac{10}{9} \quad \text{or} \quad \frac{t}{\tau} = \ln\left(\frac{10}{9}\right)$$

77. b. $Q = 0.1\,Q_0 = Q_0 e^{-t/\tau}$

or $e^{t/\tau} = 10$ or $t/\tau = \ln 10 = 2.303$

78. c. $Q = Q_0(1 - e^{-t/\pi})$

or $e^{t/\pi} = 0.9$ or $e^{t/\tau} = 10/9.$

79. b. $Q = Q_0(1 - e^{-t/t}) = 0.9\,Q_0$

or $e^{-t/\tau} = 0.1$ or $e^{t/\tau} = 10.$

80. b. $Q = Q_0 e^{-t/\tau} = Q_0/\eta$

$$\text{or} \quad e^{-t/\tau} = \frac{1}{\eta} \quad \text{or} \quad e^{t/\tau} = \eta \quad \text{or} \quad \frac{t}{\tau} = \ln \eta.$$

81. a. The current in the circuit $I = (12/12) = 1$ A.
Potential across d and e = 12 V − 3 × 1 V = 9 V
Capacitance across d and e is

$$C = \frac{1 \times 2}{1 + 2} = \frac{2}{3}\,\mu F$$

∴ charge on either capacitor is 6 μC.

82. d. No current will flow in the branch containing capacitor. Hence no energy stored on the capacitor.

83. c. $I = I_0 e^{-t/RC}$ or $\dfrac{I_0}{2} = I_0 e^{-t/RC}$

$$\text{or} \quad \frac{1}{2} = e^{-t/RC} \quad \text{or} \quad \ell n 2 = \frac{t}{RC}$$

$\therefore \quad t = RC \ln 2$

or $10^{-6} \times \ell n 4 = (2 + r) \times 0.5 \times 10^{-6} \ln 2$

or $2\ell n 2 = (2 + r) \times 0.5 \times \ell n 2$

or $4 = 2 + r$

$\therefore \quad r = 4 - 2 = 2\,\Omega$

84. c. Here, $I_1 = \dfrac{V}{R} e^{-t/RC},\ I_2 = \dfrac{V}{R} e^{-t/2RC}$

$$\therefore \quad \frac{I_1}{I_2} = e^{-t/2RC - t/RC} = e^{-t/2RC} = \frac{1}{e^{t/2RC}}$$

From this expression, it is clear that, when t increases ratio decreases.

85. d. Uncharged capacitor behaves as a short circuit just after closing the switch. But charged capacitor behaves as the battery of e.m.f $\dfrac{q_0}{C_1}$ just after closing the switch. (Fig. 5.285)

$$\therefore \quad I = \frac{q_0}{C_1(2R)} = \frac{q_0}{2RC_1} = \frac{V}{2R}$$

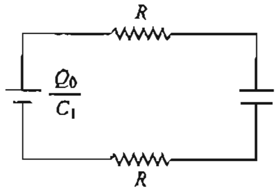


Fig. 5.285

86. c. The distribution of current is shown in Fig. 5.286.
According to the loop rule,

$2.5 - 0.5I - 2I = 0$

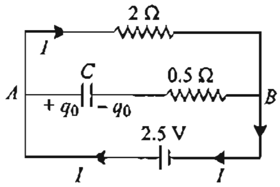


Fig. 5.286

$\therefore \quad I = 1\text{A}$

$\therefore \; V_A - V_B = \frac{q_0}{C} = 2I = 2\times 1 = 2\text{ V}$

$\therefore \quad q_0 = C\times 2 = 2\times 10^{-6}\times 2 = 4\ \mu\text{C}$

87. c. Step – I: When the switch is at position 1: Since circuit is in steady state, so the current through circuit is zero.

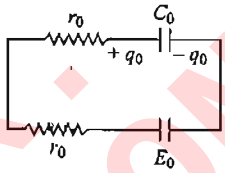


Fig. 5.287

According to the loop rule

$E_0 - \frac{q_0}{C_0} = 0 \qquad \therefore q_0 = C_0E_0$

Step – II: When the switch is at position 2: In this case, the total energy stored on the capacitor appears as heat energy in the resistor.

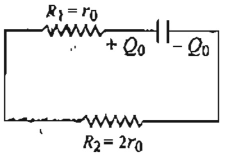


Fig. 5.288

$\therefore \quad \Delta H = I^2RT \quad \therefore \Delta H \propto R$

$\therefore \quad \frac{\Delta H_1}{\Delta H_2} = \frac{R_1}{R_2} = \frac{r_0}{2r_0} = \frac{1}{2}$

$\therefore \quad \Delta H_2 = 2\Delta H_1$

But $\quad \Delta H = \Delta H_1 + \Delta H_2$

$= \frac{\Delta H_2}{2} + \Delta H_2 = \frac{3}{2}\Delta H_2$

$\therefore \quad \Delta H_2 = \frac{3}{2}\Delta H = \frac{2}{3}\times\frac{1}{2}C_0E_0^2 = \frac{1}{3}C_0E_0^2$

88. c. When switch K_1 is open and K_2 is closed. Then, total energy stored on capacitor appears as heat in resistor. So

$$\Delta H = \frac{q_0^2}{2C} = \frac{(30\times10^{-6})^2}{2\times3\times10^{-6}} = 0.15\times10^{-3}\text{ J} = 0.15\text{ mJ}$$

89. b. In the steady state, no current is passing through capacitor. Let the charge on each capacitor be q.
Since the current through galvanometer is zero.

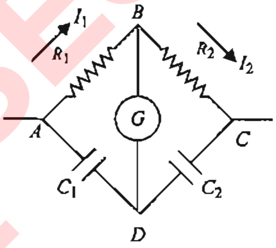


Fig. 5.289

$\therefore \quad I_1 = I_2$

The potential difference between ends of galvanometer will be zero.

$\therefore \; V_A - V_B = V_A - V_D$

$I_1R_1 = \frac{q}{C_1} \qquad$ (i)

Similarly, $V_B - V_C = V_D - V_C$

$I_2R_2 = \frac{q}{C_2} \qquad$ (ii)

Dividing equation (i) by equation (ii), we get

$$\frac{I_1R_1}{I_2R_2} = \frac{\frac{q}{C_1}}{\frac{q}{C_2}} = \frac{C_2}{C_1}$$

$$\frac{C_1}{C_2} = \frac{R_2}{R_1}$$

90. b. As the capacitors are identical, each of them finally have charge $Q/2$.

Initial energy of the system $= E_i = \frac{Q^2}{2C}$

Final energy of the system

$$= E_f = 2\left[\frac{(Q/2)^2}{2C}\right] = \frac{Q^2}{4C}$$

Heat produced = loss in energy

$$= E_i - E_f = \frac{Q^2}{4C}$$

91. c. A fully charged capacitor draws no current. If the capacitor is removed from the circuit, we can distribute the current and find the potential difference across each resistance.

92. d. $j = \frac{1}{A} = nqv$

where V is the drift velocity of charge carriers each with charge q.

93. b. Since the resistivity of a current carrying conductor carrying charge carriers each of charge q, mass is given by

$$\tau = \frac{m}{ng^2\delta} = \frac{m\sigma}{ng^2} \quad \rho = \frac{m}{nq^2\tau}$$

where n is number density of charge carriers and τ is the average relaxation time.

94. c. $\alpha(T) = \frac{1}{R_0}\frac{dR}{dT}$

$$\Rightarrow \quad (3T^2 + 2T) = \frac{1}{R_0}\frac{dR}{dT}$$

$$\Rightarrow \quad dR = R_0\,(3T^2 + 2T)\,dT$$

$$\Rightarrow \quad \int_{R_0}^{R} dR = R_0\left[3\int_0^T T^2 dT + 2\int_0^T T\,dT\right]$$

$$\Rightarrow \quad R = R_0\,[1 + T^2 + T^3]$$

95. c. $F = \ F = \frac{dp}{dt} = p$

Momentum of each charge carrier moving with a drift velocity v is mv.

Total number of charge carriers in the sample is $N = n(A\ell)$, where n is number of charge carriers per unit volume and A is area of cross-section of the conductor.

Total momentum $= p = N(mv) = nA\ell mv$

Further we have $v = \frac{I}{neA}$

$$\Rightarrow \quad p = nA\ell m\left(\frac{I}{neA}\right) \quad \Rightarrow \quad p = \ell\left(\frac{m}{e}\right)I$$

Since $\quad F = \frac{dp}{dt}$

$$\Rightarrow \quad F = \frac{\ell}{S}I \qquad [\because s = \text{specific charge} = \frac{e}{m}]$$

$$\Rightarrow \quad \frac{F}{\ell} = \frac{i}{s}$$

96. b. Let x, y, and $\frac{x}{3}$ be the dimensions of the block

$$R_{\max} = \frac{\rho x}{y\left(\frac{x}{3}\right)}, \quad R_{\min} = \frac{\rho\left(\frac{x}{3}\right)}{xy}$$

$$\Rightarrow \quad \frac{R_{\max}}{R_{\min}} = 9$$

97. c.

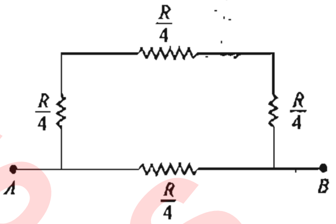


Fig. 5.290

$$\Rightarrow \quad R_{AB} = \frac{3R}{16} = \frac{3(16)}{16} = 3\Omega$$

98. c. When each element of circuit is multiplied by a factor k then equivalent resistance also becomes k times.

Let the equivalent resistance between A and B be x.

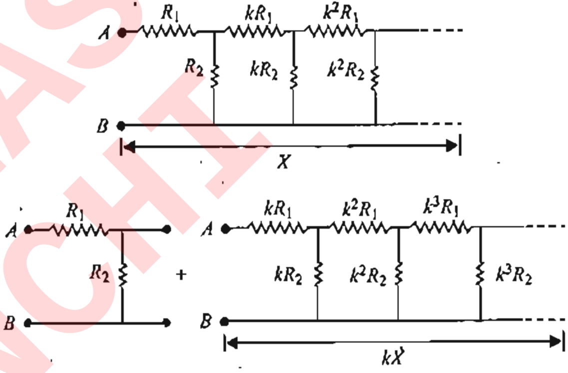


Fig. 5.291

So the equivalent circuit becomes

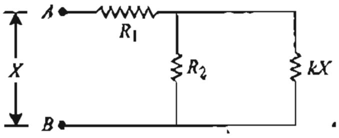


Fig. 5.292

For $\quad k = \frac{1}{2} \quad \Rightarrow \quad x = \frac{(R_1 - R_2) + \sqrt{R_1^2 + R_2^2 + 6R_1R_2}}{2}$

99. b. $R_{AB} = \frac{7}{6}R$

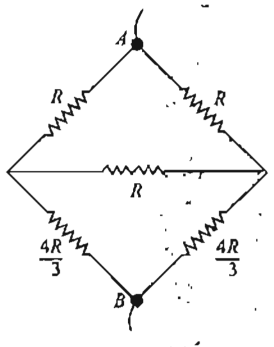


Fig. 5.293

100. c. Since net resistance is to be found between A and B. So let a current I enter at A and then exist at b.

When I enters at A then by symmetry current $\frac{1}{6}$ must flow in the branch AB from A to B for current I to exist from B, a current $\frac{I}{6}$ must flow in the branch AB from A to B.

Superimposing the two, we conclude that a current $\left(\frac{I}{6}+\frac{I}{6}\right)$ must flow in the branch AB from A to B.

According to Thevenin's theorem we have

$$I_{\text{total}}\, R_{eq} = V_{AB} = \left(\frac{I}{6}+\frac{I}{6}\right)R_0 = \frac{IR_0}{3}$$

$$\Rightarrow \quad IR_{eq} = \frac{IR_0}{3} \quad \Rightarrow \quad R_{eq} = \frac{1}{3}R_0$$

101. a. 7 Ω and 3 Ω are in parallel; 6 Ω and 4 Ω are in parallel and both in series.

$$\text{So} \qquad R_{eq} = \frac{7\times 3}{7+3} + \frac{4\times 6}{4+6}$$

$$\Rightarrow \qquad R_{eq} = 2.1 + 2.4 \quad \Rightarrow \quad R_{eq} = 4.5\ \Omega$$

102. d. At junction E or F:

$$I_1 + I_3 + I_4 = I_2 - I_3 \quad \Rightarrow \quad I_1 - I_2 + 2I_3 + I_4 = 0$$

Loop $AFBA$ or loop $ECDE$

$$rI_1 = rI_2 + RI_3 \quad \Rightarrow \quad I_1 = I_2 + I_3$$

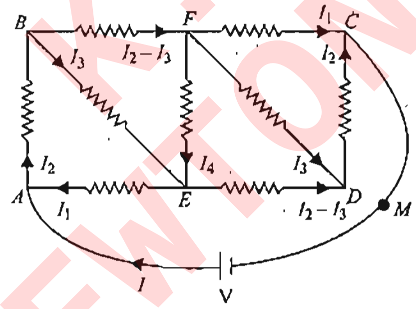


Fig. 5.294

Loop $BEFB$ or loop $EDFE$

$$r(I_2 - I_3) + rI_4 = rI_3$$

$$\Rightarrow \qquad I_2 - 2I_3 + I_4 = 0$$

Loop $AFDCMA$ or $ABECMA$

$$V = rI_1 + r(I_2 - I_3) + rI_2$$

$$\Rightarrow \qquad V = rI_1 + 2rI_2 - rI_3$$

Solve to get:

$$I_1 = 2V/5r,\ I_2 = V/3r$$

$$R_{eq} = \frac{V}{I} = \frac{V}{I_1 + I_2} = \frac{V}{2V/5r + V/3r} = \frac{15r}{11}$$

103. b. $q_g + q_d = q_0 = 10\mu\text{C} \quad \Rightarrow \quad q_d = 7\mu\text{C}$

104. d. Equivalent resistance between A and E:

$$y = \frac{x+1}{x+2}$$

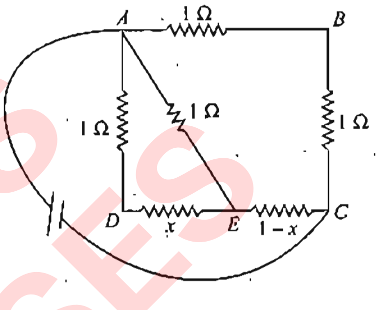


Fig. 5.295

For B and E to be equipotent

$$\frac{R_{AE}}{R_{AB}} = \frac{R_{EC}}{R_{BC}} \quad \Rightarrow \quad \frac{x+1}{(x+2)\times 1} = \frac{1-x}{1}$$

Solve to get: $x = \sqrt{2} - 1\ \Omega$

$$\text{Now} \quad \frac{CE}{ED} = \frac{1-x}{x} = \sqrt{2}\ \Omega$$

105. d. There are eight batteries.

Let current in the circuit is I,

$$\text{Then} \qquad I = \frac{8\times 5}{8\times 0.2} = 25\ \text{A}$$

p.d. across voltmeter $= E - Ir$

$$= 5 - 25\times 0.2 = 0$$

Multiple Correct Answers Type

1. a., b., d.

Let V = potential at D

$$70 - D = 10i_1.$$

$$V - 0 = 20i_2$$

$$V - 10 = 30(i_1 - i_2)$$

Solve for i_1, i_2, and V.

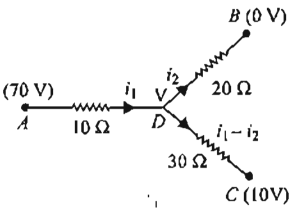


Fig. 5.296

2. a., b., d.

As C and D are joined, they must be at the same potential, and may be treated as the same point. This gives the equivalent resistance as 8 Ω. If we distribute current in the network, using symmetry.

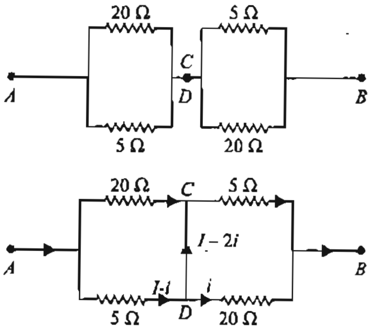


Fig. 5.297

$$V_A - V_D = V_A - V_C$$

or $\quad 20i = 5(I - i)$

or $\quad i = I/5$

$$I - 2i = I - \frac{2I}{5} = \frac{3I}{5} = \text{current flowing from } D \text{ to } C.$$

3. a., d.

The last three resistors 2 Ω, 4 Ω, and 2 Ω are in series having equivalent resistance 2 + 4 + 2 = 8 Ω. This will be in series with the 8 Ω next to them. So their equivalent resistance becomes 4 Ω. In this way net equivalent resistance of the circuit becomes R_{eq} = 9 Ω. This will be in series with r = 1 Ω. So current through 3 Ω is

$$I = e/(R + r) = 10/(9 + 1) = 1 \text{ A}.$$

Further, current will get divided at C and E into half at each point. So finally current reaching in 4 Ω will be 0.25 A.

4. a, d.

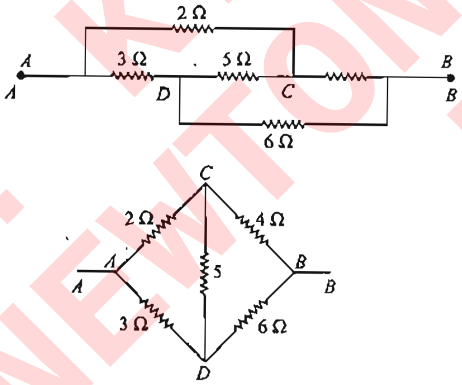


Fig. 5.298

Re-arrangement of the circuit as shown in Fig. 5.298 gives a balanced wheatstone bridge, and no current flows through the 5 Ω resistor. It can thus the removed from the circuit.

5. a., b., c.

Potential difference across resistance

= Potential difference across the terminals of the battery.

So $\quad V = E - Ir$

This is an equation of a straight line. Comparing this with the given graph, we can see that E = 10 V and r = 5 Ω.

Also $\quad I_{\max} = \dfrac{E}{r} = \dfrac{10}{5} = 2\text{A}$

When external resistance $R = 0$.

6. a., d.

2 Ω and 2 Ω are in series, 2 Ω and 4 Ω are in series then their resultant are in parallel. Producing net resistance R = 2.4 Ω

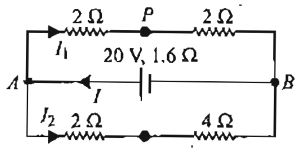


Fig. 5.299

$$I = \frac{20}{r + R} = \frac{20}{1.6 + 2.4} = 5 \text{ A}$$

$$I_1 + I_2 = 5\text{A} \qquad \text{(i)}$$

$$V_{AB} = 4I_1 = 6I_2 \;\Rightarrow\; 2I_1 = 3I_2 \qquad \text{(ii)}$$

From equations (i) and (ii), $I_1 = 3\text{A}$, $I_2 = 2\text{A}$

$$V_A - V_P = 6\text{V},\ V_A - V_Q = 2I_2 = 4\text{V}$$

Thus, $\quad V_Q - V_P = 2\text{V}$

7. a., b., d.

We know that the equivalent internal resistance is

$$R_{eq} = \frac{1}{\sum \frac{1}{R}} \Rightarrow R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$= \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

and equivalent e.m.f:

$$E_{eq} = \frac{\sum \frac{E}{R}}{\sum \frac{1}{R}} = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} + \frac{E_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Put $\quad E_3 = \dfrac{E_1R_2 + E_2R_1}{R_1 + R_2}$ to get

$$E_{eq} = \frac{E_1R_2 + E_2R_1}{R_1 + R_2} \Rightarrow E_{eq} = E_3$$

8. b., c., d.

7 Ω resistor is short-circuited, so no current will flow through it. Potential difference across each of 3 Ω and 6 Ω is 12 V, so we can find current in them.

9. a., c., d.

$$I = \frac{dQ}{dt} a - 2bt$$

$$I = 0 \text{ for } t = (a/2b) \text{ and } (dI/dt) = -2b$$

10. a., c., d.

For Loop $CEFD$: $-2(I - I_1) + 1I_1 + 9 = 0$

$\Rightarrow \; 3I_1 - 2I = -9 \qquad$ (i)

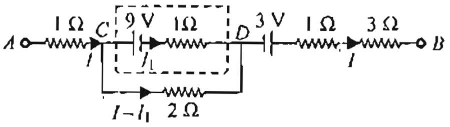


Fig. 5.300

from A to B via CD:

$$V_A - 4I - 9 + 3 - 1I_1 - 1I - 3I = V_B$$

but $\quad V_A - V_B = 16\text{V}$, so

$$\Rightarrow \quad 8I + I_1 = 10 \qquad \text{(ii)}$$

$$\Rightarrow \quad I_1 = -2\text{A}, I = 1.5\text{A}$$

$$I - I_1 = 1.5 + 2 = 3.5\text{ A}$$

$$V_C - 9 - 1I_1 = V_D \quad \Rightarrow \quad V_C - V_D = 7\text{ V}$$

11. c., d.

Note that the points a, h, g, and f have same potential. They are connected by conducting wires without any circuit elements between them. Similarly, points b, c, d, and e have the same potential. Hence the potential drop across branch e and f, and a and b is same. The two resistors (6 Ω and 4 Ω in series) are directly connected across the terminals of 12 V battery.

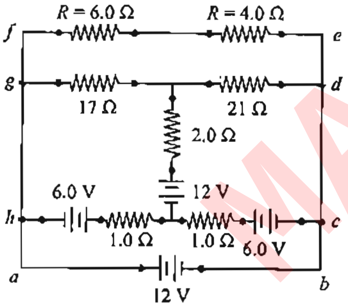


Fig. 5.301

The complex circuitry in the middle has no effect on the potential drop across the upper 10 Ω branch. If the current through it is I.

Potential drop across R_1, $V_1 = IR_1$;

Potential drop across R_2, $V_2 = IR_2$

Potential drop across branch, $V = V_1 + V_2 = I(R_1 + R_2)$

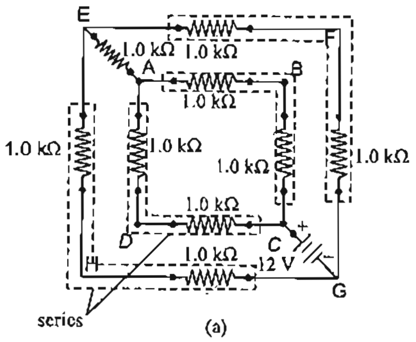
The current $\quad I = \dfrac{V}{R_1 + R_2} = \dfrac{12}{10} = 1.2$

Hence $\quad V_1 = (1.2)(6) = 7.2\text{ V}$

$$V_2 = (1.2)(4) = 4.8\text{ V}$$

12. a., b., d.

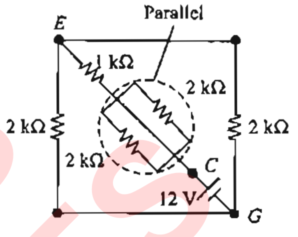
i. Imagine the wires to be flexible and lift up the inside square with the resistors and source attached [see Fig. 5.302 (a)].



(a)

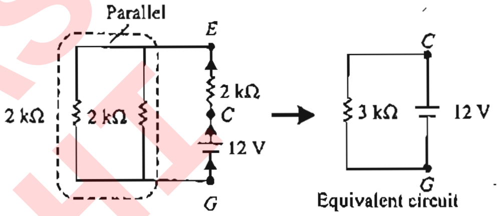
Follow Fig. 5.305 (a), (b), (c) and (d) to arrive at equivalent circuit. The equivalent resistance is 3 kΩ

ii. Since $\quad V = IR_{eq}$, $I = \dfrac{V}{R_{eq.}} = 4\text{ mA}$



(b)

iii. Start at point G, assign it a potential V_G, proceed toward E along any path. When you reach point E after adding potential drops and gains you get potential of E [see Figs. 5.302 (c) and (d)].



(c)

Equivalent circuit
(d)

Fig. 5.302

$$V_G + 12 - iR = V_E;$$

$$V_E - V_G = 12 - iR$$

$$= 12 - (4 \times 10^{-3} \times 2 \times 10^3) = 4$$

13. a., c.

When S is opened:

$$V_c - V_a = \frac{18 \times 6}{6 + 3} = 12\text{V}$$

$$V_c - V_b = \frac{18 \times 3}{6 + 3} = 6\text{V} \quad \Rightarrow \quad V_b - V_a = 12 - 6 = 6\text{ V}$$

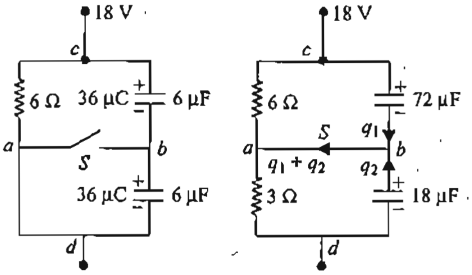


Fig. 5.303

Charges flown after S is closed:

$$q_1 = 72 - 36 = 36\,\mu\text{C}, q_2 = 36 - 18 = 18\,\mu\text{C}$$

Charges flown through S after it is closed:

$36 + 18 = 54\,\mu\text{C}$

Final potential of b is 6 V

14. a., d.

When X is joined to Y for a long time (charging), the energy stored in the capacitor = heat produced in $R = H_1$.

When X is joined to Z (discharging), the energy stored in $C(=H_1)$ reappears as heat (H_2) in R. Thus, $H_1 = H_2$.

15. c., d.

When X is joined to Y for a long time (charging), the energy stored in the capacitor = heat produced in $R = H_1$.

When X is joined to Z (discharging), the energy stored in $C(=H_1)$ reappears as heat (H_2) in R. Thus, $H_1 = H_2$.

16. b., c., d.

Disregard the capacitors and find the current through G. The potential difference across each capacitor is then found from the potential differences across the resistances in parallel with them.

17. b., c., d.

Just after closing the switch, capacitors will act like conducting wires. Find the equivalent resistance and calculate the current.

After a long time, the switch is closed, no current flows in the circuit as the capacitors act like infinite resistance.

Heat developed = $(1/2)C_{eq}V^2$.

18. a., b., c.

$$V_1 = \left(\frac{R_1}{R_1 + R_2 + R_3 + \cdots}\right)V = \left(\frac{R_1}{R_s}\right)V$$

$$\Rightarrow \quad V_2 = \left(\frac{R_2}{R_1 + R_2 + R_3 + \cdots}\right)V = \frac{R_2}{R_s}V$$

19. a., b., c., d.

Circuit A

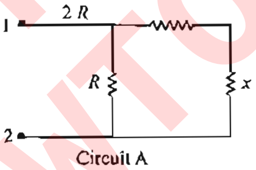


Fig. 5.304

$$X = \frac{R(2R+x)}{3R+x} \Rightarrow 3Rx + x^2 = 2R^2 + Rx$$

$$\Rightarrow \quad x^2 + 2Rx - 2R^2 = 0$$

$$\Rightarrow \quad x = -\frac{2R \pm \sqrt{4R^2 + 8R^2}}{2}$$

$$\Rightarrow \quad x = -\frac{2R + 2\sqrt{3}R}{2} \Rightarrow x = \left(\sqrt{3} - 1\right)R$$

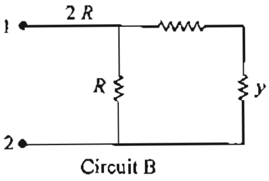


Fig. 5.305

Circuit B

$$Y = \frac{yR}{y+R} + 2R \Rightarrow y^2 + Ry = yR + 2Ry + 2R^2$$

$$\Rightarrow \quad y^2 - 2YR - 2R^2 = 0 \Rightarrow y = \left(\sqrt{3} + 1\right)R$$

20. a., b.

Resistance between diagonal corners of cube

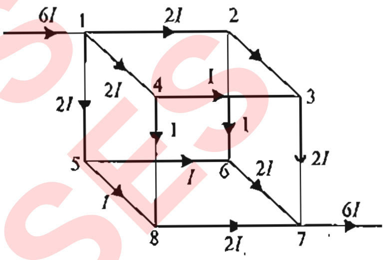


Fig. 5.306

$$\Rightarrow \quad V_{17} = (V_1 - V_5) + (V_5 - V_8) + (V_8 - V_7)$$

$$\Rightarrow \quad V_{17} = 2IR_0 + IR_0 + 2IR_0 \Rightarrow V_{17} = 5IR_0$$

Also, $V_{17} = x(6I) \Rightarrow x(6I) = 5IR_0$

$$\Rightarrow \quad x = \frac{5R_0}{6}$$

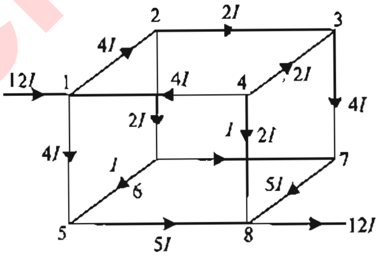


Fig. 5.307

$$\Rightarrow \quad V_{18} = (V_1 - V_s) + (V_s - V_8)$$

$$\Rightarrow \quad V_{18} = 4IR_0 + 5IR_0 \Rightarrow V_{18} = 9IR_0$$

Also $V_{18} = y(12I) \Rightarrow (121)y = 9IR_0$

$$\Rightarrow \quad y = \frac{3}{4}R_0 \Rightarrow \frac{x}{y} = \frac{\left(\frac{5}{6}\right)}{\left(\frac{3}{4}\right)} = \frac{20}{18} = \frac{10}{9}$$

$$\Rightarrow \quad x - y = \frac{5}{6}R_0 - \frac{3}{4}R_0 \Rightarrow x - y = \frac{R_0}{12}$$

Assertion-Reasoning Type

1. c. The resistance of a wire is

$$R = \rho\frac{l}{A}, \ \rho \text{ being specific resistance}$$

$$\text{or} \quad R \propto \frac{1}{A^2} \text{ or } R \propto \frac{1}{A^4} \qquad (\because A = \pi r^2)$$

Hence, when the diameter is halved the resistance of the wire is

$$R \propto \frac{1}{\left(\frac{r}{2}\right)^2} = 16R \qquad (1)$$

Hence, its resistance will become 16 times.
Again from eq. (1),

$$R \propto \frac{1}{A} \quad \text{or} \quad R \propto \frac{l^2}{Al} \quad \text{or} \quad R \propto l^2$$

Therefore, on increasing the length the resistance increases.

2. c. On increasing the temperature of metals, the resistance of metal increases. Therefore, the temperature coefficient of resistance of metals is positive.

On increasing the temperature of insulators, the resistance decreases. Therefore, temperature coefficient of resistance of insulators is negative.

3. d. Resistivity or specific resistance is a material property. So, it does not change on bending the insulated wire.

On bending, the cross-sectional area of wire charges but drift velocity of electron does not depend on area of cross-section so it does not change.

4. a. $v_d = \frac{I}{neA}$, If radius is doubled, A becomes 4 times and hence V_d becomes one fourth.

5. a. Both $R/2$ are in parallel, so their equivalent resistance is $R/4$.

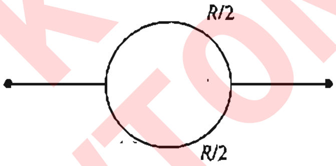


Fig. 5.308

6. b. Both the statements are correct, but independent of each other.

7. d. It is true that resistance of a wire is directly proportional to its length, but here when length is doubled, area of cross-section decreases as the volume remains constant. Finally, resistance becomes n^2 times.

8. c. Smaller is the resistance, more is the current in parallel.

9. a. In series, current in both resistances will be same. For same current, more is resistance more is the potential drop.

10. a. Copper is a conductor and germanium is a semi-conductor.

11. c. Drift velocity is directly proportional to electric field. If there is no electric field, then no drifting of electrons in a particular direction, hence no current in the conductor.

12. (a) $V = IR \Rightarrow V = neAv_d \frac{\rho\ell}{A} \Rightarrow v_d = \frac{V}{ne\rho\ell} \Rightarrow v_d \propto \frac{1}{\ell}$

Comprehension Type

For Problems 1 – 3

1. d. 2. a. 3. b.

Sol.

a. The current are as shown.

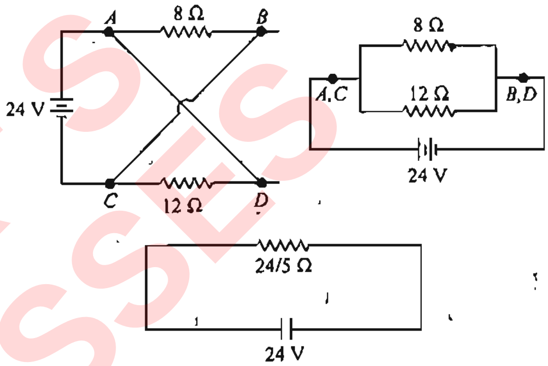


Fig. 5.309

$$I = \frac{24}{24/5} = 5\,\text{A}$$

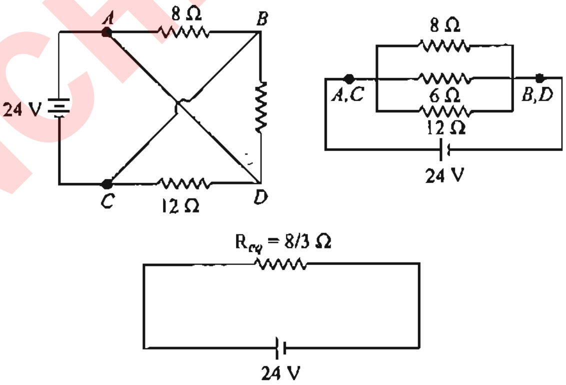


Fig. 5.310

$$\frac{1}{R_{eq}} = \frac{1}{8} + \frac{1}{6} + \frac{1}{12} \Rightarrow R_{eq} = \frac{8}{3}\,\Omega$$

$$I = \frac{24}{8/3} = 9\,\text{A}$$

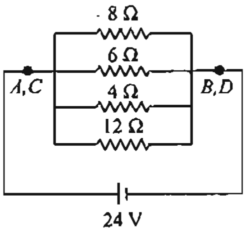


Fig. 5.311

$$\frac{1}{R_{eq}} = \frac{1}{8} + \frac{1}{6} + \frac{1}{4} + \frac{1}{12}$$

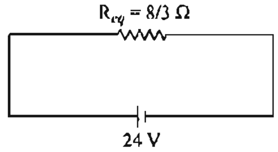


Fig. 5.312

$$R_{eq} = \frac{8}{5}\,\Omega \qquad I = \frac{24}{8/5} = 15\text{ A}$$

For Problems 4 – 6

4. c., 5. a., 6. b.

Sol.

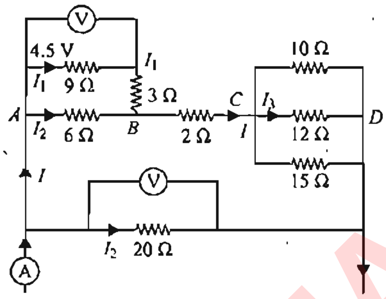


Fig. 5.313

$$I_1 = \frac{4.5}{9} = \frac{1}{2}\text{ A}$$

Potential difference across AB:

$$4.5 + 3I_1 = 6I_2 = I_2 = 1\text{A} \quad I = I_1 + I_2 = 1.5\text{ A}$$

Equivalent resistance between C and D:

$$R_1 = 4\,\Omega$$

Potential difference across

$$CD = R_1 I = 4 \times 1.5 = 6\text{ V}$$

Current through 12 Ω

$$I_3 = \frac{6}{12} = \frac{1}{2}\text{ A}$$

5. Potential difference across

$$AD = 6I_2 + 2I + 12I_3$$
$$= 6 \times 1 + 2 \times 1.5 + 12 \times 0.5 = 15\text{ V}$$

This will be equal to the reading of voltmeter across 20 Ω

6. $I_4 = \dfrac{15}{20} = 0.75\text{ A}$

Reading of ammeter $= I + I_4 = 1.5 + 0.75 = 2.25\text{ A}$

For Problems 7 – 9

7. d., 8. c., 9. d.

Sol.

7. Given $V_1 = \dfrac{V_0}{k},\ V_2 = \dfrac{V_1}{k},\ V_3 = \dfrac{V_2}{k}$

$$I = I_1 + I_2$$

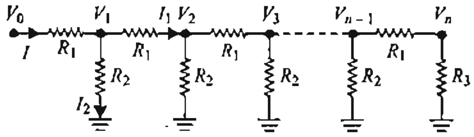


Fig. 5.314

$$\frac{V_0 - V_1}{R_1} = \frac{V_1 - V_2}{R_1} + \frac{V_1 - 0}{R_2}$$

$$\frac{V_0 - V_1/k}{R_1} = \frac{V_0/k - V_0/k^2}{R_1} + \frac{V_0/k}{R_2}$$

$$\frac{R_1}{R_2} = \frac{(k-1)^2}{k}$$

8. Current in R_1 and R_3 will be same:

$$\frac{V_{n-1} - V_n}{R_1} = \frac{V_n}{R_3}$$

$$\frac{V_{n-1} - \dfrac{V_{n-1}}{k}}{R_1} = \frac{V_{n-1}}{k\,R_3}$$

$$R_1 = R_3\,(k-1)$$

Put the value of R_1 in (i): $\dfrac{R_2}{R_3} = \dfrac{k}{k-1}$

9. Current in R_2 nearest to V_0:

$$I_2 = \frac{V_1}{R_2} = \frac{V_0/k}{R_3\left(\dfrac{k}{k-1}\right)} = \left(\frac{k-1}{k^2}\right)\frac{V_0}{R_3}$$

For Problems 10 – 12

10. a., 11. d., 12. c.

Sol.

10. $q = \int i dt =$ area of given curve $= \dfrac{1}{2} i_0 t_0$

11. $\dfrac{i}{i_0} + \dfrac{t}{t_0} = 1 \quad \Rightarrow \quad i = i_0\left(1 - \dfrac{t}{t_0}\right)$

12. Heat $= \int i^2 R\,dt = \int_0^{i_0} i^2 R\left(1 - \dfrac{t}{t_0}\right)^2 dt$

$$H = \left.\frac{i^2 R\left(1 - \dfrac{t}{t_0}\right)}{3\left(-\dfrac{t}{t_0}\right)}\right]_0^{i_0} \quad \Rightarrow \quad H = \frac{R t_0 i_0^2}{3}$$

For Problems 13 – 16

13. c., 14. a., 15. c, 16. c.

Sol.

13. c. When key K_1 is open the circuit is a balanced Wheatstone bridge.

The equivalent resistance is given by (for the bridge):

$$R_{eq} = \frac{20 \times 40}{20 + 40}\,\Omega = \frac{40}{3}\,\Omega$$

The total resistance $= R_{eq} + 10\,\Omega = 23.3\,\Omega$

The current $i = \dfrac{55 \times 3}{70}$ amp $= 33/14$ A

14 When K_1 is open, the bridge is balanced

$\Rightarrow \quad v_{BD} = 0 \quad \Rightarrow$ charge on capacitor = 0

when K_1 is closed, we have two different sources of e.m.f. Applying Kirchoff's laws, we get,

For loop $ABCDA$:

$i_1 8 + (i_1 - i_2) \,.\, 12 - (i' - i_1) \times 40 = 0$

loop $BDCB$:

$i_2 \,.\, 2 - (i_1 - i_2) \times 12 = -10$

loop $ABCYA$:

$i_1 \,.\, 8 + (i_1 - i_2) \times 12 + i' \times 10 = 55$

solving, we get: $i_1 = 2$, $i_2 = 1$, $i' = 2.7$ (amp).

Change on capacitor = 4.8 μC

15. The current i, when k_1 was open = 2.36 A

the current i', when k_1 was closed = 2.7 A.

$\therefore$ the change in the current = 0.34 A

For Problems 17 – 19

17. c., 18. b., 19. d.

Sol.

In the steady state, the equivalent circuit is

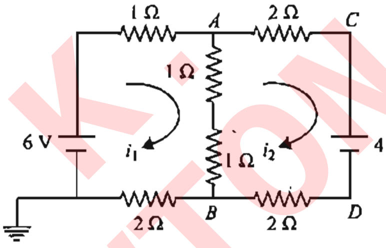


Fig. 5.315

$$5i_1 - 2i_2 = 6$$
$$6i_2 - 2i_1 = -4$$
$$13i_2 = 14$$
$$i_1 = \frac{14}{13}\text{ A},\ i_2 = -\frac{4}{13}\text{ A}$$

18. Potential difference across BC

$$V_A = 6 - \frac{14}{13} = \frac{64}{13}\text{ V}$$

$$V_0 = \frac{64}{13} - 2 \quad (-4/13) = \frac{72}{13}\text{ V}$$

$$V_B = \frac{64}{13} - 1 \times \left\{\frac{14}{13} + \frac{4}{13}\right\} = \frac{46}{13}\text{ V}$$

$$V_{DB} = \frac{72 - 46}{13} = \frac{26}{13} = 2\text{ V}$$

19. Charge on capacitor

$$= CV_{BD} = 4 \times 10^{-6} \times 2 = 8\,\mu\text{C}$$

Matching Column Type

1. i. → a., ii. → c, iii. → b., iv. → d.

Sol. The equivalent circuit is as shown in Fig. 5.316 (b).

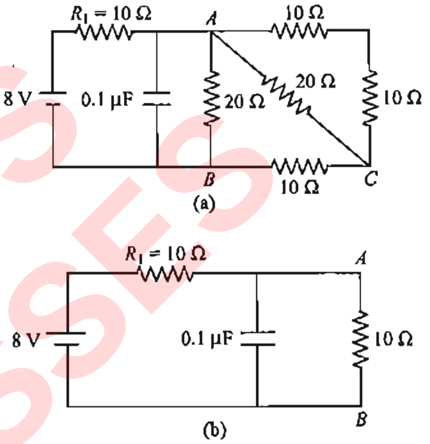


Fig. 5.316

The current through $R_1 = \dfrac{8}{20} = 0.4$ A

In the steady state, the potential difference across AB is 4 V.

Charge on capacitor in steady state is

$$q = CV = 0.4\,\mu\text{C}$$

Current through resistor R is $I = \dfrac{V}{R} = \dfrac{4}{20} = 0.2$ A

2. i. → c., ii. → b., iii. → b., iv. → a.

Sol. When a steady state is reached, no current passes through the capacitor or the branch CE.

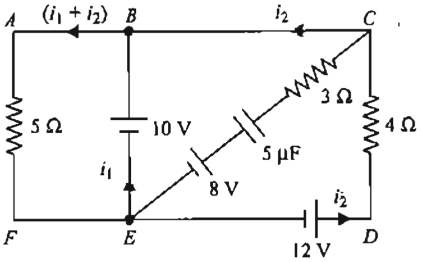


Fig. 5.317

Considering the loop $ABEFA$,

$$5 \times (i_1 + i_2) = 10 \text{ or } i_1 + i_2 = 2\text{ A} \qquad \text{(i)}$$

Considering the loop $BCDEB$

$$4i_2 = 12 - 10 = 2 \quad \Rightarrow \quad i_2 = 0.5\text{ A}$$

So, $\quad i_2 = 2 - 0.5 = 1.5$ A **[i – c]**

To find the charge on capacitors, we must know potential difference across the plates.

Consider the loop $CEDC$:

$$-12 + 4i_2 + 3 \times 0 - v_C + 8 = 0$$

Or $v_C = -2$V. So charge on capacitor $Q = cv = 10\,\mu$C

3. **i.** → **b., ii.** → **a.,b.,c., iii.** → **b., iv.** → **a.,b.,c.,d.**

Sol.

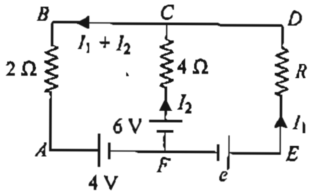


Fig. 5.318

Loop *FEDCF*: $e - 6 = RI_1 - 4I_2$ (i)

Loop *AFCBA*: $6 - 4 = 4I_2 + 2(I_1 + I_2)$

$2 = 2I_1 + 6I_2$ (ii)

Solving them we get:

$$I_1 = \frac{3e - 14}{4 + 3R},\ I_2 = \frac{R + 6 - e}{4 + 3R}$$

i. $I_2 = 0 \quad \Rightarrow \quad e = R + 6$

$e > 6\,\text{V}\ (\because \quad R \neq 0)$

ii. For current from *F* to *C* direction

$$I_2 > 0 \quad \Rightarrow \quad R + 6 > e \quad \Rightarrow \quad e < R + 6$$

possible for any finite value of *e*, because *R* is finite

iii. For current from *F* to *C* direction

$$I_2 < 0 \quad \Rightarrow \quad e > R + 6$$

iv. For current in 2 Ω from *B* to *A* direction

$$I_1 + I_2 = \frac{R - 8 + 2e}{4 + 3R} > 0$$

$$R - 8 + 2e > 0 \quad \Rightarrow \quad e > 4 - \frac{R}{2}$$

Depending upon the value of *R*, *e* can take any value from zero to infinity.

CHAPTER
6

Electrical Measuring Instruments

- Galvanometer
- Ammeter
- Voltmeter
- Potentiometer
- Meter Bridge or Slide Wire Bridge

GALVANOMETER

It is used to detect very small current. It has negligible resistance. A pivoted coil of fine wire is placed in the magnetic field of a permanent magnet (as shown in Fig. 6.1). Attached to the coil is a spring, similar to the hairspring on the balance wheel of a watch. In equilibrium position, with no current in the coil, the pointer is at zero. When there is current in the coil, the magnetic field exerts a torque on it that is proportional to the current. As the coil turns, the spring exerts a restoring torque that is proportional to the angular displacement.

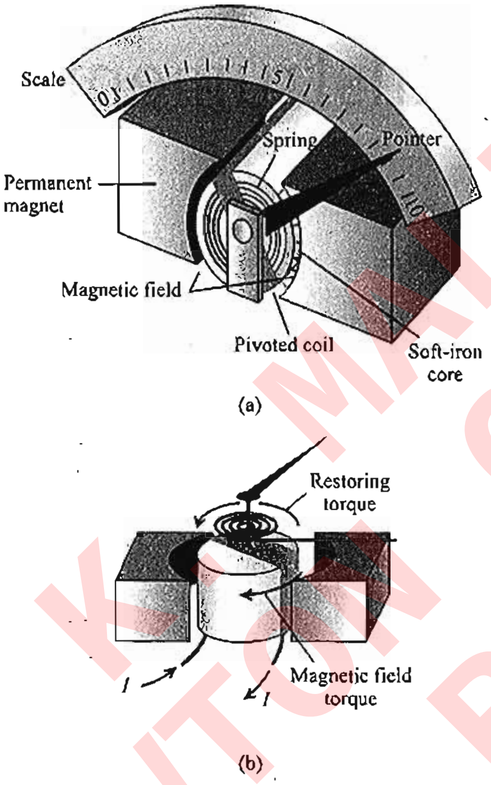


Fig. 6.1

AMMETER

It is an instrument used to measure current. It is put in series with the branch in which current is to be measured. We all know that galvanometer is a device to detect current in a circuit. The device has a needle in it. When current passes through this device, the needle deflects.

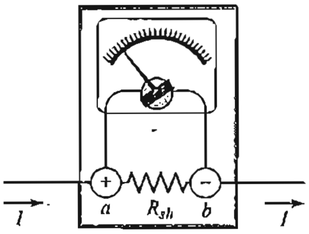


Fig. 6.2

Why Should the Needle Deflect When There is Current?

The mechanism of deflection is that of a coil and a needle, which is subject to magnetic field. Whenever there is current, torque develops in the coil. This torque is responsible for rotation of the coil and hence the needle. This is the reason the needle deflects when there is current.

The deflection of the needle is proportional to the current passing through it. By appropriate calibration, it is possible to use the galvanometer as an ammeter, a current measuring device.

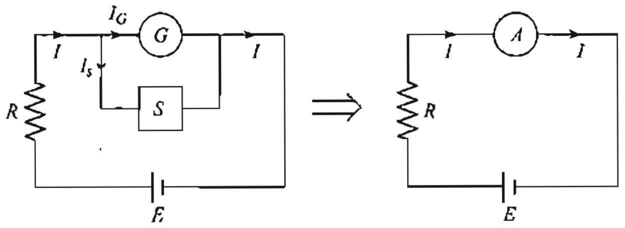
Problem in Using Galvanometer Itself as an Ammeter

As we know that the galvanometer is an electrical device, it has its own resistance. If we insert galvanometer in an electrical circuit, the total resistance of the circuit increases and hence the very current to be measured decreases. If we can bring the resistance of the galvanometer to zero, it is possible to measure the actual current.

Conversion of a Galvanometer into an Ammeter

From the concept of resistors in parallel, we know that the effective resistance of parallel resistors is less than the least value. Therefore, if we connect a very small resistance (S, generally called a shunt resistor) across the galvanometer, the effective resistance of this device can be less than S. Note that we can never make effective resistance reach zero, but can make it nearly zero. This process is called 'conversion of galvanometer into ammeter'.

Fig.6.3 depicts the above discussion.



Conversion of galvanometer into ammeter

Fig. 6.3

Let I be the current to be measured. This current gets divided into I_G and I_S. I_G is the current through the galvanometer and I_S is the current through the shunt resistor. As shunt resistance is very small, $I_S >> I_G$. The current through the galvanometer is responsible for deflection of the needle. We have to use this fraction of current (I_G) in measuring the actual current I. This is the case with all the measuring devices. We can write: $I = I_G + I_S$.

Now we have to find the relation between the current through the galvanometer (I_G) and the current (I) to be measured.

As the potential difference across G is the same as that across S, we can write $I_G G = I_S S = (I - I_G)S$

$$\text{Therefore, } I_G = \left(\frac{S}{G+S}\right) I \quad \Rightarrow \quad I_G \propto I$$

From the above relation, I_G is proportional to I. It is clear that the deflection of the needle is proportional to the current I. If the value of current I increases, the deflection increases. The scale can be graduated to read the value of I directly. Thus measuring of current becomes possible.

Maximum Current an Ammeter can Read

To know this, one must know the maximum value of I_G, the maximum current which can pass through the galvanometer. It is also known as full-scale deflection current. If we pass current of intensity more than I_G through a galvanometer, the galvano-meter may get damaged. This is a standard value for a given galvanometer.

As an example, let us suppose that when the current through the ammeter is $I = 2$ A, current through the galvanometer is 5 mA and let's suppose that this is the maximum value of I_G for full-scale deflection which can pass through the galvanometer. Therefore, we can say that the maximum current the ammeter can read is 2 A. For any other current less than 2 A, value of current through galvanometer will be less than 5 mA. For any other current greater than 2 A, deflection is not possible and hence no reading is possible. Now we can say that the range of the given ammeter is 0 – 2 A.

Modification of Ammeter to Obtain Other Range

As the maximum values of I_G and G are constant for a given galvanometer, by varying the value of shunt resistance we can vary the value of I. Therefore by selecting the value of I, we can find the value of shunt resistance. For this shunt value, the range of ammeter can be 0 to I (Fig. 6.4).

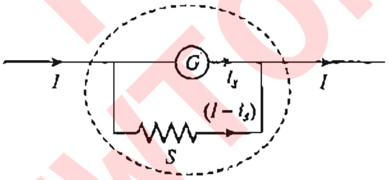


Fig. 6.4

Thus $S(I - I_G) = I_G G \Rightarrow S = \dfrac{I_G G}{I - I_G}$

Note:

1. ***The reading of an ammeter is always lesser than the actual current in the circuit.***

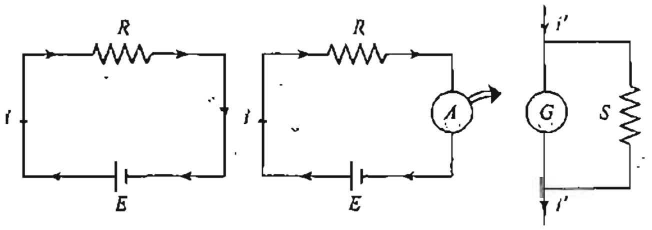


Fig. 6.5

For example, in Fig.6.5, actual current through R is

$$i = \frac{E}{R} \qquad \text{(i)}$$

while the current after connecting an ammeter of resistance $A\left(=\dfrac{GS}{G+S}\right)$ *in series with R is,*

$$i' = \frac{E}{R+A} \qquad \text{(ii)}$$

From (i) and (ii), we see that $i' < i$. To measure correct current, we should have $i' = i$. This is possible when $A = 0$. This kind of ammeter is known as ideal ammeter. So resistance of an ideal ammeter should be zero.

2. *Percentage error in measuring a current through an ammeter is*

$$\left(\frac{i-i'}{i}\right)\times 100 = \left(\frac{\frac{1}{R}-\frac{1}{R+A}}{\frac{1}{R}}\right)\times 100$$

or $$\% \text{ error} = \left(\frac{A}{R+A}\right)\times 100$$

Illustration 6.1 **A galvanometer has a resistance of 50 Ω and its full-scale deflection current is 50 μA. What resistance should be added so that the ammeter can have a range 0 – 5 mA?**

Sol. Here, the maximum value of $I_G = 50$ μA. The upper limit gives the maximum current to be measured which is $I = 5$ mA. The galvanometer resistance, $G = 50$ Ω.

From the relation:

$$S = \frac{I_G G}{I - I_G} = \frac{50\times10^{-6}\times50}{5\times10^{-3}-50\times10^{-6}} \approx 0.5\,\Omega$$

If we work out, we would understand that higher the range of ammeter, lower is the value of shunt resistance.

VOLTMETER

It is an instrument used to find the potential difference across two points in a circuit.

As we know that current and potential difference are related, we can express the current used for deflection as a function of potential difference to be measured. Hence we can use the galvanometer as a voltmeter.

Unsuitability of the Galvanometer as Voltmeter

Suppose we connect a galvanometer across a resistor R_1 as shown in Fig. 6.6 to find the potential difference across resistor R_1.

As the galvanometer has its own resistance, the total resistance of the circuit changes and hence the very potential difference to be measured becomes different. That is why we cannot use the galvanometer as a voltmeter. To keep the current same, we need to make the resistance of galvanometer very high. Hence the current through the galvanometer will be small. Ideally, it has to be infinite so that current through galvanometer is zero.

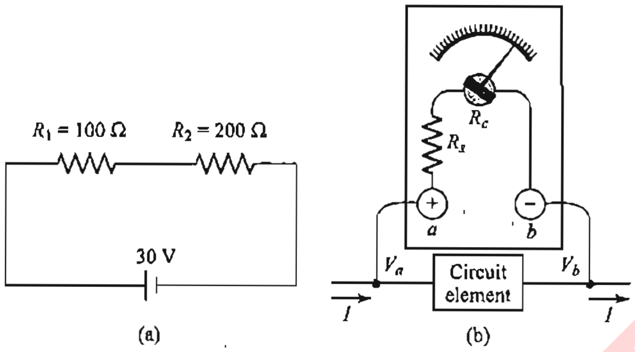


Fig. 6.6

Conversion of Galvanometer into a Voltmeter

From the concept of resistors in series, we know that the effective resistance of resistors in series is greater than the greatest value. Therefore, if we connect a very high resistance (R_h) in series with the galvanometer, the effective resistance of this device can be greater than R_h (Fig.6.7). Note that we can never make effective resistance reach infinite, but can make it nearly infinite. This process is called 'conversion of galvanometer into voltmeter'.

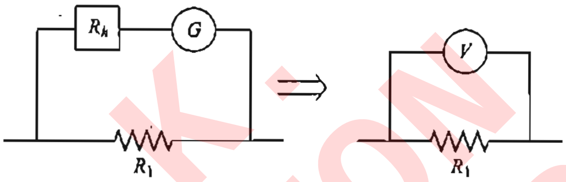


Fig. 6.7

Let V be the potential difference (p.d.) across the resistor. By parallel circuit, p.d. across voltmeter is also V. Let I_G be the current through the galvanometer.

Hence $\qquad V = I_G\,G + I_G R_h$

Therefore $\qquad I_G = \dfrac{V}{R_h + G} \;\Rightarrow\; I_G \propto V$

Note that the deflection is proportional to the current I_G and hence to V. The scale can be calibrated to read the potential difference directly.

To find the maximum potential difference (V) a voltmeter can read, one must know the maximum value of I_G for full-scale deflection of the galvanometer. This is a standard value for a given galvanometer. Let us suppose that 0.1 mA is the maximum value of I_G for full scale deflection, when the p.d. is 5 V. Therefore, we can say that the maximum p.d. a voltmeter can read is 5 × 2 V. For any other p.d. less than 5 V, I_G value is less than the maximum value. Therefore, reading is possible. For any other p.d. greater than 5 V, deflection is not possible and hence no reading is possible. Now we can say that the range of the given voltmeter is 0 – 5 V.

Modifying Voltmeter to have Desired Range (say 0 to V)

As the maximum values of I_G and G are constant for a given galvanometer; by varying the value of shunt resistance we can vary the value of V. Therefore by selecting the value of V (say V), we can find the value of shunt resistance. For this shunt resistance value, the range of voltmeter can be 0 to V.

It is essential that the resistance $R_v = G + R$ of a voltmeter be very large as compared to the resistance of any circuit element with which the voltmeter is connected. Otherwise, the metre itself becomes an important circuit element and alters the potential difference that is measured. For an ideal voltmeter $R_v = \infty$ (Fig. 6.8)

Now $\qquad I_G(G + R) = V \;\Rightarrow\; R = \dfrac{V}{I_G} - G$

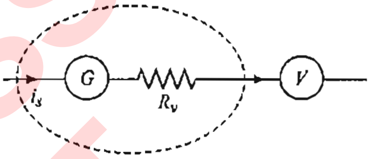


Fig. 6.8

Note:

1. *The reading of a voltmeter is always lesser than the true value.*

   For example, if a current, i, is passing through a resistance, r, the actual value is $V = ir$

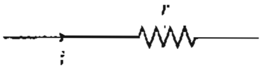


Fig. 6.9

Now if a voltmeter of resistance R_v (= G + R) is connected across the resistance r, the new value will be

$$V = ir \qquad \text{(i)}$$

$$V' = \frac{i \times (rR_v)}{r + R_v} \quad or \quad V' = \frac{ir}{1 + \dfrac{r}{R_v}} \qquad \text{(ii)}$$

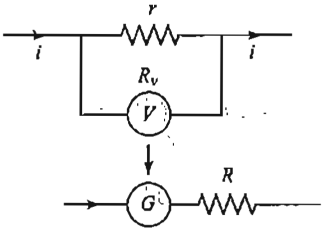


Fig. 6.10

From eqs. (i) *and* (ii), *we can see that*

$$V' < V \;\Rightarrow\; V' = V \quad if \quad R_v = \infty$$

2. *Percentage error in measuring the potential difference by a voltmeter is,*

$$\left(\frac{V-V'}{V}\right)\times 100=\left(\frac{1}{1+\frac{r}{R_V}}\right)\times 100$$

or
$$\boxed{\%\ error\left(\frac{1}{1+\frac{r}{R_V}}\right)\times 100}$$

Illustration 6.2 **A galvanometer has a resistance of 50 Ω and its full scale deflection current is 50 μA. What resistance should be added to it so that it can have a range of 0 – 5V?**

Sol. Here, the maximum value of I_G = 10 μA. The upper limit gives the maximum voltage to be measured which is $V = 5$ V. The galvanometer resistance, $G = 50\ \Omega$.
From the above relation, $R_h \approx 100$ kΩ.
If we work out, we would understand that higher the range of voltmeter, higher is the value of shunt resistance.

Illustration 6.3 **What is the value of shunt which passes 10% of the main current through a galvanometer of 99 Ω?**

Sol. A shunt is a small resistance, S, in parallel with a galvanometer (of resistance G) as shown in Fig. 6.11.

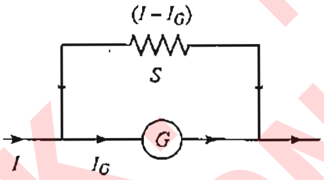


Fig. 6.11

$$(I-I_G)\,S=I_G{}^*G,\quad \text{i.e.,}\quad S=\frac{I_G G}{(I-I_G)}$$

And as here, $G = 99\ \Omega$ and $I_G = (10/100)\,I = 0.1I$

$$S=\frac{0.1I\times 99}{(I-0.1I)}=\frac{0.1}{0.9}\times 99=11\Omega$$

Illustration 6.4 **The deflection in a moving coil galvanometer falls from 50 divisions to 10 divisions when a shunt of 12 Ω is applied. What is the resistance of the galvanometer?**

Sol. In case of a galvanometer, $I \propto \theta$

So, $$\frac{I_G}{I}=\frac{10}{50}=\frac{1}{5},\quad \text{i.e.,}\quad I_G=\frac{1}{5}I$$

Now as in case of a shunted galvanometer as shown in Fig. 6.12.

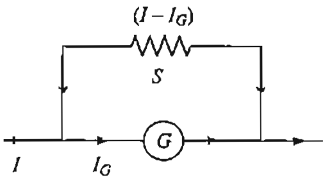


Fig. 6.12

$$(I-I_G)\,S=I_G,\quad \text{i.e.,}\quad (I-\frac{1}{5}I)\times 12=\frac{1}{5}IG$$

So, $$G=4\times 12=48\,\Omega$$

Illustration 6.5 **A galvanometer has a resistance of 30 Ω and a current of 2 mA is needed for a given full scale deflection. What is the resistance and how is it to be connected to convert the galvanometer (a) into an ammeter of 0.3 A range (b) into a voltmeter of 0.2 V range?**

Sol. As here galvanometer resistance G = 30Ω and full scale deflection current I_g = 2mA, so,

(a) To convert the galvanometer into an ammeter of range 0.3 A, a resistance of value S is connected in parallel with it such that

$$(I-I_g)\,S=I_g\,G,\ (0.3-0.002)\,S$$
$$=0.002\times 3,$$
or, $$S=\frac{0.002\times 30}{0.298}$$
$$=0.2013\,\Omega$$

(b) To convert the galvanometer into a voltmeter of range 0.2 V, a resistance R is connected in series with it such that

$$V=I_g\,(R+G),$$
i.e., $$0.2=2\times 10^{-3}(30+R)$$
i.e., $$R=100-300=70\,\Omega$$

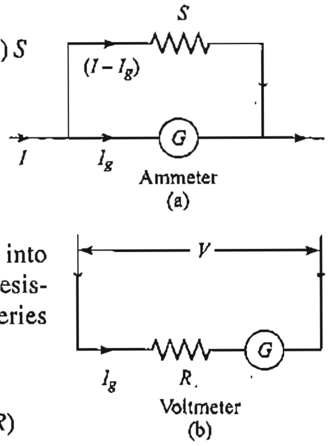


Fig. 7.13

Illustration 6.6 **The scale of a galvanometer is divided into 150 equal divisions. The galvanometer has current sensitivity of 10 divisions per mA and a voltage sensitivity of 2 divisions per mV. How can the galvanometer be designed to read (i) 6A, per division and (ii) 1 V, per division?**

Sol. As per the resistance of galvanometer,

$$G\approx\frac{\text{Full scale voltage}}{\text{Full scale current}}=\frac{75\times 10^{-3}}{15\times 10^{-3}}=5\,\Omega$$

For conversion into ammeter of range I A.

$$(I-I_g)\,S=I_g\,G$$
$$\text{S}=\frac{I_g G}{I-I_g}$$
$$=\frac{15\times 10^{-3}\times 5}{\left(150\times 6-15\times 10^{-3}\right)}$$
$$=\frac{15\times 10^{-3}\times 5}{150\times 6}$$
$$=8.3\times 10^{-5}\,\Omega$$

For conversion into voltmeter of range V volt, $I_g\,(G+R)=V$

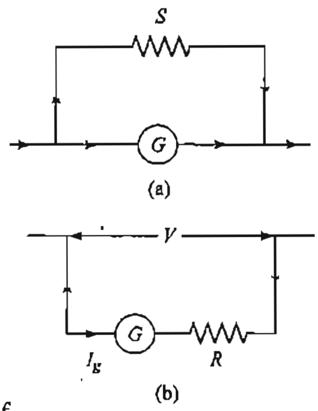


Fig. 7.14

$$R = \frac{V}{I_g} - G = \frac{150}{15\times10^{-3}} - 5 = 9995\,\Omega.$$

POTENTIOMETER

Potentiometer is an instrument that can measure the terminal potential difference with high accuracy without drawing any current from the unknown source.

It is based on the principle that if constant current is passed through a wire of uniform cross-section, the potential difference across any segment of the wire is proportional to its length.

Fig. 6.15 shows a typical arrangement to measure e.m.f. E_x of a battery.

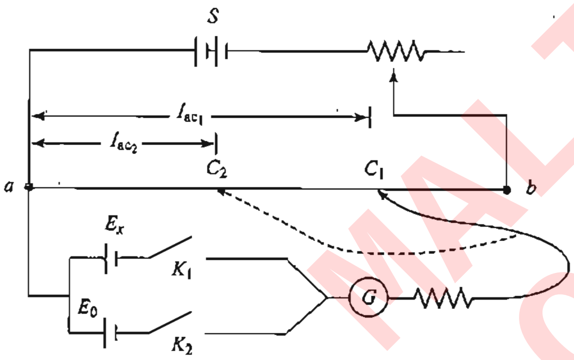


Fig. 6.15

Wire *ab* of uniform cross-section carries a constant current supplied by battery *S*. First switch K_1 is closed and K_2 is kept open. The slider is moved on the wire *ab* till we get zero deflection in the galvanometer. If C_1 is the corresponding point in the wire, $E_x = V_{ac_1}$

Now, the experiment is repeated with key K_1 open and K_2 closed. This time if the null deflection is obtained on contact with wire at C_2, $E_0 = V_{ac_2}$ (E_0 is known)

Now, $\frac{E_x}{E_0} = \frac{V_{ac_1}}{V_{ac_2}} = \frac{l_{ac_1}}{l_{ac_2}}$ where l_{ac_1} and l_{ac_2} are the lengths, of segments ac_1 and ac_2, respectively.

Comparison of e.m.f.s of Two Cells Using Potentiometer

To compare E_1 and E_2 as shown in Fig. 6.16, close the keys *K* and K_1, so that cell of e.m.f. E_1 is in the circuit. Move the jockey *J* on the wire *AB* and locate the position at which galvanometer shows no deflection. At this stage potential difference between *A* and *J* is equal to e.m.f. E_1 of the cell.

Hence, $E_1 \propto$ length AJ

$E_1 \propto l_1$, where $AJ = l_1$

or $E_1 = k\,l_1$, (i)

where *k* is a constant of proportionality.

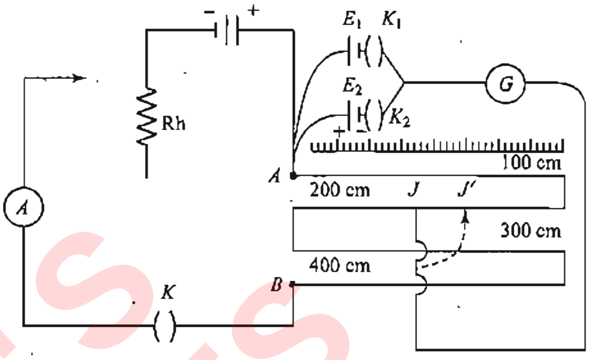


Fig. 6.16

Remove the key K_1 and close key K_2, so that cell E_2 comes in the circuit. Again find the position of the jockey on the wire, where galvanometer shows no deflection. Let this position be J'. At this stage p.d. between AJ' is equal to the e.m.f. E_2 of the cell.

Hence, $E_2 \propto$ length AJ'

$\Rightarrow$ $E_2 \propto l_2$, where $AJ' = l_2$

or $E_2 = kl_2$, (ii)

Dividing eqn. (i) and (ii), we get $\frac{E_1}{E_2} = \frac{\ell_1}{\ell_2}$

Thus knowing lengths l_1 and l_2, we can calculate the ratio of e.m.f.s of the two cells.

Determination of Internal Resistance of a Cell

Make the connections as shown in the circuit diagram in Fig. 6.17. Check the connections as explained in previous section.

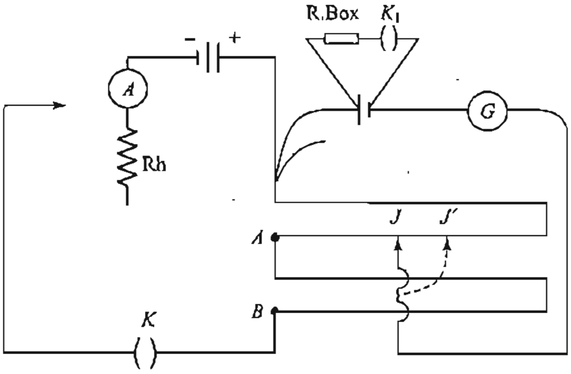


Fig. 6.17

Close the key *K* only (key K_1 is open). Move the jockey on wire *AB* and locate a point *J* at which the galvanometer shows no deflection. At this stage the p.d. across *AJ* is equal to the e.m.f. of the cell, i.e., *E* ($\because$ cell is in the open circuit)

$E \propto$ length AJ

$AJ = l_1$, then

$E \propto l_1$ or $E = kl_1$ (iii)

Now, close the key K_1 also and introduce some resistance (say *R*) from the resistance box.

Again locate the position of jockey on wire AB till galvanometer shows no deflection. Let it be at J'. At this stage p.d. across length AJ' = terminal p.d. of cell because cell is now in closed circuit.

$$V \propto \text{length } AJ'$$

$$V \propto l_2, \text{ where } AJ' = l_2$$

or $$V = Kl_2 \quad \text{(iv)}$$

Dividing equation (iii) by equation (iv), we get $\dfrac{E_1}{E_2} = \dfrac{\ell_1}{\ell_2}$ (v)

We know that the internal resistance of a cell is given by

$$r = \left(\frac{E - V}{V}\right)R \quad \text{or} \quad r = \left(\frac{E}{V} - 1\right)R$$

Using equation (v), we have $r = \left(\dfrac{\ell_1}{\ell_2} - 1\right)R$

or $$r = \left(\frac{\ell_1 - \ell_1}{\ell_2}\right)R \quad \text{(vi)}$$

Thus knowing l_1, l_2 and R, we can calculate the value of r, the internal resistance of the cell.

Illustration 6.7 **Fig. 6.18 shows a potentiometer circuit for comparison of two resistances. The balance point with a standard resistor R = 10.0 Ω is found to be 58.3 cm, while that with the unknown resistance X is 68.5 cm. Determine the value of X. What would you do if you fail to find a balance point with the given cell E?**

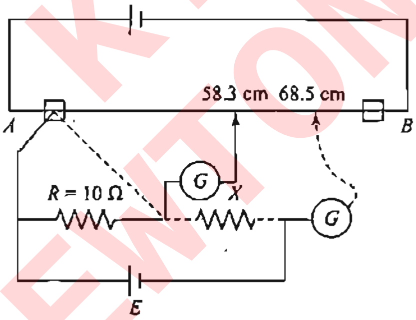


Fig. 6.18

Sol. Here, l_1 = 58.3 cm, l_2 = 68.5 cm, R ≈ 10 Ω, X = ?

Let I be the current in the potentiometer wire and E_1 and E_2 be the potential drops across R and X, respectively. Then

$$\frac{E_2}{E_1} = \frac{IX}{IR} = \frac{X}{R} \quad \text{or} \quad X = \frac{E_2}{E_1}R \quad \text{(i)}$$

But $$\frac{E_2}{E_1} = \frac{l_2}{l_1} \quad \text{From (i)}$$

$$X = \frac{l_2}{l_1}R = \frac{68.3}{58.3} \times 10.0 = 11.75\ \Omega$$

If there is no balance point with given cell of e.m.f. E, it means potential drop across R or X is greater than the potential drop across the potentiometer wire AB. In order to obtain the balance point, the potential drops across R and X are to be reduced, which is possible by reducing the current. For that, either a suitable resistance should be put in series with R and X or a cell of smaller e.m.f. should be used. Another possible way is to increase the potential drop across the potentiometer wire by increasing the voltage of driver cell.

Illustration 6.8 **Fig. 6.19 shows a 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 76.3 cm. When a resistor of 9.5 Ω is used in the external circuit of the cell, the balance point shifts to 64.8 cm, length of the potentiometer. Determine the internal resistance of the cell.**

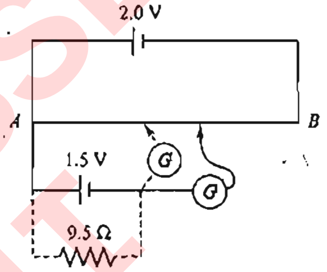


Fig. 6.19

Sol. Here, l_1 = 76.3 cm, l_2 = 64.8 cm, r = ?, R = 9.5 Ω

$$\text{Now, } r = \left(\frac{l_1 - l_2}{l_2}\right)R = \left(\frac{76.3 - 64.8}{64.8}\right) \times 9.5 = 1.68\ \Omega$$

Illustration 6.9 **A voltage V_0 is applied to a potentiometer whose sliding contact is exactly in the middle. A voltmeter V is connected between the sliding contact and one fixed end of the potentiometer. It is assumed that the resistance of the voltmeter is not very high if compared with the resistance of the potentiometer. What voltage will the voltmeter show higher than, less than, or equal to $V_0/2$?**

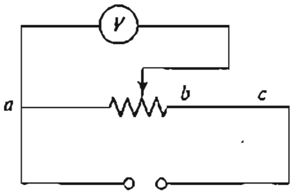


Fig. 6.20

Sol.

If R is the resistance of the whole potentiometer and R_v is the resistance of the voltmeter, the total resistance of section ab of the potentiometer is

$$R_{ab} = \frac{R_v(R/2)}{R_v(R/2)} = \frac{R}{2(1 + R/2RV)} < \frac{R}{2}$$

The resistance of section bc is equal to $R/2$. The voltage applied to the potentiometer will not be distributed evenly.

Since the resistance of *ab* is less than that of *bc*, the voltage applied to the first section is lower than that applied to the second. the higher the resistance of the voltmeter, the closer the readings of the voltmeter are to one half of the applied voltage.

Illustration 6.10 **Potentiometer wire PQ of 1 m length is connected to a standard cell E_1. Another cell, E_2, of e.m.f. 1.02 V is connected with a resistance r and a switch S as shown in the circuit diagram. With switch S open null position is obtained at a distance of 51 cm from P. Calculate**

(a) potential gradient of the potentiometer wire.

(b) e.m.f. of cell E_1.

(c) when switch S is closed, will null point move towards P or towards Q? Give reason for your answer?

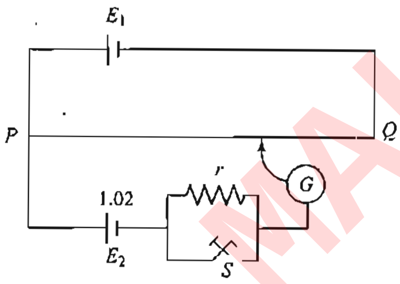


Fig. 6.21

Sol.

(a) Potential gradient, $k = \frac{V}{l} = \frac{1.02}{51} = 0.02 \text{ V/m}$

(b) The e.m.f. of cell $E_1 = k \times 100 = 0.02 \times 100 = 2$ V

(c) When switch S is closed, there is no shift in the position of null point as the position of null point depends upon the potential gradient along the potentiometer wire (which depends upon the e.m.f. of battery E_1 and resistance of potentiometer wire circuit and length of potentiometer) and e.m.f. of the cell E_2 which does not change when switch S is closed.

Illustration 6.11 **In Fig. 6.22, AB is a 1 m long uniform wire of 10 Ω resistance. Other data are shown in the diagram. Calculate (i) potential gradient along AB, (ii) length AO when galvanometer shows no deflection.**

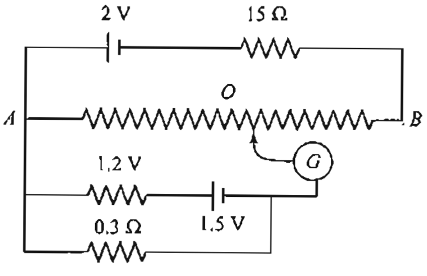


Fig. 6.22

Sol.

i. Potential gradient along

$$AB = \left(\frac{2}{15+10}\right)\frac{10}{100} = 0.008 \text{ V cm}^{-1}$$

ii. Current through 0.3 Ω $= \frac{1.5}{1.2+0.3} = 1$ A

Potential difference across 0.3 Ω $= 1 \times 0.3 = 0.3$ V

Let l be the length AO, then $0.3 = 0.008 \times l$

or $\quad l = \frac{0.3}{0.008} = 37.5$ cm

Illustration 6.12 **Cells A and B and a galvanometer G are connected to a slide wire OS by two sliding contacts C and D as shown in Fig. 6.23. The slide wire is 100 cm long and has a resistance of 12Ω. With $OD = 75$ cm, the galvanometer gives no deflection when OC is 50 cm. If D is moved to touch the end of wire S, the value of OC for which the galvanometer shows no deflection is 62.5 cm. The e.m.f. of cell B is 1.0 V.**

Calculate

a. The potential difference across O and D when D is at 75 cm mark from O.

b. The potential difference across OS when D touches S.

c. Internal resistance of cell A.

d. The e.m.f. of cell A.

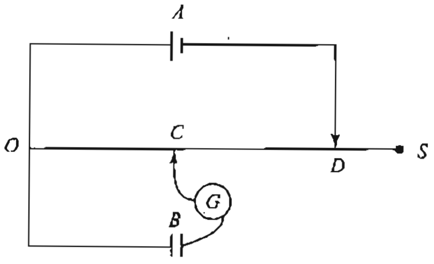


Fig. 6.23

Sol. Resistance of wire $OD = \frac{12}{100} \times 75 = 9$ Ω. Let E and r be the c.m.f. and internal resistance of cell E.

a. Potential gradient of wire = 1/50 V/cm. Therefore voltage drop across the wire OD of length 75 cm = (1/50) × 75 = 1.5 V

b. Potential gradient of wire = 1/62.5 V/cm. Therefore voltage drop across the wire OS of length 100 cm = (1/62.5) × 100 = 1.6 V

c. $\left(\frac{E}{9+r}\right) \times 9 = 1.5 \qquad$ (i)

$\left(\frac{E}{12+r}\right) \times 12 = 1.6 \qquad$ (ii)

d. On solving (i) and (ii), we get $r = 3$Ω and $E = 2$ V.

METER BRIDGE OR SLIDE WIRE BRIDGE

Slide wire bridge is a particle application of Wheatstone Bridge and is used for (1) measuring an unknown resistance (2) comparing two unknown resistances. Slide wire bridge works on the principle of Wheatstone bridge.

Construction

It consists of a uniform wire AC, usually of eureka or manganin of one metre length. It is stretched on a wooden board between two copper strips. A metre scale is fitted on the board parallel to the length of wire. Another copper strip is fitted on the wooden board in order to provide two gaps.

In one of the gaps (say left gap) a resistance box R is connected, while in other gap (right gap) an unknown resistance S is connected. A cell E and a key K are connected across the ends A and C as shown in Fig. 6.24.

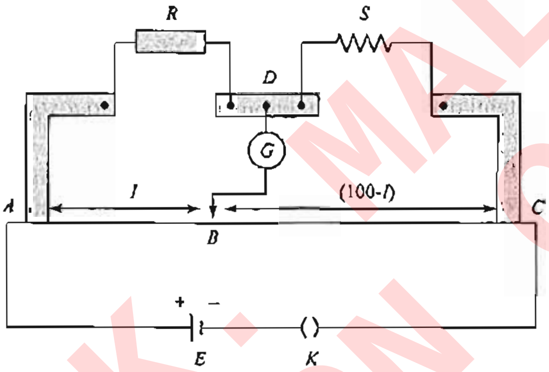


Fig. 6.24

Checking of Connections

Close the key K and put the jockey at the end A of the wire and see the direction of deflection in the galvanometer. Now remove the jockey from A and put it at the end B of the wire and note the direction of deflection in the galvanometer. If the direction of deflection reverses, the connections are correct.

Working

Close the key K and take out some suitable low resistance R from the resistance box. Now move the jockey gently on wire AC till the galvanometer shows no deflection. Let this point be B on the wire.

Let $AB = l$

$\therefore$ $BC = (100 - l)$

Let the resistance of the wire between A and B = P and the resistance of the wire between B and $C = Q$

If r = resistance of wire of unit length, then $P = lr$ and $Q = (100 - l)\,r$

According to principle of Wheatstone's bridge,

$$\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{\ell r}{(100-\ell)\,r} = \frac{R}{S}$$

$$\Rightarrow \frac{\ell}{100-\ell} = \frac{R}{S}$$

$$\Rightarrow S = \left(\frac{\ell}{100-\ell}\right)R$$

knowing λ and R, S can be calculated.

Illustration 6.13 **In the simple potentiometer circuit, where the length AB of the potentiometer wire is 1m, the resistors X and Y have values 5 Ω and 2 Ω, respectively. When X is shunted by a wire, the balance point is found to be 0.625 m from A. What is the resistance of the shunt? If the shunt wire is 0.75 m long and 0.25 mm in diameter, what is the resistivity of the material of the wire?**

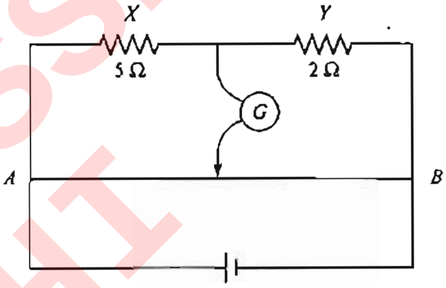


Fig. 6.25

Sol. Let R be the resistance of the shunted wire, the effective resistance of R and 5 Ω in parallel = $5 \times R/(5+R)$

At balance point, $\dfrac{5R/(5+R)}{2} = \dfrac{0.625}{1-0.625} = \dfrac{0.625}{0.0375} = \dfrac{5}{3}$

On solving we get,

$$R = 10\,\Omega$$

Now, $$\rho = \frac{Ra}{l} = \frac{R\pi r^2}{l}$$

$$= \frac{10 \times (22/7) \times (0.125 \times 10^{-3})^2}{0.75}$$

$$= 6.54 \times 10^{-7}\,\Omega\,\text{m}$$

Concept Application Exercise 6.1

1. An ammeter is always connected in series and a voltmeter is connected in parallel to the circuit element in any electrical circuit. Why?
2. By mistake a voltmeter is connected in series and an ammeter in parallel with a resistance in an electrical circuit, what will happen to the instruments?
3. A 100 V voltmeter having an internal resistance of 20 kΩ is connected in series with a large resistance R across a 110 V line. What is the magnitude of resistance R if the voltmeter reads 5 V?
4. What will be the effect on the accuracy of the result if we replace a single wire potentiometer by a potentiometer having 12 wires, the length of each wire being 1 m?

5. In the circuit shown in Fig. 6.26, a metre bridge is in its balance state. The metre bridge wire has a resistance of 1 ohm cm^{-1}. Calculate the value of unknown resistance X and the current drawn from the battery of negligible internal resistance.

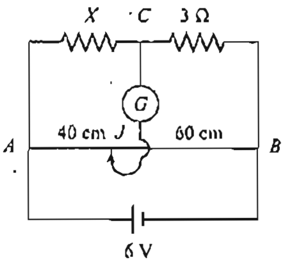
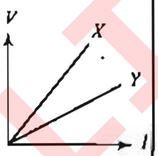


Fig. 6.26

6. The variation of potential difference V with length ℓ in case of two potentiometers X and Y is as shown in the given diagram. Which one of these two will you prefer for comparing e.m.f.s of two cells and why?



7. Fig. 6.27 shows a potentiometer circuit for comparison of two resistances. The balance point with a standard resistor R = 10.0 Ω is found to be 58.3 cm, while that with an unknown resistance X is 68.5 cm. Determine the value of X. What would you do if you fail to find a balance point with the given cell E? (Assume R and X are much more smaller than r).

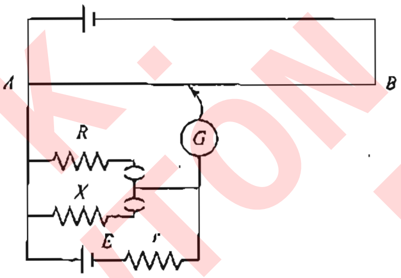


Fig. 6.27

8. Two unknown resistances X and Y are placed on the left and right gaps of a meter bridge. The null point in galvanometer is obtained at a distance of 80 cm from left. A resistance of 100 Ω is now connected in parallel across X. The null point is then found by shifting the sliding contact towards left by 20 cm. Calculate X and Y.

9. The ammeter shown in Fig. 6.28 consists of 480 Ω coil connected in parallel to 20 Ω shunt. Find the reading of the ammeter.

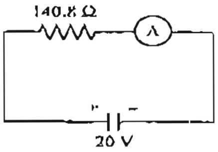


Fig. 6.28

10. A galvanometer with a coil of resistance 12.0 Ω shows full scale deflection for a current of 2.5 mA. How will you convert the meter into:

i. an ammeter of range 0 to 7.5 A?

ii. a voltmeter of range 0 to 1.0 V?

11. What shunt resistance is required to make a 1.00 mA, 20.0 Ω meter into an ammeter with a range of 0 A to 50.0 mA?

12. How can we make a galvanometer with R_G = 20.0 Ω and I_g = 0.00100 A into a voltmeter with a maximum range of 10.0 V?

13. In an experiment with a potentiometer, the null point is obtained at a distance of 60 cm along the wire from the common terminal with a Leclanche cell. When a shunt resistance of 1 Ω is connected across the cell, the null point shifts to a distance of 30 cm from the common terminal. What is the internal resistance of the cell?

14. The e.m.f. of Daniel cell gets balanced on 800 cm length of a potentiometer wire. When a 5 Ω resistance is connected at the terminals of the cell, the balancing length becomes 400 cm. Find the internal resistance of the cell.

15. A potentiometer wire of length 10 m and resistance 30 Ω is connected in series with a battery of e.m.f. 2.5 V, internal resistance 5 Ω and an external resistance R. If the fall of potential along the potentiometer wire is 50 μ V/mm, find the value of R (in Ω).

16. In the experiment of calibration of voltmeter, a standard cell of e.m.f. 1.1 V is balanced against 440 cm of potentiometer wire. The potential difference across the ends of a resistance is found to balance against 220 cm of the wire. The corresponding reading of voltmeter is 0.5 V. Find the error in the reading of voltmeter.

17. It is required to measure the resistance of a circuit operating at 120 V. There is only one galvanometer of current sensitivity 10^{-6} A per division. How should the galvanometer be connected in the circuit to operate an ohmmeter? What minimum resistance can be measured with such a galvanometer if its full scale has 40 divisions?

18. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 0.52 m of the potentiometer wire. If the cell is shunted by a resistance of 5Ω, balance is obtained when the cell is connected across 0.4 m of the wire. Find the internal resistance of the cell.

19. In the circuit shown in Fig. 6.29, a voltmeter reads 30 V when it is connected across a 400 ohm resistance. Calculate what the same voltmeter would read when it is connected across the 300 ohm resistance.

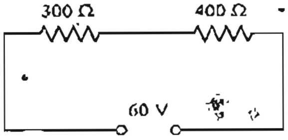


Fig. 6.29

(IIT-JEE, 1980)

20. Draw a circuit diagram to verify Ohm's law with the help of a main resistance 10^6 Ω and 10^{-3} Ω and a source of varying e.m.f. Show the correct positions of voltmeter and ammeter.

(IIT-JEE, 2004)

Solved Examples

Example 6.1 **What is the value of a shunt which passes 10% of the main current through a galvanometer of 99 Ω?**

Sol. A shunt is a small resistance S in parallel with a galvanometer (of resistance G) as shown in Fig. 6.30,

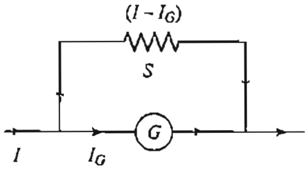


Fig. 6.30

$$(I-I_G)\,S=I_G*G,\quad \text{i.e.,}\quad S=\frac{I_GG}{(I-I_G)}$$

And as here, $G=99\ \Omega$ and $I_G=(10/100)I=0.1\,I$

$$S=\frac{0.1I\times99}{(I-0.1I)}=\frac{0.1}{0.9}\times99=11\ \Omega$$

Example 6.2 **The deflection in a moving coil galvanometer falls from 50 divisions to 10 divisions when a shunt of 12 ohm is applied. What is the resistance of the galvanometer?**

Sol. In case of a galvanometer, $I\propto\theta$

So, $$\frac{I_G}{I}=\frac{10}{50}=\frac{1}{5},\quad \text{i.e.,}\quad I_G=\frac{1}{5}I$$

Now in case of a shunted galvanometer as shown in Fig. 6.31,

$$(I-I_G)\,S=I_GG,\quad \text{i.e.,}\ (I-\frac{1}{5}I)\times12=\frac{1}{5}IG$$

So, $$G=4\times12=48\ \Omega$$

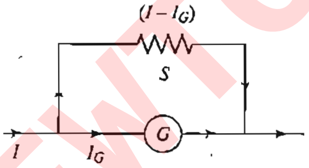


Fig. 6.31

Example 6.3 **A galvanometer of resistance 95 Ω, shunted by a resistance of 5 Ω gives a deflection of 50 divisions when joined in series with a resistance of 20 Ω and a 2 V accumulator. What is the current sensitivity of the galvanometer (in div/μA)?**

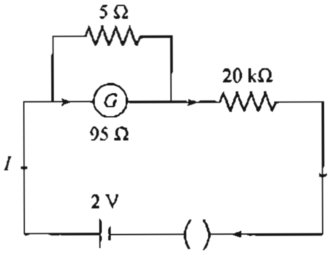


Fig. 6.32

Sol. In accordance with the given problem, the situation is depicted by the circuit diagram in the figure. As 20 kΩ is much greater than the resistance of shunted galvanometer (<5 Ω), the current in the circuit will be

$$I\simeq\frac{2}{20\times10^3}=10^{-4}\ \text{A}=100\ \mu\text{A}$$

And as this current produces a deflection of 50 divisions in the galvanometer, $\text{CS}=\frac{\theta}{I}=\frac{50\text{div}}{100\mu\text{A}}=\frac{1}{2}\frac{\text{div}}{\mu\text{A}}$

Example 6.4 **An electrical circuit is shown in Fig. 6.33. Calculate the potential difference across the resistor of 400 Ω, as will be measured by the voltmeter *V* of resistance 400 Ω, either by applying Kirchhoff's rule or otherwise.**

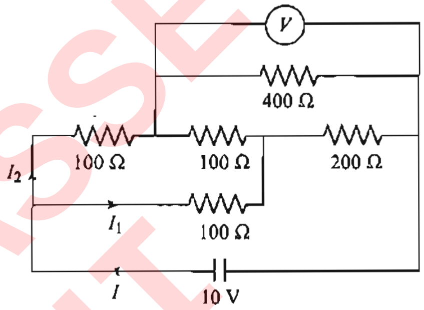


Fig. 6.33

(IIT-JEE, 1996)

Sol. Applying Kirchhoff's law in loop *JMGDJ*, we get

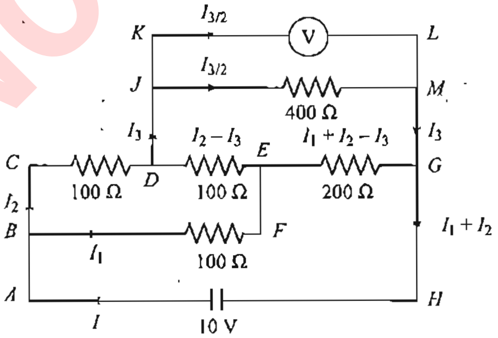


Fig. 6.34

$$-\frac{I_3}{2}\times400+(I_1+I_2-I_3)\,200+(I_2-I_3)\,100=0$$

$$-200\,I_3+200I_1+300I_2-300=\quad0$$

$$\Rightarrow\quad 2I_1+3I_2-5I_3=0 \qquad \text{(i)}$$

Applying Kirchhoff's law in *CDEFBC*:

$$100I_2-100(I_2-I_3)+100I_1=0$$

$$\Rightarrow\quad I_1-2I_3-I_3=0 \qquad \text{(ii)}$$

Multiplying eq.(ii) by 2 and subtracting from (i)

$$2I_1+3I_2-5I_3-2I_1+4I_2-2I_3=0$$

$$\Rightarrow\quad I_2=I_3 \qquad \text{(iii)}$$

Applying Kirchhoff's law in *ABFEGHA*

$$-3I_1-2I_2+2I_3+0.1=0 \qquad \text{(iv)}$$

Multiplying (ii) by 3 and adding it with (iv)

$$-3I_1-6I_2+3I_3-3I_1-2I_2+2I_3+0.1=0$$

$$\Rightarrow \quad -8I_2+5I_3+0.1=0$$

$$\Rightarrow \quad -8I_3+5I_3+0.1=0$$

$$\Rightarrow \quad 3I_3=0.1$$

$$\Rightarrow \quad I_3=\frac{0.1}{3}$$

$\therefore$ Potential difference across $JM=\frac{0.1/3}{2}\times 400$

$$=\frac{0.1}{2\times 3}\times 400=\frac{20}{3}=6.67\text{ V}$$

Alternatively: we can redraw the circuit as shown in Fig. 6.35. The equivalent resistance between G and D is

$$R_{GD}=\frac{400\times 400}{400+400}=200\,\Omega$$

Since $\dfrac{R_{GE}}{R_{GD}}=\dfrac{R_{EB}}{R_{DB}}$

$\therefore$ It is a case of balanced Wheatstone bridge.

The equivalent resistance across G and B is

$$R_{GB}=\frac{300\times 300}{300+300}=150\,\Omega$$

$\therefore$ Current $I=\dfrac{V}{R_{GB}}=\dfrac{10}{150}=\dfrac{1}{15}$ A.

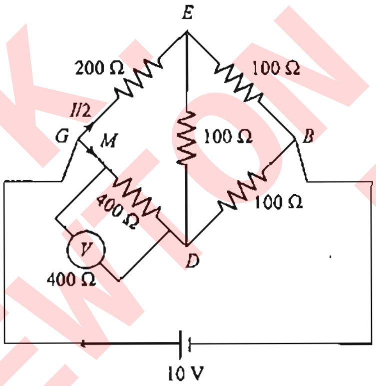


Fig. 6.35

Since $RGEB=RGDB$, the current is divided at G into two equal parts.

The current $\frac{1}{2}$ further divides into two equal parts at M. Therefore, the potential difference across the voltmeter

$$=\frac{1}{4}\times 400=\frac{1}{15}\times 100=\frac{20}{3}\text{V}$$

Example 6.5 **A thin uniform wire AB of length 1 m, an unknown resistance X and a resistance of 12 Ω are connected by thick conducting strips, as shown in Fig. 6.36. A battery and a galvanometer (with a sliding jockey connected to it) are also available. Connections are to be made to measure the unknown resistance X using the principle of Wheatstone bridge. Answer the following questions.**

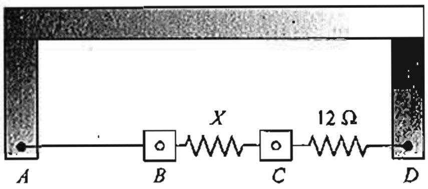


Fig. 6.36

(a) Are there positive and negative terminals on the galvanometer?

(b) Copy the figure in your answer book and show the battery and the galvanometer (with jockey) connected at appropriate points.

(c) After appropriate connections are made, it is found that no deflection takes place in the galvanometer when the sliding jockey touches the wire at a distance of 60 cm from A. Obtain the value of the resistance X.

(IIT-JEE, 2002)

Sol.

(a) No. There are no positive and negative terminals on the galvanometer.

(b)

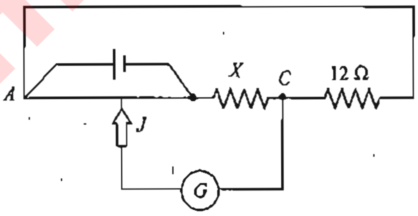


Fig. 6.37

(c) $\because$ Bridge is balanced, $\dfrac{R_{AJ}}{R_{JB}}=\dfrac{0.6\rho}{0.4\rho}=\dfrac{12\Omega}{X}\Rightarrow X=8\ \Omega$

where ρ is the resistance per unit length.

Example 6.6 **An unknown resistance is to be determined using resistances R_1, R_2 or R_3. Their corresponding null points are A, B and C. Find which of the above will give the most accurate reading and why? $R=R_1$ or R_2 or R_3**

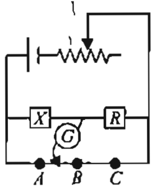


Fig. 6.38 **(IIT-JEE, 2005)**

Sol. All null point, the wheat stone bridge will be balanced

$$\therefore \frac{X}{r_1}=\frac{R}{r_2} \Rightarrow X=R\frac{r_1}{r_2}$$

where R is a constant and r_1 and r_2 are variables. The maximum fraction error is $\frac{\Delta X}{X} = \frac{\Delta r_1}{r_1} + \frac{\Delta r_2}{r_2}$

Here $\Delta r_1 = \Delta r_2 = y$ (say) then

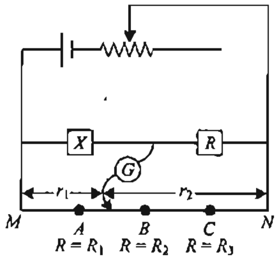


Fig. 6.39

$$\frac{\Delta X}{X} = y\left[\frac{r_2 + r_1}{r_1 r_2}\right]$$

For $\frac{\Delta X}{X}$ to be minimum, $r_1 \times r_2$ should be maximum

$[\because r_1 + r_2 = c \text{ (Constant)}]$

Let $\quad E = r_1 \times r_2 \quad \Rightarrow \quad E = r_1 \times (r_2 - c)$

$\therefore \quad \frac{dE}{dr_1} = (r_1 - c) + r_1 = 0$

$\Rightarrow \quad r_1 = \frac{c}{2} \quad \Rightarrow \quad r_2 = \frac{c}{2} \quad \Rightarrow \quad r_1 = r_2$

$\Rightarrow$ R_2 gives the most accurate value

EXERCISES

Subjective Type

Solutions on page 6.23

1. Consider the potentiometer circuit arranged as in the figure. The potentiometer wire is 600 cm long.

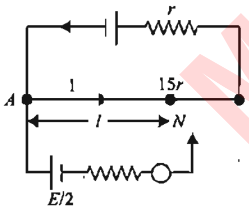


Fig. 6.40

a. At what distance from point A should the jockey touch the wire to get zero deflection in the galvanometer?

b. If the jockey touches the wire at a distance of 560 cm from A, what will be the current in the galvanometer?

2. A cell of e.m.f. 3.4 V and internal resistance 3 Ω is connected to an ammeter having resistance 2 Ω and to an external resistance of 100 Ω resistance, the ammeter reading is 0.04 A. Find the voltage read by the voltmeter and its resist-ance. Had the voltmeter been an ideal one, what would have been its reading?

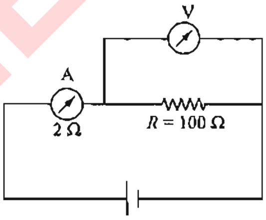


Fig. 6.41

3. Fig. 6.42 shows a metre bridge (which is nothing but a practical Wheatstone bridge), consisting of two resistors X and Y together in parallel with a metre long constantan wire of uniform cross-section.

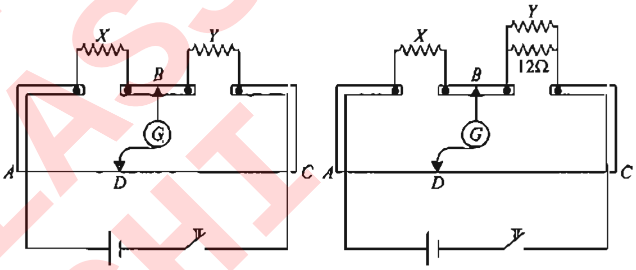


Fig. 6.42

With the help of a movable contact D, one can change the ratio of resistance of the two segments of the wire until a sensitive galvanometer G connected across B and D shows no deflection. The null point is found to be at a distance of 33.7 cm. The resistor Y is shunted by a resistance of 12 Ω and the null point is found to shift by a distance of 18.2 cm. Determine the resistance of X and Y.

4. The circuit shown in Fig. 6.43 shows the use of potentiometer to measure the internal resistance of a cell.

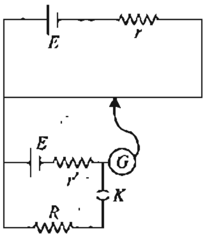


Fig. 6.43

a. When the key is open, how does the balance point change, if the current from the driver cell decreases?

b. When key is closed, how does the balance point change, if R is increased, keeping the current from the driver cell constant?

5. Let V and I represent, respectively, the readings of the voltmeter and ammeter shown in Fig. 6.44, and let R_V and R_A be their equivalent resistances. Because of the resistances of the meters, the resistance R is not simply equal to V / I.

a. When the circuit is connected as shown in Fig.6.44 (a), show that $R = \frac{V}{I} - R_A$.

Explain why the true resistance R is always less than V/I.

b. When the connections are as shown in Fig. 6.46 (b)

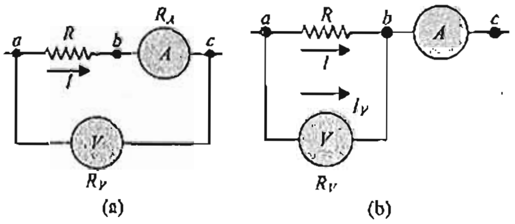


Fig. 6.44

show that $R = \frac{V}{I - (V/R_V)}$

Explain why the true resistance R is always greater than V/I.

c. Show that the power delivered to the resistor in part (i) is $IV - I^2 R_A$ and that in part (ii) is $IV - (V^2/R_V)$.

6. You are given two resistors X and Y whose resistances are to be determined using an ammeter of resistance 0.5 Ω and a voltmeter of resistance 20 kΩ. It is known that X is in the range of a few ohms, while Y is in the range of several thousand ohms. In each case, which of the following two connections (Fig. 6.45) would you choose for resistance measurement? Justify your answer quantitatively. [**Hint:** For each connection, determine the error in resistance measurement. The connection that corresponds to a smaller error (for a given range of resistance) is to be preferred.]

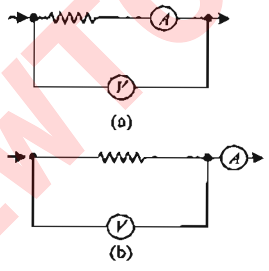


Fig. 6.45

7. Fig. 6.46 shows a potentiometer with a cell of 2.0 V and internal resistance 0.4 Ω maintaining a potential drop across the resistor wire AB. A standard cell which maintains a constant e.m.f. of 1.02 V (for very moderate currents upto a few μA) gives a balance point at 67.3 cm length of the wire. To ensure very low current is drawn from the standard cell, a very high resistance of 600 k Ω is put in series with it, which is shorted close to the balance point. The standard cell is then replaced by a cell of unknown e.m.f. and the balance point found similarly turns out to be at 82.3 cm length of the wire.

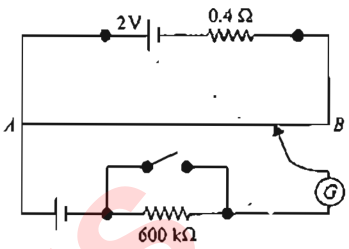


Fig. 6.46

a. What is the value of e?

b. What purpose does the high resistance of 600 kΩ have?

c. Is the balance point affected by this high resistance?

d. Is the balance point affected by internal resistance of the driver cell?

e. Would the method work in the above situation if the driver cell of the potentiometer had an e.m.f. of 1.0 V instead of 2.0 V?

f. Would the circuit work well for determining an extremely small e.m.f., say of the order of a few mV (such as the typical e.m.f. of a thermocouple)? If not, how will you modify the circuit?

8. A 5 wire potentiometer is connected to a storage cell of steady e.m.f. 2 V and 1 Ω resistance. A primary cell is balanced against 3.5 of it. What resistance will be required in series with the storage cell to push the null point to the centre of the last wire, 4.5 m? (The wire has 3 Ω resistance per meter).

9. In a meter bridge circuit, the two resistances in the gap are 5 Ω and 10 Ω. The wire resistance is 5 Ω. The e.m.f. of the cell connected at the ends of wire is 5 V. and its internal resistance is 10 Ω. What current will flow through the galvanometer of resistance 30 Ω if the contact is made at the mid-point of wire ?

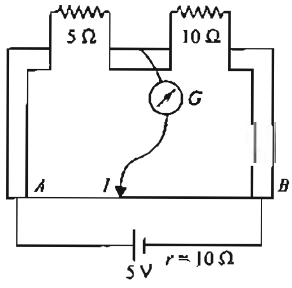


Fig. 6.47

10. In the given circuit, a meter bridge is shown in a balanced state. The bridge wire has a resistance of 1Ω/cm. Find the value of the unknown resistance X and the current drawn from the battery of negligible internal resistance.

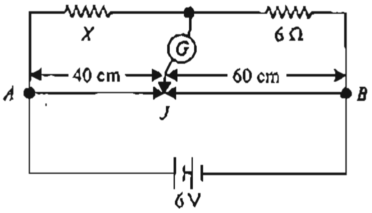


Fig. 6.48

11. An experiment with a post office box, the ratio of arms are 1000 : 10. If the value of the third resistance is 999 Ω, find the unknown resistance.

12. A voltmeter reads 5.0 V at full scale deflection and is graded according to its resistance per volt at full scale deflection as 5000 Ω/V. How will you convert it into a voltmeter that reads 20 V at full scale deflection? Will it still be graded as 5000 Ω/V? Will you prefer this voltmeter to one that is graded as 2000 Ω/V?

13. A battery of e.m.f. 1.4 V and internal resistance 2 Ω is connectd to a resistor of 100 Ω resistance through an ammeter. The resistance of the ammeter is 4/3 Ω. A voltmeter has also been connected to find the potential difference across the resistor.

(i) Draw the circuit diagram.

(ii) The ammeter reads 0.02 A. What is the resistance of the voltmeter?

(iii) The voltmeter reads 1.1 V. What is the error in the reading?

14. Fig.6.49 shows a potentiometer circuit for comparison of two resistances. The balance point with a standard resistor R = 10.0 Ω is found to be 58.3 cm, while that with the unknown resistance X is 68.5 cm. Determine the value of X. What would you do if you fail to find a balance point with the given cell E?

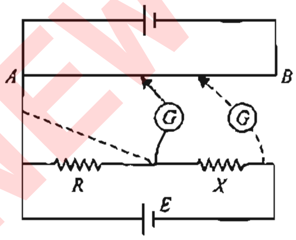


Fig. 6.49

15. A potentiometer wire has a length of 10 cm and resistance 4 Ω. An accumulator of e.m.f. 2 V and a resistance box are connected in series with it. Calculate the resistance to be introduced in the box so as to get a potential gradient of (i) 0.1 V/m and (ii) 0.1 mV/m.

16. Fig. 6.50 shows a 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 76.3 cm. When a resistor of 9.5 Ω is used in the external circuit of the cell, the balance point shifts to 64.8 cm length of the potentiometer wire. Determine the internal resistance of the cell.

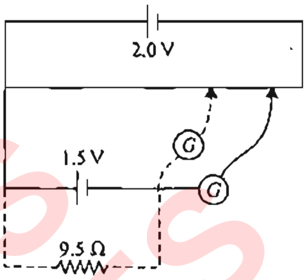


Fig. 6.50

Objective Type

Solutions on page 6.26

1. How will the reading in the ammeter A of Fig. 6.51 be affected if another identical bulb Q is connected in parallel to P as shown. The voltage in the mains is maintained at a constant value.

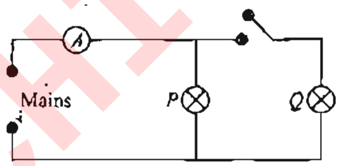


Fig. 6.51

a. The reading will be reduced to one-half
b. The reading will not be affected
c. The reading will be double of the previous one
d. The reading will be increased four-fold

2. A potentiometer is connected across A and B and a balance is obtained at 64.0 cm. When potentiometer lead to B is moved to C, a balance is found at 8.0 cm. If the potentiometer is now connected across B and C, a balance will be found at

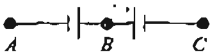


Fig. 6.52

a. 8.0 cm
b. 56.0 cm
c. 64.0 cm
d. 72.0 cm

3. In the circuit shown in Fig. 6.53, the reading of the ammeter is (assume internal resistance of the battery be zero)

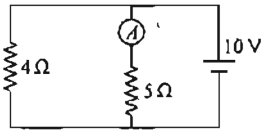


Fig. 6.53

a. $\frac{40}{29}$ A
b. $\frac{10}{9}$ A
c. $\frac{5}{3}$ A
d. 2 A

4. In the circuit shown in Fig. 6.54, resistors X and Y, each with resistance R, are connected to a 6 V battery of negligible internal resistance. A voltmeter, also of resistance R, is connected across Y.

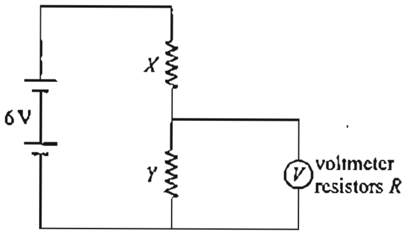


Fig. 6.54

What is the reading of the voltmeter?

a. zero
b. between zero and 3 V
c. 3 V
d. between 3 V and 6 V

5. In the circuit shown in Fig 6.55, of $P \neq R$ and the reading of the galvanometer is same with switch S open or closed, then

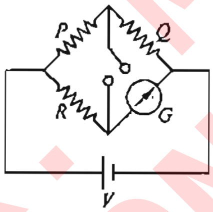


Fig. 6.55

a. $I_R = I_G$
b. $I_P = I_G$
c. $I_Q = I_G$
d. $I_Q = I_R$

6. In the shown arrangement of the experiment of a meter bridge, if AC corresponding to null deflection of galvanometer is x, what would be its value if the radius of the wire AB is doubled?

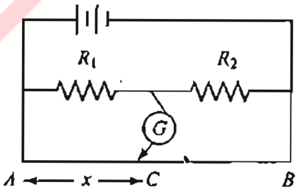


Fig. 6.56

a. x
b. $x/4$
c. $4x$
d. $2x$

7. The length of a wire of a potentiometer is 100 cm, and the e.m.f. of its standard cell is E volt. It is employed to measure the e.m.f. of a battery whose interval resistance is 0.5 Ω. If the balance point is obtained at l = 30 cm from the positive end, e.m.f. of the battery is **(AIEEE, 2003)**

a. $\frac{30\ E}{100}$
b. $\frac{30\ E}{100.5}$
c. $\frac{30\ E}{(100-0.5)}$
d. $\frac{30\ (E-0.5i)}{100}$

where i is the current in the potentiometer wire.

8. In a metre bridge experiment, null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y. If $X < Y$, then where will be the new position of the null point from the same end, if one decides to balance a resistance of $4X$ against Y? **(AIEEE, 2004)**

a. 50 cm
b. 80 cm
c. 40 cm
d. 70 cm

9. In the circuit shown in Fig. 6.57, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be **(AIEEE, 2005)**

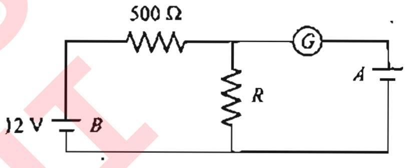


Fig. 6.57

a. 1000 Ω
b. 500 Ω
c. 100 Ω
d. 200 Ω

10. In a potentiometer experiment, the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of 2 Ω, the balancing length becomes 120 cm. The internal resistance of the cell is **(AIEEE, 2005)**

a. 2 Ω
b. 4 Ω
c. 0.5
d. 1 Ω.

11. If in the experiment of Wheatstone's bridge, the positions of cells and galvanometer are interchanged, then balance points will

a. change
b. remain unchanged
c. depend on the internal resistance of the cell and resistance of the galvanometer
d. none of these

12. Sensitivity of potentiometer can be increased by

a. increasing the e.m.f. of the cell
b. increasing the length of the potentiometer
c. decreasing the length of the potentiometer wire
d. none of the above

13. The length of a wire of a potentiometer is 100 cm, and the e.m.f. of its standard cell is E volt. It is employed to measure the e.m.f. of a battery whose internal resistance is 0.5 Ω. If the balance point is obtained at ℓ = 30 cm from the positive end, then e.m.f. of the battery is

a. $\frac{30E}{(100-0.5)}$

b. $\frac{30(E-0.5i)}{100}$ where i is the current in the potentiometer wire

c. $\frac{30E}{100}$ d. $\frac{30E}{100.5}$

14. When a 12 Ω resistor is connected with a moving coil galvanometer then its deflection reduces from 50 divisions to 10 divisions. The resistance of the galvanometer is

a. 24 Ω b. 36 Ω
c. 48 Ω d. 60 Ω

15. The resistance a galvanometer is 10 Ω. It gives full-scale deflection when 1mA current is passed. The resistance connected in series for converting it into a voltmeter of 2.5 volts will be

a. 24.9 Ω b. 249 Ω
c. 2490 Ω d. 24900 Ω

16. An ammeter of resistance 0.2 Ω reading upto 10 mA is to be used to read upto 1V potential difference. We have to connect

a. 99.8 Ω resistance in series
b. 99.8 Ω resistance in parallel
c. 0.1 Ω resistance in parallel
d. 0.1 Ω resistance in series

17. A milliammeter of range 10 mA has a coil of resistance 1 Ω. To use it as an ammeter of range 1A, the required shunt must have a resistance of

a. $\frac{1}{101}\,\Omega$ b. $\frac{1}{100}\,\Omega$
c. $\frac{1}{99}\,\Omega$ d. $\frac{1}{9}\,\Omega$

18. To use the milliammeter of the previous question as a voltmeter of range 10 V, a resistance R is placed in series with it. The value of R is

a. 9 Ω b. 99 Ω
c. 999 Ω d. 1000 Ω

19. Two cells of e.m.fs E_1 and E_2 ($E_1 > E_2$) are connected as shown in Fig. 6.58

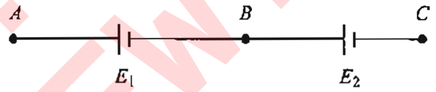


Fig. 6.58

When a potentiometer is connected between A and B, the balancing length of the potentiometer wire is 300 cm. On connecting same potentiometer between A and C, the balancing length is 100 cm. The ratio $\frac{E_1}{E_2}$ is

a. 3 : 1 b. 1 : 3
c. 2 : 3 d. 3 : 2

20. Fig. 6.59 shows a Wheatstone's net, with P = 1000 Ω, Q = 10.0 Ω, R (unknown), S variable and near 150 Ω for balance. If the connections across A, C and B, D are interchanged, the error range in R determination would

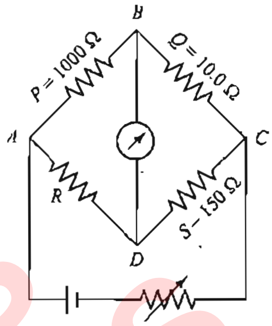


Fig. 6.59

a. remain unaffected b. increases substantially
c. increase marginally d. decreases substantially

21. An ideal ammeter (zero resistance) and an ideal voltmeter (infinite resistance) are connected as shown in Fig. 6.60. The ammeter and voltmeter readings are

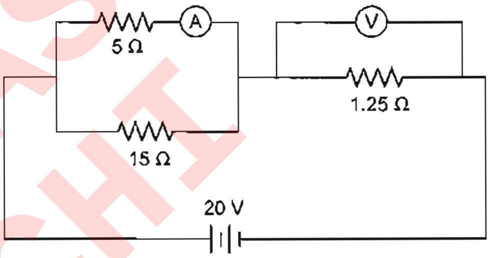


Fig. 6.60

a. 6.25 A, 3.75 V b. 3.00 A, 5 V
c. 3.00 A, 3.75 V d. 6.00 A, 6.25 V

22. A constant 60 V d.c. supply is connected across two resistors of resistance 400 kΩ and 200 kΩ. What is the reading of the voltmeter, also of resistance 200 kΩ, when connected across the second resistor as shown in the Fig. 6.61?

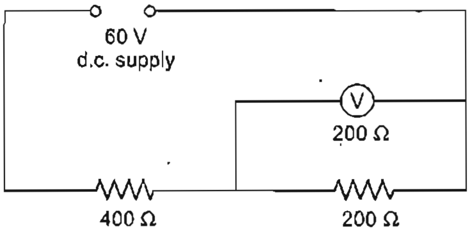


Fig. 6.61

a. 12 V b. 15 V
c. 20 V d. 30 V

23. Fig. 6.61 shows a circuit which may be used to compare the resistance R of an unknown resistor with a 100 Ω standard. The distances l from one end of the potentiometer slider wire to the balance point are 400 mm and 588 mm when X is connected to Y and Z, respectively. The length of the slide-wire is 1.00 m. What is the value of resistance R?

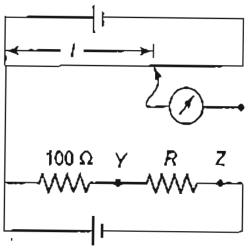


Fig. 6.61

a. 32 Ω b. 47 Ω
c. 68 Ω d. 147 Ω

24. In the circuit shown in Fig. 6.62, an ideal ammeter and an ideal voltammeter are used. When key is open, the voltmeter reads 1.53 V. When the key is closed, the ammeter reads 1.0 A and the voltmeter reads 1.03 V. The resistance R is

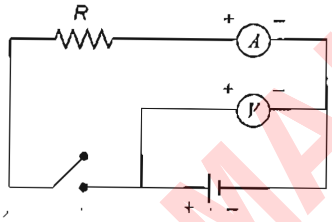


Fig. 6.62

a. 0.5 Ω b. 1.03 Ω
c. 1.53 Ω d. 0.53 Ω

25. In which one of the following arrangements of resistors does the meter M, which has a resistance of 2 Ω, give the largest reading when the same potential difference is applied between points P and Q?

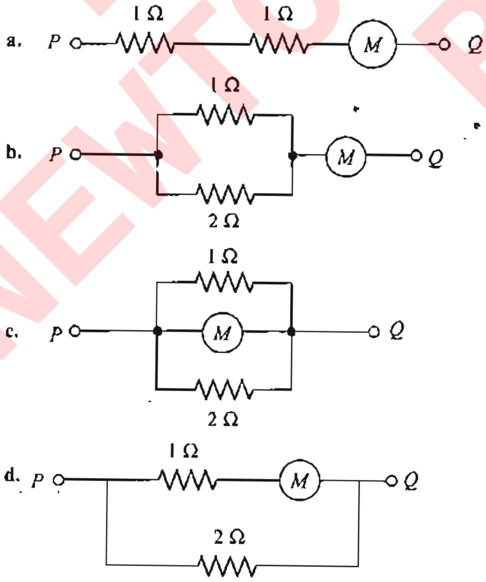


Fig. 6.63

26. Fig. 6.64 shows a simple potentiometer circuit for measuring a small e.m.f. produced by a thermocouple.

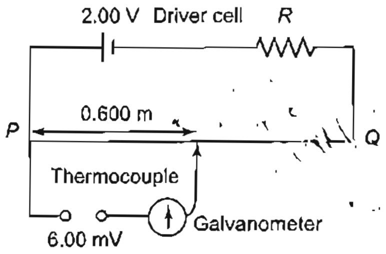


Fig. 6.65

The metre wire PQ has a resistance of 5 Ω and the driver cell has an e.m.f. of 2.00 V. If a balance point is obtained 0.600 m along PQ when measuring an e.m.f. of 6.00 mV, what is the value of resistance R?

a. 95 Ω b. 995 Ω
c. 195 Ω d. 1995 Ω

27. Fig. 6.66 shows a balanced Wheatstone's net. Now, it is disturbed by changing P to 11 Ω. Which of the following steps will not bring the bridge to balance again?

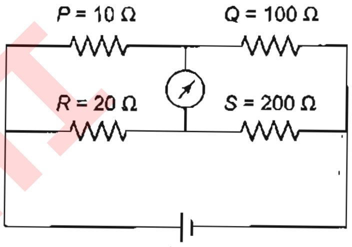


Fig. 6.66

a. Increasing R by 2 Ω
b. Increasing S by 20 Ω
c. Increasing Q by 10 Ω
d. Making product $RQ = 2200\ (\Omega)^2$

28. In the circuit (Fig. 6.67), the ammeter reading is zero. What is the value of the resistance R?

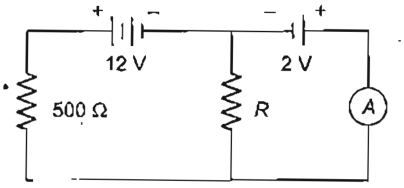


Fig. 6.67

a. 50 Ω b. 100 Ω
c. 200 Ω d. 400 Ω

29. In the above circuit, in which of the following cases, the reading of the ammeter will change if the ammeter resistance G is changed?

a. $G = 50\ \Omega$ b. $G = 100\ \Omega$
c. $G = 500\ \Omega$ d. None of these

30. In an experiment to measure the internal resistance of a cell, by a potentiometer, it is found that the balance point is at a length of 2 m when the cell is shunted by a 5 Ω resistance and is at a length of 3 m when the cell is shunted by a 10 Ω resistance, the internal resistance of the cell is then

a. 1.5 Ω b. 10 Ω
c. 15 Ω d. 1 Ω

31. When an ammeter of negligible internal resistance is inserted in series with circuit, it reads 1 A. When the voltmeter of very large resistance is connected across R_1, it reads 3 V. But when the points A and B are short circuited by a conducting wire then the voltmeter measures, 10.5 V across the battery. The internal resistance of the battery is equal to

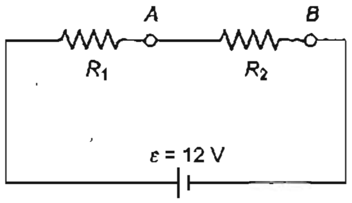


Fig. 6.68

a. $\frac{3}{7}$ Ω b. 5 Ω
c. 3 Ω d. None of these

32. When a voltmeter is connected across a 400 Ω resistance, it reads 30 V; when it is connected with 300 Ω resistance, it will read

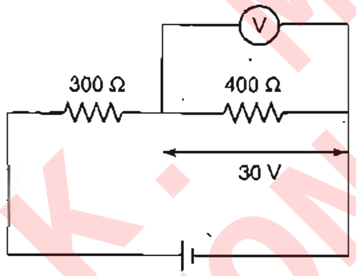


Fig. 6.69

a. 30 V b. 22.5 V
c. 20 V d. 25 V

33. An 80 Ω galvanometer deflects full scale for a potential of 20 mV. A voltmeter deflecting full scale of 5 V is to be made using this galvanometer. We must connect

a. a resistance of 19.92 kΩ parallel to the galvanometer
b. a resistance of 19.92 kΩ in series with the galvanometer
c. a resistance of 20 kΩ parallel to the galvanometer
d. a resistance of 20 kΩ in series with the galvanometer

34. A D.C. milliammeter has a resistance of 12 Ω and gives a full scale deflection for a current of 0.01 A. To convert it into a voltmeter giving a full scale deflection of 3 V, the resistance required to be put in series with the instrument is

a. 102 Ω b. 288 Ω
c. 300 Ω d. 412 Ω

35. A voltmeter having a resistance of 1800 Ω is employed to measure the potential difference across 200 Ω resistance which is connected to D.C. power supply of 50 V and internal resistance 20 Ω. What is the approximate percentage change in the p.d. across 200 Ω resistance as a result of connecting the voltmeter across it?

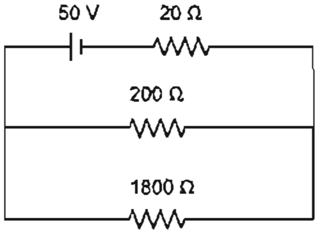


Fig. 6.70

a. 2.2% b. 5%
c. 10% d. 20%

36. In the given circuit, the voltmeter and the electric cell are ideal. Find the reading of the voltmeter

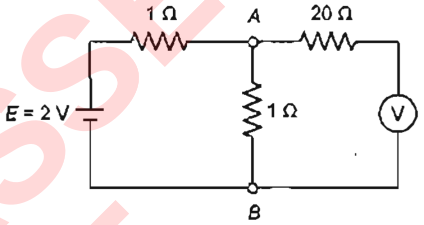


Fig. 6.71

a. 1 V b. 2 V
c. 3 V d. None of these

37. The e.m.f. of the driver cell of a potentiometer is 2V and its internal resistance is negligible. The length of the potentiometer wire is 100 cm and resistance is 5 Ω. How much resistance is to be connected in series with the potentiometer wire to have a potential gradient of 0.05 mV/cm?

a. 1990 Ω b. 2000 Ω
c. 1995 Ω d. None of these

38. In the above question, if the balancing length for a cell of e.m.f. E is 60 cm, the value of E will be

a. 3 mV b. 5 mV
c. 6 mV d. 2000 Ω

39. A, B and C are voltmeters of resistance R, $1.5R$ and $3R$, respectively. When some potential difference is applied between X and Y, the voltmeter readings are V_A, V_B and V_C, respectively. Then

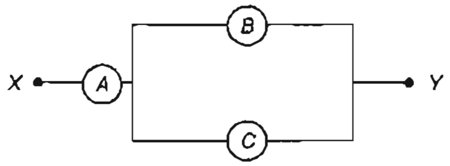


Fig. 6.71

a. $V_A = V_B = V_C$ b. $V_A \neq V_B = V_C$
c. $V_A = V_B \neq V_C$ d. $V_B \neq V_A = V_C$

40. A milliammeter of range 10 mA and resistance 9 Ω is joined in a circuit as shown in Fig. 6.72. The meter gives full-scale deflection for current I when A and B are used as its terminals, if current enters at A and leaves at B (C is left isolated), the value of I is

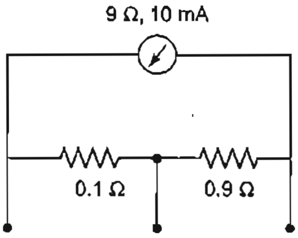


Fig. 6.72

a. 100 mA
b. 900 mA
c. 1 A
d. 1.1 A

41. A candidate connects a moving coil voltmeter *V*, a moving coil ammeter *A* and a resistor *R* as shown in Fig. 6.73. If the voltmeter reads 20 V and the ammeter reads 4 A, *R* is

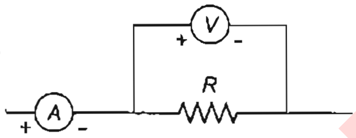


Fig. 6.73

a. equal to 5 Ω
b. greater than 5 Ω
c. less than 5 Ω
d. greater or less than 5 Ω depending upon its material

42. If a shunt of 1/10th of the coil resistance is applied to a moving coil galvanometer, its sensitivity becomes

a. 10 fold
b. 11 fold
c. $\frac{1}{10}$ fold
d. $\frac{1}{11}$ fold

43. In Fig. 6.74, when an ideal voltmeter is connected across 4000 Ω resistance, it reads 30 V. If the voltmeter is connected across 3000 Ω resistance, it will read

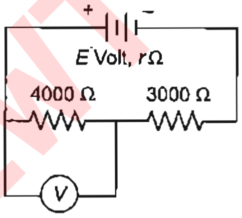


Fig. 6.73

a. 20 V
b. 22.5 V
c. 35 V
d. 40 V

44. In the given circuit, *A* and *V* are ideal ammeter and ideal voltmeter. The voltmeter reading will be

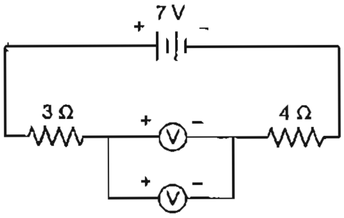


Fig. 6.74

a. 2 V
b. 3 V
c. 5 V
d. zero

45. A voltmeter has a resistance of *G* ohm and range *V* volt. The value of resistance used in series to convert it into voltmeter of range *nV* volt is

a. *nG*
b. $(n-1)G$
c. G/n
d. $G/(n-1)$

46. A galvanometer has a resistance of 3663 Ω. A shunt *S* is connected across it such that (1/34) of the total current passes through the galvanometer. The value of the shunt is

a. 3663 Ω
b. 111 Ω
c. 107.7 Ω
d. 3555.3 Ω

47. In Q.46 the combined resistance of the shunt and the galvanometer is

a. 3665 Ω
b. 111 Ω
c. 107.7 Ω
d. 3555.3 Ω

48. In Q.46, the external resistance which must be connected in series with the main circuit so that the total current in the main circuit remains unaltered even when the galvanometer is shunted is

a. 3663 Ω
b. 111 Ω
c. 107.7 Ω
d. 3555.3 Ω

49. Two moving coil galvanometers 1 and 2 are with identical field magnets and suspension torque constants, but with coil of different number of turns, N_1 and N_2, area per turn A_1 and A_2 and resistance R_1 and R_2. When they are connected in series in the same circuit, they show deflections θ_1 and θ_2 Then (θ_1/θ_2) is

a. $(A_1 N_1/A_2 N_2)$
b. $(A_1 N_2/A_2 N_1)$
c. $(A_1 R_2 N_1/A^2 R^2 N^2)$
d. $(A_1 R_1 N_1/A_2 R_2 N^2)$

50. An ammeter is obtained by shunting a 30 Ω galvanometer with a 30 Ω resistance. What additional shunt should be connected across it to double the range?

a. 15 Ω
b. 10 Ω
c. 5 Ω
d. None of these

Multiple Correct Answers Type

Solutions on page 6.30

1. Which of the following statements is/are correct for potentiometer circuit?

a. Sensitivity varies inversely with the length of the potentiometer wire
b. Sensitivity is directly proportional to the potential difference applied across the potentiometer wire
c. Accuracy of a potentiometer can be increased only by increasing the length of wire
d. Range depends upon the potential difference applied across the potentiometer wire

2. A voltmeter reads the potential difference across the terminals of an old battery as 1.40 V while a potentiometer reads its voltage to be 1.55 V. The voltmeter resistance is 280 Ω. Then

a. the e.m.f. of the battery is 1.4 V
b. the e.m.f. of the battery is 1.55 V
c. the internal resistance r of the battery is 30 Ω
d. the internal resistance r of the battery is 5 Ω

3. A circuit has an equivalent resistance R_0. A voltmeter of resistance R_v is applied across the circuit to measure the potential drop across R_0. The new equivalent resistance of the circuit is

a. R_0(for $R_0 << R_v$)
b. $\frac{R_0 R_v}{R_0 + R_v}$
c. $R_0 + R_v$
d. Data insufficient

4. In the circuit shown in Fig. 6.79, the cell is ideal with e.m.f. 9 V. If the resistance of the coil of galvanometer is 1 Ω, then

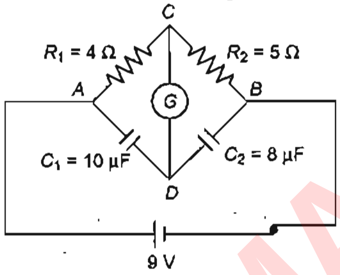


Fig. 6.75

a. No current flows in the galvanometer
b. Charge flowing through 8 μF is 40 μC
c. Potential difference across 10 μF is 5 V
d. Potential difference across 10 μF is 4 V

5. An ammeter has a resistance of 50 Ω and a full scale deflection current 50 μA. It can be used as a voltmeter or as a higher range ammeter provided that a resistance is added to it. Which of the following is/are true?

a. 10 V range with approximately 200 kΩ resistance in series
b. 30 V range with approximately 200 kΩ resistance in series
c. 1 mA range with 50 Ω resistance in parallel
d. 0.1 mA range with 50 Ω resistance in parallel

6. Fig. 6.76 shows a balanced Wheatstone's bridge

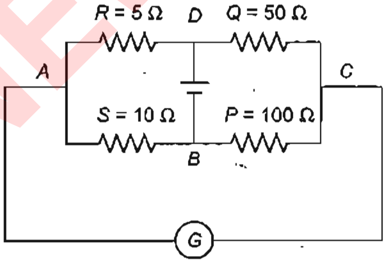


Fig. 6.76

a. If P is slightly increased, the current in the galvanometer flows from C to A
b. If P is slightly increased, the current in the galvanometer flows from A to C
c. If Q is slightly increased, the current in the galvanometer flows from C to A
d. If Q is slightly increased, the current in the galvanometer flows from A to C

Assertion-Reasoning Type

Solutions on page 6.31

In the following questions, each question contains STATEMENT I (Assertion) and STATEMENT II (Reason). Each question has four choices **a**, **b**, **c** and **d** out of which **ONLY ONE** is correct.

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
b. Statement I is True, Statement II is True; Statement II is **NOT** a correct explanation for Statement I.
c. Statement I is True, Statement II is False.
d. Statement I is False, Statement II is True.

1. **Statement I:** A potentiometer is preferred over a voltmeter for measurement of e.m.f. of a cell.
Statement II: A potentiometer is preferred, as it does not draw any current from the cell.

2. **Statement I:** The wire of a potentiometer should be of uniform area of cross-section.
Statement II: It satisfies the requirement of the principle of a potentiometer.

3. **Statement I:** In a shunted galvanometer, only, 10% of current is passing through the galvanometer. The resistor of the galvanometer is G. Then the resistance of the shunt is $\frac{G}{9}$.
Statement II: If S is the resistance of the shunt, then voltage across S and G is same.

4. **Statement I:** To increase the range of a voltmeter of resistance R to n times, its resistance should be increased by $(n - 1)$ times.
Statement II: The range of a voltmeter can be increased by connecting a low resistance in series with it.

5. **Statement I:** Higher the range greater is the resistance of an ammeter.
Statement II: To increase the range of an ammeter additional shunt is needed to be used across it.

6. **Statement I:** The resistance of an ideal voltmeter should be infinite.
Statement II: Lower resistance of voltmeters gives a reading lower than the actual potential difference across the terminals.

7. **Statement I:** Voltmeter always gives e.m.f. of a cell if it is connected across the terminals of a cell.
Statement II: Terminal potential of a cell is given by

$$V = E - ir$$

8. **Statement I :** The e.m.f. of the driver cell in the potentiometer experiment should be greater than the e.m.f. of the cell to be determined.
Statement II: The fall of potential across the potentiometer wire should not be less than the e.m.f. of the cell to be determined.

9. **Statement I:** Potential measured by a voltmeter across a wire is always less than the actual potential difference across it.
Statement II: Finite resistance of voltmeter changes current flowing through the resistance across which potential difference is to be measured.

10. **Statement I:** In a metre bridge experiment, a high resistance is always connected in series with a galvanometer.
Statement II: As resistance increases, current through the circuit increases.

Comprehension Type

Solutions on page 6.31

For Problems 1 – 3

A battery is connected to a potentiometer and a balance point is obtained at 84 cm along the wire. When its terminals are connected by a 5 Ω resistor, the balance point changes to 70 cm.

1. Calculate the internal resistance of the cell.
a. 4 Ω b. 2 Ω c. 5 Ω d. 1 Ω

2. Find the new position of the balance point when 5 Ω resistor is changed by 4 Ω resistor.
a. 26.5 cm b. 52 cm c. 67.2 cm d. 83.3 cm

3. How can we make a galvanometer with R_g = 20.0 Ω and I_{fs} = 0.00100 A into a voltmeter with a maximum range of 10.0 V?
a. By adding a resistance 9980 Ω in parallel with the galvanometer
b. By adding a resistance 9980 Ω in series with the galvanometer
c. By adding a resistance 8890 Ω in parallel with the galvanometer
d. By adding a resistance 8890 Ω in series with the galvanometer

For Problems 4 – 5

A battery of e.m.f. 1.4 V and internal resistance 2 Ω is connected to a 100 Ω resistor through an ammeter. The resistance of the ammeter is 4/3 Ω. A voltmeter is also connected to find the potential difference across the resistor.

4. The ammeter reads 0.02A. What is the resistance of the voltmeter?
a. 400 Ω b. 200 Ω
c. 2000 Ω d. 300 Ω

5. The voltmeter reads 1.10V. What is the error in the reading?
a. 0.12 V b. 0.52 V
c. 0.35 V d. 0.23 V

For Problems 6 – 7

A cell of e.m.f. 3.4 V and internal resistance 3 Ω is connected to an ammeter having resistance 2 Ω and to an external resistance of 100 Ω. When a voltmeter is connected across the 100 Ω resistance, the ammeter reading is 0.04 A.

6. Find the resistance of the voltmeter.
a. 400 Ω b. 200 Ω
c. 300 Ω d. 500 Ω

7. Had the voltmeter been an ideal one, what would have been its reading?
a. 7.2 V b. 1.8 V
c. 0.5 V d. 3.24 V

For Problems 8 – 11

A galvanometer (coil resistance 99 Ω) is converted into an ammeter using a shunt of 1 Ω and connected as shown in Fig. 6.77(a). The ammeter reads 3 A. The same galvanometer is converted into a voltmeter by connecting a resistance as shown in Fig. 6.77(b). Its reading is found to be 4/5 of the full scale reading.

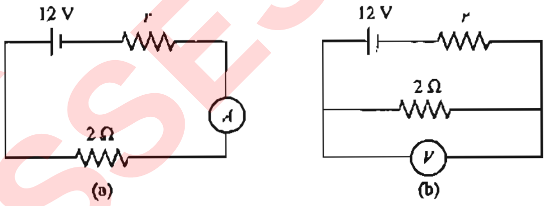


Fig. 6.77

8. Find the internal resistance of the cell (*r*).
a. 2.01 W b. 1.01 W c. 3.15 W d. 5.02 W

9. Find range of the ammeter.
a. 2 A b. 3 A c. 4 A d. 5 A

10. Voltmeter reading is
a. 9.95 V b. 7.95 V c. 9.75 V d. 8.75 V

11. Find full-scale deflection current of the galvanometer.
a. 0.12 A b. 0.5 A c. 0.15 A d. 0.05 A

For Problems 12 – 14

In the connection shown in Fig. 6.78 initially switch *K* is open and the capacitor is uncharged. Then the switch is closed and the capacitor is charged up to the steady state and the switch is opened again. Determine the values indicated by the ammeter. [Given: $V_0 = 30$ V, $R_1 = 10$ kΩ, $R_2 = 5$ kΩ]

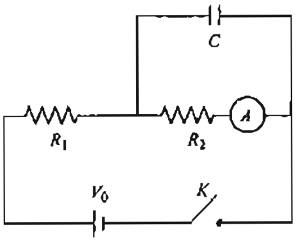


Fig. 6.78

12. Just after closing the switch
a. 2 mA b. 3 mA
c. 0 mA d. None of these

13. A long time after the switch was closed
a. 2 mA b. 3 mA
c. 6 mA d. None of these

14. Just after reopening the switch
a. 2 mA b. 3 mA
c. 6 mA d. None of these

Matching Column Type

Solutions on page 6.33

1. In the circuit shown in Fig. 6.83, battery, ammeter and voltmeter are ideal and the switch S is initially closed as shown. When switch S is opened, match the parameters of column I with the effects in column II

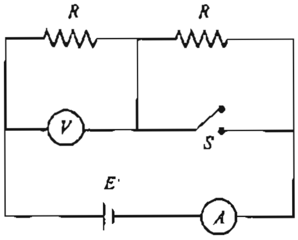


Fig. 6.78

Column I	Column II
i. Equivalent resistance across the battery	a. remains same
ii. Power dissipated by left resistance R	b. increases
iii. Voltmeter reading	c. decreases
iv. Ammeter reading	d. becomes zero

2. In a potentiometer experiment:

Column I	Column II
i. Deflection of galvanometer is in same direction at the two ends of the wire.	a. Accuracy in measurement increases
ii. A protective resistance added in series to the galvanometer	b. Accuracy in measurement decreases
iii. A short wire is used as a potentiometer	c. e.m.f. of the battery in the primary circuit is less than the e.m.f. of the cell to be measured
iv. more length of potentiometer up to null point	d. Uncertainty in the location of balance point increases.

ANSWERS AND SOLUTIONS

Subjective Type

1. a. When the jockey is not connected.

$$I=\frac{E}{16r} \qquad \text{(i)}$$

Resistance per unit length:

$$\lambda=\frac{15r}{600}\Omega/\text{cm}$$

Let ℓ be the length when we get zero deflection.

$$\left(\frac{E}{2}\right)=(\lambda\ell)\ \frac{E}{2}=\frac{E}{16r}\times\frac{15r}{600}\times\ell$$

$$\Rightarrow \quad \ell=320\,\text{cm}$$

b. Let potential at A is zero

Then apply Kirchhoff 's first law

$$\frac{x-0}{14r}+\frac{x-\frac{\varepsilon}{2}-0}{r}+\frac{(x-\varepsilon-0)}{2r}=0 \quad\Rightarrow\quad x=\frac{14\varepsilon}{22}$$

$$I_g=\frac{x-\frac{\varepsilon}{2}}{r}=\frac{\left(\frac{14\varepsilon}{22}\right)-\frac{\varepsilon}{2}}{r}=\frac{3\varepsilon}{22r}$$

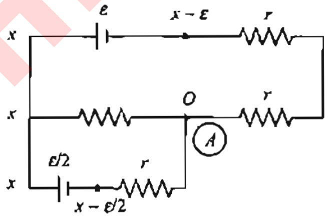


Fig. 6. 79

Alternatively:

When $\ell=500$ cm

$$r'=(560)\times\frac{15r}{600}$$

$$r'=14\text{r}$$

Using KVL in loop (1)

$$E-I_1\times 14r-Ir-Ir=0 \qquad \text{(i)}$$

And in loop (2)

$$-I_1 14r+(I-I_1)r+\frac{E}{2}=0 \qquad \text{(ii)}$$

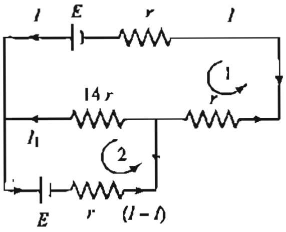


Fig. 6.80

Solving equations (i) and (ii) we have

$$(30\times I_1 r - E) = (E - 14 I_1 r)$$

$$I_1 = \frac{2E}{44\,r}$$

$$I_1 = \frac{E}{22\,r} \text{ and } I = \frac{4E}{22\,r}$$

So current in the galvanometer

$$\text{Branch} = (I - I_1) = \frac{4E}{22r} - \frac{E}{22r}$$

$$I_g = \frac{3E}{22r}$$

2. Let R_V be resistance of the voltmeter. The equivalent resistance of voltmeter and 100 V resistance

$$R' = \frac{100 R_V}{100 + R_V}$$

Net resistance of circuit,

$$R_{net} = R' = R_A + r$$

$$= R' + 3 + 2 = \frac{100 R_V}{10 + R_V} + 5$$

Current in circuit,

$$I = \frac{E}{R_{net}} = \frac{3.4}{\dfrac{100R_V}{100 + R_V} + 5}$$

But $I = 0.04$ A, $\quad \therefore 0.04 = \dfrac{3.4}{\dfrac{100R_V}{100 + R_V} + 5}$

Solving we get,

$$R_V = 400\,\Omega$$

Reading of voltmeter,

$$V = iR' = 0.04 \times \frac{100R_V}{100 + R_V} = 0.04 \times \frac{100 \times 400}{100 + 400}$$

$$= 0.04 \times 80 = 3.2\text{ V}$$

Ideal voltmeter has infinite resistance. In that case net resistance of circuit, $R'_{net} = 100 + 3 + 2 = 105\ \Omega$

$$\therefore \quad \text{Current } i' = \frac{E}{R_{net}} = \frac{3.4}{105}\text{ A}$$

New voltmeter reading $V' = i' \times R = \dfrac{3.4}{105} \times 100 = 3.24$ V

3. As the wire is of uniform cross-section, hence the resistances of the two segments AD and DC of the wire are in the ratio of the lengths of AD and DC.

According to the condition of balance of Wheatstone's bridge, $\dfrac{X}{Y} = \dfrac{\ell_1}{\ell_2}$

Here $\ell_1 = 33.7$ cm and $\ell_2 = 100 - 33.7 = 66.3$ cm

$$\frac{X}{Y} = \frac{33.7}{66.3} \qquad \text{(i)}$$

As resistance Y' is due to a parallel combination of resistance Y and a resistance of 12 Ω, hence

$$\frac{1}{Y'} = \frac{1}{Y} + \frac{1}{12} = \frac{12 + Y}{12Y} \text{ or } Y' = \frac{12Y}{12 + Y}$$

Since Y' is less than Y, hence the ratio X/Y will be greater than X/Y and the null point should shift towards end C.

$$\frac{X}{Y'} = \frac{33.7 + 18.2}{66.3 - 18.2} = \frac{51.9}{48.1}$$

$$\frac{X\,(12 + Y)}{12Y} = \frac{51.9}{48.1}$$

$$12 + Y = \frac{51.9}{48.1} \times 12 \times \frac{Y}{X} = \frac{51.9}{48.1} \times 12 \times \frac{66.3}{33.7}$$

$$= 25.47\,\Omega$$

$$Y = 25.47 - 12 = 13.47\,\Omega$$

Putting this value in equation (i),

$$X = \frac{33.7}{66.3} \times Y = \frac{33.7}{66.3} \times 13.47 = 6.85\,\Omega$$

4. a. Current in wire AB, $I = 2/25 = 0.08$ A

P.D. across $AB = 0.08 \times 20 = 1.6$ V

Potential gradient = P.D. / length

$= 1.6/200 = 0.008\text{ V cm}^{-1}$

b. Current in secondary circuit

= e.m.f./total resistance

$= 3/(2.4 + 0.6) = 1.0$ A

Current through 0.6 Ω resistance = 1.0 A

P.D. between A and O

= P.D. across 2.4 W resistance

$= 1.0 \times 2.4 = 2.4$ V

Length AO = P.D. / Potential gradient

$= 2.4\text{ V} / 0.008\text{ V cm}^{-1} = 75$ cm

5. a. $\quad V = IR + IR_A \;\Rightarrow\; R = \dfrac{V}{I} - R_A.$

The true resistance R is always less than the reading because in the circuit the ammeter's resistance causes the current to be less than the actual. Thus the smaller current requires the resistance R to be calculated larger than what it should be.

b. $\quad I = \dfrac{V}{R} + \dfrac{V}{R_V} \;\Rightarrow\; R = \dfrac{VR_V}{IR_V - V} = \dfrac{V}{I - V/R_V}$

Now the current measured is greater than that through the resistor, so $R = V/I_R$ is always greater than V/I.

c. (i) $\quad P = I^2R = I^2(V/I - R_A) = IV - I^2R_A.$

(ii) $\quad P = V^2/R = V(I - V/R_V) = IV - V^2/R_V.$

6. For X, use (b); for Y use (b).

Voltage drop across resistance and ammeter will be in the ratio of their resistances.

Arrangement (a) is preferred for Y, whose resistance is large as compared to ammeter resistance.

Arrangement (b) is preferred for X, whose resistance is not too large as compared to ammeter resistance. Voltage drop across the ammeter will be an appreciable fraction of that across the resistance and must be excluded.

7. a. By formula, $\dfrac{\varepsilon}{\varepsilon_{\text{standard}}} = \dfrac{\ell}{\ell_{\text{standard}}}$

$$\varepsilon = \frac{\ell \times \varepsilon_{\text{standard}}}{\ell_{\text{standard}}} = \frac{82.3 \times 1.02}{67.3} = 1.2474 \text{ V}$$

b. To reduce the current through the galvanometer when the movable contact is far from the balance point.

c. No.

d. No.

e. No. If ε is greater than the emf of the driver cell of the potentiometer, there will be no balance point on the wire AB.

f. The circuit, as it is, would be unsuitable, because the balance point (for ε of the order of a few mV) will be very close to the end A and the percentage error in measurement will be very large. The circuit is modified by putting a suitable resistor R in series with the wire AB so that potential drop across AB is only slightly greater than the e.m.f. to be measured. Then the balance point will be at larger length of the wire and the percentage error will be much smaller.

8. This means you will not get the balancing point.

$$i = \frac{2}{1+15} = \frac{1}{8},\ E_P = \frac{1}{8} \times 3 \times \frac{7}{2} = \frac{21}{16}$$

$$\frac{21}{16} = i \times 3 \times \frac{9}{2},\ i = \frac{7}{24 \times 3} \text{ A} = \frac{7}{12} \text{ A}$$

$$\frac{7}{72} = \frac{2}{16+R} \quad \Rightarrow \quad R = \frac{32}{7} \Omega = 4.57\,\Omega$$

9.

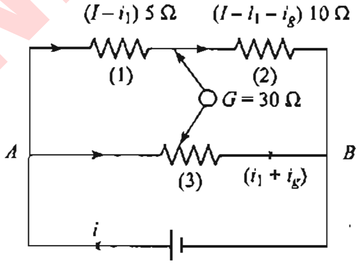


Fig. 6.81

The distribution of currents is shown in Fig. 6.86. Applying Kirchhoff's law to closed mesh (1), (2) and (3), we have

$$5(i - i_1) \times 30i_g - 2i_1 \text{ or } 5i - 7i_1 + 30i_g = 0 \qquad \text{(i)}$$

$$10(i - i_1 - i_g) - 2(i_1 + i_2) - 30i_g = 0$$

$$10i - 19i_1 - 42i_g = 0 \qquad \text{(ii)}$$

$$2i_1 + 2(i_1 + i_2) + i = 5$$

$$4i_1 + 2i_g + i = 5 \qquad \text{(iii)}$$

From equations (i) and (ii)

$$i_1 = 51i_g \qquad \text{(iv)}$$

From equations (ii) and (iii)

$$13i = 36i_g + 15 \qquad \text{(v)}$$

Substituting equation (iv) and (v) in equation (iii), we get

$$i_g = \frac{25}{1357} \text{ A}$$

10. For the balanced bridge, the ratio of the two resistances is equal to the ratio of the lengths of the two parts AJ and JB of the wire, i.e.,

$$\frac{X}{6\Omega} = \frac{40 \text{ cm}}{60 \text{ cm}} \quad \text{or} \quad X = 4\Omega$$

No current flows through the galvanometer G, the resistance of the parts AJ and JB are 40 Ω and 60 Ω, respectively. If R be the equivalent resistance between the points A and B, then we have

$$\frac{1}{R} = \frac{1}{(X+6)\Omega} + \frac{1}{(40+60)\Omega}$$

or $$R = \frac{100}{11}\Omega,\ i = \frac{V}{R} = \frac{6 \text{ V}}{(100/1)\Omega} = 0.66 \text{ A}$$

11. In the given case, the ratio arms are 1000 : 10

$$\frac{P}{Q} = \frac{1000}{10} = 100$$

Third resistance, $R = 999$ V

Let X be the unknown resistance. Then,

$$\frac{P}{Q} = \frac{R}{X} \quad \text{or} \quad \frac{Q}{P} \times R = \frac{1}{100} \times 999 = 9.99\,\Omega$$

12. Resistance per volt at full scale deflection = 5000 ΩV^{-1}

Reading of voltmeter at full scale deflection = 5 V

∴ Resistance of voltmeter

$$G = 5000 \times 5$$
$$= 25000\ \Omega$$

Also current for maximum deflection,

$$I_g = \frac{1 \text{ V}}{5000\Omega} = 0.0002 \text{ A}$$

Range of voltmeter to be changed to V = 20 V

Now, $$R = \frac{V}{I_g} - G = \frac{20}{0.0002} - 25000$$
$$= 100000 - 25000 = 75000\ \Omega$$

Thus, 7500 Ω resistor is to be connected in series.

Resistance of 20 V voltmeter

$$= 75000 + 25000 = 1{,}00{,}000\,\Omega$$

Its grading becomes $= \frac{100000}{20} = 5000\,\Omega\text{V}^{-1}$ which is same as in the earlier case. A voltmeter with grading $2000\,\Omega\,\text{V}^{-1}$ will have less resistance and is therefore not preferred.

13. (i) The circuit diagram is shown.

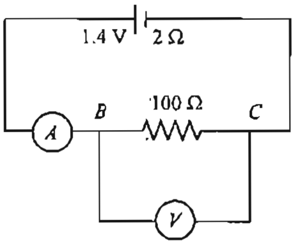


Fig. 6.82

(ii) Let resistance of the voltmeter be R ohm. The equivalent resistance of voltmeter (R ohm) and 100 Ω in parallel is

$$\frac{100 \times R}{100 + R} = \frac{100R}{100 + R}$$

Resistance of the ammeter $= \frac{4}{3}\,\Omega$.

Total resistance of the circuit $= \frac{100R}{100 + R} + \frac{4}{3} + 2\,\Omega$

Current in the circuit as read by the ammeter = 0.02 A.

Now, $\quad 0.002 = \frac{1.4}{\frac{100R}{100 + R} + \frac{4}{3} + 2} \qquad \left(\because I = \frac{V}{R}\right)$

or $\quad R = 200\ \Omega$

∴ Resistance of the voltmeter

$$= 200\ \Omega$$

(iii) Effective resistance between B and C $= \frac{100 \times 200}{100 + 200}$

$$= \frac{200}{3}\,\Omega$$

The potential drop across this resistance = circuit current $\times \frac{200}{3} = 0.02 \times \frac{200}{3} = \frac{4}{3}\text{ V} = 1.333\text{ V}$

$$= \frac{4}{3}\text{ V} = 1.333\text{ V}$$

Reading of the voltmeter = 1.1 V

Error in the reading of the voltmeter

$$= 1.1 - 1.333 = -0.233\text{ V}$$

14. For comparison of two resistances using potentiometer, we have,

$$\frac{R_1}{R_2} = \frac{\ell_1}{\ell_2}$$

Here, $\quad R_1 = R = 10\,\Omega, R_2 = X$

$$\ell = 58.3\text{ cm}, \ell_2 = 68.5\text{ cm}$$

or $\quad X = \frac{10 \times 68.5}{58.3} = 11.7\,\Omega$

If there is no balance point, it means potential drop across R or X is greater than the potential drop across the potentiometer wire AB. The obvious thing to do is to reduce the current in the outside circuit (and hence potential drops across R and X) suitably by putting resistor.

15. Let R be the resistance to be introduced in the box. Then current in the potentiometer wire is given by

$$1 = \frac{E}{R + \ell_\rho}$$

where ρ is the resistance per unit length of the wire and ℓ is the length of the wire.

Now, potential gradient

$$V = I\rho = \frac{E\rho}{R + \ell\rho}$$

Here, $\quad \ell = 10$ m

$\quad \rho = 4\ \Omega/\text{m}$

(i) For $V = 0.1$ V/m, we have

$$0.1 = \frac{2 \times 4}{R + 10 \times 4} = \frac{8}{R + 40}$$

or $\quad R = \frac{4}{0.1} = 40\ \Omega$

(ii) For $\quad V = 0.1\text{ mV/m} = 0.1 \times 10^{-3}$ V/m,

we have $0.1 \times 10^{-3} = \frac{2 \times 4}{R + 10 \times 4}$

or $\quad 10^{-4} = \frac{8}{R + 40}$

or $\quad R = 79960\ \Omega$.

Note that as there is no current through the cell and galvanometer, battery E, internal resistance r and potentiometer wire AB are in series.

16. Internal resistance of a cell using potentiometer is given by,

$$r = R \times \frac{\ell_1 - \ell_2}{\ell_2}$$

Here $\quad R = 9.5\,\Omega$

$\quad \ell_1 = 76.3$ cm

$\quad \ell_2 = 64.8$ cm

Hence $\quad r = 9.5 \times \frac{76.3 - 64.8}{64.8}$

$$9.5 \times \frac{11.5}{64.8} = 1.7\,\Omega$$

Objective Type

1. **c.** Resistance is halved. Current is doubled.
2. **b.** $E_1 \propto 64$

$E_1 - E_2 \propto 8$

$E_2 \propto l$

$\therefore \quad 64 - l = 8 \quad \text{or} \quad l = 64 - 8 = 56 \text{ cm}$

3. d. Voltage across 5 Ω = 10 V

$$\therefore \quad I = \frac{10}{5} \text{ A} = 2 \text{ A}$$

4. b. The circuit may be redrawn as follows:

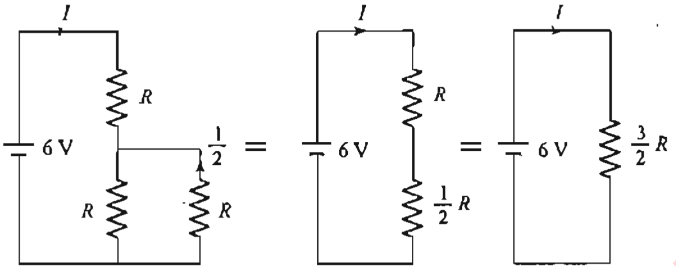


Fig. 6.83

Current is given by $I = \dfrac{6}{\frac{3}{2}R} = \dfrac{4}{R}$A

$\therefore$ Current through the voltmeter is $\dfrac{1}{2}$ or $\dfrac{2}{R}$A.

Hence, the reading of the voltmeter is $\left(\dfrac{2}{R}\right)(R)$ or 2 V.

5. a. Since the opening or closing of the switch does not affect the current through G, it means that in both the cases there is no current passing through S. This means that potential at A is equal to potential at B and it is the case of balanced wheatstone bridge. $I_P = I_Q\, I_R = I_G$ and **(a)** is the correct option.

6. a. At null point, $\dfrac{R_1}{R_2} = \dfrac{R_3}{R_4} = \dfrac{x}{100 - x}$. If radius of the wire is doubled, then the resistance of AC will change and the resistance of CB will also change. But since $\dfrac{R_1}{R_2}$ does not change, so $\dfrac{R_3}{R_4}$ should also not change at null point. Therefore point C does not change.

7. a. Using the principle of potentiometer, $V \propto l$

$$\therefore \quad \frac{V}{E} = \frac{l}{L} \quad \text{or.} \quad V = \frac{l}{L}E = \frac{30}{100}E = \frac{30\,F}{100}.$$

8. a. $\dfrac{X}{Y} = \dfrac{20}{80} = \dfrac{1}{4} \quad \text{or} \quad Y = 4X$

$$\frac{4X}{Y} = \frac{l}{100 - l} \quad \text{or} \quad \frac{4X}{4X} = \frac{l}{100 - l}$$

or $\quad l = 100 - l \quad \text{or} \quad 2l = 100 \text{ cm}$

or $\quad l = 50 \text{ cm}$

9. c. Current through $R = \dfrac{12}{500 + R}$

Voltage across $R = \dfrac{12\,R}{500 + R}$

Since galvanometer shows zero deflection,

$$= \frac{12\,R}{500 + R} = 2$$

or $\quad 12\,R = 1000 + 2R$

or $\quad 10\,R = 1000 \text{ or } R = 100\,\Omega$

10. a. $r = \dfrac{l_1 - l_1}{l_2}R = \dfrac{240 - 120}{120} \times 2\,\Omega = 2\,\Omega.$

11. b. When Wheatstone bridge is balanced, then

$$\frac{P}{Q} = \frac{R}{S} \quad \text{or} \quad \frac{P}{R} = \frac{Q}{S}$$

If the galvanometer is replaced with a cell in balanced Wheatstone bridge, the condition for balanced bridge will be $\dfrac{P}{R} = \dfrac{Q}{S}$, which is there.

Hence balance point will remain unchanged, where galvanometer shows no current.

12. b. Sensitivity of potentiometer means the smallest potential difference it can measure. It can be increased by reducing the potential gradient. The same is possible by increasing the length of the potentiometer.

13. c. $E' = \dfrac{\ell'}{\ell}E = \dfrac{30}{100}E$

14. c. $I_g = \dfrac{IS}{S + G} \quad \Rightarrow \quad 10 = \dfrac{50 \times 12}{12 + G}$

$$\Rightarrow \quad 12 + G = 60 \quad \Rightarrow \quad G = 48\,\Omega$$

15. c. $i_g = \dfrac{V}{G + R} \quad \Rightarrow \quad 10^{-3} = \dfrac{25}{10 + R}$

i.e., $\quad R = 2490\ \Omega$

16. a. $\quad V = I_g(R + G) \quad \text{or} \quad I = 10 \times 10^{-3}(r + 0.2)$

or $\quad 100 = R + 0.2, \quad \text{i.e.,} \quad R = 99.8\,\Omega$

17. c. $\quad i_g = 10 \text{ mA} = 0.01 \text{ A}$

$r = 1\,\Omega$

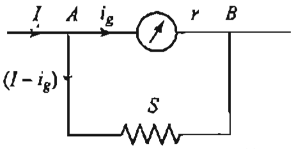


Fig. 6.84

$$I = 1 \text{ A}$$

$$V_A - V_B = i_g r = (I - i_g)\,S$$

$$S = \frac{i_g r}{(I - i_g)} = \frac{0.01 \times 1}{1 - 0.01} = \frac{1}{99}\,\Omega$$

18. c. $\quad i_g = 0.01\text{A}$

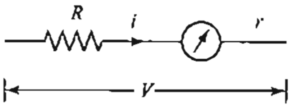


Fig. 6.85

$$r = 1\,\Omega \qquad V = 10$$

$$V = (r+R)i_g$$

$$R = \frac{V}{i_g} - r = \left(\frac{10}{0.01} - 1\right)\Omega = 999\,\Omega$$

19. d. $E_1 \propto 300$

$$E_1 - E_2 \propto 100$$

$$\frac{E_1}{E_1 - E_2} = 3 \quad \text{or} \quad E_1 = 3E_1 - 3E_2$$

$$\text{or} \qquad 3E_2 = 2E_1 \quad \text{or} \quad \frac{E_1}{E_2} = \frac{3}{2}$$

20. d. R is the order of 15,000 Ω. The junctions of the highest two and the lowest two resistances are A and C, and for better sensitivity, the galvanometer will be between these. So, error will decrease with the suggested interchange.

21. b. Total resistance $= \left(\frac{5 \times 15}{5+15} + 1.25\right)$

$$= \left(\frac{75}{20} + 1.25\right)\Omega = (3.75 + 1.25)\Omega = 5\,\Omega$$

$$I = \frac{20}{5}\,\text{A} = 4\,\text{A}$$

Current through 5 Ω $= \frac{15}{20} \times 4\,\text{A} = 3\,\text{A}$

Voltmeter reading

= Potential drop across 1.25 Ω

$= 4 \times 1.25\,\text{V} = 5\,\text{V}$

22. a. Effective resistance across the voltmeter $= \frac{200}{2} = 100\,\text{k}\Omega$

Total resistance across d.c. supply = 400 + 100 = 500 kΩ.

Thus the voltage across the voltmeter $= \frac{100}{500}(60\,\text{V}) = 12\,\text{V}.$

23. b. When X is connected to Y, the balance length l is proportional to the p.d. across the 100 Ω resistor. When X is connected to Z, the balance length is proportional to the p.d. across the 100 Ω resistor and resistor R. Assuming that the current through the 100 Ω resistor and resistor R at balance is constant and unchanged when X is reconnected, it follows that resistor ratio $\frac{100+R}{100}$ is equal to the length ratio.

24. b. A careful analysis would show that the voltage along R is 1.03 V.

$$1.03 = 1 \times R \quad \text{or} \quad R = 1.03\,\Omega$$

25. c. If V is the current difference applied across P and Q, the current through M is determined by

Circuit	Current
(a)	$V/5$
(b)	$3\,V/8$
(c)	$V/2$
(d)	$V/3$

Hence, circuit arrangement (c) gives the largest reading in ammeter M.

26. b. The voltage per unit length on the meter wire PQ is $\frac{6.00\,\text{mV}}{0.60\,\text{m}}$ or $10\,\text{mV/m}$

Hence, potential across the meter wire PQ is 10 mV/m (1m) = 10 mV.

Current drawn from the driver cell,

$$I = \frac{10\,\text{mV}}{5\,\Omega} = 2\,\text{mA}$$

Resistance of the resistor R is

$$R = \frac{2\,\text{V} - 10\,\text{mV}}{2\,\text{mA}} = \frac{1990\,\text{mV}}{2\,\text{mV}} = 995\,\Omega$$

27. b. $\frac{P}{Q} = \frac{R}{S}$. If P is increased then either P or Q should be increased or S should be decreased.

28. b. The terminal potential difference across R due to 12 V battery should be equal to 2V which is the e.m.f. of the cell in the loop containing the ammeter.

$$\text{So} \quad 12 - \frac{12}{500+R} \times 500 = 2$$

$$10 = \frac{12 \times 500}{500 + R}$$

$$\text{or} \qquad 500 + R = 60 \quad \text{or} \quad R = 100\,\Omega.$$

29. d. Reading will remain zero, whatever may be the value of ammeter resistance.

30. b. In case of internal resistance measurement by potentiometer,

$$\frac{V_1}{V_3} = \frac{\ell_1}{\ell_2} = \frac{\{ER_1/(R_1+r)\}}{\{ER_2/(R_2+r)\}} = \frac{R_1(R_2+r)}{R_2(R_1+r)}$$

Here $\ell_1 = 2\,\text{m}, \ell_2 = 3\,\text{m}, R_1 = 5\,\Omega$ and $R_2 = 10\,\Omega$

$$\therefore \quad \frac{2}{3} = \frac{5(10+r)}{10(5+r)} \quad \text{or} \quad 20 + 4r = 30 + 3r$$

$$\text{or} \qquad r = 10\,\Omega$$

31. a. Here $1 = \frac{E}{r + R_1 + R_2} = \frac{12}{r + R_1 + R_2}$

$$\therefore \qquad r + R_1 + R_2 = 12$$

$$\text{Also,} \qquad 3 = IR_1 \quad \text{or} \quad 3 = 1 \times R_1$$

$$\therefore \qquad R_1 = 3\,\Omega$$

When points A and B are connected by a conducting wire, R_2 is short circuited.

$$\therefore \quad 10.5 = I'R_1 \quad \text{or} \quad 10.5 = I' \times 3$$

$$\therefore \quad I' = \frac{10.5}{3} = 3.5 \text{ A}$$

But $\quad 10.5 = E = I'r \quad \text{or} \quad 10.5 = 12 - 3.5r$

$$\therefore \quad r = \frac{1.5}{3.5} = \frac{3}{7}\,\Omega$$

32. b. When a voltmeter is connected with 400 Ω resistance the potential difference = 30 V. Since, applied p.d. is equally shared between 300 Ω resistance and equivalent resistance (due to voltmeter and 400 Ω resistance), equivalent resistance:

$$\frac{400R}{400+R} = 300 \quad \Rightarrow \quad R = 1200\,\Omega$$

When voltmeter is connected across 300 Ω resistance, then

$$R_{eq} = \frac{300 \times 1200}{1200+300} = 240\,\Omega$$

$$\frac{V_{240}}{V_{400}} = \frac{240}{400} = \frac{3}{5}, V_{240} + V_{400} = 60$$

$$V_{240} = \left(\frac{3}{8}\right) \times 60 = 22.5 \text{ V}$$

33. b. The current through the galvanometer producing full scale deflection is

$$I = \frac{V}{R} = \frac{20 \times 10^{-3}}{80} = 2.5 \times 10^{-4} \text{ A}$$

To convert the galvanometer into a voltmeter, a high resistance is connected in series with the galvanometer

$$\therefore \quad 5\text{ V} = (2.5 \times 10^{-4})(R+80)$$

$$\therefore \quad R = 19.92\text{ k}\Omega$$

34. b. Here, $V = I(r+R)$

$$\therefore \quad 3 = I(12+R)$$

or $\quad 3 = 0.01\,(12+R) \quad \text{or} \quad R = 300 - 12 = 288\,\Omega$

35. a. $\quad V_1 = E - ir = 50 - \frac{50}{220} \times 20$

$$= 50 - 4.6 = 45.4 \text{ V}$$

Now, $\quad V_2 = 50 - \frac{50}{180} \times 20 = 44.4 \text{ V}$

Percentage change $= \frac{V_1 - V_2}{V_1} \times 100 = 2.27$ (also see the question)

36. a. The electric current through ideal voltmeter is zero. According to loop rule,

$$E - 1 \times I - 1 \times I = 0 \quad \Rightarrow \quad I = \frac{E}{2} = \frac{2}{2} = 1 \text{ A}$$

Reading of the voltmeter

$$= V_A - V_B = [1 \times I] = [1 \times 1] = 1\text{V}$$

37. c. Let the series resistance = R

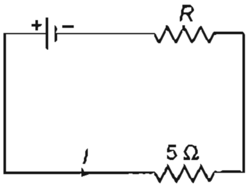


Fig. 6.86

Then the current through potentiometer wire

P.D. across 100 cm wire $= \frac{2 \times 5}{R+5}$

P.D. on 1 cm wire $= \frac{10}{(r+5)} \times \frac{1}{100}$

$$= \frac{1}{10(R+5)}$$

$$\therefore \quad R + 5 = \frac{10^3}{10 \times 0.05} = 2000$$

$$\therefore \quad R = 2000 - 5 = 1995\,\Omega$$

38. a. The value of E = Potential gradient × Length

$$= 0.05 \times 60.3 = 3.015 \text{ mV}$$

39. a. $\quad V_A = iR$

$$V_B = 2i = \frac{2i}{3} \times 1.5R = iR$$

$$V_C = (i/3)(3R) = iR$$

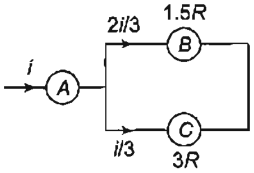


Fig. 6.87

40. c. $\quad i_g = 10 \text{ mA} = 0.01 \text{ A}$

$$V_A - V_B = (I - i_g)\,0.1 = i_g$$

Or $\quad I = \frac{10 \times 0.01}{0.1} = 1 \text{ A}$

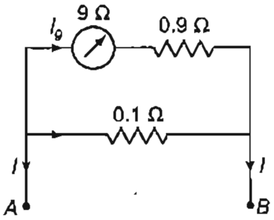


Fig. 6.88

41. b. Let a current of x ampere passes through the voltmeter; then $(4 - x)$ ampere passes through the resistance R. Therefore, voltmeter reading

$$20 = (4 - x)R$$

$$\therefore \quad R = \frac{20}{4-x}, \text{ i.e., } R > 5\,\Omega$$

42. d. $\dfrac{I_g}{I} = \dfrac{S}{S+G}$

$\therefore \quad \dfrac{I_g}{I} = \dfrac{(G/10)}{(G/10)+G} = \dfrac{1}{11}$

Initially, $\alpha_i = \theta/Ig$ (1)

Finally, after the shunt is used,

$\alpha_f = \theta/I$ (2)

$\therefore \quad \dfrac{\alpha_f}{\alpha_i} = \dfrac{\theta/I}{\theta/I_g} = \dfrac{I_g}{I} = \dfrac{1}{11}$

So, current sensitivity becomes $\dfrac{1}{11}$ fold.

43. b. Let I be the current in the circuit, then

$1 \times 4000 = 30\,\text{V}$

A voltmeter is put across 3000 resistance

$\therefore \quad 1 \times 3000 = \dfrac{30}{4000} \times 3000 = 22.5\ \text{V}$

44. d. Ideal ammeter has zero resistance. So, potential drop across it $= IR\,A = I \times 0 = 0$

45. b. We know that $R = \dfrac{V}{I_g} - G$

The voltmeter gives full-scale deflection for potential difference V. Its resistance is G.

Hence $\quad I_g = (V/G)$

Given that $\quad V = nV$

$\therefore \quad R = \dfrac{nV}{(V/G)} - G = (n-1)\,G$

46. b. $\dfrac{I_g}{I} \times \dfrac{S}{S+G}$ or $\dfrac{1}{34} = \dfrac{S}{S+G}$

$\therefore \quad S = (G/33) = (3663/33) = 111\,\Omega$

47. c. $R_s = \dfrac{SG}{S+G} = \dfrac{11 \times 3663}{111 \times 3663} = 107.7\,\Omega$

48. d. Compensation external resistance

$= G - \dfrac{SG}{S+G} = 3663 - 107.7 = 3555.3\,\Omega$

49. a. $I = \dfrac{k}{MBA}\theta$

Given that $I_1 = I_2$

$\therefore \quad \dfrac{K\theta_1}{N_1BA_1} = \dfrac{K\theta_2}{N_2BA_2}$

So $\quad \dfrac{\theta_1}{\theta_2} = \dfrac{A_1N_1}{A_2N_2}$

50. b. For ammeter, $S = \dfrac{I_g}{I - I_g} \times G$

$\therefore \quad \dfrac{S}{G} = \dfrac{I_g}{I - I_g}$ or $\dfrac{G}{S}\left(\dfrac{I}{I_g} - I\right)$

or $\quad \dfrac{30}{30} = \dfrac{I}{I_g} - I$ or $I - 2I_g$

New range is doubled, i.e., 4 I_g

Now shunt required,

$S = \dfrac{I_g}{4I_g - I_g} \times G = 10\,\Omega$

This can be obtained by shunting the earlier shunt of 30 Ω with an additional shunt of 10 Ω.

Multiple Correct Answers Type

1. a., b., c., d.

Sensitivity $\propto \dfrac{1}{\text{Length of potentiometer wire}}$

$\propto$ Potential difference across the potentiometer wire.

2. b., c.

The potential measures the exact value of e.m.f. of a battery

$\therefore \quad E = 1.55\,\text{V}$

Also $\quad 1.4 = I(280) \quad \therefore \quad I = 0.005\ \text{A}$

Also $\quad V = E - Ir \quad \therefore \quad r = \dfrac{E-V}{I} = \dfrac{1.55 - 1.40}{0.005}$

$= \dfrac{0.15}{0.005} = 30\,\Omega$

3. a., b.

Since voltmeter is a device connected in parallel across the circuit, hence $R_{\text{equivalent}} = \dfrac{R_vR_0}{R_v + R_0}$

For $\quad R_0 << R_v \quad \Rightarrow \quad R_{\text{equivalent}} = R_0$

(i.e., resistance of the circuit remains unaltered when a voltmeter of extremely high resistance is applied across the circuit)

4. a., b., d.

Since $\dfrac{R_1}{R_2} = \dfrac{C_2}{C_1}$, the Wheatstone's bridge is balanced.

Hence, $V_C = V_D$. No current passes through the galvanometer.

Hence, choice **(a)** is correct

Potential difference across R_1 = potential difference across $C_1 = 4\,\text{V}$

Potential difference across R_2 = potential difference across $C_2 = 5\,\text{V}$

Potential difference across 5 Ω is the potential difference across 8 μF capacitor. So, charge across 8 μF capacitor $Q = CV$.

$Q = 8\,\mu\text{F} \times 5\,\text{V} = 40\,\mu\text{C}$

Hence **(b)** is correct and also choice **(d)** is correct.

5. a., d.

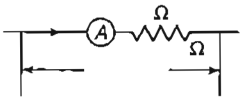


Fig. 6.89

$$I = \frac{10}{50 + 200 \times 10^3} = \frac{10}{200 \times 10^3} = 50\ \mu A$$

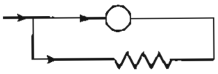


Fig. 6.90

$$50 \times 50 = 50\,(I - 50) \quad \Rightarrow \quad I = 100\,\mu A = 0.1\,\mu A$$

6. b., c.

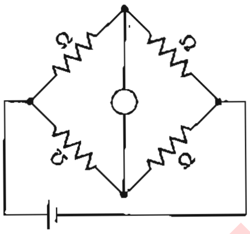


Fig. 6.91

If P is slightly increased, potential of C will decrease. Hence current will form A to C.

If Q is slightly increased, pot. of C will inc, hence current will flow from C to A.

Assertion-Reasoning Type

1. **a.** In a balanced condition, the potentiometer does not draw any current, and hence does not disturb the circuit.
2. **a.** Principle of a potentiometer states that drop of potential across any segment of the potentiometer wire is directly proportional to its length. This can be satisfied if the wire of the potentiometer has a uniform area of cross-section.
3. **a.** Potential drop across galvanometer = potential drop across shunt, i.e.,

$$I_g G = (I - I_g)S$$

$$\Rightarrow \quad S = \frac{I_g}{I - I_g} G \qquad \left(\text{Here} : I_g = \frac{1}{10}\right)$$

$$\text{Hence,} \quad S = \frac{\frac{1}{10}}{\left(I - \frac{1}{10}\right)} G = \frac{G}{9}$$

4. **c.** Initially, $V = RI_g$

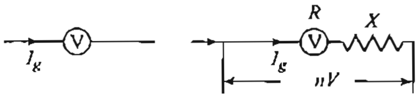


Fig. 6.92

After the range is increased,

$$nV = I_g(R + x) \quad \Rightarrow \quad x = \frac{nV}{I_g} - R$$

$$= \frac{nV}{V/R} - R = nR - R = (n-1)\,R$$

5. **d.** Lower the resistance of an ammeter, higher is the range.
6. **a.** If resistance of a voltmeter is not infinite, it will draw some current from the circuit and finally the reading will be less than actual.
7. **d.** Voltmeter gives terminal potential (V) though it can give e.m.f. if internal resistance of the cell is zero.
8. **a.** If either the e.m.f. of the driver cell or the potential difference across the whole potentiometer wire is lesser than the e.m.f. of the experimental cell, the balance point will not be obtained.
9. **a.** $i_1 = \frac{E}{R}$, $R_{eq} = \frac{RR_V}{R + R_V}$

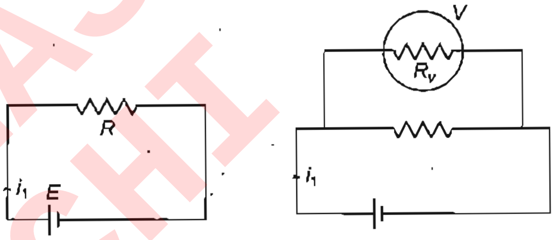


Fig. 6.93

$$i_2 = \frac{E}{(RR_V / R + R_V)}$$

10. (c) The resistance of a galvanometer is fixed. In metre bridge experiments, to protect the galvanometer from a high current, high resistance is connected to the galvanometer in order to protect it from any damage.

Comprehension Type

For Problems 1 – 3

1. d., 2. c., 3. b.

Sol. We have

$$R_s = \frac{V}{I_{fs}} - R_c = \frac{10.0\,\text{V}}{0.00100\ \text{A}} - 20.0\Omega = 9980\Omega$$

At full-scale deflection, $V_{ab} = 10.0$ V, voltage across the meter is 0.0200 V, voltage across R_s is 9.98 V, and current through the voltmeter is 0.00100 A. In this case most of the voltage appears across the series resistor.

The equivalent meter resistance is $R_{eq} = 20.0\ \Omega + 9980\ \Omega = 10{,}000\ \Omega$. Such a meter is described as a "1,000 ohms-per-volt meter" referring to the ratio of resistance to full-scale deflection. In normal operation the current through the circuit element being measured is much greater than 0.00100 A, and the resistance between points a and b in the circuit is much less than 10,000 Ω. So the voltmeter draws off only a small fraction of the current and disturbs. only slightly the circuit being measured.

For Problems 4 – 5

4. b., **5.** d.

Sol. Fig 6.99 shows the circuit diagram.

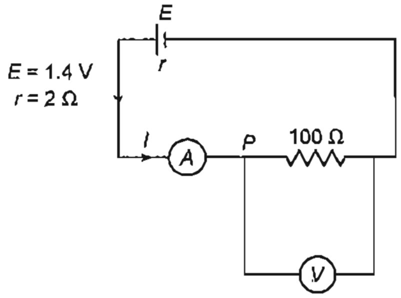


Fig. 6.94

a. Let R be the resistance of the voltmeter. Then the total resistance in the circuit is

$$\left(Y+\frac{4}{3}+2\right)$$

where Y is given by

$$\frac{1}{R}+\frac{1}{100}=\frac{1}{Y}. \quad \text{or} \quad Y=\frac{100R}{100+R}$$

Therefore,

$$Y+\frac{4}{3}+2=\frac{100R}{100+R}+\frac{10}{3}=\frac{310R+100}{3(100+R)}$$

The current I is given by

$$I=\frac{\text{e.m.f.}}{\text{total resistance}}=\frac{0.4\times3(100+R)}{310R+100}=0.02\text{ A}$$

Therefore $\quad 6.2R+20=4.2(100+R)$

which gives $\quad R=200\ \Omega$

b. The total resistance X of the 100 Ω resistor and the voltmeter is given by

$$\frac{1}{X}=\frac{1}{100}+\frac{1}{200}=\frac{3}{200}$$

$$X=\frac{200}{3}\Omega$$

Potential difference across the voltmeter

$$=\frac{200}{3}\times0.02=1.33\text{ V}$$

Voltmeter reading = 1.0 V

Error = 1.33 – 1.10 = 0.23 V

For Problems 6 – 7

6. a., **7.** d.

Sol. Let R_1 and R_2 be the resistances of the ammeter and the voltmeter, respectively. Let the external resistance be denoted by R and the internal resistance of battery by r. The equivalent resistance of the parallel combination of R and R_2 is given by

$$R'=\frac{RR_2}{R+R_2}$$

The total resistance RT of circuit then becomes

$$RT=R_1+r+\frac{RR_2}{R+R_2}$$

The current in the circuit is given by

$$I=\frac{E}{R_1+r+\dfrac{RR_2}{R+R_2}}$$

This must be equal to 0.04 A, the reading indicated by the ammeter.

$$\frac{3.4}{2+3+\dfrac{100R_2}{100+R_2}}$$

$$5+\frac{100R_2}{100+R_2}=\frac{3.4}{0.04}=85$$

$$\frac{100R_2}{100+R_2}=80$$

which on simplification gives

$$R_2=400\ \Omega$$

Total current I divides itself into I_1 along R and I_2 along R_2.

$$I_2=\left(\frac{R}{R+R_2}\right)I$$

$$=\frac{100\times0.04}{100+400}=\frac{4}{500}=0.008\text{ A}$$

Potential drop across R_2 is given by

$$V=I_2R_2=0.008\times400=3.2\text{ A}$$

The voltmeter shows this reading.

In case of an ideal voltmeter, no current flows through it. In that case current in the circuit is

$$I'=\frac{3.4}{2+3+100}=\frac{3.4}{105}=0.0324\text{ A}$$

Potential drop across the resistance R would be 100×0.0324 = 3.24 V. This should be the reading indicated by an ideal voltmeter.

For Problems 8 – 11

8. b., **9.** d., **10.** a., **11.** d.

Sol. For ammeter:

$$99I_g=(I-I_g)I \quad \text{or} \quad I=100I_0 \qquad \text{(i)}$$

I_g is the full-scale deflection current of the galvanometer and I is the range of ammeter

For the circuit in Fig. 6.81(a)

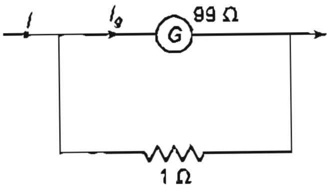


Fig. 6.95

$$\frac{12\text{ V}}{2+r+\frac{99\times 1}{99+1}}=3\text{ A} \quad \Rightarrow \quad r=1.01\,\Omega$$

For voltmeter, range:

$$V = I_g(99+101)$$

$$V = 200\,I_g \qquad \text{(ii)}$$

Also resistance of the voltmeter = 99 + 101 = 200 Ω

In Fig.6.81(b) resistance across the terminals of the battery

$$R_1 = r + \frac{200\times 2}{202} = 2.99\,\Omega$$

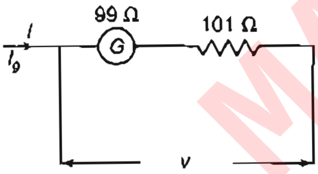


Fig. 6.96

∴ Current drawn from the battery,

$$I_1 = \frac{12}{2.99} = 4.01\text{ A}$$

∴ Voltmeter reading:

$$\frac{4}{5}V = 12 - I_1 r = 12 - 4.01\times 1.01$$

$$V = 7.96\times\frac{5}{4} = 9.95\text{ V}$$

Using (ii), $I_g = \frac{9.95}{200} = 0.05\text{ A}$

Using (i), range of the ammeter: $I = 100\,I_g = 5\text{ A}$

For Problems 12 – 14

12. c., 13. a., 14.a.

Sol. 12. c. Just after closing, capacitor behaves as a short circuit and all current flows through it, hence ammeter reads zero.

13. a. After a long time capacitor behaves like an open circuit and no current flows through it.

$$\text{Therefore } i = \frac{V_0}{R_1+R_2} = \frac{30}{10+5} = 2\text{ mA}$$

14. a. Just after reopening, potential difference across R_2 remains same as charge on the capacitor does not change initially, hence current remains same.

Matching Column Type

1. i. → b., ii. → c., iii. → c., iv. → c.

When the switch is closed, equivalent resistance is R. After opening the switch, equivalent resistance becomes $2R$. Hence equivalent resistance increases.

Also current through the battery decreases, hence ammeter reading decreases. Current through the left R also decreases. So voltmeter reading decreases and power dissipated through the left R also decreases.

2. i. → c., ii. → d., iii. → b,d., iv. → a.

i. If deflection in a galvanometer is in some direction for the position of jockey on one side of the null point, then for the position of jockey on the other side of the null point, the deflection in the galvanometer should be in the opposite direction. But if e.m.f. of the battery in the primary circuit is less than the e.m.f. of the cell to be measured, then for all positions of jockey on wire, deflection in the galvanometer will be in one direction only.

ii. Due to protective resistance, the galvanometer will show less deflection when away from the null point. Hence uncertainty in location of the null point increases.

iii. For a short potentiometer wire, accuracy is less.

iv. For a long potentiometer wire, accuracy is more.

CHAPTER

7

Heating Effects of Current

HEATING EFFECT OF CURRENT

When an electric current is passed through a conductor, it becomes hot and its temperature starts rising. This is known as *heating effect of current* or *Joule's heating effect*. Here electric energy converts into heat energy.

Various appliances, such as geyser, iron, heater, fuse wire, etc. work on this basis.

Cause of Heating

When current is passed through a conductor, the electrons start drifting towards the +ve end. They gain additional K.E. apart from thermal K.E. These electrons suffer collisions with atoms/ions more violently and transfer their K.E. to atoms/ions. It increases the amplitude of vibrations of ions/atoms. Thus average K.E of vibrations of atoms/ions increases which shows up in the form of increased temperature. Here, the electric energy supplied by the source of e.m.f. is converted into heat.

Heat Produced by an Electric Current

Let a current I is flowing in a resistor of resistance R (Fig. 7.1). Amount of charge passed through resistor in time t: $q = It$

Decreases in the potential of this charge: $V = IR$

Decreases in the potential energy of charge $= qV = I^2Rt$

Fig. 7.1

This decrease in the energy will appear in the form of heat energy.

So electric energy produced in a resistor of resistance R in time t in which a current I is flowing is given by

$$H = I^2RT \qquad \text{(This is Joule's law of heating.)}$$

Joule's law of heating: It states that amount of heat produced in a conductor is directly proportional to (i) square of current, (ii) resistance of conductor and (iii) time.

Other forms of H: $H = I^2Rt = \dfrac{V^2}{R} = VIt$

Joule's heating effect is irreversible. The resistor will become hot (and not cool down), if current is sent in any direction.

As $H \propto I^2$, heating effect of current is common to both d.c. and a.c. This is why instruments and appliances such as filament bulb, heater, geyser, press, toaster, etc. work both on d.c. and a.c.

Electric Power Produced in the Circuit

It is the energy produced in the resistor per unit time.

$$P = \frac{H}{t} \Rightarrow P = I^2R = \frac{V^2}{R} = VI$$

Units of Electric Energy and Electric Power

Unit of electric energy: J, cal, kWh, etc.

$$1 \text{ cal} = 4.18 \text{ J} \simeq 4.2 \text{ J}$$

Relation between kWh and joule: 1 kWh = 1000 W × hour = 1000 W × 3600 s = 3.6 × 10^6 Ws = 3.6 × 10^6 J.

1 kWh is the energy consumed by an appliance of power 1 kW when it runs for 1 h.

Commercial unit: 1 kWh is one unit of electricity. To calculate number of units, we can use

$$\text{Number of units} = \frac{\text{watt} \times \text{hour}}{1000}$$

The energy dissipated in kWh can be calculated using the following relation:

$$E = \frac{\text{V (in volt)} \times I \text{ (in ampere)} \times t \text{ (in hour)}}{1000}$$

Unit of electric power: W, kW, MW, hp; 1 kW = 10^3 W, 1 MW = 10^6 W, 1 hp = 746 W.

Some Important Points

1. If the resistances are connected in series, then using $P = I^2R$, power developed will be higher in the resistor of higher value as current will be same in all resistors.
2. If the resistances are connected in parallel, then using $P = \dfrac{V^2}{R}$, the power developed will be higher in the resistor of lower value as potential will be same across all resistors.

Illustration 7.1 **Two wires of same mass, having ratio of the lengths 1:2, density 1:3 and resistivity 2:1. They are connected one by one to the same voltage supply. The rate of heat dissipation in the first wire is found to be 10 W. Find the rate of heat dissipation in the second wire.**

Sol. Given $\dfrac{\ell_1}{\ell_2} = \dfrac{1}{2}, \dfrac{d_1}{d_2} = \dfrac{1}{3}, \dfrac{\rho_1}{\rho_2} = \dfrac{2}{1}; \quad m = A_1\ell_1d_1 = A_2\ell_2d_2$

$$\frac{P_2}{P_1} = \frac{R_1}{R_2} = \frac{\rho_1\ell_1A_2}{\rho_2\ell_2A_1} = \frac{\rho_1\ell_1^2d_1}{\rho_2\ell_2^2d_2}.$$

$$\Rightarrow \quad \frac{P_2}{P_1} = \frac{2}{1} \times \left(\frac{1}{2}\right)^2 \times \frac{1}{3} = \frac{1}{6}$$

$$\Rightarrow \quad P_2 = \frac{P_1}{6} = \frac{10}{6} = \frac{5}{3} \text{ W}$$

3. **To determine the resistance of a bulb (or other appliances):** Let a bulb is designed to operate on a voltage V_0 and its power indicated on it is P_0 (see Fig. 7.2). The resistance of the bulb is given by $R = \dfrac{V_0^2}{P_0}$.

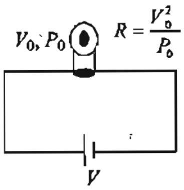


Fig. 7.2

Now let a potential difference of V is applied across this bulb, then power consumed is given by $P = \frac{V^2}{R} = \left(\frac{V}{V_0}\right)^2 P_o$.

If $V = V_0$, then $P = P_0$.

The above formula is very the convenient to calculate the power consumption when the applied voltage is different from the specified one.

An electric appliance consumes the specified power P_0 only if it runs at the specified voltage V_0. If the applied voltage V_A is greater than the specified voltage the appliance may get damaged as in this situation $I = (V_A/R)$ will exceed its current capacity $I_C = (V_0/R)$. Further, if an appliance is made to run at a voltage lower than the specified, then true power consumption will be less than the specified value.

Illustration 7.2 **A 100 W bulb is designed to operate on a potential difference of 230 V.**

- **i. Find the resistance of the bulb.**
- **ii. Find the current drawn by the bulb if it is operated at a potential difference for which it is designed.**
- **iii. Find the current drawn and power consumed by the bulb if it is connected to 200 V supply.**

Sol. Power rating of the bulb: $P_0 = 100$ W, voltage rating of the bulb: $V_0 = 230$ V.

i. Resistance of the bulb: $R = \frac{V_0^2}{P_0} = \frac{(230)^2}{100} = 529\ \Omega$

ii. Current drawn: $I = \frac{V_0}{R} = \frac{230}{529} = \frac{10}{23}$ A

iii. $I = \frac{200}{529}$ A, $P = I^2R = \left(\frac{200}{529}\right)^2 \times 529 = 75.6$ W

Illustration 7.3 **A 500 W heating unit is designed to operate from a 200 V line. By what percentage will its heat output drop if the line voltage drops to 160 V? Find the heat produced by it in 10 min.**

Sol. Actual power consumed: $P = \left(\frac{V}{V_0}\right)^2 P_0 = \left(\frac{160}{200}\right)^2 500$

$= 320$ W. Heat output drop $= 500 - 320 = 180$ W.

% Heat drop $= \frac{180}{500} \times 100 = 36\%$.

Heat produced in 10 min (600 s) is given by

$$H = 320 \times 600 = 192\,000 \text{ J} = 192 \text{ kJ}$$

4. Two bulbs connected in series: Let two bulbs of same voltage rating V_0 and power ratings P_{01} and P_{02} are connected isn series. Let $P_{01} > P_{02}$. Let potential V is applied across them as shown in Fig. 7.3.

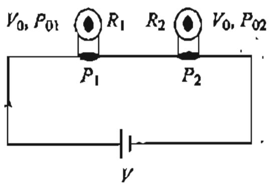


Fig. 7.3

Resistances of the bulbs:

$$R_1 = \frac{V_0^2}{P_{01}},\ R_2 = \frac{V_0^2}{P_{02}}$$

Let power produced in them are P_1 and P_2, respectively.

Then $P_1 = I^2R_1$ and $P_2 = I^2R_2$.

Now given: $P_{01} > P_{02} \Rightarrow R_1 < R_2 \Rightarrow P_1 < P_2$

It means the bulb having more power rating will consume less power.

Total power produced:

$$P = P_1 + P_2 = \frac{V^2}{R_1 + R_2}$$

$$= \frac{V^2}{\frac{V_0^2}{P_{01}} + \frac{V_0^2}{P_{02}}} = \left(\frac{V}{V_0}\right)^2 \left(\frac{P_{01}P_{02}}{P_{01} + P_{02}}\right)$$

If $V = V_0$, then $P = \frac{P_{01}P_{02}}{P_{01} + P_{02}} \Rightarrow \frac{1}{P} = \frac{1}{P_{01}} + \frac{1}{P_{02}}$

Note:

In series if any one bulb gets fused, then others will not glow.

5. Two bulbs are connected in parallel: Let two bulbs of same voltage rating V_0 and power ratings P_{01} and P_{02} are connected in parallel. Let $P_{01} > P_{02}$. Let potential V is applied across them as shown in Fig. 7.4.

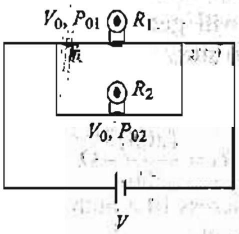


Fig. 7.4

Resistances of the bulbs:

$$R_1 = \frac{V_0^2}{P_{01}},\ R_2 = \frac{V_0^2}{P_{02}}$$

Let power produced in them are P_1 and P_2, rsespectively. Then $P_1 = \frac{V^2}{R_1}$ and $P_2 = \frac{V^2}{R_2}$.

Now given: $P_{01} > P_{02} \Rightarrow R_1 < R_2 \Rightarrow P_1 > P_2$

It means that the bulb having more power rating will consume more power.

Total power produced:

$$P = P_1 + P_2 = \frac{V^2}{R_1} + \frac{V^2}{R_2} = \frac{V^2}{V_0^2}P_{01} + \frac{V^2}{V_0^2}P_{02}$$

$$= \left(\frac{V}{V_0}\right)^2 (P_{01} + P_{02})$$

If $V = V_0$, then $P = P_{01} + P_{02}$.

Note:

In parallel if any one bulb gets fused, then others will continue to glow.

Illustration 7.4 **Two bulbs are marked 220 V–100 W and 220 V–50 W.**

i. Which bulb will produce more illumination if they are connected in parallel to a 220 V supply?

ii. Which bulb will produce more illumination if they are connected in series to a 220 V supply?

iii. Also find the total power consumed by both the bulbs in each of the two parts above.

Sol.

i. First bulb. In parallel more is power rating more is the power produced.

ii. Second bulb. In series more is power rating less is the power produced.

iii. In first part: $P = P_{01} + P_{02} = 100 + 50 = 150$ W. In second part: $P = \frac{P_{01}P_{02}}{P_{01} + P_{02}} = \frac{100 \times 50}{150} = \frac{100}{3}$ W.

Illustration 7.5 **Two bulbs are rated 30 W–200 V and 60 W–200 V. They are connected with a 400 V power supply. Find which bulb will get fused if they are connected in (i) series and (ii) parallel.**

Sol.

i. $R_1 = \frac{(200)^2}{30}\Omega,\ R_2 = \frac{(200)^2}{60}\Omega \quad \Rightarrow \quad R_1 > R_2$

Hence voltage across first bulb will be greater than 200 V. So, it will get fused.

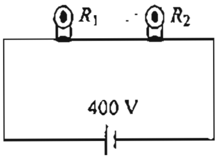


Fig. 7.5

ii. In parallel, potential across both the bulbs will be same and equal to 400 V. So both will get fused.

Illustration 7.6 **An electric tea kettle has two heating coils. When one of the coils is switched on, boiling begins in 6 min. When the other coil is switched on, the boiling begins in 8 min. In what time, will the boiling begin if both coils are switched on simultaneously (i) in series and (ii) in parallel.**

Sol. Let power of first coil is P_1 and that of second coil is P_2. Let H is the amount of heat required to boil water. Then $H = P_1t_1 = P_2t_2$ where $t_1 = 6$ min, $t_2 = 8$ min.

i. When the coils are connected in series:

$$P = \frac{P_1P_2}{P_1 + P_2}$$

$$t = \frac{H}{P} = H\left[\frac{1}{P_1} + \frac{1}{P_2}\right] = H\left[\frac{t_1}{H} + \frac{t_2}{H}\right]$$

$$= t_1 + t_2 = 6 + 8 = 14 \text{ min}$$

ii. When the coils are connected in parallel:

$$P = P_1 + P_2$$

$$t = \frac{H}{P} = \frac{H}{P_1 + P_2} = \frac{H}{\frac{H}{t_1} + \frac{H}{t_2}}$$

$$= \frac{t_1t_2}{t_1 + t_2} = \frac{6 \times 8}{6 + 8} = 3.43 \text{ min}$$

6. Let a resistance R under a potential difference V dissipates power:

$$P = \frac{V^2}{R}$$

So, if the resistance is changed from R to (R/n) keeping V same, the power consumed will be

$$P' = \frac{V^2}{(R/n)} = n\frac{V^2}{R} = nP \qquad \text{(iv)}$$

that is, if for a given voltage, resistance is changed from R to (R/n), power consumed changes from P to nP.

7. If n equal resistances are connected in series with a voltage source, then power dissipated will be

$$P_S = \frac{V^2}{nR} \qquad [\text{as } R_S = nR]$$

And if the same resistances are connected in parallel with the same voltage source,

$$P_P = \frac{V^2}{(R/n)} = \frac{nV^2}{R} \qquad [\text{as } R_P = (R/n)]$$

So, $\frac{P_P}{P_S} = n^2$ or $P_P = n^2P_S$ (v)

that is, power consumed by n equal resistors in parallel is n^2 times that of the power consumed in series, if V remains same.

Maximum Power Transfer Theorem

Suppose we want to find that for what value of the external resistance the maximum power will be drawn from a battery? For this, let in the shown network (Fig. 7.6) power developed in resistance R is

$$P = I^2R = \frac{E^2}{(R+r)^2}R \qquad \left(\text{as } I = \frac{E}{R+r}\right)$$

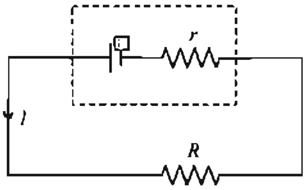


Fig. 7.6

Now, for $dP/dR = 0$ (for P to be maximum $dP/dR = 0$)

$$\Rightarrow \quad E^2\frac{(R+r)^2 - 2(R)(R+r)}{(R+r)^4} = 0$$

$$\Rightarrow \quad (r+R) = 2r \quad \Rightarrow \quad r = R$$

It means power output is maximum, when the external resistance equals the internal resistance $R = r$.

Illustration 7.7 **How will you connect (series and parallel) 24 cells each of internal resistance 1 Ω so as to get maximum power output across a load of 10 W.**

Sol. Total number of cells: $mn = 24$. For maximum power:

$$\frac{nr}{m} = R \quad \Rightarrow \quad n = 10m$$

$$\Rightarrow \quad 10m^2 = 24 \quad \Rightarrow \quad m = \sqrt{2.4} = 1.55$$

i. if $m = 1$, $I = \frac{24e}{34}$ then $P_1 = \left(\frac{24e}{34}\right)^2 \times 10 = 4.98e^2$

ii. if $m = 2$, $I = \left(\frac{12e}{16}\right)$ then $P_2 = \left(\frac{12e}{16}\right)^2 \times 10 = 5.625e^2$

So, we have two rows (m = 2) each containing 12 cells (n = 12) in series.

Illustration 7.8 **A dry cell of e.m.f. of 1.5 V and internal resistance 0.10 W is connected across a resistor in series with a very low resistance ammeter. When the circuit is switched on, the ammeter reading settles to a steady value of 2.0 A.**

- **a.** **What is the steady rate of chemical energy consumption of the cell?**
- **b.** **What is the steady rate of energy dissipation inside the cell?**
- **c.** **What is the steady rate of energy dissipation inside the resistor?**
- **d.** **What is the steady power output of the source?**

Sol.

- **a.** Rate of chemical energy consumption of the cell = $EI = 1.5 \times 2 = 3$ W.
- **b.** Rate of energy dissipation inside the cell = $I^2r = (2)^2 \times 0.1 = 0.4$ W.
- **c.** Rate of energy dissipation inside the resistor = $I^2R = EI - I^2r = 3 - 0.4 = 2.6$ W.
- **d.** Power out = $I^2R = 2.6$ W.

SOME APPLICATIONS

(a) Fusing of bulb when it is switched on

Usually filament bulbs get fused when they are switched on. This is because with rise in temperature the resistance of the bulb increases and becomes constant in steady state. So the power consumed by the bulb (V^2/R) initially is more than that in steady state and hence the bulb glows more brightly in the beginning and may get fused.

Illustration 7.9 **Two wires made of tinned copper having identical cross-section ($=10^{-6}$ m²) and lengths 10 and 15 cm are to be used as fuses. Show that the fuses will melt at the same value of current in each case.**

Sol. The temperature of the wire rises to a certain steady temperature when the heat produced per second by the current just becomes equal to the rate of loss of heat by radiation from its surface.

Heat produced per second by the current $= I^2R = I^2\frac{\rho l}{\pi r^2}$,

where l is the length, r is radius of the wire and ρ is the specific resistance. Let H = heat lost per second per unit surface area of the wire. If we neglect the loss of heat from the end faces of the wire, then heat lost per second by the wire = $H \times$ surface area of wire = $H \times 2\pi rl$.

At steady state temperature,

$$H \times 2\pi rl = \frac{I^2\rho l}{\pi r^2} \text{ or } H = \frac{I^2\rho}{2\pi^2 r^3} \qquad \text{(i)}$$

From (i) we note that the rate of loss of heat (H) which in turn depends upon the temperature of the wire is independent of length of the wire. Hence the fuses of two wires of the same values of r and ρ but of different lengths will melt for the same value of current in each case.

(b) Decrease in the brightness of bulb after long use

Also due to evaporation of metal from the filament (which deposits as black substance on the inner side of glass wall), the filament of the bulb becomes thinner and thinner with use. This increases the resistance [$R = \rho L/\pi r^2$] of the bulb and as $r = V^2/R$ the brightness of light emitted by a bulb decreases gradually with time.

(c) Decrease in brightness of a bulb in a room when a heavy current appliance is switched on

As shown in Fig. 7.7(a) if the bulb draws a current I_1 from the source, then terminal voltage of source = $V = (E - I_1 r)$

and so power consumed by the bulb is, $P = \dfrac{V^2}{R} = \dfrac{(E - I_1 r)^2}{R}$.

Now, when a heavy current appliance such as motor, heater or geyser is switched on, it will draw a heavy current, say I_2, from the source so that terminal voltage will become

$$V' = [E - (I_1 + I_2)r] = (V - I_2 r) \qquad (< v)$$

and hence power consumed by the bulb will now be

$$P' = \frac{V'^2}{R} = \frac{(V - I_2 r)^2}{R} < P$$

So, the brightness of the bulb decreases.

Note:

If the source is ideal, i.e., $r = 0$, $V' = V = E$ then $P' = P$, i.e., there will be no change in the brightness of the bulb, if the source is ideal.

(d) Fuse and its action

It is a metallic conducting wire of 75% Pb and 25% Sn with low melting point and higher resistance and is in series with an appliance [Fig. 7.7 (b)].

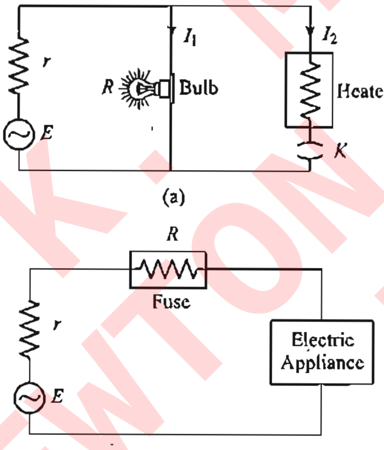


Fig. 7.7

It is a safety device which protects the appliance from getting damaged, by melting and opening the circuit, if the current in the circuit exceeds a specific predetermined value, called 'current capacity'.

(e) Long distance power transmission

When power is transmitted through a power line of resistance R, power-loss will be:

Power loss = $I^2 R$

Now, let power P_0 is transmitted at voltage V, then $P_0 = VI$,

i.e., $\quad I = \dfrac{P_0}{V}$

So, $\quad$ power loss = $\dfrac{P_0^{\,2}}{V^2} \times R$

Now, as for a given power and line, P and R are constant.

So, $\quad$ power loss $\propto \dfrac{1}{V^2}$

So, if power is transmitted at high voltage, power loss will be small and vice versa, e.g. power loss at 22 kV is 10^{-4} times than that at 220 V. This is why long distance power transmission is carried out at high voltage.

(f) The wires supplying current to a bulb are not heated while the filament of the bulb becomes hot. It is because resistance of the wires is very small in comparison to the resistance of bulb. If alone the wires are connected then whole of the potential differences will be across the wires and because their resistance is very small, a large amount of heat will be generated from $P = \dfrac{V^2}{R}$. This happens when wires are short circuited.

(g) The resistance of high electric power instrument will be smaller than that of low electric power instrument. Because for a given voltage: $P = \dfrac{V^2}{R}$. For example, iron, heater, geyser.

The heating element of these appliances are made of nichrome. It is an alloy of Ni and Cr. Its resistivity is higher in comparison to platinum, tungsten and copper, etc. Nichrome is used because

i. it has high resistivity and high melting point,
ii. it is not oxidized when heated,
iii. it can be easily drawn into wires.

Resistivity is kept higher so that smaller length can be used, as $H = \dfrac{V^2}{R} t = \dfrac{V^2 A t}{\rho \ell}$.

(h) Incandescent electric lamp

It consists of a metal filament generally made of tungsten. It is enclosed in a glass bulb with some inert gas and at suitable pressure. The filament gets heated, then it becomes white hot (known as incandescent stage) and starts emitting white light. The filament should have high melting point.

Illustration 7.10 **An electric kettle taking 3 A at 210 V brings 1 ℓ of water from 20° C to 80° C in 10 min. Find its efficiency.**

Sol. $\eta = \dfrac{OP}{IP} \times 100 = \dfrac{10^3 \times 4.2 \times 60}{210 \times 3 \times 600} \times 100 = 66.67\%$

Illustration 7.11 **A line having a total resistance of 0.2 W delivers 10 kW at 220 V to a small factory. Calculate the efficiency of transmission.**

Sol. $I = \dfrac{10000}{220} = \dfrac{500}{11}$ A. Loss $= I^2 r = \left(\dfrac{500}{11}\right)^2 \times 0.2 = 413.22.$

Efficiency: $\eta = \dfrac{10000}{10000 + 413.22} \times 100 = 96\%$

Concept Application Exercise 7.1

1. a. When is the rate at which energy being delivered to a light bulb higher: just after it is turned on, and the glow of the filament is increasing, or after it has been on for a few seconds and the glow is steady?

b. If a piece of wire were used to connect points *b* and *c* in Fig. 7.8, does the brightness of bulb R_1 increase, decrease or stay constant? What happens to the brightness of bulb R_2?

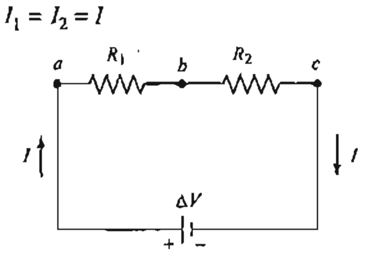


Fig. 7.8

c. Compare the brightness of four identical light bulbs in Fig. 7.9. What happens if bulb *A* fails, so that it cannot conduct? What if *C* fails? What if *D* fails?

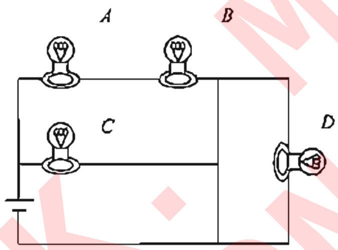


Fig. 7.9

d. If electric power is transmitted over long distances, the resistance of the wires becomes significant. Why? Which mode of transmission would result in less energy loss–high current and low voltage or low current and high voltage? Discuss.

e. In Fig. 7.10, describe what happens to the light bulb after the switch is closed. Assume the capacitor has a large capacitance and is initially uncharged and assume that the light illuminates when connected directly across the battery terminals.

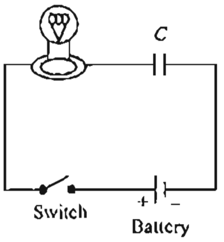


Fig. 7.10

f. A student claims that a second light bulb in series is less bright than the first, because the first bulb uses up some of the current. How would you respond to this statement?

g. If you were to design an electric heater using Nichrome wire as the heating element, what parameters of the wire could you vary to meet a specific power output, such as 1000 W?

2. A heater joined in series with a 50 W bulb is connected to the mains. If the 50 W bulb is replaced by a 100 W bulb, then will the heater now give more heat, less heat or same heat ? Why?

3. Each of the three resistors in Fig. 7.11 has a resistance of 2 Ω and can dissipate a maximum of 18 W without becoming excessively heated.

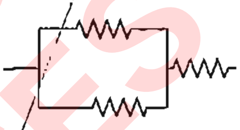


Fig. 7.11

Find the maximum power the circuit can dissipate.

4. An electric bulb rated 220 V and 60 W is connected in series with another electric bulb rated 220 V and 40 W. The combination is connected across 220 V source of e.m.f. Which bulb will glow more?

5. We have a 30 W, 6 V bulb which we want to glow by a supply of 120 V. What will have to be done for it?

6. a. Two heater coils made of the same material are connected in parallel across the mains, the length and the diameter of one coil is double that of the other. Which of them will produce more heat?

b. Three equal resistances connected in series across a source of e.m.f. together dissipate *P* watt of power. What would be the power dissipated if the same resistors are connected in parallel across the same source of e.m.f.?

7. A series circuit consists of three identical lamps connected to a battery as shown in Fig. 7.12.

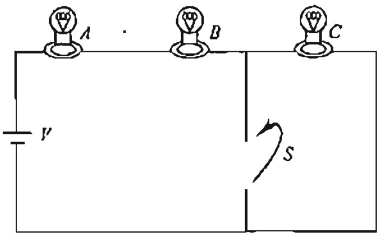


Fig. 7.12

When the switch *S* is closed, what happens

a. to the intensities of lamp *A* and *B*

b. to the intensity of lamp *C*

c. to the current in the circuit and

d. to the voltage drop across the three lamps? Does the power dissipated in the circuit increase, decrease or remain the same?

8. Two wires of the same material and having the same uniform area of cross section are connected in an electric circuit. The masses of the wires are *m* and 2*m*, respectively. When a current *I* flows through both of them connected in series, then find the ratio of heat produced in them in a given time.

9. Water boils in an electric kettle in 15 min after being switched on. Using the same main supply should the length of the heating element be increased or decreased if the water is to be boiled in 10 min? Why?

10. An electric motor operating on a 50 V d.c. supply draws a current of 12 A. If the efficiency of the motor is 30%, estimate the resistance of the windings of the motor.

11. A fuse with a circular cross-sectional radius of 0.15 mm blows at 15 A. What should be the radius of cross section of a fuse made of the same material which blows at 30 A.

12. A motor operating on 120 V draws a current of 2 A. If the heat is developed in the motor at the rate of 9 cal s^{-1}, what is its efficiency?

13. The walls of a closed cubical box of edge 40 cm are made of a material of thickness 1 mm and thermal conductivity 4×10^{-4} cals^{-1}cm^{-1} °C^{-1}. The interior of the box is maintained at 100°C above the outside temperature by a heater placed inside the box and connected across 400 V d.c. Calculate the resistance of the heater.

14. Two tungsten lamps with resistances R_1 and R_2, respectively, are connected first in parallel and then in series in a lighting circuit of negligible internal resistance. Given that $R_1 > R_2$.

a. Which lamp will glow more brightly when they are connected in parallel?

b. If the lamp of resistance R_1 now burns out, how will the net illumination produced change?

c. Which lamp will glow more brightly when they are connected in series?

d. If the lamp of resistance R_2 now burns out and the lamp R_1 alone is plugged in, will the net illumination increase or decrease?

15. *n* identical bulbs are connected in series and illuminated by a power supply. One of the bulbs gets fused. The fused bulb is removed and the remaining bulbs are again illuminated by the same power supply. Find fractional change in the illumination of **a**. all the bulbs, **b**. one bulb.

16. An electric motor is designed to work at 100 V and draws a current of 6 A. The output power supplied by the motor is 150 W and remaining goes to heat. What is the resistance of the windings of the motor and its percentage efficiency?

17. A house is fitted with certain numbers of 100 W, 230 V incandescent lamps. The power to the house is fed by a generator producing the power at 240 V. The resistance of the wires from generator to the house is 2 W. Find the maximum number of lamps that can be illuminated so that voltage across none of the lamps drops below 230 V.

18. A house is fitted with 7 tubelights of rating 220 V – 40 W each, 2 bulbs of rating 220 V – 60 W each, 5 fans each drawing a current of 0.4 A at 220 V and a heater of resistance 48.4 W. The main line power supplied to the house is at 220 V. Calculate the bill for the month of January if tubelights and bulbs are used for 6 h daily, fans for 1 h daily and heater for 10 h daily. The electricity is to cost Rs. 2 per unit.

19. Two bulbs are marked 200 V, 300 W and 200 V, 600 W. The bulbs are connected in series and the combination is connected to a 200 V supply.

a. Which bulb will produce more illumination?

b. Find the total power consumed by both the bulbs.

c. Find the total power consumed if both the bulbs were connected in parallel.

20. A voltage stablizer restricts the voltage output to 220 V 1%. If the electric bulb rated at 220 V, 100 W is connected to it, what will be the minimum and maximum power consumed by it?

21. The efficiency of a cell when connected to a resistance R is 60%. W will be its efficiency if the external resistance is increased to six times.

22. A 25 W and a 100 W bulb are joined in series and connected to the mains. Which bulb will glow brighter? **(IIT-JEE, 1979)1**

Solved Examples

Example 7.1 **A series battery of 6 cells each of e.m.f. 2 V and internal resistance 0.5 Ω is charged by a 100 V d.c. supply. What resistance should be used in the charging circuit in order to limit the charging current to 8 A. Using this relation, obtain (a) the power supplied by the d.c. source, (b) the power dissipated as heat and (c) the chemical energy stored in the battery in 15 min.**

Sol. Given: Number of cells $n = 6$; e.m.f. of each cell, $E = 2$V; internal resistance of each cell, r = 0.5 W; charging voltage $V = 100$ V. Let R be the resistance used in the series of the circuit while charging the cells. Then current in the circuit will be

$$i = \frac{V - nE}{nr + R}$$

or
$$R = \frac{V - nE}{i} - nr = \frac{100 - 6 \times 2}{8} - 6 \times 0.5$$

$$= 11 - 3 = 8\,\Omega$$

a. Power supplied by d.c. source $= V \times i = 100 \times 8 = 800$ W

b. Power dissipated as heat $= i^2(R + nr) = 8^2\,(8 + 6 \times 0.5)$
$= 704$ W

c. Rate at which the chemical energy is stored $= 800 - 704$
$= 96$ W

∴ Chemical energy stored in 15 min $= 96 \times 15 \times 60 = 86400$ J

Example 7.2 **Determine the current through the battery of internal resistance 0.5 Ω for the circuit shown in Fig. 7.13. How much power is dissipated in 6 Ω resistance?**

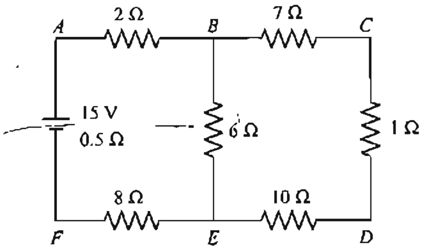


Fig. 7.13

Sol. Resistance of arm $BCDE = 7 + 1 + 10 = 18\ \Omega$. Here 18 and 6 Ω are in parallel. Their effective resistance is

$$R_P = \frac{18 \times 6}{18 + 6} = \frac{18 \times 6}{24} = 4.5\ \Omega$$

Total resistance of the circuit $= 2 + 4.5 + 8 + 0.5 = 15\ \Omega$

∴ Current through the circuit, $i = \frac{15}{15} = 1\text{ A}$

Potential difference across B and $E = i \times R_P = 1 \times 4.5 = 4.5$ V

∴ Power dissipated as heat due to resistance 6 Ω is

$$\frac{(4.5)^2}{6} = 3.375\text{ W}$$

Example 7.3 **Two uniform wires of same material each weighting 1 g but one having double the length of the other are connected in series, carrying a current of 10 A. The length of the longer wire is 20 cm. Calculate the rate of consumption of energy in each of the two wires. Which wire gets hotter? Density of the material of wire = 11 gcm^{-3}, specific resistance of the material is 20 × 10^{-5} Ω cm.**

Sol. Let a_1 and a_2 be the area of cross-section of shorter and longer wires, respectively.

As mass = volume × density = $al\rho$

$$\therefore \quad 1 = a_1 \times 10 \times 11 = a_2 \times 20 \times 11$$

$$\text{or} \quad a_1 = \frac{1}{10 \times 11}\text{ cm}^2 \quad \text{and} \quad a_2 = \frac{1}{20 \times 11}\text{ cm}^2$$

$$\therefore \quad R_1 = 20 \times 10^{-5} \times \frac{10}{1/(10 \times 11)}$$

$$= 20 \times 10^{-5} \times 10 \times 10 \times 11 = 22 \times 10^{-3}\ \Omega$$

$$R_2 = 20 \times 10^{-5} \times \frac{20}{1/(20 \times 11)} = 88 \times 10^{-2}\ \Omega$$

And rate of heat produced

$$H_1 = I^2R_1 = (10)^2 \times 22 \times 10^{-2} = 22\text{ W}$$

$$\text{and} \quad H_2 = I^2R_2 = (10)^2 \times 88 \times 10^{-2} = 88\text{ W}$$

Thus the wire of longer length gets hotter.

Example 7.4 **A series battery of 6 lead accumulators each of e.m.f. 2.2 V and internal resistance 0.05 Ω is charged by a 100 V d.c. supply. What series resistance should be used in charging circuit in order to limit the current to 7.8 A? Using the required resistor, obtain**

(a) the power supplied by the d.c. source,

(b) the chemical energy stored in the battery in 15 min.

Sol. Total resistance $= R + nr = R + 6 \times 0.5 = R + 3$

Total e.m.f. of the battery $= 6 \times 2.2 \times 13.2$ V

Effective potential difference in the circuit $= 100 - 13.2 = 86.8$ V

$$\text{Now,} \quad \text{current} = \frac{\text{effective pot. diff.}}{\text{total resistance}}$$

$$\Rightarrow \quad 7.8 = \frac{86.8}{R + 3} \text{ or } R = 8.13\ \Omega$$

a. Power supplied by d.c. source

$$= VI = 100 \times 7.8 = 780\text{ W} = I^2(R + nr)$$
$$= (7.8)^2(8.13 + 3) = 677.15\text{ W}.$$

b. Chemical energy stored in the battery in 15 min

$$= (780 - 677.15) \times 15 \times 60 = 92565\text{ J}$$

Example 7.5 **In a house having 220 V line, the following appliances are operating:**

(i) 60 W bulb, (ii) a 1000 W heater and (iii) a 40 W radio. Calculate (a) the current drawn by heater and (b) the current passing through fuse for this line.

Sol. Here, $V = 220$ V; $P_1 = 60$ W; $P_2 = 1000$ W; $P_3 = 40$ W.

a. Current drawn by heater $= \frac{P_2}{V} = \frac{1000}{220} = \frac{50}{11}$ A

b. Current drawn by bulb $= \frac{P_1}{V} = \frac{60}{220} = \frac{3}{11}$ A

Current drawn by radio $= \frac{P_3}{V} = \frac{40}{220} = \frac{2}{11}$ A

Current passing through fuse for the line

$$= \frac{50}{11} + \frac{3}{11} + \frac{2}{11} = 5\text{ A}$$

Example 7.6 **A heater is designed to operate with a power of 1000 W in a 100 V line. It is connected, in combination with a resistance R, to a 100 V mains as shown in Fig. 7.14. What should be the value of R as such that heater may operate with a power of 62.5 W?**

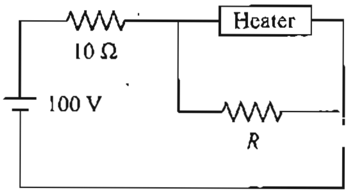


Fig. 7.14

(IIT-JEE, 1978)

Sol. The resistance of the heater is

$$R = \frac{V^2}{P} = \frac{100 \times 100}{1000} = 10\ \Omega$$

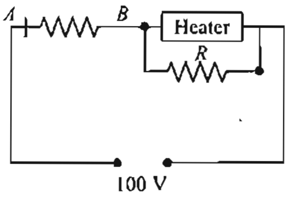


Fig. 7.15

The power on which it operates is 62.5 W.

$$\therefore \quad V = \sqrt{R \times P'} = \sqrt{10 \times 62.5} = \sqrt{625} = 25\text{ V}$$

$\Rightarrow$ The potential drop across $AB = 75$ V

$\therefore$ The current in $AB = I = \dfrac{V}{R} = \dfrac{75}{10} = 7.5$ A

This current divides into two parts. Let I_1 be the current that passes through the heater. Therefore

$$25 = I_1 \times 10 \quad \Rightarrow \quad I_1 = 2.5\text{ A}$$

$\Rightarrow$ current through R is 5 A

Applying Ohm's law across R, we get:

$$25 = 5 \times R \quad \Rightarrow \quad R = 5\ \Omega$$

Example 7.7 **(i) Find the time taken by a filament of 200 W to heat 500 ml of water from 25°C to 75°C. Specific heat of water = 1 calg^{-1}°C^{-1}. Take 1 cal = 4.2 J.**

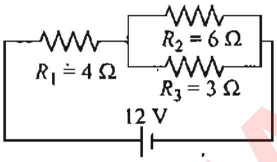


Fig. 7.16

(ii) Find the power produced by each resistor shown in Fig. 7.16. If R_1 is dipped in 1000 ml of water at 30°C, find the time taken by it to boil the water.

Sol.

(i) $200\ t = 0.5 \times 4200 \times 50 \quad \Rightarrow \quad t = 525$ s

(ii) $R_{eq} = 6\ \Omega$, $I_1 = \dfrac{12}{6} = 2$ A in 4 Ω, $I_2 = \dfrac{2}{3}$ A in 6 Ω, $I_3 = \dfrac{4}{3}$ A in 3 Ω

$$P_1 = I_1^2 R_1 = 2^2 \times 4 = 16\text{ W},\ P_2 = \left(\frac{2}{3}\right)^2 \times 6 = \frac{8}{3}\text{ W},$$

$$P_3 = \left(\frac{4}{3}\right)^2 \times 3 = \frac{16}{3}\text{ W}$$

Now, $P_1 t = 1 \times 4200 \times 70 \quad \Rightarrow \quad t = \dfrac{4200 \times 70}{16} = 18375$ s

Example 7.8 **A heating coil of 2000 W is immersed in an electric circuit. How much time will it take in raising the temperature of 1 l of water from 4°C to 100°C? Only 80% of the thermal energy produced is used in raising the temperature of water.**

Sol. Here, $P = 2000$ W, $t = ?$ (in seconds)

Volume of water = 1 l = 1000 cm^3

Mass of water, m = volume × density = 1000 × 1 = 1000 g

Rise in temperature = $\theta_2 - \theta_1 = 100 - 4 = 96$°C

We know specific heat of water, $c = 1$ cal g^{-1} °C^{-1}

$\therefore$ Heat taken by water

$$= mc(\theta_2 - \theta_1) = 1000 \times 1 \times 96 = 96000\text{ cal}$$

Energy spent in heating coil = $Pt = 2000 \times t$

Useful energy produced = 80% = $2000 \times t \times 80/100$ J

Useful heat produced = $\dfrac{2000 \times t \times 80}{100 \times 4.2}$ cal

As this heat is taken by water, hence $\dfrac{2000 \times t \times 80}{100 \times 4.2} = 96000$

or $\quad t = \dfrac{96000 \times 100 \times 4.2}{2000 \times 80} = 252$ s

Example 7.9 **Consider the following circuit (Fig. 7.17) where some resistances have been arranged in a definite order. With the given condition that heat produced by 6 Ω resistance is 60 cal s^{-1} due to the current flowing through it, find out the heat produced across 2 Ω resistance in calorie per second.**

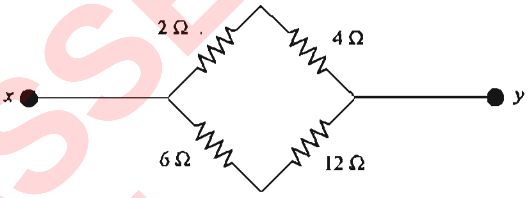


Fig. 7.17

Sol. Same current flows through resistances connected in series.

Heat produced in 6 Ω resistance = $\dfrac{I^2 R}{J}$

$$\Rightarrow \quad 60 = \frac{I^2 \times 6}{4.2} \quad \Rightarrow \quad I^2 = 42 \quad \Rightarrow \quad I = \sqrt{42}\text{ A}$$

Now the voltage drop across x and y

$$= (6 + 12)\sqrt{42} = 20\sqrt{42}\text{ V}$$

As this potential drop is same in every area of a parallel circuit, so the potential drop across upper part of the circuit is same.

$\therefore$ Current through 2 and 4 Ω resistances

$$= \frac{20\sqrt{42}}{6} = \frac{10\sqrt{42}}{3}$$

Hence the heat produced across the 4 Ω resistance

$$= \frac{I^2 R}{J} = \frac{100 \times 42 \times 4}{9 \times 4.2} = \frac{4000}{9}\text{ cal}$$

Example 7.10 **Consider a Wheatstone's bridge *PQRS* as shown in Fig. 7.18 where current *I* is in the circuit of four resistances 10, 20, 30 and 40 Ω. Find the ratio of the heat generated in the four arms *PQ*, *QR*, *PS* and *SR*.**

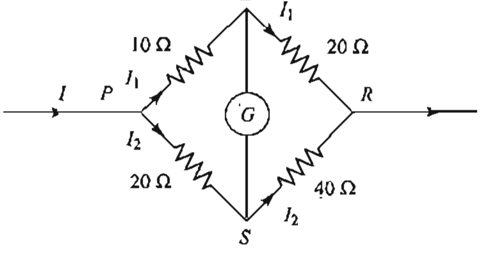


Fig. 7.18

Sol. Given: $R_1 = 10\,\Omega$, $R_2 = 20\,\Omega$, $R_3 = 20\,\Omega$ and $R_4 = 40\,\Omega$.

$$\text{Now,} \quad \frac{R_1}{R_2} = \frac{10}{20} = \frac{1}{2} \text{ and } \frac{R_3}{R_4} = \frac{20}{40} = \frac{1}{2}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Hence Wheatstone's bridge is balanced. Now as the bridge is balanced hence no current will flow through arm QS.

Let I_1 and I_2 be the currents flowing in arms PQ and PS, respectively.

Then potential difference across P and Q = potential difference across P and S

$$\Rightarrow \quad I_1 \times 10 = I_2 \times 20 \quad \Rightarrow \quad I_1 = 2I_2$$

$\therefore$ Heat produced in arm PQ is

$$H_1 = I_1^2 \times 10 = 40\, I_2^2 \text{ J}$$

Also heat produced in arm QR is

$$H_2 = I_1^2 \times 20 = 80\, I_2^2 \text{ J}$$

Similarly, heat produced in arm PS is

$$H_3 = I_2^2 \times 20 = 20\, I_2^2 \text{ J}$$

And heat produced in arm SR is

$$H_4 = I_2^2 \times 40 = 40\, I_2^2 \text{ J}$$

$\therefore H_1 : H_2 : H_3 : H_4 = 40\, I_2^2 : 80\, I_2^2 : 20\, I_2^2 : 40\, I_2^2 = 2 : 4 : 1 : 2$, which is the required ratio.

Example 7.11 **A person with body resistance between his hands of 10 kΩ accidentally grasps the terminals of a 18 kV power supply. (a) If the internal resistance of the power supply is 2000 Ω, what is the current through the person's body? (b) What is the power dissipated in his body? (c) If the power supply is to be made safe by increasing its internal resistance, what should the internal resistance be for the maximum current in the above situation to be 1.00 mA or less?**

Sol. Given: $R = 10\text{ k}\Omega$, $V = 18\text{ kV}$

a. To find out current flowing through the body, we need to sum up the resistances present in the circuit and divide the voltage by it.

$$I = \frac{V}{R + r} = \frac{18 \times 10^3}{10 \times 10^3 + 2 \times 10^3} = \frac{18 \times 10^3}{12 \times 10^3}$$

$$= \frac{3}{2} = 1.5 \text{ A}$$

b. We know power dissipated

$$= V_1 = I^2 R = (1.5)^2 (10000) = 2.25 \times 10000$$

$$= 22500 = 22.5 \text{ kW}$$

c. To find the internal resistance for the safe limit of power, we can use the formula as in part (a). The only difference is here I is given and r is to be calculated.

$$R + r = \frac{V}{I} = \frac{18 \times 10^3}{1 \times 10^{-3}} \quad \Rightarrow \quad R + r = 18 \times 10^6$$

$$\Rightarrow \quad r = 18 \times 10^6 - 10 \times 10^3 = (18\text{ M}\Omega - 10\text{ k}\Omega)$$

$$\approxeq 18\text{ M}\Omega$$

Example 7.12 **A circuit contains a 48 V battery and a single bulb whose resistance is 240 Ω. A second identical bulb can be connected either in series or parallel with the first one. Determine the power in a single bulb when the circuit contains (a) only one bulb, (b) two bulbs in series and (c) two bulbs in parallel. Assume that the battery is ideal without any internal resistance.**

Sol. Power consumed by a light bulb is related to its resistance R and the voltage across it by

$$P = \frac{V^2}{R}$$

a. When only the bulb is connected in the circuit, the power it consumes is

$$P = \frac{V^2}{R} = \frac{(48)^2}{240} = 9.6 \text{ W}$$

b. The more the power dissipated in a light bulb, the brighter it is. When identical bulbs are wired in series, each bulb receives one half the battery voltage V. The power consumed by each bulb is

$$P = \frac{(1/2V)^2}{R} = \frac{1}{4}\frac{V^2}{R}$$

The power dissipated in each bulb is reduced to only one-fourth the power dissipated in a single bulb circuit. Thus the brightness of each bulb decreases.

Also the equivalent resistance of two bulbs is $R_{eq} = R + R$.

The current in the circuit is given by $I = V/R_{eq}$.

The power consumed by one of the light bulbs can be expressed as

$$P = I^2 R = \left(\frac{V}{R + R}\right)^2 R = \frac{V^2}{4R} = \frac{(48)^2}{4(240)} = 2.4 \text{ W}$$

c. When the bulbs are connected in parallel, each one receives full battery voltage V. Thus the power consumed by each bulb remains the same as it only one bulb is present in the circuit, so the brightness does not change.

$$P = \frac{V^2}{R} = \frac{(48)^2}{240} = 9.6 \text{ W}$$

Example 7.13 **An electric kettle has two coils of same power. When one coil is switched on, it takes 15 min to boil water and when the second coil is switched on it takes 30 min. How long will it take to boil water when both the coils are used in (a) series, (b) parallel ?**

Sol. Heat produced in resistance R in time t is

$$H = Pt = \frac{V^2}{R} t$$

$$\text{For coil 1,} \qquad H_2 = \frac{V^2}{R_1}(15 \times 60) \qquad \text{(i)}$$

And for coil 2, $H_2 = \frac{V^2}{R_2}(30 \times 60)$ (ii)

But according to the given problem

$H_1 = H_2$, i.e., $\frac{15}{R_1} = \frac{30}{R_2}$ or $R_2 = 2R_1$ (iii)

a. Both the coils are used in series:

$$H_S = \frac{V^2}{(R_1 + R_2)} t_S = \frac{V^2}{3R_1} \times t_S \qquad [\text{as } R_2 = 2R_1]$$

But as here, $H_S = H_1 \; (= H_2)$

So, $\frac{V^2}{R_1}(15 \times 60) = \frac{V^2}{3R_1} t_S$

or $t_S = (45 \times 60)\text{ s} = 45\text{ min}$

b. Both the coils are used in parallel:

$$H_P = \left[\frac{V^2}{R_1} + \frac{V^2}{R_2}\right] \times t_P = \frac{3V^2}{2R_1} \times t_P \qquad [\text{as } R_2 = 2R_1]$$

According to the given problem, $H_P = H_1$

$\frac{3V^2}{2R_1} \times t_P = \frac{V^2}{R_1} \times (15 \times 30)$ or $t_P = (10 \times 60)\text{ s} = 10\text{ min}$

Example 7.14 **What amount of heat will be generated in a coil of resistance R due to a total charge q passing through it if the current in the coil:**

(a) decreases down to zero uniformly during a time interval t_0?

(b) decreases down to zero halving its value every t_0 seconds?

Sol.

a. The current decreases uniformly with time, therefore i vs t curve is a straight line as shown in Fig. 7.19 with slope $m = -i_0/t_0$. Current as function of time can be written as

$$i = i_0 - \left(\frac{i_0}{t_0}\right) \qquad \text{(i)}$$

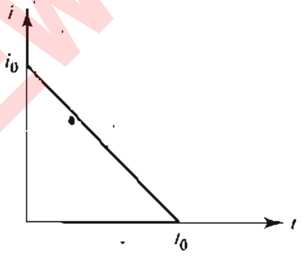


Fig. 7.19

Area under the $i-t$ graph gives the flow of charge q, therefore

$$q = \frac{1}{2}(t_0)(i_0) \Rightarrow i_0 = \frac{2q}{t_0}$$

Substituting in equation (i), we get

$$i = \frac{2q}{t_0}\left(1 - \frac{t}{t_0}\right) \Rightarrow i = \left(\frac{2q}{t_0} - \frac{2qt}{t_0^2}\right)$$

Heat produced in a time interval t_0 is

$$\int dH = \int i^2 R\,dt \text{ or } H = \int_0^{t_0}\left(\frac{2q}{t_0} - \frac{2qt}{t_0^2}\right)^2 R\,dt$$

$$= \frac{4}{3}\frac{q^2R}{t_0}$$

b. Here, current decreases from i_0 to zero exponentially with half life of t_0. The $i - t$ equation in this case is an exponential function like the radioactive decay law

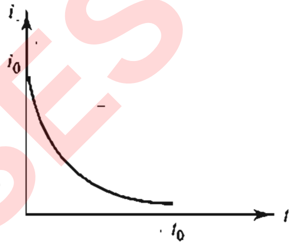


Fig. 7.20

$$i = i_0 e^{-\lambda t} \quad \text{where} \quad \lambda = \frac{\ln(2)}{t_0}$$

Total charge, $q = \int_0^\infty i\,dt = \int_0^\infty i_0 e^{-\lambda t}\,dt = \left(\frac{i_0}{\lambda}\right)$

or $i_0 = \lambda q$ or $i = (\lambda q)e^{-\lambda t}$

Heat produced in time interval dt is

$$dH = i^2R\,dt = \lambda^2 q^2 e^{-2\lambda t} R\,dt$$

or $H = \lambda^2 q^2 R \int_0^\infty e^{-2\lambda t}\,dt = \frac{q^2 \lambda R}{2} = \frac{q^2 R \ln(2)}{2t_0}$

Example 7.15 **A variable capacitor is adjusted in position of its lowest capacitance C_0 and is connected with a source of constant voltage V for a long time. Resistance of connecting wires is R. At $t = 0$, its capacitance starts to increase so that a constant current I starts to flow through the circuit. Calculate at time t:**

i. power supplied by the source,

ii. thermal power generated in the connecting wire and

iii. rate of increase of electrostatic energy stored in capacitor.

iv. What do you infer from above three results?

Sol.

i. Since voltage V of the source is constant and the circuit draws constant current I from it, therefore, power supplied by the source is $P = VI$.

ii. Thermal power generated in connecting wires is $H = I^2R$.

iii. Since initial capacitance of the capacitor was equal to C_0 and it was connected with the source for long time, therefore initial charge on capacitor was $q_0 = C_0V$.

Since a constant current I starts to flow at $t = 0$, therefore at time t, charge on capacitor becomes $q = (C_0V + It)$.

At time t, the circuit will be as shown in Fig. 7.21. Potential difference across the capacitor is

$$V_C = V_A - V_B = (V - IR) \rightarrow \text{constant}$$

∴ Electrostatic energy in capacitor at this instant is

$$U = \frac{1}{2} q V_C$$

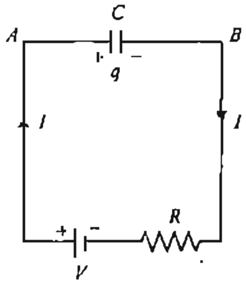


Fig. 7.21

Rate of increase of electrostatic energy

$$= \frac{dU}{dt} = \frac{1}{2} V_C \frac{dq}{dt} = \frac{1}{2}(V - IR)I$$

$$= \frac{1}{2}(VI - I^2R)$$

But power acting across the capacitor at this instant is

$$P_0 = P - H = (VI - I^2R)$$

while the rate of increase of electrostatic energy in the capacitor is half of it.

iv. In fact, a force of attraction exists between the surfaces of the capacitor. When these surfaces move towards each other capacitance increases. Hence, remaining part of the power acting across the capacitor is used to increase kinetic energy of surface (plate) of the capacitor.

EXERCISES

Subjective Type

Solutions on page 7.22

1. A circuit shown in Fig. 7.22 has resistances $R_1 = 20\ \Omega$ and $R_2 = 30\ \Omega$. At what value of the resistance R_x will the thermal power generated in it be practically independent of small variations of that resistance. The voltage between the points A and B is supposed to be constant in this case.

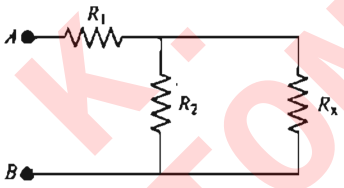


Fig. 7.22

2. A 1 kW heater is meant to operate at 220 V.
   a. What is the resistance?
   b. How much power will it consume if the line voltage drops to 100 V?
   c. How many units of electrical energy will it consume in a month (of 30 days) if it operates 10 h daily at the specified voltage?
3. Three identical resistors are connected in series. When a certain potential difference is applied across the combination, the total power dissipated is 27 W. What power would be dissipated if the three resistors were connected in parallel across the same potential difference?
4. A resistor R_1 consumes electrical power P_1 when connected to an e.m.f. e. When resistor R_2 is connected to the same e.m.f., it consumes electrical power P_2. In terms of P_1 and P_2, what is the total electrical power consumed when they are both connected to this e.m.f. source
   a. in parallel? b. in series?
5. 200 identical electrical bulbs, each having resistance 400 Ω, are connected in parallel to a d.c. source of e.m.f. 100 V and internal resistance 0.1 Ω. What is the power consumed by each bulb. Also find the percentage change is power consumed by each bulb if one bulb turns out.
6. A 200 W and a 100 W bulb, both meant for operation at 220 V, are connected to a 220 V supply. What total power will be consumed by them if they are **a.** in series, **b.** in parallel?
7. In the circuit shown in Fig. 7.23, all the resistors are rated at a maximum power of 1.00 W. What is the maximum e.m.f. e that the battery can have without burning up any of the resistors?

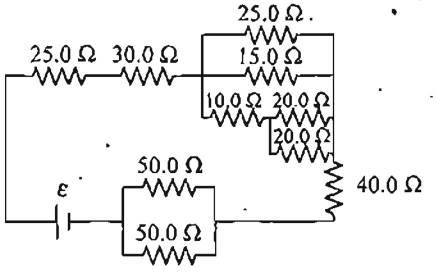


Fig. 7.23

8. In the circuit shown in Fig. 7.24,

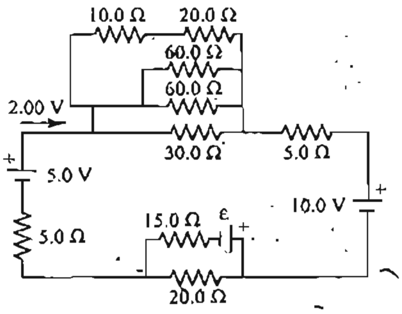


Fig. 7.24

a. What must the e.m.f. e of the battery be in order for a current of 2.00 A to flow through the 5.00 V battery, as shown? Is the polarity of the battery correct as shown?

b. How long does it take for 60.0 J of thermal energy to be produced in the 10.0 W resistor?

9. The water in an electric kettle begins to boil in 15 min after being switched on. Using the same mains supply, should the length of wire used for heating element be increased or decreased if the water is to boil in 10 min? Neglect the heat loss to the surroundings.

10. Three equal resistances connected in series across a source of e.m.f. together dissipate P watt of power. What would be the power dissipated, if the same resistors are connected in parallel across the same source of e.m.f.?

11. Two electric bulbs of 50 and 100 W are given. Which one of the bulb will be brighter when they are connected to the mains

i. in series   ii. in parallel?

12. If two bulbs of 25 and 100 W rated at 220 V are connected in series across a 440 V supply, will both the bulbs fuse? If not which one?

13. A standard 50 W electric bulb in series with a room heater is connected across the mains. If the 50 W bulb is replaced by a 100 W bulb, will the heater output be larger, smaller or remain the same?

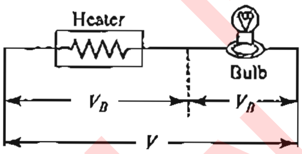


Fig. 7.25

14. Three 60 W, 120 V light bulbs are connected across a 120 V power line as shown in Fig. 7.26. Find (a) the voltage across each bulb and (b) the total power dissipated in the three bulbs.

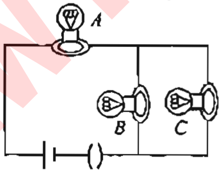


Fig. 7.26

Objective Type

Solutions on page 7.25

1. The operating temperature of the filament of lamp is 2000°C. The temperature coefficient of the material of filament is 0.005°C^{-1}. If the atmospheric temperature is 0°C, then the current in the 100 W–200 V lamp when it is switched on is nearest to

a. 2.5 A   b. 3.5 A
c. 4.5 A   d. 5.5 A

2. In the circuit below (Fig. 7.27), bulb B does not light although ammeter A indicates that the current is flowing. Why does the bulb not light?

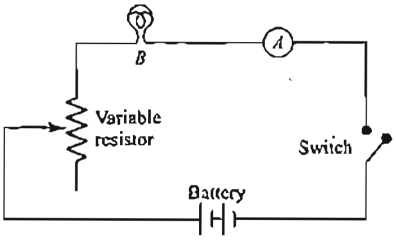


Fig. 7.27

a. The bulb is fused
b. There is break in the circuit between bulb and ammeter
c. The variable resistor has too large resistance
d. There is a break in the circuit between the bulb and the variable resistance

3. Three bulbs B_1, B_2 and B_3 are connected to the mains as shown in Fig. 7.28. How will the brightness of bulb B_1 be affected if B_2 or B_3 are disconnected from the circuit?

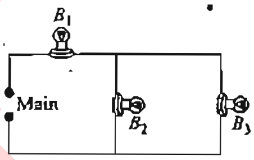
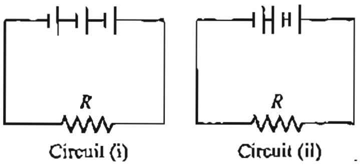


Fig. 7.28

a. Bulb B_1 becomes brighter
b. Bulb B_1 becomes dimmer
c. No change occurs in the brightness
d. Bulb B_1 becomes brighter if bulb B_2 is disconnected and dimmer if bulb B_3 is disconnected

4. Three identical cells, each having an e.m.f. of 1.5 V and a constant internal resistance of 2.0 Ω, are connected in series with a 4.0 Ω resistor R, first as in circuit (i), and secondly as in circuit (ii).



Circuit (i)   Circuit (ii)

Fig. 7.29

What is the ratio $\dfrac{\text{Power in } R \text{ circuit (i)}}{\text{Power in } R \text{ in circuit (ii)}}$?

a. 9.0   b. 7.2   c. 1.8   d. 3.0

5. All bulbs in Fig. 7.30 are indentical. Which bulb lights more brightly?

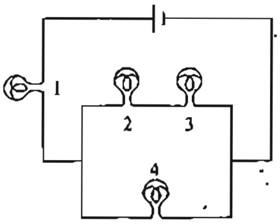


Fig. 7.30

a. 1   b. 2   c. 3   d. 4

6. Which of the two switches S_1 and S_2 shown in Fig. 7.31 will produce short-circuiting?

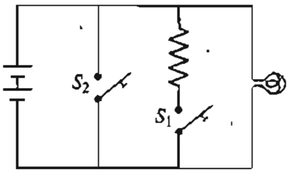


Fig. 7.31

a. S_1
b. S_2
c. Both S_1 and S_2
d. Neither S_1 nor S_2

7. Three similar light bulbs are connected to a constant voltage d.c. supply as shown in Fig. 7.32. Each bulb operates at normal brightness and the ammeter (of negligible resistance) registers a steady current.

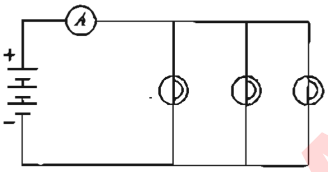


Fig. 7.32

The filament of one of the bulbs breaks. What happens to the ammeter reading and to the brightness of the remaining bulbs?

	Ammeter reading	*Bulb brightness*
a.	increases	increases
b.	increases	unchanged
c.	unchanged	unchanged
d.	decreases	unchanged

8. The circuit shown in Fig. 7.33, contains a battery, a rheostat and two identical lamps. What will happen to the brightness of the lamps if the resistance of the rheostat is increased?

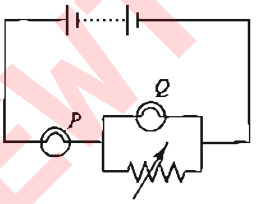


Fig. 7.33

	Lamp P	*Lamp Q*
a.	Less bright	Brighter
b.	Less brighter	Less brighter
c.	Brighter	Less brighter
d.	No change	Brighter

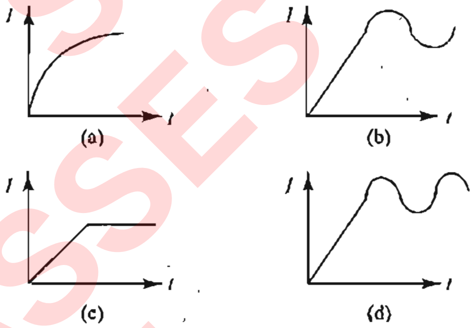
9. A cell of internal resistances r is concocted to a load of resistance R. Energy is dissipated in the load, but some thermal energy is also wasted in the cell. The efficiency of such an arrangement is found from the expression

$$\frac{\text{Energy dissipated in the load}}{\text{Energy dissipated in the complete circuit}}$$

Which of the following gives the efficiency in this case?

a. $\frac{r}{R}$
b. $\frac{R}{r}$
c. $\frac{r}{R+r}$
d. $\frac{R}{R+r}$

10. When an electric heater is switched on, the current flowing through it (i) is plotted against time (t). Taking into account the variation of resistance with temperature, which of the following best represents the resulting curve?



(a) (b) (c) (d)

Fig. 7.34

11. Two identical batteries each of e.m.f. $E = 2$ V and internal resistance $r = 1\ \Omega$ are available to produce heat in an external circuit. What is the maximum rate of production of heat that can be obtained in the external circuit?

a. 1 W
b. 2 W
c. 4 W
d. 8 W

12. Two similar head light lamps are connected in parallel to each other. Together, they consume 48 W from a 6 V battery. What is the resistance of each filament?

a. 6 Ω
b. 4 Ω
c. 3.0 Ω
d. 1.5 Ω

13. Two electric bulbs, rated for the same voltage, have powers of 200 and 100 W. If their resistances are r_1 and r_2, respectively, then:

a. $r_1 = 2r_2$
b. $r_2 = 2r_1$
c. $r_2 = 4r_1$
d. $r_1 = 4r_2$

14. The water in an electric kettle begins to boil in 15 min after being switched on. Using the same mains supply, should the length of the wire used as the heating element be increased or decreased if the water is to boil in 10 min?

a. decreased
b. increased
c. unchanged
d. none of the above

15. If the current in electric bulb decreases by 0.5% the power in the bulb decreased by approximately

a. 1%
b. 2%
c. 0.5%
d. 0.25%

16. An electric bulb rated for 500 W at 100 V is used in a circuit having a 200 V supply. The resistance R that must be put in series with the bulb, so that the bulb draws 500 W, is

a. 18 Ω
b. 20 Ω
c. 40 Ω
d. 700 Ω

17. A 2 Ω and a 2/3 Ω resistors are connected in parallel across a 3 V battery. The energy given out per minute is

a. $60 \times 2 \times 3$ J
b. $60 \times 9/2 \times 3 \times 3$ J
c. $60 \times 1/2 \times 3 \times 3$ J
d. $60 \times 3 \times 3 \times 2$ J

18. A 1°C rise in temperature is observed in a conductor by passing a certain current. If the current is doubled, then the rise in temperature is approximately

a. 2.5°C **b.** 4°C
c. 2°C **d.** 1°C

19. Two electric bulbs have tungsten filament of same length. If one of them gives 60 W and the other 100 W, then

a. 100 W bulb has thicker filament
b. 60 W bulb has thicker filament
c. both filaments are of same thickness
d. it is not possible to get different wattages unless the lengths are different.

20. n identical light bulbs, each designed to draw p power from a certain voltage supply, are joined in series across that supply. The total power which they will draw is

a. nP **b.** P
c. P/n **d.** P/n^2

21. A resistor R_1 dissipates the power P when connected to a certain generator. If a resistor R_2 is put in series with R_1, the power dissipated by R_1

a. decreases
b. increases
c. remains the same
d. any of the above depending upon the relative values of R_1 and R_2

22. How many calories of heat will be approximately developed in a 210 W electric bulb in 5 min?

a. 15,000 **b.** 1,050
c. 63,000 **d.** 80,000

23. Two bulbs of equal wattage one having carbon filament and the other having a tungsten filament are connected in series to the mains. Now, which of the following is true?

a. Carbon filament bulb glows more
b. Both bulbs glow equally
c. Tungsten filament bulb glows more
d. Carbon filament bulb glows less

24. A constant voltage is applied between the two ends of a metallic wire. If both the length and the radius of the wire are doubled, the rate of heat developed in the wire will

a. be halved **b.** be doubled
c. remain the same **d.** be quadrupled

25. The power rating of an electric motor which draws a current of 3.75 A, when operated at 200 V, is nearly

a. 54 W **b.** 1 hp
c. 500 W **d.** 750 hp

26. A cable of resistance 10 W carries electric power from a generator producing 250 kW at 10,000 V. The current in the cable is

a. 1,000 A **b.** 250 A
c. 100 A **d.** 25 A

27. In the previous problem, the power lost in the cable during transmission is

a. 3.15 kW **b.** 12.5 kW
c. 6.25 kW **d.** 25 kW

28. The heat generated through 4 and 9 Ω resistances separately, when a capacitor of 100 μF capacity charged to 200 V is discharged one by one, will be

a. 2 and 8 J, respectively
b. 8 and 2 J, respectively
c. 2 and 4 J, respectively
d. 2 and 2 J, respectively

29. If the length of the filament of a heater is reduced by 10%, the power of the heater will

a. increase by about 9% **b.** increase by about 11%
c. increase by about 19% **d.** decrease by about 10%

30. A 2 W heater used for 1 h every day consumes the following electrical energy in 30 days

a. 60 units **b.** 120 units
c. 15 units **d.** none of the above

31. Two bulbs which consume powers P_1 and P_2 are connected in series. The power consumed by the combination is

a. $P_1 + P_2$ **b.** $\sqrt{P_1 P_2}$
c. $P_1P_2/(P_1 + P_2)$ **d.** $2P_1P_2/(P_1 + P_2)$

32. Two cells, each of e.m.f. E and internal resistance r, are connected in parallel across a resistor R. The power delivered to the resistor is maximum if R is equal to

a. $r/2$ **b.** r
c. $2r$ **d.** 0

33. A constant voltage is applied between the two ends of a uniform metallic wire. Some heat is developed in it. The heat developed is doubled if

a. both the length and radius of the wire are halved
b. both the length and radius of the wire are doubled
c. the radius of the wire is doubled
d. the length of the wire is doubled

34. A given resistor cannot carry currents exceeding 20 A, without exceeding its maximum power dissipation ratings. By forced air cooling suppose that we increase the rate at which heat can be carried by a factor of 2. Now the maximum current that the resistor can carry is

a. 10 A **b.** $20\sqrt{2}$ A
c. $30\sqrt{2}$ A **d.** 40 A

35. If in the circuit, power dissipation is 150 W, then R is **(AIEEE, 2002)**

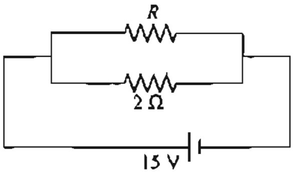


Fig. 7.35

a. 2 Ω **b.** 6 Ω
c. 5 Ω **d.** 4 Ω

36. A wire when connected to 220 V mains supply has power dissipation P_1. Now, the wire is cut into two equal pieces which are connected in parallel to the same supply. Power dissipation in this case is P. Then $P_2 : P_1$ is **(AIEEE, 2002)**

a. 1 **b.** 4
c. 2 **d.** 3

37. A 220 V, 1,000 W bulb is connected across a 110 V main supply. The power consumed will be **(AIEEE, 2003)**

a. 1000 W b. 750 W
c. 500 W d. 250 W

38. A heater coil is cut into two equal parts and only one part is now used in the heater. The heat generated will now be **(AIEEE, 2005)**

a. halved b. one-fourth
c. four times d. doubled

39. Two electric bulbs whose resistances are in the ratio of 1 : 2 are connected in parallel to a constant voltage source. The powers dissipated in them have the ratio

a. 1 : 2 b. 1 : 1 c. 2 : 1 d. 1 : 4

40. If the above two bulbs are connected in series, the powers dissipated in them have the ratio

a. 1 : 2 b. 1 : 1 c. 2 : 1 d. 1 : 4

41. Three 10 Ω, 2 W resistors are connected as in Fig. 7.36. The maximum possible voltage between points A and B without exceeding the power dissipation limits of any of the resistors is

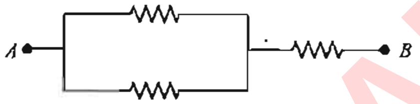


Fig. 7.36

a. $5\sqrt{3}$ V b. $3\sqrt{5}$ V
c. 15 V d. $\frac{5}{3}$ V

42. A torch bulb rated 4.5 W, 1.5 V is connected as shown in Fig. 7.37. The e.m.f. of the cell needed to make the bulb glow at full intensity is

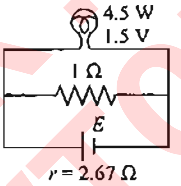


Fig. 7.37

a. 4.5 V b. 1.5 V c. 2.67 V d. 13.5 V

43. An electric bulb rated 500 W, 100 V is used in a circuit having a 200 V supply. The resistance R that must be put in series with the bulb, so that the bulb draws 500 W, is

a. 18 Ω b. 20 Ω c. 40 Ω d. 700 Ω

44. A heater is designed to operate with a power of 1000 W on a line 100 V. It is connected in combination with resistance of 10 Ω and a resistance R to line 100 V. The value of R so that heat operates with a power of 625 W is

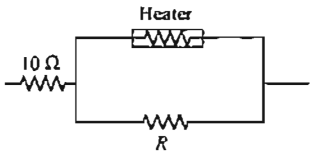


Fig. 7.38

a. 5 Ω b. 10 Ω c. 15 Ω d. 20 Ω

45. The main supply voltage to a room is 120 V. The resistance of the lead wires is 6 Ω. A 60 W bulb is already giving light. What is the decrease in voltage across the bulb when a 240 W heater is switched on?

a. no change b. 10 V
c. 20 V d. more than 10 V

46. Fig. 7.39 shows a network of three resistances. When some potential difference is applied across the network, thermal powers dissipated by A, B and C are in the ratio

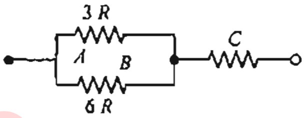


Fig. 7.39

a. 2 : 3 : 4 b. 2 : 4 : 3
c. 4 : 2 : 3 d. 3 : 2 : 4

47. Three equal resistances are connected as shown in Fig. 7.40. The maximum power consumed by each resistor is 18 W. Then maximum power consumed by the combination is

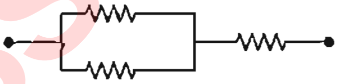
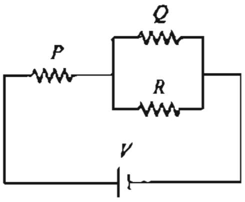


Fig. 7.40

a. 18 W b. 27 W
c. 36 W d. 54 W

48. Resistors P, Q and R in the circuit have equal resistances. If the battery is supplying a total power of 12 W, what is the power dissipated as heat in resistor R?



a. 2 W b. 6 W
c. 3 W d. 8 W

49. Three bulbs of 40, 60 and 100 W are connected in series with a 240 V source.

a. The potential difference will be maximum across the 40 W bulb.
b. The current will be maximum in the 100 W bulb.
c. The resistance of the 40 W bulb is maximum.
d. The current through the 60 W bulb will be slightly less than 0.1 A.

50. Three bulbs of 40, 60 and 100 W are connected in series with a 240 V source.

a. The potential difference will be maximum across the 40 W bulb.
b. The current will be maximum in 100 W bulb.
c. The resistance of the 40 W bulb is maximum.
d. The current through the 60 W bulb will be slightly less than 0.1 A.

51. In the circuit shown in Fig. 7.41 the heat produced in the 5 Ω resistor due to the current flowing through it is 10 cal s^{-1}. The heat generated in the 4 Ω resistor is

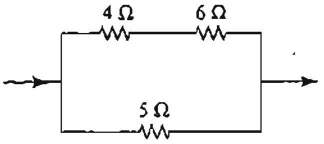


Fig. 7.41

a. 1 cal s^{-1} b. 2 cals^{-1}
c. 3 cals^{-1} d. 4 cals^{-1}

52. A battery of internal resistance 4 Ω is connected to the network of resistances as shown in Fig. 7.42. In order that the maximum power can be delivered to the network, the value of R in Ω should be

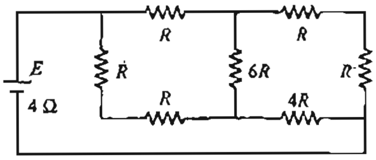


Fig. 7.42

a. $\frac{4}{9}$ b. 2 c. $\frac{8}{3}$ d. 18

53. Four resistance carrying a current shown in Fig. 7.43 are immersed in a box containing ice at 0°C. How much ice must be put in the box every 10 min to keep the average quantity of ice in the box constant? Latent heat of ice is 80 cal g^{-1}?

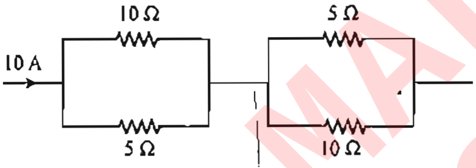


Fig. 7.43

a. 1.190 kg b. 3.20 kg
c. 4.2 kg d. 0.25 kg

54. The three resistances of equal value are arranged in the different combinations shown below. Arrange them in increasing order of power dissipation.

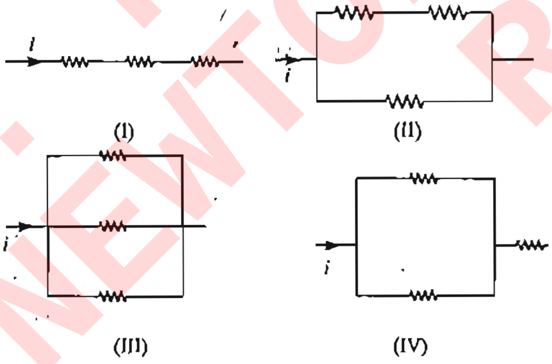


Fig. 7.44

a. III < II < IV < I b. II < III < IV < I
c. I < IV < III < II d. I < III < II < IV

55. An ideal gas is filled in a closed rigid and thermally insulated container. A coil of 100 Ω resistor carrying current 1 A for 5 min supplies heat to the gas. The change in internal energy of the gas is

a. 10 kJ b. 30 kJ c. 20 kJ d. 0 kJ

56. The resistance in which the maximum heat is produced is given by (Fig. 7.45)

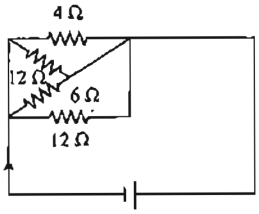


Fig. 7.45

a. 2 W b. 6 W c. 4 W d. 12 W

57. The resistance of hot tungsten filament is about 10 times the cold resistance. What will be the resistance of 100 W and 200 V lamp when not in use?

a. 14000 W b. 400 W c. 40 W d. 4 W

58. The resistance of the filament of a lamp increases with the increase in temperature. A lamp rated 100 W and 200 V is connected across 220 V power supply. If the voltage drops by 10%, then the power of the lamp will be

a. 90 W b. 81 W
c. between 90 and 100 W d. between 81 and 90 W

59. A wire of length L and three identical cells of negligible internal resistance are connected in series. Due to the current, the temperature of the wire is raised by ΔT in time t. A number N of similar cells is now connected in series with a wire of same material and cross-section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time. The value of N is

a. 4 b. 6 c. 8 d. 9

60. An electric immersion heater of 1.08 kW is immersed in water. After it has reached a temperature of 100°C, how much time will be required to produce 100 g of steam?

a. 50 s b. 420 s c. 105 s d. 210 s

61. Two electric bulbs A and B are rated 60 and 100 W, respectively. If they are connected in parallel to the same source, then

a. both the bulbs draw the same current
b. bulb A draws more current than bulb B
c. bulb B draws more current than bulb A
d. currents drawn in the bulbs are in the ratio of their resistances

62. If two bulbs of wattages 25 and 100 W, respectively, each rated by 220 V are connected in series with the supply of 440 V. Which bulb will fuse?

a. 100 W bulb b. 25 W bulb
c. none of them d. both of them

63. A 25 W–220 V bulb and a 100 W–220 V bulb are connected in series across a 220 V line; which electric bulb will glow more brightly?

a. 25 W bulb b. 100 W bulb
c. both will have equal incandescence
d. neither will give light

64. Two identical heaters rated 220 V–1000 W are placed in series with each other across 220 V line; then the combined power is

a. 1000 W b. 2000 W
c. 500 W d. 4000 W

65. A heater boils 1 kg of water in time t_1 and another heater boils the same water in time t_2. If both are connected in series, the combination will boil the same water in time:

a. $\frac{t_1 t_2}{t_1 + t_2}$ b. $\frac{t_1 t_2}{t_1 - t_2}$

c. $t_1 - t_2$ d. $2(t_1 + t_1)$

66. Fig. 7.46 shows three similar lamps L_1, L_2 and L_3 connected across a power supply. If the lamp L_3 fuses, how will the light emitted by L_1 and L_2 change?

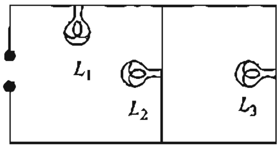


Fig. 7.46

a. no change
b. brilliance of L_1 decreases and that of L_2 increases
c. brilliance of both L_1 and L_2 increases
d. brilliance of both L_1 and L_2 decreases

67. If a wire of resistance 20 Ω is covered with ice and a voltage of 210 V is applied across the wire, then the rate of melting of ice is

a. 8.85 gs^{-1} b. 1.92 gs^{-1}
c. 6.56 gs^{-1} d. none of these

68. A factory is served by a 220 V supply line. In a circuit protected by a fuse marked 10 A, the maximum number of 100 W lamps in parallel that can be turned on is

a. 11 b. 22
c. 33 d. 66

69. It takes 16 min to boil some water in an electric kettle. Due to some defect it becomes necessary to remove 10% turns of the heating coil of the kettle. After repairs, how much time will it take to boil the same mass of water?

a. 17.7 min b. 14.4 min
c. 20.9 min d. 13.7 min

70. An electric kettle (rated accurately at 2.5 kW) is used to heat 3 kg of water from 15°C to boiling point. It takes 9.5 min. Then the amount of heat that has been lost is

a. 3.5×10^5 J b. 7×10^8 J
c. 3.5×10^4 J d. 7×10^8 J

71. How many 60 W lamps may be safely run on a 230 V circuit fitted with a 5 A fuse?

a. 2 b. 19
c. 20 d. 4

72. A condenser of capacity 5 μF is connected to a constant source of e.m.f. 200 V as shown in Fig. 7.47. What will be the amount of heat produced in R_1 when the key is thrown from contact 1 to 2?

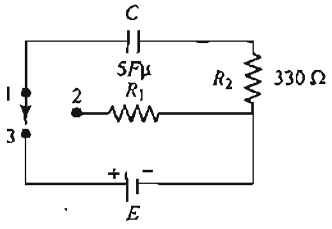


Fig. 7.47

a. 0.6 J b. 0.06 J
c. 6 J d. 20 J

73. Two electric bulbs rated P_1 watt – V volt and P_2 watt – V volt are connected in parallel and V volt are applied to it. The total power will be

a. $\frac{P_1 P_2}{P_1 + P_2}$ watt b. $\sqrt{P_1 P_2}$ watt
c. $(P_1 + P_2)$ watt d. $\frac{P_1 + P_2}{P_1 P_2}$ watt

74. If a given volume of water in a 220 V heater is boiled in 5 min, then how much time will it take for the same volume of water in a 110 V heater to be boiled?

a. 20 min b. 30 min
c. 25 min d. 40 min

75. The charge flowing through a resistance R varies with time t as $Q = at - bt^2$. The total heat produced in R is

a. $\frac{a^3 R}{6b}$ b. $\frac{a^3 R}{3b}$
c. $\frac{a^3 R}{2b}$ d. $\frac{a^3 R}{b}$

Multiple Correct Answers Type

Solutions on page 7.31

1. Two electric bulbs rated 25 W, 220 V and 100 W, 220 V are connected in series across a 220 V voltage source. The 25 and 100 W bulbs now draw P_1 and P_2 powers, respectively.

a. $P_1 = 16$ W b. $P_1 = 4$ W
c. $P_2 = 16$ W d. $P_2 = 4$ W

2. Two heaters designed for the same voltage V have different power ratings. When connected individually across a source of voltage V, they produce H amount of heat each in times t_1 and t_2, respectively. When used together across the same source, they produce H amount of heat in time t.

a. If they are in series, $t = t_1 + t_2$
b. If they are in series, $t = 2(t_1 + t_2)$
c. If they are in parallel, $t = \frac{t_1 t_2}{(t_1 + t_2)}$
d. If they are in parallel, $t = \frac{t_1 t_2}{2(t_1 + t_2)}$

3. A voltmeter and an ammeter are connected in series to an ideal cell of e.m.f. E. The voltmeter reading is V and the ammeter reading is I. Then

i. $V < E$
ii. the voltmeter resistance is V/I
iii. the potential difference across the ammeter is $E - V$
iv. voltmeter resistance plus ammeter resistance = E/I

Correct statements are

a. i and ii b. ii and iii
c. iii and iv d. all

4. In the circuit shown in Fig. 7.48,

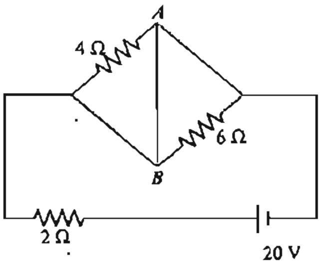


Fig. 7.48

a. power supplied by the battery is 200 W
b. current flowing in the circuit is 5 A
c. potential difference across the 4 Ω resistance is equal to the potential difference across the 6 Ω resistance
d. current in wire AB is zero.

5. Two bulbs consume same energy when operated at 200 and 300 V, respectively. When these bulbs are connected in series across a d.c. source of 500 V, then
a. ratio of potential difference across them is 3/2
b. ratio of potential difference across them is 4/9
c. ratio of potential difference across them is 4/9
d. ratio of potential difference across them is 2/3

Assertion-Reasoning Type

Solutions on page 7.31

In the following questions, each question contains STATEMENT I (Assertion) and STATEMENT II (Reason). Each question has four choices a., b., c. and d. out of which **ONLY ONE** is correct.

a. Statement I is True, Statement II is True; Statement II is a correct explanation for Statement I.
b. Statement I is True, Statement II is True; Statement II is **NOT** a correct explanation for Statement I.
c. Statement I is True, Statement II is False.
d. Statement I is False, Statement II is True.

1. **Statement I:** The wires supplying current to an electric heater are not heated appreciably.
Statement II: Resistance of connecting wires is very small and $H \propto R$.

2. **Statement I:** A 60 W bulb has greater resistance than a 100 W bulb.
Statement II: $P = \dfrac{V^2}{R}$.

3. **Statement I:** If the current of a lamp decreases by 20%, the percentage decrease in the illumination of the lamp is 40%.
Statement II: Illumination of the lamp is directly proportional to the square of the current through the lamp.

4. **Statement I:** Heater wire must have high resistance than connecting wires and high metallic point.
Statement II: If resistance is high, the electrical conductivity will be less.

5. **Statement I:** However long a fuse wire may be, the safe current that can be allowed is the same.
Statement II: The safe current that can be allowed to pass through the fuse wire depends on the radius of the wire.

6. **Statement I:** In the circuit of Fig. 7.49, both cells are ideal and of fixed e.m.f., the resistor R_1 has fixed resistance and the resistance of resistor R_2 can be varied (but R_2 is always non-zero). Then the electric power delivered to resistor of the resistance R_1 is independent of the value of resistance R_2.

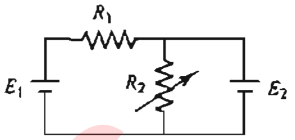


Fig. 7.49

Statement II: If potential difference across a fixed resistance is unchanged, the power delivered to the resistor remains constant.

7. **Statement I:** Two bulbs of 25 and 100 W rated at 200 V are connected in series across a 200 V supply. Ratio of powers of both the bulbs in series is 2 : 1.
Statement II: In series connection, current in both bulbs is same, therefore power depends on the resistance of the bulb.

8. **Statement I:** Since all the current coming to our house returns to power house (as current travels in a closed loop), so there is no need to pay the electricity bill.
Statement II: The electricity bill is paid for the power used, not for the current used.

9. **Statement I:** When current through a bulb is increased by 2%, power increases by 4%.
Statement II: Current passing through the bulb is $\propto \dfrac{1}{\text{resistance}}$.

10. **Statement I:** Internal resistance of battery is drawn parallel to battery in electrical circuit.
Statement II: Heat generated in battery is due to internal resistance.

Comprehension Type

Solutions on page 7.32

For Problems 1 – 2

In Fig. 7.50 circuit section AB absorbs energy at a rate of 50 W when a current i = 1.0 A passes through it in the indicated direction.

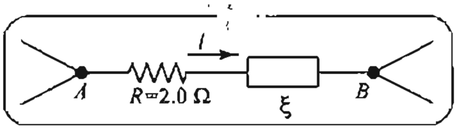


Fig. 7.50

1. What is the potential difference between A and B?
a. 10 V
b. 50 V
c. 20 V
d. 30 V

2. e.m.f. device X does not have internal resistance. What is its e.m.f.?
a. 24 V
b. 32 V
c. 48 V
d. 12 V

For Problems 3 – 4

An electric kettle has two heating coils. When one of the coils is switched on, the water in the kettle boils in 6 min, and when the other is switched on, the water boils in 8 min. In what time will the water boils if both the coils are switched on simultaneously:

3. In series?
a. 14 min **b.** 24/7 min
c. 10 min **d.** 10/3 min

4. in parallel?
a. 14 min **b.** 24/7 min
c. 10 min **d.** 10/3 min

For Problems 5 – 6

A three-way light bulb has three brightness settings (low, medium, high) but only two filaments. The two filaments are arranged in three settings, when connected across a 120 V line and can dissipate 60, 120 and 180 W. Answer the followings questions:

5. **i.** higher resistance filament only working for 60 W
ii. low resistance filament working for 120 W
iii. low resistance filament working for 60 W
iv. high resistance filament working for 120 W
v. low and high resistance filaments in parallel for 180 W
vi. low and high resistance filament in series for 180 W
a. i, ii and v are correct
b. i, ii and vi are correct
c. iii, iv and v are correct
d. iii, iv and vi are correct

6. When the filament of higher resistance burns out then intensity in
a. all three settings is 120 W
b. all three settings is 60 W
c. two settings is 60 W
d. two settings is 120 W

For Problems 7 – 9

In Fig. 7.51, each of the segments (e.g., *AE*, *GM*, etc.) has resistance *r*. A battery of e.m.f. *V* is connected between *A* and *C*. Internal resistance of the battery is negligible.

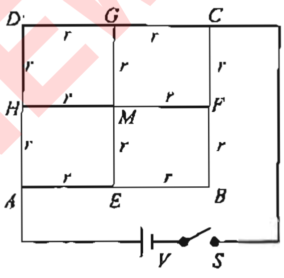


Fig. 7.51

7. What is the equivalent resistance of the system about *A* and *C*?
a. r **b.** $\frac{r}{2}$ **c.** $\frac{3r}{2}$ **d.** $2r$

8. Find the ratio of the power developed in segment *AE* to that in segment *HM*.
a. 1 **b.** 2 **c.** 3 **d.** 4

9. If a potentiometer circuit having potential gradient k is connected across the points *H* and *C*, find the balancing length shown by the potentiometer.
a. $\frac{v}{k}$ **b.** $\frac{2v}{3k}$
c. $\frac{3v}{2k}$ **d.** none of these

For Problems 10 – 12

Refer to Fig. 7.52.

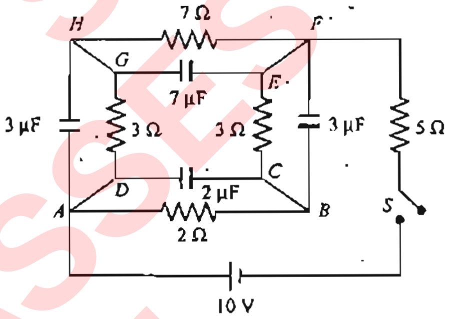


Fig. 7.52

10. At $t = 0$, the switch is closed. Just after closing the switch, find the current through the 5 Ω resistor.
a. $\frac{4}{5}$ A **b.** $\frac{2}{5}$ A **c.** $\frac{6}{5}$ A **d.** 2 A

11. Long time after closing the switch, find the current through the 5 Ω resistor.
a. $\frac{4}{5}$ A **b.** $\frac{2}{5}$ A **c.** $\frac{6}{5}$ A **d.** $\frac{8}{5}$ A

12. Now, the switch is opened after closing it for a long time. Find the total energy dissipated in the system.
a. 40.8 μJ **b.** 50.8 μJ
c. 40 μJ **d.** none of these

For Problems 13 – 15

All bulbs consume same power. The resistance of bulb 1 is 36 Ω. Answer the following questions

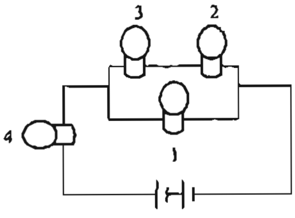


Fig. 7.53

13. What is the resistance of bulb 3?
a. 4 W **b.** 9 W **c.** 12 W **d.** 18 W

14. What is the resistance of bulb 4?
a. 4 W **b.** 9 W **c.** 12 W **d.** 18 W

15. What is the voltage output of the battery if the power of each bulb is 4 W?
a. 12 V **b.** 16 V
c. 24 V **d.** none of these

Matching Column Type

Solutions on page 7.34

Column I and Column II contain four entries each. Entries of column I are to be matched with some entries of column II. One or more than one entries of column I may have the matching with the same entries of column II and one entry of column I may have one or more than one matching with entries of column II.

1. In Fig. 7.54, the resistance R is variable, r is the internal resistance of battery of e.m.f. E.

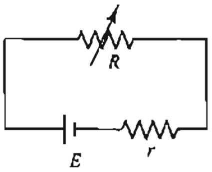


Fig. 7.54

Column I	Column II
i. Terminal potential difference across the cell to be maximum	a. $R > r$
ii. Power transferred to R is less than the maximum possible	b. $R < r$
iii. Power dissipated in the cell is maximum	c. $R = \infty$
iv. Fastest drift of ions in the electrolyte in the cell will be for	d. $R = 0$

2. Fig. 7.55 shows a charging circuit of a capacitor. At $t = 0$, S is closed.

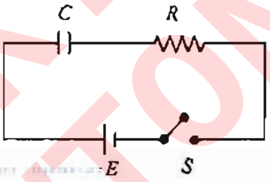


Fig. 7.55

Column I	Column II
i. When the charging rate of the capacitor is maximum, the current through R is	a. maximum
ii. When charge on the capacitor is maximum, then current through R is	b. minimum but not zero
iii. When power supplied by the battery is maximum, then charge on the capacitor is	c. zero
iv. The difference in the power supplied by battery and power consumed in R at $t = 0$ is	d. not equal to zero

3. For the circuit shown in Fig. 7.56, 4 cells are arranged. In Column I, the cell number is given while in Column II, some statement related to cells are given. Match the entries of Column I with the entries of Column II.

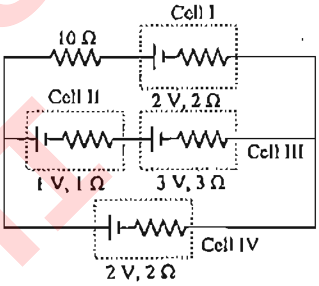


Fig. 7.56

Column I	Column II
i. Cell I	a. Chemical energy of cell is decreasing
ii. Cell II	b. Chemical energy of cell is increasing
iii. Cell III	c. Work done by cell is + ve
iv. Cell IV	d. Thermal energy developed in cell is +ve

ANSWERS AND SOLUTIONS

Subjective Type

1. The equivalent resistance between the points

$$R_0 = R_1 + \frac{R_2 R_x}{R_2 + R_x}$$

Power generated by

$$R_x = I_x^2 R_x$$

From Fig. 7.57,

$$I = \frac{V}{R_1 \dfrac{R_2 R_x}{R_2 + R_x}}$$

$$I_x = \left(\frac{V}{R_1 \dfrac{R_2 R_x}{R_2 + R_x}}\right)\left(\frac{R_2}{R_2 + R_x}\right)$$

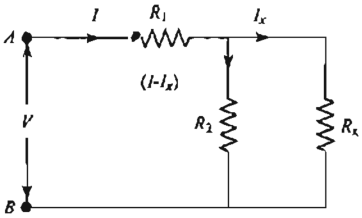


Fig. 7.57

$$P_x = I_x^2 R_x = \left[\frac{VR_2}{R_1R_2 + R_1R_x + R_2R_x}\right]^2 R_x$$

$$\frac{dP_x}{dR_x} = (VR_2)^2\,[R_1R_2 + R_1R_x + R_2R_x]^2$$
$$- 2(R_1R_2 + R_1R_x + R_2R_x)\,(R_1 + R_2)\,R_x$$

For maximum value, $\frac{dP_x}{dR_x} = 0$

$\Rightarrow \quad R_1R_2 + R_1R_x + R_2R_x = 2\,(R_1 + R_2)R_x$

$\Rightarrow \quad R_x = \frac{R_1R_2}{R_1 + R_2} = \frac{20 \times 30}{20 + 30} = 12\ \Omega$

2. **a.** The resistance of an electric appliance is given by

$$R = \frac{V_S^2}{W}$$
$$= \frac{(200)^2}{1000} = 40\Omega$$

b. The 'actual power' consumed by an electric appliance is given by

$$P = \left(\frac{V_A}{V_s}\right)^2 \times \mathrm{W}$$

$$P = \left(\frac{100}{200}\right)^2 \times 1000 = 250\,\mathrm{W}$$

c. The total electrical energy consumed by an electric appliance in a specified time is given by

$$E = \frac{\Sigma W_i h_i}{1000}\ \mathrm{kWh}$$
$$= \frac{1000 \times (10 \times 30)}{1000} = 300\ \mathrm{kWh}$$

3. For three identical resistors in series,

$$P_s = \frac{V^2}{3R}$$

If they are now in parallel over the same voltage,

$$P_p = \frac{V^2}{R_{eq}} = \frac{V^2}{R/3} = \frac{9V^2}{3R} = 9P_s = 9 \times (27\,\mathrm{W}) = 243\,\mathrm{W}.$$

4. $P_1 = \varepsilon^2/R_1, \quad \text{so} \quad R_1 = \varepsilon^2/P_1$

$P_2 = \varepsilon^2/R_1, \quad \text{so} \quad R_2 = \varepsilon^2/P_2$

a. When the resistors are connected in parallel to the e.m.f., the voltage across each resistor and the power dissipated by each resistor are the same as if only one resistor were connected.

$$P_{tot} = P_1 + P_2$$

b. When the resistors are connected in series the equivalent resistance is

$$R_{eq} = R_1 + R_2$$

$$P_{tot} = \frac{\varepsilon^2}{R_1 + R_2} = \frac{\varepsilon^2}{\varepsilon^2/P_1 + \varepsilon^2/P_2} = \frac{P_1P_2}{P_1 + P_2}$$

5. If R is the resistance of each bulb, then equivalent resistance of N bulbs in parallel,

$$R_{eq} = \frac{R}{N}$$

$\therefore$ Current supplied by battery,

$$i = \frac{E}{\left(\frac{R}{N} + r\right)}$$

This current is equally divided among all the N bulbs, as potential drop across each bulb is same.

So, power consumed by each bulb,

$$P = \left(\frac{i}{N}\right)^2 R = \frac{E^2R}{N^2\left(\frac{R}{N} + r\right)^2}$$

$$= \frac{E^2R}{(Nr + R)^2} \qquad \text{(i)}$$

With $(N - 1)$ bulbs, the power consumed by each bulb can be obtained by replacing N by $(N - 1)$ in above equation.

So, $\quad P' = \frac{E^2R}{[(N-1)r + R]^2} \qquad \text{(ii)}$

Now, percentage change in power consumption of each bulb is

$$\frac{P' - P}{P} \times 100 = \left[\frac{(rN + R)^2}{[r(n-1) + R]^2} - 1\right] \times 100$$

$$= \left[\frac{1}{\left[1 - \frac{r}{Nr + R}\right]} - 1\right] \times 100$$

As $r << (Nr + R)$

$$\left(1 - \frac{r}{Nr + R}\right)^{-2} = 1 + \frac{2r}{Nr + R}$$

$\therefore$ % change in power is

$$\frac{2r}{Nr + R} \times 100 = \frac{2 \times 0.1 \times 100}{200 \times 0.1 + 400} = 0.048\,\%$$

6. As for an electric appliance

$$R = \frac{V_S^2}{W}, \quad \therefore \quad \frac{R_{100}}{R_{200}} = \frac{200}{100},$$

i.e., $\quad R_{100} = 2R_{200} = 2R$ with $R_{200} = R$ and $\frac{V_S^2}{R} = 200\ \mathrm{W}$

a. When both the bulbs are connected in series, as

$$R_S = R + 2R = 3R,$$

$$P_S = \frac{V_A^2}{R_S} = \frac{V_S^2}{3R} = \frac{1}{3} \times 200 = 66.6\ \mathrm{W}$$

b. When both the bulbs are connected in parallel,

$$P_P = \frac{V_A^2}{R_1} + \frac{V_A^2}{R_2} = \frac{V_S^2}{R}\left[1+\frac{1}{2}\right] = \frac{3}{2}\times 200 = 300\,\text{W}$$

7. First recognize that if the 40 Ω resistor is safe, all the other resistors are also safe.

$$I^2R = P \rightarrow I^2(40\ \Omega) = 1\ \text{W}$$

$$I = 0.158\ \text{A}$$

Now, use series/parallel reduction to simplify the circuit. The upper parallel branch is 6.38 Ω and the lower one is 25 Ω. The series sum is now 126 Ω. Ohm's law gives

$$\varepsilon = (126\ \Omega)(0.158\ \text{A}) = 19.9\ \text{V}$$

8. a. First, do series/parallel reduction:
Now, apply Kirchhoff's laws and solve for e:

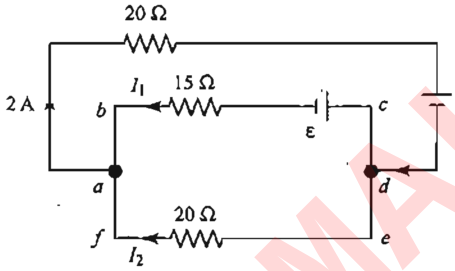


Fig. 7.58

$$\Delta V_{\text{adefa}} = 0: -(20\ \Omega)(2\ \text{A}) - 5\text{V} - (20\ \Omega)\, I_2 = 0$$

$$I_2 = 2.25\ \text{A}$$

$$I_1 + I_2 = 2\ \text{A} \rightarrow I_1 = 2\ \text{A} - (-2.25\ \text{A}) = 4.25\ \text{A}$$

$$\Delta V_{\text{adefa}} = 0: -(15\ \Omega)(4.25\ \text{A}) + \varepsilon - (20\ \Omega)(-2.22\ \text{A}) = 0$$

$$\varepsilon = -109\ \text{V}; \text{polarity should be reversed.}$$

b. Parallel branch has a 10 W resistance.

$$\Delta V_{\text{par}} = RI = (10\ \Omega)(2\ \text{A}) = 20\ \text{V}$$

Current in upper part:

$$I = \frac{\Delta V}{R} = \frac{20\ \text{V}}{30\Omega} = \frac{2}{3}\ \text{A}$$

$$Pt = U \rightarrow I^2 Rt = U$$

$$\left(\frac{2}{3}\ \text{A}\right)^2 (10\ \Omega)t = 60\ \text{J}$$

$$t = 13.5\ \text{s}$$

9. If a resistance R is connected across a source of potential difference Joule heat developed in time t is

$$H = P \times t = \frac{V^2}{R} \times t \text{ joule}$$

So, to produce same heat for same V

$$\frac{t}{R} = \text{const}, \quad \text{i.e,} \quad R \propto t$$

or $\quad \rho\dfrac{L}{A} \propto t \qquad \left[\text{as } R = \rho\dfrac{L}{A}\right]$

or $\quad L \propto t \qquad$ [as ρ and A are const.]

So, to boil the water in lesser time the resistance and hence length of the coil must be decreased to 10/15, i.e., 2/3 of its initial value.

10. The power consumed by a resistance R when connected across a sound of e.m.f. V is given by

$$P = (V^2/R)$$

Now, if r is the resistance of each resistor, the resistance of combination, in series, will be

$$R_S = r + r + r, \quad \text{i.e.,} \quad R_S = 3r$$

and in parallel,

$$\frac{1}{R_P} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r}, \quad \text{i.e.,} \quad R_P = \frac{r}{3}$$

So, power consumption in series will be

$$P_S = \frac{V^2}{3r} \qquad [\text{as } R_S = 3r]$$

and in parallell

$$P_P = \frac{V^2}{(r/3)} = 3\left[\frac{V^2}{r}\right] \qquad \left[\text{as } R_P = \frac{r}{3}\right]$$

$$\frac{P_P}{P_S} = 3\left[\frac{V^2}{r}\right] \times \left[\frac{3r}{V^2}\right] = 9$$

And as here, $P_S = 10\ \Omega$ [given]

$$P_P = 9 \times (P_S) = 9 \times 10 = 90\ \text{W}$$

11. Question is incomplete as the voltage for which the bulbs are meant to operate is not given. Assuming that both the bulbs have same specified voltage, resistance of a bulb will be given by

$$R = \frac{V_S^2}{W}, \quad \text{i.e.,} \quad R \propto \frac{1}{W}$$

$$\frac{R_{50}}{R_{100}} = \frac{100}{50}, \quad \text{i.e.,} \quad R_{50} = 2R_{100}$$

i. When the bulbs are connected in series, current through each of them will be same and hence in accordance with

$$H = I^2R, \quad \text{i.e.,} \quad H \propto R$$

the 50 W bulb will be more bright (as $R_{50} > R_{100}$).

ii. When the bulbs are connected in parallel, voltage across each of them will be same and so in accordance with

$$H = \frac{V^2}{R}, \quad \text{i.e.,} \quad H \propto \frac{1}{R}$$

the 100 W bulb will be more bright (as $R_{100} < R_{50}$).

Note:

In series, current through both the bulbs will be same but voltage across the 50 W bulb will be more while in parallel, voltage across both the bulbs will be same but current though the 100 W bulb will be more.

12. As for an electric appliance, $R = \dfrac{V_S^2}{W}$, so for same specified voltage V_S

$$\frac{R_{25}}{R_{100}} = \frac{100}{25} = 4$$

i.e., $R_{25} = 4R$ with $R_{100} = r$

Now, in series, potential divides in proportion to resistance.

So, $V_1 = \frac{R_1}{(R_1 + R_2)}$V, i.e., $V_{25} = \frac{4}{5} \times 440 = 352$ V

And $V_2 = \frac{R_1}{(R_1 + R_2)}$V, i.e., $V_{100} = \frac{1}{5} \times 400 = 88$ V

From this, it is clear that voltage across the 100 W bulb (= 88 V) is lesser than specified (220 V) while across 25 W bulb (= 352 V) is greater than specified (220 V), so 25 W bulb will fuse.

13. If a bulb is put in series with a heater of resistance R, then power dissipated by the heater is

$$H = \frac{V_H^2}{R_H} = \frac{(V - V_B)^2}{R_H}$$

Now, as for an electric applicance $R = \frac{V_S^2}{W}$. Assuming V_S to be same for all given appliance $R \propto (1/W)$ and in series, as $V_B \propto R$, $V_B \propto (1/W)$, i.e., voltage drop across the 100 W bulb will be lesser than across the 50 W bulb. Hence, the heater output will increase when the 50 W bulb is replaced by a 100 W bulb.

Note:

The question is incomplete as here it is not mentioned that all the appliances are meant to operate at same voltage (say 220 V).

14. a. As bulbs B and C are in parallel voltage across B and C will be same, i.e., $V_B = V_C$. Further, if R is the resistance of each bulb (as bulbs are identical), the resistance of bulbs B and C together (=R/2) is in series with resistance R of bulb A and as in series, potential divides in proportion to resistance,

$$V_A = \frac{R}{R + 0.5\,R} = \frac{2}{3}\,V_S = \frac{2}{3} \times 120 = 80\text{ V}$$

$$V_B = V_C = \frac{0.5\,R}{R + 0.5\,R},\ V = \frac{1}{3}\,V_S = \frac{1}{3} \times 120 = 40\text{ V}$$

[or $V_B = V_C = V - V_A = 120 - 80 = 40$ V]

b. As actual power consumed by a bulb

$$P = \frac{V_A^2}{R} = \left[\frac{V_A}{V_S}\right]^2 \times W \qquad \left[\because\ R = \frac{V_S^2}{W}\right]$$

So, total power consumption

$$P = P_A + P_B + P_C = P_A + 2\,P_B = \frac{(4+2)\,60}{9} = 40\text{ W}$$

Objective Type

1. d. $R_{200} = \frac{200 \times 200}{100} = 400\ \Omega$

So, $400 = R_0\,[1 + 0.005 \times 2000]$

$$\therefore \quad R_0 = \frac{400}{11} \approx 36\ \Omega$$

Hence, current $I = \frac{200}{36} = 5.5$ A

2. c. It is case of weak current.

3. a.

$$I = \frac{10}{\frac{15}{3} + \frac{4}{2} + 2 + 1}\text{ A} = \frac{10}{5 + 2 + 3}\text{ A} = \frac{10}{10}\text{ A} = 1\text{ A}$$

4. a. Power $P = \frac{V^2}{R}$

Since R is common, $P \propto V^2$

$$\text{Power ratio} = \left(\frac{V_1}{V_2}\right)^2 = \left(\frac{4.5}{1.5}\right)^2 = 3^2 = 9$$

5. a. Maximum current flows through bulb 1.

6. b. When the switch S_2 is closed, the whole of current shall flow through the connecting wire only which is supposed to have zero resistance.

7. d. Suppose V is the voltage of the supply and R is the resistance of each bulb.

Now, $R_p = \frac{R}{3}$ and current in ammeter, $I = \frac{V}{R_p} = 3\frac{V}{R}$,

provided all three bulbs are working properly.

If one bulb has broken down, then

$$R_p = \frac{R}{2} \quad \text{and} \quad I = 2\frac{V}{R}$$

∴ Current decreases and since current through each bulb is V/R the same as before, brightness of bulbs is not affected.

8. a. Consider two extreme cases. (i) When the resistance of the rheostat is zero, the current through Q is zero since Q is short-circuited. The circuit is then essentially a battery in series with lamp P. (ii) When the resistance of the rheostat is very large, almost no current flows through it. So, the currents through P and Q are almost equal. The circuit is essentially a battery in series with lamps P and Q.

9. d. Let current I flows through the circuit.

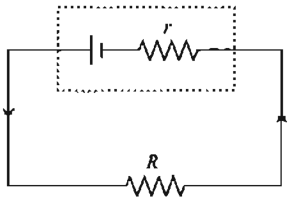


Fig. 7.59

Energy dissipated in load = I^2R

Energy dissipated in the complete circuit = $I^2\,(r + R)$

$$\therefore \quad \text{The efficiency} = \frac{I^2R}{I^2(R + r)} = \frac{R}{R + r}$$

10. b. When we move in the direction of the current in a uniform conductor, the potential decreases linearly. When we pass through the cell, from its negative to positive terminal, the potential increases by an amount equal to its potential difference. This is less than its e.m.f., as there is some potential drop across its internal resistance when the cell is driving current.

11. b. Power output is maximum when external resistance is 2Ω.

Current in the circuit = $\frac{4\text{ V}}{4\,\Omega}$ = 1A and power in the external circuit = $(1)^2 \times 2 = 2$ W.

12. d. Power consumed by each lamp = 24 W.
Hence using

$$R = (V^2/P) \text{ we find}$$

$$R = (36/24) = 1.5\,\Omega$$

13. b. $P = \frac{V^2}{R}, \quad \frac{P_1}{P_2} = \frac{R_2}{R_1}, \quad r_2 = 2r_1$

14. a. Heat given $H = \frac{V^2}{R}t$. This gives

$$\frac{t_1}{R_1} = \frac{t_2}{R_2}$$

$$\Rightarrow \quad \frac{t_2}{t_1} = \frac{R_2}{R_1} = \frac{\ell_2}{\ell_1}$$

If $t_2 < t_1$, than $\ell_2 < \ell_1$

15. a. $P = i^2R, \quad \frac{dP}{P} = 2\frac{dI}{I} = 2 \times 0.5\% = 1\%$

16. b. $P = VI, \; I = \frac{P}{V}$ or $I = \frac{500\text{ W}}{100\text{ V}} = 5\text{ A}$

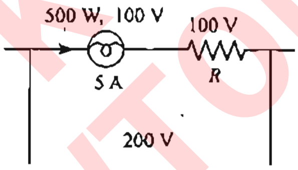


Fig. 7.60

Now, $5R = 100$ or $R = 20\,\Omega$

17. d. Combined resistance = $\frac{\frac{4}{3}}{\frac{8}{3}}\,\Omega = \frac{1}{2}\,\Omega$

$$\text{Energy} = \frac{V^2}{R}t = \frac{3 \times 3 \times 60}{1/2}\text{ J}$$

$$= 60 \times 3 \times 3 \times 2\text{ J}$$

18. b. When current is doubled, heating effect becomes four times. Now, $Q \propto \Delta T$
So, ΔT becomes four times.

19. a. $P = \frac{V^2}{R}$. If P is more, R is less. $R = \rho\frac{\ell}{a}$. For less R, 'a' is more. So, the 100 W bulb has thicker element.

20. c. Total resistance = $\frac{nV^2}{P}$. Power = $\frac{V^2P}{nV^2} = \frac{P}{n}$.

21. a. Due to increase in resistance, the current decreases. Again, $P = I^2R$. Note that I^2 is the dominant term.

22. a. Heat in calories = $\frac{210 \times 5 \times 60}{4.2} = 15000$

23. a. Think in terms of resistance.

24. b. $P = \frac{V^2}{R}$ or $P = \frac{V^2a}{\rho\ell} = \frac{V^2\pi r^2}{\rho\ell}$

25. b. $P = 200 \times 3.75\text{ W} = 750\text{ W} \approx 1\text{ hp}$

26. d. $P = VI; \; I = \frac{P}{V} = \frac{250 \times 1000}{10000}\text{ A} = 25\text{ A}$

27. c. Power lost = $25 \times 25 \times 10$ W
$= 6250\text{ W} = 6.25\text{ kW}$

28. d. $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 100 \times 10^{-6} \times 200 \times 200\text{ J} = 2\text{ J}$

29. b. $P = \frac{V^2}{\rho\frac{\ell}{a}} \quad \Rightarrow \quad P \propto \frac{1}{\ell}, \; P' \propto \frac{1}{\ell - \frac{10}{100}\ell}$

$$\frac{P'}{P} = \frac{10}{9}$$

$$\left(\frac{P'}{P} = 1\right) \times 100 = \left(\frac{10}{9} - 1\right) \times 100$$

$$\frac{P' - P}{P} \times 100 = \frac{100}{9} \approx 11$$

30. a. $2\text{ kW} \times 30\text{ h} = 60\text{ kWh} = 60$ units

31. c. $R = R_1 + R_2$

$$\frac{V^2}{P} = \frac{V^2}{P_1} + \frac{V^2}{P_2} \text{ or } \frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}$$

or $P = \frac{P_1P_2}{P_1 + P_2}$

32. a. The condition for delivering maximum power is that the external resistance is equal to internal resistance.

33. b. Heat developed, $H = \frac{V^2}{R}t$
Heat developed will be doubled when R is halved.
Further, $R = \rho\ell/(\pi r^2)$

$$H = \frac{V^2\pi r^2 t}{\rho\ell}$$

So, heat produced will be doubled when both the length and radius of the wire are doubled.

34. b. Heat produced $\propto I^2$
Initial heat produced = $k\,(20)^2$
Final heat produced = $2k\,(20)^2$
If final current is I', then

$$kI'^2 = 2 \times k \times (20)^2 \quad \Rightarrow \quad I' = 20\sqrt{2}\text{ A}$$

35. b. $150 = \frac{15 \times 15}{R}$ or $R = \frac{15 \times 15}{150}\,\Omega$

or $R = \frac{15}{10}\,\Omega = \frac{3}{2}\,\Omega$

Now, $\frac{2R}{2+R} = \frac{3}{2}$

or $4R - 3R = 6$ or $R = 6\ \Omega$.

36. b. $P = \frac{V^2}{R}$. R is reduced by a factor of 4. So, P is increased by a factor of 4.

37. d. $P = \frac{V^2}{R}$. If V is halved, P is reduced by a factor of 4. So, new power is $\frac{1000}{4}$ W, i.e., 250 W.

38. d. $Q = \frac{V^2}{R}t$. When R is halved, then Q is doubled.

39. c. Let potential difference of the voltage source be v. If resistances are R_1 and R_2 and power dissipated in them be P_1 and P_2, then

$$P_1 = \frac{V^2}{R_1},\ P_2 = \frac{V^2}{R_2},\ \text{so}\ \frac{P_1}{P_2} = \frac{R_2}{R_1}$$

$$\frac{R_2}{R_1} = \frac{2}{1},\ \text{so}\ \frac{P_1}{P_2} = \frac{2}{1}$$

40. a. With series combination current through both the bulbs is the same. Let this be I.

Then $P_1 = I^2R_1$ and $P_2 = I^2R_2$

$$\frac{P_1}{P_2} = \frac{R_1}{R_2}$$

As $\frac{R_1}{R_2} = \frac{1}{2}$, so $\frac{P_1}{P_2} = \frac{1}{2}$

41. b. Power $= \frac{V^2}{R}$

or $V = \sqrt{PR} = \sqrt{2 \times 10} = \sqrt{20} = 2\sqrt{5}$

Clearly voltage across single resistor of $10\ \Omega$ cannot exceed $2\sqrt{5}$ V. Note that the resistance of parallel combination is half of $10\ \Omega$. Thus, the maximum possible voltage between A and B is $3\sqrt{5}$ V.

42. d. Resistance of bulb $= \frac{1.5 \times 1.5}{4.5}\ \Omega = 0.5\ \Omega$

Resistance of parallel combination,

$$R = \frac{1 \times \frac{1}{2}}{1 + \frac{1}{2}}\ \Omega = \frac{1}{3}\ \Omega$$

Now, $r = \frac{E - V}{V}R$

or $\frac{8}{3} = \frac{E - 1.5}{1.5} \times \frac{1}{3}$ or $E = 13.5$ V

43. b. $P = VI$, $I = \frac{P}{V}$ or $I = \frac{500\ \text{W}}{100\ \text{V}} = 5\ \text{A}$

Now, $5R = 100$ or $R = 20\ \Omega$

44. c. Power of the heater $P = 1000$ W

Potential difference $V = 100$ V

$\therefore$ Resistance $R_1 = \frac{V^2}{P} = \frac{100 \times 100}{1000} = 10\ \Omega$

Now resistance of the circuit is

$$R_2 = 10 + \frac{10R}{10 + R} = \frac{100 + 20R}{10 + R}$$

$\therefore$ Power $= \frac{V^2}{R_2} = 625$ W

$$R_2 = \frac{625}{V^2} \Rightarrow \frac{100 + 20R}{10 + R} = \frac{625}{100 \times 100}$$

$R = 15\ \Omega$

45. d. $R_{60} = \frac{120 \times 120}{60}\ \Omega = 240\ \Omega$

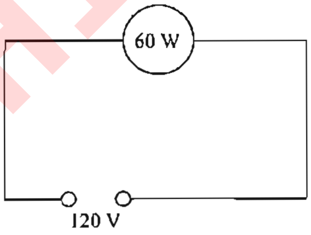


Fig. 7.61

Current $= \frac{120}{240 + 6}$ A $= \frac{120}{246}$ A

Voltage across bulb

$= \frac{120}{246} \times 240$ V $=$ [illegible]

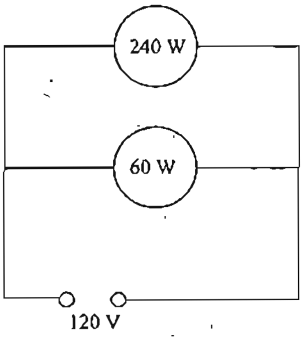


Fig. 7.62

$R_{240} = \frac{120 \times 120}{240}\ \Omega = 60\ \Omega$

Resistance of parallel combination

$$= \frac{60 \times 240}{60 + 240}\,\Omega = 48\,\Omega$$

Total resistance = (48 + 6) Ω = 54 Ω

Current $I = \frac{120}{54}$ A

Voltage across parallel combination

$$= \frac{120}{54} \times 48 \text{ V} = 106.7 \text{ V}$$

Change in voltage = (117.1 − 106.7) V = 10.4 V.

46. c. Let current flows from b to a as shown (Fig. 7.63).

Ratio is $\left(\frac{2}{3}I\right)^2 3R : \left(\frac{1}{3}I\right)^2 6R : I^2R$

or $\frac{4}{3} : \frac{2}{3} : 1$ or 4 : 2 : 3.

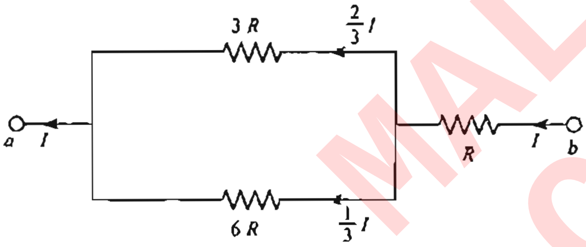


Fig. 7.63

47. b. If power consumed in resistor I is 18 W, then power consumed in each resistor II and III is $\frac{18}{4}$ W.

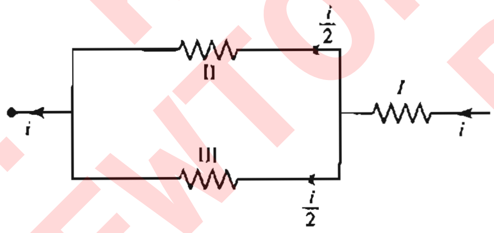


Fig. 7.64

∴ Total power consumed

$$= \left(18 + \frac{18}{4} + \frac{18}{4}\right) \text{W} = 27 \text{ W}.$$

48. a. Let resistors P, Q and R have resistance r. The effective resistance across the source is

$$R_{eff} = r + r \| r = r + \frac{(r)(r)}{r + r} = r + \frac{r}{2} = \frac{3r}{2}.$$

Current drawn from source is

$I_s^2 R_{eff} = 12$

$$\Rightarrow \quad I_s = \sqrt{\frac{12}{R_{eff}}} = \sqrt{\frac{8}{r}} \text{ A}.$$

Since Q and R have equal resistance r, each draws a current of I which is given by

$$I = \frac{1}{2} I_s = \sqrt{\frac{2}{r}} \text{ A}$$

Heat dissipation in R can now be determined and is given by

$$I^2 r = \left(\frac{2}{r}\right) r = 2 \text{ W}.$$

49. b. If power consumed in resistor I is 18 W, then power consumed in each resistor II and III is $\frac{18}{4}$ W.

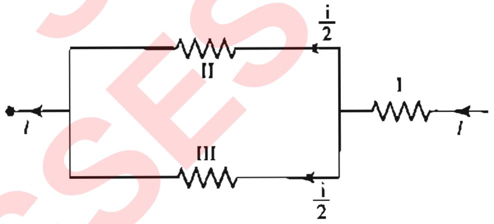


Fig. 7.65

∴ Total power consumed

$$= \left(18 + \frac{18}{4} + \frac{18}{4}\right) \text{W} = 27 \text{ W}.$$

50. a. Since all the three resistors are in series the same current will flow through them. Their resistances are given by $R = V^2/P$.

$$R_{40} = \frac{240 \times 240}{40} = 1440\,\Omega$$

$$R_{60} = 960\,\Omega \quad \text{and} \quad R_{100} = 576\,\Omega$$

Total resistance $R = 2976\,\Omega$

$$I = \frac{240}{2976} = 0.0806 \text{ A}$$

potential difference across the 40 W bulb = 1440 × 0.0806 = 116 V

51. b. Let I_1 be the current flowing in the 5 Ω resistor and $(I - I_1)$ in 4 Ω and 6 Ω resistors. The heat generated in the 5 Ω resistor is 10 cal $s^{-1} \approx 4.2 \times 10$ Js^{-1}

∴ $4.2 \times 10 = I_1^2 R$

$$I_1 = \sqrt{\frac{4.2 \times 10}{5}} = \sqrt{8.4} = 2.9 \text{ A}. \qquad \text{(i)}$$

Since AB and CD are in parallel. Therefore, the potential difference remains the same between C and D, and between A and B.∴ $(I - I_1)(4 + 6) = I_1 \times 5$

On solving using I_1 from (i) we get $(I - 2.9)\,10 = 2.9 \times 5$

∴ $I - 2.9 = 1.45$ or $I = 4.35$.

Heat released per sec and in the 4 Ω resistor will be $(4.35 - 2.9)^2 \times 4 = 8.4$ Js^{-1} = 2 cal s^{-1}.

52. b. For maximum power, external resistance is equal to internal resistance. Therefore,

$2R = 4$ or $R = 2$

53. a. Net resistance of circuit, $R = \frac{10 \times 5}{10 + 5} + \frac{10 \times 5}{10 + 5} = \frac{20}{3}\,\Omega$

Heat generated in circuit per minute

$$Q = I^2Rt = (10)^2 \times \frac{20}{3} \times (10 \times 60)$$

$$= 4 \times 10^5 \text{ J} = \frac{4 \times 10^5}{4.2 \times 10^5} \text{kilocal}$$

$$m = \frac{Q}{L} = \frac{4 \times 10^5}{4.2 \times 10^3 \times 80} = 1.19 \text{ kg}$$

54. a. $P_I = I^2(3R)$ $P_{II} = I^2\left(\frac{2R}{3}\right)$ $P_{III} = I^2\left(\frac{R}{3}\right)$

$$P_{IV} = I^2\left(\frac{3R}{2}\right)$$

$\therefore$ III < II < IV < I

55. b. The heat supplied under these conditions is the change in internal energy $Q = \Delta U$. The heat supplied $Q = i^2Rt$ $1 \times 1 \times 100 \times 5 \times 60 = 30000 \text{ J} = 30 \text{ kJ}$.

56. a. All resistances are in parallel.

So, for parallel combination $x = \frac{V^2}{R} t \Rightarrow x \propto \frac{1}{R}$

57. c. When hot,

$$R = \frac{V^2}{P} = \frac{200 \times 200}{100} = 400 \ \Omega$$

Hence, when cold, the resistance is 40 W.

58. d. Let the resistance of the lamp filament be R. Then $100 = (220)^2/R$. When the voltage drops, expected power is $P = (220 \times 0.9)^2/R'$.

Here, R' will be less than R, because now the rise in temperature will be less. Therefore, P is more than $(220 \times 0.9)^2/R = 81$ W. But it will not be 90% of the earlier value, because fall in temperature is small. Hence, option (d) is correct.

59. b. In the first case $\frac{(3E)^2}{R} r = (m) s \Delta t$ (1)

When the length of the wire is doubled, resistance and mass both are doubled. Therefore, in the second case,

$$\frac{(NE)^2}{2R} t = (2m) s \Delta t \quad (2)$$

Dividing equation (2) by (1), we get

$$\frac{N^2}{18} = 2 \quad \text{or} \quad N^2 = 36 \quad \text{or} \quad N = 6.$$

60. d. L is the latent heat of vaporization of water, the heat required for producing 1 g of steam,

$L = 540 \text{ cal} = 540 \times 4.2 = 2268 \text{ J}$.

Energy supplied = 1080 Js^{-1}.

Time required to boil 100 g of water

$$= 540 \times 4.2 \times 100/1080 = 210 \text{ s}$$

61. c. Given: $P_A = 60$ W; $P_B = 100$ W

We know that current flowing through the bulb,

$$I = \frac{P}{V}$$

We also know that as both the bulbs are connected in parallel, therefore potential difference (V) across both the bulbs is same. Thus $I \propto P$.

Since the power of bulb B is greater than that of bulb A, therefore bulb B draws more current than bulb A.

62. b. Resistance of the 25 W bulb,

$$R_1 = \frac{V^2}{P_1} = \frac{(220)^2}{25} = 1936 \ \Omega$$

Resistance of the 100 W bulb,

$$R_2 = \frac{V^2}{P_2} = \frac{(220)^2}{100} = 484 \ \Omega$$

Series resistance, $R = R_1 + R_2 = 2420 \ \Omega$

Current through the series combination,

$$I = \frac{440}{2420} = \frac{2}{11} \text{ A}$$

$\therefore$ Potential difference across the 100 W bulb,

$$V_2 = R_2 \times I = 484 \times \frac{2}{11} = 88 \text{ V}$$

Thus the bulb of 25 W will be fused because it can tolerate only 220 V while the voltage across it is 352 V.

63. a. Since power P is given by $P = V^2/R$, so $R = V^2/P$.

For the first bulb,

$$R_1 = \left(\frac{V^2}{P_1}\right)\left[\frac{(220)^2}{25}\right] = 1936 \ \Omega$$

For the second bulb,

$$R_{12} = \left(\frac{V^2}{P_2}\right)\left[\frac{(220)^2}{100}\right] = 484 \ \Omega$$

Current in series combination is the same in the two bulbs and current I is given by

$$I = \frac{V}{R_1 + R_2} = \frac{220}{1936 + 484}$$

$$= \frac{220}{2420} = \frac{1}{11} \text{ A}$$

If the actual powers in the two bulbs be P_1' and P_2', then

$$P_1' = I^2R_1 = \left(\frac{1}{11}\right)^2 \times 1936 = 16 \text{ W}$$

And $$P_2' = I^2R_2 = \left(\frac{1}{11}\right)^2 \times 484 = 4 \text{ W}$$

Since $P_1' > P_2'$, so the 25 W bulb will glow more brightly.

64. a. Let the resistance of the two heaters are denoted by R_1 and R_2, respectively.

$$R_1 = \left(\frac{V^2}{P_1}\right) \text{and } R_2 = \left(\frac{V^2}{P_2}\right)$$

If the resistance of the series combination be denoted by R_S and the corresponding power by P_S, then $R_S = R_1 + R_2$.

or $$\frac{V^2}{P_S} = \frac{V^2}{P_1} + \frac{V^2}{P_2}$$

or $$P_S = \frac{P_1 P_2}{P_1 + P_2} = \frac{1000 \times 1000}{2000} = 500 \text{ W}$$

65. c. $$H = \frac{V^2}{R_1} t_1 \text{ or } t_1 = \frac{HR_1}{V^2} \quad \text{(i)}$$

$$H = \frac{V^2}{R_2} t_2 \text{ or } t_2 = \frac{HR_2}{V^2} \quad \text{(ii)}$$

In series combination,

$$H = \frac{V^2}{R_1 + R_2} t$$

or $$t = \frac{H(R_1 + R_2)}{V^2} = \frac{HR_1}{V^2} + \frac{HR_2}{V^2} = t_1 + t_2$$

66. b. Let R be the resistance of each lamp. If E is the applied e.m.f., then the current in the circuit I_1 is given by

$$I_1 = \frac{E}{R + (R/2)} = 2E/3R$$

Current flowing through L_2 or L_3

$$= \frac{1}{2}\left[\frac{2E}{3R}\right] = \frac{E}{3R}$$

When L_3 is fused, the whole current flows through L_1 and L_2. Thus $I_2 = \left(\frac{E}{2R}\right)$

So, current through L_1 decreases and the current through L_2 increases.

67. c. The rate of heat developed is given by

$$\frac{V^2}{R} = \frac{(210)^2}{20} = 2205 \text{ Js}^{-1} = \frac{2205}{4.2} \text{ cals}^{-1}$$

Let m be the amount of ice melting per sec and, then

$$m \times 80 = \frac{2205}{4.2} \quad \text{or} \quad m = 6.56 \text{ g s}^{-1}$$

68. b. Current required by each bulb,

$$I = \frac{P}{V} = \frac{100}{220} \text{ A}$$

If n bulbs are joined in parallel, then

$$nI = I_{fuse} \quad \text{or} \quad n \times \frac{100}{220} = 10 \quad \text{or} \quad n = 22$$

69. b. If N be the initial number of turns in the coil and r be the radius of coil, then its resistance,

$$R = \rho \frac{L}{A} = \rho \frac{N 2\pi r}{A}$$

or $$\frac{V^2 t A}{4.2\, \rho N 2\pi r} = Q = ms\, d\theta$$

or $$\frac{t}{N} = \frac{ms\, d\theta \times 4.2\, \rho 2\pi r}{V^2 A} = \text{constant}$$

or $$\frac{t_1 / N_1}{t_2 / N_2} = 1$$

or $$t_2 = \frac{N_2}{N_1} t_1 = \left(\frac{\frac{9}{10} N_1}{N_1}\right) t_1 = \frac{9}{10} \times 16 \text{ min}$$

70. a. Energy consumed in 9.5 min

$$= 25 \times 1000 \times 9.5 \times 60 \text{ J} = 1425000 \text{ J}$$

Heat usefully consumed

$$= 3 \times 4.2 \times 100 \times (100 - 15) \text{ J} = 1071000 \text{ J}$$

Loss $= 3.5 \times 10^5$ J

71. b. Watt = volt × ampere

$$\Rightarrow \quad 60 = 230 \times I \quad \text{or} \quad I = \frac{6}{23} \text{ A}$$

If n lamps are used in parallel, each allowing 6/23 A, then total current,

$n \times (6/23) \leq 5$

or $n \leq 19.1$ or $n = 19$

72. b. Consider the position of key in contact with the condenser.

∴ Energy stored in the condenser $= \frac{1}{2}CE^2$. Now, the key is thrown to contact with 2. If I be the current in the circuit, then

$$H_1 = I^2 R_1 \quad \text{and} \quad H_2 = I^2 R_2$$

$$H = H_1 + H_2 = I^2(R_1 + R_2) = \frac{1}{2}CE^2$$

$$I^2(500 + 300) = \frac{1}{2} \times (5 \times 10^{-6})\,(200)^2$$

$$H_1 = \frac{5 \times 10^{-6} \times 100 \times 200}{830} \times 500 = 0.06 \text{ J}$$

73. c. $$R_1 = \frac{V^2}{P_1}, \qquad R_2 = \frac{V^2}{P_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{V^4}{P_1 P_2} \frac{P_1 P_2}{V^2(P_1 + P_2)} = \frac{V^2}{P_1 + P_2}$$

Now, $P_1 + P_2 = \frac{V^2}{R_{eq}} = P_{eq}$

74. a. $H = \frac{V^2 t}{R}$. When voltage is halved, the heat becomes one-fourth. Hence, time taken to heat the water becomes four times.

75. a. $Q = at - bt^2 \Rightarrow i = \frac{dQ}{dt} = a - 2bt$

$i = 0$ for $t = t_0 = a/2b$, i.e., current flows from $t = 0$ to $t = t_0$. The heat produced $= \int_0^{t_0} i^2 R\,dt$.

Multiple Correct Answers Type

1. a., d.
Let $V = 200$ V; R_1 and R_2 = resistance of the 25 W and 100 W bulbs.

$$P_1 = 25 = V^2/R_1 \text{ or } R_1 = V^2/25 \text{ and } R_2 = V^2/100$$

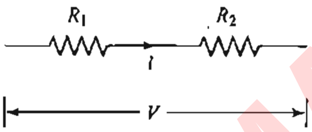


Fig. 7.66

When the bulbs are joined in series, the current

$$I = \frac{V}{R_1 + R_2}$$

Power in the 25 W bulb $= R_1 I^2$ and in the 100 W bulb $= R_2 I^2$.

2. a., c.
Let R_1 and R_2 be the resistances of the two heaters and let H be the heat produced.

$$\therefore \quad H = \left(\frac{V^2}{R_1}\right) t_1 = \left(\frac{V^2}{R_2}\right) t_2$$

When used in series,

$$H = \frac{V^2}{R_1 + R_2} t$$

When used in parallel,

$$H = \left(\frac{V^2}{R_1}\right) t_1 = \left(\frac{V^2}{R_2}\right) t_2$$

When used in series,

$$H = \frac{V_2}{R_1 + R_2} t$$

When used in parallel,

$$H = \left(\frac{V^2}{R_1} + \frac{V_2}{R_2}\right)$$

3. a., b., c., d.
Treat all voltmeters and ammeters as resistances. Draw the circuit and find the currents and potential differences for each section.

4. a., c., d.
4 Ω and 6 Ω resistor are short-circuited. Therefore, no current will flow through these resistances. Current passing through the battery is $I = (20/2) = 10$ A.
This is also the current passing in wire AB from B to A.
Power supplied by the battery

$$P = EI = (20)(10) = 200 \text{ W}$$

Potential difference across the 4 Ω resistance
= potential difference across the 6 Ω resistance

5. b., c.

$$P = \frac{V^2}{R}$$

$$\therefore \quad R = \frac{V^2}{P} \text{ or } R \propto V^2, \text{ i.e., } \frac{R_1}{R_2} = \left(\frac{200}{300}\right)^2 = \frac{4}{9}$$

When connected in series, potential drop is in the ratio of their resistances. So,

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{4}{9}. \quad \text{Now, } P = I^2 R$$

or $\quad P \propto R$ (in series I is the same)

or $\quad \frac{P_1}{P_2} = \frac{R_1}{R_2} = \frac{4}{9}$

Assertion-Reasoning Type

1. a. Resistance of the connecting wires is much smaller than the electric appliances to which current is supplied by the wires.

2. a. From $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P}$, lesser is the power rating, more is the resistance.

3. d. $H \propto I^2$

$$\Rightarrow \quad \frac{H_2}{H_1} = \left(\frac{I_2}{I_1}\right)^2 = \left(\frac{1.2 I_1}{I_1}\right)^2 = 1.44$$

There is 44% increase in illumination.

4. b. Heater wire must have high resistance than the connecting wires so that most of the potential drops across the heater wire. Because in series more is the resistances, more is the potential difference. Obviously it should have high melting point.

5. b. Safe current is independent of the length. It depends on the radius and property of the material of fuse wire.

6. a. The potential difference across the resistance is always $|E_1 - E_2|$. Hence assertion and reason are true and the reason is the correct explanation of the assertion.

7. d. Ratio of resistances will be 4:1, therefore ratio of powers produced will also be 4:1 in series.

8. d. Power used $= i^2 R$
Hence power is consumed, not the current.

9. b. $P = \Sigma^2 R$

$$100\% \times \left(\frac{dP}{P}\right) = 2\left(\frac{dI}{I}\right)100\%.$$

10. d. Internal resistance is drawn in series with battery.

Comprehension Type

For Problems 1 – 2

1. b., 2. c.

Sol. As section AB consumes 50 W at 1 A and $P = VI$,

$$V = V_A - V_B = \frac{P}{I} = \frac{50\ W}{1\ A} = 50\ \text{V}$$

As R and C are in series

$$V = V_R + V_C, \quad \text{i.e.,} \quad V_C = V - V_R$$

But here $V = 50$ V and $V_R = IR = 1 \times 2 = 2$ V

So, $V_C = 50 - 2 = 48$ V

Now as the element C absorbs energy and has no resistance, it is a source of e.m.f. with zero internal resistance (i.e., ideal battery)

So e.m.f., $E = V + Ir = 48 + 1 \times 0 = 48$ V

and as in charging positive and negative terminals of the charger are connected to the positive and negative terminals of the battery, respectively, B is connected to the negative terminal of element C.

For Problems 3 – 4

3. b., 4. b.

Sol. Heat required to begin boiling is same for every case. Let this be H. Let R_1 and R_2 be the resistances of the coils and V be the supply voltage. $t_1 = 6$ min and $t_2 = 8$ min.

Let t_s is the time when they are in series and t_p is the time when they are in parallel.

$$H = \text{power} \times \text{time}$$

$$H = \frac{V^2}{R_1}t_1 = \frac{V^2}{R_1}t_2 = \frac{V^2}{R_1 + R_2}t_s = \frac{V^2(R_1 + R_2)}{R_1R_2}t_p$$

$$R_1 = \frac{V^2t_1}{H} \quad \text{and} \quad R_2 = \frac{V^2t_2}{H}$$

Subtracting, we get $H = \dfrac{V^2t_s}{R_1 + R_2} = \dfrac{V^2t_s}{V^2(t_1 + t_2)}$

$$t_s = t_1 + t_2 = 14 \text{ min}$$

Subtracting in H, we get $H = \dfrac{V^2}{R_1R_2}(R_1 + R_2)t_p$

$$H = V^2t_p\left(\frac{H}{V^2t_1} + \frac{H}{V^2t_2}\right)$$

$$t_p = \frac{t_1t_2}{t_1 + t_2} = \frac{6 \times 8}{6 + 8} = \frac{24}{7} \text{ min}$$

For Problems 5 – 6

5. a., 6. d.

Sol. 60 W bulb will be of higher resistance. If only switch '1' is closed, 120 W bulb will glow. If only switch '2' is closed, 60 W bulb will glow. If both the switches are closed, both bulbs will glow with total illumination 180 W.

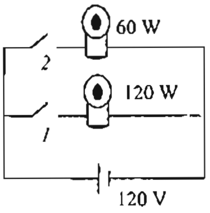


Fig. 7.67

When the filament of higher resistance (of 60 W bulb) burns out then for two settings (either '1' is closed or both are closed) power dissipated will be 120 W.

For Problems 7 – 9

7. c., 8. d., 9. b.

Sol. In loop $EBFME$:

$$V_E - ri_1 - ri_1 + r\left(\frac{i}{2} - i_1\right) + r\left(\frac{i}{2} - i_1\right) = V_E$$

$$\Rightarrow \quad 2i_1 = 2\left(\frac{i}{2} - i_1\right) \quad \Rightarrow \quad i_1 = \frac{i}{4}$$

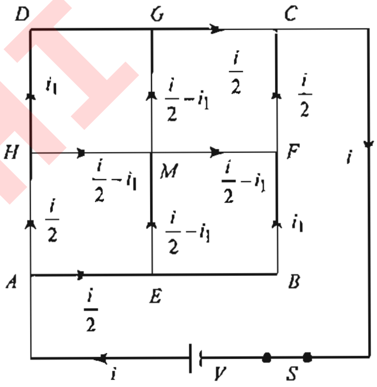


Fig. 7.68

Loop $AEBCSA$:

$$V_A - r\frac{i}{2} - ri_1 - ri_1 - r\frac{i}{2} + V = V_A$$

$$\Rightarrow \quad v = ri + 2ri_1 \quad \Rightarrow \quad v = ri + 2r\frac{i}{4}$$

$$\Rightarrow \quad V = \frac{3ri}{2}, R_{eq} = \frac{V}{i} = \frac{3r}{2}$$

$$\frac{P_1}{P_2} = \frac{\left(\frac{i}{2}\right)^2 r}{\left(\frac{i}{2} - i_1\right)^2 r} = \frac{i^2}{(i - 2i_1)^2} = 4$$

Let ℓ is the balancing length, then

$$K\ell = V_H - V_C$$

$$\Rightarrow \quad K\ell = \left(\frac{i}{2} - i_1\right)r + \left(\frac{i}{2} - i_1\right)r + \frac{i}{2}r$$

$$\Rightarrow \quad K\ell = \left(\frac{3i}{2} - 2i_1\right)r \quad \Rightarrow \quad K\ell = ir$$

$$\Rightarrow \quad K\ell = \frac{2\text{ V}}{3} \quad \Rightarrow \quad \ell = \frac{2\text{ V}}{3k}$$

For Problems 10 – 12

10. d., 11. c., 12.a.

Sol. Initially when all the capacitors are uncharged, they will act as conducting wires. Hence, all the resistances (except 5 Ω) will be short-circuited. So, no resistance occurs between A and F. So, current through the 5 Ω resistor will be $\frac{10}{5}$ A $=2$ A.

In steady state, distribution of current is shown.

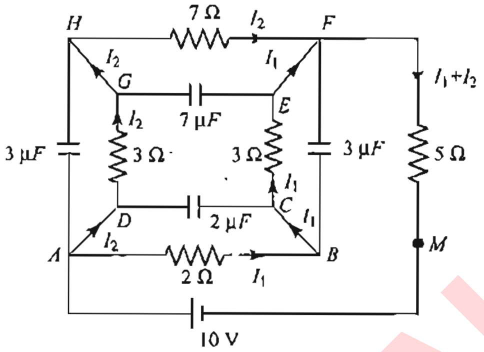


Fig. 7.69

Loop $ABCEFMA$:

$$10 = 2I_1 + 3I_1 + 5(I_1 + I_2)$$

$$10I_1 + 5I_2 = 10 \qquad \text{(i)}$$

Loop $ADGHFMA$:

$$10 = 3I_2 + 7I_2 + 5(I_1 + I_2)$$

$$5I_1 + 15I_2 = 10 \qquad \text{(ii)}$$

From (i) and (ii)

$$I_1 = \frac{4}{5}\text{ A}, \quad I_2 = \frac{2}{5}\text{ A}$$

Hence, current through the 5 Ω resistor is $I_2 + I_2 = \frac{6}{5}$ A

Potential difference across the 2 μF capacitor

$$= 2I_1 = 2\times\frac{4}{5} = \frac{8}{5}\text{ V}$$

Energy in it is given by

$$U_1 = \frac{1}{2}\times 2\left(\frac{8}{5}\right)^2 = \frac{64}{25}\ \mu\text{J}$$

Potential difference across the 3 μF capacitor is

$$3I_1 = 3\times\frac{4}{5} = \frac{12}{5}\text{ V}$$

Energy in it is given by

$$U_2 = \frac{1}{2}\times 3\left(\frac{12}{5}\right)^2 = \frac{216}{25}\ \mu\text{J}$$

Potential difference across the 3 μF capacitor is

$$3I_2 = 3\times\frac{2}{5} = \frac{6}{5}\text{ V}$$

Energy in it is given by

$$U_3 = \frac{1}{2}\times 3\left(\frac{6}{5}\right)^2 = \frac{54}{25}\ \mu\text{J}$$

Potential difference across the 7 μF capacitor is

$$7I_2 = 7\times\frac{2}{5} = \frac{14}{5}\text{ V}$$

Energy in it is given by

$$U_4 = \frac{1}{2}\times 7\left(\frac{14}{5}\right)^2 = \frac{686}{25}\ \mu\text{J}$$

This whole of the energy stored in all capacitors will be dissipated after the switch is opened. So energy dissipated

$$= U_1 + U_2 + U_3 + U_4 = 40.8\ \mu\text{J}$$

For Problems 13 – 15

13. b., 14. a., 15. b.

Sol. $i_2 = i_3 = I_b$

$V_1 = (V_2 + V_3) = V$

and $P_2 = P_3$

$\Rightarrow$ $R_2 = R_3$

and $V_2 = V_3 = V/2$

$P_1 = P_2 = P_3 = P_4$ (given)

$$P_1 = \frac{V^2}{36}; \quad P_3 = \frac{(V/2)^2}{R_3}$$

As $P_1 = P_3$, so $R_3 = 9\ \Omega$

Also $R_3 = R_2 = 9\ \Omega$

$$I_a = \frac{I}{3} \text{ and } I_4 = I, P_1 = P_4 = 4\text{ W}$$

$$\left(\frac{I}{3}\right)^2 R_1 = (I)^2 R_4 = 4\text{ W}$$

$$\Rightarrow \quad \frac{R_1}{9} = R_4' \ \Rightarrow \ R_4 = 4\ \Omega$$

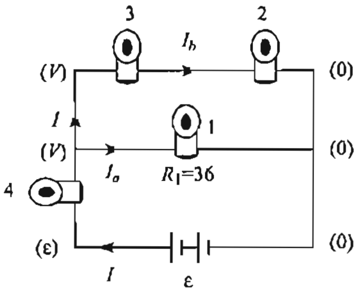


Fig. 7.70

Also $I = 1$ A

$R_{eq} = 16\ \Omega$ and $I = 1$ A, $\varepsilon = 16$ V

Matching Column Type

1.i. → c., ii. → a.,b.,c.,d., iii. → d., iv. → d.

Sol. $I = \dfrac{E}{R+r}$

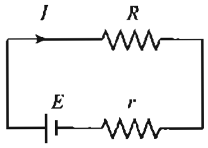


Fig. 7.71

Terminal potential difference across the cell is

$$V = E - Ir$$

This is maximum if $I = 0$, and this is possible if $R = \infty$. Hence i. → c.

Power transferred to R is maximum if $R = r$. In all other cases it will be somewhat less.

Hence ii. → a.,b.,c.,d.

Power dissipated in cell $= EI$

This is maximum if I is maximum, for this $R = 0$. Hence iii. → d.

Also if I is maximum, there will be the fastest drift of ions in electrolyte in the cell. Hence iv. → d.

2.i. → a.,d., ii. → c., iii. → c., iv. → c.

Sol. Charging rate means current $\left(\dfrac{dq}{dt} = I\right)$. So, if charging rate is maximum, then current is also maximum. Hence i. → a, d.

Charge on the capacitor is maximum in steady state. In steady state current becomes zero. Hence ii. → c.

Power supplied by the battery is maximum initially because current is maximum initially. At this time charge on the capacitor is zero. Hence iii. → c.

Difference in the power supplied by battery and power consumed in R is

$$EI - I^2R = EI - I(IR) = EI - IE = 0$$

Hence iv. → c.

3. i. → b.,d., ii. → a.,c.,d., iii. → a.,c.,d., iv. → b.,d.

Sol. We have, $I_1 = -\dfrac{1}{20}$ A, $I_2 = \dfrac{7}{20}$ A, $I_3 = -\dfrac{6}{20}$ A.

In each cell thermal energy will be dissipated due to internal resistance whether the chemical energy of the cell is increasing or decreasing.

i. Cell I is getting charged, hence its chemical energy increases.

ii. Cells II and III both are getting discharged, hence their chemical energy is decreasing. So, work done by both of them is positive.

iv. Cell IV is getting charged, hence its chemical energy increases.

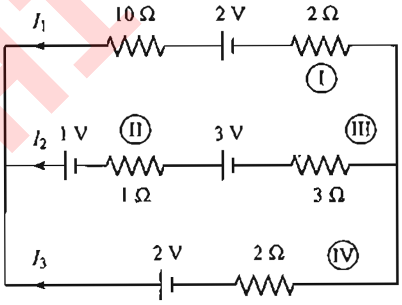


Fig. 7.72

CHAPTER
8

Faraday's Law and Lenz's Law

- Magnetic Flux
- Lenz's law
- Motional Electromotive Force
- Induced Electric Field and Inductance
- Mutual Inductance
- Self-Inductance
- Application of the Kirchhoff's Law
- Series and Parallel Combination of Inductors
- Combination of Inductors with Resistors
- Energy Stored in Magnetic Field of an Inductor
- LC Oscillations

MAGNETIC FLUX

The magnetic flux passing through a loop of area A is defined as

$$\phi_B = \vec{B}\cdot\vec{A} \qquad \text{[for uniform } \vec{B}\text{]}$$

or $$\phi = BA\cos\theta = B_{\perp}A$$

where $B_{\perp}$ is the component of the magnetic field B perpendicular to the face of the loop, and θ is the angle between the direction of the magnetic field and the vector representing area. Direction of area vector is outward normal to the face of the loop (see Fig. 8.1).

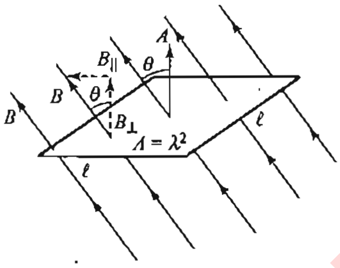


Fig. 8.1

If the surface is not plane, we can divide any surface into elements of area dA (as shown in Fig. 8.2). For each element we determine $B_{\perp}$, the component of normal to the surface at the position of that element, as shown. From the figure, $B_{\perp} = B\cos\phi$, where ϕ is the angle between the direction of B and a line perpendicular to the surface. In general, this component varies from point to point on the surface.

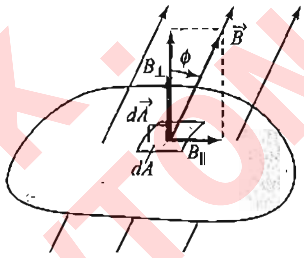


Fig. 8.2

We define the magnetic flux $d\Phi_B$ through the area element dA as

$$d\Phi_B = B_{\perp}\,dA$$

The total magnetic flux through the surface is the sum of the contributions from the individual area elements:

$\Phi_B = \int B_{\perp}dA = \int B\cos\phi\, dA = \int \vec{B}\cdot d\vec{A}$ (magnetic flux through a surface)

The unit of magnetic flux is tesla metre^2 which is called weber (Wb) in honour of Wilhelm Weber. 1 Wb = 1 Tm^2. Clearly, B can be measured in Wb m^{-2}. 1 Wb m^{-2} = 1 T. Sometimes it is referred to as *flux density*.

Illustration 8.1 **A long solenoid with radius 2 cm carries a current of 2 A. The solenoid is 70 cm long and is composed of 300 turns of wire. Assuming ideal solenoid model, calculate the flux linked with a circular surface,**

a. if it has a radius of 1 cm and is perpendicular to the axis of the solenoid:
  i. inside, ii. outside.

b. if it has a radius of 3 cm and is perpendicular to the axis of the solenoid with its centre lying on the axis of the solenoid.

c. if it has a radius greater than 2 cm and axis of the solenoid subtends an angle of 60° with the normal to the area (the centre of the circular surface being on the axis of the solenoid).

d. if the plane of the circular area is parallel to the axis of the solenoid.

Sol. For an ideal solenoid: $B_{out} = 0$ and $B_{in} = \dfrac{\mu_0}{4\pi}(4\pi nI)\,\mu_0 nI$.

So, here $B_{out} = 0$ and $B_{in} = 10^{-7}[4\pi(300/0.7)\times 2]\;1.076\times 10^{-3}\,\mathrm{T}$

and for constant field $\phi_B = \int \vec{B}\cdot d\vec{s} = BS\cos\theta$.

a. As in this situation $\theta = 0$ and $s = \theta\times(1\times 10^{-2})^2\,\mathrm{m}$

  i. $\phi_{in} = (1.076\times 10^{-3})\times(\pi\times 10^{-4})\cos 0 = 3.38\times 10^{-7}\,\mathrm{Wb}$

  ii. $\phi_{out} = 0\times(\pi\times 10^{-4})\cos 0 = 0$ (as $B_{out} = 0$)

b. In this situation $\theta = 0$ [Fig. 8.3(a)].

So, $\phi_b = \phi_{in} + \phi_{out} = B_{in}[\pi\times(2\times 10^{-2})^2] + B_{out}[\pi(3^2 - 2^2)\times 10^{-4}]$
$= (1.076\times 10^{-3})\times(\pi\times 4\times 10^{-4}) + 0[\pi\times 5\times 10^{-4}]$
$= 13.52\times 10^{-7}\,\mathrm{Wb}$

Note:

> ***This is the maximum flux which the given solenoid can produce for any surface and is independent of shape and size of the surface if $R \ge r$.***

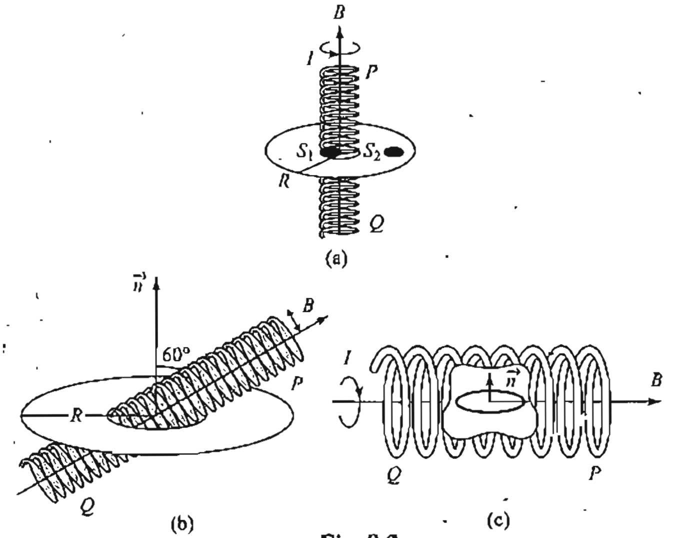
c. In this situation, $\theta = 60°$ [Fig. 8.3(b)], so

$\phi_c = \phi_{out} = B_{in}\times(\pi\times 4\times 10^{-4})\cos 60 + 0 \qquad [\text{as } B_{out} = 0]$

or $\phi_c = \phi_b\cos 60 = (13.52\times 10^{-7})\times\cos 60 = 6.76\times 10^{-7}\,\mathrm{Wb}$

d. In this situation, as $\theta = 90°$ [Fig. 8.3(c)]

$\phi_d = B_{in}\times S\times\cos 90 = 0$



(a)

(b)

(c)

Fig. 8.3

Illustration 8.2 **A long copper wire carries a current of I ampere. Calculate the magnetic flux per metre of the wire for a plane surface S inside the wire as shown in Fig. 8.4.**

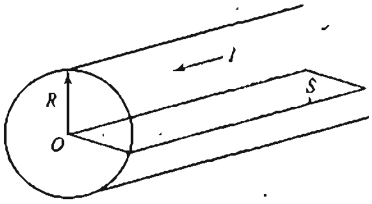


Fig. 8.4

Sol. Consider an element of width dr at a distance r from the axis of the wire. The field due to the current I in the wire at the position of element will be $B = \frac{\mu_0}{4\pi}\frac{2I'}{r}$.

But $\quad I' = \frac{I}{\pi R^2} \times \pi r^2 = \frac{Ir^2}{R^2}$

So, $\quad B = \frac{\mu_0}{4\pi}\frac{2I}{R^2}r$

and as its direction is perpendicular to the plane surface, flux linked with the element is given by

$$d\phi = B\,ds\cos\theta = \frac{\mu_0}{4\pi}\frac{2I}{R^2}r\ (l\,dr)\cos 0$$

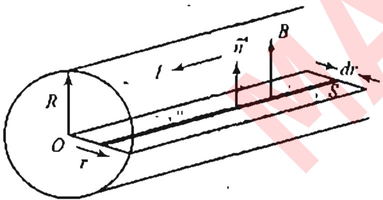


Fig. 8.5

So, the flux linked with the plane surface is given by

$$\phi = \frac{\mu_0}{4\pi}\frac{2I\ell}{R^2}\int_0^R r\,dr = \frac{\mu_0}{4\pi}(I\ell)$$

Faraday's Law of Electromagnetic Induction

On the basis of several experimental observations, Michael Faraday came to the following conclusions.

1. Whenever there is a relative motion between a magnet (source of magnetic field) and a closed conducting loop, electric current appears in the loop. It happens because of the change in magnetic flux associated with the loop.
2. Since e.m.f. causes current in a circuit, when loop and magnet are brought in relative motion current flows in the loop. This implies that an e.m.f. is set up in the loop. This e.m.f. is known as induced e.m.f. and its magnitude is directly proportional to the rate of change of magnetic flux with time.

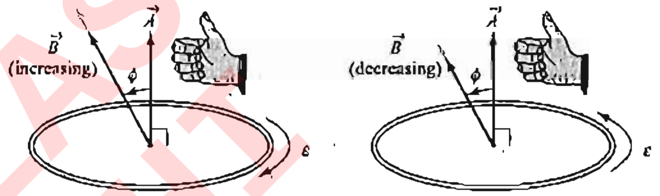
Now, we come to the Faradays's law of electromagnetic induction. In mathematical form induced e.m.f. can be given by the expression $\varepsilon = -\frac{d\varphi_B}{dt}$.

As such, Faraday's law in itself is complete to tell the magnitude and polarity of induced e.m.f. But Lenz's rule is commonly used to determine the polarity of induced e.m.f. or direction of induced current.

Direction of Induced e.m.f.

We can find the direction of an induced e.m.f. or current by using $\varepsilon = -d\Phi_B/dt$ together with some simple sign rules. Here is the procedure:

1. Define a positive direction for the area vector $\vec{A}$
2. From the directions of $\vec{A}$ and the magnetic field $\vec{B}$, determine the sign of the magnetic flux F_B and its rate of change $d\Phi_B/dt$. Fig. 8.6 shows several examples.
3. Determine the sign of the induced e.m.f. or current. If the flux is increasing, i.e., $d\Phi_B/dt$ is positive, then the induced e.m.f. or current is negative; if the flux is decreasing, $d\Phi_B/dt$ is negative, then the induced e.m.f. or current is positive.
4. Finally, determine the direction of the induced e.m.f. or current using your right hand. Curl the fingers of your right hand around the $\vec{A}$ vector, with your right thumb in the direction of $\vec{A}$. If the induced e.m.f. or current in the circuit is positive, then it is in the same direction as your curled fingers; if the induced e.m.f. or current is negative, then it is in the opposite direction.

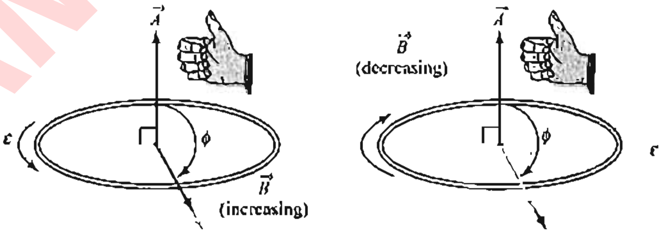


Positive flux ($\Phi_B > 0$)
Flux becoming more positive $\left(\frac{d\Phi_B}{dt} > 0\right)$
Induced e.m.f. is negative ($\varepsilon < 0$)

(a)

Positive flux ($\Phi_B > 0$)
Flux becoming less positive $\left(\frac{d\Phi_B}{dt} < 0\right)$
Induced e.m.f. is negative ($\varepsilon > 0$)

(b)



Negative flux ($\Phi_B < 0$)
Flux becoming more negative $\left(\frac{d\Phi_B}{dt} > 0\right)$
Induced e.m.f. is positive ($\varepsilon > 0$)

(c)

Negative flux ($\Phi_B < 0$)
Flux becoming less negative $\left(\frac{d\Phi_B}{dt} > 0\right)$
Induced e.m.f. is negative ($\varepsilon < 0$)

(d)

Fig. 8.6

The magnetic flux is becoming (a) more positive, (b) less positive, (c) more negative and (d) less negative. Therefore, Φ_B is increasing in (a) and (d) and decreasing in (b) and (c). In (a) and (d) the values of the e.m.f. are negative (they are opposite to the direction of the curled fingers of your right hand when your right thumb points along $\vec{A}$). In (b) and (c) the values of the e.m.f. are positive (in the same direction as the curled fingers).

LENZ'S LAW

Direction of the Induced Current in a Circuit

Lenz's law states that "when the magnetic flux through a loop changes, a current is induced in the loop such that the magnetic field due to the induced current opposes the change in the magnetic flux through the loop".

The above rule can be systematically applied as follows to determine the direction of the induced currents.

- Identify the loop in which the induced current is to be determined.
- Determine the direction of the magnetic field in this loop (i.e., in or out of the loop).
- The direction of flux is the same as the direction of the magnetic field. Determine if the flux through the loop is increasing or decreasing (because of change in area or change in B).

Choose the appropriate current in the loop that will oppose the change in flux.

- If the flux is into the paper and increasing the flux due to the induced current should be out of the paper.
- If the flux is into the paper and decreasing, the flux due to the induced current should be into the paper.
- If the flux is out of the paper and increasing, the flux due to the induced current should be into the paper.
- If the flux is out of the paper and decreasing, the flux due to the induced current should be out of the paper.

The above description is the physical interpretation of Lenz's law. We can determine the direction of the induced current mathematically by simply applying Lenz's law $\xi_{\text{ind}} = -\frac{d\Phi_B}{dt}$ with the appropriate conventions.

The right hand sign convention used is as follows.

- For counterclockwise current, e.m.f. is positive.
- For clockwise current, e.m.f. is negative.
- Magnetic flux out of the paper is positive.
- Magnetic flux into the plane of the paper is negative.
- The rate of change of an increasing positive flux is positive.
- The rate of change of a decreasing positive flux is negative.
- The rate of change of an increasing negative flux is negative.
- The rate of change of a decreasing negative flux is positive.

Let us consider some of the cases regarding application of Lenz's law.

- Suppose north pole of a bar magnet is moved towards a loop as shown in Fig. 8.7. Because of change in magnetic flux associated with the loop current is induced in it. Due to induced current, magnetic field is induced in such a way that it opposes the motion of the bar magnet. As the north pole is moving towards the loop, hence to oppose the motion of the bar magnet only the north pole will be induced on that face of the loop which faces the magnet. The induced current due to the change in $\vec{B}$ is clockwise, as seen from above the loop. The added field $\vec{B}_{\text{induced}}$ that it causes is downward, opposing the change in the upward field $\vec{B}$.

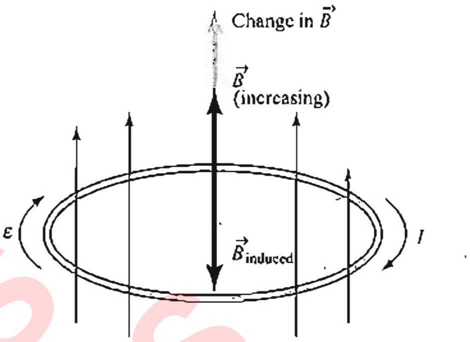


Fig. 8.7

- Consider Fig. 8.8. A rectangular loop $ABCD$ is being pulled out of the magnetic field directed into the plane of the paper. As the loop is dragged out of the field the flux associated with the loop which is directed into the plane of the paper decreases. The induced current will flow in the loop in the sense to oppose the decrease of this flux. For this to happen, magnetic field due to the induced current in the loop must be directed into the plane of the paper. Hence current in the loop must flow in the clockwise sense.

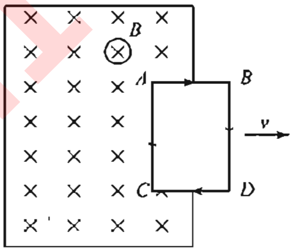


Fig. 8.8

Illustration 8.3 **A closed loop with the geometry shown in Fig. 8.9 is placed in a uniform magnetic field directed into the plane of the paper. If the magnetic field decreases with time, determine the direction of the induced e.m.f. in this loop.**

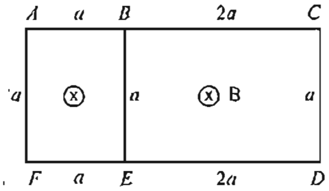


Fig. 8.9

Sol. There are two loops that are immersed in the magnetic field, namely, $ABEFA$ and $BCDEB$.

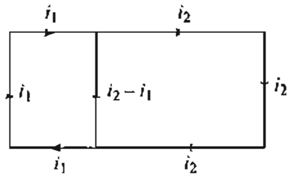


Fig. 8.10

Consider loop $ABEFA$. Magnetic flux is into the plane of the paper. For loop $BCDEB$ too, the magnetic flux is into the plane of the paper.

In both loops, the magnetic flux is decreasing with time.

Therefore the induced current in loop *ABEFA*, i_1, will be in a direction so as to induce a flux into the paper. The direction of i_1 is clockwise.

Likewise in loop *BCDEB* the current i_2 will be in a direction so as to induce a flux into the paper. The direction of i_2 is also clockwise. The final induced currents in all the arms of the loop are shown in Fig. 8.10.

According to Faraday's law, whenever there is a change in magnetic flux an induced e.m.f. is produced. It is wrong to interpret the law as follows.

When there is no change in the magnetic flux, no induced e.m.f. will be produced.

There are situations when induced e.m.f. is produced but there is no change in magnetic flux.

Note down the following points regarding the Faraday's law:

1. As we have seen, induced e.m.f. is produced only when there is a change in magnetic flux passing through a loop. The flux passing through the loop is given by $\phi = BA\cos\theta$. Thus, flux can be changed in several ways:

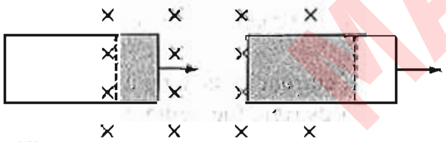


Fig. 8.11

- The magnitude of $\vec{B}$ can change with time. In the problems if magnetic field is given as a function of time, it implies that the magnetic field is changing. Thus, $B = B(t)$.
- The current produced by the magnetic field can change with time. So, the current can be given as a function of time. Hence,

$$i = i(t)$$

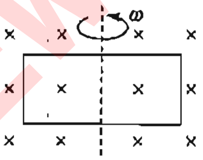


Fig. 8.12

- The area enclosed by the loop can change with time. This can be done by pulling a loop inside (or outside) a magnetic field. By doing so, the area enclosed by the loop (hatched area) can be changed.
- The angle θ between $\vec{B}$ and the normal to the loop can change with time. This can be done by rotating a loop in a magnetic field.
- Any combination of the above can occur.

2. When the magnetic field passing through a loop is changed an induced e.m.f. and hence an induced current is produced in the circuit. If R is the resistance of the circuit, then induced current is given by

$$i = \frac{e}{R} = \frac{1}{R}\left(\frac{-d\phi_B}{dt}\right)$$

Current starts flowing in the circuit, i.e., flow of charge takes place. Charge flown in the circuit in time dt will be given by

$$dq = i\,dt = \frac{1}{R}(-d\phi_B)$$

Thus, for a time interval Δt we can write

$$e = -\frac{\Delta\phi_B}{\Delta t},\; i = \frac{1}{R}\left(\frac{-\Delta\phi_B}{\Delta t}\right) \text{ and } \Delta q = \frac{1}{R}(-\Delta\phi_B)$$

From these equations we can see that e and i are inversely proportional to Δt while Δq is independent of Δt. It depends on the magnitude of change in flux, not the time taken by it.

Illustration 8.4 **A circular loop of radius a having n turns is kept in a horizontal plane. A uniform magnetic field B exists in a vertical direction as shown in Fig. 8.13. Find the e.m.f. induced in the loop if the loop is rotated with a uniform angular velocity ω about**

a. an axis passing through the center and perpendicular to the plane of the loop.

b. a diameter.

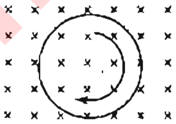


Fig. 8.13

Sol.

a. The e.m.f. induces when there is change of flux. As in this case (Fig. 8.13) there is no change of flux, hence no e.m.f. will be induced in the coil.

b. If the loop is rotated about a diameter there will be change of flux with time. In this case e.m.f. will be induced in the coil. The area of the loop is $A = \pi a^2$. If the normal of the loop makes an angle $\theta = 0$ with the magnetic field at $t = 0$, this angle will become $\theta = \omega t$ at time t. The flux of the magnetic field at this time is

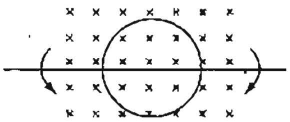


Fig. 8.14

$$\phi = nB\pi a^2\cos\theta = nB\pi a^2\cos\omega t$$

The induced e.m.f. is $\varepsilon = \frac{d\phi}{dt} = \pi n a^2 B\omega\sin\omega t$.

Illustration 8.5 Fig. 8.15(a) shows two circular rings of **radii a and b ($a > b$) joined together with wires of negligible resistance. Fig. 8.15(b) shows the pattern obtained by folding the small loop in the plane of the large loop.**

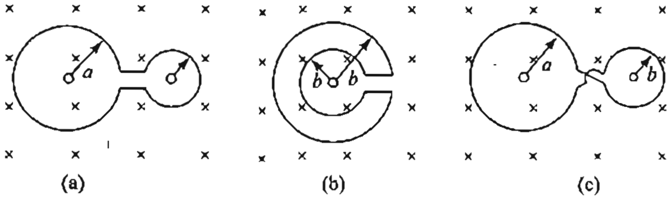


Fig. 8.15

The pattern shown in Fig. 8.15(c) is obtained by twisting the small loop of Fig. 8.15(a) through 180°.

All the three arrangements are placed in a uniform time varying magnetic field $\frac{dB}{dt} = k$, perpendicular to the plane of the loops. If the resistance per unit length of the wire is λ, then determine the induced current in each case.

Sol.

a. $$\phi_B = p\,(a^2 + b^2)B$$

$$\varepsilon = \frac{d\phi_B}{dt} = \pi(a^2 + b^2) = \frac{dB}{dt}\pi(a^2 + b^2)k$$

Induced current, $I = \frac{\varepsilon}{R} = \frac{\pi(a^2 + b^2)k}{\lambda[2\pi(a + b)]} = \frac{k(a^2 + b^2)}{2\lambda(a + b)}$

b. $$\phi_B = \pi a^2 B - \pi b^2 B = \pi B(a^2 - b^2)$$

$$\varepsilon = \frac{d\phi_B}{dt} = \frac{\pi dB}{dt} = (a^2 - b^2) = \pi k(a^2 - b^2)$$

Induced current, $I = \frac{\varepsilon}{R} = \frac{\pi k(a^2 - b^2)}{2\pi\lambda(a + b)} = \frac{k(a - b)}{2\lambda}$

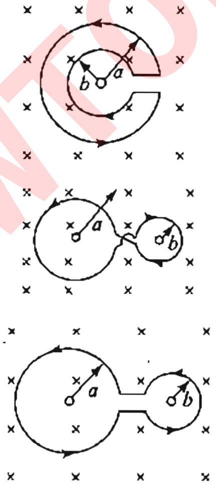


Fig. 8.16

c. $$\Phi_B = \pi a^2 B - \pi b^2 B = \pi B(a^2 - b^2)$$

$$\Rightarrow \quad \varepsilon = \frac{d\Phi_B}{dt} = \pi k\,(a^2 - b^2)$$

Induced current, $I = \frac{\varepsilon}{R} = \frac{k(a - b)}{2\lambda}$

Illustration 8.6 A conducting rod of length l slides at constant velocity v on two parallel conducting rails, placed in a uniform and constant magnetic field B perpendicular to the plane of the rails as shown in Fig. 8.17. A resistance R is connected between the two ends of the rail.

i. Identify the cause which produces change in magnetic flux.
ii. Identify the direction of current in the loop.
iii. Determine the e.m.f. induced in the loop.
iv. Compute the electric power dissipated in the resistor.
v. Calculate the mechanical power required to pull the rod at constant velocity.

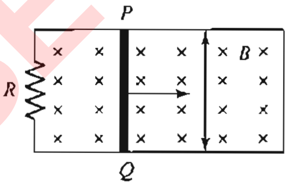


Fig. 8.17

Sol.

i. The change in area produces the change in magnetic flux, $\phi_B = Blx = Blv$.

ii. The direction of current in the loop is anticlockwise. As the rod moves towards right the number of crosses in the loop increases with time and to oppose the increasing number of crosses in the loop the current in the loop must be anticlockwise.

iii. According to Faraday's law, $|\varepsilon| = \frac{d\phi_B}{dt} = Blv$.

The magnitude of current is $I = \frac{\varepsilon}{R} = \frac{Blv}{R}$.

iv. The electric power dissipated in the resistor is

$$P_{elec} = I^2R = \frac{B^2l^2v}{R}$$

v. The mechanical power is $P_{mech} = F_{ext}v$.

The external force is equal and opposite to the Ampere's force:

$$F_{ext} = -F_{ampere}$$

The Ampere's force is given by

$$F_{ampere} = \int Id\vec{\ell} \times \vec{B} \text{ or } F_{ampere} = BI\ell$$

Now, $P_{mech} = (BI\ell)v$.

Substituting the value of I, we get $P_{mech} = \frac{B^2l^2v^2}{R}$.

Illustration 8.7 Space is divided by the line AD into two regions. Region I is field free and region II has a uniform magnetic field B directed into the plane of the paper. ACD is a semicircular conducting loop of radius r with centre at O, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity ω about an axis passing through O and the perpendicular to the plane of the paper. The effective resistance of the loop is R.

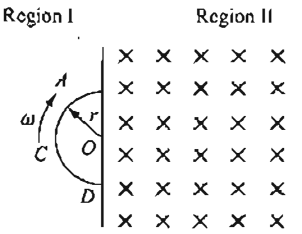


Fig. 8.18

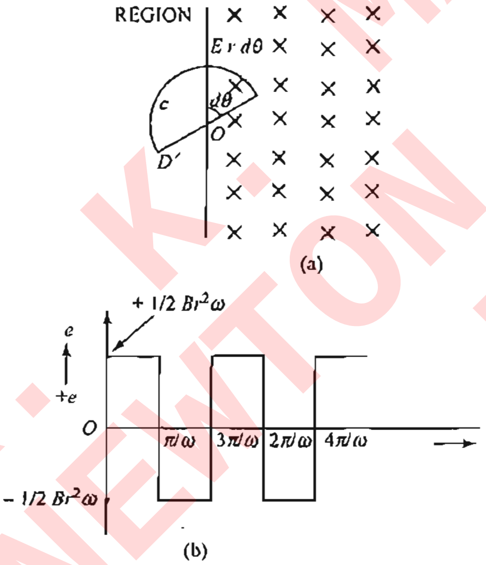
i. Obtain an expression for the magnitude of the induced current in the loop.

ii. Show the direction of the current when the loop is entering into region II.

iii. Plot a graph between the induced e.m.f. and the time of rotation for the two periods of rotation. (IIT-JEE, 1985)

Sol.

i. When the loop is rotated about an axis passing through the centre O and perpendicular to the plane of the paper, the angle between magnetic field vector $\vec{B}$ and area $\vec{A}$ is always 0°. When the loop is in region I, the magnetic flux linked with loop = BA cos 0 = 0 the (since $B = 0$ in region I).



(a)

(b)

Fig. 8.19

When the loop enters the magnetic field in region II, the magnetic flux linked with it is given by $\phi = BA$. Therefore, e.m.f. induced

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA) = -B\frac{dA}{dt} = B\frac{dA}{dt}$$

If $d\theta$ is the angle rotated by the loop in time dt, then dA is the area of triangle OEA'

$$= \frac{1}{2}r(rd\theta) = \frac{1}{2}r^2 d\theta$$

$$\therefore \quad e = B\frac{1}{2}r^2\frac{d^2\theta}{dt} = B\frac{1}{2}r^2\omega = \frac{1}{2}\omega$$

As resistance of the loop is R, the current induced is given by

$$i = \frac{e}{R} = \frac{1}{2}\frac{Br^2\omega}{R}$$

This is the required expression for current induced in the loop.

ii. According to Lenz's law the direction of current induced is to oppose the change in magnetic flux. So, when entering into region II the field produced by the current induced must be upward. For this, the current in the loop must be anticlockwise.

iii. When the loop enters the magnetic field the magnetic flux linked with it increases and the e.m.f. $e = \frac{1}{2}Br^2\omega\, l$ is induced in one direction. When the loop comes out of the field, the flux decreases and e.m.f. is induced in opposite sense. The graph for representing the e.m.f. induced versus time for two periods ($T = 2\pi/\omega$) is shown in Fig. 8.19(b).

Concept Application Exercise 8.1

1. Consider the hemispherical closed surface as shown in Fig. 8.20. If the hemisphere is in a uniform magnetic field that makes an angle θ with the vertical, calculate the magnetic flux

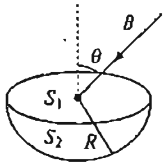


Fig. 8.20

a. through the flat surface S_1.
b. through the hemisphere surface S_2.

2. A cube of edge length ℓ = 2.50 cm is positioned as shown in Fig. 8.21. A uniform magnetic field given by $\vec{B} = (5.00\hat{i} + 4.00\hat{j} + 3.00\hat{k})$ T exists throughout the region.

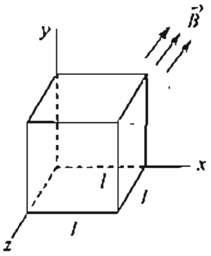


Fig. 8.21

a. Calculate the flux through the shaded face.
b. What is the total flux through the six faces?

3. A conducting ring is placed near a solenoid as shown in Fig. 8.22. Find the direction of the induced current in the ring

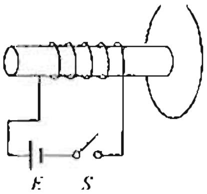


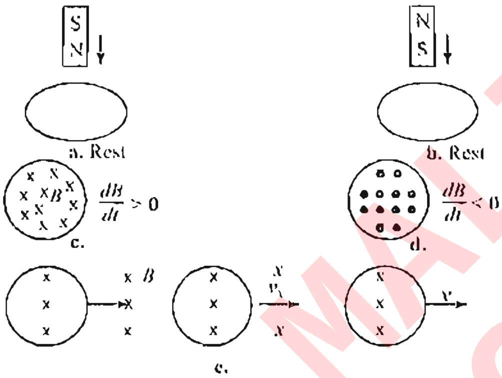
Fig. 8.22

a. at the instant the switch in the circuit containing the solenoid is closed.

b. after the switch has been closed for a long time.

c. at the instant the switch is opened.

4. Identify the direction of induced current as seen from the above in the following cases.



a. Rest b. Rest c. d. e.

Fig. 8.23

5. A circular loop of wire is in a region of spatially uniform magnetic field, as shown in Fig. 8.24. The magnetic field is directed into the plane of the paper. Determine the direction (clockwise or counterclockwise) of the induced current in the loop when

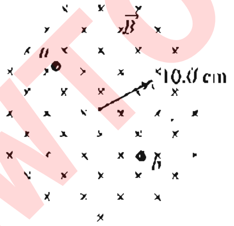


Fig. 8.24

a. B is increasing. b. B is decreasing.

c. B is constant with value B_0. Explain your reasoning.

6. Using Lenz's law, determine the direction of the current in resistor ab of Fig. 8.25 when

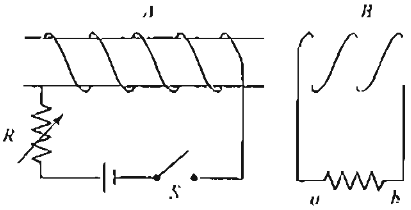


Fig. 8.25

a. switch S is opened after having been closed for several minutes.

b. coil B is brought closer to coil A with the switch closed.

c. the resistance of R is decreased while the switch remains closed.

7. A cardboard tube is wrapped with two windings of insulated wire wound in opposite directions, as shown in Fig. 8.26. Terminals a and b of winding A may be connected to a battery through a reversing switch. State whether the induced current in the resistor R is from left to right or from right to left in the following circumstances.

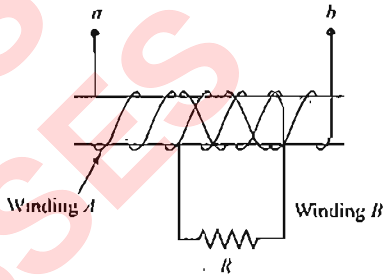


Fig. 8.26

a. The current in winding A is from a to b and is increasing.

b. The current in winding A is from b to a and is decreasing.

c. The current in winding A is from b to a and is increasing.

8. A small, circular ring is inside a larger loop that is connected to a battery and a switch, as shown in Fig. 8.27. Use Lenz's law to find the direction of the current induced in the small ring

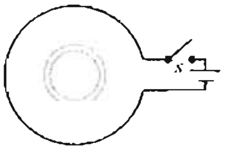


Fig. 8.27

a. just after switch S is closed;

b. after S has been closed for a long time;

c. just after S has been reopened after being closed for a long time.

9. a. Predict the polarity of the capacitor C as shown in Fig. 8.28 when S and N poles of two identical magnets approach the coil from opposite sides with equal velocity. The plane of the loop containing the capacitor is perpendicular to the plane of the paper.

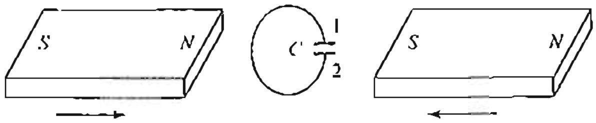


Fig. 8.28

b. A magnetic field perpendicular to the plane of a rectangular frame of wire is concentrated about O. If the field decreases, will there be any e.m.f. induced in loop 1? What about loop 2? Explain why.

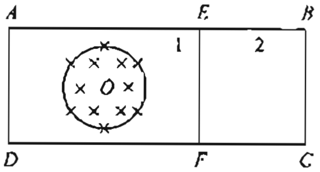


Fig. 8.29

10. A coil is placed in a constant magnetic field. The magnetic field is parallel to the plane of the coil as shown in Fig. 8.30. Find the e.m.f. induced in the coil.

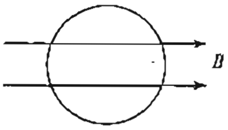


Fig. 8.30

11. Find the e.m.f. induced in the coil shown in Fig. 8.31. The magnetic field is perpendicular to the plane of the coil and is constant.

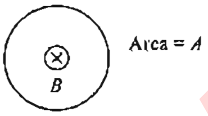


Fig. 8.31

12. Find the direction of induced current in the coil shown in Fig. 8.32. Magnetic field is perpendicular to the plane of the coil and it is increasing with time.

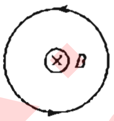


Fig. 8.32

13. Fig. 8.33 shows a coil placed in a decreasing magnetic field applied perpendicular to the plane of the coil. The magnetic field is decreasing at a rate of 10 T s^{-1}. Find out current in magnitude and direction.

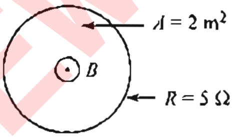


Fig. 8.33

14. Fig. 8.34 shows a coil placed in a magnetic field decreasing at a rate of 10 T s^{-1}. There is also a source of e.m.f. 30 V in the coil. Find the magnitude and direction of the current in the coil.

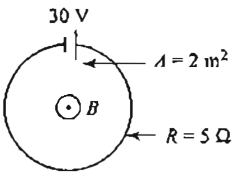


Fig. 8.34

15. A square loop of wire with resistance R is moved at a constant speed v across a uniform magnetic field confined to a square region whose sides are twice the length of those of the square loop (as shown in Fig. 8.35).

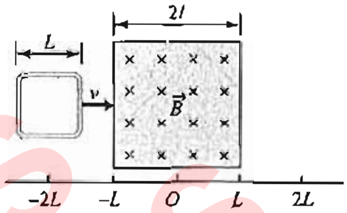


Fig. 8.35

a. Graph the external force F needed to move the loop at a constant speed as a function of the coordinate x from $x = -2L$ to $x = +2L$. (The coordinate x is measured from the centre of the magnetic field region to the center of the loop. It is negative when the center of the loop is to the left of the center of the magnetic field region. Take positive force to be to the right.)

b. Graph the induced current in the loop as a function of x. Take counterclockwise currents to be positive.

16. Two magnetic fields exist in the two regions as shown in Fig. 8.36. A loop *abcd* of 40 × 10 cm is placed in the field. The resistance per unit length of the loop is $r = 2\ \Omega\ cm^{-1}$. All of a sudden the loop is given a velocity $v_0 = 20\ cm\ s^{-1}$ towards right. What is the potential difference $V_c - V_b$?

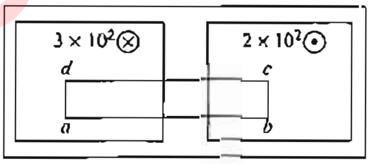


Fig. 8.36

17. The wire shown in Fig. 8.37 is bent in the shape of a tent, with $\theta = 60.0°$ and $L = 1.50$ m, and is placed in a uniform magnetic field of magnitude 0.300 T perpendicular to the tabletop. The wire is rigid but hinged at points a and b. If the "tent" is flattened out on the table in 0.100 s, what is the average induced e.m.f. in the wire during this time?

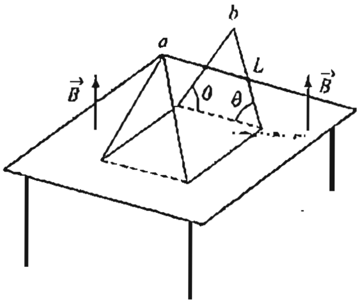


Fig. 8.37

18. The plane of a square loop of wire with edge length $a = 0.200$ m is perpendicular to the Earth's magnetic field at a point where $B = 15.0\,\mu\text{T}$, as shown in Fig. 8.38. The total resistance of the loop and the wires connecting it to a sensitive ammeter is 0.500 Ω. If the loop is suddenly collapsed by horizontal forces as shown, what is the total charge passing through the ammeter?

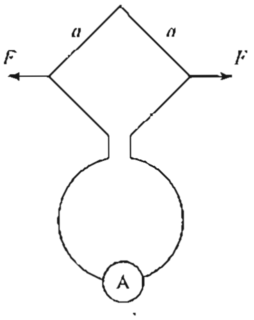
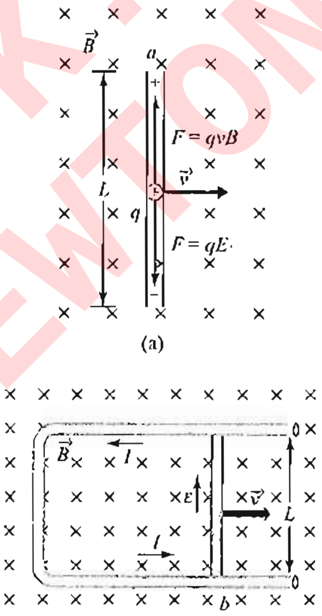


Fig. 8.38

MOTIONAL ELECTROMOTIVE FORCE

Fig. 8.39 shows a moving rod in a uniform magnetic field $\vec{B}$ directed into the page. If a straight conductor is moving in a magnetic field electrons inside it experience a force $F = e\vec{v} \times \vec{B}$ and accumulate at end of the conductor. Thus, an electric field is established across its ends. The $e\vec{v} \times \vec{B}$ is balanced by eE in the opposite direction, at equilibrium.



(a)

(b)

Fig. 8.39

$$\therefore \qquad e\vec{v} \times \vec{B} + e\vec{E} = 0$$

$$\varepsilon = -\int \vec{E} \cdot d\bar{l} = \int \left(\bar{v} \times \vec{B}\right) \cdot d\bar{l} = \int \vec{B} \cdot (d\bar{l} \times \bar{v}) \qquad \text{(i)}$$

As $d\bar{l} \times \bar{v}$ is the area swept per unit time by length $d\bar{l}$ and hence $B(d\bar{l} \times \bar{v})$ is the flux of induction through the area. Therefore, the motional e.m.f. is equal to the flux of induction cut by the conductor per unit time. The magnitude of the potential difference $V_{ab} = V_a - V_b$ is equal to the electric field magnitude E multiplied by the length L of the rod. From the above discussion, $E = VB$. So, $V_{ab} = EL = vBL$ with point a at higher potential than point b.

The e.m.f. associated with the moving rod as shown in the figure is analogous to that of a battery with its positive terminal at a and its negative terminal at b, although the origins of the two e.m.f.'s are quite different.

A motional e.m.f. is also present in the isolated moving rod as shown in the figure, in the same way that a battery has an e.m.f. even when it is not part of a circuit. The direction of the induced e.m.f. as shown in the figure can be deduced by using Lenz's law, even if the conductor does not form a complete circuit. In this case, we can mentally complete the circuit between the ends of the conductor and use Lenz's law to determine the direction of the current.

This expression looks very different from our original statement of Faraday's law, which states that $\varepsilon = -d\Phi_B / dt$. In fact, the two statements are equivalent. It can be shown that the rate of change of magnetic flux through a moving conducting loop is always given by the negative of the expression in the figure. Thus this equation gives us an alternative formulation of Faraday's law. This alternative is often more convenient than the original one in problems with moving conductors. But when we have stationary conductors in changing magnetic fields, the above equation can be used; in this case, $\varepsilon = -d\Phi_B / dt$ is the only correct way to express Faraday's law.

If a straight conductor is moving with constant velocity in a uniform magnetic field as shown in Fig. 8.39(a) then equation (i) can be written as

$$\varepsilon = \vec{B} \cdot (\vec{\ell} \times \vec{v}) \qquad \text{(ii)}$$

Equation (ii) is the scalar triple product of the vector that can be written using the property of scalar triple product.

$$e = \vec{B} \cdot (\vec{\ell} \times \vec{v}) = (\vec{B} \times \vec{\ell}) \cdot \vec{v}$$

$$= \vec{v} \cdot (\vec{B} \times \vec{\ell}) = (\vec{v} \times \vec{B}) \cdot \vec{\ell} = \vec{\ell} \cdot (\vec{v} \times \vec{B}) \qquad \text{(iii)}$$

It is clear from the above equations that if any of the vectors becomes parallel to another, induced motional e.m.f. will be zero.

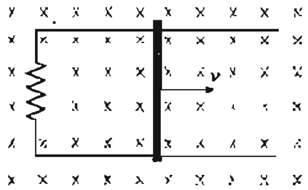
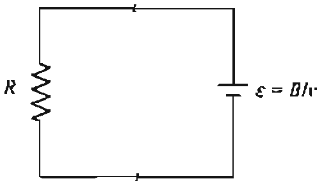
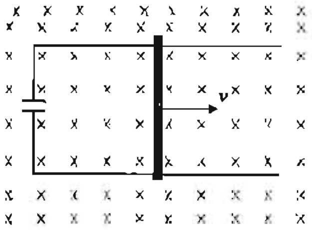
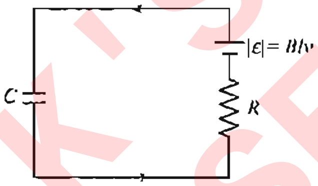
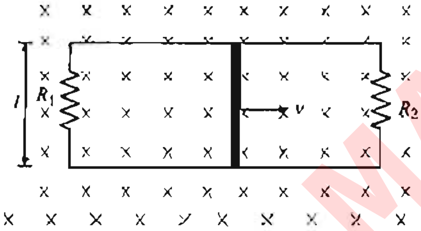
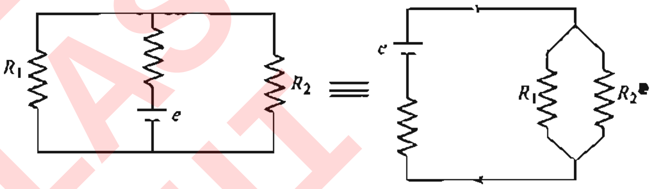
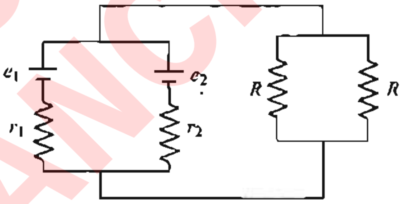
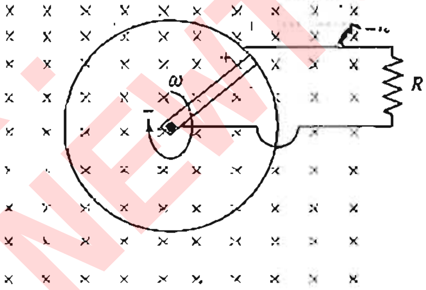
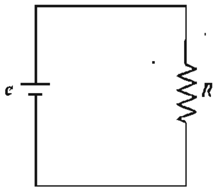
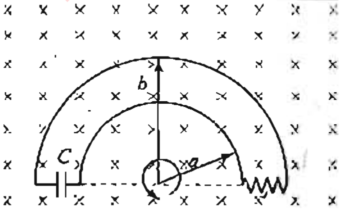
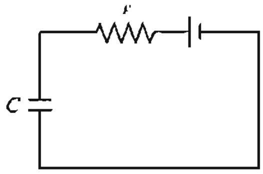
Situation	Equivalent circuit
	
	
	
	
	
 **Fig. 10.40**	 **Fig. 10.41**

Illustration 8.8 A circular loop of radius r moves with a constant velocity v in a region with uniform magnetic field B. Calculate the potential difference between two points (A, B), (C, D) and (E, F) located on the loop.

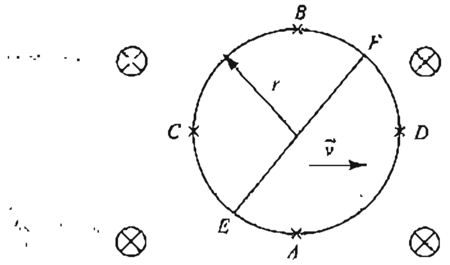


Fig. 8.42

Sol. This illustration highlights a very common misconception in students. We know that when a closed loop translates in a uniform magnetic field, the net e.m.f. induced in the loop is zero. Students jump to the conclusion that the potential difference across any two points on the loop should also be zero. This conclusion is wrong.

Consider the case of two points A and B shown in Fig. 8.43. They are on a diameter that is oriented perpendicular to the direction of motion of the loop. Since net induced e.m.f. is zero there will be no induced currents in the loop. We can therefore cut the loop along the diameter AB and divide it into two semicircular loops without affecting the physics of the problem. Each of these loops can in turn be replaced by straight conductors of length $2r$. The induced e.m.f. across each of these conductors is Brv and the potential difference across AB is Brv. Note that when we travel through the whole circuit, the net induced e.m.f. is still zero as the two e.m.f.'s cancel each other.

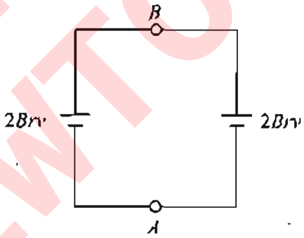


Fig. 8.43

In a similar manner we can show that the potential difference between C and D is zero and the potential difference between E and F is $2Brv \sin \theta$ where θ is the angle between line EF and the direction of motion.

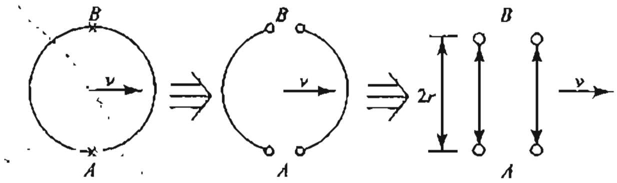


Fig. 8.44

Illustration 8.9 An angle $\angle AOB$ made of a conducting wire moves along its bisector through a magnetic field B as suggested by Fig. 8.45. Find the e.m.f. induced between the two free ends if the magnetic field is perpendicular to the plane at the angle.

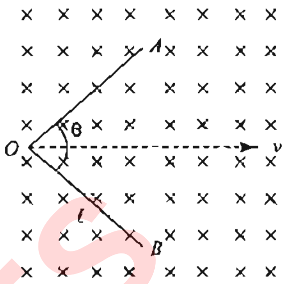


Fig. 8.45

Sol. The rod OA is equivalent to a battery of e.m.f. $vB\ell \sin \theta/2$. The positive charges of OA shift towards 'A' due to the force. The positive terminal of the battery appears towards A. Similarly, the rod OB is equivalent to a battery of e.m.f. $vB\ell \sin \theta/2$ with the positive terminal towards O. The equivalent circuit is shown in Fig. 8.46. Clearly, the e.m.f. between the point A and B is $2B\ell v \sin \theta/2$.

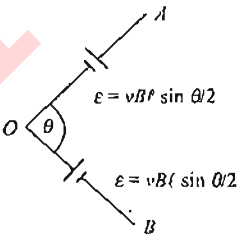


Fig. 8.46

Illustration 8.10 A conducting rod of length ℓ slides at constant velocity v on two parallel conducting rails, placed in a uniform and constant magnetic field B perpendicular to the plane of the rails as shown in Fig. 8.47. A resistance R is connected between the two ends of the rail.

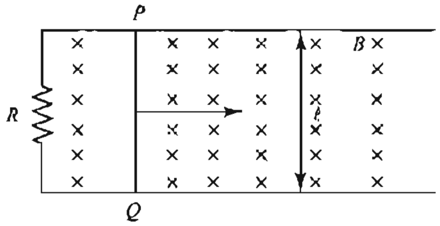


Fig. 8.47

i. Identify the cause which produces change in magnetic flux.
ii. Identify the direction of current in the loop.
iii. Determine the e.m.f. induced in the loop.
iv. Compute the electric power dissipated in the resistor.
v. Calculate the mechanical power required to pull the rod at a constant velocity.

Sol.

i. The change in area produces the change in magnetic flux $\phi_B = B\ell x = B\ell v$.

ii. The direction of current in the loop is anticlockwise. As the rod moves towards right the number of crosses in the loop increases with time and to oppose the increasing number of crosses in the loop the current in the loop must be anticlockwise.

iii. According to Faraday's law, $|\varepsilon| = \dfrac{d\phi_B}{dt} = B\ell v$.

The magnitude of current is $I = \dfrac{\varepsilon}{R} = \dfrac{B\ell v}{R}$.

iv. The electric power dissipated in the resistor is $P_{\text{elec}} = I^2R = \dfrac{B^2\ell^2 v}{R}$.

v. The mechanical power is $P_{\text{mech}} = F_{\text{ext}}v$.

The external force is equal and opposite to the Ampere's force $F_{\text{ext}} = -F_{\text{ampere}}$.

The Ampere's force is given by

$$F_{\text{ampere}} = \int I d\vec{\ell} \times \vec{B}$$

or $F_{\text{ampere}} = BI\ell$

Now, $P_{\text{mech}} = (BI\ell)v$.

Substituting the value of I, we get $P_{\text{mech}} = \dfrac{B^2\ell^2 v^2}{R}$.

Illustration 8.11 **A rod slides on a U-shaped conductor. If the resistance of the circuit at any instant is R, find the force needed to pull the wire with a constant speed v. What external power is needed to pull the rod?**

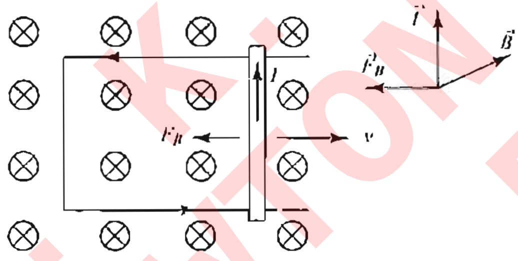


Fig. 8.48

Show that the power input equals that dissipated in the resistance at any moment.

Sol. When the rod moves to the right at a speed v, a clockwise current is set up in the circuit. From Lenz's law we can see that the induced current must oppose the increase of flux due to motion of the rod.

Thus the induced current is anticlockwise so as to produce a magnetic field out of the page (right hand thumb rule). The force experienced by a current carrying wire in a magnetic field is $I\vec{\ell} \times \vec{B}$ for a constant B. From Fleming's left hand rule this force resists out effort to move the rod to the right.

The induced current is $I = \dfrac{B\ell v}{R}$.

The magnetic force is $F_B = I\ell B = \dfrac{B^2\ell^2 v}{R}$.

In order to move the rod at a constant speed, we must exert a force equal in magnitude to F_B and directed towards right.

$$|F_{\text{external}}| = |F_B| = \frac{B^2\ell^2 v}{R}$$

The external power needed to move the rod for constant R is

$$P = Fv = \frac{B^2\ell^2 v^2}{R}$$

The power dissipated in resistor is

$$P = I^2R$$

or $$P = \left(\frac{B\ell v}{R}\right)^2 R = \frac{B^2\ell^2 v^2}{R}$$

Thus the power input equals the power lost as heat in the resistor.

Illustration 8.12 **Two long parallel horizontal rails, a distance d apart and each having a resistance λ per unit length, are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide along the rails without friction (see Fig. 8.49). There is a uniform magnetic field of induction B normal to the plane of the paper and directed into the paper. A variable force F is applied to the rod MN such that, as the rod moves, a constant current flows through R.**

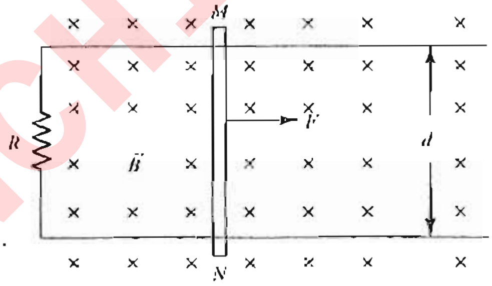


Fig. 8.49

i. Find the velocity of the rod and the applied force F as functions of the distance x of the rod from R.

ii. What fraction of the work done per second by F is converted into heat? **(IIT-JEE, 1988)**

Sol.

i. If the rod has instantaneous velocity v at a distance x from R, the induced e.m.f. = Bvd.

Instantaneous resistance of current = $R + 2\lambda x$

$\therefore$ Induced current $i = \dfrac{e}{R+2\lambda x} = \dfrac{Bvd}{R+2\lambda x}$ = constant

$\therefore$ Velocity $v = \dfrac{(R+2\lambda x)i}{Bd}$

If F is the force acting on rod,

$$F - Bid = m\frac{dv}{dt}$$

or $$F - Bid = m\frac{dv}{dx}\frac{dx}{dt} = mv\frac{d}{dt}\left\{\frac{(R+2\lambda x)i}{bd}\right\} = mv\frac{2\lambda i}{Bd}$$

$$\therefore \quad F = Bid + m\frac{(R+2\lambda x)}{B^2d^2}i^2$$

ii. Work down per second = Fv
Heat produced per second = $i^2 (R + 2\lambda x)$

$$\text{Required ratio} = \frac{i^2(R+2\lambda x)}{Fv} = \frac{i^2(R+2\lambda x)Bd}{F(R+2\lambda x)i} = \frac{iBd}{F}$$

$$= \frac{Bid}{Bid + \dfrac{2m\lambda(R+2\lambda x)i^2}{B^2d^2}}$$

$$= \frac{1}{1+\dfrac{2m\lambda(R+2\lambda x)i^2}{B^2d^2}} = \frac{1}{1+\dfrac{2m\lambda(R+2\lambda x)i}{B^3d^3}}$$

Rotation of a Conducting Rod in Constant Magnetic Field

Case I: A conducting rod of length ℓ attached to a rod of insulating material of length L is rotated with constant angular speed in a plane normal to the uniform magnetic field B, as shown in Fig. 8.50(a). The e.m.f. will be induced across the ends of the conducting rod. Consider a small elemental length dx of the rod at a distance x from the end of the rod as shown in the diagram.

The e.m.f. across the elemental rod will be $\varepsilon = \int(d\vec{x} \times \vec{v}) \cdot \vec{B}$

$$|d\vec{x}\times\vec{v}| = wr\,dx\sin\theta$$

$$(d\vec{x}\times\vec{v})\cdot\vec{B} = B\omega(r\sin\theta)dx = B\omega x dx$$

$$\therefore \qquad \varepsilon = \omega B\int x dx = \frac{1}{2}\omega B\ell^2$$

The rod AB may be replaced by a battery of e.m.f. $\frac{1}{2}B\omega\ell^2$ with positive terminal towards A.

Note:

> *A becomes the positive terminal as positive charges accumulate towards A because $\vec{F} = q\vec{v}\times\vec{B}$ (or apply Fleming's left hand rule).*

Case II: If instead of the rod it had been a disc [Fig. 8.50(b)], the potential difference between the centre of the disc and a point on the rim will be same. As the disc is equivalent to a large number of rods each having its one end at the centre while the other at the rim (like spokes in a wheel) and from electrical point of view the situation is equivalent to that of a number of sources of equal e.m.f. in parallel.

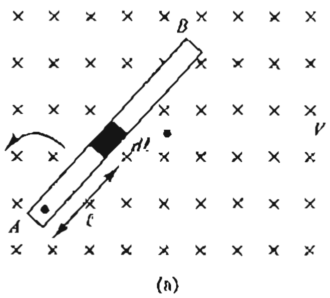
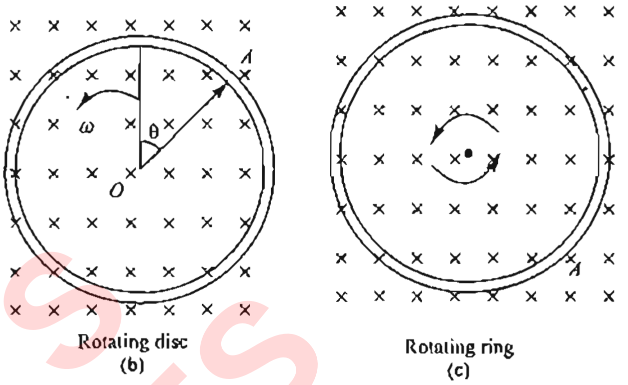


Fig. 8.50 (*Contd.*)



Rotating disc
(b)

Rotating ring
(c)

Fig. 8.50

The e.m.f. induced between centre of the disc and circumference will be

$$\varepsilon = \frac{1}{2}\omega BR^2$$

where R is the radius of the disc.

Case III: If instead of rod or disc, it had been a ring which is rotated about its own axis and the field had been perpendicular to its plane as shown in Fig. 8.50(c).

$$\phi = B\pi r^2 = \text{const, so } e = \frac{d\phi}{dt} = 0$$

Illustration 8.13 **A metal rod 1.5 m long rotates about its one end in a vertical plane at right angles to the magnetic meridian. If the frequency of rotation is 20 rev s^{-1}, find the e.m.f. induced between the ends of the rod (B_H = 0.32 G).**

Sol. When the rod rotates in a vertical plane perpendicular to the magnetic meridian, it will cut horizontal component of earth's magnetic field so that

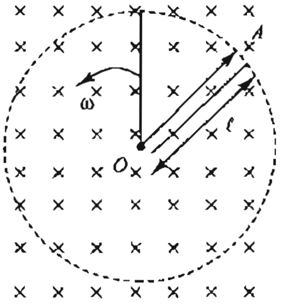


Fig. 8.51

$$e = \frac{1}{2}B_H\ell^2\omega = \pi\ell^2 fB_H \qquad (\text{as } \omega = 2\pi f$$

Substituting the given data, $e = \pi \times (1.5)^2 \times 20 \times 0.32 \times 10^{-}$
$= 4.5$ mV.

Illustration 8.14 **A wire is in the form of a semicircle o radius r. One end is attached to an axis about which is rotate with an angular speed ω. The axis is normal to the plane o the semicircle. The wire is immersed in a uniform magneti field B parallel to the axis. Find the induced e.m.f. betwee points O and P of the semicircle.**

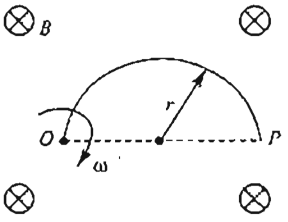


Fig. 8.52

Sol. It is given that the magnetic field is uniform.

Join the end points O and P and replace the semicircle by a straight rod of length $2r$.

We now have a straight rod rotating in a uniform magnetic field in a plane perpendicular to the magnetic field.

Therefore, the induced e.m.f. between O and P will be

$$\xi_{\text{ind}} = \frac{1}{2}B\omega(2r)^2 = B\omega r^2$$

From the right hand rule we see that electrons will accumulate at end O. Therefore, end P is at a higher potential than O.

Illustration 8.15 **A metal disc of radius R = 25 cm rotates with a constant angular velocity ω = 130 rad s^{-1} about its axis. Find the potential difference between the centre and rim of the disc if**

a. the external magnetic field is absent,

b. the external uniform magnetic field B = 5.0 mT is directed perpendicular to the disc.

Sol.

a. Centripetal force required for circular motion of electron is generated by a radial electric field caused by the redistribution of the electrons in the disc.

$$F = eE = mr\omega^2 \quad \Rightarrow \quad E = \frac{mr\omega^2}{e}$$

From $dV = -E\,dr$, we have

$$dV = -\frac{m\omega^2}{e}r\,dr$$

$$\Rightarrow \quad \int_{V_1}^{V_2} dV = -\frac{m\omega^2}{e}\int_0^R r\,dr$$

$$V_1 - V_2 = \frac{m\omega^2R^2}{2e} = \frac{(9.1\times10^{-31})(130)^2(0.25)^2}{(2)(1.6\times10^{-19})}$$

$$= 3.0\times10^{-9}\text{ V} \doteq 3.0\text{ nV}$$

$V_1 > V_2$, i.e., potential at centre is more than the potential at edge.

b. A disc may be assumed to be made up of a large number of radial, conducting, differential elements rotating with angular velocity ω about the centre of the disc O. Thus,

$$V_{\text{centre}} - V_{\text{edge}} = \frac{1}{2}BR^2\omega$$

Now, $V_{\text{centre}} > V_{\text{edge}}$ for anticlockwise rotation and $V_{\text{edge}} > V_{\text{centre}}$ for clockwise rotation.

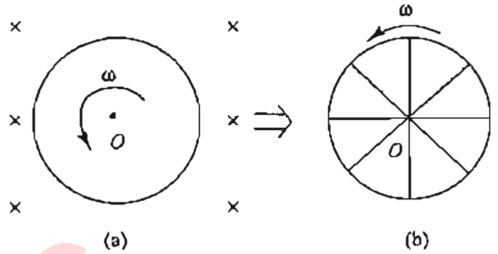


Fig. 8.53

Substituting the values, we have

$$V_{\text{centre}} - V_{\text{edge}} = \frac{1}{2}\times5.0\times10^{-3}\times0.25\times0.25\times130$$

$$= 0.02\text{ V} = 20\text{ mV}$$

Motional e.m.f. when the Magnetic Field is Non-uniform

In some of the cases motion of a conductor may be in a non-uniform magnetic field. Take the following steps while calculating motional e.m.f.

Step 1: Determine the magnetic field at all points on the rod.

Step 2: Consider a small element at some distance from one end of the rod.

Step 3: Assuming B to be uniform over this element, calculate the potential difference across this element using the procedures outlined earlier.

Step 4: Integrate over the entrie rod to calculate the total induced e.m.f.

Let us learn to calculate the induced e.m.f. through the illustrations given below.

Illustration 8.16 **A copper rod of length 0.19 m is moving with uniform velocity 10 m s^{-1} parallel to a long straight wire carrying a current of 5.0 A. The rod is perpendicular to the wire with its ends at distances 0.01 and 0.2 m from it. Calculate the e.m.f. induced in the rod.**

Sol. As shown in Fig. 8.54 consider an element of length dy at a distance y from the wire, then at this position of the element, the magnetic field due to the current-carrying wire PQ will be

$B = \dfrac{\mu_0}{4\pi}\dfrac{2I}{y}$ into the plane of the paper.

So, the e.m.f. induced in the element $d\varepsilon = Bv\,dy = \dfrac{\mu_0}{4\pi}\dfrac{2I}{y}v\,dy$

and hence the e.m.f. induced across the ends of the rod due to its motion in the field of the wire,

$$\varepsilon = \int_a^b d\varepsilon = \frac{\mu_0}{4\pi}2Iv\int_a^b\frac{dy}{y},\text{ i.e., }\varepsilon = \frac{\mu_0}{4\pi}2Iv\log_e\left(\frac{b}{a}\right)$$

Substituting the given data with $b = (a + \ell)$,

$$\varepsilon = 10^{-7}\times2\times5\times10\log_e\frac{0.20}{0.01} = 10^{-5}\times\log_e 20$$

$$= 10^{-5}\times2.3036\times1.3010 \simeq 30\ \mu\text{V}$$

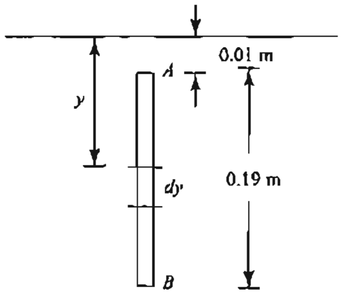


Fig. 8.54

Note:

> *If the rod is moved at a constant velocity with its length parallel to the wire as shown in Fig. 8.54, the e.m.f. in the rod will depend on its position.*

$$\varepsilon = Bv\ell = \frac{\mu_0}{4\pi}\frac{2Iv}{y}\ell = 10^{-7} \times \frac{2\times5\times10}{y} \times 0.01$$

$$= \frac{10^{-7}}{y}\text{ V}$$

Further, in accordance with Lenz's law (or Fleming's right hand rule) the direction of the induced current in the rod in both the cases is from B to A with A being at higher potential.

Illustration 8.17 **An infinite wire carries a current I. An 'S'- shaped conducting rod of two semicircles each of radius r is placed at an angle θ to the wire. The centre of the conductor is at a distance d from the wire. If the rod translates parallel to the wire with a velocity v as shown in Fig. 8.55, calculate the e.m.f. induced across the ends OB of the rod.**

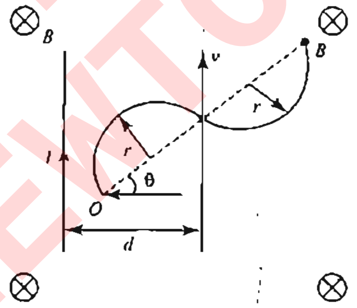


Fig. 8.55

Sol. Join the end point O and B and replace the two semicircles by a straight rod of length $4r$ [Fig. 8.56 (a)].

The effective rod is not perpendicular to the direction of motion. Projecting the rod onto a plane perpendicular to its direction of motion we find the effective length of the conductors is $4r \cos \theta$ [Fig. 8.56(b)].

We now have a rod of effective length $4r \sin \theta$ translating in a non-uniform magnetic field. Consider a small element of the wire of length dr located at a distance r from the wire. The magnetic field at this element is $B = \frac{\mu_0 I}{2\pi r}$ [Fig. 8.56(c)].

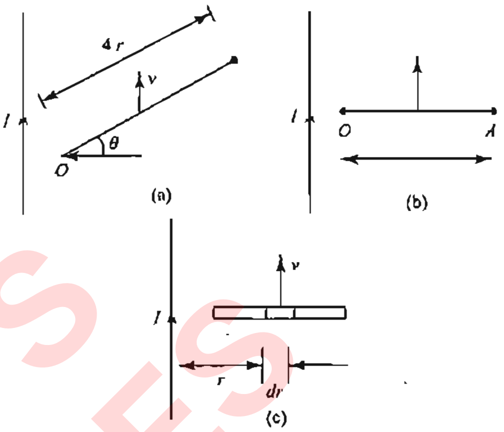


Fig. 8.56

The potential difference across this element is $dV = \frac{\mu_0 I}{2\pi r} v\, dr$.

The potential difference across the ends of the rod can be calculated by integrating over the whole end. Therefore,

$$V = \int dV = \int_{d-2r\cos\theta}^{d+2r\cos\theta} \frac{\mu_0 I}{2\pi r} v\, dr$$

or $$V = \frac{\mu_0 I}{2\pi r}\ln\left[\frac{d+2r\cos\theta}{d-2r\cos\theta}\right]$$

From the right hand rule we see that electrons will accumulate at end B. Therefore, end O is at a higher potential than B.

Concept Application Exercise 8.2

1. Fig. 8.57 shows a long current-carrying wire and two rectangular loops moving with velocity v. Find the direction of current in each loop.

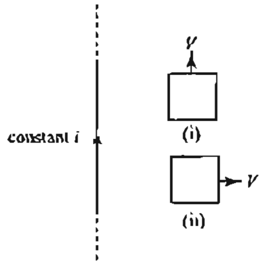
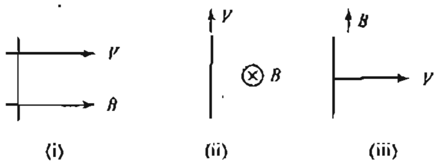


Fig. 8.57

2. A rod of length l is moving with velocity v in magnetic field B as seen in the diagram. Find the e.m.f. induced in all three cases.



3. Fig. 8.58 shows a closed coil $ABCA$ moving in a uniform magnetic field B with a velocity v.
   Find (i) e.m.f. induced in the coil.
   (ii) e.m.f. induced in curve part $\overset{\frown}{ACB}$ and straight part AB as $\vec{l} \times \vec{B}$

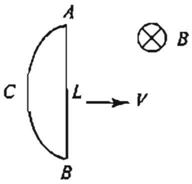


Fig. 8.58

4. Fig. 8.59 shows an irregular shaped wire *AB* moving with velocity *v*, as shown. Find the e.m.f. induced in the wire.

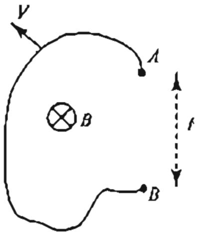


Fig. 8.59

5. A circular coil of radius *R* is moving in a magnetic field *B* with a velocity *v* as shown in Fig. 8.60. Find the e.m.f. across the diametrically opposite points *A* and *B*.

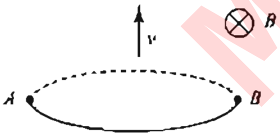


Fig. 8.60

6. Find the e.m.f. across the points *P* and *Q* which are diametrically opposite points of a semicircular closed loop moving in a magnetic field as shown in Fig. 8.61. Also draw the electrical equivalent circuit of each branch.

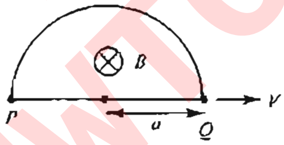


Fig. 8.61

7. Find the e.m.f. across the points *P* and *Q* which are diametrically opposite points of a semicircular closed loop moving in a magnetic field as shown in Fig. 8.62. Also draw the electrical equivalence of each branch.

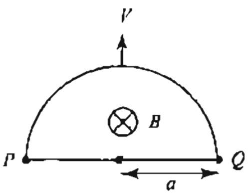


Fig. 8.62

8. Fig. 8.63 shows a rectangular loop moving in a uniform magnetic field. Show the electrical equivalence of each branch.

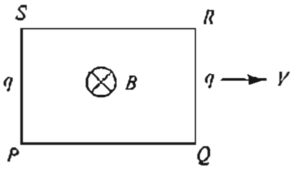


Fig. 8.63

9. Fig. 8.64 shows a rod of length ℓ and resistance r moving on two rails shorted by a resistance *R*. A uniform magnetic field *B* is present normal to the plane of rod and rails. Show the electrical equivalence of each branch.

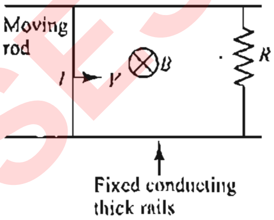


Fig. 8.64

10. A rod of length ℓ is kept parallel to a long wire carrying constant current *i*. It is moving away from the wire with a velocity *v*. Find the e.m.f. induced in the wire when its distance from the long wire is *x*.

11. A rectangular loop, as shown in Fig. 8.65, moves away from an infinitely long wire carrying a current *i*. Find the e.m.f. induced in the rectangular loop.

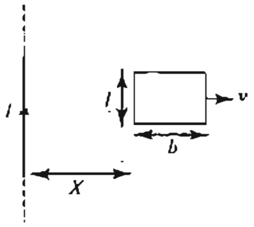


Fig. 8.65

12. A rod of length ℓ is placed perpendicular to a long wire carrying current *i*. The rod is moved parallel to the wire with a velocity *v*. Find the e.m.f. induced in the rod, if its nearest end is at a distance '*a*' from the wire.

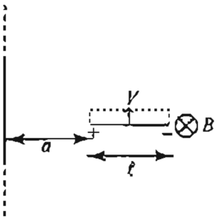


Fig. 8.66

13. A rectangular loop is moving parallel to a long wire carrying current *i* with a velocity *v*. Find the e.m.f. induced in the loop, if its nearest end is at a distance '*a*' from the wire. Draw equivalent electrical diagram.

14. A rod *PQ* of length 2ℓ is rotating about one end *P* in a uniform magnetic field *B* which is perpendicular to the

plane of rotation of the rod. Point M is the mid-point of the rod. Find the induced e.m.f. between M and Q if that between P and Q is 100 V.

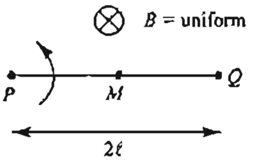


Fig. 8.67

15. A rod PQ of length 2ℓ is rotating about its mid-point C in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. Find the induced e.m.f. between PQ and PC. Draw the circuit diagram of parts PC and CQ.

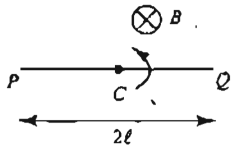


Fig. 8.68

16. A rod of length L and resistance r rotates about one end as shown in Fig. 8.69. Its other end touches a conducting ring of negligible resistance. A resistance R is connected between the centre and periphery. Draw the electrical equivalence and find the current in resistance R. There is a uniform magnetic field B directed as shown in Fig. 8.69.

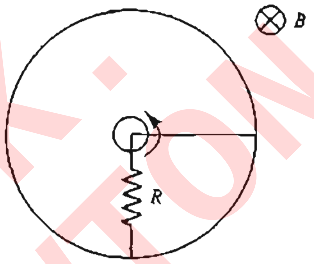


Fig. 8.69

17. Solve problem 16 if the length of rod is $2L$ and resistance $2r$ and it is rotating about its centre. Both ends of the rod now touch the conducting ring.

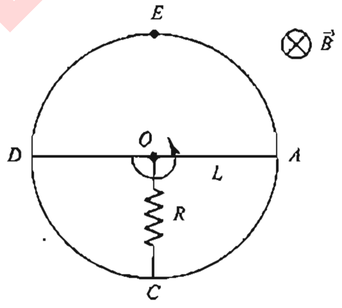


Fig. 8.70

18. A rod of length ℓ is rotating with an angular speed ω about one of its ends which is at a distance 'a' from an infinitely long wire carrying current i. Find the e.m.f. induced in the rod at the instant shown in Fig. 8.71.

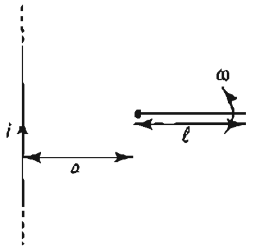


Fig. 8.71

19. A rod of length ℓ is rotating with an angular speed ω about one of its ends which is at a distance 'a' from an infinitely long wire carrying current i. Find the e.m.f. induced in the rod at the instant shown in Fig. 8.72.

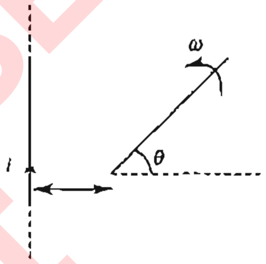


Fig. 8.72

20. A rod PQ of mass m and resistance r is moving on two fixed, resistanceless, smooth conducting rails (closed on both sides by resistances R_1 and R_2). Find the current in the rod at the instant its velocity is v.

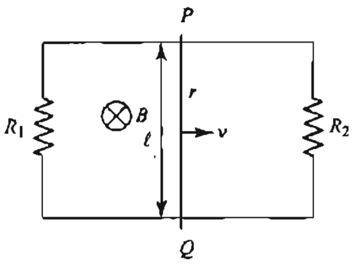


Fig. 8.73

21. A rod PQ of length ℓ is rotating about end P, with an angular velocity ω. Due to centrifugal forces the free electrons in the rod move towards the end Q and an e.m.f. is created. Find the induced e.m.f.

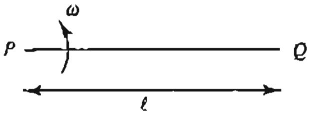


Fig. 8.74

22. A ring rotates with angular velocity ω about an axis perpendicular to the plane of the ring passing through the center of the ring. A constant magnetic field B exists parallel to the axis. Find the e.m.f. induced in the ring. Flux passing through the ring $\phi = BA$ is constant here, therefore e.m.f. induced in the coil is zero. Every point of this ring is at the same potential, by symmetry.

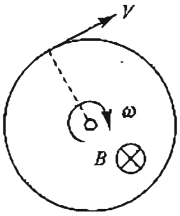


Fig. 8.75

23. A ring rotates with angular velocity ω about an axis in the plane of the ring which passes through the center of the ring. A constant magnetic field B exists perpendicular to the plane of the ring. Find the e.m.f. induced in the ring as a function of time.

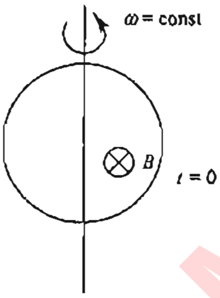


Fig. 8.76

24. In Fig. 8.77, a conducting rod of length $L = 30.0$ cm moves in a magnetic field $\vec{B}$ of magnitude 0.450 T directed into the plane of the figure. The rod moves with speed $v = 5.00\ \text{ms}^{-1}$ in the direction shown.

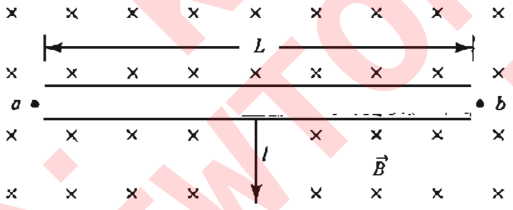


Fig. 8.77

a. What is motional e.m.f. induced in the rod?

b. What is the potential difference between the ends of the rod?

c. Which point, a or b, is at higher potential?

d. When the charges in the rod are in equilibrium, what are the magnitude and direction of the electric field within the rod?

e. When the charges in the rod are in equilibrium, which point, a or b, has an excess of positive charge?

25. The cube shown in Fig. 8.78, 50.0 cm on a side, is in a uniform magnetic field of 0.120 T, directed along the positive y-axis. Wires A, C and D move in the directions indicated, each with a speed of $0.350\ \text{ms}^{-1}$. (Wire A moves parallel to the xy plane, C moves at an angle of 45.0° below the xy plane, and D moves parallel to the xz plane.) What is the potential difference between the ends of each wire?

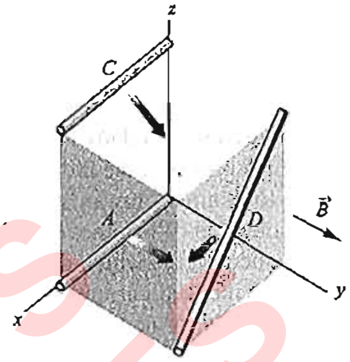


Fig. 8.78

26. A rod of mass m, length ℓ and resistance R is sliding down on a smooth inclined parallel rails with a constant velocity v. If a uniform horizontal magnetic field B exists, then find the value of B.

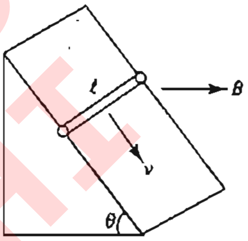


Fig. 8.79

27. A conducting rod AC of length 4ℓ is rotated about a point O in a uniform magnetic field $\vec{B}$ directed into the plane of the paper. $AO = \ell$ and $OC = 3l$. Find $V_A - V_C$.

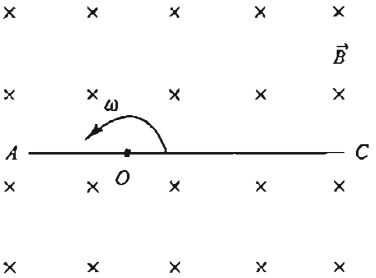


Fig. 8.80

28. Consider the sliding wire circuit shown in Fig. 8.81. The wire slides at constant speed and the plane of the circuit is perpendicular to a uniform magnetic field. Show that the induced e.m.f. is given by $E = B\ell v^2 t/D$ for $0 < t < D/v$. What is the expression for the e.m.f. for $t > D/v$?

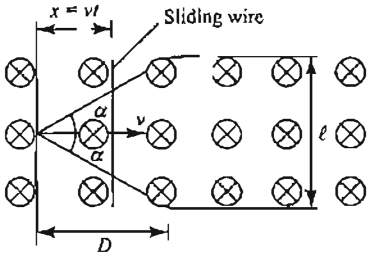


Fig. 8.81

INDUCED ELECTRIC FIELD AND INDUCTANCE

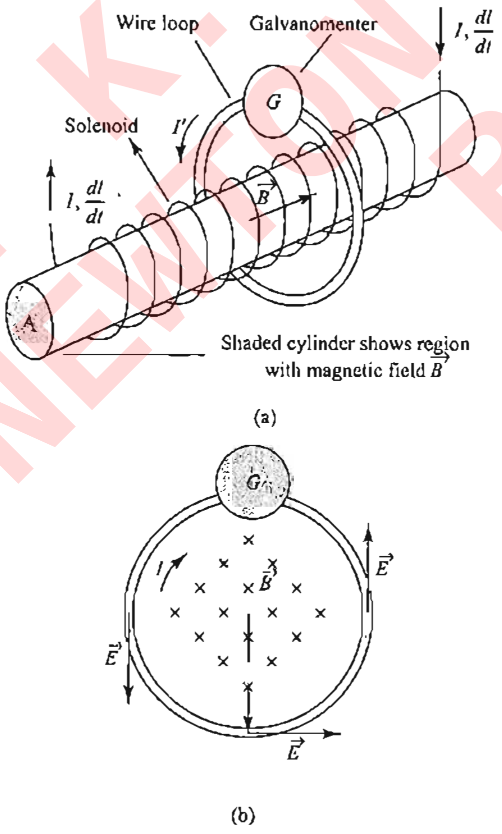
Induced Electric Field: Induced e.m.f. in a Stationary Conductor

According to Faraday's law, it is the relative motion between the loop and the magnet which produces the induced e.m.f.; it does not matter whether the loop moves towards the magnet or the magnet moves towards the loop. When the loop moves towards the magnet, it is the magnetic force which drives the charge to flow. But, what causes the induced current in a stationary loop when magnet moves towards it? A magnetic field cannot exert a force on a stationary conductor produced by the varying magnetic flux. Whenever a magnetic field is varying with time, an induced electric field E is produced in any closed path, whether in matter or in empty space.

$$\oint E d\ell = -A\frac{dB}{dt}$$

The induced electric field is different from the electrostatic field or the Coulomb field.

- Unlike electrostatic field the lines of induced field form closed loops. It is also called a circuital field or vortex field.
- It is not a conservative field, i.e., $\oint E d\ell \neq 0$.
- The line integral of the electrostatic field between any two points is the potential difference while the line integral of the induced electric field between any two points is the electromotive force.



(a)

(b)

Fig. 8.82

Fig. 8.82(a) shows the windings of a long solenoid carrying a current I that is increasing at a rate of dI/dt. The magnetic flux in the solenoid is increasing at a rate of $d\Phi_B/dt$, and this changing flux passes through a wire loop. An e.m.f. $\varepsilon = -d\Phi_B/dt$ is induced in loop, inducing a current I' that is measured by the galvanometer G. Fig. 8.82(b) shows the cross-sectional view.

Note:

There are two basic mechanisms of induced e.m.f. generation.

1. *The first one involves the motion of a conductor relative to magnetic field lines, called the motional e.m.f.*
2. *The second one involves the generation of an electric field associated with a time-varying magnetic field. In the modified form, Fraday's law may be stated as:*

$$\varepsilon = \oint\left(\vec{v}\times\vec{B}\right)\cdot d\vec{\ell} - \vec{A}\cdot\frac{d\vec{B}}{dt}$$

Time-varying Magnetic Field

Consider a conducting loop of area 'A' in a uniform but time-varying magnetic field. Rate of change of magnitude of magnetic field $= \frac{dB}{dt}$ for the loop, flux linked with it $= BA = \phi$ (say) (take area vector directed along $\vec{B}$).

Hence, rate of change of flux ϕ is $A\frac{dB}{dt}$ and hence induced e.m.f. $= -A\frac{dB}{dt}$.

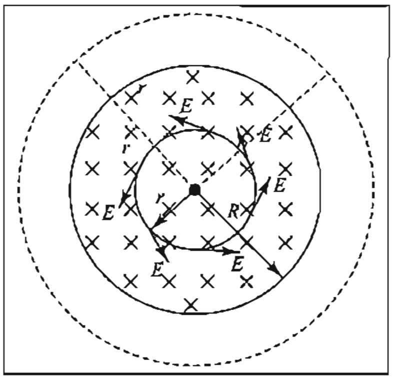


Fig. 8.83

For non-zero values of $\frac{dB}{dt}$, there could be a definite current in the loop, whose direction can be obtained using Lenz's rule. For example, if $\frac{dB}{dt} > 0$, i.e., B is increasing with time, magnetic field produced by the induced current would oppose the existing magnetic field. Hence, the induced current would be anticlockwise.

The current in the loop can be easily known if the resistance of the loop is known as $I = \varepsilon/R$.

In the case of motional e.m.f., you learnt that the electric field caused due to drifting of electrons is responsible for the induced e.m.f. Do we also have an electric field in the present case which is linked with the induced e.m.f.? The answer is partly 'Yes' and partly 'No'.

Yes, as there is a definite field and the electric field that you know. The electric field that you learnt in electrostatics is conservative and the associated lines of force never form closed loops.

On the other hand, the field associated with the induced e.m.f. in case of time-varying magnetic field is non-conservative as then only we would have non-zero value for $\oint \vec{E}_n \cdot d\vec{l}$. Here $\vec{E}_n$ denotes the induced field caused by the time-varying magnetic field.

For the path described by the loop, $\varepsilon_{\text{induced}} = -\frac{d\phi}{dt} = \oint \vec{E}_n \cdot d\vec{l}$.

Consider a magnetic field where B (magnitude of the magnetic field) is a function of r ($r < R$), the distance of the point O as shown in Fig. 8.84. For the circular path shown in the figure,

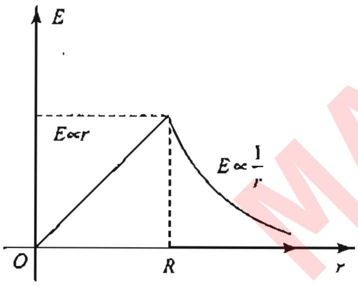


Fig. 8.84

$$E_n(2\pi r) = \pi r^2 \left|\frac{dB}{dt}\right|$$

$$\Rightarrow \quad E_n = \frac{r}{2}\left|\frac{dB}{dt}\right|$$

For $r > R$, $$E_n \pi r = \pi R^2 \left|\frac{dB}{dt}\right|$$

$$\Rightarrow \quad E_n = \frac{R^2}{2r}\left|\frac{dB}{dt}\right|$$

[From symmetry, $\vec{E}_n$ can be considered to be of the magnitude of energy and on the circular path is directed tangentially.]

Direction of $\vec{E}_n$ can be easily obtained as it would be responsible for the induced current when a conducting loop is placed on the given path. For example, in the present case, for $\frac{dB}{dt} > 0$, path is in anticlockwise sense.

Illustration 8.18 **A thin non-conducting ring of mass m carrying a charge q can freely rotate about its axis. At the initial moment the ring was at rest and no magnetic field was present. Then a uniform magnetic field was switched on, which was perpendicular to the plane of the ring and increased with time according to a certain law: $\frac{dB}{dt} = k$.**

Find the angular velocity ω of the ring as a function of k.

Sol. $$E = \frac{1}{2}R\frac{dB}{dt}$$

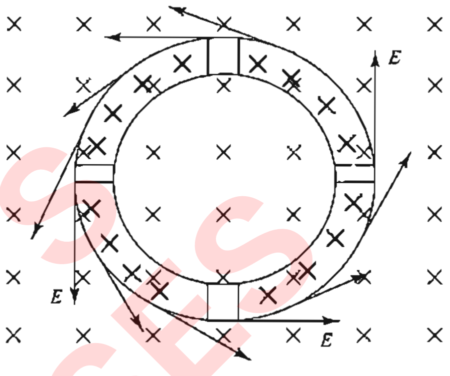


Fig. 8.85

Electric force on charge dq is given by

$$dF = Edq = \frac{1}{2}R\left(\frac{dB}{dt}\right)dq \quad \Rightarrow \quad d\tau = R\,dF$$

$$\Rightarrow \quad d\tau = \frac{1}{2}R^2\left(\frac{dB}{dt}\right)dq \quad \Rightarrow \quad \tau = \frac{1}{2}R^2 kq$$

Now, $$\tau = \frac{1}{2}R^2 kq$$

$$\Rightarrow \quad I\alpha = \frac{1}{2}R^2 kq \quad \Rightarrow \quad mR^2\alpha = \frac{1}{2}R^2 kq$$

$$\Rightarrow \quad \alpha = \frac{kq}{2m} \quad \Rightarrow \quad \omega = \frac{kq}{2m}t$$

Illustration 8.19 **A thin non-conducting ring of mass m carrying a charge q can rotate freely about its axis. At $t = 0$, the ring was at rest and no magnetic field was present. Then suddenly a magnetic field B was set perpendicular to the plane.**

Find the angular velocity acquired by the ring.

Sol. Due to the sudden charge in flux an electric field is set up and the ring experiences an impulsive torque and suddenly acquires an angular velocity.

$$E = -\frac{r}{2}\frac{dB}{dt}$$

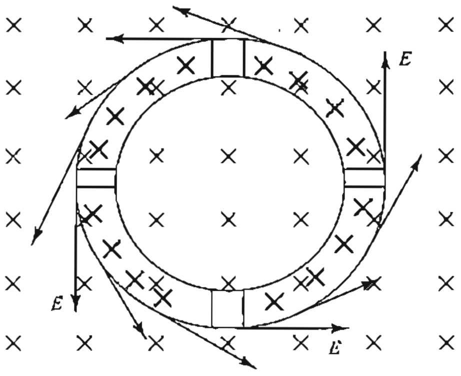


Fig. 8.86

Force experienced by an element of ring $dF = dqE$
Torque experience by ring $t = qEr$

$$\tau = qE = q\frac{r}{2}\frac{dB}{dt}r$$

angular impulse experience

$$L = \int \tau dt = \frac{qr^2}{2}\int \frac{dB}{dt}dt = \frac{qr^2}{2}B$$

$$L = \Delta I\omega = I\omega \Rightarrow mr^2\omega = \frac{qr^2}{2}B \Rightarrow \omega = \frac{qB}{2m}$$

Illustration 8.20 **A non-conducting ring of mass m and radius R has charge Q uniformaly distribute over its circumference. The ring is placed on a rough horizontal surface such that the plane of the ring is parallel to the surface. A vertical magnetic field $B = B_0t^2$ tesla is switched on. After 2 s from switching on the magnetic field, the ring is just about to rotate about vertical axis through its centre.**

a. Find friction coefficient m between the ring and the surface.
If magnetic field is switched off after 4 s, then find
b. the angular velocity of the ring just after switching off the magnetic field.
c. the angle rotated by the ring before coming to rest after switching off the magnetic field.

Sol.

a. $$E = -\frac{r}{2}\frac{dB}{dt} \Rightarrow |E| = B_0Rt$$

Force on the ring $F = QE = B_0QRt$. This force is tangential to the ring. The ring starts rotating when torque of this force is greater than the torque due to maximum friction ($f_{\max} = \mu mg$).
$\tau_F \geq \tau_{\text{fric,max}}$, $FR \geq \mu mg R \Rightarrow F > \mu mg$

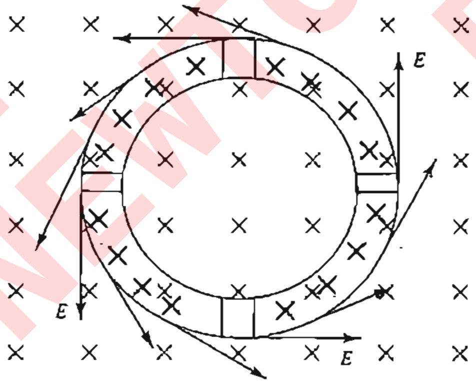


Fig. 8.87

$$B_0QRt = \mu mg$$

Hence, $$\mu = \frac{B_0QRt}{mg}$$

Given, $$t = 2\text{ s} \Rightarrow \mu = \frac{2B_0QR}{mg}$$

b. After 2 s

Net torque $\tau = \tau_F - \tau_{f\max} = B_0QR^2t - \mu mgR$

$$= B_0QR^2t - \frac{2B_0QR}{mg}mgR$$

$$\Rightarrow \quad \tau = B_0QR^2(t-2) \quad \Rightarrow \quad I\alpha = B_0QR^2(t-2)$$

$$mR^2\left(\frac{d\omega}{dt}\right) = B_0QR^2(t-2)$$

$$\Rightarrow \quad d\omega = \frac{B_0QR^2}{mR^2}(t-2)\,dt$$

$$\Rightarrow \quad \int_0^{\omega} d\omega = \frac{B_0Q}{m}\int_2^4 (t-2)\,dt$$

$$\Rightarrow \quad \omega = \frac{2B_0Q}{m}$$

If magnetic field is switched off after 4 s, only force present is frictional force which will retard the motion.
Retarding torque $\tau = \tau_{\text{friction}}$

Angular retardation $\alpha = \frac{\tau_{\text{friction}}}{I} \Rightarrow \alpha = \frac{\mu mgR}{mR^2} = \frac{\mu mg}{R} = \frac{\mu g}{R}$

Using $\omega^2 = \omega_0^2 + 2\alpha\theta \Rightarrow 0 = \left(\frac{B_0Q}{m}\right)^2 - 2\left(\frac{\mu g}{R}\right)\theta$

$$\Rightarrow \quad \theta = 2\left(\frac{B_0Q}{m}\right)^2\frac{R}{\mu g}$$

Illustration 8.21 **A line charge l is wound around an insulating disc of mass M and radius R, which is then suspended horizontally as shown in Fig. 8.88, so that it is free to rotate. In the central region, of radius a, there is a uniform magnetic field B_0, pointing up. Now the magnetic field is switched off, which causes the disc to rotate.**

Find the angular speed with which the disc starts rotating.

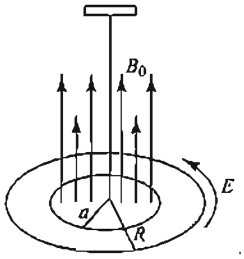


Fig. 8.88

Sol. The induced electric field E due to changing magnetic field is given by (from Faraday's law)

$$\oint \vec{E}\cdot d\vec{\ell} = \frac{-d\Phi_B}{dt} = -\pi R^2\frac{dB}{dt}$$

$$\Rightarrow \quad E\cdot 2\pi R = -\pi a^2\frac{dB}{dt} \quad \Rightarrow \quad E = \frac{-a^2}{2R}\frac{dB}{dt}$$

Hence, induced electric field is tangential to the disc as shown in Fig. 8.86 and its magnitude is $E = \frac{a^2}{2R}\frac{dB}{dt}$.

This electric field causes the disc to rotate. Now torque on the disc is $\tau = (\lambda 2\pi R)\, ER = \pi\lambda a^2 R\frac{dB}{dt}$.

If ω is the angular speed of the disc, then

$$\tau = \frac{dL}{dt} \qquad \text{(where } L \text{ is the angular momentum)}$$

$$\tau\lambda a^2 R\frac{dB}{dt} = \frac{dL}{dt}$$

$$\tau\lambda a^2 R(B_0 - 0) = \left(\frac{MR^2}{2}\omega - 0\right) \Rightarrow \omega = \frac{2\pi\lambda a^2 B_0}{MR}$$

MUTUAL INDUCTANCE

Consider Fig. 8.89 that shows two long co-axial solenoids each of length l. We denote the radius of the inner solenoid S_1 by r_1 and the number of turns per unit length by n_1. The corresponding quantities for the outer solenoid S_2 are r_2 and n_2, respectively.

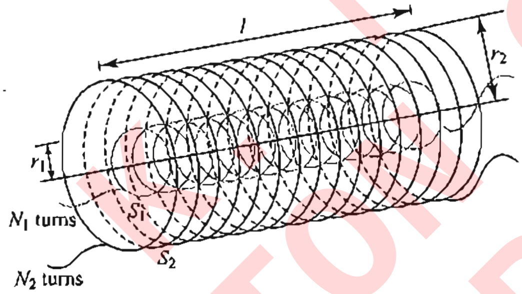


Fig. 8.89

We pass time-varying current I_2 through S_2. This sets up a time-varying magnetic flux through S_1 which we designate by Φ_1. The mutual inductance is the constant of proportionality given by

$$\Phi_1 = M_{12} I_2 \qquad \text{(i)}$$

M_{12} is called the mutual inductance of circuit 1 with respect to the current 2 sometimes also referred to as the coefficient of mutual induction.

From Faraday's law, the induced e.m.f. in S_1 is

$$\varepsilon_1 = -\frac{d\Phi_1}{dt} = M_{12} = \frac{dI_2}{dt} \qquad \text{(ii)}$$

We now consider the reverse case. A time-varying current I_1 is passed through the solenoid S_1 and the associated flux through S_2 is Φ_2, $\Phi_2 = M_{21} I_1$.

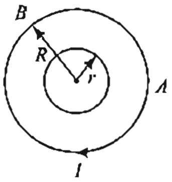
M_{21} is called the mutual inductance of the circuit 2 with respect to the current 1.

From Faraday's law, the induced e.m.f. ε_2 in S_2 is

$$\varepsilon_2 = -\frac{d\Phi_2}{dt} = M_{21} = \frac{dI_1}{dt}$$

It can be shown that $M_{21} = M_{12}$ (reciprocity theorem). Note that M is a purely geometrical quantity, depending only on the size, number of turns, relative position and relative orientation of the two coils. The S.I. unit of mutual inductance is called henry (H).

Illustration 8.22 **The figure shows the concentric coplanar circular loops A and B of radii r and R, respectively. Current I flows in the loop A. Find the magnetic flux through the loop B, assuming $r \ll R$. Also determine the coefficient of mutual induction.**



Sol. Since the magnetic field is not uniform over the big loop B, therefore, the direct calculation of the flux through this loop is not possible. The reciprocity theorem greatly simplifies the problem. According to this theorem, if the same current I passes through the loop B, then the flux through the loop A may be easily obtained.

Now, magnetic field at the centre of the loop A is $B = \frac{\mu_0 I}{2R}$ and the magnetic flux through the loop A is $\phi_A = B(pr^2) = \frac{\mu_0\pi r^2 I}{2B}$ ($\because r << R$).

According to the reciprocity theorem, $\phi_A = \phi_B = \frac{\mu_0\pi r^2 I}{2B}$. The coefficient of mutual induction is given by

$$M = \frac{\phi_B}{I} = \frac{\mu_0\pi r^2}{2R}$$

According to the reciprocity theorem, $\phi_A = \phi_B = \frac{\mu_0\pi r^2 I}{2R}$. The coefficient of mutual induction is given by

$$M = \frac{\phi_B}{I} = \frac{\mu_0\pi r^2}{2R}$$

Illustration 8.23 **A small square loop of wire of side l is placed inside a large square loop of wire of side L (>>l). The loops are coplaner and their centers coincide. What is the mutual inductance of the system?**

Sol. Considering the larger loop to be made up of four rods each of length L, the field at the centre, i.e., at a distance $L/2$ from each rod, will be

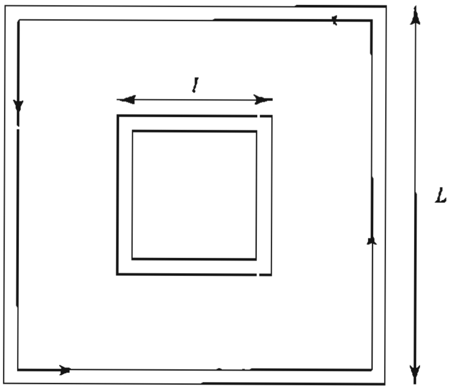


Fig. 8.90

$$B = 4\times\frac{\mu_0 I}{4\pi d}[\sin\alpha+\sin\beta]$$

i.e., $$B = 4\times\frac{\mu_0 I}{4\pi(L/2)}2\sin 45°$$

i.e., $$B_1 = 4\times\frac{\mu_0 8\sqrt{2}}{4\pi L}I$$

So the flux linked with the smaller loop is

$$\phi = B_1 S_2 = \frac{\mu_0 8\sqrt{2}}{4\pi L}\ \ell^2 I$$

Hence, $$M = \frac{\phi_2}{I} = 2\sqrt{2}\,\frac{\mu_0 l^2}{4\pi L}$$

Illustration 8.24 **What is the mutual inductance of a system of co-axial cables carrying current in opposite directions as shown in Fig. 8.91. Their radii are a and b, respectively.**

Sol. The 'B' between the space of the cables is $B = \mu_0 I/2\pi r$. The Ampere's law tells that B outside the cables is zero, as the net current through the Amperian loop would be zero.

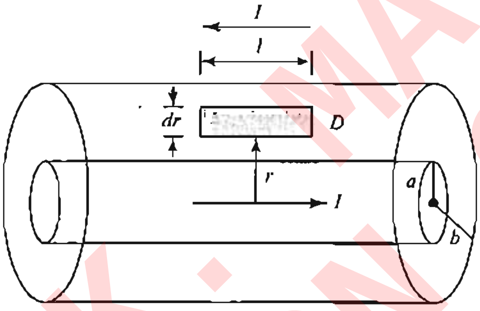


Fig. 8.91

Taking an element of length l and thickness dr, dp through it is

$$dp = \frac{\mu_0 i}{2\pi r}l\,dr \Rightarrow F = \frac{\mu_0 il}{2\pi}\int_a^b \frac{1}{r}dr = \frac{\mu_0 li}{2\pi}\ln\frac{b}{a}$$

$$L = \frac{\Phi}{i} = \frac{\mu_0 l}{2\pi}\ln\left(\frac{b}{a}\right)$$

SELF-INDUCTANCE

When current is present in a circuit, it sets up a magnetic field that causes a magnetic flux through the same circuit; this flux changes when the current changes. Thus any circuit that carries a varying current has an e.m.f. induced in it by the variation in its own magnetic field. Such an e.m.f. is called a self-induced e.m.f. By Lenz's law, a self-induced e.m.f. always opposes the change in the current that causes the e.m.f. and so tends to make it more difficult for variations in the current to occur. For this reason, self-induced e.m.f.s can be of great importance whenever there is varying current.

Self-induced e.m.f.s can occur in any circuit, since there is always some magnetic flux through the closed loop of a current-carrying circuit. But the effect is greatly enhanced if the circuit includes a coil with N turns of wire (as shown in Fig. 8.92). As a result of the current i, there is an average magnetic flux Φ_B through each turn of the coil. In analogy to equation

$$M = \frac{N_2\Phi_{B_2}}{i_1} = \frac{N_1\Phi_{B_1}}{i_2}\ \text{(mutual inductance)} \qquad \text{(i)}$$

we define the self-inductance L of the circuit as

$$L = \frac{N\Phi_B}{i}\ \text{(self-inductance)} \qquad \text{(ii)}$$

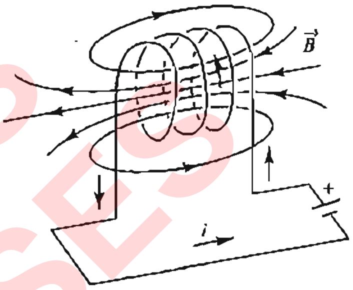


Fig. 8.92

To avoid any confusion with mutual inductance, the self-inductance is called simply called the inductance. Comparing equations (i) and (ii), we see that the unit of self-inductance is the same as that of mutual inductance; the S.I. unit of self-inductance is one henry.

If the current i in the circuit changes, so does the flux Φ_B; on rearranging equation (ii) and taking the derivative with respect to time, the rate of change is related by $N\frac{d\Phi_B}{dt} = L\frac{di}{dt}$

From Faraday's law for a coil with N turns, the self-induced e.m.f. is $\varepsilon = -\frac{N\,d\Phi_B}{dt}$, so it follows that

$$\varepsilon = -L\frac{di}{dt}\ \text{(self-induced e.m.f.)} \qquad \text{(iii)}$$

The minus sign in the equation is a reflection of Lenz's law; it says that the self-induced e.m.f. in a circuit opposes any change in the current in that circuit.

Equation (iii) also states that the self-inductance of a circuit is the magnitude of the self-induced e.m.f. per unit rate of change of current. This relationship makes it possible to measure an unknown self-inductance in a relatively simple way: Change the current in the circuit at a known rate di/dt, measure the induced e.m.f. and take the ratio to determine L.

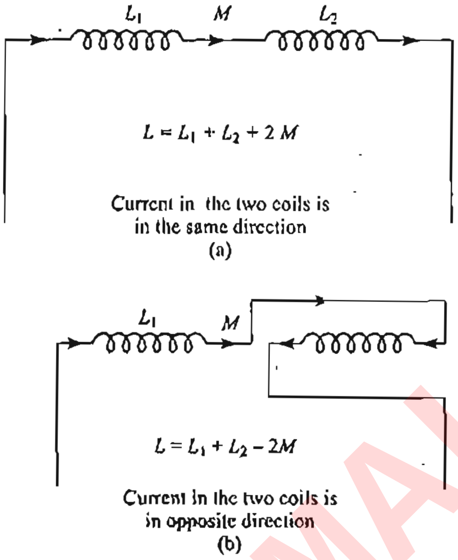
A circuit device that is designed to have a particular inductance is called an inductor. The usual circuit symbol for an inductor is .

In the previous section, we have discussed the flux in one solenoid due to the current in the other. Consider the general case of current flowing simultaneously in two nearby coils. The flux linked with one coil will be due to the sum of two fluxes which exist independently. The law of superposition applies to magnetic fields. For example, equation (i) would generalize to, $\Phi_1 = L_{11} I_1 + M_{12} I_2$.

Therefore, using Faraday's law, $\varepsilon_1 = -L_1\frac{dI_1}{dt} - M_{12}\frac{dI_2}{dt}$

Note:

In case of series grouping of two inductors, if mutual inductance is also taken into account, then $L = L_1 + L_2 \pm 2M$ *as shown in Fig. 8.93:*



Current in the two coils is in the same direction
(a)

Current in the two coils is in opposite direction
(b)

Fig. 8.93

APPLICATION OF THE KIRCHHOFF'S LAW

Suppose a current passes through an inductor from left to right and increases with time. From Lenz's law, the induced e.m.f. must oppose the change in the magnetic flux. The induced e.m.f. opposes the increasing flux by acting like a source of e.m.f. that opposes the external source of e.m.f. driving the current (as shown in Fig. 8.92). So we treat an inductor as a common circuit element labelled with polarity (+) and (–) marked according to Lenz's law.

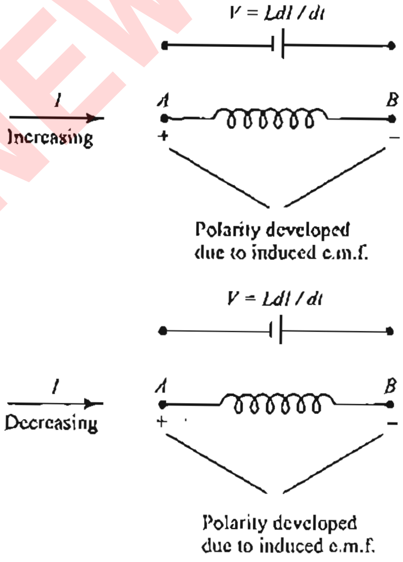


Fig. 8.94

If the current is decreasing, then by Lenz's law the induced e.m.f. acts to help the decreasing flux. The inductor acts like a source of e.m.f. reinforcing the external e.m.f. driving the current. The induced e.m.f. acts to increase I (Fig. 8.94).

Thus we can consider an inductor as a battery whose polarity is decided according to Lenz's law.

SERIES AND PARALLEL COMBINATION OF INDUCTORS

Series Combination

Fig. 8.95(a) shows a collection of inductors in series, all of them will have same current through them.

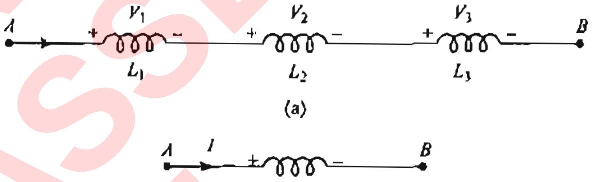


Fig. 8.95

For each inductor, we have

$$V_1 = L_1\frac{dI}{dt},\ V_2 = L_2\frac{dI}{dt},\ V_3 = L_3\frac{dI}{dt}$$

We replace the series with a single equivalent inductance L_{eq} with the same potential difference V between the terminals A and B, as shown in Fig. 8.95(b).

$$V = L_{eq}\frac{dI}{dt}$$

The potential difference V is the sum of potential differences.

$$V = V_1 + V_2 + V_3$$

$$L_{eq}\frac{dI}{dt} = L_1\frac{dI}{dt} + L_2\frac{dI}{dt} + L_3\frac{dI}{dt}$$

$$\Rightarrow \quad L_{eq} = L_1 + L_2 + L_3$$

Parallel Combination

When the circuit elements are in parallel, the potential difference across each circuit element is same.

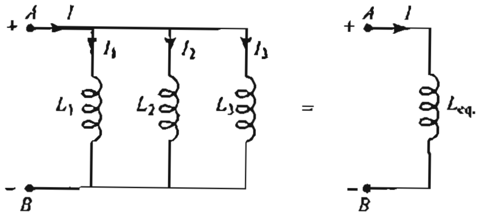


Fig. 8.96

Thus for each inductor we have

$$V = L_1\frac{dI_1}{dt},\ V = L_2\frac{dI_2}{dt},\ V = L_3\frac{dI_3}{dt} \qquad \text{(i)}$$

If we replace the combination with a single equivalent inductor L_{eq}, we get

$$V = L_{eq}\frac{dI}{dt} \qquad \text{(ii)}$$

From Kirchhoff's current law (KCL), we get $I = I_1 + I_2 + I_3$. Differentiating this equation with respect to t, we get

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} + \frac{dI_3}{dt} \qquad \text{(iii)}$$

On substituting derivatives from equations (i) and (ii) in equation (iii), we have

$$\frac{V}{L_{eq}} = \frac{V}{L_1} + \frac{V}{L_2} + \frac{V}{L_3}$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \qquad \text{(inductors in parallel)}$$

Illustration 8.25 **The equivalent inductance of two inductors is 2.4 H when connected in parallel and 10 H when connected in series. What is the value of inductance of the individual inductors?**

Sol. As inductances obey laws similar to the 'grouping of resistances',

$$L_1 + L_2 = 10\text{ H and } \frac{L_1L_2}{L_1 + L_2} = 2.4\text{ H}$$

Substituting the value of $(L_1 + L_2)$ from first expression into second, $L_1L_2 = (2.4)(L_1 + L_2) = 2.4 \times 10 = 24$

so that $(L_1 - L_2)^2 = (L_1 + L_2)^2 - 4L_1L_2$, i.e., $L_1 - L_2 = 2$ H

and as $L_1 + L_2 = 10$ H, so $L_1 = 6$ H and $L_2 = 4$ H.

Illustration 8.26 **What will happen to the inductance of a solenoid**

a. when the number of turns and the length are doubled keeping the area of cross section same?

b. when the air inside the solenoid is replaced by iron of relative permeability μ_r?

Sol. In case of a solenoid as $B = \mu_0 nI$, $f = B(nlA) = \mu_0 n^2 lAI$ and

hence $L = \frac{\phi}{I} = \mu_0 n^2 lA = \mu_0\frac{N^2}{l}A$.

a. When N and l are doubled, $L' = \mu_0$

$$L' = \mu_0\frac{(2N)^2}{2l}A = 2\mu_0\frac{N^2}{l}A = 2L$$

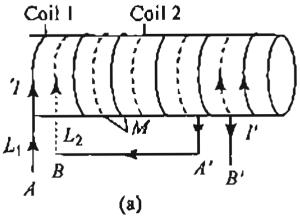
i.e, inductance of the solenoid will be doubled.

b. When air is replaced by iron, μ_0 will change to μ, so that L' $= \mu n^2 lA$ and hence, $\frac{L'}{L} = \frac{\mu}{\mu_0} = \mu_r$

i.e., $L' = \mu_r L$

So, the inductance will become μ_r times of its initial value.

Illustration 8.27 **In Fig. 8.97 (a) coil 1 and coil 2 are wound on a long cylindrical insulator. The ends A' and B are joined together and current I is passed. The self-inductance of the two coils are L_1 and L_2 their mutual inductance is M.**



(a)

Fig. 8.97

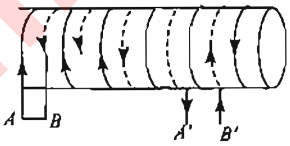
a. Show that this combination can be replaced by a single coil of equivalent inductance given by $L_{eq} = L_1 + L_2 + 2M$.

b. How could the coils be reconnected by yielding an equivalent inductance of $L_{eq} = L_1 + L_2 - 2M$.

Sol.

a. When the terminals A' and B are connected, the sense of current in both the coils is same. Let the current be changing at the rate of di/dt. The magnetic fields of both coil 1 and coil 2 point towards left. When the current increases both fields increase and both changes in flux contribute e.m.f. s in the same direction. Thus the induced e.m.f. in coil 1 is

$$E_1 = -(L_1 + M)\frac{di}{dt} \qquad \text{(i)}$$



(b)

Fig. 8.97

Similarly, the magnetic field in coil 2 due to it and field due to coil 1 point towards left. The two induced e.m.f. s are again in the same direction.

$$E_2 = -(L_2 + M)\frac{di}{dt} \qquad \text{(ii)}$$

Hence, the total e.m.f. across both coils is

$$E = E_1 + E_2 = -(L_1 + L_2 + 2M)\frac{di}{dt} \qquad \text{(iii)}$$

If these coils were replaced by a single coil, then we have

$$E = -L_{eq}\frac{di}{dt} \qquad \text{(iv)}$$

On comparing equations (iii) and (iv), we have

$$L_{eq} = L_1 + L_2 + 2M$$

b. When the terminals A and B are connected, the sense of the flow of current in coil 1 is opposite to that in coil 2. Then at the site of coil 1 the field produced by coil 2 is opposite to the field produced by coil 1 itself. An increasing current in coil 1 tends to increase the flux in that coil but an increasing current in coil 2 tends to decrease it. The e.m.f. across coil 1 is

$$E_1 = -(L_1 - M)\frac{di}{dt}$$

Similarly, e.m.f. across coil 2 is $E_2 = -(L_2 - M)\dfrac{di}{dt}$

The total e.m.f. $E = E_1 + E_2 = -(L_1 + L_2 - 2M)\dfrac{di}{dt}$

Thus, $L_{eq} = (L_1 + L_2 - 2M)$

COMBINATION OF INDUCTORS WITH RESISTORS

An *LR* circuit is analyzed in three states:

- Initial state, i.e., just after closing the switch or just after opening the switch.
- Transient state or instantaneous state, i.e., any time after closing or opening the switch.
- Steady state, i.e., a long time after closing or opening the switch. In this state, current in the inductor does not vary with time, i.e., $\dfrac{dI}{dt} = 0$.

Initial State

At $t = 0$, the current tends to increase very rapidly, therefore opposition produced by the inductor is infinite. Hence, no current flows through the circuit at the instant of closing the switch. The entire voltage is dropped across the inductor and no voltage is dropped across the resistor.

i.e., $v_L = \varepsilon_1 \quad v_R = 0 \quad \text{at } t = 0$

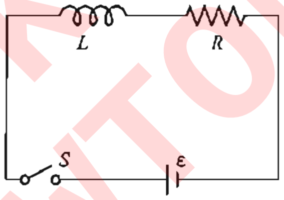


Fig. 8.98

Steady State

At $t = \infty$, the current has risen to its maximum value, and the inductor does not produce any opposition. No voltage is dropped across the inductor, the entire voltage is dropped across the resistor.

i.e., $v_L = 0; \quad v_R = \varepsilon \quad \text{at } t = \infty$

Transient State

At any instant $(0 < t < \infty)$, both inductor and resistor share the total applied voltage. Let i be the instantaneous current in the circuit as shown in Fig. 8.98.

Rise of Current

A series combination of an inductor L and a resistor R is connected across a cell of e.m.f. ε through a switch S as shown in Fig. 8.99. When switch is closed, current starts increasing in the inductor. This causes an induction of e.m.f. in the inductor. The induced e.m.f. opposes the growth of current in the circuit. Let at any time t, the current in the circuit be I_t.

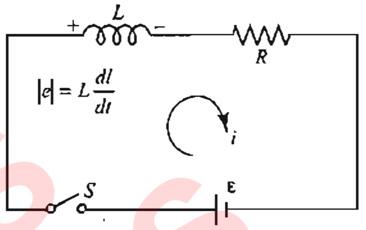


Fig. 8.99

From loop rule, we obtain

$$\varepsilon = L\frac{dI_t}{dt} + I_t R$$

$$\Rightarrow \quad \varepsilon - I_t R = L\frac{dI_t}{dt}$$

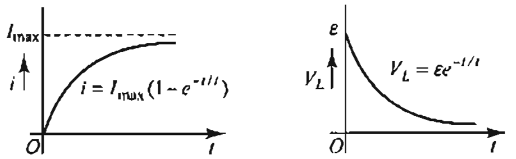
$$\Rightarrow \quad \frac{1}{L}\int_0^t dt = \int_0^{I_t} \frac{dI_t}{\varepsilon - I_t R}$$

$$\Rightarrow \quad \frac{t}{L} = \frac{\ln[\varepsilon - I_t R]}{-R}$$

$$\Rightarrow \quad -\frac{tR}{L} = \ln\frac{\varepsilon - I_t R}{\varepsilon}$$

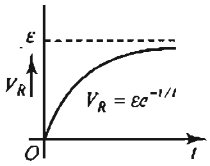
$$\Rightarrow \quad I_t = \frac{\varepsilon}{R}\left(1 - e^{-\frac{tR}{L}}\right)$$

Here, I_t represents the instantaneous current in the circuit.



(a) Current in the circuit increases with time

(b) Voltage across the inductor decreases with time



(c) Voltage across the resistor increases with time

Fig. 8.100

Note:

1. ***Final current in the circuit*** $= \dfrac{\varepsilon}{R}$, ***which is independent of*** L.

2. *After one time constant, current in the circuit = 63% of the final current (verify yourself).*
3. *More time constant in the circuit implies slower rate of change of current.*
4. *If there is any change in the circuit containing inductor, then there is no instantaneous effect on the flux of inductor, $L_1 i_1 = L_2 i_2$.*

Decay of Current

In this case, source of e.m.f. is disconnected from the circuit (Fig. 8.101).

$$\Rightarrow \quad \frac{-L\,dI_t}{dt} - I_t R = 0$$

$$\Rightarrow \quad \int_{I_0}^{I_t}\frac{dI_t}{I_t} = -\frac{R}{L}\int_0^t dt$$

$$\Rightarrow \quad I_t = I_0 e^{\frac{-tR}{L}}$$

(L/R) is called time constant as its dimension is same as that of time.

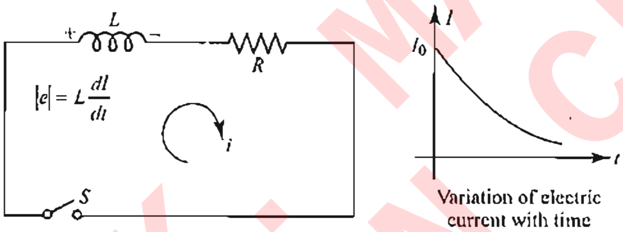


Fig. 8.101

Note:

If t = 0, the current in the circuit is $\frac{i_0}{n}$. *The current in the circuit in steady state will be again* i_0. *So it will decrease exponentially from* $\frac{i_0}{n}$ *to* i_0. *From the i–t graph, the equation can be formed without doing any calculation.*

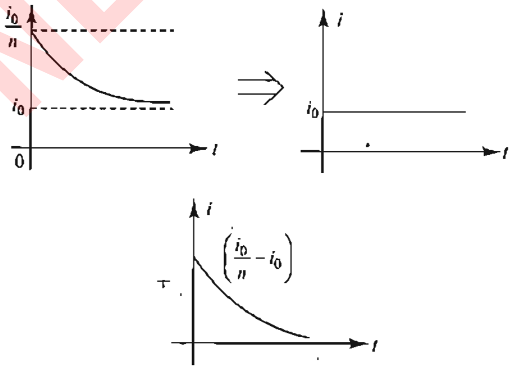


Fig. 8.102

$$i = i_0 + \left(\frac{i_0}{n} - i_0\right)e^{-t/\tau}.$$

Illustration 8.28 **The network shown in Fig. 8.103 is a part of a complete circuit. What is the potential difference $V_B - V_A$, when the current I is 5 A and is decreasing at a rate of 10^3 A/s?**

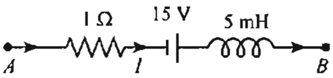


Fig. 8.103

Sol. In accordance with the law of potential distribution, for the given network, $V_A - IR + E - \frac{dI}{dt}L = V_B$.

And here I is decreasing (i.e., dI/dt is negative).

$$V_B - V_A = -5\times1 + 15 - 5\times10^{-3}(-10^3)$$
$$V_B - V_A = -5 + 15 + 5 = 15\text{ V}$$

Illustration 8.29 **In an LR circuit as shown in Fig. 8.104, when the switch is closed how much time will it take for the current to grow to a value (where $\eta < 1$).**

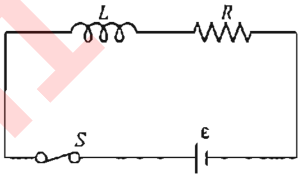


Fig. 8.104

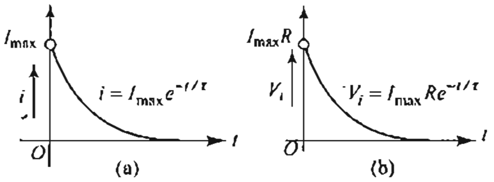
Sol. We know that $i = \frac{\varepsilon}{R}(1 - e^{-t/\tau})$

$$i = \eta\frac{\varepsilon}{R} \qquad \text{(given)}$$

$$\eta\frac{\varepsilon}{R} = \frac{\varepsilon}{R}(1 - e^{-t/\tau})$$

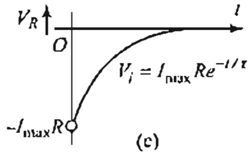
or $\quad e^{-t/\tau} = 1 - \eta$

or $\quad t = \tau\ln\left(\frac{1}{1-\eta}\right)$



(a) Current in the circuit decreases with time

(b) Voltage gain across the inductor decreases with time



(c) Voltage drop across the resistor decreases with time

Fig. 8.105

Illustration 8.30 In the circuit shown in Fig. 8.106, the initial current through the inductor at $t = 0$ is I_0. After a time $t = \frac{L}{R}$, the switch is quickly shifted to the position 2.

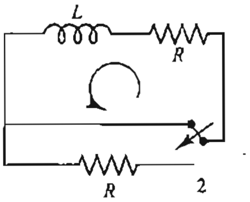


Fig. 8.106

a. Plot a graph showing the variation of current with time.

b. Calculate the value of current in the inductor at $t = \frac{3}{2}\frac{L}{R}$.

Sol.

a. Shifting the position of switch increases the resistance of the circuit. This decreases the value of time constant. Consequently, the rate of decrease of current increases. The variation of current with time is shown in Fig. 8.107.

b. The equations of the curves are $i = I_0 e^{-Rt/L}$ (for $0 \le t \le R/L$)

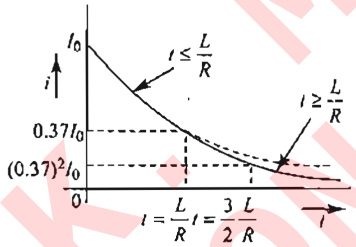


Fig. 8.107

for $\frac{R}{L} \le t \le \infty$ and $i = (0.37) I_0 e^{\frac{-2R}{L}\left(t - \frac{L}{R}\right)}$

at $t = \frac{3}{2}\frac{L}{R}$; hence $i = (0.37) I_0 e^{\frac{-2R}{L}\left(\frac{3L}{2R} - \frac{L}{R}\right)}$ or $i = (0.37)^2 I_0$.

Illustration 8.31 During the decay of current in an LR circuit, if the current d falls to η times the initial value in a time T, then determine the value of time constant.

Sol. We know that $i = I_{\max} e^{-t/T}$

At $t = T, I = \eta I_{\max}$

$$\eta I_{\max} = I_{\max} e^{-t/T}$$

or $e^{-t/T} = \eta;\quad T = \frac{t}{\ln\left|\frac{L}{\eta}\right|}$

Illustration 8.32 Fig. 8.108 shows an LR circuit.

a. Immediately after closing the switch, determine
   i. the current in the resistor;
   ii. the current in the inductor;
   iii. the potential difference across the resistor;
   iv. the potential difference across the inductor.

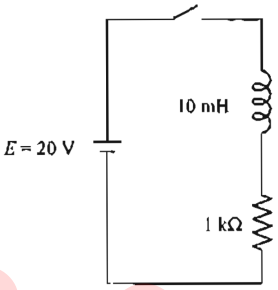


Fig. 8.108

b. Determine some quantities a long time after closing the switch, that is, after many time constants (for $t \to \infty$).

c. Show explicitly that the Kirchhoff's voltage law (KVL) is satisfied in the circuit at any time t.

d. Write the expression for potential difference across the resistor and inductor as functions of time, and plot the graphs of the potential difference across the resistor and inductor as functions of time.

Sol.

a. For a series RL circuit, the current through the inductor, the equation is given by $I(t) = \frac{E}{R}[1 - e^{-(R/L)t}]$ (i)

At initial state $t = 0$, $\frac{1}{e^{(R/L)0}} = 1$. Hence, $I(0) = 0$.

The current through the inductor and resistor is zero. The potential difference across resistance $V_R = I(0)\,R = 0$, as no current flows in the circuit at $t = 0$. Hence, potential difference across inductor will be e.m.f. of the battery $V_L = 20$ V.

The e.m.f. across inductor $E_L = -L\left(\frac{dI}{dt}\right)$ is positive because the slope dI/dt of the graph is always negative. The positive e.m.f. is generated in accordance with the Lenz's law; the flux is decreasing; the e.m.f. must oppose this decrease of flux.

b. As $\frac{dI}{dt} = \frac{E}{L} e^{-(R/L)t}$ (ii)

after long time (at $t = \infty$), $e^{-(R/L)\infty} = 0 \Rightarrow \frac{dI}{dt} = 0$.

The current is no longer changing with time; hence, the potential difference across the inductor is zero.

The current through the inductor is constant, from equation (i),

$$I = \frac{E}{R} = \frac{20}{1.0 \times 10^3} = 20 \text{ mA}$$

The same current flows through the resistor.

The potential difference across the resistor is

$$V_R = IR = (20 \times 10^{-3})(1.0 \times 10^3) = 20 \text{ V}.$$

c. The potential difference across resistor at any time $V_R(t)$ is

$$V_R(t) = I(t)R = \frac{E}{R}[1 - e^{-(R/L)t}] \times R = E(1 - e^{-(R/L)t})$$

The potential difference across the inductor $V_L(t)$ at any time t is

$$|V_L(t)| = \left|L\frac{dI}{dt}\right|$$

But we have $\frac{dI}{dt} = \frac{E}{R} e^{-(R/L)t}$

The current through the resistor is zero at $t = 0$; the potential difference across it is zero; hence, from KVL applied on the loop the potential difference on the inductor must be equal to that of the e.m.f. source, i.e, 20 V.

The potential difference on an inductor is $E = L\dfrac{dI}{dt}$

$$\therefore \quad 20\,\text{V} = (10\times10^{-3}\,\text{H})\frac{dI}{dt}$$

Solving for dI/dt, we have $\dfrac{dI}{dt} = \dfrac{20\,\text{V}}{10\times10^{-3}\,\text{H}} = 2.0\times10^{3}$ A/s.

The potential difference on the inductor depends on the rate of change of current.

The current through the resistor changes at the same rate as it is in series. Let us see another way to determine dI/dt. The current through the inductor is given by

$$I(t) = \frac{E}{R}[1 - e^{-(R/L)t}] \qquad \text{(i)}$$

Differentiating w.r.t. t, we get

$$\frac{dI}{dt} = \frac{E}{R}(-1)\,e^{-(R/L)t}\left(-\frac{R}{L}\right) = \frac{E}{L}e^{-(R/L)t} \qquad \text{(ii)}$$

At $t = 0$, $e^{-(R/L)t} = 1$, we have $\dfrac{dI}{dt} = \dfrac{E}{L}$, the same result as obtained earlier,

$$V_L = L\frac{dI}{dt} = Ee^{-(R/L)t}$$

The polarities are shown in Fig. 8.109. Now we apply KVL around the closed loop, traversing the circuit in the clockwise sense. Beginning from left corner, we get

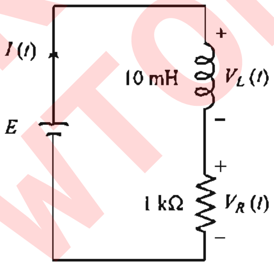


Fig. 8.109

$$E - V_L(t) - V_R(t) = 0$$

$$E - Ee^{-(R/L)t} - E[1 - e^{-(R/L)t}] = 0$$

$$0\,\text{V} = 0\,\text{V}$$

Hence, from the above discussion, we can say the KVL is satisfied at any instant.

d. The time constant of circuit is

$$\tau = \frac{L}{R} = \frac{10\times10^{-3}\,\text{H}}{1.0\times10^{3}\,\Omega} = 1.0\times10^{-5}\,\text{s}$$

The potential difference across the resistor is

$$V_R(t) = E[1 - e^{-(t/\tau)}] = 20\,\text{V}\,[1 - e^{-(1.0\times10^{-5}\,\text{s}^{-1})t}]$$

The potential difference across the inductor is

$$V_L(t) = Ee^{-(t/\tau)} = (20\,\text{V})e^{-(1.0\times10^{-5}\,\text{s}^{-1})t}$$

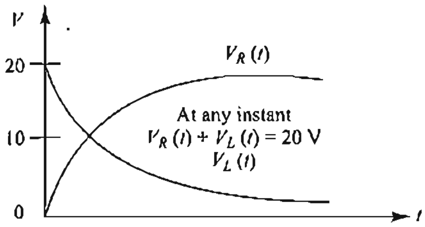


Fig. 8.110

Illustration 8.33 **In the following circuit (Fig. 8.111) the switch is closed at $t = 0$. Intially, there is no current in inductor. Find out the equation of current in the inductor coil as a function of time.**

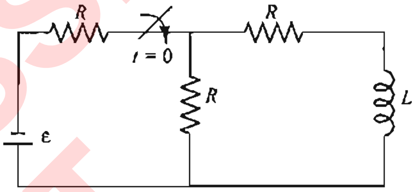


Fig. 8.111

Sol. At any time t,

$$-\varepsilon + i_1R - (i - i_1)R = 0$$

$$-\varepsilon + 2i_1R - iR = 0$$

$$i_1 = \frac{iR + \varepsilon}{2R}$$

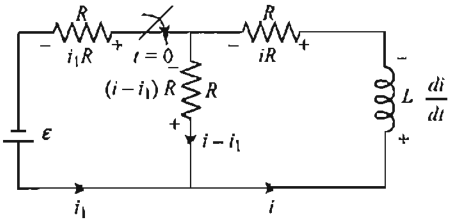


Fig. 8.112

Now,

$$-\varepsilon + i_1R + iR + L = 0$$

$$-\varepsilon + \left(\frac{iR + \varepsilon}{2}\right) + iR + i\frac{di}{dt} = 0$$

$$-\frac{\varepsilon}{2} + \frac{3IR}{2} = -L\frac{di}{dt}$$

$$\left(\frac{-\varepsilon + 3iR}{2}\right)dt = -L\,di - \frac{-dt}{2L} = \frac{di}{-\varepsilon + 3iR} - \int_0^t \frac{dt}{2L} = \int_0^i \frac{di}{-\varepsilon + 3iR}$$

$$-\frac{t}{2L} = \frac{1}{3R}\ln\left(\frac{-\varepsilon + 3iR}{-\varepsilon}\right) - \ln\left(\frac{-\varepsilon + 3iR}{-\varepsilon}\right) = \frac{3Rt}{2L}$$

$$i = +\frac{\varepsilon}{3R}\left(1 - e^{-\frac{3Rt}{2L}}\right)$$

Illustration 8.34 Fig. 8.113 shows a circuit consisting of a ideal cell, an inductor L and a resistor R, connected in series. Let the switch S be closed at $t = 0$. Suppose at $t = 0$ the current in the inductor is i_0 then find out equation of current as a function of time

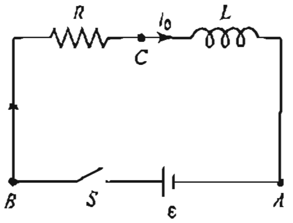


Fig. 8.113

Sol. Let at an instant t, the current in the circuit is i which is increasing at the rate di/dt.

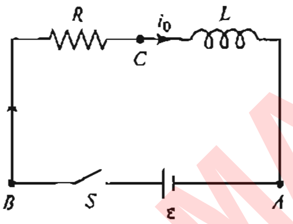


Fig. 8.114

Writing KVL along the circuit, we have $-L\dfrac{di}{dt} - iR = 0$

$$\Rightarrow \qquad L\frac{di}{dt} = \varepsilon - iR \quad \Rightarrow \quad \int_{i_0}^{i}\frac{di}{\varepsilon - iR} = \int_0^t\frac{dt}{L}$$

$$\Rightarrow \qquad \ln\left(\frac{\varepsilon - iR}{\varepsilon - i_0R}\right) = -\frac{Rt}{L} \quad \Rightarrow \quad \varepsilon - iR = (\varepsilon - i_0R)\,e^{-Rt/L}$$

$$\Rightarrow \qquad i = \frac{\varepsilon - (\varepsilon - i_0R)e^{-Rt/L}}{R}$$

Illustration 8.35 In a circuit shown in Fig. 8.115, A and B are two cells of same e.m.f. E but different internal resistances r_1 and r_2 $(r_1 > r_2)$, respectively. Find the value of R such that the potential difference across the terminals of cell A is zero a long time after the key K is closed. (IIT-JEE, 2004)

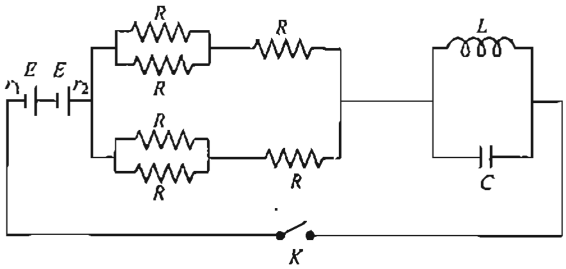


Fig. 8.115

Sol. After a long time, steady state is reached in which impedance due to induced (ωL for dc) is zero and that due to capacitance $\left(\dfrac{1}{\omega C}\right)$ becomes infinite, so equivalent circuit is shown in Fig. 8.116

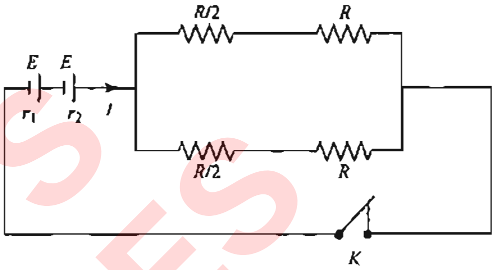


Fig. 8.116

Net external resistance $R_{ext}\ \dfrac{\dfrac{R}{2}+R}{2} = \dfrac{3}{4}R$

Net internal resistance $R_{int} = r_1 + r_2$

$\therefore$ Current in circuit $I = \dfrac{2E}{\dfrac{3}{4}R + r_1 + r_2}$

The potential difference across the terminals of cell A is zero;

so $E - Ir_1 = 0 \Rightarrow E - \dfrac{2Er_1}{\dfrac{3}{4}R + r_1 + r_2} \Rightarrow R = \dfrac{4}{3}(r_1 - r_2)$.

ENERGY STORED IN MAGNETIC FIELD OF AN INDUCTOR

As $\varepsilon = IR + \dfrac{LdI}{dt}$, $\varepsilon I = I^2R + LI\dfrac{dI}{dt}$

εI is the power supplied by the battery, I^2R is the electrical power dissipated in the resistance and $LI\dfrac{dI}{dt}$ is the rate of energy stored in the inductor.

$$\therefore \qquad \varepsilon I dt = I^2Rdt + LIdI$$

$$\Rightarrow \qquad \text{Energy stored in the inductor is } U_B = \int_0^I LIdI = \frac{1}{2}LI^2$$

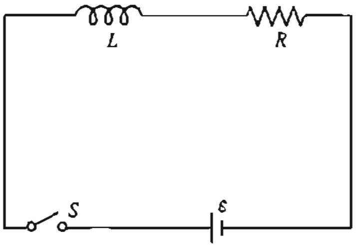


Fig. 8.117

Alternatively:

The energy of a capacitor is stored in the electric field between its plates. Similarly, an inductor has the capability of storing energy in its magnetic field.

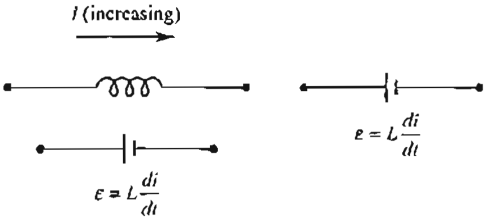


Fig. 8.118

An increasing current in an inductor causes an e.m.f. between its terminals.

The work done per unit time is power $P = \frac{dW}{dt} = -ei = -Li\frac{di}{dt}$

From $\quad dW = - dU \quad$ or $\quad \frac{dW}{dt} = -\frac{dU}{dt}$

We have $\frac{dU}{dt} = Li\frac{di}{dt}$ or $dU = Li\,di$

The total energy U supplied while the current increases from zero to a final value i is $U = L\int_0^I i\,di = \frac{1}{2}Li^2$

$$\therefore \qquad U = \frac{1}{2}Li^2$$

The energy in an inductor is actually stored in the magnetic field within the coil. We can develop relation magnetic energy density u (energy stored per unit volume) analogous to those we obtained in electrostatics. We will concentrate on one simple case of an ideal long cylindrical solenoid. For a long solenoid, its magnetic field can be assumed completely within the solenoid. The energy U stored in the solenoid when a current i is

$$U = \frac{1}{2}Li^2 = \frac{1}{2}(\mu_0 n^2 V)i^2$$

as $\qquad L = \mu_0 n^2 V$

The energy per unit volume is $u = \frac{U}{V}$

$$u = \frac{U}{V} = \frac{1}{2}\mu_0 n^2 i^2 = \frac{1}{2}\frac{(\mu_0 ni)^2}{\mu_0} = \frac{1}{2}\frac{B^2}{\mu_0}$$

as $\qquad B = \mu_0\, ni$

Thus, $\qquad u = \frac{1}{2}\frac{B^2}{\mu_0}$

This expression is similar to $u = \frac{1}{2}\varepsilon_0 E^2$ used in electrostatics. Although we have derived it for one special situation, it turns out to be correct for any magnetic field configuration.

Illustration 8.36 **Consider the *RL* circuit in Fig. 8.119. When the switch is closed in position 1 and open in position 2, electrical work must be performed on the inductor and on the resistor. The energy stored in the inductor is for the magnetic field inside it which increases as *I* increases. In the resistor energy appears as heat.**

a. What is the ratio of P_L/P_R of the rate at which energy is stored in the inductor to the rate at which work is done in the resistor?

b. Express the ratio P_L/P_R as a function of time.

c. If the time constant of circuit is t, what is the time at which $P_L = P_R$?

Sol.

a. The energy stored in the inductor is $U_L = \frac{1}{2}LI^2$

The power $P_L = \frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}LI^2\right) = LI$ (i)

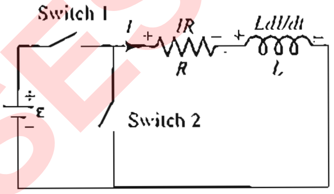


Fig. 8.119

The power of a resistor $P_R = I^2R$ (ii)

So the ratio is $\frac{P_L}{P_R} = \frac{L}{R}\frac{1}{I}\frac{dI}{dt} = \tau\frac{1}{I}\frac{dI}{dt}$ (iii)

b. As $\qquad I = \frac{E}{R}[1 - e^{-(t/\tau)}]$ (iv)

$$\frac{dI}{dt} = \frac{d}{dt}\left[\frac{E}{R}(1 - e^{-(t/\tau)})\right] = \frac{E}{R\tau}e^{-(t/\tau)} \qquad \text{(v)}$$

Inserting the value of dI/dt and I in equation (iii), we have

$$\frac{P_L}{P_R} = \tau\frac{\frac{E}{R\tau}e^{-t/\tau}}{\frac{E}{R}(1 - e^{-t/\tau})} = \frac{e^{-(t/\tau)}}{1 - e^{-(t/\tau)}} = \frac{1}{e^{(t/\tau)} - 1} \qquad \text{(vi)}$$

When $t = 0$ this ratio tends to infinity, due to large initial value of dI/dt and small initial value of I. When t tends to infinity, after many time constants, the current tends to zero. As a result, P_L vanishes and the ratio goes to zero.

c. From equation (vi), $P_L = P_R$ when $\frac{1}{e^{(t/\tau)} - 1} = 1$ which on simplification yields $e^{(t/\tau)} = 2$, $\frac{t}{\tau} = \ln 2 \Rightarrow t = 0.693\,\tau$.

Illustration 8.37 **Derive an expression for the total magnetic energy stored in two coils with inductances L_1 and L_2 and mutual inductance M_1 when the currents in the coils are I_1 and I_2, respectively.**

Sol. When the currents are increasing in the circuit, we have for e.m.f. s

$$E_1 = -L_1\frac{dI_1}{dt} \pm M\frac{dI_2}{dt}, \; E_2 = -L_2\frac{dI_2}{dt} \pm M\frac{dI_1}{dt}$$

where the ± sign appears consistently in both equations and depends on the geometry of the coils and the sense of current.

Work done in pushing charges dq_1 and dq_2 through each circuit, respectively, is $dW = -E_1\,dq - E_2\,dq_2$

$$= -L_1\frac{dI_1}{dt}dq_1 \mp M\frac{dI_2}{dt}dq_1 + L_2\frac{dI_2}{dt}dq_2 \mp M\frac{dI_1}{dt}dq_2$$

$$I_1 = \frac{dq_1}{dt},\ dq_1 = I_1\,dt,\ I_2 = \frac{dq_2}{dt},\ dq_2 = I_2\,dt$$

$$dW = L_1I_1dI_1 + L_2I_2dI_2 \mp (MI_1\,dI_2 + MI_2\,dI_1)$$

$$= L_1I_1dI_1 + L_2I_2dI_2 \mp Md(I_1I_2)$$

On integrating the above expression from 0 to final current, we have

$$U = \int dW = L_1\int_0^{I_1} I_1\,dI_1 + I_2\int_0^{I_2} I_2\,dI_2 \mp M\int_0^{I_1I_2} d(I_1I_2)$$

$$= \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 + MI_1I_2$$

LC OSCILLATIONS

A capacitor is charged to a potential difference of V_0 by connecting it across a battery and then is allowed to discharge through a pure inductor of inductance L.

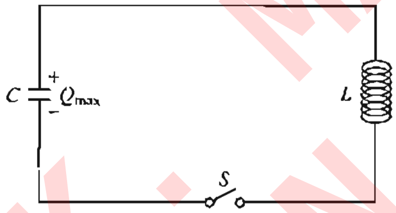


Fig. 8.120

Initial charge on the plates of the capacitor $q_0 = CV_0$.

At any instant, let the charge flown in the circuit be q and the current in the circuit be i. Applying Kirchhoff's law

$$\frac{q_0 - q}{C} - L\frac{dI}{dt} = 0$$

Differentiating w.r.t. time, we get $-\dfrac{dq}{dt} - LC\dfrac{d^2I}{dt^2} = 0$

$$\frac{d^2I}{dt^2} = -\frac{1}{LC} = -\omega,\ f = \frac{1}{2\pi\sqrt{LC}}$$

The charge q on the plates of the capacitor and current I in the circuit vary sinusoidally as

$$q = q_0\sin(\omega t + \phi) \text{ and } I = q_0\,\omega\cos(\omega t + \phi).$$

where ϕ is the initial phase and it depends on the initial situation of the circuit $\omega = \dfrac{1}{\sqrt{LC}}$.

The total energy of the system remains conserved.

$$\therefore\quad \frac{1}{2}CV^2 + \frac{1}{2}LI^2 = \text{constant} = \frac{1}{2}CV_o^2 = \frac{1}{2}LI_o^2$$

Illustration 8.38 **In an LC circuit as shown in Fig. 8.121, the switch is closed at $t = 0$: $Q_{max} = 100\ \mu C$; $L = 40$ mH; $C = 100\ \mu F$.**

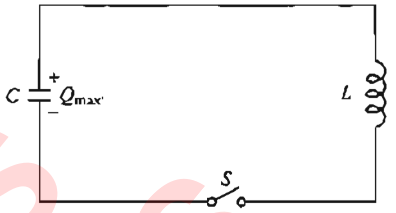


Fig. 8.121

a. Determine the equation for instantaneous charge on the capacitor.

b. Determine the equation for instantaneous current in the circuit.

c. Plot the following graphs:

- **a. q versus t**
- **b. i versus t**
- **c. U_E versus t**
- **d. U_B versus t**

Sol.

a. $q = Q_{max}\cos\omega t$

Here $Q_{max} = 100\ \mu C$; $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(40\times10^{-3})(100\times10^{-6})}}$

$= 500$ rad/s, $q = 100\cos(500t)$ (μC)

b. $i = \dfrac{dq}{dt} = -(100)(500)\sin(500t)$ (μA)

or $= -50000\sin(500t)$ (μA)

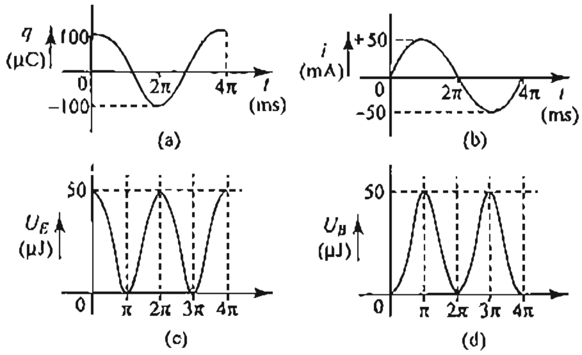


Fig. 8.122

c. $U_E = \dfrac{1}{2}\dfrac{Q_{max}^2}{C}\cos^2\omega t$; $U_B = \dfrac{1}{2}LI_{max}^2\sin^2\omega t$

Illustration 8.39 **Initially the 900 μF capacitor is charged to 100 V and the 100 μF capacitor is uncharged in Fig. 8.123. Then the switch S_2 is closed for a time t_1, after which it is opened and at the same instant switch S_1 is closed for a time t_2 and then opened. It is now found that the 100 μF capacitor is charged to 300 V. Find the minimum possible values of the time interval t_1 and t_2.**

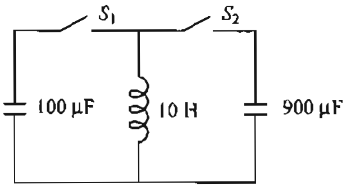


Fig. 8.123

Sol. Initial energy stored in the 900 μF capacitor is

$$U_1 = (1/2)\times 900\times 10^{-6}\times(100)^2 = 4.5\text{ J}$$

Finally, energy stored in the 100 μF capacitor is

$$U_2 = (1/2)\times 900\times 10^{-6}\times(300)^2 = 4.5\text{ J}$$

The entire energy of the 900 μF capacitor has been transferred to the 900 μF capacitor. First, electrical energy of the 900 μF capacitor is converted into magnetic energy in the inductor and then this energy is converted into electrical energy once again using S_2 and S_1 appropriately.

In an LC circuit the transfer of electrical energy into magnetic energy and vice versa takes place in a time $T/4$ where $T = 2\pi\sqrt{LC}$ is the time period of the electrical oscillations.

Thus, $\quad T_1 = 2\pi\sqrt{10\times 900\times 10^{-6}} = 0.6\text{ s}$

$$T_2 = 2\pi\sqrt{10\times 900\times 10^{-6}} = 0.2\text{ s}$$

Therefore, switch S_2 is first closed for time $0.6/4 = 0.15$ s, during which time the 900 μF capacitor gets fully discharged and the current in the inductor is fully established. Next the switch S_2 is opened and simultaneously switch S_1 is closed for time $0.2/4 = 0.05$ s during which time the current in the inductor disappear and the 100 μF capacitor gets fully charged.

After this time, the switch S_1 is also opened. The 100 μF capacitor is now charged to 300 V.

Thus, $\quad t_1 = 0.15\text{ s} \quad$ and $\quad t_2 = 0.05\text{ s}.$

Illustration 8.40 **The circuit shown in Fig. 8.124 is in the steady state with switch S_1 closed. At $t = 0$, S_1 is opened and switch S_2 is closed.**

a. Derive an expression for the charge on the capacitor C_2 as a function of time.

b. Determine the first instant t, when the energy in the inductor becomes one-third of that in the capacitor.

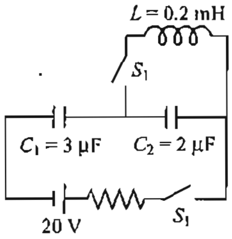


Fig. 8.124

Sol.

a. In the steady state, C_1 and C_2 are in series arrangement, their equivalent is

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} = 1.2\text{ μF}$$

Charge on the capacitor C_2, $Q_0 = C_{eq} = 1.2\times 20 = 24$ mC. When S_1 is opened and S_2 is closed. Capacitor C_2 starts discharging through the inductor and let at any time t, charge on the capacitor be Q.

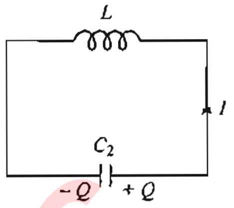


Fig. 8.125

Applying Kirchhoff's voltage law, we get

$$\frac{Q}{C_2} - L\frac{dI}{dt} = 0, \quad \frac{Q}{C_2} + L\frac{d^2Q}{dt^2} = 0 \quad \left[I = \frac{-dQ}{dt}\right]$$

The solution of this equation is $Q = Q_0\sin(\omega t + \phi)$ where

$$\omega = \frac{1}{\sqrt{LC_2}} = 5\times 10^4\text{ rad/s}$$

At $\quad t = 0,$

Hence $\quad \phi = \frac{\pi}{2}$

Thus the charge on the capacitor at any time t is

$$Q = Q_0\cos\omega t,\ U_E + U_B = \frac{Q_0^2}{2C_2}$$

At the time $t = t_1$, $U_B = \frac{1}{3}U_E$

Hence, $\quad U_E = \frac{3}{4}\left(\frac{1}{2}\frac{Q_0^2}{C_2}\right) \Rightarrow \frac{1}{2}\frac{Q^2}{C_2} = \frac{3}{4}\left(\frac{1}{2}\frac{Q_0^2}{C_2}\right)$

$$Q = \sqrt{\frac{3}{2}}Q_0 \quad \text{or} \quad Q_0\cos\omega t_1 = \sqrt{\frac{3}{2}}Q_0$$

$$\omega t_1 = \frac{\pi}{2} \quad \text{or} \quad t_1 = \frac{\pi}{6\omega} = 1.05\times 10^{-5} = 10.5\text{ μs}$$

Illustration 8.41 **In the circuit shown in Fig. 8.126, the battery has negligible internal resistance. Show that the current in the circuit through the battery rises instantly to its steady state value E/R when the switch is closed, provided that the resistance R is $\sqrt{L/C}$.**

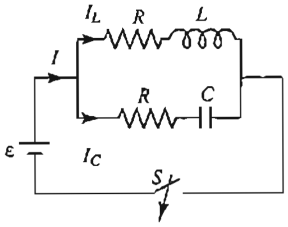


Fig. 8.126

Sol. Let the currents through inductive branch and capacitive branch be I_L and I_C respectively. Then the current through the battery, from KCL, is $I = I_L + I_C$.

Since the battery is connected in parallel to the RL and RC branches of the circuit, the current in the RL branch is unaffected by the presence of the RC branch, so

$$I_L = \frac{E}{R}[1 - e^{-(R/L)t}] \quad \text{and} \quad I_C = \frac{E}{R}e^{-(t/RC)}$$

Hence, the current through battery is $I = \frac{E}{R}[1 - e^{-(R/L)t} + e^{-(t/RC)}]$

If the current has to reach its final value E/R instantaneously, the exponential terms must cancel out, i.e., $e^{-t/\tau_L} = e^{-t/\tau_r}$, which is possible if $\tau_L = \tau_C$.

$L/R = RC, R = \sqrt{(L/C)}$

For all $t > 0$, this is the desired result.

Illustration 8.42 **An inductor of inductance 2.0 mH is connected across a charged capacitor of capacitance 5.0 μF and the resulting LC circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor and I the current in the circuit. It is found that the maximum value of charge Q is 200 μC.**

a. When $Q = 100\,\mu C$, what is the value of $\left|\frac{dI}{dt}\right|$?

b. When $Q = 200\,\mu C$, what is the value of I?

c. Find the maximum value of I.

d. When I is equal to one-half its maximum value, what is the value of $|Q|$? **(IIT-JEE, 1998)**

Sol.
$$L = 2.0\text{ mH} = 2.0 \times 10^{-3}\text{ H}$$
$$C = 5.0\,\mu\text{F} = 5.0 \times 10^{-6}\text{ F}$$
$$Q_{max} = 20\,\mu\text{C} = 200 \times 10^{-6}\text{ C}$$

In an LC circuit, energy transfer continues from inductance to capacitance and vice versa.

a. By Kirchhoff's law in an LC circuit

$$\frac{Q}{C} + L\frac{dI}{dt} = 0 \Rightarrow \frac{dI}{dt} = -\frac{Q}{LC}$$

$$\therefore \quad \left|\frac{dI}{dt}\right| = \frac{Q}{LC} = \frac{100 \times 10^{-6}}{2.0 \times 10^{-3} \times 5.0 \times 10^{-6}} = 10^4\text{ A/s}$$

b. When $Q = 200\,\mu C$, the entire energy of circuit resides in capacitance. That is, no energy is stored in inductance

$$\therefore \quad \frac{1}{2}LI^2 = 0 \Rightarrow I = 0$$

c. Maximum value of I is given by

$$\frac{1}{2}LI_{max}^2 = \frac{Q_{max}^2}{2C} \Rightarrow I_{max} = \frac{1}{\sqrt{LC}}Q_{max}$$

$$\text{or} \quad I_{max} = \frac{1}{\sqrt{(2.0 \times 10^{-3}) \times (5.0 \times 10^{-6})}} \times 200 \times 10^{-6} = 2\text{ A}$$

d. Given $\quad I = \frac{I_{max}}{2} = \frac{2}{2} = 1\text{ A}$

Then again from the conservation of energy,

$$\frac{1}{2}LI^2 + \frac{Q^2}{2C} = \frac{1}{2}LI_{max}^2;\ \frac{Q^2}{2C} = \frac{1}{2}L\left(I_{max}^2 - I^2\right)$$

$$Q = \sqrt{LC\left(I_{max}^2 - I^2\right)}$$
$$= \sqrt{(2.0 \times 10^{-3} \times 5.0 \times 10^{-6})(2^2 - 1^2)}$$
$$= 10^{-4}\sqrt{3}\text{ C} = 1.72 \times 10^{-4}\text{ C}$$
$$= 173.2\,\mu\text{C}$$

Concept Application Exercise 8.3

1. The magnetic field at all points within a circular region of radius R is uniform in space and directed into the plane of the page in Fig. 8.127. (The region could be a cross section inside the windings of a long, straight solenoid.) If the magnetic field is increasing at a rate dB/dt, what are the magnitude and direction of the force on a stationary positive point charge q located at points a, b and c? (Point a is a distance r above the centre of the region, point b is a distance r to the right of the centre and point c is at the center of the region.)

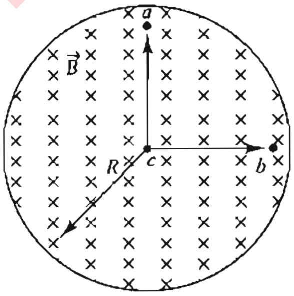


Fig. 8.127

2. Fig. 8.128 shows two circular regions R_1 and R_2 with radii r_1 = 21.2 cm and r_2 = 32.3 cm, respectively. In R_1 there is a uniform magnetic field B_1 = 48.6 mT into the page and in R_2 there is a uniform magnetic field B_2 = 77.2 mT out of the page (ignore any fringing of these fields). Both fields are decreasing at the rate 8.50 mT/s. Calculate the integral $\oint \vec{E} \cdot d\vec{s}$ for each of the three identical paths.

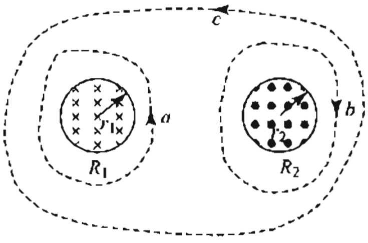


Fig. 8.128

3. Fig. 8.129 shows five lettered regions in which a uniform magnetic field extends directly either out of the page (as in region *a*) or into the page. The field is increasing in magnitude at the same steady rate in all five regions; the regions are identical in area. Also shown are four numbered paths along which $\oint \bar{E}\cdot d\bar{s}$ has the magnitudes given below in terms of a quantity mag. Determine whether the magnetic fields in regions *b* through *e* are directed into or out of the page.

Path:	1.	2.	3.	4.
$\oint \bar{E}\cdot d\bar{s}$	mag	2(mag)	3(mag)	0

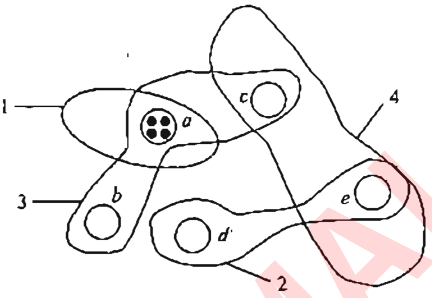


Fig. 8.129

4. A magnetic field directed into the page changes with time according to $B = (0.0300\,t^2 + 1.40)$T, where *t* is in seconds. The field has a circular cross section of radius $R = 2.50$ cm. What are the magnitude and direction of the electric field at point P_1 when $t = 3.00$ s and $r_1 = 0.0200$ m?

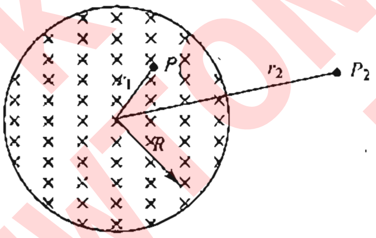


Fig. 8.130

5. Fig. 8.131 shows an *LCR* circuit. When the switch *S* is closed the current through resistor *R*, inductor *L* and capacitor *C* are I_1, I_2 and I_3, respectively. Determine the values of I_1, I_2 and I_3

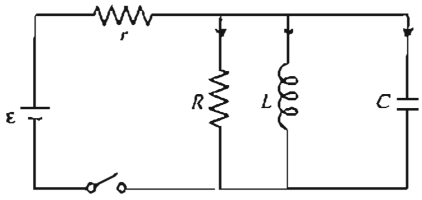


Fig. 8.131

a. at $t = 0$ **b.** at $t = \infty$

6. It has been proposed to use large inductors as energy storage devices.

a. How much electrical energy is converted to light and thermal energy by a 200-Ω light bulb in one day?

b. If the amount of energy calculated in part (a) is stored in an inductor in which the current is 80.0 A, what is the inductance?

7. A 1.00 kΩ resistor is connected in series with a 10.0-mH inductor, a 30.0 V battery and an open switch. At time $t = 0$, the switch is suddenly closed.

a. What is the maximum current in this circuit and when does it occur?

b. What are the voltage drops across the inductor and across the resistor 20.0 μs after the switch is closed ?

c. On a single set of axes, sketch the voltage across the resistor and the voltage across the inductor as functions of time. Also, sketch a graph of the current in the circuit as a function of time.

8. A capacitor with capacitance 6.00×10^{-5} F is charged by connecting it to a 12.0-V battery. The capacitor is disconnected from the battery and connected across an inductor with $L = 1.50$ H.

a. What are the angular frequency *w* of the electrical oscillations and the period of these oscillations (the time for one oscillation)?

b. What is the initial charge on the capacitor?

c. How much energy is initially stored in the capacitor?

d. What is the charge on the capacitor 0.0230 s after the connection to the inductor is made? Interpret the sign of your answer.

e. At the time given in part (d), what is the current in the inductor? Interpret the sign of your answer.

f. At the time given in part (d), how much electrical energy is stored in the capacitor and how much is stored in the inductor?

9. In the circuit shown in Fig. 8.132, $E = 10$ V, $R_1 = 5.0\ \Omega$, $R_2 = 10\ \Omega$ and $L = 5.0$ H. For the two separate conditions, (I) switch *S* is just closed and (II) switch *S* is closed for a long time, calculate

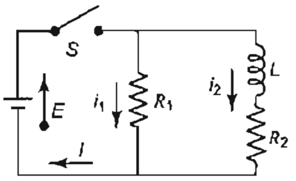


Fig. 8.132

a. the current i_1 through R_1,

b. the current i_2 through R_2,

c. the current *i* through the switchs,

d. the potential difference across R_2,

e. the potential difference across *L*,

f. di_2/dt.

10. In Fig. 8.133, the switch is closed for $t > 0$ and steady-state conditions are established. The switch is thrown open at $t = 0$.

a. Find the initial voltage E_0 across *L* just after $t = 0$. Which end of the coil is at the higher potential: *a* or *b* ?

b. Make freehand graphs of the currents in R_1 and R_2 as a function of time, treating the steady-state directions as positive. Show values before and after $t = 0$.

c. How long after $t = 0$ does the current in R_2 have the value 2.00 mA?

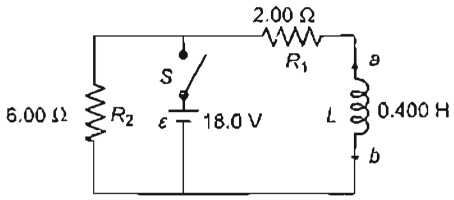


Fig. 8.133

11. The switch in Fig. 8.134 is closed at time $t = 0$. Find the current in the inductor and the current through the switch as functions of time thereafter.

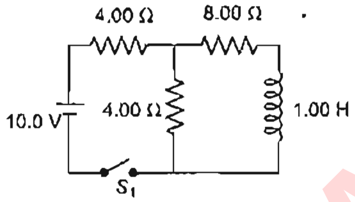


Fig. 8.134

12. AB is a part of circuit. Find the potential difference $v_A - v_B$ if

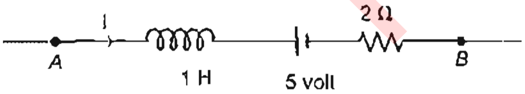


Fig. 8.135

a. current $i = 2$ A, and is constant;

b. current $i = 2$ A, and is increasing at the rate of 1 A/s;

c. current $i = 2$ A, and is decreasing at the rate 1 A/s.

13. A circuit contains an ideal cell and an inductor with a switch. Initially, the switch is open. It is closed at $t = 0$. Find the current as a function of time.

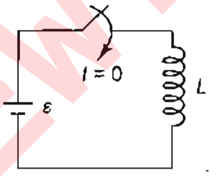


Fig. 8.136

14. In the following circuit (Fig. 8.137), the switch is closed at $t = 0$. Find the currents i_1, i_2, i_3 and $\frac{di_3}{dt}$ at $t = 0$ and at $t = \infty$. Initially, all currents are zero.

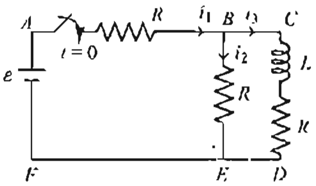


Fig. 8.137

15. In a circuit (see the solution), S_1 remains closed for a long time and S_2 remains open. Now S_2 is closed and S_1 is opened. Find out the di/dt just after that moment.

16. At $t = 0$, switch S is closed (shown in Fig. 8.138) after a long time suddenly the inductance of the inductor is made η times lesser (L/η) then its initial value, find out instant current just after the operation.

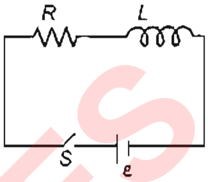


Fig. 8.138

17. Which of the two curves shown has less time constant.

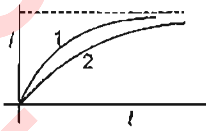


Fig. 8.139

18. Two insulated wires are wound on the same hollow cylinder, so as to form two solenoids sharing a common air-filled core. Let l be the length of the core, A the cross-sectional area of the core, N_1 the number of times the first wire is wound around the core and N_2 the number of times the second wire is wound around the core. Find the mutual inductance of the two solenoids, neglecting the end effects.

19. Find the mutual inductance of two concentric coils of radii a_1 and a_2 ($a_1 << a_2$) if the planes of the coils are same.

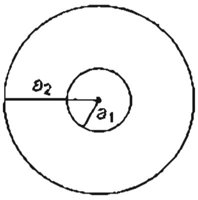


Fig. 8.140

20. Solve problem 19, if the planes of the coils are perpendicular.

21. Solve problem 19, if the planes of the coils make an angle θ with each other.

22. Fig. 8.141 shows two concentric coplanar coils with radii a and b ($a << b$). A current $i = 2t$ flows in the smaller loop. Neglecting self-inductance of the larger loop,

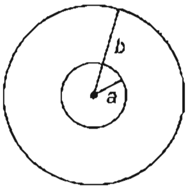


Fig. 8.141

a. find the mutual inductance of the two coils;
b. find the e.m.f. induced in the larger coil;
c. if the resistance of the larger loop is R, then find the current in it as a function of time.

23. In problem 22, if a capacitor of capacitance C is also connected in the larger loop as shown in Fig. 8.142, find the charge on the capacitor as a function of time.
24. If the current in the inner loop changes according to $i = 2t^2$ (Fig. 8.142), then find the current in the capacitor as a function of time.

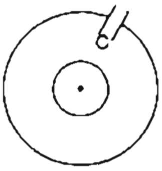


Fig. 8.142

Solved Examples

Example 8.1 **Two infinitely long parallel wires carrying currents $I = I_0 \sin \omega t$ in opposite direction are placed at a distance $3a$ apart. A square loop of side a of negligible resistance with a capacitor of capacitance C is placed in the plane of wires as shown. Find the maximum current in the square loop. Also sketch the graph showing the variation of charge on the upper plate of the capacitor as a function of time for one complete cycle taking anticlockwise direction for the current in the loop as positive.** (IIT-JEE, 2000)

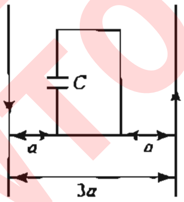


Fig. 8.143

Sol. The magnetic field due to both wires on elementary strip of width dx at a distance from left wire is

$$B = \frac{\mu_0 I}{2\pi x} + \frac{\mu_0}{2\pi}\frac{I}{(3a-x)} \text{ (upward)}$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{x} + \frac{1}{3a-x}\right]$$

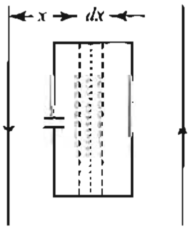


Fig. 8.144

Magnetic flux linked with this strip

$$d\phi = \vec{B}\cdot d\vec{S} = \frac{\mu_0 I}{2\pi}\left[\frac{1}{x} + \frac{1}{3a-x}\right] a\, dx$$

Total magnetic flux linked with this strip

$$\phi = \frac{\mu_0 I a}{2\pi}\int_0^{2a}\left(\frac{1}{x} + \frac{1}{3a-x}\right)dx$$

$$= \frac{\mu_0 I a}{2\pi}\left[\log_e x - \log_e(3a-x)\right]_a^{2a}$$

$$= \frac{\mu_0 I a}{\pi}\log_e^{2}$$

$$\Rightarrow \quad \phi = \frac{\mu_0 a}{\pi}\log_e 2\,(I_0 \sin \omega t) \text{ (since } I = I_0 \sin \omega t)$$

Magnitude of induced e.m.f.

$$\varepsilon = \frac{d\phi}{dt} = \frac{\mu_0 I_0 a}{\pi}\log_e 2\,(\omega \cos \omega t).$$

∴ Maximum induced e.m.f.

$$\varepsilon_0 = \frac{\mu_0 I_0 a\omega}{\pi}\log_e 2.$$

Charge on the capacitor,

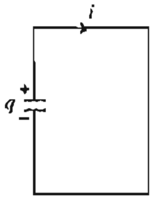


Fig. 8.145

$$q = C\varepsilon = C\varepsilon_0 \cos \omega t$$

$$= \frac{C\mu_0 I_0 a\omega \log_e 2}{\pi}\cos \omega t = q_0 \cos \omega t$$

where $\quad q_0 = \frac{\mu_0 I_0 a\omega C}{\pi}\log_e 2$

Magnetic flux linked with the loop ∝ sin ωt, i.e., at $t = 0$, the magnetic flux is increasing (B is upward); so the current will be induced to oppose the increase of current, i.e., in the loop the current will be clockwise. So upper plate will be positive. The graph of variation of charge for full cycle is shown in Fig. 8.146.

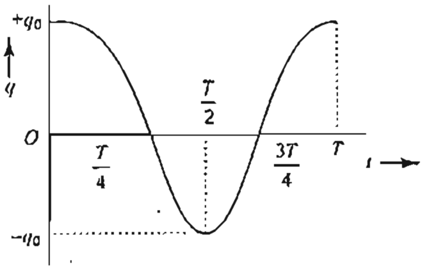


Fig. 8.146

Example 8.2 A metal bar AB can slide on two parallel thick metallic rails separated by a distance l. A resistance R and an inductance L are connected to the rails as shown in Fig 8.147. A long straight wire carrying a constant current I_0 is placed in the plane of rails and perpendicular to them as shown. The bar AB is made to slide on the rails away from the wire. Answer the following questions:

a. Find a relation among i, $\frac{di}{dt}$ and $\frac{d\varphi}{dt}$, where i is the current in the curcuit and ϕ is the flux of the magnetic field due to the long wire through the circuit.

b. It is observed that at time $t = T$, the metal bar AB is at a distance of $2x_0$ from the long wire and the resistance R carries a current i_1. Obtain an expression for the net charge that has flown through resistance R from $t = 0$ to $t = T$

c. The bar is suddenly stopped at time T. The current through resistance R is found to be $\frac{i_1}{4}$ at time $2T$. Find the value of L/R in terms of the othyer given quantities. (IIT-JEE, 2002)

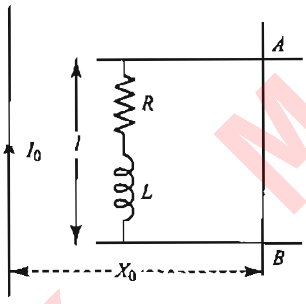


Fig. 8.147

Sol.

a. When bar AB slides away, the magnetic force acts on free electrons along negative Y-axis, so electrons move from end A to end B, making end A at positive potential relative to end B.

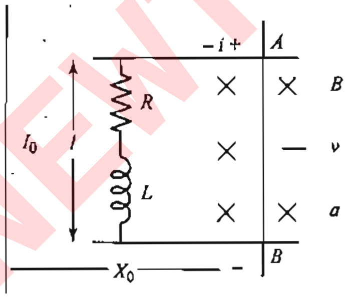


Fig. 8.148

Let ε be induced e.m.f. According to Kirchhoff's second law,

$$-Ri - L\frac{di}{dt} + \varepsilon = 0 \quad \Rightarrow \quad \varepsilon = Ri + L\frac{di}{dt}$$

But $\varepsilon = \frac{d\phi}{dt}$ (numerically)

$$\therefore \quad \frac{d\phi}{dt} = Ri + L\frac{di}{dt} \qquad \text{(i)}$$

$\frac{d\phi}{dt}$ is taken positive, because direction of e.m.f. induced has already been investigated.

Moreover, when $\frac{d\phi}{dt}$ increases, current I increases; this ensures positive sign of $\frac{d\phi}{dt}$.

b. Equation (i) can be expressed as

$$d\phi = Ri\,dt + L\,di$$

$$\Rightarrow \quad d\phi = R\,dq + Ldi$$

Integrating, we get

$$\int_0^T d\phi = R\int_0^q dq + L\int_0^{i_1} di$$

$$[\phi]_{t=0}^{T} = Rq + Li_1$$

$\therefore$ Charge flown from $t = 0$ to $t = T$ will be

$$q = \frac{1}{R}[\phi(T) - \phi(0)] - \frac{Li_1}{R} \qquad \text{(ii)}$$

Change in flux during the displacement of bar from $x = x_0$ to $x = 2x_0$ in time T is

$$\phi(T) - \phi(0) = \int_{x_0}^{2x_0} BdA = \int_{x_0}^{2x_0} \frac{\mu_0 I_0}{2\pi x} ldx$$

$$= \frac{\mu_0 I_0 l}{2\pi}\log_e 2 \qquad \text{(iii)}$$

$\therefore$ From (ii), charge flown

$$q = \frac{\mu_0 I_0 l}{2\pi R}\log_e 2 - \frac{Li_1}{R} \qquad \text{(iv)}$$

c. The equation of decay of current in an LR circuit is given by

$$i = i_0 e^{-Rt/L} \qquad \text{(v)}$$

Given $\quad i = \frac{i_1}{4}, i_0 = i_1, t = 2T - T = T$

$\therefore$ Substituting these values in equation (v), we get

$$\frac{i_1}{4} = i_1 e^{-RT/L} \quad \Rightarrow \quad e^{-RT/L} = \frac{1}{4}$$

or $\quad \frac{RT}{L} = \log_e 4 \quad \therefore \quad \frac{L}{R} = \frac{T}{\log_e 4}$

Example 8.3 A thermocole vessel contains 0.5 kg of distilled water at 30°C. A metal coil of area 5×10^{-3} m^2, number of turns 100, mass 0.06 kg and resistance 1.6 Ω is lying horizontally at the bottom of the vessel. A uniform, time varying magnetic field is set up to pass vertically through the coil at time $t = 0$. The field is first increased from zero at 0.8 T at a constant rate between 0 and 0.2 s and then decreased to zero at the same rate between 0.2 and 0.4 s. The cycle is repeated 1200 times. Make sketches of the current through the coil and the power dissipated in the coil as function of time for the first two cycles. Clearly indicate the magnitude of the quantities on the axes. Assume that no heat is lost to the vessel or the surroundings. (IIT-JEE, 2000)

Sol. e.m.f. induced

$$\varepsilon = -\frac{d\phi}{dt}$$

$$= -\frac{d}{dt}(NBA\cos 0°)$$

$$= -NA\frac{dB}{dt}$$

$$= -100 \times 5 \times 10^{-3} \times \frac{0.8}{0.2} = -2\ \text{V}$$

It is negative between 0 and 0.2 s when field is increased and positive between 0.2 and 0.4 s when field is decreased.

Current induced $i = \frac{\varepsilon}{R} = \frac{2}{1.6} = 1.25$ A

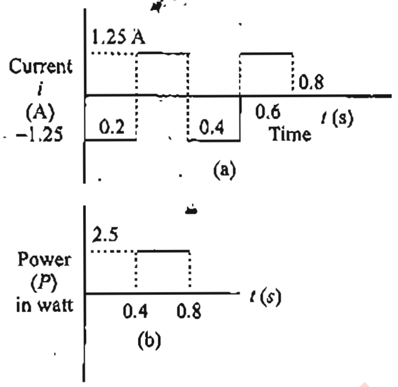


Fig. 8.149

It is negative between 0 and 0.2 s and positive between 0.2 and 0.4 s.

Power dissipated $= i^2R = (1.25)^2 \times 1.63$ W

$= 2.5$ W

Total energy supplied $= Pt$

$= 2.5 \times 12000 \times 0.4$ J

$= 12 \times 10^3$ J

Now from the conservation of energy

$$12 \times 10^3 = m_1c_1\Delta\theta + m_2c_2\Delta\theta$$

$$\Rightarrow \quad 12 \times 10^3 = (m_1c_1 + m_2c_2)\Delta\theta$$

$$\Rightarrow \quad \Delta\theta = \frac{12 \times 10^3}{m_1c_1 + m_2c_2}$$

$$= \frac{12 \times 10^3}{0.5 \times 4200 + 0.06 \times 500} = \frac{12 \times 10^3}{2130} = 5.6°$$

$$\Delta\theta = \theta_2 - \theta_1$$

$\Rightarrow$ Final temperature

$$\theta_2 = \theta_1 + \Delta\theta$$

$$= 30 + 5.6 = 35.6°\text{C}$$

Example 8.4 **A circuit containing a two position switch S is shown in Fig. 8.150.** **(IIT-JEE, 1991)**

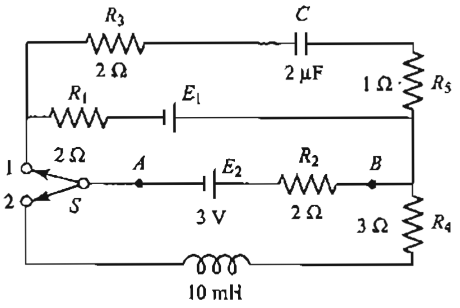


Fig. 8.150

a. The switch S is in position 1. Find the potential difference and the rate of production of heat in joule.

b. If now the switch S is put in position 2 at $t = 0$ find the steady current and the time when current is half in the steady value. Also calculate the energy stored in the inductor L at that time.

Sol.

a. The capacitor offers infinite resistance to dc in the steady state, therefore the current in capacitor branch is zero. The equivalent circuit, in steady state, when switch is in position (1) is given in Fig. 8.151.

The distribution of current according to Kirchhoff's first law is shown in Fig. 8.151. Applying Kirchhoff's second law to mesh *abcda*,

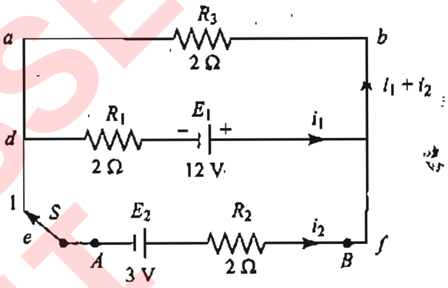


Fig. 8.151

$$i_1 \times 2 + (i_1 + i_2) \times 2 = 12$$

or $$4i_1 + 2i_2 = 12$$

or $$2i_1 + i_2 = 6 \quad \text{(i)}$$

Applying Kirchhoff's second law to mesh *abfea*,

$$i_2 \times 2 + (i_1 + i_2)\,2 = 3$$

or $$2i_1 + 4i_2 = 3$$

Solving (i) and (ii), we get

$$i_1 = 3.5\ \text{A},\ i_2 = -1\ \text{A}$$

∴ Potential difference between A and B is

$$V_B - V_A = V = E_2 - i_2R_2 = 3 - (-1) \times 2 = 5\ \text{V}$$

$$\therefore \quad V_A - V_B = -5\ \text{V}$$

b. When the switch S is put in position 2, the equivalent circuit takes the form shown in Fig. 8.152.

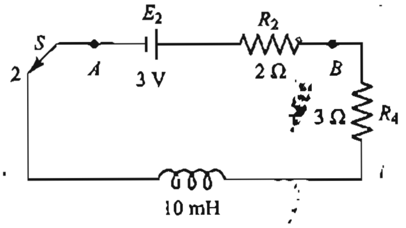


Fig. 8.152

Total resistance of the circuit,

$$R = R_2 + R_4 = 2 + 3 = 5\ \Omega$$

$$L = 10\ \text{mH} = 10 \times 10^{-3}\ \text{H}$$

In steady state, there is no role of inductor L.

$\therefore$ Steady current

$$i_0 = \frac{E}{R} = \frac{3}{5} = 0.6 \text{ A}$$

The growth of current in RL circuit is given by

$$i = i_0\left(1 - e^{-\frac{R}{L}t}\right)$$

i.e.,
$$\frac{i}{i_0} = \left(1 - e^{-\frac{R}{L}t}\right)$$

Given $i = \frac{i_0}{2}$, i.e., $\frac{i}{i_0} = \frac{1}{2}$ after time t.

$$\therefore \quad \frac{1}{2} = 1 - e^{-\frac{R}{L}t}$$

or
$$e^{-\frac{R}{L}t} = \frac{1}{2}$$

Taking log, we get

$$-\frac{R}{L}t \log_e e = \log_e 1 - \log_e 2$$

i.e.,
$$\frac{R}{L}t = \log_e 2$$

i.e.,
$$t = \frac{L}{R}\log_e 2$$

$$\therefore \quad t = \frac{10 \times 10^{-3}}{5} \times 2.3026 \times 0.3010$$
$$= 1.386 \times 10^{-3} \text{ s}$$

The current at the instant,

$$i = \frac{i_0}{2} = 0.3 \text{ A}$$

$\therefore$ Energy stored,

$$U = \frac{1}{2}Li^2$$
$$= \frac{1}{2} \times 10 \times 10^{-3} \times (0.3)^2$$
$$= 4.5 \times 10^{-4} \text{ J.}$$

Example 8.5 **Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at two ends by resistances R_1 and R_2 as shown in Fig. 8.153. A horizontal metallic bar L of mass 0.2 kg slides without friction vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6 T perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the powers dissipated in R_1 and R_2 are 0.76 and 1.2 W, respectively. Find the terminal velocity of the bar L and the values of the bar L and the values of R_1 and R_2.**

(IIT-JEE, 1994)

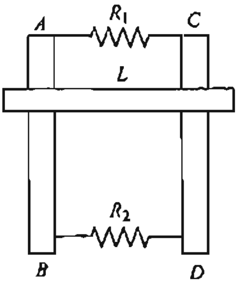


Fig. 8.153

Sol. Let e be the induced e.m.f. and i the induced current in the bar.

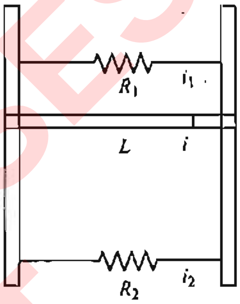


Fig. 8.154

The forces acting on the bar are

1. Weight mg acting vertically downward.
2. Magnetic force BiL acting vertically upward.

When these forces become numerically equal, the bar is in dynamical equilibrium and attains terminal velocity v (say)

i.e., $mg = BiL$

Substituting $m = 0.2$ kg, $L = 1$ m, $B = 0.6$ T, we get

$$0.2 \times 9.8 = 0.6 i \times 1$$

or
$$i = \frac{0.2 \times 9.8}{0.6} = \frac{49}{15} \text{ A} \qquad \text{(i)}$$

Let i_1 and i_2 be currents in R_1 and R_2, respectively. Then

$$i = i_1 + i_2$$

or
$$i_1 + i_2 = \frac{49}{15} \text{ A} \qquad \text{(ii)}$$

R_1 and R_2 are in parallel with the bar.

$$\therefore \quad e = R_1 i_1 = R_2 i_2 \qquad \text{(iii)}$$

Power in R_1, $P_1 = ei_1 = 0.76$ W (iv)

Power in R_2, $P_2 = ei_2 = 1.2$ W (v)

Dividing (iv) by (v), we get

$$\frac{i_1}{i_2} = \frac{0.76}{1.2} = \frac{19}{30}$$

$$i_1 = \frac{19}{30} i_2 \qquad \text{(vi)}$$

Then equation (ii) gives

$$\frac{19}{30} \times 2 = \frac{19}{15} \text{ A}$$

$$\frac{19}{30}i_2 + i_2 = \frac{49}{15} \quad \text{or} \quad \frac{49}{30}i_2 = \frac{49}{15}$$

or $\qquad i_2 = 2$ A

From (vi)

$$i_1 = \frac{19}{30} \times 2 = \frac{19}{15}\text{ A}$$

Now from (iv)

$$ei_1 = 0.76$$

or
$$e = \frac{0.76}{i_1} = \frac{0.76 \times 15}{19} = 0.6\text{ V}$$

Induced e.m.f. $e = BvL$

Terminal velocity $v = \frac{e}{BL} = \frac{0.6}{0.6 \times 1} = 1$ m/s

Again from (iii)

$$R_1 = \frac{e}{i_1} = \frac{0.6 \times 15}{19} = \frac{9}{19}\,\Omega = 0.47\,\Omega$$

$$R_2 = \frac{e}{i_2} = \frac{0.6}{2} = 0.3\,\Omega$$

Example 8.6 **A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is *L*. A conducting massless rod of resistance *R* can slide on the rails frictionlessly. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass *m* tied to the other end of the string hangs vertically. A constant magnetic field *B* exists perpendicular to the table if the system is released from rest. Calculate**

(IIT-JEE, 1997)

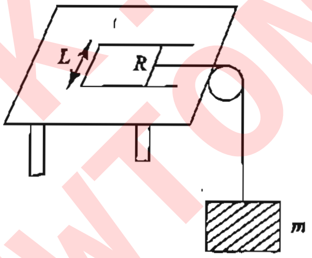


Fig. 8.155

a. the terminal velocity achieved by the rod, and

b. the acceleration of the mass at the instant when the velocity of the rod is half the terminal velocity.

Sol.

a. If v is the instantaneous velocity of the rod at any time *t*, the induced e.m.f. will be $\varepsilon = BvL$.

∴ Induced current in rod

$$i = \frac{\varepsilon}{R} = \frac{RvL}{R} \tag{i}$$

Due to this current the rod in magnetic field *B* will experience a force

$$F = BiL \text{ opposite to motion.} \tag{ii}$$

If v_T is terminal velocity of rod, and *T* the tension is string, then free-body diagrams of rod and mass *m* are shown in Fig. 8.156.

So equation of motion of mass *m* is

$$Mg - T = ma \tag{iii}$$

For terminal velocity, $a = 0$

$$Mg - T = 0 \quad \Rightarrow \quad T = mg \tag{iv}$$

Equation of motion of rod is

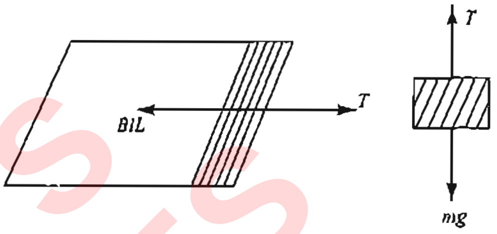


Fig. 8.156

$$T - BiL = 0 \times a = 0 \text{ (as rod is massless)}$$

$$T = BiL. \tag{v}$$

Using (iv), we get

$$mg = BiL$$

$$mg = B\left(\frac{Bv_T L}{R}\right)L$$

$$\Rightarrow \qquad v_T = \frac{mgR}{B^2L^2} \tag{vi}$$

b. When velocity of the rod is half the terminal velocity

$$v = \frac{v_T}{2} = \frac{1}{2}\frac{mgR}{B^2L^2} \tag{vii}$$

From (iii) acceleration

$$A = g - \frac{T}{m}$$

$$= g - \frac{BiL}{m} \qquad \text{(using – v)}$$

$$= g - \frac{BL}{m}\frac{BL}{R}\frac{\frac{1}{2}mgR}{B^2L^2}$$

$$= g - \frac{g}{2} = \frac{g}{2}$$

Example 8.7 **A metal rod *OA* of mass *m* and length *r* is kept rotating with a constant angular speed in a vertical plane about a horizontal axis at the end *O*. The free end *A* is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant magnetic induction is applied perpendicular and into the plane of rotation as shown in Fig. 8.157. An inductor *L* and an external resistance *R* are connected through a switch *S* between the point *O* and a point *C* on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open.**

(IIT-JEE, 1998)

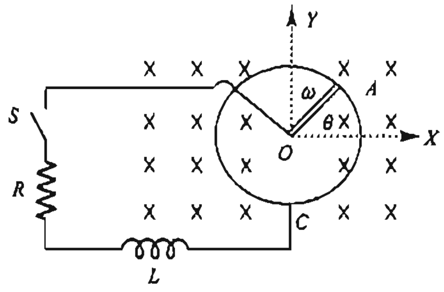


Fig. 8.157

a. What is the induced e.m.f. across the terminals of the switch?

b. The switch S is closed at time $t = 0$.

i. Obtain an expression for the current as a function of time.

ii. In the steady state, obtain the time dependence of the torque required to maintain the constant angular speed, given that the rod OA was along the positive X-axis at $t = 0$.

Sol.

a. The rod OA may be supposed to be divided into a large number of length elements. The angular velocity (ω) of each element is same, while linear velocities of elements are different. Consider one element of length dx at a distance x from centre O. The linear speed of element $v = x\omega$. As $\vec{B} \cdot d\vec{x}$ and $\vec{v}$ are mutually perpendicular, the e.m.f. induced across the element $d\varepsilon = Bv\, dx = B(x\omega)\, dx$.

∴ The e.m.f. induced across the whole rod,

$$\varepsilon = \int_0^r Bx\omega\, dx$$

$$= B\omega\left[\frac{x^2}{2}\right]_0^r = \frac{1}{2}B\omega r^2 \qquad \text{(i)}$$

b. When rod rotates anticlockwise (as shown), the end O becomes positive and A negative. As resistance of ring and rod is negligible, therefore the equivalent circuit is shown in Fig. 8.158. When switch S is closed, let i be the current and $\frac{di}{dt}$ the rate of change of current in the circuit, then from Kirchhoff's second law, the equation of e.m.f.s

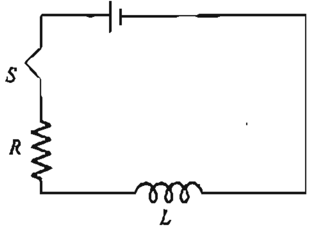


Fig. 8.158

$$E - L\frac{di}{dt} = Ri$$

$$\Rightarrow \quad L\frac{di}{dt} = E - Ri$$

$$\frac{di}{E - Ri} = \frac{dt}{L}$$

Integrating, we get

$$\int_0^I \frac{di}{E - Ri} = \int_0^t \frac{dt}{L} \qquad \text{(since at } t = 0,\ i = 0 \text{ and at } t = t,\ I = I\text{)}$$

$$\text{or} \quad \left[\frac{\log_e(E - Ri)}{-R}\right]_0^I = \left[\frac{t}{L}\right]_0^t$$

$$\Rightarrow \quad \frac{\log_e(E - RI) - \log_e E}{-R} = \frac{t}{L}$$

$$\text{or} \quad \log_e \frac{E - RI}{E} = \frac{Rt}{L}$$

$$\text{or} \quad 1 - \frac{R}{E}I = e^{-Rt/L}$$

$$\Rightarrow \quad I = \frac{E}{R}(1 - e^{-Rt/L})$$

$$\text{Using (i),} \quad I = \frac{Br^2\omega}{2R}(1 - e^{-Rt/L})$$

ii. According to Maxwell's right-hand rule, the direction of the induced current in the rod is from periphery to centre. The force on small length dx of current-carrying wire

$$d\vec{F} = -I d\vec{x} \times \vec{B}$$

Torque on element

$$d\vec{\tau}_1 = \vec{x} \times d\vec{F} = \vec{x} \times (-I d\vec{x} \times \vec{B})$$

$$= IBx\, dx$$

$$\therefore \text{Torque } \tau_1 = BI\left[\frac{x^2}{2}\right]_0^r = BI\frac{r^2}{2}$$

$$= B\frac{B\omega r^2}{2R}(1 - e^{-Rt/L})\frac{r^2}{2}$$

$$= B\frac{B^2\omega r^4}{4R}(1 - e^{-Rt/L})$$

In steady state ($t \to \infty$)

$$\text{So} \quad \tau_1 = \frac{B^2\omega r^4}{4R} \text{ (constant).}$$

Torque due to the weight of rod,

$$\tau_2 = mg\left(\frac{r}{2}\right)\cos\omega t \qquad (\because q = \omega t)$$

∴ Net torque

$$\tau = \tau_1 + \tau_2$$

$$= \frac{B^2\omega r^4}{4R} + \frac{mgr\cos\omega t}{2}$$

EXERCISES

Subjective Type

Solutions on page 8.98

1. In Fig. 8.159, a long thin wire carrying a varying current $i = i_0 \sin \omega t$ lies at a distance y above one edge of a rectangular wire loop of length L and width W lying in the x-z plane. What e.m.f. is induced in the loop?

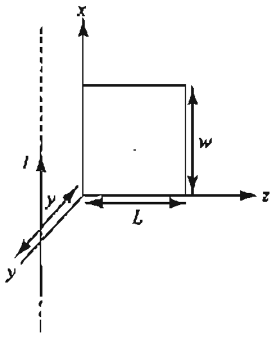


Fig. 8.159

2. A wire is bent into three circular segment of radius $r = 10$ cm as shown in Fig. 8.160. Each segment is a quadrant of a circle, ab, lying in the x-y plane, bc lying in the yz plane and ca lying in the zx plane.

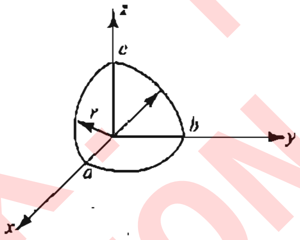


Fig. 8.160

a. If a magnetic field B points in the positive x direction, what is the magnitude of the e.m.f. developed in the wire when B increases at the rate of 3 m T/s?

b. What is the direction of the current in the segment bc.

3. Three identical wires are bent into semi-circular arcs each of radius r. These arcs are connected with each other to form a closed mesh such that one of them lies in x-y plane, one in y-z plane and the other in z-x plane as shown in Fig. 8.161. In the region of space a uniform magnetic field of induction $\vec{B} = B_o(\hat{i} + \hat{j})$ exists, whose magnitude increases at a constant rate $\frac{dB}{dt} = \alpha$. Calculate the magnitude of e.m.f. induced in the mesh and mark the direction of flow of induced current in the mesh shown in Fig. 8.159.

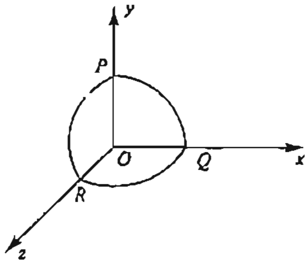


Fig. 8.161

4. In Fig. 8.162, the four rods have λ resistance per unit length. The arrangement is kept in a magnetic field of constant magnitude B and directed perpendicular to the plane of the figure and direction inwards. Initially, the sides as shown form a square. Now each wire starts moving with constant velocity v towards opposite wire.

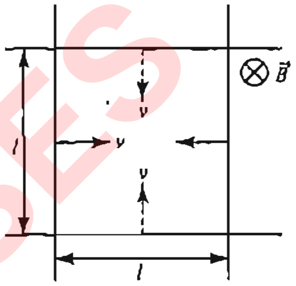


Fig. 8.162

Find as a function of time:

a. induced e.m.f. in the circuit.

b. induced current in the circuit with direction.

c. force required on each wire to keep its velocity constant.

d. total power required to maintain constant velocity.

e. thermal power developed in the circuit.

5. An electric circuit is composed of the three conducting rods MO, ON and PQ, as shown in Fig. 8.163. The resistance of the rods per unit length is λ. The rod PQ slides, as shown in the figure, at a constant velocity v, keeping its tilt angle relative to ON and MO fixed at 45°. At each instance, the circuit is closed. The whole system is embedded in a uniform magnetic field B, which is directed perpendicularly into the page. Compute the time-dependent induced electric current.

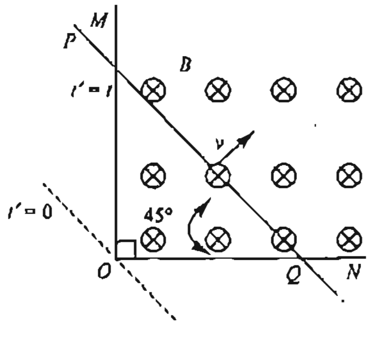


Fig. 8.163

6. The wire loop, shown in Fig. 8.164, is made by taking a flat rectangular loop of sides 10 cm and 20 cm bending the long sides at their midpoints to produce two mutually perpendicular square parts. The loop is placed in an oscillating magnetic field $B = B_0 \sin 2\pi\nu t$, with $B_0 = 1.2 \times 10^{-3}$ T and $\nu = 60$ Hz. The magnetic field is inclined at angle θ with xz plane.

a. Express the e.m.f. around the loop as a function of time and the angle θ.

b. For what angle θ does the induced e.m.f. have the largest amplitude?

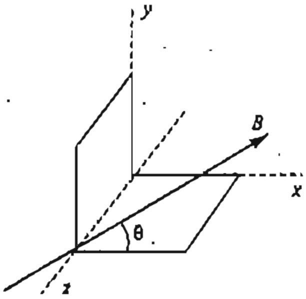


Fig. 8.164

7. In Fig. 8.165, a uniform magnetic field decreases at a constant rate $dB/dt = -K$, where K is a positive constant. A circular loop of wire of radius a containing a resistance R and a capacitance C is placed with its plane normal to the field.

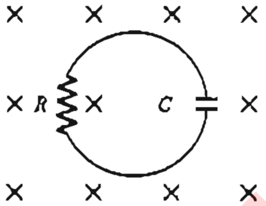


Fig. 8.165

a. Find the charge Q on the capacitor when it is fully charged.
b. Which plate is at higher potential when it is fully charged ?
c. Discuss the force that causes the separation of charges.

8. A square coil $AECD$ of side 0.1 m is placed in a magnetic field $B = 2t^2$ (Fig. 8.166). Here t is in seconds and B is in tesla. The magnetic field is into the plane of the paper. At time $t = 2$ s, find the induced electric field in DC.

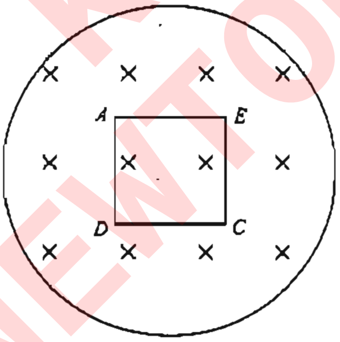


Fig. 8.166

9. A rod of mass m can rotate without friction about axis O, sliding (also without friction) along and annular conductor of radius b arranged in a vertical plane as shown in the Fig. 8.167. The entire arrangement is placed in a uniform magnetic field with the induction B perpendicular to the plane of the annular conductor. The axis and the conductor are connected to the terminals of a current source. Determine

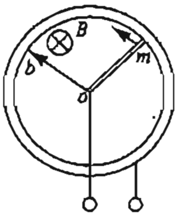


Fig. 8.167

a. according to which law the current i flowing in the rod must vary for the rod to rotate at a constant angular speed. Begin to measure the time from the instant when the rod is in its right-hand horizontal position. Consider the current to be positive when it flows from the axis of rotation to the annual conductor.
b. what the e.m.f. E of the square must be to maintain the required current? Consider the total resistance of the circuit to be constant and equal to R. Disregard the inductance of the circuit.

10. A square, conducting wire loop of side L, total mass m and total resistance R initially lies in the horizontal xy-plane, with corners at $(x, y, z) = (0, 0, 0)$, $(0, L, 0)$, $(L, 0, 0)$ and $(L, L, 0)$. There is a uniform upward magnetic field in the space within and around the loop. The side of the loop that extends from $(0, 0, 0)$ to $(L, 0, 0)$ is held in place on the x-axis; the rest of the loop is released, it begins to rotate due to the gravitational torque.

a. Find the net torque (magnitude and direction) that acts on the loop when it has rotated through an angle ϕ from its original orientation and is rotating downward at an angular speed ω.
b. Find the angular acceleration of the loop at the instant described in part (a).
c. Compared to the case with zero magnetic field, does it take the loop a longer or shorter time to rotate through 90°? Explain.
d. Is mechanical energy conserved as the loop rotates downward? Explain.

11. In Fig. 8.168, $CDEF$ is a fixed conducting smooth frame in vertical plane. A conducting uniform rod GH of mass m can move vertically and smoothly without losing contact with the frame. GH always remains horizontal and is given velocity u upwards and released. Taking the acceleration due to gravity as g and assuming that resistance is present other than R. Find out the time taken by the rod to reach the highest point.

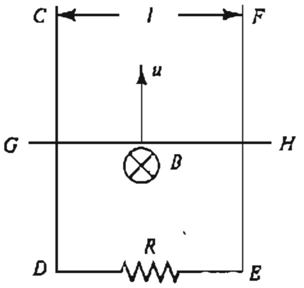


Fig. 8.168

12. In Fig. 8.169, $ABCD$ is a fixed smooth conducting frame in horizontal plane. T is a bulb of power 100 W, P is a smooth pulley and OQ is a conducting rod. Neglect the self-

inductance of the loop and resistance of any part other than the bulb. The mass M is moving down with constant velocity 10 m/s. Bulb lights at its rated power due to induced e.m.f. in the loop due to earth's magnetic field. Find the mass M of the block. (g = 10 m/s^2)

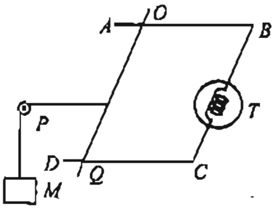


Fig. 8.169

13. Fig. 8.170 shows a conducting rod of length l, resistance R and mass m moving vertically downward due to gravity. Other parts are kept fixed. B = constant = B_0 MN and PQ are vertical, smooth, conducting rails. The capacitance of the capacitor is C. The rod is released from rest. Find the maximum current in the circuit.

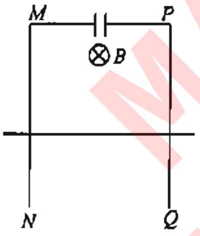


Fig. 8.170

14. The magnetic field of a cylindrical magnet that has a pole face radius 2.8 cm can be varied sinusoidally between the minimum value 16.8 T and the maximum value 17.2 T at a frequency of $\frac{60}{\pi}$ Hz. Cross section of the magnetic field created by the magnet is shown in Fig. 8.171. At a radial distance of 2 cm from the axis find the amplitude of the electric field (in mN/C) induced by the magnetic field variation.

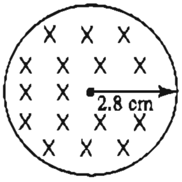


Fig. 8.171

15. A long coaxial cable consists of two thin-walled conducting cylinders with inner radius 2 cm and outer radius 8 cm. The inner cylinder carries a steady current 1A, and the outer cylinder provides the return path for that current. The current produces a magnetic field between the two cylinders. Find the energy stored in the magnetic field for length 1m of the cable. Express answer in nJ (use ln2 = 0.7).

16. A rectangular frame $ABCD$ made of a uniform metal wire has a straight connection between E and F made of the same wire, as shown in Fig. 8.172. $AEFD$ is a square of side 1m and EB = FC = 0.5 m. The entire circuit is placed in a steadily increasing, uniform magnetic field directed into the plane of paper and normal to it. The rate of change of the magnetic field is 1 T/s. The resistance per unit length of the wire is 1 W/m. Find the magnitudes and directions of the currents in the segments AE, BE and EF.

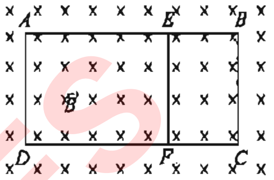


Fig. 8.172

17. Two fixed long straight wires carry the same current i in opposite directions as shown in Fig. 8.173. A square loop of side b is fixed in the plane of the wires with its length parallel to one wire at a distance a as shown in the figure.

a. Calculate the induced e.m.f. in the loop if the current in both the wires is changing at the rate di/dt.

b. What is the direction of force on the loop if di/dt is positive?

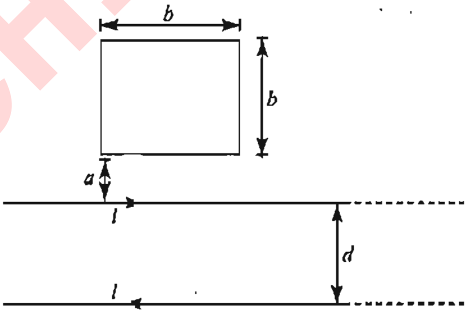


Fig. 8.173

18. Two parallel, long, straight conductors lie on a smooth plane surface. Two other parallel conductors rest on them at right angles so as to form a square of side a initially. A uniform magnetic field B exists at right angles to the plane containing the conductors. Now they start moving out with a constant velocity v.

a. Will the induced e.m.f. be time dependent?

b. Will the current be time dependent?

19. The rectangular wire frame, shown in Fig. 8.174, has a width d, mass m, resistance R and a large length. A uniform magnetic field B exists to the left of the frame. A constant force F starts pushing the frame into the magnetic field at t = 0.

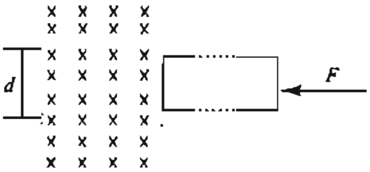


Fig. 8.174

a. Find the acceleration of the frame when its speed has increased to v.

b. Find the terminal velocity of the loop.

c. Find the velocity at time t.

20. In Fig. 8.175, $ABCEFGA$ is a square conducting frame of side 2 m and resistance 1 W/m. A uniform magnetic field B is applied perpendicular to the plane and pointing inwards. It increases with time at a constant rate of 10 T/s. Find the rate at which heat is produced in the circuit, $AB = BC = CD = BH$.

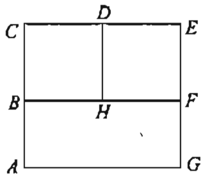


Fig. 8.175

21. Fig. 8.176 shows a part of a bigger circuit. The capacity of the capacitor is 6 mF and decreasing at the constant rate 0.5 mF s^{-1}. The potential difference across the capacitor is changing as follows:

$$\frac{dV}{dt} = 2\,\text{Vs}^{-1},\ \frac{d^2V}{dt^2} = \frac{1}{2}\text{Vs}^{-2}$$

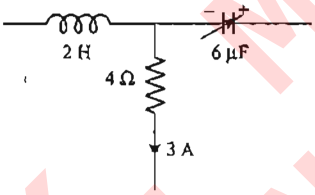


Fig. 8.176

The current in the 4 W resistor is decreasing at the rate of 1 mA s^{-1}. What is the potential difference (in micro-volts) across the inductor at this moment?

22. In the circuits shown in Figs. 8.177 and 8.178, S_1 and S_2 are switches. S_2 remains closed for a long time and S_1 is opened. Now S_1 is also closed. Just after S_1 is closed, find the potential difference (V) across R and $\frac{di}{dt}$ (with sign) in L.

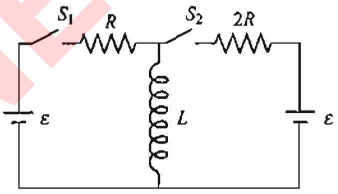


Fig. 8.177

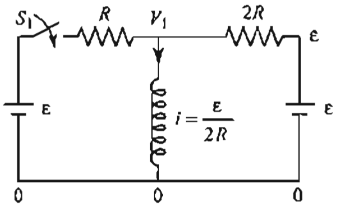


Fig. 8.178

23. In Fig. 8.179, a rod of length l and mass m moves with an initial velocity u on a fixed frame containing inductor L and resistance R. PQ and MN are smooth conducting wires. There is a uniform magnetic field of strength B. Initially, there is no current in the inductor. Find the total charge flown through the inductor by the time, the current in the inductor becomes i velocity of rod becomes v_1 and the rod has travelled a distance x.

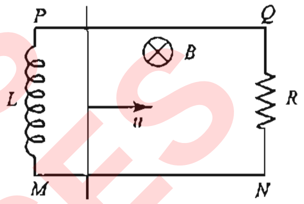


Fig. 8.179

24. A 1.00 mH inductor and a 1.00 mF capacitor are connected in series. The current in the circuit is described by $I = 20.0\ t$, where t is in seconds and I is in amperes. Initially, the capacitor has no charge. Determine

a. the voltage across the inductor as a function of time,

b. the voltage across the capacitor as a function of time,

c. the time when the energy stored in the capacitor first exceeds that in the inductor.

25. Two capacitors of capacitance $2C$ and C, respectively, are connected in series with an inductor of inductance L. Initially the capacitors have charge such that $V_B - V_A = 4V_0$ and $V_C - V_D = V_0$. Initial current in the circuit is zero.

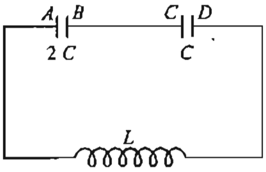


Fig. 8.180

Find

a. the maximum current that will flow in the circuit,

b. the potential difference across each capacitor at that instant,

c. the equation of current flowing towards left in the inductor.

26. Switch S is closed in the circuit at time $t = 0$. Find the current through the capacitor and the inductor at any time t.

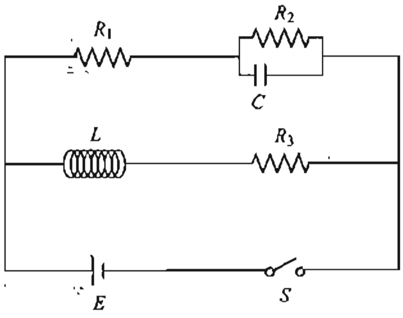


Fig. 8.181

27. In the circuit shown in Fig. 8.182, the capacitor is initially uncharged and the two-way switch is connected in the position BC. Find the current through the resistance R as a function of time t. After a time t = 4 ms, the switch is connected in the position AC. Find the frequency of oscillation of the circuit in the position, and the maximum charge on the capacitor C. At what time will the energy stored in the capacitor be one half of the total energy stored in the circuit? It is given $L = 2 \times 10^{-4}$ H, C = 5 mF, $R = \frac{\ln 2}{10}\ \Omega$ and e.m.f. of the battery = 1 V.

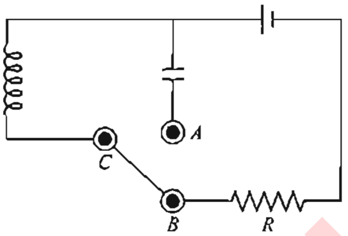


Fig. 8.182

Objective Type

Solutions on page 8.106

1. A horizontal straight conductor when placed along south-north direction falls under gravity; there is

- **a.** an induced current from south-to-north direction
- **b.** an induced current from north-to-south direction
- **c.** no induced e.m.f. along the length of the conductor
- **d.** an induced e.m.f. along the length of the conductor

2. Two circular, similar, coaxial loops carry equal currents in the same direction. If the loops are brought nearer, what will happen?

- **a.** Current will increase in each loop
- **b.** Current will decrease in each loop
- **c.** Current will remain same in each loop
- **d.** Current will increase in one and decrease in the other

3. As shown in Fig. 8.183, a magnetic is moved with a fast speed towards a coil at rest. Due to this, induced electromotive force, induced current and induced charge in the coil are E, I and Q, respectively. If the speed of magnetic is doubled, the incorrect statement is

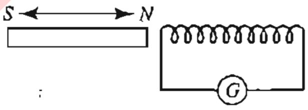


Fig. 8.183

- **a.** E increases
- **b.** I increases
- **c.** Q remains the same
- **d.** Q increases

4. A rectangular coil $ABCD$ is rotated anticlockwise with a uniform angular velocity about the axis shown in Fig. 8.184. The axis of rotation of the coil as well as the magnetic field B is horizontal. The induced e.m.f. in the coil would be minimum when the plane of the coil

- **a.** is horizontal
- **b.** makes an angle of 45° with the direction of magnetic field
- **c.** is at right angle to the magnetic field
- **d.** makes an angle of 30° with the magnetic field

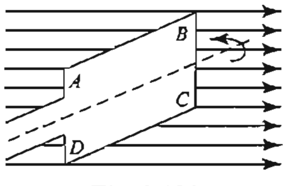


Fig. 8.184

5. A rod PQ is connected to the capacitor plates. The rod is placed in a magnetic field (B) directed downwards perpendicular to the plane of the paper. If the rod is pulled out of magnetic field with velocity $\vec{v}$ as shown in Fig. 8.185.

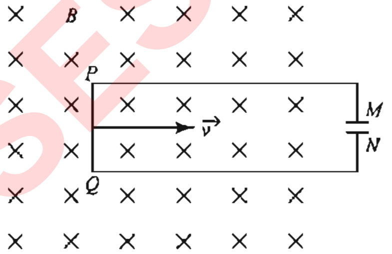


Fig. 8.185

- **a.** plate M will be positively charged
- **b.** plate N will be positively charged
- **c.** both plates will be similarly charged
- **d.** no charge will be collected on plates.

6. A mutual inductor consists of two coils X and Y as shown in Fig. 8.186 in which one quarter of the magnetic flux produced by X links with Y, giving a mutual inductance M.

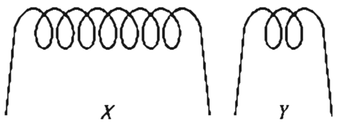
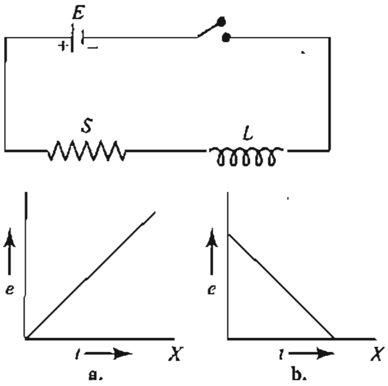


Fig. 8.186

What will be the mutual inductance when Y is used as the primary?

- **a.** $M/4$
- **b.** $M/2$
- **c.** M
- **d.** $2M$

7. Switch S of the circuit shown in Fig. 8.187 is closed at $t = 0$. If e denotes the induced e.m.f. in L and i, the current flowing through current flowing through the circuit at time t, then which of the following graphs correctly represents the variation of e with i ?



a. b.

Fig. 8.187 (*Contd.*)

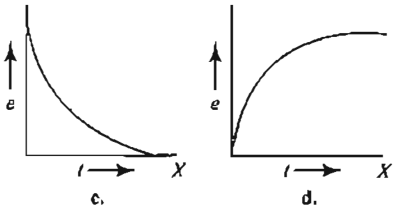


Fig. 8.187

8. A flexible wire bent in the form of a circle is place in a uniform magnetic field perpendicularly to the plane of the coil. The radius of the coil changes as shown in Fig. 8.188. The graph of magnitude induced e.m.f. in the coil is represented by

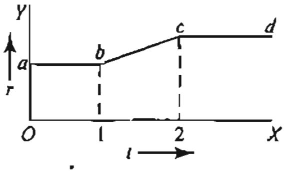


Fig. 8.188

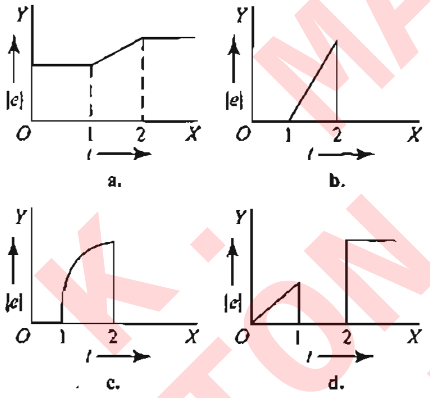


Fig. 8.189

9. A small coil of radius r is placed at the centre of a large coil of radius R, where $R >> r$. The two coils are coplanar. The mutual inductance between the coils is proportional to

a. r/R b. r^2/R

c. r^2/R^2 d. r/R^2

10. A thin circular ring of area A is perpendicular to uniform magnetic field of induction B. A small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of circuit is R. When the ring is suddenly squeezed to zero area, the charge flowing through the galvanometer is

a. $\frac{BR}{A}$ b. $\frac{AB}{R}$

c. ABR d. $\frac{B^2A}{R^2}$

11. A circuit contains two inductors of self-inductance L_1 and L_2 in series (Fig. 8.190). If M is the mutual inductance then the effective inductance of the circuit shown will be

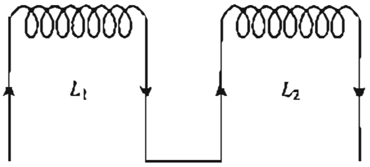


Fig. 8.190

a. L_1+L_2 b. L_1+L_2-2M

c. L_1+L_2+M d. L_1+L_2+2M

12. In the circuit (Fig. 8.191) what is potential difference V_B-V_A when the current I is 5 A and is decreasing at the rate of 10^3 A/s?

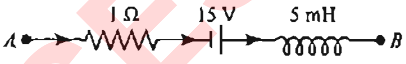


Fig. 8.191

a. 0 b. 15 V

c. 20 V d. 25 V

13. In the circuit (Fig. 8.192), the current through 30 Ω resistance when circuit is completed is

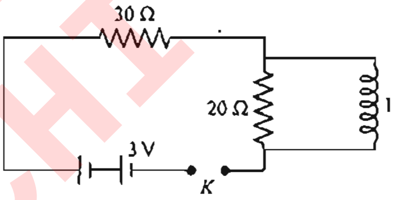


Fig. 8.192

a. 3 A b. 0.1 A

c. 5 A d. 0.5 A

14. In an a.c. sub-circuit (Fig. 8.193), the resistance $R = 0.2\ \Omega$. At a certain instant $V_A - V_B = 0.5$ V, $I = 0.5$ A, $\frac{\Delta I}{\Delta t} = 8$ A/s. Find the inductance of the coil.

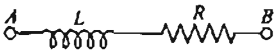


Fig. 8.193

a. 0.01 H b. 0.02 H

c. 0.5 H d. 0.5 H

15. A wire is bent to form the double loop shown in Fig. 8.194. There is a uniform magnetic field directed into the plane of the loop. If the magnitude of this field is decreasing, the current will flow from

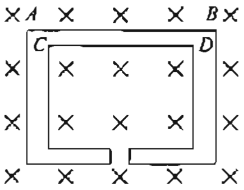


Fig. 8.194

a. a to b and c to d b. b to a and d to c

c. a to b and d to c d. b to a and c to d

16. A wire of sliding as shown in Fig. 8.195. The angle between the acceleration and the velocity of the wire is

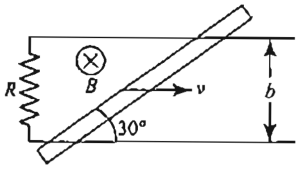


Fig. 8.195

a. 30° b. 40° c. 120° d. 90°

17. A conducting ring of radius r is rolling without slipping with a constant angular velocity ω (Fig. 8.196). If the magnetic field strength is B and is directed into the page then the e.m.f. induced across PQ is

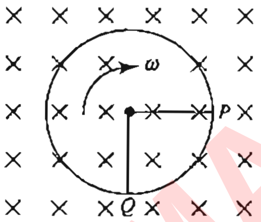


Fig. 8.196

a. $B\omega r^2$ b. $\frac{B\omega r^2}{2}$

c. $4B\omega r^2$ d. $\frac{\pi^2 r^2 B\omega}{8}$

18. A 0.1 m long conductor carrying a current of 50 A is perpendicular to a magnetic field of 1.25 mT. The mechanical power to move the conductor with a speed of 1 m s^{-1} is

a. 0.25 mW b. 6.25 mW

c. 0.625 W d. 1 W

19. An electric current I is passed through a circular loop of folded compeer wire as shown in Fig. 8.197. The magnetic induction at the centre of the loop will be

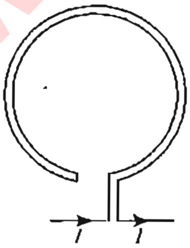


Fig. 8.197

a. 0 b. $\frac{2\mu_0 I}{r}$

c. $\frac{\mu_0 I}{r}$ d. $\frac{\mu_0 I}{2r}$

20. A conducting wire xy of length l and mass m is sliding without friction on vertical conduction rails ab and cd as shown in Fig. 8.198. A uniform magnetic field B exists perpend-icular to the plane of the rails, x moves with a constant velocity of

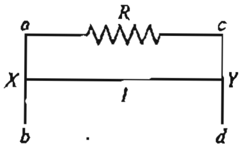


Fig. 8.198

a. $\frac{mgR}{Bl}$ b. $\frac{mgR}{Bl^2}$

c. $\frac{mgR}{B^2l^2}$ d. $\frac{mgR}{B^2l}$

21. The inner loop has an area of 5×10^{-4} m^2 and a resistance of 2 Ω (Fig. 8.199). The larger circular loop is fixed and has a radius of 0.1 m. Both the loops are concentric and coplanar. The smaller loop is rotated with an angular velocity of ω rad s^{-1} about linked its diameter. The magnetic flux with the smaller loop is

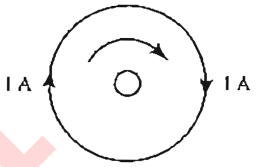


Fig. 8.199

a. $2\pi \times 10^{-6}$ weber b. $\pi \times 10^{-9}$ weber

c. $\pi \times 10^{-9} \cos \omega t$ weber d. zero

22. The coefficient of mutual inductance of two circuits A and B is 3 mH and their respective resistances are 10 and 4 Ω. How much current should change in 0.02 s in the circuit A, so that the induced current in B should be 0.0096 A?

a. 0.24 A b. 1.6 A

c. 0.18 A d. 0.16 A

23. A circuit $ABCD$ is held perpendicular to the uniform magnetic field of $B = 5 \times 10^{-2}$ T extending over the region $PQRS$ and directed into the plane of the paper. The circuit is out of the field at a uniform speed of 0.2 ms^{-1} for 1.5 s. During this time, the current in the 5 Ω resistor is

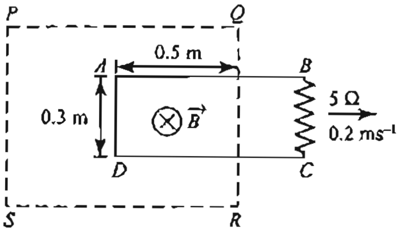


Fig. 8.200

a. 0.6 mA from B to C b. 0.9 mA from B to C

c. 0.9 mA from C to B d. 0.6 mA from C to B

24. A square metal wire loop of side 10 cm and resistances 1 Ω is moved with constant velocity v_0 in a uniform magnetic field of induction $B = 2$ Wb m^{-2}, as shown in Fig. 8.201. The magnetic field lines are perpendicular to the plane of loop and directed into the paper. The loop is connected to the network of resistances, each of value 3 Ω. The resistance of the lead wires is negligible. The speed of the loop so as to have a steady current of 1 mA in the loop is

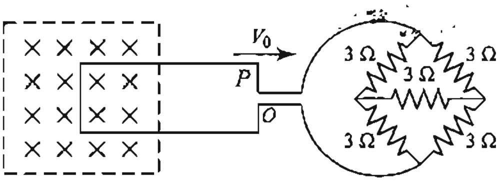


Fig. 8.201

a. 2 ms^{-1} b. 2 cms^{-1}
c. 10 ms^{-1} d. 20 ms^{-1}

25. A long solenoid of length L, cross section A having N_1 turns has wound about its centre a small coil of N_2 turns as shown in Fig. 8.202. Then the mutual inductance of two circuits is

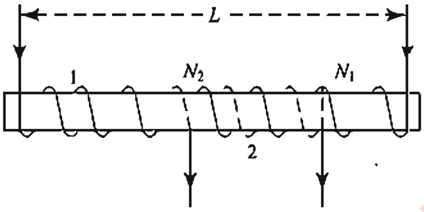


Fig. 8.202

a. $\frac{\mu_0 A(N_1/N_2)}{L}$ b. $\frac{\mu_0 A(N_1 N_2)}{L}$

c. $\mu_0 A N_1 N_2 L$ d. $\frac{\mu_0 A N_1^2 N_2}{L}$

26. A conducting wire of mass m slides down two smooth conducting bars, set at an angle θ to the horizontal as shown in Fig. 8.203. The separation between the bars is l. The system is located in the magnetic field B, perpendicular to the plane of the sliding wire and bars. The velocity of the wire is

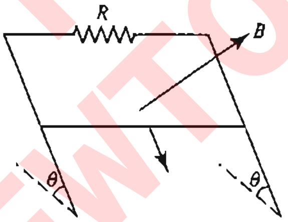


Fig. 8.203

a. $\frac{mg R \sin\theta}{B^2 l^2}$ b. $\frac{mg R \sin\theta}{B l^3}$

c. $\frac{mg R \theta}{B^2 l^5}$ d. $\frac{mg R \sin\theta}{B l^4}$

27. An e.m.f. of 15 V is applied in a circuit containing 5 H inductance and 10 Ω resistance. The ratio of the currents at time $t = \infty$ and $t = 1$ s is

a. $\frac{e^{1/2}}{e^{1/2}-1}$ b. $\frac{e^2}{e^2-1}$

c. $1-e^{-1}$ d. e^{-1}

28. In Fig. 8.204 (a) and (a), two air-cored solenoid P and Q have been shown. They are placed near each other. In Fig. 8.204 (a), when I_P, the current in P, changes at the rate of 5 A s^{-1}, an e.m.f. of 2 mV is induced in Q. The current in P is then switched off, and the current changing at 2 A s^{-1} is fed through Q as shown in the figure. What e.m.f. will be induced in P?

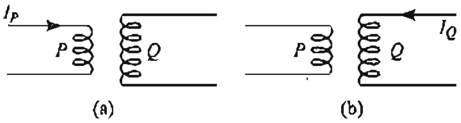


Fig. 8.204

a. 8×10^{-4} V b. 2×10^{-3} V
c. 5×10^{-3} V d. 8×10^{-2} V
e. 8×10^{-1} V

29. Two different coils have self-inductances L_1 = 8 mH and L_2 = 2 mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time, the current, the induced voltage and energy stored in the first coil are I_1, V_1 and W_1, respectively. The corresponding values for the second coil are I_2, V_2 and W_2, at the same resistant, respectively. The ratio of I_1 to I_2 is

a. $\frac{I_1}{I_2} = \frac{1}{4}$ b. $\frac{I_1}{I_2} = 4$

c. $\frac{I_1}{I_2} = \frac{2}{1}$ d. $\frac{I_1}{I_2} = 1$

30. In Q. 29, the ratio of V_1 to V_2 is

a. $\frac{V_1}{V_2} = \frac{1}{4}$ b. $\frac{V_1}{V_2} = 4$

c. $\frac{V_1}{V_2} = \frac{2}{1}$ d. $\frac{V_1}{V_2} = 1$

31. In Q. 29, the ratio of W_1 to W_2 is

a. $\frac{W_2}{W_1} = 4$ b. $\frac{W_1}{W_2} = 4$

c. $\frac{W_1}{W_2} = 1$ d. $\frac{W_1}{W_2} = 2$

32. At a place, the value of horizontal component of the earth's magnetic field H is 2×10^{-5} Wb/m^2. A metallic rod AB of length 2 m placed in east-west direction, having the end A towards east, falls vertically. Which end of the rod becomes positively charged and what is the value of induced potential difference between the two ends?

a. End A, 3×10^{-3} mV
b. End A, 3 mV
c. End B, 3×10^{-3} mV
d. End B, 2 mV

33. A coil of inductance 0.20 H is connected in series with a switch and a cell of e.m.f. 1.6 V. The total resistance of the current is 4.0 Ω. What is the initial rate of growth of the current when the switch is closed?

a. 0.050 As^{-1} b. 0.40 As^{-1}
c. 0.13 As^{-1} d. 8.0 As^{-1}
e. 0.32 As^{-1}

34. A small square loop of wire of side l is placed inside a large square loop of side L ($L \ll 0$). The loops are coplanar and their centres coincide. The mutual induction of the system is

a. $\frac{l}{L}$ b. $\frac{l^2}{L}$

c. $\frac{L}{l}$ d. $\frac{L^2}{l}$

35. The length of a wire required to manufacture a solenoid of length l and self-induction L is (cross-sectional area is negligible)

a. $\sqrt{\frac{2\pi Ll}{\mu_0}}$ b. $\sqrt{\frac{\mu_0 Ll}{4\pi}}$

c. $\sqrt{\frac{4\pi Ll}{\mu_0}}$ d. $\sqrt{\frac{\mu_0 Ll}{2\pi}}$

36. Fig. 8.205 shows a copper rod moving with velocity v parallel to a long straight wire carrying current = 100 A. Calculate the induced e.m.f. in the rod, where $v = 5\ \text{ms}^{-1}$, $a = 1$ cm, $b = 100$ cm.

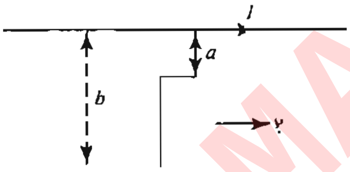


Fig. 8.205

a. 0.23 mV b. 0.46 mV

c. 0.16 mV d. 0.32 mV

37. A square loop of side a and a straight infinite conductor are placed in the same plane with the two sides of a square parallel to the conductor (Fig. 8.206). The inductance and the resistance are equal to L and R. The frame is turned through 180° about axis OO'. Find the charge that flows in the square loop.

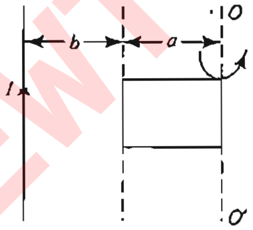


Fig. 8.206

a. $\frac{\mu_0}{\pi R}\ln\frac{a+b}{b}$ b. $\frac{\mu_0}{\pi R}\ln\left(\frac{2a+b}{b}\right)$

c. $\frac{\mu_0 Ia}{2\pi R}\ln\frac{2a+b}{b}$ d. $\ln\left(\frac{a+b}{b}\right)\frac{\mu_0 I}{2\pi R}$

38. A superconducting ring of radius a and inductance L is located in a uniform magnetic field of induction B. The plane of the ring is parallel to B and the current in the ring is zero. Then the ring is turned through 90° so that the plane is perpendicular to the field. What is the work done in turning the ring?

a. $\frac{\pi^2 a^4 B^2}{2L}$ b. $\frac{\pi^2 a^2 B^2}{2L}$

c. $\frac{\pi a^2 B^2}{2L}$ d. $\frac{\pi^2 a^4 B^2}{4L}$

39. A rectangular loop with a sliding conductor of length l is located in a uniform magnetic field perpendicular to the plane of the loop (Fig. 8.207). The magnetic induction is B. The conductor has a resistance R. The sides AB and CD have resistances R_1 and R_2, respectively. Find the current through the conductor during its motion to the right with a constant velocity v.

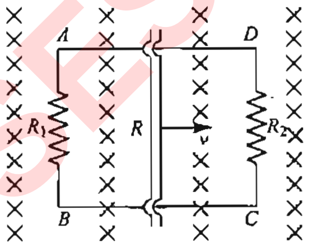


Fig. 8.207

a. $\frac{Blv(R_1+R_2)}{R_1(R_1+R_2)}$ b. $\frac{Bl^2v}{R_1+R_1R_2}$

c. $\frac{Blv(R_1+R_2)}{R_1R_2+R(R_1+R_2)}$ d. $\frac{Bl^2v}{R_1R_2+R(R_1+R_2)}$

40. A wire of length l, mass m and resistance R slides without any friction down the parallel conducting rails of negligible resistance (Fig. 8.208). The rails are connected to each other at the bottom by a resistanceless rail parallel to the wire so that the wire and the rails form a closed rectangular conducting loop. The plane of the rails makes an angle θ with the horizontal and a uniform vertical magnetic field of induction B exists throughout the region. Find the steady-state velocity of the wire.

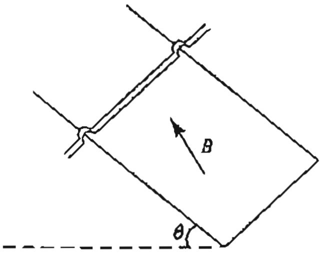


Fig. 8.208

a. $\frac{mg}{R}\frac{\sin\theta}{B^2l^2\cos^2\theta}$ b. $\frac{mg}{R}\frac{\sin^2\theta}{B^2l^2\cos^2\theta}$

c. $\frac{mgR\sin\theta}{B^2l^2\cos^2\theta}$ d. $mgR\frac{\sin^2\theta}{B^2l^2\cos\theta}$

41. A plane loop, shaped as two square of sides $a = 1$ m and $b = 0.4$ m is introduced into a uniform magnetic field ⊥ to the plane of loop (Fig. 8.209). The magnetic field varies as $B = 10^{-3}\sin 100t$. The amplitude of the current induced in the loop, if its resistance per unit length is $r = 5\ \Omega\ \text{m}^{-1}$ is

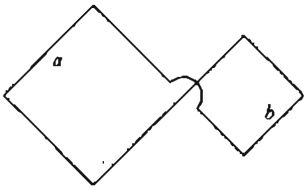


Fig. 8.209

a. 2 A b. 3 A
c. 4 A d. 5 A

42. A conductor of mass $m = \frac{1}{4}$ kg and length = 2 m can move without friction along two metallic parallel plates in a horizontal plane and connected across a capacitor $C = 1000\ \mu F$ (Fig. 8.210). The entire system is in a magnetic field of induction $B = 2$ T upwards. A constant force F is applied to the middle of conductor ⊥ to it and parallel to tracks. The acceleration of the conductor is

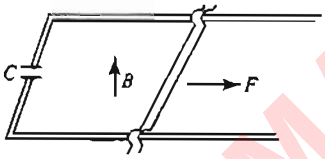


Fig. 8.210

a. 5 ms^{-2} b. 4 ms^{-2}
c. 3 ms^{-2} d. 2 ms^{-2}

43. A conductor of length l and mass m can slide without any friction along the two vertical conductors connected at the top through a capacitor (Fig. 8.211). A uniform magnetic field B is set up ⊥ to the plane of paper. The voltage across the capacitor in terms of distance x through which it falls is

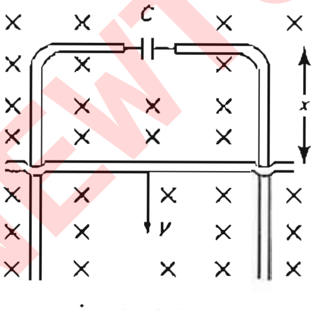


Fig. 8.211

a. $Bl2gx$ b. $Bl\sqrt{gx}$
c. $Bl\sqrt{2gx}$ d. $Bx\sqrt{2gl}$

44. The inductance L of a solenoid of length l, whose windings are made of material of density D and resistivity ρ, is (the winding resistance is R)

a. $\frac{\mu_0}{4\pi l} \frac{Rm}{\rho D}$ b. $\frac{\mu_0}{4\pi R} \frac{lm}{\rho D}$

c. $\frac{\mu_0}{4\pi l} \frac{R^2 m}{\rho D}$ d. $\frac{\mu_0}{2\pi R} \frac{lm}{\rho D}$

45. A toroid is along a coil of wire, wound over a circular core. The coefficient of self-inductance of the toroid is given by (radius = r), when the magnetic field is within it is uniform and $R \gg r$

a. $L = \frac{\mu_0 N r^2}{2R}$ b. $L = \frac{\mu_0 N r}{2R}$

c. $L = \frac{\mu_0 N r^2}{R}$ d. $L = \frac{\mu_0 N^2 r^2}{2R}$

46. A rectangular loop of sides 10 cm and 5 cm with a cut is stationary between the pole pieces of an electromagnet. The magnet field of the magnet is normal to the loop. The current feeding the electromagnet is reduced so that the field decreases from its initial value of 0.3 T at the rate of 0.02 T s^{-1}. If the cut is joined and the loop has a resistance of 2.0 Ω, the power dissipated by the loop as heat is

a. 5 nW b. 4 nW
c. 3 nW d. 2 nW

47. A straight solenoid has 5000 turns per metre in the primary and 200 turns in the secondary. If the area of crosses is 4 cm^2, the mutual induction will be

a. 503 H b. 503 mH
c. 503 μH d. 5.03 H

48. The approximate formula expressing the formula inductance of two thin co-axial loops of the same radius a when their centres are separated by a distance l with $l \gg a$ is

a. $\frac{1}{2} \frac{\mu_0 \pi a^4}{l^3}$ b. $\frac{1}{2} \frac{\mu_0 a^4}{l^2}$

c. $\frac{\mu_0}{4\pi} \frac{\pi a^4}{l^2}$ d. $\frac{\mu_0}{\pi} \frac{a^4}{l^3}$

49. The length of a thin wire required to manufacture a solenoid of length l = 100 cm and inductance L = 1 mH, if the solenoid's cross-sectional diameter is considerably less than its length is

a. 1.0 km b. 0.10 km
c. 0.010 km d. 10 km

50. Find the inductance of a solenoid of length l whose winding is made of copper wire of mass m. The winding resistance is equal to R. The solenoid is considerably less than its length.

a. $\frac{\mu_0 R}{4\pi m l \rho \rho_0}$ b. $\frac{\mu_0 l \rho \rho_0}{4\pi m R}$

c. $\frac{\mu_0}{4\pi} \frac{mR}{l \rho \rho_0}$ d. None of these

51. In a long straight solenoid with cross-sectional radius a and number of turns per unit length n, the current varies with the rate I A/s. The magnitude of the eddy current field strength as a function of distance r from the solenoid axis is

a. $\frac{1}{2} \frac{n I a^2}{\mu_0 r}$ b. $\frac{1}{2} \frac{I a^2}{\mu_0 r}$

c. $\frac{n I a^2}{\mu_0 r}$ d. $\frac{1}{2} \frac{\mu_0 n I a^2}{r}$

52. A 22 cm long solenoid, having total number of turns 1000, consists of a core of cross-sectional area 4 cm^2. Half portion of the core consists of air and other half is made of iron of relative permeability 500. The self- inductance of the solenoid is

a. 0.57 H b. 0.64 mH c. 0.057 H d. 57.0 H

53. Magnetic flux linked with a stationary loop of resistance R varies with respect to time during the time period T as follows:

$$\phi = aT(T - r)$$

The amount of heat generated in the loop during that time. (inductance of the coil is negligible) is

a. $\frac{aT}{3R}$ b. $\frac{a^2T^2}{3R}$ c. $\frac{a^2T^2}{R}$ d. $\frac{a^2T^3}{3R}$

54. A current in a coil of self-inductance 2.0 H is increasing as $i = 2 \sin t^2$. The amount of energy spent during the period when the current changes from 0 to 2 A is

a. 1 J b. 2 J c. 3 J d. 4 J

55. A thin ring of radius 10 cm carries a uniformly distributed charge. The ring rotates at a constant speed of 1200 r.p.m. about its axis ⊥ to the plane. The charge on the ring if $B = 3.14 \times 10^{-9}$ T at the centre is

a. 10^{-5} C b. 2×10^{-5} C
c. 2.5×10^{-5} C d. 5.2×10^{-5} C

56. A small magnet is along the axis of a coil and its distance from the coil is 80 cm. In this position, the flux linked with the coil is 4×10^{-5} weber. If the coil is displaced 40 cm towards the magnet in 0.08 s, then the induced e.m.f. produced in the coil will be

a. 0.5mV b. 1mV c. 7mV d. –4mV

57. In the circuit shown (Fig. 8.212), X is joined to Y for a long time and then X is joined to Z. The total heat produced in R_2 is

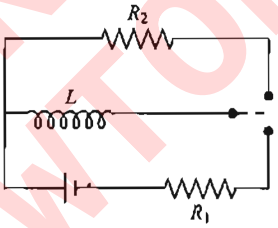


Fig. 8.212

a. $\frac{LE^2}{2R_1^2}$ b. $\frac{LE^2}{2R_2^2}$ c. $\frac{LE^2}{2R_1R_2}$ d. $\frac{LE^2R_2}{2R_1^3}$

58. A cylindrical space of radius R is filled with a uniform magnetic induction B parallel to the axis of the cylinder. If B changes at a constant rate, the graph showing the variation of induced electric field with distance r from the axis of cylinder is

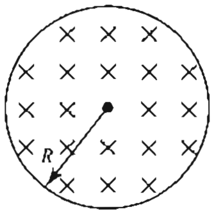


Fig. 8.213

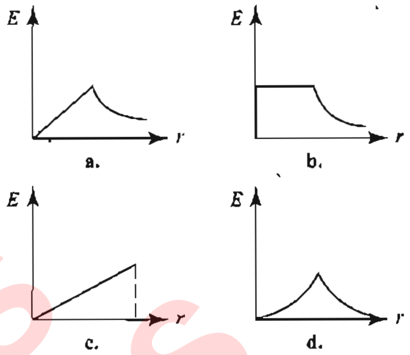


Fig. 8.214

59. A small circular loop of radius r is placed inside a circular loop of radius R ($R \gg r$). The loops are coplanar and their centres coincide. The mutual inductance of the system is proportional to

a. r/R b. r^2/R c. r/R^2 d. r^2/R^2

60. A circular coil of wire consists of exactly 200 turns with a total resistance 0.20 Ω. The area of the coil is 100 cm^2. The coil is kept in a uniform magnetic field B as shown in Fig. 8.215. The magnetic field is increased at a constant rate of 2 T/s. The current induced in the coil (state the sense of induced current) is

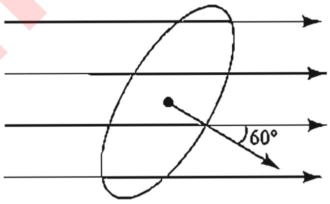


Fig. 8.215

a. 13 A b. 5 A
c. 10 A d. 20 A

61. A flip coil consists of N turns of circular coils which lie in a uniform magnetic field. Plane of the coils is perpendicular to the magnetic field as shown in Fig. 8.216. The coil is connected to a current integrator which measures the total charge passing through it. The coil is turned through 180° about the diameter. The charge passing through the coil is

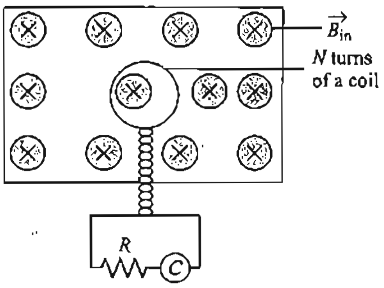


Fig. 8.216

a. $\frac{NBA}{R}$ b. $\frac{\sqrt{3}NBA}{2R}$ c. $\frac{NBA}{\sqrt{2}R}$ d. $\frac{2NBA}{R}$

62. An elasticized conducting band is around a spherical balloon (Fig. 8.217). Its plane passes through the centre of the balloon. A uniform magnetic field of magnitude 0.04 T is directed perpendicular to the plane of the band. Air is let out of the balloon at 100 cm^3/s at an instant when the radius of the balloon is 10 cm. The induced e.m.f. in the band is

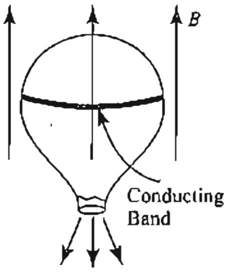


Fig. 8.217

a. 15 μV b. 25 μV c. 10 μV d. 20 μV

63. A copper rod is bent into a semi-circle of radius a and at ends straight parts are bent along diameter of the semi-circle and are passed through fixed, smooth and conducting ring O and O' as shown in Fig. 8.218. A capacitor having capacitance C is connected to the rings. The system is located in a uniform magnetic field of induction B such that axis of rotation OO' is perpendicular to the field direction. At initial moment of time ($t = 0$), plane of semi-circle was normal to the field direction and the semi-circle is set in rotation with constant angular velocity ω. Neglecting resistance and inductance of the circuit. The current flowing through the circuit in function of time is

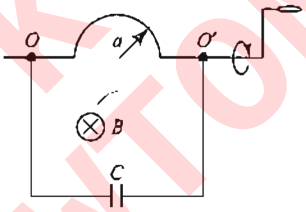


Fig. 8.218

a. $\frac{1}{4}\pi\omega^2 a^2 CB\cos\omega t$ b. $\frac{1}{2}\pi\omega^2 a^2 CB\cos\omega t$

c. $\frac{1}{4}\pi\omega^2 a^2 CB\sin\omega t$ d. $\frac{1}{2}\pi\omega^2 a^2 CB\sin\omega t$

64. A uniform magnetic field of induction B fills a cylindrical volume of radius R. A rod AB of length $2l$ is placed as shown in Fig. 8.219. If B is changing at the rate dB/dt, the e.m.f. that is produced by the changing magnetic field and that acts between the ends of the rod is

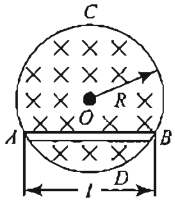


Fig. 8.219

a. $\frac{dB}{dt}l\sqrt{R^2-l^2}$ b. $\frac{dB}{dt}l\sqrt{R^2+l^2}$

c. $\frac{1}{2}\frac{dB}{dt}l\sqrt{R^2-l^2}$ d. $\frac{1}{2}\frac{dB}{dt}l\sqrt{R^2+l^2}$

65. Charge Q is uniformly distributed on a thin insulating ring of mass m which is initially at rest. To what angular velocity will the ring be accelerated when a magnetic field B, perpendicular to the plane of the ring, is switched on ?

a. $-\frac{QB}{2m}$ b. $-\frac{3QB}{2m}$ c. $-\frac{QB}{m}$ d. $-\frac{QB}{4m}$

66. The length of a thin wire required to manufacture a solenoid of inductance L and length l, if the cross-sectional diameter is considered less than its length is

a. $\sqrt{\frac{4\pi lL}{\mu_0}}$ b. $\sqrt{\frac{\pi lL}{\mu_0}}$ c. $\sqrt{\frac{2\pi lL}{\mu_0}}$ d. $\sqrt{\frac{4\pi lL}{3\mu_0}}$

67. Calculate the inductance of a unit length of a double tape line as shown in Fig. 8.220. If the tapes are separated by a distance h which is considerably less than their width b.

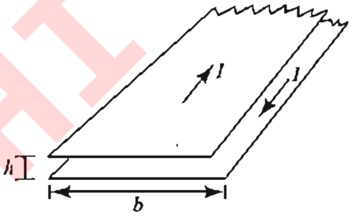


Fig. 8.220

a. $\frac{\mu_0 h}{b}$ b. $\frac{\mu_0 h}{2b}$ c. $\frac{2\mu_0 h}{b}$ d. $\frac{\sqrt{2}\,\mu_0 h}{b}$

68. Find the inductance of a unit length of two parallel wires each of radius a whose centres are a distance d apart carry equal currents in opposite directions. Neglect the flux within the wire. The wires carry current in opposite directions.

a. $\frac{\mu_0 l}{2\pi}\ln\left(\frac{d-a}{a}\right)$ b. $\frac{\mu_0 l}{\pi}\ln\left(\frac{d-a}{a}\right)$

c. $\frac{3\mu_0 l}{\pi}\ln\left(\frac{d-a}{a}\right)$ d. $\frac{\mu_0 l}{3\pi}\ln\left(\frac{d-a}{a}\right)$

69. Fig. 8.221 shows a rectangular coil near a long wire. The mutual inductance of the combination is

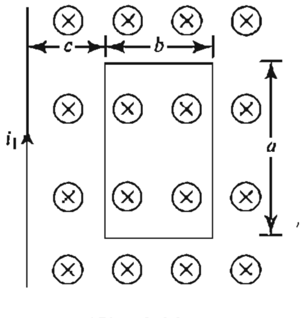


Fig. 8.221

a. $\frac{\mu_0 a}{2\pi}\ln\left(1-\frac{b}{c}\right)$ **b.** $\frac{\mu_0 a}{2\pi}\ln\left(1+\frac{b}{c}\right)$

c. $\frac{\mu_0 a}{\pi}\ln\left(1+\frac{b}{c}\right)$ **d.** $\frac{\mu_0 a}{\sqrt{2}\pi}\ln\left(1+\frac{b}{c}\right)$

70. In the given circuit (Fig. 8.222), the current through the 5 mH inductor in steady state is

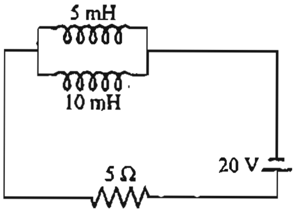


Fig. 8.222

a. $\frac{2}{3}$A **b.** $\frac{8}{3}$A **c.** $\frac{1}{3}$A **d.** $\frac{2}{3}$A

71. In the electrical network at $t<0$ (Fig. 8.223), key was placed on (1) till the capacitor got fully charged. Key is placed on (2) at $t=0$. Time when the energy in both the capacitor and the inductor will be same for the first time is

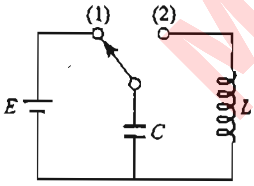


Fig. 8.223

a. $\frac{\pi}{4}\sqrt{LC}$ **b.** $\frac{3\pi}{4}\sqrt{LC}$ **c.** $\frac{\pi}{3}\sqrt{LC}$ **d.** $\frac{2\pi}{3}\sqrt{LC}$

72. The total heat produced in the resistor R in an RL circuit when the current in the inductor decreases from I_0 to 0 is

a. LI_0^2 **b.** $\frac{1}{2}LI_0^2$ **c.** $\frac{3}{2}LI_0^2$ **d.** $\frac{1}{3}LI_0^2$

73. In the circuit shown (Fig. 8.224), X is joined to Y for a long time and then X is joined to Z. The total heat produced in R_2 is

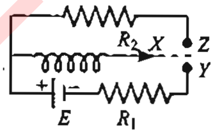


Fig. 8.224

a. $\frac{LE^2}{2R_1^2}$ **b.** $\frac{LE^2}{2R_2^2}$

c. $\frac{LE^2}{2R_1R_2}$ **d.** $\frac{LE^2R_2}{2R_1^3}$

74. In the circuit shown (Fig. 8.225), the cell is ideal. The coil has an inductance of 4 H and zero resistance. F is a fuse of zero resistance and will blow when the current through it reaches 5 A. The switch is closed at $t=0$. The fuse will blow

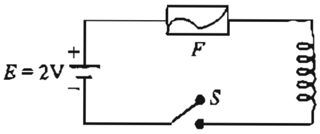


Fig. 8.225

a. almost at once **b.** after 2 s

c. after 5 s **d.** after 10 s

75. In the circuit shown (Fig. 8.226), the coil has inductance and resistance. When X is joined to Y, the time constant is t during the growth of current.

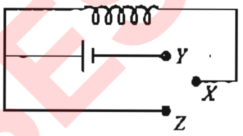


Fig. 8.226

When the steady state is reached, heat is produced in the coil at a rate P. X is now joined to Z

a. The total produced in the coil is $P\tau$

b. The total heat produced in the coil is $\frac{1}{2}P\tau$

c. The total heat produced in the coil is $2P\tau$

d. The data given are not sufficient to reach a conclusion

76. A cylindrical space of radius R is filled with a uniform magnetic induction B parallel to the axis of the cylinder (Fig. 8.227). If B changes at a constant rate, the graph showing the variation of induced electric field with distance r from the axis of cylinder is:

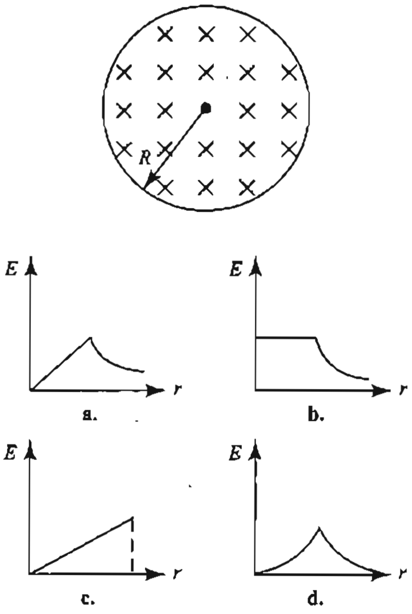


Fig. 8.227

77. A conductor PQ, with $\overrightarrow{PQ}=\vec{r}$ moves with a velocity $\vec{v}$ in a uniform magnetic field of induction $\vec{B}$. The e.m.f. induced in the rod is

a. $(\vec{v}\times\vec{B})\cdot\vec{r}$　　b. $\vec{v}\cdot(\vec{r}\times\vec{B})$

c. $\vec{B}\cdot(\vec{r}\times\vec{v})$　　d. $|\vec{r}\times(\vec{v}\times\vec{B})|$

78. A vertical ring of radius r and resistance R falls vertically. It is in contact with two vertical rails which are joined at the top. The rails are without friction and resistance. There is a horizontal uniform magnetic field of magnitude B perpendicular to the plane of the ring and the rails. When the speed of the ring is v, the current in the top horizontal of the rail section is

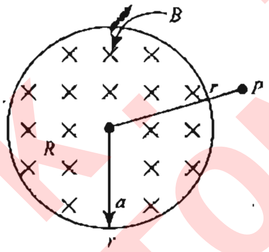
a. 0　b. $\frac{2Brv}{R}$　c. $\frac{4Brv}{R}$　d. $\frac{8Brv}{R}$

79. A long solenoid having 200 turns per cm carries a current of 1.5 A. At the centre of the solenoid is placed a coil of 100 turns of cross-sectional area 3.14×10^{-4} m^2 having its axis parallel to the field produced by the solenoid. When the direction of current in the solenoid is reversed within 0.05 s, the induced e.m.f. in the coil is

a. 0.48 V　b. 0.048 V

c. 0.0048 V　d. 48 V

80. A uniform but time-varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper as shown in Fig. 8.228. The magnitude of the induced electric field at point P at a distance from the centre of the circular region



Fgi. 8.228

a. is zero　b. decreases as $\frac{1}{r}$

c. increases as r　d. decreases as $\frac{1}{r^2}$

81. In an LR circuit connected to a battery of constant e.m.f. E switch S is closed at time $t = 0$ (Fig. 8.229). If e denotes the induced e.m.f. across the inductor and i the current in the circuit at any time t. Then which of the following graphs shows the variation of e with i?

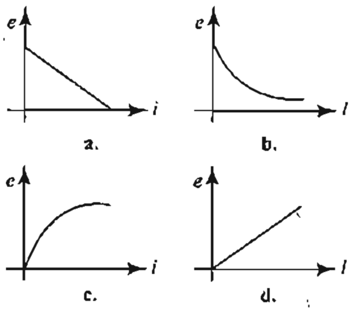


Fig. 8.229

82. A rectangular loop with a sliding connector of length $l = 1.0$ m is situated in a uniform magnetic field $B = 2$ T perpendicular to the plane of loop. Resistance of connector is $r = 2\ \Omega$. Two resistances of 6 and 3 Ω are connected as shown in Fig. 8.230. The external force required to keep the connector moving with a constant velocity $v = 2$ m/s is

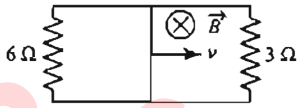


Fig. 8.230

a. 6 N　b. 4 N　c. 2 N　d. 1 N

83. A metal rod of resistance 20 Ω is fixed along a diameter of a conducting ring of radius 0.1 m and lies on x–y plane. There is a magnetic field $\vec{B} = (50\ \text{T})\hat{k}$. The ring rotates with an angular velocity $\omega = 20$ rad/s about its axis. An external resistance of 10 Ω is connected across the centre of the ring and rim. The current through external resistance is

a. $\frac{1}{4}$　b. $\frac{1}{2}$　c. $\frac{1}{3}$　d. 0

84. Some magnetic flux is changed from a coil of resistance 10 Ω. As a result an induced current is developed in it, which varies with time as shown in Fig. 8.231. The magnitude of the change in flux through the coil in webers is

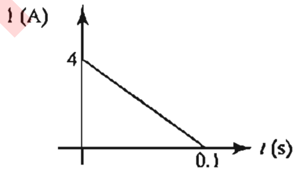


Fig. 8.231

a. 2　b. 4　c. 6　d. 8

85. In an LR circuit connected to a battery, the rate at which energy is stored in the inductor is plotted against time during the growth of current in the circuit. Which of the following best represents the resulting curve?

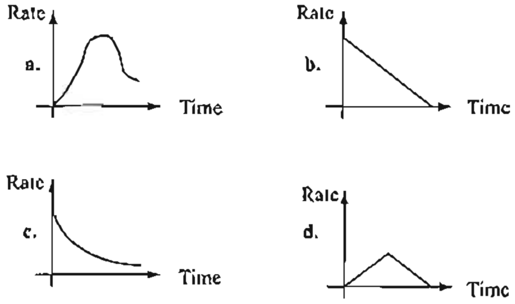


Fig. 8.232

86. Two coils are at fixed locations. When coil 1 has no current and the current in coil 2 increases at the rate 15.0 A/s the e.m.f. in coil 1 is 25.0 mV, when coil 2 has no current and coil 1 has a current of 3.6 A, the flux linkage in coil 2 is

a. 16 mWb b. 10 mWb
c. 4.00 mWb d. 6.00 mWb

87. Two concentric and coplanar coils have radii a and b (>> a) as shown in Fig. 8.233. Resistance of the inner coil is R. Current in the outer coil is increased from 0 to i, then the total charge circulating the inner coil is

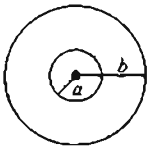


Fig. 8.233

a. $\dfrac{\mu_0 i a^2}{2Rb}$ b. $\dfrac{\mu_0 iab}{2R}$

c. $\dfrac{\mu_0 iab}{2a}\dfrac{\pi b^2}{R}$ d. $\dfrac{\mu_0 ib}{2\pi R}$

88. A current i_0 is flowing through an L-R circuit of time constant t_0. The source of the current is switched off at time $t = 0$. Let r be the value of $(-di/dt)$ at time $t = 0$. Assuming this rate to be constant, the current will reduce to zero in a time interval of

a. t_0 b. et_0

c. $\dfrac{t_0}{e}$ d. $\left(1-\dfrac{1}{e}\right)t_0$

89. A metal disc of radius a rotates with a constant angular velocity ω about its axis. The potential difference between the centre and the rim of the disc is (m = mass of electron, e = charge on electron)

a. $\dfrac{m\omega^2 a^2}{e}$ b. $\dfrac{1}{2}\dfrac{m\omega^2 a^2}{e}$

c. $\dfrac{e\omega^2 a^2}{2m}$ d. $\dfrac{e\omega^2 a^2}{m}$

90. The radius of the circular conducting loop shown in Fig. 8.234 is R. Magnetic field is decreasing at a constant rate α. Resistance per unit length of the loop is ρ.

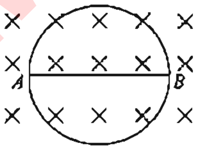


Fig. 8.234

Then, the current in wire AB is (AB is one of the diameters)

a. $\dfrac{R\alpha}{2\rho}$ from A to B b. $\dfrac{R\alpha}{2\rho}$ from B to A

c. $\dfrac{2R\alpha}{2\rho}$ from A to B d. 0

91. A current of 2 A is increasing at a rate of 4 A/s through a coil of inductance 2 H. The energy stored in the inductor per unit time is

a. 2 J/s b. 1 J/s c. 16 J/s d. 4 J/s

92. A conducting rod PQ of length L = 1.0 m is moving with a uniform speed v = 2.0 m/s in a uniform magnetic field B = 4.0 T directed into the plane of the paper.

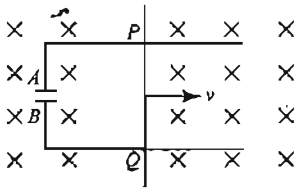


Fig. 8.235

A capacitor of capacity C = 10 μF is connected as shown in Fig. 8.233, then

a. $q_A = +80\ \mu\text{C}$ and $q_B = -80\ \mu\text{C}$
b. $q_A = -80\ \mu\text{C}$ and $q_B = +80\ \mu\text{C}$
c. $q_A = 0 = q_B$
d. charge stored in the capacitor increases exponentially with time

93. A long conducting wire AH is moved over a conducting triangular wire CDE with a constant velocity v in a uniform magnetic field $\vec{B}$ directed into the plane of the paper. Resistance per unit length of each wire is r. Then

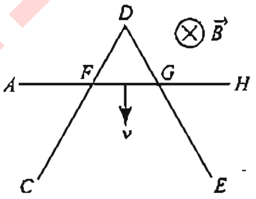


Fig. 8.236

a. a constant clockwise induced current will flow in the closed loop
b. an increasing anticlockwise induced current will flow in the closed loop
c. a decreasing anticlockwise induced current will flow in the closed loop
d. a constant anticlockwise induced current will flow in the closed loop

94. A horizontal wire is free to slide on the vertical rails of a conducting frame as shown in Fig. 8.237. The wire has a mass m and length l and the resistance of the circuit is R. If a uniform magnetic field B is directed perpendicular to the frame, the terminal speed of the wire as it falls under the force of gravity is

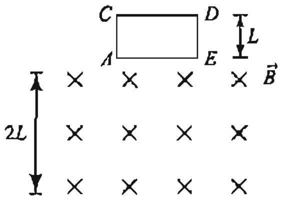


Fig. 8.237

a. $\dfrac{mgR}{Bl}$ b. $\dfrac{mgl}{BR}$

c. $\dfrac{B^2l^2}{mgR}$ d. $\dfrac{mgR}{B^2l^2}$

95. A square coil $ACDE$ with its plane vertical is released from rest in a horizontal uniform magnetic field $\vec{B}$ of length $2L$ (Fig. 8.238). The acceleration of the coil is

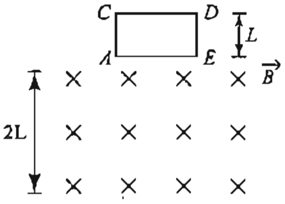


Fig. 8.238

a. less than g for all the time till the loop crosses the magnetic field completely

b. less than g when it enters the field and greater than g when it comes out of the field

c. g all the time

d. less than g when it enters and comes out of the field but equal to g when it is within the field

96. A conducting wire frame is placed in a magnetic field which is directed into the plane of the paper (Fig. 8.239). The magnetic field is increasing at a constant rate. The directions of induced currents in wires AB and CD are

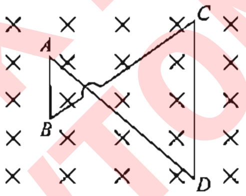


Fig. 8.239

a. B to A and D to C

b. A to B and C to D

c. A to B and D to C

d. B to A and C to D

97. An equilateral triangular loop ADC having some resistance is pulled with a constant velocity v out of a uniform magnetic field directed into the paper (Fig. 8.240). At time $t = 0$, side DC of the loop is at edge of the magnetic field.

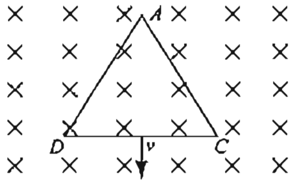


Fig. 8.240

The induced current (i) versus time (t) graph will be as

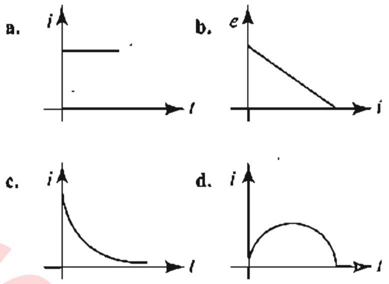


Fig. 8.241

98. In the circuit shown in Fig. 8.242, switch S is closed at time $t = 0$. The charge that passes through the battery in one time constant is

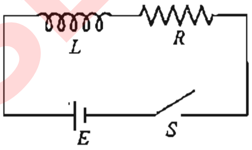


Fig. 8.242

a. $\dfrac{eR^2E}{L}$ b. $E\left(\dfrac{L}{R}\right)$ c. $\dfrac{EL}{eR^2}$ d. $\dfrac{eL}{ER}$

99. Fig. 8.243 shows a square loop of side 0.5 m and resistance 10 Ω. The magnetic field has a magnitude $B = 1.0$ T. The work done in pulling the loop out of the field slowly and uniformly in 2.0 s is

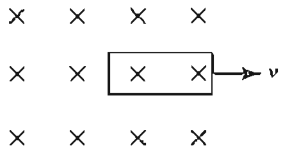


Fig. 8.243

a. 3.125×10^{-3} J b. 6.25×10^{-4} J

c. 1.25×10^{-2} J d. 5.0×10^{-4} J

100. A copper rod of mass m slides under gravity on two smooth parallel rails l distance aparted set at an angle θ to the horizontal (Fig. 8.244). At the bottom, the rails are joined by a resistance R. There is a uniform magnetic field perpendicular to the plane of the rails. The terminal velocity of the rod is

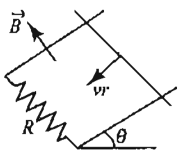


Fig. 8.244

a. $\dfrac{mgR\cos\theta}{B^2l^2}$ b. $\dfrac{mgR\sin\theta}{B^2l^2}$

c. $\dfrac{mgR\tan\theta}{B^2l^2}$ d. $\dfrac{mgR\cot\theta}{B^2l^2}$

101. A rectangular loop of wire with dimensions shown in Fig. 8.245 is coplanar with a long wire carrying current I.

The distance between the wire and the left side of the loop is r. The loop is pulled to the right as indicated. What are the directions of the induced current in the loop and the magnetic forces on the left and the right sides of the loop when the loop is pulled?

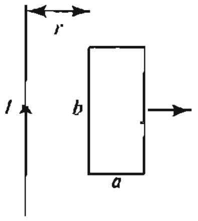


Fig. 8.245

Induced current	Force on left side	Force on right side
a. Counterclockwise	To the left	To the right
b. Counterclockwise	To the right	To the left
c. Clockwise	To the right	To the left
d. Clockwise	To the left	To the right

102. A bar magnet was pulled away from a hollow coil A as shown in Fig. 8.246. As the south pole came out of the coil, the bar magnet next to hollow coil B experienced a magnetic force

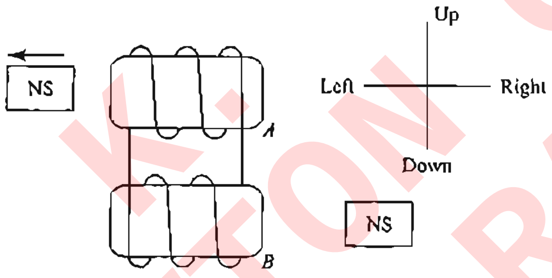


Fig. 8.246

a. to the right
b. to the left
c. upwards
d. equal to zero

103. The four wire loops shown in Fig. 8.247 have vertical edge lengths of either L, $2L$ or $3L$. They will move with the same speed into a region of uniform magnetic field $\vec{B}$ directed out of the page. Rank them according to the maximum magnitude of the induced e.m.f. greatest to least.

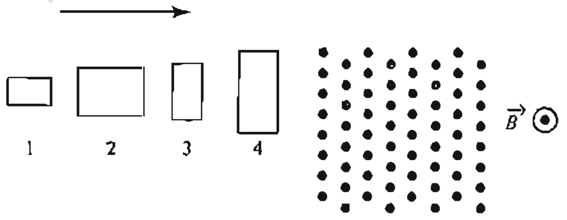


Fig. 8.247

a. 1 and 2 tie, then 3 and 4 tie
b. 3 and 4 tie, then 1 and 2 tie
c. 4, 2, 3, 1
d. 4 then, 2 and 3 tie and then 1

104. A rod lies across frictionless rails in a uniform magnetic field $\vec{B}$ as shown in Fig. 8.248. The rod moves to the right with speed v. In order for the induced e.m.f. in the circuit to be zero, the magnitude of the magnetic field should

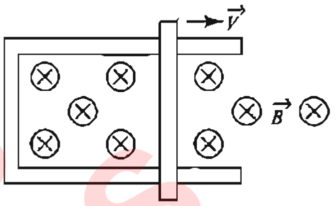


Fig. 8.248

a. not change
b. increase linearly with time
c. decrease linearly with time
d. decrease nonlinearly with time

105. In the given circuit diagram (Fig. 8.249), the key K is switched on at $t = 0$. The ratio of the current i through the cell at $t = 0$ to that at $t = \infty$ will be

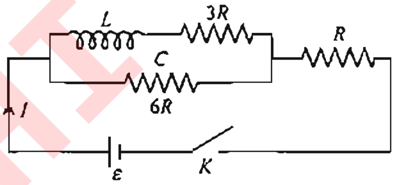
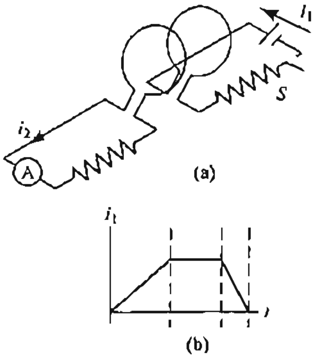


Fig. 8.249

a. 3 : 1 b. 1 : 3 c. 1 : 2 d. 2 : 1

106. The current through the coil in Fig. 8.250(a) varies as shown in Fig. 8.250(b). Which graph in best shows the ammeter A reading as a function of time?



(a)

(b)

Fig. 10.250

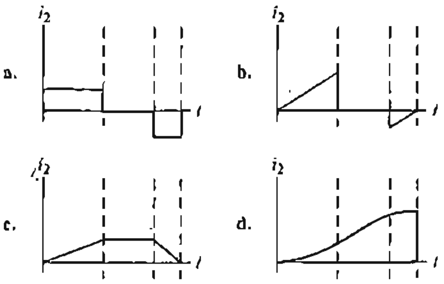


Fig. 8.251

107. The potential difference V and current I flowing through the AC circuit is given by $V = 5\cos(\omega t - \pi/6)$ volt and $I = 10\sin t$ ampere. The average power dissipated in the circuit is

a. W **b.** 12.5 W

c. 25 W **d.** 50 W

108. A closed loop of cross-sectional area 10^{-2} m^2 which has inductance L = 10 mH and negligible resistance is placed in a time-varying magnetic field. Fig. 8.252 shows the variation of B with time for the interval 4 s. The field is perpendicular to the plane of the loop (given at $t = 0$, $B = 0$, $i = 0$). The value of the maximum current induced in the loop is

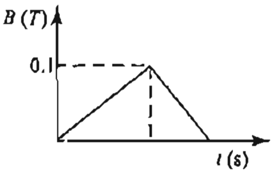


Fig. 8.252

a. 0.1 mA **b.** 10 mA

c. 100 mA **d.** Data insufficient

109. In Fig. 8.253, the key K is closed at $t = 0$. After a long time, the potential difference between A and B is zero, the value of R will be [$r_1 = r_2 = 1\,\Omega$, $E_1 = 3$ V and $E_2 = 7$ V, $C = 2\,\mu$F, $L = 4$ mH, where r_1 and r_2 are the internal resistances of cells E_1 and E_2, respectively].

a. $\frac{4}{3}\,\Omega$ **b.** $\frac{4}{9}\,\Omega$

c. $\frac{2}{3}\,\Omega$ **d.** 4 Ω

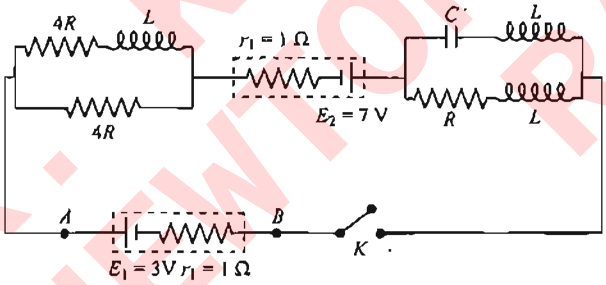


Fig. 8.253

110. An electron moves on a straight line path YY' as shown in Fig. 8.254. A coil is kept on the right such that YY' is in the plane of the coil. At the instant when the electron gets closest to the coil (neglect self-induction of the coil)

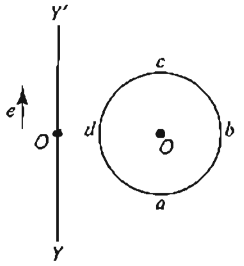


Fig. 8.254

a. the current in the coil flows clockwise

b. the current in the coil flows anticlockwise

c. the current in the coil is zero

d. the current in the coil does not change the direction as the electron crosses point O

111. A solenoid of inductance L and resistance r is connected in parallel to a resistance R. A battery of e.m.f. E and of negligible internal resistance r is connected across this parallel combination. At $t = 0$ switch s is opened.

a. Current in the inductor just after opening of the switch is $= \dfrac{E(r+R)}{rR}$

b. Total energy dissipated in the solenoid and the resistor long time after opening of the switch is $\dfrac{1}{2}L\dfrac{E^2(R+r)^2}{r^2R^2}$

c. The amount of heat generated in the solenoid due to the opening of the switch is $\dfrac{E^2L}{2r(r+R)}$

d. The amount of heat generated in the solenoid due to the opening of the switch is $\dfrac{E^2L}{2R(r+R)}$

112. The cell in the circuit shown in Fig. 8.255 is ideal. The coil has an inductance of 4 mH and a resistance of 2 mΩ. The switch is closed at $t = 0$. The amount of energy stored in the inductor at $t = 2$ s is (take $e = 3$)

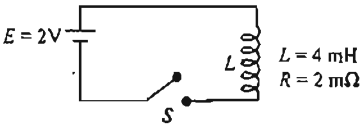


Fig. 8.255

a. $\frac{4}{3}$ J **b.** $\frac{8}{9}\times10^3$ J

c. $\frac{8}{3}\times10^{-3}$ J **d.** 2×10^3 J

113. The switch S shown in Fig. 8.256 is closed for $t < 0$ and is opened at $t = 0$. When currents through L_1 and L_2 are first equal, their common value is

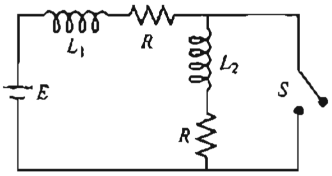


Fig. 8.256

a. $\dfrac{E}{R}$ **b.** $\dfrac{E(L_2+L_1)}{RL_1}$

c. $\dfrac{EL_1}{R(L_1+L_2)}$ **d.** $\dfrac{E}{R}\dfrac{(L_1+L_2)}{L_2}$

114. In Fig. 8.257, the mutual inductance of a coil and a very long straight wire is M, the coil has resistance R and the self-inductance L. There is an infinite wire which lies in the same plane as that of the coil. The current in the wire varies according to the law $i = at$, where a is a constant and t is the time, the time dependence of current in the coil is

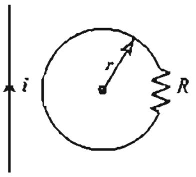


Fig. 8.257

a. $\frac{M}{aR}$
b. $MaRe^{-Rt/L}$
c. $\frac{M}{R}e^{-tR/L}$
d. $\frac{Ma}{R}(1-e^{-tR/L})$

115. In Fig. 8.258, there are two sliders and they can slide on two frictionless parallel wires in uniform magnetic field B, which is present everywhere. The mass of each slider is m, resistance R and initially these are at rest. Now, if one slider is given a velocity v_0, the velocity of other slider after considerably long time will be (neglect the self-induction)

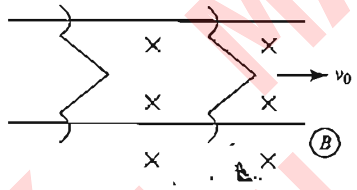


Fig. 8.258

a. $\frac{v_0}{4}$
b. $\frac{v_0}{2}$
c. v_0
d. 0

116. In Fig. 8.259, there exists a uniform magnetic field B into the plane of paper. Wire CD is in the shape of an arc and is fixed. OA and OB are the wires rotating with angular velocity ω as shown in Fig. 8.257 in the same plane as that of the arc about point O. If at some instant $OA = OB = l$ and each wire makes angle $\theta = 30°$ with y-axis, then the current through resistance R is (wires OA and OB have no resistance)

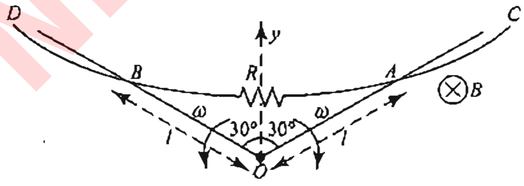


Fig. 8.259

a. 0
b. $\frac{B\omega l^2}{R}$
c. $\frac{B\omega l^2}{2R}$
d. $\frac{B\omega l^2}{4R}$

117. In the circuit shown in Fig. 8.260, the switch S is shifted to position 2 from position 1 at $t = 0$, having been in position 1 for a long time. The current in the circuit just after shifting of switch will be (battery and both the inductors are ideal)

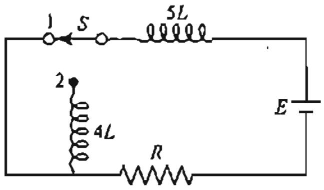


Fig. 8.260

a. $\frac{4}{5}\frac{\varepsilon}{R}$
b. $\frac{5}{4}\frac{\varepsilon}{R}$
c. $\frac{5}{9}\frac{\varepsilon}{R}$
d. $\frac{\varepsilon}{R}$

118. A cylindrical region of uniform magnetic field exists perpendicular to the plane of paper which is increasing at a constant rate $\frac{dB}{dt} = \alpha$. The diameter of the cylindrical region is l. A non-conducting rigid rod of length l having two charged particles is kept fixed on the diameter of cylindrical region w.r.t. inertial frame. If two charged particles having charges θ each is kept fixed at the ends of the non-conducting rod. The net force on any one of the charge q is

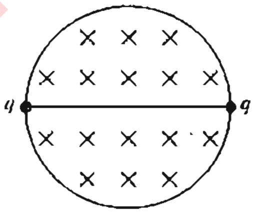


Fig. 8.261

a. $\frac{ql\alpha}{4}$
b. $\frac{ql\alpha}{2}$
c. 0
d. $q\ell\alpha$

119. In Fig. 8.262, there is a conducting ring having resistance R placed in the plane of paper in a uniform magnetic field B_0. If the ring is rotating about in the plane of paper about an axis passing through point O and perpendicular to the plane of paper with constant angular speed ω in clockwise direction, then

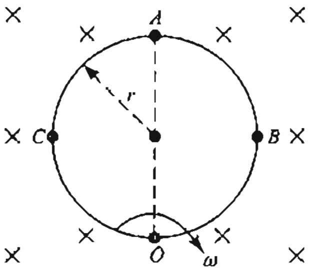


Fig. 8.262

a. point A will be at higher potential than O
b. the potential of point B and C will be same

c. the current in the ring will be zero

d. the current in the ring will be $\frac{2B_0\omega r^2}{R}$

120. An ideal coil of 20 H is joined in series with a resistance 10 Ω and an ideal battery of 10 V. After 2 s the current flowing (in A) in the circuit will be

a. e
b. e^{-1}
c. $(1 - e^{-1})$
d. $(1 - e)$

121. The capacitance in an oscillatory LC circuit is increased by 1%. The change in inductance required to restore its frequency of oscillation is to

a. decrease it by 0.5%
b. increase it by 1%
c. decrease it by 1%
d. decrease it by 2%

122. A square conducting loop of resistance 1 Ω and side 10 cm is moved with a constant velocity partly inside a magnetic field of 2 wb/m², directed into the paper as shown. The loop is connected to network of five resistors each of 4 Ω resistance. If a steady current of 0.1 A flows in the loop, then the speed of the loop is

a. 0.5 m/s
b. 2 m/s
c. 2 cm/s
d. 2.5 m/s

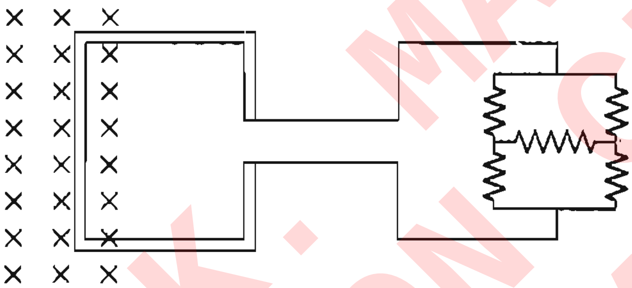


Fig. 8.263

123. A square conducting loop of side L is situated in gravity-free space. A small conducting circular loop of radius r ($r \ll L$) is placed at the centre of the square loop, with its plane perpendicular to the plane of the square loop. The mutual inductance of the two coils is

a. $\frac{2\sqrt{2}\mu_0 I}{L} r^2$
b. $\frac{\sqrt{2}\mu_0 I_0}{L} r^2$
c. 0
d. none of these

124. A vertical ring of radius r and resistance R slips vertically between two frictionless and resistanceless vertical rails (Fig. 8.264). The rails are joined at the top. There is a uniform magnetic field B perpendicular to plane of the ring and the rails. When the speed of the ring is v, the current in the section PQ is

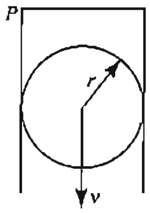


Fig. 8.264

a. 0
b. $2Brv/R$
c. $4Brv/R$
d. $8Brv/R$

125. In the circuit shown in Fig. 8.265 the switch S was initially at position 1. After sufficiently long time, the switch S was thrown from position 1 to position 2. What will be the voltage drop across the resistor after long time shifting switch from 1 to 2?

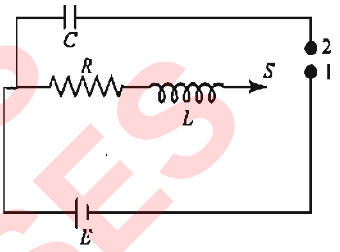


Fig. 8.265

a. 0
b. E
c. $\frac{E}{R}LC$
d. none of these

126. A rod PQ of mass m and length l can slide without friction on two vertical conducting semi-infinite rails. It is given a velocity V_0 downwards, so that it continues to move downward with the same speed V_0 on its own at any later instant of time. Assuming g to be constant everywhere, the value of V_0 is

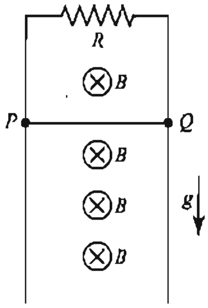


Fig. 8.266

a. $\frac{mgR}{2B^2L^2}$
b. $\frac{mgR}{B^2L^2}$
c. 0
d. any value

127. In the circuit of Fig. 8.267, (1) and (2) are ammeters. Just after key K is pressed to complete the circuit, the reading is

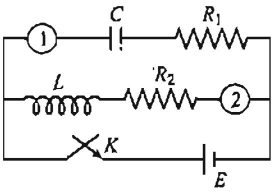


Fig. 8.267

a. maximum in both 1 and 2
b. zero in both 1 and 2
c. zero in 1, minimum in 2
d. maximum in 1, zero in 2

128. In the space shown a non-uniform magnetic field $\vec{B} = B_0(1+x)\hat{k}$ tesla is present. A closed loop of small resistance, placed in the xy plane is given velocity v_0. The force due to magnetic field on the loop is

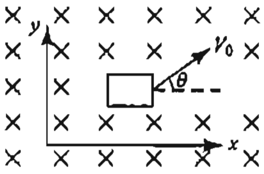


Fig. 8.268

a. zero
b. Along $+x$ direction
c. along $-x$ direction
d. along $+y$ direction

129. There is a uniform magnetic field B in a circular region of radius R as shown in Fig. 8.269 whose magnitude changes at the rate of dB/dt. The e.m.f. induced across the ends of a circular concentric conducting arc of radius R_1 having an angle θ as shown ($\angle OAO' = \theta$) is

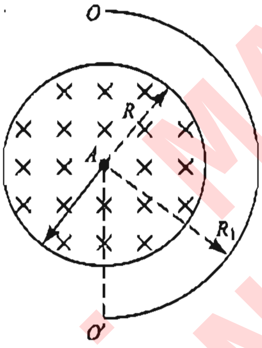


Fig. 8.269

a. $\frac{\theta}{2\pi} R_1^2 \frac{dB}{dt}$
b. $\frac{\theta}{2} R^2 \frac{dB}{dt}$
c. $\frac{\theta}{2\pi} R^2 \frac{dB}{dt}$
d. none of these

130. The power factor of the circuit in Fig. 8.270 is $1/\sqrt{2}$. The capacitance of the circuit is equal to

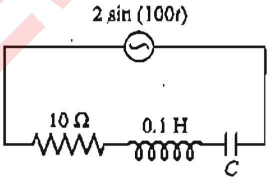


Fig. 8.270

a. 400 μF
b. 300 μF
c. 500 μF
d. 200 μF

131. Power factor of the circuit given in Fig. 8.271 will be

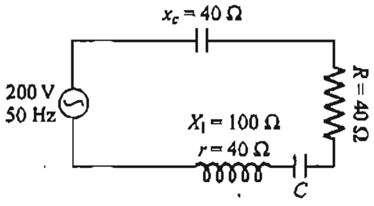


Fig. 8.271

a. 0.2
b. 0.4
c. 0.8
d. 0.6

132. A gold rod of length ℓ is accelerated in the horizontal direction with an acceleration a_0. The rod is held between two perfectly insulating clamps. Calculate the electric field set up in the rod. Take the mass of electron as m.

a. $E = \frac{ma_0}{e}$
b. $E = ma_0 \ell$
c. zero
d. none of these

133. An electron moves on a straight line path YY' as shown. A coil is kept on right hand side of the path. OO' is perpendicular to YY'. At this instant,

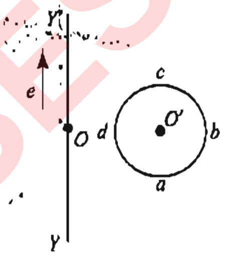


Fig. 8.272

a. the current in the coil flows clockwise
b. the current in the coil flows anticlockwise
c. the current in the coil is zero
d. the current in the coil does not change direction as the electron crosses point O

134. Two long solenoids having their radii R_1 and R_2 and number of turns N_1 and N_2 carry currents i_1 and i_2, respectively. If the ratio of $R_1/R_2 = \frac{1}{4}$, $N_1/N_2 = \frac{4}{1}$ and $i_1/i_2 = \frac{1}{2}$ the ratio of their self-inductances L_1/L_2 will be (ignore mutual inductance)

a. 1:2
b. 2:1
c. 1:4
d. 1:1

135. At any time t, $0 < t < \infty$ (excluding the cases $t \to 0$ and $t \to \infty$), the equivalent resistance between A and B is

a. $R_1 + R_2 + R_3$
b. $R_1 + R_2$
c. $R_1 + R_3$
d. none of these

136. In Fig. 8.273, a square loop $PQRS$ of side a and resistance r is placed near an infinitely long wire carrying a constant current I. The sides PQ and RS are parallel to the wire. The wire and the loop are in the same plane. The loop is rotated by 180° about an axis parallel to the long wire and passing through the mid-points of the sides QR and PS. The total amount of charge which passes through any point of the loop during rotation is

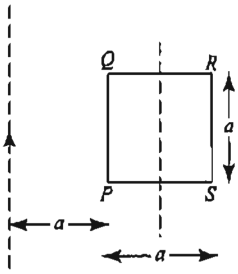


Fig. 8.273

a. $\frac{\mu_0 Ia}{2\pi r}\ln 2$ b. $\frac{\mu_0 Ia}{\pi r}\ln 2$ c. $\frac{\mu_0 Ia^2}{2\pi r}$

d. Cannot be found because time of rotation is not given.

137. A wooden stick of length 3ℓ is rotated about an end with constant angular velocity ω in a uniform magnetic field B perpendicular to the plane of motion. If the upper one-third of its length is coated with copper, the potential difference across the whole length of the stick is

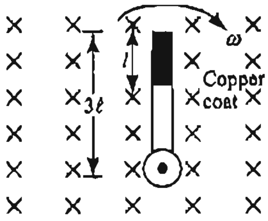


Fig. 8.274

a. $\frac{9B\omega\ell^2}{2}$ b. $\frac{4B\omega\ell^2}{2}$

c. $\frac{5B\omega\ell^2}{2}$ d. $\frac{B\omega\ell^2}{2}$

138. PQ is an infinite current-carrying conductor. AB and CD are smooth conducting rods on which a conductor EF moves with constant velocity V as shown. The force needed to maintain constant speed of EF is

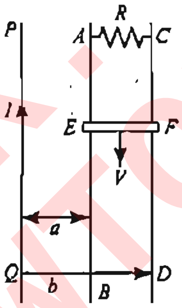


Fig. 8.275

a. $\frac{1}{VR}\left[\frac{\mu_0 IV}{2\pi}\ln\left(\frac{b}{a}\right)\right]^2$ b. $\left[\frac{\mu_0 IV}{2\pi}\ln\left(\frac{b}{a}\right)\right]^2\frac{1}{VR}$

c. $\left[\frac{\mu_0 IV}{2\pi}\ln\left(\frac{b}{a}\right)\right]^2\frac{V}{R}$ d. $\frac{V}{R}\left[\frac{\mu_0 IV}{2\pi}\ln\left(\frac{b}{a}\right)\right]^2$

139. Loop A of radius $r << R$ moves towards loop B with a constant velocity V in such a way that their planes are always parallel. What is the distance between the two loops (x) when the induced e.m.f. in loop A is maximum

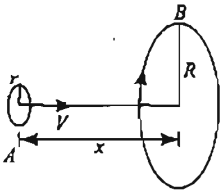


Fig. 8.276

a. R b. $\frac{R}{\sqrt{2}}$

c. $\frac{R}{2}$ d. $R\left(1-\frac{1}{\sqrt{2}}\right)$

140. Rate of increment of energy in an inductor with time in series LR circuit getting charge with a battery of e.m.f. E is best represented by (inductor has initially zero current)

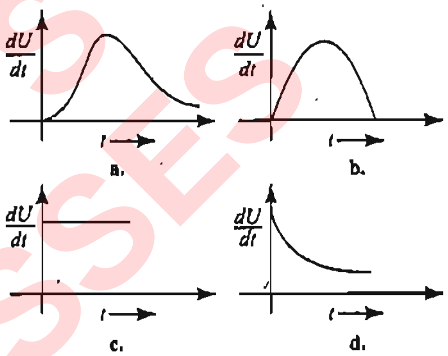


Fig. 8.277

141. A wire of fixed length is wound on a solenoid of length ℓ and radius r. Its self-inductance is found to be L. Now, if the same wire is wound on a solenoid of length $\frac{\ell}{2}$ and radius $\frac{r}{2}$, then the self-inductance will be

a. $2L$ b. L c. $4L$ d. $8L$

142. When the current in a certain inductor coil is 5.0 A and is decreasing at the rate of 10.0 $\mathrm{As^{-1}}$, the potential difference is 60 V. The self-inductance of the coil is

a. 2 H b. 4 H c. 8 H d. 12 H

143. Fig. 8.278 shows three regions of magnetic field each of area A, and in each region magnitude of magnetic field decreases at a constant rate α. If $\vec{E}$ is the induced electric field, then value of the line integral $\oint \vec{E}\cdot d\vec{r}$ along the given loop is equal to

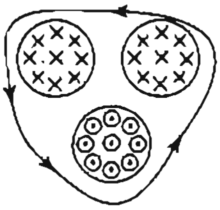


Fig. 8.278

a. αA b. $-\alpha A$ c. $3\alpha A$ d. $-3\alpha A$

144. In an ideal transformer, the voltage and the current in the primary are 200 V and 2 A, respectively. If the voltage in the secondary is 2000 V, then the value of current in the secondary will be

a. 0.2 A b. 2 A c. 10 A d. 20 A

145. A superconducting loop of radius R has self-inductance L. A uniform and constant magnetic field B is applied perpendicular to the plane of the loop. Initially current in this loop is zero. The loop is rotated by 180°. The current in the loop after rotation is equal to

a. 0
b. $\frac{B\pi R^2}{L}$
c. $\frac{2B\pi R^2}{L}$
d. $\frac{B\pi R^2}{2L}$

146. A semicircular wire of radius R is rotated with constant angular velocity about an axis passing through one end and perpendicular to the plane of the wire. There is a uniform magnetic field of strength B. The induced e.m.f. between the ends is

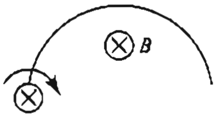


Fig. 8.279

a. $B\omega R^2/2$
b. $2B\omega R^2$
c. is variable
d. none of these

147. The frequency of oscillation of current in the inductance is

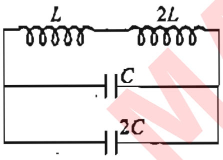


Fig. 8.280

a. $\frac{1}{3\sqrt{LC}}$
b. $\frac{1}{6\pi\sqrt{LC}}$
c. $\frac{1}{\sqrt{LC}}$
d. $\frac{1}{2\pi\sqrt{LC}}$

148. A rectangular loop of sides a and b is placed in xy plane. A very long wire is also placed in xy plane such that side of length a of the loop is parallel to the wire. The distance between the wire and the nearest edge of the loop is d. The mutual inductance of the system is proportional to
a. a
b. b
c. $1/d$
d. current in wire

149. Radius of a circular ring is changing with time and the coil is placed in a uniform magnetic field perpendicular to its plane. The variation of r with time t is shown in Fig. 8.281. Then the induced e.m.f. ε with time will be best represented by

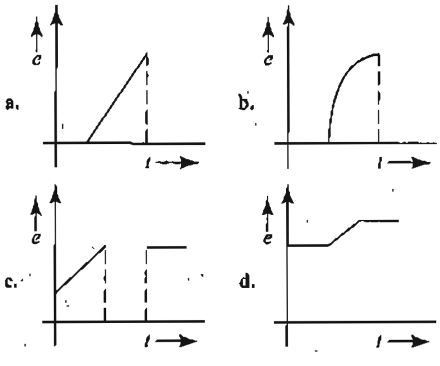


Fig. 8.281

150. Switch S is closed for a long time at $t = 0$. If it is opened, then

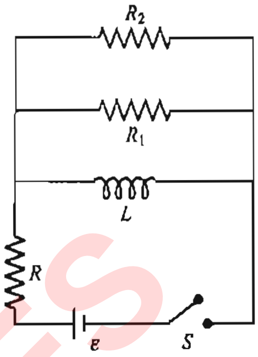


Fig. 8.282

a. total heat produced in resistor R after opening the switch is $\frac{1}{2}\frac{LV}{R^2}$
b. total heat produced in resistor R_1 after opening the switch is $\frac{1}{2}\frac{LV^2}{R^2}\left(\frac{R_1}{R_1+R_2}\right)$
c. heat produced in resistor R_1 after opening the switch is $\frac{1}{2}\frac{R_2LV^2}{(R_1+R_2)R^2}$
d. no heat will be produced in R_1

151. A rod of length ℓ with uniformly distributed charge Q is rotated about one end with a constant frequency f. Its magnetic moment is

a. $\pi fQ\ell^2$
b. $\frac{\pi fQ\ell^2}{3}$
c. $\frac{2\pi fQ\ell^2}{3}$
d. $2\pi fQ\ell^2$

152. Two identical cycle wheels (geometrically) have different number of spokes connected from center to rim. One is having 20 spokes and the other having only 10 (the rim and the spokes are resistanceless). One resistance of value R is connected between centre and rim. The current in R will be
a. double in the first wheel than in the second wheel
b. four times in the first wheel than in the second wheel
c. will be double in the second wheel than that of the first wheel
d. will be equal in both these wheels

153. When magnetic flux through a coil is changed, the variation of induced current in the coil with time is as shown in the graph. If resistance of the coil is 10 Ω, then the total change in flux of coil will be

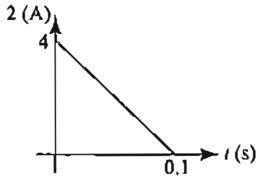


Fig. 8.283

a. 4 b. 8 c. 2 d. 6

154. A uniform magnetic field exists in a region given by $\vec{B}=3\hat{i}+4\hat{j}+5\hat{k}$. A rod of length 5 m is moved along y-axis with a constant speed of 1 m/s. Then the induced e.m.f. in the rod will be

a. 0 b. 25 V c. 20 V d. 15 V

155. In a LR growth circuit, inductance and resistance used are 1 H and 20 Ω, respectively. If at t = 50 ms, the current in the circuit is 3.165 A, then applied direct current e.m.f. is

a. 200 V b. 100 V c. 50 V

d. Data is insufficient to find out the value.

156. A square loop of area 2.5×10^{-3} m^2 and having 100 turns with a total resistance of 100 Ω is moved out of a uniform magnetic field of 0.40 T in 1 s with a constant speed. Then work done, in pulling the loop, is

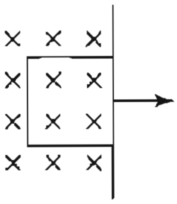


Fig. 8.284

a. 0 b. 1 mJ c. 1 μJ d. 0.1 mJ

157. A and B are two metallic rings placed at opposite sides of an infinitely long straight conducting wire as shown. If current in the wire is slowly decreased, the direction of the induced current will be

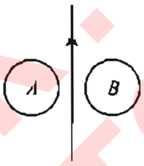


Fig. 8.285

a. clockwise in A and anticlockwise in B

b. anticlockwise in A and clockwise in B

c. clockwise in both A and B

d. anticlockwise in both A and B

158. A vertical conducting ring of radius R falls vertically with a speed V in a horizontal uniform magnetic field B which is perpendicular to the plane of the ring.

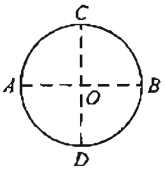


Fig. 8.286

a. A and B are at the same potential

b. C and D are at the same potential

c. current flows in clockwise direction

d. current flows in anticlockwise direction

159. Two identical conducting rings A and B of radius R are in pure rolling over two horizontal conducting planes with same speed (of centre of mass) v but in opposite direction. A constant magnetic field B is present pointing into the plane of paper. Then the potential difference between the highest points of the two rings is

Fig. 8.287

a. 0 b. $2Bvr$

c. $4Bvr$ d. none of these

160. An inductor L and a resistor R are connected in series with a direct current source of e.m.f. E. The maximum rate at which energy is stored in the magnetic field is

a. $\frac{E^2}{4R}$ b. $\frac{E^2}{R}$

c. $\frac{4E^2}{R}$ d. $\frac{2E^2}{R}$

161. In the circuit shown, switch S is connected to position 2 for a long time and then joined to position 1. The total heat produced in resistance R_1 is

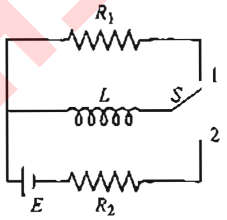


Fig. 8.288

a. $\frac{LE^2}{2R_2^2}$ b. $\frac{LE^2}{2R_1^2}$

c. $\frac{LE^2}{2R_1R_2}$ d. $\frac{LE2(R_1+R_2)^2}{2R_1^2R_2^2}$

162. In the circuit shown in Fig. 8.289, the switch S was initially at position 1. After sufficiently long time, the switch S was thrown from position 1 to position 2. The voltage drop across the resistor at that instant is

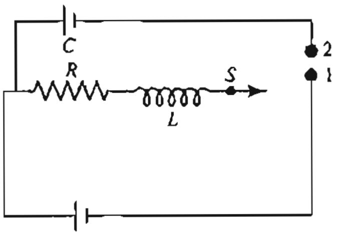


Fig. 8.289

a. zero b. E

c. $\frac{E}{R}LC$ d. none of these

163. A uniform magnetic field of induction B is confined to cylindrical region of radius R. The magnetic field is increasing at a constant rate of $\frac{dB}{dt}$ (Ts^{-1}). An electron of charge q, placed at the point P on the periphery of the field, experiences an acceleration

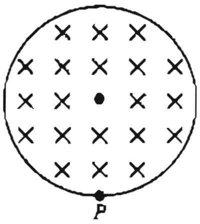


Fig. 8.290

a. $\frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$ towards left
b. $\frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$ towards right
c. $\frac{eR}{m}\frac{dB}{dt}$ towards left
d. zero

164. AB is a resistanceless conducting rod which forms a diameter of a conducting ring of radius r rotating in a uniform magnetic field B as shown. The resistors R_1 and R_2 do not rotate. Then current through the resistor R_1 is

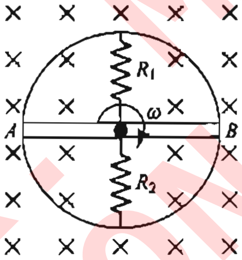


Fig. 8.291

a. $\frac{B\omega r^2}{2R_1}$
b. $\frac{B\omega r^2}{2R_2}$
c. $\frac{B\omega r^2}{2R_1 R_2}(R_1+R_2)$
d. $\frac{B\omega r^2}{2(R_2+R_2)}$

165. AB and CD are fixed conducting smooth rails placed in a vertical plane and joined by a constant current source at its upper end. PQ is a conducting rod which is free to slide on the rails. A horizontal uniform magnetic field exists in space as shown. If the rod PQ is released form rest then,

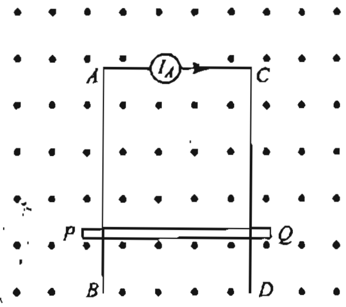


Fig. 8.292

a. the rod PQ will move downward with constant acceleration
b. the rod PQ will move upward with constant acceleration
c. the rod will move downward with decreasing acceleration and finally acquire a constant velocity
d. either a or b

166. A conducting ring of radius r with a conducting spoke is in pure rolling on a horizontal surface in a region having a uniform magnetic field B as shown, n being the velocity of the centre of the ring. Then the potential difference $V_0 - V_A$ is

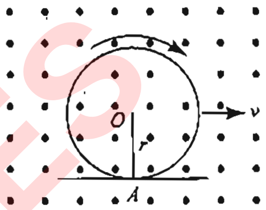


Fig. 8.293

a. $\frac{Bvr}{2}$
b. $\frac{3Bvr}{2}$
c. $\frac{-3Bvr}{2}$
d. $\frac{Bvr}{2}$

167. A metallic ring of mass m and radius r with a uniform metallic spoke of same mass m and length r is rotated about its axis with angular velocity ω in a perpendicular uniform magnetic field B as shown. The central end of the spokes is connected to the rim of the wheel through a resistor R as shown. The resistor does not rotate, its one end is always at the center of the ring and the other end is always in contact with the ring. A force F as shown is needed to maintain constant angular velocity of the wheel. F is equal to (the ring and the spoke has zero resistance)

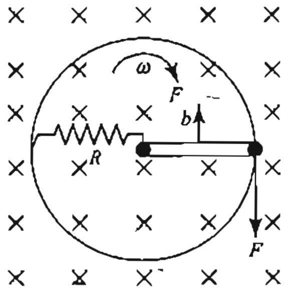


Fig. 8.294

a. $\frac{B^2\omega r^2}{8R}$
b. $\frac{B^2\omega r^2}{2R}$
c. $\frac{B^2\omega r^3}{2R}$
d. $\frac{B^2\omega r^3}{4R}$

168. A closed circuit consists of a resistor R, inductor of inductance L and a source of e.m.f. E are connected in series. If the inductance of the coil is abruptly decreased to $L/4$ (by removing its magnetic core), the new current immediately after this moment is (before decreasing the inductance the circuit is in steady state)

a. zero
b. E/R
c. $4\frac{E}{R}$
d. $\frac{E}{4R}$

169. The current generator I_g, shown in Fig. 8.295, sends constant current i through the circuit. The wire ab has length ℓ and mass m and can slide on the smooth, horizonta rails connected to i_g. The entire system lies in a vertical magnetic field B. The velocity of the wire as a function of time is

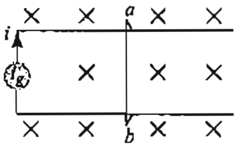


Fig. 8.295

a. $\frac{i\ell Bt}{m}$ b. $\frac{i\ell Bt}{2m}$ c. $\frac{2i\ell Bt}{m}$ d. $\frac{i\ell Bt}{3m}$

170. A vertical ring of radius r and resistance R falls vertically. It is in contact with two vertical rails which are joined at the top. The rails are without friction and resistance. There is a horizontal uniform magnetic field of magnitude B perpendicular to the plane of the ring and the rails. When the speed of the ring is v, the current in the section PQ is

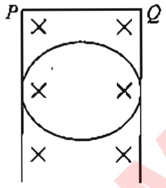


Fig. 8.296

a. zero b. $\frac{2Brv}{R}$ c. $\frac{4Brv}{R}$ d. $\frac{8Brv}{R}$

171. Given $L_1 = 1$ mH, $R_1 = 1\ \Omega$
$L_2 = 2$ mH, $R_2 = 2\ \Omega$

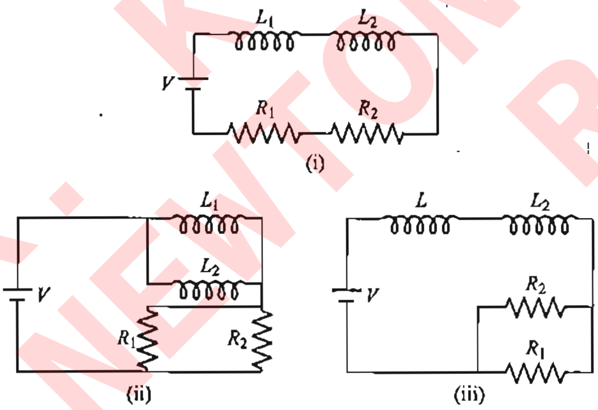


Fig. 8.297

Neglecting mutual inductance, the time constants (in ms) for the circuits (i), (ii) and (iii) are

a. $1, 1, \frac{9}{2}$ b. $\frac{9}{4}, 1, 1$ c. 1, 1, 1 d. $1, \frac{9}{4}, 1$

172. A horizontal ring of radius $r = \frac{1}{2}$ m is kept in a vertical constant magnetic field 1 T. The ring is collapsed from maximum area to zero area in 1 s. Then the e.m.f. induced in the ring is

a. 1 V b. $(\pi/4)$ V c. $(\pi/2)$ V d. π V

173. In the circuit shown, the key (K) is closed at $t = 0$, the current through the key at the instant $t = 10^{-3} \ln 2$ s is

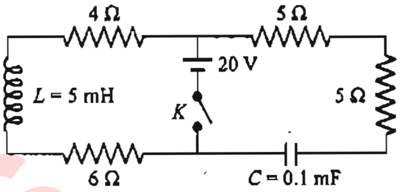


Fig. 8.298

a. 2 A b. 3.5 A c. 2.5 A d. 0

174. A rod of length ℓ rotates in the form of a conical pendulum with an angular velocity ω about its axis as shown in Fig. 8.299. The rod makes an angle θ with the axis. The magnitude of the motional e.m.f. developed across the two ends of the rod is

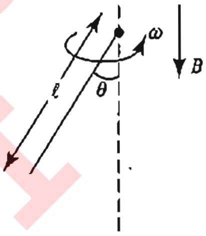


Fig. 8.299

a. $\frac{1}{2} B\omega\ell^2$ b. $\frac{1}{2} B\omega\ell^2 \tan^2\theta$

c. $\frac{1}{2} B\omega\ell^2 \cos^2\theta$ d. $\frac{1}{2} B\omega\ell^2 \sin^2\theta$

175. In the circuit shown, switch k_2 is open and switch k_1 is closed at $t = 0$. At time $t = t_0$, switch k_1 is opened and switch k_2 is simultaneously closed. The variation of inductor current with time is

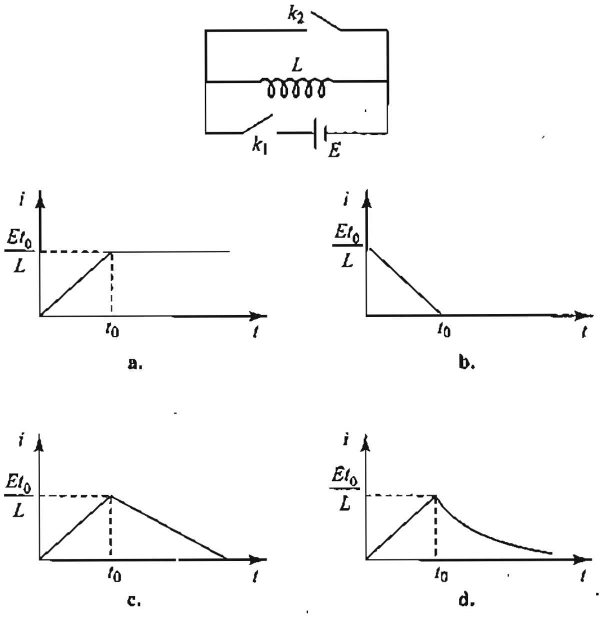


Fig. 8.300

176. In an *LC* circuit shown in Fig. 8.301, $C = 1$ F, $L = 4$ H. At time $t = 0$ charge in the capacitor is 4 C and it is decreasing at a rate of $\sqrt{5}\,\text{Cs}^{-1}$. Choose the correct statement.

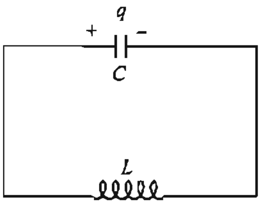


Fig. 8.301

a. Maximum charge in the capacitor can be 6 C
b. Maximum charge in the capacitor can be 8 C
c. Charge in the capacitor will be maximum after time $3\sin^{-1}(2/3)$ s
d. None of these

177. An aluminium ring hangs vertically from a thread with its axis pointing east-west. A coil is fixed near to the ring and coaxial with it.

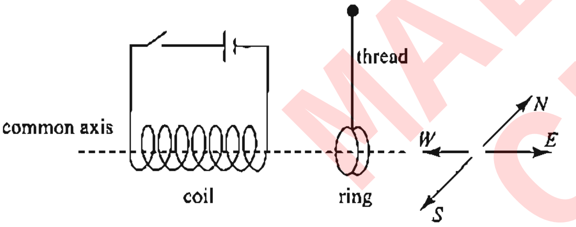


Fig. 8.302

What is the initial motion of the aluminium ring when the current in the coil is switched on?

a. moves towards *E*
b. moves towards *W*
c. moves towards *N*
d. moves towards *S*

178. The magnetic flux density *B* is changing in magnitude at a constant rate dB/dt. A given mass *m* of copper, drawn into a wire of radius *a* and formed into a circular loop of radius *r* is placed perpendicular to the field *B*. The induced current in the loop is *i*. The resistivity of copper is *r* and density is *d*. The value of the induced current *i* is

a. $\frac{m}{2\pi\rho d}\frac{dB}{dt}$
b. $\frac{m}{4\pi a^2 r}\frac{dB}{dt}$
c. $\frac{m}{4\pi a d}\frac{dB}{dt}$
d. $\frac{m}{4\pi\rho d}\frac{dB}{dt}$

179. The magnetic field in a region is given by $\vec{B} = B_0\left(1+\frac{x}{a}\right)\hat{k}$. A square loop of edge length *d* is placed with its edge along the *x*- and *y*-axes. The loop is moved with a constant velocity $\vec{v} = v_0\hat{i}$. The e.m.f. induced in the loop is

a. $\frac{v_0B_0d^2}{a}$
b. $\frac{v_0B_0d^3}{a^2}$
c. v_0B_0d
d. zero

180. A coil carrying a steady current is short-circuited. The current in it decreases α times in time t_0. The time constant of the circuit is

a. $\tau = t_0 \ln\alpha$
b. $\tau = \frac{t_0}{\ln\alpha}$
c. $\tau = \frac{t_0}{\alpha}$
d. $\tau = \frac{t_0}{\alpha-1}$

181. A solenoid has 2000 turns wound over a length of 0.3 m. Its cross-sectional area is equal to 1.2×10^{-3} m^2. Around its central cross section a coil of 300 turns is wound. If an initial current of 2 A flowing in the solenoid is reversed in 0.25 s, the e.m.f. induced in the coil is

a. 0.6 mV
b. 60 mV
c. 48 mv
d. 0.48 mV

182. Two coils *X* and *Y* are linked such that e.m.f. *E* is induced in *Y* when the current in *X* is changing at the rate $I\left(=\frac{dI}{dt}\right)$. If a current I_0 is now made to flow through *Y*, the flux linked with *X* will be

a. EI_0I
b. $\frac{I_0I}{E}$
c. $(EI)I_0$
d. $\left(\frac{E}{I}\right)I_0$

183. A conductor *AB* of length ℓ moves in *xy* plane with velocity $\vec{v} = v_0(\hat{i}-\hat{j})$. A magnetic field $\vec{B} = B_0(\hat{i}+\hat{j})$ exists in the region. The induced e.m.f. is

a. zero
b. $2B_0\ell v_0$
c. $B_0\ell v_0$
d. $\sqrt{2}B_0 l v_0$

184. The time constant of an inductance coil is 2×10^{-3} s. When a 90 Ω resistance is joined in seires, the same constant become 0.5×10^{-3} s. The inductance and resistance of the coil are

a. 30 mH; 30 W
b. 60 mH; 30 W
c. 30 mH; 60 W
d. 60 mH; 60 W

185. A line charge λ per unit length is pasted uniformly on to the rim of a wheel of mass *m* and radius *R*. The wheel has light non-conducting spokes and is free to rotate about a vertical axis as shown in Fig. 8.303. A uniform magnetic field extends over a radial region given by $B = -B_0\bar{K}$ ($r \le a$; $a < R$) $= 0$ (otherwise). What is the angular velocity of the wheel when this field is suddenly switched off?

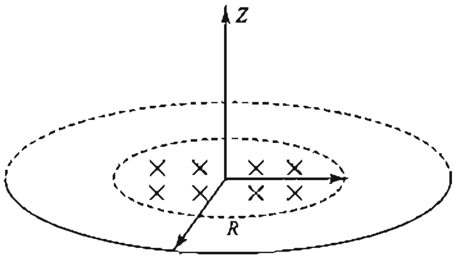


Fig. 8.303

a. $\frac{-2B\pi a^2 r}{mR}\hat{k}$
b. $\frac{-B\pi a^2 r}{3mR}\hat{k}$
c. $\frac{-B\pi a^2\lambda}{2mR}\hat{k}$
d. $\frac{-B\pi a^2\lambda}{2mR}\hat{k}$

186. The current passing through the battery immediately after key (K) is closed [it is given that initially all the capacitors are uncharged (given that $R = 6\ \Omega$ and $C = 4\ \mu F$)] is

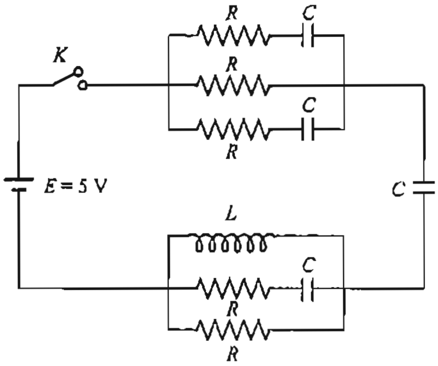


Fig. 8.304

a. 1 A b. 5 A c. 3 A d. 2 A

187. A flexible wire loop in the shape of a circle has a radius that grows linearly with time. There is a magnetic field perpendicular to the plane of the loop that has a magnitude inversely proportional to the distance from the centre of the loop, $B(r) \propto \frac{1}{r}$. How does the e.m.f. E vary with time?

a. $E \propto t^2$ b. $E \propto t$

c. $E \propto \sqrt{t}$ d. E is constant

188. A conducting wire of length ℓ and mass m is placed on two inclined rails as shown in Fig. 8.305. A current I is flowing in the wire in the direction shown. When no magnetic field is present in the region, the wire is just on the verge of sliding. When a vertically upward magnetic field is switched on, the wire starts moving up the incline. The distance traveled by the wire as a function of time t will be

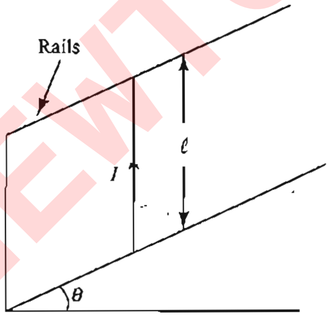


Fig. 8.305

a. $\frac{1}{2}\left[\frac{IBl}{m}-2g\right]t^2$

b. $\frac{1}{2}\left[\frac{IBl}{m}\times\frac{1}{\cos\theta}-2g\sin\theta\right]t^2$

c. $\frac{1}{2}\left[\frac{IBl}{m}-2g\sin\theta\right]t^2$

d. $\frac{1}{2}\left[\frac{IBl}{m}\frac{\cos 2\theta}{\cos\theta}-2g\sin\theta\right]t^2$

189. A pure inductor L, a capacitor C and a resistance R are connected across a battery of e.m.f. E and internal resistance r as shown in the figure. The switch S_w is closed at $t = 0$, select the correct alternative(s).

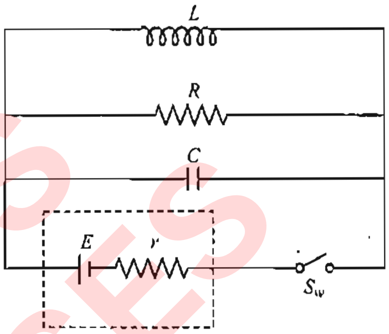


Fig. 8.306

a. current through resistance R is zero all the time
b. current through resistance R is zero at $t = 0$ and $t \to \infty$
c. maximum charge stored in the capacitor is CE
d. maximum energy stored in the inductor is equal to the maximum energy stored in the capacitor

190. A simple LR circuit is connected to a battery at time $t = 0$. The energy stored in the inductor reaches half its maximum value at time

a. $\frac{R}{L}\ln\left[\frac{\sqrt{2}}{\sqrt{2}-1}\right]$ b. $\frac{L}{R}\ln\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right]$

c. $\frac{L}{R}\ln\left[\frac{\sqrt{2}}{\sqrt{2}-1}\right]$ d. $\frac{R}{L}\ln\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right]$

191. Three identical coils A, B and C carrying currents are placed coaxially with their planes parallel to one another. A and C carry currents as shown. B is kept fixed, while A and C both are moved towards B with the same speed. Initially, B is equally separated from A and C. The direction of the induced current in the coil B is

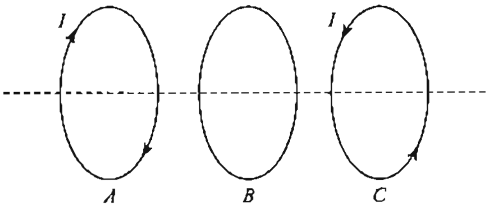


Fig. 8.307

a. same as that in coil A a. same as that in coil B

c. zero d. none of these

192. The natural frequency of the circuit shown in Fig. 8.308 is

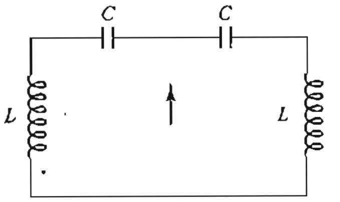


Fig. 8.308

a. $\frac{1}{\sqrt{LC}}$ b. $\frac{1}{\sqrt{2LC}}$

c. $\frac{2}{\sqrt{LC}}$ d. none of these

193. A conducting ring of radius r and resistance R rolls on a horizontal surface with constant velocity v. The magnetic field B is uniform and is normal to the plane of the loop. Choose the correct option.

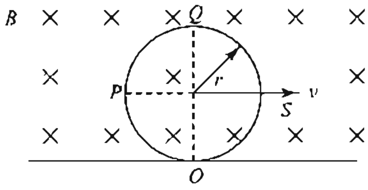


Fig. 8.309

a. The induced e.m.f. between O and Q is $2Brv$.

b. An induced current $I = \frac{2Bvr}{R}$ flows in the clockwise direction.

c. An induced current $I = \frac{2Bvr}{R}$ flows in the anticlockwise direction.

d. No current flows.

194. The magnetic flux density B is changing in magnitude at a constant rate of dB/dt. A given mass m of copper, drawn into a wire of radius a and formed into a circular loop of radius r is placed perpendicular to the field B. The induced current in the loop is i. The resistivity of copper is r and density is d. Then

a. $i = \frac{m}{4\pi\rho d}\frac{dB}{dt}$ b. $i = \frac{m}{4\pi\rho^2 r}\frac{dB}{dt}$

c. $i = \frac{m}{4\pi a d}\frac{dB}{dt}$ d. $i = \frac{m}{2\pi\rho d}\frac{dB}{dt}$

195. Two resistors of 10 Ω and 20 Ω and an ideal inductor of 10 H are connected to a 2 V battery as shown. The key K is inserted at time $t = 0$. The initial ($t = 0$) and final ($t \to \infty$) currents through the battery are

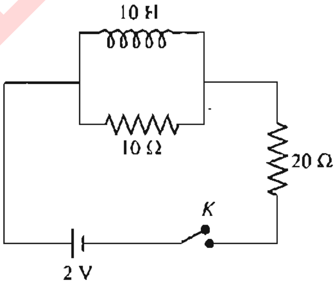


Fig. 8.310

a. $\frac{1}{15}$ A, $\frac{1}{10}$ A b. $\frac{1}{10}$ A, $\frac{1}{15}$ A

c. $\frac{2}{15}$ A, $\frac{1}{10}$ A d. $\frac{1}{15}$ A, $\frac{2}{25}$ A

196. In the circuit shown, A is joined to B for a long time, and then A is joined to C. The total heat produced in R is

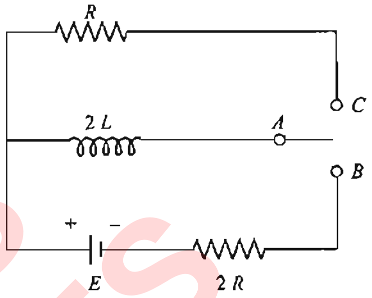


Fig. 8.311

a. $\frac{LE^2}{R^2}$ b. $\frac{LE^2}{2R^2}$ c. $\frac{LE^2}{4R^2}$ d. $\frac{LE^2}{8R^2}$

197. Two identical conductors P and Q are placed on two frictionless rails R and S in a uniform magnetic field directed into the plane. If P is moved in the direction shown in Fig. 8.312 with a constant speed, then rod Q

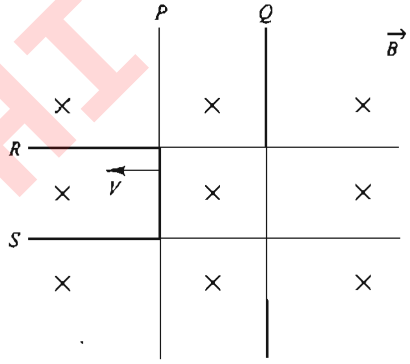


Fig. 8.312

a. will be attracted towards P

b. will be repelled away from P

c. will remain stationary

d. may be repelled away or attracted towards P

198. In the circuit shown in Fig. 8.313, a conducting wire HE is moved with a constant speed v towards left. The complete circuit is placed in a uniform magnetic field $\bar{B}$ perpendicular to the plane of the circuit inwards. The current in $HKDE$ is

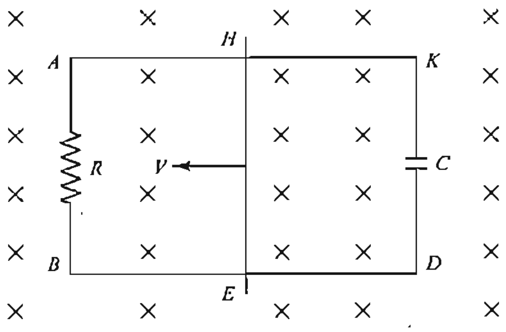


Fig. 8.313

a. clockwise b. anticlockwise

c. alternating d. zero

199. A straight wire of length $\bar{L}$ moves with constant velocity $\vec{v}$ (no rotation) through a uniform magnetic field $\bar{L}$ as shown. $\vec{L}$ is a vector directed from a to b. The induced e.m.f. ξ_i is given by

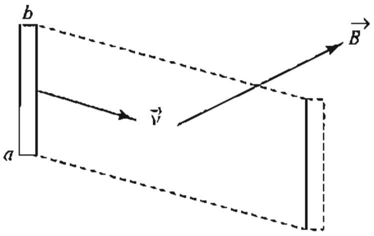


Fig. 8.314

a. $\xi_i = 0$ a. $\xi_i = \vec{B} \cdot (\bar{v} \times \vec{L})$
c. $\xi_i = \bar{L} \cdot (\bar{B} \times \vec{v})$ d. $\xi_i = \vec{B} \cdot (\vec{L} \times \bar{v})$

200. A square metal loop of side 10 cm and resistance 1 Ω is moved with a constant velocity partly inside a magnetic field of 2 Wbm^{-2}, directed into the paper, as shown in Fig. 8.315. This loop connected to a network of five resistors each of value 3 Ω. If a steady current of 1 mA flows in the loop, then the speed of the loop is

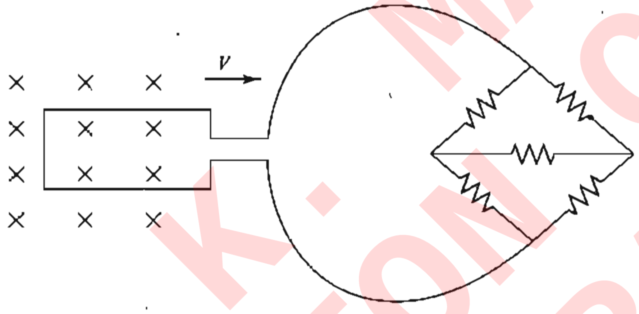


Fig. 8.315

a. 0.5 cms^{-1} b. 1 cms^{-1}
c. 2 cms^{-1} d. 4 cms^{-1}

201. A magnet is moving towards the coil along the axis and the e.m.f. induced in the coil is ε. If the coil also starts moving towards the magnet with the same speed, the induced e.m.f. will be

a. $\frac{\varepsilon}{2}$ b. ε c. 2ε d. 4ε

202. The magnetic field in a region is given by $\bar{B} = B_0 \left(1 + \frac{x}{a}\right)\hat{k}$. A square loop of edge length d is placed with its edges along the x- and y-axes. The loop is moved with a constant velocity $\hat{v} = V_0\,\hat{i}$. The e.m.f. induced in the loop is

a. zero b. $v_0 B_0 d$
c. $\frac{V_0 B_0 d^3}{a^2}$ d. $\frac{V_0 B_0 d^2}{a}$

203. A conducting rod PQ of length $\ell = 2$ m is moving at a speed of 2 ms^{-1} making an angle of 30° with its length. A uniform magnetic field $B = 2$ T exists in a direction perpendicular to the plane of motion. Then

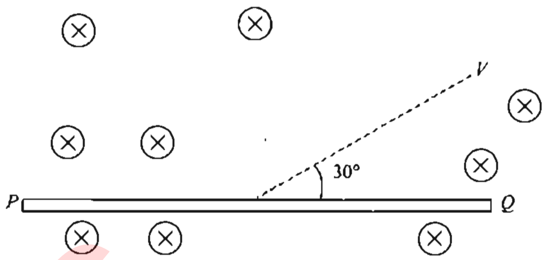


Fig. 8.316

a. $V_P - V_Q = 8$ V b. $V_P - V_Q = 4$ V
c. $V_Q - V_P = 8$ V d. $V_Q - V_P = 4$ V

204. A conductor AB of length ℓ moves in xy plane with velocity $\hat{v} = V_0\left(\hat{i} - \hat{j}\right)$. A magnetic field $\hat{B} = B_0\left(\hat{i} + \hat{j}\right)$ exists in the region. The induced e.m.f. is

a. $\sqrt{2}\,B_0 \ell_0 V_0$ a. $2\,B_0 \ell\, v_0$
c. $B_0\,\ell\,V_0$ d. zero

Multiple Correct Answers Type

Solutions on page 8.128

1. The uniform magnetic field perpendicular to the plane of a conducting ring of radius a changes at the rate of α, then
   a. all the points on the ring are at the same potential
   b. the e.m.f. induced in the ring is $\pi a^2 \alpha$
   c. electric field intensity E at any point on the ring is zero
   d. $E = \frac{a\alpha}{2}$

2. In the given circuit, switch is closed at $t = 0$. Choose the correct answers.

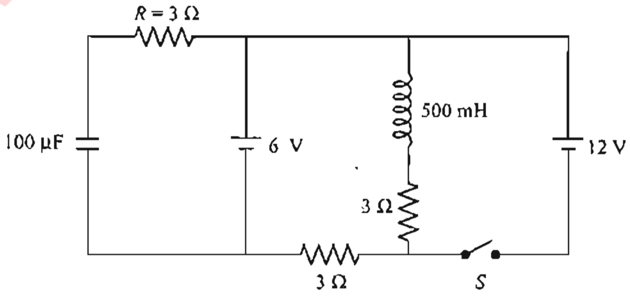


Fig. 8.317

   a. Current in the inductor when the circuit reaches the steady state is 4 A.
   b. The net change in flux in the inductor is 1.5 Wb.
   c. The time constant of the circuit after closing S is 555.55 s.
   d. The charge stored in the capacitor in steady state is 1.2 mC.

3. The magnetic flux ϕ linked with a conducting coil depends on time as $\phi = 4t^n + 6$, where n is a positive constant. The induced e.m.f. in the coil is e.
   a. If $0 < n < 1$, $e \neq 0$ and $|e|$ decreases with time.
   b. If $n = 1$, e is constant.
   c. If $n > 1$, $|e|$ increases with time.
   d. If $n > 1$, $|e|$ decreases with time.

4. A circular loop of radius r, having N turns of a wire, is placed in a uniform and constant magnetic field B. The normal of the loop makes an angle θ with the magnetic field. Its normal rotates with an angular velocity ω such that the angle θ is constant. Choose the correct statement from the following.

a. e.m.f. in the loop is $\dfrac{NB\omega r^2}{2}\cos\theta$

b. e.m.f. induced in the loop is zero

c. e.m.f. must be induced as the loop crosses magnetic lines

d. e.m.f. must not be induced as flux does not change with time

5. A uniform circular loop of radius a and resistance R is placed perpendicular to a uniform magnetic field B. One half of the loop is rotated about the diameter with angular velocity ω as shown. Then, the current in the loop is

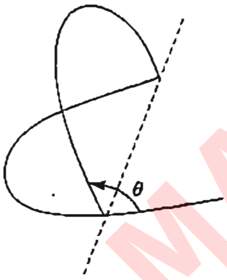


Fig. 8.318

a. zero, when θ is zero

b. $\dfrac{\pi a^2 B\omega}{2R}$, when θ is zero

c. zero, when $\theta = \pi/2$

d. $\dfrac{\pi a^2 B\omega}{2R}$, when $\theta = \pi/2$

6. A conducting wire of length ℓ and mass m can slide without friction on two parallel rails and is connected to capacitance C. The whole system lies in a magnetic field B and a constant force F is applied to the rod. Then

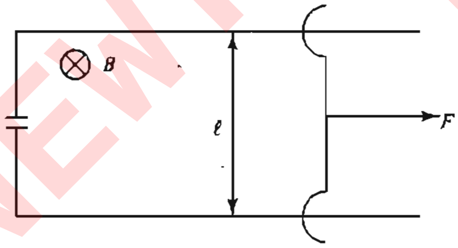


Fig. 8.319

a. the rod moves with constant velocity.

b. the rod moves with an acceleration of $\dfrac{F}{m+B^2\ell^2c}$.

c. there is constant charge on the capacitor.

d. charge on the capacitor increases with time.

7. A conducting rod of length ℓ is hinged at point O. It is free to rotate in a vertical plane. There exists a uniform magnetic field $\bar{B}$ in horizontal direction. The rod is released from the position shown in Fig. 8.320. Potential difference between the two ends of the rod is proportional to

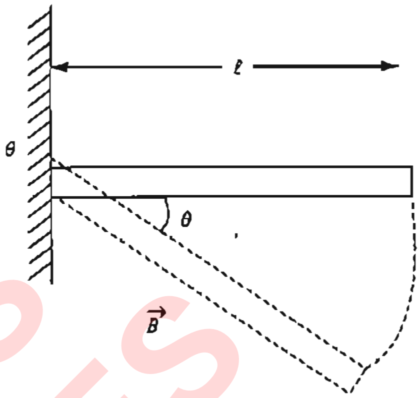


Fig. 8.320

a. $\ell^{3/2}$ b. ℓ^2 c. $\sin\theta$ d. $(\sin\theta)^{1/2}$

8. A conducting rod of length ℓ is moved at constant velocity v_0 on two parallel, conducting, smooth, fixed rails, which are placed in a uniform constant magnetic field B perpendicular to the plane of the rails as shown in Fig. 8.321. A resistance R is connected between the two ends of the rail. Then which of the following is/are correct?

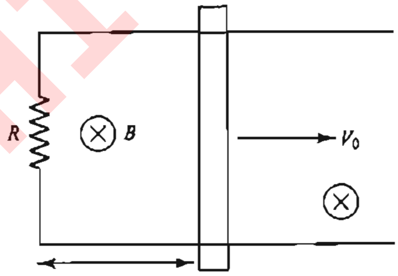


Fig. 8.321

a. The thermal power dissipated in the resistor is equal to the rate of work done by an external person pulling the rod

b. If applied external force is doubled, then a part of the external power increases the velocity of the rod

c. Lenz's law is not satisfied if the rod is accelerated by an external force

d. If resistance R is doubled, then power required to maintain the constant velocity V_0 becomes half

9. In Fig. 8.322, R is a fixed conducting ring of negligible resistance and radius a. PQ is a uniform rod of resistance r. It is hinged at the centre of the ring and rotated about this point in clockwise direction with a uniform angular velocity ω. There is a uniform magnetic field of strength B pointing inward and r is a stationary resistance. Then

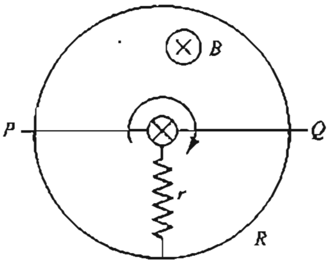


Fig. 8.322

- **a.** current through r is zero
- **b.** current through r is $\frac{2B\omega a^2}{5r}$
- **c.** direction of current in external resistance r is from centre to circumference
- **d.** direction of current in external resistance r is from circumference to centre

10. An infinite current-carrying conductor is placed along the z-axis and a wire loop is kept in the xy plane. The current in the conductor is increasing with time. Then the

- **a.** e.m.f. induced in the wire loop is zero
- **b.** magnetic flux passing through the wire loop is zero
- **c.** e.m.f. induced is zero but magnetic flux is not zero
- **d.** e.m.f. induced is not zero but magnetic flux is zero

11. An inductor and two capacitors are connected in the circuit as shown in Fig. 8.323. Initially capacitor A has no charge and capacitor B has CV charge. Assume that the circuit has no resistance at all. At t = 0, switch S is closed, then

[given $LC = \frac{2}{\pi^2 \times 10^4}$ s^2 and CV = 100 mC]

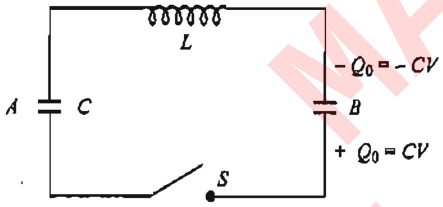


Fig. 8.323

- **a.** when current in the circuit is maximum, charge on each capacitor is same
- **b.** when current in the circuit is maximum, charge on capacitor A is twice the charge on capacitor B
- **c.** $q = 50(1 + \cos 100\,\pi t)$ mC, where q is the charge on capacitor B at time t
- **d.** $q = 50(1 - \cos 100\,\pi t)$ mC, where q is the charge on capacitor B at time t

12. The potential difference across a 2 H inductor as a function of time is shown in Fig. 8.324. At t = 0, current is zero. Choose the correct statement.

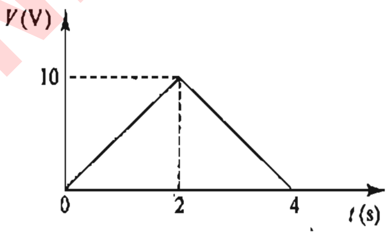


Fig. 8.324

- **a.** Current at t = 2 s is 5 A
- **b.** Current at t = 2 s is 10 A
- **c.** Current versus time graph across the inductor will be

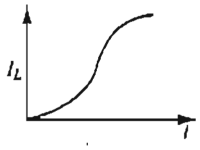


Fig. 8.325

- **d.** Current versus time graph across inductor will be

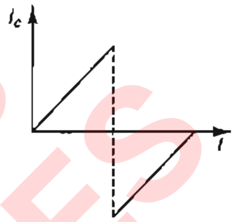


Fig. 8.326

13. A disc of radius R is rolling without sliding on a horizontal surface with a velocity of centre of mass v and angular velocity ω in a uniform magnetic field B which is perpen-dicular to the plane of the disc as shown in Fig. 8.327. O is the centre of the disc and P, Q, R and S are the four points on the disc.

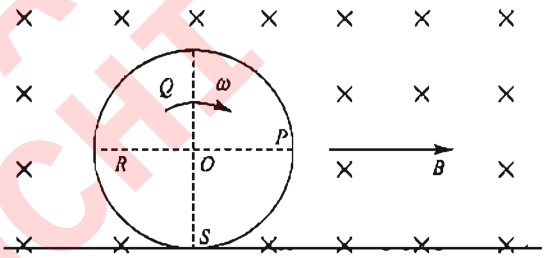


Fig. 8.327

- **a.** Due to translation, induced e.m.f. across $PS = Bvr$
- **b.** Due to rotation, induced e.m.f. across $QS = 0$
- **c.** Due to translation, induced e.m.f. across $RO = 0$
- **d.** Due to rotation, induced e.m.f. across $OQ = Bvr$

14. Two parallel resistanceless rails are connected by an inductor of inductance L at one end as shown in Fig. 8.328. A magnetic field B exists in the space which is perpendicular to the plane of the rails. Now a conductor of length ℓ and mass m is placed transverse on the rail and given an impulse J towards the rightward direction. Then choose the correct option(s).

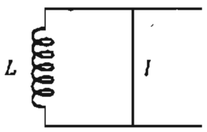


Fig. 10.328

- **a.** Velocity of the conductor is half of the initial velocity after a displacement of the conductor $d = \sqrt{\frac{3J^2L}{4B^2\ell^2m}}$
- **b.** Current flowing through the inductor at the instant when velocity of the conductor is half of the initial velocity is $i = \sqrt{\frac{3J^2}{4Lm}}$

c. Velocity of the conductor is half of the initial velocity after a displacement of the conductor $d = \sqrt{\frac{3J^2L}{B^2\ell^2m}}$

d. Current flowing through the inductor at the instant when velocity of the conductor is half of the initial velocity is $i = \sqrt{\frac{3J^2}{mL}}$

15. A conducting loop rotates with constant angular velocity about its fixed diameter in a uniform magnetic field. Whose direction is perpendicular to that fixed diameter.

a. The e.m.f. will be maximum at the moment when flux is zero

b. The e.m.f. will be '0' at the moment when flux is maximum

c. The e.m.f. will be maximum at the moment when plane of the loop is parallel to the magnetic field

d. The phase difference between the flux and the e.m.f. is $\pi/2$

16. A bar magnet is moved between two parallel circular loops A and B with a constant velocity v as shown in Fig. 8.329.

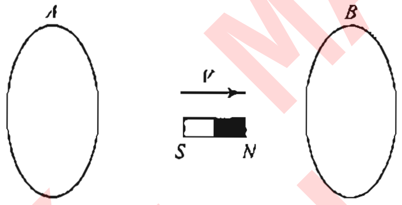


Fig. 8.329

a. The current in each loop flows in the same direction

b. The current in each loop flows in opposite directions

c. The loops will repel each other

d. The loops will attract each other

17. A bar magnet moves towards two identical parallel circular loops with a constant velocity v, as shown in Fig. 8.330.

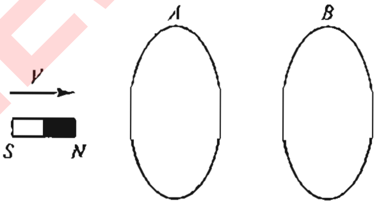


Fig. 8.330

a. Both the loops will attract each other

b. Both the loops will repel each other

c. The induced current in A is more than that in B

d. The induced current is same in both the loops

18. In the circuit shown in Fig. 8.331, the switch is closed at $t = 0$.

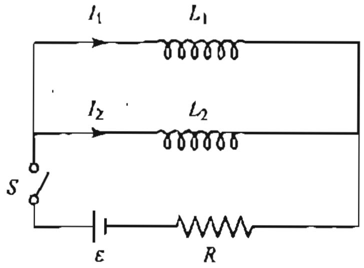


Fig. 8.331

a. At $t = 0$, $I_1 = I_2 = 0$

b. At any time t, $\frac{I_1}{I_2} = \frac{L_2}{L_1}$

c. At any time t, $I_1 + I_2 = \frac{\varepsilon}{R}$

d. At $t = \infty$, I_1 and I_2 are independent of L_1 and L_2

19. A highly conducting ring of radius R is perpendicular to and concentric with the axis of a long solenoid, as shown. The ring has a narrow gap of width δ in its circumference. The cross-sectional area of the solenoid is a. The solenoid has a uniform internal field of magnitude $B(t) = B_0 + \beta t$, where $\beta > 0$. Assume that no charge can flow across the gap, the face(s) accumulating an excess of positive charge is/are

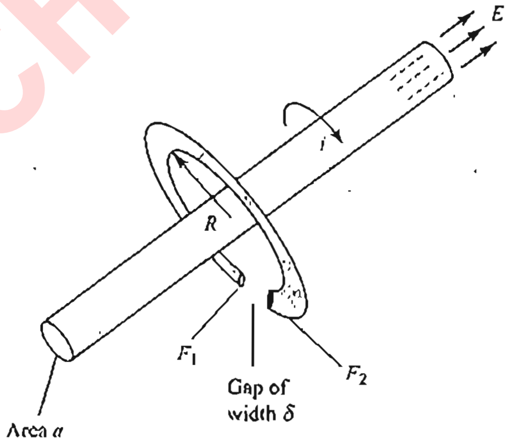


Fig. 8.332

a. F_1

b. F_2

c. F_1 and F_2 both

d. difficult to conclude as data given are insufficient

20. The accumulation of the charge on the gap faces will cease when the total electric field within the ring becomes zero. For this to happen, the electric field in the gap E_0 is

a. $E_0 = \frac{a\beta}{\delta}$

b. $E_0 = \frac{2a\beta}{\delta}$

c. E_0 is dependent on R for $R > \sqrt{\frac{a}{\pi}}$

d. E_0 is independent of R for $R > \sqrt{\frac{a}{\pi}}$

21. In the figure shown, the wires P_1Q_1 and P_2Q_2 are made to slide on the rails with same speed of 5 cm s^{-1}. In this region a magnetic field of 1 T exists. The electric current in the 9 Ω resistance is

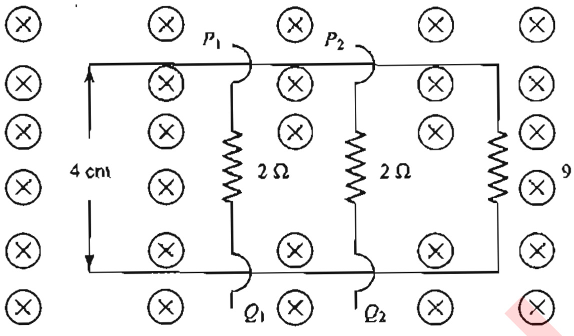


Fig. 8.333

a. zero if both wires slide towards left
b. zero if both wires slide in opposite directions
c. 0.2 mA if both wires move towards left
d. 0.2 mA if both wires move in opposite directions

22. A small magnet M is allowed to fall through a fixed horizontal conducting ring R. Let g be the acceleration due to gravity. The acceleration of M will be

a. $<g$ when it is above R and moving towards R
b. $>g$ when it is above R and moving towards R
c. $<g$ when it is below R and moving away from R
d. $>g$ when it is below R and moving away from R

23. The conductor AD moves to the right in a uniform magnetic field directed into the plane of the paper.

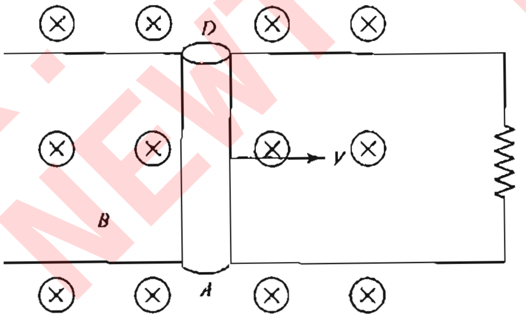


Fig. 8.334

a. The free electron in AD will move towards A
b. D will acquire a positive potential with respect to A
c. A current will flow from A to D in AD in close loop
d. The current in AD flows from lower to higher potential

24. The magnitude of the earth's magnetic field at the north pole is B_0. A horizontal conductor of length ℓ moves with a velocity v. The direction of v is perpendicular to the conductor. The induced e.m.f. is

a. zero, if v is vertical
b. $B_0\ell v$, if v is vertical
c. zero, if v is horizontal
d. $B_0\ell v$, if v is horizontal

25. A vertical conducting ring of radius R falls vertically in a horizontal magnetic field of magnitude B. The direction of B is perpendicular to the plane of the ring. When the speed of the ring is v,

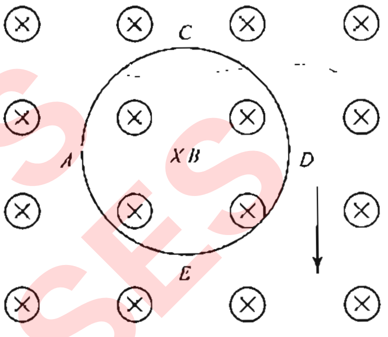


Fig. 8.335

a. no current flows in the ring
b. A and D are at the same potential
c. C and E are at the same potential
d. the potential difference between A and D is $2BRv$, with D at a higher potential

26. A flat coil, C, of n turns, area A and resistance R, is placed in a uniform magnetic field of magnitude B. The plane of the coil is initially perpendicular to B. The coil is rotated by an angle θ about the plane xy and charge of amount Q flows through it. Choose the correct alternatives.

a. $\theta = 90°$, $Q = (BAn/R)$
b. $\theta = 180°$, $Q = (BAn/R)$
c. $\theta = 180°$, $Q = 0$
d. $\theta = 360°$, $Q = 0$

27. In problem 26, the plane of the coil is initially kept parallel to B. The coil is rotated by an angle θ about the palne xy, and charge of amount Q flows through it. Choose the correct alternatives.

a. $\theta = 90°$, $Q = (Ban/R)$
b. $\theta = 180°$, $Q = (2Ban/R)$
c. $\theta = 180°$, $Q = 0$
d. $\theta = 360°$, $Q = 0$

28. In question 26, if the coil rotates about the xy plane with a constant angular velocity ω, the e.m.f. induced in it

a. is zero
b. changes non-linearly with time
c. has a constant value $= BAn\omega$
d. has a maximum value $= BAn\omega$

29. Switch S of the circuit shown in Fig. 8.336 is closed at $t = 0$. If e denotes the induced e.m.f. in L and I is the current flowing through the circuit at time t, which of the following graphs is/are correct?

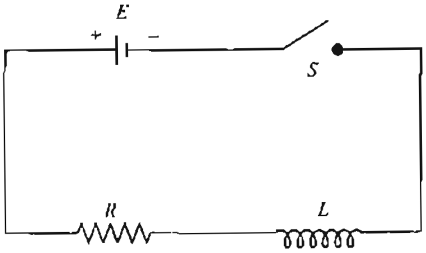


Fig. 8.336

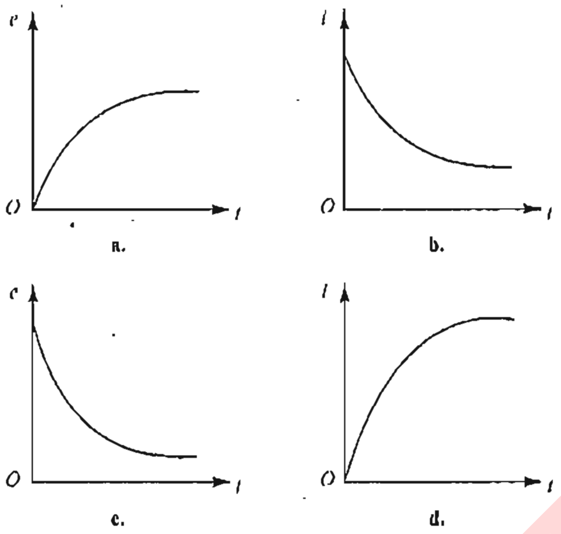


Fig. 8.337

30. For the circuit shown in Fig. 8.338, which of the following statements is/are correct?

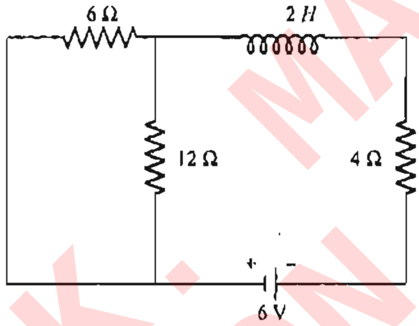


Fig. 8.338

a. Its time constant is 0.25 s
b. In steady state, current through the inductance will be equal to zero
c. In steady state, current through the battery will be equal to 0.75 A
d. None of these

Assertion-Reasoning Type

Solutions on page 8.131

a. Statement I is True, Statement II is True; Statement II is correct explanation for Statement I.
b. Statement I is True, Statement II is True; Statement II is NOT a correct explanation for statement I.
c. Statement I is True, Statement II is False.
d. Statement I is False, Statement II is True.

1. Statement I: Two coaxial conducting rings of different radii are placed in space. The mutual inductance of both the rings is maximum if the rings are also coplanar.
Statement II: For two coaxial conducting rings of different radii, the magnitude of magnetic flux in one ring due to current in the other ring is maximum when both rings are coplanar.

2. Statement I: An inductor acts as perfect conductor for d.c.
Statement II: d.c. remains constant in magnitude and direction.

3. Statement I: The magnetic flux through a loop of conducting wire of a fixed resistance changes by $\Delta\phi_B$ in a time Δt. Then $\Delta\phi_B$ is proportional to the current through the loop.
Statement II: $I = -\dfrac{\Delta\phi_B}{R}$.

4. Statement I: An e.m.f. is induced in a long solenoid by a bar magnet that moves while totally inside the solenoid along axis of the solenoid.
Statement II: As the magnet moves inside the solenoid the flux through individual turns of the solenoid changes.

5. Statement I: Lenz's law violates the principle of conservation of energy.
Statement II: Induced e.m.f. always opposes the change in magnetic flux responsible for its production.

6. Statement I: Only a change in magnetic flux will maintain an induced current in the coil.
Statement II: The presence of large magnetic flux through a coil maintains a current in the coil if the circuit is continuous.

7. Statement I: An electric lamp is connected in series with a long solenoid of copper with air core and then connected to an a.c. source. If an iron rod is inserted in the solenoid the lamp will become dim.
Statement II: If an iron rod is inserted in the solenoid, the inductance of the solenoid increases.

8. Statement I: A capacitor allows a.c. but blocks d.c.
Statement II: When a.c. passes through a capacitor, there is local oscillation of bound charges of dielectric.

9. Statement I: The self-inductance (L) is given by ϕ (magnetic flux) = L_i (current).
Statement II: When current is increased, self-inductance increases.

10. Statement I: The work done by a charge in a closed (induced) current-carrying loop is non-zero.
Statement II: Induced electric field is non-conservative in nature.

11. Statement I: No electric current will be present within a region having uniform and constant magnetic field.
Statement II: Within a region of uniform and constant magnetic field $\vec{B}$, the path integral of magnetic field $\oint \vec{B}\cdot d\vec{l}$ along any closed path is zero. Hence from Ampere circular law $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$ (where the given terms have usual meaning), no current can be present within a region having uniform and constant magnetic field.

12. Statement I: The growth of current in R L circuit is uniform.
Statement II: Inductor (L) opposes the growth of current.

13. Statement I: Magnetic flux linked to closed surface is zero.
Statement II: Direction of induced current due to change of magnetic flux is given by Faraday's law.

14. Statement I: Time-dependent magnetic field generates electric field.
Statement II: Direction of electric field generated from time variable magnetic field does not obey Lenz's law.

15. **Statement I:** Induced potential across a coil and therefore induced current is always opposite to the direction of current due to external source.
    Statement II: Lenz's law states that induced e.m.f. always opposes the cause due to which it is being produced.
16. **Statement I:** The magnetic field at the ends of a very long current-carrying solenoid is half of that at the centre.
    Statement II: If the solenoid is sufficiently long, the field within it is uniform.
17. **Statement I:** The energy of charged particle moving in a uniform magnetic field does not change.
    Statement II: Work done by magnetic field on the charge is zero.
18. **Statement I:** When two coils are wound on each other, the mutual induction between the coils is maximum.
    Statement II: Mutual induction does not depend on the orientation of the coils.
19. **Statement I:** The induced e.m.f. in a conducting loop of wire will be non-zero when it rotates in a uniform magnetic field.
    Statement II: The e.m.f. may be induced due to change in magnetic field.
20. **Statement I:** The direction of induced e.m.f. is always such as to oppose the change that causes it.
    Statement II: The direction of induced e.m.f. is given by Lenz's law.
21. **Statement I:** Capacitor serves as a block for d.c. and offers an easy path to a.c.
    Statement II: Capacitive reactance is inversely proportional to frequency.
22. **Statement I:** A resistance R is connected between the two ends of the parallel smooth conducting rails. A conducting rod lies on these fixed horizontal rails and a uniform constant magnetic field B exists perpendicular to the plane of the rails as shown in Fig.8.339. If the rod is given a velocity v and released as shown in Fig. 8.339, it will stop after some time. The total work done by magnetic field is negative.

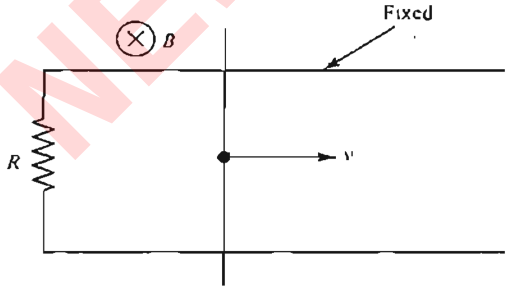


Fig. 8.339

    Statement II: If force acts opposite to direction of velocity its work done is negative.
23. **Statement I:** Consider the arrangement shown below. A smooth conducting rod, CD, lying on a smooth U-shaped conducing wire, is fixed and lies on horizontal plane. There is a uniform and constant magnetic field B in vertical direction (perpendicular to plane of page in Fig. 8.340). If the magnetic field strength is decreased, the rod moves towards right.

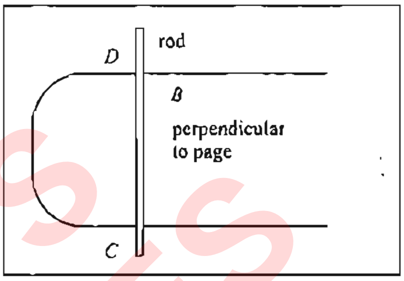


Fig. 8.340

    Statement II: In the situation of statement I, the direction in which the rod will slide is that which tends to maintain constant flux through the loop. Providing a larger loop area counteracts the decrease in magnetic flux. So the rod moves to the right independent of the fact that the direction of the magnetic field is into the page or out of the page.
24. **Statement I:** No electric current will be present within a region having uniform and constant magnetic field.
    Statement II: Within a region of uniform and constant magnetic field $\vec{B}$, the path integral of magnetic field $\oint \vec{B} \cdot d\vec{\ell}$ along any closed path is zero. Hence, from Ampere circuital law $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$ (where the given terms have usual meaning), no current can be present within a region having uniform and constant magnetic field.

Comprehension Type

Solutions on page 8.132

For Problems 1–2

A flexible circular loop 20 cm in diameter lies in a magnetic field with magnitude 1.0 T, directed into the plane of the page as shown in Fig. 8.341. The loop is pulled at the points indicated by the arrows, forming a loop of zero area in 0.314 s.

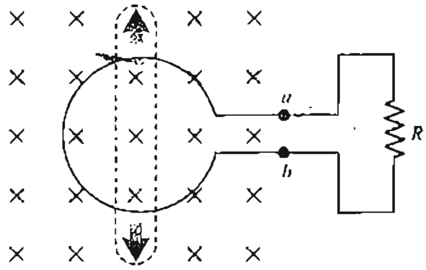


Fig. 8.341

1. The average induced e.m.f. in the circuit is
   a. 0.2 V b. 0.1 V c. 1 V d. 10V
2. If $R = 0.01\ \Omega$, the magnitude and direction of current flowing in the loop is
   a. 1 A, clockwise b. 1 A, anticlockwise
   c. 10 A, clockwise d. 10 A, anticlockwise

For Problems 3–4

The current in the long, straight wire AB shown in Fig. 8.342 is upward and is increasing steadily at a rate of $di/dt = K$.

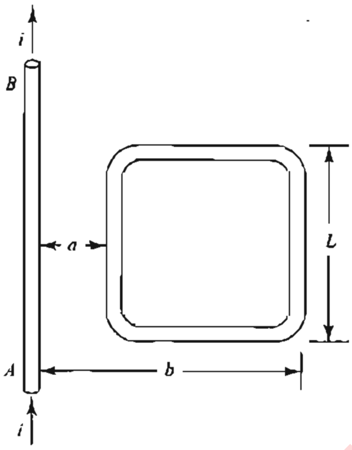


Fig. 8.342

3. The total flux through the loop is

a. $\frac{2\mu_0 iL}{\pi}\ln\frac{b}{a}$

b. $\frac{\mu_0 iL}{2\pi}\ln\frac{b}{a}$

c. $\frac{\sqrt{2}\mu_0 iL}{\pi}\ln\frac{b}{a}$

d. $\frac{\mu_0 iL}{\sqrt{3}\pi}\ln\frac{b}{a}$

4. The induced e.m.f. in the loop is

a. $\frac{\mu_0 LK}{2\pi}\ln\frac{b}{a}$

b. $\frac{2\mu_0 LK}{\pi}\ln\frac{b}{a}$

c. $\frac{\sqrt{3}\mu_0 LK}{2\pi}\ln\frac{b}{a}$

d. $\frac{\mu_0 LK}{\sqrt{3}\pi}\ln\frac{b}{a}$

For Problems 5–6

A space is divided by the line AD into two regions. Region I is field-free and region II has a uniform magnetic field B directed into the paper. ACD is a semi-circular conducting loop of radius r with centre at O, the plane of the loop rotates with a velocity ω about an axis passing through O, and perpendicular to the plane of the paper. The effective resistance of the loop is R.

5. The magnitude and direction of the induced current in the loop, when it starts to enter the magnetic field is

a. $\frac{\omega Br^2}{2R}$

b. $\frac{\omega Br^2}{R}$

c. $\frac{\omega Br^2}{\sqrt{2}R}$

d. $\frac{2\omega Br^2}{\sqrt{3}R}$

6. The graph between the induced e.m.f. and the time of rotation for two periods of rotation is

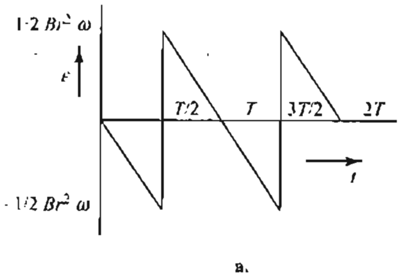


Fig. 8.343 (*Contd.*)

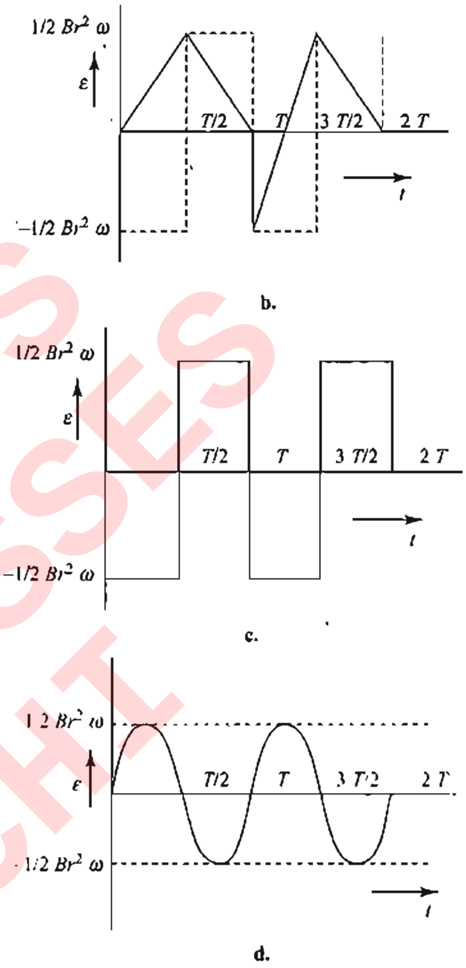


Fig. 8.343

For Problems 7–8

Fig. 8.344 shows two parallel and coaxial loops. The smaller loop (radius r) is above the larger loop (radius R), by distance $x \gg R$. The magnetic field due to current i in the larger loop is nearly constant throughout the smaller loop. Suppose that x is increasing at a constant rate of $dx/dt = v$.

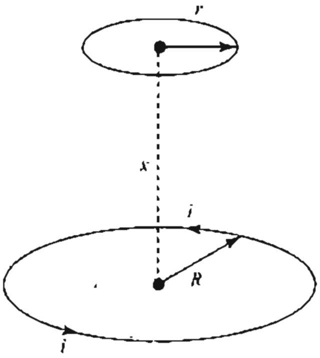


Fig. 8.344

7. Determine the magnetic flux through the smaller loop as a function of x.

a. $\frac{\mu_0 i R^2 \pi r^2}{(R^2+x^2)^{3/2}}$ b. $\frac{\mu_0 i R^2 \pi r^2}{2(R^2+x^2)^{3/2}}$

c. $\frac{2\mu_0 i R^2 \pi r^2}{(R^2+x^2)^{3/2}}$ d. $\frac{\sqrt{2}\mu_0 i R^2 \pi r^2}{(R^2+x^2)^{3/2}}$

8. The induced e.m.f. and the direction of the induced current in the smaller loop is

a. $\frac{\mu_0 \pi i R^2 r^2}{x^4} v$ b. $\frac{\mu_0 \pi i R^2 r^2}{2x^4} v$

c. $\frac{3}{2}\frac{\mu_0 \pi i R^2 r^2}{x^4} v$ d. $\frac{\mu_0 \pi i R^2 r^2}{3x^4} v$

For Problems 9–10

A wire loop enclosing a semicircle of radius R is located on the boundary of a uniform magnetic field B. At the moment $t = 0$, the loop is set into rotation with constant angular acceleration α about an axis O conducting with the line of vector on the boundary. The clockwise e.m.f. direction is taken to be positive.

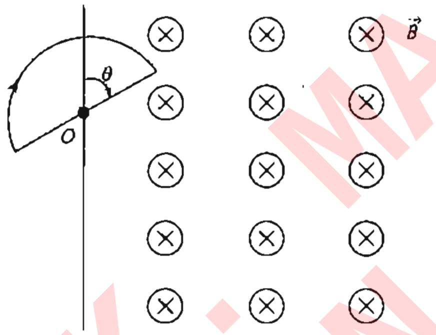
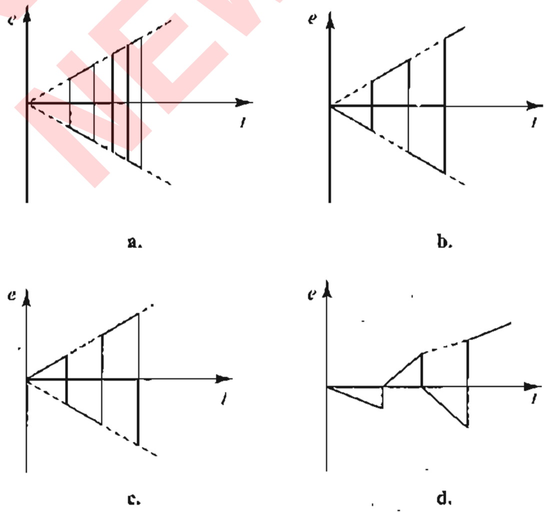


Fig. 8.345

9. The variation of e.m.f. as a function of time is

a. $\frac{1}{2}BR^2\alpha t$ b. $\frac{3}{2}BR^2\alpha t A$ c. $\sqrt{3}BR^2\alpha t$ d. $\frac{BR^2\alpha t}{\sqrt{2}}$

10. The variation of e.m.f. as a function of time is



a. b.

c. d.

Fig. 8.346

For Questions 11–13

In the circuit shown in Fig. 8.347, the capacitor has capacitance C = 20 μF and is initially charged to 100 V with the polarity shown. The resistor R_0 has resistance 10 Ω. At time $t = 0$ the switch is closed. The smaller circuit is not connected in any way to the larger one. The wire of the smaller circuit has a resistance of 1.0 Ω m^{-1} and contains 25 loops. The larger circuit is a rectangle 2.0 m by 4.0 m, while the smaller one has dimensions a = 10.0 cm and b = 20.0 cm. The distance c is 5.0 cm. (The figure is not drawn to scale.) Both circuits are held stationary. Assume that only the wire nearest the smaller circuit produces an appreciable magnetic field through it.

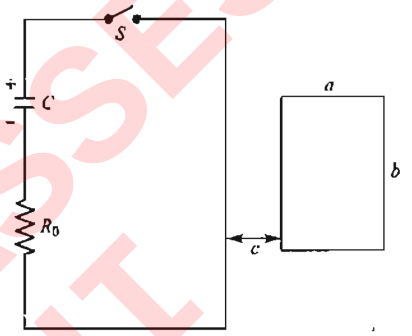


Fig. 8.347

11. The current in the larger circuit 200 ms after closing S is

a. $\frac{5}{e}$ A b. $\frac{2}{e}$ A c. $\frac{15}{e}$ A d. $\frac{10}{e}$ A

12. The current in the smaller circuit 200 μs after closing S is

a. 54 μA b. 10 μA c. 15 μA d. 36 μA

13. The direction of current in the smaller circuit is

a. clockwise b. anticlockwise

c. changes always with time d. cannot be calculated

For Questions 14–15

Two long parallel conducting rails are placed in a uniform magnetic field. On one side the rails are connected with a resistance R. Two rods MN and $M'N'$ each having resistance r are placed as shown in Fig. 8.348. Now on the rods MN and $M'N'$ forces are applied such that the rods move with constant velocity v.

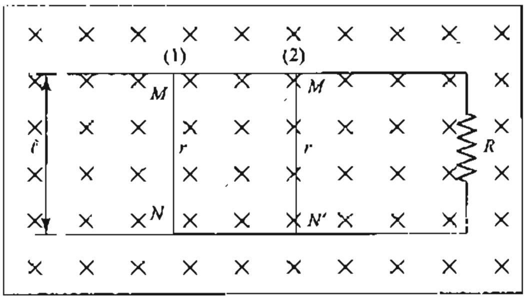


Fig. 8.348

14. The current flowing through resistance R if both the rods move with the same speed v towards right is

a. $\frac{B\ell v}{R+(r/2)}$ b. $\frac{2B\ell v}{R+r}$ c. zero d. $\frac{3B\ell v}{2(R+r)}$

15. i. The current flowing through resistance R if the rod MN moves towards left and the rod $M'N'$ moves towards the right is

a. $\frac{B\ell v}{R+(r/2)}$ b. $\frac{2B\ell v}{R+r}$ c. zero d. $\frac{3B\ell v}{2(R+r)}$

ii. If in the previous problem resistances each of value R are connected on both ends as shown in Fig. 8.349, the current I_1 flowing through resistance R_1 is given by

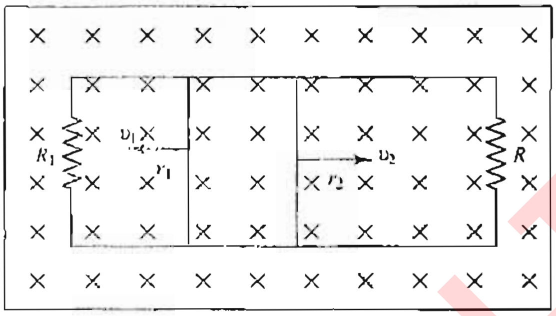


Fig. 8.349

a. $\frac{B\ell R_2(v_1r_2-v_2r_1)}{R_1R_2(r_1+r_2)+r_2r_1(R_1+R_2)}$ b. $\frac{B\ell R_2(v_1r_2+v_2r_1)}{R_1R_2(r_1+r_2)+r_2r_1(R_1+R_2)}$

c. $\frac{B\ell R_2(v_1r_2-v_2r_1)}{R_1R_2(r_1-r_2)+r_2r_1(R_1-R_2)}$ d. $\frac{B\ell R_2(v_1r_2-v_2r_1)}{R_1R_2(r_1+r_2)-r_2r_1(R_1+R_2)}$

For Problems 16–17

A metal bar is moving with a velocity of 5 cm s^{-1} over a U-shaped conductor. At $t = 0$, the external magnetic field is 0.1 T out of the page and is increasing at a rate of 0.2 T s^{-1}. Take $\ell = 5$ cm, and at $t = 0$, $x = 5$ cm.

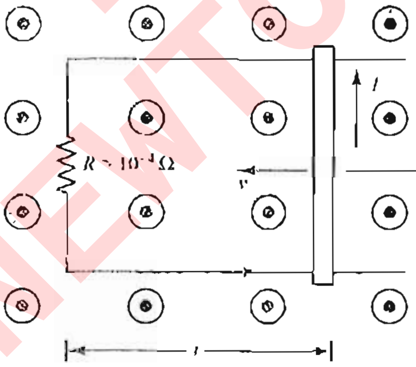


Fig. 8.350

16. The e.m.f. induced in the circuit is

a. 125 μV b. −250 μV
c. −100 μV d. 300 μV

17. The current flowing in the circuit is

a. 2.5 A b. 5 A c. 1 A d. 2 A

For Problems 18–20

In Fig. 8.351 shown, the rod has a resistance R, the horizontal rails have negligible friction. A battery of e.m.f. E and negligible internal resistance is connected between points a and b. The rod is initially at rest.

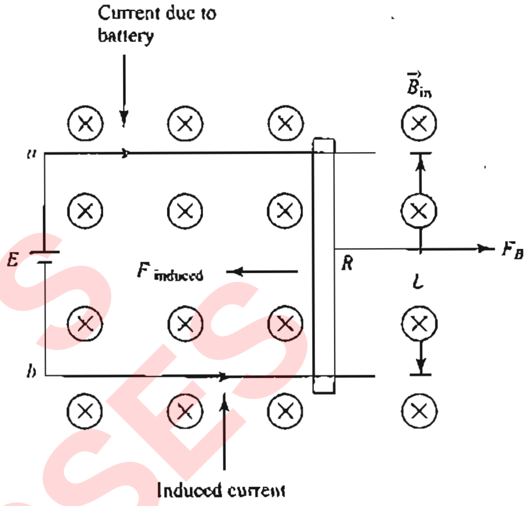


Fig. 8.351

18. The force on the rod as a function of the speed v (where $\tau = mR/B\ell^2$) is

a. $\frac{E}{B\ell}(1-e^{-t/\tau})$ b. $\frac{E}{B\ell}(1+e^{-t/\tau})$

c. $\frac{3}{2}\frac{E}{B\ell}(1-e^{-t/\tau})$ d. $\frac{E}{2B\ell}(1-e^{-t/\tau})$

19. After some time the rod will approach a terminal speed. Find an expression for it.

a. $\frac{3}{2}\frac{E}{B\ell}$ b. $\frac{E}{2B\ell}$ c. $\frac{E}{B\ell}$ d. $\frac{2E}{B\ell}$

20. The current when the rod attains its terminal speed is

a. $\frac{2E}{R}$ b. $\frac{E}{R}$ c. $\frac{3}{2}\frac{E}{R}$ d. $\frac{E}{2R}$

For Problems 21–22

A long, thin solenoid has 900 turns per meter and radius 2.50 cm. The current in the solenoid is increasing at a uniform rate of 60.0 A s^{-1}. What is the magnitude of the induced electric field at a point near the center of the solenoid and

21. 0.500 cm from the axis of the solenoid?

a. $54\pi \times 10^{-8}$ N C^{-1} b. $48\pi \times 10^{-8}$ N C^{-1}
c. $36\pi \times 10^{-8}$ N C^{-1} d. $18\pi \times 10^{-8}$ N C^{-1}

22. 1.00 cm from the axis of the solenoid?

a. $50\pi \times 10^{-8}$ N C^{-1} b. $100\pi \times 10^{-8}$ N C^{-1}
c. $81\pi \times 10^{-8}$ N C^{-1} d. $36\pi \times 10^{-8}$ N C^{-1}

For Problems 23–27

The magnetic field within a long, straight solenoid with a circular cross section and radius R is increasing at a rate of dB/dt.

23. The rate of change of flux through a circle with radius r_1 inside the solenoid, normal to the axis of the solenoid, and with center on the solenoid axis is

a. $\sqrt{2}\pi r_1^2 \frac{dB}{dt}$ b. $\frac{1}{2}\pi r_1^2 \frac{dB}{dt}$

c. $\pi r_1^2 \frac{dB}{dt}$ d. $\frac{3}{2}\pi r_1^2 \frac{dB}{dt}$

24. The magnitude of the induced electric field inside the solenoid at a distance r_1 from its axis is

a. $\frac{r_1}{2}\frac{dB}{dt}$ b. $r_1\frac{dB}{dt}$ c. $\frac{3r_1}{2}\frac{dB}{dt}$ d. $\frac{r_1}{\sqrt{2}}\frac{dB}{dt}$

25. The magnitude of the induced electric field outside the solenoid at a distance r_2 from the axis is

a. $\frac{R^2}{r_2}\frac{dB}{dt}$ b. $\frac{R^2}{2r_2}\frac{dB}{dt}$

c. $\frac{3R^2}{2r_2}\frac{dB}{dt}$ d. $\frac{R^2}{\sqrt{2}r_2}\frac{dB}{dt}$

26. Graph the magnitude of the induced electric field as a function of the distance r from the axis from $r = 0$ to $r = 2R$.

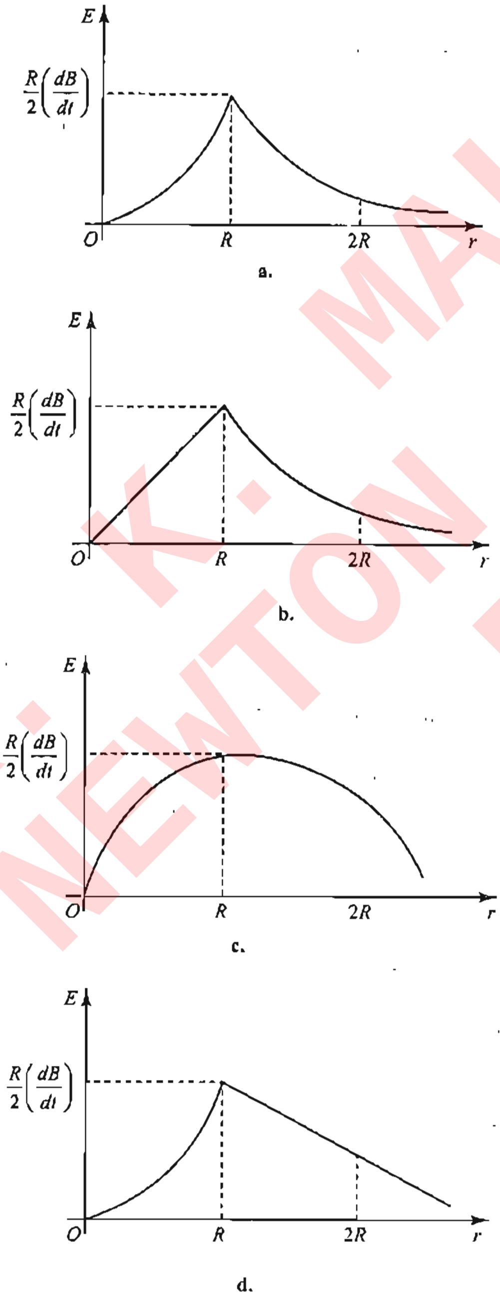


Fig. 8.352

27. The magnitude of the induced e.m.f. in a circular turn of radius $R/2$ that has its centre on the solenoid axis is

a. $\frac{R}{2}\left(\frac{dB}{dt}\right)$ b. $\frac{R}{4}\left(\frac{dB}{dt}\right)$ c. $R\left(\frac{dB}{dt}\right)$ d. $\sqrt{2}R\left(\frac{dB}{dt}\right)$

For Problems 28–29

A long solenoid of radius R has n turns of wire per unit length and carries a time varying current that varies sinusoidally as $I = I_{max}\cos\omega t$, where I_{max} is the maximum current and ω is the angular frequency of the alternating current source (shown in Fig.8.353).

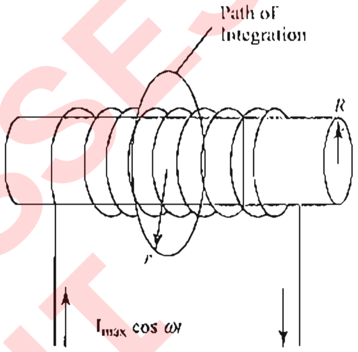


Fig. 8.353

28. The magnitude of the induced electric field inside the solenoid, a distance $r < R$ from its long central axis.

a. $\frac{3\mu_0 nI_{max}\omega}{2}r\sin\omega t$ b. $\frac{\mu_0 nI_{max}\omega}{2}r\cos\omega t$

c. $\mu_0 nI_{max}\omega r\sin\omega t$ d. $\frac{\mu_0 nI_{max}\omega}{2}r\sin\omega t$

29. The magnitude of electric field outside the solenoid at a distance $r > R$ from its long central axis is

a. $\frac{\mu_0 nI_{max}\omega R^2}{2r}\sin\omega t$ b. $\frac{2\mu_0 nI_{max}\omega R^2}{r}\sin\omega t$

c. $\frac{\mu_0 nI_{max}\omega R^2}{3r}\sin\omega t$ d. $\frac{3\mu_0 nI_{max}\omega R^2}{2r}\sin\omega t$

For Problems 30–32

A square conducting loop, 20.0 cm on a side, is placed in the same magnetic field as shown in Fig. 8.354; centre of the magnetic field region, where $dB/dt = 0.035\ \text{T s}^{-1}$.

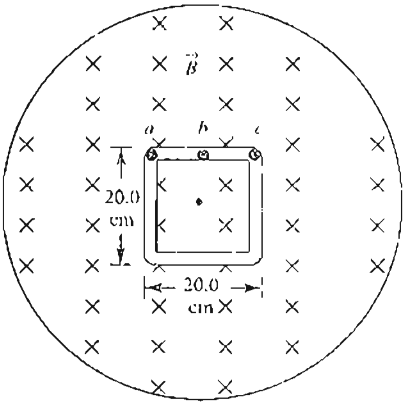


Fig. 8.354

30. The directions of induced electric field at points a, b and c:

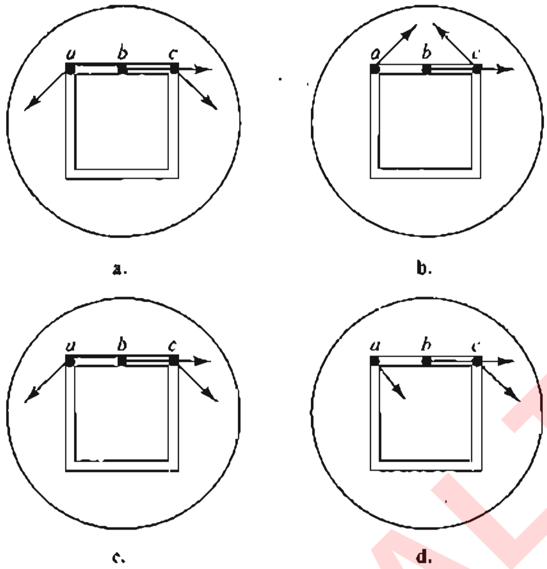


Fig. 8.355

31. The current induced in the loop if its resistance is 2.00 Ω is

a. 2.50×10^{-4} A b. 4.35×10^{-4} A
c. 1.25×10^{-4} A d. 7.00×10^{-4} A

32. The potential difference between points a and b is

a. 6 V b. 0
c. 10 V d. 12 V

For Problems 33–34

A thin non-conducting ring of mass m, radius a, carrying a charge q can rotate freely about its own axis which is vertical. At the initial moment the ring was at rest and no magnetic field was present. At instant $t = 0$, a uniform magnetic field is switched on which is vertically downwards and increases with time according to the law $B = B_0t$. Neglecting magnetism induced due to rotational motion of the ring, calculate

33. the angular acceleration of the ring and its direction of rotation as seen from above

a. $\frac{Eq}{2ma}$, anticlockwise b. $\frac{Eq}{ma}$, anticlockwise
c. $\frac{2Eq}{ma}$, clockwise d. $\frac{Eq}{ma}$, clockwise

34. the power developed by the forces acting on the ring, as a function of time

a. $\frac{E^2q^2}{2m}t$ b. $\frac{E^2q^2}{m}t$
c. $\frac{2E^2q^2}{m}t$ d. $\frac{\sqrt{2}E^2q^2}{m}t$

For Problems 35–40

In Fig. 8.356, $i_1 = 10\,e^{-2t}$ A, $i_2 = 4$ A and $V_C = 3e^{-2t}$ V.

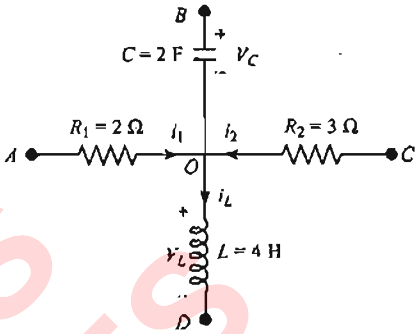


Fig. 8.356

35. The current i_L is

a. $[2 - 2(1 - e^{-2t})]$ A b. $[2 + 2(1 - e^{-2t})]$ A
c. $[3 - 2(1 - e^{-2t})]$ A d. $[2 + 3(1 - e^{-2t})]$ A

36. The variation of current in the inductor with time can be represented as

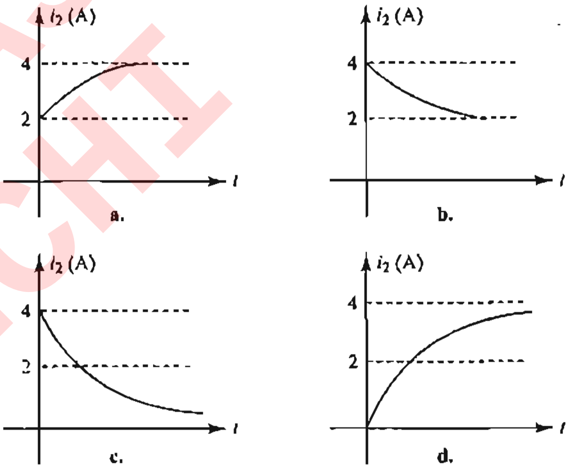


Fig. 8.357

37. The potential difference across inductor V_L is

a. $8e^{-2t}$ V b. $9e^{-2t}$ V c. $16e^{-2t}$ V d. $18e^{-2t}$ V

38. The variation of potential difference across A and C (V_{AC}) with time can be represented as

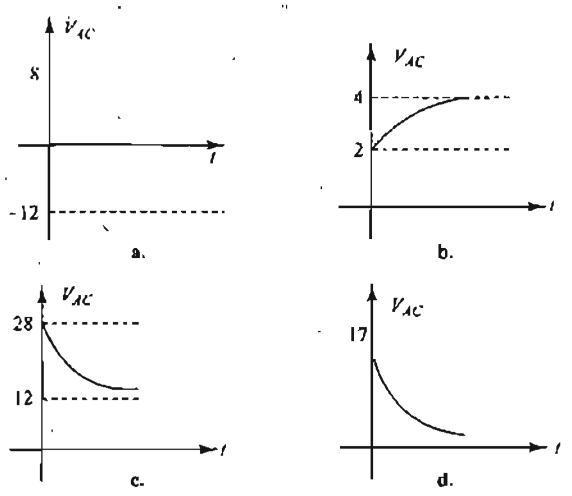


Fig. 8.358

39. The potential difference across AB (V_{AB}) is

a. $8e^{-2t}$ V **b.** $\frac{1}{2}e^{-3t}$ V **c.** $17e^{-2t}$ V **d.** $16e^{-2t}$ V

40. The variation of potential difference across C and D (V_{CD}) with time can be expressed as

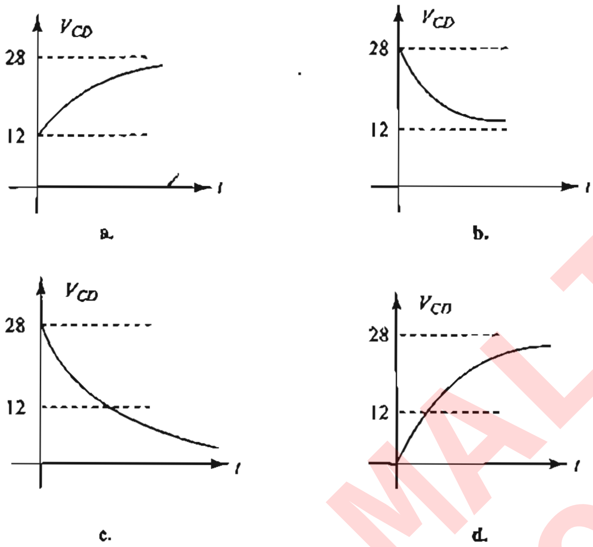


Fig. 8.359

For Problems 41–43

In the circuit shown, switches S_1 and S_3 have been closed for 1 s and S_2 remained open. Just after 1 s, switch S_2 is closed and S_1 and S_3 are opened. Find after that instant ($t = 0$):

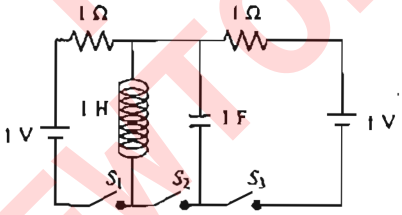


Fig. 8.360

41. the maximum current in the circuit containing inductor and capacitor only (only S_2 is closed)

a. $\sqrt{3}\left(1-\frac{1}{e}\right)$ b. $\sqrt{2}\left(1-\frac{1}{e}\right)$

c. $\sqrt{3}\left(1+\frac{1}{e}\right)$ d. $\sqrt{2}\left(1+\frac{1}{e}\right)$

42. the maximum charge on the capacitor

a. $\sqrt{3}\left(1+\frac{1}{e}\right)$ b. $\sqrt{3}\left(1-\frac{1}{e}\right)$

c. $\sqrt{2}\left(1+\frac{1}{e}\right)$ d. $\sqrt{2}\left(1-\frac{1}{e}\right)$

43. the charge on the upper plate of the capacitor as a function of time

a. $\sqrt{2}\left(1+\frac{1}{e}\right)\sin\left(t+\frac{\pi}{4}\right)$ b. $\sqrt{2}\left(1-\frac{1}{e}\right)\sin\left(t+\frac{\pi}{4}\right)$

c. $\sqrt{3}\left(1-\frac{1}{e}\right)\sin\left(t+\frac{\pi}{4}\right)$ d. $\sqrt{3}\left(1+\frac{1}{e}\right)\sin\left(t+\frac{\pi}{4}\right)$

For Problems 44–48

In the circuit shown $E = 120$ V, $R_1 = 30.0\ \Omega$, $R_2 = 50.0\ \Omega$ and $L = 0.200$ H. Switch S is closed at $t = 0$. Just after the switch is closed.

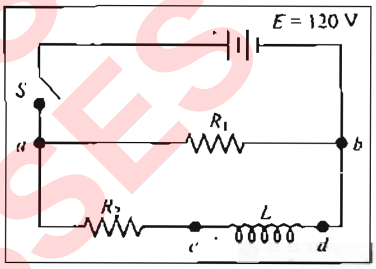


Fig. 8.361

44. The potential difference V_{ab} across the resistor R_1 is

a. 60 V b. 100 V c. 120 V d. 90 V

45. which point, a or b, is at higher potential?

a. point a
b. point b
c. points a and b both will be at the same potential
d. none of these

46. The potential difference V_{cd} across the inductor L is

a. 60 V b. 100 V c. 120 V d. 90 V

47. Now the switch s is opened, just after opening the S, what is the potential difference V_{ab} across the resistance R_1?

a. 72 V b. 36 V c. 56 V d. 90 V

48. which point, c or d, is at a higher potential?

a. point c
b. point d
c. points c and d are at the same potential
d. none of these

For Problems 49–50

In a very long solenoid of radius R, if the magnetic field changes at the rate of dB/dt,

49. The induced e.m.f. for the triangular circuit ABC shown in Fig. 8.362 is

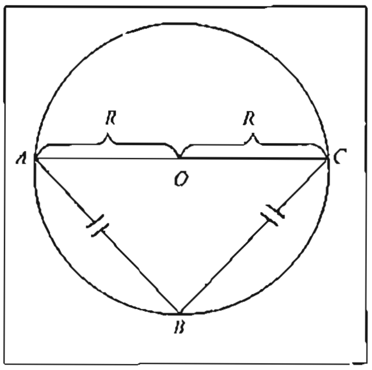


Fig. 8.362

a. $R^2\left(\frac{dB}{dt}\right)$ b. $4R^2\left(\frac{dB}{dt}\right)$

c. $\frac{1}{2}R^2\left(\frac{dB}{dt}\right)$ d. $2R^2\left(\frac{dB}{dt}\right)$

50. Calculate the induced e.m.f. between the ends of length AB, if AC and BC were removed from the circuit.

a. $R^2\left(\frac{dB}{dt}\right)$ b. $4R^2\left(\frac{dB}{dt}\right)$

c. $\frac{1}{2}R^2\left(\frac{dB}{dt}\right)$ d. $2R^2\left(\frac{dB}{dt}\right)$

For Problems 51–53

In the given circuit, all the symbols have their usual meanings. At $t = 0$, the key K is closed. Now answer the following questions.

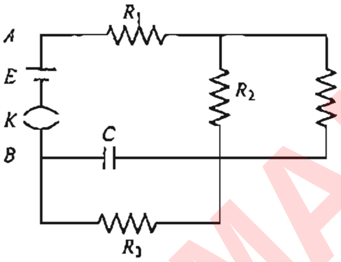


Fig. 8.363

51. At $t = 0$, the equivalent resistance between A and B is

a. $R_1 + R_2 + R_3$ b. $R_1 + R_2$

c. $R_1 + R_3$ d. indeterminate

52. At $t \to \infty$, the equivalent resistance between A and B is

a. $R_1 + R_2 + R_3$ b. $R_1 + R_2$

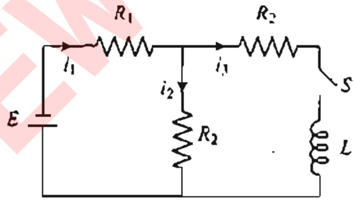
c. $R_1 + R_3$ d. none of these

53. At any time t, $0 < t < \infty$ (excluding the cases $t \to 0$ and $t \to \infty$), the equivalent resistance between A and B is

a. $R_1 + R_2 + R_3$ b. $R_1 + R_2$

c. $R_1 + R_3$ d. none of these

For Problems 54–56



In the circuit shown in the figure, $E = 15$ V, $R_1 = 1\Omega$, $R_2 = 1\ \Omega$, $R_3 = 2\ \Omega$ and $L = 1.5$ H. The currents flowing through R_1, R_2 and R_3 are i_1, i_2 and i_3 , respectively.

54. Immediately after turning the switch S on,

a. $i_1 = i_2 = 7.5$ A, $i_3 = 0$ A b. $i_1 = i_3 = 5$ A, $i_2 = 0$ A

c. $i_1 = i_2 = 9$ A, $i_3 = 0$ A d. $i_1 = i_2 = i_3 = 0$ A

55. After the circuit reaches the steady state,

a. $i_1 = 9$ A, $i_2 = 6$ A, $i_3 = 3$ A b. $i_1 = 9$ A, $i_2 = 3$ A, $i_3 = 6$ A

c. $i_1 = 6$ A, $i_2 = 6$ A, $i_3 = 0$ A d. $i_1 = 0$ A, $i_2 = 0$ A, $i_3 = 0$ A

56. Immediately after connecting switch S,

a. $i_3 = 0$ A and $\frac{di_3}{dt} = 0$ A s^{-1}

b. $i_3 = 0$ A and $\frac{di_3}{dt} \neq 0$ A s^{-1}

c. $i_3 = 0$ A and the rate at which magnetic energy stored is not zero

d. none of these

For Problems 57–59

Two resistors R_1 and R_2 can slide without friction along two parallel metal guides directed at an angle α to the horizontal and separated by a distance b. The guides are not connected at the bottom as shown. The entire system is placed in an upward magnetic field which decreases with time as $\frac{dB}{dt} = -a$, and also it decreases along the guides as $\frac{dB}{dt} = -\beta$, $x = 0$ and x-axis is along the guide as shown in Fig. 8.364. The resistors are made to slide with a constant velocity v downwards.

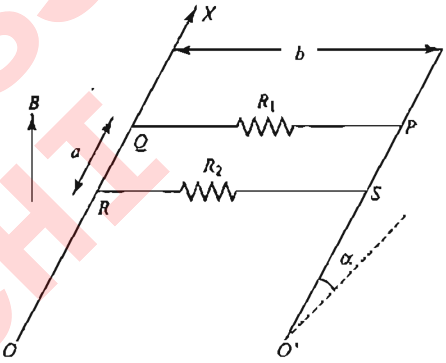


Fig. 8.364

57. The e.m.f. induced due to decrease of B with time only (and the direction of current in loop, respectively) (ACW → anticlockwise, CW → clockwise)

a. $ab\alpha$ (ACW) b. $\frac{a^2}{b}\alpha$(CW)

c. $ab\alpha$ (CW) d. $\frac{a^2}{b}\alpha$(ACW)

58. The e.m.f. induced due to decrease in B due to change in position only

a. βabv (CW) b. βabv (ACW)

c. $\beta^2 av$ (CW) d. $\beta^2 av$ (ACW)

59. The net current in loop is

a. $\frac{ab\alpha + ab\beta v}{R_1 + R_2}$ b. $\frac{ab\alpha - ab\beta v}{R_1 + R_2}$

c. $\frac{a^2 / b\alpha + \beta^2 av}{R_1 + R_2}$ d. $\frac{a^2 / b\alpha + \beta^2 av}{\frac{R_1 R_2}{R_1 + R_2}}$

For Problems 60–62

In Fig. 8.365, a square loop consisting of an inductor of inductance L and resistor of resistance R is placed between two long parallel wires. The two long straight wires have time varying current of magnitude $I = I_0 \cos \omega t$ but the directions of current in them are opposite.

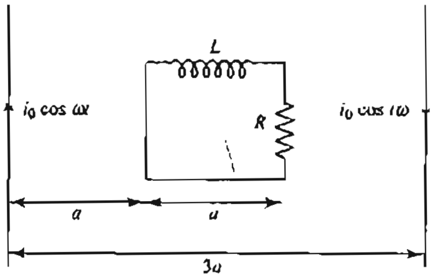


Fig. 8.365

60. Total magnetic flux in this loop is

a. $\frac{\mu_0 Ia}{\pi} \ln 2$ b. $\frac{2\mu_0 Ia}{\pi} \ln 2$

c. $\frac{4\mu_0 Ia}{\pi} \ln 2$ d. $\frac{\mu_0 Ia}{2\pi} \ln 2$

61. Magnitude of e.m.f. in this circuit only due to flux change associated with two long straight current carrying wires will be

a. $\frac{\mu_0 a \ln 2\ I_0 \omega}{\pi} \sin \omega t$ b. $\frac{2\mu_0 a \ln 2\ I_0 \omega}{\pi} \sin \omega t$

c. $\frac{\mu_0 a \ln 2\ I_0 \omega}{2\pi} \cos \omega t$ d. $\frac{\mu_0 a \ln 2\ I_0 \omega}{\pi} \cos \omega t$

62. The instantaneous current in the circuit will be

a. $\frac{2\mu_0 a \ln 2\ I_0 \omega}{\pi\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \varphi)$

b. $\frac{2\mu_0 a \ln 2\ I_0 \omega}{\pi\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \varphi)$

c. $\frac{\mu_0 a \ln 2\ I_0 \omega}{\pi\sqrt{R^2 + \omega^2 L^2}} \sin \omega t$

d. $\frac{\mu_0 a \ln 2\ I_0 \omega}{\pi\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \varphi)$

$\left(\text{where } \tan \varphi = \frac{\omega L}{R}\right)$

For Problems 63–65

In the given circuit at $t = 0$, switch S is closed.

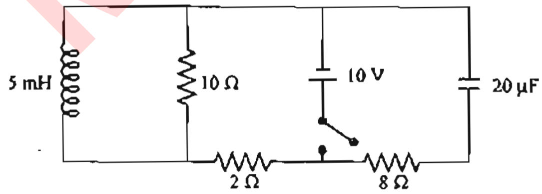


Fig. 8.366

63. The current through the 10 Ω resistor at any instant $t (0 < t < \infty)$ will be

a. $\frac{1}{6} e^{-(1000/3)t}$ b. $\frac{5}{6} e^{-(1000/3)t}$

c. $\frac{1}{6} e^{(1000/3)t}$ d. $\frac{6}{5} e^{(1000/3)t}$

64. The energy stored in the inductor at any instant t $(0 < t < \infty)$ will be

a. $\frac{1}{2}\left[5 - 5e^{-(1000/3)t}\right]^2$ mJ b. $\frac{125}{2}\left[1 - e^{-(1000/3)t}\right]^2$ mJ

c. $\frac{25}{2}\left[1 - e^{-(1000/3)t}\right]^2$ mJ d. $\frac{5}{2}\left[1 - e^{-(1000/3)t}\right]^2$ mJ

65. The energy stored in the capacitor and inductor, respectively, as $t \to \infty$ will be

a. 1 mJ and 62.5 mJ b. 62.5 mJ and 1 mJ

c. 2 mJ and 62.5 mJ d. 1 mJ and 60 mJ.

For Problems 66–68

In Fig. 8.367, there is a frame consisting of two square loops having resistors and inductors as shown. This frame is placed in a uniform but time varying magnetic field in such a way that one of the loops is placed in crossed magnetic field and the other is placed in dot magnetic field. Both magnetic fields are perpendicular to the planes of the loops.

If the magnetic field is given by $B = (20 + 10\ t)$ Wb m^{-2} in both regions [ℓ = 20 cm, b = 10 cm and R = 10 Ω, L = 10 H).

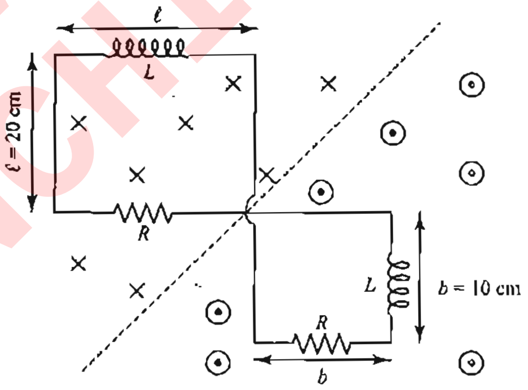


Fig. 8.367

66. The direction of induced current in the bigger loop will be

a. clockwise

b. anticlockwise

c. first clockwise for some time, then anticlockwise, and so on

d. first anticlockwise for some time, then clockwise, and so on

67. The induced e.m.f. in the frame only due to the variation of magnetic field will be

a. 0.3 V b. 0.1 V c. 0.5 V d. 0.4 V

68. The current in the frame as a function of time will be

a. $\frac{1}{20}(1 - e^{-t})$ b. $\frac{1}{40}(1 - e^{-t})$

c. $\frac{1}{20} e^{-t}$ d. $\frac{1}{10} e^{-t}$

For Problems 69–71

There is no current in any part of this circuit for time $t < 0$. The switch S is closed at $t = 0$.

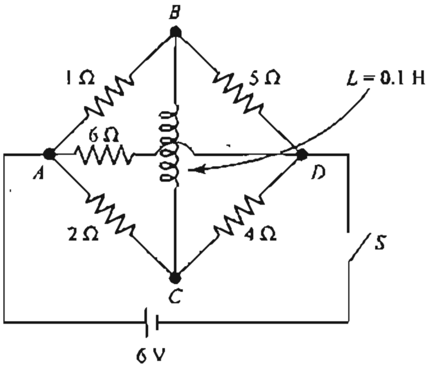


Fig. 8.368

69. The rate at which the current through the inductor increases initially is

a. zero b. 10 A s^{-1} c. 1 A s^{-1} d. 5 A s^{-1}

70. Current through the 6 Ω resistor

a. increases linearly with time

b. increases non-linearly with time

c. decreases non-linearly with time

d. remains constant

71. The current through the inductor after a long time will be

a. zero b. infinite

c. $\frac{6}{13}$ A d. none of these

For Problems 72–74

A standing wave $y = 2A \sin kx \cos \omega t$ is set up in the wire AB fixed at both ends by two vertical walls (see Fig. 8.369). The region between the walls contains a constant magnetic field B. Now, answer the following questions:

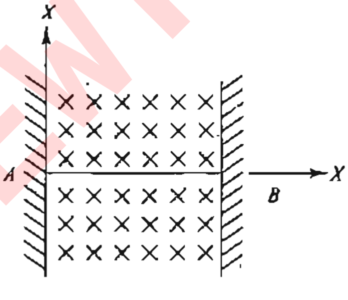


Fig. 8.369

72. The wire is found to vibrate in the third harmonic. The maximum e.m.f. induced is

a. $\frac{4(AB)\omega}{k}$ b. $\frac{3(AB)\omega}{k}$ c. $\frac{2(AB)\omega}{k}$ d. $\frac{(AB)\omega}{k}$

73. In the above question, the time when the e.m.f. becomes maximum for the first time is

a. $\frac{2\pi}{\omega}$ b. $\frac{\pi}{\omega}$ c. $\frac{\pi}{2\omega}$ d. $\frac{\pi}{4\omega}$

74. In which of the following modes the e.m.f. induced in AB is always zero?

a. fundamental mode b. second harmonic

c. second overtone d. fourth overtone

For Problems 75–77

In the given circuit, all the symbols have their usual meanings. At $t = 0$, the key K is closed. Now, answer the following questions.

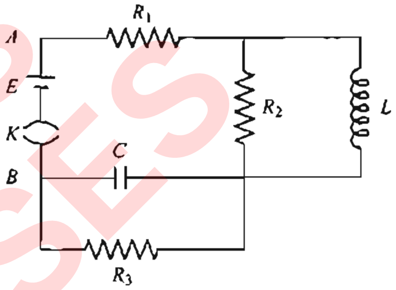


Fig. 8.370

75. At $t = 0$, the equivalent resistance between A and B is

a. $R_1 + R_2 + R_3$ b. $R_1 + R_2$

c. $R_1 + R_3$ d. indeterminate

76. At $t \to \infty$, the equivalent resistance between A and B is

a. $R_1 + R_2 + R_3$ b. $R_1 + R_2$

c. $R_1 + R_3$ d. none of these

77. At any time t, $0 < t < \infty$ (excluding the cases $t \to 0$ and $t \to \infty$), the equivalent resistance between A and B is

a. $R_1 + R_2 + R_3$ b. $R_1 + R_2$

c. $R_1 + R_3$ d. none of these

For Problems 78–80

A fan operates at 200 V (d.c.) consuming 1000 W when running at full speed. Its internal wiring has resistance 1 Ω. When the fan runs at full speed, its speed becomes constant. This is because the torque due to magnetic field inside the fan is balanced by the torque due to air resistance on the blades of the fan and torque due to friction between the fixed part and the shaft of the fan. The electrical power going into the fan is spent (i) in the internal resistance as heat, call it P_1, (ii) in doing work against internal friction and air resistance producing heat, sound, etc., call it P_2. When the coil of fan rotates, an e.m.f. is also induced in the coil. This opposes the external e.m.f. applied to send the current into the fan. This e.m.f. is called back e.m.f., call it e. Answer the following questions when the fan is running at full speed.

78. The current flowing into the fan and the value of back e.m.f. e is

a. 200 A, 5 V b. 5 A, 200 V

c. 5 A, 195 V d. 1 A, 0 V

79. The value of power P_1 is

a. 1000 W b. 975 W c. 25 W d. 200 W

80. The value of power P_2 is

a. 10000 W b. 975 W c. 25 W d. 200 W

For Problems 81–83

Fig. 8.371 shows a conducting rod of negligible resistance that can slide on a smooth U-shaped rail made of wire of resistance 1 Ω m^{-1}. Position of the conducting rod at $t = 0$ is shown. A time-dependent magnetic field $B = 2t$ tesla is switched on at $t = 0$.

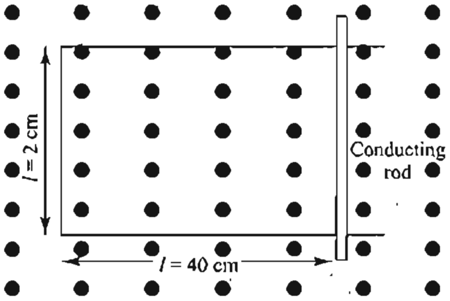


Fig. 8.371

81. The current in the loop at $t = 0$ due to induced e.m.f. is

a. 0.16 A, clockwise **b.** 0.08 A, clockwise
c. 0.08 A, anticlockwise **d.** zero

82. At $t = 0$, when the magnetic field is switched on, the conducting rod is moved to the left at a constant speed of 5 cm s^{-1} by some external means. The rod moves perpendicular to the rail. At $t = 2$ s, induced e.m.f. has magnitude

a. 0.12 V **b.** 0.08 V **c.** 0.04 V **d.** 0.02 V

83. Following situation of the previous question, the magnitude of the force required to move the conducting rod at a constant speed of 5 cm s^{-1} at the same instant $t = 2$ s is equal to

a. 0.16 N **b.** 0.12 N **c.** 0.08 N **d.** 0.06 N

For Problems 84–86

A massless rod AB of length 2ℓ is placed in a uniformly varying magnetic field confined in a cylindrical region of radius ($R > l$) as shown in Fig. 8.372. The center of the rod coincides with the center of the magnetic field. The rod can freely rotate about an axis that passes through its center and perpendicular to its length. Two particles, each of mass m and charge q are attached to the ends A and B of the rod. The time varying magnetic field in this cylindrical region is given by $B = B_0\left[1 - \frac{t}{2}\right]$ where B_0 is a constant. The magnetic field is switched on at time $t = 0$.

(Consider: $B_0 = 100$ T, $\ell = 4$ cm, $\frac{q}{m} = \frac{2\pi}{100}$ C kg^{-1}).

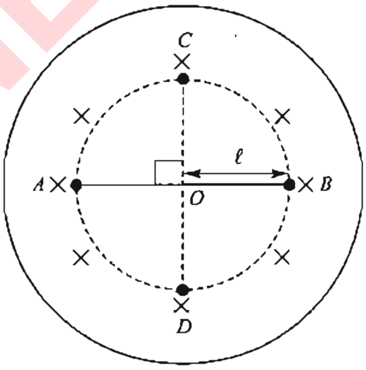


Fig. 8.372

84. The type of motion executed by the particle A is

a. uniform circular motion
b. non-uniform circular motion
c. pure rotational motion
d. none of the above

85. Two neutral particles C and D of mass $2m$ each are placed on two diametrically opposite points as shown in the figure. Consider the following statements and choose the correct option.

i. A will collide with C and B will collide with D
ii. A will collide with D and B will collide with C
iii. As magnetic force is a no-work force, so the energy of the system will remain conserved and is equal to $\frac{q^2}{8\pi\varepsilon_0\ell}$
iv. Considering the rod to be conducting, the magnitude of the potential difference between O and A is equal to the magnitude of the potential difference between O and B

a. Statements (i) and (iii) are correct
b. Statements (ii) and (iii) are correct
c. Statements (i) and (iv) are correct
d. Statements (ii) and (iv) are correct

86. The time after which A and B will collide with the $2m$ masses kept at C and D is

a. $t = 1$ s **b.** 2 s **c.** 4 s **d.** 8 s

For Problems 87–89

Two capacitors of capacitance C and $3C$ are charged to potential difference V_0 and $2V_0$, respectively, and connected to an inductor of inductance L as shown in Fig. 8.373. Initially the current in the inductor is zero. Now, the switch S is closed.

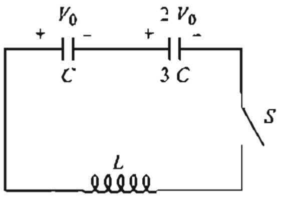


Fig. 8.373

87. The maximum current in the inductor is

a. $\frac{3V_0}{2}\sqrt{\frac{3C}{L}}$ **b.** $V_0\sqrt{\frac{3C}{L}}$
c. $2V_0\sqrt{\frac{3C}{L}}$ **d.** $V_0\sqrt{\frac{C}{L}}$

88. Potential difference across capacitor of capacitance C when the current in the circuit is maximum is

a. $\frac{V_0}{4}$ **b.** $\frac{3V_0}{4}$
c. $\frac{5V_0}{4}$ **d.** none of these

89. Potential difference across capacitor of capacitance $3C$ when the current in the circuit is maximum is

a. $\frac{V_0}{4}$ **b.** $\frac{V_0}{4}$
c. $\frac{5V_0}{4}$ **d.** none of these

For Problems 90–92

In Fig. 8.374, there is a conducting loop *ABCDEF* of resistance λ per unit length placed near a long straight current-carrying wire. The dimensions are shown in the figure. The long wire lies in the plane of the loop. The current in the long wire varies as $I = I_0(t)$.

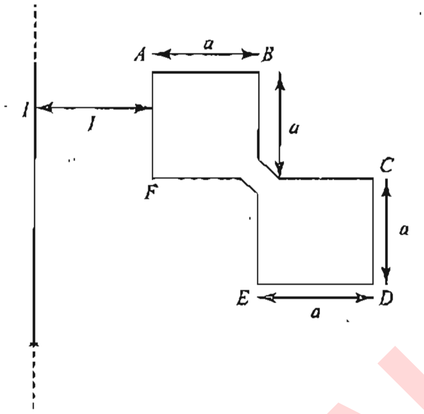


Fig. 8.374

90. The mutual inductance of the pair is

a. $\frac{\mu_0 a}{2\pi}\ln\left(\frac{2a+\ell}{\ell}\right)$ b. $\frac{\mu_0 a}{2\pi}\ln\left(\frac{2a-\ell}{\ell}\right)$

c. $\frac{2\mu_0 a}{\pi}\ln\left(\frac{a+\ell}{\ell}\right)$ d. $\frac{\mu_0 a}{\pi}\ln\left(\frac{a+\ell}{\ell}\right)$

91. The e.m.f. induced in the closed loop is

a. $\frac{\mu_0 I_0 a}{2\pi}\ln\left(\frac{2a+\ell}{\ell}\right)$ b. $\frac{\mu_0 I_0 a}{2\pi}\ln\left(\frac{2a-\ell}{\ell}\right)$

c. $\frac{2\mu_0 I_0 a}{\pi}\ln\left(\frac{a+\ell}{\ell}\right)$ d. $\frac{\mu_0 I_0 a}{\pi}\ln\left(\frac{a+\ell}{\ell}\right)$

92. The heat produced in the loop in time *t* is

a. $\frac{\left[\frac{\mu_0}{2\pi}\ln\left(\frac{a+\ell}{\ell}\right)I_0\right]^2 at}{4\lambda}$ b. $\frac{\left[\frac{\mu_0}{2\pi}\ln\left(\frac{2a+\ell}{\ell}\right)I_0\right]^2 at}{8\lambda}$

c. $\frac{\left[\frac{2\mu_0}{\pi}\ln\left(\frac{a+\ell}{\ell}\right)I_0\right]^2 at}{3\lambda}$ d. $\frac{\left[\frac{\mu_0}{2\pi}\ln\left(\frac{3a+\ell}{\ell}\right)I_0\right]^2 at}{6\lambda}$

For Problems 93–95

Initially the capacitor is charged to a potential of 5 V and then connected to position 1 with the shown polarity for 1 s. After 1 s it is connected across the inductor at position 2.

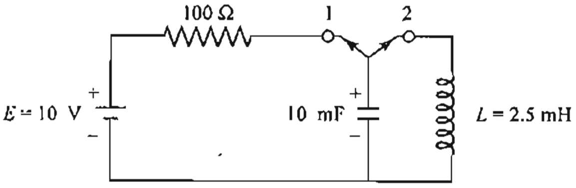


Fig. 8.375

93. The potential across the capacitor after 1 s of its connection to position 1is

a. $5\times10^3\left(2+\frac{1}{e}\right)$V b. $5\times10^3\left(2-\frac{1}{e}\right)$V

c. $5\times10^3\left(1+\frac{2}{e}\right)$V d. none of these

94. The maximum current flowing in the *LC* circuit when the capacitor is connected across the inductor is

a. $\left(2-\frac{1}{e}\right)\times10^4$ A b. $\left(1+\frac{2}{e}\right)\times10^4$ A

c. $\left(1-\frac{2}{e}\right)\times10^4$ A d. none of these

95. The frequency of *LC* oscillations is

a. $(20/\pi)$ Hz b. $(2/\pi)$ Hz

c. $(40/\pi)$ Hz d. $(17/\pi)$ Hz

For Problems 96–97

A brillant student of physics developed a magnetic balance to weigh objects. The mass *m* to be measured is hung from the centre of the bar. Bar is kept in a uniform magnetic field of 1.5 T directed into the plane of the figure. Battery voltage can be adjusted to vary the current in the circuit. The horizontal bar shown is 60 cm long and is made of extremely light weight material. It is connected to the battery via a resistance. There is no tension in the supporting wires. The magnetic force only supports the hanging weight.

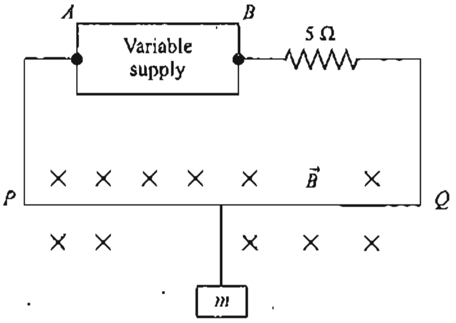


Fig. 8.376

96. Which point of battery terminal is positive?

a. *A* b. *B*

c. either of *A* or *B* d. cannot be found

97. If $V = 150$ V, what is the maximum mass *m*?

a. 1.3 kg b. 1.8 kg c. 2.2 kg d. 2.7 kg

For Problems 98–100

Consider two parallel, conducting frictionless tracks kept in a gravity-free space as shown in Fig. 8.377. A movable conductor *PQ*, initially kept at *OA*, is given a velocity 10 m s^{-1} towards right. The space contains a magnetic field which depends upon the distance moved by conductor *PQ* from the *OA* line and given by

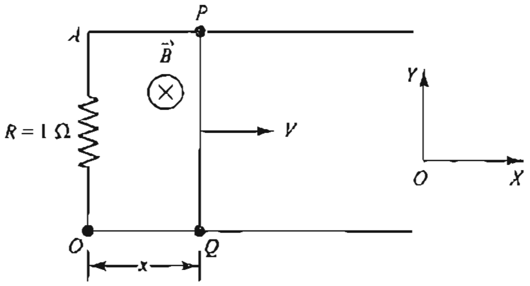


Fig. 8.377

$\vec{B} = cx(-\hat{k})$ |c = constant = 1 S.I. unit|

The mass of the conductor PQ is 1 kg and length of PQ is 1 m. Answer the following questions based on the above passage.

98. The distance travelled by the conductor when its speed is 5 m/s^{-1} is

a. $\left(\frac{15}{2}\right)^{1/3}$ b. $\left(\frac{10}{3}\right)^{1/3}$

c. $(10)^{1/3}$ d. none of the above

99. The heat loss during the time interval $t = 0$ to time t seconds, when the speed of the conductor is 5 m s^{-1} is

a. 50 J b. 30 J

c. 10 J d. none of the above

100. The work done by magnetic force acting on the conductor PQ during its motion in the time interval $t = 0$ to $t = t$ seconds when the speed of conductor is 5 m s^{-1} is

a. zero b. 50 J c. 10 J d. 30 J

Matching Column Type

Solutions on page 8.141

1. The switch S in the circuit is connected with point a for a very long time, then it is shifted to position b. The resulting current through the inductor is shown by curves in the graph for four sets of values for the resistance R and inductance L (given in column I). Which set corresponds with which curve?

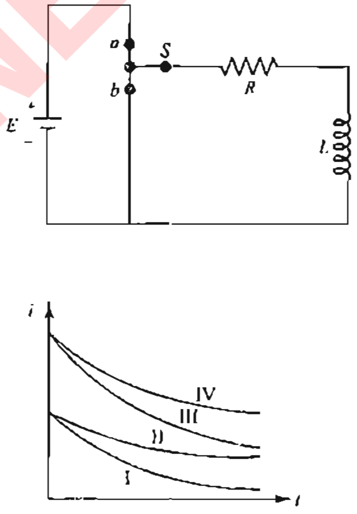


Fig. 8.378

Column I	Column II
i. R_0 and L_0	a. I
ii. $2R_0$ and L_0	b. II
iii. R_0 and $2L_0$	c. III
iv. $2R_0$ and $2L_0$	d. IV

2. The magnetic field in the cylindrical region shown in Fig. 8.379 increases at a constant rate of 10.0 mT s^{-1}. Each side of the square loop $abcd$ and $defa$ has a length of 2.00 cm and a resistance of 2.00 Ω. Correctly match the current in the wire ad in four different situations as listed in column I with the values given in column II.

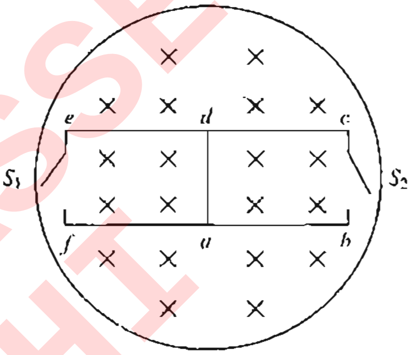


Fig. 8.379

Column I	Column II
i. The switch S_1 is closed but S_2 is open	a. 5×10^{-7} A, d to a
ii. S_1 is open but S_2 is closed	b. 5×10^{-7} A, a to d
iii. both S_1 and S_2 are open	c. 2.5×10^{-8} A, d to a
iv. both S_1 and S_2 are closed	d. no current flows

3. Match the following column.

Column I	Column II
i. Inductance of a coil	a. Depends on resistivity
ii. Capacitance	b. Depends on shape
iii. Impedance of coil	c. Depends on medium inserted
iv. Reactance of a capacitor	d. Depends on external voltage source

4. Fig. 8.380 shows a metallic solid block, placed in a way so that its faces are parallel to the coordinate axes. Edge lengths along axes x, y and z are a, b and c, respectively. The block is in a region of uniform magnetic field of magnitude 30 mT. One of the edge lengths of the block is 25 cm. The block is moved at 4 m s^{-1} parallel to each axis and in turn, the resulting potential difference V that appears across the block is measured. When the motion is parallel to the y-axis, V = 24 mV; with the motion parallel to the z-axis, V = 36 mV; with the motion parallel to the x-axis, V = 0. Using the given information, correctly match the dimensions of the block with the values given

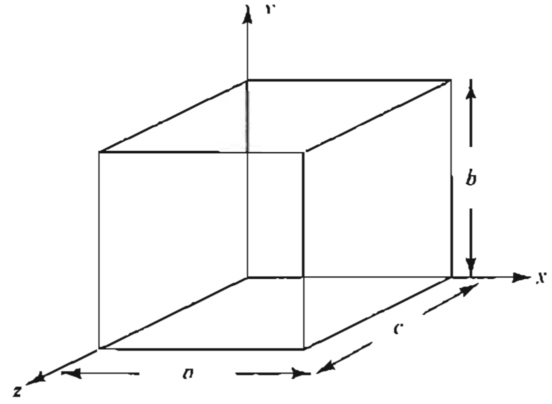


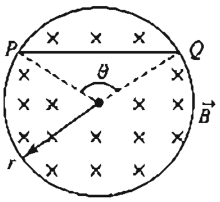
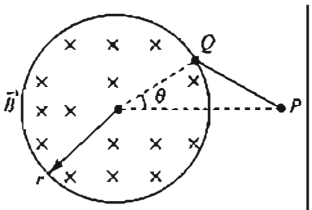
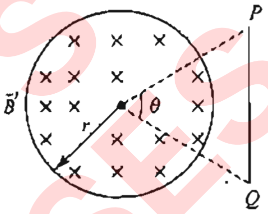
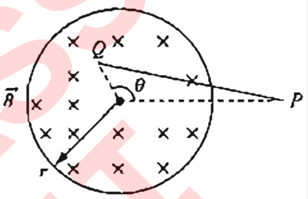
Fig. 8.380

Column I	Column II
i. a	a. 20 cm
ii. b	b. 24 cm
iii. c	c. 25 cm
iv. $\frac{bc}{a}$	d. 30 cm

5. Column I gives situations involving a charged particle which may be realized under the condition given in column II. Match the situations in column I with the conditions in column II.

Column I	Column II
i. Increase in speed of a charged particle	a. Electric field uniform in space and constant in time
ii. Exert a force on an electron initially at rest	b. Magnetic field uniform in space and constant in time
iii. Move a charged particle in a circle with uniform speed	c. Magnetic field uniform in space but varying with time
iv. Accelerate a moving charged particle	d. Magnetic field non-uniform in space but constant with time

6. Column I shows the cylindrical region of radius r where a downward magnetic field $\vec{B}$ exists, where $\vec{B}$ is increasing at the rate of $\frac{dB}{dt}$. A rod PQ is placed in different citation as shown. Match the column I with the correct statement in column II regarding the induced e.m.f. in rod.

Column I	Column II
i. 	a. Induced e.m.f. in rod PQ is $\frac{1}{2}r^2\theta\frac{dB}{dt}$.
ii. 	b. Induced e.m.f. in rod PQ is less than $\frac{1}{2}r^2\theta\frac{dB}{dt}$.
iii. 	c. End P is positive with respect to point Q.
iv. 	d. End Q is positive with respect to point P.

7. A uniform but time varying magnetic field $B(t)$ exists in a cylindrical region of radius a and is directed into the plane of the paper, as shown in Fig. 8.381. The magnetic field decreases at a constant rate inside the region. If r is the distance from the axis of the cylindrical region then match column I to column II.

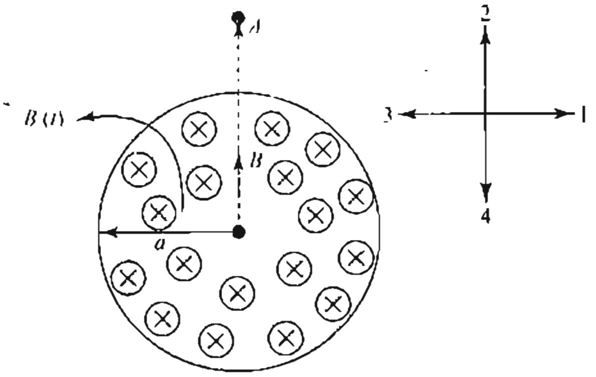
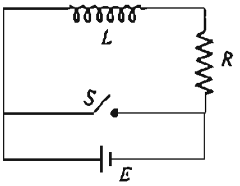
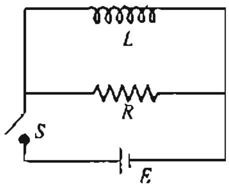
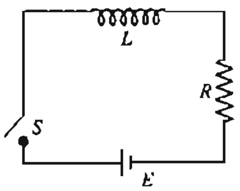
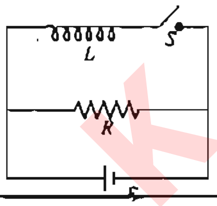


Fig. 8.381

Column I	Column II
i. Induced electric field at point A	a. Directed along 3
ii. Induced electric field at point B	b. Directed along 1
iii. Force on an electron placed at point A	c. Increases as r
iv. Force on an electron placed at point B	d. Decreases as $1/r$

8. In column I some circuits are given. In all the circuits except in (i), switch S remains closed for long time and then it is opened at $t = 0$ while for (i), the situation is reversed. column II tells something about the circuit quantities. Match the entries of column I with the entries of coumn II.

Column I	Column II
i. 	a. Induced e.m.f. can be greater than E.
ii. 	b. Induced e.m.f. would be less than E.
iii. 	c. Finally, energy stored in inductor is zero.
iv. 	d. Finally, energy stored in inductor is non-zero.

9. A rectangular loop of wire with dimensions ℓ and b has N turns and a total resistance R. The loop moves with constant velocity from AB to PQ in a region of uniform magnetic field as shown. Let x be the distance of AB from PQ.

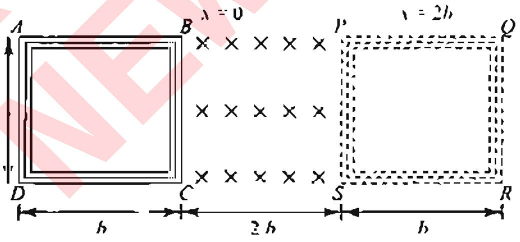


Fig. 8.382

Column I	Column II
Flux linked with the loop	E.m.f. induced in the loop
i. $0 < x < b$, $BN\ell x$	a. $Bv\ell N$
ii. $b < x < 2b$, $BN\ell b$	b. 0
iii. $2b < x < 3b$, $BN\ell(3b - x)$	c. $-Bv\ell N$
iv. $x > 3b$	d. BvN

10. Magnetic flux in a circular coil of resistance 10 Ω changes with time as shown in Fig. 8.383. Cross indicates a direction perpendicular to paper inwards. Match the following.

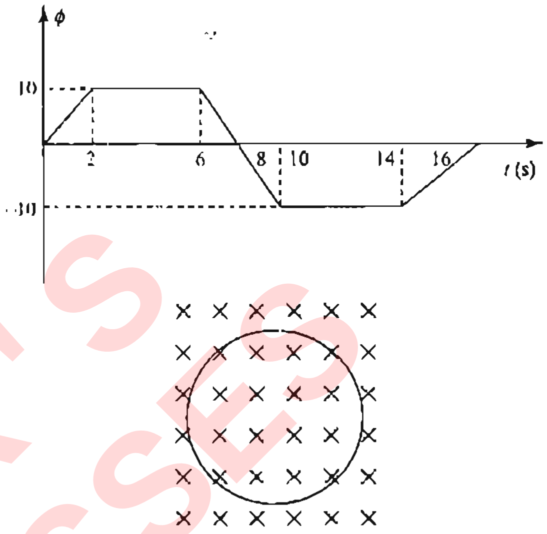
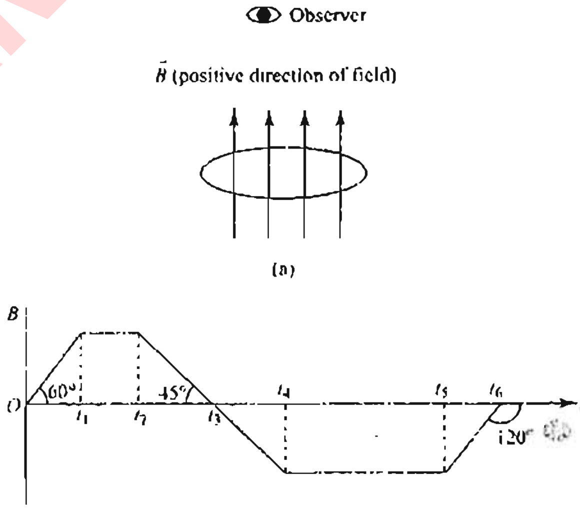


Fig. 8.383

Column I	Column II
i. At 1 s, induced current is	a. Clockwise
ii. At 5 s, induced current is	b. Anticlockwise
iii. At 9 s, induced current is	c. Zero
iv. At 15 s, induced current is	d. 2 A

11. A conducting loop is held in a magnetic field such that the field is oriented perpendicular to the area of the loop as shown in Fig. 8.384(a). At any instant magnetic flux density over the entire area has the same value but it varies with time as shown in Fig. 8.384(b).



(a)

(b)

Fig. 8.384

Column I	Column II
i. Induced current in the coil is in the clockwise sense	a. For $t_2 < t < t_3$
ii. Induced current in the coil is in the anticlockwise sense	b. For $t_3 < t < t_4$
iii. Induced current is zero	c. For $t_5 < t < t_6$
iv. Induced current is maximum	d. For $t_4 < t < t_5$

12. Let $x = E/B$, $y = \sqrt{\dfrac{1}{\mu_0 \varepsilon_0}}$ and $z = \dfrac{1}{RC}$ where in length R and C are resistance and capacitance, respectively.

Column I	Column II
i. x and y have same dimensions as	a. $M^0L^0T^{-1}$
ii. y and z have same dimensions as	b. M^0LT^{-1}
iii. z and x have same dimensions as	c. MLT^{-2}
iv. none of the pairs has same dimensions as	d. ML^2T^{-2}

13. A frame $ABCD$ is rotating with the angular velocity ω about an passing through the point O perpendicular to the plane of paper as shown in the Fig. 8.385. A uniform magnetic field $\vec{B}$ is applied into the plane of the paper in the region as shown. Match the following.

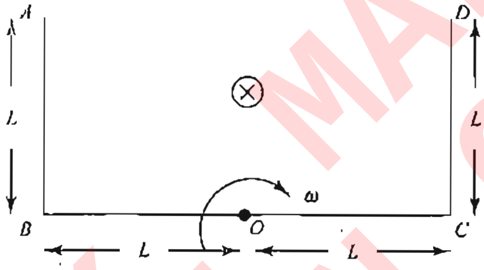


Fig. 8.385

Column I	Column II
i. Potential difference between A and O	a. zero
ii. Potential difference between O and D	b. $\dfrac{B\omega L^2}{2}$
iii. Potential difference between C and D	c. $B\omega L^2$
iv. Potential difference between A and D	d. constant

14. Fig. 8.386 shows two coaxial coils M and N. Column I is regarding some operations done with coil M and column II about induced current in coil N.

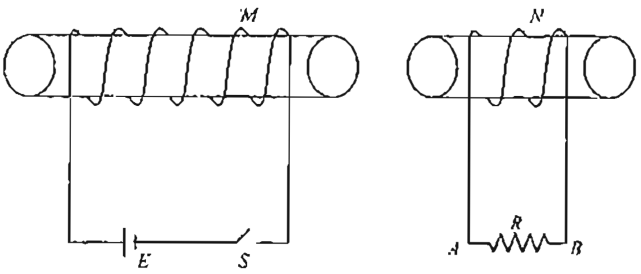


Fig. 8.386

Column I	Column II
i. Just after the switch S is closed	a. Current is induced from A to B
ii. Switch S is opened after keeping it closed for a long time.	b. Current is induced from B to A
iii. After the switch S is closed for a long time	c. No current is induced
iv. Just after the switch S is closed while moving M away from N	d. Current is induced either from A to B or from B to A

Archives

Solutions on page 8.143

Fill in the Blanks Type

1. A uniformly wound solenoidal coil of self-inductance 1.8×10^{-4} henry and resistance 6 Ω is broken up into two identical coils. These identical coils are then connected in parallel across a 15 V battery of negligible resistance. The time constant for the current in the circuit is ________ seconds and the steady state current through the battery is ________ amperes. **(IIT-JEE, 1989)**
2. If ε_0 and μ_0 are, respectively, the electric permittivity and magnetic permeability of free space, ε and μ are corresponding quantities in a medium, the index of refraction of the medium in terms of the above parameter is ________. **(IIT-JEE, 1992)**
3. In a straight conducting wire, a constant current flowing from left to right due to a source of e.m.f. When the source is switched off, the direction of the induced current in the wire will be ________. **(IIT-JEE, 1993)**
4. The network shown in Fig. 8.488 is part of a complete circuit. If at a certain instant the current (I) is 5 A, and is decreasing at a rate of 10^5 A/s then $V_B - V_A = -$V. **(IIT-JEE, 1997)**

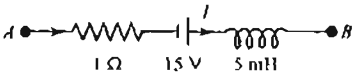


Fig. 8.488

True or False Type

1. A coil of metal wire is kept stationary in a non-uniform magnetic field. An e.m.f. is induced in the coil. **(IIT-JEE, 1986)**
2. A conducting rod AB moves parallel to the x-axis in a uniform magnetic field pointing in the positive z-direction. The end A of the rod gets positively charged. **(IIT-JEE, 1987)**

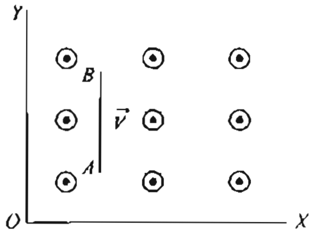


Fig. 8.489

Single Correct Answer Tpye

1. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular to and into the plane of the loop exists everywhere. **(IIT-JEE, 1989)**

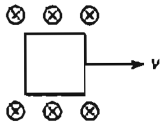


Fig. 8.490

The current induced in the loop is

a. BLv/R clockwise b. BLv/R anticlockwise

c. $2BLv/R$ anticlockwise d. zero

2. A thin circular ring of area A is held perpendicular to a uniform magnetic field of induction B. A small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of the circuit is R. When the ring is suddenly squeezed to zero area, the charge flowing through the galvanometer is **(IIT-JEE, 1995)**

a. $\frac{BR}{A}$ b. $\frac{AB}{R}$

c. ABR d. $\frac{E^2A}{R^2}$

3. A thin semicircular conducting ring of radius R is falling with its plane vertical in horizontal magnetic induction $\vec{B}$. At the position MNQ the speed of the ring is V, and the potential difference developed across the ring is **(IIT-JEE, 1996)**

a. zero

b. $BV\pi R^2/2$ and M is at higher potential

c. πRBV and Q is at higher potential

d. $2RBV$ and Q is at higher potential

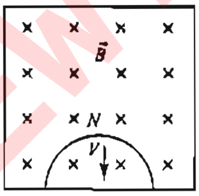


Fig. 8.491

4. A metal rod moves at a constant velocity in a direction perpendicular to its length. A constant uniform magnetic field exists in space in a direction perpendicular to the rod as well as its velocity. Select the correct statement(s) from the following: **(IIT-JEE, 1998)**

a. The entire rod is at the same electric potential

b. There is an electric field in the rod

c. The electric potential is highest at the centre of the rod and decrease towards its ends

d. The electric potential is lowest at the centre of the rod and increases towards its ends

5. A small square loop of wire of side ℓ is placed inside a large square loop of wire of side $L(L \gg \ell)$. The loops are coplanar and their centres coincide. The mutual inductance of the system is proportional to **(IIT-JEE, 1998)**

a. ℓ/L b. ℓ^2/L c. L/ℓ d. L^2/ℓ

6. Two identical circular loops of metal wire are lying on a table without touching each other. Loop-A carries a current which increases with time. In response, the loop-B **(IIT-JEE, 1999)**

a. remains stationary

b. is attracted by the loop A

c. is repelled by the loop A

d. rotates about its CM with CM fixed

7. A coil of inductance 8.4 mH and resistance is connected to a 12 V battery. The current in the coil is 1.0 A at approximately the time **(IIT-JEE, 1999)**

a. 500 s b. 25 s c. 35 ms d. 1 ms

8. A uniform but time-varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper, as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region **(IIT-JEE, 2000)**

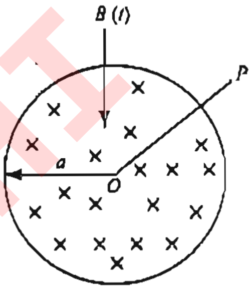


Fig. 8.492

a. is zero b. decreases as $1/r$

c. increases as r d. decreases as $1/r^2$

9. A coil of wire having inductance and resistance has a conducting ring placed coaxially within it. The coil is connected to a battery at time $t = 0$, so that a time-dependent current $I_1(t)$ starts flowing through the coil. If $I_2(t)$ is the current induced in the ring, and $B(t)$ is the magnetic field at the axis of the coil to $I_1(t)$, then as a function of time ($t > 0$), the product $I_2(t)B(t)$ **(IIT-JEE, 2000)**

a. increases with time b. decreases with time

c. does not vary with time d. passes through a maximum

10. A metallic square loop $ABCD$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the figure. An electric field is induced **(IIT-JEE, 2001)**

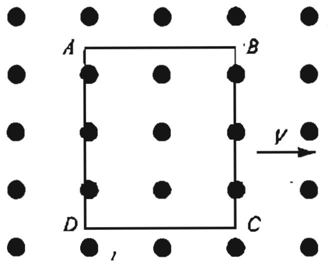


Fig. 8.493

a. in AD, but not in BC b. in BC, but not in AD
c. neither in AD nor in BC d. in both AD and BC

11. Two circular coils can be arranged in any of the three situations shown in the figure. Their mutual inductance will be: **(IIT-JEE, 2001)**

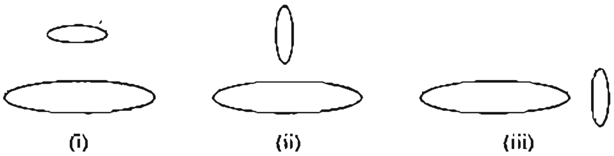


Fig. 8.494

a. maximum in situation (i) b. maximum in situation (ii)
c. maximum in situation (iii) d. the same in all situations

12. As shown in Fig. 8.495, P and Q are two coaxial conducting loops separated by some distance. When the switch S is closed, a clockwise current I_P flows in P (as seen by E) and an induced current I_{Q1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q2} flows in Q. Then the direction IQ_1 and IQ_2 (as seen by E) are **(IIT-JEE, 2002)**

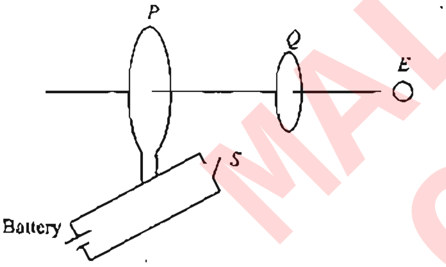


Fig. 8.495

a. respectively clockwise and anticlockwise
b. both clockwise
c. both anticlockwise
d. respectively anticlockwise and clockwise

13. A short-circuited coil is placed in a time-varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be **(IIT-JEE, 2002)**

a. halved b. the same
c. doubled d. quadrupled

14. When an a.c. source of e.m.f. sin(100 t) is connected across a circuit, the phase difference between the e.m.f. e and the current i in the circuit is observed to be $\pi/4$, as shown in Fig. 8.496. If the circuit consists possibly only of R-C or R-L or L-C in series, find the relationship between the two elements. **(IIT-JEE, 2003)**

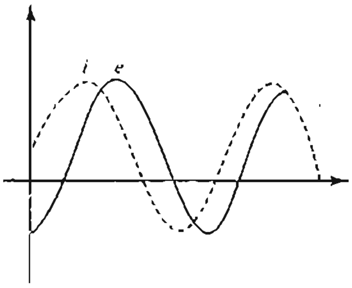


Fig. 8.496

a. $R = 1\ \text{k}\Omega, C = 10\ \mu\text{F}$ b. $R = 1\ \text{k}\Omega, C = 1\ \mu\text{F}$
c. $R = 1\ \text{k}\Omega, L = 10\ \text{H}$ d. $R = 1\ \text{k}\Omega, L = 1\ \text{H}$

15. A small bar magnet is being slowly inserted with constant velocity inside a solenoid as shown in Fig. 8.497. Which graph best represents the relationship between e.m.f. induced with time **(IIT-JEE, 2004)**

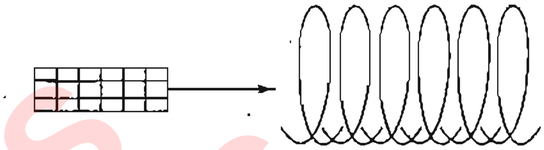
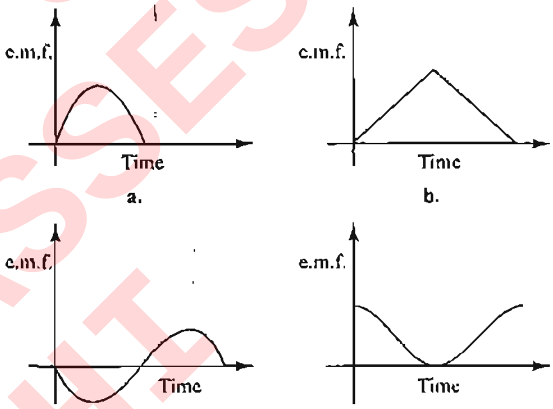


Fig. 8.497



a. b.
c. d.

Fig. 8.498

16. An infinitely long cylinder is kept parallel to a uniform magnetic field B directed along positive z-axis. The direction of induced current as seen from the z-axis will be **(IIT-JEE, 2005)**

a. zero
b. anticlockwise of the +ve z-axis
c. clockwise of the +ve z-axis
d. along the magnetic field

17. Fig. 8.599 shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments ab and cd. Then,

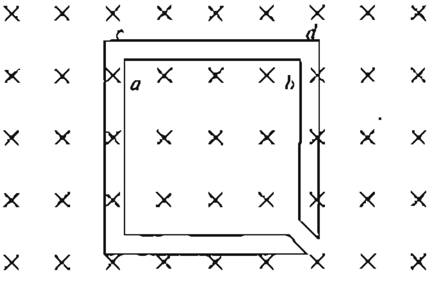


Fig. 8.499

(IIT-JEE, 2009)

a. $I_1 > I_2$
b. $I_1 < I_2$
c. I_1 is in the direction ba and I_2 is in the direction cd
d. I_1 is in the direction ab and I_2 is in the direction dc

Multiple Correct Answers Type

1. L, C and R represent the physical quantities inductance, capacitance and resistance, respectively. The combinations which have the dimensions of frequency are **(IIT-JEE, 1984)**

   a. $1/RC$ b. R/L c. $1/\sqrt{LC}$ d. C/L

2. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular and into the plane of the loop exists everywhere **(IIT-JEE, 1989)**

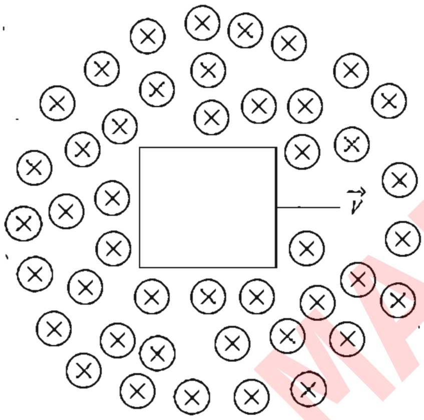


Fig. 8.500

The current induced in the loop is

a. BLv/R clockwise
b. BLv/R anticlockwise
c. $2BLv/R$ anticlockwise
d. zero

3. Two different coils have self-inductances L_1 = 8 mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1, respectively. Corresponding values for the second coil at the same instant are i_1, V_2 and W_2, respectively. Then **(IIT-JEE, 1994)**

   a. $\frac{i_1}{i_2}=\frac{1}{4}$
   b. $\frac{i_1}{i_2}=4$
   c. $\frac{W_2}{W_1}=\frac{1}{4}$
   d. $\frac{V_2}{V_1}=4$

4. A small square loop of wire of side ℓ is placed inside a large square loop of wire of side $L (L \gg \ell)$. The loops are coplanar and their centres coincide. The mutual inductance of the system is proportional to **(IIT-JEE, 1998)**

   a. ℓ/L b. ℓ^2/L c. L/ℓ d. L^2/ℓ

5. The S.I. unit of inductance, the henry, can be written as **(IIT-JEE, 1998)**

   a. weber/ampere
   b. volt-second/ampere
   c. joule/(ampere)^2
   d. ohm-second

6. A field line is shown in Fig. 8.501. This field cannot represent **(IIT-JEE, 2006)**

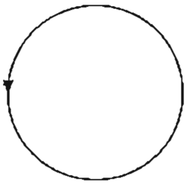


Fig. 8.501

a. Magnetic field
b. Electrostatic field
c. Induced electric field
d. Gravitational field

7. Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in Fig. 8.502. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is(are) **(IIT-JEE, 2009)**

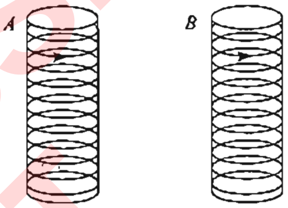


Fig. 8.502

a. $\rho_A > \rho_B$ and $m_A = m_B$
b. $\rho_A < \rho_B$ and $m_A = m_B$
c. $\rho_A > \rho_B$ and $m_A > m_B$
d. $\rho_A < \rho_B$ and $m_A < m_B$

Assertion-Reasoning Type

Mark your answer as

a. Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.
b. Statement I is true, Statement II is true; Statement II is **NOT** a correct explanation for Statement I.
c. Statement I is true, Statement II is false.
d. Statement I is false, Statement II is true.

1. **Statement I:** A vertical iron rod has coil of wire wound over it at the bottom end. An alternating current flows in the coil. The rod goes through a conducting ring as shown in the figure. The ring can float at a certain height above the coil.

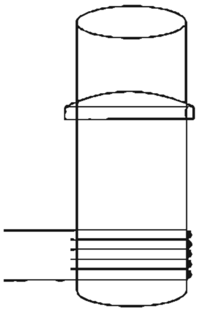


Fig. 8.503

Statement II: In the above situation, a current is induced in the ring which interacts with the horizontal component of the magnetic field to produce an average force in the upward direction. **(IIT-JEE, 2007)**

Comprehension Type

For Problems 1 – 3

In the given circuit the capacitor (C) may be charged through resistance R by a battery V by closing switch S_1. Also, when S_1 is opened and S_2 is closed the capacitor is connected in series with inductor (L). **(IIT-JEE, 2006)**

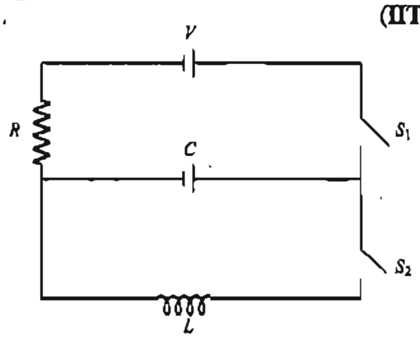


Fig. 8.504

1. At the start, the capacitor was uncharged. When switch S_1 is closed and S_2 is kept open, the time constant of this circuit is t. Which of the following is correct.
   a. After time interval t, charge on the capacitor is $\frac{CV}{2}$
   b. After time interval 2τ, charge on the capacitor of $CV(1-e^{-2})$
   c. The work done by the voltage source will be half of the heat dissipated when the capacitor is fully charged
   d. After time interval 2τ, charge on the capacitor is $CV(1-e^{-1})$
2. When the capacitor gets charged completely, S_1 is opened and S_2 is closed. Then
   a. at $t = 0$, energy stored in the circuit is purely in the form of magnetic energy
   b. at any time $t > 0$ current in the circuit is in the same direction
   c. at $t > 0$, there is no exchange in the circuit is in the same direction
   d. at any time $t > 0$, instantaneous current in the circuit may be $V\sqrt{\frac{C}{L}}$
3. Given that the total charge stored in the LC circuit is, the charge on the capacitor is
   a. $Q = Q_0\cos\left(\frac{\pi}{2}+\frac{t}{\sqrt{LC}}\right)$
   b. $Q = Q_0\cos\left(\frac{\pi}{2}-\frac{t}{\sqrt{LC}}\right)$
   c. $Q = -LC\frac{d^2Q}{dt^2}$
   d. $Q = -\frac{1}{\sqrt{LC}}\frac{d^2Q}{dt^2}$

For Problems 4 – 6

Modern trains are based on Maglev technology in which trains are magnetically levitated, which runs its EDS Maglev system.

There are coils on both sides of wheels. Due to motion of the train, current induces in the coil of track which levitate it. This is in accordance with Lenz's law. If trains lower down then due to Lenz's law, repulsive force increases due to which train gets uplifted and if it goes much higher then there is a net downward force due to gravity. The advantage of Maglev train is that there is no friction between the train and the track, thereby reducing power consumption and enabling the train to attain very high speeds.

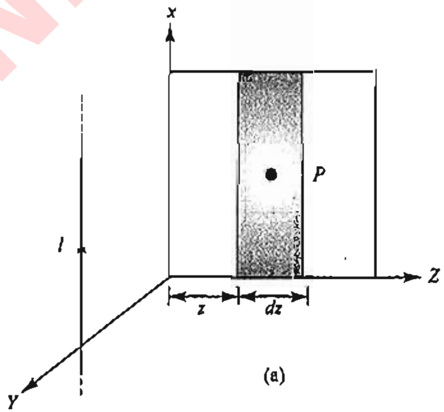
Disadvantage of Maglev train is that as it slows down the electromagnetic forces decrease and it becomes difficult to keep it levitated and as it moves forward according to Lenz's law there is an electromagnetic drag force. **(IIT-JEE, 2006)**

4. What is the advantage of this system?
   a. No friction hence no power consumption
   b. No electric power is used
   c. Gravitation force is zero
   d. Electrostatic force draws the train
5. What is the disadvantage of this system?
   a. Train experiences upward force according to Lenz's law
   b. Friction force creates a drag on the train
   c. Retardation
   d. By Lenz's law, the train experiences a drag
6. Which force causes the train to elevate up?
   a. Electrostatic force
   b. Time varying electric field
   c. Magnetic force
   d. Induced electric field

ANSWERS AND SOLUTIONS

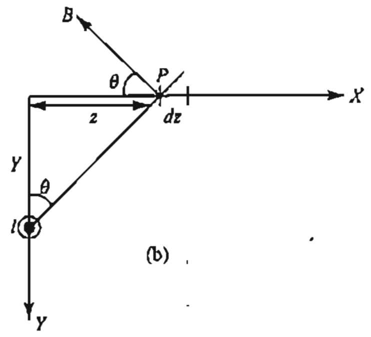
Subjective Type

1.



(a)

Fig. 8.387 (*Contd.*)



(b)

Fig. 8.387

The magnetic field at point P [Fig. 8.389 (b)] is

$$B = \frac{\mu_0}{2\pi}\frac{i}{\sqrt{y^2+z^2}}$$

The magnetic flux through the shaded strip in Fig. 8.387 (a) is

$$d\phi = (W\,dz)\,\frac{\mu_0}{2\pi}\frac{i}{\sqrt{y^2+z^2}}\sin\theta$$

where $\sin\theta\ \frac{z}{\sqrt{y^2+z^2}}$

∴ total magnetic flux through rectangular loop is

$$\theta = \int_0^L \frac{\mu_0}{2\pi}\frac{i_0 \sin \omega t\, Wz dz}{y^2+z^2}$$

$$= \frac{\mu_0}{4\pi} W \ln\left(\frac{y^2+L^2}{y^2}\right) i_0 \sin\ \omega t$$

∴ induced e.m.f. in the loop is

$$e = \frac{d\phi}{dt} = \frac{\mu_0}{4\pi} i_0 W\ \omega \cos \omega t\ \ln\left(\frac{L^2+y^2}{y^2}\right)$$

$$\phi = \frac{\mu_0 I_0 W_\omega \cos \omega t}{4\pi}\ln\left(\frac{L^2}{y^2}+1\right)$$

2. a. $\frac{dB}{dt} = 3\ \text{mTs}^{-1}$

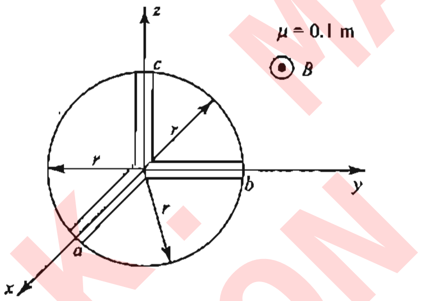


Fig. 8.388

The flux associated with one loop in y-z plane.

$$\phi = \frac{\pi r^2 B}{4}$$

$$E = \frac{d\phi}{dt} = \frac{\pi r^2}{4}\frac{dB}{dt} = \frac{\pi \times 10^{-2}}{4} \times 3 \times\ 10^{-3}$$

$$E = \frac{3\pi \times 10^{-5}}{4}\ \text{V}$$

b. Direction of current is clockwise ($c \rightarrow b$)

3. $\phi = \bar{B}.\bar{A} = B_0(\hat{i}+\hat{j}).A'\left(\hat{i}+\hat{j}+\hat{k}\right)$

where A' is area of one segment.

$$= 2R^2 + \frac{\pi R^2}{2} = R^2\left(2+\frac{\pi R^2}{2}\right)$$

∴ $\phi = B_0A' + B_0A' = 2B_0A'$

$$\frac{d\phi}{dt} = |\varepsilon| = 2A'\frac{dB}{dt}$$

$$\varepsilon = 2 \propto R^2\left(2+\frac{\pi}{2}\right)$$

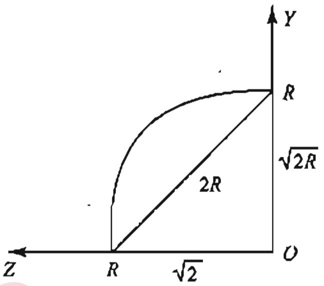


Fig. 8.389

The current flows from Q to R to P to Q.

$$Q \rightarrow R \rightarrow P \rightarrow Q.$$

4. a. $E_{net} = 4B\ell'v$

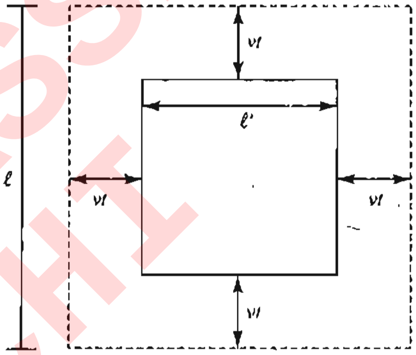


Fig. 8.390

Equivalent circuit:

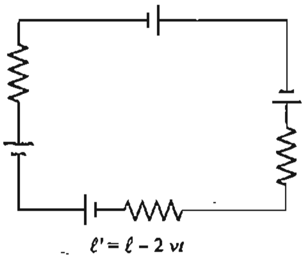


Fig. 8.391

∴ $E_{net} = 4B(\ell - 2vt)v$

$E_{net} = 4Bv(\ell - 2vt)$

b. $I = \frac{E_{net}}{4r} = \frac{4B\ell v}{4\lambda\ell} \Rightarrow I = \frac{Bv}{\lambda}$

c. Force required on such wire = $I\ell'B$

$$\text{Force} = \frac{B^2v}{\lambda}(\ell - 2vt)$$

d. Total force required to maintain constant velocity

$$= p_{ele} = 4 \times E \times I = E_{net} \times I$$

$$= 4Bv(\ell - 2vt) \times \frac{Bv}{\lambda} = \frac{4B^2v^2}{\lambda}(\ell - 2vt)$$

e. Thermal power developed in the circuit

$$= 4I^2r = 4I^2\lambda(\ell - 2vt) = 4\frac{B^2v^2}{\lambda^2}\lambda(\ell - 2vt)$$

Thermal power $= \frac{4B^2v^2}{\lambda}(\ell - 2vt)$

5. Equivalent circuit:

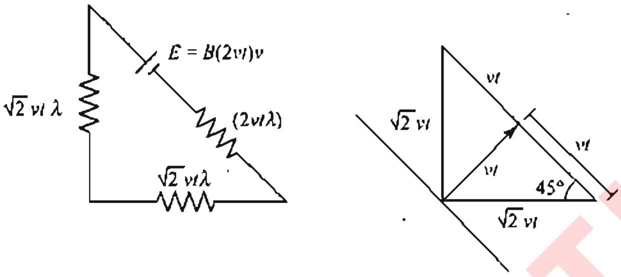


Fig. 8.392

$$I = \frac{B(2vt)\,v}{2\sqrt{2}vt\,\lambda + 2vt\,\lambda} \quad \Rightarrow \quad I = \frac{2Bv\,vt}{2vt\,\lambda(\sqrt{2}+1)}$$

$$I = \frac{Bv}{l(\sqrt{2}+1)} = \text{constant}$$

6. a.

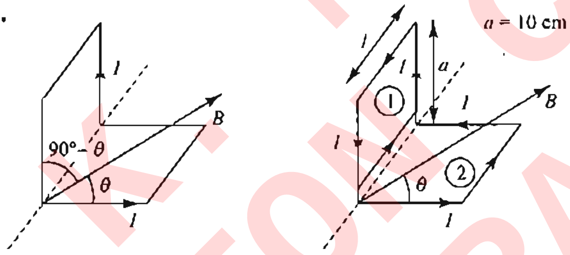


Fig. 8.393

Now flux through loop (1), $\phi_1 = Ba^2 \sin\theta$
Flux through loop (2), $\phi_2 = Ba^2 \cos\theta$
e.m.f. induced in loop (1),

$$E_1 = \frac{d\phi_1}{dt} = (2\pi v)\,B_0 \cos(2\pi vt)\,a^2 \cos\theta$$

e.m.f. induced in loop (2),

$$E_2 = \frac{d\phi_2}{dt} = (2\pi v)\,B_0 \cos(2\pi vt)\,a^2 \sin\theta$$

$\therefore$ Net induced e.m.f., $E_{net} = E_1 + E_2$

$$= (2\pi v)B_0 \cos(2\pi vt)\,a^2(\cos\theta + \sin\theta)$$
$$= (120\pi)(1.2\times 10^{-3})\cos(120\pi t)(0.01) \times (\cos\theta + \sin\theta)$$
$$= (1.44\pi\times 10^{-3})\cos(120\pi t)(\cos\theta + \sin\theta)\text{ V}$$

b. e.m.f. will be maximum when $\cos\theta + \sin\theta$ has maximum value.

$$\Rightarrow \qquad \theta = \frac{\pi}{4} \qquad (\text{as } 0 \le \theta \le 2\pi)$$

$\therefore \quad \phi''(\theta) = -\cos\theta - \sin\theta$

$\therefore$ θ is point of maxima

$\therefore$ At $\theta = \pi/4$, the induced e.m.f. is maximum.

7. $\frac{d_{B}}{dt} = -k$

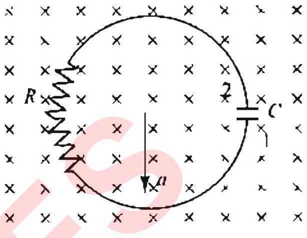


Fig. 8.394

$$\phi = \pi a^2 B$$

$$\varepsilon = \frac{-d\phi}{dt} = -\pi a^2\frac{dB}{dt} \quad \Rightarrow \quad \varepsilon = \pi a^2 k$$

a. When capacitor is fully charged, charge on capacitor is

$$q_0 = c\varepsilon$$

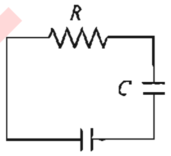


Fig. 8.395

$$q_0 = \pi a^2\,Ck$$

b. The upper plate of capacitor is at a higher potential.

c. As the area of the loop is constant when the magnetic field decreases, there is a change in the flux associated with the loop. Hence, an induced e.m.f. is developed in the loop which acts in a direction so as to restore the change in flux. This induced e.m.f. acts as a current in the loop which rotates in the separation of charges.

8. $l = 0.1$ m ; $B = 2t^2$

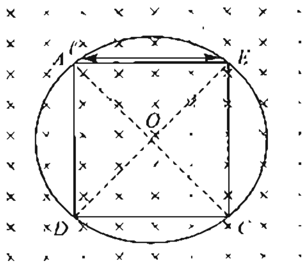


Fig. 8.396

Consider a circular region of radius $l/2$ and centre at O, i.e., centre the square loop.
C on ΔODC

$$\phi_{\Delta ODC} = B \times \frac{l}{2} \times 1 \times \left(\frac{l}{2}\right) = \frac{1}{4}Bl^2 = \frac{1}{2}l^2t^2$$

$$|\varepsilon_{\Delta ODC}| = \frac{d\phi_{\Delta ODC}}{dt} = \ell^2 t$$

The e.m.f. contribution of radial components *OD* and *OC* is zero.

Hence $\varepsilon_{\Delta ODC} = P/ir \cdot P \cdot D \cdot _{DC}$

$$V_{OC} = \int_{C}^{0} \vec{E}_{DC} \cdot d\vec{r}$$

[∴ E_{DC} is count as 11 compound of electric field is constant across *DC*.]

$$2\ell^2 = E_{DC}$$

$$E_{DC} = 2\ell \quad \Rightarrow \quad E_{DC} = 0.2\, N\, C^{-1}$$

9. a. If angular velocity = constant

$$Ei = \frac{B\omega\ell_0^2}{2}$$

$$\tau_B = F_B \times \frac{b}{2}$$

$$F_B = IbB$$

$$\tau_B = IbB \times \frac{b}{2} = IB\frac{b^2}{2}$$

$$\tau_{net} = 0 \quad \Rightarrow \quad mg\cos\theta \times \frac{b}{2} = \frac{IBb^2}{2}$$

$$\Rightarrow \qquad mg\cos\theta = IBb$$

$$\theta = \omega t$$

$$\Rightarrow \qquad I = \frac{mg\cos\omega t}{Bb}$$

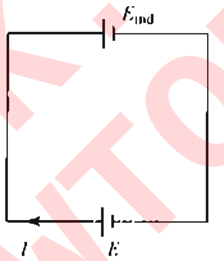


Fig. 8.397

b. $\dfrac{(E - E_{ind})}{R} = I;\ E = E_{ind} + IR$

$$E_{ind} = \frac{B\omega b^2}{2};\ E = \frac{B\omega b^2}{2} + \frac{mg R\cos\omega t}{Bb}$$

10. a. Magnetic flux passing through the position when the loop has rotated by an angle ϕ

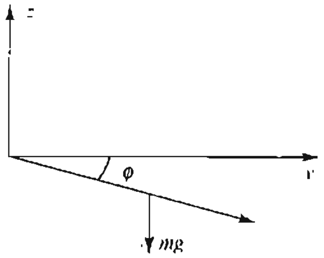


Fig. 8.398

$$\phi_{mag} = BL^2\cos\phi$$

$$|e| = \frac{d\phi}{dt} = BL^2\sin\phi\frac{d\phi}{dt}$$

$$|e| = BL^2\sin\phi\,\omega$$

Current through the loop, $I = \dfrac{|e|}{R} = \dfrac{BL^2\sin\phi\omega}{R}$

$$\tau_{mag} = MB\sin\phi = (IL^2)B\sin\phi = \frac{B^2L^2\omega\sin^2\phi}{R}$$

$$\tau_{gravity} = mg\frac{L}{2}\cos\phi$$

$$\tau_{net} = mg\frac{L}{2}\cos\phi - \frac{\omega B^2L^4\sin^2\varphi}{R}$$

b. $\tau_{net} = I\alpha$

$$\alpha = \frac{\tau_{net}}{I} = \frac{6g}{5L}\cos\phi - \frac{12\omega B^2L^2}{5mR}\sin^2\phi$$

c. Longer, because otherwise induced e.m.f. opposes the motion of the loop.

d. No, because some energy will convert into heat due to flow of current.

11.

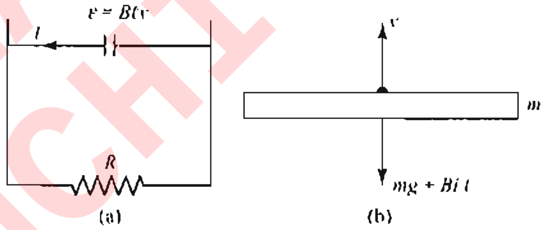


Fig. 8.399

Let v be the speed of the rod at any line. Then the equivalent and free body diagrams of rod are shown in Fig. 8.399.

Applying Newton's second law to the rod,

$$\frac{mdv}{dt} = -(mg + Bi\ell) \qquad \text{(i)}$$

Where $\qquad i = \dfrac{B\ell v}{R} \qquad$ (ii)

From equations (i) and (ii)

$$\frac{mdv}{dt} = -\left(mg + \frac{B^2\ell^2v}{k}\right)$$

Integrating between proper limits we get,

$$\int_{u}^{0} \frac{mdv}{mg + \frac{B^2\ell^2v}{R}} = \int_{0}^{t} -dt$$

$$\Rightarrow \qquad t = \frac{mR}{B^2\ell^2}\ell n\frac{mg + \frac{B^2\ell^2u}{R}}{mg}$$

12. The rate of electrical energy consumed in the bulb = rate of loss of gravitational *PE* of the mass = mgv = 100 W. Hence *M*

$$= \frac{100}{10\times10} = 1\ \text{kg}.$$

13.

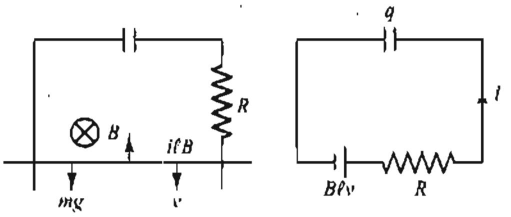


Fig. 8.400

By Newton's law, $mg - ilB$

$$= m\frac{dv}{dt} \quad \text{(i)}$$

By KVL $Blv = iR + \frac{q}{c}$ (ii)

Differentiating (ii) w.r.t. time, we get

$$Bl\frac{dv}{dt} = R\frac{di}{dt} + \frac{i}{c} \quad \text{(iii)}$$

Eliminating $\frac{dv}{dt}$ by (i) and (iii), we get

$$mg - ilB = \frac{m}{B\ell}\left[R\frac{di}{dt} + \frac{i}{c}\right]$$

$$\Rightarrow \quad mg\,B\ell - iB^2\ell^2 = m$$

$$R\frac{di}{dt} + \frac{mi}{c} \quad \text{(iv)}$$

I will be maximum when $\frac{di}{dt} = 0$. Use this in (iv)

$$\Rightarrow \quad mg\,B\ell c = i(B^2\ell^2 c + m)$$

$$\Rightarrow \quad i_{\max} = \frac{mg\,B\ell c}{m + B^2\ell^2 c}$$

14. $\int \vec{E}d\vec{\ell} = -A\frac{dB}{dt}$

As $B = 17 + (0.2)\sin(\omega t + \phi)$;

$$E(2\pi r) = -\pi r^2 (0.2)\omega\cos(\omega t + \phi)$$

$$E = -\frac{r}{2}(0.2)\omega\cos(\omega t + \phi)$$

Magnitude of the amplitude $= \frac{r}{2}(0.2)\,\omega = 240$ mN/C

15.

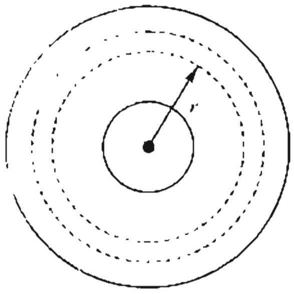


Fig. 8.401

The magnetic field inside is only due to the current of the inner cylinder.

$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic field energy density is not uniform in the space in between, the cylinders. At a distance *r* from the centre,

$$U_B = \frac{B^2}{2\mu_0} = \frac{\mu_0 i^2}{8\pi^2 r^2}$$

Energy in volume of element (length ℓ)

$$dU_B = U_B\,dV = \frac{\mu_0 i^2}{8\pi^2 r^2}(2\pi r\ell)\,dr = \frac{\mu_0 i^2\ell}{4\pi}\frac{dr}{r}$$

$$U_B = \frac{\mu_0 i^2\ell}{4\pi}\int_a^b\frac{dr}{r} = \frac{\mu_0 i^2\ell}{4\pi}\ell n\frac{b}{a}$$

Using values, we get

$$U = 140\,\text{nJ}$$

16. Considering loop *ADFEA*

$$\phi = 1\times 1\times B \rightarrow \phi = B$$

$$|\varepsilon| = \frac{d\phi}{dt} = \frac{dB}{dt} = 1\text{V}$$

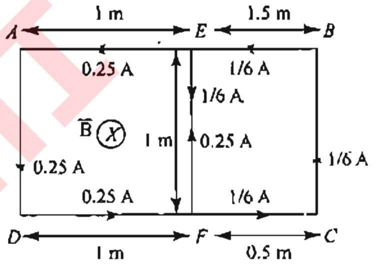


Fig. 8.402

Resistance of loop is,

$$\frac{dB}{dt} = 1\ \text{Ts}^{-1}$$

$$R = 4\mu\times 1$$

$= 4\ \Omega$ resistance per unit length, $\mu = 1\ \Omega\,\text{m}^{-1}$

current is $i_1^0 = 0.25\ \text{A} = \frac{1}{4}$ A in anticlockwise direction.

Considering loop *BEFCB*

$$\phi = 1\times\frac{1}{2}\times B = \frac{1}{2}B$$

$$|\varepsilon| = \frac{d\phi}{dt} = \frac{1}{2}\frac{dB}{dt} = \frac{1}{2}V$$

Resistance of loop is,

$$R = 2\mu\left(1 + \frac{1}{2}\right) = 3r = 3\Omega$$

current in loop is $i_2 = \frac{1}{6}$ A

Applying superposition of loops to get the original loop. Hence, current in *AE* is 0.25 A from *E* to *A*

current in *BE* is $\frac{1}{6}$ A from *B* to *E*

current in *EF* is $\frac{1}{12}$ A from *F* to *E*.

17. a.
$$\phi = \frac{\mu_0}{2\pi} i\, b \ln\left(\frac{\mu+a}{a}\right) - \frac{\mu_0 i}{2\pi} b \ln\left(\frac{\mu+u+d}{u+d}\right)$$

$$\phi = \frac{\mu_0 i}{2\pi} b \ln\left(\frac{(a+d)(b+a)}{a(b+a+d)}\right)$$

$$e = \frac{d\phi}{dt} = \frac{\mu_0 b}{2\pi} \ln\left(\frac{(a+d)(b+a)}{a(b+a+d)}\right)\frac{di}{dt}$$

b. as $\frac{di}{dt} > 0$ the direction of induced current is clockwise

∴ Force on the loop will be away from wires.

18. a. Suppose the magnetic field is into the plane of the page.

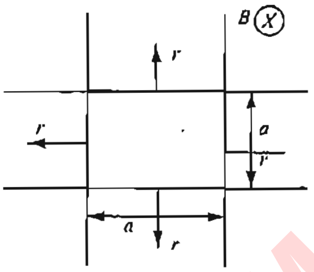


Fig. 8.403

After time t, the length of each side of the square is $l = a + 2\upsilon t$ area of square,

$$l^2 = (a + 2\upsilon t)^2$$
$$= (a^2 + 4\upsilon^2 t^2 + 4a\upsilon t)$$
$$\phi = Bl^2 = Ba^2 + 4B\upsilon^2 t^2 + 4Ba\ \upsilon t$$
$$|\varepsilon| = \frac{d\phi}{dt} = 8B\upsilon^2 t + 4B\ \upsilon a = 4B\upsilon^2(2t + a)$$
$$\varepsilon = 2B\upsilon(a + 2\upsilon t)$$

Hence, e.m.f. is time dependent.

b. The e.m.f. and resistance both are time dependent, but at any time t the resistance of loop is $\mu(a + 2\upsilon t)$, where μ is resistance per unit length of wire

$\Rightarrow \qquad i = \frac{\varepsilon}{R} = \frac{2B\mu}{r}$. current is constant

19. a. $\qquad F_{net} = F - IdB$

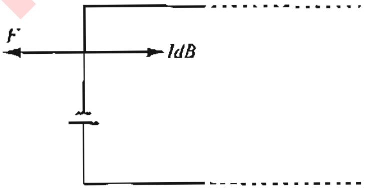


Fig. 8.404

$$m \times a = F - \frac{BdvdB}{r}; \ a = \frac{F - \frac{B^2d^2v}{R}}{m}$$

b. At the stage of terminal velocity

$$\frac{B^2d^2v_T}{R} = F; \ v_T = \frac{FR}{B^2d^2}$$

c. $$\frac{dv}{dt} = \frac{F - \frac{B^2d^2v}{R}}{m} \Rightarrow \int_0^v \frac{dv}{F - \frac{B^2d^2v}{R}} = \int_0^t \frac{dt}{m}$$

$$\frac{\ln\left(\frac{F - \frac{B^2d^2v}{R}}{F}\right)_0^v}{-\frac{B^2d^2}{R}} = \frac{t}{m}$$

$$\ln\frac{F - \frac{B^2d^2v}{R}}{F} = \frac{B^2d^2t}{mR}$$

$$1 - \frac{B^2d^2v}{RF} = e^{-\frac{B^2d^2t}{mR}}$$

$$v = \frac{RF}{B^2d^2}\left(1 - e^{-\frac{B^2d^2t}{mR}}\right)$$

20.

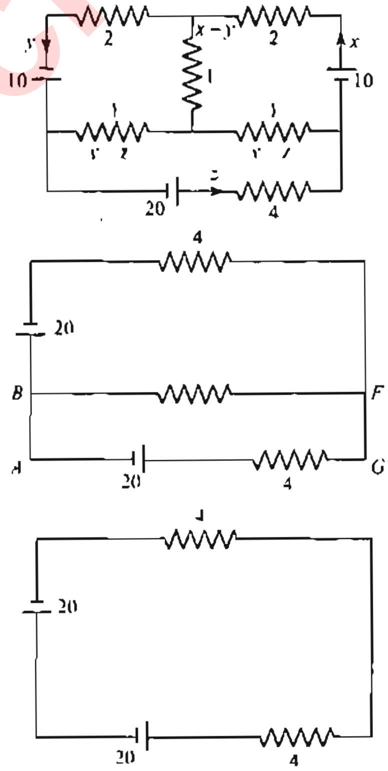


Fig. 8.405

The induced e.m.f. in loop $ABHFG$

$$= \frac{d}{dt}(BA)\ A\frac{dB}{dt} = 2 \times 10 = 20\text{ V}$$

ie induced e.m.f. in loops *BCDH* and *DEFH*

$1 \times 10 = 10$ V.

VL in top left loop,

$$10-(y-z)+(x-y)-2y=0$$

$$x-4y+z=-10 \qquad \text{(i)}$$

'L in right loop,

$$10-2-(x-y)-(x-z)=0$$

$$-4x+y+z=-10 \qquad \text{(ii)}$$

equations (i) and (ii) it is seen that $x-y=0 \Rightarrow$ no current

H (This can also be seen by symmetry)

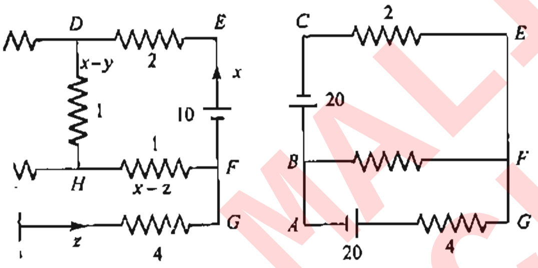
makes solution very simple, now the circuit is

ime $v_B = 0$ and $v_F = v$,

$$\frac{v+20}{4}+\frac{v-20}{4}+\frac{v-0}{2}=0$$

$$v=0 \Rightarrow \text{no current in } FB.$$

ircuit is



te of heat production $= (40)^{2/8} = 200$ W

re not able to observe the symmetry or decide 0, then write KVL in the lower loop. It will be

$x = -20$ (iii)

i), (ii) and (iii) you will get $x = +5, y = 5, z = 5$A. Heat

oe,

$$10x+10y+20z = 40 \times 5 = 200 \text{ W}$$

$$\left.-\frac{d^2q}{dt^2}\right)\Bigg| = L\left|\frac{d}{dt}i_R - \left(C\frac{d^2V}{dt^2}+\frac{2dC}{dt}\frac{dV}{dt}\right)\right|$$

closed, current in inductor remains $i = \dfrac{\varepsilon}{2R}$

$$+\frac{\varepsilon - V_1}{2R} = \frac{\varepsilon}{2R} \qquad \left(V_1 = \frac{2\varepsilon}{3}\right)$$

ial difference $(V) = \varepsilon - \dfrac{2\varepsilon}{3} = \dfrac{\varepsilon}{3}$

$$L\frac{di}{dt} = \frac{2\varepsilon}{3} \qquad \frac{di}{dt} = +\frac{2\varepsilon}{3L}$$

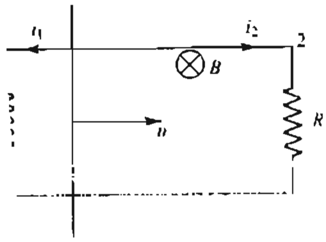


Fig. 8.406

$$I = i_1 + i_2$$

$$\frac{B\ell v}{R} = i_2 \quad \Rightarrow \quad B\ell v = L\frac{di_1}{dt}$$

$$m\frac{dv}{dt} = -i\ell B = -\left(i_1 + \frac{B\ell v}{R}\right)\ell B$$

$$\Rightarrow \quad mdv = -\ell Bi_1\,dt - \frac{B^2\ell^2}{R}x$$

$$\Rightarrow \quad m\int dv = -\ell B\int i_1\,dt - \frac{B^2\ell^2}{R}\int v\,dt$$

$$\Rightarrow \quad m(v_f - u) = -\ell BQ - \frac{B^2\ell^2}{R}x$$

(v_f = velocity, when it has moved a distance x)

$$\Rightarrow \quad Q = \frac{-\dfrac{B^2\ell^2}{R}x - m\left(v_f - u\right)}{B\ell}$$

24. a.

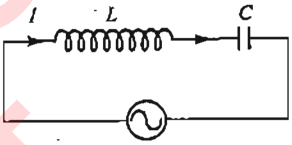


Fig. 8.407

$$I = 20t$$

$$V_{\text{inductor}} = L\frac{dI}{dt} = 1 \times 10^{-3} \times 20 = 2 \times 10^{-2} \text{ V}$$

b. $I = 20t \quad \Rightarrow \quad \int Idt = \dfrac{20t^2}{2}$

but $\quad \theta = \int_0^t Idt \quad \Rightarrow \quad \theta = 10t^2$

$$V_{\text{capacitor}} = \frac{\theta}{C} = \frac{10t^2}{10^{-6}} = 10^7 t^2 \,(v)$$

c. Energy in capacitor $= \dfrac{1}{2} \times C \times V^2$

$$U_C = \frac{1}{2} \times 10^{-6} \times 10^{14} t^4 = \frac{1}{2} \times 10^8 t^4$$

$$U_{\text{inductor}} = \frac{1}{2} \times L \times I^2 = \frac{1}{2} \times 1 \times 10^{-3} \times 400 t^2$$

$$= \frac{1}{2} \times 4 \times t^2$$

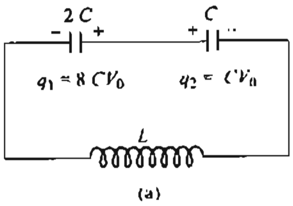
$$U_{\text{cap}} = U_{\text{ind}}$$

$$\Rightarrow \quad = \frac{1}{2} \times 10^8 \times t^2 = \frac{1}{2} \times \frac{4}{10} \times t^2$$

$$t^2 = 4 \times 10^{-3}$$

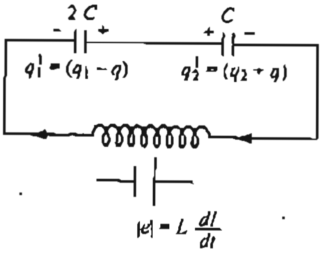
$$t = 2 \times \frac{10^{-4}}{\sqrt{10}} \quad t = 2\sqrt{10} \times 10^{-5} \text{ s}$$

25.



(a)

Fig. 8.408 (*Contd.*)



(b)

Fig. 8.408

Applying loop equation in (1)

$$\frac{q_1^1}{2C}-\frac{q_2^1}{C} = L\frac{dI}{dt} = 0$$

$$L\frac{dI}{dt} = \left(\frac{q_1-2q_2}{2C}\right)-\frac{3q}{2C} \qquad \text{(i)}$$

Differentiating (i), we get

$$L\frac{d^2I}{dt^2} = -\frac{3}{2C} \qquad \text{(ii)}$$

Solution of (ii) is $I_3 = I_0 \sin \omega t$

and $\omega^2 = \frac{3}{2CL}$ and

charge $I = \frac{dq}{dt} = I_0 \sin \omega t$ (iii)

Integrating, we get $q = \frac{I_0}{\omega}[1-\cos \omega t]$ (iv)

from (i), $\frac{3q}{2C} = \left(\frac{q_1-2q_2}{2C}\right)-L\frac{dI}{dt}$

q will be maximum when $L\frac{dI}{dt} = 0$

Hence, $q_{max} = \left(\frac{q_1-2q_2}{3}\right)$ (v)

Hence, from (iv) and (v)

$$\frac{I_0}{\omega} = \left(\frac{q_1-2q_2}{3}\right) \Rightarrow I_0 = \frac{\omega}{3}(q_1-2q_2)$$

$$I_0 = \frac{1}{3}\sqrt{\frac{3}{2CL}}(8CV_0-2CV_0)$$

$$= 2\sqrt{\frac{3}{2CL}}\ CV_0 = \sqrt{6CLV_0}$$

$$I_0 = V\sqrt{6C}$$

$$I = V_0\sqrt{6CL}\sin\sqrt{\frac{3}{2CL}}\,t$$

Potential difference across each capacitor.

$$V_1 = \frac{q_1^1}{2C} = \frac{1}{2C}\left[q_1-\left(\frac{q_1-2q_2}{3}\right)\right]$$

$$V_1 = \frac{(q_1+q_2)}{3C} = 3V_0$$

$$V_2 = \frac{q_2^1}{C} = \frac{1}{C}\left[q_2-\left(\frac{q_1-2q_2}{3}\right)\right]$$

$$V_2 = \frac{(q_1+q_2)}{3C} = 3V_0$$

26.

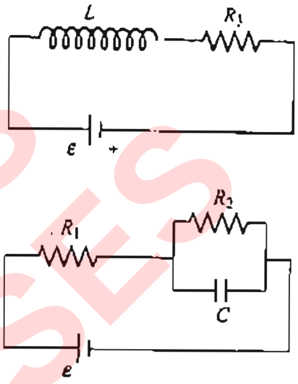


Fig. 8.409

Circuit can be analyzed separately

(i) Inductor

$$\varepsilon = I_L R_3 + L\frac{dI_L}{dt}$$

on integrating, we get

$$I_L = \frac{E}{R_3}(1-e^{\frac{R_2 t}{L}})$$

(ii)

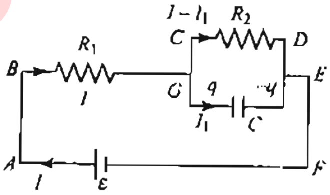


Fig. 8.410

In loop $ABCDEFA$, $\varepsilon = IR_1 + (I-I_1)R_2$,

$$\frac{\varepsilon + I_1 R_2}{R_1+R_2} = I \qquad (1)$$

In loop $ABGEFA$, $\varepsilon = IR_1 + \frac{q}{C}$

$$\varepsilon = \frac{(\varepsilon + I_1 R_2)}{R_1+R_2}R_1 + \frac{q}{C}$$

Differentiating w.r.t. time, we get

$$0 = 0 + \frac{dI_1}{dt}\times\left(\frac{R_1R_2}{R_1+R_2}\right)+\frac{dq}{dt}\times\frac{1}{C}$$

$$\frac{dI_1}{dt} = -\left(\frac{R_1+R_2}{R_1 R_2}\right)\times\frac{1}{C}\times I_1$$

$$\frac{dI_1}{I_1} = \left(\frac{-R_1+R_2}{R_1 R_2}\right)\times\frac{1}{C}\times dt$$

$$[\ln I_1]^{I_1}_{\varepsilon/R_1} = \left(\frac{-R_1+R_2}{R_1 R_2}\right)\times\frac{[t]^t_0}{C}$$

($\therefore$ at $t=0$, C acts as a conducting wire)

$$\ln\left(\frac{I_1}{\varepsilon/R_1}\right) = -\left(\frac{R_1+R_2}{R_1 R_2}\right)\times\frac{t}{C}$$

$$I_{\text{through capacitor}} = \frac{\varepsilon}{R_1}e^{-\left(\frac{R_1+R_2}{R_1R_2}\right)\frac{t}{C}}$$

27. The current through the resistance

$$I(t) = I_{\max}e^{-Rt/L} = \frac{1}{(\ln 2/10)}e^{-\frac{\ln 2\times t}{10\times 2\times 10^{-4}}}$$

$$= \left(\frac{10}{\ln 2}\right)e^{-\frac{\ln 2\times t}{2\times 10^{-3}}}$$

$$I = \left(\frac{10}{\ln 2}\right)\left(\frac{1}{2}\right)^{\left(\frac{t}{2\times 10^{-3}}\right)}$$

The current in the circuit after 4 ms,

$$I = I(4\text{ ms}) = \frac{10}{\ln 2}\times\left(\frac{1}{4}\right) = \frac{2.5}{\ln 2}\text{ A}$$

When the switch is connected in the position AC, the circuit becomes an oscillatory circuit.

The angular frequency, ω, of oscillation,

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\times 10^{-4}\times 5\times 10^{-3}}}$$

$$\omega = 10^3\text{ s}^{-1}$$

The energy stored in the inductor will go into the capacitor at some time during subsequent oscillation, so

$$\frac{1}{2}LI^2 = \frac{q^2_{\max}}{2C} \Rightarrow q_{\max} = \left(\sqrt{LC}\right)I$$

$$q_{\max} = 10^{-3}\times\frac{2.5}{\ln 2} = \frac{250\times 10^{-5}}{\ln 2}\text{ C}$$

$$q_{\max} = 3.6\times 10^{-3}\text{ C}$$

The energy stored in the capacitor become one half of its maximum value whenever the charge will be $q = \pm\frac{q_{\max}}{\sqrt{2}}$, i.e.,

at $t = 4\text{ ms} + \frac{(2n+1)\pi}{4\omega}(q = q_{\max}\sin\omega t)$.

Objective Type

1. **c.** No induced current is set up as the magnetic field line of earth is not cut by the falling conductor.
2. **b.** When the loops are brought nearer, magnetic flux linked with each loop increases. Thus the current will be induced in each loop in a direction opposite to its own current in order to oppose the increase in magnetic flux. This is in accordance with Lenz's law. So, the current will decrease in each loop.
3. **d.** With the increase in the time rate of change of flux, both induced e.m.f. and current increase. But as the induced charge does not depend upon time, it would remain the same.
4. **c.** When the plane of the coil is perpendicular to the magnetic field, then ϕ is maximum and induced e.m.f. is minimum.
5. **a.** Consider the force on an electron in PQ. This electron experiences a force towards Q. Clearly, all free electrons in PQ tend to move towards Q.
6. **c.** The mutual inductance, M, between two coils A and B is defined by the equation. e.m.f. induced in B (or A) by changing current in A (or B) $= M\times$ rate of change of current in A (or B). Hence, the mutual inductance M remains the same whether X or Y is used as the primary.
7. **b.** $e = E - IR$

   Clearly, the graph is a straight line with negative slope.
8. **b.** In the $r-t$ graph, it is clear that from a to b there is no change in radius and hence no change in area and magnetic flux. Same is the situation from c to d.

   Now, $|e| = \frac{d}{dt}(\phi)$

   $$|e| = B\frac{d}{dt}\left(\pi r^2\right)$$

   $$|e| = 2\pi B\frac{d}{dt}$$

   Since $r\propto t$, $\therefore\ \frac{dr}{dt} = \text{constant}$

   $\therefore\quad |e|\propto r$
9. **b.** Magnetic field at the centre of a large coil $E\propto\frac{\mu_0\ell}{2R}$.

   Magnetic flux linked with smaller coil

   $$= \frac{\mu_0\ell}{2R}\times\pi r^2$$

   $$M = M = \frac{\phi}{\ell} = \frac{\mu_0\pi r^2}{2R}$$

   $\therefore\quad M\propto\frac{r^2}{R}$
10. **b.** $Q = \frac{N\Delta\phi}{R} = \frac{\phi_2-\phi_1}{R} = \frac{BA-0}{R} = \frac{BA}{R}$
11. **d.** When inductances are connected like this in series, then

    $$L = L_1 + L_2 + 2M$$
12. **b.** $V_A - IR + E - \frac{dI}{dt} = V_B$
13. **b.** The current is short circuited through induction 1 H instead of passing through 20 Ω.

    Then current through 30 Ω is:

    $$I = \frac{3}{30} = 0.1\text{ A}$$

14. d. $V_A - V_B = V_L + V_R = \frac{Ldl}{dt} + IR$

$$= L \times 8 = 5.6 \times 0.2 = 0.5\text{ V}$$
$$8L = 0.5 - 0.1 = 0.4;\ L = 0.5\text{ H}$$

15. c. By Lenz's law clockwise current is induced in both loops. Greater the area, large will be the induced e.m.f. Therefore, outer loop has greater area.

16. c. $\overline{F}_m \perp \overline{v}$

$\therefore \quad \theta = 90° + 30° = 120°$

17. d. $\left|\overline{E}\right|$ = Magnitude of induced e.m.f.

$$= \frac{B\ell^2}{2}\omega \qquad \text{(i)}$$

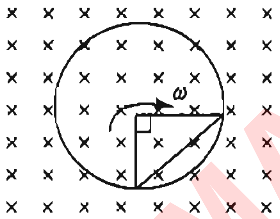


Fig. 8.411

$$\theta = \frac{\pi}{2} = \frac{\ell}{r}$$

$$\therefore \quad \left|\overline{E}\right| = \ell = \frac{\pi}{2}r \qquad \text{(ii)}$$

Using (ii) in (i), we get

$$\therefore \quad \left|\overline{E}\right| = B\frac{\left(\frac{\pi}{2}r\right)^2\omega}{2} = \frac{\pi^2 r^2 B\omega}{8}$$

18. b. $P = Fv = BI\ell_v = 1.25 \times 10^{-3} \times 50 \times 0.1 \times 1\text{ W}$

$$= 6.25 \times 10^{-3}\text{ W} = 6.25\text{ mW}$$

An iterating alternating

$$P = EI = (Blv)\,I$$

19. a. The two loops produce equal and opposite magnetic fields.

20. c. $BI\ell = mg$ or $B\frac{Bv\ell}{R}\ell = mg$ or $v = \frac{mg\ R}{B^2\ell^2}$

21. c. Magnetic field due to larger loop

$$= \frac{\mu_0 1}{2r} = \frac{4\pi \times 10^{-7} \times 1}{2 \times 0.1}\text{ T} = 2\pi \times 10^{-6}\text{ T}$$

Now, magnetic flux linked with the smaller loop, ϕ

$$= NBA\cos\omega t = 1 \times 2\pi \times 10^{-6} \times 5 \times 10^{-4}\cos\omega t$$
$$= \pi \times 10^{-9}\cos\omega t\text{ weber}$$

22. d. Induced current in $B = 0.006\text{ A} = 6 \times 10^{-3}\text{ A}$

Induced e.m.f. in $B = 6 \times 10^{-3} \times 4\text{ V} = 24 \times 10^{-3}\text{ V}$

Now, $M\frac{d\ell}{dt} = 24 \times 10^{-3}$

or $\quad d\ell = \frac{24 \times 10^{-3} \times 0.02}{3 \times 10^{-3}}\text{ A} = 0.16\text{ A}$

23. a. $\quad I = \frac{B\ell v}{R}$

$$I = \frac{5 \times 10^{-2} \times 0.3 \times 0.2}{5}\text{ A} = 0.6\text{ mA}.$$

Area and flux are decreasing. So, current flows to increase the flux. Clearly, current should be clockwise. So, it flows from B to C through 5 Ω.

24. b. Effective resistance is 4 Ω.

$$I = \frac{E}{R} = \frac{B\ell v}{R} \quad \text{or} \quad v = \frac{\ell R}{B\ell}$$

or $\quad v = \frac{1 \times 10^3 \times 4}{2 \times 10 \times 10^{-2}}\text{ ms}^{-1}$

or $\quad v = 0.02\text{ ms}^{-1} = 2\text{ cms}^{-1}.$

25. b. $\phi_2 = N_2 B_1 A$ or $\phi_2 = N_2\frac{\mu_0 N_1 I_1}{L}A$

or $\quad = \phi_2 = \frac{\mu_0 N_1 N_2 A}{L}\ell_1$

Comparing with $\phi_2 = MI_1$, we get $M_2 = \frac{\mu_0 N_1 N_1 A}{L}$.

26. a. Component of weight along the inclined place = $mg\sin\theta$

Again, $\quad F = B\ell\ell = B\frac{B\ell v}{R}\ell = \frac{B^2\ell^2 v}{R}$

Now, $\quad \frac{B^2\ell^2 v}{R} = mg\sin\theta$ or $v = \frac{mg\,R\sin\theta}{B^2\ell^2}$

27. b. $\quad I_t = \infty = I_0$

$$I_{t=1s} = I_0\left(1 - e^{-\frac{10}{5}\times 1}\right)$$

$$= I_0(1 - e^{-2}) = I_0\left(1 - \frac{1}{e^2}\right)$$

$$\therefore \quad = \frac{I_{t=\infty}}{I_{t=1s}} = \frac{I_0 e^2}{I_0(e^2 - 1)} = \frac{e^2}{e^2 - 1}$$

28. a. The mutual inductance, M. between solenoids P and Q is given by e.m.f. induced in Q due to the changing current in $P = M \times$ (rate of change of current I_Q in Q)

⇒ induced e.m.f. in $P = (4.0 \times 10^{-4}) \times (2)$

$$= 8.0 \times 10^{-4}\text{ V}$$

29. a. $\quad V_1 = L_1\frac{dI_1}{dt}$ and $V_2 = L_2\frac{dI_2}{dt}$

But $\quad \frac{dI_1}{dt} = \frac{dI_2}{dt}$ (given)

$\therefore \quad V_1 = L_1$ and $V_2 = I_2$ or $\frac{V_1}{V_2} = \frac{L_2}{L_3} = \frac{8}{2} = \frac{4}{1}$

Again, same power is given to the two coils.

$$\therefore \quad V_1I_1 = V_2I_2 \text{ or } \frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{1}{4}$$

Again, energy $= \frac{1}{2}LI^2$

$$\therefore \quad \frac{W_2}{W_1} = \frac{\frac{1}{2}L_2{I_2}^2}{\frac{1}{2}L_1{I_1}^2} = \left(\frac{L_2}{L_1}\right)^2 = \frac{2}{8}(4)^2 = \frac{4}{1}$$

30. b. $V_1 = L_1\frac{dI_1}{dt}$ and $V_2 = L_2\frac{dI_2}{dt}$

But $\frac{dI_1}{dt} = \frac{dI_2}{dt}$ (given)

$$\therefore \quad V_1 = L_1 \text{ and } V_2 = I_2 \text{ or } \frac{V_1}{V_2} = \frac{L_2}{L_3} = \frac{8}{2} = \frac{4}{1}$$

Again, same power is given to the two coils.

$$\therefore \quad V_1I_1 = V_2I_2 \text{ or } \frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{1}{4}$$

Again, energy $= \frac{1}{2}LI^2$

$$\therefore \quad \frac{W_2}{W_1} = \frac{\frac{1}{2}L_2I_2^2}{\frac{1}{2}L_1I_1^2} = \left(\frac{L_2}{L_1}\right)^2 = \frac{2}{8}(4)^2 = \frac{4}{1}$$

31. a. $V_1 = L_1\frac{dI_1}{dt}$ and $V_2 = L_2\frac{dI_2}{dt}$

But $\frac{dI_1}{dt} = \frac{dI_2}{dt}$ (given)

$$\therefore \quad V_1 = L_1 \text{ and } V_2 = I_2 \text{ or } \frac{V_1}{V_2} = \frac{L_2}{L_3} = \frac{8}{2} = \frac{4}{1}$$

Again, same power is given to the two coils.

$$\therefore \quad V_1I_1 = V_2I_2 \text{ or } \frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{1}{4}$$

Again, energy $= \frac{1}{2}LI^2$

$$\therefore \quad \frac{W_2}{W_1} = \frac{\frac{1}{2}L_2I_2^2}{\frac{1}{2}L_1I_1^2} = \left(\frac{L_2}{L_1}\right)^2 = \frac{2}{8}(4)^2 = \frac{4}{1}$$

32. d. $|E| = B_H\ell v$

$$= 2\times10^{-5}\times2\times50 \text{ V}$$
$$= 2\times10^{-3} \text{ V}$$
$$= 2\text{mV}$$

Using Fleming's left hand rule, we find that electrons shall experience force towards A. B will be deficient of electrons. So, B will be positively charged.

33. d. When the switch is closed (at t = 0s), no current flows ,voltage drop across the inductor is the same as the supply voltage of 15 V. Hence, by writing down the voltage equation for the circuit, we have

$$V = RI + L\frac{dI}{dt}$$

$$\Rightarrow \quad 1.6 = 4I + 0.20\frac{dI}{dt}$$

where I is the current drawn from the source

At $t = 0$ s, $I = 0$, we thus have

$$16 = 0.20\frac{dI}{dt} \text{ or } \frac{dI}{dt}(t = 0^+) = \frac{1.6}{0.2} = 0.8 \text{ As}^{-1}$$

34. b. Magnetic field produced due to current In large loop,

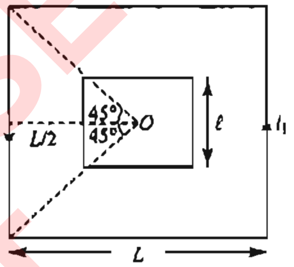


Fig. 8.412

$$B_1 = \frac{\mu_0}{4\pi}\frac{I_1}{L/2}[\sin 45° + \sin 45°]$$

$$= \frac{\mu_0 I_1}{\sqrt{2}\pi L}$$

Magnetic flux linked with smaller loop

$$\phi_1 = B_1A_2$$

$$= \frac{\mu_0 I_1 \ell^2}{\sqrt{2}\,\pi L}$$

$$M = \frac{\phi_2}{I_1} = \frac{\mu_0}{\sqrt{2}}\frac{\ell^2}{\pi L}\alpha\frac{\ell^2}{L}$$

35. c. $L = \frac{\mu_0 N^2 A}{\ell}$

If x is length of solenoid with r as radius, then

$$x = 2\pi r N, A = \pi r^2$$

$$\therefore \quad L = \mu_0\left(\frac{x^2}{4\pi^2 r^2}\right)\frac{\pi r^2}{\ell} \qquad \left[\therefore N = \frac{x}{2\pi r}\right]$$

$$\therefore \quad x = \sqrt{\frac{4\pi L\ell}{\mu_0}}$$

36. b. Let there be element dx of rod at distance x from the wire e.m.f. developed in the element, $dE = B\,dx\,v$

$$\therefore \quad dE = \left(\frac{\mu_0}{4\pi}\frac{2\pi}{x}\right)dxv$$

$$\therefore \quad E = \frac{\mu_0 I v}{2\pi}\int_a^b \frac{dx}{x} = \frac{\mu_0 Iv}{2\pi}\log_e\frac{b}{a}$$

$$\therefore \quad E=\frac{4\pi\times10^{-7}\times100\times5}{2\pi}\log_e\frac{100}{1}$$
$$=4.6\times10^{-4}\,\text{V}=0.46\,\text{mV}$$

37. c. $$I=\frac{E-L\frac{dI}{dt}}{R},\ \therefore RI=e-\frac{dI}{dt}L$$

(taking integration on both sides w.r.t. 't')

$$\therefore \int RI\,dt=\int e\,dt-\int L\frac{dI}{dt}dt$$
$$\therefore \quad Rq=\int-\frac{d\phi}{dt}dt-L[I]_i^f=\phi_i-L[I_f-I_i]$$
$$=\phi_i-\phi_f$$
$$\therefore \quad q=\frac{\phi_i-\phi_f}{R} \qquad \text{(i)}$$

But $$\phi_i=\frac{\mu_0Ia}{2\pi}\int_a^{a+b}\frac{dx}{x}=\frac{\mu_0Ia}{2\pi}\ln\frac{a+b}{b}$$
$$\phi_f=\frac{\mu_0Ia}{2\pi}\ln\frac{2a+b}{a+b}$$
$$\therefore \quad \phi_i-\phi_f=\frac{\mu_0Ia}{2\pi}\ln\frac{2a+b}{b}$$

Then $$q=\frac{\mu_0Ia}{2\pi R}\ln\frac{2a+b}{b}$$

38. a. In a superconductor, $R=0$

The induced current is $I=\dfrac{E}{R}$

$$\therefore \quad E=IR$$

since $R=0$, $E=0$, but $E=\dfrac{-d\phi}{dt}$

$$\therefore \quad \Delta\phi=0$$

i.e., $\phi_i=\phi_f$ then $\phi_f=0$ because $\phi_i=0$

Now $\phi_f=\pi a^2B-\phi_{\text{inclined}}=\pi a^2B-LI$

i.e., $\pi a^2B-LI=0$

$$\therefore \quad I=\frac{\pi a^2B}{L}$$

Work done = energy stored in coil

$$=\frac{1}{2}LI^2=\frac{\pi^2a^4B^2}{2L}$$

39. c. Induced e.m.f. $=B\ell v$. R is internal resistance of seat of e.m.f., i.e., of rod

Total resistance of circuit $=R+\dfrac{R_1R_2}{R_1+R_2}$

$$\therefore \quad I=\frac{B\ell v}{R+\dfrac{R_1R_2}{(R_1+R_2)}}$$
$$=\frac{B\ell v(R_1+R_2)}{R_1R_2+R(R_1+R_2)}$$

40. c. $BI\ell\cos\theta=mg\sin\theta$

Here induced e.m.f. across slider is $BI\ell\cos\theta$

$$\therefore \quad \text{Induced current } I=\frac{B\ell v\cos\theta}{R}$$

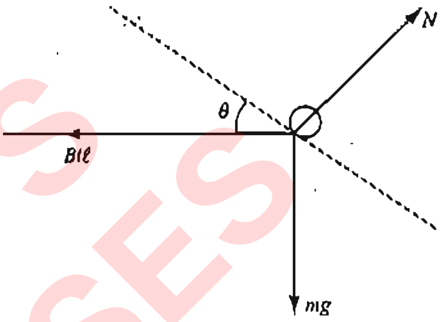


Fig. 8.413

From equation (i)

$$B\ell\cos\theta\,\frac{B\ell v\cos\theta}{R}=mg\sin\theta$$
$$\therefore \quad v=\frac{mgR\sin\theta}{B^2\ell^2\cos^2\theta}$$

41. b. ϕ(flux linked) $=a^2B\cos0°=b^2B\cos180°$

$$=(a^2-b^2)B$$
$$E=-\frac{d\phi}{dt}=-(a^2-b^2)\frac{dB}{dt}$$
$$=(a^2-b^2)B_0\,\omega\cos\omega t$$

where $B=B_0\sin\omega t$, $B_0=10^{-3}$ T, $\omega=100$

$$\therefore \quad I_{\max}=(a^2-b^2)\frac{B_0\omega}{R}$$

and $$R=(4a+4b)r=4(a+b)r$$

$$\therefore \quad I_{\max}=\frac{(a-b)B_0\omega}{4r}$$
$$=\frac{(1-0.4)\times10^{-3}\times100}{4\times5\times10^{-3}}=3\,\text{A}$$

42. a. Let q and v be the instantaneous charge and velocity, respectively.

$q=EC$ (E is induced e.m.f.).

$$\therefore \quad q=CV$$
$$\therefore \quad q=B\ell vC \qquad (\because e=B\ell v)$$
$$\therefore \quad I=\frac{dq}{dt}=B\ell C\frac{dv}{dt}=B\ell Ca$$

Now, $F-BI\ell=ma$

$$F-BI(B\ell Ca)=ma$$

or $$a=\frac{F}{m+B^2\ell^2C}=5\,\text{ms}^{-2}$$

43. c. E, the voltage across capacitor = induced e.m.f. across the slider = $B\ell v$

As capacitor does not permit flow of current, there is magnetic force on the rod to prevent its fall.

$$v^2 = 2gx, v = \sqrt{2gx}$$

$$\therefore \quad E = B\ell\sqrt{2gx}$$

44. a. For a solenoid, $L = \mu_0 N^2 \frac{A}{\ell}$. If x is length of wire and a is area of cross-section, then

$$R = \frac{\rho x}{a} \text{ and } m = axD$$

$$Rm = \frac{\rho x}{a} axD, \quad \therefore x = \sqrt{\frac{Rm}{\rho D}}$$

Also, $x = 2\pi rN$, $N = \frac{x}{2\pi r}$ $\quad \left(\therefore L = \frac{\mu_0 N^2 A}{\ell}\right)$

$$\therefore \quad L = \mu_0 \left(\frac{x}{2\pi r}\right)^2 \frac{\pi r^2}{\ell} = \frac{\mu_0}{4\pi\ell}\frac{Rm}{\rho D}$$

45. d

$$L = \frac{\phi}{I}, \phi = NAB$$

$$B = \mu_0 nI$$

where $n = \frac{R}{2\pi R}$

$$\therefore \quad \phi = N\pi r^2\left(\mu_0 \frac{N}{2\pi R} I\right)$$

$$\phi = \frac{\mu_0 N^2 r^2 I}{2R}$$

$$L = \frac{\phi}{I} = \frac{\mu_0 N^2 r^2 I}{2R \times 1} = \frac{\mu_0 N^2 r^2}{2R}$$

46. a. Here $A = 10 \times 5 = 50 \text{ cm}^2$

$$= 50 \times 10^{-4} \text{ m}^2$$

$$\frac{dB}{dt} = 0.2 \text{ T s}^{-1}$$

$$R = 2\,\Omega$$

$$E = \frac{d\phi}{dt} = A\frac{dB}{dt} = 50 \times 10^{-4} \times 0.02$$

$$= 10^{-4} \text{ V}$$

Power dissipated in the form of heat

$$= \frac{E^2}{R} = \frac{10^{-4} \times 10^{-4}}{2}$$

$$= 0.5 \times 10^{-8} \text{ W}$$

$$= 5 \times 10^{-9} \text{ W} = 5 \text{ nW}$$

47. c. $M = \frac{\mu_0 n_1 n_2 A}{\ell} a$, here $\ell = 1$ m

$$= 503 \times 10^{-6} \text{ H}$$

48. a. Let I = current in one loop. This magnetic flux at the centre of the other co-axial loop at a distance ℓ from the centre of the first loop is

$$B = \frac{\mu_0}{4\pi}\frac{2I\pi a^2}{(a^2+\ell^2)^{3/2}}$$

where $p_m = I\pi a^2$

= magnetic moment of loop.

The flux through the other loop is

$$\phi_{12} = B\pi a^2$$

$$= \frac{\mu_0}{4\pi}\frac{2\pi a^2 I}{(a^2+\ell^2)^{3/2}}\pi a^2$$

$$\text{or} \quad \frac{\phi_{12}}{I} = \frac{\mu_0}{4\pi}\frac{2\pi^2 a^4}{2(a^2+\ell^2)^{3/2}}$$

$$= \frac{\mu_0 \pi a^4}{2(a^2+\ell^2)^{3/2}}$$

$$\therefore \quad \phi_{12} = M_{12} I$$

$$\therefore \quad M_{12} = \frac{\phi_{12}}{I} = \frac{1}{2}\frac{\mu_0 \pi a^4}{\ell^3} \quad \therefore a << 1$$

49. b. Magnetic induction at centre of solenoid is

$$B = \frac{\mu_0 N}{\ell_0} I$$

Then, flux linked with whole solenoid is

$$\phi = NBA = \frac{\mu_0 N^2 \pi r^2 I}{\ell_0} \quad \text{(i)}$$

$$\text{Also,} \quad \phi = LI \quad \text{(ii)}$$

$$\therefore \quad L = \frac{\mu_0 N^2 \pi r^2}{\ell_0}$$

$$\text{or} \quad L = \mu_0 \frac{N^2 4\pi\pi r^2}{4\pi\ell_0}$$

$$= \mu_0 \frac{N^2 4\pi^2 r^2}{4\pi\ell_0}$$

$$= \mu_0 \frac{(N2\pi r)^2}{4\pi\ell_0}$$

Now, $N 2\pi r = \ell$ = length of wire used in solenoid

$$\therefore \quad L = \frac{\mu_0 \ell^2}{4\pi\ell_0}$$

$$\text{then} \quad \ell = \sqrt{\frac{4\pi\ell_0 L}{\mu_0}}$$

then $\ell = 0.10$ km

50. c. The resistance of the windings is

$$R = \rho\frac{\ell_0}{A}, \rho \text{ is specific resistance.}$$

or $$R = \rho\frac{\ell_0^2}{A\ell_0}$$

∴ volume (V). $A\ell_0 = \frac{m}{\rho_0}$

$$R = \rho\frac{\ell_0^2}{V},\ \rho_0 = \text{density of wire} = \rho\frac{\ell_0^2}{m/\rho_0}$$

$$\therefore \quad \ell_0^2 = \frac{Rm}{\rho\rho_0} \qquad \text{(i)}$$

As in solution of question 45,

$$L = \frac{\mu_0\ell_0^2}{4\pi\ell} \qquad \text{(ii)}$$

∴ ℓ_0 is equal to coil and ℓ is length of solenoid

$$\therefore \quad L = \frac{\mu_0}{4\pi\ell}\frac{Rm}{\rho\rho_0} = \frac{\mu_0}{4\pi}\left(\frac{mR}{\ell\rho\rho_0}\right)$$

51. d. Magnitude of the circular magnetic field produced is

$$-\int\vec{E}\,d\vec{l} = -\frac{d\phi}{dt}$$

$$E\,2\pi r = \pi r^2\frac{dB}{dt}$$

$$E = \frac{1}{2}r\frac{dB}{dt} \qquad \text{(i)}$$

Now $\quad B = \mu_0 nI$

$$\therefore \quad E = \frac{1}{2}r\frac{d}{dt}(\mu_0 nI)$$

$$E = \frac{1}{2}r\mu_0 nI'.\left(I' = \frac{dI}{dt}\right)$$

where $r < a$

But when $r > a$

$$-\int\vec{E}\,d\vec{l} = -\frac{d\varphi}{dt}$$

$$B\,2\pi r = \pi a^2\frac{dB}{dt}$$

$$E = \frac{1}{2}\frac{a^2}{r}\frac{d}{dt}(\mu_0 nI)$$

or $$E = \frac{1}{2}\frac{\mu_0 na^2}{r}I$$

52. a. Magnetic field at the centre of solenoid

$$B = \mu_0\frac{NI}{\ell},\ \text{here } N = 1000$$

∴ Magnetic flux linked with solenoid is

$$\varphi = NBA = \frac{\mu_0 N^2 A}{\ell}I$$

Also, $\quad \varphi = LI$

$$\therefore \quad L = \frac{\mu N^2 A}{\ell}$$

Because the core consists of two media, self-induction is

$$L = L_1 + L_2$$

$$= \frac{\mu_1 N^2 A_1}{\ell} + \frac{\mu_2 N^2 A_2}{\ell}$$

For air, $\mu_1 = \mu_0$ and for medium. $\mu_2 = \mu_r\mu_0$, where $\mu_r = 500$

$$\therefore \quad L = \frac{\mu_0 N^2}{\ell}|A_1 + \mu_r A_2| = 0.57\ \text{H}$$

53. d. Given that $\phi = at(T - t)$

Induced e.m.f.. $E = \frac{d\phi}{dt} = \frac{d}{dt}\{at(T-t)\}$

$$= at(0-1) + a(T-t)$$

$$= a(T-2t)$$

So. induced e.m.f. is also a function of time.

∴ Heat generated in time T is

$$H = \int_0^T\frac{E^2}{R}dt = \frac{a^2}{R}\int_0^T(T-at)^2\,dt$$

$$= \frac{a^2}{R}\int_0^T(T^2+4t^2-4tT)\,dt = \frac{a^2T^3}{3R}$$

54. d. Heat $L = 0.2$ H, $I = 2\sin t^2$

Induced e.m.f.,

$$E = -L\frac{dI}{dt} = -2\frac{d}{dt}(2\sin t^2)$$

$$= -4\cos t^2\,2t$$

$$= 8t\cos t^2 \text{ (numerically)}$$

Work done for increasing charge dq

$$\text{dW} = Edq = 8t\cos t^2(I\,dt)$$

$$= 8t\cos t^2\,2\sin t^2\,dt$$

$$= 8t(2\sin t^2\cos t^2)\,dt$$

i.e., $\quad dW = 8t\sin 2t^2\,dt$

∴ Total work done.

$$W = \int_0^t 8t\sin 2t^2\,dt \qquad \text{(i)}$$

When $\quad I = 0, t = 0$

$$I = 2\text{A},\ \sin t^2 = 1 = \sin\frac{\pi}{2} \qquad \therefore\ t^2\ \frac{\pi}{2}$$

To solve integral in equation (i). put $2t^2 = y$

$$4t\,dt = dy$$

$$\therefore \quad \text{W} = 2\int_0^\pi\sin y\,dy = 2|1-\cos y|_0^\pi$$

$$= -2|\cos\pi - \cos 0| = 4\ \text{J}$$

$$W = 4\ \text{J}$$

55. c. Here $I = \frac{q}{T}$ for ring to be taken as single loop

$\therefore \quad I = Nq$ (N is frequency)

$\therefore$ magnetic field produced at the centre of the ring is

$$B = \frac{\mu_0 I}{2r} = \frac{\mu_0 Nq}{2r}$$

$$\therefore \quad q = \frac{2rB}{\mu_0 B} = 2.5 \times 10^{-5}\ \text{C}$$

56. d.

$$\phi_2 = \left(\frac{r_1}{r_2}\right)^2 \phi_1 = \left(\frac{80}{40}\right)^2 \times 4 \times 10^{-5}\ \text{Wb}$$

$$= 32 \times 10^{-5}\ \text{Wb}$$

$$\therefore \quad E = \frac{-32 \times 10^{-5}\ \text{Wb}}{0.08} = -4\ \text{mV}$$

57. a. The current in L for steady state $= \frac{E}{R_1}$

$\therefore$ Energy stored in $L\,(E_1) = \frac{1}{2} L I_0^2 = \frac{1}{2} L \frac{E^2}{R_1^2}$

58. a. For $r \le R$

$|\vec{E}|$ = Magnitude of induced e.m.f. $= \frac{d\phi_B}{dt}$

$$\therefore \quad |\vec{E}| = \oint \vec{E} d\vec{\ell} = \frac{d\phi_B}{dt} \qquad (\because \quad \phi_B = \vec{B} \cdot \vec{A})$$

$$\therefore \quad E(2\pi r) = A \frac{dB}{dt} = \pi r^2 \frac{dB}{dt}$$

$\therefore \quad E \propto r \quad \Rightarrow$ shows straight line

For $\quad r \ge R$

$$E(2\pi r') = \pi R^2 \frac{dB}{dt}$$

$$\therefore \quad E \propto \frac{1}{r'}$$

$\Rightarrow$ shows rectangular hyperbola.

59. b. B_1 = Magnetic field due to outer loop of radius R at

$$O = \frac{\mu_0 I}{2R}$$

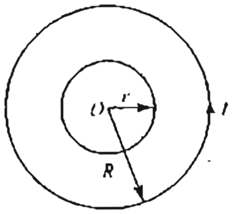


Fig. 8.414

$\therefore$ Magnetic flux linked with small loop of radius

$$r(\phi_B) = B_1 \times \pi r^2$$

$$= \frac{\mu_0 I}{2R}(\pi r^2)$$

$\therefore$ Magnetic flux linked with smaller loop $= \frac{\mu_0 I \pi r^2}{2R}$

$$\therefore \quad \phi_B = MI$$

$$\therefore \quad M = \frac{\mu_0 \pi r^2}{2R}$$

$$\therefore \quad M \propto \frac{r^2}{R}$$

60. a. The magnetic flux through single turn of the coil

$$\phi_B = \vec{B} \cdot \vec{A} = BA \cos\theta$$

The total flux through N turns of the coil.

$$\phi_{\text{total}} = NBA \cos\theta$$

According to Faraday's law of electromagnetic induction,

$$E_{\text{induced}} = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA \cos\theta)$$

$$= -(NA \cos\theta)\frac{dB}{dt}$$

Note that only the magnitude of magnetic field is changing.

$$E_{\text{induced}} = -(200)(100)(\cos 30°)(1.5)$$

The current induced in the coil,

$$I_{\text{induced}} = \frac{E_{\text{induced}}}{R} = \frac{2.59}{0.20} = 12.95\ \text{A}$$

Since the magnetic flux through the coil is increasing with time, the magnetic field due to induced current will be opposite to the primary field. Apply right hand thumb rule, point your thumb in the direction of induced field, your fingers will curl in the directional sense of induced current.

61. d. When the flip coil is rotated, the magnetic flux through it changes, resulting in an induced e.m.f. E and induced current $I = E/R$, where R is the total resistance of the circuit.

$$\text{As} \qquad I = \frac{dQ}{dt}$$

$$Q = \int dQ = \int I\, dt$$

From Faraday's law, induced e.m.f.

$$|E| \approx \frac{d\phi_B}{dt}$$

$$Q = \int I\, dt = \int \frac{E}{R} dt = \frac{1}{R} \int \frac{d\phi_B}{dt} dt$$

$$= \frac{1}{R} \int d\phi_B = \frac{\Delta\phi_B}{R}$$

This is a generalized result and can be used to determine charge through a circuit in which an e.m.f. and a current is induced.

Initial flux through the coil, $\phi_{B_i} = +NBA$

Final flux through the coil, $\phi_{B_f} = -NBA$

When the coil is turned through 180° its flux reverses; the angle between magnetic field and area vector is reversed.

$$\Delta\phi_B = \phi_{Bf} - \phi_{Bi} = NBA - (-NBA) = 2NBA$$

$$Q = \frac{2NBA}{R}$$

62. d. Volume of the balloon at any instant, when radius is r,

$$V = \frac{4}{3}\pi r^3$$

Time rate of change of volume,

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Time rate of change of radius of balloon,

$$\frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$$

Flux through rubber band at the given instant,

$$\phi = B(\pi r^2)$$

$$\text{Induced e.m.f.} = -\frac{d\phi}{dt} = -\frac{d}{dt}(B\pi r^2) = -2\pi r B\frac{dr}{dt}$$

$$= -2\pi r B\left(\frac{1}{4\pi r^2}\frac{dV}{dt}\right) = -\frac{B}{2r}\frac{dV}{dt}$$

As volume of the balloon is decreasing, $\frac{dV}{dt}$ is negative.

$$E_{\text{induced}} = -\frac{(0.4)}{2\times 10\times 10^{-2}}\times(-100\times 10^{-6})$$

$$= 20\mu\text{V}$$

63. b. When the copper rod is rotated, flux linked with the circuit varies with time.

Therefore, an e.m.f. is induced in the circuit.

At time t, plane of semi-circle makes angle ωt with the plane of rectangular part of the circuit. Hence, component of the magnetic induction normal to plane of semi-circle is equal to $B\cos\omega t$.

Flux linked with semi-circular part is

$$\phi_1 = \frac{1}{2}\pi a^2 B\cos\omega t$$

Let area of rectangular part of the circuit be A.

∴ Flux linked with this part is

$$\phi_2 = BA$$

∴ Total flux linked with the circuit is

$$\phi = \frac{1}{2}\pi a^2 B\cos(\omega t) + BA$$

∴ Induced e.m.f. in the circuit,

$$e = -\frac{d\phi}{dt} = \frac{1}{2}\pi\omega a^2 B\sin(\omega t)$$

Since resistance of the circuit is negligible, therefore potential difference across the capacitor is equal to induced e.m.f. in the circuit.

∴ Charge on the capacitor at time t is $q = Ce$

$$= \frac{1}{2}\pi\omega a^2\, CB\sin(\omega t)$$

But current $I = \frac{dq}{dt} = \frac{1}{2}\pi\omega^2 a^2 CB\cos(\omega t)$

Due to flow of current, semi-circle experiences a momentum. Therefore, power is required to keep the semi-circle rotating with constant angular velocity. In fact, power required to rotate the semi-circle is equal to electrical power generated in the circuit.

∴ Power required,

$$P = eI = \frac{1}{8}\pi^2\omega^3 a^4 CB^2\sin(2\omega t)$$

64. a. Consider a point on the circumference of a circle of radius r ($r < R$). Let E be the electric field along the tangents to the circle. Then E (electromotive force)

$$= \oint \bar{E}\cdot d\bar{l} = E\cdot 2\pi r$$

$$E = -\frac{d\phi_B}{dt} = -\frac{d}{dt}(\pi r^2 B) = -\pi r^2\frac{dB}{dt}$$

$$E\times 2\pi r = -\pi r^2\frac{dB}{dt} \Rightarrow E = -\frac{1}{2}r\frac{dB}{dt}$$

The minus sign suggests that the induced electric field acts to oppose the change in the magnetic field.

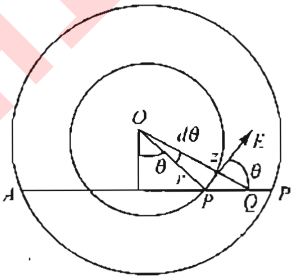


Fig. 8.415

Now consider a point P in the rod and a small distance $dl = PQ$ along AB. The electric field at P is $E = 1/2\ r\ dB/dt$ along the tangent to the circle as shown in Fig. 8.417.

Elementary work done

$$= (E\cos\theta)\,dl = E\,dl\cos\theta$$

Draw perpendicular PN from P on OQ.

Then $PN = r\,d\theta = d\ell\cos\theta$

Elementary work done

$$= Er\,d\theta = \left(\frac{1}{2}r\frac{dB}{dt}\right)r\,d\theta = \left(\frac{1}{2}r^2 d\theta\right)\frac{dB}{dt}$$

But $\frac{1}{2}r^2 d\theta$ = area of the triangle OPQ

Total work done in taking unit charge from A to B

$= \frac{dB}{dt}\times$ summation of the areas of elementary triangle

$$= \frac{dB}{dt}\times \text{area } OAB$$

$= P.D.$ between the ends of AB.

$$V = \frac{dB}{dt}\times\frac{1}{2}\times 2\times\ell\sqrt{R^2-\ell^2} = \frac{dB}{dt}\ell\sqrt{R^2-\ell^2}$$

65. a. In accordance with Faraday's law of electromagnetic induction, the changing magnetic field induces an electric field in the ring. Let us imagine the ring to be divided into differential elements of length ds and denote the tangential component of the induced electric field by E_t. The charge on element ds of the ring is $dQ = Q\frac{ds}{2\pi r}$,

where r is the radius of the ring. The force exerted on it is $dF_t = dQE_t$, and the resultant torque is $d\tau = r\,dF_t$.

Thus the total torque experienced by the ring is

$$\tau = \int d\tau = \int rQ\frac{ds}{2\pi r}E_t = \frac{Q}{2\pi}\int E_t ds$$

The induced electromotive force along the ring is directly proportional to the rate of change in the magnetic flux, we have

$$\int E_t ds = -\frac{d\Phi}{dt} = -\pi r^2\frac{dB}{dt}$$

As a result of the torque, the ring, which has a moment of inertia $I = mr^2$, starts to spin with angular acceleration α. During a time interval dt its angular velocity changes by

$$d\omega = \alpha dt = \frac{\tau}{I}dt = \frac{Q}{2\pi}\left(-\pi r^2\frac{dB}{dt}\right)\frac{1}{mr^2}dt = -\frac{Q}{2m}dB$$

Since the magnetic field strength increases from zero to B, the final angular velocity of the ring will be

$$\omega = -\frac{QB}{2m}$$

⇒ The negative sign shows that the direction of the angular velocity vector is opposite to the magnetic induction if Q is positive.

⇒ The final angular velocity does not depend on the radius of the ring, the time over which the magnetic flux changes, or even on how the magnetic flux increases with time.

⇒ In our calculation we ignored the magnetic field produced by the rotating ring.

⇒ Except in the case of a *cylindrical symmetric* uniform field, it is not possible to find the actual value of the induced electric field within the ring because the geometrical structure of the magnetic field is unknown and we do not know the position of the ring in the magnetic field. We can determine the total induced electromotive force, but not the electric field itself.

66. a. Magnetic field of a solenoid on its axis is given as

$$B = \mu_0 nI = \mu_0\left(\frac{N}{l}\right)I$$

$$N\phi = NBA = N\left[\mu_0\left(\frac{N}{l}\right)I\right]\pi r^2$$

$$\frac{N\phi}{I} = \frac{\mu_0 N^2\pi r^2}{l}$$

$$L = \frac{\mu_0 N^2 r^2\pi}{l}$$

Total length of the wire, $S = 2\pi rN$

$$L = \frac{\mu_0 N^2 r^2\pi}{\ell}\frac{4\pi}{4\pi} = \frac{\mu_0(2\pi rN)^2}{4\pi\ell}$$

$$L = \frac{\mu_0 S^2}{4\pi\ell} = \frac{\mu_0}{4\pi}\frac{S^2}{\ell}$$

$$S = \sqrt{\frac{4\pi\ell L}{\mu_0}}$$

67. a. $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$

$$Bb = \mu_0 I$$

$$B = \mu_0\frac{I}{b}$$

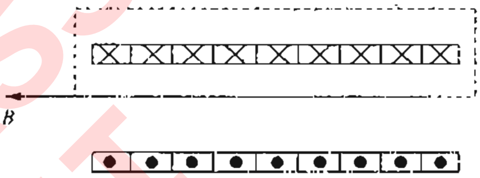


Fig 8.416

Magnetic flux passing through this double tape

$$\phi = BA = B(\ell h) \Rightarrow \phi = \frac{\mu_0 I}{b}\ell h$$

$$\frac{\phi}{I} = \frac{\mu_0\ell h}{b} \Rightarrow L = \frac{\mu_0\ell h}{b} \Rightarrow \frac{L}{\ell} = \frac{\mu_0 h}{b}$$

68. b. Flux through the strip

$$\phi = \int_a^{d-a}\frac{\mu_0 I}{2\pi r}(\ell\,dr) = \frac{\mu_0 I\ell}{2\pi}\ln\left(\frac{d-a}{a}\right)$$

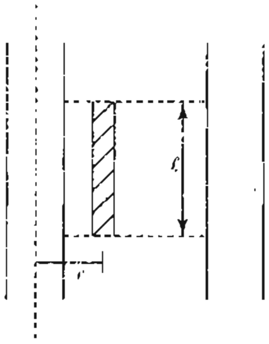


Fig 8.417

The other wire produces the same result, so the total flux through the dotted rectangle is

$$\phi_{\text{total}} = \frac{\mu_0 I\ell}{\pi}\ln\left(\frac{d-a}{a}\right)$$

The inductance of ϕ_{total}

$$L = \frac{\phi_{\text{total}}}{I} = \frac{\mu_0\ell}{\pi}\ln\left(\frac{d-a}{a}\right)$$

69. b. Let current i_1 in the straight wire be upward. Then the magnetic field due to the straight wire has magnitude $B_1 = \mu_0 i_1/2\pi r$ at a distance r. In accordance with right hand rule B_1 points inward to the plane of page. We consider a differential strip of thickness dr, area $dA_2 = a\,dr$. Magnetic flux through area dA, $d\phi_B = B_1\,(a\,dr)$.
Total flux through the loop,

$$\phi_{B_{12}} = \int B_1\, dA_2 = \int_c^{c+b} \frac{\mu_0 i_1}{2\pi r} a\, dr$$

$$= \frac{\mu_0 i_1 a}{2\pi} \int_c^{c+b} \frac{dr}{r} = \frac{\mu_0 i_1 a}{2\pi} \ln\left(\frac{c+b}{c}\right)$$

Therefore mutual inductance,

$$M = M_{12} = \frac{\phi_{12}}{i_1} = \frac{\mu_0 a}{2\pi} \ln\left(1+\frac{b}{c}\right)$$

70. b. Inductors 5 mH and 10 mH are connected in parallel, hence

equivalent inductance $L_{eq} = \frac{5\times10}{5+10} = \frac{50}{15} = \frac{10}{3}$ mH

Current at steady state, $I = \frac{20}{5} = 4$ A

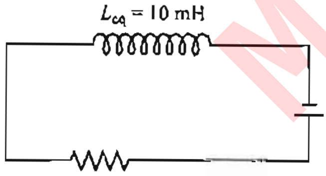


Fig 8.418

As L_1 and L_2 are in parallel

$$I_1 = I\left(\frac{L_2}{L_1+L_2}\right)$$

$$I_1 = 4\left(\frac{10}{10+5}\right)$$

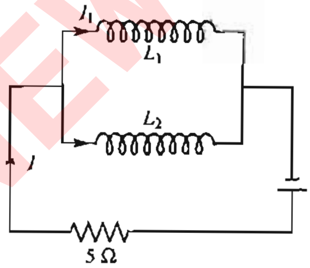


Fig 8.419

$$I_1 = 4\times\frac{10}{15} = \frac{8}{3}\text{ A}$$

$$I_2 = 4\times\frac{5}{15} = \frac{4}{3}\text{ A}$$

71. a. When switch 2 is closed, applying KCL in loop 2, we get

$$\frac{q}{C} + L\frac{di}{dt} = 0$$

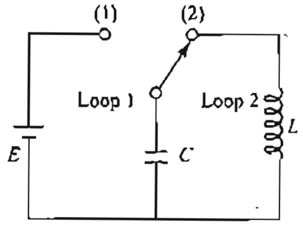


Fig 8.420

$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

$$\frac{d^2q}{dt^2} = \frac{-q}{CL} = 0 \Rightarrow \frac{d^2q}{dt^2} + \omega^2 q = 0$$

Hence, $q = q_0 \cos \omega t$, $\omega = \sqrt{\frac{1}{LC}}$

and $i = -q_0 \omega \sin \omega t$

U_C = Energy stored in the capacitor

$$= \frac{q^2}{2C} = \frac{q_0^2 \cos^2 \omega t}{2C}$$

U_L = Energy stored in the inductor

$$= \frac{1}{2}Li^2 = \frac{1}{2}Lq_0^2\omega^2 \sin^2 \omega t$$

when $U_C = U_L$

$$\frac{q_0^2 \cos^2 \omega t}{2C} = \frac{q_0^2 \omega^2 L \sin^2 \omega t}{2}$$

$$\cot^2 \omega t = \omega^2 LC = 1$$

For minimum value of t, $\omega t = \frac{\pi}{4}$

Hence $t = \frac{\pi}{4}\sqrt{LC}$

72. b. The power dissipated in the resistor,

$$P = \frac{dW}{dt} = I^2 R$$

Since the current through resistor varies with time we must integrate.
The total energy produced as heat in the resistor

$$W = \int_0^\infty I^2 R\, dt$$

The current in an RL circuit is

$$I = I_0 e^{-(R/L)t}$$

$$W = \int_0^\infty I_0^2 e^{-2R/L} R\, dt$$

We can integrate by substituting

$$W = I^2 R \frac{L}{2R} \int_0^\infty e^{-x} dx = \frac{1}{2} L I_0^2 \left[-e^{-x}\right]_0^\infty = \frac{1}{2} L I_0^2$$

Note that the total heat produced equals the energy (1/2) LI_0^2 originally stored in the conductor.

73. a. Steady-state current in $L = I_0 = \dfrac{E}{R_1}$

Energy stored in L

$$= \frac{1}{2}LI_0^2 = \frac{1}{2}L\left(\frac{E^2}{R_1^2}\right)$$

= heat produced in R_2 during discharge.

74. d. $E = L\dfrac{dI}{dt}$ or $dI = \dfrac{E}{L}\,dt$

or $I = \dfrac{2}{4}t = 0.5\,t$

For $I = 5$ A, $t = 10$ s

75. b. Let L and R be the inductance and the resistance of the coil, respectively. Let E = e.m.f. of the cell.

Steady-state current, $I_0 = E/R$

$$P = I_0^2R = \frac{E^2}{R}$$

Energy stored in the coil

$$= \frac{1}{2}LI_0^2 = \frac{1}{2}L\left(\frac{E^2}{R^2}\right); = \frac{1}{2}\left(\frac{L}{R}\right)\left(\frac{E^2}{R}\right) = \frac{1}{2}\tau P$$

76. a. For $r \le R$

$$\oint \vec{E}\cdot\vec{dl} = \left|\frac{d\phi}{dt}\right|$$

or $E(2\pi r) = (\pi R^2)\left(\dfrac{dB}{dt}\right)$

i.e., $E \propto \dfrac{1}{r}$

or $E - r$ graph is a rectangular hyperbola.

77. a. Consider a unit charge inside the conductor. As the conductor moves with a velocity $\vec{V}$, the charge inside it will experience a force $\vec{F} = \vec{V}\times\vec{B}$. If this unit charge is now moved from one end of the conductor to the other, its displacement is $\vec{r}$. Hence, the work done on it is $\vec{F}.\vec{r}$ $= (\vec{v}\times\vec{B}).\vec{r}$. This, by definition, is the e.m.f. across the conductor.

78. d. When a ring moves in a magnetic field perpendicular to its plane, replace the ring by a diameter perpendicular to the direction of motion. The e.m.f is induced across this diameter. In the question current flowing in the ring will be through the two semicircular portions, in parallel.

Induced e.m.f., $e = B(2r)\vec{v}\,I$

Resistance of each half of ring $= R/2$

As these are in parallel, the equivalent resistance $= R/4$

Current in the circuit $= \dfrac{B(2r)v}{(R/4)} = \dfrac{8Brv}{R}$.

79. b. $B = \mu_0 nI$

$= 4\pi\times 10^{-7}\times 200\times 10^{-2}\times 1.5$

$= 3.8\times 10^{-2}$ W/m^2

$\phi = BA = 3.8\times 10^{-2}\times 3.14\times 10^{-4}$

$= 1.2\times 10^{-5}$ Wb

when the current in the solenoid is reversed, the change in magnetic flux,

$d\phi = 2\times(1.2\times 10^{-5}) = 2.4\times 10^{-5}$ Wb

$\therefore$ Induced e.m.f., $e = N$

$$\frac{d\phi}{dt} = 100\times\left(\frac{2.4\times 10^{-5}}{0.05}\right) = 0.048\text{ V.}$$

80. b. For $r \ge a$

$$\oint \vec{E}.\vec{dl} = \left|\frac{d\phi}{dt}\right| = S\left|\frac{dB}{dt}\right|$$

or $E(2\pi r) = \pi a^2\left|\dfrac{dB}{dt}\right|$

$\therefore$ $E = \dfrac{a^2}{2r}\left|\dfrac{dB}{dt}\right|$

$\therefore$ Induced electric field $\propto 1/r$.

For $r \le a$

$E(2\pi r) = \pi r^2\left|\dfrac{dB}{dt}\right|$; or $E = \dfrac{r}{2}\left|\dfrac{dB}{dt}\right|$

or $E \propto r$

At $r = a$,

$$E = \frac{a}{2}\left|\frac{dB}{dt}\right|$$

Therefore, variation of E with r (distance from the centre) will be as given in Fig. 8.423.

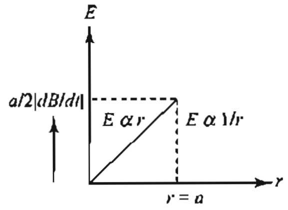


Fig 8.421

81. a. Current at any time t in $L - R$ circuit is

$$i = \frac{E}{R}\left(1 - e^{-Rt/L}\right) \qquad \text{(i)}$$

$$\therefore \quad \left(\frac{d\ell}{dt}\right) = \frac{E}{L}e^{-\frac{Rt}{L}}$$

$$e = L\frac{di}{dt} = Ee^{-\frac{Rt}{L}} \qquad \text{(ii)}$$

Eliminating t from equations (i) and (ii), we get,

$e = E - iR$

i.e., e–I graph is a straight line with a negative slope and positive intercept.

82. c. Motional e.m.f.

$e = Bv\ell$

$$e = (2)(2)(1) = 4\text{ V}$$

This acts as a cell of e.m.f. $E = 4$ V and internal resistance $r = 2\,\Omega$. The simple circuit can be drawn as follows:

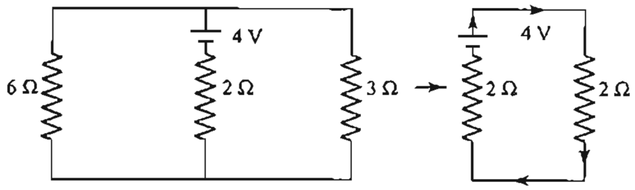


Fig 8.422

∴ Current through the connector

$$i = \frac{4}{2+2} = 1\text{ A}$$

Magnetic force on connector

$$F_m = i\ell B$$
$$= (1)(1)(2)$$
$$= 2\text{ N} \qquad \text{(towards left)}$$

Therefore, to keep the connector moving with a constant velocity, a force of 2 N will have to be applied towards right.

83. c. Potential difference between centre of the ring and the rim is

$$V = \frac{1}{2}B\omega R^2$$

$$= \frac{1}{2}(50(2)(0.1)^2 = 5\text{ V}$$

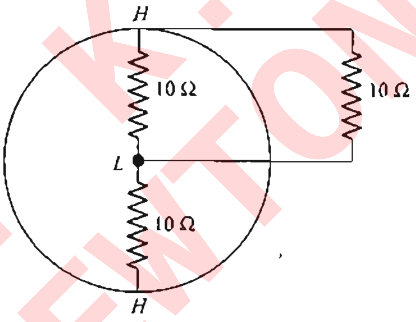


Fig 8.423

Now the circuit can be drawn as follows:

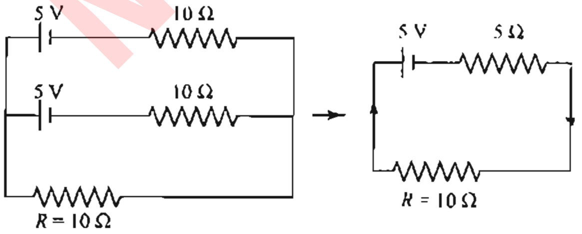


Fig 8.424

$$\therefore \quad i = \frac{5}{10+5} = \frac{1}{3}\text{A}$$

84. a. $|dq| = \dfrac{d\phi}{R} = idt$ = area under i–t graph

$$\therefore \quad d\phi = (\text{Area under } i\text{–}t \text{ graph})\,(R)$$

$$= \frac{1}{2}(4)(0.1)(10) = 2\text{ Wb}$$

85. a. $U = \dfrac{1}{2}Li^2$

$$\text{Rate} = \frac{dU}{dt} = (Li)\left(\frac{di}{dt}\right)$$

At $t = 0$, $i = 0$, ∴ rate = 0

at $t = \infty$, $I = i_0$ but $\dfrac{di}{dt} = 0$,

Therefore, rate = 0.

86. d. Coefficient of mutual induction M is given by .

$$|M| = \frac{e_1}{(di_2/dt)} = \frac{\phi_2}{i_1}$$

$$\therefore \quad \phi_2 = \frac{e_1 i_1}{(di_2/dt)} = \frac{(25.0\times10^{-3})(3.6)}{(15)}$$

$$= 6\times10^{-3} = 6\text{ mWb}$$

87. a. $dq = \dfrac{d\varphi}{R}$

$$\phi_1 = 0$$

$$\phi_f = \left(\frac{\mu_0}{2\pi}\frac{i}{b}\right)(\pi a^2) = \frac{\mu_0 i a^2}{2b}$$

$$\therefore \quad d\phi = \frac{\mu_0 i a^2}{2b}$$

So, $dq = \dfrac{\mu_0 i a^2}{2Rb}$

88. a. $i = i_0 e^{-t/t_0}$

$$\left(-\frac{di}{dt}\right) = \frac{i_0}{t_0}e^{-t/t_0}$$

$$\left(-\frac{di}{dt}\right) = \frac{i_0}{t_0} = r \qquad (\text{at } t = 0)$$

∴ The desired time is $\dfrac{i_0}{r}$ or t_0.

89. b. Let E be the electric field at a distance r from the centre of the disc. Then

$$eE = m\omega^2 r$$

or $$E = \frac{m\omega^2 r}{e}$$

$$\therefore \quad P.D. = \int_{r=0}^{r=a} E\,dr$$

$$= \int_0^a \frac{m\omega^2 r}{e} = \frac{m\omega^2 a^2}{2e}$$

90. d. According to Lenz's law, e.m.f. of same magnitude in clockwise direction is induced in the two loops into which the figure is divided. So, current is induced in the clockwise direction in the outer boundary but no current is there in wire AB.

91. c. Potential difference across the coil is $V = L\dfrac{di}{dt}$

or $\quad V = (2)(4) = 8\text{ V}$

Now energy stored per unit time

$= \text{power} = Vi$

$= (8)(2)$

$= 16\text{ J/s}$

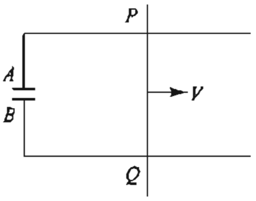


Fig 10.425

92. a.

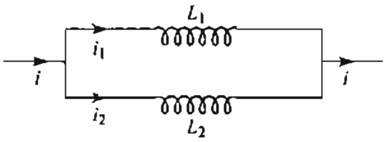


Fig 8.426

$$q = CV$$
$$= C(Bv\ell)$$
$$= (10\times10^{-6})(4)(2)(1)\text{C}$$
$$= 80\,\mu\text{C} = \text{constant}$$

Magnetic force on the electron in the conducting rod PQ is towards Q. Therefore A is positively charged and B is negatively charged.

93. d. Magnetic flux in ⊗ direction through the coil is increasing. Therefore, induced current will produce magnetic field in ⊙ direction. Thus the current in the loop is anticlockwise. Magnitude of induced current at any instant of time is

$$i = \frac{e}{R}$$
$$= \frac{Bu(FG)}{\rho(FG + GD + DF)}$$

When the wire AH moves downwards FG, GD and DF all increase in the same ratio. Therefore, i is constant.

94. d. Net force on the wire is zero when the terminal speed is attained.

$\therefore \quad F_m = mg$

or $\quad i\ell B = mg$

or $\quad \left(\dfrac{e}{R}\right)\ell B = mg$

or $\quad \left(\dfrac{Bv_T\ell}{R}\right)\ell B = mg$ (v_T = terminal speed)

or $\quad v_T = \dfrac{mgR}{B^2\ell^2}$

95. d. When the coil is within the field, there is no change in the magnetic flux passing through it. Thus no current will be induced and the acceleration will be g. But according to Lenz's law, the induced current will oppose its motion when it enters or leaves the field. Therefore, acceleration will be less than g.

96. a. Magnetic field in ⊗ direction is increasing. Therefore, induced current will produce magnetic field in ⊙ direction. Thus current in both the loops should be anticlockwise. But as the area of the loop on the right side is more, induced e.m.f. in this will be more compared to the left side loop.

$\left(e = -\dfrac{d\phi}{dt} = -S\dfrac{dB}{dt}\right)$. Therefore, net current in the complete loop will be in a direction shown below:

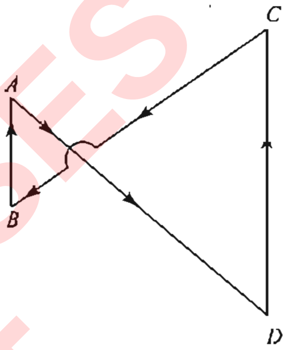


Fig 8.427

97. b. Let $2a$ be the side of the triangle and b be the length AE.

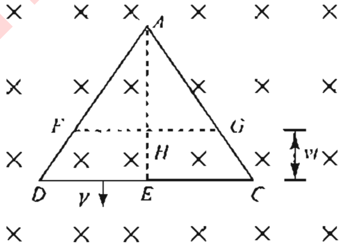


Fig 8.428

$$\frac{AH}{AE} = \frac{GH}{EC}$$

$\therefore \quad GH = \left(\dfrac{AH}{AE}\right)EC$

or $\quad GH = \dfrac{(b - vt)}{b}.a = a - \left(\dfrac{a}{b}vt\right)$

$\therefore \quad FG = 2GH = 2\left[a - \dfrac{a}{b}vt\right]$

$\therefore$ Induced e.m.f., $e = Bv(FG) = 2Bv\left(a - \dfrac{a}{b}vt\right)$

$\therefore$ Induced current, $i = \dfrac{e}{R} = \dfrac{2Bv}{R}\left[a - \dfrac{a}{b}vt\right]$

or $\quad i = k_1 - k_2 t$

Thus i-t graph is a straight line with negative slope and positive intercept.

98. c. The current at time t is given by

$$i = i_0 (1 - e^{-t/\tau})$$

Here $\quad i_0 = E/R \quad$ and $\quad \tau = \frac{L}{R}$

$$\therefore \quad q = \int_0^{\tau} i dt = \int_0^{t} i_0 (1 - e^{-t/\tau})\, dt$$

$$= \frac{i_0 \tau}{e} = \frac{\left(\frac{E}{R}\right)\left(\frac{L}{R}\right)}{e} = \frac{EL}{eR^2}$$

99. a. Speed of the loop should be

$$v = \frac{\ell}{t} = \frac{0.5}{2} = 0.25 \text{ m/s}$$

Induced e.m.f., $e = Bv\ell = (1.0)(1.0)(0.25)(0.5)$

$= 0.125$ V

$\therefore$ Current in the loop, $i = \frac{e}{R} = \frac{0.125}{10}$

$= 1.25 \times 10^{-2}$ A

The magnetic force on the left arm due to the magnetic field is

$F_m = i\ell B = (1.25 \times 10^{-2})(0.5)(1.0)$

$= 6.25 \times 10^{-3}$ N

To pull the loop uniformly an external force of 6.25×10^{-3} N towards right must be applied.

$\therefore \quad W = (6.25 \times 10^{-3}\text{N})(0.5\text{m})$

$= 3.125 \times 10^{-3}$ J

100. b. Terminal velocity is attained when magnetic force is equal to $mg \sin \theta$

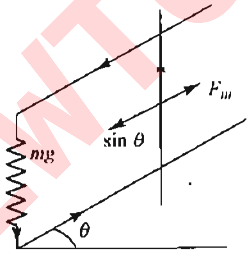


Fig. 8.429

$\therefore \quad F_m = mg \sin \theta$

or $\quad i\ell B = mg \sin \theta$

or $\quad \left(\frac{e}{R}\right) \ell B = mg \sin \theta$

or $\quad \frac{(Bv_T \ell)}{R} = \ell B = mg \sin \theta$

$\therefore \quad v_T = \frac{mgR \sin \theta}{B^2 \ell^2}$

101. d

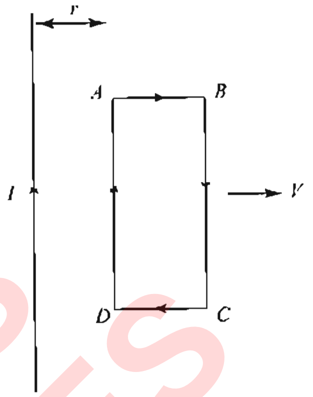


Fig 8.430

As the flux decreases, to maintain flux current in the loop is clockwise. Force on DA due to the long wire is towards left while on BC is towards right

102. a.

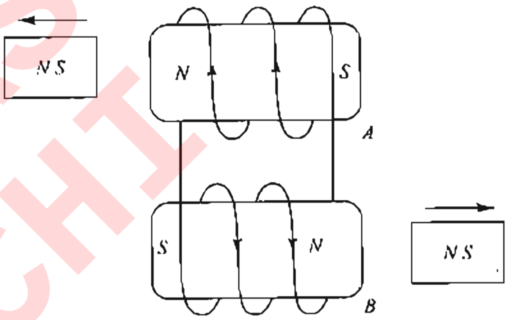


Fig 8.431

103. d. Perpendicular length is more, so induced e.m.f. is more

104. d $\quad B \ell vt =$ constant

$$B = \frac{C}{\ell vt}$$

105. a. At $t = 0$, the branch containing L will offer infinite resistance while the branch containing the capacitor will be effectively a short circuit. Hence, $(R)_{t=0} = \frac{\varepsilon}{R}$. Similarly at $t = \infty$, L will offer zero resistance whereas c will be an open circuit. Hence effective resistance $= R + \frac{6 \times 3R}{6+3} = 3R \; (i)_{t=\infty}$

$= \frac{\varepsilon}{3R}$.

$\therefore \quad$ The required ratio $= \frac{\varepsilon}{R} \times \frac{3R}{\varepsilon} = 3:1$

106. a. $\phi \propto \frac{di_1}{dt}$

Hence $i_2 = M \frac{di_1}{dt}$
(direction already indicated)
Hence (a) is correct option.

107. b $\quad V = 5 \cos (wt -)$

$i = 10 \sin wt = 10 \cos (wt -)$

$$\phi = \frac{\pi}{2} - \frac{\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$$

$$P = \frac{VI}{2}\cos\phi = \frac{5\times10}{2}\times\frac{1}{2} = 12.5\ \text{W}$$

108. c. $\phi = Li = BA$

$$i_{max} = \frac{B_{max}A}{L}; \frac{0.1\times10^{-2}}{10\times10^{-3}} = 0.1\ \text{A} = 100\ \text{mA}$$

109. b. $i = \frac{10}{3R+2}, \frac{10}{3R+2} = 3,\ R = \frac{4}{9}$

110. c. Current in the YY' direction is from Y' to Y but the current is constant and hence the magnetic flux through the coil is constant. Therefore the current in the coil is zero.

111. c. Applying KVL in the outer loop we get $I_0 r - E = 0$

$$\Rightarrow \quad I = \frac{E}{r}$$

$\therefore$ Initial energy in solenoid $= U_0 = \frac{1}{2}LI_0^2 = \frac{E^2L}{2r^2}$. This energy will be dissipated in the form of heat in r and R after opening of the switch. Since the same current flows through these resistances and the thermal power in them is i^2r and i^2R [i varies with time, respectively, therefore heat generated in each resistor is directly proportional to its resistance.

$\therefore$ Heat generated in steroid

$$= \frac{r}{r+R}U_0$$

$$= \left[\frac{r}{r+R}\right]\frac{E^2L}{2r^2} = \frac{E^2L}{2r(r+R)} \qquad \therefore \text{(C)}$$

112. b. From Kirchhoff's rule, $L\frac{di}{dt} = \Sigma$ and $U = \frac{1}{2}Li^2$

113. c. Constancy of flux implies that $\frac{E}{R}.L_1 = i(L_1 + L_2)$

i.e., $\quad i = \frac{E\,L_1}{R(L_1+L_2)}$

114. d. $E = \frac{Mdi}{dt} = Ma \qquad i = \frac{Ma}{R}(1 - e^{-tR/L})$

115. b. After long time

$$mv_0 = 2mv;\ v = \frac{v_0}{2}.$$

116. b. $i = \dfrac{\dfrac{B\omega\ell^2}{2} + \dfrac{B\omega\ell^2}{2}}{R} = \dfrac{B\omega\ell^2}{R}$

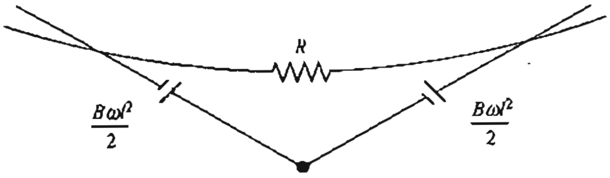


Fig 8.432

117. c. Let i_1 be the current in the circuit before shifting

$$i_1 = \frac{\varepsilon}{R} \qquad \text{(i)}$$

Since the flux associated with the inductors will be same just before and just after shifting

$\therefore \quad i_1 5L = i_2 9L$

$$i_2 = \frac{5}{9}\frac{\varepsilon}{R}$$

118. c. $\left|\int \bar{E}.d\bar{\ell}\right| = \left|\frac{d\phi}{dt}\right|;\ E2\pi\frac{\ell}{2} = \pi\frac{\ell^2}{4}\frac{dB}{dt}$

$$E = \frac{\ell}{4}\alpha$$

$F = \frac{q\ell\alpha}{4}$ is the electric force on the charge but net force of the particle is zero.

119. c. The change in magnetic flux is zero, hence the current in the ring will be zero.

120. c. $i = i_0(1 - e^{-\frac{tR}{L}})$

$$i = \frac{10}{10}(1 - e^{-\frac{2\times10}{20}})$$

$$i = (1 - e^{-1})$$

121. c. $\quad \omega_r = \text{constant}$

$\Rightarrow \quad LC = \text{constant}$

$$\Rightarrow \quad L\,dC + C\,dL = 0 \Rightarrow \frac{dL}{L} = -\frac{dC}{C} = -1\%.$$

122. d. Circuit is a balanced Wheatstone's bridge

$\therefore$ R of the whole circuit $= 1 + 4 = 5\ \Omega$

Induced e.m.f. $BLV = iR$

$$v = \frac{iR}{BL} = \frac{.1\times5}{2\times.1} = 2.5\ \text{m/s}$$

123. c. The magnetic field produced by the square loop is parallel to the plane of the circular loop. Hence the mutual inductance is zero.

124. d. $I = \frac{Bv(2r)}{R_{eq}}$, where $R_{eq} = \frac{(R/2)(R/2)}{R/2 + R/2} = R/4$

125. b. Initially, there will be no potential drop across the capacitor but only across the resistors.

126. b. $\frac{B^2V_0L^2}{R} = mg \quad \Rightarrow V_0 = \frac{mgR}{B^2L^2}$

127. d. At $t = 0$, for the purpose of current calculation in circuit, inductor can be assumed as open and capacitor as short circuited.

128. c. Use Lenz's law. Induced e.m.f. of the current opposes the change in flux through it.

129. b. Required e.m.f. = πR^2

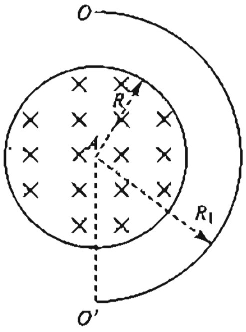


Fig 8.433

$$\frac{dB}{dt}\times\frac{\theta}{2\pi}$$

$$\therefore \quad E=\frac{R^2}{2R_1}\left(\frac{dB}{dt}\right)$$

$$\text{e.m.f.}=\frac{\theta}{2\pi}\frac{R^2}{2R_1}\left(\frac{dB}{dt}\right)2\pi R_1=\frac{\theta}{2}R^2\left(\frac{dB}{dt}\right)$$

130. c. $\cos\phi=\dfrac{R}{\sqrt{R^2+\left(\dfrac{1}{\omega C}-\omega L\right)^2}}$

Putting the values, $C=500\ \mu\text{F}$

131. a. $ma_0=eE$

$$E=\frac{ma_0}{e}$$

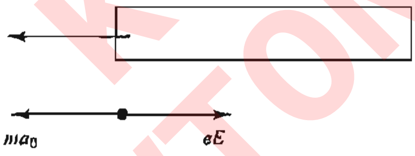


Fig 8.434

132. c. Magnetic field is out of the plane and as the electron moves, the field decreases. From Lenz's rule, the current in the coil is anticlockwise.

133. a. I is same for two values of frequency namely f_1 and f_2

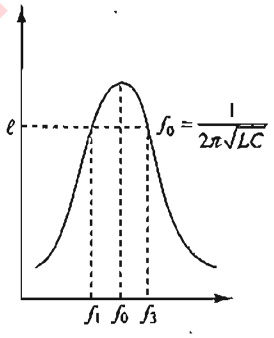


Fig 8.435

134. d. The field inside the solenoid is uniform, the magnetic flux linked with the coil will be

$$\phi=BNA=\mu_0 nI\,n\pi R^2=\mu_0 n^2\pi R^2 I$$

$$\therefore \quad \frac{L_1}{L_2}=\frac{N_1^2N_1^2}{N_2^2N_2^2}=\left(\frac{N_1}{N_2}\right)^2\times\left(\frac{R_1}{R_2}\right)^2=16\times\frac{1}{16}=1$$

135. d. At $t=0$, capacitor will behave like a short circuit and the inductor as open circuit but as $t\to\infty$, the nature is just opposite.

136. b $\quad dq=\dfrac{-d\phi}{R}$

$$=\frac{2\phi_1}{R}=\frac{2\mu_0 Ia}{R2\pi}\ln\left(\frac{2a}{a}\right)=\frac{\mu_0 Ia\ln 2}{\pi R}$$

137. c. When the rod rotates, there will be an induced current in the rod. The given situation can be treated as if a rod A of length 3 ℓ is rotating in clockwise direction. while another rod B of length 2 ℓ is rotating in the anticlockwise direction with the same angular speed ω.

As $\quad e=\dfrac{1}{2}B\omega\ell^2$

For A:

$$e_A=\frac{1}{2}B\omega(3\ell)^2 \text{ and } e_B=\frac{1}{2}B(-\omega)(2\ell)^2$$

Resultant induced e.m.f. will be:

$$e=e_A+e_B=\frac{1}{2}B\omega\ell^2\ (9-4)$$

$$e=\frac{5}{2}B\omega\ell^2$$

138. a. Induced e.m.f. $\displaystyle\int_a^b Bvdx=\int_a^b\frac{\mu_0 I}{2\pi X}Bvdx$

$\Rightarrow$ Induced e.m.f. $=\dfrac{\mu_0 Iv}{2\pi}\ln\left(\dfrac{b}{a}\right)$

$\Rightarrow$ Power dissipated $=\dfrac{E^2}{R}$

Also, power = FV $\Rightarrow$ $F=\dfrac{E^2}{VR}$

$$\Rightarrow \quad F=\frac{1}{VR}\left[\frac{\mu_0 IV}{2\pi}\ln\left(\frac{b}{u}\right)\right]^2$$

139. c $\quad \phi_A=\dfrac{\mu_0 i\pi R^2}{2\pi\left(R^2+x^2\right)^{3/2}}\pi r^2$

$$\Rightarrow \quad E_A=-\frac{d\phi}{dt}$$

$$=\frac{\mu_0 i\pi}{2}R^2r^2(-3/2)\left(R^2+x^2\right)^{-5/2}2x$$

E_A is maximum when $\dfrac{dE_A}{dx}=0$

$$\Rightarrow \qquad \frac{d}{dx}\frac{x}{\left(R^2+x^2\right)^{5/2}}=0$$

$$\text{or}\left(R^2+x^2\right)^{5/2}-\frac{5x}{2}\left(R^2+x^2\right)^{3/2}2x=0$$

$$\text{or,} \qquad R^2+x^2-5x^2=0$$

$$\text{or,} \qquad x=\frac{R}{2}$$

140. a.

Rate of increment of energy in the inductor

$$=\frac{dU}{dt}=\frac{d}{dt}\left(\frac{1}{2}Li^2\right)=Li\frac{di}{dt}$$

Current in the inductor at time is:

$$i=i_0\left(1-e^{-\frac{t}{\tau}}\right)\text{and }\frac{di}{dt}=\frac{i_0}{\tau}e^{-\frac{t}{\tau}}$$

$$\therefore \qquad \frac{dU}{dt}=\frac{Li_0^2}{\tau}e^{-\frac{t}{\tau}}\left(1-e^{-\frac{t}{\tau}}\right)$$

$$\frac{dU}{dt}=0 \text{ at } t=0 \text{ and } t=\infty$$

Hence, E is best represented by:

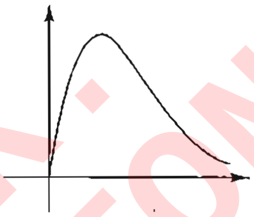


Fig 8.436

141. a. $L=\frac{\mu_0 N^2\pi r^2}{\ell}$

length of wire = $N\,2\pi r$ = Constant (= C, suppose)

$$\therefore \qquad L=\mu_0=\left(\frac{C}{2\pi r}\right)^2\frac{\pi r^2}{\ell}$$

$$\therefore \qquad L\propto\frac{1}{\ell}$$

∴ Self-inductance will become 2 L.

142. b.

Fig 8.437

$$\text{Using} \qquad V_A-V_B=RI+L\frac{dI}{dt}$$

$$\Rightarrow \qquad 140=5R+10L$$

$$60=5R-10L$$

$$\Rightarrow \qquad L=4\text{ H}$$

143. b. $\int\vec{E}d\vec{r}=-\frac{d\phi}{dt}$

And taking the sign of flux according to right hand curl rule, we get,

$$\int\vec{E}d\vec{r}=-\left(-(-\alpha A)-(-\alpha A)+(-\alpha A)\right)=-\alpha A$$

144. a. Given:

Voltage in primary, $V_P=200$ V

Current in primary, $i_p=2$ A

Voltage in secondary, $V_s=2000$ V

The relation for the current in the secondary is

$$\frac{V_s}{V_p}=\frac{i_p}{i_s}\Rightarrow\frac{2000}{200}=\frac{2}{i_s}=\frac{2\times200}{2000}=0.2\text{ A}$$

145. c. Flux cannot change in a superconducting loop.

$$\therefore \qquad \text{Finally, } Li=2\,\pi R^2\times B$$

$$i=\frac{2\times\pi R^2\times B}{L}$$

146. b.

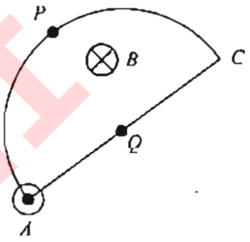


Fig 8.438

We connect a conducting wire from A to C and complete the semicircular loop.

The e.m.f. in the semicircular loop is zero because its magnetic flux does not change.

∴ e.m.f. of section APC + e.m.f. of section $CQA=0$

∴ e.m.f. of section APC = e.m.f. of section $AQC=2BR^2\omega$

147. b. Equivalent inductance

$$L_{eq}=L+2L=3L$$

$$C_{eq}=C+2C=3C$$

∴ Frequency of oscillation

$$f=\frac{1}{2\pi\sqrt{L_{eq}C_{eq}}}=\frac{1}{6\pi\sqrt{LC}}$$

148. a. The flux in the rectangular loop due to the current in the wire is

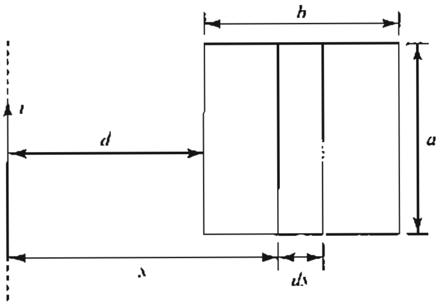


Fig 8.439

$$\phi = \int_{d}^{d+b} \frac{\mu_0 i}{2\pi x} a\, dx = \frac{\mu_0 i a}{2\pi} \ln \frac{b+d}{d}$$

Mutual inductance is

$$M = \frac{\varphi}{i} = \frac{\mu_0 a}{2\pi} \ln \frac{b+d}{d}$$

∴ Mutual inductance is proportional to a

149. a. $e = \frac{BdA}{dt} = \frac{Bd}{dt}(\pi r^2) = B 2\pi r \frac{dr}{dt}$

150. c. Just before opening the switch, the current in the inductor is ε/R. Energy stored in it $= \frac{1}{2} L\left(\frac{\varepsilon}{R}\right)^2$

This energy will dissipate in the resistors R_1 and R_2 in the ratio $\frac{1}{R_1}$ and $\frac{1}{R_2}$.

151. b.

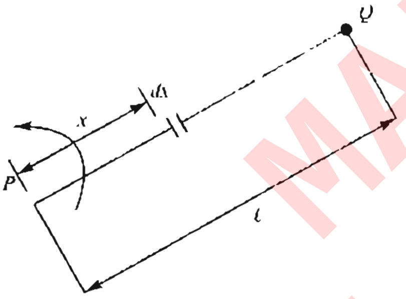


Fig 8.440

Charge on the differential element dx, $dq = \frac{Q}{l} \cdot dx$

Equivalent current, $di = f\, dq$

∴ magnetic moment of this element, $d\mu = (di) NA$ $(N = 1)$

$$= (\pi x^2) f \frac{Q}{l} dx$$

$$\Rightarrow \quad m = \int_0^{\mu} d\mu = \frac{\pi f Q}{l} \int_0^{l} x^2 dx.$$

$$\mu = \frac{1}{3} \pi f Q\, l^2$$

152. d. Since all the wires are connected between rim and axle, they will generate induced e.m.f. in parallel, hence it is same for any number of spokes.

153. c. $q = \frac{\Delta\phi}{R}$

∴ $\Delta\phi = qR$ = area of $i-t$ graph × R

154. b. $e = (\vec{v} \times \vec{B}) l$

$$e = \left[\hat{i} \times (3\hat{i} + 4\hat{j} + 5\hat{k})\right] 5\hat{j}$$

$\Rightarrow$ $e = 25\,\text{V}$

155. b. Time constant $= \frac{1}{20} = 50$ ms

So $i = 0.633\, i_{\max} = 0.633 \frac{E}{R}$

$$\Rightarrow \quad E = \frac{3.165 \times 20}{0.633} = 100\,\text{V}$$

156. d. $\omega = (BIR) S = \left[\left(\frac{Bvnl}{R}\right) nl\right] S$

$$= n^2 \frac{(B^2 v^2 S l)}{R}$$

$$= n^2 \frac{(0.4)^2 (5\times10^{-2})^2 (5\times10^{-2})(5\times10^{-2})}{100}$$

$$= m^2 (10000) \times 10^{-12}$$

$$= 1 \times 10^{-8} \times 100 \; (\because n = 100) = 10^{-4}\,\text{J} = 0.1\,\text{mJ}$$

157. b. The fields at A and B are out of the paper and inside the paper, respectively.

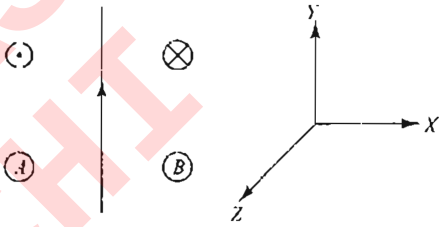


Fig 8.441

As the current in the straight wire decreases, the field also decreases.

For B

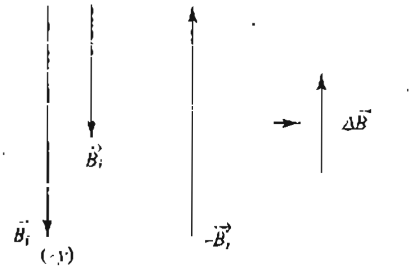


Fig 8.442

The change in the magnetic field which causes induced current $(\Delta\vec{B})$ is along (+)z direction.

Hence, induced e.m.f. and hence current should be such as to oppose this change $\Delta\vec{B}$.

Hence, induced e.m.f. should be along $-z$ direction which results in a clockwise current in B. Similarly, there will be anticlockwise current in A. Hence (b).

158. b. When the ring falls vertically, there will be an induced e.m.f. across A and B $(e = Bv(2r))$.

Note that there will be a potential difference across any two points on the ring, and the line joining these has a projected length in the horizontal plane. For example, between points P and Q there is a projected length x in the horizontal plane.

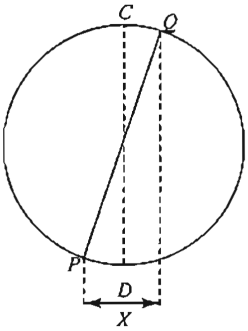


Fig 8.443

$\therefore$ p.d. across P and Q is

$$V = B\,vx$$

But for points C and D, $x = 0$

Therefore, $p.d. = 0$

Hence (b)

159. c.

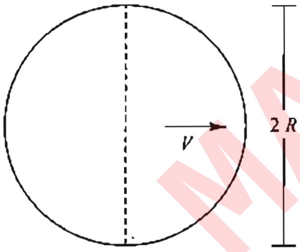


Fig 8.444

Considering a projected length $2R$ on the ring in vertical plane.

This length will move at a speed v perpendicular to the field.

This results in an induced e.m.f.:

$e = Bv\,(2R)$ in the ring.

In ring A, $eA = B\,(-v)(2R)$

In ring B, $eB = B\,(v)(2R)$

Therefore, potential difference between A and B

$$= B\,(v)\,(2R) - B\,V - v)(2R) = 4\,BvR$$

Note: ***There will be no p.d. across the diameter due to rotation.***

Alternatively: Considering rotation of diameter about lowest point:

$$e = \frac{B\omega(2r)^2}{2} = 2\,Bvr \text{ in } A \text{ (since pure rotation)}$$

and $\quad e = -2Bvr$ in B

Hence (c)

160. a. The graph of current is given by

$$I = i_0\,(1 - e^{-1/\tau}) \Rightarrow \frac{di}{dt} = \frac{i_0}{\tau}e^{-t/\tau}$$

Energy stored in the form of magnetic field energy is

$$U_B = \frac{1}{2}Li^2$$

$\therefore$ Rate of increases of magnetic field energy is

$$R = \frac{dU_B}{dt} = Li\frac{di}{dt} = \frac{Li_0^2}{\tau}\,(1 - e^{-t/\tau})e^{-1/\tau}$$

This will be maximum when $\frac{dR}{dt} = 0$

$\Rightarrow \quad e^{-t/\tau} = 1/2$

Substituting,

$$R_{\max} = \frac{Li_0^2}{\tau} = \frac{Li_{02}}{\tau}\left(1 - \frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{Li_0^2}{4\tau} = \left[\frac{L(E/R)^2}{4(L/R)}\right] = \frac{E^2}{4R}$$

161. a. When the key is at position (2) for a long time, the energy stored in the inductor is:

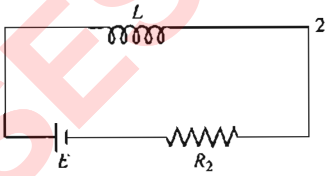


Fig 8.445

$$U_B = \frac{1}{2}Li_0^2 = \frac{1}{2}L\left(\frac{E}{R_2}\right)^2 = \frac{LE^2}{2R_2^2}$$

This whole energy will be dissipated in the form of heat when the inductor is connected to R_1 and no source is connected.

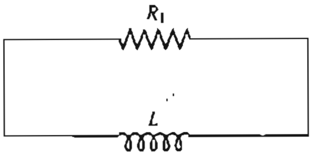


Fig 8.446

162. b. When the switch is at position 1:

$$U_B = \frac{1}{2}Li_0^2 = \frac{LE^2}{2R^2}$$

Just after the switch is shifted to position 2, current, $I = \frac{E}{R}$ is flowing across the resistance. Hence, at that instant, $p.d.$ across resistance will be

$$\Delta V = IR = \frac{E}{R}R = E$$

163. a. If we consider the cylindrical surface to be a ring of radius R, there will be an induced e.m.f. due changing field.

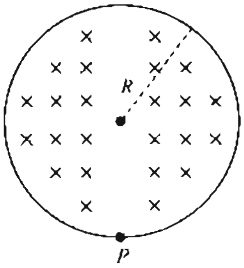


Fig 8.447

$$\int \vec{E}.\vec{d\ell} = \frac{d\phi}{dt} = -A\frac{dB}{dt}$$

$$\Rightarrow \quad E(2\pi R) = -A\frac{dB}{dt} = -\pi R^2\frac{dB}{dt} \Rightarrow E = \frac{R}{2}\frac{dB}{dt}$$

∴ Force on the electron

$$F = -Ee = -\frac{eR}{2}\frac{dB}{dt}$$

$$\Rightarrow \quad \text{Acceleration} = \frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$$

As the field is increasing and is being directed inside the paper, there will be anticlockwise induced current (in order to oppose the cause) in the ring (assumed). Hence there will be force towards left on the electron.

164. a. The equivalent diagram is

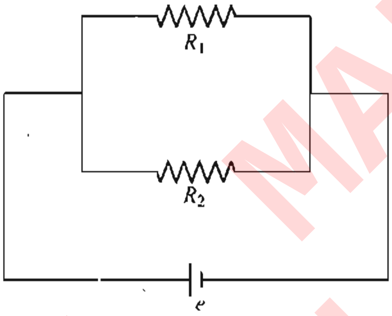


Fig 8.448

The induced e.m.f. across the centre and any point on the circumference is

$$|\bar{e}| = \frac{1}{2}B\omega\ell^2 = \frac{B\omega r^2}{2}$$

$$\therefore \quad \text{Current through } R_1 = \frac{B\omega r^2}{2R_1}$$

165. c. There is a force $\vec{F}_M = I\left(d\vec{\ell} \times \vec{B}\right)$ acting on the rod carrying a current I.

By the rule of cross produce, this force is vertically upward.

F.B.D. of the rod:

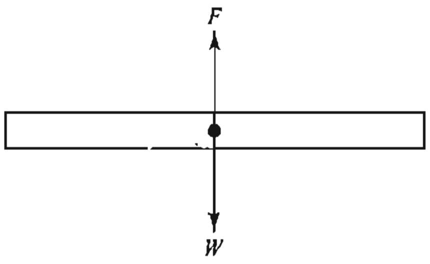


Fig 8.449

$$F - W = ma$$

$$a = \frac{F-W}{m}$$

The magnitude of acceleration will be constant, but the direction will depend on the mass of the rod.

166. c. Considering pure rolling of OA about A, the induced e.m.f. across OA will be

$$|\bar{e}| = \frac{B\omega(r)^2}{2}$$

From Lenz's law, O will be the negative end, while A will be the positive end.

$$\text{Hence, } v_0 - v_A = -\frac{B\omega r^2}{2}$$

167. d. $F_b = BIL$

Induced current:

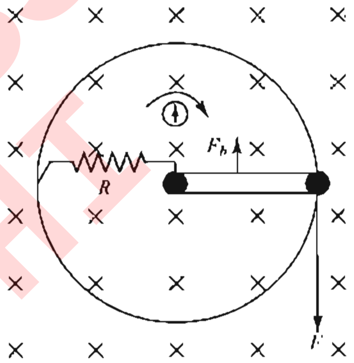


Fig 8.450

$$I = \frac{\left(B\omega r^2/2\right)}{r}$$

$$\therefore \quad F_b = B\left(\frac{B\omega r^2}{2R}\right)r = \frac{B^2\omega r^3}{2R}$$

To maintain constant angular velocity.

$$F(r) = F_b(r/2)$$

$$\Rightarrow \quad F = \frac{F_b}{2} = \frac{B^2\omega r^3}{4R}$$

168. c. Flux through a closed circuit containing an inductor does not change instantaneously.

$$\therefore \quad L\left(\frac{E}{R}\right) = \frac{L}{4}(i) \quad \Rightarrow \quad i = \frac{4E}{R}$$

169. a. Force on the wire = $i\ell B$,

$$\therefore \quad \text{Acceleration} = \frac{i\ell B}{m}$$

$$\therefore \quad \text{Velocity} = \frac{i\ell Bt}{m}$$

170. d. Equivalent circuit

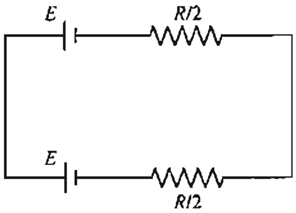


Fig 8.451

$$E = B\ell v$$
$$\ell = 2r$$
$$E = B(2r)v$$
$$i = \frac{E}{R'} = \frac{B(2r)v}{R/4} = \frac{8Brv}{R} \qquad \left(\text{As } R' = \frac{R}{4}\right)$$

171. a. Given,

$$L_1 = 1\text{ mH}, L_2 = 2\text{ mH}$$
$$R_1 = 1.2, R_2 = 2.2$$

In the first circuit,

$$L = L_1 + L_2$$
$$R = R_1 + R_2$$
$$\tau_1 = \frac{L}{R} = \frac{3\text{mH}}{3.2} = 1\text{ ms}$$

In the second circuit,

$$L = \frac{L_1 L_2}{L_1 + L_2} = \frac{2\times10^{-6}}{3\times10^{-3}} = \frac{2}{3}\times10^{-3}$$
$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{2}{3}$$
$$\tau_2 = \frac{\frac{2}{3}\times10^{-3}}{2/3} = 1\text{ ms}$$

In the third circuit,

$$L = L_1 + L_2 = 3\text{ mH}$$
$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{2}{3}$$
$$\tau_3 = \frac{L}{R} = \frac{3\times10^{-3}}{2/3} = \frac{9}{2}\text{ ms}$$

172. b.
$$|e| = B\frac{dA}{dt}$$
$$|e| = B\frac{(A_2 - A_1)}{(t_2 - t_1)}$$
$$|e| = \frac{1\left(\frac{\pi}{4} - 0\right)}{(1-0)}$$
$$e = \frac{\pi}{4}\text{ V}$$

173. c.
$$I_1 = \frac{20}{10}\left(1 - e^{-\frac{t}{5\times10^{-4}}}\right)$$

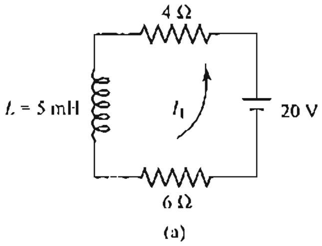


Fig 8.452

$$= \frac{3}{2} = 1.5\text{ A}$$
$$I_2 = \frac{20}{10}e^{-\frac{t}{10^{-3}}} = 1.0\text{ A}$$

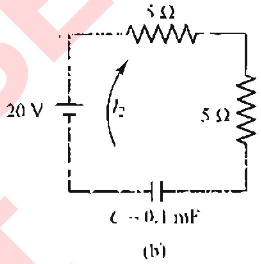


Fig 8.453

From superposition

$$I = I_1 + I_2 = 2.5\text{ A}$$

174. d. $\varepsilon = \int(\vec{v}\times\vec{B})d\vec{l} \quad \Rightarrow \quad \varepsilon = \frac{1}{2}Bl^2\omega\sin^2\theta$

175. a. $L\frac{di}{dt} = E$ or $i = \frac{Et}{L}\;\forall\; 0 \le t \le t_0$

176. a.
$$i = \sqrt{5}\text{ A}$$
$$\frac{q_m^2}{2C} = \frac{q^2}{2C} + \frac{1}{2}Li^2$$
$$\Rightarrow \quad q_{\text{max}} = 6C$$

177. a. When the switch is closed, the current flows through the coil as shown and thus sets up a magnetic flux f through the coil linking the ring. The motion of the ring will be such that, by Lenz's law, it opposes the sudden increase in flux linkage from the west. Hence, the ring moves towards E (east).

178. d. $\xi = \frac{d\Phi}{dt} \Rightarrow i = \frac{\xi}{R} = \frac{1}{R}\frac{d}{dt}(BA) = \frac{A}{R}\frac{dB}{dt}$

where πr^2 = area of the loop of radius r and R = resistance of the loop of length $(2\pi r)$ and area of cross section πa^2.

$$R = \frac{\rho l}{\pi a^2} = \frac{\rho(2\pi r)}{\pi a^2}$$

Further mass of wire is $m = (\pi a^2)(2\pi r)(d)$

$$i = \frac{(\pi a^2)(\pi r^2)}{\rho(2\pi r)}\frac{dB}{dt}$$
$$i = \frac{(\pi a^2)(2\pi r)}{4\pi\rho}\frac{dB}{dt} \quad \Rightarrow \quad i = \frac{m}{4\pi\rho d}\frac{dB}{dt}$$

179. a. $$|\varepsilon| = \frac{d\phi}{dt} \Rightarrow |\varepsilon| = A\frac{dR}{dt}$$

$$|\varepsilon| = (d^2)\frac{d}{dt}\left[B_0\left(1+\frac{x}{a}\right)\right]$$

$$|\varepsilon| = (d^2)B_0\left(0+\frac{1}{a}\frac{dx}{dt}\right) \Rightarrow |\varepsilon| = \frac{B_0d^2}{a}\left(\frac{dx}{dt}\right)$$

$$|\varepsilon| = \frac{B_0d^2v_0}{a}$$

180. b. $$I = \frac{I_0}{\alpha} \text{ at } t = t_0$$

Since $$I = I_0e^{-t_0/\tau} \Rightarrow \frac{1}{\alpha} = e^{-t_0/\tau}$$

$$\alpha = e^{t_0/\tau} \Rightarrow t_0 = \tau\log_e\alpha$$

$$\tau = \frac{t_0}{\log_e\alpha}$$

181. c. Since, $\varepsilon = \frac{d\phi}{dt} = \frac{d}{dt}(NBA)$

$$\varepsilon = \frac{d}{dt}[NA(\mu_0nI)];\ \varepsilon = NA\mu_0n\left(\frac{dI}{dt}\right)$$

where N is total number of turns in the coil and n is the number of turns per unit length in the solenoid.

$$\varepsilon = (300)(1.2\times10^{-3})(4\pi\times10^{-7})\times\frac{2000}{0.3}\times\frac{4}{0.25}$$

$$\varepsilon = 4.8\times10^{-2}\text{ V} = 48\text{ mV}$$

182. d. Let m be the mutual inductance between X and Y. By definition

$$\varepsilon_Y = M\frac{dI_x}{dt} \Rightarrow E = M\frac{dI}{dt} = MI;\ M = \frac{E}{I}$$

The flux linked with X is $\phi_X = MI_Y = \frac{E}{I}I_0$

183. a. For e.m.f. to be induced $\vec{\ell}$, $\vec{v}$ and $\vec{B}$ can never be coplanar.

184. b. $$\frac{L}{R} = 2\times10^{-3} \qquad \text{(i)}$$

$$\frac{L}{R+90} = 0.5\times10^{-3} \qquad \text{(ii)}$$

From (i) and (ii), on solving, we get

$L = 60$ mH and $R = 30$ W

185. d. $$I\vec{\omega} = 0 = -\int\frac{1}{2\pi R}\frac{d\phi}{dt}\times QRdt\,\hat{k}z = -\lambda R$$

$$\int d\phi = -B\lambda\pi a^2R\hat{k} \Rightarrow \vec{\omega} = \frac{-B\pi a^2\lambda}{mR}\hat{k}$$

186. a. $$R_{eq} = \frac{5R}{6} \Rightarrow I = \frac{6E}{5R} = 1\text{ A}$$

187. d. $$\phi = 2\pi cr$$

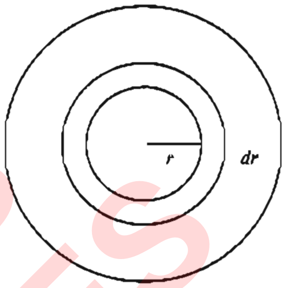


Fig 8.454

$$E = \frac{d\phi}{dt} = 2\pi c\frac{dr}{dt} \qquad [r = r_0 + kt]$$

So E is constant

188. d. The front view of the arrangement is as shown in Fig. 8.455

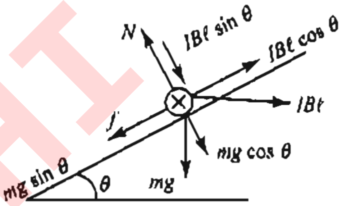


Fig 8.455

From initial condition, $mg\sin\theta = \mu mg\cos\theta$

$$\Rightarrow \quad \mu = \tan\theta$$

$$ma = IB\ell\cos\theta - mg\sin\theta - \mu N$$

$$N = mg\cos\theta + IB\ell\sin\theta$$

$$\Rightarrow \quad a = \frac{IB\ell}{m}\cos\theta - 2g\sin\theta - \frac{IB\ell}{m}\frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{IB\ell}{m}\frac{\cos2\theta}{\cos\theta} - 2g\sin\theta$$

Now, $$s = \frac{1}{2}at^2 = \frac{1}{2}\left[\frac{IB\ell}{m}\frac{\cos2\theta}{\cos\theta} - 2g\sin\theta\right]t^2$$

189. b. At $t = 0$, charge on C is zero, so $p.d.$ across C is zero, so also across R the $p.d.$ is zero. Hence there is no current in R.
At $t = \infty$, current through L is maximum and constant, so $p.d.$ across L is zero, therefore p.d. across R is zero. Hence no current in R.

190. c. $$U_{max} = \frac{1}{2}LI_0^2i,\ U = \frac{U_{max}}{2}$$

$$\Rightarrow \frac{1}{2}LI^2 = \frac{1}{2}\left[\frac{1}{2}LI_0^2\right] \Rightarrow I_0^2\left[I - e^{-t/\tau}\right]^2 = \frac{I_0^2}{2}$$

$$\Rightarrow I - e^{-t/\tau} = I - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\Rightarrow \quad -\frac{t}{\tau} = \ln\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) \Rightarrow t = \tau\ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$$

$$\Rightarrow \qquad t = \frac{L}{R}\ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$$

191. c. Because A and C are at equal distance from B, and their flux across B is in opposite direction, so at any time flux in B will be zero. Hence no e.m.f. is induced.

192. a.
$$\omega = \frac{1}{\sqrt{L_{eq}\,L_{eq}}} = \frac{1}{\sqrt{2LC/2}} = \frac{1}{\sqrt{LC}}$$

193. a.
$$\xi(t) = -\frac{d\phi_m}{dt};\ \xi(t) = -\frac{d}{dt}[Ba]$$
$$\xi(t) = -a\frac{dB}{dt}$$
$$\xi(t) = -a\beta < 0 \qquad [\because \beta > 0]$$

The e.m.f. is constant and negative, so that induced electric field points around the ring F_2 towards F_1. So face F_1 will develop an excess positive charge.

194. a.
$$\xi = \frac{d\phi}{dt} \qquad [\text{in magnitude}]$$
$$i = \frac{\xi}{R} = \frac{1}{R}\frac{d}{dt}(BA) = \frac{A}{R}\frac{dB}{dt}$$

where $A = \pi r^2$ = area of loop of radius r and R = resistance of the loop of length $(2\pi r)$ and area of cross-section πa^2.

$$R = \frac{\rho\ell}{\pi a^2} = \frac{\rho(2\pi r)}{\pi a^2}$$

Further mass of wire is $m = (\pi a^2)(2\pi r)(d)$

$$i = \frac{(\pi a^2)(\pi r^2)}{\rho(2\pi r)}\frac{dB}{dt}$$
$$i = \frac{(\pi a^2)(2\pi r)}{4\pi\rho}\frac{dB}{dt} \quad \Rightarrow \quad i = \frac{m}{4\pi\rho d}\frac{dB}{dt}$$

195. a. At $t = 0$, i.e., when the key is just pressed, no current exists inside the inductor. So 10 Ω and 20 Ω resistors are in series and a net resistance of (10 + 20) = 30 Ω exists across the circuit.

Hence, $I_1 = \frac{2}{30} = \frac{1}{15}$ A

As $t \to \infty$, the current in the inductor grows to attain a maximum value, i.e., the entire current passes through the inductor and no current passes through 10 Ω resistor.

Hence, $I_2 = \frac{2}{20} = \frac{1}{10}$ A

196. c. I_0 = Peak value $= \frac{E}{2R}$

Total heat produced across R is $H = \frac{1}{2}LI_0^2$

$$H = \frac{1}{2}(2L)\frac{E^2}{4R^2} \quad \Rightarrow \quad H = \frac{LE^2}{4R^2}$$

197. a. From Lenz's law if one rod is moved away from the second rod then the second rod will be attracted towards the first rod.

198. d. Potential difference across capacitor

$$V = Bv\ell = \text{constant}$$

Therefore, change stored in the capacitor is also constant. Thus, current through the capacitor is zero.

199. a. $v_b - v_a = \vec{B}\cdot(\vec{\ell}\times\vec{v})$

$(\vec{B}\times\vec{\ell})\cdot\vec{\ell} = \vec{v}\cdot(\vec{B}\times\vec{\ell})$

$(\vec{v}\times\vec{B})\cdot\vec{\ell} = \vec{\ell}\cdot(\vec{v}\times\vec{B})$

200. c. The equivalent resistance of five resistor is 3 Ω
(It is a balanced Wheatstone's bridge)

So $\qquad R_{total} = 3 + 1 = 4\ \Omega$

$\Rightarrow \qquad (10^{-3})(4) = (2)(0.1)V \qquad (\because \xi = IR = B\ell V)$

$\Rightarrow \qquad V = 2\ \text{cms}^{-1}$

201. c. The relative velocity of approach becomes $2v$. (i.e., doubled) so induced e.m.f. is also doubled i.e. becomes 2ε.

202. d.
$$|\varepsilon| = \frac{d\phi}{dt}$$
$$\Rightarrow \quad |\varepsilon| = A\frac{dB}{dt}$$
$$\Rightarrow \quad |\varepsilon| = (d^2)\frac{d}{dt}\left[B_0\left(1+\frac{x}{a}\right)\right]$$
$$\Rightarrow \quad |\varepsilon| = (d^2)B_0\left(0+\frac{1}{a}\frac{dx}{dt}\right)$$
$$\Rightarrow \quad |\varepsilon| = \frac{B_0 d^2}{a}\left(\frac{dx}{dt}\right)$$
$$\Rightarrow \quad |\varepsilon| = \frac{B_0 d^2 V_0}{a}$$

203. b. e.m.f. induced across the rod AB is

$$e = \vec{B}\cdot(\vec{\ell}\times\vec{v})$$
$$= B\ell v \sin\theta$$
$$= 2\times2\times2\times\sin 30$$
$$e = 4\text{V}$$

Free electrons of the rod shift towards right due to force $q(\vec{v}\times\vec{B})$

Thus end P is at higher potential

or $V_P - V_Q = 4$V

Thus, choice (b) is correct.

204. d. See the unit vector of $\vec{B}$ and $\vec{v}$ make e.m.f. zero.

Multiple Correct Answers Type

1. b., d.

$$\phi = \pi a^2 B$$
$$e = \pi a^2\frac{dB}{dt} = \pi a^2\alpha$$

Let R be the resistance of the ring. Then current in the ring is $i=\frac{e}{R}$

Consider a small element $d\ell$ on the ring.

e.m.f. induced in the element, $de=\left(\frac{e}{2\pi a}\right)d\ell$

resistance of the element, $dR=\left(\frac{R}{2\pi a}\right)d\ell$

∴ Potential difference across the element

$$=de-idR$$

$$=\left(\frac{e}{2\pi a}\right)d\ell-\left(\frac{e}{R}\right)\left(\frac{R}{2\pi a}\right)d\ell=0$$

2. a., b.

At $t<0, I_L=\frac{6}{6}=1\text{ A}$

At $t>>0, I_L=\frac{12}{3}=4\text{ A}$

∴ $|\phi|=L[i_f-i_i]=500\times10^{-3}\times3=1.5\text{ Wb}$

∴ (a) and (b) are the correct choices.

3. a., b., c.

$$\phi=4t^n+6$$

$$\frac{d\phi}{dt}=4nt^{n-1}$$

$$|e|=4nt^{n-1},\ |e|=\frac{4n}{t^{1-n}}$$

4. b., d.

Flux remains constant here, so e.m.f. Induced is zero.

5. a., d.

Flux changes due to the rotation of the semi-circle.

6. b., d.

$$i=\frac{dq}{dt}=\frac{d}{dt}(CvB\ell)=CB\ell\frac{dv}{dt}=CB\ell a$$

∴ $F-CB^2\ell a=ma$

⇒ $a=\frac{F}{M+B^2\ell^2C}$

⇒ e.m.f. increases

⇒ charge increases.

7. a., d.

$$E=\int v\vec{B}\times d\vec{\ell}$$

8. a., b., d.

Rate of work done by external agent is

$\frac{de}{dt}=\frac{BIL.dx}{dt}=BILv$ and thermal power dissipated in the resistor $=eI=(BvL)I$ clearly both are equal, hence (a).

If applied external force is doubled, the rod will experience a net force and hence acceleration. As a result velocity increases, hence (b)

Since, $I=\frac{e}{R}$

On doubling R, current and hence required power become half.

Since, $P=BILv$

Hence (d)

9. b., d.

Equivalent circuit:

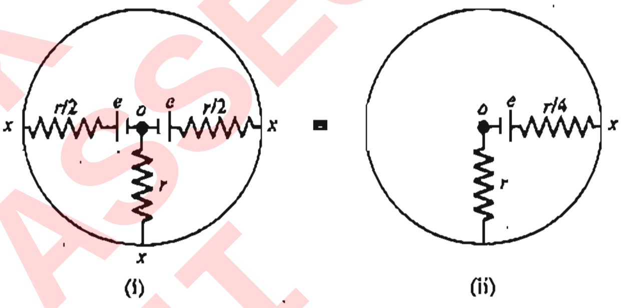


Fig 8.456

Induced e.m.f. $e=\frac{B\omega r^2}{2}=\left(\frac{B\omega a^2}{2}\right)$ (∵ radius $=a$)

By nodal equation:

$$4\left(\frac{X-e}{r}\right)+\left(\frac{X-0}{r}\right)=0$$

$$5X=4e$$

⇒ $x=\frac{4e}{5}=\frac{2B\omega a^2}{5r}$

and $I=\frac{X}{e}=\frac{2B\omega a^2}{5r}$

Also direction of current in (i) will be toward negative terminal, i.e., from rim to origin Alternating, by equivalent of cells [Fig. 8.460 (b)]:

$$I=\frac{e}{r+\frac{r}{4}}=\frac{4e}{5r}$$

10. a., b.

As $\vec{B}\perp\vec{A}$, hence $\phi=0$ and $e=0$.

11. a., c.

$$L\frac{d^2q}{dt^2}+\frac{q}{C}-\frac{CV-q}{C}=0$$

$$\frac{d^2q}{dt^2}+\frac{2q-CV}{LC}=0$$

$$\omega=\sqrt{\frac{2}{LC}}$$

$$q = \frac{CV}{2}[1 - \cos \omega t]$$

$$q' = CV - q = \frac{CV}{2}[1 + \cos \omega t]$$

$$I = +\frac{CV\omega}{2}\sin \omega t$$

12. a., c.

Area in $(v_L - t)$ graph = $L\,\Delta I$

$$\Rightarrow 2(I_f - 0) = \frac{1}{2} \times 10 \times 2$$

$$\Rightarrow \quad I_f = 5\text{A}$$

13. a., b., c., d.

Due to rotation, e.m.f. = $\dfrac{Br^2\omega}{2}$

Due to translation indeed e.m.f. = Bvr

Where r is the separation.

14. a., b.

$$L\frac{di}{dt} = Bv\ell$$

$$i = \frac{B\ell}{L}x \text{ and } \frac{B^2\ell^2 x}{L} = -mv\frac{dv}{dx}$$

$$d = \sqrt{\frac{3v_0^2\, mL}{4B^2\ell^2}} \text{ where } v_0 = \frac{J}{M}$$

15. a., b., d. $|e| = BAW \sin \omega t$

$\phi = BA \cdot \cos \omega t$

$|e|$ is maximum when $\omega t = \dfrac{\pi}{2}$

So ϕ is zero.

$|e|$ is zero then $\omega t = 0$ so $\phi =$ is maximum.

16. b., c.

17. a., c.

Current induced in both A and B will be in same direction. So they will attract each other.

A is closer to magnet, so rate of change of M in A will be more. So more current is induced in A.

18. a., b.

The coils are in parallel, so

$$L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt} \Rightarrow \int L_1 dI_1 = \int L_2 dI_2$$

$$\Rightarrow \quad \text{Initially } I_1 = 0, I_2 = 0 \quad \Rightarrow \quad C = 0$$

So $\quad L_1 I_1 = L_2 I_2$

19. a., d.

$$e = \frac{1}{2} bw(2r)^2 = Bw2r^2 = Bv2r$$

Net induced e.m.f. in the ring will be zero. Hence no current is induced.

20. a., d.

$$E = \frac{|\xi|}{\delta} = \frac{a\beta}{\delta}$$

This expression is independent of R as long as the radius of the ring exceeds the radius $\sqrt{\dfrac{a}{x}}$ of the solenoid.

21. b., c.

Each wire can be replaced by a battery whose e.m.f. is equal to

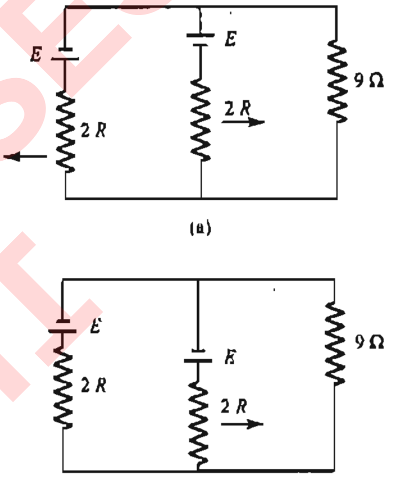
$$B\ell v = 1 \times 4 \times 10^{-2} \times 5 \times 10^{-2}$$
$$= 20 \times 10^{-4}\text{ V}$$

The polarity of the battery can be given by Fleming's right hand rule. When both wire move in opposite direction, the circuit diagram looks like as shown in Fig. 8.457 (a).

The effective e.m.f. of the two batteries shown in the diagram is zero.

So, choice (b) is correct and choice (d) is wrong.

When both wires move towards left, the circuit diagram looks like as shown in Fig. 8.457 (b)



(a)

(b)

Fig. 8.457

Effective e.m.f. of two battery shown in $E\,(= 20 \times 10^{-4}\text{ V})$ and internal resistance is 1 Ω.

Hence, current in the circuit is

$$I = \frac{20 \times 10^{-4}}{10} = 0.2\text{ mA}$$

Hence, choice (c) is correct and choice (a) is wrong.

22. a., c.

Use Lenz's law. The motion of ring will be opposed.

23. a., b., c., d.

Use concept of motional e.m.f.

24. a., d.

At the poles, the earth's magnetic field is vertical.

25. c., d.

Replace the ring by a diameter perpendicular to its direction of motion. The spin of a ring about its axis causes no e.m.f

26. a., b., d.

Charge flowing in the circuit = $\dfrac{\Delta\phi}{R}$

where, $\Delta\phi$ = change in flux

$= \phi_{\text{final}} - \phi_{\text{initial}}$

and $\quad R$ = resistance in the circuit

27. a., c., d.

Charge flowing in the circuit = $\dfrac{\Delta\phi}{R}$

where, $\Delta\phi$ = change in flux

$= \phi_{\text{final}} - \phi_{\text{initial}}$

and R = resistance in circuit

28. a., d.

If the normal to the plane of the coil makes an angle θ with the direction of B, the flux linked with the coil is

$$\phi = BAn\cos\theta$$
$$= BAn\cos(\omega t)$$

($\because$ the coil rotates with an angular velocity ω)

$$\text{e.m.f.} = e = \frac{d\phi}{dt} = BAn\omega\sin(\omega t)$$

29. c., d.

When switch is just closed in the circuit shown, at that moment current through the circuit is zero. Hence, e.m.f. induced across the inductance L will be equal to e.m.f. E of the battery.

But as the current through the circuit increases, the induced e.m.f. in the solenoid decreases. But induced e.m.f. in the solenoid is equal to $|e| = L\dfrac{di}{dt}$.

Since $\dfrac{di}{dt}$ decreases as the time passes, therefore, the graph for induced e.m.f. e and time t will be as shown in option (c). Hence (a) is wrong and (c) is correct.

The graph for current should be such that at initial moment current is zero and current increases with the time in such a way that the rate of increase of current gradually decreases. Hence, slope of the current–time curve should decrease with time. Therefore, the graph between current and time will be as shown in option (d).

Hence (b) is wrong and (d) is correct.

30. a., c.

In the circuit shown in the figure in problem, 6 Ω and 12 Ω resistances are in parallel with each other and their parallel combination is in series with 4 Ω and the inductance of 2H. Hence, equivalent resistance of these three resistances is equal to 8 Ω. Therefore, this circuit may be reduced to the circuit as shown in figure.

The time constant for the circuit is

$$\lambda = \frac{L}{R} = \frac{2}{8} = 0.25\text{ s}$$

Hence (a) is correct.

In steady state, no e.m.f. will be induced in the inductance. Hence current through the circuit will be equal to E/R where R is the equivalent resistance. Hence, the steady state current will be equal to $\dfrac{6}{8} = 0.75$ A.

Hence (c) is correct and (b) is wrong.

Assertion-Reasoning Type

1. a. It is obvious that flux linkage in one ring due to current in other coaxial ring is maximum when $x = 0$ (as shown) or the rings are also coplanar. Hence under this condition their mutual induction is maximum.

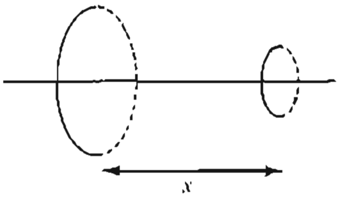


Fig. 8.458

2. a. As, $\varepsilon = -\dfrac{d\phi}{dt}$

$\therefore$ there will be no change in the flux in DC.

3. a. $I = -\dfrac{\Delta\phi_B}{R}$

As R is constant, $I \propto \Delta\phi_B$.

4. d. Even though flux through individual lines changes, it remains unchanged for the solenoid as a whole. Therefore no e.m.f. is induced in the long solenoid.

5. d. Lenz's law is based on conservation of energy and induced e.m.f. always opposes the cause of it, i.e., change in magnetic flux.

6. c. Presence of magnetic flux cannot produce current.

7. a. If inductance of solenoid increases, reactance of circuit also increases, then obviously current will decrease and light becomes dim.

8. a. When AC passes, due to local oscillations of bound charges, current is conducted but in DC, as there are no free charge carriers in dielectric, the current cannot be conducted.

9. c. L is dependent only upon geometrical parameter.

10. a. $\Delta W = q(\Delta V)$

Here ΔV = non-zero in a closed loop.

11. a. $\oint \vec{B}\cdot d\vec{l}$ along any closed path within a uniform magnetic field is always zero. Hence the closed path can be chosen of any size, even very small size enclosing a very small area. Hence we can prove that net current through each area of infinitesimally small size within region of uniform magnetic field is zero. Hence we can say no current (rather than no net current) flows through region of uniform magnetic field. Hence statement 2 is correct explanation of statement 1.

12. d. The current in R-L circuit grows exponentially

$$I = I_0\left(1 - e^{-Rt/L}\right)$$

13. c. Magnetic lines are present in closed loop.

14. c. Electric field generated from time dependent magnetic field obeys Lenz's law.

15. d. When current due to external source decreases, induced current will be in same direction.

16. b. For solenoid $B_{\text{end}} = \dfrac{1}{2}(B_{\text{in}})$

Also for long solenoid the magnetic field is uniform within it but this reason is not explaining the assertion.

17. a. The force on a charged particle moving in a uniform magnetic field always acts in direction perpendicular to the direction of motion of charge. As work done by magnetic field on the charge is zero, $W = FS\cos\theta$, so the energy of charged particle does not change.

18. c. Coefficient of coupling between them

$$M = K^2 L_1 L_2$$

When two coils are wound on each other, the coefficient of coupling is maximum and hence mutual inductance between the coils is maximum.

19. d. As the coil rotates, the magnetic flux linked with the coil (being $\vec{B}\cdot\vec{A}$) will change and e.m.f. may be induced in the loop.

20. b. According to Lenz's Law when a magnet is moved towards the coil, the direction of induced current is such that the coil repels the magnet and when the magnet moves away from the coil, the coil attracts the magnet.

21. a. The capacitive reactance of a capacitor is given by

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi fc}$$

So this is infinite for *d.c.* ($f = 0$) and has a very small value for *a.c.*, therefore a capacitor blocks *d.c.* Hence (1) is true.

22. d. Magnetic field cannot do work, hence statement 1 is false.

23. a. Obviously statement 2 is the correct explanation of statement 1.

24. a. $\oint \vec{B}\cdot d\vec{\ell}$ along any closed path within a uniform magnetic field is always zero. Hence the closed path can be chosen of any size, even very small size enclosing is very small area. Hence we can prove that net current through each area of infinitesimally small size within a region of uniform magnetic field is zero. Hence we can say no current (rather than no net current) flows through the region of uniform magnetic field. Hence statement 2 is the correct explanation of statement 1.

Comprehension Type

For Problems 1–2

1. b., 2. c.

Sol. $\varepsilon_{eq} = -\frac{\Delta\Phi_B}{\Delta t} = -B\frac{\Delta A}{\Delta t} = -B\frac{-\pi r^2}{\Delta t}$

$$= \frac{1\times\pi(0.10)^2}{0.314} = 0.1 \text{ V}$$

Since the flux through the loop is decreasing, the induced current must produce a field that goes into the page. Therefore the current flows from point in clockwise direction $I_{av} = \frac{\varepsilon_{av}}{R}$

$$= \frac{0.1}{0.01} = 10 \text{ A}$$

For Problems 3–4

3. b., 4. a.

Sol. $\Phi_B = \int_a^b d\Phi_B = \frac{\mu_0 iL}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 iL}{2\pi}\ln(b/a)$

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{\mu_0 L}{2\pi}\ln(b/a)\frac{di}{dt}$$

For Problems 5–6

5. a., 6. c.

Sol. $A = \frac{1}{2}r^2\theta \Rightarrow \frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt}$

$$\varepsilon = -\frac{d\Phi}{dt} = -B\frac{1}{2}r^2\frac{d\theta}{dt} \quad (\text{as } \phi = BA)$$

$$\varepsilon = -\frac{1}{2}\omega Br^2$$

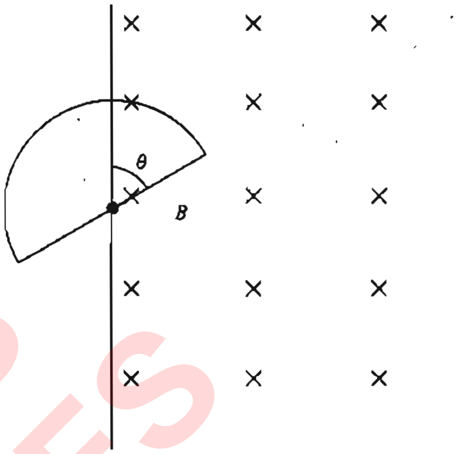


Fig. 8.459

The current is anticlockwise in the loop as it is entering region II.

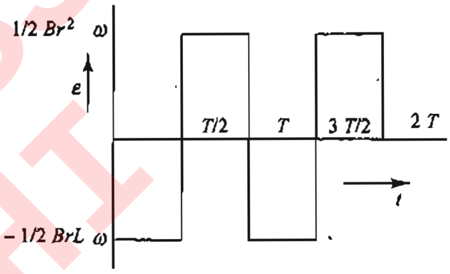


Fig. 8.460

For Problems 7–8

7. b., 8. c.

Sol. Magnetic field on the axis of a circular coil is given by

$$B = \frac{\mu_0 iR^2}{2(R^2+x^2)^{3/2}}$$

Since $x >> R$, therefore, magnetic field at the centre of the smaller loop is

$$B \approx \frac{\mu_0 iR^2}{2x^3}$$

Flux linked with coil is

$$\phi = B(\pi r^2) = \frac{\mu_0 \pi iR^2 r^2}{2x^3}$$

b. From Faraday's law we have

$$E = -\frac{d\phi}{dt} = \frac{3}{2}\frac{\mu_0\pi iR^2r^2}{x^4}v$$

As the external magnetic field at the position of smaller coil is decreasing, the induced magnetic field is parallel to external field.

Hence the direction of current is anti clockwise when seen from the top.

For Problems 9–10

9. a., 10. b.

Sol. e.m.f. is induced in the loop because area inside the magnetic field is continually changing.

From $q = 0$ to π, 2π to 3π, 4π to 5π, the loop begins to enter the magnetic field. Thus the magnetic field passing through the loop is increasing. Hence, current in the loop is anticlockwise, and for $\theta = \pi$ to 2π, 3π to 4π. 5π to 6π, etc. magnetic field passing through the loop is decreasing. Hence current in the loop is clockwise.

Angular acceleration of the loop is constant, therefore angle turned in time t is $\theta = 1/2\alpha t^2$ and time taken, $t = \sqrt{2\pi/\theta}$

t_1 = time taken to rotate angle

$$\pi = \sqrt{\frac{2\pi}{\alpha}}$$

From $t = 0$ to $t = t_1$, e.m.f. is negative,

t_2 = time taken to rotate angle $2\pi = \sqrt{\dfrac{4\pi}{\alpha}}$

From $t = t_1$ to $t = t_2$, e.m.f. is positive

t_n = time taken to rotate angle

$$n\pi = \sqrt{\frac{2n\pi}{\alpha}}$$

Area inside the field is $A = \dfrac{1}{2}R^2\theta$

$$A = \frac{1}{4}R^2\alpha t^2$$

So flux passing through the loop,

$$\phi = BA = \frac{1}{4}BR^2\alpha t^2$$

$$e = \left|\frac{d\phi}{dt}\right| = \frac{1}{2}BR^2\alpha t$$

$$e \propto t$$

i.e., e-t graph is a straight line passing through the origin. e-t equation with sign can be written as

$$e = (-1)^n\left(\frac{1}{2}BR^2\alpha t\right)$$

Hence $n = 1, 2, 3, \ldots$ is the number of half revolutions that the loop performs at the given moment.

The e-t graph is as shown in figure (b).

For Problems 11–13

11. d., 12. a., 13. b.

Sol. The large circuit is a circuit with a time constant of

$$\tau = RC = (10\ \Omega)(20 \times 10^{-6}\ \text{F}) = 200\mu\text{s}.$$

Thus, the current as a function of time is

$$i = \left(\frac{100\ \text{V}}{10\ \Omega}\right)e^{-\frac{t}{200\mu\text{s}}}$$

At $t = 200$ ms, we obtain

$$i = (10\ \text{A})(e^{-1}) = 3.7\ \text{A}.$$

Assuming that only the long wire nearest the small loop produces an appreciable magnetic flux through the small loop,

$$\Phi_B = \int_c^{c+a}\frac{\mu_0 ib}{2\pi r}dr = \frac{\mu_0 ib}{2\pi}\ln\left(1 + \frac{a}{c}\right)$$

So the e.m.f. induced in the small loop at $t = 200$ ms

$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{\mu_0 b}{2\pi}\ln\left(1 + \frac{a}{c}\right)\frac{di}{dt}$$

$$= -\frac{(4\pi \times 10^{-7}\ \frac{\text{Wb}}{\text{A} \times \text{m}^2})(0.200\ \text{m})}{2\pi}$$

$$\times \ln(3.0)\left(-\frac{3.7\ \text{A}}{200 \times 10^{-6}\ \text{s}}\right)$$

Thus, the induced current in the small loop is

$$i' = \frac{\varepsilon}{R} = \frac{0.81\ \text{mV}}{25(0.600\ \text{m})(1.0\ \Omega/\text{m})} = 54\ \mu\text{A}.$$

The induced current will act to oppose the decrease in flux from the large loop. Thus, the induced current flows counterclockwise.

Three of the wires in the large loop are too far away to make a significant contribution to the flux in the small loop—as can be seen by comparing the distance to the dimensions of the large loop.

For Problems 14–15

14. a. ,15. i., c., ii., a.

Sol. Case I

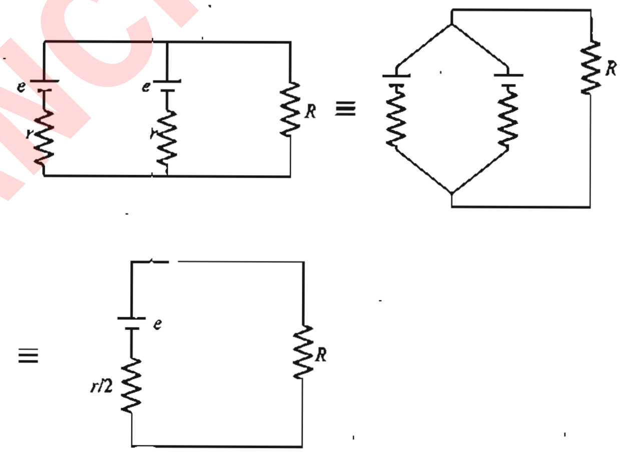


Fig. 8.461

$$I = \frac{e}{R + (r/2)} = \frac{Blv}{R + (r/2)}$$

Case II

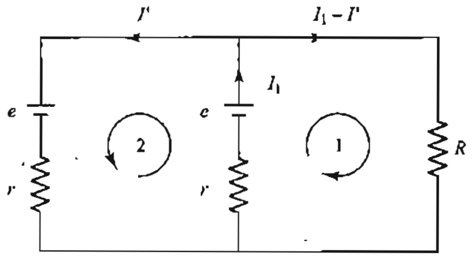


Fig. 8.462

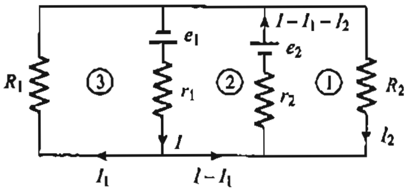
$$-(I_1 - I')R - I_1 r + e = 0 \quad \text{For loop (1)} \qquad \text{(i)}$$

$$r(I_1 + I') = 2e \quad \text{For loop (2)} \qquad \text{(ii)}$$

Solve to get, $I_1 = I' = \dfrac{e}{R}$

Hence current in 'R' is zero.

11. $e_1\ Bl\ v_1,\ e_2 = Bl\ v_2$



For (1) $\rightarrow e_2 = (I - I_1 + I_2)r_2 + I_2 R_2$

For (2) $\rightarrow e_1 + e_2 = (I - I_1 + I_2)r_2 + Ir_1$

For (3) $\rightarrow e_1 = Ir_1 + I_1 R_1$

Solve to get $I_1 = \dfrac{BlR_2(v_1 r_2 - v_2 r_1)}{R_1 R_2(r_1 + r_2) + r_2 r_1(R_1 + R_2)}$

For Problems 16–17

16. b., 17. a.

Sol. Note that the magnitude of magnetic field as well as area enclosed by the circuit is varying.

The magnetic flux is

$$\phi = BA = B\ell x$$

$$\frac{d\phi}{dt} = B\frac{dA}{dt} + \frac{dB}{dt}A \quad = Bl\frac{dx}{dt} + \frac{dB}{dt}A$$

In the given problem $\dfrac{dx}{dt} = -v$ as it tends to decrease the flux.

According to Faraday's law,

$$E_{\text{induced}} = -\frac{d\phi}{dt} = -\left[-B\ell v + \frac{dB}{dt}A\right]$$

$$= -\left[(0.1)(5\times10^{-2})(5\times10^{-2}) + (0.2)\times (5\times5\times10^{-4})\right]$$

$$= -25\times10^{-5}\ \text{V}$$

The net rate of change of flux is positive, hence the secondary magnetic field must oppose it; it is directed into the page. The induced current is clockwise in accordance with the right hand thumb rule.

Induced current $I = \dfrac{25\times10^{-5}}{10^{-4}} = 2.5$ A

For Problems 18–20

18. a., 19. c., 20. b.

Sol. a. The current of the battery at any instant, $I = E/R$.

The magnetic force due to this current

$$F_B = IBL = \frac{EB\ell}{R}$$

This magnetic force will accelerate the rod from its position of rest. The motional e.m.f. developed in the rod is $B\ell v$;

the induced current,

$$I_{\text{induced}} = \frac{B\ell v}{R}$$

The magnetic force due to the induced current,

$$F_{\text{induced}} = I_{\text{induced}} = \frac{B^2\ell^2 v}{R}$$

From Fleming's left hand rule, force F_B is to the right and F_{induced} is to the left. Net force on the rod = $F_B - F_{\text{induced.}}$

From Newton's second law,

$$F_B = F_{\text{induced}} = m\frac{dv}{dt}$$

$$\frac{EBl}{R} - \frac{B^2 l^2 v}{R} = m\frac{dv}{dt}$$

On separating variables and integrating speed from v_0 to v and time from 0 to t, we have

$$\frac{dv}{E - Bvl} = \frac{Bl}{mR}dt$$

$$\int_0^v \frac{dv}{(E - Bvl)} = \frac{Bl}{mR}\int_0^t dt$$

$$\ln\left(\frac{E - Bvl}{E}\right) = -\frac{B^2 l^2}{mR}t$$

$$\frac{E - Bvl}{E} = e - \frac{B^2 l^2}{mR}t$$

$$v = \frac{E}{Bl}(1 - e^{-t/\tau})$$

where $$t = \frac{mR}{(Bl)^2}$$

b. The rod will attain a terminal velocity at $t \rightarrow \infty$, i.e., when $e^{-t/t} = 0$, the velocity is independent of time.

$$v_T = \frac{E}{Bl}$$

c. The induced current $I_{\text{induced}} = Blv/R$. When the rod has attained terminal speed,

$$I_{\text{induced}} = \frac{Bl}{R}\times\left(\frac{E}{Bl}\right) = E/R$$

The current of battery and the induced current are of same magnitude, hence net current through the circuit is zero.

For Problems 21–22

21. a., 22. b.

Sol. $$E = \frac{r}{2}\left(\frac{dB}{dt}\right)$$

$$B = \mu nI$$

$$\frac{dB}{dt} = \mu n\frac{dI}{dt}$$

$$= \mu_0(900)\ 60 = (4\pi\times10^{-7})\times900\times60$$

$$= 216\pi\times10^{-4}\ \text{T/s}$$

a. $$E = \left(\frac{0.5}{2}\times10^{-2}\right)\times216\pi\times10^{-6}$$

$$= 54\pi\times10^{-8}\ \text{N/C}$$

b. $$E = 100\pi\times10^{-8}\ \text{N/C}$$

For Problems 23–27

23. c., 24.a., 25.b, 26.b., 27.b.

Sol. $\frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi r_1^2 \frac{dB}{dt}$

$$E = \frac{1}{2\pi r_1}\frac{d\Phi_B}{dt} = \frac{\pi r_1^2}{2\pi r_1}\frac{dB}{dt} = \frac{r_1}{2}\frac{dB}{dt}$$

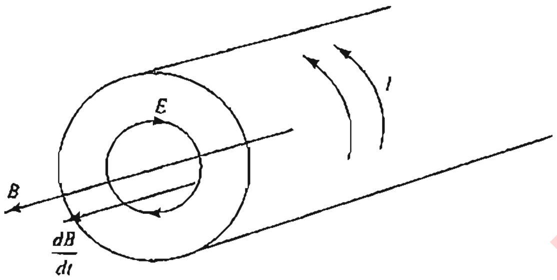


Fig. 8.463

$$E = \frac{1}{2\pi r_2}\frac{d\Phi_B}{dt} = \frac{\pi R^2}{2\pi r_2}\frac{dB}{dt} = \frac{R^2}{2r_2}\frac{dB}{dt}$$

For Problems 28–29

28. d., 29. b.

Sol. First let us consider an external point and take the path for out line integral to be a circle of radius r centered on the solenoid, as illustrated in Fig. 8.465.

By symmetry we see that the magnitude of electric field is constant on this path and that is tangent to it. The magnetic flux through the area enclosed by this path is $BA = B\pi R^2$; hence the equation gives

$$\oint \vec{E}\cdot d\vec{s} = \frac{d}{dt}(B\pi R^2) = -\pi R^2\frac{dB}{dt}$$

$$\oint \vec{E}\cdot d\vec{s} = E(2\pi r) = -\pi R^2\frac{dB}{dt} \qquad \text{(i)}$$

The magnetic field inside the long solenoid is given by $B = m_0 nI$. When we substitute $I = I_{\max}\cos\omega t$ into this equation and then substitute the result into equation (i), we find that

$$E(2\pi r) = -\pi R^2\mu_0 nI_{\max}\frac{d}{dt}(\cos\omega t) = \pi R^2\ \mu_0 nI_{\max}\sin\ \omega t$$

$$E = \frac{\mu_0 nI_{\max}\omega R^2}{2r}\sin\ \omega t \qquad (\text{for } r > R)$$

Hence, the electric field varies sinusoidally with time and its amplitude falls off as $1/r$ outside the solenoid.

Proceeding as above, we find that

$$E(2\pi r) = -\pi r^2\frac{dB}{dt} = \pi r^2\mu_0 nI_{\max}\sin\ \omega t$$

$$E = \frac{\mu_0 nI_{\max}\omega}{2r}r\sin\ \omega t \qquad (\text{for } r < R)$$

This shows that the amplitude of the electric field induced inside the solenoid by the changing magnetic flux through the solenoid increases linearly with r and varies sinusoidally with time.

For Problems 30–32

30. c. 31., d., 32. b.

Sol. a. For clarity, figure is rotated so it comes out of the page.

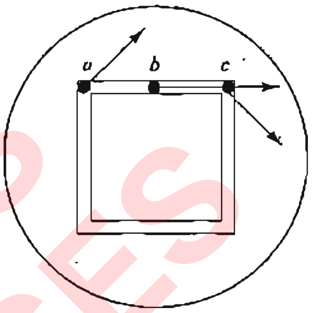


Fig. 8.464

b. $I = \frac{\varepsilon}{R} = \frac{A}{R}\frac{dB}{dt} = \frac{L^2}{R}\frac{dB}{dt}$

$$= \frac{(0.20\,\text{m})^2(0.0350\,\text{T/s})}{1.90\,\Omega}$$

$$= 7.37\times10^{-4}\,\text{A}$$

c. $\varepsilon_{ab} = \frac{1}{8}\varepsilon = \frac{1}{8}L^2\frac{dB}{dt} = \frac{(0.20\,\text{m})^2(0.0350\ \text{T/s})}{8}$

$$= 1.75\times10^{-4}\,\text{V}$$

But there is a potential drop

$$V = IR = 1.75\times10^{-4}\,\text{V},$$

so the potential difference is zero.

For Problems 33–34

33. b., 34., b.

Sol. i. $E = \frac{r}{2}\frac{dB}{dt}$

$$\frac{dB}{dt} = B_0$$

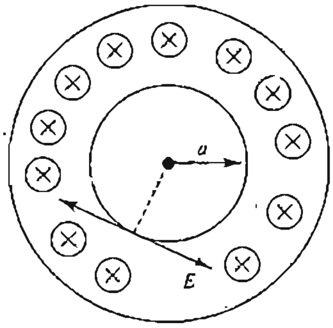


Fig. 8.465

$$E = \frac{a}{2}B_0 \qquad \text{(i)}$$

$$d\tau = dF\cdot a = (dqE)\cdot a$$

$$\tau = \int d\tau = Ea\int dq = Eaq$$

$$\tau = I\alpha$$

$$\alpha = \frac{\tau}{I} = \frac{Eaq}{(ma^2)} = \frac{Eq}{ma}$$

$$\alpha = \frac{Eq}{ma} \text{ anticlockwise}$$

ii. Power

$$P = \tau \cdot \omega$$

$$= (Eaq)\,(\alpha t) = Eaq\frac{Eq}{ma}t$$

$$P = \frac{E^2q^2}{m}t$$

For Problems 35–40

35. b. **36.** a., **37.** c., **38.** a., **39.** c., **40.** b.

Sol. Instantaneous current in the capacitor,

$$q = CV_C = (2)(3e^{-2t}) = 6e^{-2t}\text{ A}$$

$$\text{Current, } i_C = \frac{dq}{dt} = -12e^{-2t}\text{ A}$$

Current flows from B to O.

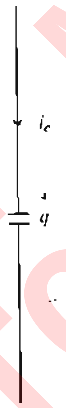


Fig. 8.466

From KVL, we have

$$i_L = i_1 + i_2 + i_C = 10e^{-2t} + 4 - 12e^{-2t}$$

$$= (4 - 2e^{-2t})\text{ A} = [2 + 2(1 - e^{-2t})]\text{ A}$$

i_L vs. time graph is as shown in Fig. 8.467.
i_L increases from 2 A to 4 A exponentially.

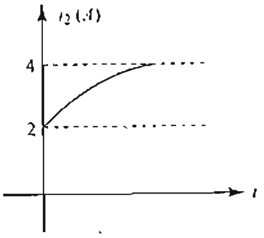


Fig. 8.467

$$V_L = L\frac{di_L}{dt}$$

$$= (4)\frac{d}{dt}(4 - 2e^{-2t}) = 16e^{-2t}\text{ V}$$

V_L decreases exponentially from 16 A to 0 as shown in Fig. 8.468.

To determine V_{AC}, we begin from A and end at C. From KVL, we have

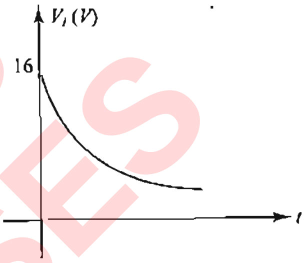


Fig. 8.468

$$V_A - i_1R_1 + i_2R_2 = V_C$$

$$V_A - V_C = i_1R_1 - i_2R_2$$

Substituting the values, we have

$$V_{AC} = (10e^{-2t})(2) - (4)(3)$$

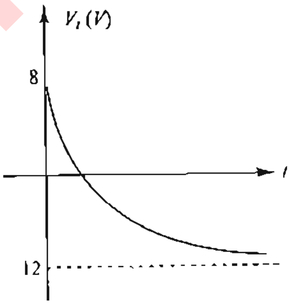


Fig. 8.469

$$V_{AC} = (20e^{-2t} - 12)\text{ V}$$

At $t = 0,\ V_{AC} = 8\text{ V}$

At $t = \infty,\ V_{AC} = -12\text{ V}$

Therefore, V_{AC} decreases exponentially from 8 V to – 12 V.

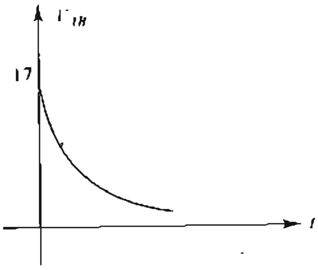


Fig. 8.470

Similarly, we have from A to B

$$V_A - i_1R_1 + V_C = V_B$$

$$V_{AB} = V_A - V_B = i_1 R_1 - V_C$$

Substituting the values we have ,

$$V_{AB} = (10e^{-2t})(2) - 3e^{-2t}$$

$$V_{AB} = 17e^{-2t}\ \text{V}$$

Thus, V_{AB} decreases exponentially from 17 V to 0.
As we move from C to D,

$$V_C - i_2 R_2 - i_L = V_D$$

$$V_{CD} = V_C - V_D = i_2 R_2 + i_L$$

Substituting the values we have ,

$$V_{CD} = (4)(3) + 16e^{-2t}$$

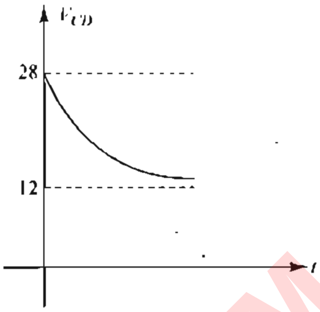


Fig. 8.471

$$V_{CD} = (12 + 16e^{-2t})\ \text{V}$$

At $t = 0$, $V_{CD} = 28$ V

and at $t = \infty$, $V_{CD} = 12$ V

i.e., V_{CD} decreases exponentially from 28 V to 12 V.

For Problems 41–43

41. b. 42. c., 43. b.

Sol. S_1 and S_2 are closed for 1 s.
Change in capacitor,

$$q = CE\left(1 - e^{-t/RC}\right)$$

$$q = 1 \times 1\left(1 - e^{-1/1\times1}\right)$$

$$q = \left(1 - \frac{1}{e}\right) \qquad \text{(i)}$$

The current in inductor,

$$I = \frac{E}{R}\left(1 - e^{-tR/L}\right)$$

$$I = \left(1 - \frac{1}{e}\right) \qquad \text{(ii)}$$

Now S_1 and S_3 are opened and S_2 is closed.
It is LC circuit.

$$q = q_{\max} \sin(\omega t + \phi) \qquad \text{(iii)}$$

$$I = (q_{\max})\omega \cos(\omega t + \phi) \qquad \text{(iv)}$$

As total energy [Magnetic + electrical] is constant

$$\frac{1}{2} L I_{\max}^2 = \frac{1}{2}\frac{q_{\max}^2}{C} = \frac{1}{2} L I^2 + \frac{1}{2}\frac{q^2}{C} \qquad \text{(v)}$$

$$q_{\max} = \sqrt{2}\left(1 - \frac{1}{e}\right) \qquad \text{(vi)}$$

$$I_{\max} = \sqrt{2}\left(1 - \frac{1}{e}\right) \qquad \text{(vii)}$$

From (iii) at $t = 0$, we get

$$\left(1 - \frac{1}{e}\right) = \sqrt{2}\left(1 - \frac{1}{e}\right)\sin\phi \Rightarrow \phi = \frac{\pi}{4}$$

$$q = \sqrt{2}\left(1 - \frac{1}{e}\right)\sin\left(\omega t + \frac{\pi}{4}\right)$$

where, $\omega = \dfrac{1}{\sqrt{LC}} = 1$

$$q = \sqrt{2}\left(1 - \frac{1}{e}\right)\sin\left(\omega t + \frac{\pi}{4}\right)$$

For Problems 44–48

44. c. 45. a., 46. c., 47. a., 48. a.

Sol. a. When switch is closed, the inductor will act as infinite resistance. Hence the current will flow through R_1 only

$$V_{ab} = E = 120\ \text{V}$$

b. Point a will be at higher potential.

c. The nature of circuit at the time of closing the switch is

$$V_{cd} = V_{ab} = E = 120\ \text{V}$$

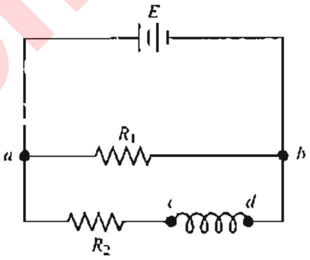


Fig. 8.472

d. c will be at higher potential.

Now the switch is closed for a long time and then opened. After just before opening the switch the current through the inductor

$$I_0 = \frac{E}{R_2} = \frac{120}{50} = \frac{12}{5}\ \text{A}$$

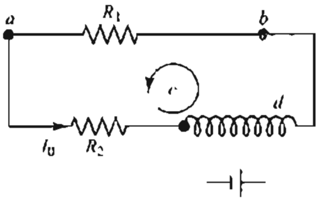


Fig. 8.473

Potential difference across $R_1 = I_0 R_1 = \dfrac{12}{5} \times 30 = 72$ V

For Problems 49–50

49. d., 50. a.

Sol. $|E| = \dfrac{d\phi}{dt} = \dfrac{2 \times 2R \times R}{2} \times \dfrac{dB}{dt} = 2R^2\dfrac{dB}{dt}$

In loop ABC, e.m.f. induced due to branch AC is zero and contribution of e.m.f. due to AB and BC are equal; hence contribution of e.m.f. for the branch is

$$|E|_{AB} = R^2\left(\frac{dB}{dt}\right)$$

For Problems 51–53

51. b., 52. c., 53. d.

Sol: At $t = 0$, capacitor will behave like a short circuit and the inductor as open circuit but as $t \to \infty$, the nature is just opposite.

At $t = 0$, capacitor will behave like a short circuit and the inductor as open circuit but as $t \to \infty$, the nature is just opposite.

At $t = 0$, capacitor will behave like a short circuit and the inductor as open circuit but as $t \to \infty$, the nature is just opposite.

For Problems 54–56

54. a., 55. a., 56. b.

Sol. At $t = 0$, circuit can be considered as follows.

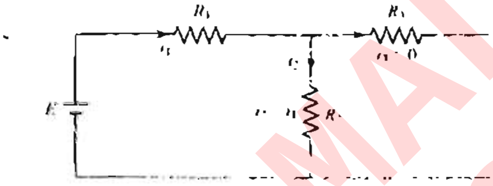


Fig. 8.474

After a long time, circuit can be considered as follows:

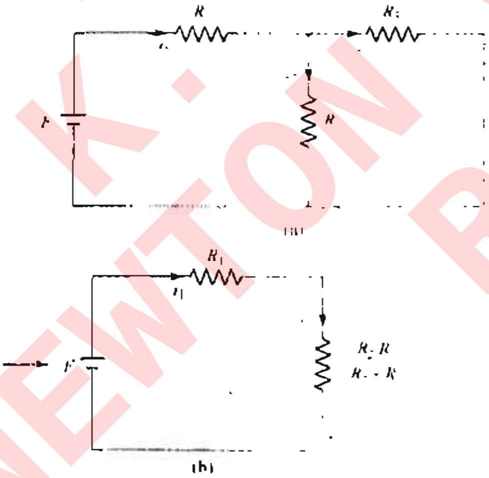


Fig. 8.475

Immediately after closing S, $\frac{di_1}{dt} \neq 0$ Because induced e.m.f. $\frac{Ldi_1}{dt} \neq 0$

For Problems 57–59

57. a., 58. b., 59. b.

Sol. $\phi = abBe = -\frac{d\phi}{dt} = ab\left(-\frac{dB}{dt}\right) = ab\alpha$, anticlockwise as seen from above.

$\varepsilon = \beta abv$ in clockwise sense

Applying principle of superposition.

For Problems 60–62

60. a., 61. b., 62. d.

Sol. $d\phi = BdA$

$$d\phi = \left[\frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(3a - x)}\right]adx$$

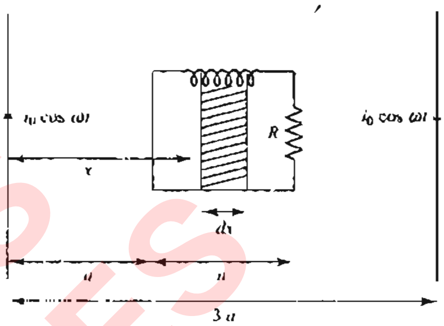


Fig. 8.476

$$\phi = \frac{\mu_0 I}{2\pi}\left[\int_a^{2a}\frac{dx}{x} + \int_a^{2a}\frac{dx}{(3a - x)}\right]a; \quad \phi = \frac{\mu_0 Ia}{\pi}\ln 2$$

Magnitude of e.m.f. in this circuit

$$\varepsilon = \left|\frac{d\phi}{dt}\right| = \frac{\mu_0 a(\ln 2)}{\pi}\left|\frac{dI}{dt}\right|$$

$$\varepsilon = \frac{\mu_0 a \ln 2}{\pi} I_0\omega \sin \omega t$$

a.c. current, $I = \frac{\mu_0 a \ln 2 I_0 \omega}{\pi\sqrt{R^2 + \omega^2 L^2}}\sin(\omega t - \phi)$

For Problems 63–65

63. b., 64. b., 65. a.

Sol. From loop, applying Kirchhoff's law,

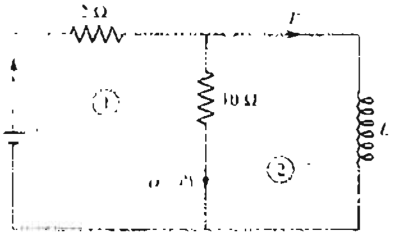


Fig. 8.477

$$12I - 10I' = 10 \quad \text{(i)}$$

From loop (ii)

$$-L\frac{dI'}{dt} + 10I - 10I' = 0$$

$$I - I' = \frac{L}{10}\frac{dI'}{dt} \quad \text{(ii)}$$

Solving simultaneously (i) and (ii), we have

$$I' = 5 - 5e^{-\frac{5t}{3L}} \quad \text{(iii)}$$

and $$I = 5 - \frac{25}{6}e^{-\frac{5t}{3L}} \quad \text{(iv)}$$

$$I - I' = \frac{5}{6}e^{-\frac{1000t}{3}}$$

$$E_L = \frac{1}{2}LI'^2; \quad E_L = \frac{125}{2}(1 - e^{-\frac{1000t}{3}})^2 \text{ mJ}$$

$$E_L(t \to \infty) = \frac{1}{2} \times 5 \times 10^{-3}(5-0)^2$$

$$E_L = 62.5 \text{ mJ}$$

$$E_C(t \to \infty) = \frac{1}{2}CV^2 = \frac{1}{2} \times 20 \times 10^{-6} \times 100 = 1 \text{ mJ}$$

For Problems 66–68

66. b., 67. c., 68. b.

Sol. Since the field is increasing in inside direction in bigger loop, so the current will be induced to oppose this increasing flux. Hence in anticlockwise direction.

$$\phi = BA = B(l^2 + b^2)$$

$$|\varepsilon| = \left|\frac{d\varphi}{dt}\right| = \frac{dB}{dt}(\ell^2 + b^2) = 0.5 \text{ V.}$$

$$i = \frac{\varepsilon}{2R}\left[1 - e^{-\frac{t2R}{2L}}\right] = \frac{0.5}{20}\left[1 - e^{-t}\right]$$

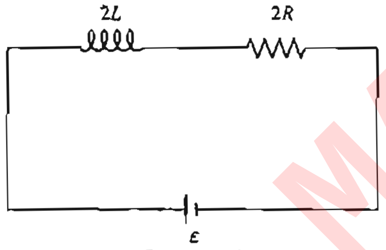


Fig. 8.478

$$i = \frac{1}{40}\left[1 - e^{-t}\right]$$

For Problems 69–71

69. b., 70. d., 71. c.

Sol. The inductor behaves as an open circuit at the initial instant ($t = 0$),

$$L\frac{dI}{dt} = \text{potential difference across the inductor}$$

$= |V_B - V_C|$, which may be determined by using Kirchhoff's rules

The 6 Ω resistor is connected directly across the battery and the current through it is constant.
After a long time ($t \to \infty$), the current through the inductor becomes constant, i.e., the potential difference across it $\left(L\frac{di}{dt}\right)$ becomes zero. Applying Kirchhoff's rules, one can find the current through the inductor.

For Problems 72–74

72. a., 73. c., 74. b.

Sol. $y = 2A \sin kx \cos \omega t$

$$v = \frac{dy}{dt} = -2A \sin kx\, \omega \sin \omega t$$

$$v_{max} = -2AS \sin kx,\ k = \frac{3\pi}{AB}$$

$$e = \int_0^{l=AB} Bv_{max}\, dx = -2\pi AB \int_0^{AB} \sin kx\, dx =$$

$$+\frac{2\omega AB}{k}\left[\cos\frac{3\pi}{AB}AB - \cos\theta\right] = \frac{-4(AB)\omega}{k}.$$

$$\omega t = \frac{\pi}{2}$$

$$t = \frac{\pi}{2\omega}$$

For second harmonic $k = \frac{2\pi}{AB}$

For Problems 75–77

75. b., 76. c., 77. d.

Sol. At $t = 0$, capacitor will behave like a short-circuit and the inductor as an open circuit but as $t \to \infty$, the nature is just opposite.
At $t = 0$, capacitor will behave like a short-circuit and the inductor as an open circuit but as $t \to \infty$, the nature is just opposite.
It will vary with time.

For Problems 78–80

78. b., 79. c., 80. b.

The fan is running at 200 V, consuming 1000 W, then

$$I = \frac{1000}{200} = 5 \text{ A}$$

But as coil resistance is 1 Ω, power dissipated by internal resistance as heat is $P_1 = I^2R = 25$ W.
If V is the net e.m.f. across the coil, then

$$\frac{V^2}{R} = 25 \text{ W} \quad \text{or} \quad V = 5 \text{ V}$$

Net e.m.f. = source e.m.f. - back e.m.f

or $V = V_S - e \Rightarrow e = 195$ V

The work done $P_2 = 1000 - 25 = 975$ W.

For Problems 81–83

81. a., 82. a., 83. c.

Sol. $\frac{dB}{dt} = 2$ T/s

$$E = -\frac{AdB}{dt} = -800 \times 10^{-4}\text{ m}^2 \times 2 = -0.16 \text{ V}$$

$$i = \frac{0.16}{1\ \Omega} = 0.16 \text{ A, clockwise}$$

At $t = 2$ s, $B = 4$ T, $\frac{dB}{dt} = 2$ T/s

$$a = 20 \times 30 \text{ cm}^2$$

$$= 600 \times 10^{-4}\text{ m}^2;\ \frac{dA}{dt} = -(5 \times 20)\text{ cm}^2/\text{s}$$

$$= -100 \times 10^{-4}\text{ m}^2/\text{s}$$

$$E = -\frac{d\phi}{dt} = -\left[\frac{d(BA)}{dt}\right] = -\left[\frac{BdA}{dt} + \frac{AdB}{dt}\right]$$

$$= -[4 \times (-100 \times 10^{-4}) + 600 \times 10^{-4} \times 2]$$

$$= -[-0.04 + 0.120] = -0.08 \text{ V}$$

When capacitor is connected at position 1.

$$E - IR - \frac{q}{c} = 0$$

$$\int_0^t \frac{1}{RC} dt = \int_{Q_0}^{q} \frac{dq}{EC - q} \quad \text{or} \quad q = 50\,[2 - e^{-t}]\text{ mC}$$

$$\text{At } t = 1\text{ s}, q = 50\,[2 - e^{-1}]$$

Voltage across the capacitor at that time

$$V = \frac{q}{c} = \frac{50(20 - 1/e)}{10 \times 10^{-3}} = 5 \times 10^3 (2 - 1/e)\text{ V}$$

$$\frac{1}{2} Li^2 = \frac{1}{2} CV^2 \Rightarrow i = \left(2 - \frac{1}{e}\right) \times 10^4 \text{ A}$$

$$\text{Frequency} = \frac{1}{2m\sqrt{LC}} = \frac{1000}{2\pi \times 25} = \frac{10^3}{50\pi}\text{ Hz}$$

$$= \frac{20}{\pi}\text{ Hz}$$

For Problems 96–97

96. a., 97. d.

Sol. Current should enter the bar from P so that magnetic force is upwards.

$$ilB = mg \text{ or } \frac{V}{5} lB = mg$$

$$\text{or} \qquad m = \frac{150 \times 0.6 \times 1.5}{5 \times 10} = 2.7\text{ kg}$$

For Problems 98–100

98. a., 99. d., 100. a.

Sol. $\phi = Blx$

$$\text{e.m.f.} = l \times \frac{dB}{dt} + Blx = lxv + xlv = 2lxv$$

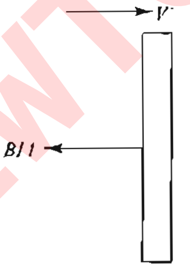


Fig. 8.482

$$I = \frac{lx}{R}(2v) = 2xv$$

$$\Rightarrow \quad -v\frac{dv}{dx} = \frac{-dv}{dx} = \frac{BIl}{m} = x^2(2v)$$

$$\Rightarrow \quad \int_{v_0}^{v} dv = -2\int_0^x x^2 dx$$

$$\Rightarrow \quad v = v_0 - \frac{2x^3}{3} \quad \Rightarrow x = \left[\frac{3(v_0 - v)}{2}\right]^{1/3}$$

Using work energy theorem.

Matching Column Type

1. i. → c., ii. → a., iii. → d., iv. → b.

When the switch is connected with a for a long time, current in the circuit would be $\frac{E}{R}$. When the switch is connected with b, the current is $\frac{F}{2R_0}$.

Next, comparison is made on the basis of time constant. Shorter time constant means faster decay of the current. Between (i) and (ii), (iii) has greater time constant, and hence slower decay of current, corresponding to graph IV.

2. i. → b., ii. → a., iii. → d., iv. → d.

i. $\frac{dB}{dt} = 10 \times 10^{-3}\text{ T/s}$

$$A = 2^2 = 4\text{ cm}^2 = 4 \times 10^{-4}\text{ m}^2$$

$$e = \frac{d\phi}{dt} = \frac{dB}{dt} = 4 \times 10^{-4} \times 10 \times 10^{-3}$$

$$= 4 \times 10^{-6}\text{ V}$$

$$I = \frac{e}{R} = \frac{4 \times 10^{-6}}{2 \times 4} = 5 \times 10^{-7}\text{ A}$$

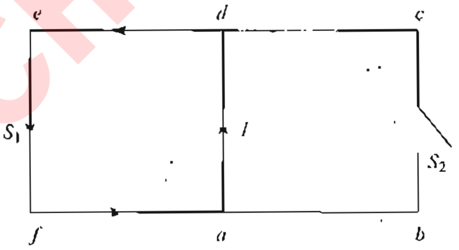


Fig. 8.483

The e.m.f. will be in anticlockwise direction, so current will be from a to d.

ii. Again, current will be in anticlockwise direction.

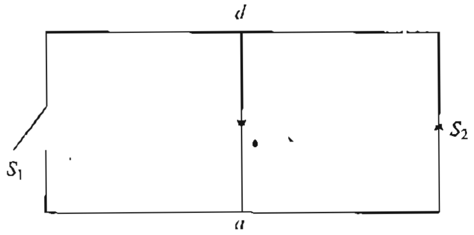


Fig. 8.484

This makes the direction of current from d to a, magnitude same as that in part (i).

iii. If both are open, induced e.m.f. will develop, but no current will flow.

iv. If both are closed, then induced e.m.f. in the left part will tend to flow current from a to d and in right part current will tend to flow from d to a. So, from the principle of superposition, no current will flow in ad.

3. i. → **b.,c., ii.** → **b.,c., iii.** → **a.,b.,c.,d., iv.** → **b.,c.,d.**

i. $L = \frac{\mu N^2 A}{\ell}$

So, L depends upon shape, size (l, A) and medium (μ) inserted.

ii. $C = \frac{\varepsilon A}{O}$

So, C depends upon shape, size (A, d) and medium (ε) inserted.

iii. $Z = \sqrt{R^2 + x_L^2}$

When $x_L = \omega L$, ω depends upon the external voltage source. So, Z depends upon all the factors in column II.

iv. $X_C = \frac{1}{\omega C}$ (independent of resistivity)

4. i. → **c., ii.** → **d., iii.** → **a., iv.** → **b.**

Magnetic field is along x axis because when the cube is moved along x-axis, there is no motional e.m.f. as $\vec{v} \times \vec{B} = 0$. When the block is moved along y-axis, force on the electrons is in direction $-(\hat{j} \times \hat{i}) = \hat{k}$

Therefore, electric field will be created along z-axis.

Now, $c\ v\ B = 24$ mV

$\Rightarrow \quad c = 20$ cm

Similarly, $bvB = 36$ mV

$\Rightarrow \quad b = 30$ cm

$\therefore \quad a = 25$ cm

5. i. → **a.,c., ii.** → **a.,c., iii.** → **b.,d., iv.** → **a.,b.,c.,d.**

a. Speed of the charged particle cannot be changed by magnetic force because magnetic force does no work on charged particle. Only electric field in case (p) and induced electric field in case (r) can change speed of the charged particle.

b. Magnetic field cannot exert force on the charged particle at rest. Only electric field in case (p) and induced electric field in case (r) can exert force on charge initially at rest. In case (r) after the charged particle starts moving, the magnetic field can exert force on the charge.

c. A charged particle can move on a circle with a uniform speed due to uniform and constant magnetic field. Even within a region of non-uniform magnetic field, at all points on the circle, the field may be uniform, for example, on any circle coaxial with a current-carying ring.

d. A moving charged particle is accelerated by electric field and also accelerated by magnetic field (provided v is not parallel to B).

6. i. → **b.,c., ii.** → **a.,d., iii** → **a.,c., iv.** → **b.,d.**

i. Area $OPMQ = \frac{1}{2} r^2 \theta$

Flux in this area, $\phi_1 = \frac{1}{2} r^2 \theta B$

Induced e.m.f. in this area,

$$e_1 = \frac{d\phi_1}{dt} = \frac{1}{2} r^2 \theta \frac{dB}{dt}$$

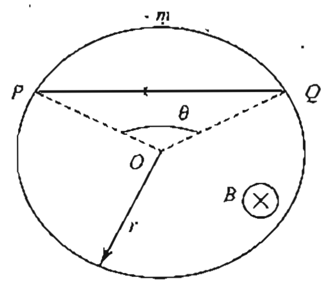


Fig. 8.485

Area $OPQ = r\sin\left(\frac{\theta}{2}\right) r\cos\left(\frac{\theta}{2}\right) = \frac{1}{2} r^2 \sin\theta$

Induced e.m.f. in OPQ,

$$e_2 = \frac{d\phi_2}{dt} = \frac{1}{2} r^2 \sin\theta \frac{dB}{dt}$$

e_2 will be only in part PQ, because in PQ and OP induced e.m.f. will be zero. Clearly, $e_2 < e_1$, because area $OPQ <$ area $OPMQ$.

Since B is increasing, so e.m.f. will be in anticlockwise direction. Hence end P will be positive w.r.t. Q.

ii. Here e.m.f. in OPQ will be due to flux changing in area OMQ. This area is $\frac{1}{2} r^2 \theta$. The entire e.m.f. will be in part PQ. End Q will be positive.

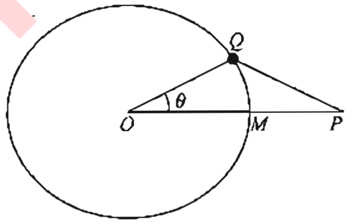


Fig. 8.486

iii. Induced e.m.f. $= \frac{1}{2} r^2 \theta \frac{dB}{dt}$. End P will be positive.

iv. Area in which flux is changing is lesss than $\frac{1}{2} r^2 \theta$. End Q will be positive.

7. i. → **b.,d., ii.** → **b.,c., iii.** → **a.,d., iv.** → **a.,c.**

Since field is decreasing, so induced electric field at both points A and B will be in clockwise direction or towards 1. Hence, force on an electron will be along 3 to both points A and B.

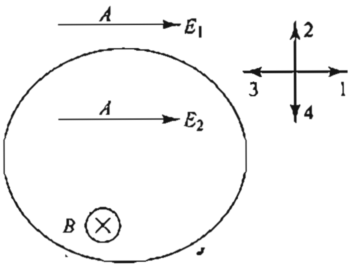


Fig. 8.487

For A: $E\,2\pi r = \pi a^2 \frac{dB}{dt}$

$$E \propto \frac{1}{r}$$

For B: $E\,2\pi r = \pi r^2 \frac{dB}{dt}$

$$E \propto r$$

8. i. → c., ii. → c., iii. → a.,c., iv. → a.,c.

i. Current in inductor when switch is open:

$$I_0 = \frac{E}{R}.$$

Initially induced e.m.f. will be equal to E and finally it is zero. So, energy stored will be zero.

ii. Same as (i).

iii., iv. Here current becomes zero suddenly.

So, $\frac{dI}{dt}$ is large.

Hence, induced e.m.f. $L\frac{dI}{dt}$ will be large. Finally, energy stored in inductor will be zero.

9. i. → c., ii. → b., iii. → a., iv. → b.

Using Faraday's law, whenever there is a change in flux linked with the coil, e.m.f. is induced the coil.

When $o < x < b$, flux lined with the coil is $BI\ell x$.

$$\therefore \quad e = \frac{d\phi}{dt} = -\frac{dBI\ell x}{dt} - BI\ell\frac{dx}{dt}$$

or $\quad e = -BI\ell v$

When $b < x \le 2b$, there is no change in flux. So, no e.m.f. is induced. When $2b < x < 3b$, there is a decrease in flux, hence $e = BI\ell v$.

When $x > 3b$, again flux linked with the coil is zero, hence no e.m.f. is induced.

10. i. → b., ii. → c., iii. → a., iv. → b.

i. At $t = 1$ s, flux is increasing in the inward direction, hence induced e.m.f. will be in anticlockwise direction.

ii. At $t = 5$ s, there is no change in flux, so induced e.m.f. is zero.

iii. At $t = 9$ s, flux is increasing in upward direction, hence induced e.m.f. will be in clockwise direction.

iv. At $t = 15$ s, flux is decreasing in upward direction, so induced e.m.f. will be in anticlockwise direction.

11. i. → c., ii. → a.,b., iii. → d., iv. → c.

We know that $e = -\frac{d\phi}{dt} = -A\frac{dB}{dt}$. If we take area vector in the upward direction, then anticlockwise direction will be + ve. From 0 to t_1 and t_5 to t_6, dB/dt is + ive. Hence induced e.m.f. e is –ve. So, induced current will be in clockwise direction. From t_2 to t_4, dB/dt is –ve. Hence induced e.m.f. e is + ve. So, induced current will be in anticlockwise direction. From t_1 to t_2 and t_4 to t_5, dB/dt is zero. Hence, no e.m.f. is induced. Induced e.m.f. or current is maximum from 0 to t_1 and t_5 to t_6, because here magnitude of dB/dt is maximum.

12. i. → c.,d., ii. → c.,d., iii. → b.,d., iv. → a.,d.

i. $e_{OA} = \frac{1}{2}B\omega(A\omega)^2 = \frac{1}{2}B\omega(\sqrt{2}\,L)^2 = B\omega L^2$

ii. $e_{OD} = \frac{1}{2}B\omega(OD)^2 = \frac{1}{2}B\omega(\sqrt{2}\,L)^2 = B\omega L^2$

iii. $e_{OC} = \frac{1}{2}B\omega L^2$ or $e_{OC} = E_{OD} - E_{OC} = \frac{1}{2}B\omega L^2$

iv. $e_A - e_0 = B\omega L^2$

$e_D - e_0 = B\omega L^2$

$e_A - e_D = 0$

$e_{AD} = 0$

13. i. → a.,d., ii. → b.,d., iii. → c., iv. → a.,d.

i. Just after switch S is closed, flux in M starts increasing in left direction, so in N also the flux starts increasing in left direction. This will induce current in N in a direction so that the flux is in right direction. This is possible if induced current in N is from A to B.

ii. In this case just reverse of (i) will happen, because after closing the switch, the flux in M starts decreasing in left direction.

iii. After a long time of closing the switch, flux becomes constant. Hence, no current is induced.

iv. Just after closing S, flux starts increasing, but because M moves away so due to this flux through N will decrease. But there will be a net increase in flux in N in left direction. This is the case similar to (i).

Archives

Fill in the Blanks Type

1. i. The coil is broken into two identical coils.

$$L_{eq} = \frac{L/2 \times L/2}{L/2 + L/2} = \frac{L}{4} = 0.45 \times 10^{-4}\ \text{H},$$

$$R_{eq} = \frac{R/2 \times R/2}{\frac{R}{2} + \frac{R}{2}} = \frac{R}{4} = 1.5\ \Omega$$

Time constant $= \frac{L_{eq}}{R_{eq}} = \frac{0.45 \times 10^{-4}}{1.5} = 0.3 \times 10^{-4}$ s. Steady current,

$$I = \frac{E}{R} = \frac{12}{1.5} = 8\ \text{A}$$

2. We know that the velocity of light in vacuum $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ and the velocity of light in a medium $v = \frac{1}{\sqrt{\mu\varepsilon}}$. Also, the refractive index

$$n = \frac{\text{Velocity of light in medium}}{\text{Velocity of light in vacuum}} = \frac{v}{c}$$

$$= \frac{1/\sqrt{\mu_0\varepsilon_0}}{1/\sqrt{\mu\varepsilon}} = \frac{\sqrt{\mu\varepsilon}}{\sqrt{\mu_0\varepsilon_0}}$$

3. Left to right.
4. $V_B + e - 15 + \ell \times 1 = V_A$

$$\Rightarrow V_B - V_A = 15 - e - I$$

Here, $I = 5\text{ A},\ e = \frac{d\phi}{dt} = L\left[\frac{dI}{dt}\right]$

$$= -5 \times 10^{-3} \times 10^{-3} = -5\text{ V}$$

$$V_B - V_A = 15\text{V}$$

True or False

1. For induced e.m.f. to develop in a coil the magnetic flux through it must change. But in this case the number of magnetic lines of force through the coil is not changing. Therefore the statement is false.
2. When conducting rod AB moves parallel to x-axis in a uniform magnetic field pointing in the positive z-direction, then according to Fleming's left hand rule, the electrons will experience a force towards B. Hence, the end A will become positive. Therefore the statement is true.

Single Correct Answer Type

1. **d.** Net charge in magnetic flux passing through the coil is zero.

$\therefore$ Current (or e.m.f.) induced in the loop is zero.

2. **b.** The individual e.m.f. produced in the coil $e = \frac{d\phi}{dt}$.

$\therefore$ The current induced will be $i = \frac{|e|}{R}$

$$\Rightarrow \quad i = \frac{1}{R}\frac{d\phi}{dt}$$

But $\quad i = \frac{dq}{dt} \Rightarrow \frac{dq}{dt} = \frac{1}{R}d\phi$

$$\Rightarrow \int dq = \frac{1}{R}\int d\phi \Rightarrow q = \frac{BA}{R}$$

3. **d.** The semicircular ring is falling vertically. If it moves a distance dx in time dt, then change in area in the magnetic field is

$$dA = 2R\,dx$$

$$\Rightarrow \quad \frac{dA}{dt} = 2R\frac{dx}{dt} = 2RV$$

Now, induced e.m.f.,

$$e = -\frac{\delta\phi}{dt} = -\frac{d}{dt}(\bar{B}\cdot\bar{A}) = -\frac{d}{dt}(BA\cos\theta)$$

$$= -\frac{d}{dt}(BA\cos 90°)$$

$$\Rightarrow \quad e = -B\frac{dA}{dt} - B(2RV)$$

$$|e| = 2RBv$$

When the semicircular ring moves out of the magnetic field, the magnetic field passing through it outward of the plane of paper decreases. By Lenz's law induced e.m.f. will be produced such that current flows in anticlockwise direction. This will create a magnetic field in the direction outward of the plane of paper. In this case, Q will be at a higher potential (as current flows from high potential to low potential).

4. **b.** A motional e.m.f., $e = B\ell v$, is induced in the rod. Or we can say a potential difference is induced between the two ends of the rod AB, with A at higher potential and B at lower potential. Due to this potential difference, there is an electric field in the rod.
5. **b.** Magnetic field produced by a current I in a large square loop at its centre,

$$B \propto \frac{i}{L} \quad \Rightarrow \quad B = K\frac{i}{L}$$

$\therefore$ Magnetic flux linked with the smaller loop,

$$\phi = BS \quad \Rightarrow \quad \phi = \left(K\frac{i}{L}\right)(l^2)$$

Therefore, the mutual inductance

$$M = \frac{\phi}{i} = K\frac{i^2}{L} \text{ or } M \propto \frac{i^2}{L}$$

6. **c.** When the current in loop A increases, the magnetic lines of force in loop B also increase as loop A is near loop B. This induces an e.m.f. in B is such a direction that current flows in opposite direction in B (as compared to A). Since currents are in opposite directions, loop B is repelled by loop A.

7. **d.** We have

$$I = I_0(1 - e^{-t/\tau})$$

But $\quad I_0 = \frac{V}{R}$ and $\tau = \frac{L}{R}$

$$\therefore \quad I = \frac{V}{R}(1 - e^{Rt/L}) = \frac{12}{6}\left[1 - e^{6t/18.4\times10^{-3}}\right]$$

$$= 1 \text{ (given)}$$

$$\therefore \quad t = 0.97 \times 10^{-3}\text{ s} = 1\text{ ms}$$

8. **b.** $E = \frac{d\phi}{dt} = \frac{d}{dt}(\bar{B}\cdot\bar{A}) = \frac{d}{dt}(BA\cos 90°)$

$$= A\frac{dB}{dt} \Rightarrow E(2\pi r) = \pi a^2\frac{dB}{dt} \text{ for } r \geq a$$

$$\Rightarrow \quad E = \frac{a^2}{2r}\frac{dB}{dt} \Rightarrow E \propto \frac{1}{r}$$

9. **d.** The magnetic field at the center of the coil $B(t) = \mu_0 n I_1$. As the current increases, B will also increase with time till it reaches a maximum value (when the current becomes steady). The induced e.m.f. in the ring

$$e = \frac{d\phi}{dt} - \frac{d}{dt}(\vec{B}\cdot\vec{A})$$

$$= -A\frac{d}{dt}(\mu_0 n I_1)$$

$\therefore$ The induced current in the ring

$$I_2(t) = \frac{|e|}{R} = \frac{\mu_0 n A}{R}\frac{dI_1}{dt}$$

[Please note that $\frac{dI_1}{dt}$ decreases with time and hence I_2.]

where $I_1 = I_{max}(1 - e^{-t/t})$. The relvant graphs are

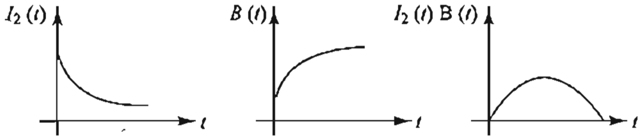


Fig. 8.488

10. c. Redrawing the figure, we find that there is a quadrilateral with four resistances R_1, R_2, R_3 and R_4, i.e., a Wheatstone bridge. If this Wheatstone bridge is balanced, then R_6 becomes ineffective which is the required condition here.

Therefore, $\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow R_1R_4 = R_2R_3$

11. a. When current flows in any of the coils, the flux linked with the other coil will be maximum in the first case. Therefore, mutual inductance will be maximum in case (a).

12. d. Apply Lenz's law.

13. d

$$P = \frac{E^2}{R} = \frac{\pi r^2}{\rho l}\left(\frac{d\phi}{dt}\right)^2 = \frac{\pi r^2}{\rho l}\left[\frac{d}{dt}(NBA)^2\right]$$

$$= \frac{\pi r^2}{\rho l} N^2 A^2 \left(\frac{dB}{dt}\right)^2 \Rightarrow P \propto \frac{N^2 r^2}{\ell}$$

Case 1: $P_1 \propto \frac{N^2 r^2}{\ell}$

Case 2: $P_2 \propto \frac{(4N)^2 (r/2)^2}{4l}$

$$\Rightarrow \frac{P_1}{P_2} = \frac{1}{4}$$

14. a. Since current leads e.m.f. (as seen from the graph), therefore this is an R–C circuit.

$$\tan\phi = \frac{X_C - X_L}{R}$$

Here, $\phi = 45°$

$\therefore X_c = R$ [$X_L = 0$ as there is no inductor]

$$\frac{1}{\omega C} = R$$

L.H.S. $= \frac{1}{100 \times 10 \times 10^{-6}} = R = 10^3\,\Omega$

15. c. Initially ϕ_B increases as the magnet approaches the solenoid. Therefore ε is –ve and increases in magnitude when the magnet moves inside the solenoid. Increase in ϕ_B slows down and finally ϕ_B starts decreasing. Therefore e.m.f. becomes positive and starts increasing. Only graph (c) shows these characteristics.

16. a. For a current to induce in the cylindrical conducting rod, the cylindrical rod should cut magnetic lines of force which will happen only when the cylindrical conducting rod is moving. Since conducting rod is at rest, no current will be induced. The magnitude and direction of the magnetic field changes. A changing magnetic field will create an electric field which can apply force on the free electrons of the conducting rod and a current will get induced. But since the magnetic field is constant, no current will be induced.

17. d. According to Lenz's law, current will be in anticlockwise sense as magnetic field is increasing into the plane of paper.

Multiple Correct Answers Type

1. a., b., c.

2. d. Since the rate of change of magnetic flux is zero, hence there will be no net induced e.m.f. and hence no current flowing in the loop.

3. a., c., d.

Rate of charge of current $= \frac{di_1}{dt} = m$ (say)

Induced e.m.f. $v_1 = -L_1\frac{di_1}{dt} = -8 \times 10^{-3} \times m$

$$\therefore \frac{V_2}{V_1} = \frac{1}{4}$$

Power, $P = V_1 i_1 = 8 \times 10^{-3} \times m \times i_1$

Rate of charge of current $= \frac{di_2}{dt} = m$ (given). Induced e.m.f.

$$V_2 = -L_2\frac{di_2}{dt} = -2 \times 10^{-3} \times m$$

Power, $P = V_2 i_2 - 2 \times 10^{-3} \times m \times i_2$

Since power is equal

$\therefore 8 \times 10^{-3} \times mi_1 = 2 \times 10^{-3} \times mi_2$

$$\Rightarrow \frac{i_1}{i_2} = \frac{1}{4} \quad \text{(i)}$$

Energy $W_1 = \frac{1}{2}L_1 i_1^2 = \frac{1}{2} \times 8 \times 10^{-3} \times i_1^2$

Energy $W_2 = \frac{1}{2}L_2 i_2^2 = \frac{1}{2} \times 2 \times 10^{-3} \times i_2^2$

$$\therefore \frac{W_1}{W_2} = \frac{10^{-3} \times i_2^2}{4 \times 10^{-3} \times i_1^2} = \frac{1}{4} \times 4 \times 4 = 4$$

4. b. The magnetic field due to a current flowing in a wire of finite length is given by

$$B = \frac{\mu_0 I}{4\pi R}(\sin\alpha + \sin\beta)$$

Applying the above formula for AB for finding the field at O is

$$B = \frac{\mu_0 I_1}{4\pi(L/2)}(\sin 45° + \sin 45°)$$

$$= \frac{\mu_0 I_1}{\sqrt{2}\pi L}$$

acting perpendicular to the plane of paper upwards. Therefore the total magnetic field due to current flowing through $ABCD$ is

$$B = 4B_1 = \frac{4\mu_0 I_1}{\sqrt{2}\pi L} = \frac{2\sqrt{2}\mu_0 I_1}{\pi L}$$

The total flux passing through the square *EFGH*

$$\phi_2 = B \times \ell^2 = \frac{2\sqrt{2}\mu_0 I_1}{\pi L} \times \ell^2 \qquad \text{(i)}$$

The flux through the small square loop is directly proportional to the current passing through big square loop.

∴ $\phi_2 \propto_2 I_1 \Rightarrow \phi_2 = M_2 I_1$

where M_2 = mutual conductance

$$\therefore \quad M_2 = \frac{\phi_2}{I_1} = \frac{\frac{2\sqrt{2}\mu_0 I_1}{\pi L} \times \ell^2}{I_1}$$

$$= \frac{2\sqrt{2}\mu_0}{\pi L} \times \ell^2 \Rightarrow M_2 \propto \frac{\ell^2}{L}$$

5. a., b., c., d.

a. $L = \frac{\phi}{i}$ or henry $= \frac{\text{weber}}{\text{ampere}}$

b. $e = -L\left(\frac{di}{dt}\right)$

$$\therefore \quad L = -\frac{e}{(di/dt)}$$

or henry $= \frac{\text{volt - second}}{\text{ampere}}$

c. $U = \frac{1}{2}Li^2$

$$\therefore \quad L = \frac{2U}{i^2}$$

or henry $= \frac{\text{joule}}{(\text{ampere})^2}$

d. $U = \frac{1}{2}Li^2$

∴ $L = Rt$ or henry = ohm-second

6. b., d.

Electrostatic and gravitational field do not make closed loops.

7. b., d.

As $\frac{d\phi}{dt}$ = e.m.f. is the same, the current induced in the ring will depend upon the resistance of the ring. Larger the resistivity smaller the current.

Assertion-Reasoning Type

1. a. The induced current in the ring will interact with horizontal component of magnetic field and both will repel each other. This repulsion will balance the weight of the ring.

Comprehension Type

1. b. Charge on capacitor at time t is:

$$q = q_0(1 - e^{-t/\tau})$$

Here, $q_0 = CV$ and $t = 2\tau$

$$q = CV(1 - e^{-2\tau/\tau}) = CV(1 - e^{-2})$$

2. d. From conservation of energy,

$$\frac{1}{2}LI_{\max}^2 = \frac{1}{2}CV^2 \Rightarrow I_{\max} = V\sqrt{\frac{C}{L}}$$

3. c. Comparing the LC oscillation with normal SHM, we get

$$\frac{d^2Q}{dt^2} = -\omega^2 Q$$

Here, $\omega^2 = \frac{1}{LC}$

$$Q = -LC\frac{d^2Q}{dt^2}$$

4. a. **5. d.** **6. c.**

CHAPTER

9

Magnetics

- Introduction
- Motion of a Charged Particle in a Magnetic Field
- Path of a Charged Particle in Both Electric and Magnetic Fields
- Force on a Current Carrying Wire
- Magnetic Dipole and Dipole Moment
- Magnetic Field Due to a Moving Charge and Current Carrying Wire
- Ampere's Law
- Field of a Long, Straight, Current Carrying Conductor
- Field of a Long Solenoid

INTRODUCTION

We represented electric interactions in two steps:

- A distribution of electric charge at rest creates an *electric field* $\vec{E}$ in the surrounding space.
- The *electric field* exerts a force $\vec{F} = q\vec{E}$ on any other charge q that is present in the field.

We can describe magnetic interactions in a similar way:

1. A moving charge or a current creates a *magnetic field* in the surrounding space (in addition to its electric field).
2. The *magnetic field* exerts a force $\vec{F}$ on any other moving charge or current that is present in the field.

Like electric field, magnetic field is a vector field—that is, a vector quantity associated with each point in space. We will use the symbol $\vec{B}$ for magnetic field. At any position, the direction of $\vec{B}$ is defined as that in which the north pole of a compass needle tends to point.

We can quantify the magnetic field $\vec{B}$ by using our model of a particle in a field. The existence of a magnetic field at some point in space can be determined by measuring the magnetic force $\vec{F}_B$ exerted on an appropriate test particle placed at that point. This process is the same one we followed in defining the electric field. Our test particle will be an electrically charged particle such as a proton. If we perform such an experiment, we find the following results:

- The magnetic force $\vec{F}_B$ is proportional to the charge q of the particle as well as to the speed v of the particle.
- When a charged particle moves parallel to the magnetic field vector, the magnetic force $\vec{F}_B$ on the charge is zero.
- When the velocity vector makes an angle θ with the magnetic field, the magnetic force acts in a direction perpendicular to both $\vec{v}$ and $\vec{B}$; that is, the magnetic force is perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$.
- The magnetic force on a negative charge is directed opposite to the force on a positive charge moving in the same direction.
- If the velocity vector makes an angle θ with the magnetic field, the magnitude of the magnetic force is proportional to $\sin\theta$.

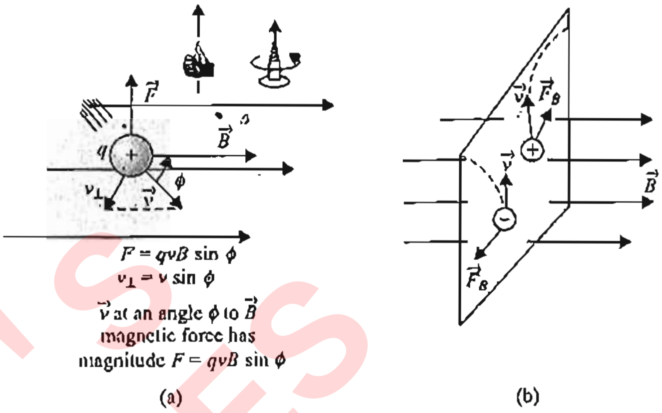


Fig. 9.1

These results show that the magnetic force on a particle is more complicated than the electric force. The magnetic force is distinctive because it depends on the velocity of the particle and because its direction is perpendicular to both $\vec{v}$ and $\vec{B}$. Despite this complicated behavior, these observations can be summarized in a compact way by writing the magnetic force in the form

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

where the direction of the magnetic force is that of $\vec{v} \times \vec{B}$ which, by definition of the cross product, is perpendicular to both $\vec{v}$ and $\vec{B}$. We can regard equation $\vec{F}_B = q\vec{v} \times \vec{B}$ as an operational definition of the magnetic field at a point in space. The S.I. unit of magnetic field is the tesla (T), where 1 T = 1 N s^{-1} m^{-1}.

Thus, magnitude of the magnetic force is $F_B = |q|vB \sin\phi$ (Fig. 9.1) where ϕ is the angle between $\vec{v}$ and $\vec{B}$. From this expression, we see that F_B is zero when $\vec{v}$ is either parallel or antiparallel to $\vec{B}$ ($\phi = 0$ or 180°). Furthermore, the force has its maximum value $F_B = |q|vB$ when $\vec{v}$ is perpendicular to $\vec{B}$ ($\phi = 90°$).

Right Hand Rules for Determining the Direction of the Magnetic Force Acting on a Moving Charged Particle

Figure 9.2 reviews to right-hand rules for determining the direction of the cross product $\vec{v} \times \vec{B}$ and determining the direction of $\vec{F}_B$. The rule in Fig. 9.2(a) depends on our right-hand rule for the cross product. Point the four fingers of your right hand along the direction of $\vec{v}$ with the palm facing $\vec{B}$ and curl them toward $\vec{B}$. The extended thumb, which is at a right angle to the fingers, points in the direction of $\vec{v} \times \vec{B}$. Because $\vec{F}_B = q\vec{v} \times \vec{B}$, $\vec{F}_B$ is in the direction of your thumb if q is positive and opposite to the direction of your thumb if q is negative.

An alternative rule is shown in Fig. 9.2 (b). Here the thumb points in the direction of $\vec{v}$ and the extended fingers in the direction of $\vec{B}$. Now, the force $\vec{F}_B$ on a positive charge extends outward from your palm. The advantage of this rule is that the force on the charge is in the direction that your would push on something with your hand-outward from your palm. The force on a negative charge is in the opposite direction.

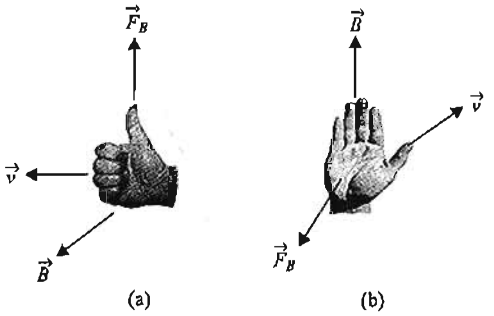


Fig. 9.2

There are important differences between electric and magnetic forces on charged particles:

- The electric force is always parallel or antiparallel to the direction of the electric field, whereas the magnetic force is perpendicular to the magnetic field.
- The electric force acts on a charged particle independent of the particle's velocity, whereas the magnetic force acts on a charged particle only when the particle is in motion and the force is proportional to the velocity.
- The electric force does work in displacing a charged particle, whereas the magnetic force associated with a constant magnetic field does no work when a charged particle is displaced.

This last statement is true because when a charge moves in a constant magnetic field, the magnetic force is always perpendicular to the displacement. Hence, the work done by the magnetic force on the particle is zero.

From the work–energy theorem, we conclude that the kinetic energy of a charged particle cannot be altered by a constant magnetic field alone. In other words, when a charge moves with a velocity $\vec{v}$, an applied magnetic field can alter the direction of the velocity vector, but it cannot change the speed of the particle.

MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

If a charged particle is projected in a magnetic field. It experiences a magnetic force.

$$\vec{F} = q\vec{v}\times\vec{B} \quad \text{or} \quad F = qBV_{\perp}$$

i. Consider a charged particle of mass m moving in a uniform magnetic field $\vec{B}$ with an initial velocity vector $\vec{v}$ perpendicular to the field (Fig. 9.3).

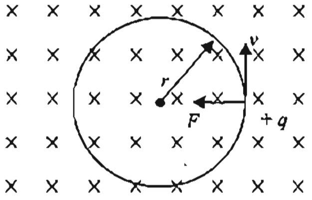


Fig. 9.3

The particle moves in a circular path with constant speed and the magnetic force provides the centripetal force. The radius r of the circular path is

$$\frac{mv^2}{r} = qvB \quad \text{or,} \quad r = \frac{mv}{qB}$$

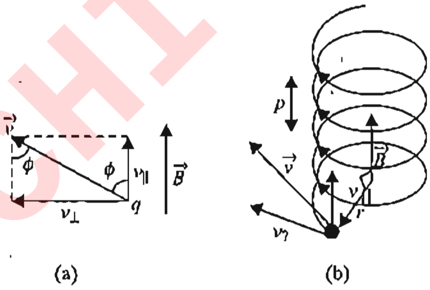
The angular speed ω of the particle is $\omega = \frac{v}{r} = \frac{qB}{m}$

The time period T of the motion is $T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$

ii. If a charge particle moves in a uniform magnetic field at some arbitrary angle θ with respect to B, then it moves in a helical path.
With perpendicular component of velocity it moves in a circular path of radius $r = \frac{mv_{\perp}}{qB}$

And, with parallel component of velocity it also moves along the field lines. The linear distance travelled (along the field line) in one revolution (time period) is called pitch (p) (Fig. 9.4).

Pitch: $p = v_{\parallel}T = \frac{2\pi m v_{\parallel}}{qB}$



(a) A charged particle moves in a uniform magnetic field, $\vec{B}$, its velocity $\vec{v}$ making an angle ϕ with the field direction.
(b) The particle follows a helical path of radius r and pitch p.

Fig. 9.4

Illustration 9.1 **A potential difference of 600 volts is applied across the plates of a parallel plate condenser. The separation between the plates is 3 mm. An electron projected vertically, parallel to the plates, with a velocity of 2×10^6 ms^{-1} moves undeflected between the plates. Find the magnitude and direction of the magnetic field in the region between the condenser plates. (Neglect the edge effects. Charge of the electron $= -1.6\times10^{-19}$ C.)** **(IIT-JEE, 1981)**

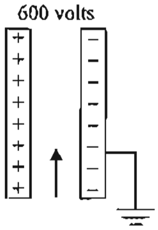


Fig. 9.5

Sol. The force on electron will be towards the left plane due to electric field and will be equal to $F_e = eE$.

For the electron to move undeflected between the plates, there should be a force (magnetic) which is equal to the electric force and opposite in direction. The force should be directed towards the right as the electric force is towards the left. On applying Fleming's left hand rule, we get the magnetic field should be directed perpendicular to the plane of paper inwards. Therefore,

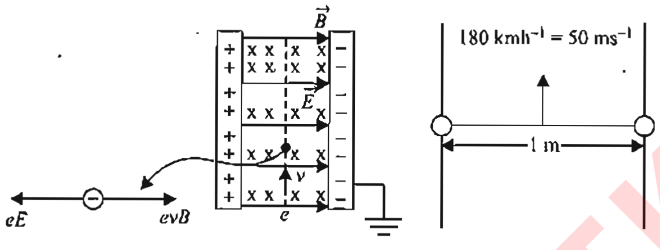


Fig. 9.6

Force due to electric field = Force due to magnetic field

$$eE = evB \qquad \left[\because E = \frac{V}{d}\right]$$

$$B = \frac{E}{v} = \frac{V/d}{v}$$

where V = potential difference between the plates, and
d = distance between the plates

$$B = \frac{600/3\times10^{-3}}{2\times10^{6}} = \frac{600}{3\times10^{-3}\times2\times10^{6}}$$

$\Rightarrow \quad B = 0.1$ tesla

Illustration 9.2 **The region between $x = 0$ and $x = L$ is filled with uniform, steady magnetic field $B_0\hat{k}$. A particle of mass m, positive charge q and velocity $v_0\hat{i}$ travels along x-axis and enters the region of magnetic field. Neglect gravity throughout the question.**

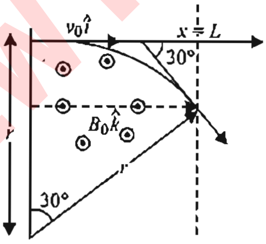


Fig. 9.7

a. Find the value of L if the particle emerges from the region of magnetic field with its final velocity at an angle 30° to the initial velocity.

b. Find the final velocity of the particle and the time spent by it in the magnetic field, if the field now extends up to $x + 2.1L$. (IIT-JEE, 1999)

Sol.

a. As the initial velocity of the particle is perpendicular to the field, the particle will move along the arc of a circle as shown (Fig. 9.8).

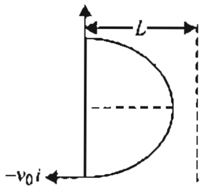


Fig. 9.8

If r is the radius of the circle, then $\dfrac{mv_0^2}{r} = qv_0B_0$

Also, from geometry, $L = r\sin 30° \Rightarrow r = 2L$

or $\qquad L = \dfrac{mv_0}{2qB_0}$

b. In this case, $L = \dfrac{2.1mv_0}{2qB_0} > r$

Hence, the particle will complete semicircular path and emerge from the field with velocity $-v_0\hat{i}$ as shown in the figure. Time spent by the particle in the magnetic field

$$T = \frac{\pi r}{v_0} = \frac{\pi m}{qB_0}$$

The speed of the particle does not change due to magnetic field.

Illustration 9.3 **A particle of mass 1×10^{-26} kg and charge $+1.6 \times 10^{-19}$ C travelling with a velocity 1.28×10^6 ms^{-1} in the $+x$-direction enters a region in which uniform electric field E and a uniform magnetic field of induction B are present such that $E_x = E_y = 0$, $E_z = -102.4$ kVm^{-1}, and $B_x = B_z = 0$, $B_y = 8 \times 10^{-2}$. The particle enters this region at time $t = 0$. Determine the location (x, y, z coordinates) of the particle at $t = 5 \times 10^{-6}$ s. If the electric field is switched off at this instant (with the magnetic field present), what will be the position of the particle at $t = 7.45 \times 10^{-6}$ s? (IIT-JEE, 1982)**

Sol. The Lorentz force on the charged particle is $F = q\left(\bar{E} + \bar{v} + \bar{B}\right)$.

The electric force on the charged particle, $F_E = qE_z$ which acts toward negative z-direction.

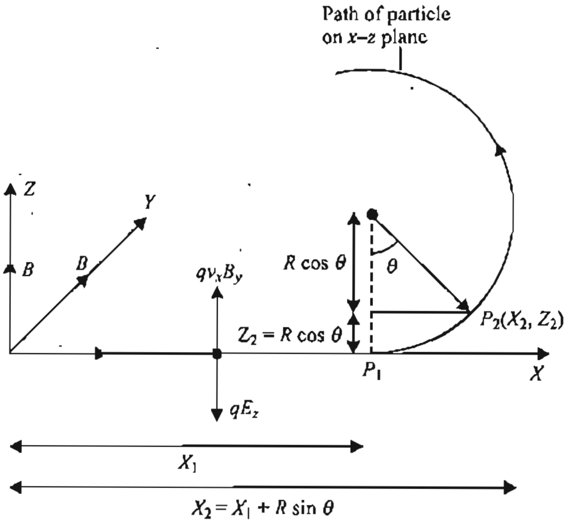


Fig. 9.9

The magnetic force on the charged particle, $F_B = qv_x B_y$.

As velocity of charge is in $+x$-direction and magnetic field is along $+y$-direction, from right hand rule the magnetic force acts along positive z-direction.

The resultant force, $F = F_E + F_B = q(E_z + v_x B_y) = q[-102.4 \times 10^3 + 1.28 \times 10^6 \times 8 \times 10^{-2}] = 0$

During time $t = 0$ to $t_1 = 5 \times 10^{-6}$, the resultant force on the particle is zero, it moves with uniform velocity v_x. The position of the particle (X_1, Y_1, Z_1) after time t_1 is

$$X_1 = v_x t_1 = (1.28 \times 10^6) \times (5 \times 10^{-6}) = 6.4 \text{ m}$$

When electric field is switched off, the particle circulates in xz-plane under the influence of magnetic field.

Radius R of circulation is

$$R = \frac{mv_x}{qB_y} = \frac{10^{-26} \times 1.28 \times 10^6}{1.6 \times 10^{-19} \times 8 \times 10^{-2}} = 1 \text{ m}$$

Let the particle rotate by an angle $\theta = \omega(t_2 - t_1)$. The arc length $P_1P_2 = 2R\,\theta = v_x(t_2 - t_1)$, as the particle circulates for

$$t_2 - t_1 = (7.45 - 5) \times 10^{-6} = 2.45 \times 10^{-6} \text{ s}$$

$$\theta = \frac{v_x(t_2 - t_1)}{R} = \frac{(1.28 \times 10^6) \times (2.45 \times 10^{-6})}{1}$$

$$= 3.316 \simeq \pi \text{ radian}$$

The coordinates of the particle are

$$X_2 = X_1 + R \sin\theta = X_1 + R \sin\pi = X_1 = 6.4 \text{ m}$$

and $\quad Z_2 = R - R\cos\theta = R - R\cos\pi = 2R = 2 \text{ m}$

Note that $\theta = \pi$ implies that $t_2 = T/2$, where T is time period of circulation. We could have written the result directly.

Illustration 9.4 **A particle of mass m and charge q is projected into a region having a perpendicular uniform magnetic field B of width d. Find the angle of deviation θ of the particle as it comes out of the magnetic field.**

Sol. The radius of the circular orbit is $r = \dfrac{mv}{qB}$

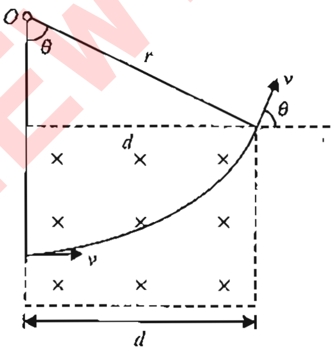


Fig. 9.10

The deviation θ may be obtained from the Fig. 9.10 as

$$\sin\theta = \frac{d}{r} = \frac{dBq}{mv} \quad \text{or} \quad \theta = \sin^{-1}\left(\frac{dBq}{mv}\right)$$

Note:

In the above illustration,

(i) *if d were slightly less than* $\dfrac{mv}{qB}$, *then the particle leaves the magnetic field at B, just before it is able to turn back. Hence the angle of deviation is 90°.*

(ii) *If d were slightly greater than* $\dfrac{mv}{qB}$, *then the particle turns around and completes a semicircle. When it leaves the magnetic field its angle of deviation is 180°.*

Illustration 9.5 **A charged particle of mass m and charge q is accelerated through a potential difference of V volts. It enters a region of uniform magnetic field which is directed perpendicular to the direction of motion of the particle. Find the radius of circular path moved by the particle in magnetic field.**

Sol. Since the particle is accelerated through V volts, therefore its kinetic energy will be equal to qV.

or $\quad \dfrac{1}{2}mv^2 = qV$. Therefore, $v = \sqrt{\dfrac{2qV}{m}}$.

Radius of circular path is given by $R = \dfrac{mv}{qB}$. Therefore, $R = \sqrt{\dfrac{2mV}{qB^2}}$.

Illustration 9.6 **A particle of mass m and charge $+q$ enters a region of magnetic field with a uniform velocity v, as shown in Fig. 9.11.**

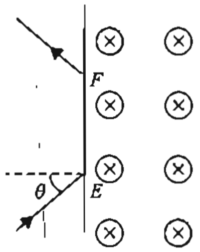


Fig. 9.11

a. Find the angle subtended by the circular arc described by it in the magnetic field.

b. How long does the particle stay inside the magnetic field?

c. If the particle enters at E, what is the intercept EF?

(IIT-JEE, 1984)

Sol.

a. The particle circulates under the influence of magnetic field. As the magnetic field is uniform, the charge comes out symmetrically. The angle subtended at the center is $(180 - 2\theta)$.

b. The length of the arc traced by the particle, $\ell = R(2\pi - \theta)$

Time spent in the field, $t = \dfrac{\ell}{v} = \dfrac{R(2\pi - \theta)}{v}$ and

$R = \frac{mv}{Bq}$ which gives $t = \frac{m}{Bq}(2\pi - \theta)$

As time period: $T = \frac{2\pi m}{Bq}$, hence $t = \frac{T}{2\pi}(2\pi - \theta)$

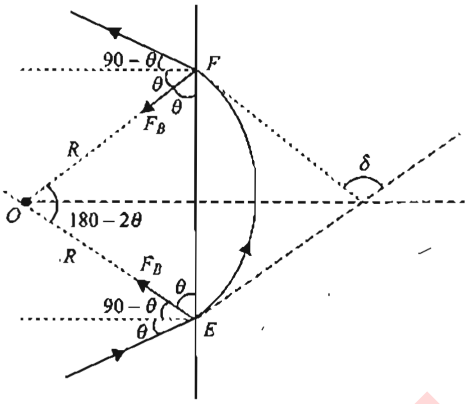


Fig. 9.12

We can generalise this result. If ϕ is the angle subtended by the arc traced by the charged particle in the magnetic field, the time spent is $t = T\left(\frac{\phi}{2\pi}\right)$

c. Intercept $EF = 2R\cos\theta$

Illustration 9.7 **A beam of protons with a velocity $4.0 \times 10^5\ \text{ms}^{-1}$. enters a uniform magnetic field of 0.3 tesla at an angle of 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also, find the pitch of the helix (which is the distance travelled by a proton in the beam parallel to the magnetic field during one period of rotation).**

(IIT-JEE, 1986)

Sol. a. The radius of helical path $r = \frac{mv_{\perp}}{Bq}$

b. $r = \frac{mv\sin\theta}{Bq} = \frac{(1.67\times10^{-27})(4\times10^5)(\sin 60°)}{(0.3)(1.6\times10^{-19})}$

$= 1.2 \times 10^{-2}\,\text{m}$

Pitch of helical path $p = v_{\parallel} \cdot T$

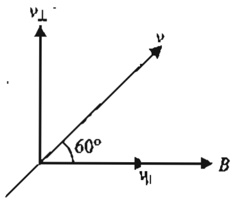


Fig. 9.13

$$p = \left(\frac{2\pi m}{Bq}\right)(v\cos\theta)$$

$$= \frac{(2\pi)(1.67\times10^{-27})(4\times10^5)(\cos 60°)}{(0.3)(1.6\times10^{-19})} = 4.37\times10^{-2}\,\text{m}$$

Illustration 9.8 **An electron gun G emits electrons of energy 2 keV travelling in the positive x-direction. The electrons are required to hit the spot S where $GS = 0.1$ m, and the line GS make an angle of 60° with the x-axis as shown in Fig. 9.14. A uniform magnetic field $\vec{B}$ parallel to GS exists in the region outside the electron gun. Find the minimum value of B needed to make the electrons hit S.** **(IIT-JEE, 1993)**

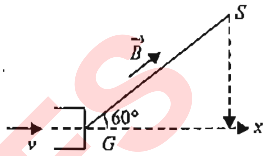


Fig. 9.14

Sol. Kinetic energy of electron, $K = \frac{1}{2}mv^2 = 2\,\text{keV}$

$\therefore$ Speed of electron, $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\times2\times1.6\times10^{-16}}{9.1\times10^{-31}}}\,\text{ms}^{-1}$

$= 2.65 \times 10^7\,\text{ms}^{-1}$

Since the velocity $(\vec{v})$ of the electron makes an angle of $\theta = 60°$ with the magnetic field $\vec{B}$, the path will be a helix.

So, the particle will hit S if $GS = np$; Here, $n = 1, 2, 3, \ldots$

p = pitch of helix $= \frac{2\pi m}{qB} v\cos\theta$

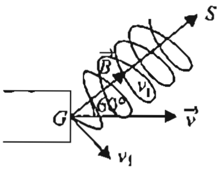


Fig. 9.15

But for B to be minimum, $n = 1$

Hence, $GS = p = \frac{2\pi m}{qB} v\cos\theta \Rightarrow B = B_{\min} = \frac{2\pi mv\cos\theta}{q(GS)}$

Substituting the values, we have

$$B_{\min} = \frac{(2\pi)(9.1\times10^{-31})(2.65\times10^7)\left(\frac{1}{2}\right)}{(1.6\times10^{-19})(0.1)} \text{ or } B_{\min} = 4.73\times10^{-3}\,\text{T}$$

Illustration 9.9 **A slightly divergent beam of charged particles accelerated by a P.D. V propagates from a point A along the axis of a solenoid. The beam is brought into focus at a distance l from the point A at two successive values of magnetic induction B_1 and B_2. Find the specific charge q/m of the particles.** **(IIT-JEE, 2007)**

Sol. Let us first calculate the velocity of the particles from the energy equation,

$$\frac{1}{2}mv^2 = Vq \quad \Rightarrow \quad v = \sqrt{\frac{2Vq}{m}}$$

Since the charged particles are slightly divergent, they will follow a helical path. Let θ be the small angle made by a particle with B.

Then, $v_{\parallel} = v\cos\theta$ and $v_{\perp} = v\sin\theta$

$$F_c = \text{centripetal force} = qv_{\perp}B = \frac{mv_{\perp}^2}{R}$$

$$qB = \frac{mv_{\perp}}{R} \quad \Rightarrow \quad qB = \frac{m\omega R}{R}$$

$$\therefore \quad \omega = \frac{qB}{m} \quad \Rightarrow \quad T = \frac{2\pi m}{qB}$$

$$\therefore\ p\ (\text{pitch of the particle}) = v_{\parallel} \times T = v\cos\theta \times \frac{2\pi m}{qB} = \frac{2\pi vm}{qB}$$

Particles are focussed if l contains integral number of pitches.

$$\ell = np \quad \Rightarrow \quad p = \ell/n = \ell,\ \ell/2,\ \ell/3, \ldots$$

$\therefore$ For two consecutive focussings,

$$\ell = p = \frac{2\pi mv}{qB_1} \quad \text{and} \quad \frac{\ell}{2} = \frac{2\pi mv}{qB_2}$$

$$\text{or} \quad B_1 = \frac{2\pi mv}{q\ell} \quad \text{and} \quad B_2 = \frac{4\pi mv}{q\ell},$$

$$\text{or} \quad B_2 - B_1 = \frac{2\pi mv}{q\ell} \quad \text{or,} \quad B_2 - B_1 = \frac{2\pi m}{q\ell}\sqrt{\frac{2Vq}{m}},$$

$$\text{or} \quad \frac{q}{m} = \frac{8\pi^2 V}{\ell^2 (B_2 - B_1)^2}$$

Illustration 9.10 **A beam of charged particle, having kinetic energy 10^3 eV, contains masses 8×10^{-27} kg and 1.6×10^{-26} kg emerge from the end of an accelerator tube. There is a plate at distance 10^{-2} m from the end of the tube and placed perpendicular to the beam. Calculate the magnitude of the smallest magnetic field which can prevent the beam from striking the plate.**

Sol. Let $\vec{B}$ be required magnetic field and E_k the kinetic energy. Maximum radius of circular path for the beam not to strike the plane

$$r = \frac{mv}{qB} = \frac{\sqrt{2mE_k}}{qB}$$

For maximum radius, mass should be maximum and magnetic field should be minimum, i.e.,

$$r_{\max} = \frac{\sqrt{2m_{\max}E_k}}{qB_{\min}}$$

$$B_{\min} = \frac{\sqrt{2m_{\max}E_k}}{qr_{\max}}$$

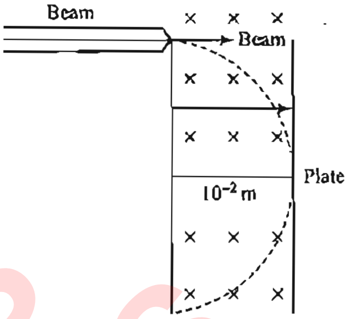


Fig. 9.16

Given $r_{\max}\ 10^{-2} m$, $m_{\max} = 1.6 \times 10^{-26}$ kg

$$q = e = 1.6 \times 10^{-19}\ \text{C (assume)}$$

$$E_k = 10^3\ \text{eV} = 10^3 \times 1.6 \times 10^{-19}\ \text{J}$$

$$B_{\min} = \frac{\sqrt{2 \times 1.6 \times 10^{-26} \times (10^3 \times 1.6 \times 10^{-19})}}{1.6 \times 10^{-19} \times 10^{-2}}$$

$$= \frac{1.6\sqrt{2} \times 10^{-21}}{1.6 \times 10^{-21}} = \sqrt{2}\ \text{T} = 1.414\ \text{T}$$

PATH OF A CHARGED PARTICLE IN BOTH ELECTRIC AND MAGNETIC FIELDS

Here, normally two cases are popular. In the first case, $\vec{E} \parallel \vec{B}$ and particle velocity is perpendicular to both of these fields. In the second case, $\vec{E} \perp \vec{B}$ and the particle is released from rest. From IIT-JEE point of view, first case is useful. Here we will discuss case 1.

Case 1: **When $\vec{E} \parallel \vec{B}$ and particle velocity is perpendicular to both of these fields.**

Consider a particle of charge q and mass m released from the origin with velocity $\vec{v} = v_0\hat{i}$ into a region of uniform electric and magnetic fields parallel to y-axis, *i.e.*, $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. The electric field accelerates the particle in y-direction, i.e., y component of velocity goes on increasing with acceleration,

$$a_y = \frac{F_y}{m} = \frac{F_e}{m} = \frac{qE_0}{m}.$$

The magnetic field rotates the particle in a circle in x–z plane (perpendicular to magnetic field). The resultant path of the particle is a helix with increasing pitch. The axis of the plane is parallel to y-axis. Velocity of the particle at time t would be,

$$\vec{v}(t) = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$\text{Here} \quad v_y = a_y t = \frac{qE_0}{m}t$$

$$\text{and} \quad v_x^2 + v_z^2 = \text{constant} = v_0^2; \quad \theta = \omega t = \frac{Bq}{m}t$$

$$v_x = v_0\cos\theta = v_0\cos\left(\frac{Bqt}{m}\right) \quad \text{and}$$

$$v_z = v_0 \sin\theta = v_0 \sin\left(\frac{Bqt}{m}\right)$$

$$\boxed{\vec{v}(t) = v_0 \cos\left(\frac{Bqt}{m}\right)\hat{i} + \left(\frac{qE_0}{m}t\right)\hat{j} + v_0 \sin\left(\frac{Bqt}{m}\right)\hat{k}}$$

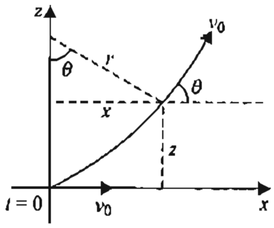


Fig. 9.17

Similarly, position vector of particle at time t can be given by,

$\vec{r}(t) = x\hat{i} + y\hat{j} + z\hat{k}$

Here, $$y = \frac{1}{2}a_y t^2 = \frac{1}{2}\left(\frac{qE_0}{m}\right)t^2$$

$$x = r\sin\theta = \left(\frac{mv_0}{Bq}\right)\sin\left(\frac{Bqt}{m}\right) \text{ and}$$

$$z = r(1-\cos\theta) = \left(\frac{mv_0}{Bq}\right)\left[\left\{1-\cos\left(\frac{Bqt}{m}\right)\right\}\right]$$

$$\vec{r}(t) = \left(\frac{mv_0}{Bq}\right)\sin\left(\frac{Bqt}{m}\right)\hat{i} + \frac{1}{2}\left(\frac{qE_0}{m}\right)t^2\hat{j}$$
$$+\left(\frac{mv_0}{Bq}\right)\left[\left\{1-\cos\left(\frac{Bqt}{m}\right)\right\}\right]\hat{k}$$

Illustration 9.11 **A particle of mass m and charge q is released from the origin in a region occupied by electric field E and magnetic field B, $\vec{B} = -B_0\hat{j}$; $\vec{E} = E_0\hat{j}$.**

Find the speed of the particle as a function of the x-coordinate.

Sol. Since the magnetic field does not perform any work, therefore whatever has been the gain in kinetic energy it is only because of the work done by electric field. Applying work–energy theorem,

$$W_E = \Delta K$$

$$qEx = \frac{1}{2}mv^2 - 0 \quad \text{or} \quad v = \sqrt{\frac{2qEx}{m}}$$

Illustration 9.12 **An electron accelerated through a potential difference of 2.5 kV, moves horizontally into a region of space in which there is a downward directed uniform electric field of magnitude 10 kV m^{-1}.**

a. In what direction must a magnetic field be applied so that the electron moves undeflected? Ignore the gravitational force. What is the magnitude of the smallest magnetic field possible in this case?

b. What happens if the charge is a proton that passes through the same combination of fields?

Sol. The Lorentz force experienced by the electron is

$$F = -e(\vec{E} + \vec{v} \times \vec{B}),$$

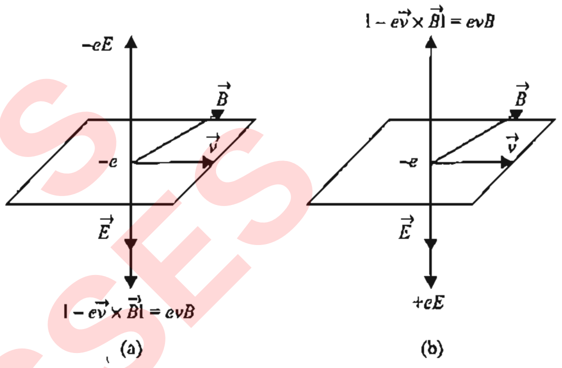


Fig. 9.18

where $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic field and v is the velocity of electron. The magnitude of magnetic force is $evB\sin\theta$, where θ is the angle between the velocity and the field. The total force vanishes, hence

$$B = E/v\sin\theta \qquad \text{(i)}$$

The magnetic field is to be smallest, therefore $\theta = 90°$. Thus, $B = E/v$.

Fig. 9.18(a) shows forces on an electron. In case of proton, both electric and magnetic forces reverse direction, but they still cancel; see Fig. 9.18(b).

Kinetic energy of electron $= eV = 2.5$ keV

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(2.5\times10^3)(1.60\times10^{-19})}{9.11\times10^{-31}}} = 2.96\times10^7\,\text{ms}^{-1}$$

$$B = \frac{E}{v} = \frac{10\times10^3}{2.96\times10^7} = 3.37\times10^{-4}\,\text{J}$$

Concept Application Exercise 9.1

1. A charged particle of mass 5 mg and charge $q = +2\,\mu$C has velocity $\vec{v} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Find out the magnetic force on the charged particle and its acceleration at this instant due to magnetic field $\vec{B} = 3\hat{j} - 2\hat{k}$. $\vec{v}$ and $\vec{B}$ are in ms^{-1} and Wbm^{-2}, respectively.
2. A charged particle has acceleration $\vec{a} = 2\hat{i} + x\hat{j}$ in a magnetic field $\vec{B} = -3\hat{i} + 2\hat{j} - 4\hat{k}$. Find the value of x.
3. A positive charge particle of charge q, mass m enters into a uniform magnetic field with velocity v as shown in Fig. 9.19. There is no magnetic field to the left of PQ.

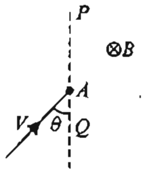


Fig. 9.19

Find (i) time spent, (ii) distance travelled in the magnetic field, (iii) impulse of magnetic force.

4. Repeat above Question 3, if the charge is −ve and the angle made by the boundary with the velocity is $\frac{\pi}{6}$.

5. A uniform magnetic field of strength 'B' exists in a region of width 'd'. A particle of charge 'q' and mass 'm' is shot perpendicularly (as shown in Fig. 9.20) into the magnetic field. Find the time speed n by the particle in the magnetic field if

i. $d > \frac{mu}{qB}$ **ii.** $d < \frac{mu}{qB}$

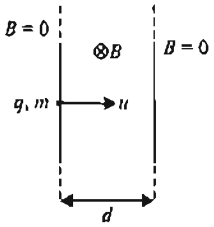


Fig. 9.20

6. In Fig. 9.21, what should be the speed of the charged particle so that it cannot collide with the upper wall? Also, find the coordinates of the point where the particle strikes the lower plate in the limiting case of velocity.

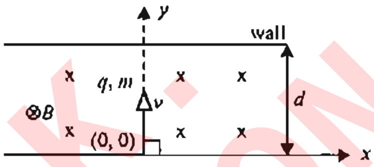


Fig. 9.21

7. A charged particle enters into a region which offers a resistance against its motion and a uniform magnetic field exists in the region. The particle traces a spiral path as shown in Fig. 9.22. State the following statements as True and False.

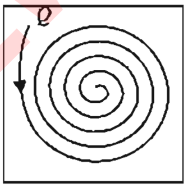


Fig. 9.22

i. Component of magnetic field in the plane of spiral is zero.

ii. Particle enters the region at Q.

iii. If magnetic field is outwards, then the particle is positively charged.

iv. If magnetic field is outwards, then the particle is negatively charged.

8. An electron moves in a uniform magnetic field and follows a spiral path as shown in Fig. 9.23. State the following statements as True and False.

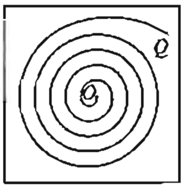


Fig. 9.23

i. Angular velocity of the electron remains constant.

ii. Magnitude of velocity of the electron decreases continuously.

iii. Net force on the particle is always perpendicular to its direction of motion.

iv. Magnitude of net force on the electron decreases continuously.

9. A charged particle moves in a gravity free space where an electric field of strength E and a magnetic field of induction B exist. State the following statements as True and False.

a. If $E \neq 0$ and $B \neq 0$, velocity of the particle may remain constant.

b. If $E = 0$, the particle cannot trace a circular path.

c. If $E \approx 0$, kinetic energy of the particle remains constant.

10. A particle with charge $+q$ and mass m, moving under the influence of a uniform electric field $E\hat{i}$ and a uniform magnetic field $B\hat{k}$, follows a trajectory from P to Q as shown (Fig. 9.24). The velocities at P and Q are $v\hat{i}$ and $-2v\hat{j}$.

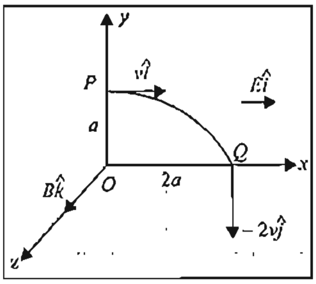


Fig. 9.24

i. Value of electric field is ________.

ii. The rate of work done by the electric field at P is ________.

iii. The rate of work done by both the fields at Q is ________.

11. A particle with charge −5.60 nC is moving in a uniform magnetic field $\vec{B} = -(1.25\text{T})\hat{k}$. The magnetic force on the particle is measured to be

$$\vec{F} = -(3.40 \times 10^{-7}\,\text{N})\hat{i} + (7.40 \times 10^{-7}\,\text{N})\hat{j}.$$

a. Calculate all components of velocity of the particle that you can from this information.

b. Are there components of the velocity that are not determined by the measurement of the force? Explain.

c. Calculate the scalar product $\vec{v} \cdot \vec{F}$. What is the angle between v and F?

12. A particle with charge 7.80 μC is moving with velocity $\vec{v} = -(3.80 \times 10^{3}\,\text{ms}^{-1})\hat{j}$. The magnetic force on the particle is measured to be $\vec{F} = +(7.60 \times 10^{-3}\,\text{N})\hat{i} - (5.20 \times 10^{-3}\,\text{N})\hat{k}$.

  a. Calculate all the components of the magnetic field you can from this information.
  b. Are there components of the magnetic field that are not determined by measurement of the force ? Explain.
  c. Calculate the scalar product $\vec{B}\cdot\vec{F}$. What is the angle between $\vec{B}$ and $\vec{F}$?

13. A particle with charge 6.40×10^{-19} C travels in a circular orbit with radius 4.68 mm due to the force exerted on it by a magnetic field with magnitude 1.65 T and perpendicular to the orbit.
  a. What is the magnitude of the linear momentum $\vec{P}$ of the particle?
  b. What is the magnitude of the angular momentum $\vec{L}$ of the particle?

14. Fig. 9.25 shows the trace of the path of a charged particle in a bubble chamber. Assume that the magnetic field is into the plane of the paper, with magnitude 0.4 T. The smooth spiral path occurs because the particle loses energy in ionizing molecules along the path.

B is into the page

Fig. 9.25

  a. Which part of the path corresponds to higher kinetic energy for the particle ?
  b. Is the charge positive or negative ?
  c. The radius of curvature ranges from 70 to 10 mm. What is the range of values of the magnitude of momentum if the magnitude of the charge is *e* ?

15. A particle of mass *m* and charge *q* is accelerated by a potential difference *V* volt and made to enter a magnetic field region at an angle θ with the field. At the same moment, another particle of same mass and charge is projected in the direction of the field from the same point. Magnetic field induction is *B*. What would be the speed of second particle so that both particles meet again and again after regular interval of time, which should be minimum? Also, find the time interval after which they meet and the distance travelled by the second particle during that interval.

16. A beam of equally charged particles after being accelerated through a voltage *V* enters into a magnetic field '*B*' as shown in Fig. 9.26. It is found that all the particles hit the plate between *C* and *D*. Find the ratio between the masses of the heaviest and lightest particles of the beam.

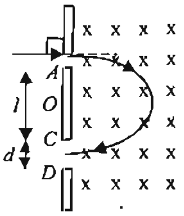


Fig. 9.26

17. A proton and an alpha particle are projected in a magnetic field which exists in the width of region *d*. Compare the angles of deviation suffered by the proton and the alpha particle if before entering the magnetic field both the particles

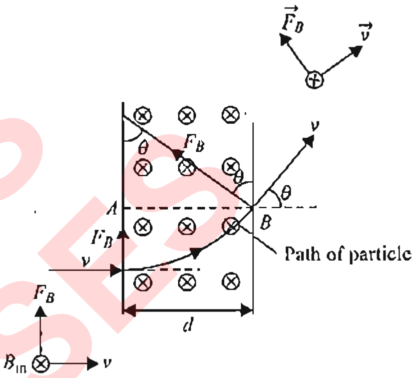


Fig. 9.27

  a. have the same momentum,
  b. have the same kinetic energy, and
  c. are accelerated through the same potential difference. Take $m_\alpha = 4m_p$, $q_\alpha = 2q_p$.

18. A beam of singly ionized atoms of carbon (each charge +*e*) all have the same speed and enter a mass spectrometer, as shown in Fig. 9.28. The ions strike the photographic plate in two different locations 5.00 cm apart. The $^{12}C_6$ isotope traces a path of smaller radius, 15.0 cm. What is the atomic mass number of other isotope?

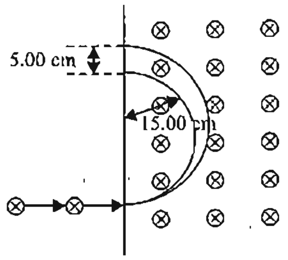


Fig. 9.28

19. A charge $q = -4$ μC has an instantaneous velocity $\vec{v} = (2\hat{i} - 3\hat{j} + \hat{k}) \times 10^6$ ms^{-1} in a uniform magnetic field $\vec{B} = (2\hat{i} + 5\hat{j} - 3\hat{k}) \times 10^{-2}$ T. What is the force on the charge?

20. When a proton has a velocity $\vec{v} = (2\hat{i} + 3\hat{j}) \times 10^6$ ms^{-1}, it experiences a force $\vec{F} = -(1.28 \times 10^{-13}\hat{k})$ N. When its velocity is along the *z*-axis, it experiences a force along the *x*-axis. What is the magnetic field?

21. The force on a charged particle moving in a magnetic field can be computed as the vector sum of the force due to each separate component of the magnetic field. As an example, a particle with charge *q* is moving with speed *v* in the –*y* direction. It is moving in a uniform magnetic field $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$.

a. What are the components of the force $\vec{F}$ exerted on the particle by the magnetic field?

b. If $q > 0$, what must the signs of the components of $\vec{B}$ be if the components $\vec{F}$ are all non-negative?

c. If $q < 0$ and $B_x = B_y = B_z > 0$, find the direction and magnitude of $\vec{F}$ in terms of $|q|$, v and B_x.

22. A particle of charge $q > 0$ is moving at speed v in the $+z$ direction through a region of uniform magnetic field. The magnetic force on the particle $\vec{F} = F_0(3\hat{i} + 4\hat{i})$, where F_0 is a positive constant.

a. Determine the components B_x, B_y and B_z or at least as many of the three components as is possible from the information given.

b. If it is given in addition that the magnetic field has magnitude $6F_0 / qv$, determine the magnitude of B_z.

23. Protons having a kinetic energy of 5.00 MeV are moving in the positive x-direction and enter a magnetic field $B = 0.0500\,\hat{k}$ T directed out of the plane of the page and extending from $x = 0$ to $x = 1.00$ m, as shown in Fig. 9.29.

a. Calculate the y-component of the protons' momentum as they leave the magnetic field.

b. Find the angle ϕ between the initial velocity vector of the proton beam and the velocity vector after the beam emerges from the field. Ignore relativistic effects and note that $1\,\text{eV} = 1.60 \times 10^{-19}$ J.

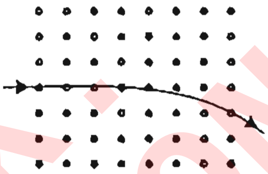


Fig. 9.29

24. Electrons in a beam are accelerated from rest through a potential difference ΔV. The beam enters an experimental chamber through a small hole. As shown in Fig. 9.30, the electron velocity vector lie within a narrow cone of half angle ϕ oriented along the beam axis. We wish to use a uniform magnetic field directed parallel to the axis to focus the beam, so that all of the electrons can pass through a small exit port on the opposite side of the chamber after they travel the length d of the chamber. What is the required magnitude of the magnetic field?

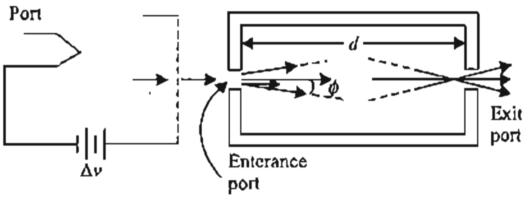
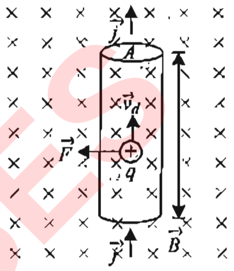


Fig. 9.30

FORCE ON A CURRENT CARRYING WIRE

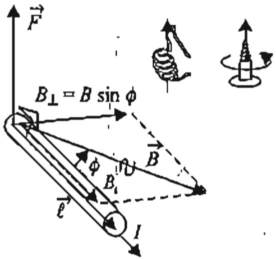
Fig. 9.31 shows a straight segment of a conducting wire, with length ℓ and cross-sectional area A; the current is from bottom to top. The wire is in a uniform magnetic field $\vec{B}$, perpendicular to the plane of the diagram and directed into the plane. Let us assume first that the moving charges are positive.



Forces on a moving positive charge in a current carrying conductor.

Fig. 9.31

The drift velocity $\vec{v}_d$ is upward, perpendicular to $\vec{B}$. The average force on each charge is $\vec{F} = q\vec{v}_d \times \vec{B}$, directed to the left as shown in the figure; since $\vec{v}_d$ and $\vec{B}$ are perpendicular, the magnitude of the force is $F = qv_d B$.



A straight wire segment of length $\vec{\ell}$ carries a current I in the direction of $\vec{\ell}$. The magnetic force on this segment is perpendicular to both $\vec{\ell}$ and the magnetic field $\vec{B}$.

Fig. 9.32

We can derive an expression for the total force on all the moving charges in a length ℓ of a conductor with cross-sectional area A. The number of charges per unit volume is n; a segment of conductor with length ℓ has volume $A\ell$ and contains a number of charges equal to $nA\ell$. The total force $\vec{F}$ on all the moving charges in this segment has magnitude

$$F = (nA\ell)(qv_d B) = (nqv_d A)(\ell B) \qquad \text{(i)}$$

The current density is $J = nqv_d$. The product JA is the total current I, so we can rewrite (i) as,

$$F = I\ell B \qquad \text{(ii)}$$

If the field $\vec{B}$ is not perpendicular to the wire but makes an angle ϕ with it, we handle the situation the same way we did for a single charge. Only the component of $\vec{B}$ perpendicular to the wire (and to the drift velocities of the charges) exerts a force; this component is $B_\perp = b \sin\phi$. The magnetic force on the wire segment is then

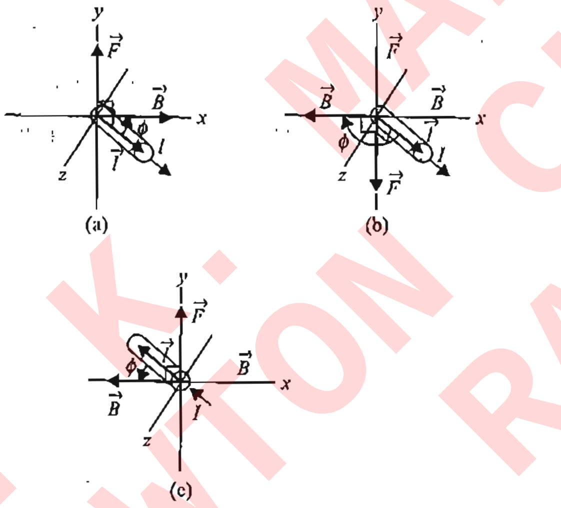
$$F = I\ell B_\perp = I\ell B \sin\phi \qquad \text{(iii)}$$

The force is always perpendicular to both the conductor and the field, with the direction determined by the same right hand rule we used for a moving positive charge (as shown in Fig. 9.32). Hence, this force can be expressed as a vector product, just like the force on a single moving charge. We represent the segment of wire with a vector $\vec{\ell}$ along the wire in the direction of the current; then the force $\vec{F}$ on this segment is

$$\vec{F} = I\vec{\ell}\times\vec{B} \text{ (magnetic force on a straight wire segment)} \qquad \text{(iv)}$$

If the conductor is not straight, we can divide it into infinitesimal segments $d\vec{\ell}$. The force $d\vec{F}$ on each segment is

$$d\vec{F} = I\,\vec{\ell}\times\vec{B} \text{ (magnetic force on an infinitesimal wire section)} \qquad \text{(v)}$$



(a) Magnetic field $\vec{B}$, length $\vec{\ell}$ and force $\vec{F}$ vectors for a straight wire carrying a current I.
(b) Reversing $\vec{B}$ reverses $\vec{F}$.
(c) Also reversing the current reverses $\vec{\ell}$ and returns $\vec{F}$ to the same directions as in (a)

Fig. 9.33

Direction of Force on a Current Carrying Wire in Magnetic Field

Left Hand Rule

If the thumb and first two fingers of the left hand are held each at right angles to the other, with the first finger pointing in the direction of the field and the second finger in the direction of the current, then the thumb predicts the direction of the thrust or force.

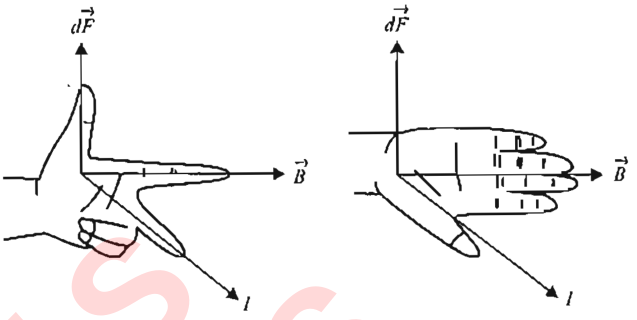


Fig. 9.34

Right Hand Palm Rule

Stretch the fingers and thumb of right hand at right angles to each other. Then if the fingers point in the direction of field $\vec{B}$ and thumb in the direction of current I, the normal to palm will point in the direction of force (or motion).

Regarding the force on a current carrying conductor in a magnetic field it is worth mentioning that:

As the force $BI\,dL \sin q$ is not a function of position r, the magnetic force on a current element is non-central [a central force is of the form $F = Kf(r)\,\hat{n}_r$]

The force $d\vec{F}$ is always perpendicular to both $\vec{B}$ and $I\,d\vec{L}$ though $\vec{B}$ and $I\,d\vec{L}$ may or may not be perpendicular to each other.

- **In case of current carrying conductor in a magnetic field if the field is uniform, i.e., $\vec{B}$ = constant,**

$$\vec{F} = \int I\,d\vec{L}\times\vec{B} = I[\int d\vec{L}]\times\vec{B}.$$

For a conductor, $\int d\vec{L}$ represents the vector sum of all the length elements from initial to final point, which in accordance with the law of vector addition is equal to the length vector $\vec{L}$ joining initial to final point. So, a current carrying conductor of any arbitrary shape in a uniform field experiences a force

$$\boxed{\vec{F} = i[\int d\vec{L}]\times\vec{B} = I\vec{L}\times\vec{B}}$$

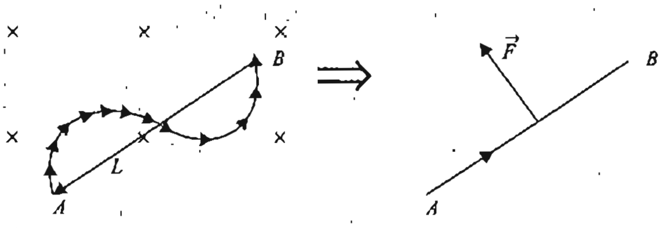


Fig. 9.35

where $\vec{L}$ is the length vector joining initial and final points of the conductor as shown in Fig. 9.35.

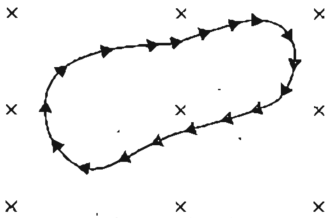
If the current carrying conductor in the form of a loop of any arbitrary shape is placed in a uniform field,

$$\vec F = \oint I d\vec L \times \vec B = I[\oint d\vec L] \times \vec B .$$

For a closed loop, the vector sum of $d\vec L$ is always zero.

So, $\vec F = 0$ [as $\oint d\vec L = 0$]

i.e., the net magnetic force on a current loop in a uniform magnetic field is always zero as shown in Fig. 9.36.

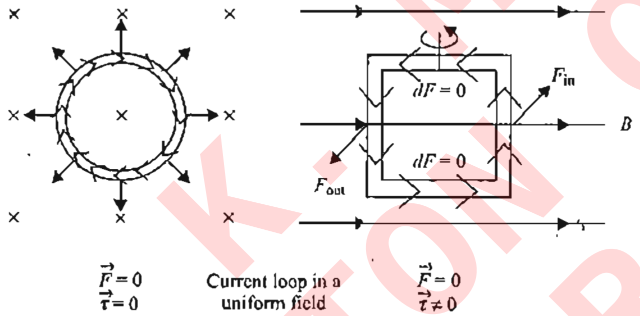


$\vec F = I(\oint d\vec L) \times \vec B = 0$

Fig. 9.36

- **A current carrying loop in a uniform magnetic field**

  Here, it must be kept in mind that in this situation different parts of the loop may experience elemental force due to which the loop may be under tension or may experience a torque as shown in Fig. 9.37.



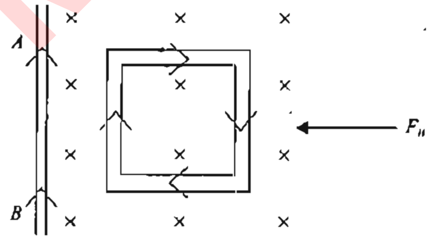
$\vec F = 0$
$\vec \tau = 0$

Current loop in a uniform field

$\vec F = 0$
$\vec \tau \neq 0$

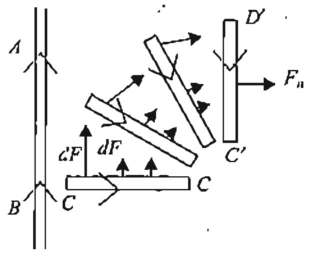
Fig. 9.37

- If a current carrying conductor is situated in a non-uniform field. Its different elements will experience different forces; so in this situation, $\vec F_R = 0$ but $\vec \tau$ may or may not be zero. If the conductor is free to move, it translates with or without rotation as shown in Fig. 9.38.



$\vec F_R = 0$
$\vec \tau = 0$

Loop in the field of current carrying wire AB



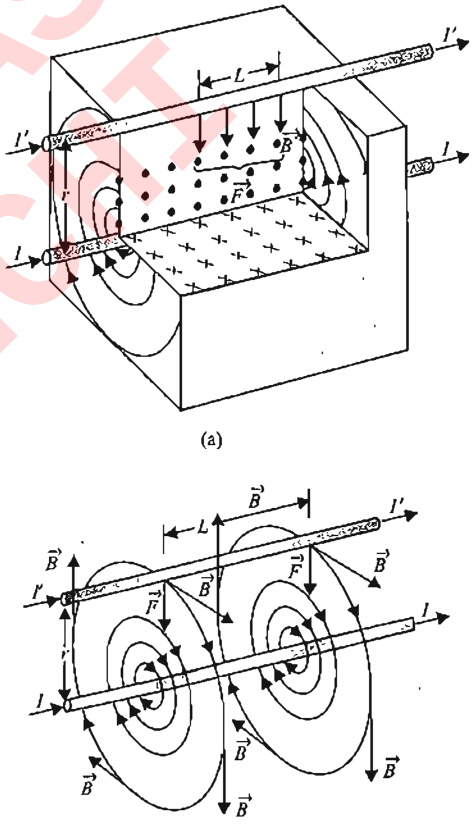
$\vec F_R \neq 0$
$\vec \tau \neq 0$

A current carrying rod in the field of wire AB

Fig. 9.38

Force Between Two Infinite Parallel Current Carrying Wires

Let two infinite parallel wires carrying currents I and I' are separated by a distance r (Fig. 9.39).



(a)

(b)

Fig. 9.39

If we take an arbitrary point on the second wire, $B_{21} = \frac{\mu_0 I}{2\pi r}$

and $\vec F_{21} = I'(\vec l \times \vec B_{21}) \Rightarrow F_{21} = \frac{\mu_0 I I'}{2\pi r}\ell_2 \Rightarrow \frac{F_{21}}{l_2} = \frac{\mu_0 I I'}{2\pi r}$

By symmetry, $\frac{F_{12}}{l_1} = \frac{\mu_0 I I'}{2\pi r}$ (Force per unit length on the first wire, due to second wire)

Note:

Force per unit length $= \frac{\mu_0 I I'}{2\pi r}$. *We note that the wires carrying current in the same direction attract.*

Illustration 9.13 **A circular loop of radius R is bent along a diameter and given a shape as shown in Fig. 9.40. One of the semicircles (KNM) lies in the x–z plane and the other one (KLM) in the y–z plane with their centers at the origin. Current I is flowing through each of the semicircles as shown in the figure.**

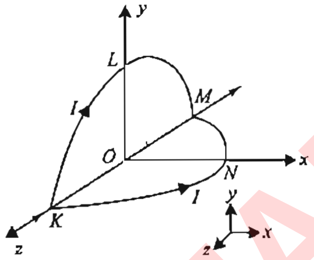


Fig. 9.40

a. **A particle of charge q is released at the origin with a velocity $v = -v_0\,\hat{j}$. Find the instantaneous force f on the particle. Assume that space is gravity free.**

b. **If an external uniform magnetic field $B\,\hat{j}$ is applied, determine the forces f_1 and f_2 on the semicircles KLM and KNM due to this field and the net force F on the loop.**

(IIT-JEE, 2006)

Sol. R = Radius of circular loop. Given that semicircle KNM lies in the x–z plane while the semicircle KLM lies in the y–z plane. Both the semicircles have their centers located at the origin.

a. Charge on the particle released at the origin = q

Velocity of the particle, $\vec{v} = v_0\hat{i}$

Magnetic field at center O due to current carrying loop KLM lying in y–z plane

$\vec{B}_1 = \frac{\mu_0 I}{4R}(-\hat{j})$. The direction of $\vec{B}_1$ will be along –ve x-axis.

Similarly, magnetic field at center O due to current carrying loop KNM lying in x–z plane, $\vec{B}_2 = \frac{\mu_0 I}{4R}(\hat{j})$.

Thus, the two fields at O are mutually perpendicular in vector form. Total field at O can be expressed as

$$\vec{B} = -\hat{i}B_1 + \hat{j}B_2 = \frac{\mu_0 I}{4R}\left(-\hat{i} + \hat{j}\right)$$

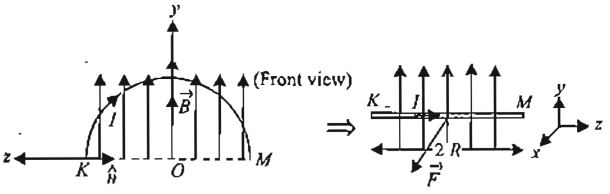


Fig. 9.41

Hence, instantaneous force acting on the charged particle, released at O

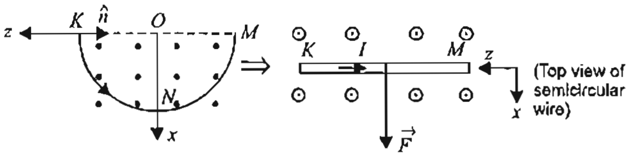


Fig. 9.42

$$\bar{F} = q\left(\bar{v} \times \vec{B}\right) = q\left[-v_0\hat{i} \times \left(-\hat{i} + \hat{j}\right)\frac{\mu_0 I}{4R}\right]$$

$$= \frac{\mu_0 I v_0}{4R}\left[\hat{i} \times \left(\hat{i} - \hat{j}\right)\right] = -\frac{\mu_0 I v_0}{4R}\hat{k},$$

$$\therefore \qquad \vec{F} = I(2R)(-\hat{k}) \times B(\hat{j}) = 2RIB\hat{i}$$

b. External uniform magnetic field $\bar{B}_{\text{ext}} = B\hat{j}$

As semicircular wires are placed in uniform magnetic field, these loops can be reduced to straight wires each of length $2R$ placed along z-axis (by joining initial point K and final point M).

Force $\vec{F}_1$ on semicircular loop KLM lying in y–z plane:

Force on a current element $I\bar{d}\ell$ in a field $\bar{B}$ is given by

$$d\bar{F} = I\left(d\vec{l} \times \bar{\mathrm{B}}\right)$$

The force on a current carrying conductor in a uniform field will be $\vec{F} = \int I\left(d\bar{l} \times \bar{\mathrm{B}}\right) = I\left[\int d\bar{l}\right] \times \vec{B} = I\left[KM \times \vec{B}\right]$

Hence net force due to both the wires

$$= \bar{F}_{\text{net}} = 2\bar{F} = 4RIB\,\hat{i}$$

Illustration 9.14 **Two long parallel wires carry currents of equal magnitude but in opposite directions. These wires are suspended from rod PQ by four chords of same length L as shown in Fig. 9.43. The mass per unit length of the wires is λ. Determine the value of θ assuming it to be small.**

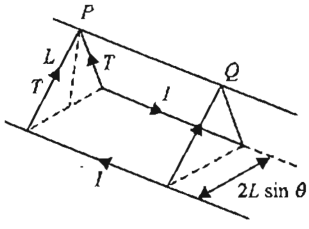


Fig. 9.43

Sol. The force per unit length between current carrying parallel wires is $\frac{dF}{dL} = \frac{\mu_0 I_1 I_2}{2\pi d}$

If two wires carry current in opposite directions the magnetic force is repulsive, due to which the parallel wires in Fig. 9.43 have moved out so that equilibrium is reached.

Fig. 9.44 shows freebody diagram of each wire. In equilibrium,

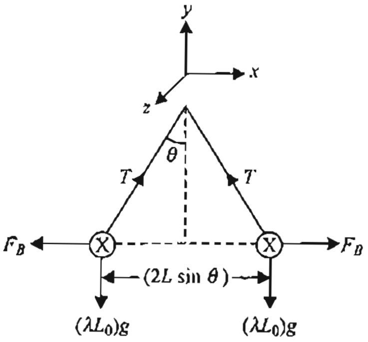


Fig. 9.44

$$\Sigma F_y = 0,\ 2T\cos\theta = (\lambda L_0)g \qquad \text{(i)}$$

$$\Sigma F_z = 0,\ 2T\sin\theta = F_B \qquad \text{(ii)}$$

Now, dividing equation (ii) by (i), we get $\tan\theta = \dfrac{F_B}{\lambda L_0 g}$ (iii)

The magnetic force $F_B = \left(\dfrac{dF}{dL}\right)\times L_0 = \dfrac{\mu_0 I^2}{4\pi \sin\theta}L_0$ (iv)

For small θ, $\tan\theta \simeq \sin\theta \simeq \theta$

On substituting equation (iv) in (iii), we get $\theta = I\sqrt{\dfrac{\mu_0}{4\pi\lambda g L}}$

Illustration 9.15 **A straight segment OC (of length L meter) of a circuit carrying a current I amp is placed along the x-axis. Two infinitely long straight wires A and B, each extending from $z = -\infty$ to $+\infty$ are fixed at $y = -a$ meter and $y = +a$ meter, respectively, as shown in Fig. 9.45. If the wires A and B each carry a current I amp into the plane of the paper, obtain the expression for the force acting on segment OC. What will be the force on OC if the current in the wire B is reversed?**

(IIT-JEE, 1992)

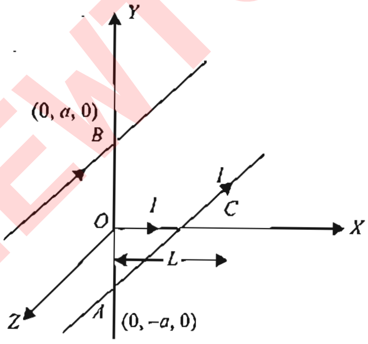


Fig. 9.45

Sol. Let us take an element of thickness dx at a distance x from origin on the wire OC. Magnetic field B_A produced at P $(x,0,0)$ due to wires placed at A and B are

$$B_A = \mu_0 I/2\pi R, \quad B_B = \mu_0 I/2\pi R$$

Components of B_A and B_B along x-axis cancel, while those along y-axis add up to give total field,

$$B = 2\left(\frac{\mu_0 I}{2\pi R}\right)\cos\theta = \frac{2\mu_0 I}{2\pi R}\,\frac{x}{R} = \frac{\mu_0 I}{\pi}\,\frac{x}{(a^2+x^2)}$$

The force dF acting on the current element is

$$d\vec{F} = I(d\vec{l}\times\vec{B})$$

$$dF = \frac{\mu_0 I^2}{\pi}\,\frac{x dx}{a^2+x^2} \qquad [\because \sin 90^\circ = 1]$$

Hence, net force on wire OC, $F = \int dF$

i.e.,
$$F = \frac{\mu_0 I^2}{\pi}\int_0^L \frac{x dx}{a^2+x^2} = \frac{\mu_0 I^2}{2\pi}\ln\frac{a^2+L^2}{a^2}$$

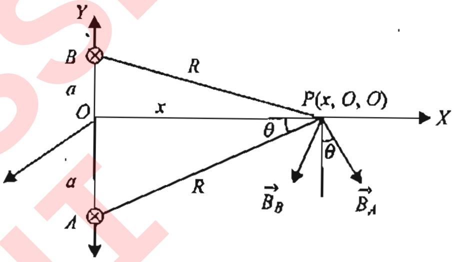


Fig. 9.46

If the current in B is reversed, the magnetic field due to the two wires would be only along x-direction and the force on the current carrying wire along x-direction will be zero.

Concept Application Exercise 9.2

1. Two long wires, carrying currents i_1 and i_2, are placed perpendicular to each other in such a way that they just avoid a contact. Find the magnetic force on a small length dl of the second wire situated at a distance λ from the first wire.

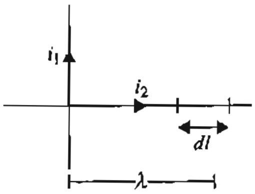


Fig. 9.47

2. A wire is bent in the form of an equilateral triangle PQR of side 10 cm and carries a current of 5.0 A. It is placed in a magnetic field B of magnitude 2.0 T directed perpendicularly to the plane of the loop. Find the forces on the three sides of the triangle.

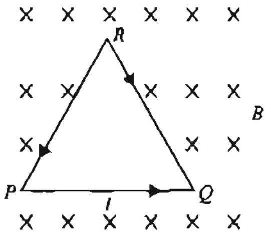


Fig. 9.48

3. Fig. 9.49 shows two long metal rails placed horizontally and parallel to each other at a separation l. A uniform magnetic field B exists in the vertically downward direction. A wire of mass m can slide on the rails. The rails are connected to a constant current source which drives a current i in the circuit. The friction coefficient between the rails and the wire is μ.
   a. What should be the minimum value of μ which can prevent the wire from sliding on the rails?
   b. Describe the motion of the wire if the value of μ is half the value found in the previous part.

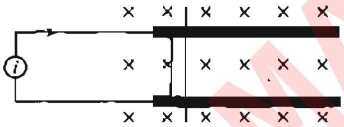


Fig. 9.49

4. In Fig. 9.50, a semicircular wire is placed in a uniform field $\vec{B}$ directed toward right. Find the resultant magnetic force and torque on it.

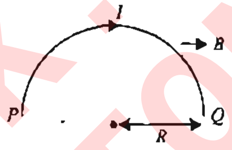


Fig. 9.50

5. In Fig. 9.51, find the resultant magnetic force and torque about 'C', and 'P'.

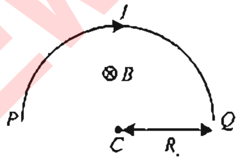


Fig. 9.51

6. Find the magnetic force on the loop '$PQRS$' due to the loop wire.

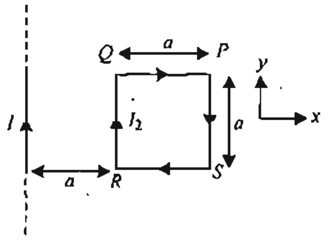


Fig. 9.52

7. A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field B. What is the magnitude of the magnetic field?

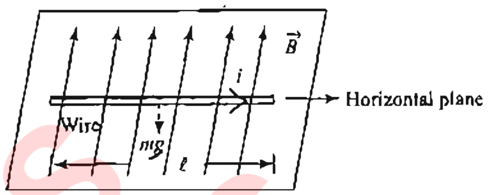


Fig. 9.53

8. Calculate the force on a current carrying wire in a uniform magnetic field as shown in Fig. 9.54.

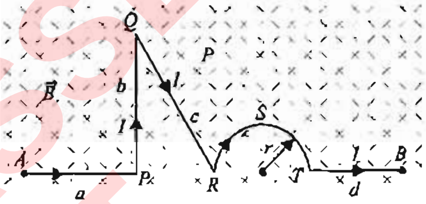


Fig. 9.54

9. The horizontal component of the earth's magnetic field at a certain place is 3×10^{-5} T and the direction of the field is from the geographic south to the geographic north. A very long straight conductor is carrying a steady current of 1 A. What is the force per unit length on it when it is placed on a horizontal table and the direction of the current is
   a. east to west;
   b. south to north?

10. The circuit in Fig. 9.55(a) consists of wires at the top and bottom and identical metal springs as the left and right sides. The wire at the bottom has a mass of 10.0 g and is 5.00 cm long. The springs stretch 0.500 cm under the weight of the wire, and the circuit has a total resistance of 12.0 Ω. When a magnetic field is turned [illegible] directed out of the page, the springs stretch an adc [illegible]. What is the magnitude of the magnetic field? (The upper portion of the circuit is fixed.)

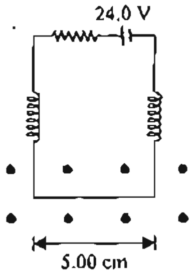


Fig. 9.55(a)

11. A long straight conductor carrying I_1 is placed in the plane of a ribbon carrying current I_2 parallel to the previous one. The width of the ribbon is b and the straight conductor is at a distance a from the near edge. Find the force of attraction between the two.

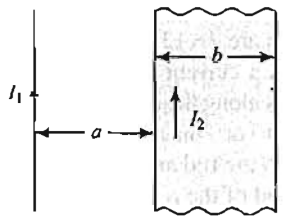


Fig. 9.55 (b)

12. The pie-shaped current loop shown in Fig. 9.56 subtends an angle of $\pi/6$ rad and lies in the xy-plane. The radius $R = 40.0$ cm and the current $I = 6.00$ A. The uniform magnetic field $\vec{B}$ is parallel to the positive z-axis and has a magnitude of 0.750 T.

- **a.** Compute the magnetic force (magnitude and direction) on the segment ab.
- **b.** Compute the magnetic force on the segment bc.
- **c.** Compute the magnetic force on the segment ca, and show that the net force on the current loop is zero.

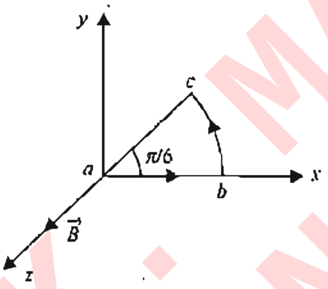


Fig. 9.56

13. A charge Q is uniformly distributed over a ring which is rotating with constant angular velocity ω about an axis passing through its center and perpendicular to the plane. A wire which carries a current I is lying perpendicular to the plane of the ring along its axis having one end at its center. Find resultant magnetic force on the wire by the ring.

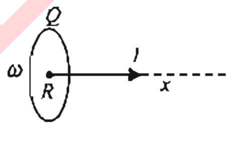


Fig. 9.57

14. Each of the lettered points at the corners of the cube as shown in Fig. 9.58 represents a positive charge q moving with a velocity of magnitude v in the direction indicated. The region in the figure is in a uniform magnetic field $\vec{B}$, parallel to the x-axis and directed toward the right. Copy the figure, find the magnitude and direction of the force on each charge and show the force in your diagram.

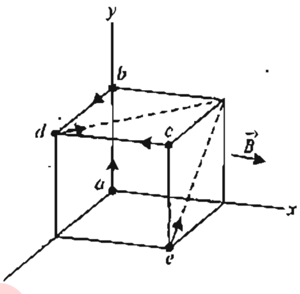


Fig. 9.58

15. A +6.00 μC point charge is moving at a constant velocity of 8.00×10^6 ms^{-1} in the $+y$-direction, relative to a reference frame. At the instant when the point charge is at the origin of this reference frame, what is the magnetic field vector it produces at the following points:

- **a.** $x = 0.500$ m, $y = 0$, $z = 0$, and
- **b.** $x = 0$, $y = -0.500$ m, $z = 0$.

16. If magnetic field calculated in Question no. 15 is B_0, then calculate the magnetic field at the points

- **a.** $x = 0$, $z = +0.500$ m, and
- **b.** $x = 0$, $y = -0.500$ m, $z = +0.500$ m.

17. The cube as shown in Fig. 9.59, 75.0 cm on a side, is in a uniform magnetic field of 0.860 T parallel to the x-axis. The wire $abcdef$ carries a current of 6.58 A in the direction indicated.

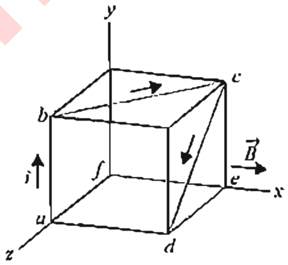


Fig. 9.59

- **a.** Determine the magnitude and direction of the force acting on the segment ab.
- **b.** Determine the magnitude and direction of the force acting on the segment bc.
- **c.** Determine the magnitude and direction of the force acting on the segment cd.
- **d.** Determine the magnitude and direction of the force acting on the segment de.
- **e.** Determine the magnitude and direction of the force acting on the segment ef.
- **f.** What are the magnitude and direction of the total force on the wire ?

18. Two long, parallel wires are separated by a distance of 0.400 m (as shown in Fig. 9.60). The currents I_1 and I_2 have the directions shown.

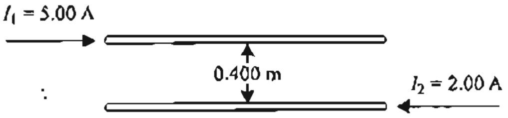


Fig. 9.60

a. Calculate the magnitude of the force exerted by each wire on a 1.20 m length of the other. Is the force attractive or repulsive?

b. Each current is doubled, so that I_1 becomes 10.0 A and I_2 becomes 4.00 A. Now, what is the magnitude of the force that each wire exerts on a 1.20 m length of the other?

19. A straight wire lies along a body diagonal of an imaginary cube of side a = 20 cm, and carries a current of 5 A (as shown in Fig. 9.61). Find the force on it due to a uniform field $\vec{B} = 0.6\hat{j}T$.

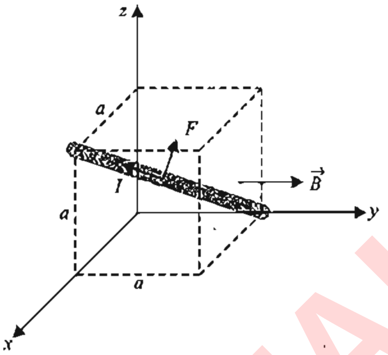


Fig. 9.61

20. In Fig. 9.62, the bar AC has a mass of 50 g. It slides freely on the metal strips 40 cm apart at the edges of the incline. A current I flows through these strips and the bar, as inclined. There is a magnetic field $|B_x| = 0.02$ T directed in the $-y$-direction. How long must I be if the rod is to remain motionless? Neglect the slight overhang of the rod.

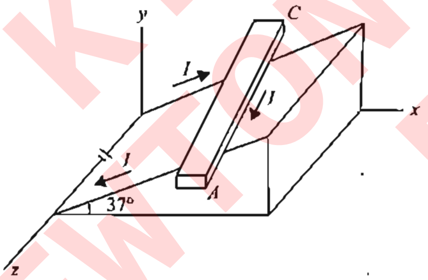


Fig. 9.62

21. In Fig. 9.63, a three sides frame is pivoted at AC and hangs vertically. Its sides are each of the same length and have a linear density of 0.10 kg m^{-1}. A current of 10.0 A is sent through the frame, which is in a uniform magnetic field of 10 mT directed upward. Through what angle will the frame be deflected?

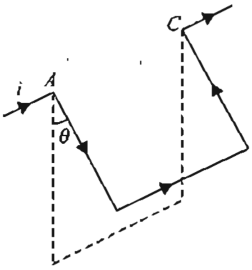


Fig. 9.63

22. A rod of mass 0.720 kg and radius 6.00 cm rests on two parallel rails that are d = 12.0 cm apart and L = 45.0 cm long. The rod carries a current of I = 48.0 A (in the direction shown) and rolls along the rails without slipping. A uniform magnetic field of magnitude 0.240 T is directed perpendicular to the rod and the rails. If it starts from rest, what is the speed of the rod as it leaves the rails.

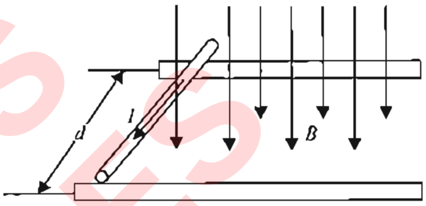


Fig. 9.64

23. Two circular loops are parallel, coaxial, and almost in contact, 1.00 mm apart. Each loop is 10.0 cm in radius. The top loop carries a clockwise current of 140 A. The bottom loop carries a counterclockwise current of 140 A. (a) Calculate the magnetic force exerted by the bottom loop on the top loop. (b) The upper loop has a mass of 0.0210 kg. Calculate its acceleration, assuming that the only forces acting on it are the force in part (a) and the gravitational force.

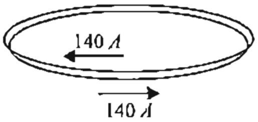


Fig. 9.65

24. Two long, parallel conductors carry currents in the same direction as shown in Fig. 9.66. Conductor A carries a current of 150 A and is held firmly in position. Conductor B carries a current I_B and is allowed to slide freely up and down (parallel to A) between a set of non-conducting guides. If the mass per unit length of conductor B is 0.100 g cm^{-1}, what value of current I_B will result in equilibrium when the distance between the two conductors is 2.50 cm?

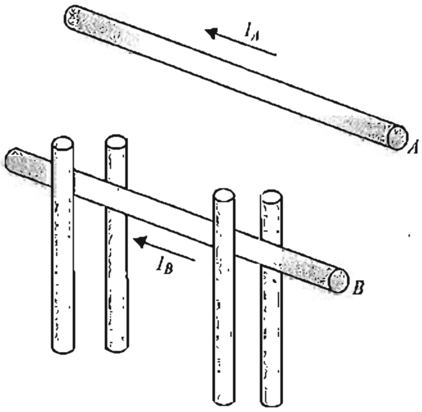


Fig. 9.66

25. An infinitely long straight wire carrying a current I_1 is partially surrounded by a loop as shown in Fig. 9.67. The loop has a length L, radius R and carries a current I_2. The axis of the loop coincides with the wire. Calculate the force exerted on the loop.

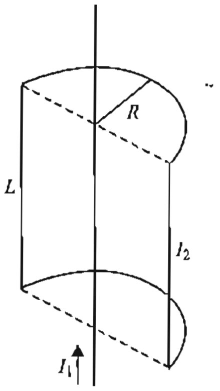


Fig. 9.67

MAGNETIC DIPOLE AND DIPOLE MOMENT

1. Magnetic dipole is the magnetic equivalent of electric dipole.
2. The magnetic field pattern produced by a small current loop is similar to a bar magnet. Therefore, it also acts like a magnetic dipole. The magnetic moment of a flat current loop is defined as the product of the current I and the area A enclosed by it, i.e., $\vec{M} = I\vec{A}$.

   The direction of the magnetic moment coincides with the direction of the area vector (which is the direction of the magnetic field).

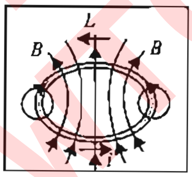


Fig. 9.68

If the loop contains N number of turns, the magnetic moment is given by $M = NIA$

- Sometimes a current carrying loop does not lie in a single plane. But by assuming two equal and opposite currents in one branch (which obviously makes no change in the given circuit) two (or more) closed loops are completed in different planes. Now, the net magnetic moment of the given loop is the vector sum of individual loops. For example, in Fig. 9.69, six sides of a cube of side l carry a current i in the directions shown. By assuming two equal and opposite currents in wire AD, two loops in two different planes (xy and yz) are completed.

$$\vec{M}_{ABCDA} = -il^2\hat{k} \quad \text{and} \quad \vec{M}_{ADGFA} = -il^2\hat{j}$$

$$\Rightarrow \quad \vec{M}_{\text{net}} = -il^2(\hat{i} + \hat{k})$$

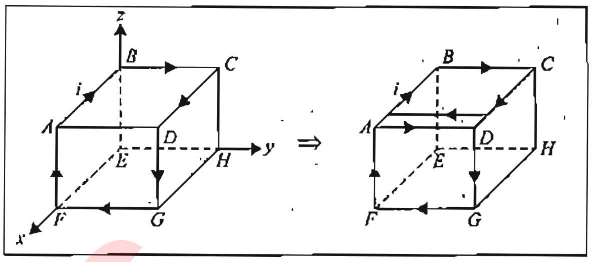


Fig. 9.69

- Sometimes a non-conducting body is related with some angular speed. In this case, the ratio of magnetic moment and angular momentum is constant which is equal to $q/2m$, where q is the charge and m is the mass of the body. For example, in case of a ring of mass m, radius R and charge q distributed on its circumference,

Angular momentum, $L = I\omega = (mR^2)(\omega)$ (i)

Magnetic moment, $M = iA = (qf)(\pi R^2)$ (ii)

Here, f = frequency $= \dfrac{\omega}{2\pi}$

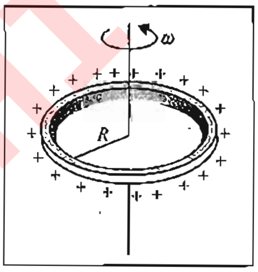


Fig. 9.70

$$\therefore \quad M = (q)\left(\frac{\omega}{2\pi}\right)(\pi R^2) = q\frac{\omega R^2}{2}$$

From equations (i) and (ii), $\dfrac{M}{L} = \dfrac{q}{2m}$

Although this expression is derived for simple case of a ring, it holds good for other bodies also. For example, for a disk or a sphere.

Illustration 9.16 **Compute the magnetic dipole moment of the loop shown in Fig. 9.71.**

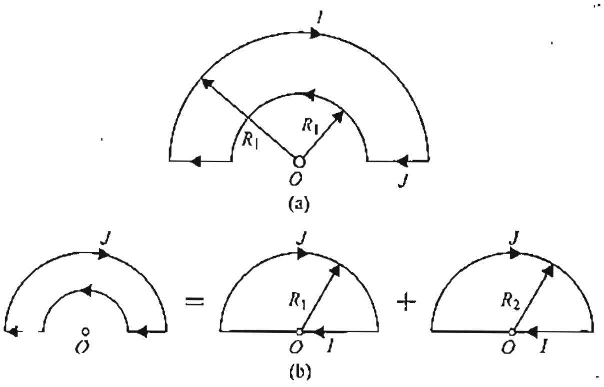


Fig. 9.71

Sol. The given loop may be considered as the superposition of the two loops, as shown in the figure.

The resultant dipole moment is $M = \frac{\pi R_1^2 I}{2} - \frac{\pi R_2^2 I}{2}$

or $\quad M = \frac{\pi I}{2}\left(R_1^2 - R_2^2\right)$ (inwards)

Illustration 9.17 **A circular loop of wire of radius R is bent about its diameter along two mutually perpendicular planes as shown in Fig. 9.72. If the loop carries a current I, then determine its magnetic moment.**

Sol. The given loop may be obtained by the superposition of two semicircular loops as shown in the figure.

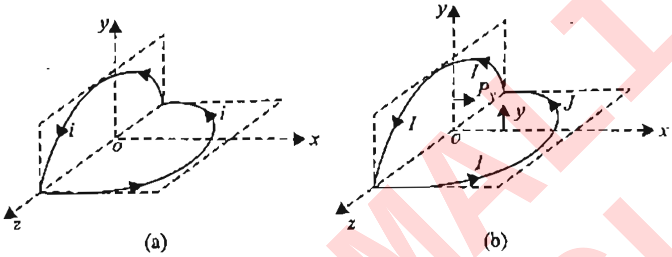


Fig. 9.72

The magnetic moment of the semicircle in the yz plane is along the x-axis and that in the xz plane is along the y-axis.

$$M_x = \frac{\pi R^2 I}{2}, \quad M_y = \frac{\pi R^2 I}{2}$$

The total magnetic moment is $\vec{M} = M_x\hat{i} + M_y\hat{j}$

or $\quad M = \frac{\pi R^2 I}{2}\left(\hat{i} + \hat{j}\right)$

Illustration 9.18 **A conductor carries a constant current I along the closed path *abcdefgha* involving 8 of the 12 edges of length l. Find the magnetic dipole moment of the closed path.**

Sol. The closed path is a superposition of three loops: *bcfgb*, *abgha* and *cdefc*.

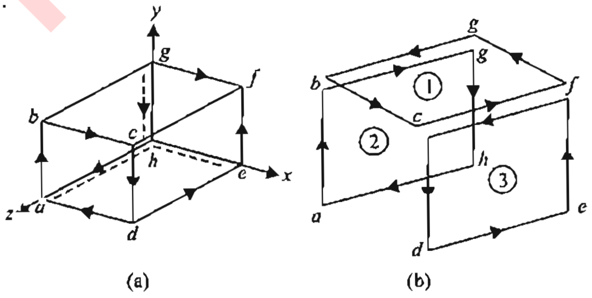


Fig. 9.73

The magnetic moments of the three loops are:

Loop $\quad$ 1 (*bcfgb*): $\vec{\mu}_1 = l^2 I\,\hat{j}$

2 (*abgha*): $\vec{\mu}_2 = -l^2 I\hat{i}$

3 (*cdefc*): $\vec{\mu}_3 = l^2 I\hat{i}$

The total magnetic moment is $\vec{\mu} = \vec{\mu}_1 + \vec{\mu}_2 + \vec{\mu}_3$ or $\vec{\mu} = l^2 I\hat{j}$.

Illustration 9.19 **A non-conducting disk of mass M and radius R has a surface charge density and rotates with an angular velocity about its axis. Show that magnetic dipole moment and angular momentum are related as $\vec{\mu} = \left(\frac{Q}{2M}\right)\vec{L}$.**

Sol. The charge is distributed on the surface of the disk.

We consider a differential ring of radius r and thickness dr.

The charge on the element is $dq = \sigma dA = \sigma(2\pi r dr)$

The magnetic moment of the ring $d\mu = (dI)A = (dI)\pi r^2$

The current in the differential ring

$$= (dq)\nu = (\sigma dA)\frac{\omega}{2\pi} = (\sigma 2\pi r dr)\frac{\omega}{2\pi} = \sigma\omega r dr$$

The magnetic moment of the differential ring,

$$d\mu = (\sigma\omega r dr)\,\pi r^2 = \pi\sigma\omega r^3 dr$$

$$\mu = \int d\mu = \int_0^R \pi\sigma\omega r^3 dr = \frac{1}{4}\pi\sigma\omega R^4$$

The magnetic moment vector $\vec{\mu}$ is parallel to $\vec{\omega}$ if it is positive.

$$\vec{\mu} = \frac{1}{4}\pi\sigma R^4\vec{\omega}$$

In terms of total charge $Q = \sigma\pi R^2$, the magnetic moment is

$$\vec{\mu} = \frac{1}{4}QR^2\vec{\omega}.$$

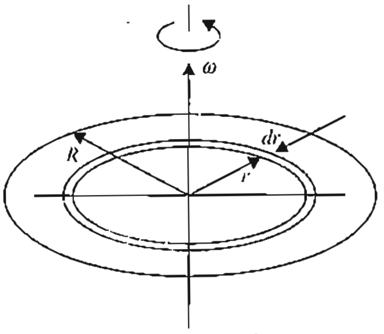


Fig. 9.74

The angular momentum of disk is $\vec{L} = \left(\frac{1}{2}MR^2\right)\vec{\omega}$ and $\vec{\mu} = \left(\frac{Q}{2M}\right)\vec{L}$.

This is a general result for any rigid body of any arbitrary shape, with mass M and charge Q.

Illustration 9.20 **A sphere of radius R, uniformly charged with the surface charge density σ, rotates around the axis passing through its center at an angular velocity. Find the magnetic induction at the center of the rotating sphere. Also, find its magnetic moment.**

Sol. Charge on the differential circular strip is

$$dq = (2\pi R \sin\theta)(R d\theta)\sigma$$

$$\Rightarrow \quad dq = 2\pi\sigma R^2 \sin\theta\, d\theta;$$

$$dI = \frac{\omega\, dq}{2\pi} = \omega\sigma R^2 \sin\theta\, d\theta$$

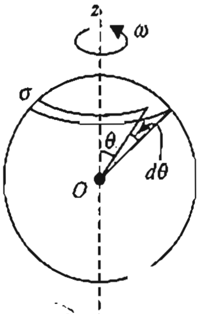


Fig. 9.75

a. Magnetic field at the center is

$$dB = \frac{\mu_0 dI\,(R^2 \sin^2\theta)}{2R^3} = \frac{\mu_0\omega\sigma R \sin^3\theta\, d\theta}{2}$$

$$B = \frac{\mu_0\omega\sigma R}{2}\int_0^{\pi}\sin^3\theta\, d\theta = \mu_0\omega\sigma R\int_0^{\pi/2}\sin^3\theta\, d\theta$$

$$\Rightarrow \quad \vec{B} = \frac{2}{3}\mu_0\omega\sigma R\,(\hat{k})$$

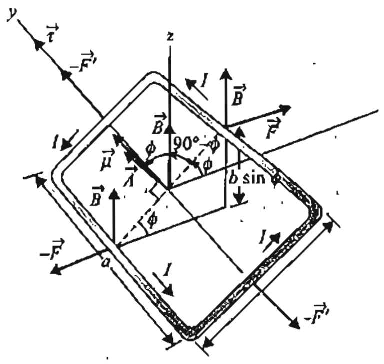
b. Magnetic moment due to elementary ring, $d\mu = (dI)\pi r^2$

$$\mu = \int (dI)\pi r^2 = 2\int_0^{\pi/2}(\omega\sigma R^2 \sin\theta)\pi R^2 \sin^2\theta\, d\theta$$

$$= 2\pi R^4\omega\sigma\int_0^{\pi/2}\sin^3\theta\, d\theta = \frac{4}{3}\pi\sigma R^4\omega$$

Torque on a Current Carrying Planer Loop in a Uniform Magnetic Field

Fig. 9.76 shows a rectangular loop of wire with side lengths a and b. A line perpendicular to the plane of the loop (i.e., a normal to the plane) makes an angle ϕ with the direction of the magnetic field $\vec{B}$, and the loop carries a current I.



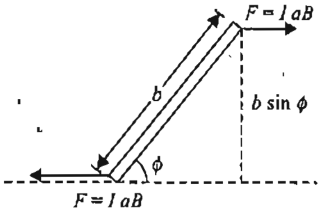


Fig. 9.76

The force $\vec{F}$ on the right side of the loop (length a) is to the right, in the $+x$-direction as shown. On this side, $\vec{B}$ is perpendicular to the current direction and the force on this side has magnitude $F = IaB \sin\phi$ (i)

A force $-\vec{F}$ with the same magnitude but opposite direction acts on the opposite side of the loop, as shown in the figure.

The lines of action of both forces lie along the y-axis.

The total force on the loop is zero because the forces on opposite sides cancel out in pairs. The net force on a current loop in a uniform magnetic field is zero. However, the net torque is not in general equal to zero. The two forces $\vec{F}$ and $-\vec{F}$ lie along different lines, and each gives rise to a torque about the y-axis. According to the right hand rule for determining the direction of torques, the vector torques due to $\vec{F}$ and $-\vec{F}$ are both in the $+y$-direction; hence the net vector torque $\vec{\tau}$ is in the $+y$-direction as well. The magnitude of the net torque is

$$\tau = F(b)\sin\phi = (IBa)(b\sin\phi) \qquad \text{(ii)}$$

The area A of the loop is equal to ab, so we can rewrite equation (ii) as

$$\tau = IBA \sin\phi$$

(magnitude of torque on a current loop) (iii)

where ϕ is the angle between the normal to the loop (the direction of the vector area $\vec{A}$) and $\vec{B}$.

The product IA is called the magnetic dipole moment or magnetic moment of the loop

$$M = IA \qquad \text{(iv)}$$

We can express this interaction in terms of the torque vector $\vec{\tau}$, which we used for electric-dipole interactions. The magnitude of $\vec{\tau}$ is equal to the magnitude of $\vec{M}\times\vec{B}$. So, we have

$$\vec{\tau} = \vec{M}\times\vec{B} \text{ (vector torque on a current loop)} \qquad \text{(v)}$$

When a magnetic dipole changes orientation in a magnetic field, the field does work on it. In an infinitesimal angular displacement $d\phi$, the work dW is given by $\tau\, d\phi$, and there is a corresponding change in potential energy.

Energy needed to rotate the loop through an angle $d\theta$ is

$$dU = \tau d\theta$$

$$\Rightarrow \quad \Delta U = \int_{\theta_1}^{\theta_2}\tau d\theta = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta$$

$$\Rightarrow \qquad \Delta U = MB(\cos\theta_1 - \cos\theta_2)$$

If we choose θ_1 such that at $\theta_1 = \theta_0$, $U_1 = 0$

$$U = -\vec{M}\cdot\vec{B}$$

This is the energy stored in the loop.

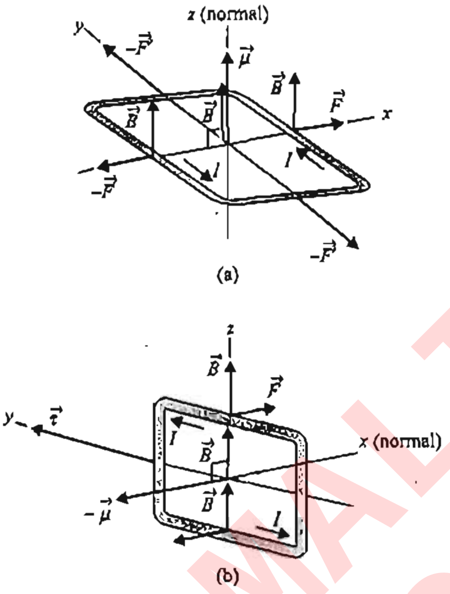


Fig. 9.77

Various forces on the sides of a current carrying loop in a uniform magnetic field are:

a. The resultant force is zero; the net torque has magnitude $\tau = IAB\sin\phi$.

b. The torque is maximum when the normal to the loop is perpendicular to $\vec{B}$.

c. When the normal to the loop is parallel to $\vec{B}$, the torque is zero and the equilibrium is stable. If the normal is antiparallel to $\vec{B}$, the torque is also zero but the equilibrium is unstable.

Illustration 9.21 **An electron in the ground state of hydrogen atom is revolving in anti-clockwise direction in a circular orbit of radius R.**

i. Obtain an expression for the orbital magnetic dipole moment of the electron.

ii. The atom is placed in a uniform magnetic induction $\vec{B}$ such that the plane-normal to the electron-orbit makes an angle of 30° with the magnetic induction. Find the torque experienced by the orbiting electron.

(IIT-JEE, 1996)

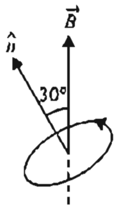


Fig. 9.78

Sol.

a. In ground state ($n = 1$) according to Bohr's theory:

$$mvR = \frac{h}{2\pi} \quad \text{or} \quad v = \frac{h}{2\pi mR}$$

Now, time period, $T = \dfrac{2\pi R}{v} = \dfrac{2\pi R}{h/2\pi mR} = \dfrac{4\pi^2 mR^2}{h}$

Magnetic moment, $M = IA$

where $\quad i = \dfrac{\text{charge}}{\text{time period}} = \dfrac{e}{\dfrac{4\pi^2 mR^2}{h}} = \dfrac{eh}{4\pi^2 mR^2}$

and $\quad A = \pi R^2$

$$\therefore \quad M = (\pi R^2)\left(\frac{eh}{4\pi^2 mR^2}\right) \quad \text{or} \quad M = \frac{eh}{4\pi m}$$

Direction of magnetic moment $\vec{M}$ is perpendicular to the plane of orbit.

b. $\vec{\tau} = \vec{M}\times\vec{B} \quad \Rightarrow \quad |\vec{\tau}| = MB\sin\theta$

where θ is the angle between $\vec{M}$ and $\vec{B}$

Given, $\quad \theta = 30°$

$$\therefore \quad \tau = \left(\frac{eh}{4\pi m}\right)(B)\sin 30° \quad \Rightarrow \quad \tau = \frac{ehB}{8\pi m}$$

The direction of $\vec{\tau}$ is perpendicular to both $\vec{M}$ and $\vec{B}$.

Illustration 9.22 **A uniform, constant magnetic field is $\vec{B}$ directed at an angle of 45° to the x-axis in the x–y plane. $PQRS$ is a rigid, square wire frame carrying a steady current I_0, with its center at the origin O. At time $t = 0$, the frame is at rest in the position shown in Fig. 9.79, with its sides parallel to the x- and y-axes. Each side of the frame is of mass M and length L.**

a. What is the torque $\vec{\tau}$ about O acting on the frame due to the magnetic field?

b. Find the angle by which the frame rotates under the action of this torque in a short interval of time Δt, and the axis about which this rotation occurs. (Δt is so short that any variation on the torque during this interval may be neglected). Given, moment of inertia of the frame about an axis through its center perpendicular to its plane is (4/3) ML^2. **(IIT-JEE, 1998)**

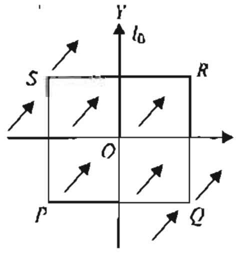


Fig. 9.79

Sol.

a. As magnetic field is in x–y plane and subtends an angle of 45° with x-axis,

$$B_x = B\cos 45° = B/\sqrt{2}$$

and $\quad B_y = B\sin 45° = B/\sqrt{2}$

so, in vector form $\vec{B} = \hat{i}\left(B/\sqrt{2}\right) + \hat{j}\left(B/\sqrt{2}\right)$

and $\quad \vec{M} = I_0 S\hat{k} = I_0 L^2\hat{k}$

$$\Rightarrow \quad \vec{\tau} = \vec{M} \times \vec{B} = I_0 L^2 \hat{k} \times \left(\frac{B}{\sqrt{2}} \hat{i} + \frac{B}{\sqrt{2}} \hat{j} \right)$$

$$\Rightarrow \quad \vec{\tau} = \frac{I_0 L^2 B}{\sqrt{2}} \times (-\hat{i} + \hat{j})$$

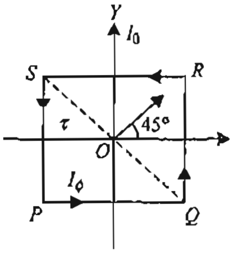


Fig. 9.80

i.e., torque has magnitude $I_0 L^2 B$ and is directed along the line QS from Q to S.

b. By the theorem of perpendicular axis, moment of inertia of the frame about QS,

$$I_{QS} = \frac{1}{2} I_z = \frac{1}{2} \left(\frac{4}{3} ML^2 \right) = \frac{2}{3} ML^2$$

$$\text{as } \tau = I\alpha \quad \Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{I_0 L^2 B \times 3}{2L^2 M} = \frac{3}{2} \frac{I_0 B}{M}$$

As here α is constant, equations of circular motion are valid.

Hence, from $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$, with $\omega_0 = 0$, we have

$$\theta = \frac{1}{2} \alpha t^2 = \frac{1}{2} \left(\frac{3}{2} \frac{I_0 B}{M} \right) (\Delta t)^2 = \frac{3}{4} \frac{I_0 B}{M} \Delta t^2$$

Illustration 9.23 **A rectangular loop $PQRS$ made from a uniform wire has length a, width b and mass m. It is free to rotate about the arm PQ, which remains hinged along a horizontal line taken as the y-axis. Take the vertically up-ward direction as the z-axis. A uniform magnetic field $\vec{B} = (3\hat{i} + 4\hat{k}) B_0$ exists in the region. The loop is held in the x–y plane and a current I is passed through it. The loop is now released and is found to stay in the horizontal position in equilibrium.**

a. What is the direction of the current I in PQ?

b. Find the magnetic force on the arm RS.

c. Find the expression for I in terms of B_0, a, b and m.

(IIT-JEE, 2002)

Sol.

a. Torque due to weight of coil,

$$\vec{\tau} = \left(\frac{a}{2} \hat{i} \right) \times (-mg\,\hat{k}) = mg\,\frac{a}{2}\,(\hat{j})$$

For the equilibrium of loop, torque on it must be along negative y-axis. Let the magnetic moment of loop be $\mu \hat{k}$. As the loop lies in x–y plane, its magnetic moment vector (from right hand thumb rule) either points up or down.

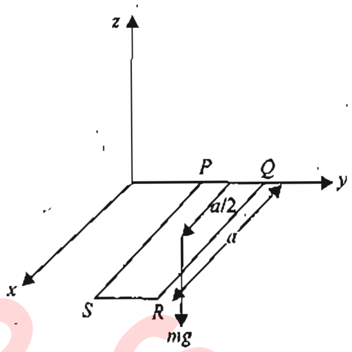


Fig. 9.81

Torque due to magnetic force,

$$\vec{\tau}_B = \vec{\mu} \times \vec{B} = \mu \hat{k} \times (3\hat{i} + 4\hat{k}) B_0 = 3\mu B_0 \hat{j}$$

If it is to be in negative direction, $\vec{\mu}$ must point downward. So, the current in the coil must be from P to Q.

b. Force acting on arm $RS = I(\vec{l} \times \vec{B}) = I[(-b\hat{j}) \times (3\hat{i} + 4\hat{k}) B_0]$

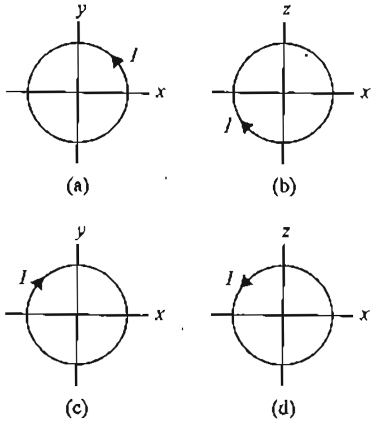
$$= IB_0 b\,(3\hat{k} - 4\hat{i})$$

c. In equilibrium $\vec{\tau}_{\text{gravity}} + \vec{\tau}_B = 0$

Hence, $3(abI)B_0 = \frac{mga}{2}$ or $I = \frac{mg}{6B_0 b}$

Concept Application Exercise 9.3

1. A circular coil with area A and N turns is free to rotate about a diameter that coincides with the x-axis. Current I is circulating in the coil. There is a uniform magnetic field $\vec{B}$ in the positive y-direction. Calculate the magnitude and direction of the torque $\vec{\tau}$ and the value of the potential energy U, when the coil is oriented as shown in parts (a) through (d) of Fig. 9.82.



(a) (b) (c) (d)

Fig. 9.82

2. A square loop $OABCO$ of side l carries a current i. It is placed as shown in Fig. 9.83. Find the magnetic moment of the loop.

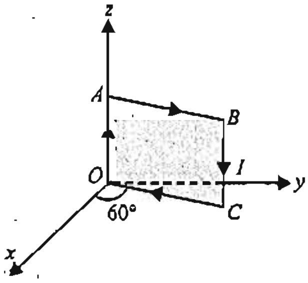


Fig. 9.83

3. Find the magnetic moment of the current carrying loop *OABCO* shown in Fig. 9.84. Given that $i = 4.0$ A, $OA = 20$ cm and $AB = 10$ cm.

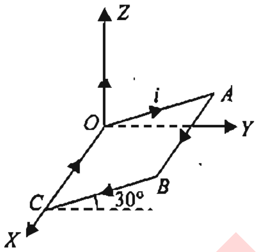


Fig. 9.84

4. Compute the magnetic dipole moment of the loop shown in Fig. 9.85.

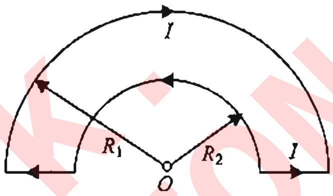


Fig. 9.85

5. Fig. 9.86 shows a bent coil with all edges of length 1 m and carrying a current of 1 A.

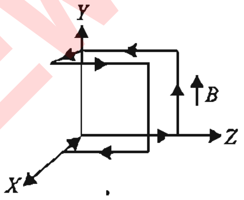


Fig. 9.86

a. Find the magnetic moment of the loop.
b. Find the torque acting on the loop.

6. An electron is in a circular orbit about the nucleus of an atom. Find the ratio between the orbital magnetic dipole moment $\vec{\mu}$ and the angular moment $\vec{L}$ of the electron about the center of its orbit.

7. A rod has a total charge Q uniformly distributed along its length L. If the rod rotates with angular velocity ω about its end, compute its magnetic moment.

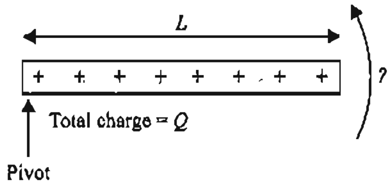


Fig. 9.87

8. The rectangular coil having 100 turns is turned in a uniform magnetic field of $(0.05/\sqrt{2})\hat{j}$ tesla as shown in Fig. 9.88. Find the torque acting on the loop.

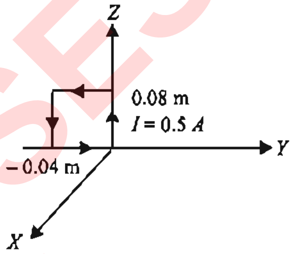


Fig. 9.88

9. The square loop in Fig. 9.89 has sides of length 20 cm. It has 5 turns and carries a current of 2 A. The normal to the loop is at 37° to a uniform field $\vec{B} = 0.5\,\hat{j}$ T.

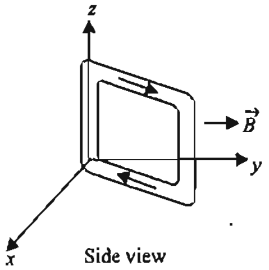


Fig. 9.89

a. Find the magnetic moment of the loop.
b. Find the torque on the loop.
c. Find the work needed to rotate the loop from its position of minimum energy to the given orientation.

10. A circular wire loop of radius R, mass m and current I lies on a rough surface (as shown in Fig. 9.90). There is a horizontal magnetic field $\vec{B}$. How large can the current I be before one edge of the loop will lift off the surface?

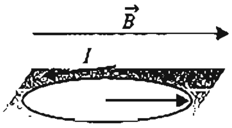


Fig. 9.90

11. A square 12-turn coil with sides of length 40 cm carries a current of 3 A. It lies in the x–y plane as shown (Fig. 9.91) in a uniform magnetic field.

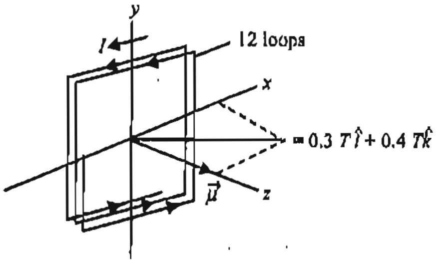


Fig. 9.91

a. Find the magnetic moment of the coil.
b. Find the torque exerted on the coil.
c. Find the potential energy of the coil.

12. Fig. 9.92 shows one quarter of a simple circular loop of wire that carries a current of 14 A. Its radius is a = 5 cm. A uniform magnetic field, B = 300 G, is directed in the +x-direction. Find the torque on the entire loop and the direction in which it will rotate.

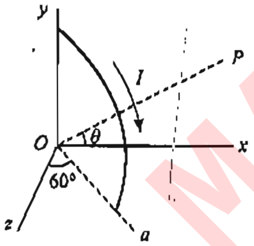


Fig. 9.92

13. The circular current loop of radius b shown in Fig. 9.93 is mounted rigidly on the axle, midway between the two supporting cords. In the absence of an external magnetic field, the tensions in the cords are equal and are T_0.

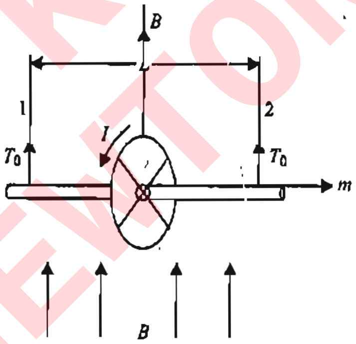


Fig. 9.93

a. What will be the tensions in the two cords when the vertical magnetic field B is present?
b. Repeat if the field is parallel to the axis.

14. A rigid circular loop of radius r and mass m lies in the xy plane on a flat table and has a current I flowing in it. At this particular place, the earth's magnetic field is $B = B_x\hat{i} + B_y\hat{j}$. How large must I be before one edge of the loop will lift from the table?

15. A circular loop of wire of radius r lies in the xy plane and carries a current I. Impinging on it is a magnetic field given by $B = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$. Find the vector torque which acts on the coil due to the magnetic field.

16. A rectangular coil consists of N = 100 closely wrapped turns and has dimensions a = 0.400 m and b = 0.300 m. The coil is hinged along the y-axis and its plane makes an angle θ = 30.0° with the x-axis. What is the magnitude of the torque exerted on the coil by a uniform magnetic field B = 0.800 T directed along the x-axis when the current is I = 1.20 A in the direction shown (Fig. 9.94). What is the expected direction of rotation of the coil?

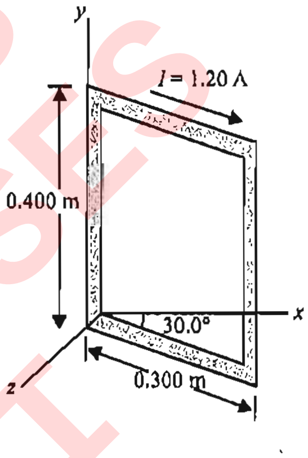


Fig. 9.94

17. A wire is formed into a circle having a diameter of 10.0 cm and placed in a uniform magnetic field of 3.00 mT. The wire carries a current of 5.00 A.
Find

a. the maximum torque on the wire, and
b. the range of potential energies of the wire–field system for different orientations of the circle.

MAGNETIC FIELD DUE TO A MOVING CHARGE AND CURRENT CARRYING WIRE

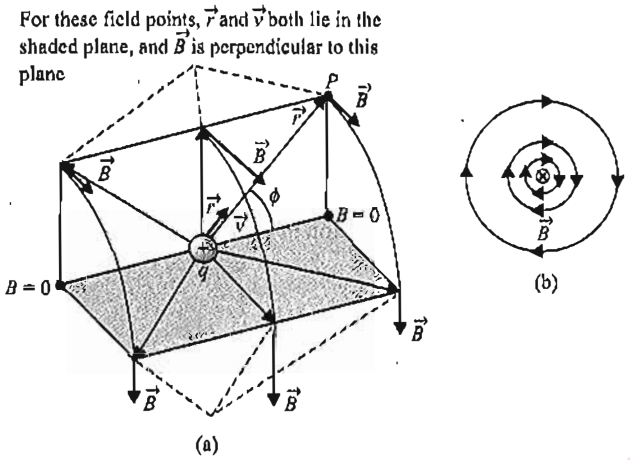
Magnetic Field of a Moving Charge

We call the location of the moving charge at a given instant the *source point* and the point P where we want to find the field the *field point*.

Experiments show that the magnitude of $\vec{B}$ is also proportional to $|q|$ and to $1/r^2$. But the direction of $\vec{B}$ is not along the line from source point to field point. Instead, $\vec{B}$ is perpendicular to the plane containing this line and the particle's velocity vector $\vec{v}$ as shown in Fig. 9.95. Furthermore, the field magnitude B is also proportional to the particle's speed v and to the sine of the angle ϕ. Thus the magnetic field magnitude at point P is given by

$$B = \frac{\mu_0}{4\pi}\frac{|q|\,v\sin\phi}{r^2} \qquad \text{(i)}$$

where $\mu_0/4\pi$ is a proportionality constant.



a. Magnetic field vectors due to a moving positive point charge q. At each point, $\vec{B}$ is perpendicular to the plane of $\vec{r}$ and $\vec{v}$ and its magnitude is proportional to the sine of the angle between them.
b. Magnetic field lines in a plane containing a moving positive charge. (×) indicates that the charge is moving into the plane of the page.

Fig. 9.95

We can incorporate both the magnitude and direction of B into a single vector equation using vector product.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \qquad \text{(ii)}$$

(magnetic field of a point charge with constant velocity)

A point charge in motion also produces an electric field, with field lines that radiate outward from a positive charge. The magnetic field lines are completely different. The above discussion shows that for a point charge moving with velocity $\vec{v}$, the magnetic field lines are circles centered on the line of $\vec{v}$ and lying in planes perpendicular to this line.

As we discussed, the unit of B is one tesla (1 T):

$$1\text{ T} = 1\text{ Ns}\,(\text{Cm}^{-1}) = 1\text{ N}\,(\text{Am}^{-1})$$

Using this with equation (i) or (ii), we find that the units of the constant μ_0 are:

$$1\text{ Ns}^2\text{C}^{-2} = 1\text{ NA}^{-2} = 1\text{ Wb}\,(\text{Am}^{-1}) = 1\text{ T mA}^{-1}$$

In S.I. units, the numerical value of μ_0 is exactly $4\pi\times 10^{-7}$. Thus,

$$\mu_0 = 4\pi\times 10^{-7}\text{ Ns}^2\text{C}^{-2} = 4\pi\times 10^{-7}\text{ Wb}\,(\text{Am}^{-1})$$
$$= 4\pi\times 10^{-7}\text{ T mA}^{-1} \qquad \text{(iii)}$$

- For points lying very far from the particle, the magnetic field is zero.
- The magnetic field due to a stationary particle is zero. As the velocity increases, the magnetic field increases.
- For points lying on a line parallel to $\vec{v}$ and passing through the charge, θ is zero and hence the magnetic field is zero. Hence, there is no magnetic field directly ahead or behind a moving charge along its line of motion.
- For points lying on a plane perpendicular to $\vec{v}$, the magnetic field is the greatest since θ is 90°.
- Magnetic field lines are concentric circles centered on the line of the velocity $\vec{v}$ and lying in planes perpendicular to this line.
- The direction of the magnetic field lines is given by the right hand rule. Grasp the velocity vector $\vec{v}$ with the right hand so that your right thumb points in the direction of $\vec{v}$. Your fingers then curl around the line of $\vec{v}$ and they point in the direction of the magnetic field at that point.

Illustration 9.24 **A point charge of magnitude $q = 4.5$ nC is moving with speed $v = 3.6\times 10^7\text{ ms}^{-1}$ parallel to the x-axis along the line $y = 3$ m. Find the magnetic field at the origin produced by this charge when the charge is at the point $x = -4$ m, $y = 3$ m, as shown in Fig. 9.96.**

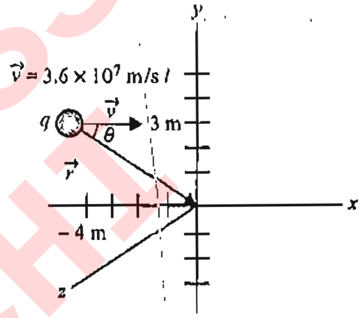


Fig. 9.96

Sol. The magnetic field is given by

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}, \text{ with } \vec{v} = v\hat{i}$$

$$\vec{r} = (4\hat{i} - 3\hat{j})\text{ m} \quad\Rightarrow\quad r = \sqrt{4^2 + 3^2}\text{ m} = 5\text{ m}$$

$$\text{Unit vector in the direction of } \vec{r} = \frac{\vec{r}}{r} = \frac{4\hat{i} - 3\hat{j}}{5} = 0.8\hat{i} - 0.6\hat{j}$$

$$\vec{v}\times\hat{r} = (v\hat{i})\times(0.8\hat{i} - 0.6\hat{j}) = -0.6v\hat{k}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{q(-0.6v\hat{k})}{r^2}$$

$$= -(10^{-7})\frac{(4.5\times 10^{-9})(0.6)(3.6\times 10^7)}{(5^2)}\hat{k}$$

$$= -3.89\times 10^{-10}\text{ T}\,\hat{k}$$

Magnetic Field of a Current Element

We begin by calculating the magnetic field caused by a short segment $d\vec{\ell}$ of a current carrying conductor, as shown in Fig. 9.97(a). The volume of the segment is $A\,d\ell$, where A is the cross-sectional area of the conductor. If there are n moving charged particles per unit volume, each of charge q, the total moving charge dQ in segment is

$$dQ = nqA\,d\ell$$

The moving charges in this segment are equivalent to a single charge dQ, travelling with a velocity equal to the drift velocity $\vec{v}_d$. (Magnetic fields due to the random motions of the charges will, on average, cancel out at every point.) From equation (i), the magnitude of the resulting field $d\vec{B}$ at any field point is

$$dB = \frac{\mu_0}{4\pi}\frac{|dQ|v_d \sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{n|q|v_d A d\ell \sin\phi}{r^2}$$

But $n|q|v_d A$ equals the current I in the element. So,

$$dB = \frac{\mu_0}{4\pi}\frac{I\,d\ell \sin\phi}{r^2} \qquad \text{(iv)}$$

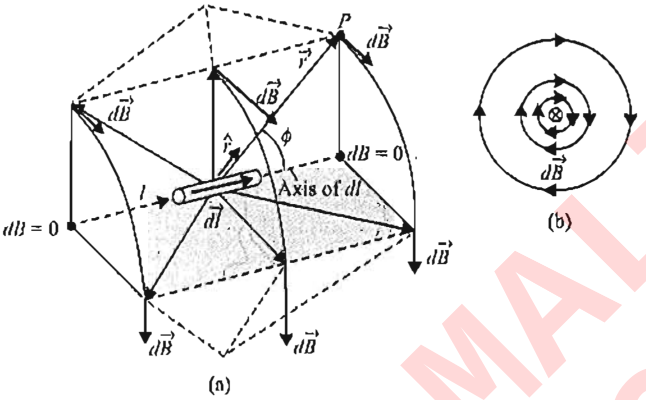


Fig. 9.97

In vector form, using the unit vector $\hat{r}$ as in 'Magnetic Field of a Moving Charge', we have

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell}\times\hat{r}}{r^2}$$

(magnetic field of a current element) (v)

Equations (iv) and (v) are called the law of Biot and Savart. We can use this law to find the total magnetic field $\vec{B}$ at any point in space due to the current in a complete circuit. To do this, we integrate equation (v) over all segments $d\vec{\ell}$ that carry current symbolically.

$$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{I\,d\vec{\ell}\times\hat{r}}{r^2} \qquad \text{(vi)}$$

As Fig. 9.97(a) shows, the field vectors $d\vec{B}$ and the magnetic field lines of a current element are exactly like those set up by a positive charge dQ moving in the direction of the drift velocity $\vec{v}_d$. The field lines are circles in planes perpendicular to $d\vec{\ell}$ and centered on the line of $d\vec{\ell}$. Their directions are given by the same right hand rule that we introduced for point charges.

Important Points of Frame Dependence of $\vec{B}$:

a. The motion of anything is a relative term. A charge may appear at rest by an observer (say O_1) and moving at some velocity $\vec{v}_1$ with respect to observer O_2 and at velocity $\vec{v}_2$ with respect to observer O_3. Then, $\vec{B}$ due to that charge w.r.t. O_1 will be zero and w.r.t. O_2 and O_3 it will be $\vec{B}_1$ and $\vec{B}_2$ (that means different).

b. In a current carrying wire electrons move in the opposite direction to that of the current and +ve ions (of the metal) are static w.r.t. the wire. Now, if some observer (O_1) moves with velocity v_d in the direction of motion of the electrons, then electrons will have zero velocity and +ve ions will have velocity v_d in the downward direction w.r.t. O_1. The density (n) of +ve ions is same as the density of free electrons and their charges are of the same magnitudes.

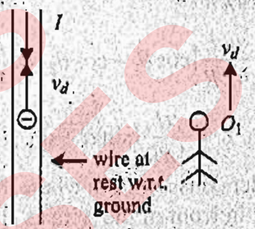


Fig. 9.98

So, w.r.t. O_1 electrons will produce zero magnetic field but +ve ions will produce the magnetic field.

c. **$\vec{B}$ due to magnet:**

$\vec{B}$ produced by the magnet does not contain the term of velocity.

So, we can say that $\vec{B}$ due to a magnet does not depend on frame.

Magnetic Field Due to Current in a Straight Line

The magnetic field due to a wire segment carrying current I at P, when the wire segment subtends angles α and β as shown (Fig. 9.99), can be determined as follows:

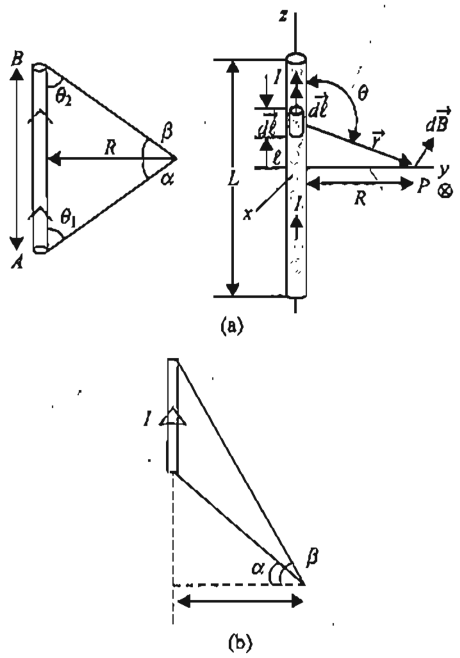


Fig. 9.99

dB at P, due to dl is: $dB = \frac{\mu_0}{4\pi}\frac{I d\ell \sin\theta}{r^2}$

Now, $\ell = R\cot(\pi-\theta)$, $d\ell = R\,\text{cosec}^2\,\theta\cdot d\theta$

$r = R\,\text{cosec}\,\theta$

$$\Rightarrow \quad dB = \frac{\mu_0}{4\pi}\frac{I(\sin\theta)(R\,\text{cosec}^2\,\theta\,d\theta)}{R^2\,\text{cosec}^2\,\theta}$$

$$B = \frac{\mu_0\ell}{4\pi R}\int_{\theta_1}^{180-\theta_2}\sin\theta\,d\theta = \frac{\mu_0\ell}{4\pi R}[\cos\theta_1+\cos\theta_2]$$

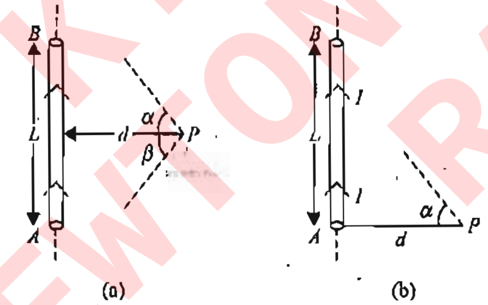
$$B = \frac{\mu_0 I}{4\pi R}[\cos\theta_1+\cos\theta_2] = \frac{\mu_0 I}{4\pi R}[\sin\alpha+\sin\beta].$$

In the vector form, $\vec{B} = \frac{\mu I}{4\pi R}[\sin\alpha+\sin\beta](-\hat{k})$ [Fig. 9.99(a)]

If the point under consideration where the magnetic field is to be calculated is not in front of the wire as shown in figure(b) the magnetic field is given by $\vec{B} = \frac{\mu I}{4\pi R}[\sin\beta-\sin\alpha](-\hat{k})$ [Fig. 9.99(b)]

Thus, it is clear that in the case of a current carrying straight wire:

- *For points along the length of the wire (but not on it), the field is always zero.*
- *The field is always perpendicular to the plane containing the wire and the point. So, in a plane perpendicular to the wire and containing the point, the lines of force are concentric circles encircling the wire as shown in Fig. 9.97.*



(a) (b)

Fig. 9.100

- *If the wire is of infinite length and the point P is not near its ends as shown in Fig. 9.100(a), $\alpha = \beta = (\pi/2)$; then*

$$B = \frac{\mu_0}{4\pi}\frac{I}{d}[1+1] \text{ i.e., } B = \frac{\mu_0}{4\pi}\frac{2I}{d}$$

- *If the point is near one end of an infinitely long wire as shown in Fig. 9.100(b), $\alpha = (\pi/2)$ and $\beta = 0$.*

So, $$B = \frac{\mu_0 I}{4\pi d}[1+0] \text{ i.e., } B = \frac{\mu_0}{4\pi}\frac{I}{d}$$

Right Hand Thumb Rule

Using this rule, the direction of magnetic field because of a current carrying wire may be obtained in the following ways:

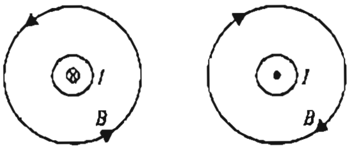


Fig. 9.101

According to this rule, if we grasp the conductor in the palm o our right hand so that the thumb points in the direction of the flo of current, then the direction in which the fingers curl gives th direction of magnetic lines of force.

- **Direction of magnetic field for straight current carryin wire**
  When the current I is coming outwards ⊙ (out of the page) the magnetic field B is a circle in the anticlockwise sense.
  When the current I is pointing inwards ⊗ (into the page), th magnetic field B is a circle in the clockwise sense.

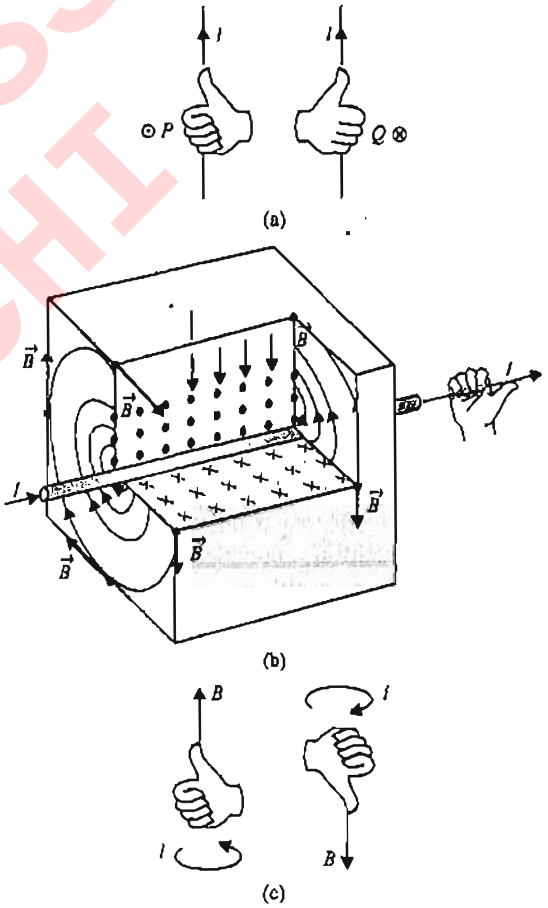


Fig. 9.102

- **Direction of magnetic field in close current carrying loop**
  When the current I is flowing in the anticlockwise sense, the magnetic field B is outwards ⊙ (out of the page)
  When the current I is flowing in the clockwise sense, the magnetic field B is inwards ⊗ (into the page).

Illustration 9.25 **A current I flows in a circuit shaped like an isosceles trapezium. The ratio of the bases is 2. The length of**

the smaller base is l. Calculate the magnetic induction at a point P located in the plane of the trapezium, but at a distance a from the midpoint of the smaller base.

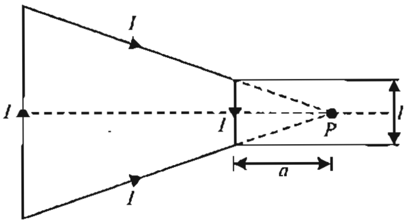


Fig. 9.103

Sol. Magnetic field at P due to wire 1 is

$$B_1 = \frac{\mu_0 I}{4\pi d}[\sin\alpha - \sin(-\alpha)] = \frac{\mu_0 I \sin\alpha}{2\pi d}$$

where α is the half-angle subtended by the wire 1 at point P and d is the perpendicular distance between the wire and point P. From similar triangles APF and CPE, we find that

$$d = 2a \quad \text{and} \quad \tan\alpha = \frac{1}{2a}$$

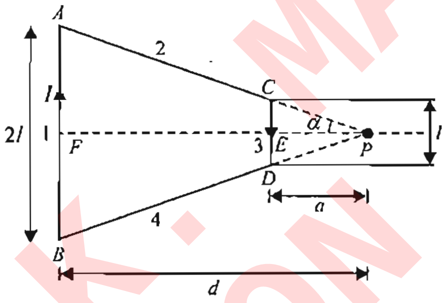


Fig. 9.104

The direction of the magnetic field is into the plane of the paper.

Magnetic field at P due to wires 2 and 4 is zero.

Magnetic field at P due to wire 3 is

$$B_3 = \frac{\mu_0 I}{4\pi a}[\sin\alpha - \sin(-\alpha)] = \frac{\mu_0 I \sin\alpha}{2\pi a}$$

And the field is directed out of the plane of the paper.

The resultant magnetic field at P is the vector sum of the four fields and is given by

$$B = \frac{\mu_0 I \sin\alpha}{2\pi a} - \frac{\mu_0 I \sin\alpha}{4\pi a} = \frac{\mu_0 I \sin\alpha}{4\pi a}$$

or
$$B = \frac{\mu_0 I}{4\pi a}\left[\frac{l}{\sqrt{l^2 + 4a^2}}\right]$$

Out of the plane of the paper.

Illustration 9.26 **Find the magnitude and direction of magnetic field at point P due to the current carrying wire as shown in Fig. 9.105.**

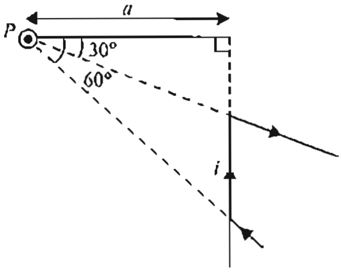


Fig. 9.105

Sol.
$$B = \frac{\mu_0 I}{4\pi R}[\sin\theta_1 + \sin\theta_2]$$

Here $q_1 = -30°$, $q_2 = 60°$. Putting these values, we get

$$B = \frac{\mu_0 I}{4\pi R}\left[-1/2 + \sqrt{3}/2\right]$$

Illustration 9.27 **A pair of stationary and infinitely long bent wires is placed in the x–y plane as shown in Fig. 9.106. Each wire carries current of 10 amp. The segments L and M are along the x-axis. the segments P and Q are parallel to the y-axis such that $OS = OR = 0.02$ m. Find the magnitude and direction of the magnetic induction at the origin O.**

(IIT-JEE, 1998)

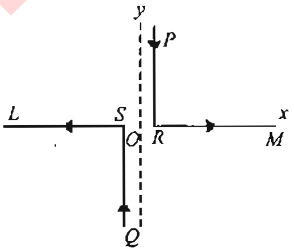


Fig. 9.106

Sol. As point O is along the length of segments L and M, so the field at O due to these segments will be zero. Also, point O is near one end of a long wire.

The resultant field at O, $B_R = B_P + B_Q$

$$\Rightarrow \quad B_R = \frac{\mu_0}{4\pi}\frac{I}{RO} + \frac{\mu_0}{4\pi}\frac{I}{SO}$$

But $RO = SO = 0.02$ m

Hence, $B_R = 2\times\frac{\mu_0}{4\pi}\times\frac{10}{0.02} = 2\times10^{-7}\frac{10}{0.02} = 10^{-4}$ Wbm^{-2}

Illustration 9.28 **A long straight wire, carrying current I, is bent at its midpoint to form an angle of 45°. Find the induction of magnetic field at point P, distant R from the point of bending (as shown in Fig. 9.107)**

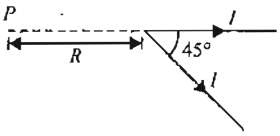


Fig. 9.107

Sol. Since point P lies on axis of straight part ab, therefore, magnetic induction due to this part is equal to zero.

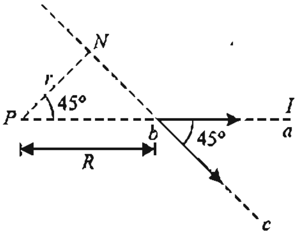


Fig. 9.108

For part bc,

From Fig. 9.108, $r = R\cos 45°$.

Since both the ends b and c are on the same side of normal PN, therefore a is negative and b is positive.

Hence $\quad a = -45°$ and $b = +90°$.

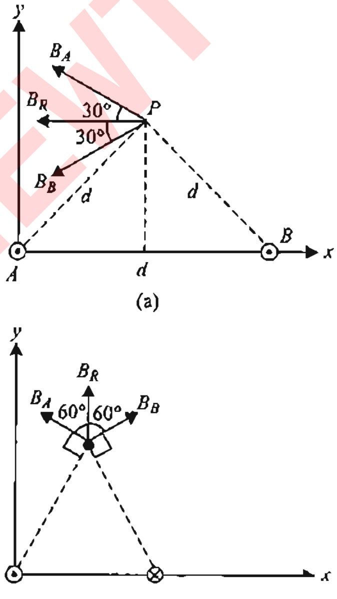
Using $\quad B = \dfrac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$, we have $B = \dfrac{(\sqrt{2}-1)\,\mu_0 I}{4\pi R}$

Illustration 9.29 **Two straight infinitely long and thin parallel wires are spaced 0.1 m apart and carry a current of 10 A each. Find the magnetic field at a point distance 0.1 m from both wires in the two cases when the currents are in the (a) same and (b) opposite directions.**

Sol. The point P is situated equidistant from the wires A and B. Hence, for the given case the magnitude of the magnetic field at P due to both the wires will be same.

$$B_A = B_B = B = \frac{\mu_0 I}{2\pi d} = 2\times 10^{-7}\times\frac{10}{0.1} = 2\times 10^{-5}\,\text{T}$$

a. If the wires carry current in the same direction, B_A and B_B will have the directions as shown in Fig. 9.109(a).



(a)

(b)

Fig. 9.109

The net magnetic field $B_R = 2B\cos 30°\,(-\hat{i}) = 2\sqrt{3}\times 10^{-5}\,\text{T}$ along negative x-axis.

b. If the wires carry current in opposite directions, the magnetic field at P due to wires A and B will be as shown in Fig. 9.109(b).

The net magnetic field $B_R = 2B\cos 60°\,(\hat{j})$.

Illustration 9.30 **In Fig. 9.110, two long wires W_1 and W_2, each carrying current I, are placed parallel to each other and parallel to z-axis. The direction of current in W_1 is outward and in W_2 it is inward. Find $\vec{B}$ at 'P' and 'Q'.**

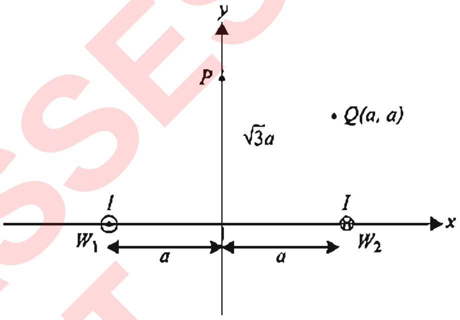


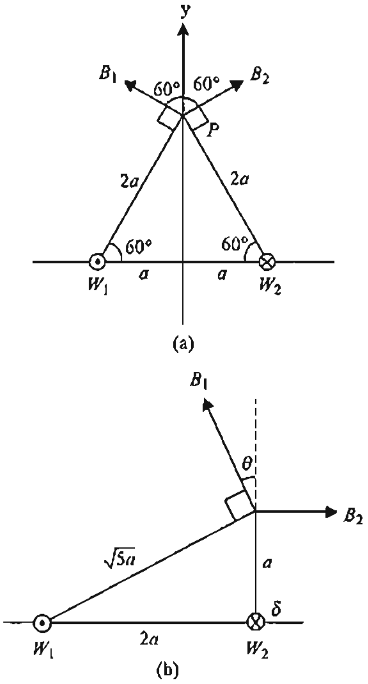
Fig. 9.110

Sol. Magnetic field at P:

Let $\vec{B}$ due to W_1 be $\vec{B}_1$ and due to W_2 be $\vec{B}_2$. By symmetry, $|\vec{B}_1| = |\vec{B}_2| = B$ (Fig 9.111(a))

$$\Rightarrow \quad B_p = 2B\cos 60° = B = \frac{\mu_0 I}{2\pi 2a} = \frac{\mu_0 I}{4\pi a}$$

$$\therefore \quad \vec{B}_P = \frac{\mu_0 I}{4\pi a}\hat{j}$$



(a)

(b)

Fig. 9.111

For magnetic field at Q (Fig. 9.111 (b)):

Magnetic field due to W_1, $B_1 = \dfrac{\mu_0 I}{2\pi\sqrt{5}a}$

Magnetic field due to W_2, $B_2 = \dfrac{\mu_0 I}{2\pi a}$

$$\tan\theta = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$$

$$\vec{B}_Q = (B_1 \cos\theta\,\hat{j}) + (B_2 - B_1 \sin\theta)\,\hat{i}$$

$$\sin\theta = \sqrt{\frac{3}{5}}$$

$$\therefore \quad \vec{B}_Q = \frac{\mu_0 I}{5\pi a}\hat{j} + \left(\frac{\mu_0 I}{2\pi\sqrt{3}a} - \frac{\sqrt{3}\mu_0 I}{10\pi a}\right)\hat{j}$$

Illustration 9.31 **A square loop of wire, edge length a, carries a current i. Compute the magnitude of the magnetic field produced at a point on the axis of the loop at a distance x from the center.**

Sol. A point on the axis of the loop is on the perpendicular bisector of each of the loop sides. Fig. 9.112 shows field of a single wire.

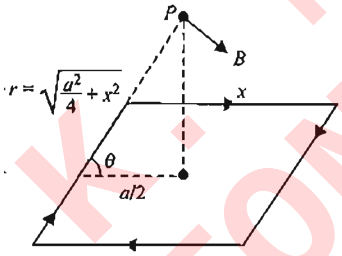


Fig. 9.112

$$B = \frac{\mu_0 ia}{\pi\sqrt{4x^2 + a^2}\sqrt{4x^2 + 2a^2}}$$

When the fields of all the four sides are considered, the horizontal components add to zero. So, the total field is given by

$$B_R = 4B\cos\theta = \frac{4Ba}{2r} = \frac{4Ba}{\sqrt{4x^2 + a^2}}$$

$$= \frac{4\pi_0 ia^2}{\pi(4x^2 + a^2)\sqrt{4x^2 + 2a^2}}$$

For $x = 0$, the expression reduces to $B_R = \dfrac{4\mu_0 ia^2}{\pi a^2\sqrt{2a}} = \dfrac{2\sqrt{2}\mu_0 i}{\pi a}$

Magnetic Field at the Center of a Current Carrying Arc

Fig. 9.113 shows a circular loop of radius r carrying a current I. Application of Biot and Savart law to a current element of length dl at angular position α with angular element $d\alpha$ is as followed.

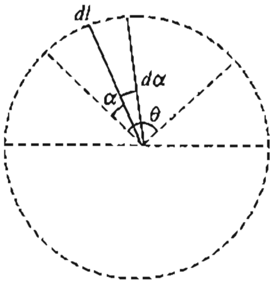
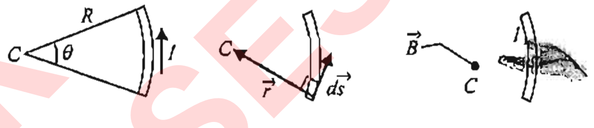


Fig. 9.113



(a) (b) (c)

(a) A wire in the shape of a circular arc with center C carries current I. (b) For any element of wire along the arc, the angle between the directions of $\vec{ds}$ and $\vec{r}$ is 90°.
(c) Determining the direction of the magnetic field at the center C due to the wire; the field is out of the page, in the direction of the fingertips, as indicated by the dot at C.

Fig. 9.114

The magnetic field due to the element dl, $d\vec{B} = \dfrac{\mu_0 I\left(d\vec{l} \times \vec{r}\right)}{4\pi r^3}$

Here, $\Rightarrow \quad dB = \dfrac{\mu_0 I}{4\pi}\dfrac{|dl|\,r\sin 90°}{r^3}$ (as $d\vec{l} \perp \vec{R}$)

$\Rightarrow \quad dB = \dfrac{\mu_0 I}{4\pi r}(d\alpha)$ (as $dl = r\alpha$)

$$B = \int dB = \frac{\mu_0 I}{4\pi r}\int_0^\theta d\alpha \quad \Rightarrow \quad B_{\text{arc}} = \frac{\mu_0 I}{4\pi r}\theta$$

where θ is the angle in the radian.
Therefore, B at the center of a circular loop of radius R is

$$B = \frac{\mu_0 I}{4\pi r}(2\pi) = \frac{\mu_0 I}{2R}$$

Magnetic Field of a Circular Current Loop

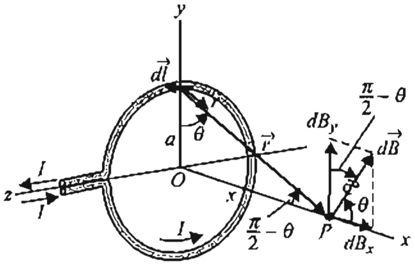


Fig. 9.115

In Fig. 9.115 the current in the segment $d\vec{\ell}$ causes the field $d\vec{B}$, which lies in the xy plane. The currents in other $d\vec{\ell}$'s cause $d\vec{B}$'s with direction components perpen-dicular to the x-axis; these components add to zero. The x-components of the $d\vec{B}$'s combine to give the total field $\vec{B}$ at point P.

We can use the law of Biot and Savart to find the magnetic field at a point P on the axis of the loop, at a distance x from the center. As Fig. 9.115 shows, $d\vec{\ell}$ and $\hat{r}$ are perpendicular and the direction of field $d\vec{B}$ caused by this particular element $d\vec{\ell}$ lies in the xy plane. Since $r^2 = x^2 + a^2$, the magnitude dB of the field due to element $d\vec{\ell}$ is

$$dB = \frac{\mu_0 I}{4\pi}\frac{d\ell}{(x^2+a^2)} \qquad \text{(i)}$$

The components of the vector $d\vec{B}$ are

$$dB_x = dB\cos\theta = \frac{\mu_0 I}{4\pi}\frac{d\ell}{(x^2+a^2)}\frac{a}{(x^2+a^2)^{1/2}} \qquad \text{(ii)}$$

$$dB_y = dB\sin\theta = \frac{\mu_0 I}{4\pi}\frac{d\ell}{(x^2+a^2)}\frac{x}{(x^2+a^2)^{1/2}} \qquad \text{(iii)}$$

The situation has rotational symmetry about the x-axis, so there cannot be a component of the total field $\vec{B}$ perpendicular to this axis. For every element $d\vec{\ell}$, there is a corresponding element on the opposite side of the loop, with opposite direction. These two elements give equal contributions to the x-component of $d\vec{B}$, given by equation (ii), but opposite components perpendicular to the x-axis. Thus, all the perpendicular components cancel out and only the x-components survive.

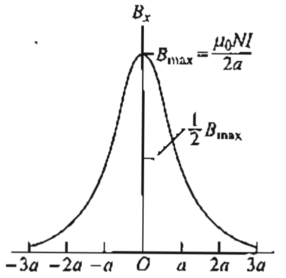
To obtain the x-component of the total field $\vec{B}$, we integrate equation (ii), including all the $d\vec{\ell}$'s around the loop. Everything in this expression except $d\ell$ is constant and can be taken outside the integral. So, we have

$$B_x = \int \frac{\mu_0 I}{4\pi}\frac{a\,d\ell}{(x^2+a^2)^{3/2}} = \frac{\mu_0 I a}{4\pi(x^2+a^2)^{3/2}}\int d\ell$$

The integral of $d\ell$ is just the circumference of the circle, i.e., $\int d\ell = 2\pi a$. So, we finally get

$$B_x = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}} \quad \text{(on the axis of a circular loop)} \qquad \text{(iv)}$$

Now suppose that instead of the single loop in Fig. 9.115, we have a coil consisting of N loops, all with the same radius. The loops are closely spaced so that the plane of each loop is essentially the same distance x from the field point P. Each loop contributes equally to the field, and the total field is N times the field of a single loop:



Graph of the magnetic field along the axis of a circular coil with N turns. When x is much larger than a, the field magnitude decreases approximately as $1/x^3$.

Fig. 9.116

$$B_x = \frac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}} \quad \text{(on the axis of } N \text{ circular loops)} \qquad \text{(v)}$$

If $x >> a$, then $B = \frac{\mu_0 I a^2}{2x^3} = \frac{\mu_0}{2\pi}\frac{I\pi a^2}{x^3} = \frac{\mu_0}{4\pi}\frac{I.2\pi a^2}{x^3}$

But $\pi a^2 = A$ = Area of cross section of the coil.

Thus, $B = \frac{\mu_0}{4\pi}\frac{2IA}{x^3} = \frac{\mu_0 2M}{4\pi x^3}$ or, $B = \frac{\mu_0}{4\pi}\frac{2M}{x^3}$

where $\vec{M} = I\vec{A}$ = magnetic dipole moment of the loop. The direction of $\vec{M}$ is same as the direction of the normal to the area of the loop.

Note:

A loop as a magnet: *The pattern of the magnetic field is comparable with the magnetic field produced by a bar magnet.*

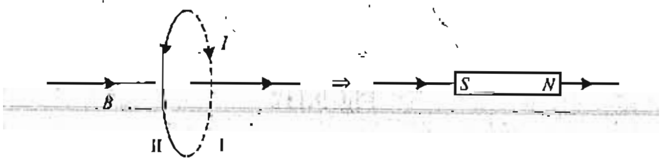


Fig. 9.1171

The side 'I' (the side from which $\vec{B}$ energes out) of the loop acts as 'NORTH POLE' and side II (the side in which $\vec{B}$ enters) acts as the 'SOUTH POLE'. It can be verified by studying force on one loop due to a magnet or a loop.

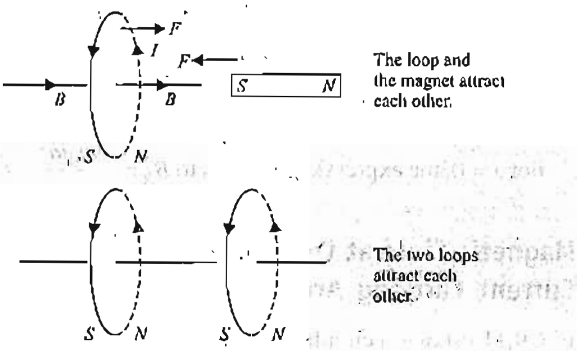


Fig. 9.118

Magnetic Field on the Axis of a Solenoid Having N Turns Per Unit Length and Carrying a Current I

The field at a point on the axis of a solenoid can be obtained by superposition of fields due to a large number of identical coils all having their center on the axis of solenoid.

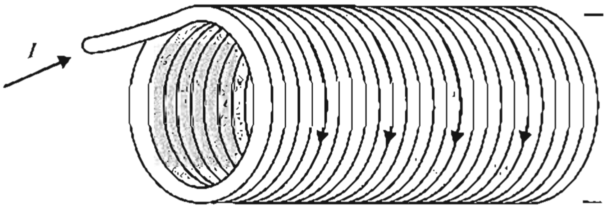


Fig. 9.119

Let us consider a coil of radius R and width dx at a distance x from the point P on the axis of solenoid as shown in Fig. 9.120.

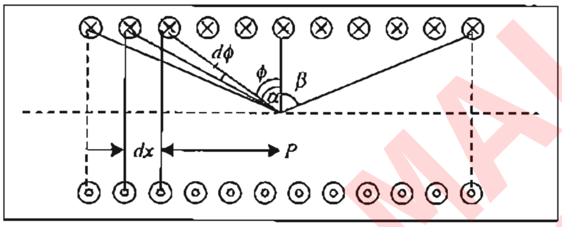


Fig. 9.120

The magnetic field due this coil

$$dB = \frac{\mu_0 NIR^2}{2(R^2+x^2)^{3/2}}$$

Here, $N = ndx$, $x = R\tan\phi$ and $dx = R\sec^2\phi\, d\phi$.

Hence, $$dB = \frac{\mu_0 ndx \times IR^2}{2(R^2+R^2\tan^2\phi)^{3/2}}$$

$$\Rightarrow \quad B = \int dB = \frac{\mu_0}{2} nI \int_{\alpha}^{\beta} \cos\phi \, d\phi$$

i.e., $$B = \frac{\mu_0 nI}{2}[\sin\alpha + \sin\beta]$$

For a point inside a long solenoid, $\alpha = \beta \approx 90°$; therefore, $B = \mu_0 nI$. At one end of a long solenoid, $\alpha = 0°, b \approx 90°$; therefore, $B = \mu_0 nI/2$.

Magnetic Field $\vec{B}$ at Point P, at a Distance R From the Center of a Flat Strip of Width 'a' Along its Perpendicular Bisector

Let us subdivide the strip into long, infinitesimal filaments of width dx, each of which may be treated as a wire carrying a current element di given by $i(dx/a)$. For the current element in the left half of the strip in Fig. 9.121, the magnitude dB of the field at P is given by $dB = \frac{\mu_0}{2\pi}\frac{di}{d} = \frac{\mu_0}{2\pi}\frac{i(dx/a)}{R\sec\theta}$, in which $d = R/\cos\theta = R\sec\theta$.

Note that the vector $d\vec{B}$ is at right angles to the line marked d.

Only the horizontal component of $d\vec{B}$ — namely, $dB\cos\theta$— is effective; the vertical component is cancelled by the contribution of a symmetrically located current element on the other side of the strip (the second shaded element in figure). Thus, B at point P is given by the (scalar) integral

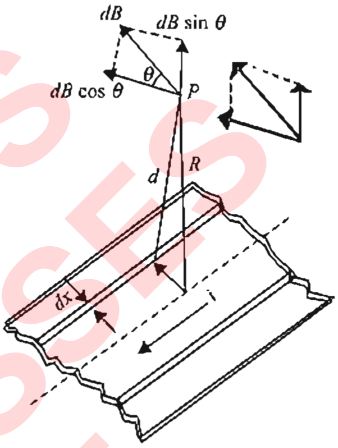


Fig. 9.121

$$B = \int dB\cos\theta = \int \frac{\mu_0 i(dx/a)}{2\pi R\sec\theta}\cos\theta$$

$$= \frac{\mu_0 i}{2\pi aR}\int \frac{dx}{\sec^2\theta}$$

A flat strip of width a carries a current i. The variable x and θ are not independent, being related by

$$x = R\tan\theta$$

or $$dx = R\sec^2\theta\, d\theta.$$

The limits on θ are $\pm\alpha$, where $\alpha = \tan^{-1}(a/2R)$. Substituting for dx in the expression for B, we find

$$B = \frac{\mu_0 i}{2\pi aR}\int \frac{R\sec^2\theta d\theta}{\sec^2\theta} = \frac{\mu_0 i}{2\pi a}\int_{-\alpha}^{+\alpha} d\theta$$

$$= \frac{\mu_0 i}{\pi a}\alpha = \frac{\mu_0 i}{\pi a}\tan^{-1}\frac{a}{2R}$$

$$B = \frac{\mu_0 i}{\pi a}\alpha = \frac{\mu_0 i}{\pi a}\tan^{-1}\frac{a}{2R}$$

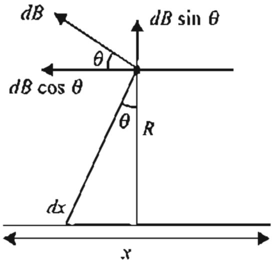


Fig. 9.122

This is the general result for the magnetic field due to the strip. At points far from the strip, α is a small angle, for which $\alpha \approx \tan\alpha = a/2R$. Thus, we have, as an approximate result,

$$B \approx \frac{\mu_0 i}{\pi a}\left(\frac{a}{2R}\right) = \frac{\mu_0}{2\pi}\,\frac{i}{R}$$

This result is expected because at distant points the strip cannot be distinguished from a thin wire.

Illustration 9.32 **Shown in the Fig. 9.123 is a conductor carrying a current *I*. Find the magnetic field intensity at the point *O*.**

Sol. The magnetic field at the center of an arc is equal to

$$B = \frac{\mu_0 I}{4\pi r}\theta$$

Magnetic field due to arc 1, $B_1 = \dfrac{\mu_0 I\theta}{4\pi \times 3r}(-\hat{k})$

Magnetic field due to arc 2, $B_2 = \dfrac{\mu_0 I\theta}{4\pi \times 2r}(\hat{k})$

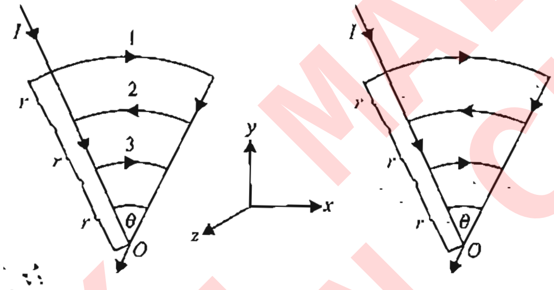


Fig. 9.123

Magnetic field due to arc 3, $B_3 = \dfrac{\mu_0 I\theta}{4\pi r}(-\hat{k})$

Net magnetic field, $B = B_1 + B_2 + B_3$

Hence, net $B = \dfrac{\mu_0 I}{4\pi}\left[-\dfrac{1}{r} + \dfrac{1}{2r} - \dfrac{1}{3r}\right]\theta(\hat{k}) = -\dfrac{5\mu_0 I\theta}{24\pi r}\hat{k}$

Illustration 9.33 **A battery is connected between two points *A* and *B* on the circumference of a uniform conducting ring of radius *r* and resistance *R* as shown in Fig. 9.124. One of the arcs *AB* of the ring subtends angle *θ* at the center. Show that the magnetic field at the center of the coil is zero and independent of *q*.**

Sol. Magnetic field at the center of an arc is given by

$$B = \frac{\mu_0 I}{2R} \times \frac{\theta}{2\pi}$$

Magnetic field due to a smaller arc, $\vec{B}_1 = \dfrac{\mu_0 I_1}{2r} \times \dfrac{\theta}{2\pi}(-\hat{k})$

Magnetic field due to larger arc, $\vec{B}_2 = \dfrac{\mu_0 I_2}{2r} \times \dfrac{(2\pi-\theta)}{2\pi}(+\hat{k})$

Resultant magnetic field $= \left[-\dfrac{\mu_0 I_1\theta}{4\pi r} + \dfrac{\mu_0 I_2(2\pi-\theta)}{4\pi r}\right](\hat{k})$ (i)

Two arcs form a parallel combination of resistors.

Here $I_1 R_1$ and $I_2 R_2 \Rightarrow \dfrac{I_1}{I_2} = \dfrac{R_1}{R_2}$ (ii)

Here R_1 amd R_2 are the resistances of smaller and bigger arc, respectively.

But $\dfrac{R_2}{R_1} = \dfrac{\ell_2}{\ell_1} = \dfrac{(2\pi-\theta)r}{\theta r} = \dfrac{2\pi-\theta}{\theta r}$ (iii)

From (ii) and (iii), we get $I_1\theta = I_2(2\pi-\theta)$ (iv)

Hence from (i) and (iv),

Net magnetic field at the centre of ring will be zero.

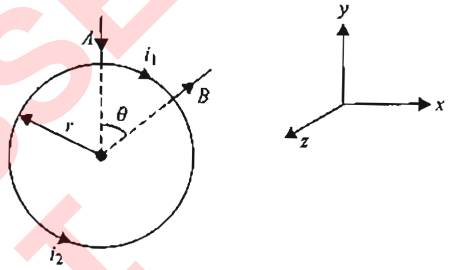


Fig. 9.124

Illustration 9.34 **A solenoid 60 cm long and of radius 4.0 cm has 3 layers of windings of 300 turns each. A 2.0 cm long wire of mass 2.5 g lies inside the solenoid near its center normal to its axis, both the wire and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of the solenoid to an external battery which supplies a current of 6.0 A in the wire. What value of current (with appropriate sense of circulation) in the windings of the solenoid can support the weight of the wire?**

Sol. Let *I* be the current in the windings of the solenoid that can support the weight of the wire. Then, magnetic field inside the solenoid along its axis is given by $B = \mu_0 nI$.

Given that: $\mu_0 = 4\pi \times 10^{-7}\ \text{TmA}^{-1}$

n = number of turns per unit length $= \dfrac{3\times300}{60\ \text{cm}} = \dfrac{3\times300}{60\times10^{-2}\,\text{m}}$

= 1500 turns/m

Hence, $B = 4\pi \times 10^{-7} \times 1500 \times I = 6\pi \times 10^{-4} I$

As the wire is placed normal to the axis of the solenoid and magnetic field inside the solenoid is along the axis, hence field will act normal to the wire. Hence, force experienced by the wire due to the magnetic field is given by: $F = BI'\ell$

Here, $I' = 6.0$ A, $\ell = 20$ cm $= 2.0 \times 10^{-2}$ m and $B = 6\pi \times 10^{-4} I$

Hence, $F = 6\pi \times 10^{-4} \times 6.0 \times 2.0 \times 10^{-2}\ \text{N} = 72\pi \times 10^{-6} I$

Current *I* will support the weight of the wire, if the force *F* equals the weight of the wire, i.e., $F = mg$

or $72\pi \times 10^{-6} I = 2.5 \times 10^{-3} \times 9.8$

$$\Rightarrow \quad I = \frac{2.5\times10^{-3}\times9.8}{72\times3.14\times10^{-6}} = 108.37\ \text{amp}$$

Illustration 9.35 **A thin insulated wire forms a plane spiral of $N = 100$ turns carrying a current $i = 8$ mA. The inner and outer radii are equal to $a = 5$ cm and $b = 10$ cm. Find the magnetic induction at the centre of the spiral.**

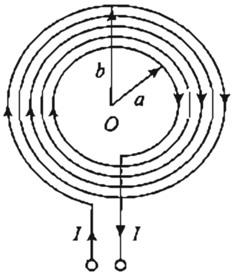


Fig. 9.125

Sol. Let n be the number of turns per unit length along the radii of the spiral. Consider a ring of radii x and $x + dx$.

Number of turns in the ring $= n\,dx$

$$\therefore \text{ Magnetic field at the centre} = \frac{\mu_0\,(ndx)\,i}{2x}$$

$$\therefore\ B\,(\text{total field}) = \int_a^b \frac{\mu_0\,nidx}{2x} = \frac{1}{2}\,\mu_0\,ni\ln\frac{b}{a}$$

$$\text{But } n = \frac{N}{b-a}$$

$$B = \frac{\mu_0\,Ni}{2(b-a)}\ln\frac{b}{a}$$

$$= \frac{4\pi\times10^{-7}\times100\times8\times10^{-3}}{2(10-5)\times10^{-2}}\ln\frac{10}{5}$$

$$B = 6.9\times10^{-6}\,T = 6.9\,\mu T$$

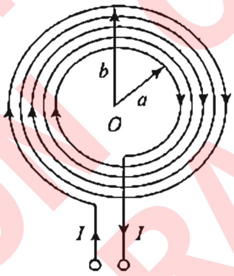


Fig. 9.126

Illustration 9.36 **A current of 10 A flows around a closed path in a circuit which is in the horizontal plane as shown in Fig. 9.127. The circuit consists of eight alternating areas of radii $r_1 = 0.08$ m and $r_2 = 0.12$m. Each arc subtends the same angle at the center.**

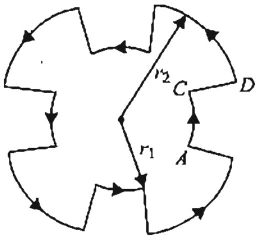


Fig. 9.127

a. Find the magnetic field produced by this circuit at the center.

b. An infinitely long straight wire carrying a current of 10 A is passing through the center of the above circuit vertically with the direction of the current being into the plane of the circuit. What is the force acting on the wire at the center due to the current in the circuit? What is the force acting on the arc AC and the straight segment CD due to the current at the center? **(IIT-JEE, 2001)**

Sol.

a. Magnetic field at the center due to the straight parts is zero. Magnetic field due to the arcs of radius

$$r_1 = 4(\mu_0 i/2r_1)(1/8) = (\mu_0 i/4r_1)$$

Similarly the magnetic field due to the arcs of radius

$$r_2 = (\mu_0 i/4r_2)$$

$\Rightarrow$ Net magnetic field

$$= (\mu_0 i/4)\ \{1/r_1 + 1/r_2\} = 6.25\times10^{-5}\,\text{T}$$

b. **i.** As the current in the wire at the center is antiparallel to the direction of magnetic field, the force on the wire everywhere will be zero.

ii. Further due to the current at the center, magnetic field at AC will be tangential and hence the force on AC will be zero.

iii. Force on CD $$= \int_{r_1}^{r_2}\frac{\mu_o i}{2\pi x}\,i\,dx = \frac{\mu_o i^2}{2\pi}\ln\left(\frac{r_2}{r_1}\right) = 8.11\times10^{-6}\,\text{N}$$

(Vertically downwards)

AMPERE'S LAW

Similar to the Gauss's law of electrostatics, this law provides us shortcut methods of finding magnetic field in cases of symmetry. According to this law, the line integral of magnetic field over the closed path ($\oint \vec{B}.d\vec{l}$) is equal to μ_0 times the net current crossing the area enclosed by that path.

$$\oint_{\text{closed loop}} \vec{B}\cdot d\vec{\ell} = \mu_0 I_{\text{enclosed}}$$

This law is called the Ampere's law and the closed loop on which it is applied is called the Amperian Loop. The integral on the left hand side is called the magnetic circulation. Thus, Ampere's law can be stated as: the magnetic circulation (C) around a closed loop is μ_0 times the net electric current enclosed by the loop.

Note:

While applying the Ampere's law, the following right hand convention is to be used

- *Current into the plane of the paper is negative.*
- *Current out of the plane of the paper is positive.*
- *Circulation taken in the counter clockwise direction is positive.*
- *Circulation taken in clockwise direction is negative.*

Illustration 9.37 **Fig. 9.128 shows three current carrying conductors and three imaginary loops. Calculate the current enclosed by each of the loops.**

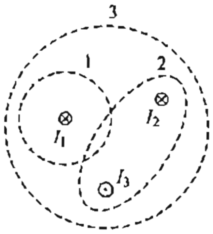


Fig. 9.128

Sol. The current enclosed by the first loop is $-I_1$. The current enclosed by the second loop is $I_3 - I_2$ and the current enclosed by the third loop is $I_3 - I_1 - I_2$.

Illustration 9.38 **Fig. 9.129 shows two current carrying wires piercing the plane of an imaginary loop. One of the wires is normal to the plane of the loop while the other is at an angle. Calculate the net current enclosed.**

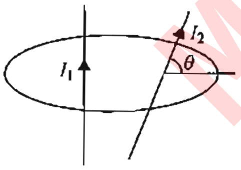


Fig. 9.129

Sol. The current enclosed is simply $I_1 + I_2$. There is no need to take any component of I_2 perpendicular to the plane of the loop. All that matters is just the magnitude of the net charge per unit time that crosses the plane of the loop.

Just as in the case of Gauss's law, the selection of the Amperian loop is critical for easy application of the law. The general rules while selecting the loop are:

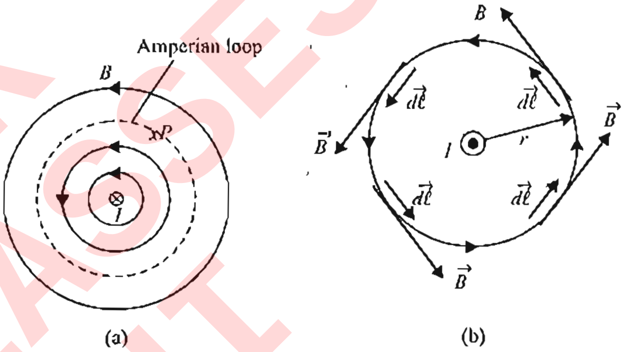
- Estimate the direction force the magnetic field times around the current carrying conductor.
- The loop should include the current carrying element whose magnetic field is to be calculated.
- The magnetic field should be constant over the whole loop or should be easy to calculate the different portions of the loop.
- The dot product should be easy to calculate.

Note:

- *If B is everywhere tangent to the integration path and has the same magnitude B at every point on the path, then its line integral is equal to B multiplied by the circumference of the path.*
- *If B is everywhere perpendicular to the path, for all or some portion of the path, that portion of the path makes no contribution to the line integral.*
- *In the integral $\oint \vec{B}\cdot d\vec{l}$, $\vec{B}$ is always the total magnetic field at each point on the path. In general, this field is caused partly by currents enclosed by the path and partly by currents outside. Even when no current is enclosed by the path, the field at points on the path need not be zero. In that case, however, $\oint \vec{B}\cdot d\vec{l}$ is always zero.*
- *Some judgment is required in choosing an integration path. Two useful guiding principles are that the point or points at which the field is to be determined must lie on the path, and that the path must have enough symmetry so that the integral can be evaluated.*

FIELD OF A LONG, STRAIGHT, CURRENT CARRYING CONDUCTOR

Selection of Ampere Loop



(a) (b)

Fig. 9.130

We know that the magnetic field lines around a current carrying conductor are concentric circles in a plane perpendicular to the conductor. A natural choice for an amperian loop is a circle concentric with the conductor as shown in Fig. 9.130. For this loop,

$\vec{B}$ is constant at all points due to symmetry.

$\vec{B}$ is always in the same direction as $d\vec{\ell}$ and so the dot product is unity.

We exploit the cylindrical symmetry of the situation by taking as our integration path a circle with radius r centered on the conductor and in a plane perpendicular to it. At each point, $\vec{B}$ is tangent to this circle. With our choice of integration path, Ampere's law becomes

$$\oint \vec{B}\cdot d\vec{\ell} = \oint \vec{B}_{\parallel}\, d\ell = B(2\pi r) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

Ampere's law determines the direction of $\vec{B}$ as well as its magnitude. Since we go around the integration path in the counterclockwise direction, the positive direction for current is out of the plane. This is the same as the actual current direction in the figure, so I is positive and the integral $\oint \vec{B}\cdot d\vec{\ell}$ is also positive. Since the $d\vec{\ell}$'s run counterclockwise, the direction of $\vec{B}$ must be counterclockwise.

Magnetic Field *B* Outside and Inside a Cylindrical Wire

A steady current I flows along an infinitely long straight wire with circular cross section of radius R.

It can be concluded from the symmetry that the field lines of B are circles with their centers at the wire axis.

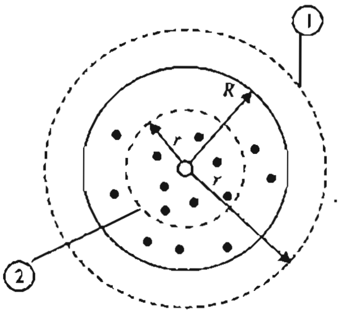


Fig. 9.131

B Outside the Wire ($r \geq R$)

Consider a circular loop of radius r ($\geq R$) as shown by loop 1 in Fig. 9.131.

Here, the integral $\oint \vec{B}\cdot d\vec{\ell} = B\oint d\ell = B(2\pi r)$

Applying Ampere's law,

$$B(2\pi r) = \mu_0 I \quad \Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$

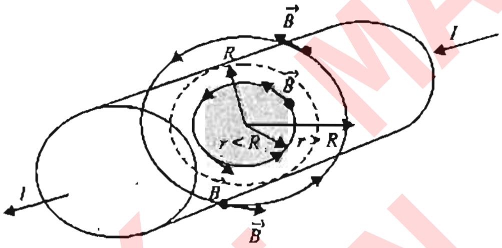


Fig. 9.132

B Inside the Wire ($r \leq R$)

Consider the loop 2 as shown in Fig. 9.131.

Using Ampere's law,

$$B(2\pi r) = \mu_0\left[\frac{I}{\pi R^2}\left(\pi r^2\right)\right] \quad \text{or,} \quad B = \frac{\mu_0 I r}{2\pi R^2}$$

The variation of B with r is shown in the Fig. 9.133.

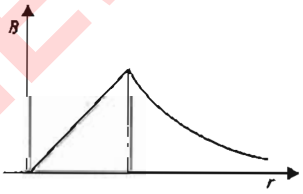
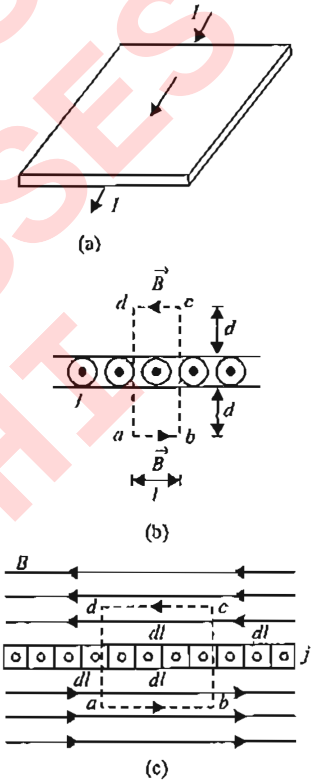


Fig. 9.133

Magnetic Field of an Infinite Sheet of Given Linear Current Density

Selection of Amperian Loop for a Sheet of Current

Once again the choice of the Amperian loop will depend on the predicted nature and shape of the magnetic lines of force for the present configuration. The infinite sheet of current can be treated as a bundle of infinite wires. The fields due to these wires will be concentric circles. The resultant magnetic field due to all these individual fields will be orientated parallel to the plate as shown in Fig. 9.134(a). The field above the plate will be directed to the left while the field below the plate will be directed towards the right. The obvious choice for an Amperian loop is a rectangle with sides parallel to the plate and enclosing a portion of the plate as shown in the Fig. 9.134(b). Furthermore, as the magnetic field intensity may vary with distance from the plate, we place the loop so that the two sides parallel to the plate are at the same distance from the plate.



(a)

(b)

(c)

Fig. 9.134

Application of Ampere's Law

Fig. 9.134(c) shows an infinite sheet of current with linear current density j (Am^{-1}). Due to symmetry, the field lines pattern above and below the sheet is uniform.

Consider a square loop of side l as shown in the figure.

According to Ampere's law,

$$\int_a^b B\cdot d\ell + \int_b^c B\cdot d\ell + \int_c^d B\cdot d\ell + \int_d^a B\cdot d\ell = \mu_0 I$$

Since $B \perp d\ell$ in the path $b \rightarrow c$ and $d \rightarrow a$, therefore

$$\therefore \quad \int_b^c B\cdot d\ell = 0; \int_d^a B\cdot d\ell = 0$$

Also, $B \parallel d\ell$ in the path $a \rightarrow b$ and $c \rightarrow d$, thus

$$\therefore \quad \int_a^b B\cdot d\ell + \int_c^d B\cdot d\ell = 2B\ell$$

The current enclosed by the loop is $I = j\ell$

Therefore, according to Ampere's law, $2B\ell = \mu_0(j\ell)$

or, $B = \dfrac{\mu_0 j}{2}$

FIELD OF A LONG SOLENOID

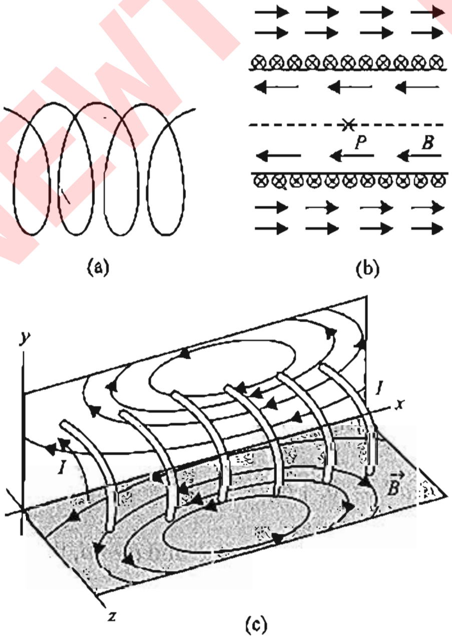
Selection of Amperian Loop for a Long Solenoid

Suppose we desire to calculate the magnetic field inside a long solenoid using Ampere's law, then what would be the choice of the Amperian loop?

To answer this, let us analyze the expected nature of the magnetic field inside and outside the solenoid using symmetry arguments:

1. We know that the magnetic field at a point on the axis of a loop is directed along the axis in the same direction on either side of the loop. The solenoid comprises of many concentric loops. Consequently, the magnetic field at a point on the axis of the solenoid will also be directed along the axis of the solenoid as shown in Fig. 9.135(a).
2. The magnetic field outside the solenoid, far from its axis will be zero since the magnetic field due to a loop far from it is similar to that of a dipole and it falls rapidly as $1/r^3$.

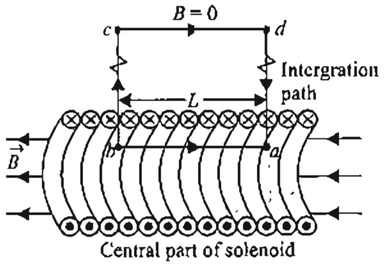
Taking consideration of above facts, what will be the optimum choice of an Amperian loop? Since the magnetic field outside the solenoid is zero, the shape of the loop in this region is immaterial as $\int \vec{B}\cdot d\vec{\ell}$ in this region is zero. Within the solenoid, as the field is parallel to the axis, it is preferable for the loop to also be directed along the axis so that the dot product can be easily calculated. Thus, the natural choice of the loop is a rectangular loop with one side inside the solenoid oriented along the axis and another side outside the solenoid as shown in Fig. 9.135(c).



(a) (b)

(c)

Fig. 9.135 *(Contd.)*



(d)

Fig. 9.135

A solenoid consists of a helical winding of wire on a cylinder, usually circular in cross section. There can be hundreds or thousands of closely spaced turns, each of which can be regarded as a circular loop. There may be several layers of windings.

The field lines near the center of the solenoid are approximately parallel, indicating a nearly uniform $\vec{B}$; outside the solenoid, the field lines are spread apart, and the magnetic field is weak. If the solenoid is long in comparison with its cross-sectional diameter and the coils are tightly wound, the internal field near the midpoint of the solenoid's length is very nearly uniform over the cross section and parallel to the axis, and the external field near the midpoint is very small.

Application of Ampere's Law

Along sides bc and da, $B_{\parallel} = 0$ because $\vec{B}$ is perpendicular to these sides; along side cd, $B_{\parallel} = 0$ because $\vec{B} = 0$.

The integral $\oint \vec{B}\cdot d\vec{\ell}$ around the entire closed path therefore reduces to BL.

The number of turns in length L is nL. Each of these turns passes once through the rectangle $abcd$ and carries a current I, where I is the current in the windings. The total current enclosed by the rectangle is then $I_{\text{encl.}} = nLI$. Ampere's law then gives the magnitude

$$BL = \mu_0 nLI$$

$$B = \mu_0 nI \quad \text{(infinite solenoid)} \qquad \text{(iii)}$$

Side ab need not lie on the axis of the solenoid, so this calculation also proves that the field is uniform over the entire cross section at the center of the solenoid's length.

Note that the direction of $\vec{B}$ inside the solenoid is in the same direction as the solenoid's vector magnetic moment $\vec{\mu}$.

For points along the axis, the field is strongest at the center of the solenoid and drops off near the ends. For a solenoid that is very long in comparison to its diameter, the field at each end is exactly half as strong as the field at the center.

Illustration 9.39 **Consider a coaxial cable which consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c, respectively. The inner wire carries an electric current i_0 and the outer shell carries an equal current in opposite direction. Find the magnetic field at a distance x from the axis where (a) $x < a$, (b) $a < x < b$, (c) $b < x < c$ and (d) $x > c$. Assume that the current density is uniform in the inner wire and also uniform in the outer shell.**

Sol.

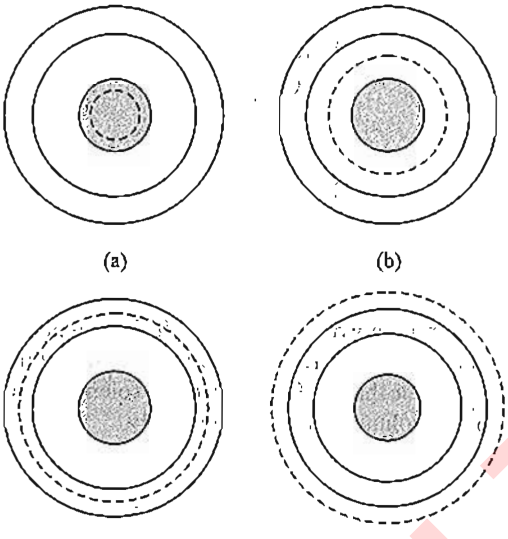

(a) (b) (c) (d)

Fig. 9.136

A cross section of the cable is shown in Fig. 9.136. Draw a circle of radius x with the center at the axis of the cable. The parts (a), (b), (c) and (d) of the figure correspond to the four parts of the problem. By symmetry, the magnetic field at each point of a circle will have the same magnitude and will be tangential to it. The circulation of B along this circle is, therefore, $\oint \vec{B}\cdot d\vec{\ell} = B2\pi x$ in each of the four parts of the figure.

a. The current enclosed within the circle in part (b) is i_0 so that

$$\frac{i_0}{\pi a^2}\pi x^2 = \frac{i_0}{a^2}x^2.$$

Ampere's law $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 i$ gives

$$B2\pi x = \frac{\mu_0 i_0 x^2}{a^2} \text{ or, } B = \frac{\mu_0 i_0 x}{2\pi a^2}$$

The direction will be along the tangent to the circle.

b. The current enclosed within the circle in part (b) is i_0 so that

$$B\,2\pi x = \mu_0 i_0 \text{ or, } B = \frac{\mu_0 i_0 x}{2\pi a^2}$$

c. Current density of outer shell: $J = \dfrac{i_0}{\pi c^2 - \pi b^2}$

So current from $x = 0$ to $x = x$:

$$I = i_0 - J(\pi r^2 - \pi b^2)$$

$$= i_0 - i_0\left(\frac{x^2-b^2}{c^2-b^2}\right) = \frac{i_0(c^2-x^2)}{c^2-b^2}$$

$$B = \frac{\mu_0 I}{2\pi x} = \frac{\mu_0 i_0 (c^2-x^2)}{2\pi x(c^2-b^2)}$$

d. For $x > c$, magnetic field will be zero, because net current is zero.

Concept Application Exercise 9.4

1. i. Find the magnetic field B at point O (center) of

a. $ABCD$, which is a square of side 'l'.
b. ABC, which is an equilateral triangle of side 'l'.

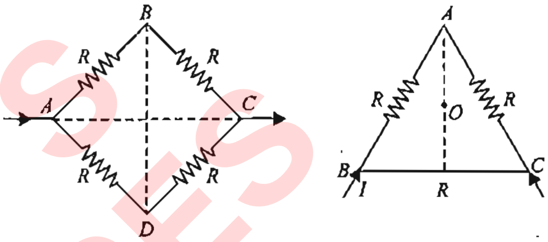


Fig. 9.137

ii. Twelve uniform wires of equal length l and each of resistance r are connected to form a skeleton cube. A battery of emf E is connected between two diagonally opposite corners of the cube. The magnetic induction at the center of the cube is ________.

iii. An infinitely long straight wire is placed at the origin along the z-axis. The current I flows along the positive z-axis.

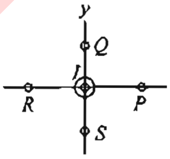
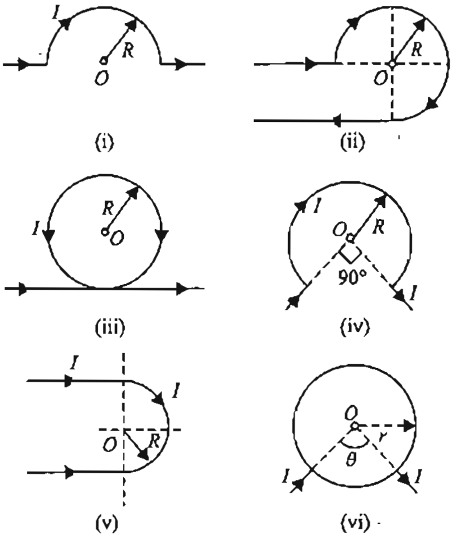


Fig. 9.138

Find the unit vectors showing the directions of magnetic field at the four points:
$P(a, 0)$, $Q(0, b)$, $R(-a, 0)$ and $S(0, -b)$

2. Calculate the magnetic field at point O in each of the following cases



(i) (ii) (iii) (iv) (v) (vi)

Fig. 9.139

3. Find resultant magnetic field at 'C' in Fig. 9.140.

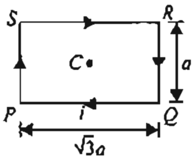


Fig. 9.140

4. Fig. 9.141 shows a square loop made from a uniform wire. Find the magnetic field at the center of the square if a battery is connected between the points A and C.

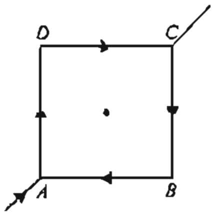


Fig. 9.141

5. In Fig. 9.142, there are two parallel long wires (placed in the plane of paper) carrying currents $2I$ and I. Consider points A, C and D on the line perpendicular to both the wires and also in the plane of the paper. The distances are mentioned. Find (i) $\vec{B}$ at A, C and D (ii) position of points on line ACD where $\vec{B}$ is 0.

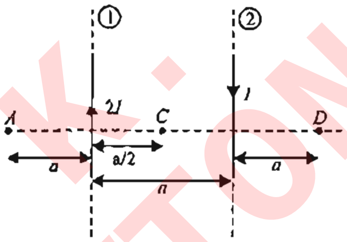


Fig. 9.142

6. In Fig. 9.143, two long wires W_1 and W_2, each carrying current I, are placed parallel to each other and parallel to z-axis. The direction of current in W_1 is outward and in W_2 it is inwards. Find $\vec{B}$ at 'P' and 'Q'.

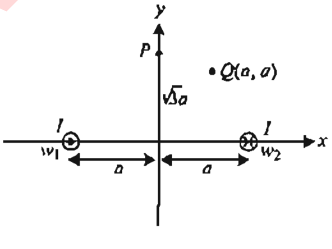


Fig. 9.143

7. In Fig. 9.144, a large metal sheet of width 'w' carries a current I (uniformly distributed in its width 'w'). Find the magnetic field at point 'P' which lies in the plane of the sheet.

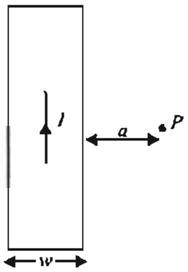


Fig. 9.144

8. Three identical long solenoids P, Q and R are connected to each other as shown in Fig. 9.145. If the magnetic field at the center of P is 2.0 T, what would be the field at the center of Q? Assume that the field due to any solenoid is confined within the volume of that solenoid only.

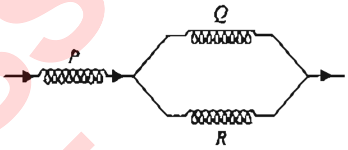


Fig. 9.145

9. Find magnetic field at point P shown in Fig. 9.146, the point P is on the bisector of angle between the wires.

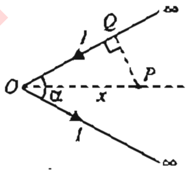


Fig. 9.146

10. Find magnetic field at O, by the system of current carrying wire.

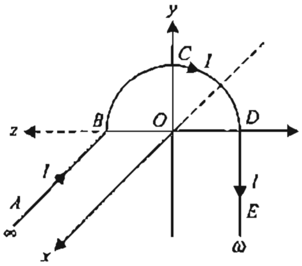


Fig. 9.147

11. A conducting ring of radius r having charge q is rotating with angular velocity ω about its axes. Find the magnetic field at the center of ring.

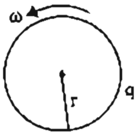


Fig. 9.148

12. Suppose that the current density in a wire of radius a varies with r according to Kr^2 where K is a constant and r is the distance from the axis of the wire. Find the magnetic field at a point at distance r from the axis when (i) $r < a$ and (ii) $r > a$.

13. In Fig. 9.149, the magnetic field at the point P.

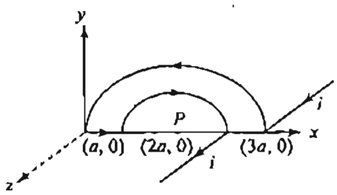


Fig. 9.149

14. An infinitely long straight wire with current I flowing along the positive z-axis is located at coordinates $P(-12 \text{ m}, 5 \text{ m})$. Determine the unit vector showing the direction of magnetic field at the origin.

15. A long, straight wire, carrying a current of 200 A, runs through a cubical wooden box, entering and leaving through holes in the centers of opposite faces (as shown in Fig. 9.150). The length of each side of the box is 20.0 cm. Consider an element $d\ell$, 0.100 cm long of the wire at the center of the box. Compute the magnitude dB of the magnetic field produced by this element at the points a, b, c, d and e as shown in the figure. Points a, c and d are at the centers of the faces of the cube; point b is at the midpoint of one edge; and point e is at a corner. Copy the figure and show the directions and relative magnitudes of the field vectors.

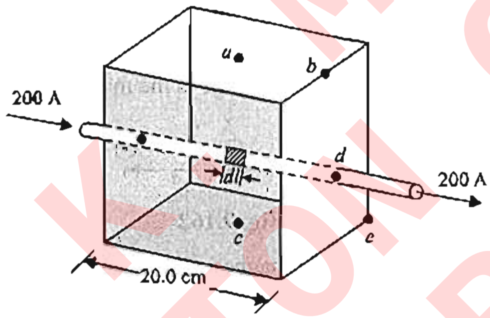


Fig. 9.150

16. The wire shown in Fig. 9.151, carries current I in the direction shown. The wire consists of a very long, straight section, a quarter-circle with radius R, and another long, straight section. What are magnitude and direction of net magnetic field at the center of curvature of quarter-circle section (point P)?

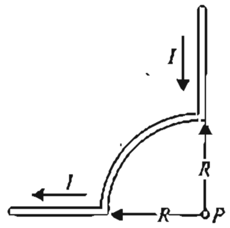


Fig. 9.151

17. A circular loop has radius R and carries current I_2 in a clockwise direction (as shown in Fig. 9.152). The center of the loop is a distance D above a long, straight wire. What are the magnitude and direction of the current I_l in the wire if the magnetic field at the center of the loop is zero ?

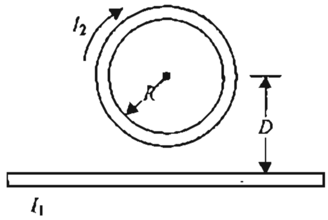


Fig. 9.152

18. For the arrangement shown in Fig. 9.153, determine the magnetic field at the center O.

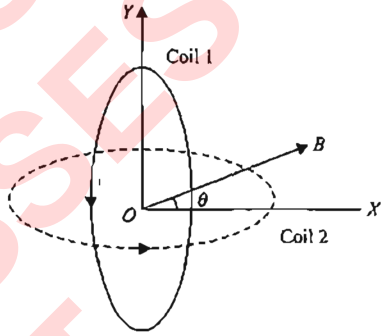


Fig. 9.153

19. Four long, parallel conductors carry equal currents of 5.0 A. The direction of the currents is into the page at points A and B and out of the page at C and D. Calculate the magnitude and direction of the magnetic field at point P, located at the center of the square.

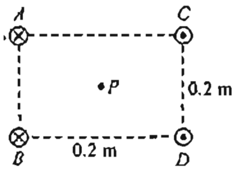


Fig. 9.154

20. A long, vertical wire carrying a current of 10 A in the upward direction is placed in a region where a horizontal magnetic field of magnitude 2.0×10^{-3} T exists from south to north. Find the point where the resultant magnetic field is zero.

21. Fig. 9.155 shows a long wire bent at the middle to form a right angle. Show that the magnitudes of the magnetic field at the points P, Q, R and S are equal and find this magnitude.

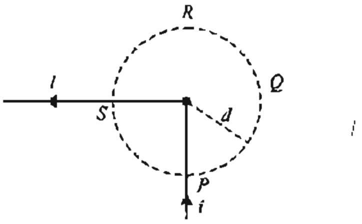


Fig. 9.155

22. In Fig. 9.156, two long, parallel wires (seen end-on) that are a distance R apart carry equal currents i in the same sense. Find the magnitude of the magnetic field at point P, which lies equidistant from the two wires at an angle θ from the plane of the wires.

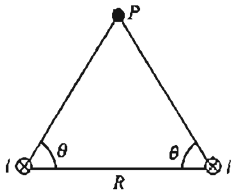


Fig. 9.156

23. Fig. 9.157 shows a square loop of edge *a* made of a uniform wire. A current *i* enters the loop at the point *A* and leaves it at the point *C*. Find the magnetic field at the point *P* which is on the perpendicular bisector of *AB* at a distance *a*/4 from it.

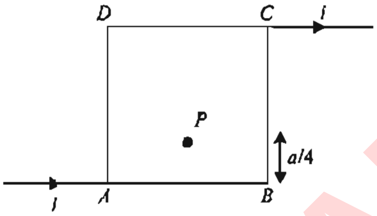


Fig. 9.157

24. Let two long parallel wires, a distance *d* apart, carry equal currents *I* in the same direction. One wire is at $x = 0$, the other at $x = d$ (as shown in Fig. 9.158). Determine between the wires as a function of *x*.

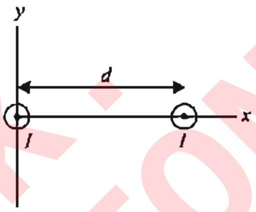


Fig. 9.158

25. In Fig. 9.159, points *P* and *Q* lie at the same small distance *R* from the current carrying wire. (By "small" we mean that *R* is very much less than the length of any segment of the wire.) What is the magnitude *B* of the magnetic field (a) at *Q* ? (b) at *P* ?

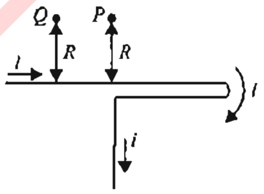


Fig. 9.159

26. A wire carrying current *i* has the configuration shown in Fig. 9.160. Two semi-infinite straight sections, each tangent to the same circle, are connected by a circular arc, of angle *θ*, along the circumference of the circle, with all sections lying in the same plane. What must *θ* be in order for *B* to be zero at the center of circle?

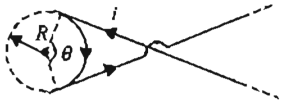


Fig. 9.160

27. A long, circular pipe, with an outside radius of *R*, carries a (uniformly distributed) current of i_0 (into the paper as shown in Fig. 9.161). A wire runs parallel to the pipe at a distance 3*R* from center to center. Calculate the magnitude and direction of the current in the wire that would cause the resultant magnetic field at the point *P* to have the same magnitude, but the opposite direction, as the resultant field at the center of the pipe.

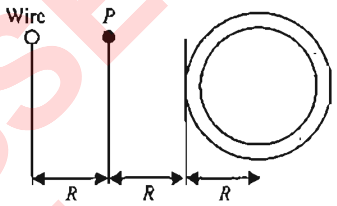


Fig. 9.161

28. Shown in Fig. 9.162, is an end-on view of three long, straight, parallel conductors spaced equal distances *a* apart. The outer conductors carry current *I* out of the page; the middle conductor carries current *I* into the page. Where in the plane of the page is the magnetic field zero?

Fig. 9.162

29. In Fig. 9.163, find the magnetic field at point *P*.

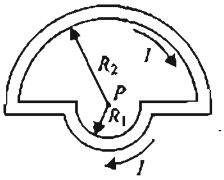


Fig. 9.163

30. Current *I* flows through a long conducting wire bent at right angle as shown in Fig. 9.164. Find the magnetic field at a point *P* on the right bisector of the angle *XOY* at distance *r* from *O*.

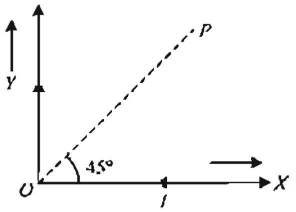


Fig. 9.164

31. Two circular coils X and Y, having equal number of turns and carrying currents in the same sense, subtend same solid angle at point O. If the smaller coil X is midway between O and Y and if we represent the magnetic induction due to bigger coil Y at O as B_y and that due to smaller coil X at O as B_x, then find the ratio B_y/B_x.

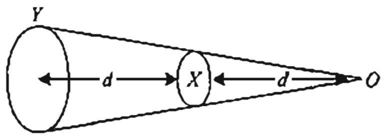


Fig. 9.165

32 Two circular coils made of similar wires but of radii 20 and 40 cm are connected in parallel. Find the ratio of the magnetic fields at their centers.

33. A wire is bent into the shape shown in Fig. 9.166 (a), and the magnetic field is measured at P_1 when the current in the wire is I. The same wire is then formed into the shape shown in Fig. 9.166 (b), and the magnetic field is measured at point P_2 when the current is again I. If the total length of wire is the same in each case, what is the ratio of B_1/B_2?

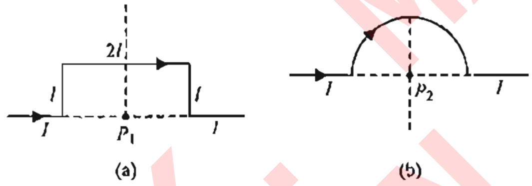


Fig. 9.166

34. Charge is sprayed onto a large non-conducting belt above the left-hand roller in Fig. 9.167. The belt carries the charge, with a uniform surface charge density s, as it moves with a speed v between the rollers as shown (Fig. 9.167). The charge is removed by a wiper at the right-hand roller. Consider a point just above the surface of the moving belt. Find an expression for the magnitude of magnetic field at this point.

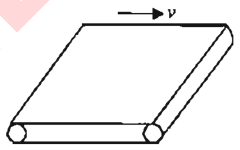


Fig. 9.167

35. An infinitely long, non-conducting cylinder of radius R lies along the z-axis. Five long, conducting wires are parallel to the cylinder and spaced equally on the upper half of its surface. Each wire carries a current I in the opposite z-direction. Find the magnetic field on the z-axis.

36. In Fig. 9.168, find the magnetic field at P? The loop is lying on $x-y$ plane.

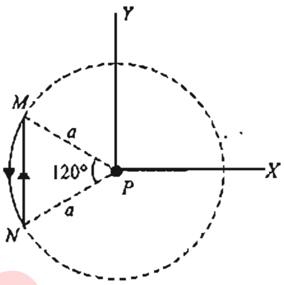


Fig. 9.168

37. Two long, straight wires, one above the other, are separated by a distance $2a$ and are parallel to the x-axis. Let the y-axis be in the plane of the wires in the direction from the lower wire to the upper wire. Each wire carries current I in the $+x$-direction. What are the magnitude and direction of the net magnetic field of the two wires at a point in the plane of wires:

a. midway between them?

b. at a distance a above the upper wire?

c. at a distance a below the lower wire?

38. A long, straight wire lies along the y-axis and carries a current $I = 8.00$ A in the $-y$-direction (as shown in Fig. 9.169). In addition to the magnetic field due to the current in the wire, a uniform magnetic field $\vec{B}_0$ with magnitude 1.50×10^{-6} T is in the $+x$-direction. What is the total field (magnitude and direction) at following points in the xz plane?

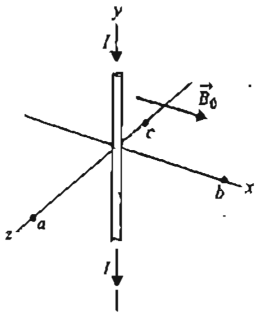


Fig. 9.169

a. $x = 0, z = 1.00$ m **b.** $x = 1.00$ m, $z = 0$

c. $x = 0, z = -0.25$ m

39. Calculate the magnitude of the magnetic field at point P as shown in Fig. 9.170, in terms of R, I_1 and I_2.

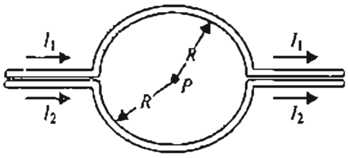


Fig. 9.170

40. The wire semicircles shown in Fig. 9.171, have radii a and b. Calculate the net magnetic field (magnitude and direction) that the current in the wires produces at point P.

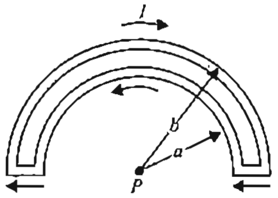


Fig. 9.171

41. Find the values of $\oint \vec{B}\cdot\overrightarrow{d\ell}$ for the loops L_1, L_2 and L_3 in Fig. 9.172.

The sense of $\overrightarrow{d\ell}$ is mentioned in the figure.

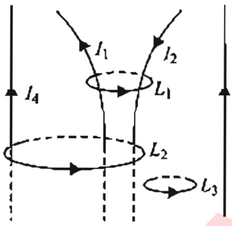


Fig. 9.172

42. A closed curve encircles several conductors. The line integral $\oint \vec{B}\cdot d\bar{\ell}$ around this curve is 3.83×10^{-4} Tm.

a. What is the net current in the conductors ?

b. If you were to integrate around the curve in the opposite direction, what would be the value of the line integral? Explain.

43. A solid conductor with radius a is supported by insulating disks on the axis of a conducting tube with inner radius b and outer radius c as shown in Fig. 9.173. The central conductor and tube carry equal currents I in opposite directions. The currents are distributed uniformly over the cross section of each conductor. Derive an expression for the magnitude of the magnetic field:

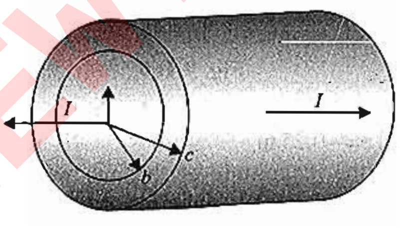


Fig. 9.173

a. at points outside the central, solid conductor, but inside the tube ($a < r < b$).

b. at points outside the tube ($r > c$).

44. Long, straight conductors with square cross sections and each carrying current I are laid side-by-side to form an infinite current sheet as shown in Fig. 9.174. The conductors lie in the xy plane, are parallel to the y-axis and carry current in the + y-direction. There are n conductors per unit length measured along x-axis.

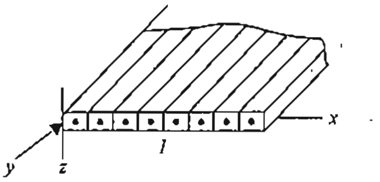


Fig. 9.174

a. What are the magnitude and direction of the magnetic field a distance a below the current sheet?

b. What are the magnitude and direction of the magnetic field a distance a above the current sheet?

Solved Examples

Example 9.1 **Two long straight parallel wires are 2m apart, perpendicular to the plane of the paper.**

The wire A carries a current of 9.6A, directed into the plane of the paper. The wire B carries a current such that the magnetic field of induction at the point P, at a distance of $\frac{10}{11}$ m from the wire B is zero.

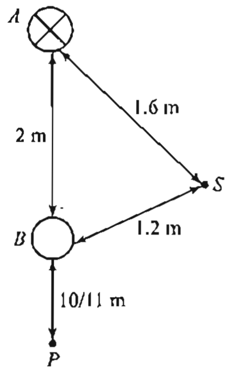


Fig. 9.175

Find:

a. The magnitude and direction of the current in B.

b. The magnitude of the magnetic field of induction at the point S.

c. The force per unit length on the wire B. **(IT-JEE, 1987)**

Sol.

a. Direction of current at B should be perpendicular to paper outwards. Let current in this wire be i_B. Then

$$\frac{\mu_0}{2\pi}\frac{i_A}{\left(2+\frac{10}{11}\right)} = \frac{\mu_0}{2\pi}\frac{i_B}{(10/11)}$$

or $$\frac{i_B}{i_A} = \frac{10}{32}$$

or $$i_B = \frac{10}{32}\times i_A = \frac{10}{32}\times 9.6 = 3\text{A}$$

b. Since $AS^2 + BS^2 = AB^2$

$\therefore \quad \angle ASB = 90°$

At S: B_1 = magnetic field due to i_A

$$= \frac{\mu_0}{2\pi}\frac{i_A}{1.6} = \frac{(2\times10^{-7})(9.6)}{1.6} = 12\times10^{-7}\,\text{T}$$

$$B_2 = \text{Magnetic field due to } iB = \frac{\mu_0}{2\pi}\frac{i_B}{1.2} = \frac{(2\times10^{-7})(3)}{1.2}$$

$$= 5\times10^{-7}\,\text{T}$$

Since B_1 and B_2 are mutually perpendicular, Net magnetic field at S would be:

$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(12\times10^{-7})^2 + (5\times10^{-7})^2}$$

$$= 13\times10^{-7}\,\text{T}$$

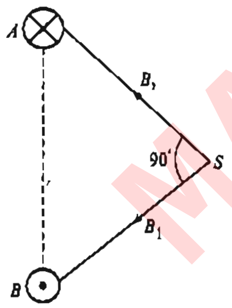


Fig. 9.176

c. Force per unit length on wire B:

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{i_A i_B}{r} \qquad (r = AB = 2\text{m})$$

$$= \frac{(2\times10^{-7})(9.6\times3)}{2} = 2.88\times10^{-6}\,\text{N/m}$$

Example 9.2 **Two long parallel wires carrying currents 2.5 and I (ampere) in the same direction (directed into the plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of paper. The points P and Q are located at a distance of 5m and 2m respectively from a collinear point R (see Fig. 9.177) (IIT-JEE, 1990)**

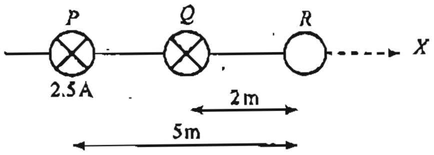


Fig. 9.177

a. An electron moving with a velocity of 4×10^5 m/s along the positive x – direction experiences a force of magnitude 2.5 A may be placed, so that the magnetic induction at R is zero.

b. Find all the positions at which a third long parallel wire carrying a current of magnitude 2.5A may be placed, so that the magnetic induction at R is zero.

Sol

a. Magnetic field at R due to both the wires P and Q will be downwards as shown in fig. Therefore, net field at R will be sum of these two.

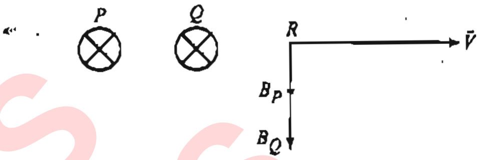


Fig. 9.178

$$B = B_P + B_Q$$

$$= \frac{\mu_0}{2\pi}\frac{I_P}{5} + \frac{\mu_0}{2\pi}\frac{I_Q}{2} = \frac{\mu_0}{2\pi}\left(\frac{2.5}{5} + \frac{I}{2}\right)$$

$$= \frac{\mu_0}{4\pi}(I+1) = 10^{-7}(I+1)$$

Net force on the electron will be.

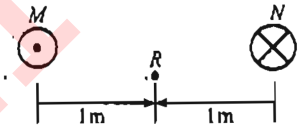


Fig. 9.179

$$Fm = Bqv\sin 90°$$

or $\quad (3.2\times10^{-20}) = (10^{-7})(I+1)(1.6\times10^{-19})(4\times10^5)$

or $\quad I + 1 = 5$

$\therefore \quad I = 4\text{A}$

b. net field at R due to wires P and Q is

$$B = 10^{-7}(I+1)\,T = 5\times10^{-7}\,\text{T}$$

Magnetic field due to third wire carrying a current of 2.5A should be 5×10^{-7} T n upward direction so, that net field at R becomes zero. Let distance of this wire from R be r. Then,

$$\frac{\mu_0}{2\pi}\frac{2.5}{r} = 5\times10^{-7} \text{ or } \frac{(2\times10^{-7})(2.5)}{r} = 5\times10^{-7}\,\text{m}$$

or $\quad \dfrac{(2\times10^{-7})(2.5)}{r} = 5\times10^{-7}\,\text{m}$

or $\quad r = 1\,\text{m}$

So, the third wire can be put at M or N as shown in fig. If it is placed at M, then current in it should be outwards and if placed at N, then current be inwards

Example 9.3 **A wire loop carrying a current is placed in the x-y plane as shown in Fig. 9.180 (a) If a particle with charge $+Q$ and mass m is placed at the centre P and given a velocity $\vec{v}$ along NP find its instantaneous acceleration. (b) If an external uniform magnetic induction field $\vec{B} = B\hat{i}$ is applied, find the force and torque acting on the loop. (IIT-JEE, 1991)**

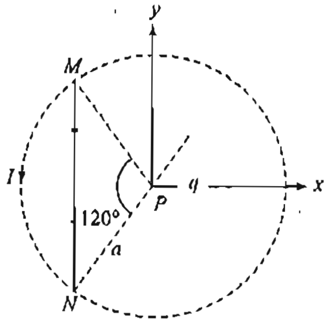


Fig. 9.180

Sol.

a. As in case of current-carrying straight conductor and arc, the magnitude of B is given by

$$B_1 = \frac{\mu_0 I}{4\pi d}\ (\sin a + \sin b) \text{ and } B_2 = \frac{\mu_0 I \phi}{4\pi r}$$

So in accordance with right hand screw rule,

$$\left(\vec{B}_w\right) = \frac{\mu}{4\pi}\frac{I}{(a\cos 60^{o})} \times 2\sin 60^{o} \text{ and}$$

$$\left(\vec{B}\right)_{MN} = \frac{\mu_0 I}{2a}\left[\frac{2\pi/3}{2\pi}\right](\hat{k})$$

and hence net $\vec{B}$ at P due to the given loop

$$\vec{B} = \vec{B}_w + \vec{B}_A \ \Rightarrow\ \vec{B} = \frac{\mu_o}{4\pi}\frac{2I}{a}\left[\sqrt{3}-\frac{\pi}{3}\right]\left(-\hat{k}\right) \quad \text{(i)}$$

Now as force on charged particle in a magnetic fields is given by $\vec{F} = q\left(\vec{v}\times\vec{B}\right)$

so here, $\vec{F} = qvB \sin 90^{o}$ along PF

i.e. $$\vec{F} = \frac{\mu_0}{4\pi}\frac{2qvI}{a}\left[\sqrt{3}-\frac{\pi}{3}\right] \text{ along } PF$$

and so $$\vec{a} = \frac{\vec{F}}{m} = 10^{-7}\frac{2qvI}{a}\left[\sqrt{3}-\frac{\pi}{3}\right] \text{a along } PF$$

b. As $d\vec{F} = I\,d\vec{L}\times\vec{B}$, so $\vec{F} = \int Id\vec{L}\times\vec{B}$

As here I and $\vec{B}$ are constant

$$\vec{F} = \left[\oint d\vec{L}\right]\times\vec{B} = 0 \qquad \left[\text{as}\oint d\vec{L} = 0\right]$$

Further as area of coil,

$$\vec{S} = \left[\frac{1}{3}\pi a^2 - \frac{1}{2}.2a\sin 60°\times a\cos 60°\right]\hat{k}$$

$$= a^2\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right]\hat{k}$$

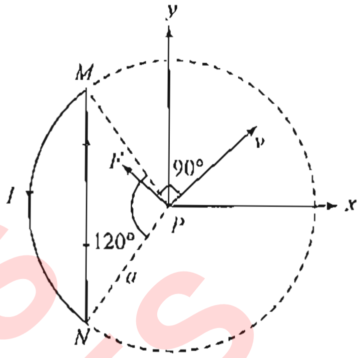


Fig. 9.181

so $$\vec{M} = I\vec{S} = Ia^2\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right]\hat{k}$$

and hence $$\vec{\tau} = \vec{M}\times\vec{B} = Ia^2B\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right]\left(\hat{k}\times\hat{i}\right)$$

$$\Rightarrow \quad \vec{\tau} = Ia^2B\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right]\hat{j} \text{ N–m}$$

Example 9.4 **A long horizontal wire AB, which is free to move in a vertical plane and carries a steady current of 20A, is in equilibrium at a height of 0.01m over another parallel long wire CD which is fixed in a horizontal plane and carries a steady current of 30A, as shown in Fig. 9.182. Show that when AB is slightly depressed. It executes simple harmonic motion. Find the period of oscillations. (IIT-JEE, 1994)**

Fig. 9.182

Sol. Let m be the mass per unit length of wire AB. At a height x above the wire CD, magnetic force per unit length on wire AB will be given by

$$F_m = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{x} \quad \text{(upwards)} \qquad \text{(i)}$$

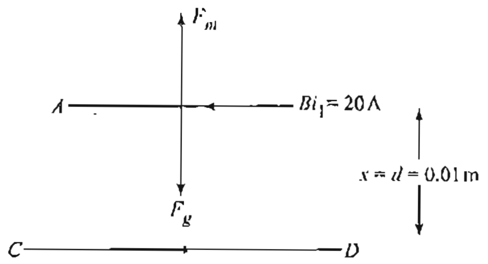


Fig. 9.183

Weight per unit length of wire AB is

$F_g = mg$ (downwards)

Here, m = mass per unit length of wire AB

At $x = d$, wire is in equilibrium i.e.

$F_m = F_g$

or $$\frac{\mu_0}{2\pi}\frac{i_1 i_2}{d} = mg$$

or $$\frac{\mu_0}{2\pi}\frac{i_1 i_2}{d^2} = \frac{mg}{d} \qquad \text{(ii)}$$

When AB is depressed, x decreases therefore F_m will increase, while F_g remains the same. Let AB is displaced by dx downwards. Differentiating Eq. (i) w.r.t x, we get

$$dF_m = -\left(\frac{mg}{d}\right).dx \qquad \text{(iii)}$$

i.e. restoring force, $F = dF_m \propto -dx$

Hence, the motion of wire is simple harmonic.

From Eqs. (ii) and (iii) we can write

$$dF_m = -\left(\frac{mg}{d}\right).dx \qquad (x = d)$$

∴ Acceleration of wire $a = -\left(\frac{g}{d}\right).dx$

Hence, period of oscillation

$$T = 2\pi\sqrt{\left|\frac{dx}{a}\right|} = 2\pi\sqrt{\left|\frac{\text{displacment}}{\text{accelertion}}\right|}$$

or $$T = 2\pi\sqrt{\frac{d}{g}} = 2\pi\sqrt{\frac{0.01}{9.8}} \text{ or } T = 0.2\text{s}$$

Example 9.5 **An electron in the ground state of hydrogen atom is revolving in anticlockwise direction in a circular orbit of radius R (see Fig. 9.184)**

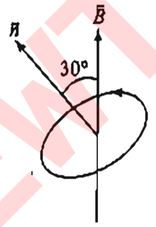


Fig. 9.184

a. Obtain an expression for the orbital magnetic moment of the electron

b. The atom is placed in a uniform magnetic induction $\vec{B}$ such that the normal to the plane of electron's orbit makes an angle of 30° with the magnetic induction. Find the torque experienced by the orbiting electron. **(IIT-JEE, 1996)**

Sol.

a. In ground state ($n = 1$) according to Bohr's theory:

$$mvR = \frac{h}{2\pi}$$

or $$v = \frac{h}{2\pi mR}$$

Now, time period, $T = \frac{2\pi R}{v} = \frac{2\pi R}{h/2\pi mR} = \frac{4\pi^2 mR^2}{h}$

Magnetic moment $M = A$

Where $I = \frac{\text{charge}}{\text{time period}} = \frac{e}{\frac{4\pi^2 mR^2}{h}} = \frac{4\pi^2 mR^2}{h}$

and $A = \pi R^2$

∴ $M = (\pi R^2)\left(\frac{eh}{4\pi^2 mR^2}\right)$ Or $M = \frac{eh}{4\pi m}$

Direction of magnetic moment $\vec{M}$ is perpendicular to the plane of orbit.

b. $\vec{\tau} = \vec{M}\times\vec{B}$

$|\vec{\tau}| = MB\sin\theta$

Where θ is the angle between $\vec{M}$ and $\vec{B}$

$\theta = 30°$

∴ $\tau = \left(\frac{eh}{4\pi m}\right)(B)\sin 30^0$

∴ $\tau = \frac{ehB}{8\pi m}$

The direction of $\vec{\tau}$ is perpendicular to both $\vec{M}$ and $\vec{B}$

Example 9.6 **Three infinitely long thin wires, each carrying current i in the same direction, are in the $x-y$ plane of a gravity free space. The central wire is along the y – axis while the other two are along $x = \pm d$.**

a. Find the locus of the points for which the magnetic field B is zero.

b. If the central wire is displaced along the z –direction by a small amount and released, show that it will execute simple harmonic motion. If the linear density of the wire is λ, find the frequency of oscillation. **(IIT-JEE, 1997)**

Sol.

a. Magnetic field will be zero on the y – axis i.e.

$x = 0 = z$

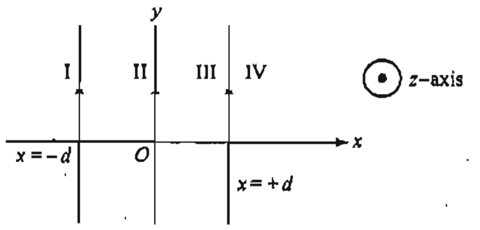


Fig. 9.185

magnetic field cannot be zero in region I and region IV because in region I magnetic field will be along positive z – direction due to all the three wires, while in region IV magnetic field will be along negative z – axis due to all the three wires. It can zero only in region II and III.

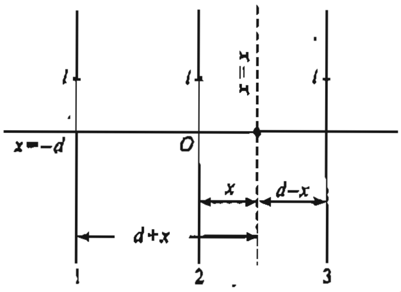


Fig. 9.186

Let magnetic field is zero on line (z = 0) and $x = x$. Then magnetic field on this line due to wires 1 and 2 will be along negative z – axis and due to wire 3 along positive z – axis. Thus

$$B_1 + B_2 = B_3$$

or $$\frac{\mu_0}{2\pi}\frac{l}{d+x} + \frac{\mu_0}{2\pi}\frac{l}{x} = \frac{\mu_0}{2\pi}\frac{l}{d-x}$$

or $$\frac{1}{d+x} + \frac{1}{x} = \frac{1}{d-x}$$

This equation gives $x = \pm\dfrac{d}{\sqrt{3}}$

Where magnetic field is zero.

b. In this part we change our coordinating axes system. Just for better understanding.

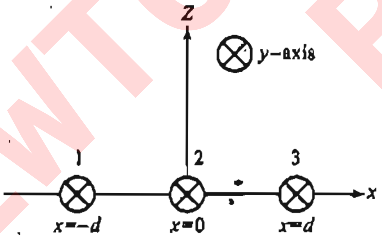


Fig. 9.187

There are three wires 1, 2 and 3 as shown in Fig. 9.188. If we displace the wire 2 towards the z – axis.

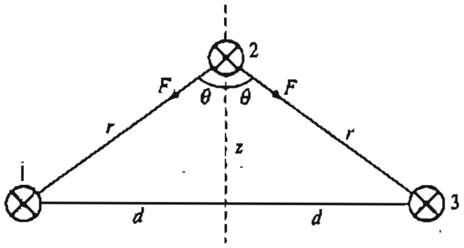


Fig. 9.188

Then force of attraction per unit length between wires (1 and 12) and (2 and 3) will be given by

$$F = \frac{\mu_0}{2\pi}\frac{i^2}{r}$$

The components of F along x – axis will be cancelled out. Net resultant force will be towards negative z – axis (or mean position) and will be given by

$$F_{net} = \frac{\mu_0}{2\pi}\frac{l^2}{r}$$

The components of F along x – axis will be cancelled out. Net result force will be towards negative z – axis. (or mean position) and will be given by

$$F_{net} = \frac{\mu_0}{2\pi}\frac{l^2}{r}(2\cos\theta)$$

$$= 2\left\{\frac{\mu_0}{2\pi}\frac{l^2}{r}\right\}\frac{z}{r}$$

$$F_{net} = \frac{\mu_0}{\pi}\frac{i^2}{(z^2+d^2)}z$$

If $z << d$, then

$$Z^2 + d^2 = d^2 \text{ and } F_{net} = -\left(\frac{\mu_0}{\pi}\frac{i^2}{\cdot d^2}\right).z$$

Negative sign implies that F_{net} is restoring in nature
Therefore $F_{net} \propto -z$
i,.e. the wire will oscillate simple harmonically.
Let a be he acceleration of wire in this position and λ is the mass per unit length of this wire then

$$F_{net} = \lambda.a = -\left(\frac{\mu_0}{\pi}\frac{l^2}{d^2}\right)z$$

or $$a = -\left(\frac{\mu_0 l^2}{\pi\lambda d^2}\right).z$$

∴ Frequency of oscillation

$$f = \frac{1}{2\pi}\sqrt{\frac{a}{z}} = \frac{1}{2\pi}\frac{i}{d}\sqrt{\frac{\mu_0}{\pi\lambda}} = f = \frac{i}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$$

Example 9.7 **A particle of mass *ma* and charge *q* is moving in a region where uniform, constant electric and magnetic field $\vec{E}$ and $\vec{B}$ are present. $\vec{E}$ and $\vec{B}$ are parallel to each other. At time $t = 0$, the velocity $\vec{v}_0$ of the particle is perpendicular to $\vec{E}$ (Assume that its speed is always $<<c$, the speed of light in vacuum). Find the velocity $\vec{v}$ of the particle at time t. You must express your answer in terms of t, q, m, the vector $\vec{v}_0$, $\vec{E}$ and $\vec{B}$ and their magnitudes v_0, E and B.** **(IIT-JEE, 1998**

Sol. $$\hat{j} = \frac{\vec{E}}{E} \text{ or } \frac{\vec{B}}{B} : \hat{i} = \frac{\vec{v}_0}{v_0}$$

$$\hat{k} = \frac{\vec{v}_0 \times \vec{B}_0}{v_0 B}$$

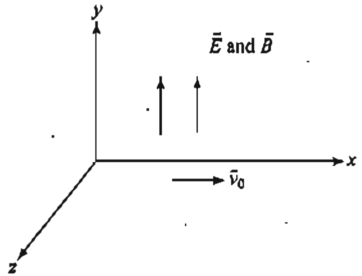


Fig. 9.189

Force due to electric field will be along y – axis. magnetic force will not affect the motion of charged particle in the direction of electric field (or y – axis) so.

$$a_y = \frac{F_e}{m} = \frac{qE}{m}.t \qquad \text{(i)}$$

The charged particle under the action of magnetic field describes a circle in x-z plane (perpendicular to $\vec{B}$) with $T = \frac{2\pi m}{Bq}$

or $\omega = \frac{2\pi}{T} = \frac{qB}{m}$

Initially ($t = 0$) velocity was along x – axis. Therefore magnetic force $\left(\vec{F}_m\right)$ will be along positive z – axis $\left[\vec{F}_m = q(\vec{v}_0 \times \vec{B})\right]$. Let it makes an angle θ with x – axis at time t, then

$$\theta = \omega t$$

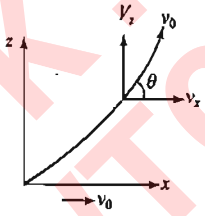


Fig. 9.190

$$\therefore \quad n_x = v_0 \cos \omega t = v_0 \cos\left(\frac{qB}{m}t\right) \qquad \text{(ii)}$$

$$v_z = v_0 \sin \omega t = v_0 \sin\left(\frac{q_B}{m}t\right) \qquad \text{(iii)}$$

From eqs (i), (ii) and (iii)

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$\therefore \quad \vec{v} = v_0 \cos\left(\frac{qB}{m}t\right)\left(\frac{\vec{v}_0}{v_0}\right) + \frac{qE}{m}t\left(\frac{\vec{E}}{E}\right) + v_0 \sin\left(\frac{qB}{m}t\right)\left(\frac{\vec{v}_0 \times \vec{B}}{v_0 B}\right)$$

$$\text{or } \vec{v} = \cos\left(\frac{qB}{m}t\right)(\vec{v}_0) + \left(\frac{q}{m}t\right)(\vec{E}) + \sin\left(\frac{qB}{m}t\right)\left(\frac{\vec{v}_0 \times \vec{B}}{B}\right)$$

Note:

The path of the particle will be a helix of increasing pitch. The axis of the helix will be along y – axis.

Example 9.8 **A ring of radius R having uniformly distributed charge Q is mounted on a rd suspended by two identical strings. The tension in strings in equilibrium is T_0. Now a vertical magnetic field is switched on and ring is rotated at constant angular velocity ω. Find the maximum ω with which the ring can be rotated if the strings can withstand a maximum tension of $\frac{3T_0}{2}$.** **[IIT-JEE, 2003]**

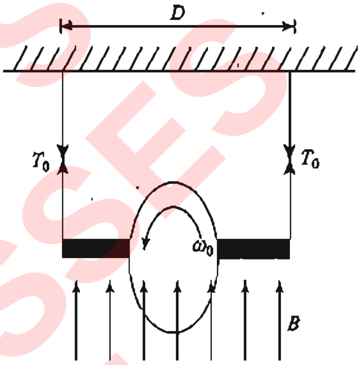


Fig. 9.191

Sol. In equilibrium,

$$2T_0 = mg$$

$$\text{or} \quad T_0 = \frac{mg}{2}$$

$$\text{Magnetic moment, } M = iA = \left(\frac{\omega}{2\pi}Q\right)(\pi R^2)$$

$$t = MB \sin 90° = \frac{\omega BQR^2}{2}$$

Let T_1 and T_2 be the tensions in the two strings when magnetic field is switched on ($T_1 > T_2$)

For translational equilibrium,

$$T_1 + T_2 = mg \qquad \text{(ii)}$$

For rotational equilibrium

$$(T_1 - T_2)\frac{D}{2} = \tau = \frac{\omega BQR^2}{2}$$

$$\text{or} \quad T_1 - T_2 = \frac{\omega BQR^2}{D} \qquad \text{(iii)}$$

Solving Eqs. (ii) and (iii) we have

$$T_1 = \frac{mg}{2} + \frac{\omega BQR^2}{2D}$$

As $T_1 > T_2$ and maximum values of T_1 can be $\frac{3T_0}{2}$, we have

$$\frac{3T_0}{2} = T_0 + \frac{\omega_{\max} BQR^2}{2D} \qquad \left(\frac{mg}{2} = T_0\right)$$

$$\therefore \quad \omega_{\max} = \frac{DT_0}{BQR^2}$$

Example 9.9 **A moving coil galvanometer has a coil of area A and number of turns N. A magnetic field B is applied on it. The torque acting on it is given by $\tau = ki$ where i is current through the coil. If moment of inertia of the coil is I about the axis of rotation** **[IIT-JEE, 2005]**

a. Find the value of k in terms of galvanometer parameters (N, B, A).

b. Find the value of torsional constant if current i_0 produces angular deflection of $\frac{\pi}{2}$ radian.

c. If a charge Q is passed almost instantaneously through coil, find the maximum angular deflection in it.

Sol.

a. The torque acting on the coil of moving coil galvanometer is

$$\tau = NiAB$$

Given $\quad \tau = ki$

$$ki = NiAB \quad \Rightarrow \quad k = NBA$$

b. If C is torsional constant of the spring of galvanometer, then

$$\tau = C\theta$$

$$Ni_0AB = C\left(\frac{\pi}{2}\right) \quad \Rightarrow \quad C = \frac{2NBAi_0}{\pi}$$

c. If θ_m is maximum deflection, then from conservation of energy

$$\frac{1}{2}C\theta_m^2 = \frac{1}{2}I\omega^2 \quad \Rightarrow \quad \theta_m = \sqrt{\frac{I}{C}}\,\omega \qquad \text{(i)}$$

We have $\tau = NiAB$

Put $\tau = \frac{dL}{dt}$ where L is angular momentum.

$$\frac{dL}{dt} = N\left(\frac{dQ}{dt}\right)AB \quad \text{or} \quad dL = NAB\,dQ$$

Integrating $\int_0^L dL = NAB\int_0^Q dQ \quad \Rightarrow \quad L = NABQ$

If ω is angular velocity.

Put $\quad L = I\omega$

$$I\omega = NABQ$$

$$\omega = \frac{NABQ}{I} \qquad \text{(ii)}$$

Substituting thus value in (i), we get

$$\theta_m = \sqrt{\frac{I}{C}}\cdot\frac{NABQ}{I} = \frac{NABQ}{\sqrt{\left\{\frac{2NAB\,i_0 I}{\pi}\right\}}} = Q\sqrt{\frac{\pi NAB}{2Ii_0}}$$

Example 9.10 **A particle of mass m and charge q is moving in a region where uniform constant electric and magnetic fields $\bar{E}$ and $\bar{B}$ are present. $\vec{E}$ and $\vec{B}$ are parallel to each other. At time $t = 0$ the velocity $\bar{v}$ of the particle is perpendicular to (Assume that its speed is always $\ll c$, the speed of light in vacuum). Find the velocity $\bar{v}$ of the particle at time t. You must express your answer in terms of t, q, m, the vectors $\bar{v}_0, \bar{E}$ and $\bar{B}$ and their magnitudes v_0, E and B.**

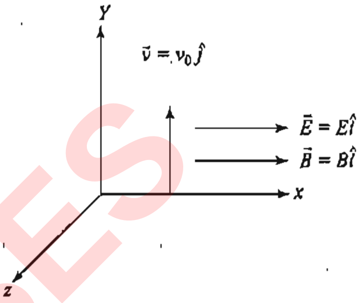


Fig. 9.192

Sol. Let the velocity v, E and B be as shown in Fig. 9.192. The Lorentz force on the charged particle is

$$q(\bar{E} + \bar{v}\times\bar{B}) = m\frac{d\bar{v}}{dt}$$

$$q[E\hat{i} + v_0\hat{j}\times B\hat{i}) = m\frac{d}{dt}[v_x\hat{i} + v_y\hat{j} + v_z\hat{k}]$$

$$qE\hat{i} + qv_0B(-\hat{k}) = m\frac{d}{dt}[v_x\hat{i} + v_y\hat{j} + v_z\hat{k}]$$

On comparing the components on both sides, we get,

$$m\frac{dv_x}{dt} = qE \qquad \text{(i)}$$

$$m\frac{dv_y}{dt} = qBv_z \qquad \text{(ii)}$$

$$m\frac{dv_z}{dt} = -qBv_y \qquad \text{(iii)}$$

From equation (i), we have

$$v_x = \frac{qE}{m}t \quad [a_x \text{ is constant}] \qquad \text{(iv)}$$

From equation (ii), we have

$$\frac{d^2v_y}{dt^2} = \frac{qB}{m}\frac{dv_z}{dt} \qquad \text{(v)}$$

We can substitute $\frac{dv_z}{dt}$ from equation (iii), in equation (v), we get

$$\frac{d^2v_y}{dt^2} = -\left(\frac{qB}{m}\right)^2 v_y \qquad \text{(vi)}$$

Similarly from equation (iii), we have

$$\frac{d^2v_z}{dt^2} = -qB\frac{dv_y}{dt} \qquad \text{(vii)}$$

Now we substitute dv_y/dt from equation (ii) in (vii) to get

$$\frac{d^2v_z}{dt^2} = -\left(\frac{qB}{m}\right)^2 v_z \qquad \text{(viii)}$$

Equations (vi) and (viii) are second order differential equtions, similar to that encountered in simple harmonic motion. Solution of equation (viii),

$$\frac{d^2v_y}{dt^2} = -\omega^2 v_y \qquad \left[\omega = \frac{qB}{m}\right]$$

$$v_y = A\cos(\omega t + \phi)$$

Hence from equation (ii),

$$v_z = \frac{m}{qB}\frac{dv_y}{dt} = -A\sin(\omega t + \phi)$$

At $t = 0$, $v_y = v_0$ and $v_z = 0$; therefore we get,

$$\phi = = 0 \text{ and } A = v_0$$

Thus the final solutions are

$$v_x = \frac{qE}{m}t,\ v_y = v_0\cos\omega t,\ v_z = -v_0\sin\omega t$$

In vector notation,

$$\vec{v} = \left(\frac{qE}{m}t\right)\hat{i} + (v_0\cos\omega t)\hat{j} + (-v_0\sin\omega t)\hat{k}$$

$$= \left(\frac{qE}{m}t\right)\frac{\vec{E}}{E} + (v_0\cos\omega t)\frac{\vec{v}_0}{v} + (-v_0\sin\omega t)\frac{\vec{B}\times\vec{v}_0}{Bv_0}$$

$$= \frac{qt}{m}\vec{E} + v_0\cos\omega t\,\vec{v}_0 + \frac{\sin\omega t}{B}\vec{v}_0\times\vec{B}$$

Example 9.11 **A charged particle of mass m and charge q is projected on a rough horizontal x-y plane surface with z-axis in the vertically upward direction. Both electric and magnetic fields are acting in the region and given by $\vec{E} = -E_0\hat{k}$ and $\vec{B} = -B_0\hat{k}$ respectively. The particle enters into the field at $(a_0, 0, 0)$ with velocity $\vec{v} = v_0\hat{j}$. The particle starts moving into a circular path on the plane. If the coefficient of friction between the particle and the plane is m. Then calculate the**

- **a. time when the particle will come to rest**
- **b. time when the particle will hit the centre**
- **c. distance travelled by the particle when it comes to rest.**

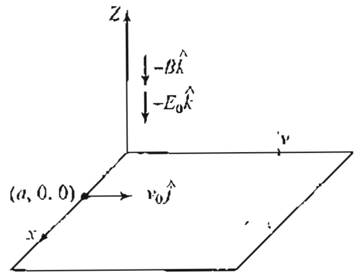


Fig. 9.193

Sol.

a. $N = mg + qE_0$ (i)

$$qB_0v = \frac{mv^2}{R} \qquad \text{(ii)}$$

And $-m\dfrac{dv}{dt} = \mu N$ (iii)

From equation (ii),

$$R = \frac{mv}{B_0q} \qquad \text{(iv)}$$

From equations (i) and (iii),

$$-m.\frac{dv}{dt} = \mu(mg + qE_0)$$

$$-m\int_{v_0}^{0} dv = -\mu(mg + qE_0)\int_0^t dt$$

Thus $$t = \frac{mv_0}{\mu(mg + qE_0)}$$

b. From equation (iv)

$$dR = \frac{m}{B_0q}dv = -\frac{\mu(mg + qE_0)dt}{qB_0}$$

$$\int_{R_i}^{0} dR = \frac{-\mu(mg + qE_0)}{qB_0}\int_0^t dt$$

$$t = \frac{qB_0R_i}{\mu(mg + qE_0)}$$

Here $$R_i = \frac{mv_0}{B_0q}$$

Thus $$t = \frac{mv_0}{\mu(mg + qE_0)}$$

(c) $-mg.\dfrac{dv}{d\ell} = \mu(mg + qE_0)$. $-m\displaystyle\int_{v_0}^{0} v\,dv = \mu(mg + qE_0)\int_0^t d\ell$

$$\ell = \frac{mv_0^2}{2\mu(mg + qE_0)}$$

Example 9.12 **A long straight wire carries a current i. A particle having a positive charge q and mass m, kept at distance x_0 from the wire is projected towards it with speed v. Find the closest distance of approach of charged particle to the wire.**

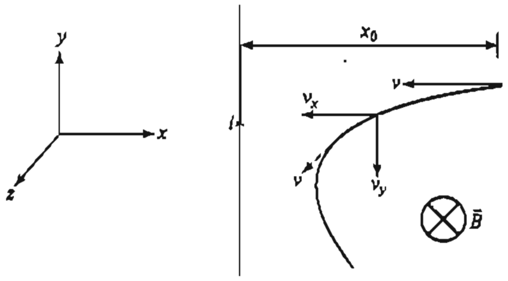


Fig. 9.194

Sol. Magnetic field due to long wire is

$$\vec{B} = -\frac{\mu_0 i\hat{k}}{2\pi x}$$

Let the velocity of the charged particle at any instant be

$$\vec{v} = (v_x\hat{i} + v_y\hat{j})$$

Magnetic force on the charge,

$$\vec{F} = q(\vec{v}\times\vec{B}) = -\frac{\mu_0 qiv_y}{2\pi x}\hat{i} + \frac{\mu_0 qiv_x}{2\pi x}\hat{j} \qquad \text{(i)}$$

From equation (i), y component of acceleration is

$$a_y = \frac{dv_x}{dt} = \frac{\mu_0 qiv_x}{2\pi mx} = \frac{\mu_0 q}{2\pi mx} i \frac{dx}{dt}$$

At the minimum separation, velocity of particle is $-v\,\hat{j}$

$$\int_0^{-v} dv_y = \frac{\mu_0 qi}{2\pi m}\int_{x_0}^{x_{min}} \frac{dx}{x}$$

$$-v = \frac{\mu_0 qi}{2\pi m}\ln\left|\frac{x_{min}}{x_0}\right|$$

Thus $\quad x_{min} = x_0 e^{-2\pi mv/\mu_0 qi}$

Example 9.13 **A loop of flexible conducting wire of length 0.5 m lies in a magnetic field of 1.0 tesla perpendicular to the plane of the loop. Show that when a current is passed through the loop, it opens into a circle. Also calculate the tension developed in the wire if the current is 1.57 A.**

Sol. The loop may be divided into a large number of small length elements. When a current I is passed through the loop placed in the magnetic field such that the plane of the loop is perpendicular to field, then force on each element is

$$Id\vec{\ell}\times\vec{B} = Id\ell B\sin 90° = I\,d\,\ell\,B$$

Perpendicular to current element $Id\vec{\ell}$ as well as magnetic field $\vec{B}$. Hence, the loop opens into a circle.

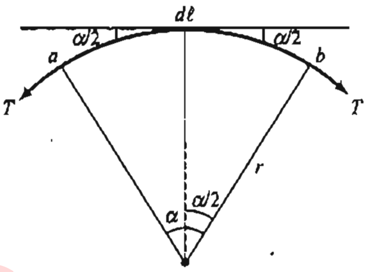


Fig. 9.195

Consider an element of length dl of circle of radius r, making an angle α at centre. If T is the tension in the wire, then force towards centre

$$2T\sin\frac{\alpha}{2} = IB\,d\ell$$

For small angle α, $\sin\frac{\alpha}{2} = \frac{\alpha}{2}$

$$2T.\frac{\alpha}{2} = IB\ d\ell$$

$$T = \frac{IB\,d\ell}{\alpha} = IBr \qquad \left(\because \alpha = \frac{d\ell}{r}\right)$$

$$= IB\left(\frac{\ell}{2\pi}\right) \qquad (\because \ell = 2\pi r)$$

$$\approx 1.57\times 10\times\left(\frac{0.5}{2\times 3.14}\right) = 1.8 = 0.125\text{ N}$$

Example 9.14 **A circular coil of 100 turns has an effective radius of 0.05 m and carries a current of 0.1 amperes. How much work is required to turn it in an external magnetic field of 1.5 weber/m² through 180° about an axis perpendicular to the magnetic field. The plane of the coil is initially perpendicular to the magnetic field.**

Sol. The potential energy of circular loop in the magnetic field $= -MB\cos\theta$ where θ is angle between normal to plane of coil and B. Initially given $\theta = 0°$.

$\therefore$ Initial potential energy,

$$U_f = -NiAB\cos 180° = -NiAB$$

When coil is turned through 180°; θ = 180°, therefore final potential energy.

$$U_f = -NiAB\cos 180° = NiAB$$

$\therefore$ Required work, W = gain in potential energy

$$= U_f - U_i = NiAB - (-NiAB) = 2NiAB$$

Here $N = 100$, $i = 0.1\,A$, $B = 1.5$ weber/m² and radius $r = 0.05$ m.

$$\therefore \quad A = \pi r^2 = 3.14\times(0.5)^2 = 7.85\times10^{-3}\text{ m}^2$$

$$\therefore \quad \text{Work, } W = 2\times100\times0.1\times7.85\times10^{-3}\times1.5$$

$$= 0.2355\text{ joule}$$

Example 9.15 A positively charged particle having charge $q_1 = 1$ coulomb and mass $m_1 = 40$ gm, is revolving along a circle of radius $R = 40$ cm with velocity $v_1 = 5\ \text{ms}^{-1}$ in a uniform magnetic field with centre of circle at origin O of a three dimensional system.

At $t = 0$, the particle was at (0,0.4m,0) and velocity was directed along positive x-direction. Another particle having charge $q_2 = 1$ coulomb and mass $m_2 = 10$ g moving uniformly parallel to positive z-direction with velocity $v_2 = \frac{40}{\pi}\text{ms}^{-1}$ collides with revolving particle at $t = 0$ and gets stuck to it. Neglecting gravitational force and coulomb force, calculate x, y and z co-ordinates of the combined particle at $= \frac{\pi}{40}$ second

Sol: In three dimensional co-ordinates system, axes are assumed according to right hand screw rule.

Consider such a system shown in figure (a). In this system positive z-direction is normal to plane of paper and is directed towards the reader.

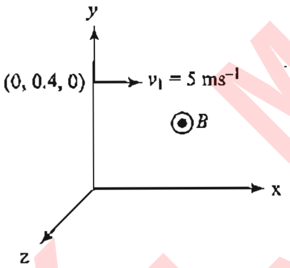


Fig. 9.196

The particle is positively charged, centre of its circular path is at origin and when the particle is on positive y-axis, its velocity is directed along positive x-direction, it means that the particle is moving clockwise in the figure (a).

Since the particle is positively charged, therefore, current associated with its motion is also clockwise or along positive x-direction at (0, 0.4, 0).

Lorentz's forces is towards origin. Therefore, according to Fleming's left hand rule, magnetic induction B must be along positive z-direction.

Radius of circular path followed by this particle is $r_1 = 0.4$ m

$$\text{But} \quad r_1 = \frac{m_1 v_1}{q_1 B} \Rightarrow B = \frac{m_1 v_1}{q_1 r_1} = 0.5\ \text{T}$$

Now the second particle collides and gets struck with it. Velocity of combined particle can be found out by applying law of conservation of momentum.

$$(m_1 + m_2)\bar{v} = m_1 \bar{v}_1 + m_2 \bar{v}_2$$

$$(40\times10^{-3} + 10\times10^{-3})\bar{v}$$

$$= (40\times10^{-3})\times5\hat{i} + (10\times10^{-3})\frac{40}{\pi}\hat{k}$$

$\therefore$ x-direction of velocity of combined particle is $v_x = 4\ \text{ms}^{-1}$

and z-direction, $v_z = \frac{8}{\pi}\text{ms}^{-1}$

Due to v_x, combined particle tries to move clockwise along a circular path and due to v_z it ties to move uniformly along z-axis.

Therefore, its ultimate path becomes helix. Radius of this helix is

$$R = \frac{(m_1 + m_2)v_x}{(q_1 + q_2)B} = 0.2\ \text{m}$$

$$\text{and period of revolution } T = \frac{2\pi(m_1 + m_2)}{(q_1 + q_2)B} = \frac{\pi}{10} \text{ seconds}$$

Axis of this helix is a straight line shown by point C in figure (b) whose equation is

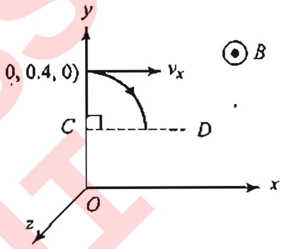


Fig. 9.197

$$y = (r_1 - R) = (0.4 - 0.2) = 0.2\text{m}$$

Angular displacement of particle, due to circular motion is

$$\theta = 2\pi\frac{t}{T}$$

$$\theta = \frac{\pi}{2}$$

$\therefore$ x-coordinate of new position of combined particle $= R\sin\theta$ $= 0.2$ m.

$$= OC + R\cos\theta = 0.2 + 0 = 0.2\ \text{m}$$

and z-coordinate $= v_z t = 0.2$ m

$\therefore$ Position of combined particle at $t = \frac{\pi}{40}$ seconds

Example 9.16 An electron accelerated by a potential difference $V = 3$ volt first enters into a uniform electric field of a parallel - plate capacitor whose plates extend over a length $l = 6$ cm in the direction of initial velocity.

The electric field is normal to the direction of initial velocity and its strength varies with time as $E = \alpha t$, where $\alpha = 3600\ \text{Vm}^{-1}\text{s}^{-1}$.

Then the electron enters into a uniform magnetic field of induction $B = \pi \times 10^{-9}$ T. Direction of magnetic field is same as that of the electric field.

Calculate pitch of helical path traced by the electron in the magnetic field. (Mass of electron, $m = 9\times10^{31}$ kg)

Sol. Since, electron is accelerated through a potential difference V, therefore its initial velocity v_0 is given by $\frac{1}{2}mv_0^2 = eV$

$$v_0 = \sqrt{\frac{2eV}{m}} \qquad \text{(i)}$$

Since, initial velocity is parallel to plates or normal to the direction of electric field, therefore component of velocity parallel to plates remains constant as v_0.

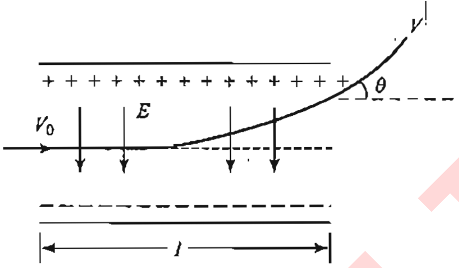


Fig. 9.198

Hence, time taken by the electron to cross electric field is

$$t_0 = \frac{l}{v_0}$$

Now consider motion of electron, normal to plates.

At some instant t, its acceleration $= \frac{eE}{m} = \frac{e\alpha t}{m}$

Let velocity component normal to plates to v_y

Then this acceleration is equal to $\frac{d}{dt}v_y$

$$\frac{d}{dt}v_y = \frac{e\alpha t}{m} \quad \text{or} \quad dv_y = \frac{e\alpha t}{m}dt$$

But at initial moment $t = 0$, $v_y = 0$ $\Rightarrow$ $\int_0^{v_y} dv_y = \frac{e\alpha}{m}\int_0^{t_0} t\,dt$

$$v_y = \frac{e\alpha}{2m}t_0^2 = \frac{\alpha l^2}{4V} \qquad \text{(ii)}$$

Angular deviation θ of electron from its initial direction of motion is shown in Fig. 9.199

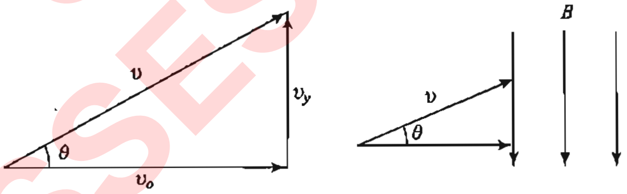


Fig. 9.199

Now electron enters into magnetic field. Pitch of its helical path.

$$P = \frac{2\pi m}{eB}v\cos(90 - \theta)$$

$$P = \frac{2\pi m}{eB}v\sin\theta = \frac{2\pi m}{eB}v_y$$

$$p = \frac{\pi m\alpha l^2}{2eBV} = 1.215\text{ cm}$$

EXERCISES

Subjective Type

Solutions on page 9.106

1. A small current carrying loop having current i_0 is placed in the plane of paper as shown (Fig. 9.200). Another semicircular loop having current i_0 is placed concentrically in the same plane as that of the small loop, the radius of semicircular loop being R ($R >> a$). Find the force applied (in newton) by the smaller ring on the bigger ring.

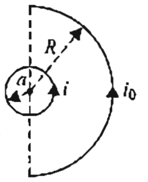


Fig. 9.200

(Given $R = 1$ m, $i = i_0 = \frac{40}{\sqrt{\mu_0}}$ A, $a = 0.1$ m)

2. A non-conducting non-magnetic rod having circular cross section of radius R is suspended from a rigid support as shown in Fig. 9.201. A light and small coil of 300 turns is wrapped tightly at the left end of the rod where uniform magnetic field B exists in vertically downward direction. Air of density ρ hits the half of the right part of the rod with velocity V as shown in the Fig. 9.201. What should be current in clockwise direction (as seen from O) in the coil so that rod remains horizontal? Give answer in mA, given

$$\frac{2}{Lv}\sqrt{\frac{\pi RB}{\rho}} = 1\text{ A}^{-1/2}$$

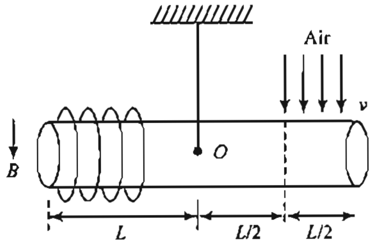


Fig. 9.201

3. A current I flows in a rectangularly shaped wire whose center lies at $(x_0, 0, 0)$ and whose vertices are located at the points A $(x_0 + d, -a, -b)$, B $(x_0 - d, a, -b)$, C $(x_0 - d, a, +b)$ and D $(x_0 + d, -a, +b)$ respectively. Assume that $a, b, d << x_0$. Find the magnitude of magnetic dipole moment vector of the rectangular wire frame in tesla. (Given: $b = 10$ m, $i = 0.01$ A, $d = 4$ m, $a = 3$ m.)

4. A small coil C with $N = 200$ turns is mounted on one end of a balance beam and introduced between the poles of an electromagnet as shown in the figure. The area of the coil is $S = 1$ cm^2, the length of the right arm of the balance beam is $l = 30$ cm. When there is no current in the coil the balance is in equilibrium. On passing a current $I = 22$ mA through the coil, equilibrium is restored by putting an additional weight of mass $m = 60$ mg on the balance pan. Find the magnetic induction field between the poles of the electromagnet, assuming it to be uniform.

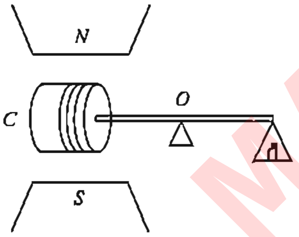


Fig. 9.202

5. A long, horizontal wire AB rests on the surface of a table and carries a current I. Horizontal wire CD is vertically above wire AB, and is free to slide up and down on the two vertical metal guides C and D (as shown in Fig. 9.203). Wire CD is connected through the sliding contacts to another wire that also carries a current I, opposite in direction to the current in wire AB. The mass per unit length of the wire CD is λ. To what equilibrium height h will the wire CD rise, assuming that magnetic force on it is due wholly to the current in wire AB ?

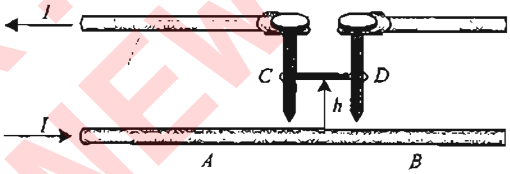


Fig. 9.203

6. An elevator carrying a charge of 0.2 C is moving down with a velocity of 4×10^3 m/s. The elevator is 10 m from the bottom and 3 m horizontally from P as shown. What magnetic field (in μT) does it produce at point P.

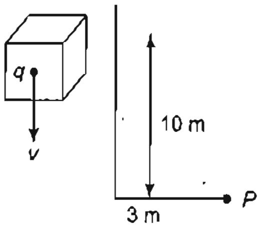


Fig. 9.204

7. In a trapeze-shaped structure, two rigid wires of negligible mass support a conducting bar of mass m and length L as shown in Fig. 9.205. A source of e.m.f. is applied to the wires so that a current I flows through the bar. A uniform magnetic field $\vec{B}$ is perpendicular to the plane of the wires and bar.

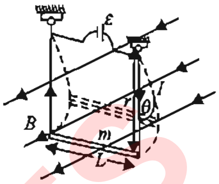


Fig. 9.205

a. Compute the current that the source of e.m.f. must provide so that there is no tension in the wires.

b. If the current is reduced to half the value computed in (a) and the plane of the structure is moved through an angle θ, compute the tension in the wires and the magnitude of the net unbalanced force on the bar at the instant it is released from this angle.

8. A strong magnet is placed under a horizontal conducting ring of radius r that carries current i as shown in Fig. 9.206. If the magnetic field makes an angle θ with the vertical at the ring's location, what are the magnitude and direction of the resultant force on the ring?

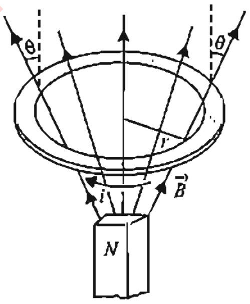


Fig. 9.206

9. A wheel with 4 spokes is placed with its plane perpendicular to a uniform magnetic field B of magnitude 0.5 T. The field is directed into the plane of the paper and is present over the entire region of the wheel as shown in Fig. 9.207. When the switch S is closed, there is initial current of 6 A between the axis and the rim and the wheel begins to rotate. Resistances of the spokes are 1, 2, 4 and 8 Ω, respectively. Resistance of rim is negligible.

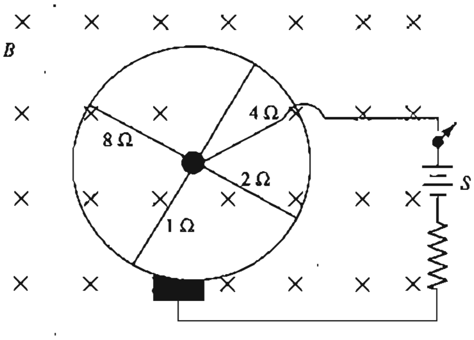


Fig. 9.207

a. What is the direction of rotation of the wheel ?

b. Radius of the wheel is 0.2 m. Calculate initial torque acting on the wheel.

10. A current $I = 10$ A flows in a ring of radius $r_0 = 15$ cm made of a very thin wire. The tensile strength of the wire is equal to $T = 1.5$ N. The ring is placed in a magnetic field, which is perpendicular to the plane of the ring so that the forces tend to break the ring. Find B at which the ring is broken.

11. A positively charged particle, having charge q, is accelerated by a potential difference V. This particle moving along the x-axis enters a region where an electric field E exists. The direction of the electric field is along positive y-axis. The electric field exists in the region bounded by the lines $x = 0$ and $x = a$. Beyond the line $x = a$ (i.e., in the region $x > a$), there exists a magnetic field of strength B, directed along the positive y-axis. Find

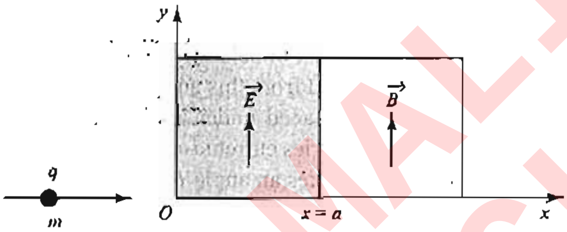


Fig. 9.208

a. at which point does the particle meet the line $x = a$.

b. the pitch of the helix formed after the particle enters the region $x \geq a$.

(Mass of the particle is m.)

12. Two long conducting rods suspended by means of two insulating threads as shown in Fig. 9.209 are connected at one end to a charged capacitor through a switch S, which in initially open. At the other end, they are connected by a loose wire. The capacitor has charge Q and mass per unit length of the rod is λ? The effective resistance of the circuit after closing the switch is R. Find the velocity of each rod when the capacitor is discharged after closing the switch. [Assume that the displacement of rods during the discharging time is negligible.]

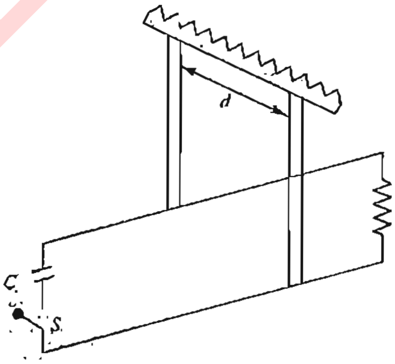


Fig. 9.209

13. A charged particle $+q$ of mass m is placed at a distance from another charged particle $-2q$ of mass $2m$ in a uniform magnetic field B as shown in Fig. 9.210. If the particles are projected towards each other with equal speed v.

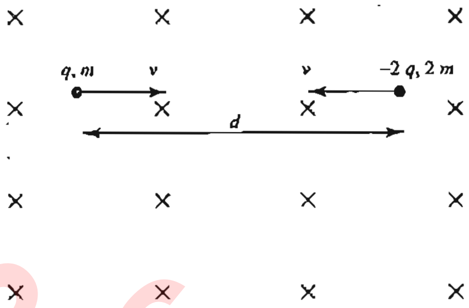


Fig. 9.210

a. find the maximum value of projection speed v_m so that the two particles do not collide.

b. find the time after which collision occurs between the particles if projection speed equals $2v_m$.

c. Assuming the collision to be perfectly inelastic find the radius of the particle in subsequent motion.

(Neglect the electric force between the charges.)

14. A conducting wire of length 'l' is placed on a rough horizontal surface, where a uniform horizontal magnetic field B perpendicular to the length of the wire exists. Least values of the forces required to move the rod when a current 'I' is established in the rod are observed to be F_1 and F_2 ($< F_1$) for the two possible directions of the current through the rod, respectively. Find the weight of the rod and the coefficient of friction between the rod and the surface.

15. A square loop of side $a = 6$ cm carries a current $I = 30$ A Calculate magnetic induction B at point P, lying on the axi of loop and at a distance $x = \sqrt{7}$ cm from the center of loop.

16. Calculate magnetic induction at point O if the wire carrying current I has the shape shown in Fig. 9.211.

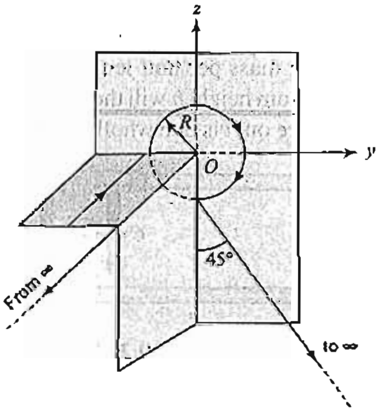


Fig. 9.211

(The radius of the curved part of the wire is equal to R an linear parts of the wire are very long.)

Objective Type

Solutions on page 9.111

1. A neutron, a proton, an electron and an α-particle enter region of uniform magnetic field with equal velocities. Th magnetic field is perpendicular to the paper and directe into it. The tracks of particles are labeled in Fig. 9.212. Th neutron follows the track

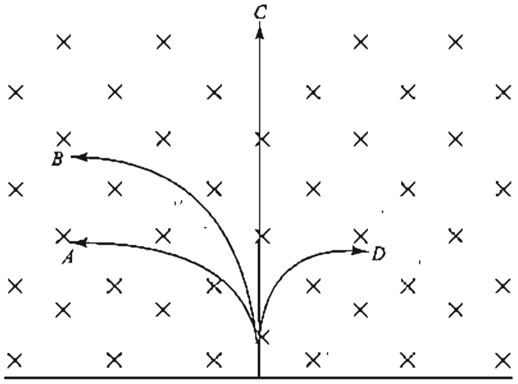


Fig. 9.212

a. A
b. B
c. C
d. D

2. A U tube of uniform square cross-sectional side a has mercury in it. Current I is passed between sealed electrodes x and y. A magnetic field B is applied across horizontal section perpendicular to the plane of diagram. The difference in mercury levels is [Given: r = density of mercury]

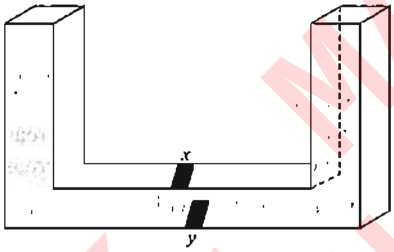


Fig. 9.213

a. $\frac{2}{3}\frac{BI}{\rho g a}$
b. $\frac{2BI}{\rho g a}$
c. $\frac{BI}{\rho g a}$
d. $\frac{4BI}{\rho g a}$

3. An electron accelerated through a potential difference V passes through a uniform transverse magnetic field and experiences a force F. If the accelerating potential is increased to $2V$, the electron in the same magnetic field will experience a force

a. F
b. $F/2$
c. $\sqrt{2}\,F$
d. $2F$

4. An electron is moving along positive x-axis. To get it moving on an anticlockwise circular path in x–y plane, a magnetic field is applied

a. along positive y-axis
b. along positive z-axis
c. along negative y-axis
d. along negative z-axis

5. Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2, respectively. The ratio of masses of X and Y is

a. $(R_1/R_2)^{1/2}$
b. (R_2/R_1)
c. $(R_1/R_2)^2$
d. (R_1/R_2)

6. A stream of electrons is projected horizontally to the right. A straight conductor carrying a current is supported parallel to the electron stream and above it. If the current in the conductor is from left to right, what will be the effect on the electron stream?

a. The electron stream will be pulled upward
b. The electron stream will be pulled downward
c. The electron stream will be retarded
d. The electron stream will be speeded up toward the right

7. An electron of mass m is accelerated through a potential difference of V and then it enters a magnetic field of induction B normal to the lines. Then, the radius of the circular path is

a. $\sqrt{\frac{2eV}{m}}$
b. $\sqrt{\frac{2Vm}{eB^2}}$
c. $\sqrt{\frac{2Vm}{eB}}$
d. $\sqrt{\frac{2Vm}{e^2B}}$

8. An electron of mass 0.90×10^{-30} kg under the action of a magnetic field moves in a circle of 2.0 cm radius at a speed of 3.0×10^6 ms^{-1}. If a proton of mass 1.8×10^{-27} kg was to move in a circle of the same radius in the same magnetic field, then its speed will be

a. 3.0×10^6 ms^{-1}
b. 1.5×10^3 ms^{-1}
c. 6.0×10^4 ms^{-1}
d. cannot be estimated from the same data

9. A charged particle of mass 10^{-3} kg and charge 10^{-5} C enters a magnetic field of induction 1 tesla. If $g = 10$ m s^{-2}, for what value of velocity will it pass straight through the field without deflection?

a. 10^{-3} ms^{-1}
b. 10^3 ms^{-1}
c. 10^6 ms^{-1}
d. 1 ms^{-1}

10. A particle of mass 2×10^{-5} kg moves horizontally between two horizontal plates of a charged parallel plate capacitor b between which there is an electric field of 200 NC^{-1} acting upward. A magnetic induction of 2.0 T is applied at right angles to the electric field in a direction normal to both $\vec{B}$ and $\vec{v}$. If g is 9.8 ms^{-2} and the charge on the particle is 10^{-6} coulomb, then its velocity is

a. 2 ms^{-1}
b. 20 ms^{-1}
c. 0.2 ms^{-1}
d. 100 ms^{-1}

11. A charged particle moves along a circle under the action of possible constant electric and magnetic fields. Which of the following are possible?

a. $E = 0, B = 0$
b. $E = 0, B \neq 0$
c. $E \neq 0, B = 0$
d. $E \neq 0, B \neq 0$

12. The instantaneous acceleration of an electron in a magnetic field $\vec{B} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ is $\vec{a} = x\hat{i} + \hat{j} - \hat{k}$. What is the value of x?

a. 0.5
b. 1
c. 2.5
d. 1.5

13. Two particles Y and Z emitted by a radioactive source at P made tracks in a could chamber as illustrated in the Fig. 9.189. A magnetic field acted downward into the paper. Careful measurements showed that both tracks were circular, the radius of Y track being half that of the Z track. Which one of the following statements is certainly true?

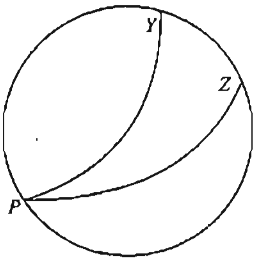


Fig. 9.214

a. Both particles Y and Z carried a positive charge

b. The mass of particle Z was one-half that of particle Y

c. The mass of particle Z was twice that of particle Y

d. The charge of particle Z was twice that of particle Y

14. A potential difference of 600 V is applied across the plates of a parallel plate capacitor, plates being separated by 3 mm. An electron projected vertically parallel to the plates with a velocity of 2×10^6 ms^{-1} moves undeflected between the plated. The magnitude of the magnetic field in the region between the condenser plates is

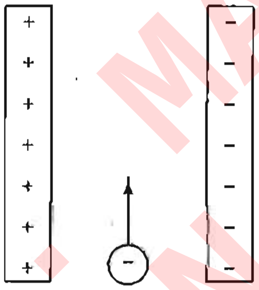


Fig. 9.215

a. 0.1 T | b. 0.2 T

c. 2×10^6 T | d. 600 T

15. A proton and an α-particle enter a uniform magnetic field perpendicular with the same speed. If the proton takes 25 µs to make 5 revolutions, then the periodic time for the α-particle would be

a. 50 µs | b. 25 µs

c. 10 µs | d. 5 µs

16. A charge q coulomb moves in a circle at n revolutions per second and the radius of the circle is r meter. Then magnetic field at the center of the circle is

a. $\frac{2\pi q}{nr} \times 10^{-7}$ N A^{-1} m^{-1} b. $\frac{2\pi q}{r} \times 10^{-7}$ N A^{-1} m^{-1}

c. $\frac{2\pi nq}{r} \times 10^{-7}$ N A^{-1} m^{-1} d. $\frac{2\pi q}{r} \times$ N A^{-1} m^{-1}

17. An electron is launched with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$. The angle θ between $\vec{v}$ and $\vec{B}$ lies between 0 and $\frac{\pi}{2}$. Its velocity vector $\vec{v}$ returns to its initial value in a time interval of

a. $\frac{2\pi m}{eB}$ b. $\frac{2 \times 2\pi m}{eB}$

c. $\frac{\pi m}{eB}$ d. depends upon angle between $\vec{v}$ and $\vec{B}$

18. Two charged particles having charges q_1 and q_2 and masses m_1 and m_2 are projected with same velocity in a region of uniform magnetic field. They follow the trajectory as shown in Fig. 9.191:

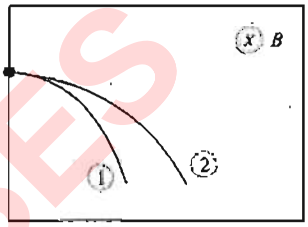


Fig. 9.216

From this, we can conclude that

a. $q_1 > q_2$ b. $q_1 < q_2$

c. $m_1 < m_2$ d. none of these

19. A charged particle is whirled in a horizontal circle on a frictionless table by attaching it to a string fixed at one end. If a magnetic field is switched on in the vertical direction, the tension in the string

a. will increase b. will decrease

c. remains same d. may increase or decrease

20. A charged particle moves with velocity $\vec{v} = a\hat{i} + d\hat{j}$ in a magnetic field $\vec{B} = A\hat{i} + D\hat{j}$. The force acting on the particle has magnitude F. Then,

a. $F = 0$, if $aD = dA$.

b. $F = 0$, if $aD = -dA$.

c. $F = 0$, if $aA = -dD$.

d. $F \mu (a^2 + b^2)^{1/2} \times (A^2 + D^2)^{1/2}$

21. An electron is ejected from the surface of a long, thick straight conductor carrying a current, initially in a direction perpendicular to the conductor. The electron will

a. ultimately return to the conductor

b. move in a circular path around the conductor

c. gradually move away from the conductor along a spiral

d. move in a helical path, with the conductor as the axis

22. A particle with a specific charge s is fired with a speed v toward a wall at a distance d, perpendicular to the wall. What minimum magnetic field must exist in this region for the particle not to hit the wall?

a. v/sd b. $2v/sd$

c. $v/2sd$ d. $v/4sd$

23. A particle with charge Q, moving with a momentum p, enters a uniform magnetic field normally. The magnetic field has magnitude B and is confined to a region of width d, where $d < \frac{p}{BQ}$. The particle is deflected by an angle θ in crossing the field. Then,

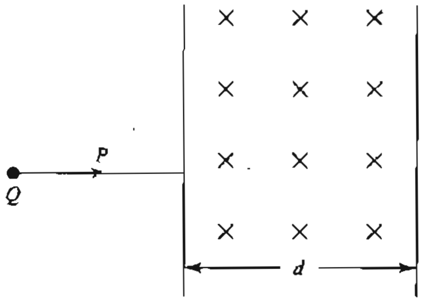


Fig. 9.217

a. $\sin\theta = \dfrac{BQd}{p}$ b. $\sin\theta = \dfrac{p}{BQd}$

c. $\sin\theta = \dfrac{Bp}{Qd}$ d. $\sin\theta = \dfrac{pd}{BQ}$

24. Electrons moving with different speeds enter a uniform magnetic field in a direction perpendicular to the field. They will move along circular paths
- a. of the same radius
- b. with larger radii for the faster electrons
- c. with smaller radii for the faster electrons
- d. either (b) or (c) depending on the magnitude of the magnetic field

25. In the previous question, time periods of rotation will be
- a. the same for all the electrons
- b. greater for the faster electrons
- c. smaller for the faster electrons
- d. either (b) or (c) depending on the magnitude of the magnetic field

26. A charged particle begins to move from the origin in a region which has a uniform magnetic field in the x-direction and a uniform electric field in the y-direction. Its speed is v when it reaches the point (x, y, z). Then, v will depend

a. only on x b. only on y

c. on both x and y, but not z d. on x, y and z

27. Two metal strips of length ℓ each are placed parallel to each other in contact with an electric circuit as shown in Fig. 9.218. The entire structure is placed on a smooth horizontal floor in a region in which uniform magnetic field is existing in vertical direction.

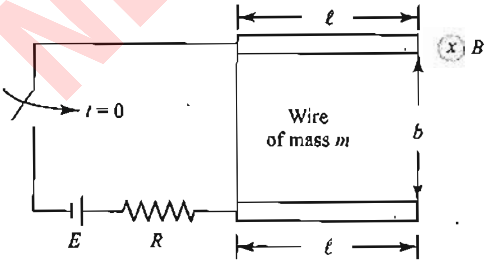


Fig. 9.218

The distance travelled by the wire as a function of time is best given by

a. $\dfrac{1}{2}\times\dfrac{EBb}{Rm}\times t^2$ for $t < t_0$ and

$\ell + \dfrac{EBb}{Rm}(t - t_0)$ for $t \geq t_0$

b. $\dfrac{1}{2}\times\dfrac{EBb}{Rm}\times t^2$ for all t

c. $\dfrac{1}{2}\times\dfrac{EBb}{Rm}\times t^2$ for $t \leq t_0$ and then the wire stops

d. $\dfrac{1}{2}\times\dfrac{EBb}{2m}\times t^2$ for $t \leq t_0$ and $\ell + \dfrac{Ebb}{Rm}\times t$ for $t > t_0$

where $t_0 = \sqrt{\dfrac{2\ell mR}{EBb}}$

28. A particle of charge q and mass m is projected with a velocity v_0 towards a circular region having uniform magnetic field B perpendicular and into the plane of paper from point P as shown in Fig. 9.219 R is the radius and O is the centre of the circular region. If the line OP makes an angle θ with the direction of v_0 then the value of v_0 so that particle passes through O is

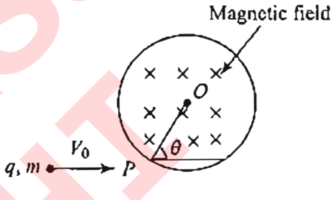


Fig. 9.219

a. $\dfrac{qBR}{m\sin\theta}$ b. $\dfrac{qBR}{2m\sin\theta}$

c. $\dfrac{2qBR}{m\sin\theta}$ d. $\dfrac{3qBR}{2m\sin\theta}$

29. A particle of charge -1.6×10^{-18} C moving with velocity 10 ms^{-1} along the x-axis enters a region where a magnetic field of induction B is along the y-axis, and an electric field of magnitude 10 Vm^{-1} is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is

a. 10^{-3} Wbm^{-2} b. 10^{3} Wbm^{-2}

c. 10^{2} Wbm^{-2} d. 10^{16} Wbm^{-2}

30. An electron moving with a speed u along the positive x-axis at $y = 0$ enters a region of uniform magnetic field which exists to the right of y-axis. The electron exits from the region after some time with the speed v at coordinate y, then

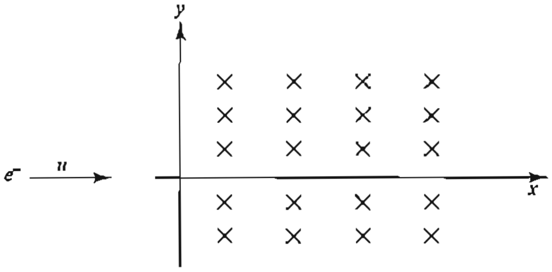


Fig. 9.220

a. $v > u, y < 0$ b. $v = u, y > 0$
c. $v > u, y > 0$ d. $v = u, y < 0$

31. An electron is accelerated from rest through a potential difference V. This electron experiences a force F in a uniform magnetic field. On increasing the potential difference to V', the force experienced by the electron in the same magnetic field becomes $2F$. Then, the ratio (V'/V) is equal to

a. $\frac{4}{1}$ b. $\frac{2}{1}$
c. $\frac{1}{2}$ d. $\frac{1}{4}$

32. A particle of charge q and mass m starts moving from the origin under the action of an electric field $\vec{E} = E_0\hat{i}$ and $\vec{B} = B_0\hat{i}$ with a velocity $\vec{v} = v_0\hat{j}$. The speed of the particle will become $2v_0$ after a time

a. $t = \frac{2mv_0}{qE}$ b. $t = \frac{2Bq}{mv_0}$
c. $t = \frac{\sqrt{3}Bq}{mv_0}$ d. $t = \frac{\sqrt{3}mv_0}{qE}$

33. A uniform magnetic field $\vec{B} = B_0\hat{j}$ exists in space. A particle of mass m and charge q is projected toward negative x-axis with speed v from a point $(d, 0, 0)$. The maximum value of v for which the particle does not hit the y–z plane is

a. $\frac{2Bq}{dm}$ b. $\frac{Bqd}{m}$
c. $\frac{Bq}{2dm}$ d. $\frac{Bqd}{2m}$

34. A conducting rod of mass and length ℓ is placed over a smooth horizontal surface. A uniform magnetic field B is acting perpendicular to the rod. Charge q is suddenly passed through the rod and it acquires an initial velocity v on the surface, then q is equal to

a. $\frac{2mv}{B\ell}$ b. $\frac{B\ell}{2mv}$
c. $\frac{mv}{B\ell}$ d. $\frac{B\ell v}{2m}$

35. A charged particle enters a magnetic field at right angles to the magnetic field. The field exists for a length of 1.5 times the radius of the circular path of the plane. The particle will be deviated from its path by

a. 90° b. $\sin^{-1}(2/3)$
c. 30° d. 180°

36. A particle of charge per unit mass α is released from origin with velocity $\vec{v} = v_0\hat{i}$ in a magnetic field

$$\vec{B} = -B_0\hat{k} \quad \text{for } x \le \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha}$$

$$\text{and } \vec{B} = 0 \quad \text{for } x > \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha}$$

The x-coordinate of the particle at time $t\left(> \frac{\pi}{3B_0\alpha}\right)$ would be

a. $\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + \frac{\sqrt{3}}{2}v_0\left(t - \frac{\pi}{B_0\alpha}\right)$

b. $\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + v_0\left(t - \frac{\pi}{3B_0\alpha}\right)$

c. $\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + \frac{v_0}{2}\left(t - \frac{\pi}{3B_0\alpha}\right)$

d. $\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + \frac{v_0 t}{2}$

37. A particle of charge per unit mass α is released from origin with a velocity $v = \vec{v}_0\hat{i}$ in a uniform magnetic field $\vec{B} = -B_0\hat{k}$. If the particle passes through $(0, y, 0)$, then y is equal to

a. $-\frac{2v_0}{B_0\alpha}$ b. $\frac{v_0}{B_0\alpha}$
c. $\frac{2v_0}{B_0\alpha}$ d. $-\frac{v_0}{B_0\alpha}$

38. Same current $i = 2$ A is flowing in a wire frame as shown in Fig. 9.221. The frame is a combination of two equilateral triangles ACD and CDE of side 1 m. It is placed in uniform magnetic field $B = 4$ T acting perpendicular to the plane of frame.

The magnitude of magnetic force acting on the frame is

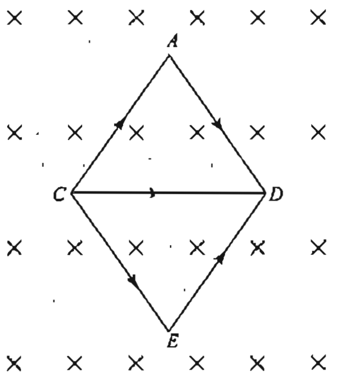


Fig. 9.221

a. 24 N b. zero
c. 16 N d. 8 N

39. An electron is moving along positive x-axis. A uniform electric field exists toward negative y-axis. What should be the direction of magnetic field of suitable magnitude so that net force on the electron is zero?

a. Positive z-axis b. Negative z-axis
c. Positive y-axis d. Negative y-axis

40. A charged particle enters a uniform magnetic field with velocity vector at an angle of 45° with the magnetic field. The pitch of the helical path followed by the particle is p. The radius of the helix will be

a. $\frac{p}{\sqrt{2}\pi}$ b. $\sqrt{2}p$

c. $\frac{p}{2\pi}$ d. $\frac{\sqrt{2}p}{\pi}$

41. A charged particle of specific charge (charge/mass) α is released from origin at time $t = 0$ with velocity $\vec{v} = v_0(\hat{i} + \hat{j})$ in uniform magnetic field $\vec{B} = B_0\hat{i}$. Coordinates of the particle at time $t = \frac{\pi}{B_0\alpha}$ are

a. $\left(\frac{v_0}{2B_0\alpha}, \frac{\sqrt{2}v_0}{\alpha B_0}, \frac{-v_0}{B_0\alpha}\right)$ b. $\left(\frac{-v_0}{2B_0\alpha}, 0; 0\right)$

c. $\left(0, \frac{2v_0}{B_0\alpha}, \frac{v_0\pi}{2B_0\alpha}\right)$ d. $\left(\frac{v_0\pi}{B_0\alpha}, 0, \frac{-2v_0}{B_0\alpha}\right)$

42. Two identical particles having the same mass m and charges $+q$ and $-q$ separated by a distance d enter in a uniform magnetic field B directed perpendicular to paper inward with speeds v_1 and v_2 as shown in Fig. 9.222. The particles will not collide if

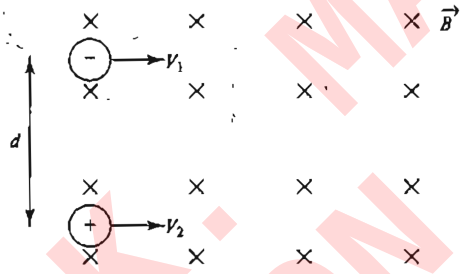


Fig. 9.222

a. $d > \frac{m}{Bq}(v_1 + v_2)$ b. $d < \frac{m}{Bq}(v_1 + v_2)$

c. $d > \frac{2m}{Bq}(v_1 + v_2)$ d. $v_1 = v_2$

43. An infinitely long straight conductor is bent into the shape as shown in Fig. 9.223. It carries a current I ampere and the radius r of the circular loop is r meter. Then, the magnetic induction at the center of the circular loop is

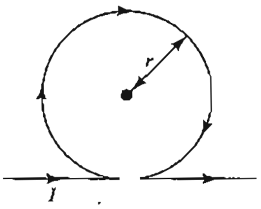


Fig. 9.223

a. zero b. ∞

c. $\frac{\mu_0}{4\pi}\frac{2I}{r}(\pi + 1)$ d. $\frac{\mu_0}{4\pi}\frac{2I}{r}(\pi - 1)$

44. A rectangular current carrying coil is placed in a uniform magnetic field B such that the sides PQ and RS are parallel to B. If the current in the coil is 2 A, then the torque on the coil is

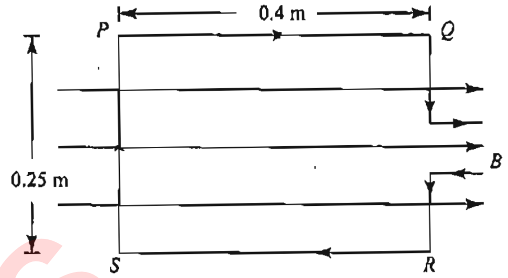


Fig. 9.224

a. 2×10^{-3} N m b. 4×10^{-2} N m

c. 4×10^{-3} N m d. 5×10^{-3} N m

45. Two very thin metallic wires placed along X- and Y-axes carry equal currents as shown in Fig. 9.225. AB and CD are lines at 45° with the axes with origin of axes at O. The magnetic fields will be zero on the line

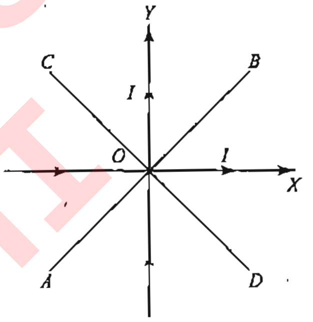


Fig. 9.225

a. AB b. CD

c. segment OB only of line AB

d. segment OC only of line CD.

46. A and B are two concentric circular conductors with center O and carrying currents I_1 and I_2 as shown in Fig. 9.226. The ratio of their radii is 1 : 2 and ratio of their flux densities at O is 1 : 3. The value of $\frac{I_1}{I_2}$ is

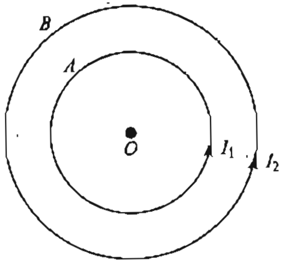


Fig. 9.226

a. $\frac{1}{6}$ b. $\frac{1}{4}$ c. $\frac{1}{2}$ d. $\frac{1}{3}$

47. In Fig. 9.227, two long parallel wires carry equal currents in opposite directions. Point O is situated midway between the wires. The X–Y plane contains the two wires and the positive Z-axis comes normally out of the plane of paper. The magnetic field B at O is non-zero along

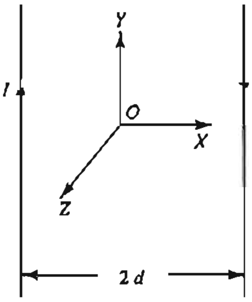


Fig. 9.227

a. X-axes, Y-axes and Z-axes b. X-axis
c. Y-axis d. $-Z$-axis.

48. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper as shown in Fig. 9.228. The variation of magnetic field B along the line XX' is given by

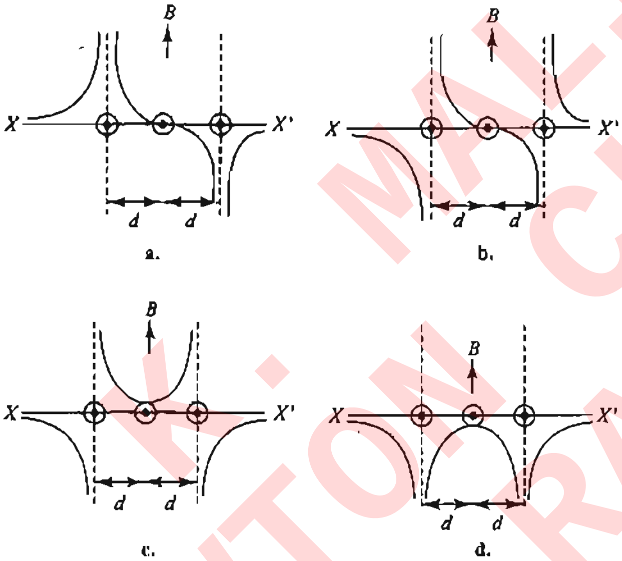


Fig. 9.228

49. A straight section PQ of a circuit lies along the x-axis from $x = -(a/2)$ to $x = +(a/2)$ and carries a steady current I. The magnetic field due to the section PQ at a point $x = +a$ will be

a. proportional to a b. proportional to a^2
c. proportional to $(1/a)$ d. equal to zero

50. A circular current carrying coil has a radius R. The distance from the center of the coil on the axis where the magnetic induction will be $(1/8)^{th}$ of its value at the center of the coil, is

a. $R/\sqrt{3}$ b. $R\sqrt{3}$ c. $2R\sqrt{3}$ d. $(2\sqrt{3})R$

51. A square conducting loop of side length L carries a current I. The magnetic field at the center of the loop is

a. independent of L b. proportional to L_2
c. inversely proportional to L d. linearly proportional to L

52. The plane of a rectangular loop of wire with sides 0.05 and 0.08 m is parallel to a uniform magnetic field of induction 1.5 $\times 10^{-2}$ T. A current of 10.0 ampere flows through the loop. If the side of length 0.08 m is normal and the side of length 0.05 m is parallel to the lines of induction, then the torque acting on the loop is

a. 6000 Nm b. zero
c. 1.2×10^{-2} Nm d. 6×10^{-4} Nm

53. A current of $1/(4\pi)$ ampere is flowing in a long straight conductor. The line integral of magnetic induction around a closed path enclosing the current carrying conductor is

a. 10^{-7} Wb m^{-1}
b. $4\pi \times 10^{-7}$ Wb m^{-1}
c. $16\pi^2 \times 10^{-7}$ Wb m^{-1}
d. zero

54. Two circular coils P and Q are made from similar wires but the radius of Q is twice that of P. What should be the value of potential difference across them so that the magnetic induction at their centers may be the same?

a. $V_q = 2\,V_p$ b. $V_q = 3\,V_p$
c. $V_q = 4\,V_p$ d. $V_q = \frac{1}{4V_p}$

55. A circular loop is kept in that vertical plane which contains the north-south direction. It carries a current that is toward north at the topmost point. Let A be a point on the axis of the circle to the east of it and B a point on this axis to the west of it. The magnetic field due to the loop

a. is toward east at A and toward west at B
b. is toward west at A and toward east at B
c. is toward east at both A and B
d. is toward west at both A and B

56. A current I is uniformly distributed over the cross section of a long hollow cylindrical wire of inner radius R_1 and outer radius R_2. Magnetic field B varies with distance r from the axis of the cylinder as

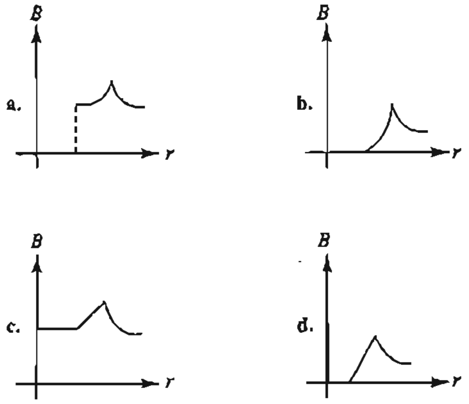


Fig. 9.229

57. Two parallel wires carry currents of 20 and 40 A in opposite directions. Another wire carrying a current antiparallel to 20 A is placed midway between the two wires. The magnetic force on it will be

a. toward 20 A b. toward 40 A c. zero
d. perpendicular to the plane of the currents

58. The resistances of three parts of a circular loop are as shown in Fig. 9.230. The magnetic field at the center O is

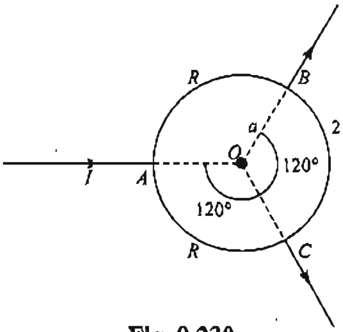


Fig. 9.230

a. $\frac{\mu_0 I}{6a}$ b. $\frac{\mu_0 I}{3a}$ c. $\frac{2}{3}\frac{\mu_0 I}{a}$ d. zero

59. Five very long, straight insulated wires are closely bound together to form a small cable. Currents carried by the wires are: $I_1 = 20$ A, $I_2 = -6$ A, $I_3 = 12$ A, $I_4 = -7$ A, $I_5 = 18$ A. [Negative currents are opposite in direction to the positive.] The magnetic field induction at a distance of 10 cm from the cable is

a. 5 μT b. 15 μT c. 74 μT d. 128 μT

60. The magnetic induction at the center O (Fig. 9.206) is

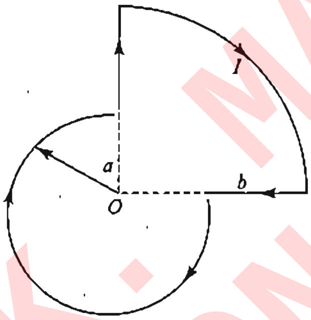


Fig. 9.231

a. $\frac{\mu_0 I}{2a}+\frac{\mu_0 I}{2b}\otimes$ b. $\frac{3\mu_0 I}{8a}+\frac{\mu_0 I}{8b}\otimes$

c. $\frac{3\mu_0 I}{8a}-\frac{\mu_0 I}{8b}\otimes$ d. $\frac{3\mu_0 I}{8a}+\frac{\mu_0 I}{8b}e$

61. The magnetic field at the center O of the arc in Fig. 9.232 is

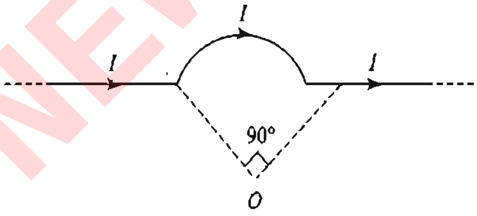


Fig. 9.232

a. $\frac{\mu_0 I}{4\pi\times r}\left[\sqrt{2}+\pi\right]$ b. $\frac{\mu I}{2\pi r}\left[\frac{\pi}{4}+1(\sqrt{2}-1)\right]$

c. $\frac{\mu_0}{4\pi}\times\frac{I}{r}\left[\sqrt{2}-\pi\right]$ c. $\frac{\mu_0}{4\pi}\times\frac{I}{r}\left[\sqrt{2}+\frac{\pi}{4}\right]$

62. In Fig. 9.233, there are two semicircles of radii r_1 and r_2 in which a current I is flowing. The magnetic induction a the center O will be

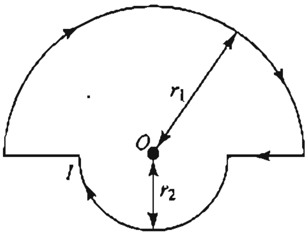


Fig. 9.233

a. $\frac{\mu_0}{r}(r_1+r_2)$ b. $\frac{\mu_0 I}{4}(r_1-r_2)$

c. $\frac{\mu_0 I}{4}\left(\frac{r_1+r_2}{r_1 r_2}\right)$ c. $\frac{\mu_0 I}{4}\left(\frac{r_2-r_1}{r_1 r_2}\right)$

63. Three long, straight and parallel wires are arranged as shown in Fig. 9.234. The force experienced by 10 cm length of wire Q is

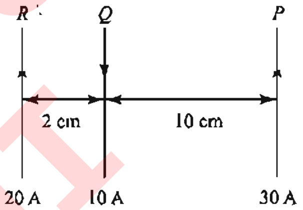


Fig. 9.234

a. 1.4×10^{-4} N toward the right
b. 1.4×10^{-4} N toward the left
c. 2.6×10^{-4} N toward the right
d. 2.6×10^{-4} N toward the left

64. Two long thin wires ABC and DEF are arrange as shown in Fig. 9.235. They carry equal currents I as shown. The magnitude of the magnetic field at O is

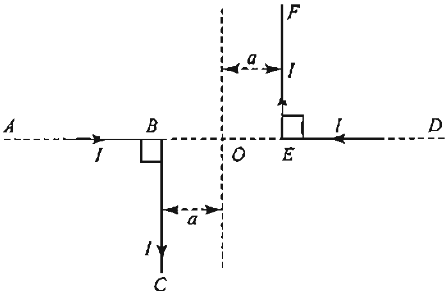


Fig. 9.235

a. zero b. $\mu_0 I/4\pi a$

c. $\mu_0 I/2\pi a$ d. $\mu_0 I/2\sqrt{2}\ \pi a$

65. The magnetic field at O due to current in the infinite wire forming a loop as shown in Fig. 9.236 is

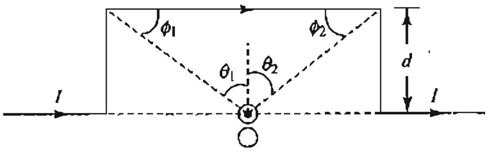


Fig. 9.236

a. $\frac{\mu_0 I}{4\pi d}(\cos\phi_1 + \cos\phi_2)$ b. $\frac{\mu_0}{4\pi}\frac{2I}{d}(\tan\theta_1 + \tan\theta_2')$

c. $\frac{\mu_0}{4\pi}\frac{I}{d}(\sin\phi_1 + \sin\phi_2)$ d. $\frac{\mu_0}{4\pi}\frac{I}{d}(\cos\theta_1 + \sin\theta_2)$

66. A current I flows a thin wire shaped as regular polygon of n sides which can be inscribed in a circle of radius R. The magnetic field induction at the center of polygon due to one side of the polygon is

a. $\frac{\mu_0 I}{\pi R}\left(\tan\frac{\pi}{n}\right)$ b. $\frac{\mu_0 I}{4\pi R}\tan\frac{\pi}{n}$

c. $\frac{\mu_0 I}{2\pi R}\left(\tan\frac{\pi}{n}\right)$ d. $\frac{\mu_0 I}{2\pi R}\left(\cos\frac{\pi}{n}\right)$

67. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii a and b, respectively. When a current i passes through the coil, the magnetic field at the center is

a. $\frac{\mu_0 NI}{b}$ b. $\frac{2\mu_0 NI}{a}$

c. $\frac{\mu_0 NI}{2(a-b)}\ln\frac{b}{a}$ d. $\frac{\mu_0 I^N}{2(b-a)}\ln\frac{b}{a}$

68. A loop of flexible conducting wire of length ℓ lies in magnetic field B which is normal to the plane of loop. A current I is passed through the loop. The tension developed in the wire to open up is

a. $\frac{\pi}{2}BI\ell$ b. $\frac{BI\ell}{2}$

c. $\frac{BI\ell}{2\pi}$ d. $BI\ell$

69. Fig. 9.237 shows three long straight wires P, Q and R normal to the plane of the paper. Wires P and R carry currents directed into the plane of the paper, and wire Q carries a current directed out of the paper. All three currents have the same magnitude.

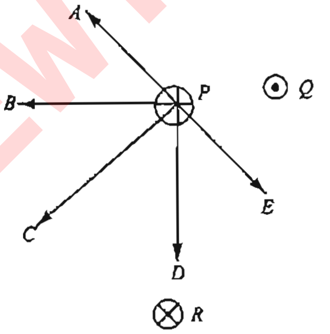


Fig. 9.237

Which arrow best shows the direction of the resultant force on wire P ?

a. A b. B

c. C d. D

70. A wire is bent in the form of a circular arc with a straight portion AB. Magnetic induction at O when current I is flowing in the wire, is

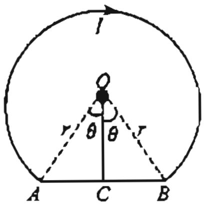


Fig. 9.238

a. $\frac{\mu_0}{2r}(\pi - \theta + \tan\theta)$ b. $\frac{\mu_0 I}{2\pi r}(\pi + \theta - \tan\theta)$

c. $\frac{\mu_0 I}{2\pi r}(\pi - \theta + \tan\theta)$ d. $\frac{\mu_0 I}{2\pi r}(-\tan\theta + \pi - \theta)$

71. The field normal to the plane of a wire of n turns and radius r which carries a current I is measured on the axis of the coil at a small distance h from the center of the coil. This is smaller than the field at the center by the fraction

a. $\frac{3}{2}\frac{h^2}{r^2}$ b. $\frac{2}{3}\frac{h^2}{r^2}$ c. $\frac{3}{2}\frac{r^2}{h^2}$ d. $\frac{2}{3}\frac{r^2}{h^2}$

72. Two identical wires A and B have the same length ℓ and carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side a. If B_1 and B_2 are the values of magnetic induction at the center of the circle and the centre of the square, respectively, then the ratio B_1/B_2 is

a. $(\pi^2/8)$ b. $\left(\pi^2/8\sqrt{2}\right)$

c. $\left(\pi^2/16\right)$ d. $\left(\pi^2/16\sqrt{2}\right)$

73. Two circular coils X and Y, having equal number of turns and carrying equal currents in the same sense, subtend same solid angle at point O. If the smaller coil X is midway between O and Y and if we represent the magnetic induction due to bigger coil Y at O as B_Y and that due to smaller coil X at O as B_X, then

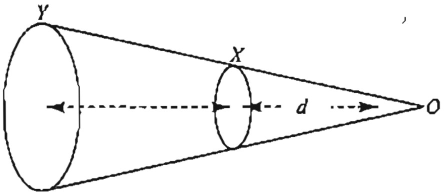


Fig. 9.239

a. $\frac{B_Y}{B_X} = 1$ b. $\frac{B_Y}{B_X} = 2$ c. $\frac{B_Y}{B_X} = \frac{1}{2}$ d. $\frac{B_Y}{B_X} = \frac{1}{4}$

74. A conducting rod of length ℓ and mass m is moving down a smooth inclined plane of inclination θ with constant velocity v. A current i is flowing in the conductor in a direction perpendicular to paper inward. A vertically upward magnetic field $\vec{B}$ exists in space. Then, magnitude of magnetic field $\vec{B}$ is

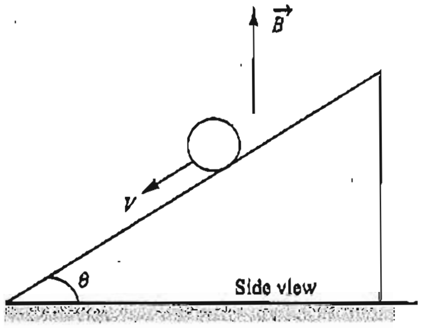


Fig. 9.240

a. $\frac{mg}{i\ell}\sin\theta$ b. $\frac{mg}{i\ell}\tan\theta$

c. $\frac{mg\cos\theta}{i\ell}$ d. $\frac{mg}{i\ell\sin\theta}$

75. A given length of wire can be bent to form a circle or a square of single turn and a current may be established in it. The ratio of magnetic induction at the center of the square to the magnetic induction at the center of circle is

a. $\frac{\pi}{2\sqrt{2}}$ b. $\frac{4\sqrt{2}}{\pi^2}$ c. $\frac{\pi^2}{8\sqrt{2}}$ d. 1 : 1

76. Four infinite thin current carrying sheets are placed in *YZ* plane. The 2*D* view of the arrangement is as shown in Fig. 9.241. Direction of current has also been shown in the figure. The linear current density, i.e., current per unit width in the four sheets are *I*, 2*I*, 3*I* and 4*I*, respectively.

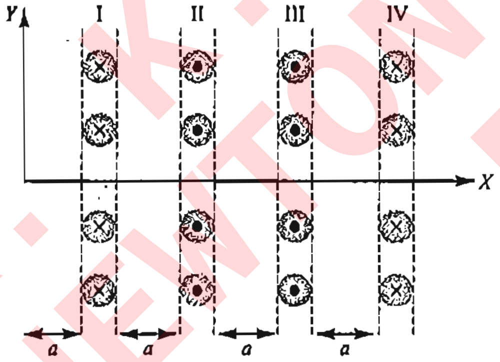
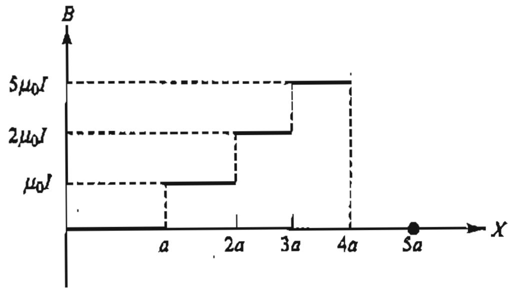
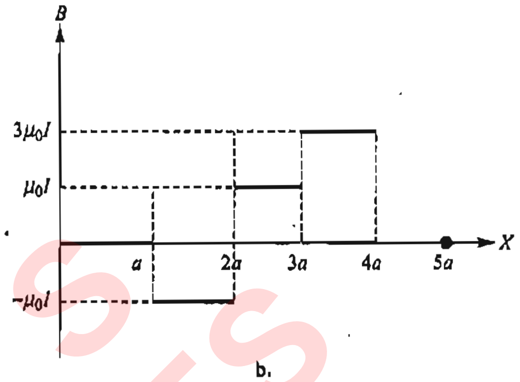


Fig. 9.241

The magnetic field as a function of *x* is best represented by

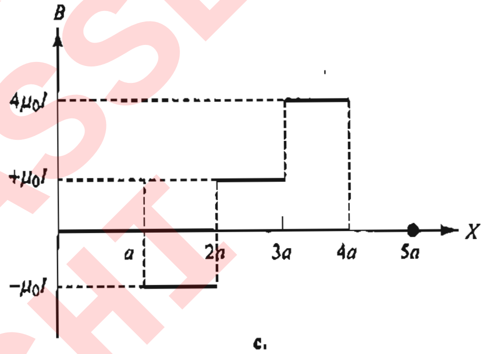


a.

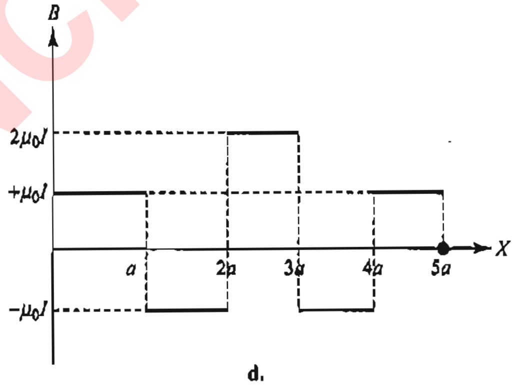


b.

Fig. 9.242 (*Contd.*)



c.



d.

Fig. 9.242

77. A square loop of wire carrying current *I* is lying in the plane of paper as shown in Fig. 9.243. The magnetic field is present in the region as shown.

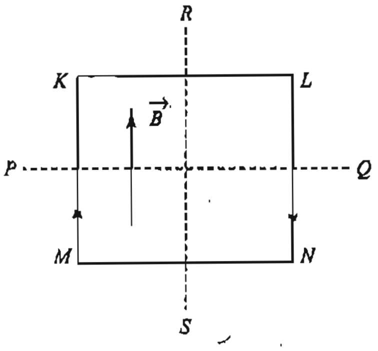


Fig. 9.243

The loop will tend to rotate

a. about PQ with KL coming out of the page
b. about PQ with KL going into the page
c. about RS with MK coming out of the page
d. about RS with MK going into the page

78. An electric current i enters and leaves a uniform circular wire of radius a through diametrically opposite points. A charged particle q moving along the axis of the circular wire passes through its center at speed v. The magnetic force acting on the particle when it passes through the center has a magnitude

a. $qv \times \frac{\mu_0 i}{2a}$ b. $qv \times \frac{\mu_0 i}{2\pi a}$ c. $qv \times \frac{\mu_0 i}{a}$ d. zero

79. A conducting loop is placed in a magnetic field (uniform) as shown in Fig. 9.219.

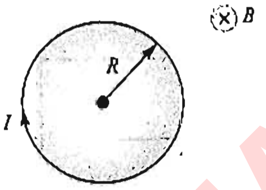


Fig. 9.244

For this situation, markout the correct statement.

a. The force of compression experienced by loop is IRB
b. The force of compression experienced by loop is $2IRB$
c. The force of expansion experienced by loop is IRB
d. The force of expansion experienced by loop is $2IRB$

80. A steady current is flowing in a circular coil of radius R, made up of a thin conducting wire. The magnetic field at the center of the loop is B_L. Now, a circular loop of radius R/n is made from the same wire without changing its length, by unfolding and refolding the loop, and the same current is passed through it. If new magnetic field at the center of the coil is B_C, then the ratio B_L/B_C is

a. $1 : n^2$ b. $n^{1/2}$
c. $n : 1$ d. none of these

81. The magnetic field at the center of the circular loop as shown in Fig. 9.245, when a single wire is bent to form a circular loop and also extends to form straight sections, is

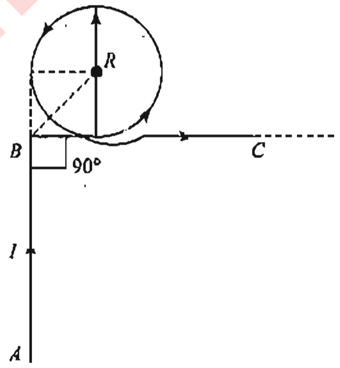


Fig. 9.245

a. $\frac{\mu_0 I}{2R} \odot$ b. $\frac{\mu_0 I}{2R}\left(1+\frac{1}{\pi\sqrt{2}}\right) \odot$

c. $\frac{\mu_0 I}{2R}\left(1-\frac{1}{\pi\sqrt{2}}\right) \otimes$ d. $\frac{\mu_0 I}{R}\left(1-\frac{1}{\pi\sqrt{2}}\right) \otimes$

82. Two parallel wires carrying equal currents in opposite directions are placed at $x = \pm a$ parallel to y-axis with $z = 0$. Magnetic field at origin O is B_1 and at P $(2a, 0, 0)$ is B_2. Then, the ratio B_1/B_2 is

a. -3 b. $-\frac{1}{2}$
c. $-\frac{1}{3}$ d. 2

83. Fig. 9.246 shows three cases: in all cases the circular part has radius r and straight ones are infinitely long. For the same current the field B at center P in the three cases $(B_1 : B_2 : B_3)$ is

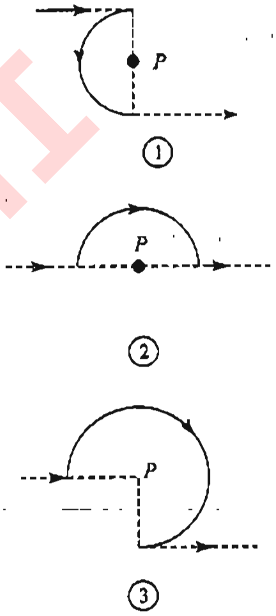


Fig. 9.246

a. $\left(-\frac{\pi}{2}\right):\left(\frac{\pi}{2}\right):\left(\frac{3\pi}{4}-\frac{1}{2}\right)$

b. $\left(-\frac{\pi}{2}+1\right):\left(\frac{\pi}{2}+1\right):\left(\frac{3\pi}{4}+\frac{1}{2}\right)$

c. $\left(-\frac{\pi}{2}\right):\left(\frac{\pi}{2}\right):\left(\frac{3\pi}{4}\right)$

d. $\left(-\frac{\pi}{2}-1\right):\left(\frac{\pi}{2}-\frac{1}{4}\right):\left(\frac{3\pi}{4}+\frac{1}{2}\right)$

84. An otherwise infinite, straight wire has two concentric loops of radii a and b carrying equal currents in opposite directions as shown in Fig. 9.247. The magnetic field at the common center is zero for

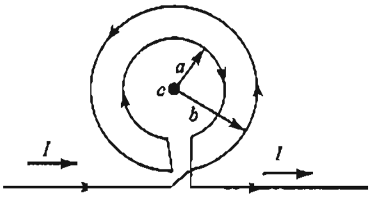


Fig. 9.247

a. $\frac{a}{b}=\frac{\pi-1}{\pi}$
b. $\frac{a}{b}=\frac{\pi}{\pi+1}$
c. $\frac{a}{b}=\frac{\pi-1}{\pi+1}$
d. $\frac{a}{b}=\frac{\pi+1}{\pi-1}$

85. Currents I_1 and I_2 flow in the wires shown in Fig. 9.248. The field is zero at distance x to the right of O. Then

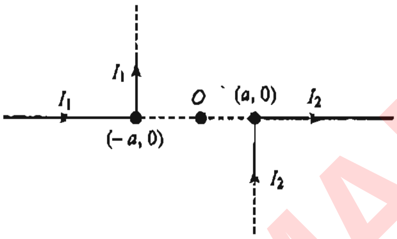


Fig. 9.248

a. $x=\left(\frac{I_1}{I_2}\right)a$
b. $x=\left(\frac{I_2}{I_1}\right)a$
c. $x=\left(\frac{I_1-I_2}{I_1+I_2}\right)a$
d. $x=\left(\frac{I_1+I_2}{I_1-I_2}\right)a$

86. For $c=2a$ and $a<b<c$, the magnetic field at point P will be zero when

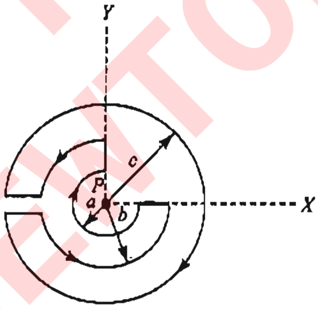


Fig. 9.249

a. $a=b$
b. $a=\frac{3}{5}b$
c. $a=\frac{5}{3}b$
d. $a=\frac{1}{3}b$

87. An infinitely long conductor PQR is bent to form a right angle as shown in Fig. 9.250. A current I flows through PQR. The magnetic field due to this current carrying conductor at the point M is B_1. Now, another infinitely long straight conductor QS, is connected at Q so that the current is $\frac{1}{2}I$ in QR as well as in QS, the current in PQ remaining unchanged. The magnetic field at M is now B_2. The ratio $\frac{B_1}{B_2}$ is given by

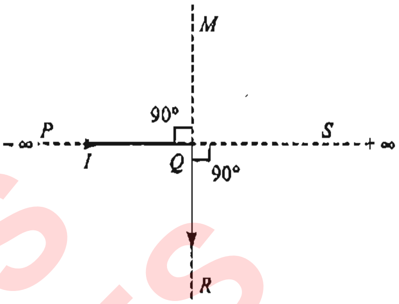


Fig. 9.250

a. 1/2
b. 1
c. 2/3
d. 2

88. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the center is 54 mT. Its value at the center of the loop will be

a. 250 μT
b. 150 μT
c. 125 μT
d. 75 μT

89. Two concentric coils, each of radius equal to 2p cm, are placed at right angles to each other. Currents of 3 and 4 A, respectively, are flowing through the two coils. The magnetic induction, in Wb m^{-2}, at the center of the coils will be [$\mu_0=4\pi\times10^{-7}$ Wb$(Am)^{-1}$]

a. 5×10^{-5}
b. 7×10^{-5}
c. 12×10^{-5}
d. 10^{-5}

90. A wire carrying current I and an other carrying current $2I$ in the same direction produce a magnetic field B at the mid-point. What will be the field when current $2I$ is switched off ?

a. $B/2$
b. $2B$
c. B
d. $4B$

91. A uniform current carrying ring of mass m and radius R is connected by a massless string as shown in Fig. 9.251. A uniform magnetic field B_0 exists in the region to keep the ring in horizontal position, then the current in the ring is (l = length of string)

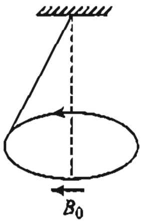


Fig. 9.251

a. $\frac{mg}{\pi RB_0}$
b. $\frac{mg}{RB_0}$
c. $\frac{mg}{3\pi RB_0}$
d. $\frac{mg\,\ell}{\pi R^2 B_0}$

92. A long, straight, hollow conductor (tube) carrying a current has two sections A and C of unequal cross sections joined by a conical section B. 1, 2 and 3 are points on a line parallel to the axis of the conductor. The magnetic fields at 1, 2 and 3 have magnitudes B_1, B_2 and B_3. Then,

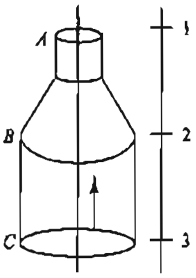


Fig. 9.252

a. $B_1 = B_2 = B_3$
b. $B_1 = B_2 \neq B_3$
c. $B_1 < B_2 < B_3$
d. B_2 cannot be found unless the dimensions of the section B are known

93. A coaxial cable consists of a thin inner conductor fixed along the axis of a hollow outer conductor. The two conductors carry equal currents in opposite directions. Let B_1 and B_2 be the magnetic fields in the region between the conductors and outside the conductor, respectively. Then,

a. $B_1 \neq 0, B_2 \neq 0$
b. $B_1 = B_2 = 0$
c. $B_1 \neq 0, B_2 = 0$
d. $B_1 = 0, B_2 \neq 0$

94. A conductor AB of length L carrying a current I', is placed perpendicular to a long straight conductor XY carrying a current I, as shown in Fig. 9.228. The force on AB has magnitude

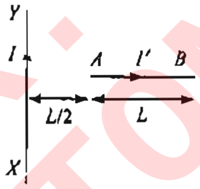


Fig. 9.253

a. $\frac{\mu_0 I'I}{2\pi}\log 2$
b. $\frac{\mu_0 I'I}{2\pi}\log 3$
c. $\frac{3\mu_0 I'I}{2\pi}\log\frac{3}{2}$
d. $\frac{2\mu_0 I'I}{3\pi}$

95. A circular coil having mass m is kept above the ground (x–z plane) at some height. The coil carries a current i in the direction shown in Fig. 9.254. In which direction a uniform magnetic field $\vec{B}$ be applied so that the magnetic force balances the weight of the coil?

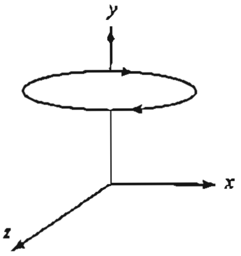


Fig. 9.254

A. Positive x-direction
B. Negative x-direction
c. Positive z-direction
d. None of these

96. The magnetic field at the center of an equilateral triangular loop of side $2L$ and carrying a current i is

a. $\frac{9\mu_0 i}{4\pi L}$
b. $\frac{3\sqrt{3}\mu_0 i}{4\pi L}$
c. $\frac{2\sqrt{3}\mu_0 i}{\pi L}$
d. $\frac{3\mu_0 i}{4\pi L}$

97. Two wires AO and OC carry equal currents i as shown in Fig. 9.255. One end of both the wires extends to infinity. Angle AOC is α.

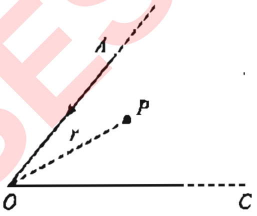


Fig. 9.255

The magnitude of magnetic field at a point P on the bisector of these two wires at a distance r from point O is

a. $\frac{\mu_0}{2\pi}\frac{i}{r}\cot\left(\frac{\alpha}{2}\right)$
b. $\frac{\mu_0}{4\pi}\frac{i}{r}\cot\left(\frac{\alpha}{2}\right)$
c. $\frac{\mu_0}{2\pi}\frac{i}{r}\frac{\left(1+\cos\frac{\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}$
d. $\frac{\mu_0}{4\pi}\frac{i}{r}\left(\frac{\alpha}{2}\right)$

98. Equal currents $i = 1$ A are flowing through the wires parallel to y-axis located at $x = +1$ m, $x = +2$ m, $x = +4$ m, etc. but in opposite directions as shown in Fig. 9.256. The magnetic field (in tesla) at origin would be

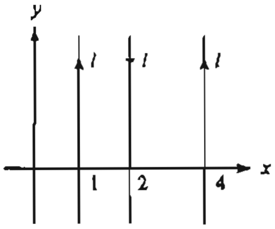


Fig. 9.256

a. $-1.33\times10^{-7}\,\hat{k}$
b. $1.33\times10^{-7}\,\hat{k}$
c. $2.67\times10^{-7}\,\hat{k}$
d. $-2.67\times10^{-7}\,\hat{k}$

99. Same current i is flowing in three infinitely long wires along positive x-, y- and z-directions. The magnetic field at a point $(0, 0, -a)$ would be

a. $\frac{\mu_0 i}{2\pi a}(\hat{j}-\hat{i})$
b. $\frac{\mu_0 i}{2\pi a}(\hat{i}+\hat{j})$
c. $\frac{\mu_0 i}{2\pi a}(\hat{i}-\hat{j})$
d. $\frac{\mu_0 i}{2\pi a}(\hat{i}+\hat{j}+\hat{k})$

100. Two very long straight parallel wires carry steady currents i and $2i$ in opposite directions. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires in the plane of the wires. Its instantaneous velocity $\vec{v}$ is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is

a. $\dfrac{\mu_0 iqv}{2\pi d}$ b. $\dfrac{\mu_0 iqv}{\pi d}$
c. $\dfrac{3\mu_0 iqv}{2\pi d}$ d. zero

101. A metallic wire is folded to form a square loop of side a. It carries a current i and is kept perpendicular to the region of uniform magnetic field B. If the shape of the loop is changed from square to an equilateral triangle without changing the length of the wire and current, the amount of work done in doing so is

a. $Bia^2\left(1-\dfrac{4\sqrt{3}}{9}\right)$ b. $Bia^2\left(1-\dfrac{\sqrt{3}}{9}\right)$
c. $\dfrac{2}{3}Bia^2$ d. zero

102. A charged particle moving along + ve x-direction with a velocity v enters a region where there is a uniform magnetic field $B_0\left(-\hat{k}\right)$, from $x = 0$ to $x = d$. The particle gets deflected at an angle θ from its initial path. The specific charge of the particle is

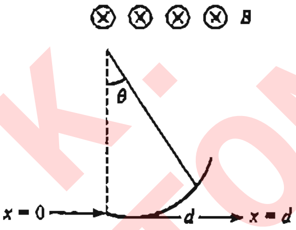


Fig. 9.257

a. $\dfrac{v\cos\theta}{Bd}$ b. $\dfrac{v\tan\theta}{Bd}$
c. $\dfrac{v}{Bd}$ d. $\dfrac{v\sin\theta}{Bd}$

103. Two infinite wires, carrying currents i_1 and i_2, are lying along x- and y-axes, as shown in the x–y plane. Then,

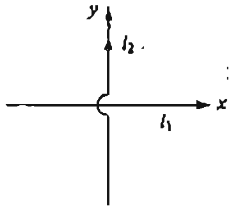


Fig. 9.258

a. locus of points where B is zero is a circle
b. locus of points where B is zero is a line
c. B decays hyperbolically along any line parallel to x-axis
d. B decays hyperbolically along any line parallel to y-axis

104. In Fig. 9.259, a coil of single turn is wound on a sphere of radius r and mass m. The plane of the coil is parallel to the inclined plane and lies in the equatorial plane of the sphere. If the sphere is in rotational equilibrium, the value of B is [Current in the coil is I]

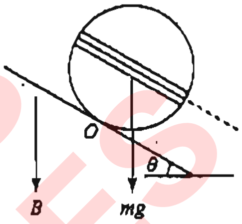


Fig. 9.259

a. $\dfrac{mg}{\pi Ir}$ b. $\dfrac{mg\sin\theta}{\pi I}$
c. $\dfrac{mg\sin\theta}{\pi I}$ d. none of these

105. Three particles, an electron (e), a proton (p) and a helium atom (He) are moving in circular paths with constant speeds in the x–y plane in a region where a uniform magnetic field B exists along z-axis. The times taken by e, p and He inside the field to complete one revolution are t_e, t_p and t_{He} respectively. Then,

a. $t_{He} > t_p = t_e$ b. $t_{He} > t_p > t_e$
c. $t_{He} = t_p = t_e$ d. none of these

106. A long cylindrical wire of radius 'a' carries a current I distributed uniformly over its cross section. If the magnetic fields at distances r and R from the axis have equal magnitude, then

a. $a = \dfrac{R+r}{2}$ b. $a = \sqrt{Rr}$
c. $a = Rr/R + r$ d. $a = R^2/r$

107. A small current carrying ring, having current i_0 and radius R, is kept in x–y plane (the plane of paper) as shown in Fig. 9.260. Another current carrying small ring having radius r ($r << R$) is kept at a distance d from the center of first ring in a plane perpendicular to x–y plane such that the centers of both rings lie on the same line. The torque acting on the second ring due to the magnetic field of first ring is

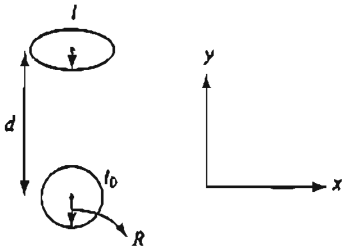


Fig. 9.260

a. $\dfrac{\mu_0}{4\pi}\dfrac{I i_0 R^2 r^2}{d^3}$ b. $\dfrac{\mu_0}{4}\dfrac{\pi i_0 i R^2 r^2}{d^3}$

c. $\frac{\mu_0}{2}\pi i_0 i R^2 r^2 d$ d. $\frac{\mu_0}{4}\frac{i_0 i R^2 r^2}{d^3}$

108. The rigid conducting thin wire frame carries an electric current I and this frame is inside a constant magnetic field $\vec{B}$ as shown in Fig. 9.261. Then,

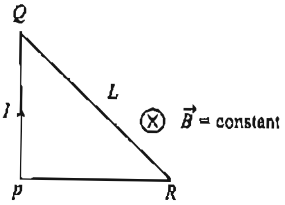


Fig. 9.261

a. the net magnetic force on the frame is zero but the torque is not zero
b. the net magnetic force on the frame and the torque due to magnetic field are both zero
c. the net magnetic force on the frame is not zero and the torque is also not zero
d. none of these

109. An electron moving in a circular orbit of radius R makes n rotations per second. The magnetic field strength at the center has magnitude

a. $\frac{2\mu_0 ne}{R}$ b. $\frac{\mu_0 ne}{2R}$
c. $\frac{\mu_0 ne}{\pi R}$ d. zero

110. Fig. 9.262 shows a conducting loop $ABCDA$ placed in a uniform magnetic field (strength B) perpendicular to its plane. The part ABC is the (3/4)th portion of the square of side length ℓ. The part ADC is a circular arc of radius R. The points A and C are connected to a battery which supplies a current I to the circuit. The magnetic force on the loop due to the field B is

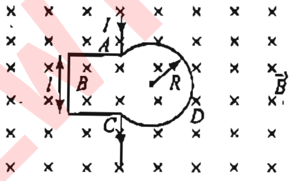


Fig. 9.262

a. zero b. $BI\ell$
c. $2BIR$ d. $\frac{BI\ell R}{I+R}$

111. Two infinite long wires, each carrying current I, are lying along x- and y-axis, respectively. A charged particle, having a charge q and mass m, is projected with a velocity u along the straight line OP. The path of the particle is (neglect gravity)

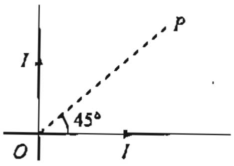


Fig. 9.263

a. straight line b. circle
c. helix d. cycloid

112. A small block of mass m, having charge q, is placed on a frictionless inclined plane making an angle θ with the horizontal. There exists a uniform magnetic field B parallel to the inclined plane but perpendicular to the length of spring. If m is slightly pulled on the incline in downward direction, the time period of oscillation will be (assume that the block does not leave contact with the plane)

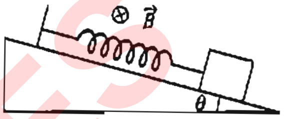


Fig. 9.264

a. $2\pi\sqrt{\frac{m}{K}}$ b. $2\pi\sqrt{\frac{2m}{K}}$
c. $2\pi\sqrt{\frac{qB}{K}}$ d. $2\pi\sqrt{\frac{qB}{2K}}$

113. A uniform conducting rectangular loop of sides ℓ, b and mass m carrying current i is hanging horizontally with the help of two vertical strings. There exists a uniform horizontal magnetic field B which is parallel to the longer side of loop. The value of tension which is least is

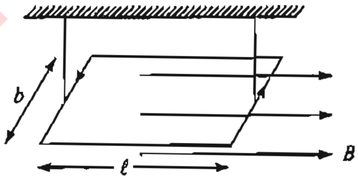


Fig. 9.265

a. $\frac{mg-Bb}{2}$ b. $\frac{mg+Bb}{2}$
c. $\frac{mg-2iBb}{2}$ d. $\frac{mg+2Bb}{2}$

114. In Fig. 9.266, infinite conducting rings each having current i in the direction shown are placed concentrically in the same plane as shown in the figure. The radii of rings are r, $2r$, 2^2r, 2^3r, …,∞. The magnetic field at the center of rings will be

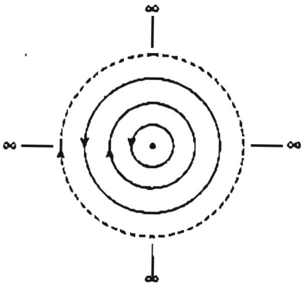


Fig. 9.266

a. zero b. $\frac{\mu_0 i}{r}$
c. $\frac{\mu_0 i}{2r}$ d. $\frac{\mu_0 i}{3r}$

115. A particle of specific charge $\frac{q}{m} = \pi$ Ckg^{-1} is projected from the origin towards positive x-axis with a velocity of 10 ms^{-1} in a uniform magnetic field $\vec{B} = -2\hat{k}$ T. The velocity $\vec{v}$ of particle after time $t = \frac{1}{12}$ s will be (in ms^{-1})

a. $5[\hat{i} + \sqrt{3}\,\hat{j}]$
b. $5[\sqrt{3}\hat{i} + \hat{j}]$
c. $5[\sqrt{3}\hat{i} - \hat{j}]$
d. $5[\hat{i} + \hat{j}]$

116. There is a conducting ring of radius R. Another ring having current I and radius r ($r << R$) is kept on the axis of bigger ring such that its center lies on the axis of bigger ring at a distance x from the center of bigger ring and its plane is perpendicular to that axis. The mutual inductance of the bigger ring due to the smaller ring is

a. $\frac{\mu_0 \pi R^2 r^2}{(R^2 + x^2)^{3/2}}$
b. $\frac{\mu_0 \pi R^2 r^2}{4(R^2 + x^2)^{3/2}}$
c. $\frac{\mu_0 \pi R^2 r^2}{16(R^2 + x^2)^{3/2}}$
d. $\frac{\mu_0 \pi R^2 r^2}{2(R^2 + x^2)^{3/2}}$

117. There is a horizontal cylindrical uniform but time-varying magnetic field increasing at a constant rate $\frac{dB}{dt}$ as shown in Fig. 9.267. A charged particle having charge q and mass m is kept in equilibrium, at the top of a spring of spring constant K, in such a way that it is on the horizontal line passing through the center of the magnetic field as shown in the figure. The compression in the spring will be

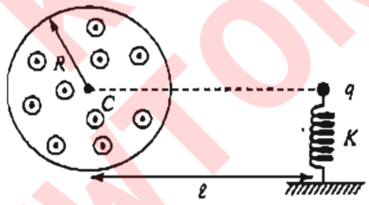


Fig. 9.267

a. $\frac{1}{K}\left[mg - \frac{qR^2}{2\ell}\frac{dB}{dt}\right]$
b. $\frac{1}{K}\left[mg + \frac{qR^2}{\ell}\frac{dB}{dt}\right]$
c. $\frac{1}{K}\left[mg + \frac{2qR^2}{\ell}\frac{dB}{dt}\right]$
d. $\frac{1}{K}\left[mg + \frac{qR^2}{2\ell}\frac{dB}{dt}\right]$

118. A charged particle of mass 2 kg and charge 2 C moves with a velocity $\vec{v} = 8\hat{i} + 6\hat{j}$ ms^{-1} in a magnetic field $\vec{B} = 2\hat{k}$ T. Then

a. the path of particle may be $x^2 + y^2 = 25$
b. the path of particle may be $x^2 + z^2 = 25$
c. the time period of particle will be 3.14 s
d. none of these

119. In Fig. 9.268, there is a uniform conducting structure in which each small square has side a. The structure is kept in a uniform magnetic field B. Then

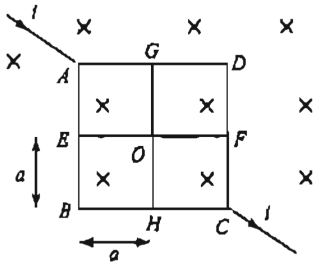


Fig. 9.268

a. the magnetic force on the structure is $2\sqrt{2}\, IBa$
b. the potential of point B = potential of point D
c. potential of point O = potential of point B
d. the magnetic force on the structure is $\sqrt{2}\, IBa$

120. A very long straight conducting wire, lying along the z-axis, carries a current of 2A. The integral $\oint \vec{B} \cdot d\vec{\ell}$ is computed along the straight line PQ, where P has the coordinates (2 cm, 0, 0) and Q has the coordinates (2 cm, 2 cm, 0). The integral has the magnitude (in S.I. units)

a. zero
b. $8\pi \times 10^{-7}$
c. $2\pi \times 10^{-7}$
d. $\pi \times 10^{-7}$

121. A long wire bent as shown in Fig. 9.269(a) carries current I. If the radius of the semicircular portion is a, the magnetic field at the center C is

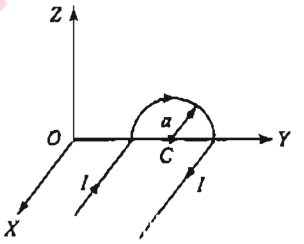


Fig. 9.269(a)

a. $\frac{\mu_0 I}{4a}$
b. $\frac{\mu_0 I}{4\pi a}\sqrt{\pi^2 + 4}$
c. $\frac{\mu_0 I}{4a} + \frac{\mu_0 I}{4\pi a}$
d. $\frac{\mu_0 I}{4\pi a}\sqrt{(\pi^2 - 4)}$

122. A long straight wire and a circular loop carry currents I_1 and I_2, respectively, and are in the same plane. If the magnetic field at the center of the loop is zero, then

a. $I_1 = 2\pi I_2$
b. $I_1 = 2I_2$
c. $I_1 = I_2$
d. $I_2 = 2\pi I_1$

123. A charged particle moves insides a pipe which is bent as shown in Fig. 9.269(b). If $R < \frac{mv}{qB}$, then force exerted by the pipe on charged particle at P is (Neglect gravity)

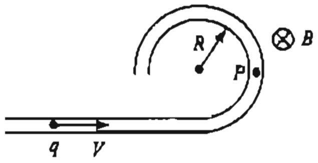


Fig. 9.269(b)

a. Towards center
b. away from center
c. zero
d. none of these

124. An infinitely long current carrying wire carries current i. A charge of mass m and charge q is projected with speed v parallel to the direction of current at a distance r from it. Then, the radius of curvature at the point of projection is

a. $\frac{2rmv}{q\mu_0 i}$
b. $\frac{2\pi rmv}{q\mu_0 i}$
c. r
d. cannot be determined

125. Two positive charges q_1 and q_2 are moving with velocities v_1 and v_2 when they are at points A and B, respectively, as shown in Fig. 9.270. The magnetic force experie-nced by charge q_1 due to the other charge q_2 is

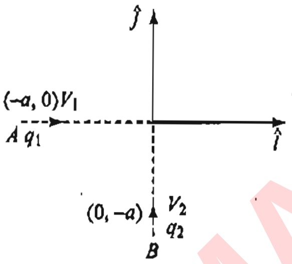


Fig. 9.270

a. $\frac{\mu_0 q_1 q_2 v_1 v_2}{8\sqrt{2}\pi a^2}$
b. $\frac{\mu_0 q_1 q_2 v_1 v_2}{4\sqrt{2}\pi a^2}$
c. $\frac{\mu_0 q_1 q_2 v_1 v_2}{2\sqrt{2}\pi a^2}$
d. $\frac{\mu_0 q_1 q_2 v_1 v_2}{\sqrt{2}\pi a^2}$

126. Consider a hypothetic spherical body. The body is cut into two parts about the diameter. One of hemispherical portion has mass distribution m while the other portion has identical charge distribution q. The body is rotated about the axis with constant speed ω. Then, the ratio of magnetic moment to angular momentum is

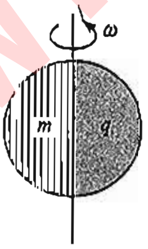


Fig. 9.271

a. $\frac{q}{2m}$
b. $> \frac{q}{2m}$
c. $< \frac{q}{2m}$
d. cannot be calculated

127. A particle of charge per unit mass α is released from origin with velocity $\vec{V} = -V_0\hat{j}$ in a magnetic field

$$\vec{B} = -B_0\hat{k} \text{ for } x \le \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha}$$

and $\vec{B} = 0$ for $x > \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha}$

The x-coordinate of the particle at time $t\left(> \frac{\pi}{3B_0\alpha}\right)$ would be

a. $\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + \frac{\sqrt{3}}{2}v_0\left(t - \frac{\pi}{B_0\alpha}\right)$
b. $\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + v_0\left(t - \frac{\pi}{3B_0\alpha}\right)$
c. $\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + \frac{v_0}{2}\left(t - \frac{\pi}{3B_0\alpha}\right)$
d. $\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + \frac{v_0 t}{2}$

128. The circular coils A and B with their centers lying on the same axis have same number of turns and carry equal currents in the same sense. They are separated by a distance, have different diameters but subtend same angle at a point P lying on their common axis. The coil B lies exactly midway between coil A and the point P. The magnetic fields at point P due to coils A and B are B_1 and B_2, respectively. The ratio B_1/B_2 is

a. $B_1 > B_2$
b. $B_1 < B_2$
c. $B_1 / B_2 = 2$
d. $B_1 / B_2 = 1/2$

129. Consider six wires coming into or out of the page, all with the same current. Rank the line integral of the magnetic field (from most positive to most negative) taken counter-clockwise around each loop shown.

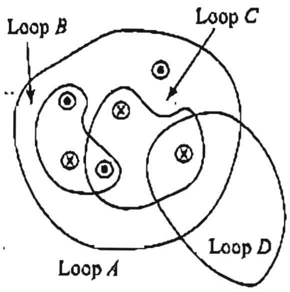


Fig. 9.272

a. $B > C > D > A$
b. $B > C = D > A$
c. $B > A > C = D$
d. $C > B = D > A$

130. A positively charged disk is rotated clockwise as shown in Fig. 9.273. The direction of the magnetic field at point A in the plane of the disk is

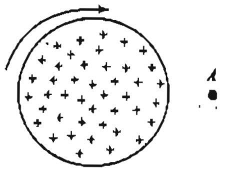


Fig. 9.273

a. ⊗ (into the page)
b. 3/4 →
c. ← 3/4
d. ⊙ (out of the page)

131. An electron and a proton each travel with equal speeds around circular orbits in the same uniform magnetic field as indicated (not to scale) in Fig. 9.274. The field is into the page on the diagram. The electron travels ______ around the ________ circle and the proton travels ________ around the _______ circle.

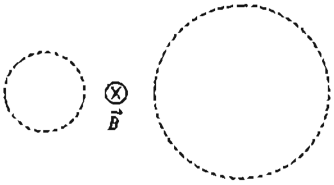


Fig. 9.274

a. clockwise, smaller, counterclockwise, larger
b. counterclockwise, larger, counterclockwise, smaller
c. clockwise, larger, counterclockwise, smaller
d. counterclockwise, larger, clockwise, smaller

132. Fig. 9.275 shows two long wires carrying equal currents I_1 and I_2 flowing in opposite directions. Which of the arrows labeled *A* to *D* correctly represents the direction of the magnetic field due to the wires at a point located at an equal distance *d* from each wire?

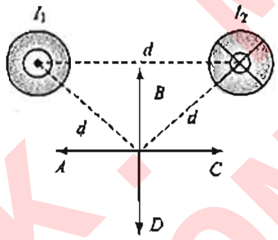


Fig. 9.275

a. *A*
b. *B*
c. *C*
d. *D*

133. Fig. 9.276 shows four different sets of wires that cross each other without actually touching. The magnitude of the current is the same in all four cases and the directions of current flow are as indicated. for which configuration will the magnetic field at the center of the square formed by the wires be equal to zero?

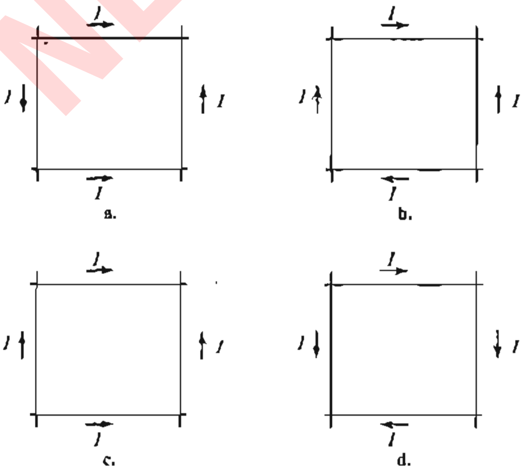


Fig. 9.276

134. When a charged particle moving with velocity *v* enters a region containing a perpendicular magnetic field, it moves along a semicircular path of radius '*r*' as shown in Fig. 9.277. Consider the following two statements:

(I) The radius '*r*' of the semicircle is proportional to the initial speed *v*.

(II) The time required for the particle to traverse the semicircle is independent of *v*.

Then,

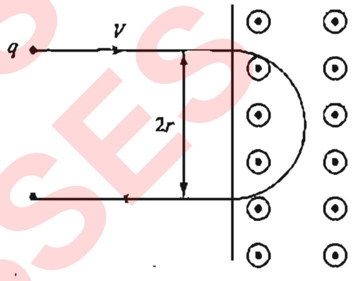


Fig. 9.277

A. I is true, but II is incorrect.
B. I is incorrect while II is true.
C. I is true, II is also true and the two statements are independent.
D. I is true, II is also true and II is the cause of I.

135. Four parallel conductors, carrying equal currents, pass vertically through the four corners of a square *WXYZ*. In two conductors, the current is flowing into the page, and in the other two out of the page. In what directions must the currents flow to produce a resultant magnetic field in the direction shown at *O*, the center of the square ?

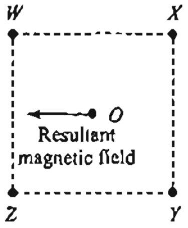


Fig. 9.278

	Into the page	Out of the page
a.	*W* and *Y*	*X* and *Z*
b.	*X* and *Z*	*W* and *Y*
c.	*W* and *Z*	*X* and *Y*
d.	*W* and *X*	*Y* and *Z*

136. In a region of space, a uniform magnetic field *B* exists in the *x*-direction. An electron is fired from the origin with its initial velocity *u* making an angle α with the *y*-direction in the *yz* plane. In the subsequent motion of the electron,

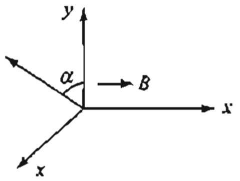


Fig. 9.279

a. y-coordinate of the electron will never be negative
b. z-coordinate of the electron will never be negative
c. x-coordinate of the electron will never be negative
d. trajectory of the electron would be helical

137. An infinitely long wire carrying current I is along Y-axis such that its one end is at point A $(0, b)$ while the wire extends upto $+\infty$. The magnitude of magnetic field strength at point $(a, 0)$ is

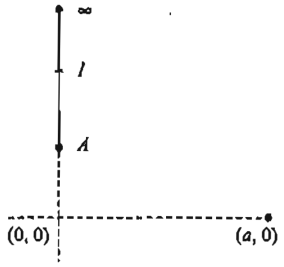


Fig. 9.280

a. $\frac{\mu_0 I}{4\pi a}\left(1+\frac{b}{\sqrt{a^2+b^2}}\right)$
b. $\frac{\mu_0 I}{4\pi a}\left(1-\frac{b}{\sqrt{a^2+b^2}}\right)$
c. $\frac{\mu_0 I}{4\pi a}\left(\frac{b}{\sqrt{a^2+b^2}}\right)$
d. none of these

138. A steady current is set up in a cubic network composed of wires of equal resistance and length d as shown in Fig. 9.281. What is the magnetic field at the center P due to the cubic network ?

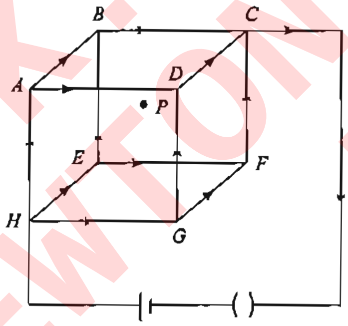


Fig. 9.281

a. $\frac{\mu_0}{4\pi}\frac{2I}{d}$
b. $\frac{\mu_0}{4\pi}\frac{3I}{\sqrt{2}d}$
c. 0
d. $\frac{\mu_0}{4\pi}\frac{6\pi I}{d}$

139. An electron moving with velocity v along the x-axis approaches a circular current carrying loop as shown in Fig. 9.282. The magnitude of magnetic force on the electron at this instant is

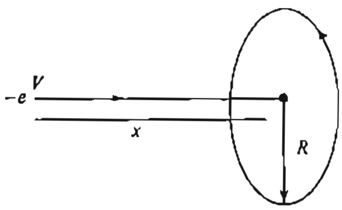


Fig. 9.282

a. $\frac{\mu_0}{2}\frac{eviR^2\,x}{(x^2+R^2)^{3/2}}$
b. $\mu_0\frac{eviR^2\,x}{(x^2+R^2)^{3/2}}$
c. $\frac{\mu_0}{4\pi}\frac{eviR^2\,x}{(x^2+R^2)^{3/2}}$
d. 0

140. If a charged particle of charge to mass ratio $\frac{q}{m}=\alpha$ enters in a magnetic field of strength B at a speed $v=(2\alpha d)(B)$, then

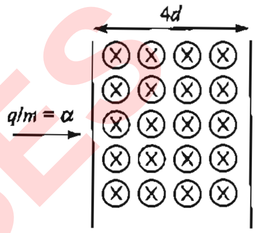


Fig. 9.283

a. angle subtended by the path of chargd particle in magnetic field at the center of circular path is 2π
b. the charge will move on a circular path and then will come out from magnetic field at some distance from the point of insertion
c. the time for which particle will be in the magnetic field is $\frac{2\pi}{\alpha B}$
d. angle subtended by the path of charged particle in magnetic field at the center of circular path is $\pi/2$

141. A conducting ring of mass 2 kg and radius 0.5 m is placed on a smooth horizontal plane. The ring carries a current of $i=4$ A. A horizontal magnetic field $B=10$ T is switched on at time $t=0$ as shown in Fig. 9.284. The initial angular acceleration of the ring will be

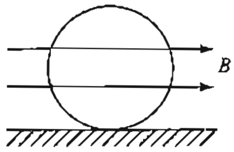


Fig. 9.284

a. 40π rad s^{-2}
b. 20π rad s^{-2}
c. 5π rad s^{-2}
d. 15π rad s^{-2}

142. A point charge is moving in clockwise direction in a circle with constant speed. Consider the magnetic field produced by the charge at a fixed point P (not at the center of circle) on the axis of the circle. Then,
a. it is constant in magnitude only
b. it is constant in direction only
c. it is constant both in direction and magnitude
d. it is constant neither in magnitude nor in direction

143. An α-particle is moving along a circle of radius R with a constant angular velocity ω. Point A lies in the same plane at a distance $2R$ from the center. Point A records magnetic field produced by the α-particle. If the minimum time interval between two successive times at which A records zero magnetic field is 't', the angular speed ω, in terms of t, is

a. $\frac{2\pi}{t}$ b. $\frac{2\pi}{3t}$

c. $\frac{\pi}{3t}$ d. $\frac{\pi}{t}$

144. Fig. 9.285 shows an equilateral triangle ABC of side ℓ carrying currents as shown, and placed in a uniform magnetic field B perpendicular to the plane of triangle. The magnitude of magnetic force on the triangle is

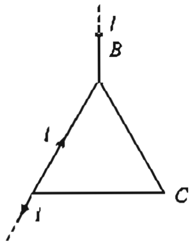


Fig. 9.285

a. $i\ell B$ b. $2i\ell B$

c. $3\, i\ell B$ d. zero

145. There exist uniform magnetic and electric fields of magnitudes 1 T and 1 Vm^{-1}, respectively, along positive y-axis. A charged particle of mass 1 kg and of charge 1 C is having velocity 1 ms^{-1} along x-axis and is at origin at $t = 0$. Then, the coordinates of the particle at time π seconds will be

a. (0, 1, 2) m b. $(0, -\pi 2, -2)$ m

c. $(2, \pi^2/2, 2)$ m d. $(0, \pi^2/2, 2)$ m

146. A uniform magnetic field of magnitude 1 T exists in region $y \ge 0$ along $\hat{k}$ direction as shown in Fig. 9.286. A particle of charge 1 C is projected from point $(-\sqrt{3}, -1)$ toward origin with speed 1 ms^{-1}. If mass of the particle is 1 kg, then coordinates of center of the circle in which the particle moves are

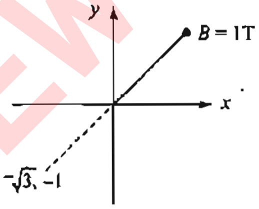


Fig. 9.286

a. $(1, \sqrt{3})$ b. $(1, -\sqrt{3})$

c. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ d. $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

147. A uniform magnetic field exists in a region which forms an equilateral triangle of side a. The magnetic field is perpendicular to the plane of the triangle. A charge q enters into this magnetic field perpendicular to a side with speed v. The charge enters from midpoint and leaves the field from midpoint of other side. Magnetic induction in the triangle is

a. $\frac{mv}{qa}$ b. $\frac{2mv}{qa}$

c. $\frac{mv}{2qa}$ d. $\frac{mv}{4qa}$

148. A particle of positive charge q and mass m enters with velocity $V\hat{j}$ at the origin in a magnetic field $B(-\hat{k})$ which is present in the whole space. The charge makes a perfectly inelastic collision with an identical particle (having same charge) at rest but free to move at its maximum positive y-coordinate. After collision, the combined charge will move on trajectory

(where $r = \frac{mV}{qB}$)

a. $y = \frac{mv}{qB} x$

b. $(x + r)^2 + (y - r/2)^2 = r^2/4$

c. $(x + r)^2 + (y - r/2)^2 = r^2/8$

d. $(x - r)^2 + (y + r/2)^2 = r^2/4$

149. In the plane mirror, the coordinates of image of a charged particle (initially at origin as shown in Fig. 9.287) after two and a half time periods are (Initial velocity of charge particle is V_0 in the xy plane and the plane mirror is perpen-dicular to the x-axis. A uniform magnetic field $B\hat{i}$ exists in the space. P_0 is pitch of helix, R_0 is radius of helix.)

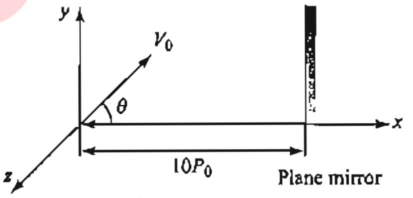


Fig. 9.287

a. $17P_0, 0, -2R_0$ b. $3P_0, 0, -2R_0$

c. $17.5P_0, 0, -2R_0$ d. $3P_0, 0, 2R_0$

150. A uniform magnetic field of 1.5 T exists in a cylindrical region of radius 10.0 cm, its direction being parallel to the axis along east to west. A current carrying wire in north–south direction passes through this region. The wire intersects the axis and experiences a force of 1.2 N downward. If the wire is turned from north south to northeast–southwest direction, then magnitude and direction of force is

a. 1.2 N, upward b. $1.2\sqrt{2}$, downward

c. 1.2 N, downward d. $\frac{1.2}{\sqrt{2}}$ N, downward

151. Three infinite current carrying conductors are placed as shown in Fig. 9.288. Two wires carry same current while current in third wire is unknown. The three wires are electrically insulated from each other and all of them are in the plane of paper. Which of the following is correct about a point 'P' which is also in the same plane?

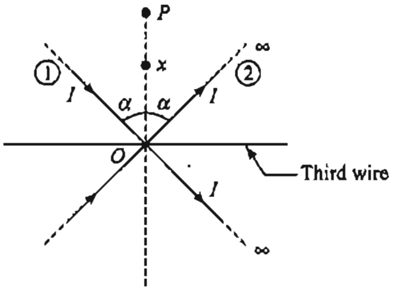


Fig. 9.288

a. Magnetic field intensity at P is zero for all values of x, whatever is the current in the third wire

b. If the current in the third wire is $\frac{2I}{\sin\alpha}$ (left to right), then magnetic field will be zero at P for all values of x

c. If the current in the third wire is $\frac{2I}{\sin\alpha}$ (right to left), then magnetic field will be zero at P for all values of x

d. None of these

152. An insulating rod of length ℓ carries a charge q distributed uniformly on it. The rod is pivoted at its mid point and is rotated at a frequency f about a fixed axis perpendicular to the rod and passing through the pivot. The magnetic moment of the rod system is

a. $\frac{1}{12}\pi q f \ell^2$ b. $\pi q f \ell^2$
c. $\frac{1}{6}\pi q f \ell^2$ d. $\frac{1}{3}\pi q f \ell^2$

153. Rank the value of $\oint \vec{B}\cdot d\vec{l}$ for the closed paths shown in Fig. 9.289 from the smallest to largest.

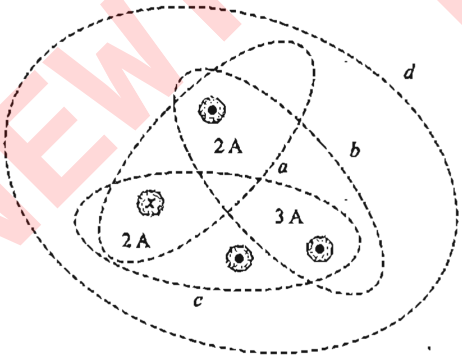


Fig. 9.289

a. a, b, c, d b. a, c, d, b
c. [illegible] d, c, b d. a, c, b, d

154. Two straight segments of wire ab and bc each carrying current I, are placed as shown in Fig. 9.290. The cube edge is 50 cm and magnetic field is uniform along Y-axis having magnitude 0.4 T. If $I = 3$ A, the force experienced by wire abc in the presence of magnetic field is

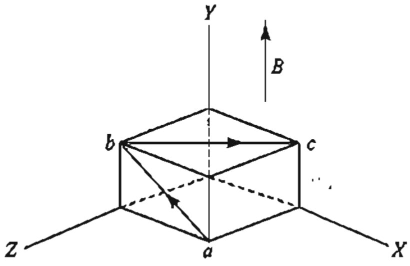


Fig. 9.290

a. $0.6\hat{i}$ b. $1.2(\hat{i}+\hat{k})$
c. $0.6(\sqrt{2}\hat{i}+\hat{j}-\sqrt{2}\hat{k})$ d. $0.6(\sqrt{2}\hat{i}-\hat{k})$

155. An equilateral triangular loop is kept near to a current carrying long wire as shown in Fig. 9.291. Under the action of magnetic force alone, the loop

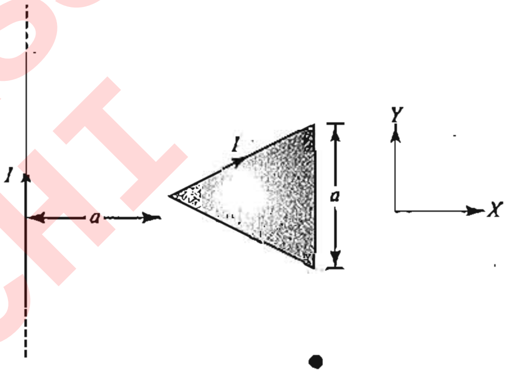


Fig. 9.291

a. must move along positive or negative X-axis
b. must move in XY plane and not along X- or Y-axis
c. does not move
d. moves but which way we cannot predict

156. A current carrying loop is placed in the non-uniform magnetic field whose variation in space is shown in Fig. 9.292. Direction of magnetic field is into the plane of paper. The magnetic force experienced by the loop is

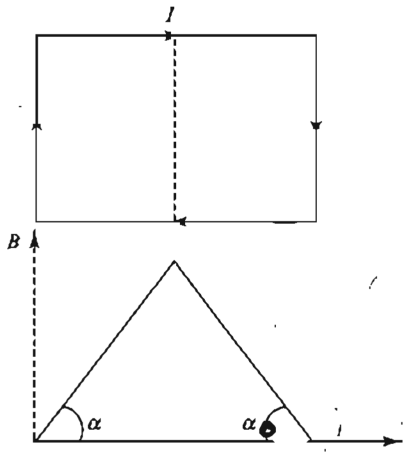


Fig. 9.292

a. non-zero
b. zero
c. cannot say anything
d. none of the above

157. A particle is moving with velocity $\vec{v} = \hat{i} + 3\hat{j}$ and it produces an electric field at a point given by $\vec{E} = 2\hat{k}$. It will produce magnetic field at that point equal to (all quantities are in S.I. units)

a. $(6\hat{i} - 2\hat{j})\mu_0 \varepsilon_0$
b. $(6\hat{i} + 2\hat{j})\mu_0 \varepsilon_0$
c. zero
d. cannot be determined from the given data

158. A current carrying loop lies on a smooth horizontal plane. Then,

a. it is possible to establish a uniform magnetic field in the region so that the loop starts rotating about its own axis
b. it is possible to establish a uniform magnetic field in the region so that the loop will tip over about any of the point
c. it is not possible that loop will tip over about any of the point whatever be the direction of established magnetic field (uniform)
d. both (a) and (b) are correct.

159. A parallel plate capacitor is moving with a velocity of 25 ms^{-1} through a uniform magnetic field of 4.0 T as shown in Fig. 9.293. If the electric field within the capacitor plates is 175 NC^{-1} and plate area is 25×10^{-7} m^2, then the magnetic force experienced by the positive charge plate is

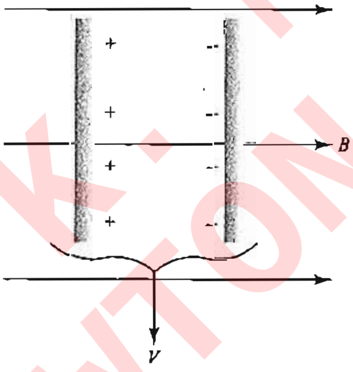


Fig. 9.293

a. 8.85×10^{-13} N directed out of the plane of the paper
b. zero
c. 8.85×10^{-15} N directed out of the plane of the paper
d. none of above

160. A semicircular wire of radius R, carrying current I, is placed in a magnetic field as shown in Fig. 9.294. On left side of $X'X$, magnetic field strength is B_0, and on right side of $X'X$, magnetic field strength is $2B_0$. The magnetic force experienced by the wire would be

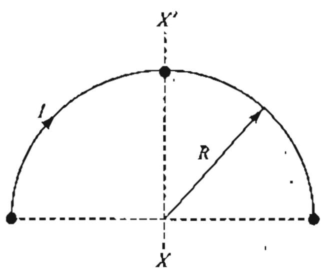


Fig. 9.294

a. $3IB_0R$
b. $2IB_0R$
c. $\sqrt{10}\ IB_0R$
d. $\sqrt{5}\ IB_0R$

161. A wire of cross-sectional area A forms three sides of a square and is free to rotate about axis OO'. If the structure is deflected by an angle θ from the vertical when current i is passed through it in a magnetic field B acting vertically upward and density of the wire is ρ, then the value of θ is given by

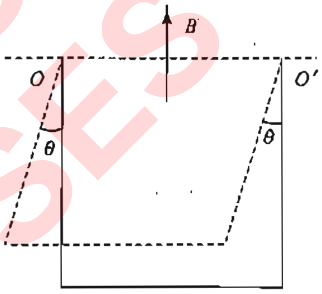


Fig. 9.295

a. $\dfrac{2A\rho g}{iB} = \cot\theta$
b. $\dfrac{2A\rho g}{iB} = \tan\theta$
c. $\dfrac{A\rho g}{iB} = \sin\theta$
d. $\dfrac{A\rho g}{2iB} = \cos\theta$

162. A proton of mass 1.67×10^{-27} kg and charge 1.67×10^{-19} C is projected with a speed of 2×10^6 ms^{-1} at an angle of 60° to the X-axis. If a uniform magnetic field of 0.10 T is applied along Y-axis, the path of proton is

a. a circle of radius 0.2 m and time period $\pi \times 10^{-7}$ s
b. a circle of radius 0.1 m and time period $2\pi \times 10^{-7}$ s
c. a helix of radius 0.1 m and time period $2\pi \times 10^{-7}$ s
d. a helix of radius 0.2 m and time period $4\pi \times 10^{-7}$ s

163. In a cylindrical region, uniform magnetic field is present as shown in Fig. 9.296. The cylinder is kept on a horizontal plane and its axis is horizontal. A charge particle of mass m and charge q is projected horizontally with velocity v through a hole normal to the axis of the cylinder as shown in the diagram. An observer states the particle moves first undeviated and subsequently.

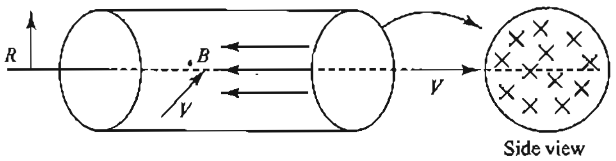


Fig. 9.296

a. oscillates inside the cylinder along a horizontal diameter passing through axis of the cylinder with time period $\dfrac{4RqB}{mg}$

b. oscillates inside the cylinder along a horizontal diameter passing through axis of the cylinder with time period $\frac{2qRB}{mg}$

c. oscillates inside the cylinder along a horizontal diameter passing through axis of the cylinder with time period $\frac{mg}{qBR}$

d. it is not possible for the particle to oscillate in this given situation

164. Current I flows around the wire frame along the edges of a cube as shown in Fig. 9.297. Point 'P' is the center of the cube. The incoming and outgoing wires have orientation towards P. Then,

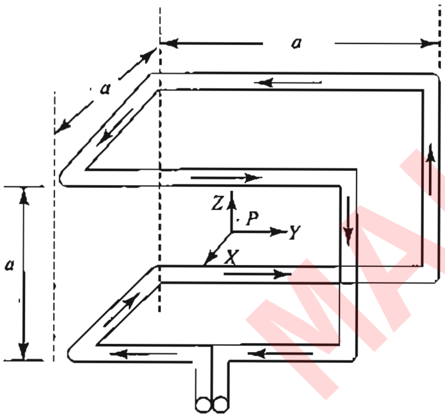


Fig. 9.297

a. the magnetic field at P is toward $+y$ direction

b. the magnetic field at P is toward $-y$ direction

c. the unit vector of magnetic field at P is $-\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})$

d. the magnitude of magnetic field at P is $\frac{4\sqrt{2}\mu_0 I}{3\pi a}$

165. In Fig. 9.298, $ABCDEFA$ was a square loop of side ℓ, but is folded in two equal parts so that half of it lies in the xz-plane and the other half lies in the yx plane. The origin 'O' is center of the frame also. The loop carries current 'i'. The magnetic field at the center is

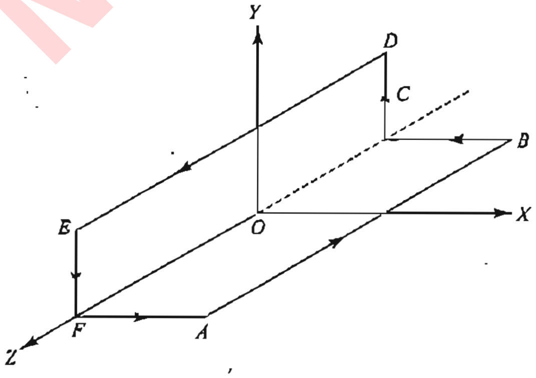


Fig. 9.298

a. $\frac{\mu_0 i}{2\sqrt{2}\pi\ell}(\hat{i}-\hat{j})$ **b.** $\frac{\mu_0 i}{4\pi\ell}(-\hat{i}+\hat{j})$

c. $\frac{\sqrt{2}\mu_0 i}{\pi\ell}(\hat{i}+\hat{j})$ **d.** $\frac{\mu_0 i}{\sqrt{2}\pi\ell}(\hat{i}+\hat{j})$

166. If the magnetic field at 'P' can be written as $K\tan\left(\frac{\alpha}{2}\right)$, then K is

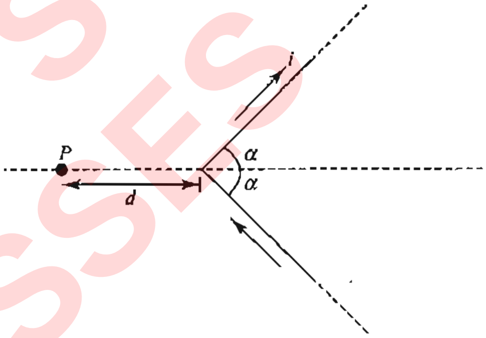


Fig. 9.299

a. $\frac{\mu_0 I}{4\pi d}$ **b.** $\frac{\mu_0 I}{2\pi d}$

c. $\frac{\mu_0 I}{\pi d}$ **d.** $\frac{2\mu_0 I}{\pi d}$

167. The magnetic field at the origin due to the current flowing in the wire is

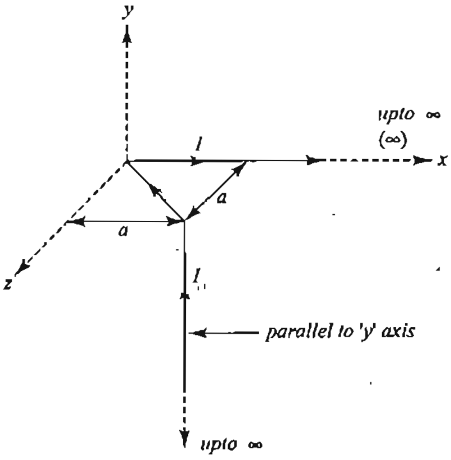


Fig. 9.300

a. $-\frac{\mu_0 I}{8\pi a}(\hat{i}+\hat{k})$ **b.** $\frac{\mu_0 I}{2\pi a}(\hat{i}+\hat{k})$

c. $\frac{\mu_0 I}{8\pi a}(-\hat{i}+\hat{k})$ **d.** $\frac{\mu_0 I}{4\pi a\sqrt{2}}(\hat{i}-\hat{k})$

168. Two infinitely long linear conductors are arranged perpendicular to each other and are in mutually perpendicular planes as shown in Fig. 9.301. If $I_1 = 2$ A along the y-axis, $I_2 = 3$ A along $-$ve x-axis and $AP = AB = 1$ cm, the value of magnetic field strength $\vec{B}$ at P is

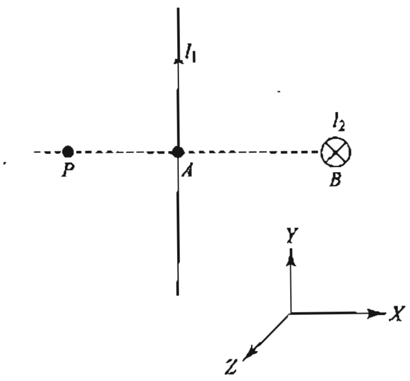


Fig. 9.301

a. $\left(3\times10^{-5}\text{T}\right)\hat{j}+\left(-4\times10^{-5}\text{T}\right)\hat{k}$

b. $\left(3\times10^{-5}\text{T}\right)\hat{j}+\left(4\times10^{-5}\text{T}\right)\hat{k}$

c. $\left(4\times10^{-5}\text{T}\right)\hat{j}+\left(3\times10^{-5}\text{T}\right)\hat{k}$

d. $\left(-3\times10^{-5}\text{T}\right)\hat{j}+\left(4\times10^{-5}\text{T}\right)\hat{k}$

169. Fig. 9.302 shows an Amperian path *ABCDA*. Part *ABC* is in vertical plane *PSTU* while part *CDA* is in horizontal plane *PQRS*. Direction of circulation along the path is shown by an arrow near point *B* and at *D*.

$\oint \vec{B}\cdot d\vec{\ell}$ for this path according to Ampere's law will be

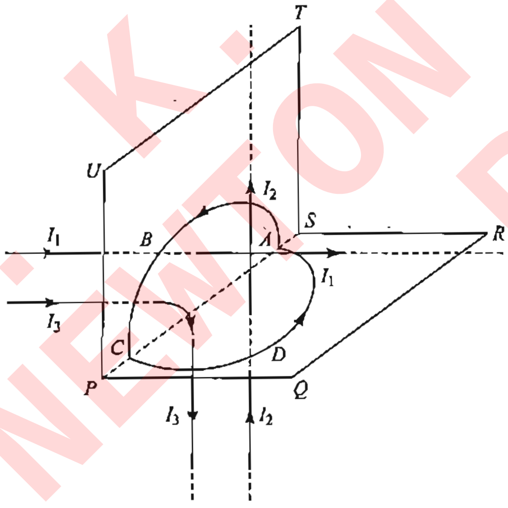


Fig. 9.302

a. $(i_1-i_2+i_3)\mu_0$

b. $(-i_1+i_2)\mu_0$

c. $i_3\mu_0$

d. $(i_1+i_2)\mu_0$

170. A coaxial cable is made up of two conductors. The inner conductor is solid and is of radius R_1 and the outer conductor is hollow of inner radius R_2 and outer radius R_3. The space between the conductors is filled with air. The inner and outer conductors are carrying currents of equal magnitudes and in opposite directions. Then, the variation of magnetic field with distance from the axis is best plotted as

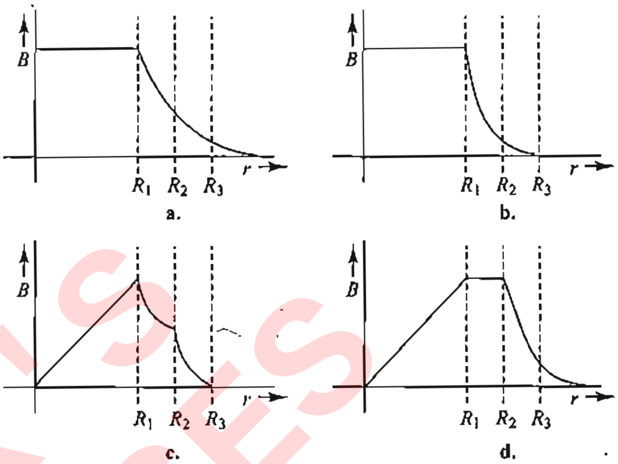


Fig. 9.303

171. From a cylinder of radius *R*, a cylinder of radius *R*/2 is removed, as shown in Fig. 9.304. Current flowing in the remaining cylinder is *I*. Then, magnetic field strength is

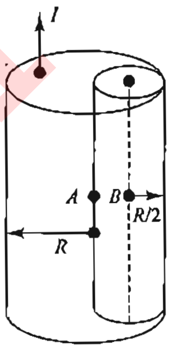


Fig. 9.304

a. zero at point *A*

b. zero at point *B*

c. $\dfrac{\mu_0 I}{3\pi R}$ at point *A*

d. $\dfrac{\mu_0 I}{3\pi R}$ at point *B*

172. A current *I* enters at *A* in a square loop of uniform resistance and leaves at *B*. The ratio of magnetic field at *E*, the centre of square, due to segment *AB* to that due to *DC* is

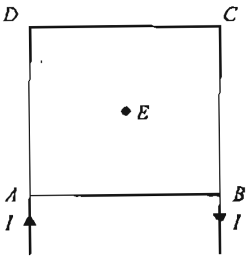


Fig. 9.305

a. 1

b. 2

c. 3

d. 4

173. A beam of mixture of α particles and protons are accelerated through same potential difference before entering into the magnetic field of strength *B*. If $r_1 = 5$ cm, then r_2 is

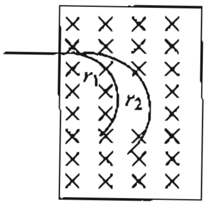


Fig. 9.306

a. 5 cm
b. $5\sqrt{2}$ cm
c. $10\sqrt{2}$ cm
d. 20 cm

174. A wire of length 'l' is used to form a coil. The magnetic field at its center for a given current in it is minimum if the coil has

a. 4 turns
b. 2 turns
c. 1 turn
d. data is not sufficient

175. The value of the electric field strength in vacuum if the energy density is same as that due to a magnetic field of induction 1 T in vacuum is

a. $3\times10^8\,\text{NC}^{-1}$
b. $1.5\times10^8\,\text{NC}^{-1}$
c. $2.0\times10^8\,\text{NC}^{-1}$
d. $1.0\times10^8\,\text{NC}^{-1}$

176. Fig. 9.307 shows a small loop carrying a current I. The curved portion is an arc of a circle of radius R and the straight portion is a chord to the same circle subtending an angle θ. The magnetic induction at the center O is

a. zero
b. always inward irrespective of the value of θ
c. inward as long as θ is less than π
d. always outward irrespective of the value of θ

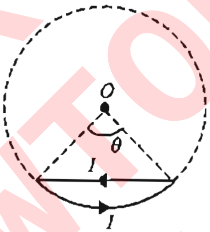


Fig. 9.307

177. An electron is projected at an angle θ with a uniform magnetic field. If the pitch of the helical path is equal to its radius, then the angle of projection is

a. $\tan^{-1}\pi$
b. $\tan^{-1}2\pi$
c. $\cot^{-1}\pi$
d. $\cot^{-1}2\pi$

178. A charged particle moves in a uniform magnetic field perpendicular to it, with a radius of curvature 4 cm. On passing through a metallic sheet it loses half of its kinetic energy. Then, the radius of curvature of the particle is

a. 2 cm
b. 4 cm
c. 8 cm
d. $2\sqrt{2}$ cm

179. Three rings, each having equal radius R, are placed mutually perpendicular to each other and each having its center at the origin of coordinate system. If current I is flowing through each ring, then the magnitude of the magnetic field at the common center is

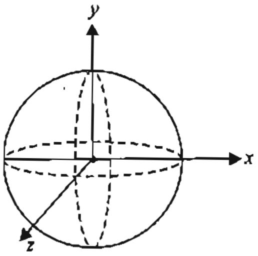


Fig. 9.308

a. $\sqrt{3}\dfrac{\mu_0 I}{2R}$
b. zero
c. $(\sqrt{2}-1)\dfrac{\mu_0 I}{2R}$
d. $(\sqrt{3}-\sqrt{2})\dfrac{\mu_0 I}{2R}$

180. Positive point charges $q = +8.00\ \mu\text{C}$ and $q' = +3.00\ \mu\text{C}$ are moving relative to an observer at point P, as shown in Fig. 9.309. The distance d is 0.120 m. When the two charges are at the locations shown in the figure, what are the magnitude and direction of the net magnetic field they produce at point P? (Take $v = 4.50\times10^6\ \text{ms}^{-1}$ and $v' = 9.00\times10^6\ \text{ms}^{-1}$.)

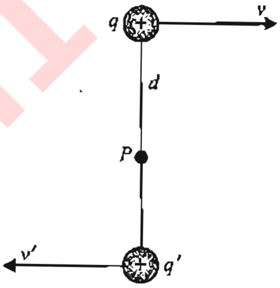


Fig. 9.309

a. 4.38×10^{-4} T, into the page
b. 4.38×10^{-4} T, out of the page
c. 2.16×10^{-4} T, into the page
d. 2.16×10^{-4} T, out of the page

181. Four very long, current carrying wires in the same plane intersect to form a square 40.0 cm on each side, as shown in Fig. 9.310. Find the magnitude and direction of the current I so that the magnetic field at the center of the square is zero.

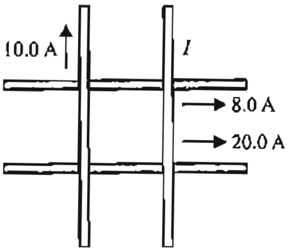


Fig. 9.310

a. 4.0 A toward the bottom of the page
b. 2.0 A toward the bottom of the page
c. 2.5 A toward the bottom of the page
d. 3.6 A toward the top of the page

182. A square loop of side a carries a current I. The magnetic induction B at point P, lying on the axis of the loop and at a distance x from the center of loop is

a. $\dfrac{4\mu_0 ia^2}{3\pi(4x^2+a^2)\sqrt{4x^2+2a^2}}$

b. $\dfrac{\sqrt{3}\,\mu_0 ia^2}{\pi(4x^2+a^2)\sqrt{4x^2+2a^2}}$

c. $\dfrac{\mu_0 ia^2}{\pi(4x^2+a^2)\sqrt{4x^2+2a^2}}$

d. $\dfrac{4\mu_0 ia^2}{\pi(4x^2+a^2)\sqrt{4x^2+2a^2}}$

183. Two very long, straight wires carry currents as shown in Fig. 9.311. Find all locations where the net magnetic field is zero.

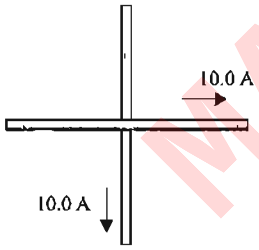


Fig. 9.311

a. $y=\sqrt{2}\,x$ b. $y=x$

c. $y=-x$ d. $y=-(x/2)$

184. A particle of specific charge α is projected from origin with velocity $\vec{v}=v_0\hat{i}-v_0\hat{k}$ in a uniform magnetic field $\vec{B}=-B_0\hat{k}$. Find time dependence of velocity and position of the particle.

a. $\vec{v}_{(t)}=v_0\cos(\alpha B_0 t)\hat{i}+v_0\sin(\alpha B_0 t)\hat{j}-v_0\hat{k}$

b. $\vec{v}_{(t)}=-v_0\cos(\alpha B_0 t)\hat{i}+v_0\sin(\alpha B_0 t)\hat{j}+v_0\hat{k}$

c. $\vec{v}_{(t)}=-v_0\cos(\alpha B_0 t)\hat{i}+v_0\sin(\alpha B_0 t)\hat{j}-v_0\hat{k}$

d. $\vec{v}_{(t)}=v_0\cos(\alpha B_0 t)\hat{i}+v_0\sin(\alpha B_0 t)\hat{j}+v_0\hat{k}$

185. A straight piece of conducting wire with mass M and length L is placed on a frictionless incline tilted at an angle θ from the horizontal (as shown in Fig. 9.312). There is a uniform, vertical magnetic field at all points (produced by an arrangement of magnets not shown in Fig. 9.312). To keep the wire from sliding down the incline, a voltage source is attached to the ends of the wire. When just the right amount of current flows through the wire, the wire remains at rest. Determine the magnitude and direction of the current in the wire that will cause the wire to remain at rest.

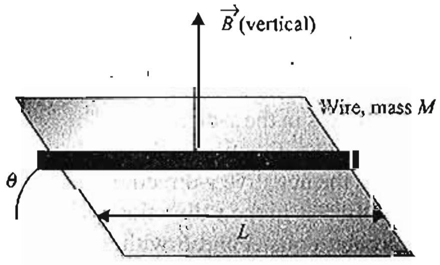


Fig. 9.312

a. $\dfrac{Mg\tan\theta}{2LB}$ to the left b. $\dfrac{Mg\tan\theta}{LB}$ to the right

c. $\dfrac{Mg\tan\theta}{LB}$ to the left d. $\dfrac{3Mg\tan\theta}{2LB}$ to the left

Multiple Correct Answers Type

Solutions on page 9.131

1. Which of the following statements are correct?

a. If a moving charged particle enters into a region of magnetic field from outside, it does not complete a circular path

b. If a moving charged particle traces a helical path in a uniform magnetic field, the axis of the helix is parallel to the magnetic field

c. The power associated with the force exerted by a magnetic field on a moving charged particle is always equal to zero

d. If in a region a uniform magnetic field and a uniform electric field both exist, a charged particle moving in this region cannot trace a circular path

2. A charged particle P leaves the origin with speed $v=v_0$ at some inclination with the x-axis. There is a uniform magnetic field B along the x-axis. P strikes a fixed target T on the x-axis for a minimum value of $B=B_0$. P will also strike T if:

a. $B=2B_0,\ v=2v_0$ b. $B=2B_0,\ v=v_0$

c. $B=B_0,\ v=2v_0$ d. $B=\dfrac{B_0}{2},\ v=2v_0$

3. A charged particle is fired at an angle θ to a uniform magnetic field directed along the x-axis. During its motion along a helical path, the particle will

a. never move parallel to the x-axis

b. move parallel to the x-axis once during every rotation for all values of θ

c. move parallel to the x-axis at least once during every rotation if $\theta=45°$

d. never move perpendicular to the x-direction

4. In previous problem, if the pitch of the helical path is equal to the maximum distance of the particle from the x-axis, then which of the following are not correct?

a. $\cos\theta=\dfrac{1}{\pi}$ b. $\sin\theta=\dfrac{1}{\pi}$

c. $\tan\theta=\dfrac{1}{\pi}$ d. $\tan\theta=\pi$

5. A particle having a mass of 0.5 g carries a charge of 2.5×10^{-8} C. The particle is given an initial horizontal velocity of 6×10^{4} ms^{-1}. To keep the particle moving in a horizontal direction

a. the magnetic field should be perpendicular to the direction of the velocity

b. the magnetic field should be along the direction of the velocity
c. magnetic field should have a minimum value of 3.27 T
d. no magnetic field is required

6. The force $\vec{F}$ experienced by a particle of charge q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by $\vec{F} = q(\vec{v} \times \vec{B})$. Which pairs of vectors are at right angles to each other?
a. $\vec{F}$ and $\vec{v}$
b. $\vec{F}$ and $\vec{B}$
c. $\vec{B}$ and $\vec{v}$
d. $\vec{F}$ and $(\vec{v} \times \vec{B})$

7. An electron (mass $= m_e$) and a proton (mass $= m_p$) initially at rest move through a certain distance in a uniform electric field in times t_1 and t_2. Neglect the effect of gravity. Then,
a. the acceleration of electron is much greater than that of proton
b. the acceleration of proton is much greater than that of electron
c. $\frac{t_1}{t_2} = \left(\frac{m_e}{m_p}\right)^{1/2}$
d. $\frac{t_1}{t_2} = \left(\frac{m_p}{m_e}\right)^{1/2}$

8. A charged particle moves in a uniform magnetic field. The velocity of the particle at some instant makes an acute angle with the magnetic field. The path of the particle will be
a. a circle
b. a helix with uniform pitch
c. a helix with non-uniform pitch
d. a helix with uniform radius

9. An electron is moving along the positive x-axis. You want to apply a magnetic field for a short time so that the electron may reverse its direction and move parallel to the negative x-axis. This can be done by applying the magnetic field along
a. y-axis
b. z-axis
c. y-axis only
d. z-axis only

10. If a charged particle goes unaccelerated in a region containing electric and magnetic fields, then
a. $\vec{E}$ must be perpendicular to $\vec{B}$
b. $\vec{v}$ must be perpendicular to $\vec{E}$
c. $\vec{v}$ must be perpendicular to $\vec{B}$
d. E must be equal to Vb

11. A proton is fired from origin with velocity $\vec{v} = v_0\hat{j} + v_0\hat{k}$ in a uniform magnetic field $\vec{B} = B_0\hat{j}$.
In the subsequent motion of the proton
a. its z-coordinate can never be negative
b. its x-coordinate can never be positive
c. its x- and z-coordinates cannot be zero at the same time
d. its y-coordinate will be proportional to its time of flight

12. Velocity and acceleration vector of a charged particle moving in a magnetic field at some instant are $\vec{v} = 3\hat{i} + 4\hat{j}$ and $\vec{a} = 2\hat{i} + x\hat{j}$. Select the correct options:
a. $x = -1.5$
b. $x = 3$
c. Magnetic field is along z-direction
d. Kinetic energy of the particle is constant

13. A charged particle goes undeflected in a region containing electric and magnetic fields. It is possible that
a. $\vec{E} \parallel \vec{B}$, $\vec{v} \parallel \vec{E}$
b. $\vec{E}$ is not parallel to $\vec{B}$
c. $\vec{v} \parallel \vec{B}$ but $\vec{E}$ is not parallel to $\vec{B}$
d. $\vec{E} \parallel \vec{B}$ but $\vec{v}$ is not parallel to $\vec{E}$

14. A charged particle with velocity $\vec{v} = x\hat{i} + y\hat{j}$ moves in a magnetic field $\vec{B} = y\hat{i} + x\hat{j}$. The force acting on the particle has magnitude F. Which one of the following statements is/are correct?
a. No force will act on charged particle if $x = y$
b. If $x > y, f \propto (x^2 - y^2)$
c. If $x > y$, the force will act along z-axis
d. If $y > x$, the force will act along y-axis

15. In the loops shown in Fig. 9.313, all curved sections are either semicircles or quarter circles. All the loops carry the same current. The magnetic fields at the centers have magnitudes B_1, B_2, B_3 and B_4. Then,

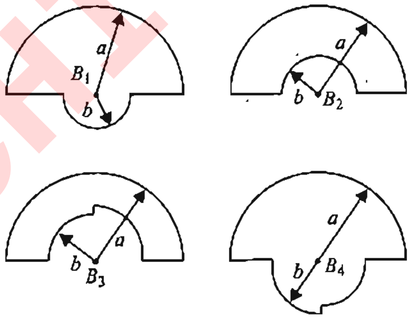


Fig. 9.313

a. B_4 is maximum
b. B_3 is minimum
c. $B_4 > B_1 > B_2 > B_3$
d. $B_1 > B_4 > B_3 > B_2$

16. A conductor $ABCDE$, shaped as shown, carries current I. It is placed in the x–y plane with the ends A and E on the x-axis. A uniform magnetic field of magnitude B exists in the region. The force acting on it will be

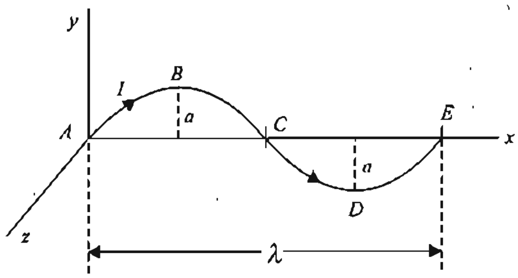


Fig. 9.314

a. zero, if B is in the x-direction
b. λBI in the z-direction, if B is in the y-direction
c. λBI in the negative y-direction, if B is in the z-direction
d. λaBI, if B is in the x-direction

17. Two circular coils A and B with their centers lying on the same axis have same number of turns and carry equal currents in the same sense. They are separated by a distance, have different diameters but subtend same angle at a point P lying on their common axis. The coil B lies exactly midway between coil A and the point P. The magnetic field at point P due to coils A and B is B_1 and B_2, respectively. Then,

a. $B_1 > B_2$
b. $B_1 < B_2$
c. $B_1 / B_2 = 2$
d. $\frac{B_1}{B_2} = \frac{1}{2}$

18. A long straight wire carries a current along the x-axis. Consider the points A (0, 1, 0), B (0,1, 1,), C (1, 0, 1) and D (1, 1, 1). Which of the following pairs of points will have magnetic fields of the same magnitude?

a. A and B
b. A and C
c. B and C
d. B and D

19. In previous problem, if the current is I and the magnetic field at D has magnitude B, then

a. $B = \frac{\mu_0 I}{2\sqrt{2}\pi}$
b. $B = \frac{\mu_0 I}{2\sqrt{3}\pi}$
c. B is parallel to the z-axis
d. B makes an angle of 45° with the $x-y$ plane

20. A straight conductor carries a current along the z-axis. Consider the points A (a, 0, 0), B (0, $-a$, 0), C ($-a$, 0, 0) and D (0, a, 0). Then,

a. all four points have magnetic fields of the same magnitude
b. all four points have magnetic fields in different directions
c. the magnetic fields at A and C are in opposite directions
d. the magnetic fields at A and B are mutually perpendicular

21. A steady electric current is flowing through a cylindrical conductor. Then,

a. the electric field at the axis of the conductor is zero
b. the magnetic field at the axis of the conductor is zero
c. the electric field in the vicinity of the conductor is zero
d. the magnetic field in the vicinity of the conductor is zero

22. AB and CD are smooth parallel rails, separated by a distance L and inclined to the horizontal at an angle θ. A uniform magnetic field of magnitude B, directed vertically upwards, exists in the region. EF is a conductor of mass m, carrying a current I. For EF to be in equilibrium,

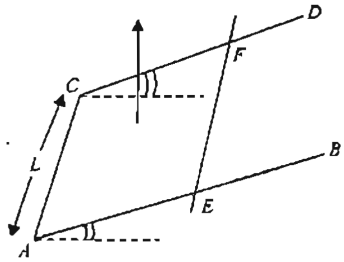


Fig. 9.315

a. I must flow from E to F
b. $BIL = mg \tan \theta$
c. $BIL = mg \sin \theta$
d. $BIL = mg$

23. A wooden cubical block $ABCDEFG$ of mass m and side a is wrapped by a square wire loop of perimeter 4a, carrying current I. The whole system is placed at frictionless horizontal surface in a uniform magnetic field $\vec{B} = B_0 \hat{j}$ as shown in Fig. 9.316. In this situation, normal force between horizontal surface and block passes through a point at a distance x from centre. Choose correct statement(s).

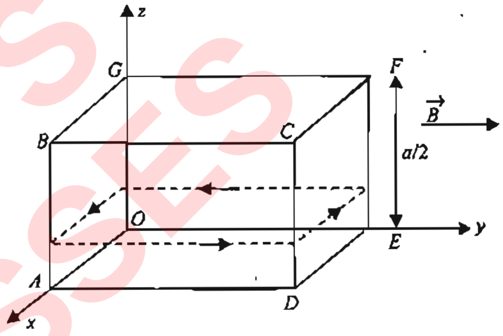


Fig. 9.316

a. The block must not topple if $I < \frac{mg}{aB_0}$
b. The block must not topple if $I < \frac{mg}{2aB_0}$
c. $x = \frac{a}{4}$ if $I = \frac{mg}{2aB_0}$
d. $x = \frac{a}{4}$ if $I = \frac{mg}{4aB_0}$

24. A particle of charge $-q$ and mass m enters a uniform magnetic field $\vec{B}$ (perpendicular to paper inward) at P with a velocity v_0 at an angle α and leaves the field at Q with velocity v at angle β as shown in Fig. 9.317.

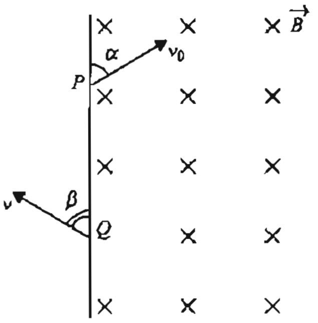


Fig. 9.317

a. $\alpha = \beta$
b. $v = v_0$
c. $PQ = \frac{2mv_0 \sin \alpha}{Bq}$
d. The particle remains in field for time $t = \frac{2m(\pi - \alpha)}{Bq}$

25. Let $\vec{E}$ and $\vec{B}$ denote the electric and magnetic fields in a certain region of space. A proton moving with a velocity along a straight line enters the region and is found to pass through it undeflected. Indicate which of the following statements are consistent with the observations:

a. $\vec{E} = 0$ and $\vec{B} = 0$
b. $\vec{E} \neq 0$ and $\vec{B} = 0$
c. $\vec{E} \neq 0$ and $\vec{B} \neq 0$ and both $\vec{E}$ and $\vec{B}$ are parallel to $\vec{v}$
d. $\vec{E}$ is parallel to $\vec{v}$ but $\vec{B}$ is perpendicular to $\vec{v}$

26. A particle of charge q and mass m moves rectilinearly under the action of an electric field $E = \alpha - \beta x$. Here, α and β are positive constants and x is the distance from the point where the particle was initially at rest. Then,

a. the motion of the particle is oscillatory
b. the amplitude of the particle is (α / β)
c. the mean position of the particle is at $x = (\alpha / \beta)$
d. the maximum acceleration of the particle is $\frac{q\alpha}{m}$

27. Two circular coils of radii 5 cm and 10 cm carry equal currents of 2 A. The coils have 50 and 100 turns respectively and are placed in such a way that their planes as well as their centres coincide. Magnitude of magnetic field at the common centre of coils is

a. $8\pi \times 10^{-4}$ T if current in the coil are in same sense
b. $4\pi \times 10^{-4}$ T if current in the coil are in opposite sense
c. zero if currents in the coils are in opposite sense
d. $8\pi \times 10^{-4}$ T if current in the coil are in opposite sense

28. A charged particle of unit mass and unit charge moves with velocity $\vec{v} = (8\hat{i} + 6\hat{j})$ m/s in a magnetic field of $\vec{B} = 2\hat{k}$ T. Choose the correct altenative(s).

a. The path of the particle may be $x^2 + y^2 - 4x - 21 = 0$
b. The path of the particle may be $x^2 + y^2 = 25$
c. The path of the particle may be $y^2 + z^2 = 25$
d. The time period of the particle will be 3.14 s

29. A proton enters in a region of uniform electric and magnetic fields $\vec{E}$ and $\vec{B}$, respectively. Velocity of the proton is $\vec{v}$. All the three vectors are mutually perpendicular. The proton is deflected along positive x-axis when either of the fields or both are switched on simultaneously. Which of the following statement(s) is/are correct ?

a. $\vec{v}$ may be along positive y-axis
b. $\vec{E}$ is along positive x-axis
c. $\vec{B}$ may be along positive z-axis
d. $\vec{B}$ may be along negative y-axis

30. A particle with charge $+q$ and mass m, moving under the influence of a uniform electric field $E\hat{i}$ and a uniform magnetic field $B\hat{k}$, follows a trajectory from P to Q as shown in Fig. 9.318. The velocities at P and Q are $v\hat{i}$ and $-2v\hat{j}$. Which of the following is correct?

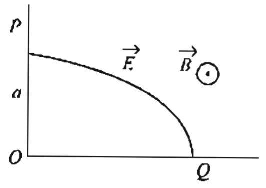


Fig. 9.318

a. $E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$
b. The rate of work done by the electric field at P is $\frac{3}{4}\left(\frac{mv^3}{a}\right)$
c. The rate of work done by the electric field at P is 0
d. The rate of work done by both the fields at Q is 0

31. A long straight wire carries a current along the Z-axis. One can find two points in the X-Y plane where

a. the magnitude of force on identical point charge are equal
b. the direction of the magnetic fields are the same
c. the magnitude of the magnetic fields are equal
d. the field at one point is opposite to that at the other point

32. A charged particle enters into a region which offers a resistance against its motion and a uniform magnetic field exists in the region. The particle traces a spiral path as shown in Fig. 9.319. Which of the following statements is/are correct?

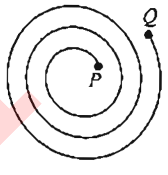


Fig. 9.319

a. Component of magnetic field in the plane of spiral is zero
b. The particle enters the region at Q
c. If magnetic field is outward, then the particle is positively charged
d. If magnetic field is outward, then the particle is negatively charged

33. An electron moves in a uniform magnetic field and follows a spiral path as shown in Fig. 9.320. Which of the following statements is/are correct?

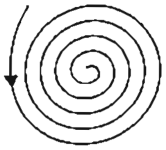


Fig. 9.320

a. Angular velocity of the electron remains constant
b. Magnitude of velocity of the electron decreases continuously
c. Net force on the particle is always perpendicular to its direction of motion
d. Magnitude of net force on the electron decreases continuously

34. A charged particle moves in a gravity free space where an electric field of strength E and a magnetic field of induction B exist. Which of the following statement is/are correct?

a. If $E \neq 0$ and $B \neq 0$, velocity of the particle may remain constant
b. If $E = 0$, the particle cannot trace a circular path

c. If $E = 0$, kinetic energy of the particle remains constant

d. None of these.

35. In a long current carrying cylindrical conductor of radius r, the current is distributed uniformly over its cross section. The magnetic field at a separation x from the axis of the conductor has magnitude B. Then,

a. $B = 0$, at the axis
b. $B \propto x$, for $0 \le x \le r$
c. $B \propto \frac{1}{x}$, for $x > r$
d. B is maximum for $x = r$

36. A charged particle of specific charge α moves with a velocity $\vec{v} = v_0\hat{i}$ in a magnetic field $\vec{B} = \frac{B_0}{\sqrt{2}}(\hat{j} + \hat{k})$. Then (specific charge = charge per unit mass)

a. path of the particle is a helix
b. path of the particle is circle
c. distance moved by the particle in time $t = \frac{\pi}{B_0\alpha}$ is $\frac{\pi v_0}{B_0\alpha}$
d. velocity of the particle after time $t = \frac{\pi}{B_0\alpha}$ is $\left(\frac{v_0}{2}\hat{i} + \frac{v_0}{2}\hat{j}\right)$

37. When a current carrying coil is placed in a uniform magnetic field with its magnetic moment anti-parallel to the field,

a. torque on it is maximum
b. torque on it is zero
c. potential energy is maximum
d. dipole is in unstable equilibrium

38. A charge particle of charge q and mass m is moving with velocity v as shown in Fig. 9.321 in a uniform magnetic field B along –ve z-direction. Select the correct alternative(s).

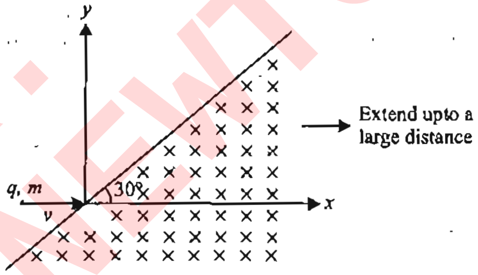


Fig. 9.321

a. Velocity of the particle when it comes out from the magnetic field is $\vec{v} = v\cos 60°\hat{i} + v\sin 60°\hat{j}$
b. Time for which the particle was in magnetic field is $\frac{\pi m}{3qB}$
c. Distance travelled in magnetic field is $\frac{\pi m v}{3qB}$
d. None of these

39. A particle of charge 'q' and mass 'm' enters normally (at point P) in a region of magnetic field with speed v. It comes out normally from Q after time T as shown in Fig. 9.322. The magnetic field B is present only in the region of radius R and is uniform. Initial and final velocities are along radial direction and they are perpendicular to each other. For this to happen, which of the following expression (s) is/are correct?

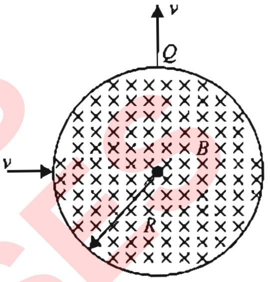


Fig. 9.322

a. $B = \frac{mv}{qR}$
b. $T = \frac{\pi R}{2v}$
c. $T = \frac{\pi m}{2qB}$
d. None of these

40. A wire of mass m and length ℓ is placed on a smooth incline making an angle θ with the horizontal, whose front view is shown in Fig. 9.323. When a finite amount of charge is passed through it in an infinitesimal time, the wire immediately acquires some velocity and then ascends the incline by a distance s. For this small duration, we can neglect the gravity force because the current can be considered very large due to small time duration. The amount of charge passed through the wire is

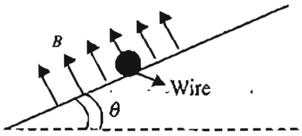


Fig. 9.323

a. $\frac{m\sqrt{2gs\sin\theta}}{B\ell}$
b. $\frac{mv}{B\ell}$
c. $\frac{m\sqrt{2gs\sin\theta}}{B\ell\cos\theta}$
d. information insufficient

41. A long straight wire carries the current along +ve x-direction. Consider four points in space $A(0, 1, 0)$, $B(0, 1, 1)$, $C(1, 0, 1)$ and $D(1, 1, 1)$. Which of the pairs will have same magnitude of magnetic field?

a. A and B
b. A and C
c. B and C
d. B and D

42. Two thin long wires carry currents I_1 and I_2 along x- and y-axes, respectively, as shown in Fig. 9.324. Consider the points only in x–y plane.

a. Magnetic field is zero at least at one point in each quadrant

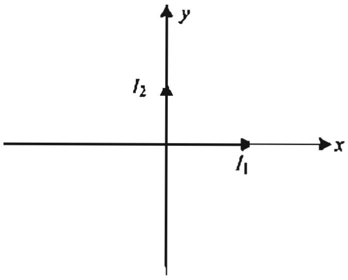


Fig. 9.324

b. Magnetic field can be zero somewhere in the first quadrant
c. Magnetic field can be zero somewhere in the second quadrant
d. Magnetic field is non-zero in second quadrant

43. In the setup of the previous question, an electron E moves from X to Y and a proton P moves from Y to Z. Both particles start from rest. Then,
  a. E reaches Y with greater energy than P
  b. P reaches Y with greater energy than E
  c. P and E reach Y with equal energies
  d. P reaches Y with greater momentum than E

44. Velocity and acceleration vector of a charged particle moving in a magnetic field at some instant are $\vec{v} = 3\hat{i} + 4\hat{j}$ and $\vec{v} = 2\hat{i} + x\hat{j}$. Select the correct alternative(s).
  a. $x = -1.5$
  b. $x = 3$
  c. magnetic field is along z-direction
  d. kinetic energy of the particle is constant

Assertion-Reasoning Type

Solutions on page 9.137

In each of the question, a statement of, Statement I is given by corresponding statement of Statement II of the statements, mark the correct answer.

a. If both Statement I and Statement II are true and the Statement II is the correct explanation of Statement I.
b. If both Statement I and Statement II are true but Statement II is not the correct explanation of Statement I.
c. If Statement I is true, but Statement II is false.
d. If Statement I is false but Statement II is true.

1. **Statement I:** Cyclotron is a device which is used to accelerate the position ions.
   Statement II: Cyclotron frequency depends upon the velocity.
2. **Statement I:** Cyclotron does not accelerate an electron.
   Statement II: Mass of the electrons is very small.
3. **Statement I:** When a charging particle is fired in a magnetic field, the radius of its circular path is directly proportional to the kinetic energy of the particle.
   Statement II: The centripetal force on the test change q_0 is q_0vB, where v is the velocity of a particle and B is the magnetic field.
4. **Statement I:** Magnetic field due to a infinite straight conductor varies inversely as the distance from it.
   Statement II: The magnetic field at the center of the circular coil in the following figure is zero
5. **Statement I:** The magnetic field at the center of the circular coil in Fig. 9.325 is zero.

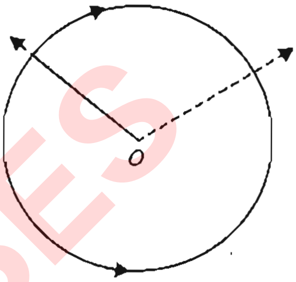


Fig. 9.325

   Statement II: $I_1 = I_2$ therefore the magnetic field due to one part balance that due to the other part of circle.
6. **Statement I:** Acceleration of a moving charged particle in a magnetic field is non-zero.
   Statement II: Inside magnetic field region, the particle may be moving on a curved path.
7. **Statement I:** Any rod of length ℓ moving with velocity v in a magnetic field B has an induced e.m.f. of $Bv\ell$.
   Statement II: Induced e.m.f. in a rod is given by $Bv\ell \sin\theta$.
8. **Statement II:** The net force on a closed circular current carrying loop placed in a uniform magnetic field is zero.
   Statement II: The torque produced in a conducting circular ring is zero when it is placed in a uniform magnetic field such that the magnetic field is perpendicular to the plane of loop.
9. **Statement I:** A rectangular current loop is in an arbitrary orientation in an external uniform magnetic field. No work is required to rotate the loop about an axis perpendicular to its plane.
   Statement II: All positions represent the same level of energy.
10. **Statement I:** Magnitude of $\vec{B}$ is constant along a magnetic field line.
    Statement II: $\vec{B}$ is tangent to a magnetic field line.
11. **Statement I:** If a proton and an α-particle enter a uniform magnetic field perpendicularly with the same speed, the time period of revolution of α-particle is double than that of proton.
    Statement II: In a magnetic field, the period of revolution of a charged particle is directly proportional to the mass of the particle and inversely proportional to the charge of particle.
12. **Statement I:** A direct current flows through a metallic rod. It produces magnetic field only outside the rod.
    Statement II: The charge carriers flow through whole of the cross section.

13. **Statement I:** A loosely bound helix made of stiff wire is suspended vertically with the lower end just touching a dish of mercury. When a current is passed though the wire, the wire executes oscillatory motion with the lower end jumping out of and into the mercury.

Statement II: When electric current is passed through a helix, a magnetic field is produced both inside and outside the helix.

14. **Statement I:** A charged particle is moving in a circular path under the action of a uniform magnetic field as shown in Fig. 9.326. During motion kinetic energy of charged particle is constant.

Statement II: During the motion, magnetic force acting on the particle is perpendicular to instantaneous velocity.

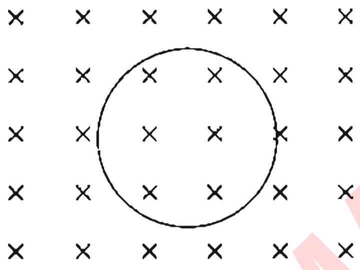


Fig. 9.326

15. **Statement I:** Consider the situation shown in the Fig. 9.327. A conductor is moved with constant velocity by an external agent. A force is required to move the conductor with constant velocity.

Statement II: As the conductor is moved, a current is induced in the circuit. A magnetic force acts on the conductor opposite to its velocity.

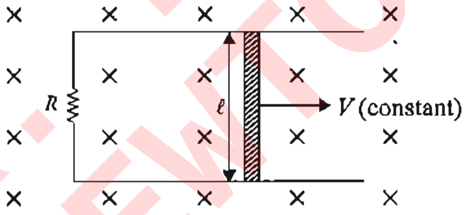


Fig. 9.327

16. **Statement I:** When radius of a circular loop carrying current is doubled, its magnetic moment becomes four times.

Statement II: Magnetic moment depends on the area of the loop.

17. **Statement I:** The poles of a magnet cannot be separated by breaking into two pieces.

Statement II: The magnetic moment will be reduced to half when a magnet is broken into two equal pieces.

18. **Statement I:** A magnetic field independent of time can change the velocity of a charged particle.

Statement II: It is not possible to change the velocity of a particle in a magnetic field as magnetic field does no work on the charged particle.

19. **Statement I:** The current constituted by electrons in a metallic wire creates only electric field while on electron beam creates both, electric and magnetic fields.

Statement II: The electron beam contains only electrons while metallic wire carries both positive and negative charges and also the wire is electrically neutral.

20. **Statement I:** The magnetic field on the closed loop in Fig. 9.328 is zero.

Statement II: Force (magnetic) on the wire is $\int d\vec{F} = \int i d\vec{\ell} \times \vec{B}$

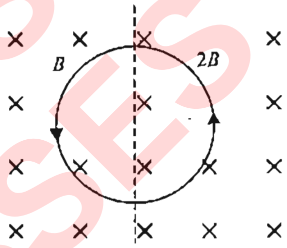


Fig. 9.328

21. **Statement I:** A closed current carrying loop behave like a magnetic dipole.

Statement II: Force and torque on the loop are zero as shown in Fig. 9.329.

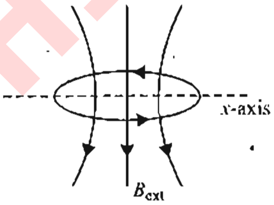


Fig. 9.329

22. **Statement I:** The magnetic field at the ends of a very long current carrying solenoid is half of that at the center.

Statement II: If the solenoid is sufficiently long, the field within it is uniform.

23. A charge is projected in a region of magnetic field (no other field is present).

Statement I: Kinetic energy of the charge particle will remain constant.

Statement II: Work done by magnetic force on the moving charge particle is zero.

24. A semicircular ring is present in a uniform magnetic field. Magnetic field is perpendicular to the loop of ring.

Statement I: Force $\vec{F}$ on each element of ring is different.

Statement II: Net force on the ring must be perpendicular to magnetic field.

25. **Statement I:** Magnetic field at a point on the surface of a long cylindrical wire is maximum.

Statement II: For any other point, closed loop perpendicular to the wire and of radius equal to the distance between the axis of wire and the given point will enclose less current.

26. **Statement I:** Magnitude of force acting on a current carrying loop placed in a uniform magnetic field will be equal to zero whether magnetic field is in the plane of the loop or perpendicular to it.

Statement II: Magnitude of force does not depend upon the direction of magnetic field.

27. Statement I: A linear solenoid carrying current is equivalent to a bar magnet.
Statement II: The magnetic field lines of both are same.

28. Statement I: Consider two small current carrying elements I and II as shown in Fig. 9.330. The magnetic force experienced by element I due to III is zero while magnetic force experie-nced by II due to I is non-zero, i.e., the force exerted by element I on element II is not equal and opposite to that exerted by element II on element I and hence Newton's action-reaction law gets violated here (for magnetic force).

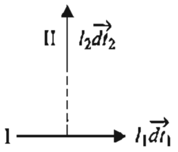


Fig. 9.330

Statement II: Law of conservation of momentum is an independent law and is not a consequence of Newton's third law.

Comprehension Type

Solutions on page 9.138

For Problems 1–2

The circuit in Fig. 9.331 consists of wires at the top and bottom and identical metal springs as the left and right sides. The wire at the bottom has a mass of 10.0 g and is 5.00 cm long. The wire is hanging as shown in the figure. The springs stretch 0.500 cm under the weight of the wire, and the circuit has a total resistance of 12.0 Ω. When a magnetic field is turned on, the springs stretch an additional 0.300 cm.

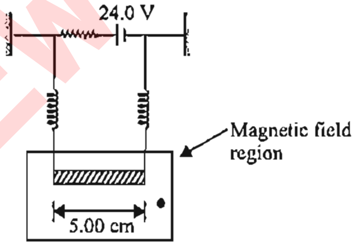


Fig. 9.331

1. From the above statements we can conclude that
   a. the magnetic field is directed into the plane of page
   b. the magnetic field is directed out of the page
   c. the magnetic field is toward left in the plane of page
   d. the magnetic field is toward right in the plane of page
2. The magnitude of magnetic field is
   a. 1.2 T
   b. 6 T
   c. 0.6 T
   d. 12 T

For Problems 3–5

A particle of mass m and charge q is accelerated by a potential difference V volt and made to enter a magnetic field region at an angle θ with the field. At the same moment, another particle of same mass and charge is projected in the direction of the field from the same point. Magnetic field of induction is B.

3. What would be the speed of second particle so that both particles meet again and again after a regular interval of time, which should be minimum?
   a. $\sqrt{\frac{qv}{m}}\cos\theta$
   b. $\sqrt{\frac{2qv}{m}}\cos\theta$
   c. $\sqrt[2]{\frac{qv}{m}}\sin\theta$
   d. $\sqrt[2]{\frac{qv}{m}}\cos\theta$
4. Find the time interval after which they meet.
   a. $\frac{2\pi m}{qB}$
   b. $\frac{\pi m}{2qB}$
   c. $\frac{\pi m}{qB}$
   d. $\frac{3\pi m}{2qB}$
5. Find the distance travelled by the second particle during that interval mentioned in the above problem.
   a. $\sqrt{\frac{vm}{q}}\frac{2\pi}{B}\cos\theta$
   b. $\sqrt{\frac{2vm}{3q}}\frac{2\pi}{B}\cos\theta$
   c. $\sqrt{\frac{2vm}{q}}\frac{2\pi}{B}\cos\theta$
   d. $\frac{2}{3}\sqrt{\frac{vm}{q}}\frac{\pi}{m}\cos\theta$

For Problems 6–7

A charged particle carrying charge $q = 10\ \mu C$ moves with velocity $v_1 = 10^6\ ms^{-1}$ at angle 45° with x-axis in the xy plane and experiences a force $F_1 = 5\sqrt{2}$ mN along the negative z-axis. When the same particle moves with velocity $v_2 = 10^6\ ms^{-1}$ along the z-axis, it experiences a force F_2 in y-direction.

6. Find the magnetic field $\vec{B}$.
   a. $(10^{-3}T)(\hat{i}+\hat{j})$
   b. $(2\times10^{-3}T)\hat{i}$
   c. $(10^{-3}T)\hat{i}$
   d. $(2\times10^{-3}T)\ (\hat{i}+\hat{j})$
7. Find the magnitude of the force F_2.
   a. 10^{-2} N
   b. 10^{-3} N
   c. 10^{-4} N
   d. 10^{-5} N

For Problems 8–10

A conducting ring of mass m and radius r has a weightless conducting rod PQ of length $2r$ and resistance $2R$ attached to it along its diameter. It is pivoted at its center C with its plane vertical, and two blocks of mass m and $2m$ are suspended by means of a light in-extensible string passing over it as shown in Fig. 9.332. The ring is free to rotate about C and the system is placed in a magnetic field B (into the plane of the ring). A circuit is now completed by connecting the ring at A and C to a battery of e.m.f. V. It is found that for certain value of V, the system remains static.

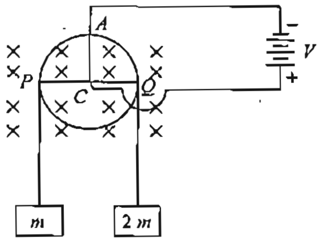


Fig. 9.332

8. In static condition, find the current through rod PC.
   a. V/R
   b. $V/2R$
   c. $4V/R$
   d. $2V/R$

9. Net torque applied by the tension in string can be related as
   a. $\frac{3BVr^2}{R}$
   b. $\frac{BVr^2}{R}$
   c. $\frac{BVr^2}{3R}$
   d. $\frac{BVr^2}{2R}$

10. The value of V can be related with m, B and r as
   a. $2mgR/Br$
   b. mgR/Br
   c. $mgR/2Br$
   d. $3mgR/Br$

For Problems 11–12

There exists a long conductor along z-axis carrying a current of I_0 along positive z-direction. A loop having total resistance R is placed symmetrical about x- and y-axes in xy plane as shown in Fig. 9.333. Potential difference $V_{BA} = V$ is applied. Radii of arcs are 'a' and 'b', respectively, as shown in the figure.

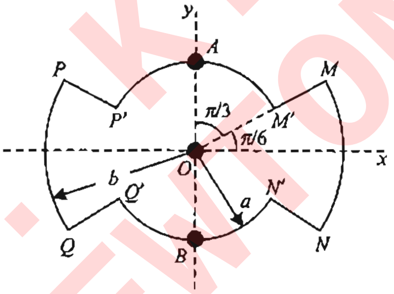


Fig. 9.333

11. The magnitude of force experienced by the arc MN is
   a. zero
   b. $\frac{\mu_0 VI_0}{\pi Rb}$
   c. $\frac{\mu_0 VI_0}{2\pi Rb}$
   d. none of these

12. The total torque acting on the loop is nearly.
   a. $\frac{\mu_0 V I_0}{\pi R}(b-a)\hat{i}$
   b. $-2\frac{\mu_0 V I_0}{\pi R}(b-a)\hat{i}$
   c. $\frac{\mu_0 V I_0}{2\pi R}(b-a)\hat{i}$
   d. none of these

For Problems 13–14

In a region, magnetic field along x-axis changes according to the graph given in Fig. 9.334:

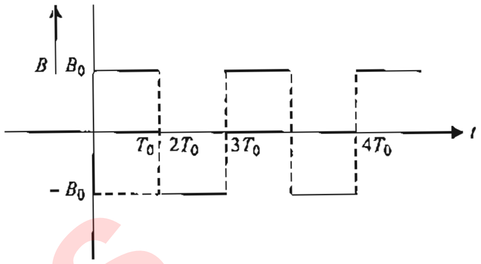


Fig. 9.334

If time period, pitch and radius of helix path are T_0, P_0 and R_0, respectively, and if the particle is projected at an angle θ_0 with the positive x-axis toward positive y-axis in x–y plane, then

13. select the correct statement:
   a. At $t = \frac{T_0}{2}$, coordinates of charge are $\left(\frac{P_0}{2}, 0, -2R_0\right)$
   b. At $t = \frac{3T_0}{2}$, coordinates of charge are $\left(\frac{3P_0}{2}, 0, 2R_0\right)$
   c. At $t = \frac{T_0}{2}$, coordinates of charge are $(P_0, 0, -2R_0)$
   d. At $t = \frac{3T_0}{2}$, coordinates of charge are $(3P_0, 0, 2R_0)$

14. select the correct statement:
   a. Two extremes (positions of charge particle during the motion) from x-axis are at a distance $2R_0$ from each other
   b. Two extremes from x-axis are at a distance $4R_0$ from each other
   c. Two extremes (positions of charge particle during the motion) from x-axis are at a distance R_0 from each other
   d. Two extremes from x-axis are at a distance $3R_0$ from each other

For Problems 15–16

The region between $x = 0$ and $x = L$ is filled with uniform constant magnetic field $25(\text{T})\hat{k}$. A particle of mass m = 50 g having positive charge q = 1 C and velocity $\vec{V}_0 = 50\sqrt{3}\,\hat{i} + 50\,\hat{j}$ ms^{-1} enters the region of the magnetic field. Neglect gravity throughout the question.

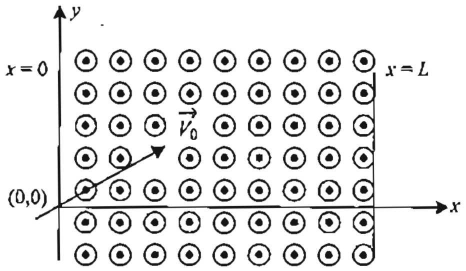


Fig. 9.335

15. The value of L if the particle emerges from the region of magnetic field with its velocity $100(\text{ms}^{-1})\hat{i}$ is

a. 20 cm
b. 10 cm
c. 30 cm
d. None of these

16. The maximum and minimum values of L such that velocity of emerging particle makes angle $\pi/3$ with the y-axis are

a. $L_{min} = 10$ cm, $L_{max} = \infty$
b. $L_{min} = 20$ cm, $L_{max} = 40$ cm
c. $L_{min} = 10$ cm, $L_{max} = 20$ cm
d. $L_{min} = 20$ cm, $L_{max} = \infty$

For Problems 17–19

A charged particle enters into a uniform magnetic field and follows a spiral path as shown in Fig. 9.336. The arrows in the diagram represent the motion of the charged particle. For this situation, answer the following questions.

Fig. 9.336

17. Mark out the correct statement(s).

a. If the charged particle is +ve, then direction of magnetic field is into the paper
b. Net force acting on the particle is not perpendicular to its direction of motion
c. The speed of particle does not remain constant
d. All of the above

18. Which of the following quantities associated with the particle continuously decreases during motion?

a. Speed
b. Angular momentum
c. Net force acting on the particle
d. All of the above

19. Which set of the quantities is remaining conserved during the motion of particle?

a. Angular velocity, angular momentum, kinetic energy
b. Angular velocity
c. Angular velocity, total mechanical energy, magnitude of net force
d. Angular momentum, mechanical energy

For Problems 20–22

Curves in the graph shown in Fig. 9.337 give, as functions of radius distance r, the magnitude B of the magnetic field inside and outside four long wires a, b, c and d, carrying currents that are uniformly distributed across the cross sections of the wires. Overlapping portions of the plots are indicated by double labels.

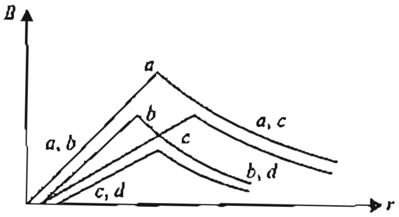


Fig. 9.337

20. Which wire has the greatest radius?

a. a
b. b
c. c
d. d

21. Which wire has the greatest magnitude of the magnetic field on the surface?

a. a
b. b
c. c
d. d

22. The current density in wire a is

a. greater than in wire c
b. less than in wire c
c. equal to that in wire c
d. not comparable to that of in wire c due to lack of information

For Problems 23–25

Ampere's law provides us an easy way to calculate the magnetic field due to a symmetrical distribution of current. Its mathematical expression is as $\oint \vec{B} \cdot d\ell = \mu_0 I_{in}$

The quantity on the left hand side is known as line integral of magnetic field over a closed Ampere's loop.

23. Only the current inside the Amperian loop contributes in

a. finding magnetic field at any point on the Ampere's loo
b. line integral of magnetic field
c. in both of the above
d. in neither of them

24. If the current density in a linear conductor of radius a varie with r according to relation $J = kr^2$, where k is a constan and r is the distance of a point from the axis of conducto find the magnetic field induction at a point distance r fro the axis when $r < a$. Assume relative permeability of th conductor to be unity.

a. $\frac{\mu_0 k \pi a^4}{4r}$
b. $\frac{\mu_0 k r^3}{2}$
c. $\frac{\mu_0 k \pi a^4}{2r}$
d. $\frac{\mu_0 k r^3}{2}$

25. In the above question, find the magnetic field induction at point distance r from the axis when $r > a$. Assume relativ permeability of the medium surrounding the conductor t be unity.

a. $\frac{\mu_0 k \pi a 4}{4r}$
b. $\frac{\mu_0 k r^3}{2}$
c. $\frac{\mu_0 k \pi a^4}{2r}$
d. $\frac{\mu_0 k r^3}{4}$

For Problems 26–27

According to Biot–Savarat's Law, magnetic field due to a straig current carrying wire at a point at a distance r from it is given

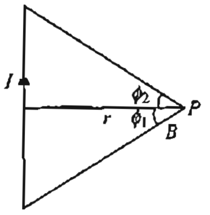


Fig. 9.338

$$B = \frac{\mu_0 I}{4\pi r}(\sin\phi_1 + \sin\phi_2)$$

The direction of magnetic field being perpendicular to the plane containing the wire and that point.

26. Fig. 9.339 shows a closed loop $AOCBA$ in which current I is flowing as shown. Given $OA = OB = OC = a$. Find the magnetic field at point B due to this loop.

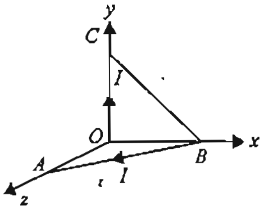


Fig. 9.339

a. $-\frac{\mu_0 I}{4\pi\sqrt{2}a}(\hat{i}+\hat{k})$ **b.** $-\frac{\mu_0 I}{4\pi\sqrt{2}a}(\hat{j}+\hat{k})$

c. $\frac{-\mu_0 I}{2\sqrt{2}\pi a}(\hat{j}+\hat{k})$ **d.** None of these

27. Find magnetic field at point O in Fig. 9.314.

a. $-\frac{\mu_0 I}{4\pi a}(\hat{i}+\hat{k})$ **b.** $-\frac{\mu_0 I}{4\pi a}(\hat{j}+\hat{k})$

c. $-\frac{-\mu_0 I}{2\pi a}(\hat{j}+\hat{k})$ **d.** $\frac{-\mu_0 I}{2\sqrt{2}\pi a}(\hat{j}+\hat{k})$

For Problems 28–29

In Fig. 9.340, the circular and the straight parts of the wire are made of same material but have different diameters. The magnetic field at the center is zero.

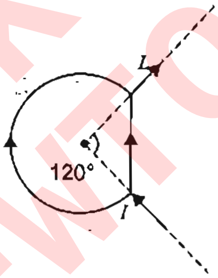


Fig. 9.340

28. Ratio of the currents I_1 and I_2 flowing through the circular and straight parts is

a. $\frac{\sqrt{3}}{2\pi}$ **b.** $\frac{2\sqrt{3}}{\pi}$ **c.** $\frac{3\sqrt{3}}{2\pi}$ **d.** $\frac{3\sqrt{3}}{2\sqrt{2}\pi}$

29. The ratio of the diameters of circular and straight parts is

a. $\frac{\sqrt{3}}{2\pi}$ **b.** $\frac{2\sqrt{3}}{\pi}$ **c.** $\frac{3\sqrt{3}}{2\pi}$ **d.** $\frac{3\sqrt{3}}{2\sqrt{2}\pi}$

For Problems 30–31

A charged particle of mass m and charge q is projected on a rough horizontal X–Y plane, both electric and magnetic fields are acting in the region and given by $\vec{E} = -E_0\hat{k}$ and $\vec{B} = -B_0\hat{k}$, respectively. At $t = 0$, the particle enters into the field at $(A_0, 0, 0)$ with velocity $\vec{v} = v_0\hat{j}$. The particle starts moving into a circular path on the plane. If coefficient of friction between the particle and the plane is μ, then calculate the

30. time when the particle will come to rest.

a. $\frac{mv_0}{\mu(mg+qE)}$ **b.** $\frac{2mv_0}{\mu(mg+2qE)}$

c. $\frac{3mv_0}{\mu(2mg+qE)}$ **d.** None of these

31. length of the path travelled by the particle when it comes to rest.

a. $\frac{mv_0^2}{\mu(mg+qE)}$ **b.** $\frac{3mv_0^2}{2\mu(mg+qE)}$

c. $\frac{mv_0^2}{2\mu(mg+qE)}$ **d.** None of these

For Problems 32–33

A charged particle carrying charge $q = 10\ \mu\text{C}$ moves with velocity $v_1 = 10^6\ \text{ms}^{-1}$ at angle 45° with x-axis in the xy plane and experiences a force $F_1 = 5\sqrt{2}$ mN along the negative z-axis. When the same particle moves with velocity $v_2 = 10^6\ \text{ms}^{-1}$ along the z-axis, it experiences a force F_2 in y-direction.

32. Find the magnetic field $\vec{B}$.

a. $(10^{-3}\text{T})(\hat{i}+\hat{j})$ **b.** $(2\times10^{-3}\text{T})\hat{i}$

c. $(10^{-3}\text{T})\hat{i}$ **d.** $(2\times10^{-3}\text{T})(\hat{i}+\hat{j})$

33. Find the magnitude of the force F_2.

a. 10^{-2} N **b.** 10^{-3} N

c. 10^{-4} N **d.** 10^{-5} N

For Problems 34–36

In a certain region of space, there exists a uniform and constant electric field of magnitude E along the positive y-axis of a co-ordinate system. A charged particle of mass 'm' and charge '$-q$' ($q > 0$) is projected from the origin with speed $2v$ at an angle of 60° with the positive x-axis in x–y plane. When the x-coordinate of particle becomes $\frac{\sqrt{3}\,mv^2}{qE}$, a uniform and constant magnetic field of strength B is also switched on along positive y-axis.

34. Velocity of the particle just before the magnetic field is switched on is

a. $v\hat{i}$ **b.** $v\hat{i}+\frac{\sqrt{3}v}{2}\hat{j}$

c. $v\hat{i}-\frac{\sqrt{3}v}{2}\hat{j}$ **d.** $2v\hat{i}-\frac{\sqrt{3}v}{2}\hat{j}$

35. x-coordinate of the particle as a function of time after the magnetic field is switched on is

a. $\frac{\sqrt{3}mv^2}{qE} - R\sin\left(\frac{qB}{m}t\right)$ **b.** $\frac{\sqrt{3}mv^2}{qE} + R\sin\left(\frac{qB}{m}t\right)$

c. $\frac{\sqrt{3}mv^2}{qE} - R\cos\left(\frac{qB}{m}t\right)$ **d.** $\frac{\sqrt{3}mv^2}{qE} + R\cos\left(\frac{qB}{m}t\right)$

36. z-coordinate of the particle as a function of time after the magnetic field is switched on is

a. $R\left[1+\cos\left(\frac{qB}{m}t\right)\right]$ **b.** $R\left[1+\sin\left(\frac{qB}{m}t\right)\right]$

c. $-R\left[1-\cos\left(\frac{qB}{m}t\right)\right]$ **d.** $-R\left[1+\cos\left(\frac{qB}{m}t\right)\right]$

For Problems 37–40

A particle of mass m and charge q enters with velocity v_0 perpendicular to a magnetic field B (coming out of the plane of the paper) as shown in Fig. 9.341. It moves in the magnetic field for $t = \frac{\pi m}{4qB}$ and then enters into a constant electric field region.

The electric and magnetic fields are present only in a rectangular region of thickness d. The length of rectangular region is ℓ, the particle enters parallel to and grazing side RQ. The particle leaves the region at P.

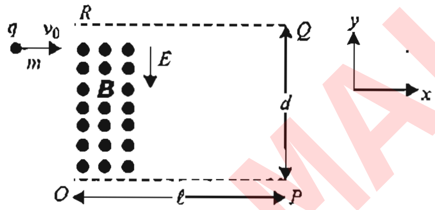


Fig. 9.341

Given $v_0 = (\sqrt{2}+1)\,\text{ms}^{-1}$,

$E/B = 8\ \text{ms}^{-1}$, $l = \left|\frac{4\sqrt{2}}{5}v_0\right|$ A metres and $\frac{m}{dB} = \frac{4}{5}$

37. Displacement of the particle in x-direction before it enters in the electric field is

a. $\frac{3}{5}\left(2+\frac{1}{\sqrt{2}}\right)$ m **b.** $\frac{4}{5}\left(1+\frac{1}{\sqrt{2}}\right)$ m

c. $\frac{4}{5}\left(1-\frac{3}{\sqrt{2}}\right)$ m **d.** none of these

38. Length of the electric field region is

a. $\frac{3}{5}\left(2+\frac{1}{\sqrt{2}}\right)$ m **b.** $\frac{4}{5}\left(1+\frac{1}{\sqrt{2}}\right)$ m

c. $\frac{4}{5}\left(1-\frac{3}{\sqrt{2}}\right)$ m **d.** none of these

39. Time taken by the particle to cross the electric field region is

a. 4/5 s **b.** 3/5 s

c. 1/5 s **d.** none of these

40. The value of ℓ–d is equal to

a. 3 m **b.** 4 m

c. 2 m **d.** 1 m

For Problems 41–43

Two long, straight, parallel wires are 1.00 m apart (as shown in Fig. 9.342). The upper wire carries a current I_1 of 6.00 A into the plane of the paper.

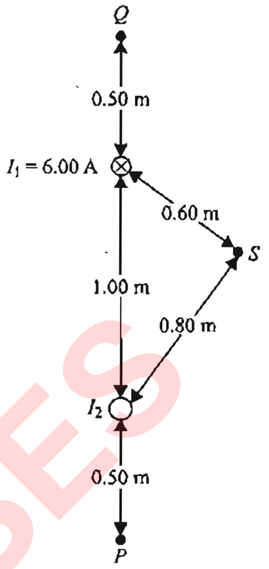


Fig. 9.342

41. What must the magnitude and direction of the current I_2 be for the net field at point P to be zero?

a. 3.00 A **b.** $\sqrt{3}$ A

c. 2.00 A **d.** 1.00 A

42. What are the magnitude and direction of the net field at Q ?

a. 2.13×10^{-6} T **b.** 4.26×10^{-6} T

c. 1.21×10^{-6} T **d.** 5.30×10^{-6} T

43. What is the magnitude of the net field at S ?

a. 4.1×10^{-6} T **b.** 1.6×10^{-6} T

c. 3.2×10^{-6} T **d.** 2.1×10^{-6} T

For Problems 44–46

Fig. 9.343 shows an end view of two long, parallel wires perpendicular to the xy plane, each carrying a current I, but in opposite directions.

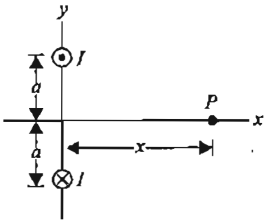


Fig. 9.343

44. Derive the expression for the magnitude of $\vec{B}$ at any point on the x-axis in terms of the x-coordinate of the point. What is the direction of $\vec{B}$?

a. $\frac{\sqrt{2}\mu_0 Ia}{\pi(x^2+a^2)}$ **b.** $\frac{3\mu_0 Ia}{\pi(x^2+a^2)}$

c. $\frac{\mu_0 Ia}{2\pi(x^2+a^2)}$ **d.** $\frac{\mu_0 Ia}{\pi(x^2+a^2)}$

45. Graph the magnitude of $\bar{B}$ at points on the x-axis.

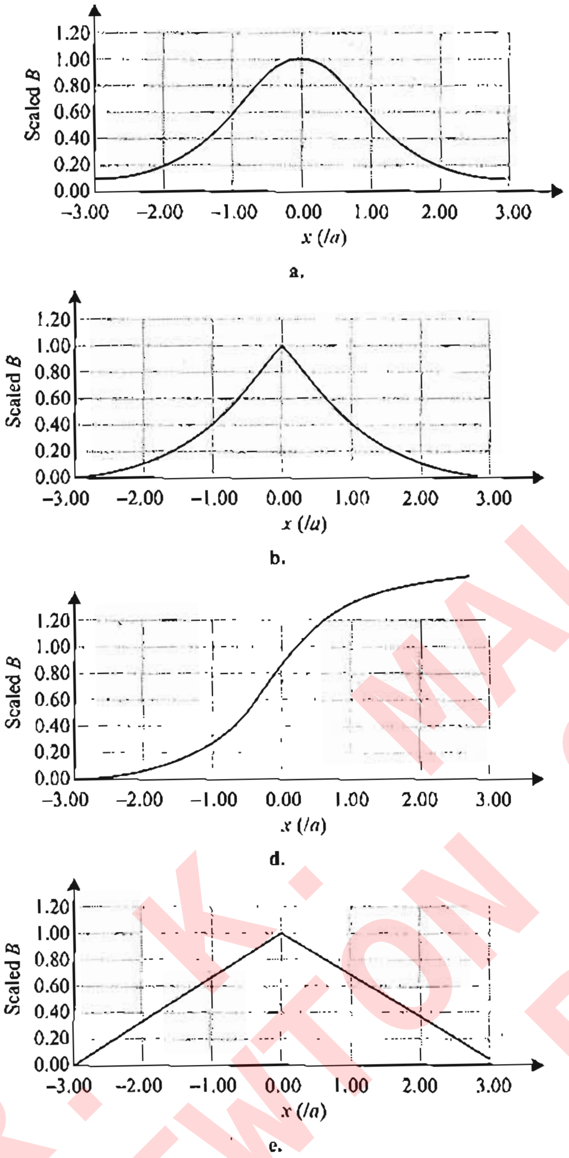


Fig. 9.344

46. At what value of x is the magnitude of $\vec{B}$ a maximum ?

a. $x=0$ **b.** $x=\sqrt{2}\ a$ **c.** $x=1/\sqrt{2}\ a$ **d.** $x=a/2$

For Problems 47–49

Repeat the above problem, but with the current in both wires shown in Fig. 9.345 directed into the plane of the figure.

47. Derive the expression for the magnitude of $\vec{B}$ at any point on the x-axis in terms of the x-coordinate of the point. What is the direction of $\vec{B}$?

a. $\dfrac{\sqrt{2}\,\mu_0 Ia}{\pi\left(x^2+a^2\right)}$ **b.** $\dfrac{3\,\mu_0 Ia}{\pi\left(x^2+a^2\right)}$

c. $\dfrac{\mu_0 Ia}{2\pi\left(x^2+a^2\right)}$ **d.** $\dfrac{\mu_0 Ia}{\pi\left(x^2+a^2\right)}$

48. Graph the magnitude of $\bar{B}$ at points on the x-axis.

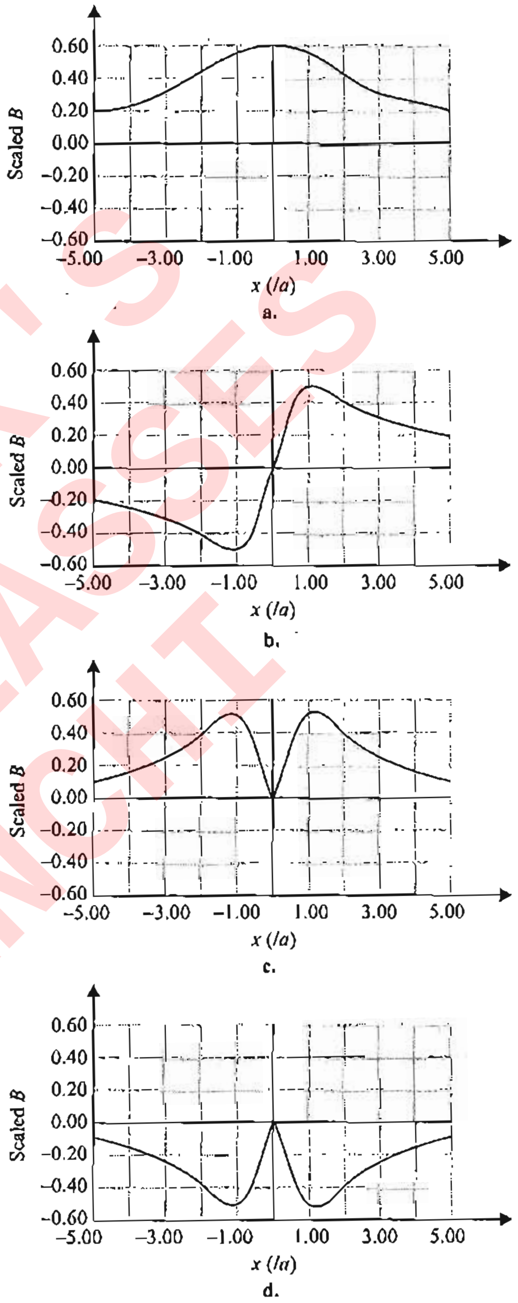


Fig. 9.345

49. At what value of x is the magnitude of $\bar{B}$ a maximum ?

a. $x=0$ **b.** $x=\sqrt{2}\ a$

c. $x=\pm a/2$ **d.** $x=\pm a$

For Problems 50–52

An electron with a speed $v_0 << c$ moves in a circle of radius r_0 in a uniform magnetic field. The time required for one revolution of the electron is T_0. The speed of the electron is now doubled to $2v_0$.

50. The radius of the circle will change to

a. $4r_0$ b. $2r_0$ c. r_0 d. $r_0/2$

51. The time required for one revolution of the electron will change to

a. $4T_0$ b. $2T_0$ c. T_0 d. $T_0/2$

52. A charged particle is projected in a magnetic field $\vec{B}=(2\hat{i}+4\hat{j})\times10^{-2}$T. The acceleration of the particle is found to be $\vec{a}=(x\hat{i}+2\hat{j})$ ms^{-2}. Find the value of x.

a. 4 ms^{-2} b. – 4 ms^{-2} c. –2 ms^{-2} d. 2 ms^{-2}

For Problems 53–54

Uniform electric and magnetic fields with strength E and induction B, respectively, are along y-axis as shown in Fig. 9.346. A particle with specific charge q/m leaves the origin O in the direction of x-axis with an initial non-relativistic velocity v_0.

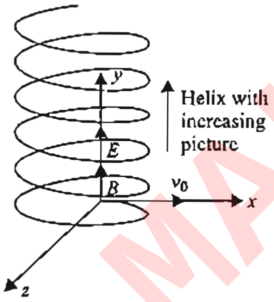


Fig. 9.346

53. The coordinate y_n of the particle when it crosses the y-axis for the n^{th} time is

a. $\dfrac{2\pi^2mn^2E}{qB^2}$ b. $\dfrac{\pi^2mn^2E}{qB^2}$

c. $\dfrac{2\pi^2mn^2E}{3qB^2}$ d. $\dfrac{\sqrt{3}\,\pi^2mn^2E}{qB^2}$

54. The angle α between the particle's velocity vector and y-axis at that moment is

a. $\tan^{-1}\left(\dfrac{3v_0B}{2\pi nE}\right)$ b. $\tan^{-1}\left(\dfrac{v_0B}{\pi nE}\right)$

c. $\tan^{-1}\left(\dfrac{v_0B}{\sqrt{2}\pi nE}\right)$ d. $\tan^{-1}\left(\dfrac{v_0B}{2\pi nE}\right)$

For Problems 55–56

A thin, 50.0 cm long metal bar with mass 750 g rests on, but is not attached to, two metallic supports in a uniform 0.450 T magnetic field, as shown in Fig. 9.347. A battery and a 25.0 Ω resistor in series are connected to the supports.

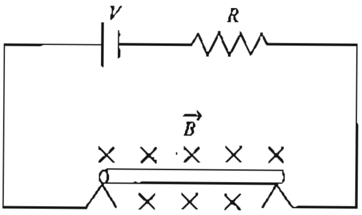


Fig. 9.347

55. What is the largest voltage the battery can have without breaking the circuit at the supports?

a. 817 V b. 412 V c. 325 V d. 160 V

56. The battery voltage has the maximum value calculated in Q.55. If the resistor suddenly gets partially short-circuited, decreasing its resistance to 2.0 Ω, find the initial acceleration of the bar.

a. 113 ms^{-2} b. 55 ms^{-2} c. 180 ms^{-2} d. 12.4 ms^{-2}

For Problems 57–58

The circuit shown in Fig. 9.323 is used to make a magnetic balance to weigh objects. The mass m to be measured is hung from the center of the bar that is in a uniform magnetic field of 1.50 T, directed into the plane of the figure. The battery voltage can be adjusted to vary the current in the circuit. The horizontal bar is 60.0 cm long and is made of extremely light-weight material. It is connected to the battery by thin vertical wires that can support no appreciable tension; all the weight of the suspended mass m is supported by the magnetic force on the bar. A resistor with R = 5.00 Ω is in series with the bar; the resistance of the rest of the circuit is much less than this.

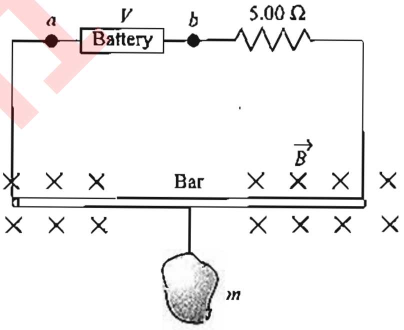


Fig. 9.348

57. Which point, a or b, should be the positive terminal of the battery?

a. a b. b

c. Either a or b

d. No conclusion can be drawn

58. If the maximum terminal voltage of the battery is 175 V, wha is the greatest mass m that this instrument can measure ?

a. 10.2 kg b. 3.21 kg c. 20.4 kg d. 5.2 kg

For Problems 59–60

A conducting bar with mass m and length L slides over horizonta rails that are connected to a voltage source. The voltage sourc maintains a constant current I in the rails and bar, and a constant uniform, vertical magnetic field $\vec{B}$ fills the region between the rail (as shown in Fig. 9.349).

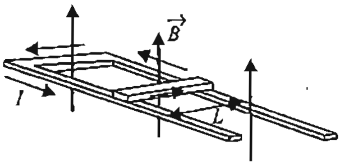


Fig. 9.349

59. Find the magnitude and direction of the net force on the conducting bar. Ignore friction, air resistance and electrical resistance.

a. ILB, to the right. b. ILB, to the left.
c. $2ILB$, to the right. d. $2ILB$, to the left.

60. If the bar has mass m, find the distance d that the bar must move along the rails from rest to attain speed v.

a. $\frac{3v^2m}{2ILB}$ b. $\frac{5v^2m}{2ILB}$ c. $\frac{v^2m}{ILB}$ d. $\frac{v^2m}{2ILB}$

For Problems 61–62

As shown in Fig. 9.350, a thin, flexible wire hangs from point P in a region where there is a uniform, horizontal, magnetic field of magnitude B directed into, and perpendicular to, the plane of the figure. A weight is attached to the bottom of the wire to provide a uniform tension T throughout the wire. (The weight of the wire itself is negligible.) When a current I flows from the top to bottom of wire, the wire curves into a circular arc of radius R.

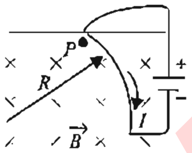


Fig. 9.350

61. By considering the forces on a small segment of wire that subtends an angle θ, find the radius of curvature of the wire.

a. $R = 2T/IB$ b. $R = T/3IB$
c. $R = \sqrt{2}\,T/B$ d. $R = T/IB$

62. The wire is now removed. A negatively charged particle of charge $-q$ and mass m is launched from the same point P from which the wire was hung, in the same direction as that in which the wire extended from P. If the trajectory of the particle follows the same circular arc as in previous problem, the speed of the particle will be

a. Tq/mI b. $2Tq/mI$
c. $Tq/3mI$ d. $\sqrt{2}Tq/mI$

For Problems 63–64

A thin, uniform rod with negligible mass and length 0.200 m is attached to the floor by a frictionless hinge at point P (as shown in Fig. 9.351). A horizontal spring with force constant $k = 4.80$ Nm^{-1} connects the other end of the rod to a vertical wall. The rod is in a uniform magnetic field $B = 0.340$ T directed into the plane of the figure. There is current $I = 6.50$ A in the rod, in the direction shown.

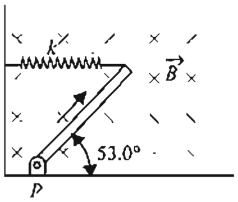


Fig. 9.351

63. Calculate the torque due to the magnetic force on the rod, for an axis at P.

a. 0.0442 Nm^{-1}, clockwise
b. 0.0442 Nm^{-1}, anticlockwise
c. 0.022 Nm^{-1}, clockwise
d. 0.022 Nm^{-1}, anticlockwise

64. When the rod is in equilibrium and makes an angle of 53.0° with the floor, is the spring stretched or compressed?

a. 0.05765 m, stretched
b. 0.05765 m, compressed
c. 0.0242 m, stretched
d. 0.0242 m, compressed

For Problems 65–66

At your laboratory's location, the earth's magnetic field has magnitude 1.0×10^{-4} T and points north. A long, straight wire (wire A) runs in a north-south direction along the laboratory floor. You know that the wire carries current from south to north, but you do not know the magnitude of the current. To find out, you place two other long, straight wires on the floor parallel to wire A. Wire B is 5.0 cm to the east of wire A and wire C is 10.0 cm to the east of wire B. You connect sources of e.m.f. to wires B and C so that a constant current of 1.0 A flows from south to north in wire B, while an adjustable amount of current flows in wire C. Wires A and C are attached rigidly to the floor, while wire B is able to slide around. You find by experiment that wire B will indeed start to slide one way or the other unless there is a 3.0 A current in wire C that flows from south to north. If this is the case, wire B remains at rest.

65. How much current is flowing in wire A?

a. 2 A b. 1.5 A
c. 2.5 A d. 3 A

66. If the current in wire C were increased to direction would wire B tend to slide?

a. Wire B will slide to the right
b. Wire B will slide to the left
c. Wire B will slide up the page
d. Wire B will slide down the page

For Problems 67–69

A wire carrying a 10 A current is bent to pass through various sides of a cube of side 10 cm as shown in Fig. 9.352. A magnetic field $\vec{B} = (2\hat{i} - 3\hat{j} + \hat{k})$T is present in the region. Then, find

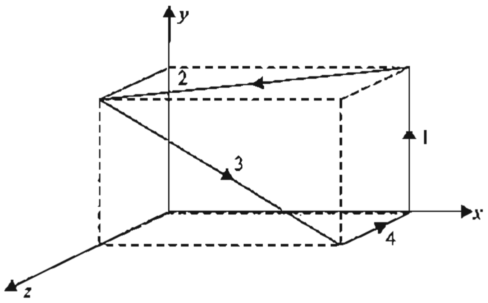


Fig. 9.352

67. the net force on the loop shown.

a. $\vec{F}_{net} = 0$ b. $\vec{F}_{net} = (0.1\hat{i} - 0.2\hat{k})$ N

c. $\vec{F}_{net} = (0.3\hat{i} + 0.4\hat{k})$ N d. $\vec{F}_{net} = (0.36\hat{k})$ N

68. the magnetic moment vector of the loop.

a. $(0.1\hat{i} + 0.05\hat{j} - 0.05\hat{k})$ Am^2

b. $(0.1\hat{i} + 0.05\hat{j} + 0.05\hat{k})$ Am^2

c. $(0.1\hat{i} - 0.05\hat{j} + 0.05\hat{k})$ Am^2

d. $(0.1\hat{i} - 0.05\hat{j} - 0.05\hat{k})$ Am^2

69. the net torque on the loop.

a. $-0.1\hat{i} + 0.4\hat{k}$ Nm b. $-0.1\hat{i} - 0.4\hat{k}$ Nm

c. $0.1\hat{i} - 0.4\hat{k}$ Nm d. $0.1\hat{i} - 0.4\hat{k}$ Nm

Matching Column Type

Solutions on page 9.145

Column I and Column II contains four entries each. Entries of Column I are to be matched with some entries of Column II. One or more than one entries of Column I may have the matching with the same entries of Column II and one entry of Column I may have one or more than one matching with entries of Column I.

1. A long current carrying wire and a loop made of conducting wire are placed in x–y plane, such that the long wire is parallel to y-axis. Column I is regarding some changes made in the position of loop and Column II indicates the resulting effects.

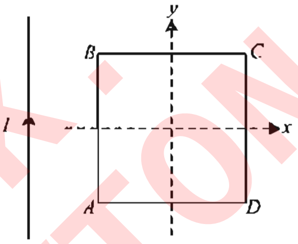
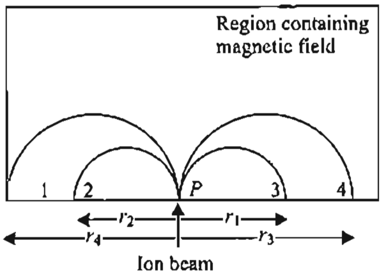


Fig. 9.353

Match the columns:

Column I	Column II
i. If loop is moved away from the wire while keeping in x–y plane,	a. current is induced in the loop in anticlockwise direction
ii. If loop is moved toward the wire while keeping in x–y plane	b. current is induced in the loop in clockwise direction
iii. If loop is rotated about x-axis, then just after this	c. no e.m.f. is induced in the loop
iv. If loop is rotated about y-axis, then just after this	d. the wire will attract or repel the loop

2. A beam consisting of four types of ions A, B, C and D enters a region at P that contains a uniform magnetic field as shown in Fig. 9.354. The field is perpendicular to the plane of the paper, but its precise direction is not given. All ions in the beam travel with the same speed. The following table shows the masses and charges of the ions.



$r_4 > r_3 = r_2 > r_1$

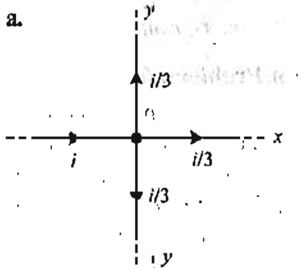
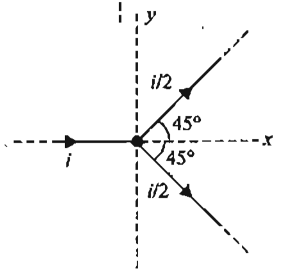
Fig. 9.354

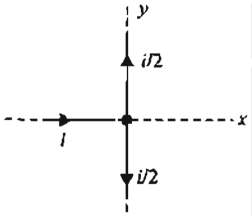
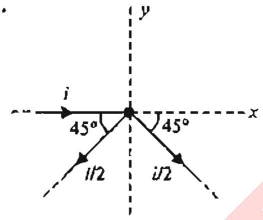
Ion	Mass	Charge
A	$2m$	e
B	$4m$	−e
C	$2m$	−e
D	m	+e

The ions fall at different positions 1, 2, 3 and 4 as shown. Correctly match the ions with their respective positions of fall.

Column I	Column II
i. A	a. 1
ii. B	b. 2
iii. C	c. 3
iv. D	d. 4

3. Column II gives four situations in which three or four semi-infinite current carrying wires are placed in xy plane as shown. The magnitude of the direction of current is shown in each figure. Column I gives statements regarding the x- and y-components of magnetic field at a point P whose coordinates are (0, 0, d). Match the statements in Column I with the corresponding figures in Column II.

Column I	Column II
i. The x-component of magnetic field at point P is zero in	a. 
ii. The z-component of magnetic field at point P is zero in	b. 

iii. The magnitude of magnetic field at point P is $\frac{\mu_0 i}{4\pi d}$ in	**c.** 
iv. The magnitude of magnetic field at point P is less than $\frac{\mu_0 i}{4\pi d}$ in	**d.**  **Fig. 9.330**

4. A square loop of uniform conducting wire is as shown in Fig. 9.355. A current I (in amperes) enters the loop from one end and exits the loop from opposite end as shown. The length of one side of the square loop is ℓ meter. The wire has uniform cross-section area and uniform linear mass density. In four situations of Column I, the loop is subjected to four different magnetic fields. Under the conditions of Column I, match the Column I with corresponding results of Column II (B_0 in Column I is a positive non-zero constant)

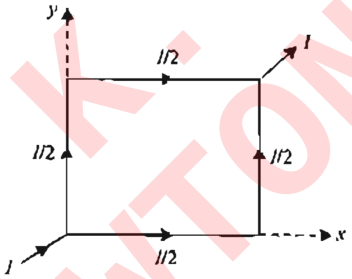


Fig. 9.355

Column I	Column II
i. $\vec{B} = B_0\,\hat{i}$ in tesla	**a.** Magnitude of net force on the loop is $\sqrt{2}\,B_0 I\ell$ newton
ii. $\vec{B} = B_0\,\hat{j}$ in tesla	**b.** Magnitude of net force on the loop is zero
iii. $\vec{B} = B_0\left(\hat{i} + \hat{j}\right)$ in tesla	**c.** Magnitude of net torque of the loop about its center is zero
iv. $\vec{B} = B_0\,\hat{k}$ in tesla	**d.** Magnitude of net force on the loop is $B_0 I\ell$ newton

5. A charged particle with some initial velocity is projected in a region where non-zero electric and/or magnetic fields are present. In column I, information about the existence of electric and/or magnetic field and direction of initial velocity of charged particle are given, while in Column II the probable path of the charged particle is mentioned. Match the entries of Column I with the entries of Column II.

Column I	Column II
i. $\vec{E} = 0, \vec{B} \neq 0$, and initial velocity is at any angle with $\vec{B}$	**a.** Straight line
ii. $\vec{E} \neq 0, \vec{B} = 0$ and initial velocity is at any angle with $\vec{E}$	**b.** Parabola
iii. $\vec{E} \neq 0, \vec{B} \neq 0, \vec{E} \parallel \vec{B}$ and initial velocity is ⊥ to both	**c.** Circular
iv. $\vec{E} \neq 0, \vec{B} \neq 0, \vec{E}$ perpendicular $\vec{B}$ to and $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$	**d.** Helical path with non-uniform pitch

6. An elementary current loop is placed in a non-uniform magnetic field as shown in Fig. 9.356. In Column I, different orientations of loop are described and in Column II, the corresponding forces experienced by the loop.

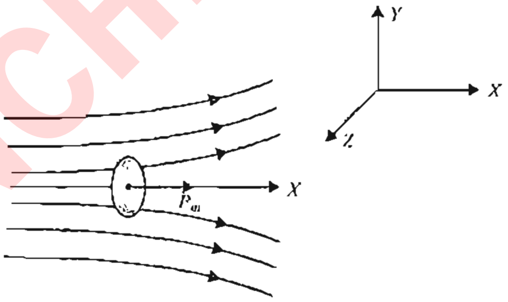


Fig. 9.356

Column I	Column II
i. In the given situation,	**a.** resultant force is acting along $\vec{P}_m$
ii. If loop is rotated such that $\vec{P}_m$ is along +ve Z-direction,	**b.** resultant force is acting opposite to $\vec{P}_m$
iii. If loop is rotated such that $\vec{P}_m$ is along −ve Z-direction	**c.** $F_x = 0, F_y = 0$
iv. If loop is rotated such that $\vec{P}_m$ is along +ve Y-direction	**d.** $F_x = 0, F_z = 0$

7. Consider a closed loop in the form of a Trapezium carrying current I. Match the following regarding the magnitude of magnetic field at point P.

Column I	Column II
i. Magnetic field due to AB	**a.** is greater than that due to DA
ii. Magnetic field due to BC	**b.** is greater than that due to CD
iii. Magnetic field due to DA	**c.** is not equal to zero
iv. Magnetic field due to complete figure	**d.** is zero

8. Match Column I with Column II

Column I	Column II
i. A charge particle is moving in uniform electric and magnetic fields in gravity free space.	**a.** Velocity of the particle may be constant.
ii. A charge particle is moving in uniform electric magnetic and gravitational fields.	**b.** Path of the particle may be straight line.
iii. A charge particle is moving in uniform magnetic and gravitational fields (where electric field is zero).	**c.** Path of the particle may be circular.
iv. A charge particle is moving in only uniform electric field.	**d.** Path of the particle may be helical.

9. A square loop is placed near a long straight current carrying wire as shown in Fig. 9.357. Match the following table:

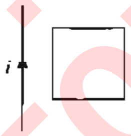


Fig. 9.357

Column I	Column II
i. If current is increased,	**a.** induced current in loop is clockwise
ii. If current is decreased,	**b.** induced current in loop is anticlockwise
iii. If loop is moved away from the wire,	**c.** wire will attract the loop
iv. If loop is moved toward the wire,	**d.** wire will repel the loop

10. A circular current carrying loop of 100 turns and radius 10 cm is placed in x–y plane as shown in Fig. 9.358. A uniform magnetic field $\vec{B} = (-\hat{i}+\hat{k})$ tesla is present in the region. If current in the loop is 5 A, then match the following:

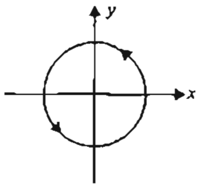


Fig. 9.358

Column I	Column II
i. Magnitude and direction of moment (in A-m) of the loop are	**a.** Zero
ii. Magnitude and direction of torque (in N-m) on the loop are	**b.** 5 π
iii. Magnitude and direction of net force (in N) on the current loop are	**c.** along positive z-axis
iv. Direction of magnetic field of loop at the center is	**d.** along negative y-axis

11. A square loop of side a and carrying current i as shown in Fig. 9.359 is placed in gravity free space having magnetic field $B = B_0\hat{j}$. Now, match the following:

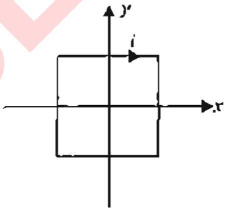


Fig. 9.359

Column I	Column II
i. Torque on loop	**a.** is zero
ii. Net force on loop	**b.** is along x-axis
iii. Potential energy of loop	**c.** is along negative z-axis
iv. Magnetic moment of loop	**d.** has maximum magnitude

12. A charged particle passes through a region that could have electric field only or magnetic field only or both electric and magnetic fields or none of the fields. Match Column I with Column II:

Column I	Column II
i. Kinetic energy of the particle remains constant	**a.** Under special conditions, this is possible when both electric and magnetic fields are present
ii. Acceleration of the particle is zero	**b.** The region has electric field only
iii. Kinetic energy of the particle changes and it also suffers deflection	**c.** The region has magnetic field only
iv. Kinetic energy of the particle changes but it suffers no deflection	**d.** The region contains no field

13. Three wires are carrying same constant current i in different directions. Four loops enclosing the wires in different manners are shown in Fig. 9.360. The direction of $d\vec{\ell}$ is shown in the figure.

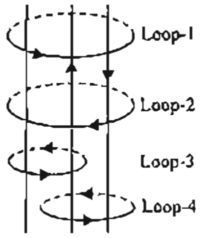


Fig. 9.360

Column I	Column II
i. Along closed Loop 1	a. $\oint \vec{B}\, d\vec{\ell} = \mu_0 i$
ii. Along closed Loop 2	b. $\oint \vec{B} \cdot d\vec{\ell} = -\mu_0 i$
iii. Along closed Loop 3	c. $\oint \vec{B} \cdot d\vec{\ell} = 0$
iv. Along closed Loop 4	d. net work done by the magnetic force to move a unit charge along the loop is zero

Archives

Solutions on page 8.132

Fill in the Blanks Type

1. A neutron, a proton, an electron and an alpha particle enter a region of constant magnetic field with equal velocities. The magnetic field is along the inward normal to the plane of the paper. The tracks of the particles are labelled in Fig. 9.361. The electron follows track ______ and the alpha particle follows track ______. **(IIT-JEE, 1984)**

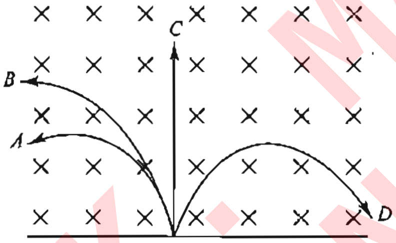


Fig. 9.361

2. A wire of length L meters, carrying a current i amperes, is bent in the form of a circle. The magnitude of its magnetic moment is ______ in M.K.S. units. **(IIT-JEE, 1987)**

3. In a hydrogen atom, the electron moves in an orbit of radius 0.5 Å making 10^{16} revolutions per second. The magnetic moment associated with the orbit motion of the electron is ______ **(IIT-JEE, 1988)**

4. The wire loop $PQRSP$ formed by joining two semicircular wires of radii R_1 and R_2 carries a current I as shown. The magnitude of the magnetic induction at the center C is ______ **(IIT-JEE, 1990)**

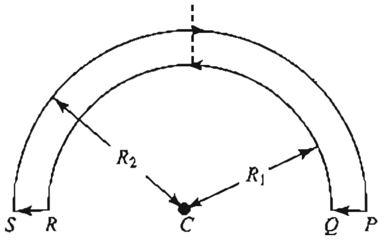


Fig. 9.362

5. A wire $ABCDEF$ (with each side of length L) bent as shown in Fig. 9.363 and carrying a current I is placed in a uniform magnetic induction B parallel to the positive y-direction. The force experienced by the wire is ______ in the ______ direction. **(IIT-JEE, 1990)**

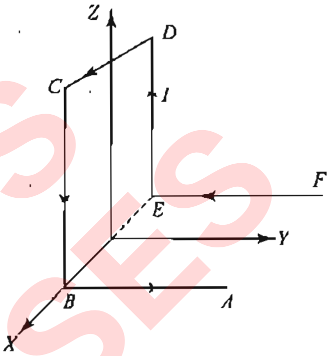


Fig. 9.363

6. A metallic block carrying current I is subjected to a uniform magnetic induction as $\vec{B}$ shown in Fig. 9.364. The moving charges experience a force $\vec{F}$ given by ______ which results in the lowering of the potential of the face ______. Assume the speed of the carriers to be v. **(IIT-JEE, 1996)**

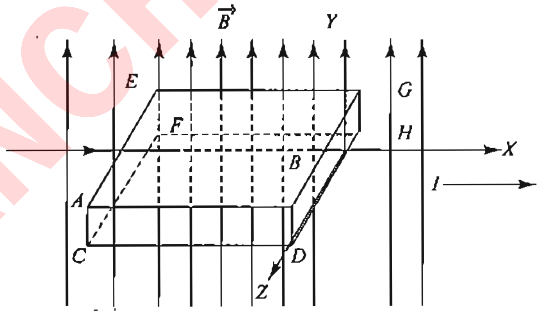


Fig. 9.364

7. A uniform magnetic field with a slit system as shown in Fig. 9.365 is to be used as a momentum filter for high-energy charged particles. With a field B tesla, it is found that the filter transmits particles each of energy 5.3 MeV. The magnetic field is increased to $2.3B$ tesla and deuterons are passed into the filter. The energy of each deuteron transmitted by the filter is ______ MeV. **(IIT-JEE, 1997)**

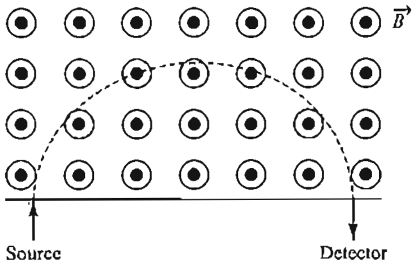


Fig. 9.365

True or False

1. No net forces act on a rectangular coil carrying a steady current when suspended freely in a uniform magnetic field. **(IIT-JEE, 1981)**
2. There is no charge in the energy of a charged particle moving in a magnetic field although a magnetic force is acting on it. **(IIT-JEE, 1983)**
3. A charged particle enters a region of uniform magnetic field at an angle to the magnetic line of force. The path of the particle is a circle. **(IIT-JEE, 1983)**
4. An electron and a proton are moving with the same kinetic energy along the same direction. When they pass through a uniform magnetic field perpendicular to the direction of their motion, they describe circular paths of the same radius. **(IIT-JEE, 1985)**

Single Correct Answer Type

1. A magnetic needle is kept in a non-uniform magnetic field. It experiences **(IIT-JEE, 1982)**
   - a. a force and a torque
   - b. a force but not a torque
   - c. a torque but not a force
   - d. neither a force nor a torque
2. A conducting circular loop of radius r carries a constant current i. It is placed in a uniform magnetic field $\vec{B}_0$ A perpendicular to the plane of the loop. The magnetic force acting on the loop is **(IIT-JEE, 1983)**
   - a. irB_0
   - b. $2\pi irB_0$
   - c. zero
   - d. πirB_0
3. A rectangular loop carrying a current i is situated near a long straight wire such that the wire is parallel to one of the sides of the loop and is in the plane of the loop. If steady current I is established in the wire as shown in the figure, the loop will: **(IIT-JEE, 1985)**

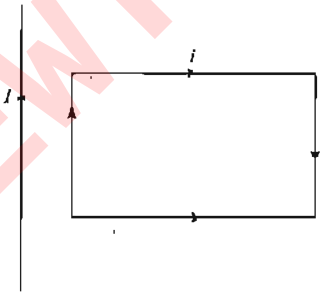


Fig. 9.366

   - a. rotate about an axis parallel to the wire
   - b. move away from the wire
   - c. move toward the wire
   - d. remain stationary
4. Two thin long parallel wires separated by a distance b are carrying a current I ampere each. The magnitude of the force per unit length exerted by one wire on the other is **(IIT-JEE, 1986)**
   - a. $\frac{\mu_0 i^2}{b^2}$
   - b. $\frac{\mu_0 i^2}{2\pi b}$
   - c. $\frac{\mu_0 i}{2\pi b}$
   - d. $\frac{\mu_0 i}{2\pi b^2}$
5. Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2, respectively. The ratio of the mass of X to that of Y is **(IIT-JEE, 1988)**
   - a. $(R_1/R_2)^{1/2}$
   - b. R_2/R_1
   - c. $(R_1/R_2)^2$
   - d. R_1/R_2
6. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then **(IIT-JFE, 1993)**
   - a. the magnetic field at all points inside the pipe is the same, but not zero
   - b. the magnetic field at any point inside the pipe is zero
   - c. the magnetic field is zero only on the axis of the pipe
   - d. the magnetic field is different at different points inside the pipe
7. A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R. One of the arcs AB of the ring subtends an angle θ at the center. The value of the magnetic induction at the centre due to the current in the ring is **(IIT-JEE, 1995)**
   - a. proportional to $2(180° - \theta)$
   - b. inversely proportional to r
   - c. zero, only if $\theta = 180°$
   - d. zero for all values of θ
8. A proton, a deuteron and an α-particle having the same kinetic energy are moving in circular trajectories in a constant magnetic field. If r_p, r_d and r_α denote, respectively, the radii of the trajectories of these particles, then **(IIT-JEE, 1997)**
   - a. $r_\alpha = r_p < r_d$
   - b. $r_\alpha > r_d > r_p$
   - c. $r_\alpha = r_d > r_p$
   - d. $r_p = r_d = r_\alpha$
9. Two particles, each of mass m and charge q, are attached to the two ends of a light rigid rod of length $2R$. The rod is rotated at constant angular speed about a perpendicular axis passing through its center. The ratio of the magnitudes of the magnetic moment of the system and its angular momentum about the center of the rod is **(IIT-JEE, 1998)**
   - a. $q/2m$
   - b. q/m
   - c. $2q/m$
   - d. $q/\pi m$
10. A metal rod moves at a constant velocity in a direction perpendicular to its length. A constant, uniform magnetic field exists in space in a direction perpendicular to the rod as well as its velocity. Select the correct statement(s) from the following: **(IIT-JEE, 1998)**
    - a. The entire rod is at the same electric potential
    - b. There is an electric field in the rod
    - c. The electric potential is highest at the center of the rod and decreases toward its ends
    - d. The electric potential is lowest at the center of the rod, and increases toward its ends
11. Two very long, straight, parallel wires carry steady currents I and $-I$, respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires, in the plane of the wires. Its

instantaneous velocity v is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is **(IIT-JEE, 1998)**

a. $\frac{\mu_0 Iqv}{2\pi d}$ b. $\frac{\mu_0 Iqv}{\pi d}$ c. $\frac{2\mu_0 Iqv}{\pi d}$ d. 0

12. A circular loop of radius R, carrying current I, lies in x–y plane with its center at origin. The total magnetic flux through x–y plane is **(IIT-JEE, 1999)**

a. directly proportional to I
b. directly proportional to R
c. inversely proportional to R
d. zero

13. A charged particle is released from rest in a region of steady and uniform electric and magnetic fields which are parallel to each other. The particle will move in a **(IIT-JEE, 1999)**

a. straight line b. circle
c. helix d. cycloid

14. A particle of charge q and mass m moves in a circular orbit of radius r with angular speed ω. The ratio of the magnitude of its magnetic moment to that of its angular momentum depends on **(IIT-JEE, 2000)**

a. ω and q b. ω, q and m
c. q and m d. ω and m

15. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper, as shown (Fig. 9.367). The variation of the magnetic field B along the line XX' is given by **(IIT-JEE, 2000)**

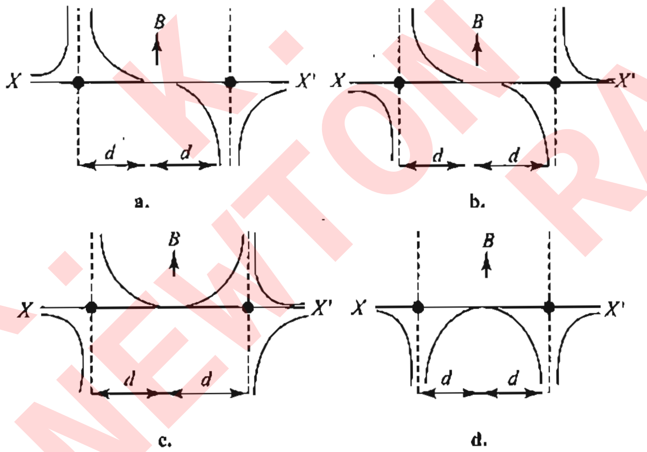


Fig. 9.367

16. An infinitely long conductor PQR is bent to form a right angle as shown in figure. A current I flows through PQR. The magnetic field due to this current at the point M is H. Now, another infinitely long straight conductor QS is connected to Q so that current is $I/2$ in QR as well as in QS, the current in PQ remaining unchanged. The magnetic field at M is now H_2. The ratio H_1/H_2 is given by **(IIT-JEE, 2000)**

a. 1/2 b. 1 c. 2/3 d. 2

17. An ionized gas contains both positive and negative ions. If it is subjected simultaneously to an electric field along the $+x$-direction and a magnetic field along the $+z$-direction, then **(IIT-JEE, 2000)**

a. positive ions deflect towards $+y$-direction and negative ions towards $-y$-direction
b. all ions deflect towards $+y$-direction
c. all ions deflect towards $-y$-direction
d. positive ions deflect towards $-y$-direction and negative ions towards $+y$-direction

18. A non-planar loop of conducting wire carrying a current I is placed as shown in Fig. 9.368. Each of the straight sections of the loop is of length $2a$. The magnetic field due to this loop at the point $P(a, 0, a)$ points in the direction **(IIT-JEE, 2001)**

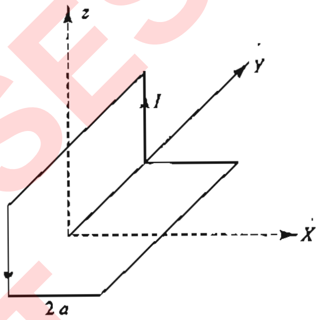


Fig. 9.368

a. $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ b. $\frac{1}{\sqrt{3}}(-\hat{j}+\hat{k}+\hat{i})$

c. $\frac{1}{\sqrt{3}}(\hat{j}+\hat{k}+\hat{i})$ d. $\frac{1}{\sqrt{2}}(\hat{i}+\hat{k})$

19. Two particles A and B of masses m_A and m_B, respectively, and having the same charge are moving in a plane. A uniform magnetic field exists perpendicular to this plane. The speeds of the particles are v_A and v_B, respectively and the trajectories are as shown in Fig. 9.369. Then **(IIT-JEE, 2001)**

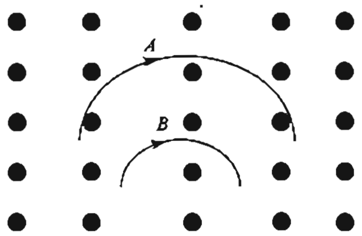


Fig. 9.369

a. $m_A v_A < m_B v_B$ b. $m_A v_A > m_B v_B$
c. $m_A < m_B$ and $v_A < v_B$ d. $m_A = m_B$ and $v_A = v_B$

20. Two circular coils can be arranged in any of the three situations shown in Fig. 9.370. Their mutual inductance will be **(IIT-JEE, 2001)**

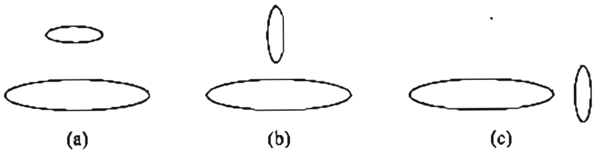


Fig. 9.370

a. maximum in situation (a)
b. maximum in situation (b)
c. maximum in situation (c)
d. the same in all situations

21. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii a and b, respectively. When a current I passes through the coil, the magnetic field at the center is **(IIT-JEE, 2001)**

a. $\frac{\mu_0 NI}{b}$ b. $\frac{2\mu_0 NI}{a}$
c. $\frac{\mu_0 NI}{2(b-a)}\ln\frac{b}{a}$ d. $\frac{\mu_0 I^N}{2(b-a)}\ln\frac{b}{a}$

22. A particle of mass m and charge q moves with a constant velocity v along the positive x-direction. It enters a region containing a uniform magnetic field B directed along the negative z-direction, extending from $x = a$ to $x = b$. The minimum value of v required so that the particle can just enter the region $x > b$ is **(IIT-JEE, 2002)**

a. $\frac{qbB}{m}$ b. $\frac{q(b-a)B}{m}$
c. $\frac{qaB}{m}$ d. $\frac{q(b+a)B}{2m}$

23. A long straight wire along the z-axis carries a current I in the negative z-direction. The magnetic vector field $\vec{B}$ at a point having coordinates (x, y) in the $z = 0$ plane is **(IIT-JEE, 2002)**

a. $\frac{\mu_0 I(y\hat{i}-x\hat{j})}{2\pi(x^2+y^2)}$ b. $\frac{\mu_0 I(x\hat{i}+y\hat{j})}{2\pi(x^2+y^2)}$
c. $\frac{\mu_0 I(x\hat{j}-y\hat{i})}{2\pi(x^2+y^2)}$ d. $\frac{\mu_0 I(x\hat{i}-y\hat{j})}{2\pi(x^2+y^2)}$

24. The magnetic field lines due to a bar magnet are correctly shown in **(IIT-JEE, 2002)**

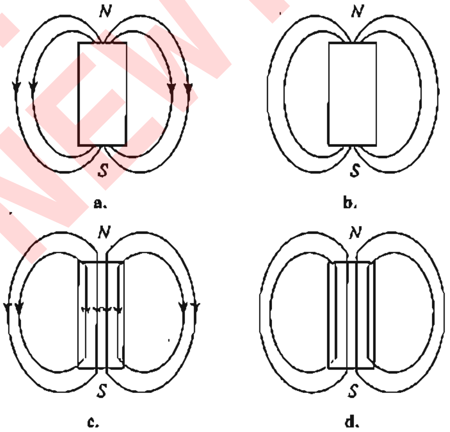


Fig. 9.371

25. For a positively charged particle moving in a x–y plane initially along the x-axis, there is a sudden change in its path due to the presence of electric and/or magnetic fields beyond P. The curved path is shown in the x–y plane and is found to be non-circular. Which one of the following combinations is possible ? **(IIT-JEE, 2003)**

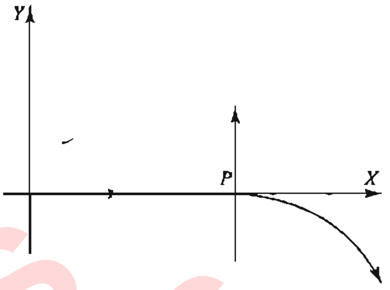


Fig. 9.372

a. $\vec{E}=0;\ \vec{B}=b\hat{i}+c\hat{k}$ b. $\vec{E}=a\hat{i};\ \vec{B}=c\hat{k}+a\hat{i}$
c. $\vec{E}=0;\ \vec{B}=c\hat{j}+b\hat{k}$ d. $\vec{E}=a\hat{i};\ \vec{B}=c\hat{k}+b\hat{j}$

26. A conducting loop carrying a current I is placed in a uniform magnetic field pointing into the plane of the paper as shown (Fig. 9.373). The loop will have a tendency to

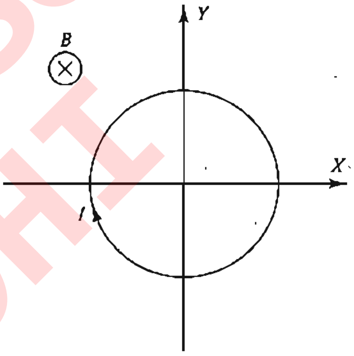


Fig. 9.373

(IIT-JEE, 2003)

a. contract
b. expand
c. move toward +ve x-axis
d. move toward −ve x-axis

27. A current carrying loop is placed in a uniform magnetic field in four different orientations, I, II, III and IV, arrange them in the decreasing order of potential energy **(IIT-JEE, 2003)**

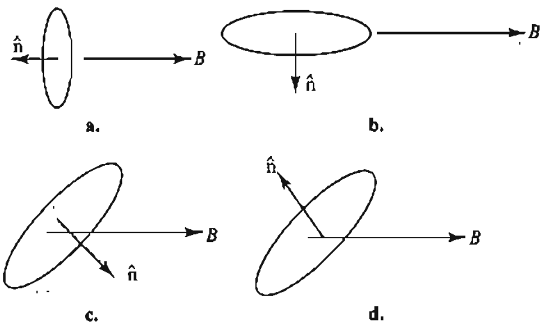


Fig. 9.374

a. I > III > II > IV b. I > II > III > IV
c. I > IV > II > III d. III > IV > I > II

28. An electron travelling with a speed u along the positive x-axis enters into a region of magnetic field where $B = -B_0\hat{k}$ ($x > 0$). It comes out of the region with speed v. Then **(IIT-JEE, 2004)**

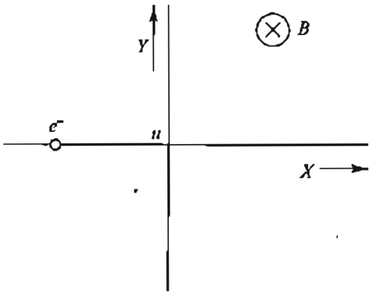


Fig. 9.375

a. $v = u$ at $y > 0$
b. $v = u$ at $y = 0$
c. $v > u$ at $y > 0$
d. $v > u$ at $y < 0$

29. A magnetic field $\vec{B} = B_0\,\hat{i}$ exists in the region $a < x < 2a$ and $\vec{B} = B_0\,\hat{j}$, in the region $2a < x < 3a$, where B_0 is a positive constant. A positive point charge moving with a velocity $\vec{v} = v_0\,\hat{i}$, where v_0 is a positive constant, enters the magnetic field at $x = a$. The trajectory of the charge in this region can be like **(IIT-JEE, 2007)**

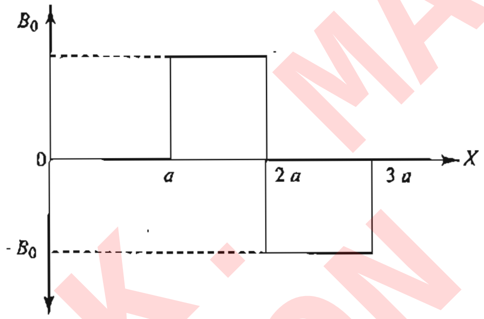
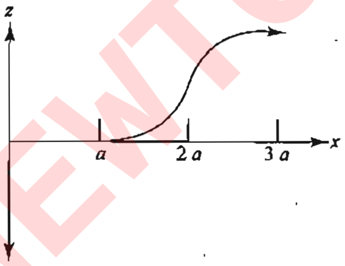
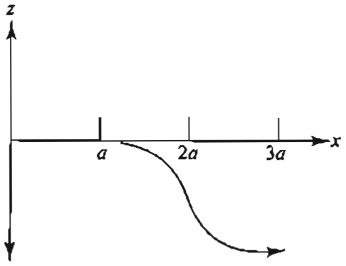


Fig. 9.376

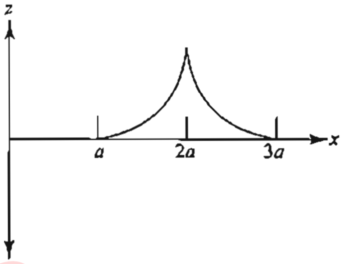


a.

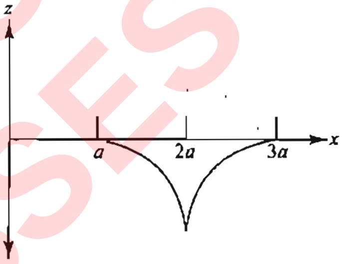


b.

Fig. 9.377 (*Contd.*)



c.



d.

Fig. 9.377

30. A steady current I goes through a wire loop PQR having shape of a right angle triangle with $PQ = 3x$, $PR = 4x$ and $QR = 5x$. If the magnitude of the magnetic field at P due to this loop is $k\left(\dfrac{\mu_0 I}{48\pi x}\right)$, the value of k is **(IIT-JEE, 2009)**

a. 5
b. 8
c. 7
d. 10

Multiple Correct Answers Type

1. A proton moving with a constant velocity passes through a region of space without any change in its velocity. If E and B represent the electric and magnetic fields respectively, this region of space may have **(IIT-JEE, 1985)**

a. $E = 0, B = 0$
b. $E = 0, B \neq 0$
c. $E \neq 0, B = 0$
d. $E = \neq 0, B \neq 0$

2. A particle of charge $+q$ and mass m moving under the influence of a uniform electric field $E\,\hat{i}$ and uniform magnetic field $B\,\hat{k}$ follows a trajectory from P to Q as shown in Fig. 9.478. The velocities at P and Q are $v\,\hat{i}$ and $-2\,\hat{j}$ which of the following statement(s) is/are correct? **(IIT-JEE, 1991)**

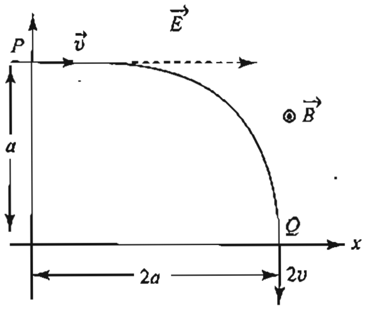


Fig. 9.378

a. $E = \frac{3}{4}\left[\frac{mv^2}{qa}\right]$

b. Rate of work done by the electric field at P is $\frac{3}{4}\left[\frac{mv^2}{a}\right]$

c. Rate of work done by the electric field at P is zero

d. Rate of work done by both the fields at Q is zero

3. A micrometer has a resistance of 100 Ω and a full scale range of 50 μA. It can be used as a voltmeter or as a higher range ammeter provided a resistance is added to it. Pick the correct range and resistance combination (s): **(IIT-JEE, 1991)**

a. 50 V range with 10 kΩ resistance in series

b. 10 V range with 200 kΩ resistance in series

c. 5 mA range with 1 Ω resistance in parallel

d. 10 mA range with 1 Ω resistance in parallel

4. H^+, He^+ and O^{2+} all having the same kinetic energy pass through a region in which there in a uniform magnetic field perpendicular to their velocity. The masses of H^+, He^+ and O^{2+} are 1, 4 and 16 amu, respectively. Then, **(IIT-JEE, 1994)**

a. H^2 will be deflected most

b. O^{2+} will be deflected most

c. He^+ and O^{2+} will be deflected equally

d. All will be deflected equally

5. The following field lines can never represent **(IIT-JEE, 2006)**

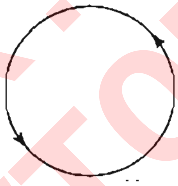


Fig. 9.379

a. induced electric field

b. magnetostatic field

c. gravitational field of a mass at rest

d. electrostatic field

6. A long current carrying wire, carrying current such that it is flowing out from the plane of paper, is placed at O. A steady state current is flowing in the loop $ABCD$. Then, **(IIT-JEE, 2006)**

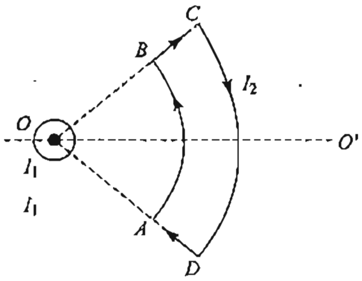


Fig. 9.380

a. the net force is zero

b. the net torque is zero

c. as seen from O, the loop will rotate in clockwise direction along axis OO'

d. as seen from O, the loop will rotate in anticlockwise direction along axis OO'

7. A particle of mass m and charge q, moving with velocity V enters Region II normal to the boundary as shown in Fig. 9.381. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the Region II is ℓ. Choose the correct choice(s). **(IIT-JEE, 2008)**

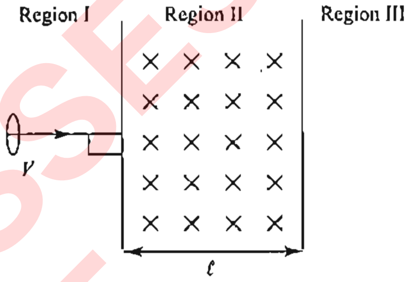


Fig. 9.381

a. The particle enters Region II only if its velocity $V > \frac{q\ell B}{m}$

b. The particle enters Region II only if its velocity $V < \frac{q\ell B}{m}$

c. Path length of the particle in Region II is maximum when velocity $V = \frac{q\ell B}{m}$

d. Time spent in Region II is same for any velocity V as long as the particle returns to Region I

Assertion-Reasoning Type

Mark your answer as **(IIT-JEE, 2008)**

a. Statement 1 is true, Statement 2 is true; Statement 2 is a correct expalantion for Statement 1.

b. Statement 1 is true, Statement 2 is true; Statement 2 is **NOT** a correct expalantion for Statement 1.

c. Statement 1 is true, Statement 2 is false.

d. Statement 1 is false, Statement 2 is true.

1. Statement 1: The sensitivity of a moving coil galvanometer is increased by placing a suitable magnetic material as a core inside the coil.

Statement 2: Soft iron has a high magnetic permeability and cannot be easily magnetized or demagnetized.

Matching Column Type

1. Some law/processes are given in Column I. Match these with the physical phenomena given in Column II: **(IIT-JEE, 2006)**

Column I	Column II
i. Dielectric ring uniformly charged	a. Time independent electrostatic field out of system
ii. Dielectric ring uniformly charged rotating with angular velocity ω	b. Magnetic field
iii. Constant current in ring i_0	c. Induced electric field
iv. $I = i_0 \cos \omega t$	d. Magnetic moment

2. Column I gives certain situations in which a straight metallic wire of resistance R is used and Column II gives some resulting effects. Match the statements in Column I with the statements in Column II: **(IIT-JEE, 2007)**

Column I	Column II
i. A charged capacitor is connected to the ends of the wire	a. A constant current flows through the wire
ii. The wire is moved perpendicular to its length with a constant velocity in a uniform magnetic field perpendicular to the plane of motion	b. Thermal energy is generated in the wire
iii. The wire is placed in a constant electric field that has a direction along the length of the wire	c. A constant potential difference develops between the ends of the wire
iv. A battery of constant emf is connected to the ends of the wire	d. Charges of constant magnitude appear at the ends of the wire

3. Two wires, each carrying a steady current I, are shown in four configuration in Column I. Some of the resulting effects are described in Column II. Match the statements in Column I with statements in Column II and indicate your answer by darkening appropriate bubbles in the 4×4 matrix given in the *ORS*. **(IIT-JEE, 2007)**

Column I

i. Point P is situated midway between the wires.

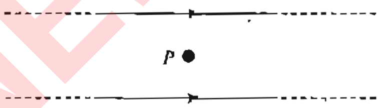


Fig. 9.382

ii. Point P is situated at the midpoint of the line joining the centers of the circular wires, which have same radii.

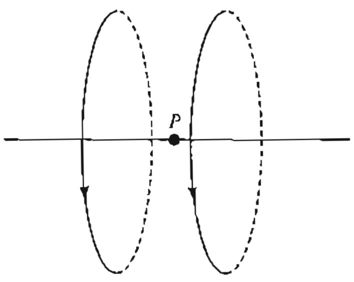


Fig. 9.383

iii. Point P is situated at the midpoint of the line joining the centers of the circular wires, which have same radii.

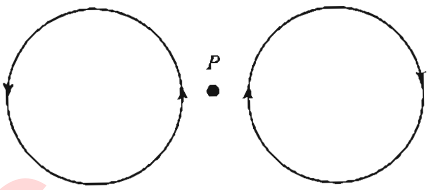


Fig. 9.384

iv. Point P is situated at the common center of the wires.

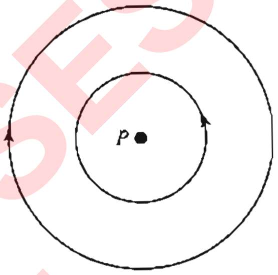


Fig. 9.385

Column II

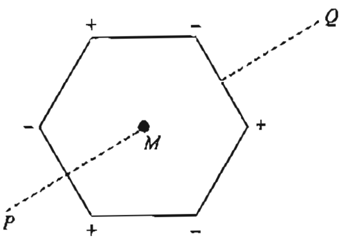
a. The magnetic fields (B) at P due to the currents in the wires are in the same direction.

b. The magnetic fields (B) at P due to the currents in the wires are in opposite directions.

c. There is no magnetic field at P.

d. The wires repel each other.

4. Six point charges, each of the same magnitude q, are arranged in different manners as shown in Column II. In each case, a point M and a line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B be the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady current. **(IIT-JEE, 2009)**

Column I	Column II
i. $E = 0$	a.  **Fig. 9.386** Charges are at the corners of a regular hexagon. M is at the center of the hexagon. PQ is perpendicular to the plane of the hexagon

ii. $V \neq 0$

b.

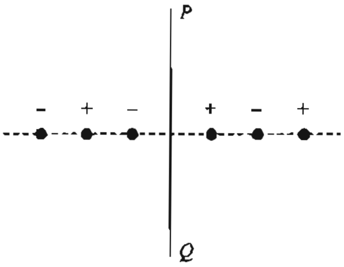


Fig. 9.387

Charges are on a line perpendicular to PQ at equal intervals. M is the mid point between the two innermost charges

iii. $B = 0$

c.

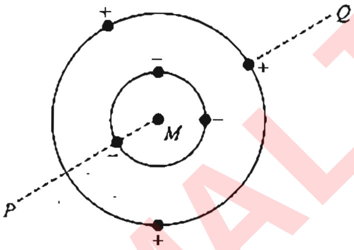


Fig. 9.388

Charges are placed on two coplanar insulating rings at equal intervals. M is the common center of the rings. PQ is perpendicular to the plane of the rings

iv. $\mu \neq 0$

d.

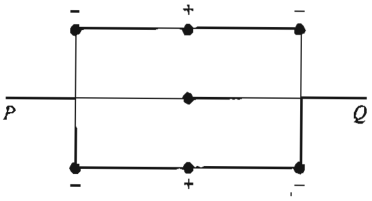


Fig. 9.389

Charges are placed at the corners of a rectangle of sides a and $2a$ and at the midpoints of the longer sides. M is at the center of the rectangle. PQ is parallel to the longer sides

e.

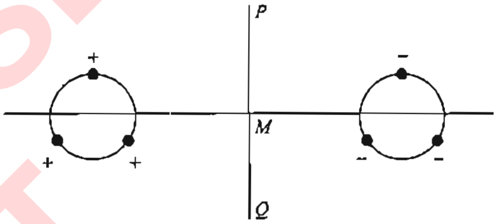


Fig. 9.390

Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the midpoint between the centres of the rings. PQ is perpendicular to the line joining the centers and coplanar to the rings.

ANSWERS AND SOLUTIONS

Subjective Type

1. Magnetic field due to smaller ring at distance R from the center $B = \frac{\mu_0}{4\pi}\frac{M}{R^3}$

where $M = i\pi a^2$

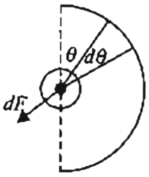


Fig. 9.391

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{i\pi a^2}{R^3} = \frac{\mu_0 i a^2}{4R^3}$$

$$dF = i_0 R d\theta \frac{\mu_0 i a^2}{4R^3}$$

$$dF_x = dF \sin\theta$$

$$= \frac{\mu_0 i i_0 a^2 \sin\theta\, d\theta}{4R^2}$$

$$F_x = \frac{\mu_0 i\, i_0 a^2}{4R^2}\int_0^{\pi}\sin\theta\, d\theta$$

$$\Rightarrow \quad F_x = \frac{\mu_0 i\, i_0 a^2}{4R^2} \times 2 = \frac{\mu_0 i\, i_0 a^2}{2R^2}$$

$$F_y = 0$$

$$\therefore \quad F_{\text{net}} = F_x = \frac{\mu_0 i\, i_0 a^2}{2R^2} = 8\text{ N}$$

2. Force exerted by air on the rod $= \left(\rho\frac{L}{2}2R\right)v^2 = \rho L R v^2$

Balancing torque about point O, $N\,1\,(\pi R^2)B = \rho L R v^2 = \frac{3L}{4}$

$$\Rightarrow 300\,\pi 1\, BR = \frac{3\rho v^2 L^2}{4} \Rightarrow 1 = \frac{\rho L^2 V^2}{400\pi BR} = 0.01\text{A} = 10\text{ mA}$$

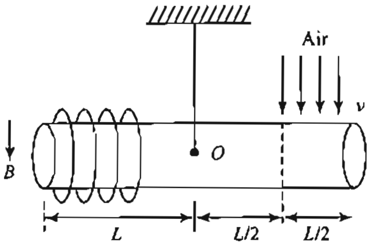


Fig. 9.392

3. $\vec{m} = I_2\vec{S}$

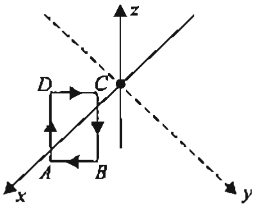


Fig. 9.393

$$\vec{S} = \overline{BA} \times \overline{AD}$$

$$\overline{BA} = 2d\hat{i} - 2a\hat{j}$$

$$\overrightarrow{AD} = 2b\hat{k}$$

$$\vec{S} = 2(d\hat{i} - a\hat{j}) \times 2b\hat{k}$$

$$\bar{M} = -4bI(d\hat{j} + a\hat{i})$$

$$|\vec{M}| = 4bI\sqrt{d^2 + a^2} = 2\,\text{T}$$

4.

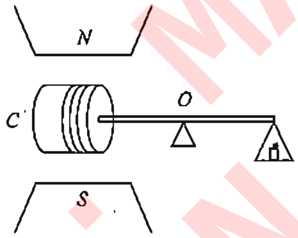


Fig. 9.394

Magnetic torque = $NISB \sin(\vec{S}, \vec{B})\ NISB \sin 90°$
Gravitational torque = $(\Delta m \times g)\,\ell$
For equilibrium $NISB = \Delta mg\ell \Rightarrow \Delta\ mg\ell\,/\,NIS$

$$B = \frac{(60\times10^{-6})\times9.8\times0.3}{200\times22\times10^{-3}\times1\times10^{-4}} = 0.4\,\text{T}$$

5. We need the magnetic and gravitational forces to cancel:

$$\lambda Lg = \frac{\mu_0 I^2 L}{2\pi h} \Rightarrow h = \frac{\mu_0 I^2}{2\pi\lambda g}$$

6. According to Biot–Savart law,

$$B = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2}$$

$$B = \frac{10^{-7}\times0.2\times4\times10^3\times3}{109\times\sqrt{109}} = 21\,\mu\text{T}$$

7. a. Writing torque equation about A

$$\tau = (mg - F_B)\frac{L}{2} - TL$$

Since the rod is in equilibrium, there is no torque in vertical direction.

$$\Rightarrow \quad TL = \frac{1}{2}(mg - F_B)$$

For wires to be tension free,

$$T = 0$$

$$\Rightarrow \quad F_B = mg \quad \Rightarrow \quad iLB = mg$$

$$\Rightarrow \quad i = \frac{mg}{LB}$$

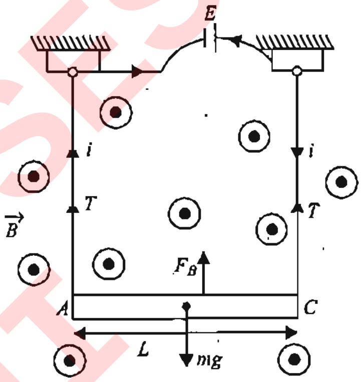


Fig. 9.395

b. If the wire is in equilibrium the forces acting on it are as shown in Fig. 9.396.

$$i = \frac{mg}{2LB} \quad \therefore\ F_B = \frac{1}{2}mg$$

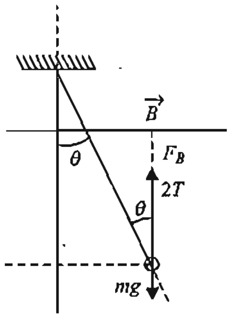


Fig. 9.396

c. For tension in the wire,
$2T + F_B\cos\theta = mg\cos\theta$

$$T = \frac{mg}{4}\cos\theta$$

Net unbalanced force on the rod will be in the direction perpendicular to the wire

$$F_{\text{net}} = mg\sin\theta - F_B\sin\theta$$

$$F_{\text{net}} = \frac{mg}{2}\sin\theta$$

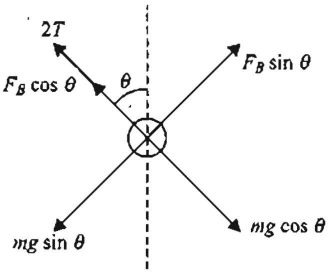


Fig. 9.397

8. The ring carries current i and the magnetic field makes an angle θ with the vertical at the ring's location.

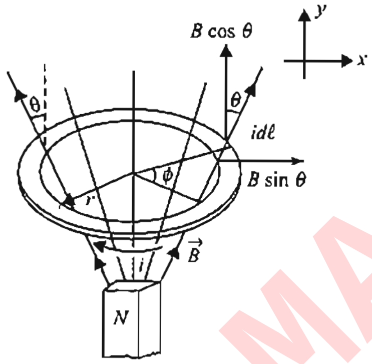


Fig. 9.398

Consider a small element subtending an angle $d\phi$ at the center of the ring.

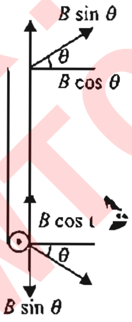


Fig. 9.399

Hence, force on the ring due to horizontal component of the field is

$$dF = i\,d\ell\,B\sin\theta\,\hat{j}$$

$$dF = i\,B\sin\theta\,r\,d\phi\,\hat{j}$$

$$\therefore \quad F_j = \int dF = r\,i\,B\sin\theta\int_0^{2\pi} d\phi\,\hat{j}$$

$$\therefore \quad \bar{F}_y = 2\pi i r\,B\sin\theta$$

Force on the ring due to vertical component of the field is

$$dF = -i\,d\ell B\cos\theta\hat{i}$$

$$F_x = \int dF = -i\int d\ell\,B\cos\theta\hat{i} = 0$$

As the ring lies in a region of constant magnetic field, $B\cos\theta \perp$ to it.

The force on it due to this component is zero as $\int d\ell = 0$

$$\therefore \qquad \bar{F} = 2\pi r\,i\,B\sin\theta\,\hat{j}$$

9. $B = 0.5$ T; $\quad i = 6$ A

$$i_1 : i_2 : i_4 : i_8 :: 1 : \frac{1}{2} : \frac{1}{4} : \frac{1}{8}$$

$$\therefore \quad x + \frac{1}{2}x + \frac{1}{4}x + \frac{1}{8}x = 6$$

$$\Rightarrow \quad x\frac{15}{8} = 6 \quad \Rightarrow \quad x = \frac{16}{5}$$

$$\therefore \quad i_1 = \frac{16}{5}\text{A},\ i_2 = \frac{8}{5}\text{A},\ i_4 = \frac{4}{5}\text{A},\ i_8 = \frac{2}{5}\text{A}$$

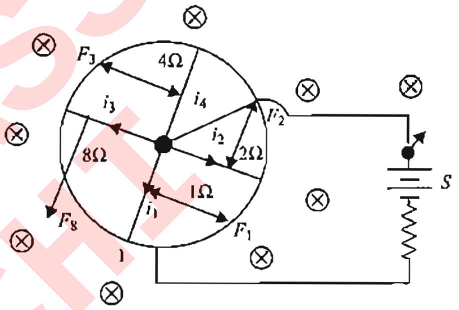


Fig. 9.400

a. The forces acting on spokes are as shown in Fig. 9.401. Hence, the wheel starts rotating in the anticlockwise direction.

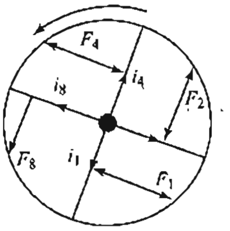


Fig. 9.401

b. $r = 0.2$ m

Suppose the torque acting on the spokes with resistances 1, 2, 4 and 8 Ω are τ_1, τ_2, τ_3 and τ_4, respectively.

Then, as all these torques act in the same direction

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \tau_4$$

$$\Rightarrow \quad \tau = i_1 B\,r\,\frac{r}{2} + i_2 B\,r\,\frac{r}{2} + i_4 B\,r\,\frac{r}{2} + i_8 B\,r\,\frac{r}{2}$$

$$= \frac{Br^2}{2}(i_1 + i_2 + i_4 + i_8) = \frac{i\,Br^2}{2}$$

$$\Rightarrow \quad \tau = \frac{6\times 0.5\times(0.2)^2}{2} = 1.5\times 0.04 = 60\times 10^{-3}$$

$$= 0.06 \text{ Nm}$$

Hence, $\tau = 0.06$ Nm ⊙

10. For the equilibrium of a small part of semicircular arc subtending an angle of $d\theta$ at the center,

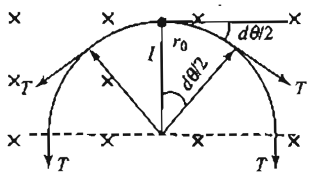


Fig. 9.402

$$\text{or, } 2T\sin\left(\frac{d\theta}{2}\right) = BI\,r_0\,d\theta$$

$$B = \frac{T}{Ir_0} = \frac{1.5}{(10)(0.15)} = 1\text{ T}$$

11. The work done by the potential differences get stored as its kinetic energy.

$$\therefore \quad \frac{1}{2}mv^2 = qV \quad \Rightarrow v = \left(\frac{2qV}{m}\right)^{1/2} \qquad \text{(i)}$$

a. When it enters the region $x \in [0, a)$, it experiences an electric field $\vec{E} = E\,\hat{j}$

Time taken to cross the region: $vt = a$

$$\Rightarrow \quad t = \frac{a}{v} = a\left(\frac{m}{2qV}\right)^{1/2} \qquad \text{(ii)}$$

The distance travelled in y-direction during this time is

$$y = \frac{1}{2}\frac{qE}{m}t^2 = \frac{1}{2}\times\frac{qE}{m}\times a^2\times\frac{m}{2qV}$$

$$\Rightarrow \quad y = \frac{1}{4}\frac{Ea^2}{V}$$

Hence, the particle meets the line $x = a$ in point

$$(x, y) = \left(a, \frac{1}{4}\frac{Ea^2}{V}\right)$$

b. Now, velocity of the particle as it crosses the line $x = a$

$$\vec{v} = \left(\frac{2qV}{m}\right)^{1/2}\hat{i} + \frac{qE}{m}\left(\frac{2qV}{m}\right)^{1/2}\hat{j}$$

Magnetic field in this region, $\vec{B} = B\vec{j}$

Hence, the time period of revolution, $t = \dfrac{2\pi m}{qB}$

$$\text{Pitch } p = v_{\|}\,t = \frac{2\pi m}{mqB}\times aE\left(\frac{mq}{2V}\right)^{1/2}$$

$$p = \frac{\pi aE}{B}\left(\frac{2m}{Vq}\right)^{1/2}$$

12.
$$F_{\text{mag}} = ma$$

$$\frac{\mu_0 I^2}{2\pi d} = \lambda\ell a \text{ (where } \ell = \text{length of each rod)}$$

$$\frac{\mu_0}{2\pi d}\left(\frac{Q}{RC}e^{-\frac{t}{RC}}\right) = \lambda\ell\frac{dv}{dt}$$

$$\frac{\mu_0}{2\pi d}\frac{Q^2}{(RC)^2}\int_0^\infty e^{-\frac{2t}{RC}}dt = \lambda\ell\int_0^v dv$$

$$\frac{\mu_0 Q^2}{2\pi d\,RC.2} = \lambda\ell v \Rightarrow \quad v = \frac{\mu_0 Q^2}{4\pi d\,RC\,\lambda\ell}$$

13. a. The particle will move in circular paths, as velocity vector is perpendicular to magnetic field. Time period of both the particles is same $\left(T = \dfrac{2\pi m}{Bq}\right)$.

So, for collision not to take place,

$$r_1 + r_2 < d$$

$$\frac{mv}{Bq} + \frac{2mv}{2Bq} < d \text{ or } v < \frac{Bqd}{2m}$$

Therefore, maximum speed should be $\dfrac{Bqd}{2m}$

i.e., $\quad v_m < \dfrac{Bqd}{2m}$

b. From symmetry, it can be concluded that collision occurs at $d/2$ if

$$v = 2v_m = \frac{qBd}{m}$$

$$r = \frac{mv}{qB} = d,\ \sin\theta = \frac{d/2}{d} = \frac{1}{2};\ \theta = \frac{\pi}{6}$$

$$t = T\left(\frac{\theta}{2\pi}\right) = \frac{2\pi m}{qB}\left(\frac{\pi/6}{2\pi}\right) = \frac{\pi m}{6qB}$$

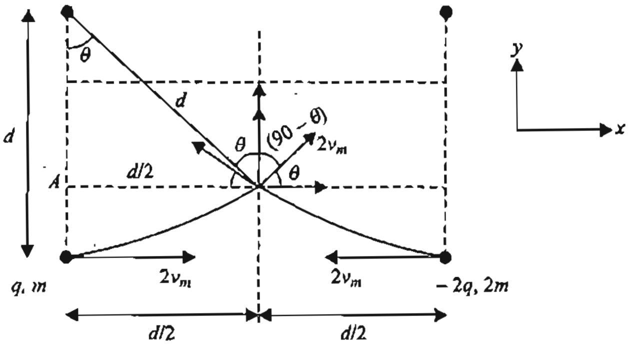


Fig. 9.403

c. After collision, charge on the combined particle $= -q$,

Mass $= 3m$

The combined particle will have velocity in y-direction just after collision. Using conservation of linear momentum

$$(mv\cos\theta i + mv\sin\theta j) + (-2mv\cos\theta i + 2mv\sin\theta j) = 3m\bar{v}$$

$$3m\bar{v} = -mv\cos\theta i + 3mv\sin\theta j$$

$$\bar{v} = \left(-\frac{v}{2\sqrt{3}}\hat{i} + \frac{v}{2}\hat{j}\right)$$

$$|\bar{v}| = v\sqrt{\left(\frac{1}{4\times 3}+\frac{1}{4}\right)} = v\sqrt{\frac{1}{4}\left(\frac{4}{3}\right)} = \frac{v}{\sqrt{3}}$$

$$|v| = \frac{1}{\sqrt{3}}\times 2\times\frac{qBd}{2m} = \frac{qBd}{\sqrt{3}m}$$

$$\therefore \quad r = 3m\frac{qBd}{\sqrt{3}mqB} = \sqrt{3}\,d$$

14. Changing the direction of current in the wire, we can change the normal reaction on the wire by the surface.
In one case, magnetic force on the wire will be in upward direction while in the other case, it will be in the downward direction. Hence, normal reaction, $N = mg \pm Bi\ell$.

$$f_{(\text{friction limiting})} = \mu(mg \pm Bi\ell)$$

As $F_1 > F_2$,

$$F_1 = \mu(mg + Bi\ell) \qquad \text{(i)}$$

$$F_2 = \mu(mg - Bi\ell) \qquad \text{(ii)}$$

From equations (i) and (ii),

$$\frac{F_1}{F_2} = \frac{mg + Bi\ell}{mg - Bi\ell}$$

$$mg = Bi\ell\left[\frac{F_1 + F_2}{F_1 - F_2}\right] \qquad \text{(iii)}$$

From equations (i) and (iii), we get $\mu = \dfrac{F_1 - F_2}{2Bi\ell}$

15. Axis of the loop means a line passing through center of loop and normal to its plane. Since distance of the point P is x from center of loop and side of square loop is a as shown in Fig. 9.404(a).

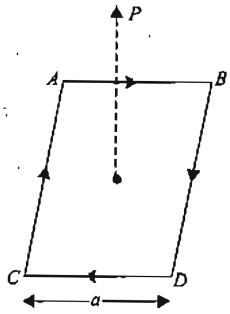


Fig. 9.404 (a)

Therefore, perpendicular distance of P from each side of the loop is,

$$r = \sqrt{x^2 + \left(\frac{a}{2}\right)^2} = 4 \text{ cm}$$

Now, consider only one side AB of the loop as shown in Fig. 9.404(b).

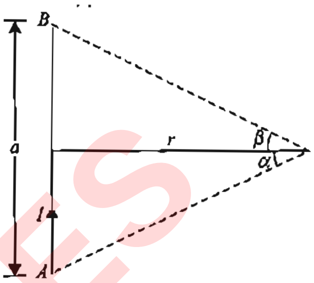


Fig. 9.404 (b)

$$\tan\alpha = \tan\beta = \frac{(a/2)}{r} = \frac{3}{4}$$

$$\alpha = \beta = \tan^{-1}\left(\frac{3}{4}\right) = 37^\circ$$

Magnitude of magnetic induction at P, due to current in this side AB, is

$$B_0 = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$$

$$= 9\times 10^{-5}\text{ T}$$

Now, consider magnetic inductions, produced by currents in two opposite sides AB and CD as shown in Fig. 9.404(c).

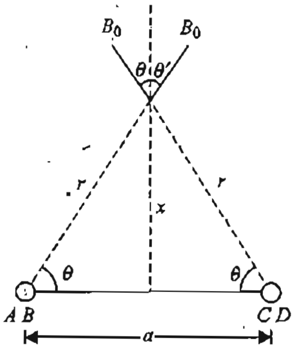


Fig. 9.404 (c)

Components of these magnetic inductions, parallel to plane of loop neutralise each other. Hence, resultant of these two magnetic inductions is $2B_0\cos\theta$ (along the axis).
Similarly, resultant of magnetic inductions produced by currents in remaining two opposite sides BC and AD will also be equal to $2B_0\cos\theta$ (along the axis in same direction).
Hence, resultant magnetic induction,

$$B = 4B_0\cos\theta$$

$$B = 4\times(9\times 10^{-5})\frac{(a/2)}{r}$$

$$= 2.7\times 10^{-4}\text{T}$$

16. Circuit segment shown in Fig. 9.405 can be considered in three parts.

1. A circular loop in y–z plane. Since this loop is made of uniform wire, therefore magnetic induction at O due to it is $B_1 = 0$.

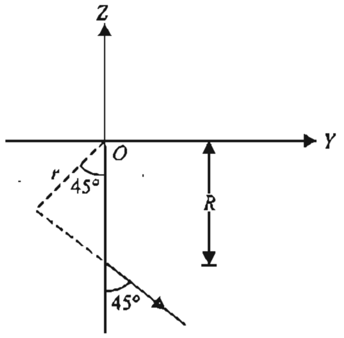


Fig. 9.405

2. A straight part, parallel to x-axis. Magnetic induction due to it is

$$B_2 = \frac{\mu_0 I}{4\pi R}(-\hat{k}) = -\frac{\mu_0 I}{4\pi R}\hat{k}$$

3. A straight part in y–z plane. Perpendicular distance of O from axis of this straight part is $r = R\cos 45°$ as shown in figure.
Angles subtended by lines joining O and ends of this straight part with perpendicular drawn from O are $\alpha = -45°$ and $\beta = 90°$.
Magnetic induction at O due to this part is

$$B_3 = \frac{\mu_0 I}{4\pi R}(\sin\alpha + \sin\beta)$$

$$\vec{B}_3 = \frac{\mu_0 I}{4\pi R}(\sqrt{2}-1)\hat{i}$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = -\frac{\mu_0 I}{4R}(\sqrt{2}-1)\hat{i} - \frac{\mu_0 I}{8\pi R}\hat{k}$$

Objective Type

1. c. The neutron follows straight path. This is because of zero force.

2. c. $(h\rho g)\,a^2 = BIa$ or $h = \dfrac{BI}{\rho g a}$

3. c. $F = Bqv$

But $\frac{1}{2}mv^2 = eV$ or $v = \sqrt{\dfrac{2eV}{m}}$

$$\therefore \quad F = Bq\sqrt{\frac{2eV}{m}}$$

$$\Rightarrow \quad F \propto \sqrt{V} \text{ and}$$

$$F' \propto \sqrt{2V}$$

$$\frac{F'}{F} = \sqrt{2} \text{ or } F' = \sqrt{2}F$$

4. b. For the particle to move along anticlockwise path, force should be along $\hat{j}$. Velocity is along $\hat{i}$.

Now, $\vec{F}_m = -e(\vec{v} \times \vec{B})$

In terms of unit vectors only,

$$\hat{j} = -(\hat{i} \times \hat{?}) \text{ or } \hat{j} = \hat{?} \times \hat{i}$$

Clearly, $\hat{?}$ is $\hat{k}$.

5. c. $\frac{1}{2}mv^2 = qV$ or $v = \sqrt{\dfrac{2qV}{m}}$

Centripetal force, $\dfrac{mv^2}{R} = qvB$

$$\therefore \quad v = \left(\frac{qB}{m}\right)R$$

Hence, $\sqrt{\dfrac{2qV}{m}} = \left(\dfrac{qB}{m}\right)R$

or $R = \left(\dfrac{2mV}{q}\right)^{1/2} \times \dfrac{1}{B}$

Here, V, q and B are constant.

Hence, $R \propto \sqrt{3}$

So, $\dfrac{m_1}{m_2} = \left(\dfrac{R_1}{R_2}\right)^2$

6. b. Current due to a stream of electrons will be in a direction opposite to the direction of current in the straight conductor, which is held parallel above the stream of electrons. The two currents are in opposite directions and hence they will repel. So, the stream of electrons is pulled downward.

7. b. $Bev = \dfrac{mv^2}{r}$ or $r = \dfrac{mv}{Be}$

As $mv\sqrt{2mT}\,(T = KE)$,

so $r = \dfrac{\sqrt{2mT}}{Be}$

As the electron has been accelerated from rest through a potential difference of V volt, therefore $T = eV$.

$$r = \sqrt{\frac{2mVe}{B^2e^2}} = \sqrt{\frac{2mV}{B^2e}}$$

8. b. $Bqv = \dfrac{mv^2}{r}$

or $Bqr = mv$

For electron as well as proton B is the same, r is the same and numerically charge q is the same: therefore mv is constant.

$$m_e v_e = m_p v_p$$

or $$v_p = \left(\frac{m_e}{m_p}\right) v_e$$

or $$v_p = \left(\frac{0.90\times10^{-30}}{1.8\times10^{-27}}\right)(3.0\times10^6)$$

$$= 1.5\times10^3$$

9. b. Since the charged particle passes straight without deflection, therefore

$$Bqv = mg$$

or $$v = \frac{mg}{Bq} = \frac{10^{-3}\times10}{1\times10^{-5}} = 10^3\ \mathrm{m\,s^{-1}}$$

10. a. Since the particle moves horizontally, the resultant force is zero,

i.e., $$mg + Bqv - qE = 0$$

or $$mg + Bqv = qE$$

or $$2\times10^{-5}\times9.8 + 2.0\times10^{-6}\times v = 10^{-6}\times200$$

Solving, we get $v = 2\ \mathrm{ms^{-1}}$

11. b. The charged particle has circular path in the case when only magnetic field is present.

12. a. $\vec{B}\cdot\vec{F} = 0$ and $\vec{B}\cdot\vec{a} = 0$

$$(2\hat{i} + 3\hat{j} + 4\hat{k})\cdot(x\hat{i} + \hat{j} - \hat{k}) = 0$$

$$2x + 3.1 + 4.(-1) = 0$$

$$2x + 3 - 4 = 0 \quad \Rightarrow \quad x = \frac{1}{2}$$

13. a. It is clear from Right Palm Rule that the charges are positively charged.

As $$r = \frac{mv}{qB}$$

The radius of path is not the only function of either m or q. For (b), (c) and (d) we cannot make clear statement, but statement (a) is certainly true.

14. a $$B = \frac{E}{v}$$

$$= \frac{600\ \mathrm{V}}{3\times10^{-3}\ \mathrm{m}}\times\frac{1}{2\times10^{6}\ \mathrm{m\,s^{-1}}}$$

$$= 10^{-1}\ \mathrm{T} = 0.1\ \mathrm{T}$$

15. c. $$T = \frac{2\pi m}{Bq} \text{ or } T \propto \frac{m}{q}$$

$$\frac{T_\alpha}{T_p} = \frac{4m}{2q}\times\frac{q}{m} = 2$$

or $$T_\alpha = 2\left[\frac{25}{5}\right]\mu\mathrm{s} = 10\ \mu\mathrm{s}$$

16. c. $$B = \frac{\mu_0 I}{2r} \text{ or } B = \frac{\mu_0 q}{2rT}$$

$$B = \frac{4\pi\times10^{-7}\times q}{2r}n$$

$$B = \frac{2\pi nq}{r}\times10^{-7}\ \mathrm{N\,A^{-1}\,m^{-1}}$$

17. a. Time interval in which $\vec{v}$ returns to its initial value is same as time period of the particle, hence the required time $= \dfrac{2\pi m}{eB}$

18. d. $R_1 < R_2$ and $R = \dfrac{mv}{qB}$ of $\left(\dfrac{m}{q}\right)_1 < \left(\dfrac{m}{q}\right)_2$

19. d. Depending on the direction of magnetic field, tension may increase or decrease.

20. a. $\vec{F} \propto (\vec{v}\times\vec{B}) = \hat{k}[aD - dA]$

21. a

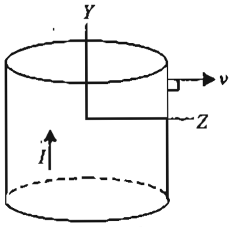


Fig. 9.406

The initial force on the electron is downward. As the electron changes direction, the force on it remains in the xy plane, with a component directed toward the conductor.

22. a

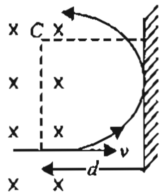


Fig. 9.407

The particle moves in a circular path with radius d if it is to just miss the wall.

$\Rightarrow$ $$mv = Bqr$$

or $$B = \frac{v}{(q/m)d} = \frac{v}{sd}$$

23. a.

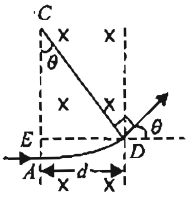


Fig. 9.408

A to D is part of a circle with center C.

$$CD = r$$

$$mv = p = BQr$$

or $$r = p/BQ$$

$$\sin\theta = \frac{ED}{CD} = \frac{d}{r} = \frac{BQd}{p}$$

24. b. $$mv = BQr$$

or $$r = \left(\frac{m}{BQ}\right)v$$

As m, Q and B are the same for all the electrons, $r \propto v$.

25. a. $T = 2\pi = \frac{m}{QB} =$ the same for all electrons, as it is independent of v.

26. b. For a particle moving in any combination of electric and magnetic fields, work is done only by the electric field. Energy of the particle = work done by the electric field = electric field × displacement in the direction of the electric field.

27. a. Use $F = IB\ell \sin\theta$ and the kinematics concepts.

28. b. If the particle passes through O, the situation can be shown in the figure. Let r the radius of circular path then from the given figure

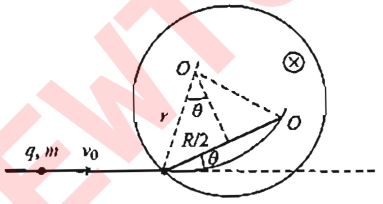


Fig. 9.409

$$\sin\theta = \frac{R/2}{r} = \frac{R}{2r}$$

$$r = \frac{mv}{qB}$$

$$\sin\theta = \frac{RqB}{2mv};\ v_0 = \frac{qBR}{2m\sin\theta}$$

29. b.

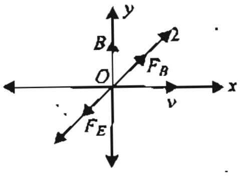


Fig. 9.410

$$E = 10\,\text{Vm}^{-1}$$

$$v = 10\,\text{cm s}^{-1}$$

$$F_E = F_B$$

$$qE = Bqv$$

$$B = \frac{E}{v} = \frac{10}{10^{-2}}$$

$$B = 10^3\,\text{T}$$

30. d. y will be less than zero.

The trajectory will be as shown in Fig. 9.411

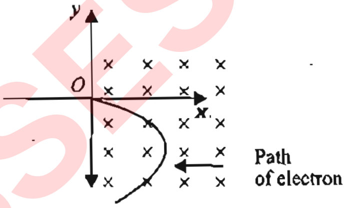


Fig. 9.411

31. a. $K = \frac{1}{2}mv^2 = eV$ or $v = \sqrt{\frac{2eV}{m}}$

Also, $F = evB = e\left[\sqrt{\frac{2eV}{m}}\right] \times B$

Therefore, $\frac{F}{2F} = \sqrt{\frac{V_1}{V_2}} = \sqrt{\frac{V}{V'}}$

$\therefore$ $\frac{V'}{V} = \frac{4}{1}$

32. d. $\vec{E}$ is parallel to $\vec{B}$ and $\vec{v}$ is perpendicular to both. Therefore, path of the particle is a helix with increasing pitch. Speed of particle at any time t is

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \quad (1)$$

Here, $v_y^2 + v_z^2 = v_0^2$

and $v = 2v_0$

Substituting the values in equation (1), we get

$$t = \frac{\sqrt{3}mv_0}{qE}$$

33. b. As the magnetic field is uniform and the particle is projected in a direction perpendicular to the field, it will describe a circular path. The particle will not hit the y–z plane, if the radius of the circle is smaller than d. For the maximum value of v, the radius is just equal to d. Thus,

$$\frac{mv}{Bq} = d$$

$\therefore$ $$v = \frac{Bqd}{m}.$$

34. c. Using, impulse = change in linear momentum, we have

$$\int F\,dt = mv$$

or $\int (iB\ell)\,dt = mv$

or $B\ell q = mv$ (as $\int i\,dt = q$)

$\therefore \quad q = \dfrac{mv}{B\ell}$

35. d. From Fig. 9.412, it is clear that deviation is 180° if $x > R$.

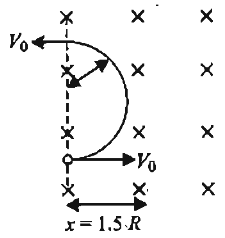


Fig. 9.412

36. c. $$r = \frac{mv_0}{B_0 q} = \frac{v_0}{B_0 \alpha}$$

$$\frac{x}{r} = \frac{\sqrt{3}}{2} = \sin\theta$$

$$\therefore \quad \theta = 60°$$

$$t_{OA} = \frac{T}{6} = \frac{\pi}{3B_0\alpha}$$

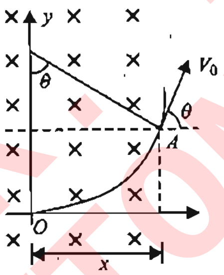


Fig. 9.413

Therefore, x-coordinate of particle at any time $t > \dfrac{\pi}{3B_0\alpha}$ will be

$$x = \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + v_0\left(t - \frac{\pi}{3B_0\alpha}\right)\cos 60°$$

$$= \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + \frac{v_0}{2}\left(t - \frac{\pi}{3B_0\alpha}\right)$$

37. c. $$y = 2r = \frac{2mv_0}{B_0 q} = \frac{2v_0}{B_0\alpha}$$

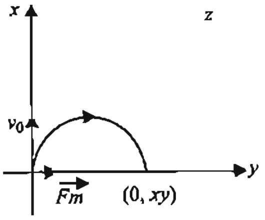


Fig. 9.414

Here, $\dfrac{q}{m} = \alpha$

38. a. $F_{CAD} = F_{CD} = F_{CAD}$

$\therefore$ Net force on the frame $= 3\,F_{CD}$

$= (3)(2)(1)(4) \quad (F = i\ell B)$

$= 24$ N

39. b. Electrostatic force on the electron should be equal and opposite to magnetic force, i.e.,

$$\vec{F}_e = -\vec{F}_m$$

or $q\vec{E} = -q(\vec{v} \times \vec{B})$

So, $\vec{B}$ should be along negative z-axis.

40. c.

$$p = \frac{2\pi m}{Bq}(v\cos 45°) = \frac{2\pi m}{Bq}(v\sin 45°)$$

$$\therefore \quad \frac{mv\sin 45°}{Bq} = \frac{p}{2\pi}$$

$=$ radius of helix

41. d. $\alpha = \dfrac{q}{m}$, path of the particle will be a helix of time period,

$$T = \frac{2\pi m}{B_0 q} = \frac{2\pi}{B_0\alpha}$$

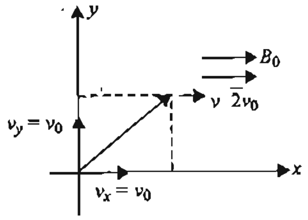


Fig. 9.415

$\because$ The given time $t = \dfrac{\pi}{B_0\alpha} = \dfrac{T}{2}$

$\therefore$ Coordinates of particle at time $t = T/2$ would be $(v_x T/2, 0, -2r)$

Here, $r = \dfrac{mv_r}{B_0 q} = \dfrac{v_0}{B_0\alpha}$

$\therefore$ The coordinate are $\left(\dfrac{v_0\pi}{B_0\alpha}, 0, \dfrac{-2v_0}{B_0\alpha}\right)$.

42. c. The particles will not collide if

$$d > 2(r_1 + r_2)$$

or $$d > 2\left(\frac{mv_1}{Bq} + \frac{mv_2}{Bq}\right)$$

or $$d > \frac{2m}{Bq}(v_1 + v_2)$$

43. d.

$$\frac{\mu_0 I}{2r}-\frac{\mu_0 I}{2\pi r}=\frac{\mu_0 I}{2r}\left(1-\frac{1}{\pi}\right)$$

$$=\frac{\mu_0 I(\pi-1)}{2\pi r}$$

44. c. No force on PQ and SR.
Force on PS or $QR = Bi\ell$

$$= 2\times 10^{-2}\times 2\times 0.25\text{ N} = 10^{-2}\text{ N}$$

$$\tau = 10^{-2}\text{ N}\times 0.4\text{ m} = 4\times 10^{-3}\text{ N m}$$

45. a. All points on AB are equidistant from the two wires. So, magnetic fields are equal and opposite. Thus, they cancel out.

46. a. $$B=\frac{\mu_0 I}{2r}$$

$$I \propto Br$$

$$\therefore \quad \frac{I_1}{I_2}=\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$$

47. d. The magnetic fields due to both the currents are along '$-Z$-axis'. So, they add up to give a net magnetic field along '$-Z$-axis'.

48. b. At the midpoint of the line joining the conductors, $\vec{B} = 0$. As we come close to the wires, the magnitude of B increases. The direction of magnitude fields on opposite sides of a wire will be opposite. Again, $\vec{B} = 0$, as $r \to \infty$.

49. d $$B=\frac{\mu_0 I}{4\pi d}[\sin 0-\sin 0]=0$$

50. b $$B_{\text{axis}}=\frac{\mu_0}{4\pi}\times\frac{2\pi IR^2}{(R^2+x^2)^{3/2}}$$

At center, $B_{\text{center}}=\dfrac{\mu_0}{4\pi}\times\dfrac{2\pi I}{R}$

In the given problem,

$$\frac{\mu_0}{4\pi}\times\frac{2\pi IR^2}{(R^2+x^2)^{3/2}}=\frac{1}{8}\left[\frac{\mu_0}{4\pi}\times\frac{2\pi I}{R}\right]$$

or $(R^2+x^2)^{3/2}=8R^3$

Solving, we get $= x = R\sqrt{3}$

51. c. Magnetic field at the center due to either arm

$$B_1=\frac{\mu_0}{4\pi}\times\frac{I}{(L/2)}[\sin 45°+\sin 45°]$$

$$=\frac{\mu_0}{4\pi}\times\frac{2\sqrt{2}\,I}{L}$$

Field at center due to the four arms of the square

$$B = 4B_1 = 4\times\frac{\mu_0}{4\pi}\times\frac{2\sqrt{2}\,I}{L}$$

i.e., $B \propto \dfrac{1}{L}$

52. d. Torque τ acting on a current carrying coil of area A placed in a magnetic field of induction B is given by

$$\tau = NIBA\sin\theta$$

where I = current in the coil, θ = angle which the normal to the plane of the coil makes with the lines of induction B.

Here, $N = 1$, $B = 1.5\times 10^{-2}$ T
$A = 0.05\times 0.08 = 40\times 10^{-4}\text{ m}^2$
$I = 10.0$ amp, $\theta = 90° = \pi/2$

$$\tau = (1.5\times 10^{-2})(10.0)\times(1)(40\times 10^{-4})\sin\frac{\pi}{2}$$

$$= 6\times 10^{-4}\text{ Nm}$$

53. a. $$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

$$= 4\pi\times 10^{-7}\times\frac{1}{4\pi}$$

$$= 10^{-7}\text{ Wb m}^{-1}$$

54. c. $$B_1=\frac{\mu_0}{4\pi}\left(\frac{2\pi I_1}{r_1}\right)$$

$$B_2=\frac{\mu_0}{4\pi}\left(\frac{2\pi I_2}{r_2}\right)$$

$r_2 = 2r_1$, $B_1 = B_2$ and $I_2 = 2I_1$

$$\frac{Vq}{Vp}=\frac{I_2\times r_2}{I_1\times r_1}=\frac{2I_1}{I_1}\times\frac{2r_1}{r_1}=\frac{4}{1}$$

55. d. Using right hand rule, the direction of magnetic field is clearly toward west at A and B

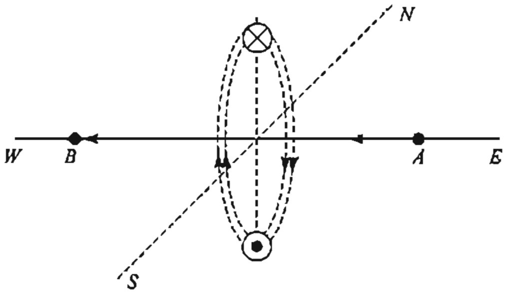


Fig. 9.416

56. b. Using Ampere's circuital law, $B = 0$ for $r \le R_1$

$$B=\frac{\mu_0 i}{2\pi(R_2^2-R_1^2)}\left(\frac{r^2-R_1^2}{r}\right)\text{ for } R_1\le r\le R_2$$

and $$B=\frac{\mu_0}{2\pi}\frac{i}{r}\text{ for } r\ge R_2$$

The corresponding B–r graph will be as shown in option (b).

57. b.

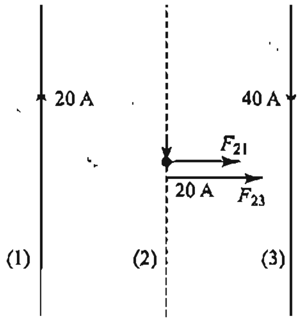


Fig. 9.417

Hence, net force on the middle wire will be toward the 40 A wire.

58. d. No current will flow through section BC as the potentials of points B and C are same. Therefore magnetic field at center O will be equal and opposite due to section AB and AC. Hence, they cancel out.

59. c. Net current is $(20-6+12-7+18)$ A, i.e., 37 A.

$$r=\frac{10}{100}\text{ m}=\frac{1}{10}\text{ m}$$

$$B=\frac{\mu_0 I}{2\pi r}=\frac{4\pi\times10^{-7}\times37\times10}{2\pi\times1}\text{T}$$

$$=74\times10^{-6}\text{ T}=74\ \mu\text{T}.$$

60. b. $$B=\frac{3}{4}\left[\frac{\mu_0 I}{2a}\right]+\frac{1}{4}\left[\frac{\mu_0 I}{2b}\right]$$

$$B=\frac{3\mu_0 I}{8a}+\frac{\mu_0 I}{8b}$$

61. b. Magnetic field due to straight wires

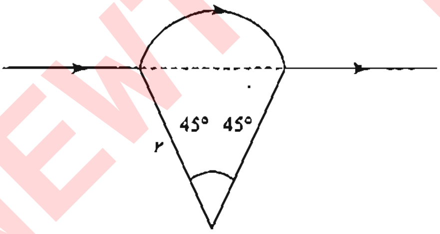


Fig. 9.418

$$B_{\text{circular}}=\frac{1}{4}\frac{\mu_0 I}{2r}=\frac{\mu_0 I}{8r}$$

$$B_{\text{straight}}=2\frac{\mu_0 I}{4\pi(r\cos45°)}[\sin90°-\sin45°]$$

$$=\frac{\sqrt{2}\mu_0 I}{2\pi r}\left[1-\frac{1}{\sqrt{2}}\right]$$

62. c. $$B=\frac{\mu_0 I}{4r_1}+\frac{\mu_0 I}{4r_2}$$

or $$B=\frac{\mu_0 I}{4}\left[\frac{1}{r_1}+\frac{1}{r_2}\right]$$

$$=\frac{\mu_0 I}{4}\left[\frac{r_2+r_1}{r_1 r_2}\right]$$

63. a. $$F=\frac{4\pi\times10^{-7}\times20\times10\times10\times10^{-2}}{2\pi\times2\times10^{-2}}$$

$$-\frac{4\pi\times10^{-7}\times10\times30\times10\times10^{-2}}{2\pi\times10\times10^{-2}}$$

$$=\frac{4\pi\times10^{-7}\times10\times10^{-2}}{2\pi\times10^{-2}}[100-30]$$

$$=20\times10^{-7}\times70=1400\times10^{-7}$$

$$=1.4\times10^{-4}\text{ N toward right.}$$

64. c. AB and DE do not give any magnetic field at Q.

Magnetic field at Q due to $BC=\dfrac{\mu_0 I}{4\pi a}$

It is directed 'up'.

Magnetic field at Q due to

$$EF=\frac{\mu_0 I}{4\pi a}$$

So, $$B=\frac{\mu_0 I}{2\pi a}$$

65. a. $$B=\frac{\mu_0 I}{4\pi d}[\sin\theta_1+\sin\theta_2]$$

But $\theta_1+\phi_1=90°$ or $\theta_1=90°-\phi_1$

$\sin\theta_1=\sin(90°-\phi_1)=\cos\phi_1$

Similarly, $\sin\theta_2=\cos\phi_2$

$$B=\frac{\mu_0 I}{4\pi d}(\cos\phi_1+\cos\phi_2)$$

66. c. $$B=\frac{\mu_0 I}{4\pi r}\left[2\sin\frac{\pi}{n}\right]$$

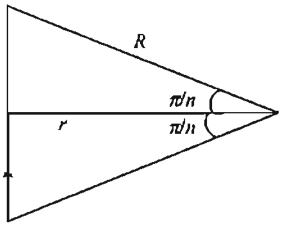


Fig. 9.419

But $\cos\dfrac{\pi}{n}=\dfrac{r}{R}$

or $r=R\cos\dfrac{\pi}{n}$

$$\therefore \quad B=\frac{\mu_0 I}{4\pi R\cos\frac{\pi}{n}}\left[2\sin\frac{\pi}{n}\right]$$

$$\text{or} \quad B=\frac{\mu_0 I}{2\pi R}\left[\tan\frac{\pi}{n}\right]$$

67. c. Consider an element of thickness dr at a distance r form the center of the spiral.

Number of turns, $dN=\frac{N}{b-a}dr$

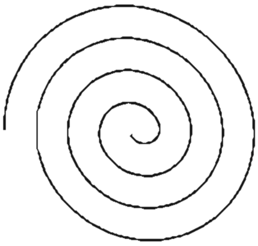


Fig. 9.420

$$dB=\frac{\mu_0\frac{N}{b-a}dr\,I}{2r}$$

$$\text{or} \quad dB=\frac{\mu_0 I}{2}\frac{N}{b-a}\frac{dr}{r}$$

$$\Rightarrow \quad B=\frac{\mu_0 I}{2}\frac{N}{b-a}\int_a^b\frac{1}{r}dr$$

$$=\frac{\mu_0 I}{2}\frac{N}{b-a}\left|\log_e r\right|_a^b$$

$$=\frac{\mu_0 NI}{2(b-a)}[\log_e b-\log_e a]$$

$$\therefore \quad B=\frac{\mu_0 NI}{2(b-a)}\log_e\frac{b}{a}$$

68. c. $BI=2T\sin\theta$

θ is small, $\sin\theta=\theta$

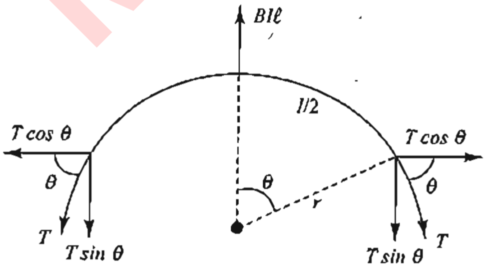


Fig. 9.421

$$\therefore \quad BI\ell=2T\sin\theta=2T\cdot\theta=2T\left(\frac{\ell/2}{r}\right)$$

$$\text{or} \quad BI=\frac{T}{r}$$

$$\text{But} \quad 2\pi r=\ell \text{ or } r=\frac{\ell}{2\pi}$$

$$\therefore \quad BI=\frac{T2\pi}{\ell} \text{ or } T=\frac{BI\ell}{2\pi}$$

69. c. The forces F_Q on the current carrying wire P due to the currents in Q and R are shown in Fig. 9.367. The resultant force is F which is best represented by vector C in the original diagram of the question.

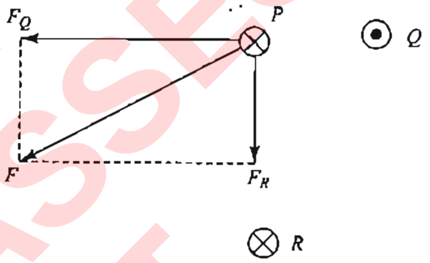


Fig. 9.422

70. c. $B_{AB}=\frac{\mu_0 I}{4\pi(OC)}[2\sin\theta]$

But $OC=r\cos\theta$

$$\text{or} \quad B_{AB}=\frac{\mu_0 I}{2\pi r}\tan\theta$$

Magnetic field due to circular portion,

$$B_{AB}=\frac{\mu_0 I}{2r}\frac{2\pi-2\theta}{2\pi}=\frac{\mu_0 I}{2\pi r}(\pi-\theta)$$

Total magnetic field

$$=\frac{\mu_0 I}{2\pi r}\tan\theta+\frac{\mu_0 I}{2\pi r}(\pi-\theta)$$

$$=\frac{\mu_0 I}{2\pi r}[\tan\theta+\pi-\theta]$$

71. a. The magnetic field on the axis of a coil carrying current I, having n turns, radius r and at a distance h from the center of the coil, is given by

$$B=\frac{\mu_0}{4\pi}\times\frac{2\pi NIr^2}{\left(r^2+h^2\right)^{3/2}} \quad (1)$$

The field at the center is given by

$$B_C=\frac{\mu_0}{4\pi}\times\frac{2\pi NI}{r} \quad (2)$$

$$\therefore \quad \frac{B}{B_C}=\frac{r^3}{\left(r^2+h^2\right)^{3/2}}=\frac{r^3}{r^3\left[1+\frac{h^2}{r^2}\right]^{3/2}}$$

$$= \frac{1}{\left[1+\frac{3}{2}\frac{h^2}{r^2}\right]} \quad \text{or} \quad B\left[1+\frac{3}{2}\frac{h^2}{r^2}\right] = B_C$$

$$\therefore \frac{(B_C - B)}{B} = \frac{3}{2}\frac{h^2}{r^2}$$

72. b. $B_1 = \frac{\mu_0}{4\pi} \times \frac{2\pi I}{R} = \frac{\mu_0}{4\pi} \times \frac{2\pi I \times 2\pi}{L}$ (1)

($\because L = 2\pi R$, for circular loop)

$$B_2 = \frac{\mu_0}{4\pi} \times \frac{1}{(a/2)}[\sin 45° + \sin 45°] \times 4$$

where $a = (L/4)$

$$\therefore \quad B_2 = \frac{\mu_0 I}{4\pi L} \times 8 \times 4 \times \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$$

$$= \frac{\mu_0 I}{4\pi L} \times \frac{64}{\sqrt{2}}$$

$$\therefore \quad \frac{B_1}{B_2} = \left(\frac{\mu_0}{4\pi}\right)\frac{4\pi^2 I}{L} \bigg/ \frac{\mu_0}{4\pi L} \times \frac{6I}{\sqrt{2}}$$

$$\text{or} \quad \frac{B_1}{B_2} = \frac{\pi^2}{8\sqrt{2}}$$

73. c.

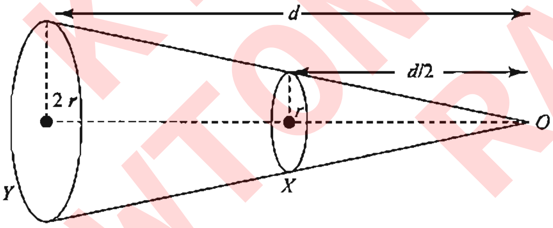


Fig. 9.423

As two coils subtend the same solid angle at O, hence area of coil $Y = 4 \times$ area of coil X

$$\left[\text{Solid angle} = \frac{\text{area}}{(\perp \text{distance})^2}\right]$$

i. e., radius of coil $Y = 2 \times$ radius of coil X

$$\therefore \quad B_y = \frac{\mu_0}{4\pi} \times \frac{2\pi I (2r)^2}{\left[(2r)^2 + (d^2)\right]^{3/2}}$$

$$B_x = \frac{\mu_0}{4\pi} \times \frac{2\pi I (r)^2}{\left[(r)^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}}$$

$$\therefore \quad \frac{B_y}{B_x} = \frac{4}{(4r^2 + d^2)^{3/2}} \times \left[\frac{4r^2 + d^2}{4}\right]^{3/2}$$

$$= \frac{4}{(4)^{3/2}} = \frac{4}{8} = \frac{1}{2}$$

74. b. Magnetic force acts in the direction shown in Fig. 9.424

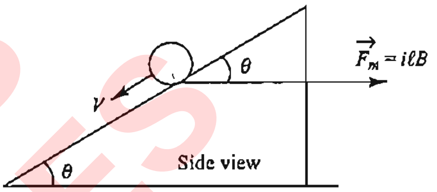


Fig. 9.424

Road will move downward with constant velocity if net force on it is zero.

or $F_m \cos\theta = mg \sin\theta$

or $i\ell B \cos\theta = mg \sin\theta$

$$\therefore \quad B = \left(\frac{mg}{i\ell}\right)\tan\theta$$

75. b. $B_{\text{square}} = \frac{\mu_0 I}{4\pi(L/8)}(\sin 45° + \sin 45°) = \frac{2\mu_0 I\sqrt{2}}{\pi L}$

Radius of circular loop $r = \frac{L}{2\pi}$

$$B_{\text{circular}} = \frac{\mu_0 I}{2r} = \frac{\mu_0 I \pi}{L}$$

$$\frac{B_{\text{cir.}}}{B_{\text{sq.}}} = \frac{\pi^2}{4\sqrt{2}}$$

76. c. Magnetic field due to infinite current carrying sheet is given by $B = \frac{\mu_0 J}{2}$, where J is linear current density.

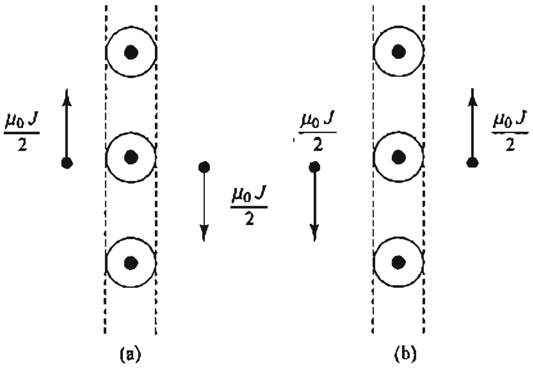


Fig. 9.425

Fig. 9.425(a) and (b) represent the direction of magnetic field due to current carrying sheets.

For $x < a$, $B_{\text{resultant}} = \frac{\mu_0 J}{2} - \frac{\mu_0(2J)}{2} - \frac{\mu_0(3J)}{2} + \frac{\mu_0(4J)}{2}$

For $a < x < 2a$,

$$B_{\text{resultant}} \approx -\frac{\mu_0 J}{2}-\frac{\mu_0(2J)}{2}-\frac{\mu_0(3J)}{2}+\frac{\mu_0(4J)}{2}=-\mu_0 J$$

For $2a<x<3a$,

$$B_{\text{resultant}} = -\frac{\mu_0 J}{2}+\frac{\mu_0(2J)}{2}-\frac{\mu_0(3J)}{2}+\frac{\mu_0(4J)}{2}=\mu_0 J$$

For $4a<x$,

$$B_{\text{resultant}} = \frac{-\mu_0 J}{2}+\frac{\mu_0(2J)}{2}+\frac{\mu_0(3J)}{2}-\frac{\mu_0(4J)}{2}=0$$

So, the required curve is

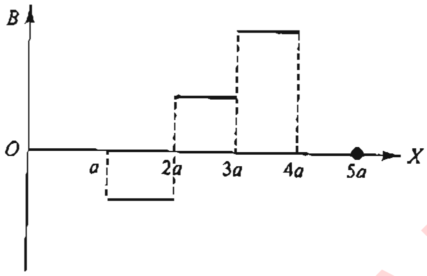


Fig. 9.426

77. a. See the direction of torque about center.

78. d. Magnetic field at the center due to the given configuration is zero.

79. c. Force is of expansive nature.

$T = IRB$

80. a $B_L = \dfrac{\mu_0 I}{2\pi R}$

If the radius is R/n; the number of turns will be n.

$$B_C = \frac{n\mu_0 I}{2\pi(R/n)} = n^2\frac{\mu_0 I}{2\pi R}$$

Hence, $\dfrac{B_L}{B_C} = \dfrac{1}{n^2}$

81. b. Magnetic field due to '*AB*';

$$\vec{B}_1 = \frac{\mu_0 I}{4\pi R}\left[\sin\frac{\pi}{2}-\sin\frac{\pi}{4}\right]\otimes$$

$$= \frac{\mu_0 I}{4\pi R}\left[1-\frac{1}{\sqrt{2}}\right]\otimes$$

Magnetic field due to circular loop,

$$\vec{B}_2 = \frac{\mu_0 I}{2R}\odot$$

Magnetic field due to straight wire BC,

$$\vec{B}_3 = \frac{\mu_0 I}{4\pi R}\left[\sin\frac{\pi}{2}+\sin\frac{\pi}{4}\right]\odot$$

$$= \frac{\mu_0 I}{4\pi R}\left[1+\frac{1}{\sqrt{2}}\right]\odot$$

$$\vec{B} = \vec{B}_1+\vec{B}_2+\vec{B}_3$$

$$= \left(\frac{\mu_0 I}{2R}+\frac{2\mu_0 I}{4\pi R}\frac{1}{\sqrt{2}}\right)\odot = \frac{\mu_0 I}{2R}\left[1+\frac{1}{\sqrt{2}\,\pi}\right]\odot$$

82. a. As shown in Fig.9.427,

$$\vec{B}_1 = -2\left(\frac{\mu_0}{2\pi}\frac{i}{a}\right)\hat{k} = -\frac{\mu_0 i}{\pi a}\hat{k}$$

= magnetic field at O

$$\vec{B}_2 = \frac{\mu_0}{2\pi}\frac{i}{a}\hat{k}-\frac{\mu_0}{2\pi}\frac{i}{3a}\hat{k}$$

= magnetic field at P

$$= \frac{\mu_0}{3\pi}\frac{i}{a}\hat{k}$$

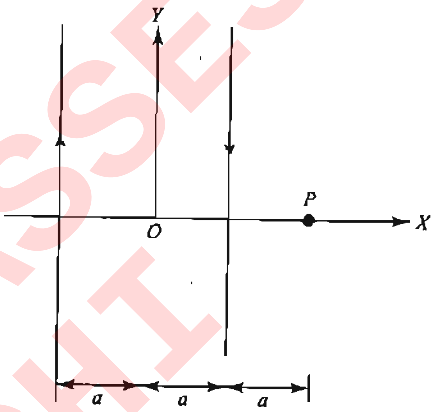


Fig. 9.427

$\therefore \quad B_1/B_2 = -3$

83. a. $\vec{B}_1 = \dfrac{\mu_0 I}{4r}\odot;\ \vec{B}_2 = \dfrac{\mu_0 I}{4r}\odot$

$$\vec{B}_3 = \frac{\mu_0 I}{2r}\left(\frac{3}{4}\right)\otimes+\frac{\mu_0 I}{4\pi r}\odot$$

$$= \frac{\mu_0 I}{4r}\left[\frac{3}{2}\otimes+\frac{1}{\pi}\odot\right]$$

$$= \frac{\mu_0 I}{4r}\left[\frac{3\pi-2}{2\pi}\right]\otimes = \frac{\mu_0 I}{4r}\left(\frac{2}{\pi}\right)\left(\frac{3\pi}{4}-\frac{1}{2}\right)\otimes$$

$$B_1 : B_2 : B_3 := -\left(\frac{\pi}{2}\right):\left(\frac{\pi}{2}\right):\left(\frac{3\pi}{4}-\frac{1}{2}\right)$$

84. b. $B_{\text{center}} = 0$

$$\frac{\mu_0 I}{4\pi b}\odot+\frac{\mu_0 I}{4\pi b}\odot+\frac{\mu_0 I}{2b}\odot+\frac{\mu_0 I}{2a}\otimes=0$$

$$\frac{\mu_0 I}{2\pi b}+\frac{\mu_0 I}{2b}-\frac{\mu_0 I}{2a}=0$$

$$\frac{1}{2\pi b}+\frac{1}{2b}=\frac{1}{2a}$$

$$\frac{a}{b}=\frac{\pi}{\pi+1}$$

85. c. $\dfrac{\mu_0 I_1}{4\pi(a+x)} = \dfrac{\mu_0 I_2}{4\pi(a-x)}$

$$\frac{a-x}{a+x} = \frac{I_2}{I_1}$$

$$I_1a - I_1x = I_2a + I_2x$$

$$x = \left(\frac{I_1 - I_2}{I_1 + I_2}\right)a$$

86. c. Magnetic field at the center

$$B = \frac{\mu_0 I}{2c}\otimes + \frac{\mu_0 I}{2b}\left(\frac{3}{4}\right)\odot + \frac{\mu_0 I}{2a}\left(\frac{3}{4}\right)\otimes$$

As per problem:

$$0 = \frac{\mu_0 I}{2}\left[\frac{1}{c} - \frac{3}{4b} + \frac{3}{4a}\right] \Rightarrow \frac{3}{4}\left[\frac{1}{b} - \frac{1}{a}\right] = \frac{1}{c}$$

If $c = 2a$,

$$\frac{3}{4}\left[\frac{1}{b} - \frac{1}{a}\right] = \frac{1}{2a} \Rightarrow a = \frac{5}{3}b$$

87. c. $\bar{B}_1 = \frac{\mu_0 I}{4\pi R}\odot,\ \bar{B}_2 = \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I/2}{4\pi R}\odot$

$$\Rightarrow \quad B_2 = \frac{\mu_0 I}{4\pi R}\left(\frac{3}{2}\right)\odot \Rightarrow \left|\frac{B_1}{B_2}\right| = \frac{2}{3}$$

88. a. $B_{\text{axial}} = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} = 54\times 10^{-6}\ \text{T}$

$$I = \frac{54\times 10^{-6} \times 2(x^2 + a^2)^{3/2}}{\mu_0 a^2}$$

$$B_{\text{center}} = \frac{\mu_0}{2}\frac{I}{a}$$

Substituting for I,

$$B_{\text{center}} = \frac{\mu_0}{2}\frac{54\times 10^{-6} \times 2(x^2 + a^2)^{3/2}}{\mu_0 a^2}$$

$$= \frac{54\times 10^{-6} \times (x^2 + a^2)^{3/2}}{a^2}$$

Putting $x = 4\times 10^{-2}$ m and $a = 3\times 10^{-2}$ m, we get

$$B_{\text{center}} = 250\times 10^{-6}\ \text{T} = 250\ \mu\text{T}$$

89. a.

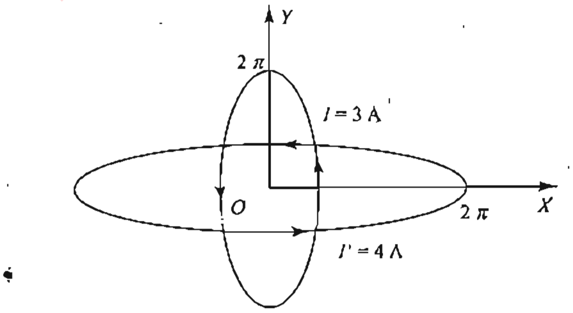


Fig. 9.428

$$B_x = \frac{\mu_0}{2}\frac{I}{2\pi\times 10^{-2}} = \frac{\mu_0}{4\pi}3\times 10^2 = 3\times 10^{-5}\ \text{T}$$

$$B_y = \frac{\mu_0}{2}\frac{I'}{2\pi\times 10^{-2}} = 4\times 10^{-5}\ \text{T}$$

$$B_{\text{net}} = \sqrt{B_x^2 + B_y^2} = \sqrt{25\times 10^{-10}} = 5\times 10^{-5}\ \text{T}$$

90. c. $B_1 = \frac{\mu_0}{4\pi}\frac{2I}{R};\quad B_2 = \frac{\mu_0}{4\pi}\frac{4I}{R}$

$$B_2 - B_1 = \frac{\mu_0}{4\pi}\frac{2I}{R} = B$$

$$B_2 = 2B_1$$

$$2B_1 - B_1 = B$$

$$\therefore \quad B_1 = B$$

91. a. Torque due to magnetic field $\tau_{\text{mag}} = MB = I\pi R^2 B$ (i)

Torque due to weight about the point where string is connected

$\tau_{\text{weight}} = mgR$ (ii)

If ring remains horizontal, then $\tau_{\text{mag}} = \tau_{\text{weight}}$

$$I\pi R^2 B = gR \Rightarrow I = \frac{mg}{\pi R B_0}$$

92. a. To find the magnetic field outside a thick conductor, the current may be assumed to flow along the axis. As points 1, 2 and 3 are equidistant from the axis.

$$B_1 = B_2 = B_3$$

93. c. Apply Ampere's circuital law to the coaxial circular loops L_1 and L_2. The magnetic field is B_1 at all points on L_1 and B_2 at all points on L_2. $\Sigma I \neq 0$ for L_1 and 0 for L_2.

Hence, $B_1 \neq 0$ but $B_2 = 0$

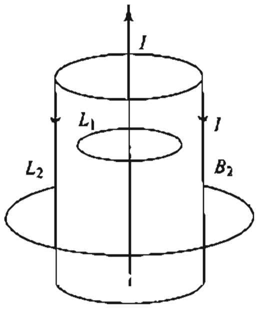


Fig. 9. 429

[As $\oint \vec{B}.\vec{dl} = \mu_0 \sum I$]

94. b. Consider an element of length dx on AB, at a distance x from XY. Force on the element,

$$dF = (\mu_0 I/2\pi x)I'x\ dx$$

Total force on AB,

$$F = \int_{L/2}^{3L/2}\frac{\mu_0 II'}{2\pi x}dx$$

or $$F = \frac{\mu_0 II'}{2\pi}\log 3$$

95. d. Net force on a current carrying loop in a uniform magnetic field is zero. So, magnetic force cannot balance its weight.

96. a. Magnetic field at O is

$$B = 3\left[\frac{\mu_0}{4\pi}\frac{i}{r}(\sin 60° + \sin 60°)\right]$$

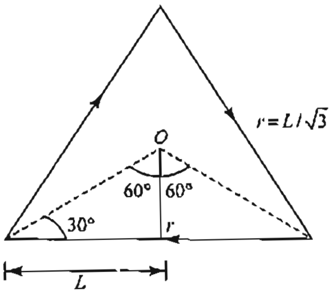


Fig. 9.430

$$= 3\left(\frac{\mu_0}{4\pi}\right)(i)\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{L}\right) = \frac{9\mu_0 i}{4\pi L}$$

97. c. $x = r\sin\dfrac{\alpha}{2}$

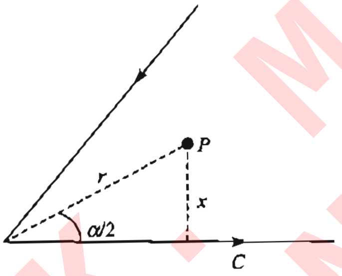


Fig. 9.431

$$\therefore \quad B_P = 2\left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{x}\right)\left[\sin\left(90° - \frac{\alpha}{2}\right) + \sin 90°\right]$$

$$= \frac{\mu_0}{2\pi}\frac{i}{r}\frac{\left(1+\cos\dfrac{\alpha}{2}\right)}{\sin\dfrac{\alpha}{2}}$$

98. b. $$\vec{B} = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+\cdots\right)\hat{k} - \left(\frac{1}{2}+\frac{1}{8}+\cdots\right)\hat{k}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{1-\frac{1}{2}}\right)\hat{k} - \frac{1}{2}\left(\frac{1}{1-\frac{1}{4}}\right)\hat{k}\right]$$

$$= \frac{2}{3}\frac{\mu_0}{2\pi}\hat{k} = \left(\frac{2}{3}\times 2\times 10^{-7}\right)\hat{k}$$

$$= 1.33\times 10^{-7}\,\hat{k}$$

99. a. Point $(0, 0, -a)$ lies on z-axis, therefore magnetic field due to current along z-axis is zero and due to rest two wires is $\dfrac{\mu_0}{2\pi}\dfrac{1}{a}$ in mutually perpendicular directions along positive y-direction and negative x-direction.

$$\therefore \quad \vec{B} = \frac{\mu_0}{2\pi}\frac{i}{a}(\hat{j} - \hat{i})$$

100. d. Magnetic field at P is perpendicular to paper inward due to both the wires. Charged particle is also projected in the same direction. So, force on the charged particle is zero as $\vec{v} \parallel \vec{B}$ and $\vec{F}_m = q(\vec{v}\times\vec{B})$

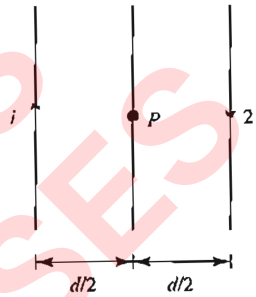


Fig. 9.432

101. a. E.m.f. induced in coil, $\varepsilon = -\dfrac{\Delta\phi}{\Delta t} = -\dfrac{(\phi_2 - \phi_1)}{t_2 - t_1}$

$$\phi_2 = BA_2 - B\,\frac{4a}{2\times 3}\times\frac{4a}{3}\times\sin 60° = \frac{4\sqrt{3}}{9}Ba^2$$

$$\phi_1 = Ba^2$$

Work done, $W = \varepsilon I\Delta t = Ba^2\left(1 - \dfrac{4\sqrt{3}}{9}\right)i$

102. d. $\sin\theta = \dfrac{d}{R}$

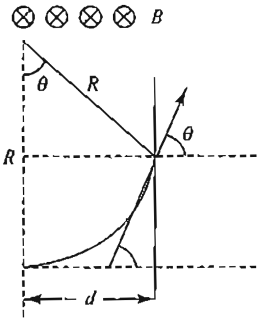


Fig. 9.433

$$d = R\sin\theta = \frac{mv}{qB}\sin\theta$$

$$\Rightarrow \quad \frac{q}{m} = \frac{v\sin\theta}{Bd}$$

103. b. Magnetic field may be zero in first and third quadrants.

$\dfrac{\mu_0 I_1}{2\pi y} = \dfrac{\mu_0 I_2}{2\pi x}$ represents a straight line passing through the origin.

104. a. The gravitational torque must be counter balanced by the magnetic torque about O, for equilibrium of the sphere. The gravitational torque $\tau_m = \pi i r^2 B \sin\theta$

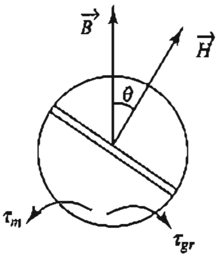


Fig. 9.434

$$\therefore \quad \pi i r^2 B \sin\theta = mgr \sin\theta$$

$$\Rightarrow \quad B = \frac{mg}{\pi i r}$$

105. b. $T = 2\pi m/qB$

$$\therefore \quad t_{He} > t_p > t_e$$

106. b. Applying Ampere's circuital law.

$$B 2\pi r = \frac{\mu_0 I \pi r^2}{\pi a^2} \quad \text{for } r < a$$

and $B 2\pi R = \mu_0 I$ for $R > a$

Taking the ratio

$$\frac{r}{R} = \frac{r^2}{a^2}$$

$$\therefore \quad a = \sqrt{Rr}$$

107. b. $B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$

$$B = \frac{\mu_0}{4\pi}\frac{i_0 \pi R^2}{d^3}$$

$$\tau = i\pi r^2 \frac{\mu_0}{4\pi}\frac{i_0 \pi R^2}{d^3}; \tau = \frac{\mu_0}{4}\frac{\pi i_0 i R^2 r^2}{d^3}$$

108. b. As we know that

$$\vec{F} = \oint I d\vec{\ell} \times \vec{B} \text{ and } \vec{\tau} = \vec{M} \times \vec{B}$$

109. b. $\omega = 2\pi n$

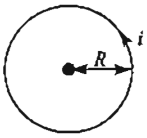


Fig. 9.435

$$i \times 2\pi R = R\omega q = R 2\pi nq$$

$$B = \frac{\mu_0 i}{2R} = \frac{\mu_0 nq}{2R}$$

110. b. $F = BI_1\ell + BI_2\ell$

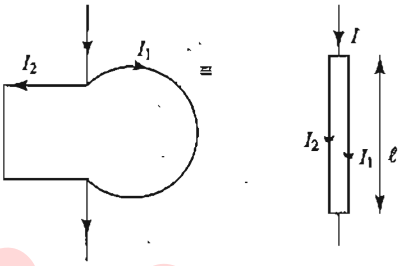


Fig. 9.436

$$\Rightarrow \quad F = B\ell(I_1 + I_2) = BI\ell$$

111. a. B at all the points lying on OP is zero so u will not be affected by the magnetic force. Hence, it moves on a straight line.

112. b. There will be no effect of magnetic force on time period because the magnetic force will be perpendicular to the inclined plane.

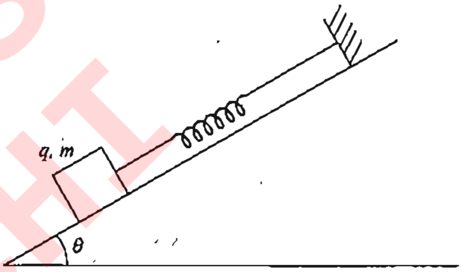


Fig. 9.437

113. c. Taking moments about point B to be zero.

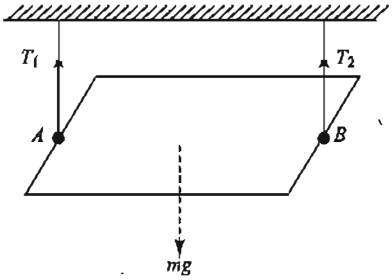


Fig. 9.438

$$T_1\ell + ib\ell B = mg\frac{\ell}{2}; \quad T_1 = \frac{mg - 2ibB}{2}$$

114. d. $B = \frac{\mu_0 i}{2r} - \frac{\mu_0 i}{2(2r)} + \frac{\mu_0 i}{2(2^2 r)} + \cdots\infty$

$$B = \frac{\mu_0 i}{2r}\left[1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \cdots\infty\right]$$

$$B = \frac{\mu_0 i}{2r}\left[\frac{1}{1-\left(-\frac{1}{2}\right)}\right]$$

$$B = \frac{\mu_0 i}{3r}$$

115. b. Time period, $T = \dfrac{2\pi m}{qB}$

$$T = \frac{2\pi}{\pi \times 2} = 1\ \text{s}$$

Thus, particle will be at point P after $t = \dfrac{1}{12}$ s

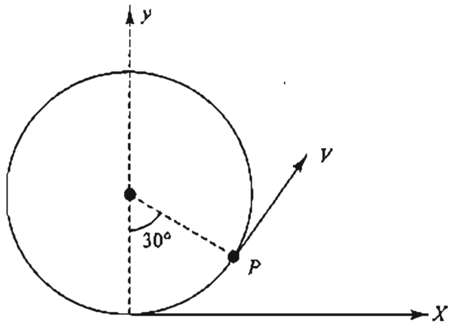


Fig. 9.439

$$\bar{v} = 10[\cos 30\hat{i} + \sin 30\hat{j}]$$

$$\bar{v} = 10\left[\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right] = 5\left[\sqrt{3}\hat{i} + \hat{j}\right] \text{ms}^{-1}$$

116. d. Let current i flows in the bigger ring, then the magnetic field on its axis

$$B = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$$

Flux linked with the smaller ring

$$\phi = B\pi r^2$$

$$\phi = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}\pi r^2 = M\,i$$

$$\therefore \qquad M = \frac{\mu_0 \pi R^2 r^2}{2(R^2 + x^2)^{3/2}}$$

117. d. $E 2\pi \ell = \pi R^2 \left(\dfrac{dB}{dt}\right);\ E = \dfrac{R^2}{2\ell}\left(\dfrac{dB}{dt}\right)$

$$qE + mg = Kx$$

$$x = \frac{qR^2}{K 2\ell}\left(\frac{dB}{dt}\right) + \frac{mg}{K};\ x = \frac{1}{K}\left[mg + \frac{qR^2}{2\ell}\frac{dB}{dt}\right]$$

118. c. $T = \dfrac{2\pi m}{qB} = 3.14\ \text{s}$

119. a. $F_R = 2\sqrt{2}\,iBa$

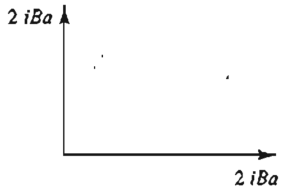


Fig. 9.440

120. d. $\displaystyle\int_{PAQ} \vec{B}\cdot d\vec{\ell} = \int_{RBQ} \vec{B}\cdot d\vec{\ell}$ $\left(\because \oint \vec{B}\cdot d\vec{\ell} = 0 \text{ and } \displaystyle\int_{PCR} \vec{B}\cdot d\vec{\ell} = 0\right)$

$$= \left(\frac{\mu_0 I}{2\pi R}\right)\left(\frac{\pi}{4}R\right)$$

$$= \frac{(4\pi \times 10^{-7})(2)}{16} = \pi \times 10^{-7}\ \text{S.I. units}$$

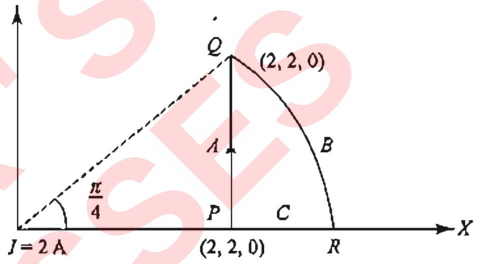


Fig. 9.441

121. b. $B_1 = \dfrac{\mu_0}{4\pi} \times \dfrac{2\pi I}{a} \times \dfrac{1}{2}$ (due to semicircular part)

$B_2 = \dfrac{\mu_0}{4\pi} \times \dfrac{2I}{a}$ (due to parallel parts of currents)

These two fields are at right angles to each other. Hence, resultant field

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0 I}{4\pi a}\sqrt{\pi^2 + 4}$$

122. a. $B_{\text{circle}} = \dfrac{\mu_{0 I_2}}{2R} \odot$

B_{wire} at the center of circle $= \dfrac{\mu_0 I_1}{2\pi(2R)} \otimes$

Total magnetic field at the center is zero.

$$= \frac{\mu_0 I_2}{2R} - \frac{\mu_0 I_1}{2\pi(2R)} = 0$$

$$I_1 = I_2(2\pi)$$

123. a. Radius of circular path of the charged particle $r = \dfrac{mV}{qB}$

Here $R < r$, hence the particle will press the outer wall of the pipe hence the force applied by the pipe on the particle should be towards the centre of the pipe.

124. b. $F = qvB = \dfrac{qv\mu_0 i}{2\pi r}$

$$\frac{mv^2}{R} = \frac{qv\mu_0 i}{2\pi r}$$

$$R = \frac{2\pi r m v}{q\mu_0 i}$$

125. a. $$\left|\bar{F}_{B/A}\right| = \left|-\frac{\mu_0}{4\pi}\frac{q_1q_2v_1v_2}{2\sqrt{2}a^2}\hat{i}\right|$$

$$\left|\vec{F}_{12}\right| = \left|-\frac{\mu_0}{4\pi}\frac{q_1q_2v_1v_2}{2\sqrt{2}a^2}\hat{j}\right|$$

126. a. The ratio M/L is always $\frac{q}{2m}$

127. c. $$r = \frac{mv_0}{B_0q} = \frac{v_0}{B_0\alpha}$$

$$\frac{x}{r} = \frac{\sqrt{3}}{2} = \sin\theta \quad \Rightarrow \quad \theta = 60^\circ$$

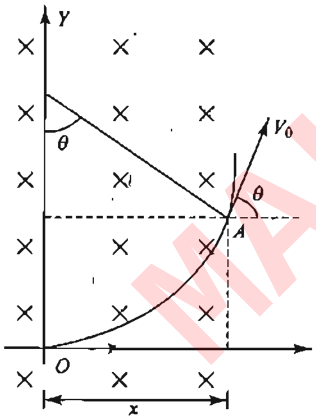


Fig. 9.442

$$t_{OA} = \frac{T}{6} = \frac{\pi}{3B_0\alpha}$$

Therefore, x-coordinate of particle at any time $t > \frac{\pi}{3B_0\alpha}$ will be

$$x = \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + v_0\left(t - \frac{\pi}{3B_0\alpha}\right)\cos 60^\circ$$

$$= \frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha} + \frac{v_0}{2}\left(t - \frac{\pi}{3B_0\alpha}\right)$$

128. d. $$B_1 = \frac{\mu_0}{4\pi}\times\frac{2\pi(2r)^2}{\left((2r)^2+d^2\right)^{3/2}}$$

$$B_2 = = \frac{\mu_0}{4\pi}\times\frac{2\pi(r)^2}{\left[r^2+\left(\frac{d}{2}\right)^2\right]^{3/2}}$$

$$\frac{B_1}{B_2} = \frac{4}{\left[4r^2+d^2\right]^{3/2}}\times\left[\frac{4r^2+d^2}{4}\right]^{3/2}$$

$$= \frac{4}{(4)^{3/2}} = \frac{4}{(64)^{1/2}} = \frac{1}{2}$$

129. c.

$$\text{Loop } B\text{: } \mu_0(2i - i) = \oint B\cdot d\ell$$

$$\text{loop } C\text{: } \mu_0(i - 2i) = \oint B\cdot d\ell$$

$$\text{Loop } A\text{: } \mu_0(3i - 3i) = \oint B\cdot d\ell$$

$$\text{Loop } D\text{: } \mu_0(0 - i) = \oint B\cdot d\ell$$

(c) $B > A < C = D$

130. d. For a circular wire

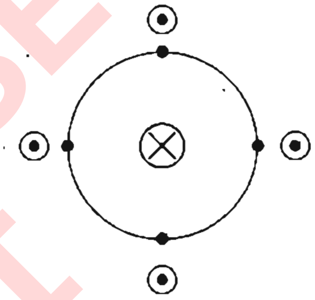


Fig. 9.443

131. a $r = \frac{mv}{qB} \Rightarrow r \propto m$ (for v same for both sense)

132. b

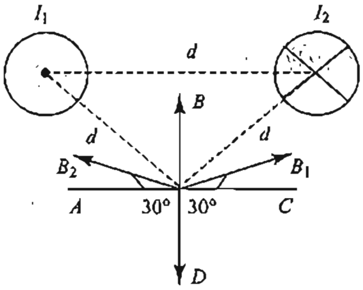


Fig. 9.444

133. c. The field due to clockwise current cancels the field due to anticlockwise current.

134. d. If r is proportional to v, t is independent of r.

135. d. To get magnetic field in resultant direction, current in X should be in and that in Z should be out; I in W should be in and that in Y should be out.

136. c. We can see from trajectory that both y- and z-coordinates can become zero.

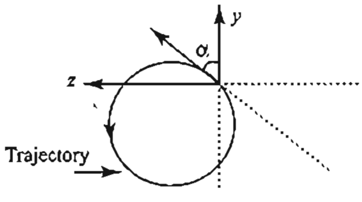


Fig. 9.445

This trajectory lies in y–z plane. So, x-coordinate is always zero.

137. b. $$B = \frac{\mu_0}{4\pi}\frac{i}{a}[\sin 90 + \sin(-\theta)] = \frac{\mu_0}{4\pi}\frac{i}{a}(1 - \sin\theta)$$

$$= \frac{\mu_0}{4\pi}\frac{i}{a}\left(1 - \frac{b}{\sqrt{a^2+b^2}}\right)$$

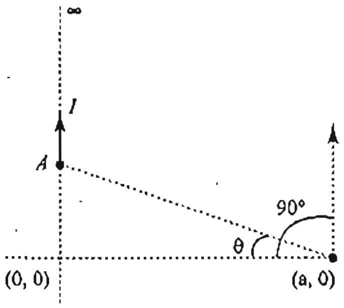


Fig. 9.446

138. c. Net magnetic field at P will be zero, because the magnetic field at center P is cancelled by opposite pairs like AD–EF, etc.

139. d. $\vec{F} = q[v\hat{i}] \times B(-\hat{i})] = 0$

Because B as well as v are along x-axis.

140. b. $$r = \frac{mv}{qB} = \frac{v}{B\alpha} = \frac{(2\alpha d)(B)}{(B\alpha)} = 2d$$

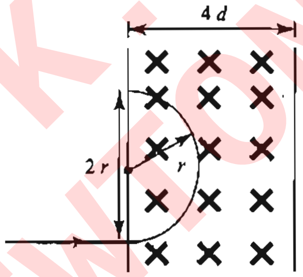


Fig. 9.447

i.e., the electron will move out after travelling on a semicircular path of radius $r = 2d$.

141. a. $\tau = I\alpha \Rightarrow MB = I\alpha$

$$\Rightarrow \quad i\pi r^2 B = \frac{1}{2}mr^2\alpha$$

$$\Rightarrow \quad \alpha = \frac{2iB\pi}{m} = \frac{2\times4\times10\pi}{2} = 40\pi \text{ rad s}^{-2}$$

142. a. The point charge moves in circle as shown in Fig. 9.393. The magnetic field vectors at a point p on the axis of circle are $\vec{B}_A$ and $\vec{B}_C$ at the instant the point charge is at A and C, respectively, as shown in Fig. 9.448.

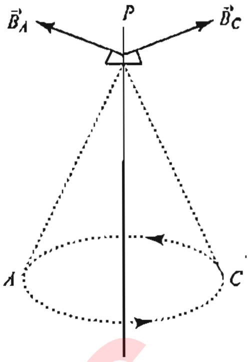


Fig. 9.448

Hence, as the particle rotates in circle, only magnitude of magnetic field remains constant at P but its direction changes.

Alternative solution:

The magnetic field at a point on the axis due to charge moving along a circular path is given by

$$\frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$$

It can be seen that he magnitude of the magnetic field at a point on the axis remains constant. But direction of the field keeps on changing.

143. b. Point A shall record zero magnetic field when the α-particle is at positions P and Q as shown in Fig. 9.449. The time taken by α-particle to go from P to Q is

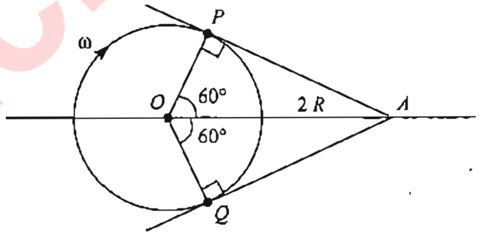


Fig. 9.449

$$t = \frac{T}{3} = \frac{1}{3}\times\frac{2\pi}{\omega} \quad \text{or} \quad \omega = \frac{2\pi}{3t}$$

144. a. The equivalent figure can be redrawn as shown in Fig. 9.450.

Force on $AB = ilB$, Force on $BCA = 2ilB$ in opposite direction to that on BA.

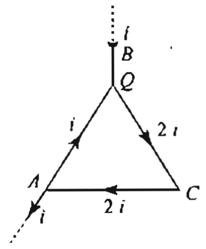


Fig. 9.450

Hence, net force $= 2ilB - ilB = ilB$

145. d. The particle will move in a non-uniform helical path with increasing pitch as shown in Fig. 9.451.

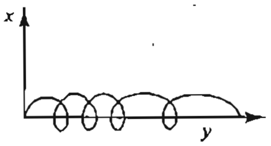


Fig. 9.451

Its time period will be: $T = \dfrac{2\pi m}{qB} = 2\pi$ s

Changing the view, the particle seems to move in a circular path in (X–Z) plane as shown in Fig. 9.452

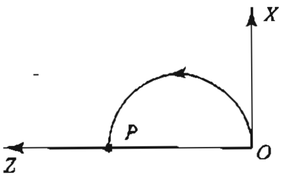


Fig. 9.452

After π seconds, the particle will be at point 'P' (after completing half circle), hence X-coordinate of P will be 0.

$$y(\pi) = 0(\pi) + \frac{1}{2}\frac{Eq}{m}(\pi)^2 = \frac{\pi^2}{2},$$

$$z = 2r = 2mv/qB = 2\text{ m}$$

Hence, the coordinates of the particle are $\left(0, \dfrac{\pi^2}{2}, 2\right)$

146. c. The center will be at 'C' as shown in Fig. 9.453

Coordinates of the center are

$(r\cos 60°, -r\sin 60°)$

where $\quad r =$ radius of circle $= \dfrac{mv}{Bq} = \dfrac{1\times 1}{1\times 1} = 1$

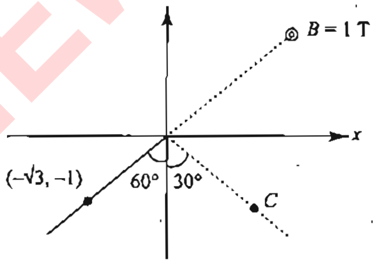


Fig. 9.453

i.e., $\quad \left(\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$

147. b. The charged particle moves in a circle of radius $a/2$.

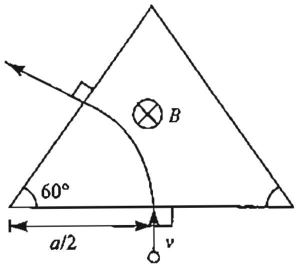


Fig. 9.454

$$qvB = \frac{mv^2}{a/2} \quad \text{or} \quad B = \frac{2mv}{qa}$$

148. b. Two particles will meet at P. After they meet they will stick together. Momentum mv will remain same. But charge is doubled, so radius is halved.

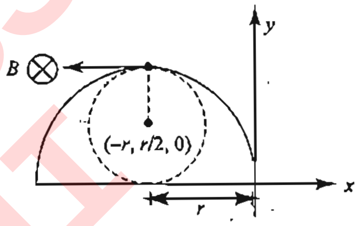


Fig. 9.455

Finally, both move in dotted circle.

149. c. After two and a half time periods, its x-coordinate will be $2.5P_0$ (so x-coordinate of image will be $17.5P_0$). It will be at distance $2R_0$ (equal to diameter) on the negative z-axis and y-coordinate will be zero.

150. c. Initially: $1.2\text{ N} = 1(\vec{\ell} \times \vec{B})$ downward

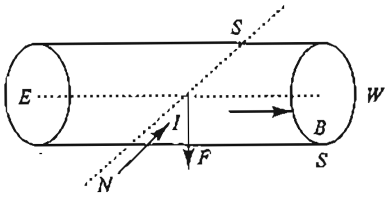


Fig. 9.456

In the given condition:

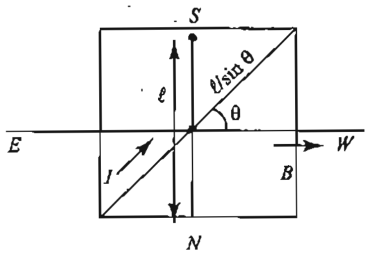


Fig. 9.457

$$F = I\frac{\ell}{\sin\theta}B\sin\theta = I\ell B = 1.2\text{ N downward}$$

151. c. Magnetic field at '*P*' due to wires (1) and (2) is

$$B_1 = \frac{\mu_0 I}{2\pi(x\sin\alpha)} + \frac{\mu_0 I}{2\pi(x\sin\alpha)} = \frac{2\mu_0 I}{2\pi(x\sin\alpha)}$$

(outside the paper)

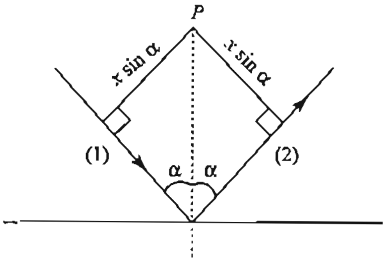


Fig. 9.458

Now, if a current of $\frac{2I}{\sin\alpha}$ is flowing in the third wire, then the magnetic field due to the same will be

$B_2 = \frac{\mu_0}{2\pi x}\left(\frac{2I}{\sin\alpha}\right)$, which will cancel B_1 if it is in the paper which is possible if the current $\frac{2I}{\sin\alpha}$ in the third wire is from right to left.

152. a. At a distance x consider a small element of width dx. Magnetic moment of the small element is:

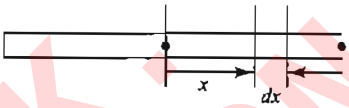


Fig. 9.459

$$dM = IA = (dq)f\pi x^2 = \frac{q}{\ell}dxf\pi x^2$$

$$M = \frac{qf\pi}{\ell}\int_{-\ell/2}^{\ell/2} x^2\,dx = \frac{q\pi f\ell^2}{12}$$

153. b. Use $\oint \bar{B}\cdot d\bar{l} = \mu_0 I$, where I is the current enclosed by the loop.

154. a. Wire *abc* can be replaced by a straight wire *ac* for the computation of force.

Length of $\overline{ac}$ can be written as,

$$\bar{\ell} = \vec{r}_C - \vec{r}_A = \left[(\hat{i}+\hat{j})-(\hat{i}+\hat{k})\right]50\times10^{-2}$$

$$= 0.50\,[\,\hat{j}-\hat{k}\,]$$

Required force, $\bar{F} = I\left(\vec{\ell}\times\vec{B}\right) = 0.6\hat{i}$

155. a. The net force acting on the loop would be along *X*-axis (to determine whether it is along +ve or –ve *X*-axis, calculation has to be carried out) as shown in Fig. 9.460.

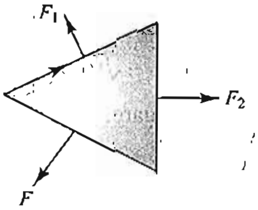


Fig. 9.460

156. b. Each and every pair of loop elements located symmetrically w.r.t. central line experiences zero net force, so total magnetic force experienced by the loop is zero.

157. a.
$$\bar{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r} = 2\hat{k},\ \hat{r} = \hat{k}$$

$$\bar{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} = \mu_0\varepsilon_0\left[\frac{q}{4\pi\varepsilon_0 r^2}\right](\hat{i}+3\hat{j})\times\hat{k}$$

$$= \mu_0\varepsilon_0 2\left[-\hat{j}+3\hat{i}\right] = \left(6\hat{i}-2\hat{j}\right)\mu_0\varepsilon_0$$

158. b. As the loop is placed in horizontal plane, so area vector is along vertical direction. From $\vec{\tau} = I\left(\vec{A}\times\vec{B}\right)$, as $\vec{A}$ is in vertical direction, $\vec{\tau}$ would be the plane of loop only. So, option (a) is wrong because for rotation of loop about its own axis $\vec{\tau}$ must be along vertical direction. (b) is correct because we can produce torque in the plane of the loop and due to this the loop can tip over.

159. a. Electric field in between the capacitor plates is given by

$$E = \frac{Q}{\varepsilon_0 A}$$

where Q is the charge on capacitor.

$$Q = e_0 A\times E = 8.85\times10^{-12}\times25\times10^{-7}\times400$$
$$= 8.85\times10^{-15}\,\text{C}$$

Magnetic force experienced by + ve plate is,

$$F_m = QvB = 8.85\times10^{-15}\times25\times4$$
$= 8.85\times10^{-13}$ C in direction out of the plane of paper.

160. c. The part *AUB* of the wire can be replaced by straight wire *AB* and *BWC* can be replaced by *BC*.

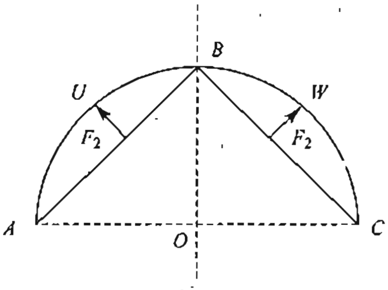


Fig. 9.461

Force experienced by *AB* is, $F_1 = IB_0\left(\sqrt{2}R\right)$

Force experienced by *BC* is, $F_2 = I\times2B_0\times\sqrt{2}R$

Resultant magnetic force acting on the wire is,

$$F = \sqrt{F_1^2 + F_2^2} = \sqrt{10}\, IB_0R$$

161. a. Let a is the side of square.

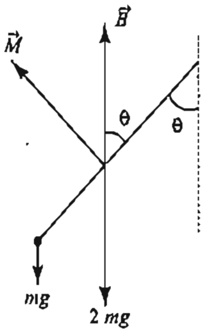


Fig. 9.462

Torque of current $= MB \sin(90 - \theta)$

$= MB \cos\theta = i\,a^2B\cos\theta$

Mass of each side $= m = \rho Aa$

Torque of gravity

$$= 2mg\,\frac{a}{2}\sin\theta + mg\,a\sin\theta$$

$$= 2\,mg\,a\sin\theta = 2\rho Aga^2\sin\theta$$

Now, both torques should be same

i.e., $Ia^2B\cos\theta = 2\rho Aga^2\sin\theta$

$$\Rightarrow \quad \cot\theta = \frac{2\rho Ag}{IB}$$

162. c.

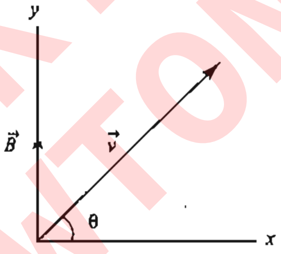


Fig. 9.463

Since the proton is entering the magnetic field at some angle other than 90°, its path is helix.

Corresponding velocity of the proton along X-axis,

$$v_x = v\cos 60^\circ \;=\; 2\times 10^6 \times \frac{1}{2} = 10^6\ \text{ms}^{-1}$$

Due to velocity component v_x, the radius of the helix is described and is given by the relation

$$r = \frac{mv_x}{qB} = \frac{1.6\times10^{-27}\times10^6}{1.6\times10^{-19}\times0.10} = 0.1\ \text{m}$$

Now, $$T = \frac{2\pi r}{v_x} = \frac{2\pi\times0.1}{10^6} = 2\pi\times10^{-7}\ \text{s}$$

163. d. Consider the 2D view of the situation. Before any collision with the walls of cylinder, the particle will move in horizontal plane if $qvB = mg$.

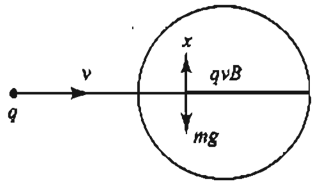


Fig. 9.464

But after 1st collision, qvB and mg start acting in the same direction along vertical and the particle is not able to move in horizontal plane.

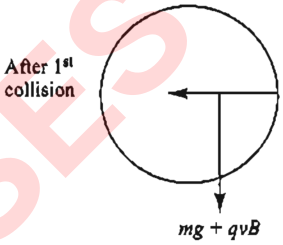


Fig. 9.465

164. a. The resultant figure can be redrawn as in Fig. 9.466.

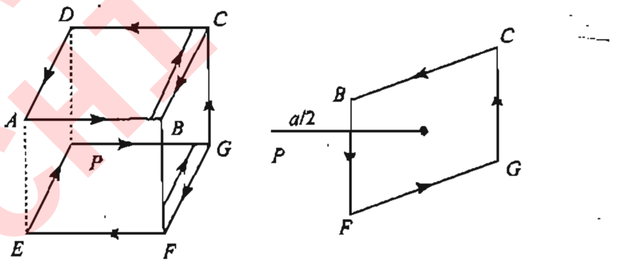


Fig. 9.466

The field at P due to upper and lower squares will cancel out. Resultant field at P will be due to the square $BCGF$ as shown in the figure.

Solve to get, $B_P = \dfrac{4\mu_0 I}{\pi\sqrt{3}a}$

165. c.

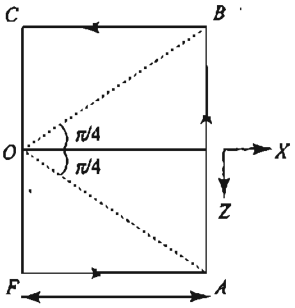


Fig. 9.467

Due to $FABC$, the magnetic field at O is along y-axis and due to $CDEF$, the magnetic field is along axis.

Hence, the field will be of the form $A\left[\hat{i} + \hat{j}\right]$

Calculating field due to $FABC$:

Due to AB:

$$\vec{B}_{AB} = \frac{\mu_0 i}{4\pi\left(\frac{\ell}{2}\right)}(\sin 45° + \sin 45°)\,\hat{j}$$

$$= \sqrt{2}\frac{\mu_0 i}{2\pi\ell}$$

Due to BC:

$$\vec{B}_{AB} = \frac{\mu_0 i}{4\pi I\left(\frac{\ell}{2}\right)}\{\sin 0° + \sin 45°\}$$

$$= \frac{\mu_0 i}{2\sqrt{2}\pi\ell}$$

$$\vec{B}_{AB} = \frac{\mu_0 i}{2\sqrt{2}\pi\ell}\hat{i}$$

Hence, $\vec{B}_{FABC} = \frac{\mu_0 i}{\pi\ell}\left[\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} + \frac{\sqrt{2}}{2}\right]\hat{i}$

$$\Rightarrow \quad \vec{E}_{FABC} = \frac{\sqrt{2}\mu_0 i}{\pi\ell}(\hat{j})$$

Similarly due to $CDEP$;

$$\vec{B}_{CDEF} = \frac{\sqrt{2}\mu_0 i}{\pi\ell}(\hat{i})$$

$$\Rightarrow \quad \vec{B}_{\text{net}} = \frac{\sqrt{2}\mu_0 i}{\pi\ell}(\hat{i} + \hat{j})$$

166. b.

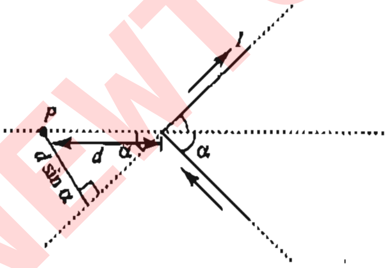


Fig. 9.468

Let us compute the magnetic field due to any one segment:

$$B = \frac{\mu_0 i}{4\pi(d\sin\alpha)}(\cos 0° + \cos(180 - \alpha))$$

$$= \frac{\mu_0 I}{4\pi(d\sin\alpha)}(1 - \cos\alpha) = \frac{\mu_0 I}{4\pi d}\tan\frac{\alpha}{2}$$

Resultant field will be:

$$B_{\text{net}} = 2B = \frac{\mu_0 I}{2\pi d}\tan\frac{\alpha}{2} \Rightarrow K = \frac{\mu_0 I}{2\pi d}$$

167. c.

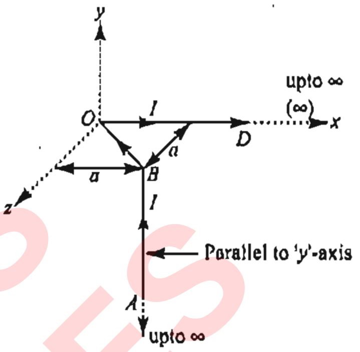


Fig. 9.469

$$B_{OD} = 0$$

$$B_{OB} = 0$$

$$B_{AB} = \frac{\mu_o I}{4\pi a\sqrt{2}}\left[\cos 45°(-\hat{i}) + \cos 45°\,\hat{k}\right]$$

$$= \frac{\mu_0 I}{8\pi a}(-\hat{i} + \hat{k})$$

168. b.

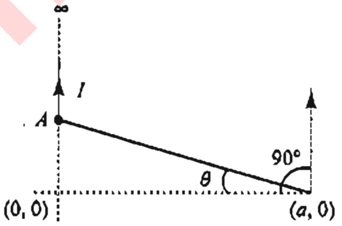


Fig. 9.470

Magnetic field strength at P due the I_1

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi(AP)}\hat{k} = \frac{4\pi\times10^{-7}\times2}{2\pi\times1\times10^{-2}}\hat{k}$$

$$= \left(4\times10^{-5}\,\text{T}\right)\hat{k}$$

Magnetic field strength at P due the I_2

$$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi(BP)}\hat{j} = \frac{4\pi\times10^{-7}\times3}{2\pi\times2\times10^{-2}}\hat{j} = \left(3\times10^{-5}\,\text{T}\right)\hat{j}$$

Hence, $\vec{B} = \left(3\times10^{-5}\,\text{T}\right)\hat{j} + \left(4\times10^{-5}\,\text{T}\right)\hat{k}$

169. d.

$$\oint\vec{B}\cdot\vec{d\ell} = \mu_0(i_1 + i_3 + i_2 - i_3) = \mu_0(i_1 + i_2)$$

[since for the given direction of circulation I_3 entering at $PSTU$ is positive while i_3 at $PQRS$ is negative]

Alternative solution:

$$\oint_{ABCDA}\vec{B}\cdot\vec{d\ell} = \oint_{ABCA}\vec{B}\cdot\vec{d\ell} + \oint_{CDAC}\vec{B}\cdot\vec{d\ell} = \mu_0 i_1 + \mu_0 i_2 = \mu_0(i_1 + i_2)$$

170. c.

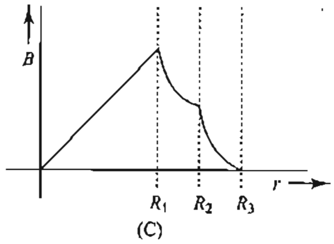


Fig. 9.471

From Ampere's law, the field at the axis is zero. From $x = 0$ to R_1, the field increases linearly as the charge enclosed increases.

From $x = R_1$ to R_2 and from $x = R_2$ to R_3, the field decreases hyperbolically but with different slopes as the media are different.

Hence, the required graph is (c).

171. d. For cylinder:

$$B = \frac{\mu_0 ir}{2\pi R^2}; \quad r < a$$

$$= \frac{\mu_0 i}{2\pi R^2}; \quad r \geq a$$

We can consider the given cylinder as a combination of two cylinders. One of radius 'R' carrying current I in one direction and other of radius $\frac{R}{2}$ carrying current $\frac{I}{3}$ in both directions.

At point A: $B = \frac{\mu_0 (I/3)}{2\pi(R/2)} + 0 = \frac{\mu_0 I}{3\pi R}$

At point B: $B = \frac{\mu_0}{2\pi}\left(\frac{4I/3}{\pi R^2}\right)\left(\frac{R}{2}\right) + 0 = \frac{\mu_0 I}{3\pi R}$

172. c. Let current in AB is I_1 and in DC, I_2. Then

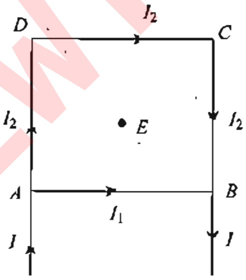


Fig. 9.472

$\frac{I_1}{I_2} = \frac{3}{1}$. It is because resistance of AB will be one-third of that of $ADCB$.

Now, $\frac{B_1}{B_2} = \frac{I_1}{I_2} = 3$

173. b. $r = \frac{mv}{qB} = \frac{\sqrt{2mE}}{qB} = \frac{\sqrt{2mqV}}{qB} = \sqrt{\frac{m}{q}} \times \frac{\sqrt{2V}}{B}$

$$\Rightarrow \quad \frac{r_2}{r_1} = \sqrt{\frac{4}{1}} \times \sqrt{\frac{1}{2}} = \sqrt{2}$$

$$\Rightarrow \quad r_2 = \sqrt{2} r_1 = 5\sqrt{2}\,\text{cm}$$

174. c. $B = \frac{\mu_0 ni}{2r}$

$$\Rightarrow \quad \ell = 2\pi rn, \; r = \frac{l}{2\pi n}$$

So, $B = \frac{\mu_0 \pi i n^2}{l} \Rightarrow B$ is minimum if $n = 1$

175. a. $\frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2\mu_0} B^2$

$$E = \sqrt{\frac{B^2}{\mu_0 \varepsilon_0}} = Bc = 3 \times 10^8\,\text{NC}^{-1}$$

where c is velocity of light.

176. c. The straight wire generates field in the inward direction and the curved wire generates field in the outward direction. The contribution of straight line is more because every part of it is closer to the point O as compared to every part on the curved wire. Furthermore, if the angle θ becomes more than or equal to π, then the magnetic induction at the point O becomes outward.

177. b. Pitch p and radius r

If $p = r$, then $\tan\theta = 2\pi$ or $\theta = \tan^{-1} 2\pi$

178. d. $R = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2(KE)}{m}} = \frac{\sqrt{2m}}{qB}\sqrt{KE}$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{\sqrt{KE}}{\sqrt{KE/2}} \Rightarrow R_2 = \frac{R_1}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\,\text{cm}$$

179. a. $\vec{B} = \frac{\mu_0 I}{2R}(\pm\hat{i}) + \frac{\mu_0 I}{2R}(\pm\hat{j}) + \frac{\mu_0 I}{2R}(\pm\hat{k})$

$$|\vec{B}| = \frac{\mu_0 I}{2R}\sqrt{3}$$

180. a. $B_{\text{total}} = B + B' = \frac{\mu_0}{4\pi}\left(\frac{qv}{d^2} + \frac{q'v'}{d^2}\right)$

$$B = \frac{\mu_0}{4\pi}\left(\frac{(8.0 \times 10^{-6}\,\text{C})(4.5 \times 10^6\,\text{ms}^{-1})}{(0.120\,\text{m})^2} + \frac{(3.0 \times 10^{-6}\,\text{C})(9.0 \times 10^6\,\text{ms}^{-1})}{(0.120\,\text{m})^2}\right)$$

$B = 4.38 \times 10^{-4}$ T, into the page.

181. b.

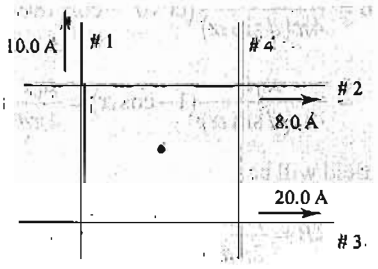


Fig. 9.473

$$\vec{B}_1 \otimes,\ \vec{B}_2 \otimes,\ \vec{B}_3$$

$$B = \frac{\mu_0 I}{2\pi r};\ r = 0.200 \text{ m for each wire}$$

$$B_1 = 1.00 \times 10^{-5} \text{ T},\ B_2 = 0.80 \times 10^{-5} \text{ T},$$

$$B_3 = 2.00 \times 10^{-5} \text{ T}$$

Let ⊙ be the positive z-direction.

$$I_1 = 10.0 \text{ A},\ I_2 = 8.0 \text{ A},\ I_3 = 20.0 \text{ A}$$

$$B_{1z} = -1.00 \times 10^{-5} \text{ T},\ B_{2z} = -0.80 \times 10^{-5} \text{ T},$$

$$B_{3z} = +2.00 \times 10^{-5} \text{ T}$$

$$B_{1z} + B_{2z} + B_{3z} + B_{4z} = 0$$

$$B_{4z} = -(B_{1z} + B_{2z} + B_{3z}) = -2.0 \times 10^{-6} \text{ T}$$

To give $\vec{B}_4$ in the ⊗ direction, the current in wire 4 must be toward the bottom of the page.

$$B_4 = \frac{\mu_0 I}{2\pi r},\ \text{so}\ I_4 = \frac{rB_4}{(\mu_0/2\pi)}$$

$$= \frac{(0.200 \text{ m})(2.0 \times 10^{-6} \text{ T})}{(2 \times 10^{-7} \text{ T mA}^{-1})} = 2.0 \text{ A}$$

182. d. A point on the axis of the loop is on the perpendicular bisector of each of the loop sides. Fig. 9.474 shows field of a single wire.

$$B = \frac{\mu_0 i a}{\pi\sqrt{4x^2 + a^2}\sqrt{4x^2 + 2a^2}}$$

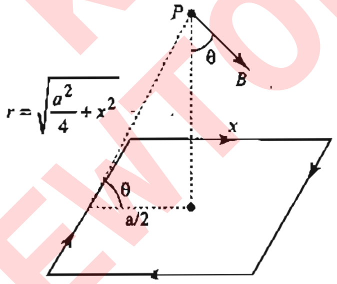


Fig. 9.474

When the fields of all the four sides are considered, the horizontal components add to zero; so the total field is given by

$$B_R = 4B\cos\theta = \frac{4Ba}{2r} = \frac{4Ba}{\sqrt{4x^2 + a^2}}$$

$$= \frac{4\mu_0 i a^2}{\pi(4x^2 + a^2)\sqrt{4x^2 + 2a^2}}$$

For $x = 0$, the expression reduces to

$$B_R = \frac{4\mu_0 i a^2}{\pi a^2 \sqrt{2a}} = \frac{2\sqrt{2}\mu_0 i}{\pi a}$$

183. c. Along the dashed line, $\vec{B}_1$ and $\vec{B}_2$ are in opposite directions. If the line has slope -1.00, then $r_1 = r_2$ and $B_1 = B_2$. So, $B_{tot} = 0$

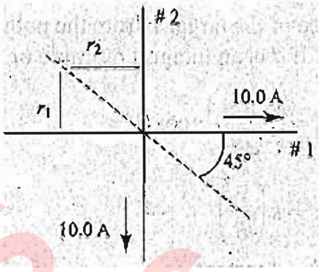


Fig. 9.475

184. c.

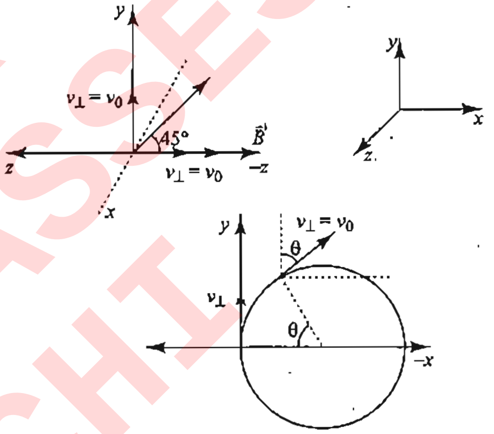


Fig. 9.476

Angle between $\vec{v}$ and $\vec{B}$,

$$\cos\theta = \frac{\vec{v} \cdot \vec{B}}{|\vec{v}|.|\vec{B}|} = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \theta = 45°$$

$$\vec{v}_{(t)} = -v_0 \cos\theta \hat{i} + v_0 \sin\theta \hat{j} - v_0 \hat{k}$$

$$\theta = \omega t = \frac{2\pi}{T} t = \frac{qB_0 t}{m} = \alpha B_0 t$$

$$\therefore \quad \vec{v}_{(t)} = -v_0 \cos(\alpha B_0 t)\hat{i} + v_0 \sin(\alpha B_0 t)\hat{j} - v_0 \hat{k}$$

185. c. The current is to the left, so the force is into the plane.

$$\sum F_y = N\cos\theta - Mg = 0$$

$$\text{and} \sum F_x = N\sin\theta - F_B = 0$$

$$F_B = Mg\tan\theta = ILB \Rightarrow I = \frac{Mg\tan\theta}{LB}$$

Multiple correct Answers Type

1. a., b., c., d.

Options (a) and (b) are theoretical facts. As in case of moving charged particle in magnetic field $\vec{F}_{mag} \perp \vec{v}$, hence power associated will be zero (option (c) is correct).

If both the electric and magnetic fields exist: If $\vec{B} \parallel \vec{E}$, the path of the charged particle will be helical.

If $\vec{B} \parallel \vec{E}$, the radius of the charged particle will not be constant. Hence, the path will not be circular (option (d) is correct).

2. a., b.

Let d = distance of the target T from the point of projection. P will strike T if d is an integral multiple of the pitch.

$$\text{Pitch} = \left(\frac{2\pi m}{QB}\right) v\cos\theta$$

$$\therefore \quad \text{Pitch} = k\left(\frac{v}{B}\right)$$

where $\quad k$ = constant

$$\text{Initially,} \quad d = k\left(\frac{v_0}{B_0}\right)$$

3. a., d.

When the particle moves in a helical path, the plane of the helical path will be slightly inclined with x-axis. Hence, options (a) and (d) are correct.

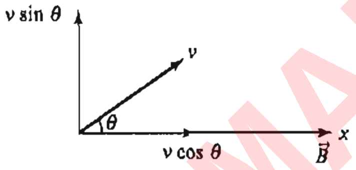


Fig. 9.477

4. a., b., c.

$$\text{Pitch} = \left(\frac{2\pi m}{QB}\right) v\cos\theta$$

Also, $mv\sin\theta = QBr$ for motion perpendicular to the magnetic field

$$\text{or} \quad r = \frac{mv\sin\theta}{QB}$$

Maximum distance of the particle from the x-axis = $2r$

$$\therefore \quad \left(\frac{2\pi m}{QB}\right) v\cos\theta = 2\frac{mv}{QB}\sin\theta$$

$$\text{or} \quad \tan\theta = \pi$$

Hence, the options (a), (b) and (c) are not correct.

5. a., c.

In the absence of a magnetic field, the particle will experience gravitational force mg. As a result, the particle will not continue moving in the horizontal direction but will describe a parabolic path. So, a magnetic field must be present and its direction must be perpendicular to the direction of the velocity. The magnetic force experienced by the particle is given by $\vec{F} = q\,(\vec{v} \times \vec{B})$.

The magnitude of the force is $F\,qvB \sin\theta$. If the particle is to move in the horizontal direction, this force must balance the force of gravity, i.e.,

$$mg = qvB\sin\theta$$

The minimum value of B corresponds to $\sin\theta = 1$ or $\theta = 90°$. Thus,

$$mg = qvB$$

$$\text{or} \quad B = \frac{mg}{qv} = \frac{0.5\times10^{-3}\times9.8}{2.5\times10^{-8}\times6\times10^{4}}$$

$$= 3.27 \text{ T}$$

Hence, the correct options are (a) and (c).

6. a., b.

The pairs $\vec{F}$, $\vec{v}$ and $\vec{F}$, $\vec{B}$ are always at right angles to each other, because $\vec{F}$ is always perpendicular to the plane containing $\vec{B}$ and $\vec{v}$. Vectors $\vec{B}$ and $\vec{v}$ have any angle between them.

7. a., c.

$$a_e = \frac{F}{m_e}, \quad a_p = \frac{F}{m_p}$$

Since $\quad m_e << m_p$, so

$$a_e >> a_p$$

$$\text{Now,} \quad d = \frac{1}{2}a_e t_1^2 \text{ and } d = \frac{1}{2}a_p t_2^2$$

$$\frac{t_1}{t_2} = \left(\frac{m_e}{m_p}\right)^{1/2}$$

8. b., d.

If a particle moves in a magnetic field in helical path, the plane of the helical path will be inclined at an acute angle with the magnetic field.

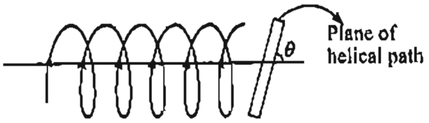


Fig. 9.478

Hence, options (b) and (d) are correct.

9. a., b.

Use right hand rule or $\vec{F} = q\vec{v} \times \vec{B}$ for explanation.

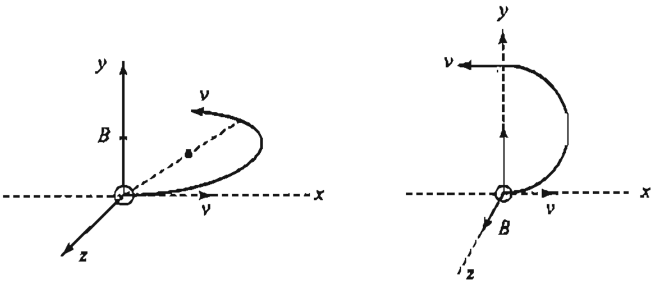


Fig. 9.479

If we apply magnetic field along y-direction, the circular path in x–z plane, the particle can move parallel to magnetic direction.

If we apply the magnetic field in z-direction, the plane of circular path will be x–y plane.

10. a., b., d.

If a charged particle goes unaccelerated in a region containing electric and magnetic fields,

$$|q\vec{E}| = |q\vec{v} \times \vec{B}| \Rightarrow \vec{E} = \vec{v} \times \vec{B}$$

$$|\vec{E}| = |vB|$$

The acceleration of the particle will be zero if $\vec{E} \perp \vec{B}$.

11. b., d.

Velocity of proton makes an angle of 45° with the direction of magnetic field. Therefore, the path of the proton is a helix. The plane of the circle of this helix is the plane formed by negative *x* and positive *z*-axis. Therefore, *x*-coordinate can never be positive. Further, *x*- and *z*-coordinates will become zero simultaneously after every pitch and *y*-coordinate of the proton at any time *t* is

$$y = v_0 t\text{, i.e., } y \propto t$$

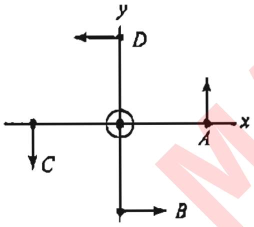


Fig. 9.480

12. a., b., c.

If $x = y$, then $\vec{v} \parallel \vec{B}$, i.e., $\vec{F}_m = 0$. Hence, the option (a) is correct.

$$\vec{F} = q(\vec{v} \times \vec{B}) = q[(x\hat{i} + y\hat{j}) \times (y\hat{i} + x\hat{j})]$$

$$= (x^2 - y^2)\hat{k}$$

Now, if $x > y$, $F \propto (x^2 - y^2)$

And *F* is along +ve *z*-axis. But if $y > x$, force will be along negative *z*-axis.

∴ The options (b) and (c) are also correct.

13. a., b.

If electric field is parallel to magnetic field, the charged particle moving parallel to $\vec{E}$ experiences a force in the direction of $\vec{E}$; due to $\vec{B}$ there will not be any force. Hence, no deflection.

The particle may go undeflected in the case when forces due to electric field and magnetic field balance each other.

14. a., b., c.

If $x = y$, then $\vec{v} \parallel \vec{B}$, i.e., $\vec{F}_{\text{mag}} = 0$.

Hence, (a) is correct.

$$\vec{F}_{\text{mag}} = q(\vec{v} \times \vec{B}) = q[(x\hat{i} + y\hat{j}) \times (y\hat{i} + x\hat{j})]$$

$$\vec{F}_{\text{mag}} = q(x^2 + y^2)\hat{k}$$

If $x > y$, then the force is along *z*-axis.

15. a., b., c.

For B_1 and B_4, the contributions due to the different sections add up. For B_2 and B_3, the contributions due to the outer sections oppose the contributions due to the inner sections. Thus, B_1 and B_4 are greater than B_2 and B_3. For B_4, there is a section with radius < *b* and hence, it contributes more than the semicircular section of radius *b* does for B_1. Thus, $B_4 > B_1$.

For B_3, there is a section with radius > *b* and hence it contributes less than the semicircular section of radius *b* does for B_2. Thus, $B_3 < B_2$.

Hence, $B_4 > B_1 > B_2 > B_3$

16. a., b., c.

To find the Ampere's force on a conductor of any shape, replace the conductor by an imaginary straight conductor joining the two ends of the given conductor. So, if *B* is in *x*-direction, then the imaginary straight conductor will be along the field and the force acting on it will be zero. If *B* is in *y*-direction, then the force will be λBI acting along the *z*-direction. Similarly, if *B* is in the *z*-direction, then the force will be λBI, acting along the negative *y*-direction.

17. b., d.

$$B = \frac{\mu_0 NIr^2}{2(r^2 + x^2)^{3/2}} \text{ along } x\text{-axis}$$

$$B = \frac{\mu_0 NI}{2r} \frac{r^3}{(r^2 + x^2)^{3/2}}$$

$$B = \frac{\mu_0 NI}{2r} \sin^3\theta$$

According to which, the coil with lesser radius will make more contribution.

18. b., d.

The perpendicular distance from wire to points (*A* and *C*) and points (*B* and *D*) are same.

19. a., d.

The magnitude of the magnetic field depends only on the distance from the *x*-axis. Points *A* and *C* are at distances of 1 unit each from the *x*-axis. Points *B* and *D* are at distances of $\sqrt{2}$ unit each from the *x*-axis. Magnetic field at point *D*,

$$B = \frac{\mu_0 I}{2\sqrt{2}\pi}$$

It is obvious that field *B* is inclined at an angle of 45° with the *x*–*y* plane.

20. a., b., c., d.

$$|\vec{B}_A| = |\vec{B}_B| = |\vec{B}_C| = |\vec{B}_D| = \frac{\mu_0 I}{2\pi a}$$

21. b., c.

If a current passes through any conductor, net charge through any section is zero. Hence, electric field in the vicinity of the conductor is zero. We can observe using Biot and Savart law that the magnetic field at the axis of the cylindrical wire will be zero. Electric field at the axis of the wire cannot be zero otherwise the current through the wire cannot flow.

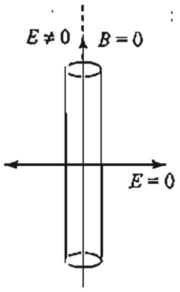


Fig. 9.481

22. a., b.

$$N\cos\theta = mg \qquad \text{(i)}$$

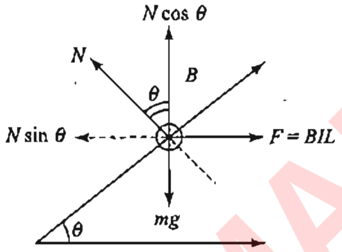


Fig. 9.482

[⊗ indicates current I is flowing into the paper]

$$N\sin\theta = BIL \qquad \text{(ii)}$$

$$\tan\theta = \frac{BIL}{mg}$$

23. b., d.

For rotation equilibrium, taking torque about the point from where normal force passes.

$$IaB_0\frac{a}{2} + IaB_0\frac{a}{2} - mgx = 0$$

$$x = \frac{Ia^2B_0}{mg}$$

For the block not to topple:

$$x < \frac{a}{2} \Rightarrow I < \frac{mg}{2aB_0}$$

24. a., b., c., d.

Option (b) is obvious

$$r = \frac{mv_0}{qB}$$

$$PQ = 2r\sin\alpha = 2\frac{mv_0}{qB}\sin\alpha$$

$$\alpha = \beta$$

$$\text{Time taken} = T = \frac{2\pi r}{v}$$

$$T = \frac{2\pi m}{qB}$$

For t time, $t = \frac{T}{2\pi}(2\pi - 2\alpha) = \frac{2m}{qB}(\pi - \alpha)$

25. a., b., c.

If a charge particle experiences no electromagnetic force, then electric field must be zero (as $\vec{F} = q\vec{E}$). The magnetic field may or may not be zero as a charge particle at rest or moving in the direction of magnetic field experiences no force.

a. When both $\vec{E}$ and $\vec{B}$ are 0, it is undeflected.

b. If $\vec{E}$ is non-zero, it will move in the direction of E and remains undeflected.

c. $\vec{E}$ is non-zero, force will be in direction of E. B is non-zero and V is parallel to $\vec{B}$.

$\vec{F} = q\vec{v}\times\vec{B}$ will become zero due to $\vec{B}$ and proton will move in a direction of $\vec{E}$ only.

26. a., b., c., d.

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{q}{m}(\alpha - \beta x) \qquad \text{(i)}$$

$$a = 0 \text{ at } x = \frac{\alpha}{\beta}$$

i.e., force on the particle is zero at $x = \frac{\alpha}{\beta}$

So, mean position of the particle is at $x = \frac{\alpha}{\beta}$

Equation (1) can be written as $v\frac{dv}{dx} = \frac{q}{m}(\alpha - \beta x)$

$$\therefore \quad \int_0^v v\,dv = \frac{q}{m}\int_0^x\left(\alpha - \frac{\beta}{2}x\right)dx$$

$$\therefore \quad v = \sqrt{\frac{2qx}{m}\left(\alpha - \frac{\beta}{2}x\right)}$$

$$v = 0 \text{ at } x = 0 \text{ and } x = \frac{2\alpha}{\beta}$$

So, the particle oscillates between $x = 0$ and $x = \frac{2\alpha}{\beta}$ with mean position at $x = \frac{\alpha}{\beta}$.

Maximum acceleration of the particle is at extreme positions (at $x = 0$ or $x = 2\alpha/\beta$) and $a_{\max} = q\alpha/m$ [from equation (i)].

27. a., c.

Using $B = \frac{\mu_0 NI}{2R}$

$|\vec{B}_1| = |\vec{B}_2| = 4\pi\times 10^{-4}$ T

If current in same sense $B_{\text{net}} = 8\pi\times 10^{-4}$ T

If current in opposite sense $B_{\text{net}} = 0$

28. a., c., d.

As magnetic field due to a long wire $B = \frac{\mu_0 I}{2r}$. The magnetic field lines are concentric with wire. The direction may be either clockwise or anticlockwise. Hence options (a), (c) and (d) are correct.

29. a., b., c.

Proton is deflected towards positive x-axis even if only electric field is switched on.

Therefore, $\vec{E}$ is along positive x-axis or option (b) is correct.

Proton is deflected along positive x-axis when only magnetic field is switched on, i.e., magnetic force $\vec{F}_m$ is along positive x-axis. This is possible in the following two cases:

a. $\vec{v}$ is along positive y-axis and $\vec{E}$ along positive z-axis.

b. $\vec{v}$ is along negative y-axis and $\vec{B}$ along negative z-axis because $\vec{F}_m = q(\vec{v} \times \vec{B})$.

Therefore options (a) and (c) are also correct.

30. a., b., d.

In going from P to Q, increase in kinetic energy

$$\frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2 = \frac{1}{2}m(3v^2) = \text{work done by electric field.}$$

or $$3mv^2 = Eq \times 2a$$

or $$E = \frac{3}{2}\left(\frac{mv^2}{qa}\right)$$

The rate of work done by E at P = force due to $E \times$ velocity.

$$= (qE)v = qv\left[\frac{3}{2}\left(\frac{mv^2}{qa}\right)\right] = \frac{3}{4}\left(\frac{mv^3}{a}\right)$$

At q, is perpendicular to and , and no work is done by either field.

31. a., c., d.

$$|\vec{v}| = \sqrt{8^2 + 6^2} = 10 \text{ m/s}$$

Radius of circular path $r = \frac{mv}{qB} = \frac{10}{2} = 5 \text{ m}$

As the magnetic field is in z-direction, hence the path of the charged particle will be circular of radius 5 m in x-y plane; hence options (a) and (b) may be possible.

Time period of circular path $T = \frac{2\pi r}{v} = \frac{2\pi \times 5}{10} = \pi \text{sec}$

32. a., b., c.

Whenever a particle moves along a spiral path, its velocity always remains in the plane of spiral or the component of the velocity normal to the plane of spiral is always equal to zero. Force exerted by magnetic field is always normal to the direction of motion of the particle.

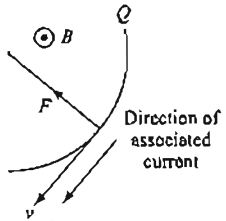


Fig. 9.483

Force exerted by the magnetic field is in the plane of spiral. Direction of magnetic field is always normal to both the velocity vector and the direction of force. It means, magnetic field is normal to the plane of spiral. Hence, (a) is correct.

Since the particle experiences a resistance against its motion, therefore its speed decreases continuously. Radius of the circular path followed by a charged particle moving in a magnetic field is given by $R = \frac{mv}{qB}$. Since v is continuously decreasing, therefore radius R also decreases continuously. So, a charged particle should follow a spiral path of decreasing radius. Hence, the particle enters the magnetic field at Q or (b) is correct.

Force experienced by the particle is towards the center and direction of the force is found out by Fleming's left hand rule. Since magnetic field is outward, therefore according to Fleming's left hand rule, the direction of current associated with its motion should be as shown in Fig. 9.428. Since the particle is moving in the same direction, therefore the particle is positively charged.

Hence, (c) is correct. Obviously, (d) is wrong.

33. a., b., d.

Period of revolution of a charged particle moving in a uniform magnetic field is given by $T = \frac{2\pi m}{qB}$. This period T does not depend upon speed of the particle. In this particular question, the moving particle is an electron. Hence, its mass and charge (q) both are constant. Magnetic field is also uniform. Hence, its period of revolution remains constant. It means, the electron moves with constant angular velocity. Hence, (a) is correct.

In previous question, we have already discussed that if a charged particle experiences a resisting force against its motion, then it follows a decreasing radius spiral path. In this question, the electron is moving along a spiral path of decreasing radius. It means, its speed is decreasing continuously. Hence, (b) is correct.

Since speed of the electron is countinuously decreasing, therefore it is experiencing a tangential retardation. It is possible only when the component of resultant force opposite to the direction of motion of electron has non-zero value. It means, net force on the electron cannot be perpendicular to its direction of motion. Hence, (c) is wrong.

Since speed of the electron is decreasing continuously, therefore the force exerted by the magnetic field ($F = qvB$) is also decreasing continuously. Hence, magnitude of net force acting on the electron is decreasing continuously.

Hence, (d) is correct.

34. a., c.

a. Velocity of the particle can be constant if the electric force balances the magnetic force, i.e., $q\vec{E} = q\vec{v} \times \vec{B}$

b. If $E = 0$, the particle can trace circular path if $v \perp B$.

c. If $E = 0$, kinetic energy remains constant, because magnetic force does not do any work.

35. a., b., c., d.

For $x \le r$, $B = \frac{\mu_0 I x}{2\pi r^2}$

Hence, $B \propto x$

At axis $x = 0 \quad \Rightarrow \quad B = 0$

For $x \ge r$, $B = \frac{\mu_0 I}{2\pi x}$

Hence, $B \propto \frac{1}{x}$

Also B is maximum at surface $(x = r)$

36. b., c. $\vec{V} \perp \vec{B}$

Therefore, path of the particle is a circle. In magnetic field, speed of the particle remains constant. Therefore, distance moved by the particle in time $t = \frac{\pi}{B_0\alpha}$ is $v_0 t$ or $\frac{\pi v_0}{B_0\alpha}$.

Magnitude of velocity is always v_0.

37. b., c., d. $\vec{\tau} = \vec{M} \times \vec{B}$ and $U = -\vec{M}\cdot\vec{B}$

Here, $\vec{M}$ and $\vec{B}$ are anti-parallel.

$\therefore \quad \vec{\tau} = \vec{O}$ and $U = +MB$ (maximum)

$\therefore$ (b), (c) and (d).

38. a., b., c.

Arc length: $AB = \frac{\pi}{3} r = \frac{\pi m v}{3qB}$

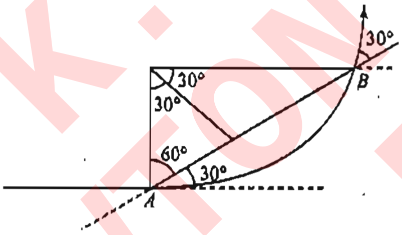


Fig. 9.484

Time: $t = \frac{\theta}{\omega} = \frac{\pi/3}{2\pi/T} = \frac{T}{6} = \frac{\pi m}{3qB}$

Distance $= vt = \frac{\pi m v}{3qB}$

39. a., b., c.

The particle will move along an arc which is part of a circle of radius: $r = \frac{mv}{Bq}$

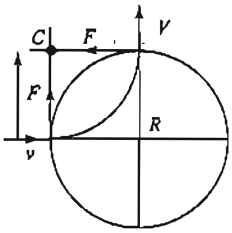


Fig. 9.485

From Fig. 9.430, we can see that $r = R$

$$\therefore \quad R = \frac{mv}{Bq}$$

$$T = \frac{\pi r/2}{v} = \frac{\pi r}{2v} = \frac{\pi R}{2v} \quad \left(\because \quad r = R = \frac{mv}{Bq}\right)$$

$$\therefore \quad T = \frac{\pi m}{2Bq}$$

40. a., b.

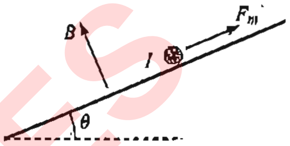


Fig. 9.486

When a charge is passed through a wire, a current flows in the wire (for a very small time). As a result of this, the wire experiences a magnetic force for a very small duration, i.e, during the time for which charge has been passed through the wire, due to which it acquires some velocity; then onwards, it moves under gravity. For the wire to move up the incline, the positive charge has to pass through the wire, due to which it acquires some velocity; then onwards, it moves under gravity. For the wire to move up the incline, the positive charge has to pass through the wire in a direction coinciding with into the plane of paper.

$$F_m = IBl = \frac{dq}{dt} \times B\ell = ma$$

[For this small duration dt, we can neglect gravity force because I would be very large due to small passage time of charge.]

$$\Rightarrow \quad m\frac{dv}{dt} = \frac{dq}{dt} \times B\ell$$

$$\Rightarrow \quad m\,dv = dq \times B\ell$$

$$\Rightarrow \quad mv = q \times B\ell$$

$$\Rightarrow \quad v = \frac{q \times B\ell}{m}$$

where q is the charge passed through the wire and v is the velocity acquired by the wire just after charge had been passed through it.

From kinematics, $v^2 = 2g \sin\theta s$

$$q = \frac{m\sqrt{2gs\sin\theta}}{B\ell}$$

41. b., d.

Distance of A and C is same from wire. Hence, magnetic field at A and C is same in magnitude.

Similarly, for points B and D.

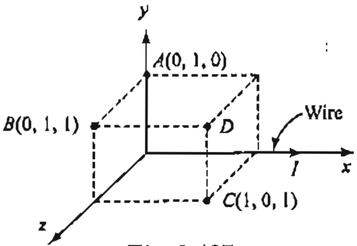


Fig. 9.487

42. b., d.
In first and third quadrants, direction of magnetic field is opposite due to the two wires. So, magnetic field can be zero only in first and third quadrants not in second and fourth. Hence, (b) and (d) are correct.

43. c., d.
The two particles have equal charges and move through the same *p.d.* They will, therefore, acquire the same energy (E).
Also, $E = p^2/2m$ or $p = \sqrt{2mE}$, where p = momentum. For the same E, the particle with greater m has greater p.

44. a., c., d.
For a charged particle moving in magnetic field $\bar{a} \perp \bar{v}$
Hence $\bar{v} \cdot \bar{a} = 0 \Rightarrow x = -1.5$
The magnetic field should be perpendicular to the $\bar{a}$, hence should be perpendicular to x-y plane.
The kinetic energy of a moving particle in the magnetic field remains unchanged as magnetic field does no work on the charged particle.

Assertion-Reasoning Type

1. c. Cyclotron is suitable for accelerating heavy charged particles such as protons, α-particles and positive ions. In a cyclotron, the positive ions cross again and again the same alternating (ratio frequency) electric field and thereby gain energy. It is achieved by making them to move along spiral circular paths under the action of a strong magnetic field. Cyclotron is also known as magnetic resonance accelerator. Cyclotron frequency is given by

$$v = \frac{1}{T} = \frac{Bq}{2\pi m}$$

It is obvious that cyclotron frequency does not depend upon velocity of the charged particle.

2. a. Cyclotron is not suitable for accelerating electrons. The reason is that due to small mass, the speed of electrons increases rapidly. Likewise, due to the quick relativistic variation in their mass, the electrons get out of step with the oscillating electric field.

3. c. If the path of the charged particle is circular, then radius of the circular path is directly proportional to the speed and mass of the particle as

$$r = \frac{mv}{q_0 B}$$

$$\therefore \quad \text{Centripetal force} = \frac{mv^2}{r} = q_0 vB$$

4. b. The magnetic field at a point due to current flowing through an infinitely long conductor is given by

$$B = \frac{\mu_0}{4\pi}\frac{2I}{a}$$

where a is the distance of that point from the conductor. Now, according to right hand thumb rule it follows that magnetic field is in the form of concentric circles, whose centers lie on the straight conductor.

5. c. Magnetic induction at O due to I_1,

$$B_1 = \frac{\mu_0}{4\pi}\frac{I_1\theta}{r} \qquad \text{(directed normally into plane)}$$

Similarly, $B_2 = \dfrac{\mu_0}{4\pi} \times \dfrac{(2\pi - \theta)}{r} I_2$

(directed normally upward to the paper)

Since $\quad I_2 = \dfrac{I_1\theta}{2\pi - \theta}, B_2 = B_1 \quad$ (but opposite in direction)

6. a. $$\frac{mv^2}{r} = qvB$$

$\dfrac{v^2}{r} = \dfrac{qvB}{m}$ is called a centripetal acceleration.

7. d. $\varepsilon = Bv\ell \sin\theta$
If $\theta = 0, \varepsilon = 0$.

8. c.

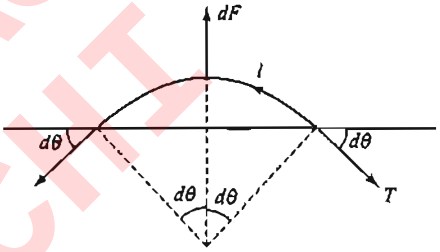


Fig. 9.488

$2T\sin d\theta = BiR\,(2d\theta)$

$T = iBR.$

9. a. $\quad U = -\bar{\mu}\cdot\bar{B} = -\mu B\cos\theta$
Since μ, B as well as θ remain constant, U does not change.

10. d. $|\bar{B}|$ is proportional to number of magnetic field lines per unit area (area should be normal to field).

11. a. The period of a charged particle in a magnetic field is given by

$$T = \frac{2\pi m}{qB}$$

12. d. Magnetic field exists both inside and outside.

13. d. The windings of helix carry currents in same direction. Hence, they experience an attractive force pulling the lower end out of mercury. As a result of this, the circuit breaks and so the force of attraction vanishes. The helix comes back to its initial position, completing the circuit again.

14. a. $\bar{F} = q\bar{V} \times \bar{B} \Rightarrow \bar{F} \perp \bar{V}$
So, power produced by magnetic force is zero.
$\Rightarrow$ Kinetic energy of particle will remain conserved.

15. a.

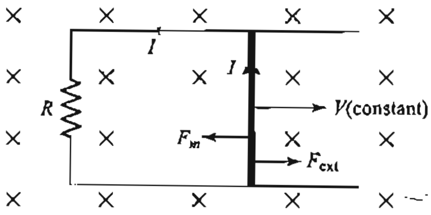


Fig. 9.489

$$\varepsilon = B\ell V$$

$$I = \frac{B\ell V}{R}$$

$$F_B = I\ell B = \frac{B^2\ell^2 V}{R}.$$

16. a. Initially, moment $M = I\pi r^2$

And afterwards $M' = I\pi(2r)^2$

$$= 4I(\pi r^2) = 4M$$

So, magnetic moment becomes four times when radius is doubled.

17. b. As we know, every atom of a magnet acts as a dipole. So, poles cannot be separated. When magnet is broken into two equal pieces magnetic moment of each part will be half of the original magnet.

18. c. Velocity is a vector quantity. Even if direction changes, velocity is said to be changing, no matter if the speed remains same or different.

19. d. Due to both positive and negative charges, the wire is electrically neutral and hence no electric field is present and only magnetic field is created.

20. d. Force on the loop is not zero; because magnetic field is not constant.

21. b. Use $\vec{F} = \oint i\, d\vec{\ell} \times B = 0$.

$$\vec{\tau} = MB\sin\theta = 0$$

22. b. For a solenoid, $B_{\text{end}} = \frac{1}{2}B_{\text{in}}$. Also, for a long solenoid magnetic field is uniform within it but this reason does not explain Statement I.

23. a. Work done by magnetic force on a moving charge is zero.

24. b. Magnetic force is always perpendicular to magnetic field and small element.

25. c. For any point outside the wire, enclosed current will be same.

26. b. $F = 0$ in both case.

27. a. The magnetic lines of force due to current carrying straight solenoid are same as that of a bar magnet.

28. b. Here, both the statements are correct but II does not explain. In general, Bio-Savart law and Ampere's forces do not obey Newtons's third law.

Comprehension Type

For Problems 1–2

1. b. **2. c.**

Sol. $$I = \frac{\varepsilon}{R} = \frac{24}{12} = 2\text{ A}$$

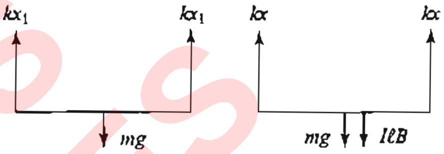


Fig. 9.490

$$\text{Magnetic force} = I\ell B\sin\frac{\pi}{2}$$

$$= 2\times5\times10^{-2}\times B = \frac{B}{10}$$

$$2kx_1 = mg;\ 2k = \frac{mg}{x_1}$$

$$2kx = mg + I\ell B$$

$$2k(x_1 + x_2) = mg + I\ell B$$

$$2kx_1 + 2kx_2 = mg + I\ell B$$

$$\frac{mx}{x_1}x_2 = I\ell B$$

$$10\times10^{-3}\times10\times\frac{0.3}{0.5} = \frac{B}{10}$$

$$B = 600\times10^{-3} = 0.6\text{ T}$$

For Problems 3–5

3. b., **4. a.,** **5. c.**

Sol. The first particle will have a helical path and the second particle will move rectilinearly along the field. For the two particles to meet again and again, $v_{||}T = v'T$ where v' is the speed of the second particle.

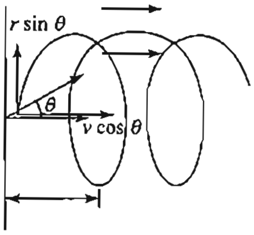


Fig. 9.491

$$\therefore \quad v' = v_{||} = v\cos\theta$$

$$\frac{1}{2}mv^2 = qV \Rightarrow v = \sqrt{\frac{2qV}{m}}$$

$$\therefore \quad v' = \sqrt{\frac{2qV}{m}}\cos\theta$$

$$T = \frac{2\pi m}{qB}$$

Distance travelled = pitch

$$\text{Distance} = \sqrt{\frac{2qV}{m}}\cos\theta \times \frac{2\pi m}{qB}$$

$$\text{Distance} = \sqrt{\frac{2Vm}{q}}\frac{2\pi}{B}\cos\theta$$

For Problems 6–7

6. c., 7. a.

Sol. $\vec{F} = q\left(\vec{v}\times\vec{B}\right)$

For the first case: $\vec{F} = q\vec{v}\times\vec{B}$

$$\Rightarrow \quad -5\sqrt{2}\times 10^{-3}\,\hat{k}$$

$$= 10^{-5}\times\frac{10^6}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)\times\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)$$

$$= \left(\frac{10}{\sqrt{2}}\right)\left[B_z\hat{i}-B_z\hat{j}+\left(B_y-B_x\right)\hat{k}\right]$$

$$\Rightarrow \quad B_z = 0,\ B_y - B_x = -10^{-3}\ \text{T} \qquad \text{(i)}$$

Similarly, for the second case:

$$F_2\,\hat{j} = (10^{-5})(10^6\,\hat{k})\times\left[\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)\right]$$

$$F_2\,\hat{j} = 10\left(B_x\hat{j}-B_y\hat{i}\right)$$

$$F_2 = 10B_x,\ B_y = 0 \qquad \text{(ii)}$$

Using (i) and (ii), we get $B_x = 10^{-3}$ T

Thus, $\vec{B} = (10^{-3}\ \text{T})\ \hat{i}$

Also, $F_2 = 10B_x = 10^{-2}$ N.

For Problems 8 – 10

8. a., 9. b., 10. b.

Sol. *PC* and *CQ* are in Parallel. So, equivalent resistance of the circuit is *R*/2. In static condition, current through the battery is $I = 2V/R$.

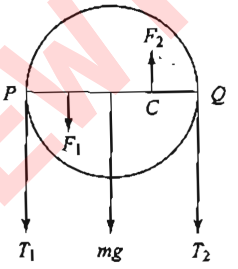


Fig. 9.492

The current through *CP* and *CQ* is $i = I/2 = V/R$

Hence,

$$F_1 = F_2 = Bir$$

Consider the FBD of ring and the blocks

FBD of ring:

Consider torque about '*C*',

$$T_1 r + F_1 r/2 + F_2 r/2 = T_2 r \qquad \text{(i)}$$

FBD of blocks: $T_1 = mg$ and $T_2 = 2mg$

So, $mgr + Bir^2/2 + Bir^2/2 = 2mgr$

or $i = \dfrac{mg}{Br}$ or $\dfrac{V}{R} = \dfrac{mg}{Br}$

or $V = \dfrac{mgR}{Br}$

Net torque of tension

$$= (T_2 - T_1)r = mgr = \frac{BVr^2}{R}$$

For Problems 11 – 12

11. a., 12. b.

Sol. Magnetic field produced by the conductor along *z*-axis will be parallel to *MN*. Hence, force on *MN* is zero.

Current from *B* to *A* (through *NM* or *QP*):

$$I = \frac{V}{R/2} = \frac{2V}{R}$$

Force due to I_0 on $N'N$ and $Q'Q$ will be out of the paper and on $M'M$ and $P'P$ will be in the paper. This will produce the torque on loop along –ve *x*-axis.

Torque on $M'M$ (along –ve *x*-axis):

$$\tau = \int_a^b\left(\frac{\mu_0 I_0}{2\pi r}I\,dr\right)r\sin\left(\frac{\pi}{6}\right) = \frac{\mu_0 I_0}{4\pi}I\,(b-a)$$

$$= \frac{\mu_0 I_0\,2V}{4R\pi}(b-a)$$

Total torque $= 4\tau = 2\dfrac{\mu_0 I_0 V}{\pi R}(b-a)$ (along negative *x*-axis.)

For Problems 13 – 14

13. a., b., 14. b.

Sol. As the magnetic field is along the *x*-axis, the magnetic force will be along (–) *z*-axis at $t = 0$. So, the particle will move in helical path along (1). At $t = T_0$, the direction of field changes, so force becomes along + *z*-direction, and now the particle will move in helical path along (2). It will be moving along *x*-axis, so that resulting path will be helical.

At $t = \dfrac{T_0}{2}$, particle will be at E_1.

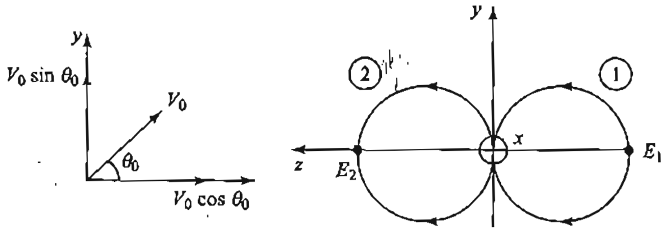


Fig. 9.493

x-coordinate $= \dfrac{P_0}{2}$ (half of pitch)

y-coordinate = 0 (from Fig. 9.438)

and z-coordinate $= -2R_0$ (from Fig. 9.438)

Hence, (a) is correct.

Similarly at $t = \dfrac{3T_0}{2}$ particle will be at E_2.

$\therefore$ The coordinates are $\left(\dfrac{3P_0}{2}, 0, 2R_0\right)$

Hence, (b) is correct.

From Fig. 9.437, we can see that distance between two extremes: E_1 and E_2 is $4R_0$.

For Problems 15 – 16

15. b., 16. d.

Sol. $\sin 30° = L/R$

$$L = \frac{mV_0}{2qB} = \frac{50\times10^{-3}\times100}{2\times1\times25} = \frac{1}{10}\text{m} = 10\,\text{cm}$$

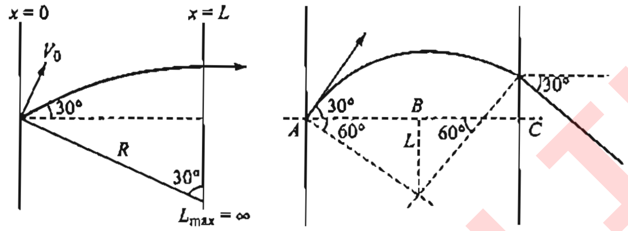


Fig. 9.494

$$L = AB + BC + \cos 60° + r\cos 60° = r$$

$$L = \frac{mv}{\varepsilon B} = 20\text{ cm}$$

For Problems 17 – 19

17. d., 18. d., 19. b.

Sol. As particle is performing a circular motion of decreasing radius (i.e., spiral path) in a magnetic field, so the speed of particle is continuously decreasing $\left(\text{from } r = \dfrac{mv}{qB}\right)$. If the speed of particle is decreasing, it means some retarding tangential force is acting on the particle in addition to the centripetal force provided by the magnetic field.

So, the net force, $\bar{F}_{net} = q(\bar{v}\times\bar{B}) + \dfrac{md\bar{v}}{dt}$

As, v is continuously decreasing, we can say $|\bar{F}_{net}|$ is also decreasing, assuming $\dfrac{md\bar{v}}{dt}$ is not increasing.

Angular velocity of the particle remains constant as it is given by $\omega = \dfrac{qB}{m}$.

As I (moment of inertia) is decreasing, $I\omega$ = (angular momentum) also decreases.

As with time, KE decreases and hence mechanical energy decreases.

For Problems 20 – 22

20. c., 21. a., 22. a.

Sol. Inside the cylinder: $B2\pi r = \mu_0 \dfrac{I}{\pi R^2}\pi r^2$

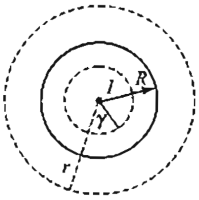


Fig. 9.495

or $\quad B = \dfrac{\mu_0 I}{2\pi R^2}r^2 \quad$ (i)

Outside the cylinder: $B\,2\pi r = \mu_0 I$

$\therefore \quad B = \dfrac{\mu_0 I}{2\pi r} \quad$ (ii)

Inside the cylinder, $B \propto r$ and outside $B \propto \dfrac{1}{r}$.

So, with surface, nature of magnetic field changes. Hence, it is clear from the graph that wire 'c' has greatest radius. Magnitude of magnetic field is maximum at the surface of wire 'a'.

Inside the wire

$$B(r) = \frac{\mu_0}{2\pi}\frac{I}{R^2}r \Rightarrow \frac{dB}{dr} = \frac{\mu_0}{2\pi}\cdot\frac{I}{R^2}$$

i.e., $\quad$ Slope $\propto \dfrac{I}{\pi R^2} \propto$ current density

It can be seen that slope of curve for wire a is greater than wire c.

For Problems 23 – 25

23. b., 24. d., 25. a.

Sol. Magnetic field at any point on Ampere's loop can be due to all currents passing through inside or outside the loop. But net contribution in the left hand side will come from inside current only.

For $r < a$, current passing through within the cylinder of radius r is given by

$$\int_0^r JdA = \int_0^r kr^2 2\pi r dr = 2\pi k\int_0^r r^3 dr$$
$$= k\pi r^4/2$$

Now using Ampere's law:

$$B\times2\pi r = \mu_0 I = \mu_0 k\pi r^4/2 \Rightarrow B = \frac{\mu_0 k r^3}{4}$$

For $r > a$, $I = k\pi a^4/2$ (by putting $r = a$)

$$\therefore \quad B = \frac{\mu_0 k\pi a^4}{4r}$$

For Problems 26 – 27

26. b., 27. d.

Sol. At B, magnetic field will be zero due to CB and AB parts.

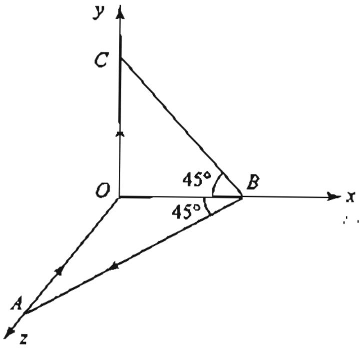


Fig. 9.496

$$\bar{B}_{AO} = \frac{\mu_0}{4\pi}\frac{I}{a}[\sin 0° + \sin 45°](-\hat{j}) = \frac{-\mu_0 I}{4\pi\sqrt{2}a}\hat{j}$$

$$\vec{B}_{OC}=\frac{\mu_0}{4\pi}\frac{I}{a}[\sin 0^\circ+\sin 45^\circ]\left(-\hat{k}\right)=\frac{-\mu_0 I}{4\pi\sqrt{2}a}\hat{k}$$

Net magnetic field at B

$$=\vec{B}_{AO}+\vec{B}_{OC}=-\frac{\mu_0 I}{4\pi\sqrt{2}a}(\hat{j}+\hat{k})$$

At O, magnetic field due to AO and OC will be zero.

Due to BC: $r=a\cos 45^\circ=\frac{a}{\sqrt{2}}$

$$\vec{B}_{CB}=\frac{\mu_0}{4\pi}\frac{I}{r}[\sin 45^\circ+\sin 45^\circ]\left(-\hat{k}\right)=\frac{-\mu_0 I}{2\pi a}\hat{k}$$

Similarly, due to BA: $\vec{B}_{BA}=\frac{-\mu_0 I}{2\pi a}\hat{j}$

Net magnetic field

$$=\vec{B}_{CB}+\vec{B}_{BA}=\frac{-\mu_0 I}{2\pi a}\left(\hat{j}+\hat{k}\right)$$

For Problems 28 – 29

28. c., 29. d.

Sol. If I_1 and I_2 be the currents in circular and straight parts, respectively, and B_1, B_2 the magnetic fields due to them, then

$$B_1=\frac{\mu_0 I_1}{2R}\times\frac{2}{3}=\frac{\mu_0 I_1}{3R}$$

$$B_2=\frac{\mu_0 I_2}{4\pi[R\cos 60^\circ]}[2\sin 60^\circ]=\frac{\sqrt{3}\mu_0 I_2}{2\pi R}$$

For the total field at 'O' to be zero,

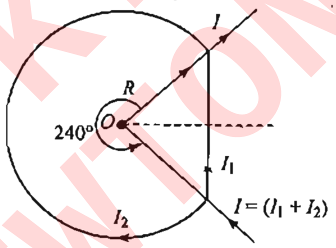


Fig. 9.497

$$\frac{\mu_0 I_1}{3R}=\frac{\sqrt{3}\mu_0 I_2}{2\pi R}\quad\Rightarrow\quad\frac{I_1}{I_2}=\frac{3\sqrt{3}}{2\pi}$$

Now, $\frac{l_1}{l_2}=\frac{R_2}{R_1}=\frac{\rho\frac{l_2}{\pi r_2^2}}{\rho\frac{l_1}{\pi r_1^2}}=\frac{l_2}{l_1}\left(\frac{r_1}{r_2}\right)^2$

Required ratio $=\sqrt{\left(\frac{l_1}{l_2}\right)\frac{l_1}{l_2}}$

$$\ell_1=2R\sin 60^\circ=\sqrt{3}\,R,$$

$$\ell_2=R\left(\frac{4\pi}{3}\right)=\frac{4}{3}\pi R\Rightarrow\frac{l_1}{l_2}=\frac{3\sqrt{3}}{4\pi}$$

$\Rightarrow$ Required ratio $=\sqrt{\left(\frac{3\sqrt{3}}{2\pi}\right)\times\left(\frac{3\sqrt{3}}{4\pi}\right)}=\frac{3\sqrt{3}}{2\sqrt{2}\pi}$

For Problems 30 – 31

30. a., 31. c.

Sol. The particle enters into the magnetic field perpendicular to it. It will start moving into a circular path. Let the initial radius of the path is R.

Free body diagram

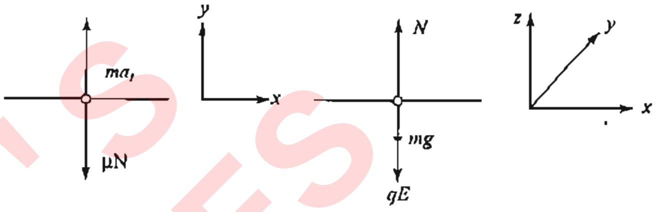


Fig 9.498

Hence, $N=mg+qE$ (i)

$$qBv=\frac{mv^2}{R}\tag{ii}$$

$$-m\frac{dv}{dt}=\mu N\tag{iii}$$

From(ii), $R=\frac{mv}{qB}$ (iv)

From (i) and (ii),

$$-m\frac{dv}{dt}=\mu(mg+qE)\tag{v}$$

$$-m\int_{v_0}^{0}\frac{dv}{dt}=\mu(mg+qE)\int_0^t dt$$

which gives $t=\frac{mv_0}{\mu(mg+qE)}$.

From (iv), $dR=\frac{m}{qB}dv=-\frac{\mu(mg+qE)dt}{qB}$

$$\int_R^0 dR=-\frac{\mu(mg+qE)}{qB}t$$

$$-R=\frac{\mu(mg+qE)}{qB}t\;;\;t=\frac{\mu(mg+qE)}{qB}t$$

From (v), $-mv\frac{dv}{d\ell}=\mu(mg+qE)$

$$-m\int_{v_0}^{0}vdv=\mu(mg+qE)\int_0^l d\ell$$

$$\frac{mv_0^2}{2}=\mu(mg+qE)\ell$$

$$\ell=\frac{mv_0^2}{2\mu(mg+qE)}$$

For Problems 32 – 33

32. c., 33. a.

Sol. $\bar{F}=q\left(\vec{v}\times\vec{B}\right)$

For the first case:

$$-(5\sqrt{2}\times10^{-3}\,\text{N})\,\hat{k}=(10^{-5}\,\text{C})$$

$$\left(\frac{10^6}{\sqrt{2}}\text{ms}^{-1}\right)(\hat{i}+\hat{j})\times(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$$

$$=\left(\frac{10}{\sqrt{2}}\right)\left[B_z\hat{i}-B_z\hat{j}+(B_y-B_x)\hat{k}\right]$$

$$\Rightarrow\quad B_z=0, B_y-B_x=-10^3\text{ T} \qquad \text{(i)}$$

For the second case:

$$F_y\,\hat{j}=(10^{-5}\text{C})\,(10^6\text{ms}^{-1})\,(\hat{k})\times\left[(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})\right]$$

$$\Rightarrow\quad F_y\hat{j}=10\,(B_x\hat{j}-B_y\hat{i})$$

$$\Rightarrow\quad F_y=10\,B_x, B_y=0$$

$\Rightarrow$ Using (i), we get $B_x=10^{-3}$ T

Thus, $\vec{B}=(10^{-3}\text{T})\,\hat{i}$

Also, $F_Y=10\,B_x=10^{-2}$ N

For Problems 34–36

34. a., 35. b., 36. c.

Sol. $\vec{E}=E\hat{j}$

$$\vec{v}_0=2v\cos 60°\hat{i}+2v\sin 60°\hat{j}$$

$$=v\hat{i}+v\sqrt{3}\hat{j}$$

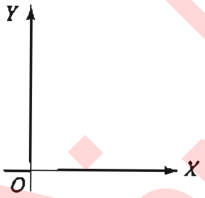


Fig. 9.499

$$\vec{a}=-\frac{qE}{m}\hat{j}$$

$$\vec{R}(t)=vt\hat{i}+\left(v\sqrt{3}t-\frac{1}{2}\frac{qE}{m}t^2\right)\hat{j}$$

$$\therefore\quad vt=\frac{\sqrt{3}mv^2}{qE}\Rightarrow t=\frac{\sqrt{3}mv}{qE}$$

For $t\le\dfrac{\sqrt{3}mv}{qE}$,

$$x(t)=vt \text{ and } v_y(t)=v\sqrt{3}-\frac{qE}{m}t$$

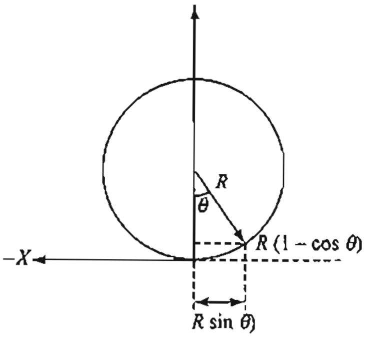


Fig. 9.500

$$\Rightarrow\quad v_y=\left(t-\frac{\sqrt{3}mv}{2E}\right)$$

$$v\sqrt{3}=\frac{qE}{m}\times\frac{\sqrt{3}mv}{qE}=0$$

For $t>\dfrac{\sqrt{3}mv}{qE}$, magnetic field is also present

The particle will start moving on helical path. The cross section of the helix will be in x–z plane.

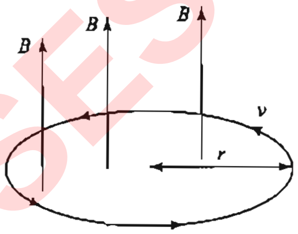


Fig. 9.501

$$R=\frac{mv}{qB};\ \omega=\frac{qB}{m};\ \theta=\omega t=\frac{qBt}{m}$$

For $t\ge\dfrac{\sqrt{3}mv}{qE}$,

$$x(t)=\frac{\sqrt{3}mv^2}{qE}+R\sin\left(\frac{qB}{m}t\right)$$

$$y(t)=\frac{3v^2m}{2qE}-\frac{1}{2}\frac{qE}{m}t^2,$$

$$Z(t)=-R\left[1-\cos\left(\frac{qB}{m}t\right)\right]$$

For Problems 37–40

37. b., 38. b., 39. a., 40. b.,

Sol. When the particle enters electric field, it has moved horizontally a distance.

$$GP=r\cos 45°=\frac{mv_0}{qB}\times\frac{1}{\sqrt{2}}$$

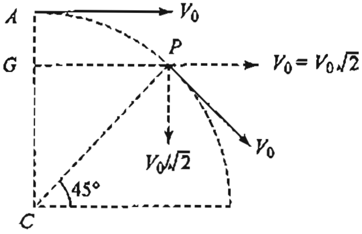


Fig. 9.502

Length of electric field is $= l-r\sin 45°$

$\therefore$ Vertical distance left

$$=d-\frac{mv_0}{\sqrt{2}qB}\left(1-\frac{1}{\sqrt{2}}\right)$$

∴ $$v_x t = l - \frac{mv_0}{\sqrt{2}qB} = v\cos 45° \times t$$

∴ $$t = \frac{\sqrt{2}}{v_0}\left\{\ell - \frac{mv_0}{\sqrt{2}qB}\right\}$$

∴ $$d - \frac{mv_0}{qB}\left(1 - \frac{1}{\sqrt{2}}\right) = v_y t - \frac{1}{2}\frac{qE}{m}\ell^2$$

$$\ell - d = \frac{mv_0}{qB}\left(\sqrt{2}-1\right) + \frac{1}{2}\left(\frac{qE}{m}\right)\left\{\frac{\sqrt{2}\ell}{v_0} - \frac{m}{qB}\right\}^2$$

∴ $\ell - d = 20/5 = 4$ m.

For Problems 41–43

41. c., 42. a., 43. d.

Sol. **a.** If the magnetic field at point P is zero, then from the figure, the current must be out of the page, in order to cancel the field from I_1. Also:

$$B_1 = B_2 \Rightarrow \frac{\mu_0 I_1}{2\pi r_1} = \frac{\mu_0 I_2}{2\pi r_2}$$

$$I_2 = I_1\frac{r_2}{r_1} = (6.00\text{ A})\frac{(0.500\text{ m})}{(1.50\text{ m})} = 2.00\text{ A}.$$

b. The field at Q points to the right and has magnitude

$$B_Q = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right) = \frac{\mu_0}{2\pi}\left(\frac{6.00\text{ A}}{0.500\text{ m}} - \frac{2.00\text{ A}}{1.50\text{ m}}\right)$$

$$= 2.13 \times 10^{-6}\text{ T}$$

c. The magnitude of the field at S is given by the sum of the squares of the two fields because they are at right angles. So,

$$B_S = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi}\sqrt{\left(\frac{I_1}{r_1}\right)^2 + \left(\frac{I_2}{r_2}\right)^2}$$

$$= \frac{\mu_0}{2\pi}\sqrt{\left(\frac{6.00\text{ A}}{0.60\text{ m}}\right)^2 + \left(\frac{2.00\text{ A}}{0.80\text{ m}}\right)^2} = 2.1 \times 10^{-6}\text{ T}$$

For Problems 44–46

44. d., 45. a., 46. a.

Sol. a.

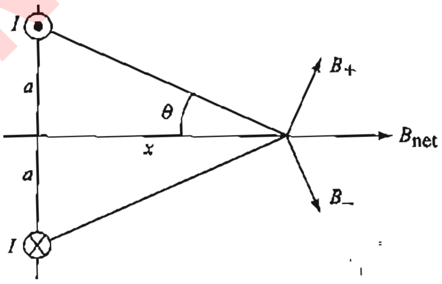


Fig. 9.503

b. At a position on the x-axis:

$$B_{\text{net}} = 2\frac{\mu_0 I}{2\pi r}\sin\theta = \frac{\mu_0 I}{\pi\sqrt{x^2+a^2}}\frac{a}{\sqrt{x^2+a^2}}$$

$$B_{\text{net}} = \frac{\mu_0 I a}{\pi\left(x^2+a^2\right)}$$

c. In the positive x-direction, as shown at left.

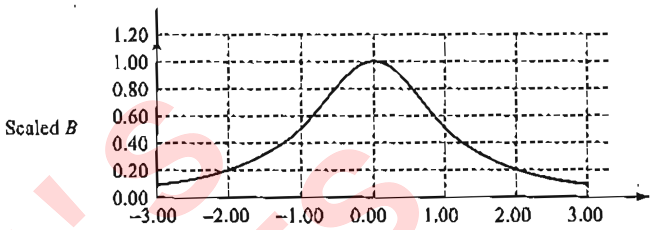


Fig. 9.504

d. The magnetic field is a maximum at origin, $x = 0$.

e. When $x >> a$, $B \approx \frac{\mu_0 I a}{\pi x^2}$

For Problems 47–49

47. c., 48. b., 49. d.

Sol. a.

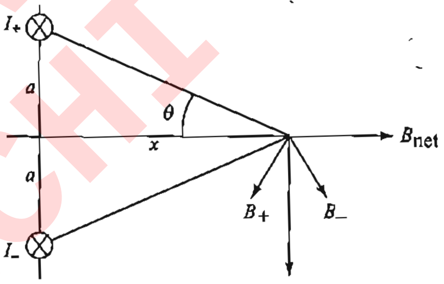


Fig. 9.505

b. At a position on the x-axis:

$$B_{\text{net}} = 2\frac{\mu_0 I}{2\pi r}\cos\theta$$

$$= \frac{\mu_0 I}{\pi\sqrt{x^2+a^2}}\frac{x}{\sqrt{x^2+a^2}}$$

$$B_{\text{net}} = \frac{\mu_0 I x}{\pi\left(x^2+a^2\right)}$$

c. In the negative y-direction, as shown at left,

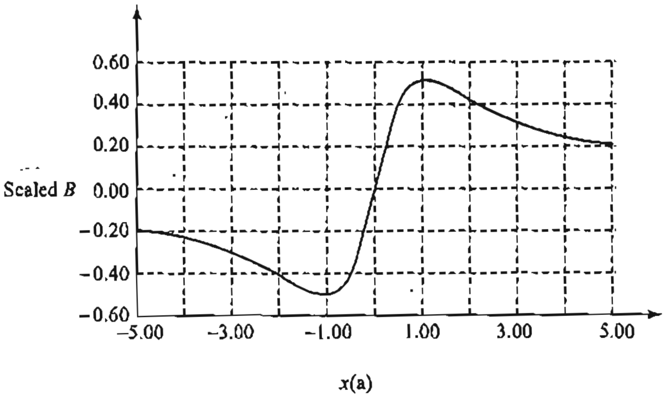


Fig. 9.506

d. The magnetic field is a maximum when:

$$\frac{dB}{dx} = 0 = \frac{C}{x^2+a^2} - \frac{2Cx^2}{\left(x^2+a^2\right)^2}$$

$$(x^2+a^2) = 2x^2 \quad \Rightarrow \quad x = \pm a$$

e. When $x >> a,\ B \approx \frac{\mu_0 I}{\pi x}$,

which is just like a wire carrying current $2I$.

For Problems 50–52

50. b., 51. c., 52. b.

Sol. $r = \frac{mv}{qB} \quad \Rightarrow \quad r = \frac{mv}{qB}$

$T = \frac{2\pi m}{qB}$ → independent of velocity. At $\bar{F} \perp \bar{B}$, hence

$\bar{a} \perp \vec{B}$

$$\therefore \qquad \bar{a}\cdot\vec{B} = 0$$

$$(x\hat{i}+2\hat{j})\ (2\hat{i}+4\hat{j}) = 0$$

$$2x+8 = 0$$

$$x = -4\ \text{ms}^{-2}$$

For Problems 53–54

53. a., 54. d.

Sol. Proceeding similar to example, we can see that the particle's acceleration is in y-direction,

$$\frac{dv_y}{dt} = \frac{qE}{m} = \text{constant}$$

The motion of the particle is equivalent to circular motion in xz plane with uniform acceleration in y-direction.

Hence, $v_0^2 = v_x^2 + v_z^2 = \text{constant}$

The magnetic force cannot change the magnitude of v_0.
The y-component of velocity,

$$v_y = \frac{qE}{m}t$$

The y-coordinate at time t, $y = \frac{1}{2}\frac{qE}{m}t^2$

The time period of circular motion in xz plane,

$$T = \frac{2\pi m}{Bq}$$

Let the particle cross y-axis after n rotations, then

$$t = nT = \frac{2\pi mn}{qB}$$

Thus, $y_n = \frac{qE}{2m} \times \left(\frac{2\pi mn}{qB}\right)^2 = \frac{2\pi^2 mn^2 E}{qB^2}$

b. As $v_y = a_y t\left(\frac{qE}{m}\right)\left(\frac{2\pi mn}{qB}\right) = \frac{2\pi nE}{B}$

Thus, $\tan\alpha = \frac{v_0}{v_y} = \frac{v_0 B}{2\pi nE}$

$$\Rightarrow \qquad \alpha = \tan^{-1}\left(\frac{v_0 B}{2\pi nE}\right)$$

For Problems 55–56

55. a., 56. a.

Sol. **a.** $F_l = mg$

When bar is just ready to levitate,

$$I\ell B = mg,\ I = \frac{mg}{\ell B} = \frac{(0.750\ \text{kg})(9.80\text{m/s}^2)}{(0.500\ \text{m})(0.450\ \text{T})} = 32.67\ \text{A}$$

$$\varepsilon = IR = (32.67\ \text{A})(25.0\ \Omega) = 817\ \text{V}$$

b. $R = 2.0\ \Omega,\ I = \varepsilon/R = (816.7\ \text{V})/(2.0\ \Omega) = 408\ \text{A}$

$F_l = I\ell B = 92\ \text{N}$

$a = (F_l - mg)/a = 113\ \text{ms}^{-2}$

For Problems 57–58

57. a., 58. b.

Sol. **a.** The magnetic force on the bar must be upward, so the current through it must be to the right. Therefore, a must be the positive terminal.

b. For balance, $I\ell B\sin\theta = mg$

$$m = \frac{I\ell B\sin\theta}{g} \quad \Rightarrow \quad F_{\text{magn}} = mg$$

$$I = \varepsilon/R = 175\ \text{V}/5.00\ \Omega = 35.0\ \text{A}$$

$$m = \frac{(35.0\ \text{A})(0.600\ \text{m})(1.50\ \text{T})}{9.80\ \text{m/s}^2} = 3.21\ \text{kg}$$

For Problems 59–60

59. a., 60. d.

Sol. **a.** $F = ILB$, to the right.

b. $v^2 = 2ad \Rightarrow d = \frac{v^2}{2a} = \frac{v^2 m}{2ILB}$

For Problems 61–62

61. d., 62. a.

Sol. **a.** By examining a small piece of the wire (Fig. 9.452), we find

$$F_B = ILB = 2T\sin(\theta/2)$$

$$ILB \approx \frac{2T\theta}{2} = \frac{2T\,L/R}{2} \Rightarrow \frac{T}{IB} = R$$

b. For a particle:

$$qvB = \frac{mv^2}{R} \Rightarrow B = \frac{mv}{Rq} = \frac{mvIB}{Tq} \Rightarrow v = \frac{Tq}{mI}$$

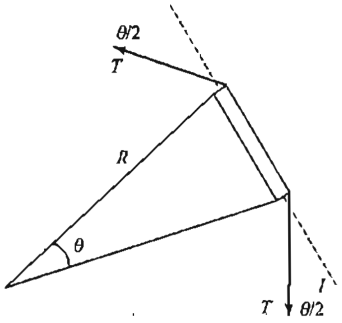


Fig. 9.507

For Problems 63–64

63. a., 64. a.

Sol. **a.** By examining a small piece of the wire (Fig. 9.508), we find:

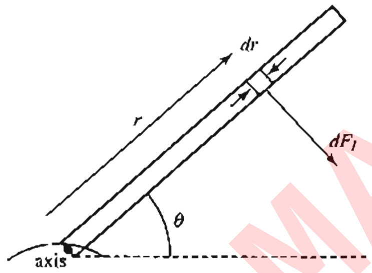


Fig. 9.508

Divide the rod into infinitesimal sections of length dr.
The magnetic force on this section is $dF_1 = IB\,dr$
and is perpendicular to the rod. The torque dt due to force on this section is $d\tau = r\,dF_1 = IBr\,dr$
The total torque is

$$\int d\tau = IB\int_0^l r\,dr = \frac{1}{2}Il^2B = 0.0442\ \text{Nm}^{-1},$$

clockwise. This is the same torque calculated from a force diagram in which the total magnetic force $F_l = ILB$ acts at the center of the rod.

b. F_1 produces a clockwise torque, so the spring force must produce a counterclockwise torque. The spring force must be to the left, the spring is stretched.
Find x, the amount the spring is stretched:
axis at hinge, counterclockwise torques positive

$$(kx)\ell\sin 53° - \frac{1}{2}I\ell^2B = 0$$

$$x = \frac{I\ell B}{2k\sin 53.0°} = \frac{(6.50\ \text{A})(0.200\ \text{m})(0.340\ \text{T})}{2(4.80\ \text{Nm}^{-1})\sin 53.0°}$$

$$= 0.05765\ \text{m}$$

$$U_s\ \frac{1}{2}kx^2 = 7.98\times 10^{-3}\ \text{J}$$

For Problems 65–66

65. b., 66. a.

Sol. **a.** Note that the Earth's magnetic field exerts no force on wire B, since the current in wire B is parallel to the Earth's magnetic field. Thus, for equilibrium, the remaining two forces that act on wire B must cancel. Assuming that the length of wire B is L and that wire A carries a current, we obtain

$$-\frac{\mu_0 I(1.0\ A)L}{2\pi(0.050\ \text{m})} + \frac{\mu_0(1.0\ \text{A})(3.0\ \text{A})L}{2\pi(0.100\ \text{m})} = 0$$

$$\text{So, } I = (3.0\ \text{A})\cdot\frac{0.050\ \text{m}}{0.100\ \text{m}} = 1.5\ \text{A}$$

b. Note that the force on wire B that is generated by wire C is to the right. Thus, if the current in wire C is increased, wire B will slide to the right.

For Problems 67–69

67. a., 68. b., 69. b.

Sol. **a.** The net force on a current carrying loop of any arbitrary shape in a uniform magnetic field is zero.

$$\vec{F}_{\text{net}} = 0$$

b. The given loop can be considered to be a superposition of three loops as shown in Fig. 9.509. The area vector of the three loops (1), (2) and (3) are

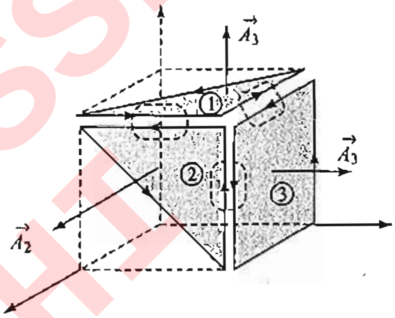


Fig. 9.509

$$\vec{A}_1 = \left(\frac{1}{2}\times 10\times 10\times 10^{-4}\right)\hat{j}\ \text{m}^2$$

$$\vec{A}_2 = \left(\frac{1}{2}\times 10\times 10\times 10^{-4}\right)\hat{k}\ \text{m}^2$$

$$\vec{A}_3 = \left(10\times 10\times 10^{-4}\right)\hat{i}\ \text{m}^2$$

Magnetic moment vector,

$$\vec{\mu} = i\vec{A} = 10(0.01\hat{i} + 0.005\hat{j} + 0.005\hat{k})\ \text{Am}^2$$

$$= (0.1\hat{i} + 0.05\hat{j} + 0.05\hat{k})\ \text{Am}^2$$

c. Torque,

$$\vec{\tau} = (0.1\hat{i} + 0.05\hat{j} + 0.05\hat{k})\times(2\hat{i} - 3\hat{j} + \hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.1 & 0.05 & 0.05 \\ 2 & -3 & 1 \end{vmatrix} = -0.1\hat{i} - 0.4\hat{k}\ \text{Nm}$$

Matching Column Type

1. i. → b., d., ii. → a., d., iii. → c., iv. → c.

i. If loop is moved away, then flux through loop decreases in –ve z-direction. To increase this decreasing flux, current induced should be in clockwise direction.
Now, induced current in AB will be parallel to I, so that there will be net attraction between wire and loop.
Hence **i. → b., d.**

ii. Explain in the similar way as in (i).

In cases (iii) and (iv), just after the rotation is started, velocity of each element of the loop will be parallel or antiparallel to the magnetic field produced by I. Hence, no e.m.f. is induced.

So **iii. → c., iv. → c.**

2. i. → d., ii. → a., iii. → b., iv. → c.

We know that $r = mv/qB$. So, larger the mass, larger is the radius. Since radius of path '1' is largest, so path '1' should correspond to ion B.

Hence, **ii. → a.**

Now path '2' should correspond to ion C, because charge on both B and C is –ve, so their paths should be on the same side.

Hence, **iii. → b.**

Now, radii of paths '2' and '4' are same. If path '2' corresponds to C, then path '4' should correspond to ion A, because masses of A and C are same.

Hence, **i. → d.**

Now, remaining path '3' should correspond to D.

Hence, **iv. – c.**

3. i. → a., b., c., ii. → a., b., c., d., iii. → c., iv. → a., b., c., d.

The magnetic field is along negative y-direction in cases a, (b) and (c). In case (d) there will be component of magnetic field along –ve x-axis.

Hence, **i. → a., b., c.**

z-component of magnetic field is zero in all cases.

Hence, **ii. → a., b., c.,d.**

The magnetic field at P is $\frac{\mu_0}{4\pi}\frac{i}{d}$ in case (c) only.

Hence **iii. → c.**

The magnetic field at P is less than $\frac{\mu_0}{2\pi}\frac{i}{d}$ for all cases.

Hence, **iv. → a.,b.,c.,d.**

4. i. → c.,d., ii. → c.,d., iii. → b.,c., iv. → a.,c.

i. Because the magnetic field is parallel to x-axis, the force on wire parallel to x-axis is zero. The force on each wire parallel to y-axis is $B_0\frac{i}{2}\ell$. Hence, net force on the loop is $B_0 i\ell$. Since force on each wire parallel to y-axis passes through center of the loop net torque about center of the loop is zero.

Similar is the situation in case (ii)

Hence, **i. → c.,d., ii. → c.,d.**

iii. Since direction of current from entry point in the loop to exit point in the loop is along the diagonal of the loop, the direction of external uniform magnetic field is also along the same diagonal. Hence, net force on the loop is zero. Since force on each wire on the loop passes through center of the loop, net torque about center of the loop is zero.

iv. The direction of current from entry point in the loop to exit point in the loop is along the diagonal (of length 2ℓ) of the loop. The direction of external uniform magnetic field is also perpendicular to the same diagonal.

Hence, magnitude of net force on the loop is $B_0 i\left(\sqrt{2}\ell\right)$.

Since force on each wire on the loop passes through center of the loop, net torque about center of the loop is zero.

5. i. → a.,c., ii. → a.,b., iii → d., iv. → a.

i. Since $\vec{E} = 0$ and $\vec{B} \neq 0$, so path will be straight line if velocity is parallel to $\vec{B}$. Or path will be circular if $\vec{V} \perp \vec{B}$.

Or path will be helical (with uniform pitch) if $\vec{V}$ is at some other angle to $\vec{B}$.

Hence, **i. → a.,c.**

ii. Since $\vec{E} \neq 0$ and $\vec{B} \neq 0$, so path will be straight line if $\vec{V} \perp \vec{B}$ or parabola otherwise.

Hence, **ii. → a.,b.**

iii. $\vec{E} \neq 0$, $\vec{B} \neq 0$, $\vec{E} \parallel \vec{B}$

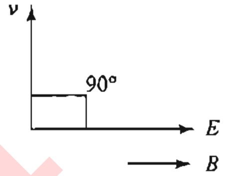


Fig. 9.510

Helical path with non-uniform pitch

Hence, **iii. → d.**

iv. Straight line path if $\vec{V} \times \vec{B} = \vec{E}$

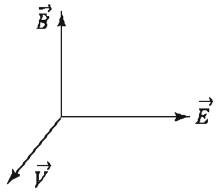


Fig. 9.511

Hence, **iv. → a.**

6. i. → b., ii. → a.,c., iii. → a.,c., iv. → a., d.

i.

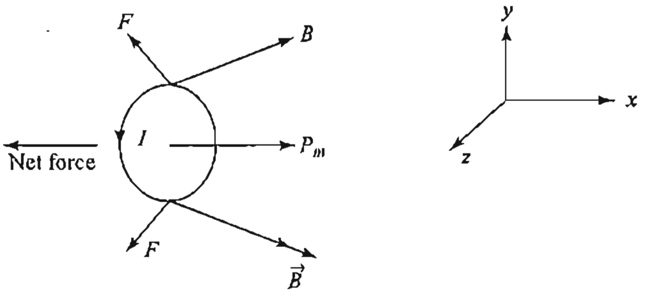


Fig. 9.512

ii. Force on left side is along z-axis and on right side is along $-z$-axis. But former is greater because of higher magnetic field on left side. Hence, net force is along z-direction or along P_m.

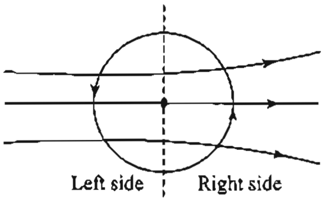


Fig. 9.513

iii. Now net force is reversed, but P_m is also reversed.

iv.

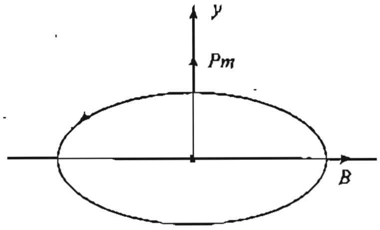


Fig. 9.514

Force on left side is more than on right side. And net force will be along y-axis or along P_m.

7. i. → **d., ii.** → **a., b., c., iii.** → **b., c., iv.** → **a., b., c.**

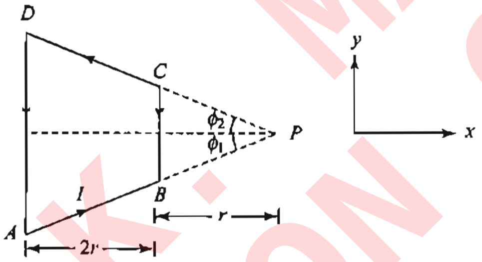


Fig. 9.515

At P, magnetic field due to AB and CD will be zero.

$$\vec{B}_{BC} = \frac{\mu_0}{4\pi}\frac{I}{r}(\sin\phi_1 + \sin\phi_2)(-\hat{k})$$

$$\vec{B}_{DA} = \frac{\mu_0}{4\pi}\frac{I}{r}(\sin\phi_1 + \sin\phi_2)(-\hat{k})$$

Net field at P:

$$\vec{B} = \vec{B}_{BC} + \vec{B}_{DA} = -\frac{2\mu_0}{4\pi}\frac{I}{3r}(\sin\phi_1 + \sin\phi_2)\hat{k}$$

Hence, we can find the answers given.

8. i. → **a., b., d., ii.** → **a., b., c., d., iii.** → **a., b., c., d., iv.** → **b.**

i. Velocity of the particle may be constant, if forces of electric and magnetic fields balance each other. Then, path of particle will be straight line. Also, path of particle may be helical if magnetic and electric fields are in same direction. But path of particle cannot be circular. Path can be circular if only magnetic field is present, or if some other force is present which can cancel the effect of electric field.

ii. Here, all the possibilities are possible depending upon the combinations of the three fields.

iii. This situation is similar to part (i).

iv. In a uniform electric field, path can be only straight line or parabolic.

9. i. → **b., d., ii.** → **a., c., iii.** → **a., c, iv.** → **b., d.**

i. If current is increased, flux in the loop will increase in inside direction, then due to Lenz's law induced emf in the loop will be in anticlockwise direction. Due to this current, the current in the nearer side of loop to the wire will be in opposite direction to that of wire. Hence, there will be repulsion.

ii. This situation is opposite to part (i).

iii. If loop is moved away, then flux decreases and this becomes similar to part (ii).

iv. Similar to part (i)

10. i. → **b., c., ii.** → **b., d., iii.** → **a., iv.** → **c.**

i. $M = NIA = 100 \times 5 \times \pi(0.1)^2 = 5\pi \text{Am}$

As the current in xy plane is along anticlockwise direction, so moment will be along z-axis by right hand thumb rule.

$\vec{M} = 5\pi\hat{k}$

ii. $\vec{\tau} = \vec{M} \times \vec{B} = 5\pi\hat{k} \times (-\hat{i} + \hat{k}) = -5\pi\hat{j}$

iii. Net force on a closed current carrying loop in a uniform magnetic field is zero.

iv. Magnetic field at center of loop due to current in loop will be along +ve z-axis from right hand thumb rule.

11. i. → **b., d., ii.** → **a., iii.** → **a., iv.** → **c.**

i. $B = B_0\hat{j}$, $\vec{M} = -i a^2\hat{k}$

$\vec{\tau} = \vec{M} \times \vec{B} = (-i a^2\hat{k}) \times B_0\hat{j} = i a^2 B_0\hat{i}$

Torque is maximum as $\vec{M}$ and $\vec{B}$ are perpendicular.

ii. Net force on a closed current carrying loop in a uniform magnetic field is zero.

iii. $U = -\vec{M}\cdot\vec{B} = 0$ because $\vec{M} \perp \vec{B}$.

iv. Magnetic moment of loop is along –ve z-axis from right hand thumb rule.

12. i. → **a., c., d., ii.** → **a., c., d., iii.** → **a., b., iv.** → **a., b**

i. Kinetic energy of the particle can remain constant, if both the fields are present. This is possible if the force due to both fields cancel each other.

Kinetic energy of the particle can also remain constant if only magnetic field is present, because magnetic field does not do any work.

Obviously K.E. will remain constant if no field is present.

ii. This is possible if either both the fields are present or no field is present. This is also possible if only magnetic field is present and the particle is at rest.

iii. This is possible if electric field is present, magnetic field may or may not be present.

iv. This is possible if only electric field is present and velocity and electric field are along the parallel lines. Magnetic field may also be present if it is parallel to velocity.

13. i. → **b., d., ii.** → **a., d., iii.** → **b., d., iv.** → **c., d**

Work done by magnetic force on a charge = 0 in any part of its motion.

'S' is matching for all parts (i), (ii), (iii), (iv).

For loop 1: $\sum I_{in} = -i + i - i = -i$

$\therefore \quad \oint \bar{B} \cdot d\vec{\ell} = \mu_0(-i)$

For loop 2: $\sum I_{in} = i - i + i = i$

$\therefore \quad \oint \bar{B} \cdot d\vec{\ell} = \mu_0(i)$

For loop 3: $\sum I_{in} = -i - i + i = -i$

$\therefore \quad \oint \bar{B} \cdot d\vec{\ell} = \mu_0(-i)$

For loop 4: $\sum I_{in} = +i + i - i = +i$

$\therefore \quad \oint \bar{B} \cdot d\vec{\ell} = \mu_0(i)$

(Note that current will be taken as positive which produces lines of magnetic field in the same sense in which $d\vec{\ell}$ is taken.)

Archives

Fill in the Blank Type

1. According to Fleming's left hand rule, the force on electrons will be toward right (*D*).

Also, by the same rule we find that the force on proton and α-particle is toward left.

Now, since the magnetic force will behave as centripetal force

$$\therefore \quad \frac{mv^2}{r} = qvB$$

$$\therefore \quad \frac{mv^2}{qB} = r$$

or $\quad r \propto \dfrac{m}{q}$

For proton $\quad r \propto \dfrac{1}{1} = 1$

For α-particle $r \propto \dfrac{4}{2} = 2$

$\therefore$ Radius will be greater for α-particle

$\therefore$ α-particle will take path *B*.

2.

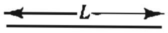


Fig. 9.516

Wire of length L is bent in the form of a circle. Then, the perimeter of the circle

$$2\pi r = L \quad \Rightarrow \quad r = \frac{L}{2\pi}$$

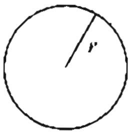


Fig. 9.517

$\therefore$ Area of the circle

$$r\pi^2 = \frac{\pi L^2}{4\pi^2} = \frac{L^2}{4\pi}$$

Magnetic moment of a loop in which i current flows is given by

$$M = iA = \frac{iL^2}{4\pi}$$

3. $i = \dfrac{q}{t} = \dfrac{ne}{t} = \dfrac{10^{16}}{1} \times 1.6 \times 10^{-19}$

$= 1.6 \times 10^{-3}$ A

$M = i \times A$

$= i \times \pi r^2 = 1.6 \times 10^{-3} \times 3.14 \times 0.5 \times 10^{-10} \times 0.5 \times 10^{-10}$

$= 1.25 \times 10^{-3}$ Am^2

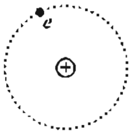


Fig. 9.518

4. The effect of current in PQ and RS for producing magnetic field at center is zero.

Magnetic field due to current in semicircular arc QAR = $\dfrac{1}{2}\left[\dfrac{\mu_0}{2}\dfrac{1}{R_1}\right]$ directed toward the reader perpendicular to the plane of paper.

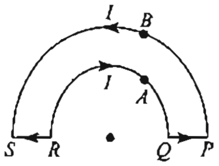


Fig. 9.519

Magnetic field due to current in semicircular arc SBP = $\dfrac{1}{2}\left[\dfrac{\mu_0}{2}\dfrac{1}{R_2}\right]$ directed away from the reader perpendicular to the plane of paper.

$\therefore$ Net magnetic field

$$= \frac{1}{2}\left[\frac{\mu_0}{2}\frac{1}{R_1}\right] - \frac{1}{2}\left[\frac{\mu_0}{2}\frac{1}{R_2}\right]$$

(directed towards the reader perpendicular to plane of paper).

$$= \frac{\mu_0 I}{4}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

5. We use the formula $\vec{F} = I(\vec{l} \times \vec{B}) \Rightarrow F = ILB \sin\theta$

   For sides FE and BA, the angle will be 180° and 0°, respectively.

   $\therefore \quad F = 0$

   For sides DE and CB, the forces will be ILB but opposite in directions, in $+Y$ and $-Y$ directions, respectively.

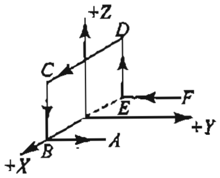


Fig. 9.520

The net force due to these two sides will be zero.

The force due to current inside DC will be $ILB \sin 90° = ILB$ in $+Z$ direction (according to Fleming's Left hand rule).

6. $\vec{F} = q(\vec{v} \times \vec{B}) \quad \Rightarrow \quad q\,vB \sin\theta = F$

   $\Rightarrow \quad F = e\,vB \sin 90° = e\,vB$

   The direction of this force is in $+Z$-direction (by Fleming's left hand rule). Since electrons move toward the side $ABDC$, therefore this side has low potential.

7. The positively charged particle enters the uniform magnetic field at right angle. Therefore, the force acting on the charged particle in the magnetic field acts as centripetal force.

$$\frac{mv^2}{r} = qvB$$

$$\Rightarrow \quad mv = qrB$$

$$\Rightarrow \quad m^2v^2 = q^2r^2B^2$$

$$\Rightarrow \quad p^2 = q^2r^2B^2$$

But $\quad \text{K.E.} = \dfrac{p^2}{2m}$

$$\therefore \quad (\text{K.E.})2m = q^2r^2B^2$$

$$\Rightarrow \quad \frac{(\text{K.E.})_\alpha 2m_\alpha}{(\text{K.E.})_d 2m_d} = \frac{q_\alpha^2 r^2 B^2}{q_d^2 r^2 (2.3B)^2}$$

$$\therefore \quad \frac{5.3}{(\text{K.E.})_d} \times \frac{4}{2} = \frac{4}{1} \times \frac{1}{2.3 \times 2.3}$$

$$\Rightarrow \quad (\text{K.E.})_d = 14.0185\,\text{eV}$$

True or False

1. **True**

   Let us consider a rectangular loop $PQRS$ $(l \times b)$ having current I in anticlockwise direction placed in a uniform magnetic field $\vec{B}$. The uniform magnetic field is directed from left to right.

   Force on section PS according to Fleming's left hand rule will be in downward direction and equal to $IlB \sin\theta$.

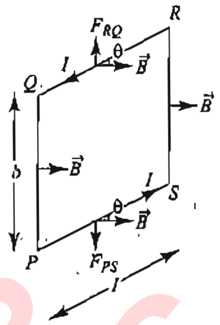


Fig. 9.521

Force on section RQ will be in upward direction and equal to $IlB \sin\theta$. These two force will cancel out. Similarly, the forces on lengths RS and QP will be equal and opposite and cancel out in pairs. The net force is zero.

2. **True.** The magnetic force acts in a direction perpendicular to the direction of velocity and hence it cannot change the speed of the charged particle. Therefore, the kinetic energy $\left(= \dfrac{1}{2}mv^2\right)$ does not change.

3. **False.** The velocity component v_2 will be responsible in moving the charged particle in a circle.

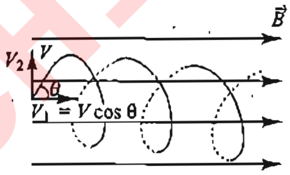


Fig. 9.522

The velocity component v_1 will be responsible in moving the charged particle in horizontal direction. Therefore, the charged particle will travel in a helical path.

4. **False.** When a charged particle passes through a uniform magnetic field perpendicular to the direction of motion, a force acts on the particle perpendicular to the velocity. This force acts as a centripetal force.

$$\therefore \quad \frac{mv^2}{r} = qvB$$

$$\therefore \quad r = \frac{mv^2}{qvB} = \frac{mv}{qB} = \frac{p}{qB} \qquad \text{(i)}$$

$$\text{K.E.} = \frac{p^2}{2m}$$

$$\therefore \quad p = \sqrt{2m\,\text{K.E.}} \qquad \text{(ii)}$$

From (i) and (ii), $r = \dfrac{\sqrt{2m\text{K.E.}}}{qB}$

$$\therefore \quad r \propto \frac{\sqrt{m}}{q} \quad [\text{for constant K.E. and } B]$$

Here, q is same for electron and proton

$\therefore$ Radius of proton will be more.

False.

Single Correct Answer Type

1. **a.** In a non-uniform magnetic field, the needle will experience both a force and a torque.

2. **c.** The magnetic field is perpendicular to the plane of the paper. Let us consider two diametrically opposite elements. By Fleming's left hand rule on element AB, the direction of force will be leftward and the magnitude will be

$$dF = Idl\,B\sin 90° = IdlB$$

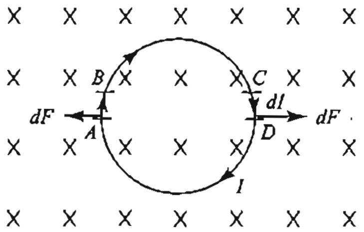


Fig. 9.523

On element CD, the direction of force will be toward right on the plane of the paper and the magnitude will be $dF = IdlB$.

These two forces will cancel out.

Similarly, all forces acting on the diametrically opposite elements will cancel out in pair. The net force on the loop will be zero.

3. **c.** is the correct option. Part AB of the rectangular loop will get attracted to the long straight wire as the currents are parallel and in the same direction whereas part CD will be repelled. But since this force $F \propto \frac{1}{r}$, where r is the distance between the wires.

Therefore, there will be a net attractive force on the rectangle loop.

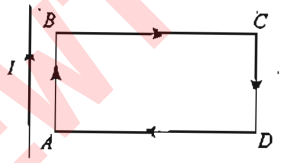


Fig. 9.524

4. **b.** Force per unit length between two wires carrying currents i_1 and i_2 at distance r is given by

$$\frac{F}{\ell} = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{r}$$

Here, $i_1 = i_2 = i$ and $r = b$

$$\frac{F}{\ell} = \frac{\mu_0 i^2}{2\pi b}$$

5. **c.** $R = \sqrt{\frac{2qVm}{Bq}}$ or $R \propto \sqrt{m}$

$$\frac{R_1}{R_2} = \sqrt{\frac{m_X}{m_Y}} \quad \text{or} \quad \frac{m_X}{m_Y} = \left(\frac{R_1}{R_2}\right)^2$$

6. **b.** Using Ampere's circuital law over a circular loop of any radius less than the radius of the pipe, we can see that net current inside the loop is zero. Hence, magnetic field at every point inside the loop will be zero.

7. **d.** Magnetic field at the center due to current in arc ABC is

$$B_1 = \frac{\mu_0}{4\pi}\frac{2\pi I_1}{r}\frac{\theta}{2\pi} \text{ (directed upwards)}$$

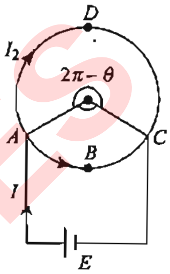


Fig. 9.525

Magnetic field at the center due to current in arc ADC is

$$B_2 = \frac{\mu_0}{4\pi}\frac{2\pi I_2}{r}\frac{2\pi\theta}{2\pi} \text{ (directed downward)}$$

Therefore, net magnetic field at the center

$$B = \frac{\mu_0}{4\pi}\frac{2\pi I_1}{r}\frac{\theta}{2\pi} - \frac{\mu_0}{4\pi}\frac{2\pi I_2}{r}\frac{(2\pi-\theta)}{2\pi}$$

Also, $$I_1 = \frac{E}{R_1} = \frac{E}{\rho l_1/A} = \frac{EQ}{\rho r\theta}$$

And $$I_2 = \frac{E}{R_2} = \frac{E}{\rho l_2/A} = \frac{EA}{\rho r(2\pi-\theta)}$$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\left[\frac{EA}{\rho r\theta}\times\frac{\theta}{r} - \frac{EA}{\rho r(2\pi-\theta)}\times\frac{(2\pi-\theta)}{r}\right] = 0$$

(d) is the correct option.

8. **a.** The centripetal force is provided by the magnetic force

$$\frac{mv^2}{r} = qvB \Rightarrow \frac{mv}{r} = qB$$

$$\Rightarrow \quad r = \frac{mv}{qB} = \frac{p}{qB} \quad [\because p = mv]$$

But $$\text{K.E.} = \frac{p^2}{2m} \Rightarrow p = \sqrt{2m\,\text{K.E.}}$$

Here, K.E. and B are same for the three particles

$$\therefore \quad r \propto \frac{\sqrt{m}}{q}$$

$$\therefore \quad r_p : r_d : r_\alpha = \frac{\sqrt{1}}{1} : \frac{\sqrt{2}}{1} : \frac{\sqrt{4}}{2} = 1 : \sqrt{2} : 1$$

$$\Rightarrow \quad r_\alpha = r_p < r_d$$

$\therefore$ (a) is the correct option.

9. a. Current, $i =$ (frequency) (charge)

$$= \left(\frac{\omega}{2\pi}\right)(2q) = \frac{q\omega}{\pi}$$

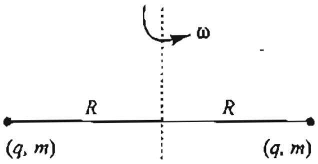


Fig. 9.526

Magnetic moment,

$$M = (I)(A) = \left(\frac{q\omega}{\pi}\right)(\pi R^2) = (q\omega R^2)$$

Angular momentum, $L = 2I\omega = 2(mR^2)\omega$

$$\frac{M}{L} = \frac{q\omega R^2}{2(mR^2)\omega} = \frac{q}{2m}$$

10. b.

A motional e.m.f., $e = Bv$ is induced in the rod. Or we can say a potential difference is induced between the two ends of the rods AB, with A at higher potential and B at lower potential. Due to this potential difference, there is an electric field in the rod.

11. d. Net magnetic field due to the wires will be downward as shown below Fig. 9.527. Since angle between $\overline{v}$ and $\overline{B}$ is 180°, therefore magnetic force $F_m = q(v \times B) = 0$

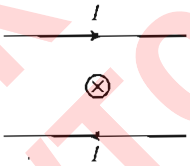


Fig. 9.527

12.d. The magnetic lines of force created due to current will be in such a way that on x–y plane these lines will be perpendicular. Further, these lines will be in circular loops. The number of lines moving downward in x–y plane will be same in number to that coming upward of the x–y plane. Therefore, the net flux will be zero. One such magnetic line is shown in Fig. 9.528.

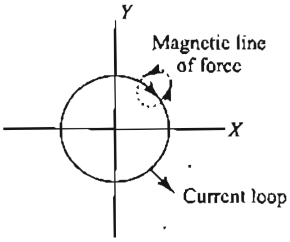


Fig. 9.528

∴ (d) is the correct option.

13. a. $F_E = qE$ (Force due to electric field)

$F_B = evB\sin\theta = qvB\sin 0 = 0$ (Force due to magnetic field)

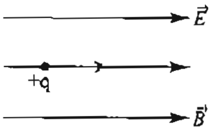


Fig. 9.529

Force due to electric field will make the charged particle released from rest to move in the straight line (that of electric field). Since the force due to magnetic field is zero, therefore the charged particle will move in a straight line.

∴ (a) is the correct option.

14. c.

The angular momentum L of the particle is given by $L = mr^2\omega$ → where $\omega = 2\pi n$

$$\therefore \text{ Frequency } n = \frac{\omega}{2\pi};$$

$$\text{Further } i = q \times n = \frac{\omega q}{2\pi}$$

$$\text{Magnetic moment, } M = iA = \frac{\omega q}{2\pi} \times \pi r^2;$$

$$\therefore \quad M = \frac{\omega q r^2}{2} \Rightarrow \frac{M}{L} = \frac{\omega q r^2}{2mr^2\omega} = \frac{q}{2m}$$

15. b. The wires are at A and B perpendicular to the plane of paper and current is toward the reader. Let us consider certain points.

Point C: The magnetic field at C due to A $(\vec{B}_{CA})$ is in upward direction but magnetic field at C due to B is in downward direction. Net field is zero.

Point E: Magnetic field due to A is upward and magnetic field due to B is downward but $|\vec{B}_{EA}| < |\vec{B}_{EB}|$

∴ Net magnetic field is in downward direction.

Point D: $|\overline{B}_{DA}| > |\overline{B}_{DB}| \Rightarrow$ Net field upward.

Similarly, other points can be considered.

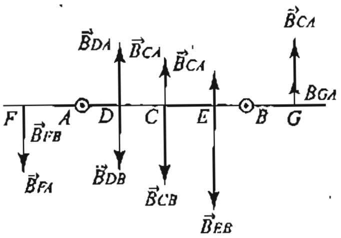


Fig. 9.530

16. c. Case 1: Magnetic field at M due to PQ and QR is

$$H_1 = \frac{1}{2}\left[\frac{\mu_0 I}{2\pi R}\right] + 0 = \frac{\mu_0 I}{4\pi R}$$

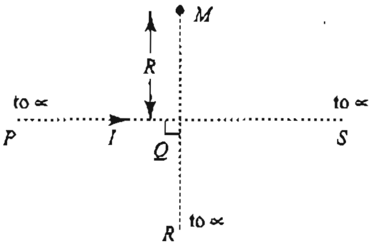


Fig. 9.531

Case 2: When wire Q is joined

H_2 = (Magnetic field at M due to PQ) + (Magnetic field at M due to QR) + (Magnetic field at M due to QS)

$$= \frac{1}{2}\left[\frac{\mu_0 I}{2\pi R}\right]+0+\frac{1}{2}\left[\frac{\mu_0 I/2}{2\pi R}\right]=\frac{3\mu_0 I}{8\pi R}$$

$$\therefore \quad \frac{H_1}{H_2}=\frac{2}{3}$$

Note:

The magnetic field due to a infinitely long wire carrying current at a distance R from the end point is half that at a distance R from the middle point.

17. b. Case of positively charged particle: Two forces are acting on the positively charged particle (i) force due to electric field in the positive x-direction, and (ii) force due to magnetic field.

$$\vec{F} = q(\vec{v}\times\vec{B})$$

$$\Rightarrow \quad \vec{F} = q\,[V\hat{i}\times B\hat{k}] \quad \Rightarrow \quad \vec{F} = qVB\,(\hat{i}\times\hat{k})$$

$$\Rightarrow \quad \vec{F} = qvB\,(-\hat{j})$$

This force will move the positively charged particle toward Y-axis.

Case of negatively charged particle:

Two forces are acting on the negatively charged particle
(i) due to electric field in the negative X-direction, and
(ii) due to magnetic field

$$\Rightarrow \quad \vec{F} = -q\,(\vec{v}\times\vec{B}) \quad \Rightarrow \quad \vec{F} = -q\,[v(-\hat{i})\times B(\hat{k})]$$

$$\Rightarrow \quad \vec{F} = q\,vB[\hat{i}\times\hat{k}] \quad \Rightarrow \quad \vec{F} = q\,vB\,(-\hat{j})$$

Same direction as that of positive charge. (b) is the correct answer.

18. d. If we take individual length for the purpose of calculating the magnetic field in a 3-dimensional figure, then it will be difficult.

Here, a smart choice is to divide the loop into two loops. One loop is $ADEFA$ in y–z plane and the other loop will be $ABCDA$ in the x–y plane.

We actually do not have any current in the segment AD. By choosing the loops, we find that in one loop we have to take current from A to D and in the other one from D to A. Hence, these two cancel out the effect of each other as far as creating magnetic field at the concerned point A is considered.

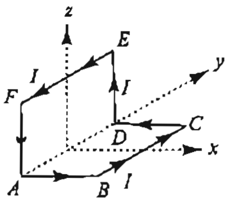


Fig. 9.532

The point $(a, 0, a)$ is in the X–Z plane.

The magnetic field due to current in $ABCDA$ will be in +ve z-direction. Due to symmetry, the y-components and x-components will cancel out each other.

Similarly, the magnetic field due to current in $ADEFA$ will be in x-direction.

$\therefore$ The resultant magnetic field will be $\vec{B} = \frac{1}{\sqrt{2}}(\hat{i}+\hat{k})$

Correct option is (d).

19. b. When a charged particle is moving at right angle to the magnetic field, then a force acts on it which behaves as a centripetal force and moves the particle in circular path.

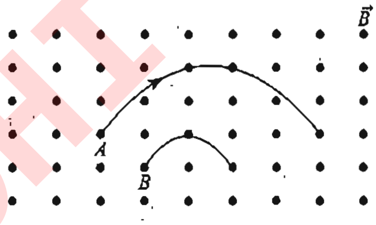


Fig. 9.533

$$\therefore \quad \frac{m_A v_A^2}{2r} = q v_A B$$

$$\therefore \quad \frac{m_A v_A}{2r} = q\,B$$

Similarly, for second particle moving with half radius as compared to the first one, we have

$$\frac{m_B v_B}{r} = q\,B$$

$$\Rightarrow \quad \frac{m_B v_B}{2r} = \frac{m_B v_B}{r} \quad \Rightarrow \quad m_A v_A = 2\,m_B v_B$$

$$\Rightarrow \quad m_A v_A > m_B v_B$$

$\therefore$ Correct option is (b).

20. a

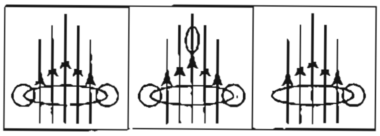


Fig. 9.534

Clearly, the flux linkage is maximum in case (a) due to the spatial arrangement of the two loops.

Correct option is (a).

21. **c.** Let us consider a thickness dx of wire. Let it be at a distance x from the center O.

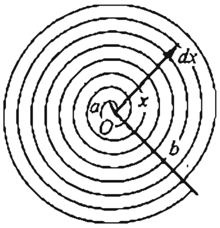


Fig. 9.535

Number of turns per unit length $= \dfrac{N}{b-a}$

Number of turns in thickness $dx = \dfrac{N}{b-a}dx$

Small amount of magnetic field produced at O due to thickness dx of the wire

$$dB = \frac{\mu_0}{2}\frac{NI}{(b-a)}\frac{dx}{x}$$

On integrating, we get

$$B = \int_a^b \frac{\mu_0}{2}\frac{NI}{(b-a)}\frac{dx}{x} = \frac{\mu_0}{2}\frac{NI}{(b-a)}\cdot\int_a^b \frac{dx}{x}$$

$$= \frac{\mu_0}{2}\frac{NI}{(b-a)}[\log_e x]_a^b$$

$$= \frac{\mu_0}{2}\frac{NI}{(b-a)}\log_e\frac{b}{a}$$

Correct option is (c).

22. **b.** Width of the magnetic field region $(b-a) \le R$; where 'R' is its radius of curvature inside magnetic field,

$$\therefore \quad R = \frac{mv}{qB}(b-a)$$

$$\Rightarrow \quad v_{\min} = \frac{(b-a)qB}{m}$$

(b) is the correct option.

23. **a.** Magnetic field $|\vec{B}| = \dfrac{\mu_0 I}{2\pi\sqrt{x^2+y^2}}$; Unit vector perpendicular to the position vector is $\dfrac{(y\hat{i}-x\hat{j})}{\sqrt{x^2+y^2}}$.

$$\therefore \quad \vec{B} = \frac{\mu_0 I}{2\pi(x^2+y^2)}(y\hat{i}-x\hat{j})$$

(a) is the correct option.

24. **d.** is the correct option. Magnetic lines of force form closed loops. Inside magnet, these are directed from south to north pole.

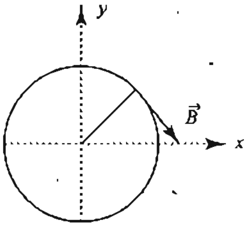


Fig. 9.536

25. **b.** The velocity at P is in the x-direction (given). Let $\vec{V} = m\hat{i}$. After P, the positively charged particle gets deflected in the x–y plane toward $-y$-direction and the path is non-circular.

Now, $\vec{F} = q(\vec{V}\times\vec{B})$

$$= q[m\hat{i}\times(c\hat{k}+a\hat{i})] \qquad \text{[for option (b)]}$$

$$= q[mc\hat{i}\times\hat{k}+ma\hat{i}\times\hat{i}] = mcq(-\hat{j})$$

Since in option (b) electric field is also present, i.e., $\overline{E} = a\hat{i}$, therefore it will also exert a force in the $+x$-direction. The net result of the two forces will be a non-circular path.

Only option (b) fits for the above logic. For other options, we get some other results.

26. **b.** Using Fleming's left hand rule, we find that a force is acting in the radically outward direction throughout the circumference of the conducting loop.

27. **a.** $U = -\vec{M}\cdot\vec{B}$

$= -MB\cos\theta$

In case I: $\theta = 180°$, $U = +MB$

In case II: $\theta = 180°$, $U = 0$

In case III: θ is acute, $U = +$ve (less than $+MB$)

In case IV: θ is obtuse, $U = -$ve

$\therefore \quad$ I > III > II > IV

28. **b.** The force acting on the electron will be perpendicular to the direction of velocity till the electron remains is the magnetic field. So, the electron will follow the path as given.

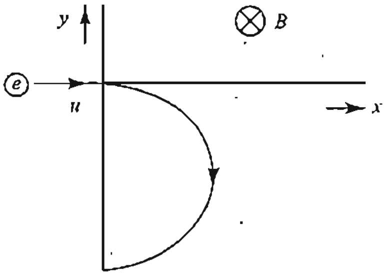


Fig. 9.537

29. **a.** $\bar{F}_m = q(\bar{v}\times\bar{B})$

30. **c.** $$B = -\frac{\mu_0 I}{4\pi\frac{12x}{5}}[\cos 53° + \cos 37°] = 7\left(\frac{\mu_0 I}{48\pi x}\right)$$

$\therefore \quad k = 7$

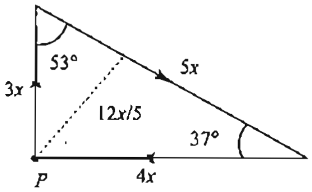


Fig. 9.538

Multiple Correct Answers Type

1. **a., b., d.**

There is no change in velocity. It can be possible when electric and magnetic fields are absent, i.e., $E = 0$, $B = 0$.

Or when electric and magnetic fields are present but force due to electric field is equal and opposite to the force due to magnetic field, (i.e., $E \neq 0$, $B \neq 0$).

Or when $E = 0$, but $B \neq 0$.

$$F = q\,vB\ \sin\theta, \text{ i.e.,}$$

$\sin\theta = 0$, i.e., $\theta = 0 \Rightarrow v$ and B are in the same direction.

2. **a., b., d.**

Considering the activity from P to Q (horizontal)

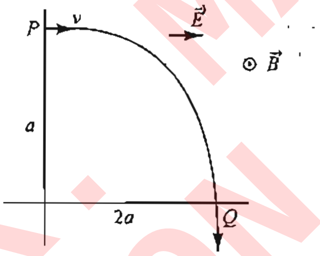


Fig. 9.539

$u = v$, $v = 2v$, $s = 2a$, Acc = ?

$$v^2 - u^2 = 2as$$

$$\Rightarrow 4v^2 - v^2 = 2A(2a) \Rightarrow A = \frac{3v^2}{4a}$$

Force acting on the horizontal direction is

$$F = qE = mA \Rightarrow E = \frac{mA}{q} = \frac{3}{4}\left[\frac{mv^2}{a}\right]$$

Rate of doing work at P = power

$$= F \times v = mA \times v = \frac{3}{4}\left[\frac{mv^3}{a}\right]$$

Rate of doing work by the magnetic field is throughout zero. The rate of doing work by electric field is zero at Q. Because at Q, the angle between force due to electric field and displacement is zero.

∴ (a), (b) and (d) are correct options.

3. **b., c.** $V = I_g(G + R)$

$$= 5 \times 10^{-5}[100 + 200{,}000] = 10\,\text{V}$$

$$I = I_g\left(\frac{G}{S} + 1\right) = 5 \times 10^{-5}\left[\frac{100}{1} + 1\right] = 5\,\text{mA}$$

∴ (b) and (c) are correct options.

4. **a., c.** When the changed particles enter a magnetic field, then a force acts on the particle which will acts as a centripetal force.

$$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB} \qquad \text{(i)}$$

Now, $$E = \frac{1}{2}mv^2 \Rightarrow v = \frac{\sqrt{2E}}{m} \qquad \text{(ii)}$$

$$\therefore \quad r = \frac{m}{qB}\sqrt{\frac{2E}{m}} = \frac{\sqrt{2Em}}{qB} = \frac{\sqrt{2E}}{B} \times \frac{\sqrt{m}}{q}$$

$$\Rightarrow \quad r \propto \frac{\sqrt{m}}{q}$$

$$\therefore \quad r_{H^+} \propto \frac{\sqrt{1}}{1};\ r_{He^+} \propto \frac{\sqrt{4}}{1};\ r_{O^+} \propto \frac{\sqrt{16}}{2}$$

$$\Rightarrow \quad r_{H^+} \propto 1;\ r_{He^+} \propto 2;\ r_{O^+} \propto 2$$

⇒ He$^+$ and O$^+$ will be deflected equally.

H$^+$ will be deflected the most since its radius is smallest.

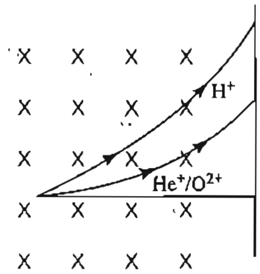


Fig. 9.540

∴ (a), (c) are correct options.

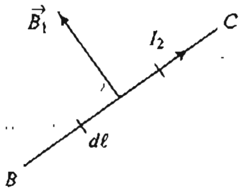
5. **c., d.** Out of the given options only induced electric field and magnetostatic field form closed loops of field lines.

6. **a., c.** Net force on the loop:

Force on AB: The magnetic field due to current I_1 is along AB.

$dF = I\,(dl \times B \times \sin 0°) = 0$

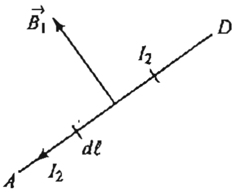
Force on CD: Similarly, the magnetic field due to current I_1 is along DC. Because here $\theta = 180°$. Therefore, force on DC is zero.



Force on BC: Consider a small element dl

$dF = I_2 dl B_1 \sin 90° \Rightarrow dF = I_2\, dl B_1$

By Fleming's left hand rule, the direction of this force is perpendicular to the plane of the paper directed outwards. force on AD: $dF = I_2 dlB_1 \sin 90° = I_2\, dlB_1$



By Fleming's left hand rule, the direction of this force is perpendicular to the plane of paper directed inwards. Since the current elements are located symmetrical to current I_1, therefore force on BC will cancel out the effect of force on AD.

$\Rightarrow$ Net force on loop $ABCD$ is zero.

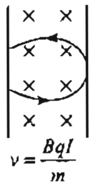
Net torque on the loop. The force on BC and AD will create a torque on $ABCD$ in clockwise direction about OO' as seen by the observer at O. Gravitational and electrostatic fields do not form closed loops.

7. **a., c., d.** $\vec{v} \perp \vec{B}$ in region II. Therefore, path of particle is circle in region II. Particle enters in region III, if radius of circular path, $r > l$

or $\quad \dfrac{mv}{Bq} > l$

or $\quad v > \dfrac{Bql}{m}$

If $v = \dfrac{Bql}{m}$, $r = \dfrac{mv}{Bq} = l$, particle will turn back and path length will be maximum. If particle returns to region I, time spent in region II will be:



$$t = \frac{T}{2} = \frac{pm}{Bq}\text{, which is independent of } v$$

$\therefore$ Corrent options are (a), (c) and (d).

$$\vec{F} = I\left[(L\hat{i}) \times (B\hat{j})\right] = ILB\hat{k}$$

$\therefore$ Magnitude of force is ILB and direction of force is positive z.

Assertion-Reasoning Type

1. **c.** $c\phi = BINA$

$$\phi = \left(\frac{BNA}{c}\right)I$$

Using iron core, value of magnetic field increases. So, deflection increases for the same current. Hence, sensitivity increases.

Soft iron can be easily magnetized or demagnetized.

$\therefore$ correct option is (c).

Matching Column Type

1. i. → a., ii. → a., b., d., iii. → b., d., iv. → b., c.
2. i. → b., ii. → c., d., iii. → d., iv. → a., b., c.
3. i. → b., c., ii. → a., iii. → b., c., iv. → b., d.
4. i. → a., c., d., ii. → c., d., iii. → a., b., e., iv. → c., d.

CHAPTER
10

Alternating Current

- Alternative Current and Voltage
- Phasor Diagrams
- Average or Mean Value of Alternating Current
- Root Mean Square (RMS) Values
- Resistance and Reactance
- Resistor in an AC Circuit
- Inductor in an AC Circuit
- Capacitor in an AC Circuit
- Resistor and Capacitor in an AC Circuit
- Comparing AC Circuit Elements
- L-R-C Series Circuit
- Meaning of Impedance and Phase Angle
- Power in Alternating-Current Circuits
- Power in a Resistor
- Power in a General AC Circuit
- Choke Coil
- Circuit Behavior at Resonance
- Transformers
- How Transformers Work

ALTERNATIVE CURRENT AND VOLTAGE

A time varying current or voltage according to its law of variation may be periodic or non-periodic. In case of periodic current or voltage, the current or voltage is said to be *alternating* if:

a. its amplitude is constant, and

b. alternative half cycle is positive and half negative.

This all is illustrated in Fig. 10.1.

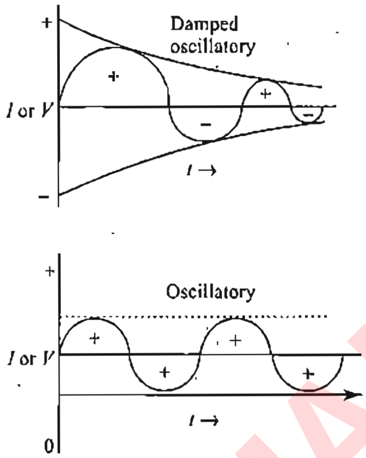


Fig. 10.1

If the current or voltage varies periodically as 'sin' or 'cos' function of time, the current or voltage is said to be sinusoidal (Fig. 10.2).

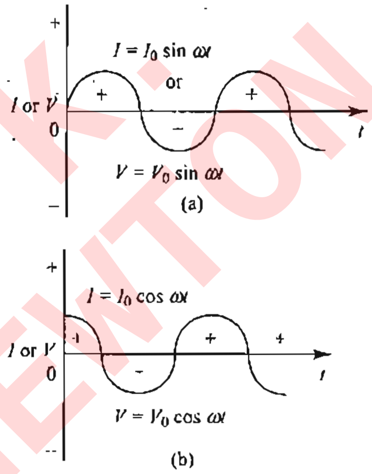


Fig. 10.2

We use the term ac source for any device that supplies a sinusoidally varying voltage (potential difference) v or current *i*. The usual circuit-diagram symbol for an ac source is shown in Fig. 10.3

Fig. 10.3

A sinusoidal voltage might be described by a function such as

$$v = V \cos \omega t \quad \text{or} \quad v = V \sin \omega t \qquad (1)$$

In this expression, v (lowercase) is the instantaneous potential difference, V (uppercase) is the maximum potential difference, which we call the voltage amplitude; and ω is the angular frequency, which is equal to 2π times the frequency f.

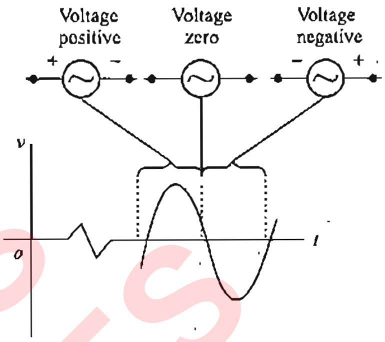


Fig. 10.4

$$i = I \cos \omega t \quad \text{or} \quad i = I \sin \omega t \qquad (2)$$

where i (lowercase) is the instantaneous current and I (uppercase) is the maximum current or current amplitude.

Points to Remember

- It is produced by a dynamo or an electronic oscillator.
- The frequency of ac in India is 50 Hz, i.e., f = 50 Hz so $\omega = 2\pi f = 100\pi$ rad/s.
- The ac can be converted into dc with the help of rectifier while dc into ac with the help of an inverter.
- It cannot produce chemical effects of current such as electroplating or electrolysis as due to large ions cannot follow the frequency of ac.
- It can be stepped up or down with the help of trans-former (while dc cannot be).
- Alternating current is measured by hot wire instruments.

PHASOR DIAGRAMS

To represent sinusoidally varying voltages and currents, we will use rotating vector diagrams similar to those we used in the study of simple harmonic motion. In these diagrams the instantaneous value of a quantity that varies sinusoidally with time is represented by the projection onto a horizontal axis of a vector with a length equal to the amplitude of the quantity. The vector rotates counterclockwise with constant angular speed ω. These rotating vectors are called phasors. The projection of the phasor onto the horizontal axis at time t is $I \cos \omega t$; this is why we chose to use the cosine function rather than the sine (see Fig. 10.5).

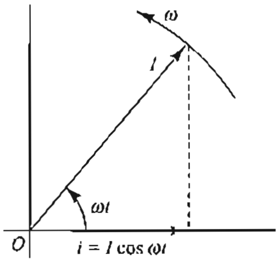


Fig. 10.5

Note:

A phasor is not a real physical quantity with a direction in space, such as velocity, momentum, or electric field. Rather, it is a geometric entity that helps us to describe and analyze physical quantities that vary sinusoidally with time. In a simple harmonic motion we use a single phasor to represent the position of a point mass undergoing simple harmonic motion. In this chapter we will use phasor to add sinusoidal voltages and currents. Combining sinusoidal quantities with the phase differences then becomes a matter of vector addition.

AVERAGE OR MEAN VALUE OF ALTERNATING CURRENT

If the average or mean value of alternating current or voltage is defined for full cycle, it will be zero as $\int_0^T \sin\omega t\, dt$ or $\int_0^T \cos\omega t\, dt = 0$ so it is defined for positive (or negative) half cycle as

$$\mathrm{I_{av}\ or\ I_{mean}} = \frac{\int_0^{T/2} I dt}{\int_0^{T/2} dt} = \frac{\int_0^{\pi/\omega} I_0 \sin\omega t\, dt}{\int_0^{\pi/\omega} dt} = \frac{2}{\pi} I_0$$

$$\Rightarrow \qquad \mathrm{I_{av}} = \frac{2I_0}{\pi}$$

ROOT MEAN SQUARE (RMS) VALUES

A more useful way to describe a quantity that can be either positive or negative I the root mean square (rms) value. We used rms values in kinetic theory of the gases in connection with the speeds of molecules in a gas. We square the instantaneous current i, take the average (mean) value of i^2, and finally take the square root of that average. This procedure defines the root mean square current, denoted as $I_{\rm rms}$. Even when i is negative, i^2 is always positive, so $I_{\rm rms}$ is never zero (unless i is zero at every instant).

Here's how we obtain $I_{\rm rms}$ for a sinusoidal current, like that shown in Fig. 10.6, if the instantaneous current is given by $i = I\cos\omega t$, then

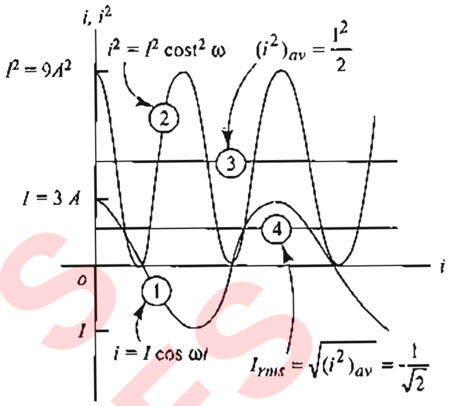
$$i^2 = I^2 \cos^2\omega t \qquad (3)$$

But we have, $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

We find $\quad i^2 = I^2\frac{1}{2}(1 + \cos 2\omega t) = \frac{1}{2}I^2 + \frac{1}{2}I^2\cos 2\omega t$

The average of $\cos 2\omega t$ is zero because it is positive half the time and negative half the time. Thus the average of i^2 is simply $I^2/2$. The square root of this is $I_{\rm rms}$:

$$I_{\rm rms} = \frac{I}{\sqrt{2}} \quad \text{(rms value of a sinusoidal current)} \quad (4)$$



Meaning of the rms value of a sinusoidal quantity (here, ac current with $I = 3$ A)

(1) Graph current i versus time
(2) Square the instantaneous current i
(3) Take the average (mean) value of i^2
(4) Take the square root of that average.

Fig. 10.6

In the same way the rms value of sinusoidal voltage with amplitude (maximum value) V is

$$V_{\rm rms} = \frac{V}{\sqrt{2}} \quad \text{(rms value of a sinusoidal voltage)} \quad (5)$$

Note:

Effective, virtual, or rms value of alternating current is defined as the square root of the average of I^2 during a complete cycle, i.e.,

$$I_{\rm rms} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right] = \left[\frac{\int_0^{2\pi/\omega} I_0^2 \sin^2\omega t dt}{\int_0^{2\pi/\omega} dt}\right];$$

$$I_{\rm rms} = \frac{I_0}{\sqrt{2}} \qquad \left[\text{as } \int_0^T \sin^2\omega t\, dt = \frac{T}{2}\right]$$

All ac instruments read this value, e.g., if we speak about 220 V alternating voltage we mean $V_{\rm rms} = 220$ V.

- *As* $\quad V_{\rm av} = \frac{2}{\pi}V_0 \quad$ *and* $\quad V_{\rm rms} = \frac{V_0}{\sqrt{2}}$

  $V_0 > V_{\rm rms} > V_{\rm av} \quad$ *and* $\quad V_0 = 1.44\, V_{\rm rms}$

  So, if $V_{\rm rms} = 220$ *V,* $\quad V_0 = \sqrt{2} \times 220 \simeq 311V$

  and $\quad V_{\rm av} = 0.9 \times 220 = 198$ *V*

- *So, for sinusoidal, ac* $= \frac{I_0}{\sqrt{2}} \times \frac{\pi}{2I_0} = \frac{\pi}{2\sqrt{2}}$

Example 10.1 **The plate on the back of a personal computer says that it draws 2.7A from a 120-V, 60 Hz line. For this computer, what is (a) the average of the square of the current, (b) the current amplitude, and (c) the average current?**

Sol.

a. The current given is the rms value: $I_{rms} = 2.7$ A. The target variable $(i^2)_{av}$ is the mean of the square of the current. The rms current is the square root of this target variable, so

$$I_{rms} = \sqrt{(i^2)_{av}} \text{ or } (i^2)_{av} = (I_{rms})^2 = (2.7 \text{ A})^2 = 7.3 \text{ A}^2$$

b. From the above eq. the current amplitude I is

$$I = \sqrt{2}\, I_{rms} = \sqrt{2}\,(2.7 \text{ A}) = 3.8 \text{ A}$$

c. We know the rms current, given by eq. ($I_{rms} = \frac{I}{\sqrt{2}}$), is the square root of the mean (average) of the square of the current. The average of any sinusoidal alternating current, over any whole number of cycles, is zero. We define the average for positive or negative half cycle $I_{av} = \frac{2I_0}{\pi} = \frac{2 \times 3.8}{\pi} = 3.42$ A

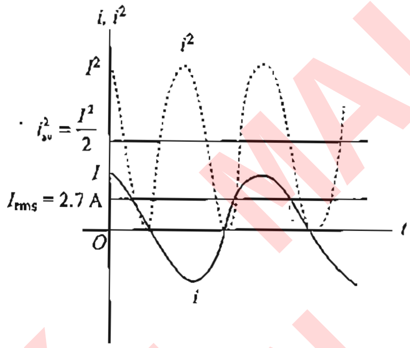


Fig. 10.7

Example 10.2 **An alternating current is given by the following equation: $I = 3\sqrt{2}\sin(100\pi t + \pi/4)$. Give the frequency and mean value of the current.**

Sol. $I = 3\sqrt{2}\sin(100\pi t + \pi/4)$

$$I_{mean} = I_{peak}/\sqrt{2} = 3\sqrt{2}/\sqrt{2} \text{ A} = 3 \text{ A}$$

and $2\pi f = 100\pi \Rightarrow f = 100\pi/\pi = 50$ Hz

Frequency $(f) = 50$ Hz

Example 10.3 **A voltage, $E = 60 \sin 314\, t$, is applied across a resistor. What will be the reading of I_{rms}**

i. in ac ammeter?

ii. ordinary moving coil ammeter in series with the resistor read?

Sol. Given that $E = 60 \sin 314\, t$

i. An ac ammeter will read the rms value

$$E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{60}{\sqrt{2}} = 42.4 \text{ V}$$

$$I_{rms} = \frac{E_v}{R} = \frac{42.4}{20} = 2.12 \text{ A}$$

Therefore ac ammeter will read 2.12 A.

ii. An ordinary moving coil ammeter will read the average value of alternating current. Since the average value of ac over one cycle is zero, this meter will give zero reading.

RESISTANCE AND REACTANCE

In this section we will derive voltage–current relationships for individual circuit elements carrying a sinusoidal current. We will consider resistors, inductors, and capacitors.

RESISTOR IN AN AC CIRCUIT

First let us consider a resistor with resistance R through which there is a sinusoidal current $i = I\cos\omega t$. The positive direction of current is counterclockwise around the circuit, as in Fig. 10.8 (a), the current amplitude (maximum current) is I. From Ohm's law the instantaneous potential v_R of point a with respect to point b (that is, the instantaneous voltage across the resistor) is

$$v_R = iR = (IR)\cos\omega t \qquad (6)$$

The maximum voltage V_R, the voltage amplitude, is the coefficient of the cosine function:

$$V_R = I_R$$

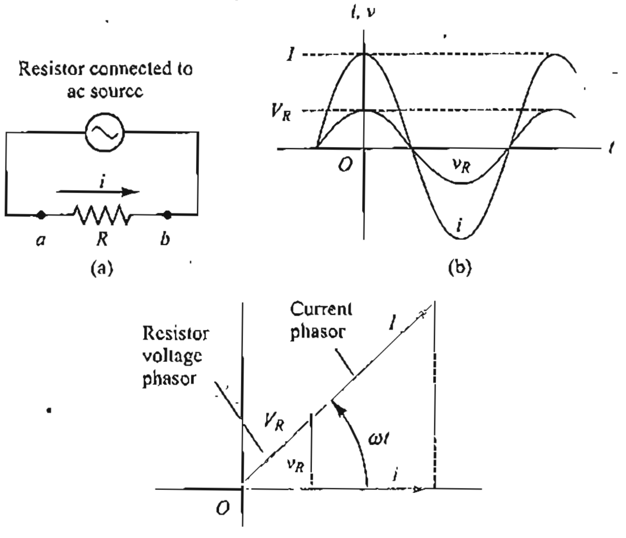
(amplitude of voltage across a resistor, ac circuit) (7)

Hence from equations (i) and (ii), we can also write

$$v_R = V_R \cos\omega t \qquad (8)$$

The current i and voltage v_R are both proportional to cos ωt, so the current is in phase with the voltage. Equation (7) shows that the current and voltage amplitude are related in the same way as in a dc circuit.

Fig. 10.8 (b) shows graphs of i and v_R as the functions of time. The vertical scales for current and voltage are different, so the relative heights of the two curves are not significant. The corresponding phasor diagram is given in Fig. 10.8 (c). Because i and v_R are in phase and have the same frequency, the current and voltage phasors rotate together; they are parallel at each instant. Their projections on the horizontal axis represent the instantaneous current and voltage, respectively.



(a) (b)

(c)

Fig. 10.8

INDUCTOR IN AN AC CIRCUIT

We replace the resistor used in the previous section and place a pure inductor with self-inductance L and zero resistance [Fig. 10.9 (a)]. Again, we assume that the current is $i = I\cos\omega t$, with the positive direction of current taken as counterclockwise around the circuit.

Although there is no resistance, there is a potential difference v_L between the inductor terminals a and b because the current varies with time. Given by $\varepsilon = -L\,di/dt$; however, the voltage v_L is not simply equal to ε. To see why, notice that if the current in the inductor is in the positive (counterclockwise) direction from a to b and is increasing, then di/dt is positive and the induced e.m.f. is directed to the left to oppose the increase in current. Hence, point a is at a higher potential than point b. Thus, the potential of point with respect to point b is positive and given by $v_L = +L\,di/dt$, the negative of the induced e.m.f. (You should convience yourself that this expression gives the correct sign of v_L in all cases, including i counterclockwise and decreasing, i clockwise and increasing, and i clockwise and decreasing);

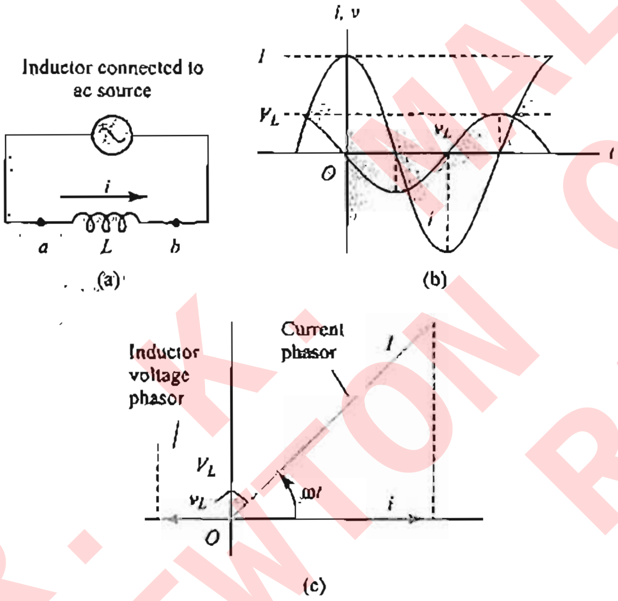


Fig. 10.9

So we have

$$V_L = L\frac{di}{dt} = L\frac{d}{dt}(I\cos\omega t) = -I\omega L\sin\omega t \qquad (9)$$

The voltage v_L across the inductor at any instant is proportional to the rate of change of the current. The points of maximum voltage on the graph correspond to maximum steepness of the current curve, and the points of zero voltage are the points where the current curve instantaneously levels off at its maximum and minimum values [Fig. 10.9 (b)]. The voltage and the current are "out of step" or out of phase by a quarter cycle. Since the voltage peaks occur a quarter cycle earlier than the current peaks, we say that the voltage leads the current by 90°. The phasor diagram in [Fig. 10.9 (c)] also shows this relationship; the voltage phasor is ahead of the current phasor by 90°.

We can also obtain this phase relationship by rewriting Eq. (9) using the identity $\cos(A + 90°) = -\sin A$

$$v_L = I\omega L\cos(\omega t + 90°) \qquad (10)$$

This result shows that the voltage can be viewed as a cosine function with a "head start" of 90° relative to the current.

As we have done in Eq. (10) we will usually describe the phase of the voltage relative to the current, not the reverse. Thus if the current i in a circuit is

$$i = I\cos\omega t$$

and the voltage v of one point with respect to another is

$$v = V\cos(\omega t + \phi)$$

We call ϕ the phase angle; it gives the phase of the voltage relative to the current. For a pure resistor $\phi = 0$, and for a pure inductor, $\phi = 90°$

From Eq. (9) and (10), the amplitude V_L of the inductor voltage is

$$V_L = I\omega L \qquad (11)$$

We define the inductive reactance X_L of an inductor as

$$X_L = \omega L \text{ (inductive reactance)} \qquad (12)$$

Using X_L, we can write Eq. (11) in a form similar to equation for a resistor ($V_R = IR$):

$$V_L = IX_L \qquad (13)$$

(amplitude of voltage across an inductor, ac circuit)

Because X_L is the ratio of a voltage and a current, its SI unit is Ohm, the same as for resistance.

Meaning of Inductive Reactance

The inductive reactance X_L is really a description of the self-inducted e.m.f that opposes any change in the current through the inductor. From equation for a given current amplitude I the voltage $v_L = +L\,di/dt$ across the inductor and the self-inducted emf $\varepsilon = -L\,di/dt$ both have an amplitude V_L that is directly proportional to X_L. According to Eq. (12), the inductive reactance and the self induced emf increase with more rapid variation in current (that is, increasing angular frequency, ω) and increasing inductance L.

If an oscillating voltage of a given amplitude V_L is applied across the inductor terminals, the resulting current will have a smaller amplitude I for larger values of X_L, since X_L is proportional to frequency, a high-frequency voltage applied to the inductor gives only a small current, while a lower-frequency voltage of the same amplitude gives rise to a larger current. Inductors are used in some circuit applications, such as power supplies and radio-interference filters, to block high frequencies while permitting lower frequencies or dc to pass through. A circuit device that uses an inductor for this purpose is called a *low-pass filter*.

Example 10.4 **Suppose you want the current amplitude in a pure inductor in a radio receiver to be 250 μA when the voltage amplitude is 3.60 V at a frequency of 1.60 MHz (corresponding to the upper is 3.60 V AM broadcast band).**

a. **What inductive reactance is needed? What inductance is required?**

b. **If the voltage amplitude through this inductor at 16.0 MHz?**

Sol. We are not told about any other elements of the circuit of which the inductor is a part of. We should also not care about those other elements, since from the perspective of this example, all they do is provide the inductor with an oscillating voltage.

Set up: We are given the current amplitude, I, and the voltage amplitude, V. Our target variables in part (a) are the inductive reactance X_L at 1.60 MHz and the inductance L. Once we know L, we use these same two equations to find the inductive reactance and current amplitude at any other frequency.

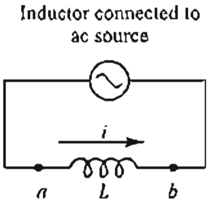


Fig. 10.10

Execute:

a. The inductive reactance

$$X_L = \frac{V_L}{I} = \frac{3.60\text{ V}}{250\times10^{-6}\text{ A}} = 1.44\times10^4\,\Omega = 14.4\text{ k}\Omega$$

But $\omega = 2\pi f$, we find

$$L = \frac{X_L}{2\pi f} = \frac{1.44\times10^4\,\Omega}{2\pi(1.60\times10^6\text{ Hz})}$$

$$= 1.43\times10^{-3}\text{ H} = 1.43\text{ mH}$$

b. We know that the current amplitude is $I = V_L/X_L = V_L/\omega L = V_L/2\pi fL$. Thus, the current amplitude is inversely proportional to the frequency f. Since $I = 250$ μA at $f = 1.60$ MHz, the current amplitude at 16.0 MHz (ten times the original frequency) will be one-tenth as great, or 2500 μA = 2.50 mA.

In general, the lower the frequency of an oscillating voltage applied across an inductor, the greater the amplitude of the oscillating current that results.

Example 10.5 **A pure inductance of 1.0 H is connected across a 110 V, 70 Hz source. Find the reactance, current, and the peak value of current.**

Sol. $L = 1.0$ H, $V = 110$ V, $f = 70$ Hz.

i. Reactance = $\omega L = 2\pi\times 70\times 1 = 439.6\ \Omega$

ii. Current = 110/439.6 = 0.25 A

iii. Peak value of current = 0.25 × 1.414 = 0.353 A

CAPACITOR IN AN AC CIRCUIT

We connect a capacitor with capacitance C to a source as shown in Fig. 10.11(a) producing a current $i = I\cos\,\omega t$ through the capacitor. Again, the positive direction of current is counterclockwise around the circuit.

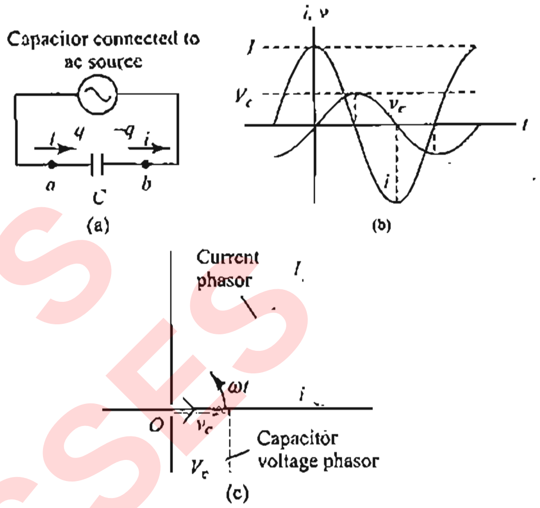


Fig. 10.11 Phasor Diagram

Caution

Alternating current through a capacitor: You may object that the charge cannot really move through the capacitor because its two plates are insulated from each other. True enough, but as the capacitor charges and discharges, there is at each instant a current i into one plate, an equal current out of the other plate, and an equal displacement current between the plates just as though the charge were being conducted through the capacitor. Thus, we often speak about alternating current through a capacitor.

To find the instantaneous voltage v_C across the capacitor, that is, the potential of point a with respect to point b – w first let q denote the charge on the left-hand plate of the capacitor in Fig. 10.11 (a) (so q is the charge on the right hand plate). The current i is related to q by $i = dq/dt$; with this definition, positive current corresponds to an increasing charge on the left-hand capacitor plate. Then

$$i = \frac{dq}{dt} = I\cos\omega t$$

Integrating this, we get

$$q = \frac{I}{\omega}\sin\omega t \qquad (14)$$

Also from $C = Q/v_{ab}$,the charge q equals the voltage v_C multiple by the capacitance $q = C\,v_C$. Using this in Eq. (14) we find

$$v_C = \frac{1}{\omega C}\sin\omega t \qquad (15)$$

The instantaneous current i is equal to the rate of change dq/dt of the capacitor charge q; since $q = Cv_C$, i is also proportional to the rate of change of voltage. (Compare to an inductor, for which the situation is reversed and v_L is proportional to the rate of change of I.) Fig. 10.11 (b) shows v_C and i as functions of t. Because $i = dq/dt = C\,d\,v_C/dt$, the current has its greatest magnitude when the v_C curve is rising or falling most steeply and is zero when the v_C curve instantaneously levels off at its maximum and minimum values.

The capacitor voltage and the current are out of phase by a quarter cycle. The peaks of voltage occur quarter cycle after the corresponding current peaks. And we say that the voltage phasor is behind the current phasor by a quarter-cycle or 90°.

We can also derive this phase difference by rewriting Eq. (15) using the identify cos $(A - 90°) = \sin A$

$$v_C = \frac{1}{\omega C}\cos(\omega t - 90°) \qquad (16)$$

This corresponds to a phase angle $\phi = 90°$. This cosine function has a "late start" of 90° compared with the current $i = I\cos\omega t$

Equations (15) and (16) show that the maximum voltage V_C (the voltage amplitude) is

$$V_C = \frac{I}{\omega C} \qquad (17)$$

To put this expression in a form similar to Eq. (7), $V_R = I_R$, for a resistor $V_R = IR$, we define a quantity X_C, called the capacitive reactance of the capacitor, as

$$X_C = \frac{1}{\omega C} \text{ (capacitive reactance)} \qquad (18)$$

Then

$$V_C = IX_C \text{ (amplitude of voltage across a capacitor, ac circuit)} \qquad (19)$$

The SI unit of X_C is Ohm, the same as for the resistance and inductive reactance, because X_C is the ratio of a voltage and a current.

Meaning of Capacitive Reactance

The capacitive reactance of a capacitor is inversely proportional both to the capacitance C and to the angular frequency 'ω'. The greater the capacitance and the higher the frequency, the smaller the capacitive reactance, X_C. Capacitors tend to pass high-frequency current and to block low-frequency currents and dc, just the opposite of inductors. A device that preferentially passes signals of high frequency is called a high-pass filter.

RESISTOR AND CAPACITOR IN AN AC CIRCUIT

Example 10.6 **A 200 Ω resistor is connected in series with a 5.0 μF capacitor. The voltage across the resistor is v_R = (1.20 V) cos (2500 rad/s)t.**

- **a. Derive an expression for the circuit current.**
- **b. Determine the capacitive reactance of the capacitor.**
- **c. Derive an expression for the voltage across the capacitor.**

Sol. This is a series circuit. The current is the same through the capacitor as through the resistor. Our target variables are the current i, capacitive reactance X_C, and capacitor voltage v_C.

Set up: Figure 10.12 shows the circuit. We find the current through the resistor, and hence through the circuit as a whole. We use Eq. (18) to find the capacitive reactance X_C, Eq. (19) to find the voltage amplitude, and Eq. (16) to write an expression for the instantaneous voltage across the capacitor.

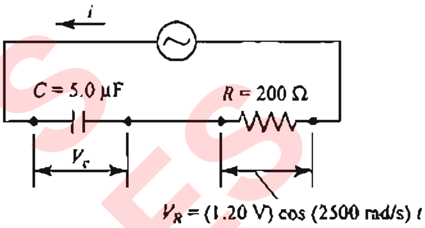


Fig. 10.12

Execute:

a. Using $v_R = iR$, we find that the current i in the resistor and through the circuit as a whole is

$$i = \frac{vR}{R} = \frac{(1.20\text{ V})\cos(2500\text{ rad/s})t}{200\,\Omega}$$
$$= (6.0 \times 10^{-3}\text{A})\cos(2500\text{ rad/s})t$$

b. From Eq. (18) the capacitive reactance at ω = 2500 rad/s is X_C

$$= \frac{1}{\omega C} = \frac{1}{(2500\text{ rad/s})(5.0\times10^{-6}\text{ F})} = 80\,\Omega$$

c. From Eq. (19), the amplitude V_C of the voltage across the capacitor is

$$V_C = IX_C = (6.0\times10^{-3}\text{A})\,(80\Omega) = 0.48\text{ V}$$

The 80 Ω reactance of the capacitor is 40% of the resistor's 200 Ω resistance, so the value of V_C is 40% of V_R. The instantaneous capacitor voltage v_C is given by Eq. (16)

$$v_C = v_C\cos(\omega t - 90°)$$
$$= (0.48\text{ V})\cos[(2500\text{ rad/s})t - \pi/2\text{ rad}]$$

Note:

Although the current through the capacitor is the same as through the resistor, the voltages across these two devices are different in both the amplitude and the phase. Note that in the expression for n_C we converted the 90° to p/2 rad so that all the angular quantities have the same units. In an ac circuit analysis, phase angles are often given in degrees, so be careful to convert to radians when necessary.

COMPARING AC CIRCUIT ELEMENTS

Table 10.1 summerizes the relationships of voltage and current amplitude for the three circuit elements we have discussed. Note again that the instantaneous voltage and the current are proportional in a resistor, where there is zero phase difference between v_R and i [see Fig. 10.13 (b)]. The instantaneous voltage and current are not proportional in an inductor or a capacitor, because there is a 90° phase difference in both the cases.

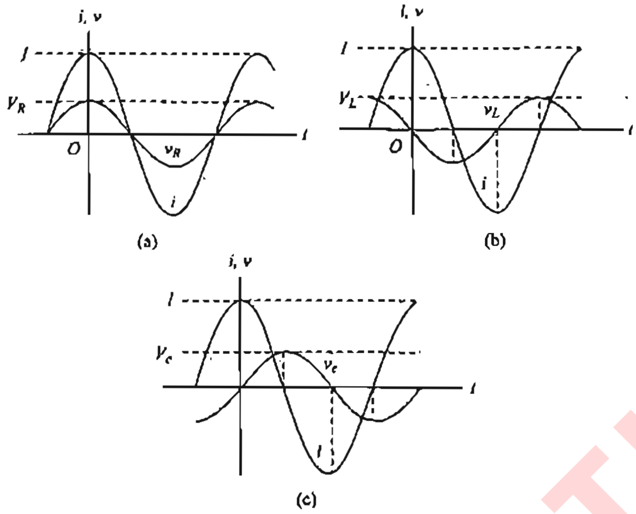


Fig. 10.13

Table 10.1 Circuit Elements with Alternating Current

Circuit Element	Amplitude Relationship	Circuit Quantity	Phase of v
Resistor	$V_R = IR$	R	In phase with i
Inductor	$V_L = IX_L$	$X_L = -\omega L$	Leads i by 90°
Capacitor	$V_C = IX_C$	$X_C = 1/\omega C$	Lags i by 90°

Figure 10.14 shows how the resistance of a resistor and the reactances of an inductor and a capacitor vary with angular frequency, ω. Resistance R is independent of frequency, while the reactances X_L and X_C are not. If $\omega = 0$, corresponding to a dc circuit, there is no current through a capacitor because $X_C \to \infty$, and there is no inductive effect because $X_L = 0$. In the limit $\omega \to \infty$, X_L also approaches infinity, and the current through an inductor becomes vanishingly small; recall that the self-induced e.m.f opposes rapid changes in the current. In this same limit, X_C and the voltage across a capacitor both approach zero; the current changes direction so rapidly that no charge can build up on either plate.

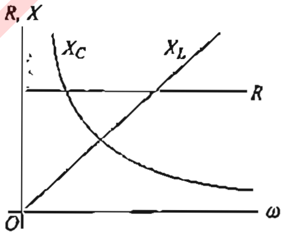


Fig. 10.14

L-R-C SERIES CIRCUIT

Many ac circuits used in practical electronic systems involve resistance, inductive reactance, and capacitive reactance. A simple example is a series circuit containing a resistor, an inductor, a capacitor, and an ac source, as shown in Fig. 10.15 (a).

To analyze this and similar circuits, we will use a phasor diagram that includes the voltage and the current phasors for each of the components. In this circuit, because of Kirchhoff's loop rule, the instantaneous total voltage v_{ad} across all the three components is equal to the source voltage at that instant. We will show that the phasor representing this total voltage is the vector sum of the phasors for the individual voltages. Complete phasor diagrams for this circuit are shown in Figs. 10.15 (b) and (c). These may appear complex, but we will explain them one step at a time.

Let us assume that the source supplies a current i given by $i = I\cos\omega t$. Because the circuit elements are connected in series, the current at any instant is the same at every point in the circuit. Thus, a single phasor I, with length proportional to the current amplitude, represents the current in all circuit elements.

We use the symbols v_R, v_L and v_C for the instantaneous voltages across R, L, C, and the symbols V_R, V_L, and V_C for the maximum voltages. We denote the instantaneous and maximum source voltages by v and V. Then, in Fig. 10.15 (a), $v = v_{ad}$, $v_R = v_{ab}$, $v_L = v_{bc}$ and $v_C = v_{cd}$.

We have shown that the potential difference between the terminals of a resistor is in phase with the current in the resistor and its maximum value V_R is given by Eq. (7).

$$V_R = I_R$$

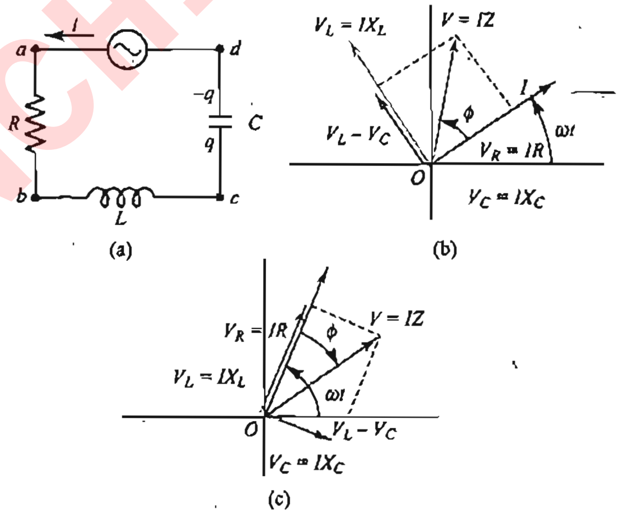


Fig. 10.15

The phasor V_R in Fig. 10.15(b), in phase with the current phasor I, represents the voltage across the resistor. Its projection onto the horizontal axis at any instant gives the instantaneous potential difference v_R.

The voltage across an inductor leads the current by 90°. Its voltage amplitude is given by Eq. (13):

$$V_L = IX_L$$

The phasor V_L in Fig. 10.15 (b) represents the voltage across the inductor, and its projection onto the horizontal axis at any instant equals v_L.

The voltage across a capacitor lags the current by 90°. Its voltage amplitude is given by Eq. (19):

$$V_C = IX_C$$

The phasor V_C in Fig. 10.15 (b) represents the voltage across the capacitor, and its projection onto the horizontal axis at any instant equals v_C.

The instantaneous potential difference v between the terminals a and d is equal at every instant to the (algebraic) sum of the potential differences v_R, v_L and v_C. But the sum of the projections of these phasors is equal to that which represents the source voltage v and the instantaneous total voltage v_{ad} across the series of elements.

To form this vector sum, we first subtract the phasor V_C from the phasor V_L. (These two phasors always lie along the same line, in opposite directions.) This gives the phasor, $V_L - V_C$. This is always at right angles to the phasor, V_R, so from the Pythagorean theorem the magnitude of the phasor V is

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

or $$V = I\sqrt{R^2 + (X_L - X_C)^2} \qquad (20)$$

We define the impedance Z of an ac circuit as the ratio of the voltage amplitude across the circuit to the current amplitude in the circuit. From Eq. (20) the impedance of the L-R-C series circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad (21)$$

So we can rewrite Eq. (20) as

$$V = IZ$$

(amplitude of voltage across an ac circuit) (22)

MEANING OF IMPEDANCE AND PHASE ANGLE

Equation 22, given in previous section, has a form similar to $V = IR$, with impedance Z in an ac circuit playing the role of resistance R in a dc circuit. Just as direct current tends to follow the path of least resistance, so alternating current tends to follow the path of lowest impedance. Note, however, that impedance is actually a function of R, L, and C, as well as of the angular frequency ω. We can see this by substituting eq. (12) for X_L and Eq. (18) for X_C into Eq. (21), giving the following complete expression for Z for a series circuit:

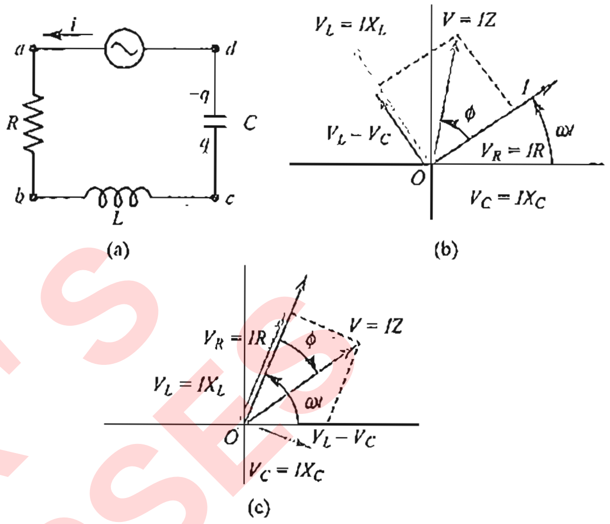
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{R^2 + [\omega L - (1/\omega C)]^2}$$

(impedance of an $L - R - C$ series circuit) (23)

Hence for a given amplitude V of the source voltage applied to the circuit, the amplitude $I = V/Z$ of the resulting current will be different at different frequencies.

In the phasor diagram shown in Fig. 10.16 (b) the angle ϕ between the voltage and the current phasors is the phase angle of the source voltage v with respect to the current I; i.e., it is the angle by which the source voltage leads the current from the diagram.



(a) (b)

(c)

Fig. 10.16

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{I(X_L - X_C)}{IR} = \frac{X_L - X_C}{R}$$

$$\tan\phi = \frac{\omega L - 1/\omega C}{R}$$

(phase angle of an $L - R - C$ series circuit) (24)

if the current is $I = I\cos\omega t$, then the source voltage v is

$$v = V\cos(\omega t + \phi) \qquad (25)$$

Figure 10.16 (b) shows the behavior of a circuit in which $X_L > X_C$. Figure 10.16 (c) shows the behavior when $X_L < X_C$; the voltage phasor V lies on the opposite side of the current phasor I and the voltage lags the current. In this case, $X_L - X_C$ is negative, and is a negative angle between θ and −90°. Since X_L and X_C depend on frequency, the phase angle ϕ depends on frequency as well.

All of the expressions that we have developed for an L-R-C series circuit are still valid if one of the circuit elements is missing. If the resistor is missing, we set $R = 0$; if the inductor is missing, we set $L = 0$, but if the capacitor is missing, we set $C = \infty$, corresponding to the absence of any potential difference ($v_C = q/C = 0$) or any capacitive reactance ($X_C = 1/\omega C = 0$).

In this entire discussion, we have described magnitudes of voltages and currents in terms of their maximum values, i.e., the voltage and the current amplitudes. These quantities are usually described in terms of rms values, not amplitudes. For any sinusoidally varying quantity, the rms value is always $1/\sqrt{2}$ times the amplitude. All the relationships between voltage and current that we have derived in this and the preceding sections are still valid if we use rms quantities throughout instead of amplitudes. For example, if we divide Eq. (22) by $\sqrt{2}$, we get

$$\frac{V}{\sqrt{2}} = \frac{I}{\sqrt{2}}Z$$

Which we can rewrite as

$$V_{rms} = I_{rms}Z \qquad (26)$$

We can translate Eq. (7), (13), and (19) in exactly the same way.

Only those circuits have been considered in which an inductor, a resistor, and a capacitor are in series. You can do a similar analysis for a parallel *L-R-C* circuit.

Finally, we remark that in this section we have been describing the steady state condition of a circuit, the state that exists after the circuit has been connected to the source for a long time. When the source is first connected, there may be additional voltages and currents, called transients, whose nature depends on the time in the cycle when the circuit is initially completed. A detailed analysis of transients is beyond our scope. They always die out after a sufficiently long time and they do not affect the steady state behavior of the circuit. But they can cause dangerous and damaging surges in power lines, which is why delicate electronic systems such as computers are often provided with power-line surge protectors.

Example 10.7 **A resistor and 1 H inductor are joined in series with an ac source of e.m.f. volt. Calculate the phase difference between *V* and *I*.**

Sol. $L = 1$ H, $R = 200\ \Omega$, $V = 10\sqrt{2}\sin(200t)$ V

Phase difference between V and $I = \phi$.

$$\tan(\phi) = \omega L/R = \frac{200\times 1}{200} = 1 \quad \therefore\ \phi = \tan^{-1}(1) = \frac{\pi}{4}$$

Example 10.8 **A lamp with a resistance of 8 W is connected to a choke coil as shown in Fig. 10.17. This arrangement is connected to an alternating source of 110 V. The current in the circuit is 11 A. The frequency of the ac is 60 Hz. Find**

i. the impendence of the circuit, and

ii. the value of inductive reactance of the choke coil.

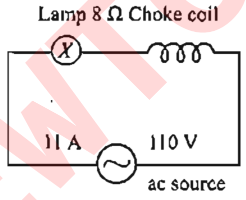


Fig. 10.17

Sol. $R = 0.8\ \Omega$, $\omega = 2\pi\times 60$ Hz, $I = 1$ A, $V = 110$ V

i. Impendence $(Z) = \sqrt{(R)^2 + X_L^2} = V/I$

X = Inductive reactance, $\quad Z = 110/11 = 10\ \Omega$

ii. $\quad Z^2 = R^2 + X_L^2$

$$10^2 = 8^2 + X_L^2 \quad \text{or} \quad X_L = \sqrt{(100-64)} = \sqrt{36} = 6\ \Omega$$

Example 10.9 **A resistor of 200 Ω and a capacitor of 15.0 µF are connected in series to a 200 V, 50 Hz source.**

a. Calculate the current in the circuit.

b. Calculate the voltage (rms) across the resistor and the inductor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

Sol. Given: $R = 100\ \Omega$, $C = 15.0\ \mu\text{F} = 15.0\times 10^{-6}$ F

$V_{\text{rms}} = 200$ V, $\nu = 450$ Hz

a. In order to calculate the current, we need the impendance of the circuit.

$$Z = \sqrt{R^2 + X_c^2} = \sqrt{R^2 + (2\pi\nu C)^{-2}}$$

$$= \sqrt{(200\ \Omega)^2 + (2\times 3.14\times 50\times 10^{-6}\ \text{F})^{-2}}$$

$$= \sqrt{(200\ \Omega)^2 + (221\ \Omega)^2} = 291.5\ \Omega$$

Therefore, the current in the circuit is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{220\ \text{V}}{291.5\ \Omega} = 0.755\ \text{A}$$

b. Since the current is the same throughout the circuit,

$$V_R = I_{\text{rms}}R = (0.755\ \text{A})\,(200\ \Omega) = 151\ \text{V}$$

$$V_C = I_{\text{rms}}X_C = (0.755\ \text{A})\,(212.3\ \Omega) = 160.3\ \text{V}$$

The algebraic sum of the two voltages V_R and V_C is 311.3 V, which is more than the source voltage of 220 V. How to resolve this paradox? You may recall that the two voltages are not in the same phase. Therefore, they cannot be added like the ordinary numbers. The two voltages are out of phase by 90°. Therefore, the total of these voltages must be obtained using the Pythagoras theorem:

$$V_{R+C} = \sqrt{V_R^2 + V_C^2}$$

$$= \sqrt{(164.6\ \text{V})^2 + (174.7\ \text{V})^2} = 220\ \text{V}$$

Thus, if the phase difference between two voltages is properly taken into account, the total voltage across the resistor and the inductor is equal to the voltage of the source.

Example 10.10 **In an *LR* series circuit, a sinusoidal $V = V_0 \sin\omega t$ is applied. It is given that $L = 35$ mH, $R = 11\ \Omega$, $V_{\text{rms}} = 220$ V, $\frac{\omega}{2\pi} = 50$ Hz and $\pi = \frac{22}{7}$. Find the amplitude of the current in steady state and obtain the phase difference between the current and the voltage. Also plot the variation of the current for one cycle on the given graph.**

Sol. Inductive reactance

$$X_L = \omega L = 2\pi\times 50\times 35\times 10^{-3}$$

$$= 2\times \frac{22}{7}\times 50\times 35\times 10^{-3} = 11\ \Omega$$

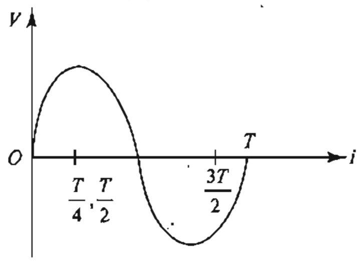


Fig. 10.18

∴ impedance of circuit

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{11^2 + 11^2} = 11\sqrt{2}\ \Omega$$

Given $V_{rms} = 220$ V

Amplitude of voltage $V_0 = \sqrt{2}\quad V_{rms} = 220\sqrt{2}$ V

Amplitude of current $i_0 = \frac{V_0}{Z} = \frac{220\sqrt{2}}{11\sqrt{2}} = 20$ A

Phase lag of current over voltage

$$\phi = \tan^{-1}\frac{\omega L}{R} = \tan^{-1}\frac{11}{11} = \tan^{-1}(1) = \frac{\pi}{4}$$

The voltage and the current variations are shown in Fig. 10.19.

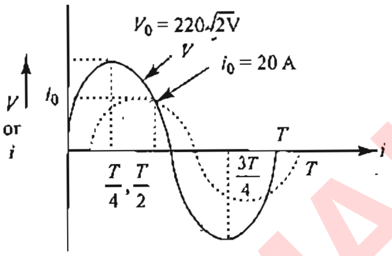


Fig. 10.19

The voltages and the current are expressed as $V = 220\sqrt{2}\sin\omega t$

$$I = 20\sin\left(\omega t - \frac{\pi}{4}\right)$$

Example 10.11 **In the series circuit of Fig. 10.20, suppose $R = 300\ \Omega$, $L = 60$ mH, $C = 0.50\ \mu$F, $V = 50$ V and $\omega = 10{,}000$ rad/s. Find the reactances X_L and X_C, the impedance Z, the current amplitude I, the phase angle ϕ, and the voltage amplitude across each circuit element.**

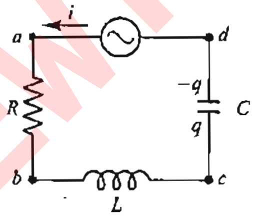


Fig. 10.20

Sol. The inductive and capacitive reactances are

$$X_L = \omega L = (10{,}000\ \text{rad/s})(60\ \text{mH}) = 600\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(10{,}000\ \text{rad/s})(0.50 \times 10^{-6}\ \text{F})} = 200\ \Omega$$

The impedance Z of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(300\ \Omega)^2 + (600\ \Omega - 200\ \Omega)^2} = 500\ \Omega$$

With source voltage amplitude $V = 50$ V the current amplitude 3 is $I = \frac{V}{Z} = \frac{50\ \text{V}}{500\ \Omega} = 0.10$ A

The phase angle ϕ is

$$\phi = \arctan\frac{X_L - X_C}{R} = \arctan\frac{400\ \Omega}{300\ \Omega} = 53°$$

The voltage amplitudes V_R, V_L and V_C across the resistor, inductor and capacitor, respectively, are

$$V_R = IR = (0.10\ \text{A})(300\ \Omega) = 30\ \text{V}$$

$$V_L = IX_L = (0.10\ \text{A})(600\ \Omega) = 60\ \text{V}$$

$$V_C = IX_C = (0.10\ \text{A})(200\ \Omega) = 20\ \text{V}$$

Evaluate: Note that $X_L > X_C$ and hence the voltage amplitude across the inductor is greater than that across the capacitor and ϕ is negative. The value $\phi = -53°$ means that the voltage leads the current by 53°; this is like the situation shown in Fig. 10.21.

Note that the source voltage amplitude $V = 50$ V is not equal to the sum of the voltage amplitude across the separate circuit elements. (That is, $50\ \text{V} \neq 30\ \text{V} + 60\ \text{V} + 0\ \text{V}$).

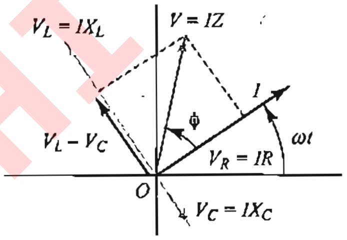


Fig. 10.21

Example 10.12 **For the *L-R-C*-series circuit described in Example 10.11, describe the time dependence of the instantaneous current and each instantaneous voltage.**

Sol. In Example 10.11, we found the amplitude of the current and voltages. Now we have to find the expressions for the instantaneous values of the current and voltages. As we learned, the voltage across a resistor is in phase with the current but the voltages across an inductor or capacitor are not. We also learned in this section that ϕ is the phase angle between the source voltage and the current.

The current and all the voltages oscillate with the same angular frequency, $\omega = 10{,}000$ rad/s and hence with the same period, $\pi/\omega = 2\pi/(10000\ \text{rad/s}) = 6.3 \times 10^{-4}\ \text{s} = 0.63$ rms.

Using Eq. (2), the current is

$$i = I\cos\omega t = (0.10\ \text{A})\cos(10{,}000\ \text{rad/s})t$$

This choice simply means that we choose $t = 0$ to be an instant when the current is maximum. The resistor voltage is in phase with the current, so

$$v_R = V_R\cos\omega t = (30\ \text{V})\cos(10{,}000\ \text{rad/s})t$$

The inductor voltage leads the current by 90°, so

$$v_L = V_L\cos(\omega t + 90°) = -V_L\sin\omega t$$

$$= -(60\ \text{V})\sin(10{,}000\ \text{rad/s})\ t$$

The capacitor voltage lags the current by 90°, so

$$v_C = V_C \cos(\omega t - 90°) = V_C \sin \omega t$$

$$= (20\text{ V}) \sin(10{,}000 \text{ rad/s})\, t$$

Finally, the source voltage (equal to the voltage across the entire combination of resistor, inductor, and capacitor) leads the current by $\phi = 53°$, so

$$v = V\cos(\omega t + \phi)$$

$$= (50\text{ V})\cos\left[(10{,}000 \text{ rad/s})t + \left(\frac{2\pi \text{ rad}}{360°}\right)(53°)\right]$$

$$= (50\text{ V})\cos[(10{,}000\text{ rad/s})t + 0.93\text{ rad}]$$

POWER IN ALTERNATING-CURRENT CIRCUITS

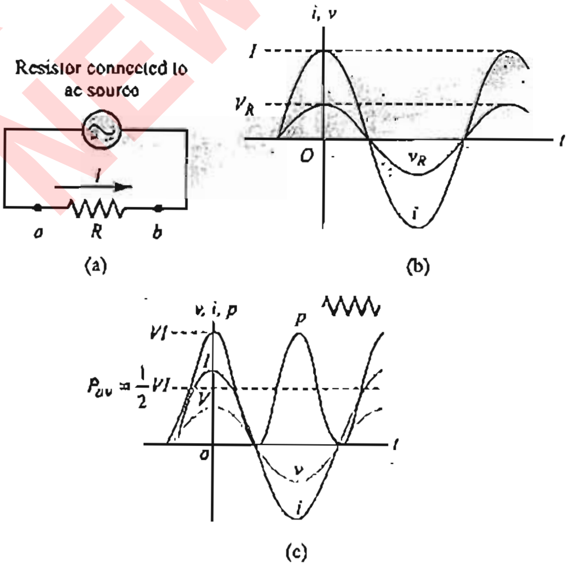
Alternating currents play a central role in the system for distributing, converting, and using electrical energy, so it is important to look at power relationships in ac circuits. For an ac circuit with instantaneous current I and current amplitude I, we will consider an element of that circuit across which the instantaneous potential difference is v with voltage amplitude V. The instantaneous power p delivered to this circuit element is

$$P = vi$$

Let us first see what this means for individual circuit elements. We will assume in each case that $I = I\cos\omega t$.

POWER IN A RESISTOR

First suppose that the circuit element is a pure resistor R, as in Fig. 10.22 (a); then $v = v_R$ and I are in phase. We obtain the graph representing p by multiplying the heights of the graphs of v and i in Fig. 10.22 (b) at each instant. This graph is shown by the back curve in Fig. 10.22 (c). The product vi is always positive because v and i are always either both positive or both negative. Hence the energy is supplied to the resistor at every instant for both directions of I, although the power is not constant.



(a) (b)

(c)

Fig. 10.22

The power curve for a pure resistor is symmetrical about a value equal to one-half its maximum value VI, so the average power P_{av} is

$$P_{av} = \frac{1}{2}VI \text{ (for a pure resistor)} \tag{27}$$

An equivalent expression is

$$P_{av} = \frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}} = V_{rms}I_{rms} \text{ (for a pure resistor)} \tag{28}$$

Also, $V_{rms} = I_{rms}R$, so we can express P_{av} by any of the equivalent forms

$$P_{av} = I_{rms}^2 R = \frac{V_{rms}^2}{R} = V_{rms}I_{rms}$$

(for a pure resistor) (29)

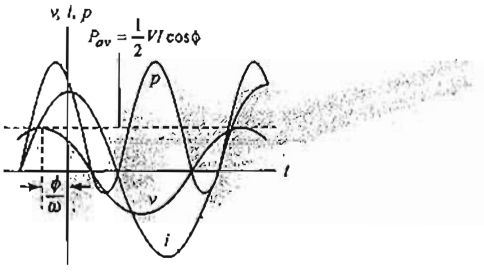
Note that the expressions in Eq. (29) have the same from as the corresponding relationships for a dc circuit, Eq. (18). Also note that they are valid only for pure resistors, not for more complicated combinations of circuit elements.

POWER IN A GENERAL AC CIRCUIT

If any ac circuit, with any combination of resistors, capacitors, and inductors, the voltage v across the entire circuit has some phase angle ϕ with respect to the current i. Then the instantaneous power p is given by

$$p = vi = [V\cos(\omega t + \phi)][I\cos\omega t] \tag{30}$$

The instantaneous power curve has the form shown in Fig. 10.23. The area between the positive loops and the horizontal axis is greater than the area between the negative loops and the horizontal axis, and the average power is positive.



Arbitrary ac circuit

Fig. 10.23

We can derive from Eq. (30) an expression for average power P_{av} by using the identity for the cosine of the sum of the two angles.

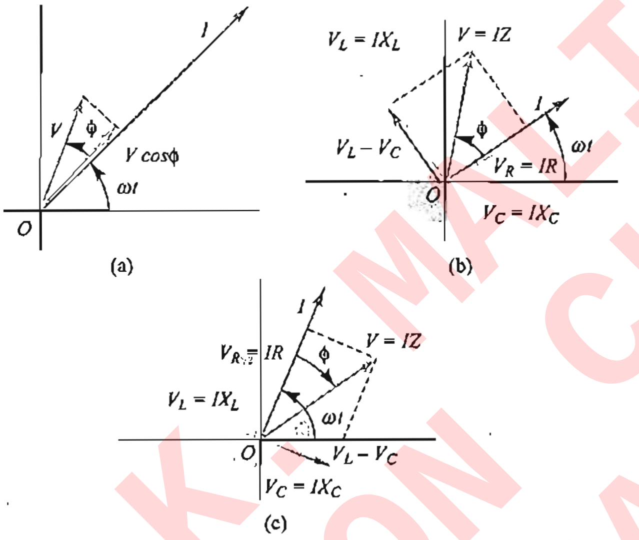
$$p = [V(\cos\omega t\cos\phi - \sin\omega t\sin\phi)]\,[I\cos\omega t]$$

$$= VI\cos\phi\cos^2\omega t - VI\sin\phi\cos\omega t\sin\omega t$$

We see that the average value of $\cos^2 \omega t$ (over one cycle) is 1/2. The average value of $\cos\omega t\, \sin\omega t$ is zero because this product is equal to $1/2(\sin 2\omega t)$, whose average over a cycle is zero. So the average power is

$$P_{av} = \frac{1}{2}VI\cos\phi = V_{rms}I_{rms}\cos\phi$$

(average power into a general ac circuit) (31) where v and I are in phase, so $\phi = 0$, the average power equals $\frac{1}{2}VI = V_{rms}I_{rms}$, when v and i are 90° out of phase, the average power is zero in the general case, when v has a phase angle ϕ with respect to i, the average power equals $\frac{1}{2}I$ multiplied by $V\cos\phi$, the component of the voltage phasor that is in phase with the current phasor. Figure 10.24 (a) shows the general relationship of the current and voltage phasors. For the *L-R-C* series circuit, Figs. 10.24 (b) and (c) show the $V\cos\phi$ equals the voltage amplitude V_R for the resistor; hence equation 30 is the average power dissipated in the resistor. On averaged there is no energy flow into or out of the inductor or capacitor, so none of P_{av} goes into either of these circuit elements.



(a) (b)

(c)

Fig. 10.24

The factor $\cos\phi$ is called the power factor of the circuit. For a pure resistance, ϕ 0, $\cos\phi = 1$ and $P_{av} = V_{rms}I_{rms}$. For a pure inductor or capacitor, $\phi = \pm 90°$, $\cos\phi = 0$ and $P_{av} = 0$. For an *L-R-C*-series circuit the power factor is equal to *R*/*Z*.

A low power factor (larger angle ϕ of lag or lead) is usually undesirable in power circuits. The reason is that for a given potential difference, a large current is needed to supply a given amount of power. This results in large i^2R losses in a client with a low power factor. Many types of ac machinery draw a logging current; i.e., the current drawn by the machinery lags the applied voltage. Hence the voltage leads the current, so $\phi > 0$ and $\cos\phi < 1$. The power factor can be corrected towards the ideal value of 1 by connecting a capacitor in parallel with the load. The current drawn by the capacitors leads the voltage (i.e., the voltage across the capacitor lags the current), which compensates for the lagging current in the other branch of the circuit. The capacitor itself absorbs no net power from the line.

CHOKE COIL

In a dc circuit, current is reduced by means of a rheostat (resistance). This results in a loss of electrical energy I^2R per second as heat in the resistance. The current in an ac circuit can however be reduced by means of a device which involves very small amount of loss of energy, This device is called *'choke coil'* or ballast and consists of a copper coil wound over a soft iron laminated core. This coil is to be reduced. As this circuit is a *L–R* circuit, the current in the circuit,

$$I = \frac{E}{Z} \quad \text{with} \quad Z = \sqrt{(R+r)^2 + (\omega L)^2}$$

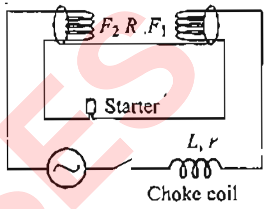


Fig. 10.25

So due to large inductance, *L*, of the coil, the current in the circuit is decreased appreciably. However, due to small resistance of the coil *r*, the power loss in the choke,

$$P_{av} = V_{rms} I_{rms} \cos\phi \to 0$$

as $$\cos\phi = \frac{r}{Z} = \frac{r}{\sqrt{r^2 + \omega^2 L^2}} \approx \frac{r}{\omega L} \to 0$$

CIRCUIT BEHAVIOR AT RESONANCE

As we vary the angular frequency ω of the source, the current amplitude $I = V/Z$ varies as shown in Fig. 10.26 the maximum value of I occurs at the frequency at which the impedance Z is minimum. This peaking of the current amplitude at a certain frequency is called resonance. The angular frequency ω_0 at which the resonance peak occurs is called the resonance angular frequency. This is the angular frequency at which the inductive and capacitive reactances are equal, so at resonance.

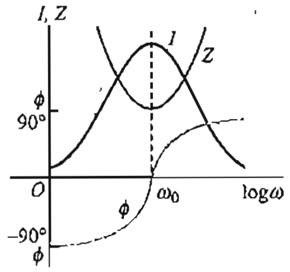


Fig. 10.26

$$X_L = X_C \; \omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{(L-R-C series circuit at resonance)} \qquad (32)$$

Note that this is equal to the natural angular frequency of the oscillation of an *L-C* circuit, which we derived in Eq. (22). The resonance frequency f_0 is $\omega_0/2\pi$. This is the frequency at which the greatest current appears in the circuit for a given source voltage amplitude; in other words, f_0 is the frequency to which the circuit is "turned".

It is instructive to look at what happens to the voltages in an *L*-*R*-*C* series circuit at resonance. The current at any instant is the same in *L* and *C*. The voltage across an inductor always leads the current by 90°, or $\frac{1}{4}$ cycle; and the voltage across the capacitor always lags the current by 90°. Therefore, the instantaneous voltages across *L* and *C* always differ in phase by 180°, or $\frac{1}{2}$ cycle; they have opposite sign at each instant. At the resonance frequency, and only at the resonance frequency, $X_L = X_C$ and the voltage amplitudes $V_L = IX_L$ and $V_C = IX_C$ are equal. Then the instantaneous voltages across *L* and *C* add to zero at each instant, and the total voltage v_{bd} across the $L-C$ combination is exactly zero. The voltage across the resistor is then equal to the source voltage, So at the resonance frequency the circuit behaves as if the inductor and the capacitor were not there at all!

The phase of the voltage relative to the current is given by Eq. (24). At frequencies below resonance, X_C is greater than X_L; the capacitive reactance dominates, the voltage lags the current, and the phase angle ϕ is between 0 and –90°. Above resonance, the inductive reactance dominates; the voltage leads the current and the phase angle is between 0 and + 90°. This variation of ϕ with angular frequency is shown .

TRANSFORMERS

One of the great advantages of ac over dc for electric-power distribution is that it is much easier to step voltage levels up and down with ac than with dc. For a long distance power transmission, it is desirable to use as high a voltage and as small a current as possible; this reduces i^2R losses in the transmission lines, and as smaller wires can be used, it saves material costs. Present day transmission lines routinely operate at rms voltages of the order of 500 kV. On the other hand, safety considerations and insulation requirements dictate relatively low voltages in generating equipment and in household and industrial power distribution. The standard voltage for household wiring is 120 V in the United States and Canada and 240 V in many other countries. The necessary voltage conservation is accomplished by the use of transformers.

HOW TRANSFORMERS WORK

Figure 10.27 shows an idealized transformer. The key components of the transformer are two coils or windings, electrically insulted from each other but wound on the same core. The core is typically made of a material, such as iron, with a very large relative permeability K_m. This keeps the magnetic field lines due to a current in one winding almost completely within the core. Hence almost all of these field lines pass through the other winding, maximizing the mutual inductance of the two windings. The winding to which power is supplied is called the primary; the winding from which power is delivered is called the secondary. The circuit symbol for a transformer with an iron core, such as those used in power distribution system is shown in Fig. 10.27.

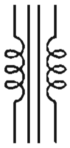


Fig. 10.27

Here it is how a transformer works. The ac source causes an alternating current in the primary, which sets up an alternating flux in the core. This induces an emf in each winding, in accordance with the Faraday's law. The induced e.m.f. in the secondary gives rise to an alternating current in the secondary, and this delivers energy to the device to which the secondary is connected. All the currents and e.m.f.s have the same frequency as the ac source.

Let us see how the voltage across the secondary can be made larger or smaller in amplitude than the voltage across the primary. We neglect the resistance of the windings and assume that all the magnetic field lines are confined to the iron core, so at any instant the magnetic flux ϕ_B is the same in each turn of the primary and secondary windings. The primary winding has N_1 turns and the secondary winding has N_2 turns. When the magnetic flux changes because of the changing currents in the two coils, the resulting induced e.m.f.s are

$$\varepsilon_1 = -N_1\frac{d\Phi_B}{dt} \quad \text{and} \quad \varepsilon_2 = -N_2\frac{d\Phi_B}{dt} \qquad (33)$$

The flux per turn F_B is the same in both the primary and the secondary, so Eq. (33) shows that the induced e.m.f. per turn is the same in each. The ratio of the secondary ε_2 to the primary e.m.f. ε_1 is therefore equal at any instant to the ratio of secondary to primary turns

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{N_2}{N_1} \qquad (34)$$

Since ε_1 and ε_2 both oscillate with the same frequency as the ac source, Eq. (33) also gives the ratio of the amplitudes or of the rms values of the induced e.m.f.s. If the windings have zero resistance, the induced e.m.f.s ε_1 and ε_2 are equal to the terminal voltages across the primary and the secondary, respectively; hence

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad (35)$$

(terminal voltages of transformer, primary and secondary)

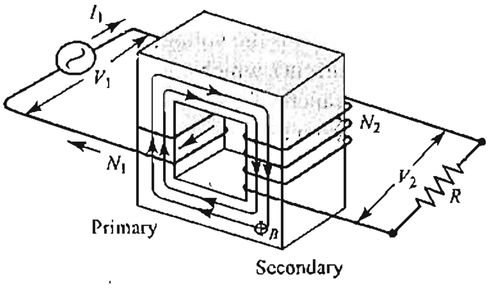


Fig. 10.28

where V_1 and V_2 are either the amplitudes or the rms values of the terminal voltages. By choosing the appropriate turns ratio N_2/N_1, we may obtain any desired secondary voltage from a given primary voltage. If $N_2 > N_1$, then $V_2 > V_1$ and we have a step-up transformer; if $N_2 < N_1$, then $V_2 < V_1$ and we have step-down transformer. At a power generating station, step-up transformer is used. The primary is connected to the power source and the secondary is connected to the transmission lines, giving the desired high voltage for transmission. Near the consumer, step-down transformers lowers the voltage to a value suitable for use in homes or industry.

EXERCISES

Subjective Type

Solutions on page 10.27

1. Find the rms and the average values of the saw tooth waveform shown in Fig. 10.29.

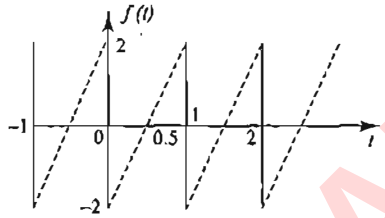


Fig. 10.29

2. Calculate the rms and the average value of the voltage wave shown in Fig. 10.30.

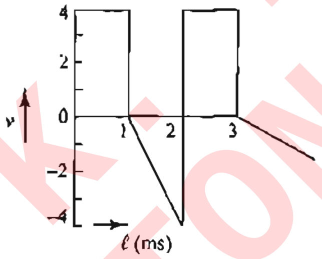


Fig. 10.30

3. An inductor 20×10^{-3} H, a capacitor 100 μF, and a resistor 50 Ω are connected in series across a source of e.m.f. $V = 10 \sin 314t$. Find the energy dissipated in the circuit in 20 min. If the resistance is removed from the circuit and the value of inductance is doubled, then find the variation of current with time in the new circuit.

4. Find the average value of current shown graphically in Fig. 10.31, from $t = 0$ to $t = 2$ s.

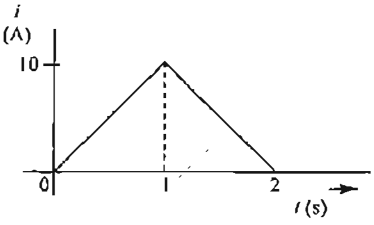


Fig. 10.31

5. Find the average value of current (see Fig. 10.32) from $t = 0$ to $t = \dfrac{2\pi}{\omega}$ if the current varies as $i = I_m \sin \omega t$.

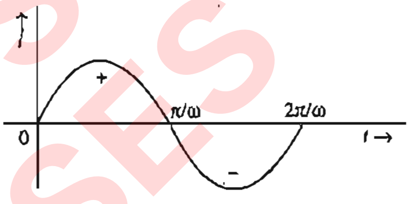


Fig. 10.32

6. Show graphically that the average of sinusoidally varying current in half cycle may or may not be zero.

7. Find the average value of current $i = I_m \sin \omega t$ from **i.** $t = 0$ to $t = \dfrac{\pi}{\omega}$ and **ii.** $t = \dfrac{\pi}{2\omega}$ to $t = \dfrac{3\pi}{2\omega}$.

8. Current in an ac circuit is given by $i = 2\sqrt{2} \sin(\pi t + \pi/4)$. Then find the average value of current during time $t = 0$ to $t = 1$ s.

9. Find the rms value of current $i = I_m \sin \omega t$ from **i.** $t = 0$ to $t = \dfrac{\pi}{\omega}$ and **ii.** $t = \dfrac{\pi}{2\omega}$ to $t = \dfrac{3\pi}{2\omega}$.

10. Find the effective value of current $i = 2 \sin 100\pi t + 2 \cos(100\pi t + 30°)$.

11. When a voltage $v_x = \sin(\omega t + 15°)$ is applied to an ac circuit, the current in the circuit is found to be $i = 2 \sin(\omega t + \pi/4)$. Find the average power consumed in the circuit.

12. An alternating voltage $E = 200\sqrt{2} \sin(100\,t)$ V is connected to a 1 μF capacitor through an ac ammeter (it reads rms value). What will be the reading of the ammeter?

13. In an RC series circuit (see Fig. 10.33), the rms voltage of source is 200 V and its frequency is 50 Hz. If $R = 100\,\Omega$ and $C = \dfrac{100}{\pi}$ μF, find

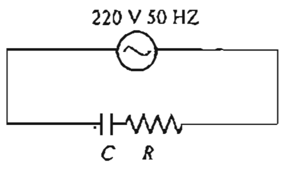


Fig. 10.33

i. Impedance of the circuit,
ii. Power factor angle,
iii. Power factor,
iv. Current,
v. Maximum current,
vi. Voltage across R.

vii. Voltage across C,
viii. Max voltage across R,
ix. Max voltage across C,
x. $< P >$,
xi. $< P_R >$, and
xii. $< P_C >$.

14. In the question 13 if $v_s(t) = 220\sqrt{2}\sin(2\pi\ 50\ t)$, find **a.** $i\,(t)$, **b.** v_R, and **c.** $v_C\,(t)$

15. An ac source of angular frequency ω is fed across a resistor R and a capacitor C in series. The current registered is I. If now the frequency of source is changed to $\omega/3$ (but maintaining the same voltage), the current in the circuit is found to be halved. Calculate the ratio of the reactance to resistance at the original frequency ω.

16. A $\dfrac{9}{100\pi}$ H inductor and a 12 Ω resistance are connected in series to a 225 V, 50 Hz ac source. Calculate the current in the circuit and the phase angle between the current and the source voltage.

17. When an inductor coil is connected to an ideal battery of e.m.f. 10 V, a constant current 2.5 A flows. When the same inductor coil is connected to an ac source of 10 V and 50 Hz then the current is 2 A. Find out the inductance of the coil.

18. A bulb is rated at 100 V, 100 W. It can be treated as a resistor. Find out the inductance of an inductor (called choke coil) that should be connected in series with the bulb to operate the bulb at its rated power with the help of an ac source of 200 V and 50 Hz.

19. A choke coil is needed to operate an arc lamp at 160 V (rms) and 50 Hz. The arc lamp has an effective resistance of 5 Ω when running of 10 A (rms). Calculate the inductance of the choke coil. If the same arc lamp is to be operated on 160 V (dc), what additional resistance is required? Compare the power losses in both the cases.

Objective Type

Solutions on page 10.31

1. The circuit given in Fig. 10.34 has a resistanceless chock coil L and a resistance R. The voltage across R and L are given in Fig. 10.34. The virtual value of the applied voltage is

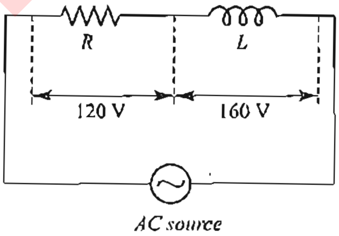


Fig. 10.34

a. 100 V **b.** 200 V **c.** 300 V **d.** 400 V

2. A series R, L, C circuit is shown in Fig. 10.35. The source frequency f varies, but the current is kept unchanged. Which of the curves showing changes of V_C and V_L are valid for the circuit under consideration?

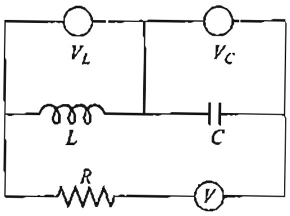
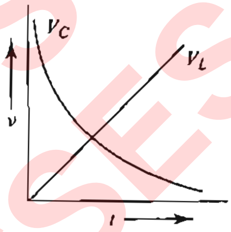
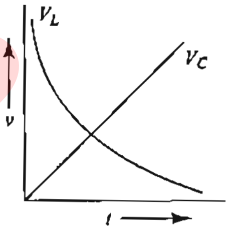
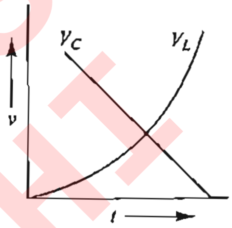
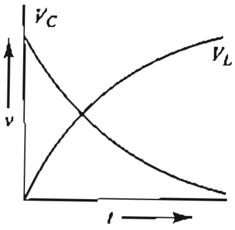


Fig. 10.35

a.


b.


c.


d.


3. Two sinusoidal voltage of the same frequency are shown in Fig. 10.36.

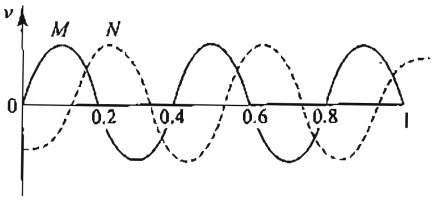


Fig. 10.36

What is the frequency and the phase relationship between the voltage?

frequency/Hz	phase lead of N N over M/rad
a. 0.4	$-\dfrac{\pi}{4}$
b. 2.5	$-\dfrac{\pi}{2}$
c. 2.5	$+\dfrac{\pi}{2}$
d. 2.5	$-\dfrac{\pi}{4}$

4. Figure 10.37 shows an iron-cored transformer assumed to be 100% efficient. The ratio of the secondary turns to the primary turns is 1:20.

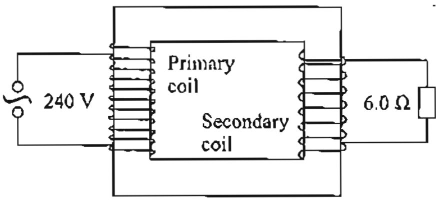


Fig. 10.37

A 240 V ac supply is connected to the primary coil and a 6.0 Ω resistor is connected to the secondary coil. What is the current in the primary coil?

a. 0.10 A
b. 0.14 A
c. 2.0 A
d. 40 A

5. In the series *LCR* circuit (Fig. 10.38), the voltmeter and ammeter readings are:

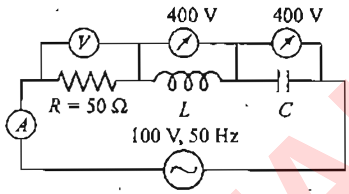


Fig. 10.38

a. $V = 100$ V, $I = 2$ A
b. $V = 100$ V, $I = 5$ A
c. $V = 1000$ V, $I = 2$ A
d. $V = 300$ V, $I = 1$ A

6. An *LCR* circuit contains resistance of 100 Ω and a supple of 200 V at 300 radian angular frequency. If only capacitance is taken out from the circuit and the rest of the circuit is joined, current lags behind the voltage by 60°. If on the other hand, only inductor is taken out the current leads by 60° with the applied voltage. The current flowing in the circuit is

a. 1 A
b. 1.5 A
c. 2 A
d. 2.5 A

7. The rms value of an AC of 50 Hz is 10 A. The time taken by an alternating current in reaching from zero to maximum value and the peak value will be

a. 2×10^{-2} s and 14.14 A
b. 1×10^{-2} s and 7.07 A
c. 5×10^{-3} s and 7.07 A
d. 5×10^{-2} s and 14.14 A

8. The peak value of an alternating emf *E* given by

$$E = E_0 \cos \omega t$$

is 10 V and frequency is 50 Hz. At time $t = (1/600)$ s, the instantaneous value of e.m.f. is

a. 10 V
b. $5\sqrt{3}$
c. 5 V
d. 1 V

9. A coil has an inductance of 0.7 H and is joined in series with a resistance of 220 Ω. When an alternating e.m.f. of 220 V at 50 cps is applied to it, then the wattless component of the current in the circuit is

a. 5 A
b. 0.5 A
c. 0.7 A
d. 7 A

10. A group of electric lamps having a total power rating of 1000 W is supplied by an AC voltage $E = 200 \sin (310t + 60°)$ Then the rms value of the electric current is

a. 10 A
b. $10\sqrt{2}$ A
c. 20 A
d. $20\sqrt{2}$ A

11. When 100 V dc is applied across a solenoid, a current of 1.0 A flows in it. When 100 V ac is applied across the same coil, the current drops to 0.5 A. If the frequency of the ac source is 50 Hz the impedance and inductance of the solenoid are

a. 200 Ω and 0.55 H
b. 100 Ω and 0.86 H
c. 200 Ω and 1.0 H
d. 100 Ω and 0.93 H

12. An inductive coil and resistance of 100 Ω. When an ac signal of frequency 1000 Hz is freed to the coil, the applied voltage leads the current by 45°. What is the inductance of the coil?

a. 2 mH
b. 3.3 mH
c. 16 mH
d. $\sqrt{5}$ mH

13. In the circuit shown in Fig. 10.39, what will be the reading of the voltmeter V_3 and ammeter *A*?

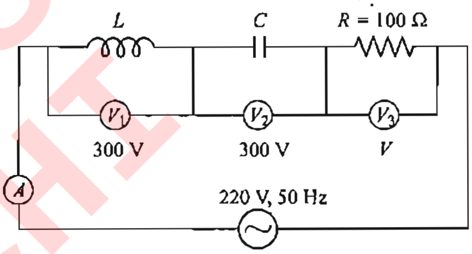


Fig. 10.39

a. 800 V, 2 A
b. 300 V, 2 A
c. 220 V, 2.2 A
d. 100 V, 2 A

14. In the circuit shown in Fig. 10.40, *R* is a pure resistor, *L* is an inductor of negligible resistance (as compared to *R*), *S* is a 100 V, 50 Hz AC source of negligible resistance. With either key K_1 alone or K_2 alone closed, the current is I_0. If the source is changed to 100 V, 100 Hz the current with K_1 alone closed and with K_2 alone closed will be respectively.

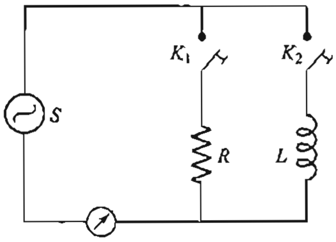


Fig. 10.40

a. $I_0, \frac{I_0}{2}$
b. $I_0, 2I_0$
c. $2I_0, I_0$
d. $2I_0, \frac{I_0}{2}$

15. For the circuit shown in Fig. 10.41 the ammeter A_2 reads 1.6 A and ammeter A_3 reads 0.4 A. Then

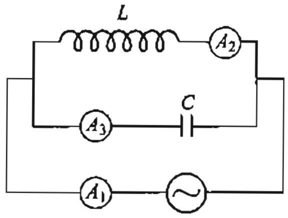


Fig. 10.41

a. $\omega_0 = \dfrac{4}{\sqrt{LC}}$

b. $f_2 = \dfrac{2\pi}{\sqrt{LC}}$

c. the ammeter A_1 reads 1.2 A

d. the ammeter A_1 reads 2A.

16. In the circuit shown in Fig. 10.42. The rims currents I_1, I, and I_3 are altered by varying the frequency f of the oscillator. The output voltage of the oscillator remains sinusoidal and has a fixed amplitude.

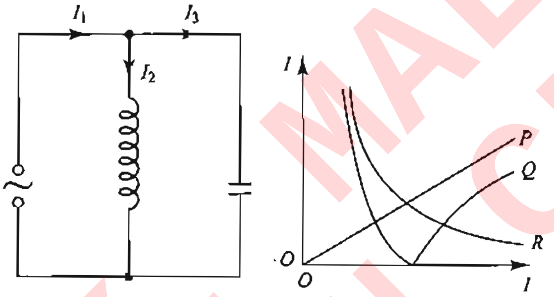


Fig. 10.42

Which curves in figure Indicate correctly the variation with frequency of the current I_1, I_2, and I_3?

	I_1	I_2	I_3
a.	Q	Q	Q
b.	R	Q	Q
c.	Q	P	R
d.	Q	R	P

17. Two resistors are connected in a series across 5 V rms source of alternating potential. The potential difference across 6 Ω resistor is 3 V_m. If R is replaced by a pure inductor L of such magnitude that current remains same, then the potential difference across L is

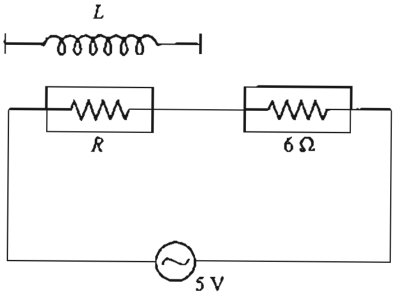


Fig. 10.43

a. 1 V
b. 2 V
c. 3 V
d. 4 V

18. In the circuit shown in Fig. 10.44, if both the bulbs B_1 and B_2 are identical

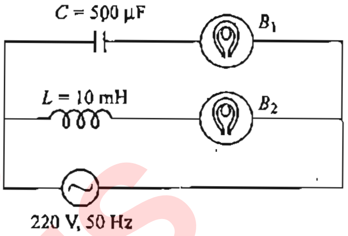


Fig. 10.44

a. their brightness will be the same

b. B_2 will be brighter than B_1

c. as frequency and that of B_2 will decrease

d. only B_2 will glow because the capacitor has infinite impedance

19. A rigid conducting wire bent as shaped is released to fall freely in a horizontal magnetic field which is perpendicular to the plane of the conductor. If the magnetic field strength is B then the emf induced across the points A and C when it has fallen through a distance h will be

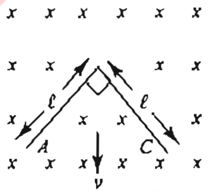


Fig. 10.45

a. $B\ell\sqrt{2gh}$
b. $B\ell\sqrt{gh}$
c. $2B\ell\sqrt{gh}$
d. $2B\ell\sqrt{2gh}$

20. Figure 10.46 shows a source of alternating voltage connected to a capacitor and a resistor. Which of the following phasor diagrams correctly describes the phase relationship between I_c, the current between the source and the capacitor, and I_R, the current in the resistor?

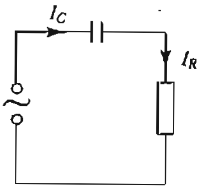
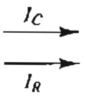
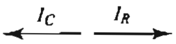


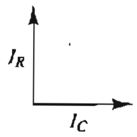
Fig. 10.46



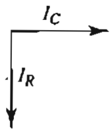
a.



b.



c.



d.

21. A sinusoidal alternating current of peak value I_0 passes through a heater of resistance R. What is the mean power output of the heater?
a. $\frac{I_2^{\,2}R}{2}$
b. $\frac{I_0^{\,2}R}{2}$
c. $I_3^{\,2}R$
d. $\sqrt{2}\,I_0^{\,2}R$

22. Power factor is one for
a. pure resistor
b. pure inductor
c. pure capacitor
d. either an inductor or a capacitor.

23. A resistance of 20 Ω is connected to a source of an alternating potential $V = 220\sin(100\,\pi t)$. The time taken by the current to change from the peak value to rms value, is
a. 0.2 s
b. 0.25 s
c. 2.5×10^{-3} s
d. 2.5×10^{-3} s

24. In *LCR* circuit current resonant frequency is 600 Hz and half power points are at 650 and 550 Hz. The quality factor is
a. $\frac{1}{6}$
b. $\frac{1}{3}$
c. 6
d. 3

25. An ac voltage is represented by

$$E = 220\sqrt{2}\cos(50\pi)t$$

How many times will the current become zero in 1 s?
a. 50 times
b. 100 times
c. 30 times
d. 25 times

26. A resistor and an inductor are connected to an ac supply of 120 volt and 50 Hz. The current in the circuit is 3 A. If the power consumed in the circuit is 108 W, then the resistance in the circuit is
a. 12 Ω
b. 40 Ω
c. $\sqrt{(52 \times 28)}$ Ω
d. 360 Ω

27. An ac ammeter is used to measure current in a circuit. When a given direct current passes through the circuit, the ac ammeter reads 3 A. When another alternating current passes through the circuit, the ac ammeter reads 4 A. Then the reading of this ammeter, if dc and ac flow through the circuit simultaneously, is
a. 3 A
b. 4 A
c. 7 A
d. 5 A

28. A transmitter transmits at a wavelength of 300 m. A condenser of capacitance 2.4 μF is being used. The value of the inductance for the resonant circuit is approximately
a. 10^{-4} H
b. 10^{-6} H
c. 10^{-8} H
b. 10^{-10} H

29. A capacitor of capacitance 1 μF is charged to a potential of 1 V. It is connected in parallel to an inductor of inductance 10^{-3} H. The maximum current that will flow in the circuit has the value
a. $\sqrt{1000}$ mA
b. 1 mA
c. 1 μA
d. 1000 mA

30. Using an ac voltmeter, the potential difference in the electrical line in a house is read to be 234 V. If the line frequency is known to be 50 cycles/second, the equation for the line voltage is
a. $V = 165\sin(100\,\pi t)$
b. $V = 331\sin(100\,\pi t)$
c. $V = 220\sin(100\,\pi t)$
d. $V = 440\sin(100\,\pi t)$

31. An inductance and a resistance are connected in series with an ac potential. In this circuit
a. The current and the p.d. across the resistance lead the p.d. across the inductance
b. The current and the p.d. across the resistance lag behind the p.d. across the inductance by an angle $\pi/2$
c. The current and the p.d. across the resistance lag behind the p.d. across the inductance by an angle π
d. The p.d. across the resistance lags behind the p.d. across the inductance by an angle $\pi/2$ but the current in resistance leads the p.d. across the inductance by $\pi/2$

32. A resistor and a capacitor are connected to an ac supply of 200 V, 50 Hz in series. The current in the circuit is 2 A. If the power consumed in the circuit is 100 watt, then the resistance in the circuit is
a. 100 Ω
b. 25 Ω
c. $\sqrt{125 \times 75}$ Ω
d. 400 Ω

33. In the Q. 32, the capacitive reactance in the circuit is
a. 100 Ω
b. 25 Ω
c. $\sqrt{125 \times 75}$ Ω
d. 400 Ω

34. In the Q. 32, the capacitance in the circuit is
a. $\frac{100}{100\pi}$ F
b. $\frac{25}{100\pi}$ F
c. $\frac{\sqrt{125 \times 75}}{100\pi}$ F
d. $\frac{1}{100\pi\sqrt{125 \times 75}}$ F

35. In a series *LCR* circuit the voltage across the resistance, capacitance and inductance is 10 V each. If the capacitance is short circuited, the voltage across the inductance will be
a. 10 V
b. $10\sqrt{}$V
c. (10/3)V
d. 20 V

36. An ideal choke takes a current of 10 A when connected to an ac supply of 125 V and 50 Hz. A pure resistor under the same conditions takes a current of 12.5 A. If the two are connected to an ac supply of 100 V and 40 Hz, then the current in series combination of above resistor and inductor is
a. 10 A
b. 12.5 A
c. 20 A
d. 25 A

37. A direct current of 5 A is superimposed on an alternating current $I = 10\sin\omega t$ flowing through a wire. The effective value of the resulting current will be
a. (15/2) A
b. $5\sqrt{3}$ A
c. $5\sqrt{5}$ A
d. 15 A

38. An ideal choke takes a current of 8 A when connected to an ac supply of 100 V and 50 Hz. A pure resistor under the same conditions takes a current of 10 A. If the two are connected to an ac supply of 150 V and 40 Hz, then the current in a series combination of the above resistor and inductor is

a. 10 A b. 8 A
c. 18 A d. $(15/\sqrt{2})$ A

39. In the Q.38, the total current through a parallel combination of the resistor and the inductor when connected to an ac supply of 150 V and 40 Hz is
a. 15 A b. 30 A
c. zero A d. $15/\sqrt{2}$ A

40. In the circuit of Fig. 10.47 the (ωt) volt with $\omega = 2000$ rad/s. The amplitude of the current will be nearest to

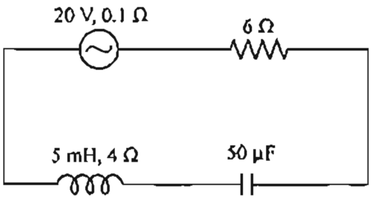


Fig. 10.47

a. 2 A b. 3.3 A
c. $2/\sqrt{5}$ A d. $\sqrt{5}$ A

41. An rms voltage of 110 V is applied across a series circuit having a resistance 11 Ω and an impedance 22 Ω. The power consumed is
a. 275 W b. 366 W
c. 550 W d. 1100 W

42. In the given circuit in Fig. 10.48, $V_C = 50$ V and $R = 50\,\Omega$. The values of C and V_R are

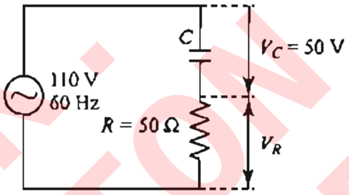


Fig. 10.48

a. 3.3 mF, 60 V b. 3.3 μF, 98 V
c. 1.6 μF, 30 V d. 2 μF, 60 V

43. A 220-V, 50 Hz ac generator is connected to an inductor and a 50 Ω resistance in series. The current in the circuit is 1.0 A. What is pd across inductor?
a. 102.2 V b. 186.4 V
c. 213.6 V d. 302 V

44. An 8 μF capacitor is connected across 220 V, 50 Hz line. What is the peak value of charge through the capacitor?
a. 2.5×10^{-3} C b. 2.5×10^{-4} C
c. 5×10^{-5} C d. 7.5×10^{-2} C

45. A dc ammeter and a hot wire ammeter are connected to a circuit in series. When a direct current is passed through circuit, the dc ammeter shows 6 A. When ac current flows through circuit, the ac ammeter shows 8 A. What will be reading of each ammeter, if dc and ac currents flow simultaneously through the circuit?
a. dc = 6 A, ac = 10 A
b. dc = 3 A, ac = 5 A
c. dc = 5 A, ac = 8 A
d. dc = 2 A, ac = 3 A

46. A coil has an inductance of 0.7 H and is joined in series with a resistance of 220 Ω. When the ac emf of 220 V is applied to it, find the wattless component of current in the circuit.
a. 0.2 A b. 0.4 A
c. 0.5 A d. 0.7 A

47. An ac is given by equation $I = I_1 \cos\omega t + I_2 \sin\omega t$. The rms value of current is given by;
a. $\dfrac{I_1 + I_2}{2}$ b. $\dfrac{(I_1 + I_2)^2}{\sqrt{2}}$
c. $\dfrac{1}{\sqrt{2}}\sqrt{I_1^2 + I_2^2}$ d. $\dfrac{I_1^2 + I_2^2}{2}$

48. A typical light dimmer used to dim the stage lights in a theatre consists of a variable induction for L (where inductance is adjustable between zero and L_{max}) connected in series with a light bulb B as shown in Fig. 10.49. The mains electrical supply is 220 V at 50 Hz, the light bulb is rated at 220 V, 1100 W. What L_{max} is required if the rate of energy dissipated in the light bulb is to be varied by a factor of 5 from its upper limit of 1100 W?

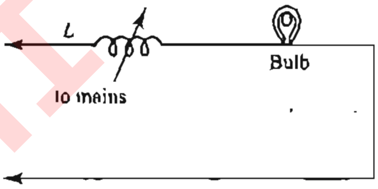


Fig. 10.49

a. 0.69 H b. 0.28 H
c. 0.38 H d. 0.56 H

49. A coil has an inductance of 0.7 H and is joined in series with a resistance of 220 Ω. When an alternating e.m.f. of 220 V at 50 cps. is applied to it, then the wattless component of the current in the circuit is
a. 5 A b. 0.5 A
c. 0.7 A d. 7 A

50. Two alternating voltage generators produce e.m.f.s of the same amplitude E_0 but with a phase difference of $\pi/3$. The resultant emf is
a. $E_0 \sin(\omega t + \pi/3)$ b. $E_0 \sin(\omega t + \pi/6)$
c. $\sqrt{3}\, E_0 \sin(\omega t + \pi/6)$ d. $\sqrt{3}\, E_0 \sin(\omega t + \pi/2)$

51. For the circuit shown in Fig. 10.50, current in inductance is 0.8 A while in capacitance is 0.6 A. What is the current drawn from the source?

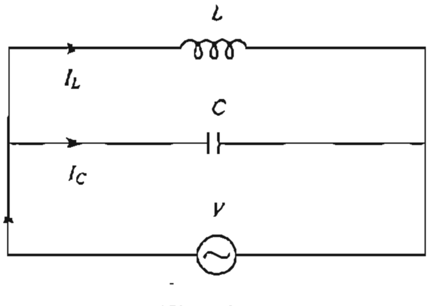


Fig. 10.50

a. 0.1 A b. 0.3 A
c. 0.6 A d. 0.2 A

52. If a direct current of value a ampere is superimposed on an alternative current $I = b \sin\omega t$ flowing through a wire, what is the effective value of the resulting current in the circuit?

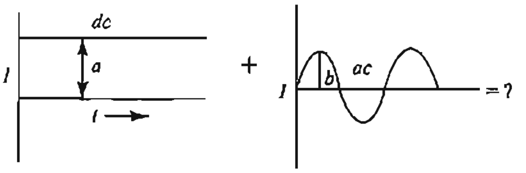


Fig. 10.51

a. $\left[a^2 - \frac{1}{2}b^2\right]^{1/2}$ b. $[a^2 + b^2]^{1/2}$
c. $\left[\frac{a^2}{2} + b^2\right]^{1/2}$ d. $\left[a^2 + \frac{1}{2}b^2\right]^{1/2}$

53. Determine the rms value of a semi-circular current wave which has a maximum value of a.

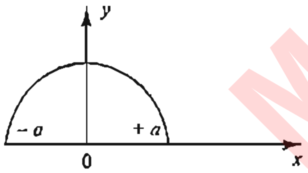
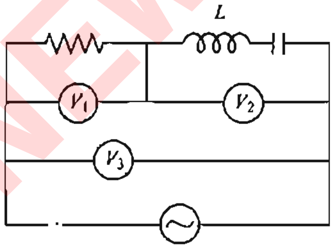


Fig. 10.52

a. $(1/\sqrt{2})a$ b. $\sqrt{(3/2)}\,a$
c. $\sqrt{(2/3)}\,a$ d. $(1/\sqrt{3})a$

54. An alternating voltage $E = 200\sqrt{2}\sin(100\,t)$ V is connected to a 1 mF capacitor through an ac ammeter. What will be the reading of the ammeter?
a. 40 mA b. 20 mA
c. 30 mA d. $10/\sqrt{2}$ mA

55. Which voltmeter will give zero reading at resonance?



a. V_1 b. V_2
c. V_3 d. None

56. A 50 W, 100 V lamp is to be connected to an ac mains of 200 V, 50 Hz. What capacitor is essential to be put in series with the lamp ?

a. $\frac{25}{\sqrt{2}}$ μF b. $\frac{50}{\pi\sqrt{3}}$ μF
c. $\frac{50}{\sqrt{2}}$ μF d. $\frac{100}{\pi\sqrt{3}}$ μF

57. A capacitor of 10 mF and an inductor of 1 H are joined in series. An ac of 50 Hz is applied to this combination. What is the impedance of the combination?
a. $28/\pi\Omega$ b. $14/\pi\Omega$
c. $10/\pi\Omega$ d. $20/\pi\Omega$

58. If $i_1 = 3\sin\omega t$ and $i_2 = 4\cos\omega t$, then i_3 is

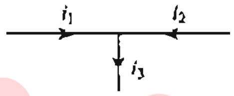


Fig. 10.53

a. $5\sin(\omega t + 53°)$ b. $5\sin(\omega t + 37°)$
c. $5\sin(\omega t + 45°)$ d. $5\sin(\omega t + 53°)$

59. An alternating e.m.f of angular frequency $\left(\omega = \frac{1}{\sqrt{LC}}\right)$ is applied to a series in LCR circuit. For this frequency of the applied e.m.f.
a. The circuit is at 'resonance'
b. The current in the circuit is in phase with the applied e.m.f. and the voltage across R equals this applied e.m.f. potential differences
c. The sum of the potential difference across the inductance and capacitance equals the applied e.m.f. which is 180° ahead of phase of the current in the circuit
d. Impedance of the circuit is less than R

60. An LCR series circuit with 100 Ω resistance is connected to an ac source of 200 V and an angular frequency 300 radians per second. When only the capacitance is removed, the current lags behind the voltage of 60°. When only the inductance is removed, the current leads the voltage by 60°. Then the current and power dissipated in LCR circuit are, respectively
a. 1A, 200 V b. 1A, 400 V
c. 2A, 200 V d. 2A, 400 V

61. When an AC source of e.m.f. $e = E_0 \sin(100t)$ is connected across a circuit, the phase difference between the e.m.f. e and the current i in the circuit is observed to be $\pi/4$ as shown in Fig. 10.54. If the circuit consists possibly only of R-C or R-C or L-C series, find the relationship between the two elements.

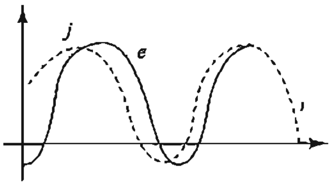


Fig. 10.54

a. $R = 1$ kΩ, $C = 10$ μF b. $R = 1$ kΩ, $C = 1$ μF
c. $R = 1$ kΩ, $L = 10$ H d. $R = 1$ kΩ, $L = 1$ H

62. A bulb is rated at 100 V, 100 W, and it can be treated as a resistor. Find out the inductance of an inductor (called choke coil) that should be connected in series with the bulb to operate the bulb at its rated power with the help of an ac source of 200 V and 50 Hz.

a. $\frac{\pi}{\sqrt{3}}$ H
b. 100 H
c. $\frac{\sqrt{2}}{\pi}$ H
d. $\frac{\sqrt{3}}{\pi}$ H

63. An ac source of angular frequency ω is fed across a resistor R and a capacitor C in series. The current registered is I. If now the frequency of source is changed to $\omega/3$ (but maintaining the same voltage), the current in the circuit is found to be halved. Then the ratio of reactance to resistance at the original frequency ω is

a. $\sqrt{3/5}$
b. $\sqrt{5/3}$
c. $\sqrt{2/3}$
d. $\sqrt{3/2}$

64. In ac circuit the potential differences across an inductance and resistance joined in series are respectively 16 V and 20 V. The total potential difference across the circuit is

a. 20 V
b. 25.6 V
c. 31.9 V
d. 53.5 V

65. Current in an ac circuit is given by $I = 3\sin\omega t + 4\cos\omega t$, then

a. rms value of current is 5 A.
b. mean value of this current in one half period will be $6/\pi$.
c. if voltage applied is $V = V_m \sin\omega t$ then the circuit must be containing resistance and capacitance.
d. if voltage applied is $V = V_m \sin\omega t$, the circuit may contain resistance and inductance.

66. A current source sends a current $I = i_0\cos(\omega t)$. When connected across an unknown load gives a voltage output of, $v = v_0\sin(\omega t + \pi/4)$ across that load. Then the voltage across the current source may be brought in phase with the current through it by

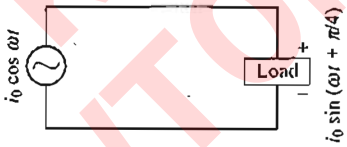


Fig. 10.55

a. connecting an inductor in series with the load
b. connecting a capacitor in series with the load
c. connecting an inductor in parallel with the load
d. connecting a capacitor in parallel with the load

67. In the circuit shown in Fig. 10.56, $X_C = 100\ \Omega$, $X_L = 200\ \Omega$ and $R = 100\ \Omega$. The effective current through the source is

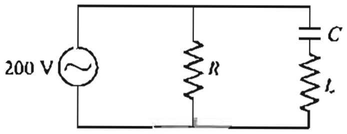
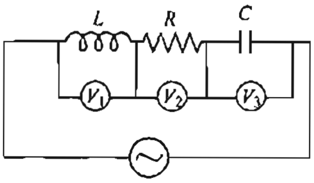


Fig. 10.56

a. 2 A
b. $2\sqrt{2}$ A
c. 0.5 A
d. $\sqrt{0.4}$ A

68. If the readings of v_1 and v_2 are 100 V each, the reading of v_3 is



200 V, 50Hz

Fig. 10.57

a. 0 V
b. 100 V
c. 200 V
d. cannot be determined by given information.

69. For an LCR series circuit with an ac source of angular frequency ω,

a. circuit will be capacitive if $\omega > \frac{1}{\sqrt{LC}}$
b. circuit will be inductive if $\omega = \frac{1}{\sqrt{LC}}$
c. power factor of circuit will be unity if capacitive reactance equals inductive reactance
d. current will be leading voltage is $\omega > \frac{1}{\sqrt{LC}}$

70. The value of current in two series LCR circuits at resonance is same. When connected across a sinusoidal voltage source. Then

a. both circuits must be having same value of capacitance and inductor
b. in both circuits ratio of L and C will be same
c. for both the circuits X_L/X_C must be same at that frequency
d. both circuits must have same impedance at all frequencies

71. In series LCR circuit voltage drop across resistance is 8 V, across inductor is 6 V and across capacitor is 12 V. Then

a. voltage of the source will be leading current in the circuit
b. voltage drop across each element will be less than the applied voltage
c. power factor of circuit will be 4/3
d. None of these

72. In a black box of unknown elements (L, or R or any other combination), an ac voltage $E = E_0\sin(\omega t + \phi)$ is applied and current in the circuit was found to be $I = i_0\sin(\omega t + \phi + \pi/4)$. Then the unknown elements in the box may be

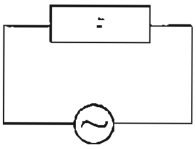


Fig. 10.58

a. only capacitor
b. inductor and resistor both
c. either capacitor, resistor and inductor or only capacitor and resistor
d. only resistor

73. The voltage time (V-t) graph for triangular wave having peak value. V_0 is as shown in Fig. 10.59.

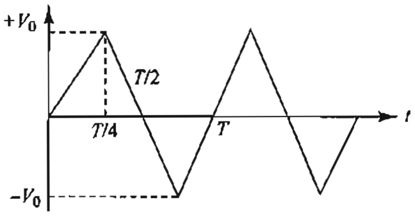


Fig. 10.59

The rms value of V in time interval from $t = 0$ to $T/4$ is

a. $\frac{V_0}{\sqrt{3}}$
b. $\frac{V_0}{2}$
c. $\frac{V_0}{\sqrt{2}}$
d. None of these

74. In the question 73, the average value of voltage (V) in one time period will be

a. $\frac{V_0}{\sqrt{3}}$
b. $\frac{V_0}{2}$
c. $\frac{V_0}{\sqrt{2}}$
d. 0

75. What reading would you expect of a square-wave current, switching rapidly between + 0.5 A and – 0.5 A, when passed through an ac ammeter?

a. 0
b. 0.5 A
c. 0.25 A
d. 1.0 A

76. The capacitor of an oscillatory circuit is enclosed in a container. When the container is evacuated, the frequency changes by 50 Hz, the dielectric constant of the gas is

a. 1.1
b. 1.01
c. 1.001
d. 1.0001

Multiple Correct Answers Type

Solutions on page 10.39

1. In an ac circuit shown below in Fig. 10.60, the supply voltage has a constant rms value V but variable frequency f. At resonance, the circuit

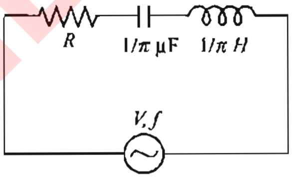


Fig. 10.60

a. has a current I given by: $I = \frac{V}{R}$
b. has a resonance frequency 500 Hz
c. has a voltage across the capacitor which is 180° out of phase with that across the inductor
d. has a current given by $I = \frac{V}{\sqrt{R^2 + \left(\frac{1}{\pi} + \frac{1}{\pi}\right)^2}}$

2. Resonance occurs in a series LCR circuit when the frequency of the applied emf is 1000 Hz. Then

a. when frequency = 900 Hz, then the current through the voltage source will be ahead of e.m.f. of the source
b. the impedance of the circuit is minimum at $f = 1000$ Hz
c. at only resonance the voltages across L and current in C differ in phase by 180°
d. if the value of C is doubled resonance occurs at $f = 2000$ Hz

3. If R, C, and L be the resistance, capacitance, and inductance in the circuit in which ac of frequency f is set up, then which of the following have the dimensions of R?

a. fC
b. fL
c. $1/fC$
d. L/f

4. Which of the following statements are true? Heat produced in a current carrying conductor depends upon

a. the time for which the current flows in the conductor
b. the resistance of the conductor
c. the strength of the current
d. the nature of current (ac or dc)

5. A choke coil of resistance 5 Ω and inductance 0.6 H is in series with a capacitance of 10 μF. If a voltage of 200 V is applied and the frequency is adjusted to resonance, the current and voltage across the inductance and capacitance are I_0, V_0, and V_1, respectively. We have

a. $I_0 = 40$ A
b. $V_0 = 9.8$ kV
c. $V_1 = 9.8$ kV
d. $V_1 = 19.6$ kV

6. In the circuit shown in the Fig. 10.61, if both the bulbs B_1 and B_2 are identical

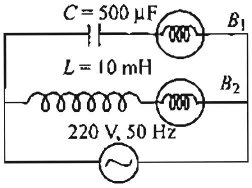


Fig. 10.61

a. their brightness will be the same
b. B_2 will be brighter than B_1
c. as frequency of supply voltage is increased, brightness of B_1 will increase and that of B_2 will decrease
d. only B_2 will glow because the capacitor has infinite impedance

7. In an RLC series circuit shown in Fig. 10.62, the readings of voltmeters V_1 and V_2 are 100 V and 120 V, respectively. The source voltage is 130 V. For this situation mark out the correct statement(s).

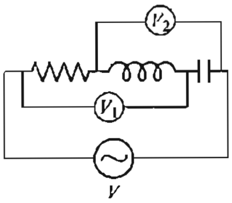


Fig. 10.62

a. Voltage across resistor, inductor and capacitor are 50 V, 86.65 V and 186.65 V, respectively
b. Voltage across resistor, inductor and capacitor are 10 V, 90 V and 30 V, respectively
c. Power factor of the circuit is $\frac{5}{13}$
d. The circuit is capacitance in nature

Assertion-Reasoning Type

Solutions on page 10.39

a. Statement I is True, Statement II is True; Statement II is correct explanation for Statement I.
b. Statement I is True, Statement II is True; Statement II is NOT a correct explanation for statement I.
c. Statement I is True, Statement II is False.
d. Statement I is False, Statement II is True.

1. **Statement I:** By only knowing the power factor for a given LCR circuit it is not possible to tell whether the applied alternating e.m.f. leads or lags the current.
Statement II:
$\cos\theta = \cos(-\theta)$

2. **Statement I:** In the purely resistive element of a series LCR, AC circuit the maximum value of rms current increases with increase in the angular frequency of the applied e.m.f.

Statement II: $I_{max} = \frac{\mathcal{E}_{max}}{z}$, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$,

where I_{max} is the peak current in a cycle.

3. **Statement I:** In a series LCR circuit, at resonance condition power consumed by circuit is maximum.
Statement II: At resonance condition the effective resistance of circuit is maximum.

4. **Statement I:** In series LR circuit voltage leads the current.
Statement II: In series LC circuit current leads the voltage.

5. **Statement I:** Average value of ac over a complete cycle is always zero.
Statement II: Average value of ac is always defined over half cycle.

6. **Statement I:** In series LCR circuit resonance can take place.
Statement II: Resonance takes place if inductance and capacitive reactance are equal.

7. **Statement I:** KVL rule is also being applied in ac circuit shown below.
Statement II:

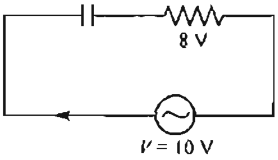


Fig. 10.63

V_C in the circuit = 2V.

8. **Statement I:** ac generators are based upon EMI principle.
Statement II: Resistance offered by capacitor for alternating current is zero.

9. **Statement I:** Rate of heat generated when resistance is connected with ac source depends on time.
Statement II: RMS voltage may be greater than maximum ac voltage.

10. An inductor, capacitor and resistance connected in series. The combination is connected across ac source.
Statement I: Peak current through each remains same.
Statement II: Average power delivered by source is equal to average power developed across resistance.

11. **Statement I:** In an alternating current, direction of motion of free electrons changes periodically.
Statement II: Alternating current changes its direction after a certain time interval.

12. **Statement I:** When frequency is greater than resonance frequency in a series LCR circuit, it will be an inductive circuit.
Statement II: Resultant voltage will lead the current.

13. **Statement I:** When capacitive reactance is smaller than the inductive reactance in LCR circuit, e.m.f. leads the current.
Statement II: The phase angle is the angle between the alternating e.m.f. and alternating current of the circuit.

14. **Statement I:** An alternating current shows magnetic effect.
Statement II: Alternating current varies with time.

15. **Statement I:** The dc and ac both can be measured by a hot wire instrument.
Statement II: The hot wire instrument is based on the principle of magnetic effect of current.

16. **Statement I:** In a series RLC circuit if V_R, V_L and V_C denote rms voltage across R, L and C respectively and V_S is the rms voltage across the source, then $V_S = V_R + V_L + V_C$.

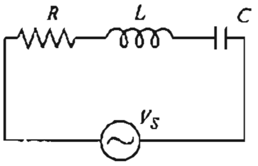


Fig. 10.64

Statement II: In ac circuits, Kirchhoff's voltage law is correct at every instant of time.

17. **Statement I:** The electrostatic energy stored in capacitor plus magnetic energy stored in inductor will always be zero in a series LCR circuit driven by ac voltage source under condition of resonance.
Statement II: The complete voltage of ac source appears across the resistor in a series LCR circuit driven by ac voltage source under condition of resonance.

Comprehension Type

Solutions on page 10.40

For Problems 1 – 3

If the voltage in an ac circuit is represented by the equation, $V = 220\sqrt{2}\sin(314t - \phi)$, calculate

1. rms value of the voltage
a. 220 V
b. 314 V
c. $220\sqrt{2}$ V
d. $200/\sqrt{2}$ V

2. average voltage
   a. 220 V b. $622/\pi$ V
   c. $220/\sqrt{2}$ V d. $220\sqrt{2}$ V

3. frequency of ac
   a. 50 Hz b. $50\sqrt{2}$ Hz
   c. $50/\sqrt{2}$ Hz d. 75 Hz

For Problems 4 – 5

Consider the Fig. 10.65. Find

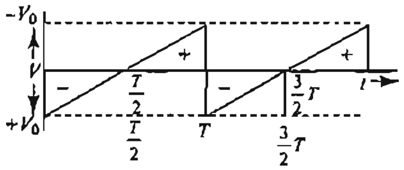


Fig. 10.65

4. the average
   a. $\left|\frac{V_0}{\sqrt{2}}\right|$ b. $\left|\frac{V_0}{2}\right|$
   c. $\left|\frac{V_0}{\sqrt{3}}\right|$ d. $\left|V_0\sqrt{2}\right|$

5. rms value for the saw–tooth voltage of peak value V_0
   a. $\frac{V_0}{\sqrt{3}}$ b. $V_0\sqrt{2}$
   c. $\frac{V_0}{\sqrt{2}}$ d. $V_0\sqrt{3}$

For Problems 6 – 7

The half cycle of an alternating signal is shown in Fig. 10.66. It increases uniformly from zero at 0° to F_m at α°, it remains constant from α° to $(180 - \alpha)$°, and decreases uniformly from F_m at 180°.

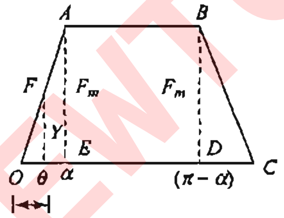


Fig. 10.66

6. The effective values of the signal is
   a. $F_m\sqrt{\left(1-\frac{4\alpha}{3\pi}\right)}$ b. $F_m\sqrt{\left(1+\frac{4\alpha}{3\pi}\right)}$
   c. $F_m\sqrt{\left(1-\frac{3\alpha}{4\pi}\right)}$ d. $F_m\sqrt{\left(1+\frac{3\alpha}{4\pi}\right)}$

7. The average values of the signal is
   a. $\frac{(\pi+\alpha)F_m}{\pi}$ b. $\frac{(\pi-\alpha)F_m}{\pi}$
   c. $\left(\frac{\pi+\alpha}{3\pi}\right)F_m$ d. $\left(\frac{2\pi+\alpha}{2\pi}\right)F_m$

For Problems 8 – 9

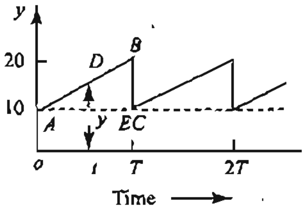


Fig. 10.67

8. The average value of the wave-form shown in Fig. 10.67.
   a. $15\sqrt{2}$ b. $10\sqrt{2}$
   c. 10 d. 15

9. The rms value of the signal is
   a. $10\sqrt{\frac{7}{3}}$ b. $\frac{10}{\sqrt{3}}$
   c. $10\sqrt{7}$ d. $10\sqrt{3}$

For Problems 10 – 11

A 0.21 H inductor and a 12 Ω resistance are connected in series to a 220 V, 50 Hz ac source.

10. The current in the circuit is
   a. $\frac{311}{\sqrt{4492}}$ A b. $\frac{220}{\sqrt{4492}}$ A
   c. $\frac{\sqrt{4492}}{220}$ A d. $\frac{\sqrt{4492}}{311}$ A

11. The phase angle between the current and the source voltage is
   a. $\tan^{-1}\left(\frac{7\pi}{4}\right)$ b. $\cos^{-1}\left(\frac{7\pi}{4}\right)$
   c. $\tan^{-1}\left(\frac{4\pi}{7}\right)$ d. $\cos^{-1}\left(\frac{4\pi}{7}\right)$

For Problems 12 – 13

When 100 V dc is applied across a coil, a current 1 A flows through it when 100 V ac of 50 Hz is applied to the same coil, only 0.5 A flows.

12. The resistance is
   a. 200 Ω b. 50 Ω
   c. 100 Ω d. $50\sqrt{3}$ Ω

13. The inductor of coil is
   a. 5.5 H b. 1.1 H
   c. 0.55 H d. 2.5 H

For Problems 14 – 17

A box P and a coil Q are connected in series with an ac source of variable frequency. The e.m.f. of the source is constant at 10 V. Box P contains a capacitance of 1 μF in series with a resistance of 32 Ω. Coil Q has a self inductance of 4.9 mH and a resistance of 68 Ω in series. The frequency is adjusted so that maximum current flows in P and Q.

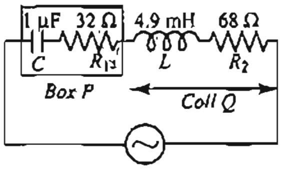


Fig. 10.68

14. The impedance of P at this frequency is
a. 77 Ω b. 36 Ω c. 40 Ω d. 125 Ω

15. The impedance of Q at this frequency is
a. 200 Ω b. $\sqrt{1350}$ Ω c. 55 Ω d. $\sqrt{9524}$ Ω

16. The voltage across P is
a. 12 V b. 7.7 V c. 10 V d. 24 V

17. The voltage across Q is
a. 20 V b. $\frac{\sqrt{1350}}{10}$ V
c. 5.5 V d. $\frac{\sqrt{9524}}{10}$ V

For Problems 18 – 20

A series *LCR* circuit containing a resistance of 120 Ω has angular resonance frequency 4×10^5 rads^{-1}. At resonance the voltages across resistance and inductance are 60 V and 40 V, respectively.

18. The value of inductance *L* is
a. 0.1 mH b. 0.2 mH c. 0.35 mH d. 0.4 mH

19. The value of capacitance *C* is
a. $\frac{1}{32}$ μF b. $\frac{1}{16}$ μF c. 32 μF d. 16 μF

20. At what frequency the current in the circuit lags the voltage by 45° ?
a. 4×10^5 rad/s b. 3×10^5 rad
c. 8×10^5 rad/s d. 2×10^5 rad/s

For Problems 21 – 22

A current of 4 A flows in a coil when connected to a 12 V dc source. If the same coil is connected to a 12 V, 50 rad/s ac source a current of 2.4 A flows in the circuit.

21. The inductance of the coil is
a. 0.02 H b. 0.04 H c. 0.08 H d. 1.0 H

22. The power developed in the circuit of a 2500 μF capacitor that is connected in series with the coil is
a. $\frac{432}{25}$ W b. $\frac{123}{11}$ W c. $\frac{230}{13}$ W d. $\frac{232}{15}$ W

For Problems 23 – 24

An inductor 20×10^{-3} H, a capacitor 100 μF and a resistor 50 Ω are connected in series across a source of e.m.f. $V = 10 \sin 314\, t$.

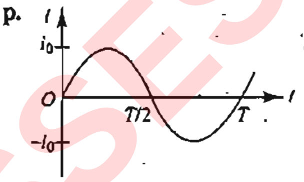
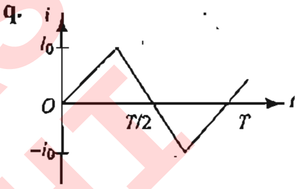
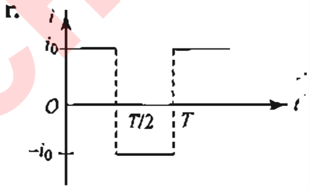
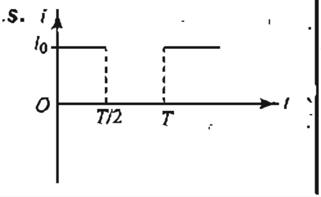
23. Then the energy dissipated in the circuit in 20 minutes is
a. 952 J b. 900 J c. 250 J d. 500 J

24. If resistance is removed from the circuit and the value of inductance is doubled, then the variation of current with time in the new circuit is
a. $0.52 \cos 314\, t$ b. $0.52 \sin 314$
c. $0.52 \sin (314\, t + \pi/3)$ d. None of these

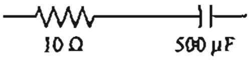
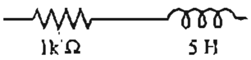
Matching Column Type

Solutions on page 10.43

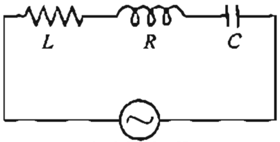
1. In column I, variation of current *I* with time *t* is given in the figure. In column II, root mean square current I_{rms} and average current is given. Match the column I with corresponding quantities given in column II

Column I	Column II
p. 	a. $I_{rms} = \frac{i_0}{\sqrt{3}}$
q. 	b. average current for positive half cycle is i_0.
r. 	c. average current for positive half cycle is $\frac{i_0}{2}$.
s. 	d. Full cycle average current is zero.

2. Four different circuit components are given in each situation of column I and all the components are connected across an ac source of same angular frequency ω = 200 rad/s. The information of phase difference between the current and source voltage in each situation of column I is given in column II. Match the circuit components in column I with corresponding results in column II

Column I	Column II
p.  10 Ω 500 μF	a. The magnitude of required phase difference is π/2.
q.  5 H	b. The magnitude of required phase difference is π/4.
r.  4 H 3 μF	c. The current leads in phase to source voltage.
s.  1k Ω 5 H	d. The current lags in phase to source voltage.

3. In series *R-L-C* circuit, $R = 100\,\Omega$, $C = \frac{100}{\pi}\,\mu F$, and $L = \frac{100}{4}$ mH, is connected to an ac source as shown in Fig. 10.69.



220 V, 50 Hz

Fig. 10.69

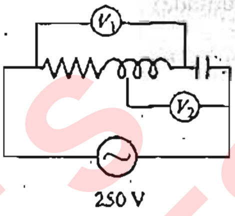
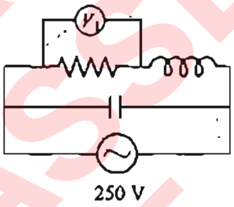
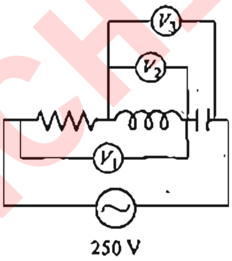
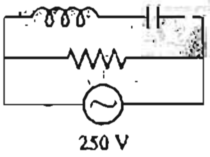
The rms value of ac voltage is 220 V and its frequency is 50 Hz. In column I some physical quantities are mentioned while in column II information about quantities are provided. Match the entries of column I with the entries of column II.

Column I	Column II
p. Average power dissipated in the resistor is	a. zero
q. Average power dissipated in the inductor is	b. non-zero
r. Average power dissipated in the capacitor is	c. 160
s. RMS voltage across the capacitor is	d. 185.6

4. Consider all possibilities (*L*, *R*, *C* are non-zero)

Column I	Column II
p. In *L-R* series AC circuit	a. current lags inductor voltage by $\frac{\pi}{2}$
q. In *R-C* series AC circuit	b. current lags voltage by an angle less than $\frac{\pi}{2}$
r. In *L-C-R* series AC circuit	c. current leads voltage by an angle less than $\frac{\pi}{2}$
s. In purely resistive AC circuit	d. current and voltage are in phase

5. In column I some ac circuits with meter readings are given and in column II some circuit quantities are given. Match the entries of column I with the entries of column II.

Column I	Column II
p.  $V_1 = 158.11$ V; $V_2 = 200$ V	a. $V_R = 150$ V
q.  $V_1 = 150$ V	b. $V_L = 50$ V
r.  $V_1 = 158.11$ V; $V_2 = 291.6$ V; $V_3 = 150$ V	c. $V_C = 250$ V
s. 	d. Power factor of the circuit is 3/5

ANSWERS AND SOLUTIONS

Subjective Type

1. The required values can be found by using either graphical method or analytical method.

Graphical method:

The average value can be found by averaging the function from $t = 0$ to $t = 1$ in parts as shown in Fig. 10.70

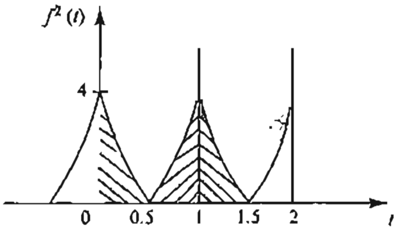


Fig. 10.70

Average value of $f(t)$

$$= \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{T} \times \text{(net area over one cycle)}$$

Now, area of a right-angled triangle

$$= \frac{1}{2} \times \text{(base)} \times \text{(altitude)}$$

Hence, area of the triangle during $t = 0$ to $t = 0.5$ s is

$$A_1 = \frac{1}{2} \times (\Delta t) \times (-2) = \frac{1}{2} \times \frac{1}{2} \times -2 = -\frac{1}{2}$$

Similarly, area of the triangle from $t = 0.5$ to $t = 1$ s is

$$A_2 = \frac{1}{2} \times (\Delta t) \times (+2) = \frac{1}{2} \times \frac{1}{2} \times 2 = \frac{1}{2}$$

Net area from $t = 0$ to $t = 0.1$ s is

$$A_1 + A_2 = -\frac{1}{2} + \frac{1}{2} = 0$$

Hence, the average value of $f(t)$ over one cycle is zero.
For finding the rms value, we will first square the ordinates of the given function and draw a new plot for $f^2(t)$ as shown in Fig. 10.70. It would be seen that the squared ordinates form a parabola.
Area under a parabolic curve = (1/3) × base × altitude. The area under the curve from $t = 0$ to $t = 0.5$ s is

$$A_1 = \frac{1}{3}(\Delta t) \times 2^2 = \frac{1}{3} \times \frac{1}{2} \times 4 = \frac{2}{3}$$

Similarly, for $t = 0.5$ to $t = 1.0$ s,

$$A_2 = \frac{1}{3}(\Delta t) \times 4 = \frac{1}{3} \times \frac{1}{2} \times 4 = \frac{2}{3}$$

$$\text{Total area} = A_1 + A_2 = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

$$\text{rms value} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt} = \sqrt{\text{average of } f^2(t)}$$

$$\text{rms value} = \sqrt{\frac{4}{3}} = 1.15$$

2. In such cases it is difficult to develop a single equation. Hence, it is usual to consider two equations, one applicable from 0 to 1 and another from 1 to 2 ms.
For t lying between 0 and 1 ms, $v_1 = 4$. For t lying between 1 and 2 ms, $v_2 = -4t + 4$.

$$V_{\text{rms}} = \sqrt{\frac{1}{2}\left(\int_0^1 v_1^2\,dt + \int_1^2 v_2^2\,dt\right)}$$

$$V_{\text{rms}}^2 = \frac{1}{2}\left[\int_0^1 4^2\,dt + \int_1^2 (-4t+4)^2\,dt\right]$$

$$= \frac{1}{2}\left[|16t|_0^1 + \left|\frac{16t^3}{3}\right|_1^2 + |16t|_1^2 - \left|\frac{32t^2}{2}\right|_1^2\right]$$

$$= \frac{1}{2}\left[16 + \frac{16 \times 8}{3} - \frac{16}{3} + 16 \times 2 - 16 \times 1 - \frac{32 \times 4}{2} + \frac{32 \times 1}{2}\right] = \frac{32}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{32}{3}} = 3.265 \text{ V}$$

$$V_{\text{av}} = \frac{1}{2}\left[\int_0^1 v_1\,dt + \int_1^2 v_2\,dt\right]$$

$$= \frac{1}{2}\left[\int_0^1 4\,dt + \int_1^2 (-4t+4)\,dt\right]$$

$$= \frac{1}{2}\left[|4t|_0^1 + \left|-\frac{4t^2}{2} + 4t\right|_1^2\right] = 1 \text{ V}$$

3. Here, time of 1 cycle $T = 1/50$ s. So, we have to calculate the average energy as time >> T.
Energy consumed in time t

$$= V_{\text{rms}} I_{\text{rms}} \cos\phi t = \frac{I_M}{\sqrt{2}} \times \frac{V_M}{\sqrt{2}} \times \frac{R}{Z} t$$

$$V = \frac{V_M^2 R}{2Z^2} t \qquad \left(\because I_M = \frac{V_M}{Z}\right)$$

$$\text{Now, } Z = \sqrt{R^2 + \left(\omega L \sim \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{(50)^2 + \left(314 \times 20 \times 10^{-3} - \frac{1}{314 \times 100 \times 10^{-6}}\right)^2}$$

$$= \sqrt{3153.6} \approx 56\ \Omega$$

$$\text{Energy consumer} = \frac{10^2 \times 50 \times 20 \times 60}{2 \times 3153.6} = 951 \text{ J}$$

When resistance is removed,

$$\cos\phi = \frac{R}{Z'} = 0 \quad \text{or,} \quad \phi = \frac{\pi}{2}$$

$$Z' = \frac{1}{\omega C} - \omega L' = \frac{1}{314 \times 10^{-4}} - 314 \times 40 \times 10^{-3}$$

$$= 19.3$$

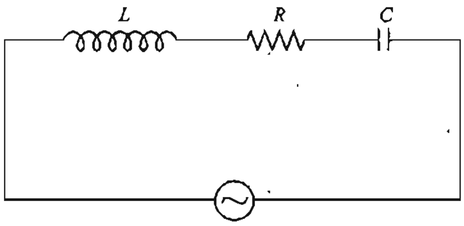


Fig. 10.71

$$I = \frac{V_M}{Z'}\sin(\omega t + \phi) = \frac{10}{19.3}\sin\left(314t + \frac{\pi}{2}\right)$$

$$= 0.52\cos 314t$$

4. From the $i - t$ graph (Fig. 10.31), area from $t = 0$ to $t = 2$ s

$$\frac{1}{2}\times 2\times 10 = 10 \text{ A s}$$

$$\therefore \text{ average current} = \frac{10}{2} = 5\text{ A}$$

5. $$<i> = \frac{\int_0^{2\pi/\omega} I_m \sin\omega t dt}{2\pi/\omega} = \frac{\frac{I_m}{\omega}\left(1-\cos\omega\frac{2\pi}{\omega}\right)}{2\pi/\omega} = 0$$

It can be seen graphically that the area of $i - t$ graph of one cycle is zero.

$\therefore$ $<i>$ in one cycle = 0.

6.

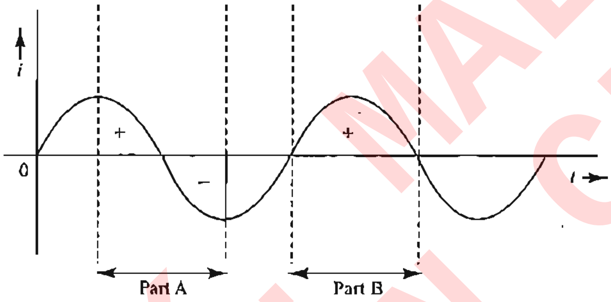


Fig. 10.72

Figure 10.72 shows two parts, A and B, each half cycle. In part A we can see that the net area is zero

$\therefore$ $<i>$ in part A = 0.

In part B, area is positive hence in this part $<i> \neq 0$.

7. **i.** $$<i> = \frac{\int_0^{\pi/\omega} I_m \sin\omega t\, dt1}{\pi/\omega} = \frac{\frac{I_m}{\omega}\left(1-\cos\omega\frac{\pi}{\omega}\right)}{\pi/\omega} = \frac{2I_m}{\pi}$$

ii. $$<i> = \frac{\int_{\pi/2\omega}^{3\pi/2\omega} I_m \sin\omega t\, dt}{\pi/\omega} = 0$$

8. $$<i> = \frac{\int_0^1 i dt}{1} = 2\sqrt{2} = \int_0^1 \sin\left(\pi t + \frac{\pi}{4}\right) = \frac{4}{\pi}$$

9. **i.** $$i_{\text{rms}} = \sqrt{\frac{\int_0^{\pi/\omega} I_m{}^2 \sin^2\omega t\, dt}{\pi/\omega}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

ii. $$<i> = \sqrt{\frac{\int_{\pi/2\omega}^{3\pi/2\omega} I_m^2 \sin^2\omega t\, dt}{\pi/\omega}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

Note:

The rms values for one cycle and half cycle (either positive half cycle or negative half cycle) is the same.

From the above two cases note that for sinusoidal functions rms value (also called effective value)

$$\frac{\textit{peak value}}{\sqrt{2}} \textit{ or } I_{rms} = \frac{I_m}{\sqrt{2}}.$$

10. The equation can be written as $i = 2\sin 100\pi t + 2\sin(100\pi t + 120°)$

so the phase difference $\phi = 120°$

$$= \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$= \sqrt{4 + 4 + 2\times 2\times 2\left(-\frac{1}{2}\right)} = 2,$$

so the effective value or rms value $= 2\sqrt{2}/ = \sqrt{2}$ A

11. $$P_{\text{av}} = v_{\text{rms}} I_{\text{rms}} \cos\phi$$

$$= \frac{200\sqrt{2}}{\sqrt{2}}\frac{2}{\sqrt{2}}\cos(30°) = 100\sqrt{6}\text{ W}$$

12. Comparing $E = 200\sqrt{2}\sin(100t)$ with $E = E_0\sin\omega t$, we find that,

$$E_0 = 200\sqrt{2}\text{ V and } \omega = 100 \text{ (rad/s)}$$

So, $$X_C = \frac{1}{\omega C} = \frac{1}{100\times 10^{-6}} = 10^4\ \Omega$$

As ac instruments reads rms value, the reading of ammeter will be,

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{X_C} = \frac{E_0}{\sqrt{2}X_C} \qquad \left[\text{as } E_{\text{rms}} = \frac{E_0}{\sqrt{2}}\right]$$

i.e., $$I_{\text{rms}} = \frac{200\sqrt{2}}{\sqrt{2}\times 10^4} = 20\text{ mA}$$

13. $$X_C = \frac{10^6}{\frac{100}{\pi}(2\pi 50)} = 100\,\Omega.$$

i. $Z = \sqrt{R^2 + Xc^2} = \sqrt{100^2 + (100)^2} = 100\sqrt{2}\,\Omega$

ii. $\tan\phi = \frac{Xc}{R} = 1 \quad \therefore\ \phi = 45°$

iii. Power factor $= \cos\phi = \frac{1}{\sqrt{2}}$

iv. Current $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{200}{100\sqrt{2}} = \sqrt{2}\ \phi$

v. Maximum current $I_{\text{rms}}\sqrt{2}=2$ A

vi. Voltage across $R=V_{R,\text{rms}}\, I_{\text{rms}}\, R=\sqrt{2}\times 100$ V

vii. Voltage across $C=V_{C,\text{rms}}=I_{\text{rms}}\, X_C=\sqrt{2}\times 100$ V

viii. Max voltage across $R=\sqrt{2}\, V_{R,\text{rms}}=200$ V

ix. Max voltage across $C=\sqrt{2}\, V_{C,\text{rms}}=200$ V

x. $<P>=V_{\text{rms}} I_{\text{rms}}\cos\phi=200\times\sqrt{2}\times\dfrac{1}{\sqrt{2}}\,200$ W.

xi. $<P_R>=I^2_{\text{rms}}\, R=200$ W

xii. $<P_C>=0$

14. a. $i(t)=I_m\sin(\omega t+\phi)$

$$=\sqrt{2}\sin(2\pi 50\,t+45°)$$

b. $v_R=i_R\times R=i(t)\,R$

$$=\sqrt{2}\times 100\sin(100\,\pi t+45°)$$

c. $v_C(t)=i_C X_C$ (with a phase lag of 90°)

$$=\sqrt{2}\times 100\sin(100\,\pi t+45-90)$$

15. According to the given problem,

$$I=\frac{V}{Z}=\frac{V}{[R^2+(1/C\omega)^2]^{1/2}} \qquad (1)$$

and, $$\frac{I}{2}=\frac{V}{[R^2+(3/C\omega)^2]^{1/2}} \qquad (2)$$

Substituting the value of I from equation (1) in (2),

$$4\left(R^2+\frac{1}{C^2\omega^2}\right)=R^2+\frac{9}{C^2\omega^2},\ \text{i.e.,}\ \frac{1}{C^2\omega^2}=\frac{3}{5}R^2$$

So that, $$\frac{X}{R}=\frac{(1/C\omega)}{R}=\frac{\left(\frac{3}{5}R^2\right)^{1/2}}{R}=\sqrt{\frac{3}{5}}$$

16. Here $X_L=\omega L=2\pi fL=2\pi\times 50\times\dfrac{9}{100\pi}=9\,\Omega$

So, $$Z=\sqrt{R^2+X_L{}^2}=\sqrt{12^2+9^2}=15\,\Omega$$

So (a) $I=\dfrac{V}{Z}=\dfrac{225}{15}=15$ A

and (b) $\phi=\tan^{-1}\left(\dfrac{X_L}{R}\right)=\tan^{-1}\left(\dfrac{9}{12}\right)=\tan^{-1}3/4=37°$

i.e., the current will lag the applied voltage by 37° in phase.

17. When the coil is connected to a dc source, the final current is decided by the resistance of the coil.

$$\therefore\quad r=\frac{10}{2.5}=4\,\Omega$$

When the coil is connected to ac source, the final current is decided by the impedance of the coil.

$$\therefore\quad Z=\frac{10}{2}=5\,\Omega$$

But $Z=\sqrt{(r)^2+(X_L)^2}$ $X_L{}^2=5^2-4^2=9$

$X_L=3\,\Omega$

$\therefore\quad \omega L=2\pi fL=3$

$\therefore\quad 2\pi 50\,L=3$

$\therefore\quad L=3/100\pi$H

18. From the rating of the bulb, the resistance of the bulb is

$$R=\frac{V^2_{\text{rms}}}{P}=100\,\Omega$$

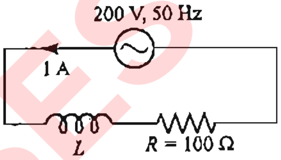


Fig. 10.73

For the bulb to be operated at its rated value, the rms current through it should be 1A

Also, $I_{\text{rms}}=\dfrac{V_{\text{rms}}}{Z}$

$$\therefore\quad 1=\frac{200}{\sqrt{100^2+(2\pi 50L)^2}}\qquad L=\frac{\sqrt{3}}{\pi}\text{H}$$

19. As for the arc lamp $V_R=IR=10\times 5=50$ V, so when it is connected to 160 V ac source through a choke in series,

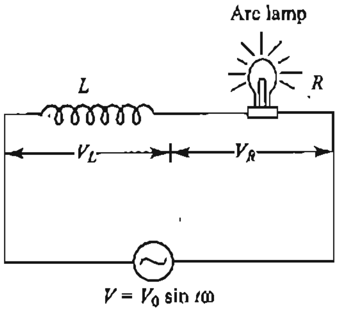


Fig. 10.74

$$V^2=V_R^2+V_L^2,\quad V_L=\sqrt{160^2-50^2}=152\text{ V}$$

and as, $V_L=IX_L=I\omega L=2\pi fLI$

So, $$L=\frac{V_L}{2\pi fI}=\frac{152}{2\times\pi\times 50\times 10}=4.84\times 10^{-2}\text{H}$$

Now the lamp is to be operated at 160 V dc; instead of choke if additional resistance r is put in series with it,

$V=I(R+r)$, i.e., $160=10(5+r)$

i.e., $r=11\,\Omega$

In case of ac, as choke has no resistance, power loss in the choke will be zero while the bulb will consume,

$$P=I^2R=10^2\times 5=500\text{ W}$$

However, in case of dc as resistance r is to be used instead of choke, the power loss in the resistance r will be.

$$PL = 10^2 \times 11 = 1100 \text{ W}$$

while the bulb will still consume 500 W, i.e., when the lamp is run on resistance r instead of choke more than double the power consumed by the lamp is wasted by the resistance r.

Objective Type

1. b. $E = \sqrt{120^2 + 160^2}$

$$10 = \sqrt{144 + 256} = 10\sqrt{400} \text{ V} = 200 \text{ V}$$

2. a. $V_C \propto \frac{1}{f}$ and $V_L \propto f$

3. c. The period of sinusoidal voltage, $T = 0.4$ s.

$$f = \frac{1}{T} = \frac{1}{0.4} = 2.5 \text{ Hz}$$

N lags M by 0.1 s which is equivalent to $\left(\frac{0.1}{0.4}\right)2\pi$ or $\frac{\pi}{2}$ rad.

Thus, the lead of N over M is $-\frac{\pi}{2}$ rad.

4. a. The equivalent primary load is

$$R_1 = \left(\frac{N_1}{N_2}\right)^2 R_2 = \left(\frac{20}{1}\right)^2 (6.0) = 2400 \ \Omega$$

Current in the primary coil

$$= \frac{220}{R_1} = \frac{240}{2400} = 0.1 \text{ A}$$

5. a. Here, $V_L = V_C$. They are in opposite phase. Hence, they will cancel each other. Now the resultant potential difference is equal to the applied potential difference

$$= 100 \text{ V}$$

$$Z = R \qquad (\therefore \quad X_L = X_C)$$

$$\therefore \quad L_{rms} = \frac{V_{rms}}{Z} = \frac{V_{vrn}}{R} = \frac{100}{50}$$

$$= 2 \text{ A}$$

6. c. According to the given question,

$$\tan 60° = \frac{\omega L}{R} \text{ and } \tan 60° = \frac{1/\omega C}{R}$$

$$\therefore \quad \omega L = (1/\omega C)(\text{case of resonance})$$

$$\text{Now} \quad Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 2$$

$$\therefore \quad I_{rms} = \frac{R_{rms}}{Z} = \frac{200 \text{ V}}{100 \Omega} = 2 \text{ A}$$

7. d. Time for reaching maximum or peak value from 0

$$= \frac{T}{4} = \frac{1}{4} \times \frac{1}{50} \text{s} = \frac{1}{200} \text{ s}$$

$$= 5 \times 10^{-3} \text{ s}$$

$$I_{min} = 10 \text{ A}, I_{min} = \frac{I_0}{\sqrt{2}} = 10$$

$$\therefore \quad I_0 = 10\sqrt{2} \text{ A}$$

$$= 14.14 \text{ A}$$

8. b. Given that $E_0 = 10$ V, $t = \frac{1}{600}$s

$$\therefore \quad E = E_0 \cos 2\pi nt$$

$$= 10 \cos \left[2\pi \times 50 \times \frac{1}{600}\right]$$

$$= 10 \cos(\pi/6) = 10(\sqrt{3}/2)$$

$$= 5\sqrt{3} \text{ V}$$

9. b. Wattless component of AC

$$= I_V \sin\theta = \frac{E_v}{Z} \sin\theta$$

$$= \frac{200}{\sqrt{R^2 + \omega^2 L^2}} \times \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}}$$

$$= \frac{200 \times \omega L}{(R^2 + \omega^2 L^2)}$$

As $\quad \omega L = 0.7 \times 2p \times 50$

Hence, wattless component of AC

$$= \frac{200 \times (0.7 \times 2\pi \times 50)}{(220^2 + 220^2)}$$

$$= \frac{1}{2} = 0.5 \text{ A}$$

10. b. $\quad$ Power $= E_0 I_0 \cos\phi$

or $\quad 1000 = (2000 \times I_0/2)\cos 60°$

Solving, we get : $I_0 = 20$ A

$$\therefore \quad I_{rms} = \frac{20}{\sqrt{2}} = 10\sqrt{2} \text{ A}$$

11. a. In case of dc, $\omega = 0$ and hence $Z = R$.

$$\therefore \quad Z = R = \frac{E}{I} = \frac{100}{1} = 100 \Omega$$

for AC: $\quad Z = [R^2 + (2\pi nL)^2]^{1/2}$

or $\quad 200 = [(100)^2 + (100\pi L)^2]^{1/2} \quad \left(\because Z = \frac{100}{0.5} = 200 \Omega\right)$

Solving, we get $L = 0.55$ H

12. c. Clearly,

$$\because \quad X_L = R$$

or $L \times 2 \times 3.14 \times 1000 = 100$

$$\text{or} \quad L = \frac{100}{2 \times 3.14 \times 1000} \text{ H}$$

$$= 15.9 \times 10^{-3} \text{ H} = 15.9 \text{ mH} \approx 16 \text{ mH}.$$

13. c. Voltage across L and C cancels out.
So, voltage across k is 220 V.

Again, $I_0 = \frac{220}{100}\text{ A} = 2.2\text{ A}$

14. a. At 50 Hz, $L\omega = R$. At 100 Hz, $L\omega = 2R$.
So the current remains unchanged in R. However, it becomes half in L.

15. c. The current of 1.6 A lags e.m.f. in phase by $\frac{\pi}{2}$. The current of 0.4 A leads e.m.f. in phase by $\frac{\pi}{2}$. So, these two currents are 180° out of phase with each other.

$\therefore$ Net current, $I_1 = (1.6 - 0.4)\text{ A} = 1.2\text{ A}$.

16. d. Reactance of the inductor O is given by $X_L = 2\pi fL$.

$\therefore$ rms current through the inductor L is I_2

$= \frac{V}{2\pi f L} \propto \frac{1}{f}$ where V is the rms of the supply voltage.

Reactance of the capacitor C is given by $X_C = \frac{1}{2\pi f C}$

$\therefore$ rms current through the capacitor is

$$I_3 = \frac{V}{X_C} = 2\pi f CV \propto f$$

The total current I_1 is given by $I_1 = I_2 + I_3$

$$= \left(\frac{V}{2\pi f L} - 2\pi f CV\right)$$

Thus, I_2 is best represented by curve R.
I_3 is best represented by curve P.
I_1 is best represented by curve Q.

17. d. $V_{\text{rms}}(6\omega) = 3\text{V} = 6I_V \quad \therefore I_V = 0.5\text{ A}$

$$I_V = \frac{1}{2} = \frac{5}{\sqrt{6^2 + X_L^2}},\ X_L = 8\ \Omega$$

Now, $V_L = I_V \cdot X_L = \frac{1}{2} \times 8 = 4\text{ V}$

18. b. Let I_1 and I_2 be currents through B_1 and B_2 then

$$I_1 \times \sqrt{R^2 \times X_L^2} = 220$$

$$\frac{I_2}{I_1} = \frac{\sqrt{R^2 \times X_C^2}}{\sqrt{R^2 + X_L^2}} = \frac{\sqrt{R^2 + \left(\frac{1}{c\omega}\right)^2}}{\sqrt{R^2 + L^2\omega^2}}$$

$$= \frac{\sqrt{R^2 + \left(\frac{1}{500 \times 10^{-6} \times 2\pi \times 50}\right)^2}}{\sqrt{R^2 + (10 \times 10^{-3} \times 2\pi \times 50)^2}}$$

$$= \sqrt{R^2 + 40} / \sqrt{R^2 + 9.87},\ I_2 > I_1$$

Bulb B_2 will be brighter. As frequency increases, X_C decreases X_L increases. I_2 becomes less, I_1 increases.
$\therefore$ brightness of B_1 will increase and that of B_2 decrease.

19. c. The effective length

$$\ell' = \sqrt{\ell^2 + \ell^2} = \sqrt{2}\,\ell$$

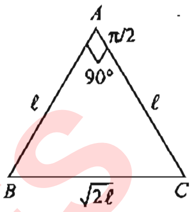


Fig. 10:75

$\therefore \quad E = B\ell v = B\sqrt{2}\,\ell\sqrt{2gh} = 2B\ell\sqrt{gh}$

20. c. Since the capacitor is connected in series to the resistor, the current I_C from the supply and I_R through the resistor is in phase as represented by choice (a).

21. b. Root mean square current of the sinusoidal waveform,

$I = \frac{I_0}{\sqrt{2}}$.

Power output of the heater,

$$P = I^2 R = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = \frac{I_0^2 R}{2}$$

22. c. Power factor $= \frac{R}{\sqrt{R^2} + L\omega - \frac{1}{C\omega}}$

For a purely resistive circuit, $L\omega - \frac{1}{C\omega} = 0$

$\therefore$ power factor = 1.

23. d. Both B and I are in the same phase. So, let us calculate the time taken by the voltage to change from peak value of rms value.

Now, $220 = 220 \sin 100\pi t_1$

or $100\pi t_1 = \frac{\pi}{2}$ or $t_1 = \frac{1}{200}$ s

Again, $\frac{200}{\sqrt{2}} = 200 \sin 100\pi t_2$

or $\frac{1}{\sqrt{2}} = \sin 100\pi t_2$ or $100\pi t_2 = \frac{\pi}{4}$

or $t_2 = \frac{1}{400}$ s

Required time $= t_1 - t_2$

$$= \frac{1}{200} - \frac{1}{400} = \frac{2-1}{400} = \frac{1}{400}\text{ s}$$

$$= 2.5 \times 10^{-3}\text{ s}$$

24. c. Quality factor

$$= \frac{f_0}{f_2 - f_1} = \frac{600}{650 - 550} = \frac{600}{100} = 6$$

25. a. $E = E_0 \cos\omega t$

$\therefore \quad \omega = 5\pi$

$2\pi\nu = 50\pi \Rightarrow \nu = 25\,\text{Hz}$

In one cycle ac current becomes zero twice.

Therefore, 50 times the current becomes zero in 1 s.

26. a. In an ac circuit, a pure inductor does not consume any power. Therefore, power is consumed by the resistor only.

$\therefore \quad P = R^2{}_v R$

or $\quad 108 = (3)^2 R$

or $\quad R = 12\,\Omega$

27. d. Quantity of heat liberated in the ammeter of resistance R

i. due to direct current of 3 A = [(3)^{2}R/J)]

ii. due to alternating current of 4 A = [(4)^{2}R/J]

Total heat produced per second

$$= \frac{(3)^2 R}{J} + \frac{(4)^2 R}{J} = \frac{25R}{J}$$

Let the equivalent alternating current be I virtual A, then

$$\frac{I^2 R}{J} = \frac{25R}{J}$$

or $\quad I = 5\,\text{A}$

28. c. $\lambda = 300\,\text{m}$

$C = 3 \times 10^8\,\text{m/s}$

Frequency $n = \frac{C}{\lambda} = \frac{3 \times 10^8}{300} = 10^6\,\text{Hz}$

Resonance frequency

$$n = \frac{1}{2\pi\sqrt{LC}}$$

or $\quad = \sqrt{LC} = \frac{1}{2\pi n}$

or $\quad LC = \frac{1}{4\pi^2 n^2}$

Here $\quad C = 2.4 \times 10^{-6}\,\text{F}$

$$\therefore \quad L = \frac{1}{4\pi^2 (10^6) \times 2.4 \times 10^{-6}}$$

$$\equiv 10^{-8}\,\text{H}$$

29. a. Change on the capacitor,

$q_0 = CV = 1 \times 10^{-6} \times 1 = 10^{-6}\,\text{C}$

Here $\quad q = q_0 \sin\omega t$

or $\quad I_0 = \omega q_0 = \text{Maximum current}$

Now $\quad \omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-9}}} = (10^9)^{1/2}$

$\therefore \quad I_0 = (10^9)^{1/2} \times (1 \times 10^{-6})$

$= \sqrt{1000}\,\text{m A}$

30. b. $E = E_0 \sin\omega t$

voltage read is rms value.

$\therefore \quad E_0 = \sqrt{2} \times 234\,\text{V} = 331\,\text{V}$

and $\quad \omega t = 2\pi n t = 2\pi \times 50 \times t$

$= 100\pi t$

Thus, the equation of the line voltage is given by

$E = 331 \sin(100\pi t)$

31. b. Here inductance and resistance are connected in series. We know that in the case of resistance, both the current and the potential difference are in the same phase. In case of inductance, when current is zero, potential difference is maximum and when current reaches its maximum value at $\omega t = \pi/2$, the potential difference becomes zero. Thus, the potential difference leads the current by $\pi/2$ or current lags behind the potential difference by $\pi/2$.

32. b. In an ac circuit, pure capacitor does not consume any power. Therefore, power is consumed by the resistor only.

$\therefore \quad P = I_v{}^2 R$

or $\quad 100 = (2)^2 R$

or $\quad R = 25\,\Omega$

33. c. Impedance $Z = \frac{E_v}{I_v}$

or $\quad Z = \frac{200}{2} = 100\,\Omega$

But $\quad Z^2 = R^2 + \left(\frac{1}{\omega C}\right)^2$

or $\quad \left(\frac{1}{\omega C}\right)^2 = Z^2 - R^2$

$= (100)^2 - (25)^2 = 125 \times 75$

or $\quad = \frac{1}{\omega C} = \sqrt{125 \times 75}$

$\therefore$ Capacitive reactance

$$\frac{1}{\omega C} = \sqrt{125 \times 75}$$

34. d. $\frac{1}{\omega C} = \sqrt{125 \times 75}$

$$\therefore \quad C \doteq \frac{1}{100\pi\sqrt{125 \times 75}}$$

Here $\quad \omega = 2\pi n$

$= 2\pi \times 50\,\text{radian/sec}$

$$\therefore \quad C = \frac{1}{100\pi\sqrt{125 \times 75}}\,\text{F}$$

35. b. Here $R = X_L = X_C$ $\quad$ ($\therefore$ voltage across them is same)

Total voltage in the circuit,

$V = I[R^2 + (X_L - X_C)^2]^{1/2}$

$= IR = 10\,\text{V}$

When capacitor is short circuited,

$$I = \frac{10}{(R^2 + X_L^2)^{1/2}} = \frac{10}{\sqrt{2}R}$$

∴ Potential drop across inductance

$$= IX_L = IR = 10\text{ V}$$

36. a. $R = \frac{125}{12.5} = 10\ \Omega$

$$X_L = \omega L = 2\pi n L = \frac{V}{I} = \frac{125}{10}$$

$$= 12.5$$

∴ $2\pi n L = 12.5$

or $2\pi L = \frac{12.5}{50} = 0.25$

∴ $X_L = 2\pi L \times n = 0.25 \times 40$

$$= 10\ \Omega$$

Impedance of the circuit

$$Z = \sqrt{R^2 + X_L^2} = 10\sqrt{2}\ \Omega$$

∴ Current $= \frac{100\sqrt{2}}{10\sqrt{2}} = 10\text{ A}$

37. b. Given $I = 5 + 10\sin\omega t$

$$I_{\text{eff}} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T (5 + 10\sin\omega t)^2\, dt\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T \left(25 + 100\sin\omega t + 100\sin^2\omega t\right)\right]^{1/2}$$

But as $\frac{1}{T}\int_0^T \sin\omega t\, dt = 0$

and $\frac{1}{T}\int_0^T \sin\omega t\, dt = \frac{1}{2}$

so $I_{\text{eff}} = \left[25 + \frac{1}{2} \times 100\right]^{1/2} = 5\sqrt{3}\text{ A}$

38. d. For an ideal choke, $I_v = \frac{E_v}{\omega L}$

or $8 = \frac{100}{100\pi \times L}$

∴ $L = \frac{1}{8\pi}\text{H}$

For a pure resistor,

$$I_v = \frac{E_v}{R} \text{ or } 10 = \frac{100}{R}$$

∴ $R = 10\Omega$

When connected in series:

$$I_v = \frac{E_v}{\sqrt{R^2 + (\omega L)^2}}$$

$$= \frac{150}{\sqrt{(10)^2 + \left(2\pi \times 40 \times \frac{1}{8\pi}\right)^2}}$$

$$= \frac{150}{\sqrt{(10)^2 + (10)^2}} = \frac{15}{\sqrt{2}}\text{ A}$$

39. d. The resistor and the inductor are connected in parallel. I_1 and I_2 are the currents through the ideal choke and the pure resistor. There is phase difference of $\pi/2$ between I_1 and I_2. Hence the resultant current I is given by

$$I = \sqrt{I_1^2 + I_2^2}$$

Now $I_1 = \frac{E_v}{\omega L} = \frac{150}{\left(\frac{1}{8\pi}\right) \times 2\pi \times 40} = 15\text{ A}$

and $I_2 = \frac{E_v}{R} = \frac{150}{10} = 15\text{ A}$

∴ $\sqrt{I_1^2 + I_2^2} = \sqrt{(15)^2 + (15)^2}\text{ A} = 15/\sqrt{2}\text{ A}$

40. a. $L\omega = 5 \times 10^{-3} \times 2000 = 10\ \Omega$

$$\frac{I}{C\omega} = \frac{1}{50 \times 10^{-6} \times 2000} = \frac{100}{10}\Omega = 10\ \Omega$$

Since $L\omega = \frac{1}{L\omega}$

∴ $Z = R = 6 + 0.10 + 4 = 10.1\ \Omega$

$$I_0 = \frac{E}{Z} = \frac{20}{10.1}\text{ A} \approx 2\text{ A}$$

41. a. $P = E_v I_v \cos\phi;\ P = E_v \frac{E_v}{R}\frac{R}{Z}$

or $P = \frac{E_v^2 R}{Z^2} = \frac{110 \times 110 \times 11}{22 \times 22}\text{ W} = 275\text{ W}$

42. b. $V_C^2 + V_R^2 = V^2$

$$50^2 + V_R^2 = 110^2$$

$$V_R^2 = 110^2 - 50^2$$

$$V_R = \sqrt{110^2 - 50^2}$$

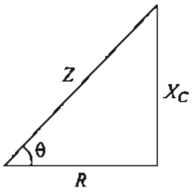


Fig. 10.76

$$V_R = \sqrt{160 \times 60} = 98\text{ V}$$

Then, $I_V = \frac{\sqrt{160 \times 60}}{50}$ (∴ $R = 50\ \Omega$)

Also, $I_V = \dfrac{110}{\sqrt{R^2 + X_C^2}}$ $\quad \therefore \dfrac{98}{50} = \dfrac{110}{\sqrt{50^2 + X_C^2}}$

Flux X_C and now using $X_C = \dfrac{1}{\omega C}$, we get

$C = 3.3\ \mu\text{F}$

43. c. $I = \dfrac{E}{Z}, I = \dfrac{220}{Z}, Z = 220\ \Omega$

$Z^2 = R^2 + X_L^2, \quad \therefore X_L = \sqrt{Z^2 - R^2}$

$L = \dfrac{1}{\omega}\sqrt{Z^2 - R^2}$

$L = \dfrac{1}{2\pi f}\sqrt{Z^2 - R^2} = 0.68\ \text{H}$

$\therefore \quad V_L = \omega LI = 2\pi \times 0.5 \times 0.68 \times 1$

$= 213.6\ \text{V}$

44. a. $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi \times 50 \times 8 \times 10^{-6}} = 398\ \Omega$

$V_{rms} = 220\ \text{V}, V_{peak} = \sqrt{2} \times 220$

$= 311\ \text{V}$

$q_{rms} = V_{rms}\ C = 1.76 \times 10^{-3}\ C$

$q_{peak} = \sqrt{2}\, q_{rms} = 1.76 \times 10^{-3} \times \sqrt{2}$

$= 2.5 \times 10^{-3}\ \text{C}$

45. a. Resultant current is superposition of two currents, i.e., I (instantaneous total current) $= 6 + I_0 \sin \omega t$

dc ammeter will read average value

$= \overline{6 + I_0 \sin \omega t} = 6 \qquad (\because\ \overline{I_0 \sin \omega t} = 0)$

ac ammeter will read

$= \sqrt{\overline{(6 + I_0 \sin^2 \omega t)^2}}$

$= \sqrt{36 + 12 I_0 \sin \omega t + I_0^2 \sin^2 \omega t}$

$(\because\ \overline{I_0 \sin \omega t} = 0)$

Since $\overline{\sin^2 \omega t} = \dfrac{1}{2}$ and $I_{rms} = 8 = \dfrac{I_0}{\sqrt{2}}$

$\therefore$ ac reading

$= \sqrt{36 + \dfrac{I_0^2}{2}} = \sqrt{36 + 64} = 10\ \text{A}$

46. c. $X_L = \omega L = 2\pi f L = 220\ \Omega$

$\phi = \tan^{-1}\left(\dfrac{X_L}{R}\right) = \tan^{-1}\dfrac{220}{200} = 45°$

$I = \dfrac{E}{\sqrt{R^2 + X_L^2}} = \dfrac{220}{\sqrt{(220)^2 + (220)^2}}$

Wattless component of current $= I \sin \phi$

$= \dfrac{1}{\sqrt{2}} \sin 45° = 0.5\ \text{A}$

47. c. $I = I_1 \cos \omega t + I_2 \sin \omega t$

$(I^2)_{mean} = I_1^2\ \overline{\cos^2 \omega t} + I_2^2\ \overline{\sin^2 \omega t} + 2 I_1 I_2\ \overline{\cos \omega t \sin \omega t}$

$= I_1^2 \times \dfrac{1}{2} + I_2^2 \times \dfrac{1}{2} + 2 I_1 I_2 \times 0$

$I_{rms} = \dfrac{(I_1^2 + I_2^2)^{1/2}}{\sqrt{2}}$

48. b. For power to be consumed at the rate of

$\dfrac{1100}{5} = 220\ \text{W}$

We have $P = E_v I_v \cos \theta$

$220 = \dfrac{220 \times 220}{\sqrt{R^2 + L^2 \omega^2}} \times \dfrac{R}{\sqrt{R^2 + L^2 \omega^2}}$

where $\quad R = \dfrac{V^2}{P} = \dfrac{220^2}{1100} = 44\ \Omega$

$220 = \dfrac{(220)^2 \times 44}{44^2 + (L\omega)^2}$

$44^2 + (L\omega)^2 = 220 \times 44$

$(L\omega)^2 = \sqrt{220 \times 44 - 44^2}$

$= \sqrt{44(220 - 44)} = \sqrt{44 \times 176} = 88\ \Omega$

$L = \dfrac{88}{2\pi \times f} = \dfrac{88}{2\pi \times 50} = \dfrac{88}{2 \times 22} \times \dfrac{7}{50}$

$= 0.28\ \text{H}$

49. b. Wattless component of ac

$= I_v \sin \theta = \dfrac{E_v}{Z} \sin \theta$

$= \dfrac{220}{\sqrt{R^2 + L^2 \omega^2}} \times \dfrac{L\omega}{\sqrt{R^2 + L^2 \omega^2}}$

$\therefore \quad L\omega = 0.7 \times 2\pi \times 50$

$= \dfrac{220 \times L\omega}{(R^2 + L^2 \omega^2)} = 0.7 \times 2 \times \dfrac{22}{7} \times 50$

$= \dfrac{220 \times (0.7 \times 2\pi \times 50)}{(220^2 + 220^2)} = 220\ \Omega$

$I = \dfrac{220 \times 220}{220^2\ (2)} = \dfrac{1}{2} = 0.5\ \text{A}$

50. c. $E_1 = E_0 \sin \omega t;\ E_2 = E_0 \sin(\omega t + \pi/3)$

$E = E_2 + E_1$

$= E_0 \sin(\omega t + \pi/3) + E_0 \sin \omega t$

$= E_0\,[2 \sin(\omega t + \pi/6) \cos(\pi/6)]$

$= \sqrt{3}\ E_0 \sin(\omega t + \pi/6)$

51. d. If an ac source $E = E_0 \sin \omega t$ is applied across an inductance and capacitance in parallel, the current in inductance will lag the applied voltage while across the capacitor will lead, and so,

$$I_L = \frac{V}{X_L} \sin\left(\omega t - \frac{\pi}{2}\right) = -0.8 \cos \omega t$$

$$I_C = \frac{V}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right) = +0.6 \cos \omega t$$

So the current drawn from the source

$$I = I_L + I_C = -0.2 \cos \omega t$$

$$|I_0| = 0.2 \text{ A}$$

52. d. As current at any instant in the circuit will be

$$I = I_{dc} + I_{ac} = a + b \sin \omega t$$

so, $$I_{\text{eff}} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{1/2} = \left[\frac{1}{T}\int_0^T (a + b\sin \omega t)^2 dt\right]^{1/2}$$

i.e., $$I_{\text{eff}} = \left[\frac{1}{T}\int_0^T (a^2 + 2ab \sin \omega t + b^2 \sin^2 \omega t)\, dt\right]^{1/2}$$

but as $$\frac{1}{T}\int_0^T \sin \omega t\, dt = 0$$

and $$\frac{1}{T}\int_0^T \sin^2 \omega t\, dt = \frac{1}{2}$$

So, $$I_{\text{eff}} = \left[a^2 + \frac{1}{2}b^2\right]^{1/2}$$

53. c. The equation of a semi-circular wave (shown in Fig. 10.52) is,

$$x^2 + y^2 = a^2 \quad \text{or,} \quad y^2 = a^2 - x^2$$

$$I_{\text{rms}} = \sqrt{\frac{1}{2a}\int_{-a}^{+a} y^2 dx}$$

$$I_{\text{rms}}^2 = \frac{1}{2a}\int_{-a}^{+a} (a^2 - x^2)\, dx$$

$$= \frac{1}{2a}\int_{-a}^{+a} (a^2 dx - x^2)\, dx = \frac{1}{2a}\left|a^2 x - \frac{x^3}{3}\right|_{-a}^{+a}$$

$$= \frac{1}{2a}\left(a^3 - \frac{a^3}{3} + a^3 - \frac{a^3}{3}\right) = \frac{2a^2}{3}$$

$$I_{\text{rms}} = \sqrt{\frac{2a^2}{3}} = 0.816a$$

54. b. Comparing $E = 200\sqrt{2} \sin (100\, t)$ with $E = E_0 \sin \omega t$, we find that

$$E_0 = 200\sqrt{2} \text{ V and } \omega = 100 \text{ (rad/s)}$$

So, $$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 10^{-6}} = 10^4\ \Omega$$

And as ac instruments read the rms value, the reading of ammeter will be

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{Z} = \frac{E_0}{\sqrt{2}\, X_C}$$

$$\left[\text{as } E_{\text{rms}} = \frac{E_0}{\sqrt{2}} \text{ and } Z = X_C\right]$$

So, $$I_{\text{rms}} = \frac{200\sqrt{2}}{\sqrt{2} \times 10^4} = 20 \text{ mA}$$

55. d. At resonance, the series combination of L and C gives zero impendance.

At resonance, the voltages across L and C are equal but opposit in phase.

56. b.

As resistance of the lamp and $R = \frac{V_S^2}{W} = \frac{100^2}{50} = 200\ \Omega$. The max. current $I = \frac{V}{R} = \frac{100}{200} = \frac{1}{2}$ A. So when the lamp is put in series with a capacitance and run at 200 V ac, from $V = IZ$, we have

$$Z = \frac{V}{I} = \frac{200}{(1/2)} = 400\ \Omega$$

Now as in case of $C - R$ circuit,

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

i.e., $$R^2 + \left(\frac{1}{\omega C}\right)^2 = 160000$$

or, $$\left(\frac{1}{\omega C}\right)^2 = 16 \times 10^4 - (200)^2 = 12 \times 10^4$$

$$\frac{1}{\omega C} = \sqrt{12} \times 10^2$$

$$C = \frac{1}{100\pi \times \sqrt{12} \times 10^2} \text{F}$$

$$C = \frac{100}{\pi\sqrt{12}}\,\mu\text{F} = \frac{50}{\pi\sqrt{3}} = 9.2\,\mu\text{F}$$

57. b. Here, $X_L = \omega L = 2\pi f L = 2\pi \times 50 \times 1 = 100\,\pi\Omega$

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}} = \frac{10^3}{\pi}\Omega$$

So,

$$X = X_L - X_C = 100\pi - \frac{10^3}{\pi} = 10^2\left[\frac{\pi^2 - 10}{\pi}\right]\Omega$$

So,

$$Z + \sqrt{R^2 + X^2} = |X| = 10^2\left[\frac{10 - 9.86}{\pi}\right]\Omega$$

i.e.,

$$Z = \frac{14}{\pi}\,\Omega = 4.47\,\Omega$$

58. a. From Kirchhoff's current law,

$$I_3 = I_1 + I_2 = 3\sin\omega t + 4\sin(\omega t + 90°)$$

$$= \sqrt{3^2 + 4^2 + 2(3)(4)\cos 90°}\,\sin(\omega t + \phi)$$

where $\tan\phi = \frac{4\sin 90°}{3 + 4\cos 90°} = \frac{4}{3}$

$$I_3 = 5\sin(\omega t + 53°)$$

59. b. $\varepsilon = \varepsilon_0 \sin\omega t$

If $I = I_m \sin(\omega t - \phi)$ then

$$vc = \left(\frac{1}{\omega C}\right) I_m \sin(\omega t - \phi - \pi/2)$$

And $v_L = (\omega L) I_m \sin(\omega t - \phi + \pi/2)$

So, $v_C + v_L + v_R = \varepsilon_0 \sin\omega t$

$$0 + v_R = \varepsilon_0 \sin\omega t$$

$$v_R = \varepsilon_0 \sin\omega t$$

also $\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R} = 0$, so $I = I_m \sin\omega t$

Hence answer is (b),

$$z = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2} = R$$

60. d. When capacitance is removed

$$\tan\theta = -\frac{\omega L}{R} \text{ or } \omega L = 100\tan 60° \quad \text{(i)}$$

When inductance is removed

$$\tan\phi = \frac{1}{(\omega C)(R)} \text{ or } = \frac{1}{\omega C} = 100\tan 60° \quad \text{(ii)}$$

So $z = R = 100\,\Omega$

$$I = v/R = 200/100 = 2\text{ A}$$

Power $P = I^2 R = 4 \times 100 = 400$ W

61. a. Current leads e.m.f. so the circuit is $R - C$.

$\tan\phi = x_C / R$, $\phi = 45°$, $R = 1000\,\Omega$, $\omega = 100$

$C = ?$

Since $\tan 45° = \frac{1}{\omega CR}$ So $C = 10\,\mu$F.

62. d. From the rating of the bulb, the resistance of the bulb can be calculated.

$$R = \frac{V_{rms}^2}{P} = 100\,\Omega$$

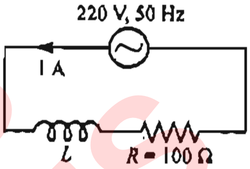


Fig. 10.77

For the bulb to be operated at its rated value the rms current through it should be 1A.

Also, $I_{rms} = \frac{V_{rms}}{Z}$

$$1 = \frac{200}{\sqrt{100^2 + (2\pi 50 L)^2}}; \quad L = \frac{\sqrt{3}}{\pi}\text{H}$$

63. a. According to the given problem,

$$I = \frac{V}{Z} = \frac{V}{[R^2 + (1/C\omega)^2]^{1/2}} \quad \text{(i)}$$

and,

$$\frac{1}{2} = \frac{V}{[R^2 + (3/C\omega)^2]^{1/2}} \quad \text{(ii)}$$

Substituting the value of I from equation (i) in (ii), we get

$$4\left(R^2 + \frac{1}{C^2\omega^2}\right) = R^2 + \frac{9}{C^2\omega^2}$$

$$\frac{1}{C^2\omega^2} = \frac{3}{5}R^2$$

So that

$$\frac{X}{R} = \frac{(1/C\omega)}{R} = \frac{\left(\frac{3}{5}R^2\right)^{1/2}}{R} = \sqrt{\frac{3}{5}}$$

64. b. $V_{rms} = \sqrt{16^2 + 20^2} = 25.6$ V

65. c. $I = 3\sin\omega t + 4\cos\omega t$

$$= 5\left[\frac{3}{5}\sin\omega t + \frac{4}{5}\cos\omega t\right] = 5[\sin(\omega t + \delta)]$$

$$\text{rms value} = \frac{5}{\sqrt{2}}$$

$$\text{Mean value} = \frac{\int_{T_1}^{T_2} I\,dt}{\int_{T_1}^{T_2} dt}$$

∴ The initial value is not given hence the mean value will be different for various time intervals. If voltage applied is $V = V_m \sin\omega t$ then I given by equation (i) indicates that it is ahead of V by δ where $0 < \delta < 90$ which indicates that the circuit contains R and C.

66. a. $v = v_0 \sin(\omega t + \pi/4) = v_0 \cos(\omega t - \pi/4)$

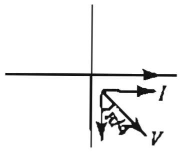


Fig. 10.78

Since V lags current, an inductor can bring it in phase with current.

67. b.

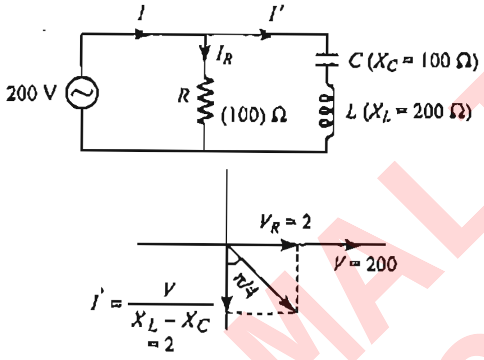


Fig. 10.79

$$I_R = \frac{V}{R} = \frac{200}{100} = 2 \text{ A}$$

$$I' = \frac{V}{X_L - X_C} = \frac{200}{100} = 2 \text{ A}$$

$$I = \sqrt{I_R^2 + I'^2} = 2\sqrt{2} \text{ A}$$

68. c. Resultant voltage = 200 V
Since V_1 and V_3 are 180° out of phase, the resultant voltage is equal to V_2.
$\therefore V_2 = 200$ V.

69. c. The circuit will have inductive nature if

$$\omega > \frac{1}{\sqrt{LC}} \left(\omega L > \frac{1}{\sqrt{LC}}\right)$$

Hence (a) is false. Also, if circuit has inductive nature the current will lag behind voltage. Hence (d) is also false.

If $\omega = \frac{1}{\sqrt{LC}} \left(\omega L = \frac{1}{\omega C}\right)$ the circuit will have resistance nature. Hence (b) is false.

$$\text{Power factor } \cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = 1$$

It $\omega L = \frac{1}{\omega C}$. Hence (c) is true.

70. c. $X_L = X_C$ at resonance

$\frac{X_L}{X_C} = 1$ for both circuits.

71. d. Since, $\cos\theta = \frac{R}{Z} = \frac{IR}{IZ} = \frac{8}{10} = \frac{4}{5}$
(Also $\cos\theta$ can never be greater than 1)
Hence (c) is wrong.
Also $IX_C > IX_L \Rightarrow X_C > X_L$
$\therefore$ current will be leading.
In an LCR circuit,
$v = \sqrt{(v_L - v_C)^2 + v_R^2} = \sqrt{(6-12)^2 + 8^2}$
$V = 10$; which is less than voltage drop across capacitor.

72. c. If we have all R, L and C then I vs. E will be
To obtain a leading phase difference of $\pi/4$.

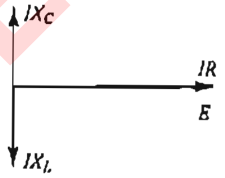


Fig. 10.80

If $X_L < X_C$ and we use all R, L and C in the circuit, then the resultant graph will be

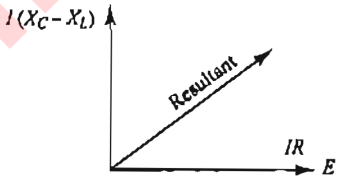


Fig. 10.81

which can give a leading phase difference of $\pi/4$.
Similarly, if we have only resistance and capacitor then we can obtain a phase difference of $\pi/4$ (leading) for suitable values of I, X_C, and R. But we cannot obtain a leading phase difference of $\pi/4$ if we use only capacitor (phase difference of $\pi/2$), or only (inductor and resistor) (phase difference $\pi/2$), or only resistor (phase difference of 0).

73. a.

1. $$V = \frac{V_0}{T/4} t; \; V = \frac{4V_0}{T} t$$

$$V_{rms} = \sqrt{<V^2>} = \frac{4V_0}{T}\sqrt{<t>}$$

$$= \frac{4V_0}{T} \left\{ \frac{\int_0^{T/4} t^2 dt}{\int_0^{T/4} dt} \right\}^{1/2} = \frac{V_0}{\sqrt{3}}$$

74. d. $<V> = \frac{\int_0^T v dt}{\int_0^T dt} = 0$

75. b.

$$(0.5)^2 R\left(\frac{T}{2}\right)+(0.5)^2 R\left(\frac{T}{2}\right)=I^2{}_{\text{rms}} RT$$

or $I_{\text{rms}}=\frac{1}{2}\text{ A}=0.5\text{ A}$

76. b.

$$f=\frac{1}{2\pi\sqrt{LC_0}} \text{ and } f-50=\frac{1}{2\pi\sqrt{LC}}$$

Given $f=100{,}000$ Hz

Hence $\frac{f-50}{f}=\sqrt{\frac{C_0}{C}}$

$$K=\frac{C}{C_0}=\left(\frac{f}{f-f_0}\right)^2=1.01$$

Multiple Correct Answers type

1. a., b., c.

$Z=R$ at resonance

$\therefore \quad E_V=I_V R \Rightarrow I_V=\frac{E_V}{R}$

$\therefore \quad X_L=X_C$

$\therefore \quad \omega=\frac{1}{\sqrt{\ell_c}}=\pi\times 10^3$

2. a., c. When $f=1000$, $\omega=\frac{1}{WC}=WL$

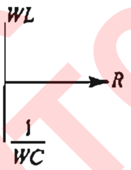


Fig. 10.82

When $f=900$, $\frac{1}{WC}>WL$ and current leads voltage hence it behaves like capacitive V_C and L_L differs by π

3. b., c.

$$\frac{R}{L}=\frac{R}{V/(di/dt)}=\frac{R}{V}\frac{di}{dt}=\frac{1}{dt}=f$$

$$\frac{1}{RC}=\frac{V}{Rq}=\frac{i}{q}=\frac{1}{t}=f$$

$$\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{[(Vdt/di)\times(dq/V)]}}$$

$$=\frac{1}{\sqrt{[(dt/di)\times(di/dt)]}}=\frac{1}{dt}=f$$

4. a., b., c.

$H=I^2Rt/4.2$ (Heat produced in a conductor is independent of the nature of current. If the current flowing in a conductor changes its direction, then also heat being produced in it will not change). Hence, (a), (b) and (c) are true.

5. a., b., c.

The frequency

$$f=\frac{1}{2\pi}\sqrt{\frac{1}{LC}}=\frac{1}{2\pi}\sqrt{\frac{1}{6\times 10^{-6}}}=65\text{ Hz}$$

The current at resonance is

$$I_0=\frac{200\text{ V}}{5\,\Omega}=40\text{ A}$$

The quantity $V_0=I\sqrt{R^2+(\omega L)^2}$

$$=40\sqrt{25+(0.6\times 2\pi\times 65)^2}=9.8\text{ kV}$$

$$V_1=\frac{I}{\omega C}=\frac{40\times 10^5}{2\pi\times 65}=9.8\text{ kV}$$

6. b., c.

Let I_1 and I_2 be currents through B_1 and B_2.

Then $I_1\times\sqrt{R^2+X_C^2}=220$

$$\therefore \quad \frac{I_2}{I_1}=\frac{\sqrt{R^2+X_C^2}}{\sqrt{R^2+X_L^2}}=\frac{\sqrt{R^2+(1/\omega C)^2}}{\sqrt{R^2+(\omega L)^2}}$$

$$=\frac{\sqrt{R^2+40}}{\sqrt{R^2+9.87}}$$

So $I_2>I_1$

Bulb B_2 will be brighter than B_1. As the frequency increases, X_C decreases while X_L increases, so I_2 becomes less than I_1. Hence the brightness of B_1 will increase and that of B_2 will decrease.

7. a., c., d.

Let voltage across resistor, inductor and capacitor be V_R, V_L and V_C, respectively.

Then, $V_R^2+V_L^2=V_1^2$

And, $V_R^2+V_2^2=V^2$

$V_R=50\text{ V}$, $V_L=86.650\text{ V}$

$|V_L-V_C|=V^2 \Rightarrow V_C=186.65\text{ V}$

Power factor of the circuit is

$$\cos\phi=\frac{V_R}{V_L}=\frac{50}{130}=\frac{5}{13}$$

As $V_C>V_L$, so the circuit is capacitative.

Assertion-Reasoning Type

1. a. For certain values of $\cos\theta$ (power factor) two values of θ are possible. One is positive the other is much negative. Accordingly, the applied e.m.f. may lead or lag.

2. d. The maximum value of rms current $= \frac{\mathcal{E}_{\text{ms}}}{Z} = \frac{\mathcal{E}_{\text{ms}}}{R}$. It does not depend upon ω.

3. c. $P_{\text{av}} \frac{VI\cos\phi}{2}$

At resonance condition $\cos\phi = 1$

But $Z = R$

Which is minimum.

4. b. *LR* circuit

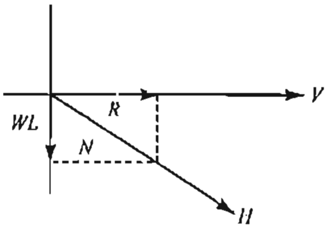


Fig. 10.83

CR circuit.

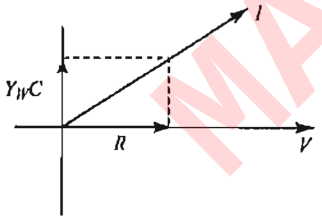


Fig. 10.84

5. b. For half cycle $I_{\text{mean}} = 0.636 I_0$

or $E_{\text{mean}} = 0.636\, E_0$

Average value is always defined over a half cycle cause in next half cycle it will be opposite in direction. Hence for one complete cycle, average value will be zero.

6. a. At resonant frequency

$$X_L = X_C \quad \therefore Z = R \text{ (minimum)}$$

Therefore, current in the circuit is maximum.

7. c. Voltage will be added vectorially.

8. c. $X_c = \frac{1}{\omega c}$

$\omega \neq 0$ area for AC.

$$I_{\text{rms}} \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{1/2} = \left[\frac{\int_0^{2\pi/\omega} I_0^2 \sin^2 \omega t\, dt}{\int_0^{2\pi/\omega} dt}\right]^{1/2} \frac{I_0}{\sqrt{2}}.$$

9. c. Rate of heat generated depends on time.

10. b. Average power consumed by capacitor or inductor is zero.

11. b. Motion of electron is random with drift velocity opposite to the direction of current.

12. a.

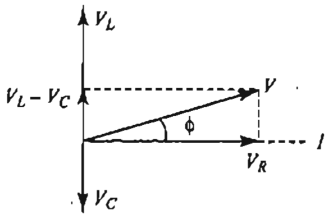


Fig. 10.85

13. c. $\tan\phi = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega c}}{R}$

When $X_L > X_C$ then $\tan\phi$ is positive, i.e., ϕ is positive (between 0 and $\pi/2$). Hence e.m.f. leads the current.

14. b. Like direct current, alternating current also produces magnetic field. But the magnitude and direction of the field goes on changing continuously with time.

15. c. Both ac and dc produce heat, which is proportional to square of the current. The reversal of direction of current in ac is immaterial so far as production of heat is concerned.

16. d. Statement I is false because the given relation is true if all voltages are instantaneous.

17. d. In resonance condition when energy across capacitor is maximum, energy stored in inductor is zero, vice versa is also true. Hence, statement I is a false.

Comprehension Type

For Problems 1 – 3

1. a., 2. b., 3. a.

Sol. 1. As in case of ac,

$$I_0 \mathrm{I} = 0.2\, A a V = V_0 \sin(\omega t - \phi)$$

The peak value $V_0 = 220\sqrt{2} = 311\,\text{V}$

and as in case of ac,

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}};\ V_{\text{rms}} = 220\,\text{V}$$

2. In case of ac,

$$V_{\text{av}} = \frac{2}{\pi} V_0 = \frac{2}{\pi} \times 311 = 198.17\,\text{V}$$

3. As $\omega = 2\pi f$, $2\pi f = 314$ i.e., $f = \frac{314}{2 \times \pi} = 50\,\text{Hz}$

For Problems 4 – 5

4. b., 5. a.

Sol. 4. As the equation of the saw-tooth wave shown in Fig. 10.65 will be

$$V = \frac{2V_0}{T} t - V_0 = V_0\left(\frac{2t}{T} - 1\right)$$

$$V_{\text{av}} = \frac{\int_0^{T/2} V\, dt}{\int_0^{T/2} dt} = \frac{2}{T}\int_0^{T/2} V_0\left(\frac{2t}{T} - 1\right) dt$$

i.e.,

$$V_{av} = \frac{2}{T}V_0\left[\frac{T}{4}-\frac{T}{2}\right] = \left|\frac{V_0}{2}\right|$$

5. $$V_{rms} = \left[\frac{\int_0^T V^2 dt}{\int_0^T dt}\right]^{1/2} = \frac{V_0}{\sqrt{T}}\left[\int_0^T\left(\frac{4t^2}{T^2}-\frac{4t}{T}+1\right)dt\right]^{1/2}$$

i.e., $$V_{rms} = \frac{V_0}{\sqrt{T}}\left[\left(\frac{4t^3}{3T^2}-\frac{4t^2}{2T}+t\right)_0^T\right]^{1/2}$$

i.e., $$V_{rms} = \frac{V_0}{\sqrt{T}}\left[\frac{4T}{3}-2T+T\right]^{1/2} = \frac{V_0}{\sqrt{3}}$$

For Problems 6 – 7

6. a., 7. b.

Sol. For finding the average value, we would find the total area of the trapezium and divide it by π (as shown in Fig. 10.66).

$$\text{Area} = 2\times\Delta OAE + \text{rectangle } ABDE$$

$$= 2\times(1/2)\times F_m\alpha + (\pi-2\alpha)F_m = (\pi-\alpha)F_m$$

$$\text{Average value} = (\pi-\alpha)F_m/\pi$$

rms value : From similar triangles, we get

$$\frac{y}{\theta} = \frac{F_m}{\alpha} \quad \text{or,} \quad y^2 = \frac{F_m^2}{\alpha^2}\theta^2$$

This gives the equation of the signal over the two triangles OAE and DBC. The signal remains constant over the angle α to $(\pi - \alpha)$, i.e., over an angular distance of $(\pi - \alpha) - \alpha = (\pi - 2\alpha)$.

Sum of the square

$$= \frac{2F_m^2}{\alpha^2}\int_0^\alpha \theta^2 d\theta + F_m^2(\pi-2\alpha) = F_m^2\left(\pi-\frac{4\alpha}{3}\right).$$

The mean value of the squares

$$= \frac{1}{\pi}F_m^2\left(\pi-\frac{4\alpha}{3}\right) = F_m^2\left(1-\frac{4\alpha}{3\pi}\right)$$

$$\text{rms value} = F_m\sqrt{\left(1-\frac{4\alpha}{3\pi}\right)}$$

For Problems 8 – 9

8. d., 9. a.

Sol. 8. The slope of the curve AB is $BC/AC = 20/T$. Next, consider the function y at any time t. It is seen that

$$\frac{DE}{AE} = \frac{BC}{AC} = \frac{10}{T}$$

$$\frac{(y-10)}{t} = \frac{10}{T}$$

$$Y = 10 + \left(\frac{10}{T}\right)t$$

This gives us the equation for the function of one cycle.

$$Y_{avl} = \frac{1}{T}\int_0^T y\,dt = \frac{1}{T}\int_0^T\left(10+\frac{10}{T}t\right)dt$$

$$= \frac{1}{T}\int_0^T\left[10dt+\frac{10}{T}dt\right] = \frac{1}{T}\left|10t+\frac{5t^2}{T}\right|_0^T = 15$$

9. Mean square value

$$= \frac{1}{T}\int_0^T y^2 dt = \int_0^T\left(10+\frac{10}{T}t\right)^2 dt$$

$$= \frac{1}{T}\int_0^T\left(100+\frac{100}{T^2}t^2+\frac{200}{T}t\right)$$

$$= \frac{1}{T}\left|100t+\frac{100t^3}{3T^2}+\frac{100t^2}{T}\right|_0^T = \frac{700}{3}$$

$$\text{rms value} = 10\sqrt{7/3} = 15.2$$

For Problems 10 – 11

10. b., 11. a.

Sol. Here,

$$X_L = \omega L = 2\pi fL = 2\pi\times50\times0.21 = 21\pi\,\Omega$$

So, $$Z = \sqrt{R^2+X^2} = \sqrt{12^2+(21\pi)^2}$$

$$= \sqrt{144+4348}$$

So, (a) $$I = \frac{V}{Z} = \frac{220}{67.02} = 3.28\text{A}$$

and (b) $$\phi = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{21\pi}{12}\right)$$

$$= \tan^{-1}(5.5) = 79.7^\circ$$

i.e., the current will lag the applied voltage by 79.7° in phase.

For Problems 12 – 13

12. a., 13. c.

Sol. In case of a coil, i.e., L–R circuit, $I = \dfrac{V}{Z}$

with $$Z = \sqrt{R^2+X^2} = \sqrt{R^2+(\omega L)^2}$$

So when dc is applied,

$\omega = 0$, so, $Z = R$

and hence, $I = \dfrac{V}{R}$

i.e., $$R = \frac{V}{I} = \frac{100}{1} = 100\,\Omega$$

and when ac of 50 Hz is applied

$$I = \frac{V}{Z} \quad \text{i.e.,} \quad Z = \frac{V}{I} = \frac{100}{0.5} = 200\,\Omega$$

but, $Z = \sqrt{R^2 + \omega^2 L^2}$

i.e., $\omega^2 L^2 = Z^2 - R^2$

i.e., $(2\pi f L)^2 = 200^2 - 100^2 = 3 \times 10^4$ (as $\omega = 2\pi f$)

So, $L = \dfrac{\sqrt{3} \times 10^2}{2\pi \times 50} = \dfrac{\sqrt{3}}{\pi} \text{H} = 0.55 \text{H}$

For Problems 14 – 17

14. a., 15. d., 16. b., 17. d.,

Sol. 14. As this circuit is a series LCR circuit, current will be maximum at resonance, i.e.,

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(4.9 \times 10^{-3})(10^{-6})}} = \frac{10^5}{7} \text{ rad/s}$$

with, $I_m = \dfrac{V_M}{R} = \dfrac{10}{(32 + 68)} = \dfrac{1}{10} \text{A}$

So the impedance, $Z_P = [R_1^2 + (1/\omega C)^2]^{1/2}$

$$= \sqrt{5924} = 77 \,\Omega$$

15. $Z_Q = [R_2^2 + (\omega L)^2]^{1/2}$

$$= [(68)^2 + (4.9 \times 10^{-3} \times 10^5 / 7)^2]^{1/2}$$

$$= \sqrt{9524} = 97.6 \,\Omega$$

16. $V_P = IZ_P = \dfrac{1}{10} \times (77) = 7.7 \text{V}$

17. $V_Q = IZ_Q = \dfrac{1}{10} \times (97.6) = 9.76 \text{V}$

For Problems 18 – 20

18. b., 19. a., 20. c.

Sol. 18. At resonance as $X = 0$,

$$I = \frac{V}{R} = \frac{60}{120} = \frac{1}{2} \text{A}$$

as $V_L = IX_L = I\omega L$, $L = \dfrac{V_L}{I\omega}$

So, $L = L = \dfrac{40}{(1/2) \times 4 \times 10^5} = 0.2 \text{mH}$

$$\omega_0 = \frac{1}{\sqrt{LC}}, \; C = \frac{1}{L\omega_0^2}$$

i.e., $C = \dfrac{1}{0.2 \times 10^{-3} \times (4 \times 10^5)^2} = \dfrac{1}{32} \mu\text{F}$

Now in case of series LCR circuit,

$$\tan \phi = \frac{X_L - X_C}{R}$$

So current will lag the applied voltage by 45° if,

$$\tan 45° = \frac{\omega L - \dfrac{1}{\omega C}}{R}$$

$$1 \times 120 = \omega \times 2 \times 10^{-4} - \frac{1}{\omega (1/32) \times 10^{-6}}$$

$$\omega^2 - 6 \times 10^5 \omega - 16 \times 10^{10} = 0$$

i.e., $\omega = \dfrac{6 \times 10^5 \pm \sqrt{(6 \times 10^5)^2 + 64 \times 10^{10}}}{2}$

i.e., $\omega = \dfrac{6 \times 10^5 + 10 \times 10^5}{2} = 8 \times 10^5 \text{rad/s}$

For Problems 21 – 22

21. c., 22. b.

Sol. 21. In case of a coil as $Z = \sqrt{R^2 + \omega^2 L^2}$

i.e., $I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + \omega^2 L^2}}$

So when dc is applied as $\omega = 0$,

$$I = \frac{V}{R}, \quad \text{i.e.,} \quad R = \frac{12}{4} = 3 \,\Omega$$

and when ac is applied,

$$I = \frac{V}{Z}, \quad \text{i.e.,} \quad Z = \left(\frac{V}{I}\right) = \left(\frac{12}{2.4}\right) = 5 \,\Omega$$

$R^2 + X_L^2 = 5^2$ (as $Z = \sqrt{R^2 + X_L^2}$)

So,

$X_L^2 = 5^2 - R^2 = 5^2 - 3^2 = 4^2$, i.e., $X_L = 4 \Omega$

but as, $X_L = \omega L$, $L = \dfrac{X_L}{\omega} = \dfrac{4}{50} = 0.08 \text{ H}$

22. Now when the capacitor is connected to the above circuit in series,

$$X_c = \frac{1}{\omega C} = \frac{1}{50 \times 2500 \times 10^{-6}} = \frac{10^3}{125} = 8 \,\Omega$$

so, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{3^2 + (4 - 8)^2} = 5 \,\Omega$$

and hence, $I = \dfrac{V}{Z} = \dfrac{12}{5} = 2.4 \text{A}$

So, $P_{av} = V_{rms} I_{rms} \cos \phi = (I_{rms} \times Z) \times I_{rms} \times \left(\dfrac{R}{Z}\right)$

i.e., $P_{av} = I_{rms}^2 R = (2.4)^2 \times 3 = 17.28 \text{ W}$

For Problems 23 – 24

23. a., 24. a.

Sol. $Z^2 = (x_C - x_L)^2 + R^2 = (31.85 - 6.28)^2 + (50)^2$

$$= 3153.675$$

$$P = \left(\frac{v_{\max}^2}{z^2}\right) R = \frac{100 \times 50}{2 \times 3153.675}$$

Heat produced in 20 min = $(P)(20 \times 60) = 951.27$ J

24. $x_C - x_L = 31.85 - 2(6.28) = 19.29$

$$I_m = \frac{10}{19.29} = 0.52$$

Hence, $I = 0.52 \sin(314t + \pi/2)$

$$= 0.52 \cos 314t$$

Matching Column Type

1. p. → c.; q. → a.; r. → d.; s. → b.

a. For sinusoidal curve $I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$

b. $$I_{\text{rms}}^2 = \frac{\int_0^T i^2 dt}{T} = \frac{4\int_0^{T/4} i^2 dt}{T}$$

$$= \frac{\int_0^{T/4} i^2 dt}{T/4} = \frac{\int_0^{T/4} \left(\frac{i_0 t}{T/4}\right)^2 dt}{T}$$

$$= \frac{i_0^2}{(T/4)^3} \int_0^{T/4} t^2 dt = \frac{i_0^2}{3} = I_{\text{rms}} = \frac{I_0}{\sqrt{3}}$$

For positive half cycle average current

$$= \frac{\int i\,dt}{\int dt} = \frac{1/2(i_0)(T/2)}{(T/2)} = \frac{i_0}{2}$$

Full cycle average current is zero.

c. For positive half cycle average current

$$= \frac{\int i\,dt}{\int dt} = \frac{i_0(T/2)}{T/2} = i_0$$

d. For full cycle average current

$$= \frac{\int i\,dt}{\int dt} = \frac{i_0(T/2) + 0}{T} = \frac{i_0}{2}$$

2. p → b., c.; q. → a., d.; r → a.; c., s. → b., d.

a. $\tan\phi = \frac{1/\omega C}{R} \Rightarrow \phi = \frac{\pi}{4}$, current leads source voltage because reactance is capacitive.

b. Pure inductive circuit $\phi = \frac{\pi}{2}$, current lags behind source voltage because reactance is inductive

c. as $R = 0$, $\tan\phi = \infty$

$\phi = \pi/2$, current leads source voltage because reactance is capacitive

d. $\tan\phi = \frac{\omega L}{R} = 1 \Rightarrow \phi = \frac{\pi}{4}$, current lags behind source voltage because reactance is inductive

3. p. → b., d.; q. → a.; r. → a.; s. → b., c.

Inductive reactance,

$$X_L = \frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times \frac{100}{\pi} \times 10^{-6}}\ \Omega = 100\ \Omega$$

Impendence of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 137.93\ \Omega$$

RMS value of the current through circuit is

$$I = \frac{220}{z} = 1.59\ A \simeq 1.6\ \text{A}$$

RMS value of voltage drop across the capacitor is

$$v_c = IX_C = 160\ \text{V}$$

Average power dissipated in the resistor is

$$P_{\text{av}} = V_R I \cos\phi = (IR) I \times \frac{R}{Z} = 185.6\ \text{W}$$

Average power dissipated in inductor and capacitor would be zero.

4. p. → a., b.; q. → c.; r. → a., b., c., d.; s. → d.

Depending on the value of L, C and R, circuit would be either capacitance, inductive or purely resistive.

For LR series circuit, the phasor diagram is as shown below

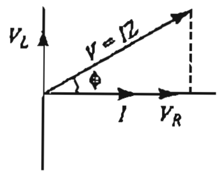


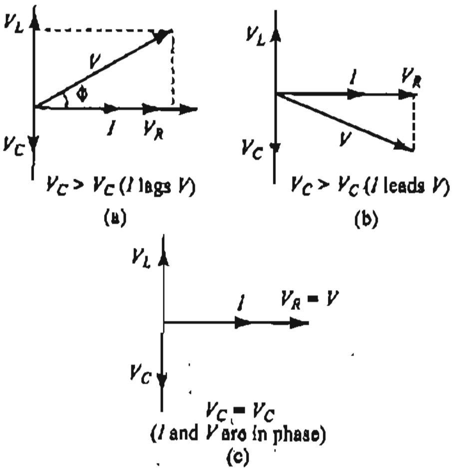
Fig. 10.86

I lags voltage by an angle $\phi(< \pi/2)$.

I lags V_L by an angle $\pi/2$.

For RC series AC circuit, I leads V by an angle less than $\pi/2$.

For LCR series AC circuit, the phasor of diagram is as:



$V_C > V_C$ (I lags V)
(a)

$V_C > V_C$ (I leads V)
(b)

$V_C = V_C$
(I and V are in phase)
(c)

Fig. 10.87

5. p. → a., b., c., d.; q. → a., c.; r. → a., b., c., d.; s. → c.

a. $V_R^2 + V_2^2 = V^2 \Rightarrow V_R = 150\text{ V}$

$V_R^2 + V_L^2 = V_1^2 \Rightarrow V_L = 50\text{ V}$

$|V_L - V_C| = V_2$

$V_C = 250\text{ V}$

$$\cos\phi = \frac{V_R}{V} = \frac{150}{250} = \frac{3}{5}$$

b. $V_C = V = 250\text{ V}$

And $V_R^2 + V_L^2 = 250^2$

$V_R = V_1 = 150\text{ V}$

$V_L = 200\text{ V}$

Power factor can be computed by determining the net reactance.

c. $V_R = V_3 = 150\text{ V}$

$V_1^2 = V_R^2 + V_L^2 \Rightarrow V_L = 50\text{ V}$

$V_2^2 = V_C^2 + V_R^2 \Rightarrow V_C = 250\text{ V}$

d. $\cos\phi = \frac{150}{250} = \frac{3}{5}$

$V_R = V = 250\text{ V}$

APPENDIX

A2

Miscellaneous Assignments and Archives on Chapters 5-10

EXERCISES

Objective Type

Solutions on page A2.31

1. A source of constant potential difference is connected across a conductor having irregular cross section as shown in Fig. A2.1.

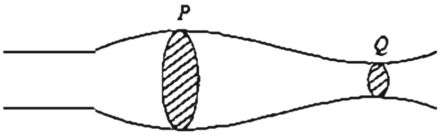


Fig. A2.1

- **a.** Electric field intensity at P is greater than that at Q
- **b.** Rate of electrons crossing per unit area of cross section at P is less than that at Q
- **c.** The rate of generation of heat per unit length at P is greater than that at Q
- **d.** Mean kinetic energy of free electrons at P is greater than that at Q

2. Current-voltage characteristics of two elements A and B are as shown in Figs. A.2 and A1.3

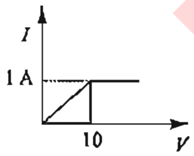


Fig. A1.2

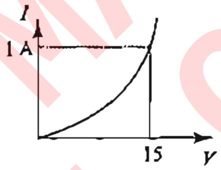
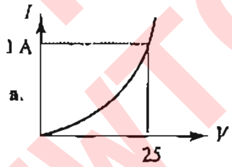
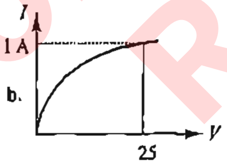
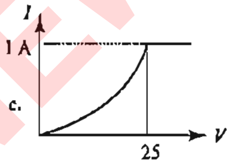


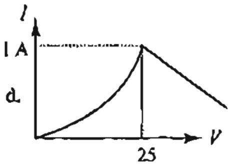
Fig. A1.3

Which of the following graphs represents current–voltage characteristics for their series combination?









3. In the given network (Fig. A2.4) the batteries getting charged are

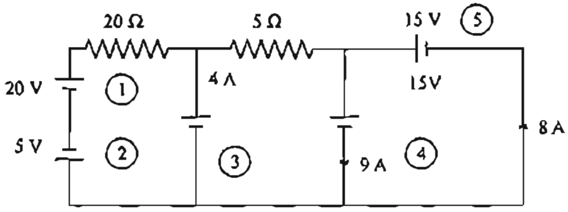


Fig. A2.4

- **a.** 1 and 3 **b.** 1, 3 and 5
- **c.** 1 and 4 **d.** 1, 2 and 5

4. When a galvanometer is shunted with a 4 Ω resistance the deflection is reduced to 1/5. If the galvanometer is further shunted with a 2 Ω wire the new deflection will be (assuming the main current remains the same)

- **a.** 5/13 of the deflection when shunted with 4 Ω only
- **b.** 8/13 of the deflection when shunted with 4 Ω only
- **c.** 3/4 of the deflection when shunted with 4 Ω only
- **d.** 3/13 of the deflection when shunted with 4 Ω only

5. When the key k is pressed at time t = 0, which of the following statements about the current I in the resistor AB of the given circuit is true?

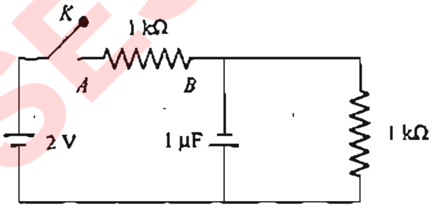


Fig. A2.5

- **a.** 2 mA at all time
- **b.** oscillates between 1 mA and 2 mA
- **c.** 1 mA at all time
- **d.** At $t = 0$, $I = 2$ mA and with time it finally reduces to 1 mA

6. Two electric bulbs rated P_1 and P_2 watt at V volt are connected in series across V volt mains, then their total power consumption P is

- **a.** $(P_1 + P_2)$ **b.** $\sqrt{P_1P_2}$
- **c.** $P_1 P_2/(P_1 + P_2)$ **d.** $(P_1 + P_2)/P_1P_2$

7. In the given circuit (Fig. A2.6), with steady current, the potential drop across the capacitor must be

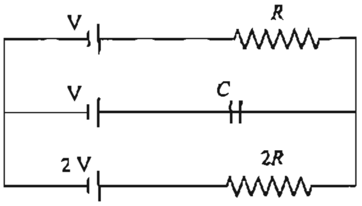


Fig. A2.6

- **a.** V **b.** V/2
- **c.** V/3 **d.** 2V/3

8. The potential difference between points A and $A2.$ in Fig. A2.7 is

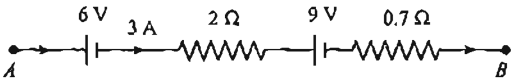


Fig. A2.7

- **a.** 3 V **b.** 15 V
- **c.** –5.1 V **d.** +5.1 V

9. The resistances in Wheatstone's bridge circuit as shown in the Fig. A2.8 have different values. The current through the galvanometer is zero. If all thermal effects are negligible, the current through the galvanometer may not be zero, when

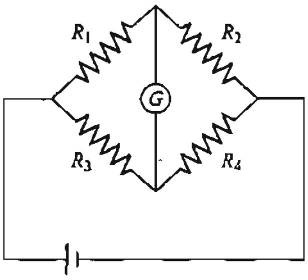


Fig. A2.8

a. the battery e.m.f. is doubled
b. the battery and galvanometer are interchanged
c. all resistances in the circuit are doubled
d. all resistances in the circuit are interchanged

10. In the circuit shown in Fig. A2.9, $P \neq R$. The reading of galvanometer is same with switch S is opened or closed. Then

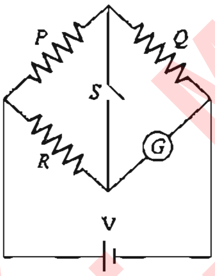


Fig. A2.9

a. $I_R = I_G$ b. $I_P = I_G$
c. $I_Q = I_G$ d. $I_Q = I_R$

11. Two cells A and B of electromotive forces 1.3 V and 1.5 V, respectively, are arranged as shown in Fig. A2.10. The voltmeter (assumed ideal) reads 1.45 V, the internal resistances of cells A and B are r_A and r_B, respectively. Which of the following is correct ?

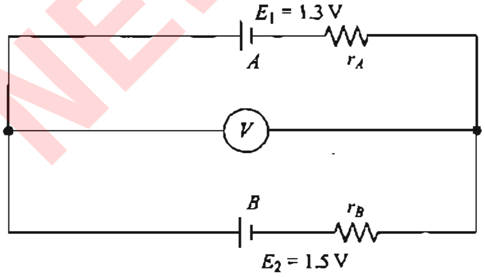


Fig. A2.10

a. $r_A = 2r_B$ b. $r_A = 3r_B$
c. $r_B = 2r_A$ d. $r_B = 3r_A$

12. In the circuit shown in Fig. A2.11, the battery E_1 has an e.m.f. of 12 V and zero internal resistance, while the battery E_2 has an e.m.f. of 2 V. If the galvanometer G reads zero, then the value of the resistance Y is

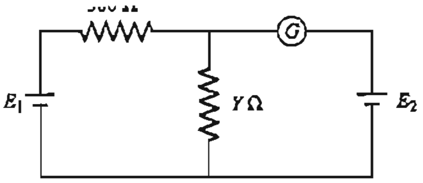


Fig. A2.11

a. 10 Ω b. 100 Ω
c. 500 Ω d. 200 Ω

13. The circuit shown in Fig. A2.12 is used to compare the e.m.f.s of two cells, E_1 and E_2 ($E_1 > E_2$). The null point is at C when the galvanometer is connected to E_1. When the galvanometer is connected to E_2, the null point will be

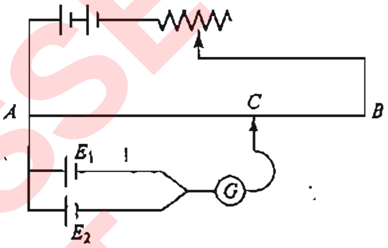


Fig. A2.12

a. to the left of C b. to the right of C
c. at C itself d. nowhere on AB

14. Two cells of e.m.f.s E_1 and E_2 and of negligible internal resistances are connected with two variable resistors as shown in the Fig. A2.13. When the galvanometer shows no deflection, the values of the resistances are P and Q.

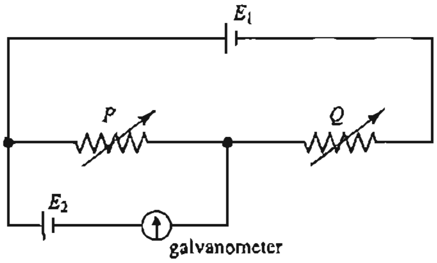


Fig. A2.13

What is the value of the ratio E_2/E_1?

a. $\frac{P}{Q}$ b. $\frac{P}{P+Q}$ c. $\frac{Q}{P+Q}$ d. $\frac{P+Q}{P}$

15. The Wheatstone's bridge shown in the Fig. A2.14 is balanced. If the positions of the cell C and the galvanometer G are now interchanged, G will show zero deflection

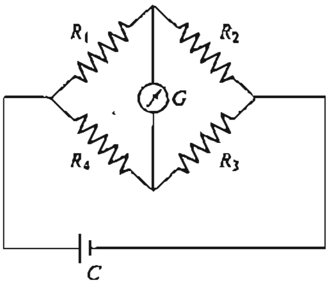


Fig. A2.14

a. in all cases
b. only if all the resistances are equal
c. only if $R_1 = R_3$ and $R_2 = R_4$
d. only if $R_1/R_3 = R_2/R_4$

16. In Fig. A2.15, two cells have equal e.m.f. E but internal resistances are r_1 and r_2. If the reading of the voltmeter is zero, then relation between R, r_1 and r_2 is

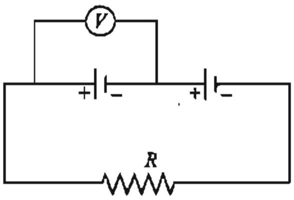


Fig. A2.15

a. $R = r_1 - r_2$
b. $R = r_1 + r_2$
c. $2r_1 - r_2$
d. $r_1 r_2$

17. A constant voltage dc source is connected, as shown in Fig. A2.16, across two resistors of resistances 400 kΩ and 100 kΩ.

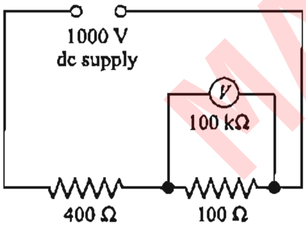


Fig. A2.16

What is the reading of the voltmeter, also of resistance 100 kΩ, when connected across the second resistor as shown?

a. 111 V
b. 250 V
c. 125 V
d. 333 V

18. In the given circuit (Fig. A2.17) in which case will the ammeter reading not change when R_2 is varied?

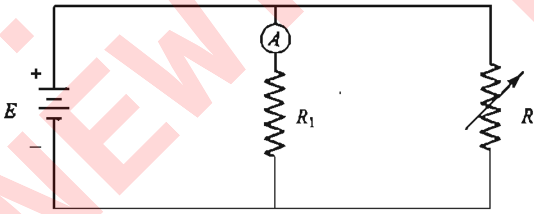


Fig. A2.17

a. $R_1 = r$
b. $R_1 = 2r$
c. $R_1 > R_2$
d. $r = 0$

19. An ammeter and a voltmeter are joined in series to a cell. Their readings are A and V, respectively. If a resistance is now joined in parallel with the voltmeter, then

a. both A and V will increase
b. both A and V will decrease
c. A will decrease, V will increase
d. A will increase, V will decrease

20. $AA2.$ is a wire of uniform resistance. The galvanometer G shows in Fig. A2.18 that no current flows when the length $AC = 20$ cm and $CB = 80$ cm. The resistance R is equal to

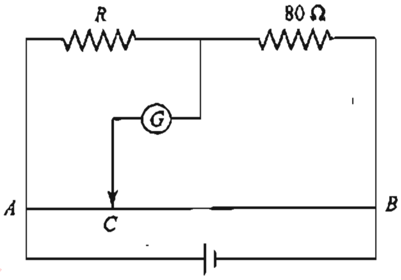


Fig. A2.18

a. 2 Ω
b. 8 Ω
c. 20 Ω
d. 40 Ω

21. A capacitor of capacitance C is connected to two voltmeters A and B (Fig. A2.19). A is ideal, having infinite resistance, while B has resistance R. The capacitor is charged and then the switch S is closed. The readings of A and B will be equal

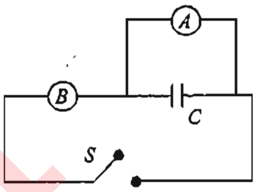


Fig. A2.19

a. at all times
b. after time RC
c. after time RC in 2
d. only after a very long time

22. The equivalent resistance of the combination across AB (Fig. A2.20) is

a. $\left(\frac{3+\sqrt{17}}{4}\right)$
b. $3+\sqrt{17}$
c. $\frac{3+\sqrt{17}}{2}$
d. $2(3+\sqrt{17})$.

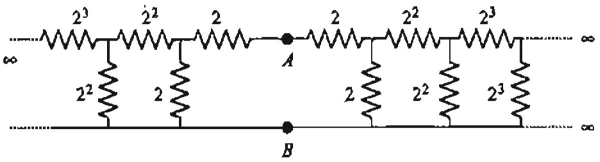


Fig. A2.20

23. The capacitor shown in Fig. A2.21 is in steady state. The energy stored in the capacitor is

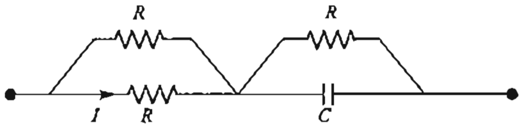


Fig. A2.21

a. CI^2R^2
b. $2CI^2R^2$
c. $4CI^2R^2$
d. None of the above

24. Charge on the capacitor having capacitance C_2 in steady state (Fig. A2.22) is

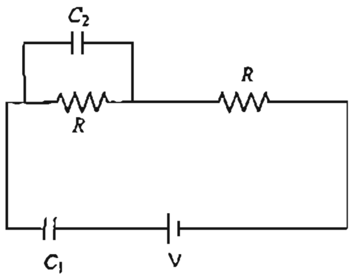


Fig. A2.22

a. Zero
b. (C_1+C_2)V
c. C_2V
d. C_1V

25. An 80 μC charge is given to the 4 μF capacitor in the circuit shown in Fig. A2.23 so that the upper plate A is positively charged. An unknown resistance R is connected in the left limb. As soon as the switch S in the central limb is closed, a current of 2 A flows through the 2 Ω resistor in the central limb. The capacitive time constant for the circuit is

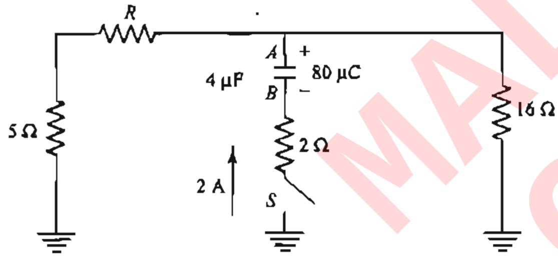


Fig. A2.23

a. 56 μs
b. 8 μs
c. 200 μs
d. 40 μs

26. In the given circuit (Fig. A2.24), when key K is open, reading of ammeter is I. Now key K is closed then the correct statement is

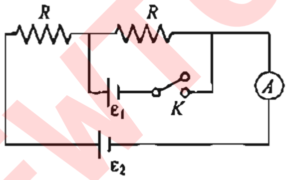


Fig. A2.24

a. If $\varepsilon_1 < IR$, reading of the ammeter is less than I
b. If $IR < \varepsilon_1$, reading of the ammeter is greater than I
c. If $\varepsilon_1 < 2IR$, reading of the ammeter will be zero
d. Reading of ammeter will not change

27. In the circuit shown (Fig. A2.25), the batteries have e.m.f. $E_1 = E_2 = 1$ V, $E_3 = 2.5$ V and the resistance $R_1 = 10$ Ω, $R_2 = 20$ Ω, Capacitance $C = 10$ μF. The charge on the left plate of the capacitor C at steady state is

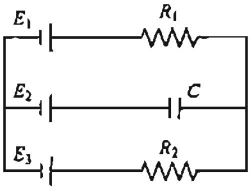


Fig. A2.25

a. +2 μC
b. −4 μC
c. −5 μC
d. +12 μC

28. In the circuit (Fig. A2.26), the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be

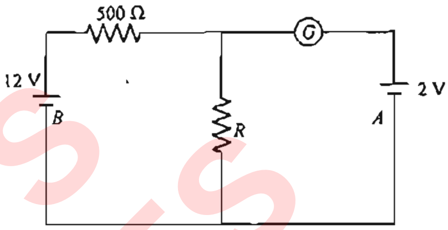


Fig. A2.26

a. 500 Ω
b. 1000 Ω
c. 200 Ω
d. 100 Ω

29. A voltmeter with resistance $R_V = 2500$ Ω indicates a voltage of 125 V in the circuit shown in Fig. A2.27. What is the series resistance (R) to be connected with voltmeter in this circuit so that it indicates 100 V?

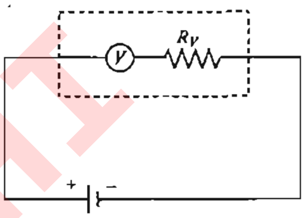


Fig. A2.27

a. 625 Ω
b. 120 Ω
c. 550 Ω
d. Data are insufficient

30. Fig. A2.28 shows a battery with e.m.f. 15 V in a circuit with $R_1 = 30$ Ω, $R_2 = 10$ Ω, $R_3 = 20$ Ω and capacitance $C = 10$ μF. The switch S is initially in the open position and is then closed at time $t = 0$. What will be the final steady-state charge on capacitor?

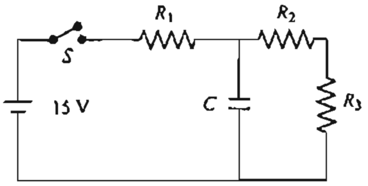


Fig. A2.28

a. 75 μC
b. 50 μC
c. 10 μC
d. None of these

31. A conductor of resistivity ρ and resistance R, as shown in the figure. is connected across a battery of e.m.f. V. Its radius varies from a at left end to b at right end. The electric field at a point P at distance x from left end of it is

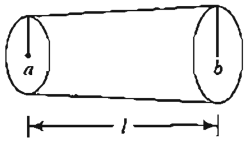


Fig. A2.29

a. $\dfrac{V\ell^2\rho}{\pi R\left(\ell a+(b-a)x\right)^2}$
b. $\dfrac{2V\ell^2\rho}{\pi R\left(\ell a+(b+a)x\right)^2}$
c. $\dfrac{V\ell^2\rho}{2\pi R\left(\ell a+(b-a)x\right)^2}$
d. None of these

32. A piece of conducting wire of resistance R is cut into $2n$ equal parts. Half the parts are connected in series to form a bundle and remaining half in parallel to form another bundle. These bundles are then connected to give the maximum resistance. The maximum resistance of the combination is

a. $\frac{R}{2}\left(1+\frac{1}{n^2}\right)$ b. $\frac{R}{2}\left(1+n^2\right)$

c. $\frac{R}{2(1+n^2)}$ d. $R\left(n+\frac{1}{n}\right)$

33. In the circuit shown in Fig. A2.30, if the switch S is closed at $t = 0$, the capacitor charges with a time constant

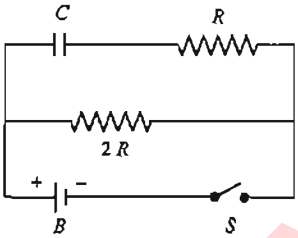


Fig. A2.30

a. RC b. $3RC$

c. $\frac{2}{3}RC$ d. $RC \ln\left(\frac{2}{3}\right)$

34. A metal wire of length L and radius r is made of copper and aluminium contributing equal lengths (Fig. A2.31). This wire is now coated by nickel till the radius of wire becomes R. If specific resistances of these materials are ρ_{Cu}, ρ_{Al} and ρ_{Ni} the equivalent conductance of the system across the length will be

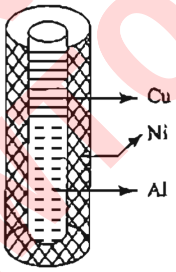


Fig. A2.31

a. $\frac{\pi r^2}{L}\left[\frac{1}{\rho_{Cu}}+\frac{1}{\rho_{Al}}+\frac{R^2}{r^2\rho_{Ni}}\right]$

b. $\frac{\pi}{L}\left[\frac{r^2}{\rho_{Cu}}+\frac{r^2}{\rho_{Al}}+\frac{2(R^2-r^2)}{r\rho_{Ni}}\right]$

c. $\frac{\pi}{L}\left[\frac{2(R^2-r^2)}{\rho_{Cu}+\rho_{Al}}-\frac{r^2}{\rho_{Ni}}\right]$

d. $\frac{\pi}{L}\left[\frac{2r^2}{\rho_{Cu}+\rho_{Al}}+\frac{R^2-r^2}{\rho_{Ni}}\right]$

35. In the given circuit (Fig. A2.32), the potential difference across the capacitor is 12 V. Each resistance is of 3 Ω. The cell is ideal. The e.m.f. of the cell is

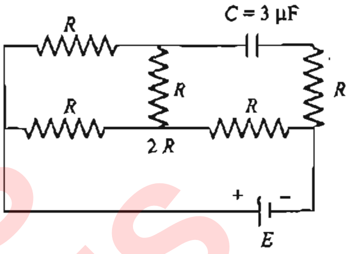


Fig. A2.32

a. 15 V b. 9 V

c. 12 V d. 24 V

36. A battery of internal resistance 4 Ω is connected to the network of the resistance as shown in Fig. A2.33. If the maximum power can be delivered to the network, the magnitude of resistance in Ω should be

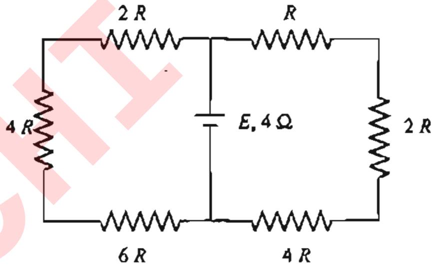


Fig. A2.33

a. $\frac{19}{21}$ Ω b. $\frac{84}{19}$ Ω

c. 12 Ω d. 7 Ω

37. Find the effective resistance between A and $A2$.

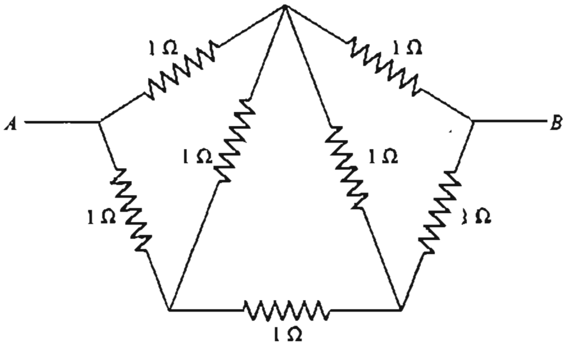


Fig. A2.34

a. 2 Ω b. 1 Ω

c. 8/7 Ω d. 7 Ω

38. A milli-ammeter of range 10 mA and resistance 9 Ω are joined in a circuit as shown in the Fig. A2.35. The meter gives full scale deflection, when current in the main circuit is I and A and D are used as terminals. The value of I is

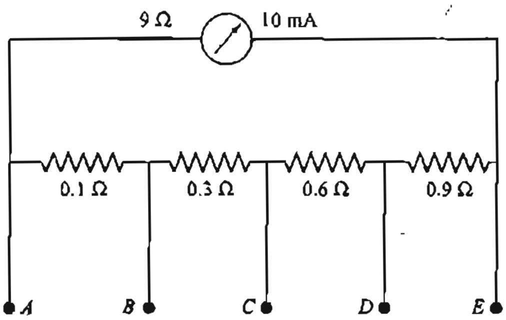


Fig. A2.35

a. 1.09 A b. 10.9 A c. zero d. 0.109 A

39. A potential of 400 V is applied at the point A. The value of resistance $R_1 = 1000\ \Omega$, $R_2 = 2000\ \Omega$ and $R_3 = 1000\ \Omega$ are connected between points A and G as shown in Fig. A2.36 Point G is earthed. The measured potential difference by an ideal voltmeter connected across R_2 is

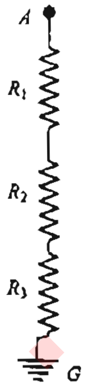


Fig. A2.36

a. 100 V b. 200 V c. 300 V d. 400 V

40. In the given circuit (Fig. A2.37), the potential difference across the 6 μF capacitor in steady state is

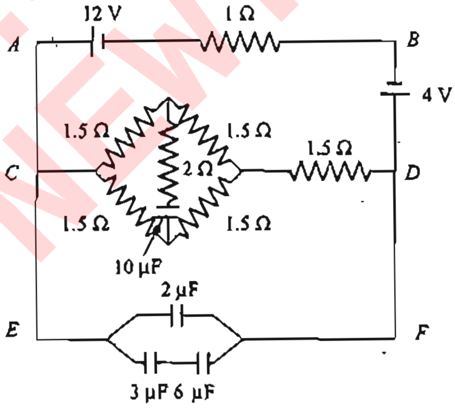


Fig. A2.37

a. 1 V b. 6 V c. 3 V d. 2 V

41. In the circuit shown (Fig. A2.38) the cells are ideal and have equal e.m.f., the capacitance of the capacitor is C and the resistance of the resistor is R. The switch X is first connected to Y and then to Z. After a long time, the total heat produced in the resistor will be

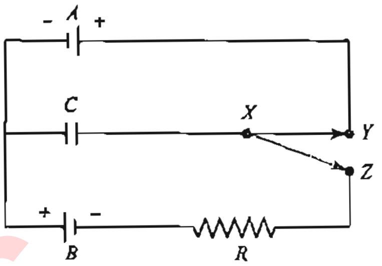


Fig. A2.38

a. equal to the energy finally stored in the capacitor
b. half the energy finally stored in the capacitor
c. twice the energy finally stored in the capacitor
d. Four times the energy finally stored in the capacitor

42. In the arrangement shown in Fig. A2.39 when the switch S_2 is open, the galvanometer shows no deflection for $\ell = L/2$. When the switch S_2 is closed, the galvanometer shows no deflection for $\ell=\frac{5}{12}L$. The internal resistance (r) of 6 V cell, and the e.m.f E of the other battery are respectively

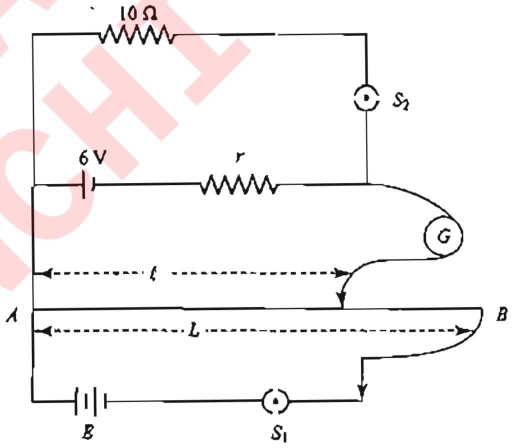


Fig. A2.39

a. 3 Ω, 8 V b. 2 Ω, 12 V c. 2 Ω, 24 V d. 3 Ω, 12 V

43. Calculate the energy stored in the capacitor of capacitance 2 μF. The voltmeter gives a reading of 15 V and the ammeter A reads 15 mA

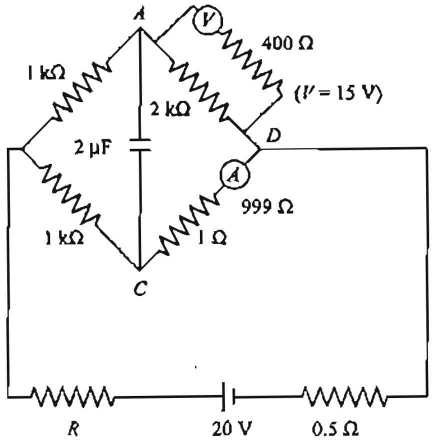


Fig. A2.40

a. 5 μJ b. 10 μJ
c. 0.5 μJ d. zero

44. In the given circuit (Fig. A2.41), $R_1 \neq R_2$ and the reading of the voltmeter is same, irrespective of whether the switch S is open or closed. Then, which of the following is correct ?

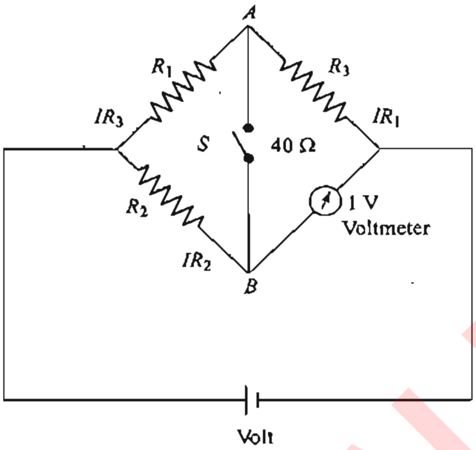


Fig. A2.41

a. $I_{R_2} = I_V$ b. $I_{R_1} = I_{R_2}$
c. $I_{R_3} = I_V$ d. None of the above

45. $V_A - V_B$ for Fig. A2.42 in steady state is

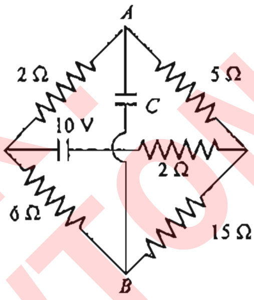


Fig. A2.42

a. 4 V b. 6 V
c. 5 V d. zero

46. A hemi-spherical network of radius a is made by using a conducting wire of resistance per unit length r (Fig. A2.43). The equivalent resistance across OP is

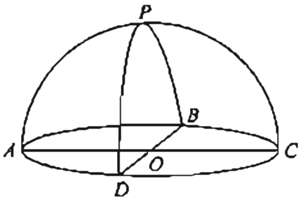


Fig. A2.43

a. $\dfrac{ra(\pi+4)}{8}$ b. $\dfrac{ra(\pi+2)}{8}$
c. $\dfrac{ra(\pi+4)}{4}$ d. $\dfrac{ra(\pi+1)}{8}$

47. In Fig. A2.44, the charge that flows from P to Q when the switch S is closed is

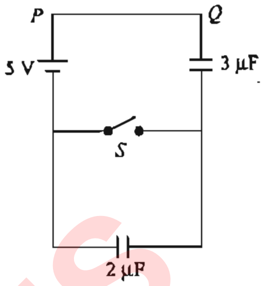


Fig. A2.44

a. 3 μC b. 6 μC
c. 9 μC d. 15 μC

48. In the circuit shown (Fig. A2.45), each resistance is 2 Ω. The potential V_1, as indicated in the circuit, is equal to

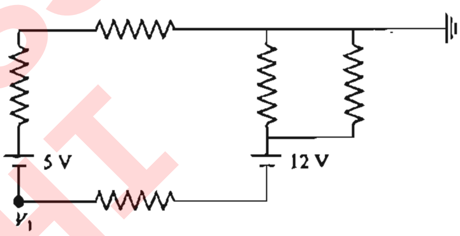


Fig. A2.45

a. 11 V b. −11 V
c. 9 V d. −9 V

49. In the circuit shown (Fig. A2.46), the value of R in ohm that will result in no current through the 30 V battery is

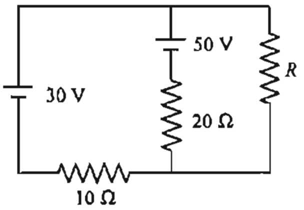


Fig. A2.46

a. 10 Ω b. 25 Ω
c. 30 Ω d. 40 Ω

50. In Fig. A2.47 the current flowing through $2R$ is

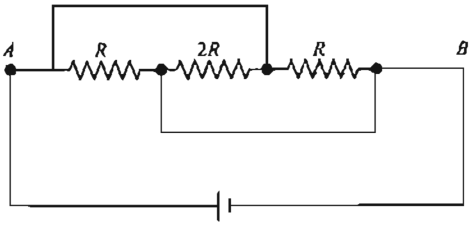


Fig. A2.47

a. from left to right b. from right to left
c. no current d. None of these

51. In the circuit shown (Fig. A2.48), switch S_2 is closed first and is kept closed for a long time. Now S_1 is closed. Just after that instant the current through S_1 is

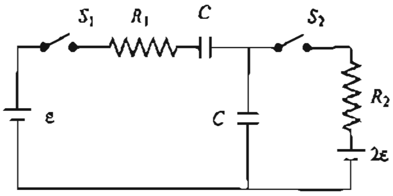


Fig. A2.48

a. $\frac{\varepsilon}{R_1}$ towards right b. $\frac{\varepsilon}{R_1}$ towards left

c. zero d. $\frac{2\varepsilon}{R_1}$

52. Each resistor in the following circuit (Fig. A2.49) has a resistance of 2 mΩ and the capacitors have capacitances of 1 μF. The battery voltage is 3 V. The voltage across the resistor A in the following circuit in steady state is

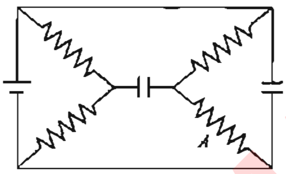


Fig. A2.49

a. 0 V b. 0.5 V

c. 0.75 V d. 1.5 V

53. Initially, switch S is connected to position 1 for a long time (Fig. A2.50). The net amount of heat generated in the circuit after it is shifted to position 2 is

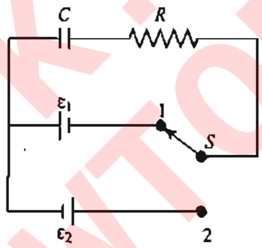


Fig. A2.50

a. $\frac{C}{2}(\varepsilon_1+\varepsilon_2)\varepsilon_2$ b. $C(\varepsilon_1+\varepsilon_2)\varepsilon_2$

c. $\frac{C}{2}(\varepsilon_1+\varepsilon_2)^2$ d. $C(\varepsilon_1+\varepsilon_2)^2$

54. The switch S in the circuit diagram (Fig. A2.51) is closed at $t = 0$. The charge on capacitors at any time t is

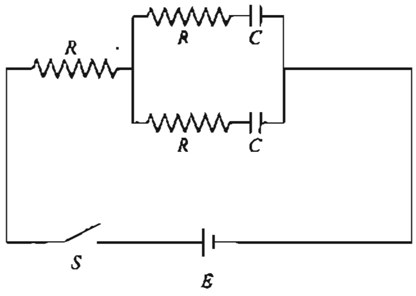


Fig. A2.51

a. $q_{(t)}=EC(1-e^{-2t/3RC})$ b. $q_{(t)}=EC(1-e^{-t/2RC})$

c. $q_{(t)}=EC(1-e^{-t/3RC})$ d. $q_{(t)}=EC(1-e^{-3t/2RC})$

55. The charge on the capacitor in steady state in the circuit shown (Fig. A2.52) is

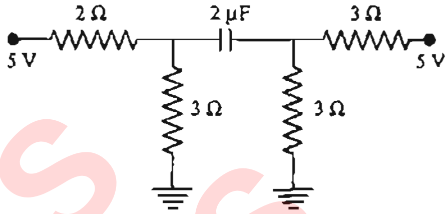


Fig. A2.52

a. 0.5 μC b. 1 μC c. 2 μC d. 4 μC

56. In the circuit shown in Fig. A2.53, XY is a potentiometer wire 100 cm long. The circuit is connected up as shown. With switches S_2 and S_3 open, a balance point is found at Z. After switch S_1 has remained closed for some time, it is found that contact at Z must be moved towards Y to maintain a balance. Which of the following is the most likely reason for this?

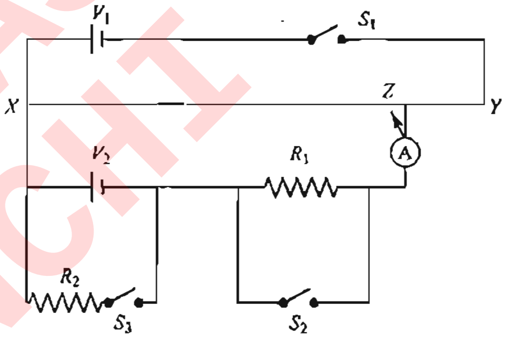


Fig. A2.53

a. The cell V_1 is running down
b. The cell V_2 is running down
c. The wire XZ is getting warm and its resistance is increasing
d. The resistor R_1 is getting warm and increasing in value

57. In the circuit shown (Fig. A2.54), if switches S_1 and S_2 have been closed for a long time, then the charge on the capacitor

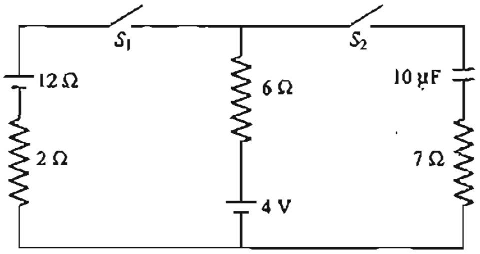


Fig. A2.54

a. is 100 μC
b. increases to 120 μC if one-third of the gap of the capacitor's plates is filled with a dielectric ($K = 2$) of same area
c. both a. and b.
d. charge on the capacitor remains unchanged if one-third of the gap of the capacitor's plates is filled with a dielectric ($K = 2$) of same area

58. The circuit shown in Fig. A2.55 consists of a battery of e.m.f. $\varepsilon = 10$ V; a capacitor of capacitance $C = 1.0\ \mu$F and three resistors of values $R_1 = 2\ \Omega$, $R_2 = 2\ \Omega$ and $R_3 = 1\ \Omega$. Initially, the capacitor is completely uncharged and the switch S is open. The switch S is closed at $t = 0$. Then

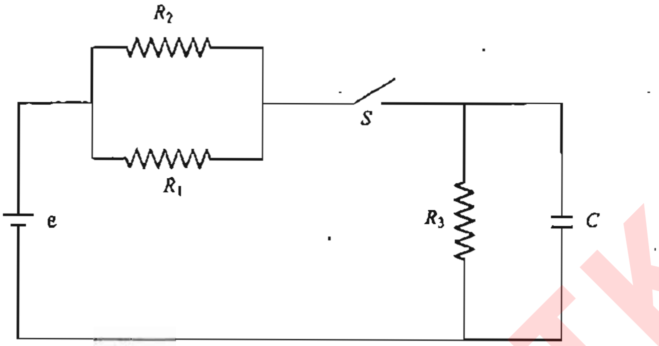
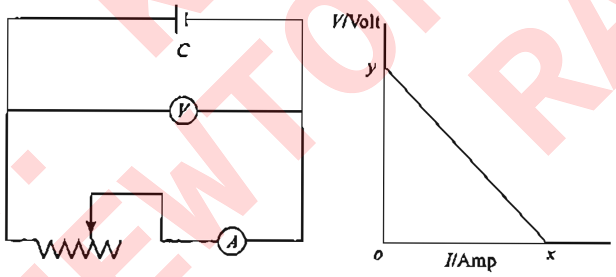


Fig. A2.55

a. the current through resistor R_3 at the moment the switch closed is zero
b. the current through resistor R_3 a long time after the switch closed is 5 A
c. the ratio of current through R_1 and R_2 is always constant
d. all of these

59. Fig. A2.56 (a) shows a circuit used in an experiment to determine the e.m.f. and internal resistance of the battery C. A graph was plotted of the potential difference V between the terminals of the battery against the current I, which was varied by adjusting the rheostat. The graph is shown in Fig. A2.56 (b); x and y are the intercepts of the graph with the axes as shown. What is the internal resistance of the battery?



(a) (b)

Fig. A2.56

a. x b. y c. x/y d. y/x

60. In the circuit shown (Fig. A2.57) what is the charge of total electrical energy stored in the capacitors when the key is pressed?

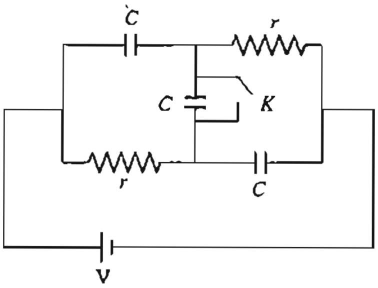


Fig. A2.57

a. $\dfrac{CV^2}{12}$ b. $\dfrac{7CV^2}{8}$
c. $\dfrac{5CV^2}{4}$ d. $\dfrac{3CV^2}{8}$

61. In the circuit shown (Fig. A2.58), calculate the current through 6 V battery.

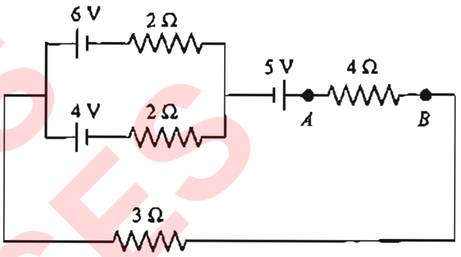


Fig. A2.58

a. (1/4) A b. (1/8) A
c. (1/2) A d. none of these

62. In Fig. A2.59, points A and $A2$. are connected by a perfectly conducting wire. Calculate the current through AB

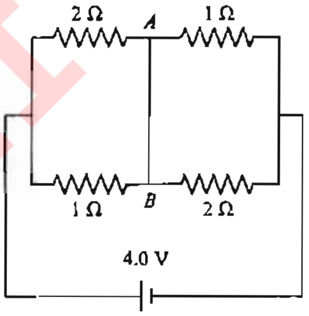


Fig. A2.59

a. 2 A b. 1 A
c. 1.5 A d. 2.5 A

63. In the potentiometer arrangement (Fig. A2.60), the driving cell A has e.m.f. ε and internal resistance r. The e.m.f. of the cell B is to be rechecked has e.m.f. $\dfrac{\varepsilon}{2}$ and internal resistance $2r$. The potentiometer wire CD is 100 cm long. If balance is obtained, the length $CJ = l$ is

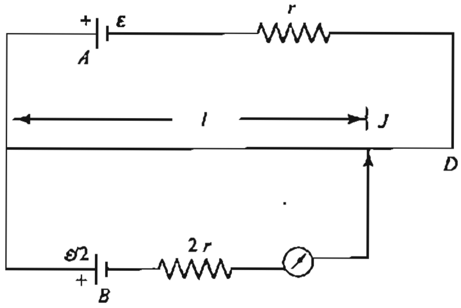


Fig. A2.60

a. $l = 50$ cm b. $l > 50$ cm
c. $l < 50$ cm d. Balance cannot be obtained

64. In a practical Wheatstone's bridge circuit (Fig. A2.61), when one more resistance of 100 Ω is connected in parallel with unknown resistance x, then the ratio l_1 / l_2 becomes 2. l_1 is the balance length. AB is a uniform wire. Then value of x must be

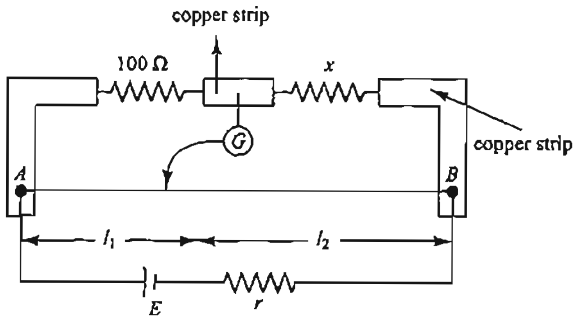


Fig. A2.61

a. 50 Ω b. 100 Ω c. 200 Ω d. 400 Ω

65. To get the maximum current through a resistance of 2.5 Ω, one can use m rows of cells, each row having n cells. The internal resistance of each cell is 0.5 Ω. What are the values of n and m, if the total number of cells is 45?

a. 3, 15 b. 5, 9 c. 9, 5 d. 15, 3

66. Two circular rings of identical radii and resistance of 36 Ω each are placed in such a way that they cross each other's centre C_1 and C_2 as shown in Fig. A2.62. Conducting joints are made at intersection points A and B of the rings. An ideal cell of e.m.f. 20 V is connected across AB. The power delivered by cell is

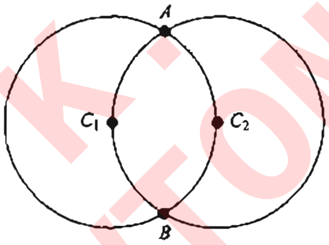


Fig. A2.62

a. 80 W b. 100 W c. 120 W d. 200 W

67. Circuit for the measurement of resistance by potentiometer is shown in Fig. A2.63. The galvanometer is first connected at point A and zero deflection is observed at length PJ = 10 cm. In second case, it is connected at point C and zero deflection is observed at a length 30 cm from P, then the unknown resistance X is

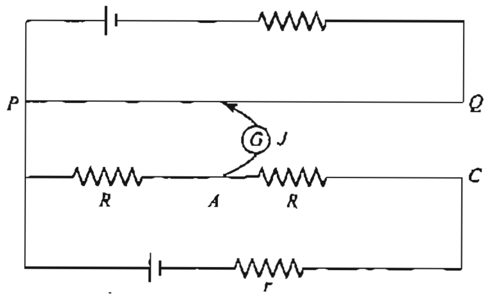
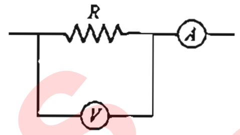


Fig. A2.63

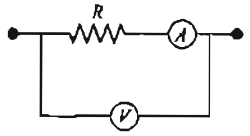
a. $2R$ b. $R/2$ c. $R/3$ d. $3R$

68. Which of the following circuit (Fig. A2.64) gives the correct value of resistance, when computed by using $R = \frac{V}{I}$ where V and I are voltmeter and ammeter readings, respectively? The meters are not ideal.

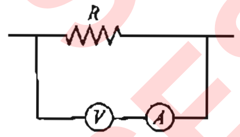
a.



b.



c.



d. None of these

Fig. A2.64

69. Two ideal batteries having e.m.f. E_1 and E_2 are connected as shown in Fig. A2.65. The value of resistances are chosen in such a way that ammeter reading is zero. The reading of voltmeter will be (consider the meters to be ideal)

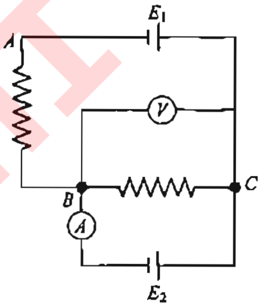


Fig. A2.65

a. E_1 b. E_2

c. In between E_1 and E_2

d. Nothing can be predicted about voltmeter reading from the given information

70. A cell of e.m.f. ε and internal resistance r is charged by a current i, then

a. the cell stores chemical energy at the rate of εi

b. the cell stores chemical energy at the rate of $i^2 r$

c. the cell stores chemical energy at the rate of $\varepsilon i - i^2 r$

d. storage of chemical energy rate cannot be calculated

71. Two long coaxial and conducting cylinders of radius a and b are separated by a material of conductivity σ and a constant potential difference V is maintained between them by a battery. Then the current per unit length of the cylinder flowing from one cylinder to the other is

a. $\frac{4\pi\sigma}{\ell n(b/a)}V$ b. $\frac{4\pi\sigma}{(b/a)}V$

c. $\frac{2\pi\sigma}{\ell n(b/a)}V$ d. $\frac{2\pi\sigma}{(b+a)}V$

72. 50 V battery is supplying current of 10 A when connected to a resistor. If the efficiency of the battery at this current is 25%. Then the internal resistance of the battery is

a. 2.5 Ω **b.** 3.75 Ω
c. 1.25 Ω **d.** 5 Ω

73. A battery is supplying power to a tape recorder by cable of resistance of 0.02 Ω. If the battery is generating 50 W power at 5 V, then power received by the tape recorder is

a. 50 W **b.** 45 W
c. 30 W **d.** 48 W

74. In a Wheatstone's bridge, resistance P, Q and R are connected in the three arms and the fourth arm is formed by two resistances S_1 and S_2 connected in parallel. The condition for the bridge to be balanced will be

(AIEEE, 2006)

a. $\frac{P}{Q}=\frac{R(S_1+S_2)}{2S_1S_2}$ **b.** $\frac{P}{Q}=\frac{R}{S_1+S_2}$

c. $\frac{P}{Q}=\frac{2R}{S_1+S_2}$ **d.** $\frac{P}{Q}=\frac{R(S_1+S_2)}{S_1S_2}$

75. Fig. A2.66 shows a meter-bridge set up with null deflection in the galvanometer. The value of the unknown resistor R is

(AIEEE, 2008)

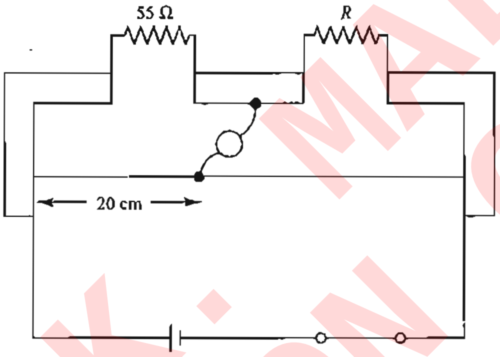


Fig. A2.66

a. 110 Ω **b.** 55 Ω
c. 13.75 Ω **d.** 220 Ω

76. A number of resistors are connected as shown in the figure. The equivalent resistance between A and B is

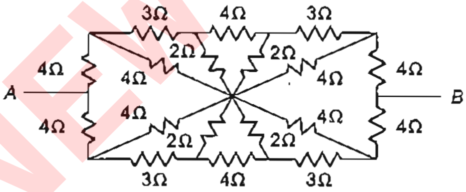


Fig. A2.67

a. 6 Ω **b.** 12 Ω
c. 9 Ω **d.** 15 Ω

77. In the circuit shown what is the change of total electrical energy stored in the capacitors when the key is pressed?

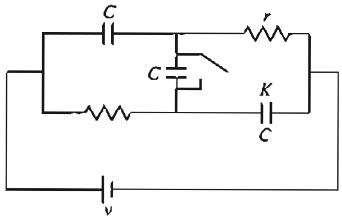


Fig. A2.68

a. $\frac{3CV^2}{8}$ **b.** $\frac{5CV^2}{4}$

c. $\frac{7CV^2}{8}$ **d.** $\frac{CV^2}{12}$

78. In the given arrangement, the reading of ammeter is same in each case when either K_1 or K_2 is closed. The reading of the ammeter is

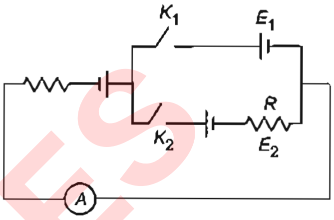


Fig. A2.69

a. $\frac{E_1+E_2}{R}$ **b.** $\frac{E_1-E_2}{R}$

c. data given is not sufficient **d.** none of these

79. The charge on the capacitor as in figure is

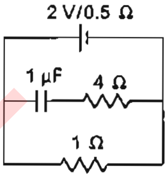


Fig. A2.70

a. 2 μC **b.** $\frac{2}{3}$ μC

c. $\frac{4}{3}$ μC **d.** zero

80. Find the potential drop across the capacitor in the given circuit.

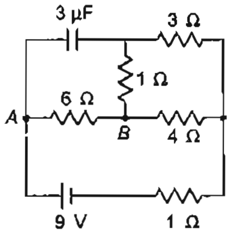


Fig. A2.71

a. 6 V **b.** 6.5 V
c. 7 V **d.** none of these

81. Find X so that ammeter reads zero in the circuit shown below

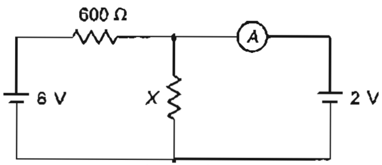


Fig. A2.72

a. 600 Ω **b.** 300 Ω
c. 200 Ω **d.** 150 Ω

82. Current flowing in a wire of a circuit is (2.5 ± 0.05) A whereas the potential difference across the wire is (20 ± 1) V. Then the resistance of the wire is

a. (8 ± 1) Ω **b.** (8 ± 20) Ω
c. (8 ± 0.56) Ω **d.** 8 Ω

83. A galvanometer may be converted into ammeter or voltmeter. In which of the following cases the resistance of the device will be largest ? (Assume maximum range of galvanometer = 1 mA)

a. an ammeter of range 10 A
b. a voltmeter of range 5 A
c. an ammeter of range 5 A
d. a voltmeter of range 10 V

84. The current-voltage graphs for a given metallic wire at two different temperature T_1 and T_2 are shown in the figure. Then

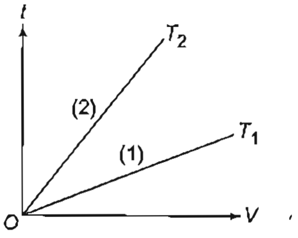
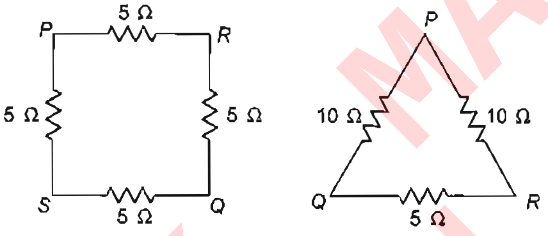


Fig. A2.73

a. $T_2 > T_1$
b. $T_2 < T_1$
c. $T_2 = T_1$
d. none of these

85. In which of the following circuits the net resistance across terminals P and Q is maximum



Circuit (i) Circuit (ii)

Fig. A2.74

a. circuit (i)
b. circuit (ii)
c. both the circuits have the same resistance
d. insufficient data to decide

86. In the diagrams, all right bulbs are identical, all cells are ideal and identical. In which circuit (a, b, c, d) will the bulbs be dimmest?

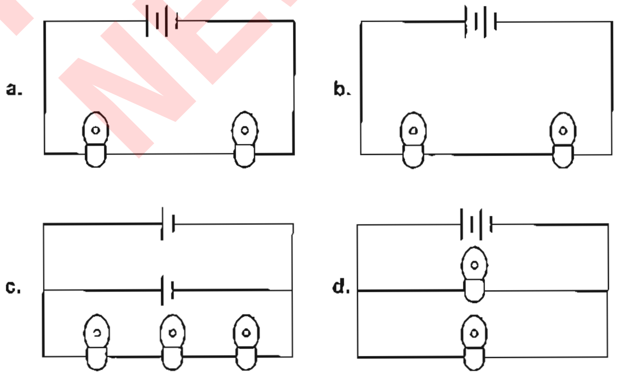


Fig. A2.75

87. For the circuit shown, a shorting wire of negligible resistance is added to the circuit between points A and $A2$. When this shorting wire is added, bulb 3 goes out. Which bulbs (all identical) in the circuit brighten?

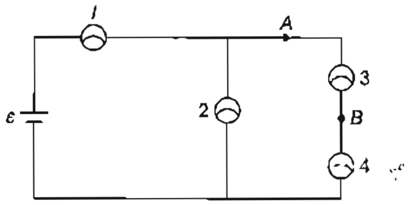


Fig. A2.76

a. only bulb 2
b. only bulb 4
c. only bulbs 1 and 4
d. only bulbs 2 and 4

88. A wire has linear resistance ρ (in Ω/m). Find the resistance R between points A and B if the side of the big square is d.

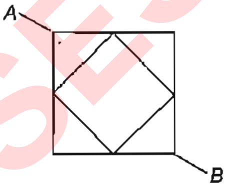


Fig. A2.77

a. $\frac{\rho d}{\sqrt{2}}$
b. $\sqrt{2}\,\rho d$
c. $2\rho d$
d. none of these

89. The equivalent resistance of the circuit across points A and B is equal to **(IIT-JEE, 2007)**

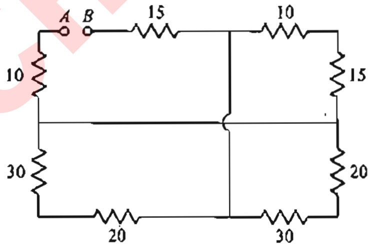


Fig. A2.78

a. 22.5 Ω
b. 25 Ω
c. 37.5 Ω
d. 75 Ω

90. The equivalent resistance between A and B in the arrangement of resistances as shown is

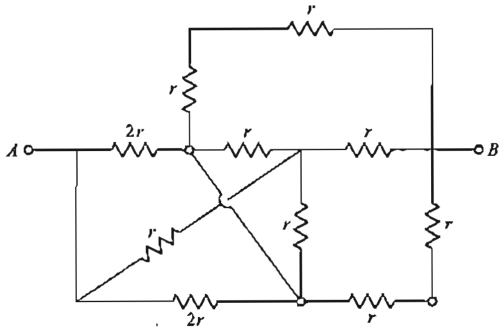


Fig. A2.79

a. $4r$
b. $3r$
c. $2.5r$
d. r

91. A capacitor of capacitance 10 μF is charged up to a potential difference of 2 V and then the cell is removed. Now it is connected to a cell of e.m.f. 4 V and is charged fully. In both cases the polarities of the two cells are in the same directions. Total heat produced in the complete charging process is

a. 10 mJ b. 20 μJ
c. 40 μJ d. 80 mJ

92. In the shown wire frame, each side of a square (the smallest square) has a resistance R. The equivalent resistance of the circuit between the points A and B is

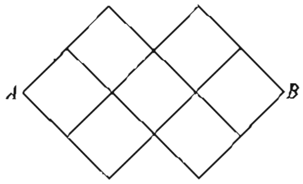


Fig. A2.80

a. R b. $2R$
c. $4R$ d. $8R$

93. A spherical shell, made of material of electrical conductivity $\frac{10^9}{\pi}(\Omega\ \text{m})^{-1}$, has thickness $t = 2$ mm and radius $R = 10$ cm. In an arrangement, its inside surface is kept at a lower potential than its outside surface.

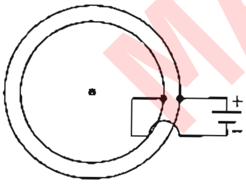


Fig. A2.81

The resistance offered by the shell is equal to

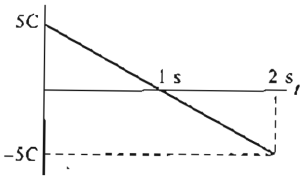
a. $5\pi\times10^{-12}\,\Omega$ b. $2.5\times10^{-11}\,\Omega$
c. $5\times10^{-12}\,\Omega$ d. $5\times10^{-11}\,\Omega$

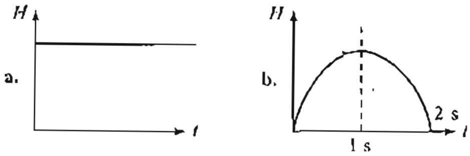
94. Two cylindrical rods of uniform cross-sectional area A and $2A$, having free electrons per unit volume $2n$ and n, respectively, are joined in series. A current I flows through them in steady state. Then the ratio of drift velocity of free electron in left rod to drift velocity of electron in the right rod $\left(\frac{V_L}{V_R}\right)$ is

$$2n \xrightarrow{I} n$$

a. 1/2 b. 1
c. 2 d. 4

95. A charge passing through a resistor is varying with time as shown in the figure. The amount of heat generated in time 't' is best represented (as a function of time) by





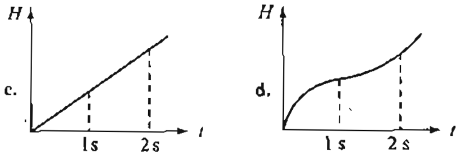
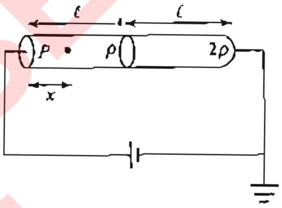


Fig. A2.82

96. Two cylindrical rods of same cross-section area and same length are connected in series to an ideal cell as shown. The resistivity of left rod is ρ and that of right rod is 2ρ. Then the variation of potential at any point P distant x from left end of combined rod system is given by



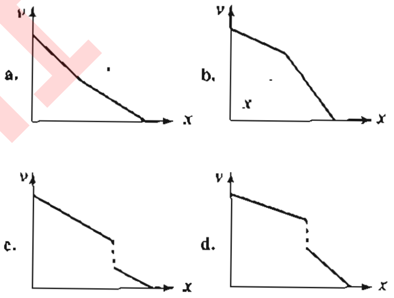


Fig. A2.83

97. The effective resistance between A and B of the shown network, where resistance of each resistor is R, is

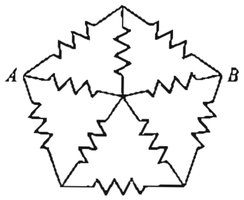


Fig. A2.84

a. $\frac{8R}{11}$ b. $\frac{6R}{11}$
c. $\frac{6R}{5}$ d. none of these

98. A potentiometer wire AB as shown is 40 cm long of resistance 50 Ω/m free end of an ideal voltmeter is touching the potentiometer wire. What should be the velocity of the jockey as a function of time so that reading in voltmeter is varying with time as ($2 \sin \pi t$).

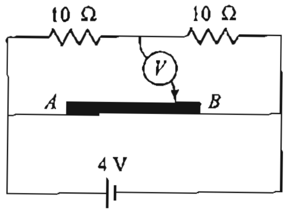


Fig. A2.85

a. $10\,\pi \sin \pi t$ cm/s b. $10\,\pi \cos \pi t$ cm/s
c. $20\,\pi \sin \pi t$ cm/s d. $20\,\pi \cos \pi t$ cm/s

99. The relation between R and r (internal resistance of the battery) for which the power consumed in the external part of the circuit is maximum.

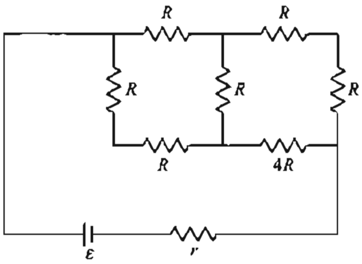


Fig. A2.86

a. $R = r$
b. $R = \frac{r}{2}$
c. $R = 2r$
d. $R = 1.5\,r$

100. For the potentiometer arrangement shown in the figure, length of wire AB is 100 cm and its resistance is 9 Ω. Find the length AC for which the galvanometer G will show zero deflection.

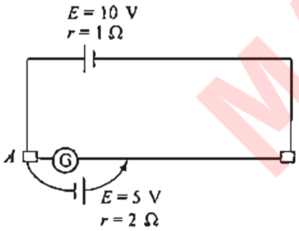


Fig. A2.87

a. 66.7 cm
b. 60 cm
c. 50 cm
d. 33.3 cm

101. A resistance $R = 12$ Ω is connected across a source of e.m.f. as shown in the figure. Its e.m.f. changes with time as shown in the graph. What is the heat developed in the resistance in the first 4 s ?

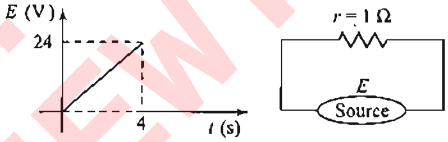


Fig. A2.88

A. 72 J
B. 64 J
C. 108 J
D. 100 J

102. If the switch at point P is opened (shown in the figure) choose the correct option

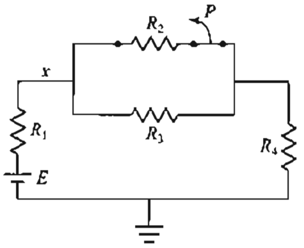


Fig. A2.89

a. the current in R_1 would not change
b. the potential difference between point X and the ground would increase
c. the current provided by the battery would increase
d. the e.m.f. produced by the battery (assumed to have no internal resistance) would change

103. Figure shows a circuit model for the transmission of an electrical signal, such as cable TV, to a large number of subscribers. Each subscriber connects a load resistance R_L between the transmission line and the ground. Assume the ground to be at zero potential and to have negligible resistance. The resistance of the transmission line between the connection points of different subscribers is modeled as the constant resistance R_T. The equivalent resistance across the signal source is

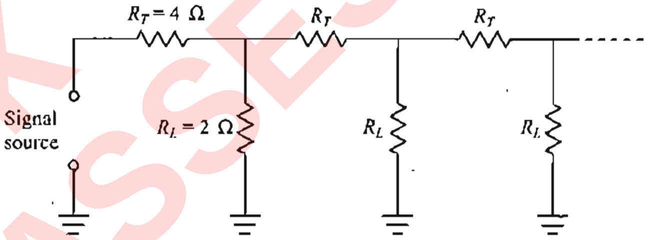


Fig. A2.90

a. 10 Ω
b. 5 Ω
c. $\sqrt{55}$ Ω
d. $\sqrt{65}$ Ω

Multiple Correct Answers Type

Solutions on page A2.41.

1. The electron in a hydrogen atom moves in a circular orbit of radius 5×10^{-11} m with a speed of $0.6\pi\times10^6$ m/s then
a. the frequency of the electron is 6×10^{15} rev/s
b. the electron carries -1.6×10^{-19} C around the loop
c. the current in the orbit is 0.96 mA
d. the current flow is in the opposite direction to the direction of the motion of electron

2. Two cells of unequal e.m.f.s, ε_1 and ε_2, and internal resistances r_1 and r_2 are joined as shown in Fig. A2.91. V_A and V_B are the potentials at A and B, respectively

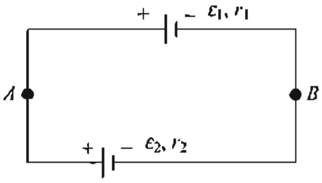


Fig. A2.91

a. one cell will continuously supply energy to the other
b. the potential difference across both the cells will be equal
c. the potential difference across one cell will be greater than its e.m.f.
d. $V_A - V_B = \frac{(\varepsilon_1 r_2 + \varepsilon_2 r_1)}{(r_1 + r_2)}$

3. A capacitor of capacitance C is connected to two voltmeters A and B (Fig. A2.92). A is an ideal voltmeter having infinite resistance, while B has resistance R. The capacitor is uncharged and then the switch S_ω is closed at $t = 0$,

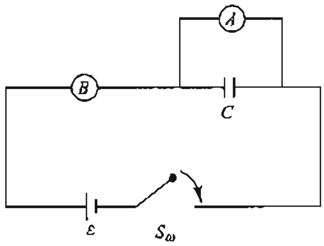


Fig. A2.92

a. readings of B and A will be ε and zero respectively at $t = 0$
b. during time interval $(0 \le t < \infty)$ readings of B and A change
c. reading of A and B will be equal at $t = RC \ln 2$
d. None of these

4. A voltmeter of resistance 600 Ω when connected in turn across resistances R_1 and R_2 gives readings of V_1 and V_2, respectively. If the battery is ideal, then

a. $V_1 = 60$ V
b. $V_2 = 30$ V
c. $V_1 = 45$ V
d. $V_2 = 75$ V

5. For the circuit shown in Fig. A2.93, select the correct statements from the following

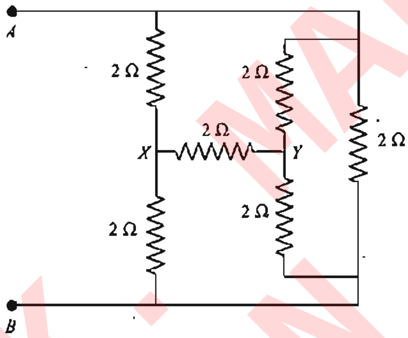


Fig. A2.93

a. x and y are equipotential points
b. Effective resistance between A and B is 2 Ω
c. Effective resistance between A and B is 1 Ω
d. None of the above

6. A uniform current I is flowing in a long wire of radius R. If the current is uniformly distributed across the cross-sectional area of the wire, then

a. magnetic field increases linearly from centre to surface
b. magnetic field decays exponentially with distance r from the centre of wire for $r > R$
c. magnetic field at the centre of wire is zero
d. None of the above

7. Two bulbs 25 W, 100 V (upper bulb in figure) and 100 W, 200 V (lower bulb in figure) are connected in the circuit as shown in Fig. A2.94. Choose the correct answer(s).

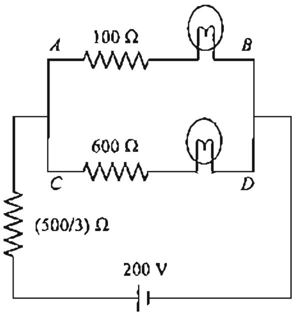


Fig. A2.94

a. Heat lost per second in the circuit will be 80 J
b. Ratio of heat produced per second in bulb will be 1:1
c. Ratio of heat produced in branch AB to branch CD will be 1:2
d. Current drawn from the cell is 0.4 A

8. In Fig. A2.95, battery of e.m.f. E has internal resistance r and a variable resistor. At an instant, current flowing through the circuit is i, potential difference between the terminals of cells is V, thermal power developed in external circuit is P and thermal power developed in the cell is equal to fraction η of total electrical generated in it. Which of the following graphs is/are correct?

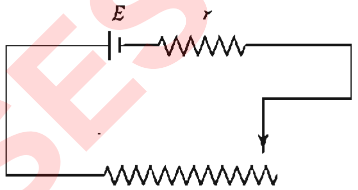


Fig. A2.95

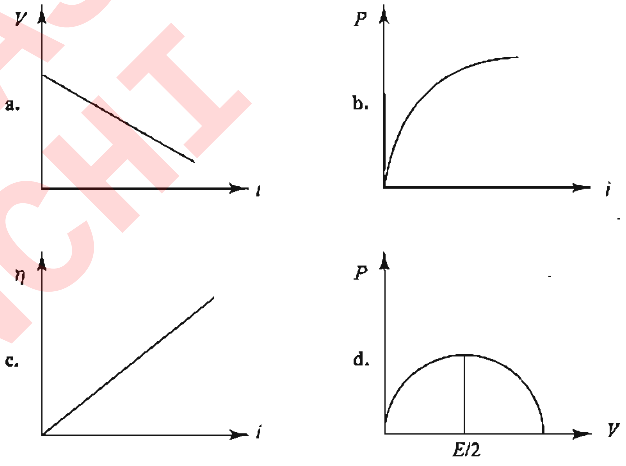


Fig. A2.96

9. In the given circuit (Fig. A2.97),

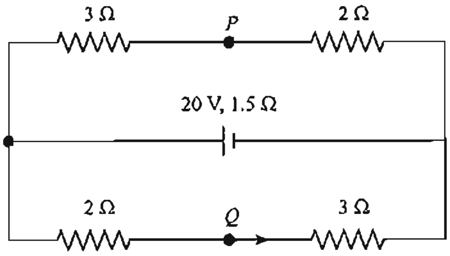


Fig. A2.97

a. the current through the battery is 5.0 A
b. P and Q are at the same potential
c. P is 2.5 V higher than Q
d. Q is 2.5 V higher than P

10. In the circuit shown (Fig. A2.98), the cell is ideal with e.m.f. = 2 V. The resistance of the coil of the galvanometer G is 1 Ω. Then

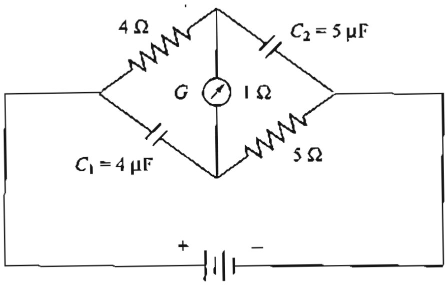


Fig. A2.98

a. no current flows in G
b. 0.2 A current flows in G
c. potential difference across C_1 is 1 V
d. potential difference across C_2 is 1.2 V

11. Which of the following statements are correct?

a. If bulbs of difference wattages are joined in parallel, then lowest wattage bulb glows with the maximum brightness.
b. If bulbs of difference wattages are joined in parallel, then highest wattage bulb glows with the maximum brightness.
c. If bulbs of difference wattages are joined in series, then the lowest wattage bulb glows with maximum brightness.
d. If bulbs of difference wattages are joined in series, then height wattage bulb glows with the maximum brightness.

12. Three ammeters A, B and C of resistances R_A, R_B and R_C, respectively, are joined as shown in Fig. A2.99. When some potential difference is applied across the terminals T_1 and T_2, their readings are I_A, I_B and I_C, then respectively, then

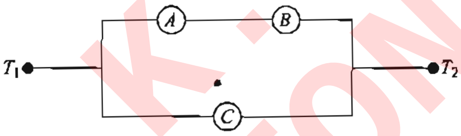


Fig. A2.99

a. $I_A = I_B$

b. $I_A R_A + I_B R_B = I_C R_C$

c. $\frac{I_A}{I_C} = \frac{R_C}{R_A}$

d. $\frac{I_B}{I_C} = \frac{R_C}{R_A + R_B}$

13. A battery of e.m.f. 2 V and initial resistance 1 Ω is connected across terminals A and B of the circuit shown in the Fig. A2.100.

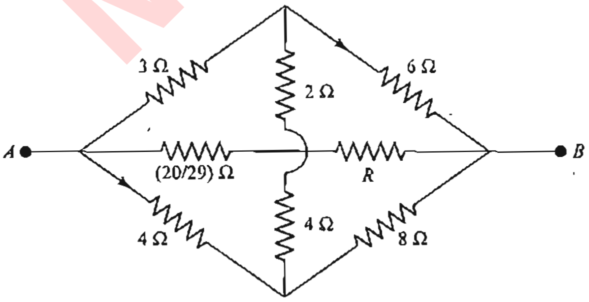


Fig. A2.100

a. Thermal power generated in the external circuit will be maximum possible when $R = \frac{16}{25}\Omega$

b. Maximum possible thermal power generated in external circuit is equal to 4 W
c. Ratio of current through 3 Ω to that through 8 Ω is independent of R
d. None of above

14. Two electric bulbs rated at 25 W–220 V and 100 W–220 V are connected in series across a 220 V voltage source. The 25 W and 100 W bulbs now draw P_1 and P_2 powers, respectively, therefore

a. $P_1 = 16$ W
b. $P_1 = 4$ W
c. $P_2 = 16$ W
d. $P_2 = 4$ W

15. In the circuit shown in Fig. A2.101

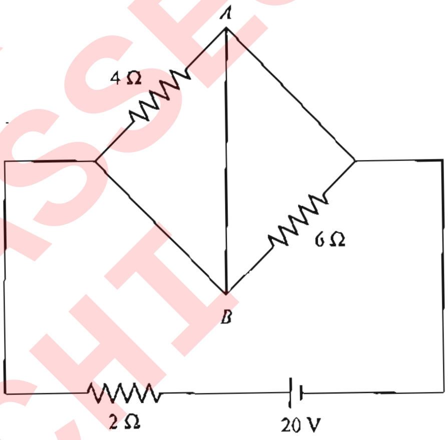


Fig. A2.101

a. Power supplied by the circuit is 200 W
b. Current flowing in the circuit is 5 A
c. Potential difference across 4 Ω resistance is equal to the potential difference across 6 Ω resistance
d. Current in wire AB is zero

16. Two cells of unequal e.m.f.s E_1 and E_2 and internal resistances r_1 and r_2 are joined as shown in Fig. A2.102. V_P and V_Q are identical at P and Q, respectively.

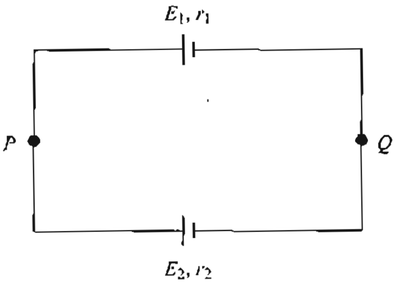


Fig. A2.102

a. The potential difference across both the cells will be equal
b. One of the cells, will supply energy to the other cell
c. The potential difference across one of the cells will be greater than its e.m.f.
d. $V_P - V_Q = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$

17. In the circuit shown in Fig. A2.103

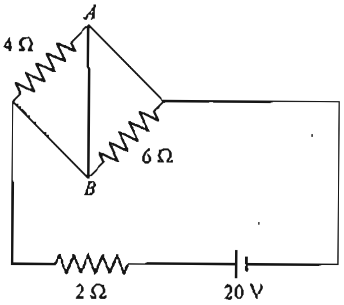


Fig. A2.103

a. Power supplied by the battery is 200 W

b. Current flowing in the circuit is 5 A

c. Potential difference across 4 Ω resistance is equal to the potential difference across 6 Ω resistance

d. Current in wire AB is zero

18. A battery of e.m.f. $\varepsilon_0 = 5$ V and internal resistance 5 Ω is connected across a long uniform wire AB of length 1 m and resistance per unit length 5 Ωm^{-1}. Two cells of $\varepsilon_1 = 1$ V and $\varepsilon_2 = 2$ V are connected as shown in the figure.

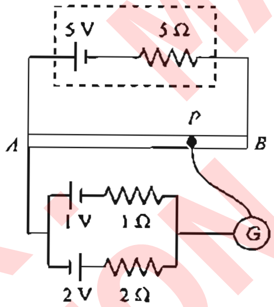


Fig. A2.104

a. The null point is at A.

b. If the jockey is touched to point B, the current in the galvanometer will be going towards B.

c. When jockey is connected to point A, no current is flowing through 1 V battery.

d. The null point is at distance of 8/15 m from A_2.

19. The diagram shows a modified meter bridge, which is used for measuring two unknown resistance at the same time. When only the first galvanometer is used, for obtaining the balance point, it is found at point C. Now the first galvanometer is removed and the second galvanometer is used, which gives balance point at D. Using the details given in the diagram, find out the value of R_1 and R_2.

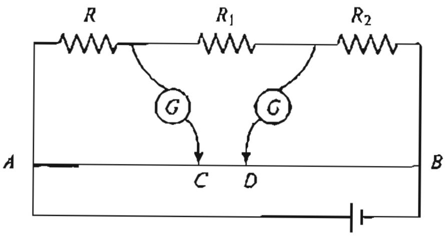


Fig. A2.105

a. $R_1 = 5R/3$ b. $R_2 = 4R/3$

c. $R_1 = 4R/3$ d. $R_2 = 5R/3$

20. A parallel-plate capacitor of capacitance 10 mF is connected to a cell of e.m.f 10 V and fully charged. Now a dielectric slab ($k = 3$) of thickness equal to the gap between the plates is completely filled in the gap, keeping the cell connected. During the filling process

a. the increase in charge on the capacitor is 200 μC

b. the heat produced is zero

c. energy supplied by the cell = increase in stored potential energy + work done on the person who is filling the dielectric slab

d. energy supplied by the cell = increase in stored potential energy + work done on the person who is filling the dielectric slab + heat produced

21. The galvanometer shown in the figure has resistance 10 Ω. It is shunted by a series combination of a resistance $S = 1$ Ω and an ideal cell of e.m.f. 2 V. A current 2 A passes as shown.

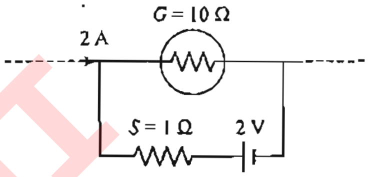


Fig. A2.106

a. The reading of the galvanometer is 1 A.

b. The reading of the galvanometer is zero.

c. The potential difference across the resistance S is 1.5 V.

d. The potential difference across the resistance S is 2 V.

22. In the circuit shown in figure, E_1 and E_2 are two ideal sources of unknown e.m.f.s. Some currents are shown. Potential difference appearing across 6 Ω resistance is $V_A - V_B = 10$ V.

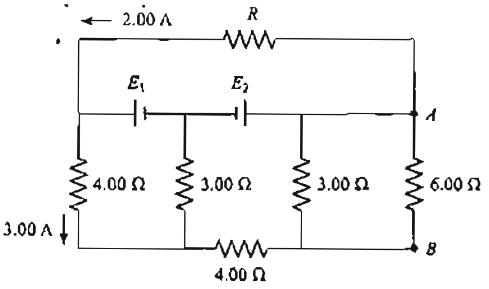


Fig. A2.107

a. The current in the 4.00 Ω resistor is 5 A

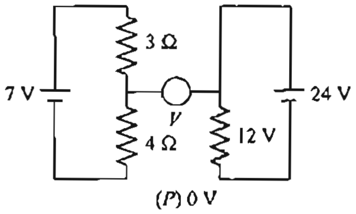
b. The unknown e.m.f. E_1 is 36 V

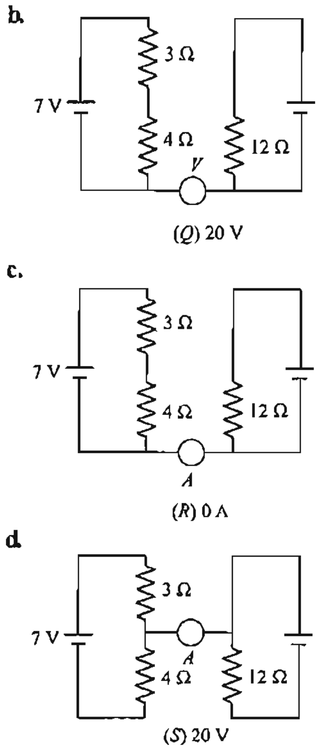
c. The unknown e.m.f. E_2 is 54 V

d. The resistance R is equal to 9 Ω

23. Match the readings of the voltmeter and ammeter, respectively shown in the figures.

a.





(*Q*) 20 V

(*R*) 0 A

(*S*) 20 V

Fig. A2.108

24. In the figure shown (all batteries are ideal),

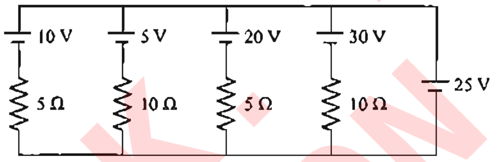


Fig. A2.109

- **a.** current through 25 V cell is 20 A
- **b.** current through 25 V cell is 12.5 A
- **c.** power supplied by 20 V cell is 20 W
- **d.** power supplied by 20 V cell is –20 W

25. Consider a resistor of uniform cross-section area connected to a battery of internal resistance zero, if the length of the resistor is doubled by stretching it then

- **a.** current will become four times
- **b.** the electric field in the wire will become half
- **c.** the thermal power produced by the resistor will become one fourth
- **d.** the product of the current density and conductance will become half

26. A variable current flows through a 1 Ω resistor for 2 s. Time dependence of the current is shown in the graph.

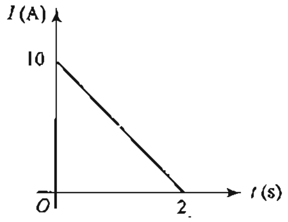


Fig. A2.110

- **a.** Total charge flown through the resistor is 10 C
- **b.** Average current through the resistor is 5 A
- **c.** Total heat produced in the resistor is 50 J
- **d.** Maximum power during the flow of current is 100 W

27. For the circuit shown in the figure

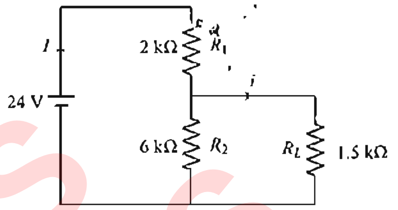


Fig. A2.111

- **a.** the current I through the battery is 7.5 mA
- **b.** the potential difference across R_L is 18 V
- **c.** ratio of powers dissipated in R_1 and R_2 is 3
- **d.** If R_1 and R_2 are interchanged, magnitude of the power dissipated in R_L will decrease by a factor of 9

(IIT-JEE, 2009)

Assertion-Reasoning Type

Solutions on page A2.45

- **a.** Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.
- **b.** Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.
- **c.** Statement I is true, Statement II is false.
- **d.** Statement I is false, Statement II is true.

1. Statement I: If an electric field is applied to a metallic conductor, the free electrons experience a force but do not accelerate; they only drift at a constant speed.

Statement II: The force exerted by the electric field is completely balanced by the Coulomb force between electrons and protons.

2. Statement II: In the meter bridge experiment shown in Fig. A2.112, the balance length AC corresponding to null deflection of the galvanometer is x. If the radius of the wire AB is doubled, the balanced length becomes $4x$.

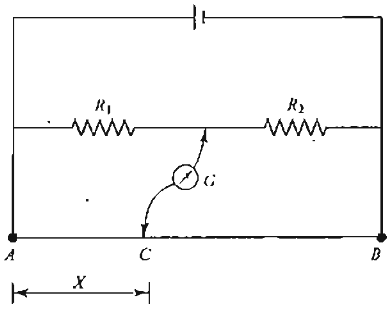


Fig. A2.112

Statement II: The resistance of a wire is inversely proportional to the square of its radius.

3. Statement I: In the potentiometer circuit shown in Fig. A2.113, E_1 and E_2 are the e.m.f. of cells C_1 and C_2, respectively, with $E_1 > E_2$. Cell C_1 has negligible internal resistance. For a given resistor R, the balance length is x. If the diameter of the potentiometer wire AB is increased, the balance length x will decrease.

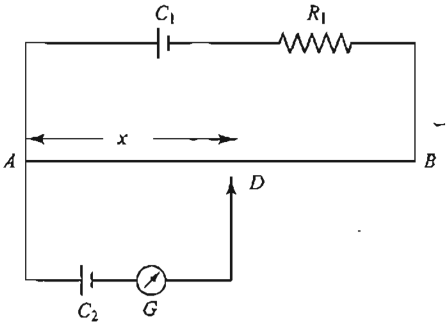


Fig. A2.113

Statement II: At the balance point, the potential difference between AD due to cell $C_1 = E_2$, the e.m.f. of cell C_2.

4. **Statement I:** When an external resistor of resistance R (connected across a cell to internal resistance r) is varied, power consumed by resistance R is maximum when $R = r$.
Statement II: Power consumed by a resistor of constant resistance R is maximum when current through it is maximum.

5. **Statement I:** The current density $\vec{J}$ at any point in ohmic resistor is in the direction of electric field $\vec{E}$ at that point.
Statement II: A point charge when released from rest in a region having only electrostatic field always moves along electric lines of force.

6. **Statement I:** A wire of uniform cross section and uniform resistivity is connected across an ideal cell. Now the length of the wire is doubled keeping volume of the wire constant. The drift velocity of electrons after stretching the wire becomes one fourth of what it was before stretching the wire.
Statement II: If a wire (of uniform resistivity and uniform cross section) of length l_0 is stretched to length nl_0 then its resistance becomes n^2 times of what it was before stretching the wire (the volume of wire is kept constant in stretching process). Further at constant potential difference, current is inversely proportional to resistance. Finally, drift velocity of free electron is directly proportional to current and inversely proportional to cross-sectional area of current carrying wire.

7. **Statement I:** If there is current in a wire, potential drop has to be there.
Statement II: If potential drop is zero, the resistance may be zero.

8. **Statement I:** Kirchhoff's laws cannot be applied in circuits with inductors.
Statement II: Kirchhoff's laws can be applied in circuits with capacitors.

9. **Statement I:** The switch S shown in Fig. A2.114 is closed at $t = 0$. Initial current flowing through battery is $\dfrac{E}{R+r}$.

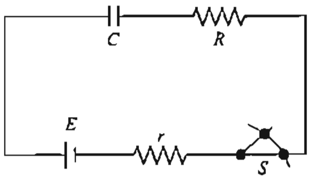
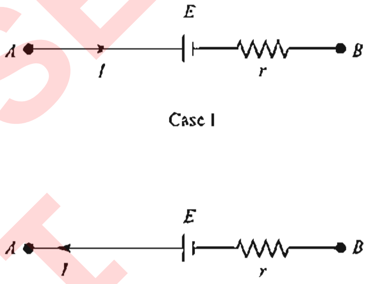


Fig. A2.114

Statement II: Initially, capacitor was uncharged, so resistance offered by capacitor at $t = 0$ is zero.

10. **Statement I:** Consider the two situations shown in Fig. A2.115 Potential difference between points A and B in Case I is more as compared to Case II.
Statement II: In Case I, $V_A - V_B = E + Ir$
In Case II, $V_A - V_B = E - Ir$



Case I

Case II

Fig. A2.115

11. **Statement I:** Electric field outside the conducting wire which carries a constant current is zero.
Statement II: Net charge on conducting wire is zero.

12. **Statement I:** A conductor carrying electric current becomes electrically charged.
Statement II: A conductor carrying electric current contains same number of positive and negative charges and thus conductor is electrically neutral.

13. **Statement II:** When the length of a conductor is doubled; its resistance will also get doubled.
Statement II: Resistance is directly proportional to the length of a conductor.

14. **Statement I:** In the following circuit, e.m.f. is 2 V internal resistance of the cell is 1 Ω and R = 1 Ω the reading of the voltmeter is 1 V.
Statement II: $V = E - ir$, where $E = 2$V, $i = 1$ A and $R = 1$ Ω

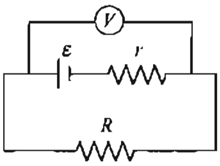


Fig. A2.116

15. **Statement I:** Direction of current cannot be from negative potential.
Statement II: Direction of current is opposite to the flow of electrons.

16. **Statement I:** When a cell is charged by connecting its positive electrode with positive terminal of the charger battery, then potential difference across the electrodes of cell will be smaller to the e.m.f. of cell.

Statement II: Potential difference across electrodes in providing electric current is $E - Ir$, where E is e.m.f. and r internal resistance.

17. Statement I: The drift velocity of electrons in a metallic wire will decrease, if the temperature of the wire is increased.

Statement II: On increasing temperature, conductivity of metallic wire decreases.

18. Statement I: In a simple battery circuit, the point at the lowest potential is positive terminal of the battery.

Statement II: The current flows towards the point of lowest potential for battery, as it does in a circuit from positive to the negative terminal.

19. Statement I: Insulators do not allow flow of current through them.

Statement II: Insulators have no free-charge carrier.

20. Statement I: In a wire of non-uniform cross section, the current is the same everywhere.

Statement II: The current in a wire is due to the drift of electrons along the wire.

21. Statement I: When two conducting wires of different resistivity having same cross-section area are joined in series, the electric field in them would be equal when they carry current.

Statement II: When wires are in series they carry equal current.

Comprehension Type

Solutions on page A2.46

For Problems 1–5

Ram and Shyam purchased two electric teakettles A and B of same size, same thickness and same volume of 0.4 L. They studied the specification of kettles as under

Kettle A:
Specific heat capacity = 1680 J/kg-K
Mass = 200 g
Cost = Rs. 400
Kettle B:
Specific heat capacity = 2450 J/kg-K
Mass = 400 g
Cost = Rs. 400

When kettle A is switched on with constant potential source, the tea begins to boil in 6 min. When kettle B is switched on with the same source separately then tea begins to boil in 8 min. The efficiency of kettle is defined as

$$= \frac{\text{Energy used for liquid heating}}{\text{Total energy supplied}}$$

They made discussion on specification and efficiency of kettles and subsequently prepared a list of questions to draw the conclusions. Some of them are as under (Assume specific heat of tea liquid as 4200 J/kg-K and density 1000 kg/m^3.)

1. Efficiency of kettle A is
   a. 63.34% b. 83.34% c. 93.34% d. 73.34%
2. Efficiency of kettle B is
   a. 82.5% b. 72.5% c. 92.5% d. 62.5%
3. Ratio of efficiency consumed charges for one time boiling of tea in kettle A to that in kettle B
   a. 3 : 5 b. 2 : 3 c. 3 : 4 d. 1 : 1
4. If resistances of coil of kettles A and B are R_A and R_B respectively, then we can say
   a. $R_A > R_B$ b. $R_A = R_B$ c. $R_A < R_B$
   d. cannot be ascertained by above date
5. If both the kettles are joined with the same source in series one after the other. Then boiling starts in kettle A and kettle B after
   a. 4 times of their original time
   b. equal to their original time
   c. 2 times of their original time
   d. cannot be ascertained by the above data

For Problems 6–8

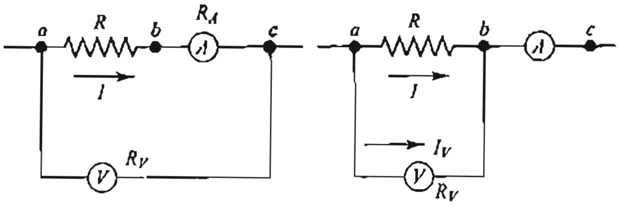
A car battery with a 12 V e.m.f. and an internal resistance of 0.04 Ω is being charged with a current of 50 A.

6. The potential difference V across the terminals of the battery is
   a. 10 V b. 12 V c. 14 V d. 16 V
7. The rate at which energy is being dissipated as heat inside the battery is
   a. 100 W b. 500 W c. 600 W d. 700 W
8. The rate of energy conservation from electrical to chemical is
   a. 100 W b. 500 W c. 600 W d. 700 W

For Problems 9–13

By ammeters and voltmeters in combination

A voltmeter and an ammeter can be used together to measure resistance and power. The resistance R of a resistor equals the potential difference V_{ab} between its terminals divided by the current I; that is, $R = V_{ab}/I$. The power input P to any circuit element is the product of the potential difference across it and the current through it; that is, $P = V_{ab}\ I$. In principle, the most straightforward way to measure R or P is to measure V_{ab} and I simultaneously.



(a) (b)

Fig. A2.117

By Ohmmeters

An alternative method for measuring resistance is to use a d' Arsonval meter in an arrangement called an ohmmeter. It consists of a meter, a resistor and a source (often a flashlight battery) connected in series (Fig. A2.118). The resistance R to be measured is connected between terminals X and Y.

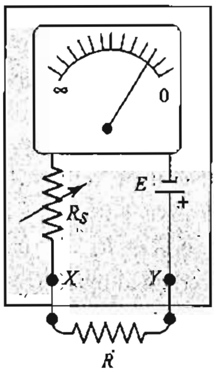
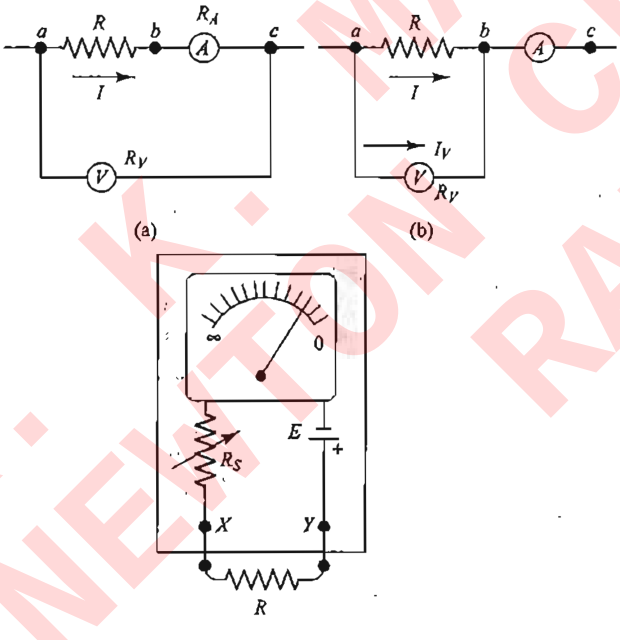


Fig. A2.118

The series resistance R_s is variable; it is adjusted so that when terminals X and Y are short-circuited (that is, when $R = 0$), the meter deflects full-scale. When nothing is connected to terminals X and Y, so that the circuit between X and Y is open (that is, when $x \to \infty$), there is no current and no deflection. For any intermediate value of R the meter deflection depends on the value of R, and the meter scale can be calibrated to read the resistance R directly. Larger currents correspond to smaller resistance, so this scale reads backward compared to the scale showing the current.

Measurement of Resistances



(a) (b)

(c)

Fig. A2.119

9. Suppose we want to measure an unknown resistance R, using the circuit in Fig. A2.119 (a). The voltmeter resistance and ammeter resistance are respectively $R_v = 10{,}000\ \Omega$ and $R_A = 2.00\ \Omega$. If the voltmeter reads 12.0 V and the ammeter reads 0.100 A, then resistance R is
   a. 120 Ω **b.** 118 Ω **c.** 121 Ω **d.** 108 Ω
10. Suppose the meters in the above problem are connected to a different resistor in the circuit of Fig. A2.119 (b), and the readings obtained on the meters are the same as in the above problem. The value of this new resistance is
   a. 120 Ω **b.** 118 Ω **c.** 121 Ω **d.** 108 Ω
11. In the ohmmeter in Fig. A2.119 (c), the coil of the meter has resistance $R_C = 50\ \Omega$, and the current required for full-scale deflection is 1.50 mA, the source is a flash light battery with $E = 0.75$ V having negligible internal resistance. The ohmmeter is to show a meter deflection of 1/2 of full scale when it is connected to a resistor with $R = 500$ W. The series resistance R_S required is equal to
   a. 50 Ω **b.** 500 Ω **c.** 450 Ω **d.** 45 Ω
12. In Fig. A2.120, 1.50 mA ohmmeter having a resistance of 100 Ω (at current of 1.50 m meter deflects full scale). The battery has an e.m.f. of 0.9 V and negligible internal resistance R is chosen so that when the terminals a and b are shorted, the meter reads 3/4 of full scale. When a and b are open the meter reads zero. The corresponding value of R is

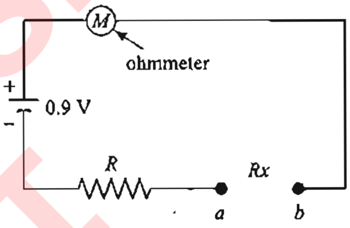


Fig. A2.120

   a. 100 Ω **b.** 800 Ω **c.** 700 Ω **d.** 900 Ω
13. In Fig. A2.120, if $R = 200\ \Omega$ and $R = 300\ \Omega$ then meter gives
   a. full-scale deflection
   b. half of full-scale deflection
   c. one fourth of deflection
   d. three fourth of deflection

For Problems 14–16

Electric fuse is a protective device used in series with an electric circuit or an electric appliance to save it from damage due to overheating produced by strong current in the circuit or application. Fuse wire is generally made from an alloy of lead and tin which has high resistance and low melting point. It is connected in series in an electric installation. If a circuit gets accidentally short circuited, a large current flows, then fuse wire melts away which causes a break in the circuit. The power through fuse (F') is equal to heat energy lost per unit area per unit time (h) (neglecting heat loses from ends of the wire).

$$P = I^2 R = h \times 2\pi r l \quad \left[R = \frac{\rho l}{\pi r^2}\right]$$

r and l are the length and radius of fuse wire, respectively.

A battery is described by its e.m.f. (E) and internal resistance (r). Efficiency of a battery (η) is defined as the ratio of the output power to the input power

$$\eta = \frac{\text{Output power}}{\text{Input power}} \times 100\%$$

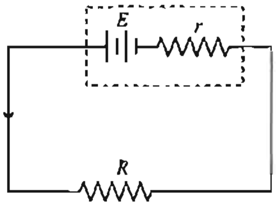


Fig. A2.121

but $I = \frac{E}{R+r}$, input power $= EI$

Output power $= EI - I^2 r$

Then $$\eta = \left(\frac{EI - I^2 r}{EI}\right)\times 100 \left(1 - \frac{Ir}{E}\right)\times 100$$

$$= 1 - \left(\frac{E}{R+r}\right)\left(\frac{r}{E}\right)\times 100$$

$$\eta = \left(\frac{R}{R+r}\right)\times 100$$

We know that output power of a source is maximum when the external resistance is equal to internal resistance, i.e., $R = r$.

14. Two fuse wires of same potential material are having length ratio 1 : 2 and ratio 4 : 1. Then respective ratio of their current rating will be
    a. 8 : 1 b. 2 : 1 c. 1 : 8 d. 4 : 1

15. The maximum power rating of a 20.0 Ω fuse wire is 2.0 kW, then this fuse wire can be connected safely to a D.C. source (negligible internal resistance) of
    a. 300 V b. 190 V c. 250 V d. 220 V

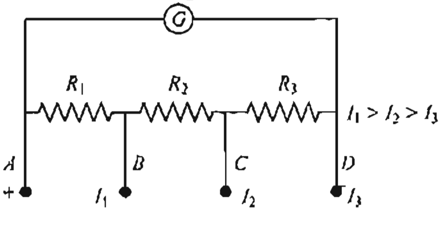
16. Efficiency of a battery (non-ideal) when delivering the maximum power is
    a. 100% b. 50% c. 90% d. 40%

For Problems 17–21

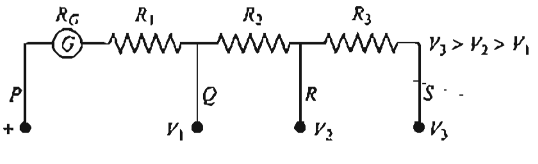
Multirange ammeter and voltmeter are used for measuring current and potential in different ranges. For three-scale multirange ammeter, the meter is connected with different resistance R_1, R_2 and R_3 as per shown in Fig. A2.122(a). The resistance of galvanometer is small and fixed and it gives full-scale deflection for certain current I_G.

When the meter is connected to the circuit being measured, one connection is made to the post marked (+) and the other to the post marked with the desired range. The values of R_1, R_2 and R_3 are so released in order to meet the requirement. Suppose connection is made between A and B, then the total current up to I_1 enters and distributes through first branch R_1 and other branch combination of R_G, R_3 and R_2. The circuit equation can be written as the potential drop is same

$$I_G[R_G + R_2 + R_3] = [I_1 - I_G]\,R_1$$



(a)



(b)

Fig. A2.122

In the similar way for other connections (between A and C, between A and D), equations can be written and R_1, R_2 and R_3 can be evaluated according to given range of I_1, I_2 and I_3.

In case of multirange voltmeter, the galvanometer is connected with three resistances R_1, R_2 and R_3 as per Fig. A2.122(b) When the meter is connected to the circuit being measured, one connection is made to the post marked + and the other to the post marked with the desired voltage range. The resistance of galvanometer is fixed and it gives full-scale deflection for certain current I_G. The resistances R_1, R_2 and R_3 are calculated according to given range of V_1, V_2 and V_3.

17. For $I_3 = 0.100$ A and $R_G = 36$ Ω, the value of $R_1 + R_2 + R_3$ should be (if meter gives full-scale deflection)
    a. 4 Ω b. 3.6 Ω c. 0.4 Ω d. 0.36 Ω

18. In the above problem, if $I_2 = 1$ A, then the value of $R_1 + R_2$ should be
    a. 4 Ω b. 3.6 Ω c. 0.4 Ω d. 0.36 Ω

19. In the above problem, if $I_1 = 10$ A, then the value of R_1 should be
    a. 4 Ω b. 3.6 Ω c. 0.4 Ω d. 0.36 Ω

20. For $V_1 = 3.00$ V, $V_2 = 15.0$ and $V_3 = 150$ V, the value of $R_1 + R_2$ and R_3 are respectively (if meter gives full-scale deflection for $I_G = 1.5$ mA)
    a. 5 kΩ, 10 kΩ, 100 kΩ b. 2 kΩ, 8 kΩ, 88 kΩ
    c. 1.975 kΩ, 8 kΩ, 88 kΩ d. 1.975 kΩ, 8 kΩ, 90 kΩ

21. If in Fig. A2.122(a) three scales are:
    $I_1 = 10$ A, $I_2 = 1$ A, $I_3 = 0.1$ A and in Fig. A2.122(b) three scales are $V_1 = 3$ V, $V_2 = 15$ V, $V_3 = 150$ V.

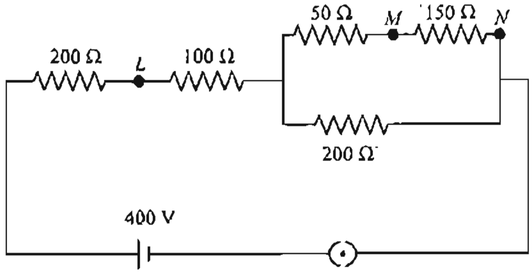


Fig. A2.123

In Fig. A2.123, the potential difference between L and M and the current through resistance of 150 Ω are to be measured, then the connections should be for multivoltmeter and multimeter are as:

a. $L \to P$, $M \to S$ and $M \to A$, $N \to D$
b. $L \to P$, $M \to Q$ and $M \to A$, $N \to B$
c. $L \to P$, $M \to R$ and $M \to A$, $N \to D$
d. $L \to P$, $M \to S$ and $M \to A$, $N \to C$

For Problems 22–24

In the connection shown in the figure, initially the switch K is open and the capacitor is uncharged. Then the switch is closed and the capacitor is charged up to the steady state and the switch is opened again. Determine the values indicated by the ammeter. [Given: $V_0 = 30\ V, R_1 = 10\,\text{k}\Omega, R_2 = 5\,\text{k}\Omega$]

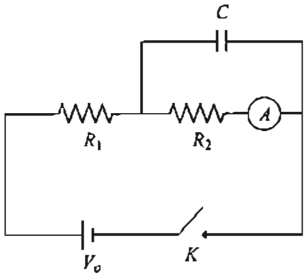


Fig. A2.124

22. Just after closing the switch
  a. 2 mA **b.** 3 mA **c.** 0 mA **d.** None of these

23. A long time after the switch was closed
  a. 2 mA **b.** 3 mA **c.** 6 mA **d.** none of these

24. Just after reopening the switch
  a. 2 mA **b.** 3 mA **c.** 6 mA **d.** None of these

For Problems 25–27

Resistance value of an unknown resistor is calculated using the formula $R = \frac{V}{I}$ where V and I be the readings of the voltmeter and the ammeter, respectively. Consider the circuits below. The internal resistances of the voltmeter and the ammeter (R_V and R_G, respectively) are finite and non-zero.

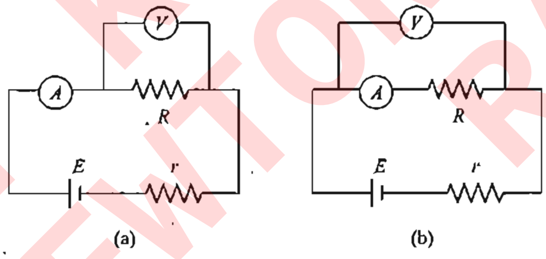


Fig. A2.125

Let R_A and R_B be the calculated values in the two cases A and B, respectively.

25. The relation between R_A and the actual value of R is

a. $R > R_A$ **b.** $R < R_A$
c. $R = R_A$ **d.** dependent upon E and r

26. The relation between R_B and the actual value of R is

a. $R < R_B$ **b.** $R > R_B$
c. $R = R_B$ **d.** dependent upon E and r

27. If the resistance of voltmeter is $R_V = 1\,\text{k}\Omega$ and that of ammeter is $R_G = 1\ \Omega$, the magnitude of the percentage error in the measurement of R (the value of R is nearly $10\ \Omega$) is

a. zero in both cases
b. non zero but equal in both cases
c. more in circuit A
d. more in circuit B

For Problems 28–30

In the circuit given below, both batteries are ideal. e.m.f. E_1 of battery 1 has a fixed value, but e.m.f. E_2 of battery 2 can be varied between 1.0 V and 10.0 V. The graph gives the currents through the two batteries as a function of E_2, but are not marked as which plot corresponds to which battery. But for both plots, current is assumed to be negative when the direction of the current through the battery is opposite the direction of that battery's e.m.f. (direction of e.m.f. is from negative to positive).

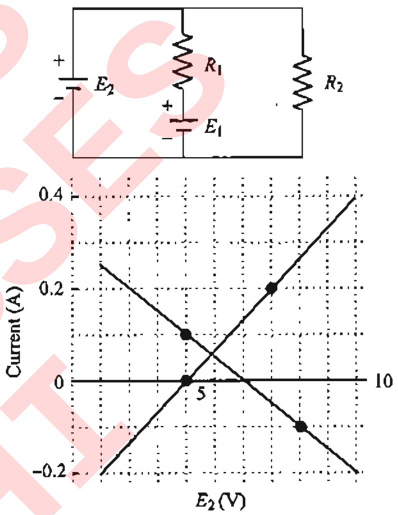


Fig. A2.126

28. The value of e.m.f. E_1 is

a. 8 V **b.** 6 V
c. 4 V **d.** 2V

29. The resistance R_1 has value

a. 10 Ω **b.** 20 Ω
c. 30 Ω **d.** 40 Ω

30. The resistance R_2 is equal to

a. 10Ω **b.** 20Ω
c. 30Ω **d.** 40Ω

For Problems 31–33

The circuit shown in steady state:

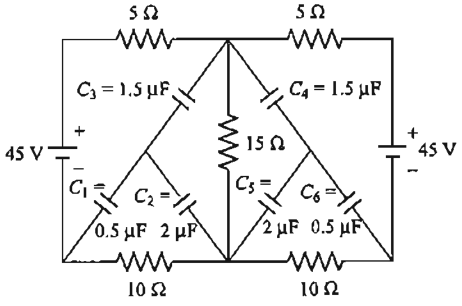


Fig. A2.127

31. The charge in capacitor C_1 is

a. 20 μC **b.** 30 μC
c. 40 μC **d.** 10 μC

32. The charge in capacitor C_2 is

a. 30μC **b.** 10μC
c. 20μC **d.** 40μC

33. The charge in capacitor C_3 is

a. 10 μC
b. 30 μC
c. 20 μC
d. 40 μC

Matching Column Type

Solutions on page A2.47

Column I and column II contains four entries each. Entries of column I are to be matched with some entries of column II. One or more than one entries of column I may have the matching with the same entries of column II and one entry of column I may have one or more than one matching with entries of column II.

1. A battery of e.m.f. E is connected across a conductor as shown in Fig. A2.128. As one observes from A to B, match the following:

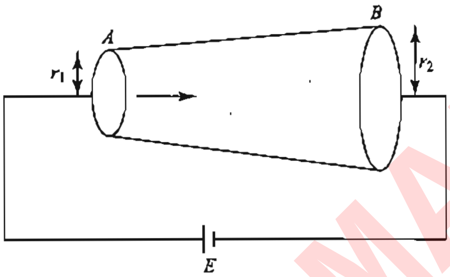


Fig. A2.128

Column I	Column II
a. Current	p. increases
b. Drift velocity of electron	q. decreases
c. Electric field	r. remains same
d. Potential drop across the length	s. cannot be determined

2. Column I gives physical quantities based on a situation in which an ideal cell of e.m.f. V is connected across a cylindrical rod of uniform cross-sectional area and conductivity (s) as shown in figure. E, J, ϕ and i are electric field at, current density through, electric flux through and current through the shaded cross section, respectively, as shown in Fig. A2.129 Physical quantities in column II are related to those in column I. Match the expressions in column I with the statements in columns II.

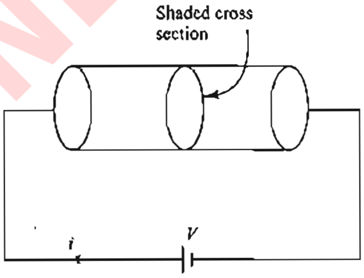


Fig. A2.129

Column I	Column II
a. $\frac{\phi}{i}$	p. Conductivity of the rod
b. $\frac{E}{J}$	q. Resistance of the rod
c. $\sigma\phi V$	r. Resistivity of the rod
d. $\frac{V}{\sigma\phi}$	s. Power delivered to the rod

3. In the circuit shown in Fig. A2.130, battery, ammeter and voltmeter are ideal and the switch S is initially closed as shown. When switch S is opened, match the parameter of column I with the effects in column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

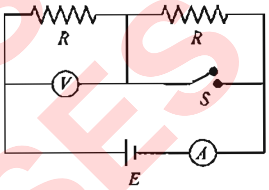


Fig. A2.130

Column I	Column II
a. Equivalent resistance across the battery	p. remains same
b. Power dissipated by left resistance R	q. increases
c. Voltmeter reading	r. decreases
d. Ammeter reading	s. becomes zero

4. Column I gives physical quantities of a situation in which a current i passes through two rods I and II of equal length that are joined in series. The ratio of free electron density (η), resistivity (ρ) and cross-section area (A) of both are in ratio $n_1:n_2 = 2:1$; $\rho_2 = 2:1$ and $A_1:A_2 = 1:2$ respectively. Column II gives corresponding results. Match the ratios of column I with the values in column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

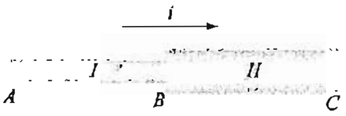


Fig. A2.131

Column I	Column II
a. $\frac{\text{Drift velocity of free electron in rod I}}{\text{Drift velocity of free electron in rod II}}$	p. 0.5
b. $\frac{\text{Electric field in rod I}}{\text{Electric field in rod II}}$	q. 1
c. $\frac{\text{Potential difference across rod I}}{\text{Potential difference across rod II}}$	r. 2
d. $\frac{\text{Average time taken by free electron to move from } A \text{ to } B}{\text{Average time taken by free electron to move from } B \text{ to } C}$	s. 4

5. Match the statements in column I with the result in column II

Column I	Column II
a. A variable resistor is connected across a non-ideal cell. As the resistance of the variable resistor is continuously increased from zero to a very large value, the electric power consumed by the variable resistor	**p.** first increases for some time and then decreases
b. A circular ring lies in space having uniform and constant magnetic field. Initially the direction of magnetic field is parallel to the plane of the ring. Keeping the centre of ring fixed, the ring is rotated by 180° about one of its diameter with constant angular speed. For the duration the ring rotates, the magnitude of induced e.m.f in the ring	**q.** first decreases for some time and then increases
c. A thin rod of length 1 cm lies along principal axis of a convex lens of focal length 5 cm. One end of rod is at a distance 1 cm from optical centre of the lens. The convex lens is moved (without rotation) perpendicular to initial principal axis by 5 mm and brought back to its initial position. The length of the image of the rod	**r.** is always constant
d. A bulb (of negligible inductance) and a capacitor in series are connected across an ideal ac source of constant peak voltage and variable frequency. As frequency of ac source is continuously increased, the brightness of bulb	**s.** increases or may increae over some time interval

6. In the circuit shown, all the ammeters are ideal. Match the following based on the circuit

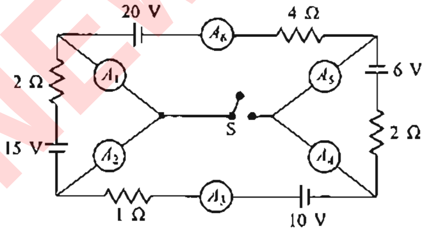


Fig. A2.132

Column I	Column II
a. If the switch S is open, the ammeter(s) that read(s) less than 10 A	**p.** A_1 and A_2
b. If the switch S is open, the ammeter(s) that read(s) equal current	**q.** A_4 and A_5
c. If the switch S is closed, the ammeter(s) that read(s) more than 5 A	**r.** A_3 and A_4
d. If the switch S is closed, the ammeter(s) that show(s) increase in the reading	**s.** A_6 and A_5

7. Consider two identical cells each of e.m.f. E and internal resistance r connected to a load resistance R

Column I	Column II
a. Maximum power transferred to load if cells are connected in series	**p.** $\frac{4E^2}{9r}$
b. Maximum power transferred to load if cells are connected in parallel	**q.** $\frac{E^2}{2r}$
c. Power transferred to load if cells are connected in series and $R = r$	**r.** $E_{eq} = E, r_{eq} = \frac{r}{2}$
d. Power transferred to load if cells are connected in parallel and $R = r$	**s.** $E_{eq} = 2E, r_{eq} = 2r$

8. In an experiement for comparing e.m.f.s of two primary cells using potentiometer, some observations are given in column I.

Column I	Column II
a. Deflection of galvanometer is in same direction at two ends of the wire	**p.** Accuracy in measurement increase
b. A series protective resistance added in series to the galvanometer	**q.** The positive terminals of all batteries/cells are not connected at a point
c. A short wire is used in potentiometer	**r.** e.m.f. of battery in primary circuit is less than the e.m.f. of cell to be measured
d. More length of wire up to null point	**s.** Uncertainity in the location of balance point increases

Archives

Solutions on page A2.49

Fill in the Blanks Type

1. An electric bulb rated for 500 watts at 100 volts is used in a circuit having a 200 volts supply. The resistance R that must be put in series with the bulb, so that the bulb delivers 500 watts is ______ ohms. **(IIT-JEE, 1987)**

2. The equivalent resistance between points A and B in the circuit (Fig. A2.133) is ______ Ω.

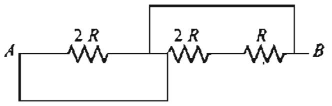


Fig. A2.133

(IIT-JEE, 1997)

3. In the circuit shown (Fig. A2.134), each battery is 5 V and has an internal resistance of 0.2 ohm.

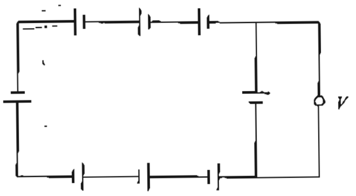


Fig. A2.134

The reading in the ideal voltmeter V is______V.

True or False

1. In an electrolyte solution, the electric current is mainly due to movement of free electrons. (IIT-JEE, 1980)

2. Electrons in a conductor have no motion in the absence of a potential difference across it. (IIT-JEE, 1982)

3. The current-voltage graphs for a given metallic wire at two different temperature T_1 and T_2 are shown in Fig. A2.135

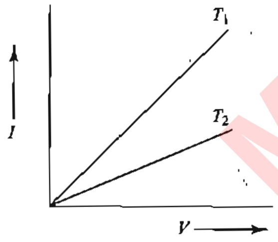


Fig. A2.135

The temperature T_1 is greater than T_2. (IIT-JEE, 1993)

Single Correct Answer Type

1. The temperature coefficient of resistance of a wire is 0.00125 per °C. At 300K, its resistance is 1 ohm. This resistance of the wire will be 2 ohms at

a. 1154K **b.** 1100K
c. 1400K **d.** 1127K (IIT-JEE, 1980)

2. A constant voltage is applied between the two ends of a uniform metallic wire. Some heat is developed in it. The heat developed is doubled if

a. both the length and the radius of the wire are halved
b. both the length and the radius of the wire are doubled
c. the radius of the wire is doubled
d. the length of the wire is doubled (IIT-JEE, 1980)

3. The electrostatic field due to a point charge depends on the distance r as $\frac{1}{r^2}$. Indicate which of the following quantities shows same dependence on r.

a. Intensity of light from a point source
b. Electrostatic potential due to a point charge
c. Electrostatic potential at a distance r from the centre of a charged metallic sphere. Given $r <$ radius of the sphere
d. None of these (IIT-JEE, 1980)

4. In the circuit shown in Fig. A2.136 the heat produced in the 5 Ω resistor due to the current flowing through it is 10 calories per second. The heat generated in the 4 Ω resistor is

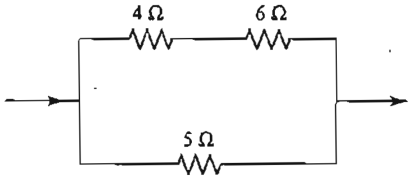


Fig. A2.136

a. 1 calorie/s **b.** 2 calories/s
c. 3 calories/s **d.** 4 calories/s

(IIT-JEE, 1981)

5. The current i in the circuit is

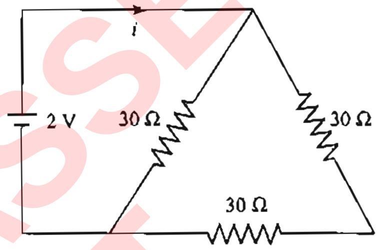


Fig. A2.137

a. 1/45 A **b.** 1/15 A
c. 1/10 A **d.** 1/5 A (IIT-JEE, 1983)

6. A piece of copper and another of germanium are cooled from room temperature to 80° k. The resistance of

a. each of them increases
b. each of them decreases
c. copper increases and germanium decreases
d. copper decreases and germanium increases

(IIT-JEE, 1981)

7. A battery of internal resistance 4 Ω is connected to the network of resistances as shown. In order that the maximum power can be delivered to the network, the value of R in Ω should be

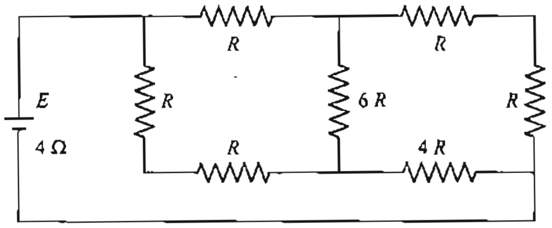


Fig. A2.138

a. 4/9 **b.** 2
c. 8/3 **d.** 18 (IIT-JEE, 1995)

8. A steady current flows in a metallic conductor of non-uniform cross-section. The quantity/quantities constant along the length of the conductor is/are

a. current, electric field and drift speed
b. drift speed only
c. current and drift speed
d. current only (IIT-JEE, 1997)

9. A parallel combination of 0.1 mΩ resistor and a 10 μF capacitor is connected across a 1.5 V source of negligible resistance. The time required for the capacitor to get charged up to 0.75 V is approximately (in seconds)

a. ∞ **b.** $\log_e 2$ **c.** $\log_{10} 2$ **d.** zero

(IIT-JEE, 1997)

10. In the circuit $P \neq R$, the reading of the galvanometer is same with switch S open or closed. Then

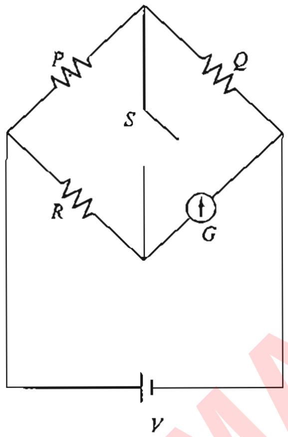


Fig. A2.139

a. $I_R = I_G$ **b.** $I_P = I_G$
c. $I_Q = I_G$ **d.** $I_Q = I_R$ **(IIT-JEE, 1999)**

11. In the given circuit, with steady current, the potential drop across the capacitor must be

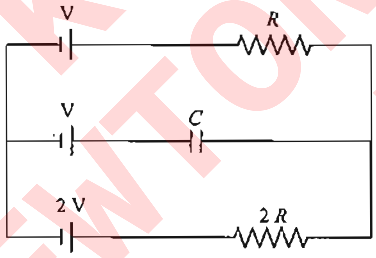


Fig. A2.140

a. V **b.** $V/2$
c. $V/3$ **d.** $2V/3$ **(IIT-JEE, 2001)**

12. A wire of length L and three identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raised by ΔT in a time t. A number N of similar cells is now connected in series with a wire of the same material and cross section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t, the value of N is

a. 4 **b.** 6
c. 8 **d.** 9 **(IIT-JEE, 2001)**

13. In the given circuit (Fig. A2.141), it is observed that the current I is independent of the value of the resistance R_6. Then the resistance values must satisfy

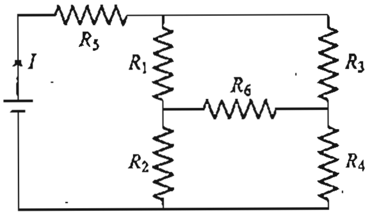


Fig. A2.141

a. $R_1 R_2 R_5 = R_3 R_4 R_6$

b. $\frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}$

c. $R_1 R_4 = R_2 R_3$

d. $R_1 R_3 = R_2 R_4 = R_5 R_6$ **(IIT-JEE, 2001)**

14. The effective resistance between point P and Q of the electrical circuit shown in Fig. A2.142 is

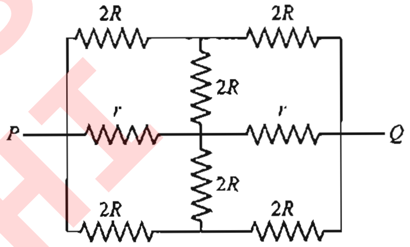


Fig. A2.142

a. $\frac{2Rr}{R+r}$ **b.** $\frac{8R(R+r)}{3R+r}$

c. $2r + 4R$ **d.** $\frac{5R}{2} + 2r$ **(IIT-JEE, 2002)**

15. A 100 W bulb B_1, and two 60 W bulbs B_2 and B_3 are connected to a 250 V source, as shown in Fig. A2.143. Now W_1, W_2 and W_3 are the output powers of the bulbs B_1 and B_2, and B_3, respectively. Then

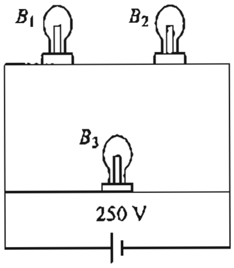


Fig. A2.143

a. $W_1 > W_2 = W_3$ **b.** $W_1 > W_2 > W_3$
c. $W_1 < W_2 = W_3$ **d.** $W_1 < W_2 < W_3$

(IIT-JEE, 2002)

16. Express which of the following setups can be used to verify Ohm's law?

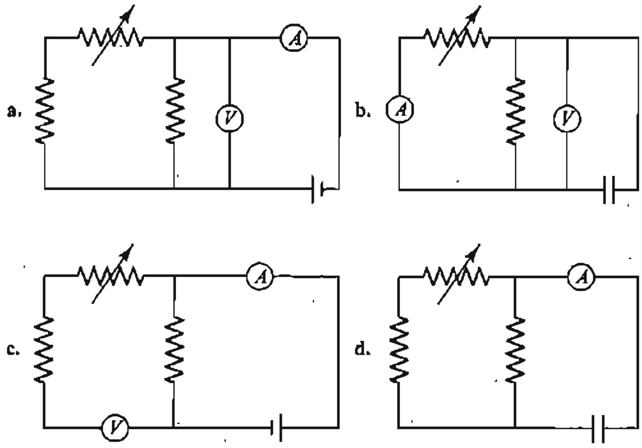


Fig. A2.144

(IIT-JEE, 2003)

17. In the shown arrangement of the experiment of the meter bridge (Fig. A2.145) if AC corresponding to null deflection of galvanometer is x, what would be its value if the radius of the wire AB is doubled?

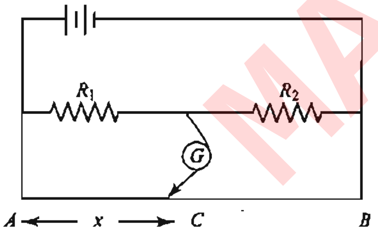


Fig. A2.145

a. x b. $x/4$
c. $4x$ d. $2x$ (IIT-JEE, 2003)

18. The three resistances of equal value are arranged in the different combinations (Fig. A2.146). Arrange them in the increasing order of power dissipation.

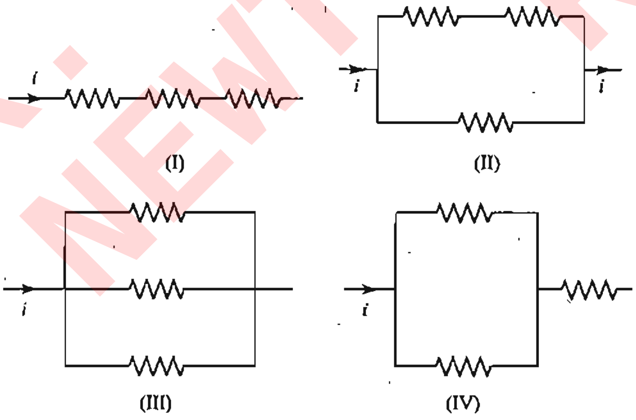


Fig. A2.146

a. III < II < IV < I b. II < III < IV < I
c. I < IV < III < II d. I < III < II < IV

(IIT-JEE, 2003)

19. Fig. A2.147 shows a Post Office box. In order to calculate the value of external resistance, it should be connected between

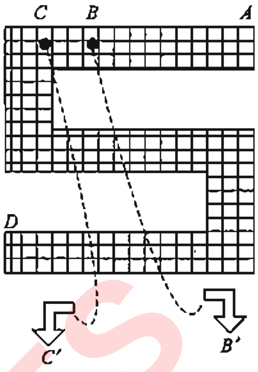


Fig. A2.147

a. B' and C' b. A and D
c. C and D d. B and D (IIT-JEE, 2004)

20. Six identical resistors are connected as shown in Fig. A2.148. The equivalent resistance will be

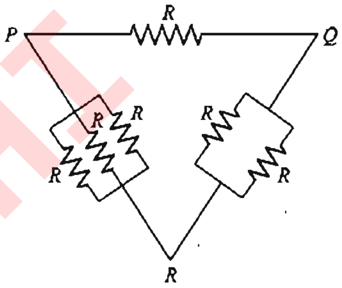


Fig. A2.148

a. Maximum between P and R
b. Maximum between Q and R
c. Maximum between P and Q
d. All are equal (IIT-JEE, 2004)

21. A capacitor is charged using an external battery with a resistance x in series. The dashed line shows the variation of $\ln I$ with respect to time. If the resistance is changed to $2x$, the new graph will be

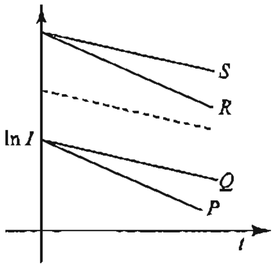


Fig. A2.149

a. P b. Q
c. R d. S (IIT-JEE, 2004)

22. Find out the value of current through 2 Ω resistance for the given circuit (Fig. A2.150).

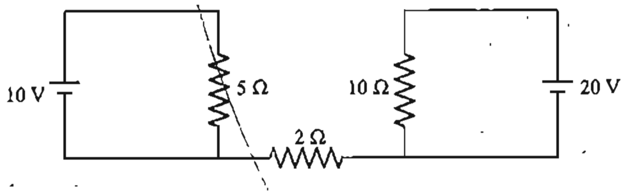


Fig. A2.150

a. 0 b. 2 A
c. 5 A d. 4 A **(IIT-JEE, 2005)**

23. A 4 µF capacitor and a resistance of 2.5 mΩ are in series with 12 V battery. Find the time after which the potential difference across the capacitor is 3 times the potential difference across the resistor. [ln2 = 0.693]

a. 13.86 s b. 6.93 s
c. 7 s d. 14 s **(IIT-JEE, 2005)**

24. A moving coil galvanometer of resistance 100 Ω is used as an ammeter using a resistance 0.1 Ω. The maximum deflection current in the galvanometer is 100 µA. Find the minimum current in the circuit so that the ammeter shows maximum deflection.

a. 100.1 mA b. 1000.1 mA
c. 10.01 mA d. 1.01 mA

(IIT-JEE, 2005)

25. An ideal gas is filled in a closed rigid and thermally insulated container. A coil of 100 Ω resistor carrying current 1 A for 5 min supplies heat to the gas. The change in internal energy of the gas

a. 10 kJ b. 30 kJ
c. 20 kJ d. 0 kJ **(IIT-JEE, 2005)**

26. In Fig. A2.151, $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $C_1 = 4\ \mu F$, $C_2 = 2\ \mu F$, the time constants (in µs) for the circuits I, II, III are respectively.

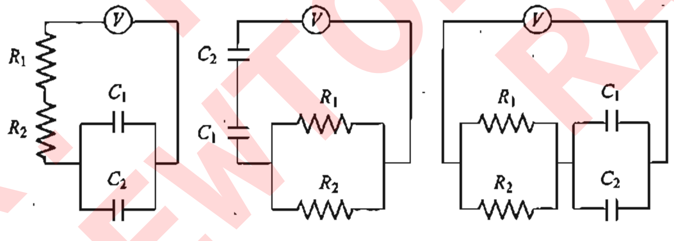


Fig. A2.151

a. 18, 4, 8/9 b. 18, 8/9, 4
c. 4, 18, 8/9 d. 4, 8/9, 18

(IIT-JEE, 2006)

27. Two bars of radius r and $2r$ are kept in contact as shown in Fig. A2.152. An electric current I is passed through the bars. Which of the following is correct?

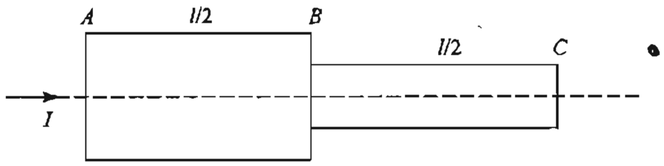


Fig. A2.152

a. $V_{AB} = 2V_{BC}$
b. Power across BC is 4 times the power across AB
c. Current density in AB and BC is equal
d. Electric field due to current inside AB and BC is equal

(IIT-JEE, 2006)

28. A resistance of 2 Ω is connected across one gap of a metre-bridge (the length of the wire is 100 cm) and an unknown resistance, greater than 2 Ω, is connected across the other gap. When these resistances are interchanged, the balance point shifts by 20 cm. Neglecting any corrections, the unknown resistance is

a. 3 Ω b. 4 Ω
c. 5 Ω d. 6 Ω

(IIT-JEE, 2007)

29. A circuit is connected as shown in Fig. A2.153 with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is

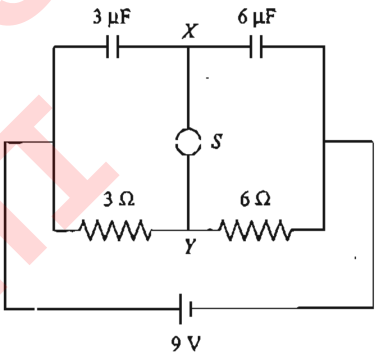


Fig. A2.153

a. 0 b. 54 µC
c. 27 µC d. 81 µC **(IIT-JEE, 2007)**

30. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant $K = 2$. The level of liquid is $d/3$ initially. Suppose the liquid level decreases at a constant speed V, the time constant as a function of time t is

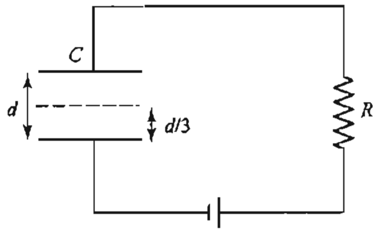


Fig. A2.154

a. $\dfrac{6\varepsilon_0 R}{5d + 3Vt}$ b. $\dfrac{(15d + 9Vt)\varepsilon_0 R}{2d^2 - 3dVt - 9V^2t^2}$

c. $\dfrac{6\varepsilon_0 R}{5d - 3Vt}$ d. $\dfrac{(15d - 9Vt)\varepsilon_0 R}{2d^2 + 3dVt - 9V^2t^2}$

(IIT-JEE, 2008)

31. Fig. A2.155 shows three resistor configuration R_1, R_2, R_3 kkconnected to 3 V battery. If the power dissipated by the configuration R_1, R_2 and R_3 is P_1, P_2 and P_3 respectively, then

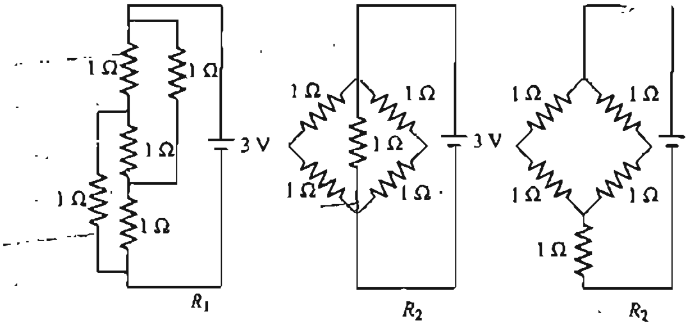


Fig. A2.155

a. $P_1 < P_2 < P_3$
b. $P_1 > P_2 > P_3$
c. $P_2 > P_1 > P_3$
d. $P_3 > P_2 > P_1$

(IIT-JEE, 2008)

32. Statement 1: In a meter-bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.

Statement 2: Resistance of a metal increases with the increase in temperature.

a. Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for statement 1.
b. Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1.
c. Statement 1 is true, Statement 2 is false.
d. Statement 1 is false, Statement 2 is true. (IIT JEE, 2008)

Multiple Correct Answers Type

1. Capacitor C_1 of capacitance 1 micro-farad and capacitor C_2 of capacitance 2 micro-farad are separately charged fully by a common battery. The two capacitors are then separately allowed to discharge through equal resistors at time $t = 0$.

a. The current in each of the two discharging circuits is zero at $t = 0$
b. The currents in the two discharging circuits at $t = 0$ are equal but not zero.
c. The currents in the two discharging circuits at $t = 0$ are unequal
d. Capacitors C_1 loses 50% of its initial charge sooner than C_2 loses 50 % of its initial charge. (IIT-JEE, 1989)

2. Read the following statements carefully:

Y: The resistively of a semiconductor decreases with the increase in temperature.

Z: In a conducting solid, the rate of collisions between free electrons and ions increases with the increase in temperature

Select the correct statement(s) from the following:

a. *Y* is true but *Z* is false
b. *Y* is false but *Z* is true
c. Both *Y* and *Z* are true
d. *Y* is true and *Z* is the correct reason for *Y* (IIT-JEE, 1993)

3. In the circuit shown in Fig. A2.156 the current through the

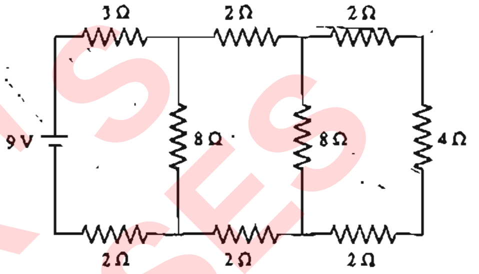


Fig. A2.156

a. 3 Ω resistor is 0.50 A
b. 3 Ω resistor is 0.25 A
c. 4 Ω resistor is 0.50 A
d. 4 Ω resistor is 0.25 A

(IIT-JEE, 1998)

4. When a potential difference is applied across, the current passing through

a. an insulator at 0K is zero
b. a semiconductor at 0K is zero
c. a metal at 0K is finite
d. a p–n dipole at 300K is finite, if it is reverse biased

(IIT-JEE, 1999)

5. For the circuit shown in the figure,

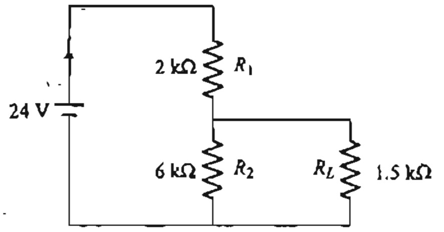


Fig. A2.157

a. the current *I* through the battery is 7.5 mA
b. the potential difference across R_L is 18 V
c. ratio of powers dissipated in R_1 to that in R_2 is 3
d. If R_1 and R_2 are interchanged, magnitude of the power dissipated in R_L will decrease by a factor of 9

(IIT-JEE, 2009)

ANSWERS AND SOLUTIONS

Objective Type

1. **b.** When a source is connected, a current starts to flow through the conductor. Let it be *I*. Then current density at a section is equal to *I*/*A*, where *A* = cross-sectional area. Since cross-sectional area at *P* is maximum, therefore current density at *P* is minimum. Hence (b) is correct.

Since electric field is $\frac{J'}{\sigma}\left(-\frac{1}{A\sigma}=\frac{I\rho}{A}\right)$, therefore at *P* electric field is minimum while that at *Q* it is maximum.
Rate of generation of heat per unit length at a section will be equal to $\frac{I^2\rho}{A}$. It is minimum at *P* and maximum at *Q*.

The mean kinetic energy of free electrons = $\frac{1}{2}mv^2_d$ which is minimum at P and maximum at Q.

2. c. In element A, the resistance remains constant up to the potential drop of 10 V. Further increase in the voltage does not increase this current (which is constant at 1 A). This means that the ratio V/R_A = constant and this resistance R_A increases linearly with voltage.
In element B, the resistance decreases gradually upto 15 V and afterwards the resistance R_B increases linearly with voltage. When both A and B are in series, the current in the circuit will increase nonlinearly upto 1 A when the total voltage drop across A and B becomes $10 + 15 = 25$ V. Further increase in this voltage does not bring about any change in the current as shown in solution (c). The voltage drop across A will go on increasing while that across B remains fixed at 15 V.

3. c. Applying Kirchhoff's law at A, C, D, the direction of the currents in each branch will be as shown in Fig. A2.158. Now, it is clear from the figure that the batteries 1 and 4 are getting charged.

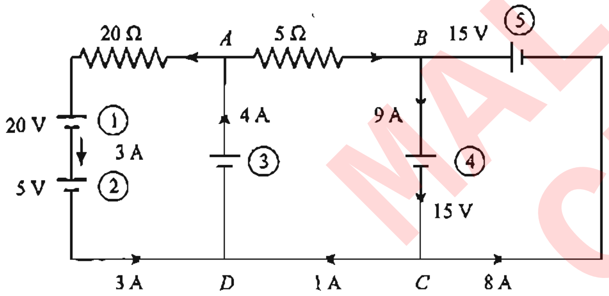


Fig. A2.158

4. a. When only 4 Ω resistance is shunted $(I_g) = i/5$

$G \times i/5 = 4 \times (4/5) \Rightarrow G = 16\,\Omega$

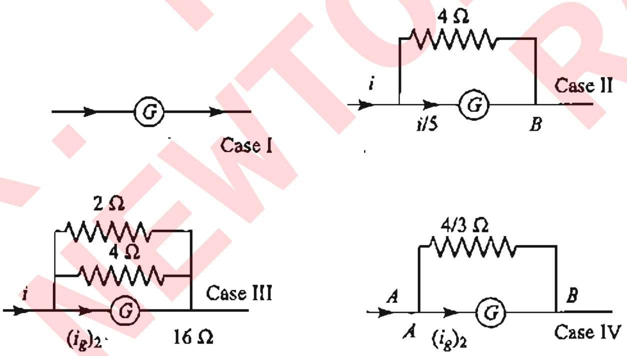


Fig. A2.159

5. d. Initially the capacitance acts as short-circuit and at steady state it acts as open circuit.

Hence, at $t = 0$, $i = \dfrac{2}{I \times 10^3} = 2$ mA

At steady state $i = \dfrac{2}{2 \times 10^3} = 1$ mA

6. c. Resistance of the first bulb, $R_1 = \dfrac{V^2}{P_1}$

Resistance of the second bulb, $R_1 = \dfrac{V^2}{P_2}$

When both bulbs are connected in series,

$$R_{eq} = V^2\left[\frac{1}{P_1} + \frac{1}{P_2}\right] = \frac{V^2(P_1 + P_2)}{P_1P_2}$$

Hence power consumed is

$$P = \frac{V^2}{R} = \frac{V^2}{V^2\left(\dfrac{P_1 + P_2}{P_1P_2}\right)}$$

7. c. In steady state, no current will pass through the capacitor.
In the outer loop

$2V - 2iR - iR - V = 0$

$\Rightarrow \quad i = V/3R$

For the upper loop, $V - V_C - iR - V = 0$

$\Rightarrow \quad |V_C| = iR = V/3$

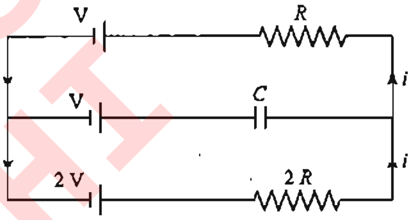


Fig. A2.160

8. c. $(V_A - V_B) - 6 - 3 \times 2 + 9 - 3 \times 0.7 = 0$

or $V_A - V_B = 5.1$

or $V_B - V_A = -5.1$ V

9. d. In (a), (b) and (c), the condition for balanced Wheatstone's bridge is satisfied. However, this condition is not satisfied if R_1 and R_2 are interchanged.

10. a. The reading of the galvanometer is same whether the switch S is open or closed. So, there is no current in the diagonal arm of the bridge. So, the bridge is clearly balanced.

$\therefore \quad \dfrac{P}{Q} = \dfrac{R}{G}$

Since $P \neq R$, $\therefore$ $Q \neq G$
In the balanced position $I_P = I_Q$ and $I_R = I_G$
But $I_P \neq I_R$ and $I_Q \neq I_G$
So, (a) is the right choice.

11. b. $1.45 = 1.3 + Ir_A$
$1.45 = 1.5 - Ir_B$
$Ir_B = 0.05$
$Ir_A = 0.15$

Dividing, $\dfrac{r_B}{r_A} = \dfrac{1}{3}$

or $r_A = 3r_B$

12. b. Clearly, $E_2 = IY$

$2 = \dfrac{12}{500 + Y}Y$ or $500 + Y = 6Y$

or $5Y = 500$ or $Y = 100\,\Omega$

13. a. Since $E_2 < E_1$, $\therefore$ $l_2 < l_1$.

14. b. Potential difference V_P across P as determined from E_1 is given by $V_P = \left(\frac{E_1}{P+Q}\right)P$.

Potential V_P across P as determined from E_2 is same as E_2 because no current is drawn, i.e., $V_P = E_2$.

Therefore,

$$E_2 = E_1\left(\frac{P}{P+Q}\right) \Rightarrow \qquad \frac{E_2}{E_1} = \frac{P}{P+Q}$$

15. a. In a Wheatstone's bridge, the deflection in the galvanometer does not change if the battery and galvanometer are interchanged.

16. a. $V = E - Ir_1, V = 0$

$\therefore$ $E = Ir_1$

Total e.m.f. $= Ir_1 = Ir_1 = 2\,Ir_1$

Total resistance $= R + r_1 + r_2$

Now, $I = \frac{2Ir_1}{R + r_1 + r_2}$

or $R + r_1 + r_2 = 2r_1$

or $R = 2r_1 - r_1 - r_2$ or $R = r_1 - r_2$

17. a. Effective resistance across voltmeter = 50 kΩ. Total resistance across the dc supply = 450 kΩ.

Current drawn form supply = $\frac{1000\text{ V}}{450\text{ k}\Omega} = \frac{1}{450}$ A. Potential difference across voltmeter

$$= \frac{50 \times 1000}{450}\text{ V} = \frac{1000}{9}\text{ V} = 111\text{ V}$$

18. d. When $r = 0$ the terminal potential difference in the arm containing R_1 remains unchanged, so the ammeter reading does not change.

19. d. When a resistance is joined in parallel with the voltmeter, the total resistance of the circuit decreases. Current will increase, so ammeter reading will increase. The equivalent resistance across the voltmeter decreases and hence its reading will decrease.

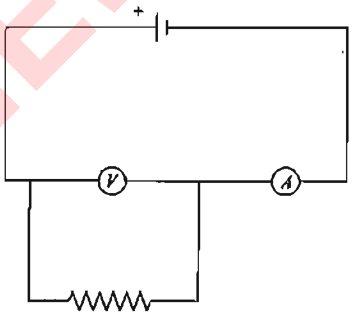


Fig. A2.161

20. c. $\frac{R}{80} = \frac{20}{80}$ or $R = 20\,\Omega$

21. a. A and B are effectively in parallel and hence give the same reading at all times.

22. a. The equivalent circuit can be redrawn as shown in Fig. A2.162.

Let $R = \frac{2R \times 2}{2R + 2} + 2$

or $R = \frac{2R}{R+1} + 2$

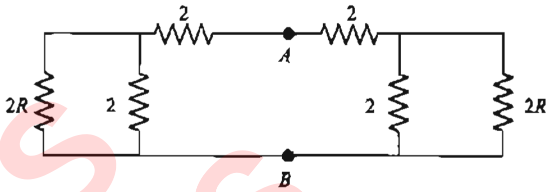


Fig. A2.162

or $R = \frac{2R + 2R + 2}{R+1} \Rightarrow \quad R = \frac{2 + 4R}{R+1}$

$$R^2 - 3R - 2 = 0 \Rightarrow R = \frac{3 + \sqrt{17}}{2}$$

$$R_{AB} = \frac{R}{2} = \frac{3 + \sqrt{17}}{4}$$

23. b. $U = \frac{1}{2}CV^2 = \frac{1}{2}C(2IR)^2 = 2I^2R^2C$

24. a. Potential difference = 0. $\therefore$ Charge = 0.

25. d. Potential difference across the central limb = 16 V = potential difference across 16 Ω = potential difference across the left limb.

⇒ current through 16 Ω = 1 A

⇒ current through the left limb = 1 A and $R = 11\,\Omega$

$\therefore$ $\tau_C = \left(\frac{16}{2} + 2\right)\Omega \times (4 \times 10^{-6})\text{ F} = 40\,\mu\text{s}$.

26. b. $R(I_1 + i_2) = E_1$

$i_2R + (i_1 + i_2)R = E_2$

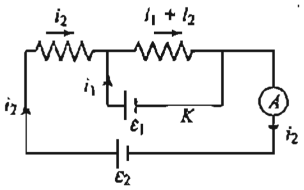


Fig. A2.163

$$i_2 = \frac{E_2 - E_1}{R}$$

[Initially $E = E_2/R$]

27. c. $i = \frac{2.5 - 1}{30} = 0.05$ A

$-20i + 2.5 = V_{\text{capacitor}} + 1$

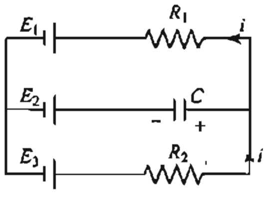


Fig. A2.164

$$V_{\text{capacitor}} = 0.5\,\text{V}$$

$$q^{(-)} = CV = 10 \times 0.5 = 5$$

28. d $i = \dfrac{12}{500+R}$

$$V_{AB} = iR = \frac{12}{500+R}R = 2$$

$\therefore$ $R = 100\,\Omega$

29. a. $R_0 = 2500\,\Omega$

$E = 125\,\text{V}$

When r is connected in series

$$i = \frac{E}{R_0 + r}$$

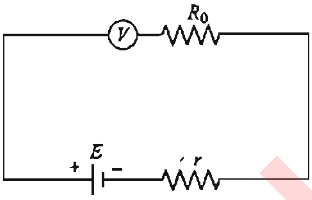


Fig. A2.165

Reading in voltmeter $= E - ir = 100$

$\therefore$ $100 = 125 - \dfrac{125}{2500+r} \times r$

$\Rightarrow$ $r = 625\,\Omega$

30. a. $i = \dfrac{15}{60} = \dfrac{1}{4}\,\text{A}$

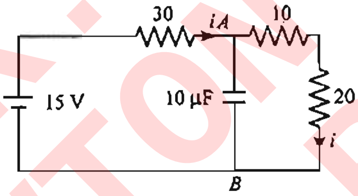


Fig. A2.166

$$V_{AB} = \frac{1}{4} \times 30 = 7.5\,\text{V}$$

$$q = eV = 10 \times 7.5 = 75\,\mu\text{C}$$

31. a $E = j\rho$ [j = current density]

$j = \dfrac{\ell}{\pi r^2}$ [r = radius of cross section at distance 'x' from left end]

$$r = \left[a + \frac{(b-a)}{l}x\right]$$

Hence, $E = \dfrac{Vl^2\rho}{\pi R\{al + (b-a)x\}^2}$

32. a. Resistance of each part $= \dfrac{R}{2n}$.

For 'n' such parts connected in series, equivalent resistances, say

$$R_1 = n = \left[\frac{R}{2n}\right] = \frac{R}{2}$$

Similarly, equivalent resistance say R_2 for another set o n identical, respectively, in parallel resistances would b

$$= \frac{1}{n}\left(\frac{R}{2n}\right) = \frac{R}{2n^2};$$

For getting maximum of R_1 and R_2, they resistances should b connected in series and hence,

$$R_{eq} = R_1 + R_2 = \frac{R}{2}\left(1 + \frac{1}{n^2}\right)$$

33. a. Either apply Kirchhoff's law or solve analytically.

34. d. Use resistance in parallel and series combination.

35. a. In the steady state, no current flows through the capacito

36. a. Recall the condition for maximum power flow through circuit.

37. c. $R_{\text{equal}} = \dfrac{\dfrac{8}{3} \times 2}{\dfrac{8}{3} + 2} = \dfrac{16}{14} = \dfrac{8}{7}\,\Omega$

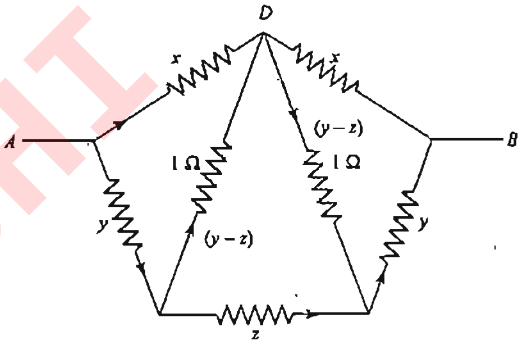


Fig. A2.167

The charge x passing between A and D will entirely pass to DA
So, we can detach the circuit at point D and shown in Fig. A2.16

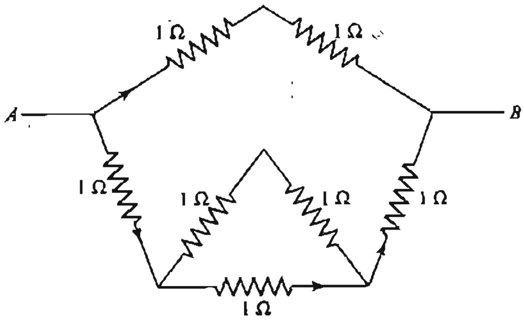


Fig. A2.168

38. d. $(0.1 + 0.3 + 0.6)i_1 = (9 + 0.9) \times 10$

$$i = 99\,\text{mA}$$

$$I = (99 + 10)\,\text{mA} = 0.109\,\text{A}$$

39. b. The potential of point G should be kept zero.

40. d. The given circuit in steady state reduces to

$$I = \frac{12-4}{4} = 2\,\text{A}$$

41. d. Charge flow through the battery will be $2CV$, while the energy stored in capacitor will be the same.

42. b. When switch S_1 is open

$$\frac{6}{E}=\frac{L}{2}\Big/L=\frac{1}{2}$$

$$\therefore \quad E = 12\text{ V}$$

When switch S_2 is closed

$$\frac{6\times10}{10+r}=\frac{5L}{12L}E=\frac{5}{12}\times12=5$$

$$\therefore \quad 10+r=12 \quad \text{or} \quad r=2\,\Omega$$

43. d. From Fig. A2.169

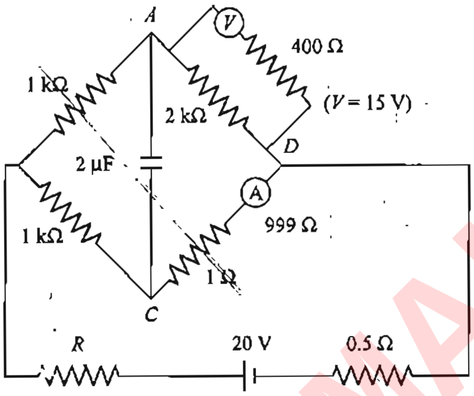


Fig. A2.169

$$V_A-V_D=15\text{V}$$

and $$V_C-V_D=iR$$

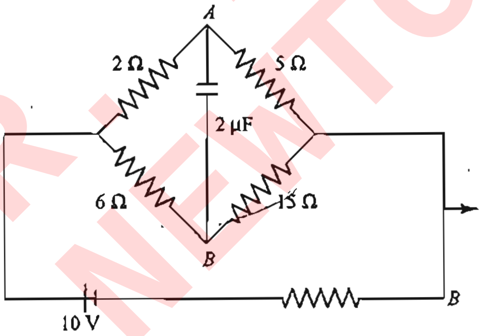
$$=15\times10^{-3}\times(1+999)=15\text{ V}$$

$$V_A-V_C=15-15=0$$

Energy stored = 0

44. a. This is a case of balanced Wheatstone's bridge.

45. d. $V_A-V_B=0$



It is a balanced Wheatstone bridge

Fig. A2.170

46. b. Points A, B, C and D are at same potential, therefore no current will be flowing through AB, BC, CD and DA.

$$\therefore \quad R_{\text{eq}}=(R_{OA}+R_{AP})/4=(r/4)\left(\frac{\pi a}{2}+a\right)$$

47. b. The equivalent capacitance C_{eq}

$$\frac{1}{C_{\text{eq}}}=\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$$

$$=C_{\text{eq}}=\frac{6}{5}\ \mu\text{F}$$

The charge that flows from P to Q

$$=Q=C_{\text{eq}}V$$

$$=\frac{6}{5}\times5$$

$$=6\ \mu\text{C}$$

48. d. $i=\dfrac{7\text{ V}}{7\,\Omega}=1\text{ A}$

Current flows in anticlockwise direction in the loop.

From earthing to v_1: $0-1\times2-1\times2-5=V_1$

$$V_1=-9\text{ V}$$

49. c. $i=\dfrac{50}{20+R}$

Potential drop across R = potential drop across AB

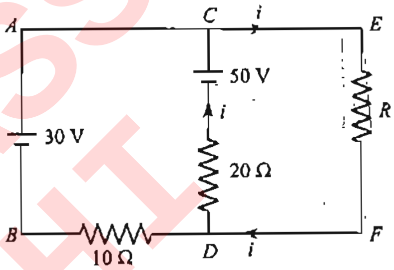


Fig. A2.171

$$\frac{50}{20+R}R=30 \quad \Rightarrow \quad R=30\ \Omega$$

50. b. In Fig. A2.172, all resistances are connected in parallel.

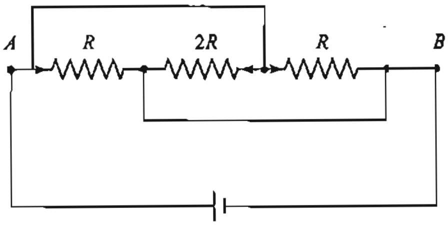


Fig. A2.172

So, $R_{\text{eq}}=\dfrac{2R\times R/2}{2R+R/2}$ and current in all resistances flows from positive terminal of battery (means A end) to negative terminal of battery (means B end).

51. b. Just before S_1 is closed the potential difference across capacitor 2 is 2ε.

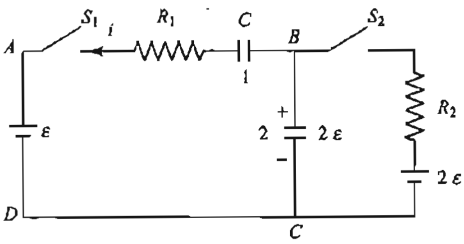


Fig. A2.173

Just after S_1 is closed the potential differences across capacitors 1 and 2 are 0 and 2ε respectively. Applying KVL to loop $ABCD$ immediately after S_1 is closed,

$$\varepsilon = -iR_1 + 0 + 2\varepsilon, \quad i = \frac{\varepsilon}{R_1} \text{ towards left}$$

52. d. This is a d.c. circuit because the battery is the only source of voltage. Hence, the capacitors behave like open circuits. An equivalent circuit is then two parallel sets of two identical series resistors (see Fig. A2.174). The voltage drop across each parallel branch must be the battery voltage of 3 V. Since the resistors are identical there is an equal voltage drop of 1.5 V across each resistor. In particular, there is a drop of 1.5 V across resistor A.

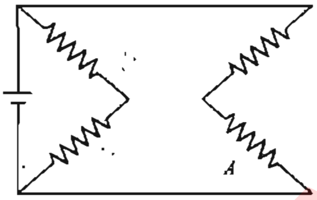


Fig. A2.174

53. c. $U_i = \frac{1}{2}C\varepsilon_1^2$

$$U_f = \frac{1}{2}C\varepsilon_2^2 \quad \Delta U = \frac{1}{2}C\left(\varepsilon_2^2 - \varepsilon_1^2\right)$$

$$Q_{\text{in}} = +C\varepsilon_1,\ Q_f = -C\varepsilon_2,\ \Delta Q = |Q_f - Q_i|$$

$$= C(\varepsilon_2 + \varepsilon_1)$$

Work done by battery $W_b = \varepsilon_2\,\Delta Q$

$$= C(\varepsilon_2 + \varepsilon_1)\varepsilon_2$$

Heat generated $= W_b - \Delta U$

$$= C\varepsilon_2^2 + C\varepsilon_1\varepsilon_2 - \frac{1}{2}C\left(\varepsilon_2^2 - \varepsilon_1^2\right)$$

$$= \frac{1}{2}C\left(\varepsilon_2^2 + \varepsilon_1^2 + 2\varepsilon_1\varepsilon_2\right) = \frac{1}{2}C(\varepsilon_1 + \varepsilon_2)^2$$

54. c. Apply KVL,

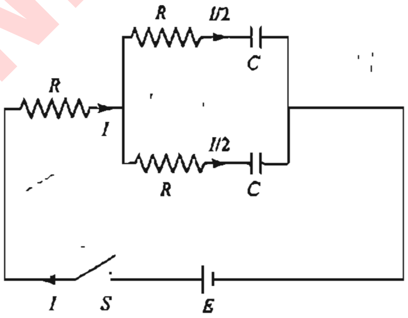


Fig. A2.175

$$\frac{I}{2} = \frac{dq}{dt} \Rightarrow I = 2\frac{dq}{dt}$$

$$E - IR - \frac{I}{2}R - \frac{q}{C} = 0$$

$$E - \frac{3}{2}IR - \frac{q}{C} = 0$$

$$E - \frac{3}{2}\times 2\frac{dq}{dt}R - \frac{q}{C} = 0$$

$$q_{(t)} = EC\left(1 - e^{-t/3RC}\right)$$

55. b. $I_1 = \dfrac{5}{2+3} = 1$ A

$$I_2 = \frac{5}{3+3} = \frac{5}{6} \text{ A}$$

$$V_A - 0 = 3I_1 \quad \Rightarrow \quad V_A = 3\text{ V}$$

$$V_B - 0 = 3I_2 \quad \Rightarrow \quad V_B = 2.5\text{ V}$$

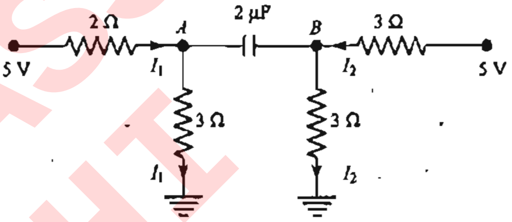


Fig. A2.176

Potential difference across capacitor

$$= 3\text{ V} - 2.5\text{ V} = 0.5\text{ V} = 1/2\text{ V}$$

$$q = CV = 2 \times 1/2 = 1\ \mu\text{C}$$

56. a. If V_1 decreases, then potential difference across xz will decrease. So, to have same potential difference, i.e., V_2 across xz, length xz has to be increased or z should be moved towards y.

b. is incorrect, because for this z should have been moved towards x.

c. will be incorrect because xz can get warm up if current through it increases which is not possible.

d. is incorrect, because there is no current in R_1.

57. c. After long time, current through the capacitor = 0.

$\therefore$ Current through the 6 Ω resistor, $i = \dfrac{12-4}{8} = 1$ A

Voltage across capacitor, $V = 4 + (6)(1) = 10$ V

Charge on the capacitor, $Q = (10)(10) = 100\ \mu$C.

After the insertion of the dielectric

$$C' = \frac{A\varepsilon_0}{\dfrac{d}{3} + \dfrac{d}{3} + \dfrac{d}{3(2)}} = \frac{A\varepsilon_0}{d}\left[\frac{1}{\left(\dfrac{1}{3} + \dfrac{1}{3} + \dfrac{1}{6}\right)}\right]$$

$$= (10)\left(\frac{1}{(5/6)}\right) = \frac{5}{6}(10) = 12\ \mu\text{F}$$

Hence, voltage across the capacitor remains same.

Charge on the capacitor, $Q' = (12)(10) = 120\ \mu$C.

58. d. At $t = \infty$, the equivalent circuit is

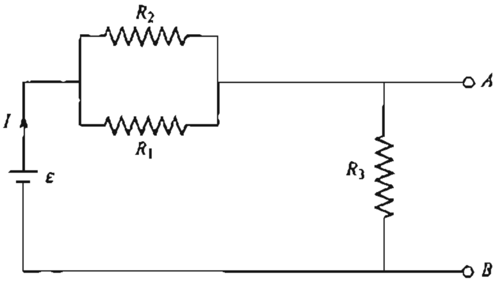


Fig. A2.177

$$\therefore \quad I = \frac{\varepsilon}{R_3 + \frac{R_1R_2}{R_1+R_2}} = \frac{10}{1+\frac{(2)(2)}{2+2}} = \frac{10}{1+1} = 5\,\text{A}$$

Also, $I_{R_1}R_1 = I_{R_2}R_2 \Rightarrow \frac{I_{R_1}}{I_{R_2}} = \frac{R_2}{R_1}$

$\therefore \quad V_{AB} = IR_3 = (5)(1) = 5\,\text{V}$

$\therefore \quad Q_{\text{max}} = CV_{AB} = 5\,\mu\text{C}.$

At $t = 0$, the equivalent circuit is

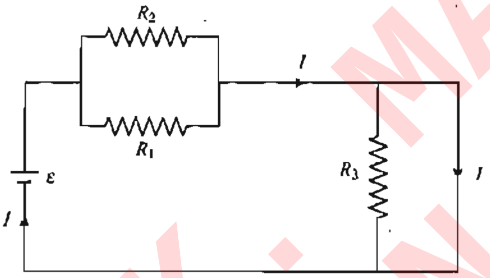


Fig. A2.178

$\therefore \quad I_{R_3} = 0$

and $I_{R_1}R_1 = I_{R_2}R_2 \Rightarrow \frac{I_{R_1}}{I_{R_2}} = \frac{R_2}{R_1}$

59. d. As $V = \varepsilon - Ir$

from graph it is clear that slope $= -r = -y/x$

or $r = y/x$

60. c. Initially all three capacitors are in parallel

$$E_i = \frac{1}{2} \times 3C\,V^2 = \frac{3}{2}CV^2$$

When key is closed two capacitors are in series.

$$E_f = \frac{1}{2}C\left(\frac{V}{2}\right)^2 \times 2 = \frac{CV^2}{4}$$

$$\Delta E = E_i - E_f = \frac{5}{4}CV^2$$

61. c. Equivalent circuit:

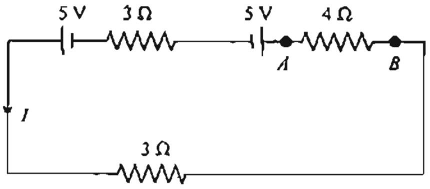


Fig. A2.179

$$I = \frac{5-5}{8} = 0 \Rightarrow V_A - V_B = 0$$

So, current will be only in the smaller loop containing the 6 V and 4 V battery. So, current through the 6 V battery,

$$I' = \frac{6-4}{4} = \frac{1}{2}\,\text{A}$$

62. b. Using KVL,

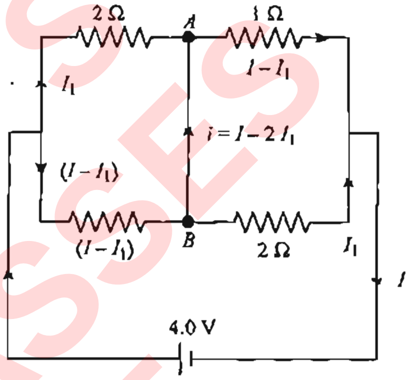


Fig. A2.180

$2I_1 - (I - I_1) = 0 \Rightarrow I = 3I_1$

and $(I - I_1) + 2I_1 = 4$

Solving, we get: $I_1 = 1$ A and $I = 3$ A

$\therefore$ current through $AB = I - 2I_1 = (3-2)\,\text{A} = 1\,\text{A}$

63. b. As potential gradient k in the potentiometer wire will be less than $(\varepsilon/100)$ (V cm^{-1}) because of the presence of r. So for the battery of e.m.f. $\varepsilon/2$ the balance point l will be greater than 50 cm as shown.

$$l = \frac{\varepsilon/2}{k(<\varepsilon/100)} \Rightarrow l > 50\,\text{cm}$$

64. b.

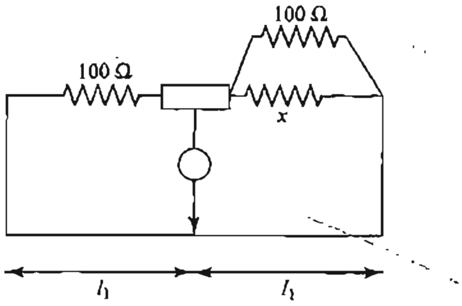


Fig. A2.181

Wheatstone's bridge is in balanced condition,

$$\text{so} \quad \frac{100}{\ell_1} = \frac{\frac{100x}{100+x}}{\ell_2}$$

$$\therefore \quad \frac{\ell_1}{\ell_2} = 2 \Rightarrow x = 100\,\Omega$$

65. d. For maximum current, net resistance of cells must be equal to 2.5 Ω.

i.e., $\frac{n(0.5)}{m} = 25$ (1)

and $m \times n = 45$ (2)

Solving, we get $n = 15$, $m = 3$

66. b.

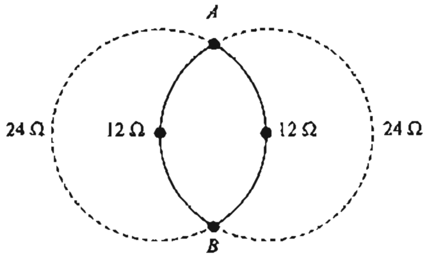


Fig. A2.182

From Fig. A2.182,

$AC_1 = AC_2 = C_1C_2 =$ radius

$\angle AC_1B = 120°$

The resistances of the four sections are 24, 12, 12 and 24 Ω.

Hence, equivalent resistance R across AB is

$$\frac{1}{R} = \frac{1}{24} + \frac{1}{12} + \frac{1}{12} + \frac{1}{24} \text{ or } R = 4\,\Omega$$

$$\therefore \quad \text{Power} = \frac{v^2}{R} = \frac{(20)^2}{4} = 100\text{ W}.$$

67. a. In potentiometer wire potential difference is directly proportional to length. Let the potential drop of unit length a potentiometer wire be K.

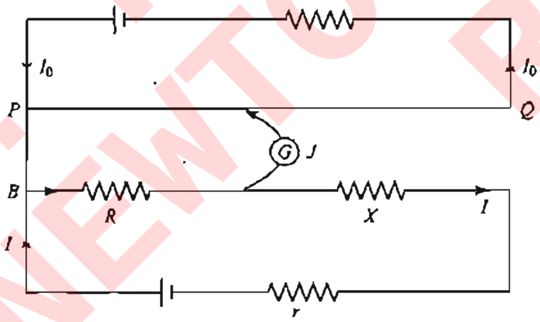


Fig. A2.183

For zero deflection the current will flow independently in two circles:

$IR = K \times 10$ (1)

$IR + IX = K \times 30$ (2)

Subtracting equation (2) from (1), we get

$IX = k \times 20$ (3)

Dividing equation (1) by (2), we have

$$\frac{R}{X} = \frac{1}{2}$$

68. d. In first two circuit diagrams, corrections have to be made while third one is absolutely wrong. So, none of the circuit diagrams is giving correct value of R.

69. b. The ammeter is not showing any reading. It means the potential drop across CB due to E_1 is equal to that of the e.m.f. E_2. And this is the reading of the voltmeter.

70. c. From work energy theorem

$$\Delta = \varepsilon i - i^2 r$$

71. c. $E = \frac{\lambda}{2\pi\varepsilon_0 r}$, where λ is the linear charge density of the inner cylinder.

And $$V = \int_a^b E\,d\ell = \frac{\lambda}{2\pi\varepsilon_0}\,\ell n\left(\frac{b}{a}\right) \quad \text{(i)}$$

Now, $$I = \int \vec{J}\cdot d\vec{A} = \sigma \int \vec{E}\cdot d\vec{A}$$

$$= \sigma \int \frac{\lambda}{2\pi\varepsilon_0 r}\,2\pi r\,dr$$

Current per unit length will be

$$I = \frac{\sigma\lambda}{\varepsilon_0} \quad \text{(ii)}$$

From (i):

$$I = \frac{2\sigma\pi\varepsilon_0}{\varepsilon_0\,\ell n(b/a)}\,v = \frac{2\pi\sigma}{\ell n(b/a)}\,v$$

Alternatively,

$$I = \frac{V}{R} \Rightarrow R = \int_{x=a}^{b} \frac{1}{\sigma}\frac{dx}{2\pi x\,1} = \frac{1}{2\pi r}\,\ell n\left(\frac{b}{a}\right)$$

$$\therefore \quad I = \frac{2\pi\sigma V}{\ell n(b/a)}$$

72. b. $50 = 10[R + r]$

$R + r = 5\,\Omega$

$$\eta = \frac{R}{R+r} \Rightarrow 0.25 = \frac{R}{R+r}$$

$R + r = 4R$

$\Rightarrow \quad r = 3R$

Then $R = \frac{5}{4} = 1.25\ \Omega$ and $r = 3.75\ \Omega$

73. d. $P = VI \Rightarrow 50 = 5 \times I \Rightarrow I = 10$ A

Power lost in cable is

$$= I^2R = 10 \times 10 \times 0.2 = 2\text{ W}$$

Power supplied to tape recorder = 50 W − 2 W = 48 W

74. d. Using the condition for Wheatstone's bridge to be balanced,

$$\frac{P}{Q} = \frac{R}{\frac{S_1S_2}{S_1+S_2}} \quad \text{or} \quad \frac{P}{Q} = \frac{R(S_1+S_2)}{S_1S_2}.$$

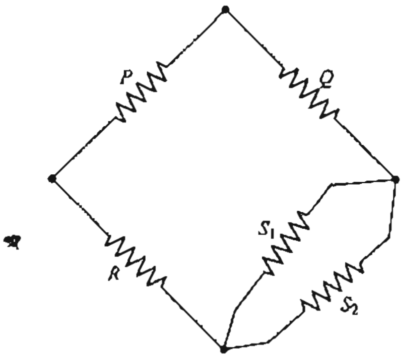


Fig. A2.184

75. d. For balanced meter bridge (null deflection)

$$\frac{55}{R} = \frac{20}{80} \quad \text{or} \quad R = 220\ \Omega$$

76. a. Across the dotted line the circuit is symmetric

$R_{eq} = 6\,\Omega$

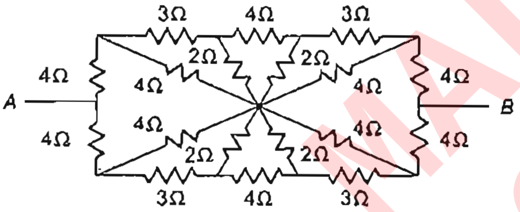


Fig. A2.185

77. b. $E_i = \frac{1}{2} \times 3C \times V^2 = \frac{3}{2}CV^2$

$$E_f = \frac{1}{2} \times C \times \left(\frac{V}{2}\right)^2 \times 2 = \frac{CV^2}{4}$$

$$\Delta E = \frac{5}{4}CV^2$$

78. b. $I = \frac{E - E_1}{R'} = \frac{E - E_2}{R + R'}$

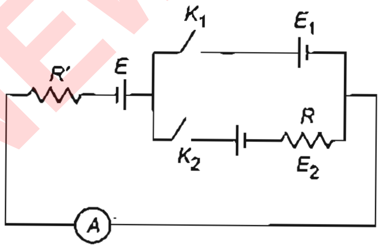


Fig. A2.186

$$\frac{E - E_2}{E - E_2} = \frac{R'}{R + R'}$$

$$\frac{E - E_1}{E_1 - E_2} = \frac{R}{R'}$$

$$I = \frac{E_1 - E_2}{R}$$

79. c. Potential drop across $1\,\Omega = \frac{2 \times 1}{1.5}$ V. This potential drop exists across capacitor.

$$\therefore \quad Q = CV = \frac{4}{3}\,\mu\text{C}$$

80. b. The current will pass through only resistance part of the circuit. Hence $I = \frac{9}{1 + 6 + 2} = 1$ A

Potential drop across 3 μF capacitance is

$$V_{AB} = V_{AB} + V_{BC} = 1 \times 6 + \frac{1}{2} \times 1 = 6.5\,\text{V}$$

(Apply current division for current in branch *BC* which is 1/2 *A*.)

81. b. If potential drop across *X* is 2 V, then no current will pass through ammeter.

$$\therefore \quad \frac{6X}{600 + X} = 2$$

or $\quad 4X = 1200$

or $\quad X = 300\ \Omega$

82. c. $R = \frac{V}{I} = \frac{20 \pm 1}{2.5 \pm 0.5} = 8 \pm \Delta R$

But, $\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{1}{20} + \frac{0.05}{2.5} = 0.07$

$\Rightarrow \quad \Delta R = (0.07)\,8 = 0.56\,\Omega$

$\therefore \quad R = (8 \pm 0.56)\,\Omega$

83. d. With these diagrams we can easily find that maximum equivalent resistance will be across voltmeter of range 10 V.

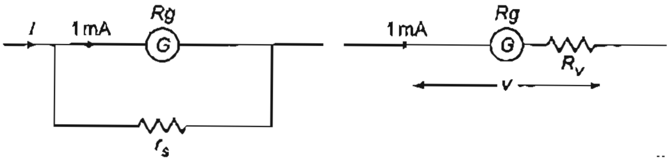


Fig. A2.187

84. b. Slope of line (1) is less than that of (2) therefore resistance of 1 is more than resistance of 2 and resistance increases with temperature.

85. c. Across *P* and *Q*

In circuit (i)

$$R_{net} = \frac{10 \times 10}{10 + 10} = 5\ \Omega$$

In circuit (ii)

$$R_{net} = \frac{10 \times (5 + 5)}{10 + (5 + 5)} = 5\ \Omega$$

86. c. Power $= I^2R$.

87. c. I_1 and I_4 increase

88. a. The circuit is equivalent to

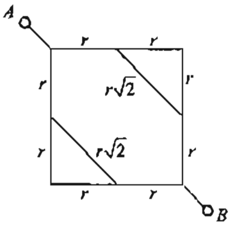


Fig. A2.188

Let each half side has resistance
$\rho(r\delta/2)$

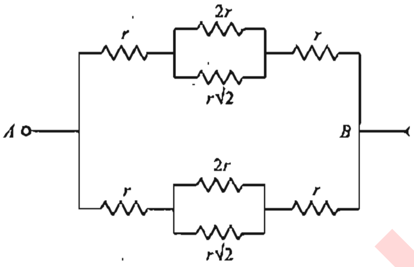


Fig. A2.189

on solving,

$$\therefore \quad R = \frac{1}{2}\left[2r + \frac{(2r)(r\sqrt{2})}{(2+\sqrt{2})r}\right] = r\sqrt{2}$$

$$\therefore \quad R = \rho d/\sqrt{2}$$

89. c. Equivalent circuit is

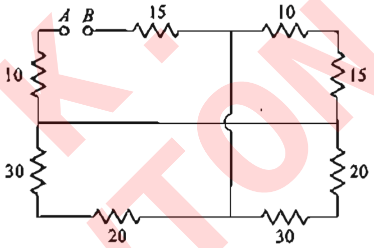


Fig. A2.190

Equivalent resistance = 37.5 Ω

90. d

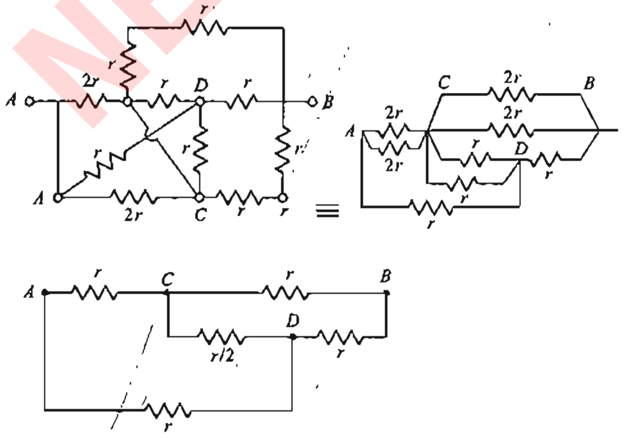


Fig. A2.191

≡ r (By Wheatstone's balanced bridge concept.)

91. b. Energy stored in capacitor when it is charged up to 2 V

$= \frac{1}{2} \times 10 \times 2^2 = 20\ \mu\text{J} = u_1$ (suppose)

Energy stored in capacitor when it is charged up to 4 V

$= \frac{1}{2} \times 10 \times 4^2 = 80\ \mu\text{J} = u_2$ (suppose)

Increase in charge = 40 – 20 = 20 μC

Energy drawn from cell = 20 × 4 = 80 μJ = u (suppose)

Heat produced $= u_1 + u - u_2$

$= 20 + 80 - 80 = 20\ \mu\text{J}$

92. b. The circuit can be folded about B and redrawn as

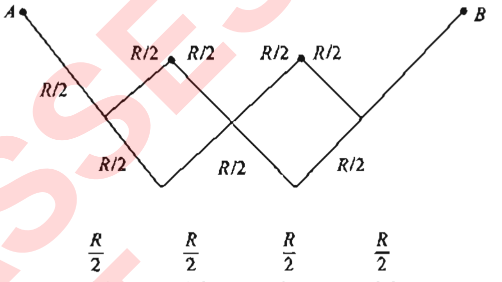


Fig. A2.192

Hence equivalent resistance between A and B is $2R$.

93. d. $R = \frac{1}{\sigma} \times \frac{t}{4\pi R^2}$

Using values, $R = 5 \times 10^{-11}\ \Omega$

94. b. Since current $I = neAv_d$ through both rods is same

$2(n)e\,Av_L = ne(2A)v_R$

$\Rightarrow \quad \frac{v_L}{v_R} = 1$

95. c. $i = \frac{dq}{dt}$ = slope of q–t graph

= – 5 (which is constant)

Amount of heat generated in time $t = i^2RT$

$\propto t$

96. b. From relation $E = \rho J$, the magnitude of electric field is greater in right rod as compared to left rod. There force magnitude of potential gradient in the right rod is greater (remember potential is continuous).

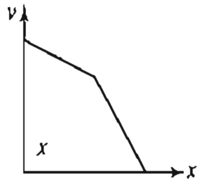


Fig. A2.193

Therefore, the variation is shown by the figure.

97. a. Equivalent circuit, $R_{\text{net}} = \frac{8R}{11}$

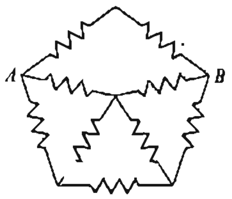


Fig. A2.194

98. d. $v_P = 4\left[\frac{20+x}{40}\right]$

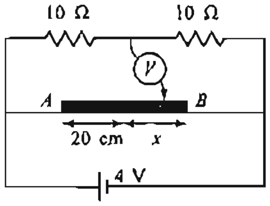


Fig. A2.195

$$v_Q = 2$$

$$v_P - v_Q = \frac{x}{10}$$

$$2\sin\pi t = \frac{x}{10}$$

$$x = 20\sin\pi t$$

$$\frac{dx}{dt} = (20\pi\cos\pi t)\text{ cm/s}$$

99. b. Circuit is forming a Wheatstone's bridge. So $R_{eq} = 2R$. For maximum power transfer $2R = r$.

100. a. Potential difference across AC is zero as $I_{AC} = 0$

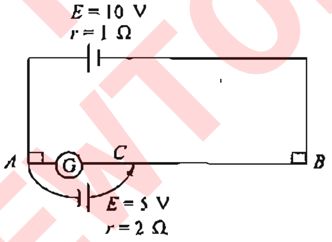


Fig. A2.196

$$5 - 2I = 0$$

$$I = 2.5\text{ A}$$

Let the resistance of part BC be r

Applying KVL

$10 + 5 - 2I - Ir - I = 0$

$2.5r = 7.5 \Rightarrow r = 3\,\Omega$

As resistance of part $AB = 9\,\Omega$

∴ Length $AC = 66.7$ cm

101. b. $H = \int_0^4 \frac{E^2}{R}dt = \int_0^4 \frac{(6t)^2\,dt}{12} = 64$ J

102. b. If P is disconnected, R_{eq} of circuit increases hence less current is drawn.

∴ $0 + -iR_1 = x$

⇒ as i decreases, x increases

103. b. From the hint, the equivalent resistance of the circuit

That is, $R_T + \frac{1}{1/R_L + 1/R_{eq}} = R_{eq}$

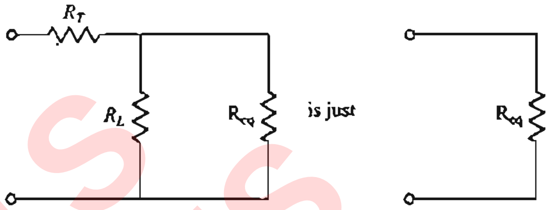


Fig. A2.197

$$R_T + \frac{R_L R_{eq}}{R_L + R_{eq}} = R_{eq}$$

$$R_T R_L + R_T R_{eq} + R_L R_{eq} = R_L R_{eq} + R_{eq}^2$$

$$R_{eq}^2 - R_T R_{eq} - R_T R_L = 0$$

$$R_{eq} = \frac{R_T \pm \sqrt{R_T^2 - 4(1)(-R_T R_L)}}{2(1)}$$

Only the positive sign is physical.

$$R_{eq} = \frac{1}{2}\left(\sqrt{4R_L R_T + R_T^2} + R_T\right)$$

For example, if $R_T = 1\,\Omega$ and $R_L = 20\,\Omega$, $R_{eq} = 5\,\Omega$

Multiple Correct Answers Type

1. a., b., c., d.

Electric current at a point on the circle

$i = fe$, where f = frequency

$$= \frac{\omega}{2\pi} = \frac{V}{2\pi r} = \frac{0.6\times10^6\times\pi}{2\pi\times5\times10^{-11}} = 6\times10^{15}\text{ rev/s}^{-1}$$

$$i = 6\times10^{15}\times1.6\times10^{-19}$$

$$= 0.96\times10^{-3}\text{ A} = 0.96\text{ mA}$$

2. a., b., d.

$$I = \frac{\varepsilon_1 - \varepsilon_2}{r_1 + r_2}$$

$$V_A - V_B = \varepsilon_2 + Ir_2 = \varepsilon_1 - Ir_1 = \frac{\varepsilon_1 r_2 + \varepsilon_2 r}{r_1 + r_2}$$

3. b., c.

Use relations $Q = Q_0\left(1 - e^{-t/RC}\right)$ and $i = i_0 e^{-t/RC}$.

4. a., b.

Connected the voltmeter in parallel.

5. a., c.

Use symmetry.

6. a., c.

Use Ampere law.

7. a., d.

Resistance of upper bulb,

$R_1 = 100^2/25 = 400\,\Omega$

Resistance of lower bulb,

$R_2 = 200^2/100 = 400\,\Omega$

Equivalent resistance of the circuit: $R_{eq} = 500\,\Omega$.

Heat lost per second in the circuit is

$e^2/R_{eq} = 200^2/500 = 80\text{ J s}^{-1}$

Potential difference across branches AB and CD will be same. Hence, ratio of heat generated in them is

$$\frac{\text{Heat}_{AB}}{\text{Heat}_{CD}} = \frac{R_{CD}}{R_{AB}} = \frac{1000}{500} = 2:1$$

Hence (c) is incorrect.

Since potential difference across branches AB and CD will be same but their resistances are different, so current in them will be different. As the resistances of the bulbs are same so heat generated in them will be different. Hence, (b) is incorrect. Current drawn from the cell is

$e/R_{eq} = 200/500 = 0.4\text{ A}$

8. a., b., c., d.

Potential difference across the terminals of the cell is given by $V = E - ir$. Hence, the graph between V and i will be a straight line having negative slope and positive intercept. Hence, (a) is correct.

Total electrical power generated is Ei and thermal power generated in the cell is i^2r. Hence, thermal power generated in the external circuit is $P = Ei - i^2r$. So, the graph between P and i is a parabola passing through the origin. Hence, choice (b) is correct.

When terminal potential difference across the cell is V, current flowing through the circuit will be $i = \dfrac{E-V}{r}$ and thermal power generated in the external circuit will be equal to $P = Vi = V\left(\dfrac{E-V}{r}\right)$.

$$= P = \frac{EV}{r} - \frac{V^2}{r}$$

Hence, the graph between P and V will be a parabola passing through the origin and having a maximum value of P at $V = \dfrac{E}{2}$. Hence, choice (d) is correct.

At an instant, thermal power generated in the cell is equal to i^2r and total electrical power generated in the cell is equal to Ei. Hence, the fraction $\eta = \dfrac{i^2r}{Ei} = \left(\dfrac{r}{E}\right)i$.

Hence, the graph between η and i will be a straight line passing through the origin and having a positive slope. Hence, (c) is correct.

9. a., d. Equivalent resistance of circuit = (2.5 + 1.5) = 4 Ω

Current through battery: $I = \dfrac{20}{4} = 5\text{ A}$

Current through P = Current through $Q = \dfrac{5}{2} = 2.5\text{ A}$

$V_A - V_Q = 3 \times 2.5 = 7.5\text{ V}$ (i)

$V_A - V_Q = 2 \times 2.5 = 5\text{ V}$ (ii)

From (i) and (ii), we get $V_Q - V_P = 2.5\text{ V}$.

10. b., c., d.

Disregard capacitors and find the current through G. The potential difference across each capacitor is then found from the potential difference across the resistances in parallel with them.

11. b., c.

As $R \propto \dfrac{1}{W}$ and actual power consumed $P = \dfrac{V^2}{R}$, lesser the resistance (i.e., higher the wattage), greater the power consumed. If the bulbs are connected in series, actual power $P = i^2R \Rightarrow P \propto R$. Hence, in series connection low wattage bulbs glow more.

12. a., b., d.

As A and B are connected in series, hence $IA = IB$. The potential difference across both the branches will be same.

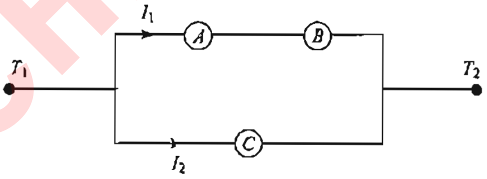


Fig. A2.198

$$I_AR_B + I_BR_B = I_CR_C \text{ or } \frac{I_B}{I_C} = \frac{R_C}{R_A + R_B}$$

13. a., b., c.

In the simplified circuit, the circuit is a balanced Wheatstone's bridge and a branch of 20/29 Ω and R is parallel with this balanced bridge for maximum power.

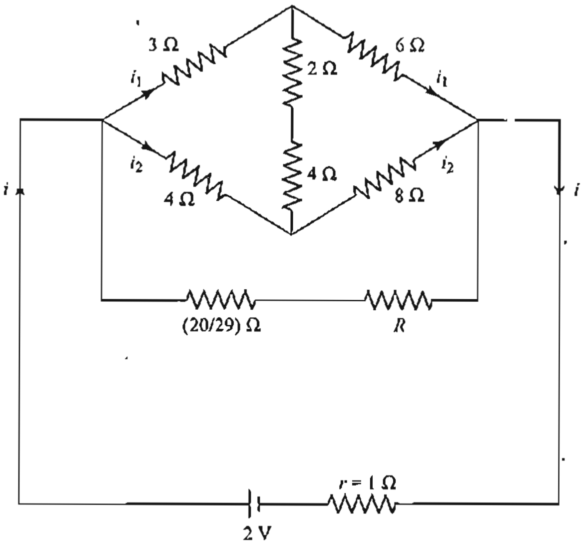


Fig. A2.199

$$r = R_{\text{external}}$$

$$1 = \frac{1}{\frac{1}{(3+6)} + \frac{1}{(20/29+R)} + \frac{1}{(4+8)}}$$

or $$R = \frac{16}{25}\ \Omega$$

Maximum power developed in the external circuit is

$$P_{\max} = i^2R = \left(\frac{2}{1+1}\right)^2 \times 1 = 1\ \text{W}$$

Current through the upper branch

$$i_1 = i\left[\frac{\left(\frac{20}{29}+R\right)(4+8)}{9+\left(\frac{20}{29}+R\right)+12}\right]$$

$$i_2 = i\left[\frac{\left(\frac{20}{29}+R\right)(3+6)}{9+\left(\frac{20}{29}+R\right)+12}\right]$$

$\therefore$ $\frac{i_1}{i_2}$ is independent of R.

14. a., d.

$$R_1 = \frac{V^2}{W_1} = \frac{(220)^2}{25} = (22)^2 \times 4\ \Omega$$

$$R_2 = \frac{V^2}{W_2} = \frac{(220)^2}{100} = (22)^2\ \Omega$$

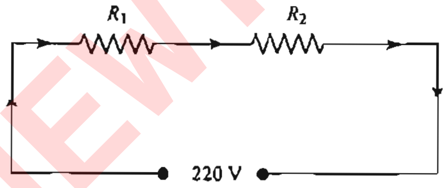


Fig. A2.200

Current in the circuit when the bulbs are connected in series,

$$I = \frac{220}{(22)^2 \times 4 + (22)^2} = \frac{10}{22 \times 5} = \frac{1}{11}\ \text{A}$$

Hence, $P_1 = I^2R_1 = \left(\frac{1}{11}\right)^2 \times (22)^2 \times 4 = 16$ W.

And $P_2 = I^2R_2 = \left(\frac{1}{11}\right)^2 \times (22)^2 = 4$ W.

15. a., c.

4 and 6Ω resistances are short-circuited. Therefore, no current will flow through these two resistances. Current passing through the battery is $I = (20/2) = 10$ A.

This is also the current passing in wire AB from B to A.

Power supplied by the battery

$$P = EI = (20)(10) = 200\ \text{W}$$

Potential difference across the 4 Ω resistance

= potential difference across the 6 Ω resistance = 0

16. a., b., c., d.

a. Potential difference across each cell = $V_P - V_Q$.

b. If it is clockwise then E_2 is supplying and for the reverse case E_1 supplies the energy.

c. Potential difference = $E = ir$, when battery supplies energy. It is $E + ir$, when battery consumes energy.

By KVL, $i = \frac{E_1 - E_2}{r_1 + r_2}$ (anticlockwise)

$$V_P - V_Q = E_1 - ir_1 = \frac{E_1r - E_2r}{r_1 + r_2}$$

17. a., c.

There is zero potential difference across the 4 and 6 Ω resistances.

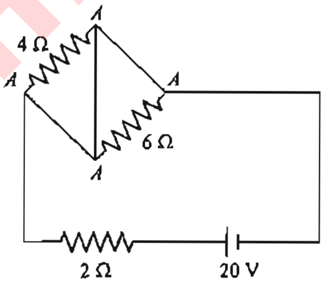


Fig. A2.201

$$i = \frac{20}{2} = 10\ \text{A}$$

Power by battery

$$P_b = \varepsilon_1 = 20 \times 10 = 200\ \text{W}$$

18. a., b.

For null point, current flows in the loop CD only.

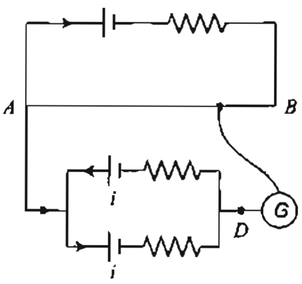


Fig. A2.202

$$i = \frac{3\ \text{V}}{2\Omega + 1\Omega} = 1\text{A}$$

$$V_{CD} = 1\ \text{V} - 1(1) = 0$$

$\therefore$ Option (a) is correct. That is, $V_A > V_B$

When jockey touches B, current from A to B increases the P.D. across the secondary circuit.

$\therefore$ Option (b) is correct.

19. a., b.

$$\frac{R}{(R_1+R_2)} = \frac{L/4}{3L/4}$$

$$3R = (R_1+R_2)$$

$$\frac{R+R_1}{R_2} = \frac{2/3}{1/3}$$

$$3R = R_2 + R_1$$

$$R = 2R_2 - R_1$$

$$4R = 3R_2$$

$$R_1 = \frac{5R}{3}$$

$$R_2 = \frac{4R}{3}$$

20. a., b., c. Initial charge (before filling the dielectric slab) $= 10 \times 10 = 100\ \mu C$.

Final charge (after filling the dielectric slab) $= 10 \times 30 = 300\ \mu C$.

Increase in charge $= 20\ \mu C$

21. a., b., d. Let the currents be as shown in the figure

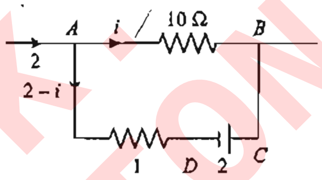


Fig. A2.203

Applying KVL along $ABCDA$, we get

$$\Rightarrow \quad -10\,i - 2 + (2-i)1 = 0$$

$$\therefore \quad i = 0$$

Potential difference across

$$s = (2-i)1 = 2 \times 1 = 2\text{ V}$$

22. a., b., c., d.

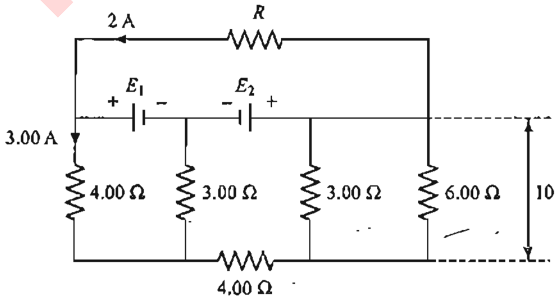


Fig. A2.204

After redrawing the circuit

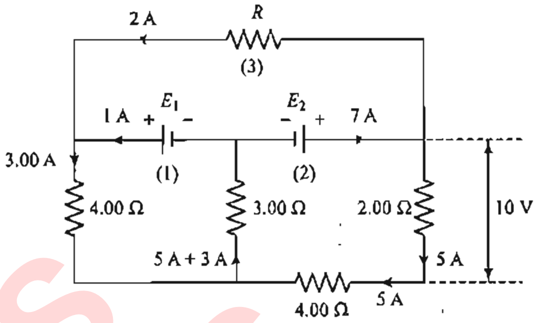


Fig. A2.205

a. $I_4 = 5$ A

b. From loop (1)

$$-8(3) + E_1 - 4(3) = 0 \quad \Rightarrow \quad E_1 = 36\text{ V}$$

From loop (2)

$$+4(5) + 5(2) - E_2 + 8(3) = 0$$

$$E_2 = 54\text{V}$$

c. From loop (3)

$$-2R - E_1 + E_2 = 0$$

$$R = \frac{E_2 - E_1}{2} = \frac{54}{2} - 36 = 9\text{ W}$$

23. a. P, b. P, c. R, d. R

In each case, there will be no current in the branch containing voltmeter or ammeter.

Therefore, reading of the instruments will be zero.

24. b., d.

Power supplied by 20 V cell $= (-1)(20) = -20$ W

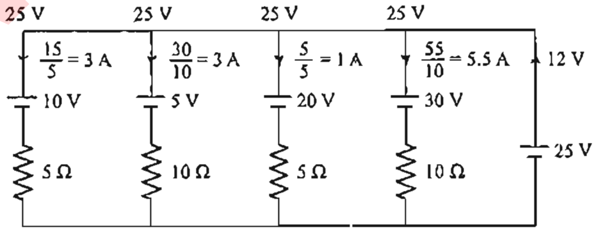


Fig. A2.206

As the cell is not supplying the power, it is eating the power, thereby getting charged.

25. b., c.

(moderates) As the length is doubled, the cross-section area of the wire becomes half, thus the resistance of the wire $R = \rho\frac{L}{A}$ becomes four times the previous value. Hence after the wire is elongated the current becomes one fourth. Electric field is potential difference per unit length and hence becomes half the initial value. The power delivered to resistance is $P = \frac{V^2}{R}$ and hence becomes one-fourth.

26. a., b., d.

Total charge $= \int I\,dt$ = Area under the curve = 10 C

Average current $= \frac{\int I\,dt}{\int dt} = 5$ A

Total heat produced $= \int I^2 R dt$

$$= \int_0^2 (-5t+10)^2 1 dt = \frac{200}{3} \text{ J}$$

Maximum power $= I^2 R$ when I is maximum current $= 100 \times 1$ $= 100$ W.

27. **a., d.**

$$24 - 2\times 10^3 I - 6\times 10^3 (I - i) = 0$$

$$24 - 2\times 10^3 I - 1.5\times 10^3 i = 0$$

Hence $\quad I = 7.5$ mA

$\quad\quad i = 6$ mA

$$24 - 6\times 10^3 I' - 2\times 10^3 (I' - i') = 0$$

$$24 - 6\times 10^3 I' - 1.5\times 10^3 i' = 0$$

$I' = 3.5$ mA

$i' = 2$ mA

$$\therefore \quad \frac{P_1}{P_2} = \frac{6^2}{2^2} = 9$$

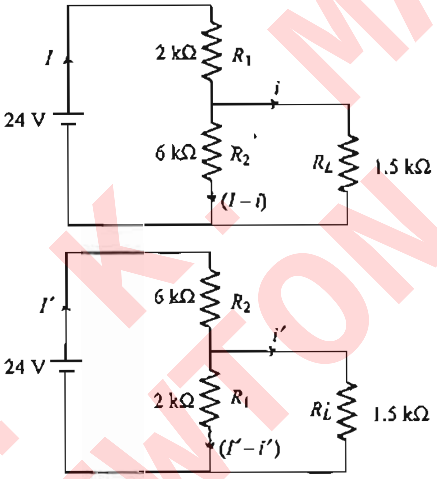


Fig. A2.207

Assertion-Reasoning Type

1. **c.** The electrons suffer a large number of collisions with the positive ions of the conductor. Although the electric field accelerates an electron between two collisions, it is decelerated by collision. The net acceleration averages out to zero and the electron acquires a constant average speed. The gain in speed between collisions is lost in the next collision.
2. **d.** The condition for no deflection of the galvanometer is

$$\frac{R_1}{R_2} = \frac{R_{AC}}{R_{CB}}$$

where R_{AC} and R_{CB} are the resistances of the bridge wire of length AC and CB, respectively. If the radius of the wire AB is doubled, the ratio R_{AC}/R_{CB} will remain unchanged. Hence, the balance length will remain the same.

3. **d.** If the diameter of wire AB is increased, its resistance will decrease. Hence, the potential difference between A and B due to cell C_1 will decrease. Therefore, the null point will be obtained at a higher value of x.
4. **A2.** Both statements 1 and 2 are true. In statement 1, R is varied while in statement 2, R is kept constant. Hence, both statements are independent.
5. **c.** From relation $\vec{J} = \sigma \vec{E}$, the current density $\vec{J}$ at any point in ohmic resistor is in direction of electric field $\vec{E}$ at that point. In space having non-uniform electric field, charges released from rest may not move along EL. Hence statement 1 is true while statement 2 is false.
6. **d.** As the length of the wire is doubled, the cross-sectional area of the wire becomes half. Therefore, resistance of the wire becomes four times and the current becomes one-fourth of the initial value.

Also, $V_d = \dfrac{I}{neA}$

Since current becomes one-fourth and cross-sectional area of the wire becomes half, therefore, from the above equation the drift velocity of electron becomes half. Hence, statement 1 is false.

7. **d.** $V = iR$.
8. **d.** Conceptual.
9. **a.** Charge on capacitor $q = CE(1 - e^{-t/CR_{eq}})$

$$\therefore \quad I = \frac{dq}{dt} = \frac{E}{R_{eq}} e^{-t/CR}$$

at $\quad t = 0,\ I = \dfrac{E}{R_{eq}} = \dfrac{E}{R+r}$

$\Rightarrow$ resistance offered by the capacitor is zero.

10. **a.** $V_A - E - Ir + V_B \quad \Rightarrow \quad V_A - V_B = E + Ir$

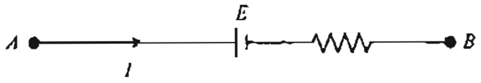


Fig. A2.208

Similarly, $V_A - V_B = E - Ir$.

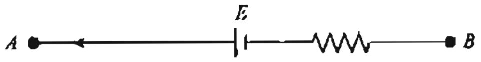


Fig. A2.209

11. **a.** When current flows through a conductor it always remains uncharged. Hence, no electric field is produced outside it.
12. **d.** The electrons are in motion which constitute electric current in a conductor but numbers of positive and negative charges are same.
13. **a.** $R = \rho \dfrac{l}{A}$ Where $l = 2l$, $A = A/2$

Because volume $V_1 = V_2$.

14. a. Here $E = 2\text{ V}, I = \frac{2}{2} = 1\text{ A}$ and $r = 1\ \Omega$

$$V = E - ir = 2 - (1)(1) = 1\text{ V}.$$

15. d. Direction of flow of current is from higher potential to lower potential.

16. d. $V = E + ir$ when charging of cell takes place.

17. b. On increasing temperature of the wire the K.E. of electrons increases and so they collide more rapidly with each other and hence their drift velocities decrease. Resistivity also increases and resistivity is inversely proportional to conductivity of material.

18. d. In a simple battery circuit the point at the lowest potential is the negative terminal of battery. The current flows in the circuit from positive terminal to negative terminal.

19. a. Since current arises due to continuous flow of charged particles. There is no free charge in insulator. Hence, no flow of charges is possible. Therefore, current does not flow through insulators.

20. b. Current is independent of area of cross section.

21. d

$\vec{E} = \rho\vec{J}$, as area and current are same, J is same but ρ is different.

$\therefore$ $\vec{E}$ is different.

Comprehension Type

For Problems 1–5

1.b., 2. b., 3. c., 4. b., 5. a.

Sol. 1. b. $\eta = \dfrac{0.4 \times 4200(\theta - \theta_1)}{420 \times 4.8(\theta - \theta_1)} \times 100 = 250/3 = 83.34\%$

2. d. $\eta = \dfrac{0.4 \times 4200(\theta - \theta_1)}{420 \times 6.4(\theta - \theta_1)} \times 100 = 62.5\%$

3. c. $H_A = 0.2 \times 1680(\theta - \theta_1) + 0.4\,'\,4200(\theta - \theta_1)$

$= 420 \times [0.8 + 4](\theta - \theta_1)$

$H_B = 0.4 \times 2450(\theta - \theta_1) + 0.4 \times 4200(\theta - \theta_1)$

$= 420 \times (2.4 + 4)(\theta - \theta_1)$

$$\frac{H_A}{H_B} = \frac{4.8}{6.4} = \frac{3}{4}$$

4. b. $H_A = \dfrac{V^2}{R_A} \times 6$ (i)

$H_B = \dfrac{V^2}{R_B} \times 8$ (ii)

$\dfrac{H_A}{H_B} = \dfrac{R_B}{R_A} \times \dfrac{3}{4}$ but $\dfrac{H_A}{H_B} = \dfrac{3}{4}$

Then $R_B = R_A$

5. a. (i) $\dfrac{V^2}{R_A} \times 6 = \dfrac{V^2}{(R_A + R_B)^2} R_A \times t_A$

But $R_A = R_B \Rightarrow \dfrac{V^2}{R_A} \times 6 = \dfrac{V^2}{4R_A^2} R_A \times t_A$

$t_A = 24$ min

(ii) $\dfrac{V^2}{R_B} \times 8 = \dfrac{V^2}{(R_A + R_B)^2} R_B \times t_B$

$\dfrac{8}{R_B} = \dfrac{R_B}{4R_{B^2}} t_B \Rightarrow t_B = 32$ min.

For Problems 6–8

6.c., 7. a., 8. c.

Sol. 6. c. $V = E + ir$ (during charging) = 14 V

7. a. $P = I^2 r$ (due to internal resistance) = $50^2 \times 4 \times 10^{-2}$ = 100 W

8. c. Rate of charging = EI = 12 V × 50 A = 600 W

For Problems 9–13

9.b., 10. c., 11. c., 12.c., 13.a.

Sol. 9. b. As per the figure $V_{bc} = 0.10 \times 2 = 0.2\text{ V}$

So actual drop = 12 – 0.2 = 11.8 V

Then $R = \dfrac{11.8}{0.1} = 118\ \Omega$

10. c. $I_V = \dfrac{V}{R_V} = \dfrac{12}{10000} = 1.2\text{ mA}$

Then the actual current I in the resistor is $I = 0.100 - 0012 = .0988$ A

$R = \dfrac{V_{ab}}{I} = \dfrac{12.0}{0.0988} = 121\ \Omega$

11. c. $\dfrac{1.50 \times 10^{-3}}{2} = \dfrac{0.75}{50 + R_S + 500}$

$R_S = 450\ \Omega$

12. c. $\dfrac{3}{4} \times 1.50 \times 10^{-3} = \dfrac{0.9}{R + 100}$

$R + 100 = 800$

$R = 700\ \Omega$

13. a. $I = \dfrac{0.9}{300 + 200 + 100} = 1.5$ mA full scale

For Problems 14–16

14.a., 15. b., 16. b.

Sol. 14. a. $I \propto r^{3/2}$

$$\frac{I_1}{I_2} = \frac{(4r)^{3/2}}{r^{3/2}} = \frac{8}{1}$$

15. b. $P = \dfrac{V^2}{R} \Rightarrow 2 \times 10^3 = \dfrac{V^2}{20}$

$V = 200$ V or applied voltage ≤ 200 V

16. b. $\eta = \dfrac{R}{R + r} \times 100$, where $R = r$, efficiency will be maximum.

Hence $\eta = 50\%$

For Problems 17–21

17.a., 18. c., 19. c.

Sol. 17. a. $I_G \times 36 = (I_3 - I_G)(R_1 + R_2 + R_3)$

$0.01 \times 36 = 0.09(R_1 + R_2 + R_3)$

$R_1 + R_2 + R_3 = 4\ \Omega$ (i)

18. c. $I_G(36 + R_3) = (I_2 - I_G)(R_1 + R_2)$

$0.1(36 + R_3) = 0.9(R_1 + R_2)$

$36+R_3=99R_1+99R_2$ (ii)

But from (i) and (ii), we get

$100R_1+100R_2=40$

$R_1+R_2=0.40\ \Omega$

19. c. $I_G(36+R_2+R_3)=(I_2-I_G)\times R_1$

$0.1(36+R_2+R_3)=09.9\times R_1$

Solving, we get $R_1=0.040\ \Omega$

20. d. $3.00\times1.5\times10^{-3}(R_G+R_1)$

$2000=25+R_1, R_1=1975\ \Omega$

$15.0=1.5\times10^{-3}(R_G+R_1+R_2)$

$R_2=8000\ \Omega$

$150=1.5\times10^{-3}(R_G+R_1+R_2+R_3)$

$10^5=10000+R_3$

$R_3=90,000\ \Omega$

21. d.

For Problems 22–24

22.c., 23. a., 24. a.

Sol. 22. c. Just after closing, capacitor behaves as short circuit and all current flows through it; hence ammeter reads zero.

23.a. After long time capacitor behaves as open circuit and no current flows through it.

Therefore after long time capacitor behaves as an open circuit and no current flows through it.

Therefore $i=\dfrac{v_0}{R_1+R_2}=\dfrac{30}{10+5}=2$ mA

24.a. Just after reopening, potential difference across R_2 remains same initially as charge on capacitor does not change initially. Hence current remains same.

For Problems 25–27

25.a., 26. a., 27. d.

Sol. 25. a. $R_A=R_A=\dfrac{R\times R_V}{R+R_V}<R$

26. a. $R_B=R+R_0>R$

27. d. (Tough)% error in case A.

$$\frac{R_A-R}{R}\times100=\left(\frac{R_V}{R+R_V}-1\right)\times100$$

$$=\frac{R}{R+R_V}\times100\approx-1\%$$

% error in case B

$$\frac{R_B-R}{R}\times100=\frac{R_G}{R}\times100\approx10\%$$

Hence percentage error in circuit B is more than that in A.

For Problems 25–27

28.b., 29. b., 30. d.

Sol. $i_0=0.1$ A, $E_2=4$ V, $i_2=0$

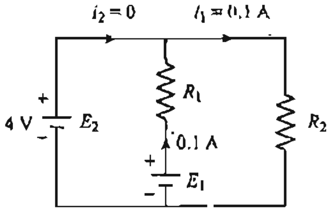


Fig. A2.210

As $0.1\ R_1+0.1\ R_2-E_1=0$

$0.1\ R_2-4\text{ V}=0$

$R_2=40\ \Omega$

Now: $i_2=0.3$ A, $i_1=0.1$ A, $E_2=8$ A

Now: $0.1\ R_1+E_1-8=0$

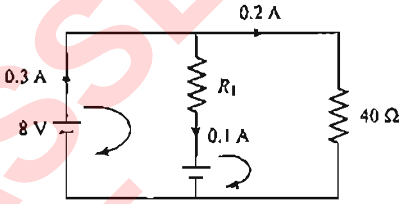


Fig. A2.211

$0.1+4-E_1=0$

$0.2\ R_1-4=0$

$R_1=\dfrac{4}{0.2}=20\ \Omega$

$E_1=2+4=6$ V

For Problems 31–33

31.d., 32. c., 33. b.

Sol. At steady state, the circuit will be simplified to

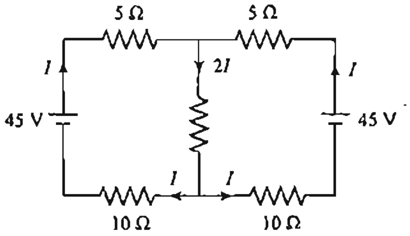


Fig. A2.212

Using Kirchhoff's law, we can calculate $I=1$ A.

Further using Kirchhoff's law in original circuit, we can calculate charges on different capacitors.

Matching Column Type

1. a. → **r., b.** → **q., c.** → **q., d.** → **q.**

Current through any cross section should remain same.

$I=neAv_d$, as area increases, drift velocity decreases.

E is directly proportional to v_d, so E also decreases.

If E decreases then *p.d.* across a segment of fixed length also decreases.

2. a. → **r., b.** → **r., c.** → **s., d.** → **q.**

From relation $J=\sigma E$

Multiplying both sides by cross-section area of rod A, we get

$JA=\sigma EA$ or $i=\sigma\phi$

$\frac{\phi}{i}=\frac{1}{\sigma}$ = resistivity of the rod

$\frac{E}{J}=\frac{1}{\sigma}$ = resistivity of the rod

$\sigma V = IV$ = power delivered to the rod

$\frac{V}{\sigma\phi}=\frac{V}{i}$ = resistance of the rod

3. a. → q., b. → r., c. → r., d. → r.

When the switch is closed, equivalent resistance is R. After opening the switch, equivalent resistance becomes $2R$. Hence, equivalent resistance increases.

Also current through battery decreases, hence ammeter reading decreases. Current through left R also decreases. So voltmeter reading decreases and power dissipated through left R also decreases.

4. a. → p., b. → s., c. → s., d. → q.

a. Since current in both rods is same.

$$\therefore \quad n_1 e v_1 A_1 = n_2 e v_2 A_2$$

$$\therefore \quad \frac{v_1}{v_2}=\frac{n_2 A_2}{n_1 A_1}=\frac{1}{2}\times\frac{2}{1}=1$$

b. $\therefore \quad E=\rho J=\rho\frac{I}{A}$

$$\therefore \quad \frac{E_1}{E_2}=\frac{\rho_1}{\rho_2}\times\frac{A_2}{A_1}=\frac{2}{1}\times\frac{2}{1}=4$$

c. $\frac{\text{p.d. across rod I}}{\text{p.d. across rod II}}=\frac{E_1\times AB}{E_2\times BC}=4$

d. $\frac{\text{Average time taken by free electron to move from } A \text{ to } B}{\text{Average time taken by free electron to move from } B \text{ to } C}$

$$=\frac{AB}{V_1}\times\frac{V_2}{BC}=1$$

5. a. → p., b. → q., c. → p. s., d. → s.

a. We know that power developed by R is maximum for $R = r$.

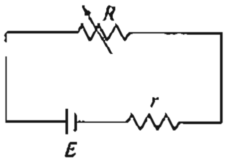


Fig. A2.213

b. Initially, angle between area vector of loop and magnetic field is 90°, it becomes zero and then again 90° after 180° rotation.

$$\phi = BA\cos\theta$$

$$e=-\frac{d\phi}{dt}=BA\sin\theta\left(\frac{d\theta}{dt}\right)=BA\,\omega\sin\theta$$

c. AB is the angle of rod before the lens is displaced. CD is the image of rod after displacing the lens perpendicular to the principle axis.

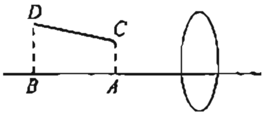


Fig. A2.214

d. $Z=\sqrt{R^2+X_C^2}=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}$

As ω in increased, z is decreased, so current through bulb will increase thus increasing the brightness.

6. a. → p., q., r., s., b. → p., q., c. → q., r., d. → q., r., s.

When S is open:

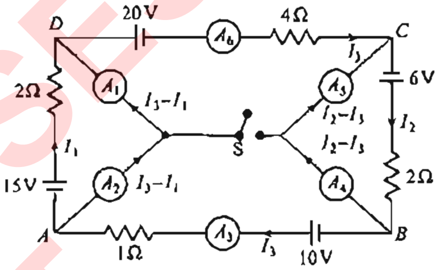


Fig. A2.215

$$I_1=\frac{15}{2}=7.5\text{ A}$$

$$I_2=\frac{6}{2}=3\text{ A}$$

Loop $ADCBA$:

$$15-20+6+10=2I_1+4I_3+2I_2+1I_3$$

$$I_3=-2\,A$$

Current in A_1 and A_2:

$$I_3-I_1=-2-7.5=-9.5\text{ A}$$

Current in A_4 and A_5:

$$I_2-I_3=3-(-2)=5\text{ A}$$

After closing S:

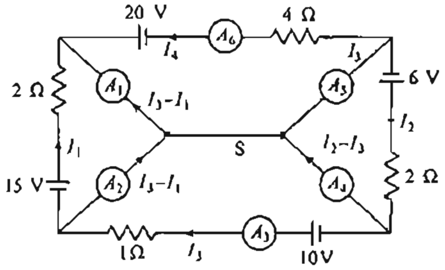


Fig. A2.216

$$I_1=\frac{15}{2}=7.5\text{ A}$$

$$I_2=\frac{6}{2}=3\text{ A}$$

$$I_3=\frac{10}{1}=10\text{ A}$$

$$I_4=\frac{20}{4}=5\text{ A}$$

Current in A_1:

$I_1 + I_4 = 7.5 + 5 = 12.5$ A

Current in A_2:

$I_3 - I_1 = 10 - 7.5 = 2.5$ A

Current in A_4:

$I_2 - I_3 = 3 - 10 = -7$ A

Current in A_5:

$I_4 - I_2 = 5 + 3 = 8$ A

7. a. → q., s., b. → q., r., c. → p., s., d. → p., r.

a. For $P_{\max} = R = 2r$

$$P_{\max} = \left(\frac{2E}{R+2r}\right)^2 R = \frac{E^2}{2r}$$

b. For $P_{\max}$: $R = \frac{r}{2}$

$$P_{\max} = \left(\frac{E}{R+R/2}\right)^2 R = \frac{E^2}{2r}$$

c. $P = \left(\frac{2E}{r+2r}\right)^2 r = \frac{4E^2}{9r}$

d. $P = \left(\frac{2E}{r+r/2}\right)^2 r = \frac{4E^2}{9r}$

8. a. → q., r., b. → s., c. → s., d. → p.

a. It is possible that, less length of potentiometer wire is used for positive terminal of e.m.f. to be measured may not be connected at extreme ends of the potentiometer wire.

b. Sensitivity of potentiometer decreases.

c. Sensitivity of potentiometer decreases

d. Accuracy of the potentiometer increases.

Archives

Fill in the Blanks Type

1. We know that $P = \frac{V^2}{R}$

$\therefore\ R = \frac{V^2}{P} = \frac{100 \times 100}{500} = 20\ \Omega$. For the bulb to deliver 500 W, it should have a potential difference of 100 V across it. This would be possible only when $R = 20\ \Omega$. Because in that case both resistances will share equal potential difference and the potential difference across the bulb will be 200 V. Hence, the answer is 20 Ω.

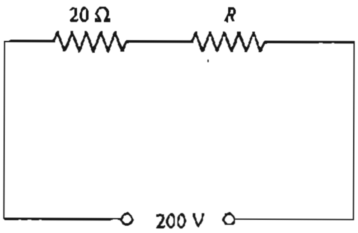


Fig. A2.217

2. The given circuit may be redrawn as shown in Fig. A2.218. Thus, the resistance 2R, 2R and R are in parallel.

Hence, $\frac{1}{R_{AB}} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{R} = \frac{2}{R}$. Hence, $R_{AB} = \frac{R}{2}\ \Omega$.

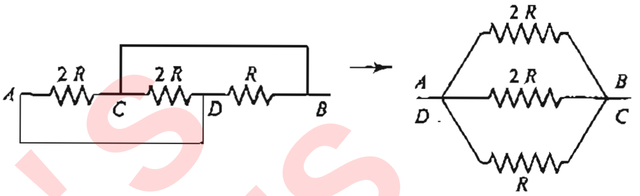


Fig. A2.218

3. Let a current I flow through the circuit. Net e.m.f. of the circuit = 8 (5 V) = 40 V. Net resistance in the circuit = 8(0.2 Ω) = 1.6 Ω.

Current flowing through the circuit, $I = \frac{40\text{ V}}{1.6\Omega} = 25$ A The voltmeter reading would be $V = E - IR$ = (5 V) − (25 A) (0.2 Ω) = 5 V − 5 V = 0.

Alternatively: we can apply Kirchhoff's law in the loop and find the current.

True/False

1. **False:** An electrolyte solution is formed by mixing an electrolyte in a solvent. The electrolyte on dissolution furnishes ions. The preferred movement of ions under the influence of electric field is responsible for electric current.
2. **False:** The electrons in a conductor are free and have thermal velocities. Thus electrons will be in motion even in the absence of potential difference.
3. **True:** For a given voltage, current is more in case of T_1 (Fig. A2.219)

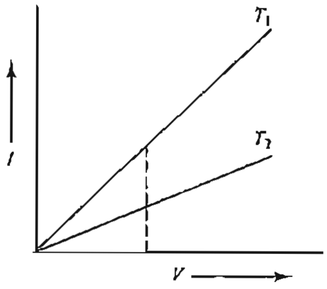


Fig. A2.219

Since, $V = IR$; $R = \frac{V}{I}$. Resistance is less in case of T_1. For a metallic wire, resistance increases with temperature, therefore $T_2 > T_1$.

Single Correct Answer Type

1. b. $R_2 = R_1 (1 + \alpha\Delta T)$

$2 = 1(1 + 0.00125 \times \Delta T) \Rightarrow \Delta T = \frac{1}{0.00125} = 800$

$\Rightarrow T_2 - T_1 = 800 \Rightarrow T_2 = 800 + T_1 = 800 + 300 = 1100\text{K}$

2. d. $H = I^2Rt = I^2 r \frac{l}{\pi r^2} t$

3. a. $I \propto \frac{1}{r^2}; V \propto \frac{1}{r}$ for metallic sphere, at inside point

4. b. Let I_1 be the current flowing in 5 Ω and $(I - I_1)$ in 4 Ω and 6 Ω.

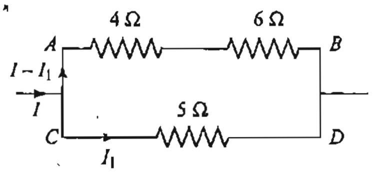


Fig. A2.220

The heat generated in 5 Ω resistor is 10 cal/s = 4.2 × 10 J/s

$\therefore \quad 4.2 \times 10 = I_1^2 R$

$\therefore \quad I_1 = \sqrt{\frac{4.2 \times 10}{5}} = \sqrt{8.4} = 2.9$ A. (i)

Since AB and CD are in parallel.

$\therefore$ The potential difference remains the same between C and D, and between A and B.

$\therefore \quad (I - I_1)(4 + 6) = I_1 \times 5$ on solving using I_1 from (i)

we get $(I - 2.9)10 = 2.9 \times 5$

$\therefore \quad I - 2.9 = 1.45$

$\therefore \quad I = 4.35$. Heat released/sec in 4 Ω will be $(4.35 - 2.9)^2 \times 4 = 8.4$ J/s = 2 cal/s

5. c. $\frac{1}{R_{eq}} = \frac{1}{30} + \frac{1}{60} = \frac{90}{30 \times 60}$, $R_{eq} = 20\,\Omega$ $V = IR$

$I = \frac{2}{20} = 0.1$ A

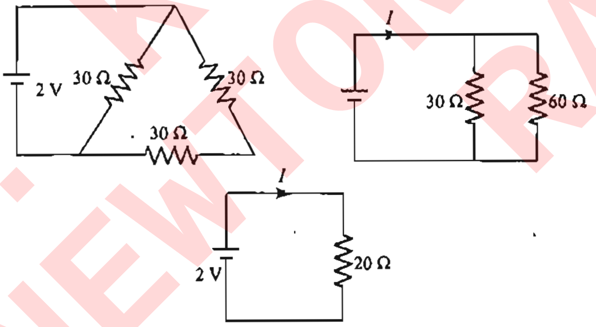
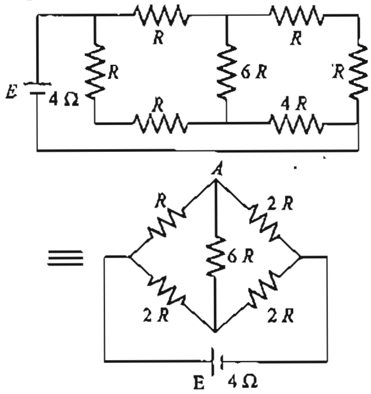


Fig. A2.221

6. d. Copper is a metal, whereas Germanium is a semiconductor.

7. b. For max. power



(*Contd.*)

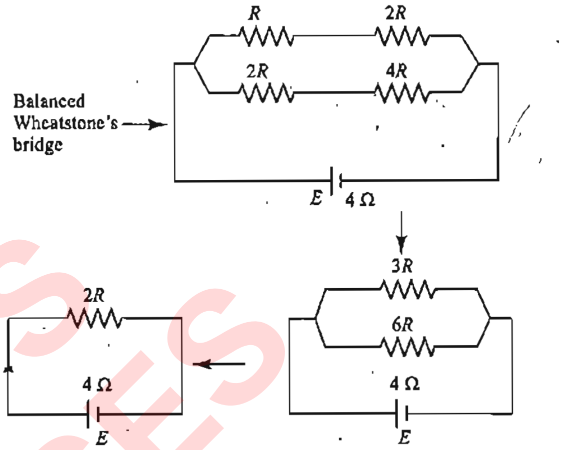


Fig. A2.222

External resistance = Internal resistance $\therefore 2R = 4, R = 2$.

8. d. When a steady current flows in a metallic conductor of non-uniform cross section, then drift speed $V_d = \frac{I}{n_e A}$ and Electric field $E = \frac{I}{\sigma A} \Rightarrow V_d \propto \frac{1}{A}$ and $E \propto \frac{1}{A} \Rightarrow$ Only current remains is constant.

9. A2. For the capacitor to get charged up to 0.75 V, the charge on the plates should be

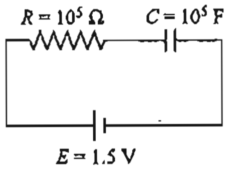


Fig. A2.223

$q = CE[1 - e^{-t/RC}]$

$0.75 \times 10^{-5} = 10^{-5} \times 1.5\left[1 - e^{\frac{-t}{10^5 \times 10^{-5}}}\right]$

$\frac{1}{2} = [1 - e^{-t}] \Rightarrow e^{-t} = \frac{1}{2}$

Taking log on both side, we get $-t = -\ln 2 \Rightarrow t = 0.693$ s.

10. a. Since opening or closing the switch does not affect the current through G, it means that in both the cases there is no current passing through S. This means that potential at A is equal to potential at B and it is the case of balanced Wheatstone's bridge.

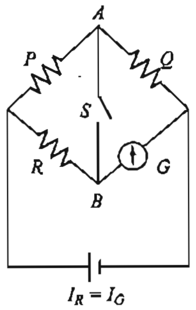


Fig. A2.224

$I_P = I_Q \therefore I_R = I_G$ and (a) is the correct option.

11. c. Method I: In this there will be no current flowing in branch BE in steady condition. Let I be the current flowing in the loop $ABCDEFA$. Applying Kirchhoff's law in the loop moving in anticlockwise direction starting from C + 2 V $-I(2R)-I(R)-V=0 \Rightarrow V=3IR \Rightarrow I=V/3R$ (1)
Applying Kirchhoff's law in the circuit $ABEFA$ we get on moving in anticlockwise direction starting from B

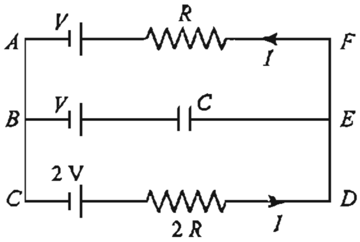


Fig. A2.225

$+V+V_{cap}-IR-V=0$ where V_{cap} is the potential difference across capacitor

$$\therefore \quad V_{cap}=IR=\left(\frac{V}{3R}\right)\times R=\frac{V}{3}$$

Method II:

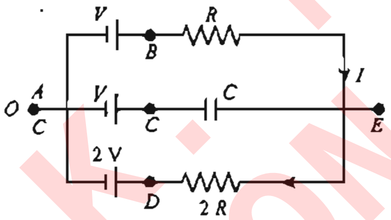


Fig. A2.226

Let us consider A to be at OV. Then points B, C and D will be at V, V and 2V, respectively. Let the current be flowing in clockwise direction. Applying Kirchhoff's law in the outer loop, we get $V-IR-I(2R)-2V=0$ $\therefore I=-V/3R$. The minus sign here indicates that the current is in the opposite direction to what we have assumed. Applying Kirchhoff's law from A to E via B we get $V_A+V+IR=V_E$ $\therefore 0+V+\frac{V}{3R}\times R=V_E=\frac{4V}{3}$. Again applying Kirchhoff's law from A to E via C, we get $V_A+V+V_{cap}=V_c$ $\therefore V_{cap}=\frac{V}{3}$

12. b. Let R be the resistance of rod

$$\text{Energy released in } t \text{ second } =\frac{\left(3V^2\right)}{R}\times t$$

$$\therefore Q=\frac{\left(9V^2\right)}{R}\times t$$

But $Q=mc\Delta T$ $\therefore$ $mc\Delta T=\dfrac{\left(9V^2\right)}{R}\times t$ (i)

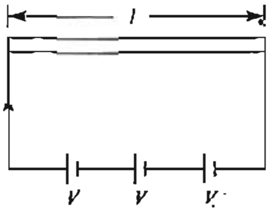


Fig. A2.227

Let R' be the resistance of rod

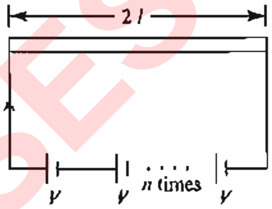


Fig. A2.228

$\Rightarrow$ $R'=2R$ ($\therefore$ Length is twice)

$$\therefore \text{ Energy released in } t \text{ seconds}=\frac{\left(NV^2\right)}{2R}\times t$$

$$\therefore \quad Q'=\frac{\left(N^2V^2\right)}{2R}\times t$$

Applying $Q'=m'c\Delta T$

$$2mc\Delta T=\frac{\left(N^2V^2\right)}{2R}\times t \qquad \text{(ii)}$$

Dividing (ii) by (i), we get $\dfrac{mc\Delta T}{2mc\Delta T}=\dfrac{9V^2\times t/R}{N^2V^2\, t/2R}$

$$\therefore \frac{1}{2}=\frac{9\times 2}{N^2} \Rightarrow N^2=18\times 2 \;\therefore N=6.$$

13. c. Since current I is independent of R_6, it follows that the resistances R_1, R_2, R_3 and R_3 must form a balanced Wheatstone's bridge.

14. a. The circuit is symmetrical about the axis PQ. Therefore, the equivalent circuit is

$$\therefore \quad \frac{1}{R_{PQ}}=\frac{1}{4R}+\frac{1}{4R}+\frac{1}{2r}=\frac{1}{2R}+\frac{1}{2r}\frac{R+r}{2Rr}$$

$$\Rightarrow \quad R_{PQ}=\frac{2Rr}{R+r}$$

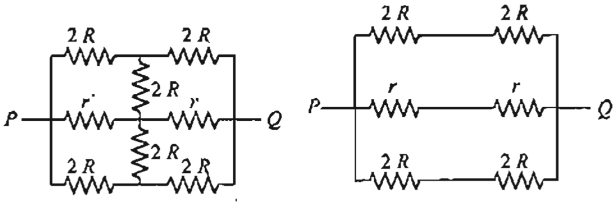


Fig. A2.229

15. d. We know that

$$R = \frac{V^2}{\rho}$$

$$\therefore\ R_1 = \frac{V^2}{100},\ R_2 = \frac{V^2}{60} = R_3;$$

$$W_1 = \frac{V_1^2}{R_1} = \frac{V^2 R_1}{(R_1+R_2)},\ W_2 = \frac{V_2^2}{R_2},\ W_3 = \frac{V^2}{R_3}$$

$$W_3 : W_2 : W_1 :: \frac{V^2}{R_3} : \frac{V^2 R_2}{(R_1+R_2)^2} : \frac{V^2 R_1}{(R_1+R_2)^2}$$

$$= 60 : \frac{(250)^2 V^2/60}{\left(\frac{V^2}{100}+\frac{V^2}{60}\right)} : \frac{(250)^2 V^2/100}{\left(\frac{V^2}{100}+\frac{V^2}{60}\right)}$$

$$= 64 : 25 : 15$$

Hence $W_3 > W_2 > W_1$

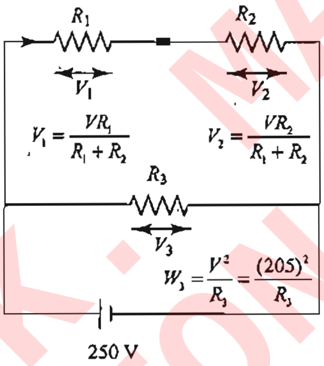


Fig. A2.230

16. a. In Ohm's law, we check $V = IR$ where I is the current flowing through a resistor and V is the potential difference across that resistor. Only option (*A*) fits the above criteria. Remember that ammeter is connected in series with resistance and voltmeter in parallel with the resistance.

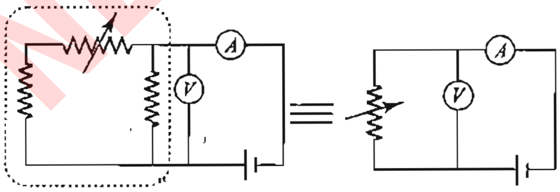


Fig. A2.231

17. a. At null point $\frac{R_1}{R_2} = \frac{R_3}{R_4} = \frac{x}{100-x}$. If radius of the wire is doubled, then the resistance of AC will change and also the resistance of CB will change. But since $\frac{R_1}{R_2}$ does not change, so $\frac{R_3}{R_4}$ should also not change at null point.

Therefore, the point C does not change.

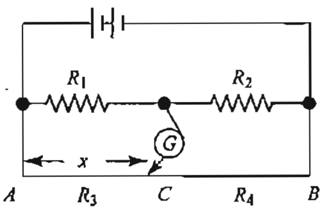
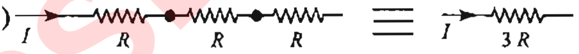


Fig. A2.232

$R \propto \frac{1}{A}$, $\frac{R_3}{R_4} = \frac{A_4}{A_3} = 1$, Ratio does not change.

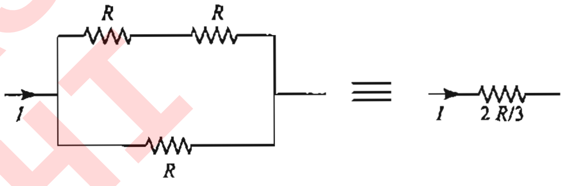
18. a.

(I)



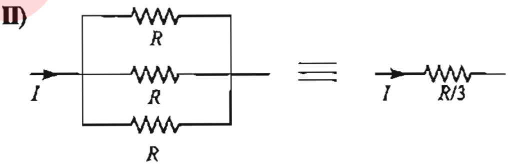
$P_I = I^2(3R)$

(II)



$$P_{II} = I^2\left(\frac{2R}{3}\right)$$

(III)



$$P_{III} = I^2\left(\frac{R}{3}\right)$$

(IV)

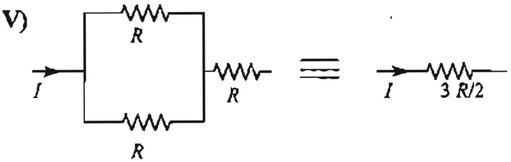


Fig. A2.233

$$P_{IV} = I^2\left(\frac{3R}{2}\right)$$

III < II < IV < I.

19. b. Total external resistance will be the total resistance of whole length of the box. It should be connected between A and D.

20. c. For various combinations, equivalent resistance is maximum between P and Q.

21. b. The current in RC circuit is given by $I = I_0 e^{-t/RC}$

$$\Rightarrow \ln I = \ln I_0 - \frac{t}{RC} \text{ or } \ln I = \left(\frac{-t}{RC}\right) + \ln I_0$$

$$\ln I = \left(-\frac{t}{RC}\right) + \ln\left(\frac{E_0}{R}\right)$$

On comparing with $y = mx + C$

Intercept $= \ln\left(\frac{E_0}{R}\right)$ and slope $= -\frac{1}{RC}$

When R is changed to $2R$, then slope decreases in magnitude. New graph is Q.

22. a. The current in 2 Ω resistor will be zero because it is not a part of any closed loop.

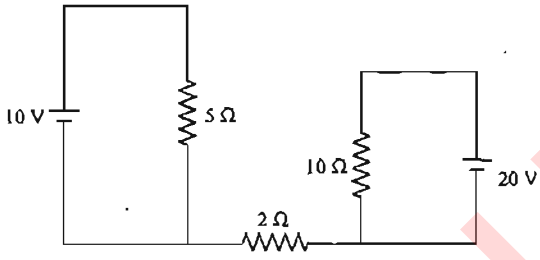


Fig. A2.234

23. a. Let at any instant of time t during charging process, the transient current in the circuit be I. $I = \frac{V_0}{R} e^{-t/RC}$ ∴ Potential difference across resistor R is $\left[\frac{V_0}{R} e^{-t/RC}\right] \times R = V_0 e^{-t/RC}$

∴ Potential diff. across

$C = V_0 - V_0 e^{-t/RC} = V_0(1 - e^{-t/RC})$

$\Rightarrow 1 - e^{-t/RC} = 3e^{-t/RC} \Rightarrow 1 = 4e^{-t/RC}$ Taking log on both sides, we get $\log_e 1 = 2\log_e 2 + \left(-\frac{t}{RC}\right)$

$\Rightarrow 0 = 2 \times 2.303 \log_{10} 2 - \frac{t}{RC}$

$\Rightarrow t = [2 \times 2.303 \log_{10} 2] \times 2.5 \times 10^6 \times 4 \times 10 = 13.86\ \text{s}.$

24. a. $I_g G = (I - I_g)S$, Here, $I_g \approx 100 \times 10^{-6}$ A

$G = 100\ \Omega, S = 0.1\ \Omega$

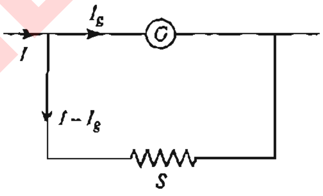


Fig. A2.235

$$\therefore I = I_g\left(\frac{G}{S} + 1\right) \approx 100 \times 10^{-6}\left(\frac{100}{0.1} + 1\right) = 100.1\ \text{mA}$$

25. b. The heat supplied under these conditions is the change in internal energy $Q = \Delta U$. The heat supplied $Q = i^2 Rt = 1 \times 1 \times 100 \times 5 \times 60 = 30{,}000\ \text{J} = 30\ \text{kJ}$

26. b. We know that time constant of an RC circuit is RC. Find equivalent resistance and capacitance in each case and then determine the time constant.

27. b

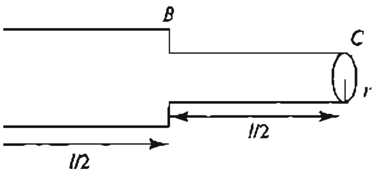


Fig. A2.236

a. $\frac{V_{AB}}{V_{BC}} = \frac{I_{AB} R_{AB}}{I_{BC} R_{BC}} = \frac{R_{AB}}{R_{BC}} = \frac{\rho \frac{l}{2[\pi \times 4r^2]}}{\rho \frac{l}{2[\pi r^2]}} = \frac{1}{4}$

[since $I_{AB} = I_{BC}$, wire is of same material]
Therefore, option (a) is wrong.

b. $\frac{P_{BC}}{P_{AB}} = \frac{I_2 R_{BC}}{I_2 R_{AB}} = \frac{\rho \frac{l}{2[\pi \times 4r^2]}}{\rho \frac{l}{2[\pi r^2]}} = \frac{1}{4}$

$\Rightarrow P_{AB} = 4P_{BC}$. Therefore (b) is correct.

c. $\frac{J_{AB}}{J_{BC}} = \frac{\frac{I}{\pi \times 4r^2}}{\frac{I}{\pi \times r^2}} = \frac{1}{4}$. Therefore (c) is incorrect.

d. $\frac{E_{AB}}{E_{BC}} = \frac{\left[\frac{V_{AB}}{l/2}\right]}{\left[\frac{V_{BC}}{l/2}\right]} = \frac{1}{4}$. Therefore (d) is incorrect.

28. a. Given X is greater than 2 Ω when the bridge is balanced

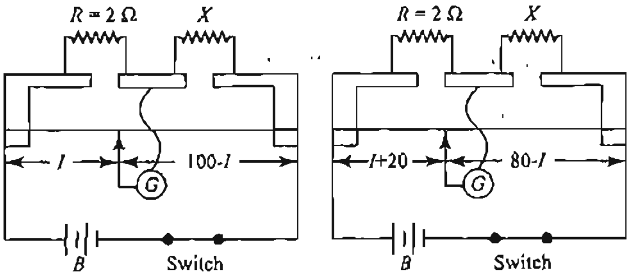


Fig. A2.237

$\frac{R}{l} = \frac{X}{100 - l} \Rightarrow 100R - Rl = lX$

$\Rightarrow\ 200 - 2l = lX \Rightarrow\ l = \frac{200}{X + 2}$

When the resistance are interchanged the jockey shift 20 cm, therefore $\frac{X}{l + 20} = \frac{2}{80 - l}$

$\Rightarrow\ 80X = \frac{200}{(x+2)} + 40 \Rightarrow\ X = \frac{240}{80} = 3\ \Omega$

29. c. When steady state is reached, the current I from the battery is

$$9 = I(3+6) \Rightarrow I = I\text{A}$$

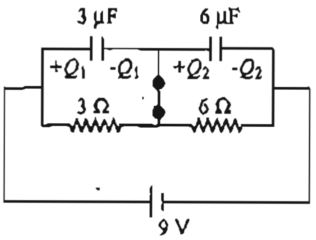


Fig. A2.238

⇒ Potential difference across 3 Ω resistance = 3 V and potential difference across 6 Ω resistance = 6 V

⇒ p.d. across 3 μF capacitor = 3 V

and p.d. across 6 μF capacitor = 6 V

∴ Charge on 3 μF capacitor

$$Q_1 = 3 \times 3 = 9\,\mu\text{C}.$$

Charge on 6 μF capacitor

$$Q_2 = 6 \times 6 = 36\,\mu\text{C}$$

⇒ Charge $(-Q_1)$ is shifted from the positive plate of 6 μF capacitor. The remaining charge on the positive plate of 6 μF capacitor is shifted through the switch.

∴ Charge passing through the switch

$$= 36 - 9 = 27\,\mu\text{C}$$

30. a. $C_{eq} = \dfrac{\dfrac{2\varepsilon_0}{d/3 - Vt} \times \dfrac{\varepsilon_0}{2d/3 + Vt}}{\dfrac{2\varepsilon_0}{d/3 - Vt} + \dfrac{\varepsilon_0}{2d/3 + Vt}} = \dfrac{6\varepsilon_0}{5d + 3Vt}$

Now $\tau = C_{eq}R = \dfrac{6\varepsilon_0 R}{5d + 3Vt}$

31. c. Here, $R_1 = 1\,\Omega$, $R_2 = 1/2\,\Omega$, $R_3 = 3\,\Omega$

As $P = \dfrac{V^2}{R}$, so $P_2 > P_1 > P_3$

32. d. Unknown resistance is directly proportional to standard resistance. On increasing the temperature, the value of unknown resistance will increase, hence standard resistance is also to be increased.

Multiple Correct Answers Type

1. b., d. During the decay of charge in RC circuit $I = I_0 e^{-t/RC}$

where $I_0 = \dfrac{q_0}{RC}$

When $t = 0$, $I = I_0 = \dfrac{q_0}{RC}$

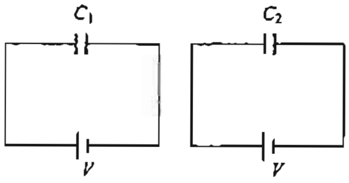


Fig. A2.239

Since potential difference between the plates is initially the same, therefore I is same in both the cases at $t = 0$ and is equal to $I = \dfrac{q_0}{RC} = \dfrac{V}{R}$. Also $q = q_0 e^{-t/RC}$

When $q_0 = \dfrac{q_0}{2}$ then $\dfrac{q_0}{2} = q_0 e^{-t/RC}$

$\Rightarrow e^{+t/RC} = 2 \quad \dfrac{t}{RC} = \log_e 2 \Rightarrow t = RC \log_e 2 \Rightarrow t \propto C$

Therefore, time taken for the first capacitor (1 μE) for discharging 50% of Initial charge will be less.

2. c. Both are basic facts.

3. d. The net resistance of the circuit is 9 Ω as shown in Fig. A2.240.

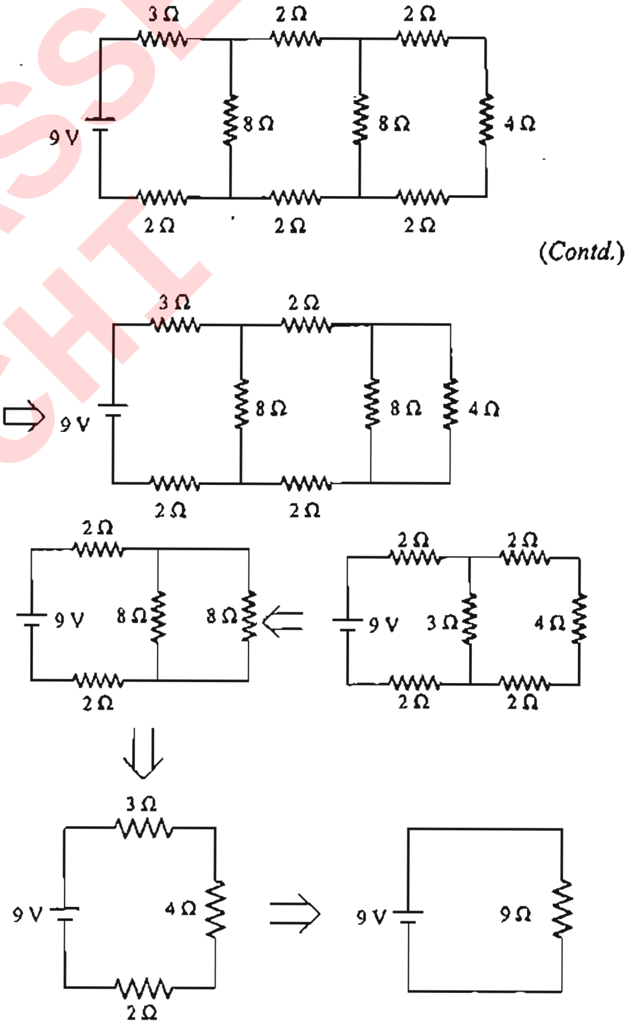


Fig. A2.240

The current flowing in the circuit is $I = \dfrac{V}{R} = \dfrac{9\text{V}}{9\Omega} = 1.0$ A.

The flow of current in the circuit is as follows. Please note that the current divides into two equal parts if it passes through two equal resistances.

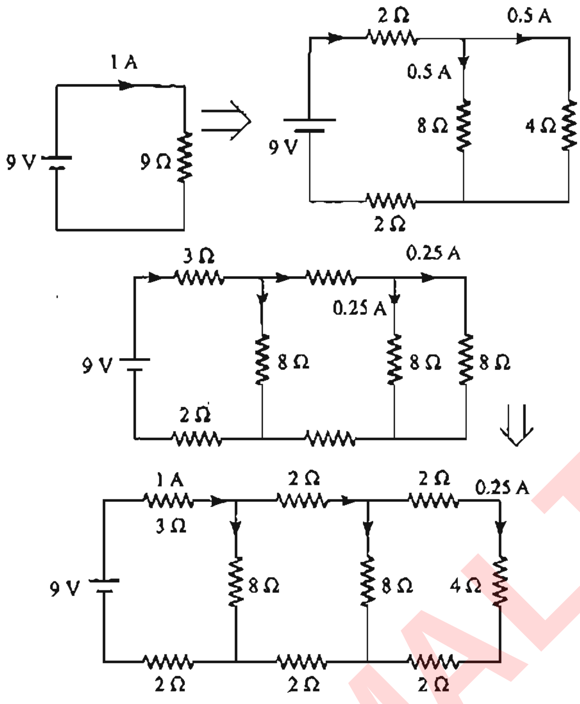


Fig. A2.241

4. a., b., d. In case of metal, the current flowing will be very-very high.

5. a., d.

$$24 - 2\times 10^3 I - 6\times 10^3 (I - i) = 0$$

$$24 - 2\times 10^3 I - 1.5\times 10^3 i = 0$$

Hence $I = 7.5\,\text{mA}$

$i = 6\,\text{mA}$

$$24 - 6\times 10^3 I' - 2\times 10^3 (I' - i') = 0$$

$$24 - 6\times 10^3 I' - 1.5\times 10^3 i' = 0$$

$I' = 3.5\,\text{mA}$

$i' = 2\,\text{mA}$

$$\frac{P_1}{P_2} = \frac{6^2}{2^2} = 9$$

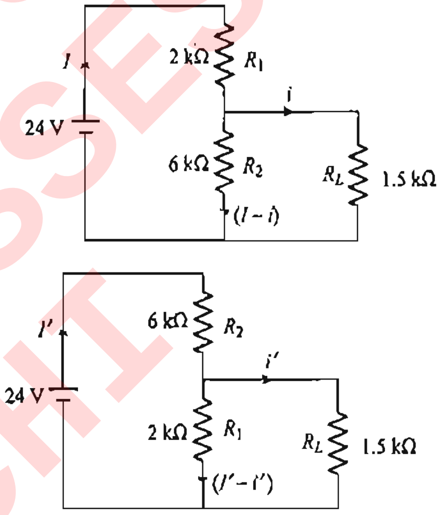


Fig. A2.242

Appendix 3

Solutions to
Concept Application Exercises

CHAPTER 1

Exercise 1.1

1. a. $q = ne$

$$n = \frac{q}{e} = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$$

b. Repulsion. As a charged body can attract uncharged body.

c. No, as charge is quantized.

2. $F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$

$$\varepsilon_0 = \frac{F \times 4\pi r^2}{q_1 q_2}$$

$$\varepsilon_0 = \frac{[MLT^{-2}][L^2]}{[A^2T^2]} = [ML^3T^{-4}A^{-2}]$$

3. By induction.

4. a. Yes. Two bodies are placed close to each other where one has much more charge than the other. Then due to induction, force of attraction becomes more than force of repulsion.

b. Yes, due to induction effect.

c. Yes, mass of negatively charged sphere will be more.

5. a. Since the particle has positive charge so it has deficiency of electrons. So it has less number of electrons compared to the neutral state.

b. $q = ne$

$$10^{-12} = n \times 1.6 \times 10^{-19}$$
$$n = 6.25 \times 10^6$$

6. a. $q = ne$

$$3.2 \times 10^{-8} = n \times 1.6 \times 10^{-19}$$
$$n = 2 \times 10^{11}$$

b. Charge on fur should be equal and opposite to that on ebonite, i.e., $+3.2 \times 10^{-8}$ C.

7. Protons cannot be transferred, as they reside in nucleus. To remove protons, a great amount of energy is required.

8. When the paper comes in contact with the rod, the paper also get similarly charged. Hence, repulsion takes place.

9. Conductor will not get completely discharged, because the person is standing on the insulating stool. Some charge of the rod may transfer to the person to make the potential of the both same.

10.

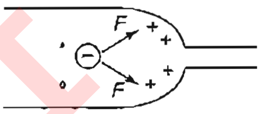


Fig. A3-1.1

Velocity increases due to induced charges on the neck.

11. There should be other charged particles to feel the effect of a charge particle, otherwise the concept of electric charge would be meaningless.

Exercise 1.2

1. a. i. There is always a restoring force towards its original position.The particle (negative charge) will perform oscillation. (The case of stable equilibrium.)

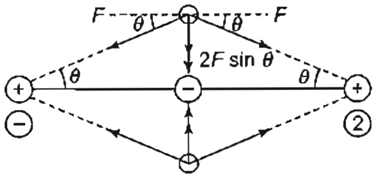


Fig. A3-1.2

ii. $\vec{F}_1$ will be more than $\vec{F}_2$. The negative charge will experience net force towards charge (1). The negative charge is under stable equilibrium.

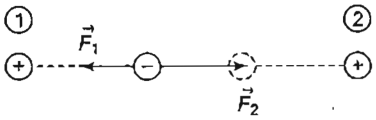


Fig. A3-1.3

b. No

2. a. No. A charged particle cannot apply force on itself.

b. Net field is $2E\cos\theta$ which is parallel to the line joining the charges.

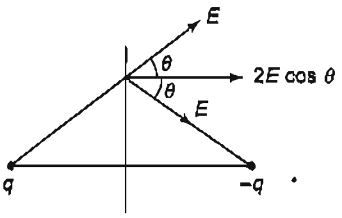


Fig. A3-1.4

3. For no position of the positive charge the system will be in equilibrium. (However, only 'q' can be in equilibrium if it is placed at the middle of the negative charges.)

4.

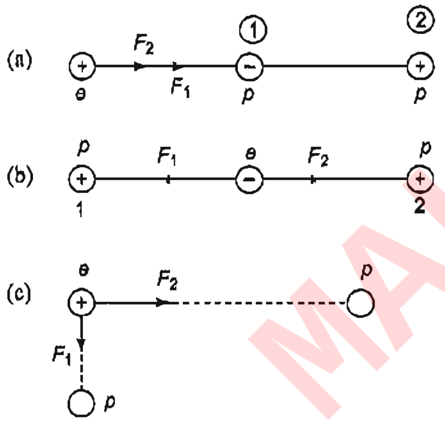


Fig. A3-1.5

a. It is clear from the diagram

$$F_a < F_c < F_b$$

b. Since $F_1 > F_2$; $\tan\theta = \frac{F_1}{F_2}$

$$\theta > 45°$$

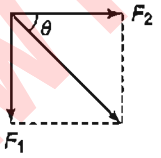


Fig. A3-1.6

5. a. Right, because third particle should be placed near the charge smaller in magnitude, and not between the charges.
   b. The third particle should be negatively charged, only then net force on any charge due to other two charges can be zero.
   c. Equilibrium is unstable, because if we displace any of the charged particles from its equilibrium position, it may not return to its initial position for all directions of displacement.

6. Magnitude of force on $-q$ at centre

$$F_n = \sqrt{F^2+F^2} = \sqrt{2}F = \sqrt{2}\times\frac{1}{4\pi\varepsilon}\frac{2q\times q}{r^2}$$

$$F_n = \frac{q^2}{\sqrt{2}\pi\varepsilon_0 r^2}$$

The direction is at angle 45° from x-axis in IV-quadrant.

7.

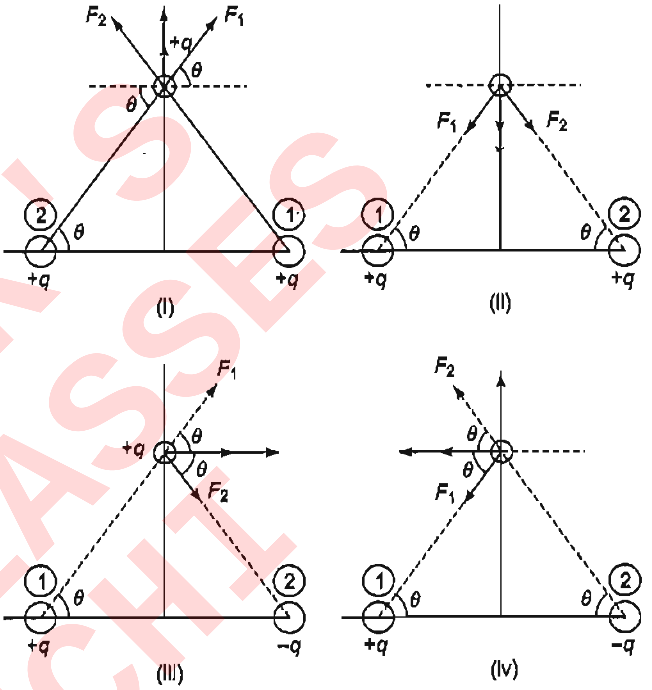


Fig. A3-1.7

a. It is clear from the diagrams that the forces $F(F_1$ or $F_2)$ in case (i) and (ii), (iii) and (iv) will be same.

b. less than $2F_0$

c. cancel in situation (i) and (ii) and add in situation (iii) and (iv)

d. Add in situation (i) and (ii) and cancel in situation (iii) and (iv).

e. Obviously, the direction of net force will be that of adding components.

f. i. $+y$ ii. $-y$ iii. $+x$ iv. $-x$

8. When charges are brought in the medium, force between them will decrease by a factor of K, where K is known as dielectric constant of that medium. Hence,

$$\frac{F'}{F} = \frac{1}{\text{Dielectric constant}} = \frac{1}{81}$$

9. Because of the free fall, there will be weightlessness in the elevator. The balls will go maximum away from each other and finally angle between them will be 180^0.

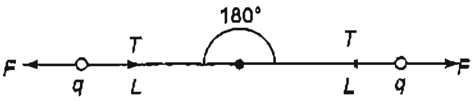


Fig. A3-1.8

Repulsion force F between them will be balanced by tension T. So

$$T = F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2L)^2} = \frac{q^2}{16\pi\varepsilon_0 L^2}$$

10. **a.** Since the size of group is small, so the group of n small charged particles must behave as single point charge, so that it can have a separation of 10 cm from the particle in question. Obviously, force on this particle due to the group of n particles is n times the force due to a single particle. Hence, force due to group of n particles, $F = n \times 3 \times 10^{-10}$ N

b. Here, $F = 6 \times 10^{-6}$ N

$$n = \frac{F}{3\times10^{-10}} = \frac{6\times10^{-6}}{3\times10^{-10}} = 2\times10^4$$

Exercise 1.3

1. The angular momentum of q will be constant, as no external torque is acting on q.

2. **a.** Top (+); middle (–) bottom (+)

Electric field lines are away from the positive charge and towards the negative charge.

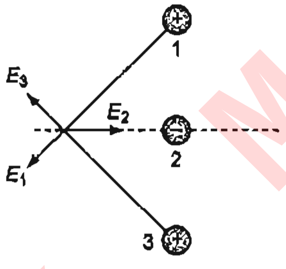


Fig. A3-1.8

b. We can get two neutral points, left and right side of middle charge, i.e., electric field will be zero at these locations. It is clear from the diagram that the electric field may be zero at left and right of charge 2.

3.

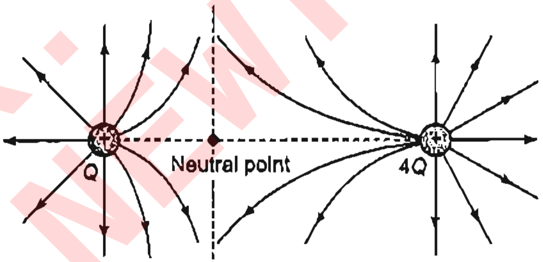


Fig. A3-1.9

For its location,

$$\frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{(\ell-x)^2}$$

$$\left(\frac{\ell-x}{x}\right)^2 = 4 \quad \Rightarrow x = \frac{\ell}{3} = \frac{12}{3}\,\text{cm}$$

Hence, neutral point will be at 4 cm from charge Q.

4. No, in given hypothetical field lines electric field is strong at bottom and weak at top. If we move in a close path some work will be done, but in an electric field no work is done in close path.

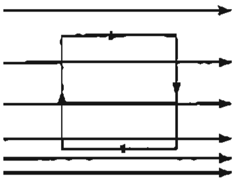


Fig. A3-1.10

5. **a.** Tangents at points A and B.

b. At A, as intensity of electric field is more at A.

6. **a.** No force will be experienced by the charge at the centre as it is electrostatically shielded.

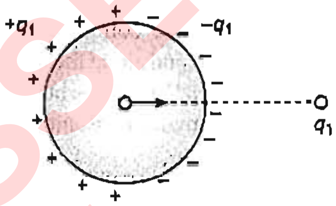


Fig. A3-1.11

b. There will not be any field at centre due to the charges Q and q. The charge induced at the surface of spherical shell due to q will produce the field towards right. This field will be responsible for producing force at centre towards right.

7. **a.** Yes.

b. Towards.

c. No.

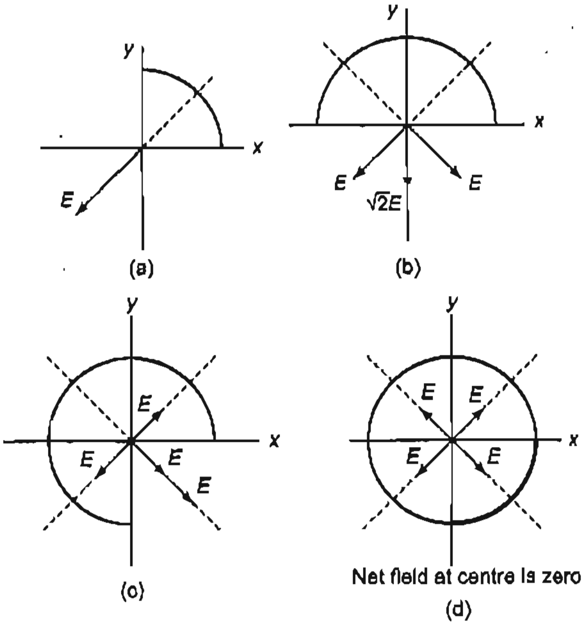
d. Cancel.

e. Add.

f. Yes.

g. Negative y.

8. Make use of symmetry property.



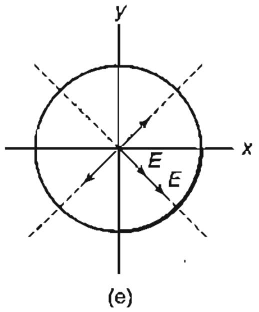


Fig. A3-1.12

It is clear from diagrams above that order of fields will be (greatest first): (e), (b), (a,c), (d) [$2E, \sqrt{2}E, E$ and zero respectively]

9. It is clear from diagram the components perpendicular to normal to plate will cancel and along the normal $dE \cos\theta$ adds. As the distance decreases dE increases but $\cos\theta$ also decreases in such a way $dE \cos\theta$ remains constant for all positions.

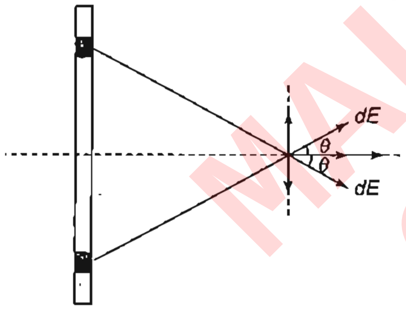


Fig. A3-1.13

It can be observed that near an infinite sheet electric field is constant everywhere.

10. (1) and (2) are negative while (3) is positive.
Acceleration of each particle,

$$a = \frac{qE}{m}$$

Deflection suffered by a particle,

$$y = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\frac{qE}{m}t^2 \quad \Rightarrow \quad y \propto \frac{q}{m}$$

Hence, (3) will have highest q/m ratio.

11. a. Along C.
 b. Path D. As x starts moving the magnitude of electric field due to charge on y decreases while the magnitude of electric field due to charge on z increases. The resultant electric field changes the direction as shown in the figure. Hence, we can say the charge particle follows path D.

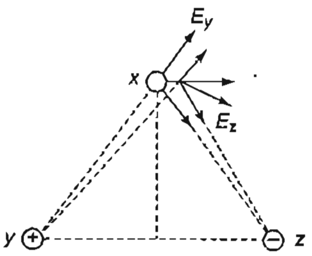


Fig. A3-1.14

12. Here we have assumed that the particle has started from the positive side of x axis.

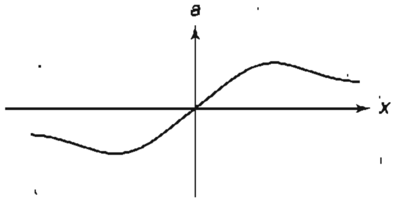


Fig. A3-1.15

Acceleration at infinity will be zero. When the particle comes towards origin, first its acceleration will increase, becomes maximum and again becomes zero at origin and similarly on the negative side of x-axis.

13. b. Magnitude of electric field will remain same but direction will be opposite.

14. We have to find intensity at P.

$$E = \frac{kQ}{a^2}, \quad \frac{a/2}{x} = \cos 30° \quad \Rightarrow x = \frac{a}{\sqrt{3}}$$

$$y = \sqrt{a^2 - x^2} = \sqrt{a^2 - \frac{a^2}{3}} = \left(\sqrt{\frac{2}{3}}\right)a$$

$$\cos\theta = \frac{y}{a} = \sqrt{\frac{2}{3}}$$

Net intensity at P :

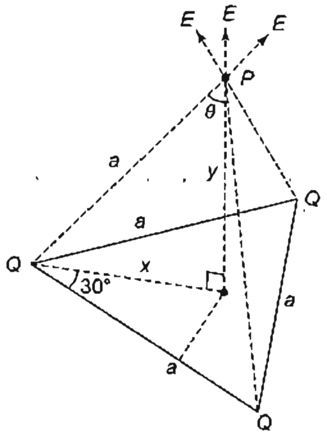


Fig. A3-1.16

$$E_0 = 3E\cos\theta = \frac{3kQ}{a^2}\sqrt{\frac{2}{3}}$$

$$= \frac{3}{4\pi\varepsilon_0}\frac{Q}{a^2}\frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{3}\,Q}{2\sqrt{2}\,\pi\varepsilon_0 a^2}$$

15. Let electric field at point P is zero, then

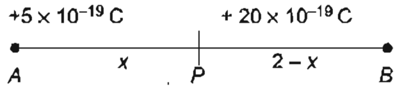


Fig. A3-1.17

$$\frac{1}{4\pi\varepsilon_0}\frac{5\times10^{-19}}{X^2}=\frac{1}{4\pi\varepsilon_0}\frac{20\times10^{-19}}{(2-X)^2}$$

or $$\frac{2-X}{X}=2 \quad \text{or} \quad X=\frac{2}{3}\text{m}$$

16. The acceleration experienced by a charged particle under the action of an electric field E is

$$a=\frac{eE}{m}$$

If it falls through a distance h, starting from rest,

then $$h=\frac{1}{2}at^2 \qquad (\text{as } u=0)$$

or $$t^2=\frac{2h}{a}=\frac{2h}{eE/m}=\frac{2mh}{eE}$$

or $$t\propto\sqrt{m} \qquad \text{(i)}$$

From equation (i) the time of fall will be

$\frac{t_{\text{electron}}}{t_{\text{proton}}}=\sqrt{\frac{m_e}{m_p}}$. Now since $m_p > m_e$, so $t_p > t_e$, i.e., time of fall through the same distance is greater for a heavier particle (i.e., proton), which is in contrast with the situation of free fall under gravity where the time of fall is independent of mass of the body.

17. **a.** The force F experienced by the electron towards the positive plate is given by

$$F=qE=1.6\times10^{-19}\times(45/16)\times10^3$$
$$=4.5\times10^{-16}\text{ N}$$

Hence, the acceleration experienced by the electron is given by

$$a=\frac{F}{m}=\frac{4.5\times10^{-16}\text{ N}}{9.0\times10^{-31}\text{ kg}}=5.0\times10^{14}\text{ m/s}^2$$

Now, the electron is released from the negative plate and is travelling towards the positive plate. Hence we have

$$u=0, x=0.10\text{ m and } a=5.0\times10^{14}\text{ m/s}^2$$

According to equation $x=ut+\frac{1}{2}at^2$,

$$t=\sqrt{\frac{2x}{a}}=\sqrt{\frac{2\times0.10}{5.0\times10^{14}}}=2\times10^{-8}\text{ s}$$

b. $v=u=at=0+5.0\times10^{14}\times2.0\times10^{-8}=10^7$ m/s

18. **a.** As $q=ne$, hence number of electrons transferred are

$$n=\frac{q}{e}=\frac{3.2\times10^{-7}\text{C}}{1.6\times10^{-19}\text{C}}=2\times10^{12}$$

The electrons are transferred from wool to polythene.

b. Since the electrons are transferred from wool to polythene and electrons have mass, obviously mass is transferred from wool to polythene. But since the mass of an electron is negligibly small, transferred mass will also be negligibly small.

19. Since q is in equilibrium with Q and Q,

$$k\frac{Qq}{x^2}=k\frac{Qq}{(r-x^2)} \quad \Rightarrow x=\frac{r}{2}$$

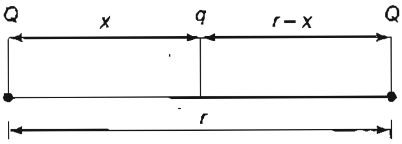


Fig. A3-1.18

Since Q is also in equilibrium with Q and q

$$k\frac{QQ}{r^2}=k\frac{Qq}{x^2} \quad \Rightarrow q=\frac{Q}{4}$$

The sign of q should be opposite to that of Q.

20. Becomes zero, thin metal plate will provide the shielding between the charges due to which electric field of one charge will not be able to reach the other charge.

21. **i.** Both the charges cannot be of same sign.

ii. The observation point (where electric field intensity is zero) has to be closer to the charge smaller in magnitude.

22. **a.** **i.** When we bring a charge q at the centre of sphere, charges $-q$ and $+q$ will be induced on inner surface and outer surface, respectively, as shown in Fig. A3-1.19(a).

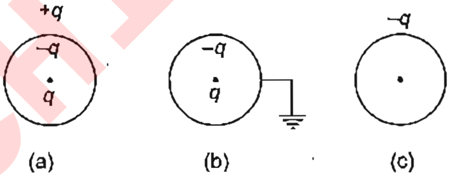


Fig. A3-1.19

ii. After earthing, $+q$ will be transferred to earth as shown in Fig. A3-1.19(b).

iii. After removing the earthing, $-q$ will be distributed uniformly on the external surface of the sphere as shown in Fig. A3-1.19(c). So finally the sphere will have charge $-q$ distributed uniformly on the external surface.

b. The intensity of the electric field inside the sphere will be zero. An electric field similar to the field of the point charge q situated at the centre of the sphere will be set up outside the sphere. Direction of the field will be radially inward.

23. Electric field at the centre of the full ring will be zero, because field at center due to some element will be cancelled by the opposite element of same length.

But in half ring there will be a net field at the centre of the ring.

24. $E=F/q=3.2\times10^{-4}/1.6\times10^{-10}=2\times10^6$ N/C

25. Here, $q=\pm8\times10^{-9}\text{C}=\pm8\times10^{-9}\text{C}$; $2a=4\text{ cm}=0.04\text{ m}$

$$\tau=4\sqrt{3}\text{ Nm and } \theta=60°$$

a. Now, $p=q(2a)=8\times10^{-9}\times0.04=3.2\times10^{-10}$ Cm

Torque on the electric dipole, $\tau=pE\sin\theta$

or $$E=\frac{\tau}{p\sin\theta}=\frac{4\sqrt{3}}{3.2\times10^{-10}\times\sin60°}$$

$$= \frac{4\sqrt{3}\times 2}{3.2\times 10^{-10}\times\sqrt{3}} = 2.5\times 10^{10}\ \text{NC}^{-1}$$

b. Potential energy of the electric dipole,

$$U = -pE\cos\theta$$
$$= -3.2\times 10^{-10}\times 2.5\times 10^{10}\times\cos 60^\circ$$
$$= -3.2\times 2.5\times 0.5 = -4\ \text{J}$$

26. Here, $E = 10^5\ \text{NC}^{-1}$; $q = 1\ \mu\text{C} = 10^{-6}$ C

$$2a = 2\ \text{cm} = 0.02\ \text{m}$$
$$p = q(2a) = 10^{-6}\times 0.02 = 2\times 10^{-8}\ \text{Cm}$$

a. The torque acting on a dipole is given by

$$\tau = pE\sin\theta$$

Torque is maximum when $\sin\theta = 1$

$$\tau_{\text{max}} = 2\times 10^{-8}\times 10^5\times 1 = 0.002\ \text{Nm}$$

b. Now, work done in rotating the dipole from $\theta = \theta_1$ to $\theta = \theta_2$ is given by

$$W = pE(\cos\theta_1 - \cos\theta_2)$$

Here, $\theta_1 = 0^\circ$, $\theta_2 = 180^\circ$

$\therefore$ $W = 2\times 10^{-8}\times 10^5(\cos 0^\circ - \cos 180^\circ)$

$$= 0.002(1-(-1)) = 0.004\ \text{J}$$

Exercise 1.4

1. a. False. Net force will be zero because equal and opposite forces will act on positive and negative charges. Torque $= pE\sin\theta$.
   b. True. In non-uniform electric field forces on both charges may not be equal and opposite.
   c. True. In a non-uniform electric field, a dipole can experience a force as well as a torque.
2. $E \propto \frac{1}{r^3}$, so $n = -3$.
3. As p is directed from negative charge to the positive charge, so angle made by E with p is $\theta + \alpha$.
4. $E = \frac{kp}{r^3} = \frac{9\times 10^9\times 1\times 10^{-6}\times 100\times 10^{-10}}{(0.1)^3} = 0.09$ N/C
5. Maximum torque $= pE = 1.2\times 10^{-2}$ Nm
6. $p = qd$

$$P_{\text{net}} = 2p\cos 30^\circ$$

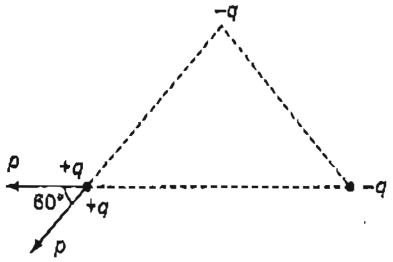


Fig. A3-1.20

$$= 2qd\frac{\sqrt{3}}{2} = \sqrt{3}qd$$

7. $\alpha = 90 - \theta$

We know that $\tan\alpha = \frac{\tan\theta}{2}$

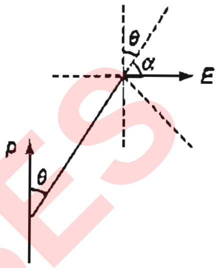


Fig. A3-1.21

$$\Rightarrow \quad \tan(90-\theta) = \frac{\tan\theta}{2} \quad \Rightarrow \tan\theta = \sqrt{2}$$
$$\Rightarrow \quad \theta = \tan^{-1}(\sqrt{2})$$

8. Net flux though yz plane is $2\left(\frac{q}{2\varepsilon_0}\right) = \frac{q}{\varepsilon_0}$.

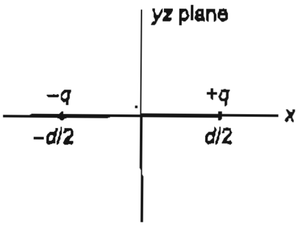


Fig. A3-1.22

So flux depends upon q but is independent of d.

9. Suppose that the dipole axis makes an angle θ with the electric field at an instant. The magnitude of the torque on it is $\tau = pE\sin\theta$. This torque will tend to rotate the dipole back towards the electric field. Also, for small angular displacement $\sin\theta \approx \theta$ so that

$$\tau = -q\ell E\theta$$

Negative sign is because torque applied by field will be opposite to angular displacement θ.
The moment of inertia of the system about the axis of rotation is

$$I = 2\times m\left(\frac{\ell}{2}\right)^2 = \frac{m\ell^2}{2}$$

Thus, the angular acceleration is

$$\alpha = \frac{\tau}{I} = -\frac{2qE}{m\ell}\theta = -\omega^2\theta$$

where $$\omega^2 = \frac{2qE}{m\ell}$$

Thus, the motion is angular simple harmonic and the time period is

$$T = 2\pi\sqrt{\frac{m\ell}{2qE}}$$

10. $x_1 = 2$ m, $x_2 = 4$ m

$$\tau = I\alpha; \quad \alpha = -\frac{pE}{I}\theta$$

$$T = 2\pi\sqrt{\frac{I}{pE}} = 2\pi\sqrt{\frac{2(2)^2+1(4)^2}{8\times10^4\times2\times10^{-6}\times6}}$$

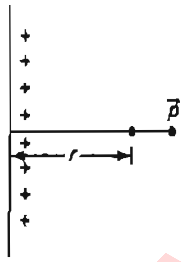


Fig. A3-1.23

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{16\times6\times10^{-2}}{24}} = 0.2 \text{ rad/s}$$

11. x components will be cancelled.

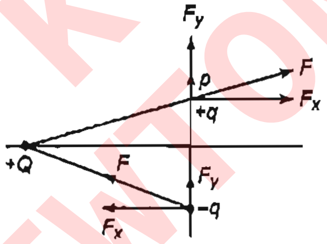


Fig. A3-1.24

y components will be added, giving net force along U.

CHAPTER 2

Exercise 2.1

1. As $\theta = 120°$, hence charge enclosed $= Q/3$.

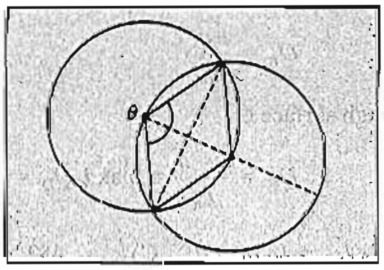


Fig. A3-2.1

$$\phi = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = \frac{Q}{3\varepsilon_0}$$

2. As the rod enters the box the charge increases; hence flux increases linearly. When rod enters the cube completely, the charge within the cube becomes constant for some time and hence the flux also becomes constant.
When the rod starts moving out of the cube, the charge inside the cube starts decreasing and hence the flux will also start decreasing linearly. Hence, graph d is the correct graph.

3. **a.** Number of field lines coming to the curved surface is equal to the field lines moving away from the surface. Hence the flux linked with curved surface is zero.
   b. Flux passing through the plane surface = the flux passing through the curved surface.
   $$\phi = E\pi R^2$$
   (positive as field lines are away from the surface)
   c. $\phi = E2\pi R^2$

4. Zero. No, as the charge resides on the outer surface of a conductor.

5. The charge induced on left side is positive.

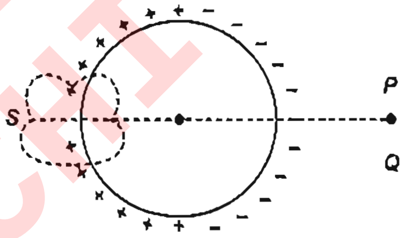


Fig. A3-2.2

Hence, charge enclosed by Gaussian surface is positive. Thus the sign of the flux is '+ve'.

6. If the charge is placed at A or D, any electric field line crosses the curved surface twice.

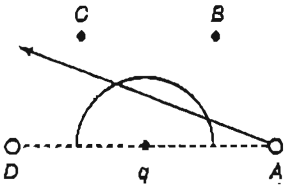


Fig. A3-2.3

Hence, the flux passing through the curved surface should be unchanged. If the charge is placed at B, some of the electric field lines may cut the surface once thereby charging the flux through the surface.

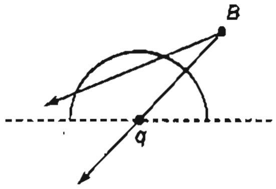


Fig. A3-2.4

7. If we close the charge by an imaginary cube the flux passing through the surface

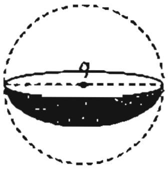


Fig. A3-2.5

a. and b. Since the point charge Q is located just above the centre of the flat face then almost half the lines emitting from Q will pass through the flat surface. Any line passing through the flat surface will also pass through the curved surface below it. Hence the required flux is $\phi = \frac{q}{2\varepsilon_0}$.

c. If the charge is placed exactly at the centre, the any line emitting from the charge will be parallel to the flat surface. Hence, no flux through the flat surface. But the flux through the curved surface will remain same as $\phi = \frac{q}{2\varepsilon_0}$.

8. Effective area of the cone through which flux passes is

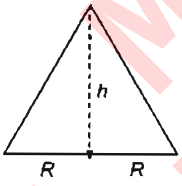


Fig. A3-2.6

$$A = 2\left[\frac{1}{2}Rh\right] = Rh$$

Flux $\quad \phi = EA = ERh$

9. $\vec{E} = a\hat{i} + b\hat{j}$

a. $\vec{A} = A\hat{i} \quad \Rightarrow \phi = \vec{E}\cdot\vec{A} = Aa$

b. $\vec{A} = A\hat{j} \quad \Rightarrow \phi = \vec{E}\cdot\vec{A} = Ab$

c. $\vec{A} = A\hat{k} \quad \Rightarrow \phi = \vec{E}\cdot\vec{A} = 0$

10. a. Any line emitting from charge q to the right of line AB will pass through the infinite plane. It means half flux will pass through the plane. So flux passing through the plane is $\frac{q}{2\varepsilon_0}$.

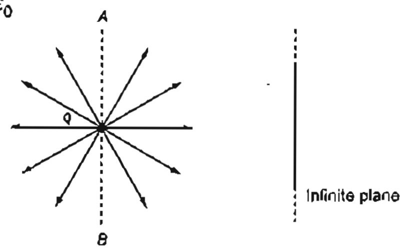


Fig. A3-2.7

b. q is above the centre at a small distance. So this becomes similar to the case (a).

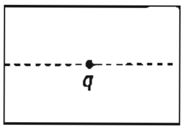


Fig. A3-2.8

11. Effective area $= A = \pi r^2$

$\therefore \quad \phi = E_0 A = E_0 \pi r^2$

12. a. Since Q_1 is inside the surface, so flux due to Q_1 is Q_1/e_0.
b. Since Q_2 is outside, so there is no contribution to the flux due to Q_2.

13. Let Coulomb's law involves $\frac{1}{r^3}$. Now, let us find flux due to a point charge.

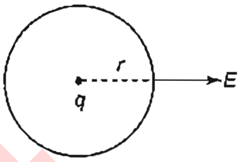


Fig. A3-2.9

$$E = \frac{K}{r^3}$$

$$\phi = \int \vec{E}\cdot d\vec{s} = \int \frac{K}{r^3} ds$$

$$\frac{K}{r^3}\int ds = \frac{K}{r^3} 4\pi r^2$$

$$\phi = \frac{4\pi K}{r}$$

It means flux depends upon r, which is not true. So Gauss's law will not be true. Because in Gauss's law, flux through a closed surface depends upon the charge present inside and not on the shape or size of the Gaussian surface.

Exercise 2.2

1. Flux through close surface A,

$$\phi_A = \frac{Q_{in}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

Flux through close surface B,

$$Q_B = \frac{-q}{\varepsilon_0}$$

Flux through surface C,

$$Q_C = \frac{q-q}{\varepsilon_0} = 0 \text{ and } Q_D = \frac{Q_{in}}{\varepsilon_0} = 0$$

2. a. $\Phi_{S_1} = q_1/\varepsilon_0 = (4.00\times10^{-9}\ \text{C})/\varepsilon_0 = 452\ \text{Nm}^2/\text{C}$

b. $\Phi_{S_2} = q_2/\varepsilon_0 = (-7.80\times 10^{-9}\text{ C})/\varepsilon_0 = -881\text{ Nm}^2/\text{C}$

c. $\Phi_{S_3} = (q_1+q_2)/\varepsilon_0 = ((4.00-7.80)\times 10^{-9}\text{ C})/\varepsilon_0$

$= -429\text{ Nm}^2/\text{C}$

d. $\Phi_{S_4} = (q_1+q_2)/\varepsilon_0 = ((4.00+2.40)\times 10^{-9}\text{ C})/\varepsilon_0$

$= 723\text{ Nm}^2/\text{C}$

e. $\Phi_{S_5} = \dfrac{(q_1+q_2+q_3)}{\varepsilon_0} = \dfrac{(4.00-7.80+2.40)\times 10^{-9}\text{ C}}{\varepsilon_0}$

$= -158\text{ Nm}^2/\text{C}$

All that matters for Gauss's law is the total amount of charge enclosed by the surface, not its distribution within the surface.

3. We know that charge on facing surface is equal and opposite.

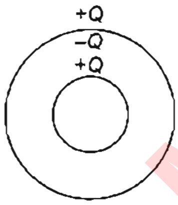


Fig. A3-2.10

a. Net charge on inner surface is $-Q$.
b. Net charge on the inner surface will remain $-Q$.
c. Net charge on inner surface is $-(Q+q)$
d. Yes, as location of charge does not matter.

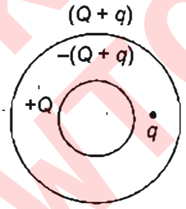


Fig. A3-2.11

4. The distribution of charge is as shown in Fig. A3-2.12.

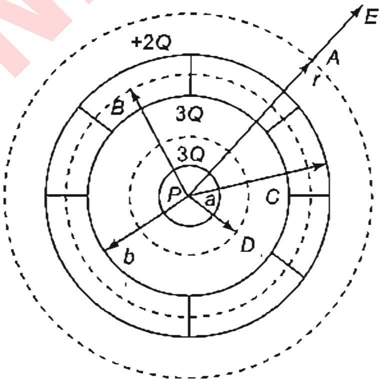


Fig. A3-2.12

a. Net charge enclosed by Gaussian surface ($r > c$) is $3Q - 3Q + 2Q = 2Q$.

b. For $r > c$, the direction of electric field is radially outward as shown in point A.

c. $E = \dfrac{k\,q_{in}}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2Q}{r^2}$

d. For $c < r < b$, we have point B. If we draw a Gaussian surface through B, net charge enclosed in it will be zero. So electric field at B will be zero.

e. As explained above, net charge enclosed is zero.

f. For $b > r > a$, we have point D. If we draw a Gaussian surface through D, it will have net charge enclosed $3Q$.

g. $E = \dfrac{k\,q_{in}}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3Q}{r^2}$

h. $\rho = \dfrac{3Q}{\dfrac{4}{3}\pi a^3}$ (volume charge density)

So, charge within radius $r\,(< a)$ is $q = \rho\dfrac{4}{3}\pi r^3 = \dfrac{3Qr^3}{a^3}$

i. $E = \dfrac{k\,q_{in}}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3Qr^3}{a^3 r^2}$

j. As shown in figure, the charge on the inner surface of shell is $-3Q$.

k. As shown in figure, the charge on outer surface of shell is $+2Q$.

l.

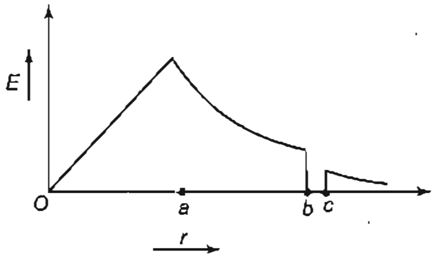


Fig. A3-2.13

5.

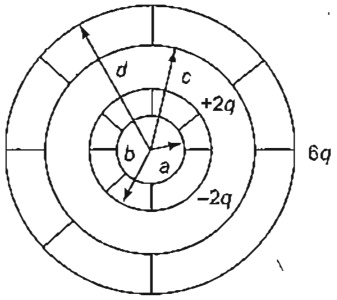


Fig. A3-2.14

a. **i.** For $r > b$, electric field is zero, because there is no charge inside.

ii. For $c > r > b$, electric field will be similar to that as if a point charge $2q$ is placed at the centre.

iii. For $d > r > c$, electric field will be zero, as there is no charge inside.

iv. For $r > d$, electric field will be similar to that as if a point charge $6q$ is placed at the centre.

The resulting plot is as shown in Fig. A3-2.15

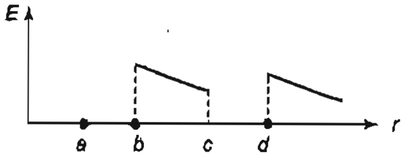


Fig. A3-2.15

b. **i.** 0

ii. 0

iii. $\dfrac{q}{2\pi\varepsilon_0 r^2}$, radially outward

iv. 0

v. $\dfrac{3q}{2\pi\varepsilon_0 r^2}$, radially outward

c. **i.** 0

ii. $+2q$

iii. $-2q$

iv. $+6q$

6. **a.** False. Electric field is due to all charges present inside or outside.

b. False. Gauss's law is applicable for any distribution of charge. But it is more useful when the charge is distributed symmetrically.

c. True. Net flux is due to the charges inside only.

7. **a.** True. If E is zero at each point, then net flux will be zero.

b. True. If net flux is zero, then net charge enclosed will be zero.

c. False. Charge may reside on the outer surface.

8. **a.** Yes, as there is no charge for $r < R_1$.

b. Volume charge density $\rho = \dfrac{Q}{\frac{4}{3}\pi\left(R_2^3 - R_1^3\right)}$.

Charge within radius r is $q = \rho\,\dfrac{4}{3}\pi(r^3 - R_1^3)$

$$= \frac{Q(r^3 - R_1^3)}{R_2^3 - R_1^3}$$

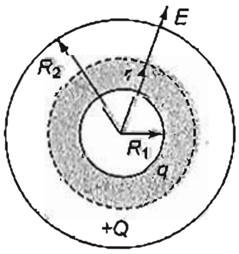


Fig. A3-2.16

$$E = \frac{kq}{r^2} = \frac{Q}{4\pi\varepsilon_0 r^2}\left[\frac{r^3 - R_1^3}{R_2^3 - R_1^3}\right]$$

c. For $r > R_2$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

9. $\phi = EA$; $\qquad \phi = E\dfrac{\pi d^2}{4}$

$$\Rightarrow \qquad E = \frac{4\phi}{\pi d^2}$$

10. **a.** $E = 0$, electric field will be cancelled due to both.

b. $\dfrac{\sigma}{\varepsilon_0}$ to the right, field of both will be added.

c. $E = 0$, electric field will be cancelled due to both.

11. **a.** According to Gauss' law,

$$\phi_1 = \frac{Q}{\varepsilon_0},\ \phi_2 = \frac{(Q+2Q)}{\varepsilon_0}$$

$$\therefore \qquad \frac{\phi_1}{\phi_2} = \frac{Q/\varepsilon_0}{3Q/\varepsilon_0} = \frac{1}{3}$$

b. If sphere S_1 is filled with a medium of dielectric constant $K = 5$, then again according to Gauss's law,

$$\phi'_1 = \frac{Q}{\varepsilon} = \frac{Q}{K\varepsilon_0} = \frac{\phi_1}{5} = \frac{Q}{5\varepsilon_0}$$

12. The flux passing through curved surface

= the flux passing through the frame

$= E\,2R\,l$

13. **a.** The charge on the inner sphere induces equal magnitude of charge, but opposite in sign, on the inner surface of the outer sphere. Sum of all the induced charges is always zero. Therefore, an equal amount of charge must appear on the outer surface. Thus outer and inner surface of the outer sphere have charges $-5Q$ and $-2Q$, respectively.

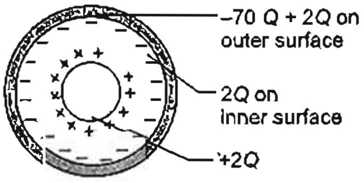


Fig. A3-2.17

b. When outer and inner spheres are connected by a wire, the entire charge is transferred to the outer surface from the inner sphere. In electrostatic equilibrium charge does not reside inside a conductor. Total charge on the outer surface of the outer sphere is $-5Q$.

Total charge on the inner surface is 0. The electric field at the surface of the inside sphere goes to zero after connection. Consider a Gaussian surface just on the surface of the inner sphere.

$$\phi_E = E(4\pi r^2) = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = 0, \text{ as } Q_{\text{enclosed}} = 0$$

Thus, we have $E = 0$

c. When the outer sphere is grounded the charge on the surface is transferred to ground, thus charge is reduced to zero. The final charge distribution is shown in Fig. A3-2.18.

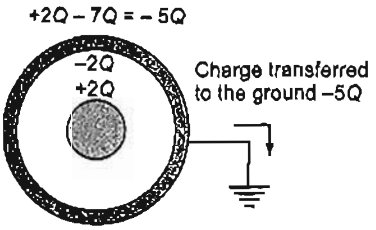


Fig. A3-2.18

CHAPTER 3

Exercise 3.1

1. True, because induced charges cancel the external electric field.
2. Electric field intensity E in that region should be zero, if potential is constant in a given region. This we can check by putting a charged particle in that region. If it experiences no force, E should be zero.
3. Yes, **a.** Potential will increase by 100 V. **b.** No effect on potential difference.
4. We know E in certain region, then we can find potential difference between any two points in that region. To find potential at one point, we should know the potential at some other points.
5. Yes, if shape and size of the conductors are different.

Exercise 3.2

1. It will loose the energy of $eV = 1.6 \times 10^{-19} \times 9 = 14.4 \times 10^{-19}$ J
2. $eV = 4.3 \times 10^{-15} \Rightarrow 1.6 \times 10^{-19}\,\text{V} = 4.3 \times 10^{-15} \Rightarrow \text{V} = 26875$ V
3. $V = \dfrac{kQ}{q}$, for same distance r from a point potential is same.

   This is possible only for a spherical surface. So shape of equipotential surface should be spherical.
4. **a.** Points B and C lie in equipotential surface.

   Change in potential while moving from B to C along circumference will be zero.

   b. As $(V_B - V_A) = |(V_C - V_A)|$

   $$(V_B - V_A) < 0 \text{ and } (V_A - V_C) > 0$$

   $$W_{ext} = DU = q(V_f - V_i)$$

   The work done from $B \rightarrow A$ is positive. The work done from $A \rightarrow C$ is negative.

   $$|W_{BA}| = |W_{AC}|$$

   Same amount of work will be done.
5. Electric field is a conservative field and in any conservative field work done in a close path must be zero.

   $W_{12} = -W_{34} \Rightarrow W_{41} = W_{23} = 0 \Rightarrow W_{12} + W_{23} + W_{34} + W_{41} = 0$

6. Let us consider the work done in a closed path $A \rightarrow B \rightarrow C \rightarrow D$

   $$W_{AB} = E_1 \cdot R\theta$$

   Path $C \rightarrow D$, $W_{CD} = -E_2 \cdot r'\theta$

   $$W_{DA} = W_{BC} = 0 \Rightarrow W_{AB} + W_{CD} = 0 \Rightarrow E_1 R\theta - E_2 r'\theta = 0$$

   $$\Rightarrow \frac{E_1}{E_2} = \frac{r'}{R} \Rightarrow E \propto \frac{1}{r}$$

7. $V_f - V_i = -\int_i^f \vec{E}.d\ell$

   In a closed path, $V_i = V_f$. Hence, $\int_i^f \vec{E}.d\ell$ in a close path will be zero.

Exercise 3.3

1. For 2 and 4, $\tau = pE \sin\theta$, $U = -pE \cos\theta$

   For 1 and 3, $\tau = pE \sin(180 - \theta) = pE \sin\theta$

   $$U = -pE \cos(180 - \theta) = pE \cos\theta$$

   Hence torque is same for all and U is greater for 1 and 3 from 2 and 4.
2. **a.** Electric field will do positive work, if dipole rotates such that its angle decreases with E.

   b. 2 and 4 orientations are identical.
3. **a.** $U = -pE \cos\theta = pE \cos(180 - \theta)$

   More is U, more is θ

   b. If angle between p and E is more closer to 90°, then more is torque. If angle is closer to 0° or 180°, then torque is less. It means lesser is the magnitude of potential energy more is torque and vice-versa.
4. **a.** As $p = 2l \times q$

   $$l = \frac{p}{2q} = \frac{6.2 \times 10^{-30}}{2q}$$

   b. $\tau_{max} = p = 6.2 \times 10^{-30} \times 1.5 \times 10^4 = 9.3 \times 10^{-26}$ Nm

   c. $W = 2\,\text{pE} = 2 \times 9.3 \times 10^{-26} = 1.86 \times 10^{-25}$ J
5. Force on any q by dipole: $F = qE_{\text{dipole}}$

   $$= \frac{q}{4\pi\varepsilon_0} \frac{p}{a^3} \text{ downward}$$

   So from third law, force on dipole due to both charges

   $$= 2\,F = \frac{qp}{2\pi\varepsilon_0 a^3} \text{ upward}$$
6. i. True ii. True
7. $V = \dfrac{kp\cos\theta}{r^2}$

   For A, $\theta = 1.35°$; $V_A = \dfrac{kp\cos 135°}{r^2} = \dfrac{-kp}{\sqrt{2}\,r^2}$

   For B, $\theta = 45°$, $V_B = \dfrac{kp\cos 45°}{r^2} = \dfrac{Kp}{\sqrt{2}\,r^2}$

   $$W = q(V_B - V_A) = \frac{q\sqrt{2}\,kp}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}\,qp}{r^2}$$

CHAPTER 4

Exercise 4.1

1.

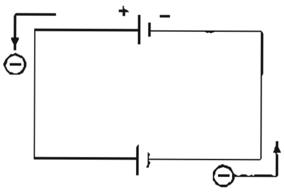


Fig. A3-4.1

Yes, total number of electrons from the plate attached to positive plate of the battery will be pumped to the other plate through the battery.

2. This energy is stored as electrostatic potential energy between the plates.

3. Electric field between the plates of a capacitor,

$$E = \frac{\sigma}{\varepsilon_0}$$

When the capacitor is submerged in a tank of oil,

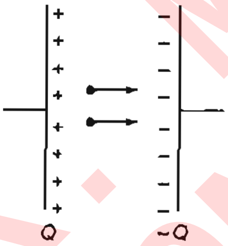


Fig. A3-4.2

the electric field between the plates is

$$E = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0 \varepsilon_r}$$

Hence electric field decreases.

The field can be measured by measuring potential difference between the plates (V) and then it can be calculated using

$$E = \frac{V}{d}$$

4. **a.** $U = Q^2/2C$

$$Q = \sqrt{2UC} = \sqrt{2(25.0\text{ J})(5.00 \times 10^{-9}\text{ F})} = 5.00 \times 10^{-4}\text{ C}$$

The number of electrons N that must be removed from one plate and added to the other is

$$N = Q/e = (5.00 \times 10\text{ C})/(1.602 \times 10^{-19}\text{ C})$$

$$= 3.12 \times 10^{15}\text{ electrons}$$

b. To double U while keeping Q constant, decrease C by a factor of 2, $C = \varepsilon_0 A/d$, halve the plate area or double the plate separation.

5. After disconnection the charge on plates will be eV and $-eV$. Now, positive plate is given a charge Q, the charge distribution on different surface are shown in Fig. A3-4.3.

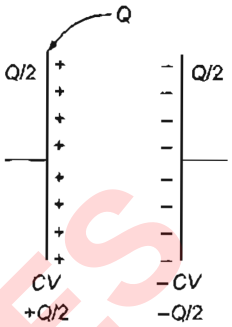


Fig. A3-4.3

Hence, new charge on capacitor is $\left(CV + \frac{Q}{2}\right)$; the potential difference between the plates will be

$$V' = \frac{CV + \frac{Q}{2}}{C} = V + \frac{Q}{2C}$$

6. **a.** The charge from one plate gets transferred to another plate through the battery. The battery pumps the charge from one plate to another.

b. Yes.

7. On connecting with earth, the charge on central plate will shift to its right face. $-Q$ charge will be induced on the left face of the earthed plate. Thus $+Q$ charge will flow to the earth.

8. $q = CV$, C decreases in both cases.

i. In the first case V remains same, hence q decreases due to which force of attraction will decrease.

ii. But in the second case q will remain same, hence force of attraction remain same. So more work will be done in the second case.

9. No, because electric field is conservative, so net work done in a closed path should be zero.

10. At point 1 distance between the plates is less, so attraction between the charges here can be more due to which concentration of charge at point 1 will be more.

11. **a.** True

b. True

c. True

Exercise 4.2

1. As $C = \frac{\varepsilon_0 K A}{d}$, the capacity of a capacitor whose plates are connected together becomes infinite.

Hence $K = \infty$

Hence dielectric constant for metal is ∞.

2. **a.** $Q = CV_0$

b. They must have equal potential difference, and their combined charge must add up to the original charge.

Therefore,

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \text{ and also } Q_1 + Q_2 = Q = CV_0$$

$$C_1 = C \text{ and } C_2 = \frac{C}{2}, \text{ so } \frac{Q_1}{C} = \frac{Q_2}{(C/2)}$$

$$\Rightarrow \qquad Q_2 = \frac{Q_1}{2}$$

$$Q = \frac{3}{2}Q_1 \quad \Rightarrow Q_1 = \frac{2}{3}Q$$

So $$V = \frac{Q_1}{C} = \frac{2}{3}\frac{Q}{C} = \frac{2}{3}V_0$$

c. $U = \frac{1}{2}\left(\frac{Q_1{}^2}{C_1} + \frac{Q_2{}^2}{C_2}\right) = \frac{1}{2}\left[\frac{(\frac{2}{3}Q)^2}{C} + \frac{2(\frac{1}{3}Q)^2}{C}\right]$

$$= \frac{1}{3}\frac{Q^2}{C} = \frac{1}{3}CV_0{}^2$$

d. The original U was

$$U = \tfrac{1}{2}CV_0{}^2 \quad \Rightarrow \Delta U = \tfrac{-1}{6}CV_0{}^2$$

e. Thermal energy of capacitor, wires, etc., and electromagnetic radiation.

3. **a.** $U_0 = \frac{Q^2}{2C} = \frac{xQ^2}{2\varepsilon_0 A}$

b. Increase the separation by

$dx \qquad \Rightarrow \quad U = \frac{(x+dx)Q^2}{2\varepsilon_0 A} = U_0(1 + dx/x)$

The change is then $\frac{Q^2}{2\varepsilon_0 A}dx$

c. The work done in increasing the separation is given by:

$$dW = U - U_0 = \frac{dxQ^2}{2\varepsilon_0 A} = Fdx \Rightarrow F = \frac{Q^2}{2\varepsilon_0 A}.$$

d. The reason for the difference is that E is the field due to both plates. The force is QE if E is the field due to one plate is Q is the charge on the other plate.

4. Let the applied voltage be V. Let each capacitor have capacitance C.

$$U = \frac{1}{2}CV^2$$

for a single capacitor with voltage V.

a. Series:

Voltage across each capacitor is $V/2$. The total energy stored is

$$U_s = 2\left(\frac{1}{2}C[V/2]^2\right) = \frac{1}{4}CV^2$$

Parallel:

Voltage across each capacitor is V. The total energy stored is

$$U_p = 2\left(\frac{1}{2}CV^2\right) = CV^2 \text{ or } U_p = 4U_s$$

b. $Q = CV$ for a single capacitor with voltage V.

$$Q_s = 2(C[V/2]) = CV; \quad Q_p = 2Q_s$$

$$Q_p = 2(CV) = 2CV; \quad Q_p = 2Q_s$$

c. $E = V/d$ for a capacitor with voltage V.

$$E_s = V/2d; \quad E_p = V/d; \quad E_p = 2E_s$$

5.

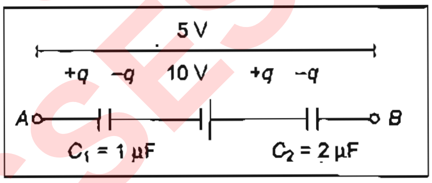


Fig. A3-4.4

Let the charge on the left plate of the capacitor is q, hence the charge on the right plate of the capacitor C_2 will be $-q$ as charge supplied by the negative plate of the capacitor will be $-q$. Moving from point A in the section of circuit AB,

$$V_A - \frac{q}{1} + 10 - \frac{q}{2} = V_B \quad \Rightarrow V_A - V_B = q + \frac{q}{2} - 10$$

$$5 = \frac{3}{2}q - 10 \quad \Rightarrow q = 10\,\mu\text{C}$$

6. The slab will undergo oscillatory motion.
7. External agent will have to do negative work.
8. **a.**

Fig. A3-4.5

b.

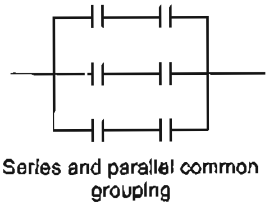


Fig. A3-4.6

9. In series, potentials of all capacitors will be added.
10. Since charges on outer surfaces will be half of the total charge of all the three plates: so $q_1 = q_6 = \frac{60}{2} = 30\,\mu\text{C}$

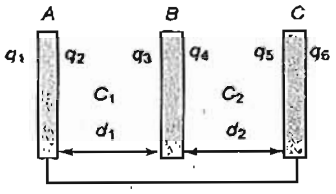


Fig. A3-4.7

$$q_3 = -q_2, q_4 = -q_5 \quad \Rightarrow V_{AB} = V_{CB}$$

$$\Rightarrow \qquad \frac{q_2}{C_1} = \frac{q_5}{C_2}$$

$$q_2 d_1 = q_5 d_2 \quad \Rightarrow q_2 2d_2 = q_5 d_2$$

$$q_5 = 2q_2 \quad \Rightarrow q_4 = 2q_3$$

And $q_3 + q_4 = 60$

$$q_3 = 20\ \mu C \quad \Rightarrow q_4 = 40\ \mu C$$

And $q_2 = -20\ \mu C \quad \Rightarrow q_5 = -40\ \mu C$

11. **a.** $q_1 = 80\ \mu C, q_2 = 20\ \mu C$

$q_3 = -20\ \mu C, q_4 = 80\ \mu C$

b. $q_1 = 0, q_2 = -60\ \mu C$

$q_3 = 60\ \mu C, q_4 = 0$

c. $q_1 = q_2 = q_3 = q_4 = 0$

12. $V_C = 0$

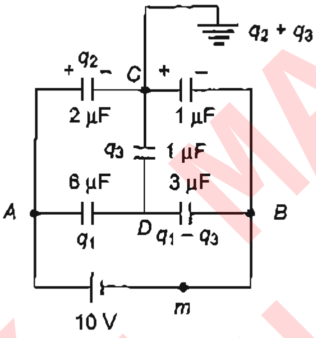


Fig. A3-4.8

$$\text{Loop ADBMA} \rightarrow \frac{q_1}{6} + \frac{q_1 - q_3}{3} = 10 \qquad \text{(i)}$$

$$\text{Loop ACBMA} \rightarrow \frac{q_2}{2} + \frac{q_2 + q_3}{1} = 10 \qquad \text{(ii)}$$

$$\text{Loop CDAC} \rightarrow -\frac{q_2}{2} + \frac{q_3}{1} + \frac{q_1}{6} = 0 \qquad \text{(iii)}$$

Solving, we get $q_1 = 20\ \mu C \quad \Rightarrow q_2 = \frac{20}{3}\ \mu C\ , q_3 = 0$

a. $V_C - V_B = \frac{q_2 + q_3}{1} \quad \Rightarrow 0 - V_B = \frac{20}{3} + 0$

$$V_B = -\frac{20}{3} V$$

b. $V_C - V_D = \frac{q_3}{1} \quad \Rightarrow V_D = 0$

c. $q_2 = \frac{20}{3}\ \mu C$

13. Both 3 µF capacitors will be in parallel, and similarly upper two 1 µF capacitors will be in parallel. The circuit may be redrawn as follows:

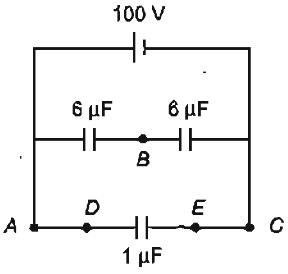


Fig. A3-4.9

a. $V_A - V_B = \frac{100 \times 2}{6 + 2} = 25\ V$

b. $V_B - V_C = \frac{100 \times 6}{6 + 2} = 75\ V$

c. $V_D - V_E$ 100 V

d. $C_{eq} = \frac{6 \times 2}{6 + 2} + 1 = \frac{5}{2} \mu F; u = \frac{1}{2} C_{eq} V^2 = \frac{1}{2}\left(\frac{5}{2} \times 10^{-6}\right) \times (100)^2$

$= 125 \times 10^{-4}$ J

14. **a.** False

b. True

c. True

15. Capacitors 4 and 5 are short-circuited. So, they are useless. '1', '2' and '3' will be in parallel and then 6 will be in series with them.

CHAPTER 5

Exercise 5.1

1. We know that $I = \frac{q}{t} = \frac{ne}{t}$ $\therefore n = \frac{It}{e} = \frac{0.7 \times 1}{1.6 \times 10^{-19}}$

So number of electrons $n = 0.44 \times 10^{19}$

2. **a.** $q = it = (7.5\ A)\ (45\ s) = 337.5\ C$

b. The number of electrons N is given by

$$N = \frac{q}{e} = \frac{337.5 C}{1.6 \times 10^{-19} C} \approx 2.1 \times 10^{21}$$

where $e = 1.6 \times 10^{-19}$ C is the charge of an electron.

3. **a.** The number N of electrons in 0.6 mol is

$N = (0.6\ \text{mol})(6.02 \times 10^{23}\ \text{electrons/mol}) = 3.6 \times 10^{23}$ electrons

$q = Ne = (3.6 \times 10^{23})(1.6 \times 10^{-19}) = 5.78 \times 10^4$ C

b. $t = (45\ \text{min})(60\ \text{s/min}) = 2.7 \times 10^3$ s

$$I = \frac{q}{t} = \frac{5.78 \times 10^4 C}{2.7 \times 10^3 s} = 21.4\ A$$

4. Number per second = (number/length) (velocity)

$= (2 \times 10^{21})(0.05) = 1 \times 10^{20}$ electrons/s

$I = Q/t = (1 \times 10^{20})(1.6 \times 10^{-19}) = 16$ A

5. $R = R_0 [1 + \alpha(T - T_0)]$ or $\alpha = \Delta R/(R_0 \Delta T)$, with $\Delta R = R - R_0 = 0.17\,\Omega$ and $\Delta T = T - T_0 = 15°\text{C}$. Then $\alpha = (0.17)/(25.00 \times 15) = 4.5 \times 10^{-4}\ \text{C}^{-1}$.

6. $R_{150°\text{C}} = 2.415 = R_0(1 + 150\alpha)\ R_{20°\text{C}} = 1.6424 = R_0(1 + 20\alpha)$
Solving these relations simultaneously, $\alpha = 3.9 \times 10^{-3}\ \text{C}^{-1}$ and $R_0 = 1.6236\ \Omega$.
From $R_0 = \rho_0(L/A)$

$$1.5236 = \frac{\rho_0(300)}{\pi(2\times10^{-3})^2/4}$$

or $$r_0 = 1.596\ 10^{-8}\ \text{Wm}$$

Then, $\rho_{20°\text{C}} = \rho_0(1 + 20\alpha) = (1.596 \times 10^{-8})[1 + (3.9 \times 10^{-3})(20)] = 1.720 \times 10^{-8}\ \Omega\text{m}$
This indicates that the material is copper.

7. We need $R_1(1 + \alpha_1 \Delta t) + R_2(1 + \alpha_2 \Delta t) = 20$. Because $R_1 + r_2 = 20$ when $\Delta t = 0$, we must have $R_1\alpha_1 = -R_2\alpha_2$ with $\alpha_1 = -0.5 \times 10^{-3}$ and $\alpha_2 = 5 \times 10^{-3}$. Solving the two equations $R_1 + R_2 = 20$ and $R_1 = 10R_2$ simultaneously leads to $R_1 = 18.18\ \Omega$ and $R_2 = 1.82\ \Omega$.

8. We assume that α is constant over the needed temperature range. Then $\Delta R = \alpha R \Delta t$ leads to $(35 - 10) = 0.0036(10)\ \Delta t$. Solving we get $\Delta t = 25/0.036 = 694°\text{C}$. Finally $694 + 20 = 714°\text{C}$ (furnace temperature).

9. When switch is made on, the electric field $\vec{E}$ responsible for setting current is transmitted through the wire from switch to bulb at speed of light $c = 3 \times 10^8$ m/s. So in time L/c, i.e., immediately after the field is set up in the circuit and electrons start drifting producing current and hence light. However, the electron at the switch will take time L/v_d, i.e., hours to reach the bulb if it reaches the bulb at all.

10. a. As by definition, current and current density are related to each other through the relation $J = I/S$, so for steady current (i.e., I = constant) but nonuniform cross section (i.e., S constant), J will not be constant but will vary with cross-sectional area S as

$$J \propto \frac{I}{S} \qquad [I \text{ being constant}]$$

Now, as J is related to electric field E and drift velocity v_d through the relations

$$J = sE \quad \text{and} \quad J = nev_d$$

$$E = \frac{1}{\sigma S} \quad \text{and} \quad v_d = \frac{1}{neS} \qquad \left[\text{as } J = \frac{I}{S}\right]$$

But as for a given metal s and n are constants,

so $$E = \frac{1}{S} \quad \text{and} \quad v_d \propto \frac{I}{S} \quad [\text{as } I = \text{constant}]$$

i.e., in case of steady current flow through a metallic conductor of non-uniform cross section, current is constant while current density, electric field and drift velocity are not constant and all vary inversely with area of cross section.

b. Yes, current flows in a conductor only when electric field established within the conductor exerts force on the electrons.

11. According to 'electron theory of metals' the drift speed of an electron inside a metal in presence of an electric field E is given by

$$v_d = \left(\frac{e\tau}{m}\right)E = \left(\frac{e\tau}{m}\right)\left(\frac{V}{L}\right) \qquad \left[\text{as } E = \frac{V}{L}\right]$$

a. As $v_d \propto V$, on doubling V, drift velocity will be doubled.
b. As $v_d \propto (1/L)$, on doubling L, drift velocity will be halved.
c. As drift velocity is independent of diameter d, it will not change on doubling the diameter.

12. a. As at a given temperature I-V curve is linear, i.e., $V \propto I$, the specimen is Ohmic.
b. As for I-V curve, reciprocal of the slope gives the resistance

i.e., $$R = \frac{V}{I} = \frac{1}{\tan\theta}$$

And as here $\theta_1 > \theta_2$
i.e., $\tan\theta_1 > \tan\theta_2$ so, $R_1 < R_2$, i.e., $R_2 > R_1$

c. In case of Ohmic conductors

$$R = R_0(1 + \alpha\Delta\theta)$$

i.e., resistance increases with temperature, i.e., higher the temperature higher will be the resistance and as here

$$R_2 > R_1,\ \therefore\ T_2 > T_1$$

13. In case of a resistance wire

$$R = \rho\frac{L}{S} = \rho\frac{L}{\pi r^2} \qquad [\text{as } S = \pi r^2]$$

And in changing dimensions without any change in mass, volume remains constant, i.e.,

$$V = V' \quad \text{or} \quad SL = S'L'$$

a. When the wire is doubled on itself (as shown in Fig. A3-5.1(b)) its length will be halved, i.e., $L' = (L/2)$ and as

$$S \times L = S \times (L/2), \quad \text{i.e., } S' = 2S$$

$$\therefore \qquad R' = \rho\left[\frac{L'}{S'}\right] = \rho\frac{(L/2)}{(2S)} = \frac{1}{4}\rho\frac{L}{S} = \frac{1}{4}R$$

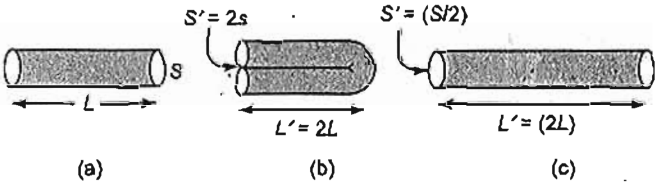


Fig. A3-5.1

b. i. When the length of the wire is doubled, i.e., $L' = 2L$ (as shown in Fig. A3-5.1(c))

$$SL = 2LS', \quad \text{i.e., } S' = S/2$$

$$\therefore \qquad R' = \rho\left[\frac{L'}{S'}\right] = \rho\frac{(2L)}{(S/2)} = 4\rho\left[\frac{L}{S}\right] = 4R$$

ii. When the radius of the wire is halved, i.e., $r' = r/2$

$$S' = \pi r'^2 = \pi(r/2)^2 = \pi r^2/4 = S/4$$

And hence from

$$SL = S'L', \quad \text{i.e.,} \quad SL = (S/4)L'$$

we have $L' = 4L$

$$R' = \rho\frac{L'}{S'} = \rho\frac{(4L)}{(S/4)} = 16\rho = 16R$$

14. Let dq be the charge which has passed in a small interval of time dt, then $dq = i\,dt = (4 + 2t)\,dt$.
Hence, total charge passed between t = 2 s and t = 6 s is

$$q = \int_2^6 (4 + 2t)\,dt = 48\text{ C}$$

15. **a.** $E_1 = \frac{V}{L},\ E_2 = \frac{2}{2L} = \frac{V}{L}$

$$E_3 = \frac{2V}{3L}$$

Hence $E_1 = E_2 = E_3$

c. $V_d \propto E$

Hence $V_{d_1} = V_{d_2} > V_{d_3}$

b. $I = neAv_d = \frac{ne\pi d^2}{4}v_d$

$$= \frac{ne\pi}{4}v_d d^2$$

$\Rightarrow$ $I \propto v_d d^2$

$\Rightarrow$ $I \propto Ed^2$

$\Rightarrow$ $I = KEd^2$

$$I_1 = \frac{KV}{L}(3d)^2 = \frac{9KVd^2}{L}$$

$$I_2 = K\frac{V}{L}d^2$$

$$I_3 = K\frac{2V}{3L}(2d)^2 = \frac{8KVd^2}{3L}$$

Hence $I_1 > I_3 > I_2$

16. We know $R = \frac{V}{I}$ and slope of I V curve $= \frac{I}{R}$

or $R = \frac{1}{\text{slope}}$

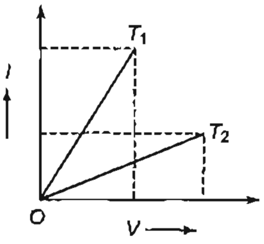


Fig. A3-5.2

Since the slope of OT_2 is smaller than OT_1, hence the resistance of wire at T_2 is greater than that at T_1 as resistance increases with temperature, so temperature T_2 is greater than T_1.

17. $R = \frac{\rho\ell}{A}$

$\therefore$ $\rho \propto R$

But $R_Y > R_X$

Because for same V, current in Y is less

So, $\rho_Y > \rho_X$

18. We know $I = neAv_d$ (i)

n = number of electrons per unit volume of copper wire

$$= \frac{N}{V} = \frac{N_A}{M}\frac{m}{V} = \frac{N_A\rho}{M} \quad \text{(ii)}$$

From (i) and (ii),

$$I = \left(\frac{N_A\rho}{M}\right)eAv_d$$

$$\Rightarrow \quad v_d = \frac{IM}{N_A\rho eA} = 4.9\times10^{-7}\text{ m/s}$$

19. **a.** As current $I = q/t$

Current density $\left(\frac{I}{A}\right) = |\bar{J}| = \frac{q}{At}$ (i)

Taking 'l', the length of beam, $v = \frac{\ell}{t}$ (ii)

Multiplying (i) with l/l, we get

$$|\bar{J}| = \frac{q\ell}{At\ell} = \frac{q}{(A\ell)}\left(\frac{\ell}{t}\right) = \left(\frac{q}{L}\right)v \quad \text{(iii)}$$

$$|\bar{J}| = \frac{2\times2\times10^8\times1.6\times10^{-19}}{10^{-6}}\times1\times10^5$$

$$= 6.4\text{ A/m}^2$$

In vector form, (iii) can be written as

$$\bar{J} = \left(\frac{q}{V}\right)\bar{v}$$

As velocity of positive ions is towards north, hence the direction of $\bar{J}$ will also be towards north.

b. No, we need area of cross section of the wire.

20. $dI = J2\pi r\,dr$

$$dI = J_0\left(1 - \frac{r}{R}\right)2\pi r\,dr$$

$$dI = 2\pi J_0\left(1 - \frac{r}{R}\right)r\,dr$$

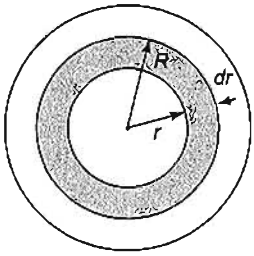


Fig. A3-5.3

$$I = \int dI = 2\pi J_0 \int_0^R \left(1-\frac{r}{R}\right) r\,dr$$

$$= 2\pi J_0 \left[\int_0^R r\,dr - \frac{1}{R}\int_0^R r^2 dr\right]$$

$$= 2\pi J_0 \left[\frac{R^2}{2} - \frac{R^3}{3}\right] = 2\pi J_0 \frac{R^2}{6}$$

$$= \frac{J_0 A}{3}$$

21. Let at distance x, the radius is r. Then,

$$r = a + \Delta r = a + \left(\frac{b-a}{\ell}\right)x$$

$$R = \int dR = \int \frac{\rho\,dx}{\pi r^2} = \int_0^\ell \frac{\rho\,dx}{\pi\left[a+\left(\frac{b-a}{\ell}\right)x\right]^2}$$

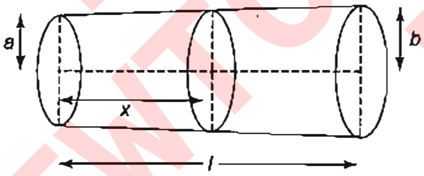


Fig. A3-5.4

Let $a+\left(\frac{b-a}{\ell}\right)x = z$

$$\left(\frac{b-a}{\ell}\right)dx = dz$$

If z goes from a to b,

$$R = \int dR = \int_a^b \frac{\rho\ell}{\pi(b-a)}\frac{dz}{z^2}$$

$$= \frac{\rho\ell}{\pi(b-a)}\int_a^b \frac{dz}{z^2}$$

$$= \frac{\rho\ell}{\pi(b-a)}\left[\frac{1}{a}-\frac{1}{b}\right]$$

$$= \frac{\rho\ell}{\pi ab}$$

$$= \frac{\rho\ell}{\pi ab}$$

22. $$\frac{R_f - R_i}{R_i}\times 100 = \frac{\rho\frac{l_f}{A_f} - \rho\frac{l_i}{A_i}}{\rho\frac{l_i}{A_i}} = \frac{\frac{l_f}{A_f}-\frac{l_i}{A_i}}{\frac{l_i}{A_i}} \quad \text{(i)}$$

Let the initial length of the wire be 100 cm, then the new length is $100+\frac{0.1}{100}\times 100$.

$$l_f = 100.1 \text{ cm} \quad \text{(ii)}$$

Let A_i and A_f be the initial and find area of cross section. Then $100 \times A_i = 100.1\ A_f$

$$\Rightarrow \quad A_f = \frac{100}{100.1}A_i \quad \text{(iii)}$$

From (i), (ii) and (iii)

$$\frac{R_f - R_i}{R_i}\times 100 = \frac{\frac{(100.1)^2}{100A_i} - \frac{100}{A_i}}{\frac{100}{A_i}}\times 100$$

$$= \frac{(100.1)^2 - (100)^2}{(100)2}\times 100$$

$$= \frac{200.1\times 0.1}{100\times 100}\times 100 = 0.2\%$$

23. $a = 0.5 \text{ mm}^2 = 0.5\times 10^{-6} \text{ m}^2$,
$l = 0.1$ m, $T_1 = 25°\text{C}$, $I = 10$ A, $T_2 = 1075$K

a. $I^2Rt = ms\Delta T \Rightarrow I^2\rho\frac{l}{a}\times t = ms\,\Delta T$

$$\Rightarrow \quad t = \frac{ms\Delta T\times a}{I^2\rho l} = \frac{(d\times a\times l)s\,\Delta T\times a}{I^2\rho l}$$

$$= \frac{da^2 s\Delta T}{I^2\rho}$$

$$= \frac{9\times 10^3\times(0.5\times 10^{-6})^2\times 9\times 10^{-2}\times 10^3\times 4.18\times 1050}{10\times 10\times .16\times 10^{-8}}$$

$= 555.5$ s $= 9$ min 16 s.

b. Since length does not occur in the expression of time, the melting does not depend on the length.

Exercise 5.2

1. $$E_{eq} = \frac{\frac{6}{3}+\frac{12}{5}}{\frac{1}{3}+\frac{1}{5}} = \frac{33}{4}\text{ V}$$

$$r_{eq} = \frac{1}{\frac{1}{3}+\frac{1}{5}} = \frac{15}{8}\ \Omega$$

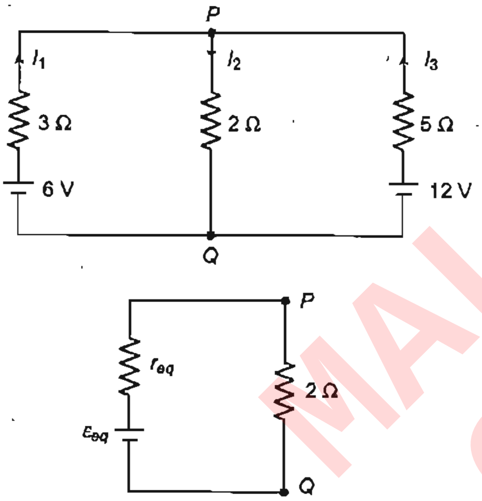


Fig. A3-5.5

Hence, $$I_2 = \frac{\frac{33}{4}}{2+\frac{15}{8}}$$

$$= \frac{66}{31}\text{ A}$$

Potential difference across PQ

$$V_{PQ} = I_2 R = \frac{66}{31}\times 2 = \frac{132}{31}\text{ V}$$

Current through 6 V battery:

$$\frac{132}{31} = 6 - 3I_1$$

$$\Rightarrow \qquad I_1 = \frac{18}{31}\text{ A}$$

Hence, $$I_3 = \frac{132}{31} - \frac{18}{31} = \frac{114}{31}\text{ A}$$

2. R_3 and R_4 are in series. The combined resistance is 400 Ω. Now, R_2 = 100 Ω and 400 Ω resistances are in parallel.

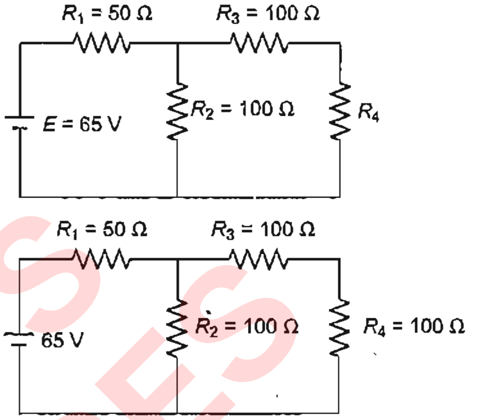


Fig. A3-5.6

The combined resistance is

(100 × 400)/500 = 80 Ω

Total resistance

$$R = 80 + 50 = 130\ \Omega$$

$$I = 65/130 = 1/2\text{ A}$$

So, $$V = IR_1 = 1/2 \times 50 = 25\text{ V}$$

3. Equivalent resistance R' of the parallel combination of 12 Ω, 6 Ω and 4 Ω resistances is given by

$$\frac{1}{R'} = \frac{1}{12}+\frac{1}{6}+\frac{1}{4} = \frac{1}{2} \quad \text{or} \quad R' = 2\ \Omega$$

Total resistance of the circuit is

$$R = 1 + 5 + 2 + 2 = 10\ \Omega$$

$$E = 20 - 8 = 12\text{ V}$$

a. Current in the circuit,

$$I = \frac{E}{R} = \frac{12}{10} = 1.2\text{ A}$$

b. Combined resistance of 6 Ω and 4 Ω in parallel

$$= \frac{6\times 4}{6+4} = 2.4\ \Omega$$

Current in resistor of 12 Ω coil

$$= \frac{2.4}{12+2.4}\times 1.2 = 0.2\text{ A}$$

c. p.d. across the 20 V battery

$$= (20 - 1.2 \times 1) = 18.8\text{ V}$$

p.d. across the 8 V battery

$$= (8 + 1.2 \times 2) = 10.4\text{ V}$$

4. **a.** When no current is drawn from a cell, potential difference across the terminals of the cell is equal to its e.m.f. From the graph it is clear that e.m.f. = 1.4 V

b. Further, $V = \varepsilon - Ir$

$$\Rightarrow \qquad r = \frac{\varepsilon - V}{I}$$

Here, $\varepsilon = 1.4$ V. Consider any given value of potential difference from graph, say $V = 1.2$ V.

Current corresponding to this p.d. is = 0.04 A

$$\therefore \quad r = \frac{1.4 - 1.2}{0.04} = 5\ \Omega$$

5. The figure drawn shows the directions of currents.

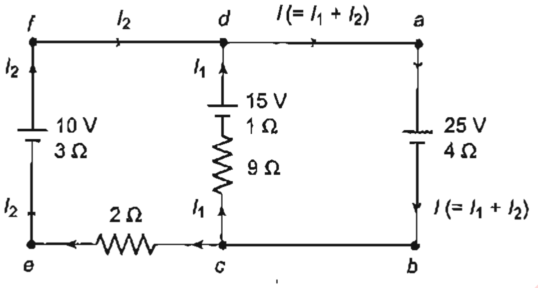


Fig. A3-5.7

Applying Kirchhoff's second law to the mesh *abcda*, we get

$$-4(I_1 + I_2) + 25 - 9I_1 - 1 \times I_1 + 15 = 0$$

$$\text{or} \quad 7I_1 + 2I_2 = 20$$

Applying Kirchhoff's second law to the mesh *abcefa*,

$$25 - 4(I_1 + I_2) - 2I_2 - 3I_2 + 10 = 0$$

$$\text{or} \quad 4I_1 + 9I_2 = 35$$

6. Since the galvanometer shows no deflection therefore Wheatstone bridge is balanced. Applying condition for balanced Wheatstone bridge,

$$\frac{100(100+R)}{100R} = \frac{200}{40} = 5$$

$$\text{or} \quad 5R = 100 + R \quad \text{or} \quad 4R = 100$$

$$\text{or} \quad R = 25\ \Omega$$

7. $0.25 = \dfrac{10E}{59+10r}$ and $25 = \dfrac{E}{0.05 + \dfrac{r}{10}}$

8. $nm = ?,\ 30 = \dfrac{n \times 1}{m}$ or $n = 30$ m

$$1 = \frac{nE}{2 \times 30} \quad \text{or} \quad \frac{3}{2} = \frac{n \times 15}{60}$$

$$n = \frac{90}{1.5} = 60 \quad \text{or} \quad m = \frac{n}{30} = \frac{60}{30} = 2$$

9. If V is the applied potential difference and R is the original resistance, then

$$\frac{V}{R} = 5 \quad \text{and} \quad \frac{V}{R+2} = 4$$

$$\text{Dividing,} \quad \frac{R+2}{R} = \frac{5}{4} \quad \text{or} \quad R = 8\ \Omega$$

10. An equivalent of the given network is shown in Fig. A3-5.8. It is clear from the network of the figure that the resistances of 2 Ω, 4 Ω and 2 Ω are connected in parallel with each other. If R_p is the total resistance, then

$$\frac{1}{R_p} = \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{2}$$

$$\text{or} \quad \frac{1}{R_p} = \frac{2+1+1+2}{4} = \frac{3}{2} \quad \text{or} \quad R_p = \frac{2}{3}\Omega$$

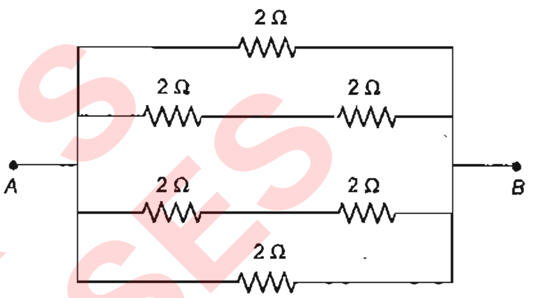


Fig. A3-5.8

11. Net e.m.f. = 8 – 4 = 4 V

$$R_{AB} = 6 \times 3/9 = 2\ \Omega$$

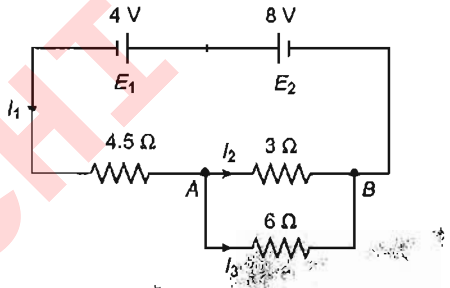


Fig. A3-5.9

Total resistance of the circuit = 8 Ω

$$\text{So,} \quad I_1 = 4/8 = 0.5\ \text{A}$$

$$V_{AB} = 0.5 \times 2 = 1\ \text{V}$$

$$\text{So,} \quad I_2 = 1/3\ \text{A and } I_3 = 1/6\ \text{A}$$

12. Traversing the upper and lower loops anticlockwise, we get

$$2I_1 + 6I_2 = 24 - 27$$

$$\text{or} \quad 2I_1 + 6I_2 = -3 \qquad \text{(i)}$$

$$\text{and} \quad 4I_3 - 6I_2 = 27$$

$$\text{or} \quad 4(I_1 - I_2) - 6I_2 = 27$$

$$\text{or} \quad 4I_1 - 10I_2 = 27 \qquad \text{(ii)}$$

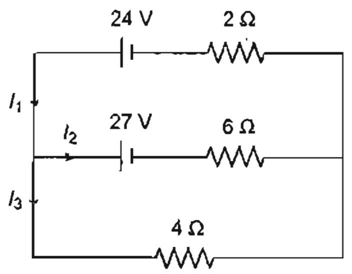


Fig. A3-5.10

Solving equations (i) and (ii), we get

$$I_1 = 3\ \text{A},\ I_2 = -1.5\ \text{A}$$

$$I_3 = I_1 - I_2 = 3 + 1.5 = 4.5 \text{ A}$$

13. The given circuit can be redrawn as follows:

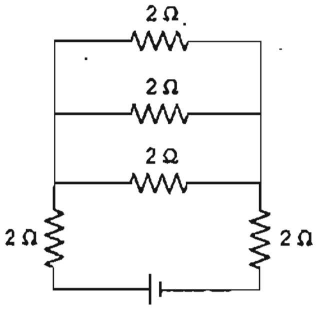


Fig. A3-5.11

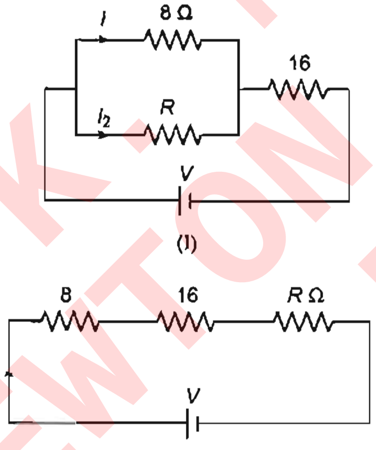
$$\frac{1}{R'} = \frac{1}{2}+\frac{1}{2}+\frac{1}{2}$$

$$\Rightarrow \quad R' = \frac{2}{3}\Omega$$

$$\therefore \quad R_{\text{Total}} = 2+\frac{2}{3}+2 = \frac{14}{3}\Omega$$

Hence, $$i = \frac{V}{R} = \frac{14\times3}{14} = 3 \text{ A}$$

14.



(I)

(II)

Fig. A3-5.12

In (I), $$V_1 = V_2$$

$$i\times 8 = R\times i_2$$

$$i_2 = \frac{8i}{R}$$

$$I = i + I_2 = i + \frac{8i}{R}$$

$$V = I(R_{\text{Total}})$$

$$V = \left(i+\frac{8i}{R}\right)\left(\frac{8R}{R+8}+16\right)$$

$$V = i\left(\frac{8+R}{R}\right)\left(\frac{8R}{R+8}+16\right) \quad \text{(i)}$$

In (II),

$$V = i\,(24 + R) \quad \text{(ii)}$$

Now, from (i) and (ii), we get

$$\frac{24R+128}{R} = 24 + R$$

$$\Rightarrow \quad R = \sqrt{128}\ \Omega$$

15. $V_{ab} = I_1\left(6+\dfrac{4\times6}{4+6}\right)$

$$V_{ab} = I_1(8.4) \quad \text{(i)}$$

$$V_{ab} = I_2\left(4+\frac{8\times8}{8+8}\right)$$

$$V_{ab} = I_2\,(8) \quad \text{(ii)}$$

From (i),

a. $I_1 = \dfrac{12}{8.4} = \dfrac{10}{7}$ A

I_1 is the current in the 6 Ω resistor.

b. $I_{2,4} = \dfrac{4}{6+4}\times\dfrac{10}{7} = \dfrac{4}{7}$ A

c. From (ii), $I_2 = \dfrac{3}{2}$ A

Current in the 4 Ω resistor is $\frac{3}{2}$ A.

It will equally divide into the 8 Ω and 8 Ω resistors which are in parallel.

$$\therefore \quad I_8 = \frac{3}{2\times2} = \frac{3}{4} \text{ A}$$

16.

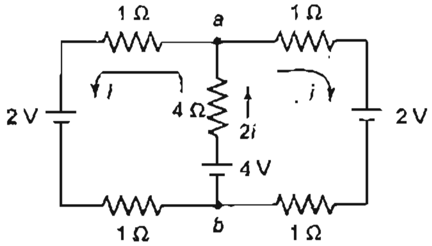


Fig. A3-5.13

Applying Kirchhoff's law in any branch,

$$4 - 4\,(2x) - x - 2 - x = 0$$

$$\Rightarrow \quad x = 0.4 \text{ A}$$

$$V_a - V_b = 4 - (4.4) = 2.4 \text{ V}$$

17. Applying Kirchhoff's current law,

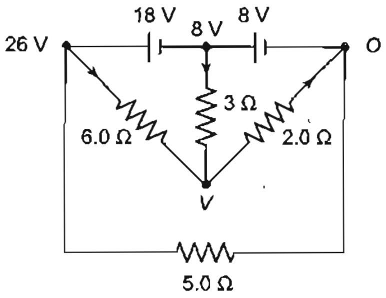


Fig. A3-5.14

At node V,

$$\frac{26-V}{6}+\frac{8-V}{3}=\frac{V-0}{2}$$

$$\frac{26}{6}+\frac{8}{3}=\frac{V}{2}+\frac{V}{3}+\frac{V}{6}$$

$$\frac{26+16}{6}=\frac{3V+2V+V}{6}$$

$$V=7\text{ V}$$

a. Current through the 6 Ω resistor is $I_6=\dfrac{26-7}{6}=\dfrac{19}{6}$ A

b. Current through the 3 Ω resistor is $I_3=\dfrac{8-7}{3}=\dfrac{1}{3}$ A

c. Current through the 2 Ω resistor is $I_2=\dfrac{7}{2}$ A

d. Current through the 5 Ω resistor is $I_5=\dfrac{26}{5}$ A

18.

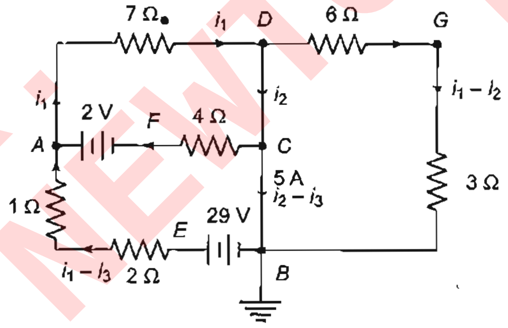


Fig. A3-5.15

Given $\qquad i_2-i_3=5 \qquad$ (A)

In ADC, $-27+3i_1-7i_3=0 \qquad$ (i)

In $FAEBC$,

$2+3(i_1-i_3)-29-4i_3=0$

$-27+3i_1-7i_3=0 \qquad$ (ii)

$(i)\times 3+(ii)\times 7 \Rightarrow i_3=-3$ A

Puting in (i), $\qquad i_1=2$ A

Puting in (A), $\qquad i_2=2$A

In ACB, $\qquad V_A-V_B=-2+(-3\times 4)$

$V_A-0=-14$ V

a. $|V_A|=14$ V

$$V_A-V_D=7\times 2=14$$

b. $V_D=14-14=0$

$$V_D-V_G=6\times(i_1-i_2)=6(2-2)$$

$$V_G=0+V_D$$

c. $V_G=0$ V

19. **a.** At $t=0$ capacitor acts as short-circuit.

There will not be any current in the 40 Ω resistance.

$$I_0=\frac{10}{20}=0.5\text{ A}$$

b. Capacitor acts as an open circuit,

$$I=\frac{10}{60}=\frac{1}{6}\text{A}$$

c. Voltage across capacitor

$$V_{40}=I\times R=\frac{1}{6}\times 40$$

$$Q=CV=0.50\times\frac{1}{6}\times 40=\frac{10}{3}\mu\text{C}$$

d. $q=q_0(e^{-t/RC})$

Capacitor will discharge through the 40 Ω resistor when S is open

$$\frac{20}{100}q_0=q_0\left(e^{-\frac{t}{40\times 0.5\times 10^{-6}}}\right)$$

$$\Rightarrow \qquad \frac{1}{5}=\frac{1}{e^{\frac{t}{20\times 10^{-6}}}}$$

or $\ln(5)\times 20\times 10^{-6}=t$

20. Here internal resistance is given by the slope of graph, i.e., $\dfrac{x}{y}$

but conductance $=\dfrac{1}{\text{resistance}}=\dfrac{y}{x}$

21. In parallel combination, each resistor has same potential difference, i.e., 200 V. The circuit therefore reduces to

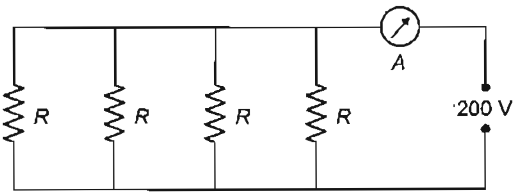


Fig. A3-5.16

$$\therefore \qquad R_{eq}=\frac{2000}{4}=500\ \Omega;\ I=\frac{200}{500}=0.4\text{ A}$$

22. From the circuit given in question, current through

$$R_2=i-\frac{i}{10}=\frac{9i}{10}$$

Potential difference across R_2 = potential difference across R

$$\therefore \qquad R_2 \times \frac{9}{10} I = R \times \frac{i}{10}$$

i.e.,
$$R_2 = \frac{R}{9} = \frac{11}{9}\,\Omega$$

$$\therefore \qquad R_{eq} = \frac{R_2 \times R}{R_2 + R} = \frac{\frac{11}{9} \times \frac{11}{1}}{\frac{11}{9} + \frac{11}{1}} = \frac{11}{10}\,\Omega$$

$$\text{Total circuit resistance} = \frac{11}{10} + R_1 = R = 11$$

$$\therefore \qquad R_1 = 11 - \frac{11}{10} = 9.9\,\Omega$$

23. Loop $AFMN$: $20 = 5I_1 + 12(I_1 - I_2)$

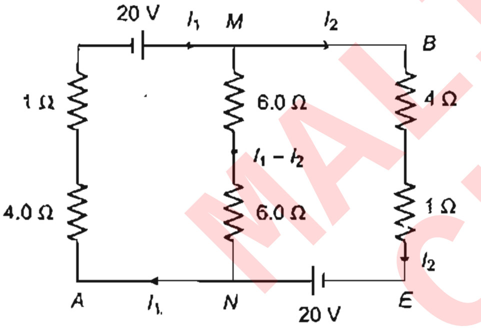


Fig. A3-5.17

$$20 = 17I_1 - 12I_2 \qquad \text{(i)}$$

Loop $AFBEA$: $20 + 20 = 5I_1 + 5I_2$

$$8 = I_1 + I_2 \qquad \text{(ii)}$$

Solving, we get $I_1 = 4$ A, $I_2 = 4$ A

So current through $MN = I_1 - I_2 = 4 - 4 = 0$

And current through each of the batteries is 4 A.

24. In the given circuit A, B, C and D are at same potential by symmetry.

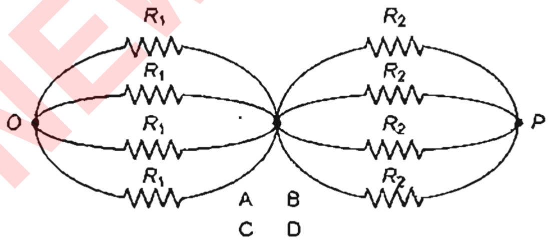


Fig. A3-5.18

$$R_{eq} = \frac{R_1 + R_2}{4},\ R_1 = ar;\ R_2 = \frac{\pi a}{2} r$$

$$R_{eq} = \frac{ar}{4}\left(1 + \frac{\pi}{2}\right) = \frac{ar}{8}(2 + \pi)$$

25. The current will flow from the positive terminal to the negative terminal in the battery. During charging the potential difference

$$= V + Ir = 2 + 5 \times 0.1 = 2.5 \text{ V}$$

26. Battery should be connected across A and B. Output can be taken across the terminals A and C or B and C

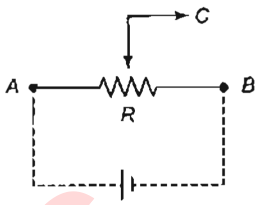


Fig. A3-5.19

Exercise 5.3

1. a. At $t = 0$, C will behave as a short-circuit, so no current passes through R_2. And $I = I_1 = e/R_1$.

   b. At $t = \infty$, C will block the current. So $I_1 = 0$.

And
$$I = I_2 = \frac{\varepsilon}{R_1 + R_2}$$

2. a. In each case, in steady state (equilibrium), potential difference across capacitor will be E. So charge in each case will be finally $q = CE$.

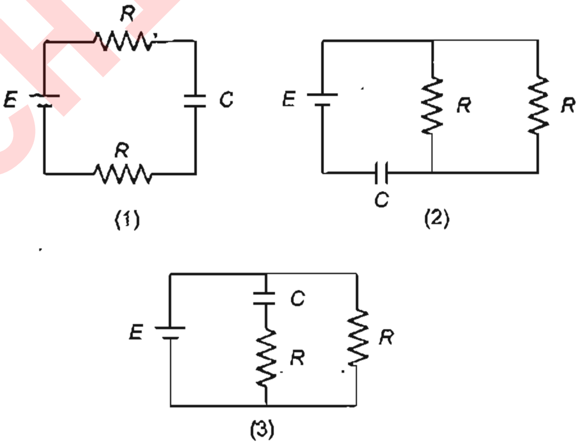


Fig. A3-5.20

   b. $\tau_1 = 2RC$, $\tau_1 = \frac{R}{2}C$, $\tau_3 = RC$

   For (1), time constant is the largest so it will take maximum time. For (2), it is the smallest. So time taken will be in the order; (1) > (3) > (2).

3. a. Immediately after the switch is closed, C_1 will act as a simple wire due to which R_2 and R_3 will be short-circuited.

So,
$$I = \frac{E}{R_1}$$

   b. After a long time, C_1 and C_2 will block the current. Current will pass through only R_1 and R_3.

$$I = \frac{E}{R_1 + R_3}$$

4. For $t = t_0$ to $t = 2t_0$,

$$V = at - b$$

Charge on capacitor $q = CV$

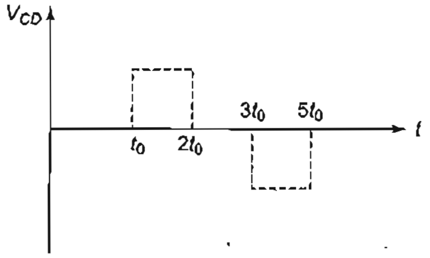


Fig. A3-5.21

$$q = C[at + b]$$

Current, $\quad I = \dfrac{dq}{dt} = Ca \rightarrow$ constant

Potential across $CD = IR = CaR \rightarrow$ constant

From $2t_0$ to $3t_0$, V is constant, so no current flows through the capacitor or resistor. Hence V_{CD} is zero.

5. **a.** S_1 is closed, S_2 is open.

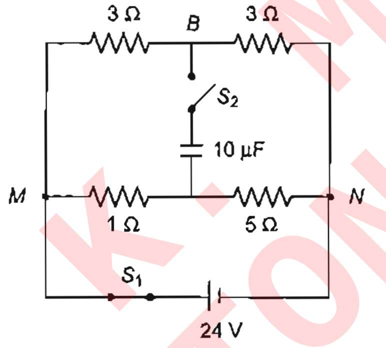


Fig. A3-5.22

$$V_M - V_A = \frac{1\times 24}{1+5} = 4\text{ V}$$

$$V_M - V_B = \frac{3\times 24}{3+3} = 12\text{ V}$$

$$V_A - V_B = 8\text{ V}$$

b. (i) Just after closing S_2, A and B will come to the same potential, so $V_A - V_B = 0$.

(ii) After a long time, no current will flow through AB, so $V_A - V_B = 8$ V

6. $$I = \frac{E}{R_1 + R_2} = \frac{6}{10+20} = \frac{1}{5}\text{A}$$

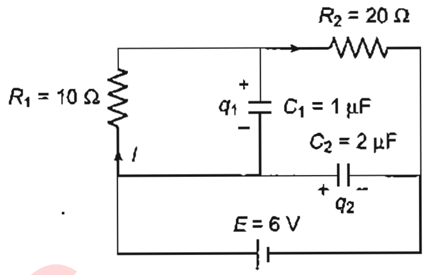


Fig. A3-5.23

$$q_1 = C_1V_1 = C_1I\,R_1 = 1\times\frac{1}{5}\times 10 = 2\ \mu\text{F}$$

$$q_2 = C_2V_2 = 2\times 6 = 12\ \mu\text{F}$$

7. $q_1 = 400\ \mu$C

$R = 1000\ \Omega$, $t = RC = 500\times 10^{-6}\times 1000$

$= 5\times 10^{-2}$

$$q = q_0e^{-t/\tau} = 400\,e^{-\frac{1}{5\times10^{-2}}}$$

$$= \frac{400}{e^{+20}}\ \mu\text{C}$$

8. $$q = \frac{q_0}{3}$$

$$q = q_0\,e^{-t/\tau}$$

$$\frac{q_0}{3} = q_0\,e^{-t/\tau}$$

$$t = \tau\,\ell n_3 = RC\,\ell n_3$$

$$R = \frac{t}{C\,\ell n_3} = \frac{4.4}{2\,\ell n_3} = \frac{2.2}{\ell n_3}$$

9. $$q = q_0(1 - e^{-t/\tau})$$

Put $\quad t = \tau,\ \dfrac{q}{q_0} = 1 - e^{-1}$

$$\therefore \frac{q}{q_0}\times 100 = \frac{e-1}{e}\times 100 = 63.2\%$$

10. $$\frac{q^2}{2C} = \frac{1}{2}\frac{q_0^2}{2C} \Rightarrow q = \frac{q_0}{\sqrt{2}}$$

So, $\quad \dfrac{q_0}{\sqrt{2}} = q_0[1 - e^{-t/\tau}] \Rightarrow t = 1.23\tau$

CHAPTER 6

Exercise 6.1

1. In series I is same for all elements, so if an ammeter is connected in series with a resistance R, its reading will be equal to the current through R (as shown in Fig. A3-6.1).

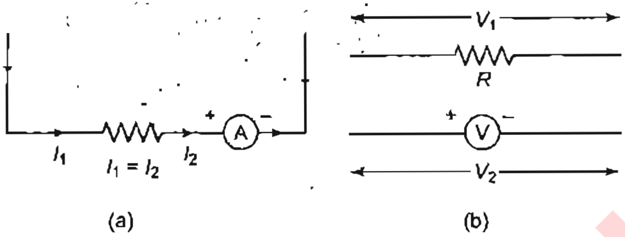


Fig. A3-6.1

Similarly, in parallel V is same for all elements, so if a voltmeter is connected in parallel with a resistance R, potential difference across it will be same as that across R. This is why an ammeter is always connected in series while a voltmeter is connected in parallel with the element to measure I (through it) and V (across it).

2. By presence of a voltmeter in series with resistance R in a circuit, due to voltmeter's high resistance, the current in the circuit I ($\approx V/R$) will decrease appreciably to I' ($\approx V/R$) and as this I' will further divide in resistance R and ammeter, the current through ammeter I_2 will be even lesser than I.

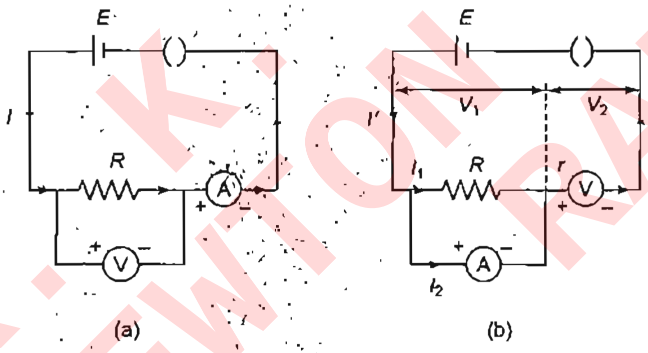


Fig. A3-6.2

3. As shown in Fig. A3-6.3 the voltmeter is in series with R, so $V = V_1 + V_R$, i.e., $110 = 5 + V_R$

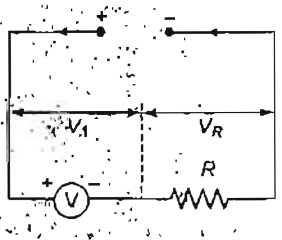


Fig. A3-6.3

i.e., $V_R = 110 - 5 = 105$ V

And as in series potential divides in proportion to resistance,

$$\frac{V_R}{V_1} = \frac{R}{R_1}, \text{ i.e., } \frac{105}{5} = \frac{R}{20\ \text{k}\Omega} \text{ or } R = 420$$

kΩ

4. Let the length of the potentiometer wire is ℓ, and V is the potential difference across it, then potential gradient $K = \frac{V}{\ell}$.

Let balance point is obtained at l_1, then measured potential difference $= K\ell_1 = \frac{V\ell_1}{\ell}$

Let us make error Δl in measuring l_1, then percentage error

$$= \frac{V\ell}{\ell_1} \times 100.$$

If l is increased, then l_1 also increases and hence percentage error will decrease. So accuracy increases.

5. For balanced Wheatstone's bridge $\frac{X}{40} = \frac{3}{60}$

$$X = 2\Omega$$

Resistance of wire $= 1 \times 100 = 100\ \Omega$

Current drawn from battery,

$$I = \frac{6}{100} + \frac{6}{2+3} = 1.26 \text{ A}$$

6. A potentiometer is said to be sensitive if fall of potential per unit length, i.e, potential gradient (dV/dl) is small. The slope of V-I graph gives potential gradient, which is smaller for potentiometer with Y than for potentiometer wire X. Therefore, potentiometer Y will be preferred for comparing e.m.f.s of the two cells.

7. In this case, $\frac{R}{X} = \frac{\ell_1}{\ell_2}$

$$X = R\frac{\ell_2}{\ell_1} = \frac{10 \times 68.5}{58.3} = 11.75\ \Omega$$

In case of failure, a high resistance is put in series with the cell E. This will reduce current through R and X and potential difference across them will be reduced to values lower than the potential difference across wire AB.

8.

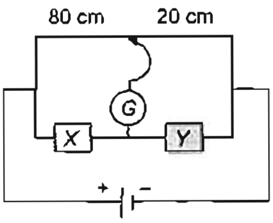


Fig. A3-6.4

From first null point

$$\frac{X}{Y} = \frac{80}{20} \qquad \text{(i)}$$

From second null point

$$\frac{\left(\dfrac{100X}{100+X}\right)}{Y} = \frac{60}{40} \qquad \text{(ii)}$$

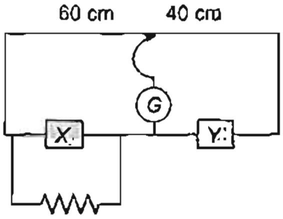


Fig. A3-6.5

From equations (i) and (ii), we get

$$X = \frac{500}{3}\ \Omega \text{ and } Y = \frac{125}{3}\ \Omega$$

9. Equivalent resistance of the ammeter

$$= \frac{(480\ \Omega)(20\ \Omega)}{480\ \Omega + 20\ \Omega} = 9.2\ \Omega$$

The equivalent resistance of the circuit is 140.8 Ω + 19.2 Ω = 160 Ω.

$$\therefore \qquad \text{Current } i = \frac{20\text{ V}}{160\ \Omega} = 0.125\text{ A}$$

This current goes through the ammeter and hence the reading of the ammeter is 0.125 A.

10. Given, $G = 12.0\ \Omega$,

$$I_g = 2.5\text{ mA} = 2.5 \times 10^{-3}\text{ A}$$

i. $I = 7.5$ A, $S = ?$

$$S = G \times \frac{I_g}{I - I_g}$$

$$= 12 \times \frac{2.5 \times 10^{-3}}{7.5 - 2.3 \times 10^{-3}}\ \Omega = 4.0 \times 10^{-3}\ \Omega$$

Resistance of ammeter is given by

$$\frac{1}{R_a} = \frac{1}{G} + \frac{1}{S} = \frac{S+G}{SG}$$

$$\text{or} \qquad R_a = \frac{SG}{S+G} = \frac{4.0 \times 10^{-3} \times 12.0}{4.0 \times 10^{-3} + 12.0}\ \Omega$$

$$= 4.0 \times 10^{-3}\ \Omega$$

ii. $V = 10.0$ V, $R = ?$

$$\therefore \qquad R = \frac{V}{I_g} - G = \frac{10.0}{2.5 \times 10^{-3}} - 12.0$$

$$= 3988\ \Omega$$

Resistance of voltmeter,

$$R_V = R + G = + 12 = 4000\ \Omega$$

11. We have $I_b = 1.00\text{ mA} = 1.00 \times 10^{-3}$ A and $R_A = 20.0\ \Omega$, and the ammeter should be able to handle maximum current, $I = 50.0 \times 10^{-3}$ A.

Solving equation $I_g R_g = (i - I_g) R_s$ for R_s, we get

$$R_s = \frac{I_g R_g}{I - I_g} = \frac{(1.00 \times 10^{-3}\text{ A})(20.0\ \Omega)}{50.0 \times 10^{-3}\text{ A} - 1.00 \times 10^{-3}\text{ A}}$$

$$= 0.408\ \Omega$$

The equivalent resistance R_{eq} of the instrument is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_G} + \frac{1}{R_S} = \frac{1}{20.0\ \Omega} + \frac{1}{0.408\ \Omega}$$

$$R_{eq} = 0.400\ \Omega$$

The shunt resistance is so small in comparison to the meter resistance that the equivalent resistance is very nearly equal to the shunt resistance. This shunt resistance gives us a low resistance instrument with the desired range of 0 to 50.0 mA. At full scale deflection, $I = 50.0$ mA, the current through the galvanometer is 1.00 mA, the current through the shunt resistor is 49.0 mA and $V_{ab} = 0.0200$ V. If the current I is less than 50.0 mA, the coil current and the deflection are proportionally less, but the resistance R_{eq} is still 0.400 Ω.

12. Solving equation for R_S, we have

$$R_s = \frac{V_V}{I_g} - R_G = \frac{10.0\,V}{0.00100\,A} - 20.0\ \Omega$$

$$= 9980\ \Omega$$

At full scale deflection, $V_{ab} = 10.0$ V, the voltage across the meter is 0.0200 V, the voltage across R_S is 9.98 V and the current through the voltmeter is 0.00100 A. In this case most of the voltage appears across the series resistor. The equivalent meter resistance is $R_{eq} = 20.0 + 9980 = 10000\ \Omega$. Such a meter is described as a "1000 ohms per volt meter", referring to the ratio of resistance to full scale deflection. So the voltmeter draws off only a small fraction of the current and disturbs only slightly the circuit being measured.

13. Internal resistance of the cell is given by $r = R\left(\dfrac{\ell_1 - \ell_2}{\ell_2}\right)$

Here, $\qquad R = 1\ \Omega,\ \ell_1 = 60\text{ cm},\ \ell_2 = 30\text{ cm}$

Hence, $\qquad r = 1 \times \dfrac{60 - 30}{30}\ \Omega = 1\ \Omega$

14. $r = \dfrac{\ell_1 - \ell_2}{\ell_2} \times R = \dfrac{800 - 400}{400} \times 5 = 5\ \Omega$,

15. Potential gradient, $X = \dfrac{ER_P}{(R + R_P + r)L}$

(where R_P is resistance of the wire)

$$\Rightarrow \qquad 50 \times 10^{-3} = \frac{2.5 \times 30}{(R + 30 + 5) \times 10}$$

$$\Rightarrow \qquad R = 115\ \Omega$$

16. $X = \frac{E_B}{L} = \frac{1.1}{440} = 0.0025$ V/cm

Potential difference across R is

$$0.0025 \times 220 = 0.550 \text{ V}$$

Error in the reading of voltmeter

= reading of voltmeter -- reading of potentiometer

$$= 0.5 - 0.55 = -0.05 \text{ V}$$

17. We should connect this galvanometer in series with the given resistor. In general resistance of galvanometer is small, so it can be neglected. If R is less, I is high, so

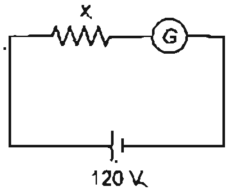


Fig. A3-6.6

$$(i_g)_{\max} = 40 \times 10^{-6} \text{ A}, R_{\min} = \frac{120}{40 \times 10^{-6}} = 3 \text{ M}\Omega$$

18. $r = R\left[\frac{E}{V} - 1\right] = 5\left[\frac{0.52}{0.4} - 1\right] = 5[1.3 - 1] = 1.5\ \Omega$

19. According to Ohm's law, the current flowing in a metallic wire is directly proportional to the potential difference applied across the ends of the wire provided other physical conditions like temperature, strain, etc. remain constant.
For the circuit of Fig. A3-6.7:
Let the resistance of the voltmeter be R_V and the current flowing through it be I_V.

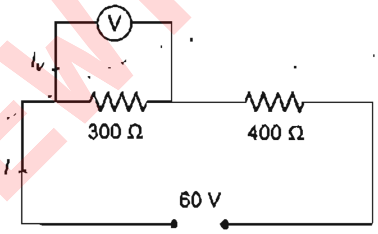


Fig. A3-6.7

Then $\quad I_V R_V = 30$ and $(I - IV) \times 400 = 30$

Also, $\quad 300 \times I = 30 \quad \Rightarrow I = 0.1$ A, $\therefore\ 0.1 - I_V = \frac{3}{40}$

$\Rightarrow \quad I_V = 0.1 - \frac{3}{40} \quad \Rightarrow I_V = \frac{4-3}{40} = \frac{1}{40}$ A

$\therefore \quad \frac{1}{40} \times R_V = 30 \Rightarrow R_V = 1200\ \Omega$

For the circuit of Fig. A3-6.8:

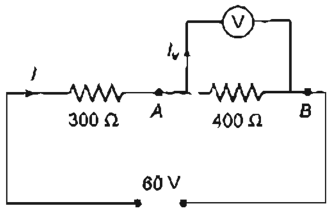


Fig. A3-6.8

$$I_V \times R_V = (I - I_V)\, 300$$

$$I_V \times 1200 = (I - I_V)\, 300$$

$$4I_V = I - I_V \quad \Rightarrow I = 5I_V \qquad (1)$$

Also, $300(I - I_V) + 400\, I = 60 \quad \Rightarrow 700\, I - 300\, I_V = 60$

$$3500\, I_V - 300\, I_V = 60 \quad \Rightarrow I_V = \frac{60}{3200} = \frac{3}{160} \text{ A}$$

$\therefore$ Reading of voltmeter $= \frac{3}{160} \times 1200 = 22.5$ V

20. For the experimental verification of Ohm's law, ammeter and voltmeter should be connected as shown in Fig. A3-6.9.

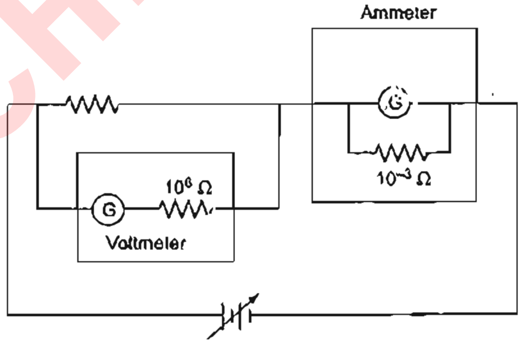


Fig. A3-6.9

CHAPTER 7

Exercise 7.1

1. a. Just after it is turned on, the energy supplied to the bulb is higher. Because initially resistance is low and it increases with increase in temperature.
   b. After connecting b and c, R_2 will be short-circuited. Hence it will not glow. Now, entire potential difference will be on R_1. So brightness of R_1 increases.
   c. D is short-circuited, so D will not glow. Potential difference across C is V. Potential difference across the series combination of A and B is V, so potential difference across each of A and B is $V/2$. So, brightness of C is the greatest and that of A and B are equal.
   If A fails: B will not glow. No effect on C and D.
   If C fails: C will not glow. No effect on A, B and D.
   If D fails: No effect anywhere.

d. Because resistance is directly proportional to length. At low current and high voltage energy loss will be less. Because if current is less, then loss of energy = I^2R will be less.

e. Initially, current will be high in the circuit, so bulb will glow with maximum possible brightness. But as the time increases, current will decrease and brightness of the bulb falls and finally the bulb goes out.

f. This is not true. Because in series, current through all the bulbs is same. The current is simultaneously set up in series through all the bulbs.

g. $P = \frac{V^2}{R} = \frac{V^2 A}{P\ell}$

Here, P, V and ℓ are fixed. We can vary A or ℓ.

2. Resistance of a bulb is given by $R = \frac{V^2}{P}$. Now, 100 W bulb will have less resistance. So we are connecting less resistance in series with the heater. It means potential difference across the heater will increase. So the heater will give more heat.

3. $P_{\max} = (i_{\max})^2 R \Rightarrow 18 = i_{\max}^2 \times 2 \Rightarrow i_{\max} = 3$ A

Here $R_{eq} = 3\ \Omega$.

So maximum power of circuit $= i_{\max}^2\ R_{eq} = 27$ W

4. We know that $R = \frac{V^2}{P}$. Therefore, resistance of the first bulb is $R_1 = \frac{V^2}{P_1}$.

And resistance of the second bulb is $R_2 = \frac{V^2}{P_2}$.

In series same current will pass through each bulb.

$\therefore$ Power developed across the first bulb is $P_1' = I^2 \frac{V^2}{P_1}$

and that across the second bulb is $P_2' = I^2 \frac{V^2}{P_1}$.

$$\frac{\text{Power in bulb 1}}{\text{Power in bulb 2}} = \frac{P_2}{P_1} \Rightarrow \frac{\text{Power in bulb 1}}{\text{Power in bulb 2}} < 1 \text{ as } P_2 < P_1$$

Hence, the bulb rated 220 V and 40 W will glow more.

5. Here, $P = 30$ W, $V = 6$ V

$\therefore$ Resistance of the bulb, $R_1 = \frac{V^2}{P} = \frac{(6)^2}{30} = 1.2\ \Omega$

Current capacity of the bulb, $I = \frac{P}{V} = \frac{30}{6} = 5$ A

Supply voltage, $V' = 120$ V

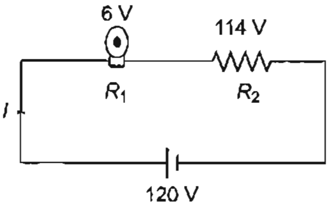


Fig. A3-7.1

Let R_2 be the resistance used in series with the bulb to have a current of 5 A in the circuit.

Total resistance $= R_2 + R_1 = R_2 + 1.2$

$\therefore$ Current $I = V'/(R_2 + 1.2)$

or $5 = \frac{120}{R' + 1.2}$ or $R' = \frac{120}{5} - 1.2 = 22.8\ \Omega$ in series

Alternatively:

Potential difference across the bulb should not exceed 6 V.

For this: $\frac{6}{114} = \frac{R_1}{R_2} \Rightarrow R_2 = \frac{114}{6} R_1 = \frac{114}{6} \times 1.2 = 22.8\ \Omega$

6. a. According to Joule heating, the rate at which heat is produced in a resistance is given by $P = I^2 R = \frac{V^2}{R}$. And in parallel V is same, i.e., $V_1 = V_2$.

So, $\frac{P_2}{P_1} = \frac{R_1}{R_2} = \frac{L_1}{L_2}\left[\frac{r_2}{r_1}\right]^2$ $\left[\text{as } R = \rho \frac{L}{\pi r^2}\right]$

$\frac{P_2}{P_1} = \frac{L_1}{2L_1}\left[\frac{2r_1}{r_1}\right]^2$ [as $L_2 = 2L_1$ and $r_2 = 2r_1$]

i.e., $P_2 = 2P_1$

i.e., heat produced in the thicker coil is more (i.e., double) than that produced in the other coil.

b. For three equal resistances each of value R,

$$R_S = 3R;\ P_S = \frac{V^2}{3R}$$

And when the given resistances are connected in parallel:

$$\frac{1}{R_P} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R}$$

i.e., $R_p = \frac{R}{3}$; $P_p = \frac{V^2}{(R/3)} = \frac{3V^2}{R}$

So, $\frac{P_P}{P_S} = \frac{3V^2}{R} \times \frac{3R}{V^2} = 9$, i.e., $P_P = 9P_S$, i.e., power dissipated in parallel is 9 times that in series.

7. As the three bulbs are in series and identical, initially when switch S is open, $V_A = V_B = V_C = \frac{1}{3}$V

Also $P_A = P_B = P_C = \frac{(V/3)^2}{R} = \frac{V^2}{9R} = P$

Now, the switch S is closed,

a. The bulb C is short-circuited and hence potential difference across it $V_C' = 0$ and so $V_A' = V_B' = V/2$ with $V_C' = 0$. And

hence, $P_A' + P_B' = \frac{(V/2)^2}{R} = \frac{V^2}{4R}$

i.e., $P_A' = P_B' = \frac{9}{4}P$, i.e., intensities of bulbs A and B will increase and become 2.25 of their initial values.

b. As $V'_C = 0$, no current will pass through bulb C, so it will give no light, i.e., $P'_C = 0$.

c. As R_T changes from $3R$ to $2R$, so the current in the circuit will change from

$$I = \frac{V}{3R} \text{ to } I' = \frac{V}{2R}, \quad \text{i.e.,} \quad I' = \frac{3}{2}I$$

i.e., current in the circuit will increase and will become 1.5 times its initial value.

d. As explained above, initially the voltage V divides equally across A, B and C, i.e., $V_A = V_B = V_C = V/3$. Now when the switch S is closed, bulb C is short-circuited, i.e., $V'_C = 0$, so voltage V now will divide equally across A and B, i.e., $V'_A = V'_B = V/2$. So voltage across bulbs A and B will change from $V/3$ to $V/2$ while across C from $V/3$ to zero, i.e., voltage drop across A and B will increase while across C, it will decrease and become zero.

Further as R_T changes from $3R$ to $2R$, so power consumed in the circuit changes from

$$P_T = \frac{V}{3R} \text{ to } P'_T = \frac{V^2}{2R}, \quad \text{i.e.,} \quad P'_T = \frac{3}{2}P_T$$

i.e., power dissipated in the circuit increases and becomes 1.5 times its initial value.

8. Since $m = \rho Al$, the lengths and hence the resistances of the wires are in the ratio 1:2. Also when they are connected in series, heat produced will be in the ratio 1:2, because $H = I^2R$.

9. $P = \dfrac{V^2}{R} = \dfrac{V^2 A}{P\ell}$

To boil water in 10 min, power should be increased. To increase power, length should be decreased.

10. Power drawn by motor:

$$P = VI = 50 \times 12 = 600 \text{ W}$$

$$\text{Power loss} = \frac{70}{100} \times 600 = 420 \text{ W}.$$

This power is lost in the form of heat in the resistance.

$$I^2R = 420$$

$$(12)^2R = 420 \Rightarrow R = 2.9\ \Omega$$

11. Heat produced = Heat loss through surface of wire

$$I^2R = H\,2\pi R\ell$$

Where H is the heat loss per unit surface area of the wire.

$$I^2\frac{\rho\ell}{a} = H\,2\pi R\ell$$

Put $\quad a = \pi r^2$

$\therefore \quad I^2 \propto r^3$

$$\Rightarrow \left(\frac{15}{30}\right)^2 = \left(\frac{0.50}{r}\right)^3 \Rightarrow r = 0.238 \text{ mm}$$

12. $P_{in} = 12 \times 2 = 240$ W

Heat low = 9 cals/s = 9 × 4.20 = 378 J/s

$$P_{out} = 240 - 37.8 = 202.2 \text{ W}$$

$$\text{Efficiency } \eta = \frac{P_{out}}{P_{in}} \times 100 = 84.25\%$$

13. Heat produced = Heat loss

$$\frac{V^2}{R} = \frac{KA\Delta T}{\ell}$$

Here $V = 400$ V, $K = 4 \times 10^{-4}$ cal/cm s°C

$= 168 \times 10^{-4}$ J/cm s°C

$\Delta T = 100$, $\ell = 1$ mm $= 0.1$ cm

$A = 6(40)^2$

$$\frac{400^2}{R} = \frac{16.8 \times 10^{-4} \times 6 \times (40)^2 \times 100}{0.1}$$

$$R = 9.92\ \Omega$$

14. a. When the lamps are connected in parallel, then potential difference V across each lamp will be same and will be equal to potential difference necessary for full brightness of each bulb.

Because illumination produced by a lamp is proportional to electric power consumed in it, and power consumed is

$$P_1 = \frac{V^2}{R_1} < \frac{V^2}{R_2} = P_2$$

hence illumination produced by the second bulb will be higher than that produced by the first bulb, i.e., the bulb having lower resistance will shine more brightly.

b. When R_1 burns out, then power is dissipated in R_2 only. Because internal resistance is quite low in lighting circuit, potential difference is still equal to V, hence power dissipated in the second lamp, i.e.,

$$\frac{V^2}{R_2} < \left(\frac{V^2}{R_1} + \frac{V^2}{R_2}\right), \text{ net power consumed initially.}$$

In other words, net illumination will now decrease.

c. When the two lamps are connected in series, the potential difference across each lamp will be different but current I flowing through each lamp will be same.

Hence, illumination produced by the first lamp will be higher as compared to that produced by the second lamp, i.e., the lamp having higher resistance will glow more brightly.

d. When lamp of resistance R_2 burns out and only lamp of resistance R_1 is connected in the circuit, then current flowing through the circuit will change. Let new current is i'.

Because potential difference still remains same (due to low internal resistance), hence

$$i'R_1 = i(R_1 + R_2) \quad \Rightarrow \quad i' = [i(R_1 + R_2)]/R_1$$

If P' is the power consumed, then

$$P' = i'^2 R_1 = [i^2(R_1 + R_2)(R_1 + R_2)]/R_1$$

When both the lamps were present then total power consumed was given by

$P_S = P_1 + P_2 = i^2(R_1 + R_2)$, i.e., illumination gets increased when only bulb is used.

15. Let the power supply is V. Let resistance of each bulb is R. Total resistance of circuit = nR

Power illuminated by all bulbs, $P_1 = \frac{V^2}{nR}$

Power illuminated by one bulb, $P_2 = \frac{P_1}{n} = \frac{V^2}{n^2 R}$

After one bulb is fused, the powers are

$$P_1' = \frac{V^2}{(n-1)R},\ P_2' = \frac{V^2}{(n-1)^2 R}$$

a. Fractional change in the illumination of all the bulbs:

$$f_1 = \frac{P_1' - P_1}{P_1} = \frac{P_1'}{P_1} - 1 = \frac{n}{n-1} - 1 = \frac{1}{n-1}$$

b. Fractional change in the illumination of one bulb:

$$f_1 = \frac{P_2' - P_2}{P_2} = \frac{n^2}{(n-1)^2} - 1 = \frac{2n-1}{(n-1)^2}$$

16. Power input = $P_i = VI = 100 \times 6 = 600$ W
Power output = 150 W

Efficiency: $n = \frac{150}{600} \times 100 = 25\%$

Heat produced is 600 − 150 = 450 W

$\Rightarrow I^2 R = 450$ where R is the resistance of windings

$$\Rightarrow \quad R = \frac{450}{6^2} = \frac{450}{36} = 12.5\,\Omega$$

17. Voltage drop across line = 10 V, current = $\frac{10}{2} = 5$ A.

Current drawn by one lamp = $\frac{100}{230}$ A

So number of lamps = $\frac{5 \times 230}{100} = 11.5$

18. Total wattage consumed (units)

$$= \frac{7 \times 40 \times 6}{1000} + \frac{2 \times 60 \times 6}{1000} + \frac{5 \times (220 \times 0.4) \times 1}{1000} + \frac{(220)^2}{48.4} \times \frac{10}{1000}$$

$= 12.84$ per day

No. of units consumed in one month (Jan) = 12.84 × 31 = 398.04.
Total bill = 2 × 398.04 = Rs. 796.08

19. a. $R_1 = \frac{200^2}{300},\ R_2 = \frac{200^2}{600}$

The resistance of the first bulb is more. Hence in series, the first bulb will produce more illumination.

b. $P = \frac{P_1 P_2}{P_1 + P_2} = \frac{300 \times 600}{300 + 600} = 200$ W

c. $P = P_1 + P_2 = 300 + 600 = 900$ W

20. Let stabilizer gives voltage V, then power produced by the bulb is

$$P = \left(\frac{V}{220}\right)^2 100$$

$$\frac{dP}{P} = 2\frac{dV}{V}$$

If $\frac{dV}{V} = \pm 1\%$, then $\frac{dP}{P} = \pm 2\%$

Hence maximum power produced,

$P_{max} = 100 + 2\% = 102$ W

Minimum power produced

$P_{min} = 100 - 2\% = 98$ W

21. Efficiency $= \eta = \frac{\text{output power}}{\text{input power}}$

$\eta = \frac{i^2 R}{\varepsilon i}$ where $i = \frac{\varepsilon}{R + r}$

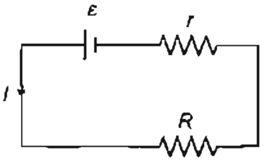


Fig. A3-7.2

$\eta = \frac{R}{R+r} \Rightarrow 0.6 = \frac{R}{R+r} \Rightarrow 2R = 3r$

∴ New efficiency $\eta_1 = \frac{6R}{6R + r} = 0.9 = 90\%$

22. 25 W bulb will glow brighter. This is because $P = I^2 R$ where I is the current flowing and R is the resistance of the appliance. When I is same, $P \propto R$. The resistance of 25 W bulb is more and therefore P is more.

CHAPTER 8

Exercise 8.1

1. a. $BA \cos\theta$ where A is area of the flat surface.
b. $BA \cos\theta$.

2. a. $\frac{1}{320}$ W or 3.125×10^{-3} W
b. 0 (zero)

2. a.

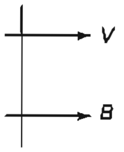


Fig. A3-8.8

e.m.f. = 0 as $\vec{B} \parallel \vec{v}$

b.

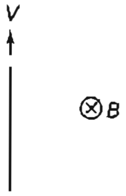


Fig. A3-8.9

e.m.f. = 0 as $\vec{\ell} \parallel \vec{v}$

c.

Fig. A3-8.10

e.m.f. = 0 as $\vec{\ell} \times \vec{B}$

3. Consider rod AB, which is a part of the coil. e.m.f. induced in the rod = BLv. Suppose the e.m.f. induced in part ACB is E, as shown.
Since the e.m.f. in the coil is zero, e.m.f. (in ACB) + e.m.f. (in BA) = 0

or $\quad -E + vBL = 0$

or $\quad E = vBL$

Thus e.m.f. induced in any path joining A and B is same, provided the magnetic field is uniform. Also, the equivalent e.m.f. between A and B is BLv (here the two e.m.f.s are in parallel).

4. The same e.m.f. will be induced in the straight imaginary wire joining A and B, whi[illegible] is $Bv\ell \sin\theta$.

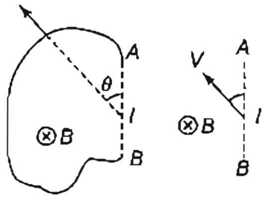


Fig. A3-8.11

5. $E = 2RvB$

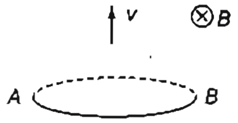


Fig. A3-8.12

6. Induced e.m.f. = 0

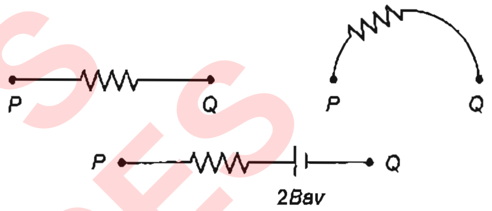


Fig. A3-8.13

7. Induced e.m.f. = $2Bav$

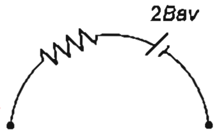


Fig. A3-8.14

8.

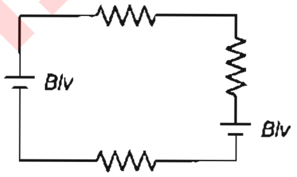


Fig. A3-8.15

9.

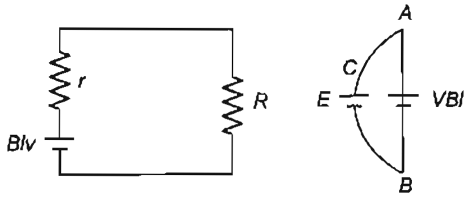


Fig. A3-8.16

10. $E = Blv = \dfrac{\mu_0 i l v}{2\pi x}$

Alternatively:

e.m.f. is equal to the rate with which magnetic field lines are cut. In dt time the area swept by the rod is $lv\,dt$. The magnetic field lines cut in dt time = $Blv\,dt = \dfrac{\mu_0 i l v\,dt}{2\pi x}$.

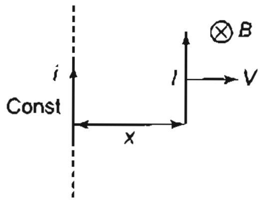


Fig. A3-8.17

∴ The rate with which magnetic field lines are cut $= \dfrac{\mu_0 i l v}{2\pi x}$.

11. $E = B_1 lv - B_2 lv = \dfrac{\mu_0 i}{2\pi x} lv - \dfrac{\mu_0 i}{2\pi(x+b)} lv = \dfrac{\mu_0 i l b y}{2\pi x(x+b)}$

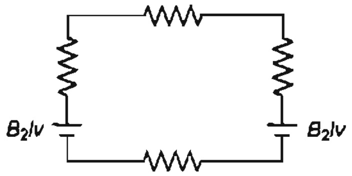


Fig. A3-8.18

Alternatively:

Consider a small segment of width dy at a distance y from the wire. Let flux through the segment be $d\phi$.

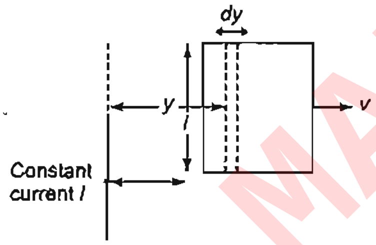


Fig. A3-8.19

$$\therefore \quad d\phi = \frac{\mu_0 i}{2\pi y} l dy$$

$$\therefore \quad \phi = \frac{\mu_0 il}{2\pi} \int_x^{x+b} \frac{dy}{y}$$

$$= \frac{\mu_0 il}{2\pi}(\ln(x+b) - \ln x)$$

Now,

$$\frac{d\phi}{dt} = \frac{\mu_0 il}{2\pi}\left[\frac{1}{x+b}\frac{dx}{dt} - \frac{1}{x}\frac{dx}{dt}\right]$$

$$= \frac{\mu_0 il}{2\pi}\left[\frac{(-b)}{x(x+b)}\right] v = \frac{-\mu_0 iblv}{2\pi x(x+b)}$$

$$\therefore \quad \text{Induced e.m.f.} = \frac{\mu_0 iblv}{2\pi x(x+b)}$$

12. Consider a segment of rod of length dx, at a distance x from the wire. e.m.f. induced in the segment

$$d\varepsilon = \frac{\mu_0 i}{2\pi x} dxv$$

$$\therefore \quad \varepsilon = \int_a^{a+\ell} \frac{\mu_0 iv\,dx}{2\pi x} = \frac{\mu_0 iv}{2\pi}\ln\left(\frac{\ell + a}{a}\right)$$

13. e.m.f. = 0

The equivalent circuit is

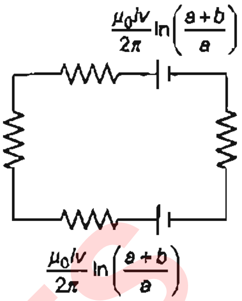


Fig. A3-8.20

$$i = \frac{\varepsilon}{R+r} = \frac{\mu_0 iv}{2\pi(R+r)}\ln\left(\frac{x+\ell}{\ell}\right)$$

14. $E_{MQ} + E_{PM} = E_{PQ}$ corner $\rightarrow \dfrac{B\omega\ell^2}{2} = 100$

$$E_{MQ} + \frac{B\omega\left(\frac{\ell}{2}\right)^2}{2} = \frac{B\omega\ell^2}{2}$$

$$E_{MQ}\,\frac{3}{8} = B\omega\ell^2 = \frac{3}{4} \times 100 \text{ V} = 75 \text{ V}$$

15. e.m.f.$_{PQ} = 0$; e.m.f.$_{PC} = \dfrac{B\omega\ell^2}{2}$

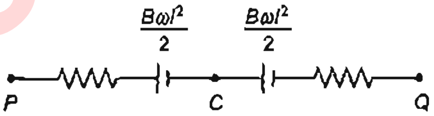


Fig. A3-8.21

16.

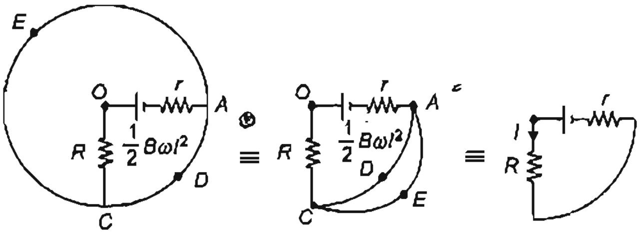
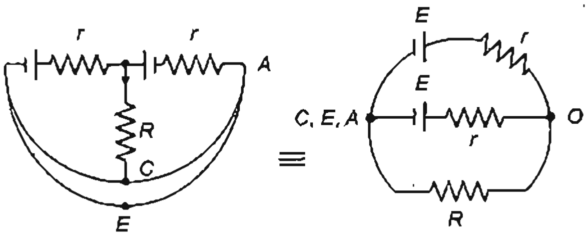


Fig. A3-8.22

$$\text{Current } i = \frac{\frac{1}{2}B\omega\ell^2}{R+r}$$

17.



$$= -1.2\times10^{-3}$$

For loop *b*:

$$\oint \vec{E}\cdot d\vec{s} = -\pi r_2^2\times\frac{dB}{dt}$$

$$= -\pi\left(\frac{32.3}{100}\right)^2\times8.5\times10^{-3}$$

$$= -2.7\times10^{-3}$$

For loop *c*:

$$\oint \vec{E}\cdot d\vec{s} = \pi(r_2^2-r_1^2)\frac{dB}{dt}$$

$$= \pi\left(\left(\frac{32.3}{100}\right)^2-\left(\frac{21.2}{100}\right)^2\right)\times8.5\times10^{-3}$$

$$= 1.5\times10^{-3}$$

3. $\oint_1 \vec{E}\cdot d\vec{s} = mag$

$$\oint_3 \vec{E}\cdot d\vec{s} = 3(mag)$$

It means field in *b* and *c* should be out of the page.

$$\oint_4 \vec{E}\cdot d\vec{s} = 0$$

It means field in *c* will be opposite to that in *e*, i.e., into the page.

4. $\frac{dB}{dt} = 0.060\,t$

$$E_{\text{at }P_1} = \frac{r}{2}\frac{dB}{dt}$$

$$= \frac{0.020}{2}\times0.060 = 6\times10^{-4}\text{ V/m}$$

Here magnetic field is increasing with time, hence electric field will be in anticlockwise direction.

5. a. At $t=0$, C will act as a simple wire, so R and L both will be short-circuited. So, there is no current in R and L.

$$I_1=0, I_2=0, I_3=\frac{\varepsilon}{r}$$

b. At $t=\infty$, L will act as a simple wire so, R and C will be short-circuited.

$$I_1=0, I_2=\frac{\varepsilon}{r}, I_3=0$$

6. a. $U=Pt=(200\text{ W})(24\text{ h/day}\times3600\text{ s/h}) = 1.73\times10^7\text{ J}$

b. $U=\frac{1}{2}LI^2 \Rightarrow L=\frac{2U}{I^2}=\frac{2(1.73\times10^7\text{ J})}{(80.0\text{ A})^2} = 5406\text{ H}$

7. a. $I_{\max} = \frac{30\text{ V}}{1000\,\Omega} = 0.030\text{ A} = 30\text{ mA}$, long after closing the switch.

b. $i = i_{\max}(1-e^{-t/(L/R)}) = 0.030\text{ A}\left(1-e^{-\frac{20\,\mu s}{10\,\mu s}}\right)$

$$= 0.0259\text{ A}$$

$$V_R = Ri = (1000\,\Omega)(0.0259\text{ A}) = 26\text{ V}$$

$$V_L = \varepsilon_{\text{Battery}} - V_R = 30\text{ V} - 26\text{ V} = 4.0\text{ V}$$

(or could use $V_L = L\frac{di}{dt}$ at $t = 20\,\mu\text{s}$)

c.

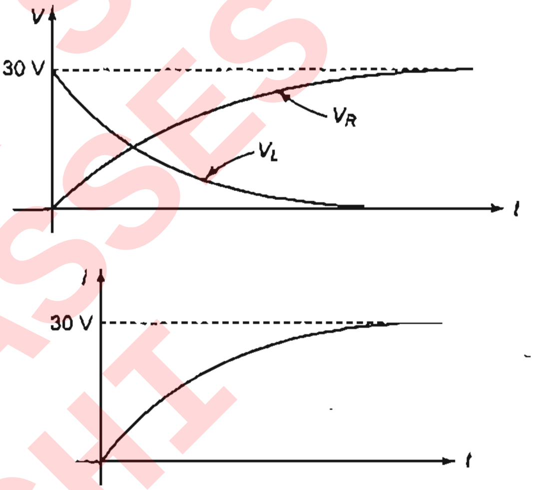


Fig. A3-8.31

8. a. $T=\frac{2\pi}{\omega} = 2\pi\sqrt{LC} = 2\pi\sqrt{(1.50\text{ H})(6.00\times10^{-5}\text{ F})}$

$$= 0.0596\text{ s}, \omega = 105\text{ rad/s}$$

b. $Q=CV=(6.00\times10^{-5}\text{ F})(12.0\text{ V}) = 7.20\times10^{-4}\text{ C}$

c. $U_0=\frac{1}{2}CV^2=\frac{1}{2}(6.00\times10^{-5}\text{ F})(12.0\text{ V})^2 = 4.32\times10^{-3}\text{ J}$

d. At $t=0$, $q=Q\cos(\omega t+\phi) \Rightarrow \phi=0$

$$t=0.030\text{ s}, q=Q\cos(\omega t)$$

$$= (7.20\times10^{-4}\text{ C})$$

$$\times\cos\left(\frac{0.0230\text{ s}}{\sqrt{(1.50\text{ H})(6.00\times10^{-5}\text{ F})}}\right)$$

$$= -5.43\times10^{-4}\text{ C}$$

Negative sign indicates that signs on plates are opposite to those at initials.

e. $t=0.0230\text{ s}, i=\frac{dq}{dt}=-\omega Q\sin(\omega t)$

$$i = -\frac{7.20\times10^{-4}\text{ C}}{\sqrt{(1.50\text{ H})(6.00\times10^{-5}\text{ H})}}$$

$$\times \sin\left(\frac{0.0230 \text{ s}}{\sqrt{(1.50 \text{ H})(6.00\times10^{-5}\text{H})}}\right) = -0.05 \text{ A}$$

Negative sign indicates positive charge flowing away from plate which had positive charge at the given time.

f. Capacitor: $U_C = \frac{q^2}{2C} = \frac{(5.43\times10^{-4} \text{ C})^2}{2(6.00\times10^{-5} \text{ F})} = 2.46\times10^{-3}$ J

$$\text{Inductor: } U_L = \frac{1}{2}Li^2 = \frac{1}{2}(1.50 \text{ H})(0.0499 \text{ A})^2$$

$$= 1.87\times10^{-3}\text{J}$$

9. At $t = 0$, inductor acts as open circuit:

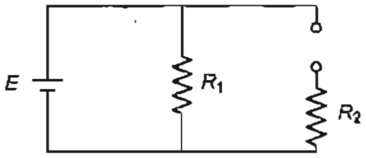


Fig. A3-8.32

a. $i_1 = \frac{E}{R_1} = 2$ A

b. 0 A

c. 2 A

d. V across resistor $R_2 = 0$

e. V across $L = 10$ V

f.

$$L\frac{di_2}{dt} = 10$$

$$\frac{di_2}{dt} = \frac{10}{5} = 2 \text{ A/s}$$

At $t = \infty$, inductor acts as a conducting wire:

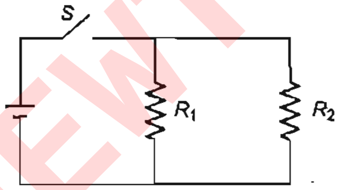


Fig. A3-8.33

a. $I_1 = \frac{\varepsilon}{R_1} = 2$ A

b. $I_2 = \frac{\varepsilon}{R_2} = \frac{10}{10} = 1$ A

c. $I_{\text{switch}} = I_1 + I_2 = 3$ A

d. $V_{R_2} = 10$ V

e. $V_L = 0$ V

f. Current in R_2 is not changing with time, so

$$\frac{di_2}{dt} = 0$$

10. **a.** b shall be at a higher potential

$$E_0 = I_0(R_1 + R_2) = 9\times8 = 72 \text{ V}$$

b. $I = I_0\, e^{-\frac{(R_1+R_2)t}{L}}$

I through R_1

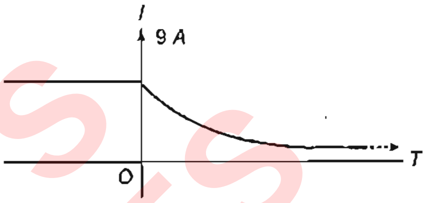


Fig. A3-8.34

I through R_2

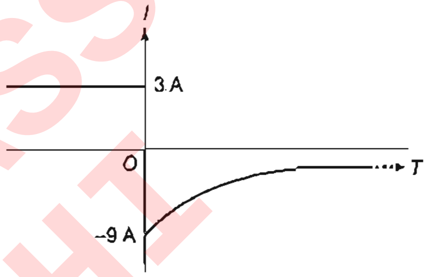


Fig. A3-8.35

c. $I = 9\, e^{-\frac{8t}{0.4}}$

$$2\times10^{-3} = 9\, e^{-20t} \quad \Rightarrow e^{20t} = \frac{9\times10^3}{2}$$

$$20t = \ln 4.5\times10^3$$

$$t = \frac{1}{20}\ln(4.5\times10^3) \text{ s}$$

11. $10 - 4I - 8I_1 - \frac{dI_1}{dt} = 0$ (i)

$I_1 + I_2 = I$ (ii)

$$10 - 4I - 4I_2 = 0$$

$$I_2 = \left(\frac{10-4I}{4}\right)$$

$$I_1 + \frac{10}{4} = 2I$$

$$4I = 2I_1 + 5$$

$$10 - 2I_1 - 5 - 8I_1 = \frac{dI_1}{dt}$$

$$dt = \frac{dI_1}{5 - 10I_1}$$

i. Time spent by the charge in magnetic field

$$\omega t=\theta \quad \Rightarrow \frac{qB}{m}t=\theta \quad \Rightarrow t=\frac{m\theta}{qB}$$

ii. Distance travelled by the charge in magnetic field

$$=r(2\theta)=\frac{mv}{qB}\times 2\theta$$

iii. Impulse = change in momentum of the charge

$$(-mv\sin\theta\hat{i}+mv\cos\theta\hat{j})-(mv\sin\theta\hat{i}+mv\cos\theta\hat{j})$$

$$=-2mv\sin\theta\hat{i}$$

4. i. $2\pi-2\theta=2\pi-2\times\frac{\pi}{6}=2\pi-\frac{\pi}{3}=\frac{5\pi}{3}=\omega t=\frac{qB}{m}$

$$\Rightarrow \quad t=\frac{5\pi m}{3qB}$$

ii. Distance travelled $s=r(2\pi-2\theta)=\frac{5\pi r}{3}$

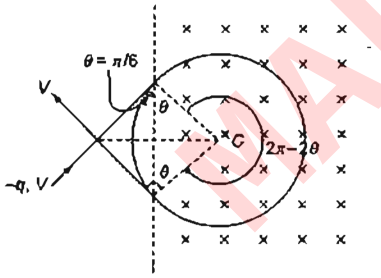


Fig. A3-9.2

iii. Impulse = charge in linear momentum

$$=m(-v\sin\theta\hat{i}+v\cos\theta\hat{j})$$

$$-m(v\sin\theta\hat{i}+v\cos\theta\hat{j})$$

$$=-2mv\sin\theta\hat{i}=-2mv\sin\frac{\pi}{6}\hat{i}=-mv\hat{j}$$

5. i. $d>\frac{mu}{qB}$ means $d>R$

$$\therefore \quad t=\frac{T}{2}=\frac{\pi m}{qB}$$

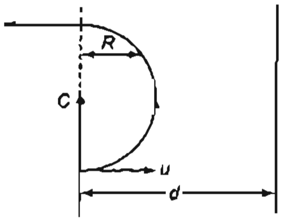


Fig. A3-9.3

ii. $\sin\theta=\frac{d}{R}$

$$\theta=\sin^{-1}\left(\frac{d}{R}\right)$$

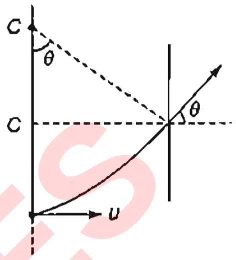


Fig. A3-9.4

$$\omega t=\theta \quad \Rightarrow t=\frac{m}{qB}\sin^{-1}\left(\frac{d}{R}\right)$$

6. The path of the particle will be circular. Larger the velocity, larger will be the radius.

For particle not to strike, $R<d$

$$\therefore \quad \frac{mv}{qB}<d \quad \Rightarrow v<\frac{qBd}{m}$$

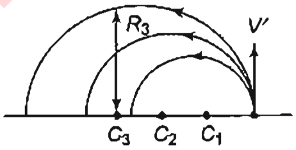


Fig. A3-9.5

For limiting case, $v;\frac{qBd}{m}$

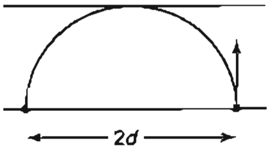


Fig. A3-9.6

$R;d$

∴ Coordinate = $(-2d,0,0)$

7. i. True, ii. True, iii. True, iv. False

Sol. Force exerted by the magnetic field is in the plane of spiral. Direction of magnetic field is always normal to both the velocity vector and direction of force. It means magnetic field is normal to plane of the spiral. Hence (a) is correct.

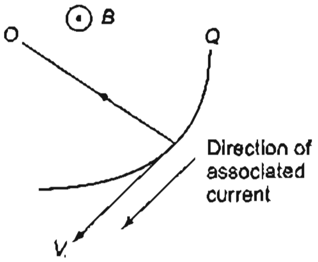


Fig. A3-9.7

Since the particle experiences a resistance against its motion, therefore its speed decreases continuously. Radius of the circular path followed by a charged particle moving in a magnetic field is given by $R = mv/qB$. Since v is continuously decreasing, therefore radius R also decreases continuously. Hence, a charged particle should follow a spiral path of decreasing radius. Hence the particle enters the magnetic field at Q. Hence (b) is correct.

Force experienced by the particle is towards the centre and direction of the force is found by Fleming's left hand rule. Since magnetic field is outwards, therefore according to Fleming's left hand rule, the direction of current associated with its motion should be as shown in Fig. A3-9.7. Since the particle is moving in the same direction, therefore the particle is positively charged.

Hence (c) is correct. Obviously (d) is wrong.

8. **i.** True, **ii.** True, **iii.** False, **iv.** True

Period of revolution of a charged particle moving in a uniform magnetic field is given by $T = \dfrac{2\pi m}{qB}$.

This period T does not depend upon speed of the particle. In this particular question, the moving particle is an electron. Hence its mass and charge (q) both are constant. Magnetic field is also uniform. Hence, its period of revolution remains constant. It means electron moves with a constant angular velocity. Hence (i) is correct.

In previous question we have already discussed that if a charged particle experiences a resisting force against motion then it follows a decreasing radius spiral path. In this question, electron is moving along a spiral path of decreasing radius. It means its speed is decreasing continuously. Hence (ii) is correct.

Since the speed of the electron is countinuously decreasing, therefore it is experiencing a tangential retardation. It is possible only when the componant of resultant force opposite to the direction of motion of electron has non-zero value. It means, net force on electron cannot be perpendicular to its direction of motion. Hence (iii) is wrong.

Since the speed of the electron is decreasing continuously, therefore the force exerted by the magnetic field ($F = qvB$) is also decreasing continuously. Hence, magnitude of net force acting on the electron is decreasing continuously.

Hence (iv) is correct.

9. **a.** True, **b.** False, **c.** True

A uniform magnetic field B and a uniform electric field E exist perpendicular to each other and the particle moves along a direction perpendicular to both these fields, then forces exerted by these two fields may be opposite to each other. If magnitudes of these forces are equal, then the resultant force on the particle will becomes equal to zero.

Hence, the particle will move with constant velocity. Hence (a) is correct. Obviously (b) is wrong.

If E is equal to zero then the particle will experience a force due to magnetic field alone. But the force exerted by the magnetic field is always perpendicular to the direction of its motion. Hence, no power is associated with this force. In other words, no work is done by the magnetic field on the particle. Therefore, K.E. of the particle will remain constant.

Hence (c) is correct.

10. **i.** $\dfrac{3}{4}\dfrac{mv^2}{a}$, **ii.** $\dfrac{3}{4}\dfrac{mv^3}{a}$, **iii.** zero

In going from P to Q, increase in kinetic energy

$$= \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2 = \frac{1}{2}m(3v^2)$$

= work done by the electric field

$$\frac{3}{2}mv^2 = Eq \times 2a \text{ or } E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$$

The rate of work done by E at P = force due to E × velocity

$$= (qE)v = qv\left[\frac{3}{4}\left(\frac{mv^2}{qa}\right)\right] = \frac{3}{4}\left(\frac{mv^3}{a}\right)$$

At q, $\hat{v}$ is perpendicular to $\bar{E}$ and $\vec{B}$, and no work is done by either field.

11.

a. $\overline{F} = q\hat{v} \times \overline{B} = qB_z[v_x(\hat{i} \times \hat{k}) + v_y(\hat{j} \times \hat{k})$

$$+ v_z(\hat{k} \times \hat{k})] = qB_z[v_x(-\hat{j}) + v_y(\hat{i})]$$

Set this equal to the given value of $\vec{F}$ to obtain

$$v_x = \frac{F_y}{-qB_z} = \frac{(7.40 \times 10^{-7}\text{ N})}{-(-5.60 \times 10^{-9}\text{ C})(-1.25\text{ T})}$$

$$= -106\text{ m/s}$$

$$v_y = \frac{F_x}{qB_z} = \frac{-(3.40 \times 10^{-7}\text{ N})}{(-5.60 \times 10^{-9}\text{ C})(-1.25\text{ T})}$$

$$= -48.6\text{ m/s}$$

b. The value of v_x is indeterminate.

c. $\bar{v} \cdot \overline{F} = v_xF_x + v_yF_y + v_zF_z = \dfrac{F_y}{-qB_z}F_x + \dfrac{F_x}{qB_z}F_y = 0;$

$\theta = 90°$

$$r = \frac{mv}{eB}$$

$$1 = r\sin\phi \quad \Rightarrow \sin\phi = \frac{1}{r} = \frac{eB}{mv}$$

y-component of the velocity is $v\sin\theta$.
Hence y-component of momentum is $Mv\sin\theta = eB$

24. $d = (v\cos\phi)\ T = (v\cos\phi)\ \dfrac{2\pi m}{qB}$

where $\quad \Delta V = \dfrac{1}{2}mv^2$

$$v = \sqrt{\frac{2\Delta V}{m}}$$

Putting the value, we get $B = \dfrac{2\pi m}{qd}\left(\sqrt{\dfrac{2\Delta V}{m}}\right)\cos\phi$

Exercise 9.2

1. The situation is shown in the figure provided. The magnetic field at the site of $d\ell$ due to the first wire is $B = \dfrac{\mu_0 i_1}{2\pi\ell}$.

   This field is perpendicular to the plane of the figure going into it. The magnetic force on the length dl is

   $$dF = i_2\, d\ell\, B\sin 90^\circ = \frac{\mu_0 i_1 i_2 d\ell}{2\pi\ell}$$

   This force is parallel to the current i_1.
2. Suppose the field and the current have directions as shown in the figure provided.
   The force on PQ is

   $$\vec{F} = i\vec{\ell}\times\vec{B}$$

   or $\quad F_1 = 5.0\text{ A}\times 10\text{ cm}\times 2.0\text{ T} = 1.0\text{ N}$

   The rule of vector product shows that the force F_1 is perpendicular to PQ and is directed towards the inside of the triangle.

   The forces $\vec{F}_2$ and $\vec{F}_3$ on QR and RP can also be obtained similarly. Both the forces are 1.0 N directed perpendicularly to the respective sides and towards the inside of the triangle.

   The three forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ will have zero resultant, so that there is no net magnetic force on the triangle. This result can be generalised. Any closed current loop, placed in a homogeneous magnetic field, does not experience a net magnetic force.
3. a. The force on the wire due to the magnetic field is

   $$\vec{F} = i\vec{\ell}\times\vec{B} \text{ or } F = i\ell B$$

   It acts towards right in the given figure provided. If the wire does not slide on the rails, the force of friction by the rails should be equal to F. If μ_0 be the minimum coefficient of friction which can prevent sliding, this force is also equal to $\mu_0\, mg$. Thus,

   $$\mu_0 mg = i\ell B \text{ or } \mu_0 = \frac{i\ell B}{mg}$$

   b. If the friction coefficient is $\mu = \dfrac{\mu_0}{2} = \dfrac{i\ell B}{2mg}$, the wire will slide towards right. The frictional force by the rails is

   $$f = \mu mg = \frac{i\ell B}{2} \text{ towards left}$$

   The resultant force is $I\ell B - \dfrac{i\ell B}{2} = \dfrac{i\ell B}{2}$ towards right. The acceleration will be $a = \dfrac{i\ell B}{2m}$. The wire will slide towards right with this acceleration.
4. The wire is equivalent to

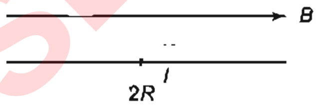


Fig. A3-9.10

$$\because \quad \theta = 0, \therefore F_{res} = 0$$

Forces on individual parts are marked in Fig. 9.11 by ⊗ and ⊙. By symmetry there will be pair of forces forming couples.

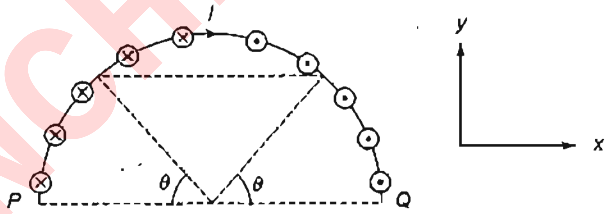


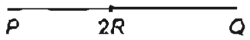
Fig. A3-9.11

$$\tau = \int_0^{\pi/2} i(Rd\theta)B\sin(90-\theta)2R\cos\theta$$

$$\tau = \frac{i\pi R^2}{2}B$$

or $\quad \vec{\tau} = \tau\left(-\hat{j}\right) = \dfrac{i\pi R^2}{q}B(-\hat{j})$

5. $\vec{F}_{nett} = I2R\,B$. The wire is equivalent to



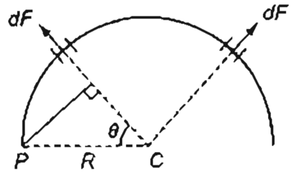


Fig. A3-9.12

Force on each element is radially outward, $\tau_C = 0$

Torque about point $P = \tau_P = \int_0^{\pi} [i(Rd\theta)B\sin 90^0]R\sin\theta$

$$= 2IBR^2$$

6. $F_{\text{res}} = \dfrac{\mu_0 I_1 I_2}{2\pi a} a\ (-\hat{i}) + \dfrac{\mu_0 I_1 I_2}{2\pi(2a)} a(\hat{i}) = \dfrac{\mu_0 I_1 I_2}{4\pi}(-\hat{i})$

7. We have for mid-air suspension,

$$mg = i\ell B$$

$$\Rightarrow \qquad B = \frac{mg}{i\ell} = \frac{0.2\times 9.81}{2\times 1.5} = 0.65\ \text{T}$$

8. The net force from A to B is $d\vec{F} = I(d\vec{L}\times\vec{B})$

$$\int_A^B d\vec{F} = \int_A^P I[d\vec{L}_1\times\vec{B}] + \int_P^Q I[d\vec{L}_2\times\vec{B}]$$

$$+\int_Q^R I[d\vec{L}_3\times\vec{B}] + \int_R^T I[d\vec{L}_4\times\vec{B}]$$

$$+\int_T^B I[d\vec{L}_5\times\vec{B}]$$

The entire path can be broken down into elemental vectors joined to each other in sequence. We know, from polygon law of addition of vectors, that vector joining the tail of the first vector to the head of the last vector is the resultant.

$$\vec{F} = I(\vec{L}\times\vec{B})$$

where $\qquad |\vec{L}| = a + \sqrt{c^2 - b^2} + 2r + d$

$F_{\text{net}} = IB(a + \sqrt{c^2 - b^2} + 2r + d)$ and its direction is upwards on the plane of paper.

9. $\vec{F} = I\vec{\ell}\times\vec{B}$

$$F = I\ell B\sin\theta$$

a. When the current is flowing from east to west,

$$\theta = 90°$$

Hence $\qquad F = I\ell B = (1\ \text{A})(1\ \text{m})(3\times 10^{-5}\ \text{T})$

The direction of the force is downwards. This direction may be obtained by either Fleming's left hand rule or the directional property of cross product of vectors.

b. When the current is flowing from south to north,

$$\theta = 0° \quad \Rightarrow F = 0$$

Hence, there is no force per unit length on the conductor.

10.

$$I = \frac{\varepsilon}{R} = \frac{24}{12} = 2\ \text{A}$$

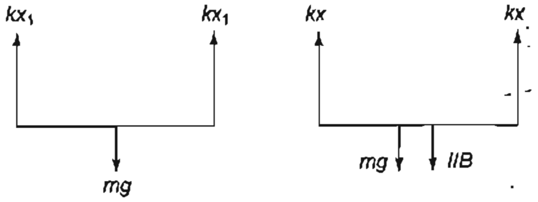


Fig. A3-9.13

Magnetic force $= I\ell B\sin\dfrac{\pi}{2}$

$$= 2\times 5\times 10^{-2}\times B = \frac{B}{10}$$

$$2kx_1 = mg;\ 2k = \frac{mg}{x_1}$$

$$2kx = mg + I\ell B$$

$$2k(x_1 + x_2) = mg + I\ell B$$

$$2kx_1 + 2kx_2 = mg + I\ell B$$

$$\frac{mx}{x_1}x_2 = I\ell B$$

$$0.10\times 10^{-3}\times 10\times\frac{0.3}{0.5} = \frac{B}{10}$$

$$\therefore \quad B = 600\times 10^{-3} = 0.6\ \text{T}$$

11. Consider a thin strip at a distance x and of the thicknes dx. It is equivalent to a long straight conductor carryin, (I_2dx/b) current.

$$dF\ (\text{force of attraction}) = \frac{\mu_0 I_1}{2\pi x}\times\frac{I_2 dx}{b} = \frac{\mu_0 I_1 I_2}{2\pi b}\times\frac{dx}{x}$$

$$\therefore \qquad F = \frac{\mu_0 I_1 I_2}{2\pi b}\int_x^{a+b}\frac{dx}{x} = \frac{\mu_0 I_1 I_2}{2\pi b}\ln\frac{a+b}{b}$$

12.

a.

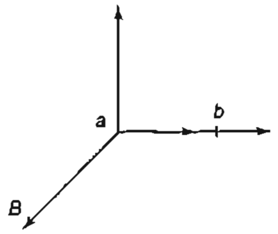


Fig. A3-9.14

$$\vec{B} = 0.75\ (\hat{k})$$

$$F_{ab} = BIl = 0.75\times 6\times 40\times 10^{-2}$$

$$F_{ab} = 1.8\,\text{N}$$

$$\Rightarrow \quad \vec{F}_{ab} = 1.8\,(-\hat{j})$$

b. $$\vec{F}_{bc} = \vec{F}_{bd} + \vec{F}_{dc}$$

$$= BI(bd)\hat{j} + BI(dc)\hat{i}$$

$$= 0.75\times6\left[(R - R\cos53°)\hat{j} + R\sin30°\,\hat{i}\right]$$

$$= 1.8\left(1 - \frac{\sqrt{3}}{2}\right)\hat{j} + 0.9\,\hat{j}\,\text{N}$$

c. $$\vec{F}_{ca} = \vec{F}_{cd} + \vec{F}_{da}$$

$$= -0.9\hat{j} + 0.9\sqrt{3}\,\hat{j}$$

We can see that net force is zero on the loop.

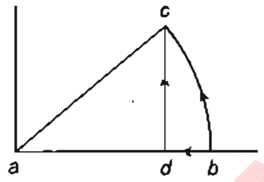


Fig. A3-9.15

13. As $d\vec{x}$ and $\vec{B}$ are parallel,

$$dF = I\,d\vec{x}\times\vec{B} = 0$$

$$F_{\text{net}} = 0$$

14. Fig. A3-9.16. Let $F_0 = qvB$, then:

$F_a = F_0$ in the $-\hat{k}$ direction

$F_b = F_0$ in the $+\hat{j}$ direction

$F_c = 0$, since magnetic field and velocity are parallel

$F_d = F_0 \sin 45°$ in the $-\hat{j}$ direction

$F_e = F_0$ in the $-(\hat{j}+\hat{k})$ direction

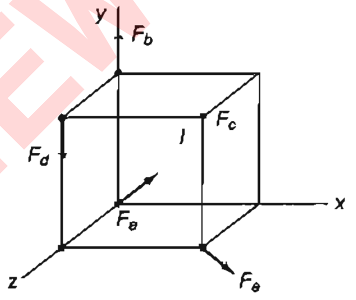


Fig. A3-9.16

15. For a charge with velocity

$$\vec{v} = (8.00\times10^6\ \text{m/s})\hat{j},$$

the magnetic field produced at a position away from the particle is $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\hat{r}}{r^2}$.

a. $\vec{r} = (+0.500\,m)\hat{i} \quad \Rightarrow \hat{v}\times\hat{r} = -\hat{k},\ r_0^2 = \frac{1}{4}$

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{qv}{r_0^2}\hat{k}$$

$$= -\frac{\mu_0}{4\pi}\frac{(6.0\times10^{-6}\,\text{C})(8.0\times10^6\,\text{m/s})}{(0.50\,\text{m})^2}\hat{k}$$

$$= -(1.92\times10^{-5}\ \text{T})\hat{k} \equiv -B_0\hat{k}$$

b. $\vec{r} = (-0.500\,m)\hat{j} \quad \Rightarrow \hat{v}\times\hat{r} = 0 \quad \Rightarrow \vec{B} = 0$

16. a. $\vec{r} = (0.500\,\text{m})\hat{k} \quad \Rightarrow \hat{v}\times\hat{r} = +\hat{i},\ r_0^2 = \dfrac{1}{4}$

$$\vec{B} = +\frac{\mu_0}{4\pi}\frac{qv}{r_0^2}\hat{i} = B_0\hat{i}$$

b. $\vec{r} = -(0.500\,\text{m})\hat{j} + (0.500\,\text{m})\hat{k}$

$$\hat{v}\times\hat{r} = -\hat{i},\quad r^2 = \frac{1}{2} = 2r_0$$

$$\vec{B} = +\frac{\mu_0}{4\pi}\frac{qv}{r^2}\frac{\hat{i}}{\sqrt{2}} = +\frac{B_0}{2}\frac{\hat{i}}{\sqrt{2}} = +\frac{B_0\hat{i}}{2\sqrt{2}}$$

17. a. $\vec{F} = I\,\vec{l}_{ab}\times\vec{B} = I(l_{ab}B)\hat{j}\times\hat{i}$

$$= -(6.58\ \text{A})\,(0.750\ \text{m})\,(0.860\ \text{T})\,\hat{k}$$

$$= (-4.24\ \text{N})\hat{k}$$

b. $\vec{F} = I\,\vec{l}_{bc}\times\vec{B} = I(l_{bc}B)\left[\dfrac{(\hat{i}-\hat{k})}{\sqrt{2}}\times\hat{i}\right]$

$$= -(6.58\ \text{A})(0.750\ \text{m})(0.860\ \text{T})\hat{j}$$

$$= (-4.24\ \text{N})\hat{j}$$

c. $\vec{F} = I\,\vec{l}_{cd}\times\vec{B} = I(l_{cd}B)\left[\dfrac{(\hat{k}-\hat{j})}{\sqrt{2}}\times\hat{i}\right]$

$$= -(6.58\,\text{A})(0.750\,\text{m})(0.860\,\text{T})\left[\hat{j}+\hat{k}\right]$$

$$\Rightarrow \quad \vec{F} = (4.24\,\text{N})\left[\hat{j}+\hat{k}\right]$$

d. $\vec{F} = I\,\vec{l}_{de}\times\vec{B} = Il_{de}B\left[-\hat{k}\times\hat{i}\right]$

$$= -(6.58\ \text{A})(0.750\ \text{m})(0.860\ \text{T})\hat{j}$$

$$= (-4.24\ \text{N})\hat{j}$$

e. $\vec{F} = I\,\vec{l}_{ef}\times\vec{B} = I(l_{ef}B)(-\hat{i})\times\hat{i} = 0$

f. Summing all the forces in parts (a)–(e), we have

$$\vec{F}_{\text{total}} = (-4.24\ \text{N})\hat{j}$$

18. a.

$$F = \frac{\mu_0 I_1 I_2 L}{2\pi r}$$

$$= \frac{\mu_0 (5.00\ A)\ (2.00\ A)\ (1.20\ m)}{2\pi(0.400\ m)}$$

$$= 6.00 \times 10^{-6}\ \text{N}$$

The force is repulsive since the currents are in opposite directions.

b. Doubling the currents makes the force increase by a factor of 4 to

$$F = 2.40 \times 10^{-5}\ \text{N}$$

19. In order to use equation $\vec{F} = I\vec{\ell} \times \vec{B}$, we would need to find the angle between $\vec{\ell}$ and $\vec{B}$. We avoid this task by using unit vector notation. From the figure provided, we see that

$$\vec{\ell} = a\hat{i} - a\hat{j} + a\hat{k}$$

Thus, the force is $\vec{F} = I\vec{\ell} \times \vec{B} = IaB(\hat{i} - \hat{j} + \hat{k}) \times (\hat{j})$

$$= IaB(-\hat{i} + \hat{k})$$

The force lies in the xz plane and has a magnitude

$$F = \sqrt{2}\, IaB = 0.85\ \text{N}$$

20. Magnetic force is along the positive x-axis. If motion is to occur along the incline, $\sum F = 0$: $IL\,|\,B_y\,|\cos 37°$ $= mg\sin 37°$ from which

$$I = \frac{mg\tan 37°}{L\,|\,B_y\,|} = \frac{(0.050)(9.8)(0.75)}{(0.40)(0.20)}$$

$$= 4.6\ \text{A}$$

21. Referring to the figure in question, we note that the magnetic forces on the slanting sides are parallel to AC and therefore do not produce any torque about the axis AC. The magnetic force on the horizontal side of the frame is

$$F_{\text{mag}} = iL \times B \qquad \text{(i)}$$

This force has magnitude $F_{\text{mag}} = iLB\sin 90° = iLB$ and points horizontally to the right on this plane of this figure, perpendicular to L and B. The magnetic torque about AC has magnitude

$$T_m = ILB(L\cos\theta) = iL^2 B\cos\theta \qquad \text{(ii)}$$

and tends to increase the angle θ. The gravitational torque about the axis AC is

$$T_g = -[(\lambda L)g(L\sin\theta) + 2(\lambda L)]$$

$$g\left(\frac{1}{2}L\sin\theta\right) \qquad \text{(iii)}$$

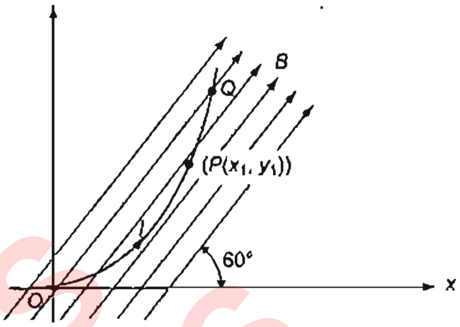


Fig. A3-9.17

where the minus sign indicates that gravity tends to turn the frame in the direction of decreasing θ. We have let λ denote the mass per unit length of the wire. In equilibrium, the torques evaluated in equations (ii) and (iii) must cancel, which implies that $\tan\theta = (Bi)/(2\lambda g)$.

Substituting the data, we obtain $\theta = \tan^{-1}\left(\frac{Bi}{2\lambda g}\right)$.

22. $F = IdB$

$$a = \frac{F}{m}$$

$$v^2 = 2\,aL = \frac{2I\,d\,B\,L}{m}$$

$$v = \sqrt{\frac{2I\,d\,B\,L}{m}}$$

23. Model the two wires as straight parallel wires.

a. $F_B = \dfrac{\mu_0 I^2 \ell}{2\pi a}$

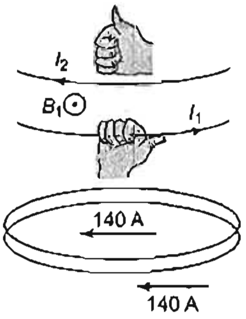


Fig. A3-9.18

$$F_B = \frac{(4\pi \times 10^{-7})(140)^2(2\pi)(0.100)}{2\pi(1.00 \times 10^{-3})}$$

$$= 2.46\ \text{N upward}$$

b. $a_{\text{loop}} = \dfrac{2.46\ \text{N} - m_{\text{loop}}g}{m_{\text{loop}}} = 107\ \text{m/s}^2$ upward

24. $\frac{\mu_0}{4\pi}=\frac{2\,I_A I_B \ell}{r}=mg$

where $r = 2.5\,\text{cm}$

$m = (0.100\,g/\text{cm})\ell$

25. Net force on circular parts will be zero because magnetic field produced by I_1 on these parts will be tangent at any point. On straight parts, forces will be in same direction.

$$\text{Net force} = 2\left[\frac{\mu_0}{4\pi}\frac{2I_1I_2}{R}L\right]$$

$$=\frac{\mu_0 I_1 I_2 L}{\pi R}$$

Exercise 9.3

1. a. $\phi = 90°$: $\tau = NIAB\sin(90°) = NIAB$

Direction: $\hat{k}\times\hat{j} = -\hat{i}$, $U = -N\mu B\cos\phi = 0$

b. $\phi = 0°$: $\tau = NIAB\sin(0) = 0$, no direction,

$U = -N\mu B\cos\phi = -NIAB$

c. $\phi = 90°$: $\tau = NIAB\sin(90°) = NIAB$

Direction: $-\hat{k}\times\hat{j} = -\hat{i}$, $U = -N\mu B\cos\phi = 0$

d. $\phi = 180°$: $\tau = NIAB\sin(180°) = 0$, no direction,

$U = -N\mu B\cos\phi = -NIAB$

2. Magnitude of dipole moment $\left|\vec{M}\right| = IA = Il^2$

Direction of magnetic moment is found by right hand rule.

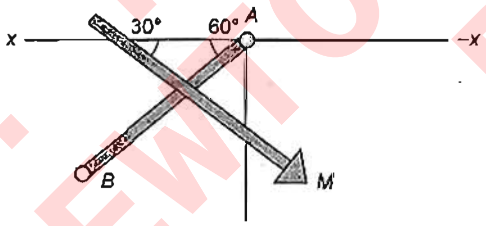


Fig. A3-9.19

To find unit vector of magnetic dipole moment, it is easier to make a two-dimensional view.
Direction of unit vector of magnetic moment:

$$\hat{M} = \cos 30°(-\hat{i}) + \sin 30°\,(\hat{j})$$

$$= -\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$$

$$\vec{M} = \frac{Il^2}{2}(-3\hat{i}+\hat{j})$$

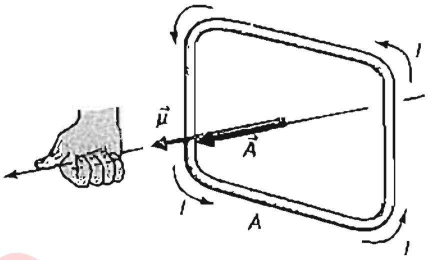


Fig. A3-9.20

The right hand rule determines the direction of the magnetic moment of a current carrying loop. This is also the direction of the loop's area vector.

- Sometimes a current carrying loop does not lie in a single plane.
- But by assuming two equal and opposite currents in one branch (which obviously makes no change in the given circuit) two (or more) closed loops are completed in different planes.
- Now the net magnetic moment of the given loop is the vector sum of individual loops.

3. Magnetic moment of the loop

$$\left|\vec{M}\right| = IA = 4.0\times 20\times 10\times 10^{-4}$$

$$\left|\vec{M}\right| = 8\times 10^{-2}\,\text{Am}^2$$

For direction:

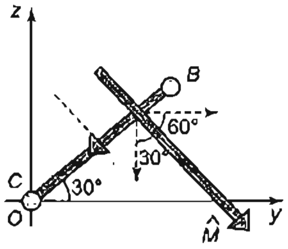


Fig. A3-9.21

Unit vector in the direction of magnetic moment

$$\hat{M} = \cos 60°\hat{j} - \cos 30\hat{k}$$

$$= \frac{\hat{j}}{2} - \frac{\sqrt{3}}{2}\hat{k}$$

$$\therefore \quad \vec{M} = \left|\vec{M}\right|\hat{M} = 4\times 10^{-2}(\hat{j}-\sqrt{3}\hat{k})\,\text{Am}^2$$

4. The given loop may be considered as the superposition of the two loops, as shown in the figure.

The resultant dipole moment is $M = \frac{\pi R_1^2 I}{2} - \frac{\pi R_2^2 I}{2}$

$$M = \frac{\pi I}{2}(R_1^2 - R_2^2) \text{ (inwards)}$$

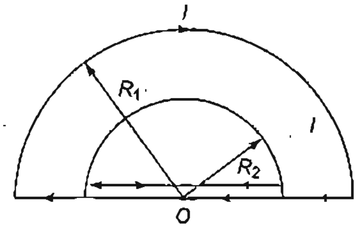


Fig. A3-9.22

5. Join additional wires at AH and DF carring same current in opposite directions. It will not affect the overall loop. Loop $ABGH$ and $DCEF$ will cancel as their magnetic moment is in opposite directions.

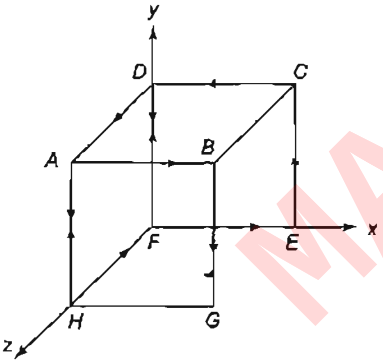


Fig. A3-9.23

a. Net magnetic moment of the loop = Magnetic moment of

$ADFH = -I\ell^2\hat{i} = \hat{i}$ Am^2

b. $\vec{B} = B\hat{j}$

$$\vec{\tau} = \vec{m}\times\vec{B} = -i\ell^2\hat{i}\times B\hat{j}$$

$$= -i\ell^2 B\hat{k} = B\hat{k} \text{ Nm}$$

6. Let v be the speed of the electron and r the radius of its orbit. The orbiting electron creates an electric current opposite to the velocity. The current is the charge that passes any given point along the circumferential path during one second. Thus current is charge e times the frequency n of the orbital motion.

$$I = ev = \frac{e}{T} = \frac{ev}{2\pi r}$$

The magnitude of magnetic moment,

$$\mu = IA = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2}$$

The direction of the magnetic dipole moment is perpendicular to the loop from right hand thumb rule.

The orbital angular momentum $\vec{L}_{\text{orbit}} = \vec{r}\times\vec{p}$

$$|\vec{L}_{\text{orbit}}| = rp\sin 90° = mvr$$

The direction of the orbital angular momentum is determined from the vector product right hand rule.

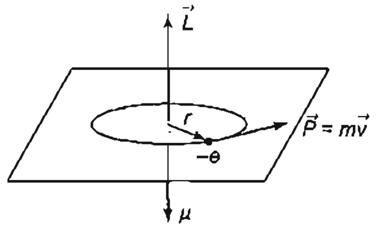


Fig. A3-9.24

The directions of angular momentum and magnetic dipole moment are opposite to each other.

$$\vec{\mu} = -\frac{e}{2m}\vec{L}$$

7. We can visualize the rod to consist of differential elements dQ, which constitute a series of concentric current loops. The charge per unit length of the rod is λ,

$$\lambda = \frac{Q}{L}$$

So the charge on a differential element of length dl,

$$dq = \lambda d\ell$$

The current dI due to rotation of this charge is given by

$$dI = \frac{dq}{(2\pi/\omega)} = \frac{\omega}{2\pi}dq = \frac{\omega}{2\pi}\lambda d\ell$$

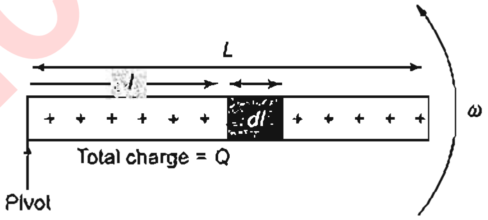


Fig. A3-9.25

The magnetic moment of this differential current loop,

$$d\mu = dI(\mu\ell^2) = \left(\frac{\omega}{2\pi}\lambda d\ell\right)\pi\ell^2$$

$$= \frac{\omega\lambda}{2}\ell^2 d\ell$$

To find total magnetic moment, we integrate

$$\mu = \frac{\omega\lambda}{2}\int_0^L \ell^2 d\ell = \frac{\omega\lambda L^3}{6}$$

Substituting for λ, we obtain $\mu = \dfrac{Q\omega L^2}{6}$

8. The magnetic dipole moment of the current carrying coil is given by $\vec{M} = NIA\hat{n}$

$$= 100\times0.5\times0.08\times0.04\,\hat{i}$$

$$= 1.6\times10^{-2} \text{ Am}^2$$

The torque acting on the coil is $\vec{\tau} = \vec{M} \times \vec{B}$

$$= M(\hat{i} \times \hat{j})$$

$$= 1.6 \times 10^{-2} \times \frac{0.05}{\sqrt{2}} \hat{k}$$

$$= 5.66 \times 10^{-4} \text{ (Nm)} \hat{k}$$

9. a. From Fig. A3-9.26 (b) we see that the unit vector normal to loop

$$\hat{n} = -\sin 37° \hat{i} + \cos 37° \hat{j} = -0.6\hat{i} + 0.8\hat{j}$$

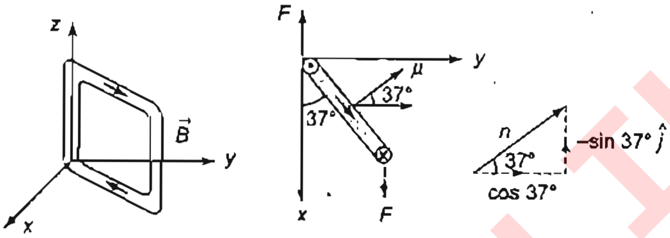


Fig. A3-9.26

The magnetic moment is

$$\mu = NIA\hat{n} = (5)(2)(0.2)^2(-0.6\hat{i} + 0.8\hat{j})$$

$$= -0.24\hat{i} + 0.32\hat{j} \text{ Am}^2$$

b. The torque, $\tau = \mu \times \vec{B} = (-0.24\hat{i} + 0.32\hat{j}) \times (0.5\hat{j})$

$$= -0.12\hat{k} \text{ Nm}$$

c. The potential energy of the loop is $U = -mB \cos \theta$ where $\mu = NIA = 0.4 \text{ Am}^2$ and the position of minimum energy is $\theta = 0$.

Thus, the external work, $W_{ext} = + \Delta U$, needed to rotate it to the given orientation, is given by

$$U_f - U_i = (-\mu B \cos 37°) - (-\mu B \cos 0°)$$

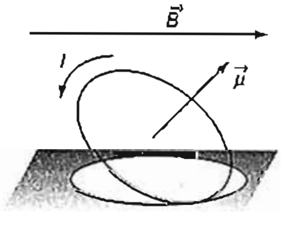
$$= (0.4)(0.5)(1 - 0.8) = 0.04 \text{ J}$$

The external work is positive since the dipole moment is rotated away from alignment with the field.

10. The loop will start to lift off when the magnetic torque equals the gravitational torque as shown in Fig. A3-9.27

The magnetic torque acting on the loop, $\tau_m = \mu B = I\pi R^2 B$.

The gravitational torque exerted on the loop, $\tau_g = mgR$.



(b)

Fig. A3-9.27

11. The magnetic moment of the loop is in the positive z-direction (right hand thumb rule).

a. The magnetic moment of the loop is given by

$$\vec{\mu} = NIA\hat{k} = (12)(3)(0.40)^2 \hat{k}$$

$$= 5.76 \text{ Am}^2 \hat{k}$$

b. The torque on the current loop is given by

$$\vec{\tau} = \vec{\mu} \times \vec{B} = (5.76\hat{k}) \times (0.3\hat{i} + 0.4\hat{k})$$

$$= 1.73 \text{ Nm } \hat{j}$$

c. The potential energy is the negative dot product of $\vec{\mu}$ and $\vec{B}$:

$$U = -\vec{\mu} \cdot \vec{B} = -(5.76\hat{k}) \cdot (0.3\hat{i} + 0.4\hat{k})$$

$$= -2.30 \text{ J}$$

12. The normal to the loop OP makes an angle $\theta = 60°$ with the $+x$ direction, the field direction. Hence,

$$\tau = NIAB \sin \theta$$

$$= (1)(14 A)(\pi \times 25 \times 10^{-4} \text{ m}^2)$$

$$(0.03 \text{ T}) \sin 60°$$

$$= 2.9 \times 10^{-3} \text{ Nm}$$

The right hand rule shows that the loop will rotate about the y-axis, so as to decrease the angle labelled 60°.

13. a. $\tau = \mu B = \pi b^2 IB$, directed out of the page, so that the loop starts to rotate, decreasing the tension in cord 2 by an amount ΔT. But taking torques about A, obtain $L \Delta T = \mu B$, so $\Delta T = (\pi b^2 IB)/L$, and thus $T_1 = T_0 = + \Delta T$ and $T_2 = T_0 = - \Delta T$.

b. Since $\mu \times B = 0$, the tensions are the same as in the absence of a field.

14. Torque on the loop must be equal to the gravitational torque exerted about an axis tangent to the loop. The gravitational torque $= mgr$, while magnetic torque $= |\mu \times B| = \mu B \sin 90° = \pi r^2 IB$. Equating gives

$$I = (mg)/[\pi r(B_x^2 + B_y^2)^{1/2}]$$

Only B_x causes a torque, so $I = (mg)/(\pi r B_x)$.

15. Torque is $\mu \times B = I\pi r^2(\pm k) \times B = I\pi r^2(\pm k) \times (B_x\hat{i} + B_y\hat{j})]$

$= \pm \pi r^2 I(B_x\hat{j} - B_y\hat{i})$. The ± arises since current can circulate in either direction.

16. $M = NIab$

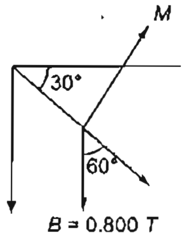


Fig. A3-9.28

$$\tau = MB\sin(90+60°)$$
$$= NIabB\sin 150°$$
$$= 100\times1.2\times0.4\times0.3\times0.8\times\frac{1}{2}$$
$$= 5.76 \text{ Nm}$$

Torque will be about negative y-axis.

17. $$\tau_{\max} = mB \quad = IAB$$
$$= I\pi r^2 B$$
$$= 5\times\pi\times(0.8)^2\times3\times10^{-3}$$
$$= 375\ \pi\times10^{-7}\text{ Nm}$$
$$U_{\max} = +MB\ ,\ U_{\min} = -\vec{M}\cdot\vec{B}$$

Exercise 9.4

1. i. a. Magnetic field due to the upper branch will cancel out that due to the lower branch and will be zero at centre.

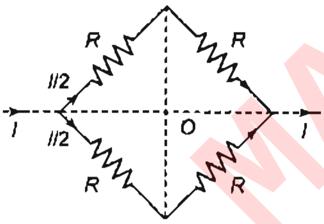


Fig. A3-9.29

b. Magnetic field due to the upper branch will cancel out the magnetic field due to the lower branch.

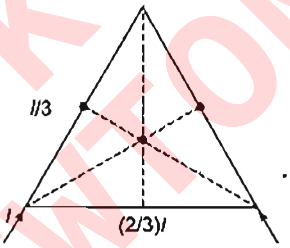


Fig. A3-9.30

ii. Magnetic field due to each pair of wire will cancel out. Net magnetic field will be zero.

iii.

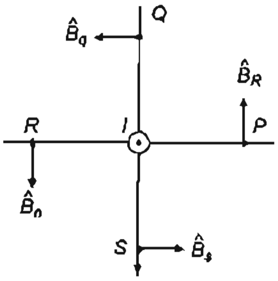


Fig. A3-9.31

2. i.

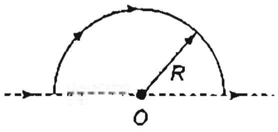


Fig. A3-9.32

Magnetic field due to straight wires = 0.

Magnetic field due to a semicircular wire

$$B_{\text{semicircular}} = \frac{\mu_0 I}{2R}\left(\frac{\theta}{2\pi}\right) = \frac{\mu I}{2R}\left(\frac{\pi}{2\pi}\right)$$

or $$B_{\text{semicircular}} = \frac{\mu_0 I}{4R}$$

The direction can be verified from the right hand rule that will be downward into the plane of the page.

ii.

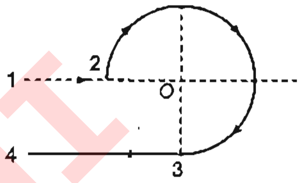


Fig. A3-9.33

Magnetic field due to straight wire 1–2 will be zero as its direction is passing through centre of the circular wire.

Magnetic field due to circular part

$$B_{\text{circular}} = \frac{\mu_0 I}{2r}\left(\frac{\theta}{2\pi}\right)$$
$$= \frac{\mu_0 I}{2r}\left(\frac{(3/2)\pi}{2\pi}\right)$$
$$B_{\text{circular}} = \frac{3\mu_0 I}{8r}\otimes$$

Magnetic field due to semi-infinite wire 3, 4,

$$B_{\text{straight}} = \frac{\mu_0 I}{4\pi R}\otimes$$

Net magnetic field at O

$$B_0 = \frac{3\mu_0 I}{8R} + \frac{\mu_0 I}{4\pi R}$$
$$B_0 = \frac{\mu_0 I}{4R}\left(\frac{3}{2}+\frac{1}{\pi}\right)\otimes$$

iii.

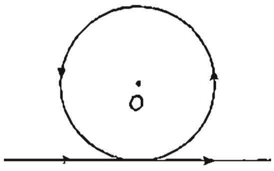


Fig. A3-9.34

Magnetic field at O

$$\vec{B}_0 = \vec{B}_{\text{circle}} + \vec{B}_{\text{straight}}$$

$$= \frac{\mu_0 I}{2R}\odot + \frac{\mu_0 I}{2\pi R}\odot$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2R}\left(1+\frac{1}{\pi}\right)\odot$$

iv.

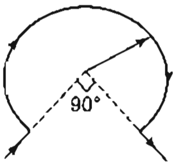


Fig. A3-9.35

The magnetic field due to straight wires will be zero. Magnetic field due to circular part

$$B_{\text{circular}} = \frac{\mu_0 I}{2R}\left(\frac{\theta}{2\pi}\right) = \frac{\mu_0 I}{2R}\left(\frac{(3/2)\pi}{2\pi}\right)$$

$$= \frac{3\mu_0 I}{8R}\otimes$$

Magnetic field at the centre of an arc is given by

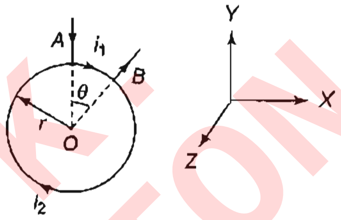


Fig. A3-9.36

$$B = \frac{\mu_0 I}{2R}\times\frac{\theta}{2\pi}$$

Magnetic field due to the smaller arc,

$$\vec{B}_1 = \frac{\mu_0 I_1}{2r}\times\frac{\theta}{2\pi}(-\hat{k})$$

Magnetic field due to the larger arc,

$$\vec{B}_2 = \frac{\mu_0 I_2}{2r}\times\frac{(2\pi-\theta)}{2\pi}(+\hat{k})$$

Resultant magnetic field

$$= \left[-\frac{\mu_0 I_1\theta}{4\pi r} + \frac{\mu_0 I_2(2\pi-\theta)}{4\pi r}\right](\hat{k}) \quad \text{(i)}$$

Two arcs form a parallel combination of resistors. Thus $I_1R_1 = I_2R_2$, where R_1 and R_2 are resistances of respective segments. As the wire is uniform,

$$\frac{R_1}{R_2} = \frac{L_1}{L_2} = \frac{R\theta}{R(2\pi-\theta)}$$

Thus, $$\frac{I_1}{I_2} = \frac{\theta}{(2\pi-\theta)},\ I_2 = \frac{I_1\theta}{2\pi-\theta} \quad \text{(ii)}$$

On substituting I_2 in equation (i), we get

$$\vec{B}_R = \left(\frac{\mu_0 I\theta}{4\pi r} - \frac{\mu_0 I\theta}{4\pi r}\right)\hat{k} = 0$$

Hence the field is independent of θ.

v. $$\vec{B}_R = \left(\frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4R} + \frac{\mu_0 I}{4\pi R}\right)(-\hat{k})$$

$$= \frac{\mu_0 I}{2R}\left(\frac{1}{\pi}+\frac{1}{2}\right)\hat{k}$$

vi. Magnetic field at the centre of an arc is given by

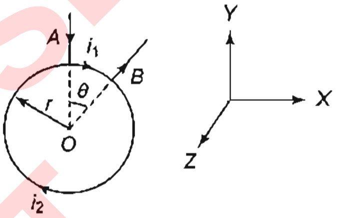


Fig. 9.37

$$\vec{B} = \frac{\mu_0 I}{2R}\times\frac{\theta}{2\pi}$$

Magnetic field due to a smaller arc,

$$\vec{B}_1 = \frac{\mu_0 I_1}{2r}\times\frac{\theta}{2\pi}(-\hat{k})$$

Magnetic field due to a larger arc,

$$\vec{B}_2 = \frac{\mu_0 I_2}{2r}\times\frac{(2\pi-\theta)}{2\pi}(+\hat{k})$$

Resultant magnetic field

$$= \left[\frac{\mu_0 I_1\theta}{4\pi r} + \frac{\mu_0 I_2(2\pi-\theta)}{4\pi r}\right](\hat{k}) \quad \text{(i)}$$

Two arcs form a parallel combination of resistors. Thus $I_1R_1 = I_2R_2$, where R_1 and R_2 are resistances or respective segments.
As the wire is uniform.

$$\frac{R_1}{R_2} = \frac{L_1}{L_2} = \frac{R\theta}{R(2\pi-\theta)}$$

Thus, $$\frac{I_1}{I_2} = \frac{\theta}{(2\pi-\theta)},\ I_2 = \frac{I_1\theta}{2\pi-\theta} \quad \text{(ii)}$$

On substituting I_2 in equation (i), we get

$$\vec{B}_R = \left(\frac{\mu_0 I\theta}{4\pi r} - \frac{\mu_0 I\theta}{4\pi r}\right)\hat{k} = 0$$

Hence the field is independent of q.

3. It is clear that 'B' at 'C' due all the wires is directed ⊗. Also B at 'C' due PQ and SR is same.
Also the magnetic field due to QR and PS is same

$$\therefore \quad B_{res} = 2(B_{PQ} + B_{SP})$$

$$B_{PQ} = \frac{\mu_0 i}{4\pi \frac{a}{2}} (\sin 60° + \sin 60°)$$

$$B_{SP} = \frac{\mu_0 i}{4\pi \frac{\sqrt{3}a}{2}} (\sin 30° + \sin 30°)$$

$$B_{res} = 2\left(\frac{\sqrt{3}\,\mu_0 i}{2\pi a} + \frac{\mu_0 i}{2\pi a\sqrt{3}}\right) = \frac{4\mu_0 i}{\sqrt{3}\pi a}$$

4. The current will be equally divided at A. The fields at the centre due to the currents in the wires AB and DC will be equal in magnitude and opposite in direction. The resultant of these two fields will be zero. Similarly, the resultant of the fields due to the wires AD and BC will be zero. Hence, the net field at the centre will be zero.

5. **i.** Let us call $\vec{B}$ due to (1) and (2) as $\vec{B}_1$ and $\vec{B}_2$, respectively.

At A, $\vec{B}_1$ is ⊙ and $\vec{B}_2$ is ⊗

$$B_1 = \frac{\mu_0 2I}{2\pi a} \text{ and } B_2 = \frac{\mu_0 I}{2\pi 2a}$$

$$\therefore \quad B_{res} = B_1 - B_2 = \frac{3}{4}\frac{\mu_0 I}{\pi a} \odot$$

At C, $\vec{B}_1$ is ⊗ and $\vec{B}_2$ is also ⊗

$$\therefore \quad B_{res} = B_1 + B_2 = \frac{\mu_0 2I}{2\pi \frac{a}{2}} + \frac{\mu_0 I}{2\pi \frac{a}{2}} = \frac{6\mu_0 I}{2\pi a}$$

$$= \frac{3\mu_0 I}{\pi a} \otimes$$

At D, $\vec{B}_1$ is ⊗ and $\vec{B}_2$ is ⊙ and both are equal in magnitude.

$$\therefore \quad B_{res} = 0$$

ii. It is clear from the above solution that $B = 0$ at point 'D'.

6. Let magnetic field due to W_1 be $\vec{B}_1$ and that due to W_2 be $\vec{B}_2$. By symmetry,

$$|\vec{B}_1| = |\vec{B}_2| = B$$

$$B_p = 2B \cos 60° = B = \frac{\mu_0 I}{2\pi 2a} = \frac{\mu_0 I}{4\pi a}$$

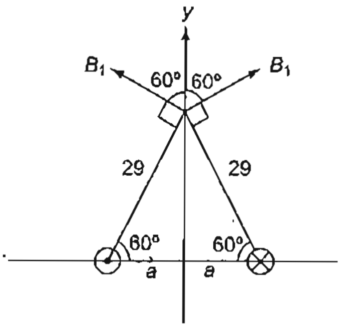


Fig. A3-9.38

$$\therefore \quad \vec{B}_p = \frac{\mu_0 I}{4\pi a}\hat{j}$$

$$\text{Now,} \quad B_1 = \frac{\mu_0 I}{2\pi\sqrt{5}a}, B_2 = \frac{\mu_0 I}{2\pi a}$$

$$\tan\theta = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$$

$$\bar{B} = (B_1 \cos\theta\,\hat{j}) + (B_2 - B_1 \sin\theta)\hat{i}$$

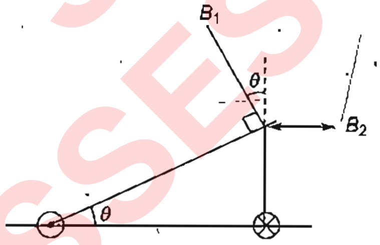


Fig. A3-9.39

$$\sin\theta = \sqrt{\frac{3}{5}}$$

$$= \frac{\mu_0 I}{5\pi a}\hat{j} + \left(\frac{\mu_0 I}{2\pi\sqrt{3}a} - \frac{\sqrt{3}\mu_0 I}{10\pi a}\right)\hat{j}$$

$$\cos\theta = \frac{2}{\sqrt{5}}$$

7. To find 'B' at 'P' the sheet can be considered as collection of large number of infinitely long wires. Take a long wire distance 'x' from 'P' and of width 'dx'. Due to this the magnetic field at 'P' is 'dB'

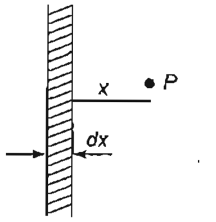


Fig. A3-9.40

$$dB = \frac{\mu_0 \left(\frac{I}{w}dx\right)}{2\pi x} \otimes$$

due to each such wire will be directed inwards

$$\therefore \quad B_{res} = \int dB = \frac{\mu_0 I}{2\pi w}\int_{x=a}^{a+w} \frac{dx}{x} = \frac{\mu_0 I}{2\pi w}$$

$$= \ln\frac{a+w}{a}$$

8. As the solenoids are identical, the currents in Q and R will be the same and will be half the current in P. The magnetic field within a solenoid is given by $B = \mu_0 ni$. Hence the field in Q will be equal to the field in R and will be half the field in P. i.e., 1.0 T.

9. Assume $PO = x$, so $PQ = x \sin(\alpha/2)$

Magnetic field B at P due to either segment of wire is

$$B = \frac{\mu_0 I}{4\pi x \sin(\alpha/2)}\left[1+\cos\frac{\alpha}{2}\right]$$

Net magnetic field at P is

$$B_{\text{net}} = \frac{\mu_0 I}{2\pi x \sin(\alpha/2)}[1+\cos(\alpha/2)]$$

10. From the figure in question, $\vec{B}_0 = \vec{B}_{AB} + \vec{B}_{BCD} + \vec{B}_{DE}$

$$\Rightarrow \qquad \vec{B}_0 = \left(\frac{\mu_0 I}{4\pi R}\right)(-\hat{j}) + \left[\frac{\mu_0 I}{4R} + \frac{\mu_0 I}{4\pi R}\right](-\hat{i})$$

11. Current in the ring $I = \dfrac{\omega q}{2\pi}$

Magnetic field $\quad B = \dfrac{\mu_0 I}{2r} = \dfrac{\mu_0 \omega q}{2\pi \times 2r}$

$$\Rightarrow \qquad B = \frac{\mu_0 \omega q}{4\pi r}$$

12. Choose a circular path centred on the conductor's axis and apply Ampere's law.

i. To find the current through the area enclosed by the path

$$I dA = (Kr^2)(2\pi r dr)$$

$$\therefore \qquad I = K\int_0^r 2\pi r^3 dr = \frac{\pi K r^4}{2}$$

Since $\quad \oint \vec{B}\cdot d\vec{\ell} = \mu_0 I \quad \Rightarrow B\,2\pi r = \mu_0 \dfrac{\pi K r^4}{2}$

$$\Rightarrow \qquad B = \frac{\mu_0 K r^3}{4}$$

ii. If $r > a$, then net current through the Amperian loop is

$$I' = \int_0^a Kr^2 2\pi r dr = \frac{\pi K a^4}{2}$$

Therefore $\quad B = \dfrac{\mu_0 K a^4}{4r}$

13. Consider Fig. A3-9.41

$$\vec{B}_p = \left(\vec{B}_1\right)_p + \left(\vec{B}_2\right)_p + \left(\vec{B}_3\right)_p + (\vec{B}_4)_P + (\vec{B}_5)_P$$

where $\left(\vec{B}_1\right)_p = \dfrac{\mu_0 i}{4\pi\left(\dfrac{3a}{2}\right)}(-\hat{j})$ (semi-infinite wire)

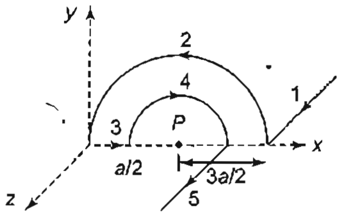


Fig. A3-9.41

$$(\vec{B}_2)_p = \frac{\mu_0 i}{4\left(\dfrac{3a}{2}\right)}(+\hat{k})$$

$$(\vec{B}_3)_p = 0$$

$$(\vec{B}_4)_p = \frac{\mu_0 i}{4\left(\dfrac{a}{2}\right)}(-\hat{k}) \Rightarrow (\vec{B}_5)_p = \frac{\mu_0 i}{4\pi\left(\dfrac{a}{2}\right)}(-\hat{j})$$

$$\vec{B}_p = \frac{\mu_0 i}{2a}\left[-\left(\frac{1}{3\pi}+\frac{1}{\pi}\right)\hat{j} - \left(1-\frac{1}{3}\right)\hat{k}\right]$$

$$\bar{B}_p = \frac{2\mu_0 i}{3a}\left[\frac{1}{\pi}\hat{j} - \hat{k}\right]$$

$$\bar{B}_p = \frac{\mu_0 i}{3\pi a}\sqrt{1+\pi^2}$$

14.

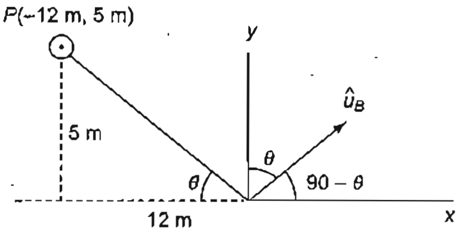


Fig. A3-9.42

Unit vector of $\hat{u}_B$:

$$\hat{u}_B = \hat{i}\cos(90-\theta) + \hat{j}\cos\theta$$

$$= \sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$$\sin\theta = \frac{5}{\sqrt{5^2+12^2}} = \frac{5}{13}$$

$$\cos\theta = \frac{12}{\sqrt{5^2+12^2}} = \frac{12}{13}$$

$$\hat{u}_B = \frac{5}{13}\hat{i} + \frac{12}{13}\hat{j} = \frac{1}{13}(5\hat{i}+12\hat{j})$$

15. The magnetic fields at the given points are

$$dB_a = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{(200\text{ A})(0.000100\text{ m})}{(0.100\text{ m})^2}$$

$$= 2.00\times10^{-6}\text{ T}$$

$$dB_b = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{(200\text{ A})(0.000100\text{ m})\sin 45^\circ}{2(0.100\text{ m})^2}$$

$$= 0.705\times10^{-6}\text{ T}$$

$$dB_c = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{(200\text{ A})(0.000100\text{ m})}{(0.100\text{ m})^2}$$

$$= 2.00\times10^{-6}\text{ T}$$

$$dB_d = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,dl\sin(0^\circ)}{r^2} = 0$$

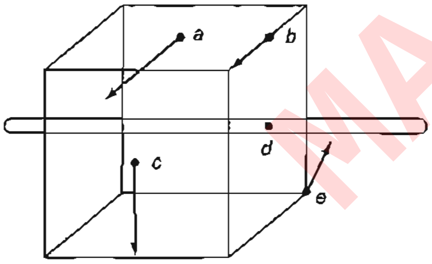


Fig. A3-9.43

$$dB_e = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{(200\text{ A})(0.00100\text{ m})}{3(0.100\text{ m})^2}\frac{\sqrt{2}}{\sqrt{3}}$$

$$= 0.545\times10^{-6}\text{ T}$$

16. The contributions from the straight segments is zero since

$$d\vec{l}\times\vec{r} = 0$$

The magnetic field from the curved wire is just one quarter of a full loop:

$$B = \frac{1}{4}\left(\frac{\mu_0 I}{2R}\right),$$

and is out of the page.

17. At the centre of the circular loop the current I_2 generates a magnetic field that is into the page. So the current I_1 must point to the right. For complete cancellation the two fields must have the same magnitude

$$\frac{\mu_0 I_1}{2\pi D} = \frac{\mu_0 I_2}{2R}$$

Thus $$I_1 = \frac{\pi D}{R}I_2$$

18. Let current flowing through coil 1 be i_1 and that through coil 2 be i_2.

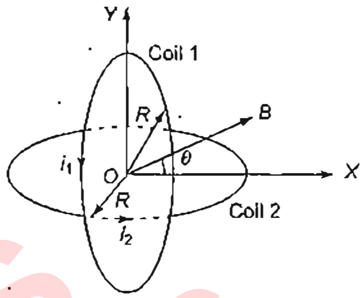


Fig. A3-9.44

Radius of each coil = R. Hence, magnetic field at O due to coil 1 is

$$\vec{B}_1 = \frac{\mu_0 i_1}{2R}\hat{i}$$

and magnetic field at O due to coil 2 is

$$\vec{B}_2 = \frac{\mu_0 i_2}{2R}\hat{j}$$

Therefore, net magnetic field $\vec{B} = \vec{B}_1 + \vec{B}_2$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 i_1}{2R}\hat{i} + \frac{\mu_0 i_2}{2R}\hat{j}$$

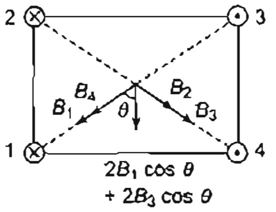
$$\Rightarrow \quad |\vec{B}| = \frac{\mu_0}{2R}\sqrt{(i_1^2 + i_2^2)}$$

19. $B_{\text{net}} = 4B\cos 45^\circ = \frac{4}{\sqrt{2}}\frac{\mu_0 I}{2\pi r}$

$$= \frac{2\sqrt{2}\mu_0 5}{2\pi\sqrt{2}\times10^{-1}\text{ m}}$$

$$B_{\text{net}} = \frac{5\mu_0}{\pi}\times10T(-\hat{j})$$

$$B_{\text{net}} = 2\times10^{-5}T(-\hat{j})$$



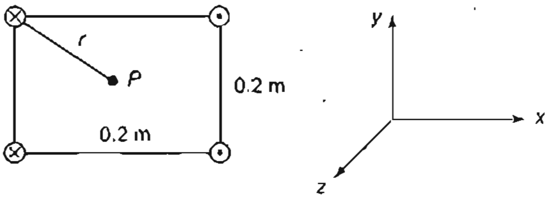


Fig. A3-9.45

20. Given, $\vec{B} = 2\times10^{-3}$ T

Now, direction of $\vec{B}$ due to the wire is ⊙ in the left half and ⊗ in the right half.

$\therefore$ $B_{net} = 0$ in the left half

$|B_{wire}| = |\vec{B}|$ in the zy plane

$$\frac{\mu_0 10}{2\pi r} = 2\times10^{-3}\text{ T}$$

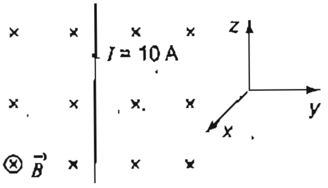


Fig. A3-9.46

$$r = \frac{4\times10^{-7}\times5}{\pi\times2\times10^{-3}}$$
$$= 10^{-3}\text{ m}$$

Thus magnetic field is zero at $r = 1$ mm to the left of the wire in the indicated yz plane.

21.
$$|B_P| = \frac{\mu_0 i}{4\pi d}$$
$$|B_R| = \frac{\mu_0 i}{4\pi d}$$

At P and R, the field due to vertical part is zero and the points are equidistant from the horizontal wire. Similarly, the points S and Q are equidistant from the vertical wire and field due to horizontal wire is zero.

Hence $|B_Q| = |B_S| = \dfrac{\mu_0 i}{4\pi d}$

$\Rightarrow$ $|B_P| = |B_Q| = |B_R| = |B_S| = \dfrac{\mu_0 i}{4\pi d}$

22. $B_{net} = 2B\cos\phi = 2\times\dfrac{\mu_0 I}{2\pi r}\cos\phi$

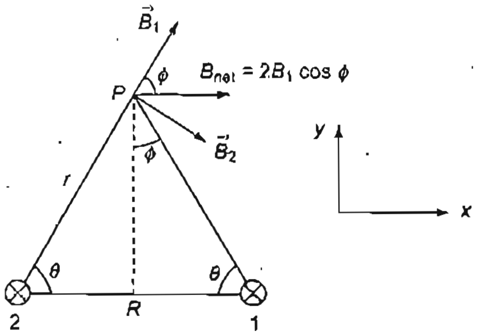


Fig. A3-9.47

Let $r = \dfrac{R}{2\cos\theta}$ and $\phi = 90 - \theta$

Then $B_{net} = \dfrac{\mu_0 I}{\pi \dfrac{R}{2}}\cos\theta\cos(90-\theta)$

$\Rightarrow$ $\bar{B}_{net} = \dfrac{2\mu_0 I}{\pi R}\cos\theta\sin\theta(\hat{i})$ towards right

23. Magnetic field at point P due to part AD = – magnetic field due to part BC

$\Rightarrow$ $\vec{B}_{AD} = -\bar{B}_{BC}$

$\Rightarrow$ $\vec{B}_{AD} + \bar{B}_{BC} = 0$

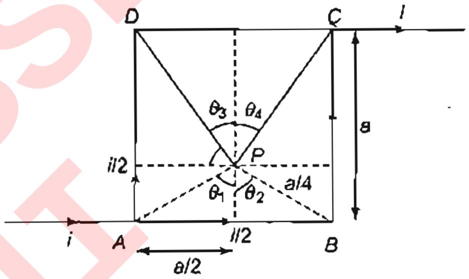


Fig. A3-9.48

$$\bar{B}_{AB} = \frac{\mu_0(i/2)}{4\pi(a/4)}(\sin\theta_1+\sin\theta_2)\odot$$
$$= \frac{\mu_0 i}{2\pi a}\left(\frac{a/2}{a\sqrt{5}/4}+\frac{a/2}{a\sqrt{5}/4}\right)\odot$$
$$= \frac{\mu_0 i}{2\pi a}\times2\times\frac{a}{2}\times\frac{4}{a\sqrt{5}}\odot = \frac{2\mu_0 i}{\sqrt{5}\pi a}\odot$$

$\therefore$ $$\bar{B}_{AB} = \frac{2\sqrt{5}\mu_0 i}{5\pi a}\odot$$

$$\bar{B}_{CD} = \frac{\mu_0(i/2)}{4\pi(3a/4)}(\sin\theta_3+\sin\theta_4)\otimes$$
$$= \frac{\mu_0(i/2)}{3\pi a}\times2\times\frac{a}{2}\times\frac{4}{a\sqrt{3}}\otimes$$
$$= \frac{2\mu_0 i}{3\sqrt{13}\pi a}\otimes = \frac{4\sqrt{13}\mu_0 i}{39\pi a}\otimes$$

$\therefore$ $$\bar{B}_{net} = \frac{\sqrt{5}\times2\mu_0 i}{5\pi a}\odot + \frac{2\sqrt{13}\mu_0 i}{39\pi a}\otimes$$
$$= \frac{2\mu_0 i}{\pi a}\left(\frac{1}{\sqrt{5}}-\frac{1}{3\sqrt{13}}\right)\odot$$

24. Let us consider a point P at a distance (x, y) from the wire 1.

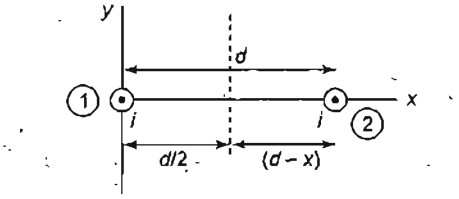


Fig. A3-9.49

$$0 \le x \le d$$

Hence $\quad \bar{B}_1 = \dfrac{\mu_0 i}{2\pi x}\hat{j}$

The point is at a distance $d - x$ from wire 2.

$$\therefore \quad \bar{B}_2 = -\frac{\mu_0 i}{2\pi(d-x)}\hat{j}$$

$$\therefore \quad \bar{B}_{\text{net}} = \vec{B}_1 + \bar{B}_2 = \frac{\mu_0 i}{2\pi x}\hat{j} - \frac{\mu_0 i}{2\pi(d-x)}\hat{j}$$

$$= \frac{\mu_0 i(d-2x)}{2\pi x(d-x)}\hat{j}$$

25. Magnetic field at Q:
As any segment of wire is considered as infinite,

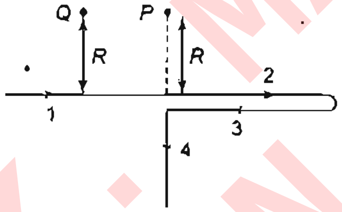


Fig. A3-9.50

$$\bar{B}_Q = \frac{\mu_0 I}{2\pi R}\odot$$

(The wire segments (2), (3) and (4) are considered at large distances from P.)
Magnetic field at P:
The magnetic fields due to segments (2) and (3) get cancelled and the field due to segment (4) will be zero. Hence magnetic field at P will be due to semi-infinite segment (1).

$$\bar{B}_P = \frac{\mu_0 I}{4\pi R}\odot$$

26. Magnetic field at O due to wire 1 is

$$\bar{B}_1 = \frac{\mu_0 i}{4\pi R}\odot$$

and that due to wire 2 is

$$\bar{B}_2 = \frac{\mu_0 i}{4\pi R}\odot \Rightarrow \bar{B} = \bar{B}_1 + \bar{B}_2 = \frac{\mu_0 i}{2\pi R}\odot$$

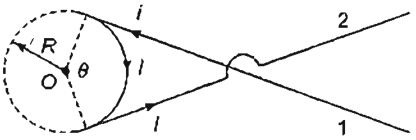


Fig. A3-9.51

Magnetic field at the centre due to circular arc subtending angle θ at the centre is given by

$$\bar{B}_{\text{arc}} = \frac{\mu_0 i}{2R}\left(\frac{\theta}{2\pi}\right)\otimes$$

If $\quad \bar{B}_{\text{net}} = 0 \Rightarrow \bar{B} + \bar{B}_{\text{arc}} = 0$

$$\Rightarrow \quad \bar{B} = -\bar{B}_{\text{arc}}$$

$$\Rightarrow \quad \frac{\mu_0 i}{2\pi R}\odot = -\frac{\mu_0 i}{2R}\left(\frac{\theta}{2\pi}\right)\otimes$$

$$\Rightarrow \quad \frac{\mu_0 i}{2\pi R} = \frac{\mu_0 i}{2R}\left(\frac{\theta}{2\pi}\right) \Rightarrow \frac{1}{\pi} = \frac{\theta}{2\pi}$$

$$\Rightarrow \quad \theta = 2 \text{ rad}$$

27. Suppose wire carries current i_1 and in the same direction as that of the pipe.

Magnetic field at the centre of the pipe $= -\dfrac{\mu_0 i_1}{6\pi R}\hat{j} = \dfrac{\mu_0 i_1}{6\pi R}(-\hat{j})$

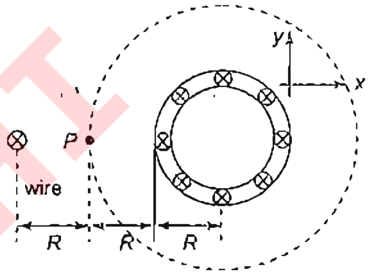


Fig. A3-9.52

Magnetic field at point P:

$$\frac{\mu_0 i_0}{2\pi(2R)}\hat{j} - \frac{\mu_0 i_1}{2\pi R}\hat{j} = \left(\frac{\mu_0 i_0}{4\pi R} - \frac{\mu_0 i_1}{2\pi R}\right)\hat{j}$$

Now magnitudes are equal:

$$\frac{\mu_0 i_1}{6\pi R} = \frac{\mu_0 i_0}{4\pi R} - \frac{\mu_0 i_1}{2\pi R} \Rightarrow i_1 = \frac{3i_0}{8}$$

28. For magnetic field to be zero at P,

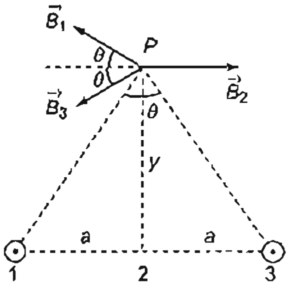


Fig. A3-9.53

$$\bar{B}_P = 0; \; 2B_1\cos\theta = B_2$$

$$2\frac{\mu_0 I}{2\pi\sqrt{a^2+y^2}}\frac{y}{\sqrt{a^2+y^2}} = \frac{\mu_0 I}{2\pi y}$$

$$\Rightarrow \quad y = \pm a$$

29. $\vec{B}_P = \frac{\mu_0}{4\pi}\frac{\pi I}{R_1}\otimes + \frac{\mu_0}{4\pi}\frac{\pi I}{R_2}\otimes$

$$= \frac{\mu_0}{4\pi}\pi I\left[\frac{1}{R_1}+\frac{1}{R_2}\right]\otimes$$

30. $B = \frac{\mu_0}{4\pi}\frac{I}{d}\ (\sin\theta_1 + \sin\theta_2)$

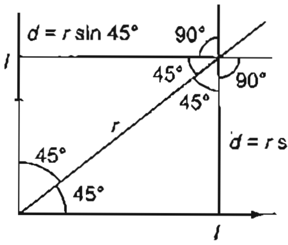


Fig. A3-9.54

Here, $d = r\sin 45^\circ = \frac{r}{\sqrt{2}}$

$$B = 2\times\frac{\mu_0}{4\pi}\frac{I}{\left(\frac{r}{\sqrt{2}}\right)}(\sin 45^\circ + \sin 90^\circ)$$

$$= 2\sqrt{2}\frac{\mu_0}{4\pi}\frac{I}{r}\left(\frac{1}{\sqrt{2}}+1\right) = \frac{\mu_0}{2\pi}\frac{I}{r}(\sqrt{2}+1)$$

31. $B_y = \frac{\mu_0}{4\pi}\frac{2\pi I(2r)^2}{[(2r)^2+d^2]^{3/2}}$, $B_x = \frac{\mu_0}{4\pi}\frac{2\pi I r^2}{\left[r^2+\left(\frac{d}{2}\right)^2\right]^{3/2}}$

$$\Rightarrow \quad \frac{B_y}{B_x} = \frac{4}{\left[4r^2+d^2\right]^{3/2}}\times\left[\frac{4r^2+d^2}{4}\right]^{3/2} = \frac{1}{2}$$

32. When the coils are connected in parallel, the ratio of currents flowing in these coils will be equal to reciprocal of the ratio of their resistances.

$\therefore \quad \frac{i_1}{i_2} = \frac{R_2}{R_1} = \frac{\rho(\ell_2/A)}{\rho(\ell_1/A)}$ (as both the coils are made of similar wires)

$\Rightarrow \quad \frac{\ell_2}{\ell_1} = \frac{2\pi r_2}{2\pi r_1} = \frac{r_2}{r_1} \quad \therefore \quad \frac{B_1}{B_2} = \frac{(\mu_0 i_1)/2r_1}{(\mu_0 i_2)/2r_2}$

$\therefore \quad \frac{B_1}{B_2} = \frac{i_1}{i_2}\times\frac{r_2}{r_1} \qquad \left[\because \frac{i_1}{i_2} = \frac{r_2}{r_1}\right]$

$\therefore \quad \frac{B_1}{B_2} = \frac{r_2^2}{r_1^2} = \left(\frac{40}{20}\right)^2 = \frac{4}{1} = 4$

33.

Case I: $\vec{B}_{net} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$

$$= \left[\frac{\mu_0 I}{4\pi\ell}[0+\sin 45^\circ] + \frac{\mu_0 I}{4\pi\ell}[\sin 45^\circ + \sin 45^\circ] + \frac{\mu_0 I}{4\pi\ell}[0+\sin 45^\circ]\right]\otimes$$

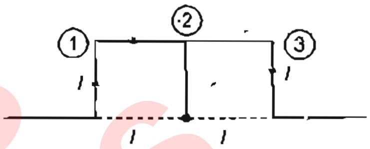


Fig. A3-9.55

$\therefore \quad |\vec{B}_{net}| = \frac{\mu_0 I}{\sqrt{2}\pi\ell}$

Case II : $\quad 2\pi r = 4\ell \quad \Rightarrow r = \frac{2\ell}{\pi}$

Magnetic field at the centre of semicircle,

$$B_{semicircle} = \frac{\mu_0 I}{4r} = \frac{\mu_0 I\pi}{8\ell}$$

$$\frac{B_1}{B_2} = \frac{\mu_0 I}{\sqrt{2}\pi\ell}\times\frac{8\ell}{\mu_0 I\pi} = \frac{4\sqrt{2}}{\pi^2}$$

34. Equivalent current in the belt $= sv$ (current per unit width)

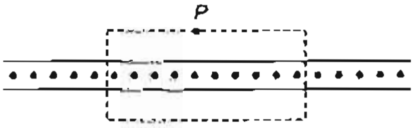


Fig. A3-9.56

$\therefore$ Linear current density of belt is $j = sv$

Using Ampere's law we can calculate magnetic field near the belt as

$$B = \frac{\mu_0 j}{2} = \frac{\mu_0 sv}{2}$$

35. The magnetic field due to $\vec{B}_1 < \vec{B}_5$ will be

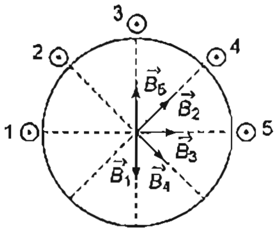


Fig. A3-9.57

$$\vec{B}_{net} = \vec{B}_3 + 2\vec{B}_2\cos 45^\circ$$

$$= \frac{\mu_0 I}{2\pi r}\hat{i} + 2\frac{\mu_0 I}{2\pi r}\frac{1}{\sqrt{2}}\hat{i}$$

$$= \frac{\mu_0 I}{2\pi r}(1+\sqrt{2})\,\hat{j}$$

36. Magnetic field due to circular loop, $\vec{B}_1 = \frac{\mu_0 i}{2a}\frac{120}{360}(\hat{k}) = \frac{\mu_0 i}{6a}\hat{k}$.

Due to straight wire,

$$\vec{B}_2 = \frac{\mu_0 i 2}{4\pi a} \times 2\sin 60(-\hat{k})$$

$$= \frac{\sqrt{3}}{2}\frac{\mu_0 i}{\pi a}(-\hat{k})$$

$$\therefore \quad \vec{B}_{\text{net}} = \frac{\sqrt{3}}{2}\frac{\mu_0 i}{\pi a}(-\hat{k}) + \frac{\mu_0 i}{6a}(\hat{k})$$

$$= \frac{\mu_0 i}{2\pi a}\left\{\sqrt{3} - \frac{\pi}{3} - (-\hat{k})\right\}$$

37. **a.** At the point exactly midway between the wires, the two magnetic fields are in opposite directions and cancel.

b. At a distance a above the top wire, the magnetic fields are in the same direction and add up:

$$\bar{B} = \frac{\mu_0 I}{2\pi r_1}\hat{k} + \frac{\mu_0 I}{2\pi r_2}\hat{k} = \frac{\mu_0 I}{2\pi a}\hat{k} + \frac{\mu_0 I}{2\pi(3a)}\hat{k}$$

$$= \frac{2\mu_0 I}{3\pi a}\hat{k}$$

c. At a distance a below the lower wire, magnitude of magnetic field is same as part (b) but is in the opposite direction:

$$\bar{B} = -\frac{2\mu_0 I}{3\pi a}\hat{k}$$

38. The total magnetic field is the vector sum of the constant magnetic field and the wire's magnetic field.

a. At (0, 0, 1 m):

$$\bar{B} = \bar{B}_0 - \frac{\mu_0 I}{2\pi r}\hat{i}$$

$$= (1.50 \times 10^{-6}\ \text{T})\hat{i} - \frac{\mu_0(8.00\ \text{A})}{2\pi(1.00\ \text{m})}\hat{i}$$

$$= -(1.0 \times 10^{-7}\ \text{T})\hat{i}$$

b. At (1 m, 0, 0):

$$\bar{B} = \bar{B}_0 + \frac{\mu_0 I}{2\pi r}\hat{k}$$

$$= (1.50 \times 10^{-6}\ \text{T})\hat{i} + \frac{\mu_0(8.00\ \text{A})}{2\pi(1.00\ \text{m})}\hat{k}$$

$$\bar{B} = (1.50 \times 10^{-6}\ \text{T})\hat{i} + (1.6 \times 10^{-6}\ \text{T})\hat{k}$$

$$= 2.19 \times 10^{-6}\ \text{T}$$

at $\theta = 46.8°$ from x to z.

c. At (0, 0, – 0.25 m):

$$\bar{B} = \bar{B}_0 + \frac{\mu_0 I}{2\pi r}\hat{i}$$

$$= (1.50 \times 10^{-6}\ \text{T})\hat{i} + \frac{\mu_0(8.00\ \text{A})}{2\pi(0.25\ \text{m})}\hat{i}$$

$$= (7.9 \times 10^{-6}\ \text{T})\hat{i}$$

39. There is no contribution from the straight wires, and now we have two oppositely oriented contributions from the two semicircles:

$$B = (B_1 - B_2) = \frac{1}{2}\left(\frac{\mu_0}{2R}\right)|I_1 - I_2|,$$

into the page. Note that if the two currents are equal, the magnetic field goes to zero at the centre of the loop.

40. The forces on the top and bottom segments cancel, leaving the left and right sides:

$$B = B_a - B_b = \frac{1}{2}\left(\frac{\mu_0 I}{2}\right)\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$B = \frac{\mu_0 I}{4a}\left(1 - \frac{a}{b}\right), \text{out of the page.}$$

41. For L_1, $\oint \vec{B}\cdot d\vec{\ell} = \mu_0(I_1 - I_2)$. Here I_1 is taken positive because magnetic lines of force produced by I_1 is anti-lockwise as seen from top. I_2 produces lines of $\vec{B}$ in clockwise sense as seen from top. The sense of $d\vec{\ell}$ is anticlockwise as seen from top.

$$\text{For} \quad L_2: \oint \vec{B}\cdot d\vec{\ell} = \mu_0(I_1 - I_2 + I_4)$$

$$\text{For} \quad L_3: \oint \vec{B}\cdot d\vec{\ell} = 0$$

42. **a.** $\oint \bar{B}\cdot d\bar{l} = \mu_0 I_{\text{encl}} = 3.83 \times 10^{-4}$ Tm

$$I_{\text{encl}} = 305\ \text{A}$$

b. -3.83×10^{-4} since $d\vec{\ell}$ points opposite to $\vec{B}$ everywhere.

43. Consider a coaxial cable where the currents run in opposite directions.

a. For $a < r < b$, $I_{\text{encl}} = I$

$$\oint \bar{B}\cdot d\bar{l} = \mu_0 I \quad \Rightarrow B 2\pi r = \mu_0 I \quad \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

b. For $r > c$, the enclosed current is zero, so the magnetic field is also zero.

44. **a.** Below the sheet, all the magnetic field contributions from different wires add up to produce a magnetic field that points in the positive x-direction. (Components in the z-direction cancel.) Using Ampere's law, where we use the fact that the field is antisymmetrical above and below the current sheet, and that the legs of the path perpendicular provide nothing to the integral. So, at a distance beneath the sheet the magnetic field is

$$I_{\text{encl}} = nLI \quad \Rightarrow \oint \vec{B}\cdot d\vec{l} = B2L = \mu_0 nLI$$

$$B = \frac{\mu_0 nI}{2},$$

in the positive x-direction. (Note that there is no dependence on a.)

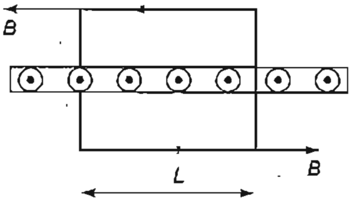


Fig. A3-9.58

b. The field has the same magnitude above the sheet, but points in the negative x-direction.

IIT-JEE 2010 Solved Paper

Physics

Paper 1

Objective Type

Multiple choice questions with one correct answer

1. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistances R_{100}, R_{60} and R_{40}, respectively, the relation between these resistances is

a. $\frac{1}{R_{100}} = \frac{1}{R_{40}} + \frac{1}{R_{60}}$ **b.** $R_{100} = R_{40} + R_{60}$

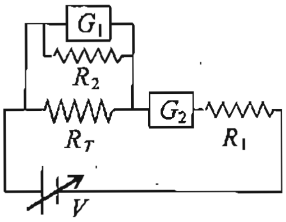
c. $R_{100} > R_{60} > R_{40}$ **d.** $\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

Sol. d.

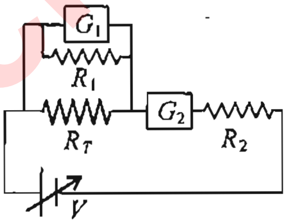
Power $\propto 1/R$

2. To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is

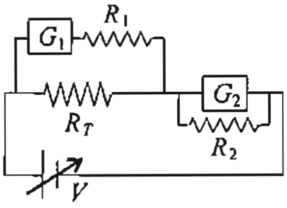
a.



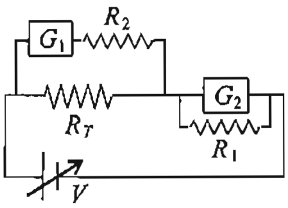
b.



c.



d.



Sol. c.

G_1 is acting as voltmeter and G_2 is acting as ammeter.

3. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased

a. the bulb glows dimmer

b. the bulb glows brighter

c. total impedance of the circuit is unchanged

d. total impedance of the circuit increases

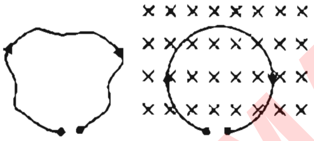
Sol. b.

Impedance $Z = \sqrt{\frac{1}{(\omega C)^2} + R^2}$

as ω increases, Z decreases.

Hence, bulb will glow brighter.

4. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is



a. IBL **b.** $\frac{IBL}{\pi}$

c. $\frac{IBL}{2\pi}$ **d.** $\frac{IBL}{4\pi}$

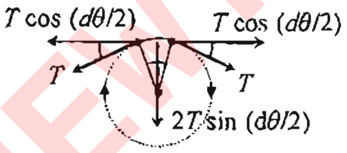
Sol. c.

$2T\sin\frac{d\theta}{2} = BiRd\theta$

Td

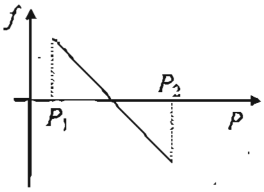
$\theta = BiRd\theta$ (for θ small)

$T = BiR = \frac{BiL}{2\pi}$

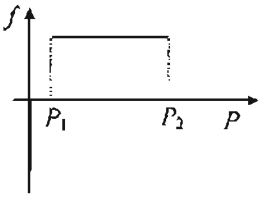


5. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the fictional force f versus P graph will look like

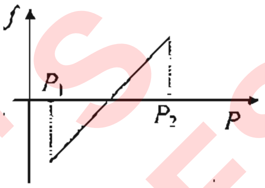
a.



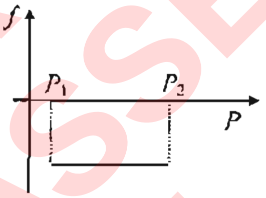
b.



c.



d.



Sol. a.

Initially, the frictional force is upwards as P increases frictional force decreases.

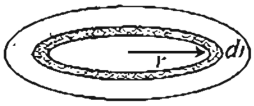
6. A thin uniform annular disc (as shown in the figure) of mass M has outer radius $4R$ and inner radius $3R$. The work required to take a unit mass from point P on its axis to infinity is

a. $\frac{2GM}{7R}(4\sqrt{2} - 5)$ **b.** $-\frac{2GM}{7R}(4\sqrt{2} - 5)$

c. $\frac{GM}{4R}$ **d.** $\frac{2GM}{5R}(\sqrt{2} - 1)$

Sol. a.

$$V = -\int_{3R}^{4R} \frac{\sigma 2\pi r dr G}{\sqrt{r^2 + 16R^2}}$$



7. Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure is

a. directly proportional to L

b. directly proportional to t

c. independent of L

d. independent of t

Sol. c.

We see that $R = \frac{\rho L}{Lt}$.

8. A real gas behaves like an ideal gas if its

a. pressure and temperature are both high

b. pressure and temperature are both low

c. pressure is high and temperature is low

d. pressure is low and temperature is high

Sol. d. Self-explaintory

Multiple choice questions with one or more than one correct answers

9. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses?

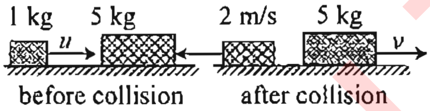
a. Total momentum of the system is 3 kg ms^{-1}

b. Momentum of 5 kg mass after collision is 4 kg ms^{-1}

c. Kinetic energy of the centre of mass is 0.75 J

d. Total kinetic energy of the system is 4J

Sol. a, c.



By conservation of linear momentum

$u = 5v - 2$ (i)

By Newton's experimental law of collision

$u = v + 2$ (ii)

Using Eqs. (i) and (ii), we have

$v = 1$ m/s and $u = 3$ m/s

Kinetic energy of the centre of mass $= \frac{1}{2} m_{\text{system}} v^2_{\text{cm}}$ = 0.75 J.

10. One mole of an ideal gas in initial state A undergoes a cyclic process $ABCA$, as shown in the figure. Its pressure at A is P_0. Choose the correct option(s) from the following:

a. Internal energies at A and B are the same

b. Work done by the gas in process AB is P_0V_0 ln4

c. Pressure at C is $P_0/4$

d. Temperature at C is $T_0/4$

Sol. a, b.

It is found that process AB is isothermal process.

11. A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 sec for this and records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) is (are) true?

a. Error ΔT in measuring T, the time period, is 0.05 seconds

b. Error ΔT in measuring T, the time period, is 1 second

c. Percentage error in the determination of g is 5%

d. Percentage error in the determination of g is 2.5%

Sol. a, c.

$$\frac{\Delta T}{T} = \frac{\Delta t}{t} = \frac{1}{40}$$

$\Delta T = 0.05$ sec

$$g = \frac{4\pi^2 Ln^2}{t^2}$$

$$\frac{\Delta g}{g} = \frac{2\Delta t}{t}$$

$$\% \text{ Error} = \frac{2\Delta t}{t} \times 100 = 5\%$$

12. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that

a. $|Q_1| > |Q_2|$

b. $|Q_1| < |Q_2|$

c. at a finite distance to the left of Q_1, the electric field is zero

d. at a finite distance to the right of Q_2, the electric field is zero

Sol. a, d.

Number of electric field lines of forces emerging from Q_1 are larger than terminating at Q_2.

13. A ray OP of monochromatic light is incident on the face AB of prism $ABCD$ near vertex B at an incident angle of 60°. If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct?

a. The ray gets totally internally reflected at face CD

b. The ray comes out through face AD

c. The angle between the incident ray and the emergent ray is 90°

d. The angle between the incident ray and the emergent ray is 120°

Sol. a, b, c.

Using Snell's law, we get

$$\sin^{-1}\frac{1}{\sqrt{3}} < \sin^{-1}\frac{1}{\sqrt{2}}$$

Net deviation is 90°

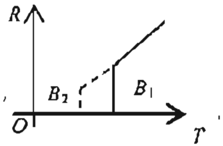
Linked comprehension type

Problems 14–15:

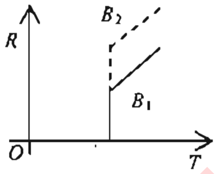
Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_c(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_c(0)$ if they are placed in a magnetic field, i.e., the critical temperature $T_c(B)$ is a function of the magnetic field strength B. The dependence of $T_c(B)$ on B is shown in the figure.

14. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1 which of the following graphs shows the correct variation of R with T in these fields?

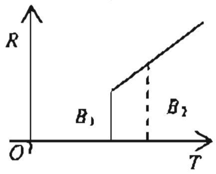
a.



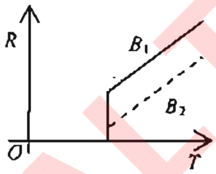
b.



c.



d.



Sol. a.

Larger the magnetic field, smaller the critical temperature.

15. A superconductor has $T_C(0) = 100$ K. When a magnetic field of 7.5 T is applied, its T_C decreases to 75 K. For this material, one can definitely say that when

a. $B = 5$ T, $T_C(B) = 80$ K

b. $B = 5$ T, 75 K $< T_C(B) <$ 100 K

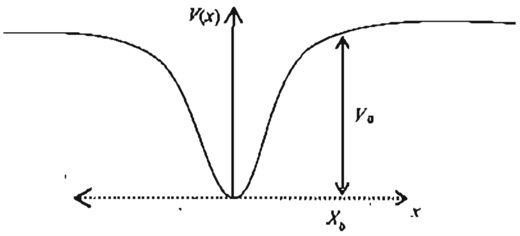
c. $B = 10$ T, 75 K $< T_C <$ 100 K

d. $B = 10$ T, $T_C = 70$ K

Sol. b. Self-explainatory

Problems 16–18:

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$ it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{m/k}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| > X_0$ (see figure).



16. If the total energy of the particle is E, it will perform periodic motion only if

a. $E < 0$ **b.** $E > 0$

c. $V_0 > E > 0$ **d.** $E > V_0$

Sol. c.

Energy must be less than V_0.

17. For periodic motion of small amplitude A, the time period T of this particle is proportional to

a. $A\sqrt{\frac{m}{\alpha}}$ **b.** $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$

c. $A\sqrt{\frac{\alpha}{m}}$ **d.** $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

Sol. b.

$[\alpha] = ML^{-2}T^{-2}$

Only option (b) has dimension of time. Alternatively,

$$\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + kx^4 = kA^4$$

$$\left(\frac{dx}{dt}\right)^2 = \frac{2k}{m}(A^4 - x^4)$$

$$4\sqrt{\frac{m}{2k}}\int_0^A \frac{dx}{\sqrt{A^4 - x^4}} = \int dt = T$$

$$4\sqrt{\frac{m}{2k}}\,\frac{1}{A}\int_0^1 \frac{du}{\sqrt{1 - u^4}} = T$$

Substituting, we get $x = Au$.

18. The acceleration of this particle for $|x| > X_0$ is

a. proportional to V_0 **b.** proportional to V_0/mX_0

c. proportional to $\sqrt{V_0/mX_0}$ **d.** zero

Sol. d.

As potential energy is constant for $|x| > X_0$, the force on the particle is zero. Hence, acceleration is zero.

Integer type

This section contains ten questions. The answer to each question is a single-digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

19. Gravitational acceleration on the surface of a planet is $\sqrt{6}/11g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is 2/3 times that of the earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be ______.

Sol. (3)

$$\frac{g'}{g} = \frac{\sqrt{6}}{11}, \frac{\rho'}{\rho} = \frac{2}{3}$$

Hence, $\frac{R'}{R} = \frac{3\sqrt{6}}{22}$

$$\frac{v'_{esc}}{v_{esc}} \propto \sqrt{\frac{R'^2\rho'}{R^2\rho}} = \frac{3}{11}$$

$v'_{esc} = 3$ km/s

20. A piece of ice (heat capacity = 2100 J kg^{-1} °C^{-1} and latent heat = 3.36×10^5 J kg^{-1}) of mass m grams is at –5°C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally, when the ice-water mixture is in equilibrium, it is found that 1 g of ice has melted. Assuming there is no other heat exchange in the process, the value of m is______.

Sol. (8)

$$420 = (m \times 2100 \times 5 + 1 \times 3.36 \times 10^5) \times 10^{-3}$$

where m is in gm.

21. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer ? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}.

Sol. (7)

$$f_{app} = f_0 \frac{c+v}{c-v}$$

$$df = \frac{2f_0c}{(c-v)^2}dv$$

where c is speed of sound

$$df = \frac{1.2}{100}f_0$$

Hence, $dv \approx 7$ km/hr.

22. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio m_{25}/m_{50} is______.

Sol. (6)

$$m = \frac{f}{f+u}$$

23. An α-particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio λ_p/λ_α, to the nearest integer, is______.

Sol. (3)

$$\frac{1}{2}mv^2 = qV$$

$$\lambda = \frac{h}{mv}$$

$$\lambda = \sqrt{8} \simeq 3$$

24. When two identical batteries of internal resistance 1 Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25\,J_2$, then the value of R in Ω is______.

Sol. (4)

$$J_1 = \left(\frac{2E}{R+2}\right)^2 R$$

$$J_2 = \left(\frac{E}{R+1/2}\right)R \text{ since } J_1/J_2 = 2.25$$

$R = 4$ Ω

25. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperature T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B?

Sol. (9)

$\lambda_m T$ = constant

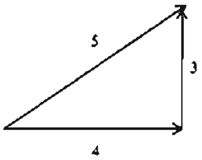
$\lambda_A T_A = \lambda_B T_B$

Rate of total energy radiated $\propto AT^4$

26. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is______.

Sol. (5)

Two waves have phase difference $\pi/2$.



27. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1 m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9$ Nm^{-2}, the value of n is______.

Sol. (4)

$$\omega = \sqrt{\frac{YA}{mL}}$$

28. A binary star consists of two stars A (mass $2.2M_s$) and B (mass $11M_s$), where M_s is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is______.

Sol. (6)

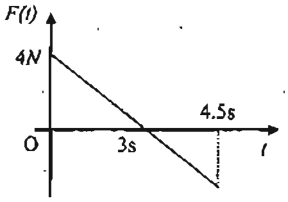
$$\frac{L_{total}}{L_B} = \frac{m_1 r_1^2}{m_2 r_2^2} + 1$$

Paper 2

Objective Type

Multiple choice questions with one correct answer

1. A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 seconds is



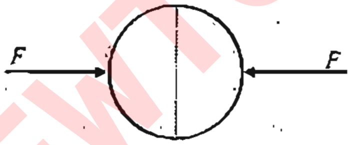
a. 4.50 J   b. 7.50 J
c. 5.06 J   d. 14.06 J

Sol. c.

Area under F-t curve = 4.5 kg-m/sec

$$\text{K.E.} = \frac{1}{2}(2)\left(\frac{4.5}{2}\right)^2 = 5.06 \text{ J}$$

2. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F. F is proportional to



a. $\frac{1}{\varepsilon_0}\sigma^2 R^2$ b. $\frac{1}{\varepsilon_0}\sigma^2 R$
c. $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ d. $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

Sol. a.

Pressure = $\frac{\sigma^2}{2\varepsilon_0}$ and force = $\frac{\sigma^2}{2\varepsilon_0} \times \pi R^2$

3. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $(81\pi/7) \times 10^5 \text{ Vm}^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity $2 \times 10^{-3} \text{ ms}^{-1}$. Given $g = 9.8 \text{ ms}^{-2}$, viscosity of the air $= 1.8 \times 10^{-5} \text{ Ns m}^{-2}$ and the density of oil = 900 kg m^{-3}, the magnitude of q is

a. 1.6×10^{-19} C   b. 3.2×10^{-19} C
c. 4.8 × 10¯19 C   d. 8.0×10^{-19} C

Sol. d.

$$\frac{4}{3}\pi R^3 \rho g = qE = 6\rho\eta R v_T$$

$\therefore q = 8.0 \times 10^{-19}$ C

4. A vernier callipers has 1 mm marks on the main scale. It has 20 equal divisions on the vernier scale which match with 16 main scale divisions. For this vernier callipers, the least count is

a. 0.02 mm   b. 0.05 mm
c. 0.1 mm   d. 0.2 mm

Sol. d.

L.C. = 1 M.S.D − 1 V.S.D

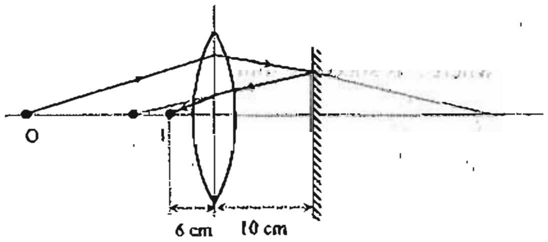
$\left(1 - \frac{16}{20}\right)$ M.S.D

$\left(1 - \frac{4}{5}\right)(1 \text{ mm}) = 0.2$ mm

5. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is

a. virtual and at a distance of 16 cm from the mirror
b. real and at a distance of 16 cm from the mirror
c. virtual and at a distance of 20 cm from the mirror
d. real and at a distance of 20 cm from the mirror

Sol. b.



6. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is

a. 5 g   b. 10 g
c. 20 g   d. 40 g

Sol. b.

$$\frac{V_s}{4L_P} = \frac{2\sqrt{\frac{T}{\mu}}}{2l_s}$$

Integer type

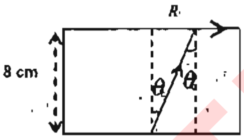
This section contains five questions. The answer to each question is a single-digit integer, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

7. A large glass slab ($\mu = 5/3$) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R?

Sol. (6)

$\sin\theta_c = 3/5$

$\therefore R = 6$ cm



8. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from 25/3 m to 50/7 m in 30 seconds. What is the speed of the object in km per hour ?

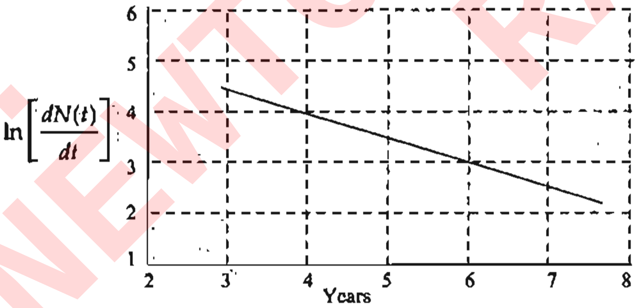
Sol. (3)

For $v_1 = \frac{50}{7}$ m, $u_1 = -25$ m

$v_2 = \frac{25}{3}$ m, $u_2 = -50$ m

Speed of object $= \frac{25}{30} \times \frac{18}{5} = 3$ kmph.

9. To determine the half-life of a radioactive element, a student plots a graph of $\ln\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is____.



Sol. (8)

$N = N_0 e - \lambda t$

$\ln|dN/dt| = \ln(N_0\lambda) - \lambda t$

From graph, $\lambda = \frac{1}{2}$ per year

$t_{1/2} = \frac{0.693}{1/2} = 1.386$ year

4.16 yrs $= 3t_{1/2}$

$\therefore p = 8$

10. A diatomic ideal gas is compressed adiabatically to 1/32 of its initial volume. If the initial temperature of the gas is T_i (in kelvin) and the final temperature is aT_i, the value of a is______.

Sol. (8)

$TV^{\gamma-1} = \text{constant}$

$TV^{7/5-1} = aT\left(\frac{V}{32}\right)^{7/5-1}$

$\therefore a = 4.$

11. At time $t = 0$, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them becomes 4 volt? [take ln 5 = 1.6, ln 3 = 1.1]

Sol. (2)

$4 = 10\,(1 - e^{-t/4})$

$\therefore t = 2$ sec

Linked comprehension type

Problems 12–14:

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When the force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

12. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming $r \ll R$) is___.

a. $2\pi rT$ **b.** $2\pi RT$

c. $\frac{2\pi r^2 T}{R}$ **d.** $\frac{2\pi R^2 T}{r}$

Sol. c.

Surface tension force $= 2\pi rT\frac{r}{R} = \frac{2\pi r^2 T}{R}$

13. If $r = 5 \times 10^{-4}$ m, $\rho = 10^3$ kg m^{-3}, $g = 10$ m/s^2, $T = 0.11$ Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately______.

a. 1.4×10^{-3} m **b.** 3.3×10^{-3} m

c. 2.0×10^{-3} m **d.** 4.1×10^{-3} m

Sol. a.

$\frac{2\pi rT}{R} = mg = \frac{4}{3}\pi R^3 \rho g$

14. After the drop detaches, its surface energy is_____

a. 1.4×10^{-6} J **b.** 2.7×10^{-6} J

c. 5.4×10^{-6} J **d.** 8.1×10^{-6} J

Sol. b.

Surface energy = $T(4\pi R^2) = 2.7 \times 10^{-6}$ J

Problems 15–17:

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition.

15. A diatomic molecule has moment of inertia I. By Bohr's quantization condition its rotational energy in the n^{th} level ($n = 0$ is not allowed) is_____.

a. $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ **b.** $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$

c. $n\left(\frac{h^2}{8\pi^2 I}\right)$ **d.** $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

Sol. d.

$$L = \frac{nh}{2\pi}$$

$$K.E. = \frac{L^2}{2I} = \left(\frac{nh}{2\pi}\right)\frac{1}{2I}$$

16. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $4/\pi \times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its centre of mass is close to_____. (Take $h = 2\pi \times 10^{-34}$ Js)

a. 2.76×10^{-46} kg m^2

b. 1.87×10^{-46} kg m^2

c. 4.67×10^{-47} kg m^2

d. 1.17×10^{-47} kg m^2

Sol. b.

$$h\nu = k.E_{n=2} - kE_{n=1}$$

$$I = 1.87 \times 10^{-46} \text{ kg m}^2$$

17. In a CO molecule, the distance between C (mass = 12 a.m.u) and O (mass = 16 a.m.u.), where 1 a.m.u. = $\frac{5}{3} \times 10^{-27}$ kg, is close to_______.

a. 2.4×10^{-10} m **b.** 1.9×10^{-10} m

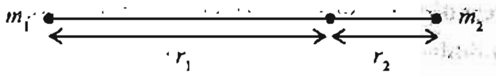
c. 1.3×10^{-10} m **d.** 4.4×10^{-11} m

Sol. c.

$$r_1 = \frac{m_2 d}{m_1 + m_2} \text{ and } r_2 = \frac{m_1 d}{m_1 + m_2}$$

$$I = m_1 r_1^2 + m_2 r_2^2$$

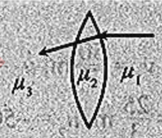
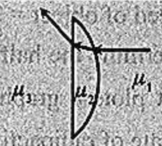
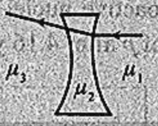
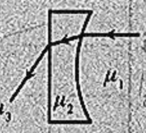
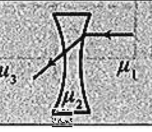
$$\therefore d = 1.3 \times 10^{-10} \text{ m}$$



Match the column type

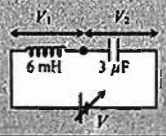
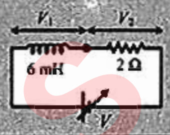
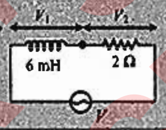
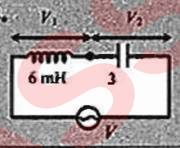
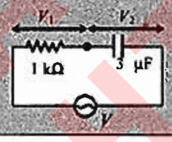
Match the entries in Column I with appropriate options in Column II.

18. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in Column II. A ray traversing these media is also shown in the figures. In Column I different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagram shown in Column II.

Column I	Column II
a. $\mu_1 < \mu_3$	**p.** 
b. $\mu_1 < \mu_2$	**q.** 
c. $\mu_2 < \mu_3$	**r.** 
d. $\mu_2 < \mu_3$	**s.** 
	t. 

Sol. **a.**→ **p., r.; b.**→ **q., s., t.; c.**→ **p., r., t.; d.** → **q., s.**

19. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in Column II. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2 (indicated in circuits) are related as shown in Column I. Match the two

Column I	**Column II**
a. $I \neq 0$, V_1 is proportional to I	**p.** 
b. $I \neq 0$, $V_2 > V_1$	**q.** 
c. $V_1 = 0$, $V_2 = V$	**r.** 
d. $I \neq 0$, V_2 is proportional to I	**s.** 
	t. 

Sol. a.→ r., s., t.; b.→ q., r., s., t.; c.→ p., q.; d.→ q., r., s., t.

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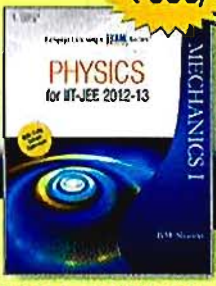

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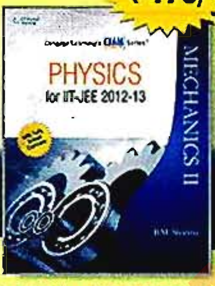

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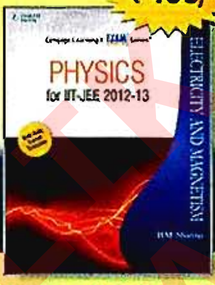

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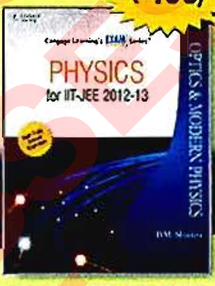

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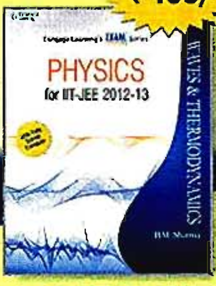

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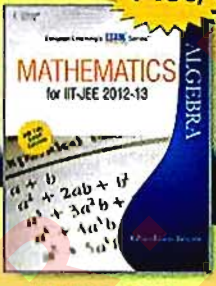

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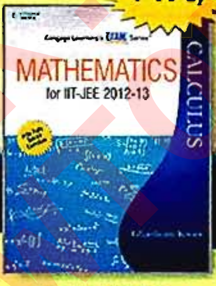

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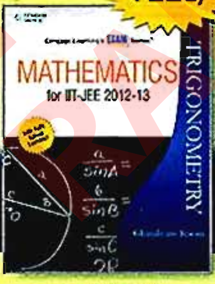

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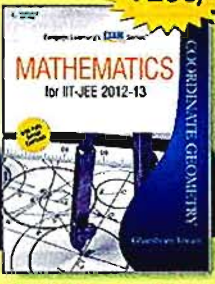

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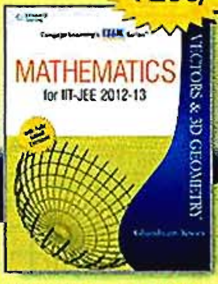

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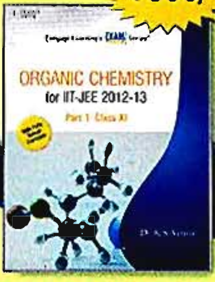

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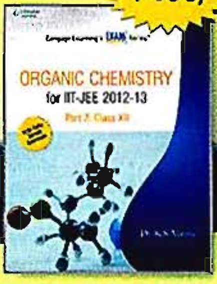

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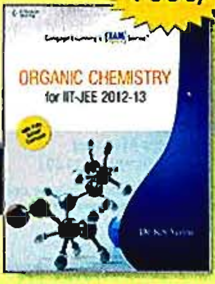

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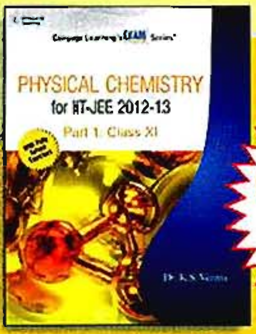

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